Reactions Theory II

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Non-elastic Cross Sections
Non-elastic Cross Sections

How?

A Code and a Book
Non-elastic Cross Sections

How?

A Code and a Book

Physics of Nuclear Reactions

Elastic Scattering

Inelastic Scattering

Transfer Reactions

Breakup Reactions

Fusion Reactions

Compound Nucleus Decays (after fusion)
Multi-channel Scattering

Use for inelastic, transfer, breakup channels (etc) in addition to elastic.

Two channel (1=elastic, 2=reaction) make coupled channels:

\[
\begin{align*}
[T_1 + U_1 - E_1] \psi_1(r) + V_{12} \psi_2(r) &= 0 \\
[T_2 + U_2 - E_2] \psi_2(r) + V_{21} \psi_1(r) &= 0.
\end{align*}
\]  

(1)

Forward coupling:

\[ V_{21} \psi_1(r) \] gives effect of channel 1 on channel 2,

Back coupling:

\[ V_{12} \psi_2(r) \] gives effect of channel 2 on channel 1.

These equations can be solved as coupled channels.
Simplified Multi-channel Scattering gives DWBA

If channel 2 is weak, we can neglect the $V_{12}\psi_2(r)$ term: the back effect on channel 1.

\[
\begin{align*}
[T_1 + U_1 - E_1]\psi_1(r) &= 0 \\
[T_2 + U_2 - E_2]\psi_2(r) + V_{21}\psi_1(r) &= 0.
\end{align*}
\] (2)

This equals the Born Approximation:

\[
\begin{align*}
[T_1 + U_1 - E_1]\psi_1(r) &= 0 \\
\psi_2(r) &= -[T_2 + U_2 - E_2]^{-1}V_{21}\psi_1(r)
\end{align*}
\] (3)

So the DWBA scattering amplitude in channel 2 is

\[
f_{21}(\theta) = -\frac{m_2}{2\pi\hbar^2} \langle k_2 | V_{21} | \psi_1 \rangle
\] (4)

DWBA is often useful for non-elastic channels.
Coupled Channels Calculations

Fresco
Coupled Reaction Channels Calculations
www.fresco.org.uk

About Fresco

Fresco is a program developed by Ian Thompson over the period 1983 - 2006, to perform coupled-reaction channels calculations in nuclear physics. It uses Fortran 90 or Fortran 95 on Unix, Linux, Vax and Windows machines.

Sfresco is an additional version of Fresco, to provide Chi-squared searches of potential and coupling parameters, and to fit additional R-matrix terms in hybrid models.

Free!
Nuclear Reactions for Astrophysics
Principles, Calculation and Applications of Low-Energy Reactions

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Physics of Nuclear Reactions

- Halo Scattering: Elastic
- Halo Total Reaction Cross Section
- Transfer Reactions
- Breakup Reactions
- Halo Fusion Reactions
Halo Scattering: Elastic

Depends on

- Folded potential from densities
- Halo breakup effects, i.e.
- Polarisation potential from breakup channel
Four- and Six-body Scattering

\[ n=4 \] 
\[ ^6\text{He}+p \] 

\[ n=6 \] 
\[ ^8\text{He}+p \]

\[ \text{Al-Khalili and Tostevin, PRC 57 (1998) 1846} \]
\[ \text{Tostevin et al., PRC 56 (1997) R2929} \]
Halo Total Reaction Cross Section

Depends on

- Densities and NN scattering, as usual
- But: effects of Halo Breakup (virtual and real) are big!
- Use few-body Glauber, not Optical Limit Glauber
- Do we scatter from average positions? Or average scattering from positions?

New radii are larger.
Inelastic Scattering

Need a structure model for the couplings: rotational or vibrational model. Consider here the rotational model with excitation energies

\[ \epsilon_I = \frac{\hbar^2}{2\mathcal{M}} [I(I+1) - K(K+1)] \] (5)

The coupling interaction of multipole \( \lambda \) depends on the derivative of the optical potential \( U(r) \) as

\[ V_{fi}^\lambda(r) = -\frac{\beta_{\lambda} R_0}{\sqrt{4\pi}} U'(r) \hat{I}_i \langle I_i K, \lambda 0 | I_f K \rangle \] (6)
Example of Inelastic Scattering

$\alpha$-particle scattering on $^{20}$Ne.

Choose here a rotational model: $\beta_2 = 0.205$.

Theory options:

- First order
- Second order
- All orders

This is an all-order calculation.
Transfer Interaction

From $\phi_p(r)$ projectile bound state ($p=n+c$), to $\phi_t(r')$ target bound state ($t=n+c'$):

\[
[H_p - \varepsilon_p]\phi_p(r) = 0 \quad H_p = T_r + V_p(r)
\]

\[
[H_t - \varepsilon_t]\phi_t(r') = 0 \quad H_t = T_{r'} + V_t(r') \quad (7)
\]

The transfer interaction has two forms:

\[
V_{\text{prior}}(R, r) = V_t(r') + U_{c'c}(R_c) - U_i(R)
\]

\[
V_{\text{post}}(R', r') = V_p(r) + U_{c'c}(R_c) - U_f(R') \quad (8)
\]

These should give the same cross sections.
Transfer Reactions to Probe Single-Particle Structure

- Weak, so use DWBA
- One-nucleon transfers, (p,d) shape shows L-value of orbital magnitude gives spectroscopic factor
- Two-neutron transfers, (p,t) Magnitude depends on s-wave pairing in halo Only relative magnitudes reliably modeled.
- But: full analysis requires multi-step calculations
Stripping (Breakup) Reactions: Measuring Momentum

Probing the momentum content of bound states by breakup reactions

consider momentum components $p_\parallel$ of the heavy residue parallel to the beam direction. In the projectile rest frame ..............
Stripping Reactions: Nuclear Structure

Glauber (eikonal) theory of breakup:

Stripping Reactions: Removing a Neutron

Reaction $^9\text{Be}(^{17}\text{C}, ^{16}\text{C}\gamma)X$

Measured $\gamma$ from core decays helps to fix the final state

\[ \ell = 0, 2 \]
admixture

\[ \ell = 0, 2 \]
admixture

pure $\ell = 2$
but large!!

[Image showing graphs and energy levels, with $E_\gamma$ values and $P_{\|}(^{16}\text{C})$ distributions.]
Halo Fusion: an Unsolved Problem

In low-energy Halo Fusion (near the Coulomb barrier):
Halo neutrons should affect fusion:

- **Increase fusion**, from neutron attractions & neutron flow
- **Decrease complete fusion**, from breakup
- **Increase fusion**, from molecular states & resonances

So: need experiments + good theories!
Some experiments already performed with $^6\text{He}$ and $^9\text{Be}$, but theoretical interpretations are still unclear.
Compound Nucleus Decays (after fusion)

Flux does not ‘disappear’ the nuclei fuse together, but reappears as mixture of narrow resonances of the compound system.

- Narrow resonances $\Rightarrow$ long-lived $\Rightarrow$ many oscillations to decay
- **Bohr hypothesis**: decay independent of production method
- So decay by all possible means $\alpha$: emission of $\gamma$, n, p, $\alpha$, maybe fission.
- **Average** the cross sections over (say) 0.1 MeV, $\langle \sigma_{\alpha'\alpha} \rangle$ to cover many resonances
- **Hauser-Feshbach theory** gives the statistical branching ratios between the channels $\alpha$.

So we can calculate residual nuclear ground states after all emissions are finished.
Transmission coefficients for CN production

‘Transmission coefficient’ \( T_\alpha(E) = 1 - |S_\alpha(E)|^2 \) is the probability of CN production for scattering at energy \( E \).

Transmission coefficients for neutrons incident on \(^{90}\text{Zr}\) in various partial waves \( L \), using a global optical potential:
Decay paths and Branching Probabilities

- So consider all possible exit channels $\alpha''$ and normalize to total
- Hauser-Feshbach cross section $\alpha \rightarrow \alpha'$ (simple form):
  \[
  \langle \sigma_{\alpha'\alpha}(L; E) \rangle = \frac{\pi}{k^2}(2L+1) \frac{T_{\alpha} T_{\alpha'}}{\sum_{\alpha''} T_{\alpha''}}
  \]
- The same $T_{\alpha}$ are used for producing as for decaying.
- If we do not know all the $\alpha$, average over a level density $\rho(E)$
Decay paths starting from neutron + $^A X$:

This is the framework for Hauser-Feshbach calculations. They ignore interference effects between successive steps, so are more semi-classical than quantum.
Result of a Hauser-Feshbach Calculation

Using the code **TALYS**:

![Graph](image)

- $n + ^{181}\text{Ta}$ Hauser–Feshbach calculation
- (n,3n)
- (n,2n) to $^{180}\text{Ta}$ (gs)
- (n,2n) to $^{180}\text{Ta}^m$
- (n,2n) sum
- (n,n')
- (n,γ) capture to $^{181}\text{Ta}$
Evaluated Data for Nuclear Reactions

National libraries available, such as ENDF: Evaluated Nuclear Data File, at NNDC.

Evaluated Nuclear Data File (ENDF)

Core nuclear reaction database containing evaluated (recommended) cross sections, spectra, angular distributions, fission product yields, thermal neutron scattering, photo-atomic and other data, with emphasis on neutron-induced reactions. All data are stored in the internationally adopted format (ENDF-B) maintained by CSEWG. Due to performance issues with the ENDF/B-VII.0 decay data sublibrary we recommend ENDF/B-VII.1 decay data.

Target 56Fe; Fe-56; 26-Fe-56; Fe*
Reaction n, n, n, g; n, f; n, inf; n, nu*
Quantity sig, de, da, da/dE, res, cov*