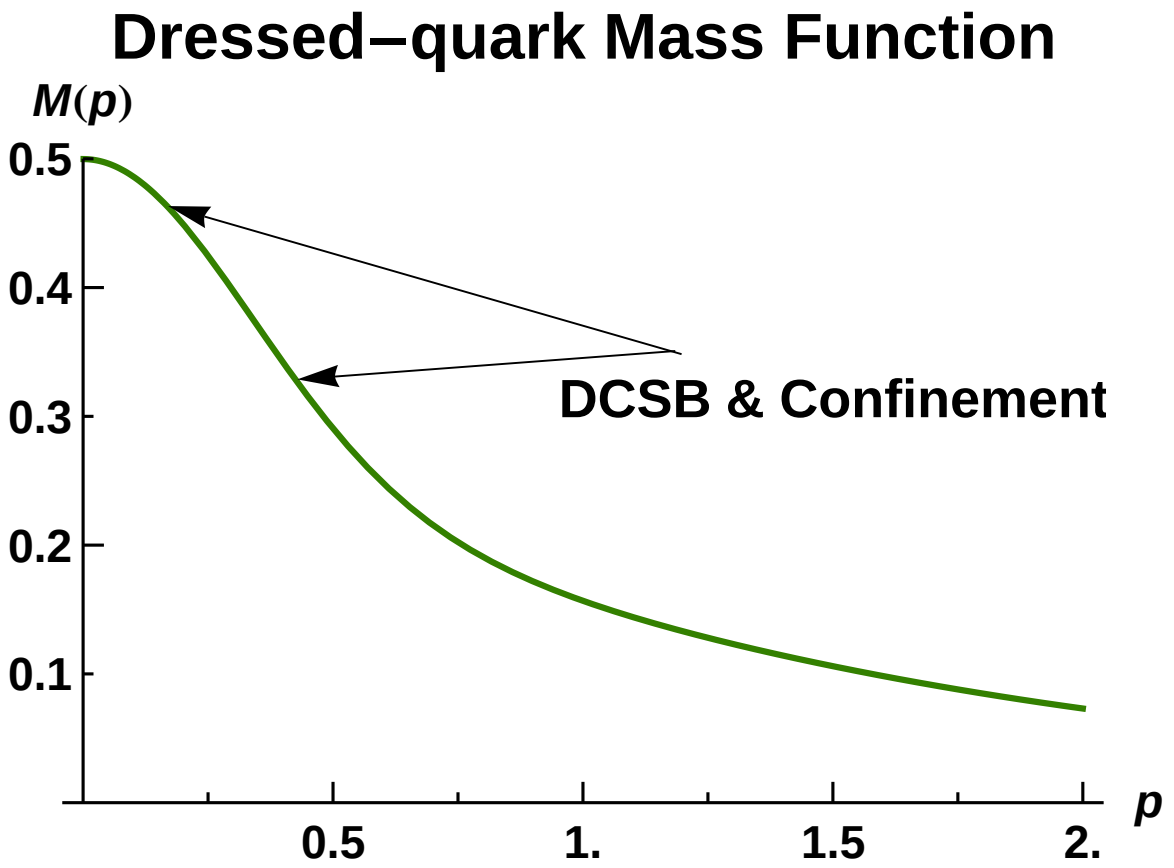
The Essence of Matter



Craig D. Roberts
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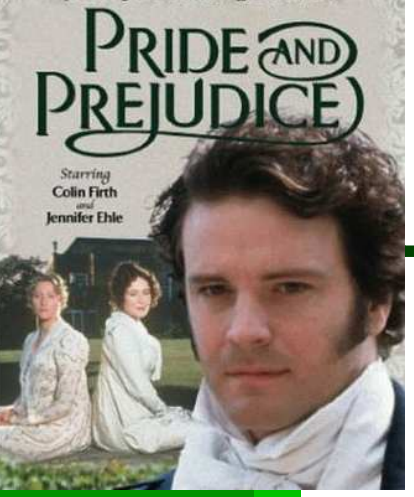
Physics Division & **School of Physics**

Argonne National Laboratory **Peking University**

http://www.phy.anl.gov/theory/staff/cdr.html

Universal Truths



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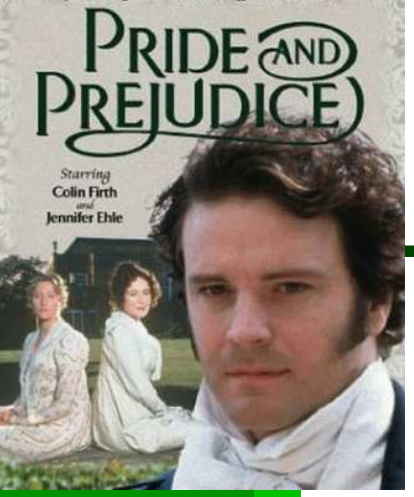

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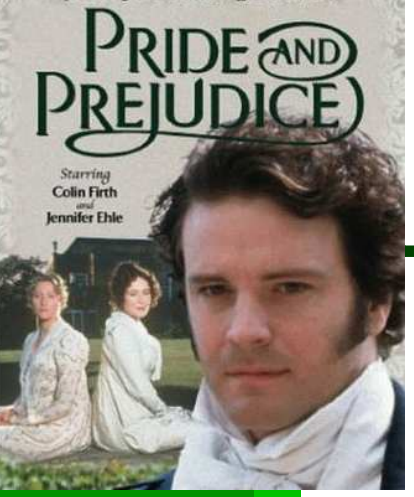

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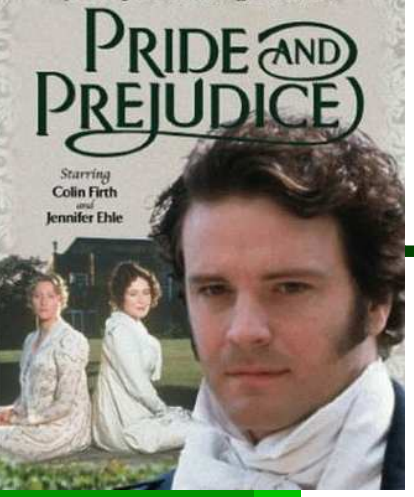

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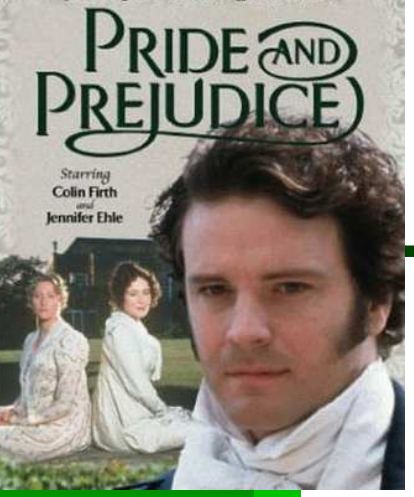

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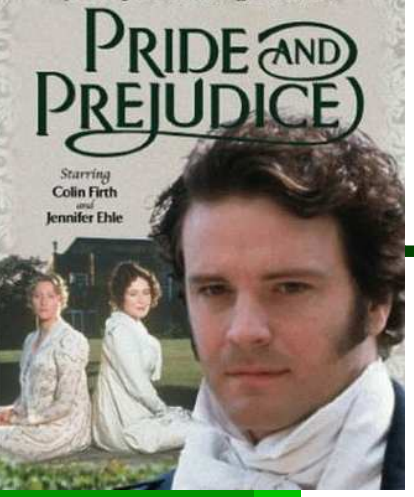

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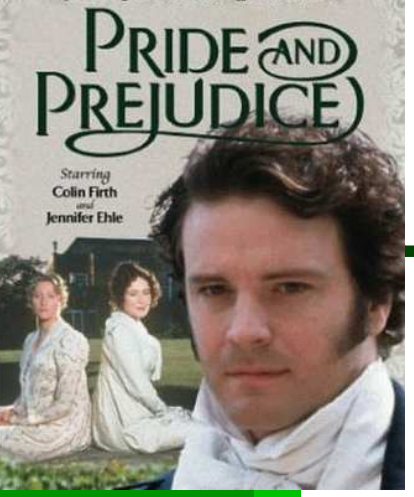

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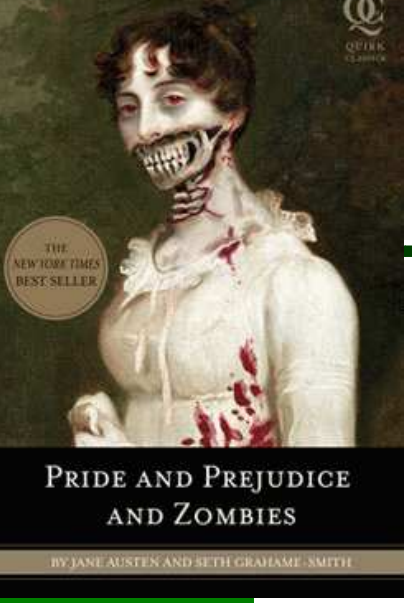

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QCD's Challenges

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- QCD – Complex behaviour arises from apparently simple rules

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Highly Nontrivial

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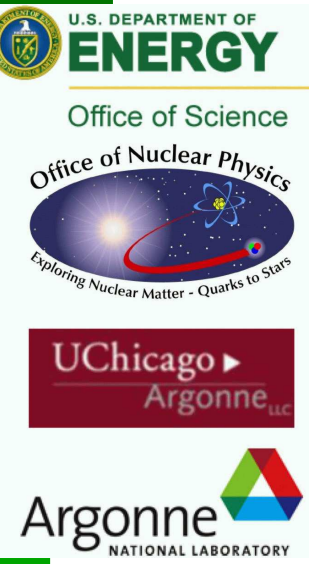

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Intranucleon Interaction

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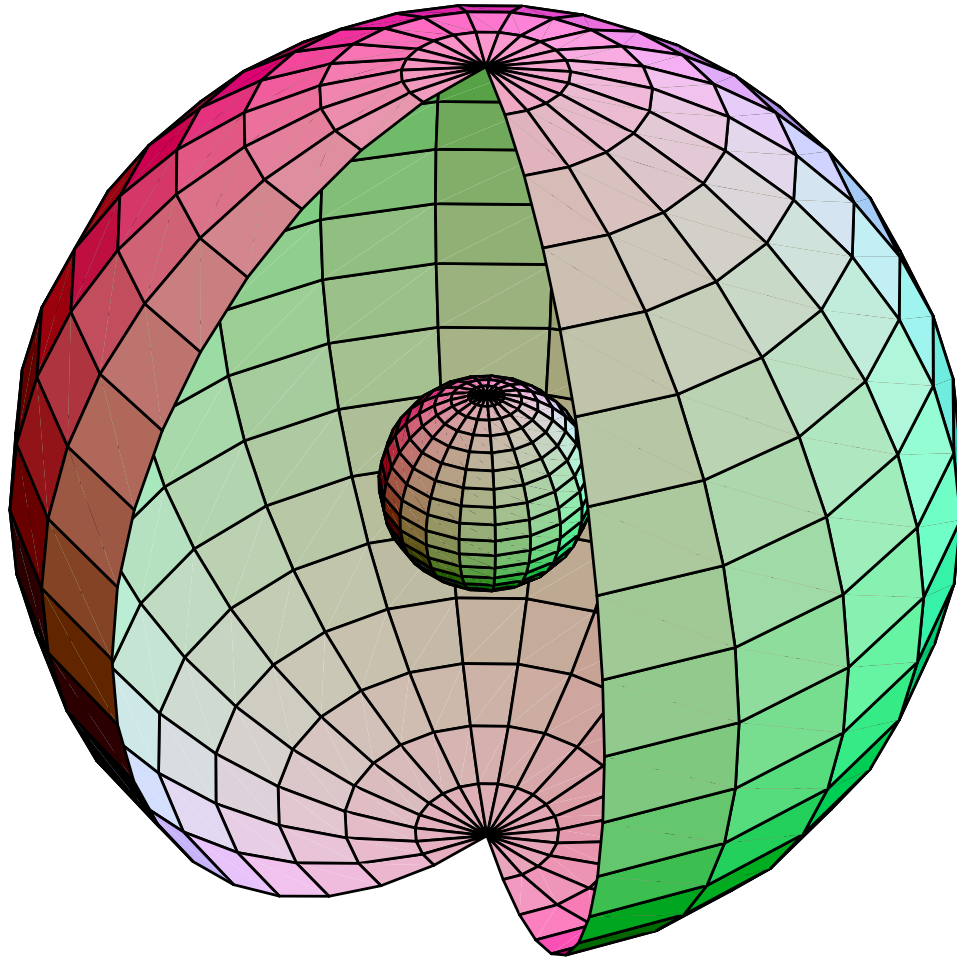

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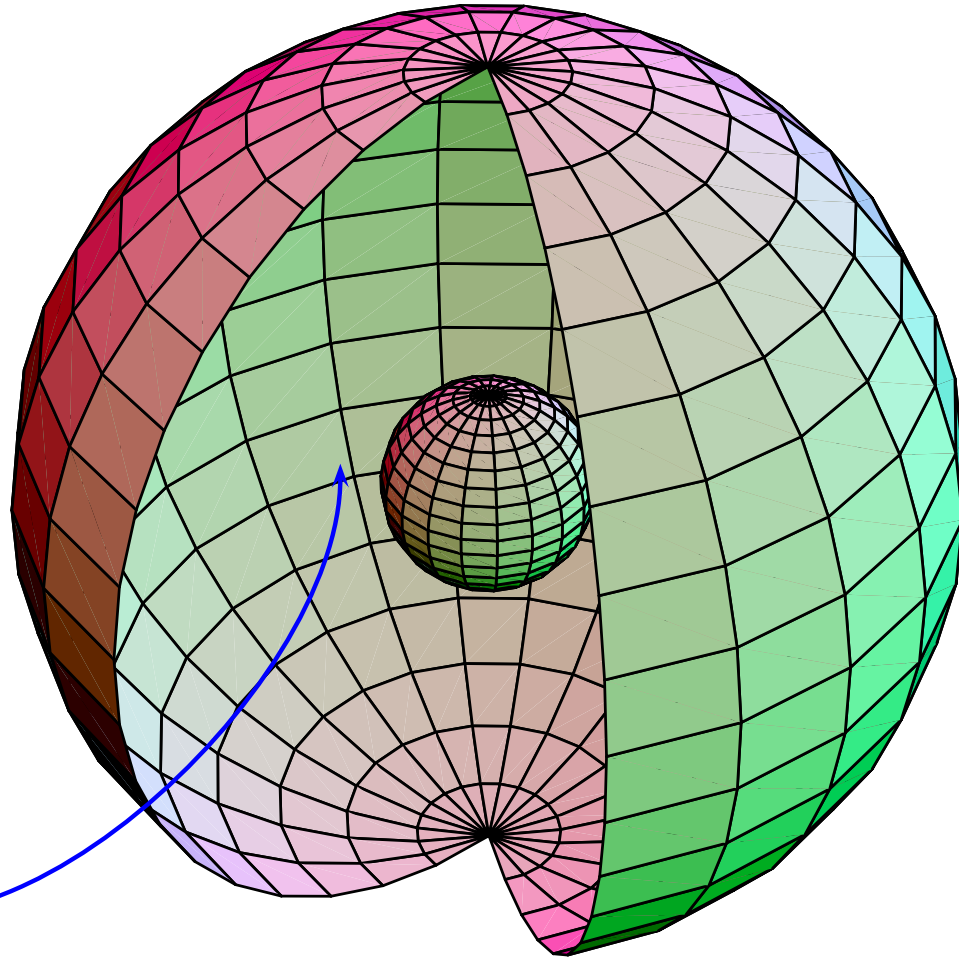
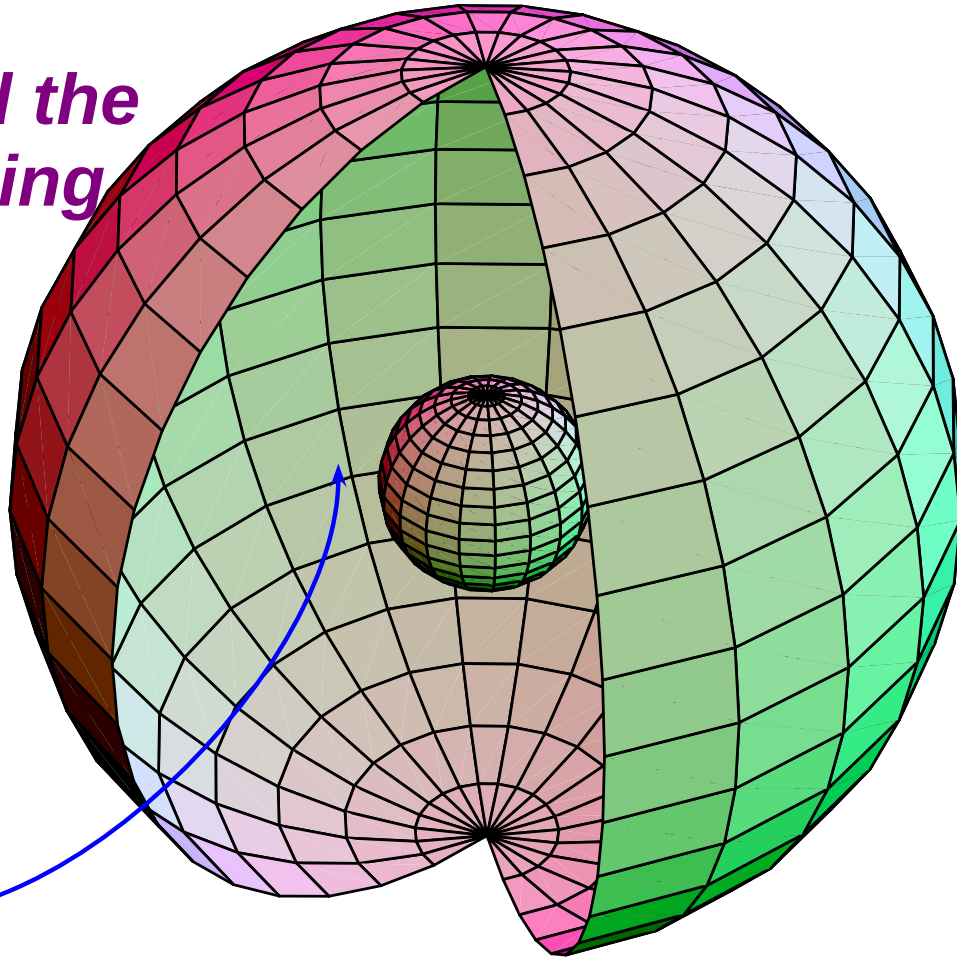


98% of the volume

What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



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Confinement

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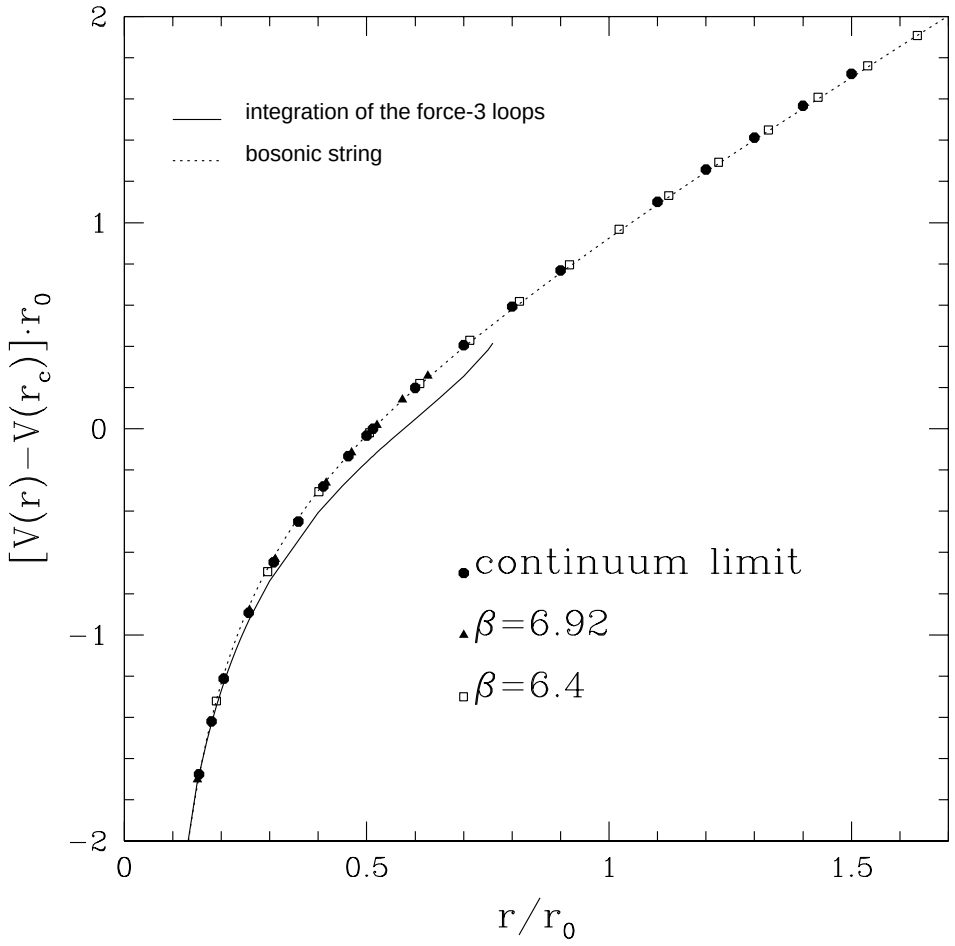
- Infinitely Heavy Quarks . . . Picture in Quantum Mechanics



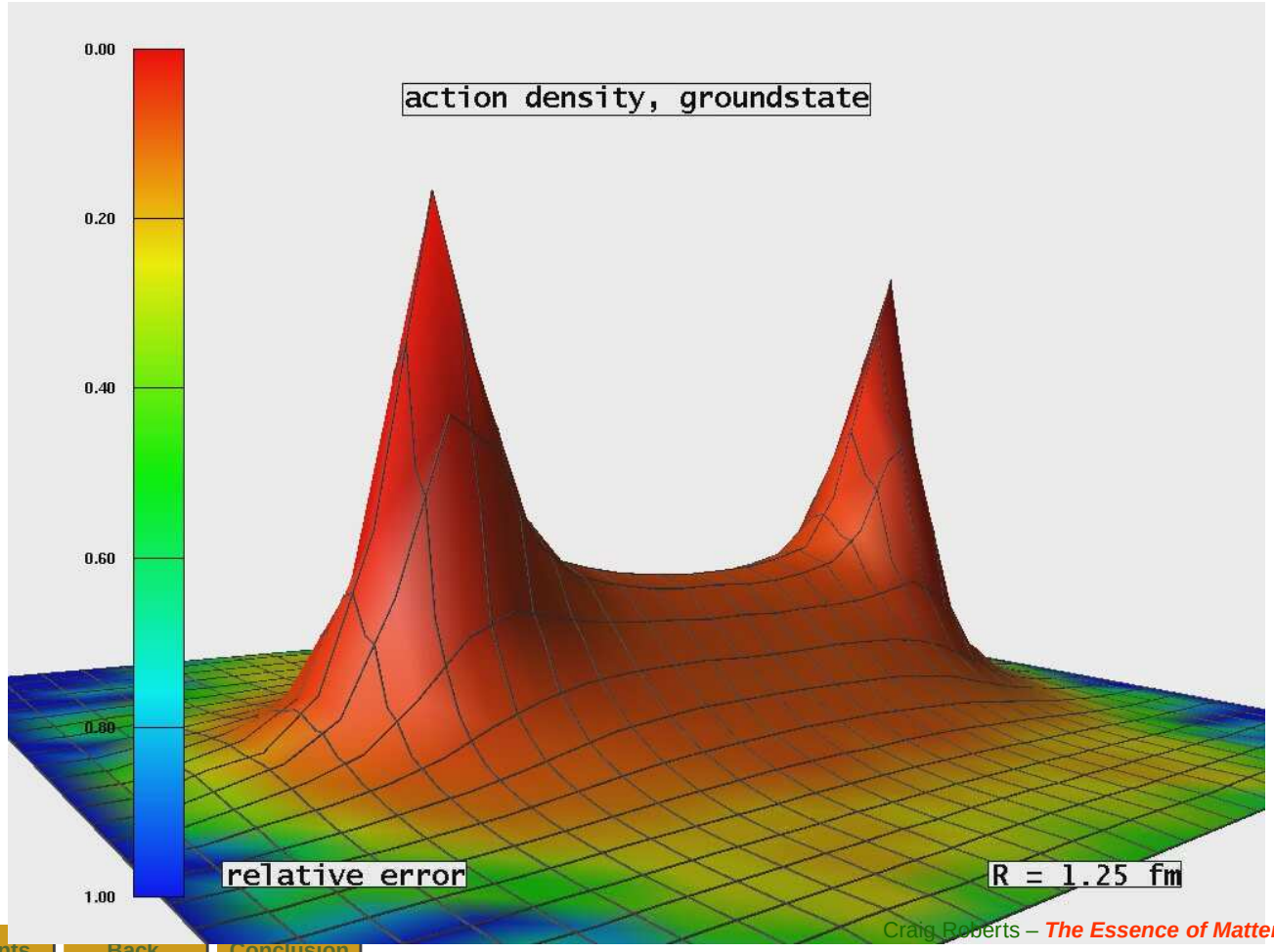
$$V(r) = \sigma\, r - \frac{\pi}{12}\,\frac{1}{r}$$

$$\sqrt{\sigma} \sim 470\,\text{MeV}$$

Necco & Sommer
he-la/0108008

Confinement

- Illustrate this in terms of the action density . . . analogous to plotting the Force $= F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12}\frac{1}{r^2}$



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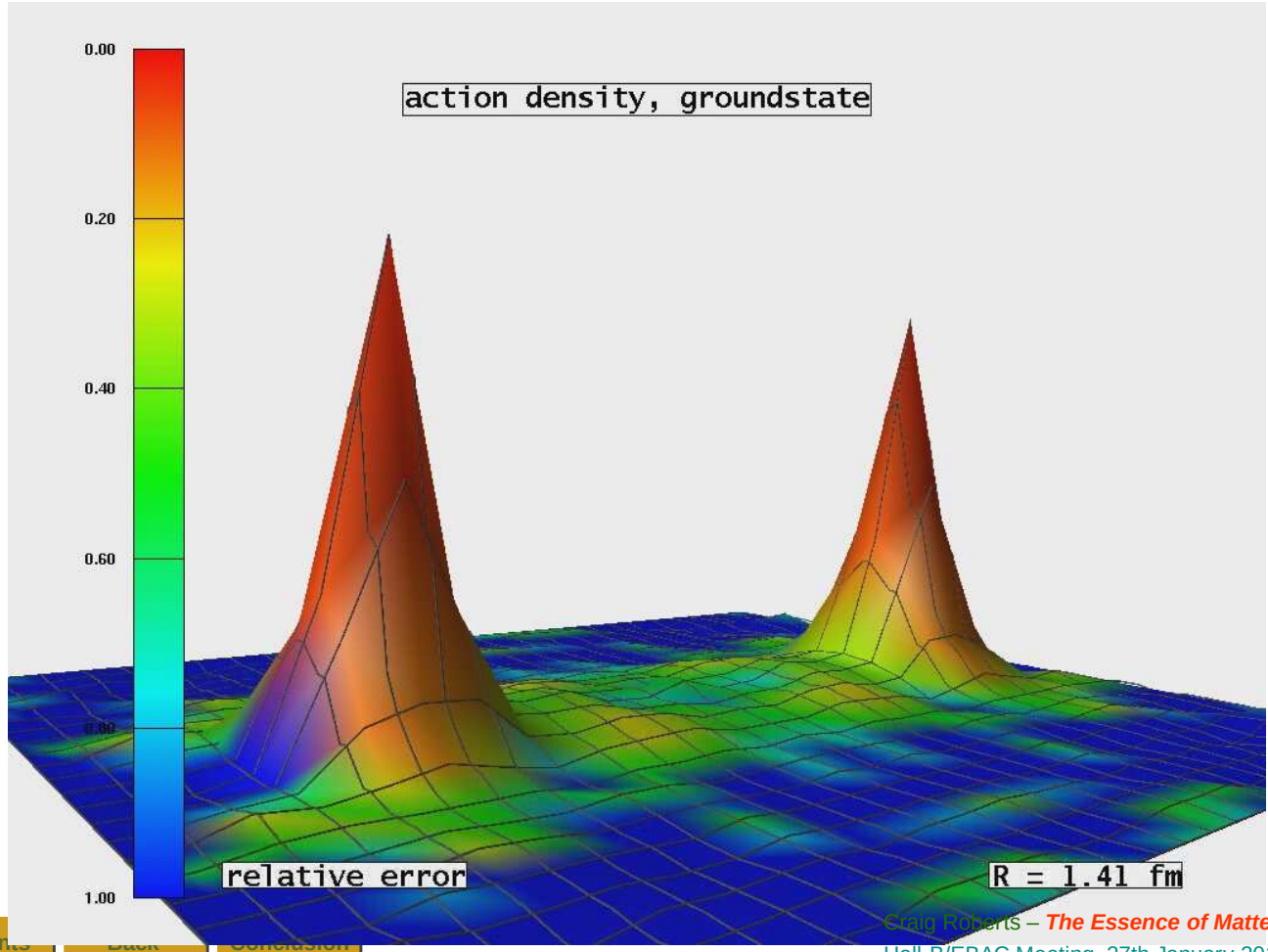

Bali, *et al.*
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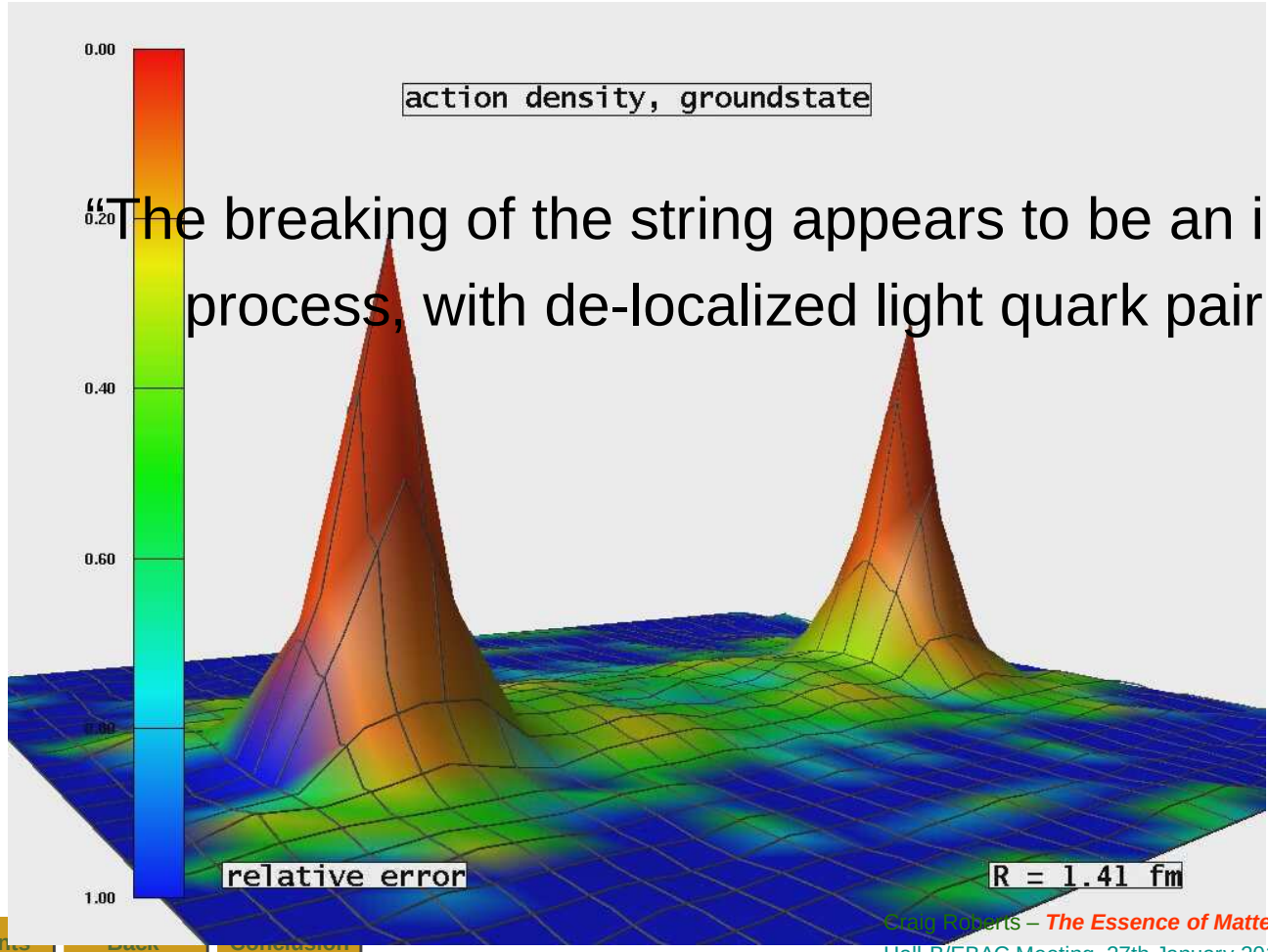

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"The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation."

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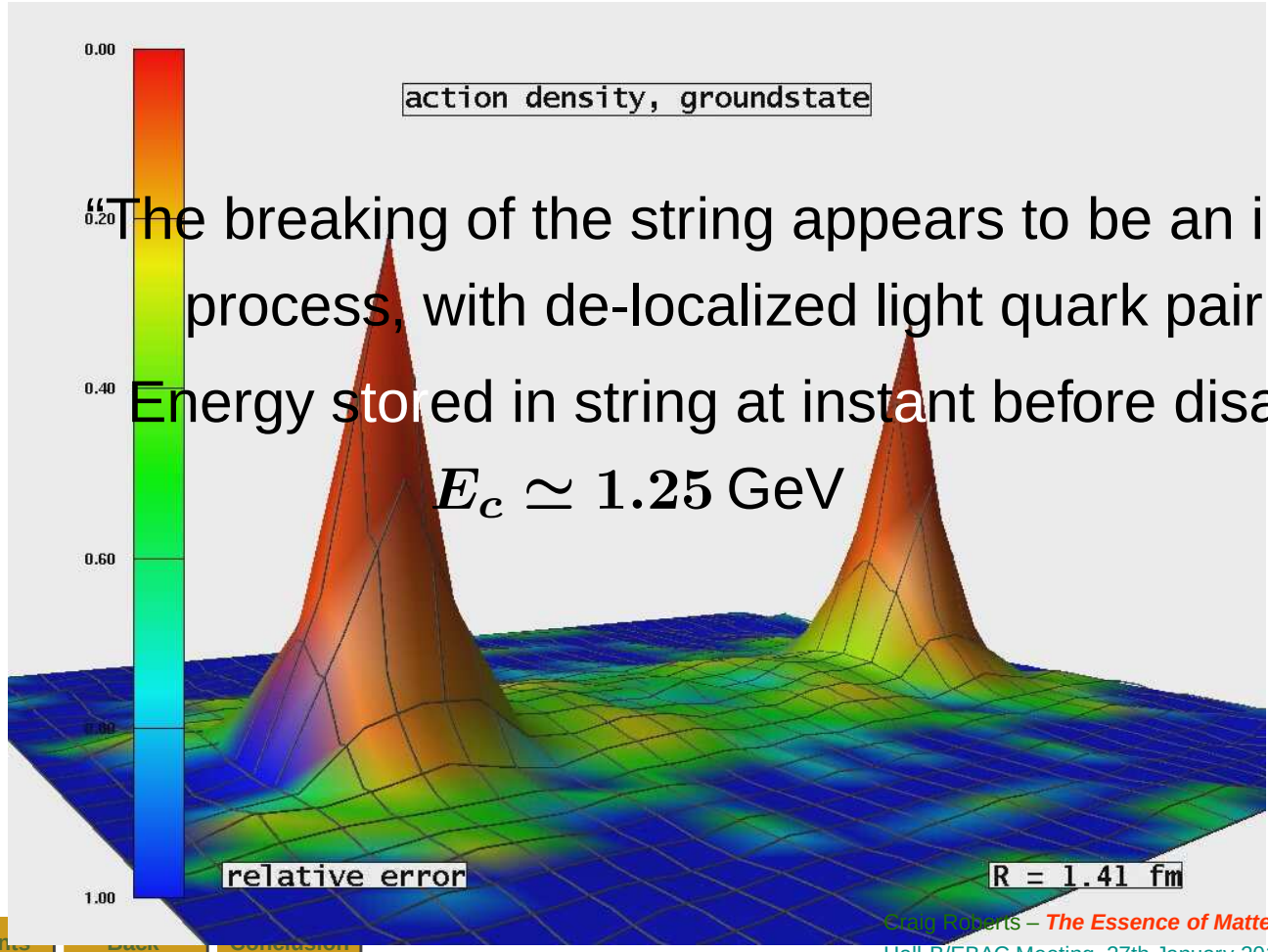

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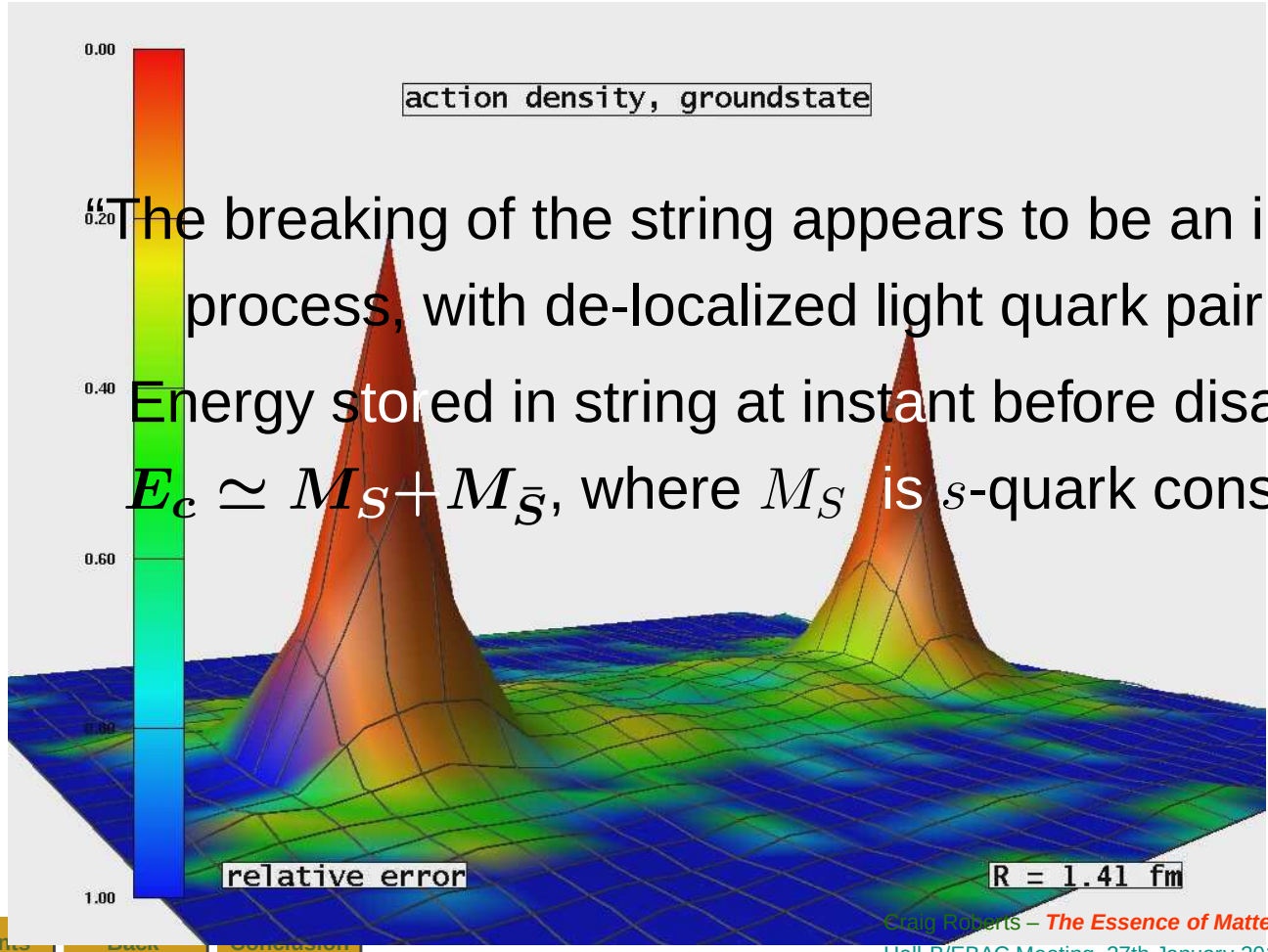

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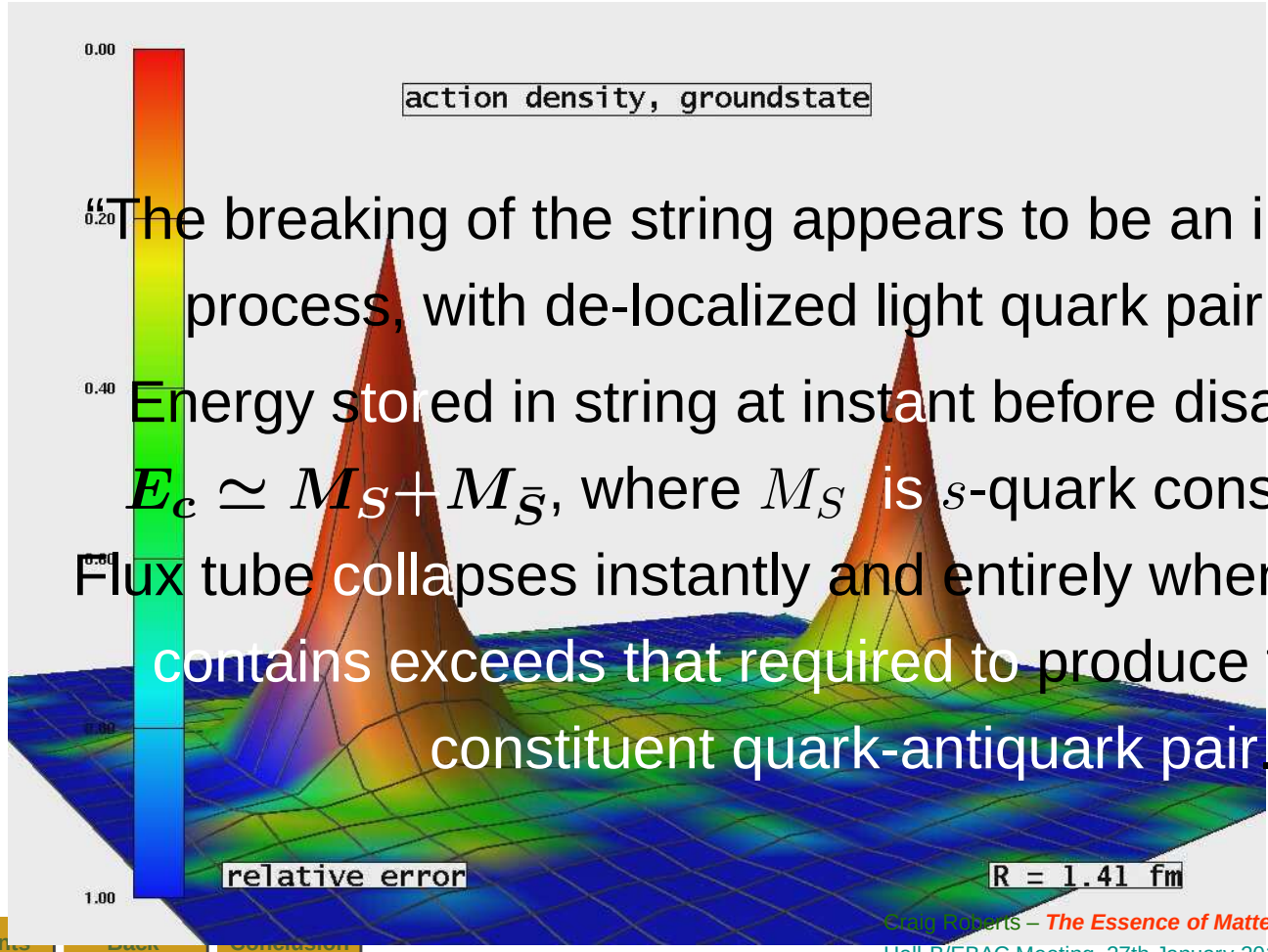

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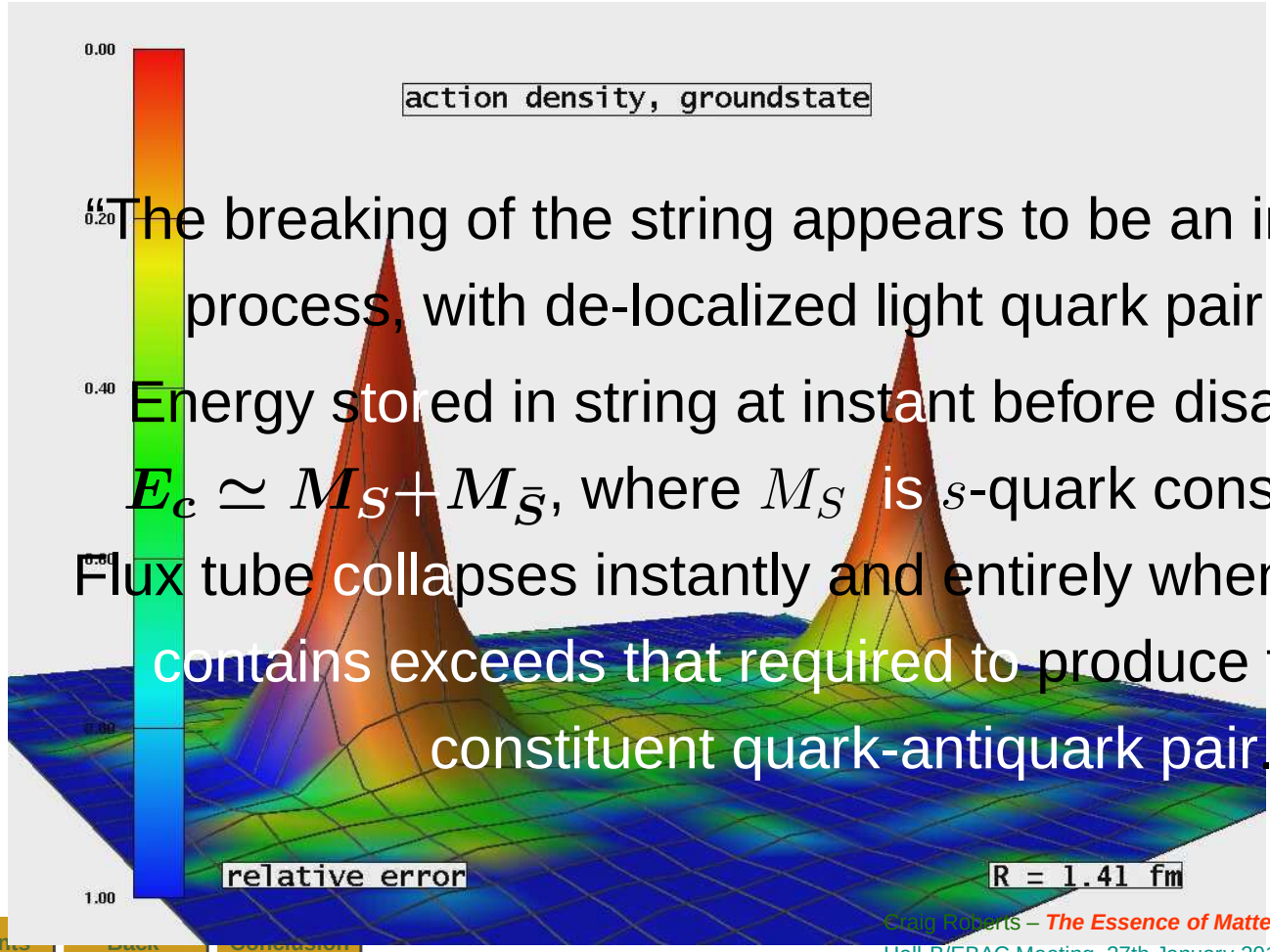

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Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

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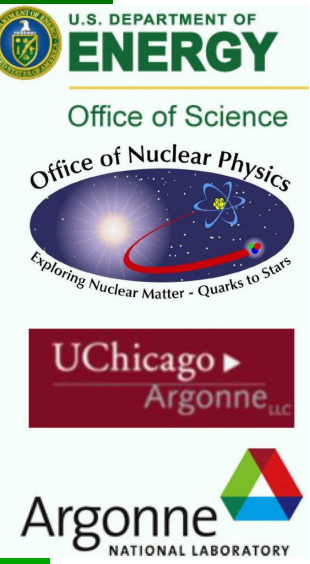

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Cross-Sections built from Schwinger Functions

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Schwinger Functions

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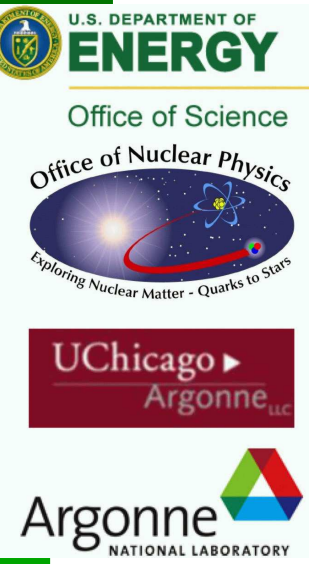

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- Opportunity for comparisons at pre-experimental level . . .
  - cross-fertilisation proving fruitful.

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- This is a well-posed problem whose solution is an elemental goal of modern hadron physics.

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- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations can be used to chart β-function's long-range behaviour
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  - Steady quantitative progress is being made with a scheme that is systematically improvable

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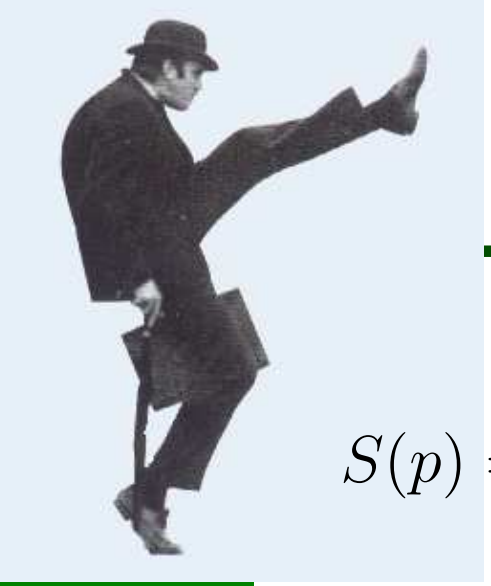
  - On other hand, at present significant qualitative advances possible with symmetry-preserving kernel *Ansätze* that express important additional nonperturbative effects – $M(p^2)$ – difficult/impossible to capture in any finite sum of contributions

Perturbative Dressed-quark Propagator



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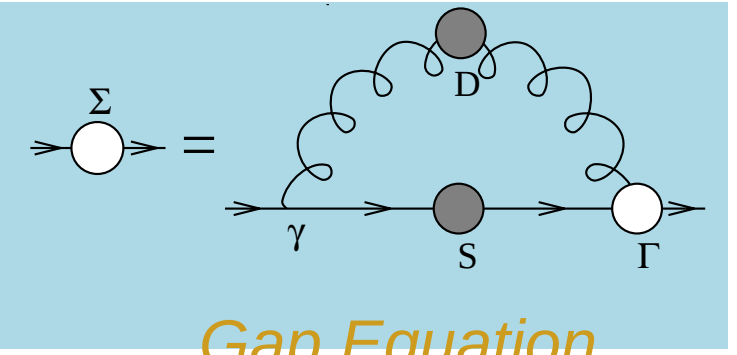
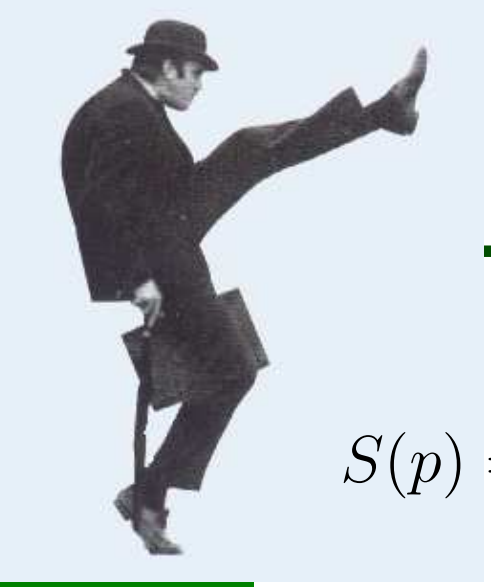
$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$



Gap Equation



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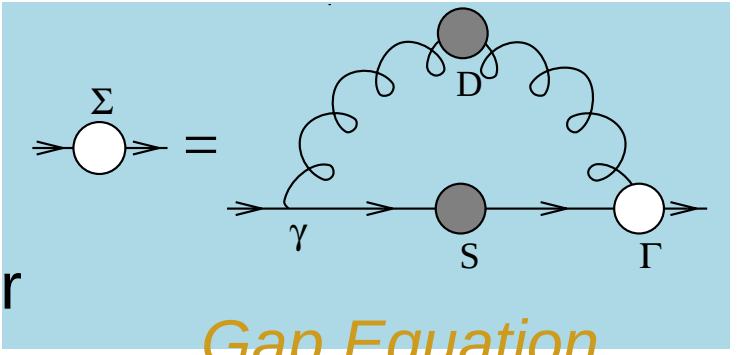

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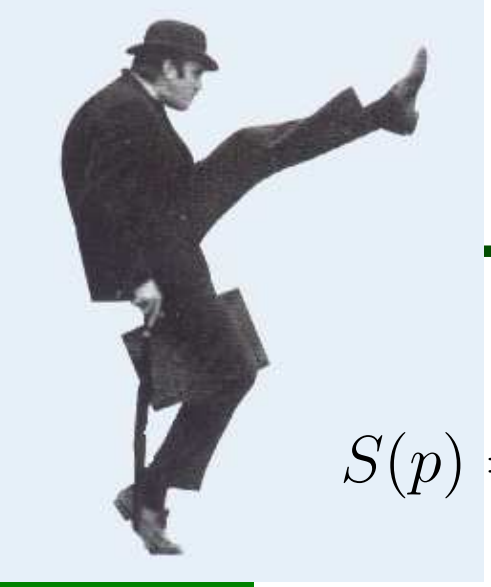

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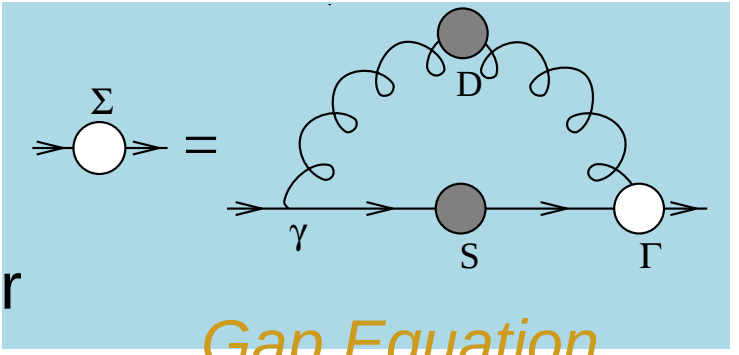

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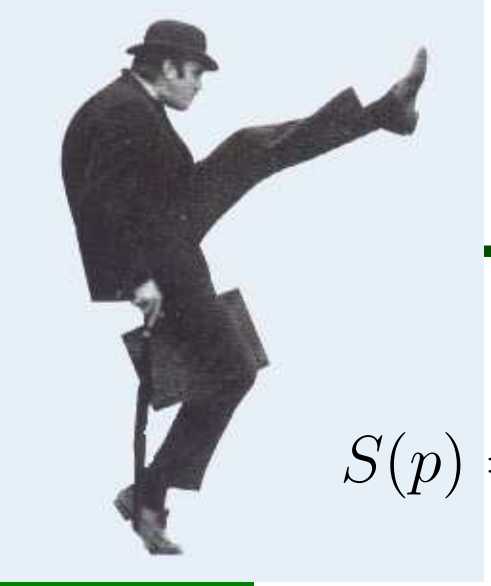
- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory



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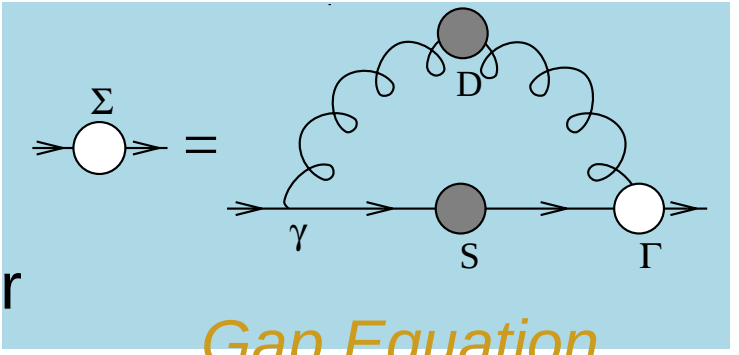

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$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\to} 0$$

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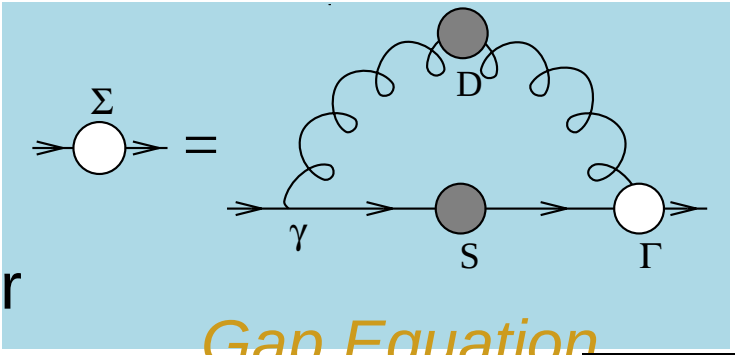

Gap Equation

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No DCSB Here!

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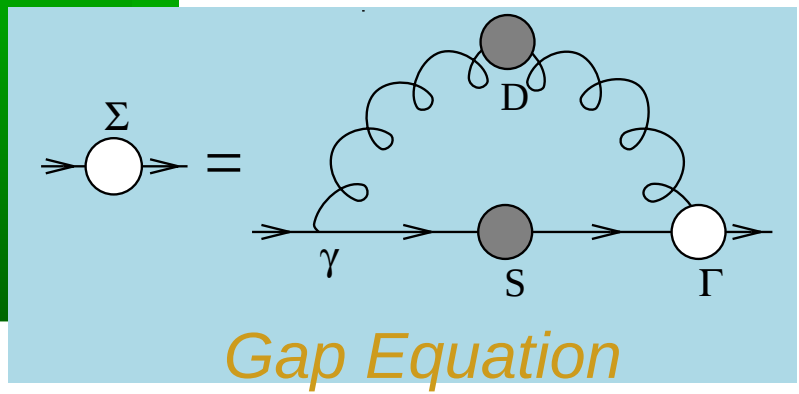
Frontiers of Nuclear Science: A Long Range Plan (2007)

Frontiers of Nuclear Science: Theoretical Advances



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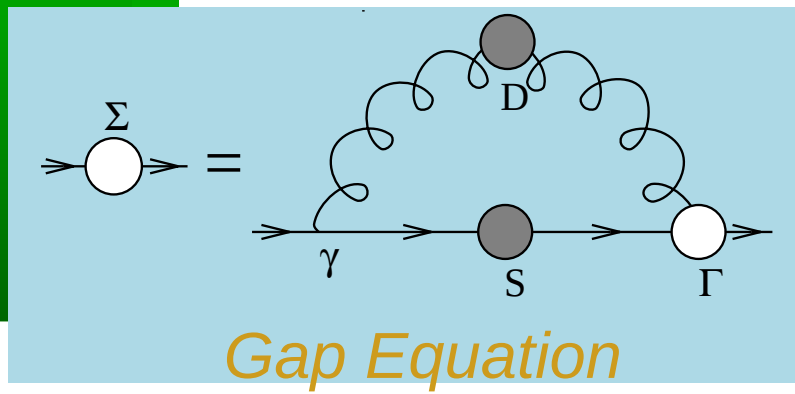

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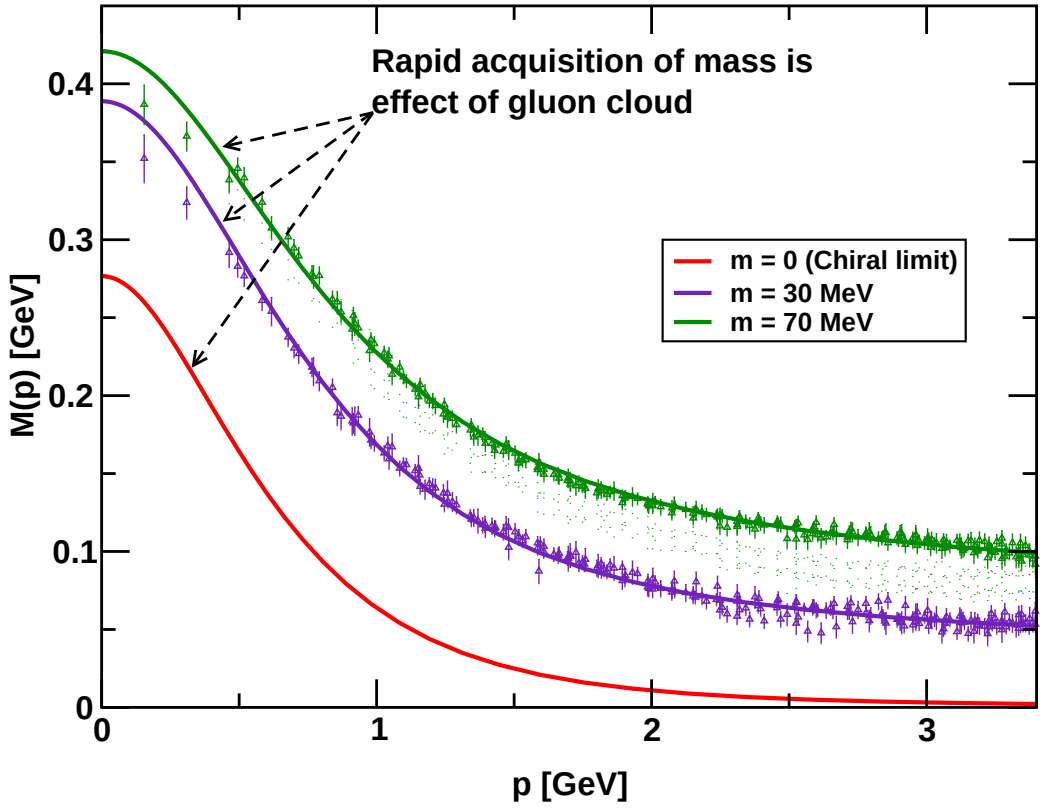

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Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.

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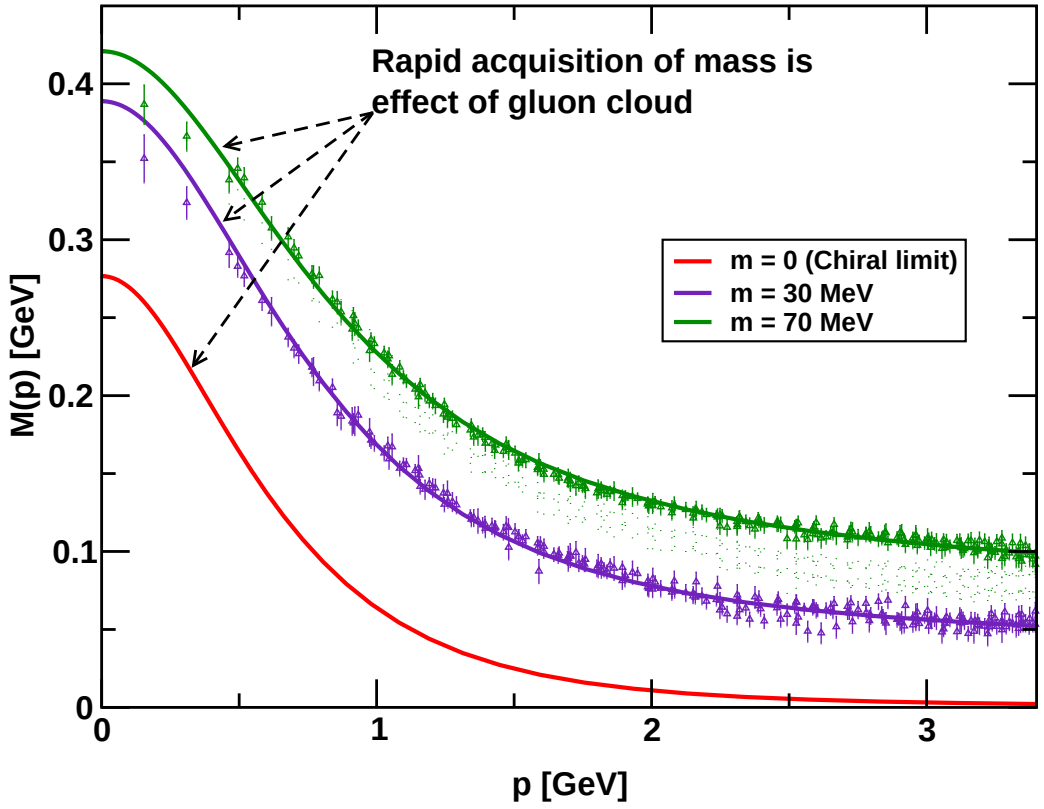

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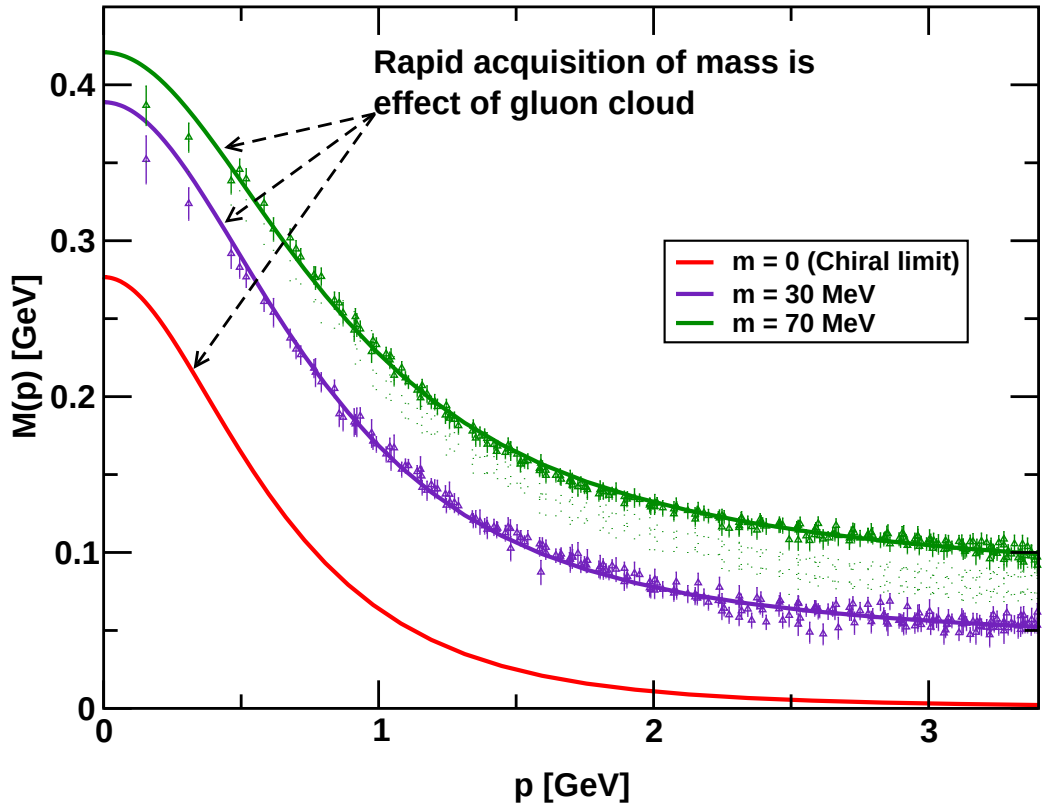

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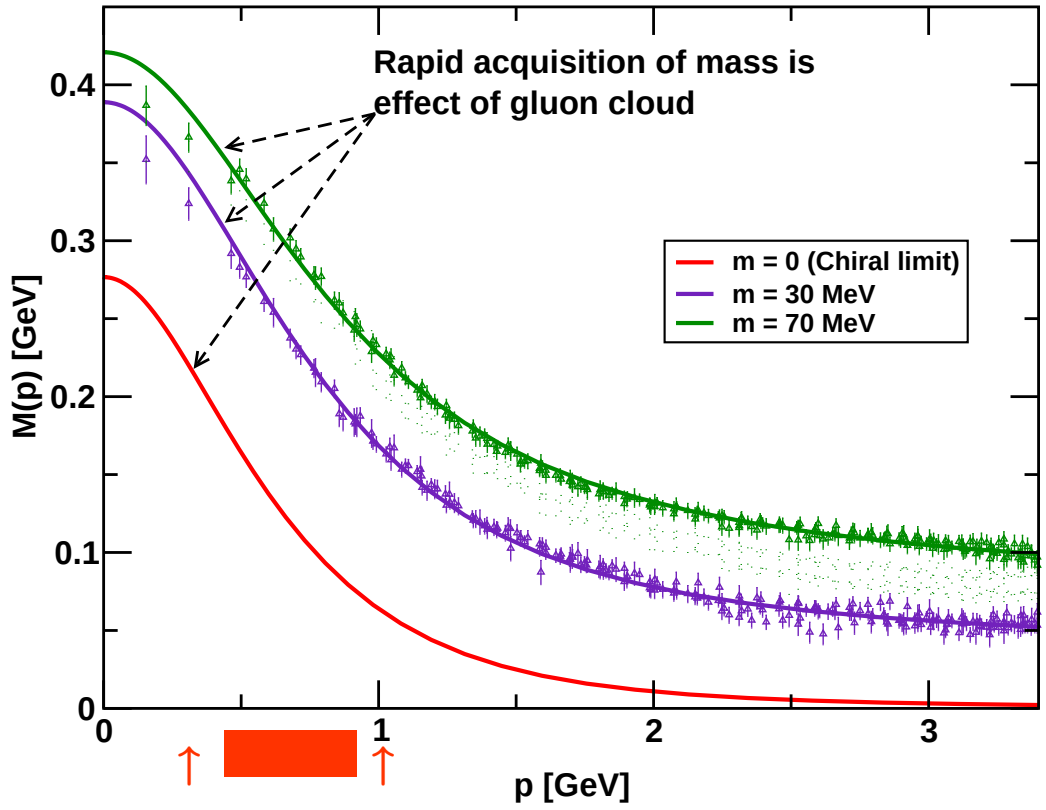

Scanned by $Q^2 \in [2, 9]\ \mathrm{GeV}^2$ Baryon Elastic and Transition Form Factors

Frontiers of Nuclear Science: Theoretical Advances

In QCD

a quark's mass must depend on

its momentum

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ALWAYS LOOK
ON THE
BRIGHT SIDE

- Established understanding of two- and three-point functions

Hadrons

- Established understanding of two- and three-point functions
- What about bound states?

Hadrons

- Without bound states, Comparison with experiment is impossible

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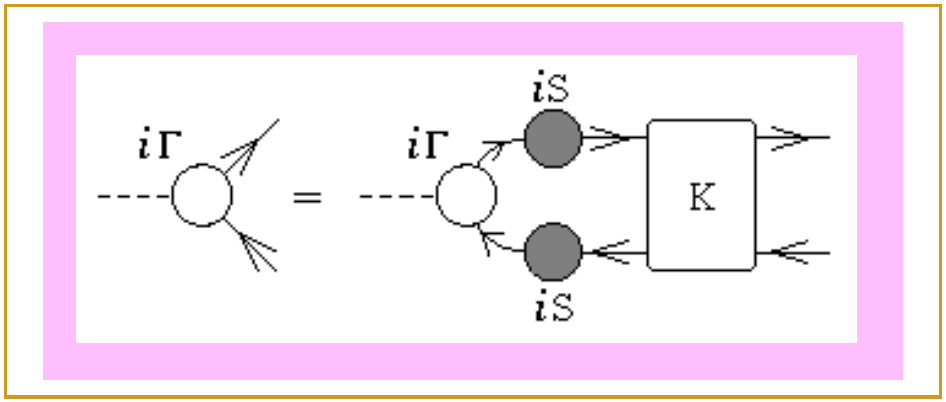
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

Hadrons

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

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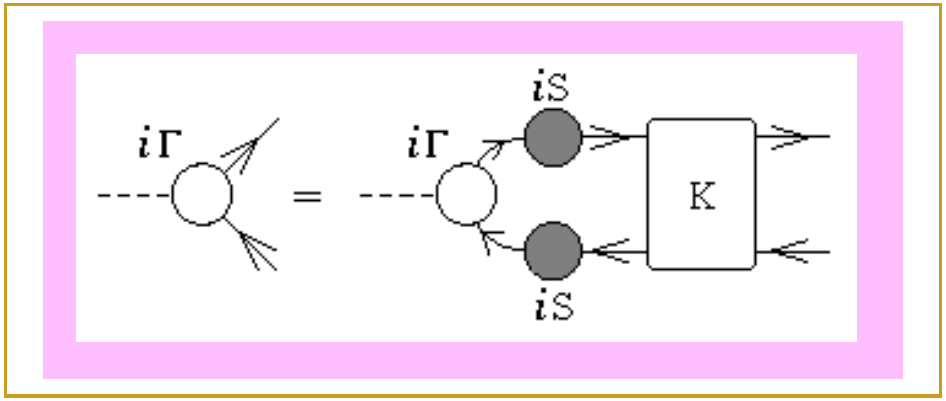

QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

  or What is the long-range potential in QCD?

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu\ \Gamma^l_{5\mu}(k;P) \;=\; \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\ \ \mathcal{S}^{-1}(k_-)$$

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QFT Statement of Chiral Symmetry

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Satisfies BSE — Satisfies DSE

Kernels very different

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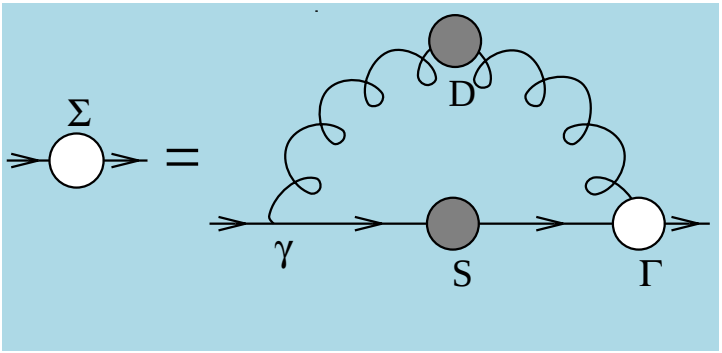
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Persistent Challenge

Persistent Challenge

- Infinitely Many Coupled Equations



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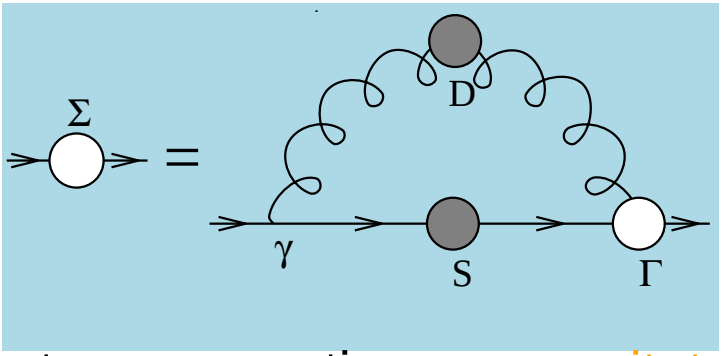

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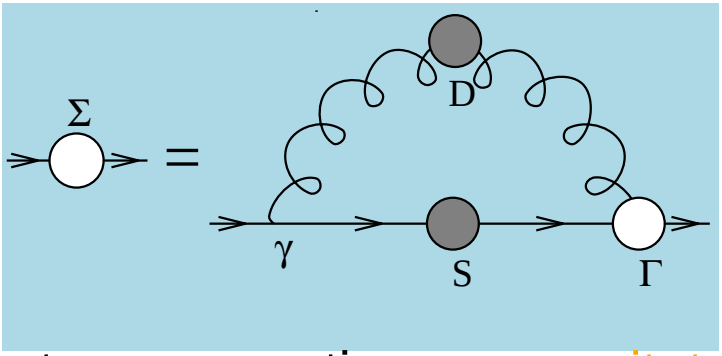

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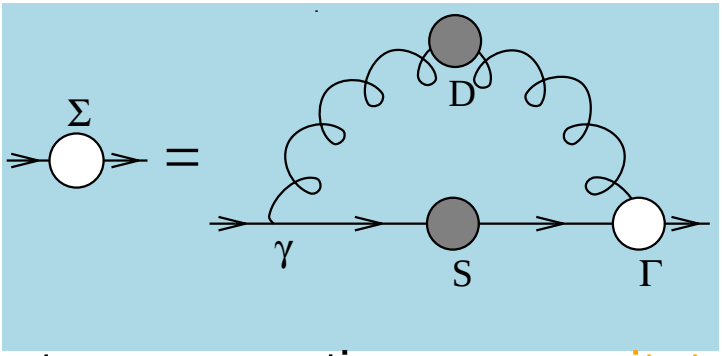

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    Not useful for the nonperturbative problems
    in which we're interested

Persistent Challenge

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  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

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- Examples:

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web.mit.edu/redingtn/www/netadv/Xdysonschw.html

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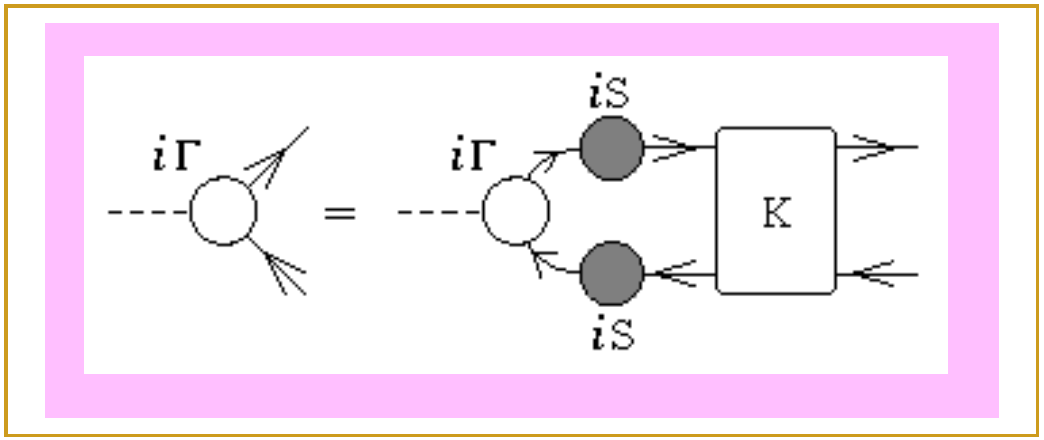
Bound-state DSE

Bound-state DSE
Bethe-Salpeter Equation

- Standard form, familiar from textbooks

$$\left[\Gamma^j_\pi(k;P)\right]_{tu} = \int_q^\Lambda \left[S(q+P/2)\Gamma^j_\pi(q;P)S(q-P/2)\right]_{sr} K^{rs}_{tu}(q,k;P)$$



$K(q,k;P)$: Fully-amputated, 2-particle-irreducible, quark-antiquark scattering kernel

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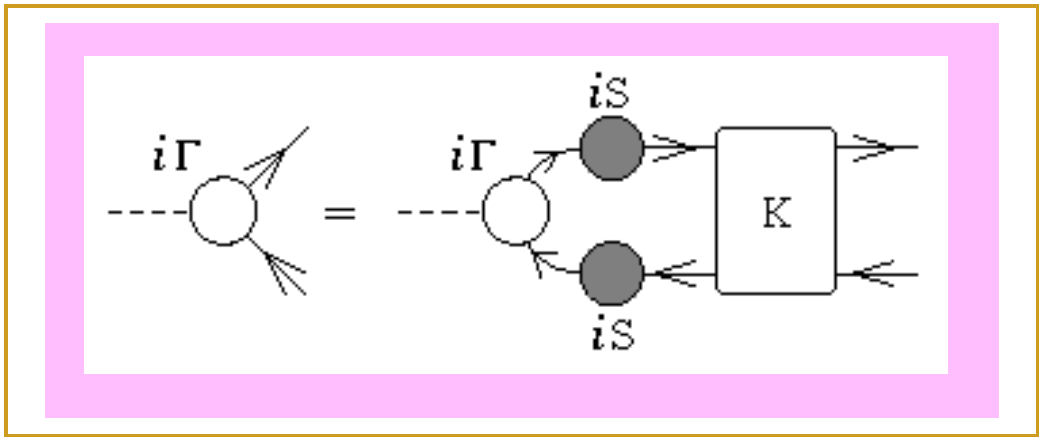

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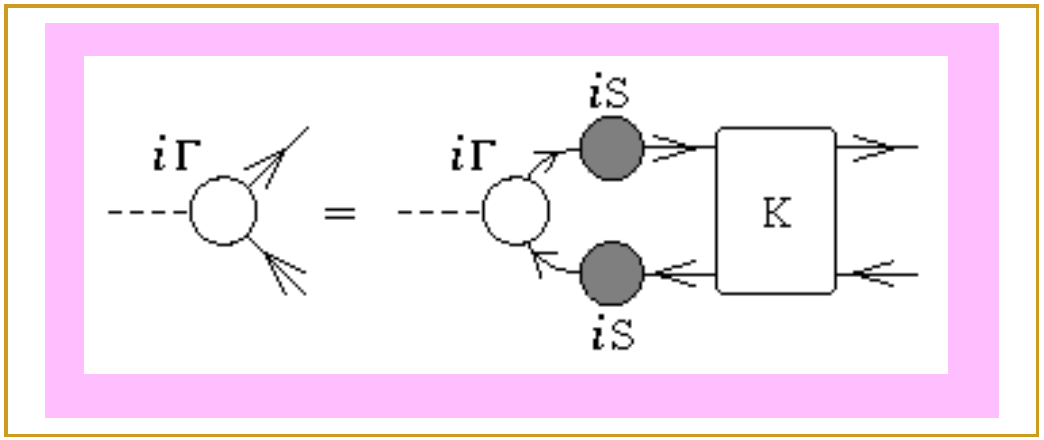

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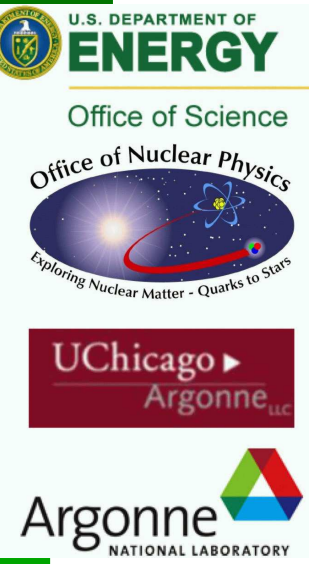
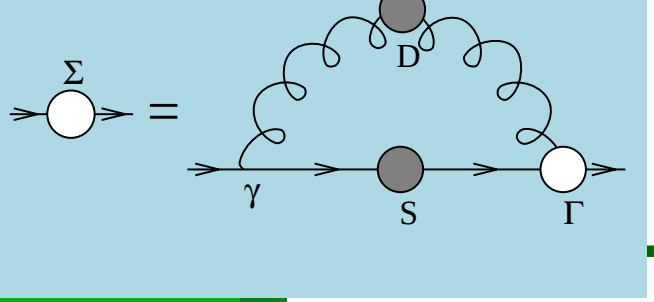
- Blocked progress for more than 60 years.

Gap Equation
General Form





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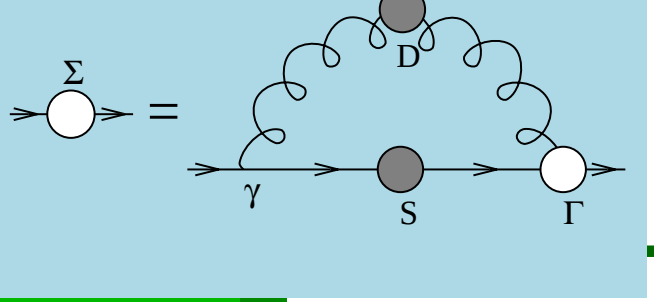

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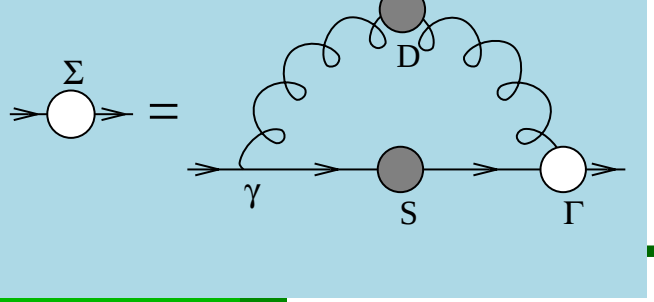
- $Z_{1,2}(\zeta^2,\Lambda^2)$ are respectively the vertex and quark wave function renormalisation constants, with ζ the renormalisation point
- $m^{\rm bm}(\Lambda)$ is the Lagrangian current-quark bare mass
- $D_{\mu\nu}(k)$ is the dressed-gluon propagator
- $\Gamma^f_\nu(q,p)$ is the dressed-quark-gluon vertex



Gap Equation
General Form

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p),$$

- $Z_{1,2}(\zeta^2,\Lambda^2)$ are respectively the vertex and quark wave function renormalisation constants, with ζ the renormalisation point
- $m^{\rm bm}(\Lambda)$ is the Lagrangian current-quark bare mass
- $D_{\mu\nu}(k)$ is the dressed-gluon propagator
- $\Gamma^f_\nu(q,p)$ is the dressed-quark-gluon vertex
- Suppose one has in-hand the exact form of $\Gamma^f_\nu(q,p)$

What is the associated

Symmetry-preserving Bethe-Salpeter Kernel?

Bethe-Salpeter Equation
General Form

L. Chang and C. D. Roberts
0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

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Bethe-Salpeter Equation
General Form

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- Equivalent exact form:

$$\Gamma^{fg}_{5\mu}(k;P) = Z_2\gamma_5\gamma_\mu$$
$$- \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\Gamma^{fg}_{5\mu}(q;P)S_g(q_-)\frac{\lambda^a}{2}\,\Gamma^g_\beta(q_-,k_-)$$
$$+ \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

(Poincaré covariance, hence $q_\pm = q \pm P/2$, etc., without loss of generality.)

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(Poincaré covariance, hence $q_\pm = q \pm P/2$, etc., without loss of generality.)

- In this form . . . $\Lambda^{fg}_{5\mu\beta}$
  is completely defined via the dressed-quark self-energy

Bethe-Salpeter Kernel & Symmetries

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- E.g., in any reliable study of light-quark hadrons, axial-vector vertex must satisfy Ward-Takahashi identity

$$P_\mu \Gamma^{fg}_{5\mu}(k;P) = S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-) - i\left[m_f(\zeta) + m_g(\zeta)\right] \Gamma^{fg}_5(k;P)\,,$$

Expresses chiral symmetry & pattern by which it's broken

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- The condition ($\Lambda^{fg}_{5\beta}$ pseudoscalar analogue of $\Lambda^{fg}_{5\mu\beta}$)

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a new Ward-Takahashi identity, is Necessary & Sufficient to ensure $\mathbf{\Gamma^{fg}_{5\mu}(k;P)}$ Ward-Takahashi identity satisfied.

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- Rainbow-ladder . . .
  - $\Gamma^f_\beta(q,k) = \gamma_\mu$
    $\Rightarrow \Lambda^{fg}_{5\mu\beta}(k,q;P) = 0 = \Lambda^{fg}_{5\beta}(k,q;P)$

Bethe-Salpeter Kernel

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- Bethe-Salpeter equation introduced in 1951

Bethe-Salpeter Kernel

60 year problem

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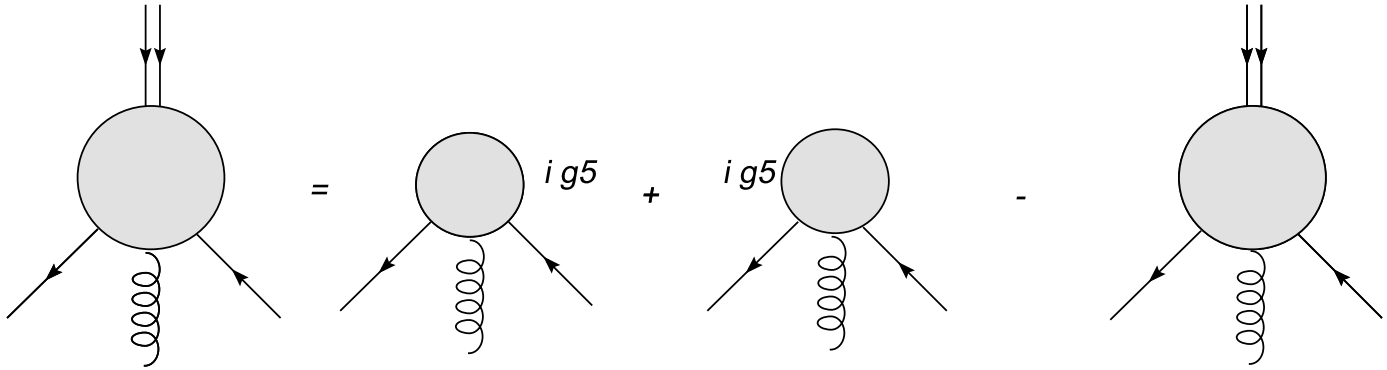

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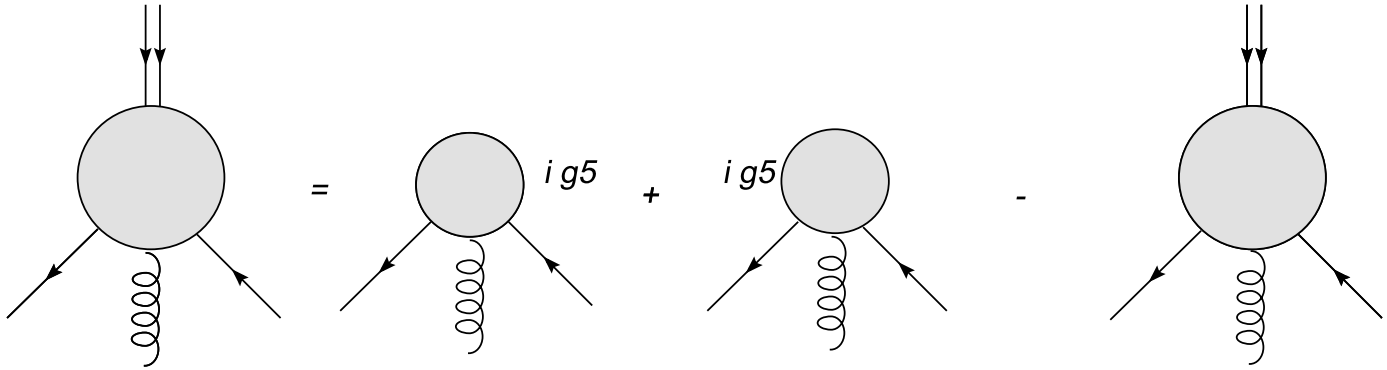

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- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
  - Consistent means - all symmetries preserved!

Bethe-Salpeter Kernel

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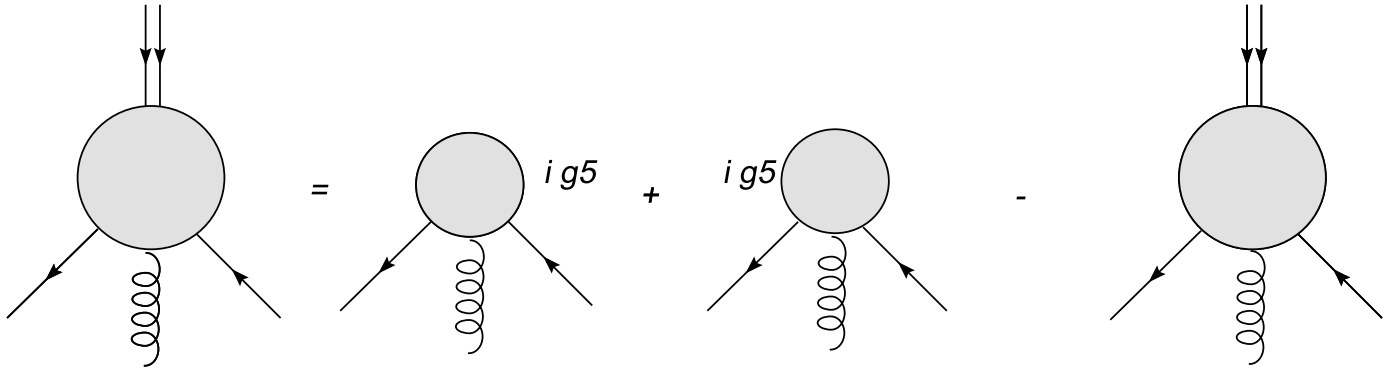

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- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
- Exemplified the procedure and results to expect . . .

Numerical Illustration

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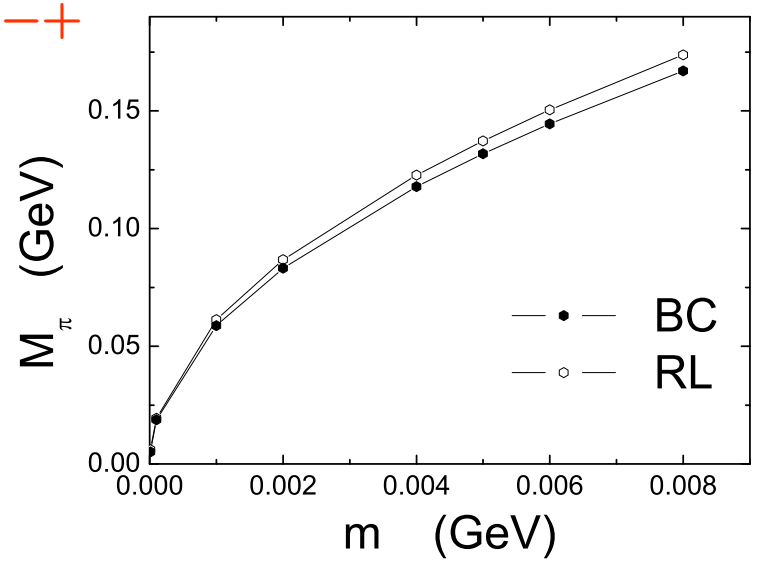
π cf. σ

0^{-+}



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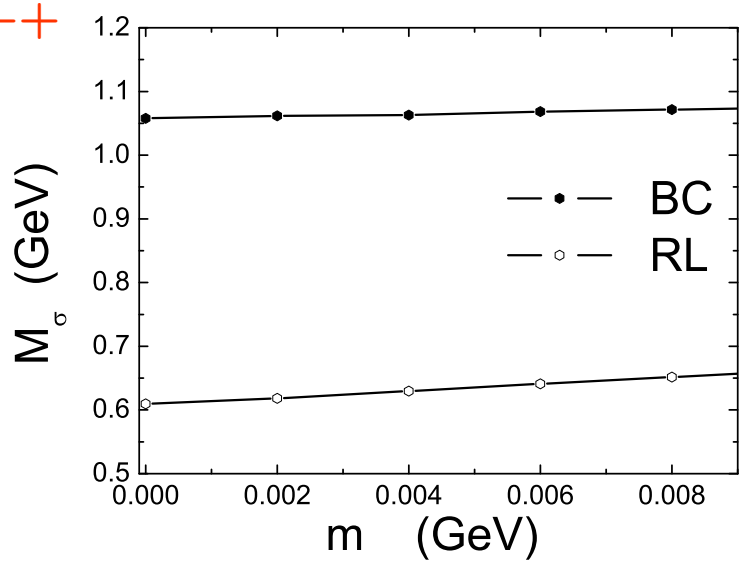

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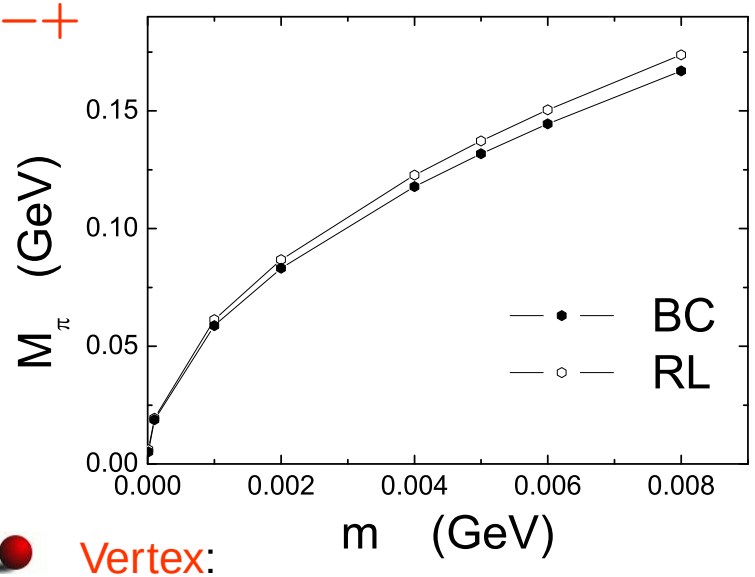

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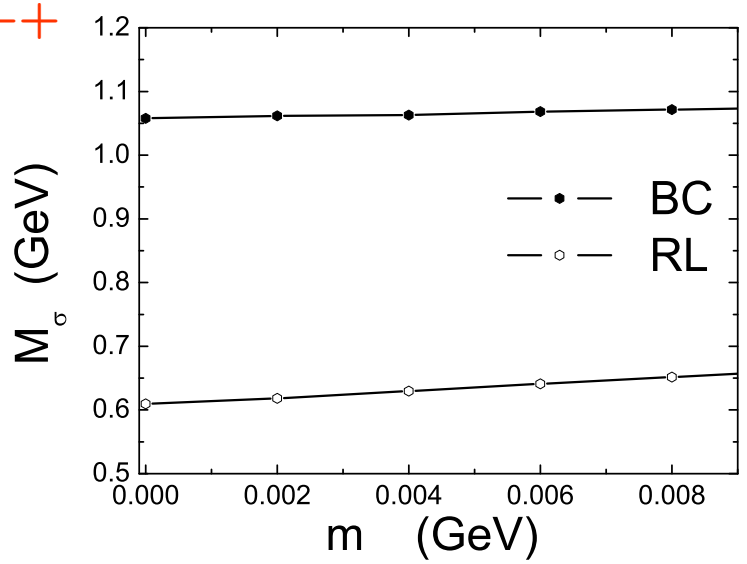

- Vertex:
  - leading-order rainbow-ladder truncation
  - cf. Ball-Chiu–consistent *Ansatz* – Essentially nonperturbative content; Expresses DCSB; Consistent with lattice-QCD simulations; Diagrammatic content unknown

Same interaction. One mass-scale in both truncations: $1/\omega = 0.4\,$fm, defining border between IR & UV.

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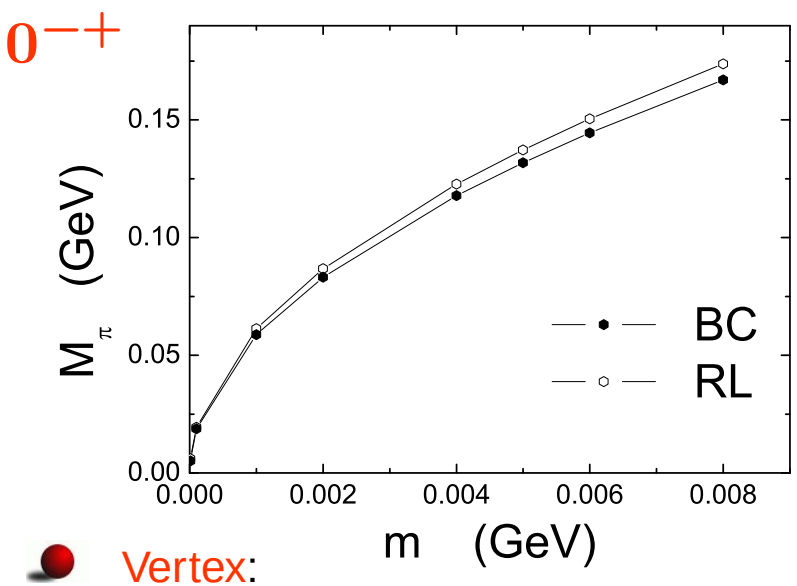

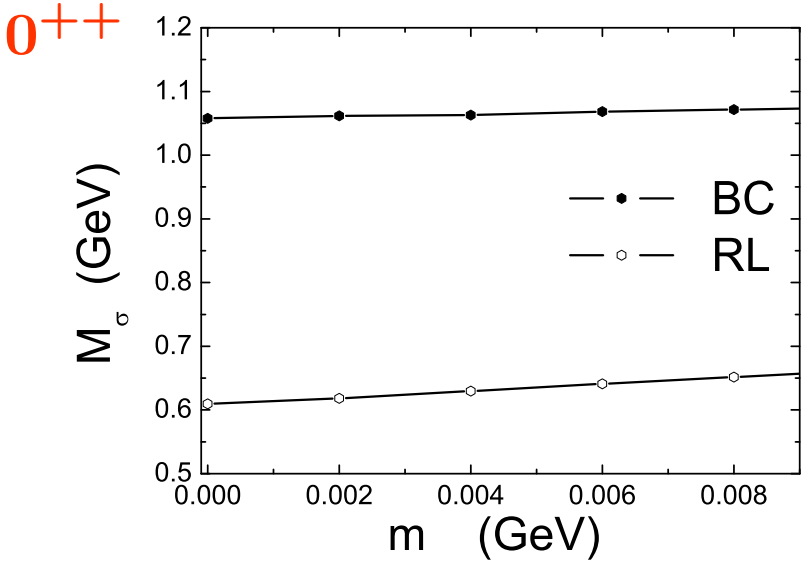

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- GMOR . . . plainly satisfied by both truncations
- A little attraction introduced in pseudoscalar channel
- Enormous repulsion introduced in scalar channel

Spin-orbit Interaction

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- Rainbow-ladder DSE truncation, $\varepsilon_\sigma^{\rm RL} := \left.\dfrac{2M(0)-m_\sigma}{2M(0)}\right|_{\rm RL} = (0.3 \pm 0.1)\,.$
- BC-consistent Bethe-Salpeter kernel; viz., $\varepsilon_\sigma^{\rm BC} \lesssim 0.1\,.$

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- Scalar mesons $= {}^3P_0$ states: Constituents' spins aligned and one unit of constituent orbital angular momentum
  - From this viewpoint,
    scalar is a spin and orbital excitation of a pseudoscalar meson

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- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit interaction.
  - Effect owes to influence of quark's dynamically-enhanced scalar self-energy in the Bethe-Salpeter kernel.

Impossible to demonstrate effect without our new procedure

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- Expect this feature to have material impact

  Especially on mesons with mass greater than 1 GeV.

prima facie . . . can overcome longstanding shortcoming of systematic, symmetry-preserving truncations;
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- Promise of realistic meson spectroscopy . . . First time, also for mass > 1 GeV

- That was where things stood in March/09

$[m_{a_1} - m_\rho]$ I ...

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- Now, we've solved inhomogeneous vector and axial-vector Bethe-Salpeter equation at spacelike total momentum
  $\Rightarrow \Gamma_{qq}(k=0, P^2)$
- $$\boxed{\frac{1}{\Gamma_{qq}(k=0, P^2)}}$$ Exhibits a zero at ground-state mass-squared
- Padé approximant extrapolation to locate zero

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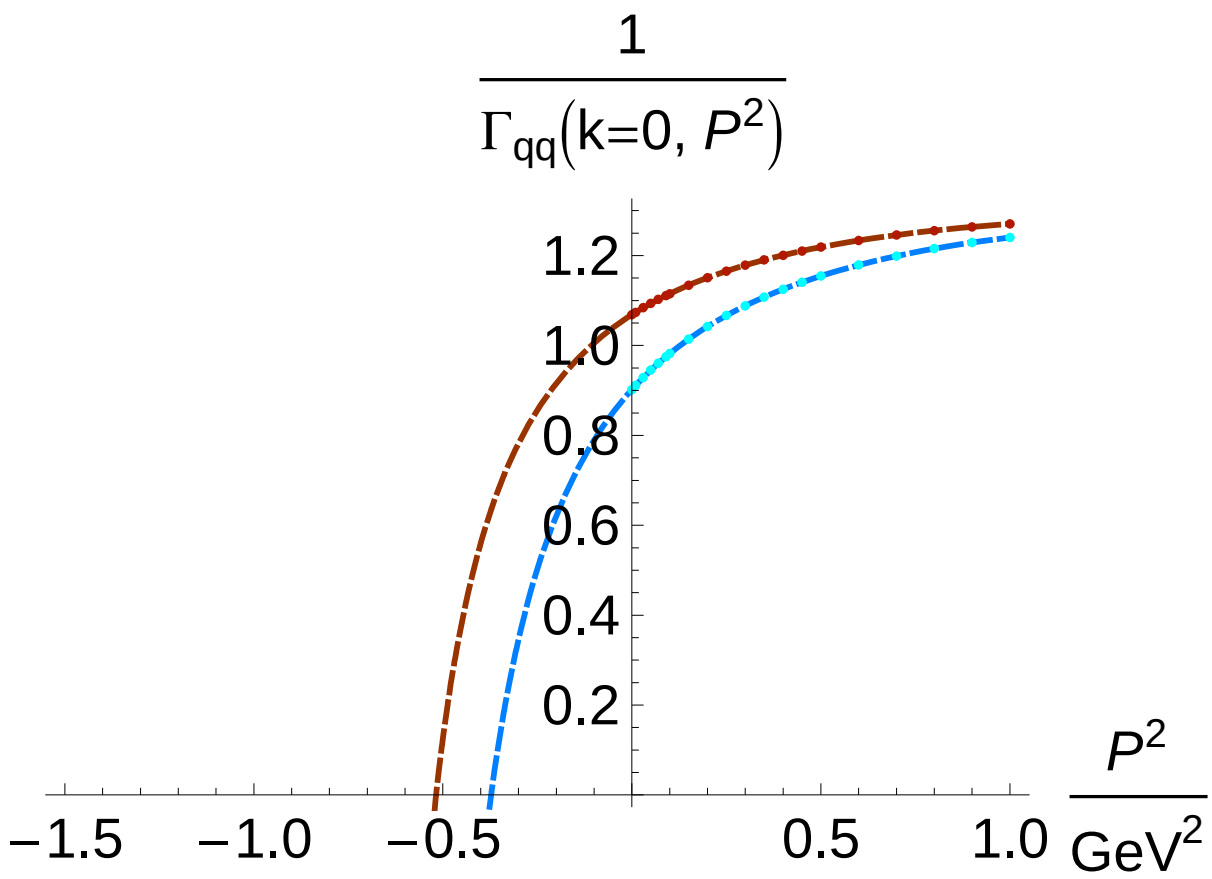
  - Almost precisely method used for ground-state masses in lattice-QCD
  - Intelligent use gives dependable results
    "Schwinger functions and light-quark bound states"
    Bhagwat, Höll, Krassnigg, Roberts & Wright,
    Few Body Syst. **40** (2007) 209, nucl-th/0701009

$\ldots\ [m_{a_1} - m_\rho]$ II $\ldots$

Rainbow-Ladder

$\Gamma_\mu(q,k) = \gamma_\mu$



$\ldots\ [m_{a_1} - m_\rho]$ II $\ldots$

$m_{a_1}\ (759\,\text{MeV}) - m_\rho\ (644\,\text{MeV})$
$= 115\,\text{MeV}\ \ldots$ expt.$= 455\,\text{MeV}$

Rainbow-Ladder
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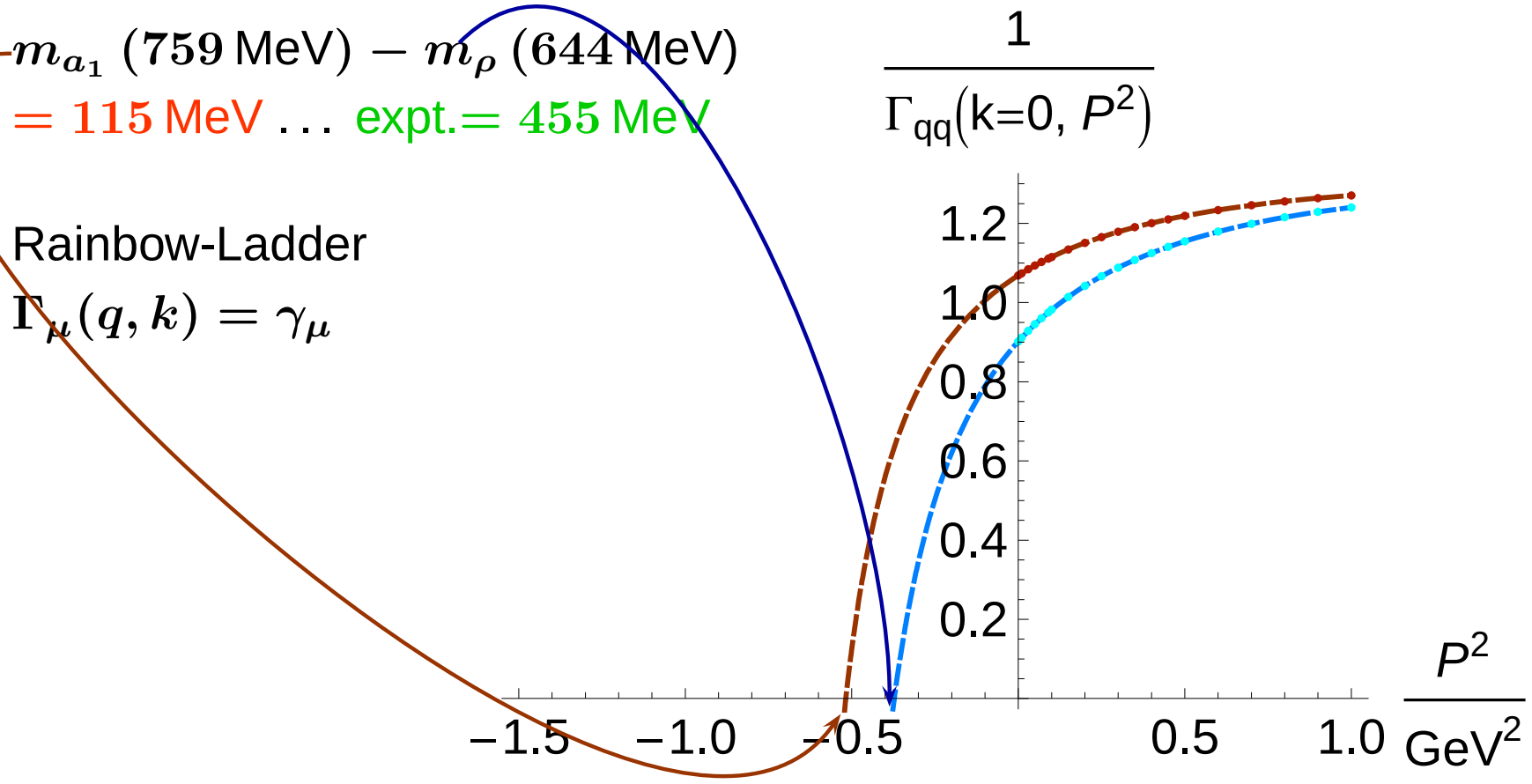
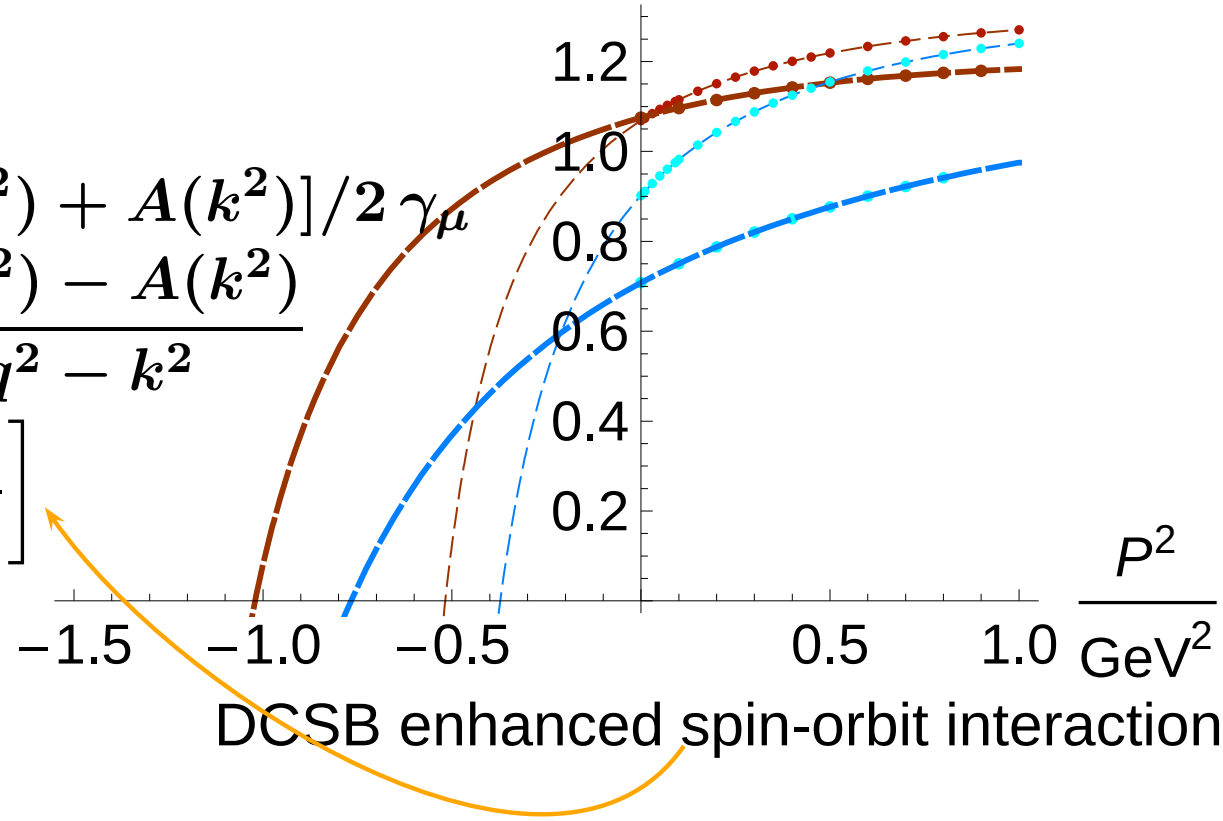


… $[m_{a_1} - m_\rho]$ III …

$m_{a_1}\,(1066\,\text{MeV}) - m_\rho\,(924\,\text{MeV})$
$= 142\,\text{MeV}$ … expt.$= 455\,\text{MeV}$

Ball-Chiu

$$\Gamma_\mu(q,k) = i[A(q^2)+A(k^2)]/2\,\gamma_\mu + 2k_\mu\left[i\gamma\cdot k\,\frac{A(q^2)-A(k^2)}{q^2-k^2} + \frac{B(q^2)-B(k^2)}{q^2-k^2}\right]$$



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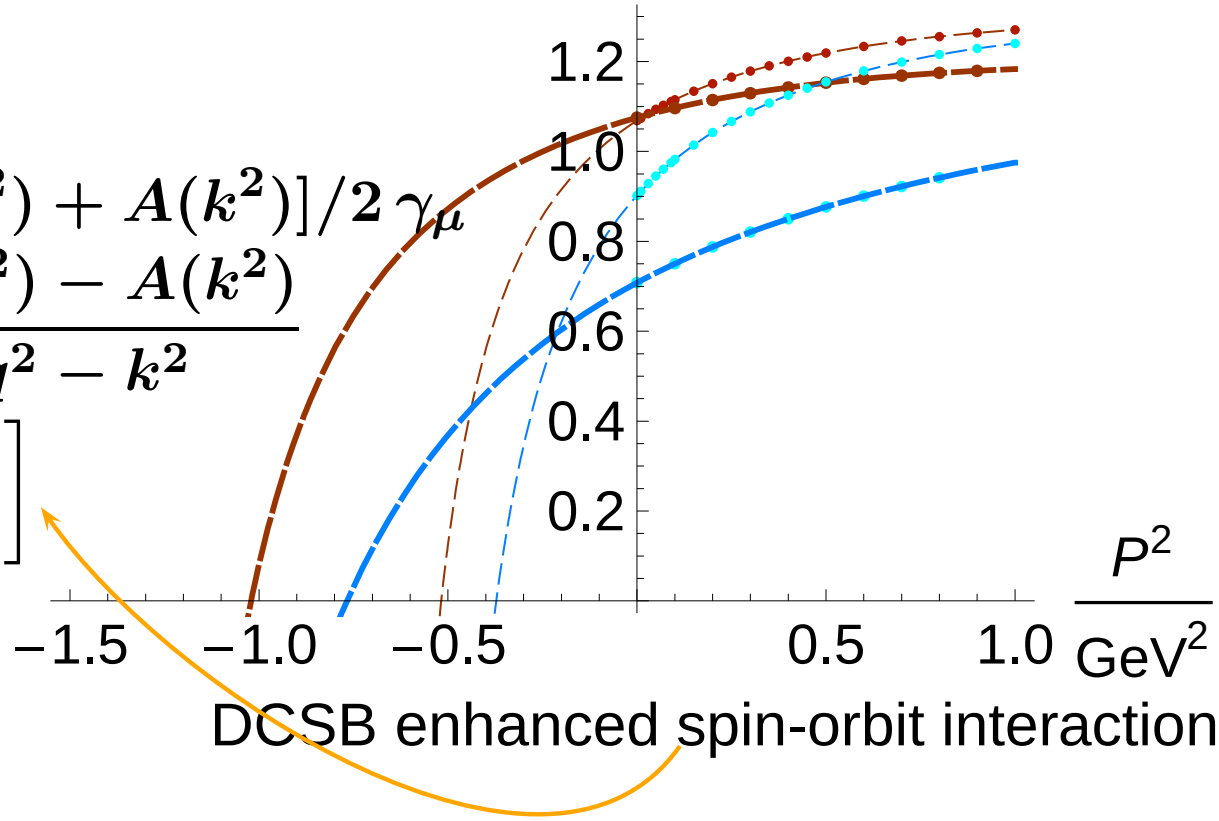
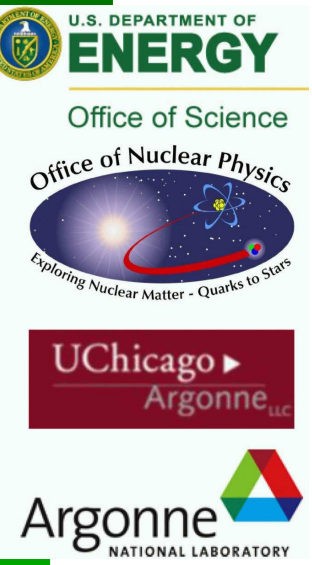


What's missing?

Missing Link

$[m_{a_1} - m_\rho]$

Chang Lei & CDR, in-preparation



Missing Link

$[m_{a_1} - m_\rho]$

Chang Lei & CDR, in-preparation

$m_{a_1}\,(1230\,\text{MeV}) - m_\rho\,(745\,\text{MeV})$
$= 485\,\text{MeV}\ldots$ expt.$= 455\,\text{MeV}$

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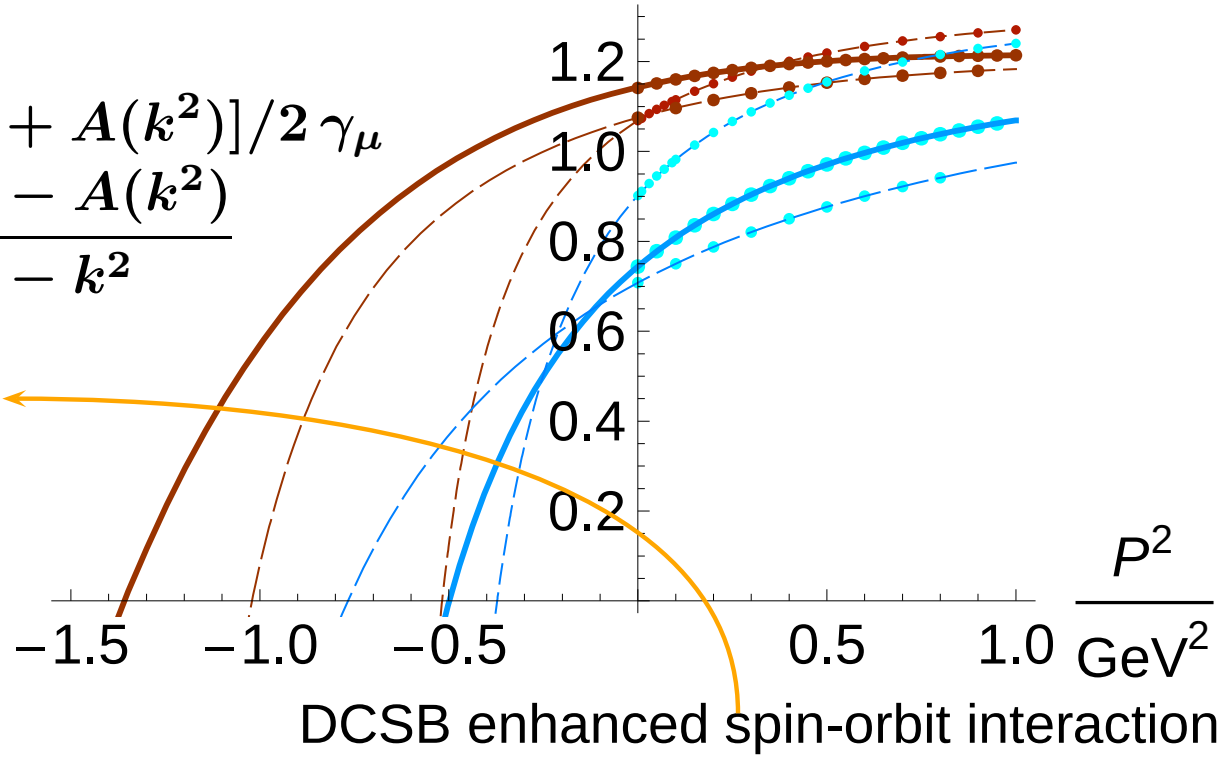

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Ball-Chiu + an. mag. mom.

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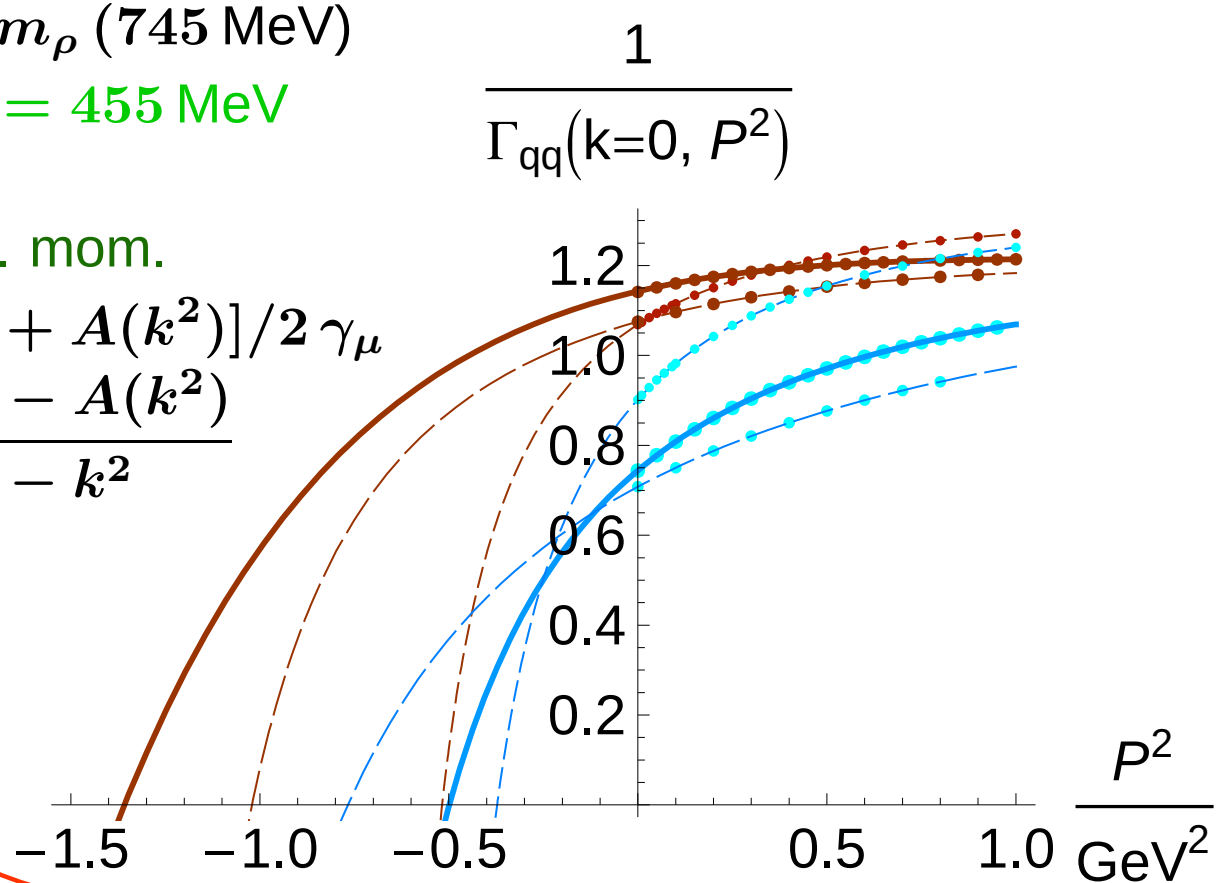

DCSB enhanced anomalous chromomagnetic moment

Missing Link

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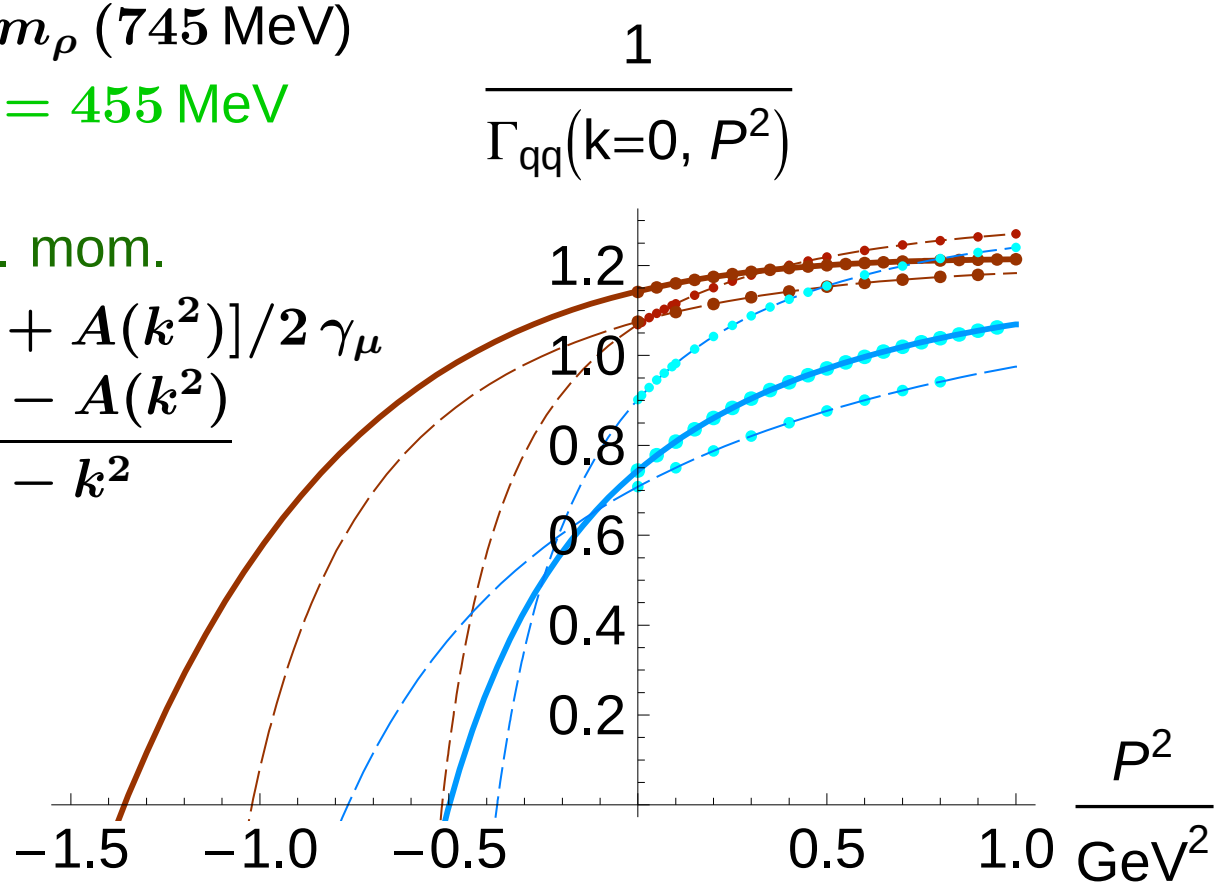

Inextricably connected with DCSB.

DCSB enhanced anomalous chromomagnetic moment

Can't appear in chirally symmetric theory

Paves way for truly reliable light-quark meson spectrum

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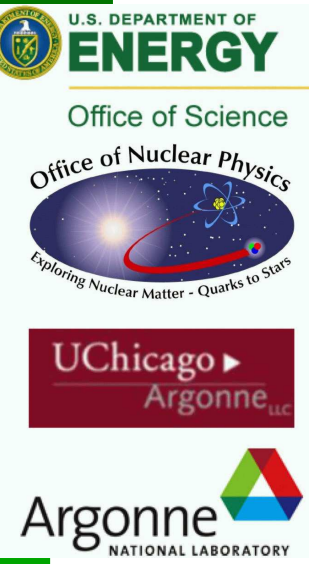
Unifying Study of Mesons and Baryons



Unifying Study of Mesons and Baryons

- How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons? Behaviour of $M(p^2)$ is essentially a quantum field theoretical effect.

Unifying Study of Mesons and Baryons

- How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons? Behaviour of $M(p^2)$ is essentially a quantum field theoretical effect.
- In quantum field theory a nucleon appears as a pole in a six-point quark Green function.
  - Residue is proportional to nucleon's Faddeev amplitude
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Unifying Study of Mesons and Baryons

- How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons? Behaviour of $M(p^2)$ is essentially a quantum field theoretical effect.
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  - Residue is proportional to nucleon's Faddeev amplitude
  - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
  - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour-$\overline{3}$ (antitriplet) channel

Faddeev equation

R. T. Cahill *et al*. Austral. J. Phys. **42** (1989) 129

Faddeev equation

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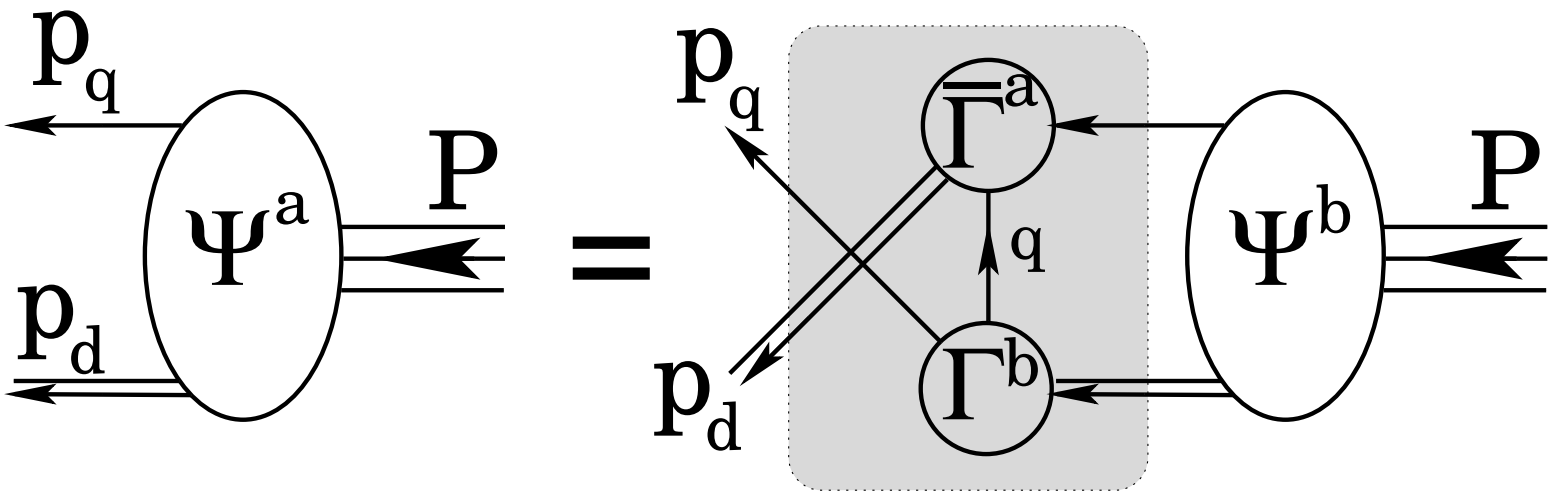

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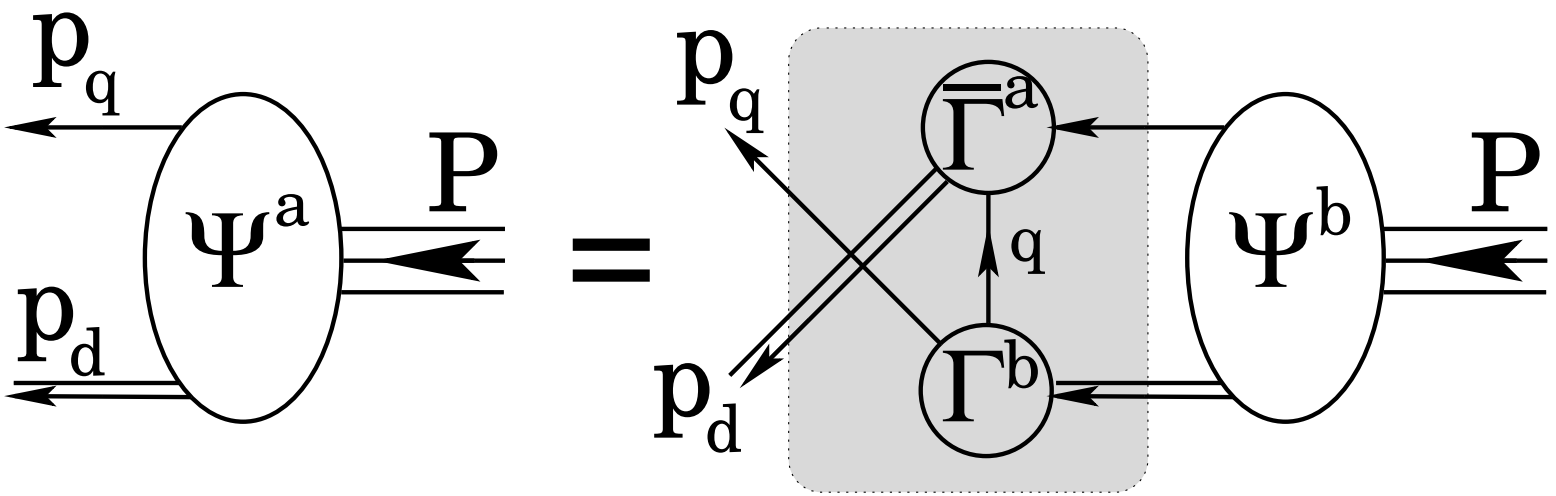
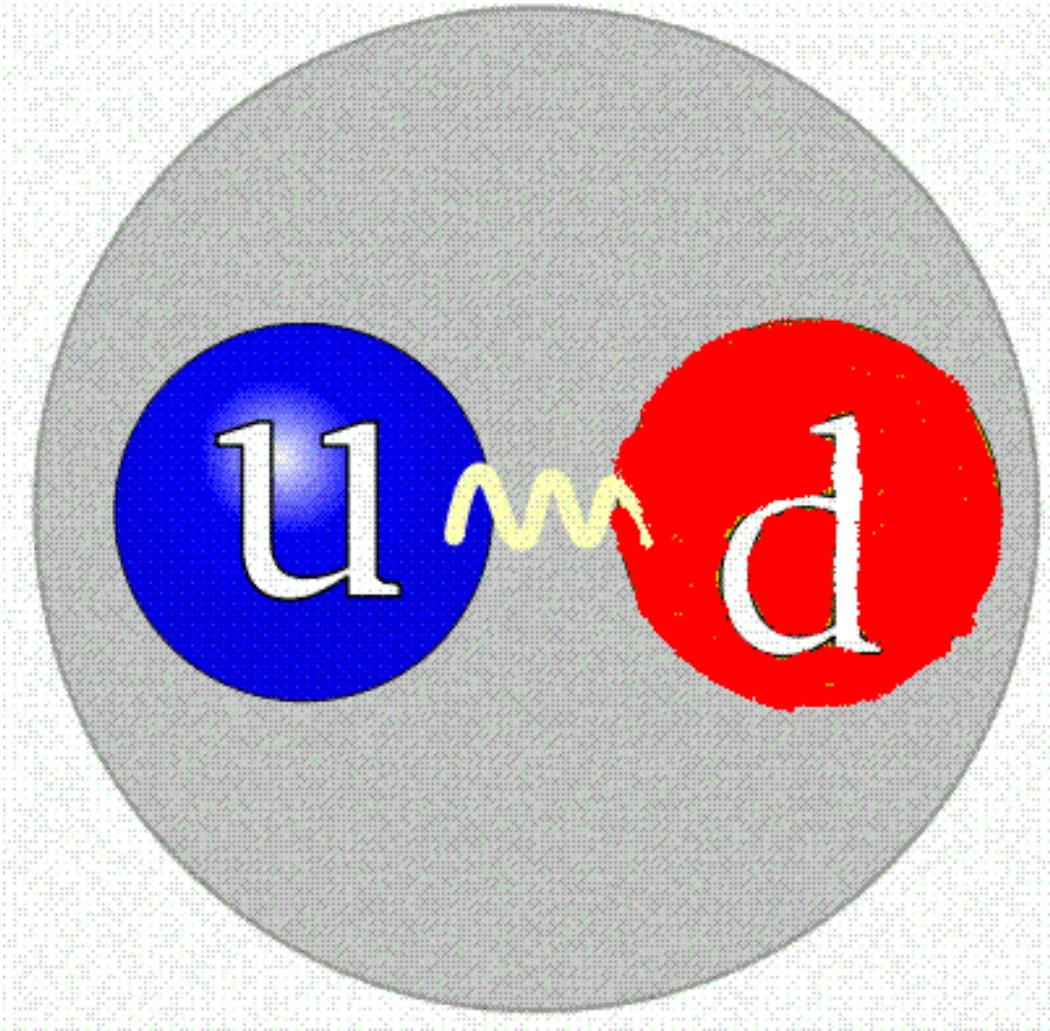


- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Diquark correlations



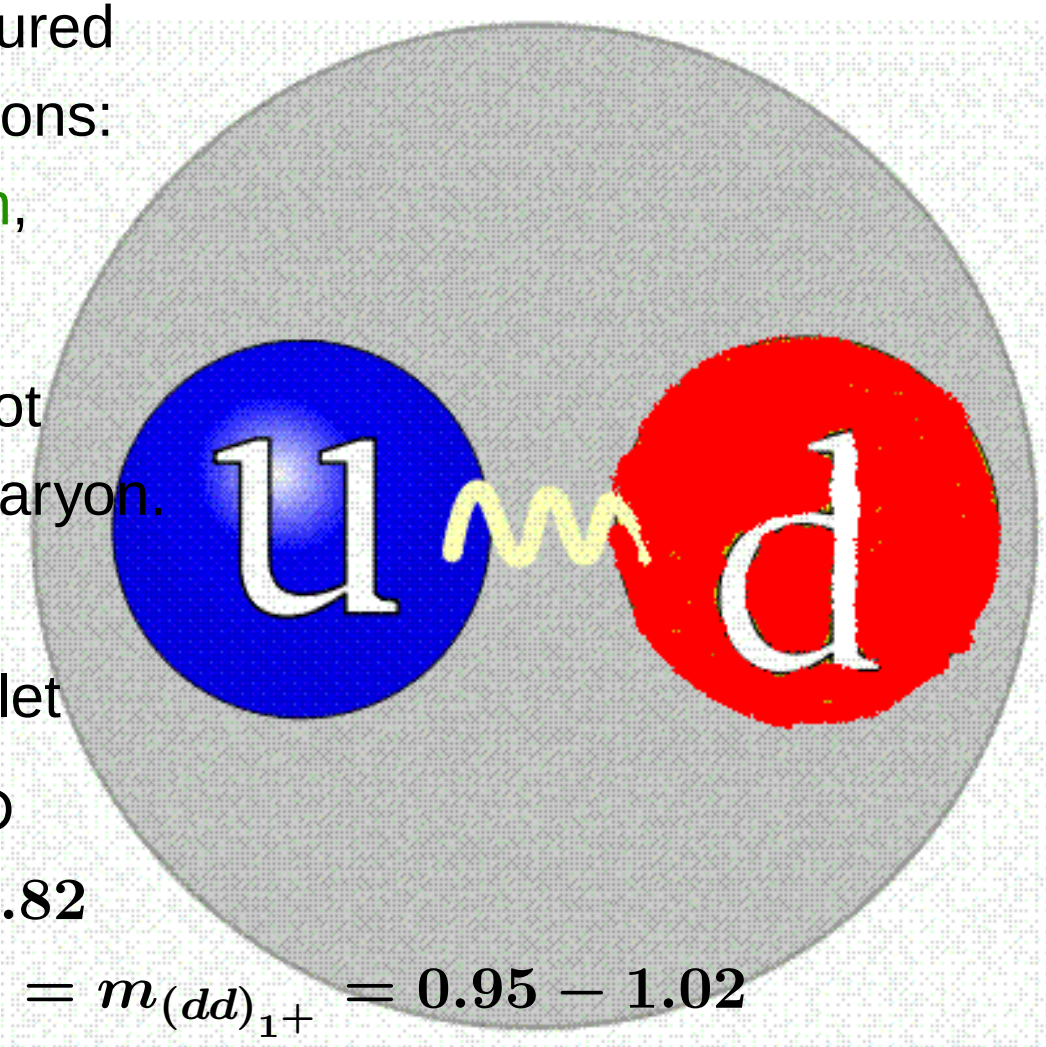
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined . . . Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

for on-shell nucleons described by Faddeev Amplitude

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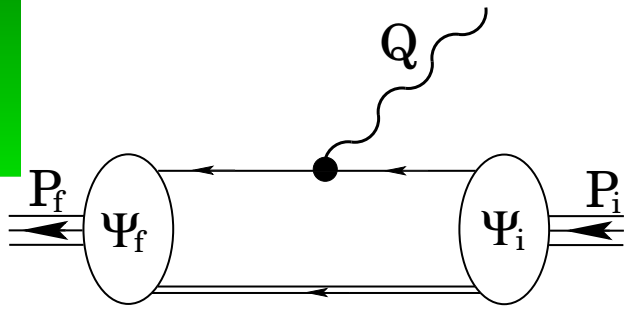

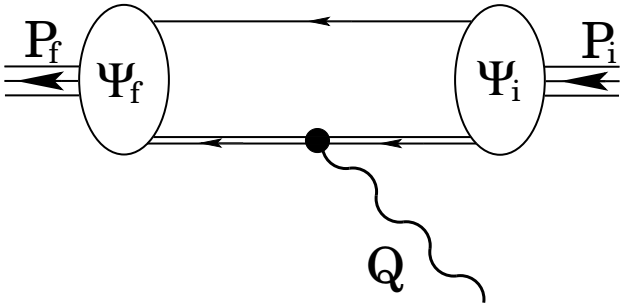

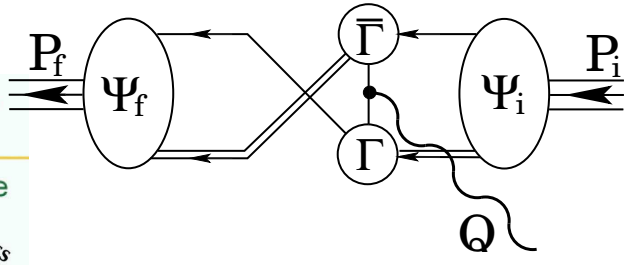

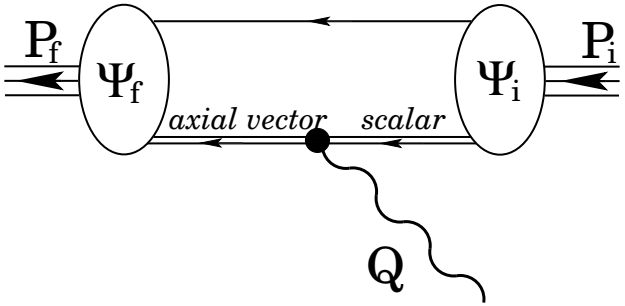

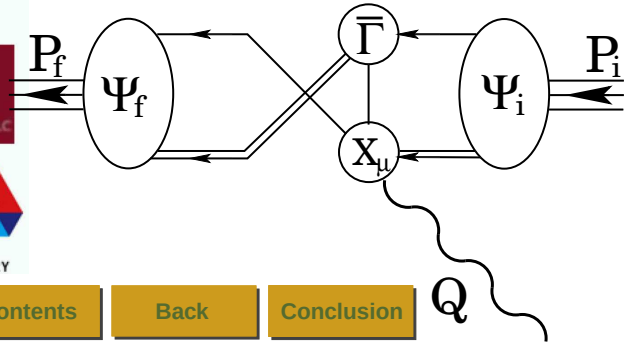

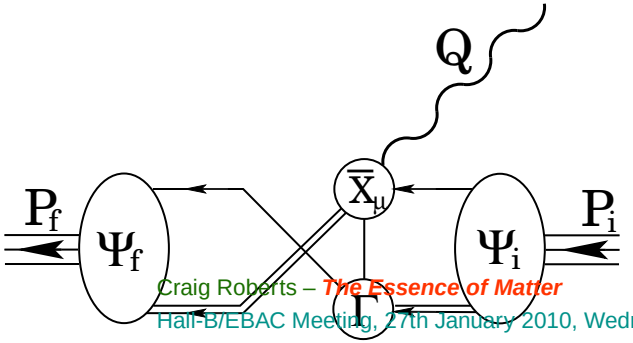

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– arXiv:0804.3118 [nucl-th]
– arXiv:0812.0416 [nucl-th] – *Survey of nucleon EM form factors*

$$\frac{\mu_n G_E(Q^2)}{G_M(Q^2)}$$

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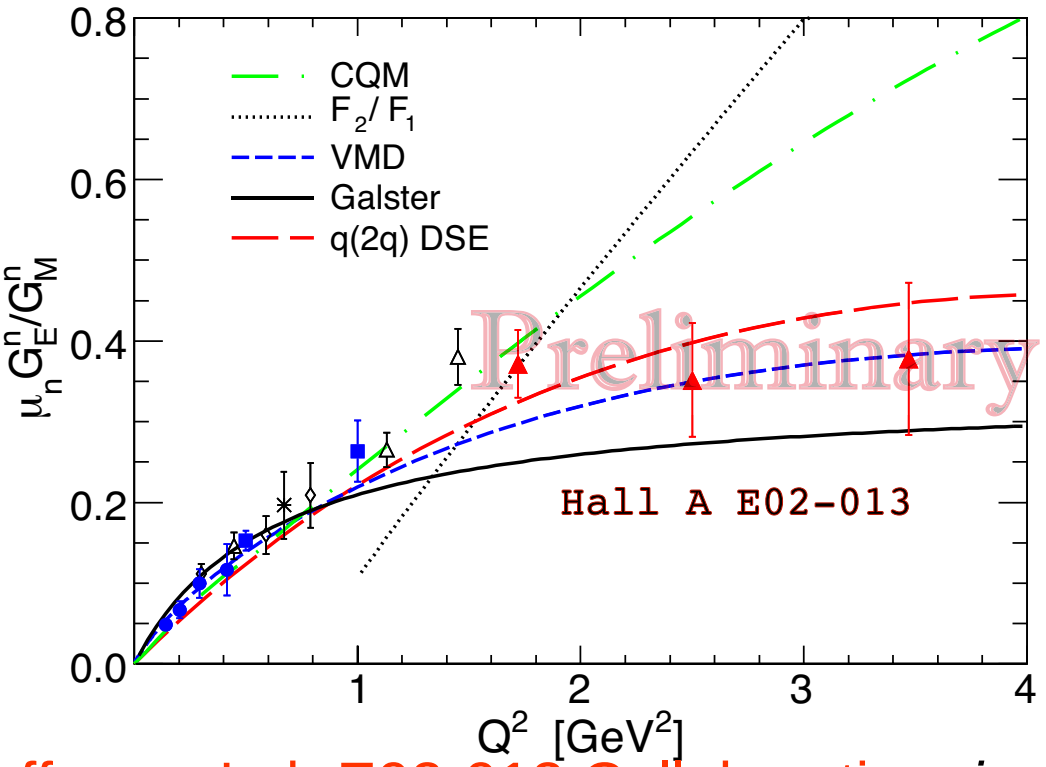

Jefferson Lab E02-013 Collaboration, *in preparation*.

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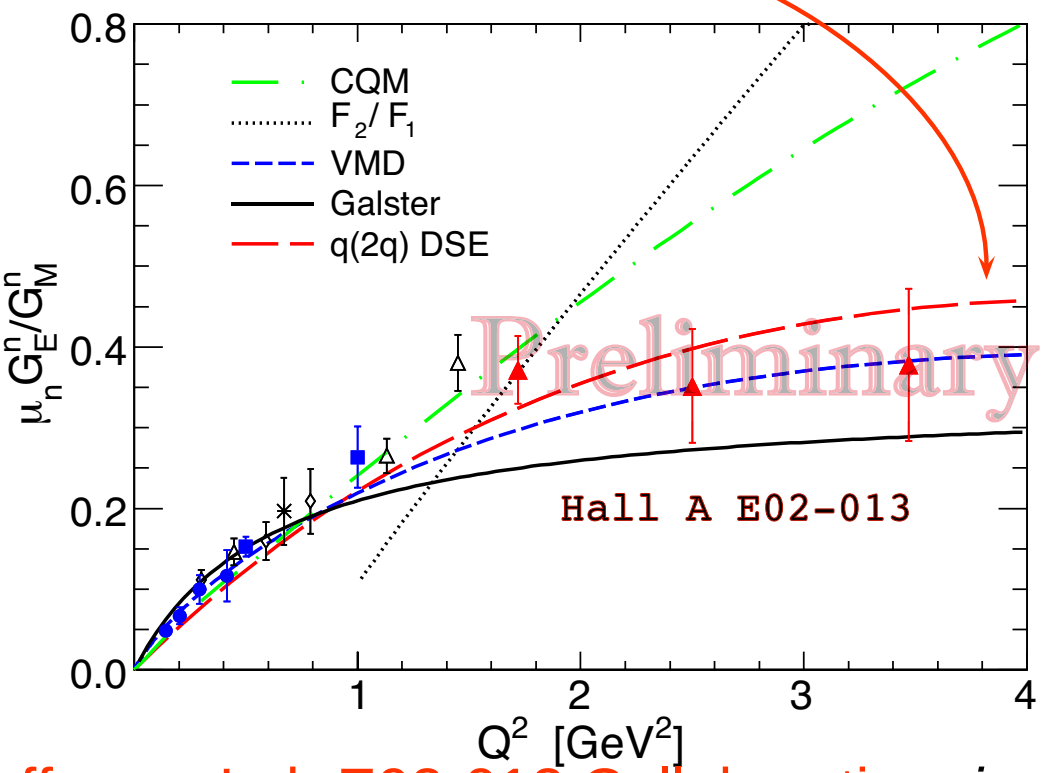
Red long-dashed curve



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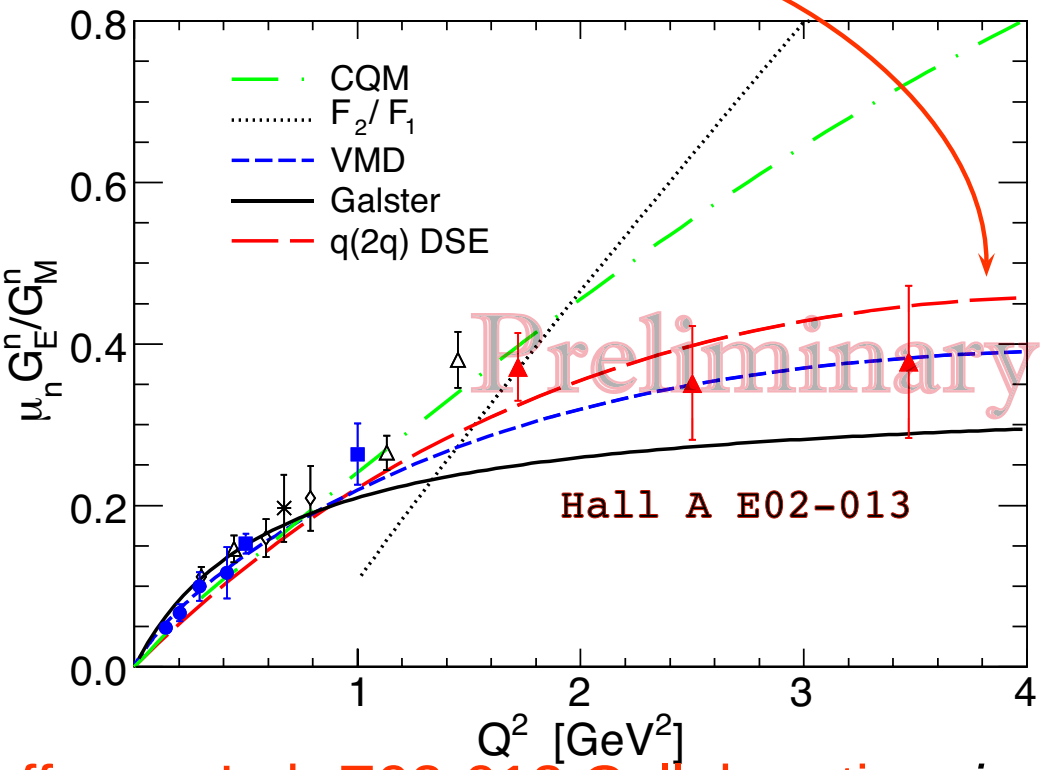

This evolution very sensitive to momentum -dependence dressed-quark propagator

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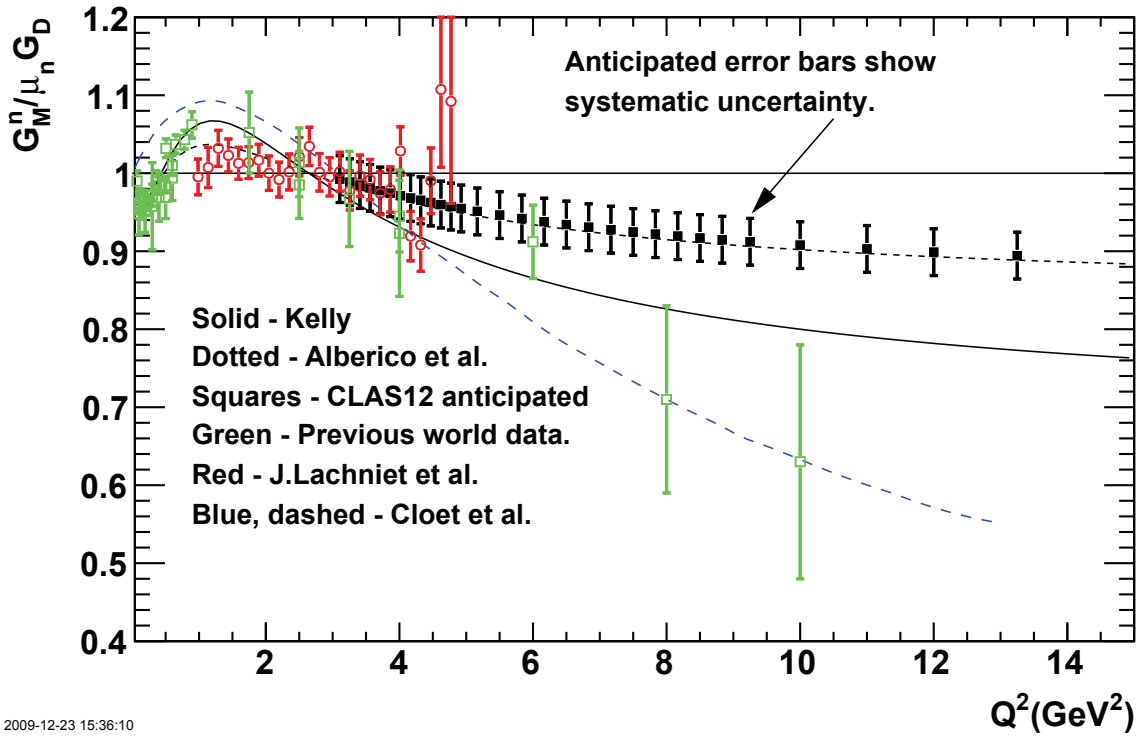

Jefferson Lab E12-07-104, *12GeV Proposal*.

Gilfoyle, Brooks, Hafidi for CLAS Collaboration

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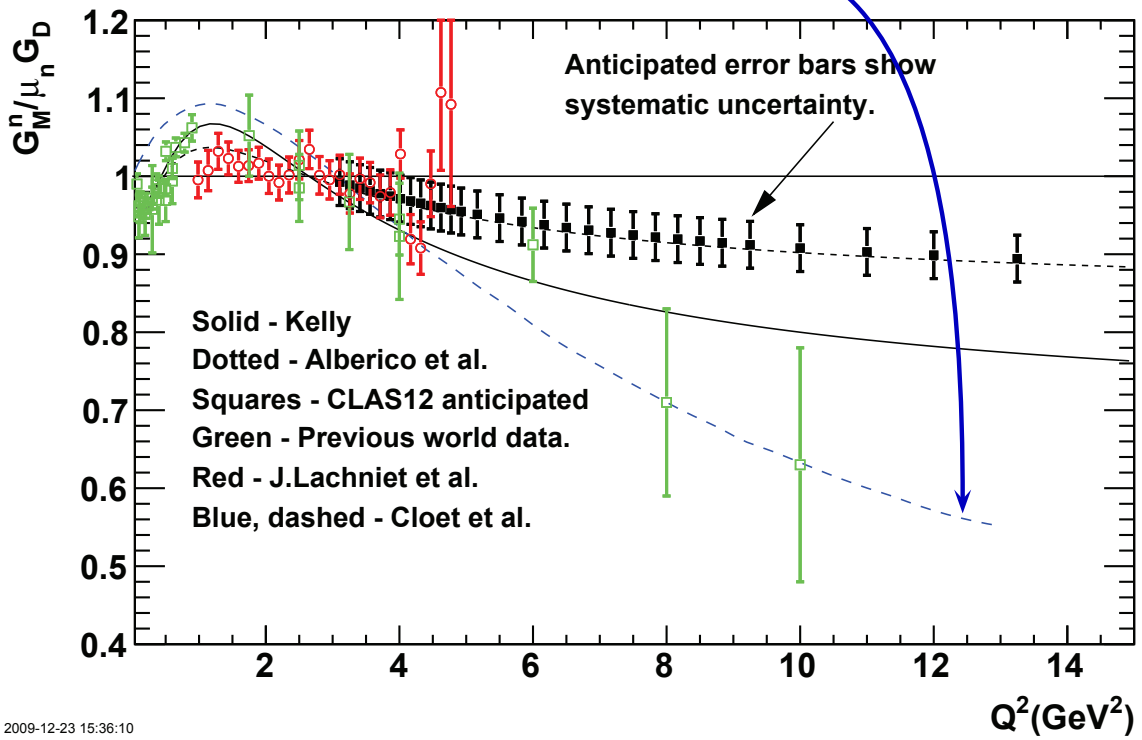

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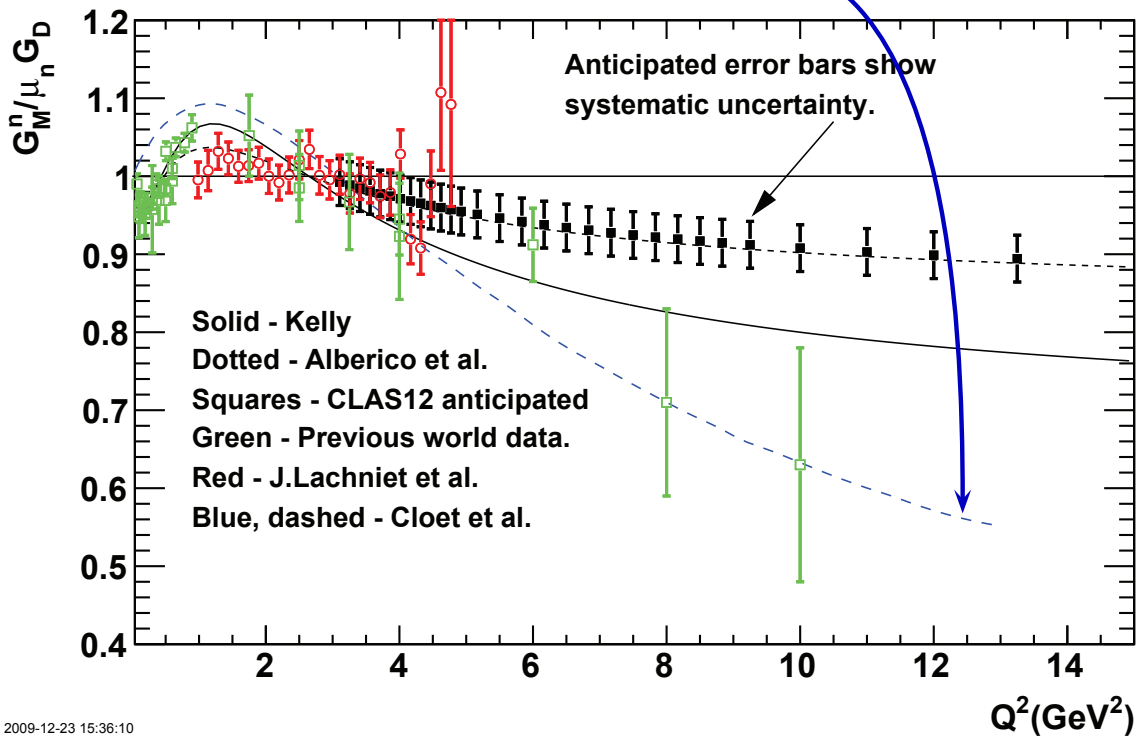

Sensitivity to $M(p^2)$ means experiments probe IR behaviour of strong running coupling

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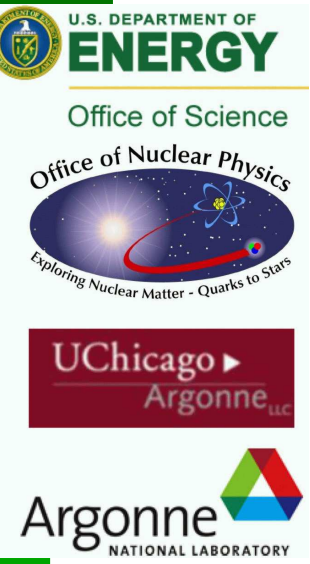
Goldberger-Treiman for pion



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Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

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$$\Rightarrow \qquad f_\pi E_\pi(k;P=0) = B(p^2)$$

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$\Rightarrow$

$$\begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

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Pseudovector components necessarily nonzero

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$\Rightarrow$ Exact in Chiral QCD

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GT for pion – QCD and $F_\pi^{\rm em}(Q^2)$

Maris, Roberts
nucl-th/9804062

- What does this mean for observables?

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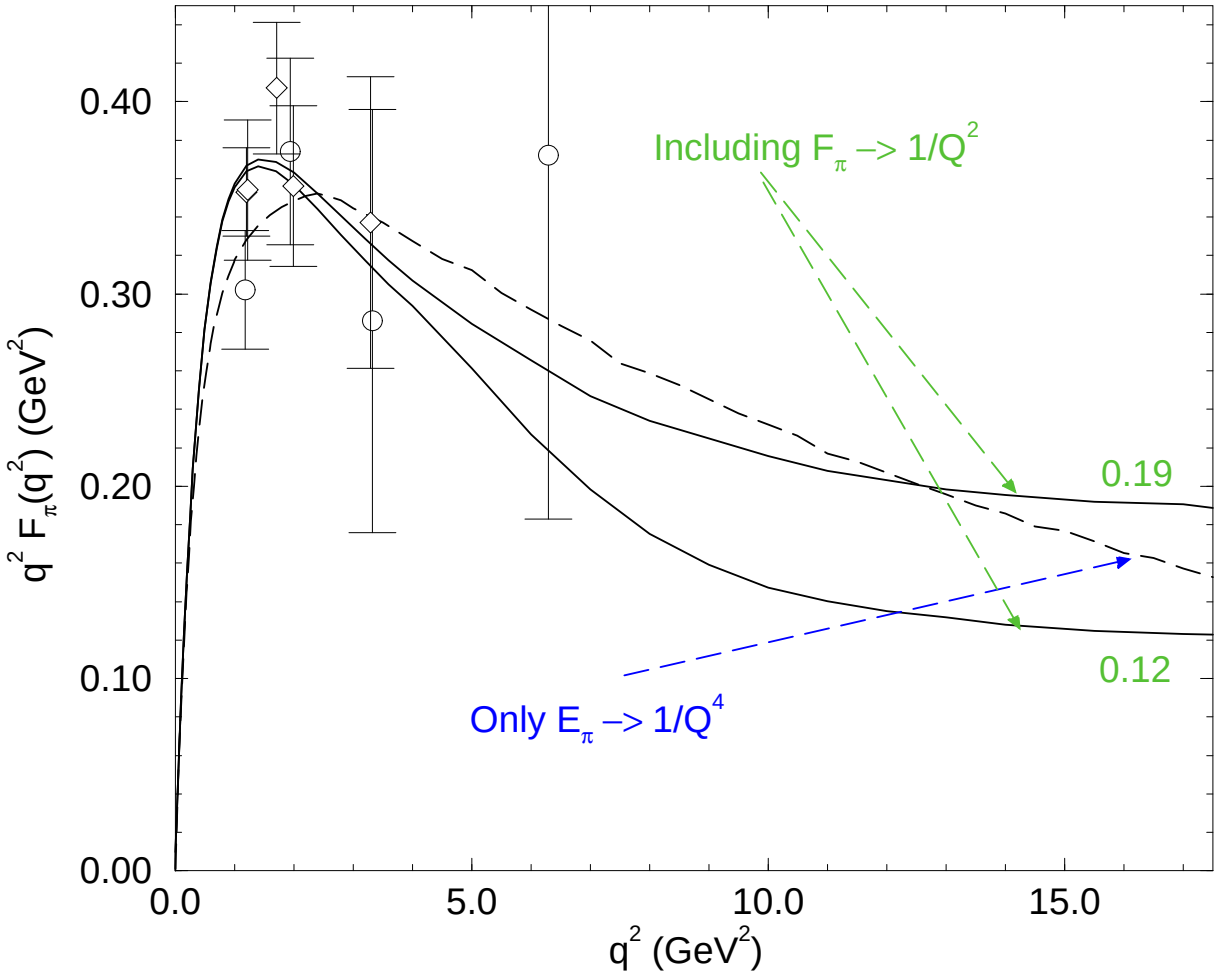

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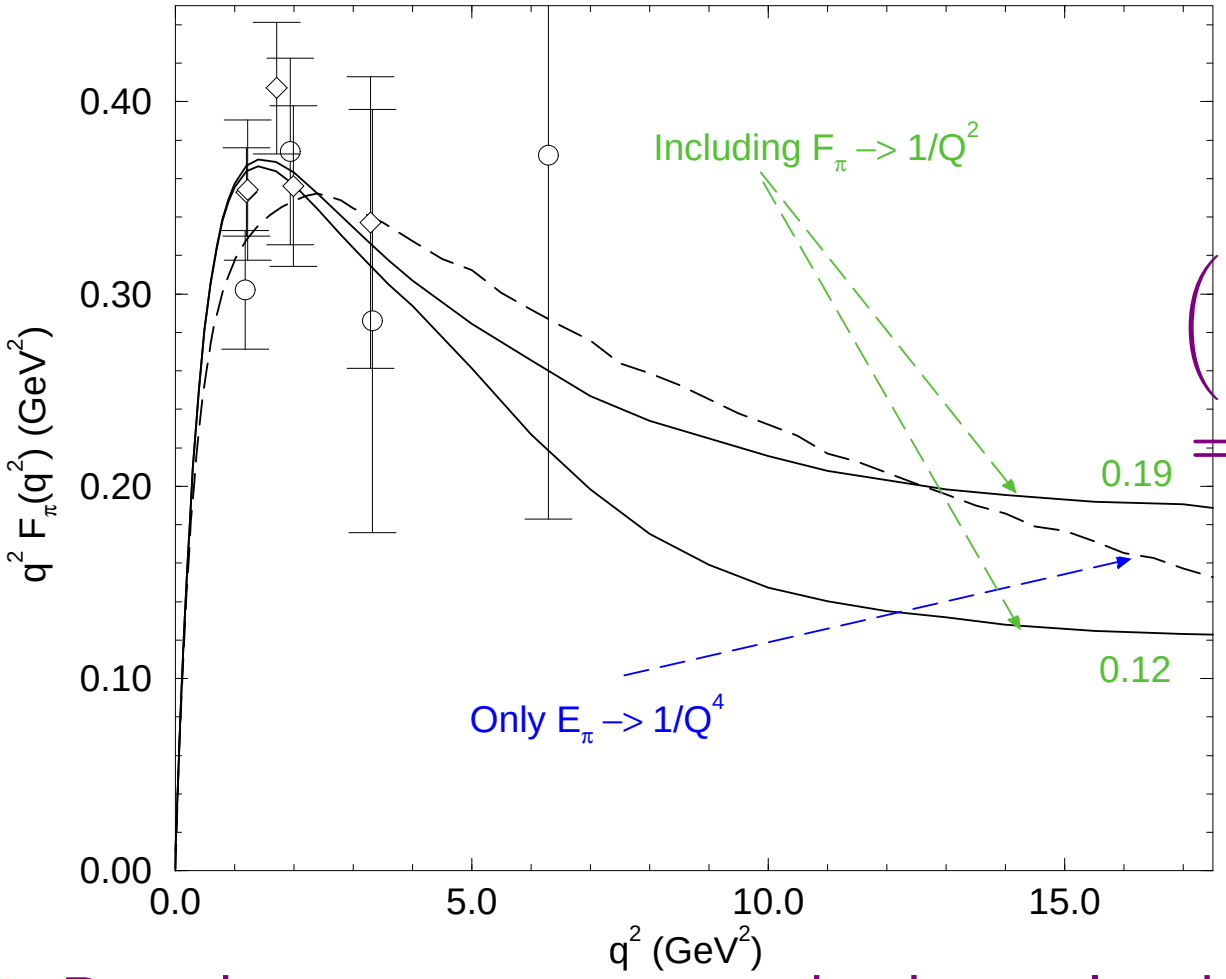

$$\left(\frac{Q}{2}\right)^2 = 2\,\mathrm{GeV}^2$$
$$\Rightarrow Q^2 = 8\,\mathrm{GeV}^2$$

- Pseudovector components dominate ultraviolet behaviour of electromagnetic form factor

Guttierez, Bashir, Cloët, Roberts:
in progress

GT for pion – NJL

Guttierez, Bashir, Cloët, Roberts:
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GT for pion – NJL

- Bethe-Salpeter amplitude can't depend on relative momentum

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    $\gamma_\mu S(k+P/2) i\gamma_5 E_\pi S(k-P/2)\gamma_\mu$
  - Has pseudovector component

    $\sim E_\pi[\sigma_S(k_+)\sigma_V(k_-) + \sigma_S(k_-)\sigma_V(k_+)]$

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  - Hence F_π on LHS is forced to be nonzero because E_π on RHS is nonzero owing to DCSB

Guttierez, Bashir, Cloët, Roberts:
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Guttierez, Bashir, Cloët, Roberts:
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Guttierez, Bashir, Cloët, Roberts:
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Guttierez, Bashir, Cloët, Roberts:
in progress

GT for pion – NJL

- Bethe-Salpeter amplitude: General Form

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- This behaviour dominates for $Q^2 \gtrsim M_Q^2 \dfrac{E_\pi}{F_\pi} > 0.8\,{\rm GeV}^2$

Computation: Elastic Pion Form Factor

Guttierez, Bashir, Cloët, Roberts:
in progress

- DSE prediction: $M(p^2)$; i.e., interaction $\dfrac{1}{|x-y|^2}$
- cf. $M(p^2) =$ Constant; i.e., interaction $\delta^4(x-y)$

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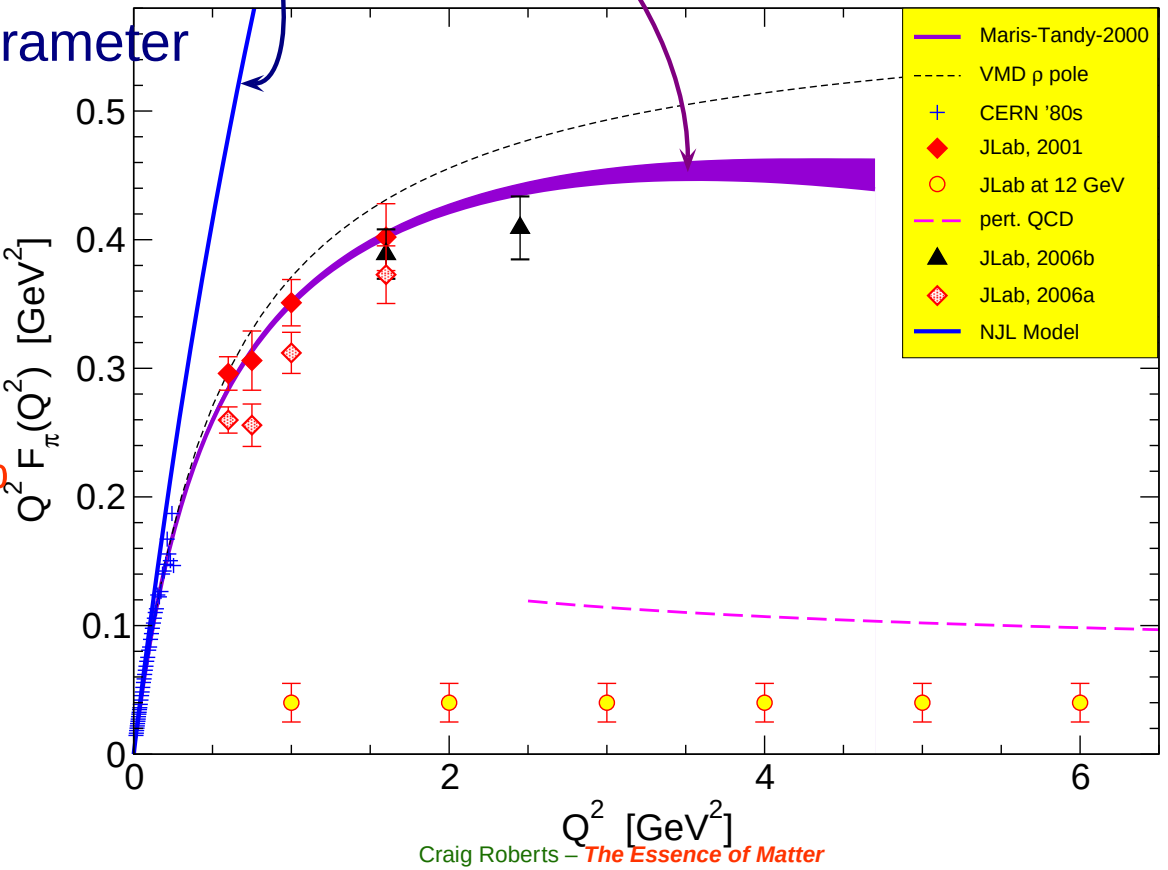
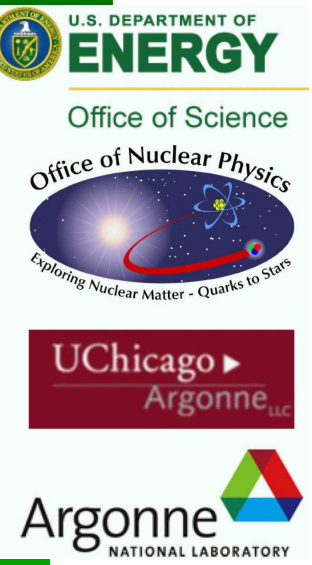
Same predictions for $Q^2 = 0$ properties

Disagreement $> 20\%$ for $Q^2 > M^2$



Epilogue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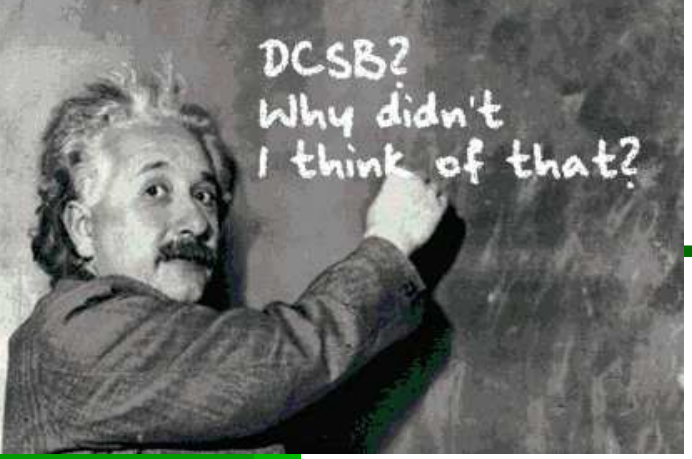

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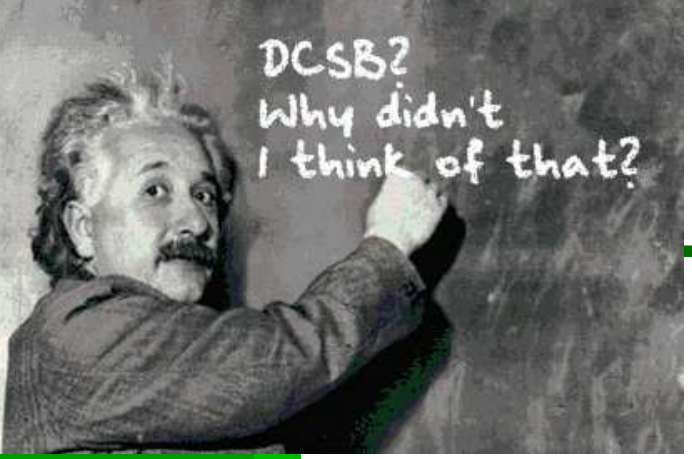

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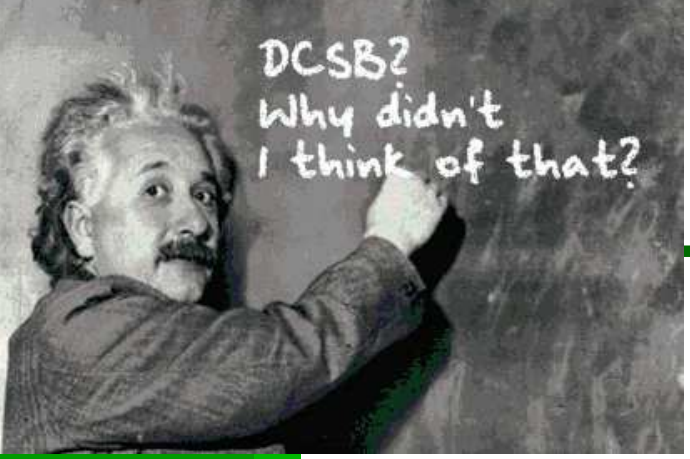
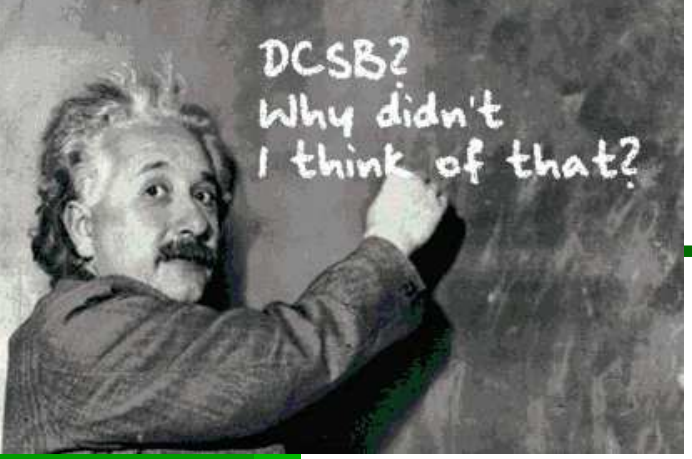


Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
  - It predicts, amongst other things, that
    - light current-quarks become heavy constituent-quarks: $4 \to 400\,$MeV
    - pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140\,$MeV
    - pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi\bar{q}q} \approx 4.3$
    - pseudscalar mesons couple unnaturally strongly to the lightest baryons $g_{\pi\bar{N}N} \approx 12.8 \approx 3 g_{\pi\bar{q}q}$



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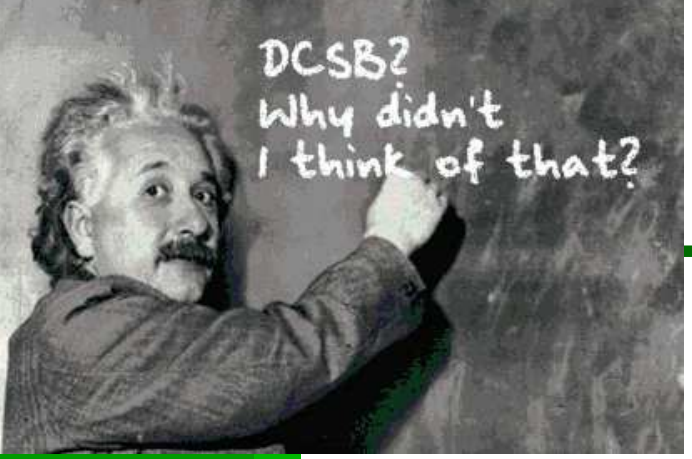
- DCSB impacts dramatically upon observables



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- DCSB impacts dramatically upon observables
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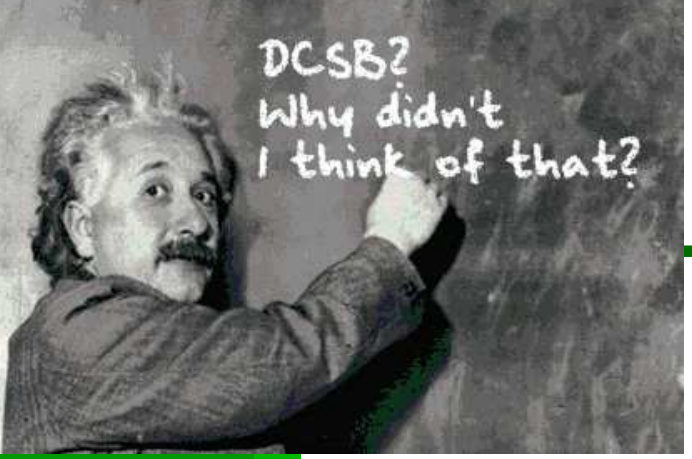

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  - Exposing & elucidating its effect in hadron physics requires nonperturbative, symmetry preserving framework; i.e., Poincaré covariance, chiral and e.m. current conservation, etc.

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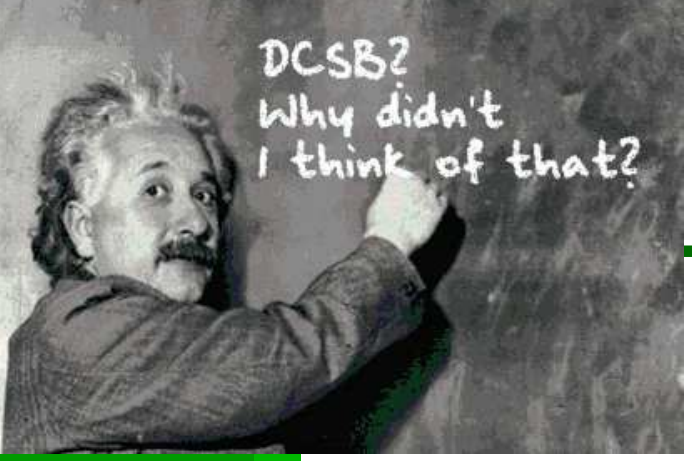

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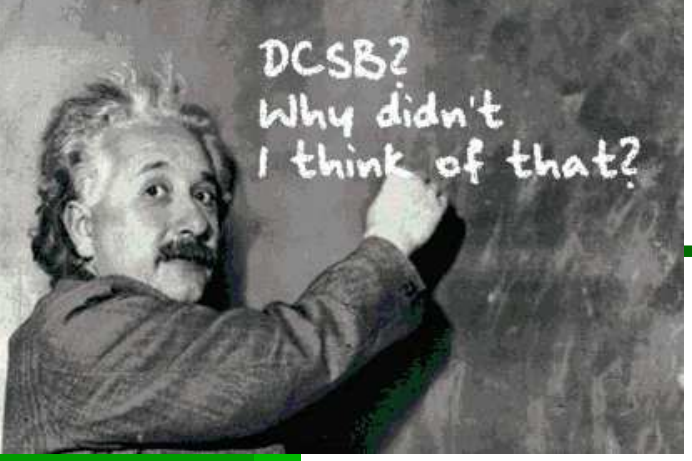
  - Studies underway will identify observable signals of $M(p^2)$, the most important mass-generating mechanism for visible matter in the Universe



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- DSEs: Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks

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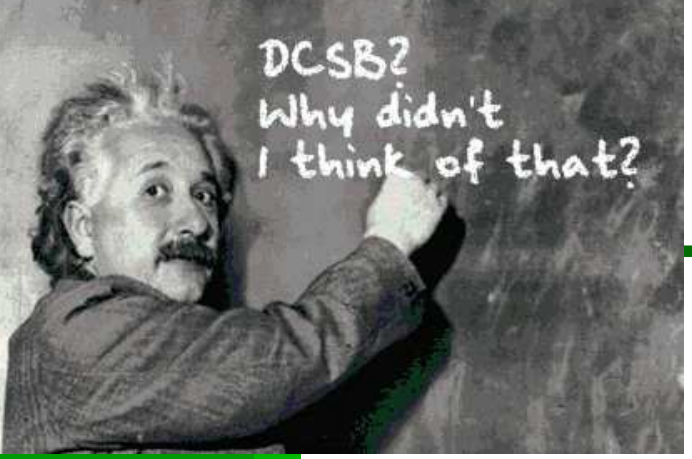


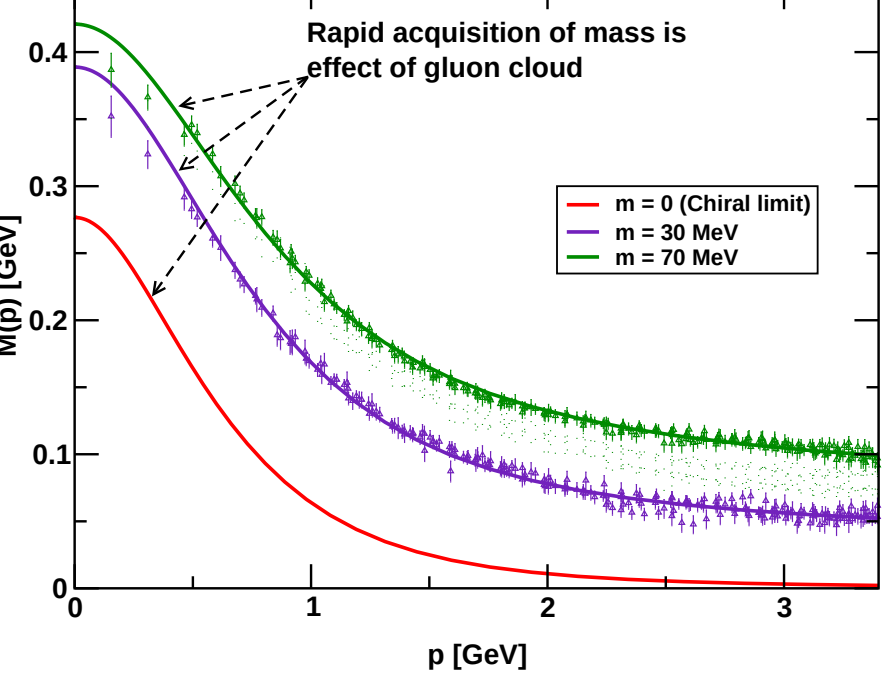
Epilogue

Now is an exciting time . . .

Positioned to unify phenomena as apparently disparate as

- Hadron spectrum
- Elastic and transition form factors, from small- to large-Q^2
- Parton distribution functions

Epilogue



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Key: an understanding of both the fundamental origin of nuclear mass and the far-reaching consequences of the mechanism responsible; namely, Dynamical Chiral Symmetry Breaking

Contents

DSE-based Faddeev Equation

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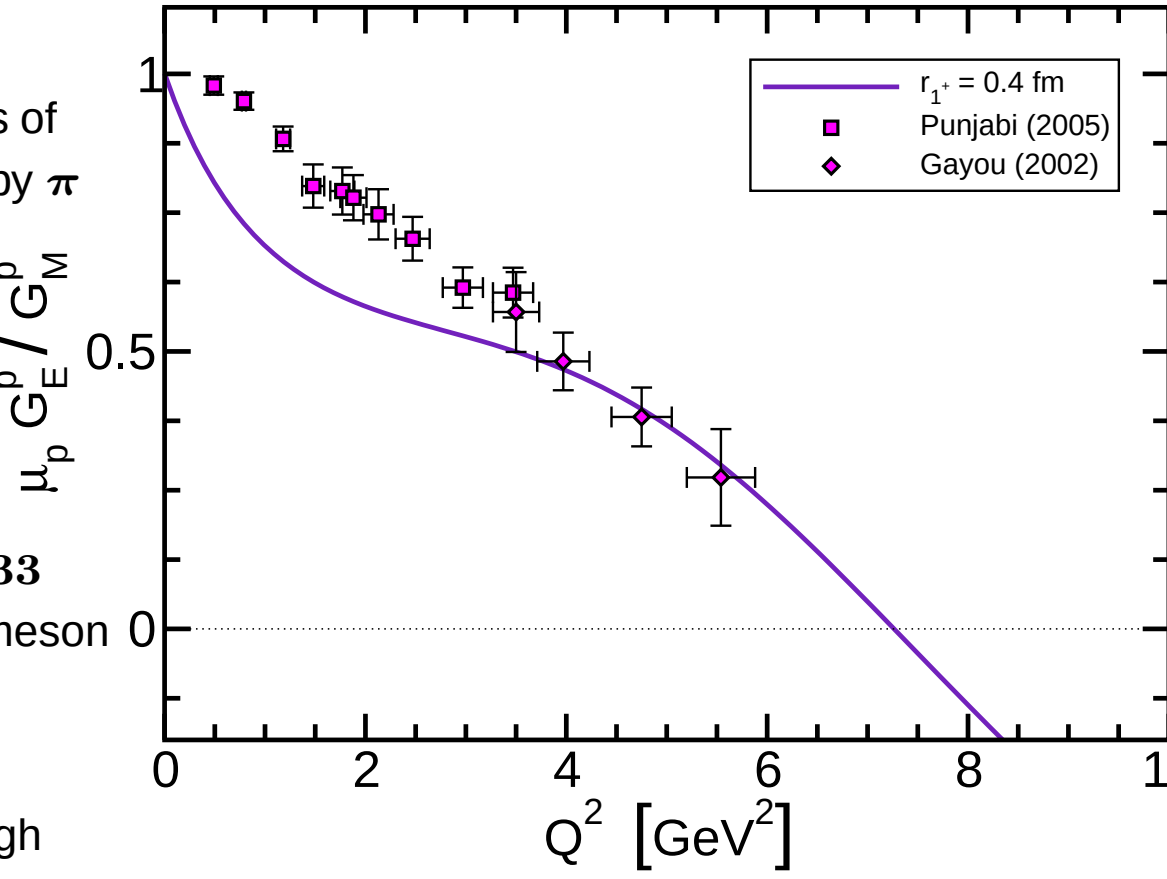

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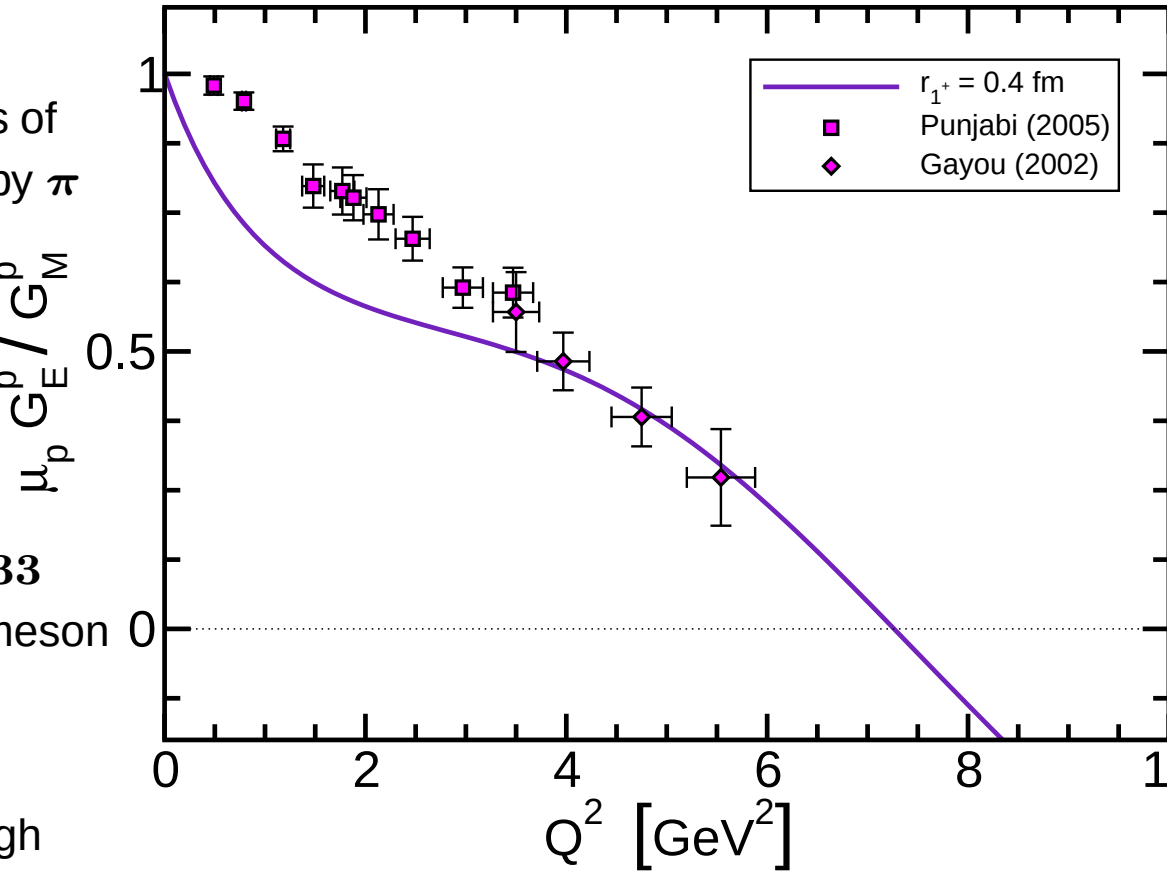

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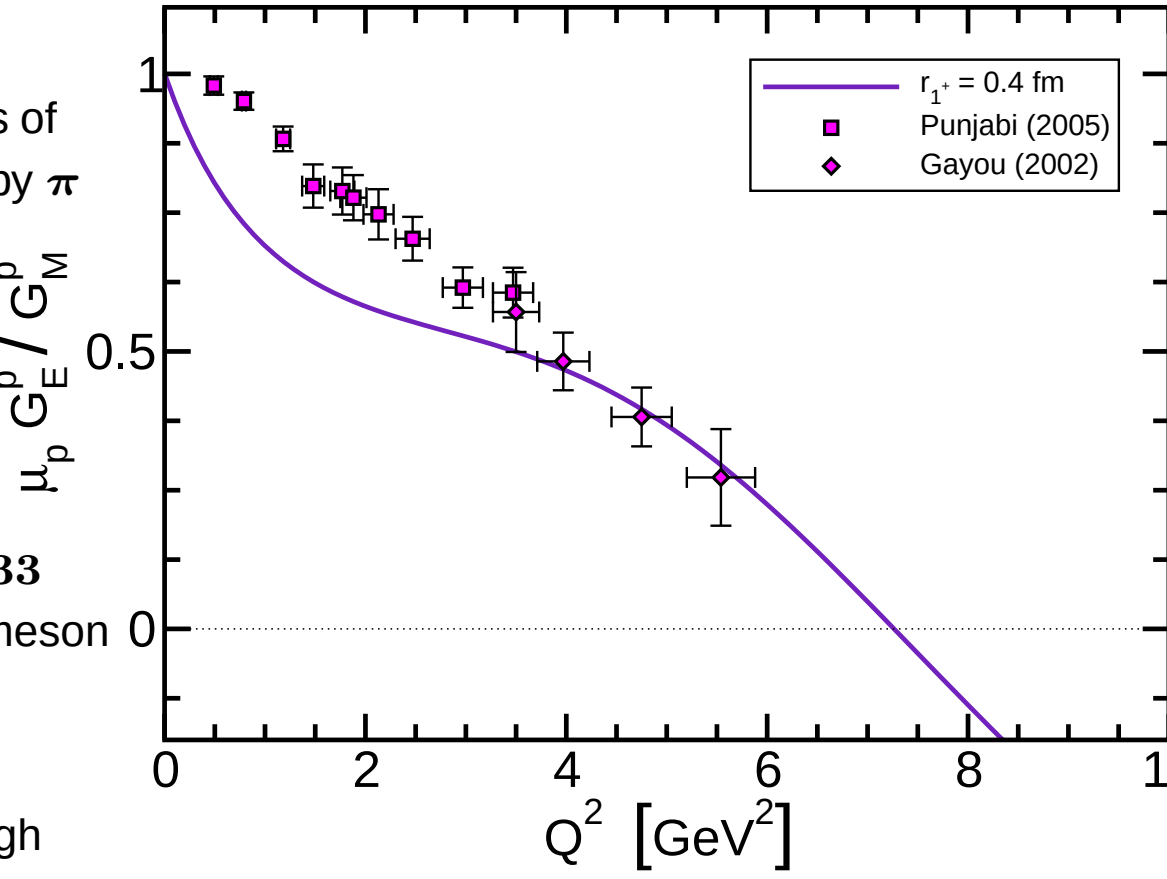

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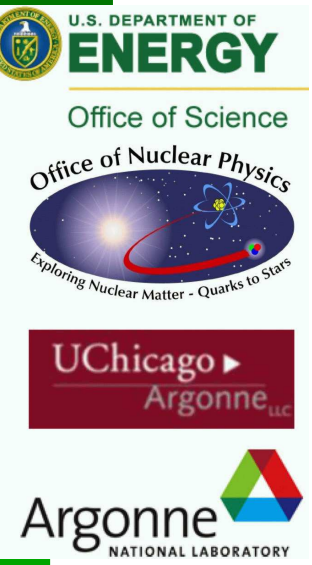
- Always a zero but position depends on details of current

Harry Lee

Pions and Form Factors



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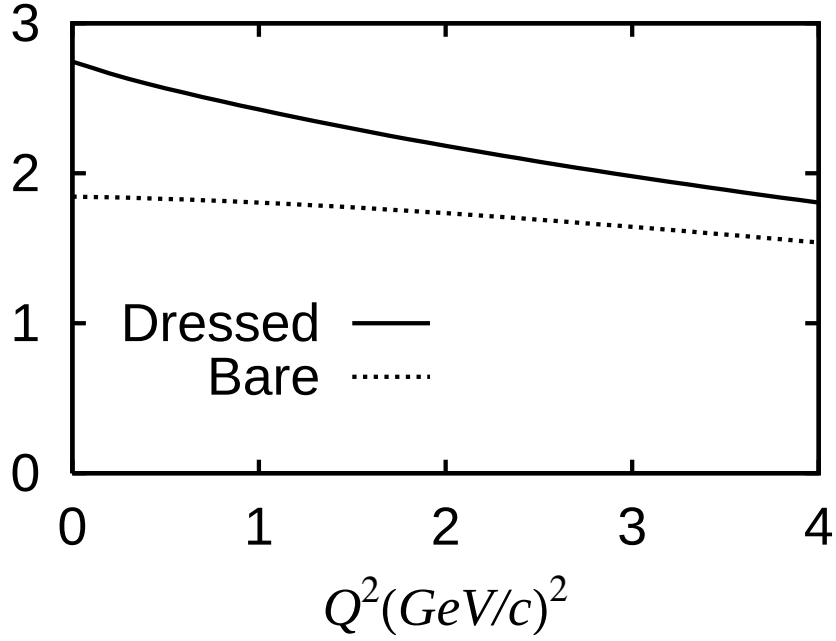

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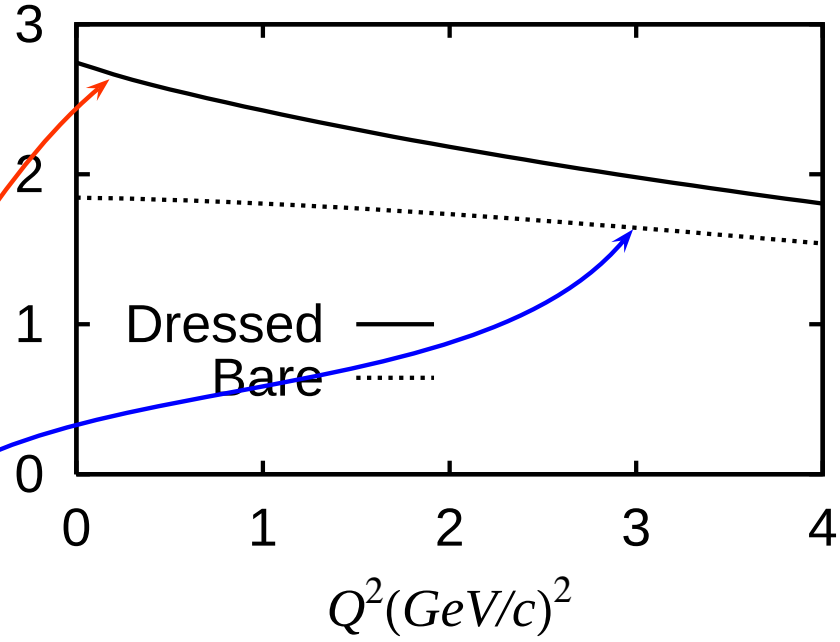

Quark Core

- Responsible for only 2/3 of result at small Q^2
- Dominant for $Q^2 > 2 - 3\,\mathrm{GeV}^2$

Pion Cloud

Pion Cloud

F2 – neutron

Pion Cloud
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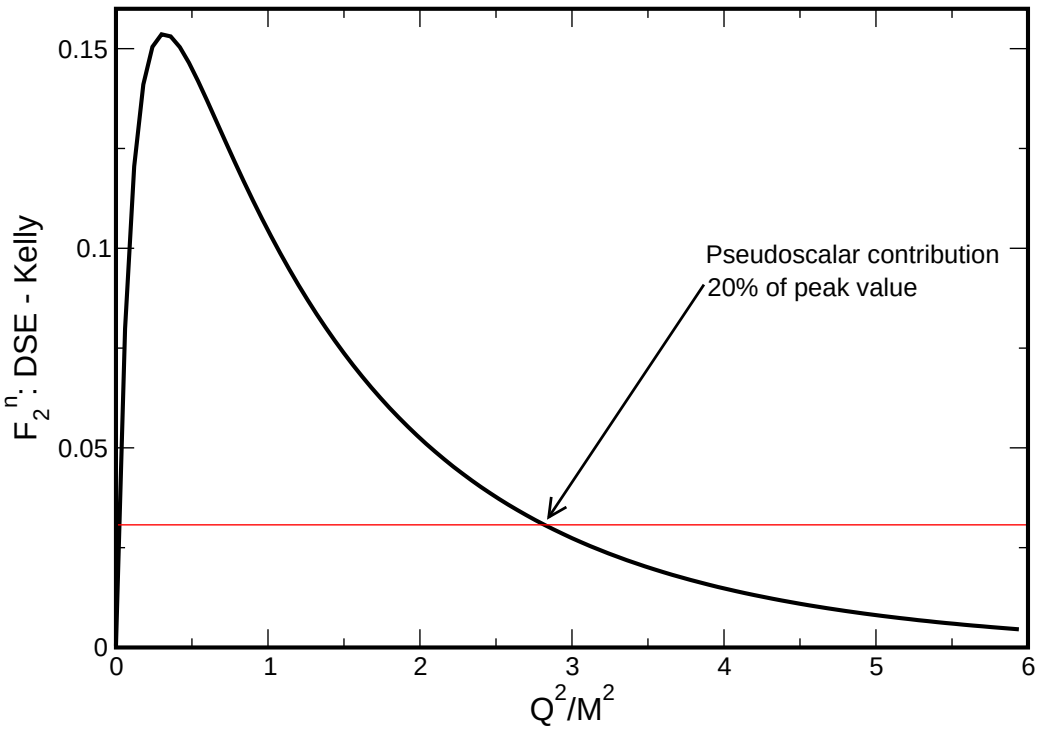
- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core



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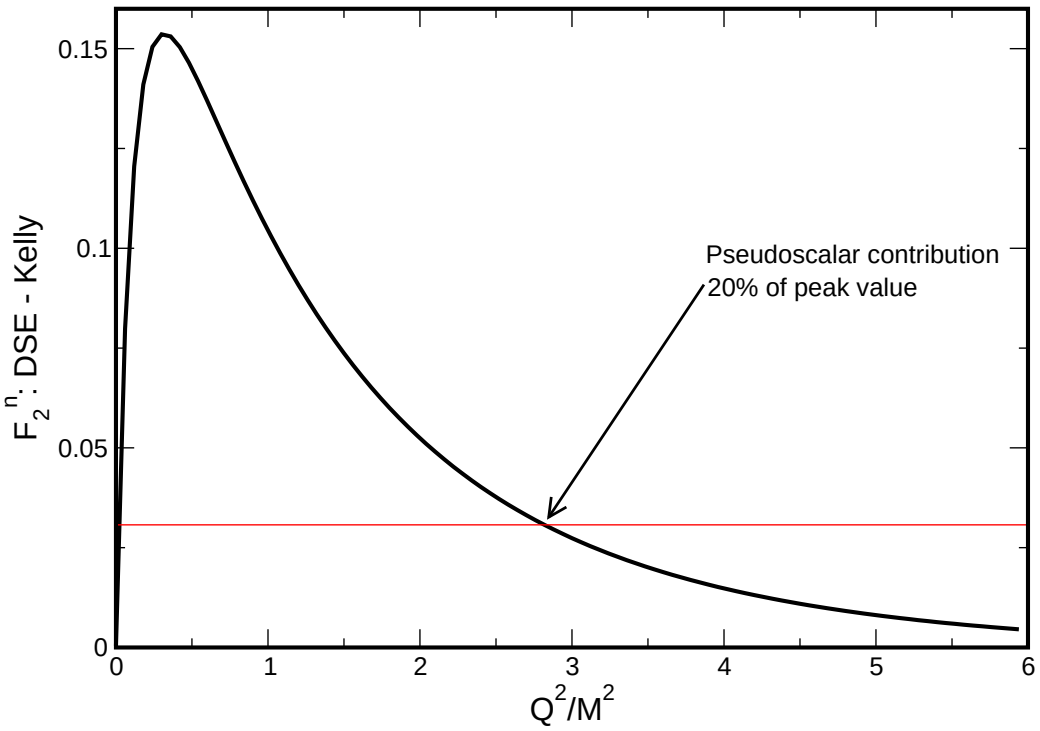

- Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3-4\,M_N^2$