Nuclear Astrophysics: Lecture 1

Carla Fröhlich
North Carolina State University
cfrohli@ncsu.edu
Lecture plan

• Lecture 1
  • Solar system abundances
  • A tiny little bit of BBN
  • Hydrostatic nuclear burning
  • Thermonuclear reaction rates

• Lecture 2
  • Explosive nuclear burning
  • Heavy element synthesis
  • Spectroscopy and metal-poor stars
Origin of elements

Astrophysical sites:
- Stellar evolution of low-mass and massive stars
- AGB stars (main s-process) core He-burning of massive stars (weak s-process)
- Supernovae
- Core-collapse supernovae
- Neutrino-driven winds in SNe?NS mergers
- X-ray bursts
Origin of elements

Solar system abundances

Hydrogen
Helium
Iron Group
α-elements ($^{12}\text{C}$, $^{16}\text{O}$, ...)

log $N$ (solar system)

Atomic Mass

$^{4}\text{He}$ elements

Origin of elements
Big Bang Nucleosynthesis (BBN)

- After $t \sim 180$ s nuclear reactions begin to occur in earnest.
- There are essentially just 11 reactions.
- The most important 7 are:
  
  $n + p \rightarrow D + \gamma$
  $D + p \rightarrow ^3\text{He} + \gamma$
  $D + D \rightarrow ^3\text{He} + n$
  $D + D \rightarrow ^3\text{H} + p$
  $^3\text{H} + p \leftrightarrow ^3\text{He} + n$
  $D + ^3\text{H} \rightarrow ^4\text{He} + n$
  $D + ^3\text{He} \rightarrow ^4\text{He} + p$
  
- The other 4 reactions that occur are:
  
  $^4\text{He} + ^3\text{H} \rightarrow ^7\text{Li} + \gamma$
  $^4\text{He} + ^3\text{He} \rightarrow ^7\text{Li} + \gamma$
  $^7\text{Be} + n \rightarrow ^7\text{Li} + p$
  $^7\text{Li} + n \rightarrow 2^4\text{He}$
Big Bang Nucleosynthesis (BBN)

- After $t \sim 15$ minutes, BBN is over
- What is produced are lots of leftover free protons, $^4$He and trace amounts of D, $^3$H + $^3$He, and $^7$Li + $^7$Be.
  - essentially every neutron ended up in $^4$He.
- The first stars were born with this composition!
Origin of elements

Hydrogen

Helium

Big Bang

Solar system abundances

Iron Group

\( \alpha \)-elements (\(^{12}\text{C}, {^{16}\text{O}}, \ldots \) )
Stars (structure and evolution)

Not to scale!
H-burning

- Typical temperature: $10^7$ K
- Net reaction: $4 \text{ p} \rightarrow ^4\text{He}$
  - **Fuel**: hydrogen
  - **Main product**: helium
  - Bottle neck: $\text{p} + \text{p} \rightarrow \text{d} + \text{e}^+ + \nu_e$
    (Q-value: 0.42 MeV)
  - Lower mass stars: pp-chains
  - Higher mass stars: CNO cycle

- Duration:
  12 billion (our Sun) to 10 million ($25M_{\text{sun}}$ star)
H-burning

pp-chain

CNO-cycle
He-burning

- **Typical conditions:**
  - Temperature: (1-2) $10^8$ K
  - Density: a few $10^2 - 10^4$ g/cm$^3$

- **Net reaction:** $^4\text{He} (2\alpha, \gamma) ^{12}\text{C}$
  - **Fuel:** helium
  - **Main products:** carbon, oxygen
  - $^4\text{He} + ^4\text{He} \leftrightarrow ^8\text{Be} + \gamma$
  - $^8\text{Be} + ^4\text{He} \leftrightarrow ^{12}\text{C} + \gamma$
  - And $^{12}\text{C} + ^4\text{He} \rightarrow ^{16}\text{O} + \gamma$
  - **Difficulty:** lifetime of $^8\text{Be} \sim 10^{-16}$ s
    $\rightarrow$ Hoyle state (resonance in $^{12}\text{C}$ at E=7.68 MeV)
  - **Other products:** $^{21,22}\text{Ne}$, $^{25,26}\text{Mg}$, $^{36}\text{S}$, $^{37}\text{Cl}$, $^{40}\text{K}$, $^{40}\text{Ar}$
  - $^{14}\text{N} (\alpha,\gamma) ^{18}\text{F} (e^+,\nu)$ $^{18}\text{O} (\alpha,\gamma) ^{22}\text{Ne} (\alpha,n) ^{25}\text{Mg}$
He-burning

\[ ^4\text{He} \rightarrow ^8\text{Be} \rightarrow ^{12}\text{C} \]

\( \tau \sim 10^{-16} \) s

Ekström+2010
Low-mass stars

- End their life after He-burning
- Eg the Sun

- If WD is in a binary system → type Ia supernova
Stellar lifetimes

Mass

Time
C-burning

- **Typical conditions:**
  - Temperature: (6-8) $10^8$ K
  - Density: $10^5$ g/cm$^3$

- **Net reaction:** $^{12}\text{C} + ^{12}\text{C}$
  - **Fuel:** carbon
  - **Main products:** neon, magnesium, oxygen
  - $^{12}\text{C} + ^{12}\text{C} \rightarrow \alpha + ^{20}\text{Ne}$ (Q=4.62 MeV)
  - $^{12}\text{C} + ^{12}\text{C} \rightarrow p + ^{23}\text{Na}$ (Q=2.24 MeV)
  - **Other reactions:** $^{23}\text{Na} + p \rightarrow \alpha + ^{20}\text{Ne}$
    - $^{20}\text{Ne} + \alpha \rightarrow ^{24}\text{Mg}$
  - $^{12}\text{C} + \alpha \rightarrow ^{16}\text{O} + \gamma$ (Q=4.73 MeV)
Neutrino-losses

- At temperatures above $\sim 10^9$ K: pair-production
  \[ \gamma \leftrightarrow e^+ + e^- \leftrightarrow \nu_e + \bar{\nu}_e \]

- Luminosity of photons and neutrinos
Ne-burning

- Typical conditions:
  - Temperature: (1-2) $10^9$ K
  - Density: $10^6$ g/cm³

- Reactions:
  - **Fuel**: neon
  - **Main products**: oxygen, silicon
  - $^{20}$Ne ($\gamma,\alpha$) $^{16}$O
  - Other reactions:
    - $^{20}$Ne ($\alpha,\gamma$) $^{24}$Mg ($\alpha,\gamma$) $^{28}$Si ($\alpha,\gamma$) $^{32}$S
    - $^{21}$Ne ($\alpha,n$) $^{24}$Mg ($n,\gamma$) $^{25}$Mg ($\alpha,n$) $^{28}$Si
    - $^{23}$Na ($\alpha,p$) $^{25}$Mg ($\alpha,n$) $^{28}$Si
      - $^{25}$Mg($p,\gamma$) $^{25}$Al
    - $^{23}$Na ($p,\alpha$) $^{20}$Ne
O-burning

- Typical conditions:
  - Temperature: (1.5-2.2) $10^9$ K
  - Density: $10^7$ g/cm$^3$

- Reaction:
  - **Fuel**: oxygen
  - **Main products**: silicon
  - $^{16}$O + $^{16}$O $\rightarrow$ p + $^{31}$P (Q=7.676 MeV)
  - $^{16}$O + $^{16}$O $\rightarrow$ α + $^{28}$Si (Q=9.593 MeV)
  - $^{16}$O + $^{16}$O $\rightarrow$ n + $^{31}$S (Q=1.459 MeV)
  - Other reactions: $^{31}$P (p,α) $^{28}$Si
    $^{33}$S (e$^-$,ν$e$) $^{33}$P
    $^{35}$Cl (e$^-$,ν$e$) $^{35}$P
C-, Ne-, O-burning reactions (details)

(a) basic energy generation
\[ 12\text{C}(^{12}\text{C}, \alpha)^{20}\text{Ne} \quad 12\text{C}(^{12}\text{C}, p)^{23}\text{Na} \]
\[ 23\text{Na}(p, \alpha)^{20}\text{Ne} \quad 23\text{Na}(p, \gamma)^{24}\text{Mg} \quad 12\text{C}(\alpha, \gamma)^{16}\text{O} \]

(b) fluxes > $10^{-2} \times (a)$
\[ 20\text{Ne}(\alpha, \gamma)^{24}\text{Mg} \quad 23\text{Na}(\alpha, p)^{26}\text{Mg}(p, \gamma)^{27}\text{Al} \]
\[ 20\text{Ne}(n, \gamma)^{21}\text{Ne}(p, \gamma)^{22}\text{Na}(e^+ \nu)^{22}\text{Ne}(\alpha, n)^{25}\text{Mg}(n, \gamma)^{26}\text{Mg} \]
\[ 21\text{Ne}(\alpha, n)^{24}\text{Mg} \quad 22\text{Ne}(p, \gamma)^{23}\text{Na} \quad 25\text{Mg}(p, \gamma)^{26}\text{Al}(e^+ \nu)^{20}\text{Mg} \]

(c) low temperature, high density burning
\[ 12\text{C}(p, \gamma)^{13}\text{N}(e^+ \nu)^{12}\text{C}(\alpha, n)^{16}\text{O}(\alpha, \gamma)^{20}\text{Ne} \]
\[ 24\text{Mg}(p, \gamma)^{25}\text{Al}(e^+ \nu)^{25}\text{Mg} \]
\[ 21\text{Ne}(n, \gamma)^{22}\text{Ne}(p, \gamma)^{23}\text{Ne}(e^- \nu)^{23}\text{Na}(n, \gamma)^{24}\text{Na}(e^- \nu)^{24}\text{Mg} \]

(b) high temperature burning
\[ 32\text{S}(\alpha, \gamma)^{36}\text{Ar}(\alpha, p)^{39}\text{K} \]
\[ 36\text{Ar}(n, \gamma)^{37}\text{Ar}(e^+ \nu)^{37}\text{Cl} \]
\[ 35\text{Cl}(\gamma, p)^{34}\text{S}(\alpha, \gamma)^{38}\text{Ar}(p, \gamma)^{39}\text{K}(p, \gamma)^{40}\text{Ca} \]
\[ 35\text{Cl}(e^-, \nu)^{35}\text{S}(\gamma, p)^{34}\text{S} \]
\[ 38\text{Ar}(\alpha, \gamma)^{42}\text{Ca}(\alpha, \gamma)^{46}\text{Ti} \]
\[ 42\text{Ca}(\alpha, p)^{45}\text{Sc}(p, \gamma)^{46}\text{Ti} \]

(c) low temperature, high density burning
\[ 31\text{P}(e^- \nu)^{31}\text{S} \quad 31\text{P}(n, \gamma)^{32}\text{P} \]
\[ 32\text{S}(e^-, \nu)^{32}\text{P}(p, n)^{32}\text{S} \]
\[ 33\text{P}(p, \alpha)^{30}\text{Si} \]

Thielemann+2010

See Lecture 2
Si-burning

- **Typical temperature:** \((3-4) \times 10^9\) K

- **Net reaction:** \(^{28}\text{Si} + \, ^{28}\text{Si}\)
  - Fuel: silicon
  - Main products: Fe-group elements (\(A = 50-60\) nuclei)
  - Other reactions: \(^{28}\text{Si} + \gamma \rightarrow p + ^{27}\text{Al}\)
    \(^{28}\text{Si} + \gamma \rightarrow \alpha + ^{24}\text{Mg}\)
    \(^{28}\text{Si} + \gamma \rightarrow n + ^{27}\text{Si}\)

- **Balance between forward and reverse reactions for increasing number of processes:** \(a + b \leftrightarrow c + d\)
  \(\rightarrow\) Nuclear statistical equilibrium (NSE)
Nuclear Statistical Equilibrium (NSE)

- Chemical equilibrium (for species i):
  \[ Z_i \mu_p + N_i \mu_n = \mu_i \]

- Abundances given by:
  \[ Y (Z, N) = G_{Z,N} (\rho N_A)^{A-1} (A^{2/3} / 2^A) \left( \frac{2\pi\hbar}{m_n kT} \right)^{3/2(A-1)} Y_n^N Y_p^Z e^{B_{Z,N}/kT} \]
Nuclear Statistical Equilibrium (NSE)
Weak Interactions

- Become relevant in later burning stages
- Lead to neutronization
- Electron captures:

\[
p + e^- \rightarrow v_e + n \quad \text{or} \quad p(e^-, v_e)n
\]
\[
(A, Z) + e^- \rightarrow v_e + (A, Z - 1) \quad \text{or} \quad A(Z(e^-, v_e)A(Z-1)
\]
\[
E_F(\rho Y_e = 10^7 \text{g cm}^{-3}) = 0.75 \text{ MeV}
\]
\[
E_F(\rho Y_e = 10^9 \text{g cm}^{-3}) = 4.70 \text{ MeV}
\]

- Electron fraction:
Central evolution
Pre-supernova stage

Composition
Pre-supernova stage
Origin of elements

- Big Bang
- Hydrogen
- Helium

Solar system abundances

Stellar burning

Iron Group

$\alpha$-elements ($^{12}\text{C, }^{16}\text{O, ...}$)

Graph showing the distribution of elements with atomic mass and their abundances in the solar system.
Nuclear binding energy

H → He → C → O → .... → Fe

Nuclear fusion in stellar cores

Need mechanisms other than charged-particle fusion:
E.g. neutrons, photons, neutrinos
Origin of elements

How are nuclei made? Where? Through what processes?
Nuclear physics

• Need to know the relevant nuclear physics:
  • Properties of nuclei (mass, half-life, spin, levels, etc)
  • Properties of reactions between nuclei (and leptons, photons)
Reaction rates

Consider:

- \( n_i \): number density of particles of type \( i \) \( \text{cm}^{-3} \)
- \( n_j \): number density of particles of type \( j \) \( \text{cm}^{-3} \)
- \( \sigma \): cross section (effective area for reaction) \( \text{cm}^2 \)

Reactions per time per volume

\[
 r = n_i v n_j \sigma(v)
\]

\( \text{cm}^{-3} \text{cm} \text{s}^{-1} \)
Reaction rates

• Previously: particles i move at constant \( v \)
• For constant relative velocity between particles i and j

\[ \rightarrow \text{reacts / vol / time: } r_{i:j} = \int \sigma \cdot |\vec{v}_i - \vec{v}_j|dn_i dn_j \]

• General: projectiles and targets follow velocity distribution

\[ r_{i:j} = n_i n_j \int \sigma(|\vec{v}_i - \vec{v}_j|)|\vec{v}_i - \vec{v}_j|\phi(\vec{v}_i)\phi(\vec{v}_j)d^3v_i d^3v_j \]

Integral depends on type of particles and distribution
Maxwell-Boltzmann distribution

- Nuclei in astrophysical plasma are not mono-energetic
- They obey MB distribution

\[ \Phi \sim E \]

\[ \Phi \sim E \exp(-kT/E) \]

\[ \Phi \sim \exp(-kT/E) \]

\[ E_{mp} = kT \]
Reaction rates

- Use center-of-mass coordinates, carry out integration, and remember that \( \int \phi(\vec{V})d^3V = 1 \)

reaction rate becomes

\[ r_{i;j} = n_i n_j \langle \sigma v \rangle_{i;j} \]

with the thermonuclear cross section \( \langle \sigma v \rangle \)

\[ \langle \sigma v \rangle (T) = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E \sigma(E) \exp(-E/kT)dE \]

- Only depends on temperature
- If we know \( \sigma(E) \), we can get \( \langle \sigma v \rangle \)
Astrophysical S-factor

- Use known energy dependence of $\sigma(E)$

- For charged particles: $\sigma(E)$ is proportional to:
  - Coulomb barrier penetration $\sim \exp(-E^{1/2})$
  - Nuclear size $\sim 1/E$

- All other energy dependencies are lumped together into astrophysical S-factor $S(E)$

- Why?
  - For non-resonant reactions: $S(E)$ is slowly varying
    $\Rightarrow$ better to work with $S(E)$ if extrapolations are needed
Astrophysical S-factor

- Cross section \( \sigma = E^{-1} \times \exp(-E^{1/2}) \times S(E) \)

- Reaction rate becomes

\[
\langle \sigma v \rangle = \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty E \sigma(E) \exp(-E/kT) dE \]

\[
= \left( \frac{8}{\mu \pi} \right)^{1/2} \frac{1}{(kT)^{3/2}} \int_0^\infty S(E) \exp(-bE^{-1/2}) \exp(-E/kT) dE.
\]

- \( S(E) \) is slowly varying with \( E \), so integral is dominated by the two exponentials
Gamow peak

Most effective stellar energy
Nuclear reaction networks

• Turn number of reactions per volume and time into differential equation, for a reaction $i(j,o)m$

$$r_{i;j} = \frac{1}{1 + \delta_{ij}} n_i n_j \langle \sigma v \rangle$$

$$\left( \frac{\partial n_i}{\partial t} \right)_\rho = \left( \frac{\partial n_j}{\partial t} \right)_\rho = -r_{i;j}$$

$$\left( \frac{\partial n_o}{\partial t} \right)_\rho = \left( \frac{\partial n_m}{\partial t} \right)_\rho = +r_{i;j}.$$

• Total rate of change of number density:

$$\dot{n}_i = \left( \frac{\partial n_i}{\partial t} \right)_\rho + n_i \dot{\rho} / \rho$$

• Includes changes due to density change (we are not interested in those)
Abundances, mass fractions

- Matter density $\rho$ (g cm$^{-3}$)
- Number density $n$ depends on matter density

- Can we separate dependence on matter density?
  → Define abundance $Y = n / \rho N_A$
- Units of abundance: mole g$^{-1}$

- Mass fraction $X_i = A_i Y_i$ with normalized sum
Nuclear reaction networks

- Use abundance

\[ Y_i = \frac{n_i}{\rho N_A} \quad \dot{Y}_i = \frac{\dot{n}_i}{\rho N_A} - \frac{n_i}{\rho N_A} \frac{\dot{\rho}}{\rho} \]

- Derivative becomes:

\[ \dot{Y}_i = \frac{1}{\rho N_A} \left( \frac{\partial n_i}{\partial t} \right) \rho = -\frac{r_{i;j}}{\rho N_A} = -\frac{1}{1 + \delta_{ij}} \rho N_A \langle \sigma v \rangle_{i;j} Y_i Y_j \]

- For decays (and reactions with photons and leptons):
  - “decay rate” \( \lambda \)
  - Derivate becomes

\[ \dot{Y}_i = -\lambda_i Y_i \]
Inverse reactions

• Many reactions are the inverse of an other reaction

• Forward and inverse reactions are linked by time reversal invariance

• For reaction $i(j,o)m$ the thermonuclear cross section depends on
  • Q-value (energy difference between products and reactants)
  • Partition functions (Energy weighted density of states)

$$\langle \sigma v \rangle_{i;j,o} = \frac{1 + \delta_{ij} G_m g_o (\mu_{om} / \mu_{ij})^{3/2}}{1 + \delta_{om} G_i g_j} \exp(-Q_{o,j} / kT) \langle \sigma v \rangle_{m;o,j}$$
Nuclear reaction networks

- Set of coupled differential equations

\[
\dot{Y}_i = \sum_j N_j^i \lambda_j Y_j + \sum_{j,k} N_{jk}^i \rho N_A \langle \sigma v \rangle_{jk} Y_j Y_k + \sum_{j,k,l} N_{jkl}^i \rho^2 N_A^2 \langle \sigma v \rangle_{jkl} Y_j Y_k Y_l
\]

Specify number of particles created or destroyed; take into account reactions between the same (indistinguishable) species

Thermonuclear cross section

\(Y\) .. Abundance
\(\lambda\) ... decay rate
Nuclear reaction networks

- Set of coupled differential equations

\[
\dot{Y}_i = \sum_j N^i_j \lambda_j Y_j + \sum_{j,k} N^i_{jk} \rho N_A \langle \sigma v \rangle_{jk} Y_j Y_k + \sum_{j,k,l} N^i_{jkl} \rho^2 N_A^2 \langle \sigma v \rangle_{jkl} Y_j Y_k Y_l
\]

- Decays, photodisintegrations, reactions with leptons (e\(^{-}\), e\(^{+}\), \(\nu\))
- Two-particle reactions
- Three-particle reactions (e.g. triple-\(\alpha\) reaction)

- Right-hand side is sum of all reactions either creating or destroying species \(i\)
- Group into 1-body, 2-body, and 3-body reactions programming reasons
Nuclear reaction networks

- Set of coupled differential equations

\[ \dot{Y}_i = \sum_j N^i_j \lambda_j Y_j + \sum_{j,k} N^i_{jk} \rho N_A \langle \sigma v \rangle_{jk} Y_j Y_k + \sum_{j,k,l} N^i_{jkl} \rho^2 N_A^2 \langle \sigma v \rangle_{jkl} Y_j Y_k Y_l \]

- One equation for each species followed
  - \(~2000\) for supernova nucleosynthesis
  - \(~6000\) for r-process nucleosynthesis

- Matrix for system of DE is sparse
  - Use special solvers and matrix storage format
How to Model Nucleosynthesis

In principle: need 3D hydro in order to follow convection, mixing, explosion

Problems:
• Coupling of hydro to reaction networks (nucleosynthesis, energy generation)
• Explosions

Compromise:
• (1D) hydro with reduced energy generation network
• Mixing length theory, convection criteria
• Parameterized explosions (mass cut and/or explosion energy as free parameters)

Nevertheless: mostly reliable nucleosynthesis expected (except for nuclides dependent on explosion mechanism)
Implementation of Networks

- Fully coupled
  - Energy feedback + abundances

- Operator splitting
  - Reduced network for energy generation
  - Abundances in full network (mixing, convection)

- Post-processing
  - Reduced network for energy generation
  - Other abundances from post-processing
Explosive burning

- Similar to hydrostatic burning, but
  - Shorter timescales
  - Higher temperatures

- H-burning:
  - Hot CNO-cycle (pp-chains are too slow), where $^{13}\text{N}(\beta)$ becomes $^{13}\text{N}(p,\gamma)$

- He-burning:
  - N-rich isotopes $^{15}\text{O}$, $^{18}\text{O}$, $^{19}\text{F}$, $^{21}\text{Ne}$

- C- and Ne-burning:
  - Simultaneously occurring
Explosive burning

- **O-burning:**
  - Quasi-equilibrium (regions of equilibrium, connected by individual reactions)

- **Si-burning:**
  - Complete destruction of silicon
  - Details depend on peak temperature and density:
    - Complete Si-burning
    - Incomplete Si-burning (p-rich)
    - Incomplete Si-burning (α-rich)
Explosive burning