Baryon Properties from Continuum-QCD

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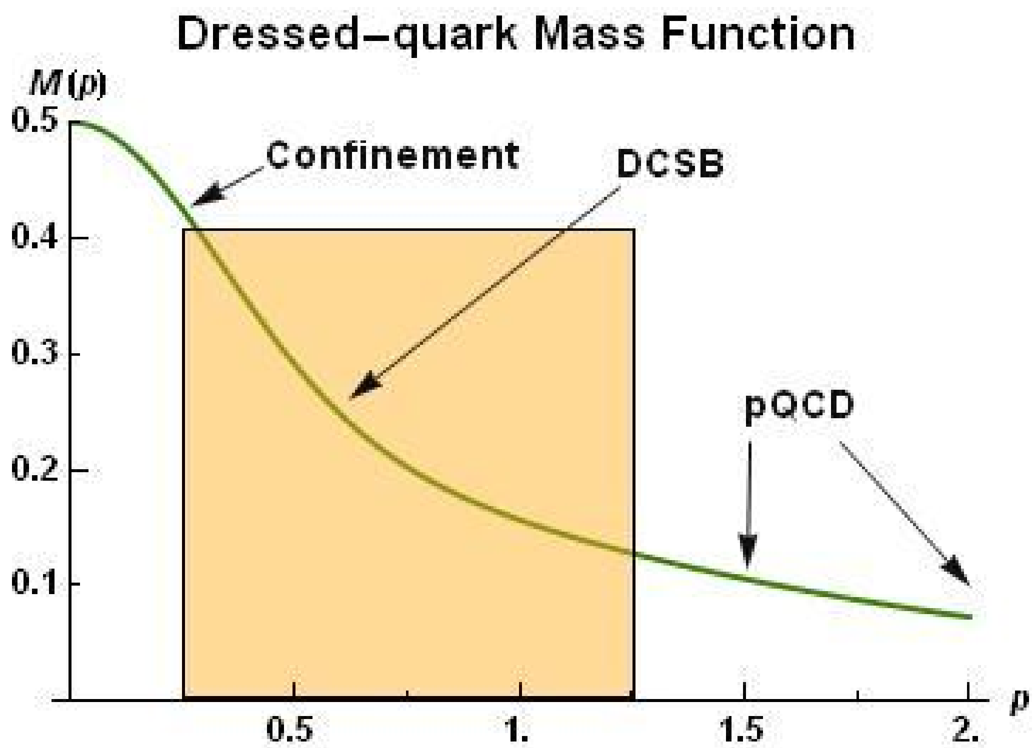
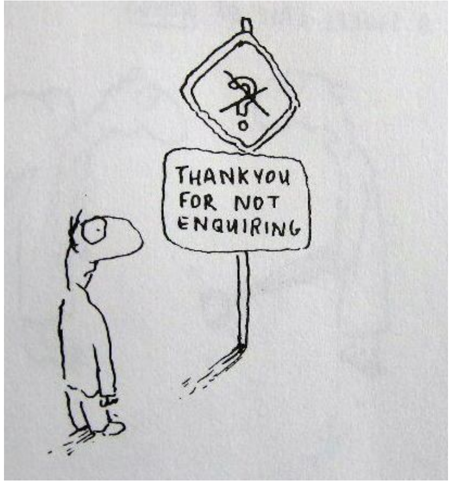
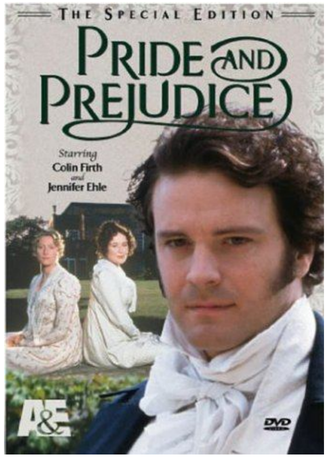




QCD's Challenges

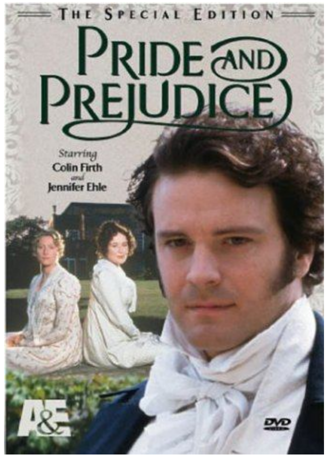
Understand emergent phenomena

- Quark and Gluon Confinement
  No matter how hard one strikes the proton,
  one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  Very unnatural pattern of bound state masses;
  e.g., Lagrangian (pQCD) quark mass is small but
  . . . no degeneracy between J^P=+ and J^P=− *(parity partners)*
- Neither of these phenomena is apparent in QCD's Lagrangian
  Yet they are the dominant determining characteristics of real-world QCD.
- QCD
  – Complex behaviour arises from apparently simple rules.



Universal Truths

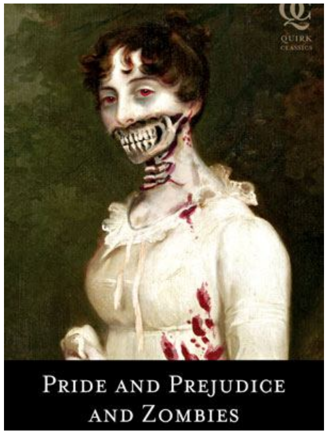
- **Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.**
- **Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.**
  Higgs mechanism is (*almost*) irrelevant to light-quarks.
- **Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach.**
  Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.
- **Confinement is expressed through a violent change of the propagators for coloured particles & can almost be read from a plot of a states' dressed-propagator.**
  It is intimately connected with DCSB.



Universal *Conventions*

- Wikipedia: (http://en.wikipedia.org/wiki/QCD_vacuum)

  "The QCD vacuum is the vacuum state of quantum chromodynamics (QCD). It is an example of a non-perturbative vacuum state, characterized by many non-vanishing condensates such as the gluon condensate or the quark condensate. These condensates characterize the normal phase or the confined phase of quark matter."



Universal Misapprehensions

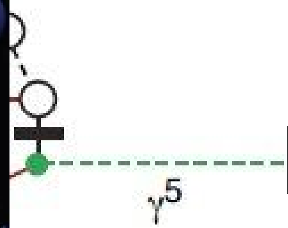
- Since 1979, DCSB has commonly been associated *literally* with a spacetime-independent mass-dimension-three "vacuum condensate."
- Under this assumption, "condensates" couple directly to gravity in general relativity and make an enormous contribution to the *cosmological constant*

$$\Omega_{QCD-condensates} = 8\pi\, G_N \frac{\Lambda_{QCD}^4}{3H_0^2} \simeq 10^{46}$$

- Experimentally, the answer is

$$\Omega_{\text{cosm. const.}} = 0.76$$

- This mismatch is a bit of a problem.



Paradigm shift: In-Hadron Condensates

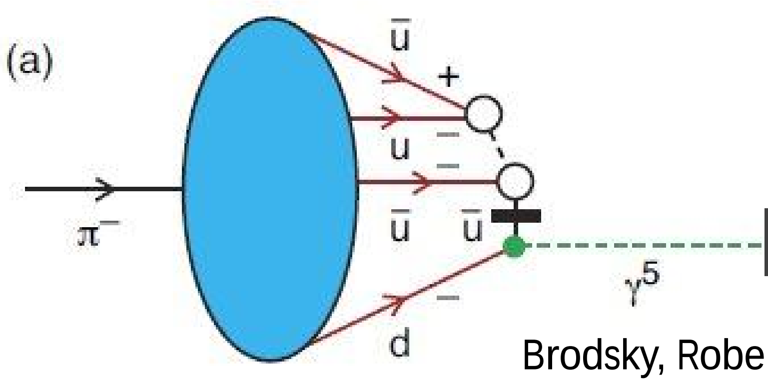
Brodsky, Roberts, Shrock, Tandy, Phys. Rev. C**82** (Rapid Comm.) (2010) 022201
Brodsky and Shrock, arXiv:0905.1151 [hep-th], to appear in PNAS

- **Resolution**
  - **Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, owing to confinement "condensates" do not exist as spacetime-independent mass-scales that fill all spacetime.**
  - ***So-called*** **vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.**
  - **GMOR**
    cf.

$$f_\pi^2 m_\pi^2 = -2\, m(\zeta) \langle \bar{q} q \rangle_0^\zeta$$

QCD

$$f_\pi m_\pi^2 = 2\, m(\zeta) \rho_\pi^\zeta$$



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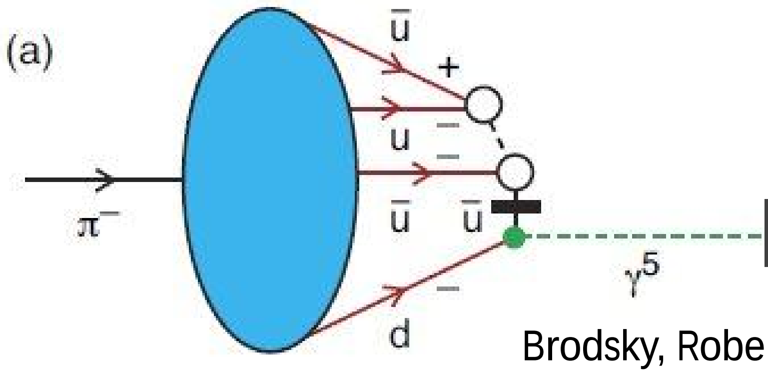
  - ***So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.**
  - **No qualitative difference between f_π and ρ_π**
  - **Both are *equivalent order parameters* for *DCSB***

$$i f_\pi P_\mu = \langle 0|\bar{q}\gamma_5\gamma_\mu q|\pi\rangle = Z_2(\zeta,\Lambda)\,\mathrm{tr}_{\mathrm{CD}}\int^\Lambda \frac{d^4q}{(2\pi)^4} i\gamma_5\gamma_\mu S(q_+)\Gamma_\pi(q;P)S(q_-), \quad (5)$$

And $|\pi\rangle \to |0\rangle$ matrix elements

$$i\rho_\pi = -\langle 0|\bar{q}\, i\gamma_5 q|\pi\rangle = Z_4(\zeta,\Lambda)\,\mathrm{tr}_{\mathrm{CD}}\int^\Lambda \frac{d^4q}{(2\pi)^4}\gamma_5 S(q_+)\Gamma_\pi(q;P)S(q_-). \quad (6)$$



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  - **No qualitative difference between f_π and ρ_π**
  - **And** $-\langle \bar{q}q \rangle_\zeta^\pi \equiv -f_\pi \langle 0|\bar{q}\gamma_5 q|\pi\rangle = f_\pi \rho_\pi(\zeta) =: \kappa_\pi(\hat{m};\zeta).$

Chiral limit

$$\kappa_\pi(0;\zeta) = -<\bar{q}q>_\zeta^0$$

ONLY expression related to the condensate that is rigorously defined in QCD for nonzero current-quark mass

Paradigm shift: In-Hadron Condensates

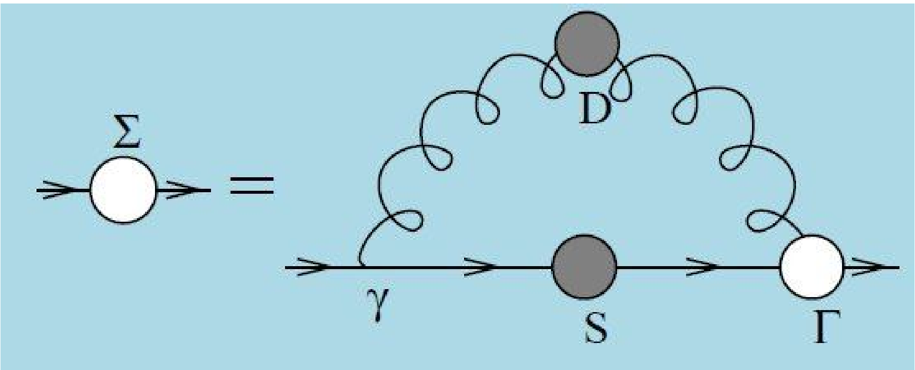
"Void that is truly empty solves dark energy puzzle"
Rachel Courtland, New Scientist 4th Sept. 2010

"EMPTY space may really be empty. Though quantum theory suggests that a vacuum should be fizzing with particle activity, it turns out that this paradoxical picture of nothingness may not be needed. A calmer view of the vacuum would also help resolve a nagging inconsistency with dark energy, the elusive force thought to be speeding up the expansion of the universe."

$$\Omega_{QCD-condensates} = \frac{8\pi G_N \Lambda_{QCD}^4}{3H_0^2} \simeq 10^{45}$$

Cosmological Constant:

- ✓ ***Putting QCD condensates back into hadrons reduces the mismatch between experiment and theory by a factor of 10^{46}***
- ✓ ***Possibly by far more, if technicolour-like theories are the correct paradigm for extending the Standard Model***



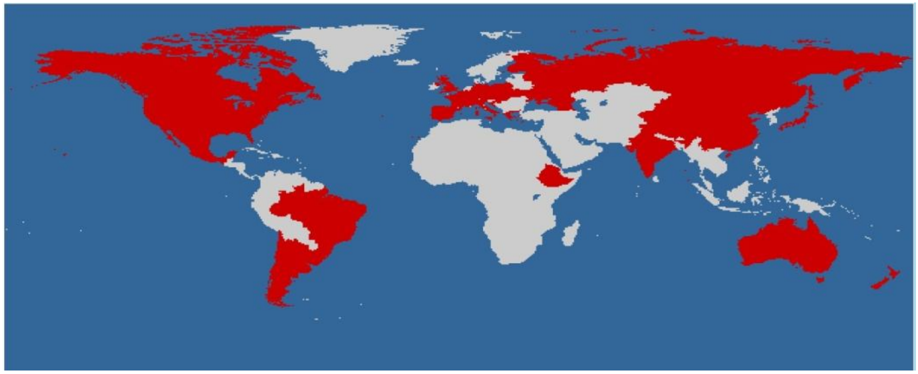
Gap equation

QCD and Baryons

- Nonperturbative tools are needed
  - This conference provides a snapshot of those that we currently have at our disposal
- Dyson-Schwinger equations
  - Nonperturbative symmetry-preserving tool

    for the study of Continuum-QCD
- DSEs provide complete and compelling understanding of the pion as *both* a bound-state & *Nambu-Goldstone mode* in QCD

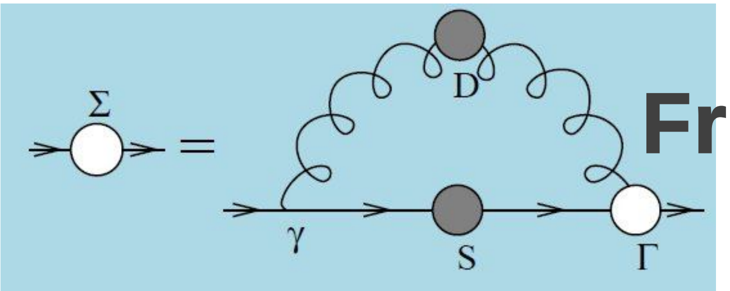
Pion mass and decay constant, P. Maris, C.D. Roberts and P.C. Tandy
nucl-th/9707003 Phys. Lett. B**20** (1998) 267-273



Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory . . . Materially Reduces Model-Dependence … Statement about long-range behaviour of quark-quark interaction
- NonPerturbative, Continuum approach to QCD
- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from *nothing*
  - Quark & Gluon Confinement
    - Coloured objects not detected, *Not detectable?*

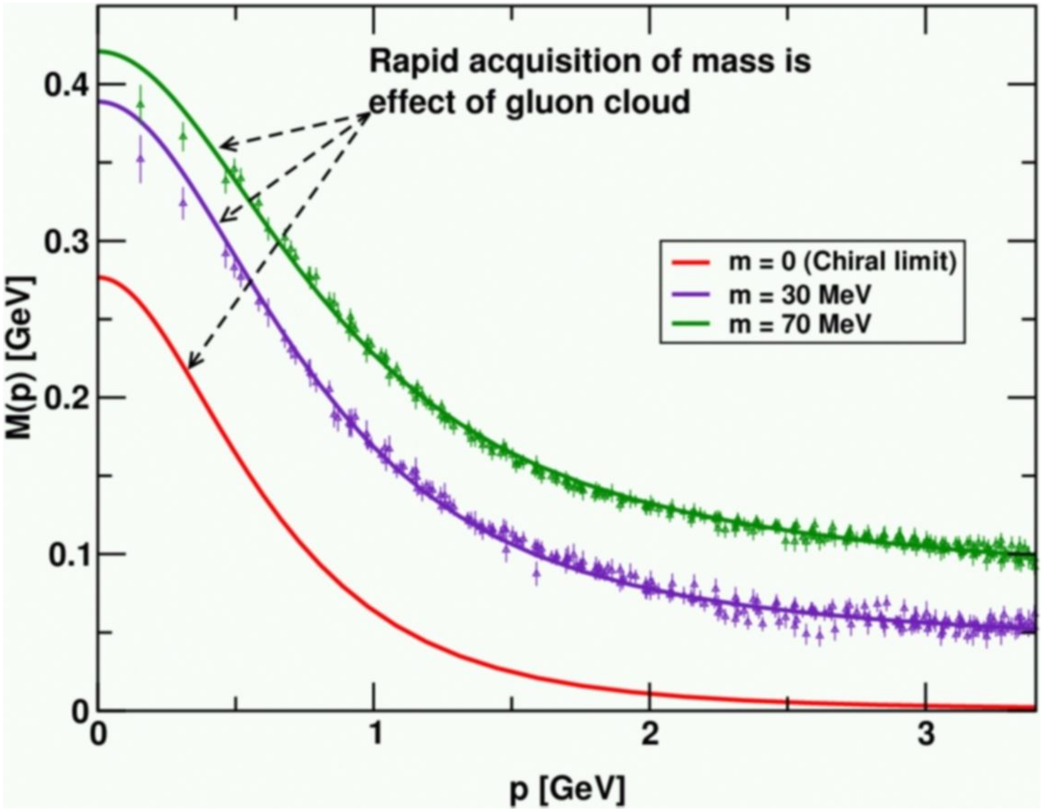
- Approach yields Schwinger functions; i.e., propagators and vertices
- Cross-Sections built from Schwinger Functions
- Hence, method connects observables with long-range behaviour of the running coupling
- Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling

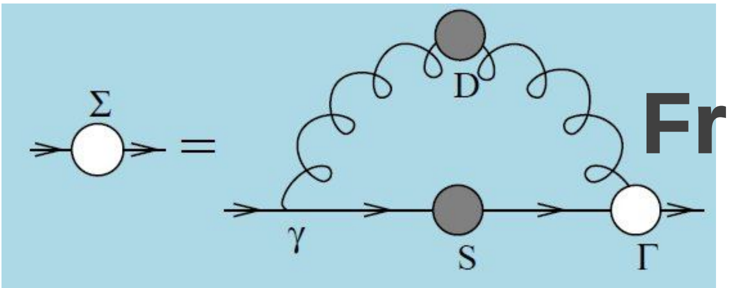


Frontiers of Nuclear Science: Theoretical Advances

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.





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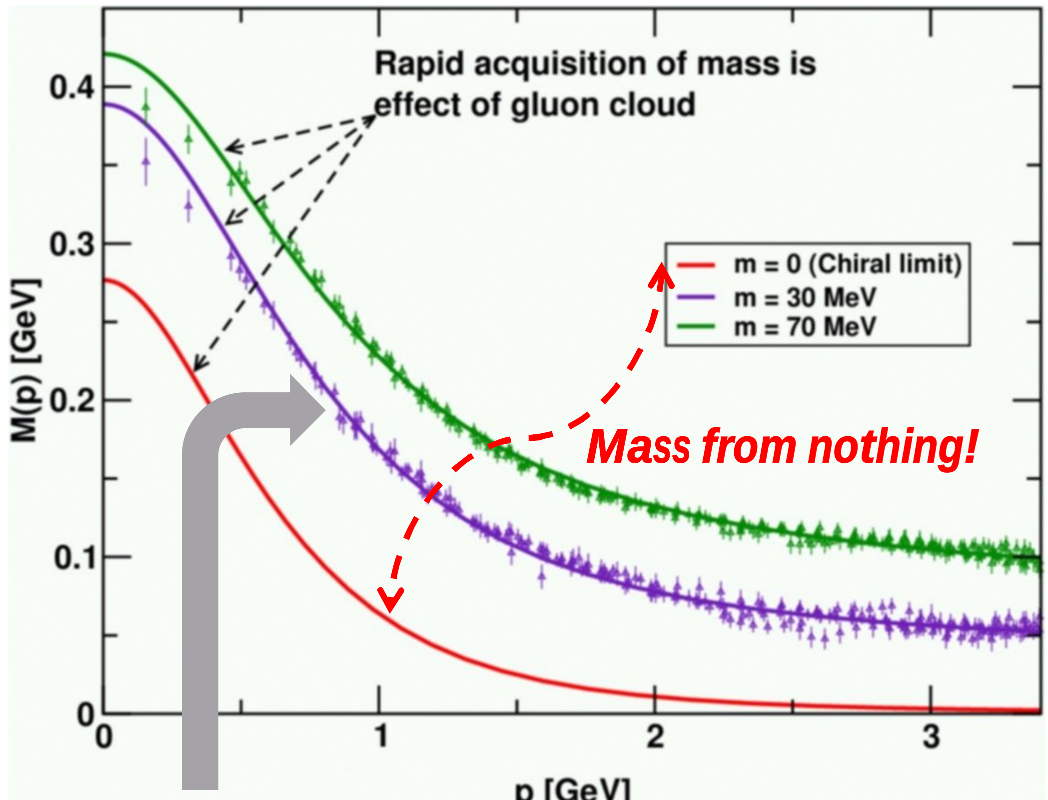
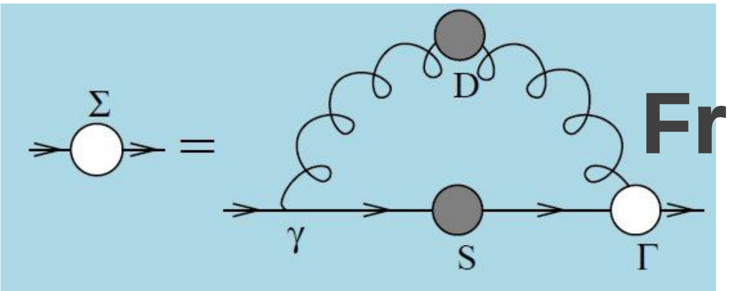


DSE prediction of DCSB confirmed



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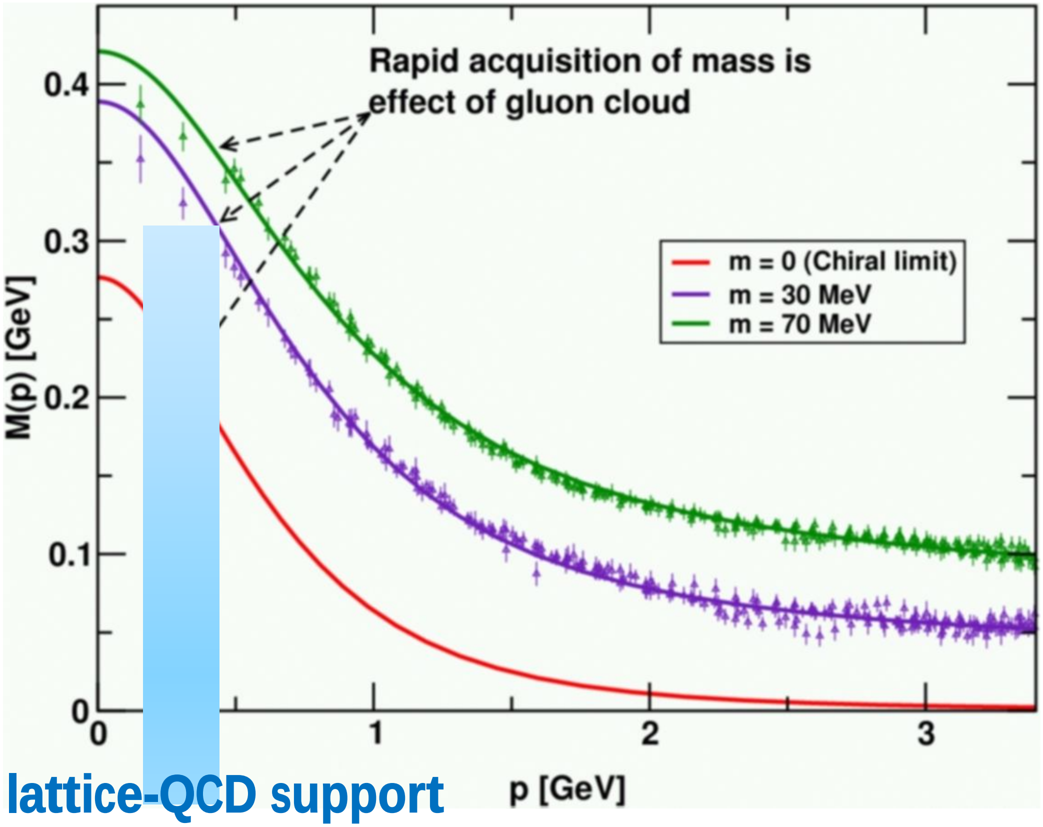
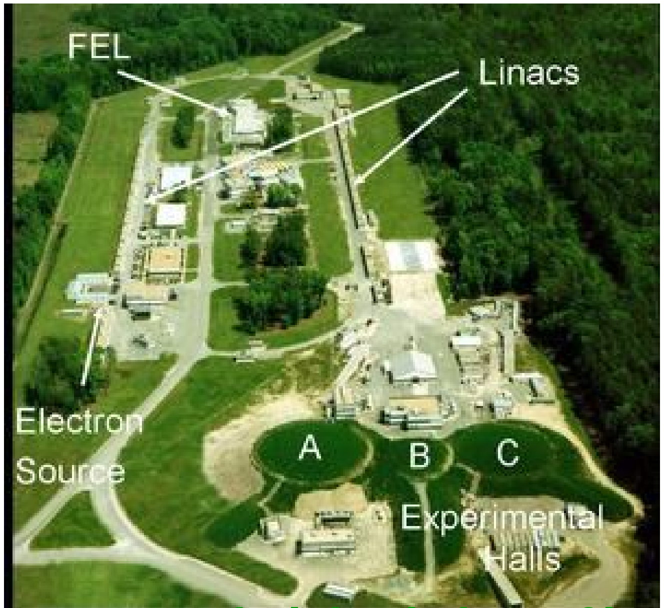


Hint of lattice-QCD support for DSE prediction of violation of reflection positivity



12GeV The Future of JLab

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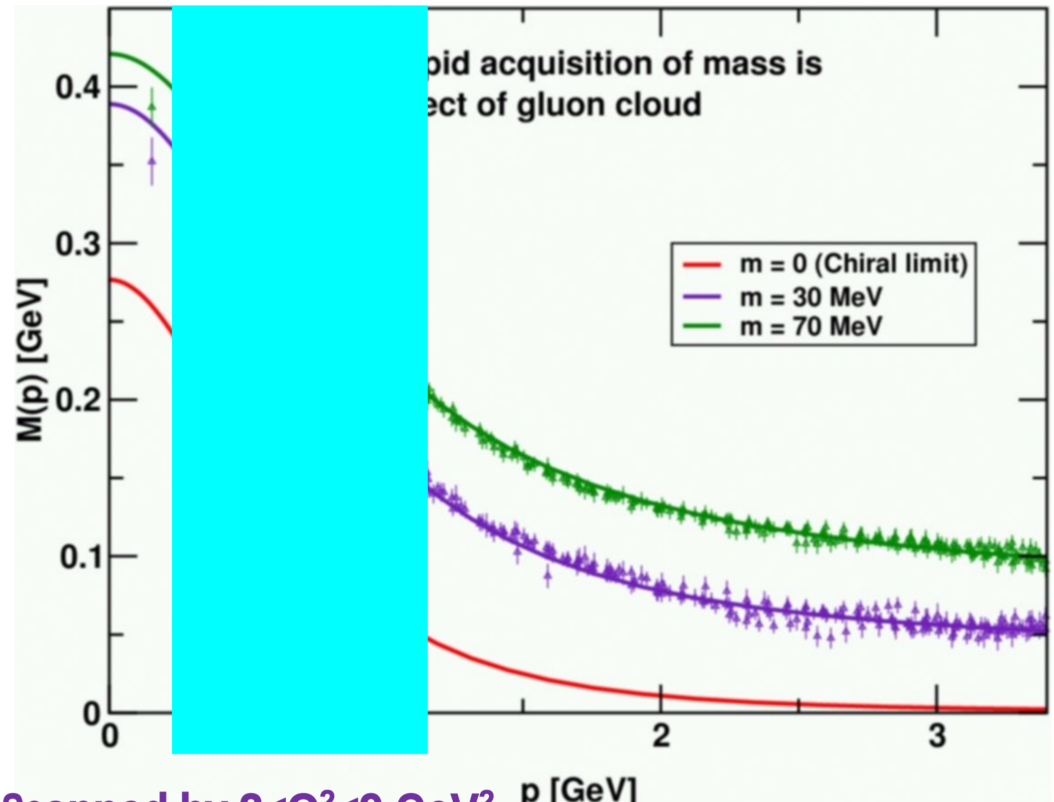
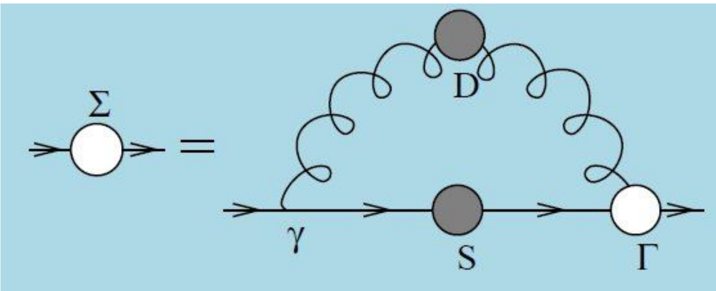


Jlab 12GeV: Scanned by $2<Q^2<9$ GeV^2 elastic & transition form factors.



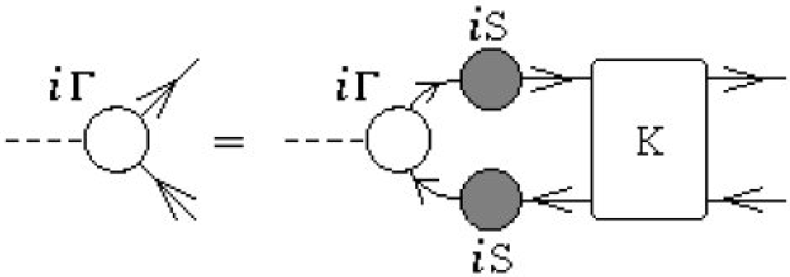
Gap Equation General Form

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p)$$

- ***$D_{\mu\nu}(k)$* – dressed-gluon propagator**
- ***$\Gamma_\nu(q,p)$* – dressed-quark-gluon vertex**
- **Suppose one has in hand – from anywhere – the exact form of the dressed-quark-gluon vertex**

What is the associated symmetry-preserving Bethe-Salpeter kernel?!



Bethe-Salpeter Equation
Bound-State DSE

$$[\Gamma_\pi^j(k;P)]_{tu} = \int_q^\Lambda [S(q+P/2)\Gamma_\pi^j(q;P)S(q-P/2)]_{sr}\, K_{tu}^{rs}(q,k;P)$$

- ***K(q,k;P) – fully amputated, two-particle irreducible, quark-antiquark scattering kernel***
- **Textbook material.**
- **Compact. Visually appealing. Correct**

Blocked progress for more than 60 years.

Lei Chang and C.D. Roberts
0903.5461 [nucl-th]
Phys. Rev. Lett. 103 (2009) 081601

Bethe-Salpeter Equation General Form

$$\Gamma^{fg}_{5\mu}(k;P) = Z_2 \gamma_5 \gamma_\mu$$

$$- \boxed{\int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\Gamma^{fg}_{5\mu}(q;P)S_g(q_-)\frac{\lambda^a}{2}\,\Gamma^g_\beta(q_-,k_-)}$$

$$+ \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

- **Equivalent exact bound-state equation *but* in this form**

  K(q,k;P) → Λ(q,k;P)

  which is *completely determined by dressed-quark self-energy*
- **Enables derivation of a Ward-Takahashi identity for Λ(q,k;P)**

Ward-Takahashi Identity Bethe-Salpeter Kernel

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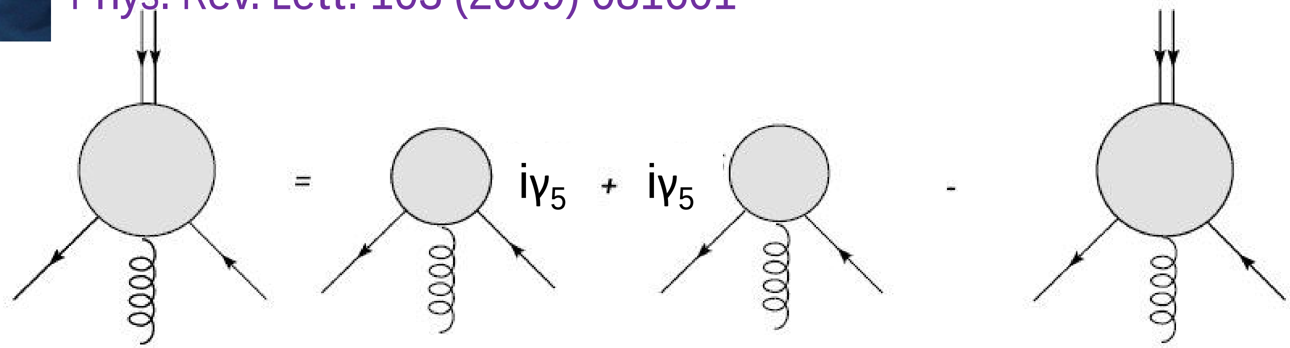
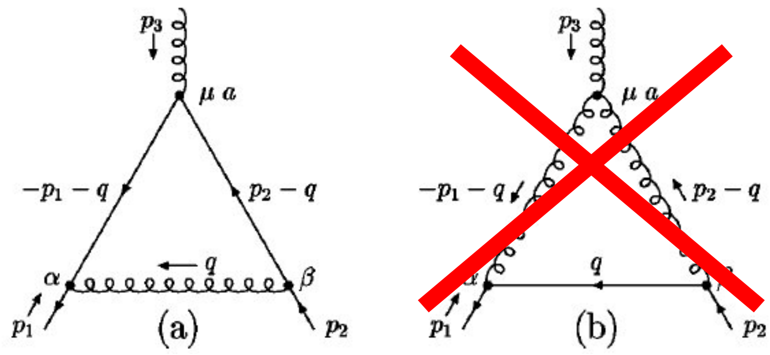


$$P_\mu \Lambda^{fg}_{5\mu\beta}(k,q;P) = \Gamma^f_\beta(q_+,k_+)\, i\gamma_5 + i\gamma_5\, \Gamma^g_\beta(q_-,k_-) - i[m_f(\zeta)+m_g(\zeta)]\Lambda^{fg}_{5\beta}(k,q;P),$$

- **Now, for first time, it's possible to formulate an *Ansatz* for Bethe-Salpeter kernel given *any form* for the dressed-quark-gluon vertex by using this identity**
- **This enables the identification and elucidation of a wide range of novel consequences of DCSB**



Dressed-quark anomalous magnetic moments

- **Schwinger's result for QED:** $\frac{q}{2m} \to \left(1+\frac{\alpha}{2\pi}\right)\frac{q}{2m}$
- **pQCD: two diagrams**
  - **(a) is QED-like**
  - **(b) is only possible in QCD – involves 3-gluon vertex**
- **Analyse (a) and (b)**
  - **(b) vanishes identically: the 3-gluon vertex does *not* contribute to a quark's anomalous chromomag. moment at leading-order**
  - **(a) Produces a finite result: " – ⅙ $\alpha_s/2\pi$ "**
    ~ (– ⅙) QED-result
- **But, in QED and QCD, the *anomalous chromo- and electro-magnetic moments vanish identically in the chiral limit*!**

Dressed-quark anomalous magnetic moments

$$\int d^4x \, \tfrac{1}{2} q \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$$

- **Interaction term that describes magnetic-moment coupling to gauge field**
  - **Straightforward to show that it mixes left ↔ right**
  - **Thus, explicitly violates chiral symmetry**
- **Follows that in fermion's e.m. current**

  $\gamma_\mu F_1$ does cannot mix with $\sigma_{\mu\nu} q_\nu F_2$

  No Gordon Identity
  - **Hence *massless fermions cannot not possess a measurable chromo- or electro-magnetic moment***
- **But what if the chiral symmetry is dynamically broken, strongly, as it is in QCD?**

Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1009.3458 [nucl-th]

Dressed-quark anomalous magnetic moments

- DCSB → **Three strongly-dressed and essentially-nonperturbative contributions to dressed-quark-gluon vertex:**

Ball-Chiu term
- Vanishes if no DCSB
- Appearance driven by STI

$$\lambda^3_\mu(p,q) = 2(p+q)_\mu \Delta_B(p,q)$$

$$\Delta_F(p,q) = \frac{F(p^2)-F(q^2)}{p^2-q^2}$$

Anom. chrom. mag. mom. contribution to vertex
- Similar properties to BC term
- Strength commensurate with lattice-QCD
  Skullerud, Bowman, Kizilersu *et al.*
  hep-ph/0303176

$$\Gamma^5_\mu(p,q) = \eta\, \sigma_{\mu\nu}(p-q)_\nu \Delta_B(p,q)$$

$$\Gamma^4_\mu(p,q) = [\ell^{\rm T}_\mu \gamma\cdot k + i\gamma^{\rm T}_\mu \sigma_{\nu\rho}\ell_\nu k_\rho]\tau_4(p,q)$$

$$\tau_4(p,q) = \mathcal{F}(z)\left[\frac{1-2\eta}{M_E}\Delta_B(p^2,q^2) - \Delta_A(p^2,q^2)\right]$$

$\mathcal{F}(z) = (1-\exp(-z))/z$, $z = (p_i^2 + p_f^2 - 2M_E^2)/\Lambda^2_{\mathcal{F}}$, $\Lambda_{\mathcal{F}} = 1\,\mathrm{GeV}$, Simplifies numerical analysis;

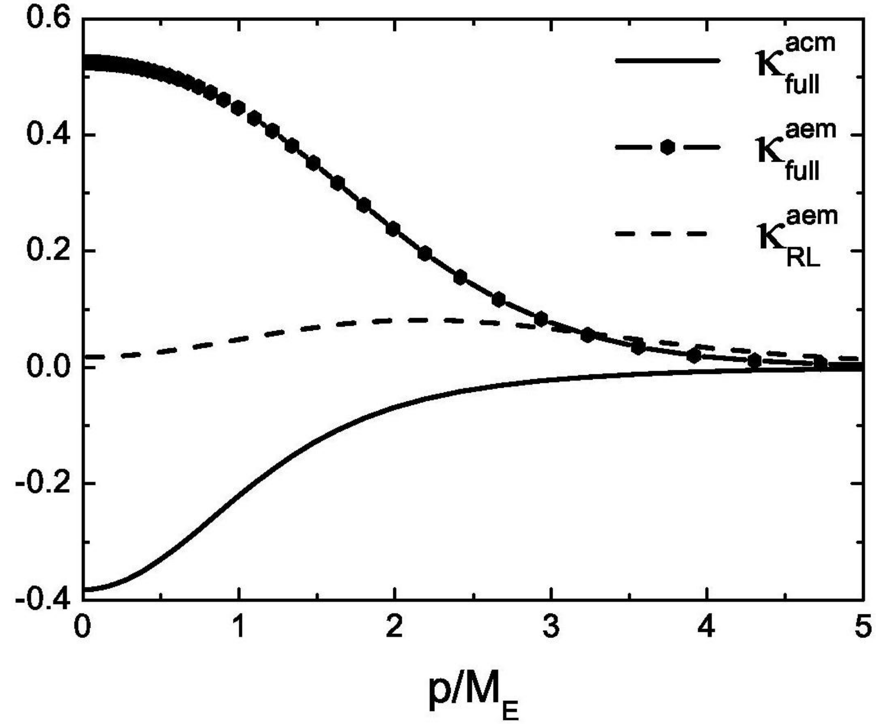
$M_E = \{s | s>0, s = M^2(s)\}$ is the Euclidean constituent-quark mass

Role and importance is Novel discovery
- Essential to recover pQCD
- Constructive interference with Γ^5

Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1009.3458 [nucl-th]

Dressed-quark anomalous magnetic moments

- Formulated and solved general Bethe-Salpeter equation
- Obtained dressed electromagnetic vertex
- Confined quarks don't have a mass-shell
  - o Can't unambiguously define magnetic moments
  - o But can define *magnetic moment distribution*



- AEM is opposite in sign but of roughly equal magnitude as ACM
  - o Potentially important for elastic & transition form factors, etc.
  - o Muon *g*-2 ?

	M^E	κ^{ACM}	κ^{AEM}
Full vertex	0.44	-0.22	0.45
Rainbow-ladder	0.35	0	0.048

R.T. Cahill *et al*.,
Austral. J. Phys. 42 (1989) 129-145

DSEs and Baryons

- *$M(p^2)$* – effects have enormous impact on meson properties.
  - *Must be included in description and prediction of baryon properties*.
- *$M(p^2)$* is essentially a quantum field theoretical effect. In quantum field theory
  - Meson appears as pole in four-point quark-antiquark Green function → Bethe-Salpeter Equation
  - *Nucleon appears as a pole in a six-point quark Green function → Faddeev Equation*.
- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates *nonpointlike* quark-quark (diquark) correlations in the colour-antitriplet channel

$r_{qq} \approx r_{\pi}$

Faddeev Equation

R.T. Cahill *et al.*,
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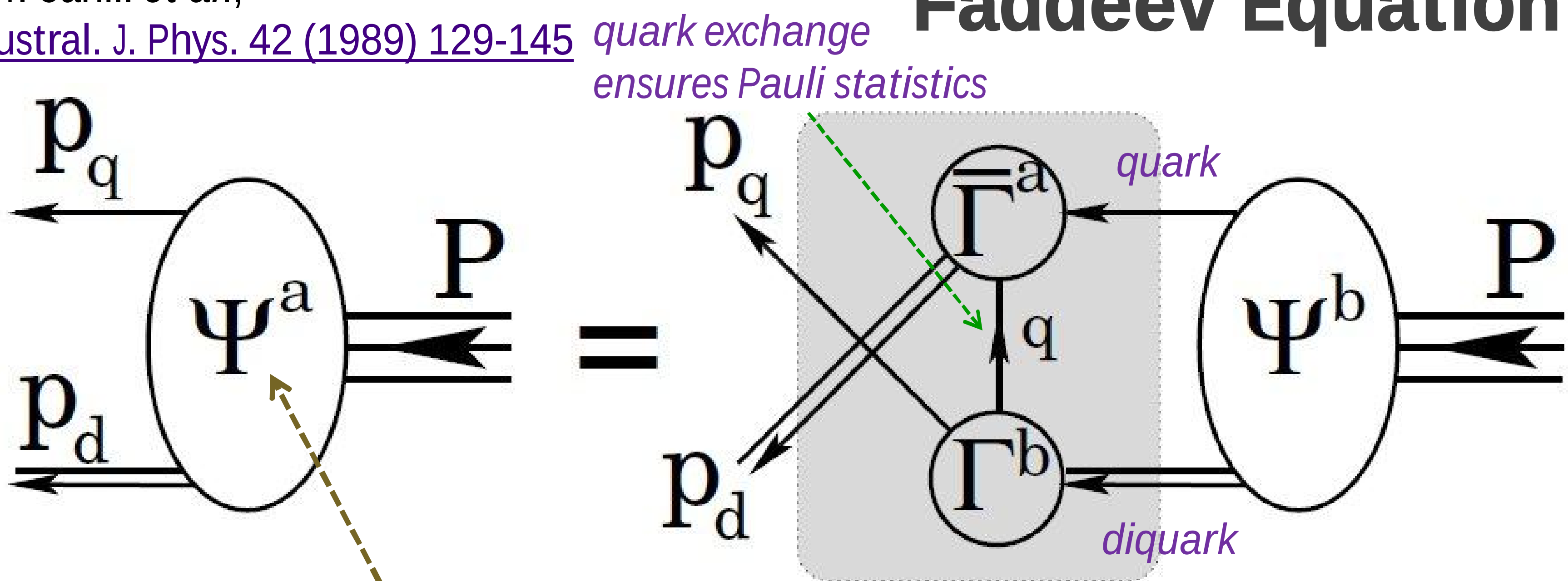


- Linear, Homogeneous Matrix equation
  - Yields *wave function (Poincaré Covariant Faddeev* Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . .
  - Both have "*correct*" parity and "*right*" masses
  - In Nucleon's Rest Frame Amplitude has

    s−, p− & d−wave correlations

H.L.L. Roberts, L. Chang and C.D. Roberts
arXiv:1007.4318 [nucl-th]
H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts
arXiv:1007.3566 [nucl-th]

Spectrum of some known *u*- & *d*-quark baryons

- Baryons: ground-states and 1st radial exciations

1st radial Excitation of N(1535)?

	m_N	m_{N*}	$m_N(½^-)$	$m_{N*}(½^-)$	m_Δ	$m_{\Delta*}$	$m_\Delta(3/2^-)$	$m_{\Delta*}(3/2^-)$
DSE	1.05	1.73	1.86	2.09	1.33	1.85	1.98	2.16
EBAC		1.76	1.80		1.39		1.98	

- *mean-|relative-error| = 2%-Agreement*

DSE dressed-quark-core masses cf. Excited Baryon Analysis Center bare masses is significant because no attempt was made to ensure this.

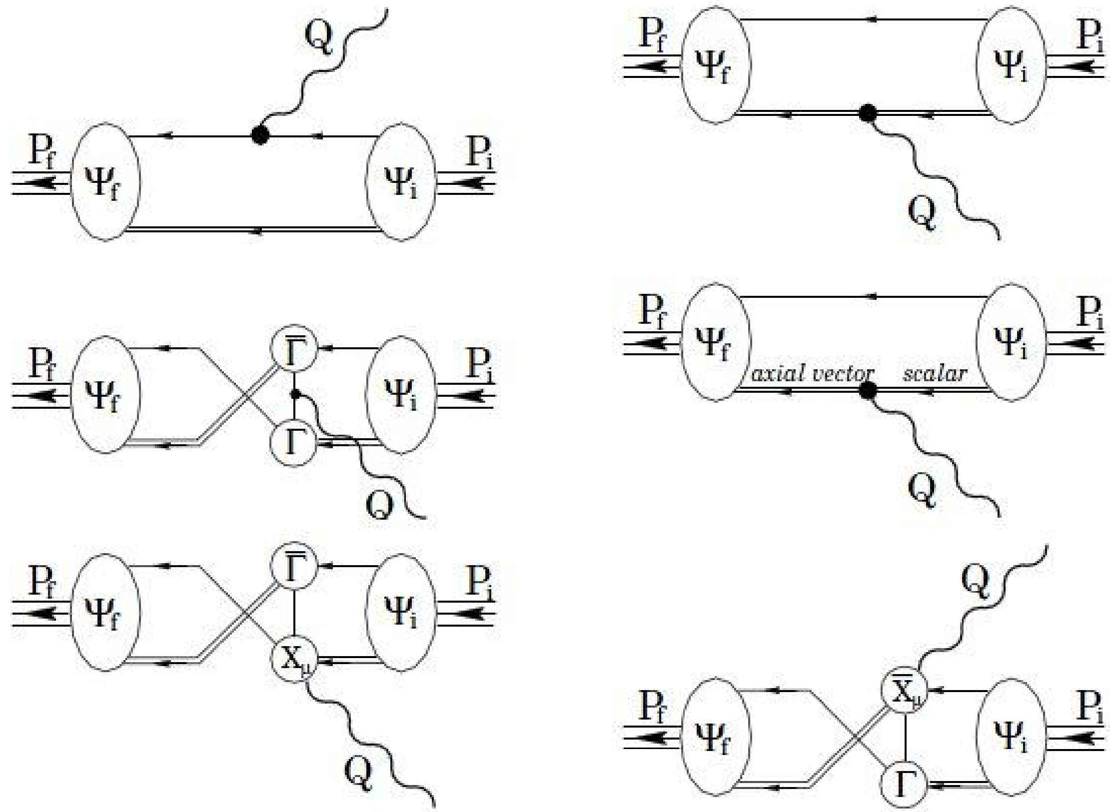
I.C. Cloët, C.D. Roberts, *et al*.
arXiv:0812.0416 [nucl-th]

Nucleon Elastic Form Factors

- Photon-baryon vertex

  Oettel, Pichowsky and von Smekal, nucl-th/9909082

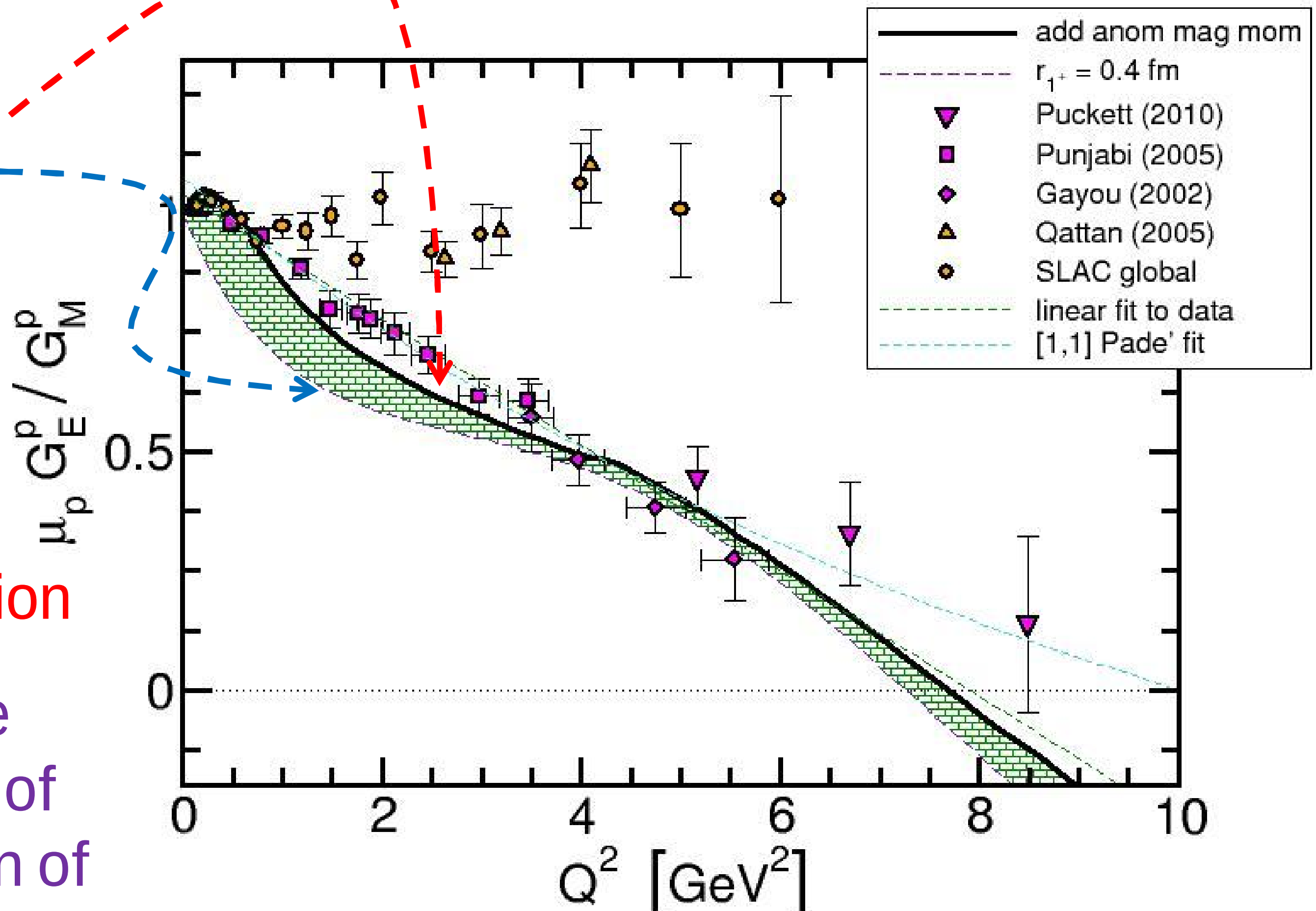
- "*Survey of nucleon electromagnetic form factors*"
  – unification of meson and baryon observables; and prediction of nucleon elastic form factors to 15 GeV^2



I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th]
I.C. Cloët, C.D. Roberts, *et al.*
In progress

$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

- ➢DSE result Dec 08
- ➢DSE result
  – including the anomalous magnetic moment distribution
- ➢Highlights again the critical importance of DCSB in explanation of real-world observables.



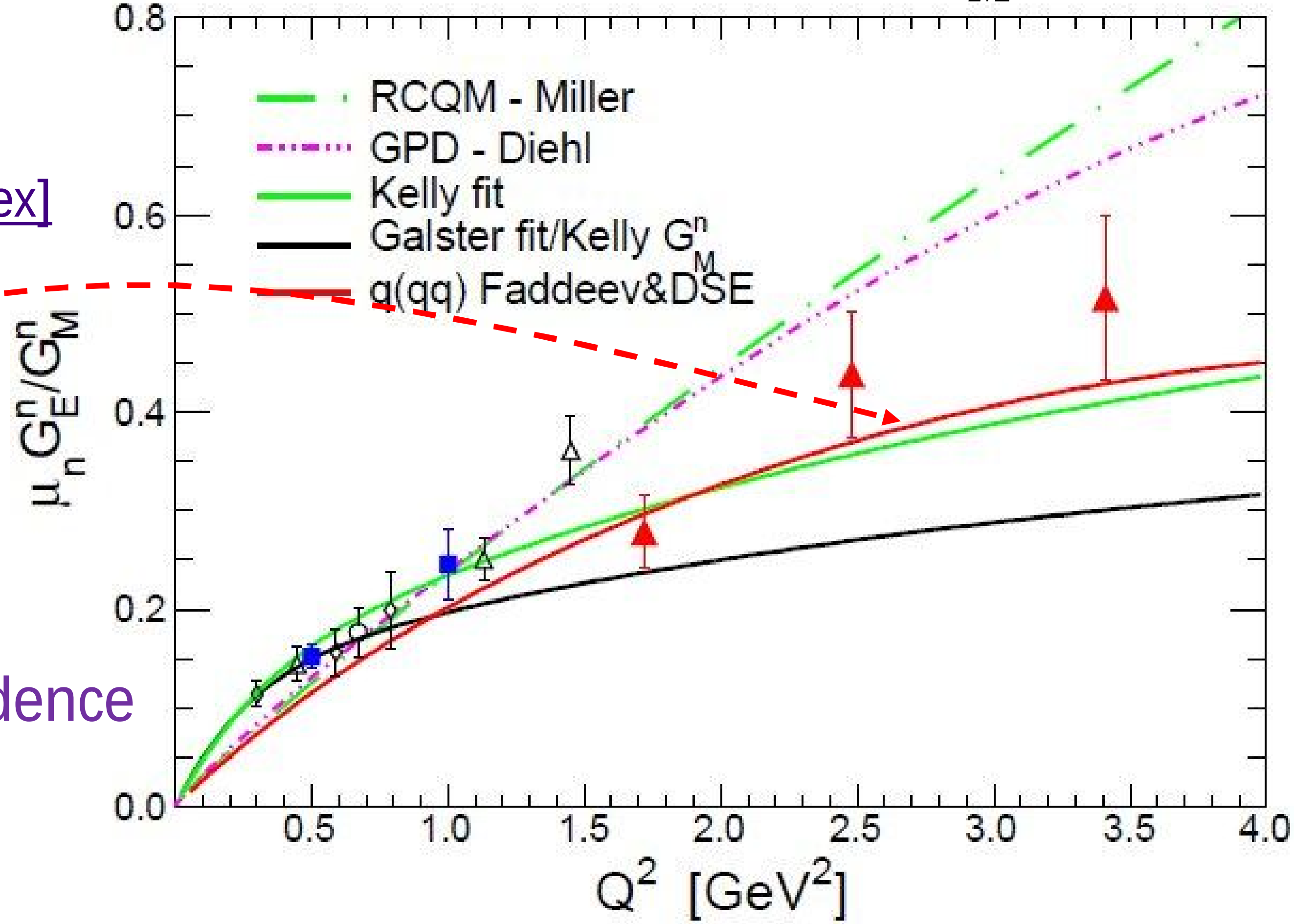
I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th]

$$\frac{\mu_n G_E^n(Q^2)}{G_M^n(Q^2)}$$

- New JLab data:
  S. Riordan *et al.*, ▲
  arXiv:1008.1738 [nucl-ex]

- DSE-prediction

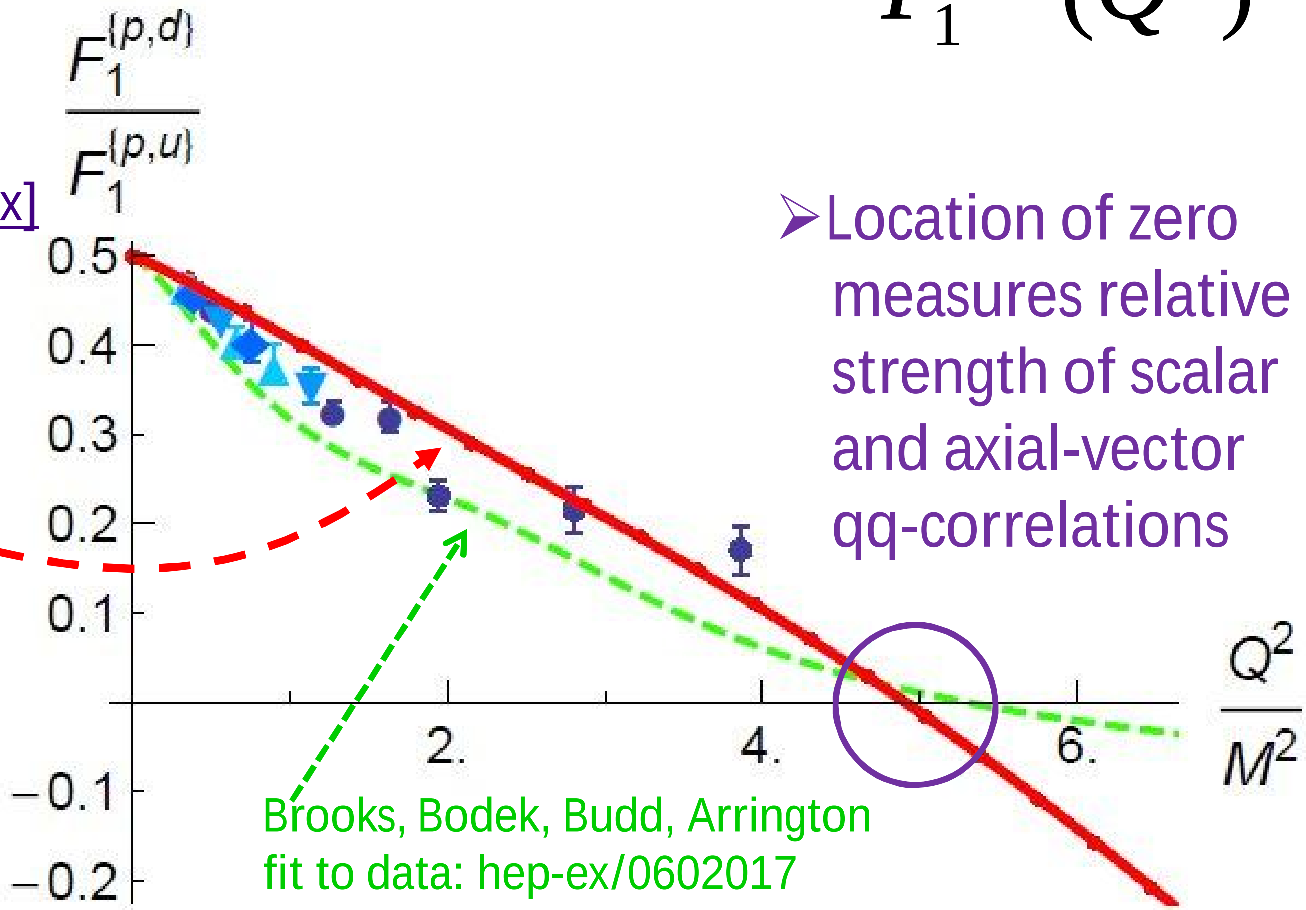
- This evolution is very sensitive to momentum-dependence of dressed-quark propagator



I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th]

$$\frac{F_1^{p,d}(Q^2)}{F_1^{p,u}(Q^2)}$$

- New JLab data: S. Riordan *et al.*, arXiv:1008.1738 [nucl-ex]
- DSE-prediction



- Location of zero measures relative strength of scalar and axial-vector qq-correlations

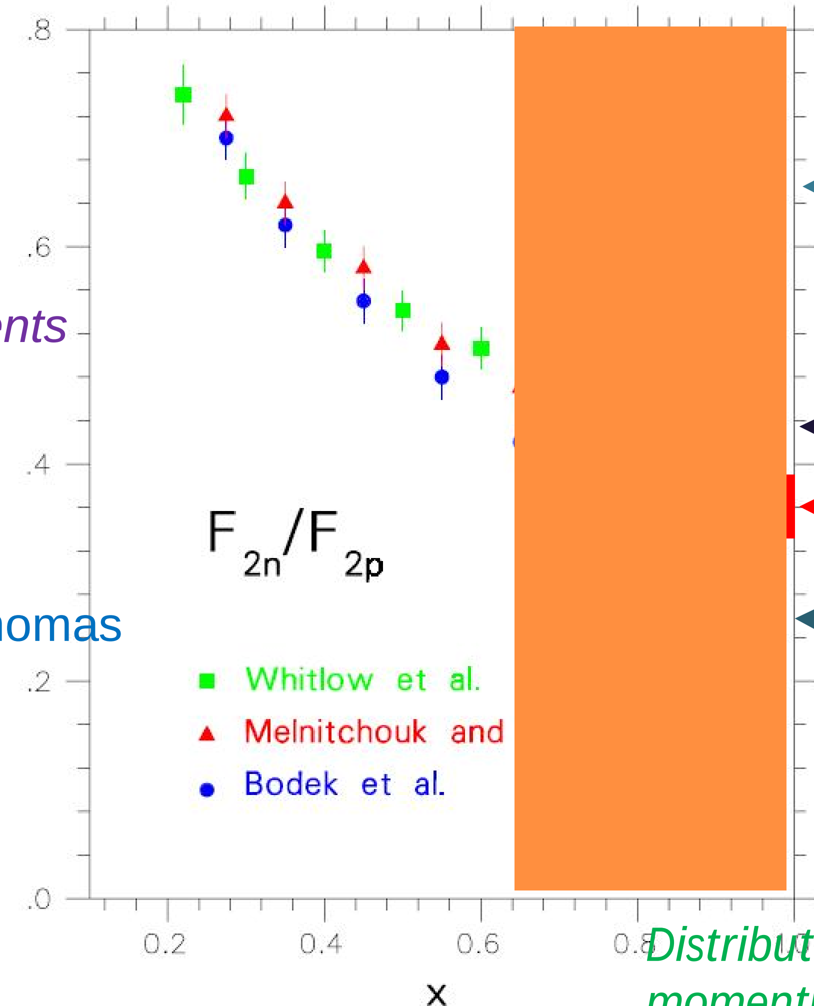
I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th]

Neutron Structure Function at high *x*

Deep inelastic scattering – the Nobel-prize winning quark-discovery experiments

Reviews:

- S. Brodsky *et al.* NP B441 (1995)
- W. Melnitchouk & A.W.Thomas PL B377 (1996) 11
- R.J. Holt & C.D. Roberts RMP (2010) 2991



SU(6) symmetry

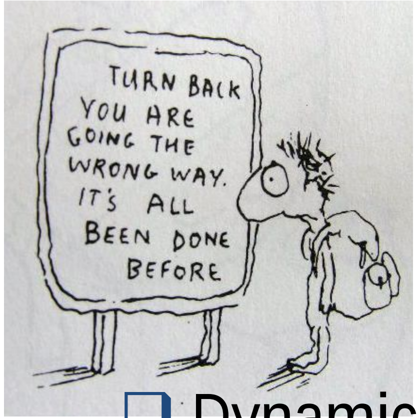
pQCD

DSE: 0^+ & 1^+ qq

0^+ qq only

Related measure of $1^+/0^+$ strength

Distribution of neutron's momentum amongst quarks on the valence-quark domain



Epilogue

- Dynamical chiral symmetry breaking (DCSB) – *mass from nothing for 98% of visible matter* – is a reality
  - Expressed in $M(p^2)$, with observable signals in experiment

Confinement is almost Certainly the origin of DCSB

- Poincaré covariance

  Crucial in description of contemporary data

- Fully-self-consistent treatment of an interaction

  Essential if experimental data is *truly* to be *understood*.

- Dyson-Schwinger equations:
  - single framework, with IR model-input turned to advantage, *"almost unique in providing unambiguous path from a defined interaction → Confinement & DCSB → Masses → radii → form factors → distribution functions → etc."*

McLerran & Pisarski
arXiv:0706.2191 [hep-ph]