Baryon Properties from Continuum-QCD

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QCD’s Challenges

Understand emergent phenomena

- Quark and Gluon Confinement
  No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

- Dynamical Chiral Symmetry Breaking
  Very unnatural pattern of bound state masses;
  e.g., Lagrangian (pQCD) quark mass is small but
  \[ J^P = + \] and \[ J^P = - \] (parity partners)

- Neither of these phenomena is apparent in QCD’s Lagrangian
  Yet they are the dominant determining characteristics of real-world QCD.

- QCD
  - Complex behaviour arises from apparently simple rules.
Universal Truths

- Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.

- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.
  
  Higgs mechanism is *almost* irrelevant to light-quarks.

- Running of quark mass entails that calculations at even modest $Q^2$ require a Poincaré-covariant approach.
  
  Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.

- Confinement is expressed through a violent change of the propagators for coloured particles & can almost be read from a plot of a states’ dressed-propagator.
  
  It is intimately connected with DCSB.
Universal Conventions


  "The QCD vacuum is the vacuum state of quantum chromodynamics (QCD). It is an example of a non-perturbative vacuum state, characterized by many non-vanishing condensates such as the gluon condensate or the quark condensate. These condensates characterize the normal phase or the confined phase of quark matter."
Since 1979, DCSB has commonly been associated *literally* with a spacetime-independent mass-dimension-three “vacuum condensate.”

Under this assumption, “condensates” couple directly to gravity in general relativity and make an enormous contribution to the cosmological constant

$$\Omega_{QCD-\text{condensates}} = 8\pi \, G_N \, \Lambda_{QCD}^4 \, \frac{3H_0^2}{G_N} \approx 10^{46}$$

Experimentally, the answer is

$$\Omega_{\text{cosm. const.}} = 0.76$$

This mismatch is a bit of a problem.
Paradigm shift: In-Hadron Condensates

Resolution

- Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, owing to confinement "condensates" do not exist as spacetime-independent mass-scales that fill all spacetime.

- So-called vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

- GMOR

\[ f_\pi^2 m_\pi^2 = -2 \, m(\zeta) \langle \bar{q}q \rangle_0^\zeta \]

\[ f_\pi^2 m_\pi^2 = 2 \, m(\zeta) \rho_\pi^\zeta \]


Craig Roberts, Physics Division: Baryon Properties from Continuum-QCD
Paradigm shift: In-Hadron Condensates

Resolution
- Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, owing to confinement “condensates” do not exist as spacetime-independent mass-scales that fill all spacetime.
- So-called vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.
- No qualitative difference between $f_\pi$ and $\rho_\pi$
- Both are equivalent order parameters for DCSB

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Baryons 2010: 11 Dec 2010
Paradigm shift: In-Hadron Condensates

Resolution

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- So-called vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

- No qualitative difference between $f_\pi$ and $\rho_\pi$.

- And \[-\langle \bar{q} q \rangle^\pi_\zeta \equiv - f_\pi \langle 0 | \bar{q} \gamma_5 q | \pi \rangle = f_\pi \rho_\pi (\zeta) =: \kappa_\pi (\hat{m}; \zeta).\]

ONLY expression related to the condensate that is rigorously defined in QCD for nonzero current-quark mass
Paradigm shift: In-Hadron Condensates

“Void that is truly empty solves dark energy puzzle”
Rachel Courtland, New Scientist 4th Sept. 2010

“EMPTY space may really be empty. Though quantum theory suggests that a vacuum should be fizzing with particle activity, it turns out that this paradoxical picture of nothingness may not be needed. A calmer view of the vacuum would also help resolve a nagging inconsistency with dark energy, the elusive force thought to be speeding up the expansion of the universe.”

Cosmological Constant:
✓ Putting QCD condensates back into hadrons reduces the mismatch between experiment and theory by a factor of $10^{46}$
✓ Possibly by far more, if technicolour-like theories are the correct paradigm for extending the Standard Model
Nonperturbative tools are needed
- This conference provides a snapshot of those that we currently have at our disposal

Dyson-Schwinger equations
- Nonperturbative symmetry-preserving tool for the study of Continuum-QCD

DSEs provide complete and compelling understanding of the pion as both a bound-state & Nambu-Goldstone mode in QCD

Pion mass and decay constant, P. Maris, C.D. Roberts and P.C. Tandy
Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory . . . Materially Reduces Model-Dependence ... Statement about long-range behaviour of quark-quark interaction
- NonPerturbative, Continuum approach to QCD
- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from nothing
  - Quark & Gluon Confinement
    - Coloured objects not detected,
      Not detectable?

- Approach yields Schwinger functions; i.e., propagators and vertices
- Cross-Sections built from Schwinger Functions
- Hence, method connects observables with long-range behaviour of the running coupling
- Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling
In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (\(m = 0\), red curve) acquires a large constituent mass at low energies.
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**DSE prediction of DCSB confirmed**
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$S_f(p)^{-1} = Z_2 \left( i \gamma \cdot p + m_f^{\text{bm}} \right) + \Sigma_f(p),$ 

\[ \Sigma_f(p) = Z_1 \int q g^2 D_{\mu\nu}(p-q) \frac{\gamma^a}{2} \gamma_{\mu} S_f(q) \frac{\gamma^a}{2} \Gamma_f(q,p) \]

- $D_{\mu\nu}(k)$ – dressed-gluon propagator
- $\Gamma_f(q,p)$ – dressed-quark-gluon vertex
- Suppose one has in hand – from anywhere – the exact form of the dressed-quark-gluon vertex

What is the associated symmetry-preserving Bethe-Salpeter kernel?!
Bethe-Salpeter Equation
Bound-State DSE

\[
\left[ \Gamma^j_\pi(k; P) \right]_{tu} = \int_q^\Lambda \left[ S(q + P/2) \Gamma^j_\pi(q; P) S(q - P/2) \right]_{sr} K^{rs}_{tu}(q, k; P)
\]

- \( K(q, k; P) \) – fully amputated, two-particle irreducible, quark-antiquark scattering kernel
- Textbook material.
- Compact. Visually appealing. Correct

Blocked progress for more than 60 years.
Bethe-Salpeter Equation
General Form

\[ \Gamma_{5\mu}^{fg}(k; P) = Z_2 \gamma_5 \gamma_\mu \]

\[ - \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \Gamma_{5\mu}^{fg}(q; P) S_g(q_-) \frac{\lambda^a}{2} \Gamma_{\beta}^g(q_-, k_-) \]

\[ + \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P), \]

- Equivalent exact bound-state equation \textbf{but} in this form
  \[ K(q, k; P) \rightarrow \Lambda(q, k; P) \]
  which is \textit{completely determined by dressed-quark self-energy}
- Enables derivation of a Ward-Takahashi identity for \( \Lambda(q, k; P) \)
Now, for first time, it’s possible to formulate an Ansatz for Bethe-Salpeter kernel given any form for the dressed-quark-gluon vertex by using this identity

This enables the identification and elucidation of a wide range of novel consequences of DCSB
Dressed-quark anomalous magnetic moments

- Schwinger’s result for QED:
  \[ \frac{q}{2m} \rightarrow \left( 1 + \frac{\alpha}{2\pi} \right) \frac{q}{2m} \]

- pQCD: two diagrams
  - (a) is QED-like
  - (b) is only possible in QCD – involves 3-gluon vertex

- Analyse (a) and (b)
  - (b) vanishes identically: the 3-gluon vertex does not contribute to a quark’s anomalous chromomagnetic moment at leading-order
  - (a) Produces a finite result: “ – ⅙ \( \alpha_s/2\pi \)”
    \[ \sim (- \frac{1}{6}) \text{ QED-result} \]

- But, in QED and QCD, the anomalous chromo- and electro-magnetic moments vanish identically in the chiral limit!
Dressed-quark anomalous magnetic moments

- Interaction term that describes magnetic-moment coupling to gauge field
  - Straightforward to show that it mixes left ↔ right
  - Thus, explicitly violates chiral symmetry
- Follows that in fermion’s e.m. current
  \[ \gamma_\mu F_1 \text{ does not mix with } \sigma_{\mu\nu}q_\nu F_2 \]
  No Gordon Identity
  - Hence massless fermions cannot possess a measurable chromo- or electro-magnetic moment
- But what if the chiral symmetry is dynamically broken, strongly, as it is in QCD?
Dressed-quark anomalous magnetic moments

Three strongly-dressed and essentially-nonperturbative contributions to dressed-quark-gluon vertex:

\[ \lambda^3_{\mu}(p, q) = 2(p + q)_\mu \Delta_B(p, q) \]
\[ \Delta_F(p, q) = \frac{F(p^2) - F(q^2)}{p^2 - q^2} \]

\[ \Gamma^5_{\mu}(p, q) = \eta \sigma_{\mu\nu}(p - q)_\nu \Delta_B(p, q) \]
\[ \Gamma^4_{\mu}(p, q) = \left[ \gamma^T \gamma \cdot k + i \gamma^T \sigma_{\mu\nu} \gamma^T \cdot k \right] \tau_4(p, q) \]
\[ \tau_4(p, q) = \mathcal{F}(z) \left[ \frac{1 - 2\eta}{M_E} \Delta_B(p^2, q^2) - \Delta_A(p^2, q^2) \right] \]

\[ \mathcal{F}(z) = (1 - \exp(-z))/z, \quad z = (p_i^2 + p_j^2 - 2M_E^2)/\Lambda_F^2, \quad \Lambda_F = 1 \text{ GeV}, \]

Simplifies numerical analysis;

\[ M_E = \{ s | s > 0, s = M^2(s) \} \text{ is the Euclidean constituent-quark mass} \]
Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1009.3458 [nucl-th]

- Formulated and solved general Bethe-Salpeter equation
- Obtained dressed electromagnetic vertex
- Confined quarks don’t have a mass-shell
  - Can’t unambiguously define magnetic moments
  - But can define magnetic moment distribution
- AEM is opposite in sign but of roughly equal magnitude as ACM
  - Potentially important for elastic & transition form factors, etc.
  - Muon $g-2$?

<table>
<thead>
<tr>
<th></th>
<th>$M^E$</th>
<th>$\kappa^{ACM}$</th>
<th>$\kappa^{AEM}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full vertex</td>
<td>0.44</td>
<td>-0.22</td>
<td>0.45</td>
</tr>
<tr>
<td>Rainbow-ladder</td>
<td>0.35</td>
<td>0</td>
<td>0.048</td>
</tr>
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</table>
DSEs and Baryons

- $M(p^2)$ – effects have enormous impact on meson properties.
  - *Must be included in description and prediction of baryon properties.*
- $M(p^2)$ is essentially a quantum field theoretical effect. In quantum field theory
  - Meson appears as pole in four-point quark-antiquark Green function
    $\rightarrow$ Bethe-Salpeter Equation
  - *Nucleon appears as a pole in a six-point quark Green function*
    $\rightarrow$ Faddeev Equation.
- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates *nonpointlike* quark-quark (diquark) correlations in the colour-antitriplet channel
  \[ r_{qq} \approx r_{\pi} \]
Faddeev Equation

- Linear, Homogeneous Matrix equation
  - Yields wave function \((Poincaré Covariant Faddeev Amplitude)\)
    that describes quark-diquark relative motion within the nucleon

- Scalar and Axial-Vector Diquarks . . .
  - Both have “correct” parity and “right” masses
  - In Nucleon’s Rest Frame Amplitude has
    \(s-, p- \& d-\) wave correlations

R.T. Cahill et al.,

_quark exchange ensures Pauli statistics_
### Spectrum of some known u- & d-quark baryons

#### Baryons: ground-states and 1\(^{st}\) radial excitations

<table>
<thead>
<tr>
<th></th>
<th>(m_N)</th>
<th>(m_{N^*})</th>
<th>(m_{N^{(1/2)}})</th>
<th>(m_{N^{*(1/2)}})</th>
<th>(m_\Delta)</th>
<th>(m_{\Delta^*})</th>
<th>(m_{\Delta^{(3/2)}})</th>
<th>(m_{\Delta^{*(3/2)}})</th>
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<tbody>
<tr>
<td>DSE</td>
<td>1.05</td>
<td>1.73</td>
<td>1.86</td>
<td>2.09</td>
<td>1.33</td>
<td>1.85</td>
<td>1.98</td>
<td>2.16</td>
</tr>
<tr>
<td>EBAC</td>
<td>1.76</td>
<td>1.80</td>
<td></td>
<td></td>
<td>1.39</td>
<td>1.98</td>
<td></td>
<td></td>
</tr>
</tbody>
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- Mean \(|\text{relative-error}| = 2\%\)-Agreement

DSE dressed-quark-core masses cf. Excited Baryon Analysis Center bare masses is significant because no attempt was made to ensure this.
Photon-baryon vertex
Oettel, Pichowsky and von Smekal, nucl-th/9909082

“Survey of nucleon electromagnetic form factors”
- unification of meson and baryon observables;
and prediction of nucleon elastic form factors to 15 GeV²
Highlights again the critical importance of DCSB in explanation of real-world observables.
New JLab data: S. Riordan et al., arXiv:1008.1738 [nucl-ex]

DSE-prediction

This evolution is very sensitive to momentum-dependence of dressed-quark propagator
arXiv:0812.0416 [nucl-th]

- New JLab data:  
  S. Riordan et al.,  
arXiv:1008.1738 [nucl-ex]

- DSE-prediction

\[
\frac{F_1^{p,d}(Q^2)}{F_1^{p,u}(Q^2)}
\]

- Location of zero measures relative strength of scalar and axial-vector qq-correlations

Brooks, Bodek, Budd, Arrington  
fit to data: hep-ex/0602017

Craig Roberts, Physics Division: Baryon Properties from Continuum-QCD
Deep inelastic scattering – the Nobel-prize winning quark-discovery experiments

Reviews:
- S. Brodsky et al. NP B441 (1995)


Neutron Structure Function at high $x$

Distribution of neutron’s momentum amongst quarks on the valence-quark domain

SU(6) symmetry
pQCD
DSE: $0^+ & 1^+ \text{qq}
0^+ \text{qq only}$

Related measure of $1^+/0^+$ strength
Dynamical chiral symmetry breaking (DCSB) – mass from nothing for 98% of visible matter – is a reality
- Expressed in \( M(p^2) \), with observable signals in experiment

Poincaré covariance

Crucial in description of contemporary data

Fully-self-consistent treatment of an interaction

Essential if experimental data is truly to be understood.

Dyson-Schwinger equations:
- single framework, with IR model-input turned to advantage,
  “almost unique in providing unambiguous path from a defined interaction \( \rightarrow \) Confinement & DCSB \( \rightarrow \) Masses \( \rightarrow \) radii \( \rightarrow \) form factors \( \rightarrow \) distribution functions \( \rightarrow \) etc.”

McLerran & Pisarski

[arXiv:0706.2191 [hep-ph]]