Experimental Overview of Drell-Yan Process

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Outline

• Brief history of Drell-Yan experiments
• Results from Fermilab Drell-Yan experiments
• What are the remaining puzzles and crucial future experiments?
First Dimuon Experiment

\[ p + U \rightarrow \mu^+ + \mu^- + X \quad 29 \text{ GeV proton} \]

Lederman et al. PRL 25 (1970) 1523

- Experiment originally designed to search for intermediate weak boson
- Missed the J/Ψ signal!
- “Discovered” the Drell-Yan process
The Drell-Yan Process

MASSIVE LEPTON-PAIR PRODUCTION IN HADRON-HADRON COLLISIONS AT HIGH ENERGIES*

Sidney D. Drell and Tung-Mow Yan
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 25 May 1970)

On the basis of a parton model studied earlier we consider the production process of large-mass lepton pairs from hadron-hadron inelastic collisions in the limiting region, $s \to \infty$, $Q^2/s$ finite, $Q^2$ and $s$ being the squared invariant masses of the lepton pair and the two initial hadrons, respectively. General scaling properties and connections with deep inelastic electron scattering are discussed. In particular, a rapidly decreasing cross section as $Q^2/s \to 1$ is predicted as a consequence of the observed rapid falloff of the inelastic scattering structure function $\nu W_2$ near threshold.

The Drell-Yan Process: $pN \to \mu^+\mu^- X$

\[
\left( \frac{d^2 \sigma}{dx_1 dx_2} \right)_{D.Y.} = \frac{4\pi\alpha^2}{9sx_1x_2} \sum_a e_a^2 \left[ q_a(x_1)\bar{q}_a(x_2) + \bar{q}_a(x_1)q_a(x_2) \right]
\]
Naive Drell-Yan and Its Successor*

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February 1, 2008

Abstract

We review the development in the field of lepton pair production since proposing parton-antiparton annihilation as the mechanism of massive lepton pair production. The basic physical picture of the Drell-Yan model has survived the test of QCD, and the predictions from the QCD improved version have been confirmed by the numerous experiments performed in the last three decades. The model has provided an active theoretical arena for studying infrared and collinear divergences in QCD. It is now so well understood theoretically that it has become a powerful tool for new physics information such as precision measurements of the W mass and lepton and quark sizes.

*Talk given at the Drell Fest, July 31, 1998, SLAC on the occasion of Prof. Sid Drell's retirement.

• “… our original crude fit did not even remotely resemble the data. Sid and I went ahead to publish our paper because of the model’s simplicity…”

• “… the successor of the naïve model, the QCD improved version, has been confirmed by the experiments…”

• “The process has been so well understood theoretically that it has become a powerful tool for precision measurements and new physics.”
Success and difficulties of the “naïve” Drell-Yan

(From Yan’s 1998 article)

Success:

• Scaling of the cross sections (depends on $x_1$ and $x_2$ only)
• Nuclear dependence (cross section depends linearly on the mass $A$)
• Angular distributions ($1+\cos^2\Theta$ distributions)

Difficulties:

• Absolute cross sections ($K$-factor is needed)
• Transverse momentum distributions (much larger $<p_T>$ than expected)
Complimentarity between DIS and Drell-Yan

Both DIS and Drell-Yan process are tools to probe the quark and antiquark structure in hadrons (factorization, universality)
Lepton-pair production provides unique information on parton distributions

\[ p + W \rightarrow \mu^+ \mu^- X \]
800 GeV/c

\[ \pi^- + W \rightarrow \mu^+ \mu^- X \]
194 GeV/c

\[ \bar{p} + p \rightarrow l^+ l^- X \]
1.8 TeV

Probe antiquark distribution in nucleon
Probe antiquark distribution in pion
Probe antiquark distributions in antiproton

Unique features of D-Y: antiquarks, unstable hadrons…
1) Fermilab E772 (proposed in 1986 and completed in 1988)  
"Nuclear Dependence of Drell-Yan and Quarkonium Production"

2) Fermilab E789 (proposed in 1989 and completed in 1991)  
"Search for Two-Body Decays of Heavy Quark Mesons"

3) Fermilab E866 (proposed in 1993 and completed in 1996)  
"Determination of $\bar{d}/\bar{u}$ Ratio of the Proton via Drell-Yan"

4) Fermilab E906 (proposed in 1999, will run in 2011-2013)  
"Drell-Yan with the FNAL Main Injector"
Nuclear dependence of the Drell-Yan process

- As an electromagnetic process, the Drell-Yan cross section is expected to depend linearly on the nuclear mass number $A$.

\[ \sigma = \sigma_0 A^\alpha \]

$\sigma_0$: cross section on a nucleon

(From review article of Kenyon in 1982)

$\alpha$ is consistent with 1
Modification of Parton Distributions in Nuclei

EMC effect observed in DIS

How are the antiquark distributions modified in nuclei?

$F_2$ contains contributions from quarks and antiquarks

How are the antiquark distributions modified in nuclei?

Drell-Yan on nuclear targets

The Drell-Yan Process: \( pN \rightarrow \mu^+ \mu^- X \)

The \( x \)-dependence of \( \frac{\sigma^{pA}}{\sigma^{pd}} \) can be directly measured as:

\[
\frac{\sigma^{pA}}{\sigma^{pd}} \approx \frac{\bar{u}_A(x)}{\bar{u}_N(x)}
\]

The \( x \)-dependence of \( \frac{\bar{u}_A(x)}{\bar{u}_N(x)} \) can be directly measured.
Drell-Yan on nuclear targets

No evidence for enhancement of antiquark in nuclei!?
E906 will extend the measurement to larger $x$

PRL 64 (1990) 2479
PRL 83 (1999) 2304
EMC-SRC Correlation

From D. Gaskell’s talk

J. Seely, et al., PRL 103, 202301 (2009)
Possible tests for EMC-SRC correlation with Drell-Yan

• Expect to see the same EMC-SRC correlation for Drell-Yan nuclear dependence (effect should be identical for valence and sea quarks)
  – Can be tested at Fermilab E906
  – \(^9\)Be target should be measured

• Expect no up-down quark flavor dependence for SRC (since SRC is dominated by isoscalar p-n correlation)
  – Can be tested by pion or antiproton induced Drell-Yan
Isovector mean-field generated in $Z \neq N$ nuclei can modify nucleon’s $u$ and $d$ PDFs in nuclei

Cloet, Bentz, and Thomas, arXiv:0901.3559

How can one check this prediction?

- SIDIS (JLab proposal) and PVDIS (P. Souder)
- Pion-induced Drell-Yan
Pion-induced Drell-Yan and the flavor-dependent EMC effect

\[ \frac{\sigma_{DY}^{A} (\pi^- + A)}{\sigma_{DY}^{D} (\pi^- + D)} \approx \frac{u_A(x)}{u_D(x)} \]

Red (blue) curves correspond to flavor-dependent (independent) EMC

(D. Dutta, JCP, Cloet, Gaskell, arXiv: 1007.3916)
Pion-induced Drell-Yan and the flavor-dependent EMC effect

\[
\frac{\sigma^{DY}(\pi^+ + A)}{\sigma^{DY}(\pi^- + A)} \approx \frac{d_A(x)}{4u_A(x)}; \quad \frac{\sigma^{DY}(\pi^- + A)}{\sigma^{DY}(\pi^- + D)} \approx \frac{u_A(x)}{u_D(x)}
\]

160 GeV pion beam

Drell-Yan data from COMPASS with pion beams could provide important new information
W-production at LHC and the flavor-dependent EMC effect

\[ R_{A/D}^+ \equiv \frac{d\sigma(p + A \to W^+ + x)}{d\sigma(p + D \to W^+ + x)} \]
\[ \approx \frac{u_A(x_2)}{u_D(x_2)} \]

\[ R_{A/D}^- \equiv \frac{d\sigma(p + A \to W^- + x)}{d\sigma(p + D \to W^- + x)} \]
\[ \approx \frac{d_A(x_2)}{d_D(x_2)} \]

\[ R_{A}^{\pm} \equiv \frac{d\sigma(p + A \to W^\pm + x)}{d\sigma(p + A \to W^\mp + x)} \]
\[ \approx \frac{\bar{d}_p(x_1) u_A(x_2)}{\bar{u}_p(x_1) d_A(x_2)} \]

(Chang, Cloet, Dutta, JCP, 1109.3108)
$\overline{d} / \overline{u}$ flavor asymmetry from Drell-Yan

\[
\left( \frac{d^2 \sigma}{dx_1 dx_2} \right)_{D.Y.} = \frac{4\pi\alpha^2}{9sx_1x_2} \sum_a e_a^2 \left[ q_a(x_1)\overline{q}_a(x_2) + \overline{q}_a(x_1)q_a(x_2) \right]
\]

at $x_1 > x_2$ : Drell-Yan:

\[
\sigma^{pd} / 2\sigma^{pp} \sim \frac{1}{2} \left( 1 + \frac{d(x_2)}{u(x_2)} \right)
\]
Meson Cloud Models

\[ \pi^+ \text{ meson} \]

Chiral-Quark Soliton Model

- nucleon = chiral soliton
- Quark degrees of freedom in a pion mean-field
- expand in \( 1/N_c \)

Theory: Thomas, Miller, Kumano, Ma, Londergan, Henley, Speth, Hwang, Melnitchouk, Liu, Cheng/Li, etc.

(For reviews, see Speth and Thomas (1997), Kumano (hep-ph/9702367), Garvey and Peng (nucl-ex/0109010))

Meson cloud has significant contributions to sea-quark distributions

Theses models also have implications on

- asymmetry between \( s(x) \) and \( \bar{s}(x) \)
- flavor structure of the polarized sea

\[ \text{Meson cloud has significant contributions to sea-quark distributions} \]
Search for the “intrinsic” quark sea

In 1980, Brodsky, Hoyer, Peterson, Sakai (BHPS) suggested the existence of “intrinsic” charm

\[ |p\rangle = P_{3q} |uud\rangle + P_{5q} |uudQ\bar{Q}\rangle + \cdots \]

The "intrinsic"-charm from \( |uudcc\rangle \) is "valence"-like and peak at large \( x \) unlike the "extrinsic" sea (\( g \rightarrow c\bar{c} \))

The \( |uudcc\rangle \) intrinsic-charm can lead to large contribution to charm production at large \( x \)
“Evidence” for the “intrinsic” charm (IC)

Gunion and Vogt (hep-ph/9706252)

“Evidence” appears to be rather weak

(subject to the uncertainties of charmed-quark parametrization in the PDF)
A global fit by CTEQ to extract intrinsic-charm

Blue band corresponds to CTEQ6 best fit, including uncertainty

Red curves include intrinsic charm of 1% and 3% ($\chi^2$ changes only slightly)

We find that the range of IC is constrained to be from zero (no IC) to a level 2–3 times larger than previous model estimates. The behaviors of typical charm distributions within this range are described, and their implications for hadron collider phenomenology are briefly discussed.

No conclusive evidence for intrinsic-charm
Search for the lighter “intrinsic” quark sea

\[ |p\rangle = P_{3q} |uud\rangle + P_{5q} |uudQ\bar{Q}\rangle + \cdots \]

No conclusive experimental evidence for intrinsic-charm so far

Are there experimental evidences for the intrinsic 5-quark states?  
\[ |uuduu\bar{u}\rangle, |uuddd\bar{d}\rangle, |uudss\bar{s}\rangle \text{ 5-quark states?} \]

\[ P_{5q} \sim 1 / m_Q^2 \]

The 5-quark states for lighter quarks have larger probabilities!
**$x$-distribution for “intrinsic” charm**

$$ | p \rangle = P_{3q} | uud \rangle + P_{5q} | uudQ\bar{Q} \rangle + \cdots $$

Brodsky et al. (BHPS) give the following probability for quark $i$ (mass $m_i$) to carry momentum $x_i$

$$ P(x_1, \ldots, x_5) = N_s \delta(1 - \sum_{i=1}^{5} x_i) \left[ m_p^2 - \sum_{i=1}^{5} \frac{m_i^2}{x_i} \right]^{-2} $$

In the limit of large mass for quark Q (charm):

$$ P(x_5) = \frac{1}{2} \bar{N}_s x_5^2 [(1 - x_5)(1 + 10x_5 + x_5^2) - 2x_5 (1 + x_5) \ln(1/x_5)] $$

An analytical expression for $P(x)$ is obtained
x-distribution for “intrinsic” light-quark sea

\[ |p\rangle = P_{3q} |uud\rangle + P_{5q} |uudQ\bar{Q}\rangle + \cdots \]

Brodsky et al. (BHPS) give the following probability for quark \( i \) (mass \( m_i \)) to carry momentum \( x_i \)

\[
P(x_1, \ldots, x_5) = N_5 \delta(1 - \sum_{i=1}^{5} x_i) \left[ m_p^2 - \sum_{i=1}^{5} \frac{m_i^2}{x_i} \right]^{-2}
\]

In the limit of large mass for quark \( Q \) (charm):

\[
P(x_5) = \frac{1}{2} \tilde{N}_5 x_5^2 \left[ (1 - x_5)(1 + 10x_5 + x_5^2) - 2x_5(1 + x_5)\ln(1/x_5) \right]
\]

One can calculate \( P(x) \) for antiquark \( \bar{Q} \) \((\bar{c}, \bar{s}, \bar{d})\) numerically
How to separate the “intrinsic sea” from the “extrinsic sea”?

- Select experimental observables which have no contributions from the “extrinsic sea”
- “Intrinsic sea” and “extrinsic sea” are expected to have different $x$-distributions
  - Intrinsic sea is “valence-like” and is more abundant at larger $x$
  - Extrinsic sea is more abundant at smaller $x$
How to separate the “intrinsic sea” from the “extrinsic sea”?

• Select experimental observables which have no contributions from the “extrinsic sea”

\[ \bar{d} - \bar{u} \] has no contribution from extrinsic sea (\( g \rightarrow \bar{q}q \)) and is sensitive to "intrinsic sea" only
Comparison between the $\bar{d}(x) - \bar{u}(x)$ data with the intrinsic 5-\(q\) model

The data have very different shape compared to the BHPS 5-\(q\) model

E866 data measured at $\langle Q^2 \rangle = 54$ GeV$^2$

Need to evolve the 5-\(q\) model prediction from the initial scale $\mu$ to $Q^2=54$ GeV$^2$
Comparison between the $\bar{d}(x) - \bar{u}(x)$ data with the intrinsic 5-q model

The data are in good agreement with the 5-q model after evolution from the initial scale $\mu$ to $Q^2 = 54 \text{ GeV}^2$

The difference in the two 5-quark components can also be determined

(W. Chang and JCP, PRL 106, 252002 (2011))

$$P_{5uud\bar{d}} - P_{5uudd\bar{u}} = 0.118$$
Comparison between the $s(x) + \bar{s}(x)$ data with the intrinsic $5-q$ model

$s(x) + \bar{s}(x)$ from HERMES kaon SIDIS data at $\langle Q^2 \rangle = 2.5$ GeV$^2$

The data appear to consist of two different components (intrinsic and extrinsic?)

Comparison between the $s(x) + \bar{s}(x)$ data with the intrinsic 5-$q$ model

$s(x) + \bar{s}(x)$ from HERMES kaon SIDIS data at $\langle Q^2 \rangle = 2.5$ GeV$^2$

Assume $x > 0.1$ data are dominate by intrinsic sea (and $x < 0.1$ are from QCD sea)

This allows the extraction of the intrinsic sea for strange quarks

(W. Chang and JCP, PL B704, 197(2011))

$$P_5^{uudss} = 0.024$$
Comparison between the $\bar{u}(x) + \bar{d}(x) - s(x) - \bar{s}(x)$ data with the intrinsic 5-q model

$\bar{d}(x) + \bar{u}(x)$ from CTEQ6.6
$s(x) + \bar{s}(x)$ from HERMES

$\bar{u} + \bar{d} - s - \bar{s}$ has no contribution from extrinsic sea

A valence-like $x$-distribution is observed
Is the “extrinsic” sea flavor-symmetric for \( u, d, \) and \( s \)?

\[
\begin{align*}
\text{Determination of the strange quark density of the proton from} \\
\text{ATLAS measurements of the} \ W \rightarrow \ell \nu \text{ and} \ Z \rightarrow \ell \ell \text{ cross} \\
\text{sections}
\end{align*}
\]

\[
\begin{align*}
\text{The ATLAS Collaboration}
\end{align*}
\]

\[
\begin{align*}
\left. r_s \right|_{\text{Bj. } x = 0.023} &= 1.00^{+0.25}_{-0.28} \\
\end{align*}
\]
Comparison between the $\bar{u}(x) + \bar{d}(x) - s(x) - \bar{s}(x)$ data with the intrinsic 5-\(q\) model

$\bar{d}(x) + \bar{u}(x)$ from CTEQ6.6

$s(x) + \bar{s}(x)$ from HERMES

$\bar{u} + \bar{d} - s - \bar{s}$

$\sim P^{\text{uudu}}_{5} + P^{\text{uudd}}_{5} - 2P^{\text{uudss}}_{5}$

(not sensitive to extrinsic sea)

(W. Chang and JCP, PL B704, 197(2011))

\[
P^{\text{uudu}}_{5} + P^{\text{uudd}}_{5} - 2P^{\text{uudss}}_{5} = 0.314
\]
Extraction of the various five-quark components for light quarks

\[ P_5^{uudd\bar{d}} - P_5^{uudu\bar{u}} = 0.118 \]

\[ P_5^{uuds\bar{s}} = 0.024 \]

\[ P_5^{uudu\bar{u}} + P_5^{uudd\bar{d}} - 2P_5^{uuds\bar{s}} = 0.314 \]

\[ P_5^{uudd\bar{d}} = 0.240; \quad P_5^{uudu\bar{u}} = 0.122; \quad P_5^{uuds\bar{s}} = 0.024 \]
Possible implications on the intrinsic charm

\[
P^{\text{uudd}}_5 = 0.240; \quad P^{\text{uudd}} = 0.122; \quad P^{\text{uudds}} = 0.024
\]

Assuming

\[
P^{\text{uuddQ\bar{Q}}}_5 \sim 1 / m_Q^2
\]

then

\[
P^{\text{uudcc\bar{c}}}_5 \sim 0.1P^{\text{uudds\bar{s}}}_5 \sim 0.003
\]

- Probability of intrinsic-charm is smaller than expected
- Evolution would shift the intrinsic-charm distribution to smaller-\(x\)
Future prospect on intrinsic sea

- Kaon production in SIDIS at COMPASS and 12 GeV JLab for additional information on s and s-bar?
- Kaon-induced Drell-Yan at COMPASS for probing s and s-bar?
- Open-charm production at forward rapidity at RHIC and LHC.
- Intrinsic sea for hyperons and mesons?
- Connection between intrinsic sea and meson-cloud?
- Spin dependence of intrinsic sea?
Lattice QCD on $A_{20}$ and flavor asymmetry

$$A_{20} = \int_0^1 x [u(x) - d(x) + \bar{u}(x) - \bar{d}(x)] dx$$

$A_{20} \sim 0.20$ from lattice QCD; $A_{20} \sim 0.15$ from data
Lattice QCD on $A_{20}$ and flavor asymmetry

$$A_{20} = \int_{0}^{1} x[u(x) - d(x) + \bar{u}(x) - \bar{d}(x)]dx$$

$A_{20} \sim 0.20$ from lattice QCD; $A_{20} \sim 0.15$ from data

$u(x) - d(x)$ is well known

$\bar{u}(x) - \bar{d}(x)$ is less well known (especially at $x > 0.25$)

The apparent discrepancy in $A_{20}$ will be reduced if $\bar{d}(x) - \bar{u}(x) < 0$ at large $x$

(K. Liu, JCP, W. Chang, H. Cheng, preprint)
Revisit the NMC measurement of the Gottfried Sum rule

The Gottfried Sum Rule

\[ S_G = \int_0^1 \left[ \frac{(F_2^p(x) - F_2^n(x))}{x} \right] dx \]

\[ = \frac{1}{3} + \frac{2}{3} \int_0^1 (\bar{u}_p(x) - \bar{d}_p(x)) \, dx \]

\[ = \frac{1}{3} \quad (\text{if} \, \bar{u}_p = \bar{d}_p) \]

New Muon Collaboration (NMC) obtains

\[ S_G = 0.235 \pm 0.026 \]

(Significantly lower than 1/3!) \[ \Rightarrow \bar{d} \neq \bar{u} \]
Extracting $\bar{d}(x) - \bar{u}(x)$ from the NMC data

$$\bar{d}(x) - \bar{u}(x) = [u(x) + d(x)]_{CT10} - 3 \times \frac{F_2^p(x)}{x} - \frac{F_2^n(x)}{x}$$

The NMC data, together with the recent PDF, already suggest that $\bar{d}(x) - \bar{u}(x) < 0$ at large $x$!
\[ \frac{d}{d\bar{u}} \text{ from W production at RHIC} \]

\[ R(x_F) = \frac{d\sigma / dx_F (pp \to W^+ x)}{d\sigma / dx_F (pp \to W^- x)} \text{ at } \sqrt{s} = 500 \text{ GeV} \]

\[ R(x_F = 0) \approx \frac{u(x = 0.16) \bar{d}(x = 0.16)}{d(x = 0.16) \bar{u}(x = 0.16)} \approx 2 \frac{\bar{d}(x = 0.16)}{\bar{u}(x = 0.16)} \]

Confirms \( \frac{d}{\bar{u}} \) asymmetry at \( x \sim 0.15 \)
Drell-Yan angular distribution

Decay Angular Distribution of “naïve” Drell-Yan:

\[
\frac{d\sigma}{d\Omega} = \sigma_0 (1 + \cos^2 \theta)
\]

Data from Fermilab E772

800 GeV p+Cu

11 GeV/c^2 < M(\mu^+\mu^-) < 17 GeV/c^2

fit to \(1 + \lambda \cos^2\theta\)

\(\lambda = 0.96 \pm 0.04 \pm 0.06\)
A general expression for Drell-Yan decay angular distributions:

\[
\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right) = \left[\frac{3}{4\pi}\right] \left[ 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right]
\]

\(\lambda\) can differ from 1, but should satisfy \(1 - \lambda = 2\nu\) (Lam-Tung)

- Reflect the spin-1/2 nature of quarks
  (analog of the Callan-Gross relation in DIS)
- Insensitive to QCD - corrections
A simple geometric derivation of the generalized Lam-Tung relation (a la Oleg Teryaev)

\[
\frac{1}{\sigma} \left( \frac{d\sigma}{d\Omega} \right) = \left[ \frac{3}{4\pi} \right] \left[ 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{v}{2} \sin^3 \theta \cos 2\phi \right]
\]

In the $\gamma^*$ rest frame:
\(\hat{z}\) signifies the Collins-Soper frame
\(\hat{z}_0\) is along the collinear $q - \bar{q}$ axis
Leptons are emitted with uniform azimutual distribution, and with $\theta_0$ dependence:
\[d\sigma \sim 1 + \lambda_0 \cos^2 \theta_0\]
($\lambda_0 = 1$ for spin-1/2 quark;
$\lambda_0 = -1$ for spin-0 quark)

\[
\cos \theta_0 = \cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos \phi
\]

\[
d\sigma \sim 1 + \lambda_0 (\cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos \phi)^2
\]

\[
= [1 + (\lambda_0 / 2) \sin^2 \theta_1] + \cos^2 \theta [\lambda_0 \cos^2 \theta_1 - (\lambda_0 / 2) \sin^2 \theta_1]
+ \sin 2\theta \cos \phi [(\lambda_0 / 2) \sin 2\theta_1] + \sin^2 \theta \cos 2\phi [(\lambda_0 / 2) \sin^2 \theta_1]
\]
A simple geometric derivation of the generalized Lam-Tung relation (a la Oleg Teryaev)

\[
\left( \frac{1}{\sigma} \right) \left( \frac{d\sigma}{d\Omega} \right) = \left[ \frac{3}{4\pi} \right] \left[ 1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi \right]
\]

Therefore, we have
\[
\lambda = \lambda_0 \frac{2-3\sin^2 \theta_1}{2 + \lambda_0 \sin^2 \theta_1}
\]
\[
\mu = \lambda_0 \frac{\sin^2 \theta_1}{2 + \lambda_0 \sin^2 \theta_1}
\]
\[
\nu = \lambda_0 \frac{2\sin^2 \theta_1}{2 + \lambda_0 \sin^2 \theta_1}
\]

and
\[
\lambda_0 = \frac{\lambda + \frac{3}{2} \nu}{1 - \frac{1}{2} \nu} \quad \text{(Generalized Lam-Tung relation)}
\]

If \( \lambda_0 = 1 \), we have \( 2\nu = 1 - \lambda \) (Lam-Tung relation).
If \( \lambda_0 = -1 \) (spin-0 quark), we have \( -\nu = 1 + \lambda \)

\[
d\sigma \sim 1 + \lambda_0 (\cos \theta \cos \theta_1 + \sin \theta \sin \theta_1 \cos \phi)^2
\]
\[
= \left[ 1 + \left( \frac{\lambda_0}{2} \right) \sin^2 \theta_1 \right] + \cos^2 \theta \left[ \lambda_0 \cos^2 \theta_1 - \left( \frac{\lambda_0}{2} \right) \sin^2 \theta_1 \right]
\]
\[
+ \sin 2\theta \cos \phi \left[ \left( \frac{\lambda_0}{2} \right) \sin 2\theta_1 \right] + \sin^2 \theta \cos 2\phi \left[ \left( \frac{\lambda_0}{2} \right) \sin^2 \theta_1 \right]
\]
Decay angular distributions in pion-induced Drell-Yan

$\nu \neq 0$ and $\nu$ increases with $p_T$

NA10 $\pi^- + W$

Z. Phys.

37 (1988) 545

Dashed curves are from pQCD calculations
Decay angular distributions in pion-induced Drell-Yan

Is the Lam-Tung relation violated?

Violation of the Lam-Tung relation suggests new mechanisms with non-perturbative origin

- $q - \bar{q}$ spin correlation in QCD color field (Nachtmann et al.)
Boer-Mulders function $h_1^\perp$

- $h_1^\perp$ represents a correlation between quark's $k_T$ and transverse spin in an unpolarized hadron
- $h_1^\perp$ is a time-reversal odd, chiral-odd TMD parton distribution
- $h_1^\perp$ can lead to an azimuthal $\cos(2\phi)$ dependence in Drell-Yan

$$\left(\frac{1}{\sigma}\right)\left(\frac{d\sigma}{d\Omega}\right) = \left[\frac{3}{4\pi}\right]\left[1 + \lambda \cos^2 \theta + \mu \sin 2\theta \cos \phi + \frac{\nu}{2} \sin^2 \theta \cos 2\phi\right]$$

- Observation of large $\cos(2\Phi)$ dependence in Drell-Yan with pion beam
  - $\nu \propto h_1^\perp(x_q) h_1^\perp(x_{\bar{q}})$
- B-M functions have same signs for pion and nucleon

Boer, PRD 60 (1999) 014012

194 GeV/c
\(\pi + W\)
Three parton distributions describing quark’s transverse momentum and/or transverse spin

1) Transversity

Three transverse quantities:
1) Nucleon transverse spin
\( \vec{S}^{N} \)
2) Quark transverse spin
\( \vec{S}^{q} \)
3) Quark transverse momentum
\( \vec{k}^{q} \)

\( \Rightarrow \) Three different correlations

Correlation between \( \vec{s} \) and \( \vec{S}^{N} \)

Correlation between \( \vec{s}^{q} \) and \( \vec{k}^{q} \)

2) Sivers function

3) Boer-Mulders function
Azimuthal \( \cos 2\Phi \) Distribution in \( p+p \) and \( p+d \) Drell-Yan

E866 Collab., Lingyan Zhu et al.,

With Boer-Mulders function \( h_{1}^{\perp} \):

\[
\nu(\pi^{-} W \rightarrow \mu^{+}\mu^{-} X) \sim [\text{valence } h_{1}^{\perp}(\pi)] \ast [\text{valence } h_{1}^{\perp}(p)]
\]

\[
\nu(pd \rightarrow \mu^{+}\mu^{-} X) \sim [\text{valence } h_{1}^{\perp}(p)] \ast [\text{sea } h_{1}^{\perp}(p)]
\]

Small \( \nu \) is observed for \( p+d \) and \( p+p \) D–Y

Sea-quark BM functions are much smaller than valence quarks
Sea-quark Boer-Mulders Functions

1) Use quark-spectator-antiquark model to calculate pion B-M functions. Pion-induced Drell-Yan data are well reproduced.

(Lu and Ma, hep-ph/0504184)

2) Use pion-cloud model convoluted with the pion B-M function to calculate sea-quark B-M for proton.

\( h_{1,\bar{u}}^{\perp} (x, k_t^2) \quad \text{and} \quad h_{1,\bar{d}}^{\perp} (x, k_t^2) \)

(Lu, Ma, Schmidt, hep-ph/0701255)
Results on $\cos 2\Phi$ Distribution in $p+p$ Drell-Yan

L. Zhu et al., PRL 102 (2009) 182001

More data are anticipated from Fermilab E906 and COMPASS
Recent result from CDF on Lam-Tung relation

\[ p + \bar{p} \rightarrow e^+ + e^- + X \text{ at } \sqrt{s} = 1.96 \text{ TeV} \]

66 \text{ GeV} < M_{ee} < 116 \text{ GeV}

arXiv:1103.5699
Recent result from CDF on Lam-Tung relation

\[ p + \bar{p} \rightarrow e^+ + e^- + X \text{ at } \sqrt{s} = 1.96 \text{ TeV} \]

\[
\frac{d\sigma}{d \cos \theta} \propto (1 + \cos^2 \theta) + \frac{1}{2} A_0 (1 - 3 \cos^2 \theta) + A_4 \cos \theta
\]

\[
\frac{d\sigma}{d \phi} \propto 1 + \beta_3 \cos \phi + \beta_2 \cos 2\phi + \beta_7 \sin \phi + \beta_5 \sin 2\phi
\]

\[ \beta_3 = \frac{3\pi A_3}{16}, \beta_2 = \frac{A_2}{4}, \beta_7 = \frac{3\pi A_7}{16} \]

\[ \langle A_0 - A_2 \rangle = 0.02 \pm 0.02 \]
Polarized Drell-Yan with polarized proton beam?

- Polarized Drell-Yan experiments have never been done before
- Provide unique information on the quark (antiquark) spin

\[ q(x) \quad \Delta q(x) \quad h_1(x) \]

**Quark helicity distribution**

**Quark transversity distribution**
Transversity and Transverse Momentum Dependent PDFs are also probed in Drell-Yan

a) Boer-Mulders functions:
   - Unpolarized Drell-Yan: \( d\sigma_{DY} \propto h_1^\perp(x_q) h_1^\perp(x_{\bar{q}}) \cos(2\phi) \)

b) Sivers functions:
   - Single transverse spin asymmetry in polarized Drell-Yan:
     \[ A_N^{DY} \propto f_{1T}^\perp(x_q) f_{\bar{q}}(x_{\bar{q}}) \]

c) Transversity distributions:
   - Double transverse spin asymmetry in polarized Drell-Yan:
     \[ A_{TT}^{DY} \propto h_1(x_q) h_1(x_{\bar{q}}) \]

- Drell-Yan does not require knowledge of the fragmentation functions
- T-odd TMDs are predicted to change sign from DIS to DY (Boer-Mulders and Sivers functions)

Remains to be tested experimentally!
Outstanding questions to be addressed by future Drell-Yan experiments

- Does Sivers function change sign between DIS and Drell-Yan?
- Does Boer-Mulders function change sign between DIS and Drell-Yan?
- Are all Boer-Mulders functions alike (proton versus pion Boer-Mulders functions)
- Flavor dependence of TMD functions
- Independent measurement of transversity with Drell-Yan

Can be studied at COMPASS, RHIC, FAIR, JPARC, JINR, etc
What do we know about the quark and gluon intrinsic transverse momentum distributions?

- Does the quark $k_T$ distribution depend on $x$?
- Do valence quarks and sea quarks have different $k_T$ distributions?
- Do $u$ and $d$ quarks have the same $k_T$ distribution?
- Do nucleons and mesons have different quark $k_T$ distribution?
- Do gluons have $k_T$ distribution different from quarks?

- Important for extracting the TMD parton distributions
- Interesting physics in its own right
What do Drell-Yan data tell us about the quark intrinsic transverse momentum distribution?

- $\langle P_T^2 \rangle$ increases linearly with $s$ (expected from QCD)
- Proton-induced D-Y has smaller mean $P_T$ than pion (expected from the uncertainty principle, reflecting the larger size of the proton)
Comparison of the mean $P_T$ of proton, pion, and kaon induced Drell-Yan

Drell-Yan with proton beam:

$$\langle P_T \rangle = (0.43 \pm 0.03) + \sqrt{s} (0.026 \pm 0.001) \text{ GeV/c}$$

Drell-Yan with pion beam:

$$\langle P_T \rangle = (0.59 \pm 0.05) + \sqrt{s} (0.028 \pm 0.003) \text{ GeV/c}$$

NA3 data also show that $\langle P_T \rangle$ for D-Y with kaon beam is larger than Drell-Yan with pion beam:

$$\langle P_T^2 \rangle = 1.51 \pm 0.08 (\text{GeV/c})^2 \text{ for kaon beam}$$

$$\langle P_T^2 \rangle = 1.44 \pm 0.02 (\text{GeV/c})^2 \text{ for pion beam}$$

with 150 GeV/c beams

The data suggest:

$$\langle k_{T_{\text{kaon}}} \rangle > \langle k_{T_{\text{pion}}} \rangle > \langle k_{T_{\text{proton}}} \rangle$$

We know

$$\langle r \rangle_{\text{kaon}}^{1/2} < \langle r \rangle_{\text{pion}}^{1/2} < \langle r \rangle_{\text{proton}}^{1/2}$$

$$\langle r \rangle_{\text{kaon}}^{1/2} = 0.58 \pm 0.02 \text{ fm for kaon}$$

$$\langle r \rangle_{\text{pion}}^{1/2} = 0.67 \pm 0.02 \text{ fm for pion}$$

$$\langle r \rangle_{\text{proton}}^{1/2} = 0.81 \text{ fm for proton}$$

New Drell-Yan data with meson and antiproton beams are essential
Flavor and $x$-dependent $k_T$-distributions?
(Bourrely, Buccella, Soffer, arXiv:1008.5322)

- $\langle k_T \rangle$ increases when $x$ increases
- $\langle k_T \rangle$ for sea quarks is smaller than for valence quarks
Test of possible $x$-dependent $k_T$-distributions

E866 p+d D-Y data (800 GeV beam)

$5.2 < M < 6.2$ GeV

$7.2 < M < 8.7$ GeV

Data from thesis of J. Webb
Possible $x$-dependent $k_T$-distributions

E866 p+d D-Y data (800 GeV beam)

$\langle p_T \rangle$ scale with $x_2$?

Analysis is ongoing (A. Ghalsasi, E. McClellan, JCP)
Summary

• The Drell-Yan process is a powerful experimental tool complimentary to the DIS for exploring quark structures in nucleons and nuclei.

• Unique information on flavor structures of sea-quark has been obtained with Drell-Yan experiments. First results on TMD have also been extracted.

• Future Drell-Yan experiments can address many important unresolved issues in the spin and flavor structures of nucleons and nuclei.