T(r)opical Dyson Schwinger Equations

Craig D. Roberts

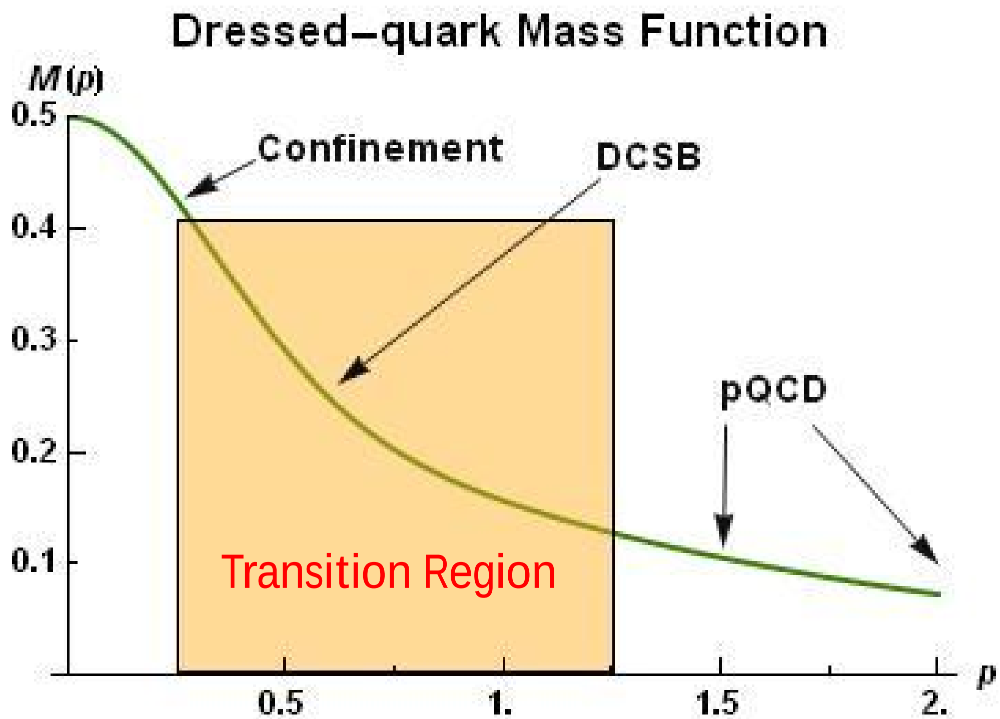
Physics Division
Argonne National Laboratory

&

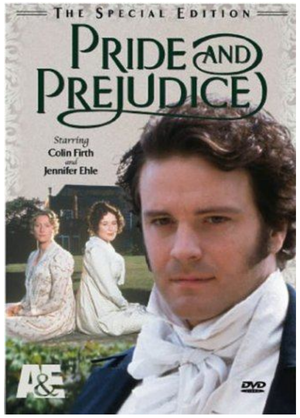
School of Physics
Peking University

&

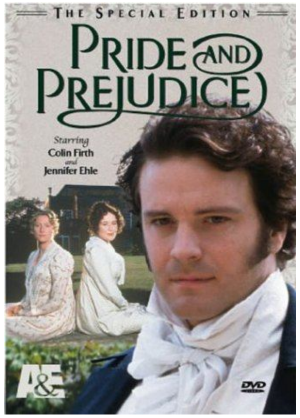
Department of Physics
ILLINOIS INSTITUTE OF TECHNOLOGY



Universal Truths

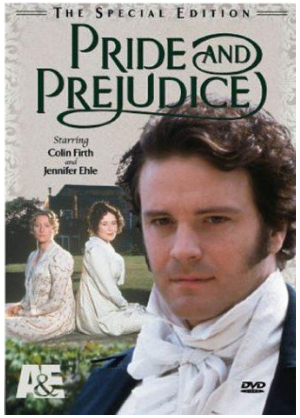


- **Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.**



Universal Truths

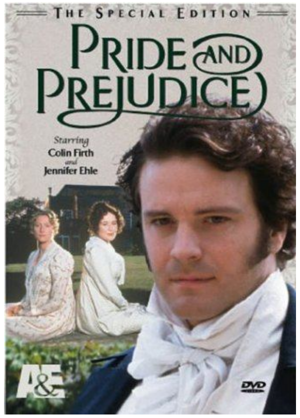
- Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.



Universal Truths

- Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.

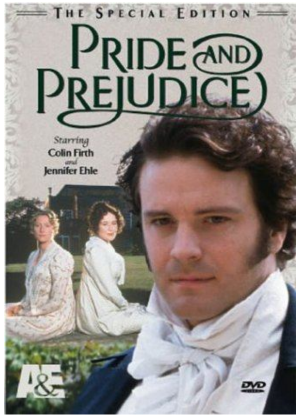
  Higgs mechanism is irrelevant to light-quarks.



Universal Truths

- Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.

  Higgs mechanism is irrelevant to light-quarks.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.



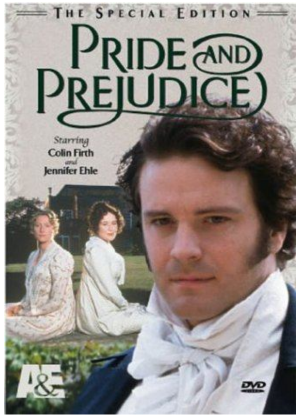
Universal Truths

- Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.

  Higgs mechanism is irrelevant to light-quarks.

- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.

  Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.

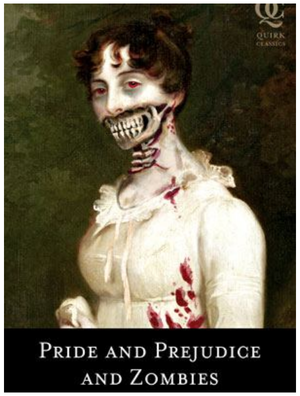


Universal Truths

- Spectrum of hadrons (ground, excited and exotic states), and hadron elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.

  Higgs mechanism is irrelevant to light-quarks.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.

  Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.
- Confinement is expressed through a violation of reflection positivity; and can almost be read-off from a plot of a states' dressed-propagator. It is intimately connected with DCSB.



Universal Truths

- **Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.**

In-Hadron Condensates

- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.
- One problem: DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not yet been realised in the light-front formulation.

In-Hadron Condensates

- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.
- One problem: DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not yet been realised in the light-front formulation.
- Resolution
  - Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, “condensates” do not exist as spacetime-independent mass-scales that fill all spacetime.
  - *So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.

In-Hadron Condensates

Brodsky, Roberts, Shrock, Tandy, Phys. Rev. C**82** (Rapid Comm.) (2010) 022201

- **Resolution**
  - **Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, "condensates" do not exist as spacetime-independent mass-scales that fill all spacetime.**
  - ***So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.**
  - **No qualitative difference between f_π and ρ_π**

$$\begin{aligned} i f_\pi P_\mu &= \langle 0|\bar{q}\gamma_5\gamma_\mu q|\pi\rangle \\ &= Z_2(\zeta,\Lambda)\,\mathrm{tr}_{\mathrm{CD}}\int^\Lambda \frac{d^4q}{(2\pi)^4} i\gamma_5\gamma_\mu S(q_+)\Gamma_\pi(q;P)S(q_-), \end{aligned} \tag{5}$$

$$\begin{aligned} i\rho_\pi &= -\langle 0|\bar{q} i\gamma_5 q|\pi\rangle \\ &= Z_4(\zeta,\Lambda)\,\mathrm{tr}_{\mathrm{CD}}\int^\Lambda \frac{d^4q}{(2\pi)^4}\gamma_5 S(q_+)\Gamma_\pi(q;P)S(q_-). \end{aligned} \tag{6}$$

In-Hadron Condensates

Brodsky, Roberts, Shrock, Tandy, Phys. Rev. C**82** (Rapid Comm.) (2010) 022201

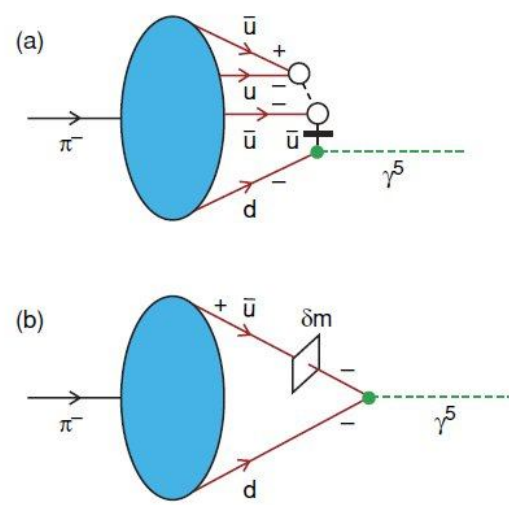
- **Resolution**
  - **Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, "condensates" do not exist as spacetime-independent mass-scales that fill all spacetime.**
  - ***So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example,**
    in their Bethe-Salpeter or
    light-front wavefunctions.
  - **No qualitative difference**
    between f_π and ρ_π
  - **And**

$$-\langle \bar{q}q\rangle_\zeta^\pi \equiv -f_\pi \langle 0|\bar{q}\gamma_5 q|\pi\rangle = f_\pi \rho_\pi(\zeta) =: \kappa_\pi(\hat{m};\zeta).$$

In-Hadron Condensates

Brodsky, Roberts, Shrock, Tandy, Phys. Rev. C**82** (Rapid Comm.) (2010) 022201

- **Resolution**
  - **Whereas it might sometimes be convenient in computational truncation schemes to imagine otherwise, “condensates” do not exist as spacetime-independent mass-scales that fill all spacetime.**
  - ***So-called* vacuum condensates can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wavefunctions.**
  - ***Conjecture*: Light-Front DCSB obtained via coherent contribution from countable infinity of higher Fock-state components in LF-wavefunction**



In-Hadron Condensates

"Void that is truly empty
solves dark energy puzzle"
Rachel Courtland, New Scientist 1st Sept. 2010

"*EMPTY space may really be empty. Though quantum theory suggests that a vacuum should be fizzing with particle activity, it turns out that this paradoxical picture of nothingness may not be needed. A calmer view of the vacuum would also help resolve a nagging inconsistency with dark energy, the elusive force thought to be speeding up the expansion of the universe.*"

- **Cosmological Constant:**
  - **Putting QCD condensates back into hadrons reduces the mismatch between experiment and theory by a factor of 10^{45}**
  - **Possibly by far more, if technicolour-like theories are the correct paradigm for extending the Standard Model**

Charting the interaction between light-quarks

- **Confinement can be related to the analytic properties of QCD's Schwinger functions.**

Charting the interaction between light-quarks

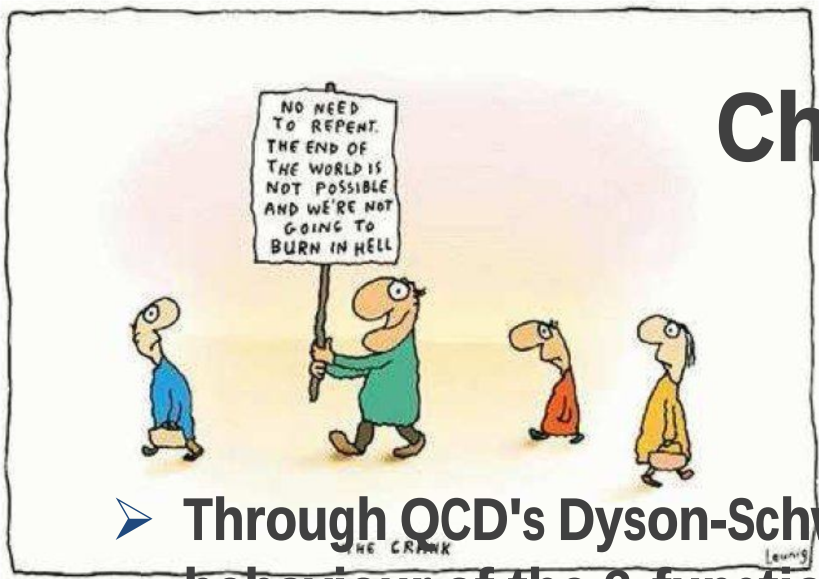
- Confinement can be related to the analytic properties of QCD's Schwinger functions.
- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's **universal** β-function
  - This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.

Of course, the behaviour of the β-function on the perturbative domain is well known.

Charting the interaction between light-quarks

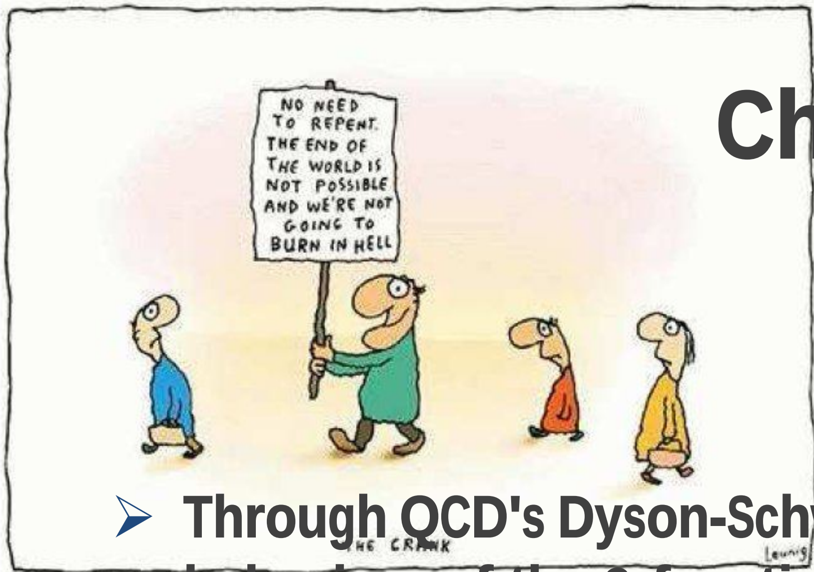
- **Confinement can be related to the analytic properties of QCD's Schwinger functions.**
- **Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's universal β-function**
  - **This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.**

  Of course, the behaviour of the β-function on the perturbative domain is well known.
- **This is a well-posed problem whose solution is an elemental goal of modern hadron physics.**



Charting the interaction between light-quarks

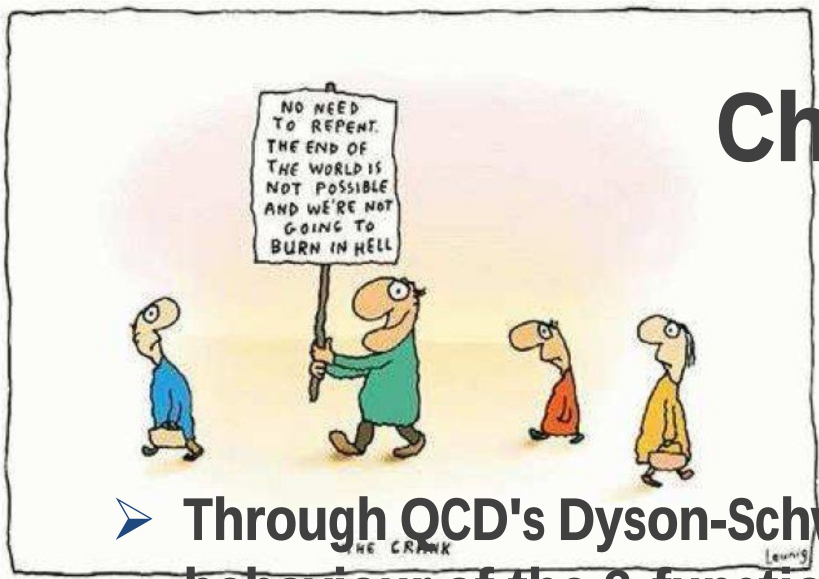
- **Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking.**



Charting the interaction between light-quarks

- **Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking.**
- **DSEs connect β-function to experimental observables. Hence, comparison between computations and observations of**
  - **Hadron mass spectrum**
  - **Elastic and transition form factors**

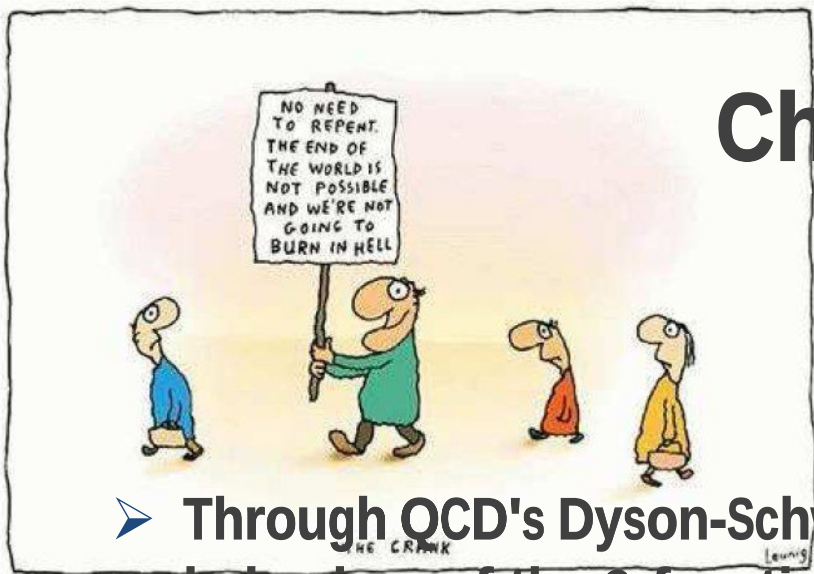
  can be used to chart β-function's long-range behaviour.



Charting the interaction between light-quarks

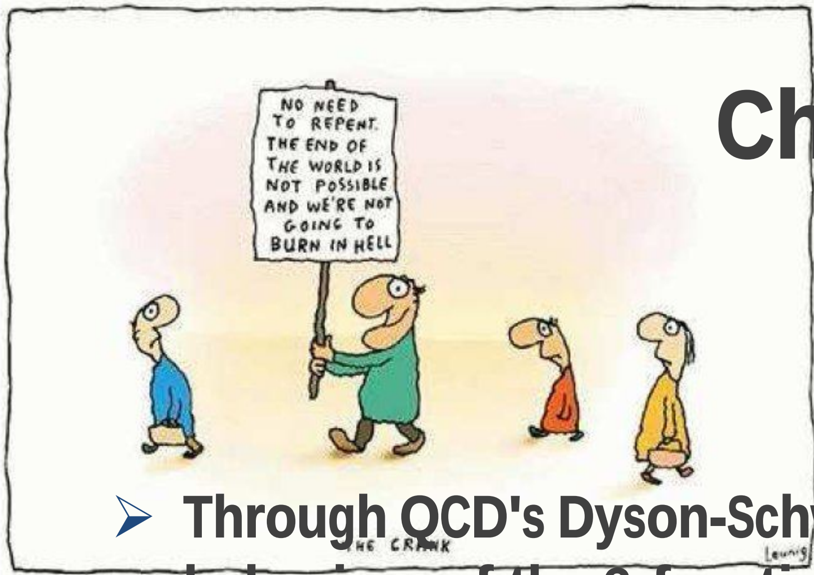
- Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking.
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations of
  - Hadron mass spectrum
  - Elastic and transition form factors

  can be used to chart β-function's long-range behaviour.
- Extant studies of mesons show that the properties of hadron excited states are a great deal more sensitive to the long-range behaviour of the β-function than those of the ground states.



Charting the interaction between light-quarks

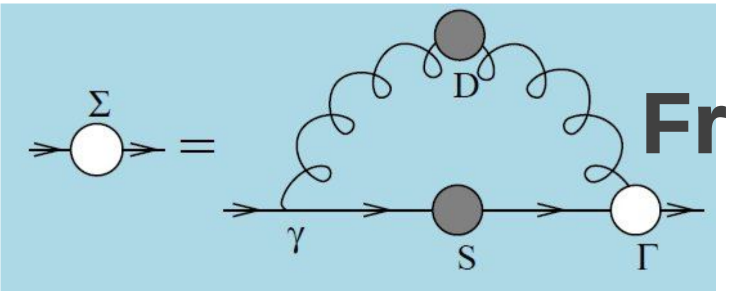
- **Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking.**
- **DSEs connect β-function to experimental observables. Hence, comparison between computations and observations can be used to chart β-function's long-range behaviour.**
- **To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary:**
  - **Steady quantitative progress is being made with a scheme that is systematically improvable (Bender, Roberts, von Smekal – nucl-th/9602012)**



Charting the interaction between light-quarks

- **Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines pattern of chiral symmetry breaking.**
- **DSEs connect β-function to experimental observables. Hence, comparison between computations and observations can be used to chart β-function's long-range behaviour.**
- **To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary:**
  - **On the other hand, at significant qualitative advances are possible with symmetry-preserving kernel Ansätze that express important additional nonperturbative effects – M(p^2) – *difficult/impossible to capture in any finite sum of contributions*.**

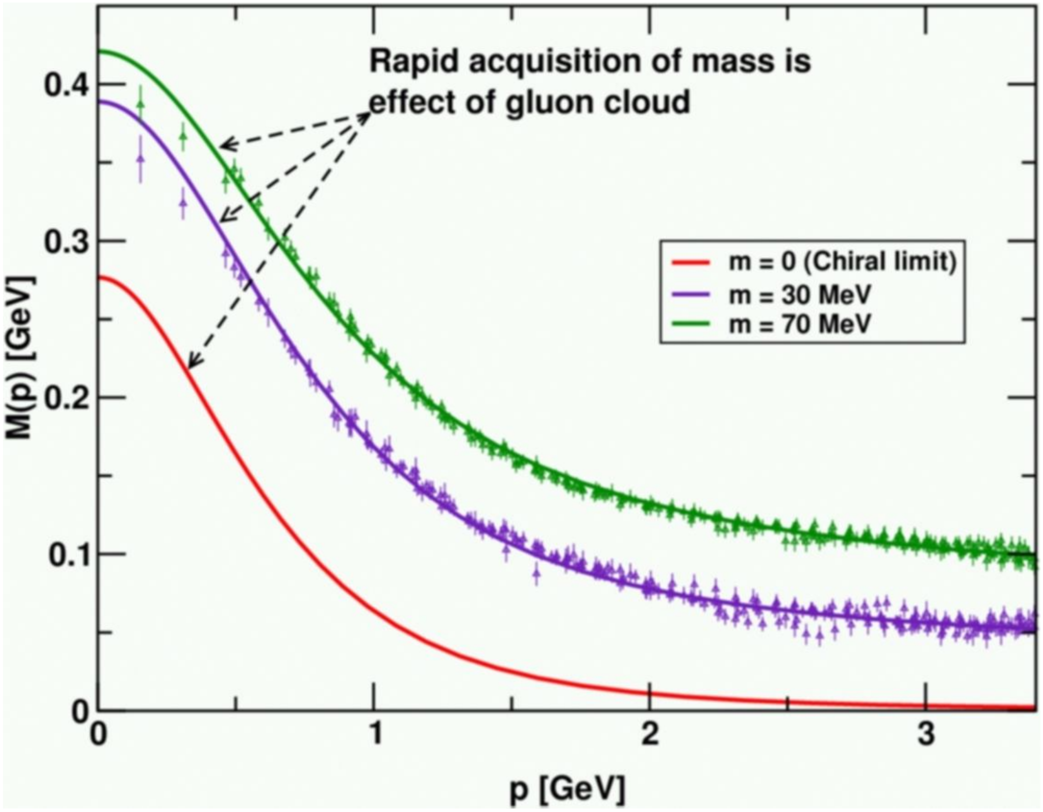
Can't walk beyond the rainbow, but must leap!

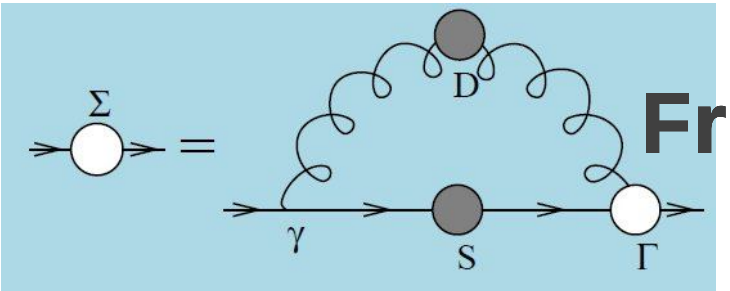


Frontiers of Nuclear Science: Theoretical Advances

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.

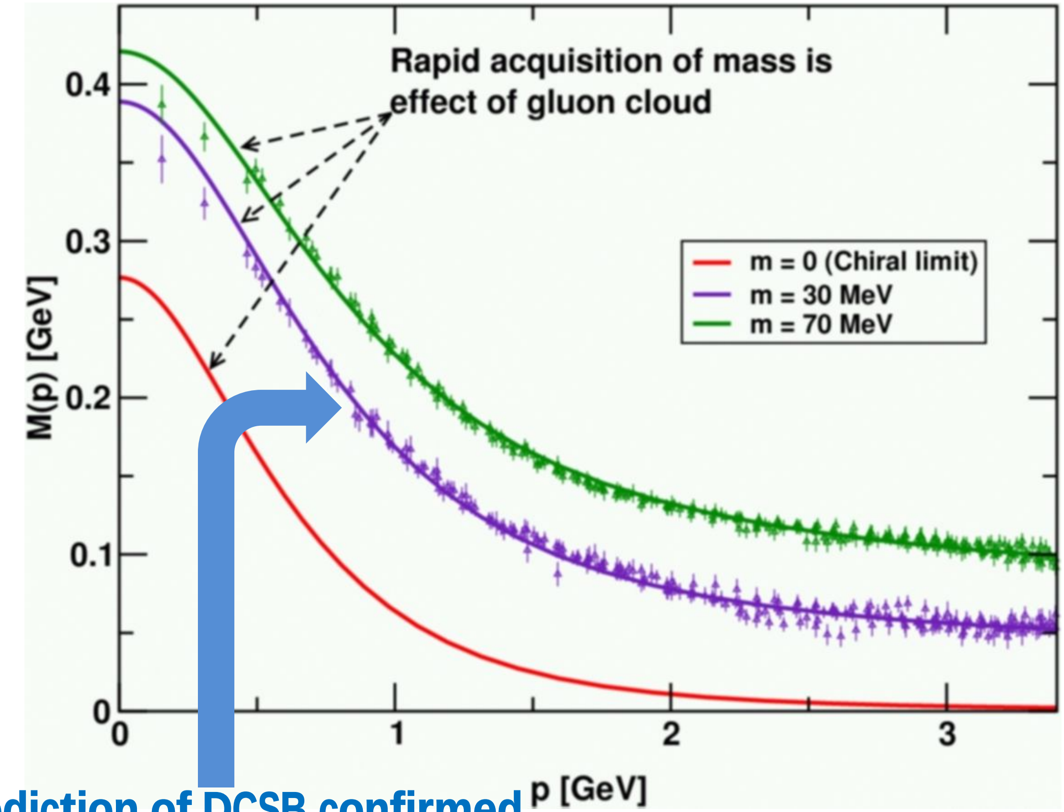




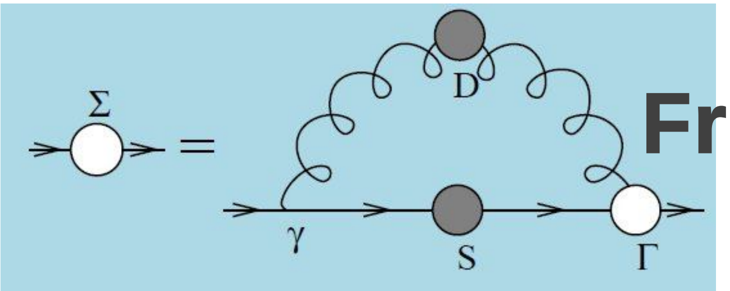
Frontiers of Nuclear Science: Theoretical Advances

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.



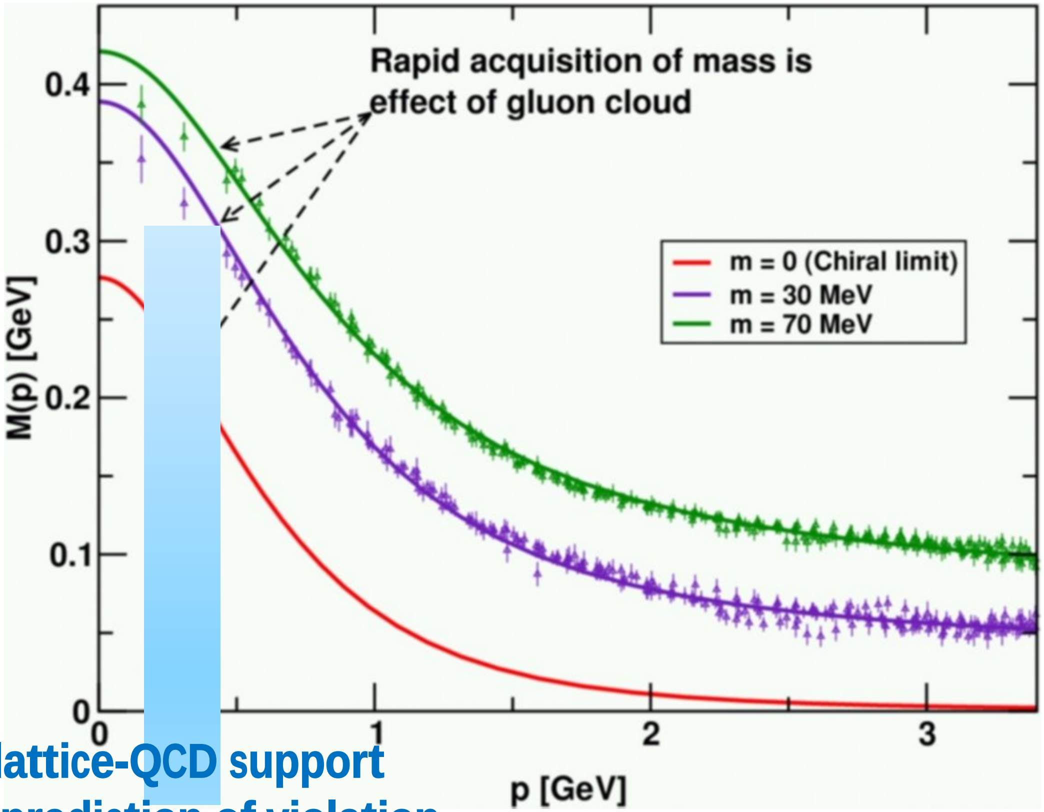
DSE prediction of DCSB confirmed



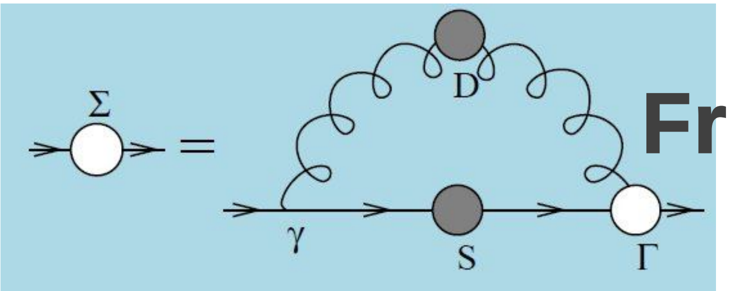
Frontiers of Nuclear Science: Theoretical Advances

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.



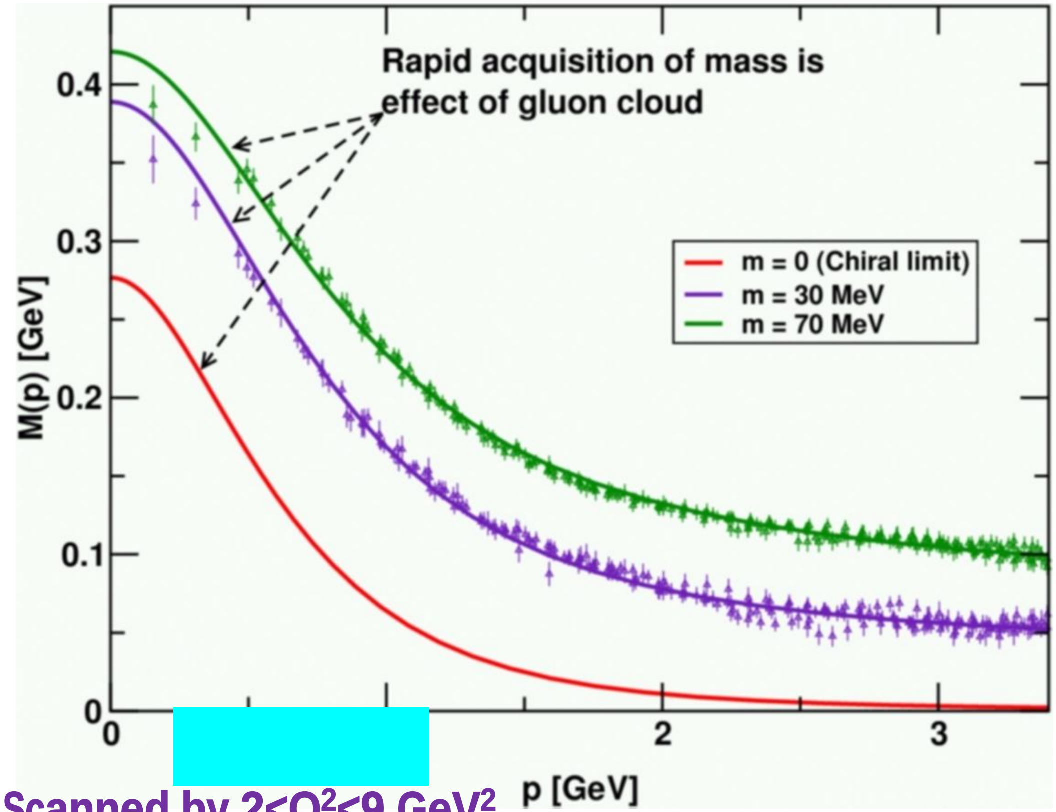
Hint of lattice-QCD support for DSE prediction of violation of reflection positivity



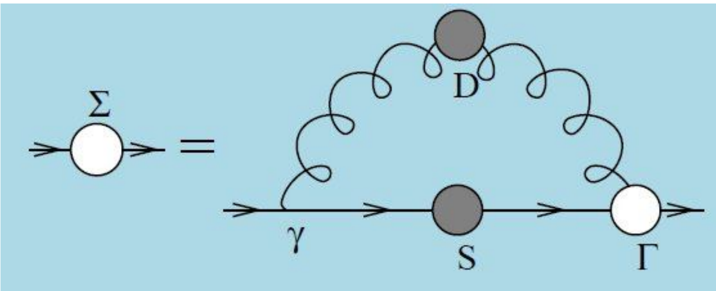
Frontiers of Nuclear Science: Theoretical Advances

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.



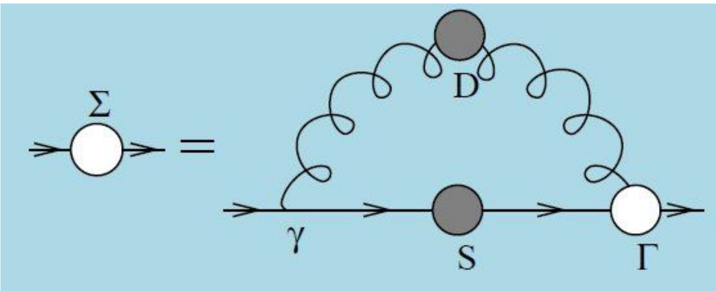
Jlab 12GeV: Scanned by $2<Q^2<9$ GeV^2 elastic & transition form factors.



Gap Equation General Form

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p)$$

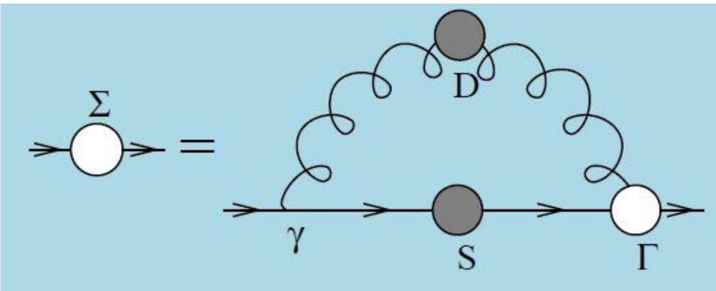


Gap Equation General Form

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p)$$

- ***$D_{\mu\nu}(k)$ – dressed-gluon propagator***
- ***$\Gamma_\nu(q,p)$ – dressed-quark-gluon vertex***



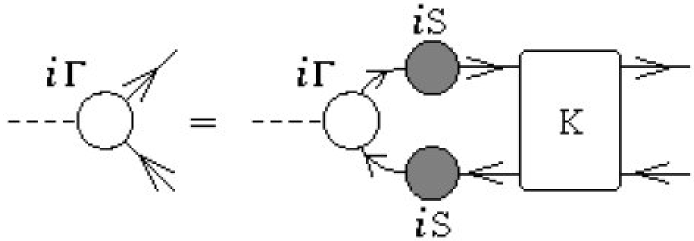
Gap Equation General Form

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p)$$

- ***$D_{\mu\nu}(k)$ – dressed-gluon propagator***
- ***$\Gamma_\nu(q,p)$ – dressed-quark-gluon vertex***
- **Suppose one has in hand – from anywhere – the exact form of the dressed-quark-gluon vertex**

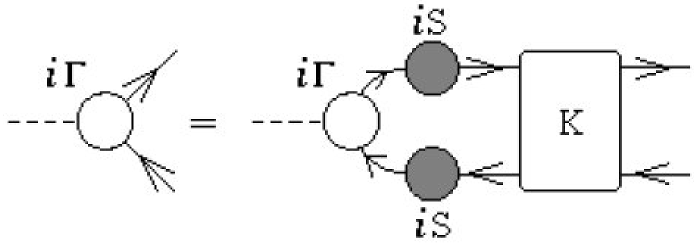
What is the associated symmetry-preserving Bethe-Salpeter kernel?!



Bethe-Salpeter Equation
Bound-State DSE

$$\left[\Gamma_\pi^j(k;P)\right]_{tu} = \int_q^\Lambda [S(q+P/2)\Gamma_\pi^j(q;P)S(q-P/2)]_{sr}\, K_{tu}^{rs}(q,k;P)$$

- ***K(q,k;P) – fully amputated, two-particle irreducible, quark-antiquark scattering kernel***
- **Textbook material.**
- **Compact. Visually appealing. Correct**



Bethe-Salpeter Equation
Bound-State DSE

$$[\Gamma_\pi^j(k;P)]_{tu} = \int_q^\Lambda [S(q+P/2)\Gamma_\pi^j(q;P)S(q-P/2)]_{sr}\, K_{tu}^{rs}(q,k;P)$$

- ***K(q,k;P) – fully amputated, two-particle irreducible, quark-antiquark scattering kernel***
- **Textbook material.**
- **Compact. Visually appealing. Correct**

Blocked progress for more than 60 years.

Bethe-Salpeter Equation
General Form

Lei Chang and C.D. Roberts
0903.5461 [nucl-th]
Phys. Rev. Lett. 103 (2009) 081601

$$\Gamma^{fg}_{5\mu}(k;P) = Z_2\gamma_5\gamma_\mu$$
$$- \boxed{\int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\gamma_\alpha S_f(q_+)\Gamma^{fg}_{5\mu}(q;P)S_g(q_-)\frac{\lambda^a}{2}\,\Gamma^g_\beta(q_-,k_-)}$$
$$+ \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

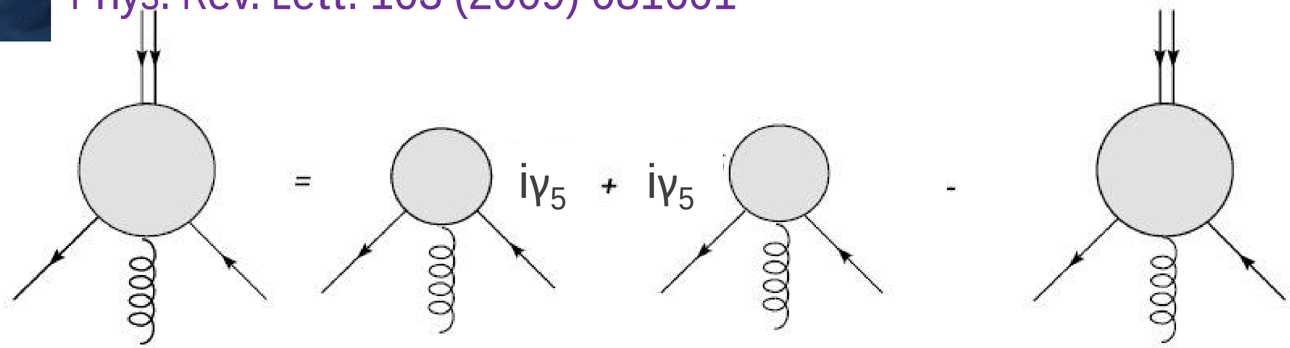
- **Equivalent exact bound-state equation *but* in this form**

  K(q,k;P) → Λ(q,k;P)

  which is *completely determined by dressed-quark self-energy*
- **Enables derivation of a Ward-Takahashi identity for Λ(q,k;P)**

Ward-Takahashi Identity Bethe-Salpeter Kernel

Lei Chang and C.D. Roberts
0903.5461 [nucl-th]
Phys. Rev. Lett. 103 (2009) 081601

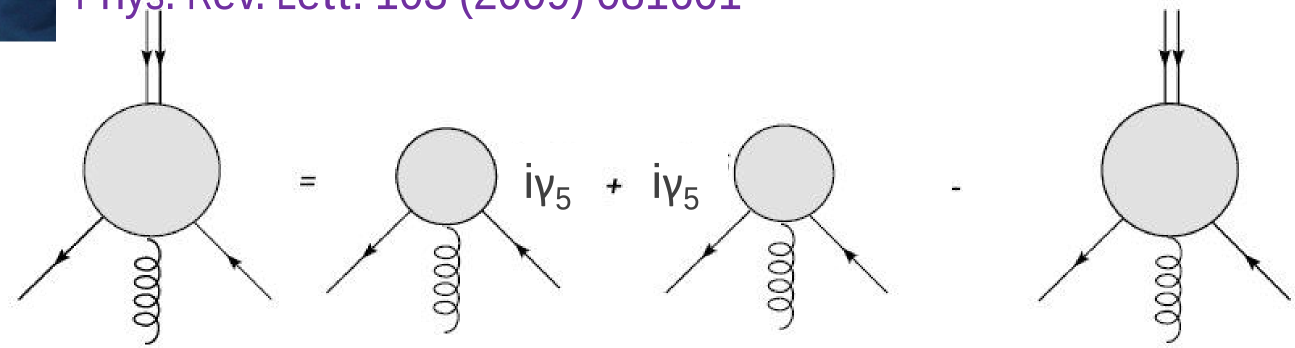


$$P_\mu \Lambda^{fg}_{5\mu\beta}(k,q;P) = \Gamma^f_\beta(q_+,k_+)\, i\gamma_5 + i\gamma_5\, \Gamma^g_\beta(q_-,k_-) - i[m_f(\zeta)+m_g(\zeta)]\Lambda^{fg}_{5\beta}(k,q;P),$$

- **Now, for first time, it's possible to formulate an *Ansatz* for Bethe-Salpeter kernel given *any form* for the dressed-quark-gluon vertex by using this identity**

Ward-Takahashi Identity Bethe-Salpeter Kernel

Lei Chang and C.D. Roberts
0903.5461 [nucl-th]
Phys. Rev. Lett. 103 (2009) 081601

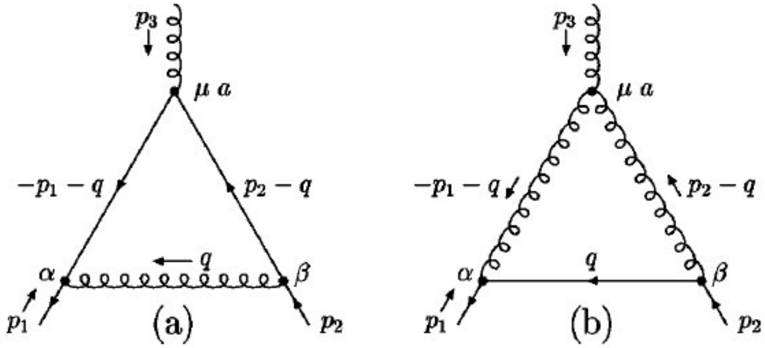


$$\begin{aligned} P_\mu \Lambda^{fg}_{5\mu\beta}(k,q;P) &= \Gamma^f_\beta(q_+,k_+)\, i\gamma_5 + i\gamma_5\, \Gamma^g_\beta(q_-,k_-) \\ &\quad - i[m_f(\zeta)+m_g(\zeta)]\Lambda^{fg}_{5\beta}(k,q;P), \end{aligned}$$

- **Now, for first time, it's possible to formulate an *Ansatz* for Bethe-Salpeter kernel given *any form* for the dressed-quark-gluon vertex by using this identity**
- **This enables the identification and elucidation of a wide range of novel consequences of DCSB**

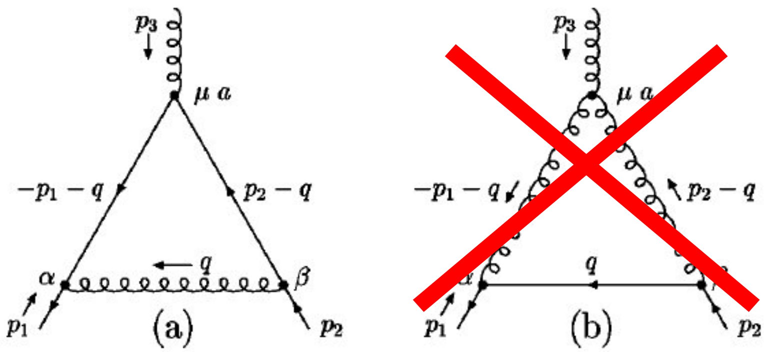
Dressed-quark anomalous magnetic moments

- **Schwinger's result for QED:** $\frac{q}{2m} \to \left(1 + \frac{\alpha}{2\pi}\right)\frac{q}{2m}$



Dressed-quark anomalous magnetic moments

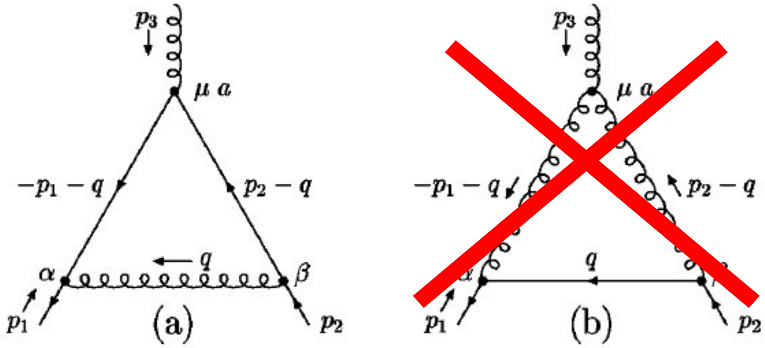
- **Schwinger's result for QED:** $\frac{q}{2m} \to \left(1+\frac{\alpha}{2\pi}\right)\frac{q}{2m}$
- **pQCD: two diagrams**
  - **(a) is QED-like**
  - **(b) is only possible in QCD – involves 3-gluon vertex**



Dressed-quark anomalous magnetic moments

- Schwinger's result for QED: $\frac{q}{2m} \to \left(1+\frac{\alpha}{2\pi}\right)\frac{q}{2m}$
- pQCD: two diagrams
  - (a) is QED-like
  - (b) is only possible in QCD – involves 3-gluon vertex
- Analyse (a) and (b)
  - (b) vanishes identically: the 3-gluon vertex does *not* contribute to a quark's anomalous chromomag. moment at leading-order
  - (a) Produces a finite result: " – ⅙ $\alpha_s/2\pi$ "

    ~ (– ⅙) QED-result



Dressed-quark anomalous magnetic moments

- Schwinger's result for QED: $\frac{q}{2m} \to \left(1+\frac{\alpha}{2\pi}\right)\frac{q}{2m}$
- pQCD: two diagrams
  - (a) is QED-like
  - (b) is only possible in QCD – involves 3-gluon vertex
- Analyse (a) and (b)
  - (b) vanishes identically: the 3-gluon vertex does ***not*** contribute to a quark's anomalous chromomag. moment at leading-order
  - (a) Produces a finite result: " – ⅙ $\alpha_s/2\pi$ "
    ~ (– ⅙) QED-result
- But, in QED and QCD, the ***anomalous chromo- and electro-magnetic moments vanish identically in the chiral limit***!

Dressed-quark anomalous magnetic moments

$$\int d^4x \, \tfrac{1}{2} q \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$$

- **Interaction term that describes magnetic-moment coupling to gauge field**
  - **Straightforward to show that it mixes left ↔ right**
  - **Thus, explicitly violates chiral symmetry**

Dressed-quark anomalous magnetic moments

$$\int d^4x \, \tfrac{1}{2} q \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$$

- **Interaction term that describes magnetic-moment coupling to gauge field**
  - **Straightforward to show that it mixes left ↔ right**
  - **Thus, explicitly violates chiral symmetry**
- **Follows that in fermion's e.m. current**

  $\gamma_\mu F_1$ does cannot mix with $\sigma_{\mu\nu} q_\nu F_2$

  No Gordon Identity
  - **Hence *massless fermions cannot not possess a measurable chromo- or electro-magnetic moment***

Dressed-quark anomalous magnetic moments

$$\int d^4x \, \tfrac{1}{2} q \, \bar{\psi}(x) \sigma_{\mu\nu} \psi(x) F_{\mu\nu}(x)$$

- **Interaction term that describes magnetic-moment coupling to gauge field**
  - Straightforward to show that it mixes left ↔ right
  - Thus, explicitly violates chiral symmetry
- **Follows that in fermion's e.m. current**

  $\gamma_\mu F_1$ **does cannot mix with** $\sigma_{\mu\nu} q_\nu F_2$

  No Gordon Identity
  - Hence ***massless fermions cannot not possess a measurable chromo- or electro-magnetic moment***
- **But what if the chiral symmetry is dynamically broken, strongly, as it is in QCD?**

Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1009.3458 [nucl-th]

Dressed-quark anomalous magnetic moments

- **DCSB** **Three strongly-dressed and essentially-nonperturbative contributions to dressed-quark-gluon vertex:**

$$\lambda^3_\mu(p,q) = 2(p+q)_\mu \Delta_B(p,q)$$

$$\Delta_F(p,q) = \frac{F(p^2)-F(q^2)}{p^2-q^2}$$

$$\Gamma^5_\mu(p,q) = \eta\,\sigma_{\mu\nu}(p-q)_\nu \Delta_B(p,q)$$

$$\Gamma^4_\mu(p,q) = [\ell^{\rm T}_\mu \gamma\cdot k + i\gamma^{\rm T}_\mu \sigma_{\nu\rho}\ell_\nu k_\rho]\tau_4(p,q)$$

$$\tau_4(p,q) = \mathcal{F}(z)\left[\frac{1-2\eta}{M_E}\Delta_B(p^2,q^2) - \Delta_A(p^2,q^2)\right]$$

$\mathcal{F}(z) = (1-\exp(-z))/z$, $z = (p_i^2 + p_f^2 - 2M_E^2)/\Lambda_{\mathcal{F}}^2$, $\Lambda_{\mathcal{F}} = 1\,\text{GeV}$,
Simplifies numerical analysis;

$M_E = \{s|s>0, s = M^2(s)\}$ is the Euclidean constituent-quark mass

Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1009.3458 [nucl-th]

Dressed-quark anomalous magnetic moments

- DCSB ⇒ **Three strongly-dressed and essentially-nonperturbative contributions to dressed-quark-gluon vertex:**

Ball-Chiu term
•Vanishes if no DCSB
•Appearance driven by STI

$$\lambda^3_\mu(p,q) = 2(p+q)_\mu \Delta_B(p,q)$$

$$\Delta_F(p,q) = \frac{F(p^2)-F(q^2)}{p^2-q^2}$$

$$\Gamma^5_\mu(p,q) = \eta\,\sigma_{\mu\nu}(p-q)_\nu \Delta_B(p,q)$$

$$\Gamma^4_\mu(p,q) = [\ell^{\rm T}_\mu \gamma\cdot k + i\gamma^{\rm T}_\mu \sigma_{\nu\rho}\ell_\nu k_\rho]\tau_4(p,q)$$

$$\tau_4(p,q) = \mathcal{F}(z)\left[\frac{1-2\eta}{M_E}\Delta_B(p^2,q^2) - \Delta_A(p^2,q^2)\right]$$

$\mathcal{F}(z) = (1-\exp(-z))/z$, $z = (p_i^2 + p_f^2 - 2M_E^2)/\Lambda^2_{\mathcal{F}}$, $\Lambda_{\mathcal{F}} = 1\,\text{GeV}$,
Simplifies numerical analysis;

$M_E = \{s | s > 0, s = M^2(s)\}$ is the Euclidean constituent-quark mass

Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1009.3458 [nucl-th]

Dressed-quark anomalous magnetic moments

- DCSB ⇒ **Three strongly-dressed and essentially-nonperturbative contributions to dressed-quark-gluon vertex:**

Ball-Chiu term

•Vanishes if no DCSB

•Appearance driven by STI

$$\lambda^3_\mu(p,q) = 2(p+q)_\mu \Delta_B(p,q)$$

$$\Delta_F(p,q) = \frac{F(p^2)-F(q^2)}{p^2-q^2}$$

Anom. chrom. mag. mom. contribution to vertex

•Similar properties to BC term

•Strength commensurate with lattice-QCD

Skullerud, Bowman, Kizilersu *et al.* hep-ph/0303176

$$\Gamma^5_\mu(p,q) = \eta\, \sigma_{\mu\nu}(p-q)_\nu \Delta_B(p,q)$$

$$\Gamma^4_\mu(p,q) = [\ell^{\rm T}_\mu \gamma\cdot k + i\gamma^{\rm T}_\mu \sigma_{\nu\rho}\ell_\nu k_\rho]\tau_4(p,q)$$

$$\tau_4(p,q) = \mathcal{F}(z)\left[\frac{1-2\eta}{M_E}\Delta_B(p^2,q^2) - \Delta_A(p^2,q^2)\right]$$

$\mathcal{F}(z) = (1-\exp(-z))/z$, $z = (p_i^2 + p_f^2 - 2M_E^2)/\Lambda_{\mathcal{F}}^2$, $\Lambda_{\mathcal{F}} = 1\,\text{GeV}$,
Simplifies numerical analysis;

$M_E = \{s | s > 0, s = M^2(s)\}$ is the Euclidean constituent-quark mass

Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1009.3458 [nucl-th]

Dressed-quark anomalous magnetic moments

➢ DCSB ⇒ **Three strongly-dressed and essentially-nonperturbative contributions to dressed-quark-gluon vertex:**

Ball-Chiu term
- Vanishes if no DCSB
- Appearance driven by STI

$$\lambda^3_\mu(p,q) = 2(p+q)_\mu \Delta_B(p,q)$$

$$\Delta_F(p,q) = \frac{F(p^2)-F(q^2)}{p^2-q^2}$$

Anom. chrom. mag. mom. contribution to vertex
- Similar properties to BC term
- Strength commensurate with lattice-QCD
  Skullerud, Bowman, Kizilersu *et al.* hep-ph/0303176

$$\Gamma^5_\mu(p,q) = \eta\, \sigma_{\mu\nu}(p-q)_\nu \Delta_B(p,q)$$

$$\Gamma^4_\mu(p,q) = [\ell^{\rm T}_\mu \gamma\cdot k + i\gamma^{\rm T}_\mu \sigma_{\nu\rho}\ell_\nu k_\rho]\tau_4(p,q)$$

$$\tau_4(p,q) = \mathcal{F}(z)\left[\frac{1-2\eta}{M_E}\Delta_B(p^2,q^2) - \Delta_A(p^2,q^2)\right]$$

$\mathcal{F}(z) = (1-\exp(-z))/z$, $z = (p_i^2+p_f^2-2M_E^2)/\Lambda^2_{\mathcal{F}}$, $\Lambda_{\mathcal{F}} = 1\,\mathrm{GeV}$,
Simplifies numerical analysis;

$M_E = \{s|s>0, s=M^2(s)\}$ is the Euclidean constituent-quark mass

Role and importance is Novel discovery
- Essential to recover pQCD
- Constructive interference with Γ^5

Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1009.3458 [nucl-th]

Dressed-quark anomalous magnetic moments

- Formulated and solved general Bethe-Salpeter equation
- Obtained dressed electromagnetic vertex
- Confined quarks don't have a mass-shell
  - Can't unambiguously define magnetic moments
  - But can define *magnetic moment distribution*

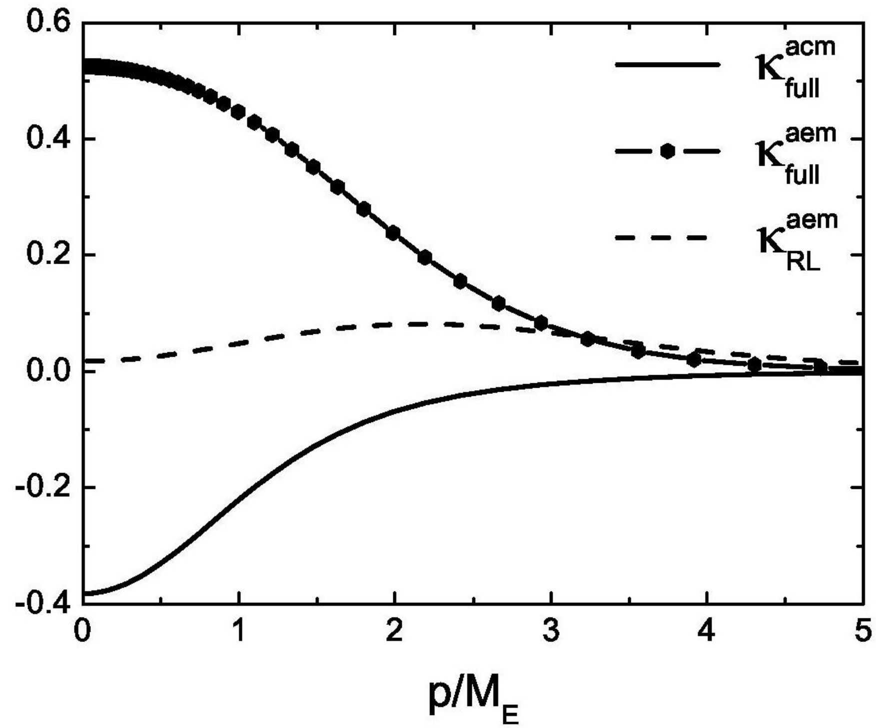
Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1009.3458 [nucl-th]

Dressed-quark anomalous magnetic moments

- Formulated and solved general Bethe-Salpeter equation
- Obtained dressed electromagnetic vertex
- Confined quarks don't have a mass-shell
  - o Can't unambiguously define magnetic moments
  - o But can define *magnetic moment distribution*
- AEM is opposite in sign but of roughly equal magnitude as ACM

o Potentially important for transition form factors, etc.

o Muon *g-2* ?



	M^E	κ^{ACM}	κ^{AEM}
Full vertex	0.44	-0.22	0.45
Rainbow-ladder	0.35	0	0.048

Dressed Vertex & Meson Spectrum

	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230				
ρ	770				
Mass splitting	455				

- **Splitting known experimentally for more than 35 years**
- **Hitherto, no explanation**

Dressed Vertex & Meson Spectrum

	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230	759	885		
ρ	770	644	764		
Mass splitting	455	115	121		

- **Splitting known experimentally for more than 35 years**
- **Hitherto, no explanation**
- **Systematic symmetry-preserving, Poincaré-covariant DSE truncation scheme of nucl-th/9602012.**
  - **Never better than ~ ¼ of splitting**
- **Constructing kernel skeleton-diagram-by-diagram, DCSB cannot be faithfully expressed:**

Full impact of $M(p^2)$ cannot be realised!

Dressed Vertex & Meson Spectrum

	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230	759	885	1066	
ρ	770	644	764	924	
Mass splitting	455	115	121	142	

- **Fully consistent treatment of Ball-Chiu vertex**
  - **Retain λ_3 – term but ignore Γ^4 & Γ^5**
  - **Some effects of DCSB built into vertex & Bethe-Salpeter kernel**
    - **Big impact on σ – π complex**
    - **But, clearly, not the complete answer.**

Dressed Vertex & Meson Spectrum

	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230	759	885	1066	1230
ρ	770	644	764	924	745
Mass splitting	455	115	121	142	485

- **Fully consistent treatment of Ball-Chiu vertex**
  - **Retain λ_3 – term but ignore Γ^4 & Γ^5**
  - **Some effects of DCSB built into vertex & Bethe-Salpeter kernel**
    - **Big impact on σ – π complex**
    - **But, clearly, not the complete answer.**
- **Fully-consistent treatment of complete vertex *Ansatz***

Dressed Vertex & Meson Spectrum

	Experiment	Rainbow-ladder	One-loop corrected	Ball-Chiu	Full vertex
a1	1230	759	885	1066	1230
ρ	770	644	764	924	745
Mass splitting	455	115	121	142	485

- **Fully-consistent treatment of complete vertex *Ansatz***
- ***Subtle interplay between competing effects, which can only now be explicated***
- **Promise of first reliable prediction of light-quark hadron spectrum, including the so-called hybrid and exotic states.**

Maris, Roberts and Tandy
nucl-th/9707003

Pion's Golderberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

Maris, Roberts and Tandy
nucl-th/9707003

Pion's Golderberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P)+\gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P)+\sigma_{\mu\nu}\,k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark propagator $S(p)=\dfrac{1}{i\gamma\cdot p\,A(p^2)+B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

Exact in Chiral QCD

$$\begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2)\\ F_R(k;0)+2\,f_\pi F_\pi(k;0) &= A(k^2)\\ G_R(k;0)+2\,f_\pi G_\pi(k;0) &= 2A'(k^2)\\ H_R(k;0)+2\,f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

Maris, Roberts and Tandy
nucl-th/9707003

Pion's Golderberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

Pseudovector components necessarily nonzero. Cannot be ignored!

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

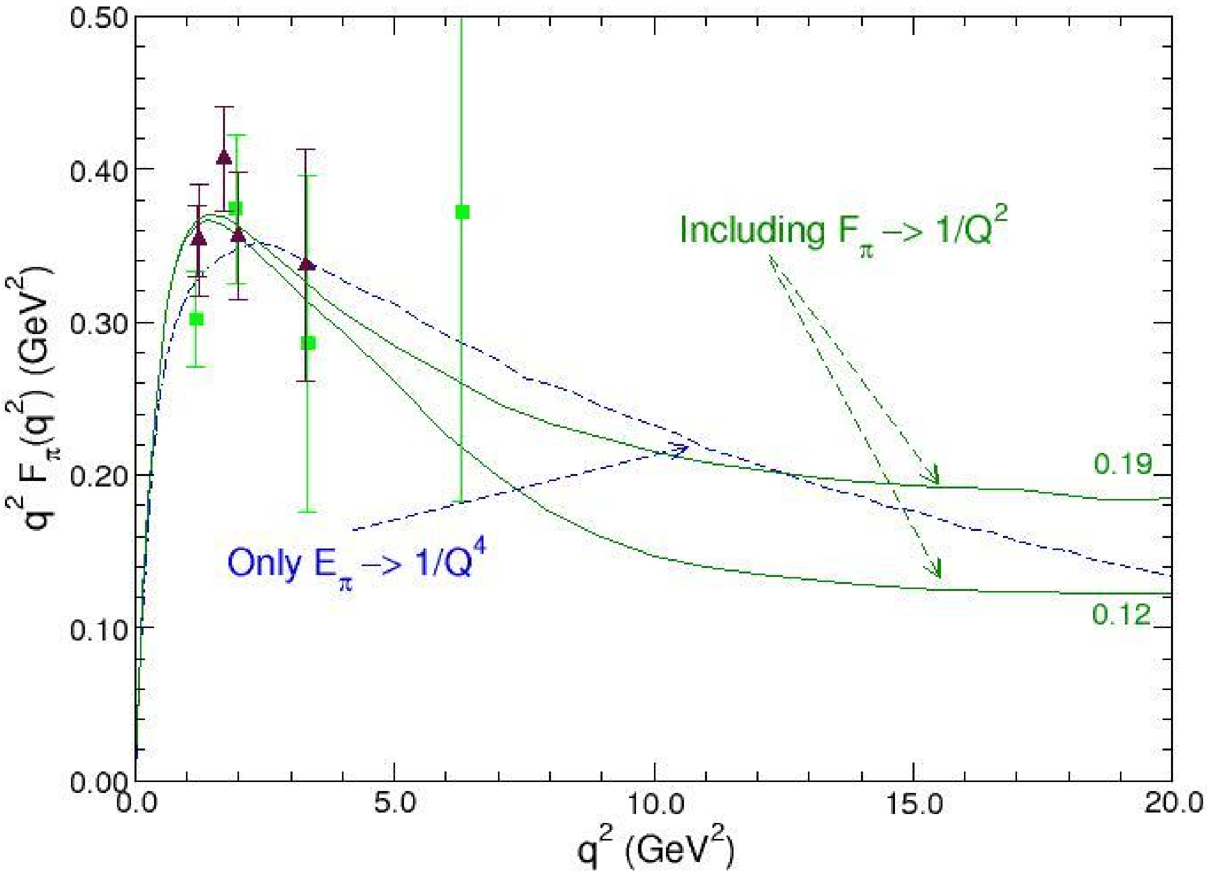
- Axial-vector Ward-Takahashi identity entails

Exact in Chiral QCD

$$\left\{\begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}\right.$$

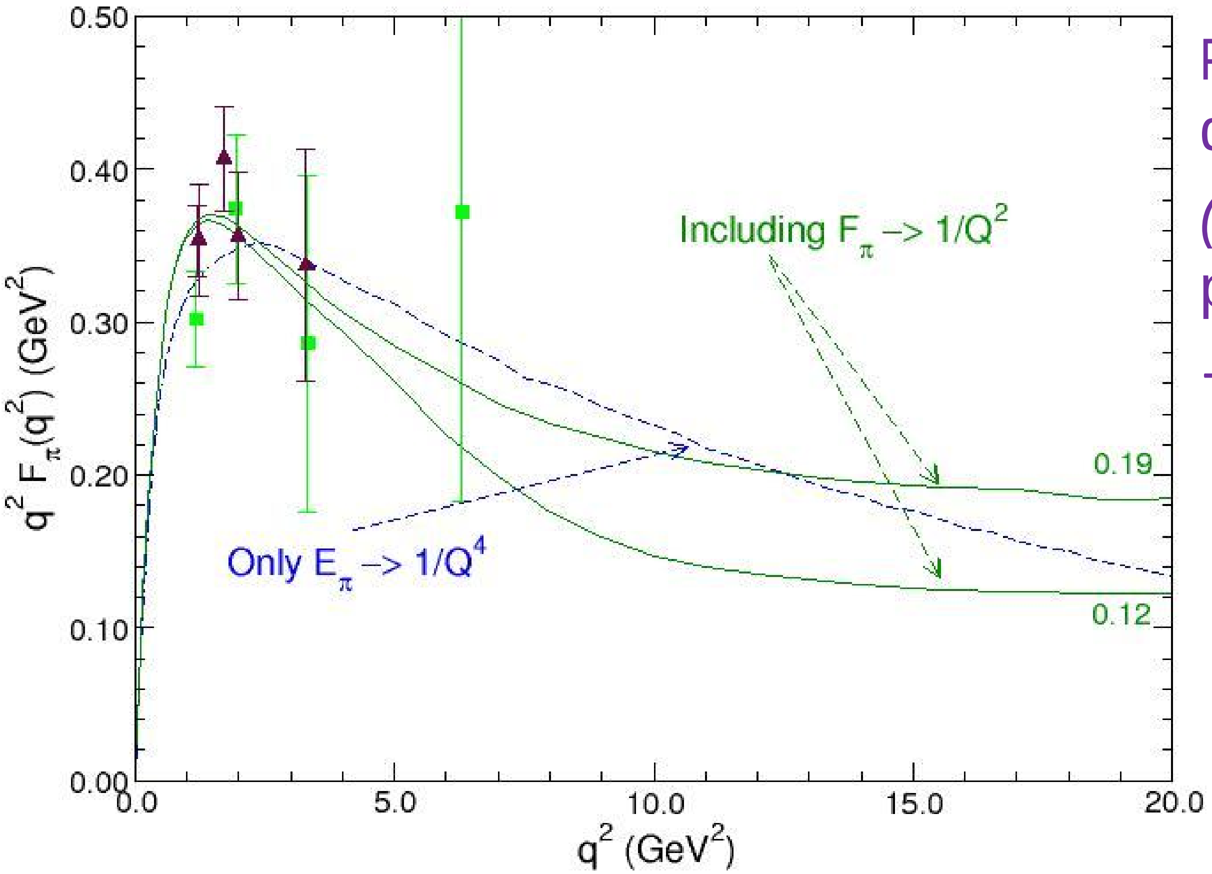
Maris and Roberts
nucl-th/9804062

Pion's GT relation
Implications for observables?



Maris and Roberts
nucl-th/9804062

Pion's GT relation
Implications for observables?



Pseudovector components dominate in ultraviolet:

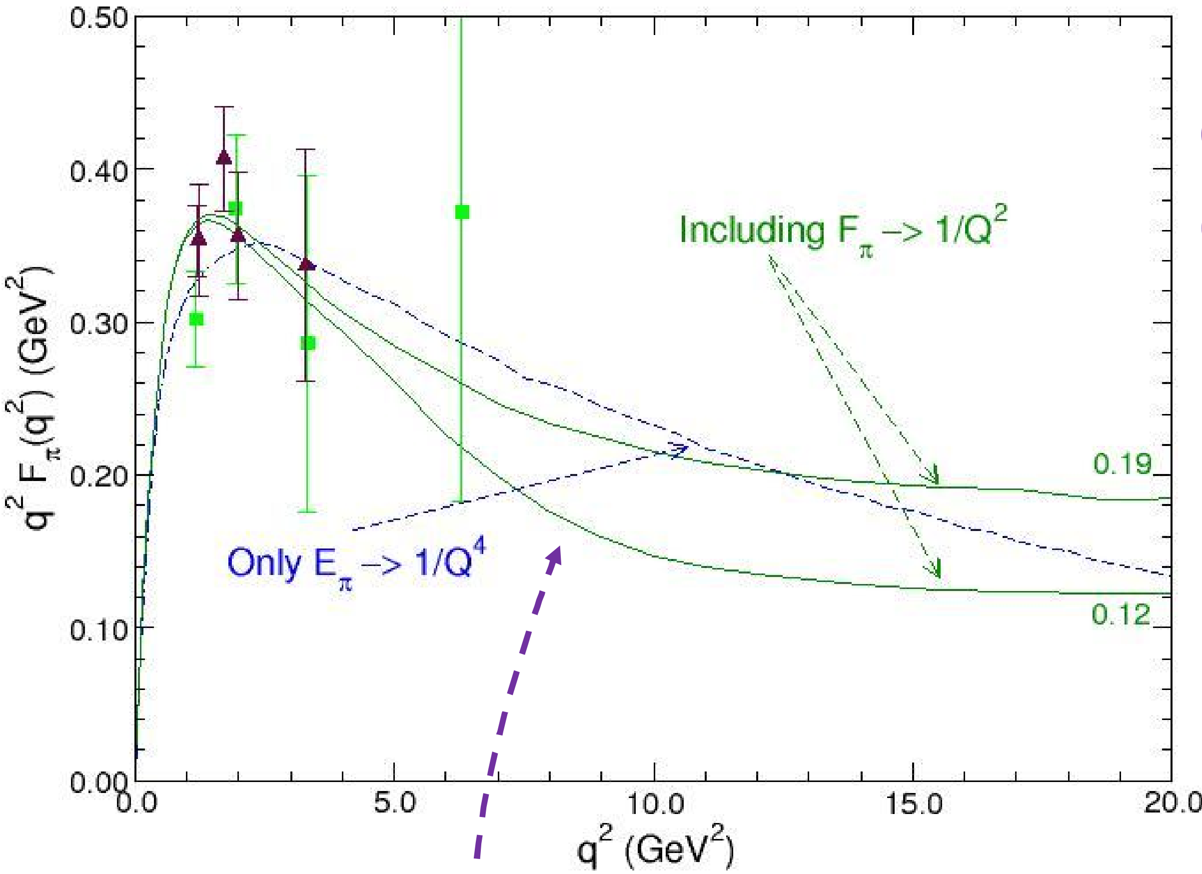
$(Q/2)^2 = 2\ GeV^2$

pQCD point for $M(p^2)$

$\rightarrow$ pQCD at $Q^2 = 8 GeV^2$

Maris and Roberts
nucl-th/9804062

Pion's GT relation
Implications for observables?



Pseudovector components dominate in ultraviolet:

$(Q/2)^2 = 2\ GeV^2$

pQCD point for $M(p^2)$

$\to$ pQCD at $Q^2 = 8GeV^2$

Guttierez, Bashir, Cloët, Roberts
arXiv:1002.1968 [nucl-th]

Pion's GT relation

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

Guttierez, Bashir, Cloët, Roberts
arXiv:1002.1968 [nucl-th]

Pion's GT relation Contact interaction

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) \\ \cancel{+\, \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H_\pi(k;P)}\Big]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\,\underbrace{A(p^2)}_{1} + \underbrace{B(p^2)}_{M_Q}}$

- Bethe-Salpeter amplitude can't depend on relative momentum; propagator can't be momentum-dependent

Guttierez, Bashir, Cloët, Roberts
arXiv:1002.1968 [nucl-th]

Pion's GT relation Contact interaction

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) \\ \cancel{+\,\gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H_\pi(k;P)}\Big]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\,\underset{1}{A(p^2)} + \underset{M_Q}{B(p^2)}}$

- Bethe-Salpeter amplitude can't depend on relative momentum; propagator can't be momentum-dependent
- Solved gap and Bethe-Salpeter equations

$P^2=0$: $M_Q=0.4$GeV, $E_\pi=0.098$, $F_\pi=0.5M_Q$

Guttierez, Bashir, Cloët, Roberts
arXiv:1002.1968 [nucl-th]

Pion's GT relation Contact interaction

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \cancel{\gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H_\pi(k;P)}\Big]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

$$A(p^2) \to 1, \quad B(p^2) \to M_Q$$

- Bethe-Salpeter amplitude can't depend on relative momentum; propagator can't be momentum-dependent
- Solved gap and Bethe-Salpeter equations

$P^2=0$: $M_Q=0.4$GeV, $E_\pi=0.098$, $F_\pi=0.5M_Q$

Nonzero and significant

Guttierez, Bashir, Cloët, Roberts
arXiv:1002.1968 [nucl-th]

Pion's GT relation
Contact interaction

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P)\right.$$
$$\left.+ \cancel{\gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H_\pi(k;P)}\right]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

$1 \qquad M_Q$

- Asymptotic form of $F_\pi(Q^2)$

$E_\pi^{\,2}(P) \rightarrow F_\pi^{em}(Q^2) = M_Q^{\,2}/Q^2$

For 20+ years it was imagined that contact-interaction produced a result that's indistinguishable From pQCD counting rule

Guttierez, Bashir, Cloët, Roberts
arXiv:1002.1968 [nucl-th]

Pion's GT relation Contact interaction

- Pion's Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) \cancel{+ \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}k_\mu P_\nu H_\pi(k;P)}\Big]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\,\underbrace{A(p^2)}_{1} + \underbrace{B(p^2)}_{M_Q}}$

- Asymptotic form of $F_\pi(Q^2)$

$E_\pi^2(P) \to F_\pi^{em}(Q^2) = M_Q^2/Q^2$ ~~For 20+ years it was imagined that contact interaction produced a result that's indistinguishable From pQCD counting rule~~

$E_\pi(P)\,F_\pi(P)$ – cross-term

$\to F_\pi^{em}(Q^2) = (Q^2/M_Q^2) * [E_\pi(P)/F_\pi(P)] * E_\pi^2(P)\text{-term} =$ CONSTANT!

Guttierez, Bashir, Cloët, Roberts
arXiv:1002.1968 [nucl-th]

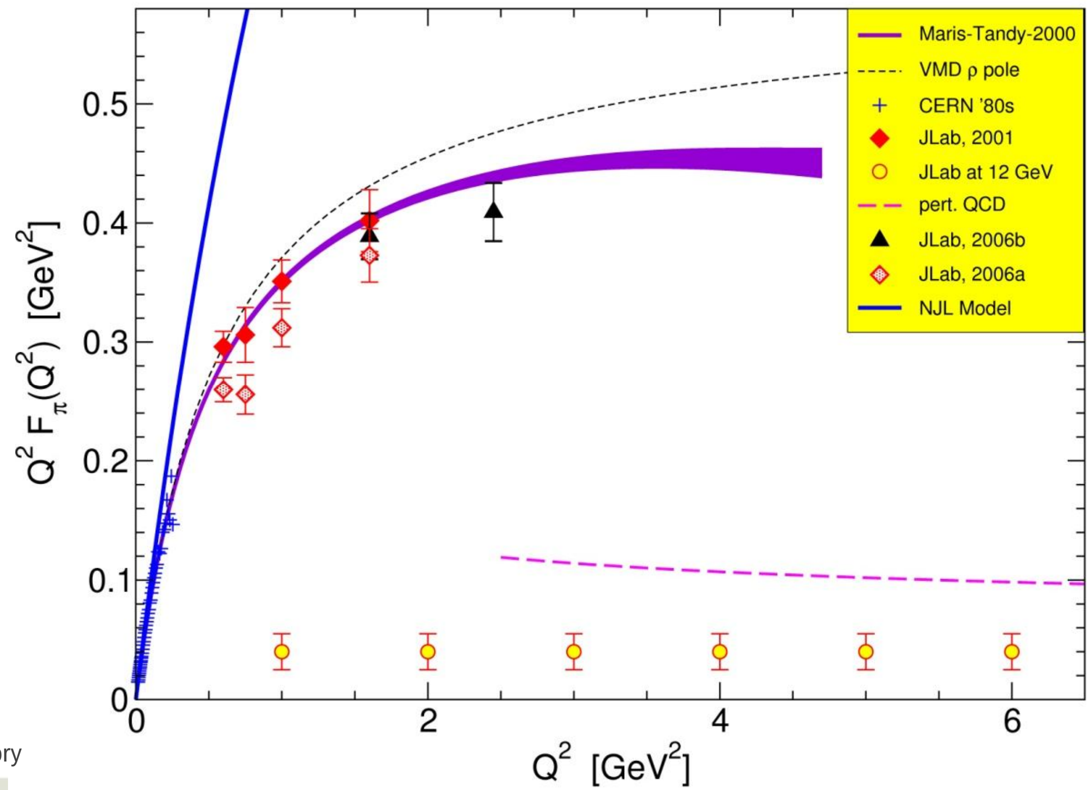
Pion's Electromagnetic Form Factor

- QCD-based DSE prediction: $D(x-y) = \dfrac{1}{(x-y)^2}$ produces $M(p^2) \sim 1/p^2$
- cf. contact-interaction: $D(x-y) \sim \delta^4(x-y)$ produces $M(p^2)$=constant

Guttierez, Bashir, Cloët, Roberts
arXiv:1002.1968 [nucl-th]

Pion's Electromagnetic Form Factor

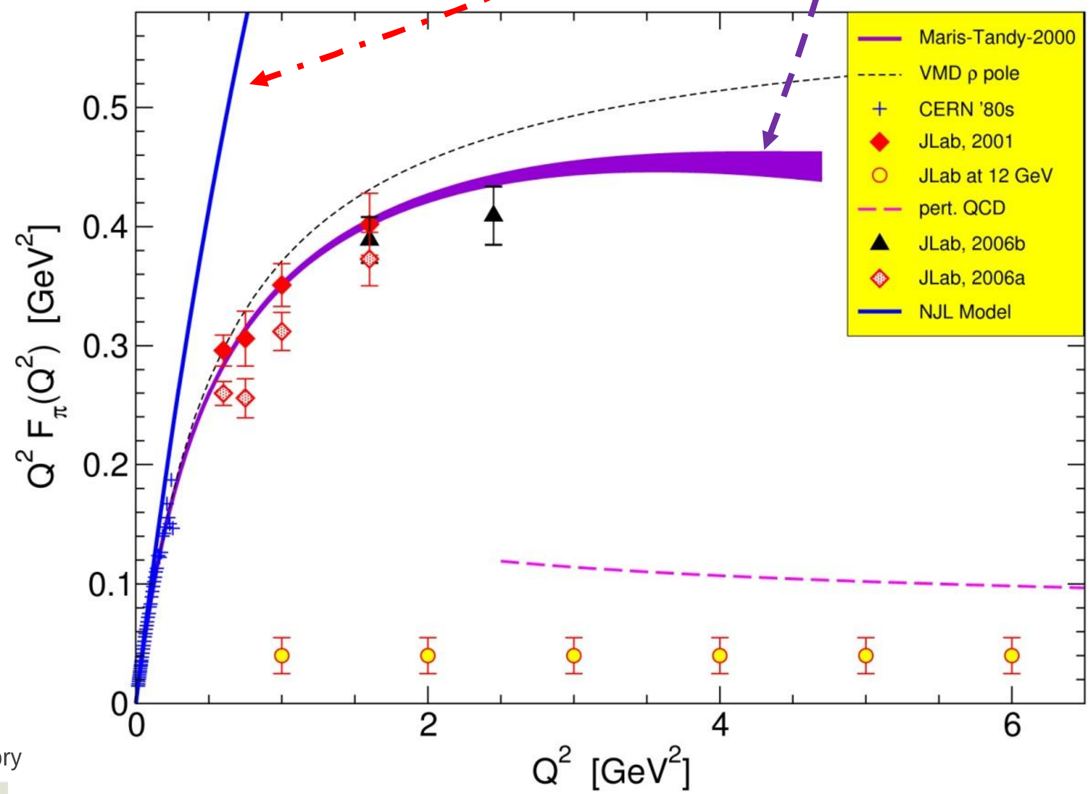
- QCD-based DSE prediction: $D(x\text{-}y) = \dfrac{1}{(x-y)^2}$ produces $M(p^2)\sim 1/p^2$
- cf. contact-interaction: $D(x-y) \sim \delta^4(x-y)$ produces $M(p^2)$=constant



Guttierez, Bashir, Cloët, Roberts
arXiv:1002.1968 [nucl-th]

Pion's Electromagnetic Form Factor

- QCD-based DSE prediction: $D(x\text{-}y) = \dfrac{1}{(x-y)^2}$ produces $M(p^2)\sim 1/p^2$
- cf. contact-interaction: $D(x-y) \sim \delta^4(x-y)$ produces $M(p^2)$=constant



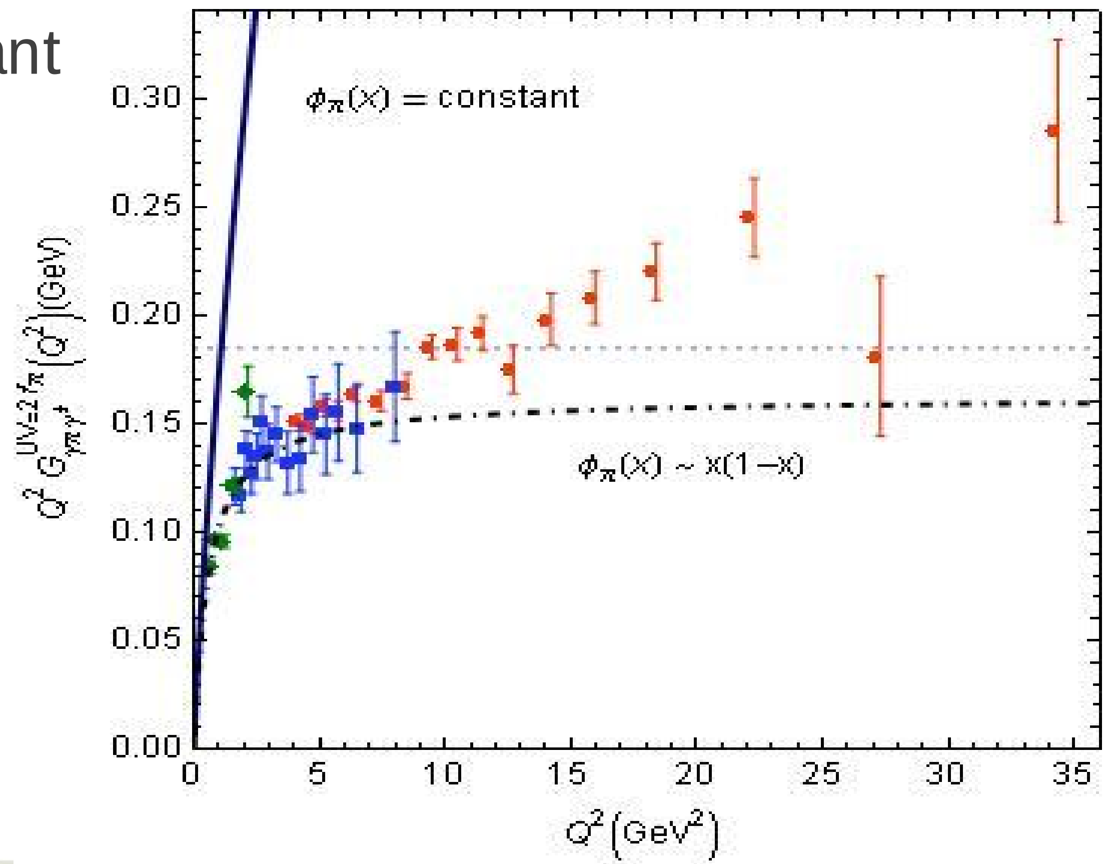
- Single mass parameter in both studies
- Same predictions for $Q^2=0$ observables
- *Disagreement >20% for $Q^2>M_Q^2$*

BaBar *Anomaly*

$\gamma^* \gamma \rightarrow \pi^0$

H.L.L. Roberts, C.D. Roberts, Bashir, Guttierez, Tandy
arXiv:1009.0067 [nucl-th]

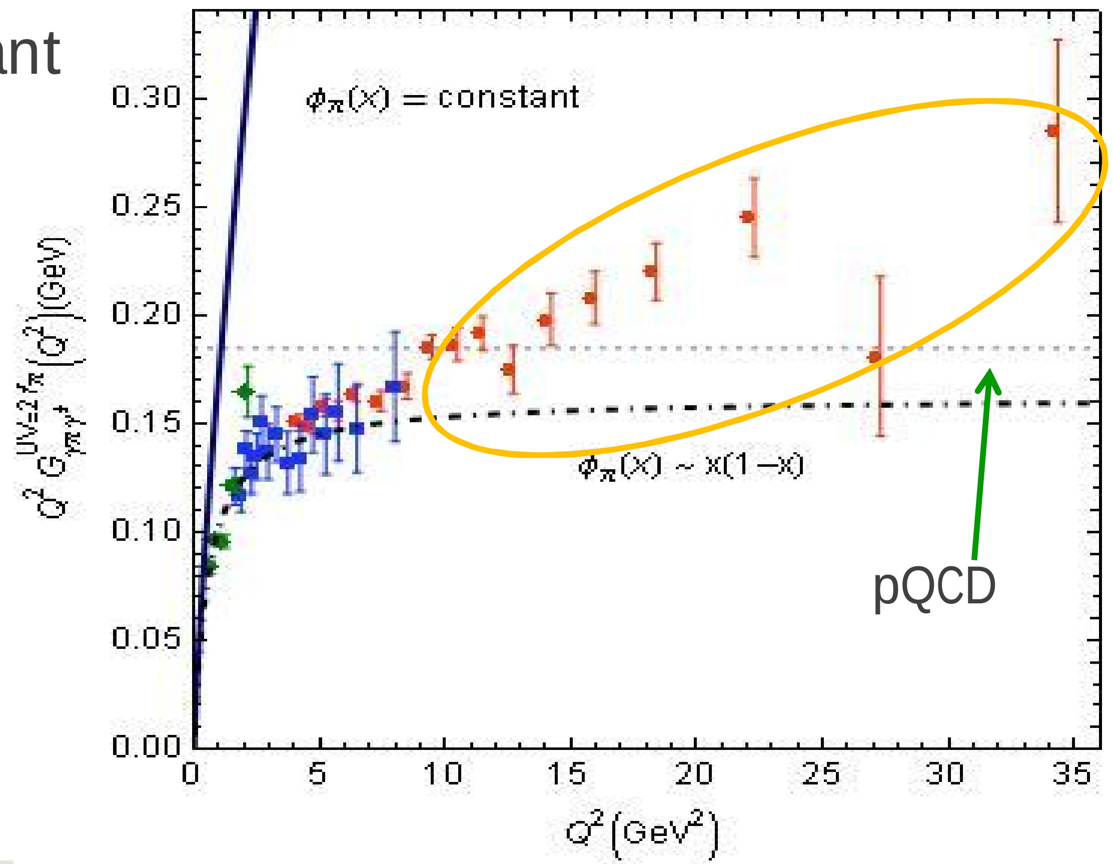
- QCD-based DSE prediction: $D(x\text{-}y) = \dfrac{1}{(x-y)^2}$ produces $M(p^2)\sim 1/p^2$
- cf. contact-interaction: $D(x-y) \sim \delta^4(x-y)$ produces $M(p^2)$=constant



H.L.L. Roberts, C.D. Roberts, Bashir, Guttierez, Tandy
arXiv:1009.0067 [nucl-th]

BaBar *Anomaly*
$\gamma^* \gamma \rightarrow \pi^0$

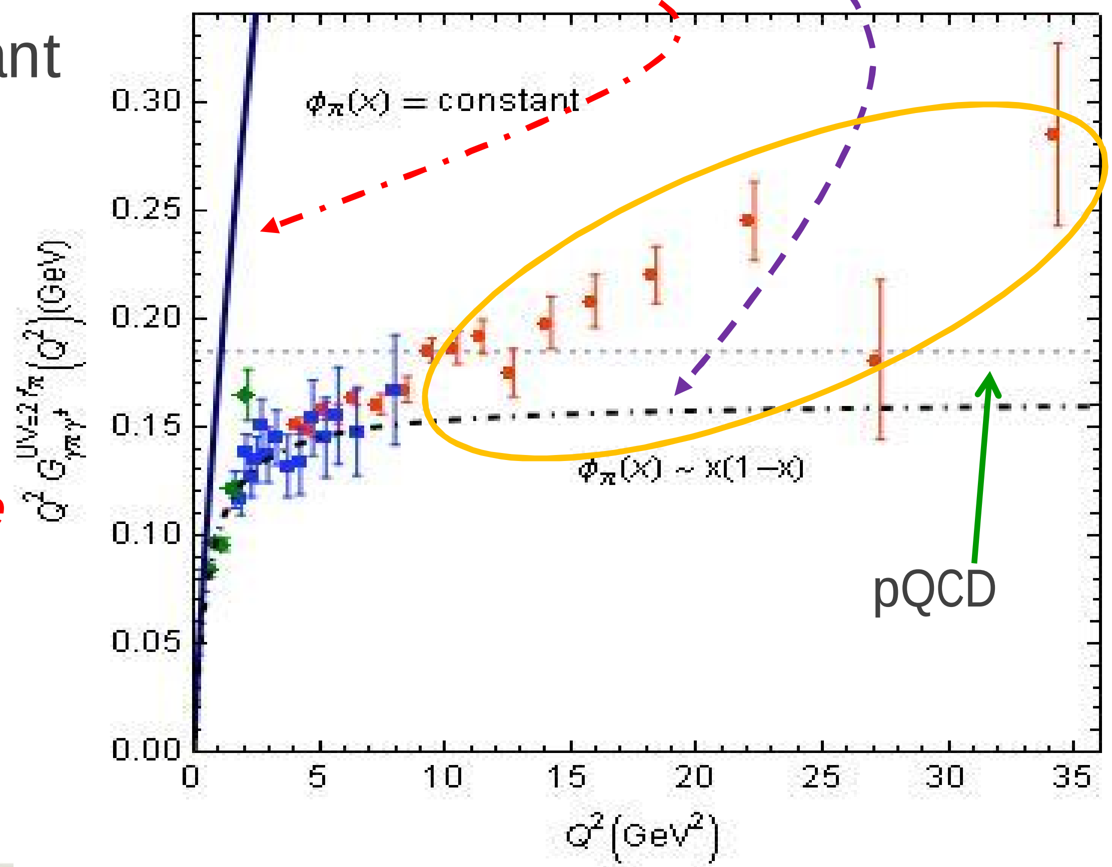
- QCD-based DSE prediction: $D(x\text{-}y) = \dfrac{1}{(x-y)^2}$ produces $M(p^2) \sim 1/p^2$
- cf. contact-interaction: $D(x-y) \sim \delta^4(x-y)$ produces $M(p^2)$=constant



H.L.L. Roberts, C.D. Roberts, Bashir, Guttierez, Tandy
arXiv:1009.0067 [nucl-th]

BaBar *Anomaly*
$\gamma^* \gamma \rightarrow \pi^0$

- QCD-based DSE prediction: $D(x\text{-}y) = \frac{1}{(x-y)^2}$ produces $M(p^2)\sim 1/p^2$
- cf. contact-interaction: $D(x-y) \sim \delta^4(x-y)$ produces $M(p^2)$=constant



- No fully-self-consistent treatment of the pion can reproduce the BaBar data.
- All produce monotonically-increasing concave functions.
- *BaBar data not a true measure of $\gamma^* \gamma \rightarrow \pi^0$*
- Likely source of error is misidentification of $\pi^0\,\pi^0$ events where 2nd π^0 isn't seen.

Unifying Baryons and Mesons

- $M(p^2)$ – effects have enormous impact on meson properties.
  - *Must be included in description and prediction of baryon properties*.

Unifying Baryons and Mesons

- $M(p^2)$ – effects have enormous impact on meson properties.
  - *Must be included in description and prediction of baryon properties*.
- $M(p^2)$ is essentially a quantum field theoretical effect. In quantum field theory
  - Meson appears as pole in four-point quark-antiquark Green function → Bethe-Salpeter Equation
  - *Nucleon appears as a pole in a six-point quark Green function → Faddeev Equation*.

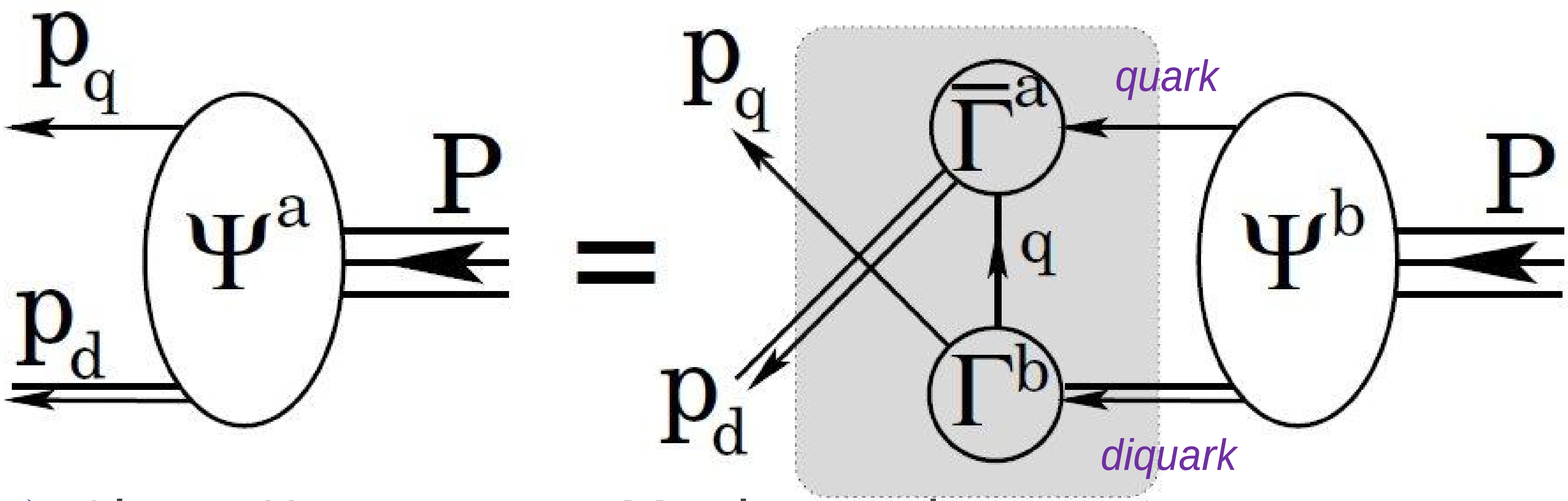
R.T. Cahill *et al.*,
Austral. J. Phys. 42 (1989) 129-145

Unifying Baryons and Mesons

- *M(p²)* – effects have enormous impact on meson properties.
  - *Must be included in description and prediction of baryon properties*.
- *M(p²)* is essentially a quantum field theoretical effect. In quantum field theory
  - Meson appears as pole in four-point quark-antiquark Green function → Bethe-Salpeter Equation
  - *Nucleon appears as a pole in a six-point quark Green function → Faddeev Equation*.
- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates *nonpointlike* quark-quark (diquark) correlations in the colour-antitriplet channel

Faddeev Equation

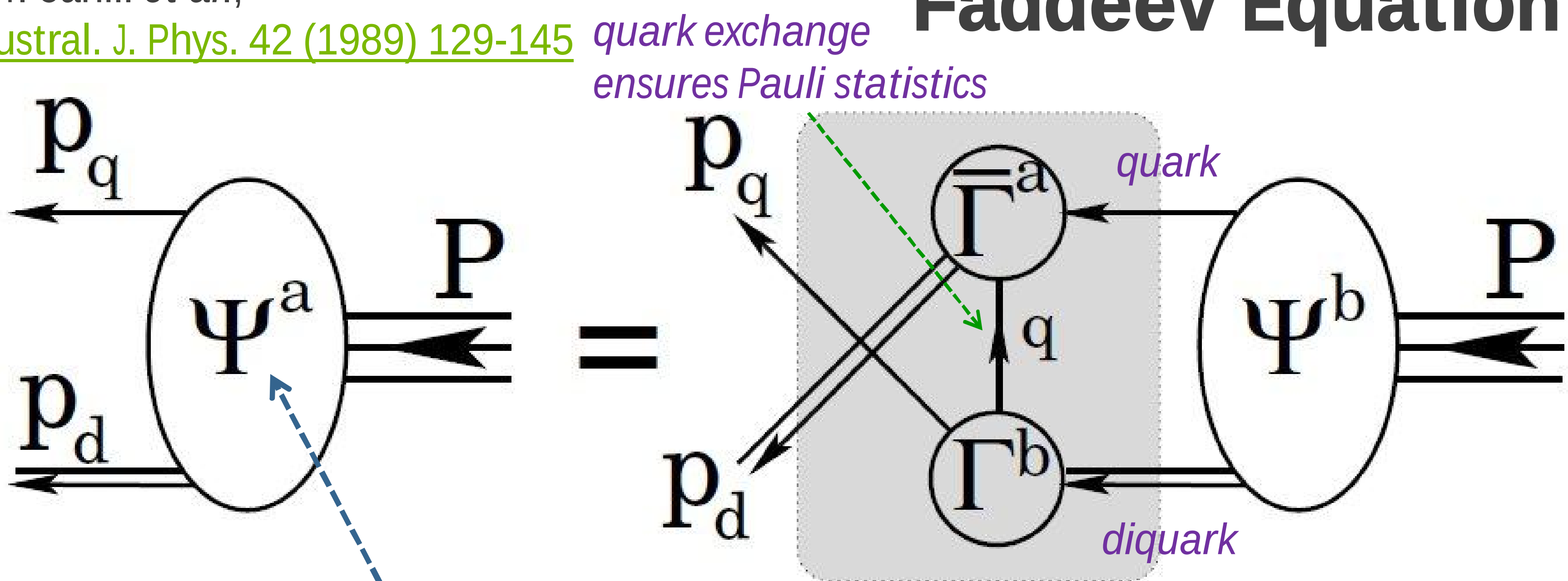
R.T. Cahill *et al*.,
Austral. J. Phys. 42 (1989) 129-145



- Linear, Homogeneous Matrix equation

R.T. Cahill *et al*.,
Austral. J. Phys. 42 (1989) 129-145

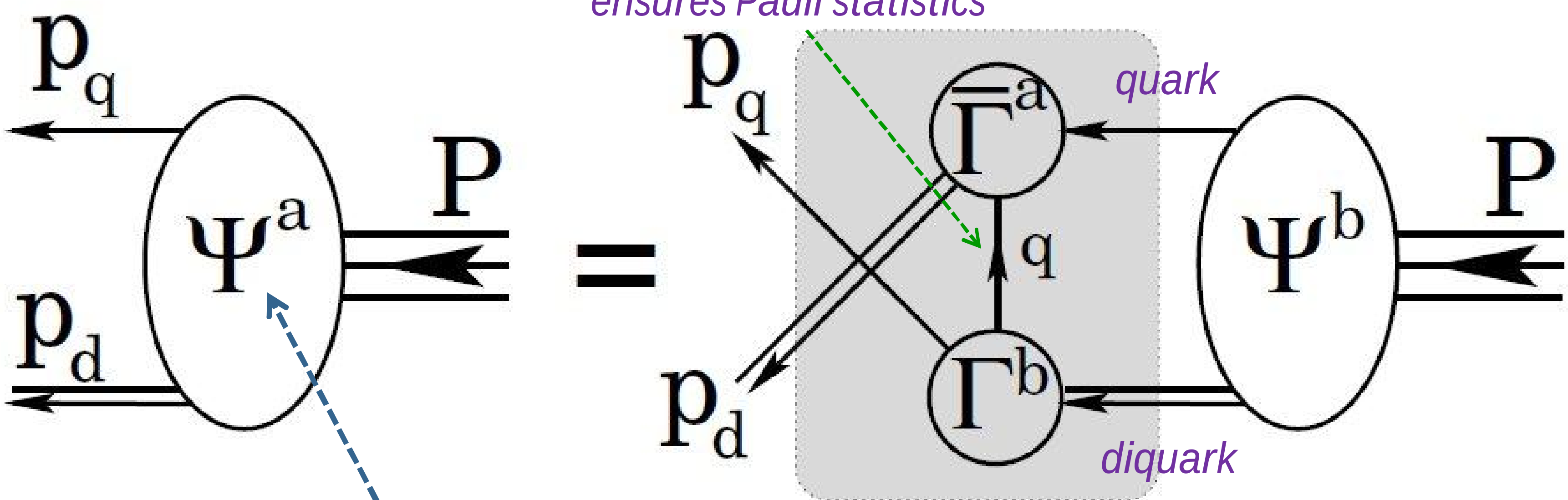
Faddeev Equation



- Linear, Homogeneous Matrix equation
  - Yields *wave function (Poincaré Covariant Faddeev* Amplitude) that describes quark-diquark relative motion within the nucleon

R.T. Cahill *et al.*,
Austral. J. Phys. 42 (1989) 129-145

Faddeev Equation



- Linear, Homogeneous Matrix equation
  - Yields *wave function (Poincaré Covariant Faddeev* Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . .
  - Both have "*correct*" parity and "*right*" masses
  - In Nucleon's Rest Frame Amplitude has

    s−, p− & d−wave correlations

H.L.L. Roberts, L. Chang and C.D. Roberts
arXiv:1007.4318 [nucl-th]
H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts
arXiv:1007.3566 [nucl-th]

Spectrum of some known *u*- & *d*-quark baryons

- Mesons & Diquarks

m_{0^+}	m_{1^+}	m_{0^-}	m_{1^-}	m_π	m_ρ	m_σ	m_{a1}
0.72	1.01	1.17	1.31	0.14	0.80	1.06	1.23

H.L.L. Roberts, L. Chang and C.D. Roberts
arXiv:1007.4318 [nucl-th]
H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts
arXiv:1007.3566 [nucl-th]

Spectrum of some known *u*- & *d*-quark baryons

- Mesons & Diquarks

Cahill, Roberts, Praschifka: Phys.Rev. D36 (1987) 2804

Proof of mass ordering: diquark-m_J+ > meson-m_J-

m_{0^+}	m_{1^+}	m_{0^-}	m_{1^-}	m_π	m_ρ	m_σ	m_{a1}
0.72	1.01	1.17	1.31	0.14	0.80	1.06	1.23

H.L.L. Roberts, L. Chang and C.D. Roberts
arXiv:1007.4318 [nucl-th]
H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts
arXiv:1007.3566 [nucl-th]

Spectrum of some known *u*- & *d*-quark baryons

- Mesons & Diquarks

Cahill, Roberts, Praschifka: Phys.Rev. D36 (1987) 2804

Proof of mass ordering: diquark-m_J+ > meson-m_J-

m_{0^+}	m_{1^+}	m_{0^-}	m_{1^-}	m_π	m_ρ	m_σ	m_{a1}
0.72	1.01	1.17	1.31	0.14	0.80	1.06	1.23

- Baryons: ground-states and 1st radial exciations

	m_N	m_{N*}	$m_N(½)$	$m_{N*}(½^-)$	m_Δ	$m_{\Delta*}$	$m_\Delta(3/2^-)$	$m_{\Delta*}(3/2^-)$
DSE	1.05	1.73	1.86	2.09	1.33	1.85	1.98	2.16
EBAC		1.76	1.80		1.39		1.98	

H.L.L. Roberts, L. Chang and C.D. Roberts
arXiv:1007.4318 [nucl-th]
H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts
arXiv:1007.3566 [nucl-th]

Spectrum of some known *u*- & *d*-quark baryons

- Mesons & Diquarks

  Cahill, Roberts, Praschifka: Phys.Rev. D36 (1987) 2804

  Proof of mass ordering: diquark-m_J+ > meson-m_J-

m_{0^+}	m_{1^+}	m_{0^-}	m_{1^-}	m_π	m_ρ	m_σ	m_{a1}
0.72	1.01	1.17	1.31	0.14	0.80	1.06	1.23

- Baryons: ground-states and 1st radial exciations

	m_N	m_{N*}	$m_N(½^-)$	$m_{N*}(½^-)$	m_Δ	$m_{\Delta*}$	$m_\Delta(3/2^-)$	$m_{\Delta*}(3/2^-)$
DSE	1.05	1.73	1.86	2.09	1.33	1.85	1.98	2.16
EBAC		1.76	1.80		1.39		1.98	

- *mean-|relative-error| = 2%-Agreement*

  DSE dressed-quark-core masses cf. Excited Baryon Analysis Center (JLab) bare masses is significant 'cause no attempt was made to ensure this.

H.L.L. Roberts, L. Chang and C.D. Roberts
arXiv:1007.4318 [nucl-th]
H.L.L. Roberts, L. Chang, I.C. Cloët and C.D. Roberts
arXiv:1007.3566 [nucl-th]

Spectrum of some known *u*- & *d*-quark baryons

- Mesons & Diquarks

  Cahill, Roberts, Praschifka: Phys.Rev. D36 (1987) 2804

  Proof of mass ordering: diquark-m_J+ > meson-m_J-

m_{0^+}	m_{1^+}	m_{0^-}	m_{1^-}	m_π	m_ρ	m_σ	m_{a1}
0.72	1.01	1.17	1.31	0.14	0.80	1.06	1.23

- Baryons: ground-states and 1st radial exciations

1st radial Excitation of N(1535)?

	m_N	m_{N*}	$m_N(\frac{1}{2}^-)$	$m_{N*}(\frac{1}{2}^-)$	m_Δ	$m_{\Delta*}$	$m_\Delta(\frac{3}{2}^-)$	$m_{\Delta*}(\frac{3}{2}^-)$
DSE	1.05	1.73	1.86	2.09	1.33	1.85	1.98	2.16
EBAC		1.76	1.80		1.39		1.98	

- *mean-|relative-error| = 2%-Agreement*

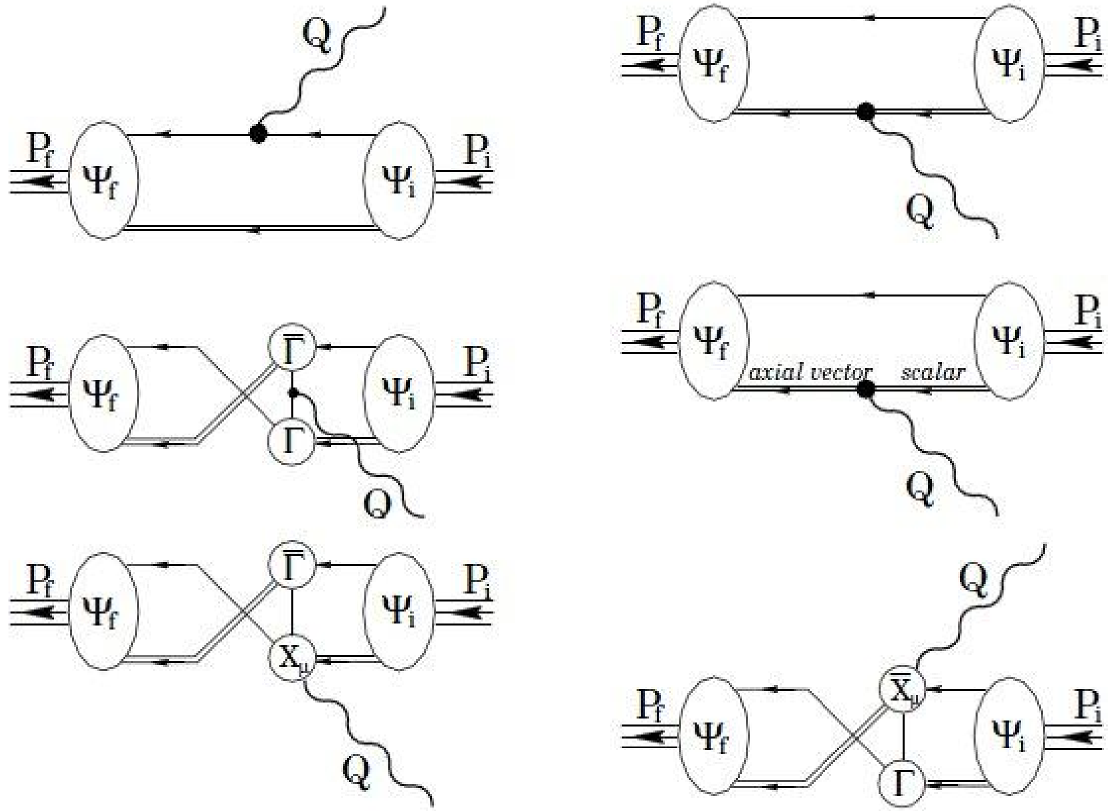
  DSE dressed-quark-core masses cf. Excited Baryon Analysis Center (JLab) bare masses is significant 'cause no attempt was made to ensure this.

I.C. Cloët, C.D. Roberts, *et al*.
arXiv:0812.0416 [nucl-th]

Nucleon Elastic Form Factors

- Photon-baryon vertex

Oettel, Pichowsky and von Smekal, nucl-th/9909082



I.C. Cloët, C.D. Roberts, *et al*.
arXiv:0812.0416 [nucl-th]

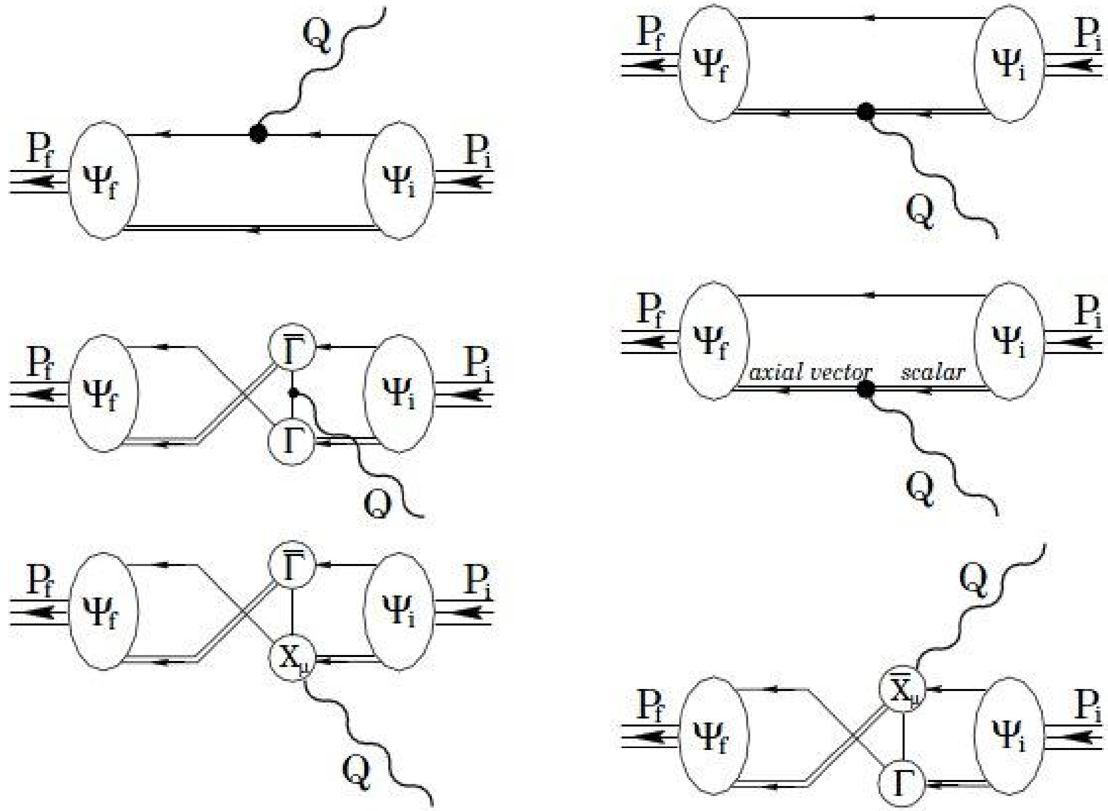
Nucleon Elastic Form Factors

- Photon-baryon vertex

  Oettel, Pichowsky and von Smekal, nucl-th/9909082

- "*Survey of nucleon electromagnetic form factors*"

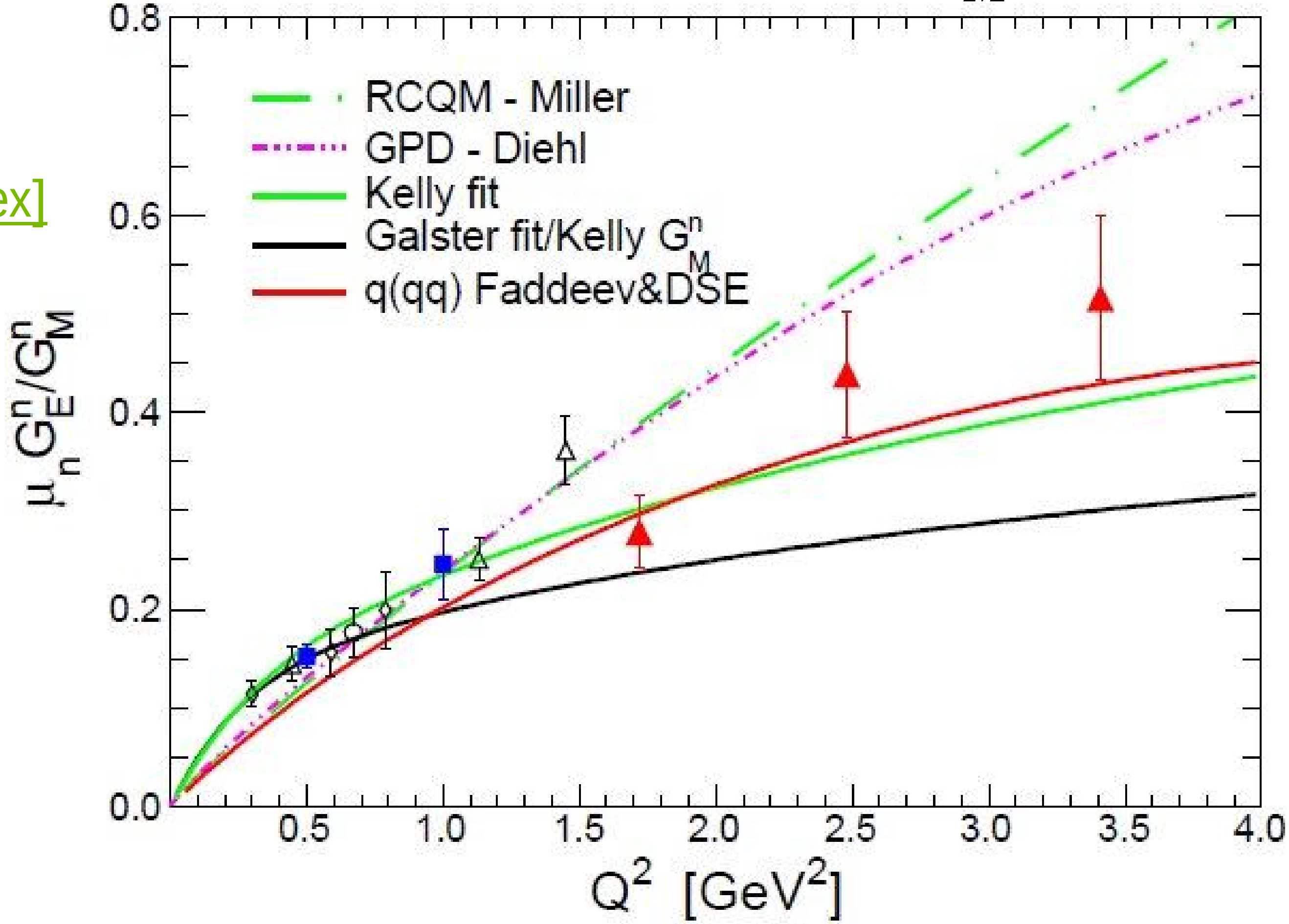
  – unification of meson and baryon observables; and prediction of nucleon elastic form factors to 15 GeV^2



I.C. Cloët, C.D. Roberts, *et al*.
arXiv:0812.0416 [nucl-th]

- New JLab data:
  S. Riordan *et al*.,
  arXiv:1008.1738 [nucl-ex]

$$\frac{\mu_n G_E^n(Q^2)}{G_M^n(Q^2)}$$



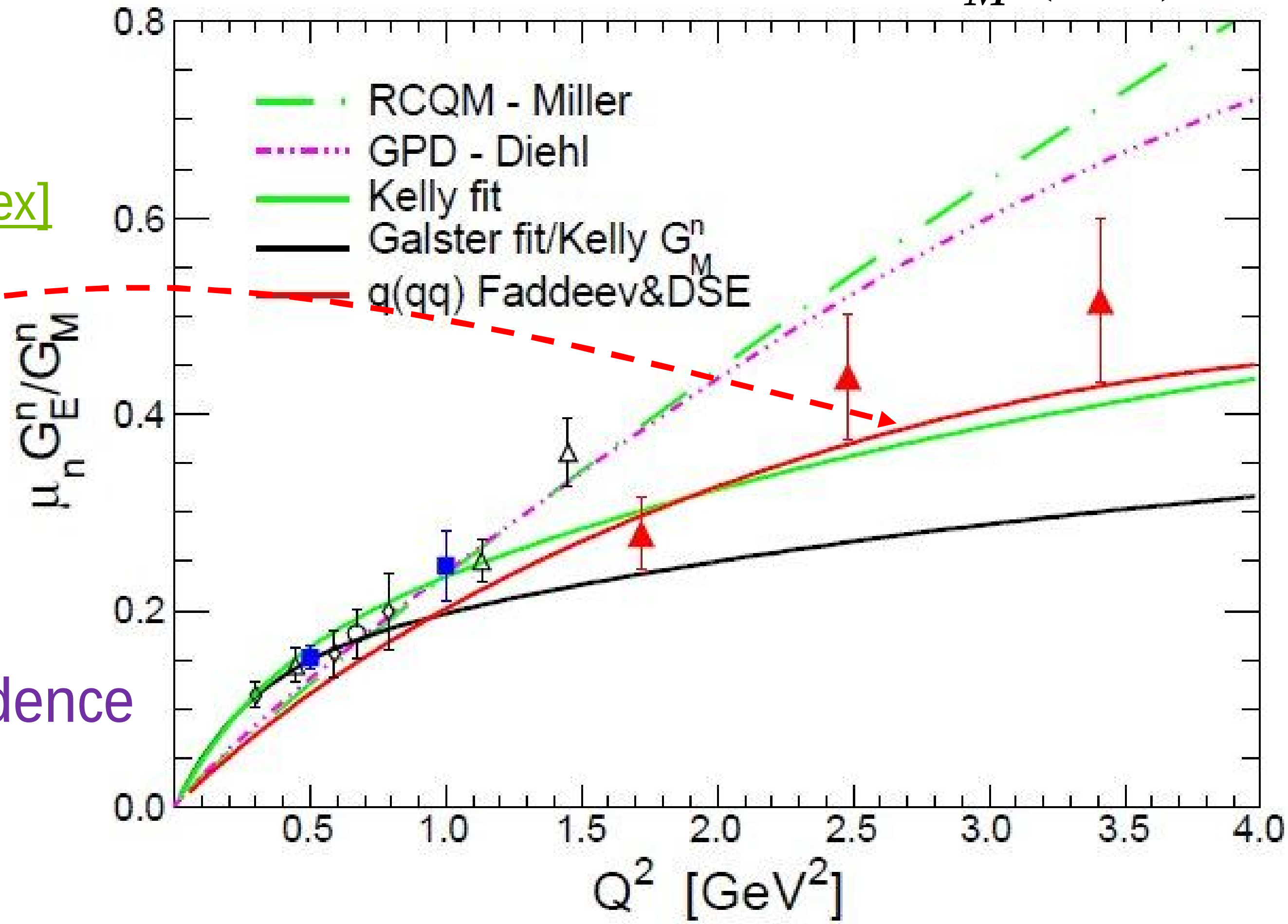
I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th]

$$\frac{\mu_n G_E^n(Q^2)}{G_M^n(Q^2)}$$

- New JLab data:
  S. Riordan *et al.*,
  arXiv:1008.1738 [nucl-ex]

- DSE-prediction

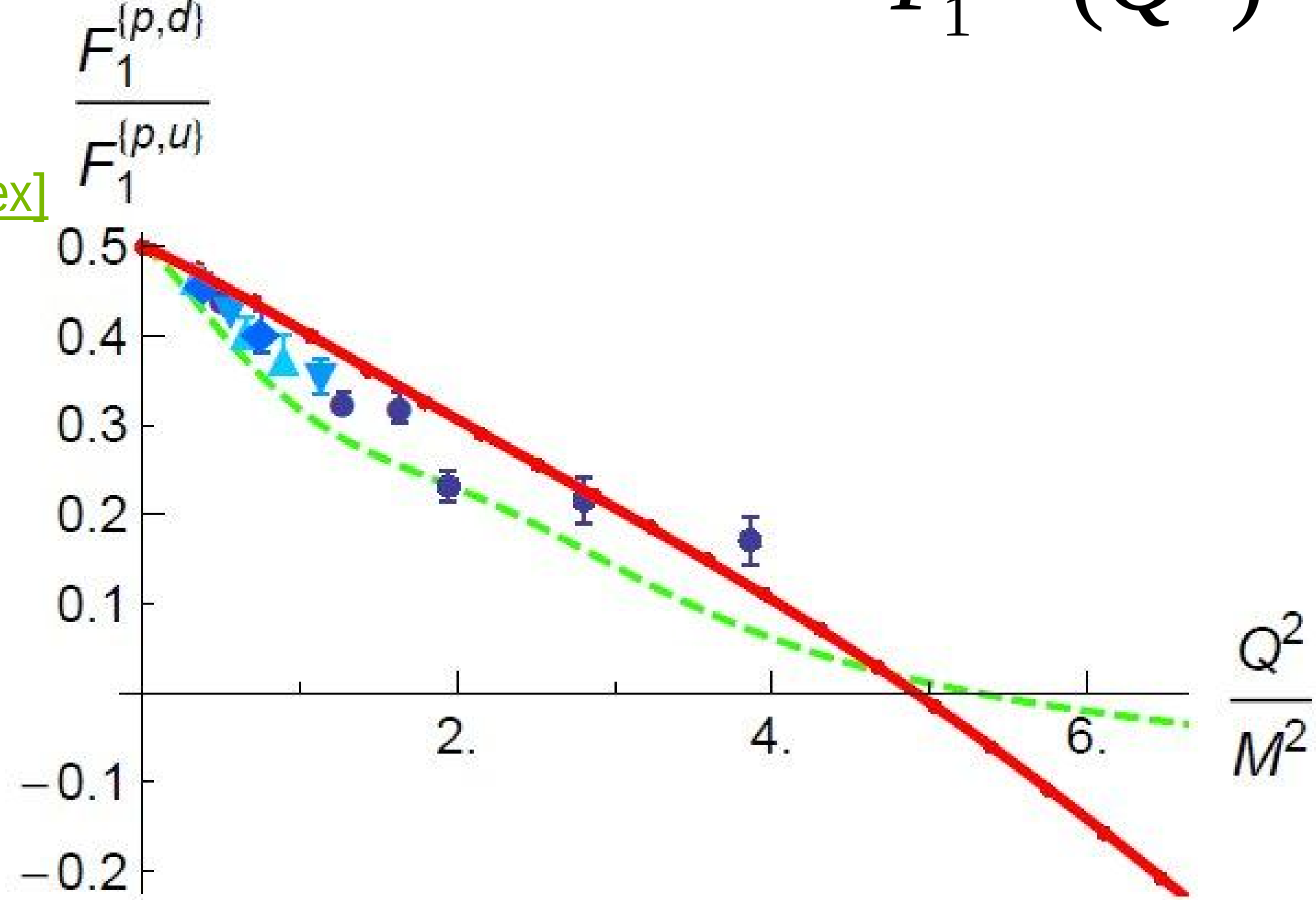
- This evolution is very sensitive to momentum-dependence of dressed-quark propagator



I.C. Cloët, C.D. Roberts, *et al.*
[arXiv:0812.0416 [nucl-th]]

$$\frac{F_1^{p,d}(Q^2)}{F_1^{p,u}(Q^2)}$$

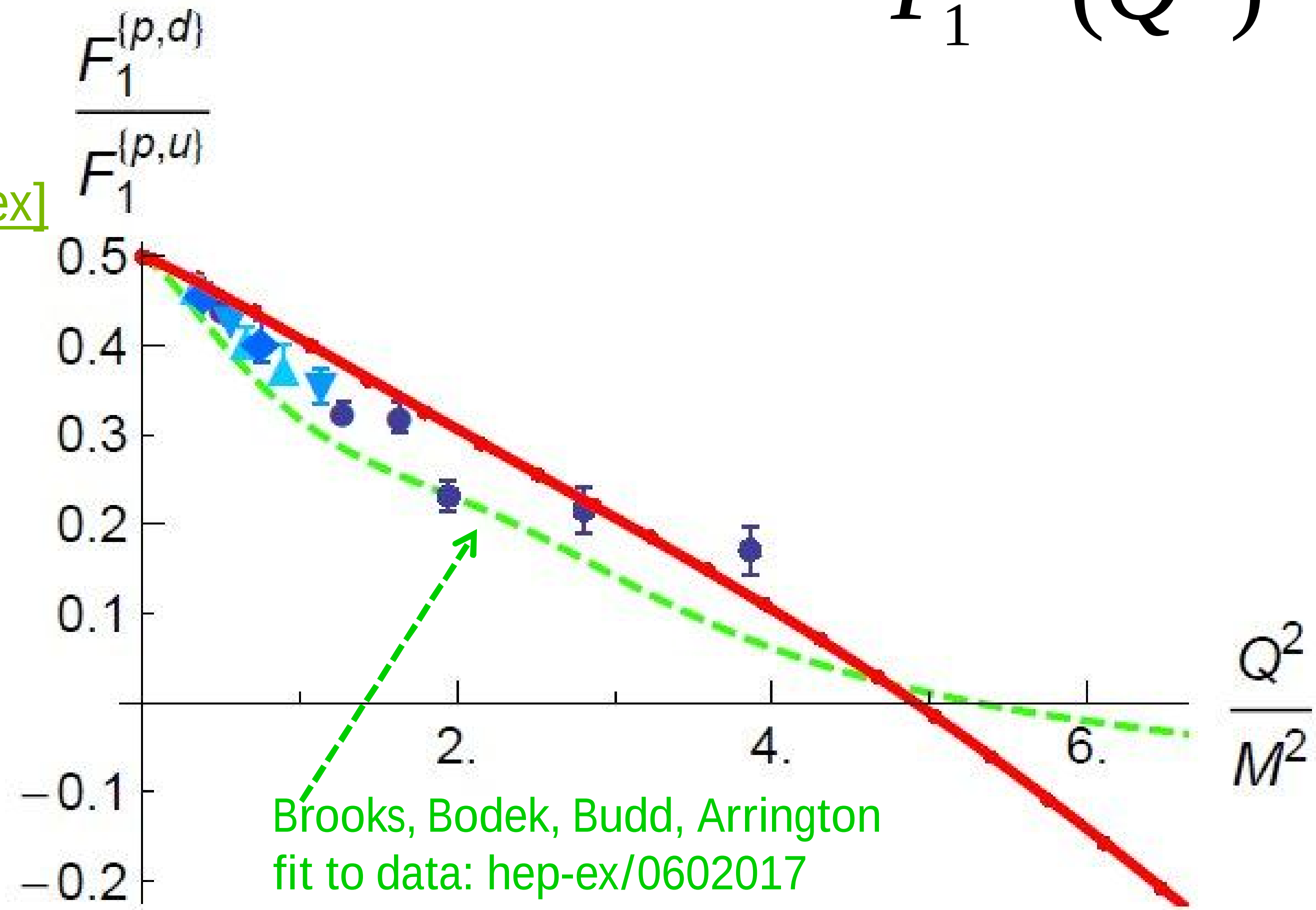
- New JLab data:
  S. Riordan *et al.*,
  [arXiv:1008.1738 [nucl-ex]]



I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th]

$$\frac{F_1^{p,d}(Q^2)}{F_1^{p,u}(Q^2)}$$

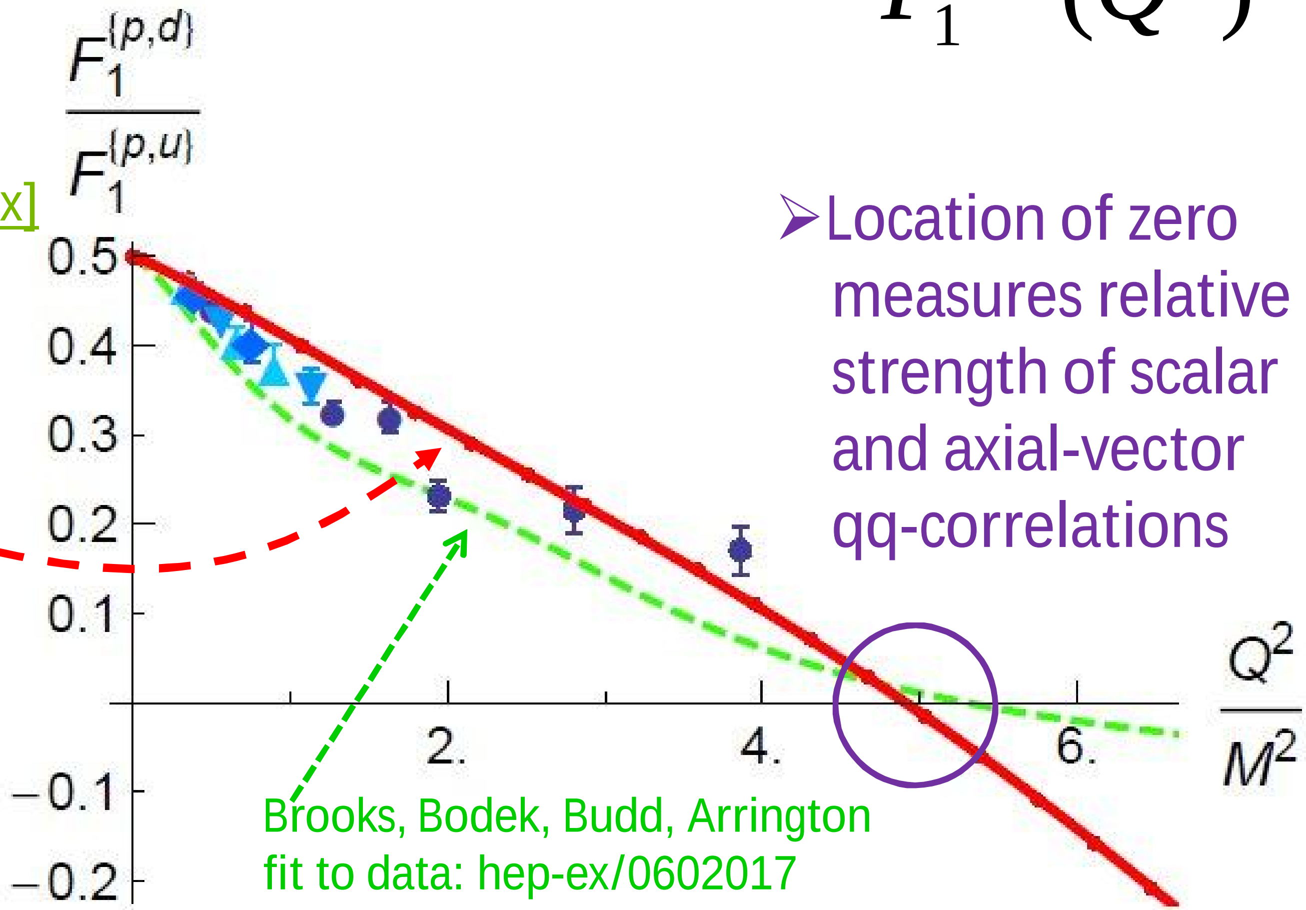
- New JLab data: S. Riordan *et al.*, arXiv:1008.1738 [nucl-ex]



I.C. Cloët, C.D. Roberts, *et al.*
arXiv:0812.0416 [nucl-th]

$$\frac{F_1^{p,d}(Q^2)}{F_1^{p,u}(Q^2)}$$

- New JLab data: S. Riordan *et al.*, arXiv:1008.1738 [nucl-ex]
- DSE-prediction

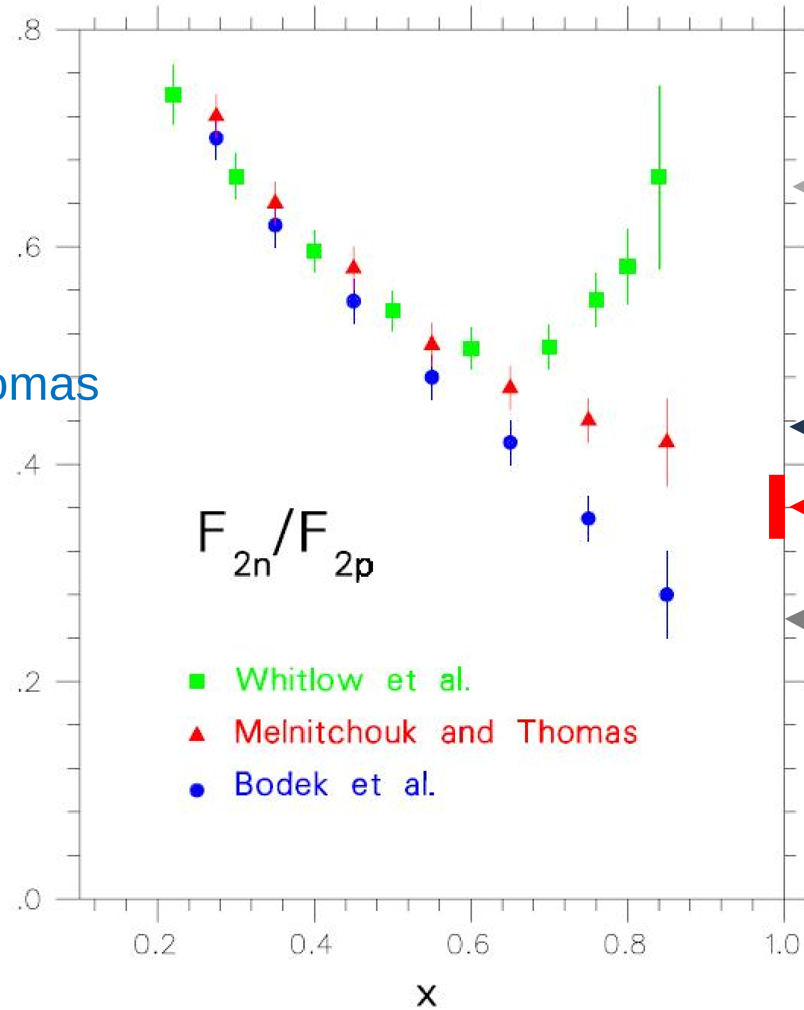


- Location of zero measures relative strength of scalar and axial-vector qq-correlations

I.C. Cloët, C.D. Roberts, *et al.*
[arXiv:0812.0416 [nucl-th]]

Neutron Structure Function at high x

Reviews:
- S. Brodsky *et al.*
  NP B441 (1995)
- W. Melnitchouk & A.W.Thomas
  PL B377 (1996) 11
- N. Isgur, PRD 59 (1999)
- R.J. Holt & C.D. Roberts
  RMP (2010)

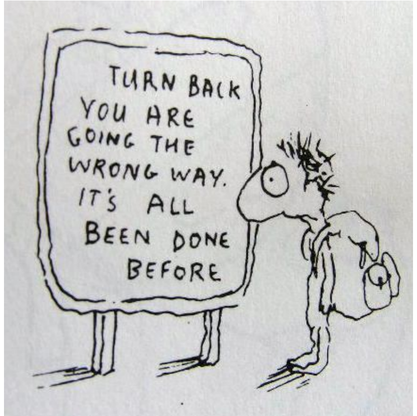


SU(6) symmetry

pQCD

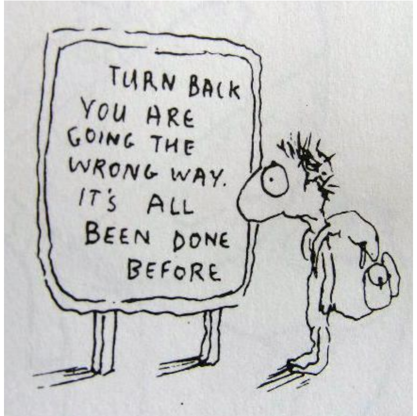
DSE: 0^+ & 1^+ qq

0^+ qq only



Epilogue

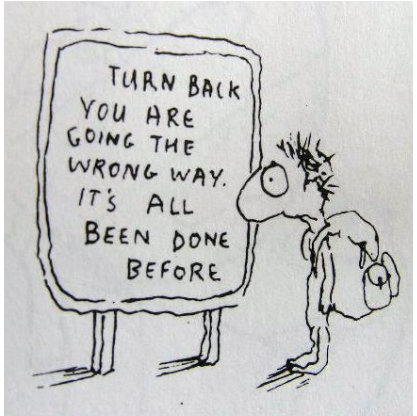
- Dynamical chiral symmetry breaking (DCSB) is a reality
  - Expressed in $M(p^2)$, with observable signals in experiment



Epilogue

- Dynamical chiral symmetry breaking (DCSB) is a reality
  - Expressed in $M(p^2)$, with observable signals in experiment
- Poincaré covariance

  Crucial in description of contemporary data

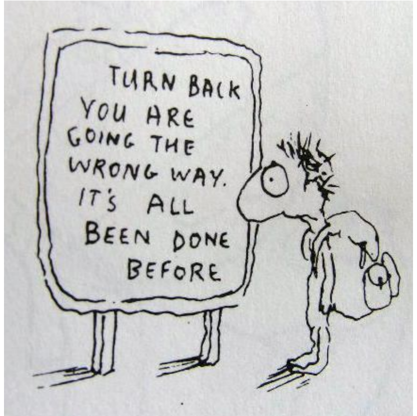


Epilogue

- Dynamical chiral symmetry breaking (DCSB) is a reality
  - Expressed in $M(p^2)$, with observable signals in experiment
- Poincaré covariance

  Crucial in description of contemporary data
- Fully-self-consistent treatment of an interaction

  Essential if experimental data is *truly* to be *understood*.



Epilogue

- Dynamical chiral symmetry breaking (DCSB) is a reality
  - Expressed in $M(p^2)$, with observable signals in experiment
- Poincaré covariance

  Crucial in description of contemporary data
- Fully-self-consistent treatment of an interaction

  Essential if experimental data is *truly* to be *understood*.
- Dyson-Schwinger equations:
  - single framework, with IR model-input turned to advantage, *"almost unique in providing unambiguous path from a defined interaction → Confinement & DCSB → Masses → radii → form factors → distribution functions → etc."* McLerran & Pisarski arXiv:0706.2191 [hep-ph]