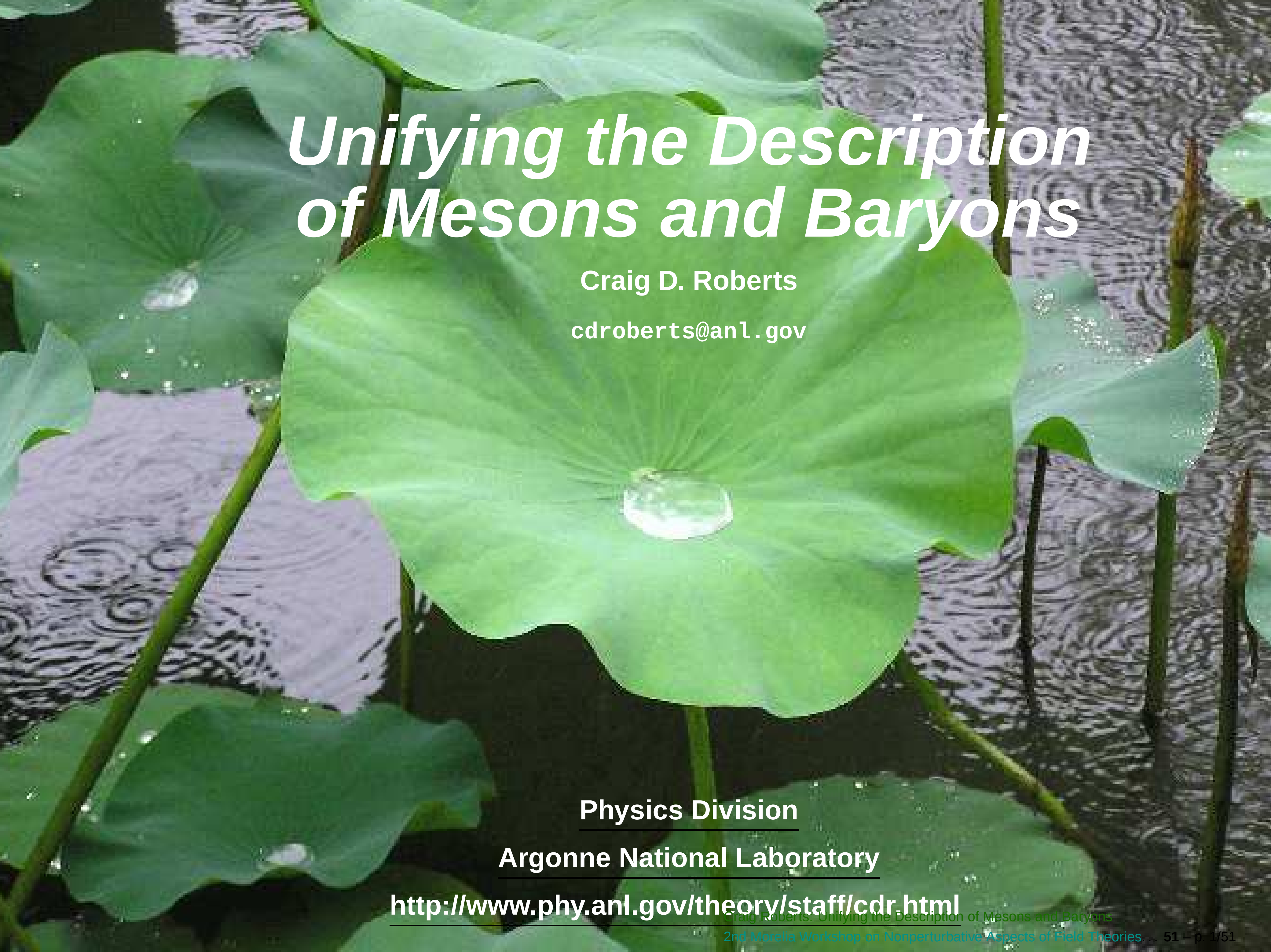

Unifying the Description of Mesons and Baryons

Craig D. Roberts

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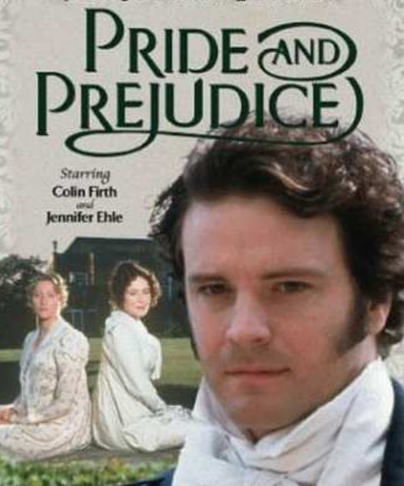
Physics Division

Argonne National Laboratory

http://www.phy.anl.gov/theory/staff/cdr.html

Universal Truths



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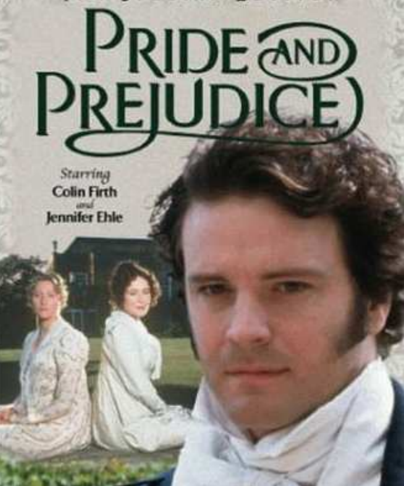

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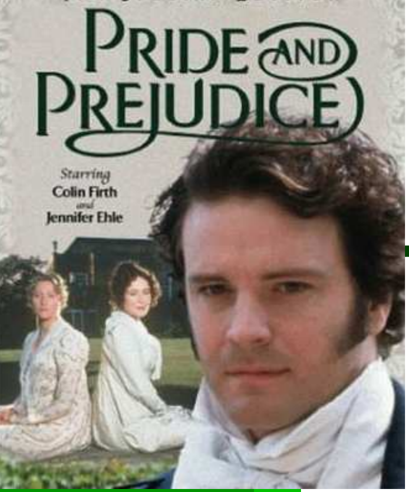

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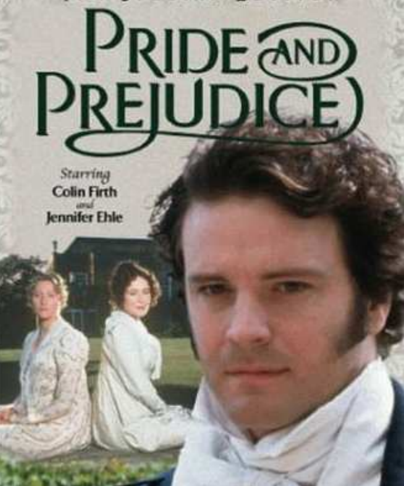

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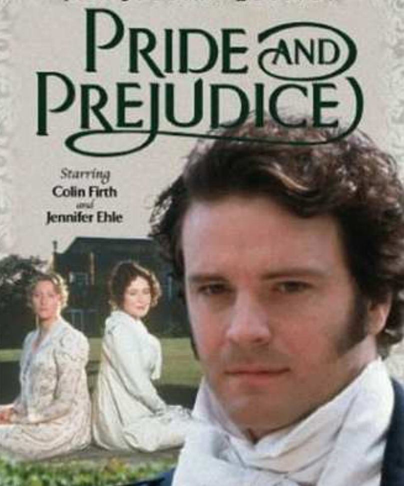

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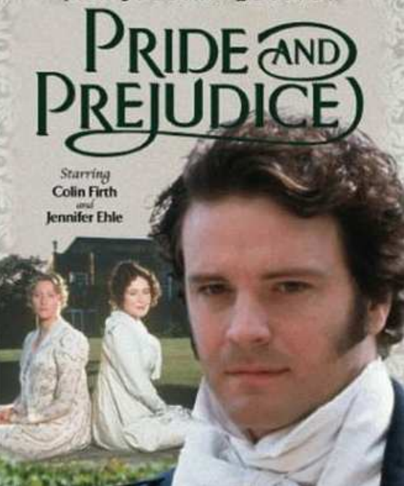

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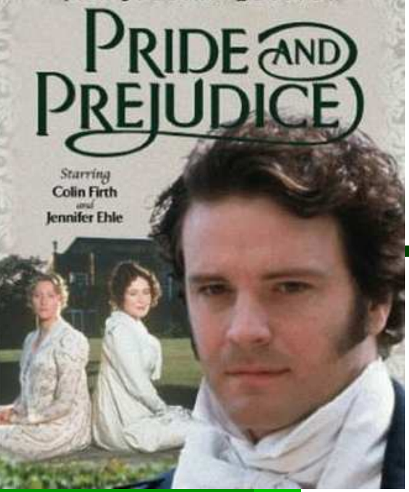

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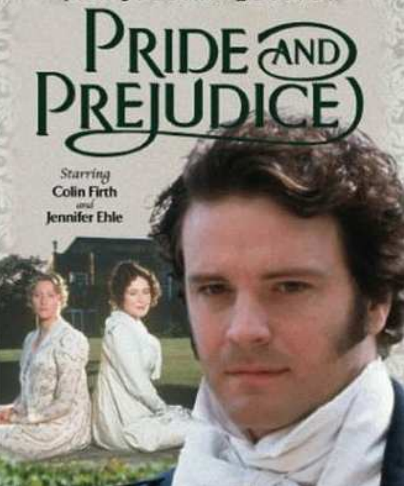

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- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.

QCD's Challenges

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- Quark and Gluon Confinement
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QCD's Challenges

Understand Emergent Phenomena

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- Neither of these phenomena is apparent in QCD's Lagrangian ***yet*** they are the dominant determining characteristics of real-world QCD.
- QCD – Complex behaviour

  arises from apparently simple rules

Dichotomy of Pion
– *Goldstone Mode* and *Bound state*

- How does one make an almost massless particle
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Current Algebra . . . 1968

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Highly Nontrivial

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  - Interaction between quarks – the Interquark "Potential" – unknown throughout $>$ 98% of a hadron's volume

Intranucleon Interaction

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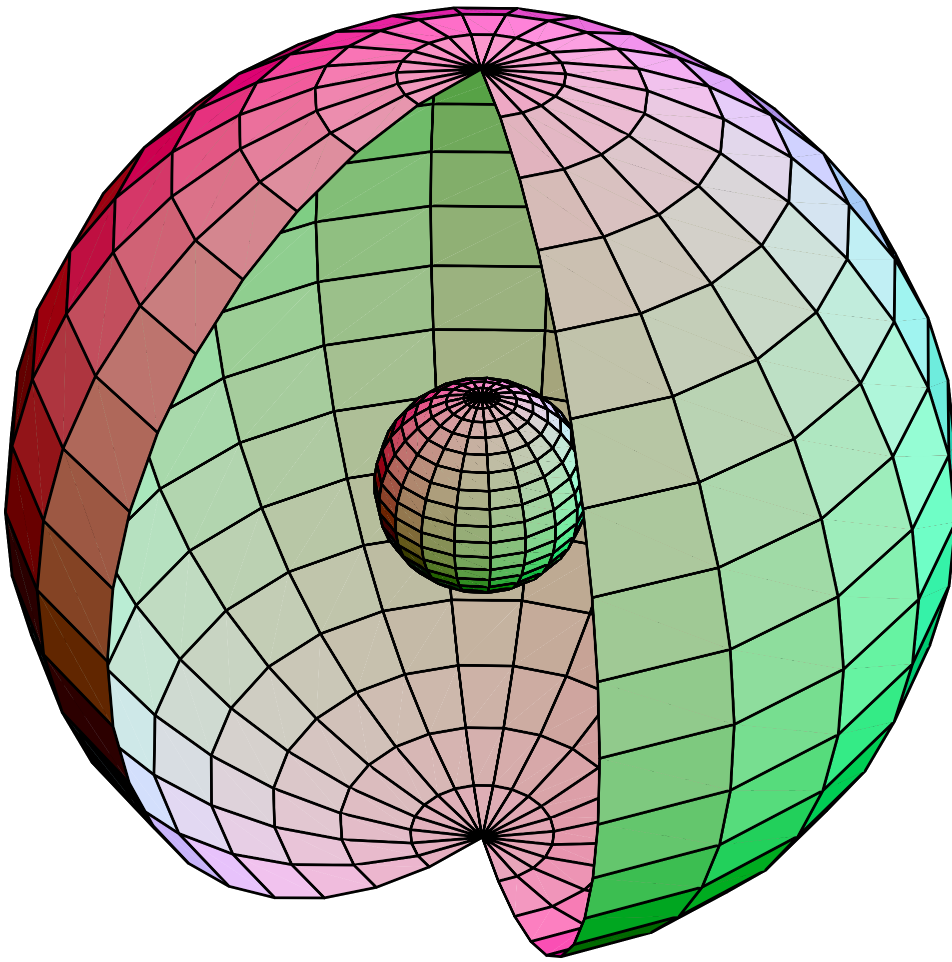

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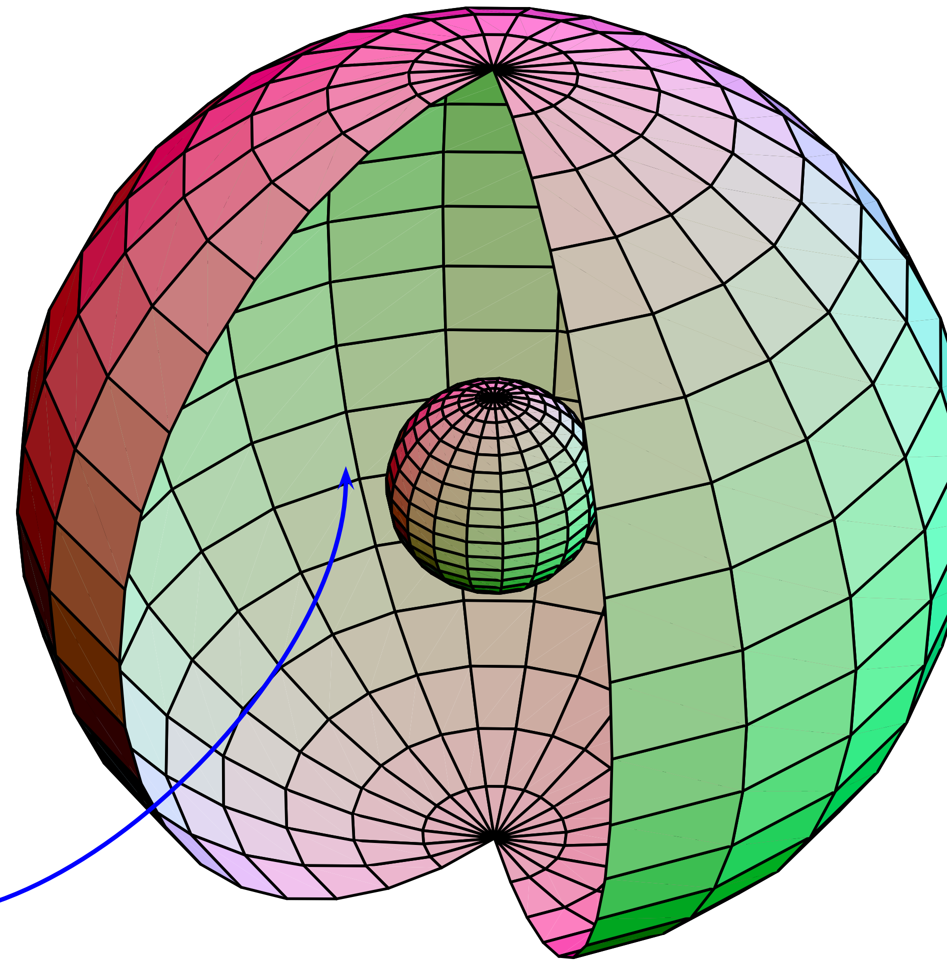
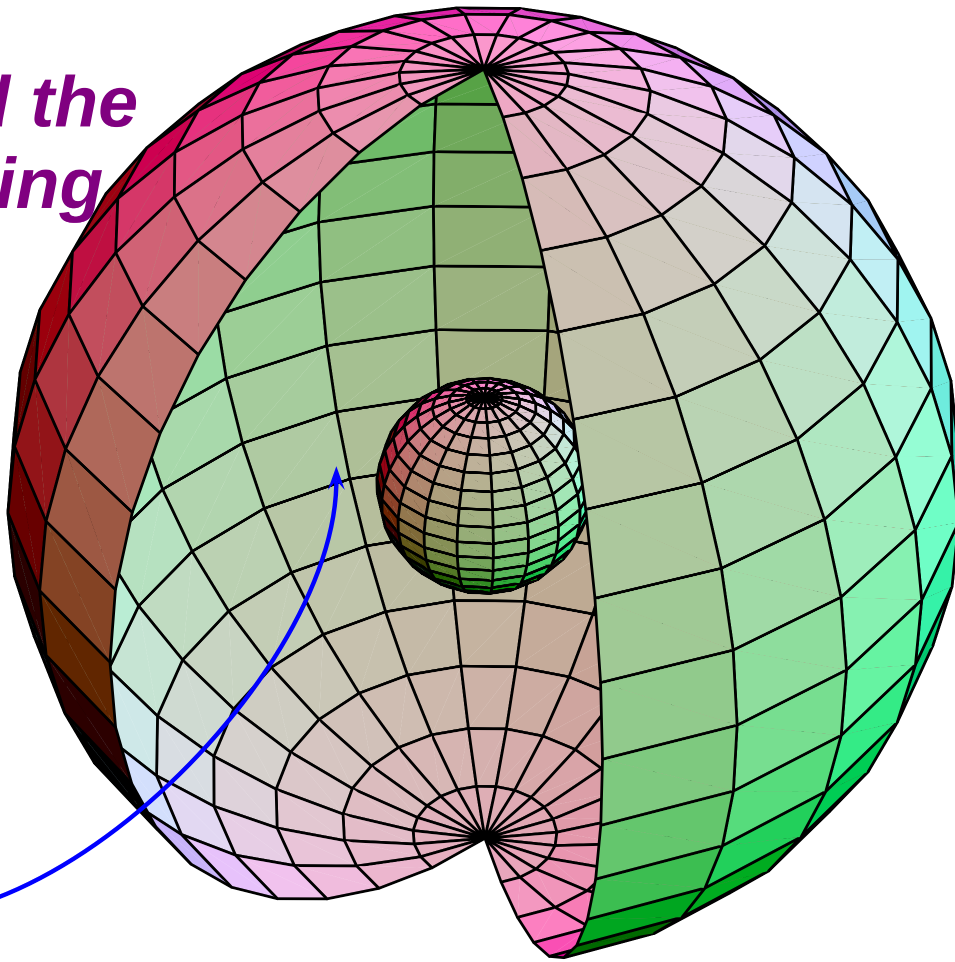


98% of the volume

What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



Confinement

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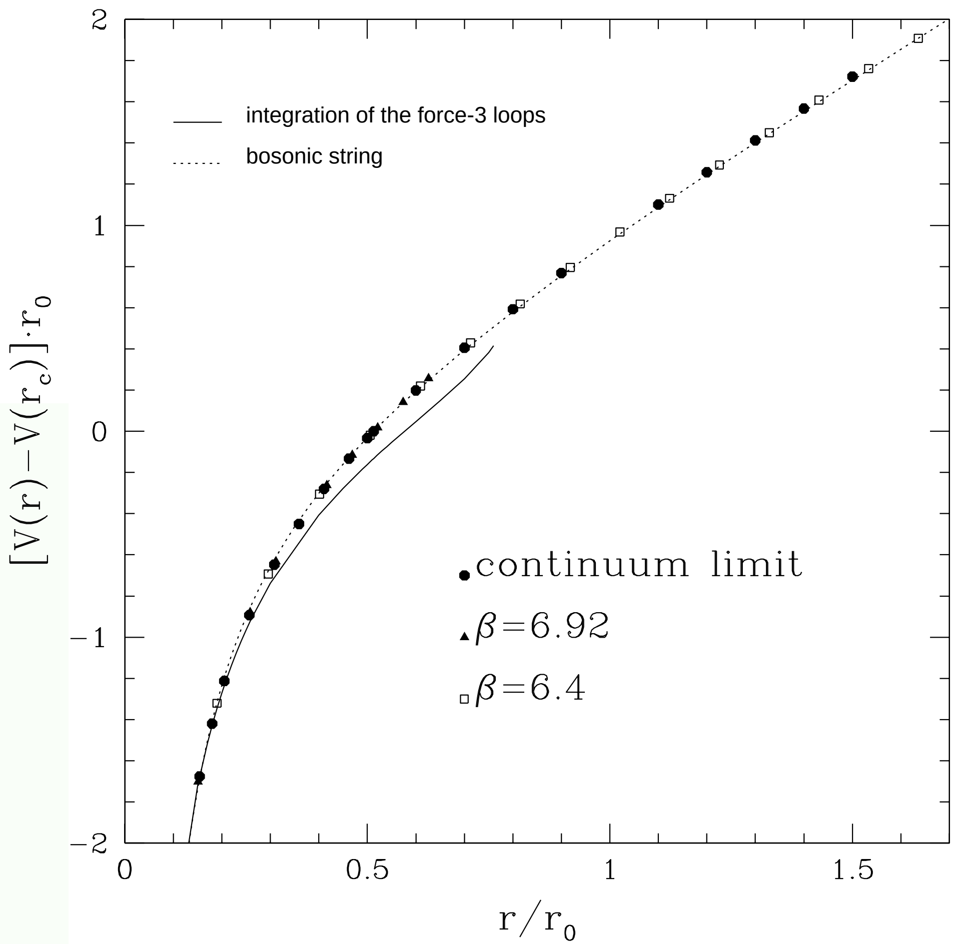
Confinement

- Infinitely Heavy Quarks . . . Picture in Quantum Mechanics



$$V(r) = \sigma\, r - \frac{\pi}{12}\,\frac{1}{r}$$

$\sigma \sim 470$ MeV

Necco & Sommer
he-la/0108008

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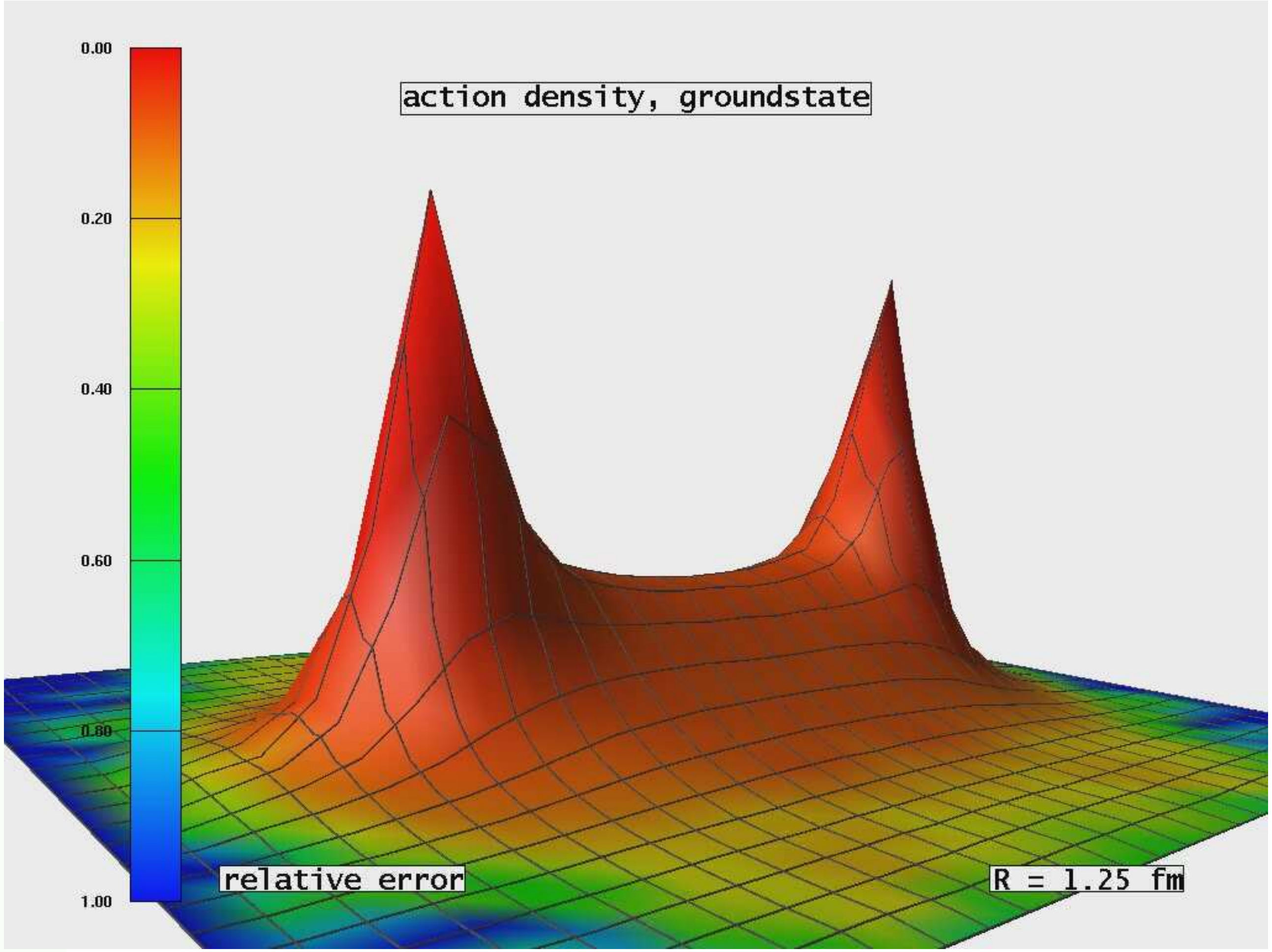
Confinement

- Illustrate this in terms of the action density ... analogous to plotting the Force $= F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12}\frac{1}{r^2}$



Bali, *et al*.
he-la/0512018

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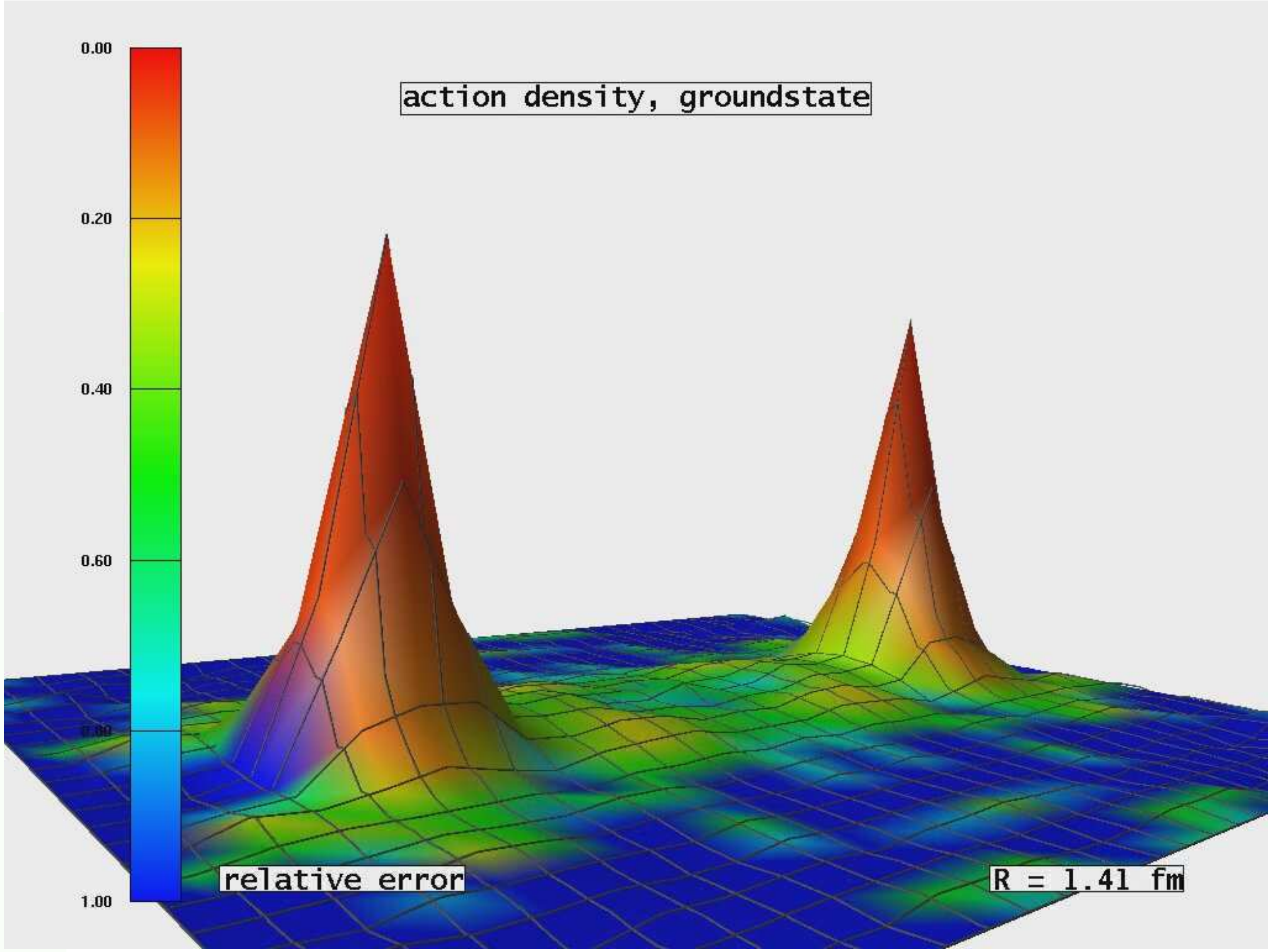

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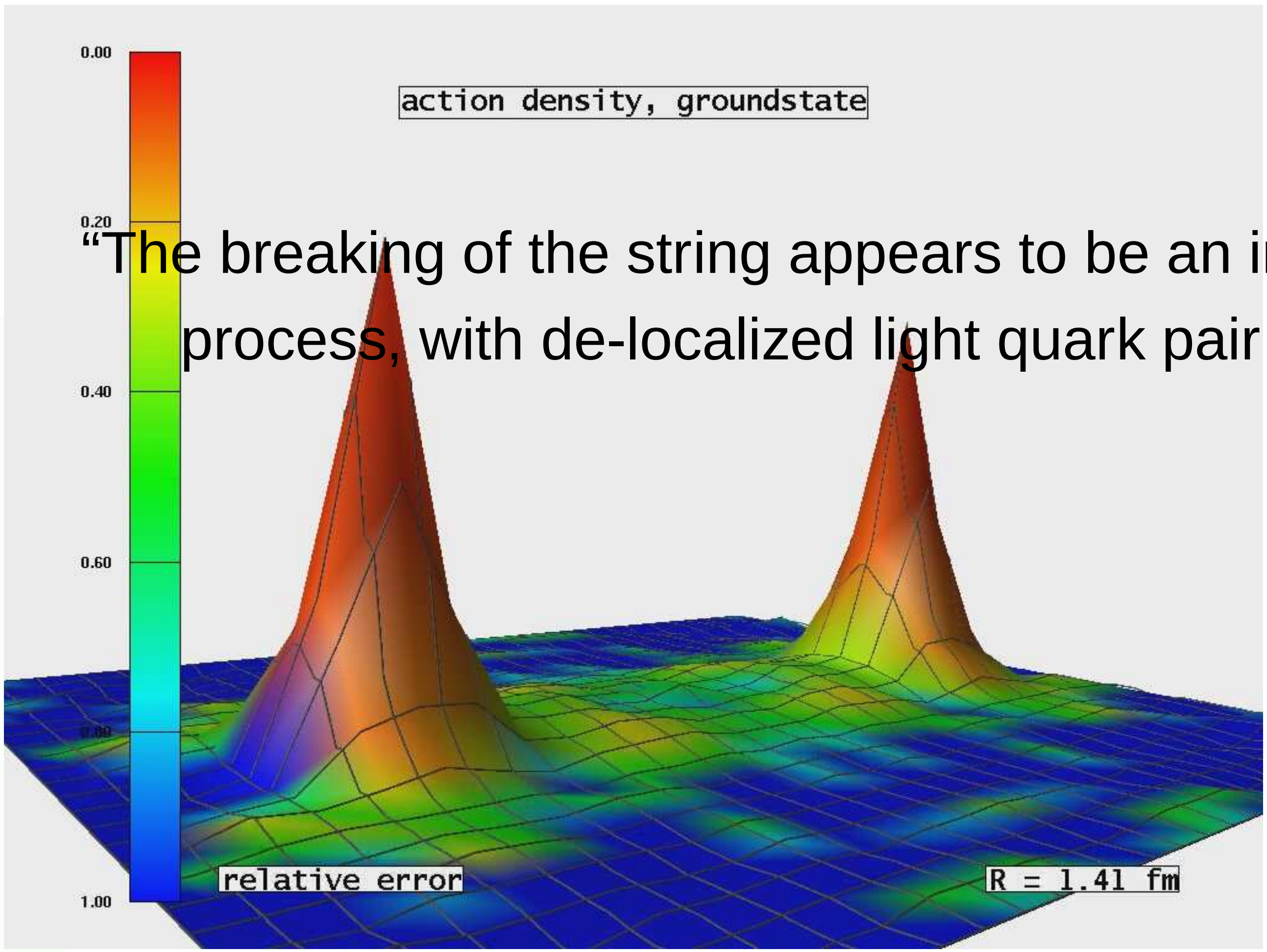

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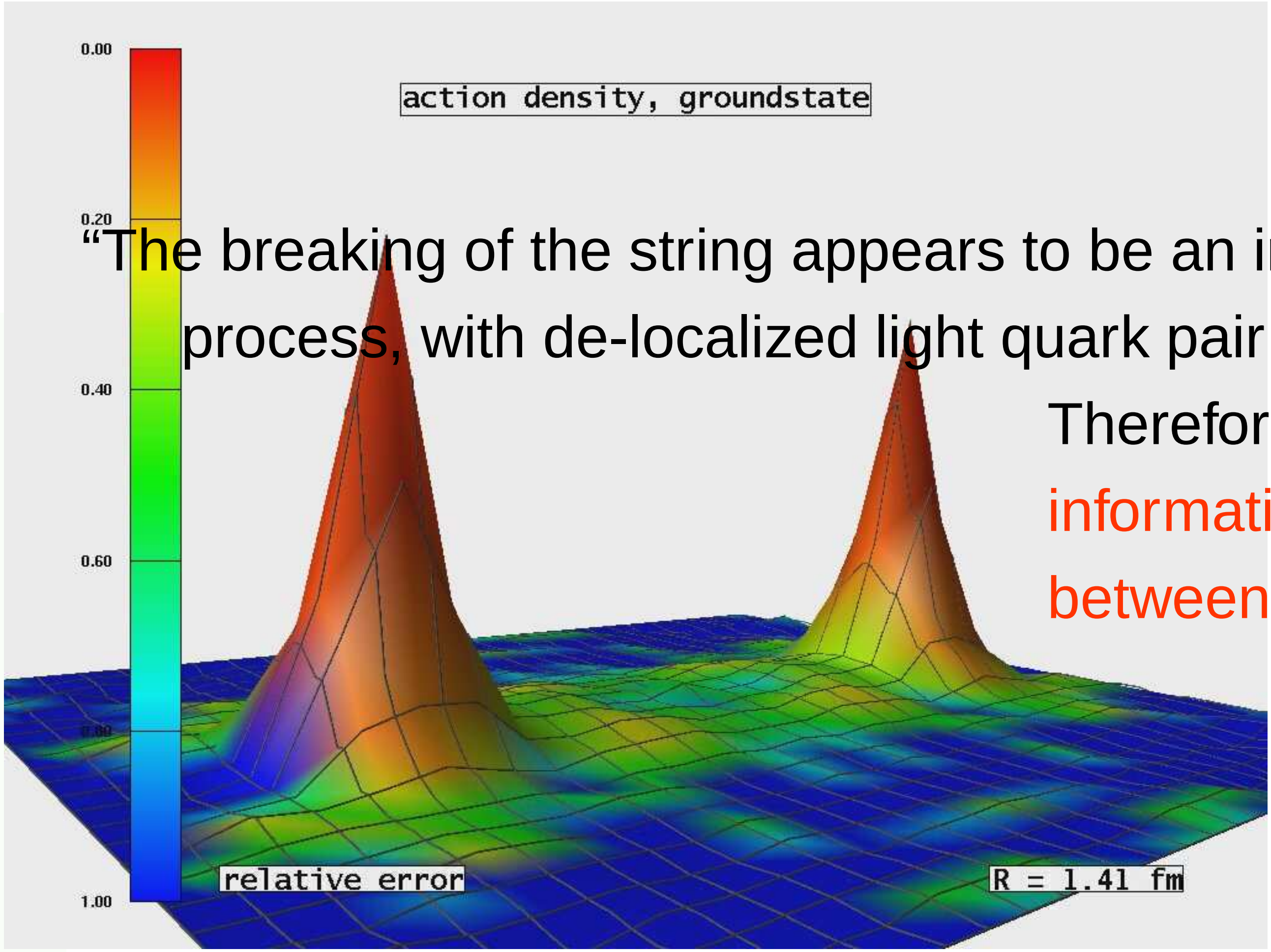

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Therefore … No information on *potential* between light-quarks.

What is the light-quark Long-Range Potential?

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What is the light-quark Long-Range Potential?

Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

Charting the Interaction between light-quarks

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- This is a well-posed problem whose solution is an elemental goal of modern hadron physics.

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- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
  - On other hand, at present significant qualitative advances possible with symmetry-preserving kernel *Ansätze* that express important additional nonperturbative effects, difficult to capture in any finite sum of contributions

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- $\Rightarrow$ Understanding InfraRed (long-range)
  .... behaviour of $\alpha_s(Q^2)$

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- Method yields Schwinger Functions $\equiv$ Propagators

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Cross-Sections built from Schwinger Functions

Perturbative
Dressed-quark Propagator

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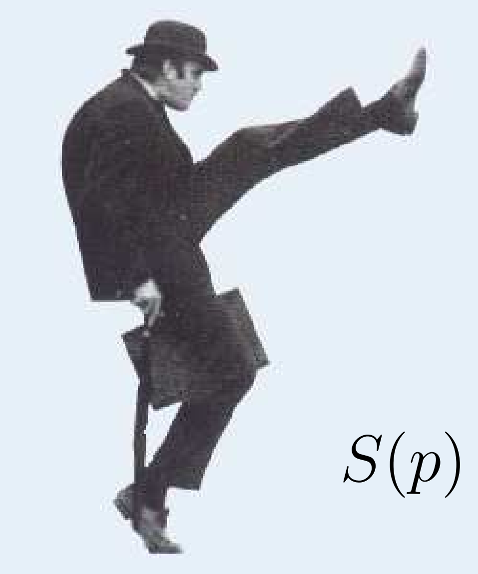
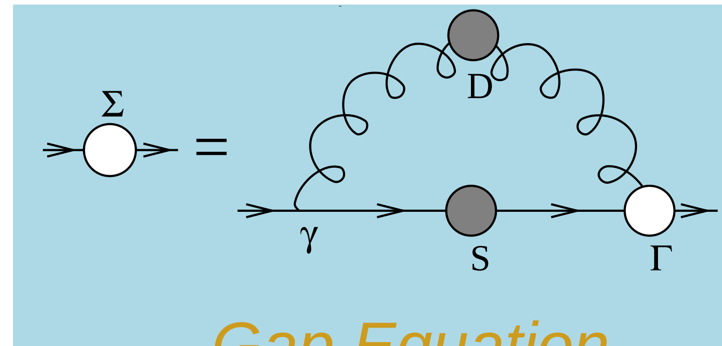
Perturbative Dressed-quark Propagator

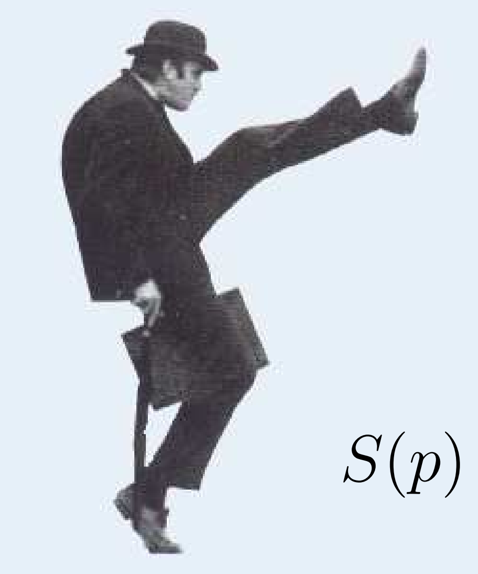


$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

Perturbative Dressed-quark Propagator



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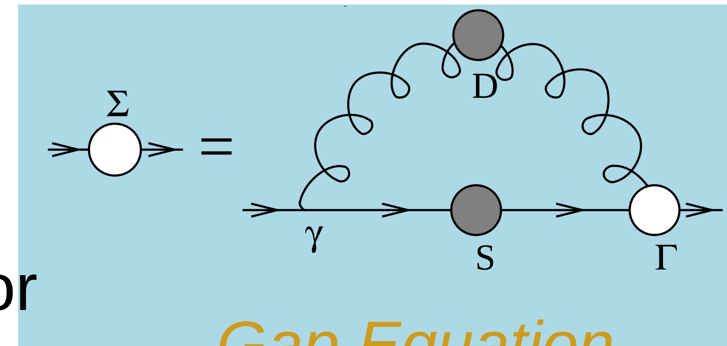

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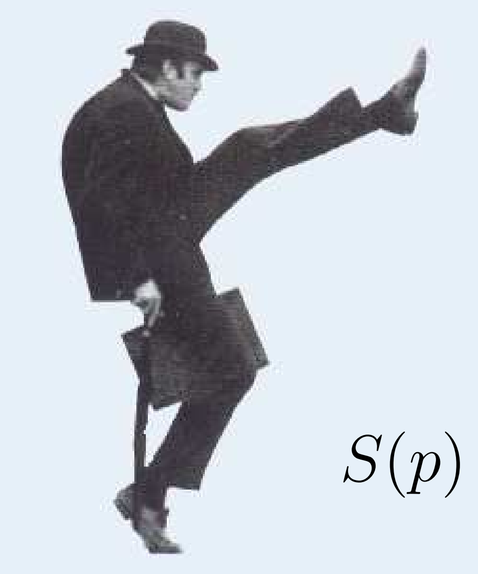

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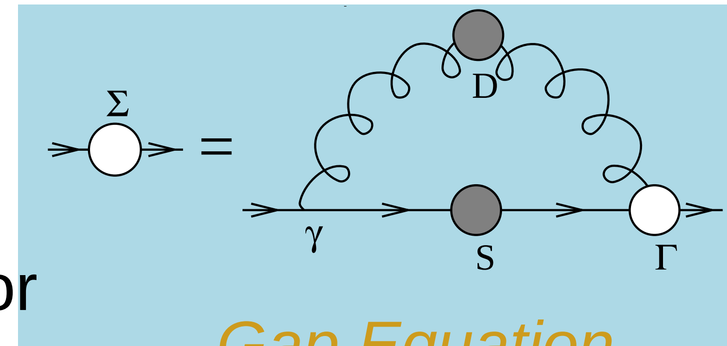

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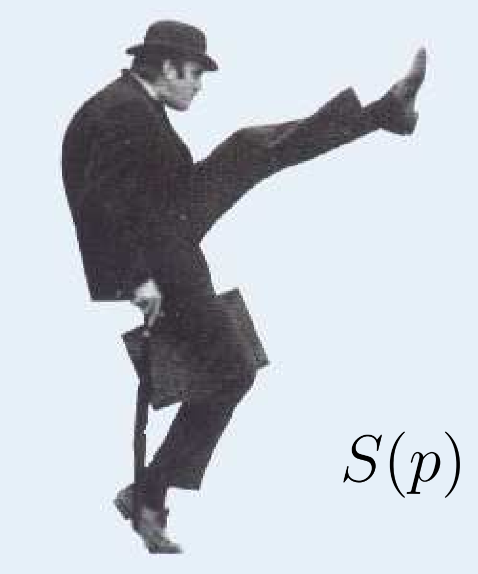

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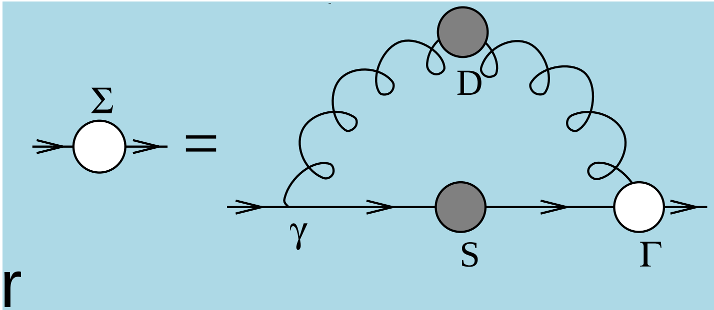

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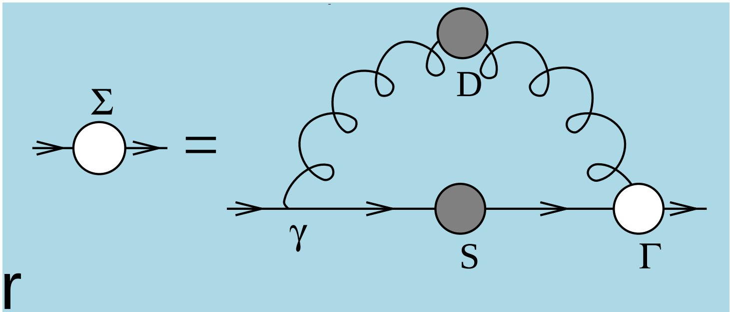

Gap Equation

No DCSB Here!

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Frontiers of Nuclear Science: A Long Range Plan (2007)

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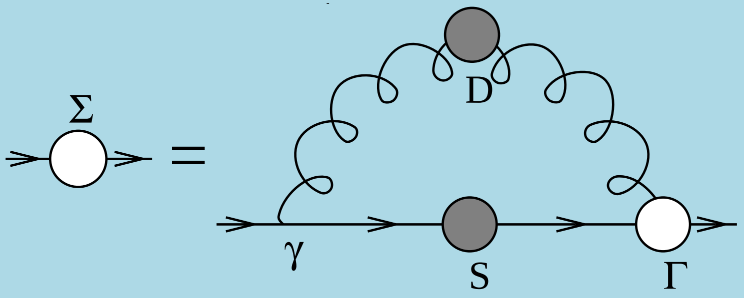

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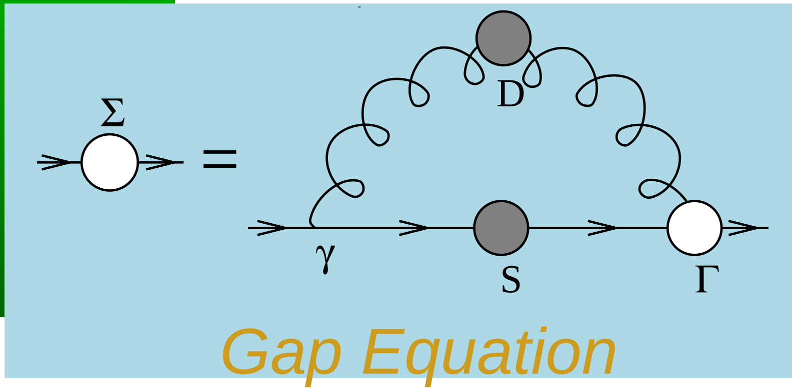

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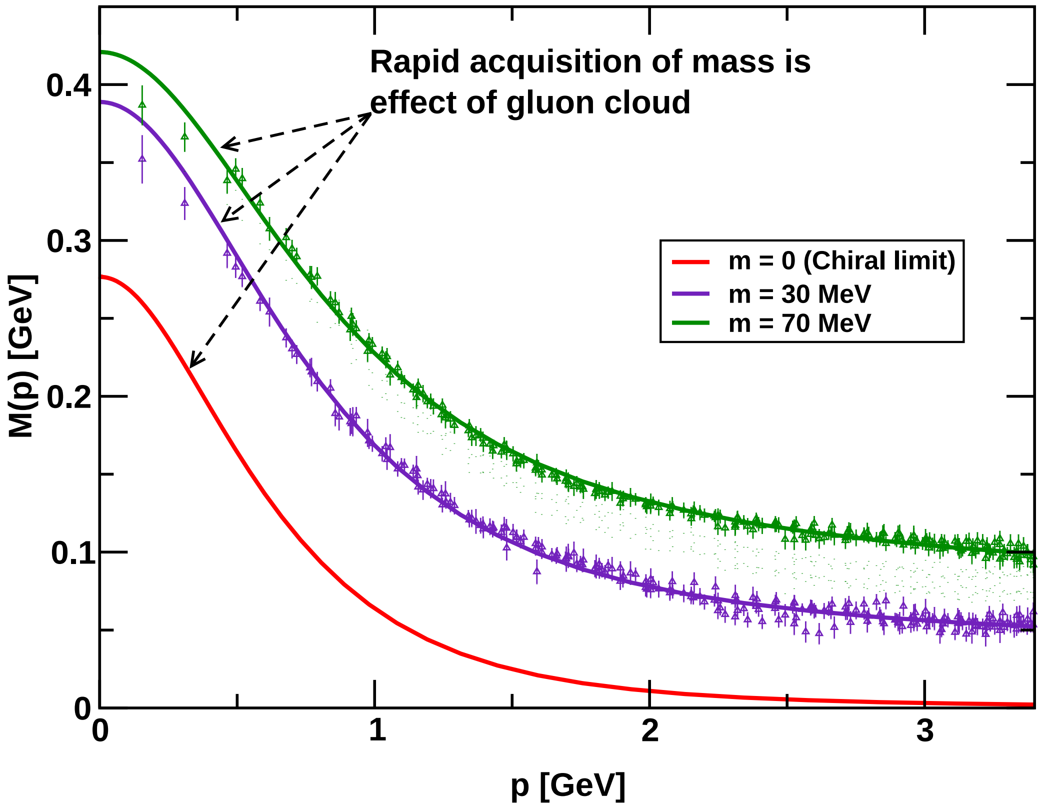

Mass from nothing.

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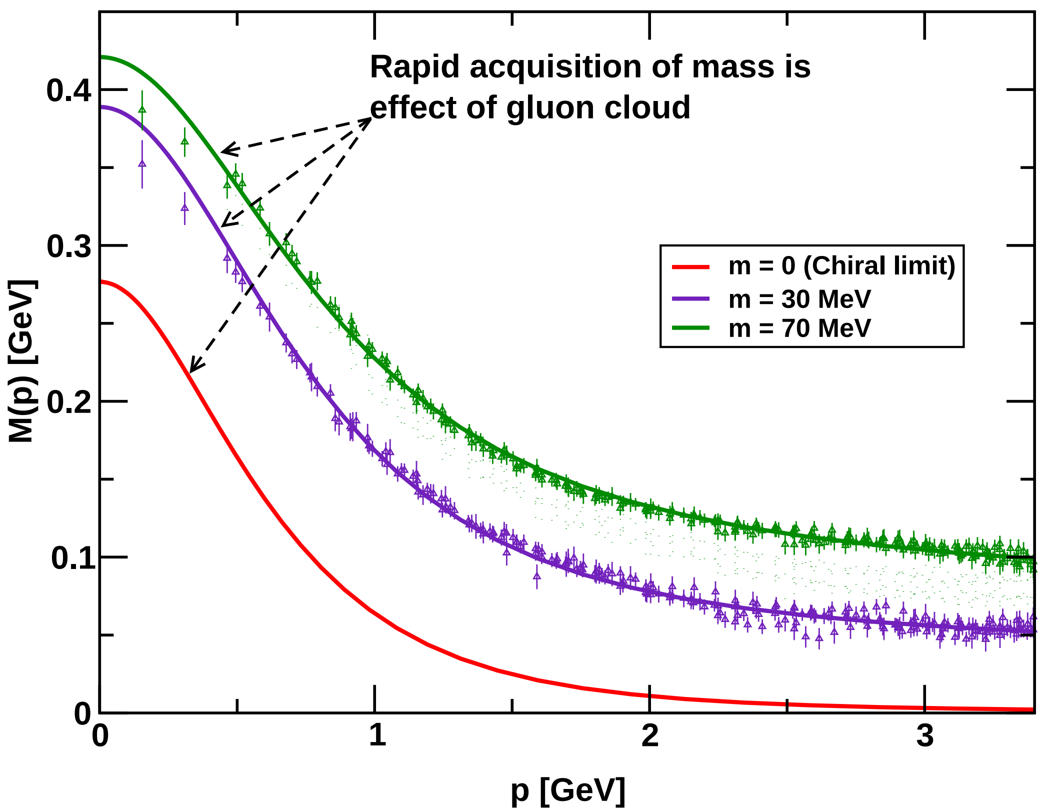
In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.



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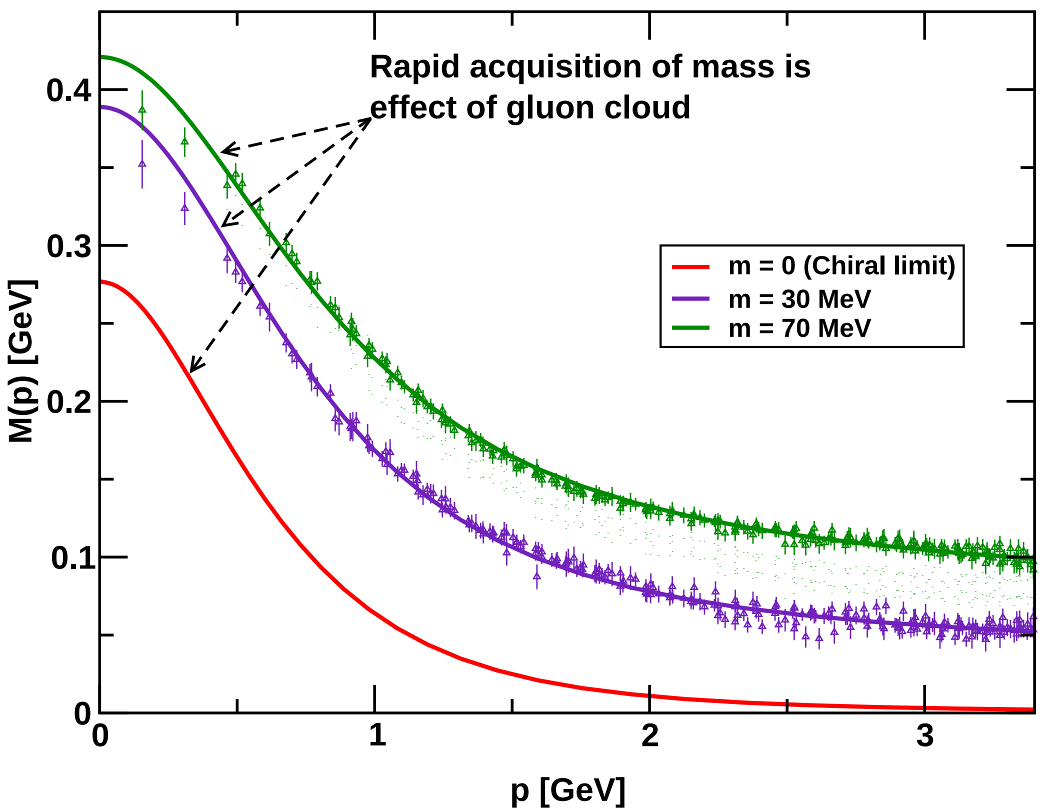

In QCD

a quark's mass must depend on

its momentum

俳句


ALWAYS LOOK
ON THE
BRIGHT SIDE

- Established understanding of two- and three-point functions

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Hadrons

- Established understanding of two- and three-point functions
- What about bound states?

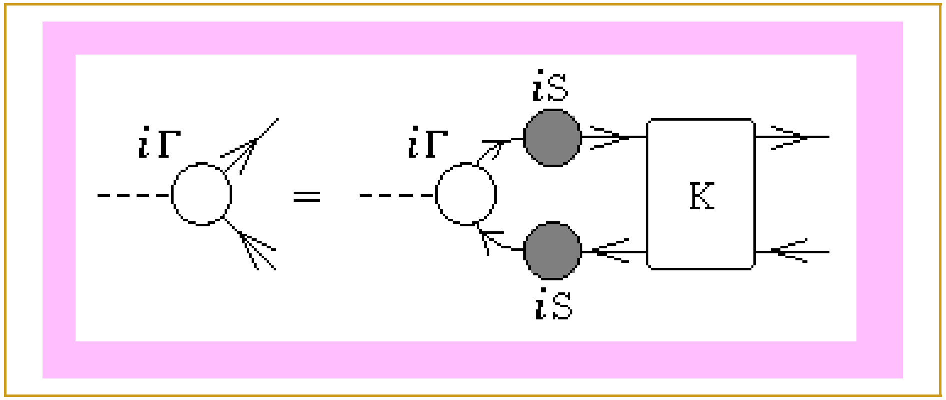
- Without bound states, Comparison with experiment is impossible

Hadrons

- Without bound states, Comparison with experiment is impossible
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

- Without bound states, Comparison with experiment is impossible

- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

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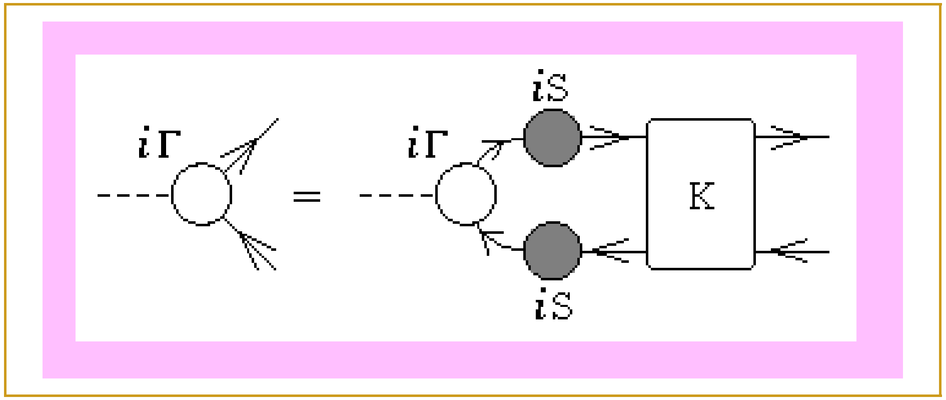

QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

or What is the long-range potential in QCD?

Bethe-Salpeter Kernel

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Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

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Satisfies DSE

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Kernels very different

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Satisfies BSE

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- Nontrivial constraint

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Satisfies BSE — Satisfies DSE

Kernels very different

but must be intimately related

- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

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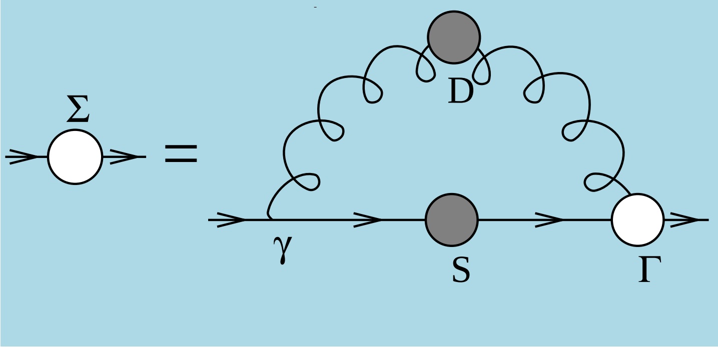
Persistent Challenge

Persistent Challenge

- Infinitely Many Coupled Equations



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Persistent Challenge

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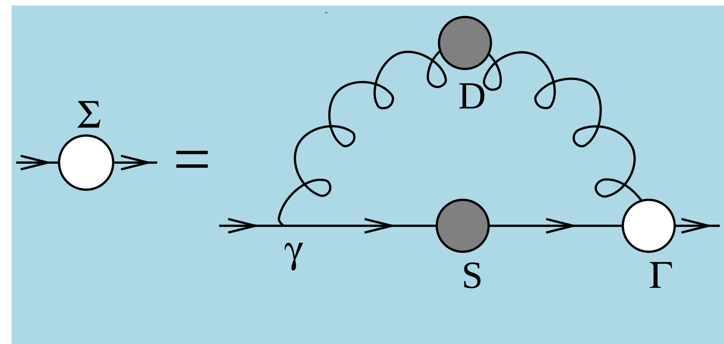

- Coupling between equations necessitates truncation

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Persistent Challenge

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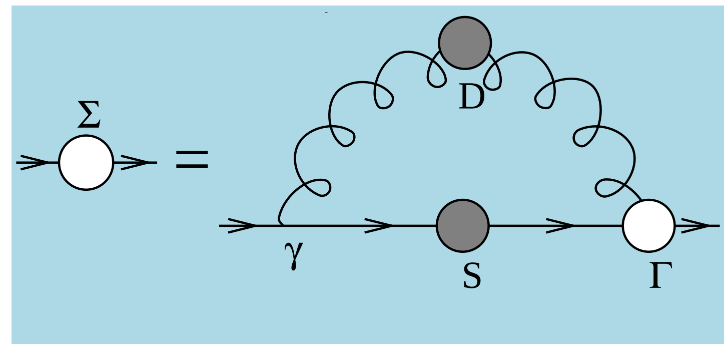

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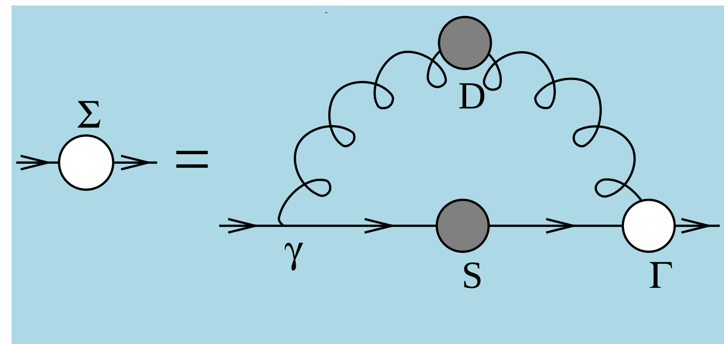

- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
    Not useful for the nonperturbative problems
    in which we're interested

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Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

Persistent Challenge

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Radial Excitations
& Chiral Symmetry

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

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Radial Excitations & Chiral Symmetry

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

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Radial Excitations & Chiral Symmetry

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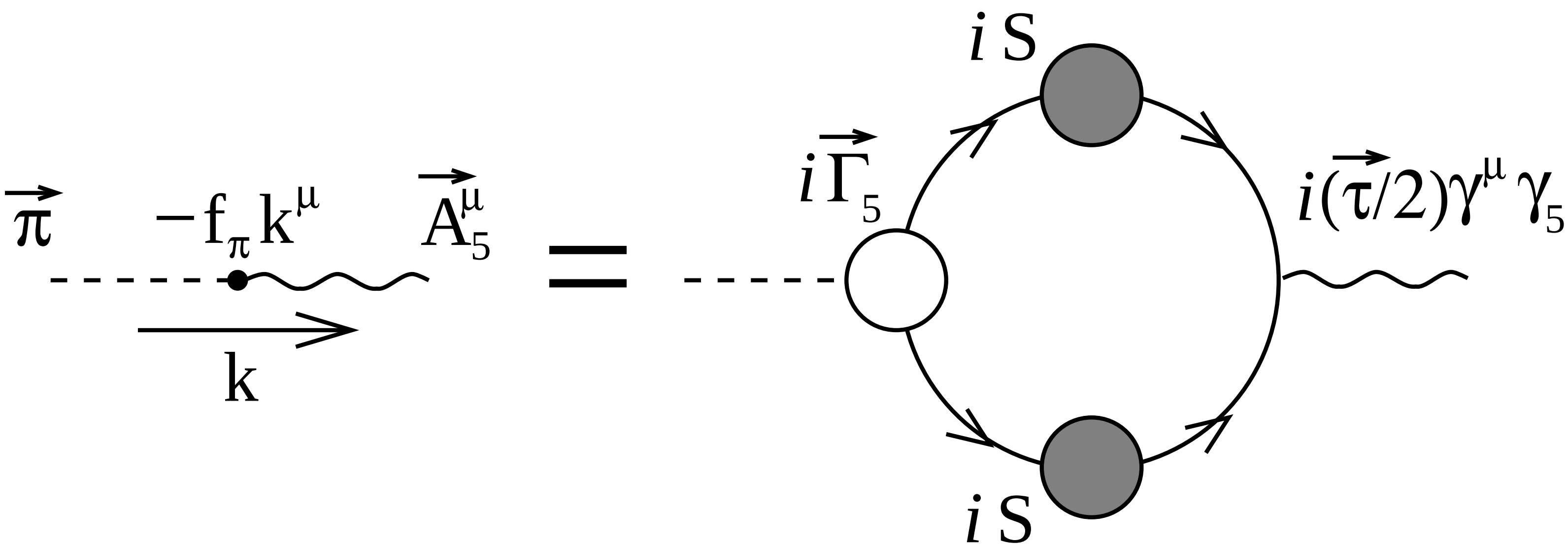
$$f_H \, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2} \mathrm{tr} \left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5 \gamma_\mu \boxed{\mathcal{S}(q_+) \Gamma_H(q;P) \mathcal{S}(q_-)} \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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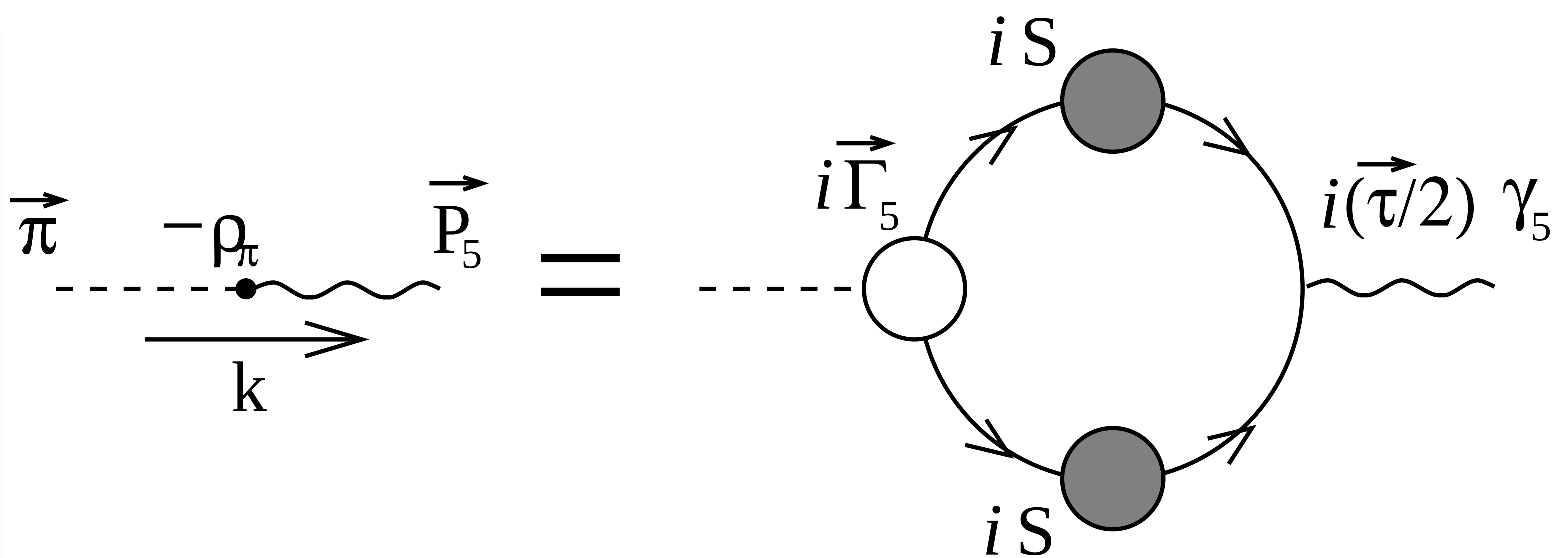

Radial Excitations & Chiral Symmetry

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Hence $\boxed{m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q}$ …GMOR relation, a corollary

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- Heavy-quark + light-quark

  $\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

  Hence, $\boxed{m_H \propto m_q}$

  . . . QCD Proof of Potential Model result

Radial Excitations & Chiral Symmetry

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Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

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  – Goldstone's Theorem –

  impacts upon *every* pseudoscalar meson

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Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

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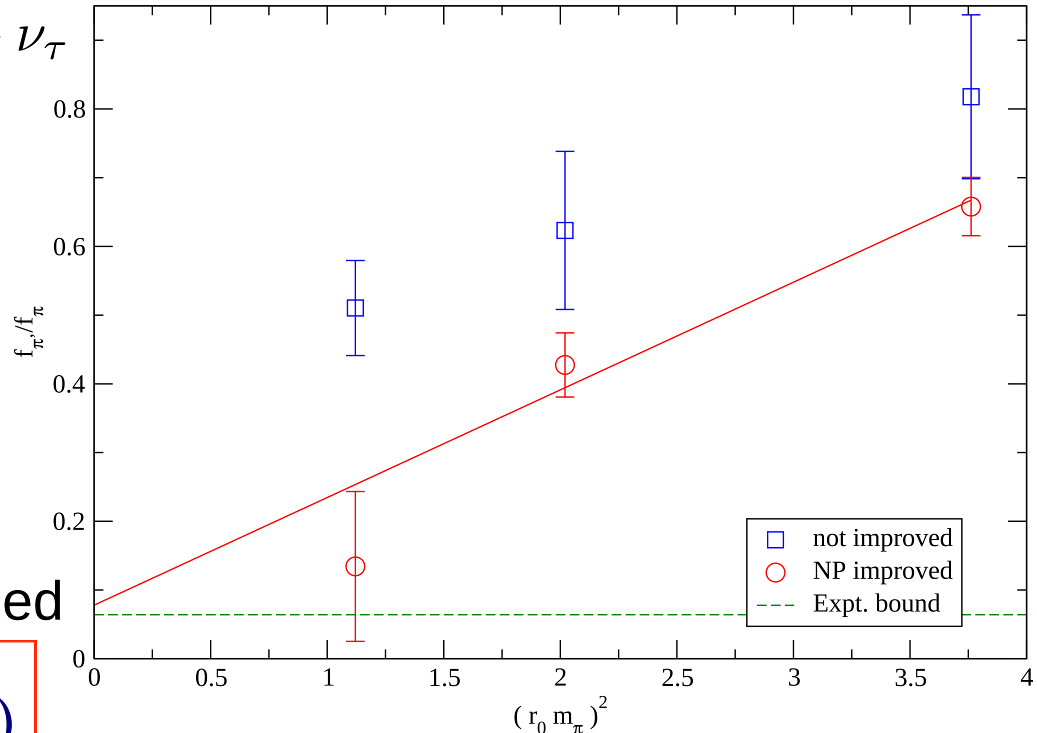

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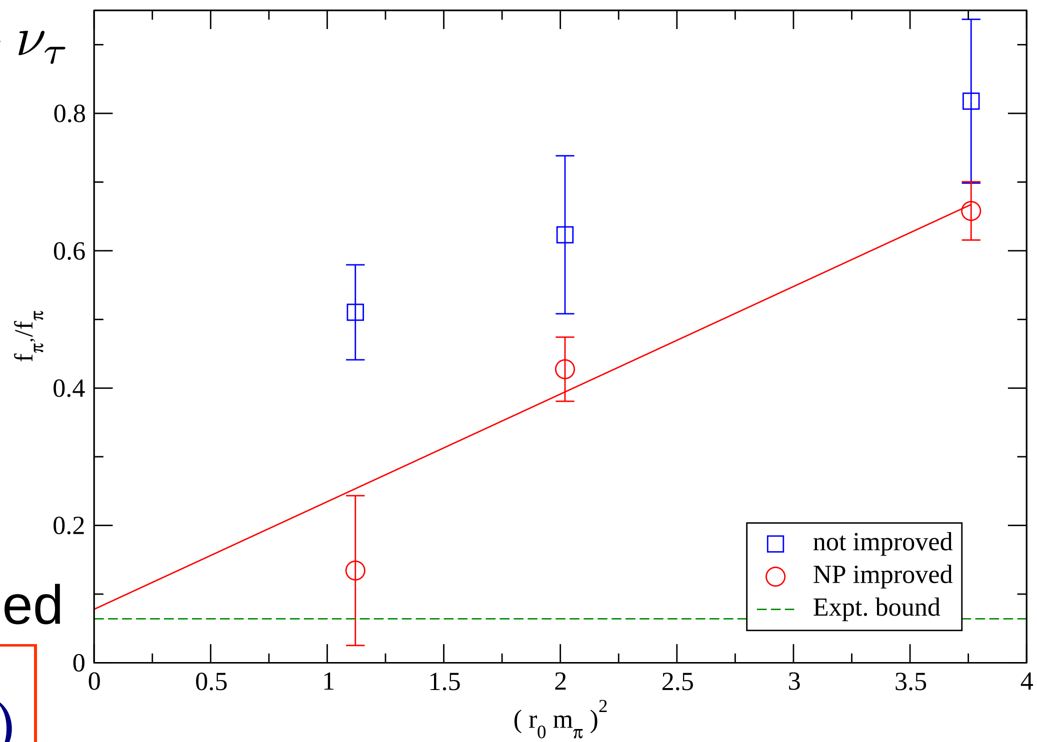

- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

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Radial Excitations & Lattice-QCD

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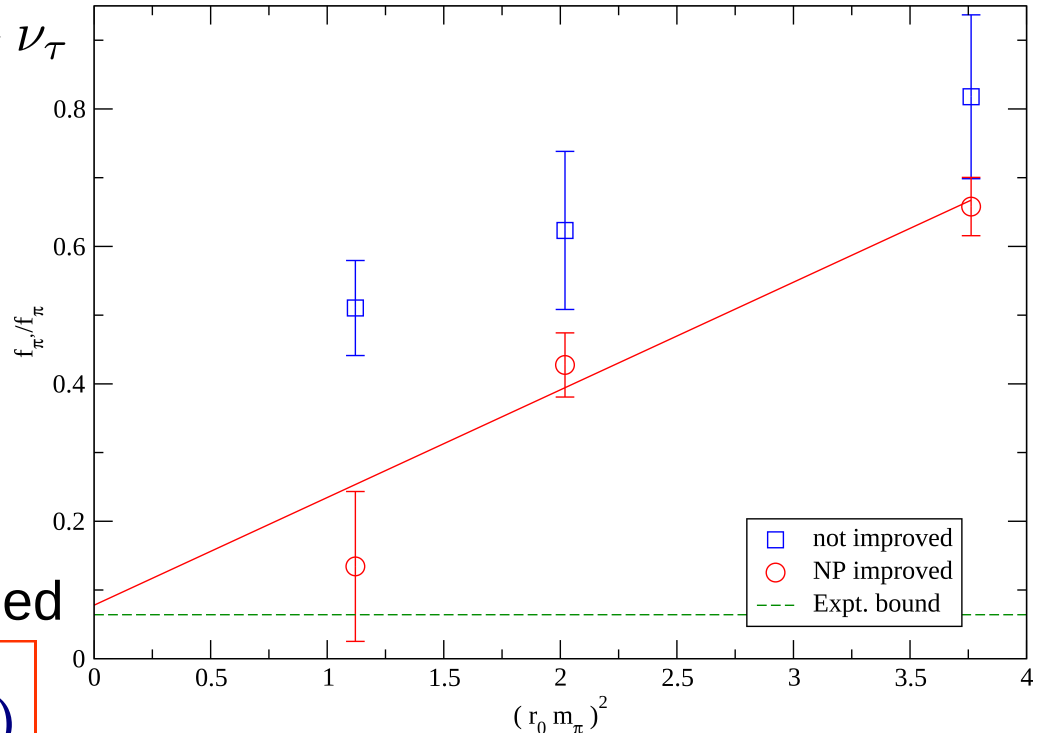

- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

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Goldberger-Treiman for pion

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Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\begin{aligned}\Gamma_{\pi^j}(k;P) &= \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P)+\gamma\cdot P F_\pi(k;P)\\ &\quad + \gamma\cdot k\, k\cdot P\, G_\pi(k;P)+\sigma_{\mu\nu}\,k_\mu P_\nu\, H_\pi(k;P)\Big]\end{aligned}$$

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- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

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$\Rightarrow$
$$f_\pi E_\pi(k;P=0) = B(p^2)$$

Maris, Roberts, Tandy
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$\Rightarrow$

$$\begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

Maris, Roberts, Tandy
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Goldberger-Treiman for pion

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Pseudovector components necessarily nonzero

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$\Rightarrow$ Exact in Chiral QCD

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GT for pion – QCD and $F_\pi^{\mathrm{em}}(Q^2)$

- What does this mean for observables?

GT for pion – QCD and $F_\pi^{\rm em}(Q^2)$

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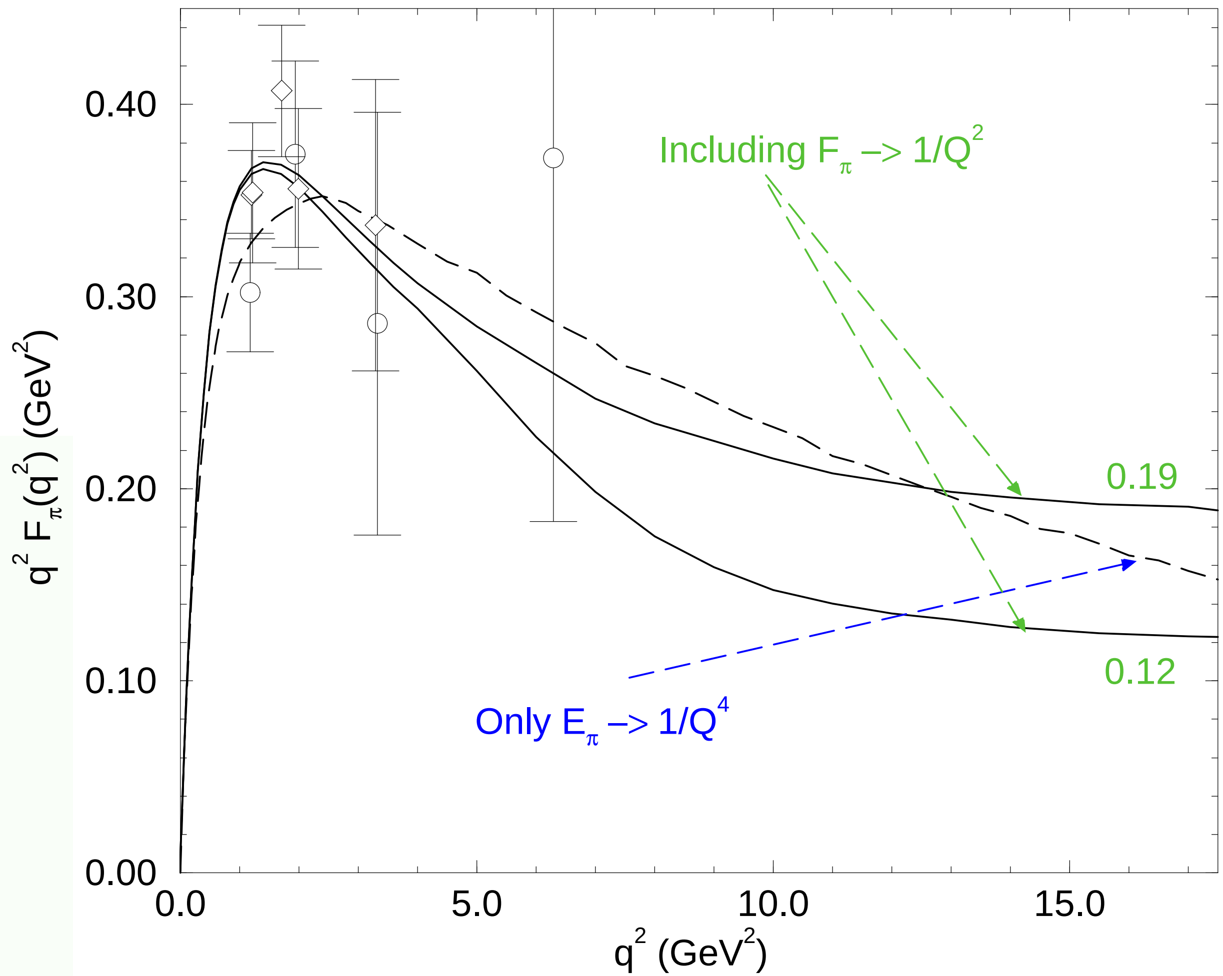

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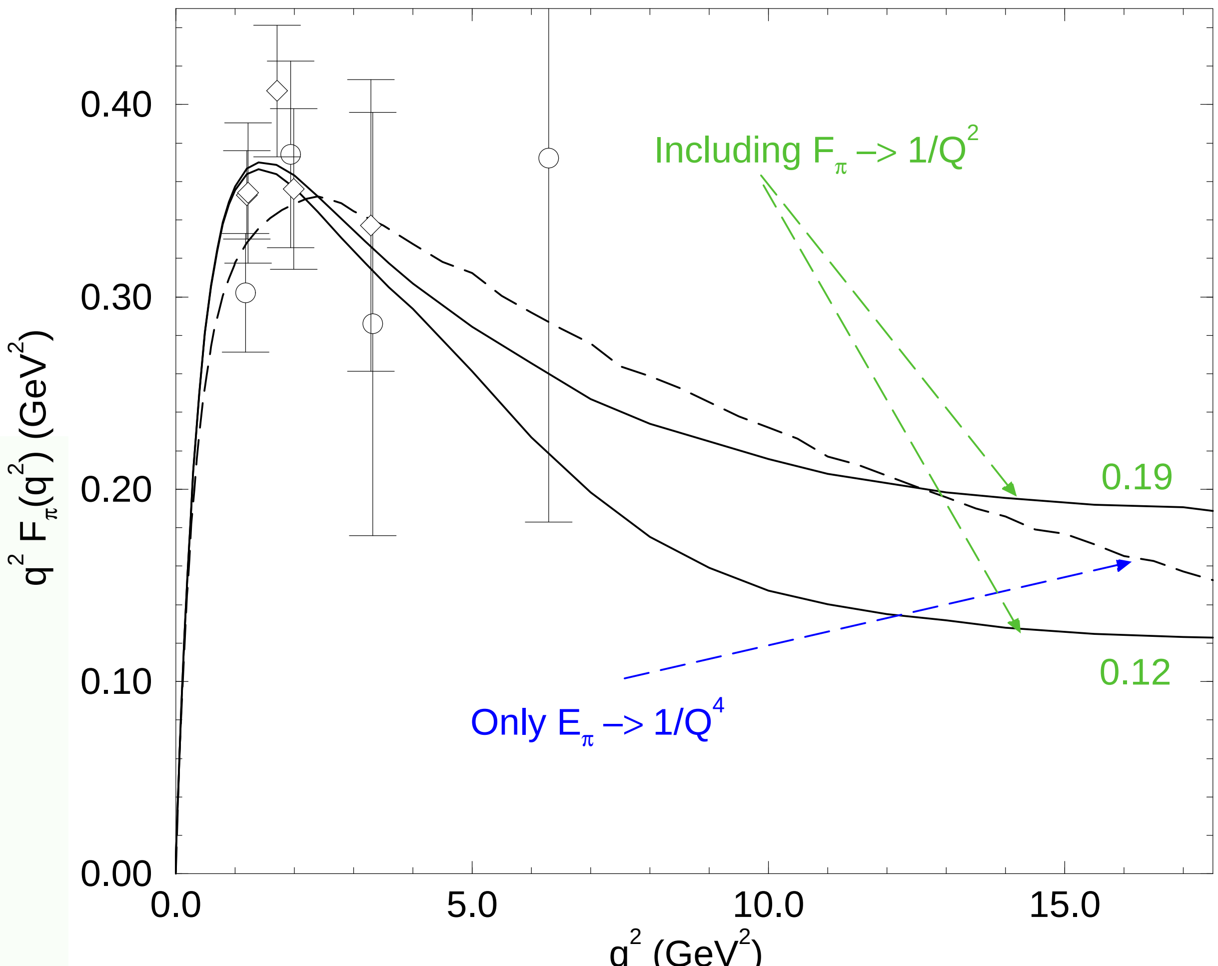

- Pseudovector components dominate ultraviolet behaviour of electromagnetic form factor

GT for pion – NJL

GT for pion – NJL

- Bethe-Salpeter amplitude can't depend on relative momentum

  $\Rightarrow$ General Form $\boxed{\Gamma_\pi(P) = i\gamma_5 E_\pi(P) + \frac{1}{M_Q}\gamma\cdot P F_\pi(P)}$

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GT for pion – NJL

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- Solve chiral-limit gap and Bethe-Salpeter equations

  $$P^2 = 0 : \; M_Q = 0.327\,, \; E_\pi = 0.994\,, \; \frac{F_\pi}{M_Q} = 0.344$$

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  - Hence F_π on LHS is forced to be nonzero because E_π on RHS is nonzero owing to DCSB

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- This Behaviour dominates for $Q^2 \gtrsim M_Q^2 \dfrac{E_\pi^2}{F_\pi^2} > 10\,\mathrm{GeV}^2$

Gap Equation
General Form

First Contents Back Conclusion

- Return to general bound-state problem . . .

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- To study the Poincaré covariant bound-state problem for mesons, one must first solve the gap equation

$$
\begin{aligned}
S_f(p)^{-1} &= Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,\\
\Sigma_f(p) &= Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma_\nu^f(q,p),
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- $D_{\mu\nu}(k)$ is the dressed-gluon propagator;
- $\Gamma^f_\nu(q,p)$ is the dressed-quark-gluon vertex;
- $m^{\rm bm}(\Lambda)$ is the Lagrangian current-quark bare mass;
- $Z_{1,2}(\zeta^2,\Lambda^2)$ are respectively the vertex and quark wave function renormalisation constants, with ζ the renormalisation point.

Bethe-Salpeter Equation

General Form

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$$\Gamma^{fg}_{5\mu}(k;P) = Z_2\gamma_5\gamma_\mu - \int_q g^2 D_{\alpha\beta}(k-q) \times \frac{\lambda^a}{2}\gamma_\alpha S_f(q_+)\Gamma^{fg}_{5\mu}(q;P)S_g(q_-)\frac{\lambda^a}{2}\Gamma^g_\beta(q_-,k_-) + \int_q g^2 D_{\alpha\beta}(k-q)\frac{\lambda^a}{2}\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

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  - $\Lambda^{fg}_{5\mu\beta}$ is defined completely via the dressed-quark self-energy and, owing to Poincaré covariance, one can employ, e.g., $q_\pm = q \pm P/2$, etc., without loss of generality

Ward-Takahashi Identity
Bethe-Salpeter Kernel

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- In any reliable study of light-quark hadrons, axial-vector vertex must satisfy

$$\begin{aligned} P_\mu \Gamma_{5\mu}^{fg}(k;P) &= S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-) \\ &\quad - i\left[m_f(\zeta) + m_g(\zeta)\right] \Gamma_5^{fg}(k;P)\,, \end{aligned}$$

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- Rainbow-ladder . . .
  - $\Gamma^f_\beta(q,k) = \gamma_\mu$

    $\Rightarrow \Lambda^{fg}_{5\mu\beta}(k,q;P) = 0 = \Lambda^{fg}_{5\beta}(k,q;P)$

Solving the Kernel's Ward-Takahashi Identity

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- Kernel's Ward-Takahashi identity provides means by which to construct a symmetry preserving kernel of the Bethe-Salpeter equation that is matched to any reasonable *Ansatz* for the dressed-quark-gluon vertex
- With this powerful capacity they realise a longstanding goal.

Solving the Kernel's WTI – Illustration

Chang Lei, to appear

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- Suppose that in the gap equation one employs an *Ansatz* for the dressed-quark-gluon vertex which satisfies

$$P_\mu i\Gamma_\mu^f(k_+,k_-) = \mathcal{B}(P^2)\left[S_f^{-1}(k_+) - S_f^{-1}(k_-)\right] \qquad (*)$$

with $\mathcal{B}$ flavour-independent.

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- NB. While true quark-gluon vertex doesn't satisfy this identity, owing to form of Slavnov-Taylor identity which it does satisfy, it's plausible that solution of Eq. $(*)$ can provide reasonable pointwise approximation to true vertex.

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- Given Eq. $(*)$, then Kernel's WTI entails

$$P_\mu(q-k)_\beta i\Lambda^{fg}_{5\mu\beta}(k,q;P) = P_\mu \mathcal{B}((k-q)^2)\left[\Gamma^{fg}_{5\mu}(q;P) - \Gamma^{fg}_{5\mu}(k;P)\right],$$

$$(q-k)_\beta i\Lambda^{fg}_{5\beta}(k,q;P) = \mathcal{B}((k-q)^2)\left[\Gamma^{fg}_5(q;P) - \Gamma^{fg}_5(k;P)\right]. \qquad (\#)$$

Solving the Kernel's WTI – Illustration

- Solution to Eq. (#)

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- with (BC construction)

$$\overline{\Lambda}_{\beta}^{fg}(k,q;P) = 2\ell_\beta\,[i\Delta_{E_5}(q,k;P) + \gamma\cdot P\Delta_{F_5}(q,k;P)]$$
$$+\gamma_\beta\,\Sigma_{G_5}(q,k;P) + 2\ell_\beta\,\gamma\cdot\ell\,\Delta_{G_5}(q,k;P) + [\gamma_\beta,\gamma\cdot P]$$
$$\times\Sigma_{H_5}(q,k;P) + 2\ell_\beta[\gamma\cdot\ell,\gamma\cdot P]\Delta_{H_5}(q,k;P)\,,$$

  - $\ell = (q+k)/2$
  - $\Sigma_\Phi(q,k;P) = [\Phi(q;P)+\Phi(k;P)]/2$
  - $\Delta_\Phi(q,k;P) = [\Phi(q;P)-\Phi(k;P)]/[q^2-k^2]$

Symmetry-preserving Ansatz

- At this point . . .
  - Began with $\Gamma_\mu(q,p)$, whose diagrammatic content is unknown, but which expresses important additional nonperturbative effects that are difficult to capture in any finite sum of contributions

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- This system and its predictions can smoothly be connected with those obtained, e.g., in a rainbow-ladder or kindred symmetry-preserving truncation of the DSEs.

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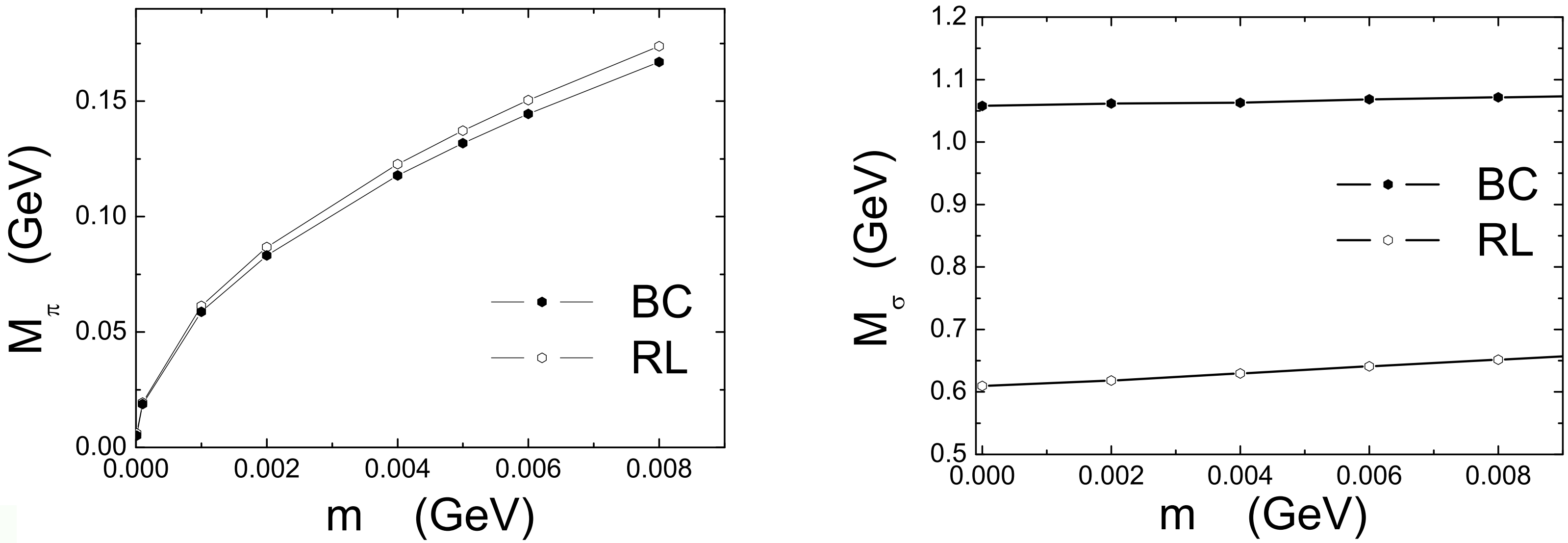
  - Given that $\Gamma_\mu(q,p)$ satisfies Eq. ($*$), the equations which follow provide symmetry-preserving closed system whose solution yields predictions for the properties of pseudoscalar mesons
- The system can be used to anticipate, elucidate and understand the impact on hadron properties of the rich nonperturbative structure expected of the fully-dressed quark-gluon vertex in QCD

Numerical Illustration



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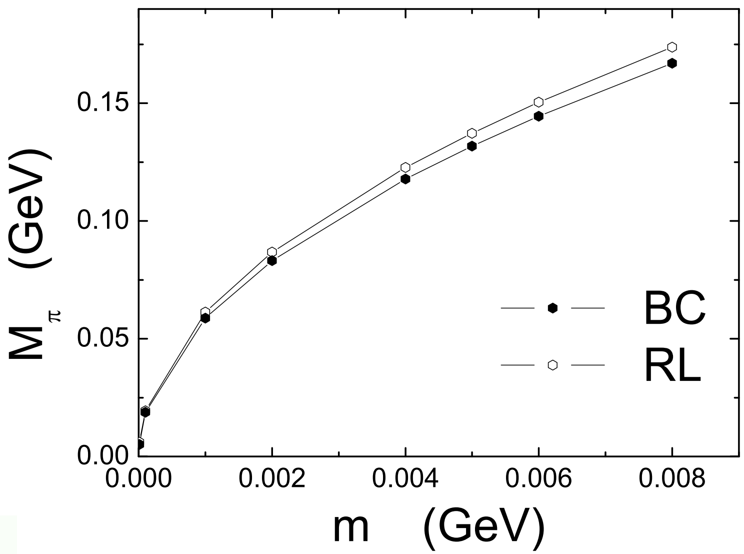

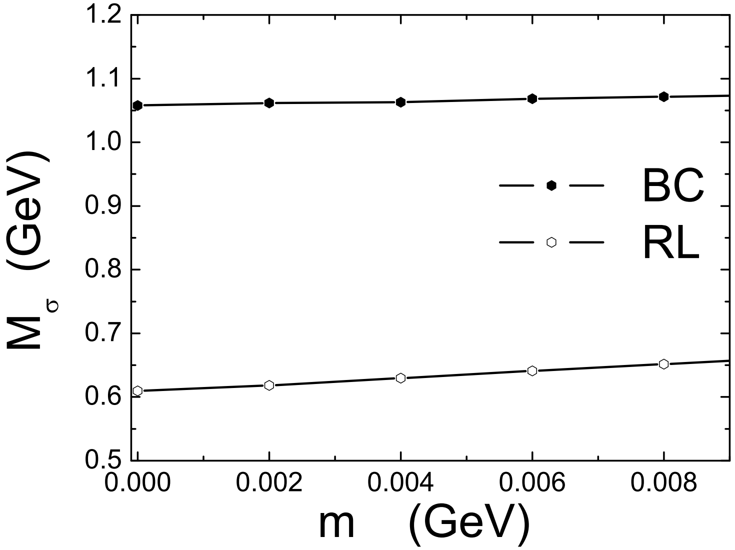

- Single interaction, common mass scale: rainbow-ladder cf. BC-consistent truncation

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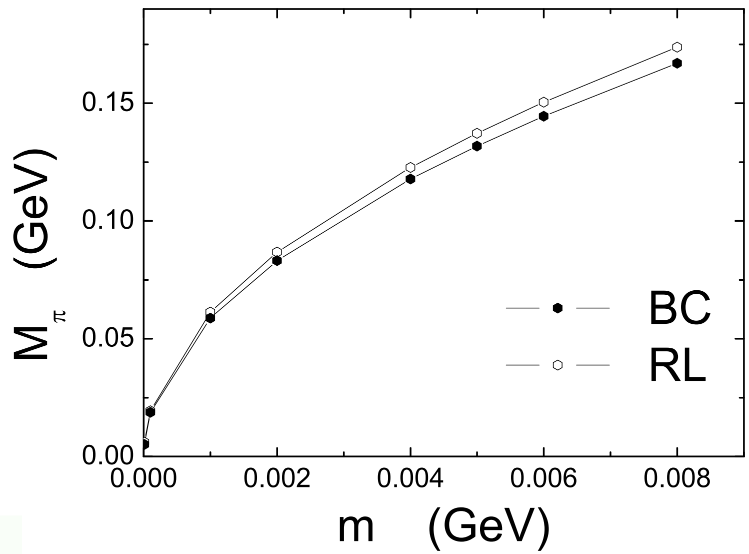

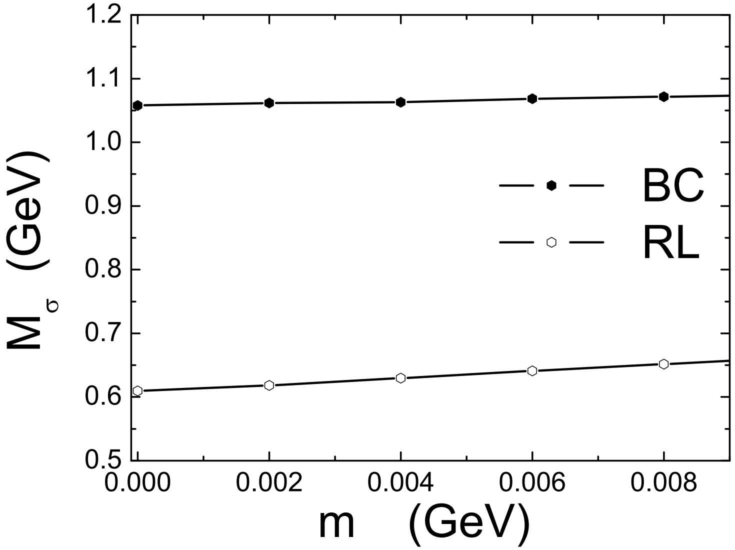

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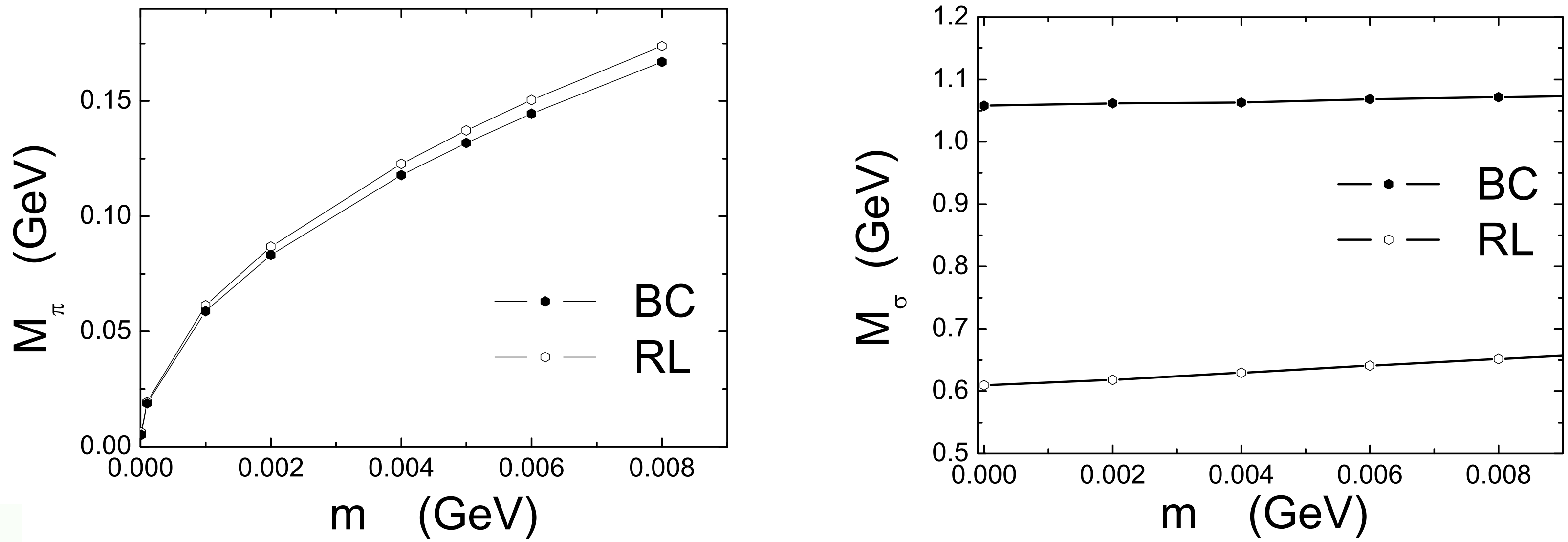

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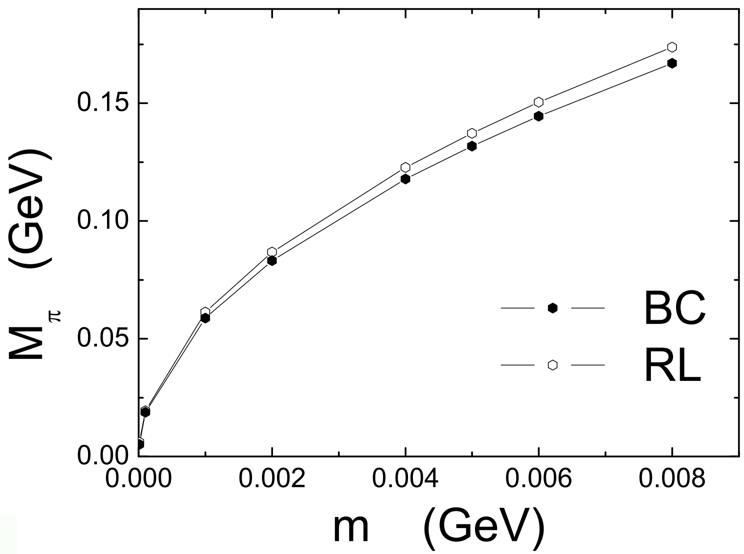

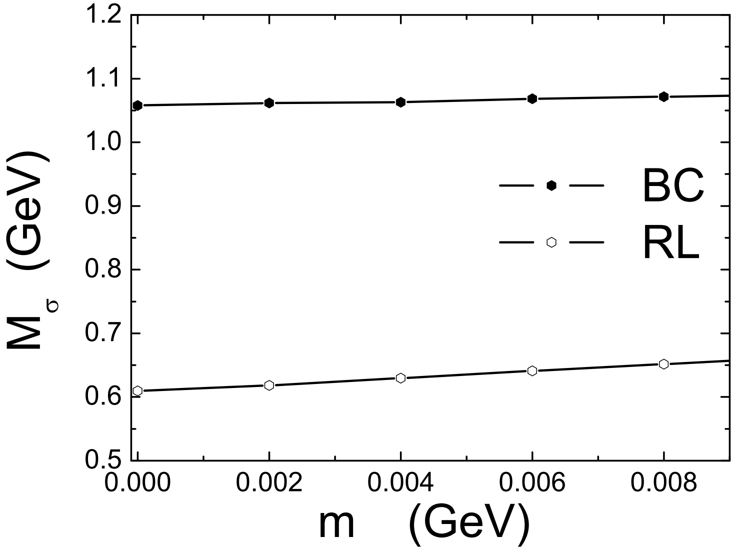

- Single interaction, common mass scale: rainbow-ladder cf. BC-consistent truncation
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  - Added attraction in pseudoscalar channel
  - Added repulsion in scalar channel

Spin-orbit Interaction

- Rainbow-ladder DSE truncation,

$$\varepsilon_\sigma^{\rm RL} := \left. \frac{2M(0) - m_\sigma}{2M(0)} \right|_{\rm RL} = (0.3 \pm 0.1)\,.$$

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- From this viewpoint scalar is a spin and orbital excitation of a pseudoscalar meson

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- Extant studies of realistic corrections to the rainbow-ladder truncation show that they reduce hyperfine splitting

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- Scalar mesons $= {}^3P_0$ states: Constituents' spins aligned and one unit of constituent orbital angular momentum
- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit splitting. Effect owes to influence of quark's dynamically-enhanced scalar self-energy in the Bethe-Salpeter kernel.
Impossible to demonstrate effect without our new procedure

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- Expect this feature to have material impact on mesons with mass greater than 1 GeV.
  prima facie … can overcome longstanding shortcoming of RL truncation; viz., splitting between vector & axial-vector mesons is too small

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- Promise of realistic meson spectroscopy
  First time, also for mass > 1 GeV


ARE WE THERE YET?

New Challenges

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- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.

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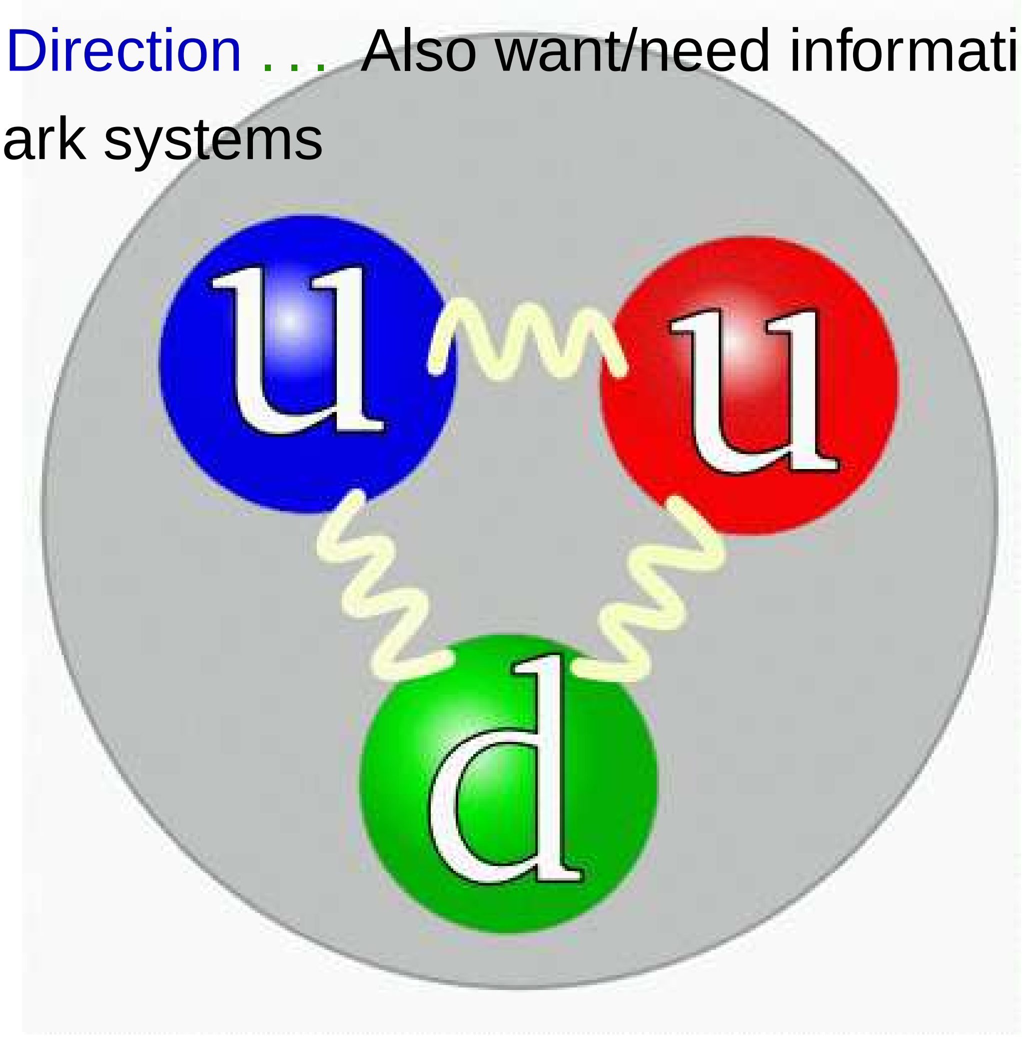
- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

New Challenges

- Another Direction . . . Also want/need information about three-quark systems



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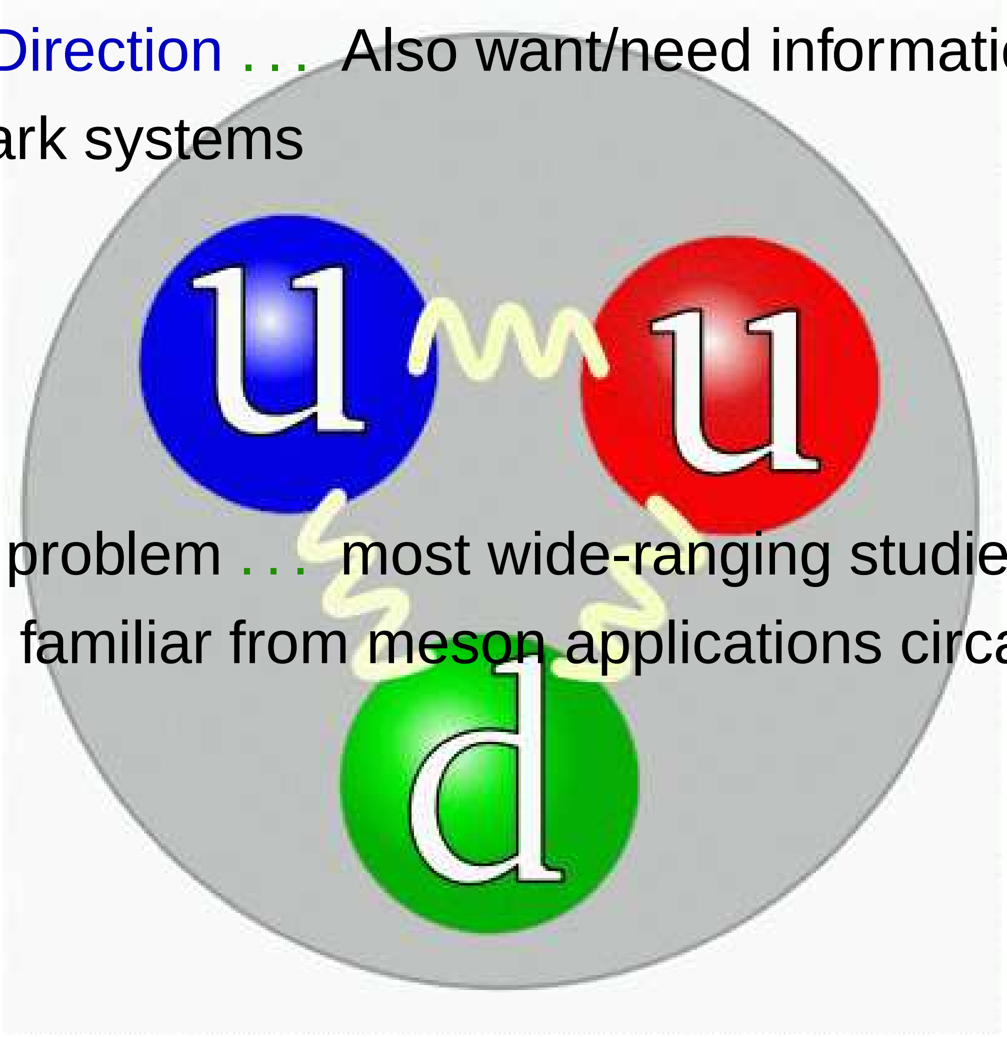

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New Challenges

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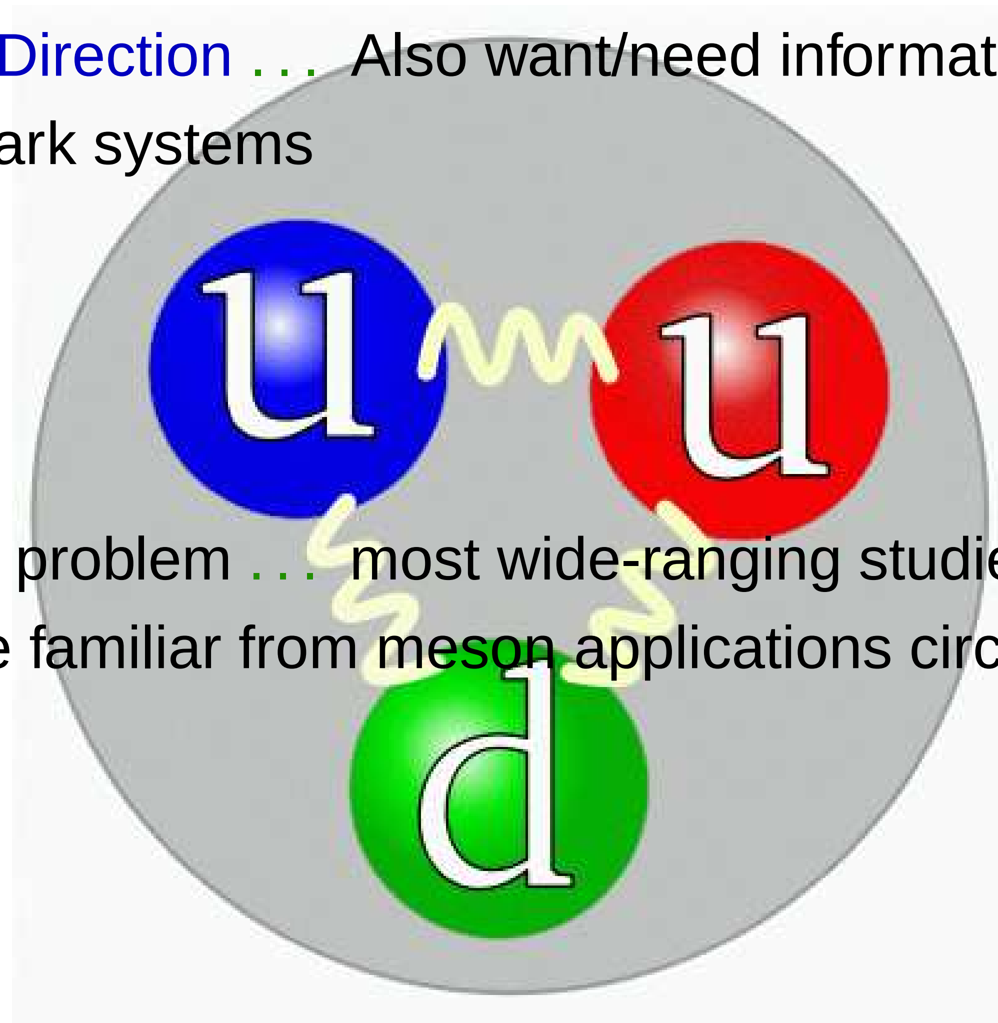

- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa ~ 1995.

- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.

New Challenges

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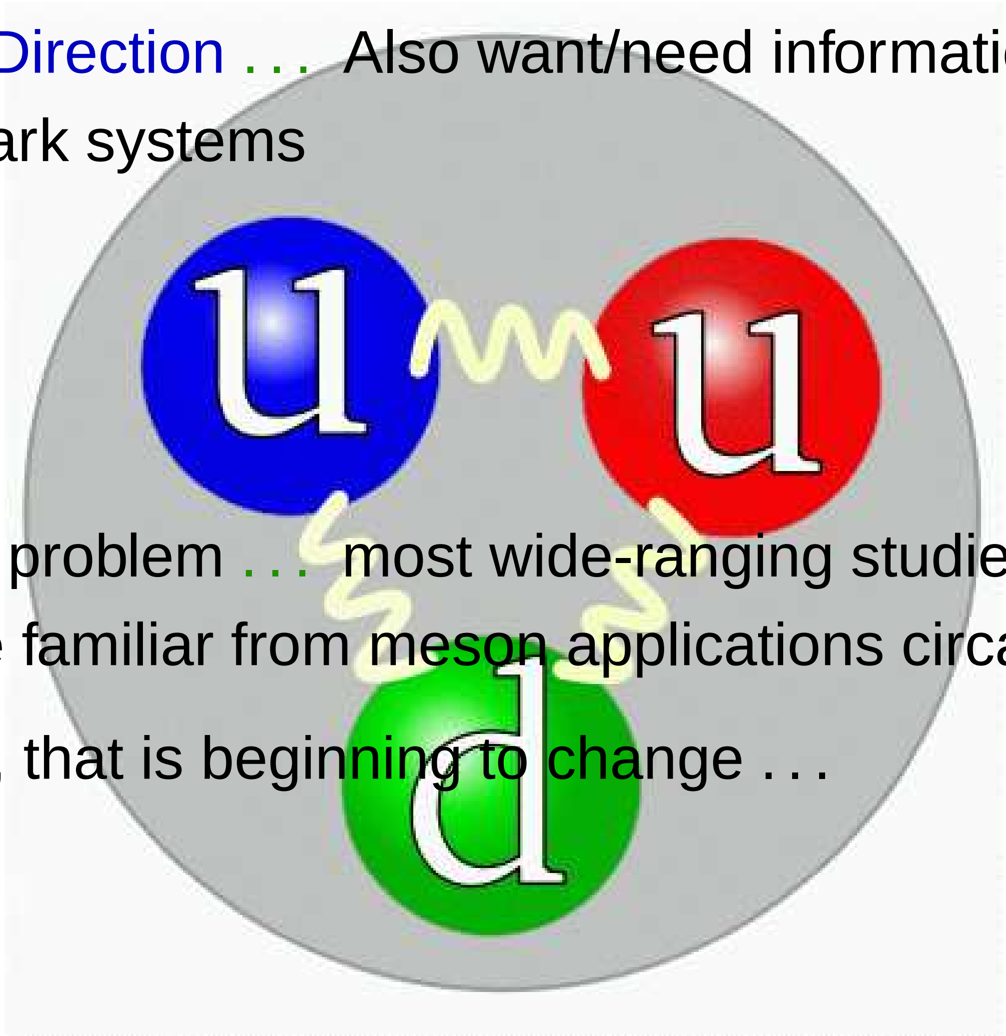

- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa $\sim$1995.
- However, that is beginning to change . . .

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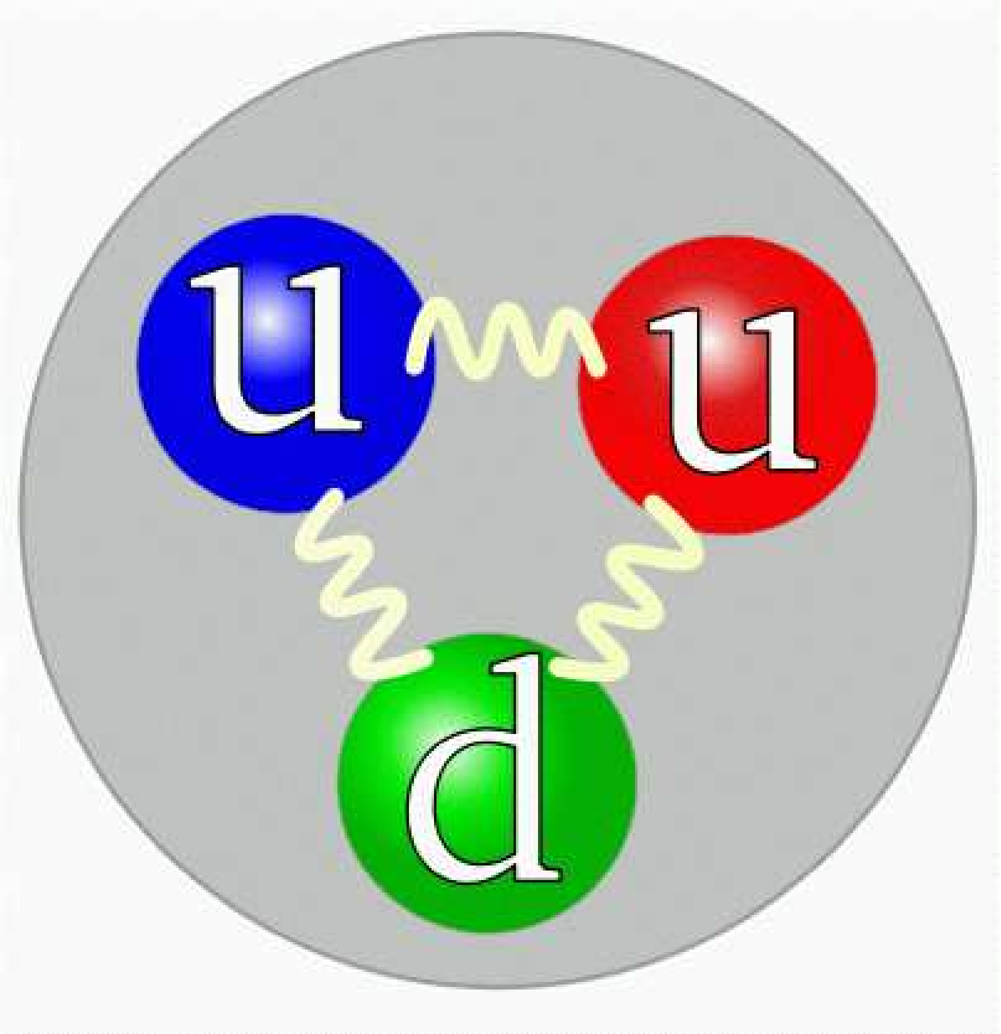
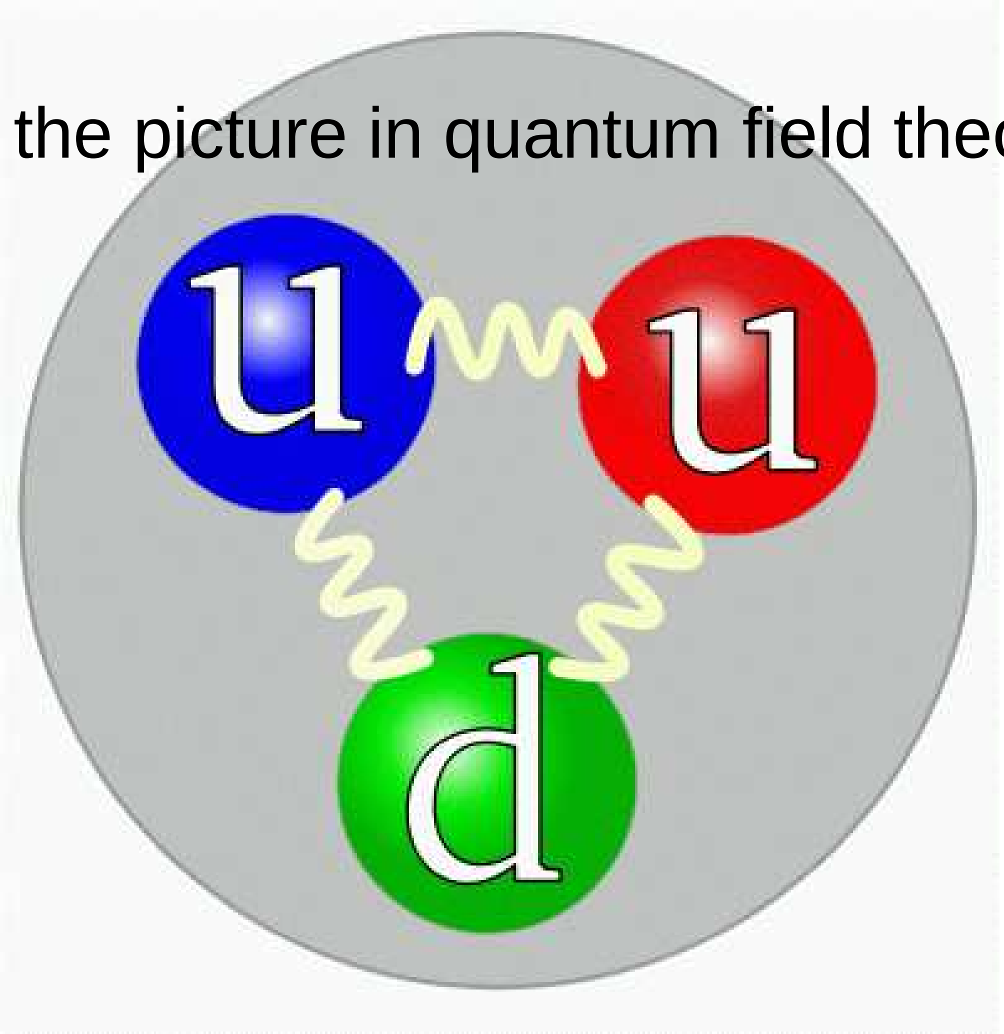
Nucleon . . .
Three-body Problem?



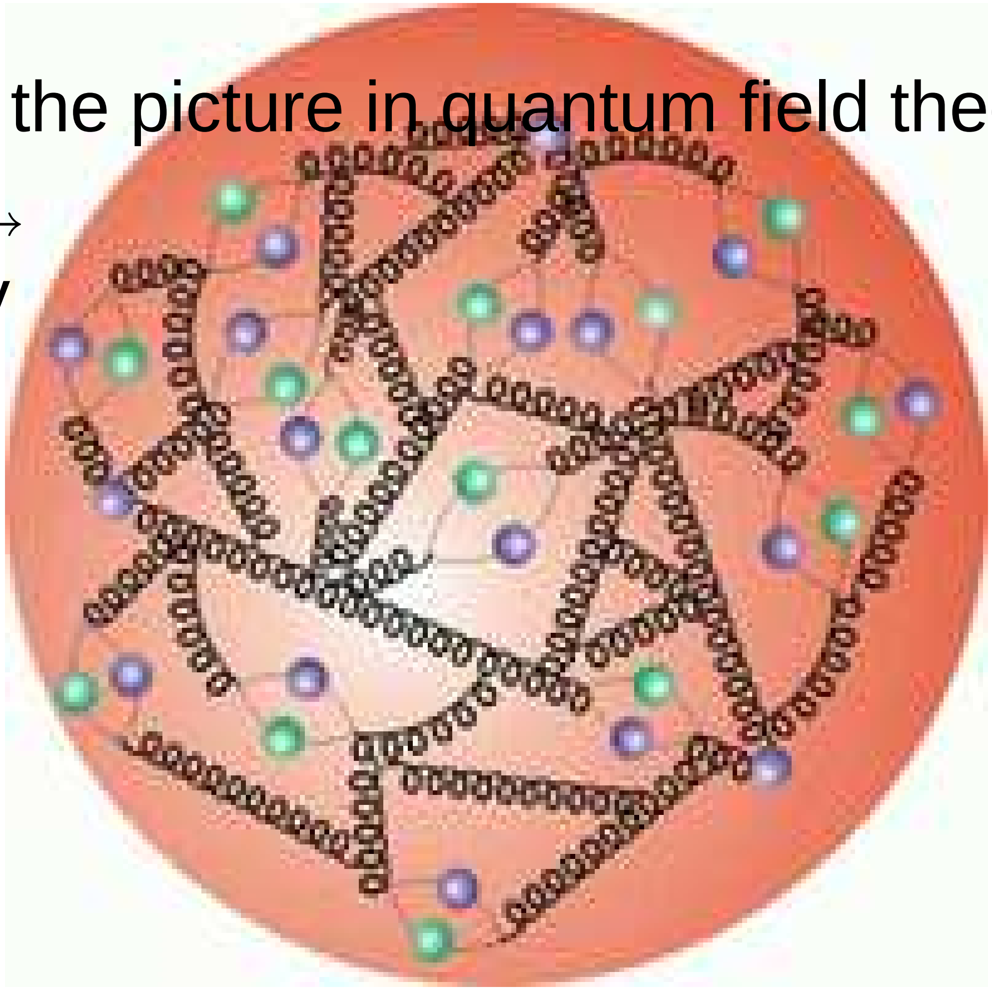
Three-body Problem?

- What is the picture in quantum field theory?



Three-body Problem?

- What is the picture in quantum field theory?
- Three → infinitely many!



Unifying Study of Mesons and Baryons

- How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons?

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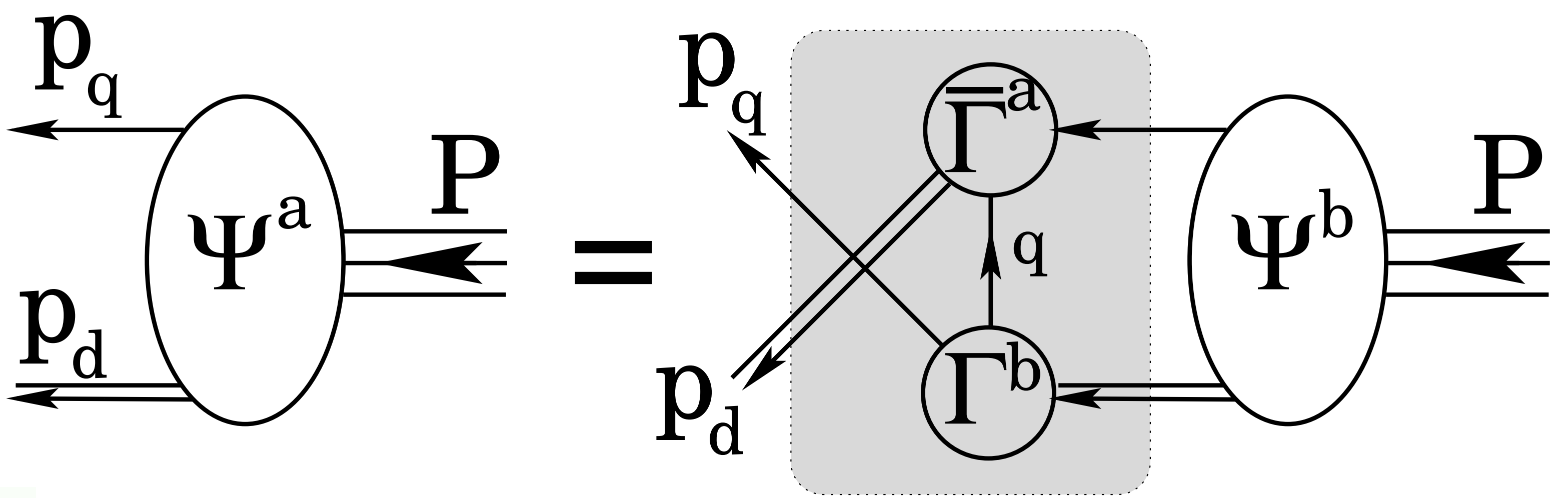
  - Residue is proportional to nucleon's Faddeev amplitude
  - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
  - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour-$\bar{3}$ (antitriplet) channel

Faddeev equation

Faddeev equation



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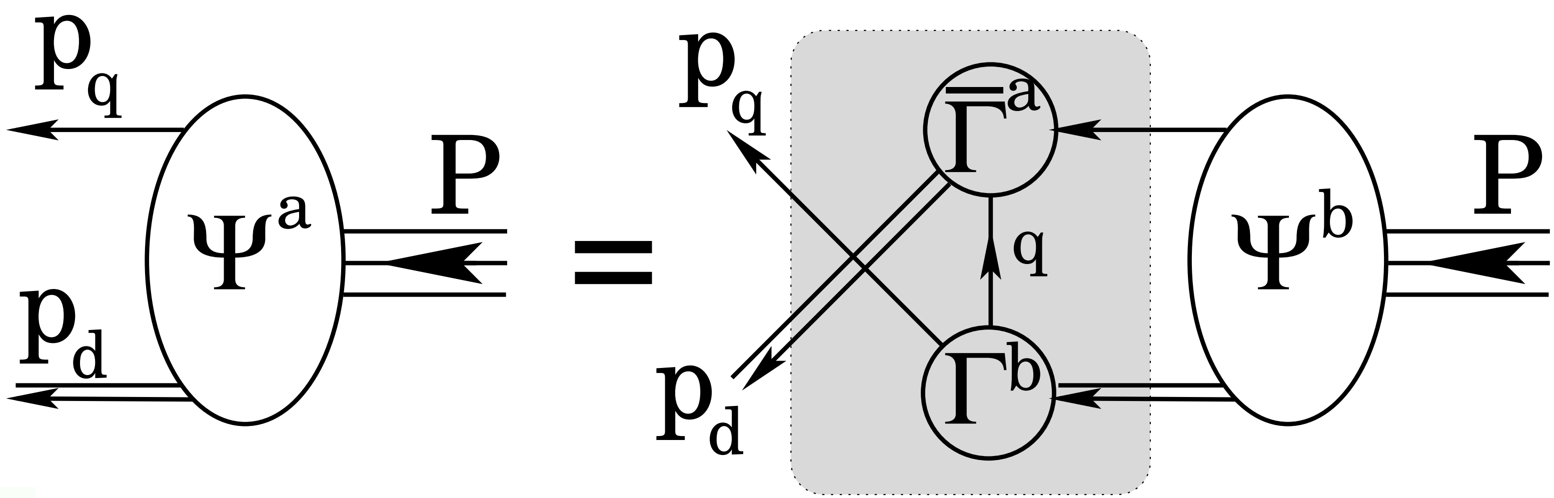
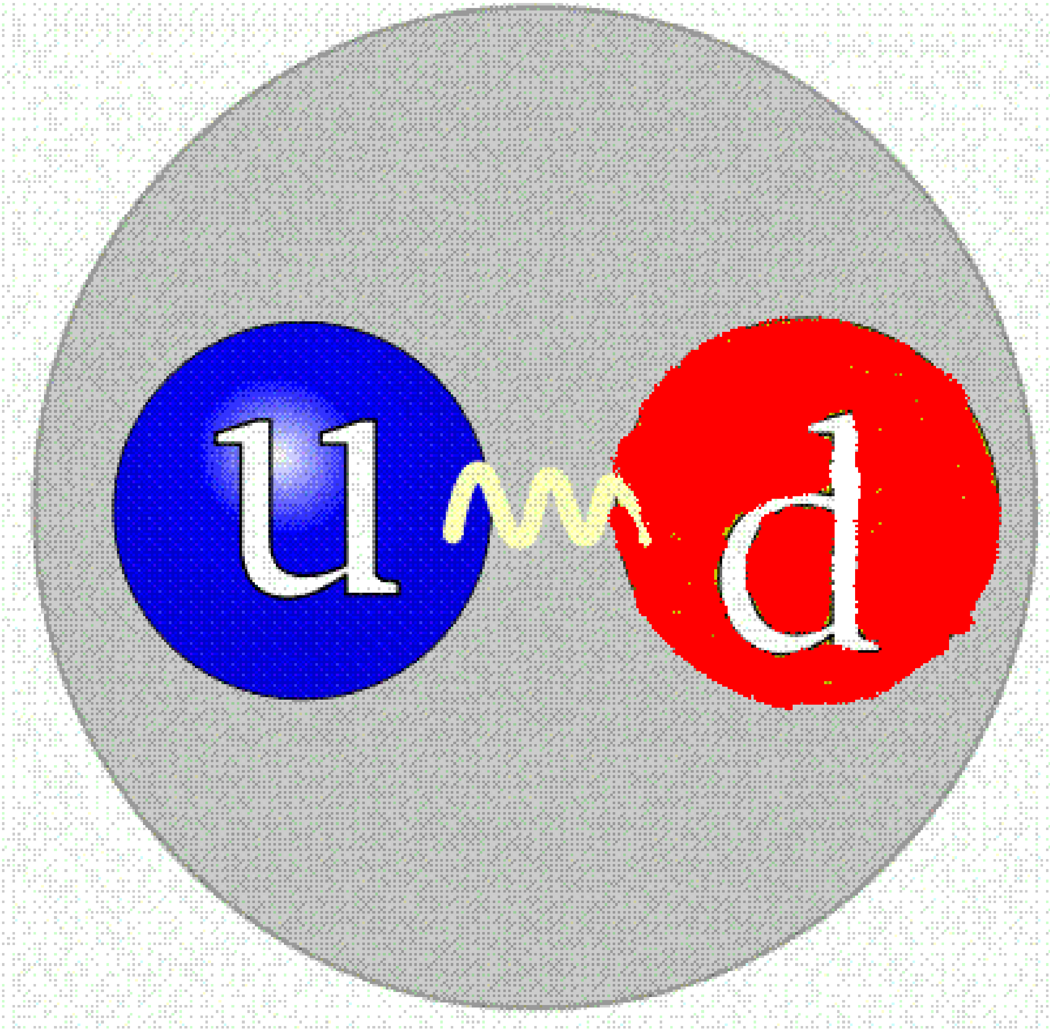


- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame Amplitude has ... $s-$, $p-$ & $d-$wave correlations

Diquark correlations

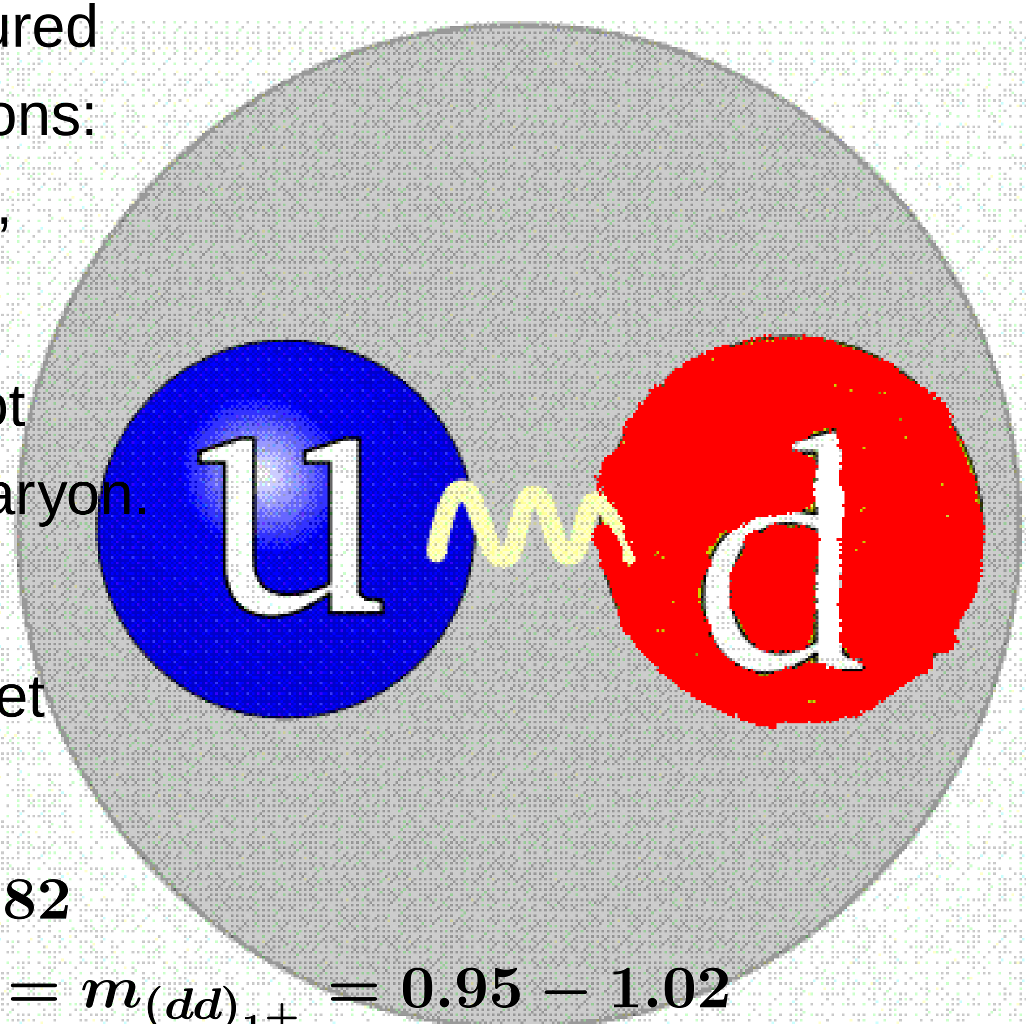


QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined … Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$
$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

Ab-initio study of mesons & nucleons

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- Leading-order truncation of DSEs – rainbow-ladder

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- Symmetry preserving and systematic approach can elucidate and account for these effects
  - Use this knowledge to constrain interaction in infrared
  - Interaction in ultraviolet predicted by perturbative expansion of DSEs

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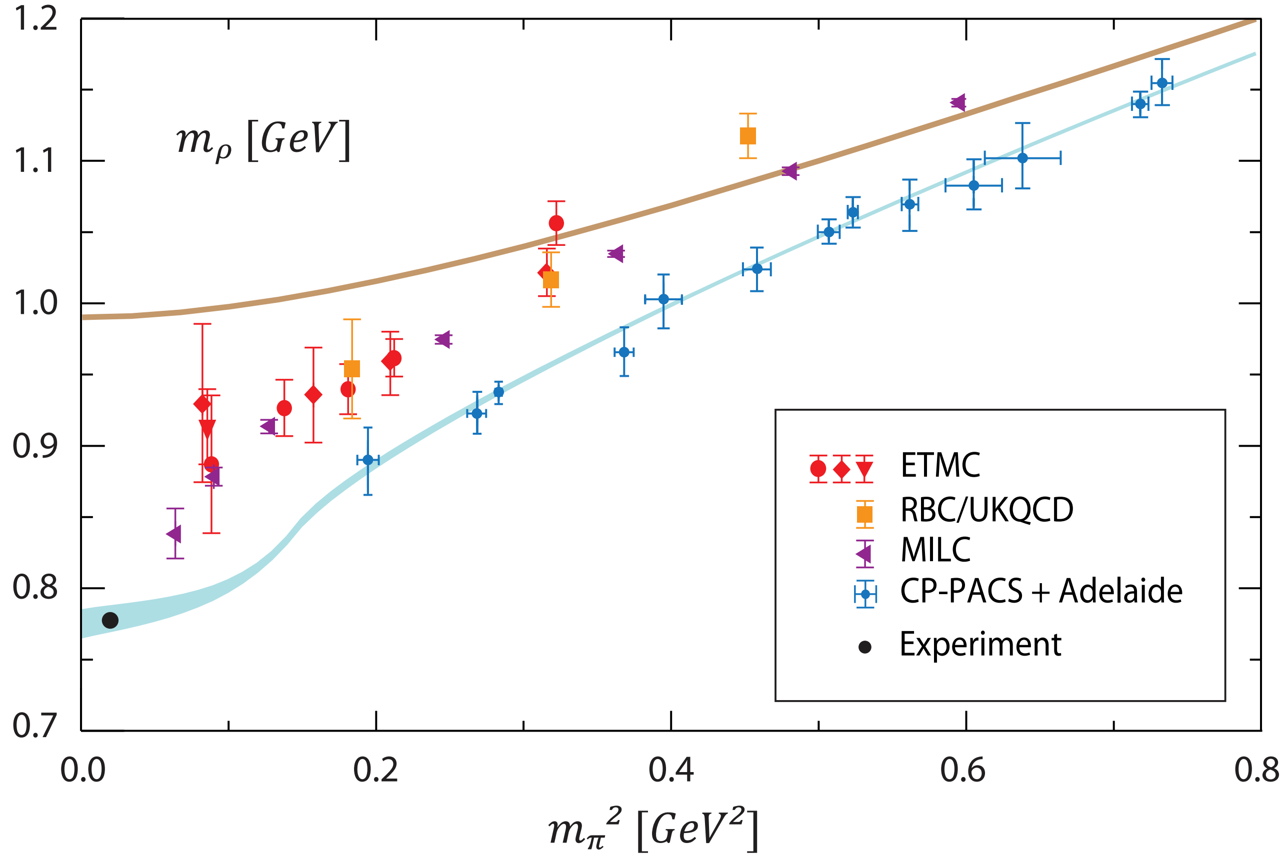

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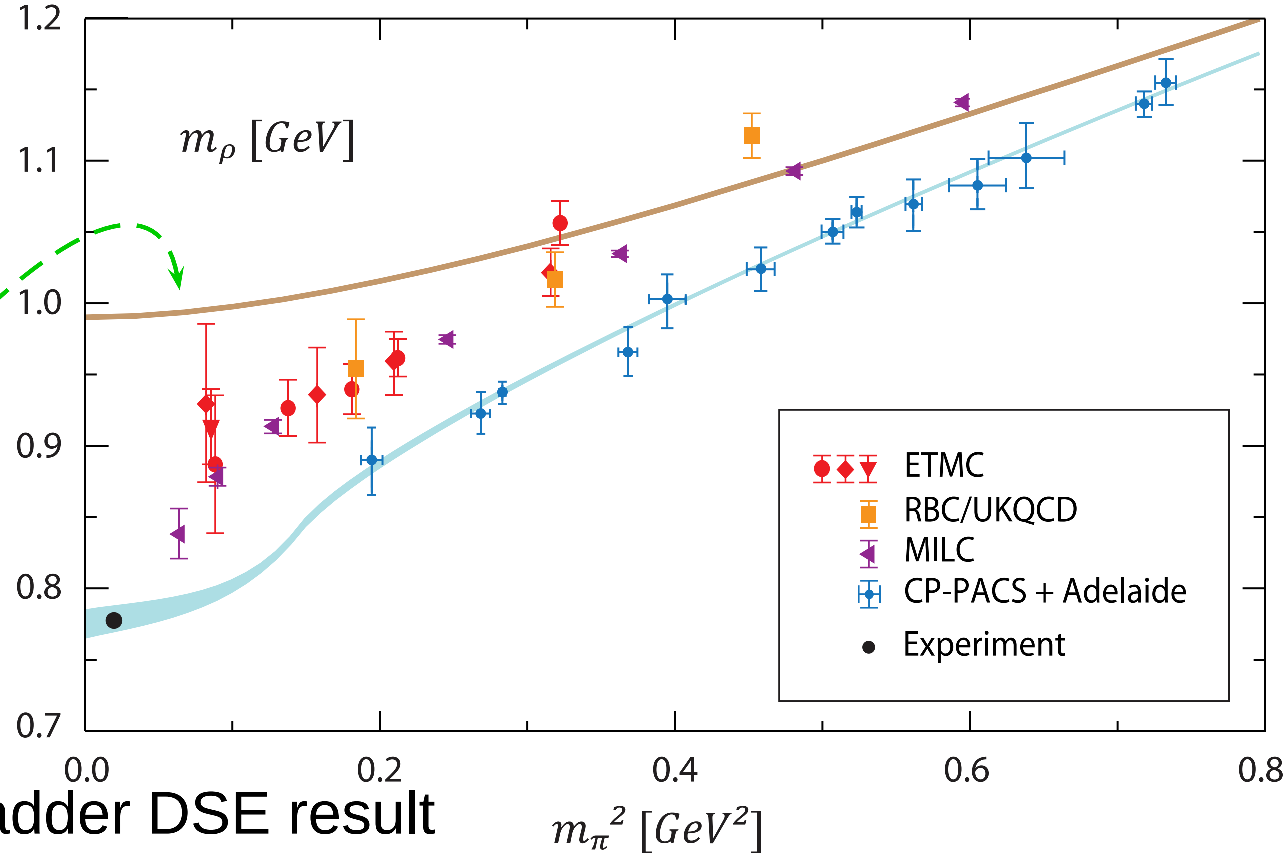

- Rainbow-Ladder DSE result

one parameter for IR – "confinement radius"

Results insensitive to value on material domain

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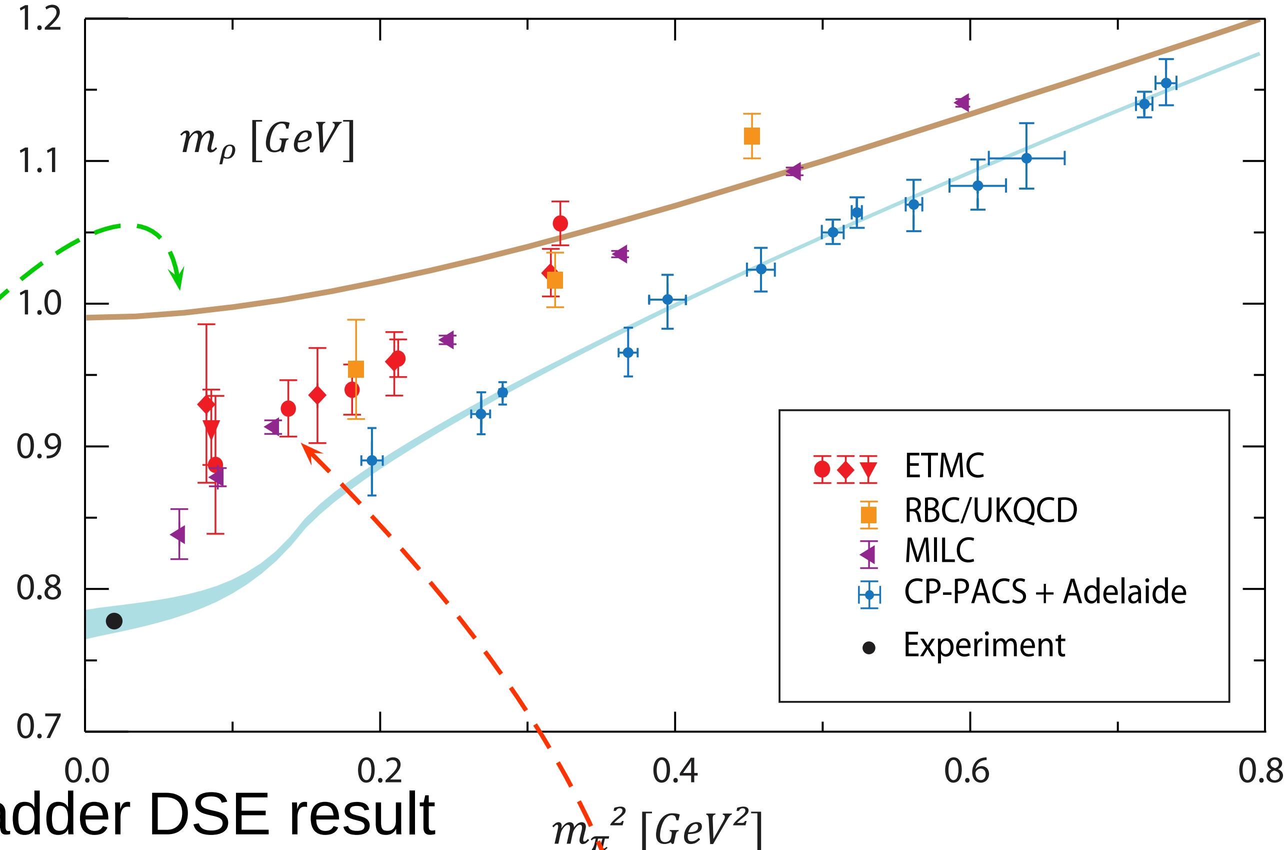

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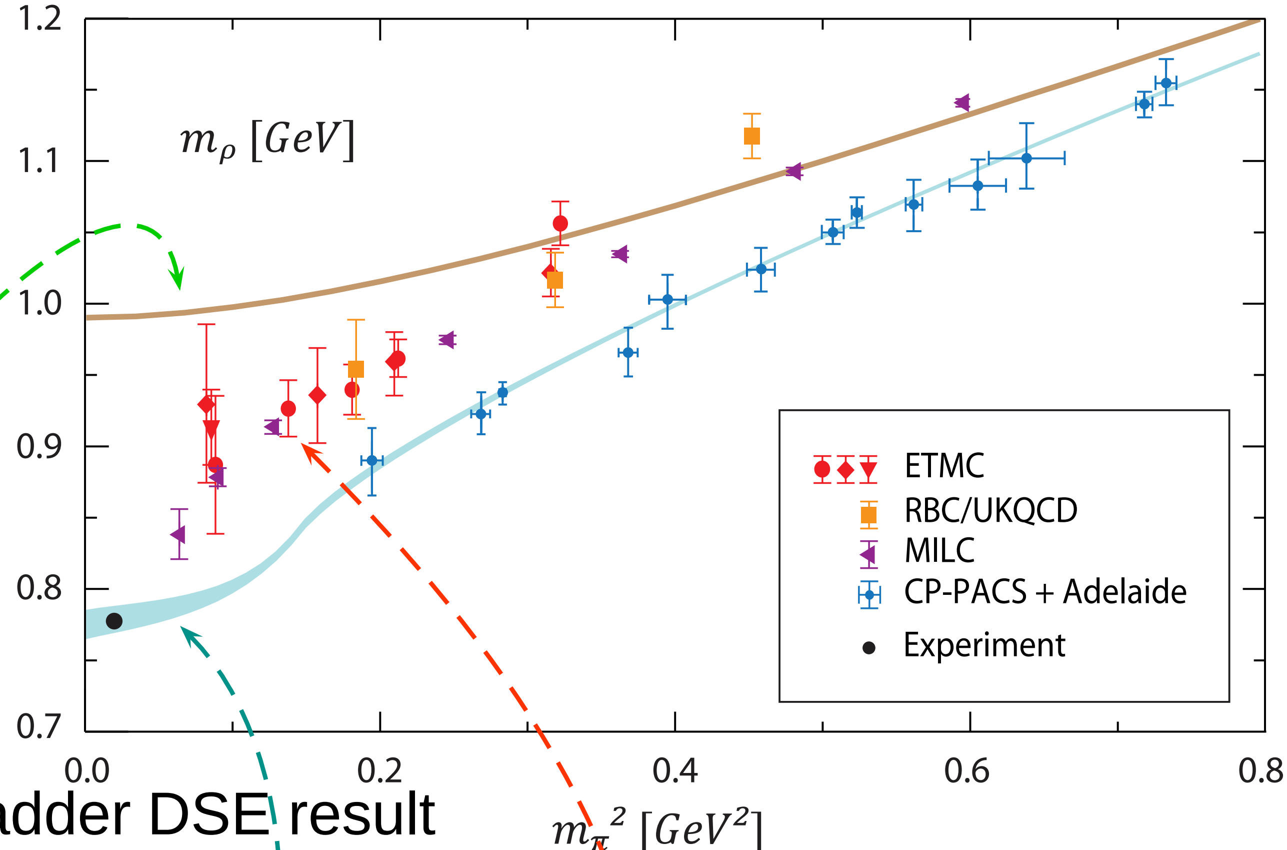

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Ab-initio study of mesons & nucleons

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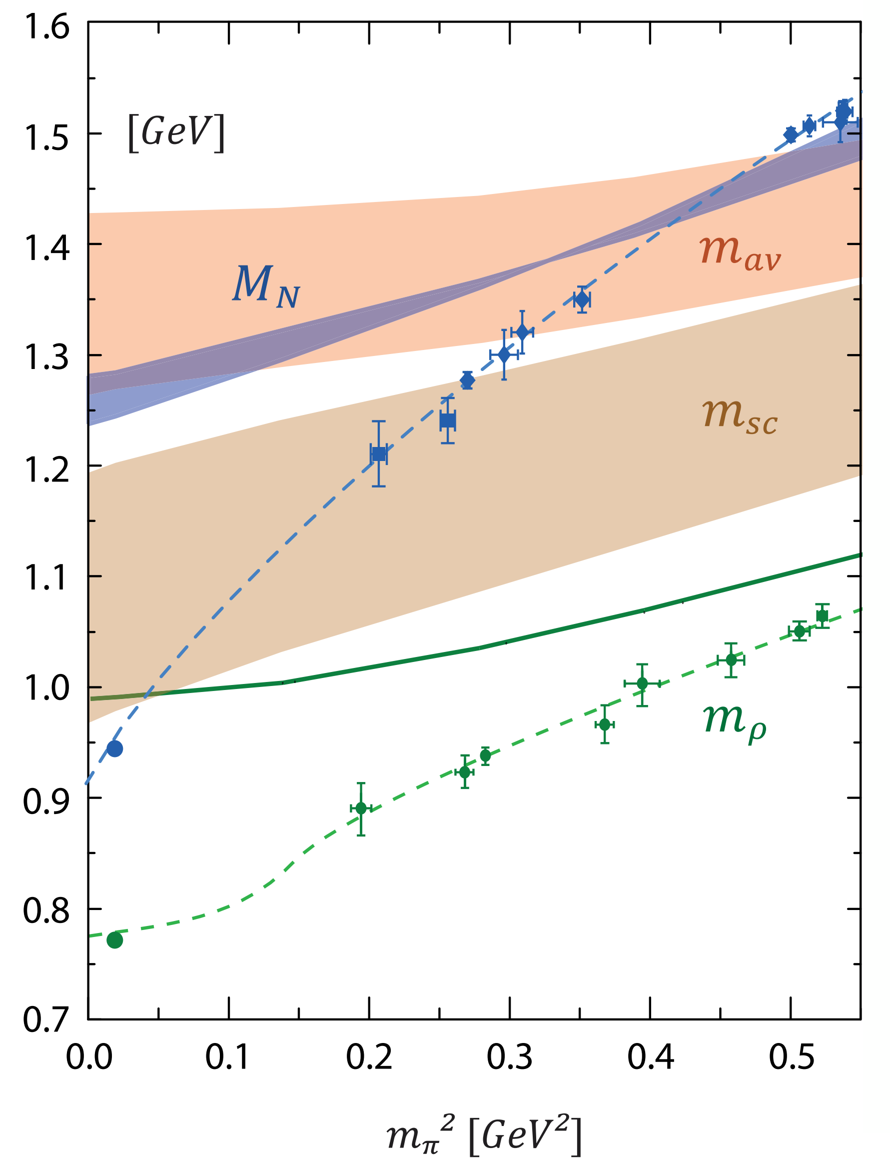

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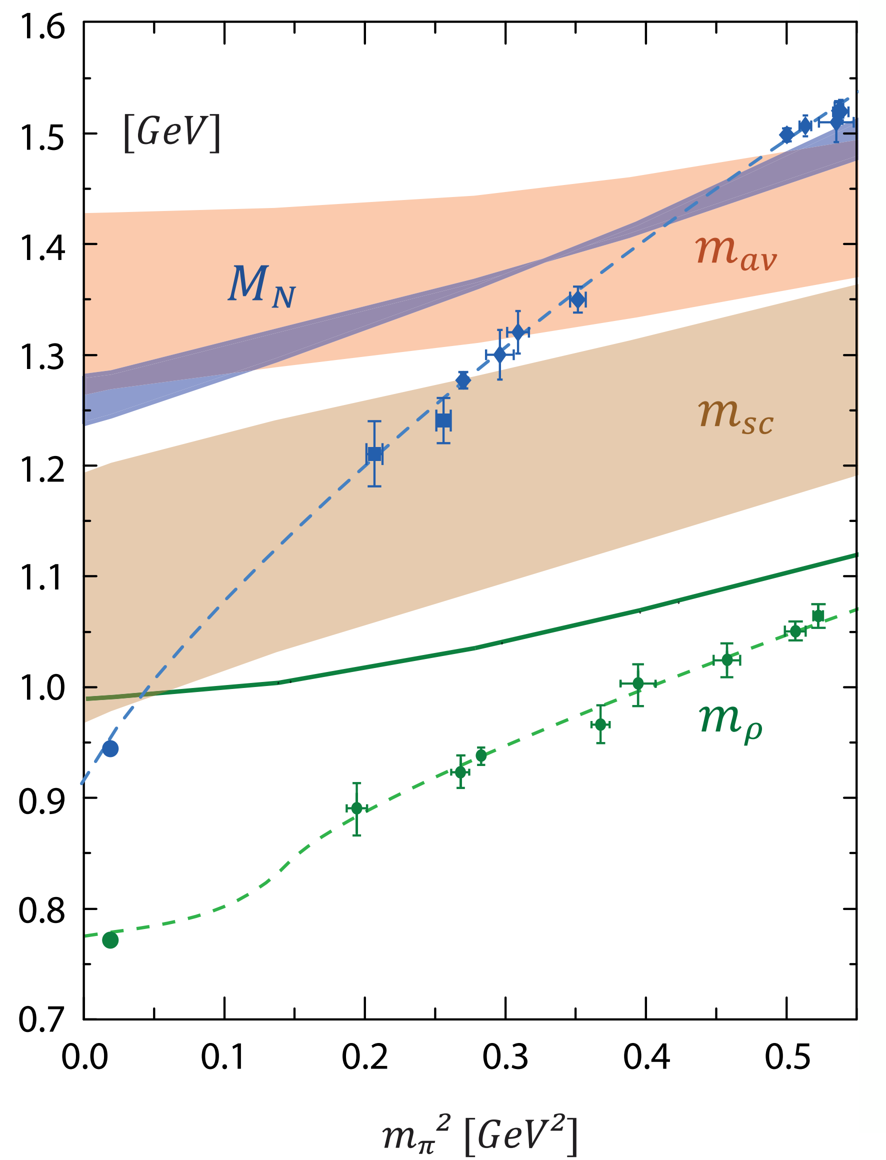

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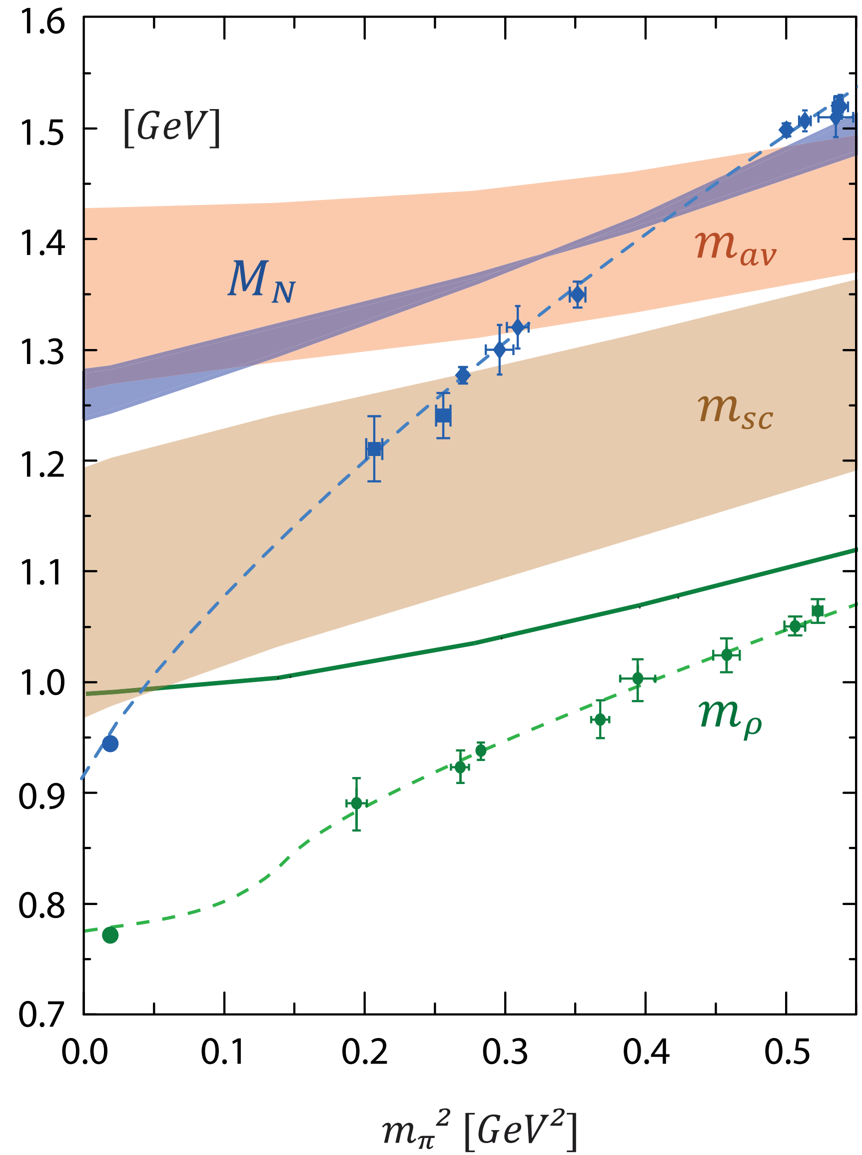
- But … $[m_{av} - m_{sc}]$, m_ρ & M_N … are *independent* of that parameter



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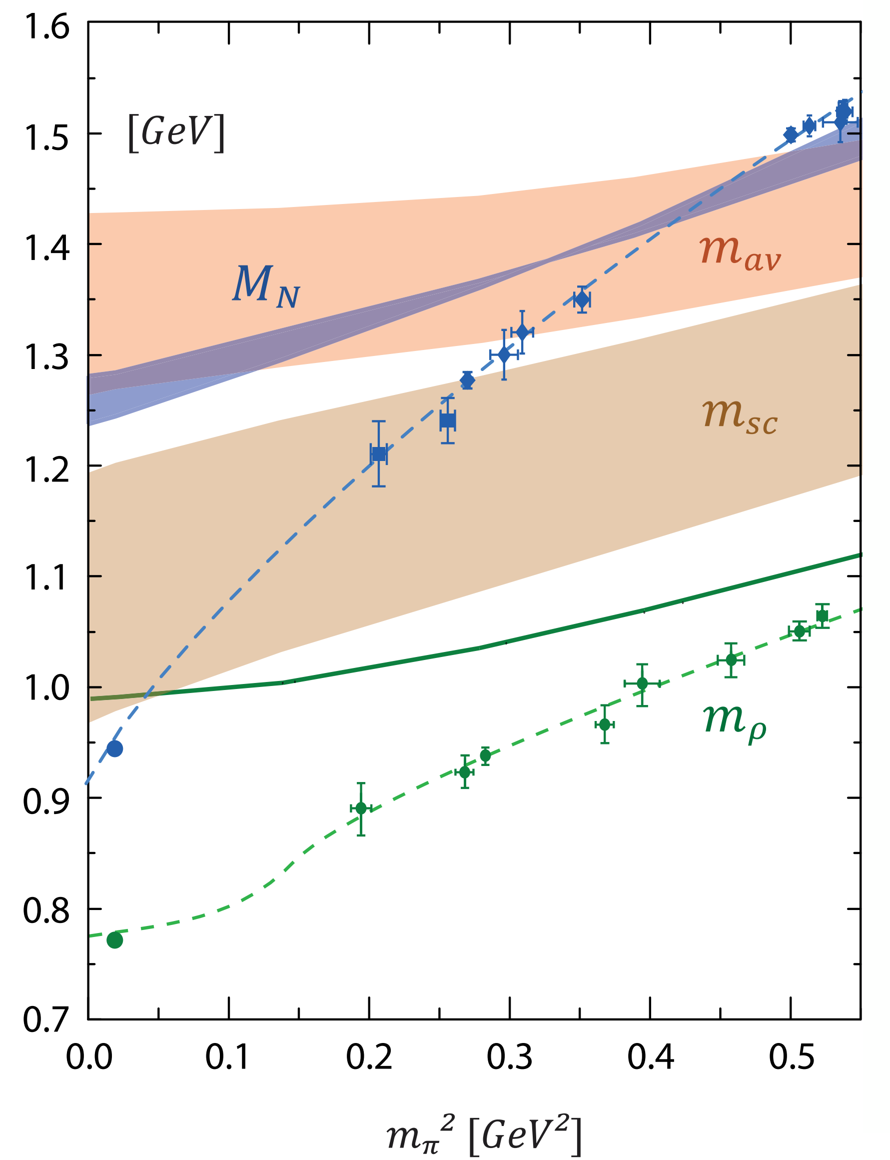

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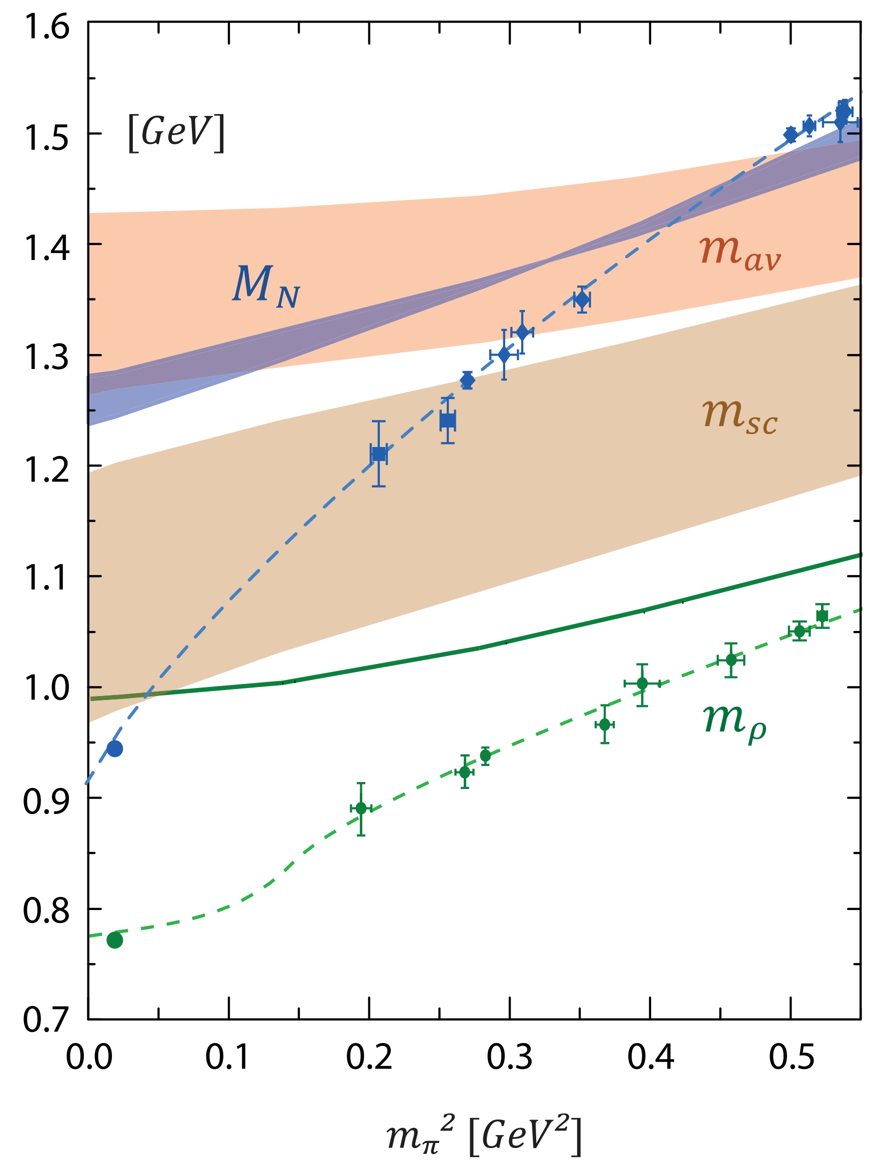

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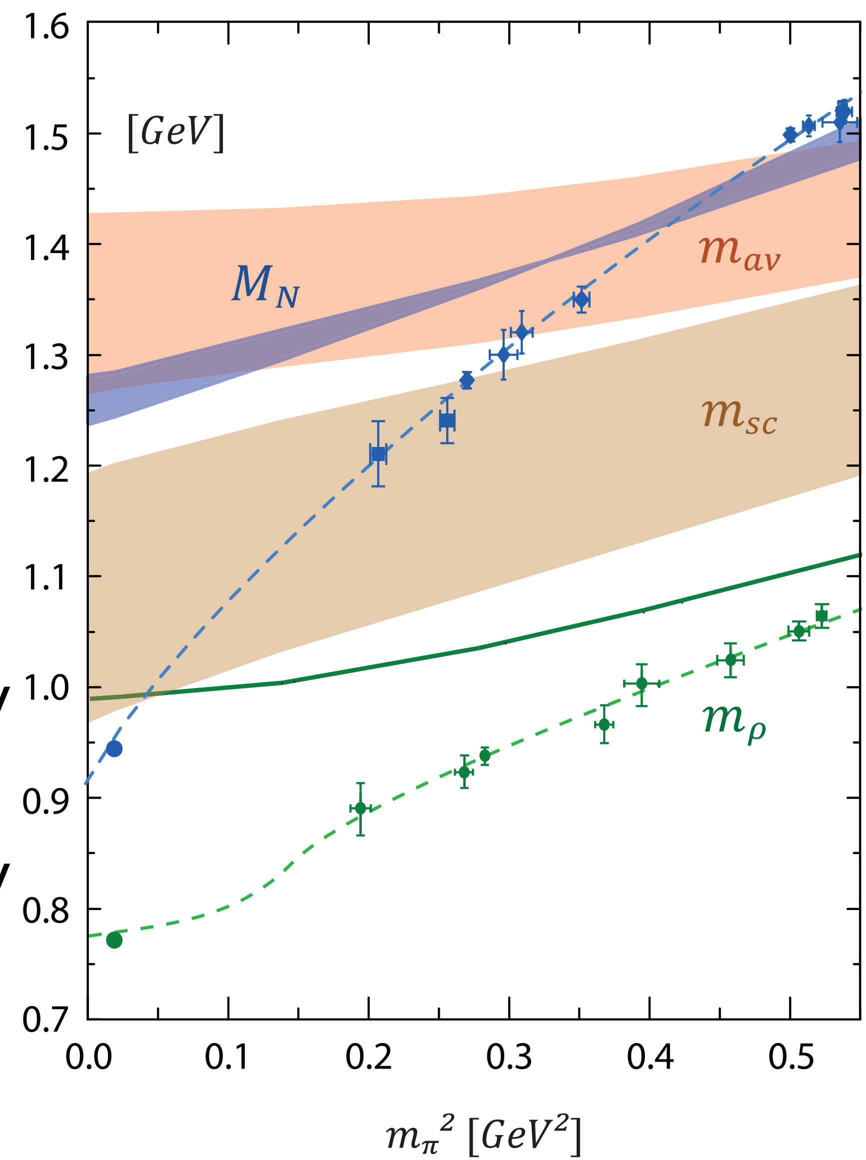
- Prediction: at physical m_π^2, $M_N^{\text{quark-core}} = 1.26(2)$ GeV cf. FRR+lattice-QCD, $M_N^{\text{quark-core}} = 1.27(2)$ GeV

$\Rightarrow$ subleading corrections, including 0^--meson loops, $\delta M_N = -320$ MeV, $\delta m_\rho = -220$ MeV



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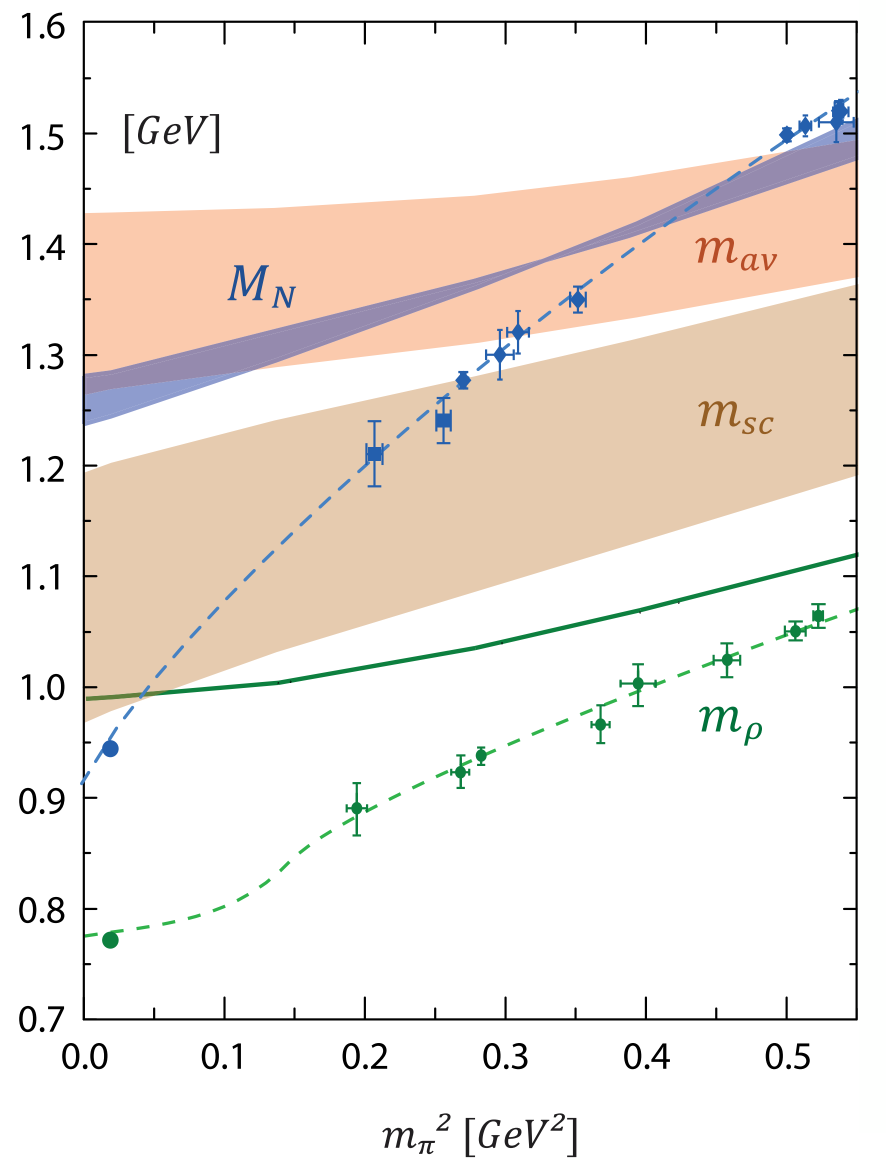

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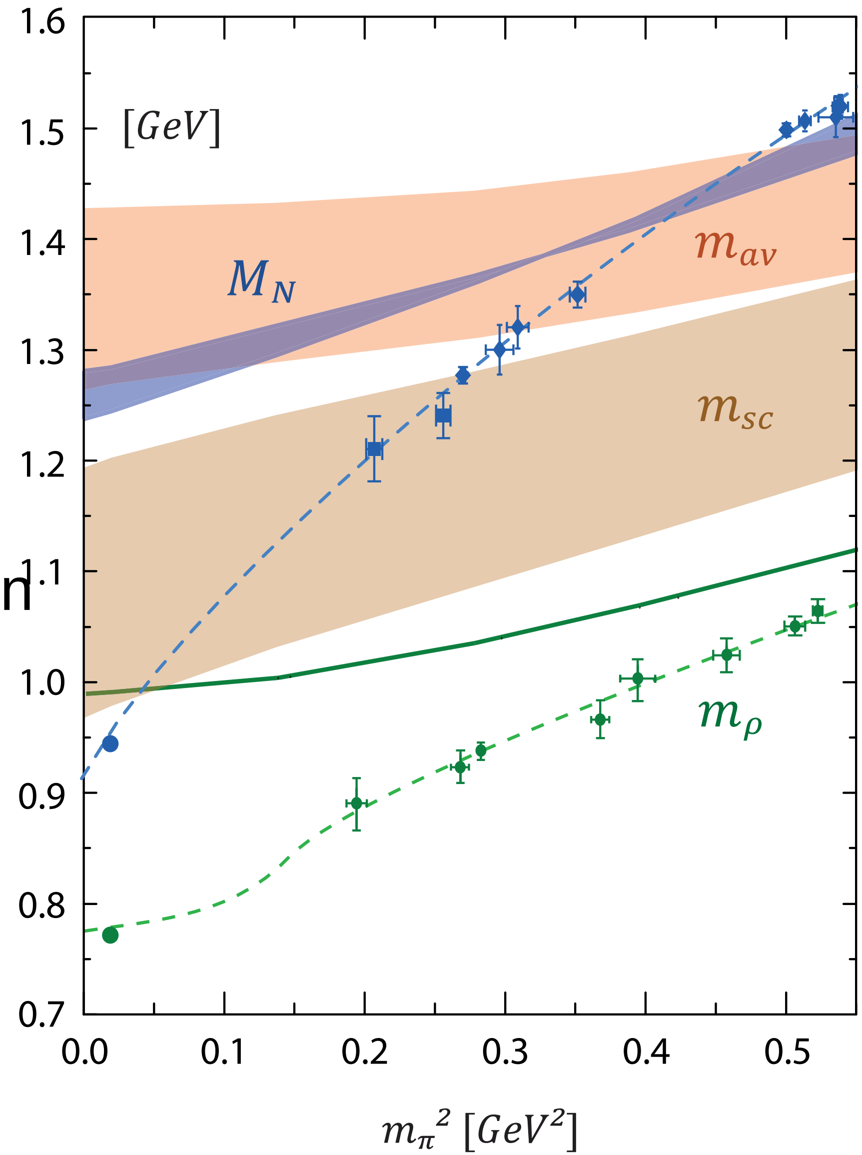

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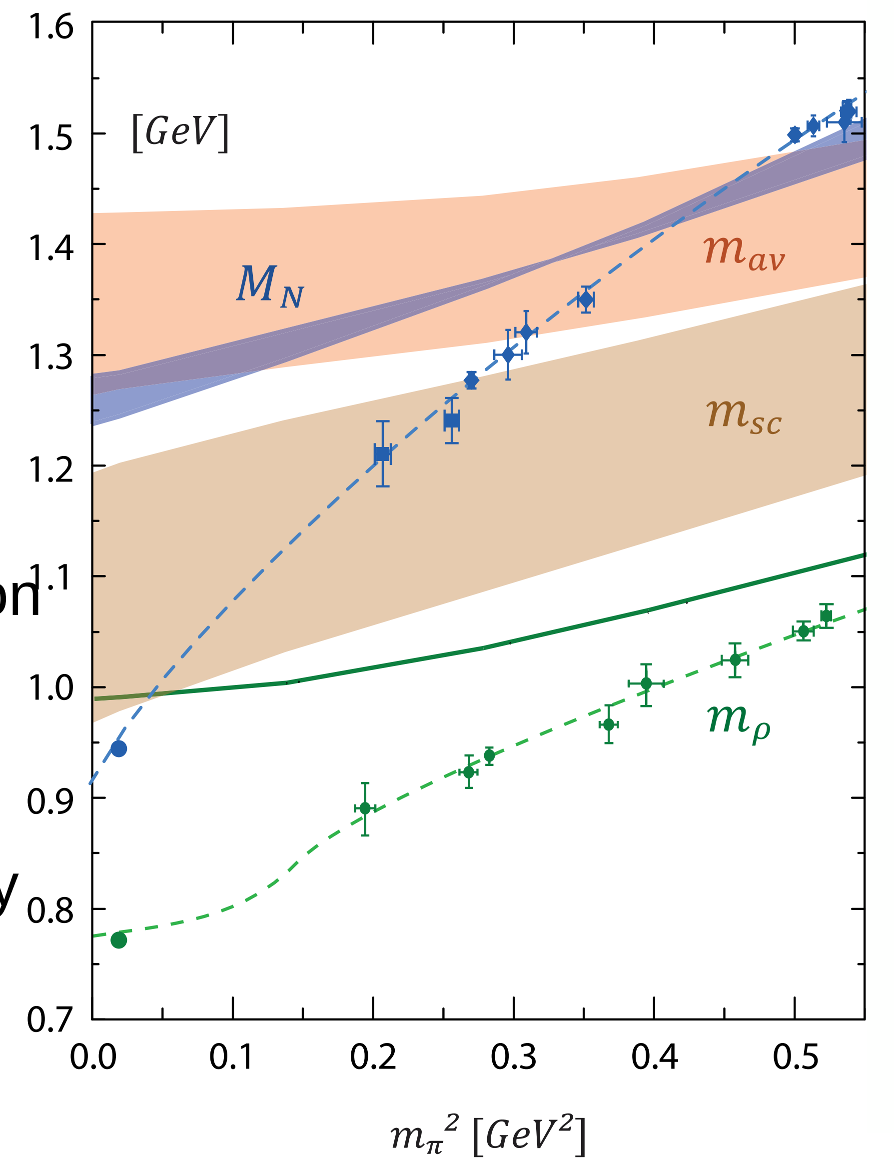

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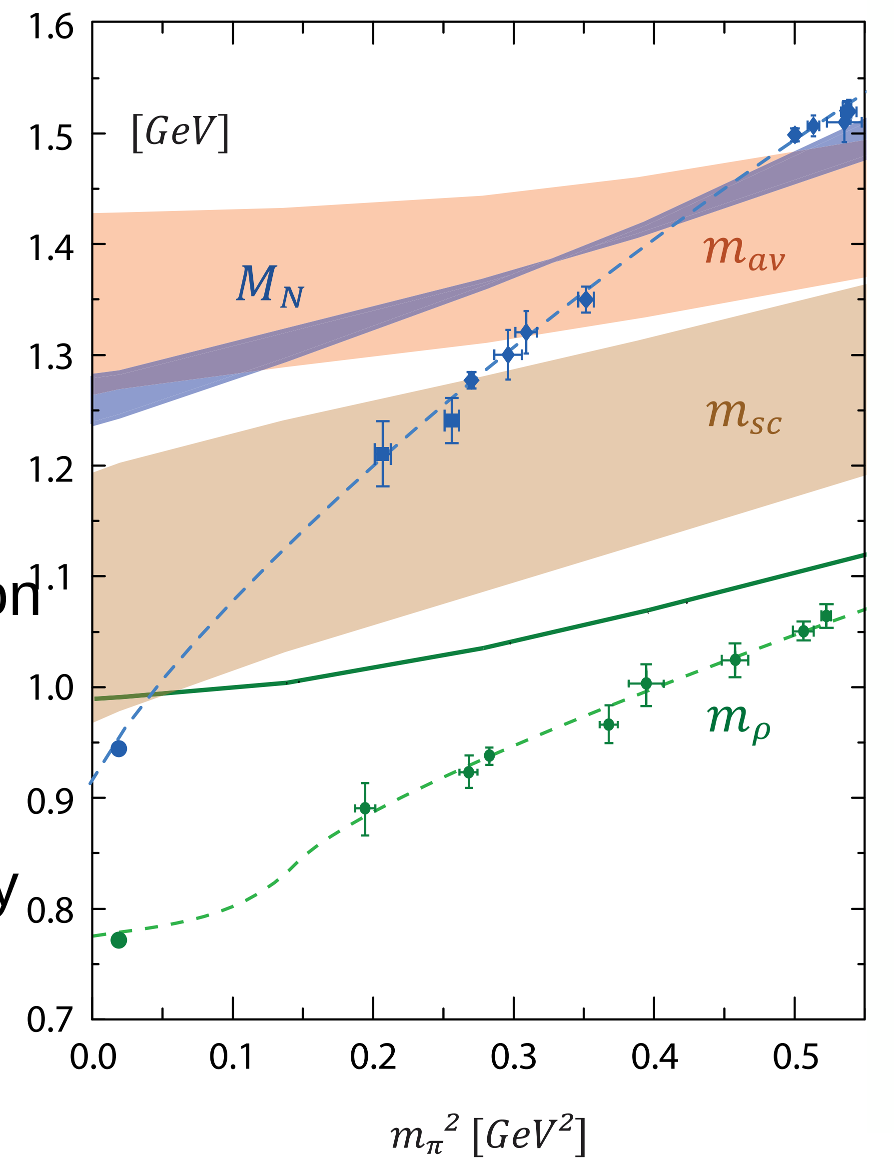
- *Systematically improvable*



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ab initio
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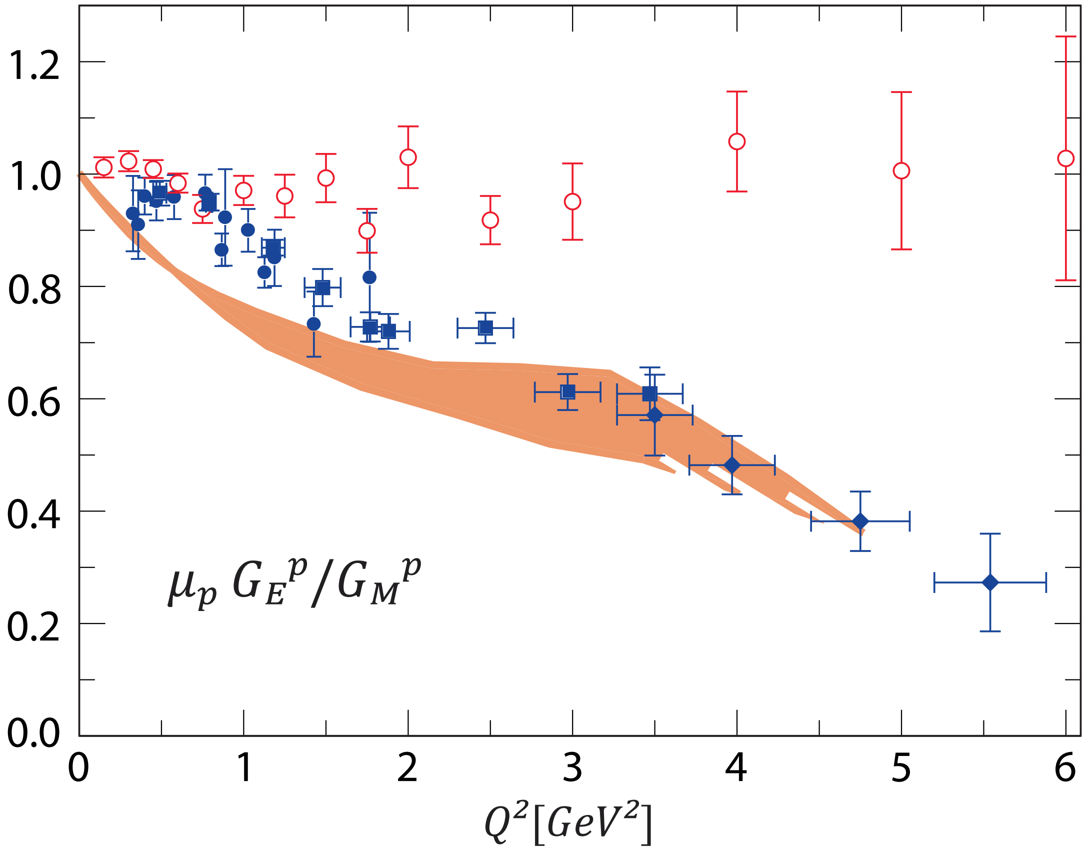

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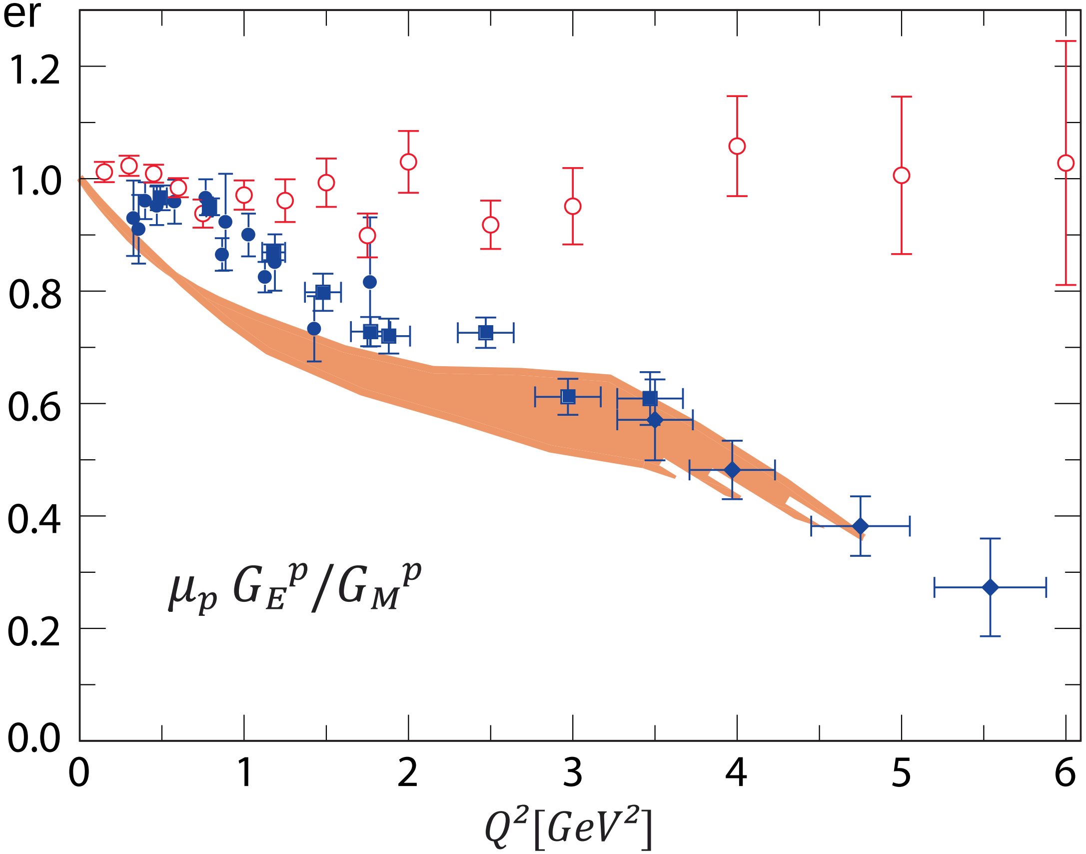

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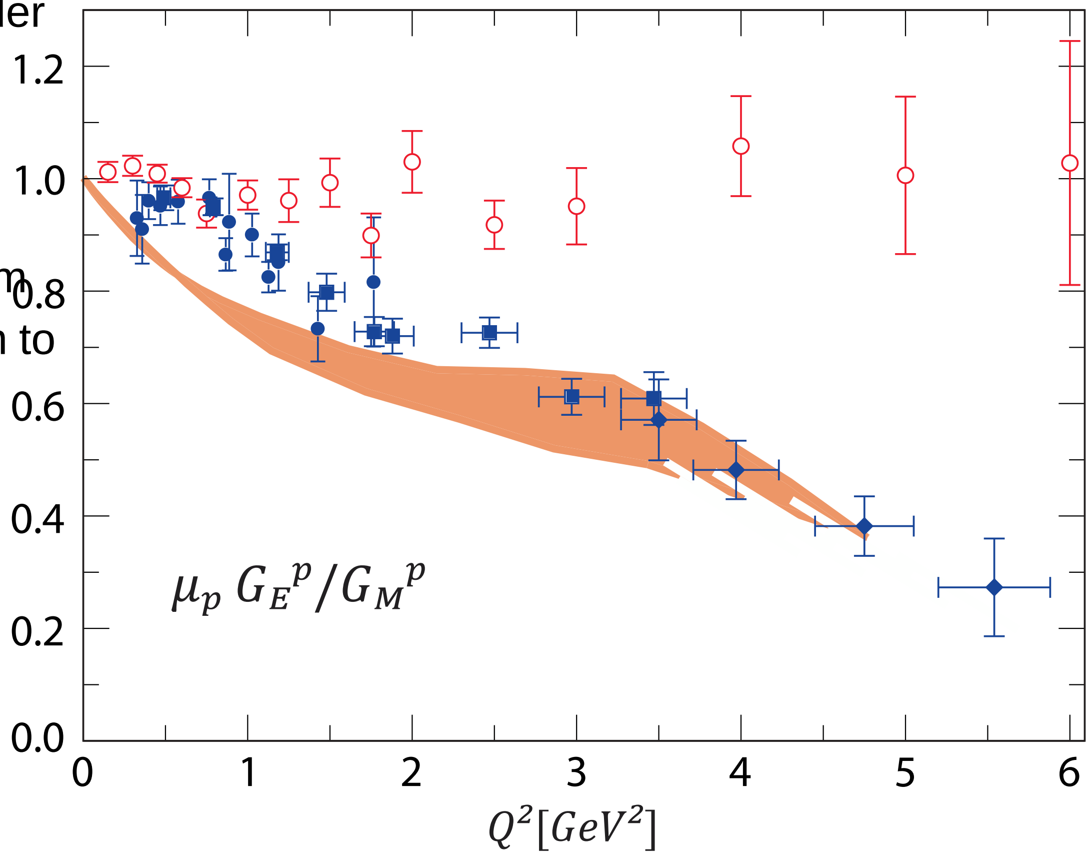

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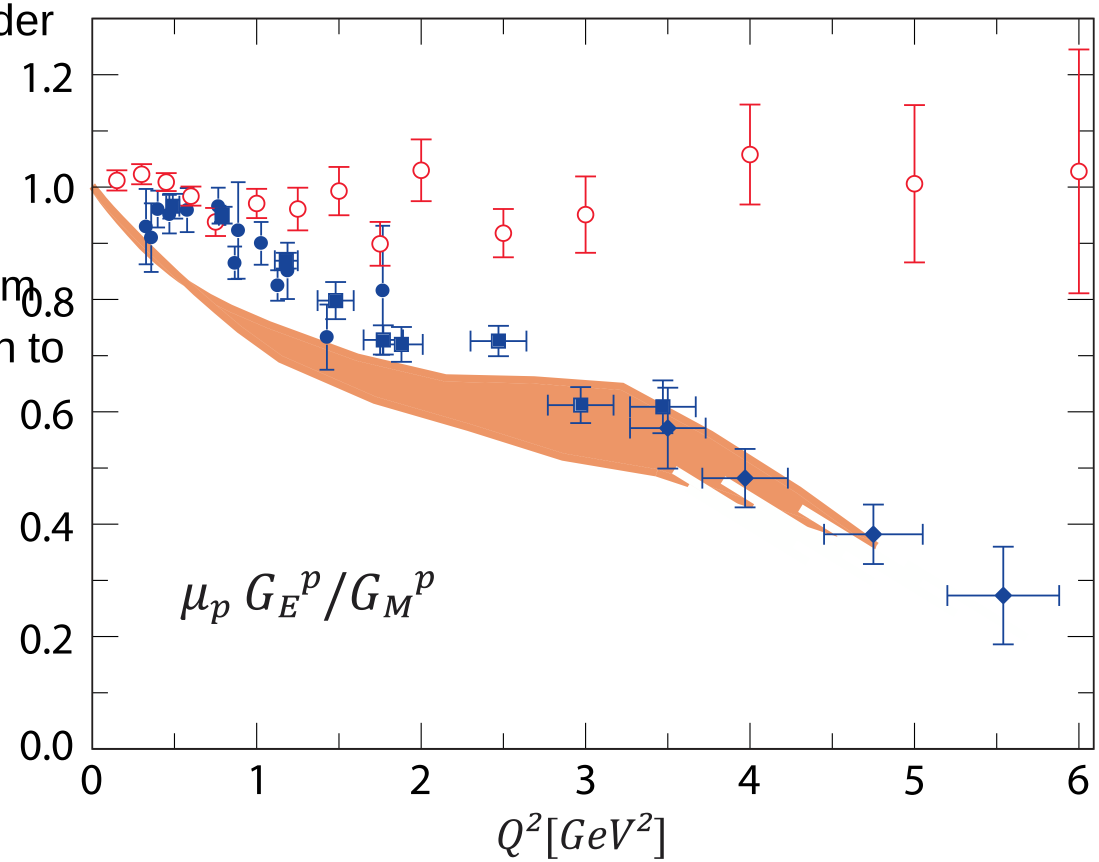

- Calculation unifies π, ρ and nucleon properties – keystone is behaviour of dressed-quark mass function and hence veracious description of QCD's Goldstone mode

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Ratio of Neutron Pauli & Dirac Form Factors

- $$\frac{\hat{Q}^2}{(\ln \hat{Q}^2/\hat{\Lambda})^2} \frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)}$$

  $$\hat{\Lambda} = \Lambda/M_N = 0.44$$

  Ensures proton ratio constant for $\hat{Q}^2 \geq 4$

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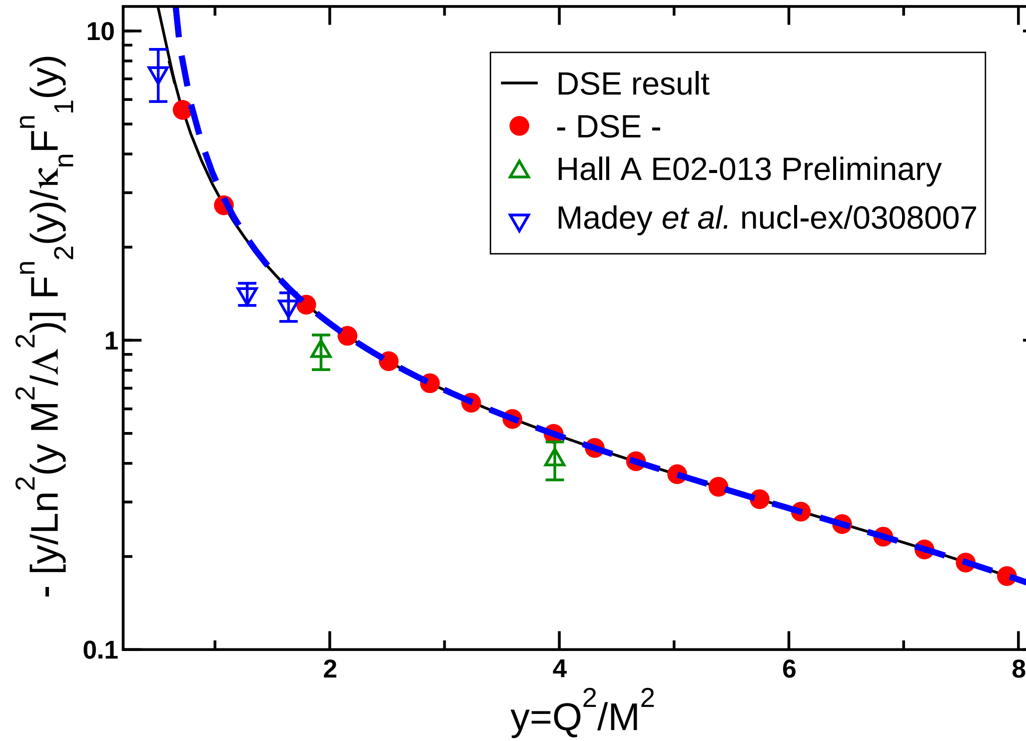

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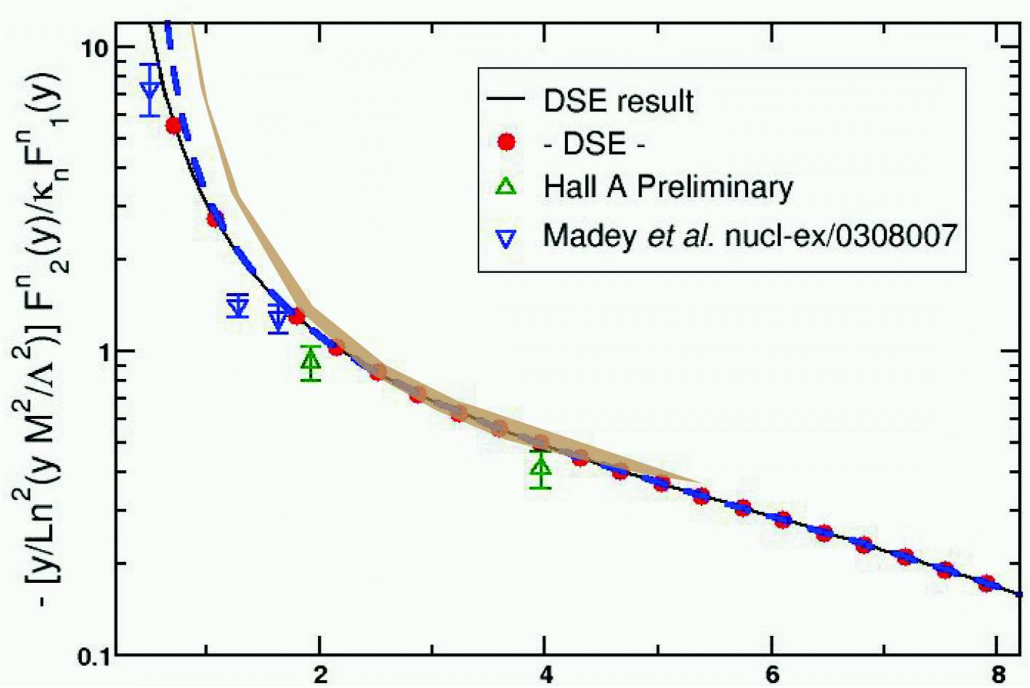
  Ensures proton ratio constant for $\hat{Q}^2 \geq 4$
- Brown band
  – *ab initio* RL result



Pion Cloud

Pion Cloud
F2 – neutron

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Pion Cloud
F2 – neutron

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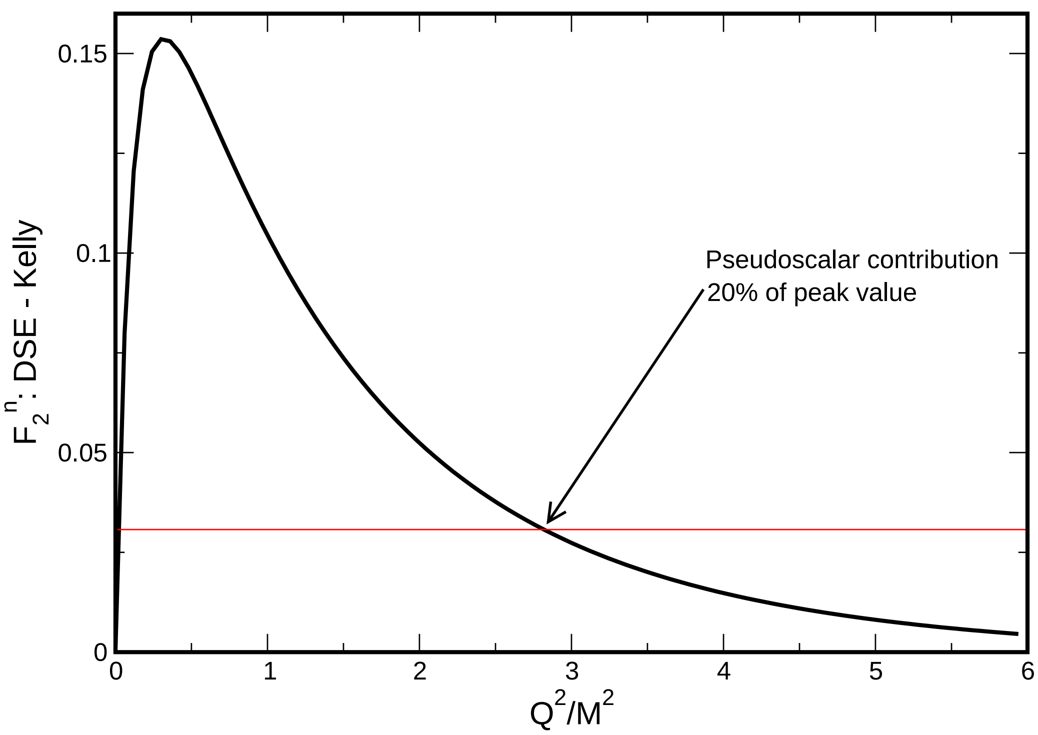

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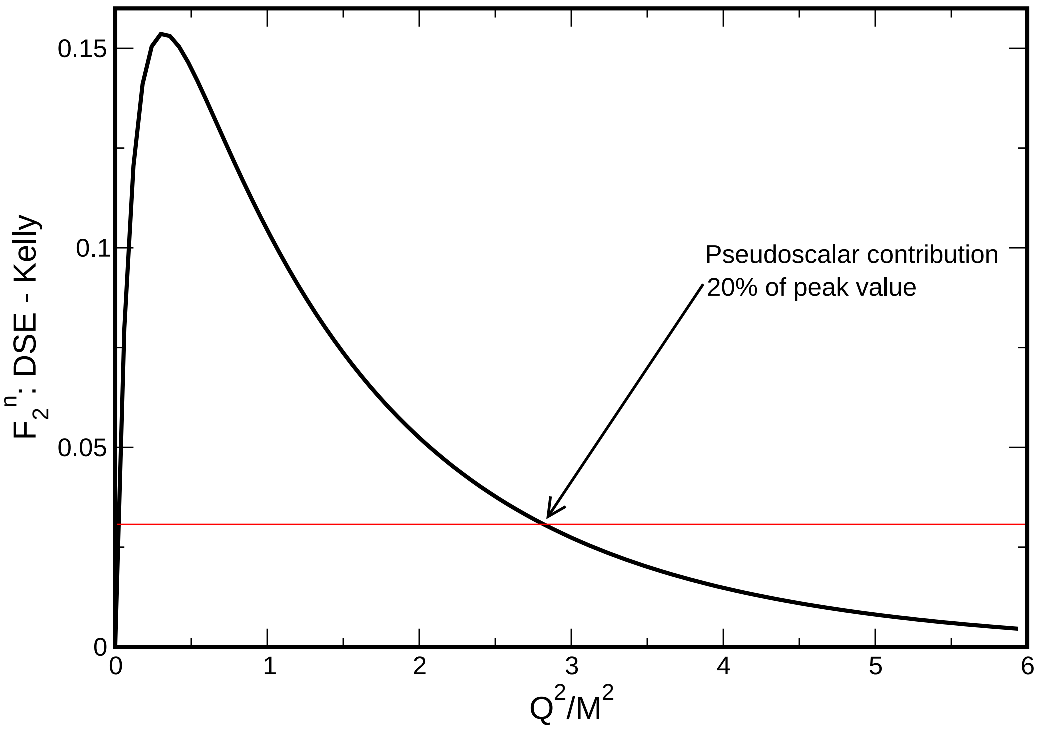

- Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4\, M_N^2$

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Epilogue

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Epilogue

- DCSB exists in QCD.

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    - light current-quarks become heavy constituent-quarks: 4 $\to$ 400 MeV
    - pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140$ MeV
    - pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi \bar{q} q} \approx 4.3$
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