Overview of phenomenology of (un)polarized Drell-Yan

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If I'm allowed to set

Un-unpolarized $\equiv$ Polarized

then a better title would be

Overview of phenomenology of (un)unpolarized Drell-Yan

$\boxed{\text{Outline}}$

- Generalities on DY
- Double-polarized DY (integrated over transverse momenta)
- Azimuthal asymmetries in unpolarized DY
- Future DY measurements
- Predictions for single-polarized DY
- Conclusions

<u>Limitations</u>:

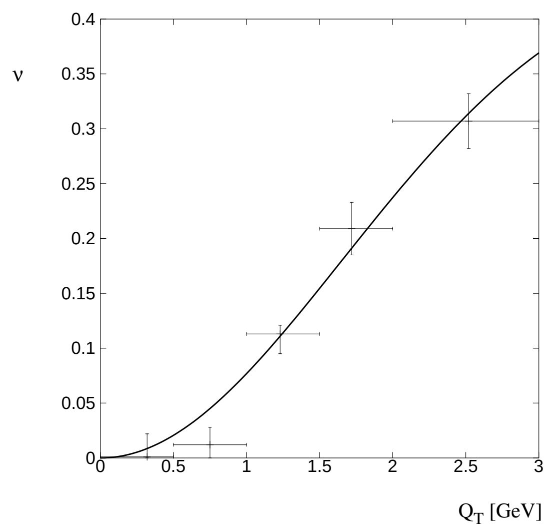
Only transverse polarization (no longitudinal)

TMD framework: $h_1^\perp, f_{1T}^\perp, h_{1T}^\perp$

Incomplete coverage of works (I apologize)

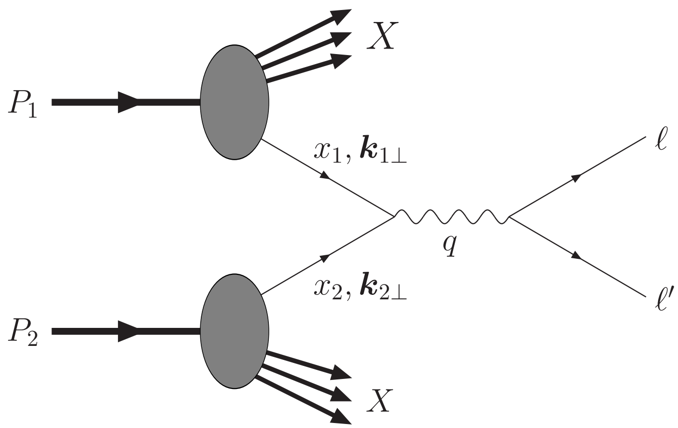
Drell-Yan processes have played a key rôle in the physics of transverse spin and transverse momentum distributions (TMDs)

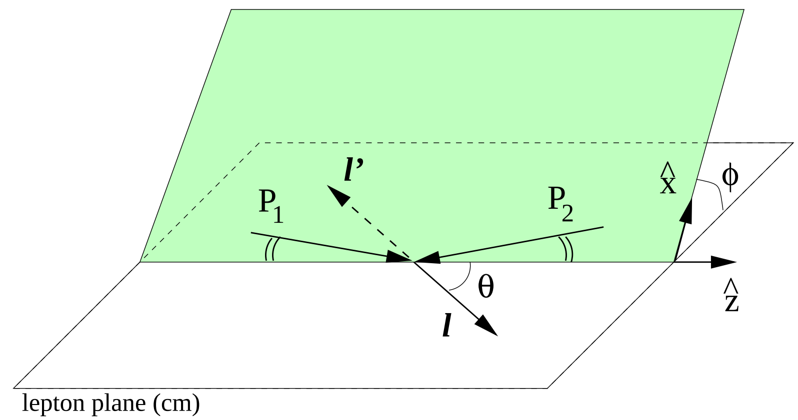
- Ralston & Soper (1979) introduced the transversity distribution h_1 in the context of transversely polarized DY. The same process was studied in the 90s by Artru & Mekhfi, Jaffe & Ji, Cortes, Pire & Ralston.
- The first phenomenological indication of the possible existence of a T-odd TMD, the Boer-Mulders function $h_1^{\perp}$, came from Boer's (1999) study of the $\cos 2\phi$ asymmetry in DY (the ν parameter)



$\boxed{\text{General features of DY}}$

Drell-Yan production: $H_1(P_1) + H_2(P_2) \to l^+(\ell) + l^-(\ell') + X(P_X)$





Invariants: $s^2 = (P_1 + P_2)^2 \qquad x_1 = \frac{Q^2}{2P_1 \cdot q} \qquad x_2 = \frac{Q^2}{2P_2 \cdot q}$

ϕ angle between lepton and hadron plane (in a given frame), $\mathbf{q}_T$ transverse momentum of virtual photon

Transverse kinematics: $\mathbf{k}_{1T} + \mathbf{k}_{2T} = \mathbf{q}_T$

In the Collins–Soper frame the z axis points in the direction bisecting the angle between $\mathbf{P}_1$ and $-\mathbf{P}_2$

The DY cross section contains 48 structure functions [Arnold, Metz & Schlegel 2009]

$$
\begin{aligned}
\frac{d^6\sigma}{d^4q\, d\Omega} = \frac{\alpha_{em}^2}{6sQ^2} \Big\{ & \Big[(1+\cos^2\theta)\, W_{UU}^1 + \sin^2\theta\, W_{UU}^2 + \sin 2\theta\, \cos\phi\, W_{UU}^{\cos\phi} + \sin^2\theta\, \cos 2\phi\, W_{UU}^{\cos 2\phi}\Big] \\
& + S_{1T}\Big[\sin\phi_{S_1}\Big((1+\cos^2\theta)\, W_{TU}^1 + \sin^2\theta\, W_{TU}^2 + \sin 2\theta\, \cos\phi\, W_{TU}^{\cos\phi} + \sin^2\theta\, \cos 2\phi\, W_{TU}^{\cos 2\phi}\Big) \\
& + \cos\phi_{S_1}\,(\sin 2\theta\, \sin\phi\, W_{TU}^{\sin\phi} + \sin^2\theta\, \sin 2\phi\, W_{TU}^{\sin 2\phi})\Big] + (1 \leftrightarrow 2, T \leftrightarrow U) \\
& + S_{1T}\, S_{2T}\, \Big[\cos(\phi_{S_1} + \phi_{S_2})\, \Big((1+\cos^2\theta)\, W_{TT}^1 + \sin^2\theta\, W_{TT}^2 \\
& + \sin 2\theta\, \cos\phi\, W_{TT}^{\cos\phi} + \sin^2\theta\, \cos 2\phi\, W_{TT}^{\cos 2\phi}\Big) \\
& + \cos(\phi_{S_1} - \phi_{S_2})\, \Big((1+\cos^2\theta)\, \overline{W}_{TT}^1 + \sin^2\theta\, \overline{W}_{TT}^2 + \sin 2\theta\, \cos\phi\, \overline{W}_{TT}^{\cos\phi} + \sin^2\theta\, \cos 2\phi\, \overline{W}_{TT}^{\cos 2\phi}\Big) \\
& + \sin(\phi_{S_1} + \phi_{S_2})\, (\sin 2\theta\, \sin\phi\, W_{TT}^{\sin\phi} + \sin^2\theta\, \sin 2\phi\, W_{TT}^{\sin 2\phi}) \\
& + \sin(\phi_{S_1} - \phi_{S_2})\, (\sin 2\theta\, \sin\phi\, \overline{W}_{TT}^{\sin\phi} + \sin^2\theta\, \sin 2\phi\, \overline{W}_{TT}^{\sin 2\phi})\Big] + \ldots \Big\}\,.
\end{aligned}
$$

Integrating upon q_T only three structure functions survive:

W_{UU}^1, W_{LL}^1 and $W_{TT}^{\cos(2\phi - \phi_{S_1} - \phi_{S_2})} \equiv \frac{1}{2}(W_{TT}^{\sin 2\phi} + W_{TT}^{\cos 2\phi})$

24 structure functions appear at leading twist

Hadronic tensor in the extended parton model

$$W^{\mu\nu} = \frac{1}{3}\sum_a e_a^2 \int \mathrm{d}^2\mathbf{k}_{1T} \int \mathrm{d}^2\mathbf{k}_{2T}\, \delta^2(\mathbf{k}_{1T}+\mathbf{k}_{2T}-\mathbf{q}_T)\mathrm{Tr}\,[\Phi(x_1,\mathbf{k}_{1T})\,\gamma^\mu\,\bar{\Phi}(x_2,\mathbf{k}_{2T})\gamma^\nu]+[1\leftrightarrow 2]$$

At leading twist:

$$W_{UU}^{1} = \mathcal{C}\,[f_1\bar{f}_1]$$

$$W_{UU}^{\cos 2\phi} = \mathcal{C}\left[\frac{2(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})-\mathbf{k}_{1T}\cdot\mathbf{k}_{2T}}{M_1M_2}\,h_1^\perp\bar{h}_1^\perp\right]$$

$$W_{UT}^{1} = \mathcal{C}\left[\frac{\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T}}{M_2}\,f_1\,\bar{f}_{1T}^\perp\right]$$

$$W_{UT}^{\sin(2\phi-\phi_{S_2})} = -\mathcal{C}\left[\frac{\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T}}{M_1}\,h_1^\perp\bar{h}_1\right]$$

$$W_{UT}^{\sin(2\phi+\phi_{S_2})} = -\mathcal{C}\left[\frac{2(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})[2(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})-\mathbf{k}_{1T}\cdot\mathbf{k}_{2T}]-\mathbf{k}_{1T}^2(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})}{2M_1M_2^2}\,h_1^\perp\bar{h}_{1T}^\perp\right]$$

$$W_{TT}^{\cos(2\phi-\phi_{S_1}-\phi_{S_2})} = \mathcal{C}\,[h_1\bar{h}_1]$$

Differences between dilepton c.m. frames, being $\mathcal{O}(Q_T/Q)$, affect only higher twists

The extended parton model is the zeroth-order approximation of the TMD factorization theorem, valid for $q_T \ll Q$ [Ji, Ma, Yuan (2005)]:

$$\mathrm{d}\sigma \sim \sum_a e_a^2 \int \mathrm{d}^2\mathbf{k}_{1T} \int \mathrm{d}^2\mathbf{k}_{2T} \int \mathrm{d}^2\mathbf{l}_T \, \delta^2(\mathbf{k}_{1T} + \mathbf{k}_{2T} + \mathbf{l}_T - \mathbf{q}_T)$$
$$\times f_a(x_1, \mathbf{k}_{1T}^2) \, \mathrm{d}\hat{\sigma} \; U(\mathbf{l}_T^2) \, \bar{f}_a(x_2, \mathbf{k}_{2T}{}^2)$$

At order $1/Q$ quark-gluon interactions give rise to "tilde" distributions and 16 more structure functions appear [Lu & Schmidt 2011], for instance $W_{UU}^{\cos\phi}$, which contains

$$x f^\perp = x\tilde{f}^\perp + f_1, \qquad x h = x\tilde{h} + \frac{k_T^2}{M^2} h_1^\perp$$

However TMD factorization is not proven beyond leading twist

$\boxed{\text{Double-polarized Drell-Yan}}$

Doubly polarized DY production (integrated over transverse momenta)

Collinear factorization applies:

$$\mathrm{d}\sigma = \sum_a \sum_{\{\lambda\}} \int \mathrm{d}\xi_1 \int \mathrm{d}\xi_2 \, \rho^{(1)}_{\lambda_1' \lambda_1} \, f_a(\xi_1, \mu^2) \, \rho^{(2)}_{\lambda_2' \lambda_2} \, \bar{f}_a(\xi_2, \mu^2) \, \mathrm{d}\hat{\sigma}_{\{\lambda\}}(Q^2, \mu^2, \alpha_s(\mu^2)) \,,$$

ξ_1 and ξ_2 are the momentum fractions of the quark (from hadron H_1) and antiquark (from hadron H_2)

At leading order, i.e. $\mathcal{O}(\alpha_s^0)$, the only contributing subprocess is $q\bar{q} \to \ell^+\ell^-$ and $\xi_1 = x_1$, $\xi_2 = x_2$

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\Omega} = \frac{\alpha_{\mathrm{em}}^2}{12 Q^2} \sum_a e_a^2 \Big[(1 + \cos^2\theta) \, f_1^a(x_1, Q^2) \, \bar{f}_1^a(x_2, Q^2)$$

$$+ S_{1T} S_{2T} \, \sin^2\theta \, \cos(2\phi - \phi_{S_1} - \phi_{S_2}) h_1^a(x_1, Q^2) \bar{h}_1^a(x_2, Q^2) \Big] + [1 \leftrightarrow 2] \,.$$

Double transverse asymmetry:

$$A_{TT}^{DY} = \frac{d\sigma^{\uparrow\uparrow} - d\sigma^{\uparrow\downarrow}}{d\sigma^{\uparrow\uparrow} + d\sigma^{\uparrow\downarrow}}$$

At leading order ($q\bar{q}$ annihilation):

$$A_{TT}^{DY} = a_{TT} \, \frac{\sum_q e_q^2 \, h_{1q}(x_1, Q^2) \, \bar{h}_{1q}(x_2, Q^2) + [1 \leftrightarrow 2]}{\sum_q e_q^2 \, f_{1q}(x_1, Q^2) \, \bar{f}_{1q}(x_2, Q^2) + [1 \leftrightarrow 2]}$$

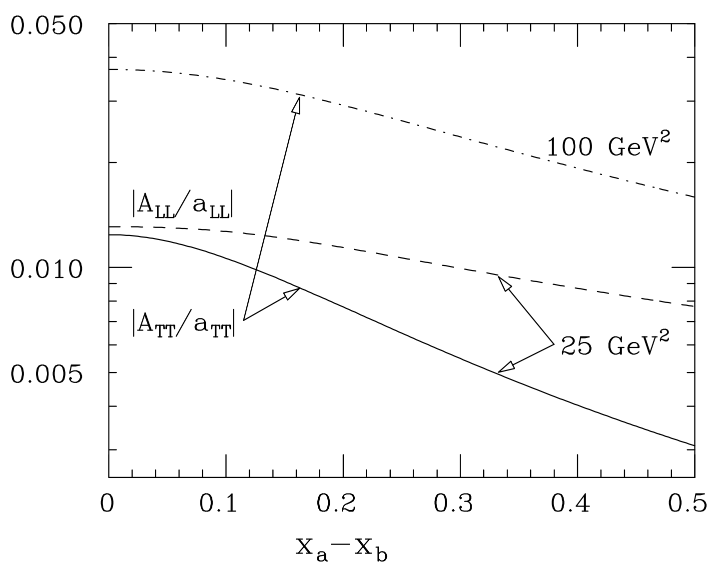
The asymmetry is completely determined by the transversity of quarks and antiquarks

Predictions for $A_{TT}^{DY}(pp)$ at RHIC

LO at $\sqrt{s} = 100$ GeV

$h_1 = g_1$ at $Q_0^2 = 0.23$ GeV^2

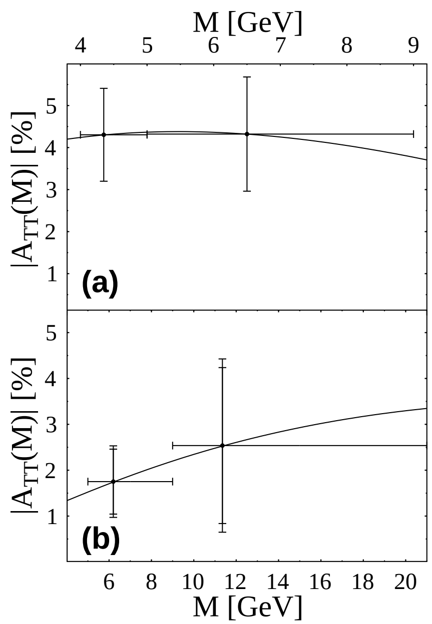
[VB, Calarco & Drago 1997]



NLO at $\sqrt{s} = 200$ GeV

Soffer bound saturated at Q_0

[Martin et al. 1998]



At RHIC energies $A_{TT}^{DY}(pp)$ is expected to be small: $\sim 2 - 3\%$

RHIC transverse asymmetries are expected to be small:

- $\sqrt{s} = 200$ GeV, $Q < 10$ GeV $\Rightarrow$ $x_1 x_2 = Q^2/s < 2.5 \times 10^{-3}$: low-$x$ region is probed
- Sea transversity distributions are small. The evolution of transversity is suppressed at low x

Two ways to improve the situation [VB, Calarco & Drago 1997]:

- Moderate energies: with $s \sim 100$ GeV^2 and $Q > 4$ GeV, one has $x_1 x_2 > 0.15$ (intermediate-x region)
- Proton–antiproton scattering probes valence $\times$ valence

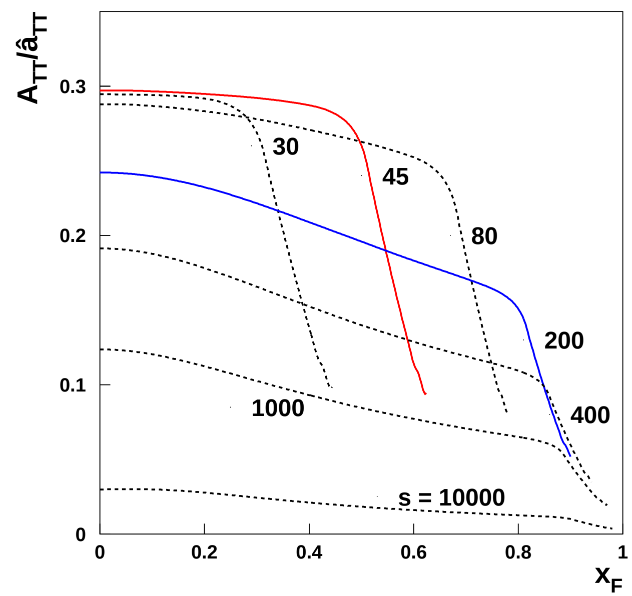
PAX: polarized $\bar{p}$ colliding on polarized p at GSI-HESR [PAX, hep-ex/0505054]

$$s = 45, 200 \text{ GeV}^2 \,, \quad Q > 2 \,\text{GeV} \,, \quad \mathcal{L} > 10^{30} \text{cm}^{-2} \text{s}^{-1}$$

A_{TT} turns out to be large

LO calculation ($Q = 4$ GeV) $\Rightarrow$

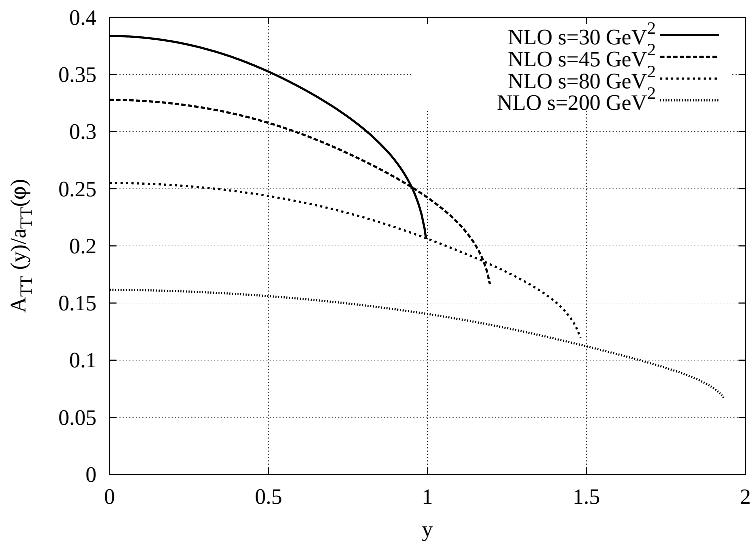
[Anselmino, VB, Drago, Nikolaev 2004]



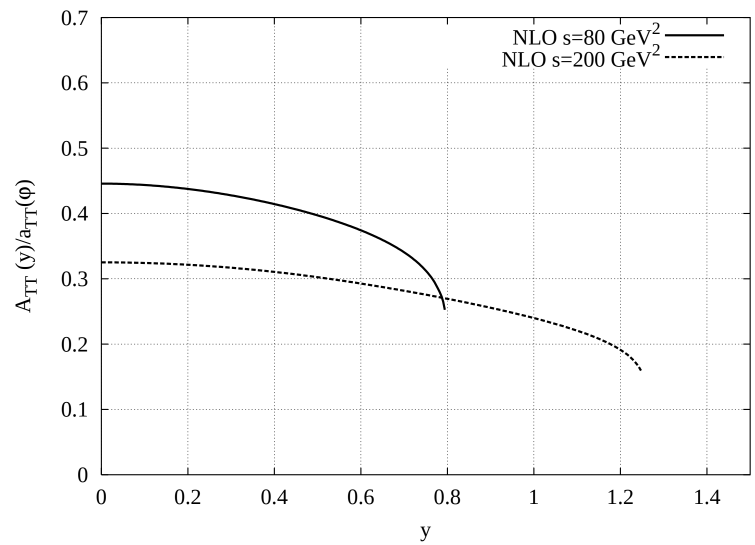
In $p^\uparrow p^\uparrow$ DY at $\sqrt{s} = 10$ GeV (J-PARC energy), asymmetries are expected to be smaller, but still sizable ($A_{TT}/a_{TT} \sim 0.1 - 0.2$). Important information on antiquark distributions

NLO predictions of $A_{TT}^{DY}(\bar{p}p)$ [VB et al. 2005]

Perturbative corrections to the cross sections largely cancel in the ratio. Asymmetries are almost unaffected



Q integrated from 2 to 3 GeV



Q integrated from 4 to 7 GeV

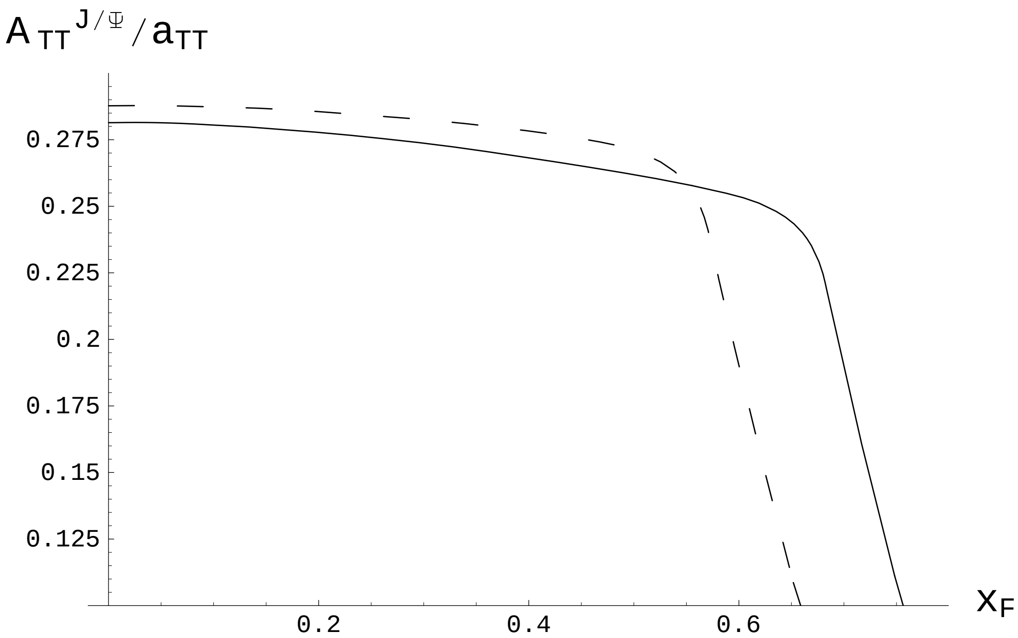
Double transverse DY asymmetry is large, but the production rate falls down rapidly for $M > 3$ GeV

$\Rightarrow$ J/ψ production [Anselmino, VB, Drago, Nikolaev 2004]

- Comparison of J/ψ production in $\bar{p}p$ and pp collisions at $s = 80$ GeV^2 (SPS data) shows dominance of $\bar{q}q$ annihilation: $\sigma(\bar{p}p) \gg \sigma(pp)$
- The helicity structure of $q\bar{q}J/\psi$ is the same as $q\bar{q}\gamma^*$
- Since the u sector dominates, the J/ψ coupling factorizes out

$$A_{TT}^{J/\psi} \sim \frac{h_{1u}(x_1, M_\psi^2)\, h_{1u}(x_2, M_\psi^2)}{f_{1u}(x_1, M_\psi^2)\, f_{1u}(x_2, M_\psi^2)}$$

[Anselmino, VB, Drago, Nikolaev 2004]



$A_{TT}^{J/\psi} \sim 0.3$ (similar results by [Efremov, Goeke & Schweitzer 2004])

$\boxed{\text{Azimuthal asymmetries in unpolarized Drell-Yan}}$

Unpolarized DY cross section

$$\frac{d^6\sigma_{UU}}{d^4q\, d\Omega} = \frac{\alpha_{\rm em}^2}{6sQ^2}\,\Big\{(1+\cos^2\theta)\,W_{UU}^1 + \sin^2\theta\, W_{UU}^2 + \sin 2\theta\,\cos\phi\, W_{UU}^{\cos\phi} + \sin^2\theta\cos 2\phi\, W_{UU}^{\cos 2\phi}\Big\}$$

Another common parametrization of the angular distribution of dileptons

$$\frac{1}{N_{\rm tot}}\frac{dN}{d\Omega} = \frac{3}{4\pi}\,\frac{1}{\lambda+3}\,\left(1+\lambda\,\cos^2\theta + \mu\,\sin 2\theta\,\cos\phi + \frac{\nu}{2}\,\sin^2\theta\cos 2\phi\right)$$

with the correspondence

$$\lambda = \frac{W_{UU}^1 - W_{UU}^2}{W_{UU}^1 + W_{UU}^2}\,, \quad \mu = \frac{W_{UU}^{\cos\phi}}{W_{UU}^1 + W_{UU}^2}\,, \quad \nu = \frac{2\,W_{UU}^{\cos 2\phi}}{W_{UU}^1 + W_{UU}^2}$$

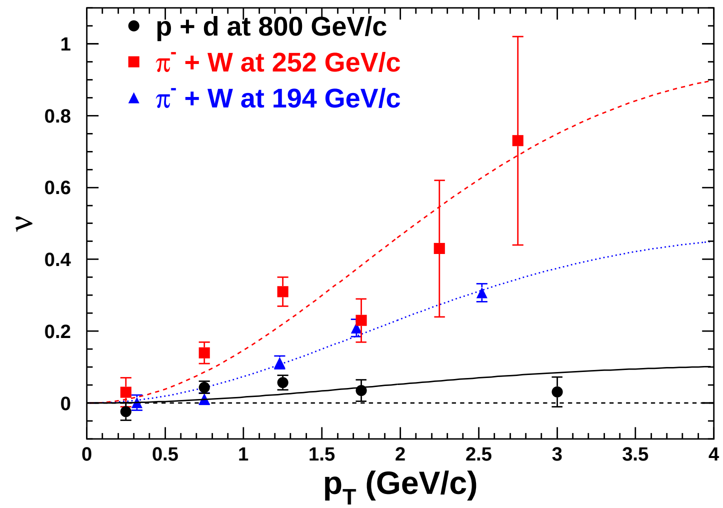
The Lam-Tung relation $\lambda + 2\nu = 1$, corresponding to $W_{UU}^2 = 2W_{UU}^{\cos 2\phi}$, is valid in collinear QCD at order α_s and slightly violated at order α_s^2

NA10 ($E_\pi = 194$ GeV) and E615 ($E_\pi = 252$ GeV) results for $\pi^- N \to \mu^+\mu^- X$:

ν increasing with Q_T and large

μ non-zero (~ 0.1) for E615, compatible with zero for NA10

Big violation of the Lam-Tung relation observed by E615 (but not by NA10)



	pd (E866)	$\pi^- W$ (NA10)	$\pi^- W$ (E615)
$\langle\lambda\rangle$	1.07 ± 0.07	0.83 ± 0.04	1.17 ± 0.06
$\langle\mu\rangle$	0.003 ± 0.013	0.008 ± 0.010	0.09 ± 0.02
$\langle\nu\rangle$	0.027 ± 0.010	0.091 ± 0.009	0.169 ± 0.019
$\langle 2\nu - (1-\lambda)\rangle$	0.12 ± 0.07	0.01 ± 0.04	0.51 ± 0.07

Possible sources of a $\cos 2\phi$ asymmetry:

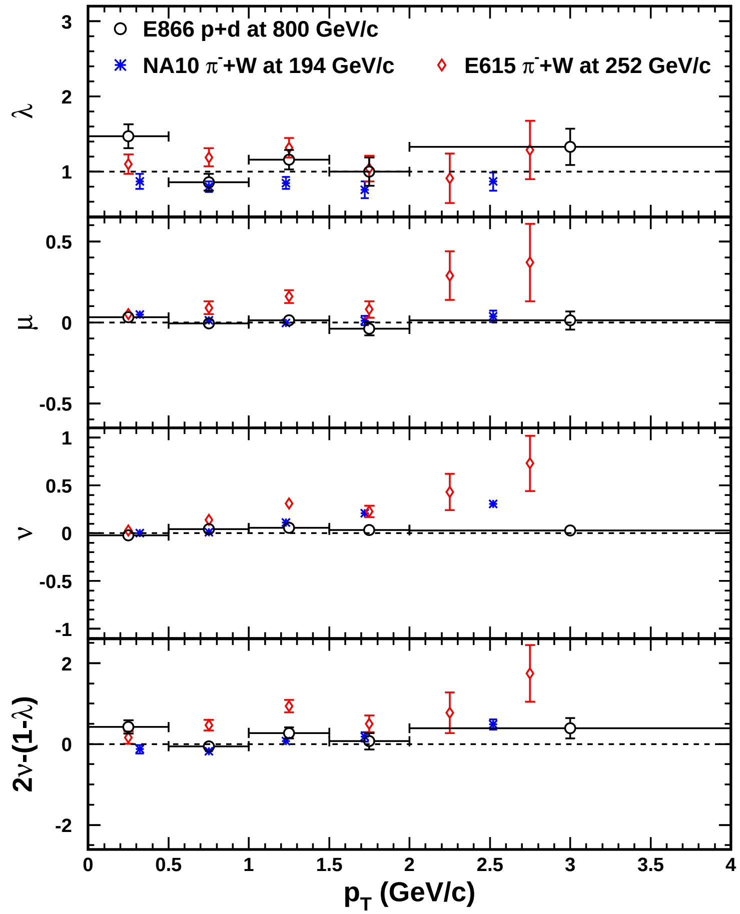
- Order-α_s pQCD processes:
  - Quark-antiquark annihilation with gluon bremsstrahlung, $q\bar{q} \to \gamma^* g$
  - QCD Compton process, $qg \to \gamma^* q$

  Relevant at high Q_T. Preserve the Lam-Tung relation

- Boer–Mulders mechanism (correlation between transverse spin and transverse momentum of quarks)

  Relevant at low Q_T. Breaks the Lam-Tung relation

E866 data: pp and pd DY at 800 GeV



Small deviation of ν from zero, μ compatible with zero, Lam-Tung relation fulfilled

The contribution of the process $q\bar{q} \to \gamma^* g$ in the Collins-Soper frame is

$$\frac{1}{N_{\rm tot}} \frac{\mathrm{d}N}{\mathrm{d}\Omega} = \frac{3}{16\pi} \left(\frac{Q^2 + \frac{3}{2} Q_T^2}{Q^2 + Q_T^2} + \frac{Q^2 - \frac{1}{2} Q_T^2}{Q^2 + \frac{1}{2} Q_T^2} \cos^2\theta + \frac{1}{2} \frac{Q_T^2}{Q^2 + Q_T^2} \sin^2\theta \cos 2\phi + \ldots \right)$$

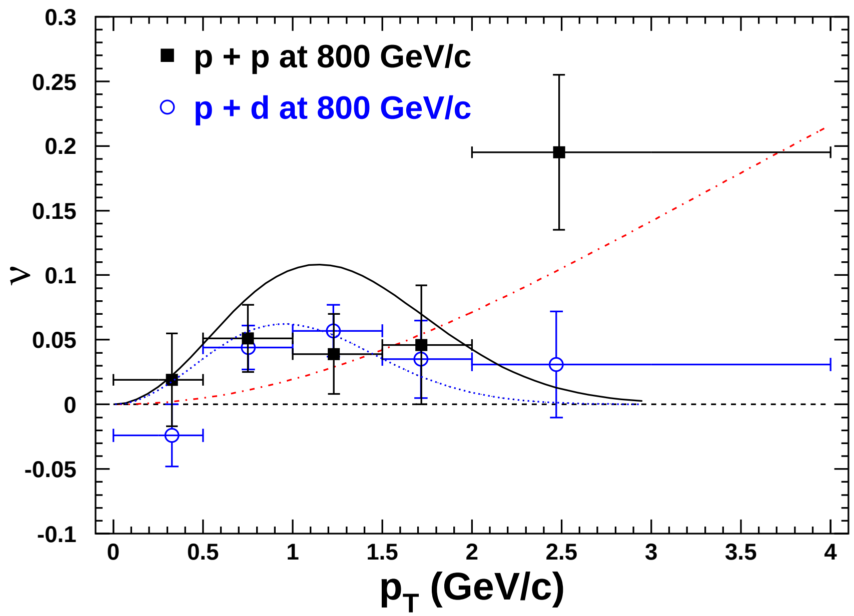
Parameters of the unpolarized cross section

$$\lambda = \frac{Q^2 - \frac{1}{2} Q_T^2}{Q^2 + \frac{3}{2} Q_T^2}\,, \quad \nu = \frac{Q_T^2}{Q^2 + \frac{3}{2} Q_T^2}$$

In the Collins-Soper frame, resummation of large $\ln(Q^2/Q_T^2)$ logarithms does not sensibly change ν, but strongly affects μ

The pQCD effects are too small to explain the large ν observed in πN DY. However, they describe at least in part the E866 findings

Perturbative QCD calculations (red line) compared with Boer-Mulders fits (by [Zhang, Lu, Ma, Schmidt 2008])



In the TMD framework, at leading twist:

$$W_{UU}^{1} = \mathcal{C}\,[f_1\,\bar{f}_1]$$
$$W_{UU}^{\cos 2\phi} = \mathcal{C}\left[\frac{2(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})-\mathbf{k}_{1T}\cdot\mathbf{k}_{2T}}{M_1\,M_2}\,h_1^{\perp}\,\bar{h}_1^{\perp}\right]$$

and therefore

$$\nu_{\mathrm{BM}} = \frac{2\,\mathcal{C}\left[(2(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})-\mathbf{k}_{1T}\cdot\mathbf{k}_{2T})\,h_1^{\perp}\,\bar{h}_1^{\perp}\right]}{M_1\,M_2\,\mathcal{C}\left[f_1\,\bar{f}_1\right]}$$

If we adopt a Wandzura-Wilczek approximation (neglect tilde distributions) the twist-3 $\cos\phi$ structure function is given by [Lu & Schmidt 2011]

$$\begin{aligned} W_{UU}^{\cos\phi} &= \frac{1}{Q}\,\mathcal{C}\left[[(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})-(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})]\,f_1\,\bar{f}_1\right] \\ &\quad +\frac{1}{Q}\,\mathcal{C}\left[\frac{(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})\mathbf{k}_{2T}^2-(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})\mathbf{k}_{1T}^2}{2M_1\,M_2}\,h_1^{\perp}\,\bar{h}_1^{\perp}\right] \end{aligned}$$

The first line is the Cahn term, a kinematic contribution not involving any new distribution, the second line is the Boer-Mulders contribution, containing $h_1^{\perp}$

There is also a $1/Q^2$ Cahn correction to $W_{UU}^{\cos 2\phi}$ [Schweitzer, Teckentrup, Metz 2010]

$$W_{UU}^{\cos 2\phi} = \frac{1}{Q^2}\,\mathcal{C}\left[\left\{\frac{1}{2}[(\hat{\mathbf{q}}_T \cdot \mathbf{k}_{1T}) - (\hat{\mathbf{q}}_T \cdot \mathbf{k}_{2T})]^2 + 2\mathbf{k}_{1T}^2\mathbf{k}_{2T}^2\right\} f_1\, \bar{f}_1\right]$$

Remember however that this is an incomplete kinematic contribution

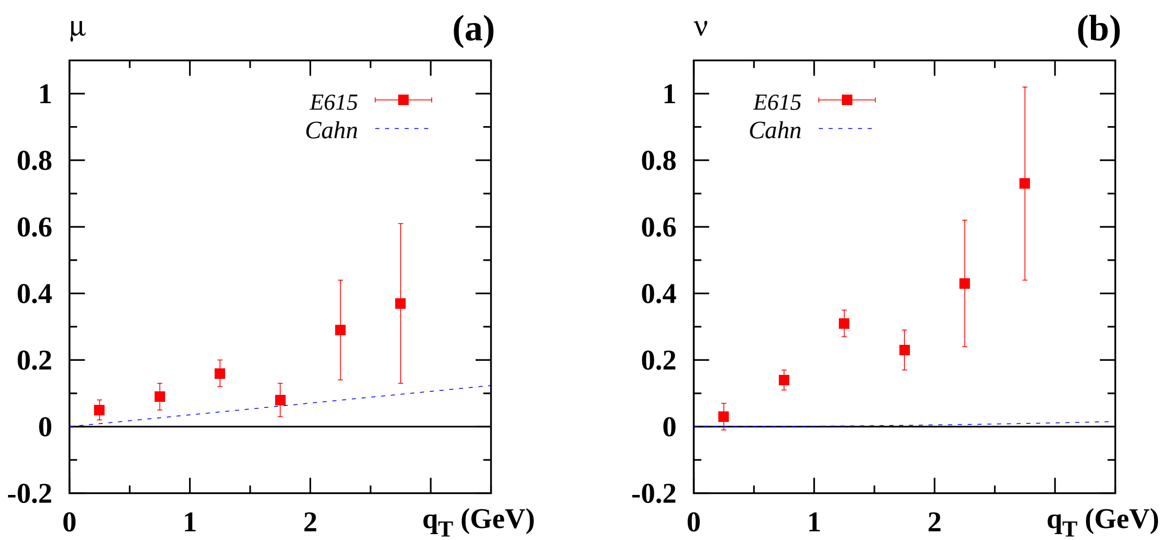
The full $1/Q^2$ kinematic correction is more complicated

Moreover, there are unknown dynamical twist-4 terms

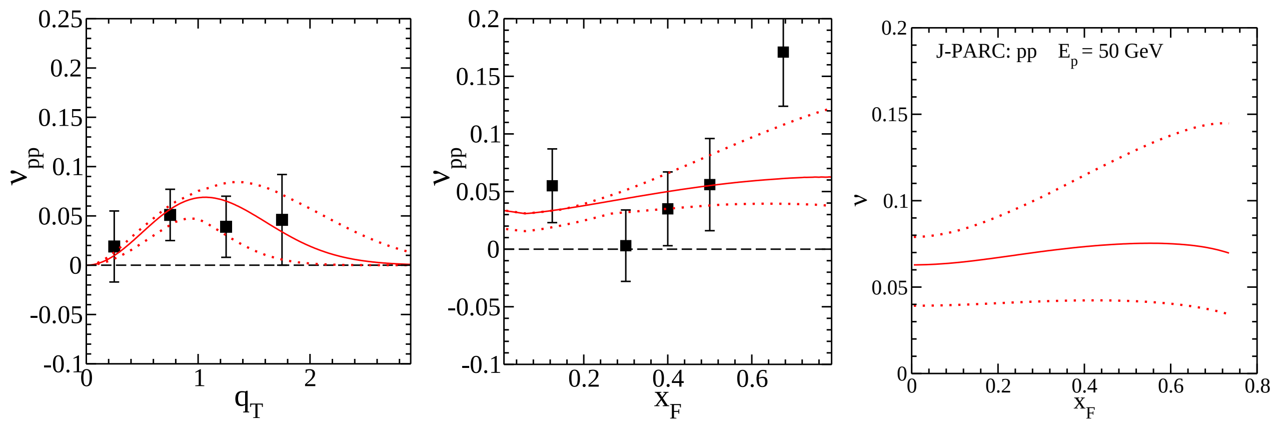
The Cahn $\cos 2\phi$ and $\cos\phi$ contributions in DY are expected to be small (at least for nucleon-nucleon DY)

In the Gaussian model one can work them out analytically [Arnold, Metz, Schlegel 2009]:

$$\nu_{\text{Cahn}} \sim \frac{Q_T^2}{Q^2}\left(\frac{\langle \mathbf{k}_{1T}^2\rangle - \langle \mathbf{k}_{2T}^2\rangle}{\langle Q_T^2\rangle}\right)^2 \qquad \mu_{\text{Cahn}} \sim \frac{Q_T}{Q}\,\frac{\langle \mathbf{k}_{1T}^2\rangle - \langle \mathbf{k}_{2T}^2\rangle}{\langle Q_T^2\rangle}$$



Extracting the Boer-Mulders distribution from E866 data [Lu, Schmidt, Ma, Zhiang]



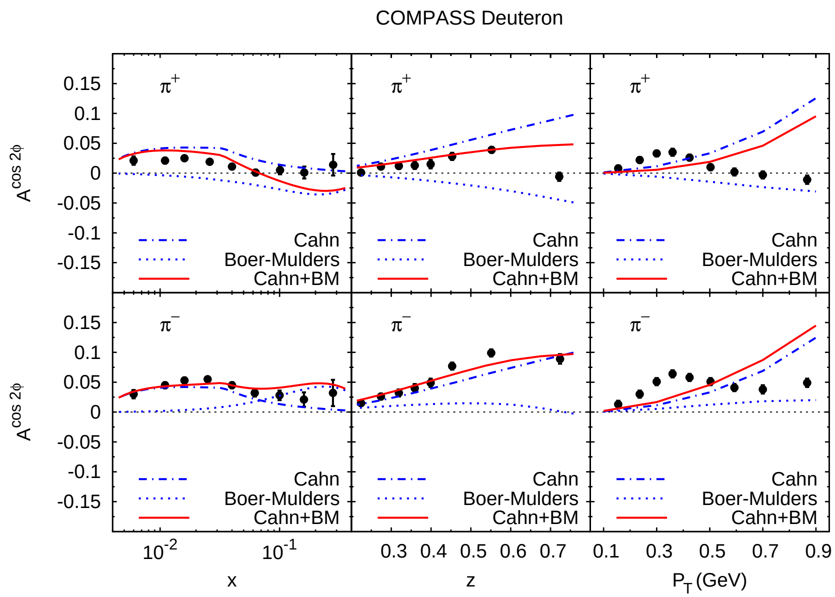
$$h_1^{\perp q}(x, \mathbf{k}_T^2) = A_q x^{a_q} (1-x)^b f_1^q(x) \frac{\exp\left(-\mathbf{k}_T^2/k_{BM}^2\right)}{\pi k_{BM}^2}$$

As DY depends on products $h_1^\perp \bar{h}_1^\perp$, the absolute normalization of the distributions and their sign are not fixed by the data

There is a factor 4 uncertainty (constrained by positivity bounds)

The $\langle \cos 2\phi_h \rangle$ asymmetry in SIDIS

[VB, Ma, Melis, Prokudin (2008, 2010)]

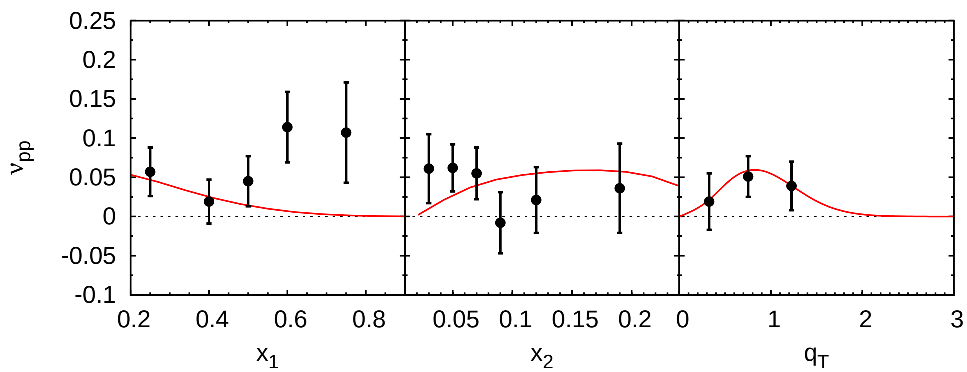


- Signs and magnitudes of the Boer-Mulders function ($h_1^{\perp u} \sim 2 f_{1T}^{\perp u}$, $h_1^{\perp d} \sim -f_{1T}^{\perp d}$) in agreement with theoretical expectations (impact-parameter + lattice, large N_c)
- Signature of the Boer-Mulders effect: $\langle \cos 2\phi_h \rangle_{\pi^-} > \langle \cos 2\phi_h \rangle_{\pi^+}$ (as a consequence of $H_1^{\perp \text{fav}} \approx -H_1^{\perp \text{unf}}$)
- Cahn contribution relatively large in spite of being $\mathcal{O}(1/Q^2)$
- Since $h_1^{\perp u}$ has the same sign as $h_1^{\perp d}$, the BM effect is not suppressed in deuteron

Use BM quark distributions from SIDIS and extract antiquarks from E866

[VB, Melis, Prokudin 2010]

(Note: SIDIS is quite insensitive to antiquark distributions)

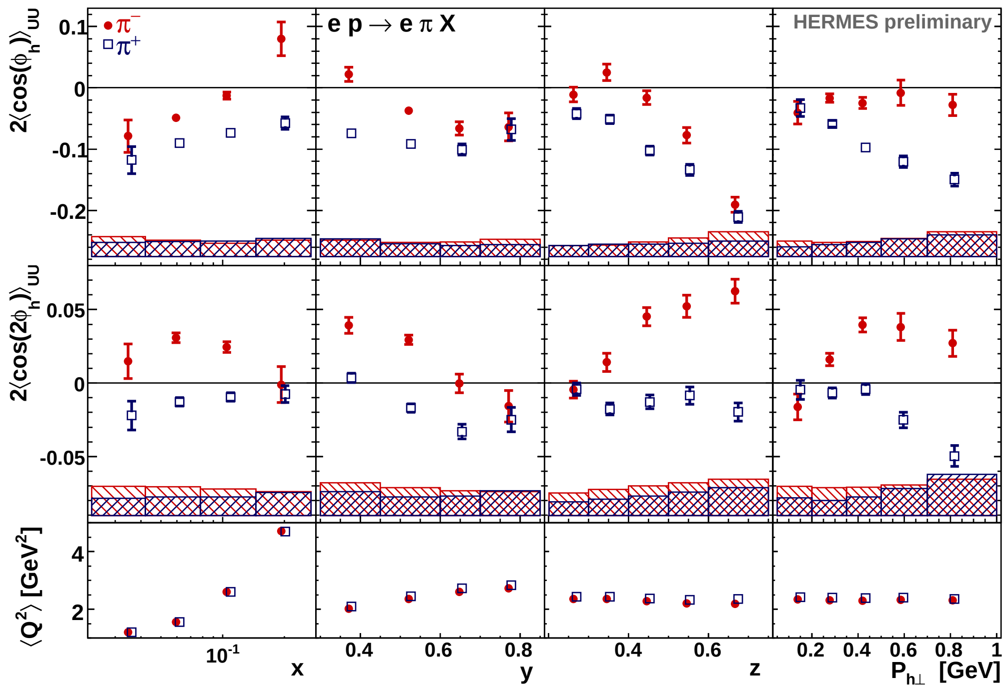


Two fits have been performed with different average transverse momenta: 0.25 GeV^2 and 0.64 GeV^2. The agreement with data is the same, but the normalization of antiquark distributions changes

Considering only the low Q_T region (< 1.5 GeV), the dataset is quite limited

HERMES preliminary results: $\langle \cos \phi_h \rangle$ and $\langle \cos 2\phi_h \rangle$ for $\pi^{\pm}$

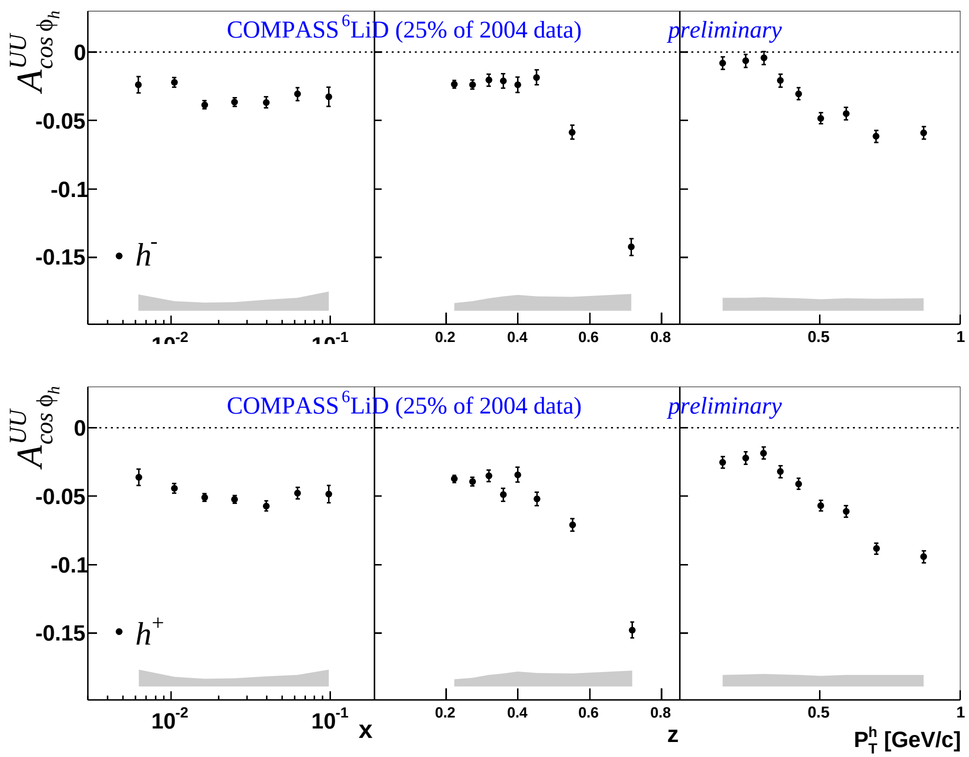
[Giordano & Lamb, HERMES (2010)]



- $\langle \cos \phi_h \rangle_{\pi^-}$ remarkably big in magnitude
- $\langle \cos 2\phi_h \rangle_{\pi^-}$ much larger than $\langle \cos 2\phi_h \rangle_{\pi^+}$
- $\langle \cos 2\phi_h \rangle_{\pi^+}$ slightly negative

COMPASS preliminary results: $\langle \cos \phi_h \rangle$ for $h^{\pm}$

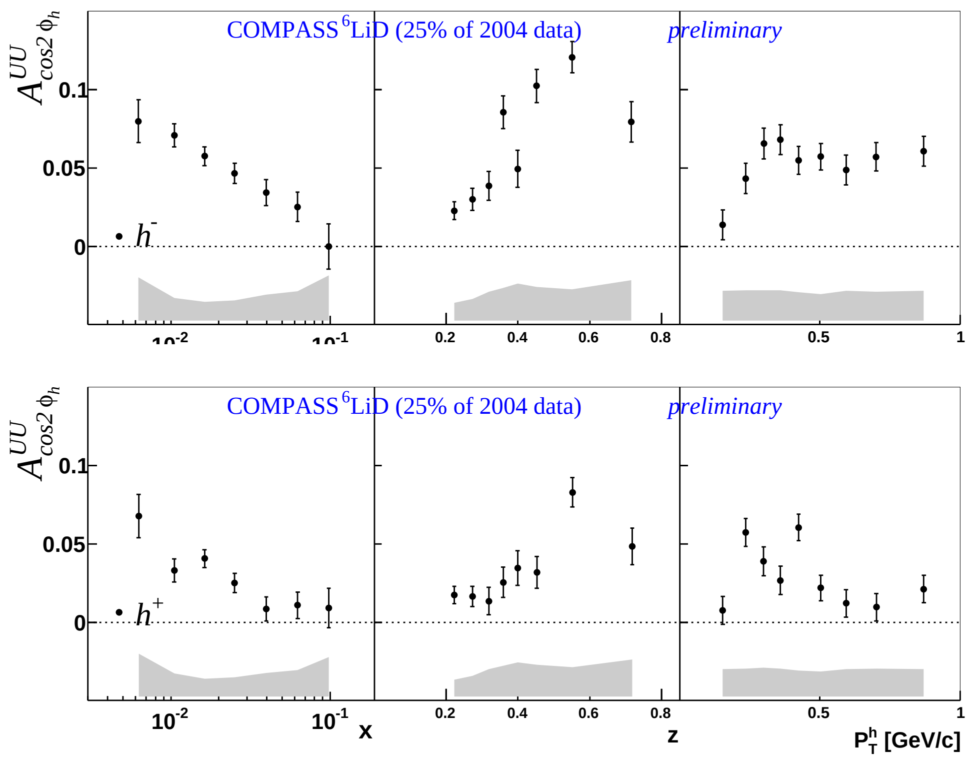
[Sbrizzai, COMPASS (2011)]



- Both $\langle \cos \phi_h \rangle_{\pi^+}$ and $\langle \cos \phi_h \rangle_{\pi^-}$ negative
- $\langle \cos \phi_h \rangle_{h^-}$ slightly larger than $\langle \cos \phi_h \rangle_{h^+}$

COMPASS preliminary results: $\langle \cos 2\phi_h \rangle$ for $h^{\pm}$

[Sbrizzai, COMPASS (2011)]



- Both $\langle \cos 2\phi_h \rangle_{\pi^+}$ and $\langle \cos 2\phi_h \rangle_{\pi^-}$ positive
- $\langle \cos 2\phi_h \rangle_{\pi^-}$ larger than $\langle \cos 2\phi_h \rangle_{\pi^+}$

- The typical scheme for SIDIS is

$$\langle \cos \phi_h \rangle = \frac{1}{Q}\,\text{Cahn} + \frac{1}{Q}\,\text{BM} \qquad \langle \cos 2\phi_h \rangle = \text{BM} + \frac{1}{Q^2}\,\text{Cahn}$$

- If we extrapolate the previous analyses:
  - Cahn contribution to $\langle \cos \phi_h \rangle$ huge (largely overshoots the data)
  - BM contribution to $\langle \cos 2\phi_h \rangle$ small and unable to account for both $\langle \cos \phi_h \rangle$ and $\langle \cos 2\phi_h \rangle$
- Corrections to the scheme above:
  - "Genuine" twist-3 contributions (quark-gluon correlations $\Rightarrow$ tilde TMDs)
  - Further $1/Q^2$ kinematic terms and dynamical twist-4 contributions
- A cutoff on k_T largely suppresses the Cahn contribution [Boglione, Melis, Prokudin 2011]

$\boxed{\text{Two brief digressions}}$

The Gaussian Ansatz for distributions

Transverse-momentum dependent distribution functions:

$$f_1(x, \mathbf{k}_\perp^2) = N\, f_1(x)\, \mathrm{e}^{-\mathbf{k}_\perp^2/\overline{\mathbf{k}_\perp^2}}$$

Average squared momenta:

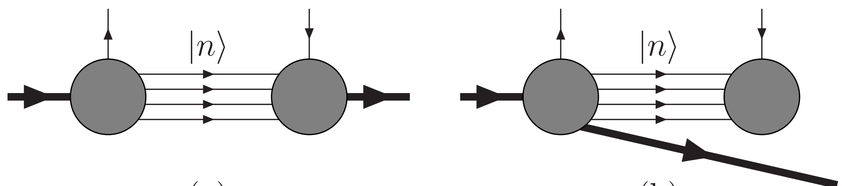
$$\langle \mathbf{k}_\perp^2 \rangle \equiv \int \mathrm{d}^2\mathbf{k}_\perp\, \mathbf{k}_\perp^2\, f_1(x, \mathbf{k}_\perp^2)$$

Average values coincide with Gaussian widths, $\langle \mathbf{k}_\perp^2 \rangle = \overline{\mathbf{k}_\perp^2}$, if one integrates over $\mathbf{k}_\perp$ between 0 and ∞

The factorization in x and k_T has no solid physical basis

In other approaches (e.g., spectator models) this factorization does not hold

“Intrinsic” quark momentum



(a) (b)

Bounds on $x \equiv k^-/P^-$: lessons from parton model [Jaffe 1983]

(b) No semi-connected diagrams $\Rightarrow x > 0$

(a) Physical intermediate states, $P_n^- \geq 0 \Rightarrow x \leq 1$

Momentum of the intermediate states $P_n^\mu = (\frac{xM^2 - \mathbf{k}_\perp^2}{2xP^-}, (1-x)P^-, -\mathbf{k}_\perp)$

The condition $M_n^2 \geq 0$ implies [Sheiman 1980]

$$\mathbf{k}_\perp^2 \leq x(1-x)M^2$$

This is the “intrinsic” quark transverse momentum

For recent discussions see [D'Alesio, Leader & Murgia (2010); Zavada (2011)]

Numerically the upper limit on the intrinsic transverse momentum of quarks is $\mathbf{k}_\perp^2 < 0.25$ GeV^2. The average value $\langle \mathbf{k}_\perp^2 \rangle$ must be smaller

On the other hand, various phenomenological analyses point to larger values, $\langle \mathbf{k}_\perp^2 \rangle \sim 0.25 - 0.40$ GeV^2

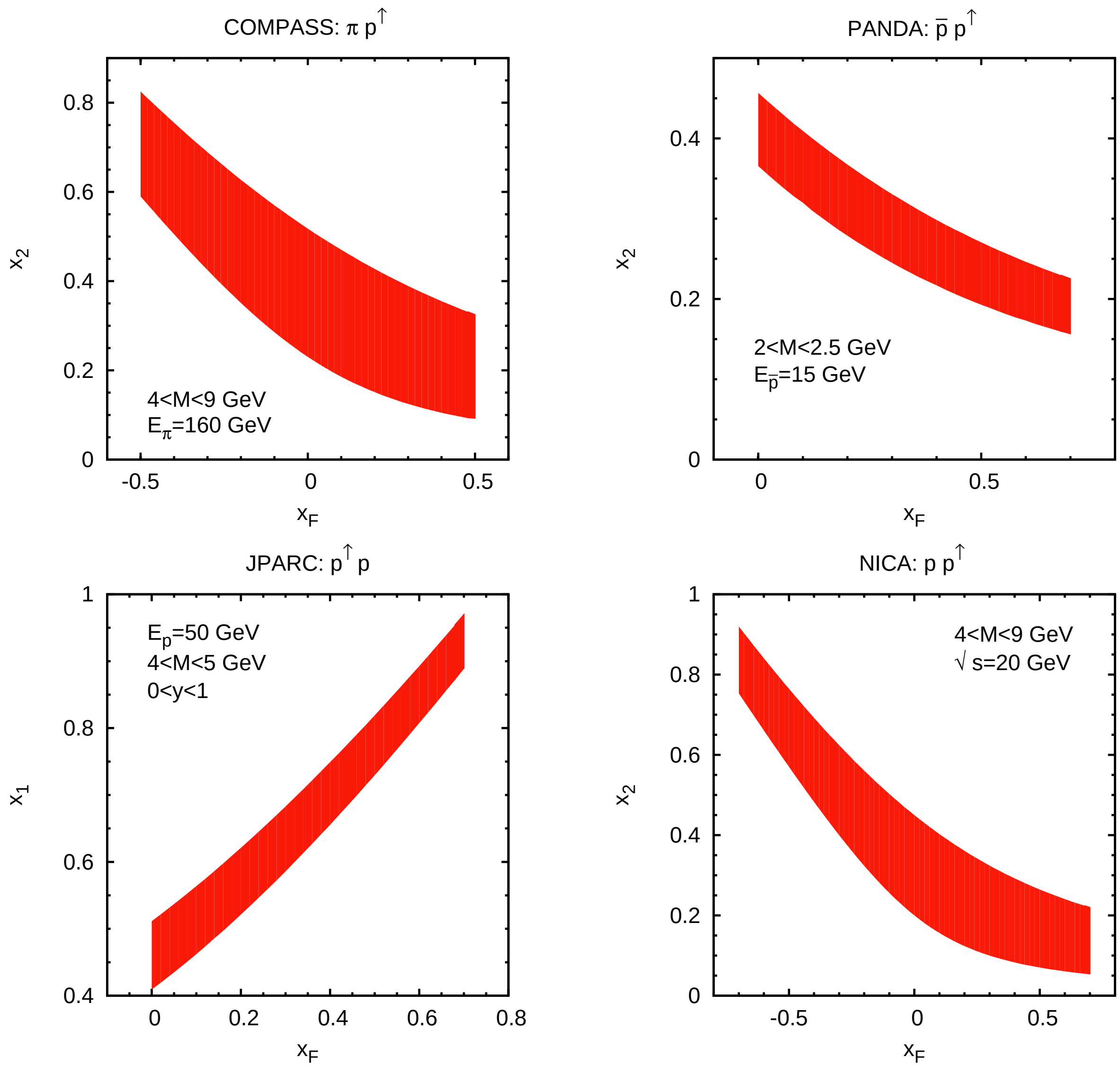
Is there a contradiction?

- The bound $\mathbf{k}_\perp^2 < x(1-x)M^2$ refers to a "static" nucleon $(Q^2 = 0)$
- The value of $\langle \mathbf{k}_\perp^2 \rangle$ extracted from experiments effectively accounts for "non-intrinsic" transverse momentum generated at a given Q^2

  $\langle \mathbf{k}_\perp^2 \rangle$ is not a fixed, universal, quantity; it must be taken as a free parameter for each dataset

$\boxed{\text{Future Drell-Yan experiments}}$

Experiment	Particles	Beam	$\sqrt{s}$ (GeV)	x_1 or x_2 range	$\mathcal{L}$ (cm^{-2} s^{-1})
COMPASS	$\pi^\pm + p^\uparrow$	160 GeV	17.4	$0.2 - 0.3$	$2 \cdot 10^{33}$
PAX	$p^\uparrow + \bar{p}^\uparrow$	collider	14	$0.1 - 0.9$	$2 \cdot 10^{30}$
PANDA	$\bar{p} + p$	15 GeV	5.5	$0.2 - 0.4$	$2 \cdot 10^{32}$
J–PARC	$p^{(\uparrow)} + p$	50 GeV	10	$0.5 - 0.9$	10^{35}
NICA	$p^{(\uparrow)} + p(d)$	collider	20	$0.1 - 0.8$	10^{30}
E906	$p + p^\uparrow$	120 GeV	15	$0.1 - 0.7$	
RHIC	$p^\uparrow + p$	collider	500	$0.05 - 0.1$	$2 \cdot 10^{32}$
RHIC IT	$p^\uparrow + p$	250 GeV	22	$0.25 - 0.4$	$10^{33} - 10^{34}$


COMPASS: π p↑
4<M<9 GeV
Eπ=160 GeV
x2
xF
PANDA: p̄ p↑
2<M<2.5 GeV
Ep̄=15 GeV
x2
xF
JPARC: p↑ p
Ep=50 GeV
4<M<5 GeV
0<y<1
x1
xF
NICA: p p↑
4<M<9 GeV
√ s=20 GeV
x2
xF

- Unpolarized DY:
  - $\bar{p}p$: PANDA can provide information on the valence BM distribution, but Q is small, unless a collider mode is realized
  - pp and pd: NICA will probe the quark and antiquark BM distributions in a wide x range, and explore the flavor content. The main disadvantage is the low luminosity
- Single (transversely) polarized DY:
  - $pp^{\uparrow}$: J-PARC, NICA, E906, RHIC-IT, RHIC will probe the Sivers asymmetry in different kinematic regions (J-PARC: large x, NICA, E906, RHIC-IT: intermediate x, RHIC: small x)
  - $\pi^{\pm}p^{\uparrow}$: COMPASS is expected to exhibit a large asymmetry, being valence-driven. The SIDIS = - DY relation can be easily tested, since the sign of the asymmetry for $\pi^{-}(\pi^{+})$ is determined by the sign of the $u(d)$ Sivers function

$\boxed{\text{Predictions for single-polarized Drell-Yan}}$

In the TMD framework, at leading twist:

$$W_{UT}^{\sin(\phi-\phi_{S_2})} = \mathcal{C}\left[\frac{\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T}}{M_2}\, f_1\, \bar{f}_{1T}^{\perp}\right]$$

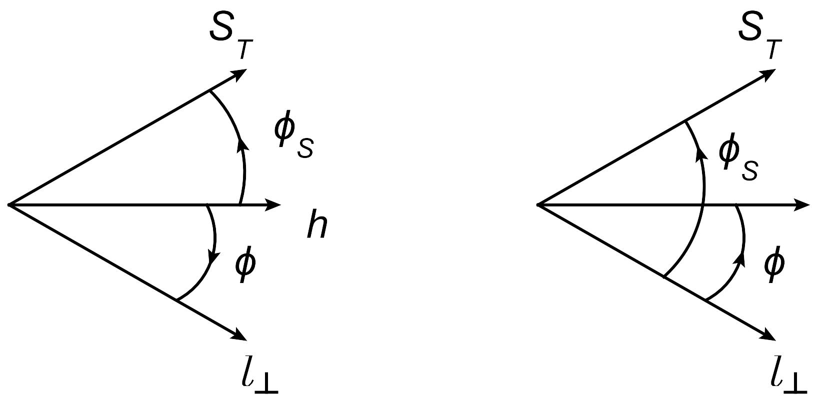
$$W_{UT}^{\sin(\phi+\phi_{S_2})} = \mathcal{C}\left[\frac{\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T}}{M_1}\, h_1^{\perp}\, \bar{h}_1\right]$$

$$W_{UT}^{\sin(3\phi-\phi_{S_2})} = \mathcal{C}\left[\frac{2(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T})[2(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{2T}) - \mathbf{k}_{1T}\cdot\mathbf{k}_{2T}] - \mathbf{k}_{2T}^2(\hat{\mathbf{q}}_T\cdot\mathbf{k}_{1T})}{2M_1 M_2^2}\, h_1^{\perp}\, \bar{h}_{1T}^{\perp}\right]$$

Single transversely polarized DY probes:

- the Sivers distribution via $\sin(\phi-\phi_S)$
- the Boer-Mulders distribution in combination with the transversity or the "pretzelosity", via $\sin(\phi+\phi_S)$ and $\sin(3\phi-\phi_S)$ respectively

Warning: Two definitions of angles



Arnold, Metz, Schlegel   Bacchetta et al., Anselmino et al.

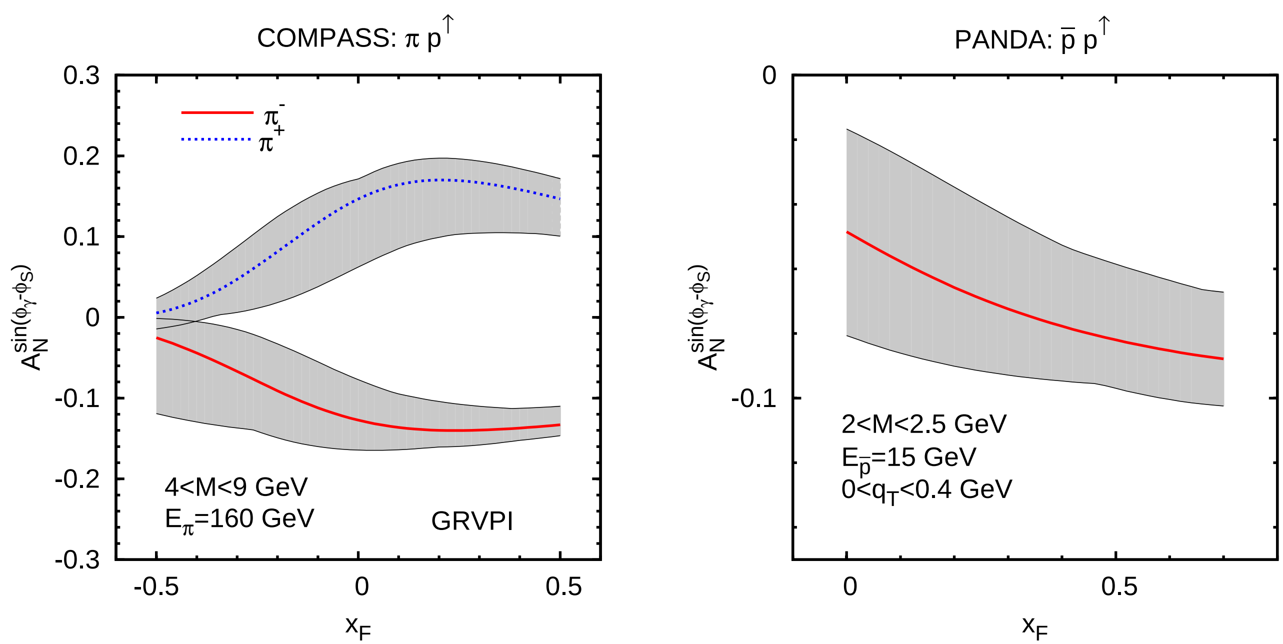
The two definitions are related by $\phi \to -\phi$ and $\phi_S \to \phi_S - \phi$

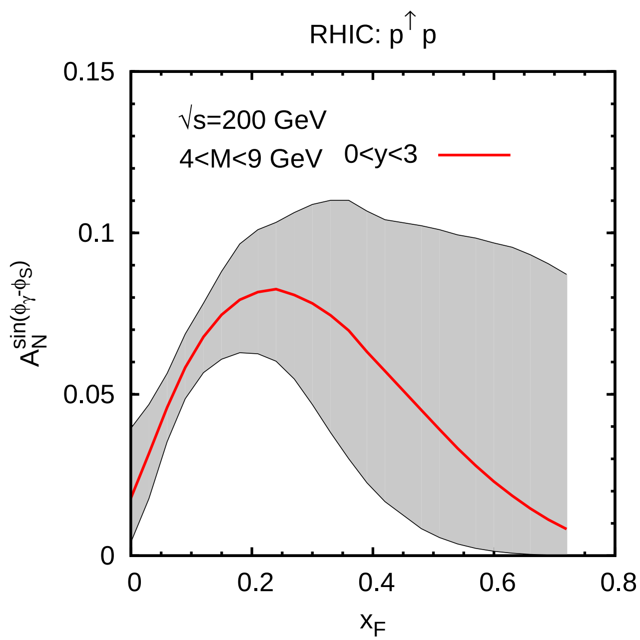
$$W_{UT}^{1}, W_{UT}^{\sin(2\phi-\phi_S)}, W_{UT}^{\sin(2\phi+\phi_S)} \to W_{UT}^{\sin(\phi-\phi_S)}, W_{UT}^{\sin(\phi+\phi_S)}, W_{UT}^{\sin(3\phi-\phi_S)}$$

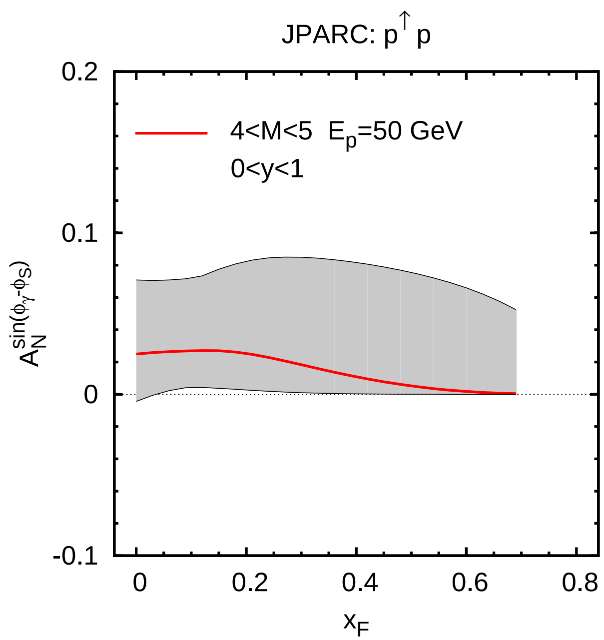
Predictions for the Sivers asymmetry [Anselmino et al. 2009]

Asymmetry defined as $\mathcal{A}^{\sin(\phi-\phi_S)} \equiv \dfrac{2\int d\phi\, \sin(\phi-\phi_S)\,(d\sigma^{\uparrow}-d\sigma^{\downarrow})}{\int d\phi\,(d\sigma^{\uparrow}+d\sigma^{\downarrow})}$

Sivers function obtained from a fit of HERMES and Compass SIDIS data



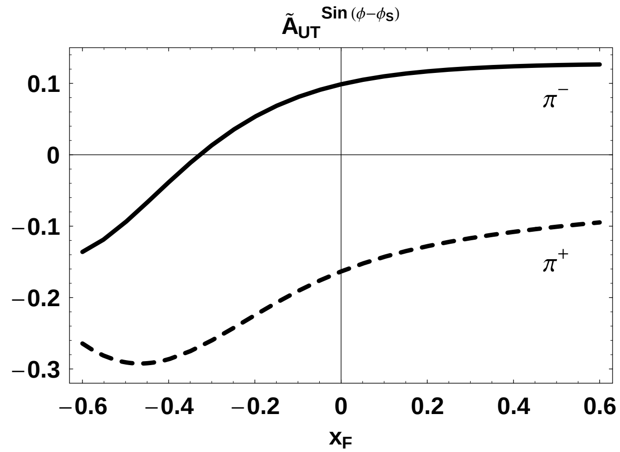

RHIC: p↑ p
√s=200 GeV
4<M<9 GeV 0<y<3
$A_N^{\sin(\phi_\gamma-\phi_S)}$
x_F
0 0.05 0.1 0.15
0 0.2 0.4 0.6 0.8


JPARC: p↑ p
4<M<5 E_p=50 GeV
0<y<1
$A_N^{\sin(\phi_\gamma-\phi_S)}$
x_F
-0.1 0 0.1 0.2
0 0.2 0.4 0.6 0.8

Weighted Sivers asymmetry [Bacchetta et al. 2010]

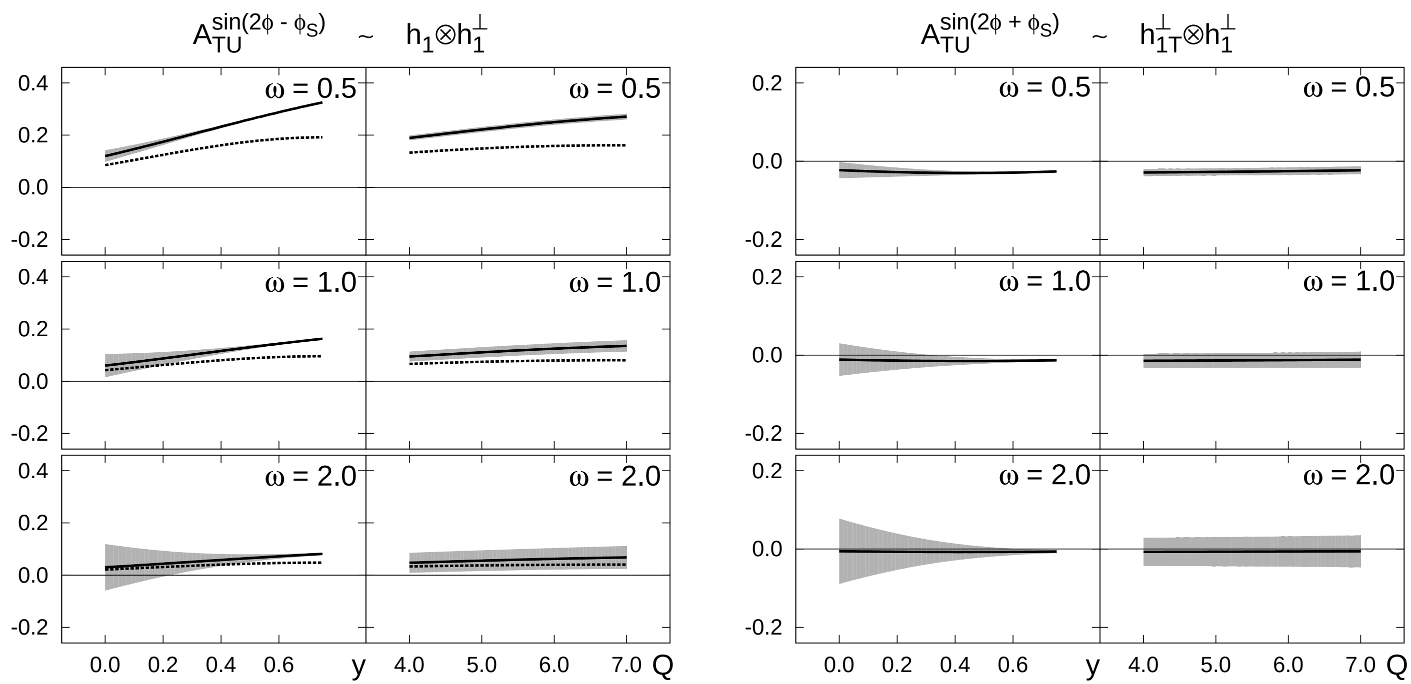
$$\tilde{A}^{\sin(\phi-\phi_S)} \equiv \frac{2\int d\phi\, d\mathbf{q}_T^2\, \frac{Q_T}{M}\, \sin(\phi-\phi_S)\,(d\sigma^{\uparrow}-d\sigma^{\downarrow})}{\int d\phi\, d\mathbf{q}_T^2 (d\sigma^{\uparrow}+d\sigma^{\downarrow})} \sim \frac{\sum_a e_a^2 \bar{f}_1^a(x_1) f_{1T}^{\perp(1)a}(x_2)}{\sum_a e_a^2 \bar{f}_1^a(x_1) f_1^a(x_2)}$$

$$\pi^- p^{\uparrow}: \quad \frac{f_{1T}^{\perp(1)u}}{f_1^u}\ \text{(proton)} \qquad \pi^+ p^{\uparrow}: \quad \frac{f_{1T}^{\perp(1)d}}{f_1^d}\ \text{(proton)}$$



Sivers function obtained from a diquark spectator model (no x, k_T factorization)

Predictions for Boer-Mulders asymmetries [Lu, Ma, Zhu 2011]



Asymmetries are defined as structure function ratios and shown for E906

BM function from DY fit (with the uncertainty on normalization parametrized by ω).
The other distributions from a light-cone model

Conclusions

- Drell-Yan production is a very rich, but so far little exploited, source of knowledge on transverse spin and transverse momentum distributions of quarks and antiquarks
- The new SIDIS data on $\langle \cos\phi \rangle$ and $\langle \cos 2\phi \rangle$ should be combined with DY data in a state-of-art analysis, to extract as much information as possible on the Boer-Mulders distribution
- The planned DY measurements are complementary to each other
  - $\bar{p}p \; \Rightarrow \; h_1^{\perp}$
  - $\bar{p}p^{\uparrow}, pp^{\uparrow}, \pi p^{\uparrow} \; \Rightarrow \; f_{1T}^{\perp}$ via $\sin(\phi - \phi_S)$ and test of $f_{1T}^{\perp}$ (DY) = - $f_{1T}^{\perp}$ (SIDIS) Also $h_1^{\perp}, h_1$ via $\sin(\phi + \phi_S)$
  - $p^{\uparrow}p^{\uparrow}, \bar{p}^{\uparrow}p^{\uparrow} \; \Rightarrow h_1$
- Perspectives appear to be very promising: hopefully, the phenomenology of polarized DY will soon become reality