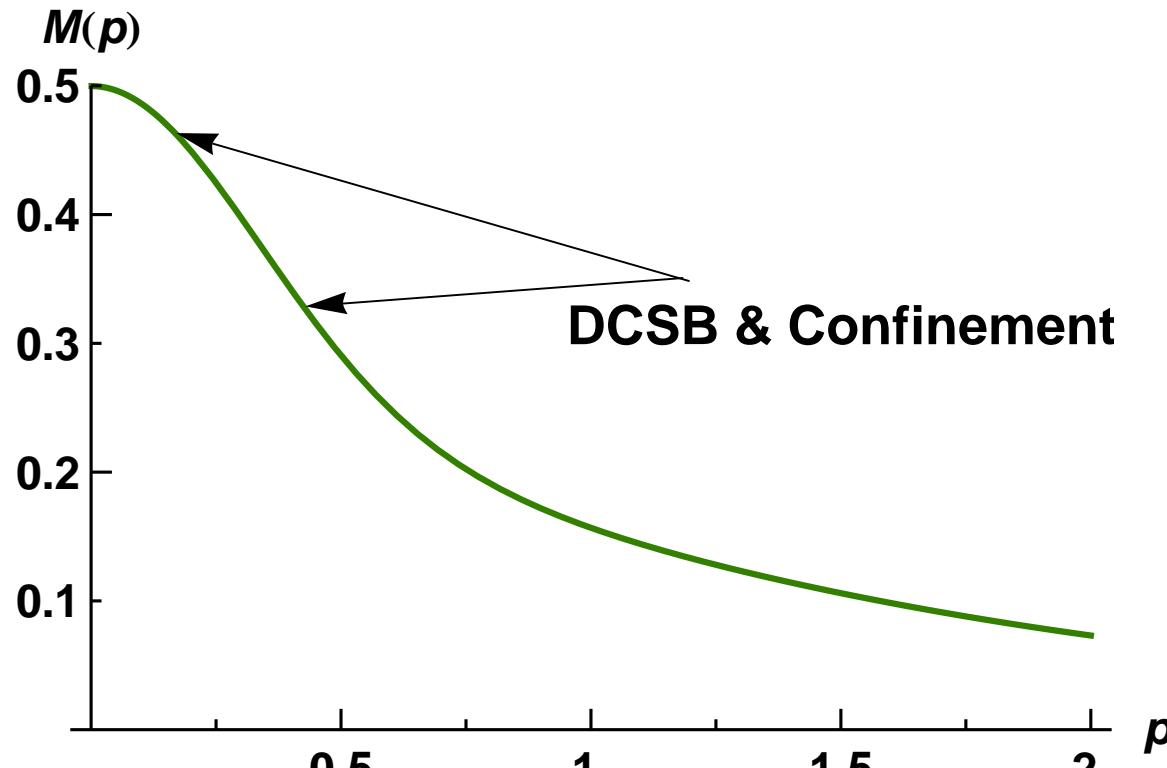


Dyson-Schwinger equations: Recent successes & future perspective

Dressed-quark Mass Function

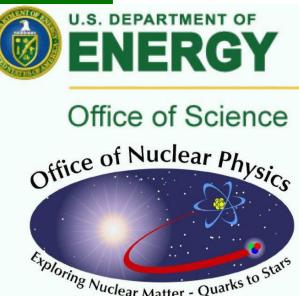


Craig D. Roberts
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Physics Division & **School of Physics**

Argonne National Laboratory **Peking University**

<http://www.phy.anl.gov/theory/staff/cdr.html>



Universal Truths



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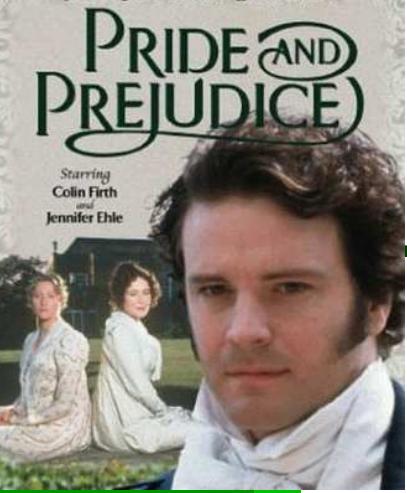
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Universal Truths



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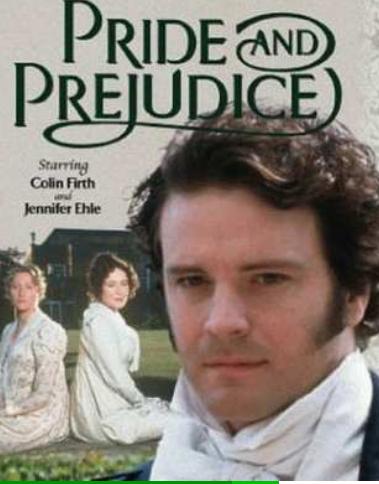
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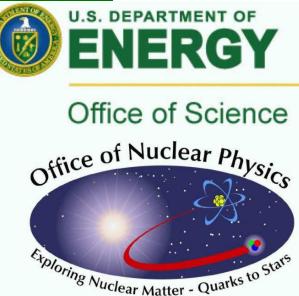
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Universal Truths

- Spectrum of excited states, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.

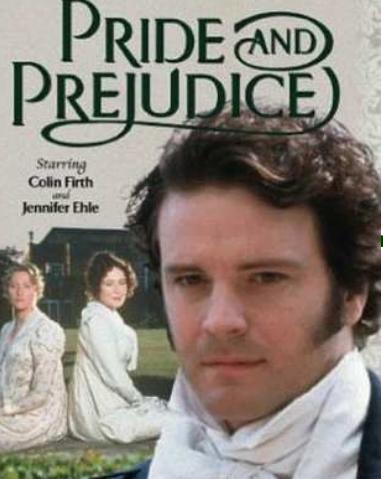


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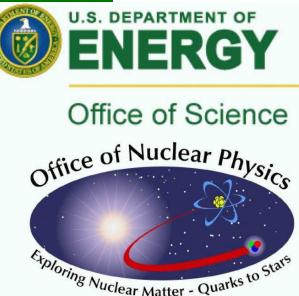
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Universal Truths

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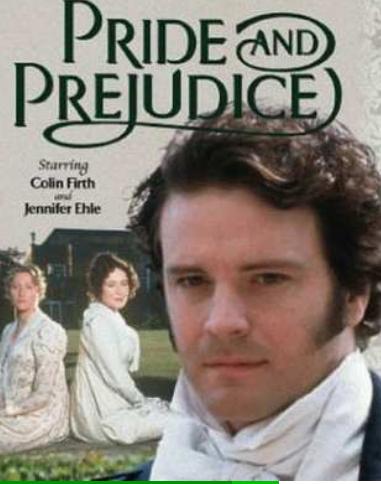


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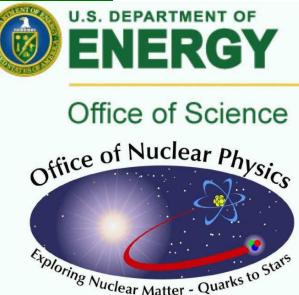
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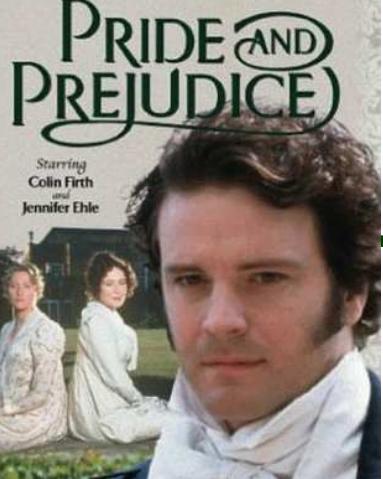


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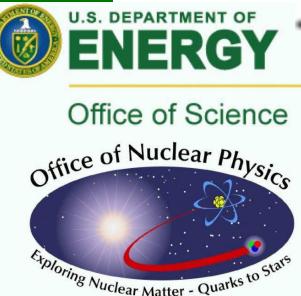
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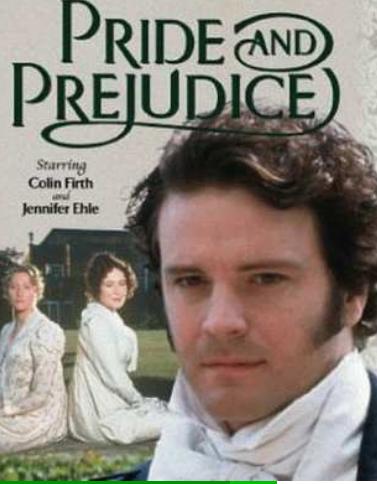


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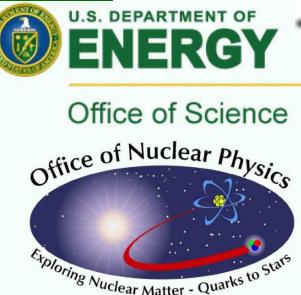
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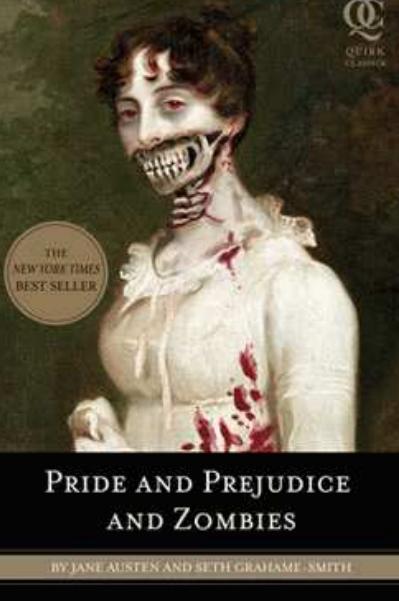
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Universal Truths

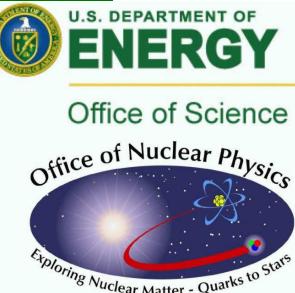
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- Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.





Universal Truths

- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.

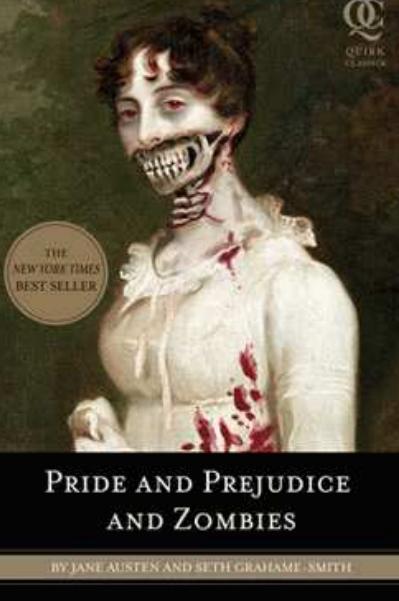


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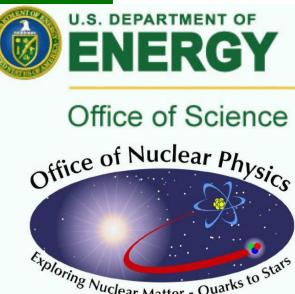
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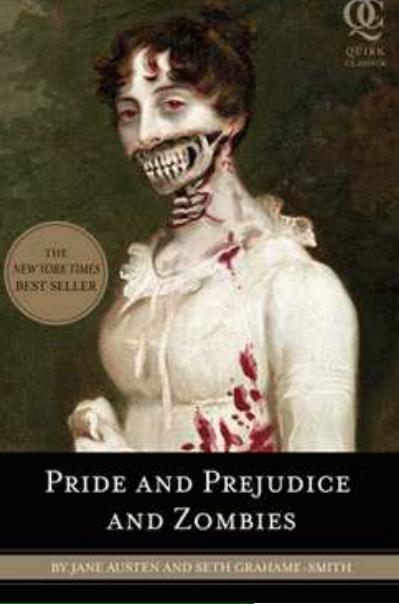
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Universal Truths

- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.
- E.g., one problem: DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not yet been realised in the light-front formulation.

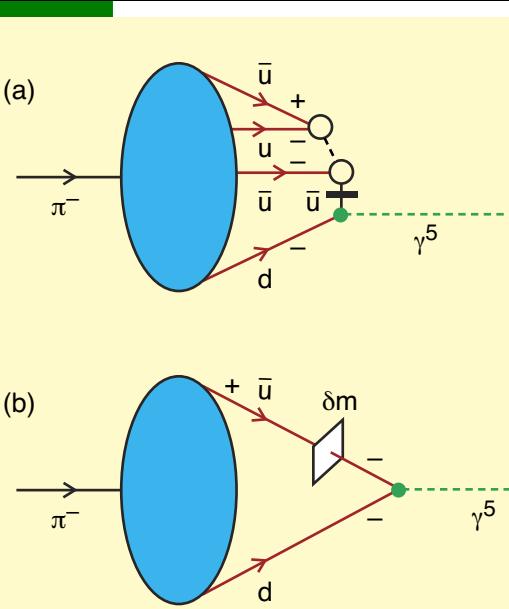




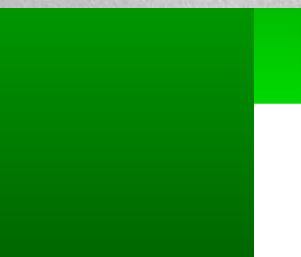
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- E.g., one problem: DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not yet been realised in the light-front formulation.

- Resolution
 - So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.
 - DCSB obtained via coherent contribution from countable infinity of higher Fock-state components in LF-wavefunction.
- Brodsky, Roberts, Shrock, Tandy – arXiv:1005.4610 [nucl-th].



QCD's Challenges



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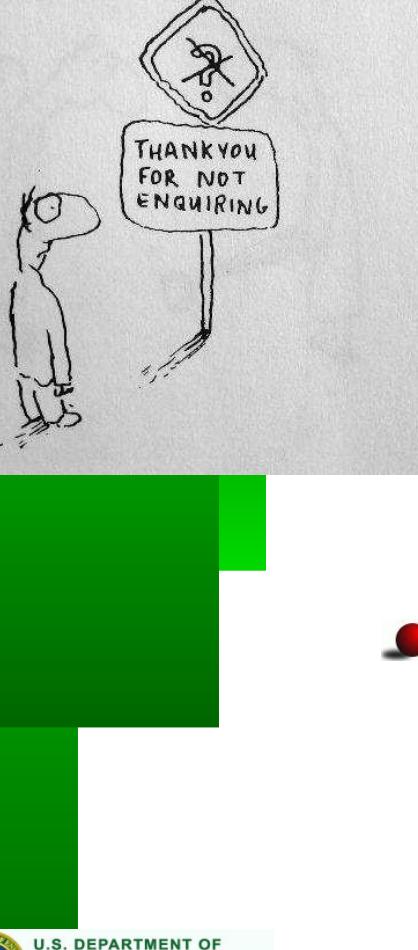


- Quark and Gluon Confinement
 - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon



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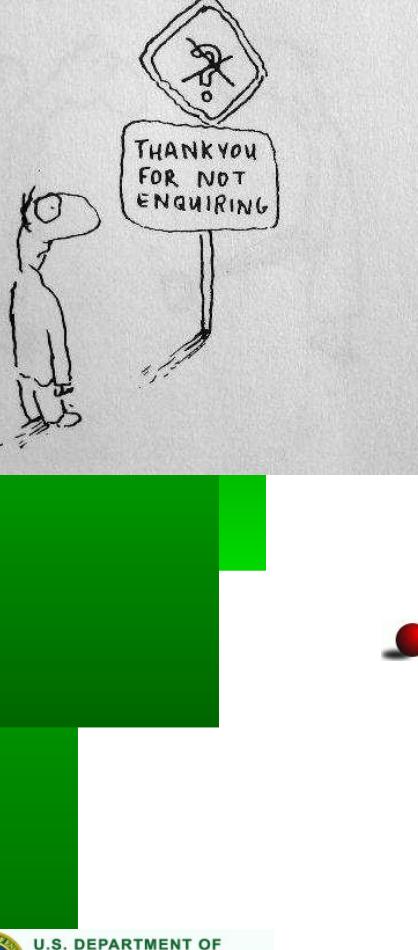


- Quark and Gluon Confinement
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- Dynamical Chiral Symmetry Breaking
 - Very unnatural pattern of bound state masses
 - e.g., Lagrangian (pQCD) quark mass is small but . . .
no degeneracy between $J^{P=+}$ and $J^{P=-}$

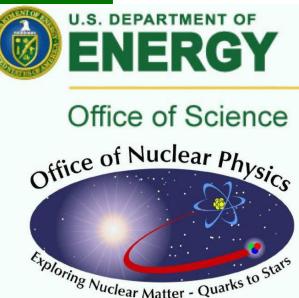


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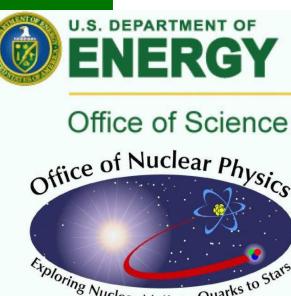




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- Neither of these phenomena is apparent in QCD's Lagrangian **yet** they are the dominant determining characteristics of real-world QCD.



Understand Emergent Phenomena



- Quark and Gluon Confinement
 - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
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no degeneracy between $J^{P=+}$ and $J^{P=-}$
- Neither of these phenomena is apparent in QCD's Lagrangian **yet** they are the dominant determining characteristics of real-world QCD.
- QCD – Complex behaviour
arises from apparently simple rules

Confinement



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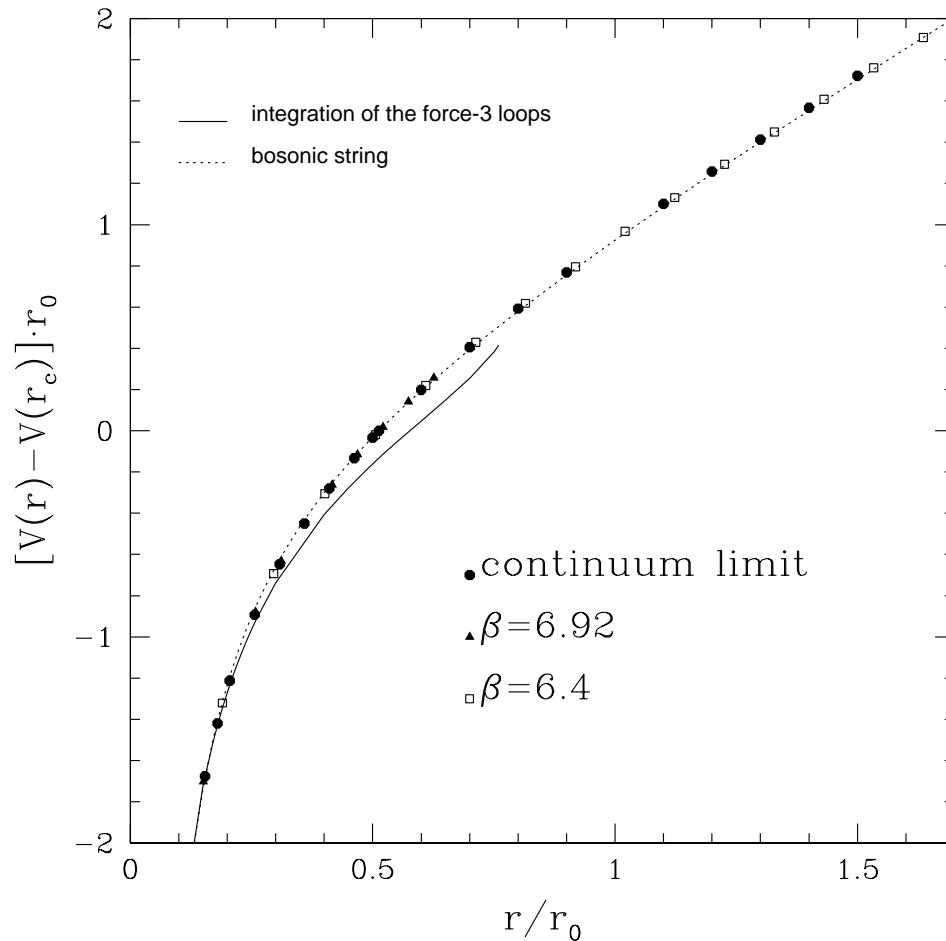
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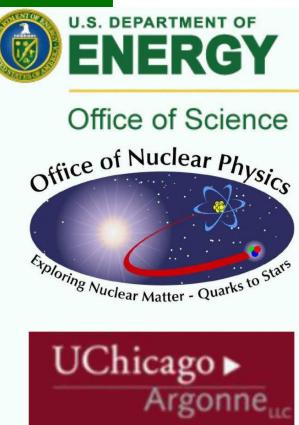
- Infinitely Heavy Quarks . . . Picture in Quantum Mechanics



$$V(r) = \sigma r - \frac{\pi}{12} \frac{1}{r}$$

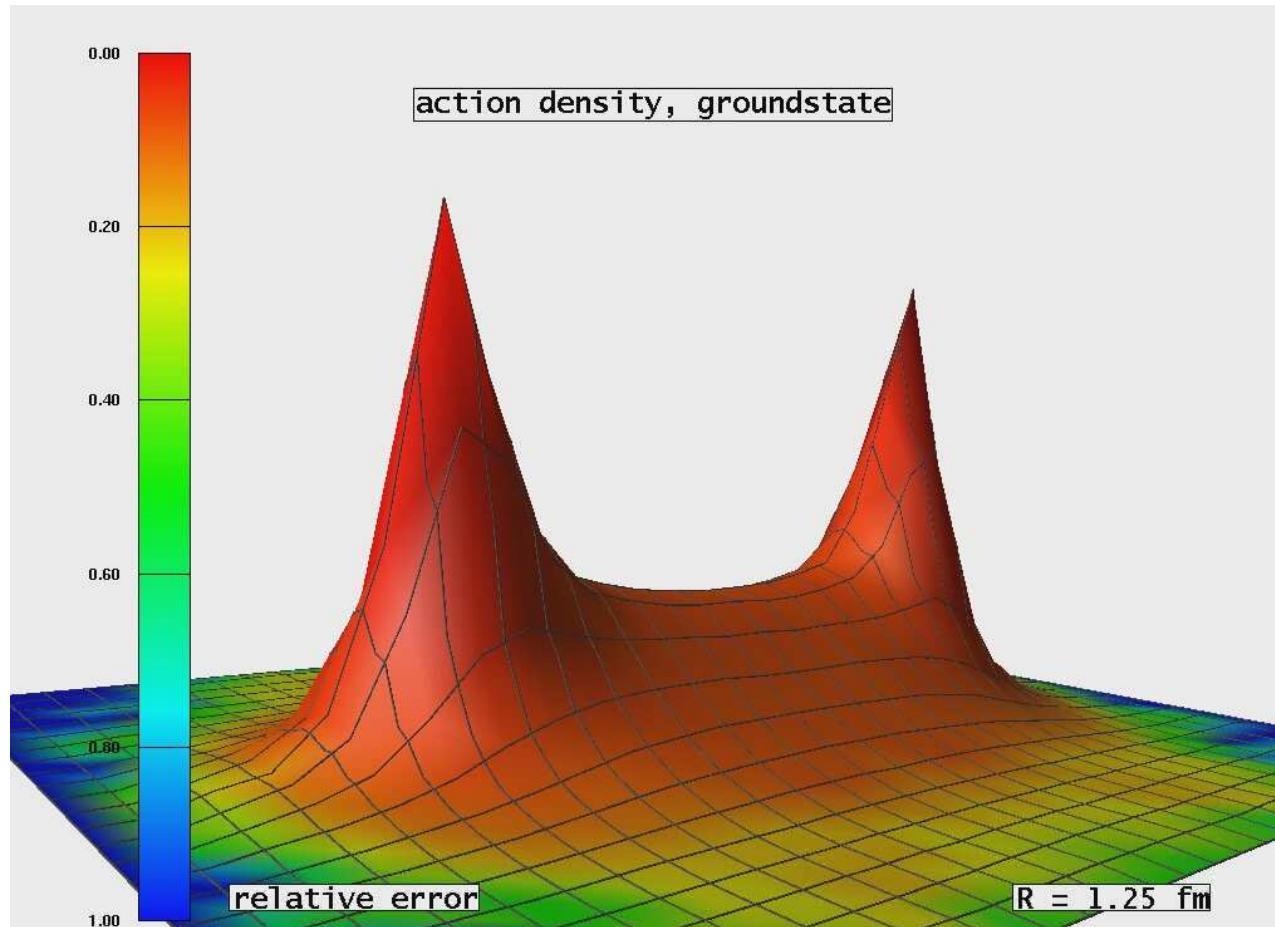
$$\sqrt{\sigma} \sim 470 \text{ MeV}$$

Necco & Sommer
he-la/0108008



Confinement

- Illustrate this in terms of the action density ... analogous to plotting the Force = $F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12} \frac{1}{r^2}$



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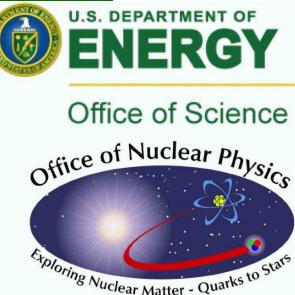


UChicago ▶ Argonne



Confinement

- What happens in the real world; namely, in the presence of light-quarks?



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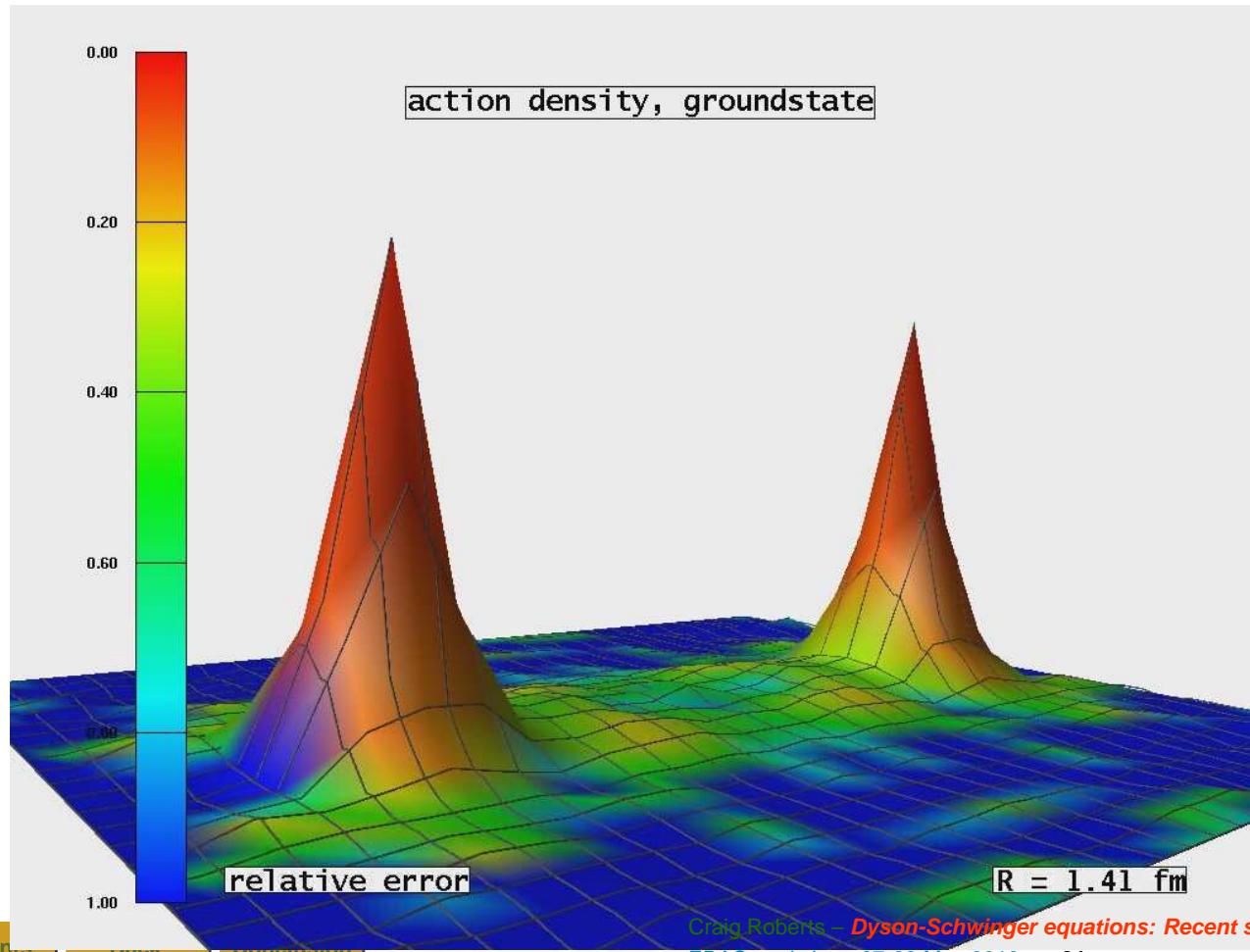
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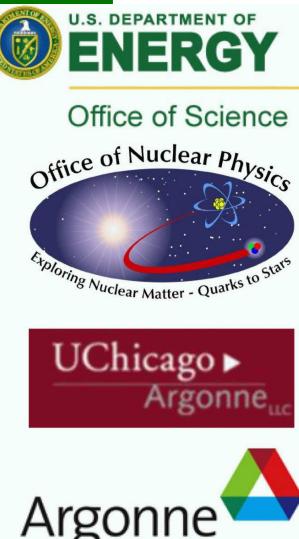
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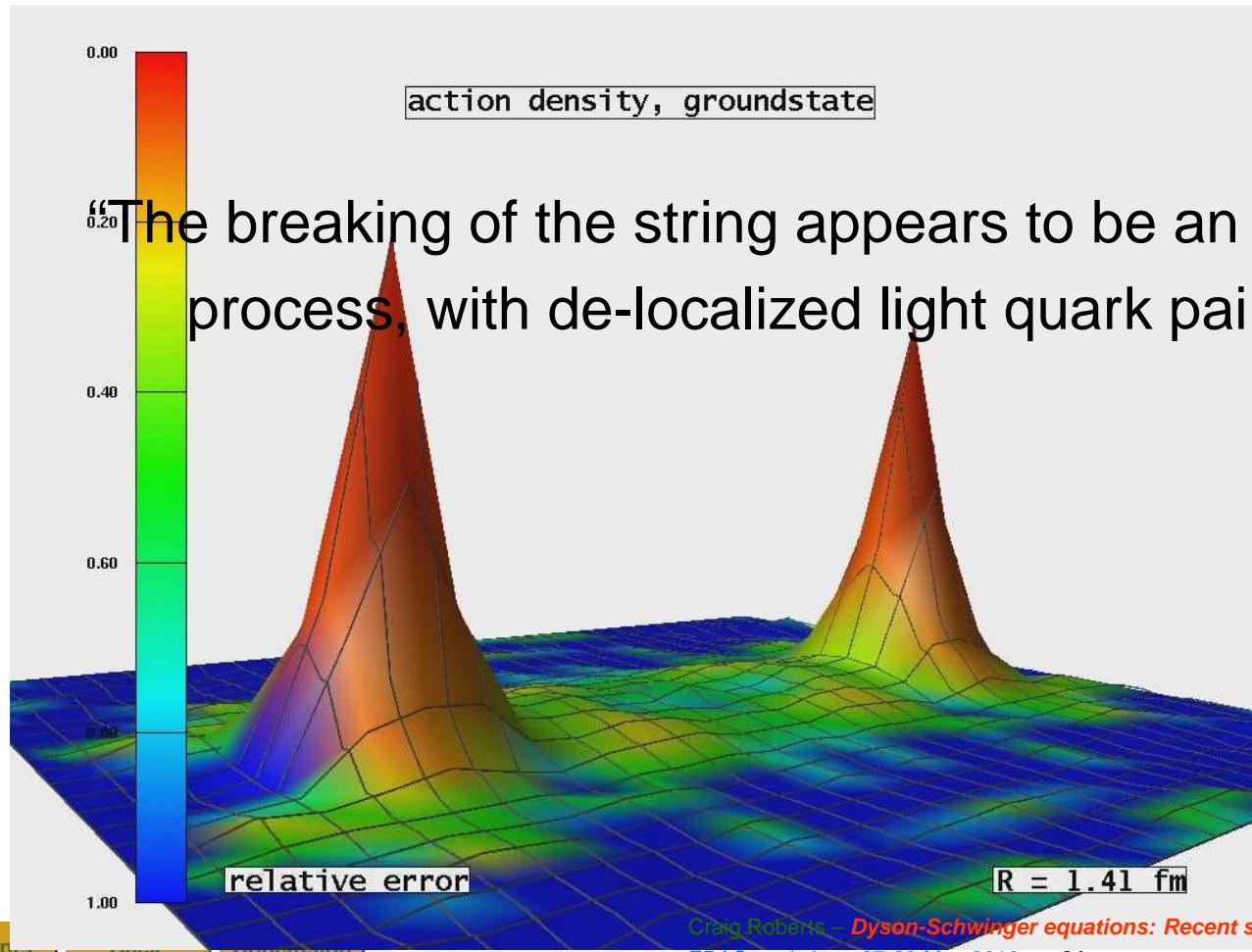


Bali, et al.
he-la/0512018

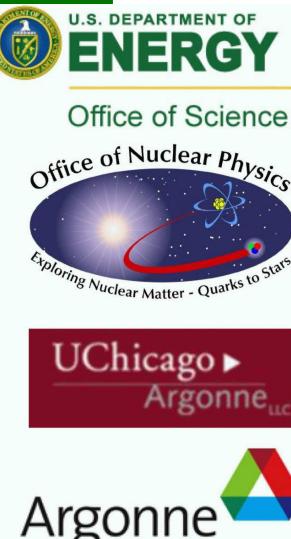


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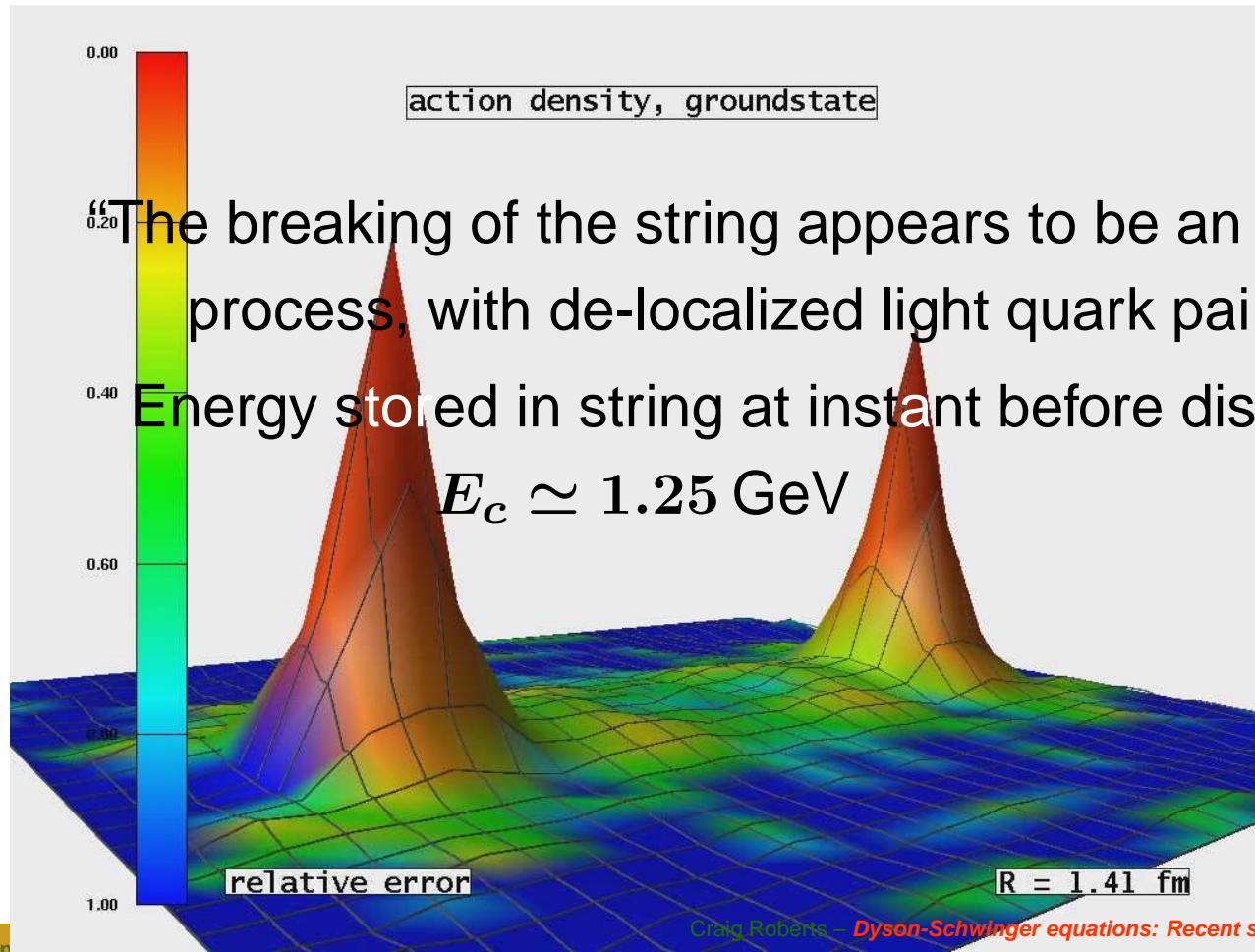


“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”



Confinement

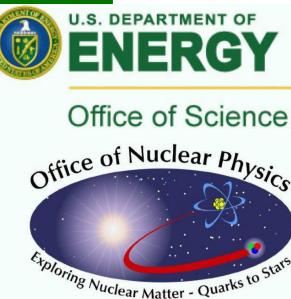
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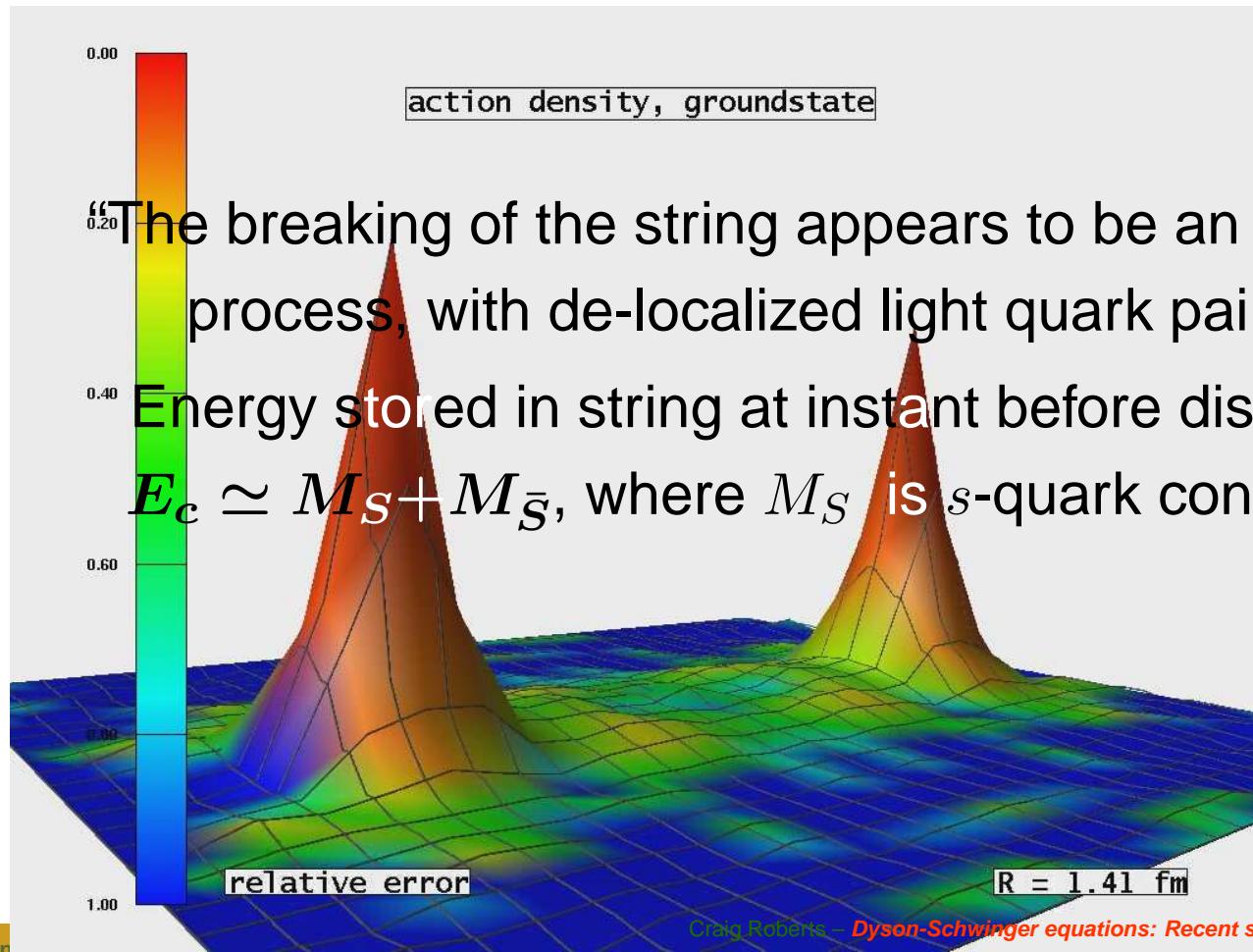
Energy stored in string at instant before disappearance:

$$E_c \simeq 1.25 \text{ GeV}$$



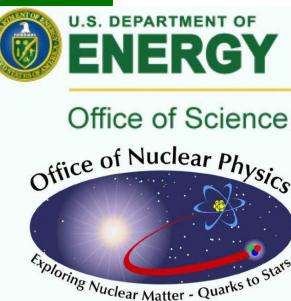
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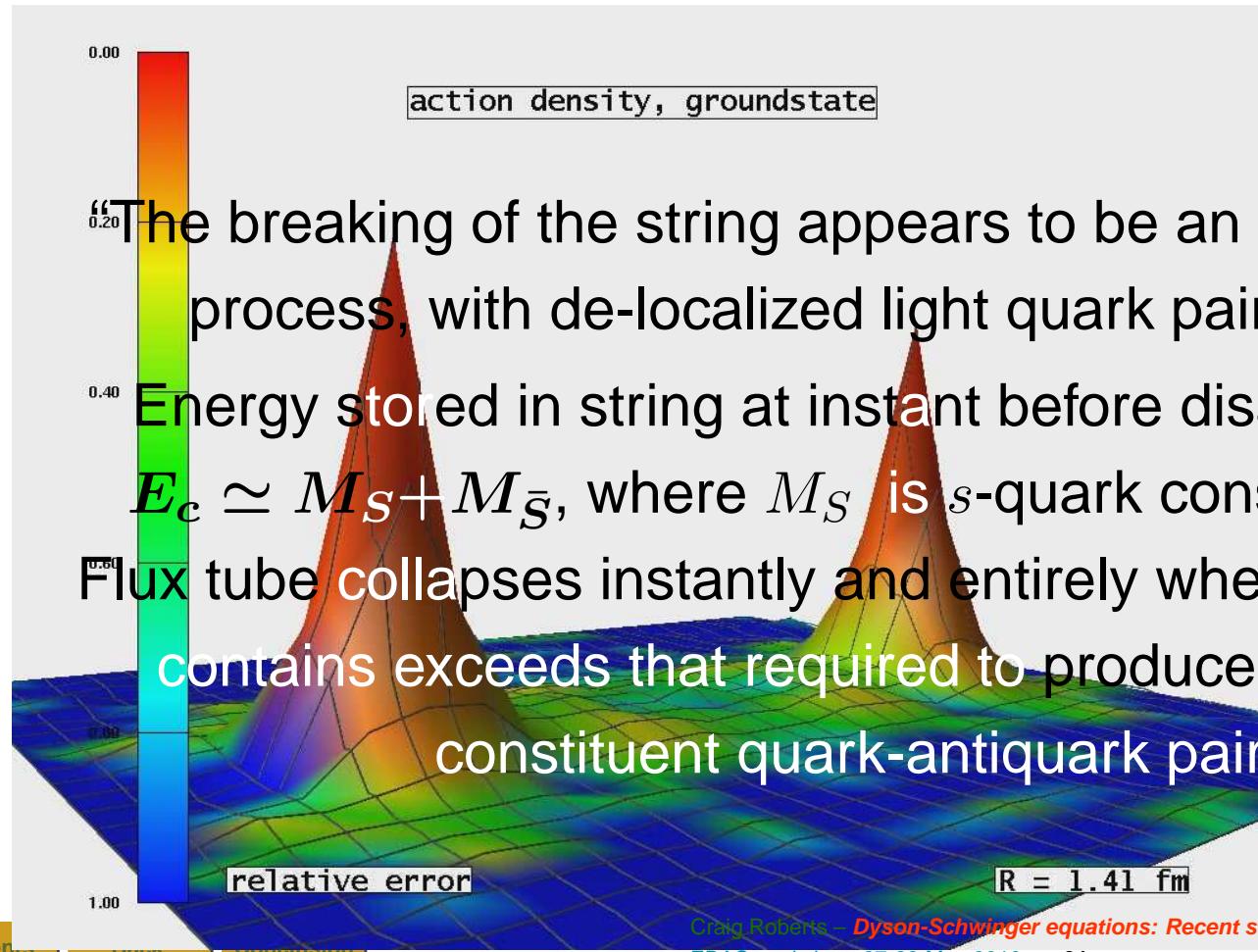
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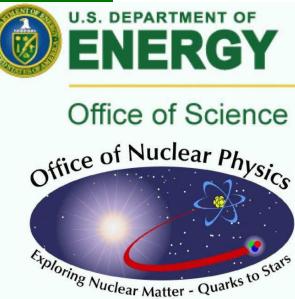
Bali, et al.

he-la/0512018

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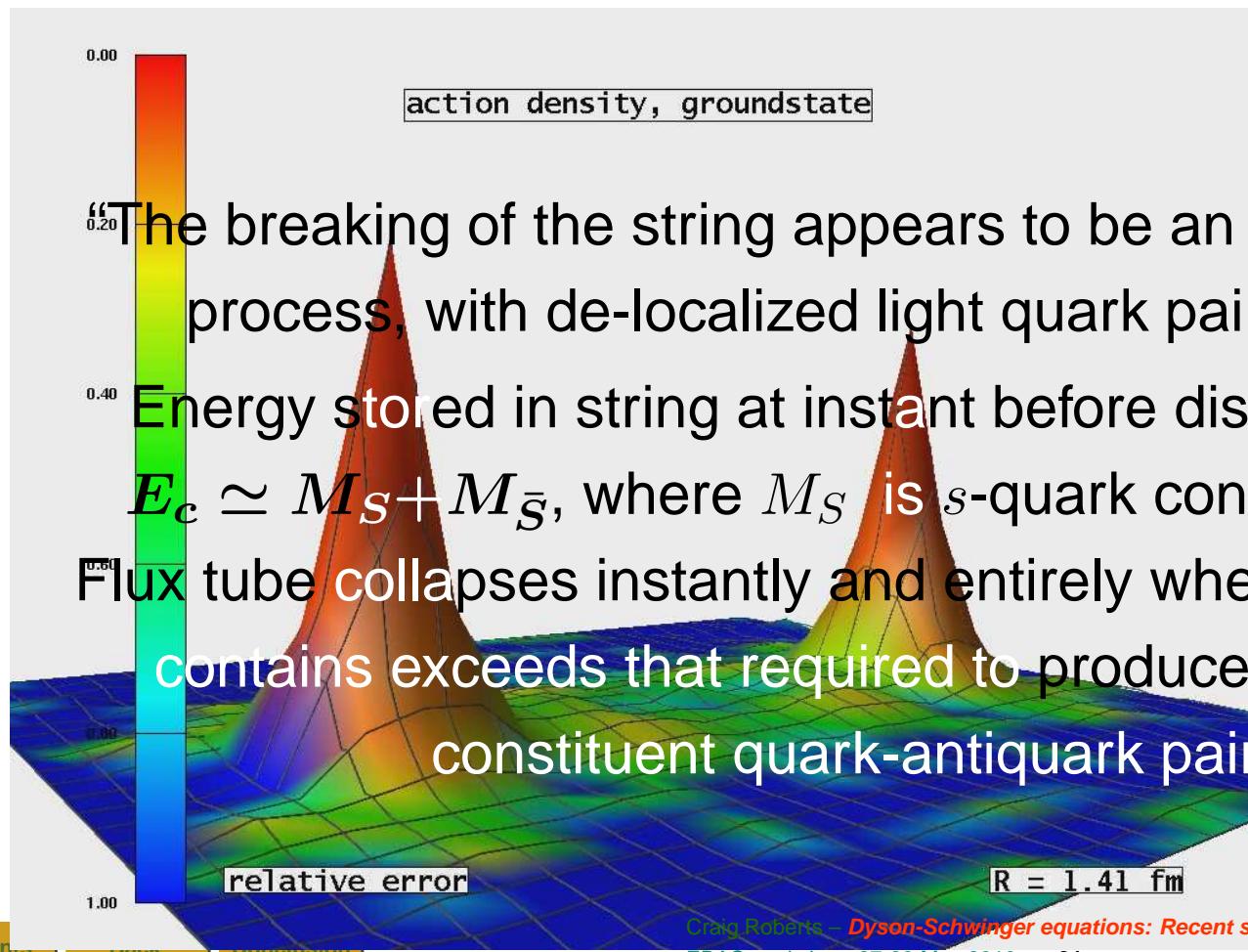
Energy stored in string at instant before disappearance: $E_c \simeq M_S + M_{\bar{S}}$, where M_S is s -quark constituent-mass

Flux tube collapses instantly and entirely when the energy it contains exceeds that required to produce the lightest constituent quark-antiquark pair.



Therefore . . . No information on *potential* between light-quarks. **Confinement**

- What happens in the real world; namely, in the presence of light-quarks? No one knows . . . but $\bar{Q}Q + 2 \times \bar{s}s$



Charting the Interaction between light-quarks



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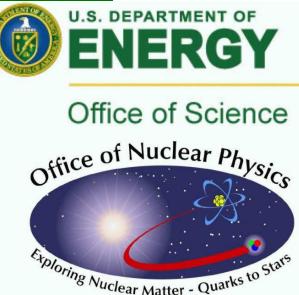
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- Confinement can be related to the analytic properties of QCD's Schwinger functions



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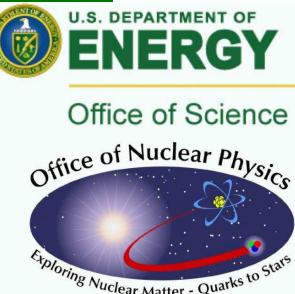
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Charting the Interaction between light-quarks

- Confinement can be related to the analytic properties of QCD's Schwinger functions
- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's *universal* β -function



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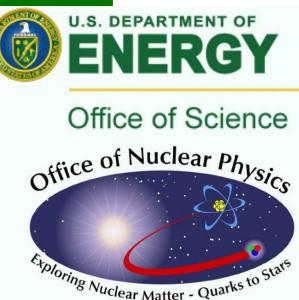
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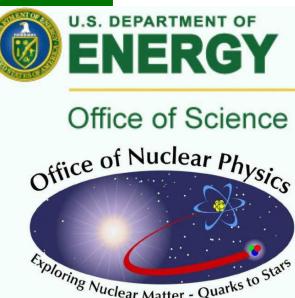
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- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's *universal* β -function
 - This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.



Charting the Interaction between light-quarks

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Of course, the behaviour of the β -function on the perturbative domain is well known.

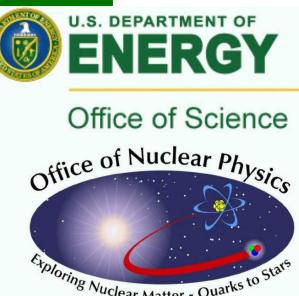


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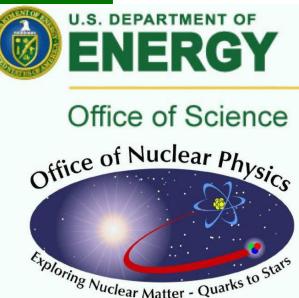
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Of course, the behaviour of the β -function on the perturbative domain is well known.

- This is a well-posed problem whose solution is an elemental goal of modern hadron physics.



What is the light-quark Long-Range Potential?



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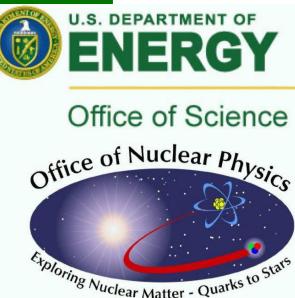
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What is the light-quark Long-Range Potential?

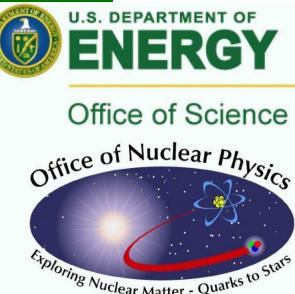


Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any known way to the light-quark interaction.



Charting the Interaction between light-quarks

- Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β -function determines pattern of chiral symmetry breaking



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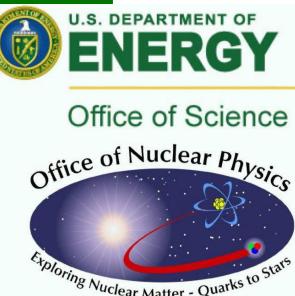
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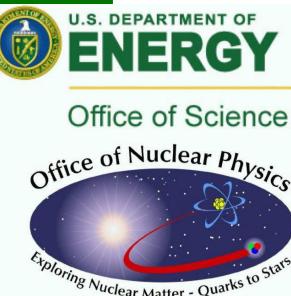
Charting the Interaction between light-quarks

- Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β -function determines pattern of chiral symmetry breaking
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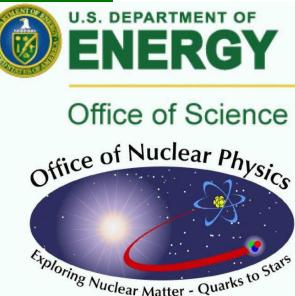
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 - hadron mass spectrum;
 - elastic and transition form factorscan be used to chart β -function's long-range behaviour
- E.g.: Extant studies of mesons show that the properties of hadron excited states are a great deal more sensitive to the long-range behaviour of β -function than those of the ground state



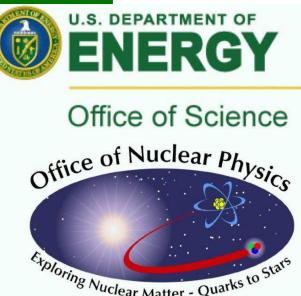
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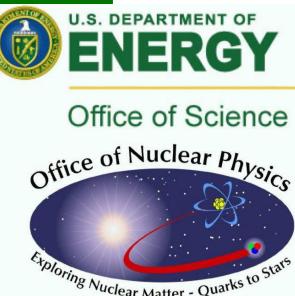
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- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary



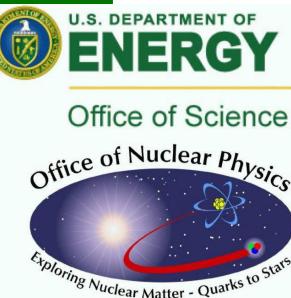
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- DSEs connect β -function to experimental observables. Hence, comparison between computations and observations can be used to chart β -function's long-range behaviour
- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
 - Steady quantitative progress is being made with a scheme that is systematically improvable
(See nucl-th/9602012 and references thereto)



Charting the Interaction between light-quarks

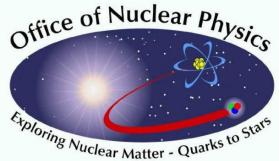
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- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
 - On other hand, at present significant qualitative advances possible with symmetry-preserving kernel Ansätze that express important additional nonperturbative effects – $M(p^2)$ – difficult/impossible to capture in any finite sum of contributions



Gap Equation General Form



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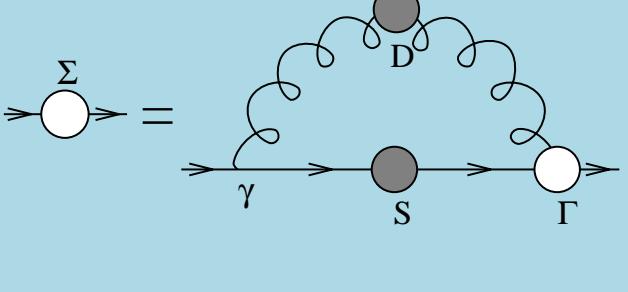
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Gap Equation

General Form



$$S_f(p)^{-1} = Z_2 (i\gamma \cdot p + m_f^{\text{bm}}) + \Sigma_f(p),$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \frac{\lambda^a}{2} \Gamma_\nu^f(q, p),$$

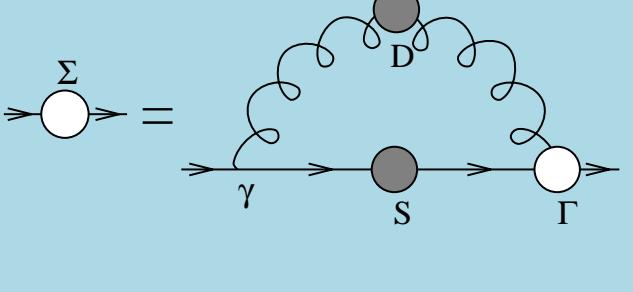


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Gap Equation

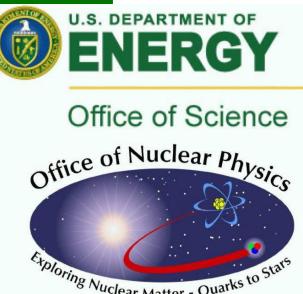
General Form



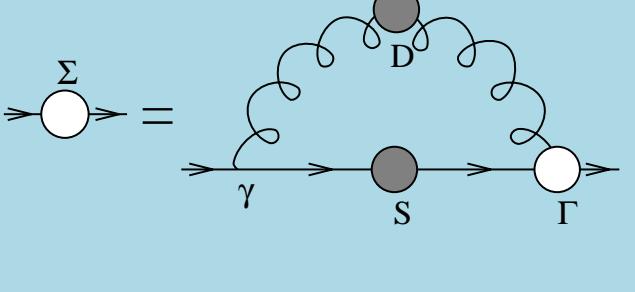
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- $Z_{1,2}(\zeta^2, \Lambda^2)$ are respectively the vertex and quark wave function renormalisation constants, with ζ the renormalisation point
- $m^{\text{bm}}(\Lambda)$ is the Lagrangian current-quark bare mass
- $D_{\mu\nu}(k)$ is the dressed-gluon propagator
- $\Gamma_\nu^f(q, p)$ is the dressed-quark-gluon vertex



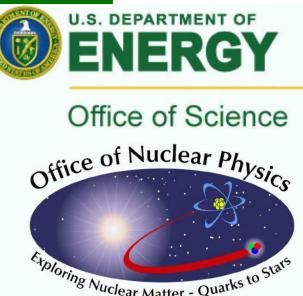
Gap Equation General Form



$$S_f(p)^{-1} = Z_2 (i\gamma \cdot p + m_f^{\text{bm}}) + \Sigma_f(p),$$

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- $D_{\mu\nu}(k)$ is the dressed-gluon propagator
- $\Gamma_\nu^f(q, p)$ is the dressed-quark-gluon vertex
- Suppose one has in-hand the exact form of $\Gamma_\nu^f(q, p)$
What is the associated Symmetry-preserving Bethe-Salpeter Kernel?





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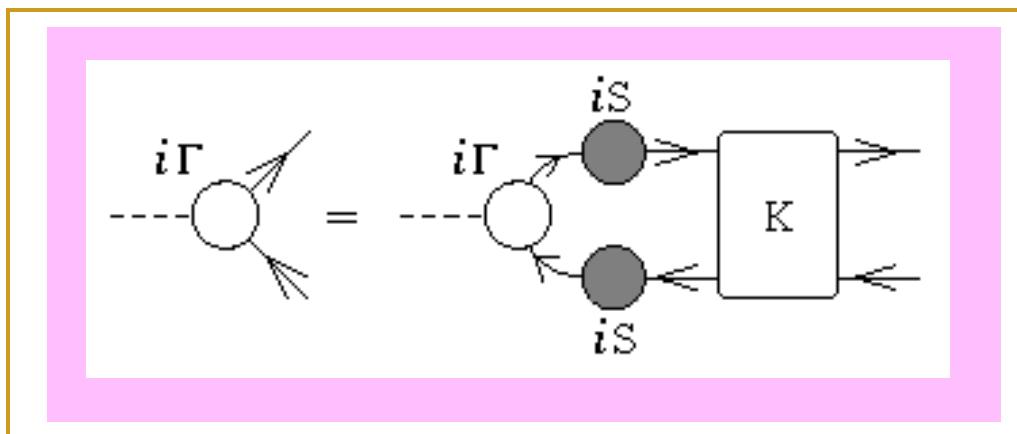
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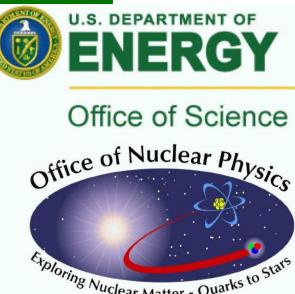
Bethe-Salpeter Equation

- Standard form, familiar from textbooks

$$[\Gamma_\pi^j(k; P)]_{tu} = \int_q^\Lambda [S(q + P/2)\Gamma_\pi^j(q; P)S(q - P/2)]_{sr} K_{tu}^{rs}(q, k; P)$$



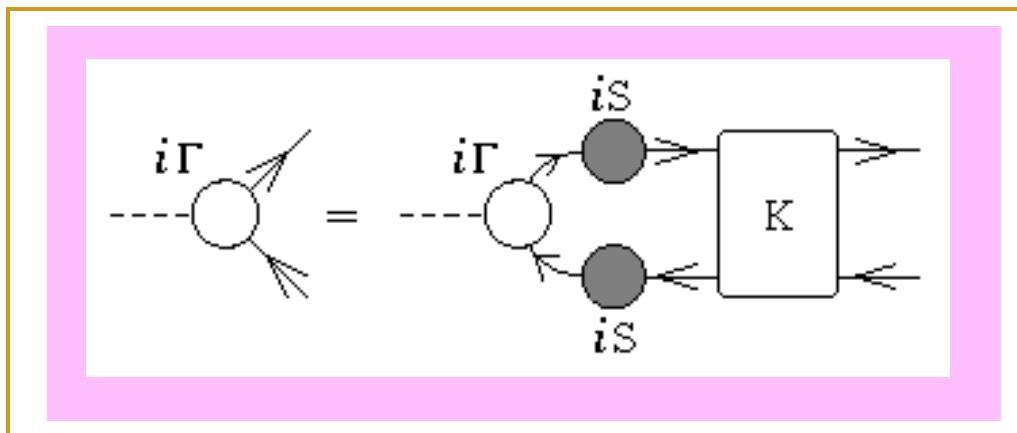
$K(q, k; P)$: Fully-amputated, 2-particle-irreducible,
quark-antiquark scattering kernel



Bethe-Salpeter Equation

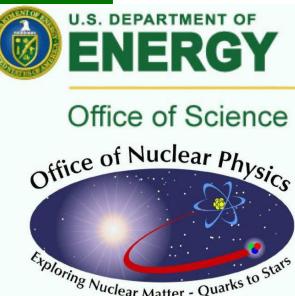
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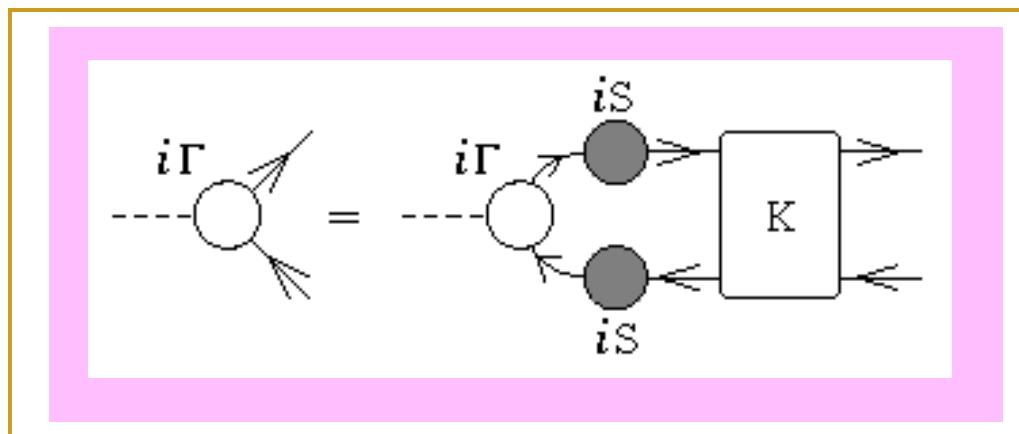
- Compact. Visually appealing. Correct.



Bethe-Salpeter Equation

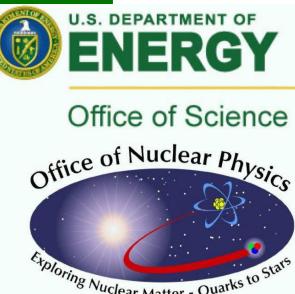
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$K(q, k; P)$: Fully-amputated, 2-particle-irreducible,
quark-antiquark scattering kernel

- Compact. Visually appealing. Correct.
- Blocked progress for more than 60 years.



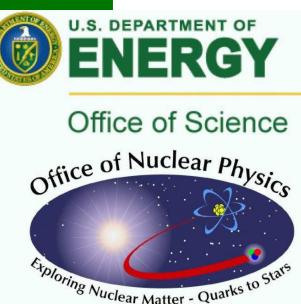


Bethe-Salpeter Equation

L. Chang and C. D. Roberts

0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

General Form



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Bethe-Salpeter Equation



L. Chang and C. D. Roberts

0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

General Form

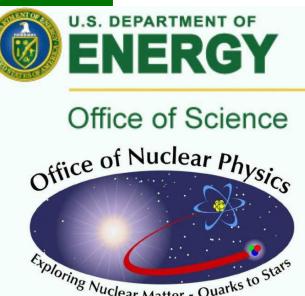
- Equivalent exact form:

$$\Gamma_{5\mu}^{fg}(k; P) = Z_2 \gamma_5 \gamma_\mu$$

$$- \int_q g^2 D_{\alpha\beta}(k-q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \Gamma_{5\mu}^{fg}(q; P) S_g(q_-) \frac{\lambda^a}{2} \Gamma_\beta^g(q_-, k_-)$$

$$+ \int_q g^2 D_{\alpha\beta}(k-q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P),$$

(Poincaré covariance, hence $q_\pm = q \pm P/2$, etc., without loss of generality.)



Bethe-Salpeter Equation



L. Chang and C. D. Roberts

0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

General Form

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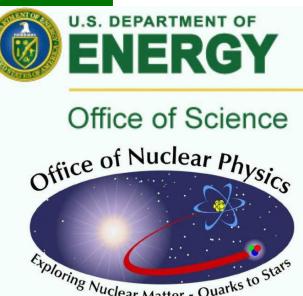
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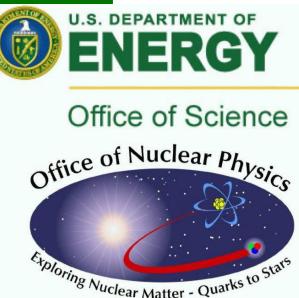
- In this form . . . $\Lambda_{5\mu\beta}^{fg}$
is completely defined via the dressed-quark self-energy



Bethe-Salpeter Kernel

L. Chang and C. D. Roberts
0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

- Bethe-Salpeter equation introduced in 1951



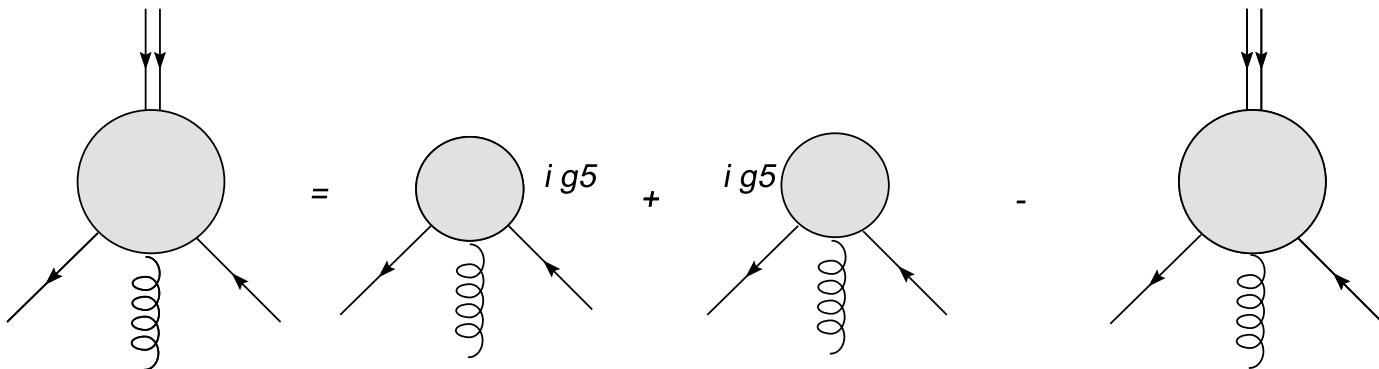
Bethe-Salpeter Kernel

L. Chang and C. D. Roberts

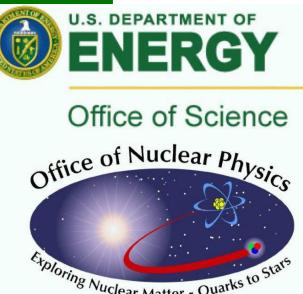
0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

60 year problem

- Bethe-Salpeter equation introduced in 1951
- Newly-derived Ward-Takahashi identity



$$\begin{aligned} P_\mu \Lambda_{5\mu\beta}^{fg}(k, q; P) &= \Gamma_\beta^f(q_+, k_+) i\gamma_5 + i\gamma_5 \Gamma_\beta^g(q_-, k_-) \\ &\quad - i[m_f(\zeta) + m_g(\zeta)] \Lambda_{5\beta}^{fg}(k, q; P), \end{aligned}$$



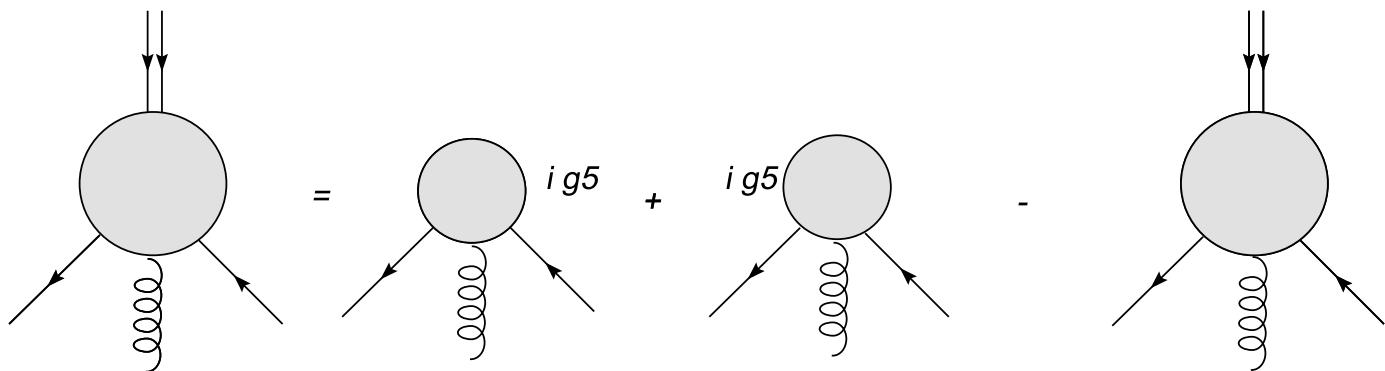
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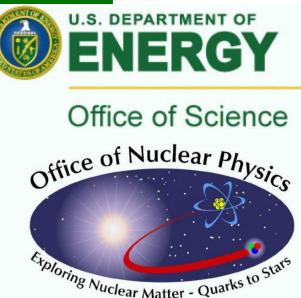
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- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
 - Consistent means - all symmetries preserved!



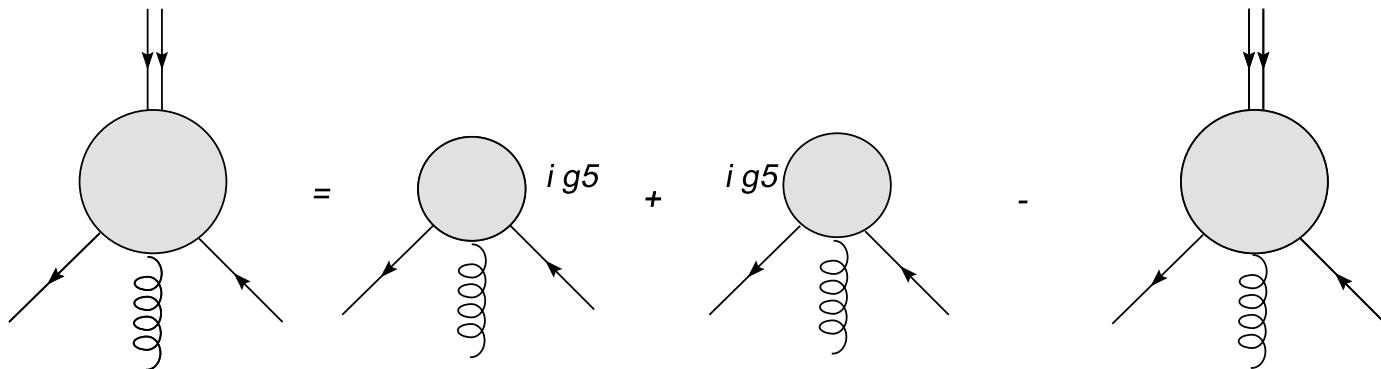
Bethe-Salpeter Kernel

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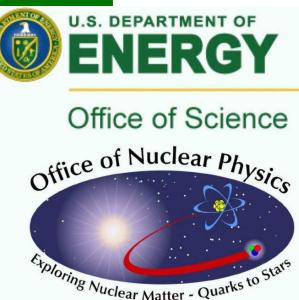
$$P_\mu \Lambda_{5\mu\beta}^{fg}(k, q; P) = \Gamma_\beta^f(q_+, k_+) i\gamma_5 + i\gamma_5 \Gamma_\beta^g(q_-, k_-) - i[m_f(\zeta) + m_g(\zeta)] \Lambda_{5\beta}^{fg}(k, q; P),$$

- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
- Procedure & results to expect ...

see arXiv:1003.5006 [nucl-th]

Craig Roberts – *Dyson-Schwinger equations: Recent successes & future perspective*

EBAC workshop, 27-28 May 2010 ... 31



$a_1 - \rho$

	exp.			
mass a_1	1230			
mass ρ	775			
mass-splitting	455			

- Splitting known experimentally for more than 35 years.
- Hitherto, no explanation.



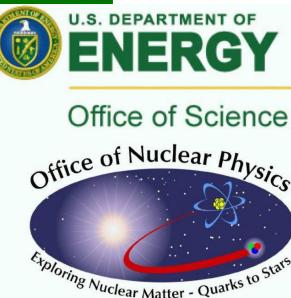
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$$a_1 - \rho$$

	exp.	rainbow-ladder	one-loop		
mass a_1	1230	759	885		
mass ρ	775	644	764		
mass-splitting	455	115	121		

- Systematic, symmetry-preserving, Poincaré-covariant DSE truncation scheme of nucl-th/9602012.
- Never better than $\sim \frac{1}{4}$ of splitting.
- Constructing kernel skeleton-diagram-by-diagram, DCSB cannot be faithfully expressed: $M(p^2)$ is absent!



$$a_1 - \rho$$

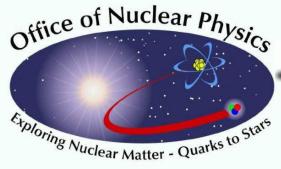
	exp.	rainbow-ladder	one-loop	Ball-Chiu consistent	
mass a_1	1230	759	885	1066	
mass ρ	775	644	764	924	
mass-splitting	455	115	121	142	

- New nonperturbative, symmetry-preserving Poincaré-covariant Bethe-Salpeter equation formulation of arXiv:0903.5461 [nucl-th]
- Ball-Chiu Ansatz for quark-gluon vertex

$$\Gamma_\mu^{\text{BC}}(k, p) = \dots + (k + p)_\mu \frac{B(k) - B(p)}{k^2 - p^2}$$
 - Some effects of DCSB built into vertex
 - Explains $\pi - \sigma$ splitting but **not** this problem



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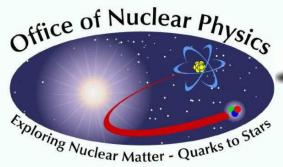


Chang & Roberts arXiv:1003.5006 [nucl-th]

$a_1 - \rho$

	exp.	rainbow-ladder	one-loop	Ball-Chiu consistent	Ball-Chiu plus anom. cm mom.
mass a_1	1230	759	885	1066	1230
mass ρ	775	644	764	924	745
mass-splitting	455	115	121	142	485

- New nonperturbative, symmetry-preserving Poincaré-covariant Bethe-Salpeter equation formulation of arXiv:0903.5461 [nucl-th]
- Ball-Chiu augmented by *quark anomalous chromomagnetic moment* term: $\Gamma_\mu(k, p) = \Gamma_\mu^{\text{BC}} + \sigma_{\mu\nu}(k - p)_\nu \frac{B(k) - B(p)}{k^2 - p^2}$

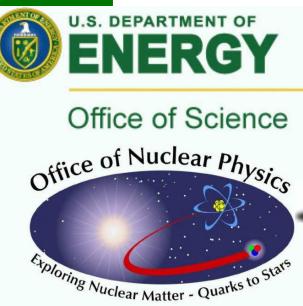


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- New nonperturbative, symmetry-preserving Poincaré-covariant Bethe-Salpeter equation formulation of arXiv:0903.5461 [nucl-th]
- DCSB is the answer. Subtle interplay between competing effects, which can only now be explicated
- Promise of first reliable prediction of light-quark meson spectrum, including the so-called hybrid and exotic states.



Quark Anomalous Magnetic Moments

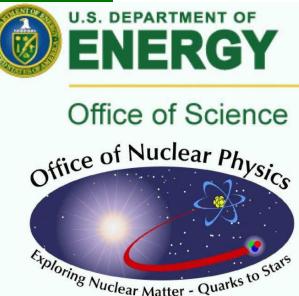
Chang & Roberts, in progress

- Massless fermion **can't** possess an anomalous magnetic moment

- Interaction term

$$\int d^4x \frac{1}{2}g \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$$

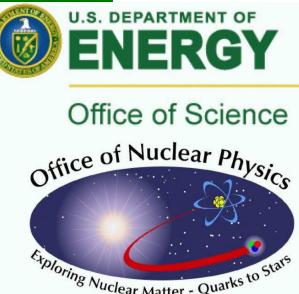
explicitly breaks chiral symmetry



Quark Anomalous Magnetic Moments

Chang & Roberts, in progress

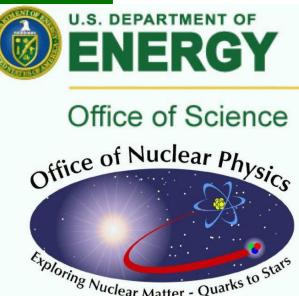
- Massless fermion **can't** possess an anomalous magnetic moment
 - Interaction term $\int d^4x \frac{1}{2}g \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$ explicitly breaks chiral symmetry
- However, DCSB can generate a large anomalous chromomagnetic moment even in chiral limit
 - This explains the a_1 - ρ mass-splitting



Quark Anomalous Magnetic Moments

Chang & Roberts, in progress

- Massless fermion **can't** possess an anomalous magnetic moment
 - Interaction term $\int d^4x \frac{1}{2}g \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$ explicitly breaks chiral symmetry
- New BSE formulation (arXiv:0903.5461 [nucl-th]) **enables** computation of dressed-quark electromagnetic moment given dressed-quark-gluon vertex with ACM-term



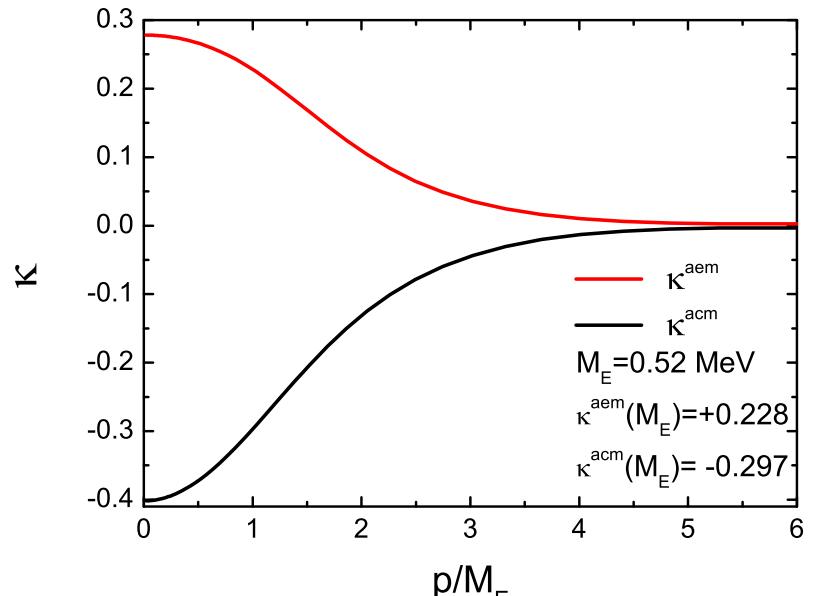
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$$M(p^2) \Rightarrow \kappa(p^2)$$

- Preliminary result for μ distributions



Quark Anomalous Magnetic Moments

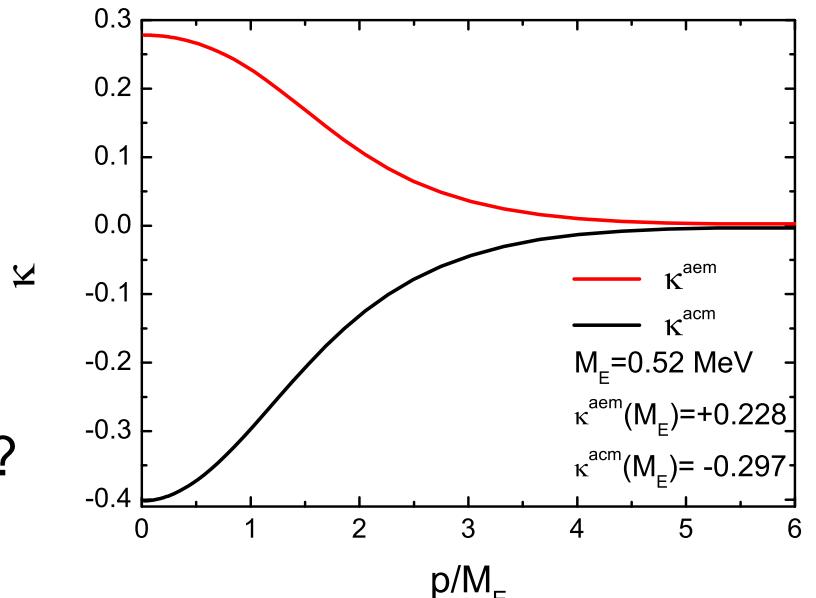
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$$M(p^2) \Rightarrow \kappa(p^2)$$

- Preliminary result for μ distributions

- Cloët & Roberts
Effect on hadron form factors?



Frontiers of Nuclear Science: A Long Range Plan (2007)



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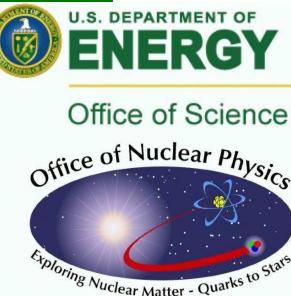
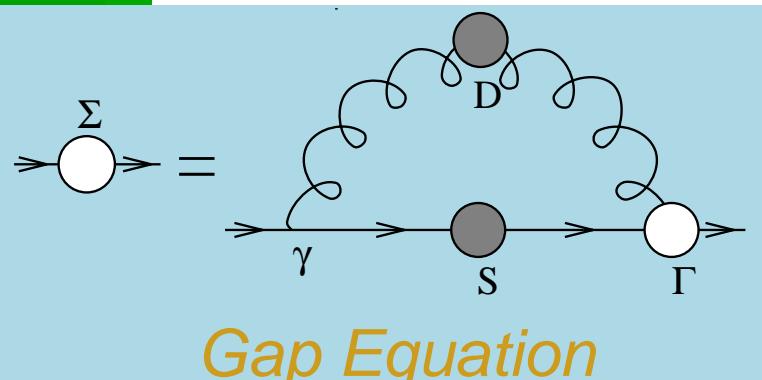
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Frontiers of Nuclear Science: Theoretical Advances



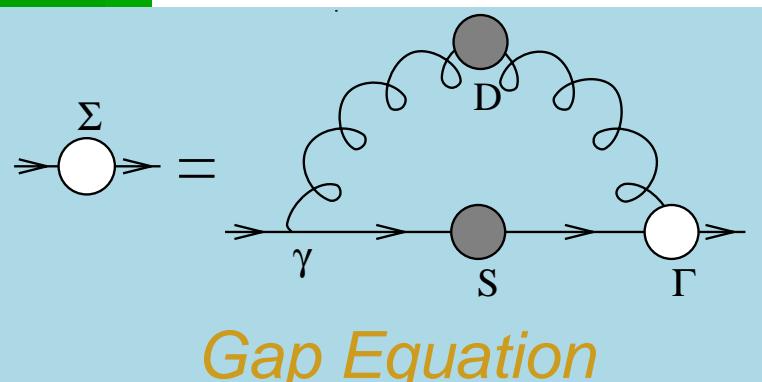
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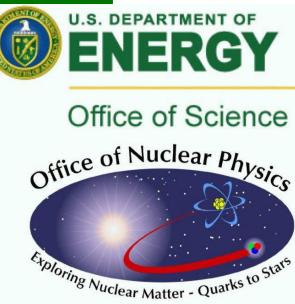
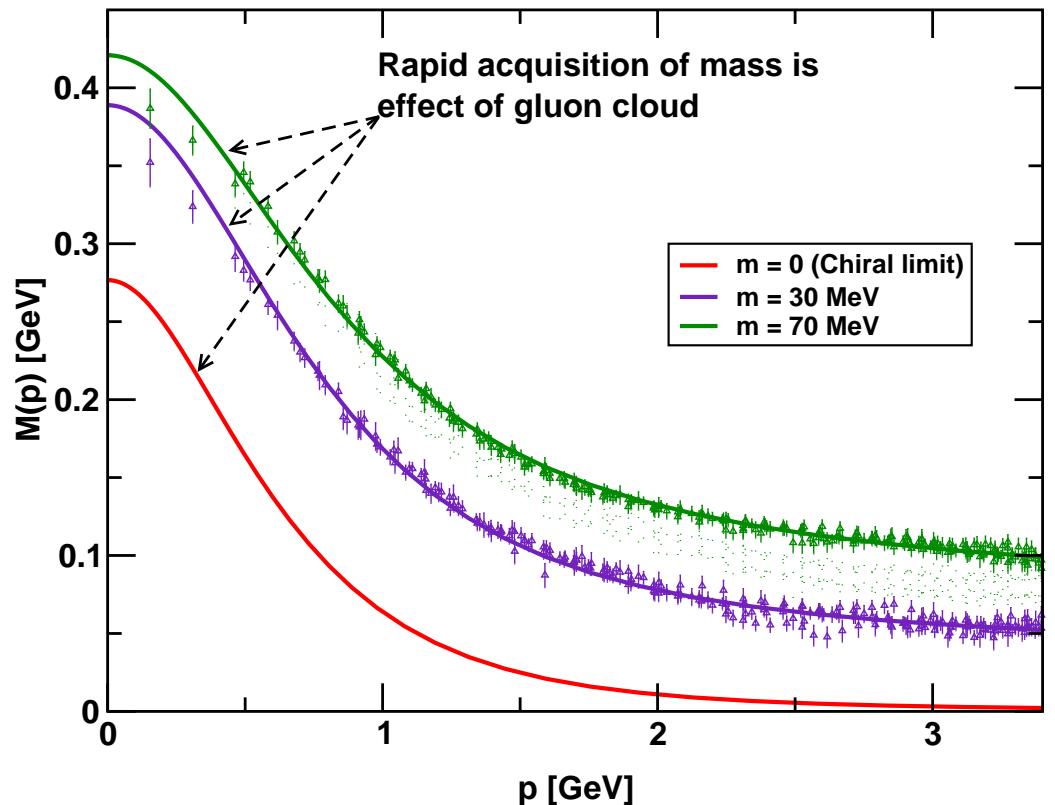
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Frontiers of Nuclear Science: Theoretical Advances



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

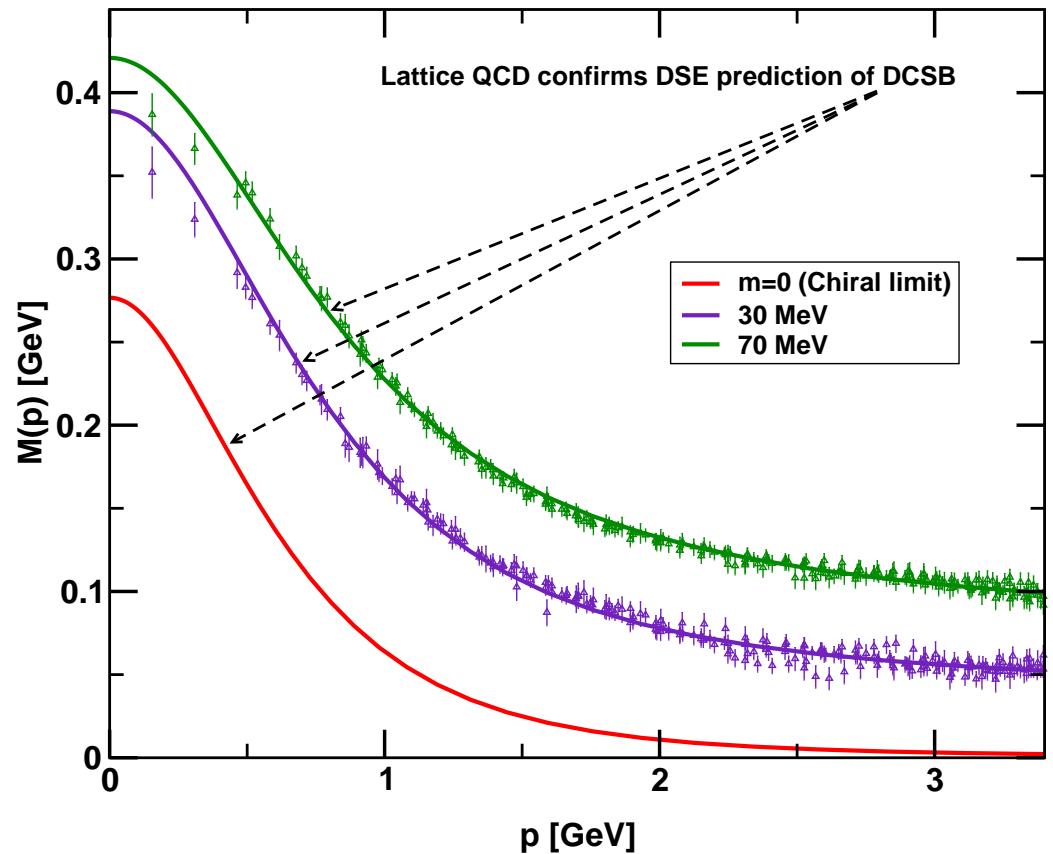


Frontiers of Nuclear Science: Theoretical Advances

Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have **confirmed model predictions** (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it **propagates**. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.

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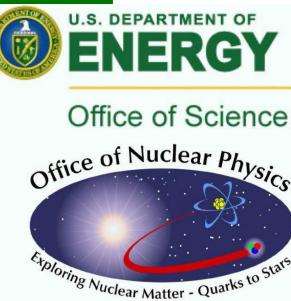
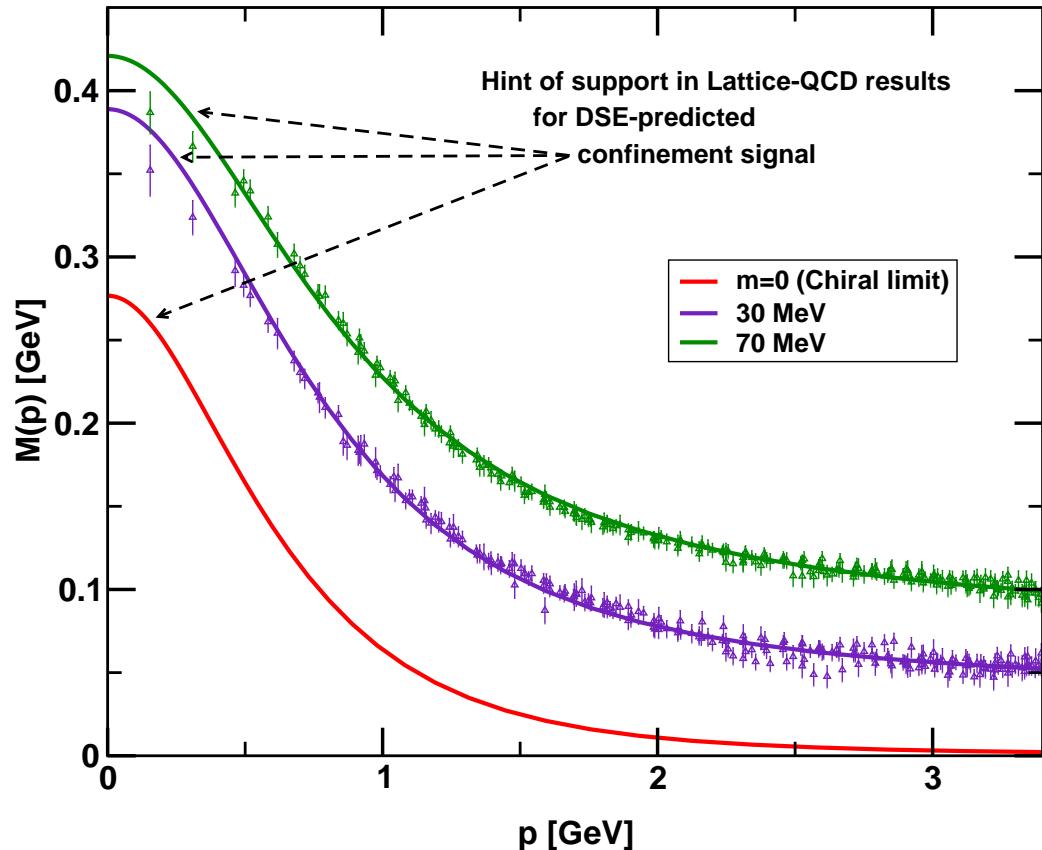


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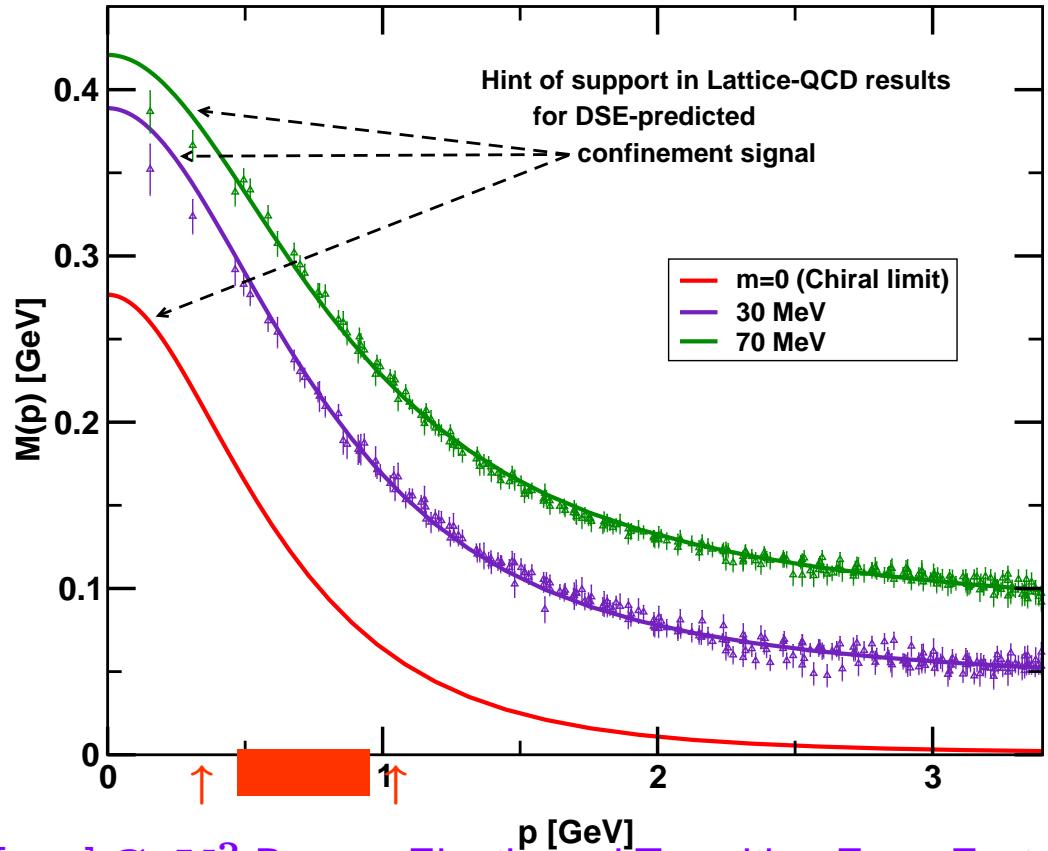


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$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Scanned by $Q^2 \in [2, 9] \text{ GeV}^2$ Baryon Elastic and Transition Form Factors

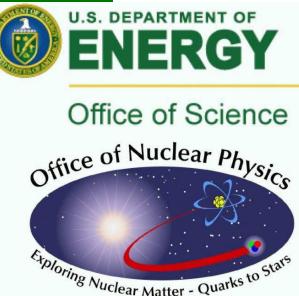
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Maris, Roberts, Tandy
nucl-th/9707003

Goldberger-Treiman for pion



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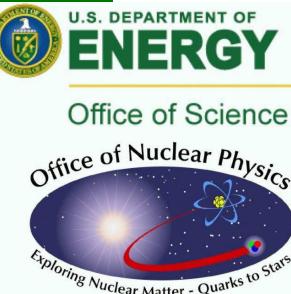
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- Pseudoscalar Bethe-Salpeter amplitude

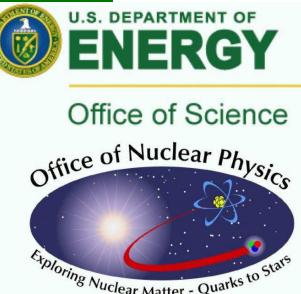
$$\begin{aligned}\Gamma_{\pi^j}(k; P) = & \tau^{\pi^j} \gamma_5 \left[iE_\pi(k; P) + \gamma \cdot P F_\pi(k; P) \right. \\ & \left. + \gamma \cdot k \, k \cdot P G_\pi(k; P) + \sigma_{\mu\nu} k_\mu P_\nu H_\pi(k; P) \right]\end{aligned}$$



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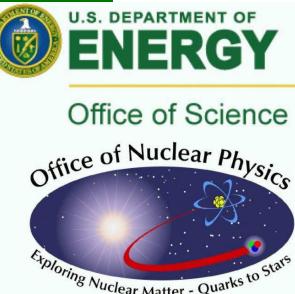
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⇒

$$f_\pi E_\pi(k; P=0) = B(p^2)$$



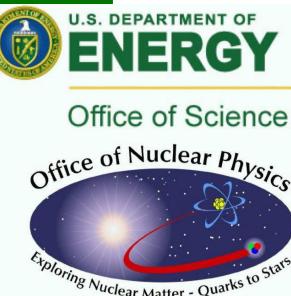
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$$\begin{aligned}f_\pi E_\pi(k; P=0) &= B(p^2) \\ F_R(k; 0) + 2 f_\pi F_\pi(k; 0) &= A(k^2) \\ G_R(k; 0) + 2 f_\pi G_\pi(k; 0) &= 2A'(k^2) \\ H_R(k; 0) + 2 f_\pi H_\pi(k; 0) &= 0\end{aligned}$$



Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k; P) = \tau^{\pi^j} \gamma_5 \left[i E_\pi(k; P) + \gamma \cdot P F_\pi(k; P) \right]$$

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Pseudovector components necessarily nonzero

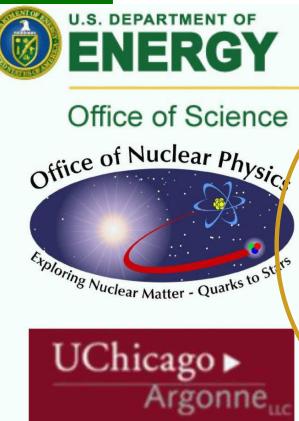
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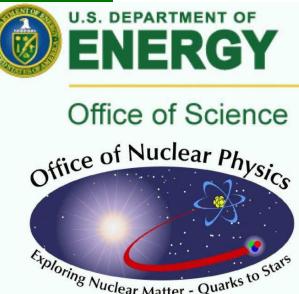
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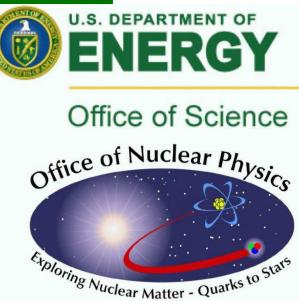
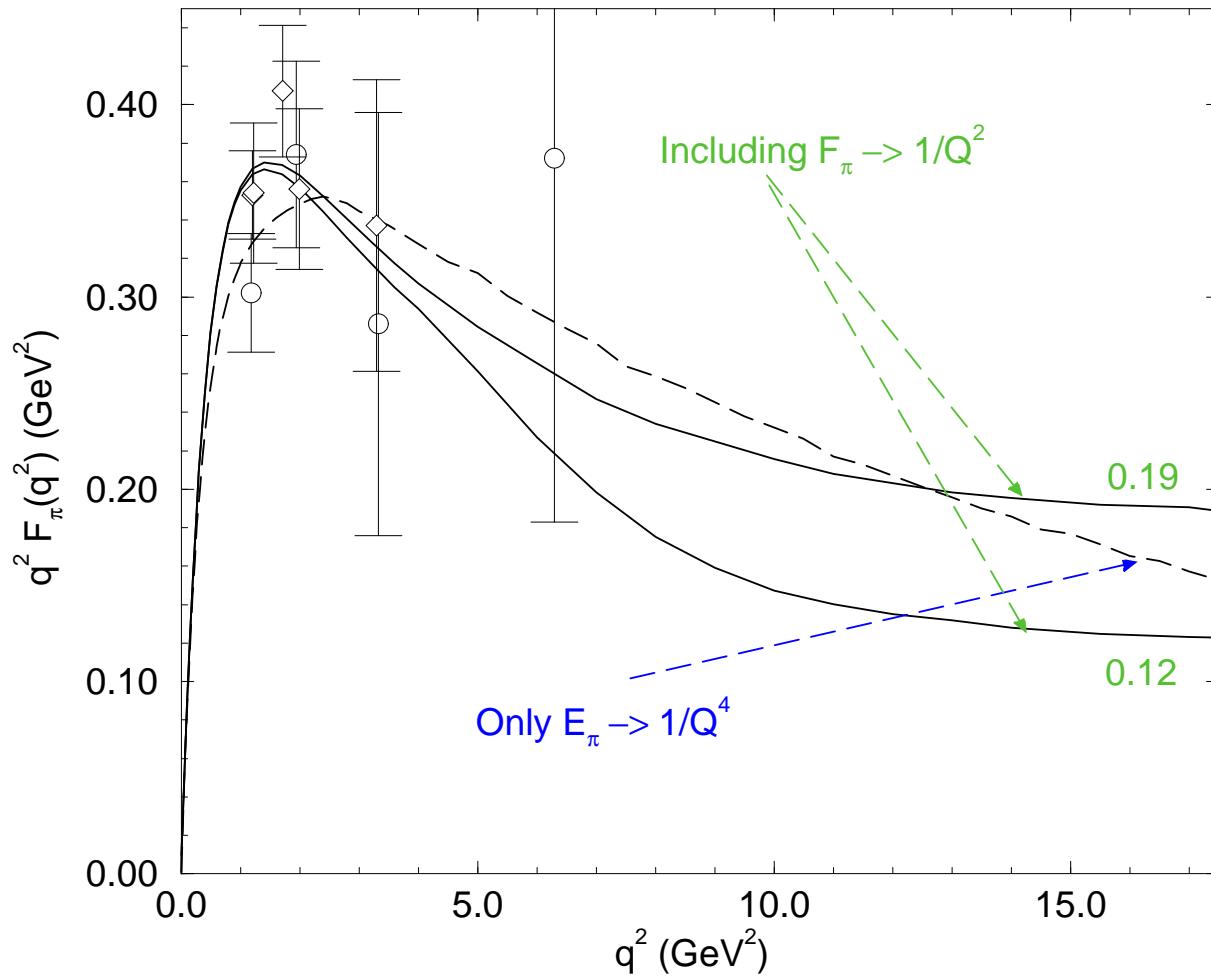
Maris, Roberts
nucl-th/9804062

– QCD and $F_\pi^{\text{em}}(Q^2)$

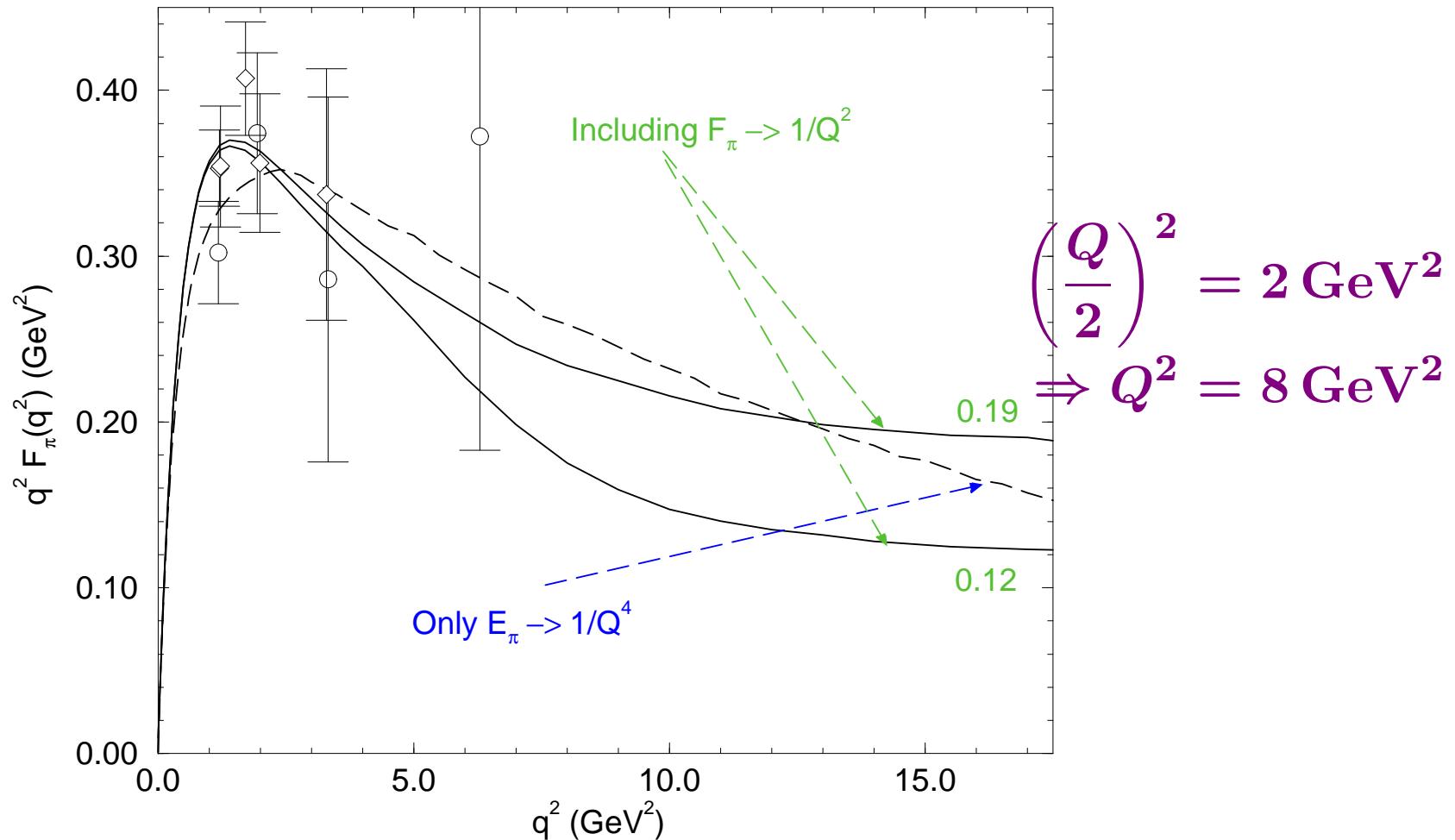
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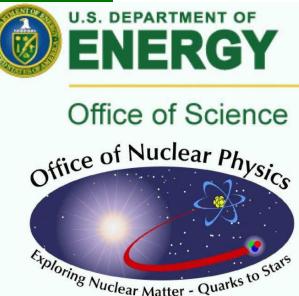
- What does this mean for observables?



- Pseudovector components dominate ultraviolet behaviour of electromagnetic form factor

Gutierrez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

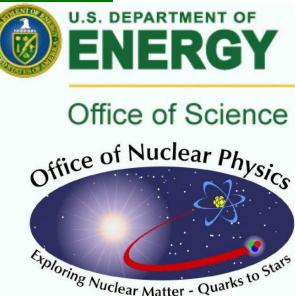
– Contact Interaction



- Bethe-Salpeter amplitude can't depend on relative momentum

⇒ General Form

$$\Gamma_\pi(P) = \gamma_5 [iE_\pi(P) + \frac{1}{M_Q} \gamma \cdot P F_\pi(P)]$$



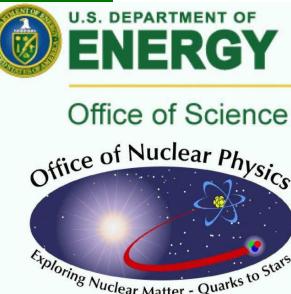
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$$P^2 = 0 : M_Q = 0.40, E_\pi = 0.98, \frac{F_\pi}{M_Q} = 0.50$$



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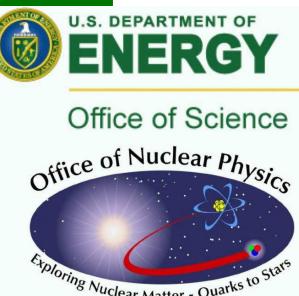
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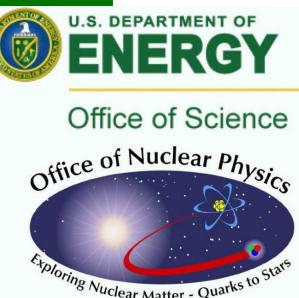
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- Has pseudovector component

$$\sim E_\pi [\sigma_S(k_+) \sigma_V(k_-) + \sigma_S(k_-) \sigma_V(k_+)]$$



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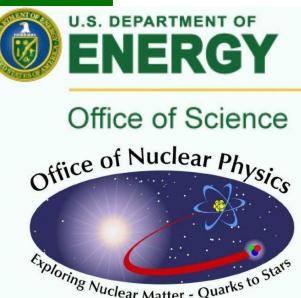
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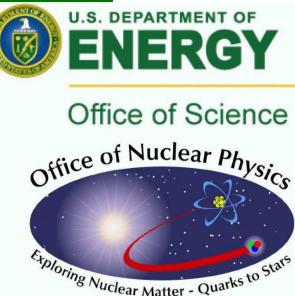
$$\gamma_\mu S(k + P/2) i\gamma_5 E_\pi S(k - P/2) \gamma_\mu$$

- Hence F_π on LHS is forced to be nonzero because E_π on RHS is nonzero owing to DCSB



- Bethe-Salpeter amplitude: General Form

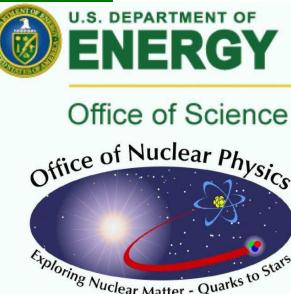
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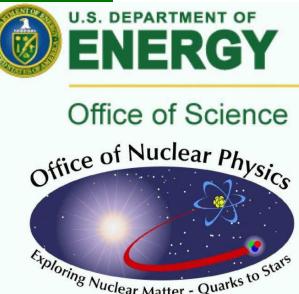
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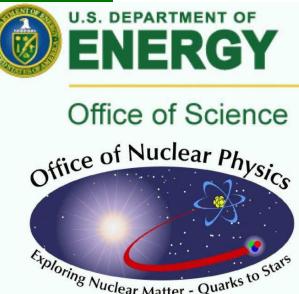
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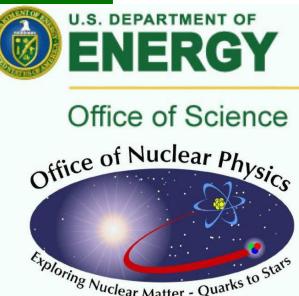
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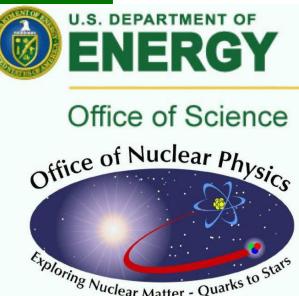
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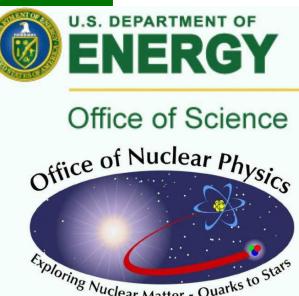
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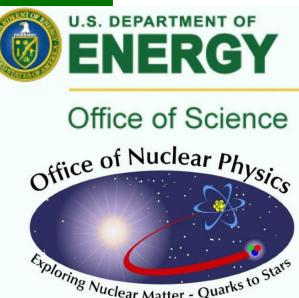
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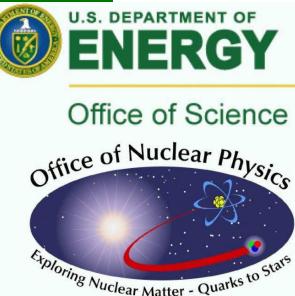
- This behaviour dominates for $Q^2 \gtrsim M_Q^2 \frac{E_\pi}{F_\pi} > 0.8 \text{ GeV}^2$



Computation: Elastic Pion Form Factor

Gutierrez, Bashir, Cloët, Roberts:
arXiv:1002.1968 [nucl-th]

- DSE prediction: $M(p^2)$; i.e., interaction $\frac{1}{|x - y|^2}$
- cf. $M(p^2) = \text{Constant}$; i.e., interaction $\delta^4(x - y)$

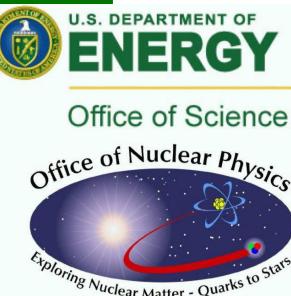


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Single mass-scale parameter
in both studies



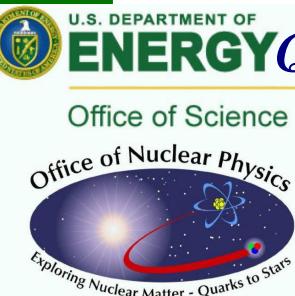
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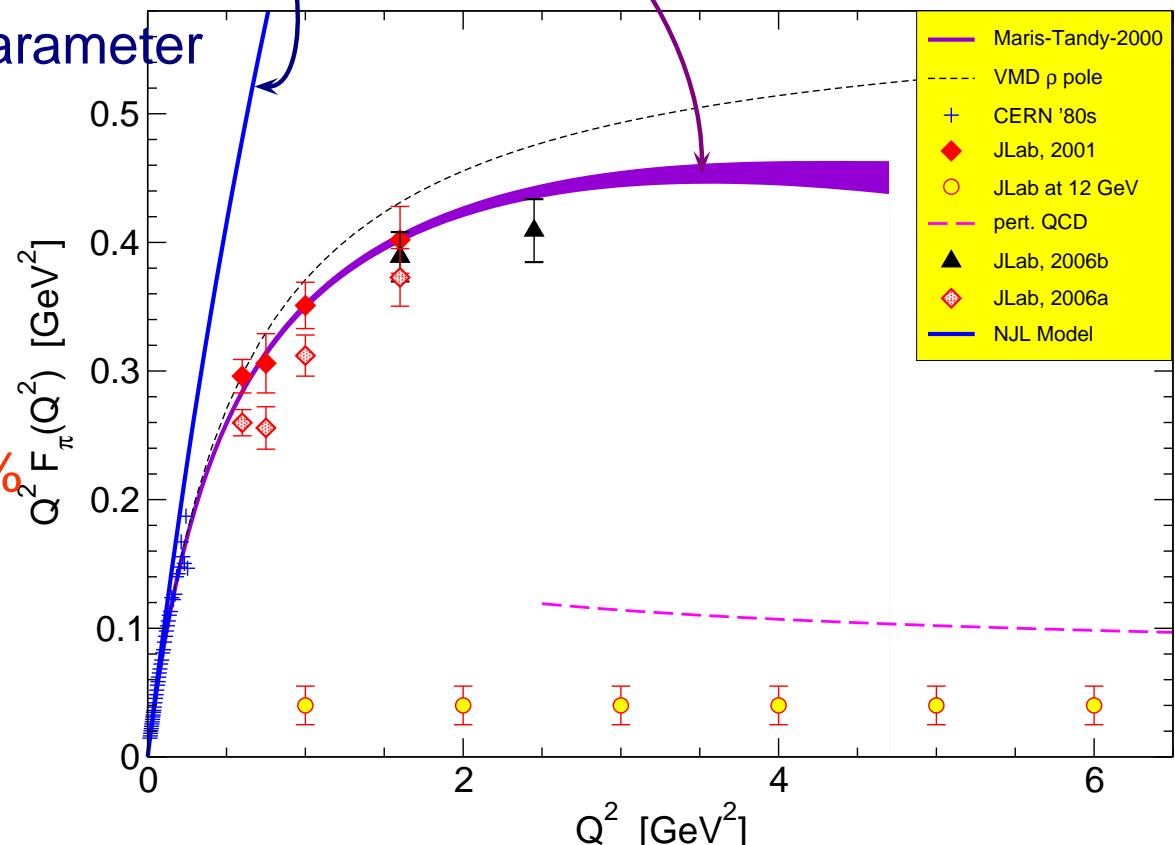
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Disagreement > 20%
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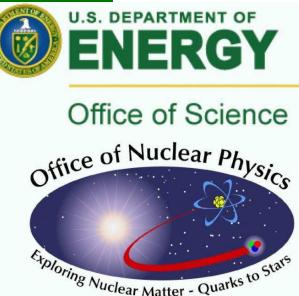
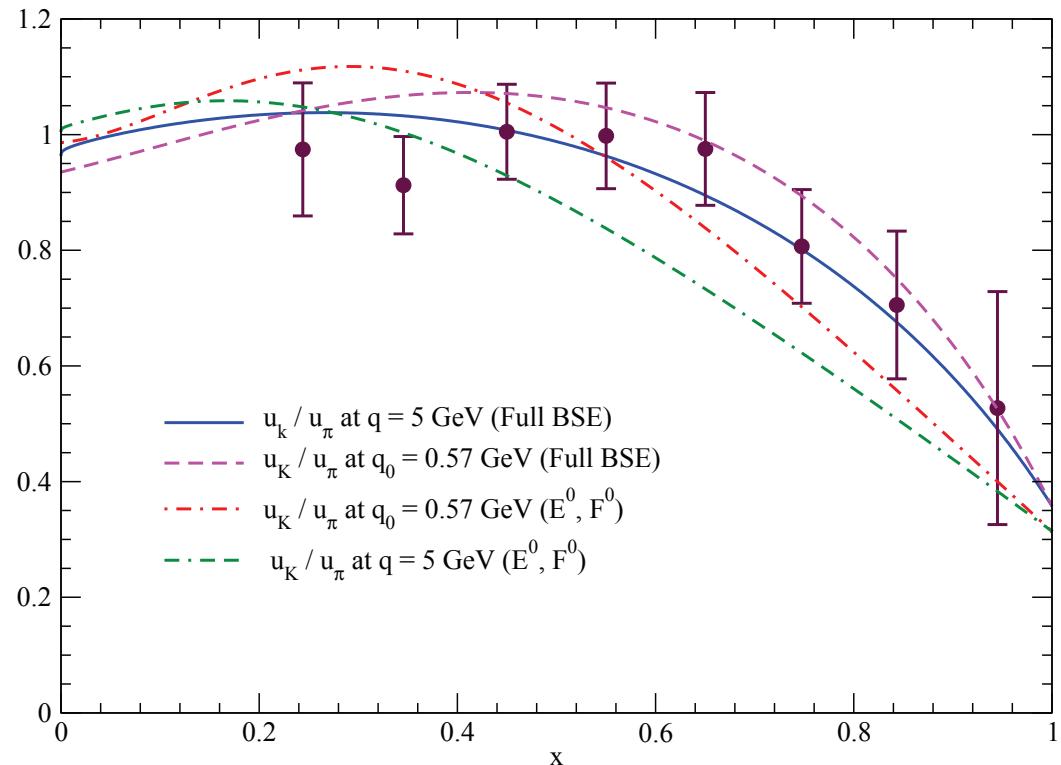
Trang: PhD Thesis (Kent State U.)

Trang, Tandy, Bashir, Roberts, in progress

Holt & Roberts: arXiv:1002.4666 [nucl-th]

u-valence distribution

data: Badier, et al. , Phys. Lett. B 93 (1980) 354



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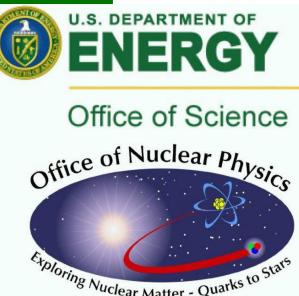
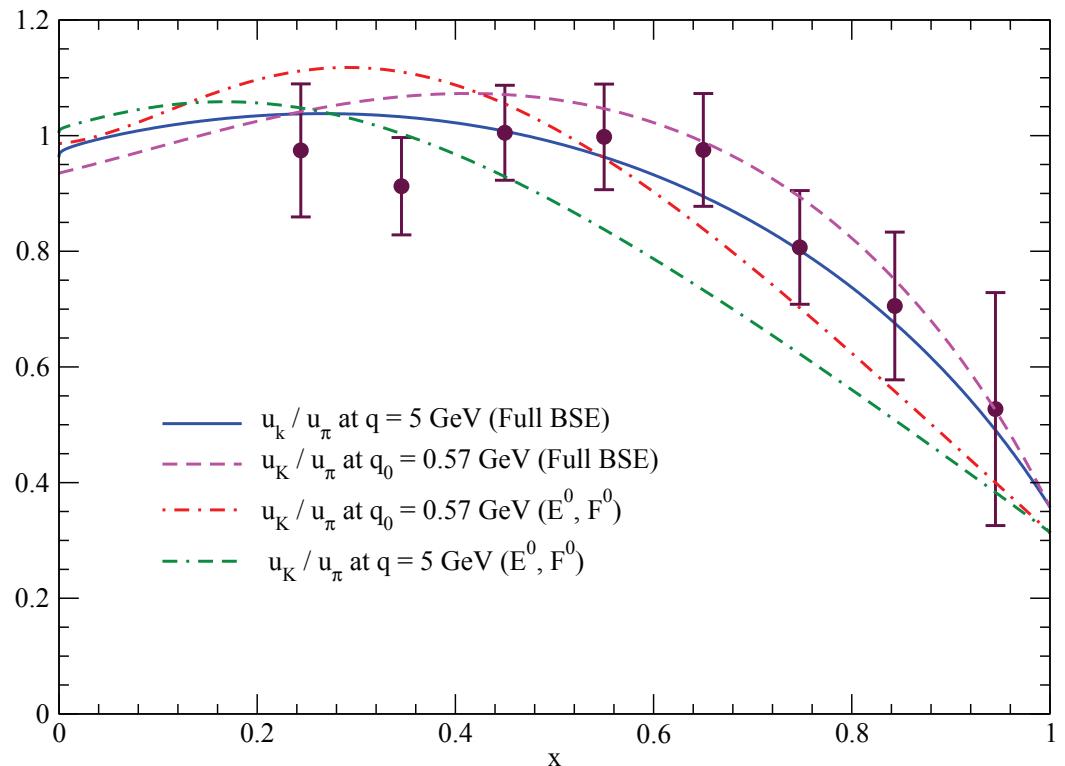
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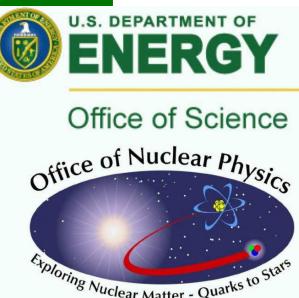
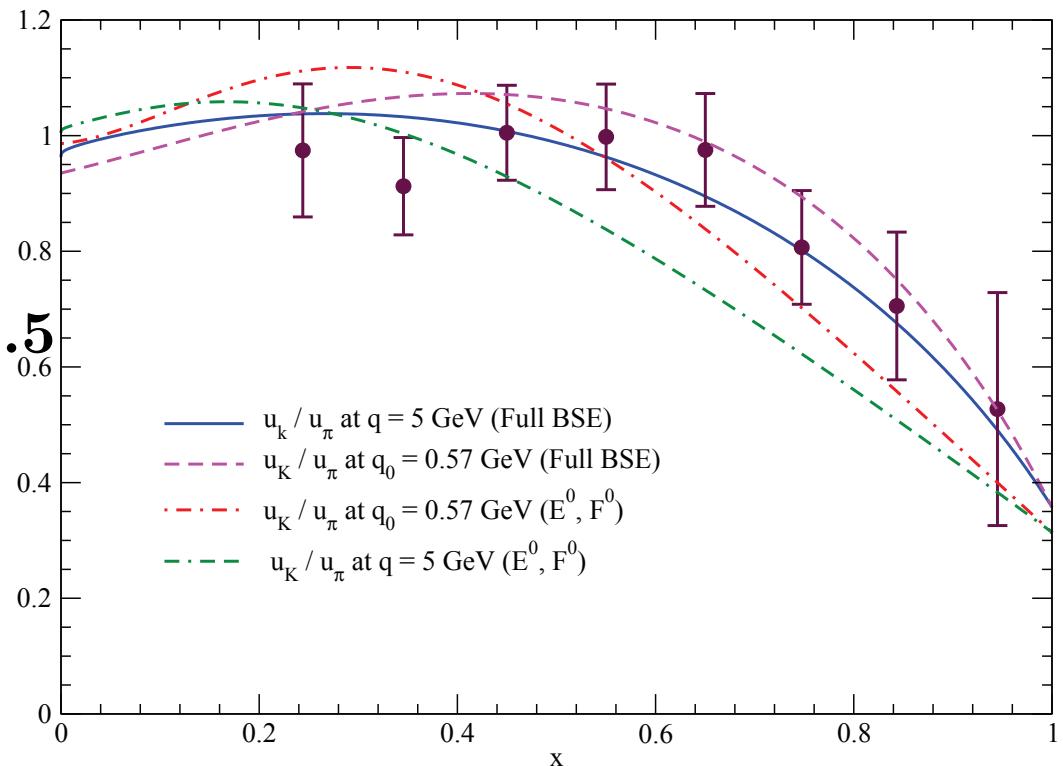
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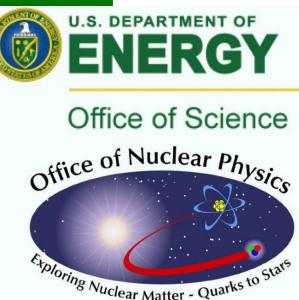
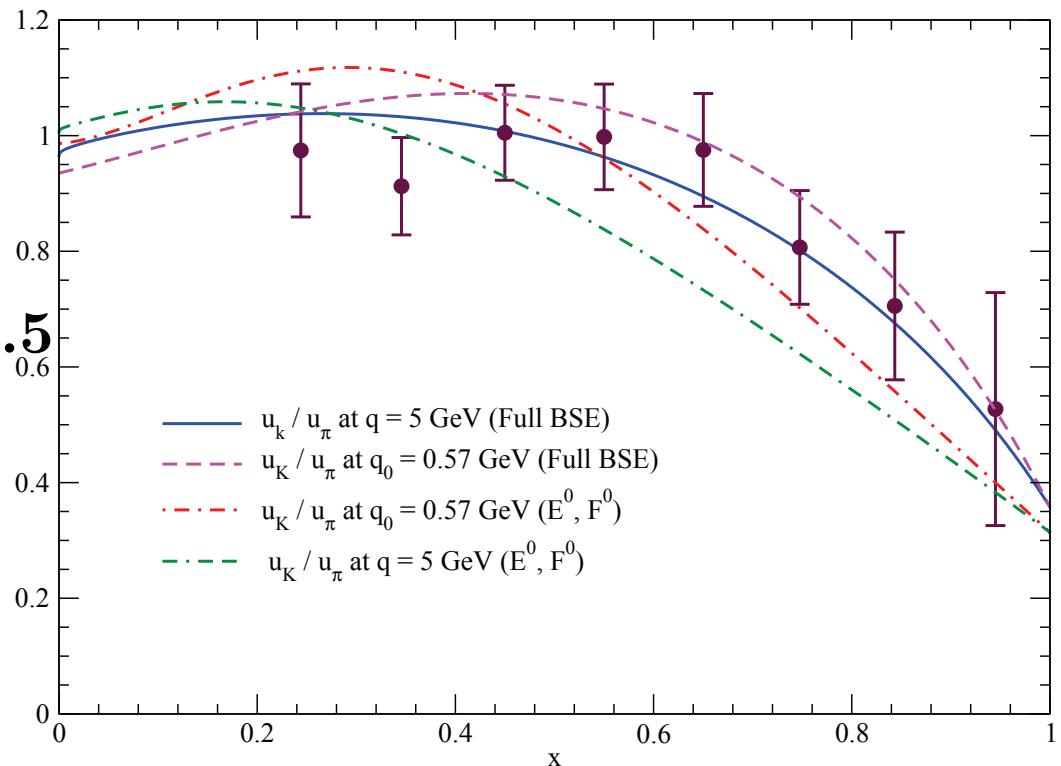
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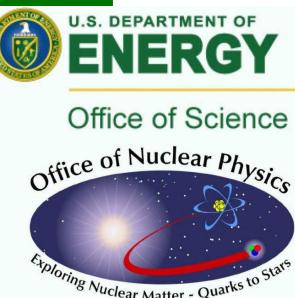
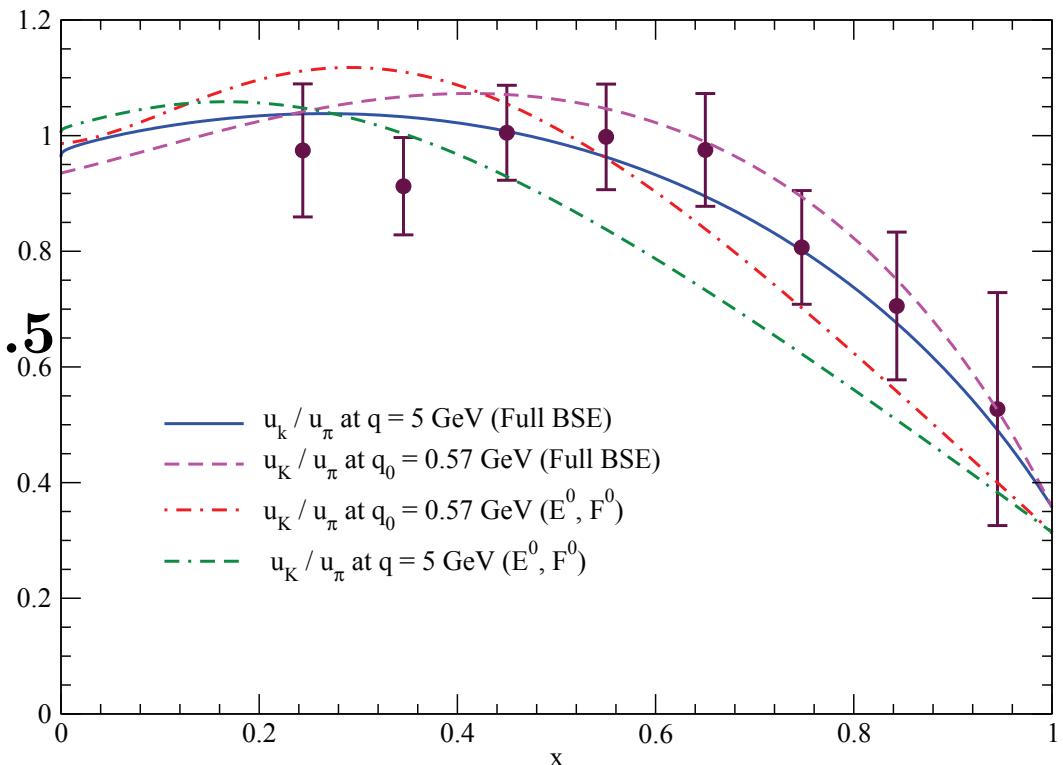
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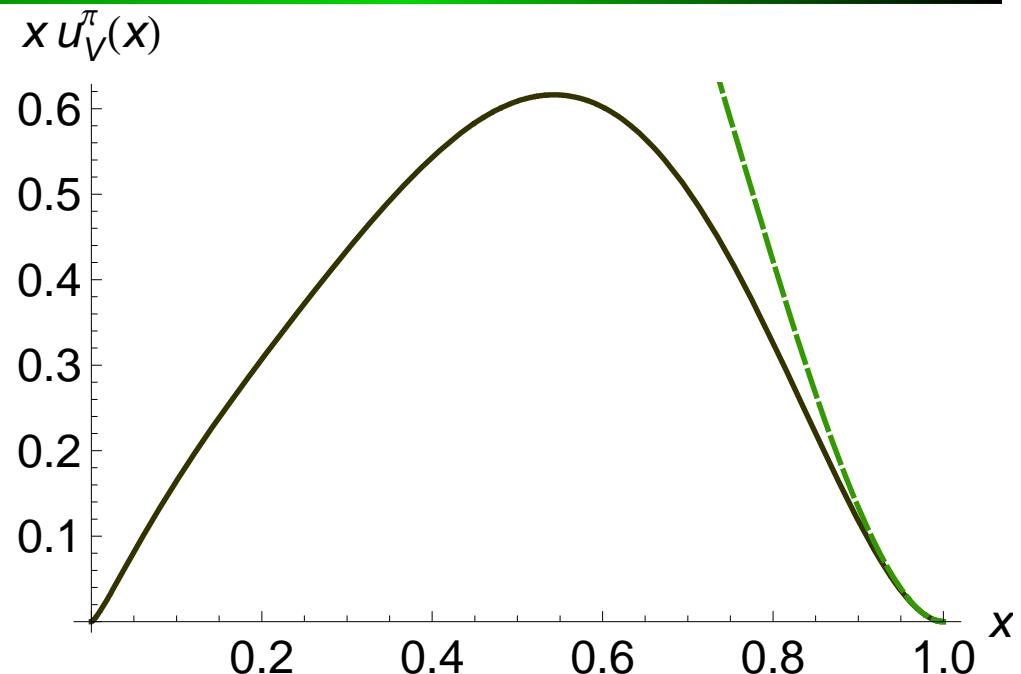
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valence distribution

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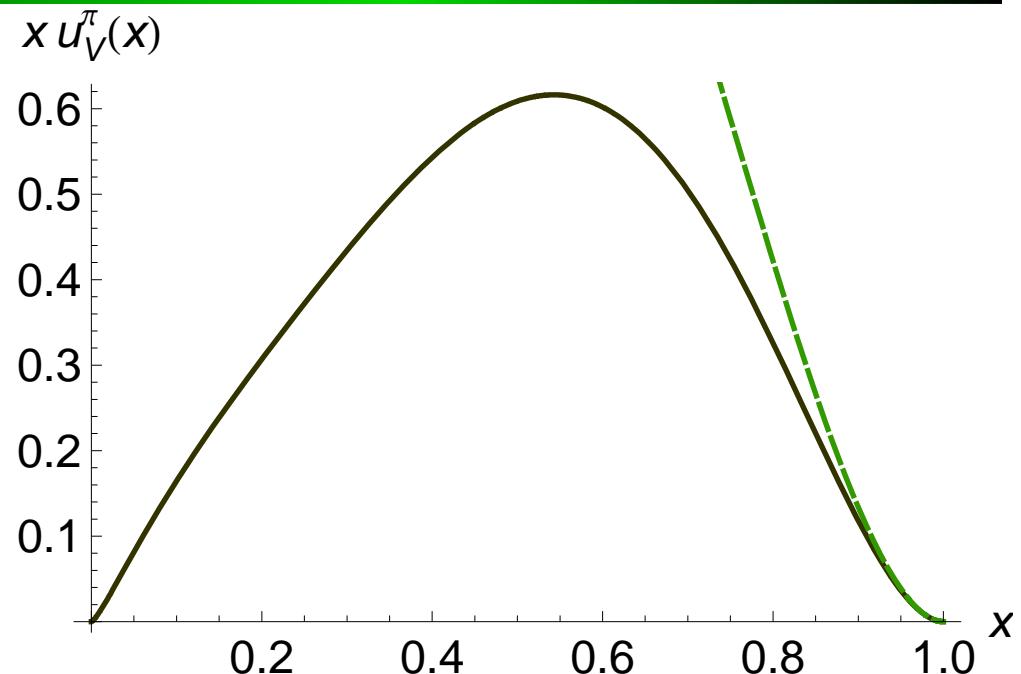
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valence distribution

Holt & Roberts: arXiv:1002.4666 [nucl-th]

$$\frac{\alpha(q^2)}{q^2} \underset{q^2 \gg M_D^2}{\propto} \left(\frac{1}{q^2} \right)^{1+\kappa}$$



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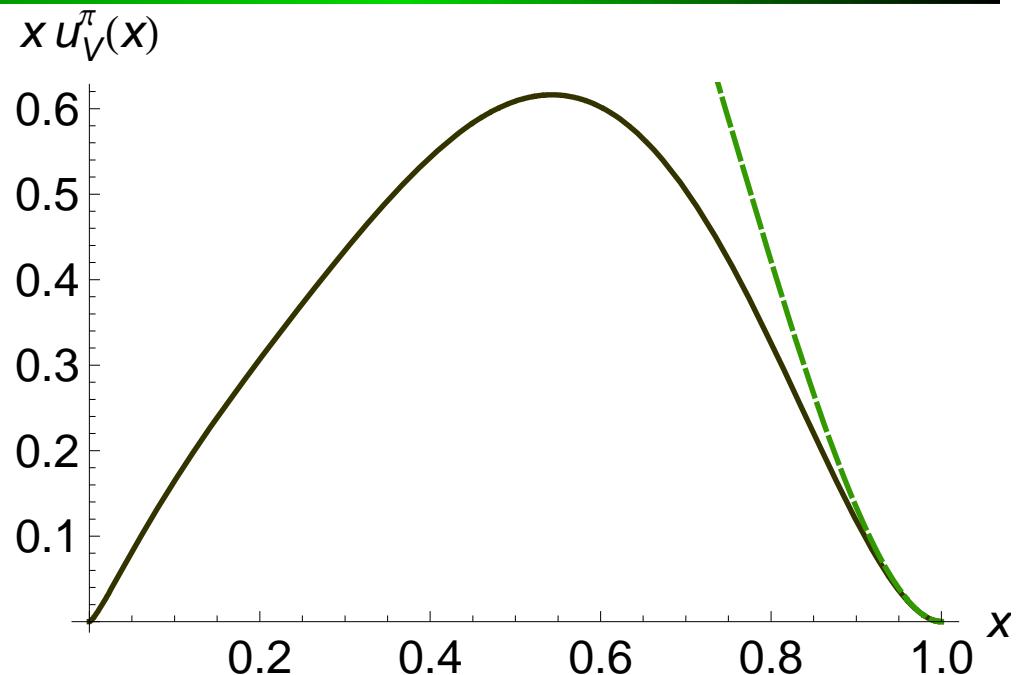


valence distribution

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$$\frac{\alpha(q^2)}{q^2} \underset{q^2 \gg M_D^2}{\propto} \left(\frac{1}{q^2}\right)^{1+\kappa}$$

$$\Rightarrow B(k^2) \underset{k^2 \gg M_D^2}{\propto} \left(\frac{1}{k^2}\right)^{1+\kappa}$$



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First

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Conclusion

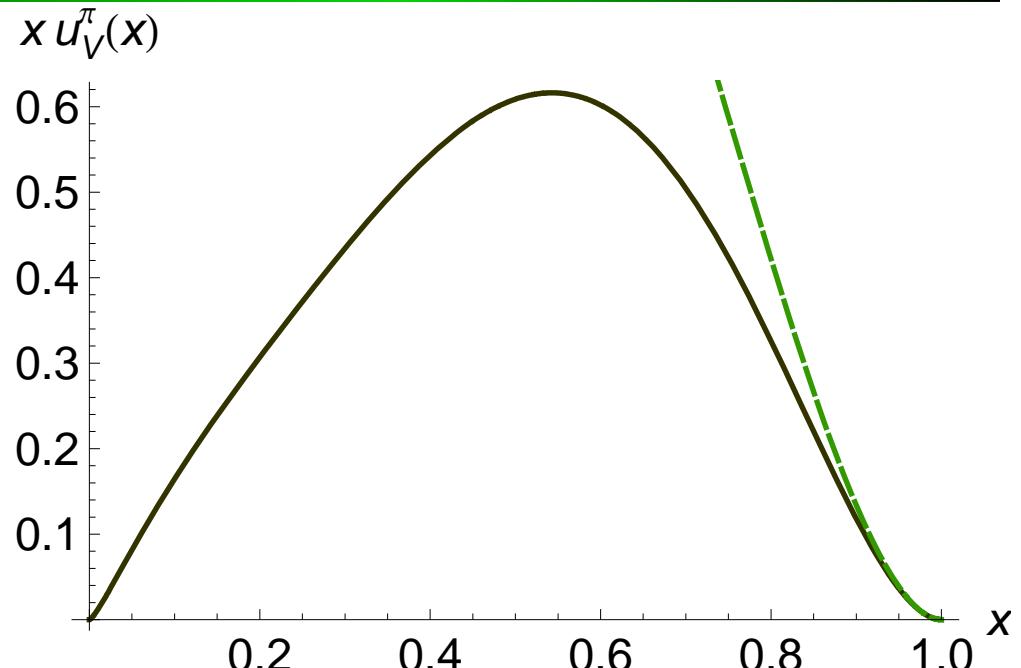
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$$\frac{\alpha(q^2)}{q^2} \underset{q^2 \gg M_D^2}{\propto} \left(\frac{1}{q^2}\right)^{1+\kappa}$$

$$\Rightarrow B(k^2) \underset{k^2 \gg M_D^2}{\propto} \left(\frac{1}{k^2}\right)^{1+\kappa}$$

$$\Rightarrow q_v^\pi(x; Q_0) \underset{x \sim 1}{\propto} (1-x)^{2(1+\kappa)} \quad \text{at } Q_0 \gg M_D .$$



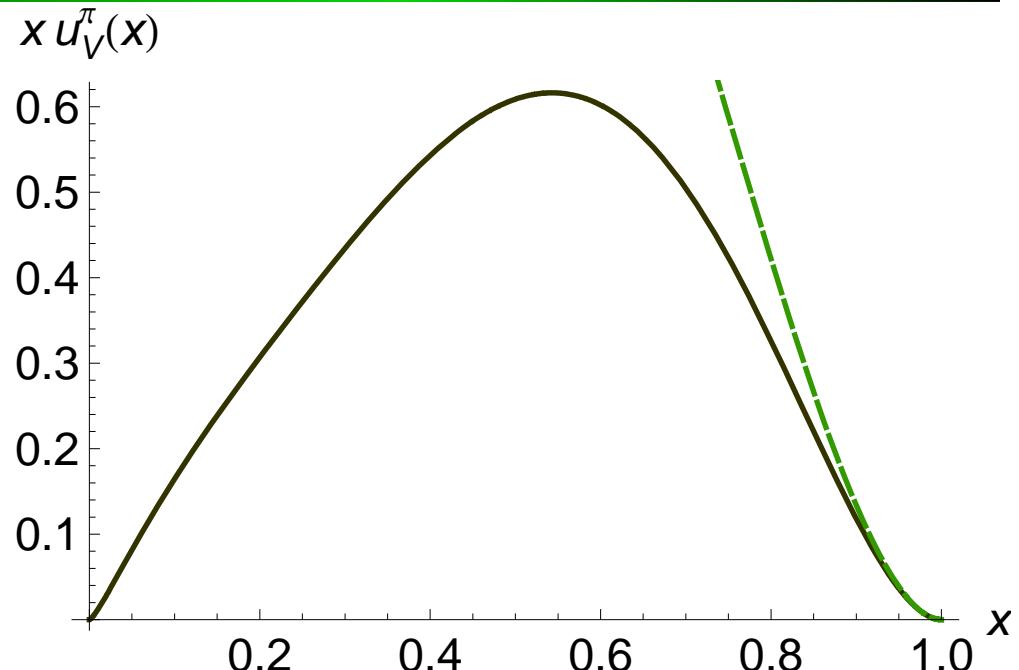
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$$\begin{aligned} \frac{\alpha(q^2)}{q^2} \xrightarrow{q^2 \gg M_D^2} & \left(\frac{1}{q^2} \right)^{1+\kappa} \\ \Rightarrow B(k^2) \xrightarrow{k^2 \gg M_D^2} & \left(\frac{1}{k^2} \right)^{1+\kappa} \\ \Rightarrow q_v^\pi(x; Q_0) \xrightarrow{x \sim 1} & (1-x)^{2(1+\kappa)} \quad \text{at } Q_0 \gg M_D. \end{aligned}$$



- $M_D \approx 0.4 \text{ GeV}$
– location of largest- p^2 inflection point in $M(p^2)$



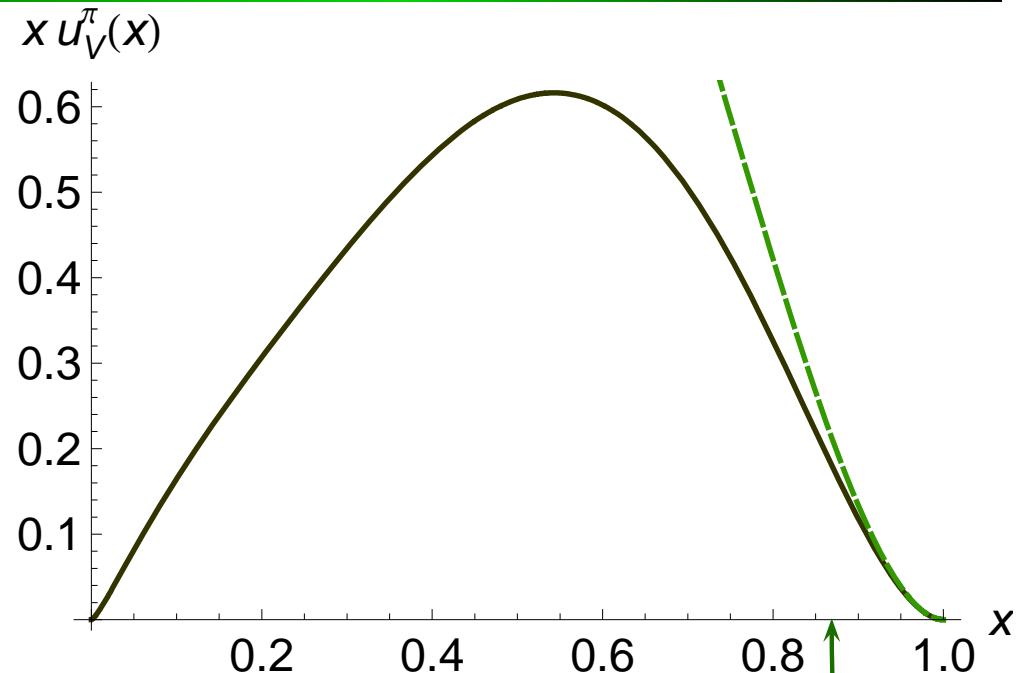
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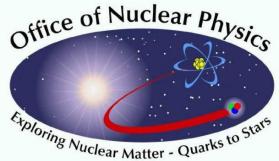


- $M_D \approx 0.4 \text{ GeV}$
– location of largest- p^2 inflection point in $M(p^2)$
- $\kappa_{\text{QCD}} = 0 \Rightarrow q_v^\pi(x; 1 \text{ GeV}^2) \propto (1-x)^2$
- DSE calculation shows this valid for $\mathcal{L}_x = \{x | x > 0.86\}$

Unifying Study of Mesons and Baryons



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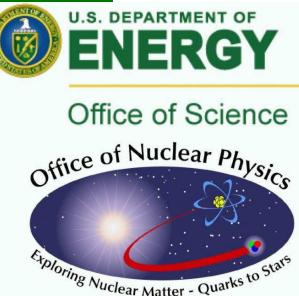
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Unifying Study of Mesons and Baryons

- How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons? Behaviour of $M(p^2)$ is essentially a quantum field theoretical effect.



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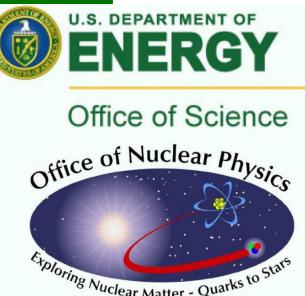
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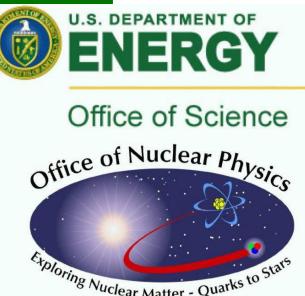
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- In quantum field theory a nucleon appears as a pole in a six-point quark Green function.
 - Residue is proportional to nucleon's Faddeev amplitude
 - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks



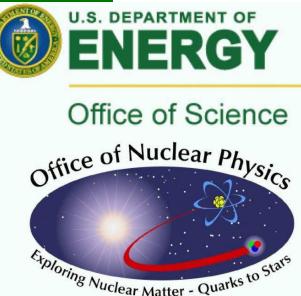
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 - Residue is proportional to nucleon's Faddeev amplitude
 - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
 - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour-3 (antitriplet) channel



Faddeev equation

R. T. Cahill *et al.* Austral. J. Phys. 42 (1989) 129



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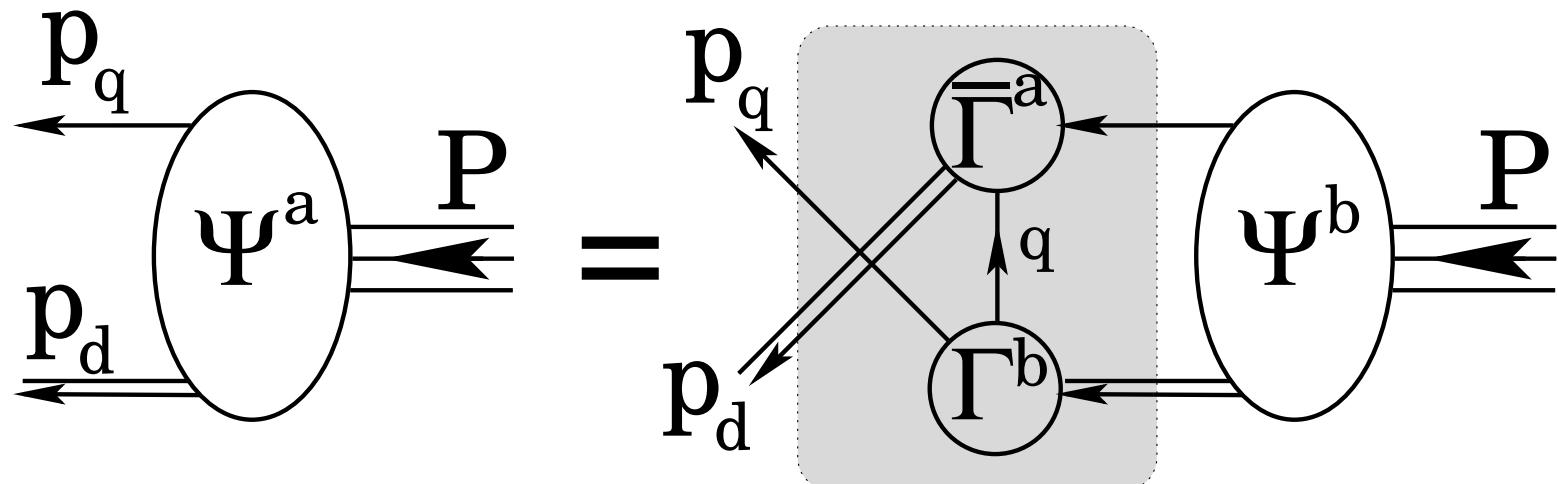
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R. T. Cahill et al. Austral. J. Phys. 42 (1989) 129



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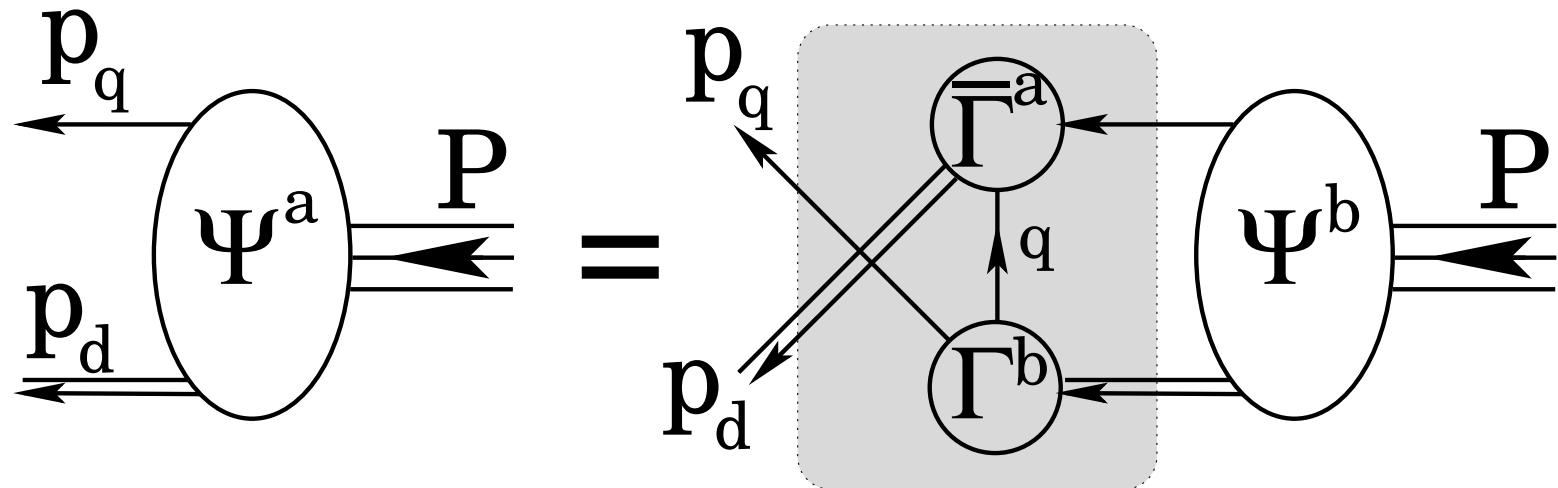
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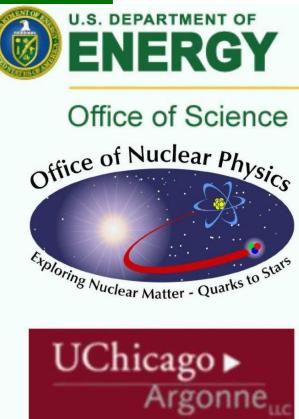
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Faddeev equation

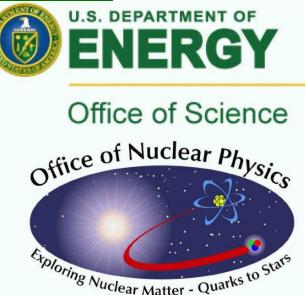
R. T. Cahill et al. Austral. J. Phys. 42 (1989) 129



- Linear, Homogeneous Matrix equation
 - Yields *wave function* (*Poincaré Covariant Faddeev Amplitude*) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame *Amplitude* has ... *s-*, *p-* & *d-*wave correlations



Nucleon-Photon Vertex



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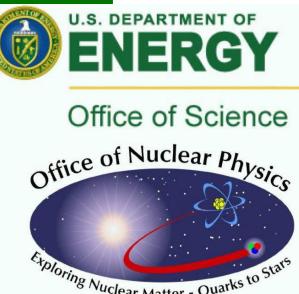
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M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

6 terms ...

Nucleon-Photon Vertex

constructed systematically ... current conserved automatically
for on-shell nucleons described by Faddeev Amplitude



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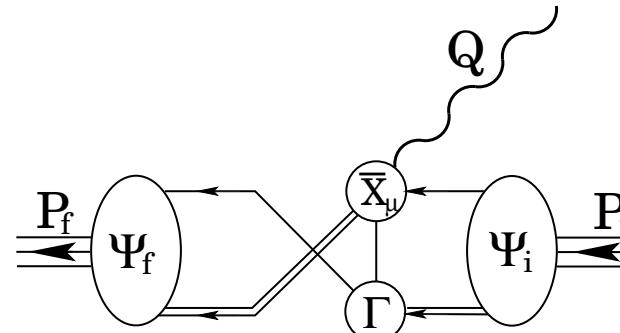
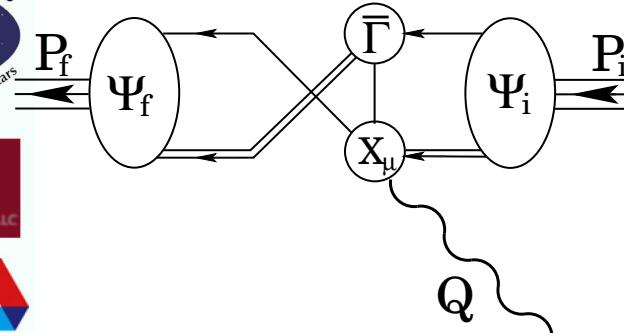
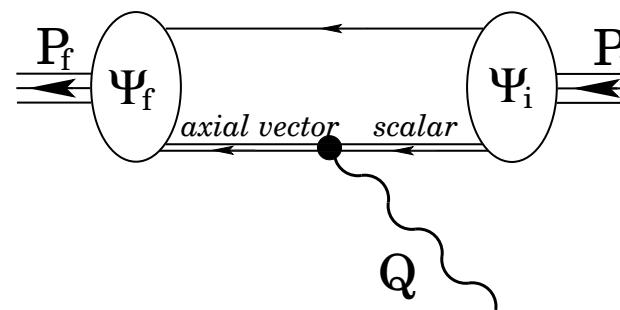
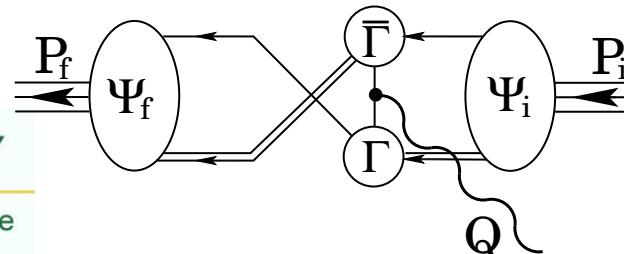
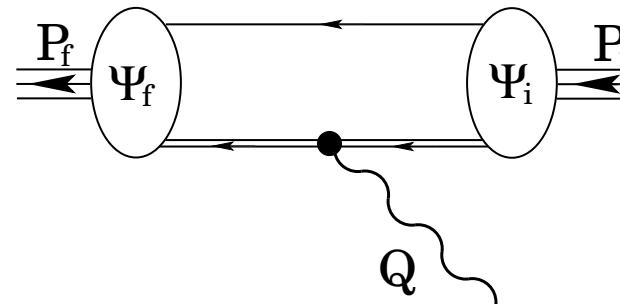
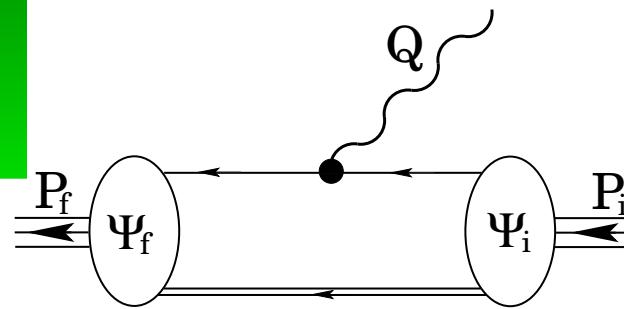
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6 terms ...

Nucleon-Photon Vertex

constructed systematically ... current conserved automatically
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Cloët, Roberts *et al.*

- arXiv:0710.2059 [nucl-th]
- arXiv:0710.5746 [nucl-th]
- arXiv:0804.3118 [nucl-th]

$$\frac{\mu_n G_E(Q^2)}{G_M(Q^2)}$$

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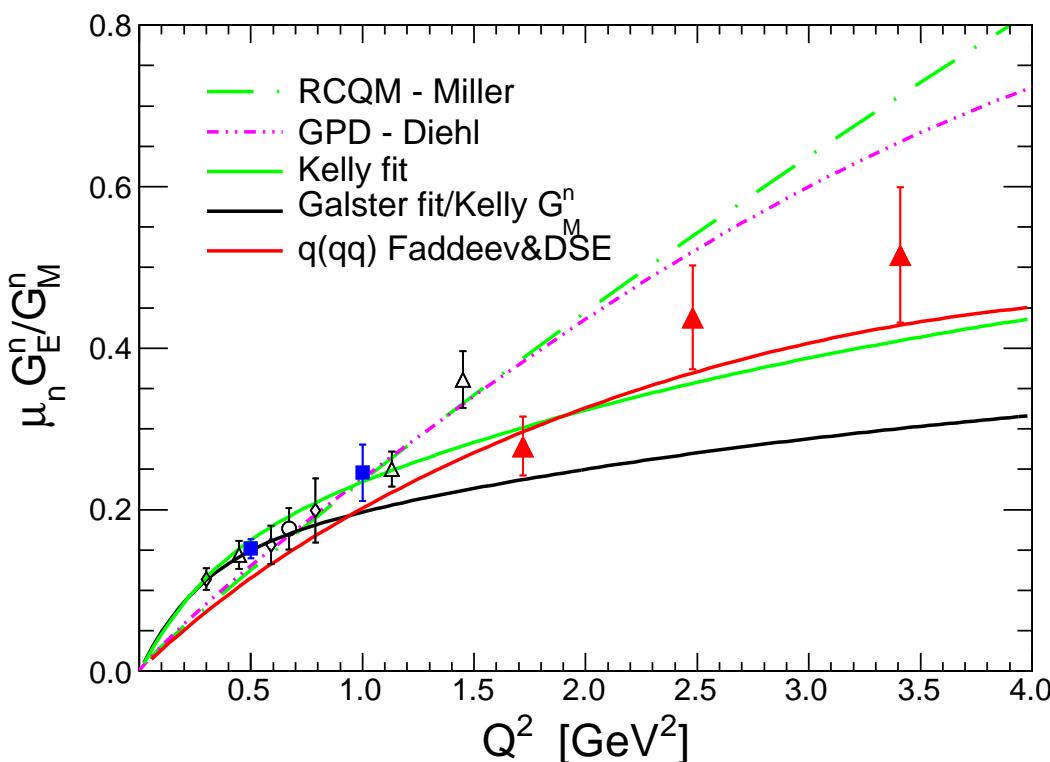
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● DSE-Faddeev Equation prediction



B. Wojtsekhowski, Jefferson Lab E02-013 Collaboration, *in preparation.*

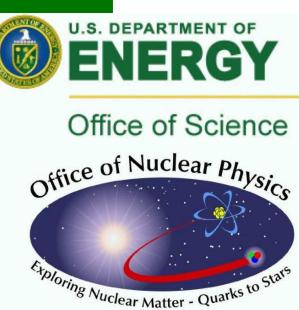


Figure courtesy S. Riordan



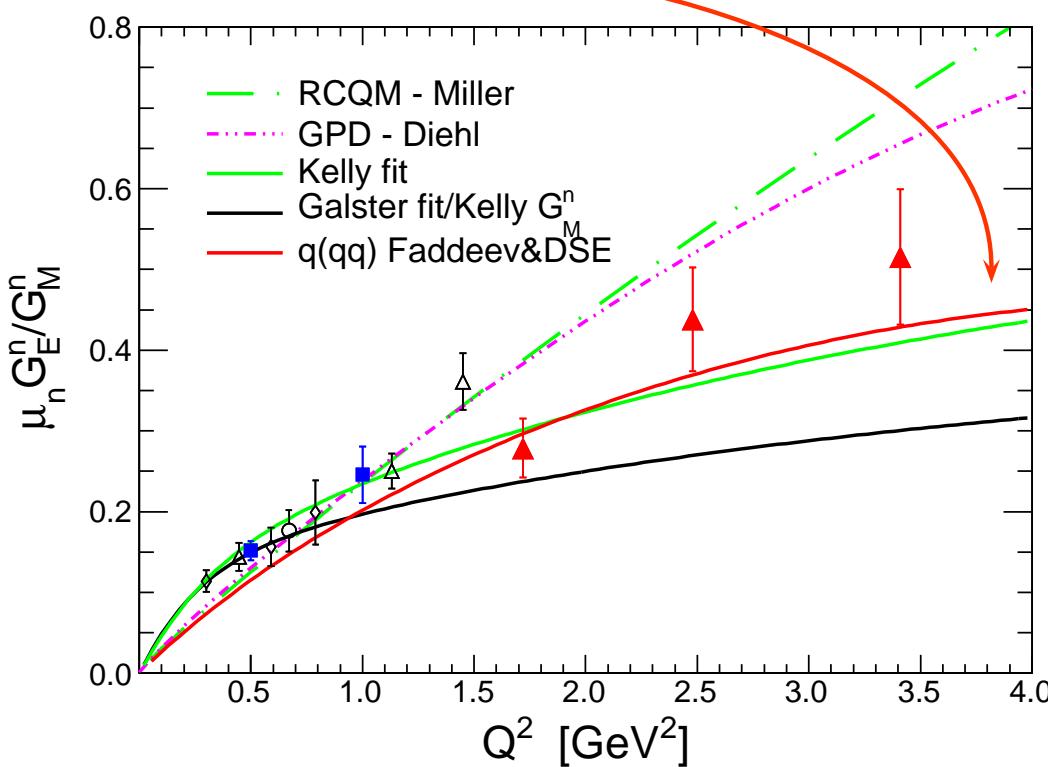
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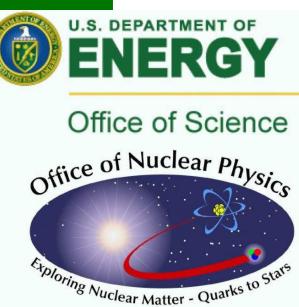
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- DSE-Faddeev Equation prediction
Red solid curve



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Figure courtesy S. Riordan





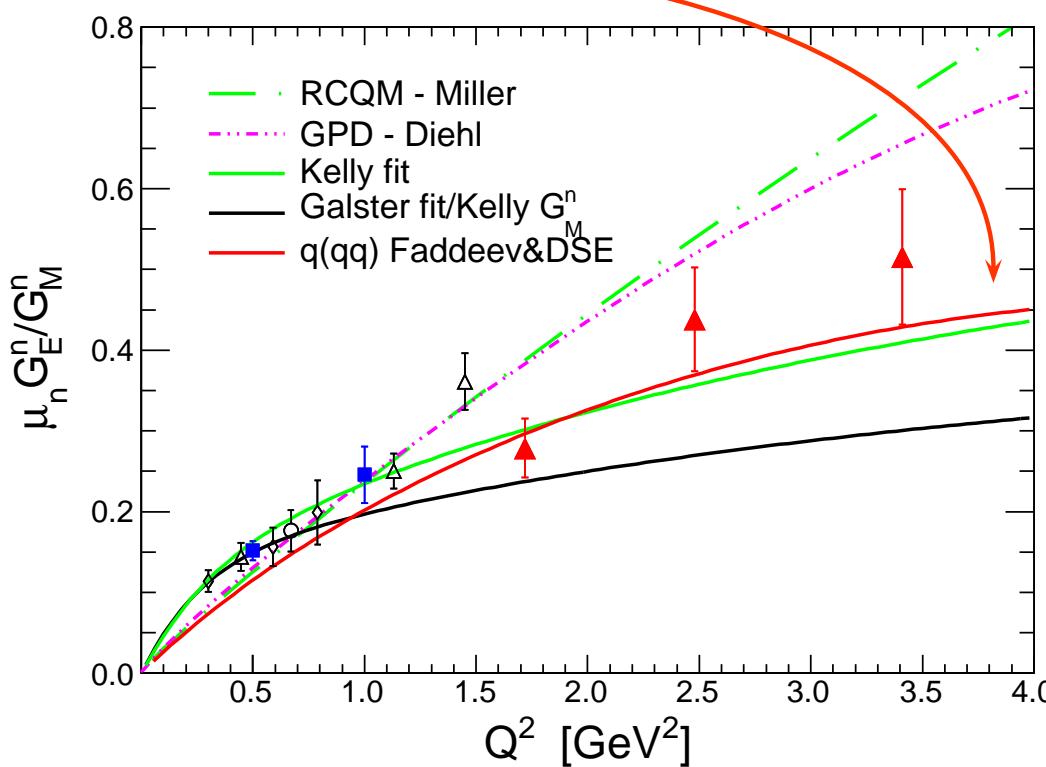
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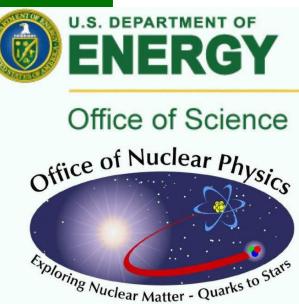
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- DSE-Faddeev Equation prediction
Red solid curve



This evolution
is very sensitive
to momentum
-dependence
of dressed-quark
propagator

B. Wojtsekhowski, Jefferson Lab E02-013 Collaboration, *in preparation.*
Figure courtesy S. Riordan





Cloët, Roberts *et al.*

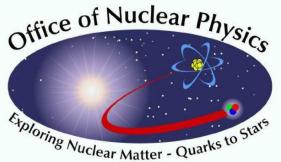
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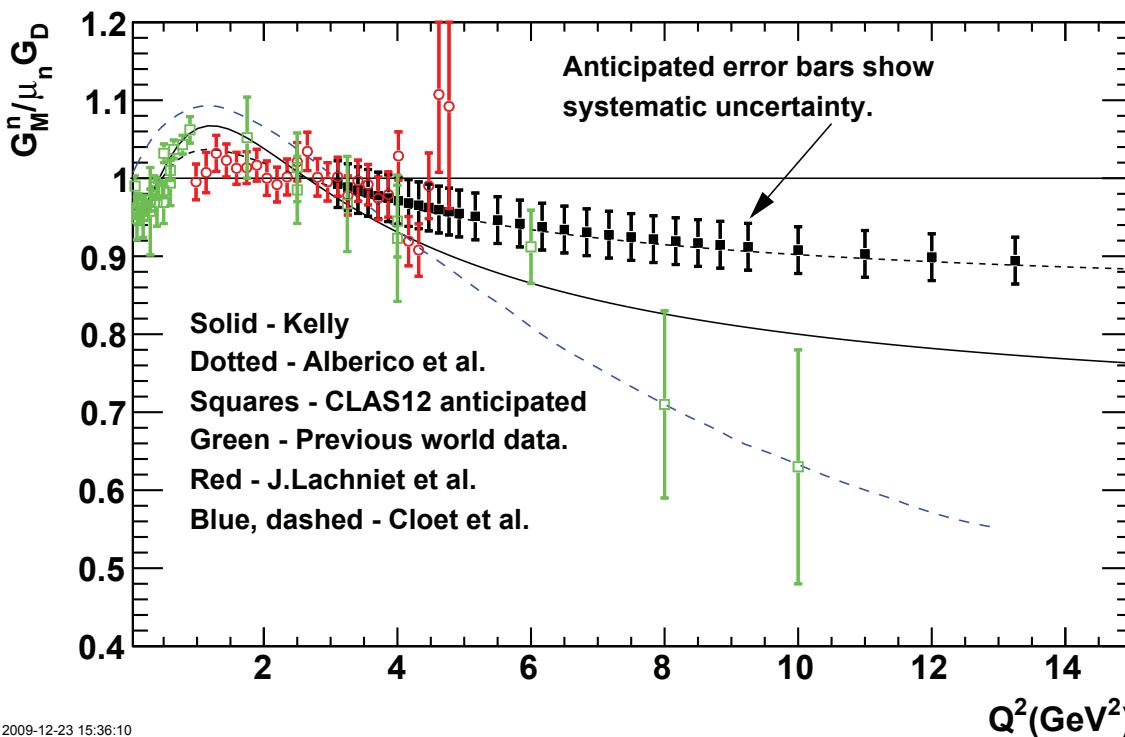
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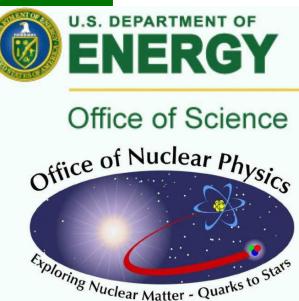
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Jefferson Lab E12-07-104, 12GeV Proposal.

Gilfoyle, Brooks, Hafidi for CLAS Collaboration





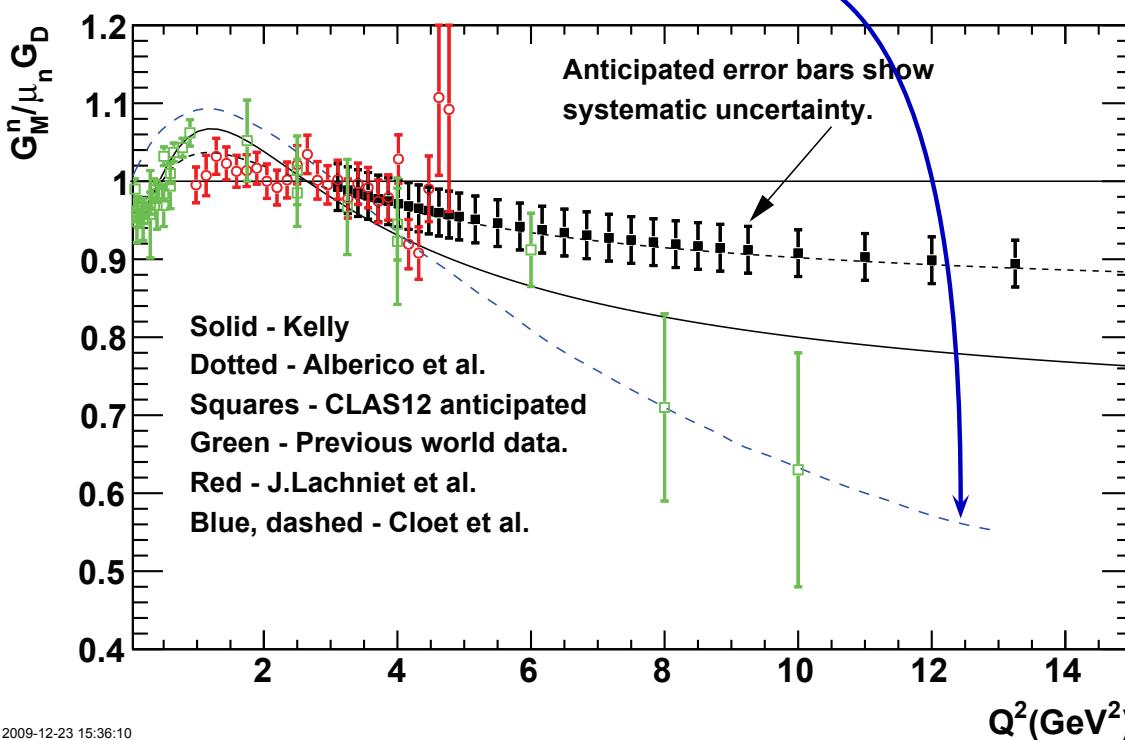
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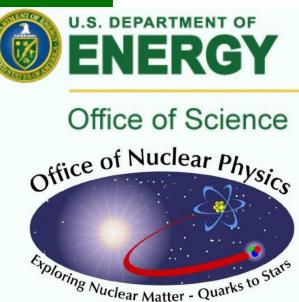
- DSE-Faddeev Equation prediction
Blue long-dashed curve



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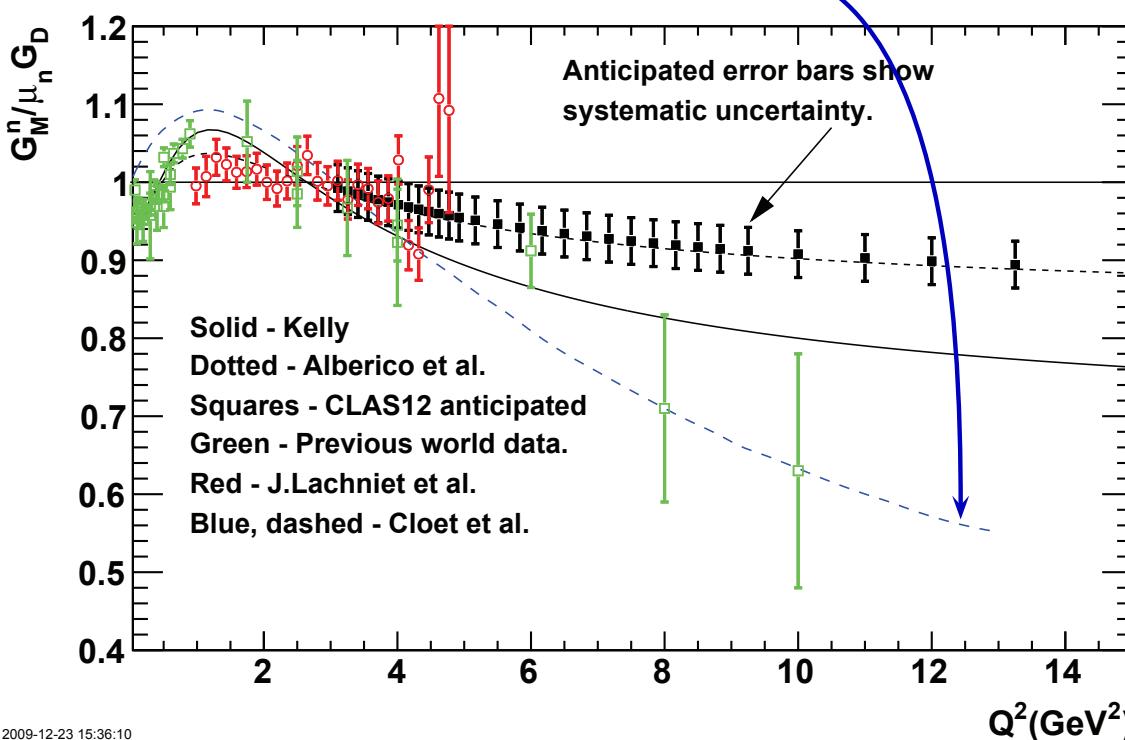
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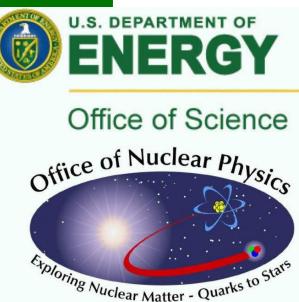
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Sensitivity to $M(p^2)$ means experiments probe IR behaviour of strong running coupling

Jefferson Lab E12-07-104, 12GeV Proposal.

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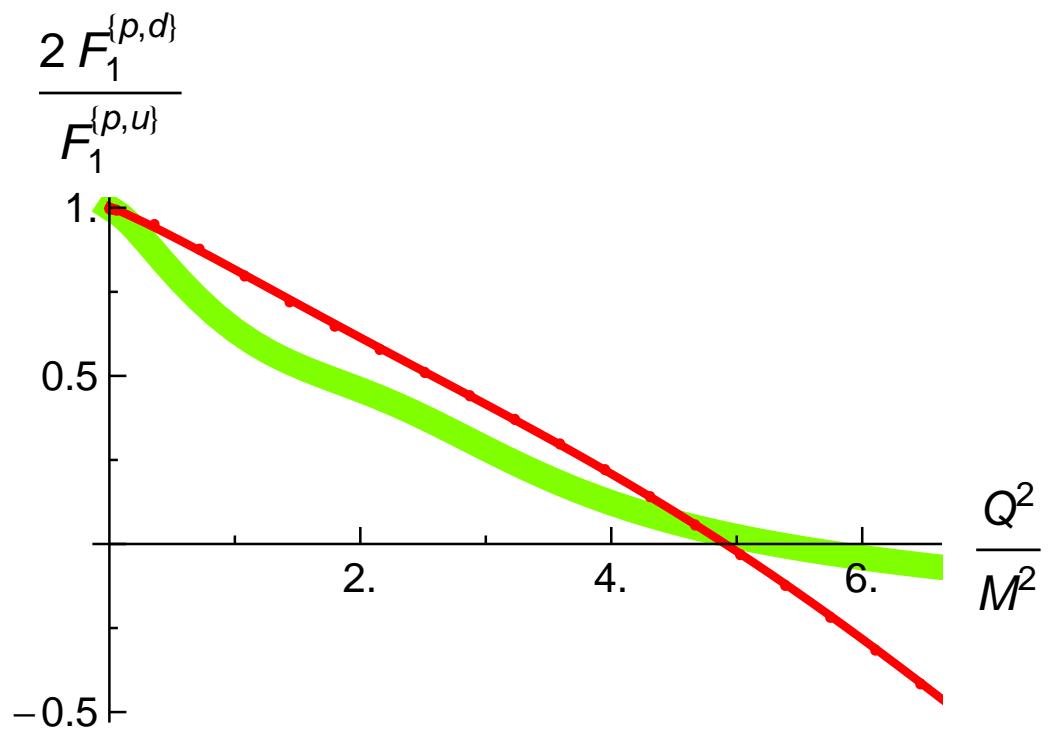
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$$\frac{2F_1^{p,d}}{F_1^{p,u}}$$

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● DSE-Faddeev Equation prediction



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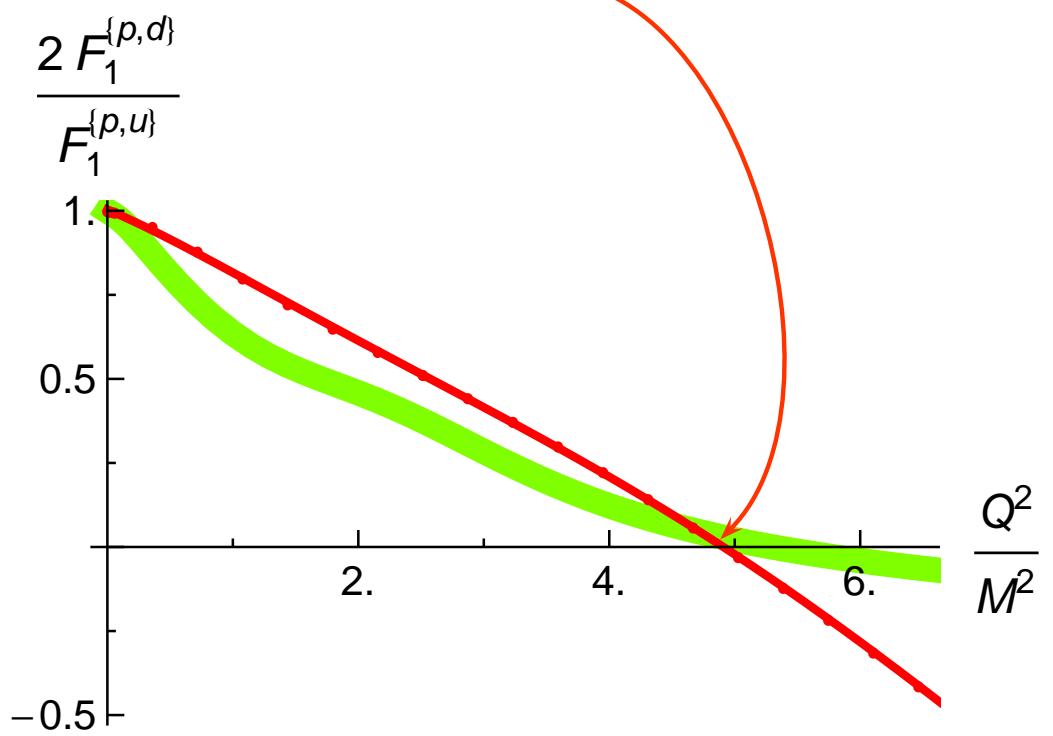


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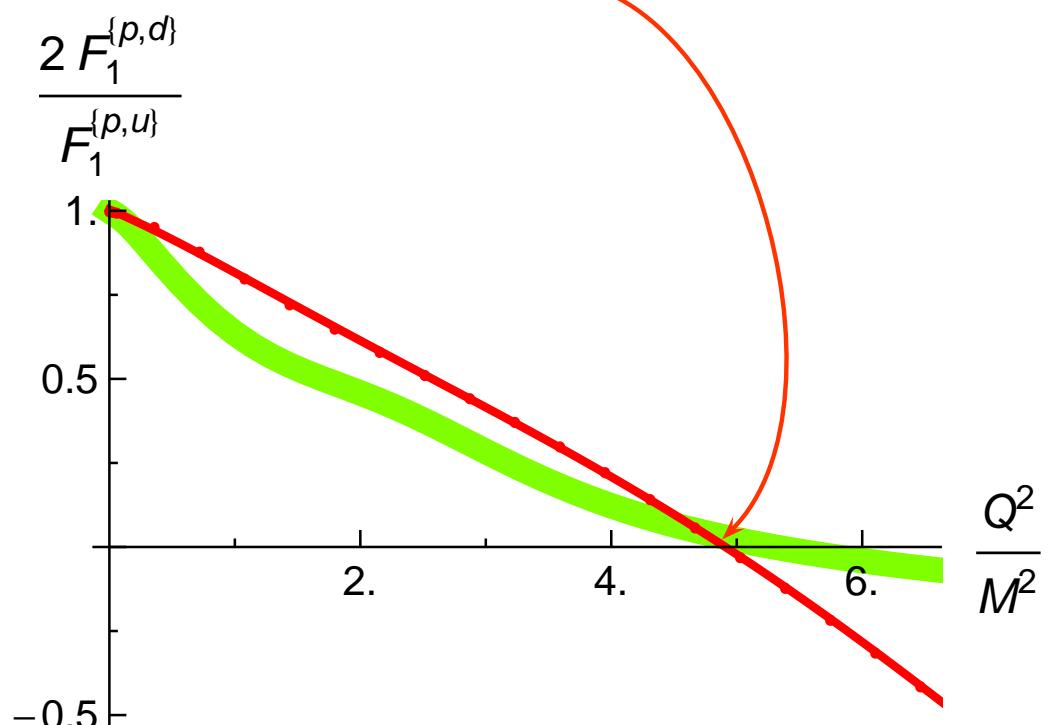
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Brooks, Bodek, Budd, Arrington fit to data
hep-ex/0602017.



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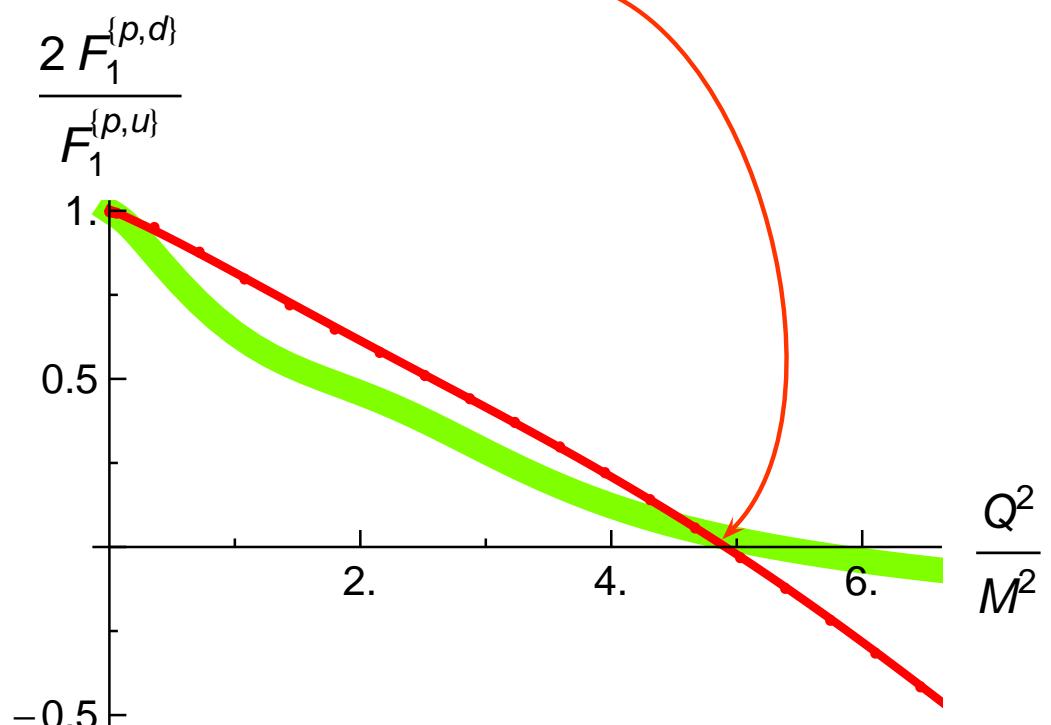
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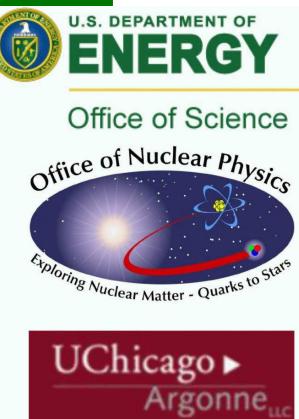
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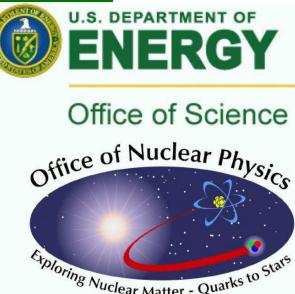
Location of zero
measures
relative strength
of scalar
and axial-vector
 qq -correlations

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Some current 12 GeV-related projects

- Elucidate signals of $M(p^2)$ in Q^2 -evolution of nucleon elastic and transition form factors; viz.,
 - $N \rightarrow \Delta$
 - $N \rightarrow P11(1440)$
 - $\kappa(p^2)$
- (M. Bhagwat, L. Chang, I. Cloët, H. Roberts)



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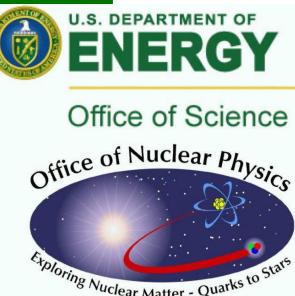
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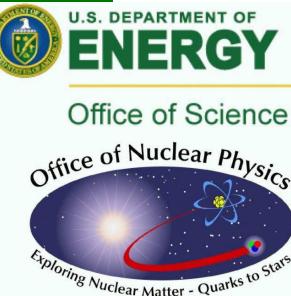
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- (*M. Bhagwat, L. Chang, I. Cloët, H. Roberts*)
- Elucidate effects of DCSB in
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 - hadron valence-quark distribution functions (*A. Bashir, P.C. Tandy*)



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 - hadron valence-quark distribution functions (*A. Bashir, P.C. Tandy*)
- Incorporate “resonant contributions” (pion cloud) in kernels of bound-state equations (e.g., *arXiv:0802.1948 [nucl-th]* & *arXiv:0811.2018 [nucl-th]*; and *C.S. Fischer et al.*)



Epilogue



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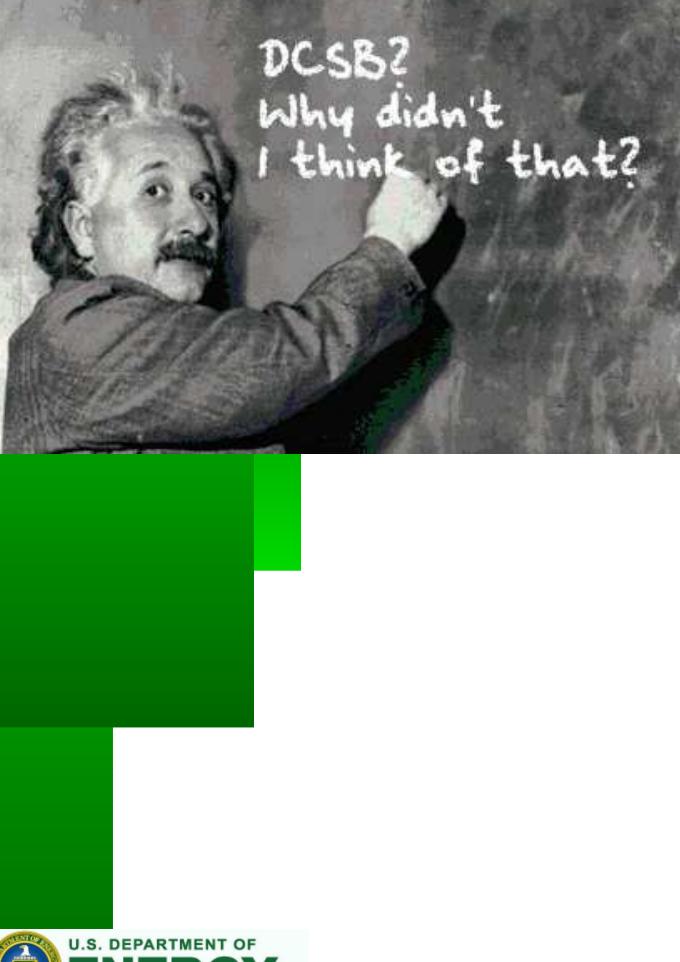


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- DCSB exists in QCD.

Epilogue



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Epilogue

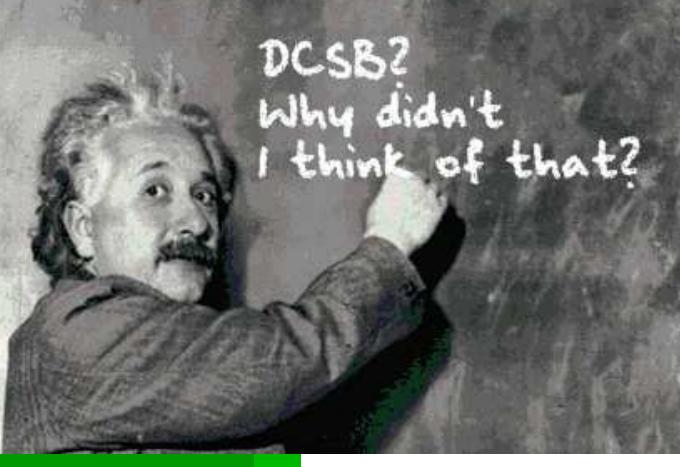
- DCSB exists in QCD.

- It is manifest in dressed propagators and vertices



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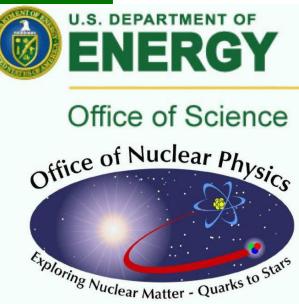


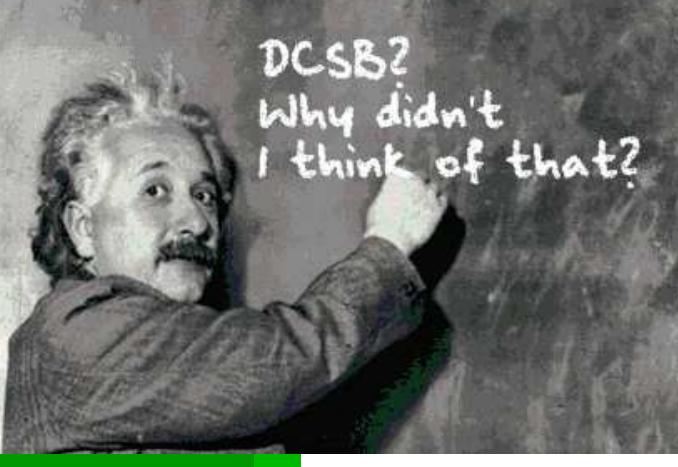


Epilogue

- DCSB exists in QCD.

- It is manifest in dressed propagators and vertices
- It predicts, amongst other things, that
 - light current-quarks become heavy constituent-quarks: $4 \rightarrow 400 \text{ MeV}$
 - pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140 \text{ MeV}$
 - pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi \bar{q}q} \approx 4.3$
 - pseudoscalar mesons couple unnaturally strongly to the lightest baryons
$$g_{\pi \bar{N}N} \approx 12.8 \approx 3g_{\pi \bar{q}q}$$





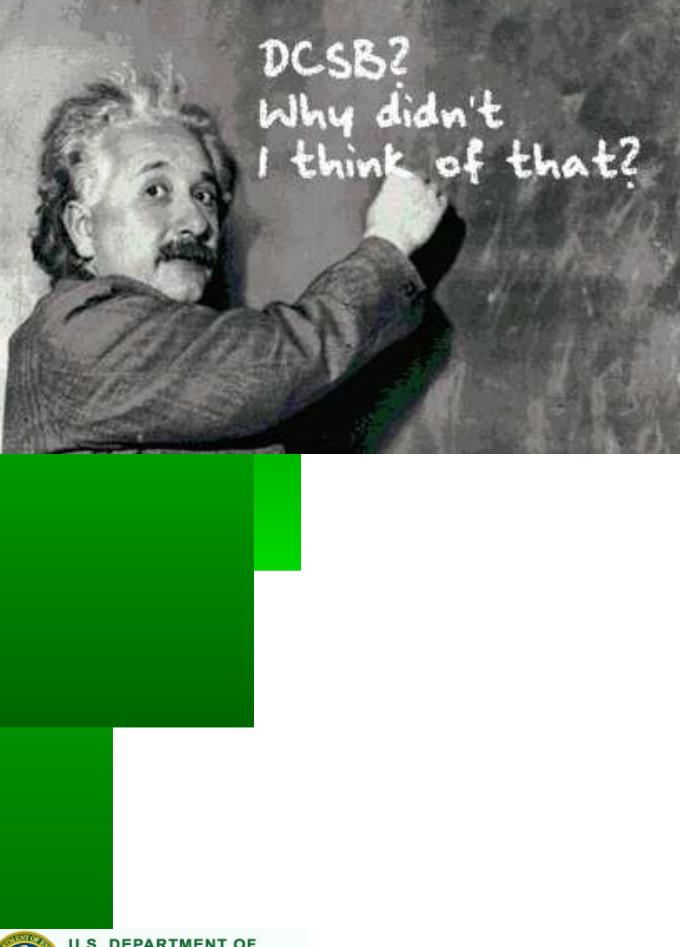
Epilogue

- DCSB impacts dramatically upon observables



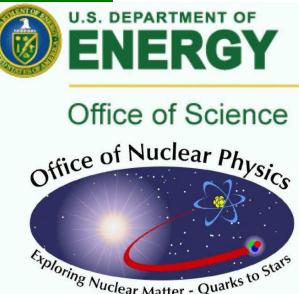
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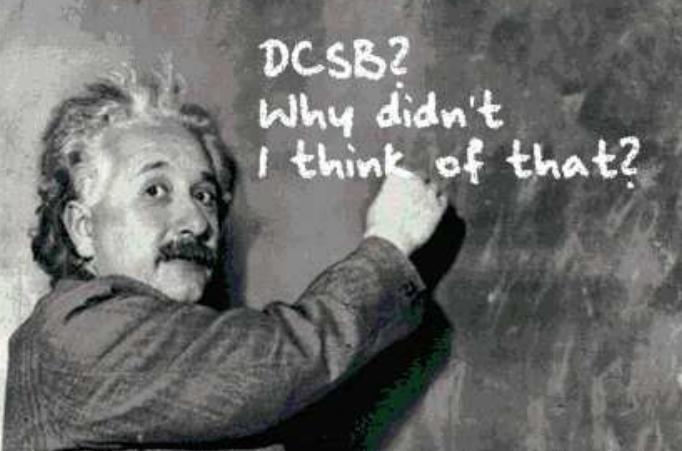




Epilogue

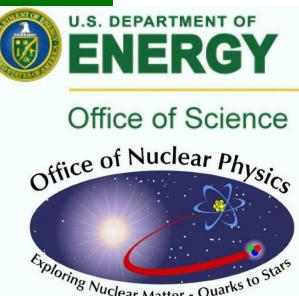
- DCSB impacts dramatically upon observables
 - Spectrum; e.g., splittings: $\sigma-\pi$ & $a_1-\rho$
 - Elastic and Transition Form Factors

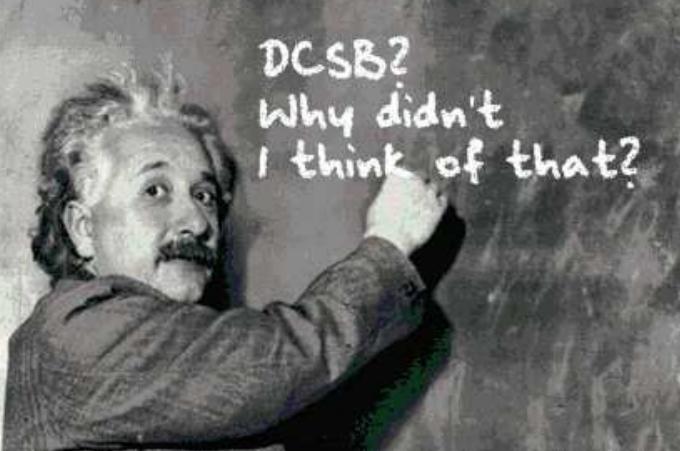




Epilogue

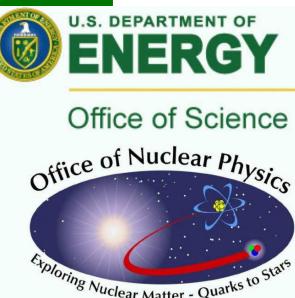
- DCSB impacts dramatically upon observables
 - Spectrum; e.g., splittings: $\sigma-\pi$ & $a_1-\rho$
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- But $M(p^2)$ is an *essentially* quantum field theoretical effect
 - Exposing & elucidating its effect in hadron physics requires nonperturbative, symmetry preserving framework; i.e., Poincaré covariance, chiral and e.m. current conservation, etc.

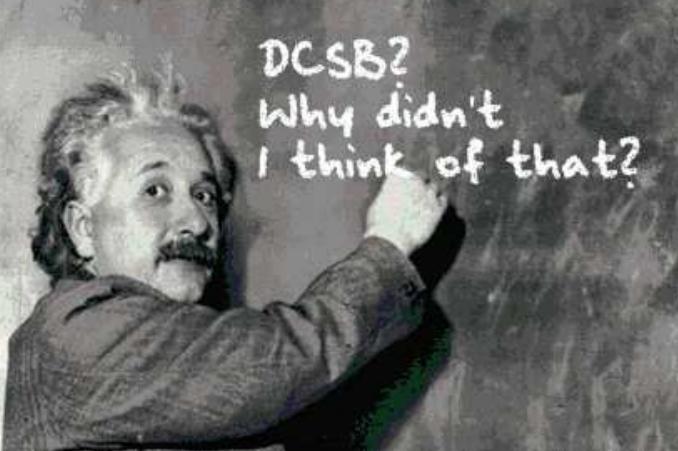




Epilogue

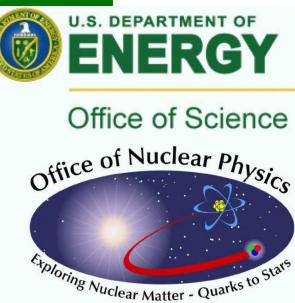
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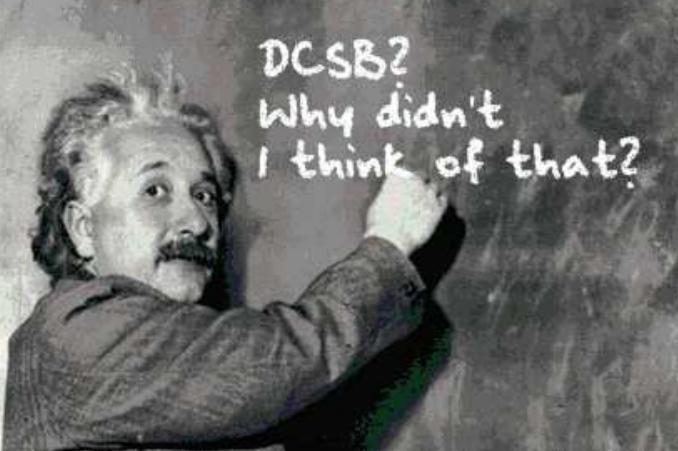




Epilogue

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- DSEs: Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks

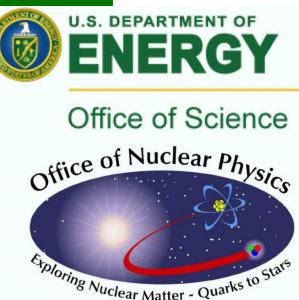




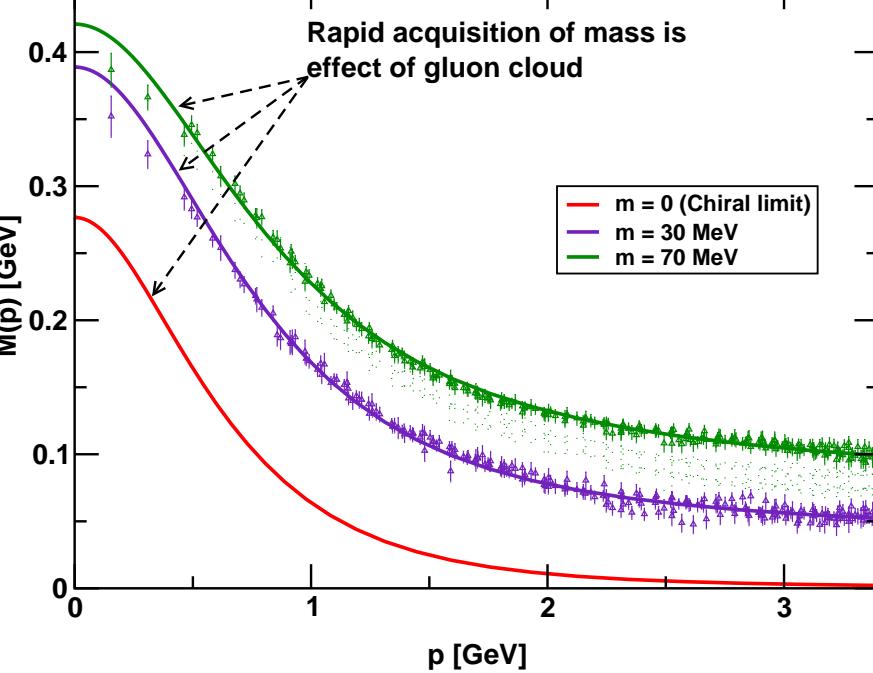
Epilogue

Now is an exciting time . . .
Positioned to unify phenomena as apparently disparate as

- Hadron spectrum
- Elastic and transition form factors, from small- to large- Q^2
- Parton distribution functions



Epilogue

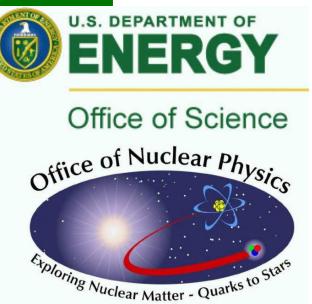


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Key: an understanding of both the fundamental origin of nuclear mass and the far-reaching consequences of the mechanism responsible; namely, **Dynamical Chiral Symmetry Breaking**



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18. $\frac{\mu_n G_E(Q^2)}{G_M(Q^2)}$
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20. $\frac{2F_1^{p,d}}{F_1^{p,u}}$
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