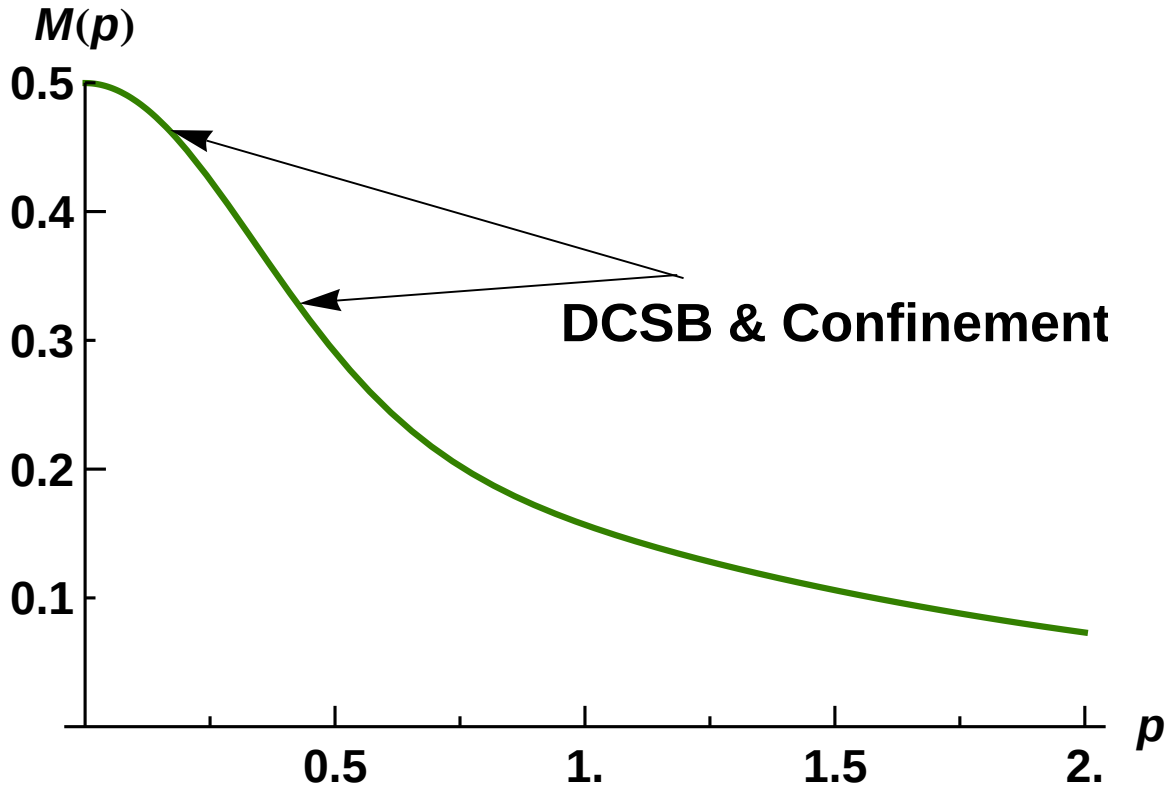
Impact of DCSB on Meson Structure and Interactions

Dressed–quark Mass Function



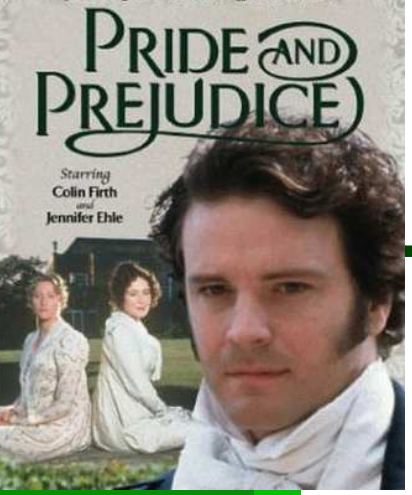
Craig D. Roberts
cdroberts@anl.gov

Physics Division & **School of Physics**

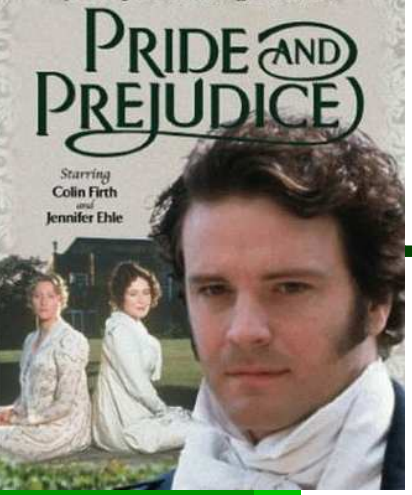
Argonne National Laboratory **Peking University**

http://www.phy.anl.gov/theory/staff/cdr.html

Universal Truths

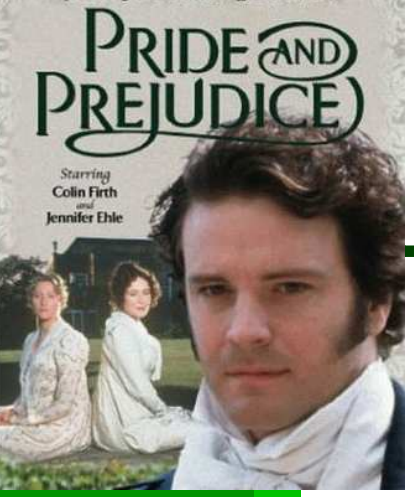


Universal Truths



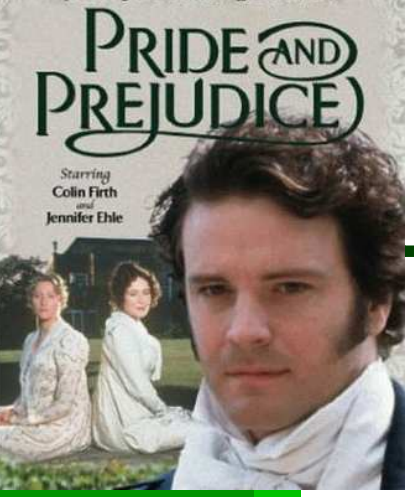
Universal Truths

- Spectrum of excited states, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.



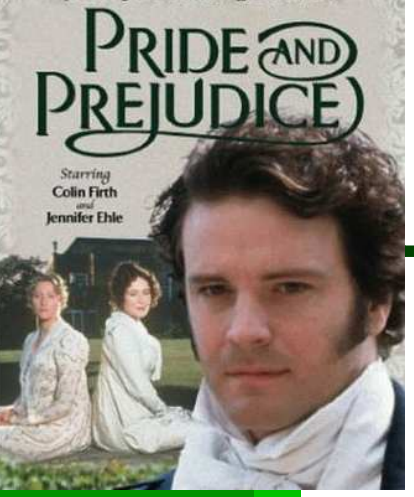
Universal Truths

- Spectrum of excited states, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.



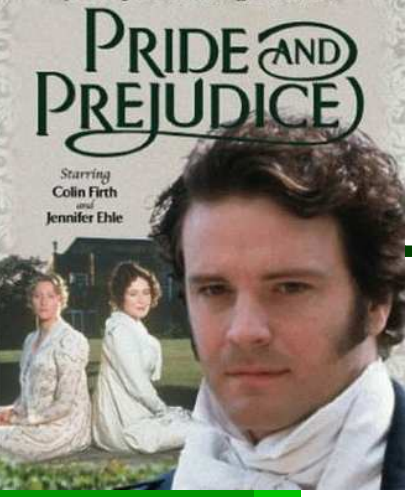
Universal Truths

- Spectrum of excited states, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe. Higgs mechanism is irrelevant to light-quarks.



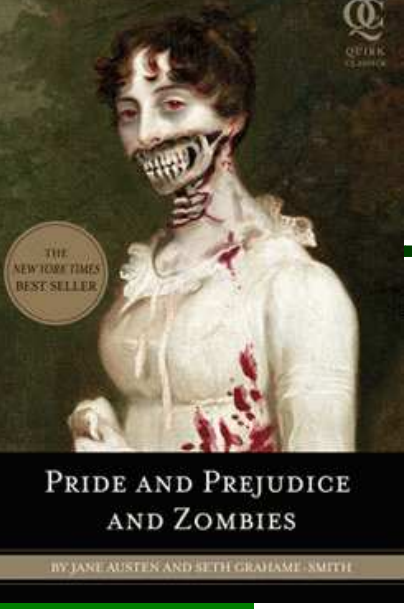
Universal Truths

- Spectrum of excited states, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe. Higgs mechanism is irrelevant to light-quarks.
- Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach.



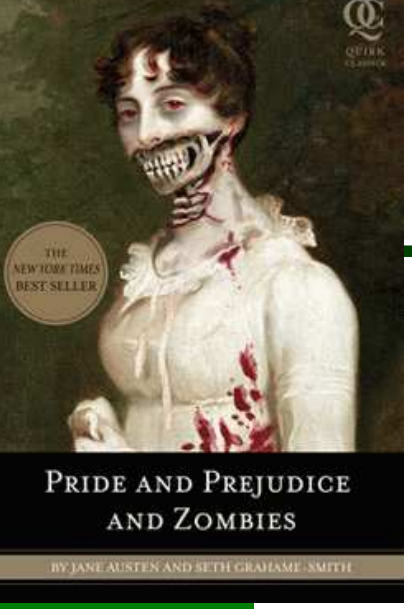
Universal Truths

- Spectrum of excited states, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe. Higgs mechanism is irrelevant to light-quarks.
- Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.



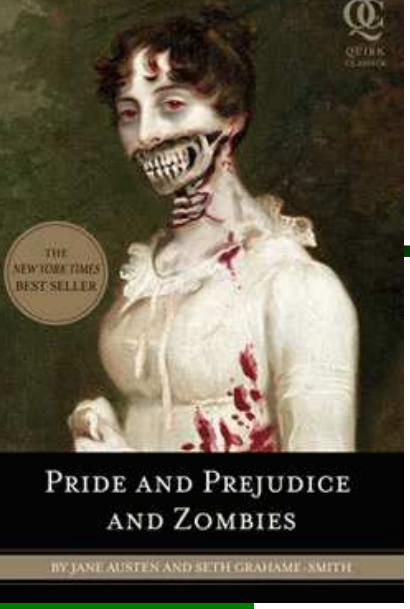
Universal Truths

- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.



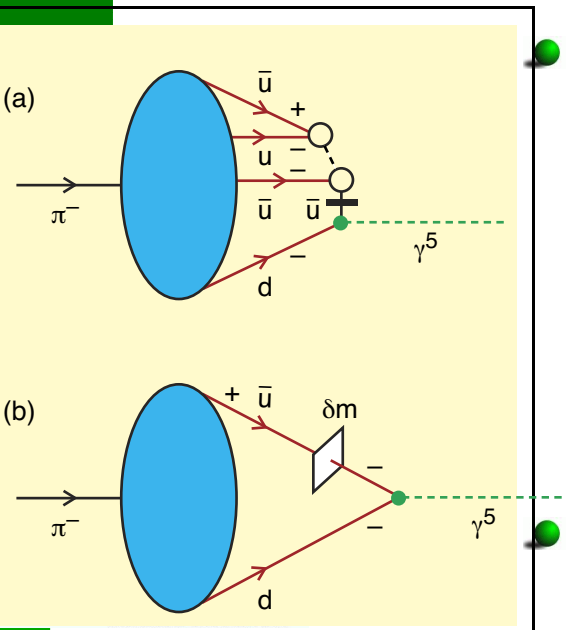
Universal Truths

- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.
- E.g., one problem: DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not yet been realised in the light-front formulation.



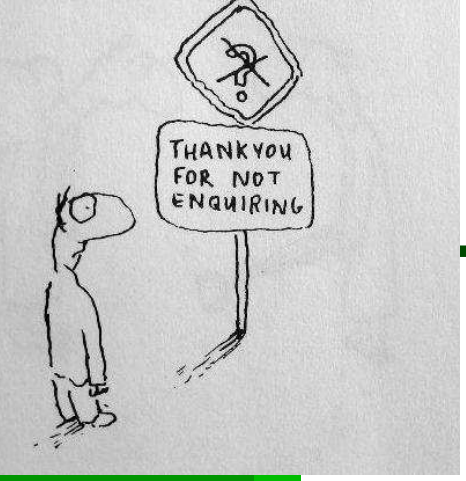
Universal Truths

- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.
- E.g., one problem: DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not yet been realised in the light-front formulation.



- Resolution
  – *So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.
  – DCSB obtained via coherent contribution from countable infinity of higher Fock-state components in LF-wavefunction.
- Brodsky, Roberts, Shrock, Tandy – arXiv:1005.4610 [nucl-th].

QCD's Challenges

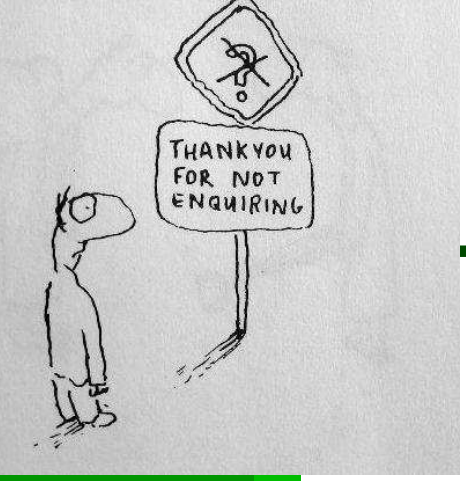


QCD's Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

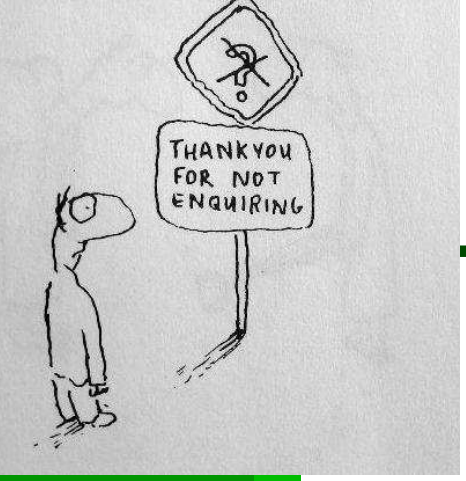
QCD's Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^{P=+}$ and $J^{P=-}$



QCD's Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^{P=+}$ and $J^{P=-}$
- Neither of these phenomena is apparent in QCD's Lagrangian ***yet*** they are the dominant determining characteristics of real-world QCD.



QCD's Challenges

Understand Emergent Phenomena

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^{P=+}$ and $J^{P=-}$
- Neither of these phenomena is apparent in QCD's Lagrangian ***yet*** they are the dominant determining characteristics of real-world QCD.
- QCD – Complex behaviour arises from apparently simple rules

Charting the Interaction between light-quarks

Charting the Interaction between light-quarks

- Confinement can be related to the analytic properties of QCD's Schwinger functions

Charting the Interaction between light-quarks

- Confinement can be related to the analytic properties of QCD's Schwinger functions
- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's *universal* β-function

Charting the Interaction between light-quarks

- Confinement can be related to the analytic properties of QCD's Schwinger functions
- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's *universal* β-function
  - This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.

Charting the Interaction between light-quarks

- Confinement can be related to the analytic properties of QCD's Schwinger functions
- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's *universal* β-function
  - This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.

Of course, the behaviour of the β-function on the perturbative domain is well known.

Charting the Interaction between light-quarks

- Confinement can be related to the analytic properties of QCD's Schwinger functions
- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's *universal* β-function
  - This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.

  Of course, the behaviour of the β-function on the perturbative domain is well known.
- This is a well-posed problem whose solution is an elemental goal of modern hadron physics.

What is the light-quark Long-Range Potential?

What is the light-quark Long-Range Potential?

Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any known way to the light-quark interaction.

Charting the Interaction between light-quarks

- Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking

Charting the Interaction between light-quarks

- Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations of, e.g.,
  - hadron mass spectrum;
  - elastic and transition form factors

  can be used to chart β-function's long-range behaviour

Charting the Interaction between light-quarks

- Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations of, e.g.,
  - hadron mass spectrum;
  - elastic and transition form factors

  can be used to chart β-function's long-range behaviour
- E.g.: Extant studies of mesons show that the properties of hadron excited states are a great deal more sensitive to the long-range behaviour of β-function than those of the ground state

Charting the Interaction between light-quarks

- Through DSEs the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations can be used to chart β-function's long-range behaviour

Charting the Interaction between light-quarks

- Through DSEs the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations can be used to chart β-function's long-range behaviour
- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary

Charting the Interaction between light-quarks

- Through DSEs the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations can be used to chart β-function's long-range behaviour
- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
  - Steady quantitative progress is being made with a scheme that is systematically improvable (See nucl-th/9602012 and references thereto)

Charting the Interaction between light-quarks

- Through DSEs the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations can be used to chart β-function's long-range behaviour
- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
  - Steady quantitative progress is being made with a scheme that is systematically improvable (See nucl-th/9602012 and references thereto)
  - Enabled proof or exact results in QCD: e.g., BRST – arXiv:1005.4610 [nucl-th]; and . . .

Radial Excitations & Chiral Symmetry

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

Radial Excitations & Chiral Symmetry

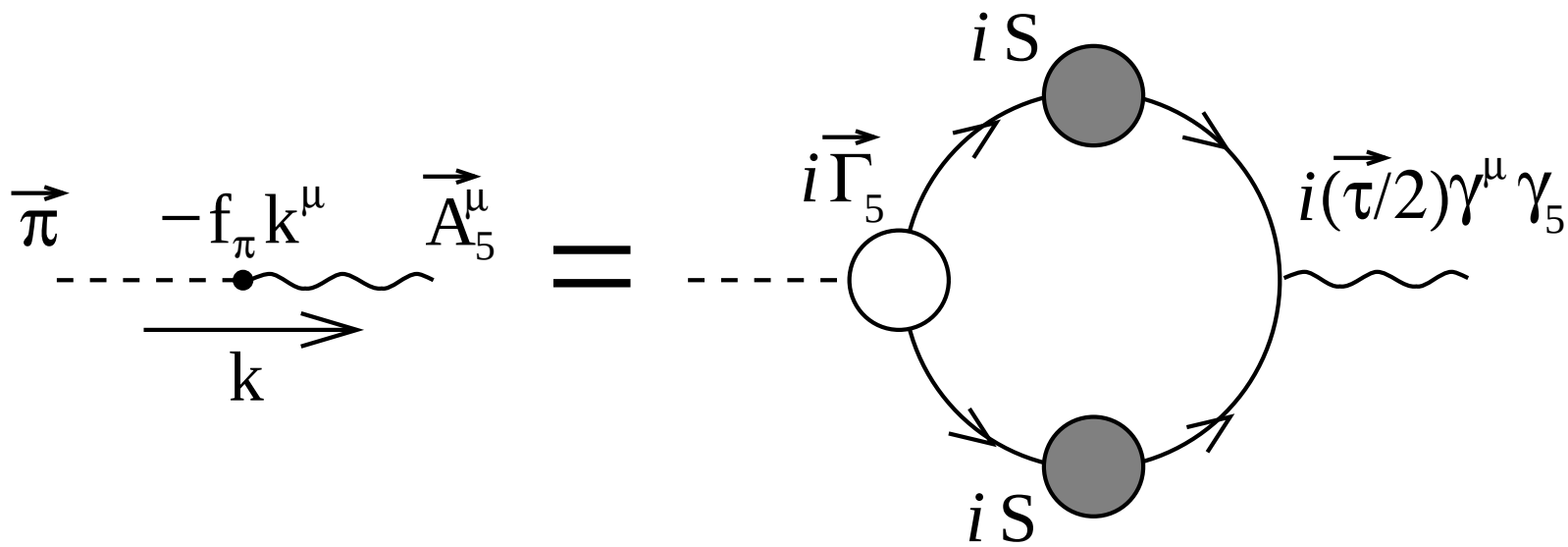
(Maris, Roberts, Tandy nu-th/9707003)

$-i\langle 0|\bar{q}\gamma_5\gamma_\mu q|\pi\rangle$

$$f_H\, m_H^2 = -\ \rho_\zeta^H\ \mathcal{M}_H$$

$$f_H\, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5\gamma_\mu \boxed{\mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-)} \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



Radial Excitations & Chiral Symmetry

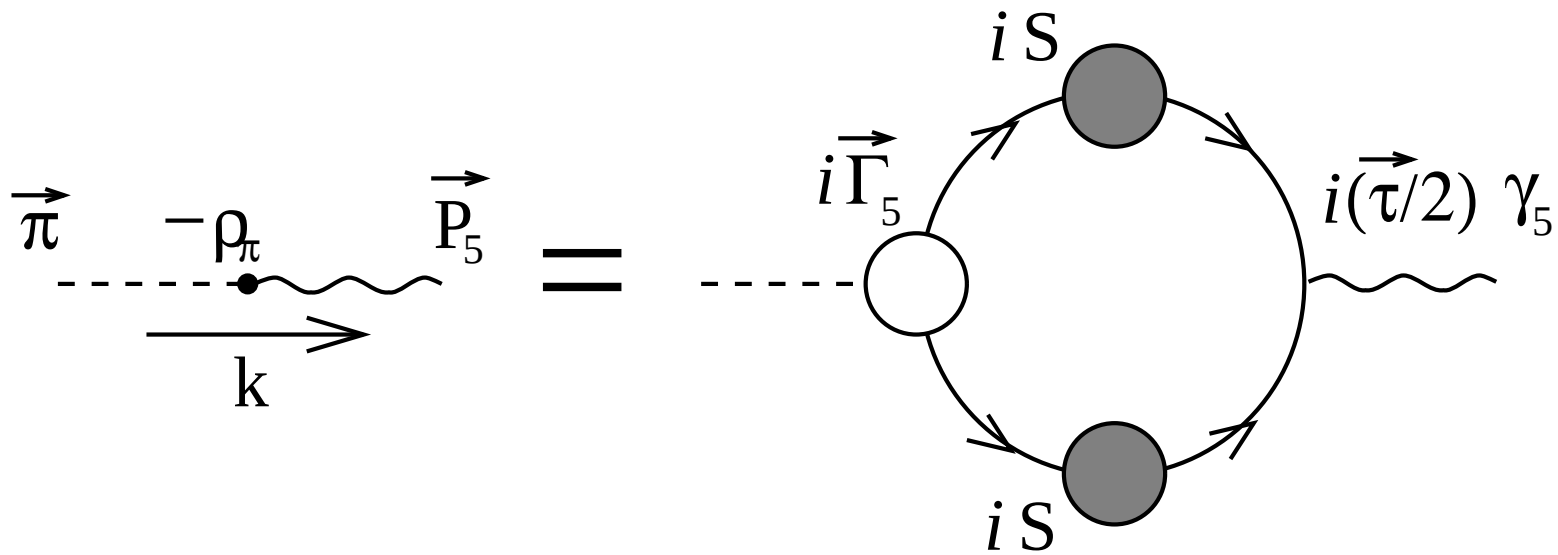
(Maris, Roberts, Tandy nu-th/9707003)

$-\langle 0|\bar{q}\gamma_5 q|\pi\rangle$

$$f_H \; m_H^2 = -\rho_\zeta^H \mathcal{M}_H$$

$$i\rho_\zeta^H = Z_4 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5 \mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-) \right\}$$

- Pseudoscalar projection of BS wave function at $x = 0$



Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ ...GMOR relation, a corollary

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ ...GMOR relation, a corollary

- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $m_H \propto m_q$

... QCD Proof of Potential Model result

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson, not the ground state, so $m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0$, in chiral limit

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson, not the ground state, so $m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0$, in chiral limit
- $\Rightarrow f_H = 0$
  ALL pseudoscalar mesons except $\pi(140)$ in chiral limit

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson, not the ground state, so $m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0$, in chiral limit
- $\Rightarrow f_H = 0$
  ALL pseudoscalar mesons except $\pi(140)$ in chiral limit
- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

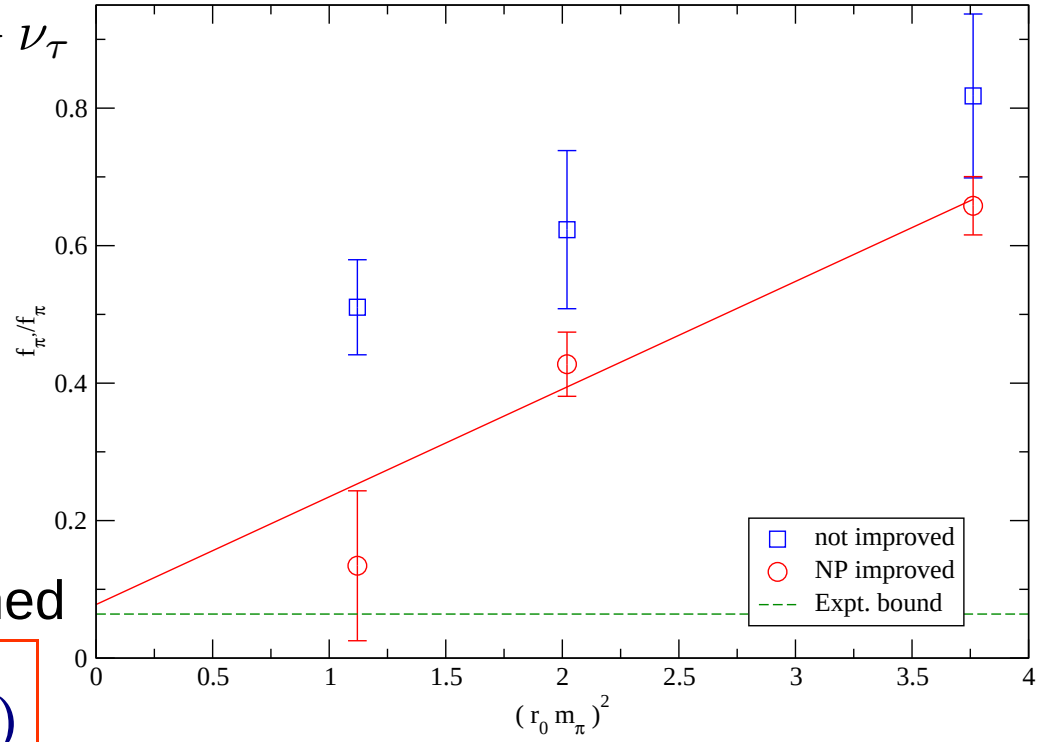
- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,$MeV
  Diehl & Hiller
  he-ph/0105194

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$

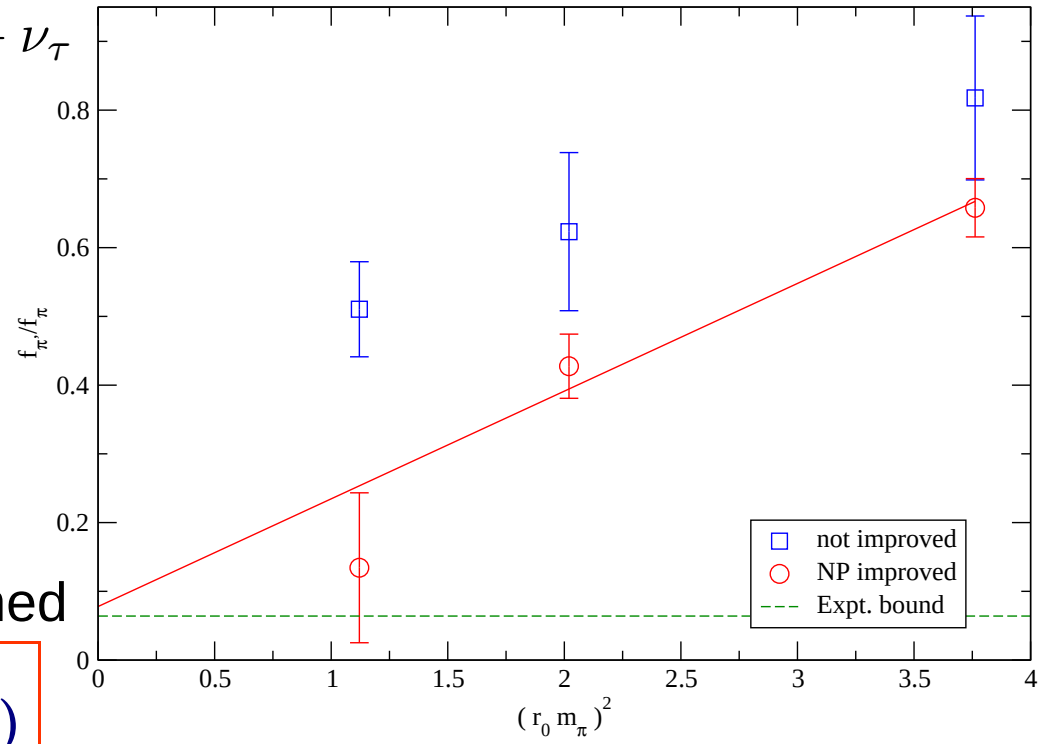


Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$



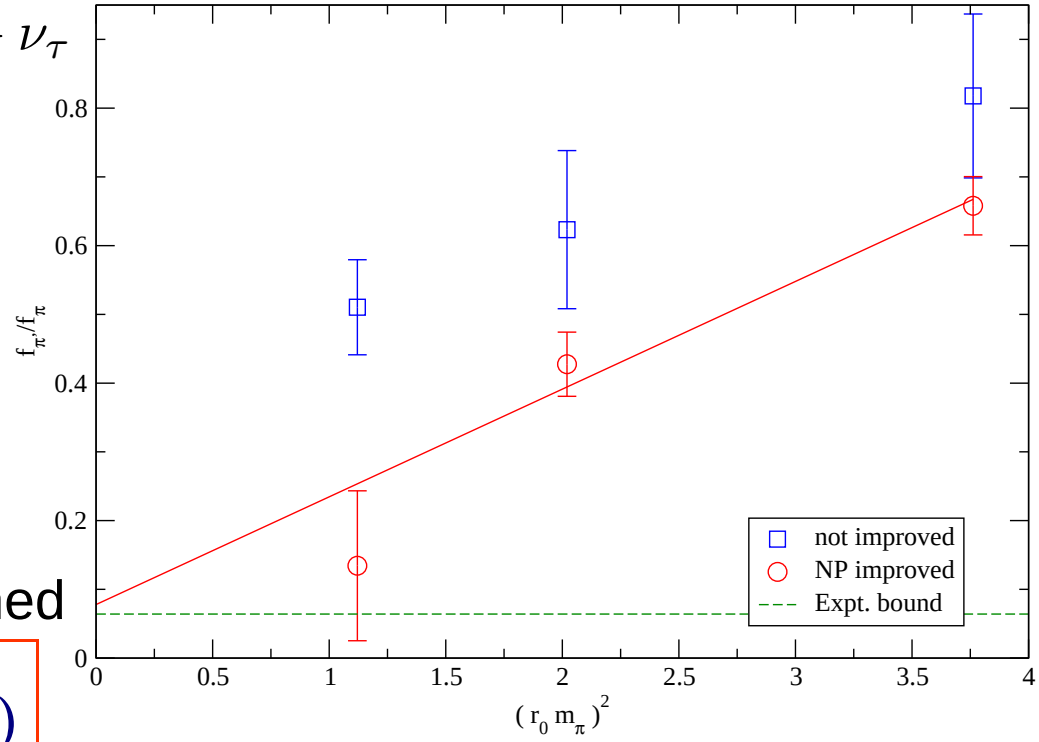
- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,$MeV
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,$fm,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$



- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

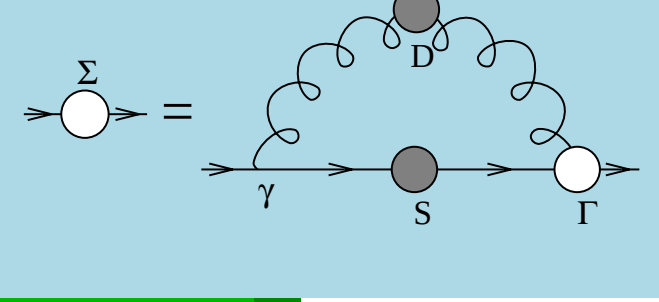
Charting the Interaction between light-quarks

- Through DSEs the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations can be used to chart β-function's long-range behaviour
- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
  - Steady quantitative progress is being made with a scheme that is systematically improvable (See nucl-th/9602012 and references thereto)
  - Enabled proof or exact results in QCD, as we've just seen

Charting the Interaction between light-quarks

- Through DSEs the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations can be used to chart β-function's long-range behaviour
- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
  - On other hand, at present significant qualitative advances possible with symmetry-preserving kernel *Ansätze* that express important additional nonperturbative effects – $M(p^2)$ – difficult/impossible to capture in any finite sum of contributions

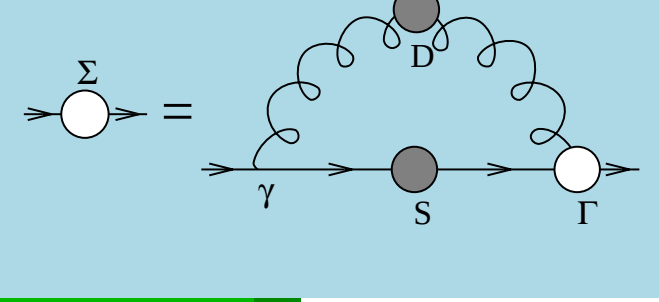
Gap Equation
General Form



Gap Equation
General Form

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p),$$

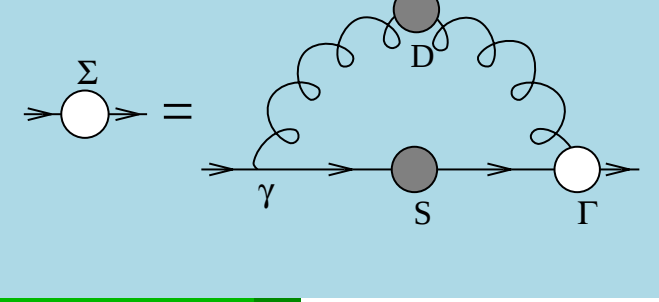


Gap Equation General Form

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p),$$

- $Z_{1,2}(\zeta^2,\Lambda^2)$ are respectively the vertex and quark wave function renormalisation constants, with ζ the renormalisation point
- $m^{\rm bm}(\Lambda)$ is the Lagrangian current-quark bare mass
- $D_{\mu\nu}(k)$ is the dressed-gluon propagator
- $\Gamma^f_\nu(q,p)$ is the dressed-quark-gluon vertex



Gap Equation General Form

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p),$$

- $Z_{1,2}(\zeta^2,\Lambda^2)$ are respectively the vertex and quark wave function renormalisation constants, with ζ the renormalisation point
- $m^{\rm bm}(\Lambda)$ is the Lagrangian current-quark bare mass
- $D_{\mu\nu}(k)$ is the dressed-gluon propagator
- $\Gamma^f_\nu(q,p)$ is the dressed-quark-gluon vertex
- Suppose one has in-hand the exact form of $\Gamma^f_\nu(q,p)$

What is the associated

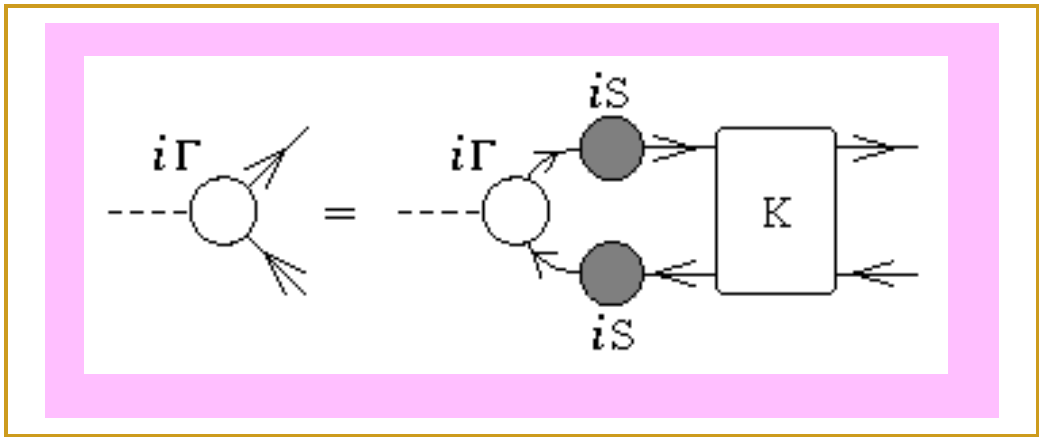
Symmetry-preserving Bethe-Salpeter Kernel?

Bound-state DSE

Bound-state DSE
Bethe-Salpeter Equation

- Standard form, familiar from textbooks

$$\left[\Gamma_\pi^j(k;P)\right]_{tu} = \int_q^\Lambda \left[S(q+P/2)\Gamma_\pi^j(q;P)S(q-P/2)\right]_{sr} K_{tu}^{rs}(q,k;P)$$

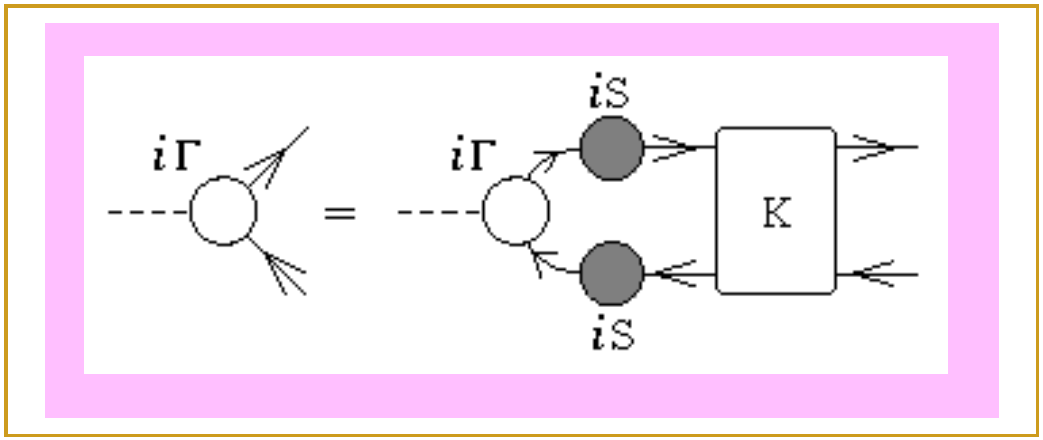


$K(q,k;P)$: Fully-amputated, 2-particle-irreducible, quark-antiquark scattering kernel

Bound-state DSE
Bethe-Salpeter Equation

- Standard form, familiar from textbooks

$$\left[\Gamma_\pi^j(k;P)\right]_{tu} = \int_q^\Lambda \left[S(q+P/2)\Gamma_\pi^j(q;P)S(q-P/2)\right]_{sr} K_{tu}^{rs}(q,k;P)$$



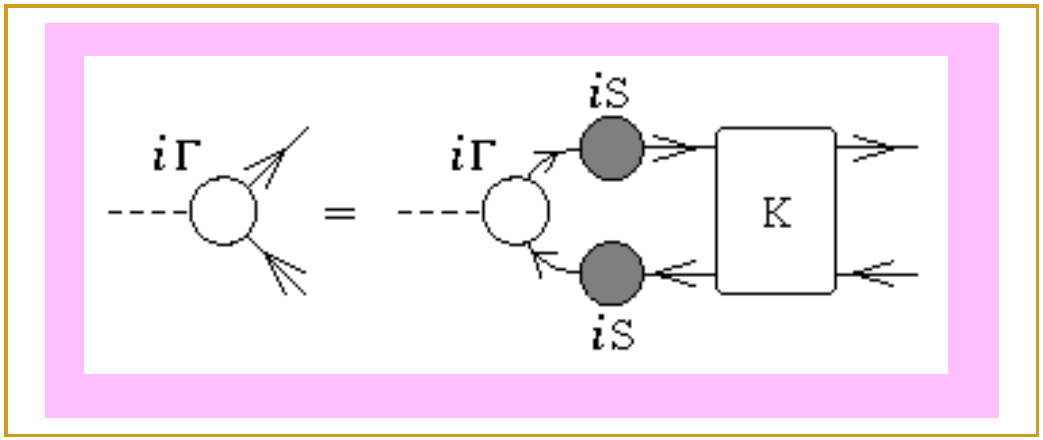
$K(q,k;P)$: Fully-amputated, 2-particle-irreducible, quark-antiquark scattering kernel

- Compact. Visually appealing. Correct.

Bound-state DSE
Bethe-Salpeter Equation

- Standard form, familiar from textbooks

$$\left[\Gamma_\pi^j(k;P)\right]_{tu} = \int_q^\Lambda \left[S(q+P/2)\Gamma_\pi^j(q;P)S(q-P/2)\right]_{sr} K_{tu}^{rs}(q,k;P)$$



$K(q,k;P)$: Fully-amputated, 2-particle-irreducible, quark-antiquark scattering kernel

- Compact. Visually appealing. Correct.
- Blocked progress for more than 60 years.

Bethe-Salpeter Equation General Form

L. Chang and C. D. Roberts
0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

Bethe-Salpeter Equation General Form

L. Chang and C. D. Roberts
0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

- Equivalent exact form:

$$\Gamma^{fg}_{5\mu}(k;P) = Z_2\gamma_5\gamma_\mu$$

$$- \boxed{\int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\Gamma^{fg}_{5\mu}(q;P)S_g(q_-)\frac{\lambda^a}{2}\,\Gamma^g_\beta(q_-,k_-)}$$

$$+ \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

(Poincaré covariance, hence $q_\pm = q \pm P/2$, etc., without loss of generality.)

Bethe-Salpeter Equation General Form

L. Chang and C. D. Roberts
0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

- Equivalent exact form:

$$\Gamma^{fg}_{5\mu}(k;P) = Z_2\gamma_5\gamma_\mu$$
$$- \boxed{\int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\Gamma^{fg}_{5\mu}(q;P)S_g(q_-)\frac{\lambda^a}{2}\,\Gamma^g_\beta(q_-,k_-)}$$
$$+ \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

(Poincaré covariance, hence $q_\pm = q \pm P/2$, etc., without loss of generality.)

- In this form . . . $\Lambda^{fg}_{5\mu\beta}$
  is completely defined via the dressed-quark self-energy

Bethe-Salpeter Kernel

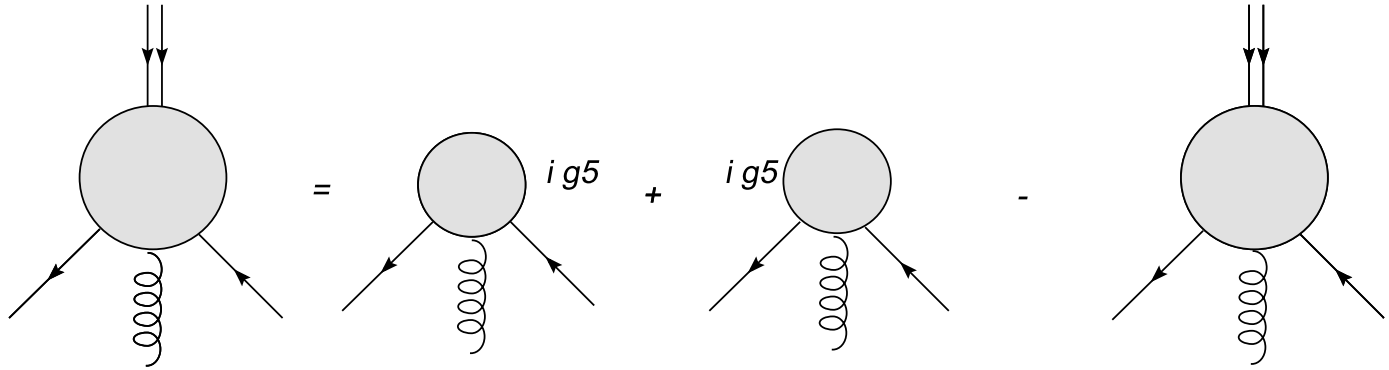
L. Chang and C. D. Roberts
0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

- Bethe-Salpeter equation introduced in 1951

Bethe-Salpeter Kernel
60 year problem

L. Chang and C. D. Roberts
0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

- Bethe-Salpeter equation introduced in 1951
- Newly-derived Ward-Takahashi identity

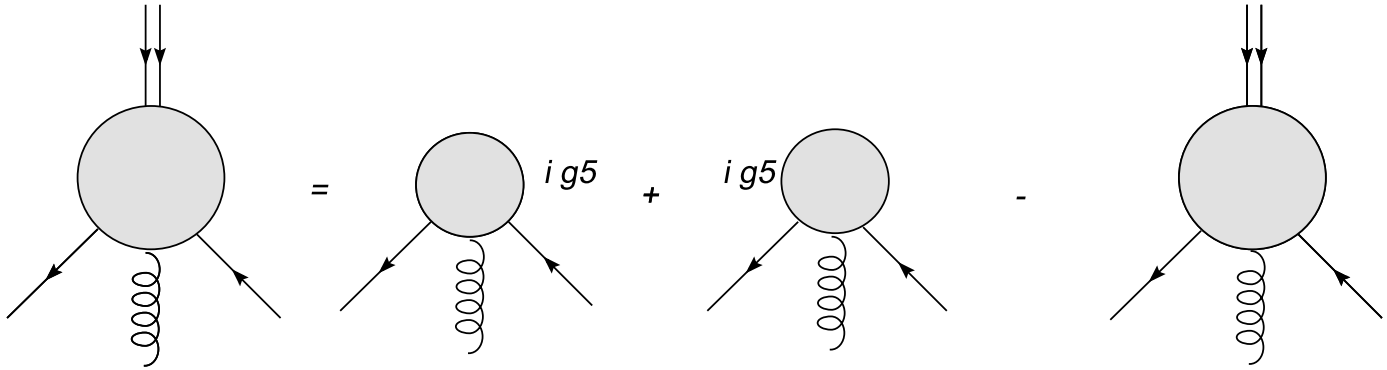


$$P_\mu \Lambda^{fg}_{5\mu\beta}(k,q;P) = \Gamma^f_\beta(q_+,k_+)\, i\gamma_5 + i\gamma_5\, \Gamma^g_\beta(q_-,k_-) - i[m_f(\zeta)+m_g(\zeta)]\Lambda^{fg}_{5\beta}(k,q;P),$$

Bethe-Salpeter Kernel
60 year problem

L. Chang and C. D. Roberts
0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

- Bethe-Salpeter equation introduced in 1951
- Newly-derived Ward-Takahashi identity



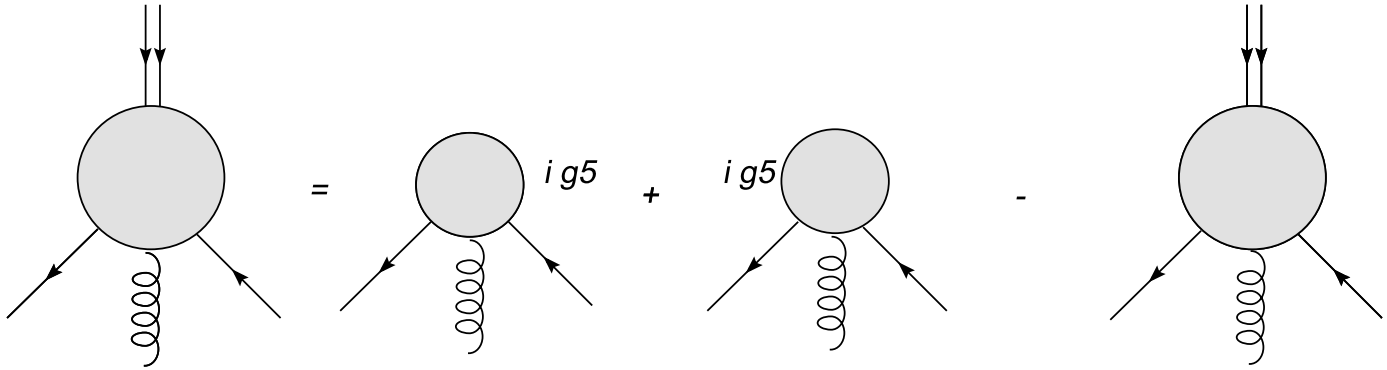
$$P_\mu \Lambda^{fg}_{5\mu\beta}(k,q;P) = \Gamma^f_\beta(q_+,k_+)\, i\gamma_5 + i\gamma_5\, \Gamma^g_\beta(q_-,k_-) - i[m_f(\zeta)+m_g(\zeta)]\Lambda^{fg}_{5\beta}(k,q;P),$$

- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
  - Consistent means - all symmetries preserved!

Bethe-Salpeter Kernel
60 year problem

L. Chang and C. D. Roberts
0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

- Bethe-Salpeter equation introduced in 1951
- Newly-derived Ward-Takahashi identity



$$P_\mu \Lambda^{fg}_{5\mu\beta}(k,q;P) = \Gamma^f_\beta(q_+,k_+)\, i\gamma_5 + i\gamma_5\, \Gamma^g_\beta(q_-,k_-) - i[m_f(\zeta)+m_g(\zeta)]\Lambda^{fg}_{5\beta}(k,q;P),$$

- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
- Procedure & results to expect . . .

see arXiv:1003.5006 [nucl-th]

Mass Splitting

$a_1 - \rho$

	exp.			
mass a_1	1230			
mass ρ	775			
mass-splitting	455			

- Splitting known experimentally for more than 35 years.
- Hitherto, no explanation.

Mass Splitting

$a_1 - \rho$

	exp.	rainbow-ladder	one-loop		
mass a_1	1230	759	885		
mass ρ	775	644	764		
mass-splitting	455	115	121		

- Systematic, symmetry-preserving, Poincaré-covariant DSE truncation scheme of nucl-th/9602012.
- Never better than $\sim \frac{1}{4}$ of splitting.
- Constructing kernel skeleton-diagram-by-diagram, DCSB cannot be faithfully expressed: $M(p^2)$ is absent!

Mass Splitting

$a_1 - \rho$

	exp.	rainbow-ladder	one-loop	Ball-Chiu consistent	
mass a_1	1230	759	885	1066	
mass ρ	775	644	764	924	
mass-splitting	455	115	121	142	

- New nonperturbative, symmetry-preserving Poincaré-covariant Bethe-Salpeter equation formulation of arXiv:0903.5461 [nucl-th]
- Ball-Chiu *Ansatz* for quark-gluon vertex
  $\Gamma_\mu^{\rm BC}(k,p) = \ldots + (k+p)_\mu \frac{B(k)-B(p)}{k^2-p^2}$
  - Some effects of DCSB built into vertex
  - Explains $\pi - \sigma$ splitting but not this problem

Mass Splitting

$a_1 - \rho$

Chang & Roberts arXiv:1003.5006 [nucl-th]

	exp.	rainbow-ladder	one-loop	Ball-Chiu consistent	Ball-Chiu plus anom. cm mom.
mass a_1	1230	759	885	1066	1230
mass ρ	775	644	764	924	745
mass-splitting	455	115	121	142	485

- New nonperturbative, symmetry-preserving Poincaré-covariant Bethe-Salpeter equation formulation of arXiv:0903.5461 [nucl-th]
- Ball-Chiu augmented by *quark anomalous chromomagnetic moment* term: $\Gamma_\mu(k,p) = \Gamma_\mu^{\rm BC} + \sigma_{\mu\nu}(k-p)_\nu \frac{B(k)-B(p)}{k^2-p^2}$

Mass Splitting

$a_1 - \rho$

Chang & Roberts arXiv:1003.5006 [nucl-th]

	exp.	rainbow-ladder	one-loop	Ball-Chiu consistent	Ball-Chiu plus anom. cm mom.
mass a_1	1230	759	885	1066	1230
mass ρ	775	644	764	924	745
mass-splitting	455	115	121	142	485

- New nonperturbative, symmetry-preserving Poincaré-covariant Bethe-Salpeter equation formulation of arXiv:0903.5461 [nucl-th]
- DCSB is the answer. Subtle interplay between competing effects, which can only now be explicated
- Promise of first reliable prediction of light-quark meson spectrum, including the so-called hybrid and exotic states.

Quark Anomalous Magnetic Moments

Chang & Roberts, in progress

- Massless fermion can't possess an anomalous magnetic moment
  - Interaction term $\boxed{\int d^4x \, \frac{1}{2} g \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)}$ explicitly breaks chiral symmetry

Quark Anomalous Magnetic Moments

Chang & Roberts, in progress

- Massless fermion can't possess an anomalous magnetic moment
  - Interaction term $\boxed{\int d^4x \, \frac{1}{2} g \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)}$ explicitly breaks chiral symmetry
- However, DCSB can generate a large anomalous chromomagnetic moment even in chiral limit
  – This explains the a_1-ρ mass-splitting

Quark Anomalous Magnetic Moments

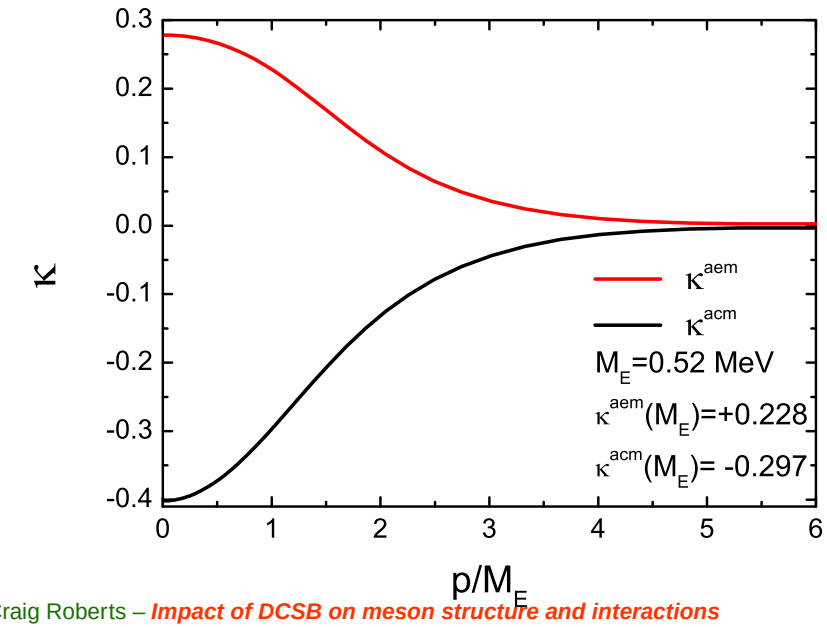
Chang & Roberts, in progress

- Massless fermion can't possess an anomalous magnetic moment
  - Interaction term $\boxed{\int d^4x \, \frac{1}{2} g \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)}$ explicitly breaks chiral symmetry
- New BSE formulation (arXiv:0903.5461 [nucl-th]) *enables* computation of dressed-quark electromagnetic moment given dressed-quark-gluon vertex with ACM-term

Quark Anomalous Magnetic Moments

Chang & Roberts, in progress

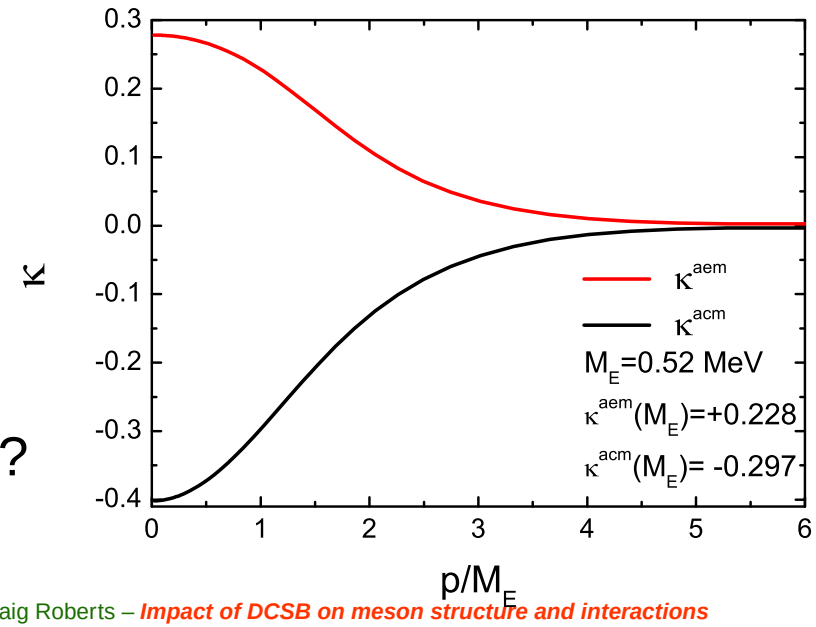
- Massless fermion can't possess an anomalous magnetic moment
  - Interaction term $\int d^4x \, \frac{1}{2} g \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$ explicitly breaks chiral symmetry
- New BSE formulation (arXiv:0903.5461 [nucl-th]) *enables* computation of dressed-quark electromagnetic moment given dressed-quark-gluon vertex with ACM-term
- $M(p^2) \Rightarrow \kappa(p^2)$
  - Preliminary result for μ distributions



Quark Anomalous Magnetic Moments

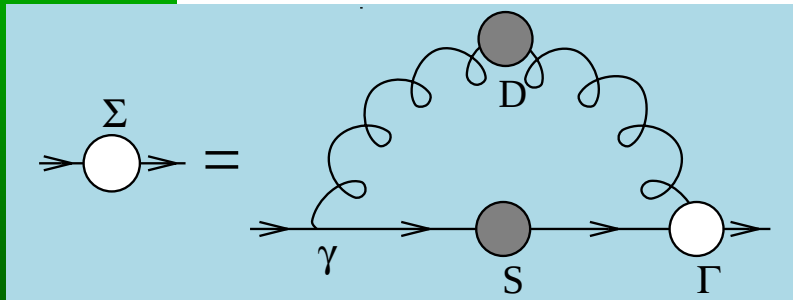
Chang & Roberts, in progress

- Massless fermion can't possess an anomalous magnetic moment
  - Interaction term $\int d^4x \, \frac{1}{2} g \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$ explicitly breaks chiral symmetry
- New BSE formulation (arXiv:0903.5461 [nucl-th]) *enables* computation of dressed-quark electromagnetic moment given dressed-quark-gluon vertex with ACM-term
- $M(p^2) \Rightarrow \kappa(p^2)$
  - Preliminary result for μ distributions
- Cloët & Roberts
  Effect on hadron form factors?



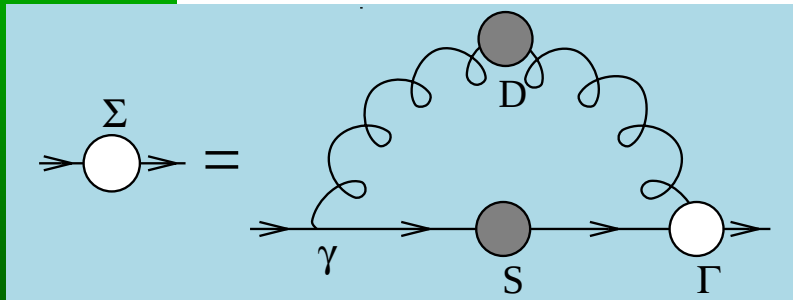
Frontiers of Nuclear Science: A Long Range Plan (2007)

Frontiers of Nuclear Science: Theoretical Advances



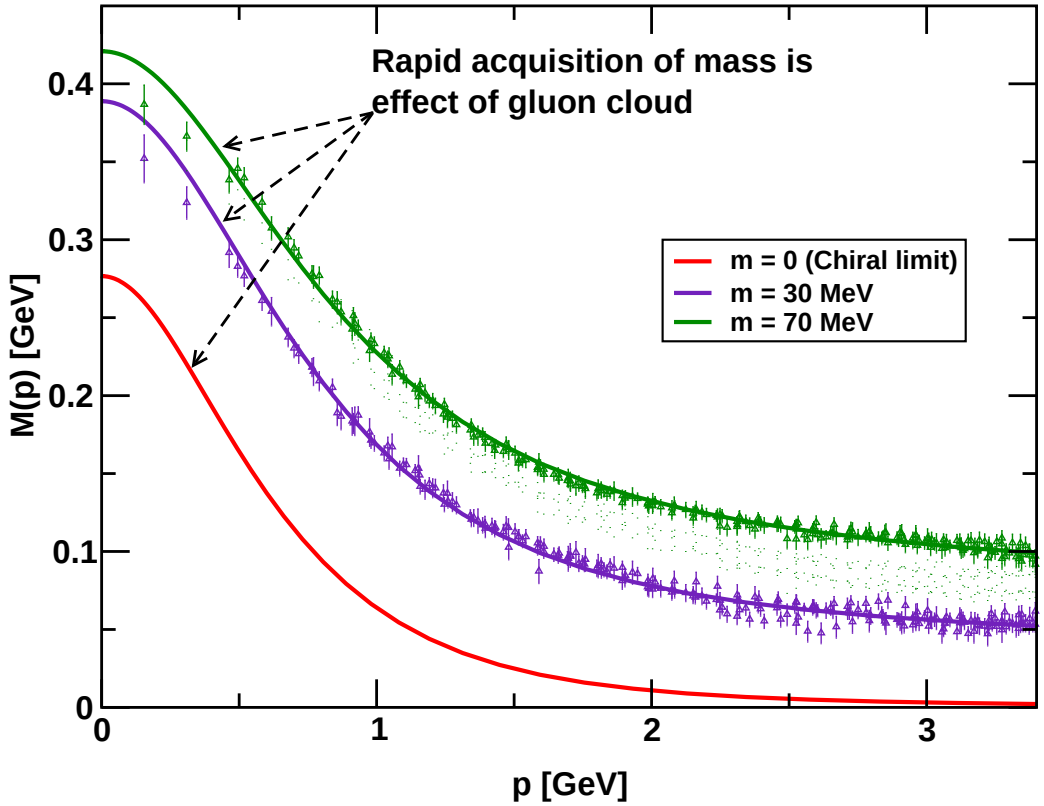
Gap Equation

Frontiers of Nuclear Science: Theoretical Advances



Gap Equation

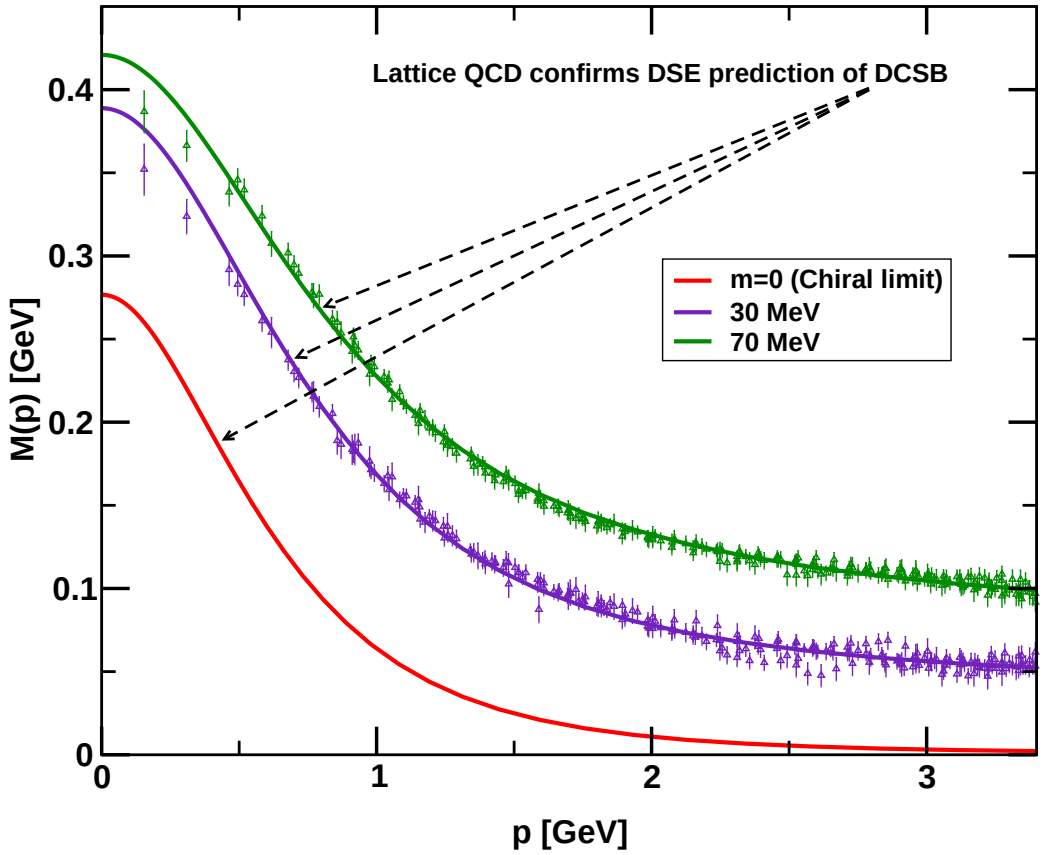
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.

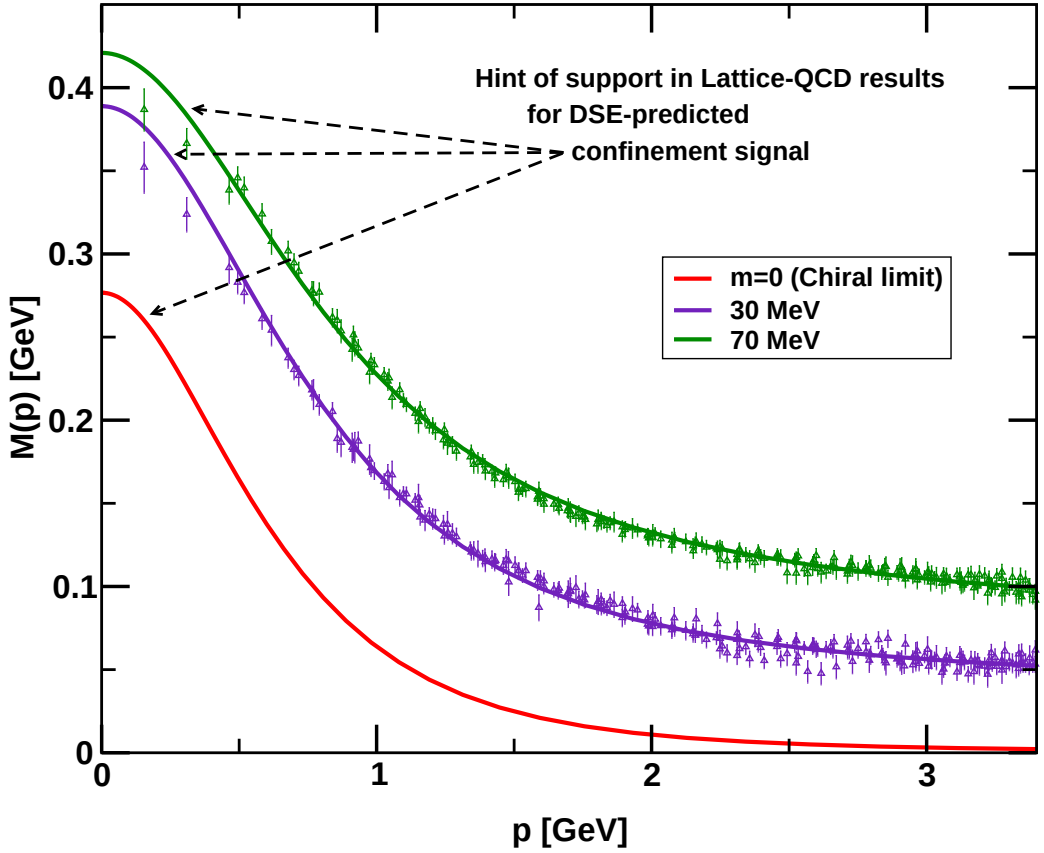
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.

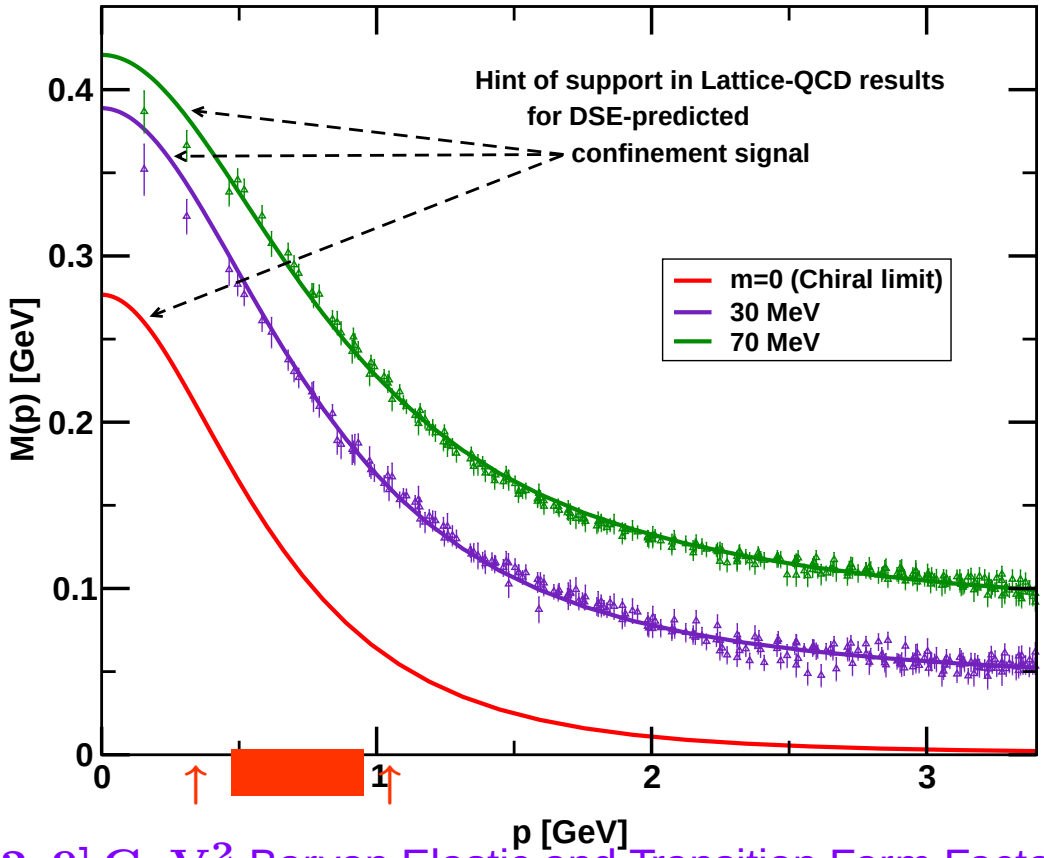
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Scanned by $Q^2 \in [2, 9]\ \mathrm{GeV}^2$ Baryon Elastic and Transition Form Factors

Maris, Roberts, Tandy
nucl-th/9707003

Goldberger-Treiman for pion

Maris, Roberts, Tandy
nucl-th/9707003

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

Maris, Roberts, Tandy
nucl-th/9707003

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

Maris, Roberts, Tandy
nucl-th/9707003

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$
- Axial-vector Ward-Takahashi identity

$\Rightarrow$ $$f_\pi E_\pi(k;P=0) = B(p^2)$$

Maris, Roberts, Tandy
nucl-th/9707003

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$
- Axial-vector Ward-Takahashi identity

$$\Rightarrow \quad \begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

Maris, Roberts, Tandy
nucl-th/9707003

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

Pseudovector components necessarily nonzero

- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$
- Axial-vector Ward-Takahashi identity

$$\begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

$\Rightarrow$ Exact in Chiral QCD

GT for pion – QCD and $F_\pi^{\rm em}(Q^2)$

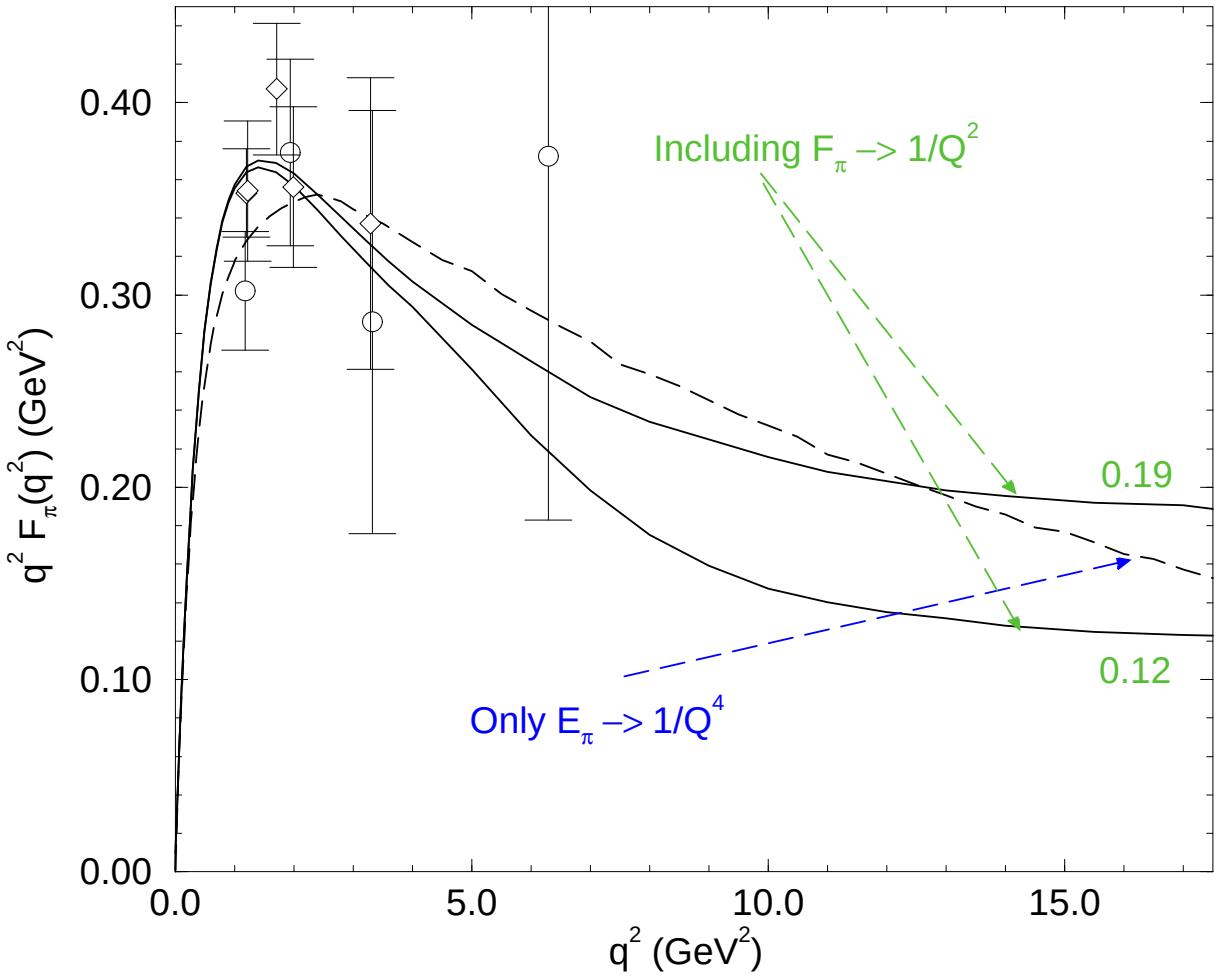
Maris, Roberts
nucl-th/9804062

- What does this mean for observables?

GT for pion – QCD and $F_\pi^{\rm em}(Q^2)$

Maris, Roberts
nucl-th/9804062

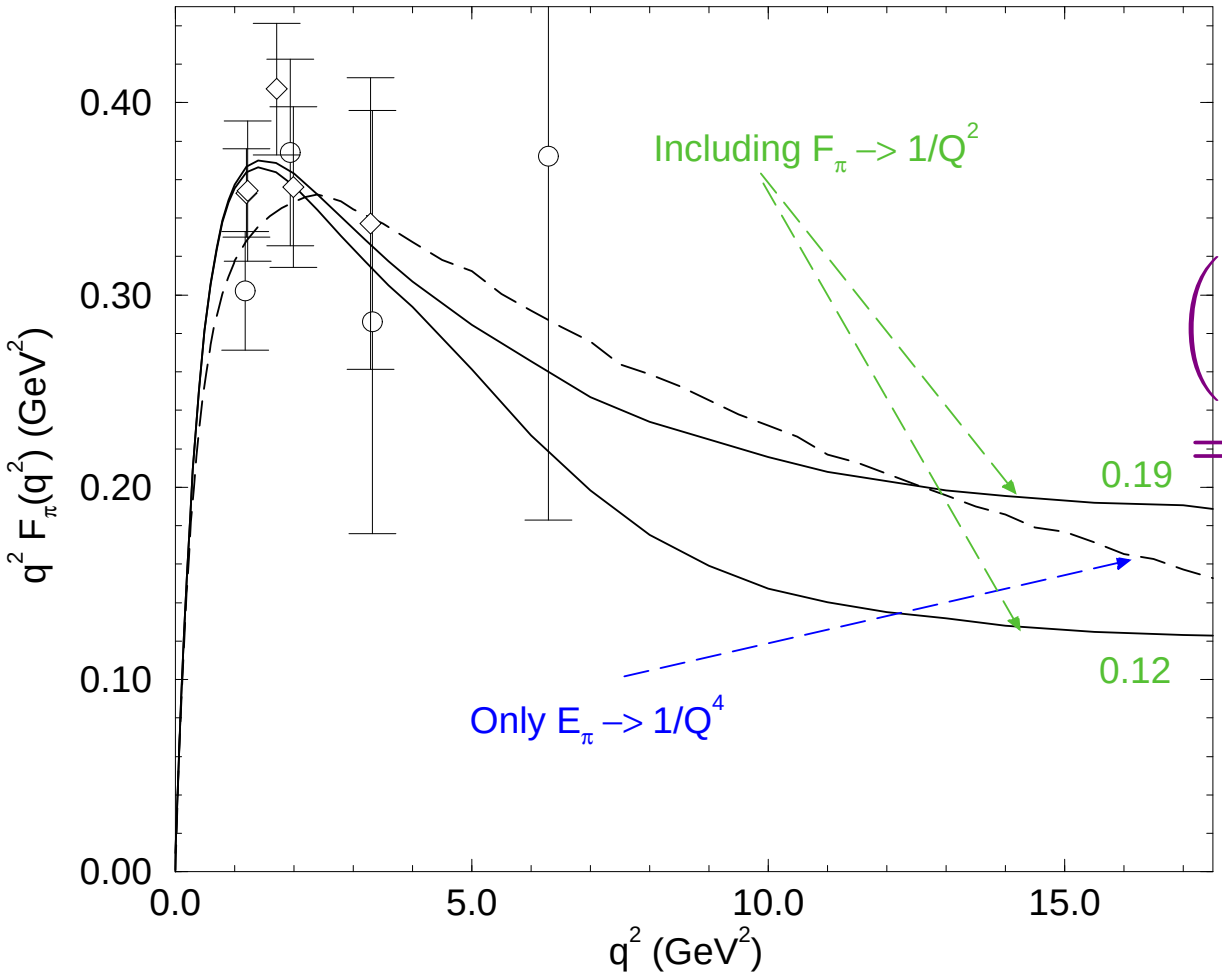
- What does this mean for observables?



GT for pion – QCD and $F_\pi^{\rm em}(Q^2)$

Maris, Roberts
nucl-th/9804062

- What does this mean for observables?



$$\left(\frac{Q}{2}\right)^2 = 2\,\mathrm{GeV}^2$$
$$\Rightarrow Q^2 = 8\,\mathrm{GeV}^2$$

- Pseudovector components dominate ultraviolet behaviour of electromagnetic form factor

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude can't depend on relative momentum

$\Rightarrow$ General Form $\boxed{\Gamma_\pi(P) = \gamma_5[iE_\pi(P) + \frac{1}{M_Q}\gamma\cdot P F_\pi(P)]}$

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude can't depend on relative momentum

$\Rightarrow$ General Form $\boxed{\Gamma_\pi(P) = \gamma_5[iE_\pi(P) + \frac{1}{M_Q}\gamma\cdot P F_\pi(P)]}$

- Solve chiral-limit gap and Bethe-Salpeter equations

$$P^2 = 0: \; M_Q = 0.40\,, \; E_\pi = 0.98\,, \; \frac{F_\pi}{M_Q} = 0.50$$

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude can't depend on relative momentum

  $\Rightarrow$ General Form $\boxed{\Gamma_\pi(P) = \gamma_5[iE_\pi(P) + \frac{1}{M_Q}\gamma\cdot P F_\pi(P)]}$

- Solve chiral-limit gap and Bethe-Salpeter equations

  $$P^2 = 0: \; M_Q = 0.40\,, \; E_\pi = 0.98\,, \; \frac{F_\pi}{M_Q} = 0.50$$

- Origin of pseudovector component: E_π drives F_π
  - RHS Bethe-Salpeter equation:

    $$\gamma_\mu S(k+P/2) i\gamma_5 E_\pi S(k-P/2)\gamma_\mu$$

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude can't depend on relative momentum

  $\Rightarrow$ General Form $\boxed{\Gamma_\pi(P) = \gamma_5[iE_\pi(P) + \frac{1}{M_Q}\gamma\cdot P F_\pi(P)]}$

- Solve chiral-limit gap and Bethe-Salpeter equations

  $$P^2 = 0: \; M_Q = 0.40\,, \; E_\pi = 0.98\,, \; \frac{F_\pi}{M_Q} = 0.50$$

- Origin of pseudovector component: E_π drives F_π
  - RHS Bethe-Salpeter equation:

    $$\gamma_\mu S(k+P/2) i\gamma_5 E_\pi S(k-P/2)\gamma_\mu$$

  - Has pseudovector component

    $$\sim E_\pi[\sigma_S(k_+)\sigma_V(k_-) + \sigma_S(k_-)\sigma_V(k_+)]$$

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude can't depend on relative momentum

  $\Rightarrow$ General Form $\boxed{\Gamma_\pi(P) = \gamma_5[iE_\pi(P) + \frac{1}{M_Q}\gamma\cdot P F_\pi(P)]}$

- Solve chiral-limit gap and Bethe-Salpeter equations

  $$P^2 = 0: \; M_Q = 0.40\,, \; E_\pi = 0.98\,, \; \frac{F_\pi}{M_Q} = 0.50$$

- Origin of pseudovector component: E_π drives F_π
  - RHS Bethe-Salpeter equation:
    $\gamma_\mu S(k+P/2) i\gamma_5 E_\pi S(k-P/2)\gamma_\mu$
  - Hence F_π on LHS is forced to be nonzero because E_π on RHS is nonzero owing to DCSB

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude: General Form

$$\Gamma_\pi(P) = \gamma_5 [iE_\pi(P) + \frac{1}{M_Q}\gamma \cdot P F_\pi(P)]$$

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude: General Form

$$\Gamma_\pi(P) = \gamma_5 [iE_\pi(P) + \frac{1}{M_Q}\gamma \cdot P F_\pi(P)]$$

- Asymptotic form of electromagnetic pion form factor

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude: General Form

$$\boxed{\Gamma_\pi(P) = \gamma_5[iE_\pi(P) + \frac{1}{M_Q}\gamma\cdot P F_\pi(P)]}$$

- Asymptotic form of electromagnetic pion form factor
  - E_π^2-term $\Rightarrow F_{\pi E}^{\rm em}(Q^2) \sim \frac{M^2}{Q^2}$, photon$(Q)$

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude: General Form

$$\boxed{\Gamma_\pi(P) = \gamma_5[iE_\pi(P) + \frac{1}{M_Q}\gamma\cdot P F_\pi(P)]}$$

- Asymptotic form of electromagnetic pion form factor
  - E_π^2-term $\Rightarrow F_{\pi E}^{\rm em}(Q^2) \sim \frac{M^2}{Q^2}$, photon$(Q)$
  - $E_\pi F_\pi$-term.

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude: General Form

$$\boxed{\Gamma_\pi(P) = \gamma_5[iE_\pi(P) + \frac{1}{M_Q}\gamma\cdot P F_\pi(P)]}$$

- Asymptotic form of electromagnetic pion form factor
  - E_π^2-term $\Rightarrow F_{\pi\,E}^{\rm em}(Q^2) \sim \frac{M^2}{Q^2}$, photon$(Q)$
  - $E_\pi F_\pi$-term. Breit Frame:
    pion$(P = (0,0,-Q/2,iQ/2))$

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude: General Form

$$\Gamma_\pi(P) = \gamma_5[iE_\pi(P) + \frac{1}{M_Q}\gamma\cdot P F_\pi(P)]$$

- Asymptotic form of electromagnetic pion form factor
  - E_π^2-term $\Rightarrow F^{\rm em}_{\pi\,E}(Q^2) \sim \frac{M^2}{Q^2}$, photon$(Q)$
  - $E_\pi F_\pi$-term. Breit Frame:
    pion$(P = (0,0,-Q/2,iQ/2))$
    $F^{\rm em}_{\pi\,EF}(Q^2) \sim 2\,S\gamma\cdot(P+Q)F_\pi S\gamma_4 S E_\pi$

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude: General Form

$$\Gamma_\pi(P) = \gamma_5 [i E_\pi(P) + \frac{1}{M_Q} \gamma \cdot P F_\pi(P)]$$

- Asymptotic form of electromagnetic pion form factor
  - E_π^2-term $\Rightarrow F^{\rm em}_{\pi\, E}(Q^2) \sim \frac{M^2}{Q^2}$, photon$(Q)$
  - $E_\pi F_\pi$-term. Breit Frame:
    pion$(P = (0, 0, -Q/2, iQ/2))$
    $F^{\rm em}_{\pi\, EF}(Q^2) \sim 2\, S\gamma \cdot (P + Q) F_\pi S\gamma_4 S E_\pi$

$$\Rightarrow F^{\rm em}_{\pi\, EF}(Q^2) \propto \frac{Q^2}{M_Q^2} \frac{F_\pi}{E_\pi} \times E_\pi^2 - \text{term} = \text{constant!}$$

GT for pion – Contact Interaction

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- Bethe-Salpeter amplitude: General Form

$$\Gamma_\pi(P) = \gamma_5 [iE_\pi(P) + \frac{1}{M_Q}\gamma\cdot P F_\pi(P)]$$

- Asymptotic form of electromagnetic pion form factor
  - E_π^2-term $\Rightarrow F^{\rm em}_{\pi\,E}(Q^2) \sim \frac{M^2}{Q^2}$, photon$(Q)$
  - $E_\pi F_\pi$-term. Breit Frame:
    pion$(P = (0,0,-Q/2,iQ/2))$
    $F^{\rm em}_{\pi\,EF}(Q^2) \sim 2\,S\gamma\cdot(P+Q)F_\pi S\gamma_4 S E_\pi$

$$\Rightarrow F^{\rm em}_{\pi\,EF}(Q^2) \propto \frac{Q^2}{M_Q^2}\frac{F_\pi}{E_\pi} \times E_\pi^2\text{−term} = \text{constant!}$$

- This behaviour dominates for $Q^2 \gtrsim M_Q^2 \frac{E_\pi}{F_\pi} > 0.8\,\mathrm{GeV}^2$

Computation: Elastic Pion Form Factor

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- DSE prediction: $M(p^2)$; i.e., interaction $\dfrac{1}{|x-y|^2}$
- cf. $M(p^2) =$ Constant; i.e., interaction $\delta^4(x-y)$

Computation: Elastic Pion Form Factor

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- DSE prediction: $M(p^2)$; i.e., interaction $\dfrac{1}{|x-y|^2}$
- cf. $M(p^2) =$ Constant; i.e., interaction $\delta^4(x-y)$

Single mass-scale parameter
in both studies

Computation: Elastic Pion Form Factor

Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- DSE prediction: $M(p^2)$; i.e., interaction $\dfrac{1}{|x-y|^2}$
- cf. $M(p^2) =$ Constant; i.e., interaction $\delta^4(x-y)$

Single mass-scale parameter
in both studies

Same predictions for
$Q^2 = 0$ properties

Computation: Elastic Pion Form Factor

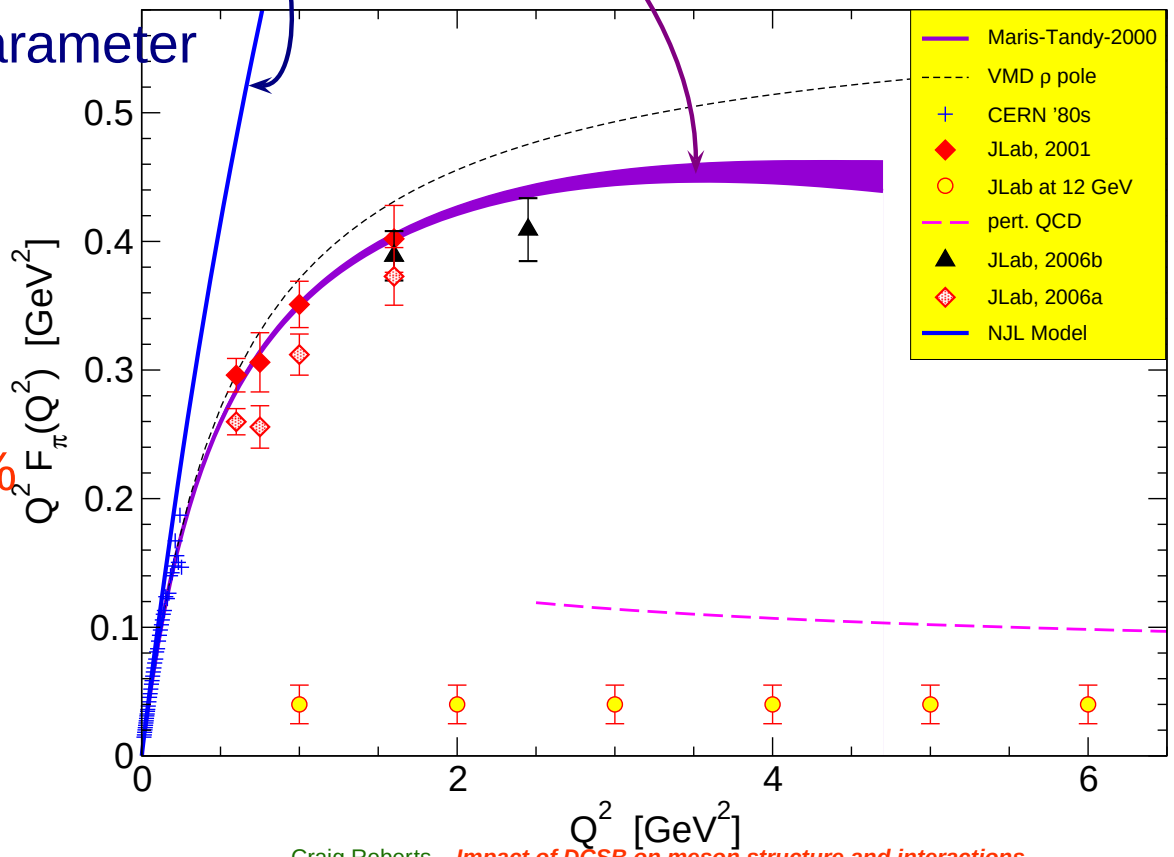
Guttierez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- DSE prediction: $M(p^2)$; i.e., interaction $\dfrac{1}{|x-y|^2}$
- cf. $M(p^2) =$ Constant; i.e., interaction $\delta^4(x-y)$

Single mass-scale parameter in both studies

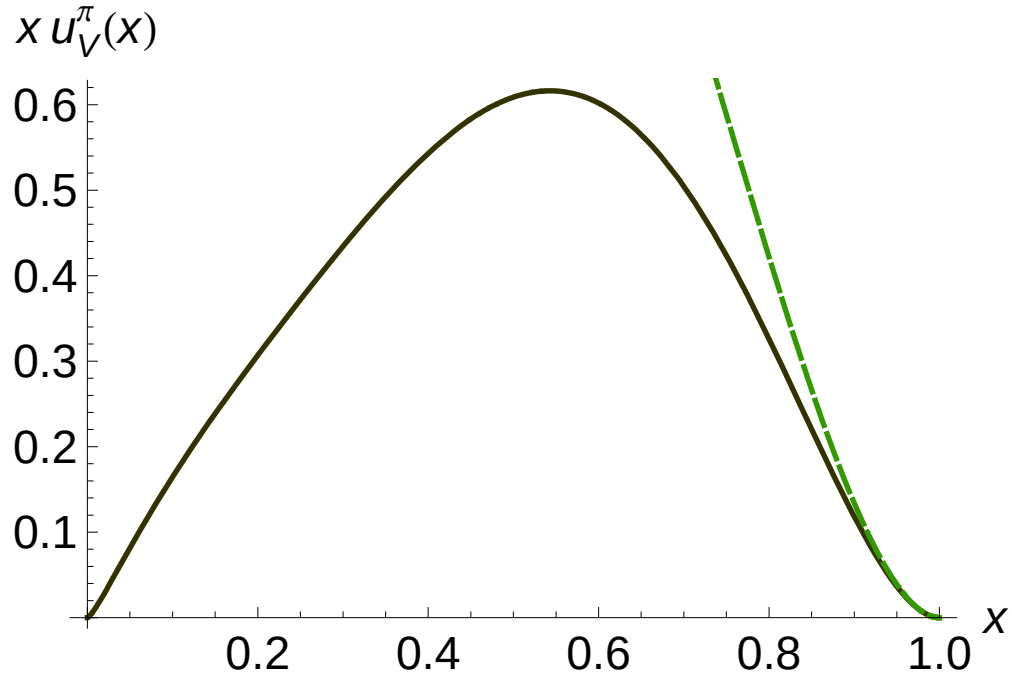
Same predictions for $Q^2 = 0$ properties

Disagreement $> 20\%$ for $Q^2 > M^2$



Pion's valence distribution

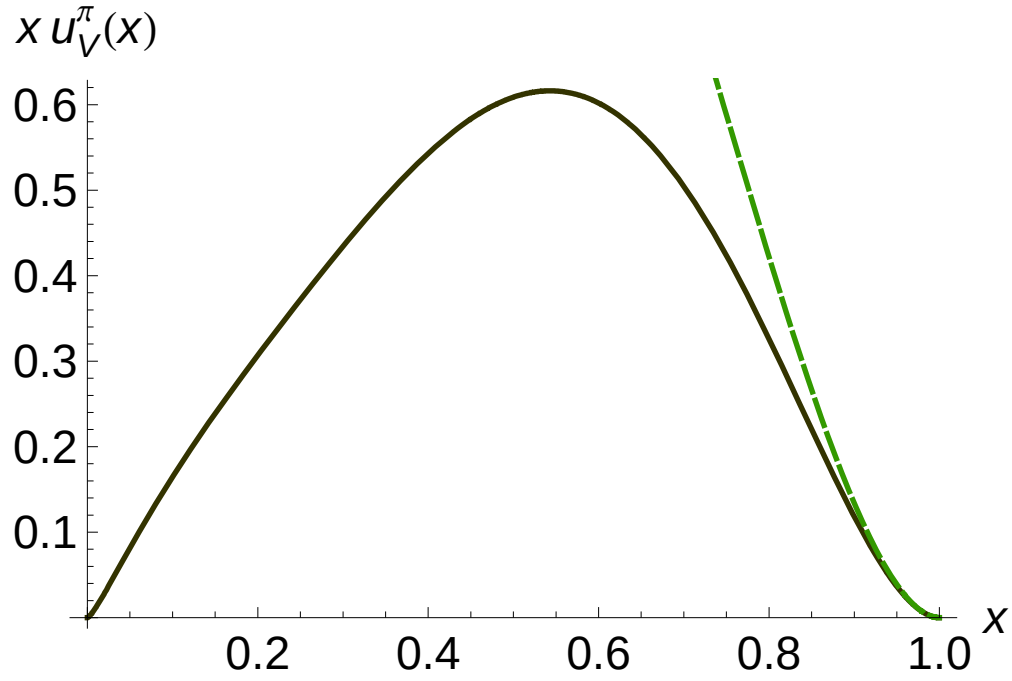
Holt & Roberts: arXiv:1002.4666 [nucl-th]



Pion's valence distribution

Holt & Roberts: arXiv:1002.4666 [nucl-th]

$$\frac{\alpha(q^2)}{q^2} \underset{\propto}{\overset{q^2 \gtrsim M_D^2}{}} \left(\frac{1}{q^2}\right)^{1+\kappa}$$

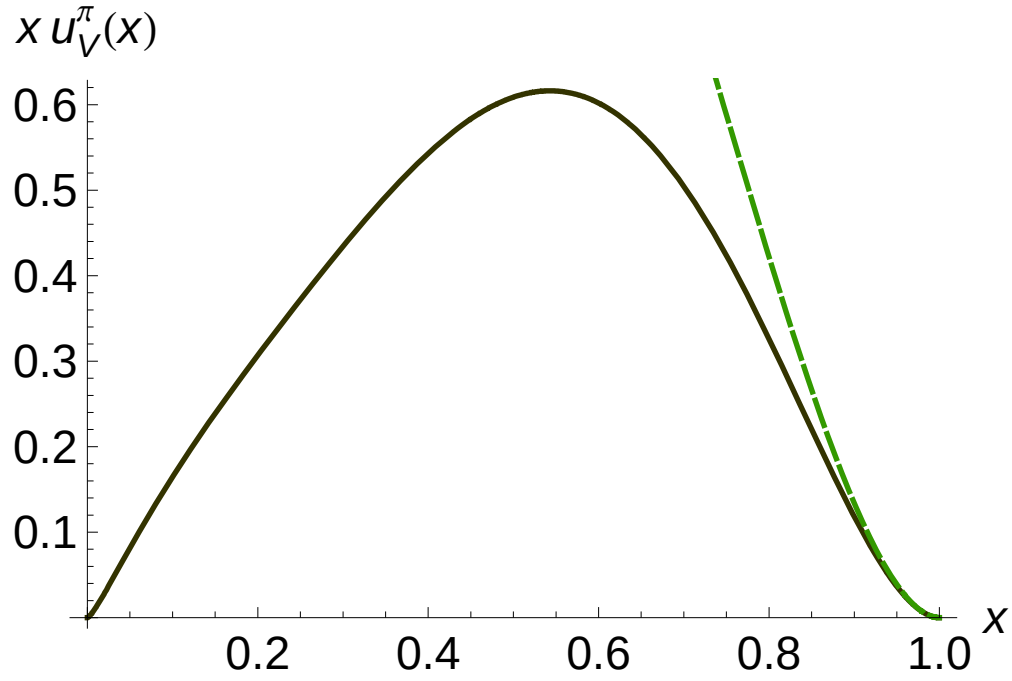


Pion's valence distribution

Holt & Roberts: arXiv:1002.4666 [nucl-th]

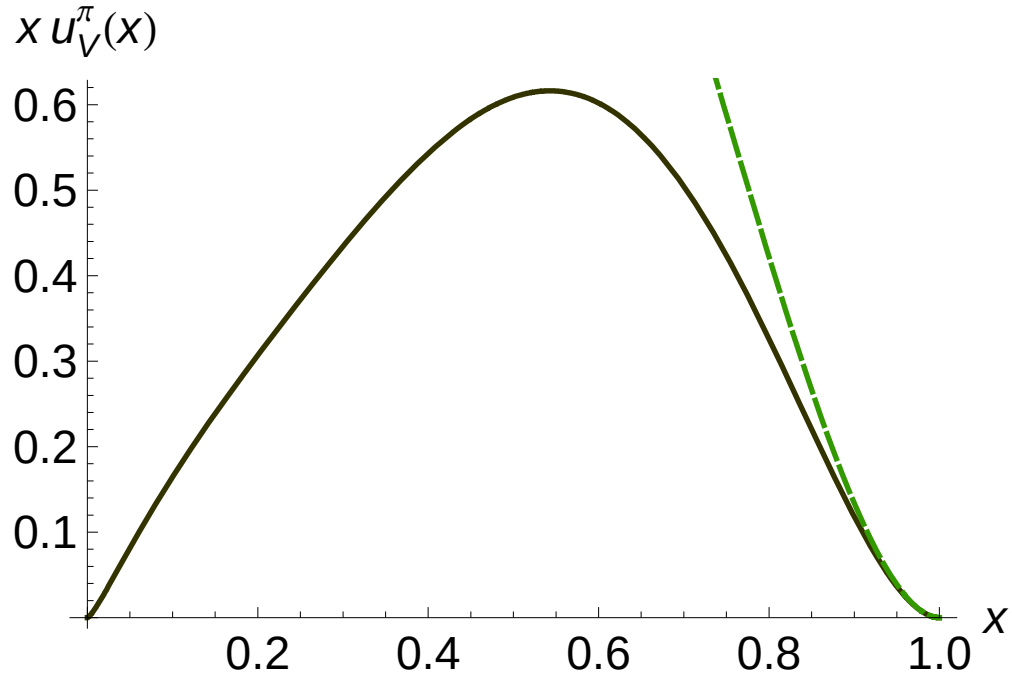
$$\frac{\alpha(q^2)}{q^2} \overset{q^2 \gtrsim M_D^2}{\propto} \left(\frac{1}{q^2}\right)^{1+\kappa}$$

$$\Rightarrow B(k^2) \overset{k^2 \gtrsim M_D^2}{\propto} \left(\frac{1}{k^2}\right)^{1+\kappa}$$



Pion's valence distribution

Holt & Roberts: arXiv:1002.4666 [nucl-th]



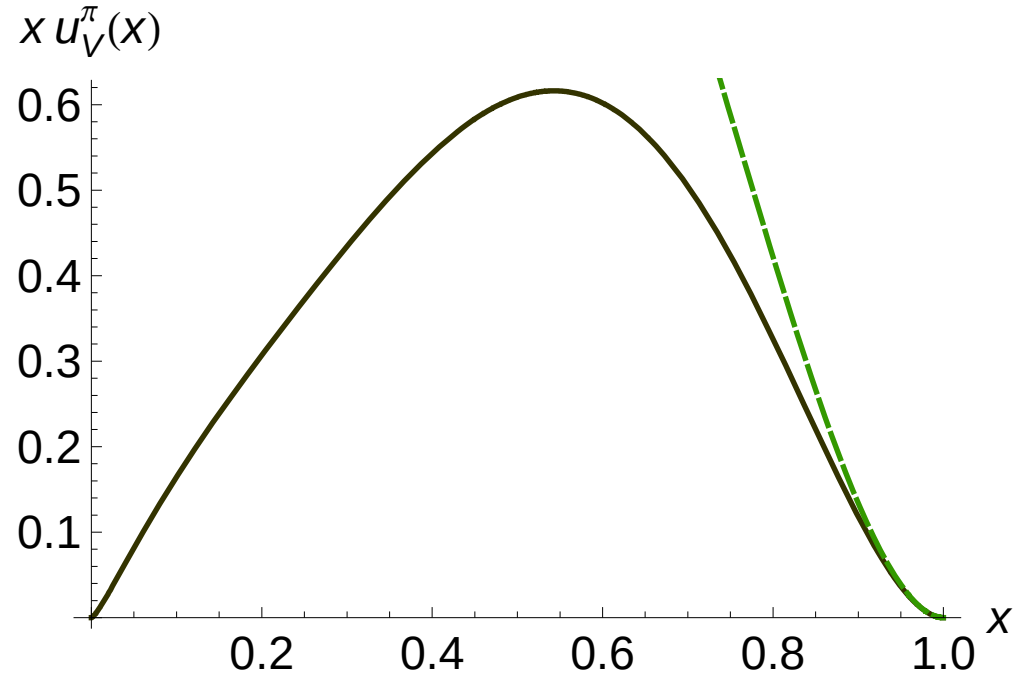
$$\frac{\alpha(q^2)}{q^2} \overset{q^2 \gtrsim M_D^2}{\propto} \left(\frac{1}{q^2}\right)^{1+\kappa}$$

$$\Rightarrow B(k^2) \overset{k^2 \gtrsim M_D^2}{\propto} \left(\frac{1}{k^2}\right)^{1+\kappa}$$

$$\Rightarrow q_v^\pi(x;Q_0) \overset{x \sim 1}{\propto} (1-x)^{2(1+\kappa)} \quad \text{at } Q_0 \gtrsim M_D\,.$$

Pion's valence distribution

Holt & Roberts: arXiv:1002.4666 [nucl-th]



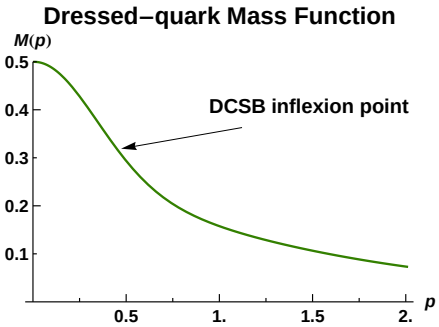
$$\frac{\alpha(q^2)}{q^2} \overset{q^2 \gtrsim M_D^2}{\propto} \left(\frac{1}{q^2}\right)^{1+\kappa}$$

$$\Rightarrow B(k^2) \overset{k^2 \gtrsim M_D^2}{\propto} \left(\frac{1}{k^2}\right)^{1+\kappa}$$

$$\Rightarrow q_v^\pi(x;Q_0) \overset{x\sim 1}{\propto} (1-x)^{2(1+\kappa)} \quad \text{at } Q_0 \gtrsim M_D\,.$$

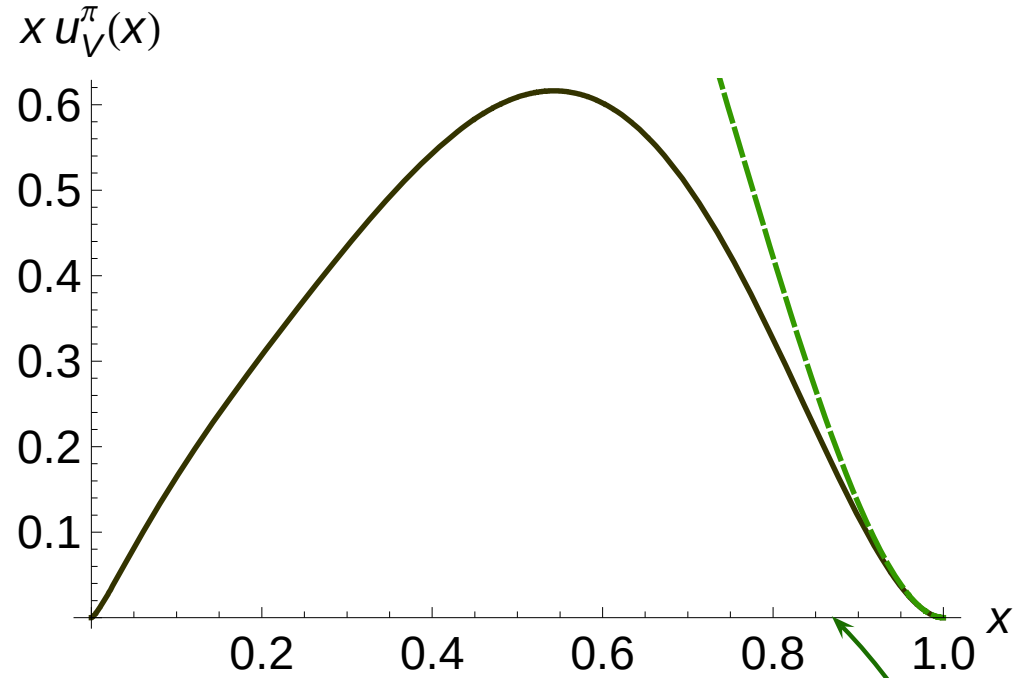
$M_D \approx 0.4\,$GeV

– p^2-location of DCSB inflexion point in $M(p^2)$



Pion's valence distribution

Holt & Roberts: arXiv:1002.4666 [nucl-th]

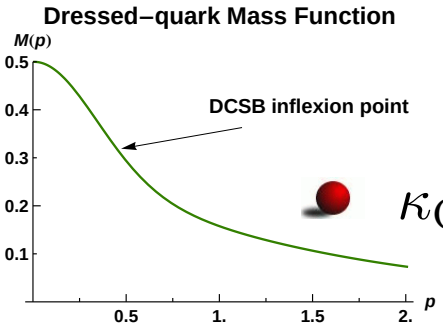


$$\frac{\alpha(q^2)}{q^2} \overset{q^2 \gtrsim M_D^2}{\propto} \left(\frac{1}{q^2}\right)^{1+\kappa}$$

$$\Rightarrow B(k^2) \overset{k^2 \gtrsim M_D^2}{\propto} \left(\frac{1}{k^2}\right)^{1+\kappa}$$

$$\Rightarrow q_v^\pi(x;Q_0) \overset{x\sim 1}{\propto} (1-x)^{2(1+\kappa)} \quad \text{at } Q_0 \gtrsim M_D\,.$$

- $M_D \approx 0.4\,\text{GeV}$
  - p^2-location of DCSB inflexion point in $M(p^2)$



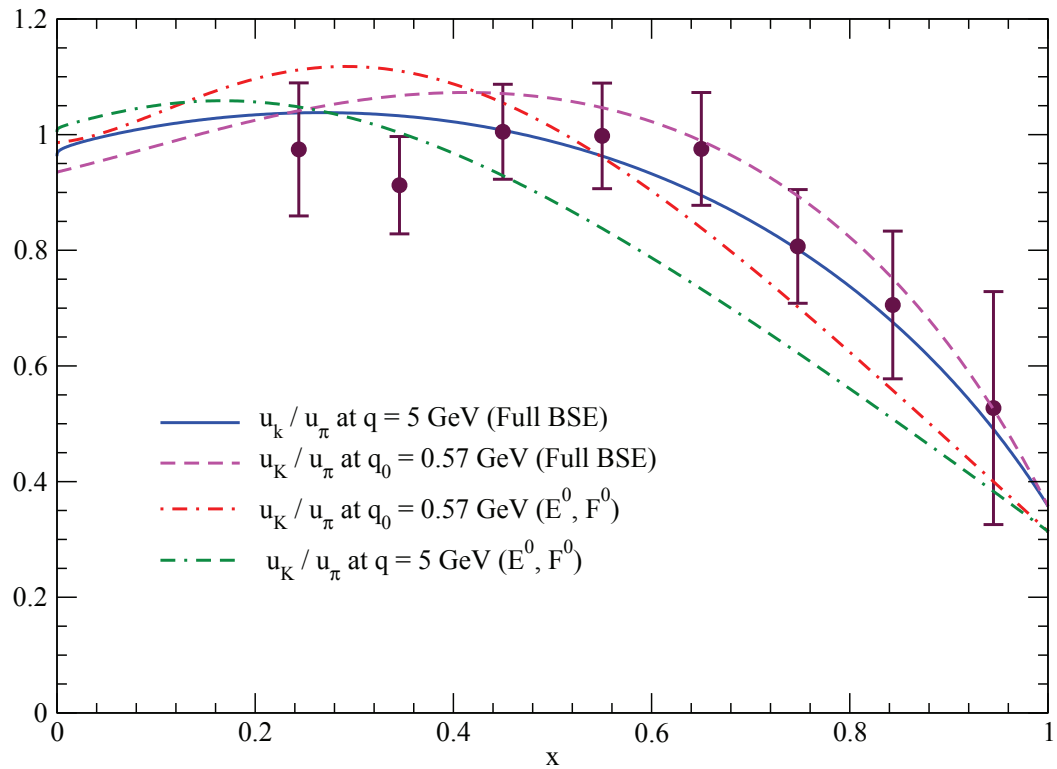
- $\kappa_{\rm QCD} = 0 \Rightarrow q_v^\pi(x;1\,{\rm GeV}^2) \propto (1-x)^2$

  DSE calculation shows this valid for $\mathcal{L}_x = \{x | x > 0.86\}$.

T. Nguyen: PhD Thesis (Kent State U.)
Nguyen, Tandy, Bashir, Roberts, in progress
Holt & Roberts: arXiv:1002.4666 [nucl-th]

Ratio – Kaon/Pion u-valence distribution

data: Badier, *et al.* , Phys. Lett. **B 93** (1980) 354

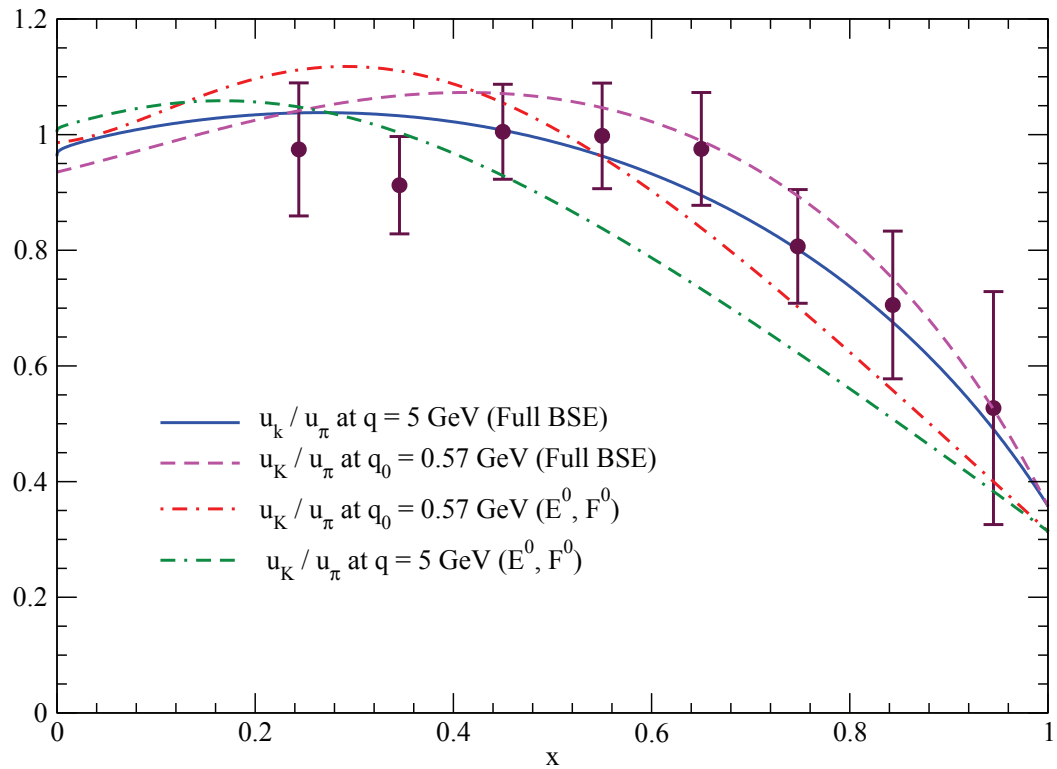


T. Nguyen: PhD Thesis (Kent State U.)
Nguyen, Tandy, Bashir, Roberts, in progress
Holt & Roberts: arXiv:1002.4666 [nucl-th]

Ratio – Kaon/Pion u-valence distribution

- DSE–result obtained using interaction that predicted $F_\pi(Q^2)$

data: Badier, *et al.* , Phys. Lett. **B 93** (1980) 354

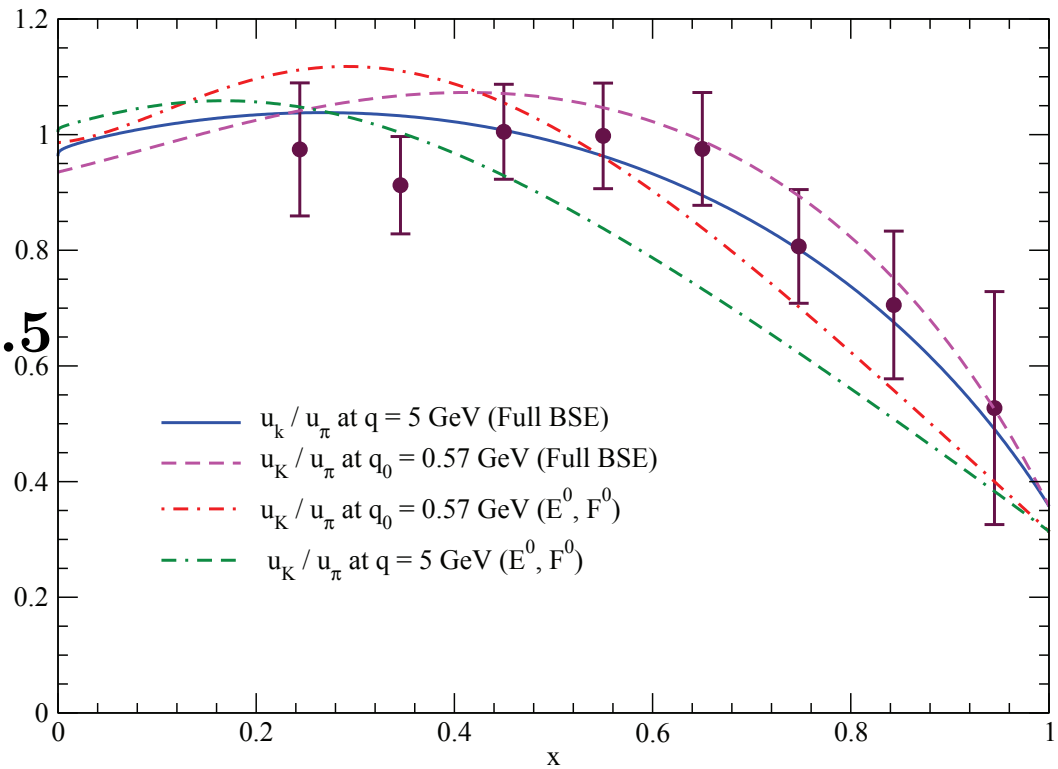


T. Nguyen: PhD Thesis (Kent State U.)
Nguyen, Tandy, Bashir, Roberts, in progress
Holt & Roberts: arXiv:1002.4666 [nucl-th]

Ratio – Kaon/Pion u-valence distribution

- DSE–result obtained using interaction that predicted $F_\pi(Q^2)$
- Influence of $M(p^2)$ felt strongly for $x > 0.5$
- QCD-$M(p^2) \Rightarrow$ prediction: $u_{\pi,K}(x) \propto (1-x)^2$ at resolving-scale $Q_0 = 0.6$ GeV

data: Badier, *et al.* , Phys. Lett. **B 93** (1980) 354

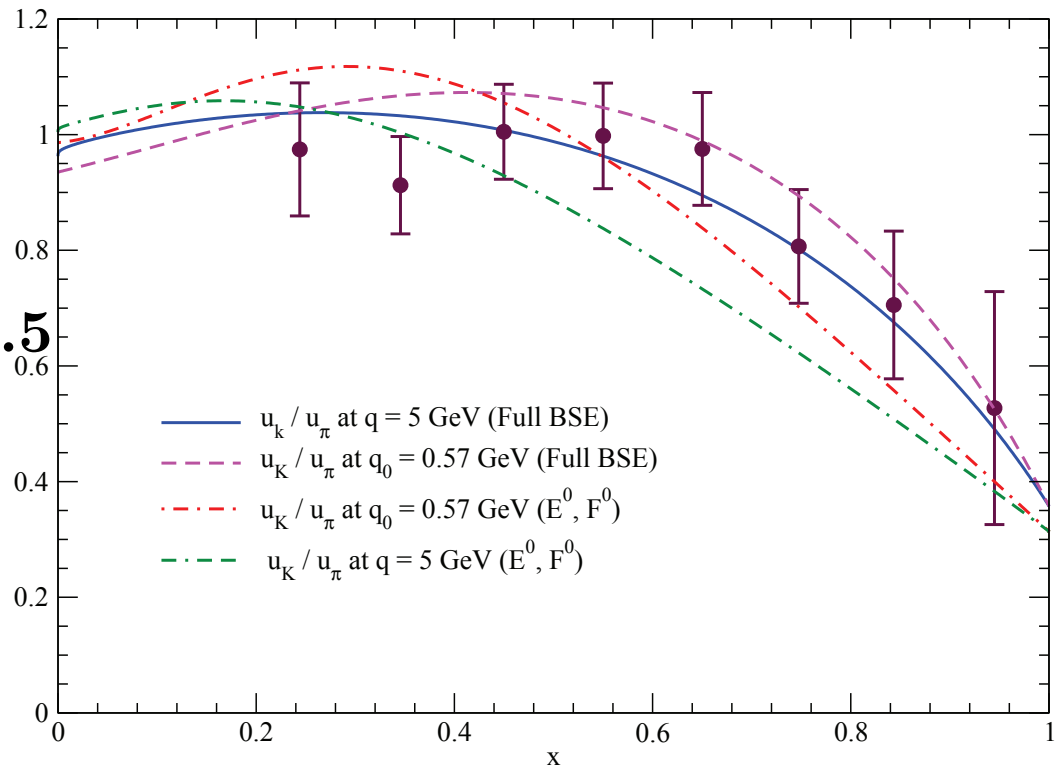


T. Nguyen: PhD Thesis (Kent State U.)
Nguyen, Tandy, Bashir, Roberts, in progress
Holt & Roberts: arXiv:1002.4666 [nucl-th]

Ratio – Kaon/Pion u-valence distribution

- DSE–result obtained using interaction that predicted $F_\pi(Q^2)$
- Influence of $M(p^2)$ felt strongly for $x > 0.5$
- QCD-$M(p^2) \Rightarrow$ prediction: $u_{\pi,K}(x) \propto (1-x)^2$ at resolving-scale $Q_0 = 0.6$ GeV

data: Badier, *et al.* , Phys. Lett. **B 93** (1980) 354



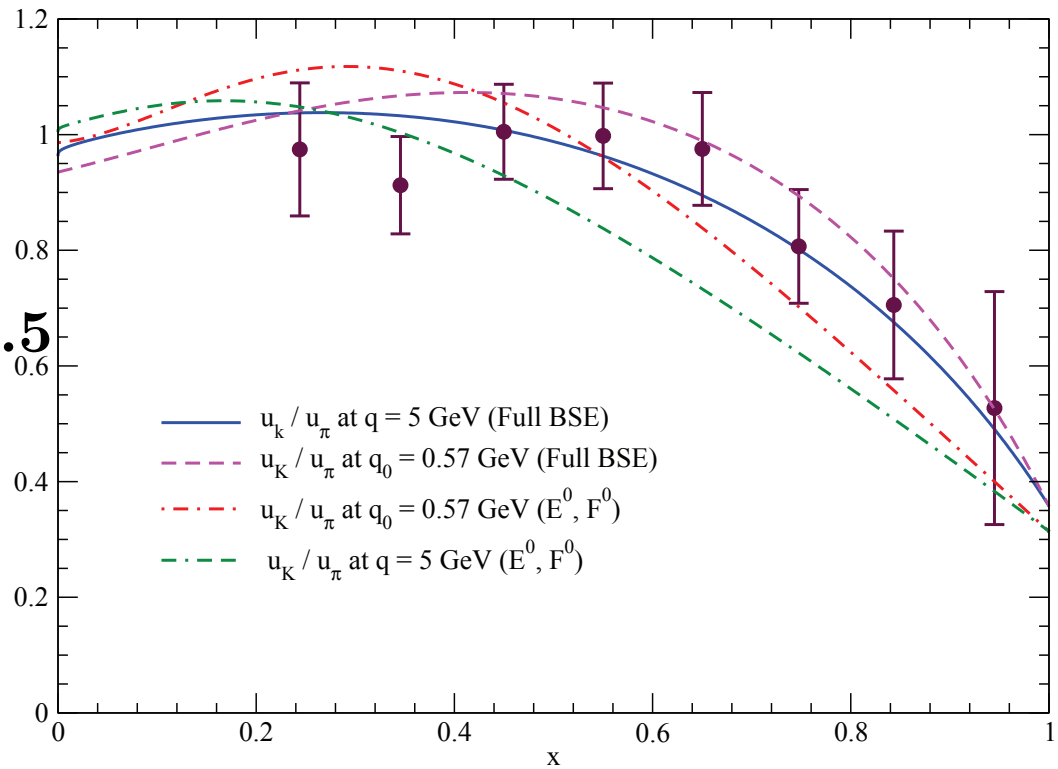
- $u_{\pi,K}(x=1)$ invariant under DGLAP-evolution

T. Nguyen: PhD Thesis (Kent State U.)
Nguyen, Tandy, Bashir, Roberts, in progress
Holt & Roberts: arXiv:1002.4666 [nucl-th]

Ratio – Kaon/Pion u-valence distribution

- DSE–result obtained using interaction that predicted $F_\pi(Q^2)$
- Influence of $M(p^2)$ felt strongly for $x > 0.5$
- QCD-$M(p^2) \Rightarrow$ prediction: $u_{\pi,K}(x) \propto (1-x)^2$ at resolving-scale $Q_0 = 0.6$ GeV

data: Badier, *et al.* , Phys. Lett. **B 93** (1980) 354



- $u_{\pi,K}(x=1)$ invariant under DGLAP-evolution
- Accessible at Upgraded JLab & Electron-Ion Collider

Some projects currently underway

- Elucidate effects of confinement & DCSB in
  - light-quark meson spectrum, including so-called hybrids and exotics, using Poincaré-covariant symmetry-preserving Bethe-Salpeter equation (*L. Chang, arXiv:0903.5461 [nucl-th]*)
  - hadron valence-quark distribution functions (*A. Bashir, P.C. Tandy*)

Some projects currently underway

- Elucidate effects of confinement & DCSB in
  - light-quark meson spectrum, including so-called hybrids and exotics, using Poincaré-covariant symmetry-preserving Bethe-Salpeter equation (*L. Chang, arXiv:0903.5461 [nucl-th]*)
  - hadron valence-quark distribution functions (*A. Bashir, P.C. Tandy*)
- Following extensive study of nucleon elastic electromagnetic form factors,

  Survey of nucleon electromagnetic form factors
  I.C. Cloët *et al.*, arXiv:0812.0416 [nucl-th], Few Body Syst. 46 (2009) pp. 1-36

  with numerous predictions, either verified by experiment or serving to motivate new experiments, studies are underway to elucidate signals of $M(p^2)$ in Q^2-evolution of nucleon elastic and transition form factors; viz.,
  - $N \to \Delta$
  - $N \to$ P11(1440)
  - $\kappa(p^2)$

  e.g., $F_1^{\rm d}(Q^2) = 0$ at $Q^2/M^2 \approx 5$

  (*M. Bhagwat, L. Chang, I. Cloët, H. Roberts*)

Some projects currently underway

- Elucidate effects of confinement & DCSB in
  - light-quark meson spectrum, including so-called hybrids and exotics, using Poincaré-covariant symmetry-preserving Bethe-Salpeter equation (*L. Chang, arXiv:0903.5461 [nucl-th]*)
  - hadron valence-quark distribution functions (*A. Bashir, P.C. Tandy*)
- Following extensive study of nucleon elastic electromagnetic form factors,

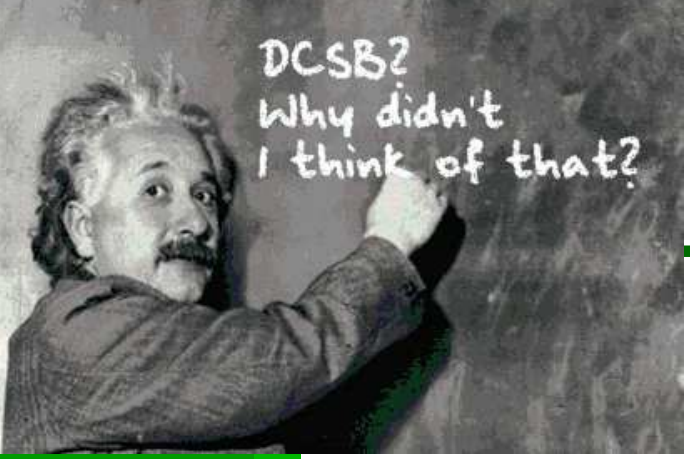
  Survey of nucleon electromagnetic form factors
  I.C. Cloët *et al.*, arXiv:0812.0416 [nucl-th], Few Body Syst. 46 (2009) pp. 1-36

  with numerous predictions, either verified by experiment or serving to motivate new experiments, studies are underway to elucidate signals of $M(p^2)$ in Q^2-evolution of nucleon elastic and transition form factors; viz.,
  - $N \to \Delta$
  - $N \to \mathrm{P11}(1440)$
  - $\kappa(p^2)$

  e.g., $F_1^{\mathrm{d}}(Q^2) = 0$ at $Q^2/M^2 \approx 5$

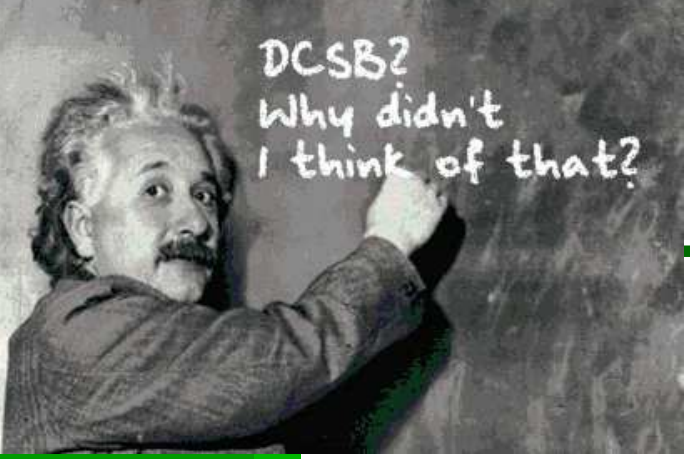
  (*M. Bhagwat, L. Chang, I. Cloët, H. Roberts*)
- Incorporate "resonant contributions" (pion cloud) in kernels of bound-state equations (e.g., Eichmann, Roberts *et al.* – 0802.1948 [nucl-th] & Cloët, Roberts – 0811.2018 [nucl-th]; and *Fischer, Williams* – 0808.3372 [hep-ph])

Epilogue



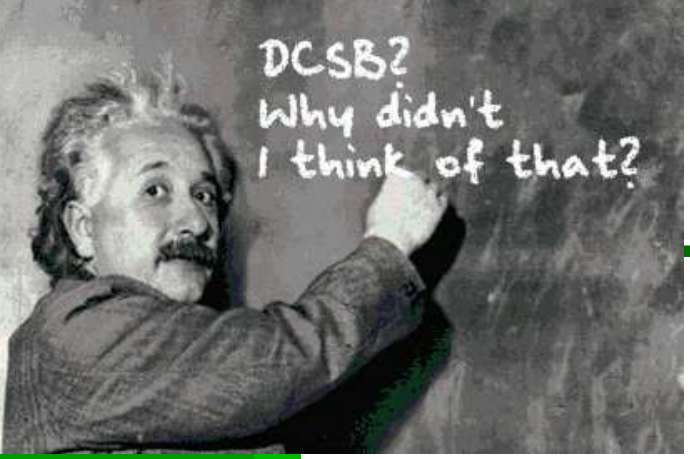
Epilogue

- DCSB exists in QCD.



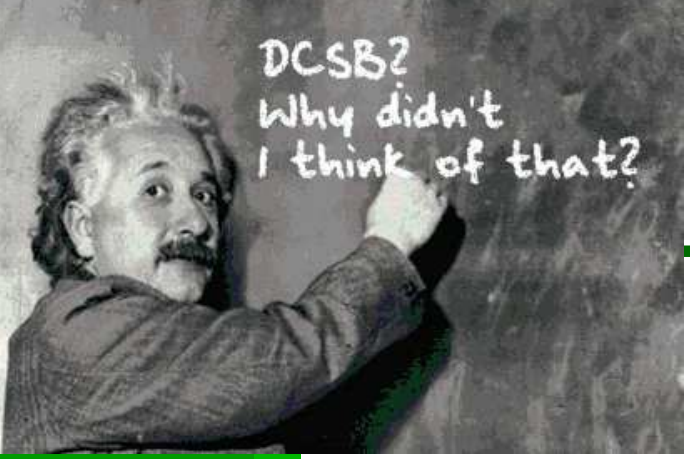
Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices



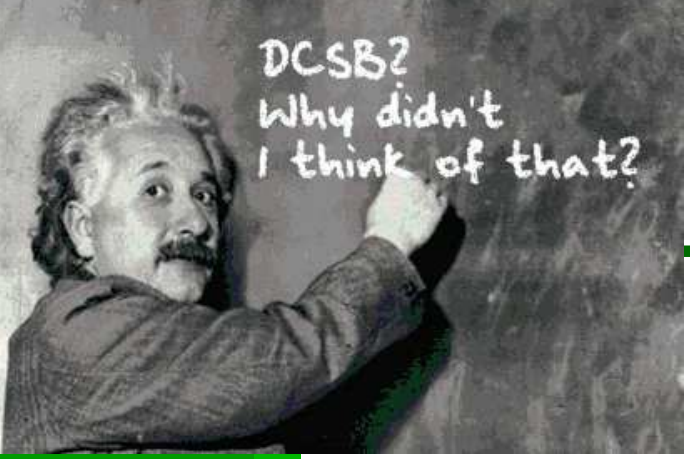
Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
  - It predicts, amongst other things, that
    - light current-quarks become heavy constituent-quarks: 4 $\rightarrow$ 400 MeV
    - pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140$ MeV
    - pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi\bar{q}q} \approx 4.3$
    - pseudscalar mesons couple unnaturally strongly to the lightest baryons $g_{\pi\bar{N}N} \approx 12.8 \approx 3 g_{\pi\bar{q}q}$



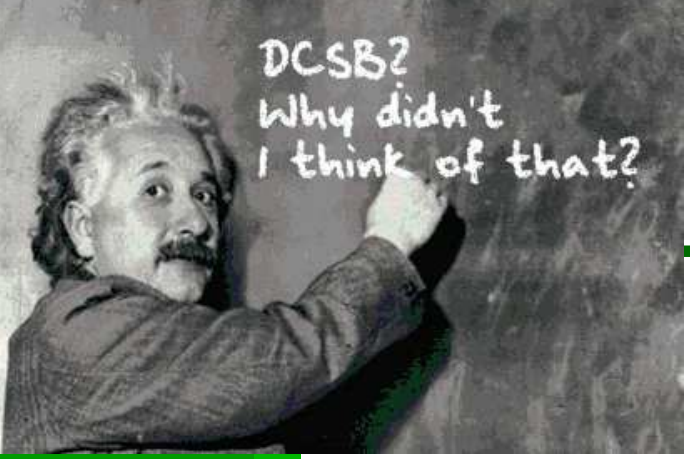
Epilogue

- DCSB impacts dramatically upon observables



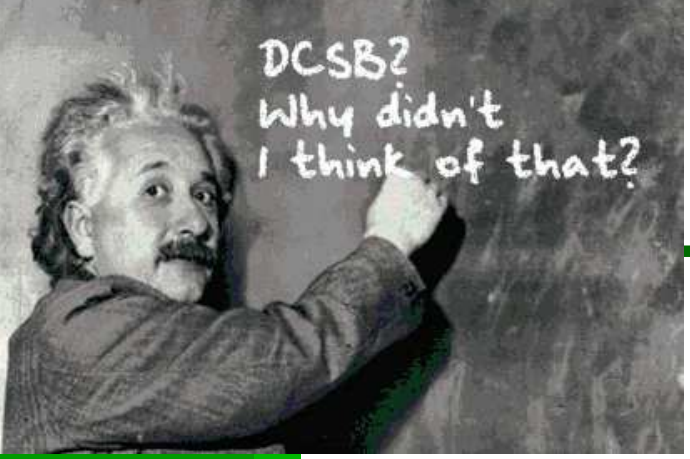
Epilogue

- DCSB impacts dramatically upon observables
  - Spectrum; e.g., splittings: σ–π & a_1–ρ
  - Elastic and Transition Form Factors



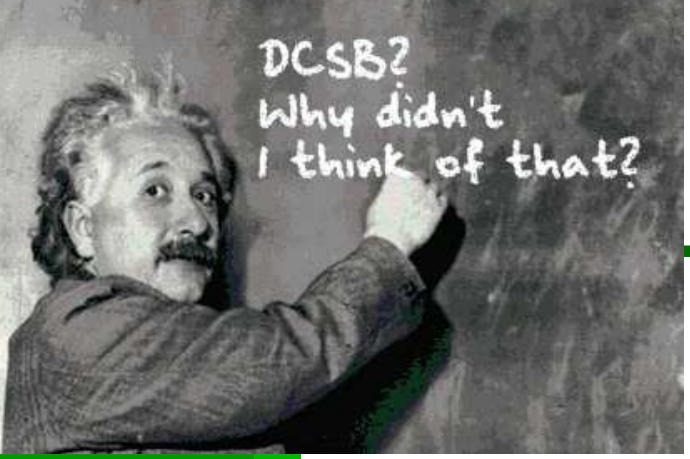
Epilogue

- DCSB impacts dramatically upon observables
  - Spectrum; e.g., splittings: σ–π & a_1–ρ
  - Elastic and Transition Form Factors
- But $M(p^2)$ is an *essentially* quantum field theoretical effect
  - Exposing & elucidating its effect in hadron physics requires nonperturbative, symmetry preserving framework; i.e., Poincaré covariance, chiral and e.m. current conservation, etc.



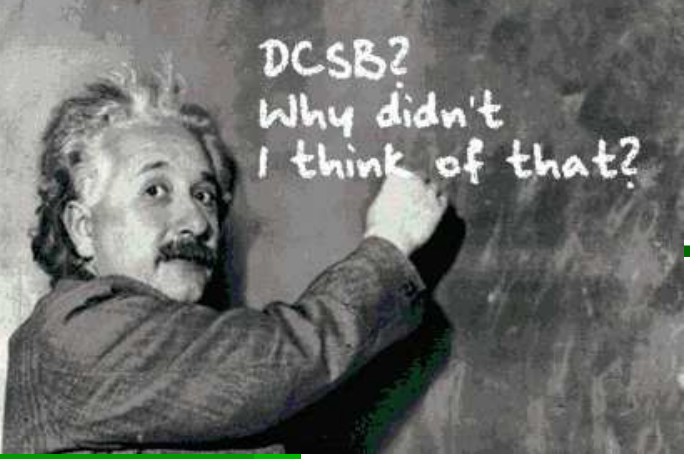
Epilogue

- DCSB impacts dramatically upon observables
  - Spectrum; e.g., splittings: σ–π & a_1–ρ
  - Elastic and Transition Form Factors
- But $M(p^2)$ is an *essentially* quantum field theoretical effect
  - Exposing & elucidating its effect in hadron physics requires nonperturbative, symmetry preserving framework; i.e., Poincaré covariance, chiral and e.m. current conservation, etc.
- DSEs provide such a framework.
  - Studies underway will identify observable signals of $M(p^2)$, the most important mass-generating mechanism for visible matter in the Universe



Epilogue

- DCSB impacts dramatically upon observables
  - Spectrum; e.g., splittings: σ–π & a_1–ρ
  - Elastic and Transition Form Factors
- But $M(p^2)$ is an *essentially* quantum field theoretical effect
  - Exposing & elucidating its effect in hadron physics requires nonperturbative, symmetry preserving framework; i.e., Poincaré covariance, chiral and e.m. current conservation, etc.
- DSEs provide such a framework.
  - Studies underway will identify observable signals of $M(p^2)$, the most important mass-generating mechanism for visible matter in the Universe
- DSEs: Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks

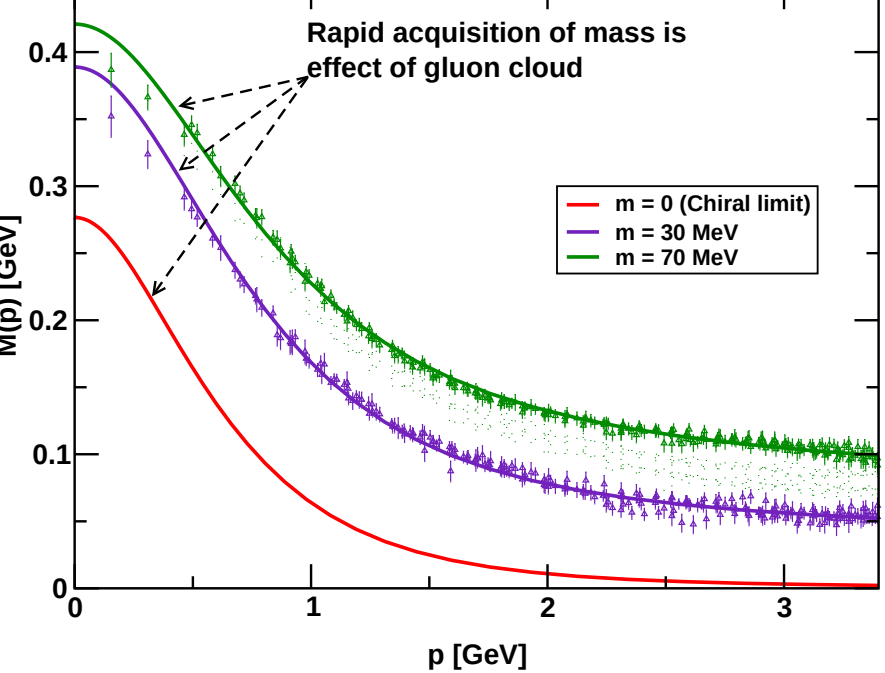


Epilogue

Now is an exciting time . . .

Positioned to unify phenomena as apparently disparate as

- Hadron spectrum
- Elastic and transition form factors, from small- to large-Q^2
- Parton distribution functions



Epilogue

Now is an exciting time . . .

Positioned to unify phenomena as apparently disparate as

- Hadron spectrum
- Elastic and transition form factors, from small- to large-Q^2
- Parton distribution functions

Key: an understanding of both the fundamental origin of nuclear mass and the far-reaching consequences of the mechanism responsible; namely, Dynamical Chiral Symmetry Breaking

Contents

1. Universal Truths
2. QCD's Challenges
3. Charting the Interaction
4. Radial Excitations & χ-Symmetry
5. Radial Excitations & χ-Symmetry II
6. Radial Excitations& Lattice-QCD
7. Bound-state DSE
8. BSE – General Form
9. $\boldsymbol{a_1 - \rho}$
10. Quark Anom. Mag. Moms.
11. Frontiers of Nuclear Science
12. Goldberger-Treiman for pion
13. GT – Contact Interaction
14. Computation: $F_\pi(Q^2)$
15. Pion's valence distribution
16. Kaon/Pion u-valence distribution
17. Current Projects