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Some Fundamentals of Modern Particle Accelerators

Day Two

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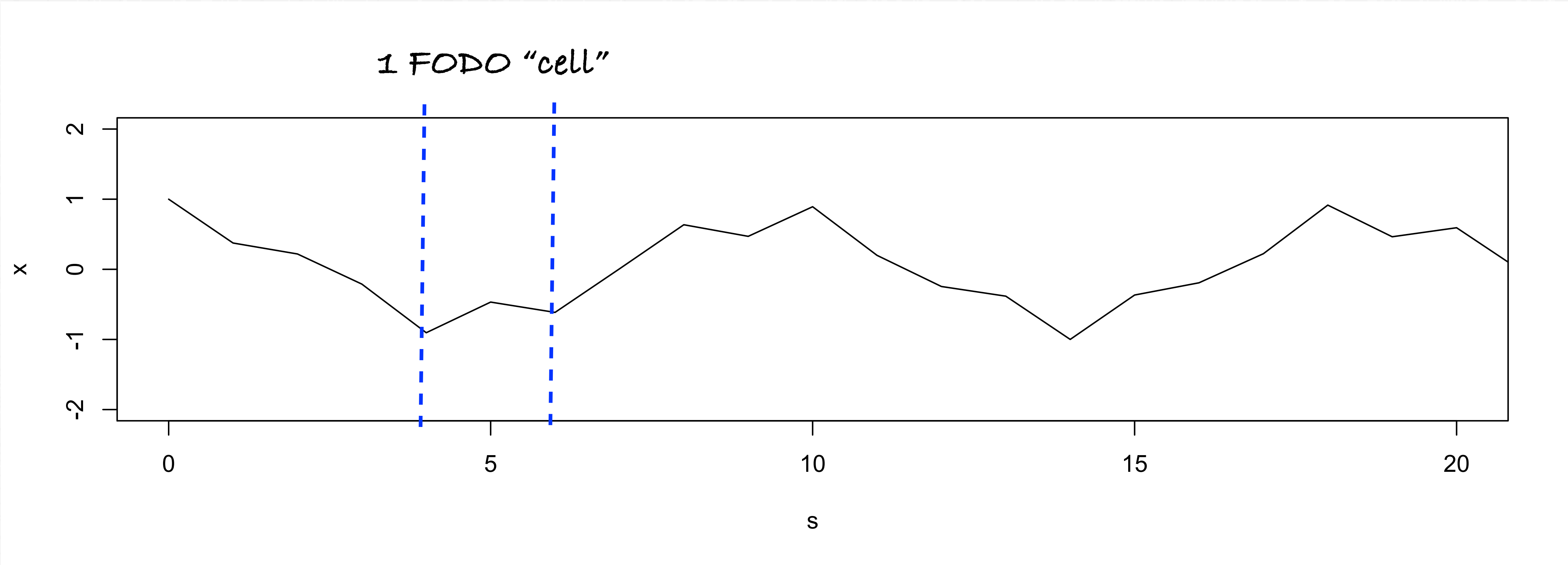
Facility for Rare Isotope Beams

Particle Trajectories



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  - Equation of Motion: (Hill's Equation)

$$x'' + K(s)x = 0$$

  - Nearly simple harmonic; so, assume soln.: $x(s) = A\sqrt{\beta(s)}\sin[\psi(s) + \delta]$

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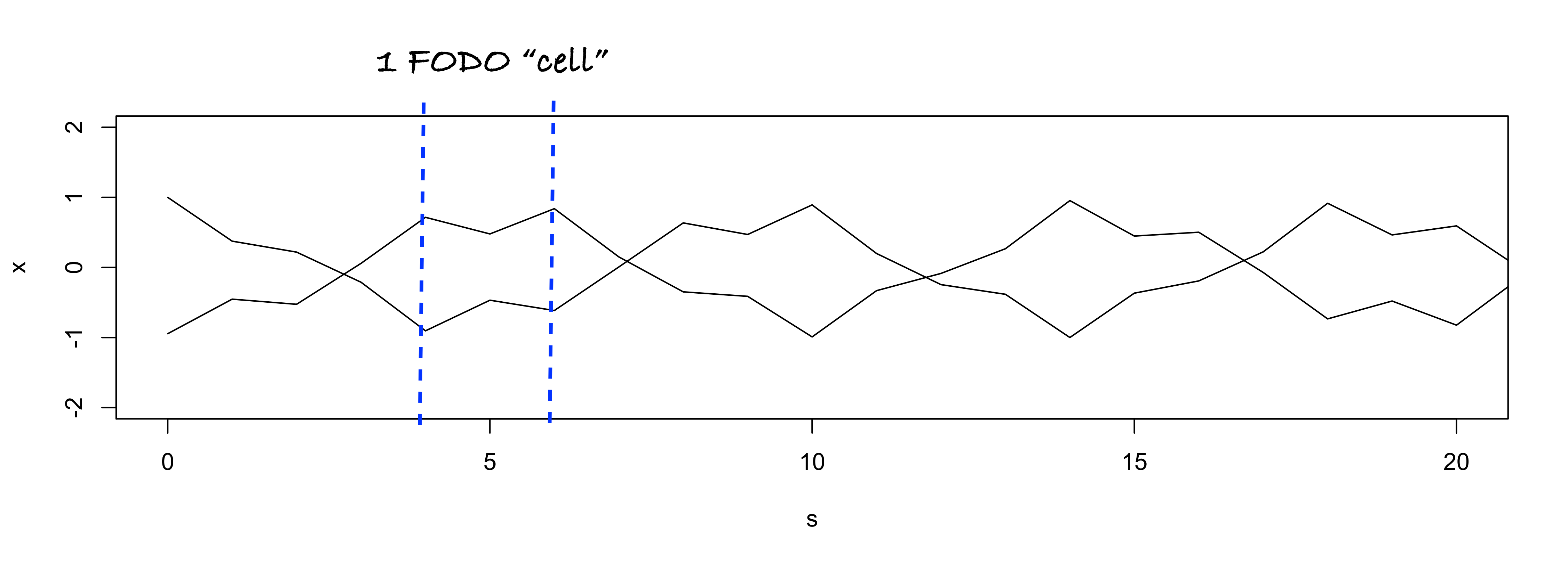

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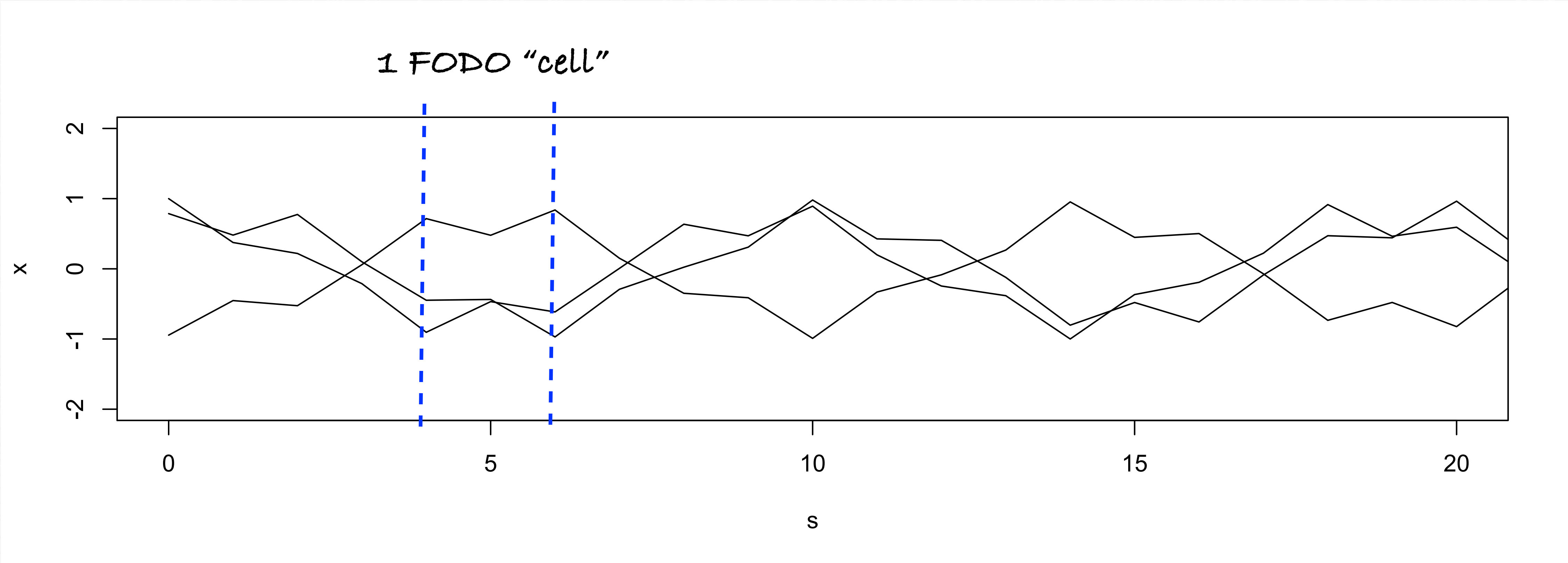

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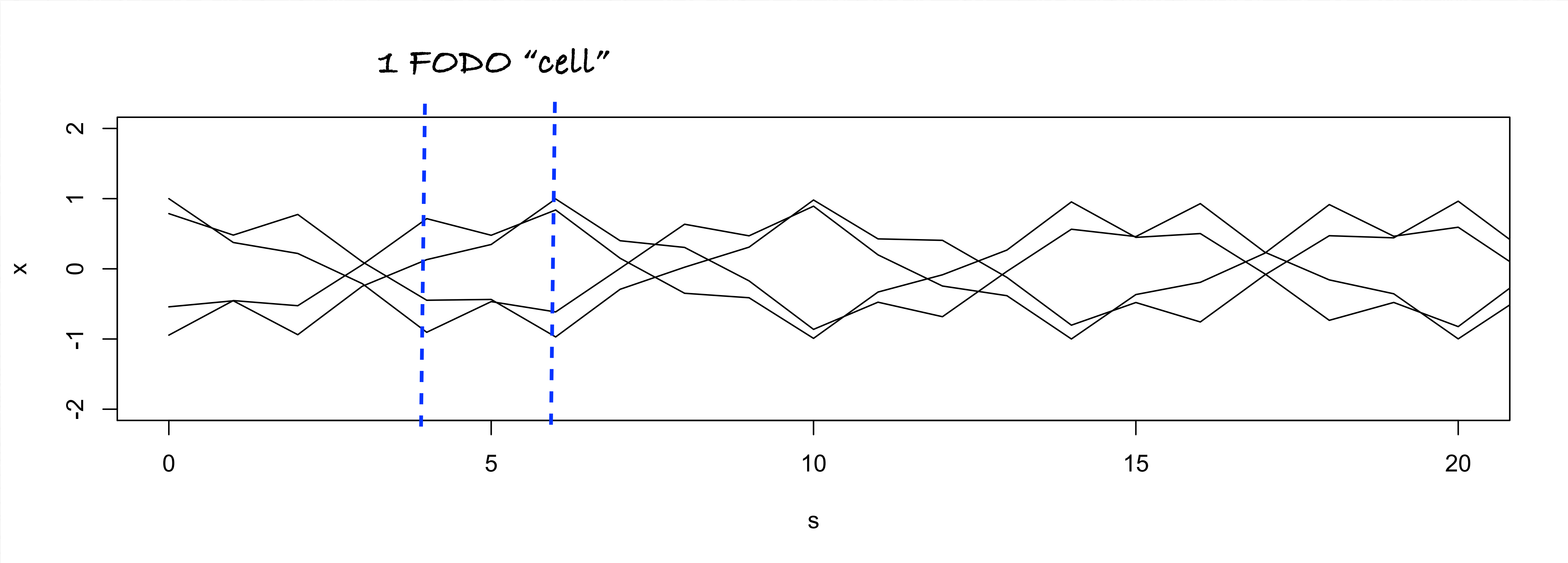

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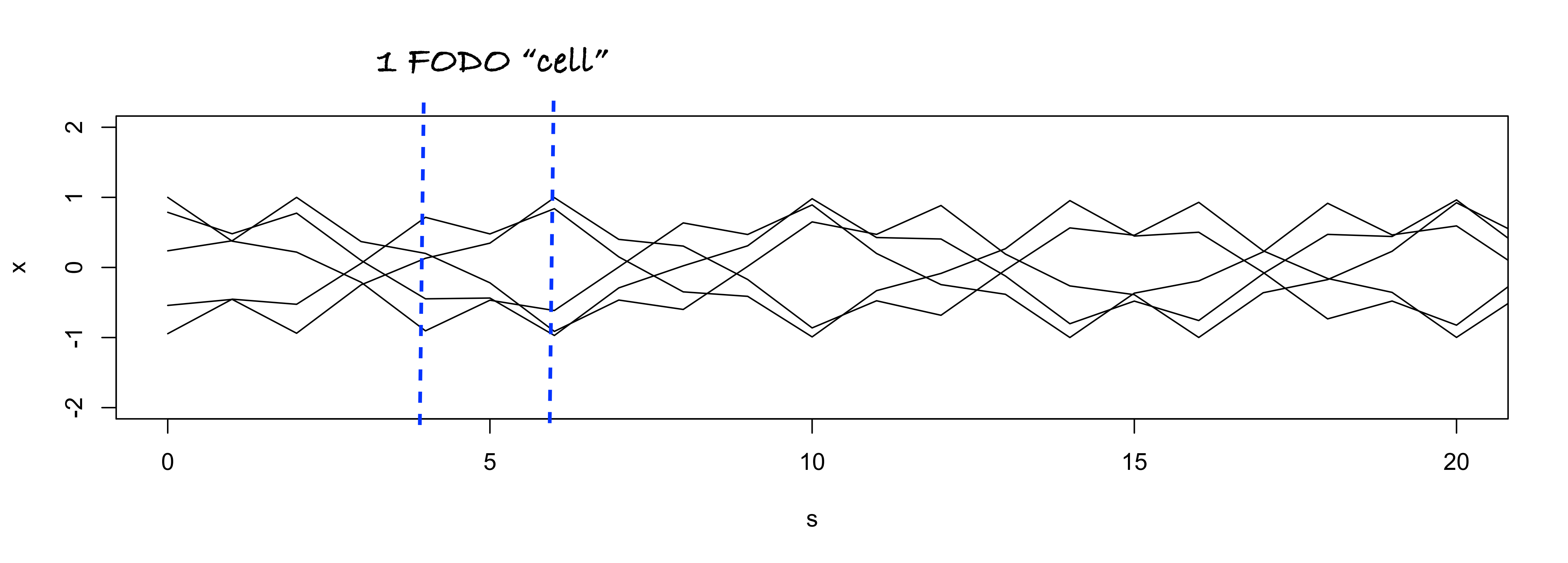

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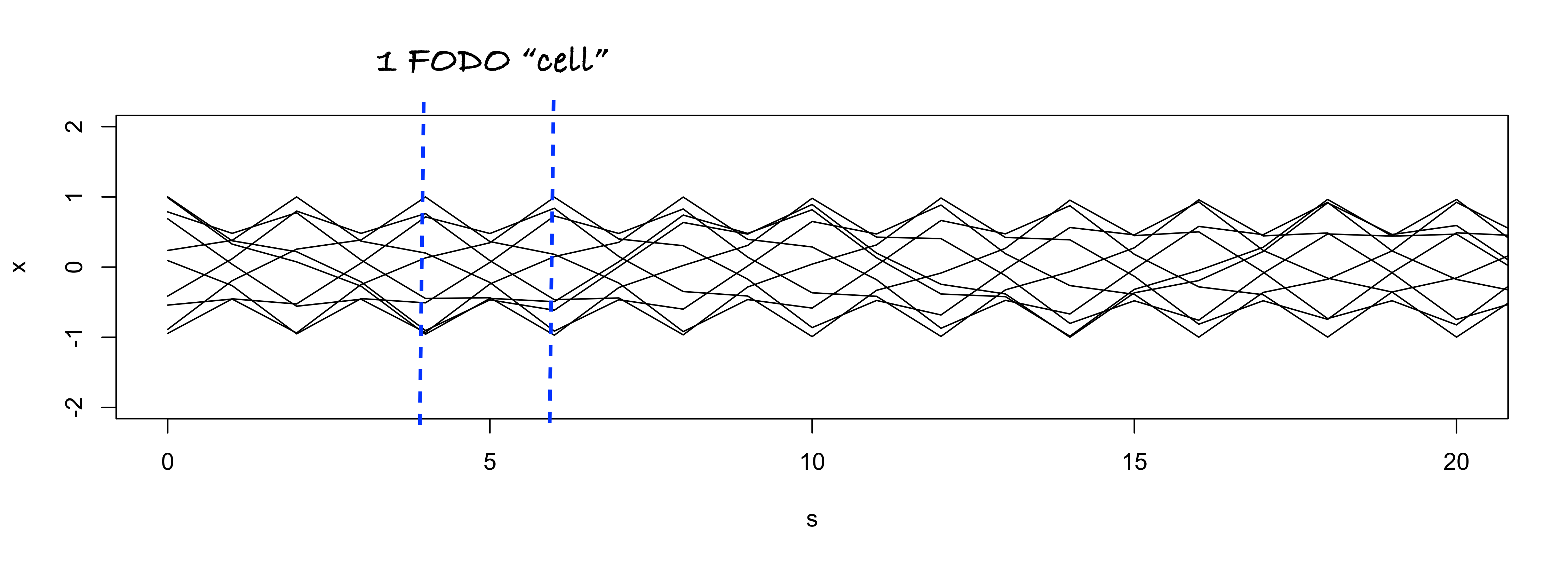

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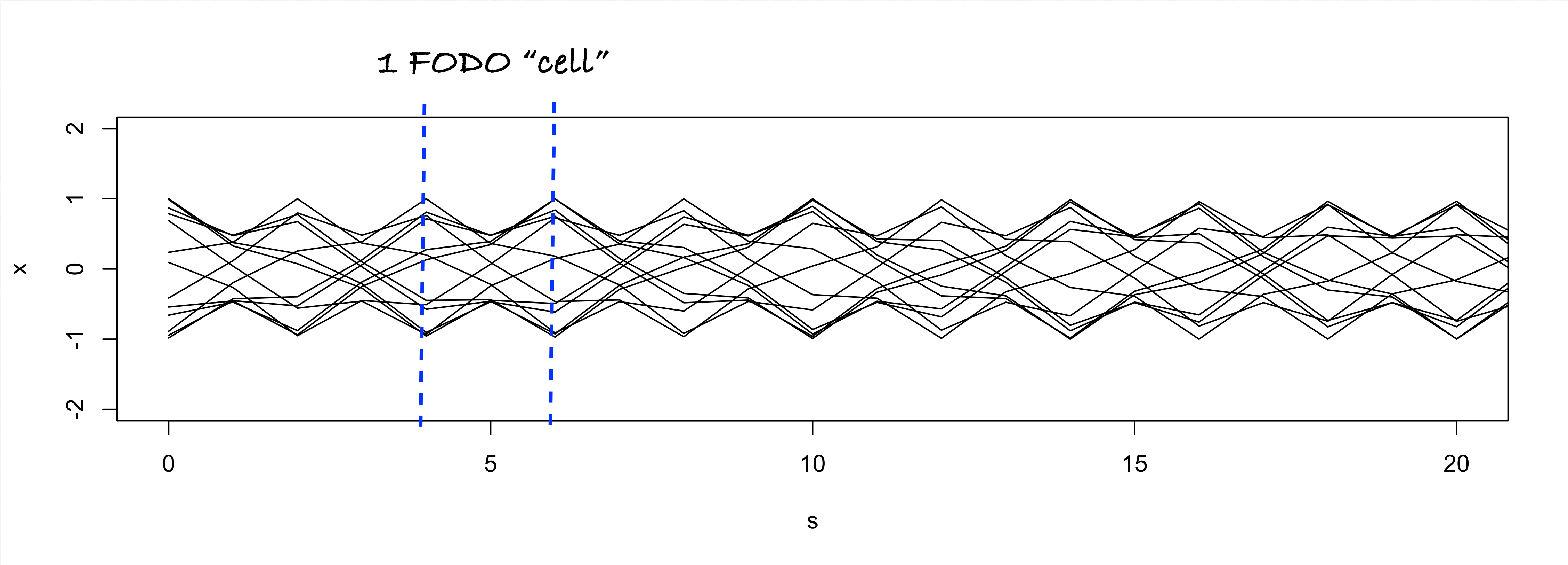

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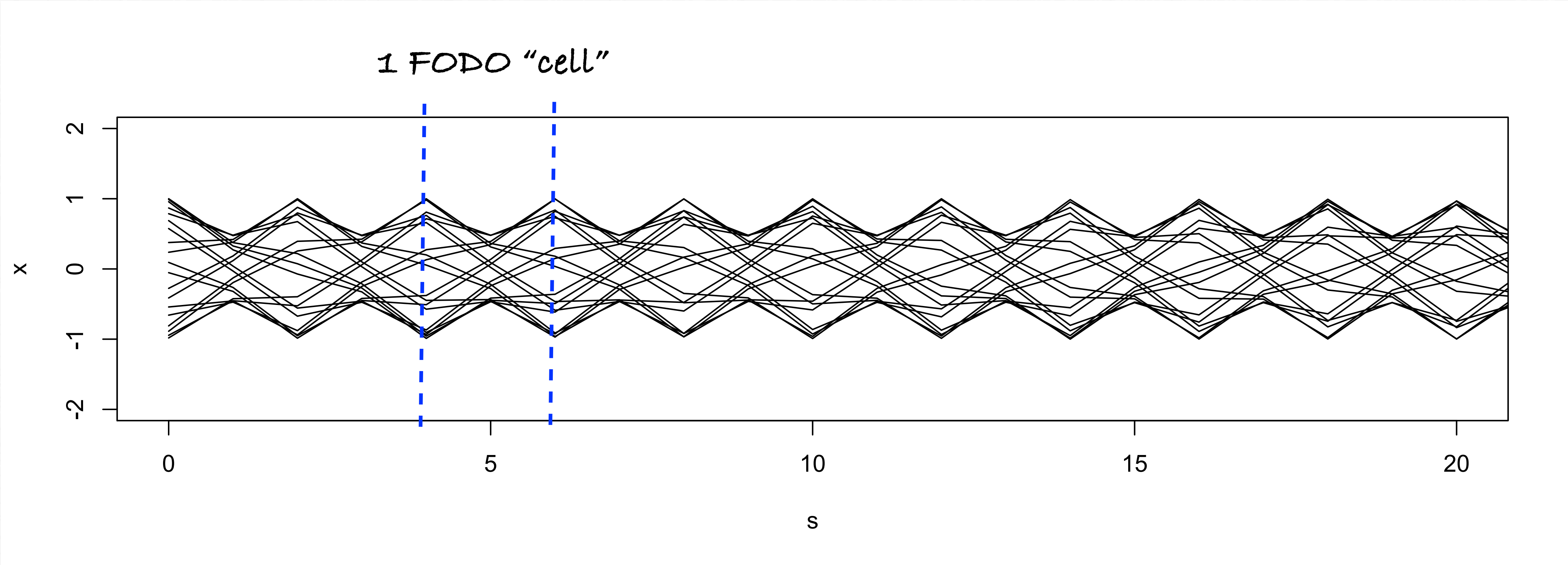

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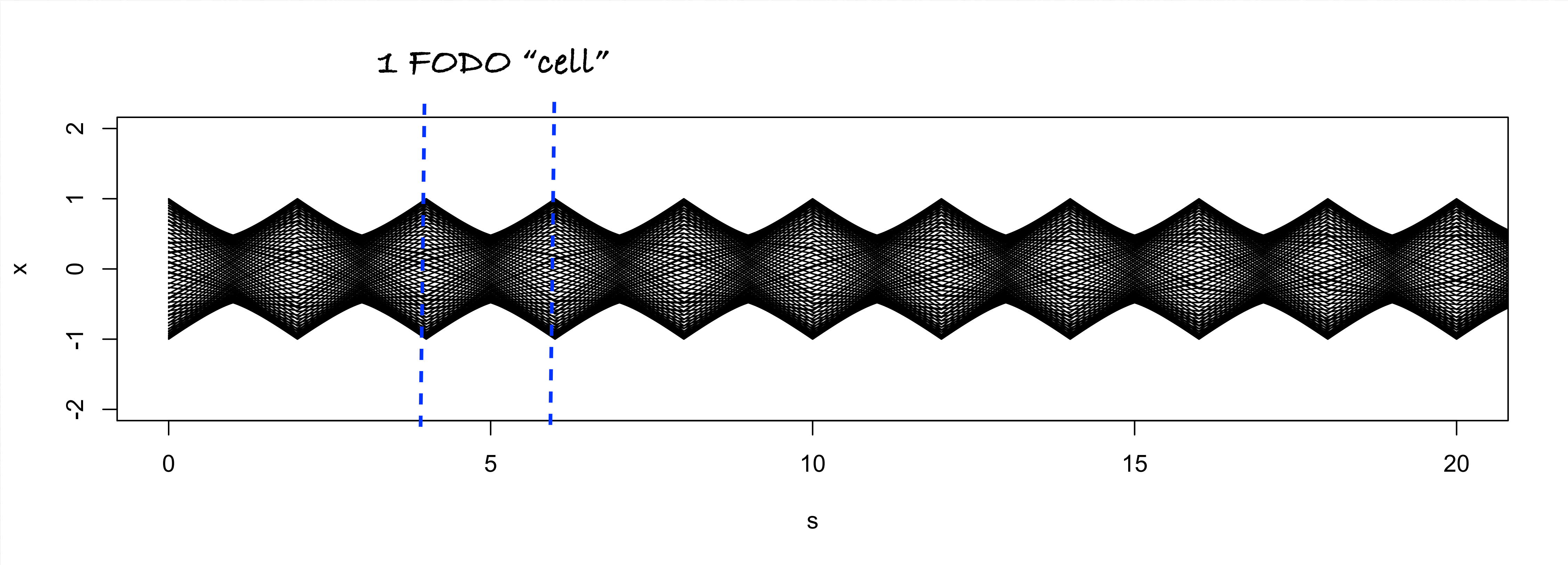

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Analytical Solution

- assumption:
  - take 1st, 2nd derivatives..

$$
\begin{aligned}
x(s) &= A\sqrt{\beta(s)}\sin[\psi(s)+\delta] \\
x' &= \frac{1}{2}A\beta^{-\frac{1}{2}}\beta'\sin[\psi(s)+\delta] + A\sqrt{\beta}\cos[\psi(s)+\delta]\psi' \\
x'' &= \ldots
\end{aligned}
$$

Plug into Hill's Equation, and collect terms...

$$
\begin{aligned}
x'' + K(s)x &= A\sqrt{\beta}\left[\psi'' + \frac{\beta'}{\beta}\psi'\right]\cos[\psi(s)+\delta] \\
&\quad + A\sqrt{\beta}\left[-\frac{1}{4}\frac{(\beta')^2}{\beta^2} + \frac{1}{2}\frac{\beta''}{\beta} - (\psi')^2 + K\right]\sin[\psi(s)+\delta] = 0
\end{aligned}
$$

A and δ are constants of integration, defined by the initial conditions (x_0, x_0') of the particle. For arbitrary A, δ, must have contents of each [ ] = 0 simultaneously.

Analytical Solution (cont'd)

- Thus, we must have ...

$$\psi'' + \frac{\beta'}{\beta}\psi' = 0$$

$$\beta\psi'' + \beta'\psi' = 0$$

$$(\beta\psi')' = 0$$

$$\beta\psi' = const$$

$$\psi' = 1/\beta$$

and

$$-\frac{1}{4}\frac{(\beta')^2}{\beta^2} + \frac{1}{2}\frac{\beta''}{\beta} - (\psi')^2 + K = 0$$

$$2\beta\beta'' - (\beta')^2 - 4\beta^2(\psi')^2 + 4K\beta^2 = 0$$

$$2\beta\beta'' - (\beta')^2 + 4K\beta^2 = 4$$

Note: the phase advance is an observable quantity. So, while could choose different value of *const*, then β would just scale accordingly; thus, valid to choose *const* = 1.

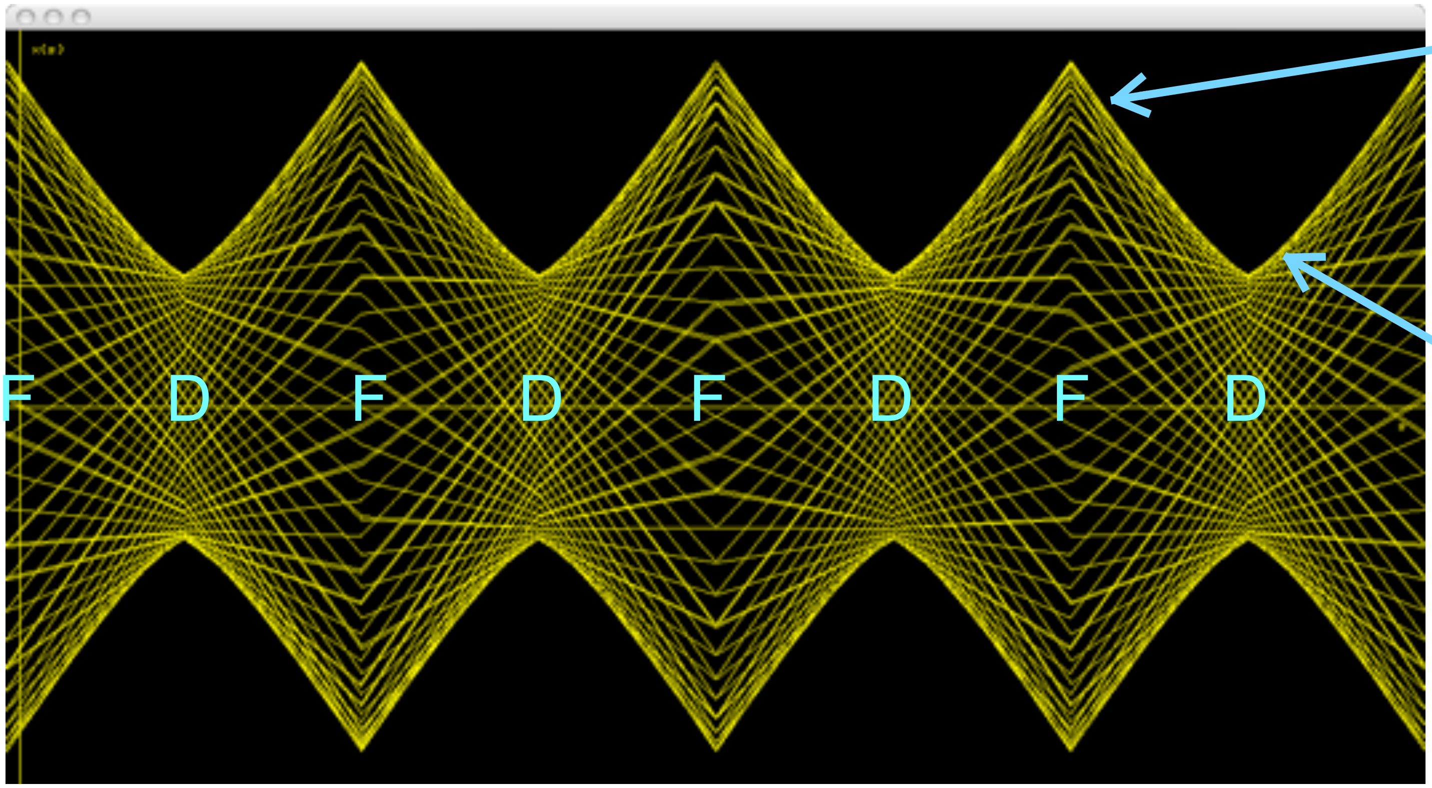
The function $\beta(s)$ is the local wavelength $(\lambda/2\pi)$ of the oscillatory motion.

Differential equation that the amplitude function must obey

Some Comments

- We chose the amplitude function to be a positive definite function in its definition, since we want to describe real solutions.
- The square root of the amplitude function determines the shape of the envelope of a particle's motion. But it also is a local wavelength of the motion.
- This seems strange at first, but ...
  - Imagine a particle oscillating within our focusing lens system; if the lenses are suddenly spaced further apart, the particle's motion will grow larger between lenses, and additionally it will travel further before a complete oscillation takes place. If the lenses are spaced closer together, the oscillation will not be allowed to grow as large, and more oscillations will occur per unit distance travelled.
  - Thus, the spacing and/or strengths (i.e., $K(s)$) determine both the rate of change of the oscillation phase as well as the maximum oscillation amplitude. These attributes must be tied together.

The Amplitude Function, β



Higher β--

smaller phase advance rate
larger beam size

Lower β--

greater phase advance rate
smaller beam size

- Since the amplitude function is a wavelength, it will have numerical values of many meters, say. However, typical particle transverse motion is on the scale of mm. So, this means that the constant *A* must have units of $m^{1/2}$, and it must be numerically small. More on this subject coming up...

Equation of Motion of Amplitude Function

From

$$2\beta\beta'' - (\beta')^2 + 4K\beta^2 = 4$$

we get

$$2\beta'\beta'' + 2\beta\beta''' - 2\beta'\beta'' + 4K'\beta^2 + 8K\beta\beta' = 0$$

$$\beta''' + 4K\beta' + 2K'\beta = 0.$$

Typically, $K'(s) = 0$, and so

$$(\beta'' + 4K\beta)' = 0$$

or

$$\boxed{\beta'' + 4K\beta = const.}$$

is the general equation of motion for the amplitude function, β.

(in regions where K is either zero or constant)

Piecewise Solutions

- $K = 0$:

$$\beta'' = const \longrightarrow \beta(s) = \beta_0 + \beta_0' s + \frac{1}{2}\beta_0'' s^2$$

Parabola!

- since $\beta > 0$, then from original diff. eq.: $2\beta\beta'' - (\beta')^2 = 4$
- the parabola is always concave up $\beta'' > 0$

- $K > 0$, $K < 0$:

$$\beta(s) \sim \sin / \cos \quad \text{or} \quad \sinh / \cosh \; + \; const$$

Courant-Snyder Parameters, & Connection to Matrix Approach

- Suppose, for the moment, that we know the value of the amplitude function and its slope at two points along our particle transport system.
- Have seen how to write the motion of a single particle in one degree of freedom between two points in terms of a matrix. We can now recast the elements of this matrix in terms of the local values of the amplitude function.
- Define two new variables,

$$\alpha \equiv -\frac{1}{2}\beta', \quad \gamma \equiv \frac{1+\alpha^2}{\beta}$$

- Collectively, β, α, γ are called the Courant-Snyder Parameters (sometimes called "Twiss parameters" or "lattice parameters")

$$2\beta\beta'' - (\beta')^2 + 4K\beta^2 = 4 \qquad == \qquad K\beta = \gamma + \alpha'$$

The Transport Matrix

- We can write: $$x(s) = a\sqrt{\beta}\sin\Delta\psi + b\sqrt{\beta}\cos\Delta\psi$$

- Solve for *a* and *b* in terms of initial conditions and write in matrix form
  - we get:

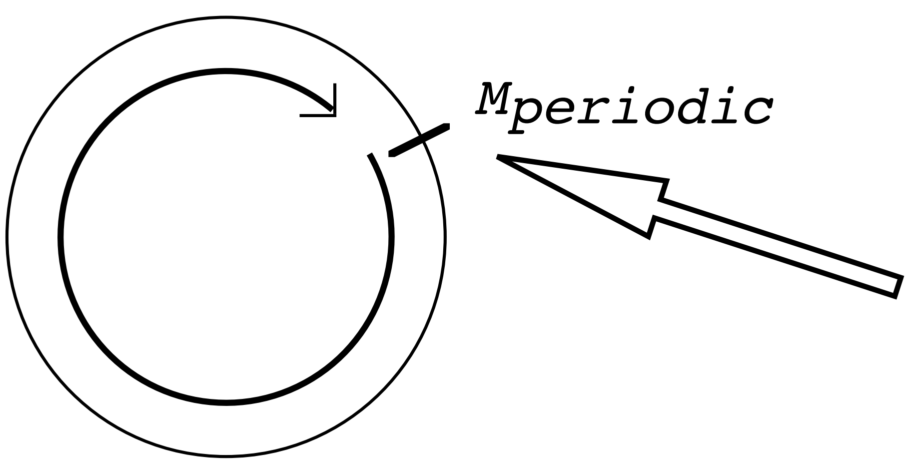
$$\begin{pmatrix} x \\ x' \end{pmatrix} = \begin{pmatrix} \left(\frac{\beta}{\beta_0}\right)^{1/2}(\cos\Delta\psi + \alpha_0\sin\Delta\psi) & \sqrt{\beta_0\beta}\sin\Delta\psi \\ -\frac{1+\alpha_0\alpha}{\sqrt{\beta_0\beta}}\sin\Delta\psi - \frac{\alpha-\alpha_0}{\sqrt{\beta_0\beta}}\cos\Delta\psi & \left(\frac{\beta_0}{\beta}\right)^{1/2}(\cos\Delta\psi - \alpha\sin\Delta\psi) \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

$\Delta\psi$ is the phase advance from point s_0 to point s in the beam line

Periodic Solutions

- Within a system made up of periodic sections it is natural to want the beam envelope to have the same periodicity.
- Taking the previous matrix to be that of a periodic section, and demanding the C-S parameters be periodic yields...

$$M_{periodic} = \begin{pmatrix} \cos\Delta\psi + \alpha\sin\Delta\psi & \beta\sin\Delta\psi \\ -\gamma\sin\Delta\psi & \cos\Delta\psi - \alpha\sin\Delta\psi \end{pmatrix}$$



Natural choice in a circular accelerator, when values of β, α above correspond to one particular point in the ring

Propagation of Courant-Snyder Parameters

- We can write the matrix of a *periodic* section as:

$$
\begin{aligned}
M_0 &= \begin{pmatrix} \cos\Delta\psi + \alpha\sin\Delta\psi & \beta\sin\Delta\psi \\ -\gamma\sin\Delta\psi & \cos\Delta\psi - \alpha\sin\Delta\psi \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\cos\Delta\psi + \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}\sin\Delta\psi \\
&= I\cos\Delta\psi + J\sin\Delta\psi \qquad = e^{J\Delta\psi}
\end{aligned}
$$

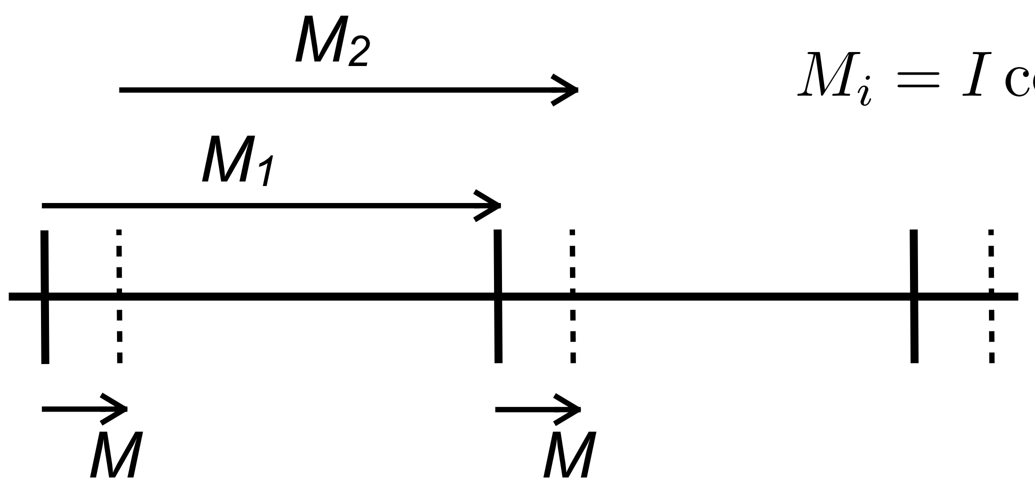
- where

$$
J = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix} \qquad det J = 1, \quad trace(J) = 0; \quad J^2 = -I
$$

α, β are values at the beginning/end of the periodic section described by matrix M

Tracking β, α, γ ...

- Let M_1 and M_2 be the "periodic" matrices as calculated at two points, and M propagates the motion between them. Then,



$$M_i = I \cos \Delta\psi + J_i \sin \Delta\psi$$

$$M_2 = M \; M_1 \; M^{-1}$$

- Or, equivalently,
  - if know C-S parameters (i.e., J) at one point, can find them at another point if given the matrix for motion in between:

$$J = \begin{pmatrix} \alpha & \beta \\ -\gamma & -\alpha \end{pmatrix}$$

$$J_2 = M \; J_1 \; M^{-1}$$

- Doesn't have to be part of a periodic section; valid between any two points of an arbitrary arrangement of elements

Evolution of the Phase Advance

- Again, if know parameters at one point, and the matrix from there to another point, then

$$M_{1\to 2} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Longrightarrow \frac{b}{a\beta_1 - b\alpha_1} = \tan \Delta\psi_{1\to 2}$$

- So, from knowledge of matrices, can “transport” phase *and* the Courant-Snyder parameters along a beam line from one point to another

Simple Examples

- Propagation through a Drift:

$$M = \begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}$$

$$\Longrightarrow \quad \Delta\psi = \tan^{-1}\left(\frac{L}{\beta_1 - L\alpha_1}\right)$$

$$\beta = \beta_0 - 2\alpha_0 L + \gamma_0 L^2$$

$$\alpha = \alpha_0 - \gamma_0 L$$

$$\gamma = \gamma_0$$

- Propagation through a Thin Lens:

$$M = \begin{pmatrix} 1 & 0 \\ -1/F & 1 \end{pmatrix}$$

$$\Longrightarrow \quad \Delta\psi = 0$$

$$\beta = \beta_0$$

$$\alpha = \alpha_0 + \beta_0/F$$

$$\gamma = \gamma_0 + 2\alpha_0/F + \beta_0/F^2$$

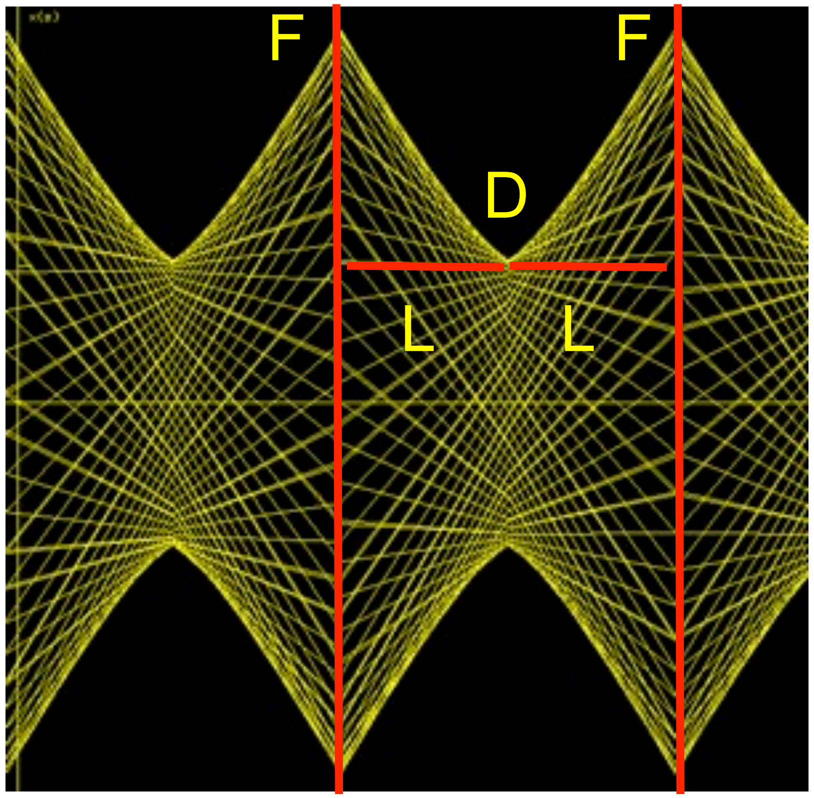
- Given *α, β* at one point, can calculate *α, β* at all downstream points

Choice of Initial Conditions

- Have seen how *β* can be propagated from one point to another. Still, have the choice of initial conditions...
- If periodic system, like a "ring," then natural to choose the periodic solution for *β, α*
- If beam line connects one ring to another ring, or a ring to a target, then we take the periodic solution of the upstream ring as the initial condition for the beam line
- In a system like a linac, wish to "match" to desired initial conditions at the input to the system (somewhere after the source, say) using an arrangement of focusing elements

Computation of Courant-Snyder Parameters

- As an example, consider again the FODO system



$$
\begin{aligned}
M &= \begin{pmatrix} 1 & 0 \\ -1/F & 1 \end{pmatrix}\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1/F & 1 \end{pmatrix}\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix} \\
&= \begin{pmatrix} 1 & L \\ -1/F & 1-L/F \end{pmatrix}\begin{pmatrix} 1 & L \\ 1/F & 1+L/F \end{pmatrix} \\
&= \begin{pmatrix} 1+L/F & 2L+L^2/F \\ -L/F^2 & 1-L/F-L^2/F^2 \end{pmatrix}
\end{aligned}
$$

- Thus, use above matrix of the periodic section to compute functions at the *exit* of the F quad..

FODO Cell

- From the matrix:

$$M = \begin{pmatrix} 1 + L/F & 2L + L^2/F \\ -L/F^2 & 1 - L/F - L^2/F^2 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

call $\mu = \Delta\psi$

Here, μ is phase advance through one periodic cell

$$traceM = a + d = 2 - L^2/F^2 = 2\cos\mu \Rightarrow \sin\frac{\mu}{2} = \frac{L}{2F}$$

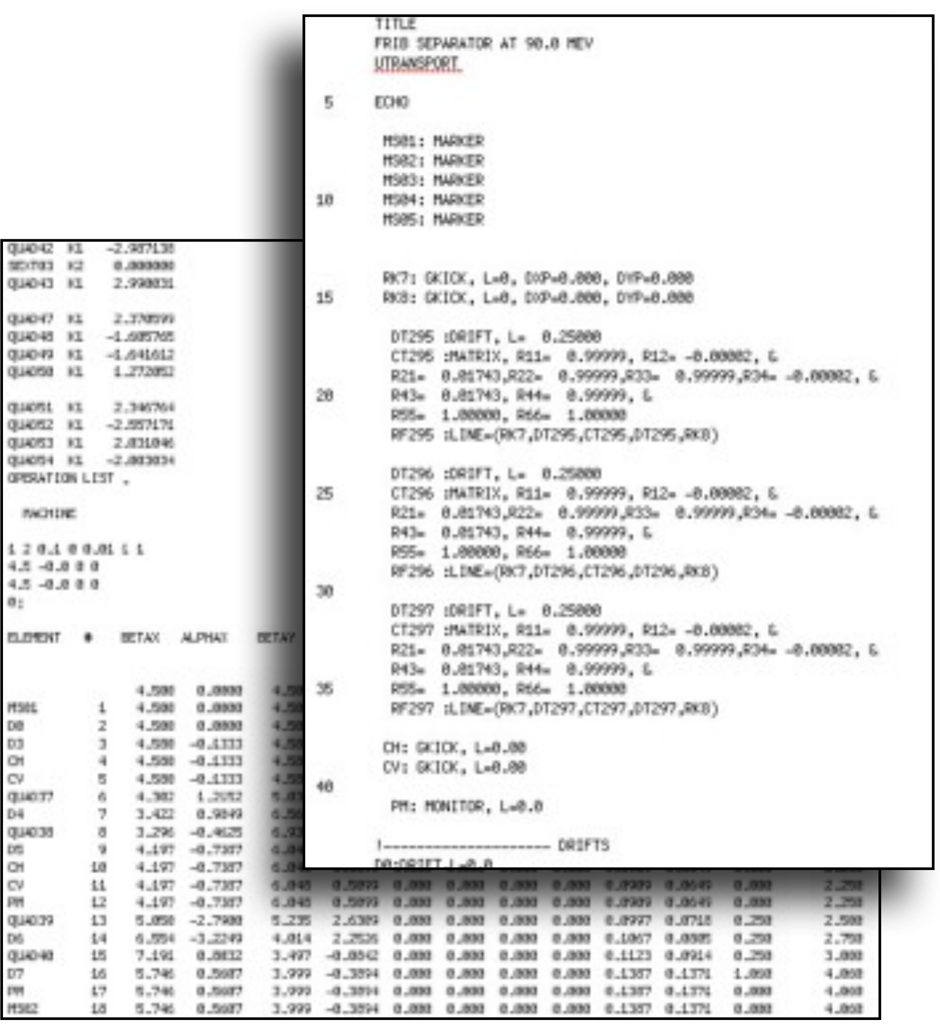
$$\beta = \frac{b}{\sin\mu} = 2F\sqrt{\frac{1+L/2F}{1-L/2F}} \qquad \alpha = \frac{a-d}{2\sin\mu} = \sqrt{\frac{1+L/2F}{1-L/2F}}$$

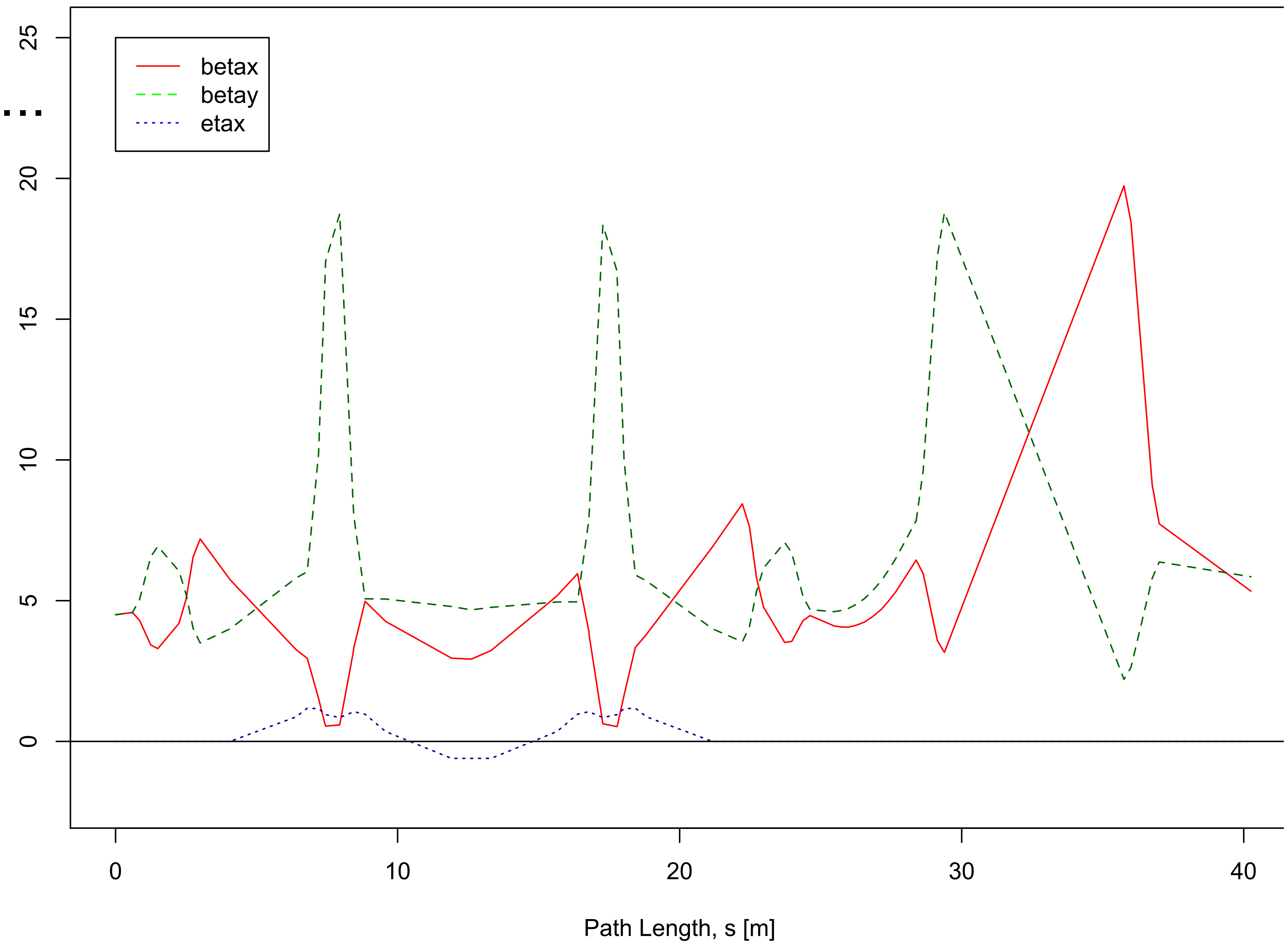
- If go from D quad to D quad, simply replace *F* --> *-F* in matrix *M*
  - at exit:

$$\beta = 2F\sqrt{\frac{1-L/2F}{1+L/2F}} \qquad \alpha = -\sqrt{\frac{1-L/2F}{1+L/2F}}$$

Computer Codes

- Complicated arrangements can be fed into now-standard computer codes for analysis
  - TRANSPORT, MAD, DIMAD
  - TRACE, TRACE3D, COSY
  - SYNCH, CHEF, many more ...





Review

- Found analytical solution to Hill's Equation:

$$x(s) = A\sqrt{\beta(s)}\sin[\psi(s) + \delta]$$

- So far, discussed amplitude function, *β*

- What about *A*?
  - Given *β(s)*, how big is the beam at a particular location? mm? cm? m?
  - If perturb the beam's trajectory, how much will it move downstream?

- Want to go from discussing single particle behavior to discussing a "beam" of particles

Courant-Snyder Invariant

- In general,

$$\begin{aligned} x &= A\sqrt{\beta}\sin\psi \\ x' &= \frac{A}{\sqrt{\beta}}[\cos\psi - \alpha\sin\psi] \\ \beta x' &= A\sqrt{\beta}[\cos\psi - \alpha\sin\psi] \\ &= A\sqrt{\beta}\cos\psi - \alpha x \\ \boxed{\beta x' + \alpha x = A\sqrt{\beta}\cos\psi} \end{aligned}$$

$$\begin{aligned} x^2 + (\beta x' + \alpha x)^2 &= A^2\beta \\ A^2 &= \frac{x^2 + (\beta x' + \alpha x)^2}{\beta} \\ &= \frac{x^2 + \alpha^2 x^2 + 2\alpha\beta x x' + \beta^2 x'^2}{\beta} \end{aligned}$$

$$\boxed{A^2 = \gamma x^2 + 2\alpha x x' + \beta x'^2}$$

While C-S parameters evolve along the beam line, the combination above remains constant.

Properties of the Phase Space Ellipse

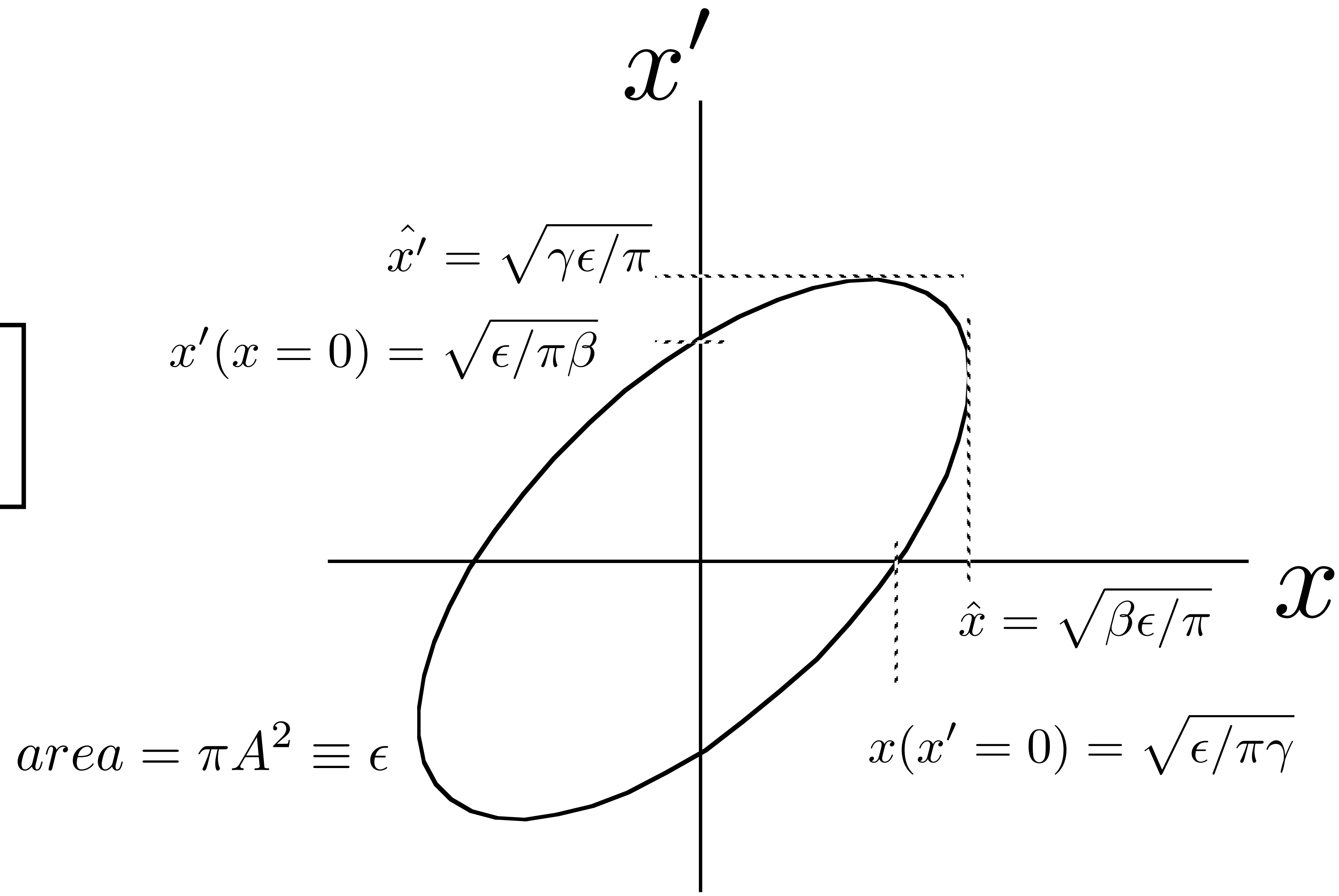
- The eqn. for the C-S invariant is that of an ellipse.
- If compute the area of the ellipse, find that

$$area = \pi A^2$$

i.e., while the ellipse changes shape along the beam line, its area remains constant

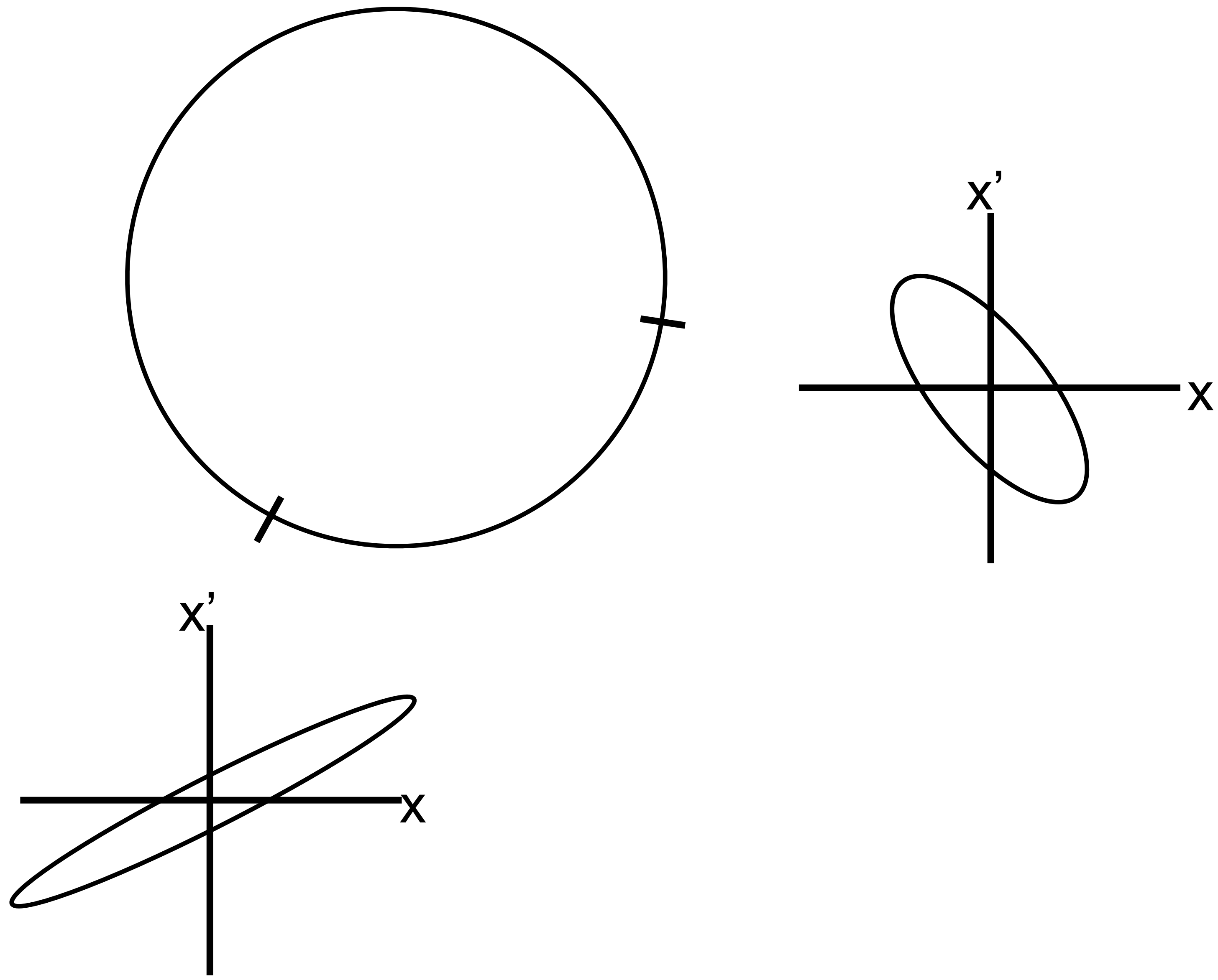
Emittance = area within a phase space trajectory

$$\gamma x^2 + 2\alpha x x' + \beta x'^2 = A^2$$



Motion in Phase Space

- Follow phase space trajectory...



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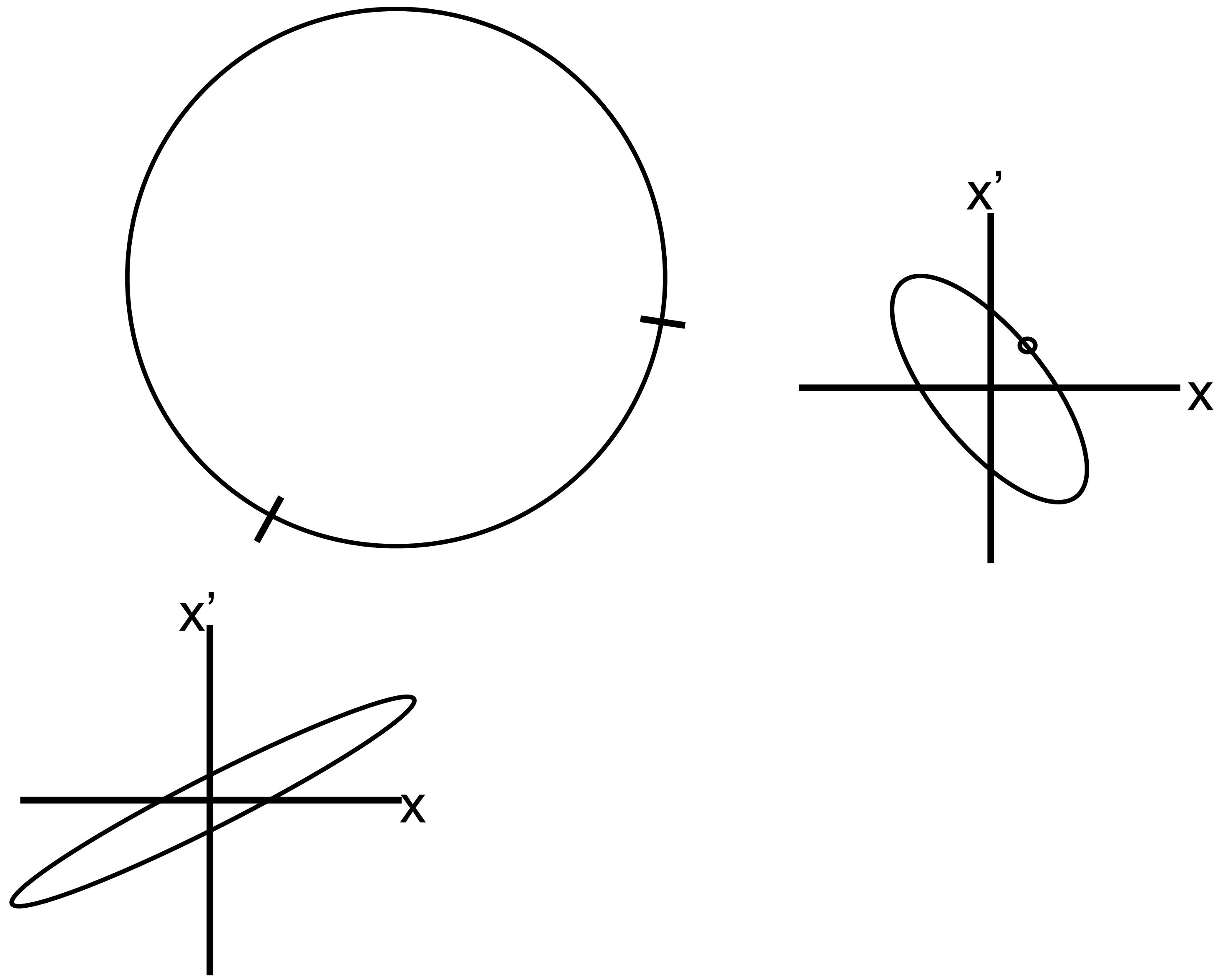

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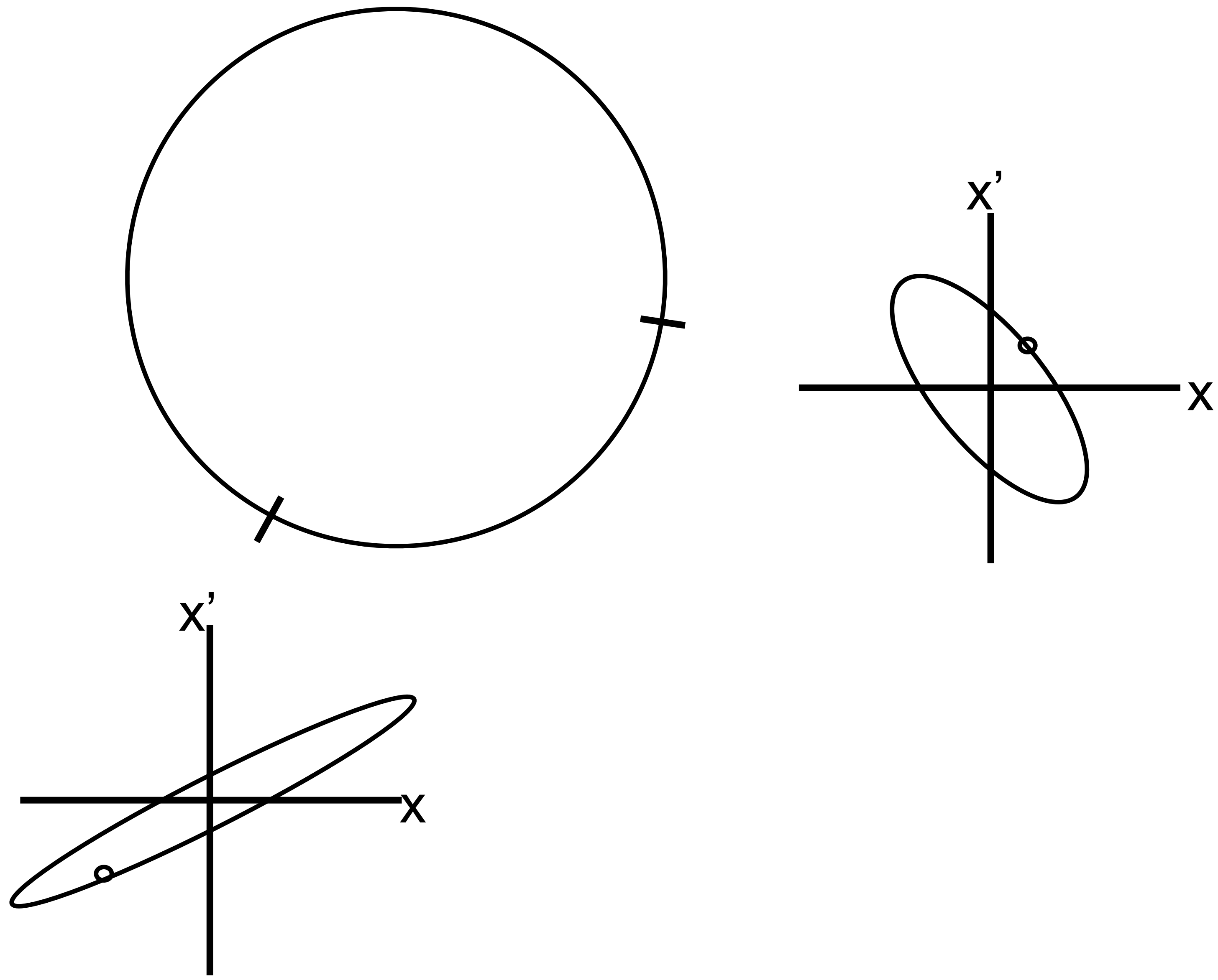

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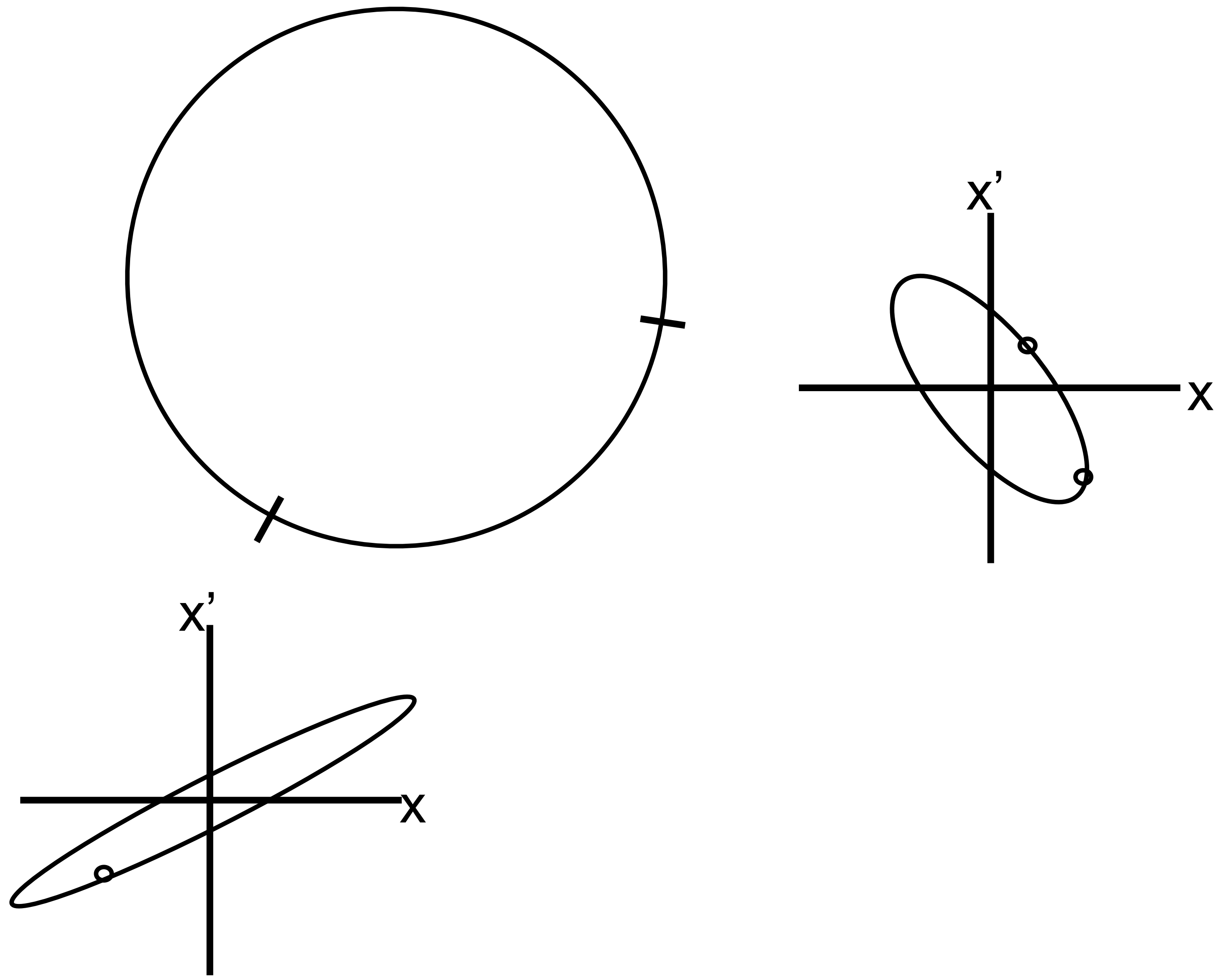

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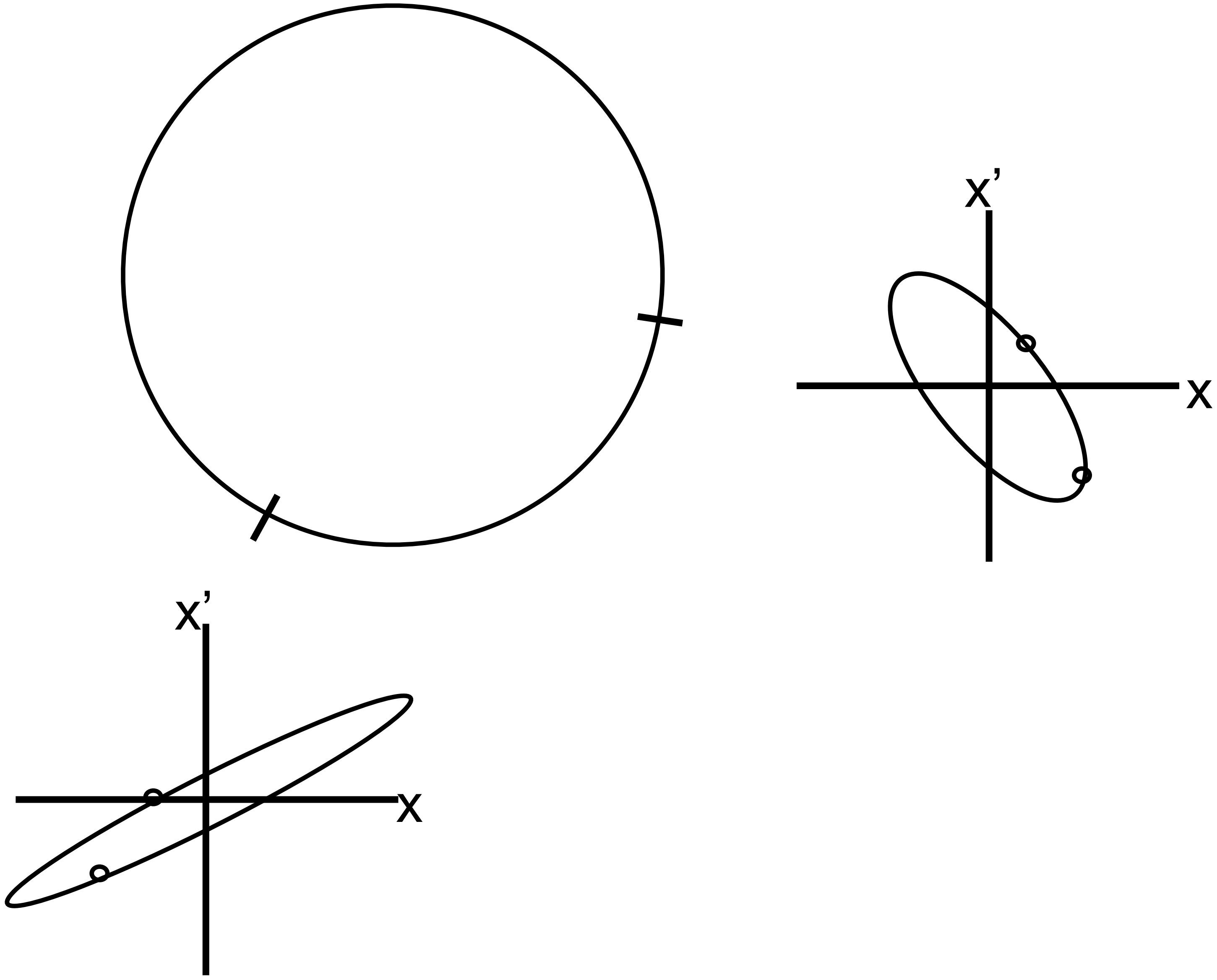

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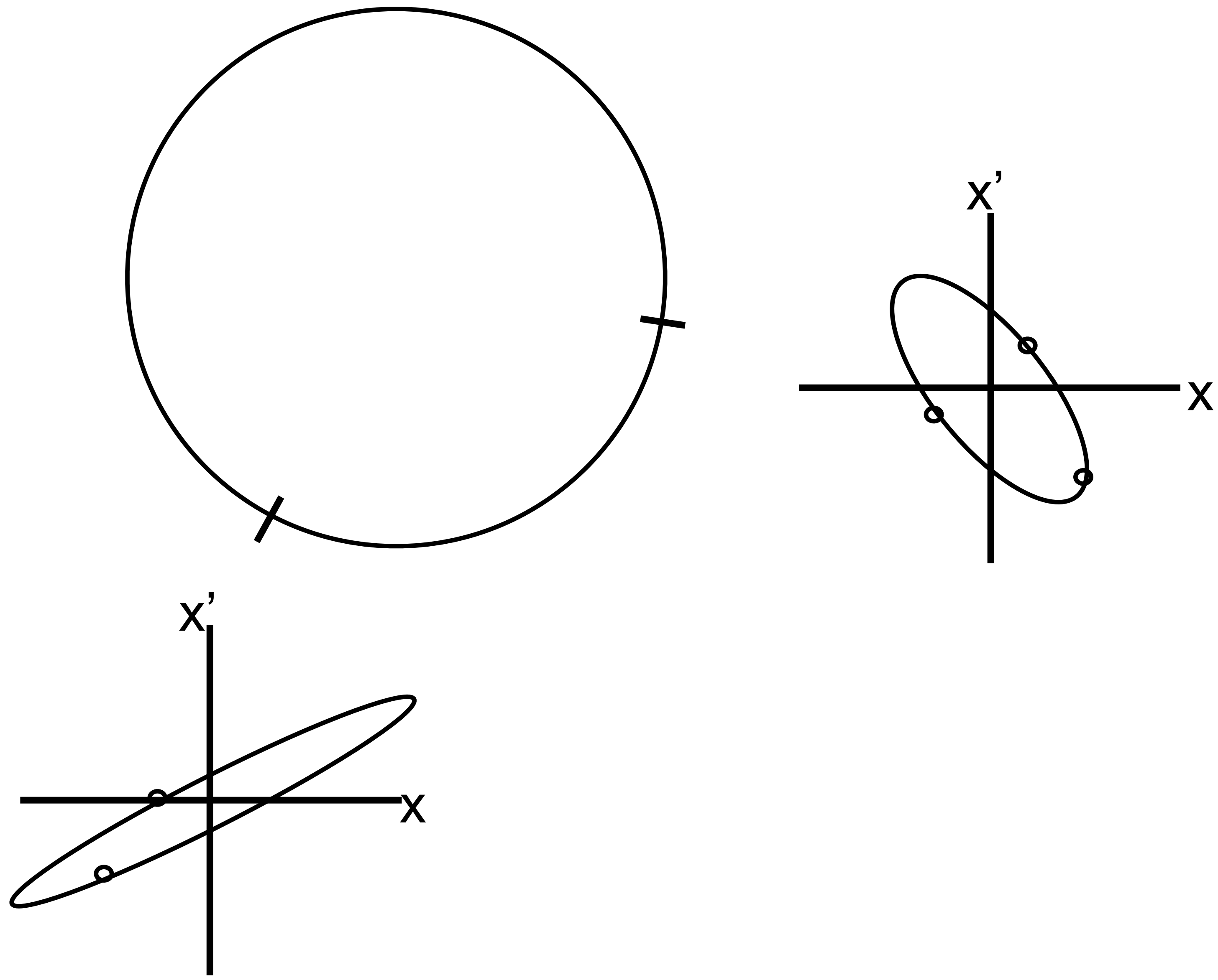

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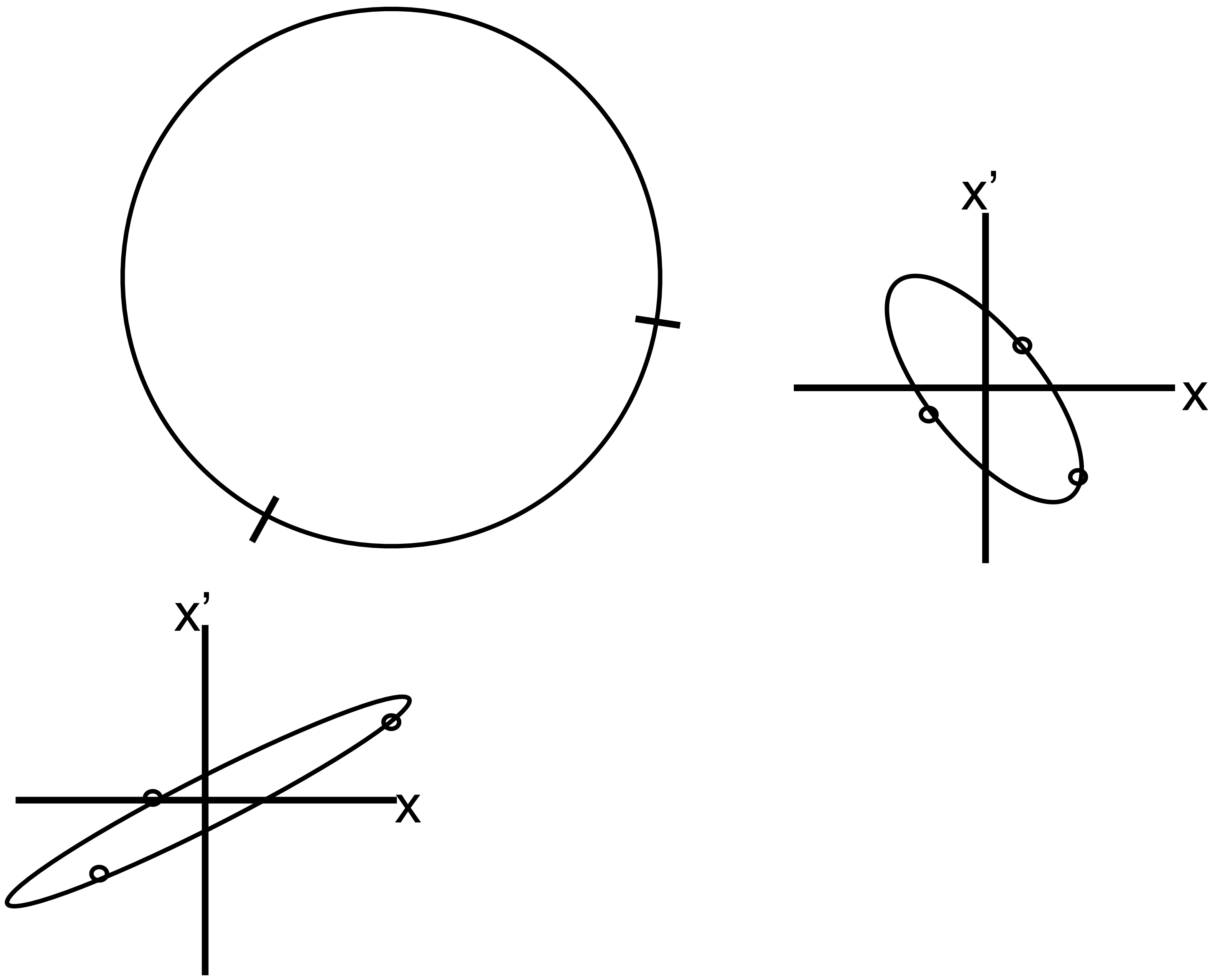

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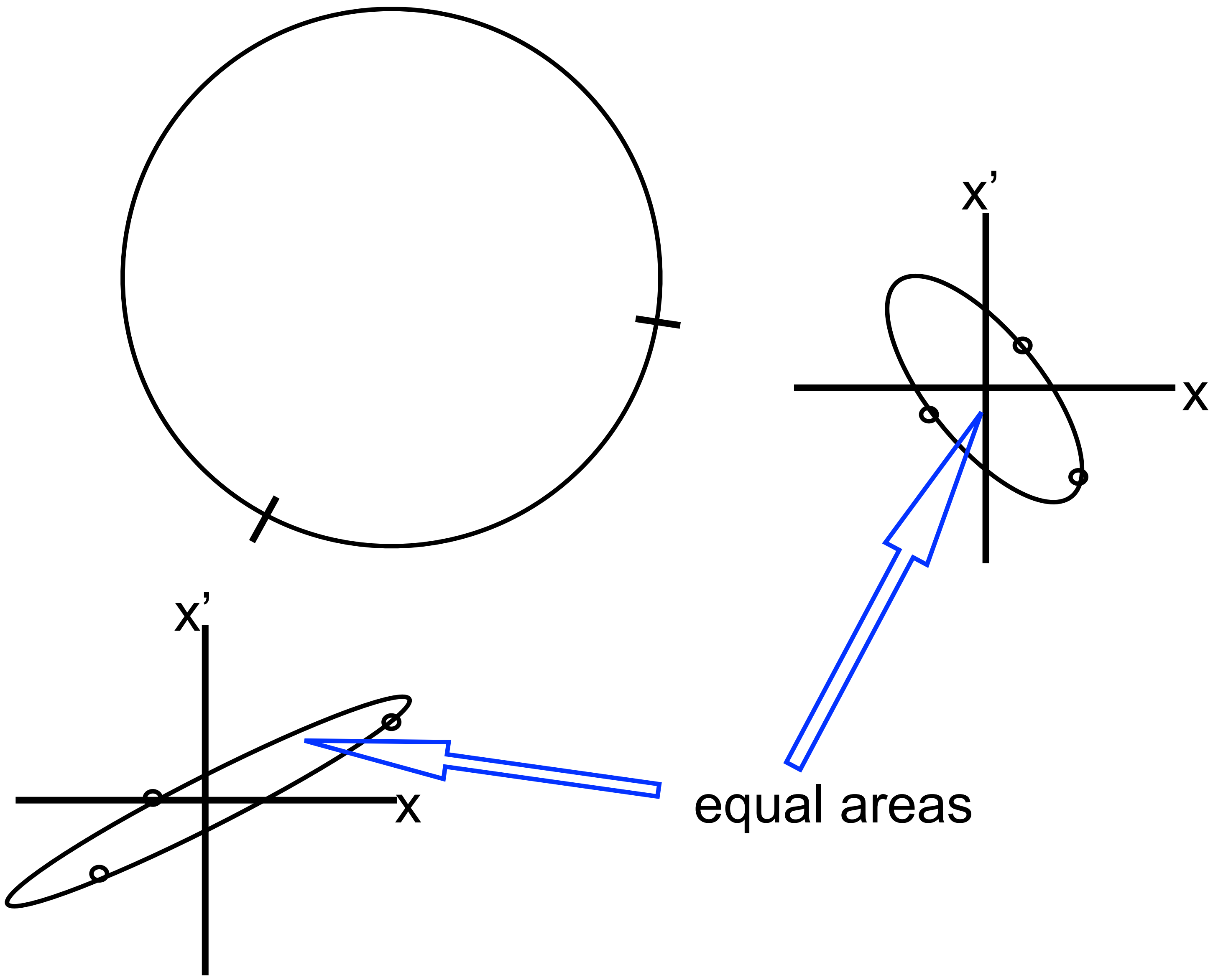

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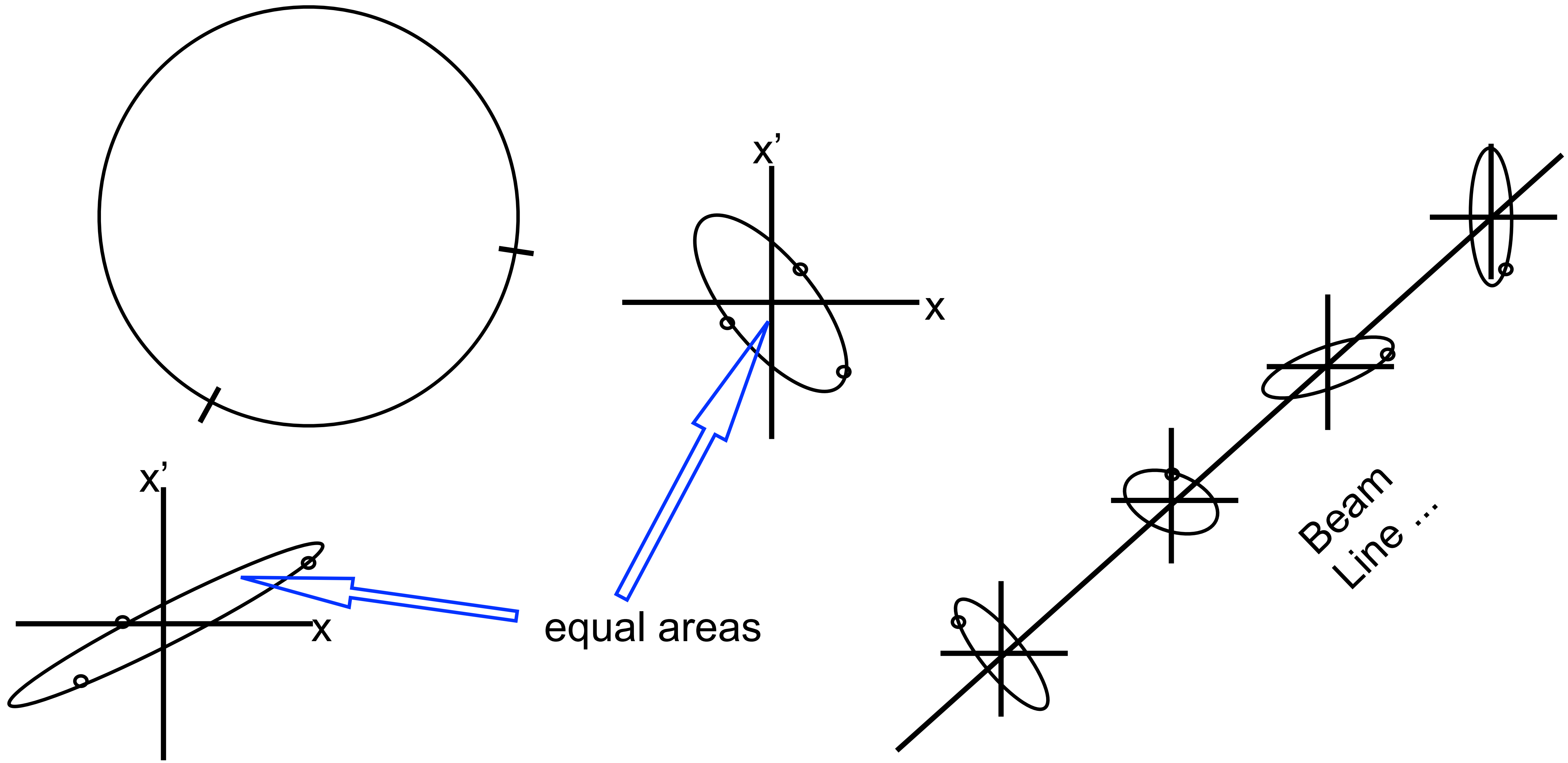
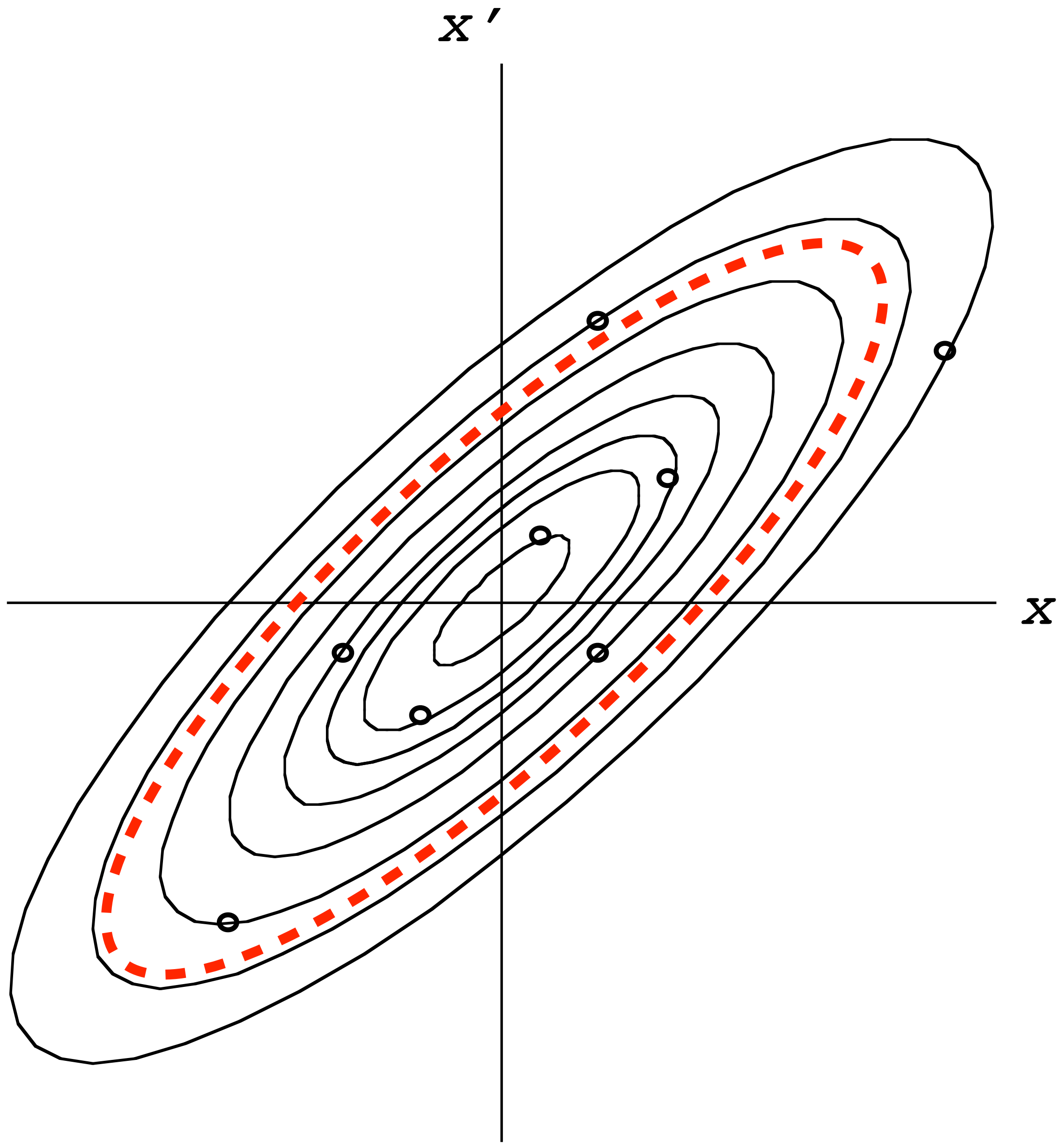


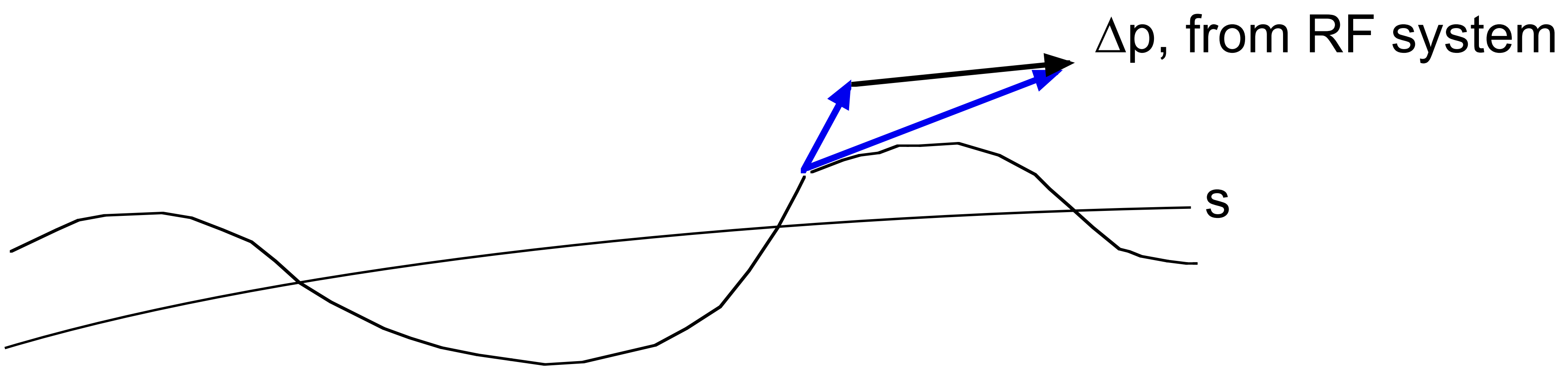
Beam Emittance

- Phase space area which contains a certain fraction of the beam particles
- Popular Choices:
  - 95%
  - 39%
  - 15%



Adiabatic Damping from Acceleration

- Transverse oscillations imply transverse momentum. As accelerate, momentum is “delivered” in the longitudinal direction (along the *s*-direction). Thus, on average, the angular divergence of a particle will decrease, as will its oscillation amplitude, during acceleration.



- The coordinates *x*-*x’* are not canonical conjugates, but x-p_x are; thus, the area of a trajectory in x-p_x phase space is invariant for adiabatic changes to the system.

Normalized Beam Emittance

- Hence, as particles are accelerated, the area in *x-x'* phase space is not preserved, while area in x-p_x is preserved. Thus, we define a "normalized" beam emittance, as

$$\epsilon_N \equiv \epsilon \cdot (\beta\gamma)$$

- In principle, the normalized beam emittance should be preserved during acceleration, and hence along the chain of accelerators from source to target. Thus it is a measure of beam quality, and its preservation a measure of accelerator performance.
- In practice, it is not preserved -- non-adiabatic acceleration, especially at the low energy regime; non-linear field perturbations; residual gas scattering; charge stripping; field errors and setting errors; *etc.* -- all contribute at some level to increase the beam emittance. Best attempts are made to keep this as small as possible.

Gaussian Emittance

- So, normalized emittance that contains a fraction *f* of a Gaussian beam is:

$$\epsilon_N = \frac{-2\pi \ln(1-f)\sigma^2(s)}{\beta(s)}(\beta\gamma)$$

Lorentz!

- Common values of *f* :

f	$\epsilon_N/(\beta\gamma)$
95%	$6\pi\sigma^2/\beta$
86.5%	$4\pi\sigma^2/\beta$
39%	$\pi\sigma^2/\beta$
15%	σ^2/β

Perhaps most commonly used, sometimes called the "rms" emittance; but, always ask if not clear in context!

Emittance in Terms of Moments

- For each particle, $x = A\sqrt{\beta}\sin\psi \qquad x' = \frac{A}{\sqrt{\beta}}(\cos\psi - \alpha\sin\psi)$
- Average over the distribution...

$$x^2 = A^2\beta\sin^2\psi \qquad x'^2 = \frac{A^2}{\beta}(\cos^2\psi + \alpha^2\sin^2\psi - \alpha\sin 2\psi)$$

$$\langle x^2\rangle = \frac{1}{2}\langle A^2\rangle\beta \qquad \langle x'^2\rangle = \frac{\langle A^2\rangle}{2\beta}(1+\alpha^2) = \frac{1}{2}\langle A^2\rangle\gamma$$

$$\beta\gamma - \alpha^2 = 1$$

and ... $$xx' = A^2(\frac{1}{2}\sin 2\psi - \alpha\sin^2\psi)$$

$$\langle xx'\rangle = -\frac{1}{2}\langle A^2\rangle\alpha$$

From which the average of all particle emittances will be $\pi\langle A^2\rangle = 2\pi\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$

and the "normalized rms emittance" can be defined as:

$$\epsilon_N = \pi(\beta\gamma)\sqrt{\langle x^2\rangle\langle x'^2\rangle - \langle xx'\rangle^2}$$

TRANSPORT of Beam Moments

- For simplicity, define $\tilde{\epsilon} \equiv \frac{1}{2}\langle A^2\rangle$; then,

- Note that:

$$\tilde{\epsilon}J = \begin{pmatrix} \tilde{\epsilon}\alpha & \tilde{\epsilon}\beta \\ -\tilde{\epsilon}\gamma & -\tilde{\epsilon}\alpha \end{pmatrix} = \begin{pmatrix} -\langle xx'\rangle & \langle x^2\rangle \\ -\langle x'^2\rangle & \langle xx'\rangle \end{pmatrix}$$

- Correlation Matrix:

$$\Sigma \equiv \begin{pmatrix} \langle x^2\rangle & \langle xx'\rangle \\ \langle xx'\rangle & \langle x'^2\rangle \end{pmatrix}$$

$$\boxed{\Sigma_2 = M\ \Sigma_1\ M^T}$$

Here, *M* is from point 1 to point 2 along the beam line (same *M* as previously)

Summary

- So, can look at propagation of amplitude function through beam line given matrices of individual elements. Beam size at a particular location determined by

$$x_{rms}(s) = \sqrt{\beta(s)\epsilon_N/\pi(\beta\gamma)}$$

- Or, given an initial particle distribution, can look at propagation of second moments (of position, angle) given the same element matrices, and hence the propagation of the beam size, $\sqrt{\langle x^2\rangle(s)}$.

- Either way, can separate out the inherent properties of the beam distribution from the optical properties of the hardware arrangement

Effects due to Momentum Distribution

- Beam will have a distribution in momentum space
- Trajectories of individual particles will spread out when pass through magnetic fields
  - B is constant; thus $\Delta\theta/\theta \sim -\Delta p/p$
  - path is also altered by the gradient fields...
- These trajectories are described by the Dispersion Function:

$$D(s) \equiv \Delta x_{c.o.}(s)/(\Delta p/p)$$

- Consequently, affects beam size:

$$\langle x^2 \rangle = \epsilon_N \beta(s)/(\pi\gamma v/c) + D(s)^2 \langle (\Delta p/p)^2 \rangle$$

- as well as trajectories of particles of various rigidities (A/Q)(p_u/e)

Chromaticity

- Focusing effects from the magnets will also depend upon momentum:

$$x'' + K(s,p)x = 0 \qquad K = e(\partial B_y(s)/\partial x)/p$$

- To give all particles the similar optics, regardless of momentum, need a "gradient" which depends upon momentum. Orbits spread out horizontally (or vertically) due to dispersion, can use a sextupole field:

$$\vec{B} = \frac{1}{2}B''[2xy\ \hat{x} + (x^2 - y^2)\ \hat{y}]$$

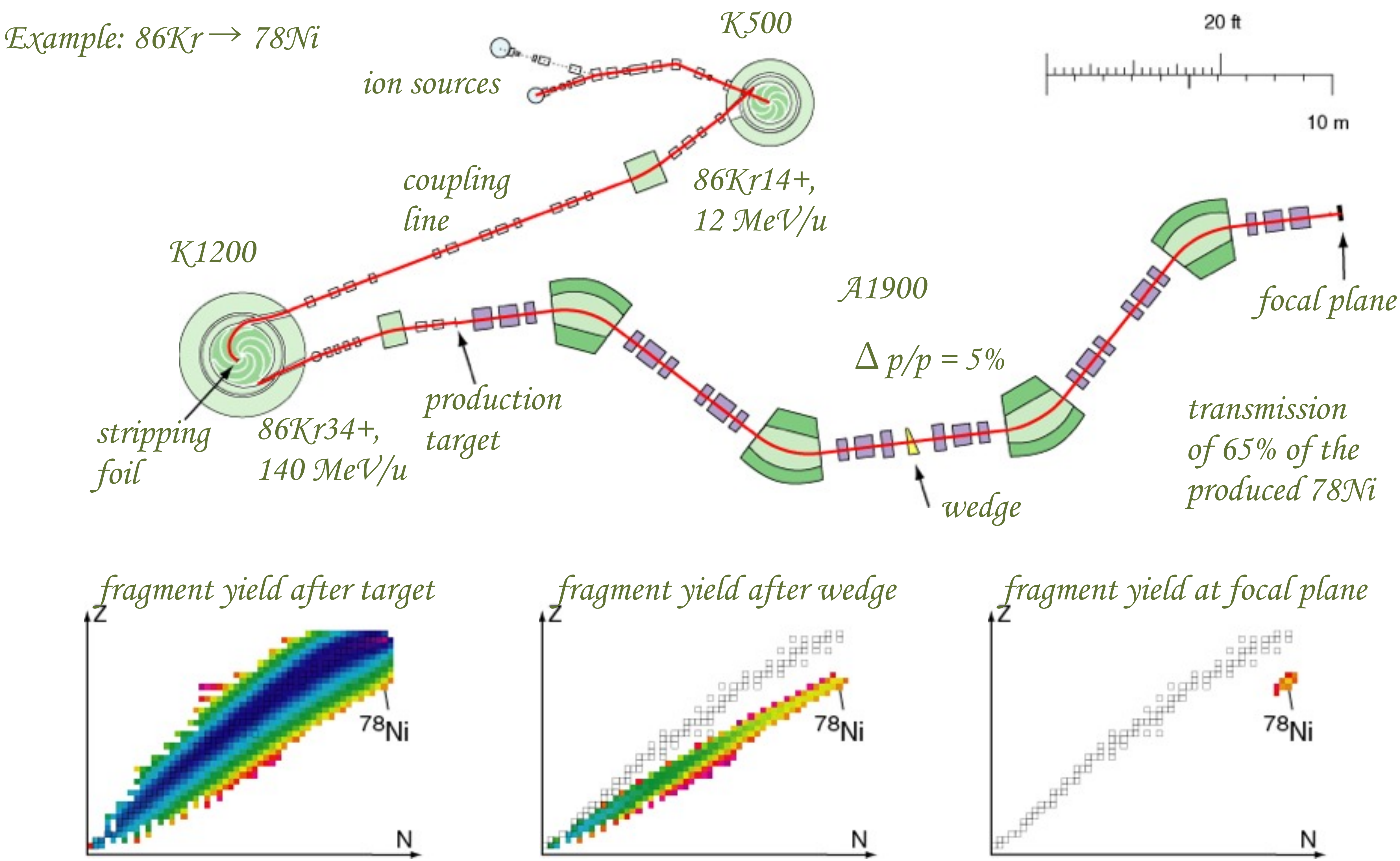
- which gives $\partial B_y/\partial x = B''x = B''D(\Delta p/p)$
  -
  - i.e., a field gradient which depends upon momentum

- Chromaticity* is the variation of optics with momentum; use sextupole magnets to control/adjust; but, now introduces a nonlinear transverse field ...
  - can have a transverse dynamic aperture!

*In a synchrotron, "the" chromaticity is the variation of the transverse oscillation frequency with momentum

In-Flight Production Example: NSCL's CCF

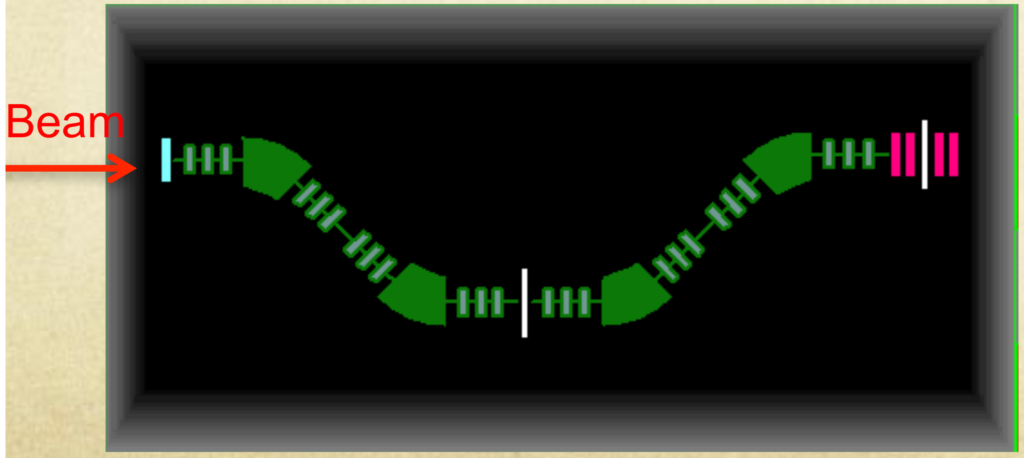
D.J. Morrissey, B.M. Sherrill, Philos. Trans. R. Soc. Lond. Ser. A. Math. Phys. Eng. Sci. 356 (1998) 1985.



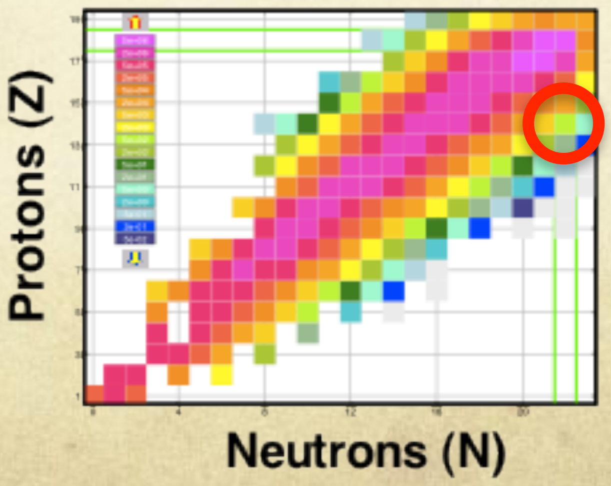
Principle of Fragment Separator [1]

M. Hausmann, FRIB

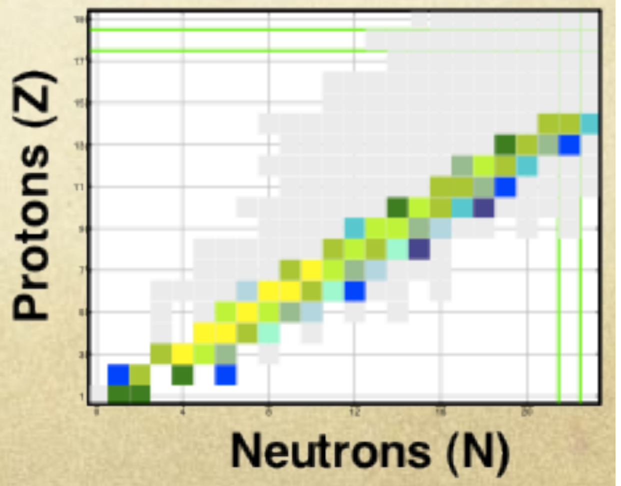
- Magnetic separation
  - Dipole magnet disperses beam according to magnetic rigidity $B\rho = p / q$
    - "Momentum" separation, similar to charge states in linac front end or after stripper
  - Velocities of different fragments after target somewhat similar
    ⟶ selection by mass-over-charge ratio
- Quadrupole magnets to focus beam
  - Small image of beam spot on target ⟶ good selection with slits at focal plane
  - Plus aberration correction with sextupoles/octupoles
- Example: ^{36}Si from ^{40}Ar primary beam in A1900 (NSCL)



After target



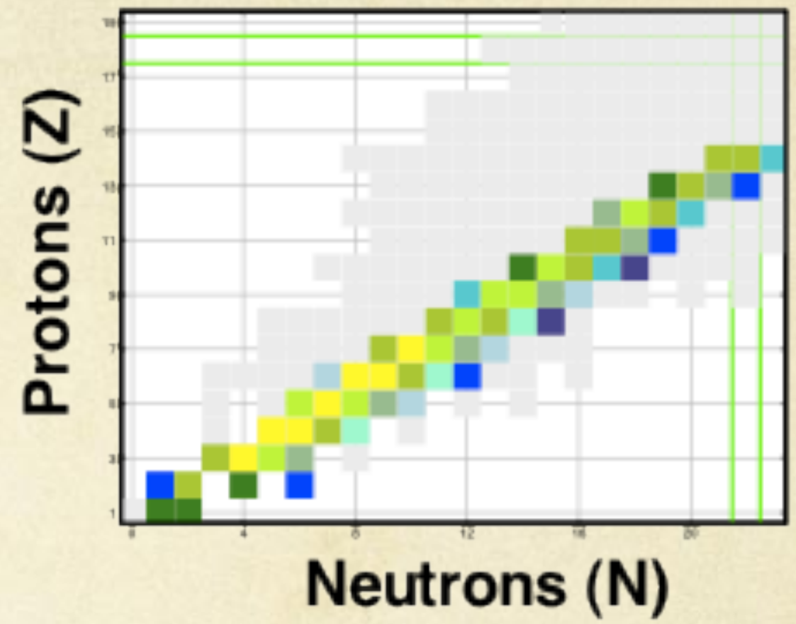
Bρ selected



Principle of Fragment Separator [2]

M. Hausmann, FRIB

- Magnetic separation alone insufficient
- Numerous nuclides with similar A/Z
- But with different proton number (Z)
- Energy loss in matter is Z dependent
  - Bethe formula

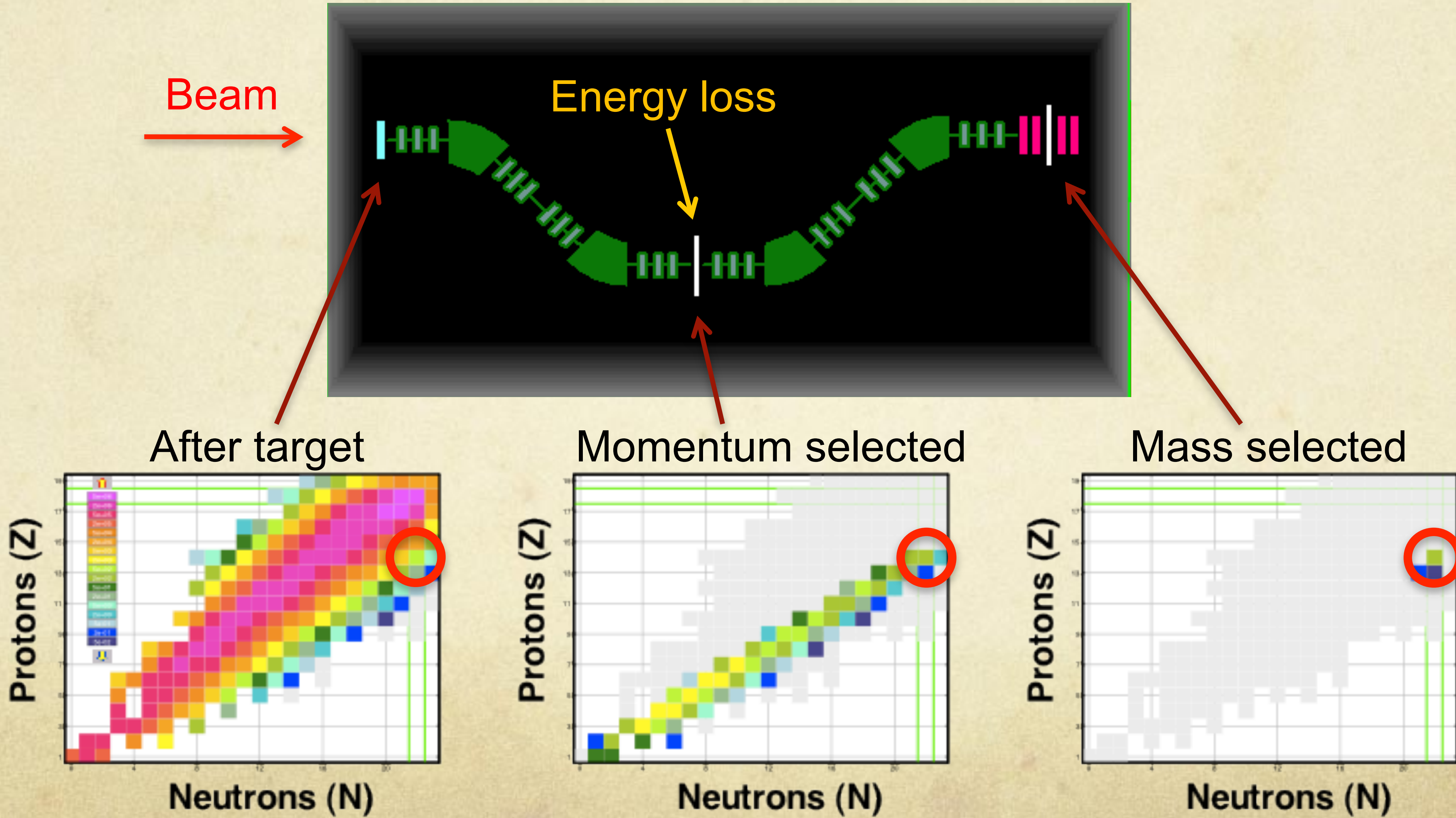


- Interaction of beam with degrader (a piece of metal) leads to different velocity changes for different fragments
  - Previously similar magnetic rigidities get “dispersed”
- This allows to separate these by magnetic rigidity ⟶ mass selection

Principle of Fragment Separator [3]

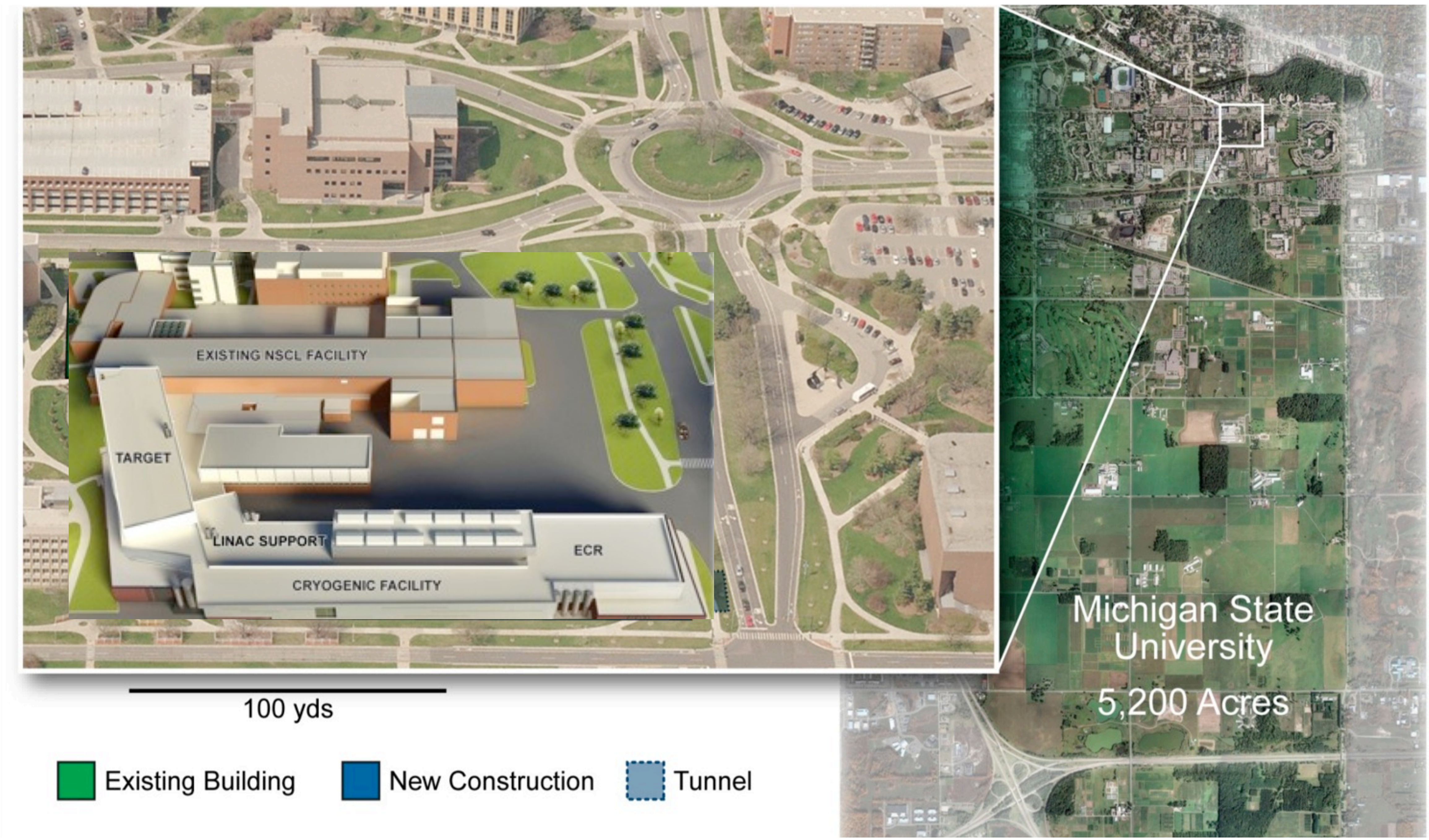
M. Hausmann, FRIB

- Combination: magnetic separation and matter-induced energy loss
- Result: purification of the desired rare isotope beam



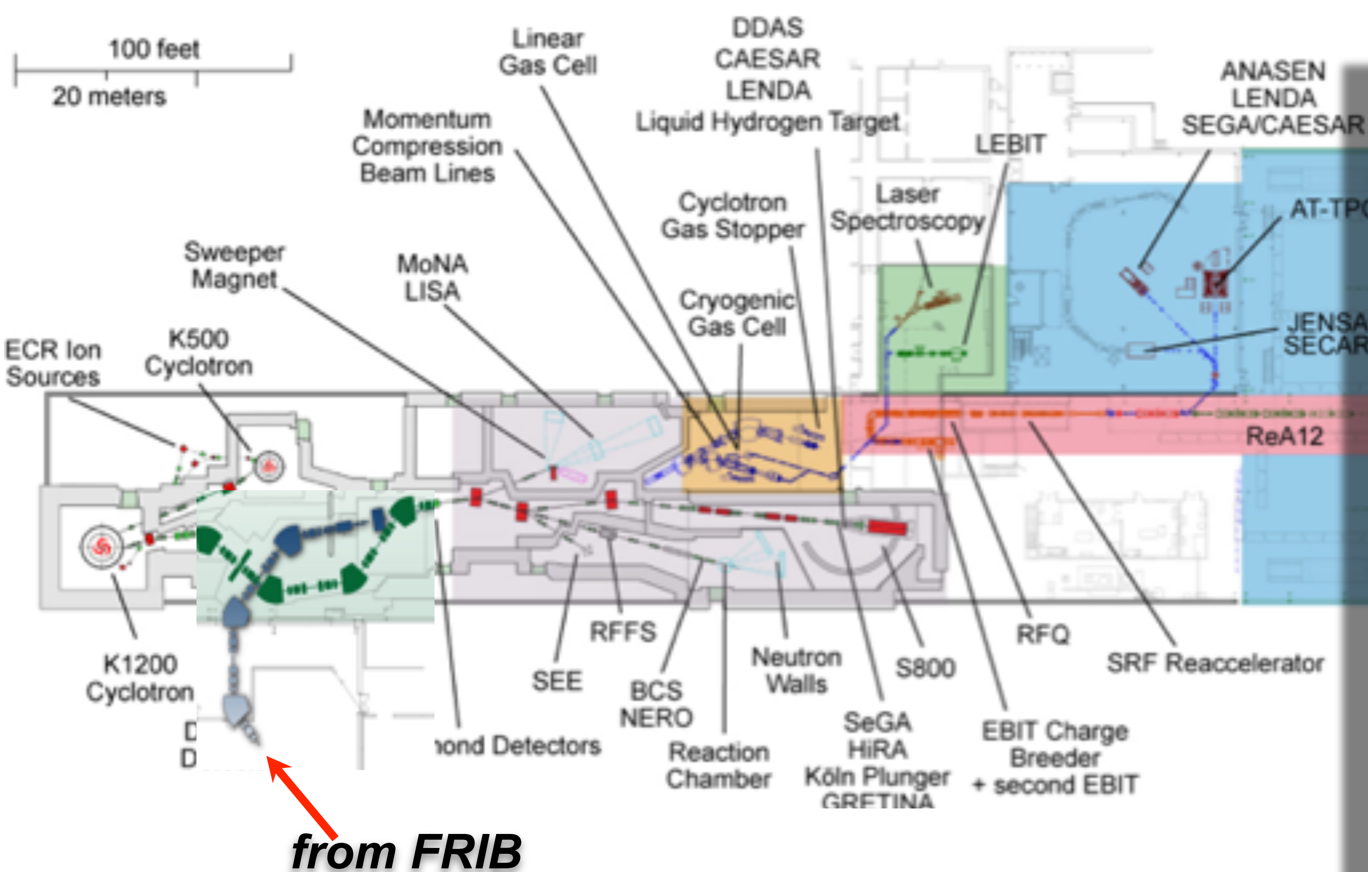
MSU's Facility for Rare Isotope Beams (FRIB)

- Facility for Rare Isotope Beams (FRIB) (usually pronounced F-RIB)



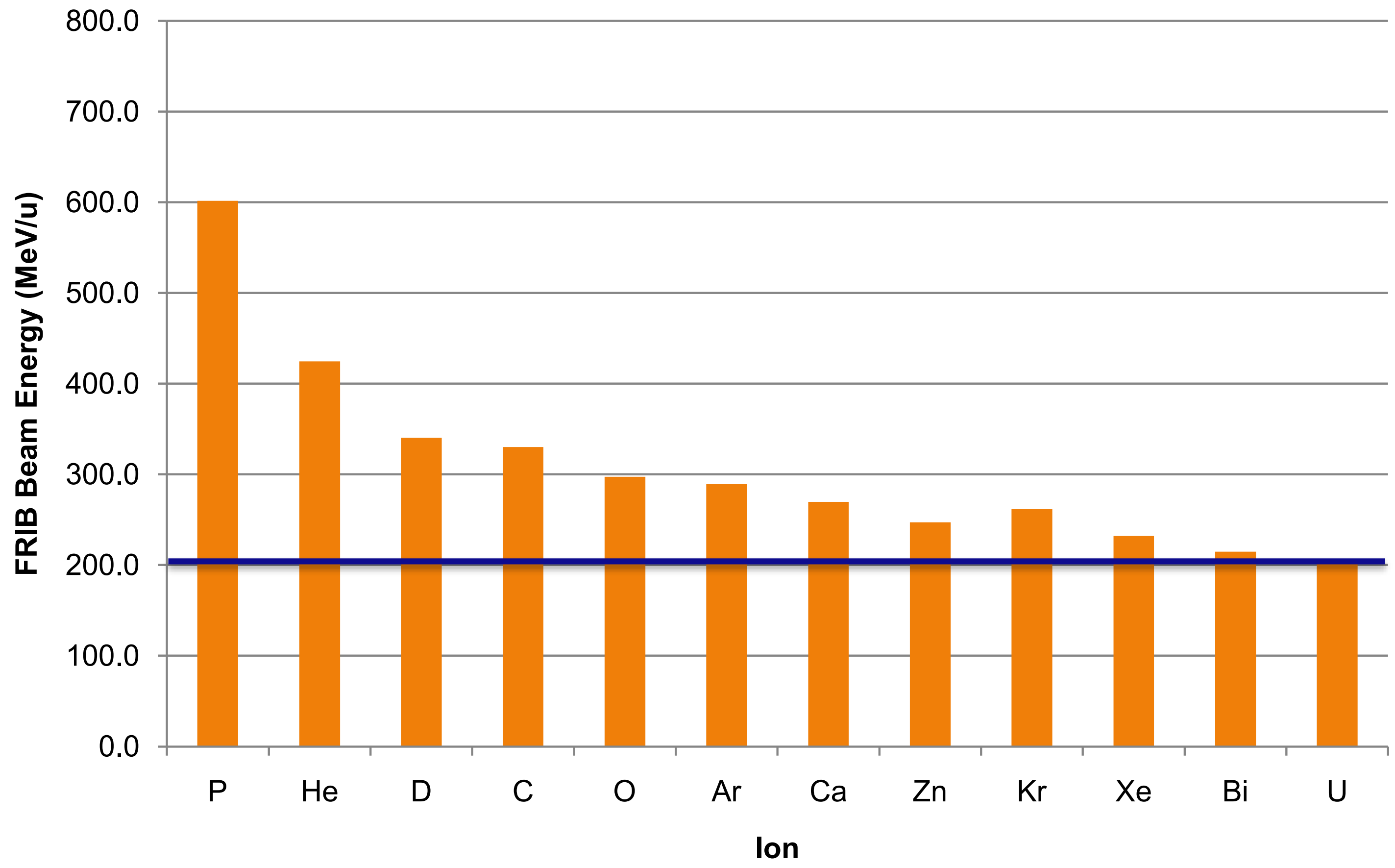
MSU's Facility for Rare Isotope Beams (FRIB)

- CW linac -- SRF
  - 100% duty factor; flexible cavity tuning; large acceptance
- high charge states -- multi-charge-state acceleration
- compact -- keep on campus; tie into existing facility; upgradeable



Major FRIB Goals

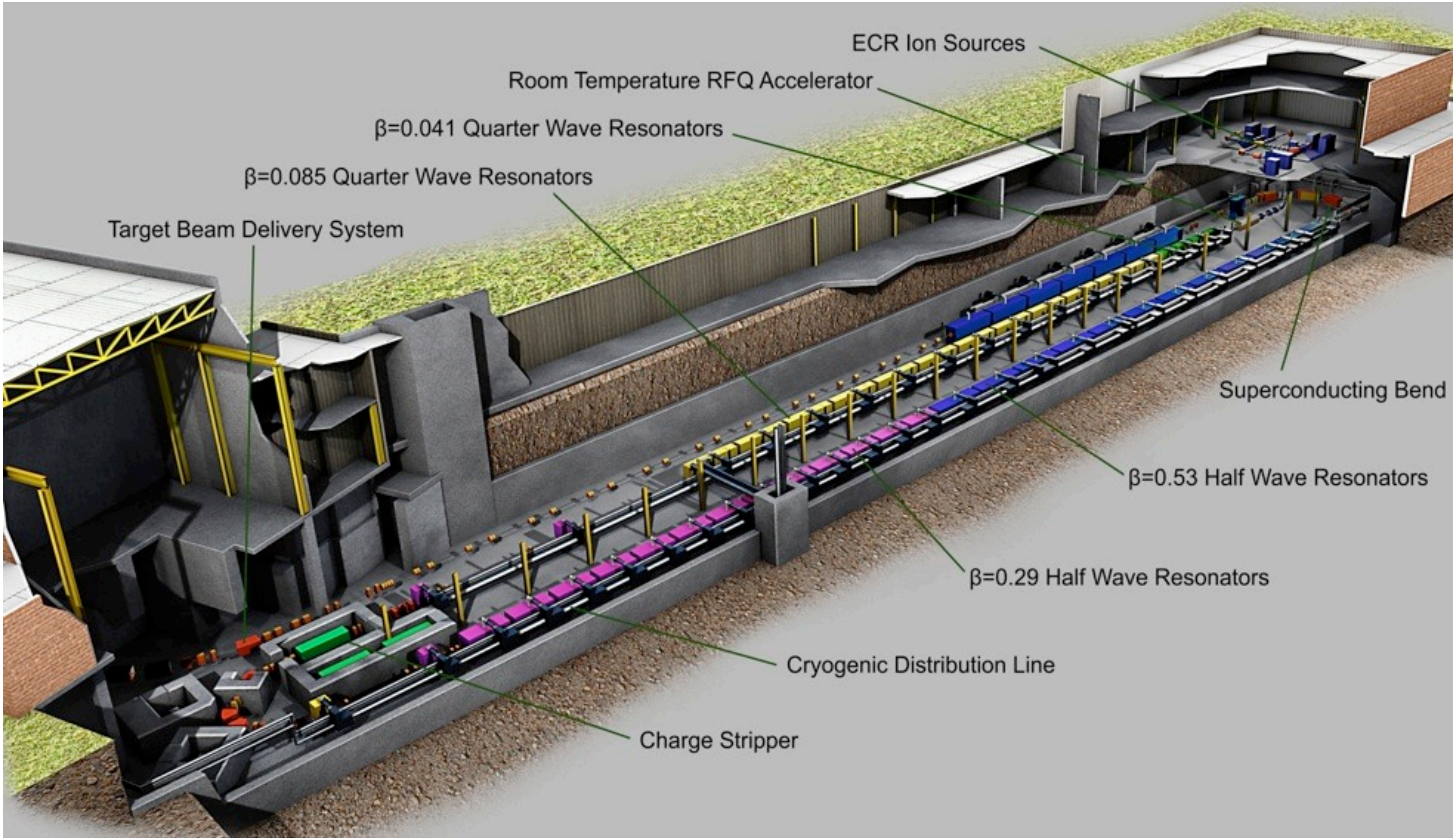
- Accelerate ions -- protons through ^{238}U -- to K.E. >200 MeV/nucleon
- Produce beam spot size on target of ~1 mm diameter
- Produce average beam power on target up to 400 kW
- Provide layout that allows for future enhancements to the facility
  - possibilities: higher beam energies, Isotope Separation On-Line (ISOL) facility, multi-user capabilities



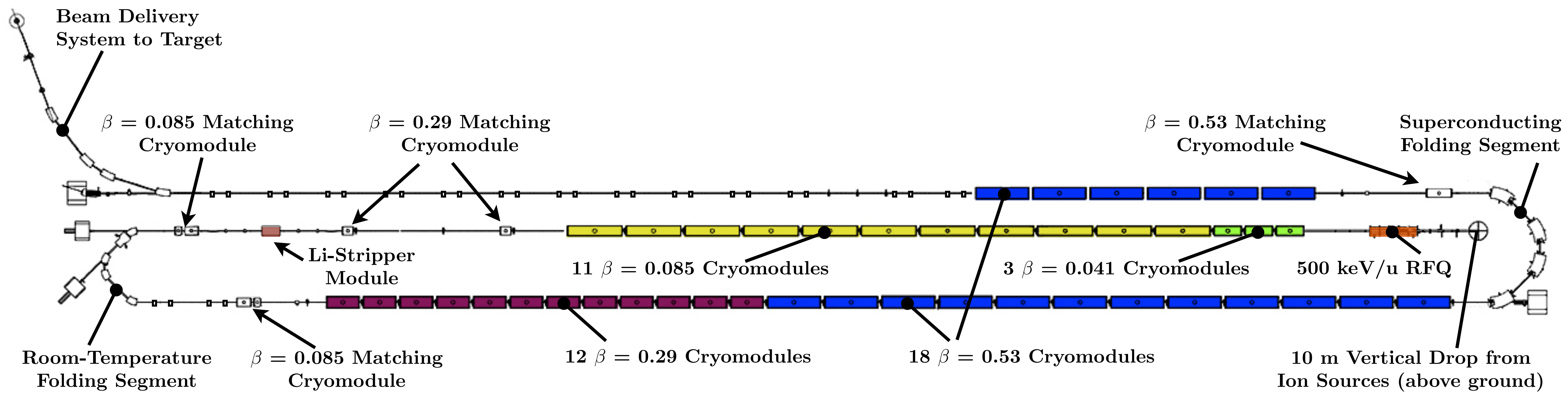
Many Interesting Accelerator Science Aspects

- Superconducting RF cavities, tailored for low-velocity (i.e., 0.02 - 0.7*c)* heavy ions
- Electron stripping to high charge state ions for higher acceleration
- Multi-charge-state operation to attain high beam power on target
- High intensity Electron Cyclotron Resonance ion sources
- High power, cw Radio Frequency Quadrupole for early acceleration
- Compact design to minimize footprint, tie into existing facility on campus
- Flexible enough to accommodate *many* ion species
- High-power beam targeting, up to 400 kW for all ion species
- Beam stopping systems -- e.g., cyclotron gas stopper
- Re-acceleration (with SRF cavities) of stopped/trapped beams to well-defined energies (~0.3 - 3 MeV/u for U, eventually up to 6-12 MeV/u)
- ...

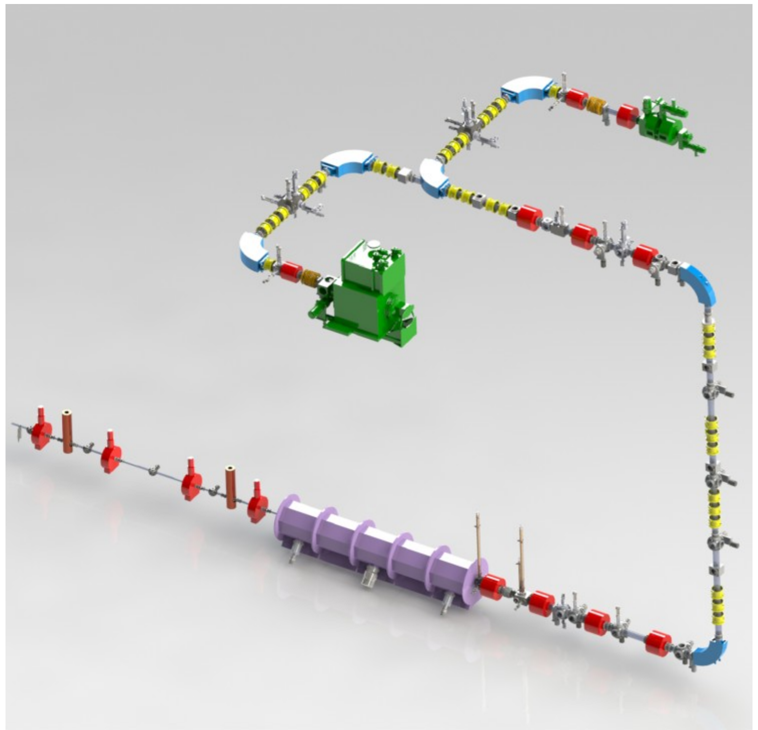
Design is Maturing Quickly



Accelerator Physics Design

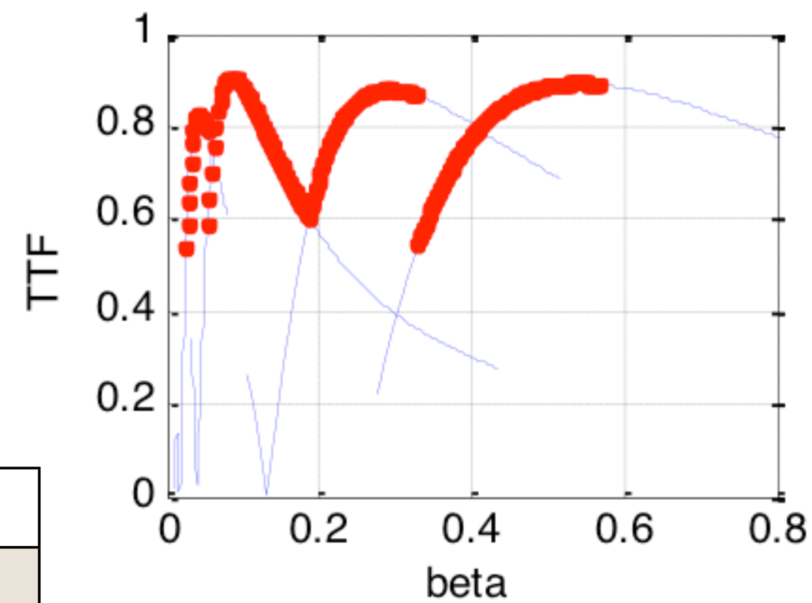


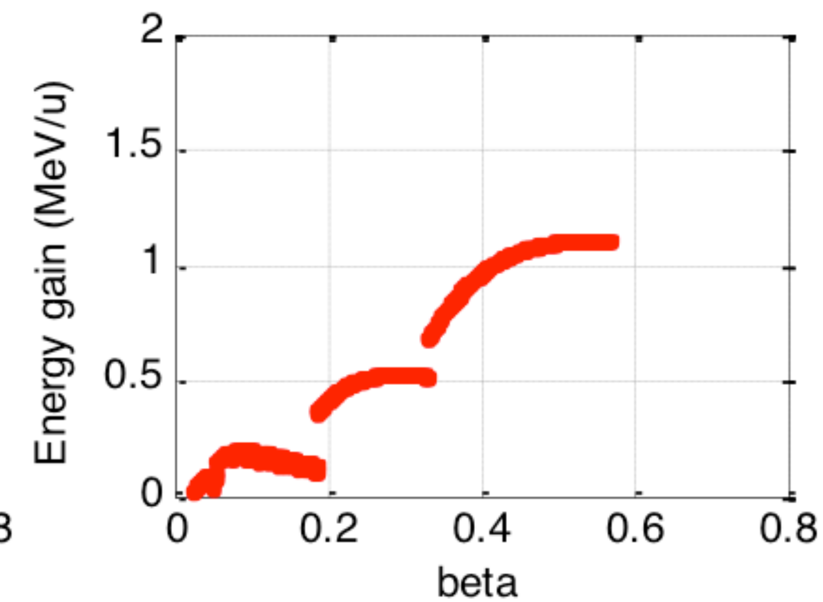
- Double-folded design
  - allows for charge stripping, selection at first fold,
    - » but provides compact cost-saving design at NSCL site
  - free space for upgrades, or for meeting performance goals
  - 4 cavity types, 2 frequencies
    - » large longitudinal admittance for multi-charge-state operation



SRF Cavities

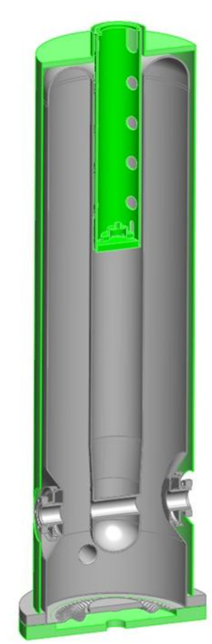
- Quarter-wave and Half-wave resonators for acceleration
  - 80.5 MHz (QWR) and
  - 322 MHz (HWR)
- Transit-Time Factors:





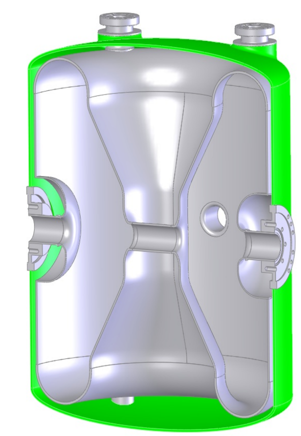
Type	λ/4	λ/4	λ/2	λ/2
β_{opt}	**0.041**	**0.085**	**0.29**	**0.53**
f(MHz)	**80.5**	**80.5**	**322**	**322**
Aperture (mm)	34	34	40	40
V_a (MV)	0.81	1.8	2.1	3.7
E_p (MV/m)	30.8	32.8	33.3	26.5
B_p (mT)	54.6	70	60	63
T(K)	2.0	2.0	2.0	2.0
Number	**12**	**94**	**76**	**148**

QWR:



β =0.085

HWR:

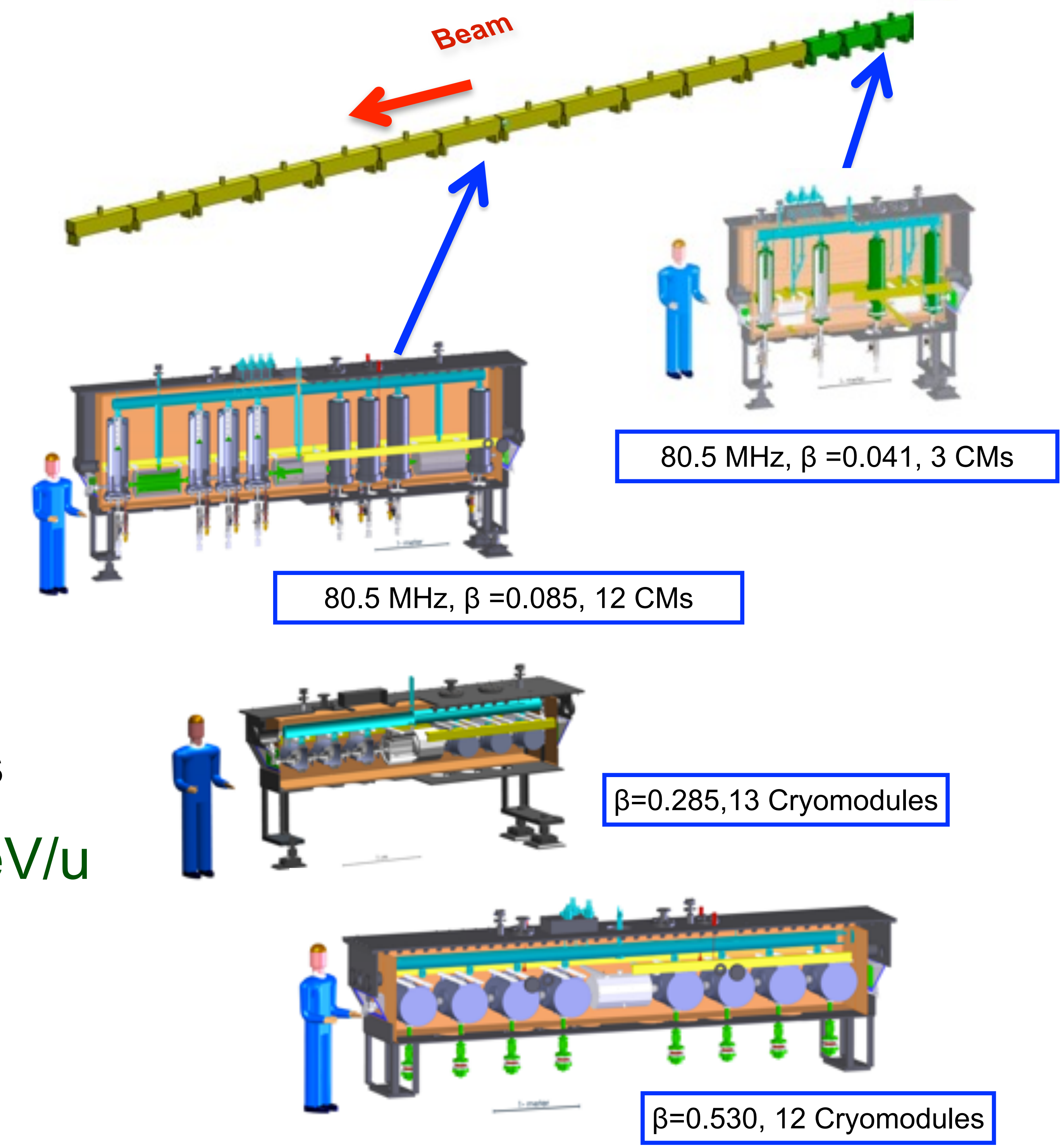


β =0.53

2-gap structures provide broad transit time range, enabling fewer cavity types

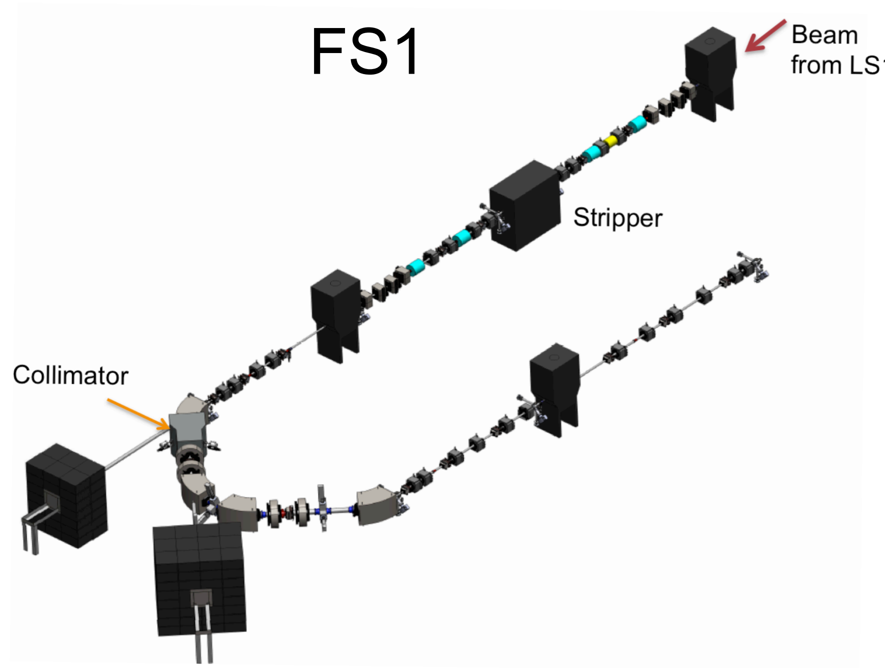
Linac Segments

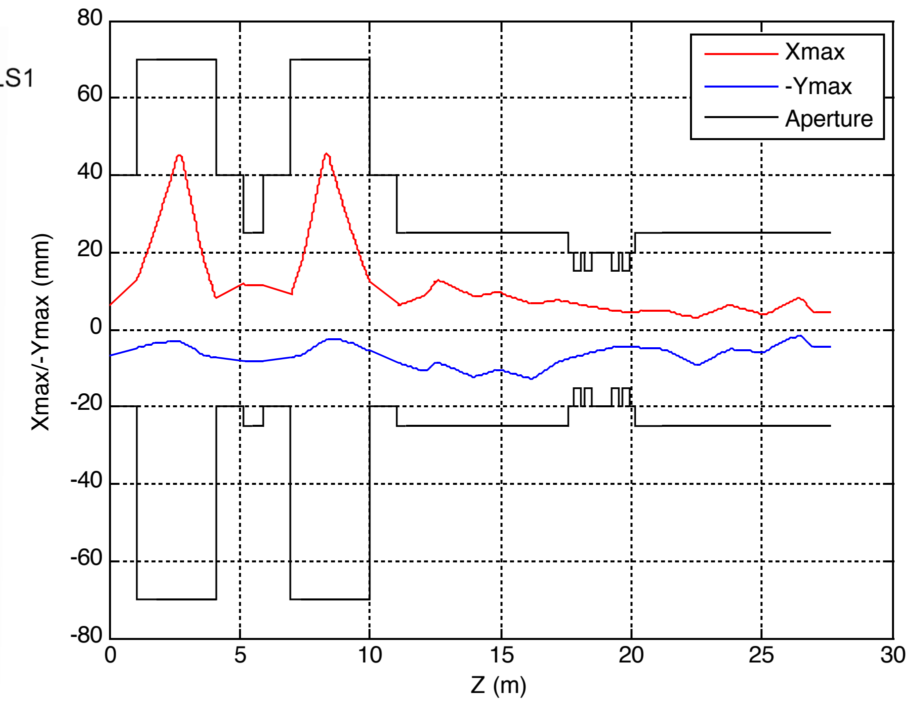
- First Linac Segment
  - emerge from RFQ @ 0.5 MeV/u
  - accelerate to 16.6 MeV/u ($U^{33+,\,34+}$)
  - λ/4 cavities – 80.5 MHz
    - »2 types; 2 cryomodule types
  - solenoid focusing
  - cold BPMs, steerers at solenoids
- Second Linac Segment
  - λ/2 cavities – 322 MHz
    - »2 types; 2 cryomodule types
  - From ~16.4 MeV/u to ~149 MeV/u (^{238}U)
  - Warm regions between CMs: diagnostics
- Third Linac Segment: β=0.29 to 200 MeV/u



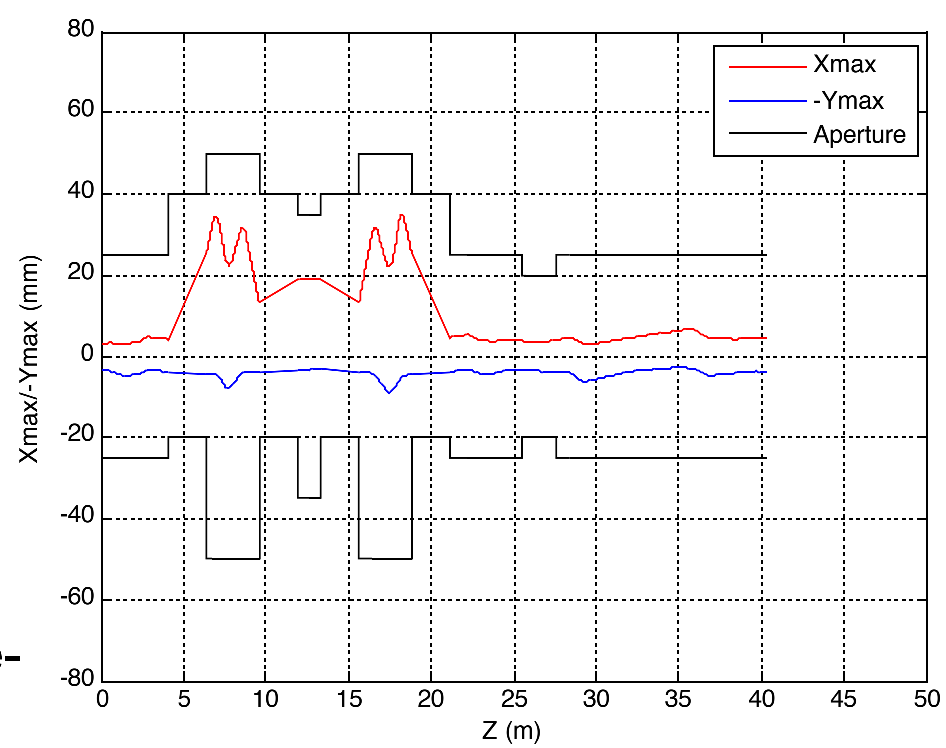
Folding Segments

- Folding Segment 1:
  - Charge selection
  - up to 5 charge states
    - »for U: 76-80+
- 6-D phase space matching to the next Linac Segment
- Frequency change going into next linac -- 322 MHz

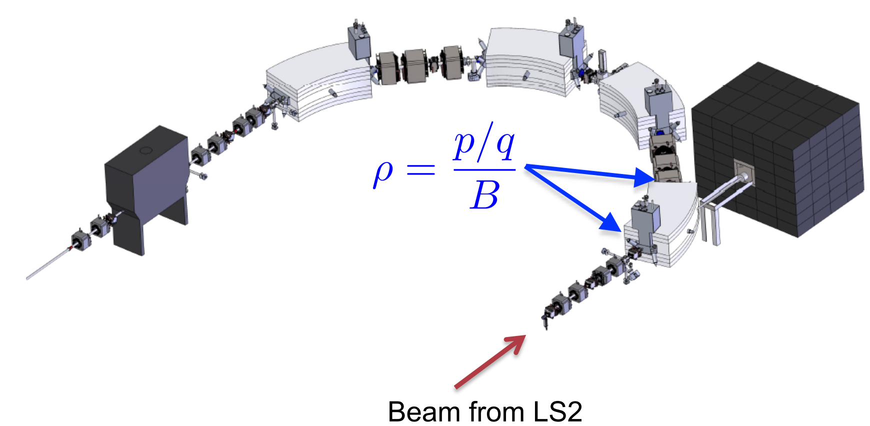




- Folding Segment 2:
  - No charge stripping
  - Challenges:
    - Tunnel width set by bend diameter
    - Minimize bend diameter while retaining beam quality
    - High quality field for multi-charge-state bending



FS2

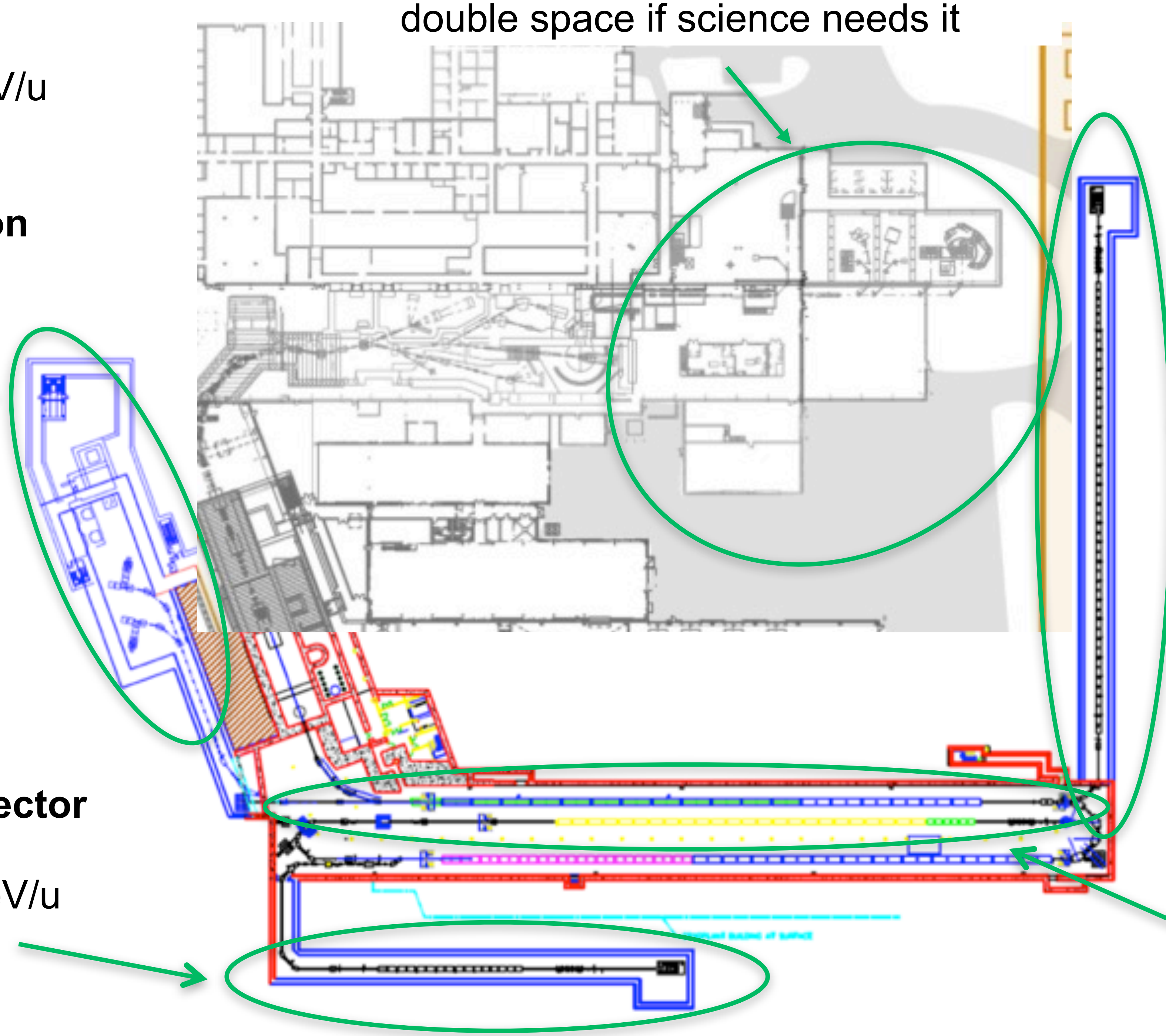


Science-Driven Upgrade Options

Experimental Area
double space if science needs it

ISOL targets
^{3}He, 400 MeV/u
Multiuser capability using light ion injector

Light ion injector upgrade (b)
$^{3}He^{+}$, ~195 MeV/u



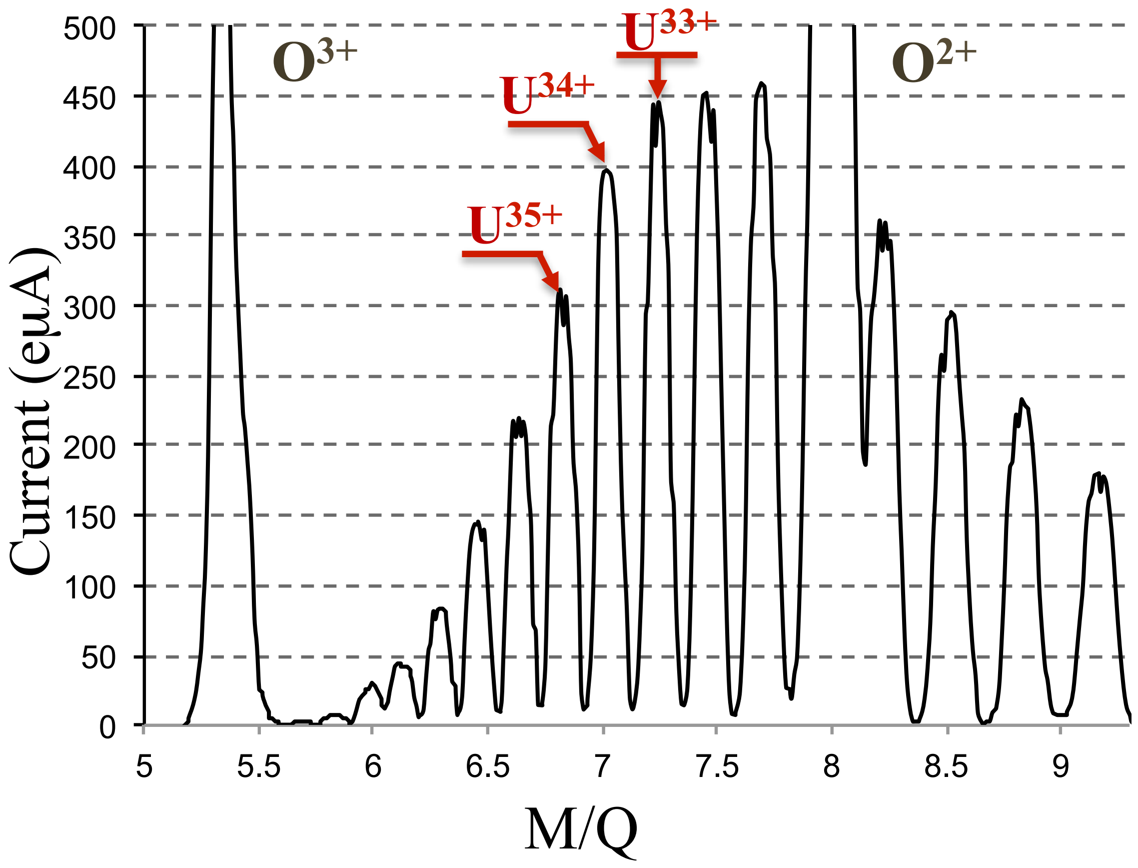
Light ion injector upgrade (a)
$^{3}He^{+}$, ~17 MeV/u

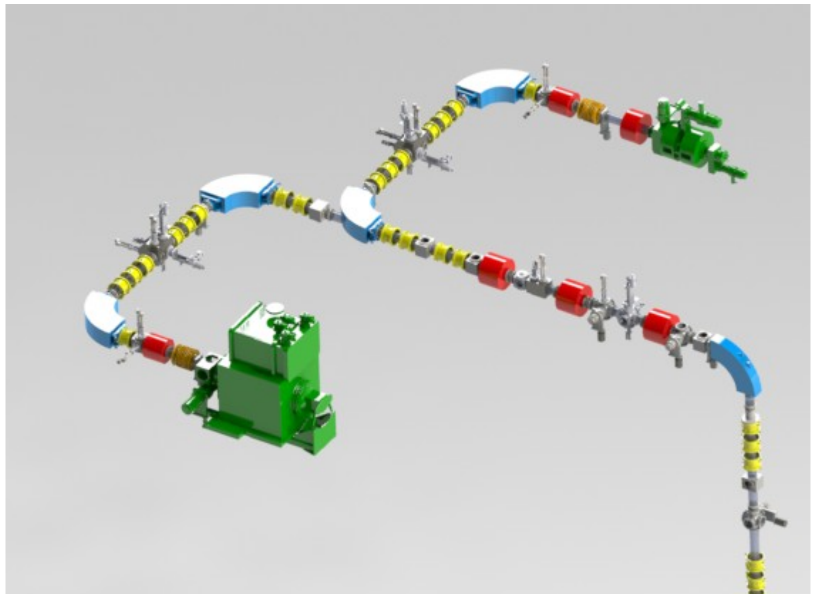
Energy upgrade to
≥ 400 MeV/u for all ions
(high performance λ/2 cryomodules)

Multi-Charge-State Acceleration

^{238}U Intensity

Machicoane, Lyneis, Leitner





- Record-setting intensities from VENUS ion source at LBNL
- VENUS-type source to be used for FRIB; experts now at MSU

- **FRIB Requirement for 400kW**

Q ECR	I ECR (eμA)	I ECR (pμA)
33	216	6.55
34	222	6.55

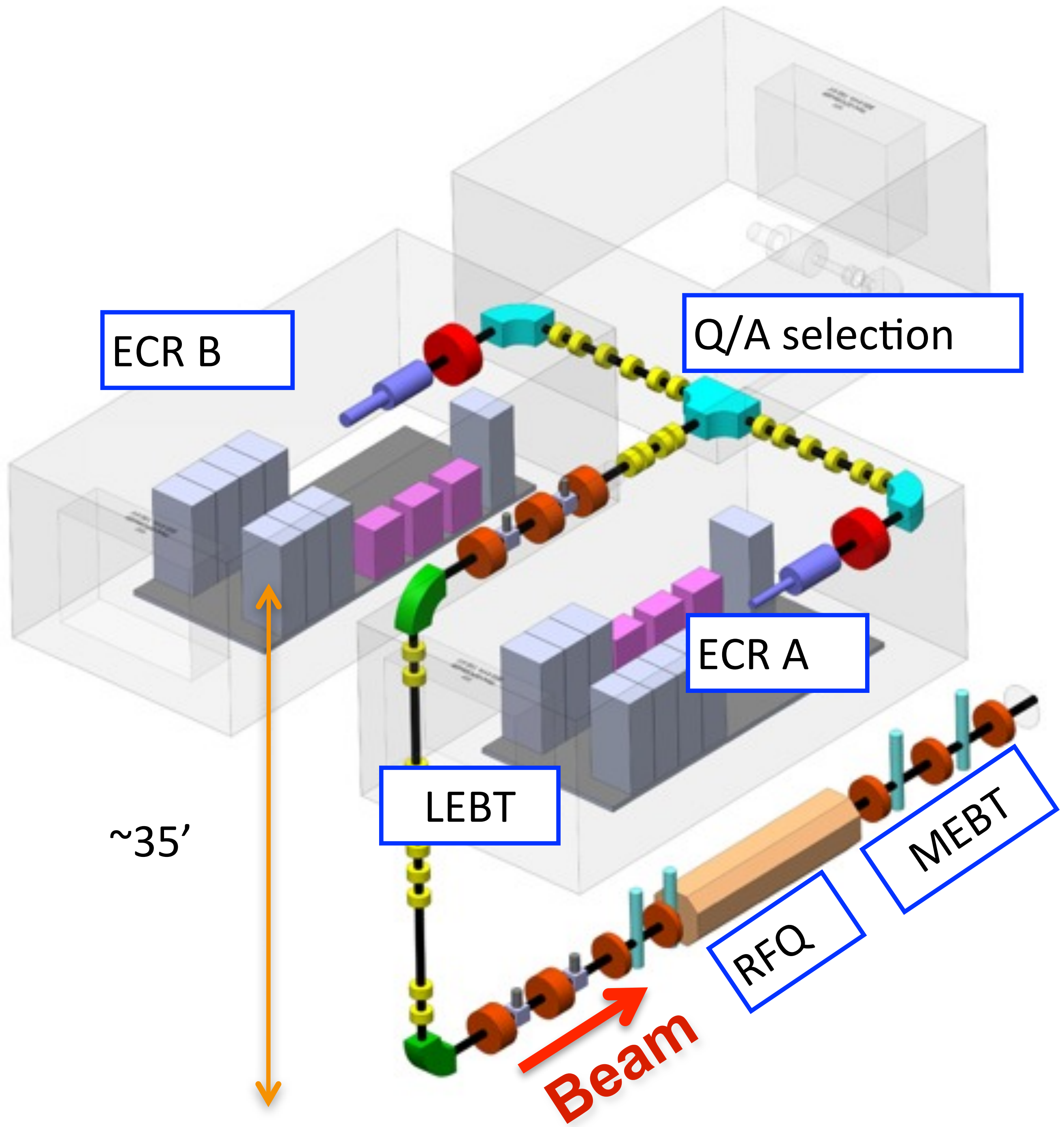
- **Beam Measurements with VENUS**

Q ECR	I ECR (eμA)	I ECR (pμA)
33	443	13.42
34	400	11.76

I_{U33+}	% of beam in 0.6 pi.mm.mrad	% of beam in 0.9 pi.mm.mrad
Horizontal	86	95
Vertical	95	99

Front-End Beam Transport

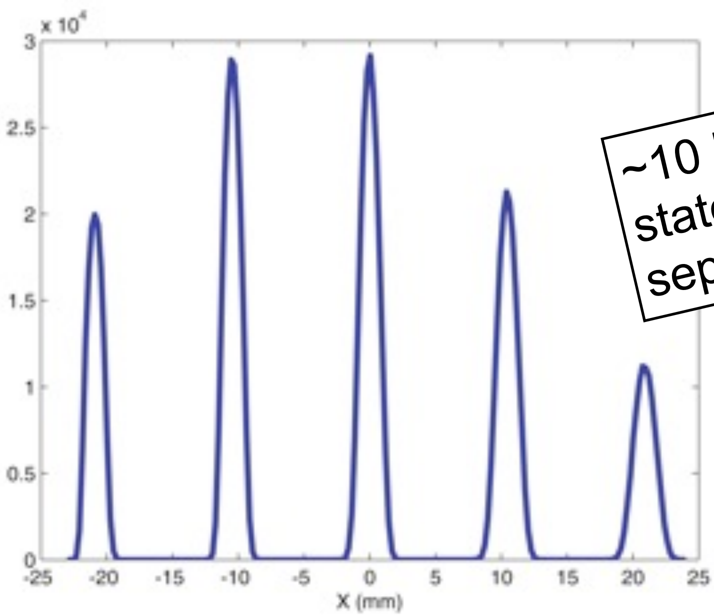
- Heavy ion currents sufficient for 400 kW
- Two charge-states for heavier ions (~>Xe) (*e.g.* 33+ & 34+ for U)
- Multi-charge state beams increase effective longitudinal emittance
- Create, maintain low longitudinal emittance by
  - Bunching in LEBT – external to RFQ
  - MEBT providing 6-D Match into superconducting linac



Higher-Energy Charge Stripping Options

F. Marti, J. Nolen

- Liquid Lithium
  - thickness measurements, stability measurements provide input to simulations and particle tracking



~10 kW per charge state (U -- +76 to +80) separated by ~1 cm

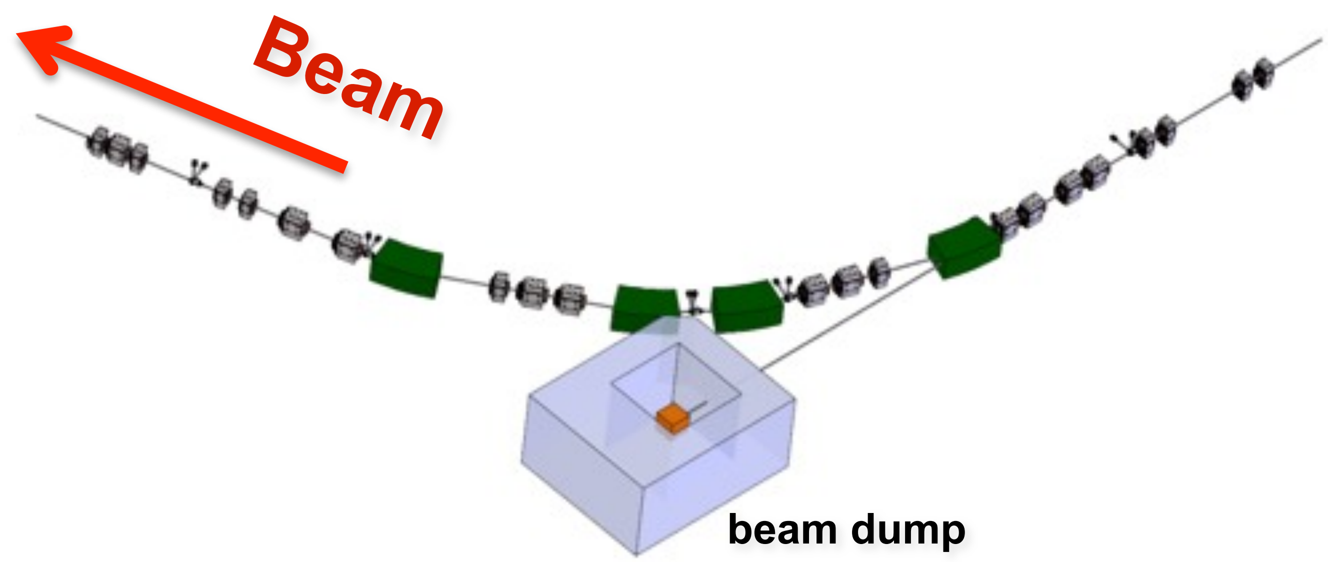
see: Y. Momozaki, et al., JInst 4 (2009) P04005

ANL tests

- Flexible optics design allows for alternative schemes
  - Helium Gas stripping -- larger energy spread, requires rebunching cavities for proper capture into second linac segment
    - » layout supports options -- space for alternative charge stripping, rebunchers

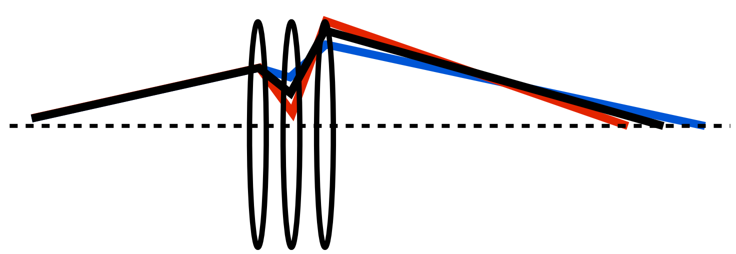
Final Focus and Targeting

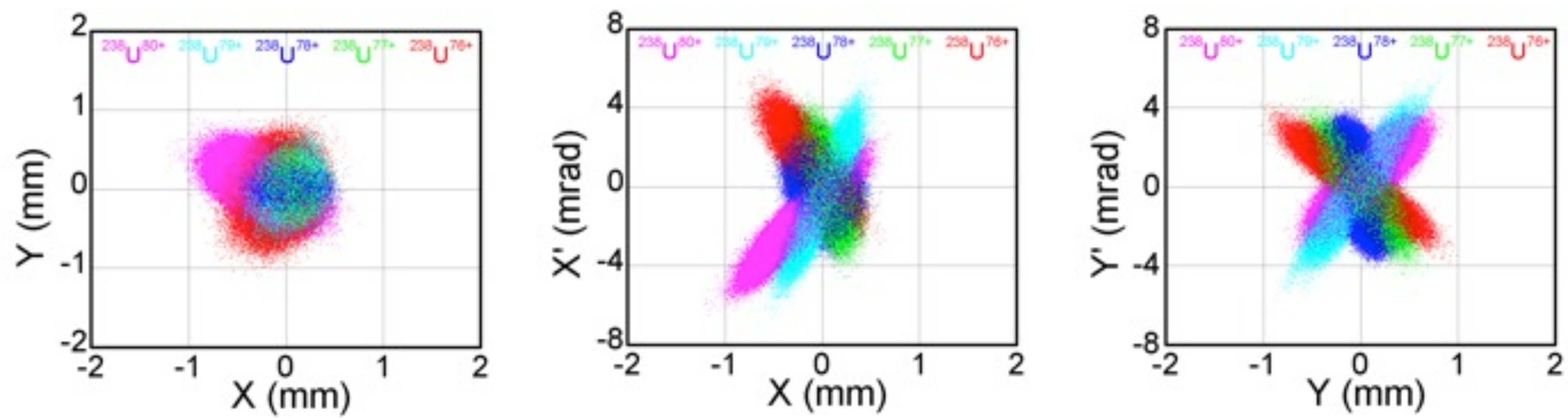
- Doublet optics w/ same period as CMs transport to the final bend
- Achromatic 70^o bend, followed by tuning quads and final focus triplet
- Deliver multi-charge state beams to a single fragmentation target
- Beam size required on fragmentation target ~ 1mm
- Satisfy possible upgrade path
  - Higher beam energy
  - Multiple targets
- Challenge
  - 90% of particles within 1mm spot
  - with large spread in rigidity
    - » $dQ/Q = \pm 2.5/78 = \pm 3.2\%$

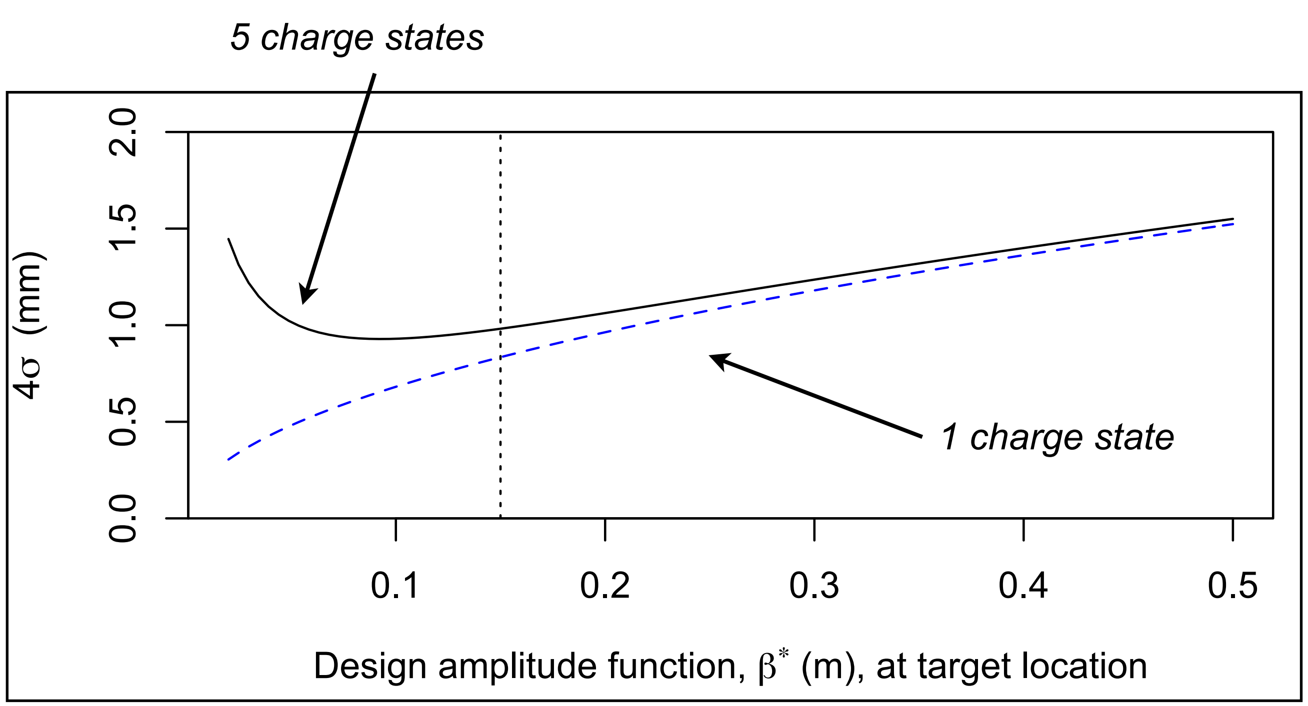


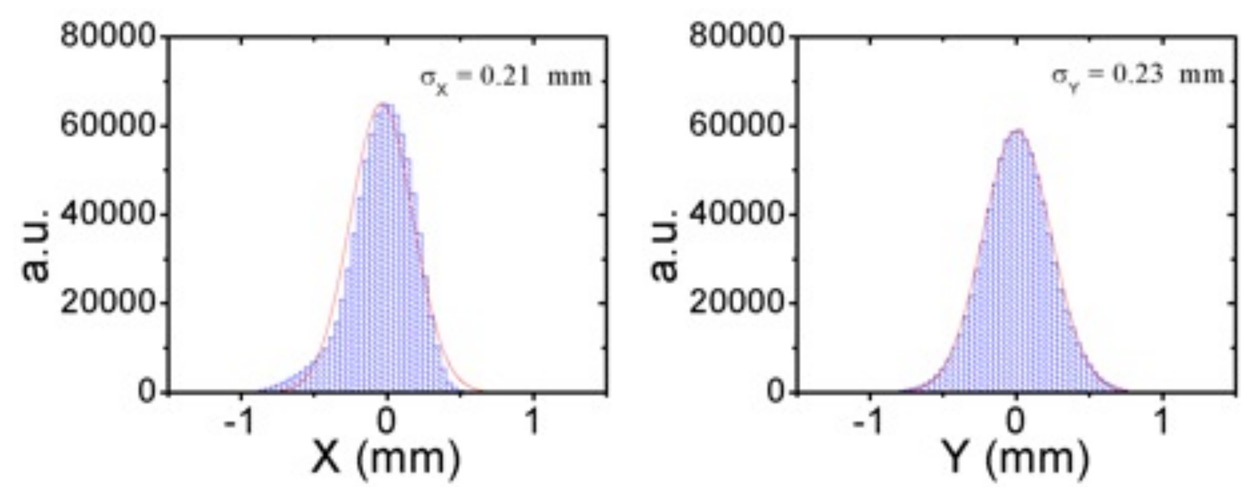
Beam Spot Size on Target
Multi-charge-state Uranium

- Can meet beam spot size requirement on target
- >90% of beam within φ=1 mm ;   rms beam size ~0.22 mm







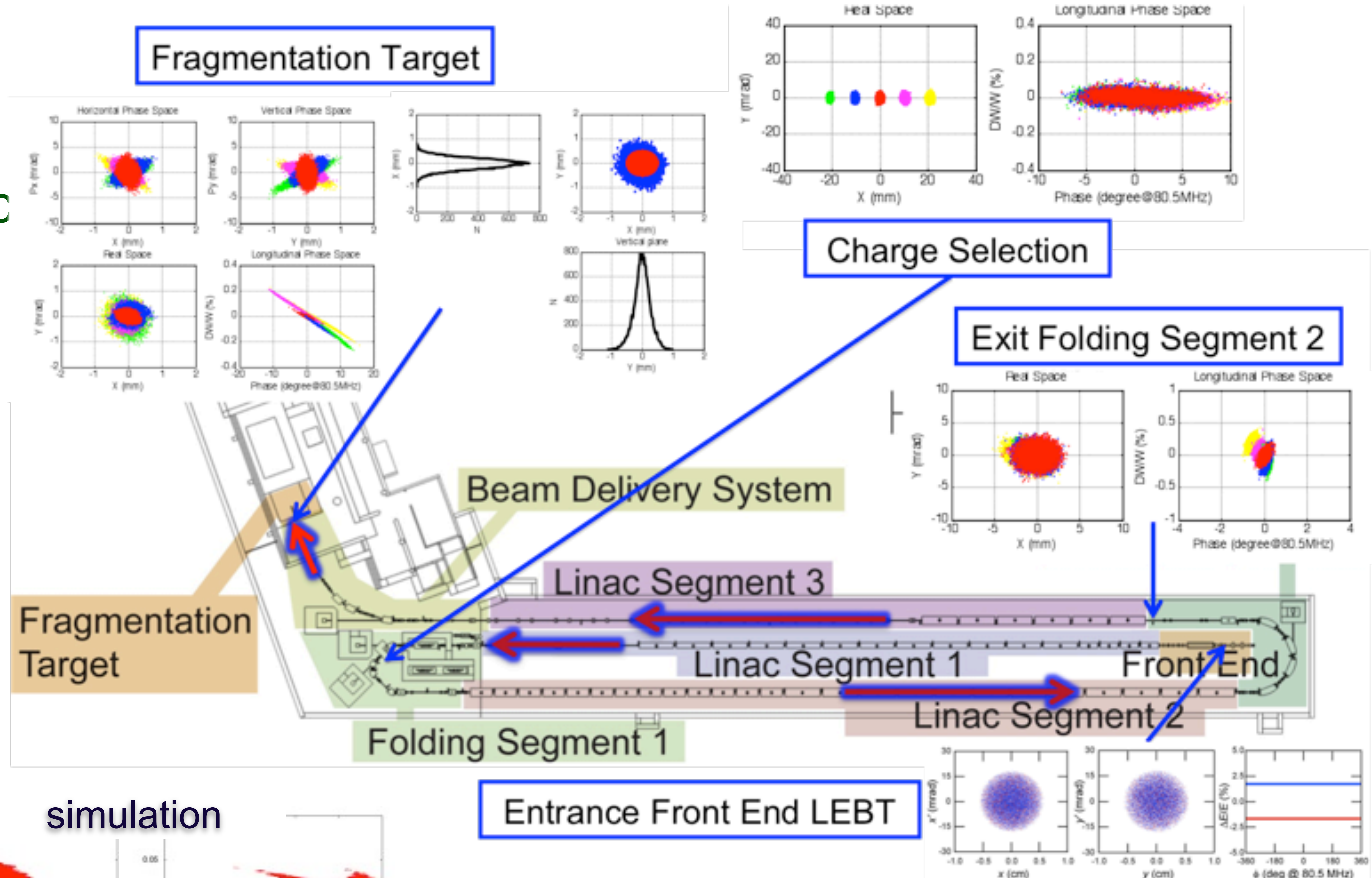


Pozdeyev, Syphers, Wu, Zhao

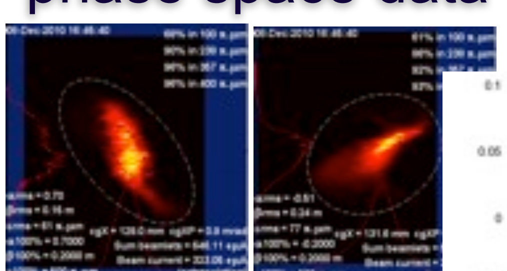
End-to-End Simulations

Q. Zhao

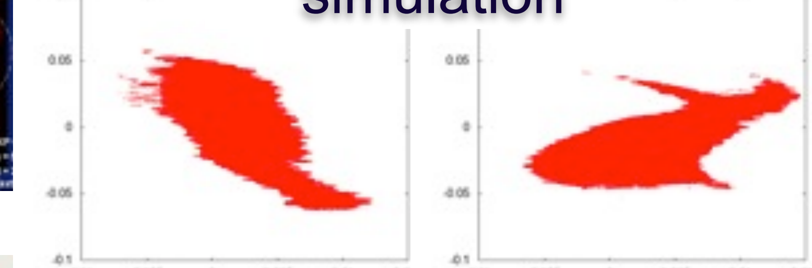
- Realistic beam distributions from ion source
- Include all realistic errors
  - rf phase, ampl. jitter
  - misalignments
  - some runs, use realistic source output:



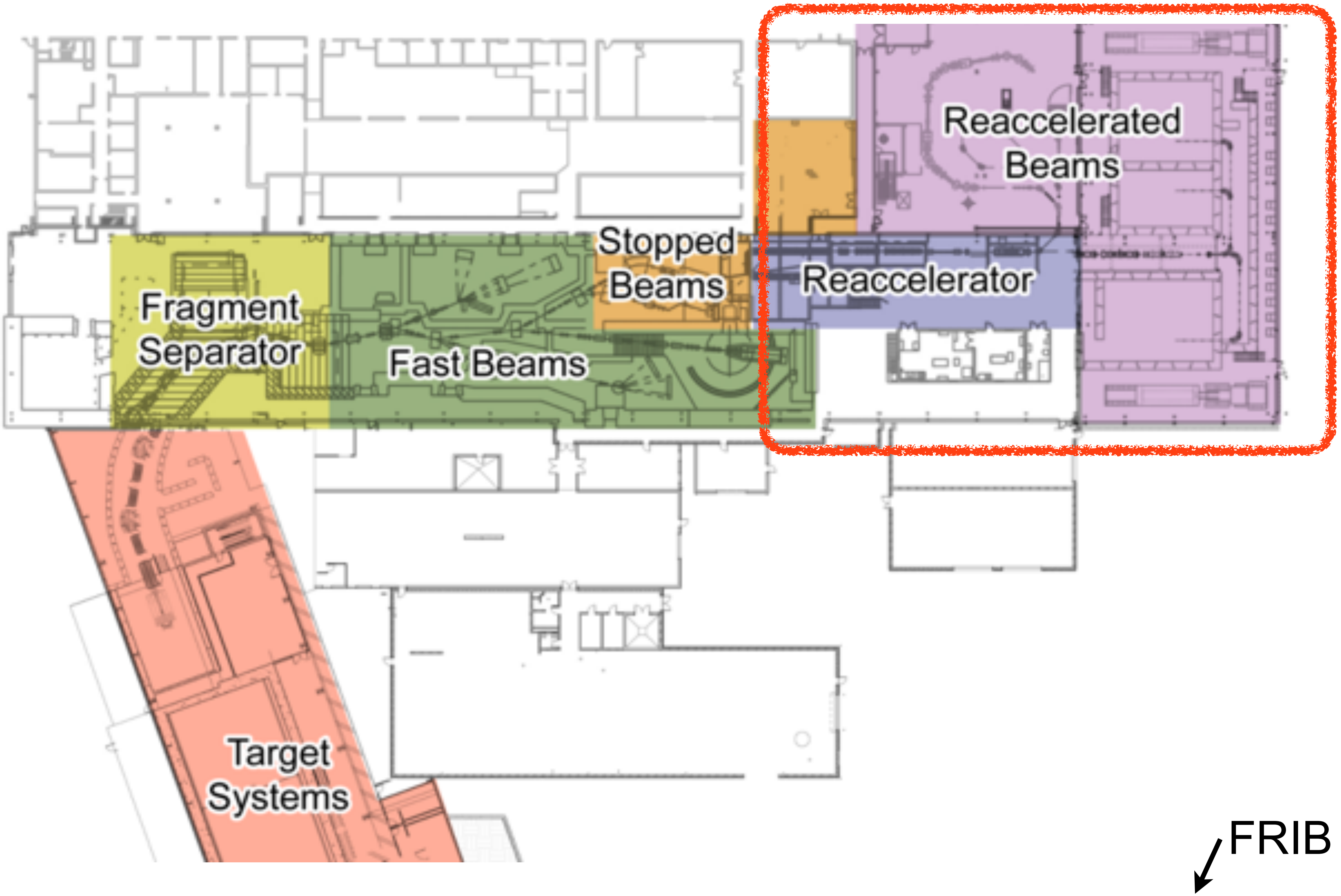
phase space data



simulation

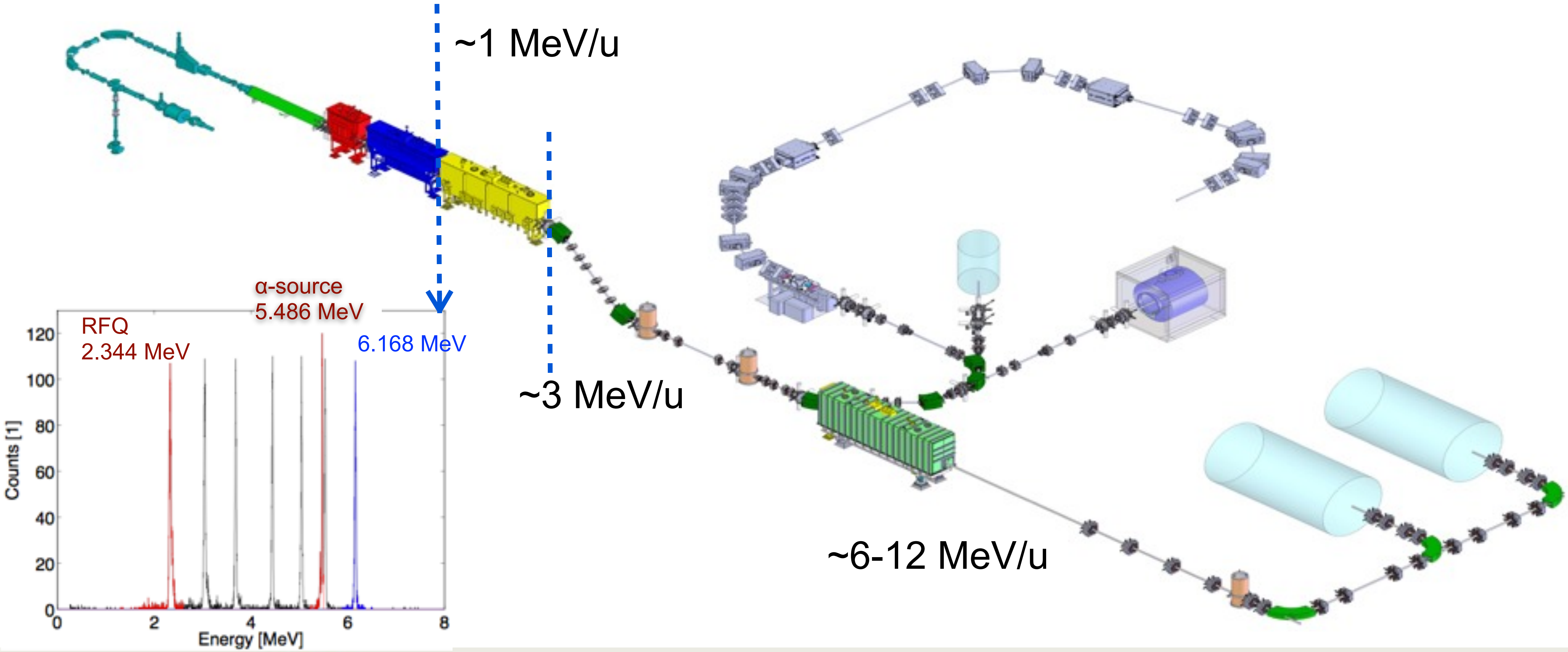


The MSU Re-Accelerator



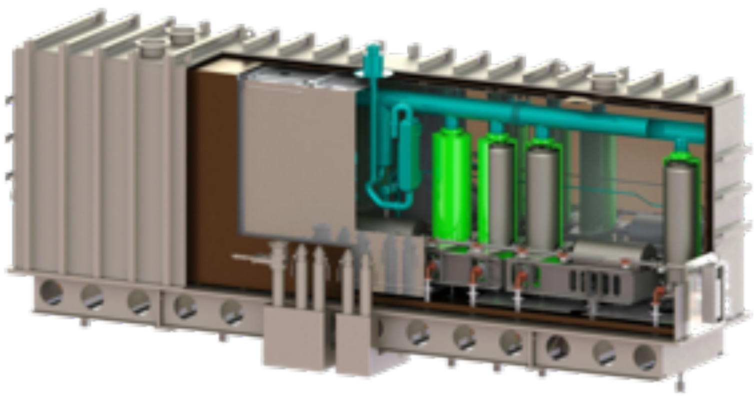
The MSU Re-Accelerator

- Facility for re-acceleration of stopped/trapped beams to variable energies (~0.3 - 3 MeV/u for U, eventually up to 6-12 MeV/u)
- Utilizes FRIB SRF technologies; test bed for the project
- Is being commissioned; beams available to users in 2013

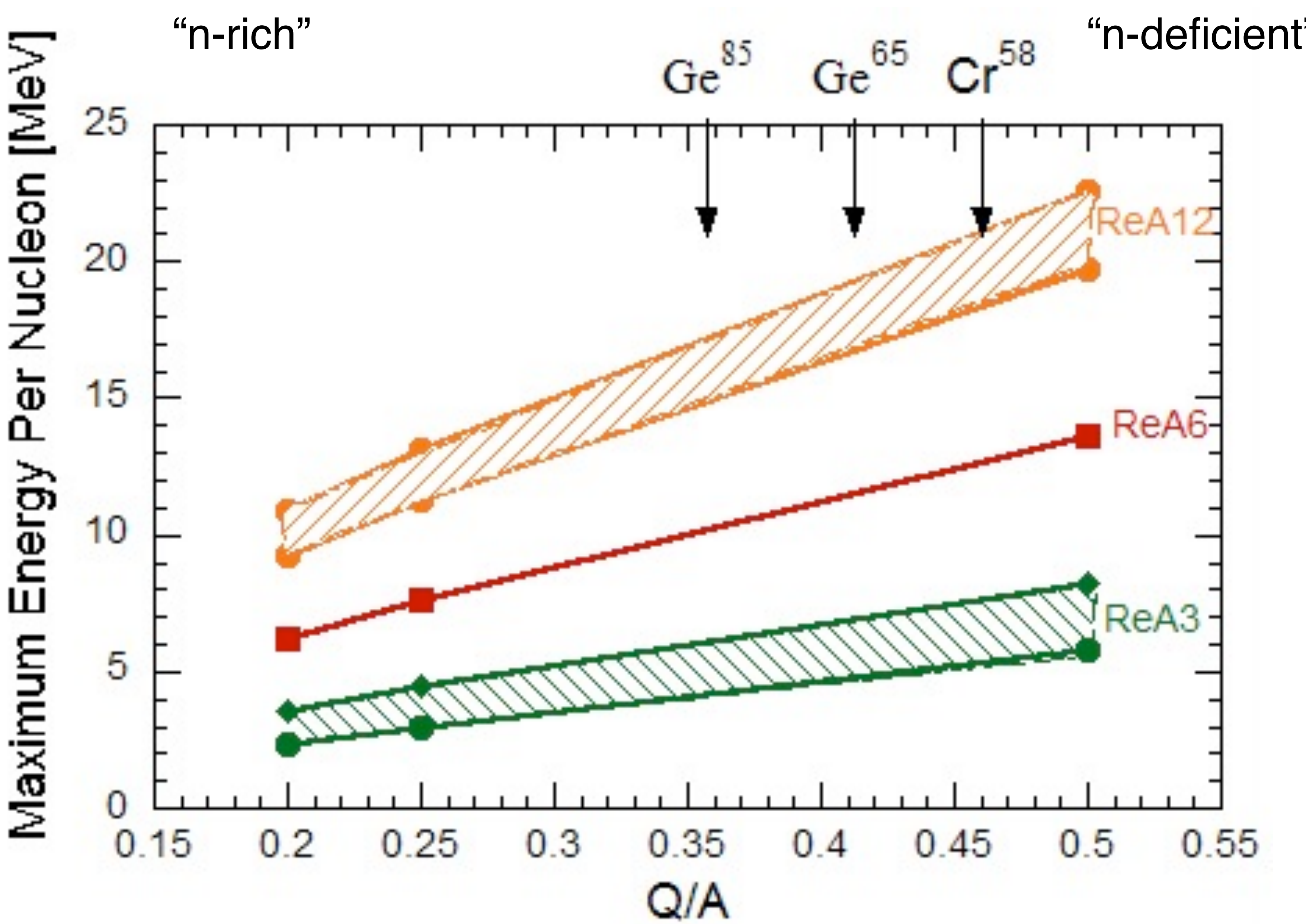


Re-Accelerator Top Energies

- Space for upgrades to “ReA6” and/or “ReA12”

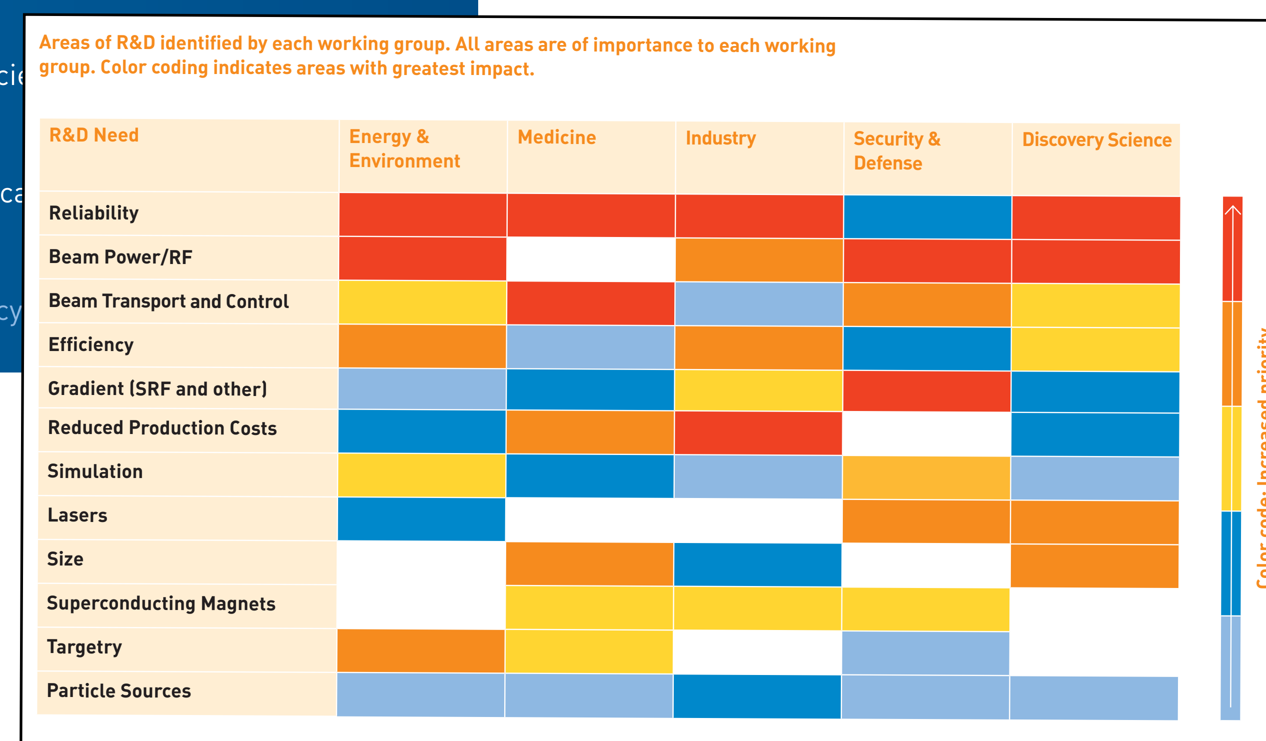


FRIB-style prototype cryomodule(s)



Accelerators for America's Future

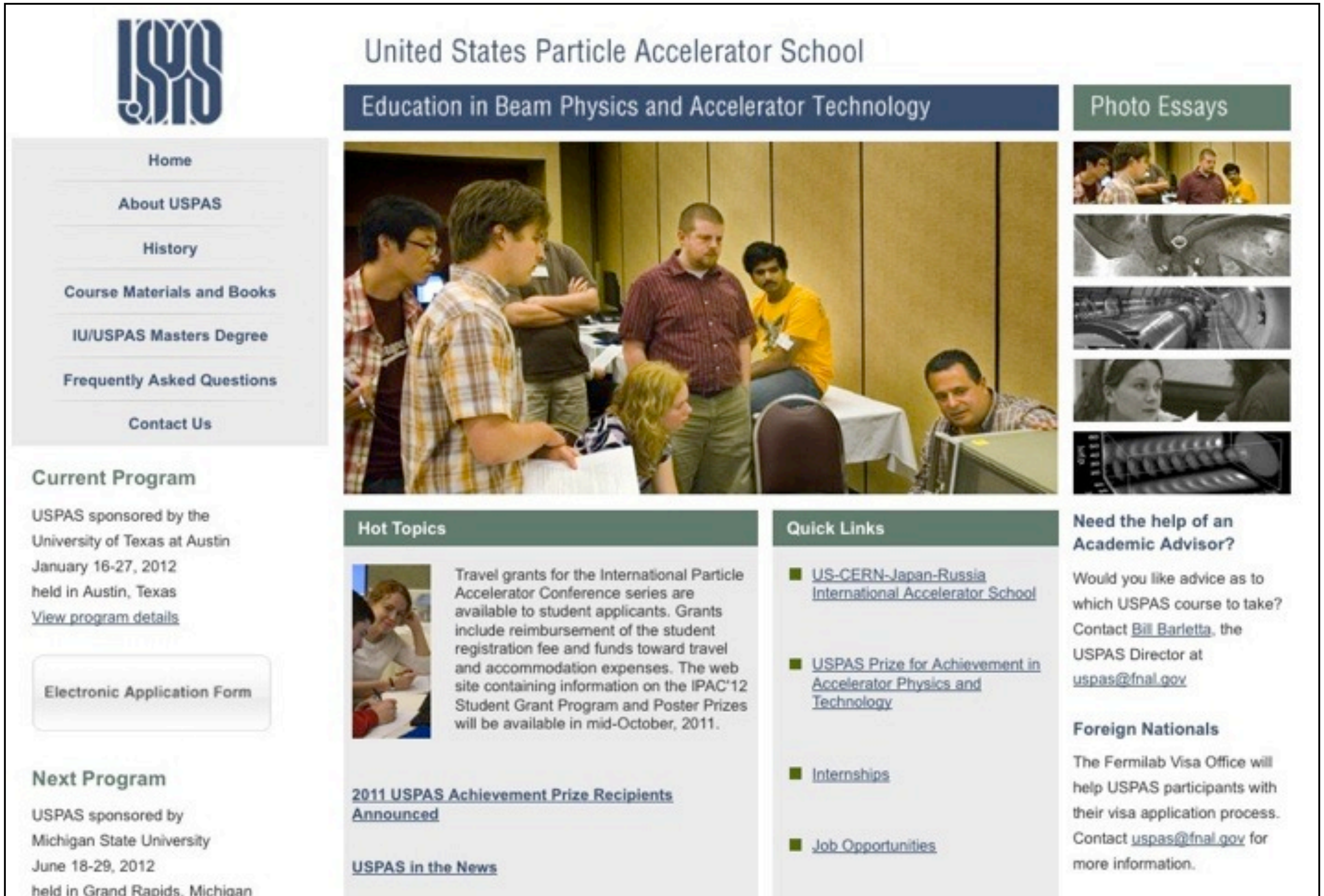
- Symposium and workshop held in Washington, D.C., October 2009
- 100-page Report available at web site



http://www.acceleratorsamerica.org/

US Particle Accelerator School

- Held twice yearly at venues across the country; offers graduate credit at major universities for courses in accelerator physics and technology



http://uspas.fnal.gov

Some Recent Schools:

June 5-16, 2000	SUNY at Stony Brook
January 15-26, 2001	Rice University
June 4-15, 2001	University of Colorado at Boulder
January 14-25, 2002	UCLA
June 10-21, 2002	Yale University
January 6-17, 2003	Indiana University (held in Baton Rouge, LA)
June 16-27, 2003	University of California, Santa Barbara
January 19-30, 2004	The College of William and Mary
June 21 - July 2, 2004	University of Wisconsin - Madison
January 10-21, 2005	University of California, Berkeley
June 20 - July 1, 2005	Cornell University
January 16-27, 2006	Arizona State University
June 12-23, 2006	Boston University
January 15-26, 2007	Texas A&M University
June 4-15, 2007	Michigan State University
January 14-25, 2008	University of California, Santa Cruz
June 16-27, 2008	University of Maryland
January 12-23, 2009	Vanderbilt University
June 15-26, 2009	University of New Mexico
January 18-29, 2010	University of California, Santa Cruz
June 14-25, 2010	MIT
January 17-28, 2011	Old Dominion University
June 13-24, 2011	Stony Brook University

See also, CERN schol:
http://cas.web.cern.ch/cas/

A "Final" word...

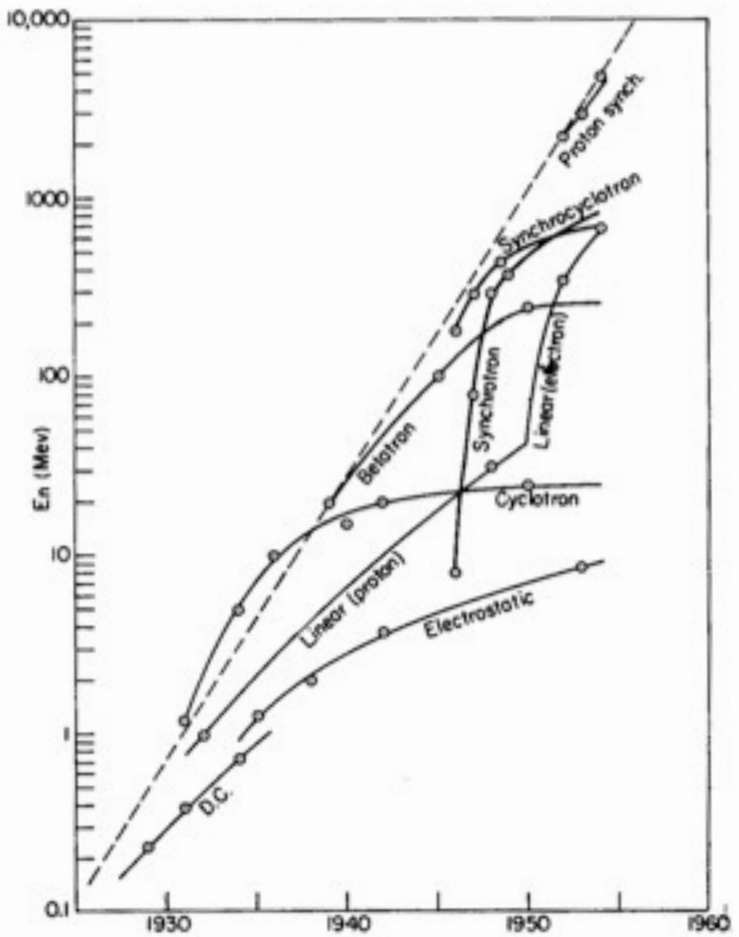


FIG. 7-8. Exponential rise in energy attained with accelerators during the past 25 years.

of the plot is the approximately linear slope of this envelope, which means that energy has in fact increased exponentially with time. The rate of rise is such that the energy has increased by a factor of 10 every six years, from a start at 100 kv in 1929 to 3 billion volts in 1952.

It is interesting to extrapolate this curve into the future, to predict the energy of accelerators after another six years. We have reason to hope that either the Brookhaven or the CERN A-G proton synchrotrons will have reached 25 Bev by that

time. Further extrapolation of this exponentially rising curve would predict truly gigantic accelerators which would exceed any possible budgets, even those of government laboratories. So we will postpone such speculation until the present machines can demonstrate their value to science.

Those of us in the accelerator field are frequently asked, "When will this development of higher-and-higher-energy accelerators stop?" Yet it must be recognized that it is not the urge to higher voltage which inspires this growth, but the pressure of the continuously expanding horizons of science. As long as there are unsolved problems in Nature which might be answered by higher-energy particles, and as long as the scientific urge to know the answers continues, there will be a steady and persistent demand to develop the tools and instruments required.

A “Final” word...

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M. Stanley Livingston, 1954

A-G proton synchrotrons will have reached 25 Bev by that

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THANKS!

- Further reading:

  - D. A. Edwards and M. J. Syphers, *An Introduction to the Physics of High Energy Accelerators*, John Wiley & Sons (1993)
  - T. Wangler, *RF Linear Accelerators*, John Wiley & Sons (1998)
  - H. Padamsee, J. Knobloch, T. Hays, *RF Superconductivity for Accelerators*, John Wiley & Sons (1998)
  - S. Y. Lee, *Accelerator Physics*, World Scientific (1999)
  - and many others…

- Conference Proceedings --
  - Particle Accelerator Conference (2011, 2009, 2007, …)
  - European Particle Accelerator Conference (2010, 2008, 2006, …)

visit http://www.jacow.org