Unifying the Description of Mesons and Baryons

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Universal Truths
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- Running of quark mass entails that calculations at even modest $Q^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron’s rest-frame wave function.
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Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.
QCD’s Challenges
Quark and Gluon Confinement

- No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon.
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  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but ... no degeneracy between $J^P=+$ and $J^P=-$.
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  - Neither of these phenomena is apparent in QCD’s Lagrangian yet they are the dominant determining characteristics of real-world QCD.
**QCD’s Challenges**

**Understand Emergent Phenomena**

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- **QCD – Complex behaviour**
  - Arises from apparently simple rules.
Dichotomy of Pion
– Goldstone Mode and Bound state
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\textbf{\ldots \ldots \ldots \ldots from two massive} constituent-quarks? \\
\textbf{Not Allowed} to do it by \textbf{fine-tuning} a potential \\
\textbf{Must exhibit} $m^2_\pi \propto m_q$

Current Algebra \ldots 1968
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The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a
- well-defined and valid chiral limit;
- and an accurate realisation of dynamical chiral symmetry breaking.
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**Highly Nontrivial**
What’s the Problem?
Minimal requirements

- detailed understanding of connection between 
  Current-quark and Constituent-quark masses;
- and systematic, symmetry preserving means of realising 
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Differences!
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Relativistic QFT!

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- Differences!
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  - Interaction between quarks – the Interquark “Potential” – unknown throughout > 98% of a hadron’s volume
Intranucleon Interaction
Intranucleon Interaction

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Intranucleon Interaction

98% of the volume
Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.

98% of the volume
Confinement

Infinitely Heavy Quarks . . . Picture in Quantum Mechanics

\[ V(r) = \sigma r - \frac{\pi}{12} \frac{1}{r} \]

\[ \sigma \sim 470 \text{ MeV} \]

Necco & Sommer
he-la/0108008
Illustrate this in terms of the action density . . . analogous to plotting the Force $F_{QQ}(r) = \sigma + \frac{\pi}{12} \frac{1}{r^2}$.
What happens in the real world; namely, in the presence of light-quarks?
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What happens in the real world; namely, in the presence of light-quarks? No one knows . . . but $\bar{Q}Q + 2 \times \bar{q}q$

Therefore . . . No information on potential between light-quarks.

“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”

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What is the light-quark Long-Range Potential?
Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD \textit{is not related} in any simple way to the light-quark interaction.
Charting the Interaction between light-quarks
Confinement can be related to the analytic properties of QCD’s Schwinger functions
Charting the Interaction between light-quarks

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- This is a well-posed problem whose solution is an elemental goal of modern hadron physics.
Through DSEs the pointwise behaviour of the $\beta$-function determines pattern of chiral symmetry breaking
Charting the Interaction between light-quarks

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DSEs connect $\beta$-function to experimental observables. Hence, comparison between computations and observations of, e.g., hadron mass spectrum can be used to chart $\beta$-function’s long-range behaviour.
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To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary.
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Steady quantitative progress is being made with a scheme that is systematically improvable.
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On other hand, at present significant qualitative advances possible with symmetry-preserving kernel Ansätze that express important additional nonperturbative effects, difficult to capture in any finite sum of contributions.
Dyson-Schwinger Equations
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Well suited to Relativistic Quantum Field Theory
**Dyson-Schwinger Equations**

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- Simplest level: **Generating Tool for Perturbation Theory**
  
  ................. **Materially Reduces** Model Dependence
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- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from nothing
  - Quark & Gluon Confinement
    - Coloured objects not detected, not detectable?
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- Understanding InfraRed (long-range)
- behaviour of $\alpha_s(Q^2)$
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- Method yields Schwinger Functions \( \equiv \) Propagators
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Cross-Sections built from Schwinger Functions
Perturbative Dressed-quark Propagator
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\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

\[ \Sigma = \Gamma \gamma S \]

Gap Equation
Perturbative Dressed-quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

\[ \Sigma = D \gamma \Gamma \]

\[ S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \]

dressed-quark propagator

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Weak Coupling Expansion
Reproduces Every Diagram in Perturbation Theory
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**Weak Coupling Expansion**

Reproduces Every Diagram in Perturbation Theory

But in Perturbation Theory

\[ B(p^2) = m \left( 1 - \frac{\alpha}{\pi} \ln \left[ \frac{p^2}{m^2} \right] + \ldots \right) \quad m \to 0 \]
Perturbative

Dressed-quark Propagator

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

dressed-quark propagator

Gap Equation

\[ \Sigma = \nabla \cdot \nabla = \gamma S \Gamma \]

Weak Coupling Expansion

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But in Perturbation Theory

\[ B(p^2) = m \left( 1 - \frac{\alpha}{\pi} \ln \left[ \frac{p^2}{m^2} \right] + \ldots \right) \rightarrow m \rightarrow 0 \]

No DCSB Here!

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\[ \Sigma = D \gamma \Gamma S \]

Gap Equation
Theoretical Advances

\[ S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)} \]

Gap Equation

\[ \Sigma = \begin{array}{c}
\gamma \\
S \\
\Gamma 
\end{array} \]

Rapid acquisition of mass is the effect of gluon cloud

\[ m = 0 \text{ (Chiral limit)} \]
\[ m = 30 \text{ MeV} \]
\[ m = 70 \text{ MeV} \]
Mass from nothing

In QCD a quark’s effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies \((m = 0, \text{ red curve})\) acquires a large constituent mass at low energies.

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In QCD

a quark’s mass must depend on
its momentum
• Established understanding of two- and three-point functions
Hadrons

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- What about bound states?
• Without bound states, Comparison with experiment is impossible
Hadrons

- Without bound states, Comparison with experiment is impossible
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions
• Without bound states, Comparison with experiment is impossible

• Bethe-Salpeter Equation

QFT Generalisation of Lippmann-Schwinger Equation.
• Without bound states, Comparison with experiment is **impossible**

• **Bethe-Salpeter Equation**

QFT Generalisation of Lippmann-Schwinger Equation.

• **What is the kernel, \( K \)?**

or **What is the long-range potential in QCD?**
Bethe-Salpeter Kernel
Axial-vector Ward-Takahashi identity

\[ P_{\mu} \, \Gamma_{5\mu}^l (k; P) = S^{-1}(k_+) \left( \frac{1}{2} \lambda^l_f i \gamma_5 + \frac{1}{2} \lambda^l_f i \gamma_5 \right) S^{-1}(k_-) \]

\[ -M_\zeta i \Gamma_{5}^l (k; P) - i \Gamma_{5}^l (k; P) M_\zeta \]

QFT Statement of Chiral Symmetry
Axial-vector Ward-Takahashi identity

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Satisfies BSE          Satisfies DSE

- Kernels very different
- but must be \textit{intimately} related
- Relation \textit{must} be preserved by truncation
- Nontrivial constraint
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Satisfies BSE \hspace{2cm} Satisfies DSE

Kernels very different \hspace{2cm} but must be intimately related

- Relation must be preserved by truncation
- Failure \implies Explicit Violation of QCD’s Chiral Symmetry
Persistent Challenge

- Infinitely Many Coupled Equations

\[ \Sigma = D \]

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Persistent Challenge

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- Coupling between equations necessitates truncation
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- Weak coupling expansion $\Rightarrow$ Perturbation Theory
Persistent Challenge

- Infinitely Many Coupled Equations

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- Weak coupling expansion $\Rightarrow$ Perturbation Theory
  Not useful for the nonperturbative problems in which we’re interested
**Persistent Challenge**

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme


*Dynamical chiral symmetry breaking, Goldstone’s theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*


*Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*
Infinitely Many Coupled Equations

There is at least one systematic nonperturbative, symmetry-preserving truncation scheme

Has Enabled Proof of EXACT Results in QCD
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  - Make Predictions with Readily Quantifiable Errors
Radial Excitations & Chiral Symmetry

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Radial Excitations & Chiral Symmetry

\[ f_H \ m_H^2 = - \rho_H \ M_H \]
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

\[ f_H \ m_H^2 = - \ \rho_H^\zeta \ M_H \]

- Mass\(^2\) of pseudoscalar hadron
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
nu-th/9707003 )

\[ f_H \quad m_H^2 = - \rho H \zeta \quad M_H \]

\[ M_H := \text{tr}_{\text{flavour}} \left[ M(\mu) \begin{cases} T^H, (T^H)^t \end{cases} \right] = m_{q_1} + m_{q_2} \]

- Sum of constituents’ current-quark masses
- e.g., \( T^{K^+} = \frac{1}{2} (\lambda^4 + i\lambda^5) \)
Radial Excitations & Chiral Symmetry

\[ f_H \rho_H^2 = - \rho_H^2 M_H \]

\[ f_H p_\mu = Z_2 \int_\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 \gamma_\mu S(q_+) \Gamma_H(q; P) S(q_-) \right\} \]

- Pseudovector projection of BS wave function at \( x = 0 \)
- Pseudoscalar meson’s leptonic decay constant

\[ \pi \rightarrow f_\pi k^\mu A_5^\mu \]

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Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

\[ f_H \ m_H^2 = - \rho_H \ M_H \]

\[ i \rho_H^H = Z_4 \int_\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 S(q+) \Gamma_H(q;P)S(q-) \right\} \]

- Pseudoscalar projection of BS wave function at \( x = 0 \)

\[ \vec{\pi} \rightarrow -\rho_\pi \rightarrow \vec{P}_5 \]

\[ \rightarrow k \]

\[ i \Gamma_5 \]

\[ i (\tau/2) \gamma_5 \]
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
nu-th/9707003)

\[ f_H \ m_H^2 = - \ \rho^H_\zeta \ M_H \]

- Light-quarks; i.e., \( m_q \sim 0 \)
- \( f_H \rightarrow f^0_H \) & \( \rho^H_\zeta \rightarrow -\langle \bar{q}q \rangle^0_\zeta f^0_H \), Independent of \( m_q \)

Hence \( m_H^2 = \frac{-\langle \bar{q}q \rangle^0_\zeta}{(f^0_H)^2} m_q \) … GMOR relation, a corollary
Radial Excitations & Chiral Symmetry

\( f_H \quad m^2_H = - \quad \rho^H_\zeta \quad M_H \)

- Light-quarks; i.e., \( m_q \sim 0 \)
  
  \[
  f_H \rightarrow f^0_H \quad \text{and} \quad \rho^H_\zeta \rightarrow \frac{-\langle \bar{q}q \rangle^0_\zeta}{f^0_H}, \quad \text{Independent of} \quad m_q
  \]

  Hence
  
  \[
  m^2_H = \frac{-\langle \bar{q}q \rangle^0_\zeta}{(f^0_H)^2} \quad m_q
  \]

  ...GMOR relation, a corollary

- Heavy-quark + light-quark

  \( f_H \propto \frac{1}{\sqrt{m_H}} \) and \( \rho^H_\zeta \propto \sqrt{m_H} \)

  Hence, \( m_H \propto m_q \)

  ...QCD Proof of Potential Model result
Radial Excitations & Chiral Symmetry

\[ f_H \ m_H^2 = - \ \rho^H_\zeta \ M_H \]

Valid for ALL Pseudoscalar mesons
Radial Excitations & Chiral Symmetry

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- \( \rho_H \Rightarrow \) finite, nonzero value in chiral limit, \( M_H \rightarrow 0 \)
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**ALL** pseudoscalar mesons except \( \pi(140) \) in **chiral limit**
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- **ALL** pseudoscalar mesons except \( \pi(140) \) in chiral limit
- **Dynamical Chiral Symmetry Breaking**
  - Goldstone’s Theorem
  - impacts upon **every** pseudoscalar meson
When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.
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CLEO: \( \tau \rightarrow \pi(1300) + \nu_\tau \)
\[ f_{\pi_1} < 8.4 \text{ MeV} \]

Diehl & Hiller

he-ph/0105194
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Lattice-QCD check:

$16^3 \times 32$,

$a \sim 0.1 \text{ fm}$,

two-flavour, unquenched

$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078 (93)$
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Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)
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The suppression of $f_{\pi_1}$ is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.
Goldberger-Treiman for pion

Maris, Roberts, Tandy
nucl-th/9707003
Pseudoscalar Bethe-Salpeter amplitude

\[
\Gamma_{\pi^j}(k; P) = \tau^{\pi^j} \gamma_5 \left[ iE_{\pi}(k; P) + \gamma \cdot PF_{\pi}(k; P) \\
+ \gamma \cdot k \cdot P G_{\pi}(k; P) + \sigma_{\mu\nu} k_\mu P_\nu H_{\pi}(k; P) \right]
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- Dressed-quark Propagator: \[ S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} \]
Goldberger-Treiman for pion

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\[ \Rightarrow f_\pi E_{\pi}(k; P = 0) = B(p^2) \]
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  \[ \Gamma_{\pi j}(k; P) = \tau_{\pi j} \gamma_5 \left[ i E_{\pi}(k; P) + \gamma \cdot P F_{\pi}(k; P) + \gamma \cdot k \cdot P G_{\pi}(k; P) + \sigma_{\mu \nu} k_{\mu} P_{\nu} H_{\pi}(k; P) \right] \]

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Exact in Chiral QCD
What does this mean for observables?
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![Graph showing the relationship between $q^2 F_\pi(q^2)$ and $q^2$ (GeV$^2$)].

- Including $F_\pi \rightarrow 1/Q^2$
- Only $E_\pi \rightarrow 1/Q^4$
What does this mean for observables?

Pseudovector components dominate ultraviolet behaviour of electromagnetic form factor.
GT for pion – NJL
Bethe-Salpeter amplitude can’t depend on relative momentum

⇒ General Form

\[ \Gamma_\pi(P) = i\gamma_5 E_\pi(P) + \frac{1}{M_Q} \gamma \cdot PF_\pi(P) \]
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\[ \Gamma_\pi(P) = i\gamma_5 E_\pi(P) + \frac{1}{M_Q} \gamma \cdot P F_\pi(P) \]

Solve chiral-limit gap and Bethe-Salpeter equations

\[ P^2 = 0 : \quad M_Q = 0.327, \quad E_\pi = 0.994, \quad \frac{F_\pi}{M_Q} = 0.344 \]
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Origin of pseudovector component: \( E_\pi \) drives \( F_\pi \)

RHS Bethe-Salpeter equation:

\[ \gamma_\mu S(k + P/2)i\gamma_5 E_\pi S(k - P/2)\gamma_\mu \]
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Has pseudovector component

\[ \sim E_\pi [\sigma_S(k_+) \sigma_V(k_-) + \sigma_S(k_-) \sigma_V(k_+)] \]
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\[ \gamma_\mu S(k + P/2)i\gamma_5 E_\pi S(k - P/2)\gamma_\mu \]

- Hence \( F_\pi \) on LHS is forced to be nonzero because \( E_\pi \) on RHS is nonzero owing to DCSB
Bethe-Salpeter amplitude: General Form

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Bethe-Salpeter amplitude: General Form

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Asymptotic form of electromagnetic pion form factor
Bethe-Salpeter amplitude: General Form

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Asymptotic form of electromagnetic pion form factor

- \( E_{\pi}^2 \)-term \( \Rightarrow F_{\pi}^{em}(Q^2) \sim \frac{1}{Q^2} \), photon(\(Q\))
Bethe-Salpeter amplitude: General Form

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  - \( F_\pi^2 \)-term. Breit Frame:
    pion \((P = (0, 0, -Q/2, iQ/2))\)
GT for pion – NJL

Bethe-Salpeter amplitude: General Form

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  - \( F_{\pi F}(Q^2) \sim S\gamma \cdot (P + Q) F_\pi S\gamma_4 S\gamma \cdot P F_\pi \)
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  \]
  \[
  \Rightarrow F_{\pi F}^{\text{em}}(Q^2) \propto \frac{Q^2}{M_Q^2} \times E_\pi^2 - \text{term} = \text{constant}!
  \]
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  \[ F_{\pi F}^{em}(Q^2) \sim S\gamma \cdot (P + Q)F_{\pi}S\gamma_4S\gamma \cdot PF_\pi \]

  \( \Rightarrow F_{\pi F}^{em}(Q^2) \propto \frac{Q^2}{M_Q^2} \times E_\pi^2 \)-term = constant!

This Behaviour dominates for \( Q^2 \gtrsim M_Q^2 \frac{E_\pi^2}{F_{\pi}^2} > 10 \text{ GeV}^2 \)
Gap Equation
General Form
Return to general bound-state problem . . .
To study the Poincaré covariant bound-state problem for mesons, one must first solve the gap equation

\[
S_f(p)^{-1} = Z_2 \left( i\gamma \cdot p + m_{f_{\text{bm}}} \right) + \Sigma_f(p),
\]

\[
\Sigma_f(p) = Z_1 \int_0^\Lambda \left( g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \frac{\lambda^a}{2} \Gamma^f_{\nu}(q, p) \right),
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\]

- \(D_{\mu\nu}(k)\) is the dressed-gluon propagator;
- \(\Gamma^f_\nu(q, p)\) is the dressed-quark-gluon vertex;
- \(m_f^{\text{bm}}(\Lambda)\) is the Lagrangian current-quark bare mass;
- \(Z_{1,2}(\zeta^2, \Lambda^2)\) are respectively the vertex and quark wave function renormalisation constants, with \(\zeta\) the renormalisation point.
Bethe-Salpeter Equation
General Form
Pseudoscalar and axial-vector mesons appear as poles in the inhomogeneous Bethe-Salpeter equation.
Bethe-Salpeter Equation
General Form

- Pseudoscalar and axial-vector mesons appear as poles in the inhomogeneous Bethe-Salpeter equation.

**Exact form:**

\[
\Gamma_{5\mu}^{fg}(k; P) = Z_2 \gamma_5 \gamma_\mu - \int_q g^2 D_{\alpha\beta}(k - q) \times \frac{\lambda^a}{2} \gamma_\alpha S_f(q+) \Gamma_{5\mu}^{fg}(q; P) S_g(q-) \frac{\lambda^a}{2} \Gamma_{\beta}^{g}(q-, k_-) \\
+ \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P),
\]
Pseudoscalar and axial-vector mesons appear as poles in the inhomogeneous Bethe-Salpeter equation.

Exact form:

\[
\Gamma_{5\mu}^{fg}(k; P) = Z_2 \gamma_5 \gamma_\mu - \int_q \frac{g^2}{2} D_{\alpha\beta}(k - q) \times \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \Gamma_{5\mu}^{fg}(q; P) S_g(q_-) \frac{\lambda^a}{2} \Gamma_{\beta}^g(q_-, k_-) 
\]

\[
+ \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P),
\]

\(\Lambda_{5\mu\beta}^{fg}\) is defined completely via the dressed-quark self-energy and, owing to Poincaré covariance, one can employ, e.g., \(q_\pm = q \pm P/2\), etc., without loss of generality.
Ward-Takahashi Identity
Bethe-Salpeter Kernel
In any reliable study of light-quark hadrons, axial-vector vertex must satisfy

\[ P_\mu \Gamma_{5\mu}^{fg}(k; P) = S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-) \]

\[ - i [m_f(\zeta) + m_g(\zeta)] \Gamma_5^{fg}(k; P), \]

expresses chiral symmetry & pattern by which it’s broken
In any reliable study of light-quark hadrons, axial-vector vertex must satisfy

\[ P_\mu \Gamma^f_{g5\mu}(k; P) = S^{-1}_f(k_+)i\gamma_5 + i\gamma_5 S^{-1}_g(k_-) \]

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expresses chiral symmetry & pattern by which it’s broken

The condition \( \Lambda^f_{g5\beta} \) pseudoscalar analogue of \( \Lambda^f_{g5\mu\beta} \)

\[ P_\mu \Lambda^f_{g5\mu\beta}(k, q; P) = \Gamma^f_{g\beta}(q_+, k_+) i\gamma_5 + i\gamma_5 \Gamma^g_{\beta}(q_-, k_-) \]

\[ -i [m_f(\zeta) + m_g(\zeta)] \Lambda^f_{g5\beta}(k, q; P) , \]

NECESSARY & SUFFICIENT to ensure Ward-Takahashi identity satisfied.
The condition \((\Lambda_{5\beta}^{fg} \text{ pseudoscalar analogue of } \Lambda_{5\mu\beta}^{fg})\)

\[
P_\mu \Lambda_{5\mu\beta}^{fg}(k, q; P) = \Gamma_{\beta}^{f}(q_+, k_+) i\gamma_5 + i\gamma_5 \Gamma_{\beta}^{g}(q_-, k_-)
- i [m_f(\zeta) + m_g(\zeta)] \Lambda_{5\beta}^{fg}(k, q; P),
\]

**NECESSARY & SUFFICIENT**

to ensure Ward-Takahashi identity satisfied.

Rainbow-ladder . . .

\[
\Gamma_{\beta}^{f}(q, k) = \gamma_\mu
\]

\[
\Rightarrow \Lambda_{5\mu\beta}^{fg}(k, q; P) = 0 = \Lambda_{5\beta}^{fg}(k, q; P)
\]
Ward-Takahashi identity is far more than merely a device for checking a truncation’s consistency.
Solving the Kernel’s Ward-Takahashi Identity

- Ward-Takahashi identity is far more than merely a device for checking a truncation’s consistency.
- Remember vector-vertex Ward-Takahashi identity . . . long been used to build Ansätze for the dressed-quark-photon vertex
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Remember vector-vertex Ward-Takahashi identity . . . long been used to build Ansätze for the dressed-quark-photon vertex

Kernel’s Ward-Takahashi identity provides means by which to construct a symmetry preserving kernel of the Bethe-Salpeter equation that is matched to any reasonable Ansatz for the dressed-quark-gluon vertex
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Kernel’s Ward-Takahashi identity provides means by which to construct a symmetry preserving kernel of the Bethe-Salpeter equation that is matched to any reasonable *Ansatz* for the dressed-quark-gluon vertex

With this powerful capacity they realise a longstanding goal.
Suppose that in the gap equation one employs an Ansatz for the dressed-quark-gluon vertex which satisfies

$$P_\mu i \Gamma^f_{\mu}(k_+, k_-) = B(P^2) \left[ S_f^{-1}(k_+) - S_f^{-1}(k_-) \right]$$  \hspace{1cm} (*)

with \( B \) flavour-independent.
Suppose that in the gap equation one employs an Ansatz for the dressed-quark-gluon vertex which satisfies

\[ P_\mu i \Gamma^f_\mu(k_+, k_-) = B(P^2) \left[ S_f^{-1}(k_+) - S_f^{-1}(k_-) \right] \]  \( (*) \)

with \( B \) flavour-independent.

NB. While true quark-gluon vertex doesn’t satisfy this identity, owing to form of Slavnov-Taylor identity which it does satisfy, it’s plausible that solution of Eq. \((*)\) can provide reasonable pointwise approximation to true vertex.
Suppose that in the gap equation one employs an \textit{Ansatz} for the dressed-quark-gluon vertex which satisfies

\[ P_\mu i \Gamma^f_\mu (k_+, k_-) = B(P^2) \left[ S_f^{-1}(k_+) - S_f^{-1}(k_-) \right] \]

with $B$ flavour-independent.

Given Eq. (*), then Kernel’s WTI entails

\[
\begin{align*}
P_\mu (q - k)_\beta i \Lambda^{fg}_{5\mu \beta}(k, q; P) &= P_\mu B((k - q)^2) \left[ \Gamma^{fg}_{5\mu}(q; P) - \Gamma^{fg}_{5\mu}(k; P) \right], \\
(q - k)_\beta i \Lambda^{fg}_{5\beta}(k, q; P) &= B((k - q)^2) \left[ \Gamma^{fg}_{5}(q; P) - \Gamma^{fg}_{5}(k; P) \right].
\end{align*}
\]
Solution to Eq. (#)

\[ \Lambda_{5\beta}^{fg}(k, q; P) := B((k - q)^2) \gamma_5 \overline{\Lambda}_{\beta}^{fg}(k, q; P), \]
Solution to Eq. (#)

\[ \Lambda_{5\beta}^{fg}(k, q; P) := \mathcal{B}((k - q)^2) \gamma_5 \Lambda_{\beta}^{fg}(k, q; P), \]

with (BC construction)

\[ \Lambda_{\beta}^{fg}(k, q; P) = 2\ell_{\beta} \left[ i\Delta_{E5}(q, k; P) + \gamma \cdot P \Delta_{F5}(q, k; P) \right] + \gamma_\beta \Sigma_{G5}(q, k; P) + 2\ell_{\beta} \gamma \cdot \ell \Delta_{G5}(q, k; P) + [\gamma_{\beta}, \gamma \cdot P] \]
\[ \times \Sigma_{H5}(q, k; P) + 2\ell_{\beta} [\gamma \cdot \ell, \gamma \cdot P] \Delta_{H5}(q, k; P), \]

\[ \ell = (q + k)/2 \]

\[ \Sigma_{\Phi}(q, k; P) = [\Phi(q; P) + \Phi(k; P)]/2 \]

\[ \Delta_{\Phi}(q, k; P) = [\Phi(q; P) - \Phi(k; P)]/[q^2 - k^2] \]
Symmetry-preserving Ansatz

At this point . . .

Began with $\Gamma_\mu(q, p)$, whose diagrammatic content is unknown, but which expresses important additional nonperturbative effects that are difficult to capture in any finite sum of contributions.
Symmetry-preserving Ansatz

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- Began with $\Gamma_\mu(q,p)$, whose diagrammatic content is unknown, but which expresses important additional nonperturbative effects that are difficult to capture in any finite sum of contributions.

- Given that $\Gamma_\mu(q,p)$ satisfies Eq. (*), the equations which follow provide symmetry-preserving closed system whose solution yields predictions for the properties of pseudoscalar mesons.
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Given that $\Gamma_\mu(q,p)$ satisfies Eq. (\*), the equations which follow provide symmetry-preserving closed system whose solution yields predictions for the properties of pseudoscalar mesons.

This system and its predictions can smoothly be connected with those obtained, e.g., in a rainbow-ladder or kindred symmetry-preserving truncation of the DSEs.
Symmetry-preserving Ansatz

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Given that $\Gamma_\mu(q, p)$ satisfies Eq. (*), the equations which follow provide symmetry-preserving closed system whose solution yields predictions for the properties of pseudoscalar mesons.

The system can be used to anticipate, elucidate and understand the impact on hadron properties of the rich nonperturbative structure expected of the fully-dressed quark-gluon vertex in QCD.
Numerical Illustration

![Graphs showing numerical data](image-url)
Single interaction, common mass scale: rainbow-ladder cf. BC-consistent truncation
Numerical Illustration

Single interaction, common mass scale: rainbow-ladder cf. BC-consistent truncation

GMOR ... plainly satisfied by both truncations
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Added **attraction** in pseudoscalar channel
Single interaction, common mass scale: rainbow-ladder cf. BC-consistent truncation

- GMOR . . . plainly satisfied by both truncations
- Added attraction in pseudoscalar channel
- Added repulsion in scalar channel
Rainbow-ladder DSE truncation,

$$\varepsilon_{\sigma}^{RL} := \left. \frac{2M(0) - m_{\sigma}}{2M(0)} \right|_{RL} = (0.3 \pm 0.1) .$$

BC-consistent Bethe-Salpeter kernel; viz., $\varepsilon_{\sigma}^{BC} \lesssim 0.1$. 
Spin-orbit Interaction

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Scalar mesons = \( ^3P_0 \) states: Constituents’ spins aligned and one unit of constituent orbital angular momentum.
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From this viewpoint scalar is a spin and orbital excitation of a pseudoscalar meson
Spin-orbit Interaction

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Extant studies of realistic corrections to the rainbow-ladder truncation show that they reduce hyperfine splitting.
Spin-orbit Interaction

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Scalar mesons = \( ^3P_0 \) states: Constituents’ spins aligned and one unit of constituent orbital angular momentum

Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit splitting.
Effect owes to influence of quark’s dynamically-enhanced scalar self-energy in the Bethe-Salpeter kernel.

Impossible to demonstrate effect without our new procedure.
Spin-orbit Interaction

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\[ \epsilon_{\sigma}^{\text{RL}} := \left. \frac{2M(0) - m_\sigma}{2M(0)} \right|_{\text{RL}} = (0.3 \pm 0.1). \]

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Expect this feature to have material impact on mesons with mass greater than 1 GeV. *Prima facie* . . . can overcome longstanding shortcoming of RL truncation; viz., splitting between vector & axial-vector mesons is too small.
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Promise of realistic meson spectroscopy
First time, also for mass > 1 GeV
ARE WE THERE YET?
New Challenges
Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
New Challenges

- **Next Steps** . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.

- Move on to the problem of a *symmetry preserving* treatment of hybrids and exotics.
New Challenges

- Another Direction . . . Also want/need information about three-quark systems
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- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa \(
\sim 1995\).
New Challenges

- Another Direction . . . Also want/need information about three-quark systems

- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa ~ 1995.

- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.
New Challenges

- Another Direction . . . Also want/need information about three-quark systems

- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa ∼ 1995.

- However, that is beginning to change . . .
Nucleon …

Three-body Problem?

Craig Roberts: Unifying the Description of Mesons and Baryons
2nd Morelia Workshop on Nonperturbative Aspects of Field Theories… 51 – p. 37/51
What is the picture in quantum field theory?
What is the picture in quantum field theory?

Three → infinitely many!
Unifying Study of Mesons and Baryons

Craig Roberts: Unifying the Description of Mesons and Baryons
2nd Morelia Workshop on Nonperturbative Aspects of Field Theories... 51 – p. 38/51
How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons?
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Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks

Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour-$\bar{3}$ (antitriplet) channel
Faddeev equation
Faddeev equation

\[ \Psi^a \rightarrow P \rightarrow \Psi^b \]

\[ p_q \rightarrow p_d \]

\[ \Gamma^a \rightarrow \Gamma^b \]
Faddeev equation

\[
\Psi^a_{pq} \Psi^b_{pd} = \Psi^a_{pq} \Gamma^a_{q} \Gamma^b_{p} \Psi^b_{pd}
\]

Linear, Homogeneous Matrix equation

- Yields wave function (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon

- Scalar and Axial-Vector Diquarks . . . In Nucleon’s Rest Frame Amplitude has . . . \(s-, p-\) & \(d-\)wave correlations
Diquark correlations
Diquark correlations

Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red

Confined . . . Does not escape from within baryon.

Scalar is isosinglet, Axial-vector is isotriplet

DSE and lattice-QCD

\[ m_{[ud]}^{0+} = 0.74 - 0.82 \]

\[ m_{(uu)^1+} = m_{(ud)^1+} = m_{(dd)^1+} = 0.95 - 1.02 \]
Ab-initio study of mesons & nucleons
Eichmann et al.

Ab-initio study
of mesons & nucleons
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Ab-initio study
of mesons & nucleons

Leading-order truncation of DSEs – rainbow-ladder
Eichmann et al.

- Leading-order truncation of DSEs – rainbow-ladder
- Corrections vanish with increasing current-quark mass
  ⇒ rainbow-ladder exact in heavy-quark limit
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**Ab-initio study of mesons & nucleons**

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- Corrections vanish with increasing current-quark mass
  - $\Rightarrow$ rainbow-ladder exact in heavy-quark limit
- However, at physical light-quark mass, corrections to observables not protected by symmetries: uniformly $\approx 35\%$
  - Roughly 50/50-split between nonresonant and resonant (pseudoscalar meson loop) contributions
Eichmann et al.

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- Symmetry preserving and systematic approach can elucidate and account for these effects
  - Use this knowledge to constrain interaction in infrared
  - Interaction in ultraviolet predicted by perturbative expansion of DSEs
Eichmann et al.

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Craig Roberts: Unifying the Description of Mesons and Baryons
2nd Morelia Workshop on Nonperturbative Aspects of Field Theories
Ab-initio study of mesons & nucleons

Rainbow-Ladder DSE result

-one parameter for IR – “confinement radius”
-Results insensitive to value on material domain
Eichmann et al.

Rainbow-Ladder DSE result
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Numerical simulations of lattice-QCD
Rainbow-Ladder DSE result

one parameter for IR – “confinement radius”

Results insensitive to value on material domain

Numerical simulations of lattice-QCD

ERR extrapolation of lattice CP-PACS result
Eichmann *et al.*

Precisely the same interaction
Eichmann et al.
- arXiv:0810.1222 [nucl-th]

- Precisely the same interaction
- Same $\rho$-meson curve
Eichmann et al.

Precisely the same interaction

Same $\rho$-meson curve

$m_\pi^2$-dependence of $0^+$ and $1^+$ diquark masses

“unobservable” – show marked sensitivity to single model parameter; viz., confinement radius
Eichmann et al.

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- Same $\rho$-meson curve
- $m_\pi^2$-dependence of $0^+$ and $1^+$ diquark masses
- “unobservable” – show marked sensitivity to single model parameter; viz., confinement radius

But . . . $[m_{av} - m_{sc}]$, $m_\rho$ & $M_N$ . . . are independent of that parameter
Ab-initio study of mesons & nucleons

Eichmann et al.

Parameter-independent RL-DSE predictions, with veracious description of Goldstone mode
Eichmann et al.

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- DSE and lattice agree on heavy-quark domain

Craig Roberts: Unifying the Description of Mesons and Baryons
2nd Morelia Workshop on Nonperturbative Aspects of Field Theories... 51 – p. 44/51
Eichmann et al.

Parameter-independent RL-DSE predictions, with veracious description of Goldstone mode

DSE and lattice agree on heavy-quark domain

Prediction: at physical \( m_\pi^2 \),
\[
M_{N}^{\text{quark-core}} = 1.26(2) \text{ GeV}
\]
cf. FRR+lattice-QCD,
\[
M_{N}^{\text{quark-core}} = 1.27(2) \text{ GeV}
\]
⇒ subleading corrections, including \( 0^- \)-meson loops,
\[
\delta M_{N} = -320 \text{ MeV},
\]
\[
\delta m_\rho = -220 \text{ MeV}
\]
Eichmann et al.

Bethe-Salpeter & Faddeev equations built from same RG-improved rainbow-ladder interaction

Ab-initio study of mesons & nucleons

Craig Roberts: Unifying the Description of Mesons and Baryons
2nd Morelia Workshop on Nonperturbative Aspects of Field Theories... 51 – p. 45/51
Ab-initio study of mesons & nucleons

- Bethe-Salpeter & Faddeev equations built from same RG-improved rainbow-ladder interaction
- Simultaneous calculation of baryon & meson properties, & prediction of their correlation

Eichmann et al.
Ab-initio study of mesons & nucleons

- Eichmann et al.
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Systematically improvable
Eichmann et al.

Eichmann et al.
Parameter-free rainbow-ladder Faddeev equation – result qualitatively identical and in semiquantitative agreement.
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Improved numerical algorithm needed to extend calculation to larger $Q^2$
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Parameter-free rainbow-ladder Faddeev equation – result qualitatively identical and in semiquantitative agreement

Improved numerical algorithm needed to extend calculation to larger $Q^2$

Calculation unifies $\pi$, $\rho$ and nucleon properties – keystone is behaviour of dressed-quark mass function and hence veracious description of QCD’s Goldstone mode
Ratio of Neutron Pauli & Dirac Form Factors

\[ \frac{\hat{Q}^2}{(\ln \hat{Q}^2 / \hat{\Lambda})^2} \frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)} \]

\[ \hat{\Lambda} = \Lambda / M_N = 0.44 \]

Ensures proton ratio constant for \( \hat{Q}^2 \geq 4 \)
\[
\frac{\hat{Q}^2}{(\ln \hat{Q}^2/\hat{\Lambda})^2} \frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)}\]

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Ensures proton ratio constant for \(\hat{Q}^2 \geq 4\)

Brown band

– \textit{ab initio} RL result
Pion Cloud

F2 – neutron
Comparison between Faddeev equation result and Kelly’s parametrisation

Faddeev equation set-up to describe dressed-quark core

![Graph showing comparison between Faddeev equation result and Kelly's parametrisation](image)
Comparison between Faddeev equation result and Kelly’s parametrisation

Faddeev equation set-up to describe dressed-quark core

Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4 M_N^2$
Epilogue
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DCSB exists in QCD.

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- It is manifest in dressed propagators and vertices
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- It predicts, amongst other things, that
  - light current-quarks become heavy constituent-quarks: $4 \rightarrow 400$ MeV
  - pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140$ MeV
  - pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi \bar{q}q} \approx 4.3$
  - pseudoscalar mesons couple unnaturally strongly to the lightest baryons
    
    
    
    
    
    Craig Roberts: Unifying the Description of Mesons and Baryons
    2nd Morelia Workshop on Nonperturbative Aspects of Field Theories... 51 – p. 50/51
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- Pseudoscalar mesons couple unnaturally strongly to the lightest baryons $g_{\pi \bar{NN}} \approx 12.8 \approx 3g_{\pi \bar{q}q}$
- It impacts dramatically upon observables.

Epilogue
Dyson-Schwinger Equations

- Poincaré covariant unification of meson and baryon observables
Epilogue

Dyson-Schwinger Equations

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- All global and pointwise corollaries of DCSB are manifested naturally without fine-tuning
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Excited states:
- Mesons already being studied
- Baryons are within practical reach
Dyson-Schwinger Equations

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Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks
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