Covariance, Dynamics and Symmetries, and Hadron Physics

Craig D. Roberts

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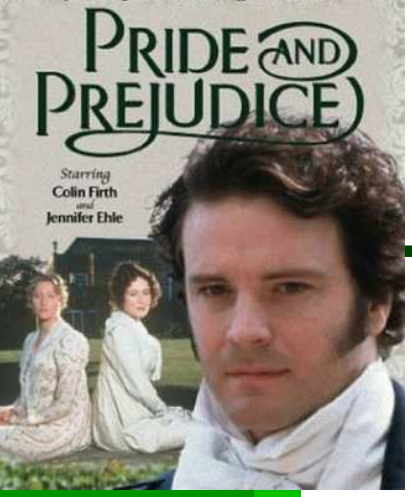
Universal Truths



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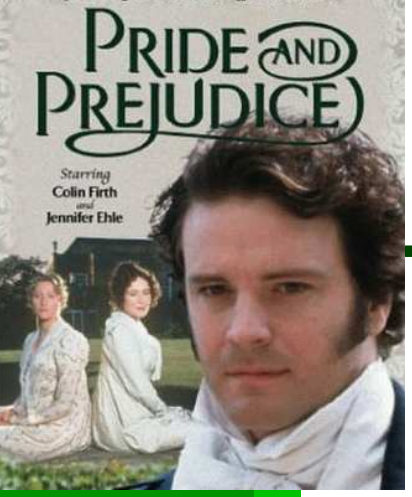

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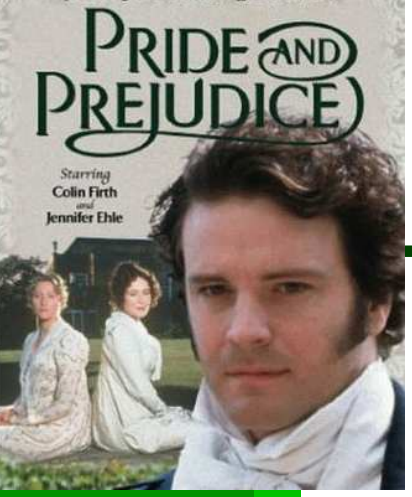

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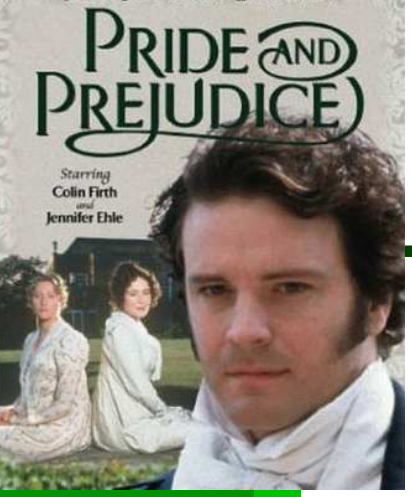

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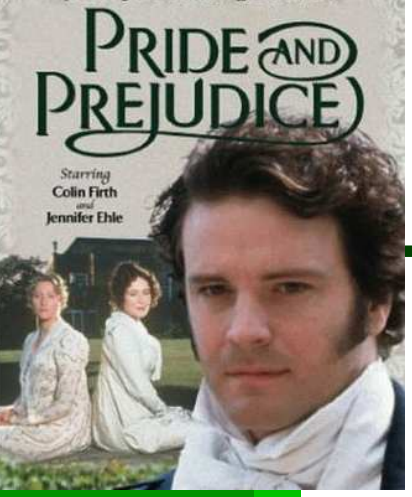

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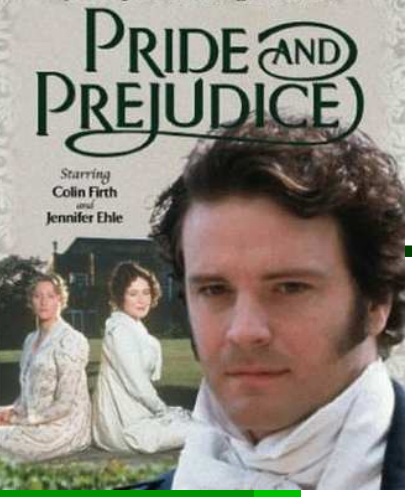

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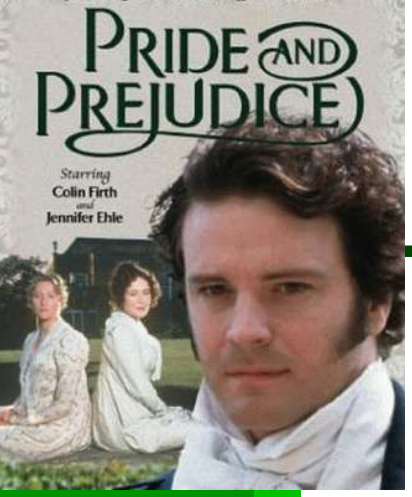
- Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.



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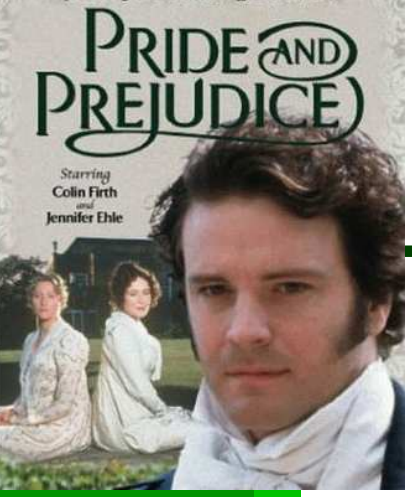

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- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.

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Highly Nontrivial

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  - Interaction between quarks – the Interquark "Potential" – unknown throughout $> 98\%$ of a hadron's volume

Intranucleon Interaction

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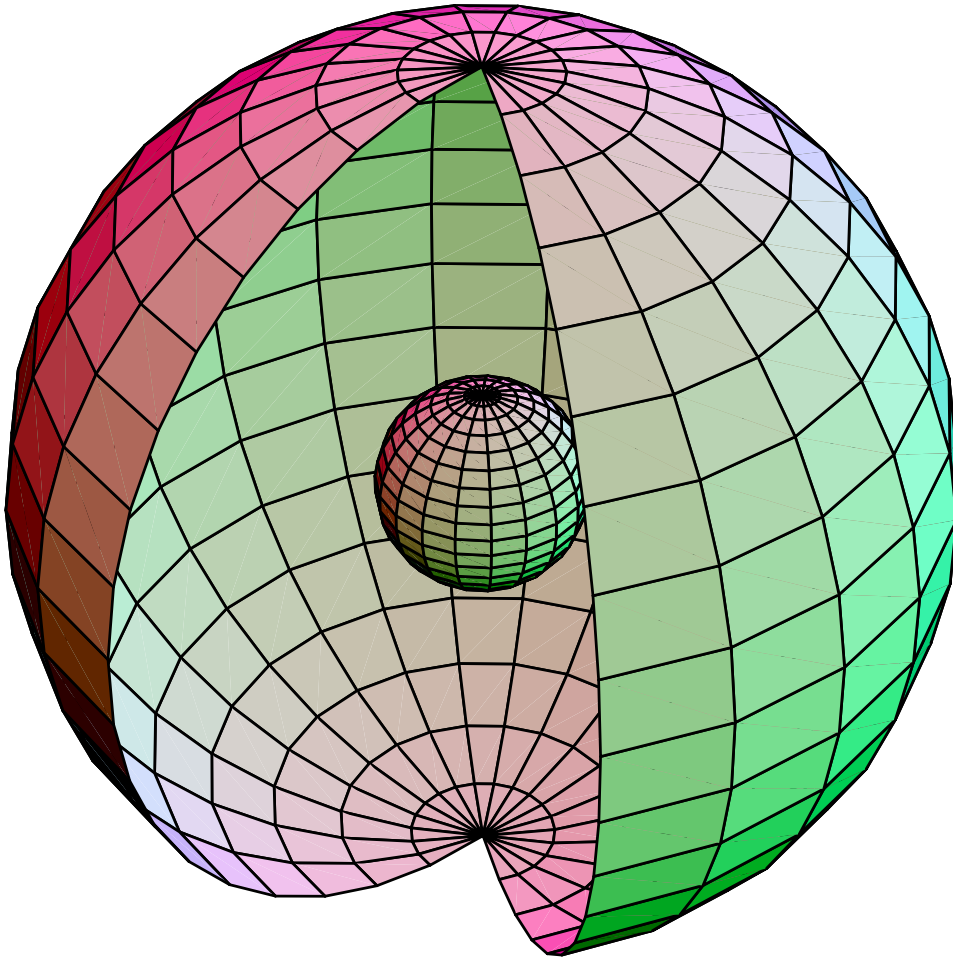

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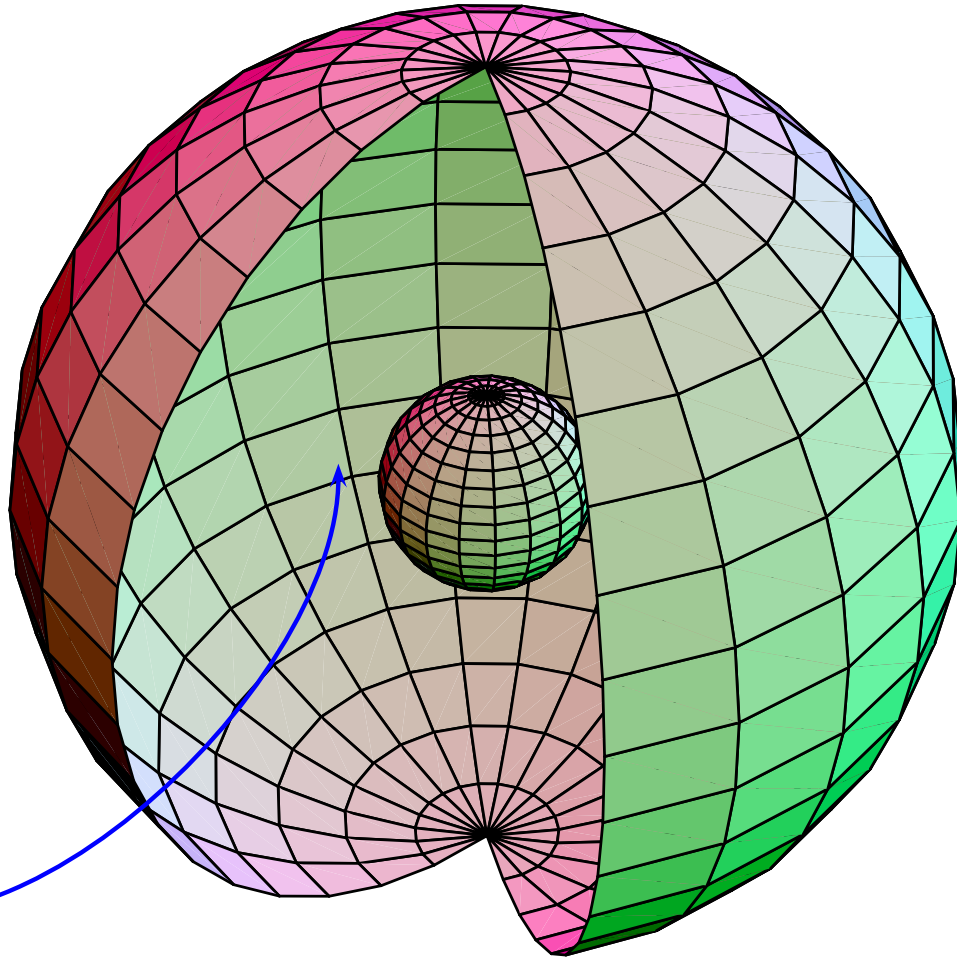
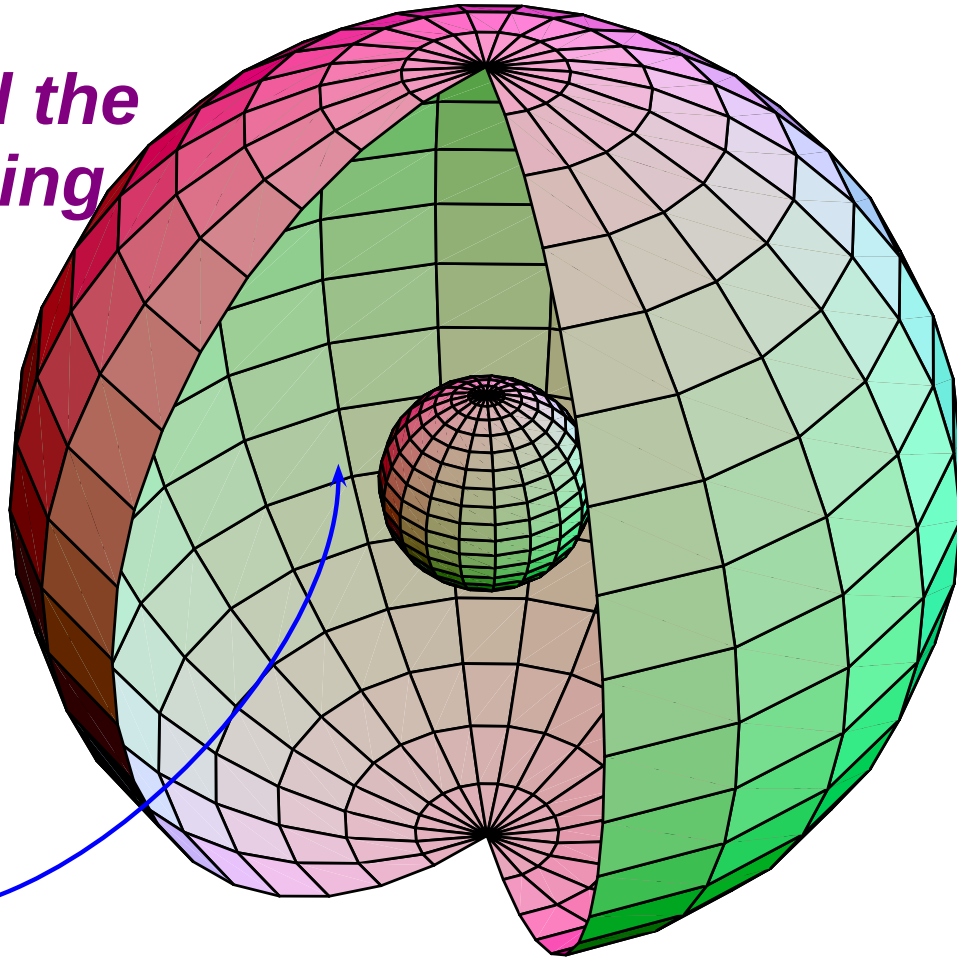


98% of the volume

What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



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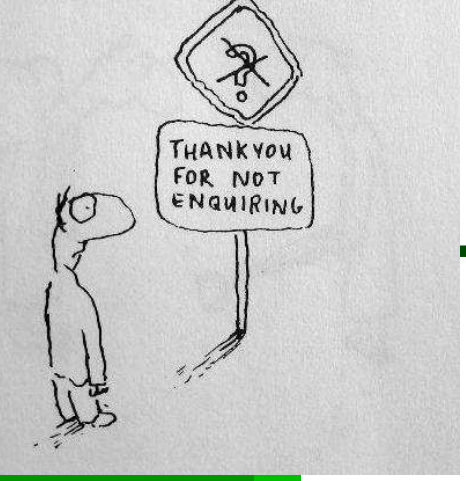
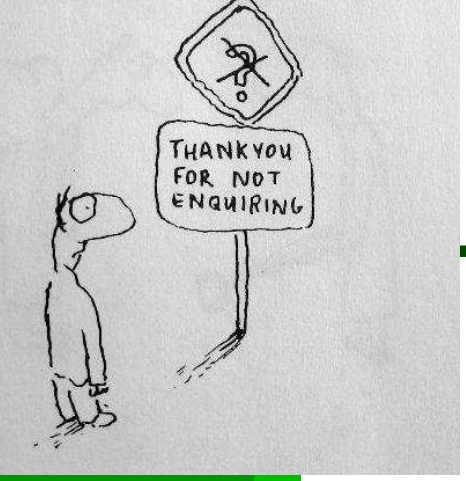


QCD's Challenges



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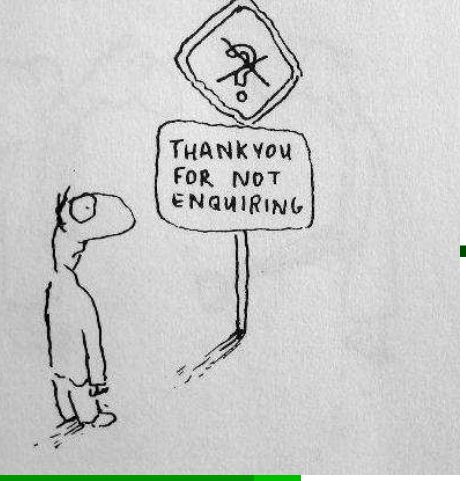

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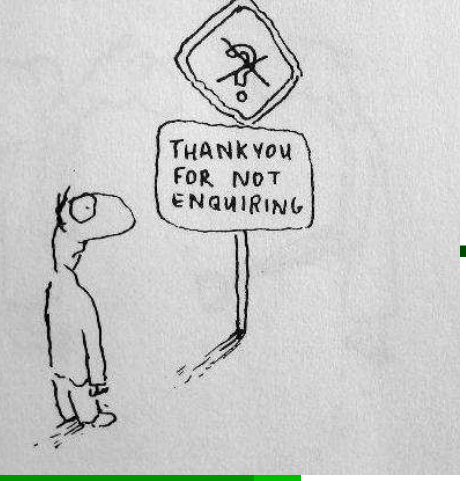

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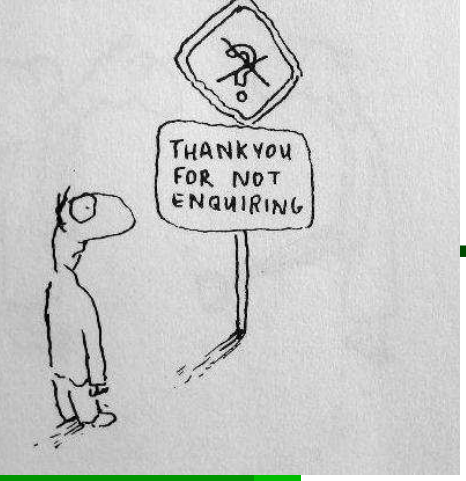

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Understand Emergent Phenomena

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- QCD – Complex behaviour arises from apparently simple rules

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- $\Rightarrow$ Understanding InfraRed (long-range)
  ..... behaviour of $\alpha_s(Q^2)$

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- Method yields Schwinger Functions $\equiv$ Propagators

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Cross-Sections built from Schwinger Functions

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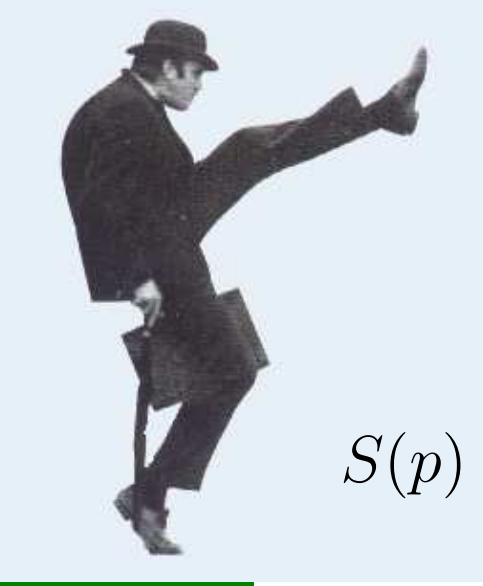

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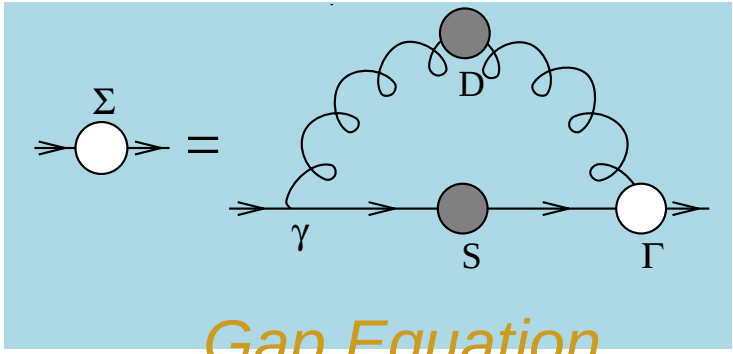

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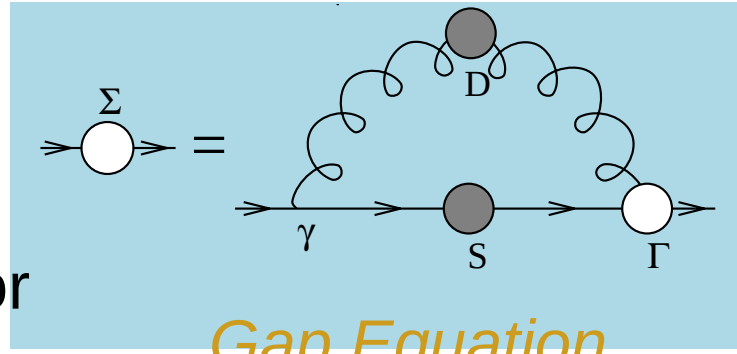

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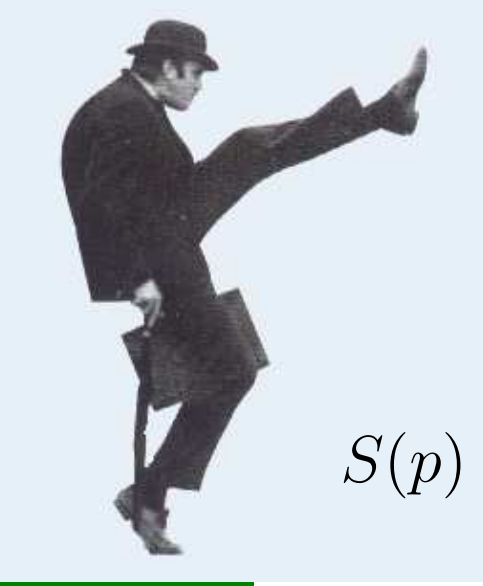

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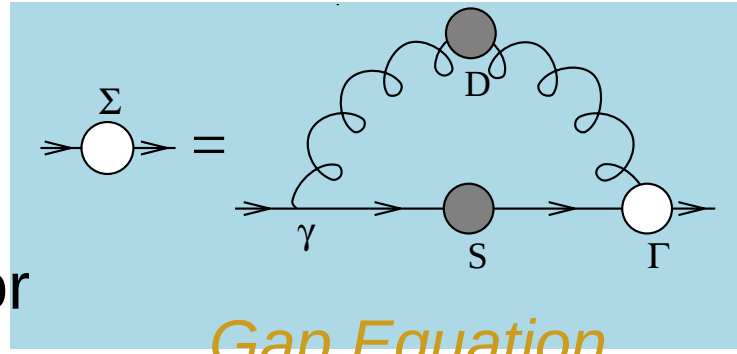

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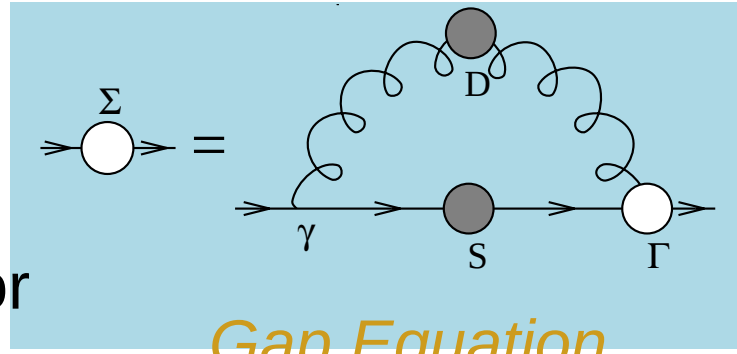

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$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\longrightarrow} 0$$

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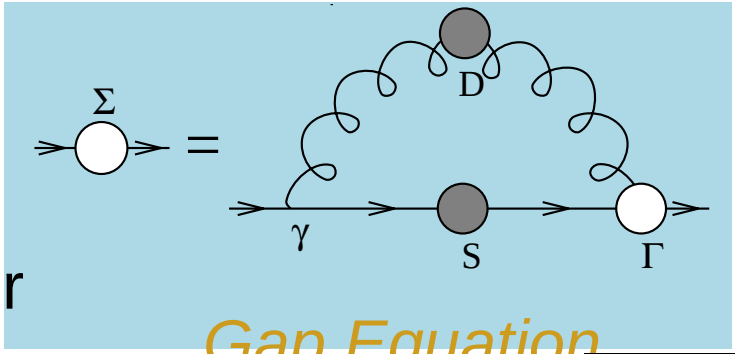

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QCD & Interaction Between Light-Quarks

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  Dressed-gluon propagator and dressed-quark-gluon vertex
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- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
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- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

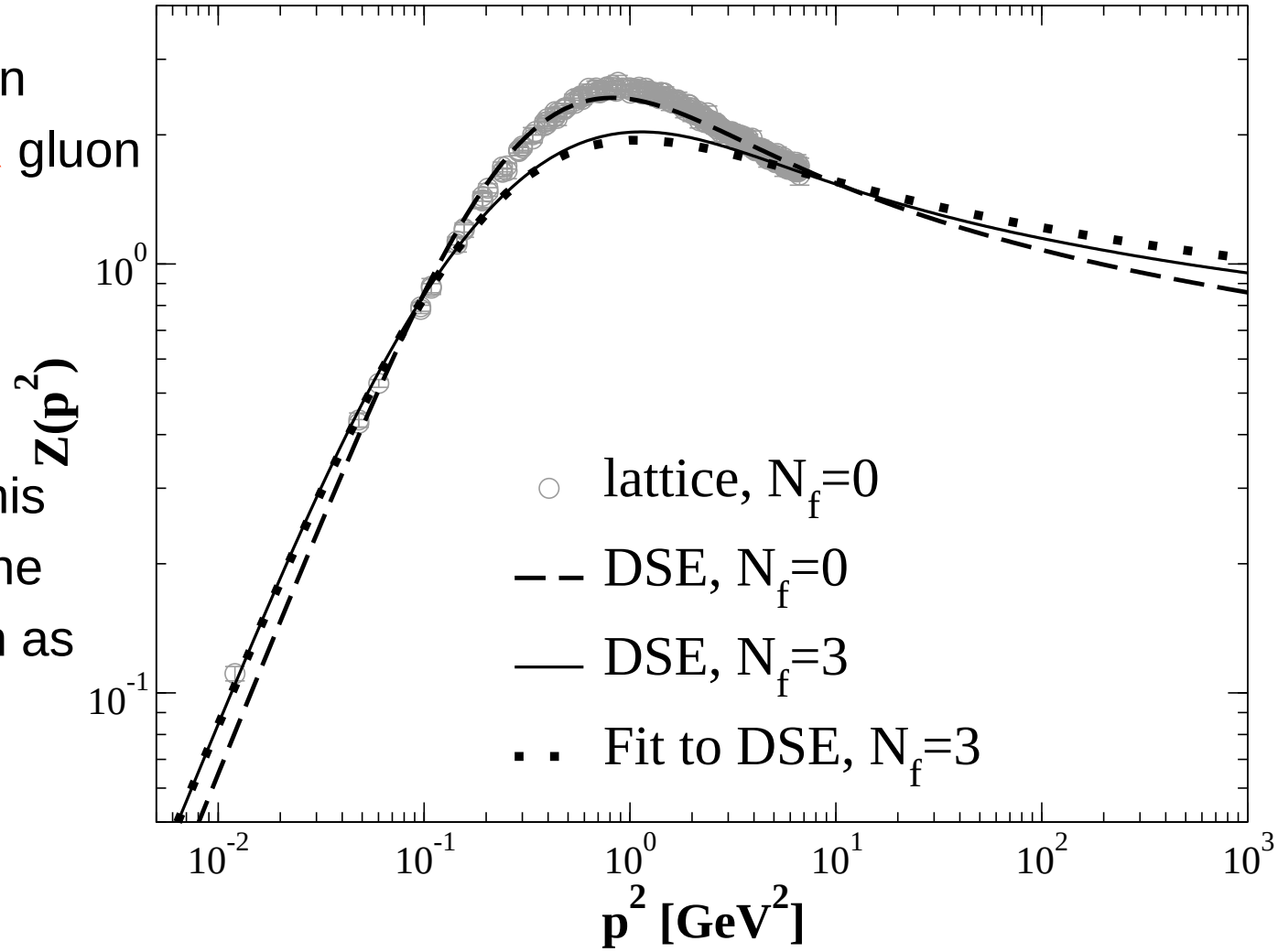
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



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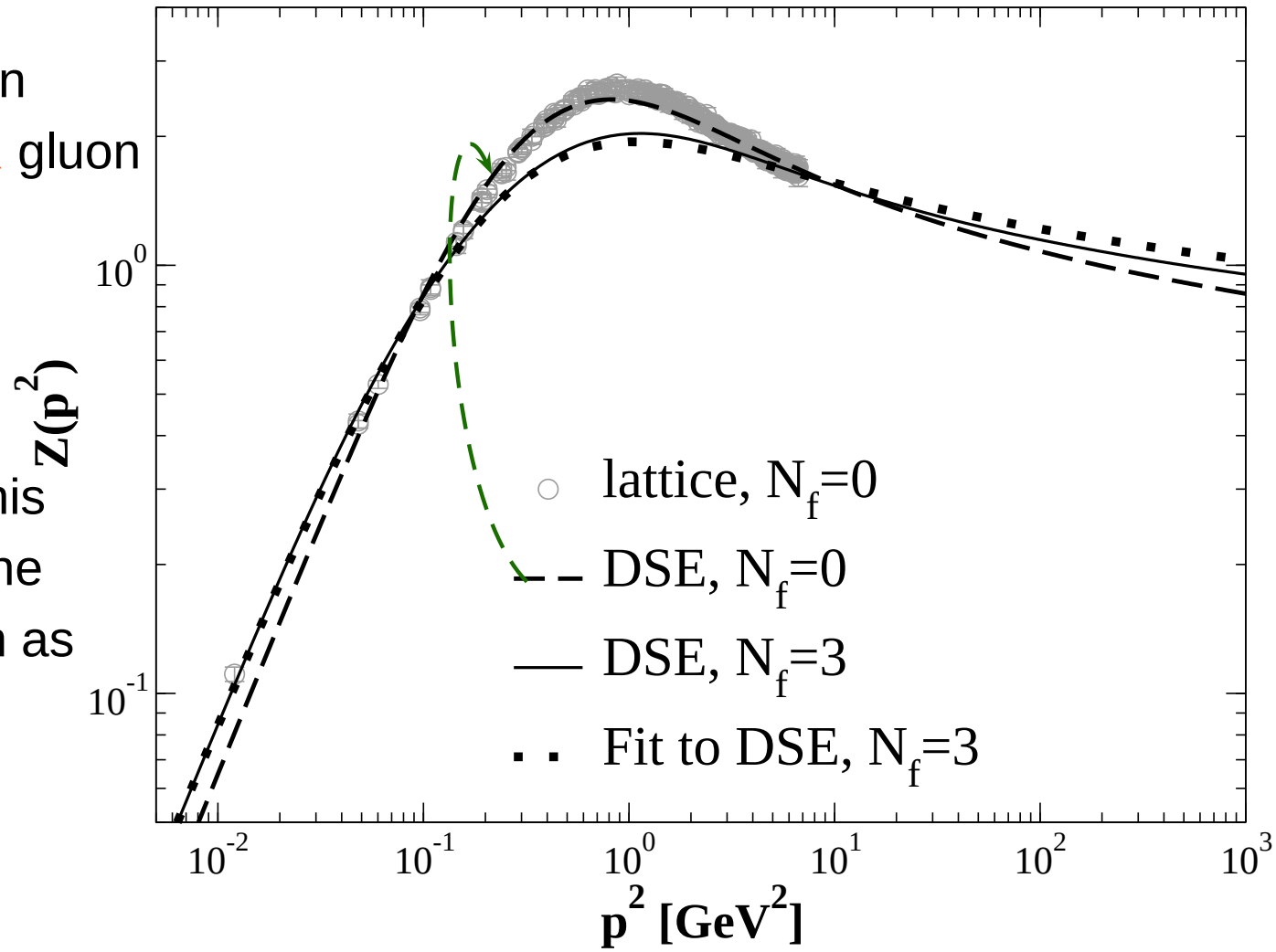

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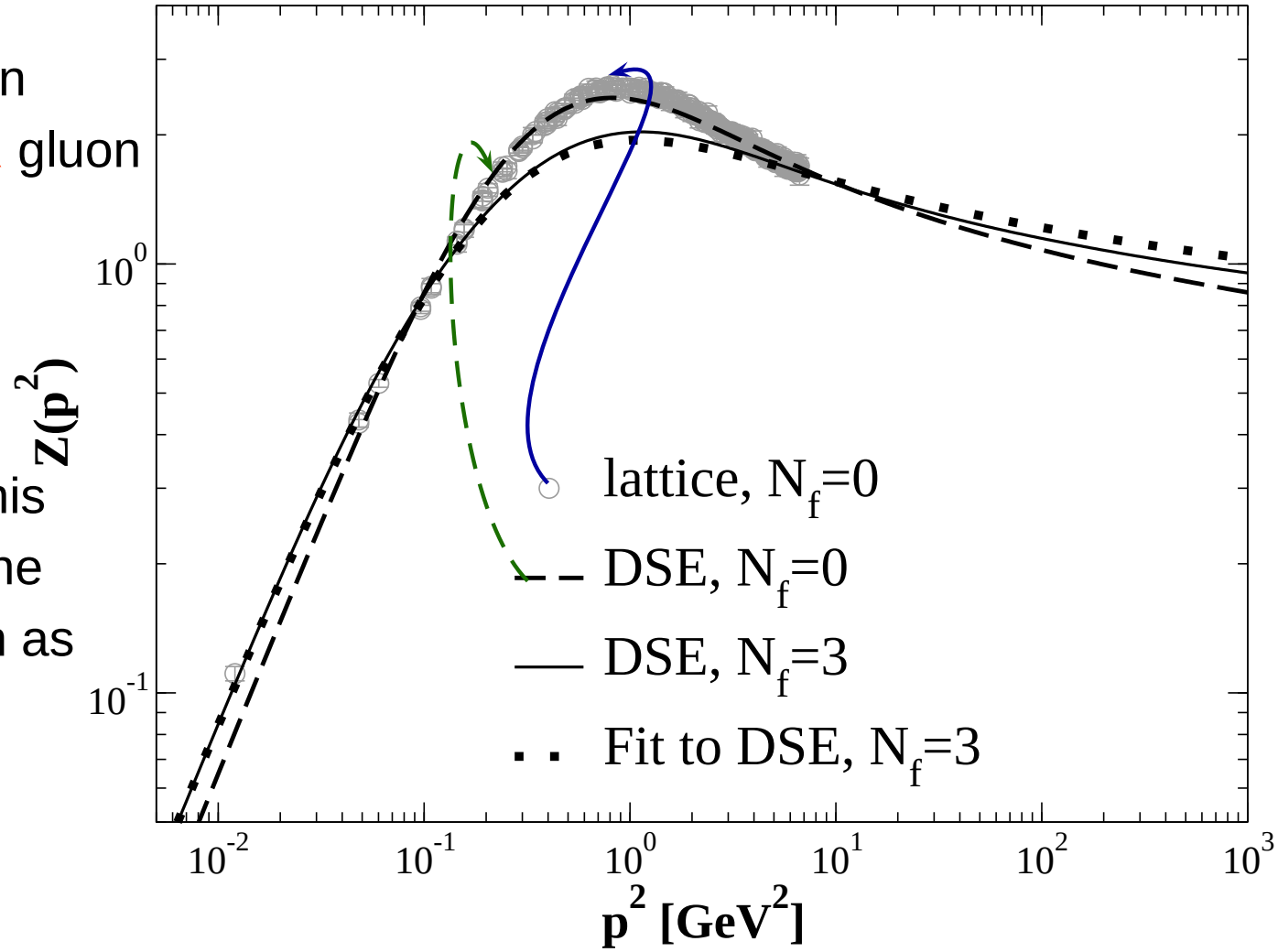

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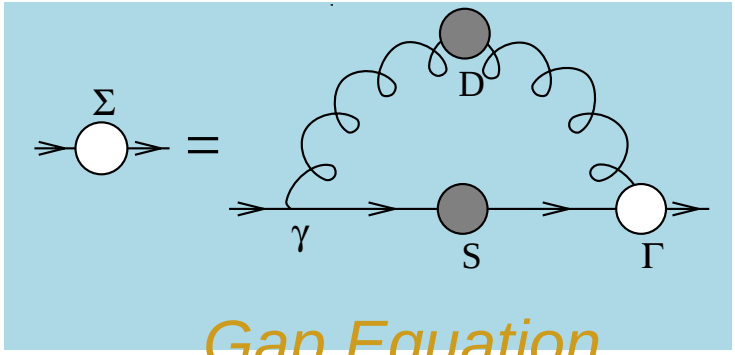

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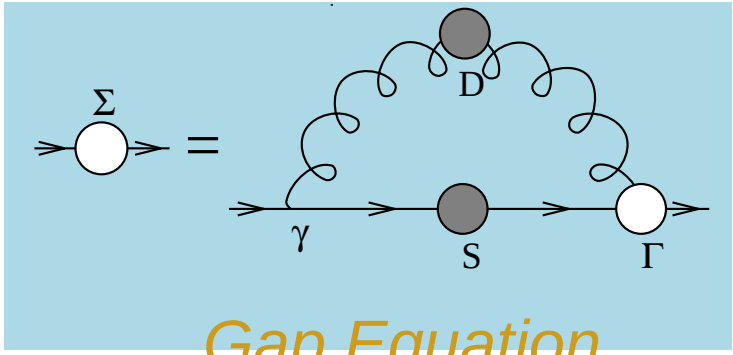
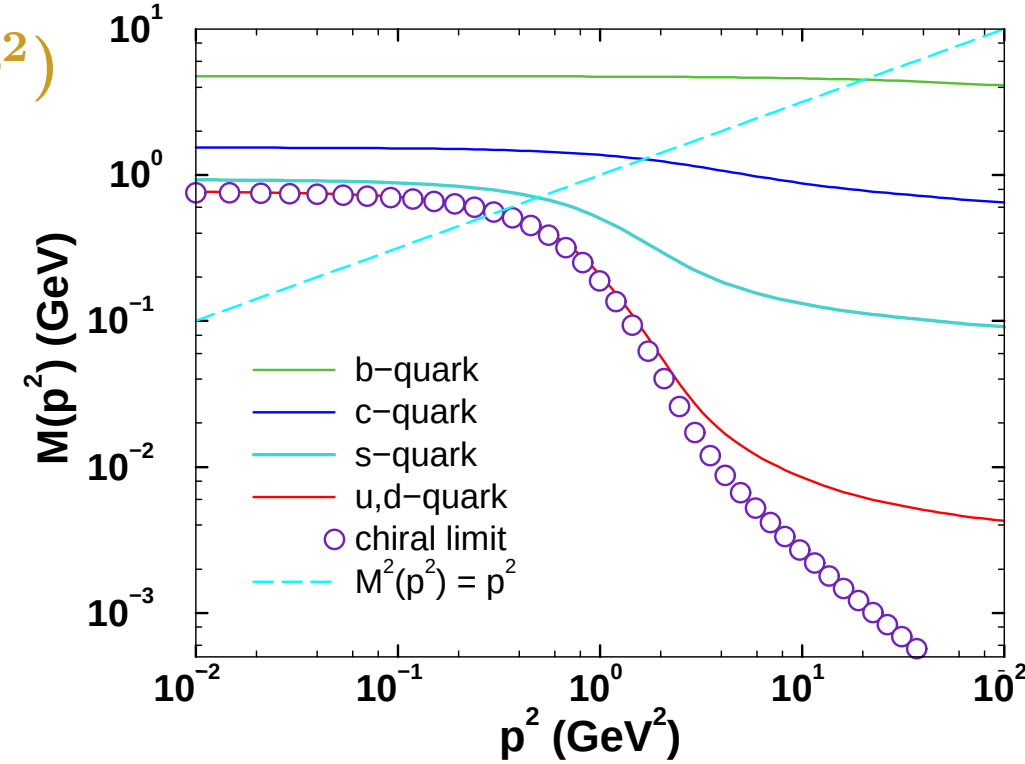


Gap Equation

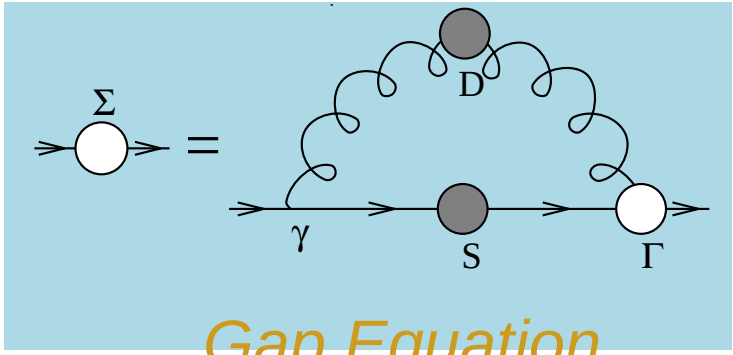
- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



Dyson-Schwinger Equations
Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



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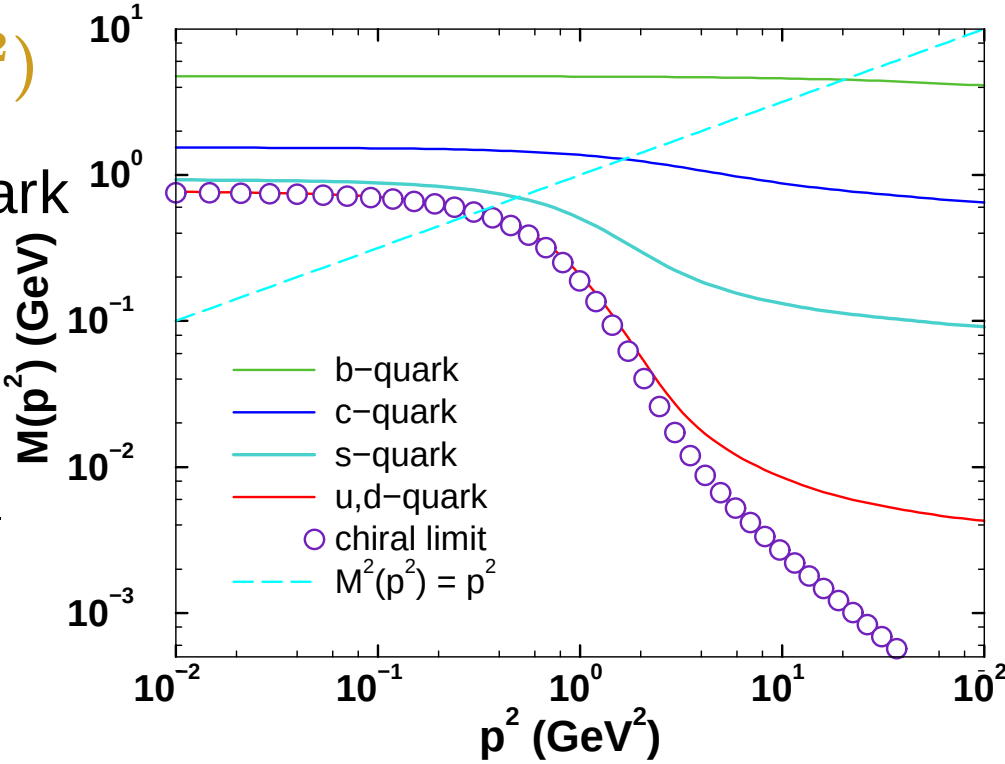
- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



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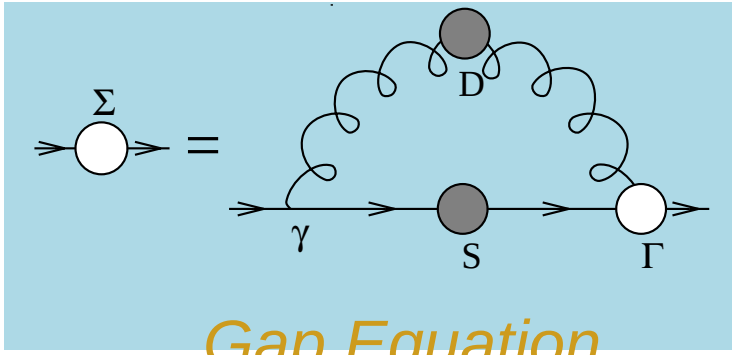

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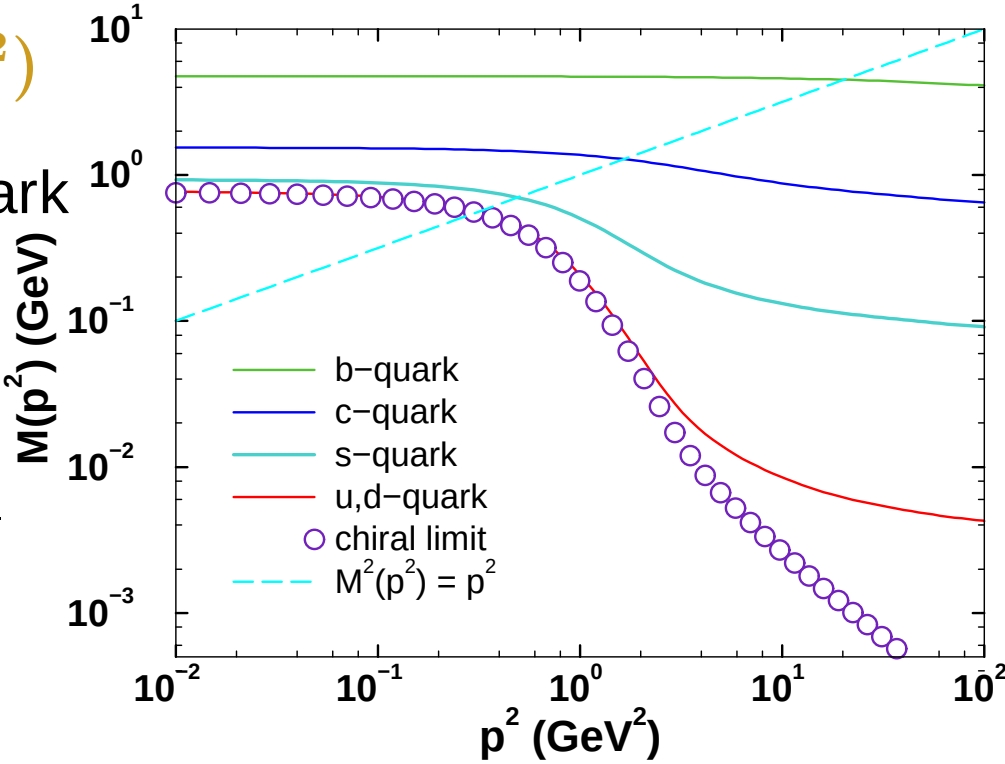
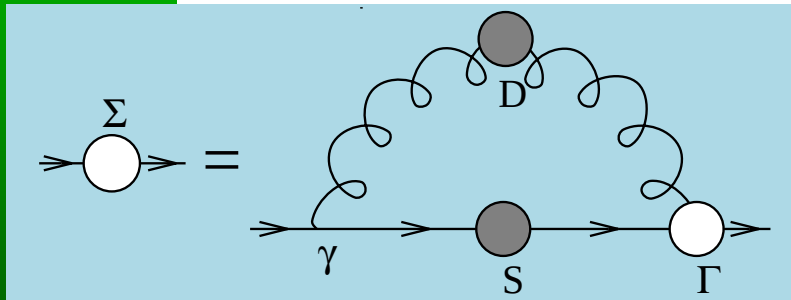
Predictions confirmed in numerical simulations of lattice-QCD



Frontiers of Nuclear Science: A Long Range Plan (2007)

Frontiers of Nuclear Science: Theoretical Advances



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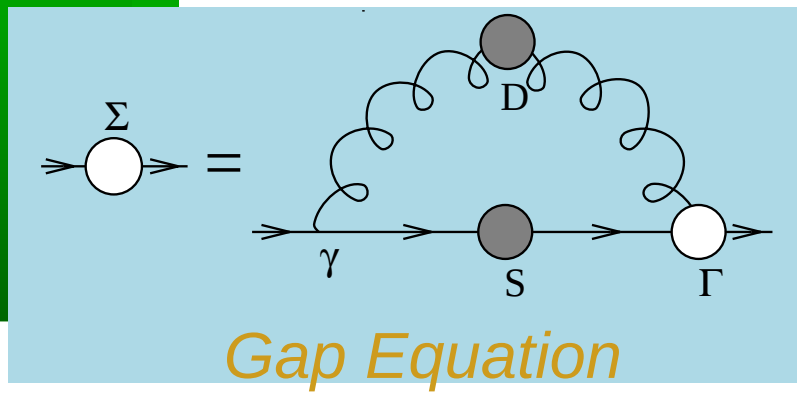
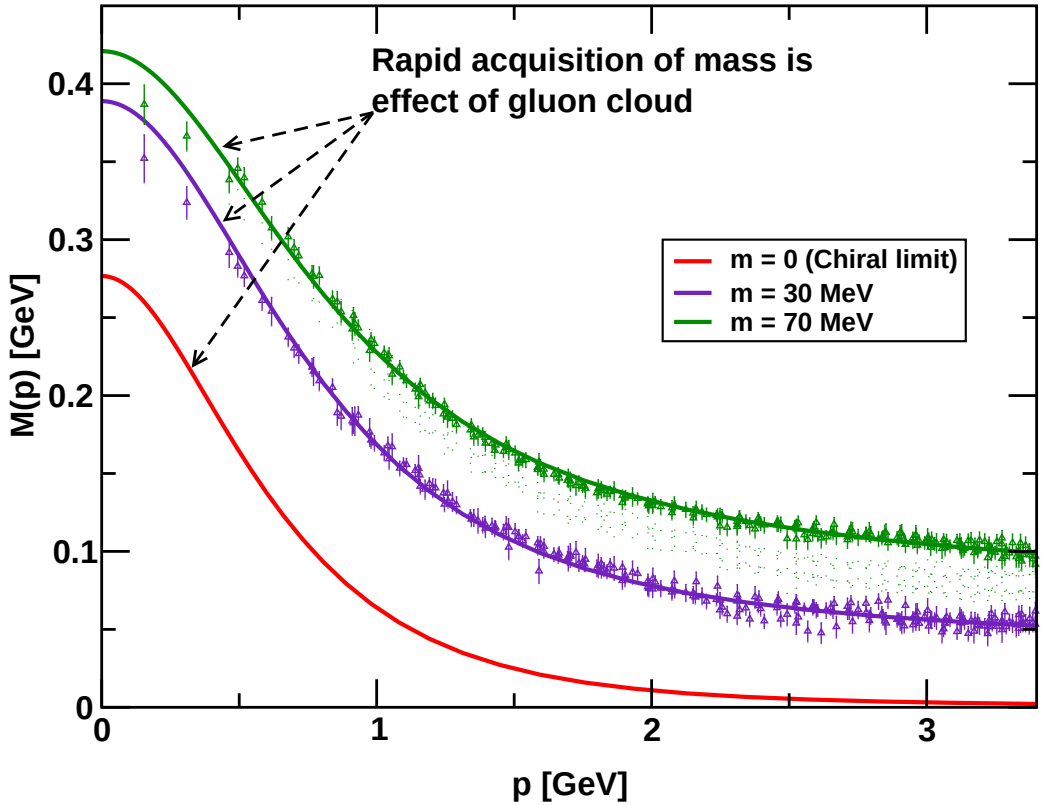
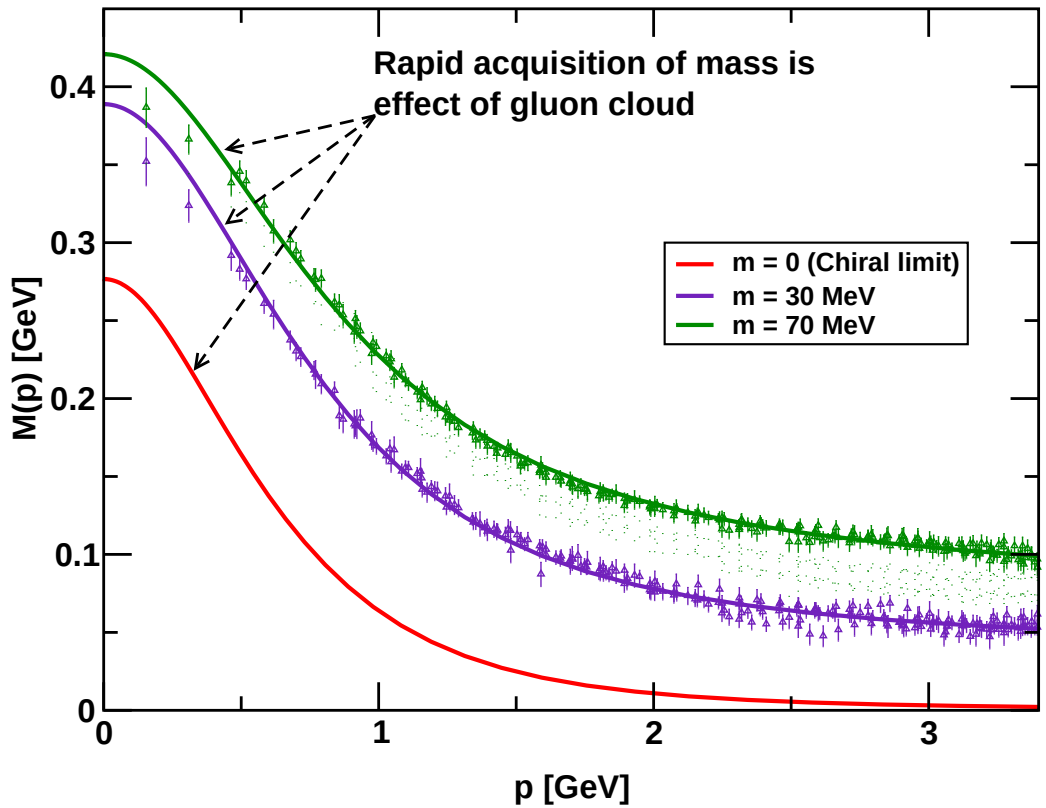




Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.



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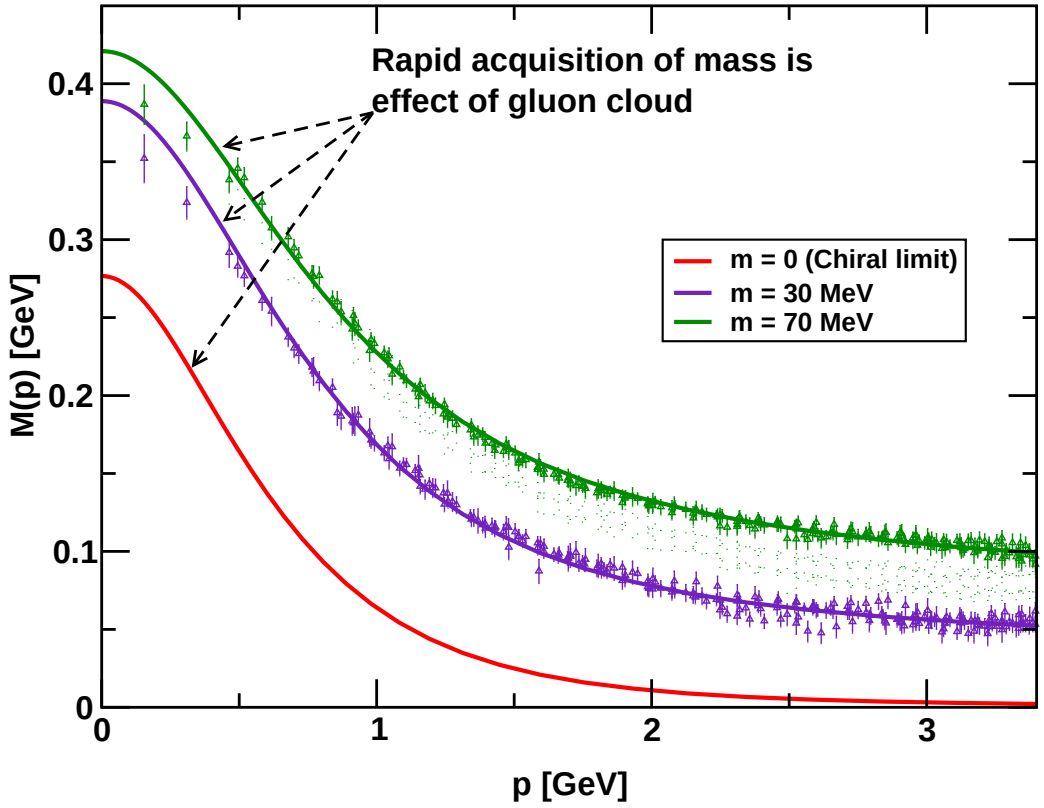

ALWAYS LOOK
ON THE
BRIGHT SIDE

- Established understanding of two- and three-point functions

Hadrons

- Established understanding of two- and three-point functions
- What about bound states?

Hadrons

- Without bound states, Comparison with experiment is impossible

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- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

Hadrons

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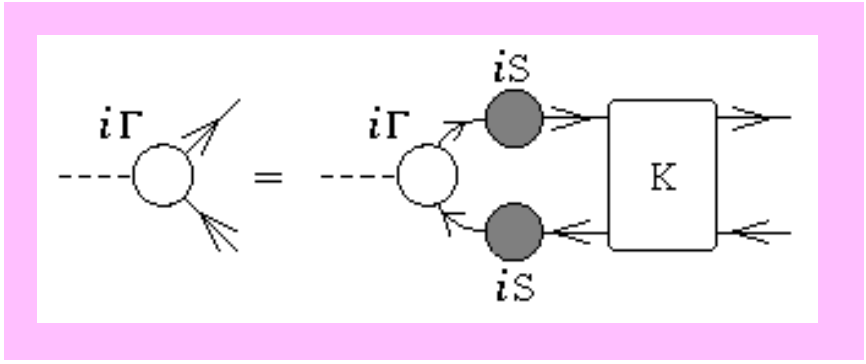
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

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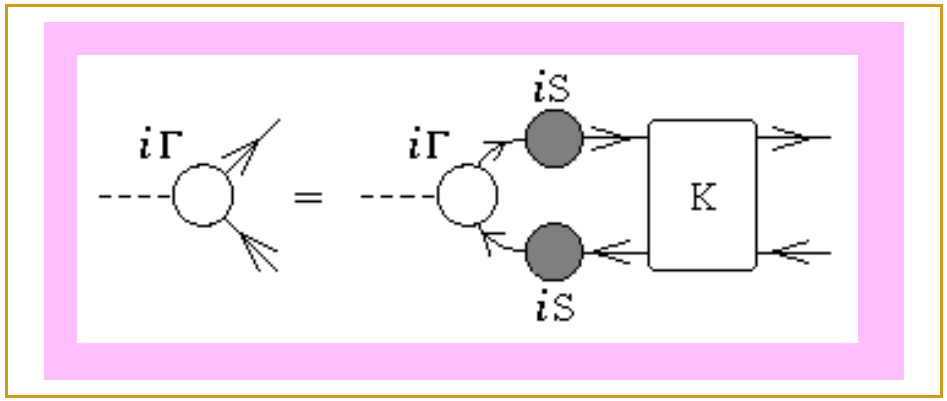


QFT Generalisation of Lippmann-Schwinger Equation.

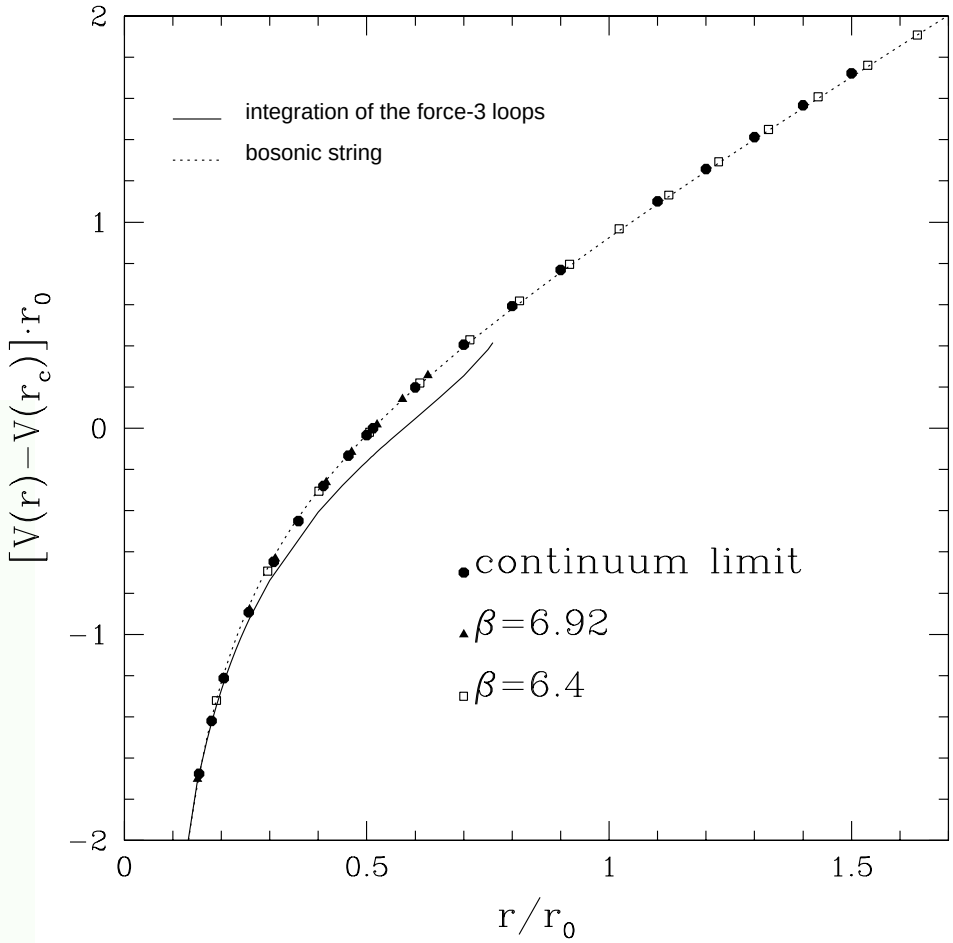
- What is the kernel, K?

or What is the long-range potential in QCD?

Confinement

Confinement

- Infinitely Heavy Quarks . . . Picture in Quantum Mechanics



$$V(r) = \sigma\, r - \frac{\pi}{12}\,\frac{1}{r}$$

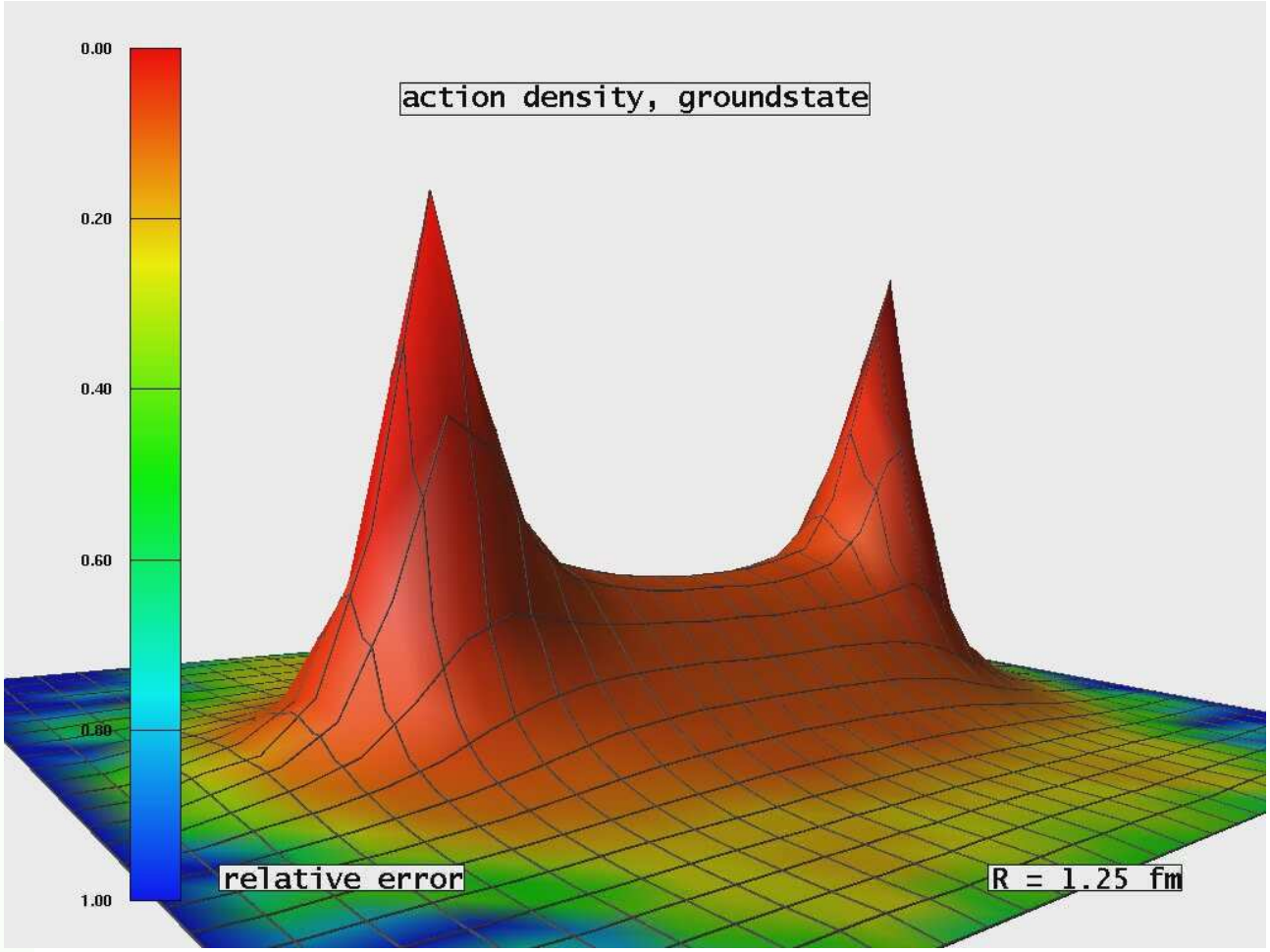
$\sigma \sim 470\,\text{MeV}$

Necco & Sommer

he-la/0108008

Confinement

- Illustrate this in terms of the action density . . . analogous to plotting the Force $= F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12}\frac{1}{r^2}$



Bali, *et al.*
he-la/0512018

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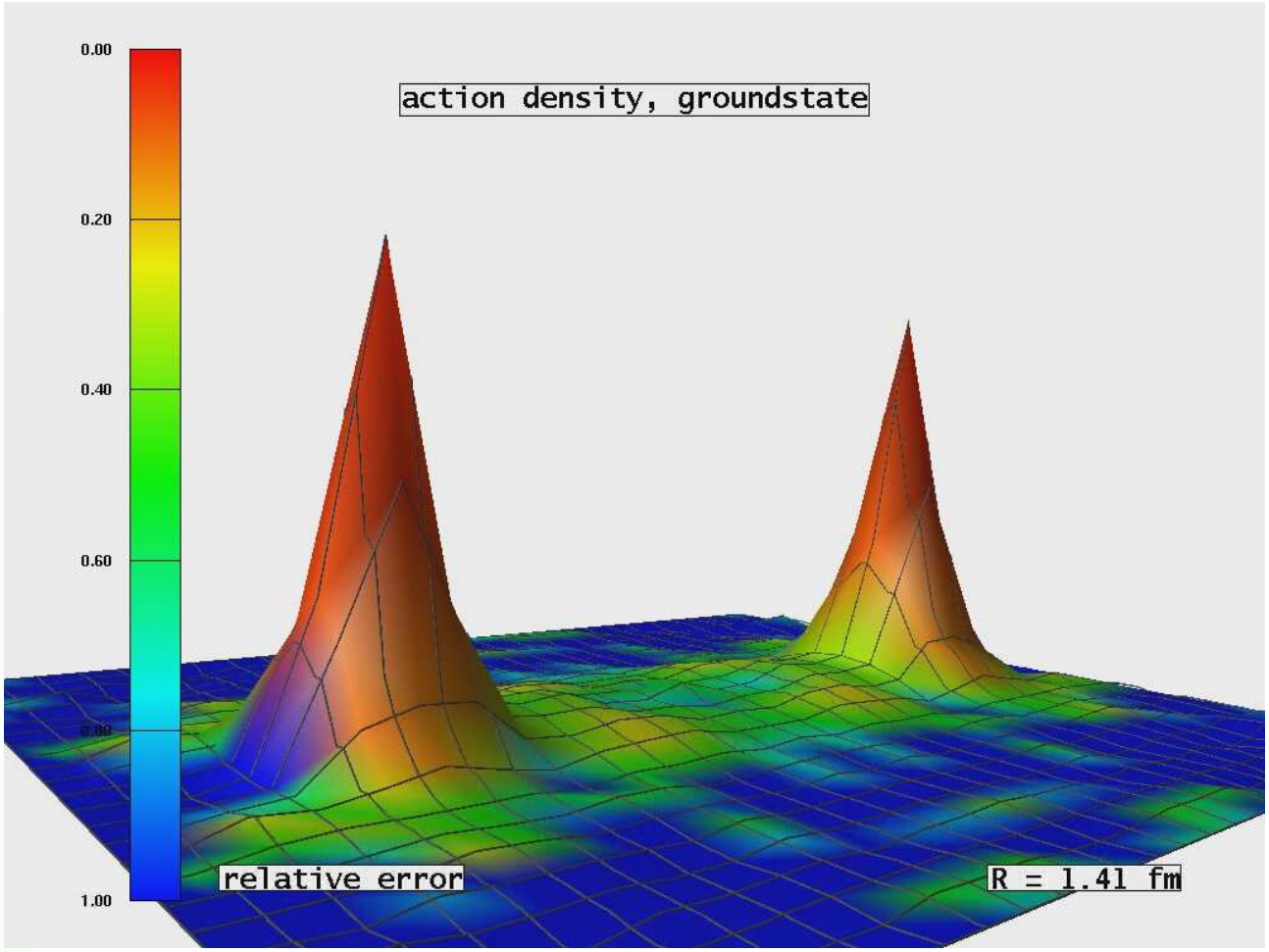

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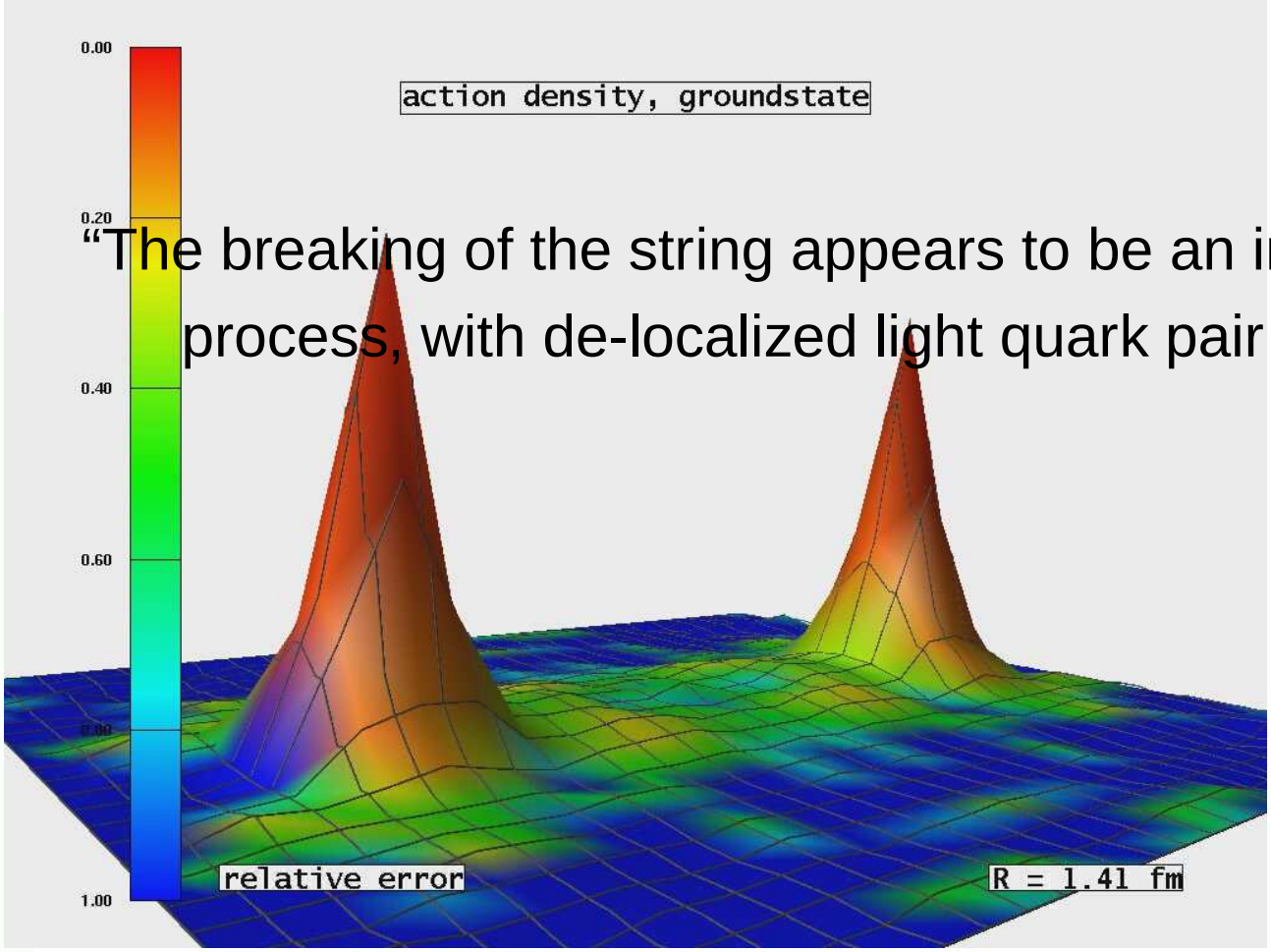

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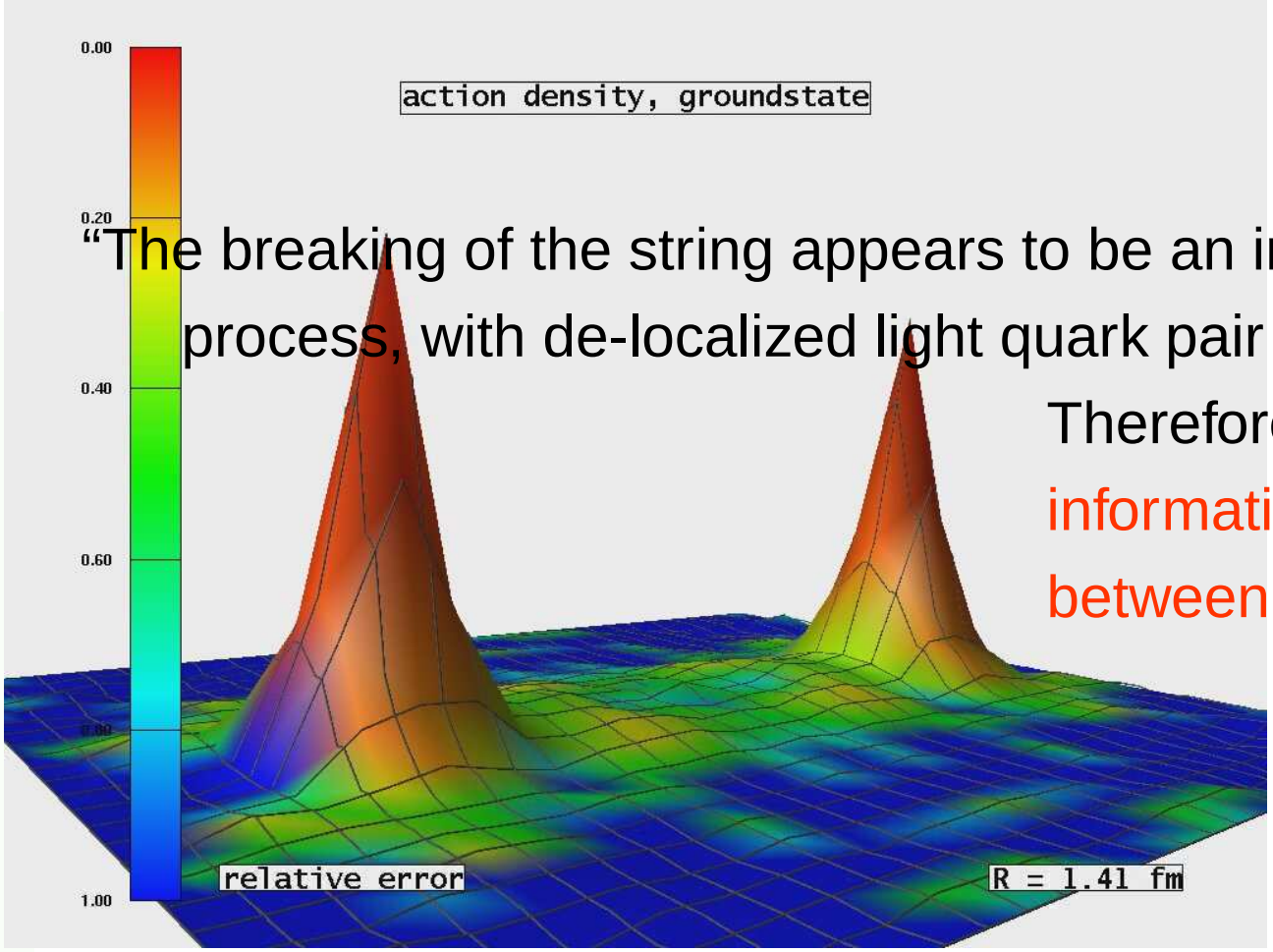

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Therefore . . . No information on *potential* between light-quarks.

What is the light-quark Long-Range Potential?

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Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

Bethe-Salpeter Kernel

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- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

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- Nontrivial constraint

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- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Persistent Challenge

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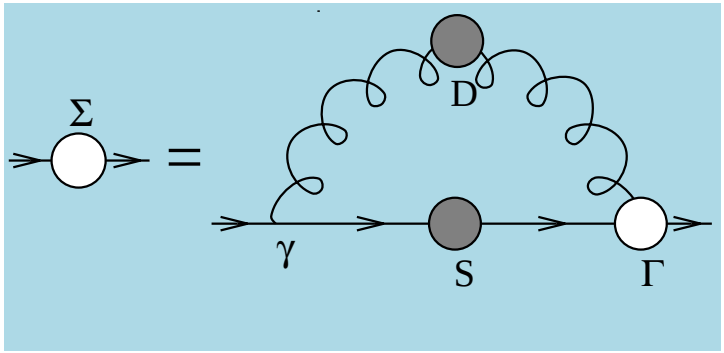
- Infinitely Many Coupled Equations



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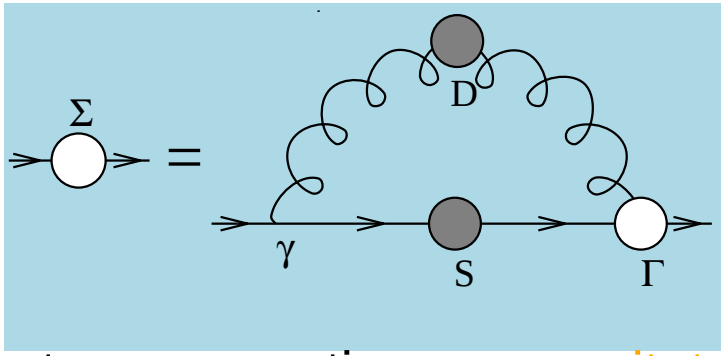

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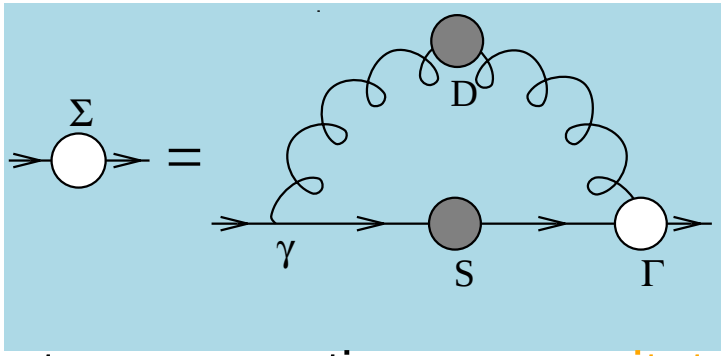

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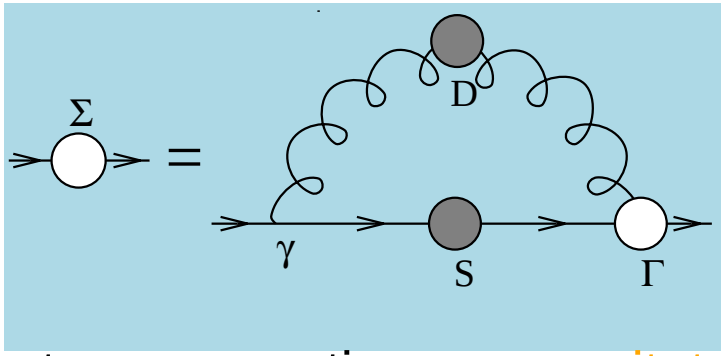

- Coupling between equations necessitates truncation
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    Not useful for the nonperturbative problems
    in which we're interested

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- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

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Radial Excitations & Chiral Symmetry

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(Maris, Roberts, Tandy nu-th/9707003)

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- Mass^2 of pseudoscalar hadron

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$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

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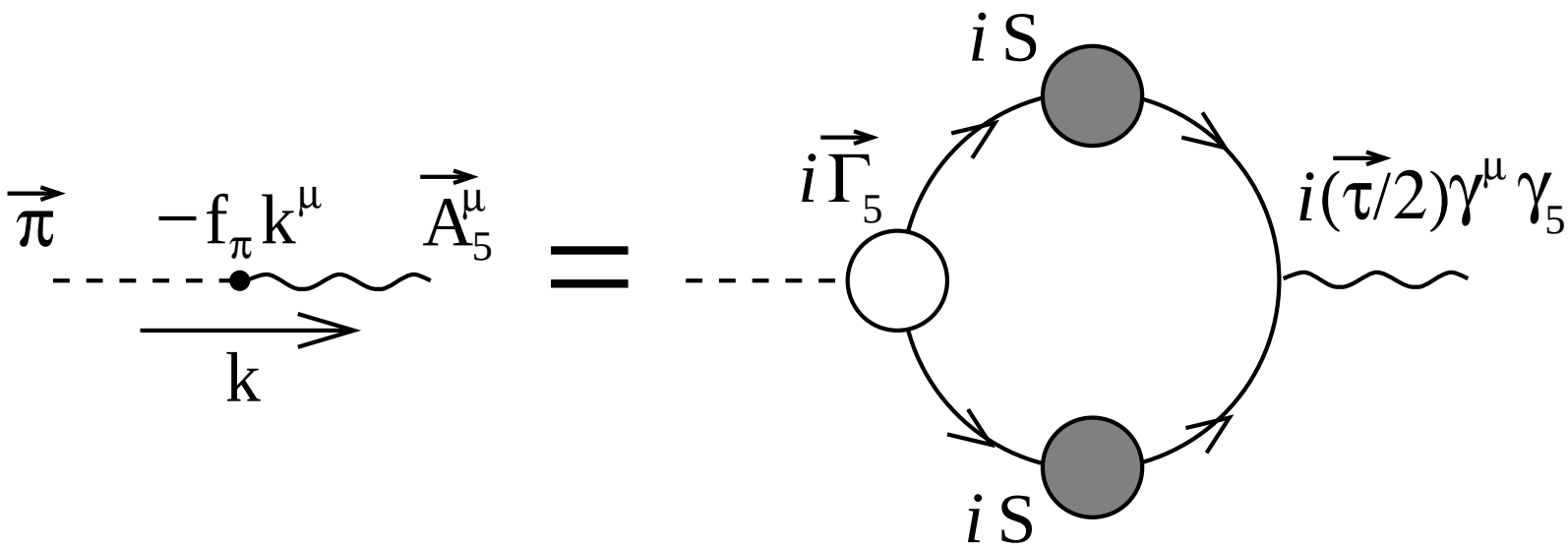
- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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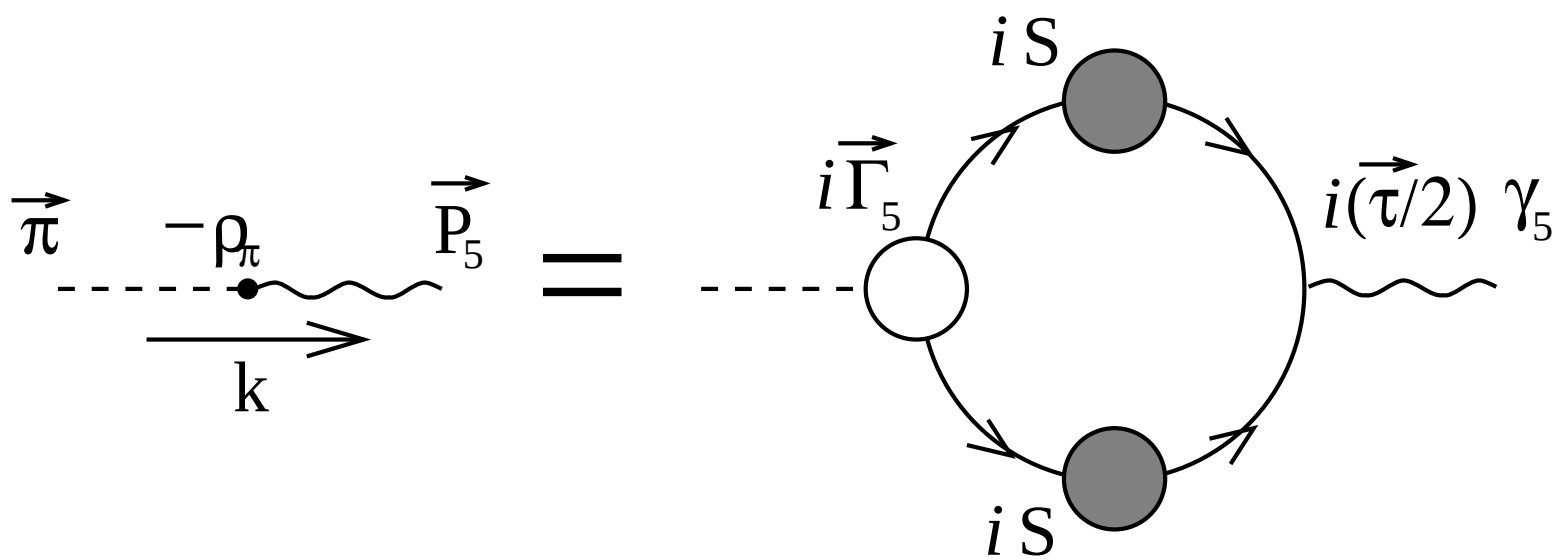

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- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $\boxed{m_H \propto m_q}$

... QCD Proof of Potential Model result

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Höll, Krassnigg, Roberts
nu-th/0406030

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- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

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  Diehl & Hiller
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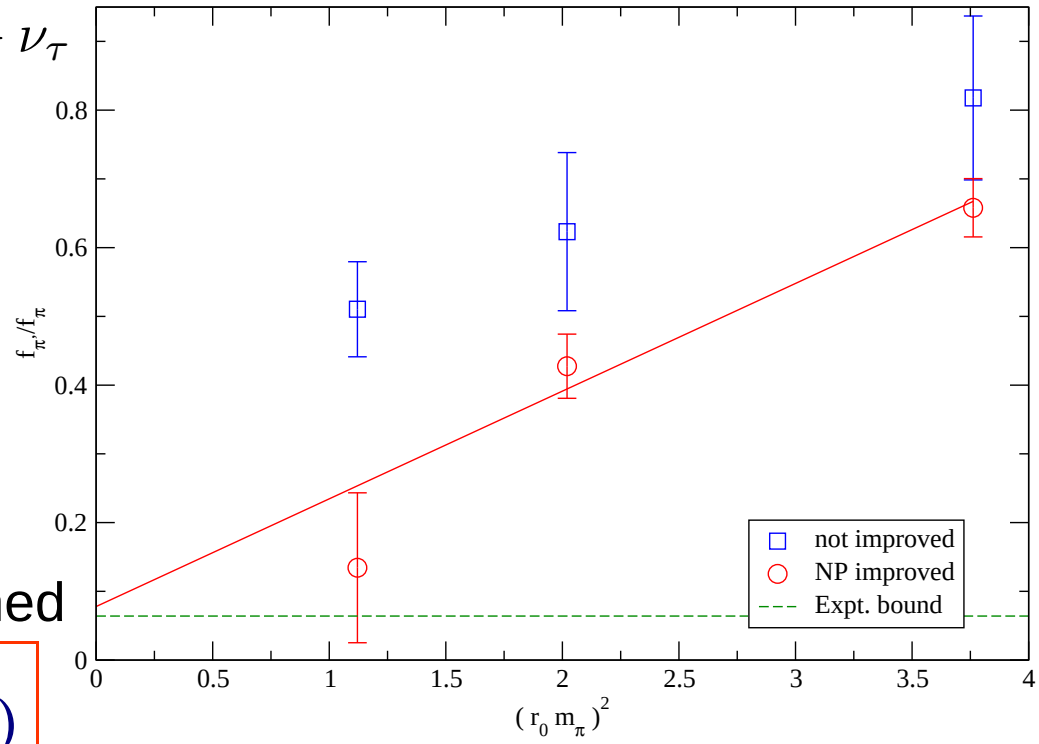
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$



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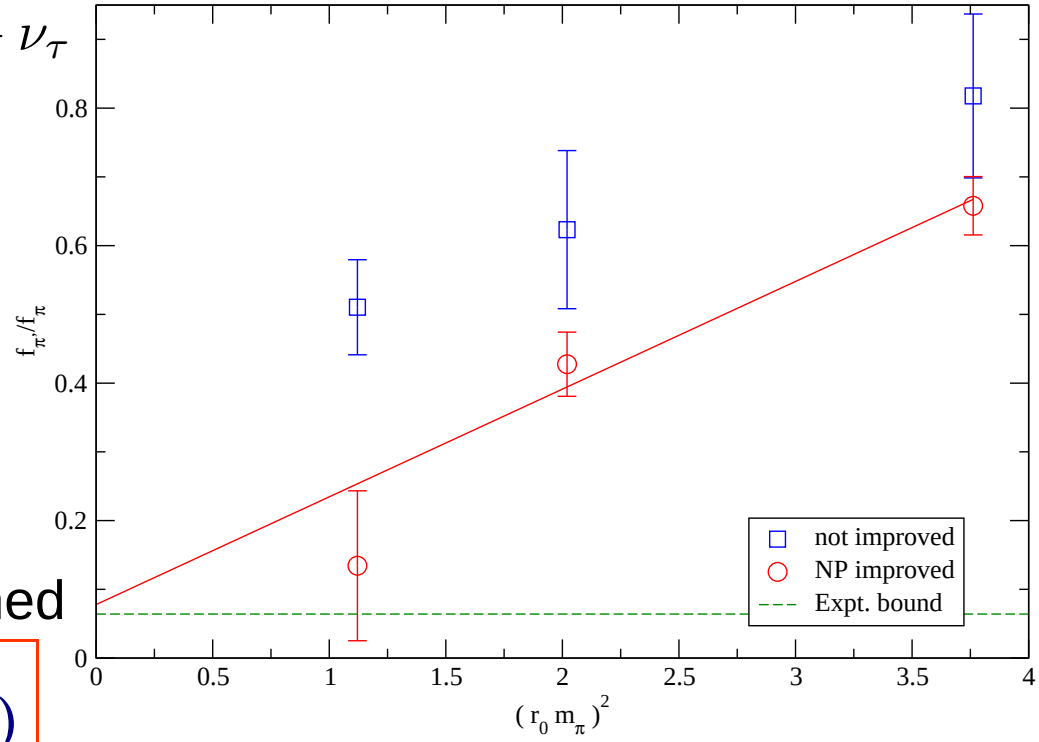
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$



- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

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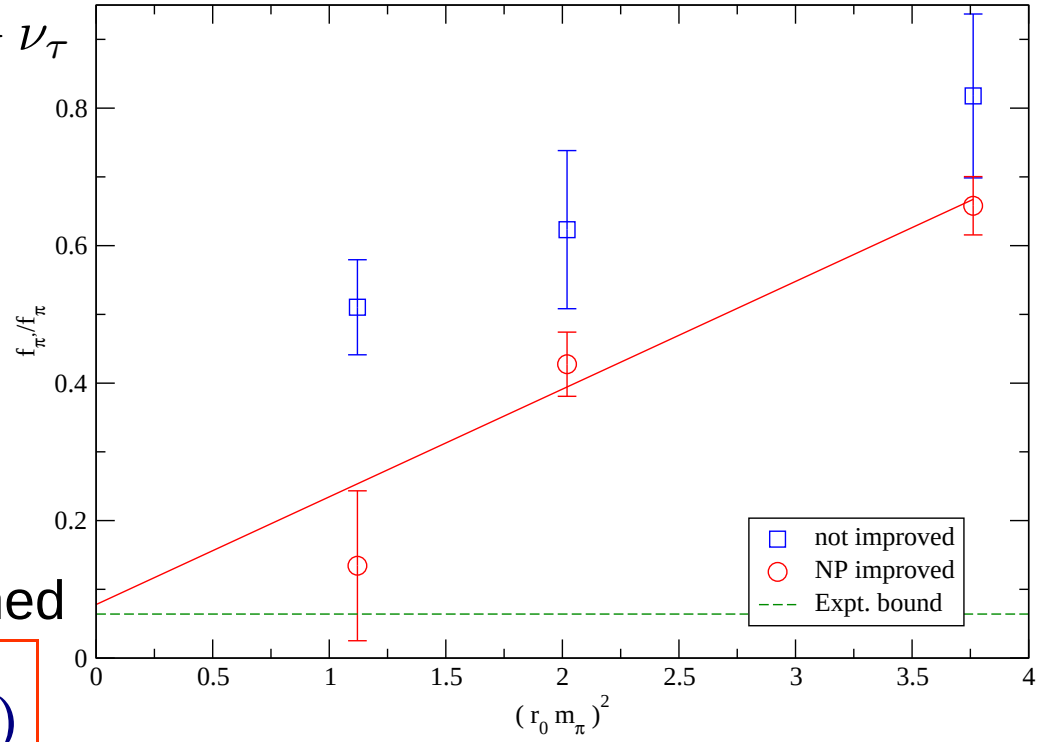


- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

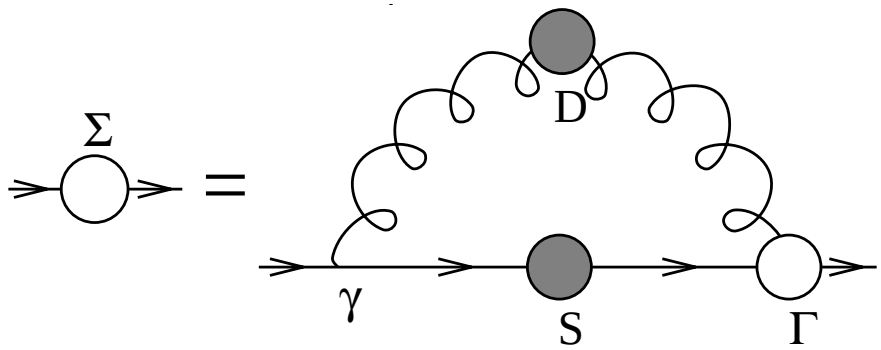
Pion Form Factor

Procedure Now Straightforward

Pion Form Factor

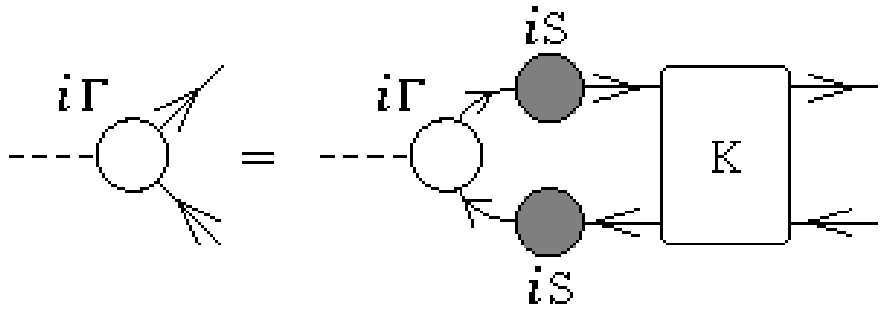
- Solve Gap Equation

  $\Rightarrow$ Dressed-Quark Propagator, $S(p)$



Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, K
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π



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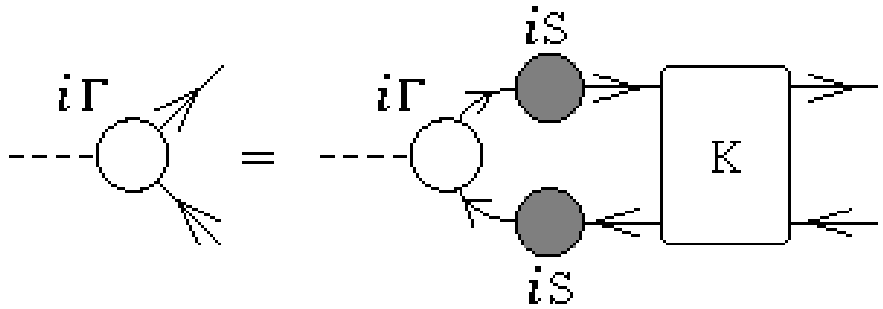
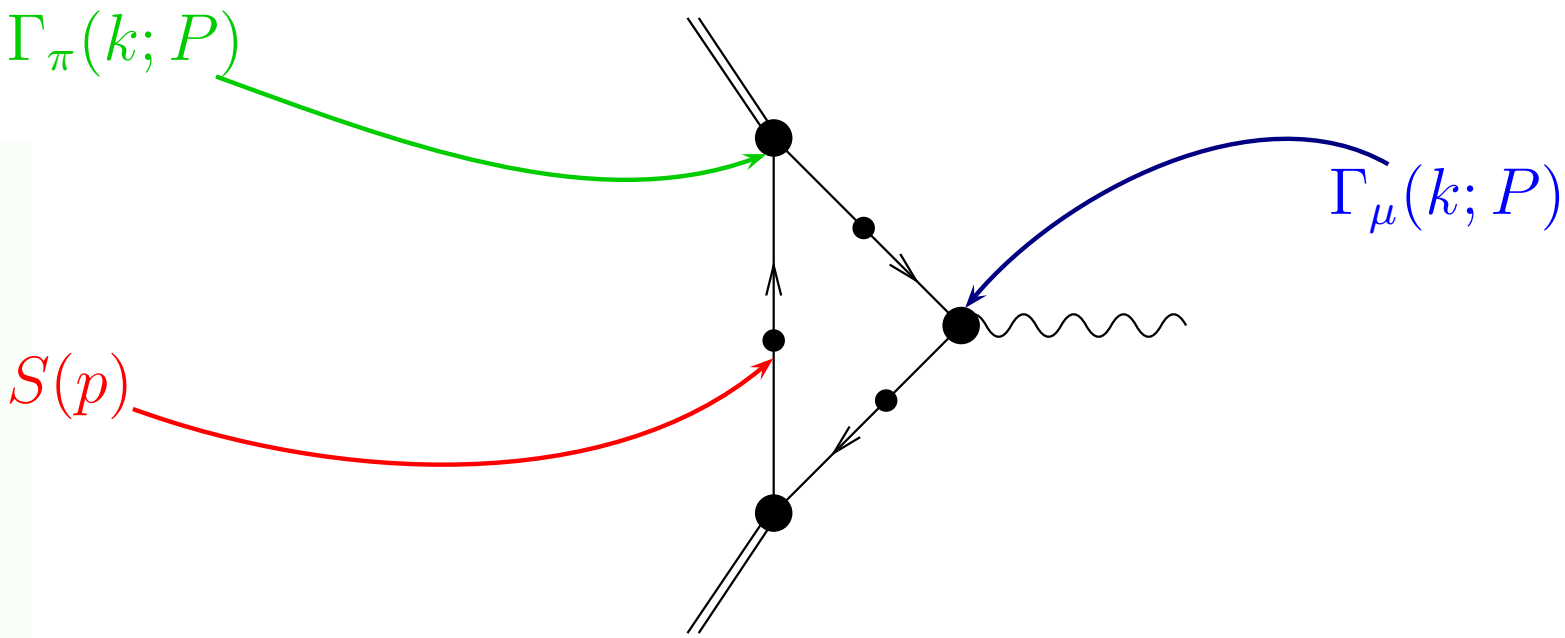


- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Photon Vertex, Γ_μ

Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



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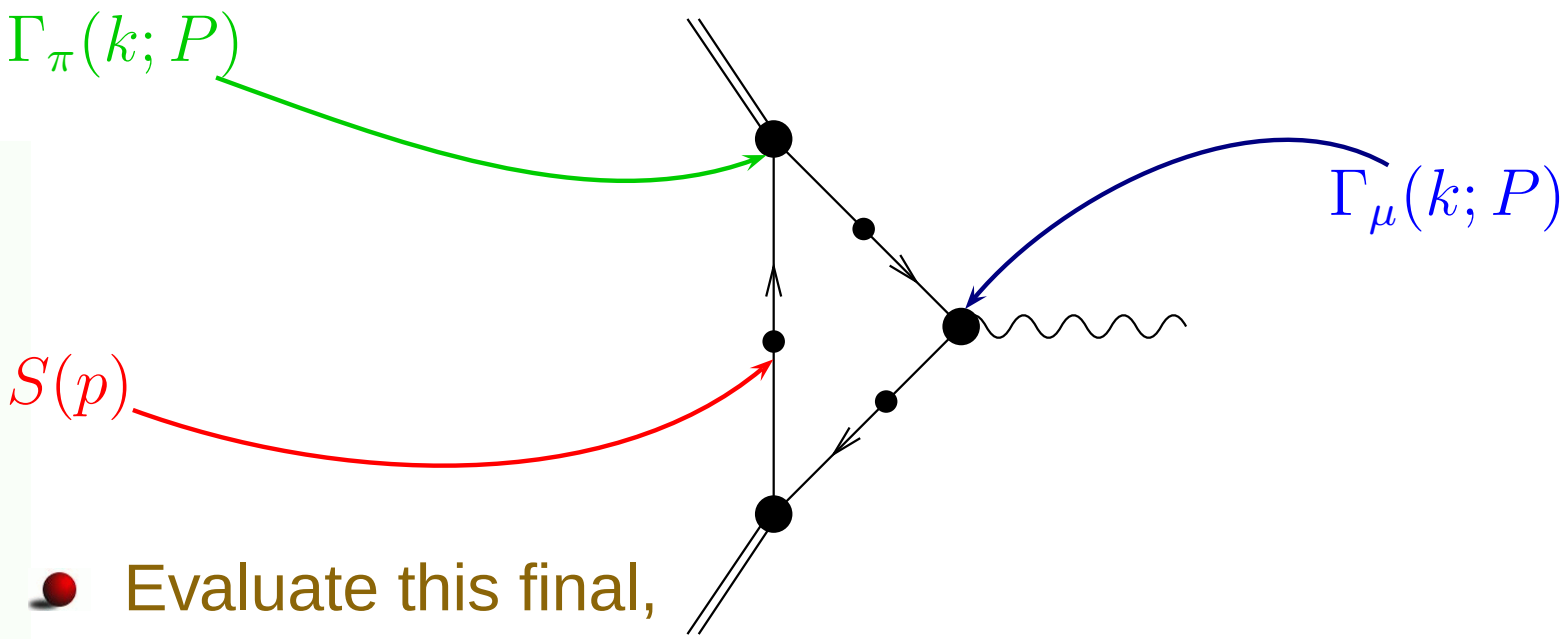

- Evaluate this final, three-dimensional integral

Timelike Pion Form Factor

Ab initio calculation into timelike region. Deeper than ground-state ρ-meson pole

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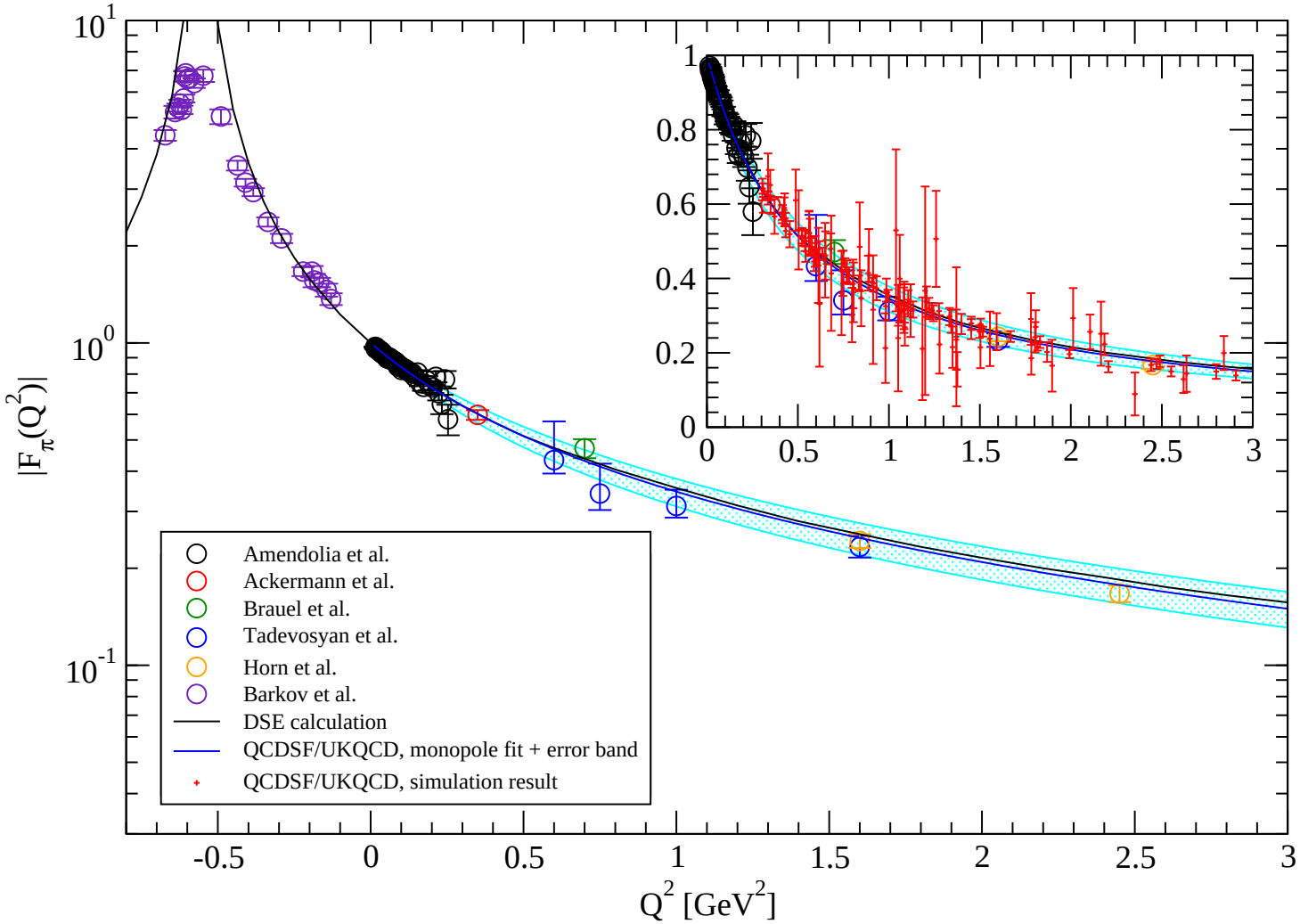
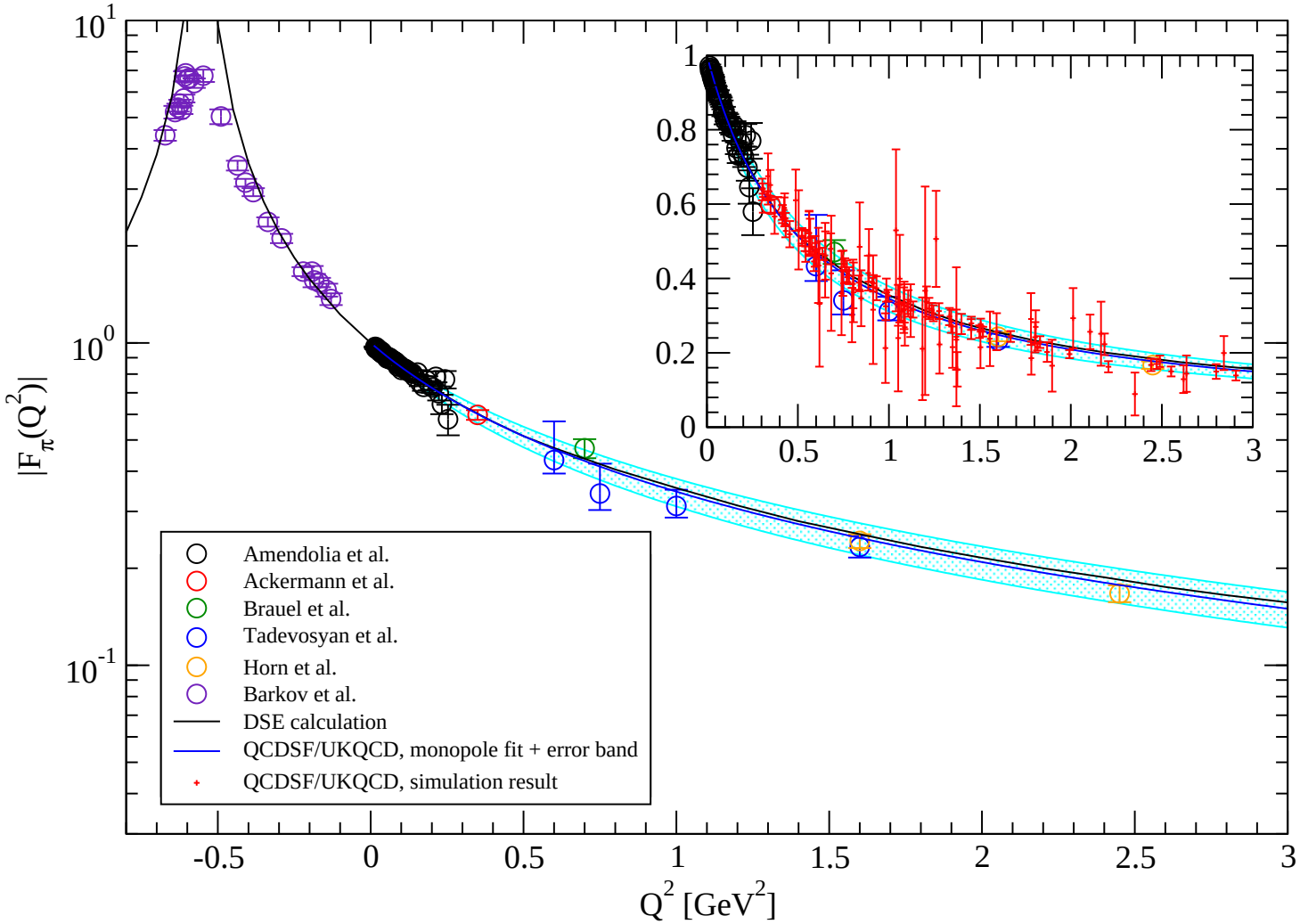


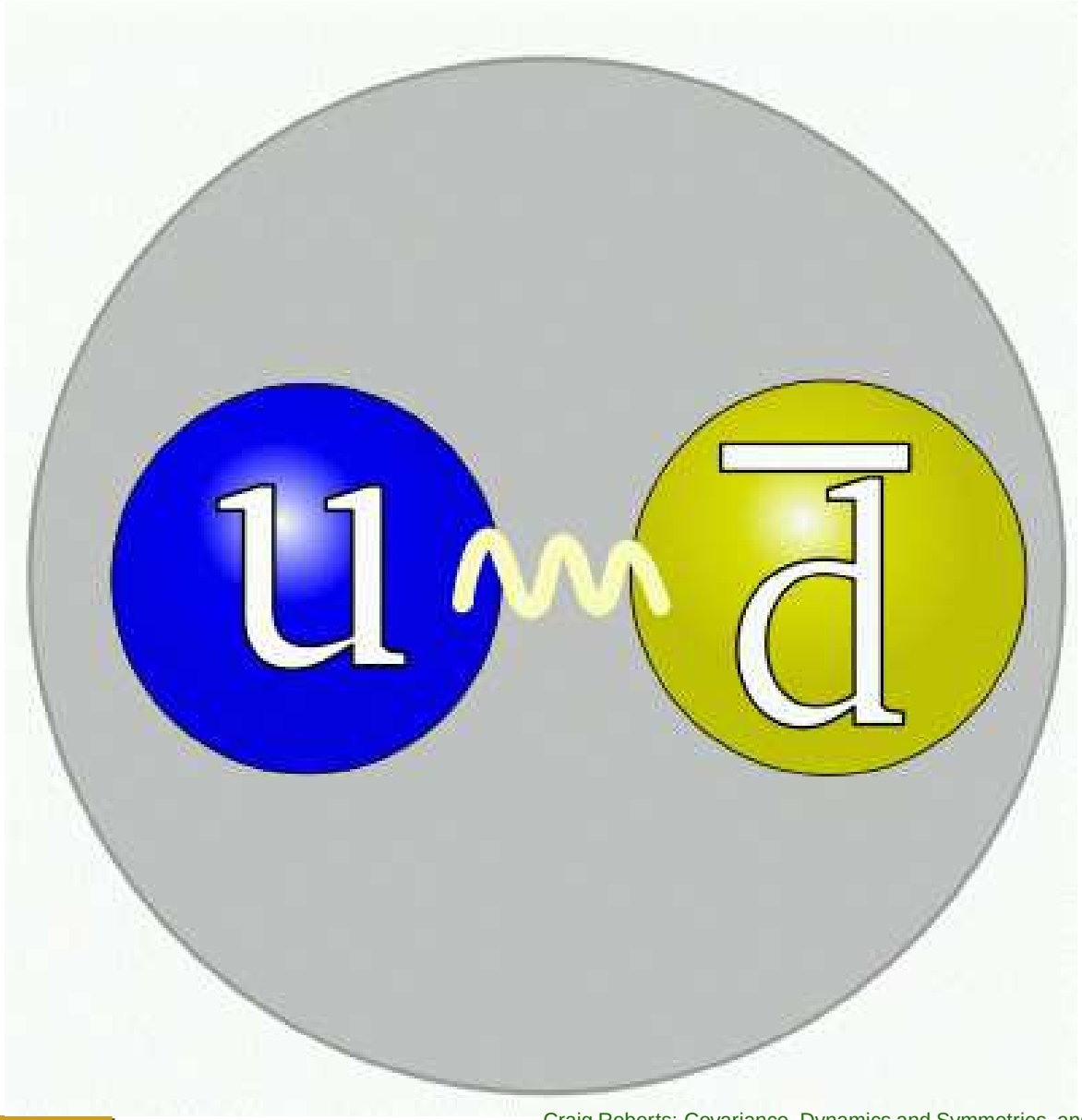
Ab initio calculation into timelike region. Deeper than ground-state ρ-meson pole

Timelike Pion Form Factor

ρ-meson not put in "by hand" – generated dynamically as a bound-state of dressed-quark and dressed-antiquark

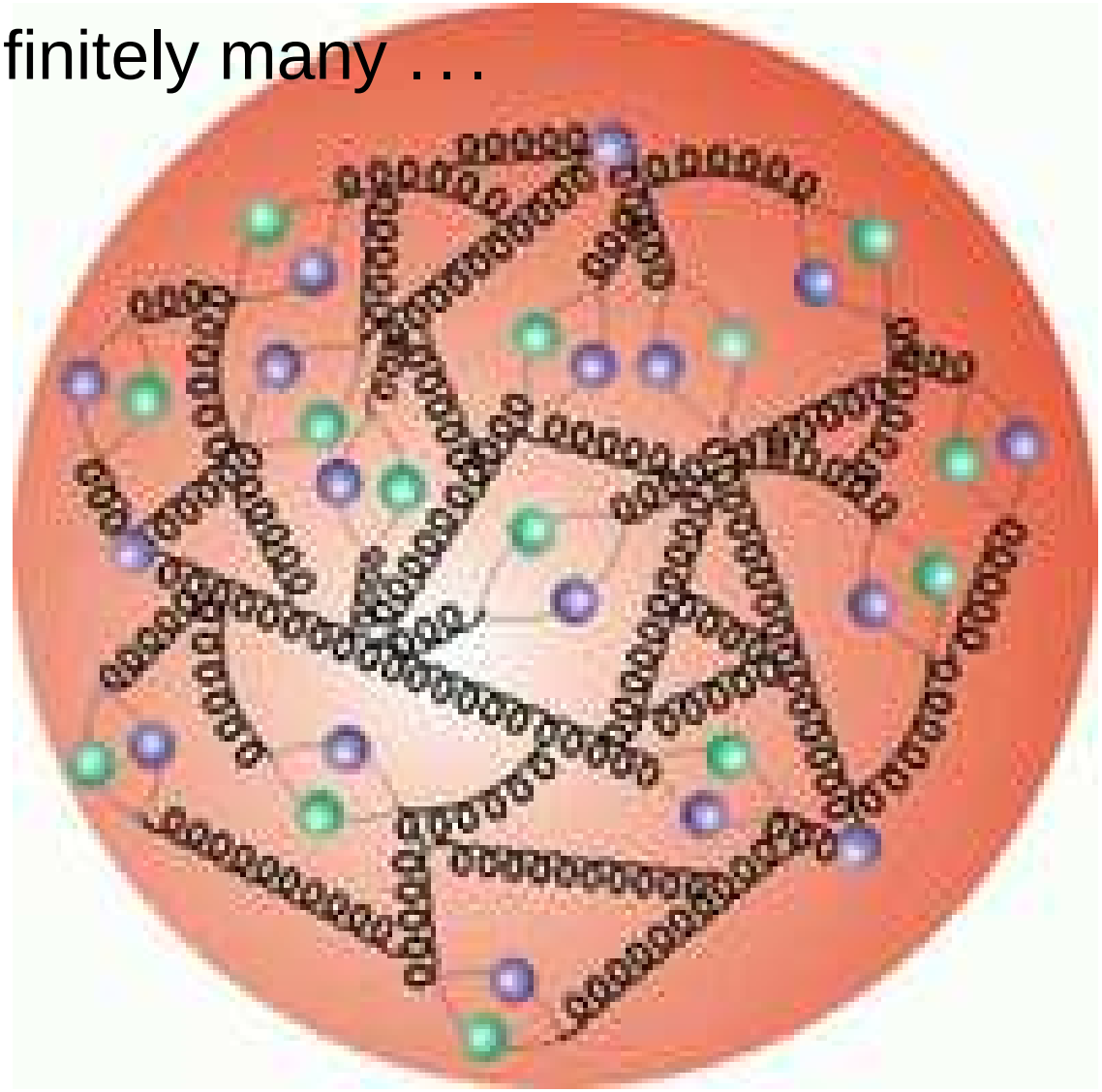


Answer for the pion



Answer for the pion

Two → Infinitely many . . .



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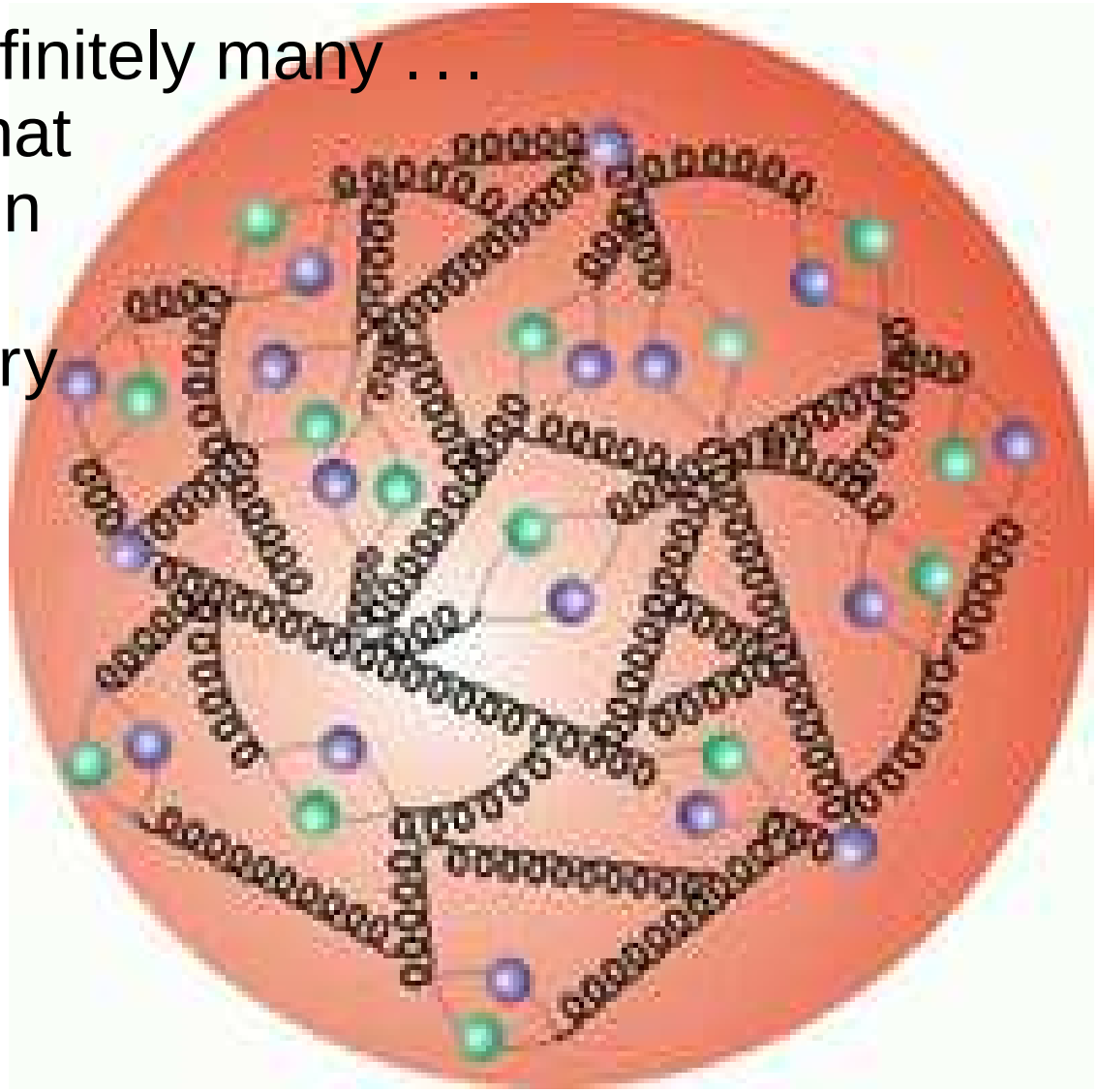

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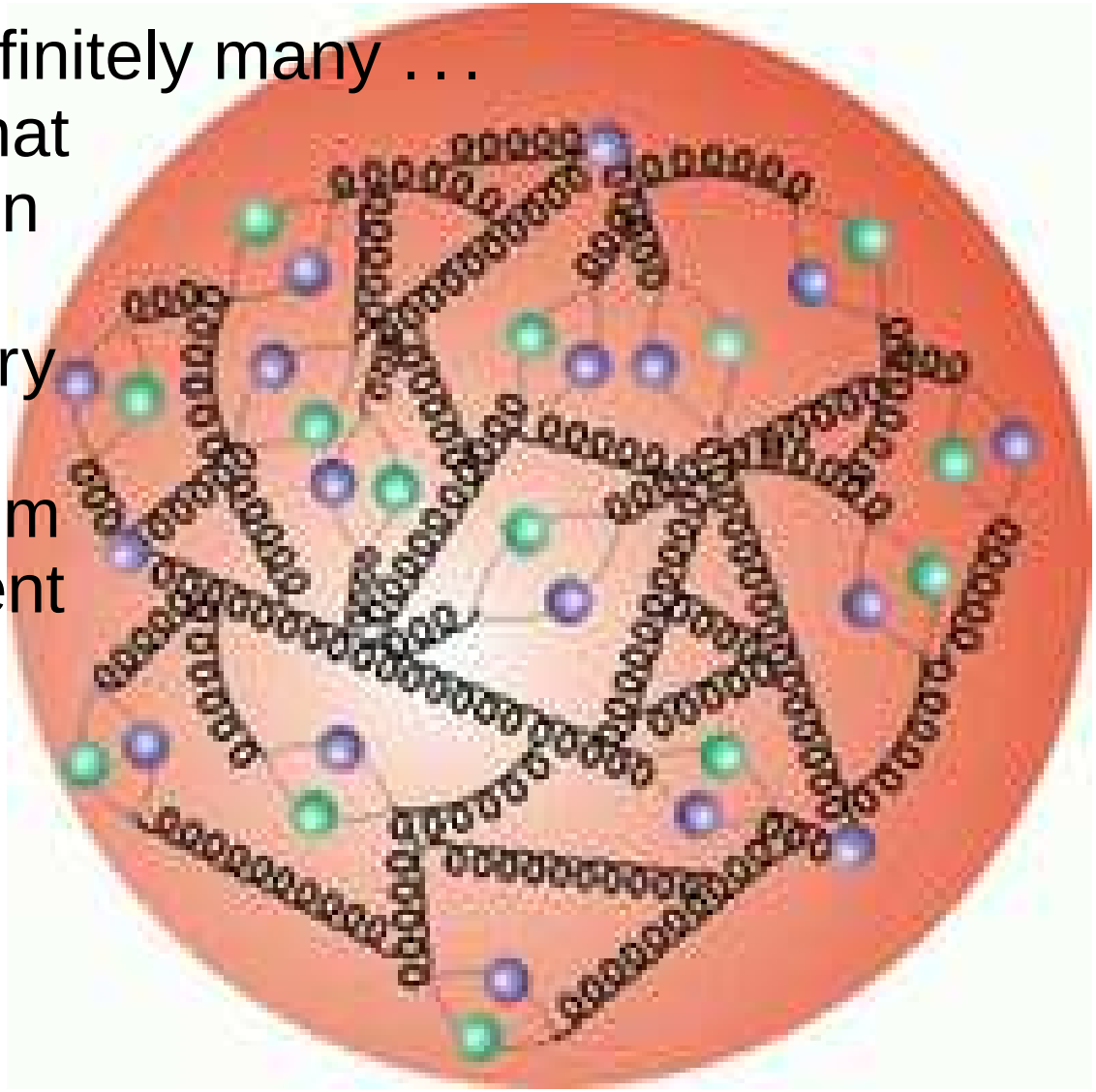

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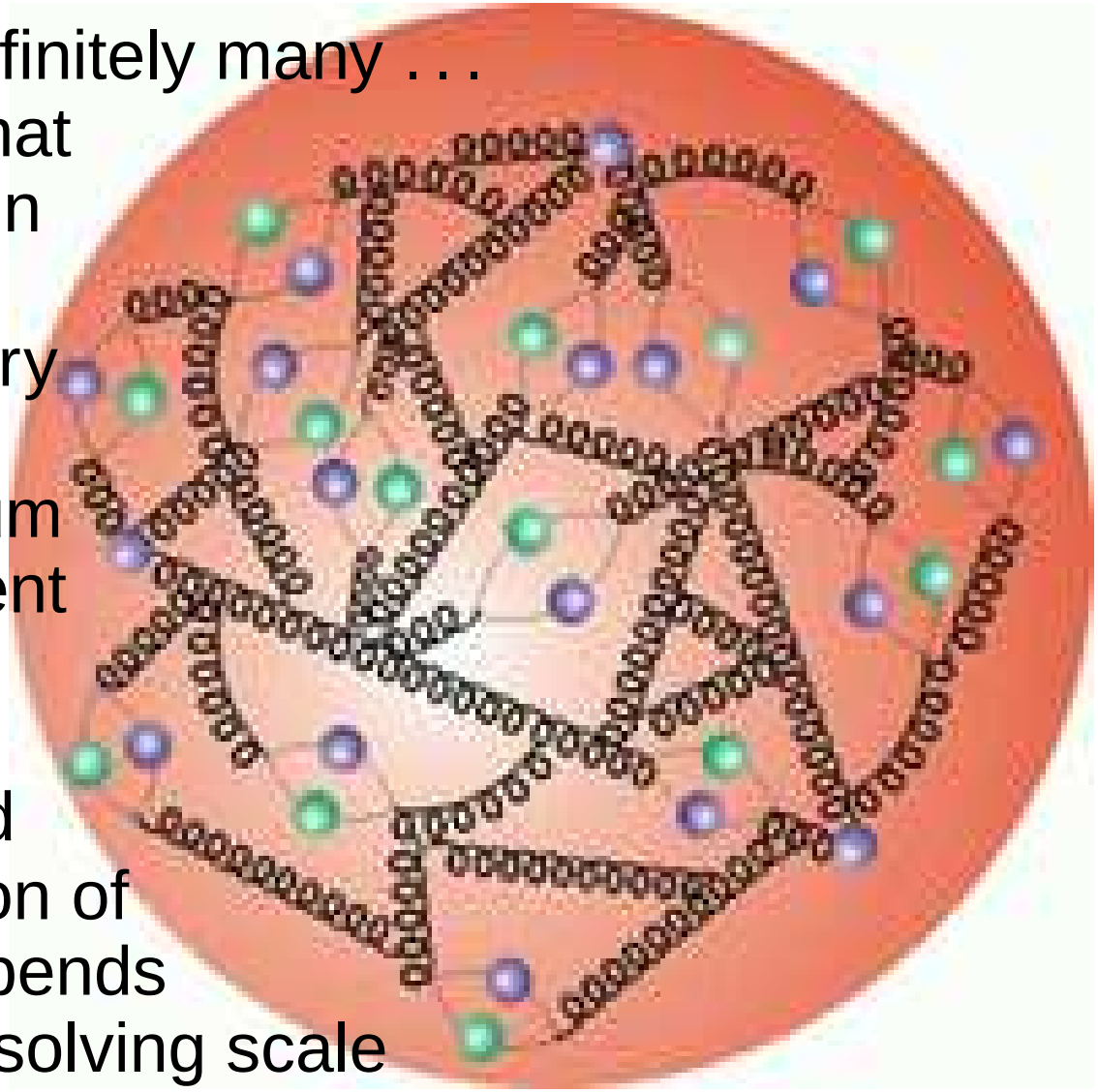
perceived distribution of mass depends on the resolving scale



ARE WE THERE YET?

New Challenges

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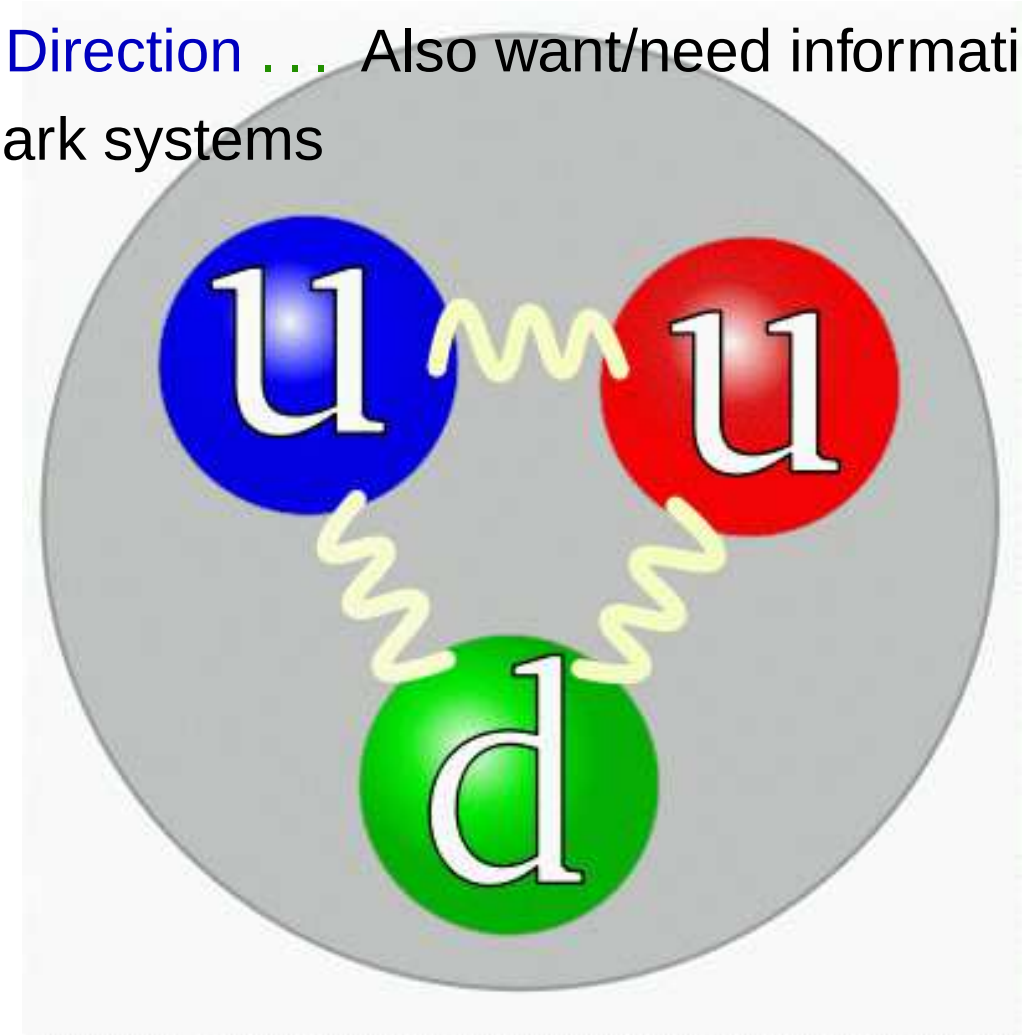
- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

New Challenges

- Another Direction . . . Also want/need information about three-quark systems



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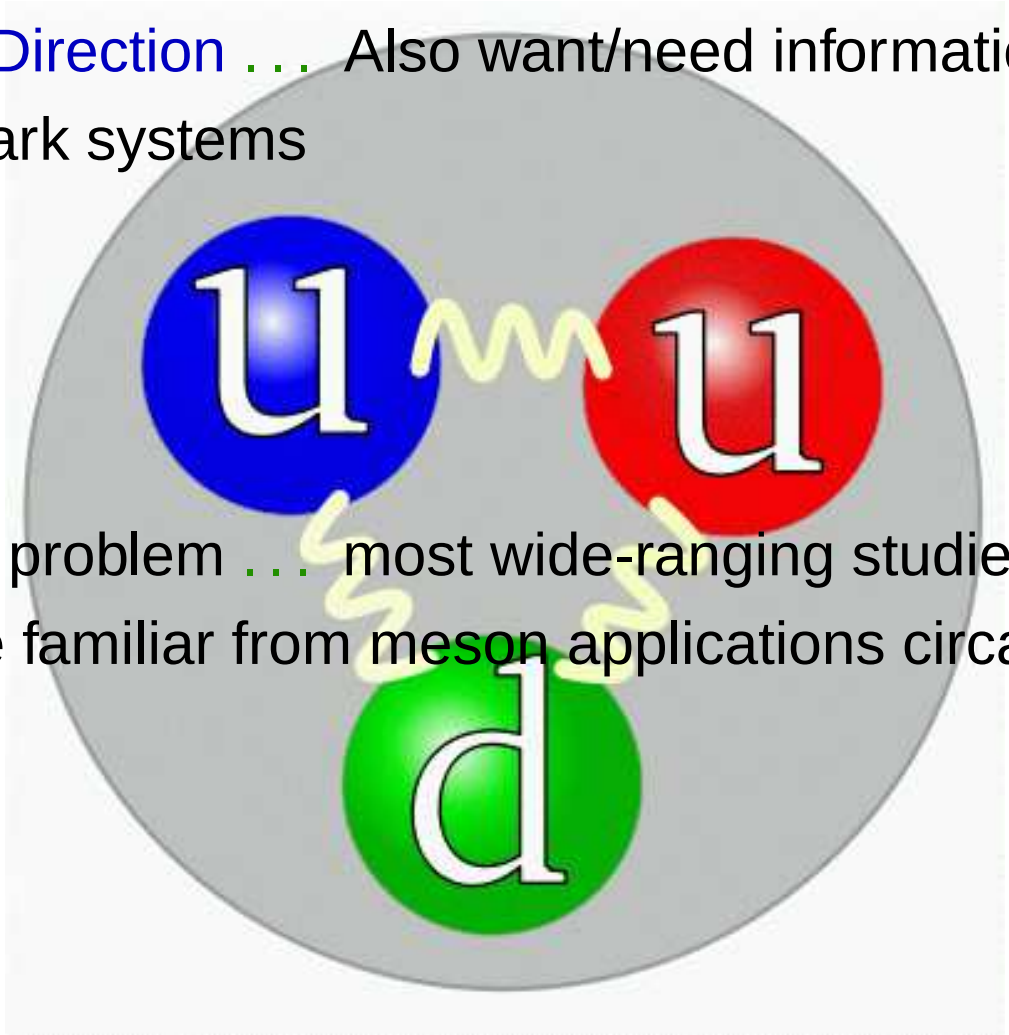

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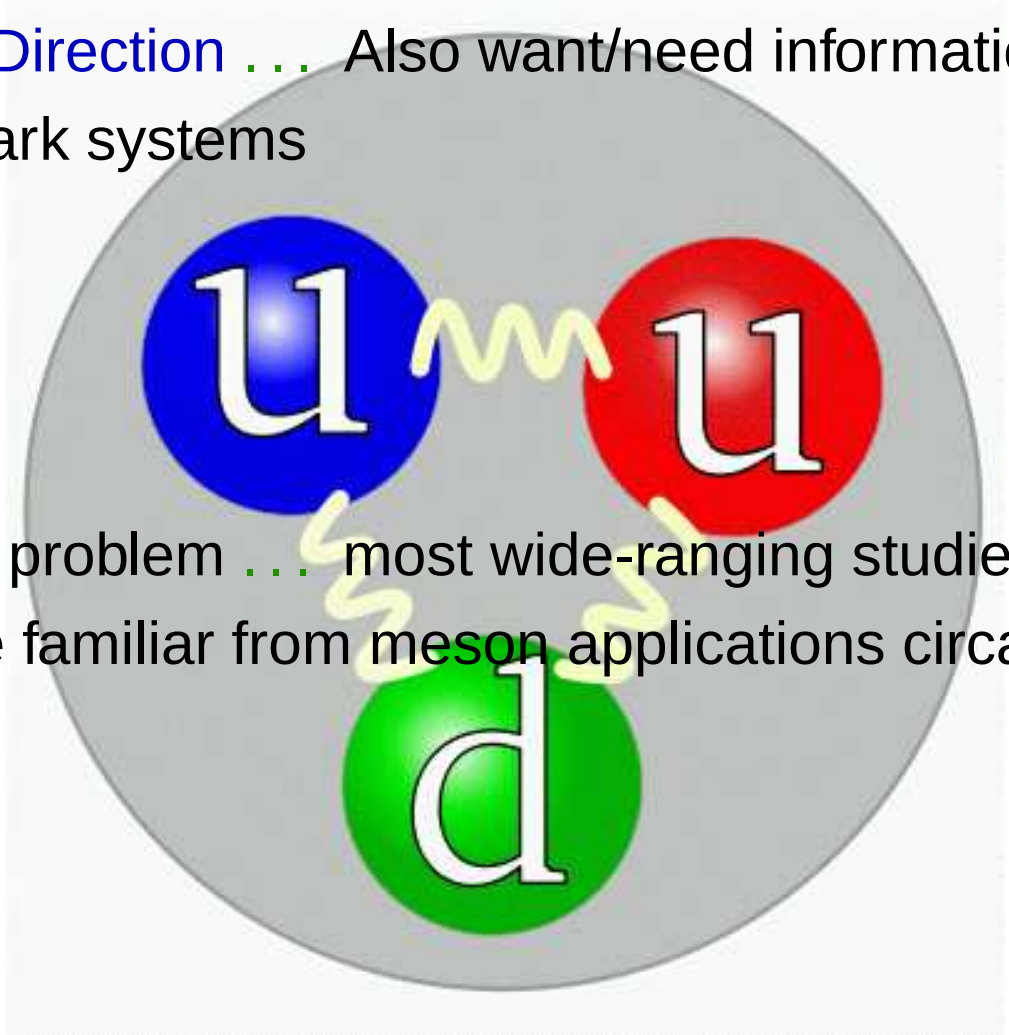

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- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.

New Challenges

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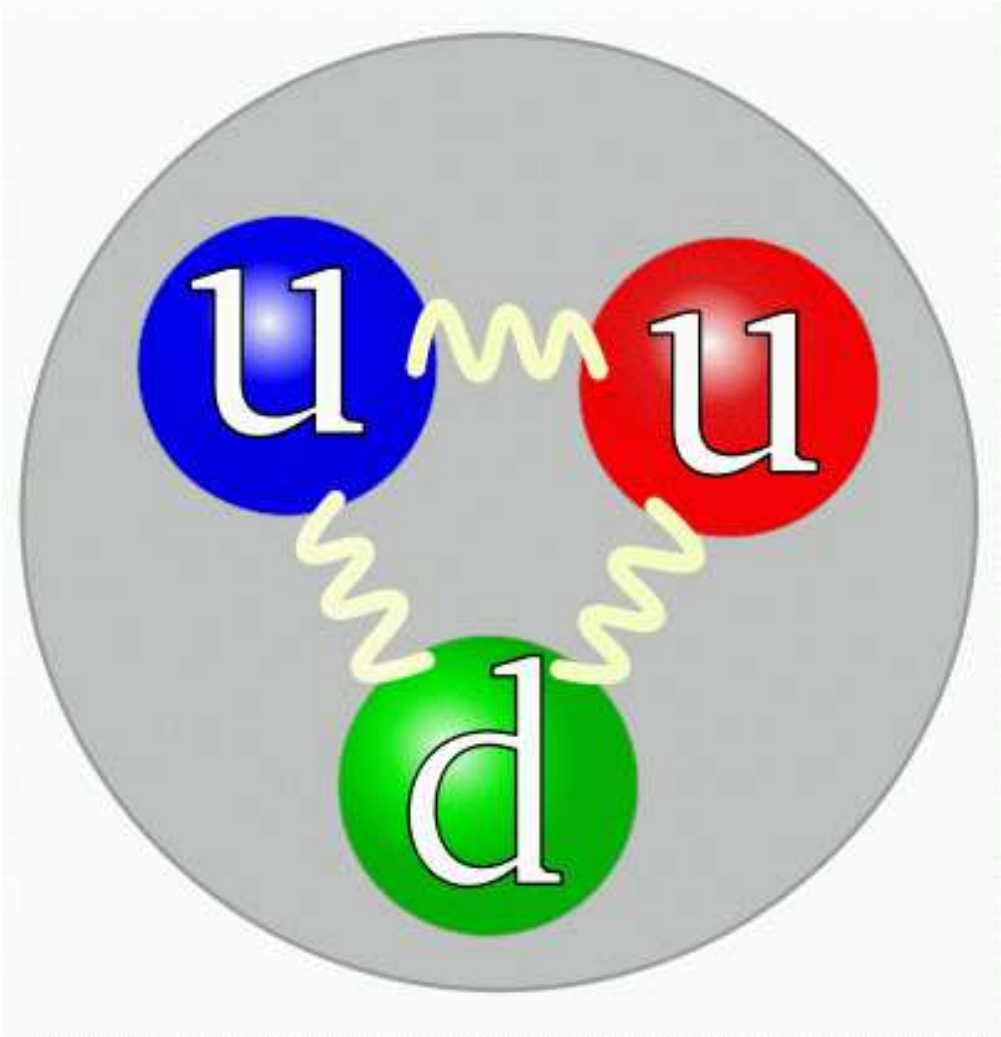
- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa $\sim$ 1995.
- However, that is beginning to change . . .

Nucleon . . .
Three-body Problem?



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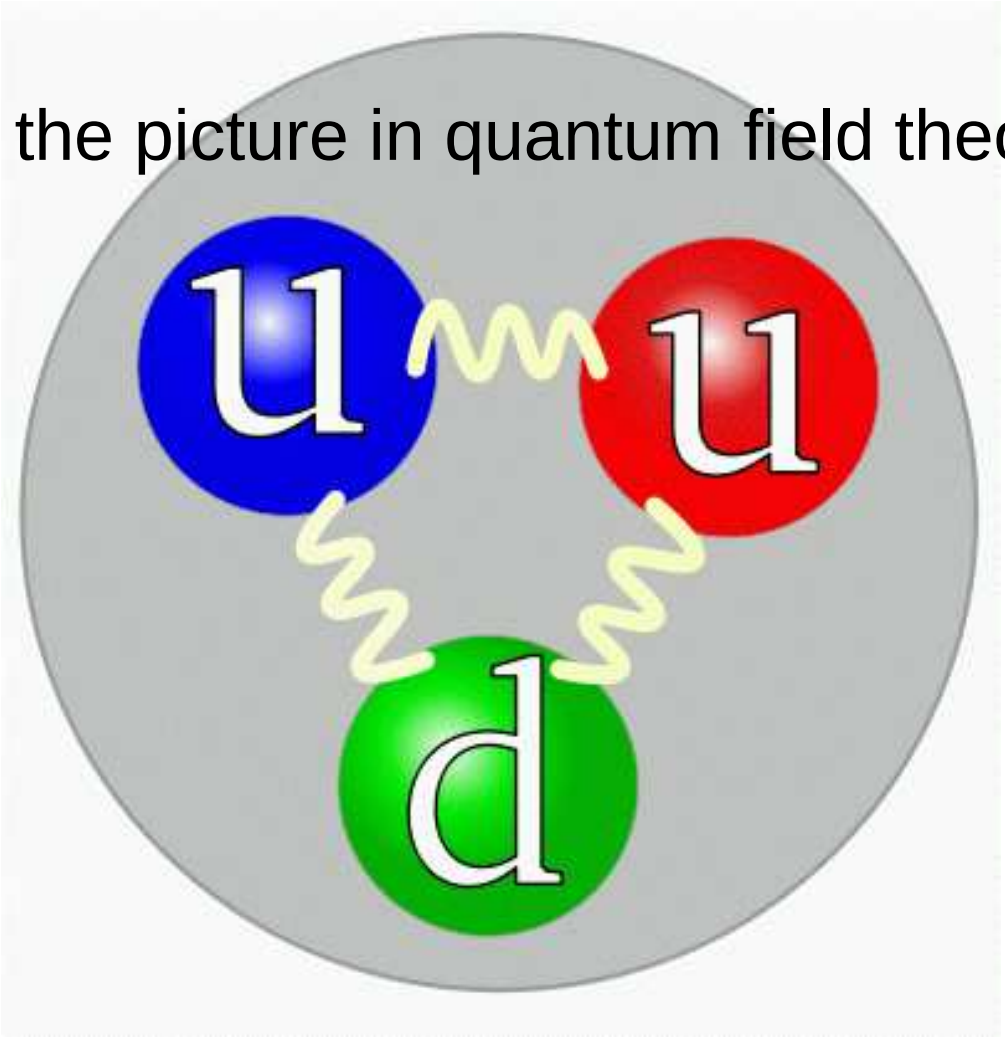

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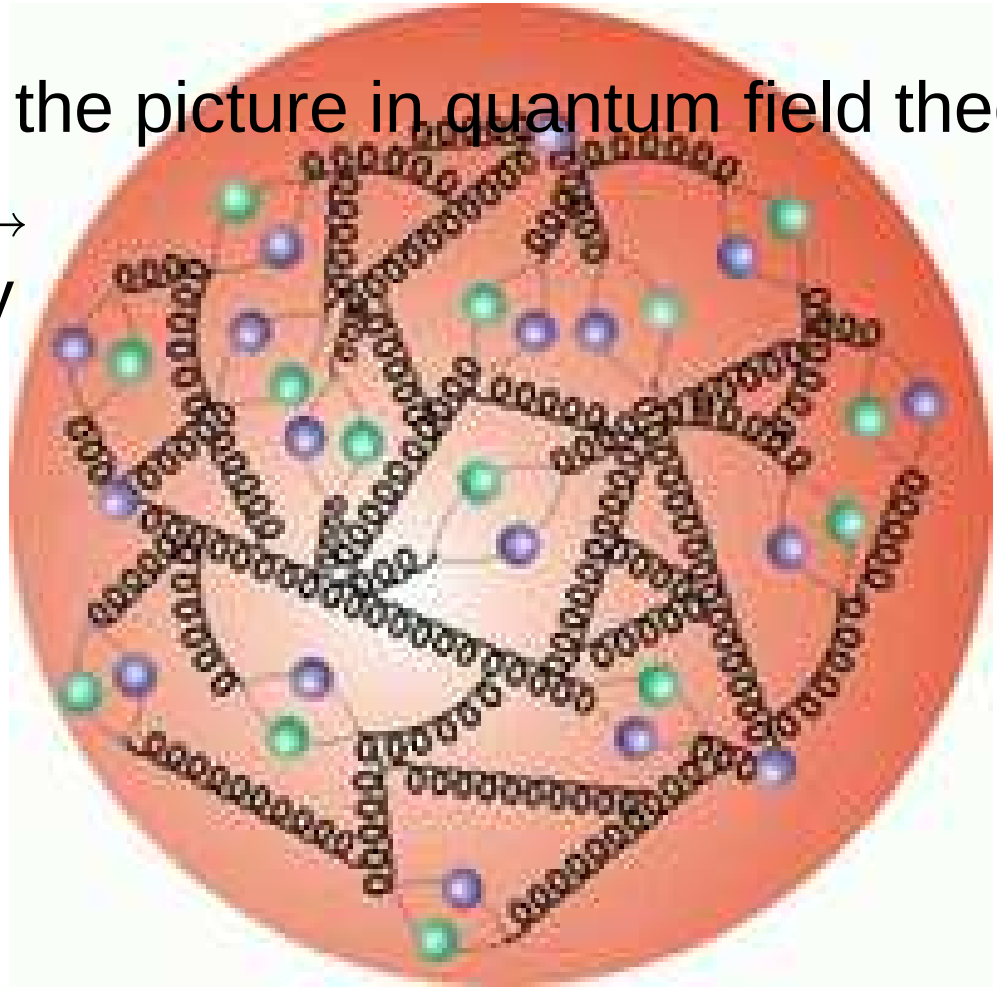
- Three → infinitely many!



Unifying Study of Mesons and Baryons

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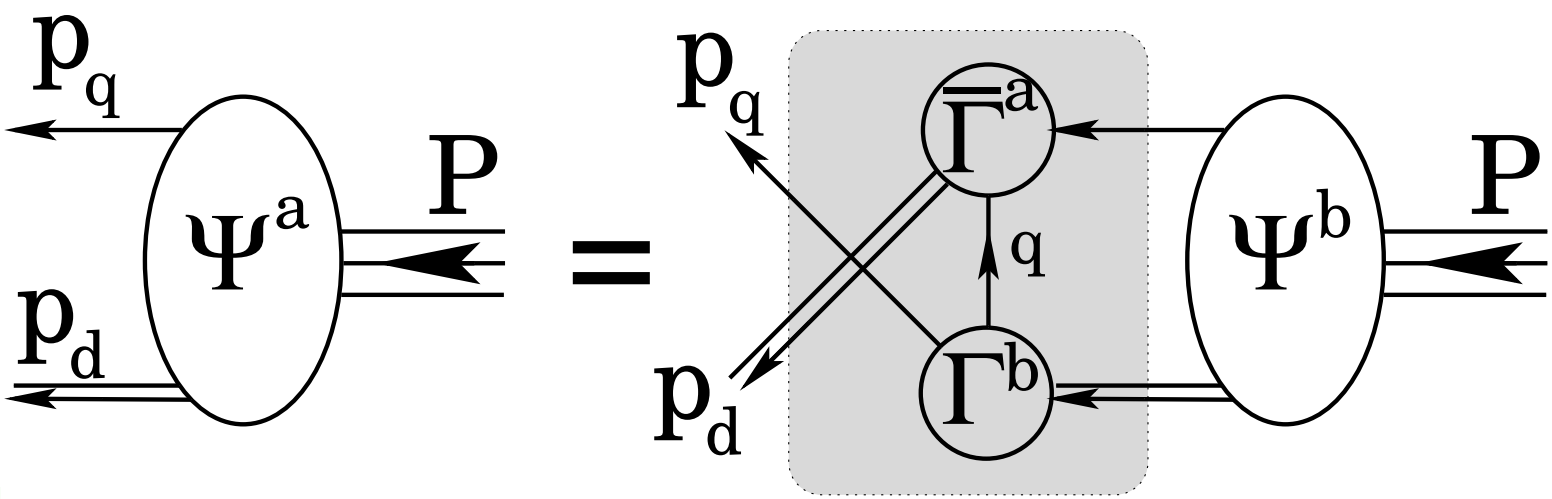
  - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
  - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour-$\bar{3}$ (antitriplet) channel

Faddeev equation

Faddeev equation



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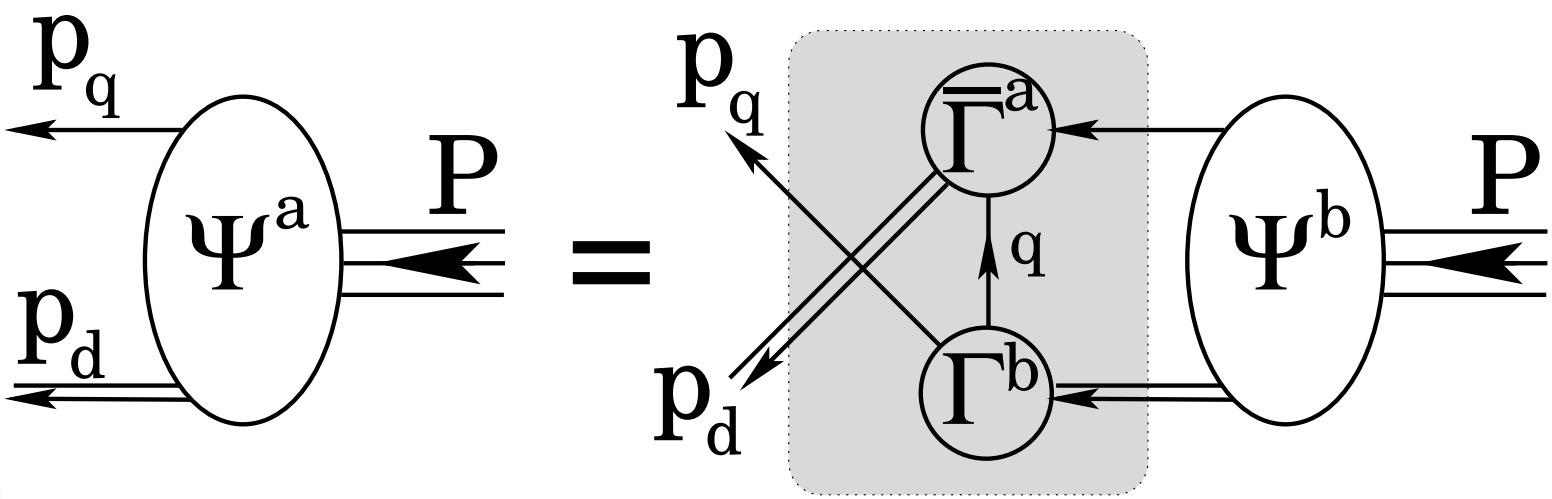
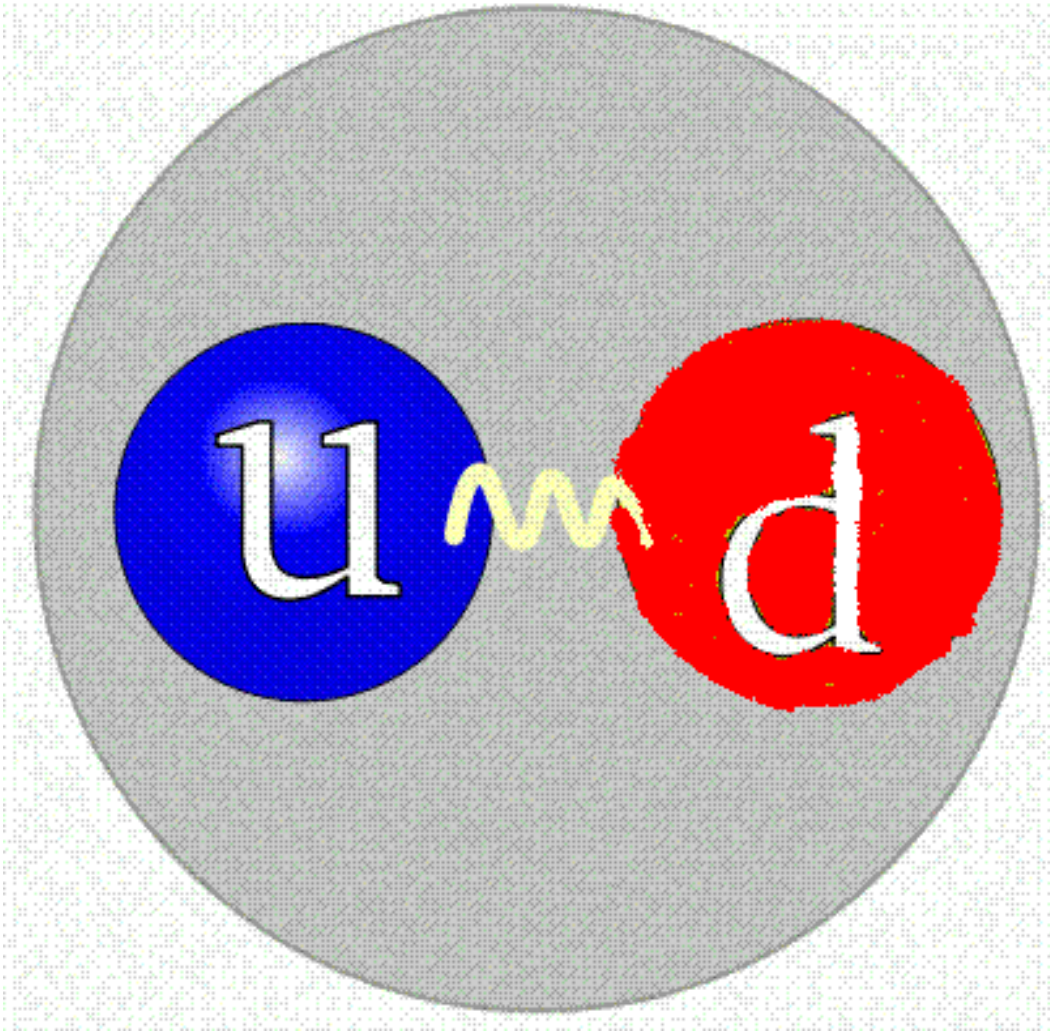


- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Diquark correlations



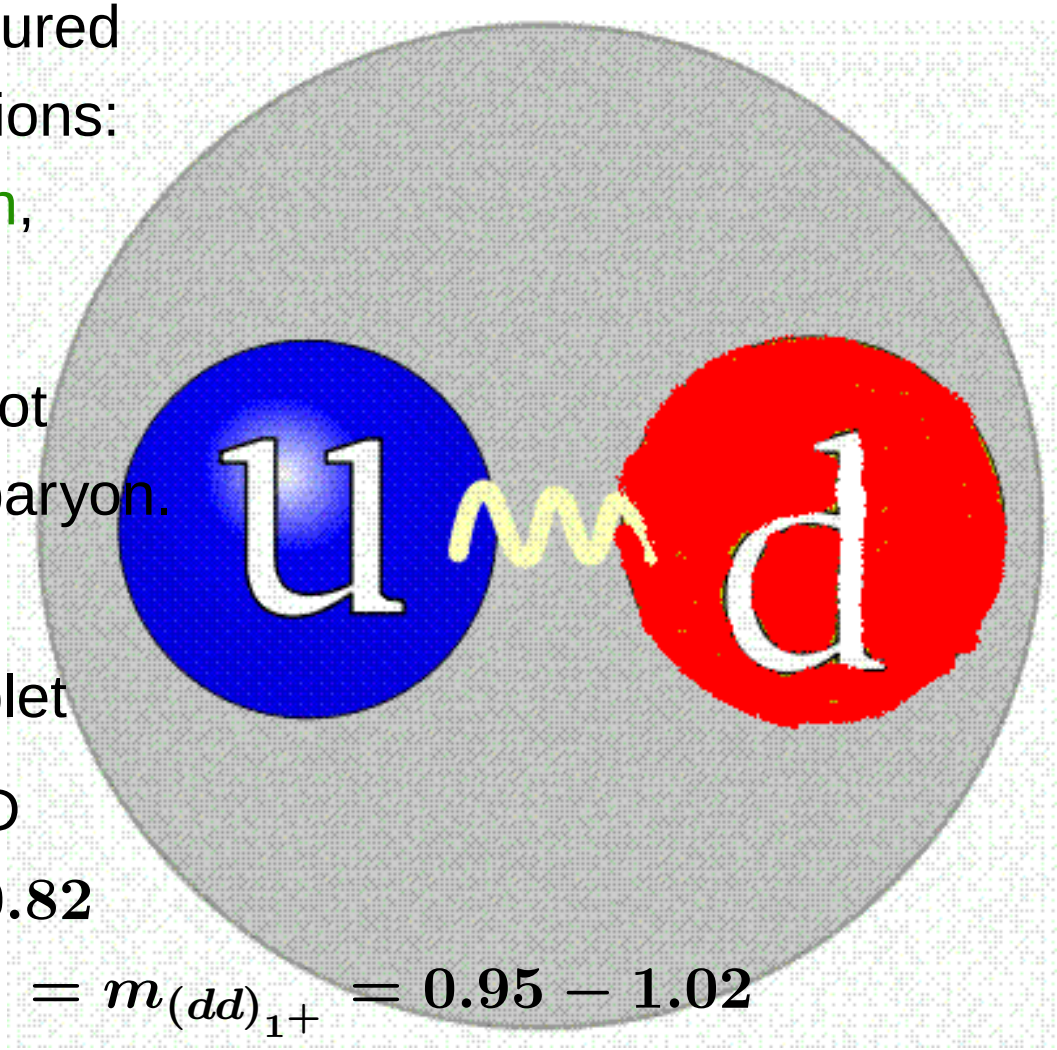
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined . . . Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

Ab-initio study of mesons & nucleons

Eichmann *et al*.
– arXiv:0802.1948 [nucl-th]
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  - Use this knowledge to constrain interaction in infrared
  - Interaction in ultraviolet predicted by perturbative expansion of DSEs

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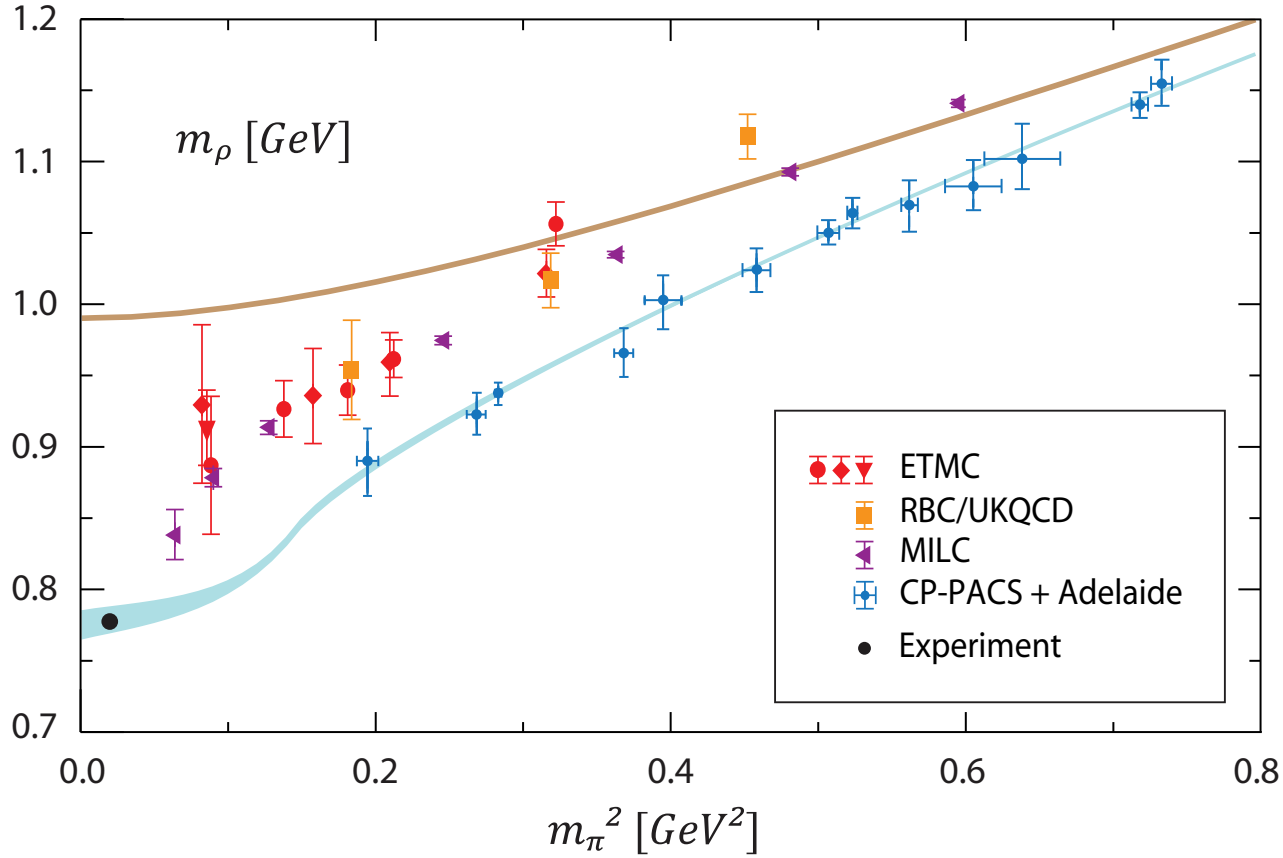

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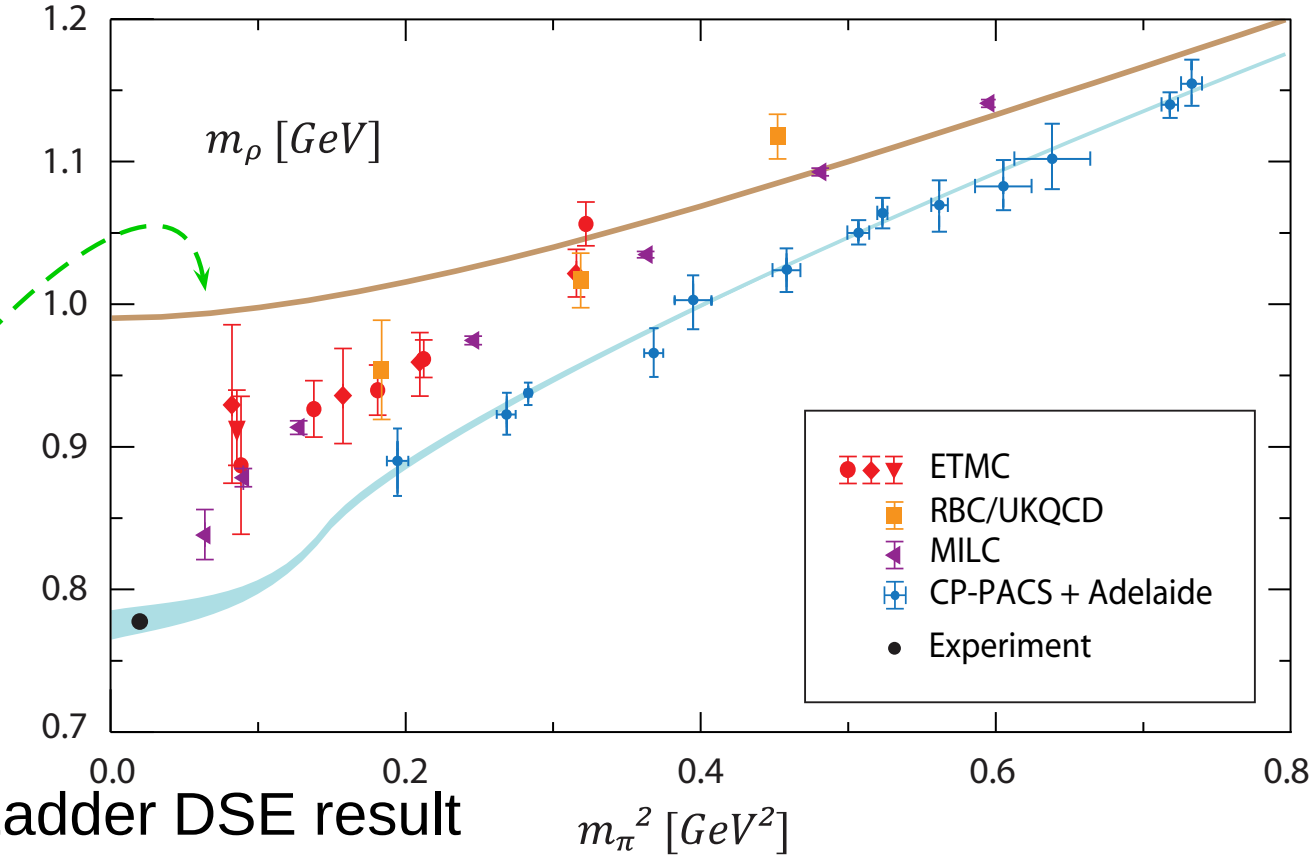

- Rainbow-Ladder DSE result
  one parameter for IR – “confinement radius”
  Results insensitive to value on material domain

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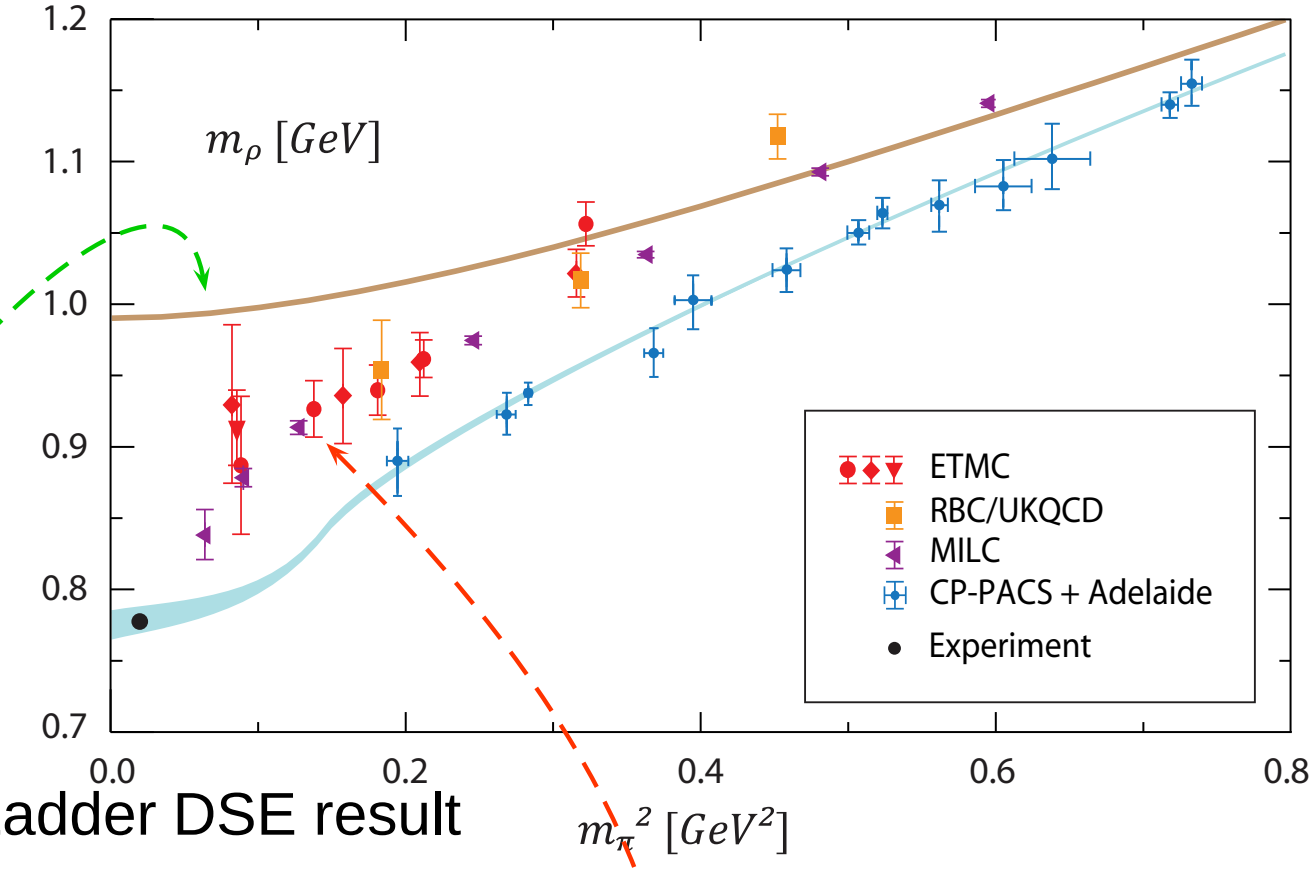

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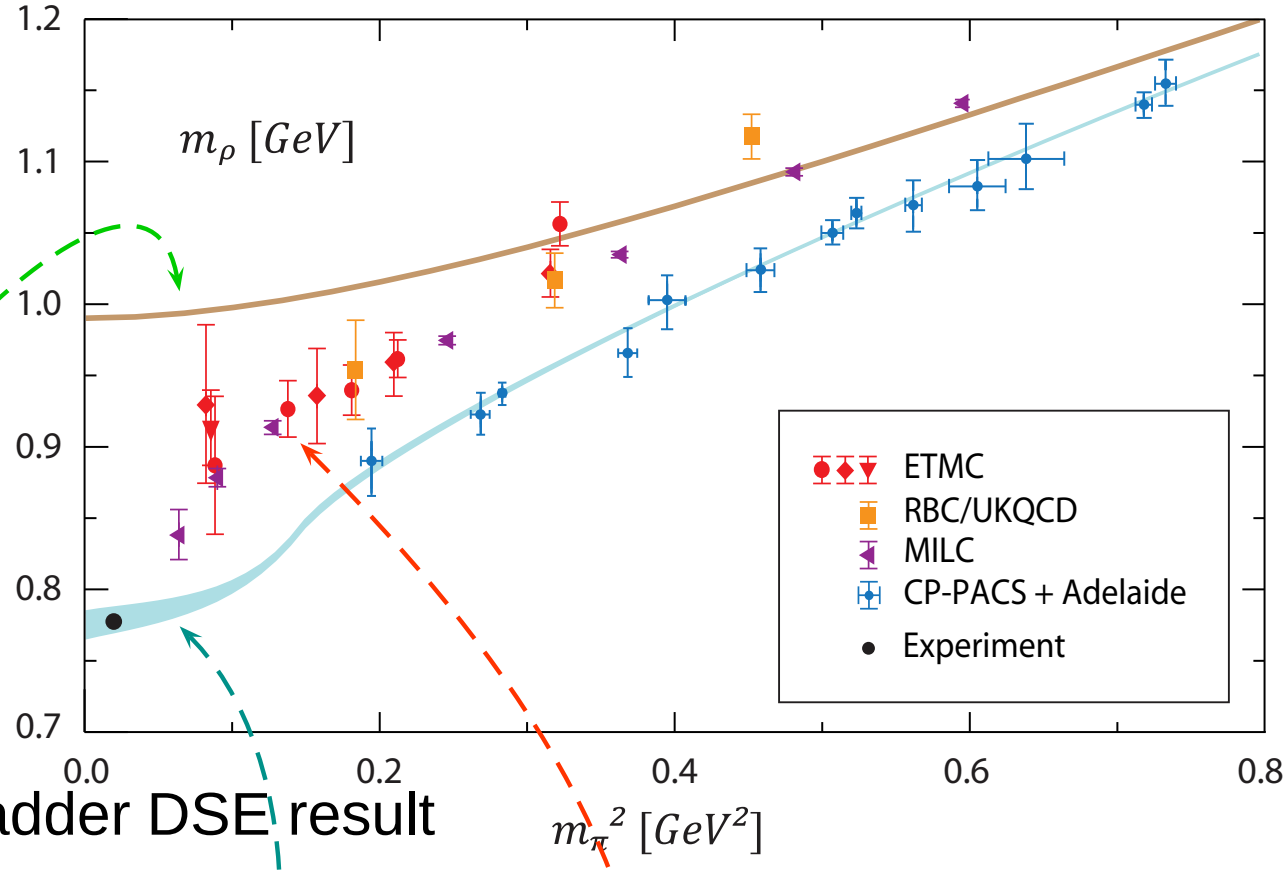

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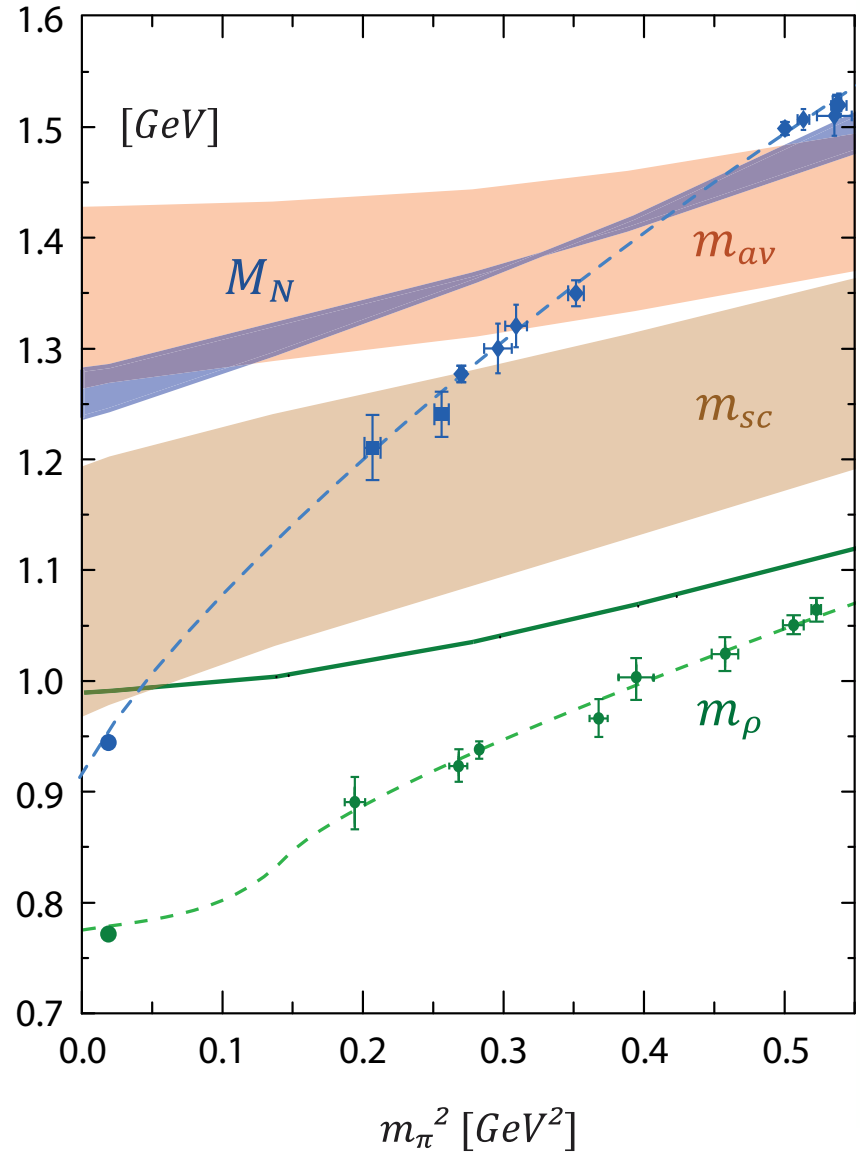

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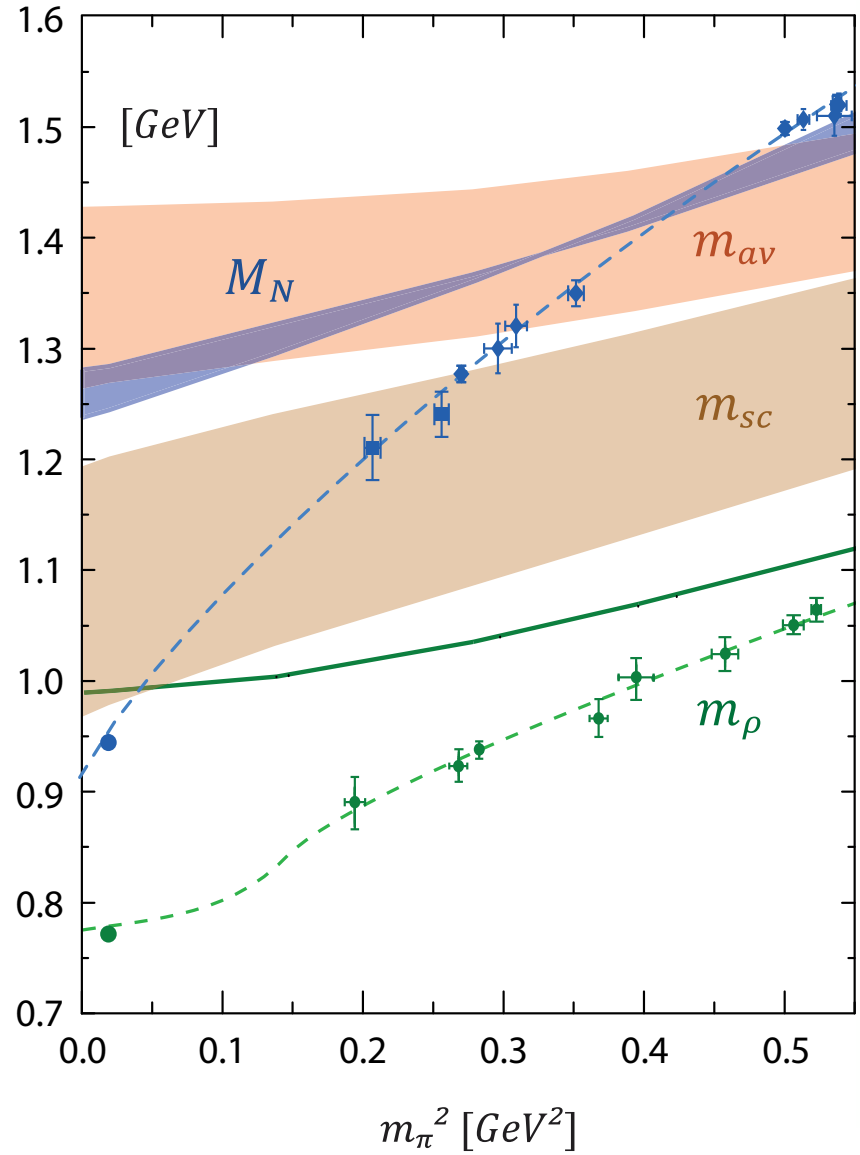
  - "unobservable" – show marked sensitivity to single model parameter; viz., confinement radius



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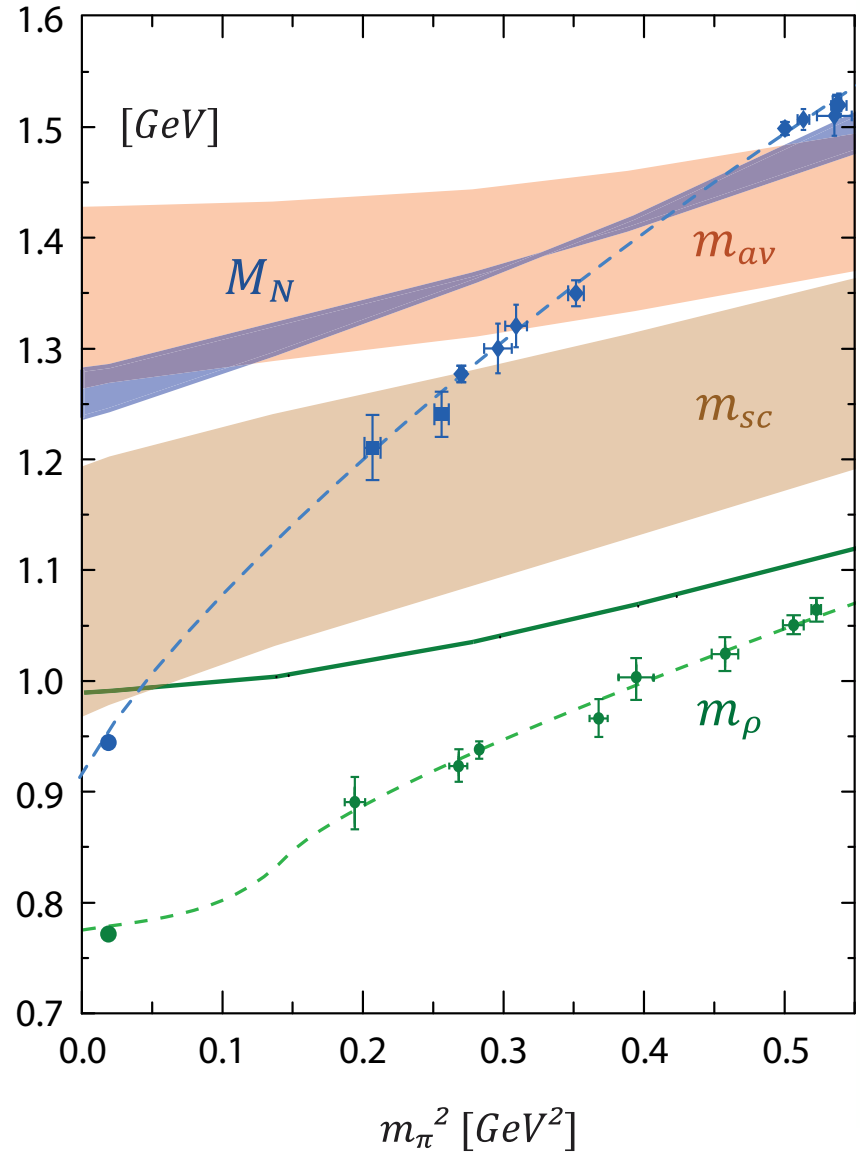
- But . . . $[m_{av} - m_{sc}]$, m_ρ & M_N . . . are *independent* of that parameter



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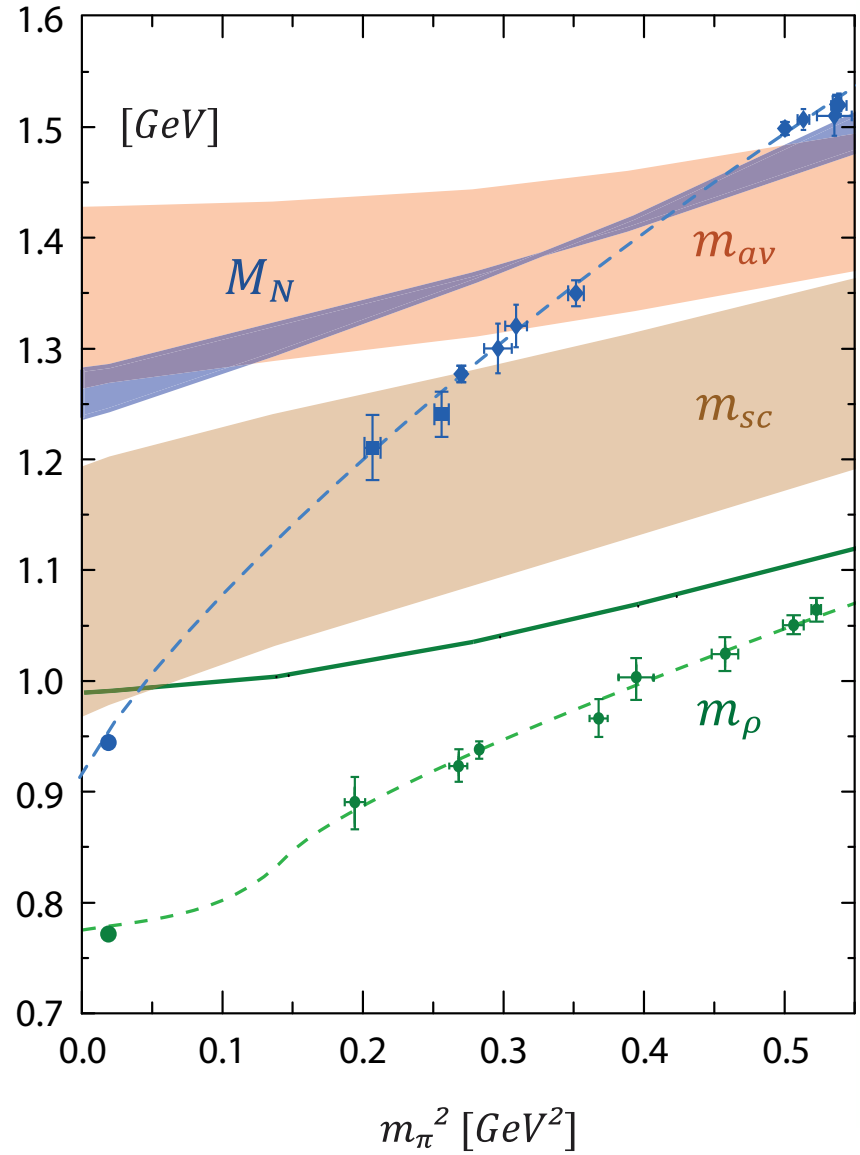

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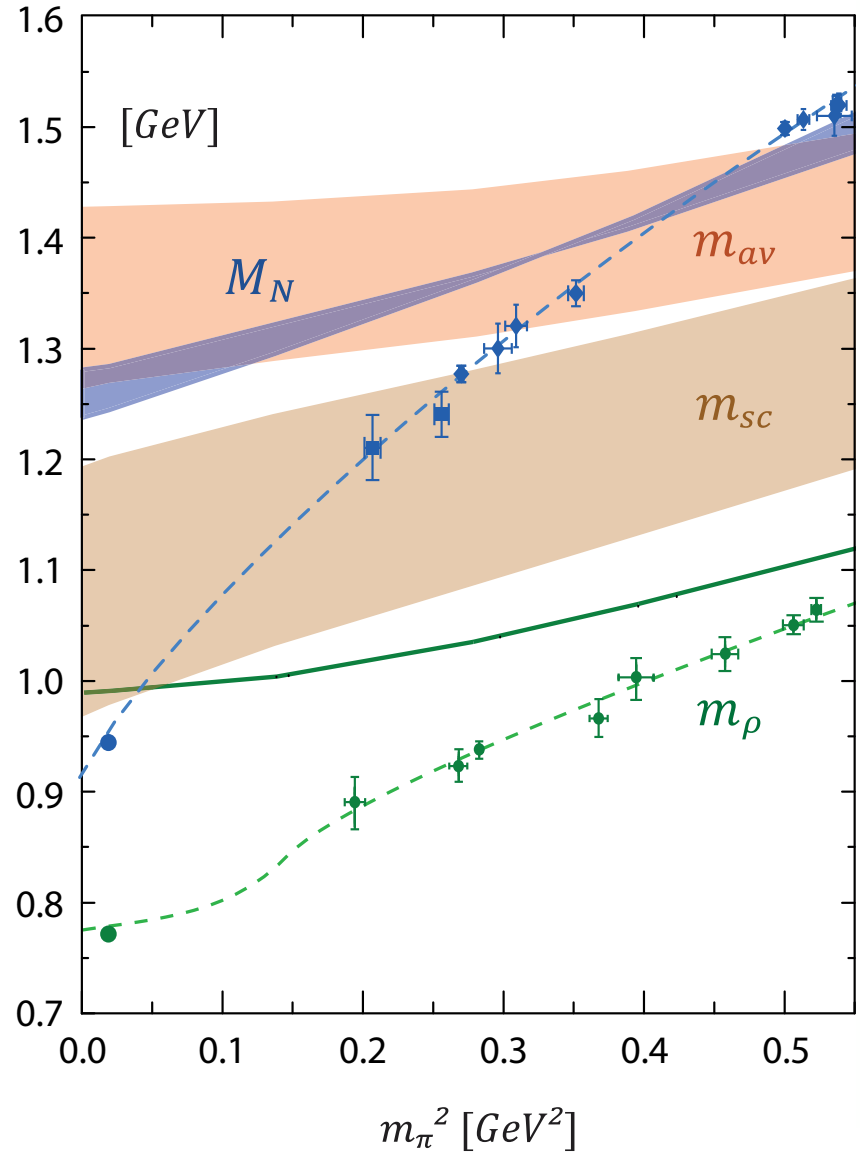

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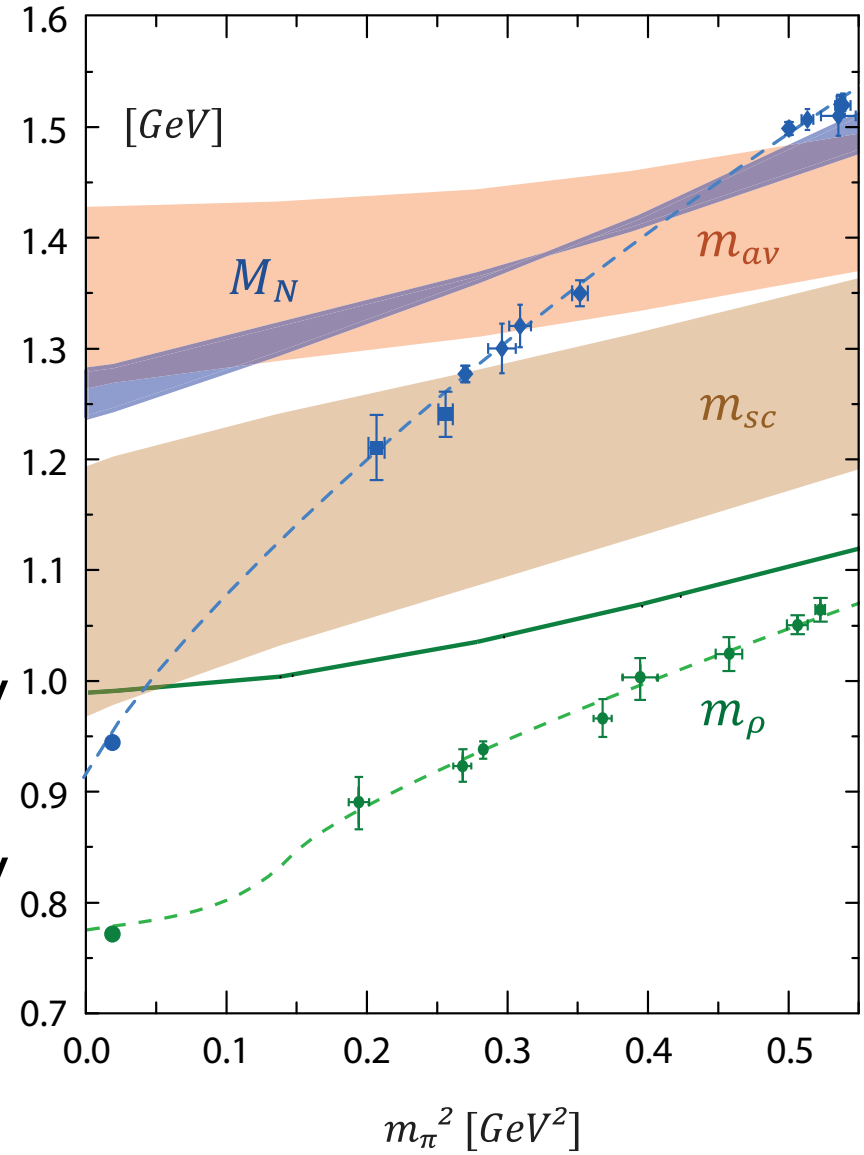
- Prediction: at physical m_π^2,
  $M_N^{\mathrm{quark-core}} = 1.26(2)$ GeV
  cf. FRR+lattice-QCD,
  $M_N^{\mathrm{quark-core}} = 1.27(2)$ GeV

$\Rightarrow$ subleading corrections, including 0^--meson loops,
$\delta M_N = -320$ MeV,
$\delta m_\rho = -220$ MeV



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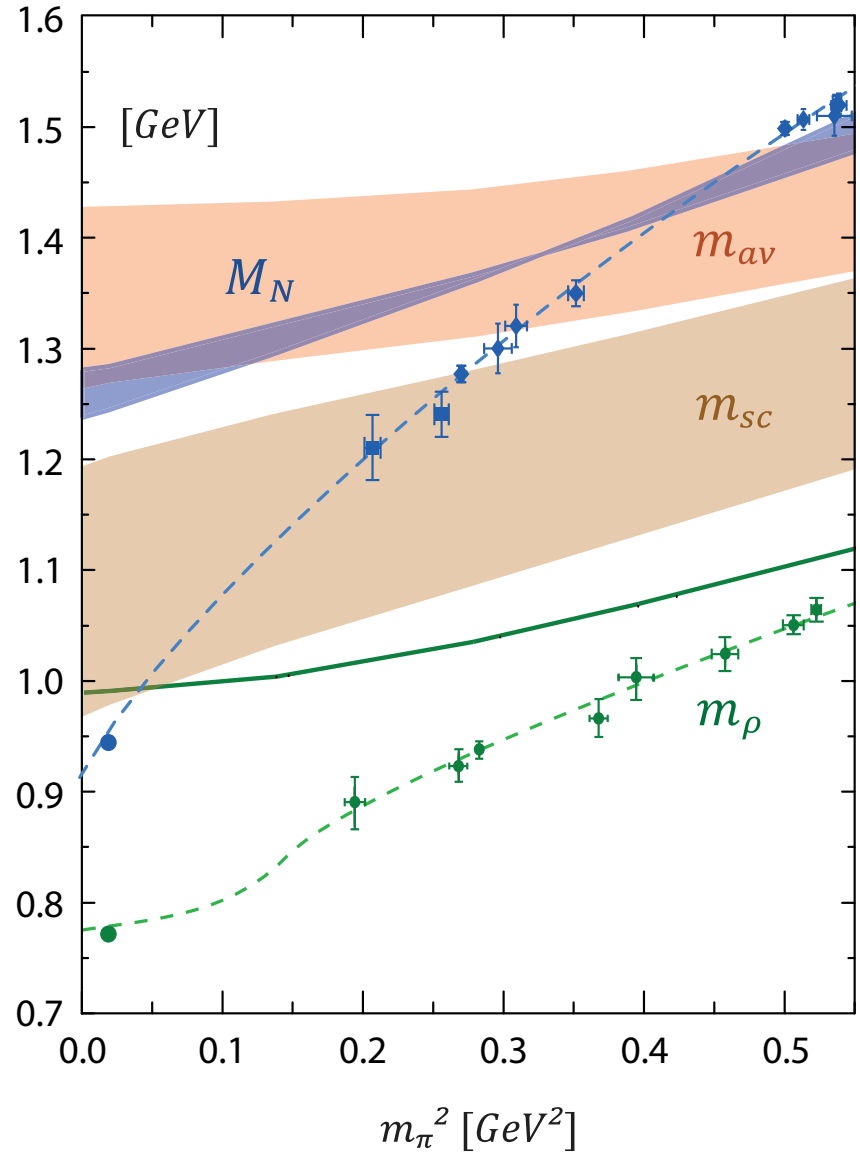

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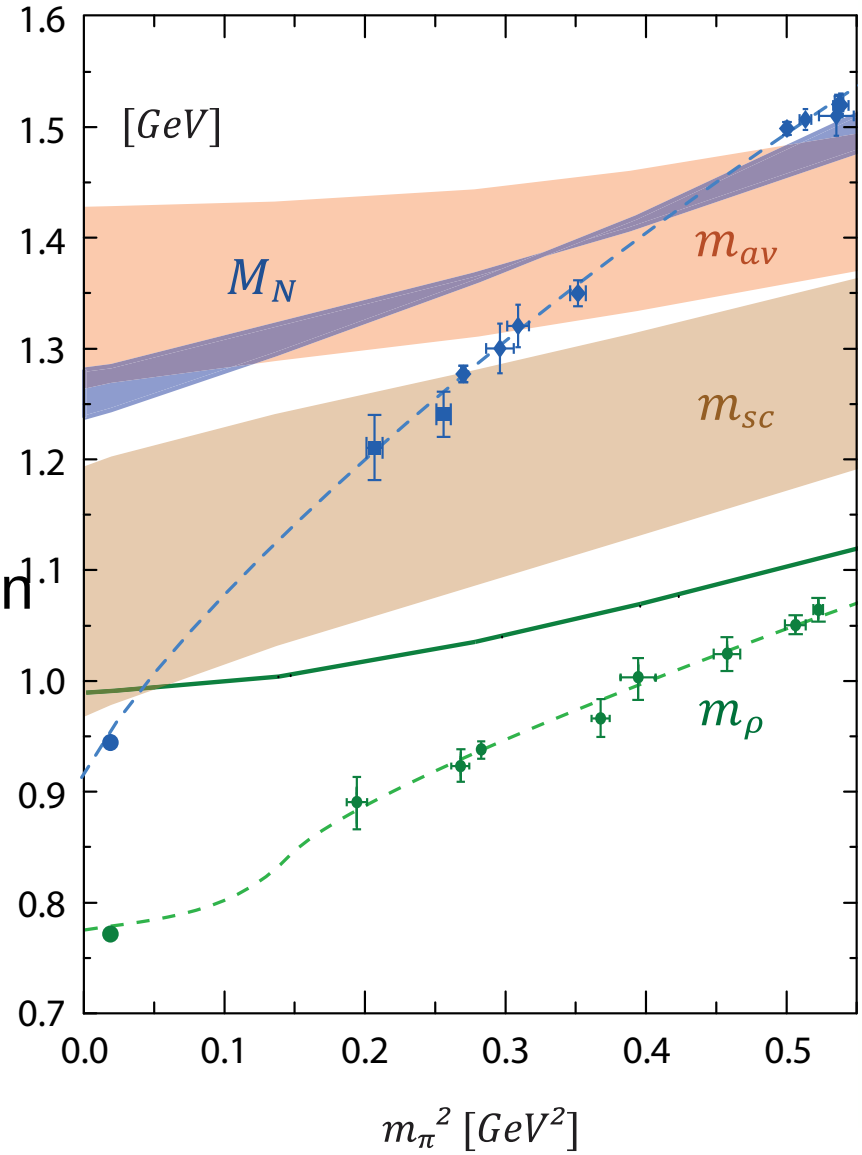

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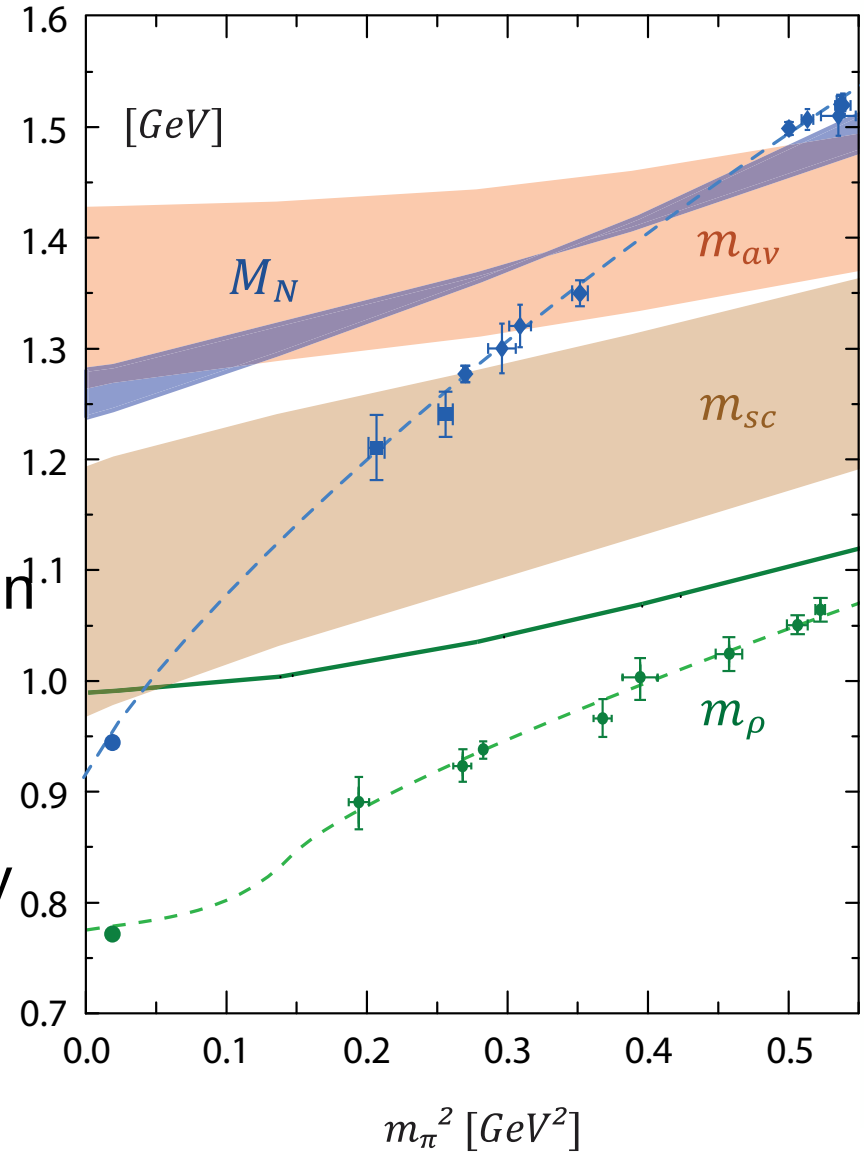

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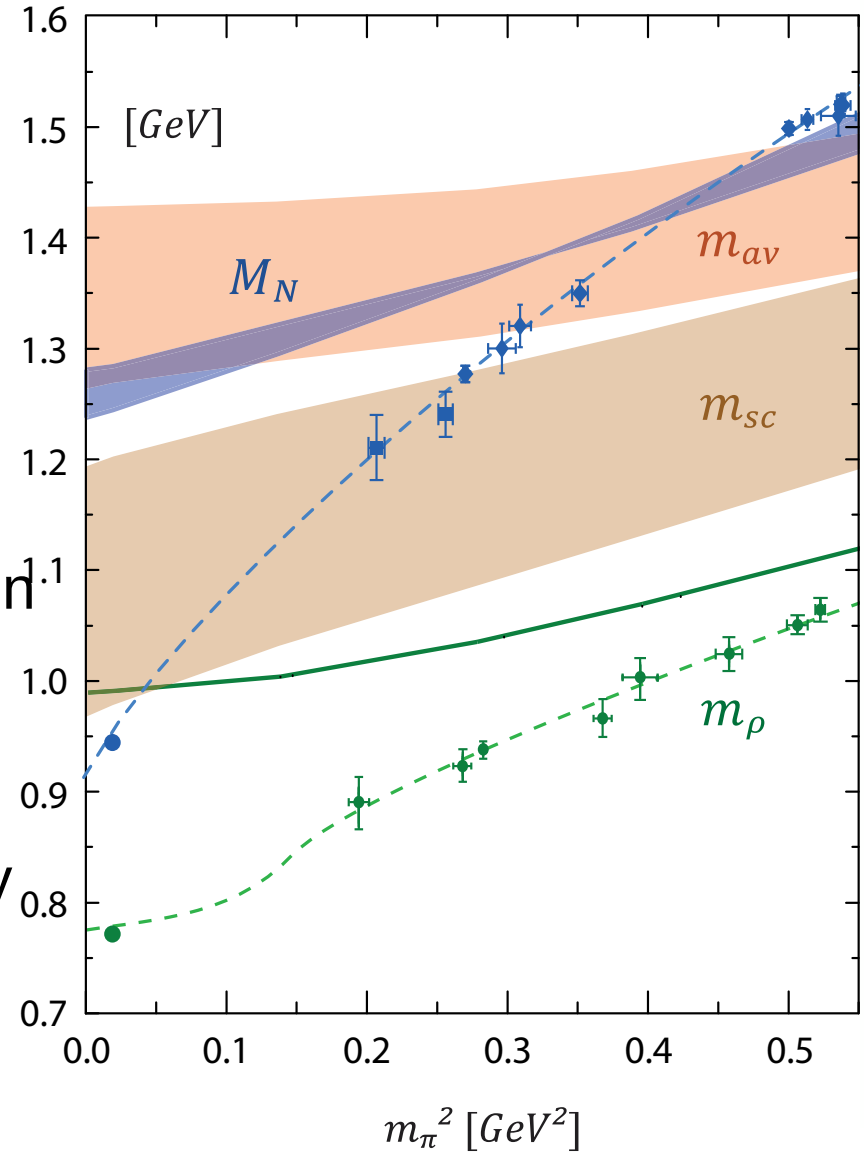
- *Systematically improvable*



Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

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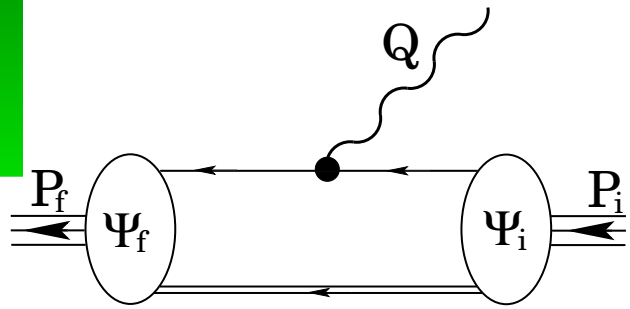

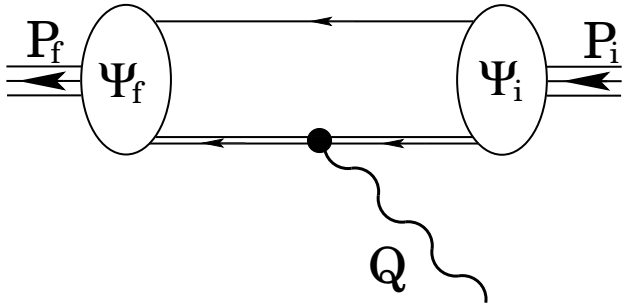

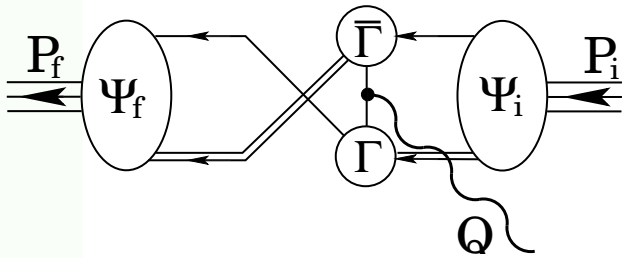

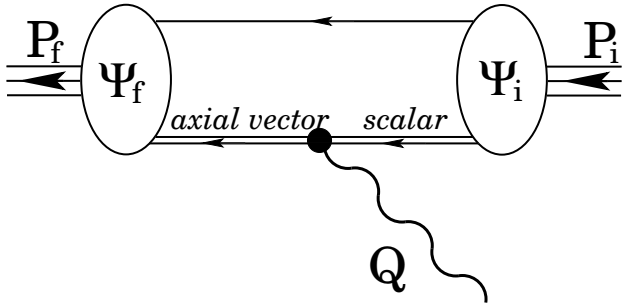

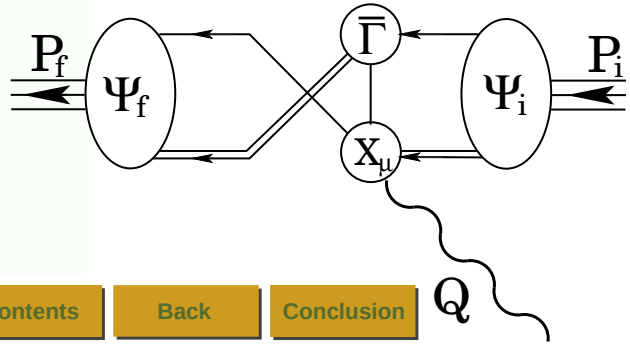

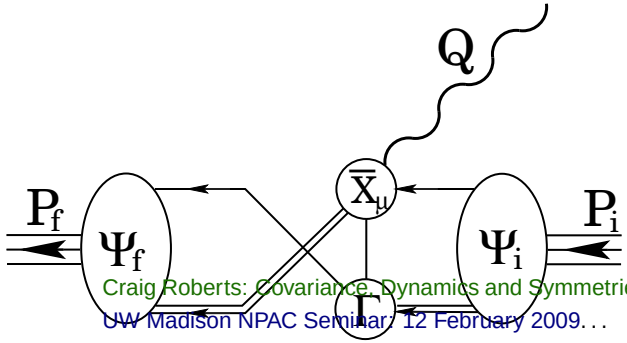

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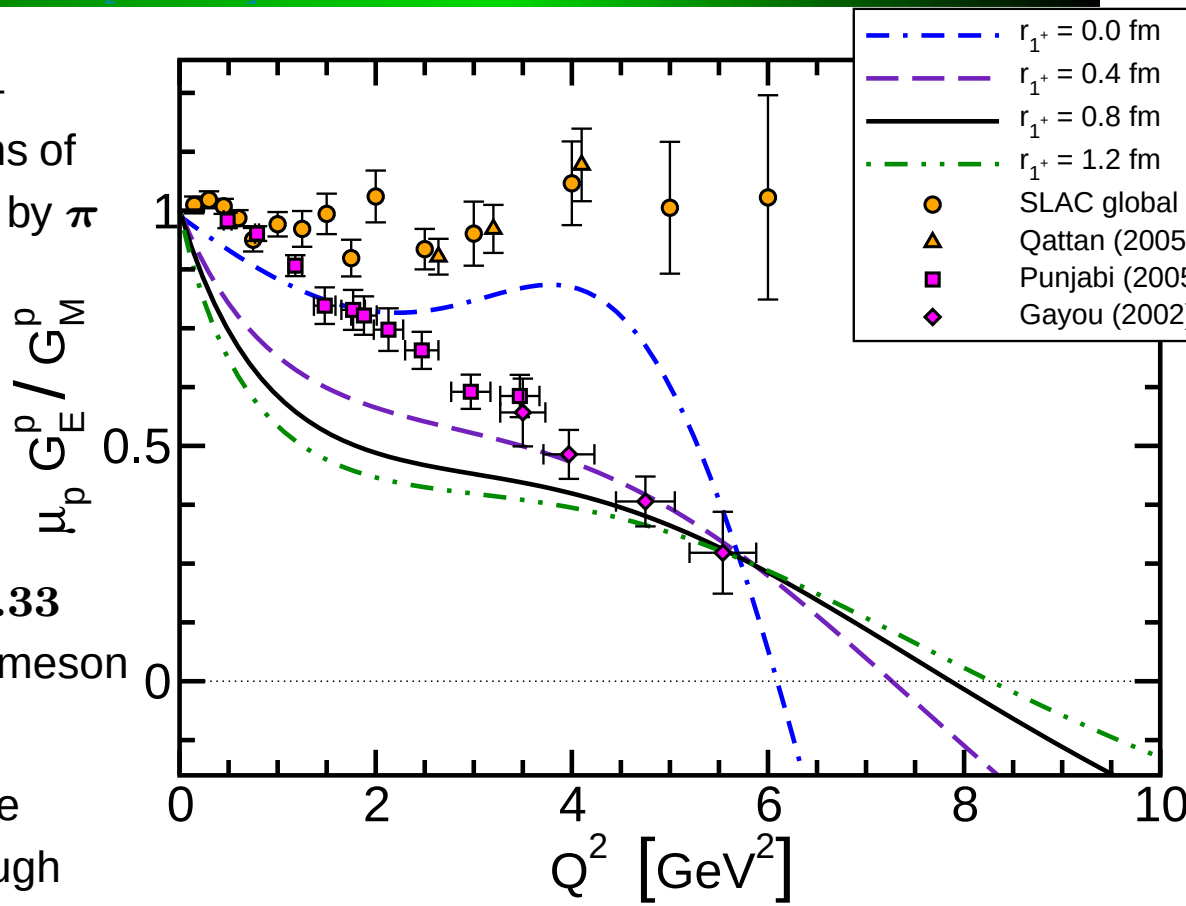

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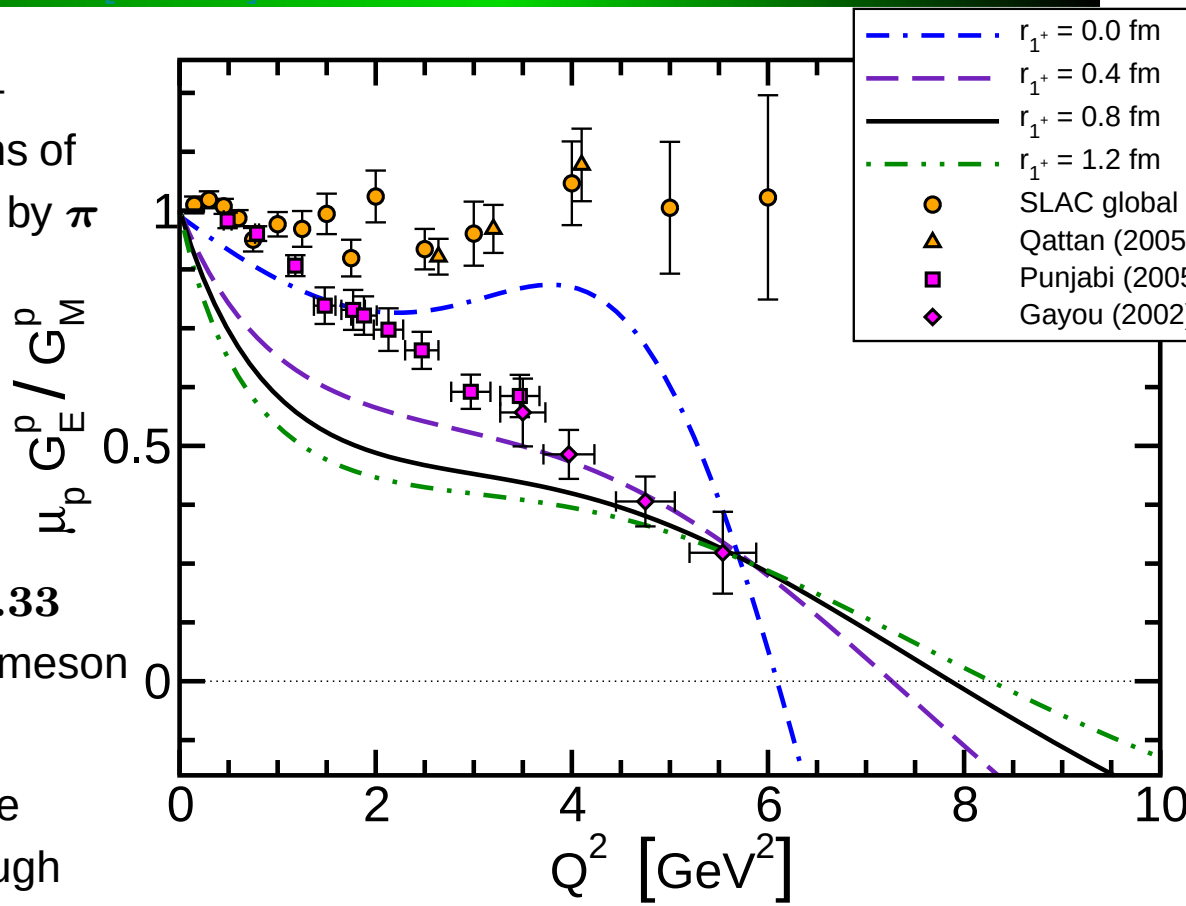

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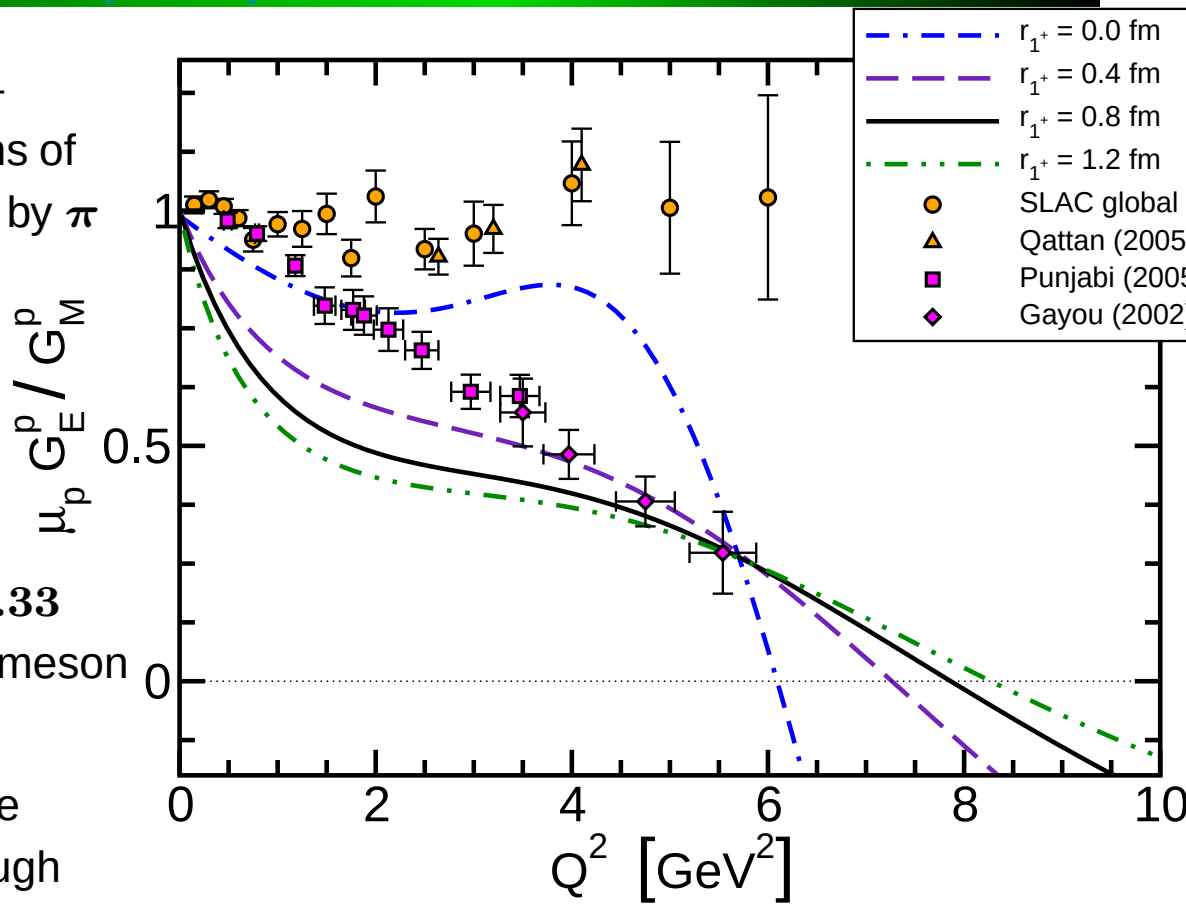

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- Always a zero but position depends on details of current

ab initio

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ab initio
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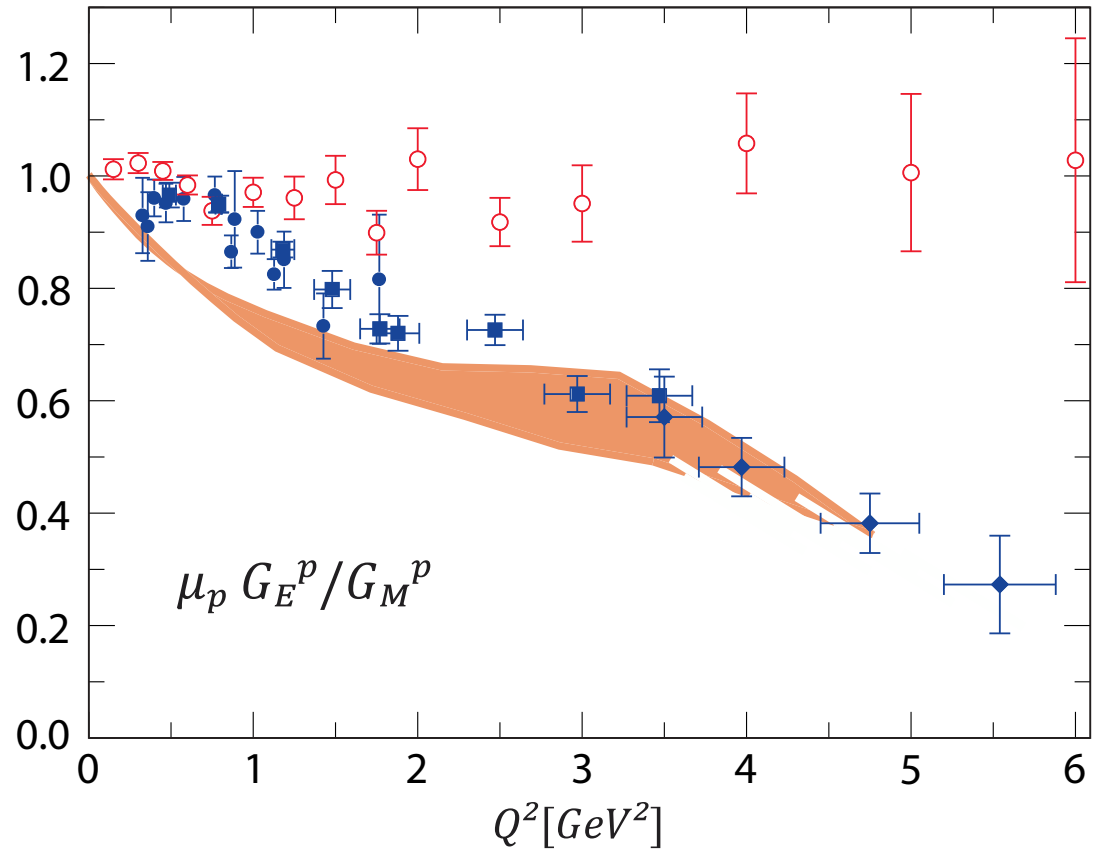

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ab initio
Faddeev Equation

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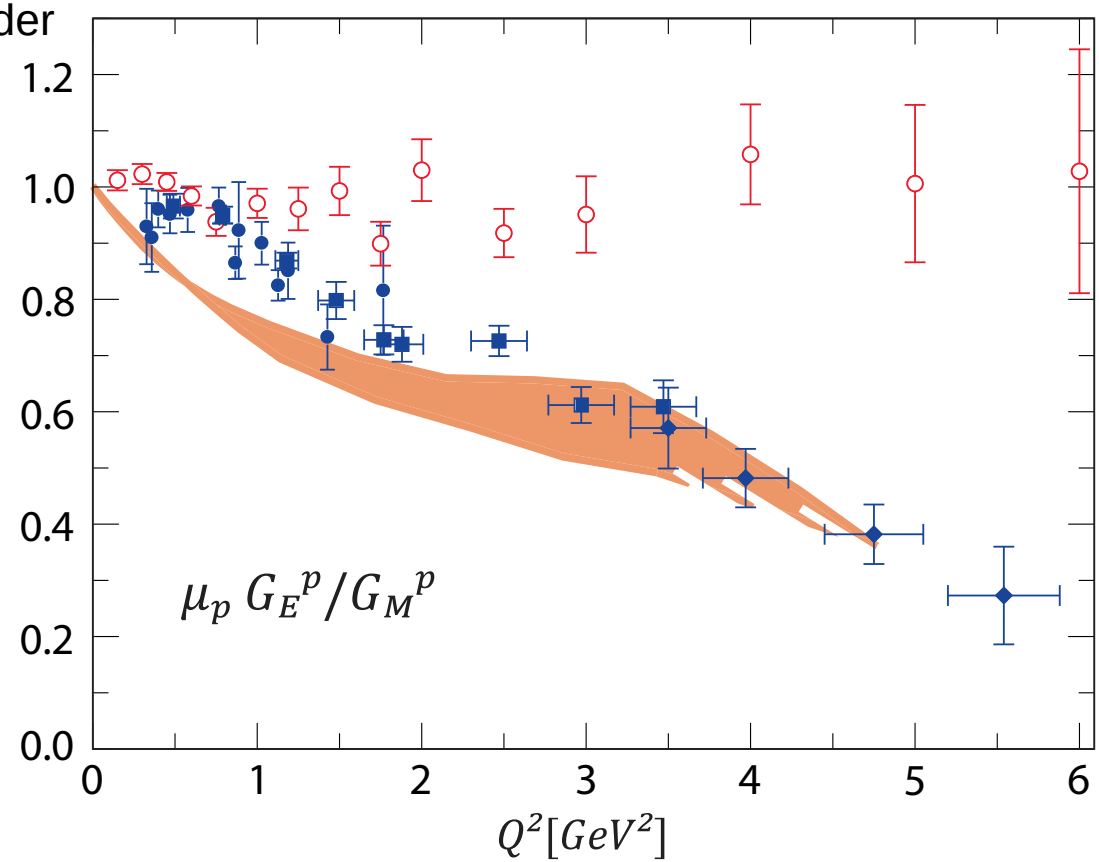

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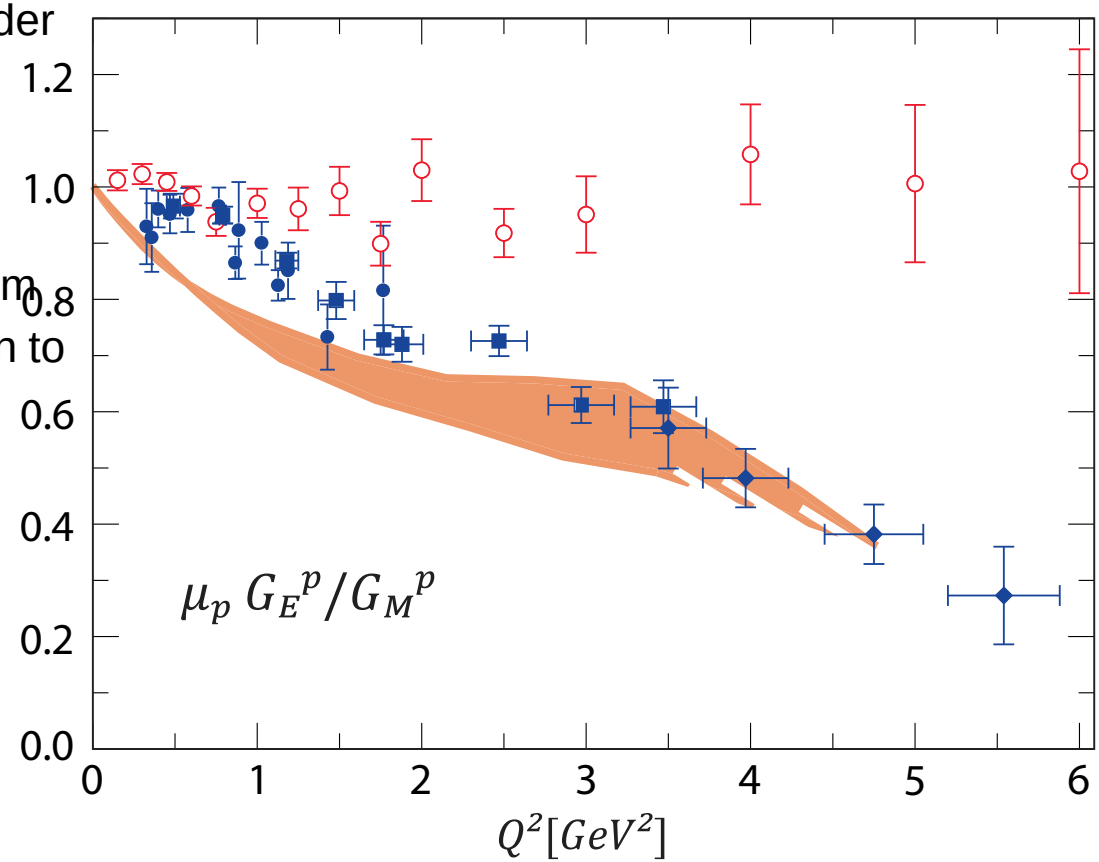

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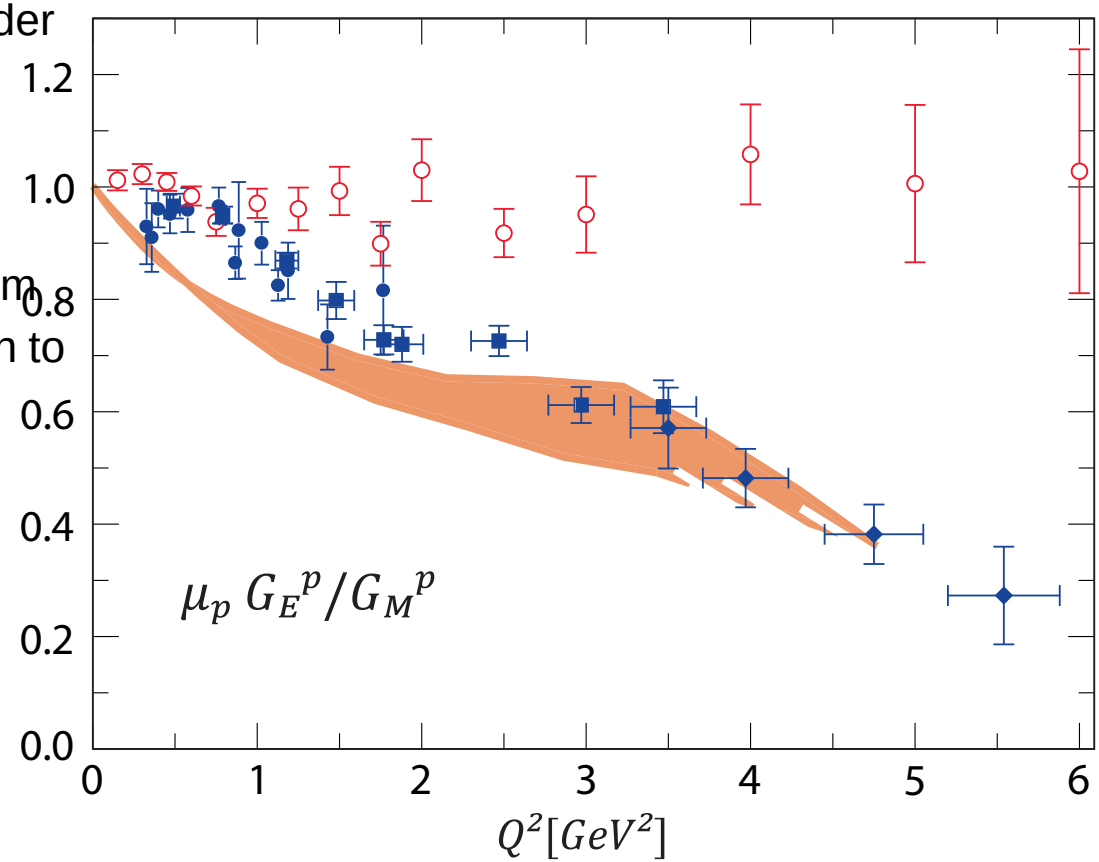

- Calculation unifies π, ρ and nucleon properties – keystone is behaviour of dressed-quark mass function and hence veracious description of QCD's Goldstone mode

Ratio of Neutron Pauli & Dirac Form Factors

- $$\frac{\hat{Q}^2}{(\ln \hat{Q}^2/\hat{\Lambda})^2} \frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)}$$

  $$\hat{\Lambda} = \Lambda/M_N = 0.44$$

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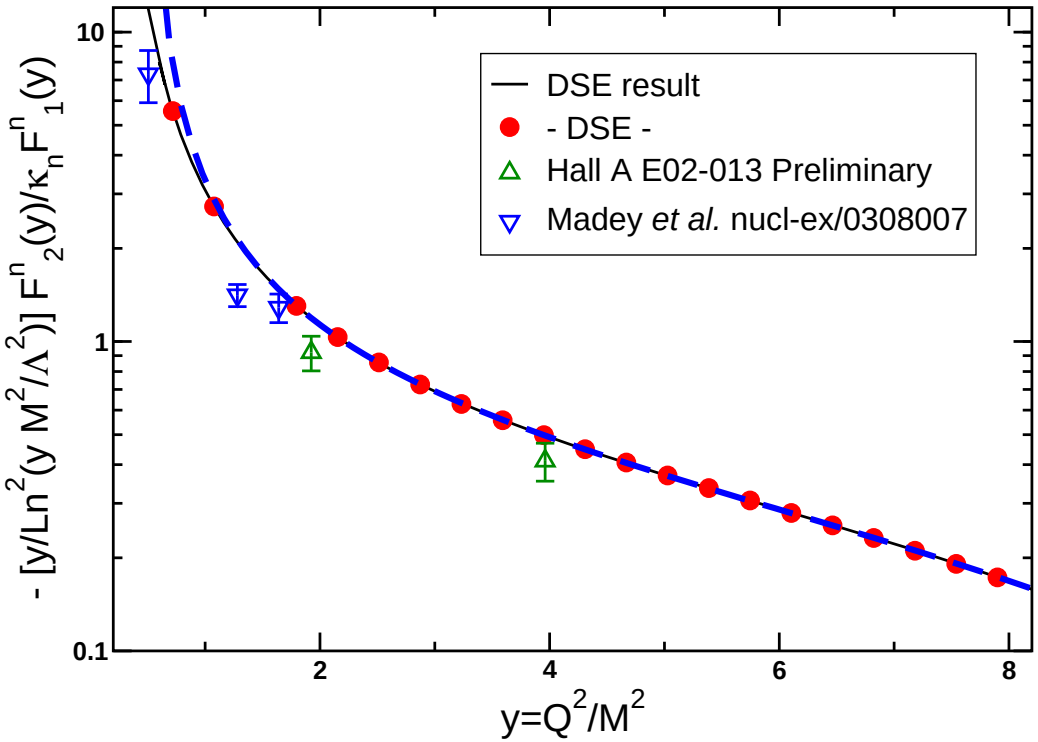

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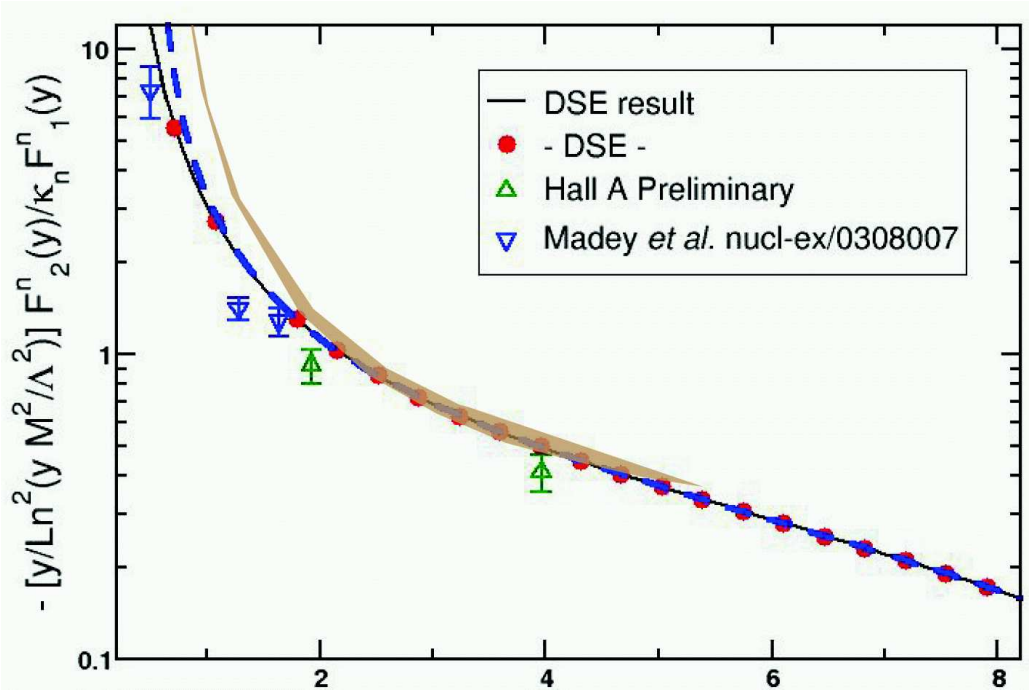
- Brown band

  – *ab initio* RL result



Pion Cloud

Pion Cloud

F2 – neutron

Pion Cloud
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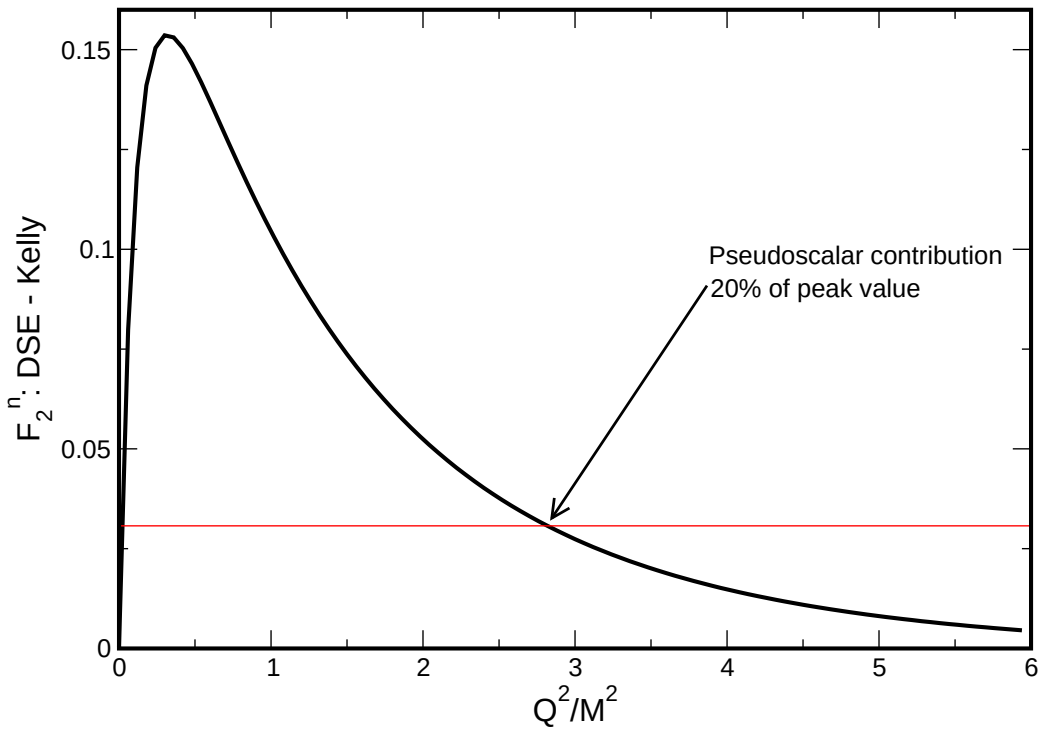
- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core



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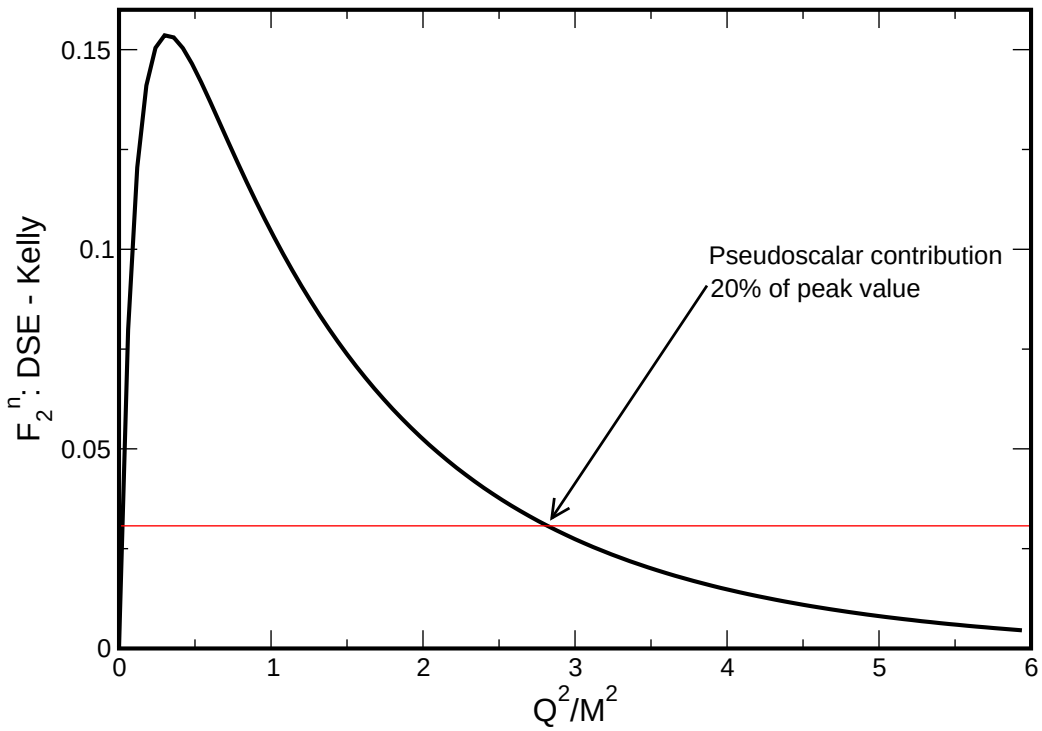
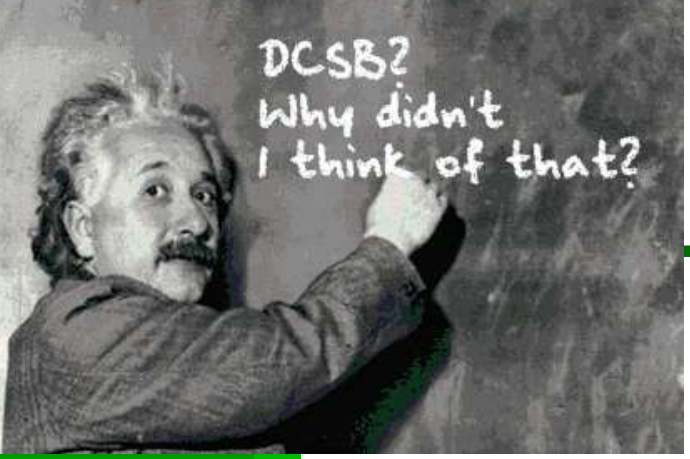


- Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4\, M_N^2$

Epilogue



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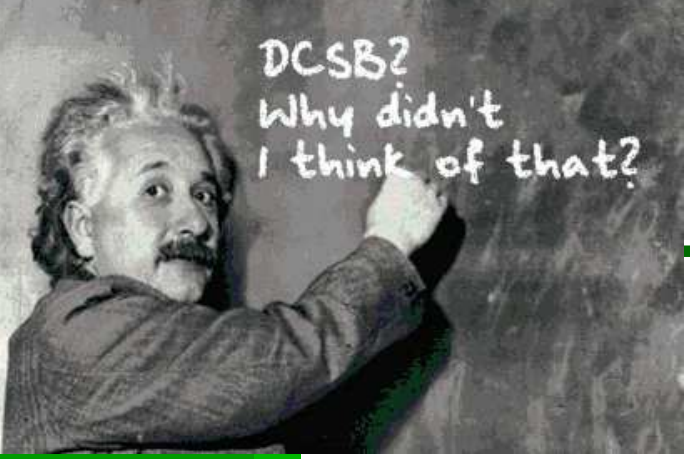

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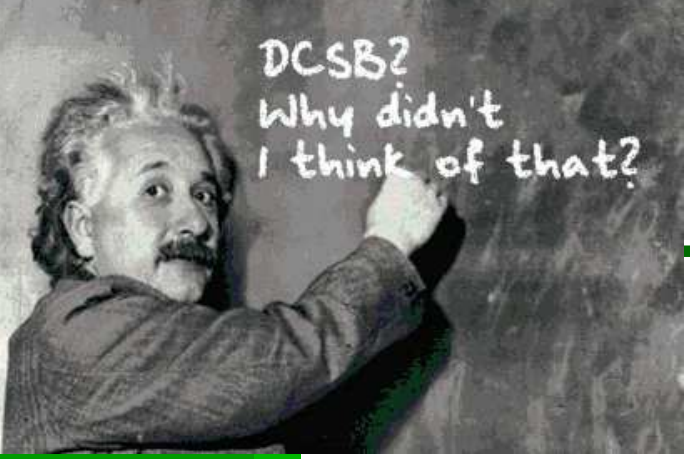

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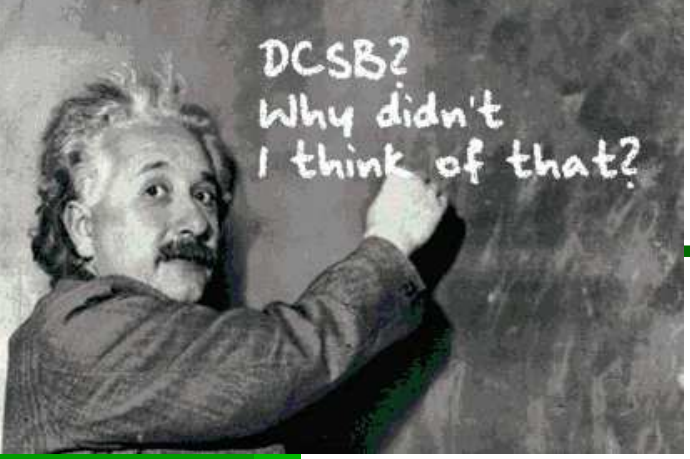

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- DCSB exists in QCD.
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  - It predicts, amongst other things, that
    - light current-quarks become heavy constituent-quarks: 4 $\rightarrow$ 400 MeV
    - pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140$ MeV
    - pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi\bar{q}q} \approx 4.3$
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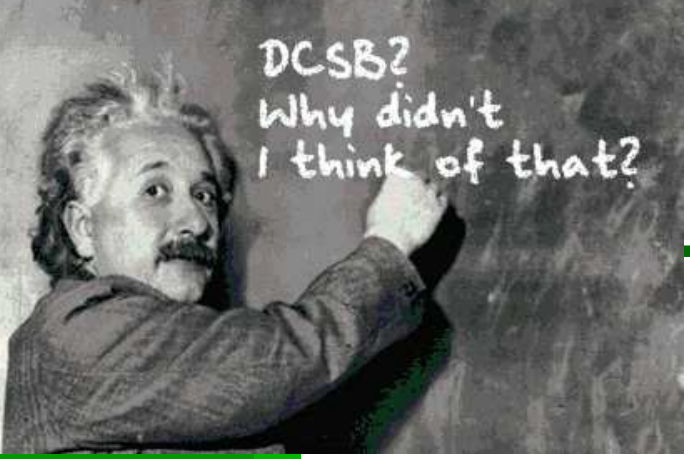

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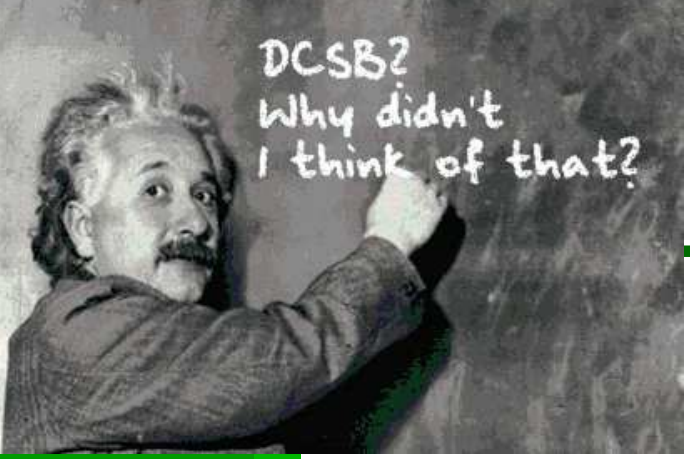
  - It impacts dramatically upon observables.



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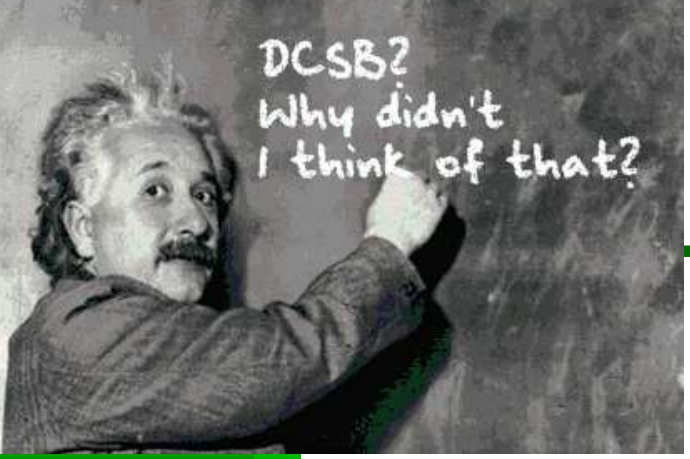

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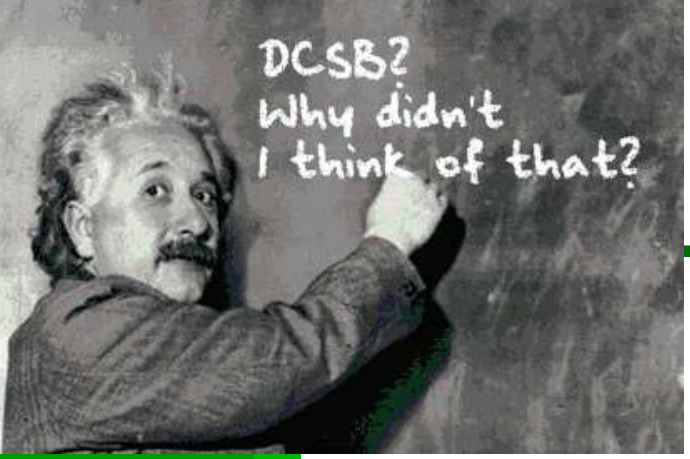

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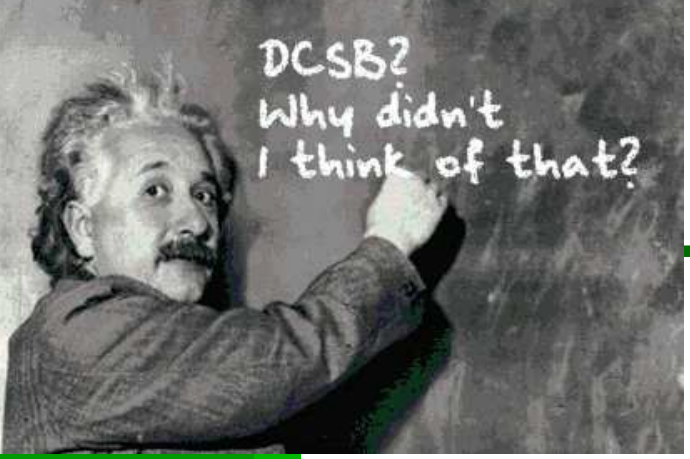

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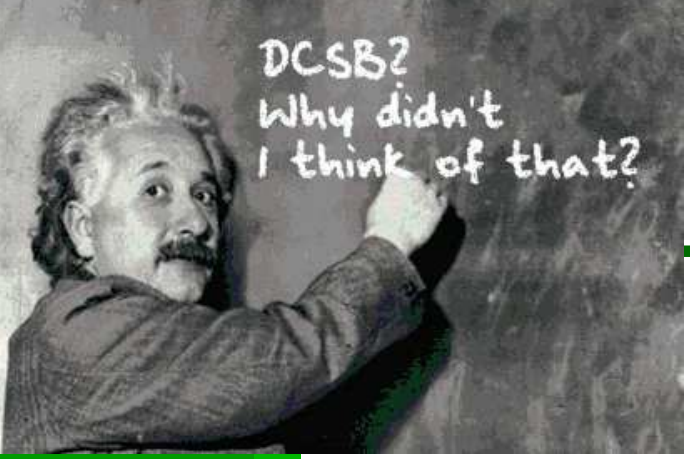

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- Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks

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