NUCLEI: A TESTING GROUND FOR FUNDAMENTAL CONCEPTS IN PHYSICS

Francesco Iachello

Yale University

Argonne National Laboratory, September 21, 2006

Dedicated to John P. Schiffer
A. DYNAMIC SYMMETRIES AND SUPERSYMMETRIES
B. QUANTUM PHASE TRANSITIONS
The construction of the Interacting Boson Model, upon which the dynamic symmetries and quantum phase transitions discussed here are based, was strongly influenced by a paper of John, to whom I am very grateful.

A. DYNAMIC SYMMETRIES AND SUPERSYMMETRIES

Quantal systems: Spectral properties

The spectroscopic problem:
Given a spectrum, understand its properties:
Classification scheme

Important tool in developing a classification scheme: Dynamic symmetry.
Dynamic symmetries (symmetries of the interactions): Situations in which the Hamiltonian operator of a physical system can be written in terms of invariant (Casimir) operators of a symmetry algebra, called Spectrum generating algebra.
All properties given in terms of quantum numbers.

In the last 80 years, dynamic symmetries and spectrum generating algebras have provided classification schemes for several physical systems, and promise to provide schemes for others.
Large

1µm

Scale 1Å

1fm

Small

Bio-polymers
Polymers

Atomic clusters
Macromolecules

Complexity

Molecules
Atoms

Nuclei
Hadrons

Quarks
Leptons
Dynamic symmetries were implicitly introduced by Pauli (1926) to classify the spectrum of the non-relativistic hydrogen atom.

Pauli argument:
Hamiltonian of a particle in a Coulomb field

\[ H = \frac{p^2}{2m} - \frac{e^2}{r} \]

\( H \) has symmetry higher than rotational invariance.
This symmetry, \( SO(4) \), is generated by

\[ \vec{A}, \vec{L} \]

\( H \) can be rewritten in terms of \( A^2 + L^2 \) (Casimir operator)

Hence, the energy eigenvalues are given in closed form

Classification scheme

\( SO(4) \supset SO(3) \supset SO(2) \)

\[ n = \omega + 1, \omega = 0, 1, \ldots, \infty \]

\[ \ell = \omega, \omega - 1, \ldots, 1, 0 \]
Dynamic symmetries attained prominence in physics with the introduction of flavor symmetry $SU_f(3)$ to classify hadron spectra (Gell-Mann, 1962; Ne’eman, 1962).

Gell-Mann argument:
Generalize Heisenberg isospin to

$$SU_f(3) \supset SU_I(2) \otimes U_Y(1) \supset SO_I(2) \otimes U_Y(1)$$

Write down the mass operator in terms of Casimir operators

$$M = M_0 + aC_1(U_Y(1)) + b\left[C_2(SU_I(2)) - \frac{1}{4}(C_I(U_Y(1)))^2\right]$$

Hence, the energy eigenvalues can be given in closed form

$$M(I, I_z, Y) = M_0 + aY + b\left[I(I + 1) - \frac{1}{4}Y^2\right]$$
Dynamic symmetries and spectrum generating algebras are particularly useful for complex systems, where a direct calculation of spectral properties is very difficult or not possible at all. For example, the configuration valence space in $^{154}\text{Sm}$ has dimension $346,132,052,934,889$ for $J=2$. And even if we were able to do the calculation, what is the meaning of a wave function with $10^{14}$ components?

Two areas of physics where they have been very useful are nuclear physics and molecular physics.

In nuclear physics, dynamic symmetries have been used to classify both single-particle motion (Elliott, 1958) and collective motion (Iachello, 1974; Arima and Iachello, 1975).

In molecular physics, they have been used to classify both rotational-vibrational motion (Iachello, 1980; Iachello and Levine, 1981) and electronic motion (Frank, Lemus and Iachello, 1986).

Only dynamic symmetries of the collective motion in nuclei will be discussed here.


Dynamic symmetries in nuclei

Dynamic symmetries in even-even nuclei have been extensively investigated within the framework of the **Interacting Boson Model** with spectrum generating algebra $U(6)$

This is a model in terms of correlated s- and d-pairs of nucleons treated as bosons

- **S** ($J=0$) pairing $\Rightarrow$ s-boson
- **D** ($J=2$) pairing $\Rightarrow$ d-boson
Dynamic symmetries of the Interacting Boson Model

\[ U(5) \supseteq SO(5) \supseteq SO(3) \supseteq SO(2) \] (I) Spherical

\[ U(6) \longrightarrow SU(3) \supseteq SO(3) \supseteq SO(2) \] (II) Deformed with axial symmetry

\[ SO(6) \supseteq SO(5) \supseteq SO(3) \supseteq SO(2) \] (III) Deformed gamma-unstable

Energy formulas

\[ E^{(I)}(N, n_d, \nu, n_\Delta, L, M) = E_0 + \varepsilon n_d + \alpha n_d (n_d + 4) + \beta \nu (\nu + 3) + \gamma L(L + 1) \]

\[ E^{(II)}(N, \lambda, \mu, K, L, M) = E_0 + \kappa \left( \lambda^2 + \mu^2 + \lambda \mu + 3 \lambda + 3 \mu \right) + \kappa' L(L + 1) \]

\[ E^{(III)}(N, \sigma, \tau, n_\Delta, L, M) = E_0 + A \sigma (\sigma + 4) + B \tau (\tau + 3) + C L(L + 1) \]
All three types of symmetries have been found:

\[ ^{110}\text{Cd} \quad N = 7 \]
Symmetry I: U(5)

\[ ^{156}\text{Gd} \quad N = 12 \]
Symmetry II: SU(3)

\[ ^{196}\text{Pt} \quad N = \bar{6} \]
Symmetry III: SO(6)
An important question is the energy scale at which the dynamic symmetry is broken and regularities in spectra disappear. It has been found that the regularity extends to much higher excitation energies than originally thought, both for rotational and for vibrational motion.

\[ ^{112}\text{Cd} \quad N=8 \]

Symmetry I: U(5)

In odd-even nuclei at least one particle is unpaired and in even-even nuclei, at some excitation energy $\sim 2\Delta$, pairs break, and the system becomes a mixed system of bosons and fermions.

To describe these situations new symmetry concepts have been introduced in physics, called Bose-Fermi symmetry and supersymmetry. The concepts were introduced initially for applications to particle physics (Volkov and Akulov, 1974; Wess and Zumino, 1974). Dynamic supersymmetries and spectrum generating superalgebras were introduced in nuclear physics in 1980 (Iachello, 1980; Balantekin, Bars and Iachello, 1981). They provide classification schemes for odd-even nuclei (Bose-Fermi symmetries) and for the combined set of even-even, even-odd, odd-even nuclei and odd-odd nuclei (supersymmetries).
Dynamic supersymmetries in nuclei have been investigated within the framework of the Interacting Boson-Fermion Model with spectrum generating superalgebra $U(6/\Omega)$

\[ \Omega = \sum_i (2j_i + 1) \]

A model in terms of correlated pairs treated as bosons and unpaired fermions
Dynamic supersymmetries of the Interacting Boson-Fermion Model

Several classes can occur, depending on the symmetry of the bosons and their associated fermionic partners. Two classes have been extensively investigated: (i) $j=3/2$, $U(6/4)$ and (ii) $j=1/2,3/2,5/2$, $U(6/12)$. Some examples have been found.

$^{190}$Os-$^{191}$Ir

Supersymmetry III$_1$: $U(6/4)$

$N_B = \overline{9}, \overline{8}$
$N_F = 0,1$
$N = N_B + N_F = 9$
The Interacting Boson Model, the Interacting Boson-Fermion Model, the Interacting Boson Fermion-Fermion Model provide a classification scheme for Even-Even, Even-Odd and Odd-Odd Nuclei, of which the dynamic symmetries and supersymmetries are the cornerstones. Together with the Shell Model, they provide a comprehensive classification scheme for all nuclei.
Symmetry classification of nuclei

A global parametrization has been given recently by McCutchan, Zamfir and Casten, 2004, by making use of a simple three-parameter Hamiltonian (Casten and Warner, 1983)

\[
H = \varepsilon \hat{n}_d + \kappa \hat{Q}^x \cdot \hat{Q}^x
\]

\[
\hat{Q}^x = (s^\dagger \tilde{d} + d^\dagger s) + \chi (d^\dagger \tilde{d})^{(2)}
\]

Overall agreement with experiment 5% or better
Protons and Neutrons: Dynamic F-spin symmetries

In a more elaborate description one distinguishes between proton and neutron pairs, and introduces a two-valued variable, called F-spin. (Arima, Otsuka, Iachello and Talmi, 1977)

Framework for even-even nuclei: Proton-neutron Interacting Boson Model $U_{\pi}(6) \otimes U_{\nu}(6)$

In this model new types of collective modes appear, for example, angular oscillations of proton versus neutron deformations, the so-called scissor modes.

Also, new dynamic symmetries appear

$I)$ Spherical

$I)$ Axially deformed

$III)$ Gamma-unstable deformed

$IV)$ Triaxially deformed
Framework for even-even, even-odd, odd-even and odd-odd nuclei: The Proton-Neutron Interacting Boson-Fermion Model with superalgebra $U_\pi(6/\Omega) \otimes U_\nu(6/\Omega)$

Dynamic supersymmetries of the Interacting Boson-Fermion Model-2

An example has been recently discovered $U_\pi(6/4) \otimes U_\nu(6/12)$

The most complex example of symmetry found so far! (5 bosons+2 fermions)
B. QUANTUM PHASE TRANSITIONS

Thermodynamic phase transitions were discussed in the 19th Century. A general theory was given by Landau in the 1930’s.

Quantum phase transitions are the same as thermodynamic phase transitions but with control parameter equal to a coupling constant, $g$, rather than the temperature, $T$.

$$H = (1 - g)H_1 + gH_2$$

Associated with phase transitions there are order parameters. For quantum phase transitions, the order parameter is the expectation value of some suitably chosen operator, $\langle O \rangle$. Quantum phase transitions occur at zero-temperature and hence are often called ground state phase transitions.

An early example is the Ising (1925) model in a transverse field

Hamiltonian

$$H = -V \left( g \sum_i S_i^x - \sum_{\langle ij \rangle} S_i^z S_j^z \right)$$

Order parameter: magnetization

$$\langle M \rangle$$

This model has a second order quantum phase transition at $g = g_c = \frac{1}{2}$
Quantum phase transitions in nuclei

Several types of quantum phase transitions can occur in nuclei, some at or around nuclear matter density and some at much larger densities.

Phase diagram of nuclear matter in the $\rho$-T plane

- Critical point
- Quark-gluon plasma
- Hadronic gas
- Color superconductor
- Liquid
- Gas
Only phase transitions at nuclear matter density, shape phase transitions, will be discussed here.
A general theory was developed by Gilmore, 1978, and applied to nuclei by Dieperink, Scholten and Iachello, 1980 and Feng, Gilmore and Deans, 1982

Phase diagram of nuclei in the Interacting Boson Model

\[ H = H_1 + g_2 H_2 + g_3 H_3 \]

\[ \uparrow \quad \uparrow \quad \uparrow \]

U(5)  SU(3)  SO(6)
Examples of both second and first order transitions in nuclei have been found

2nd order transitions Ba-Xe U(5)-SO(6)
1st order transitions Sm-Gd U(5)-SU(3)
Critical symmetry

Dynamic symmetries provide spectral signatures for the analysis of experimental data for pure phases, \( g = 0 \) or \( 1 \).
Are there spectral signatures of critical behavior?
An intriguing (and surprising) recent results: Contrary to expectations, the structure of physical systems at the critical point of a second order transition and along the critical line of a first order transition is simple.

It has been suggested that the simple features arise from special properties of the potential at the critical point [in Landau theory, for 2nd order transitions, \( V(\beta) = \beta^4 \)]

An approximation to a flat potential is a square-well.

\[ V(\beta) \]

\[ \beta \]
A new concept has been introduced, called critical symmetry, related to scale invariance in non-relativistic systems. (Conformal invariance in quantum field theory) (2nd order transitions, Iachello, 2000; 1st order transitions, Iachello, 2001).

Critical symmetries are best studied within the framework of the Geometric Collective Model (Bohr, 1952; Bohr and Mottelson, 1953), in particular by analyzing solutions of the Bohr Hamiltonian.

**Critical symmetries in the Geometric Collective Model**

L. Wilets and M. Jean, Phys. Rev. 102, 788 (1956)
In critical symmetries, the energy eigenvalues are given in terms of zeros of Bessel functions (a novel class of dynamic symmetries) and the symmetry applies to a bounded domain.

\[ E_\nu = A \left( x_{s,\nu} \right)^2 \]  
\[ x_{s,\nu} \text{ s-th zero of } J_\nu(z) \quad \nu = \tau + \frac{3}{2} \]

[X(5)] Square-well in \( \beta \) and \( \gamma \)

\[ E(s, L, s', K, M_L) = A \left( x_{s,L} \right)^2 + B \left( x_{s',K} \right)^2 \]
\[ x_{s,L} \text{ s-th zero of } J_\nu(z) \quad \nu = \left( \frac{L(L+1)}{4} + \frac{9}{4} \right)^{1/2} \]
\[ x_{s',K} \text{ s’-th zero of } J_{\nu'}(z) \quad \nu' = \frac{K}{2} \]


Examples have been found


Also here a question is the energy at which simple analytic descriptions break down. It appears to be much higher than originally thought. An example is $^{156}\text{Dy}$, which sits close to the critical value of the 1$^\text{st}$ order U(5)-SU(3) transition.

The collective motion appears to be very stable, no matter whether vibrational, rotational or critical!

\[
\frac{E(J)}{E(2)}
\]

<table>
<thead>
<tr>
<th>J</th>
<th>Exp</th>
<th>[X(5)]</th>
<th>Rotor</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>45.91</td>
<td>49.53</td>
<td>100.00</td>
</tr>
<tr>
<td>22</td>
<td>40.43</td>
<td>42.85</td>
<td>84.33</td>
</tr>
<tr>
<td>20</td>
<td>35.25</td>
<td>36.61</td>
<td>70.00</td>
</tr>
<tr>
<td>18</td>
<td>30.31</td>
<td>30.80</td>
<td>57.00</td>
</tr>
<tr>
<td>16</td>
<td>25.56</td>
<td>25.43</td>
<td>45.00</td>
</tr>
<tr>
<td>14</td>
<td>20.35</td>
<td>20.51</td>
<td>35.00</td>
</tr>
<tr>
<td>12</td>
<td>16.58</td>
<td>16.04</td>
<td>26.00</td>
</tr>
<tr>
<td>10</td>
<td>12.51</td>
<td>12.03</td>
<td>18.33</td>
</tr>
<tr>
<td>8</td>
<td>8.82</td>
<td>8.48</td>
<td>12.0</td>
</tr>
<tr>
<td>6</td>
<td>5.59</td>
<td>5.43</td>
<td>7.0</td>
</tr>
<tr>
<td>4</td>
<td>2.93</td>
<td>2.90</td>
<td>3.3</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
Very recently, the concept of critical symmetry has been extended to critical Bose-Fermi symmetry and supersymmetry (Iachello, 2005). This extension allows one to study odd-even nuclei at the critical point of a phase transition.

E(5/4)

Search for critical Bose-Fermi symmetries has already started ($^{135}$Ba)

The future

Several facilities are being built which will provide a vast pool of new nuclei: REX-ISOLDE@CERN, ISAC2@TRIUMF, RIB@RIKEN, SPIRAL2@GANIL, FAIR@GSI

1 Light neutron-rich nuclei
2 Heavy neutron-rich nuclei
3 Heavy proton-rich nuclei
4 Super-heavy nuclei
Questions that can be answered with the new data:
Is the classification scheme provided so far sufficient to describe all data or do we need to extend it?
If the extension is not needed, are there other examples of the known dynamic symmetries, supersymmetries and critical symmetries?
If the extension is needed what are its symmetries?

A particularly interesting region to study the symmetries discussed here is 2
Conclusions

Despite their complexity, nuclei display simple properties. The concepts of dynamic symmetry and dynamic supersymmetry provide an important tool to study simplicity in complex bosonic systems, fermionic systems and their mixtures. They give benchmarks for comparison with experiments. The concepts of spectrum generating algebra and superalgebra provide maps for navigating into unknown regions. A classification scheme (map) for the collective motion in nuclei has been given.

Dedicated to Eugene Wigner and Giulio Racah who pioneered the symmetry approach

With best wishes to John Schiffer for many more productive years of research! and many thanks for his support of the symmetry approach to physics.
Addendum 1

A separate (theoretical) question is:
Can we derive the simplicity observed in nuclei from \textit{ab initio} calculations?
The logic scheme for this derivation is in place

\begin{itemize}
\item Quarks and gluons (QCD)
\item Free nucleon-nucleon interaction
\item Nucleon-nucleon interaction in the medium
\item Effective interaction in the shell model
\item Interaction between correlated pairs (bosonization)
\end{itemize}

How well can be implement this scheme?
Derivation of the Interacting Boson Model from the Shell Model

T. Otsuka, A. Arima and F. Iachello,
Addendum 2

Classification scheme of nuclei in the Interacting Boson Model-2