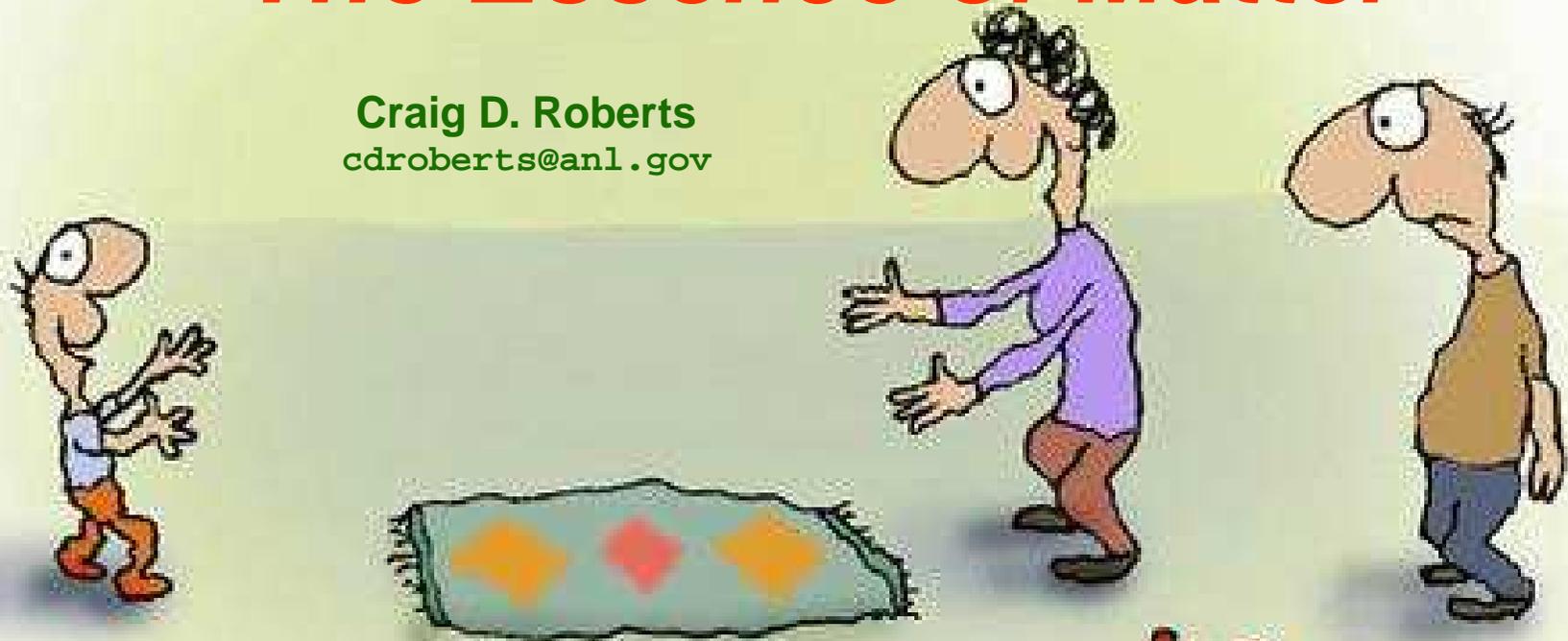


# Hadron Physics

How wonderful. He's moving  
with the times. He just took  
his first step backwards...

## The Essence of Matter

Craig D. Roberts  
cdroberts@anl.gov



Physics Division

Argonne National Laboratory

School of Physics

Peking University

<http://www.phy.anl.gov/theory/staff/cdr.html>

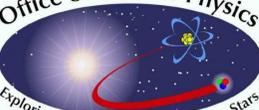
# Universal Truths



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# PRIDE AND PREJUDICE

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Colin Firth  
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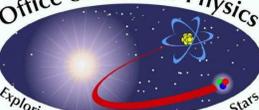
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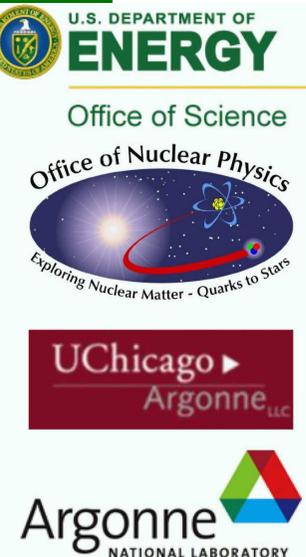
# PRIDE AND PREJUDICE

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## Universal Truths

- Spectrum of excited states, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.



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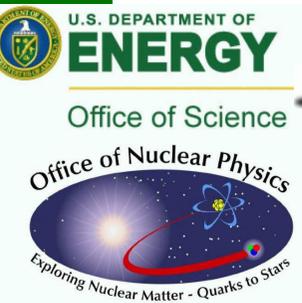
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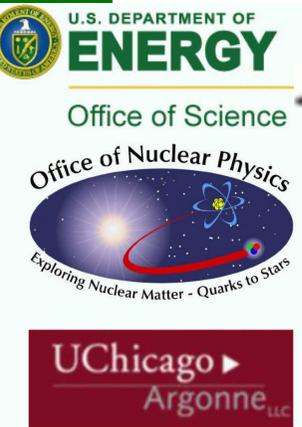
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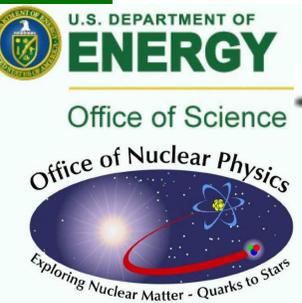
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- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.

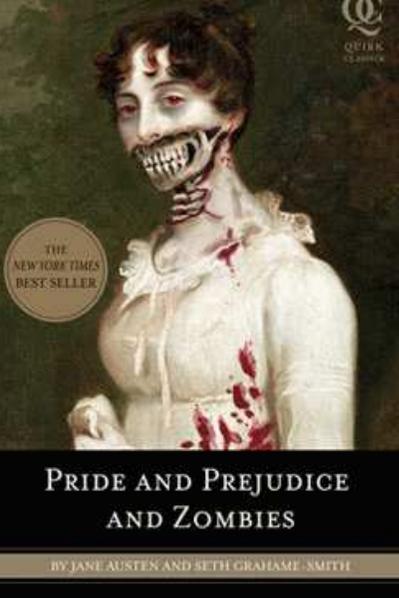


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- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe. **Higgs mechanism is irrelevant to light-quarks.**
- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. **Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.**



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# QCD's Challenges



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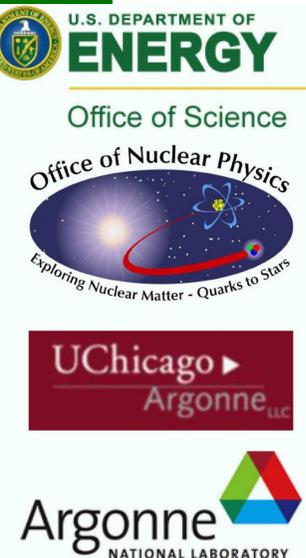
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- Quark and Gluon Confinement
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- Quark and Gluon Confinement
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  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but ... no degeneracy between  $J^{P=+}$  and  $J^{P=-}$



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- Neither of these phenomena is apparent in QCD's Lagrangian **yet** they are the dominant determining characteristics of real-world QCD.



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## Understand Emergent Phenomena

- Quark and Gluon Confinement
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- Neither of these phenomena is apparent in QCD's Lagrangian **yet** they are the dominant determining characteristics of real-world QCD.
- QCD – Complex behaviour  
arises from apparently simple rules



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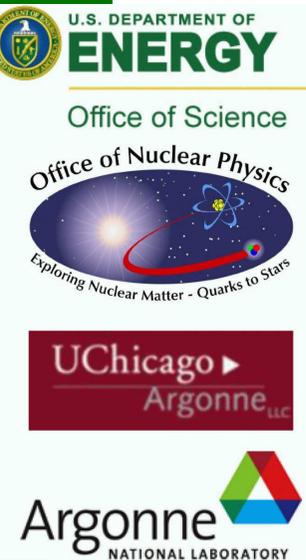
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# *Dichotomy of Pion*

## *– Goldstone Mode and Bound state*

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# Dichotomy of Pion

## – Goldstone Mode and Bound state

- How does one make an almost massless particle ..... from two massive constituent-quarks?



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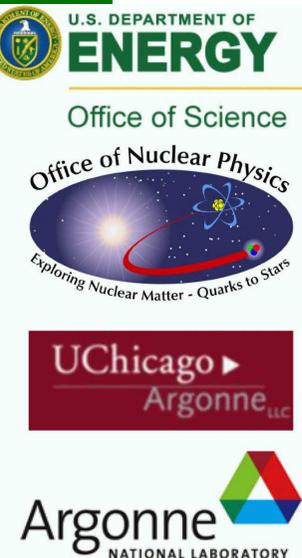
# Dichotomy of Pion

## – Goldstone Mode and Bound state

- How does one make an almost massless particle ..... from two massive constituent-quarks?
- **Not Allowed** to do it by fine-tuning a potential

Must exhibit  $m_\pi^2 \propto m_q$

Current Algebra ... 1968



# Dichotomy of Pion

## – Goldstone Mode and Bound state



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Must exhibit  $m_{\pi}^2 \propto m_q$

Current Algebra ... 1968

The **correct understanding** of pion observables; e.g. **mass**, **decay constant** and **form factors**, **requires** an approach to contain a

- **well-defined** and **valid chiral limit**;
- and an **accurate realisation** of dynamical chiral symmetry breaking.



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# Dichotomy of Pion

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**Highly Nontrivial**



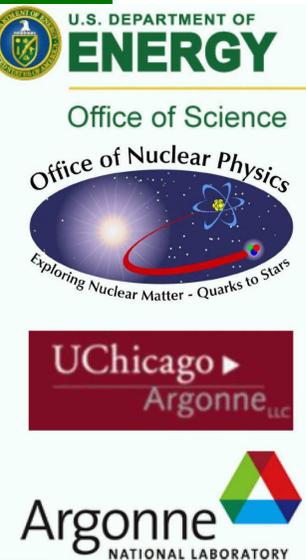
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# What's the Problem?



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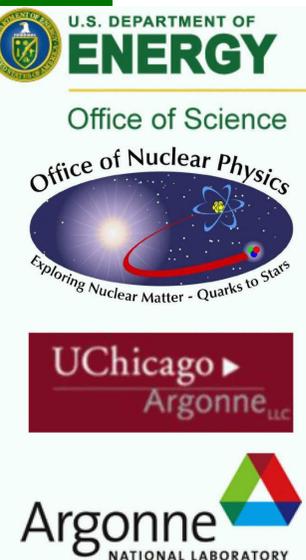
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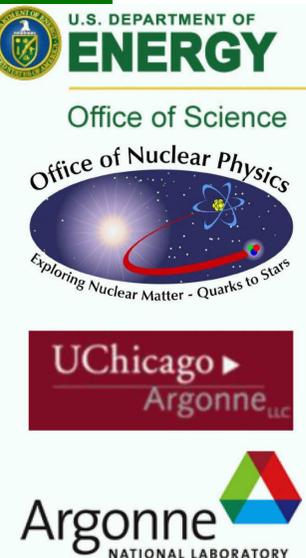
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- Minimal requirements
  - detailed understanding of connection between **Current-quark** and **Constituent-quark** masses;
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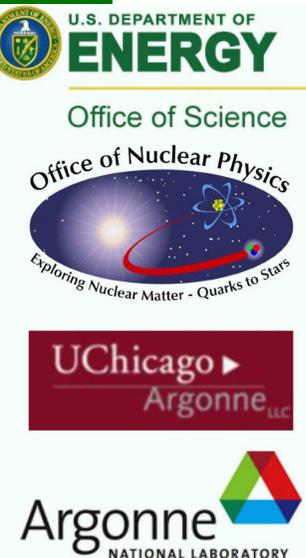
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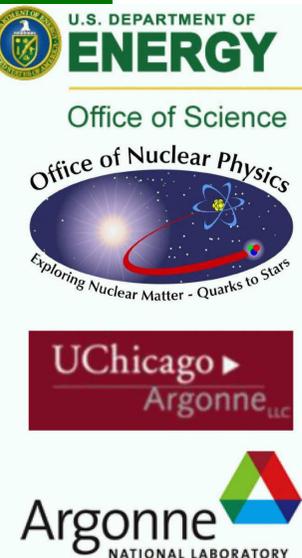
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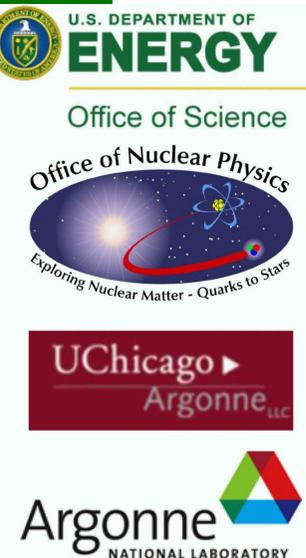
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- Differences!



# What's the Problem?

## Relativistic QFT!

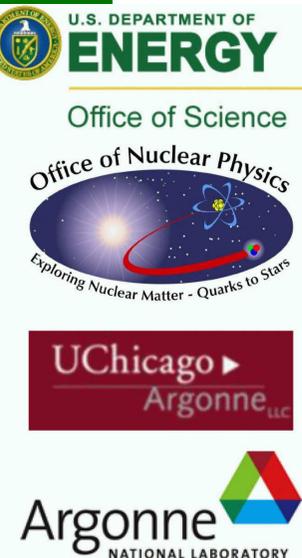
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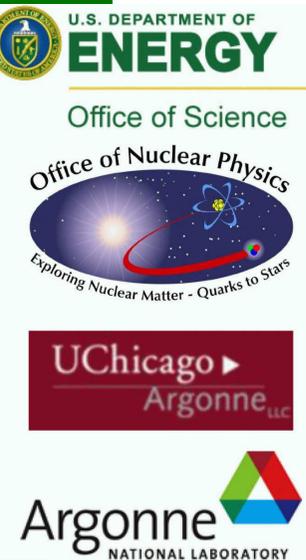
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- Differences!
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  - Interaction between quarks – the **Interquark “Potential”** – **unknown** throughout **> 98%** of a hadron's volume



# Intranucleon Interaction



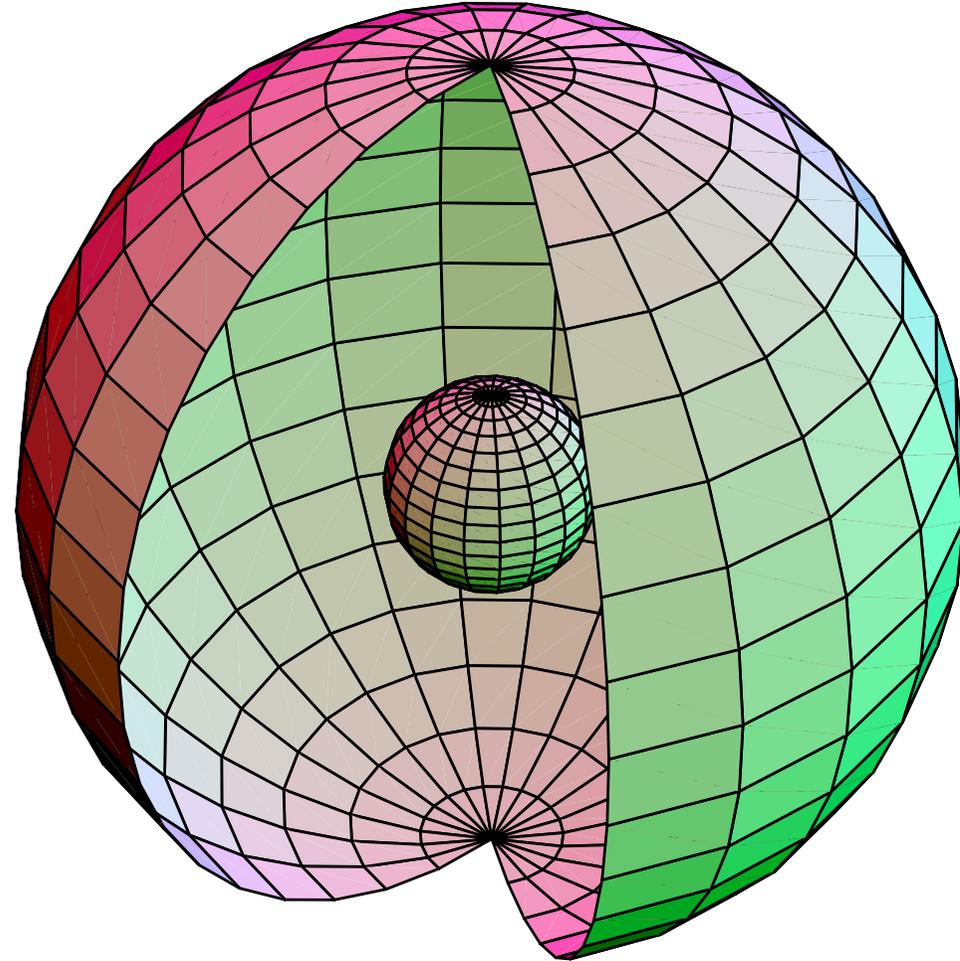
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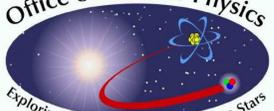
# Intranucleon Interaction



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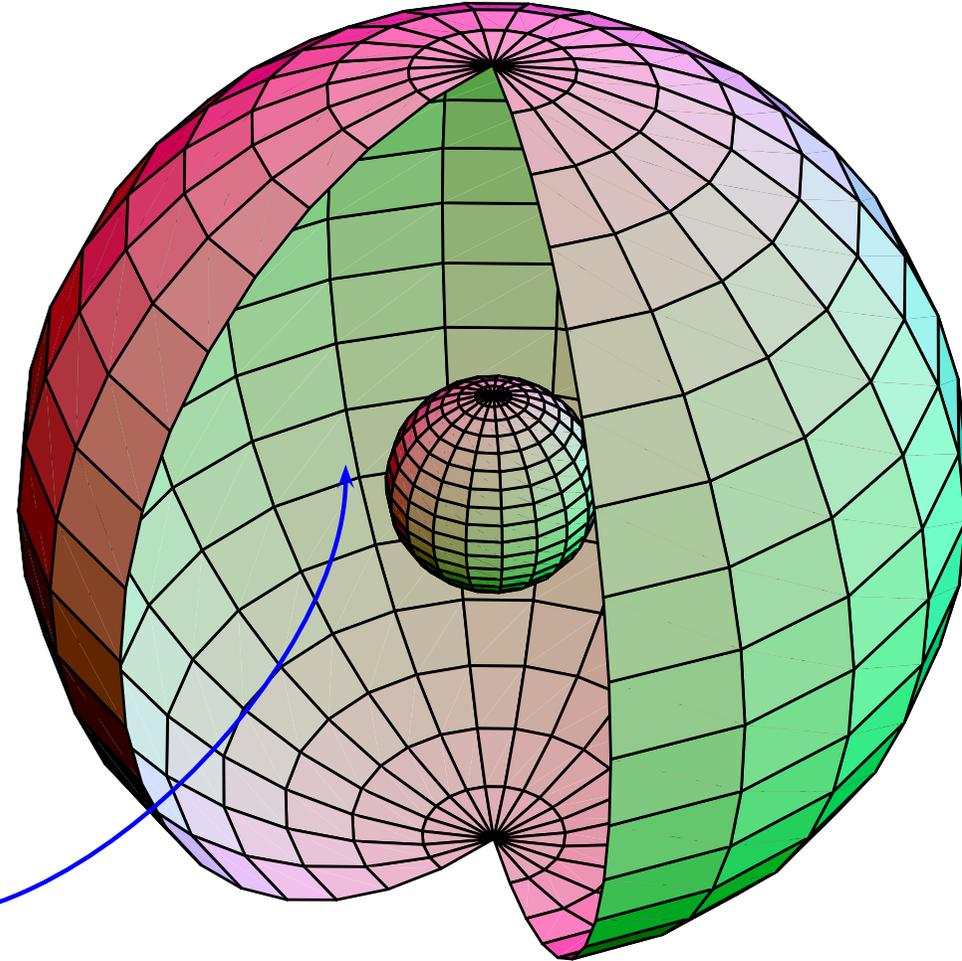
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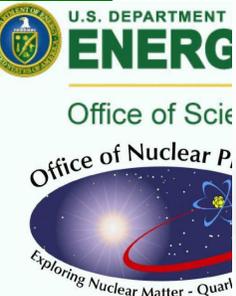
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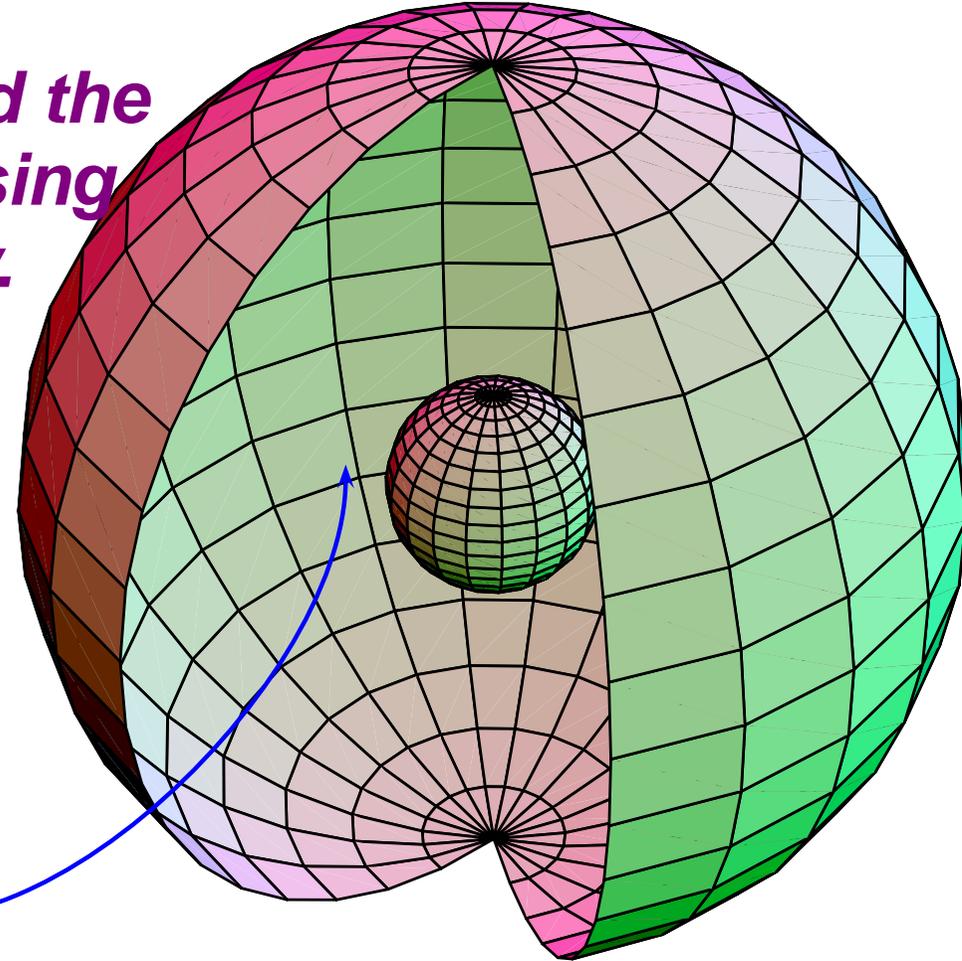


98% of the volume



# What is the Intranucleon Interaction?

*The question must be rigorously defined, and the answer mapped out using experiment and theory.*



98% of the volume



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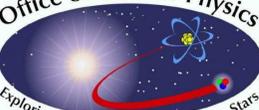
# Confinement



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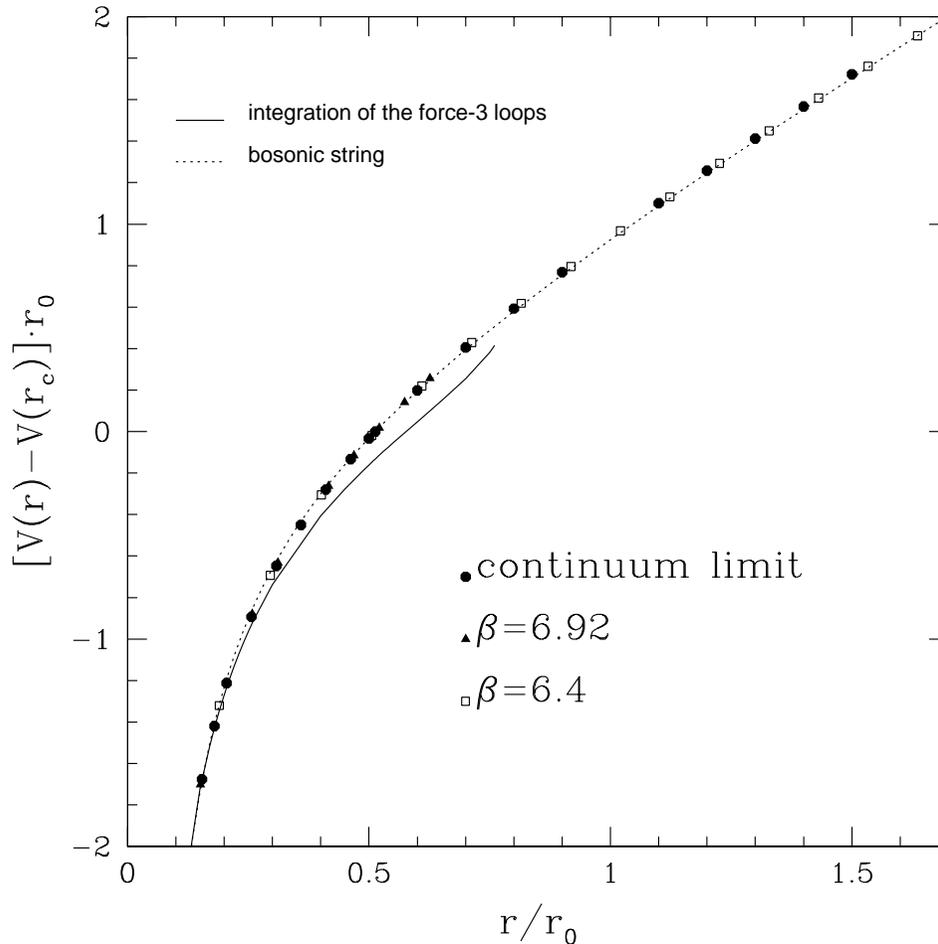
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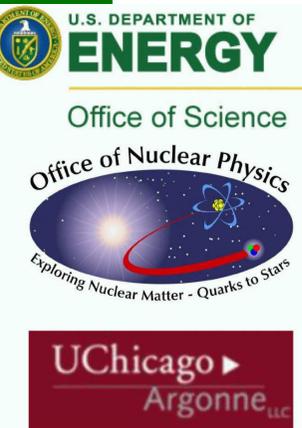
## ● Infinitely Heavy Quarks ... Picture in Quantum Mechanics



$$V(r) = \sigma r - \frac{\pi}{12} \frac{1}{r}$$

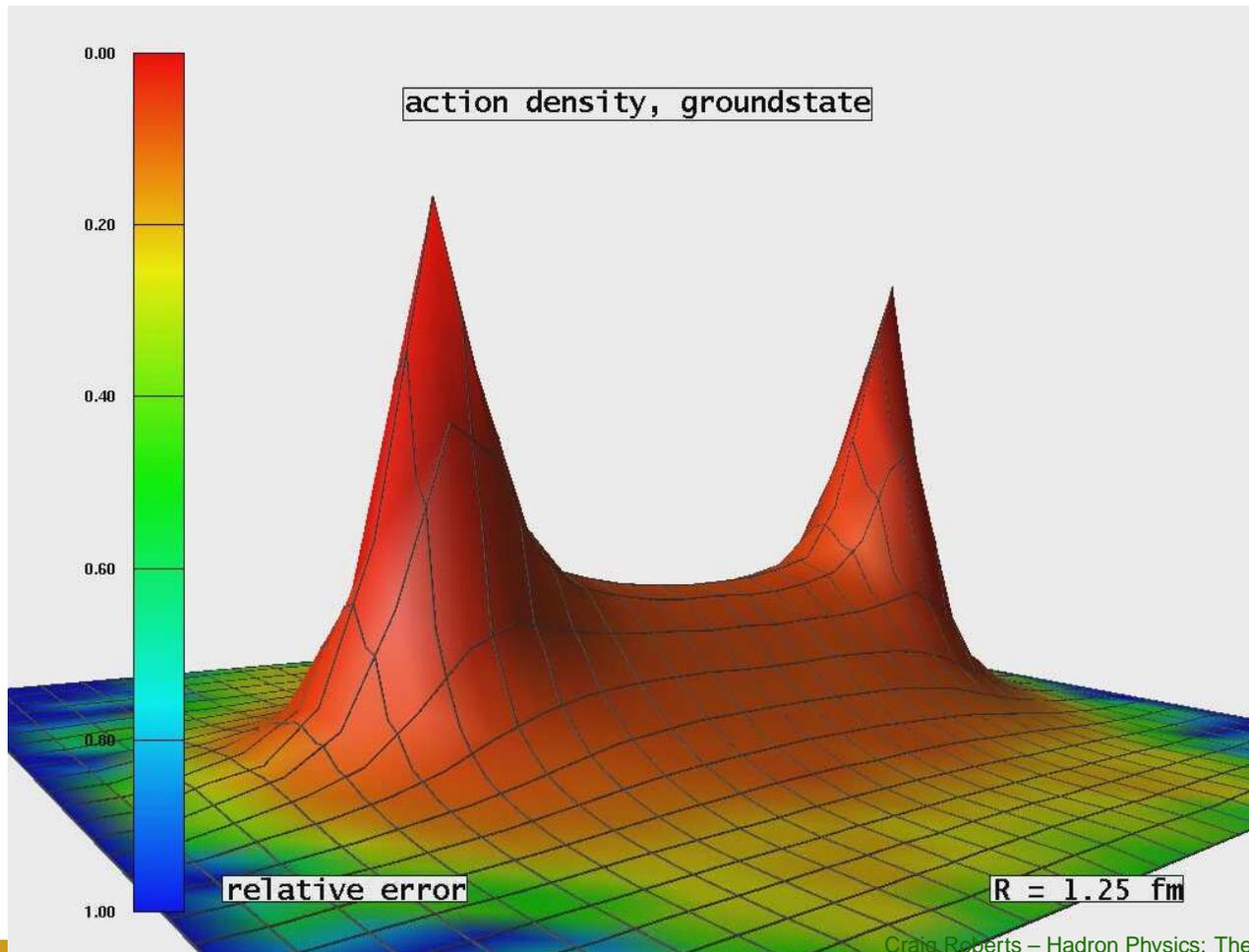
$$\sqrt{\sigma} \sim 470 \text{ MeV}$$

Necco & Sommer  
he-la/0108008



# Confinement

- Illustrate this in terms of the action density ... analogous to plotting the Force =  $F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12} \frac{1}{r^2}$



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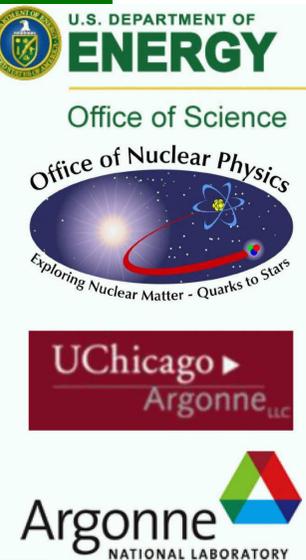
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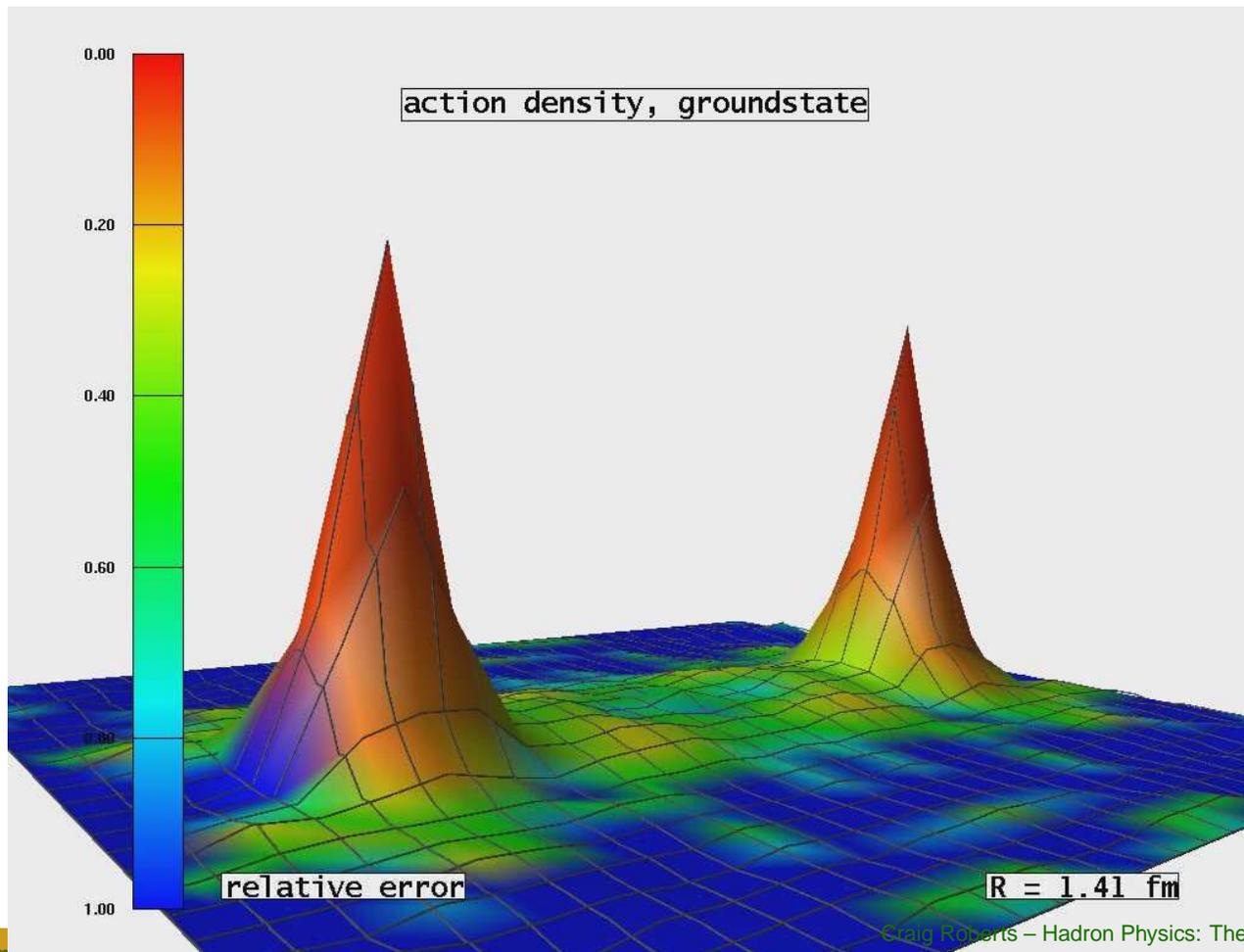
- What happens in the real world; namely, in the presence of light-quarks?



# Confinement

- What happens in the real world; namely, in the presence of light-quarks? No one knows ... but  $\bar{Q}Q + 2 \times \bar{s}s$

Bali, *et al.*  
he-la/0512018

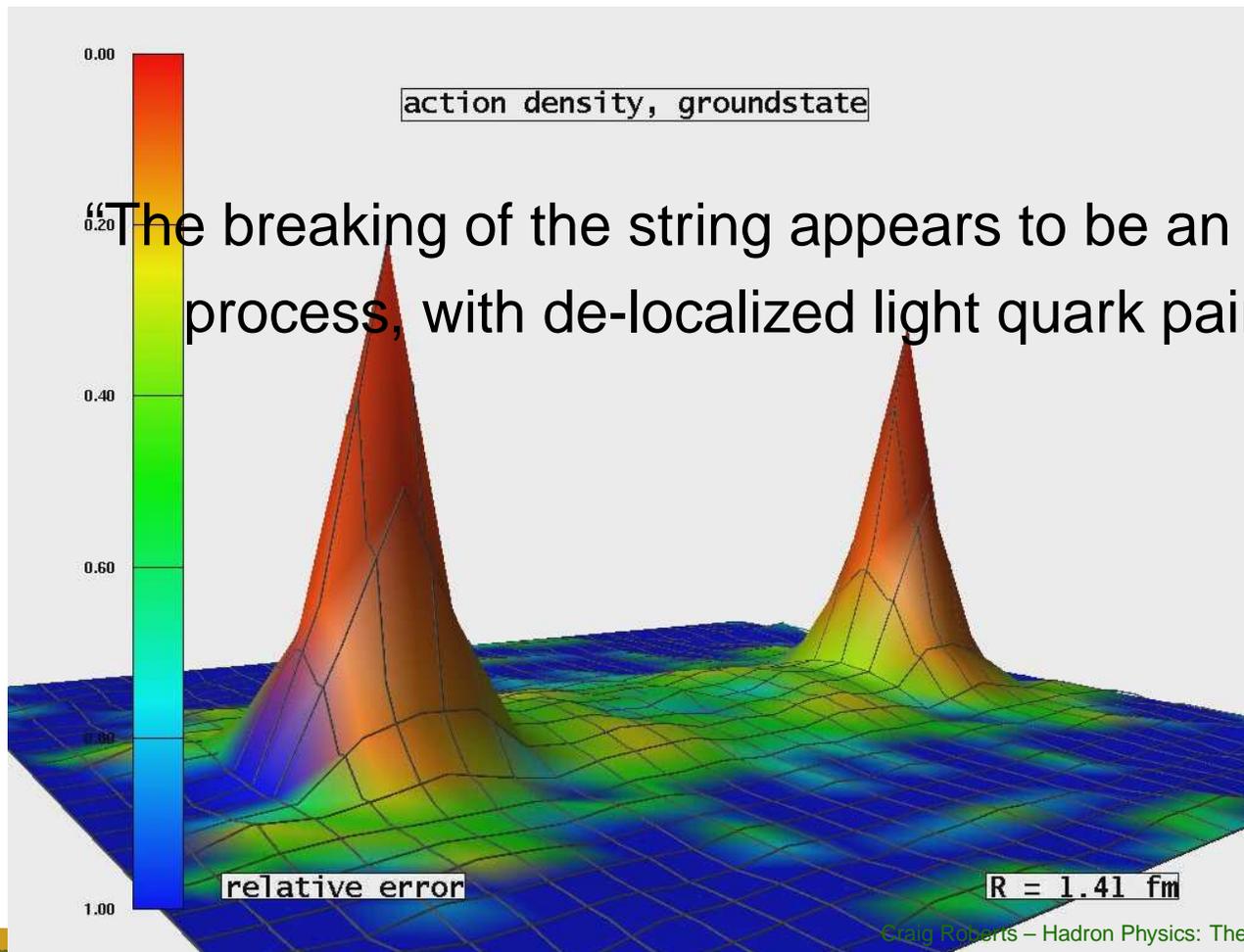


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# Confinement

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“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”

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he-la/0512018



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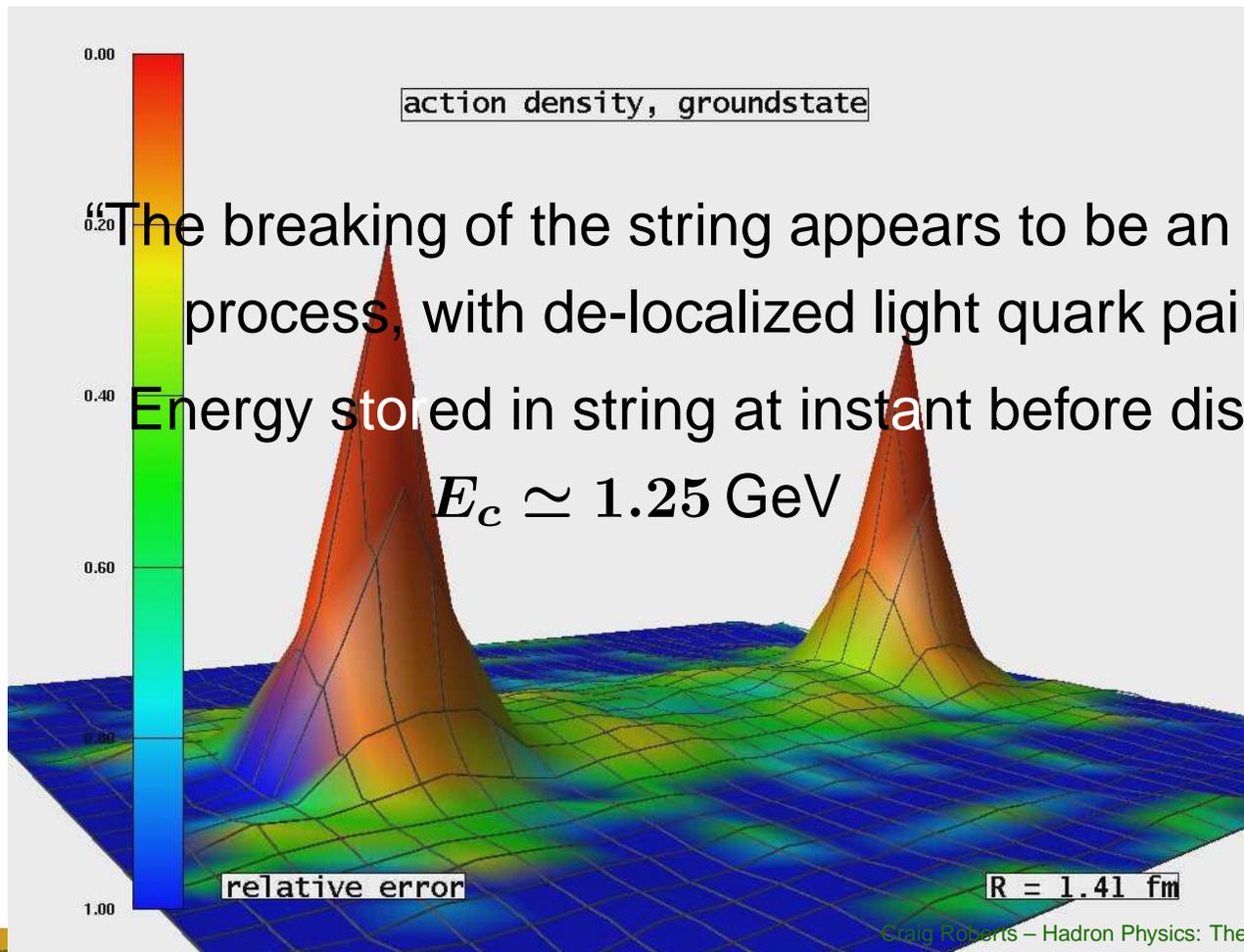
Bali, *et al.*

he-lq/0512018

“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”

Energy stored in string at instant before disappearance:

$$E_c \simeq 1.25 \text{ GeV}$$



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# Confinement

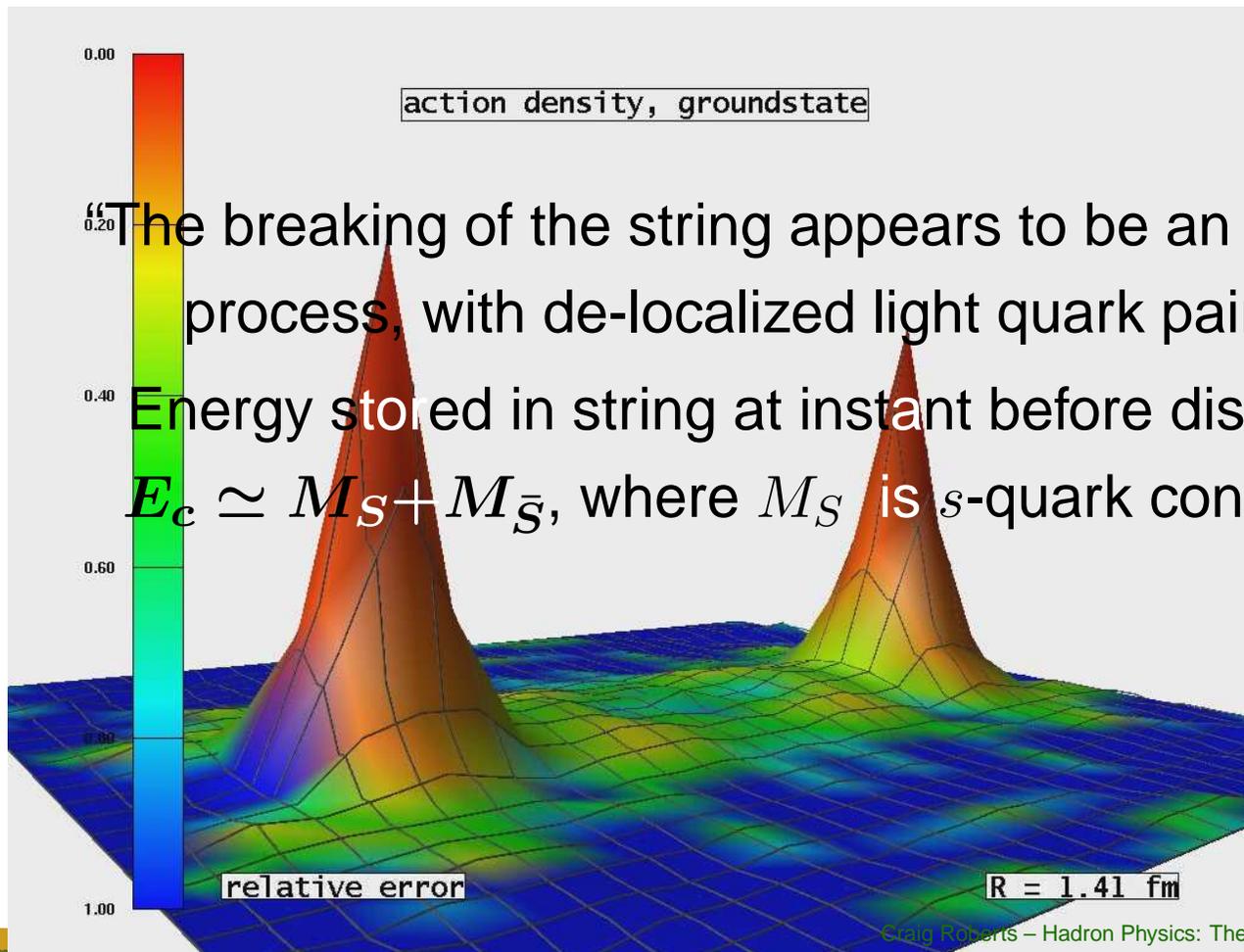
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Energy stored in string at instant before disappearance:  
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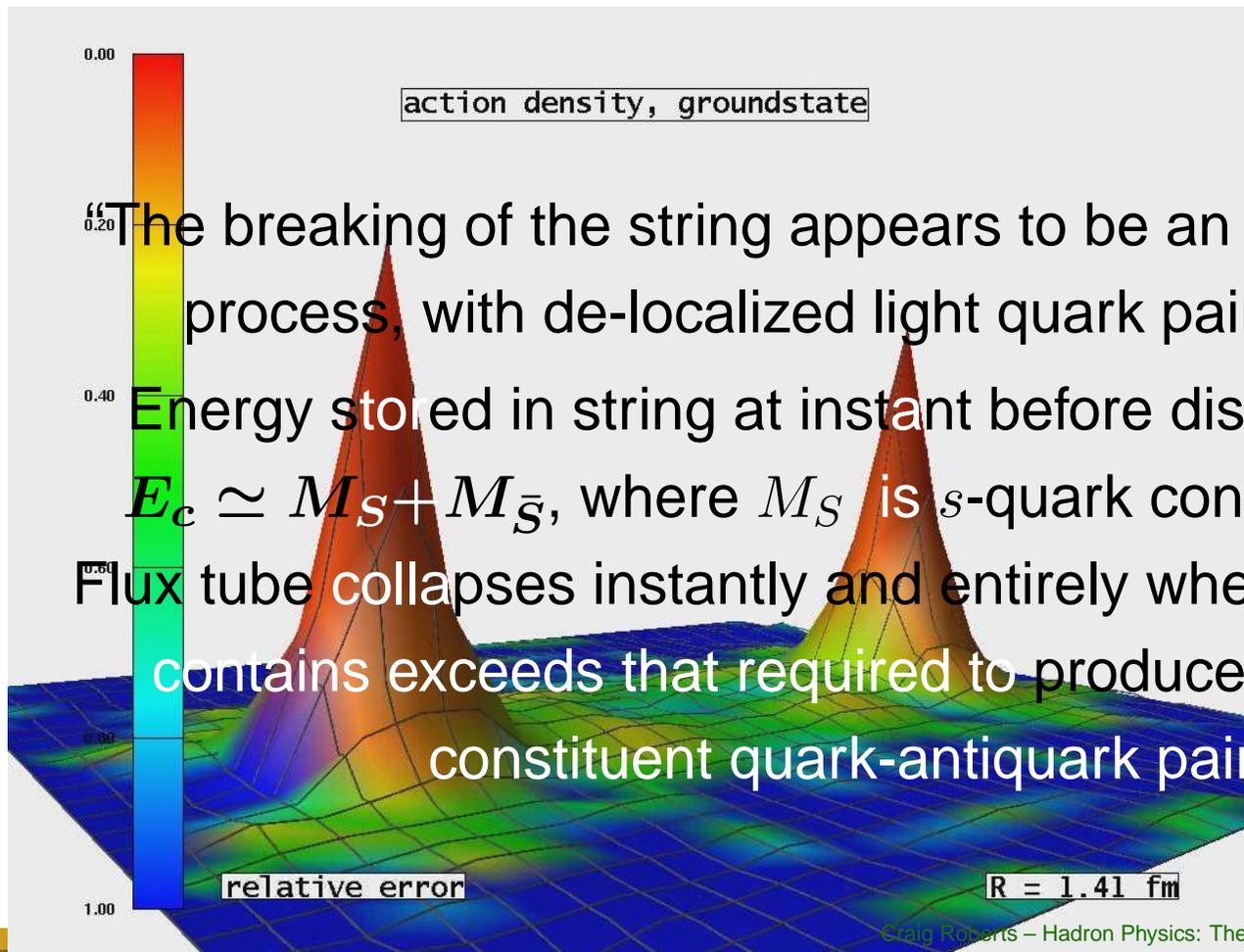


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Flux tube collapses instantly and entirely when the energy it contains exceeds that required to produce the lightest constituent quark-antiquark pair.



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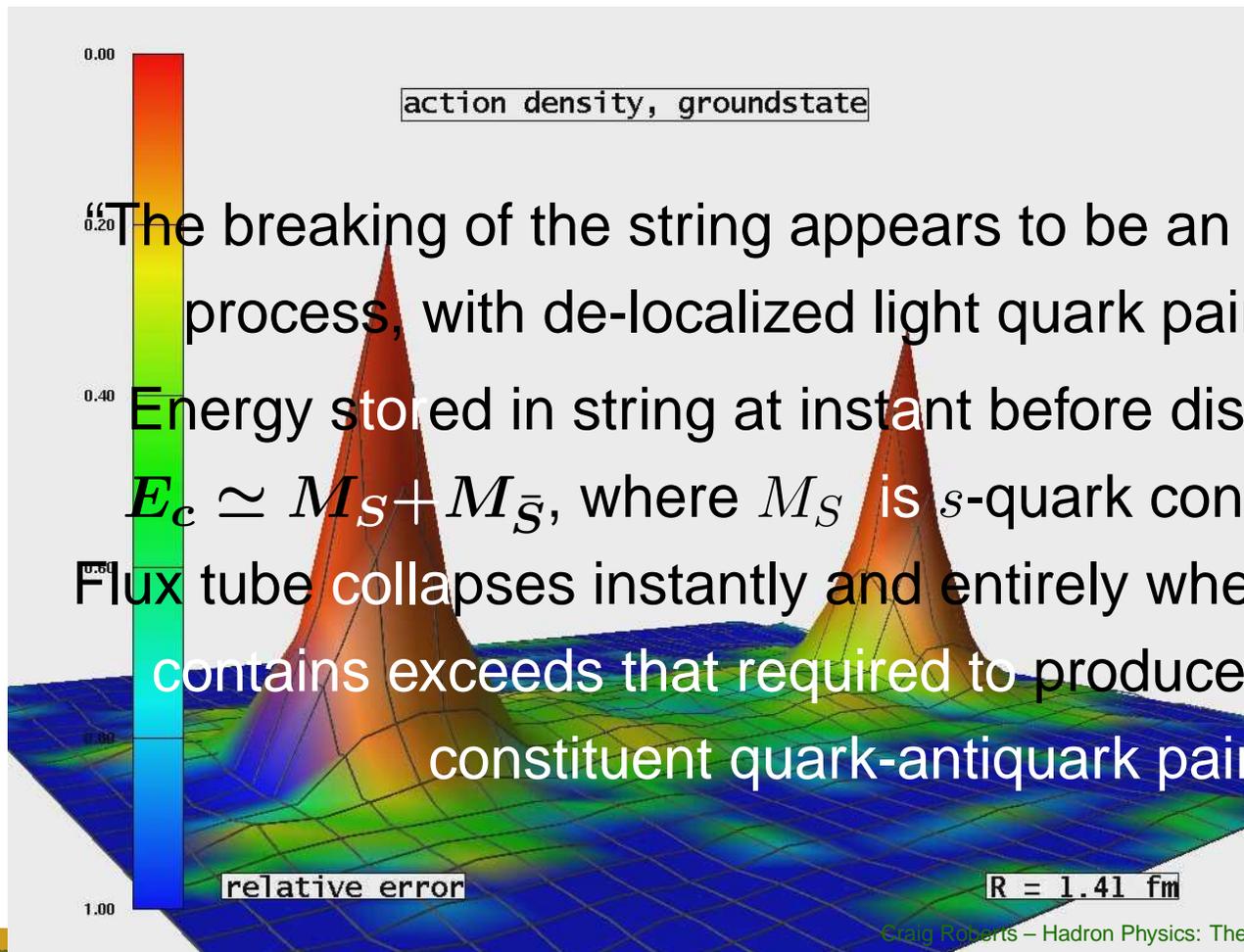


# Therefore ... No information on *potential* between light-quarks. **Confinement**

- What happens in the real world; namely, in the presence of light-quarks? No one knows ... but  $\bar{Q}Q + 2 \times \bar{s}s$

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# What is the light-quark Long-Range Potential?



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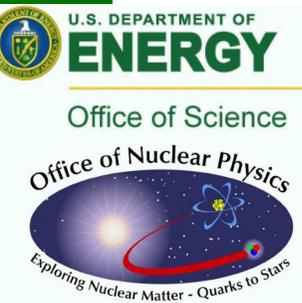
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# What is the light-quark Long-Range Potential?



Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD **is not related** in any simple way to the light-quark interaction.



# Dyson-Schwinger Equations

## *Euler-Lagrange equations for quantum field theory*

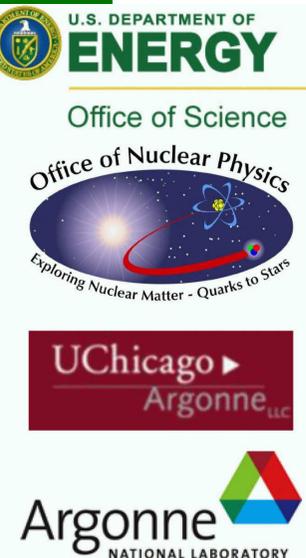
- Well suited to Relativistic Quantum Field Theory



# Dyson-Schwinger Equations

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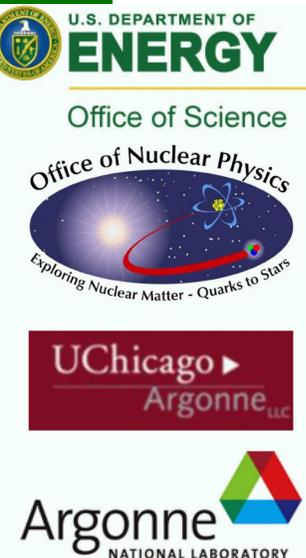
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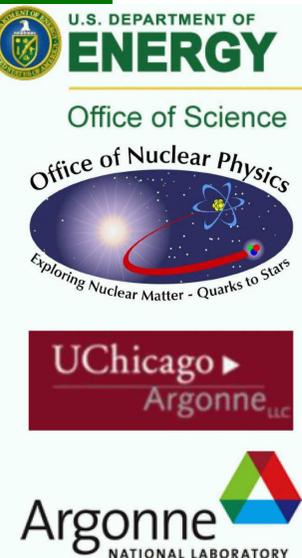
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- **NonPerturbative, Continuum approach to QCD**



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## Euler-Lagrange equations for quantum field theory

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- Simplest level: **Generating Tool for Perturbation Theory**  
..... **Materially Reduces** Model Dependence
- **NonPerturbative, Continuum approach to QCD**
  - Hadrons as Composites of **Quarks** and **Gluons**
  - Qualitative and Quantitative Importance of:
    - Dynamical Chiral Symmetry Breaking
      - Generation of fermion mass from *nothing*
    - **Quark & Gluon Confinement**
      - Coloured objects not detected, not detectable?



# Dyson-Schwinger Equations

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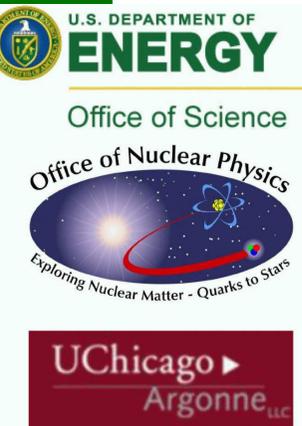


# Dyson-Schwinger Equations

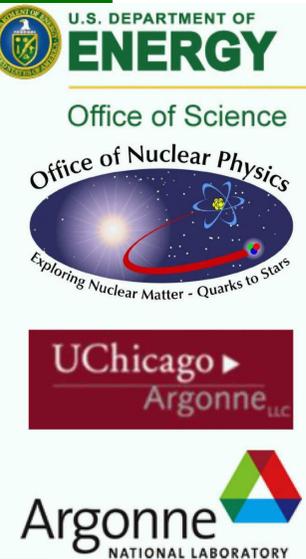
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- Method yields Schwinger Functions  $\equiv$  Propagators

**Cross-Sections built from Schwinger Functions**



# Schwinger Functions



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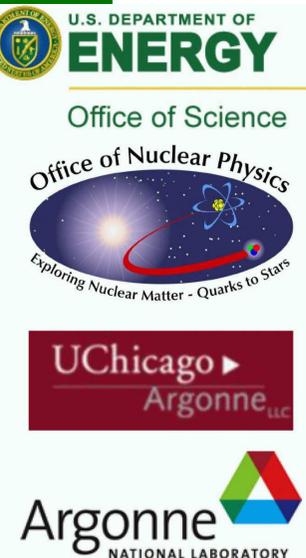
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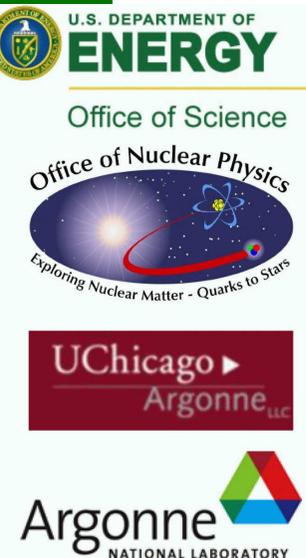
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- Solutions are Schwinger Functions  
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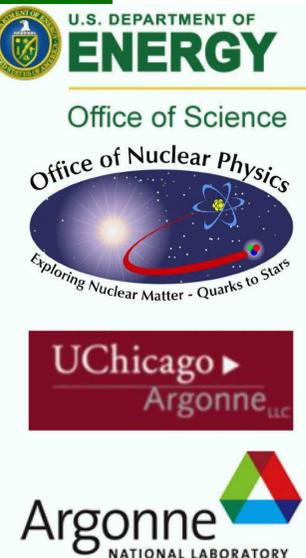
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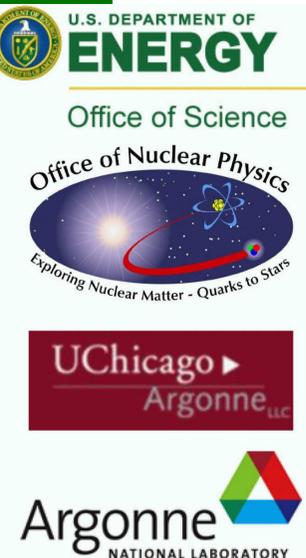
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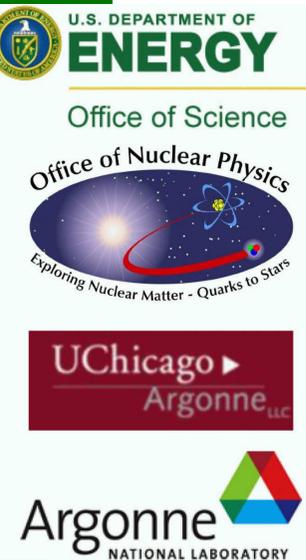


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- Proving fruitful.



# Charting the Interaction between light-quarks



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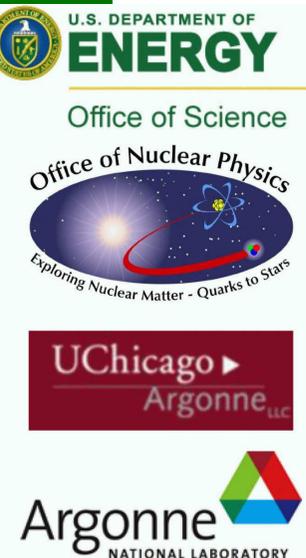
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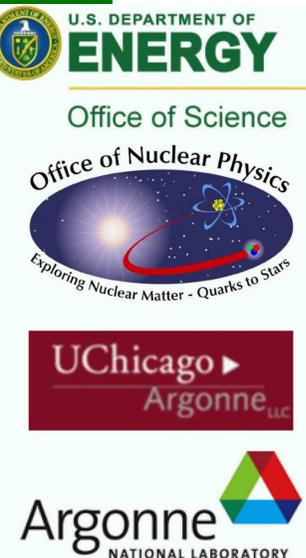
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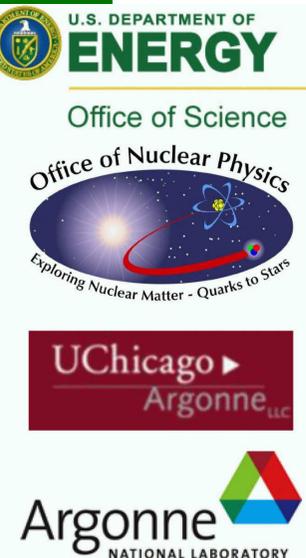
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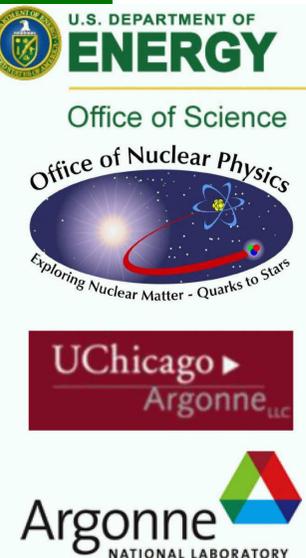
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Of course, the behaviour of the  $\beta$ -function on the perturbative domain is well known.

- This is a well-posed problem whose solution is an elemental goal of modern hadron physics.



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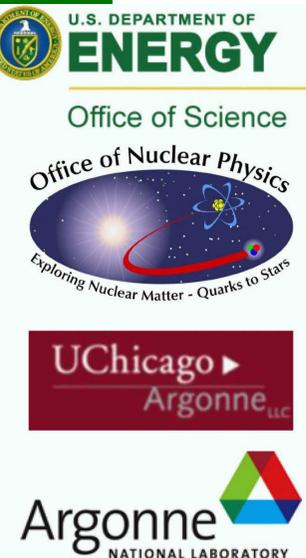
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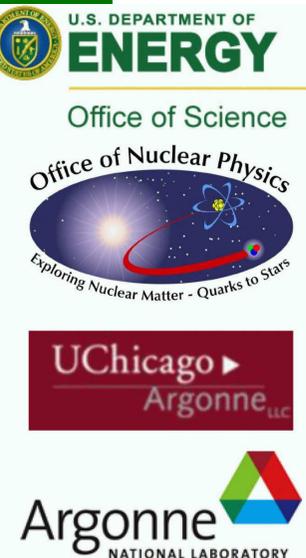
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- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
  - Steady quantitative progress is being made with a scheme that is systematically improvable



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# Charting the Interaction between light-quarks

- Through DSEs the pointwise behaviour of the  $\beta$ -function determines pattern of chiral symmetry breaking
- DSEs connect  $\beta$ -function to experimental observables. Hence, comparison between computations and observations of, e.g., hadron mass spectrum can be used to chart  $\beta$ -function's long-range behaviour
- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
  - On other hand, at present significant qualitative advances possible with symmetry-preserving kernel *Ansätze* that express important additional nonperturbative effects –  $M(p^2)$  – difficult/impossible to capture in any finite sum of contributions



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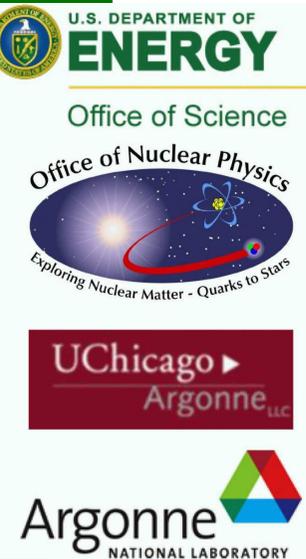
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# *Perturbative Dressed-quark Propagator*



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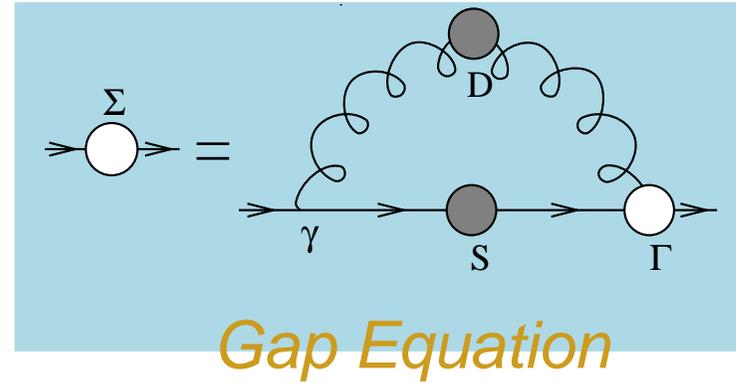
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# Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



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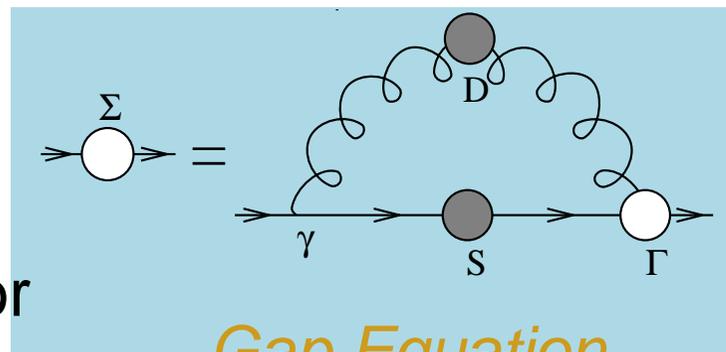
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- dressed-quark propagator



Gap Equation

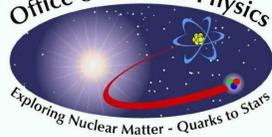
$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$



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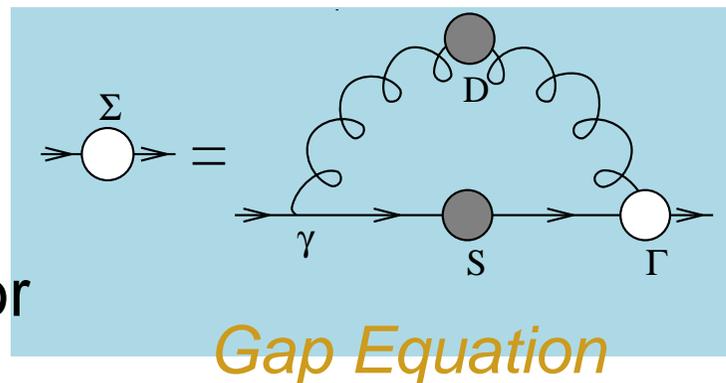
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- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

- Weak Coupling Expansion  
Reproduces **Every** Diagram in **Perturbation Theory**



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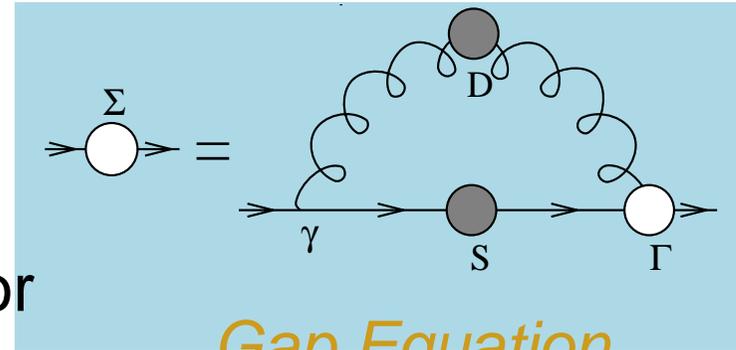


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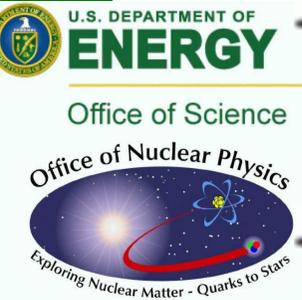
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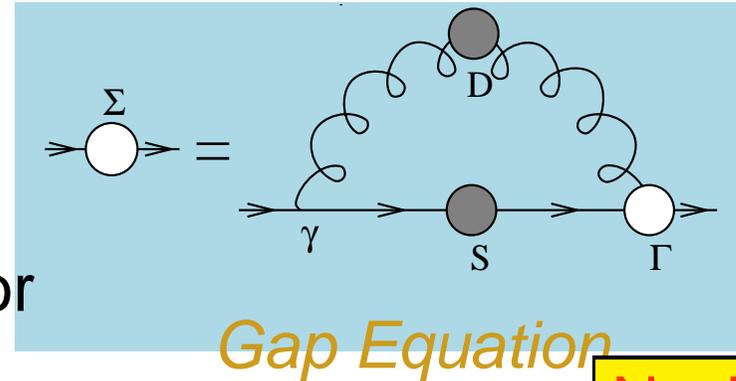
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$$B(p^2) = m \left( 1 - \frac{\alpha}{\pi} \ln \left[ \frac{p^2}{m^2} \right] + \dots \right) \xrightarrow{m \rightarrow 0} 0$$



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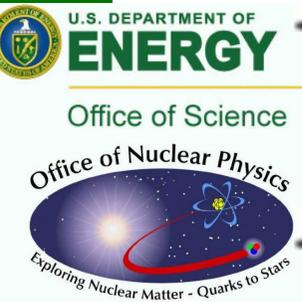
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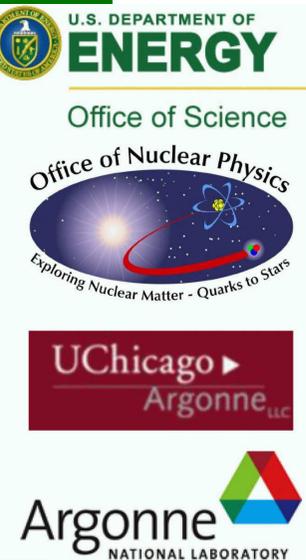
No DCSB  
Here!

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# Frontiers of Nuclear Science: A Long Range Plan (2007)



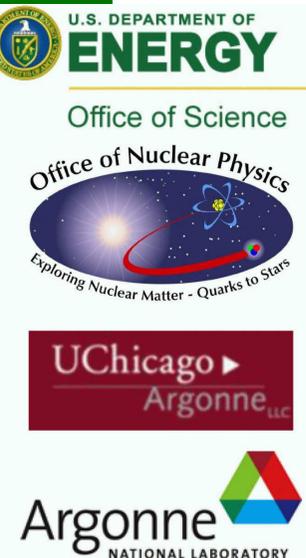
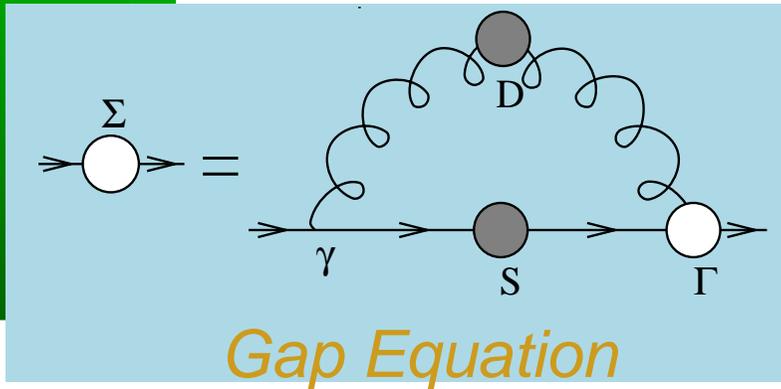
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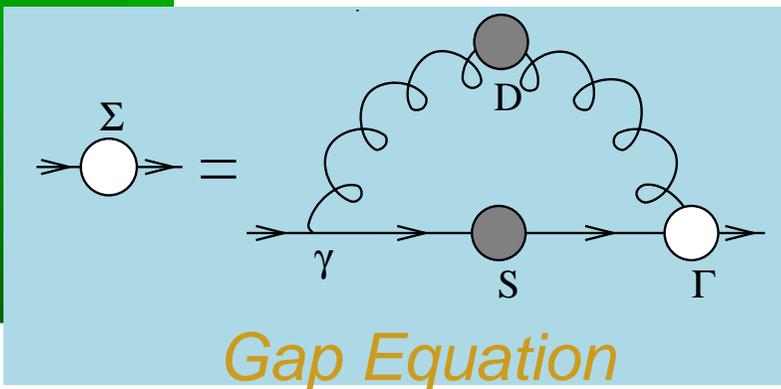
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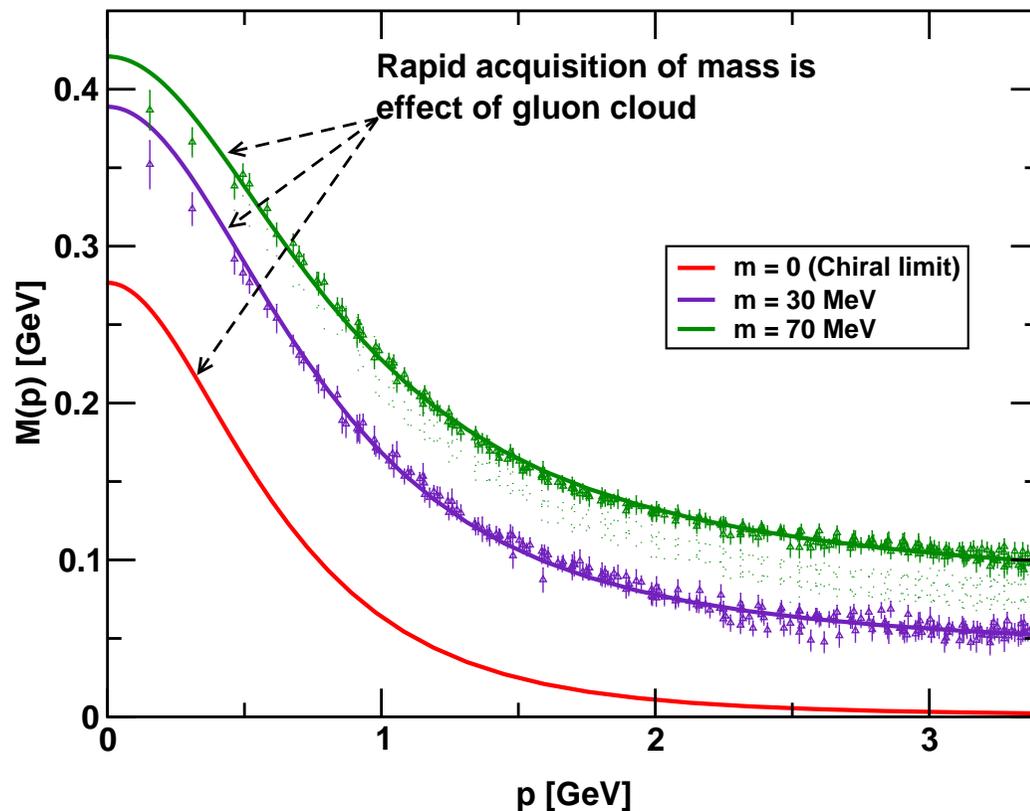
# Frontiers of Nuclear Science: Theoretical Advances



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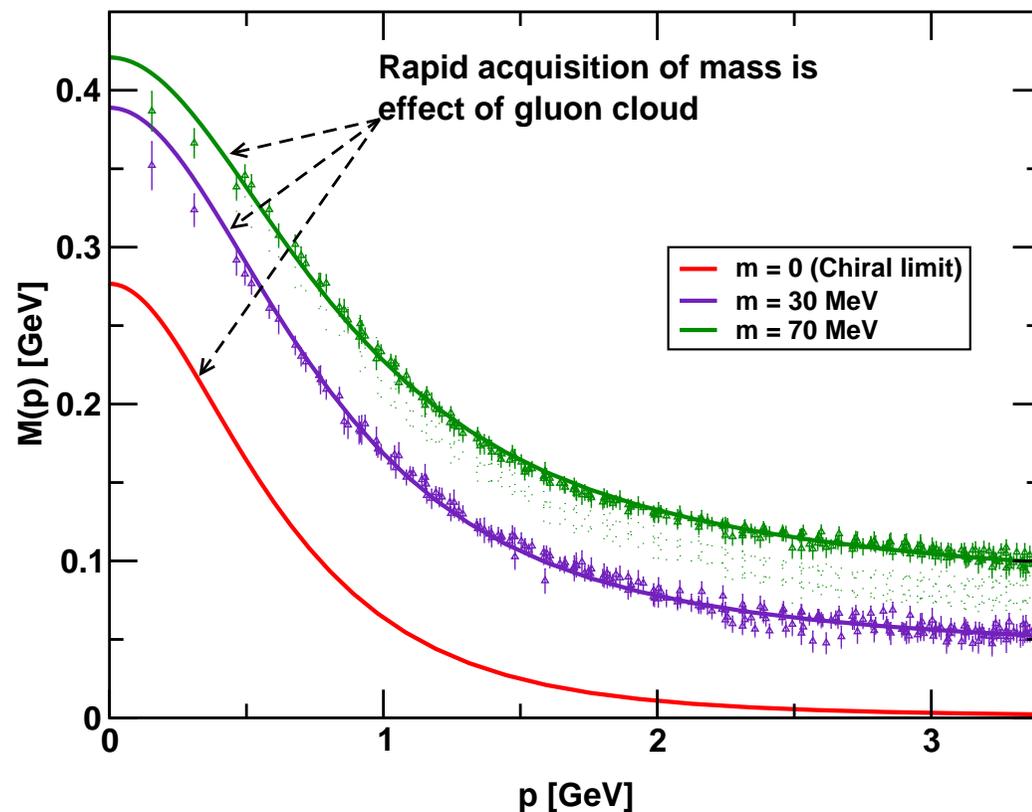
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## Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ( $m = 0$ , red curve) acquires a large constituent mass at low energies.

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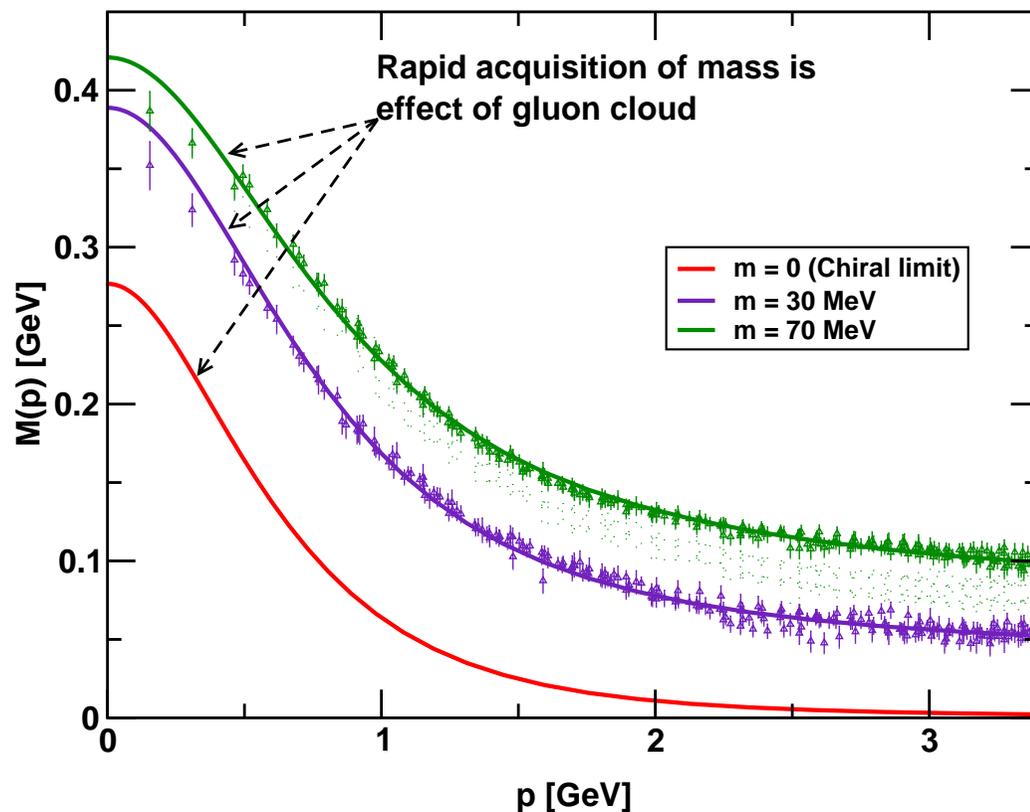
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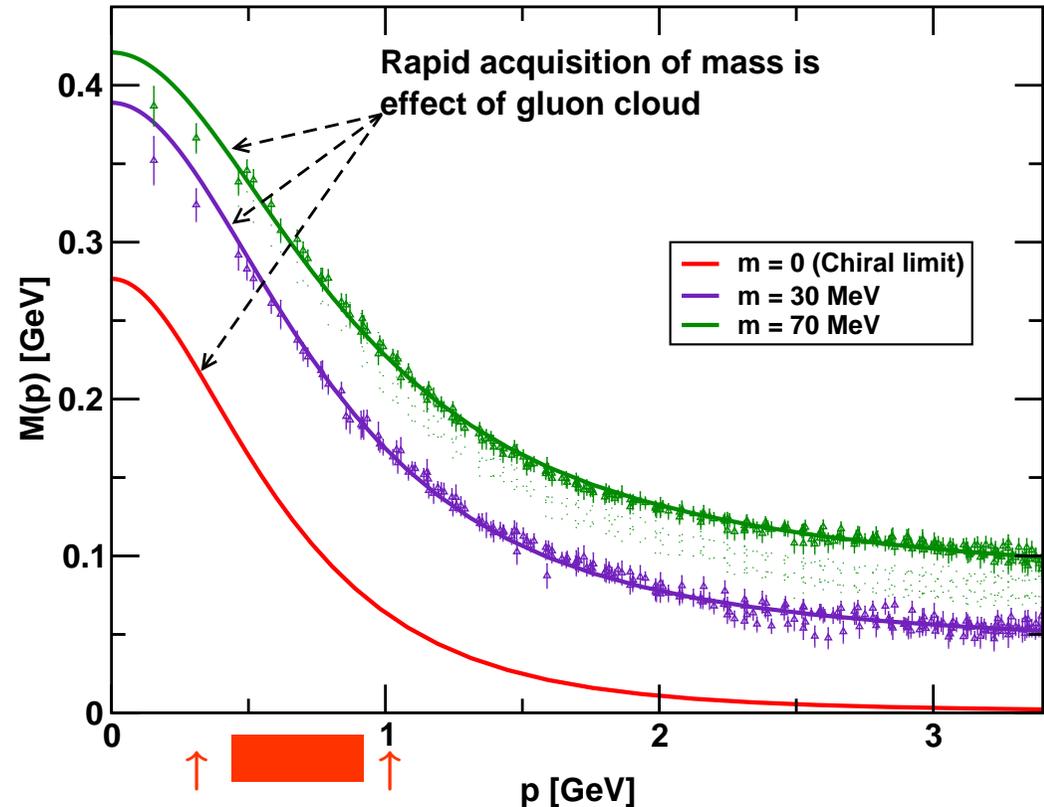
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Scanned by  $Q^2 \in [2, 9] \text{ GeV}^2$  Baryon Form Factors

Craig Roberts – Hadron Physics: The Essence of Matter

XII Mexican Workshop on Particles and Fields, 9-14 Nov. 2009... 45 – p. 14/50



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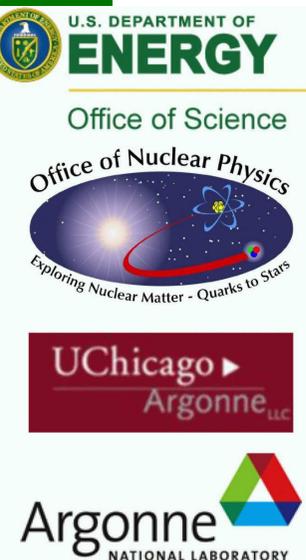
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# Frontiers of Nuclear Science: Theoretical Advances

In QCD  
a quark's mass must depend on  
its momentum

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- Established understanding of two- and three-point functions



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# Hadrons



- Established understanding of two- and three-point functions
- What about bound states?



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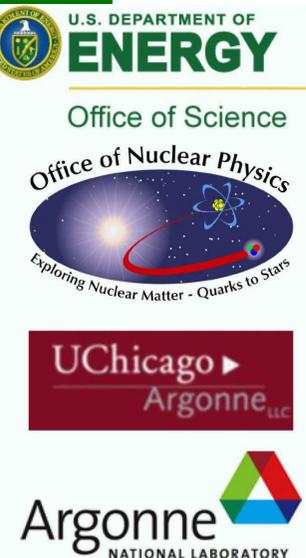
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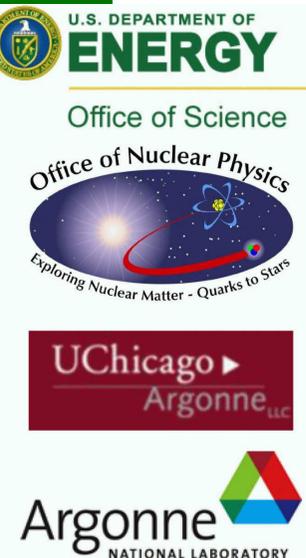
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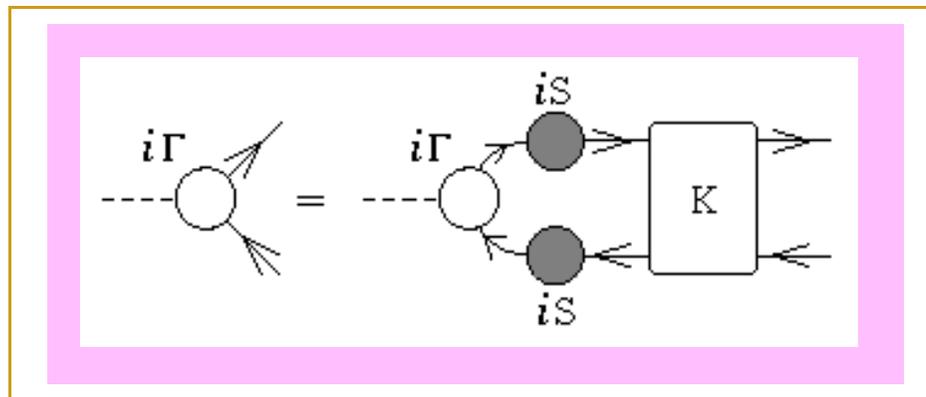
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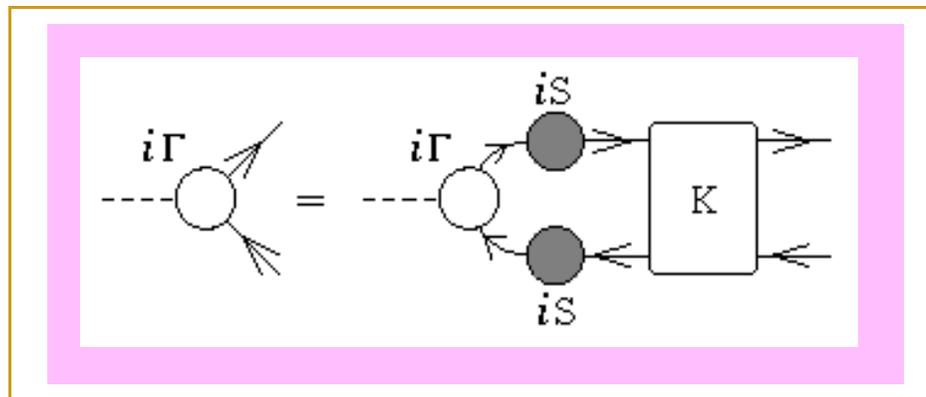


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- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

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QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel,  $K$ ?
- or What is the **long-range** potential in QCD?

# Bethe-Salpeter Kernel



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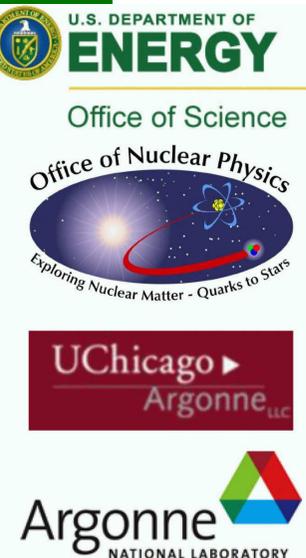
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# Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^l(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f^l i\gamma_5 + \frac{1}{2} \lambda_f^l i\gamma_5 \mathcal{S}^{-1}(k_-) \\ - M_\zeta i\Gamma_5^l(k; P) - i\Gamma_5^l(k; P) M_\zeta$$

## QFT Statement of Chiral Symmetry



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Satisfies BSE

Satisfies DSE



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- Relation **must** be preserved by truncation
- **Nontrivial** constraint



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- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^l(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f^l i\gamma_5 + \frac{1}{2} \lambda_f^l i\gamma_5 \mathcal{S}^{-1}(k_-) - M_\zeta i\Gamma_5^l(k; P) - i\Gamma_5^l(k; P) M_\zeta$$

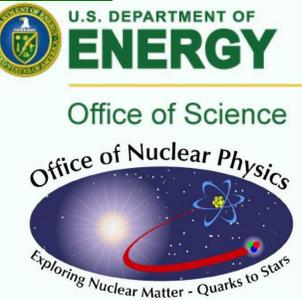
Satisfies BSE

Satisfies DSE

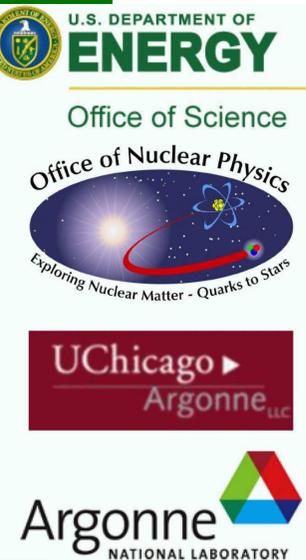
Kernels very different

but must be *intimately* related

- Relation **must** be preserved by truncation
- **Failure**  $\Rightarrow$  Explicit Violation of QCD's Chiral Symmetry



# Persistent Challenge



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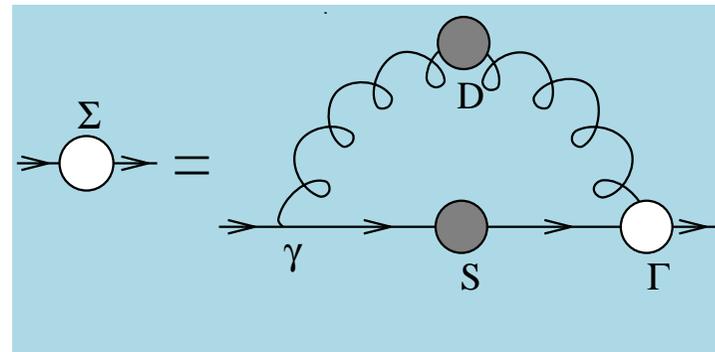
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# Persistent Challenge

- Infinitely Many Coupled Equations



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Exploring Nuclear Matter - Quarks to Stars

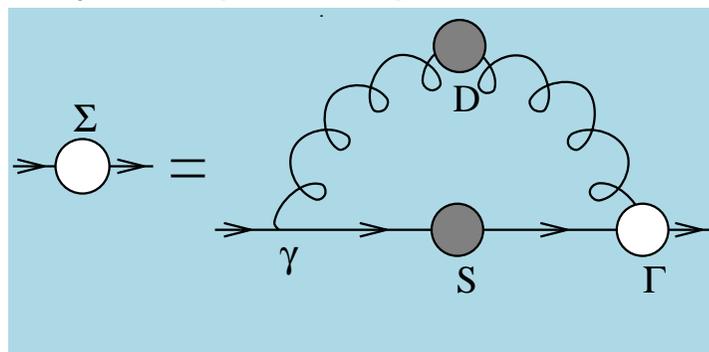
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# Persistent Challenge

- Infinitely Many Coupled Equations



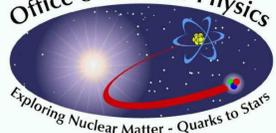
- Coupling between equations **necessitates** truncation



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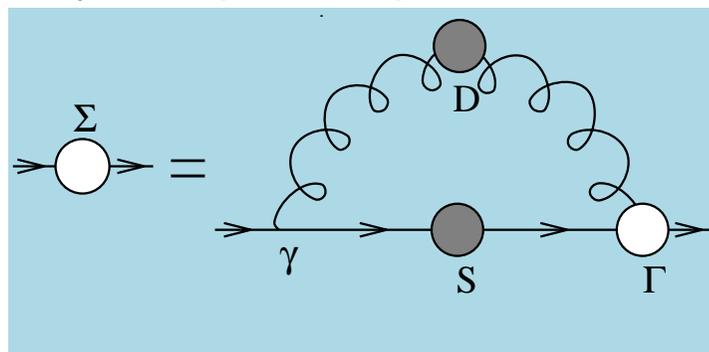
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# Persistent Challenge

- Infinitely Many Coupled Equations



- Coupling between equations **necessitates** truncation
  - Weak coupling expansion  $\Rightarrow$  Perturbation Theory



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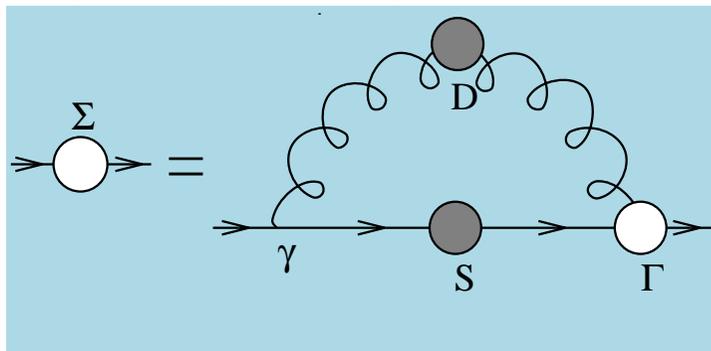
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# Persistent Challenge

- Infinitely Many Coupled Equations



- Coupling between equations **necessitates** truncation
  - Weak coupling expansion  $\Rightarrow$  Perturbation Theory  
**Not useful** for the nonperturbative problems in which we're interested



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# Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme

H.J. Munczek Phys. Rev. D **52** (1995) 4736

*Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*

A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7

*Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*



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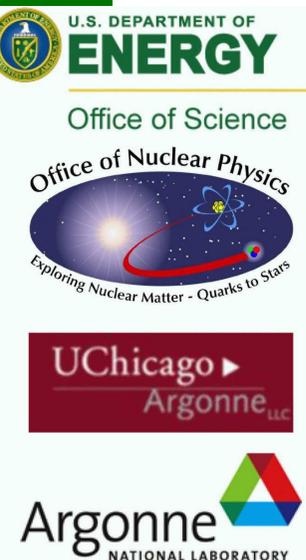
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# Persistent Challenge

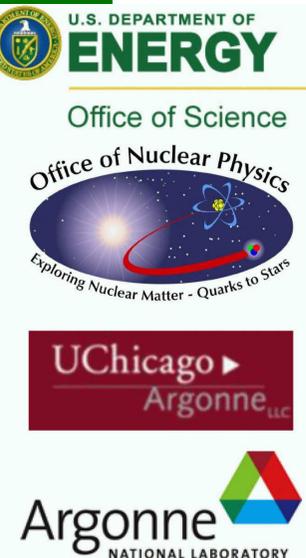
- Infinitely Many Coupled Equations
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- Has Enabled Proof of **EXACT** Results in QCD





# Persistent Challenge

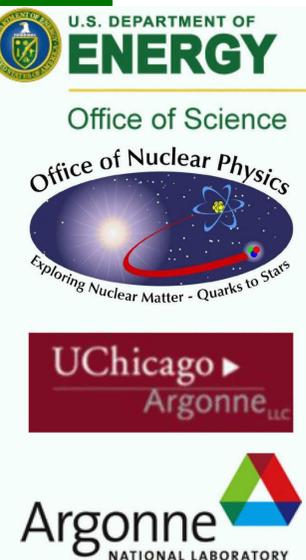
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- And Formulation of Practical Phenomenological Tool to
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# Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme
- Has Enabled Proof of **EXACT** Results in QCD
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# Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme
- Has Enabled Proof of **EXACT** Results in QCD
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  - Make Predictions with Readily Quantifiable Errors
- Examples:

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[web.mit.edu/readingtn/www/netadv/Xdysonschw.html](http://web.mit.edu/readingtn/www/netadv/Xdysonschw.html)



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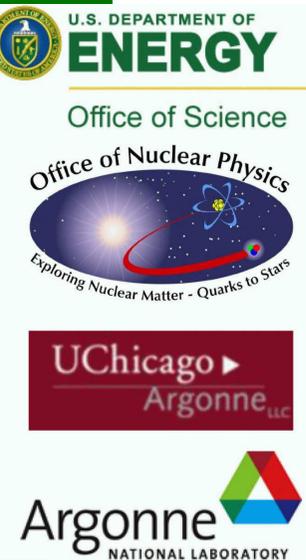
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# Radial Excitations & Chiral Symmetry



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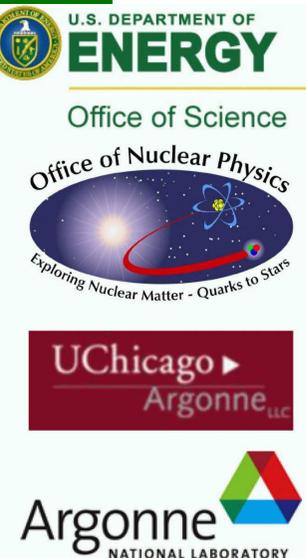
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# Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy  
nu-th/9707003 )

$$f_H m_H^2 = - \rho_{\zeta}^H \mathcal{M}_H$$

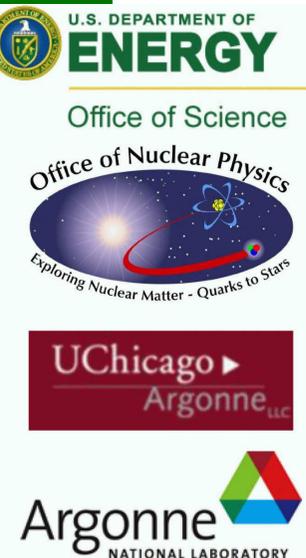


# Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy  
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$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

- Mass<sup>2</sup> of pseudoscalar hadron



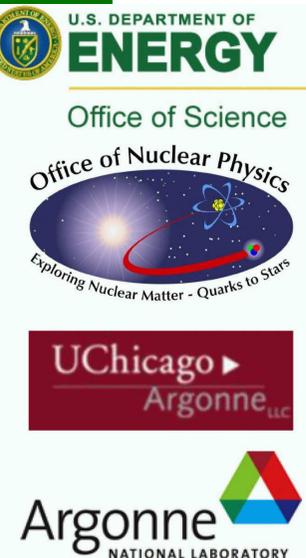
# Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy  
nu-th/9707003)

$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

$$\mathcal{M}_H := \text{tr}_{\text{flavour}} \left[ M_{(\mu)} \left\{ T^H, (T^H)^t \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g.,  $T^{K^+} = \frac{1}{2} (\lambda^4 + i\lambda^5)$



# Radial Excitations & Chiral Symmetry

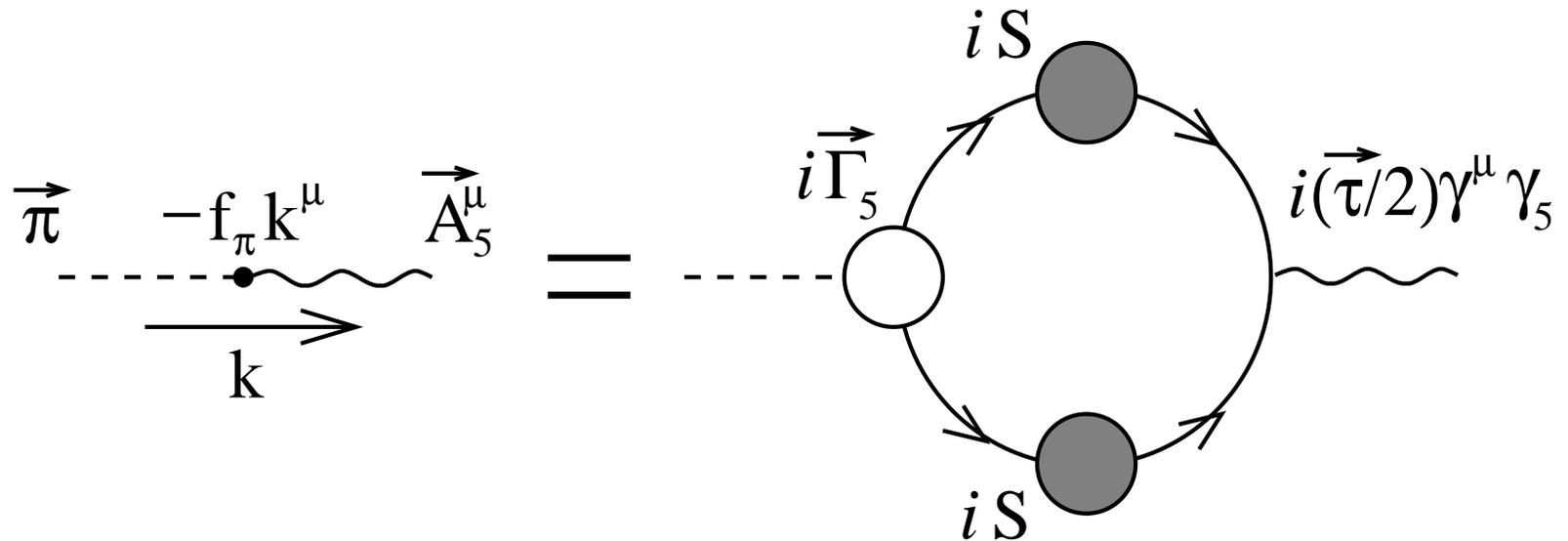
(Maris, Roberts, Tandy  
nu-th/9707003)

$$\langle 0 | \bar{q} \gamma_5 \gamma_\mu q | \pi \rangle$$

$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

$$f_H p_\mu = Z_2 \int_q^\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 \gamma_\mu \mathcal{S}(q_+) \Gamma_H(q; P) \mathcal{S}(q_-) \right\}$$

- Pseudovector projection of BS wave function at  $x = 0$
- Pseudoscalar meson's leptonic decay constant



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# Radial Excitations & Chiral Symmetry

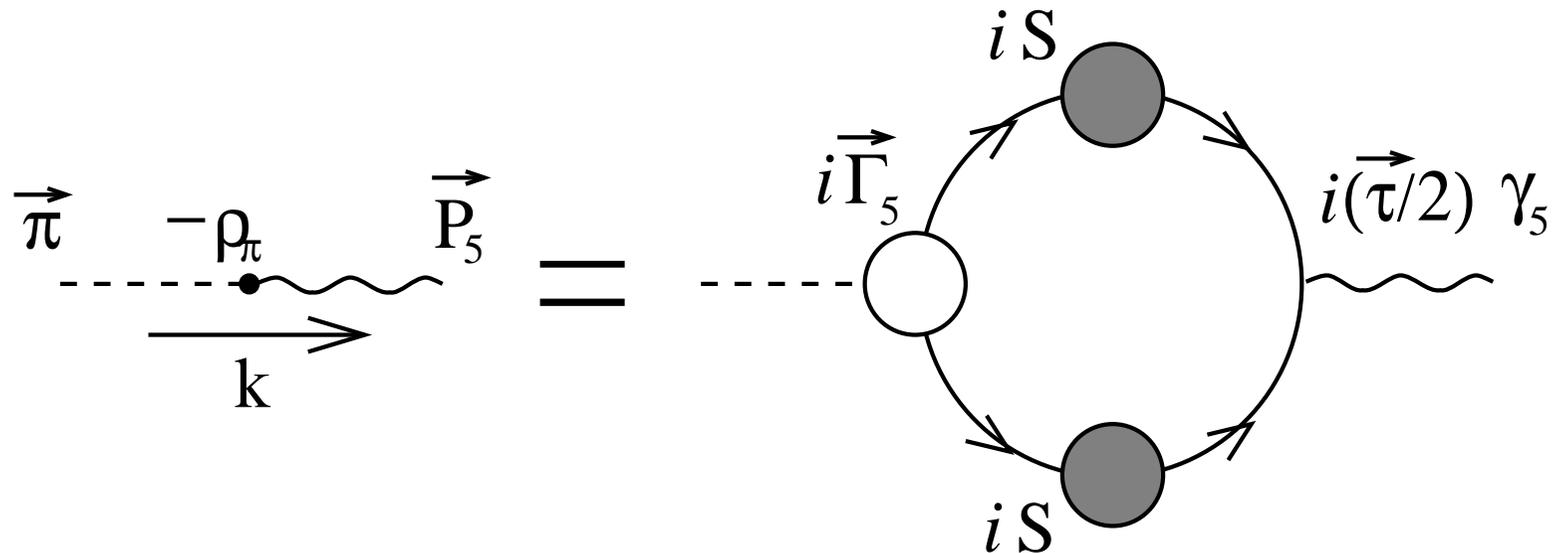
(Maris, Roberts, Tandy  
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$$\langle 0 | \bar{q} \gamma_5 q | \pi \rangle$$

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# Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy  
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$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

- Light-quarks; i.e.,  $m_q \sim 0$

- $f_H \rightarrow f_H^0$  &  $\rho_\zeta^H \rightarrow \frac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$ , Independent of  $m_q$

Hence  $m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q \dots$  GMOR relation, a corollary



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Hence  $m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q \dots$  GMOR relation, a corollary

- Heavy-quark + light-quark

$$\Rightarrow f_H \propto \frac{1}{\sqrt{m_H}} \text{ and } \rho_\zeta^H \propto \sqrt{m_H}$$

Hence,  $m_H \propto m_q$

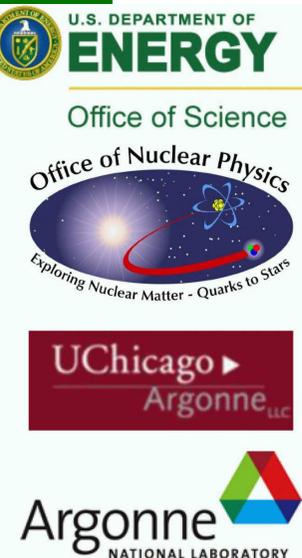
... QCD Proof of Potential Model result

# Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts  
nu-th/0406030

$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

- Valid for **ALL** Pseudoscalar mesons

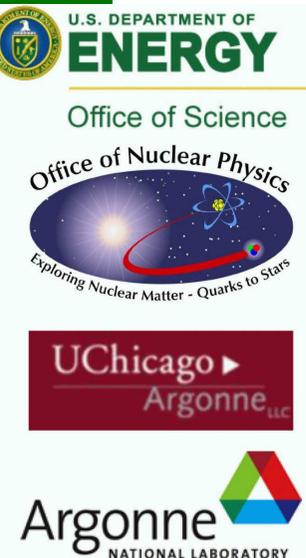


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- $\rho_H \Rightarrow$  finite, nonzero value in chiral limit,  $\mathcal{M}_H \rightarrow 0$



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- “radial” excitation of  $\pi$ -meson, not the ground state, so  $m_{\pi_{n \neq 0}}^2 > m_{\pi_{n=0}}^2 = 0$ , in **chiral limit**



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- $\Rightarrow f_H = 0$   
**ALL** pseudoscalar mesons **except  $\pi(140)$**  in **chiral limit**



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# Radial Excitations & Chiral Symmetry

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- $\Rightarrow f_H = 0$   
**ALL** pseudoscalar mesons **except  $\pi(140)$**  in **chiral limit**
- **Dynamical Chiral Symmetry Breaking**  
– Goldstone’s Theorem –  
impacts upon **every pseudoscalar meson**



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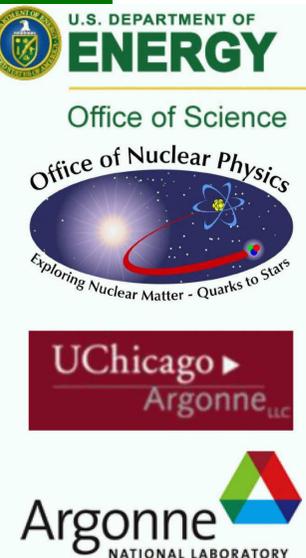
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# Radial Excitations & Lattice-QCD

McNeile and Michael  
he-la/0607032



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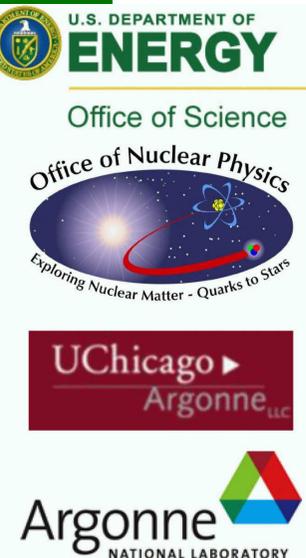
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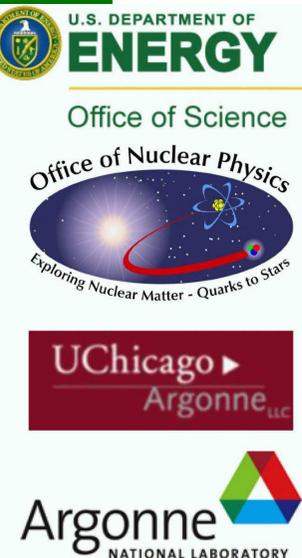
- *When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.*



# Radial Excitations & Lattice-QCD

McNeile and Michael  
he-la/0607032

- When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.
- CLEO:  $\tau \rightarrow \pi(1300) + \nu_\tau$   
 $\Rightarrow f_{\pi_1} < 8.4 \text{ MeV}$   
*Diehl & Hiller*  
*he-ph/0105194*



# Radial Excitations & Lattice-QCD

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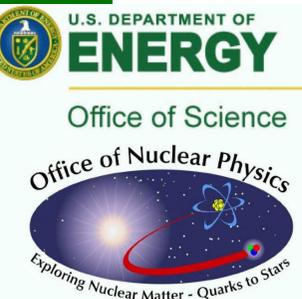
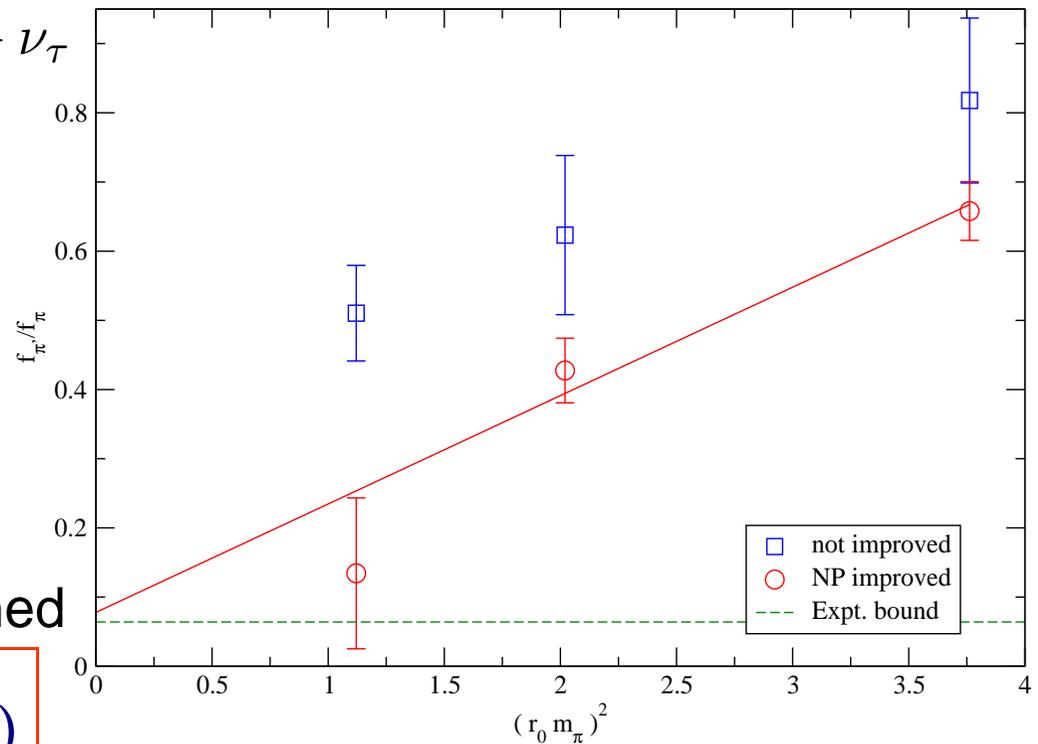
- Lattice-QCD check:

$$16^3 \times 32,$$

$$a \sim 0.1 \text{ fm},$$

two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078 (93)$$



# Radial Excitations & Lattice-QCD

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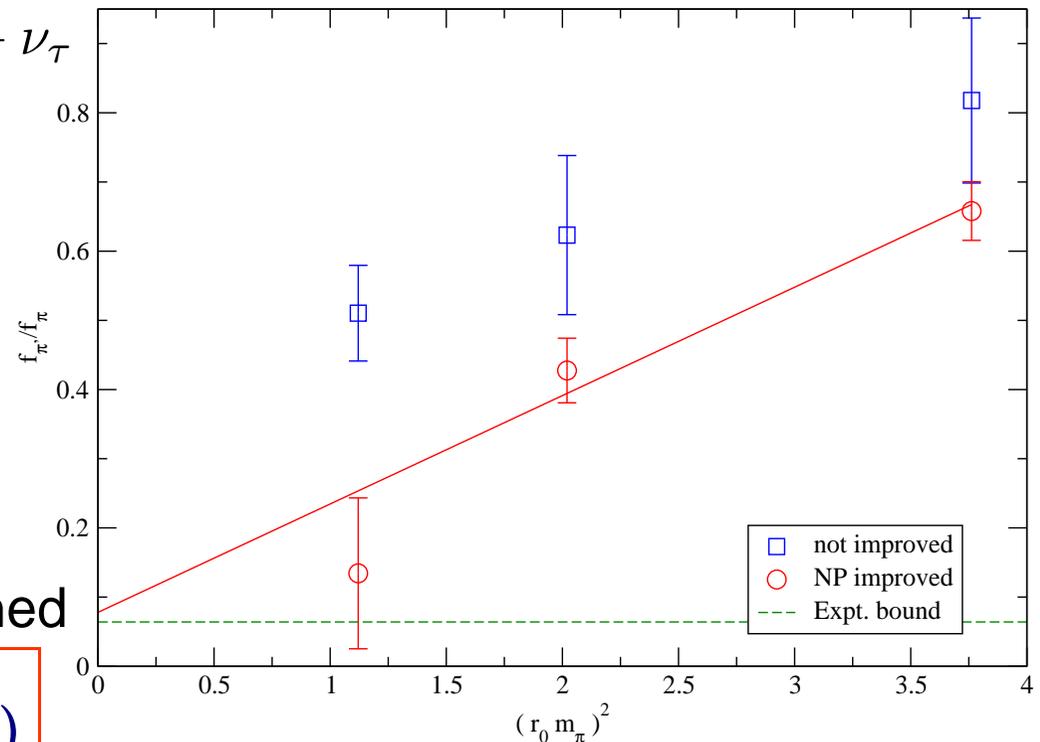
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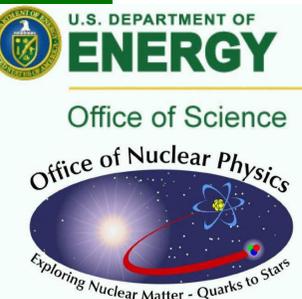
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- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)



# Radial Excitations & Lattice-QCD

McNeile and Michael  
he-la/0607032

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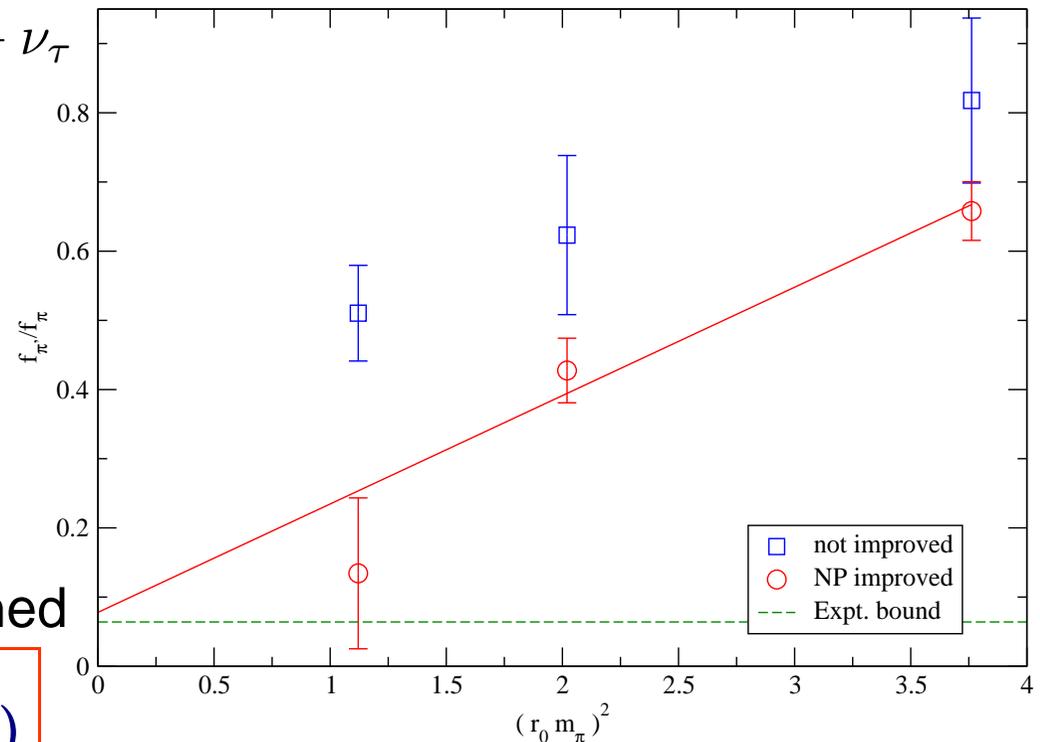
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- The suppression of  $f_{\pi_1}$  is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.



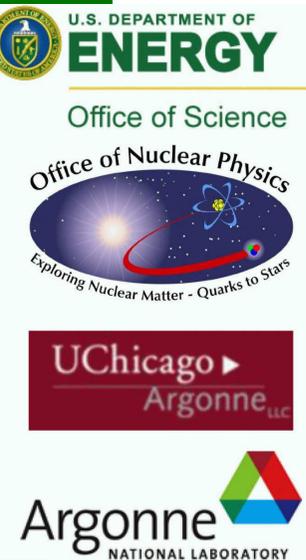
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# Bound-state DSE



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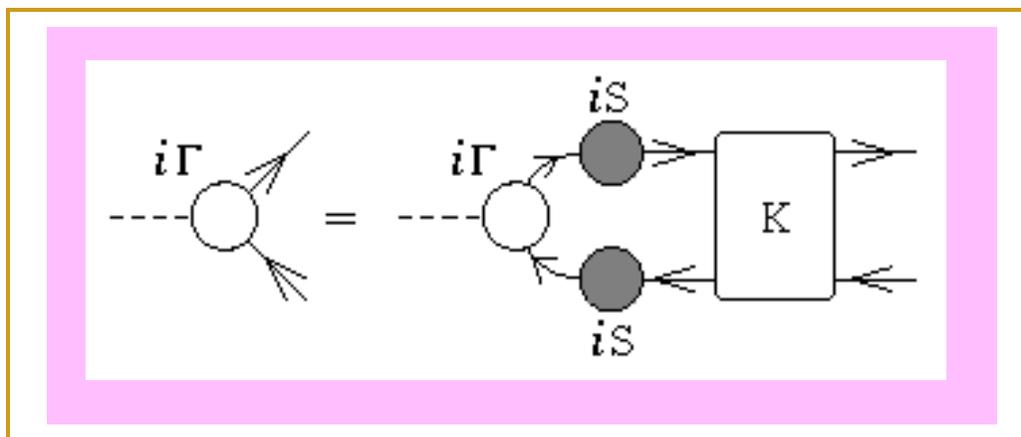
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# Bound-state DSE

## Bethe-Salpeter Equation

- Standard form, familiar from textbooks

$$[\Gamma_{\pi}^j(k; P)]_{tu} = \int_q^{\Lambda} [S(q + P/2)\Gamma_{\pi}^j(q; P)S(q - P/2)]_{sr} K_{tu}^{rs}(q, k; P)$$



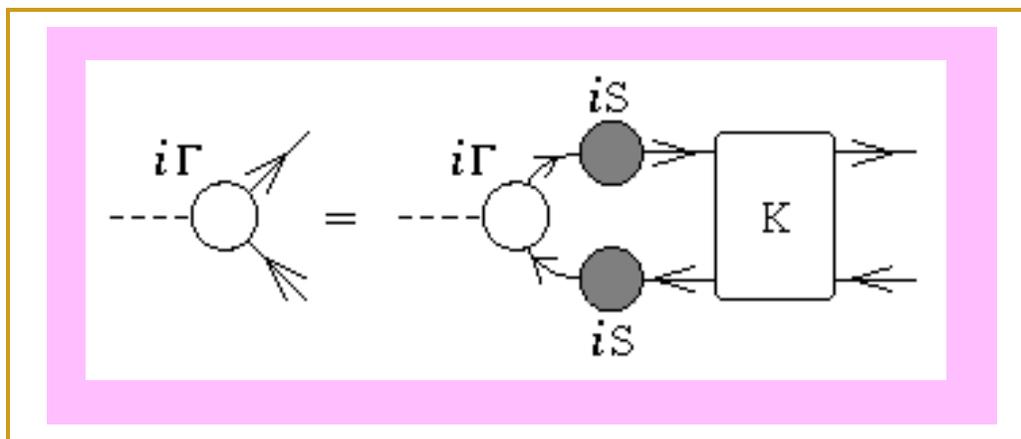
$K(q, k; P)$ : Fully-amputated, 2-particle-irreducible, quark-antiquark scattering kernel

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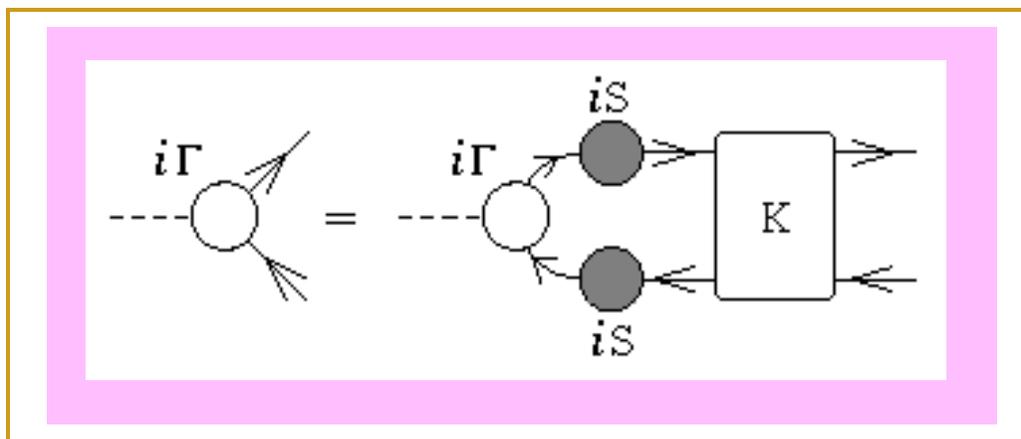
- Compact. Visually appealing. Correct.

# Bound-state DSE

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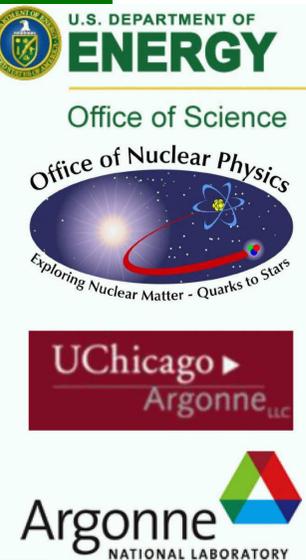
$K(q, k; P)$ : Fully-amputated, 2-particle-irreducible, quark-antiquark scattering kernel

- Compact. Visually appealing. Correct.
- Blocked progress for more than 60 years.

# Gap Equation

## General Form

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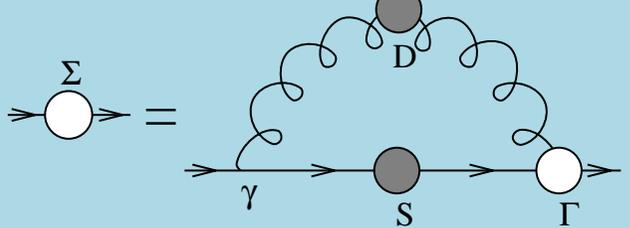
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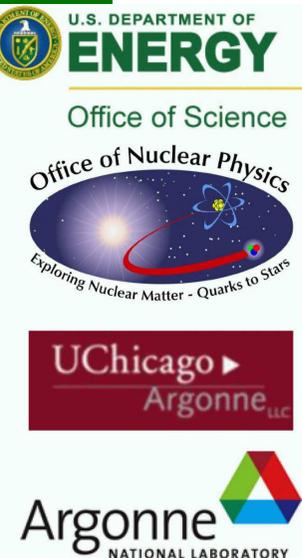
# Gap Equation

## General Form



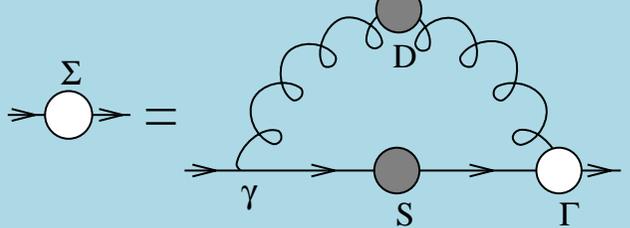
$$S_f(p)^{-1} = Z_2 (i\gamma \cdot p + m_f^{\text{bm}}) + \Sigma_f(p),$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \frac{\lambda^a}{2} \Gamma_\nu^f(q,p),$$



# Gap Equation

## General Form



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- $Z_{1,2}(\zeta^2, \Lambda^2)$  are respectively the vertex and quark wave function renormalisation constants, with  $\zeta$  the renormalisation point
- $m^{\text{bm}}(\Lambda)$  is the Lagrangian current-quark bare mass
- $D_{\mu\nu}(k)$  is the dressed-gluon propagator
- $\Gamma_\nu^f(q,p)$  is the dressed-quark-gluon vertex



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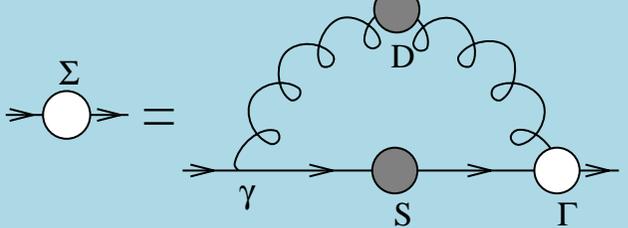
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# Gap Equation

## General Form



$$S_f(p)^{-1} = Z_2 (i\gamma \cdot p + m_f^{\text{bm}}) + \Sigma_f(p),$$

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- $Z_{1,2}(\zeta^2, \Lambda^2)$  are respectively the vertex and quark wave function renormalisation constants, with  $\zeta$  the renormalisation point
- $m^{\text{bm}}(\Lambda)$  is the Lagrangian current-quark bare mass
- $D_{\mu\nu}(k)$  is the dressed-gluon propagator
- $\Gamma_\nu^f(q,p)$  is the dressed-quark-gluon vertex
- Suppose one has in-hand the exact form of  $\Gamma_\nu^f(q,p)$

What is the associated

Symmetry-preserving Bethe-Salpeter Kernel?



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# Bethe-Salpeter Equation

## General Form

L. Chang and C. D. Roberts

0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

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# Bethe-Salpeter Equation

## General Form

L. Chang and C. D. Roberts

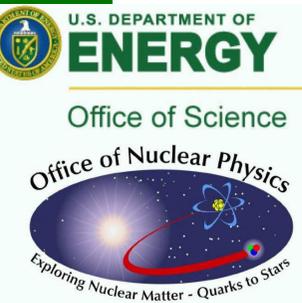
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● Equivalent exact form:

$$\begin{aligned}\Gamma_{5\mu}^{fg}(k; P) &= Z_2 \gamma_5 \gamma_\mu \\ &- \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \Gamma_{5\mu}^{fg}(q; P) S_g(q_-) \frac{\lambda^a}{2} \Gamma_\beta^g(q_-, k_-) \\ &+ \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P),\end{aligned}$$

(Poincaré covariance, hence  $q_\pm = q \pm P/2$ , etc., without loss of generality.)



# Bethe-Salpeter Equation

## General Form

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$$\Gamma_{5\mu}^{fg}(k; P) = Z_2 \gamma_5 \gamma_\mu$$

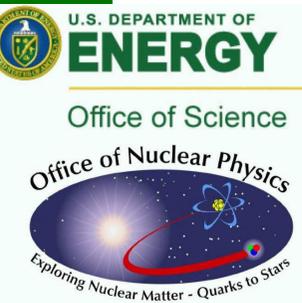
$$- \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \Gamma_{5\mu}^{fg}(q; P) S_g(q_-) \frac{\lambda^a}{2} \Gamma_\beta^g(q_-, k_-)$$

$$+ \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P),$$

(Poincaré covariance, hence  $q_\pm = q \pm P/2$ , etc., without loss of generality.)

- In this form ...  $\Lambda_{5\mu\beta}^{fg}$

is completely defined via the dressed-quark self-energy

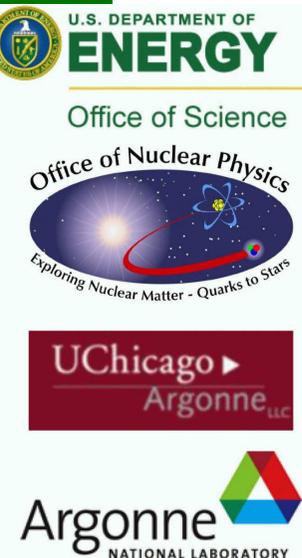


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- E.g., in any reliable study of light-quark hadrons, axial-vector vertex must satisfy Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^{fg}(k; P) = S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-) - i [m_f(\zeta) + m_g(\zeta)] \Gamma_5^{fg}(k; P),$$

Expresses chiral symmetry & pattern by which it's broken



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Expresses chiral symmetry & pattern by which it's broken

- The condition ( $\Lambda_{5\beta}^{fg}$  pseudoscalar analogue of  $\Lambda_{5\mu\beta}^{fg}$ )

$$P_\mu \Lambda_{5\mu\beta}^{fg}(k, q; P) = \Gamma_\beta^f(q_+, k_+) i\gamma_5 + i\gamma_5 \Gamma_\beta^g(q_-, k_-) - i[m_f(\zeta) + m_g(\zeta)] \Lambda_{5\beta}^{fg}(k, q; P),$$

a new Ward-Takahashi identity, is **Necessary & Sufficient** to ensure  $\Gamma_{5\mu}^{fg}(k; P)$  Ward-Takahashi identity satisfied.



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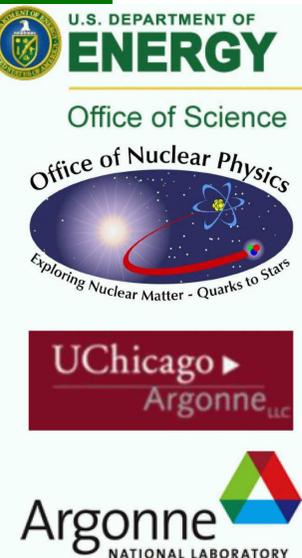
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- Rainbow-ladder ...

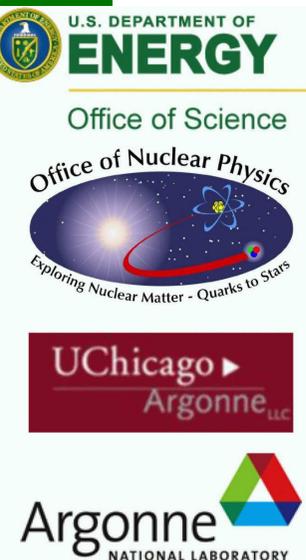
- $\Gamma_\beta^f(q, k) = \gamma_\mu$   
 $\Rightarrow \Lambda_{5\mu\beta}^{fg}(k, q; P) = 0 = \Lambda_{5\beta}^{fg}(k, q; P)$



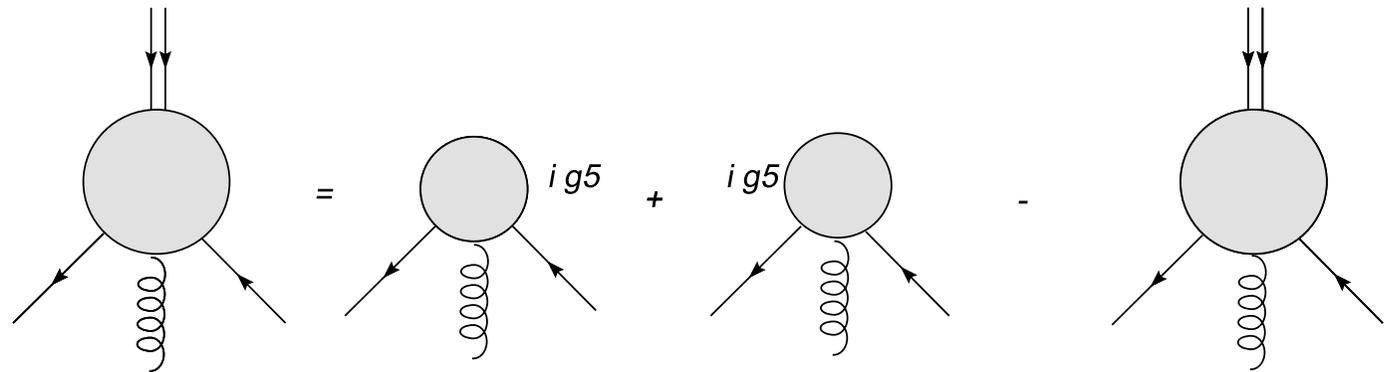
# Bethe-Salpeter Kernel

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- Bethe-Salpeter equation introduced in 1951

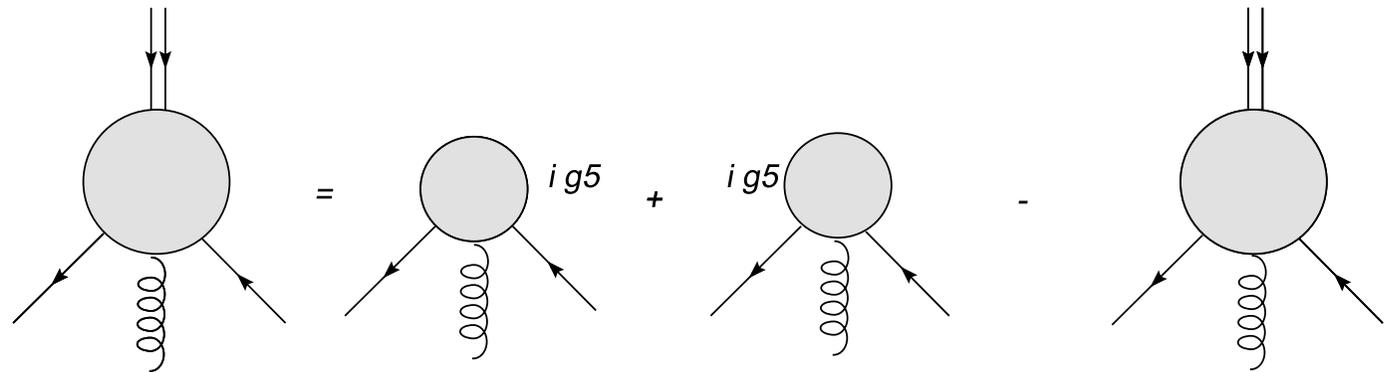


- Bethe-Salpeter equation introduced in 1951
- Newly-derived Ward-Takahashi identity



$$P_\mu \Lambda_{5\mu\beta}^{fg}(k, q; P) = \Gamma_\beta^f(q_+, k_+) i\gamma_5 + i\gamma_5 \Gamma_\beta^g(q_-, k_-) - i[m_f(\zeta) + m_g(\zeta)] \Lambda_{5\beta}^{fg}(k, q; P),$$

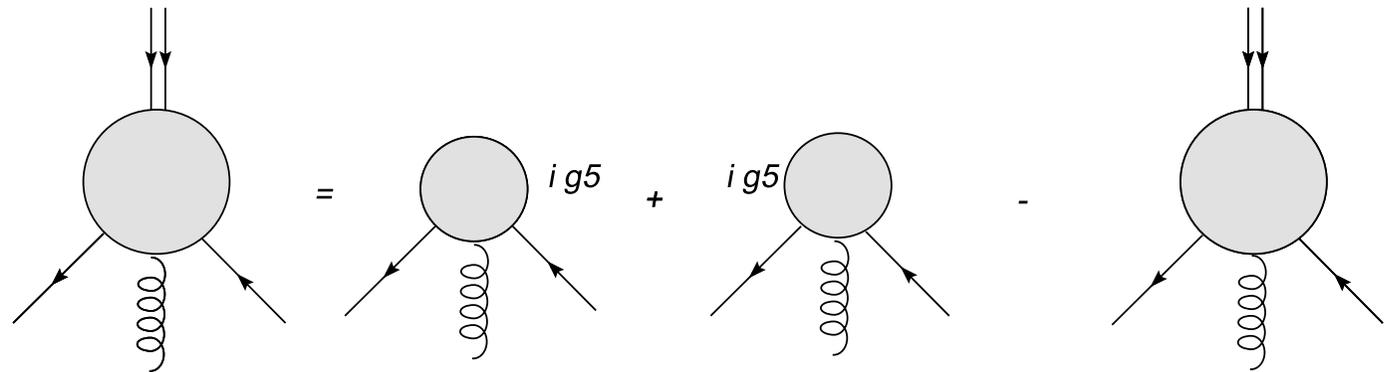
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- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
  - Consistent means - all symmetries preserved!

- Bethe-Salpeter equation introduced in 1951
- Newly-derived Ward-Takahashi identity



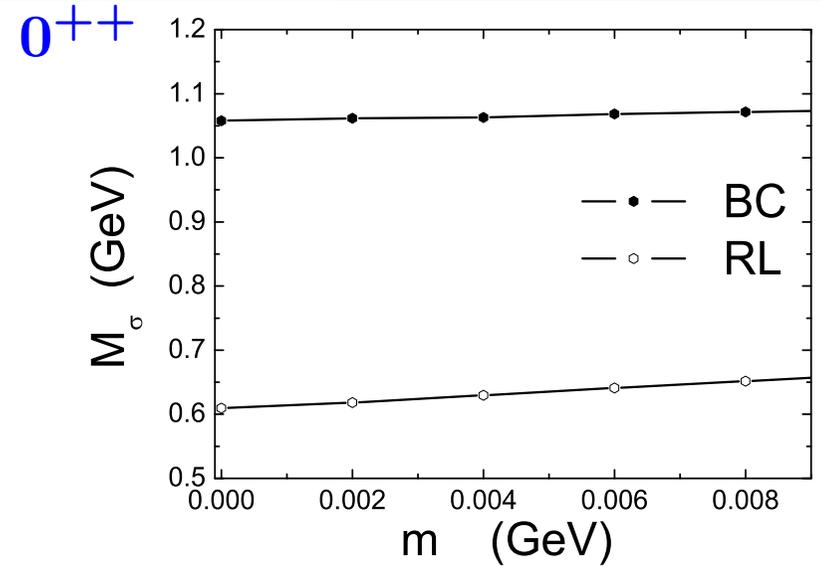
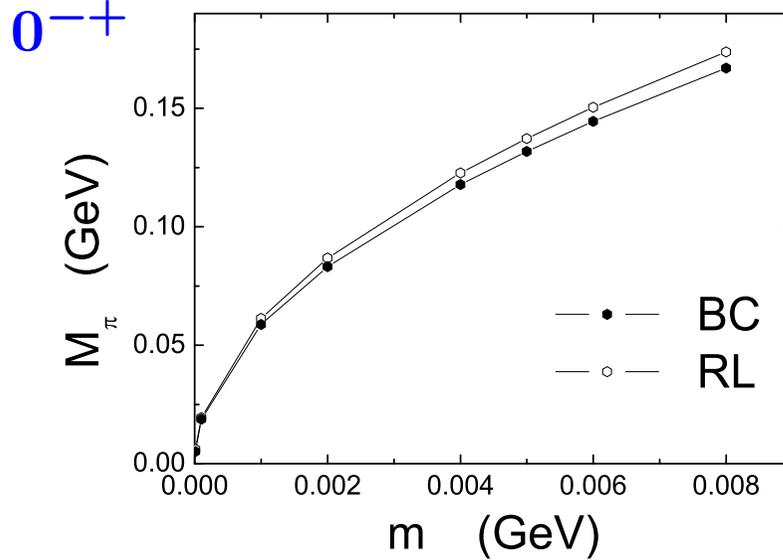
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- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
- Exemplified the procedure and results to expect . . .

# Numerical Illustration

L. Chang and C. D. Roberts  
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$\pi$  *cf.*  $\sigma$



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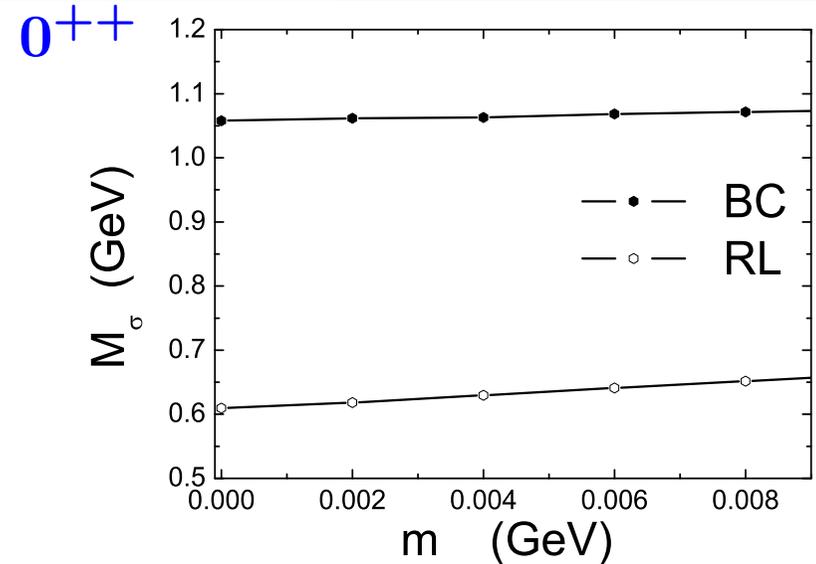
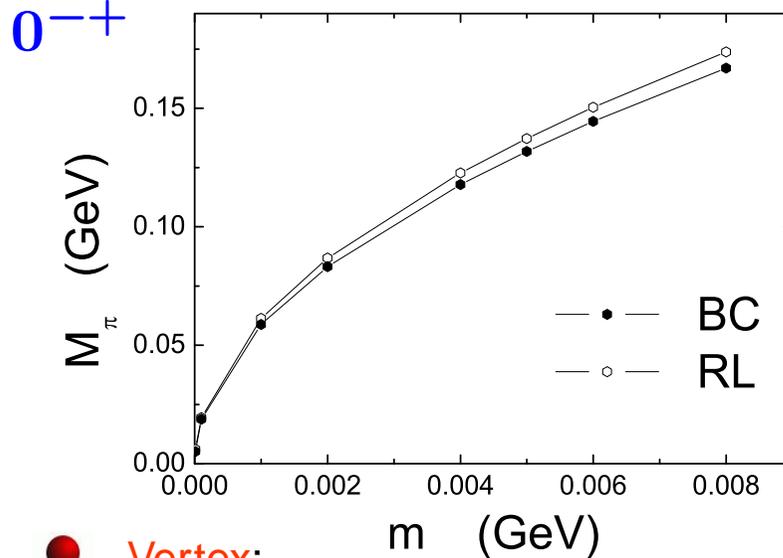
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$\pi$  cf.  $\sigma$



**Vertex:**

- leading-order rainbow-ladder truncation
- cf. Ball-Chiu-consistent *Ansatz* – Essentially nonperturbative content; Expresses DCSB; Consistent with lattice-QCD simulations; Diagrammatic content unknown

Same interaction. One mass-scale in both truncations:  $1/\omega = 0.4$  fm, defining border between IR & UV.



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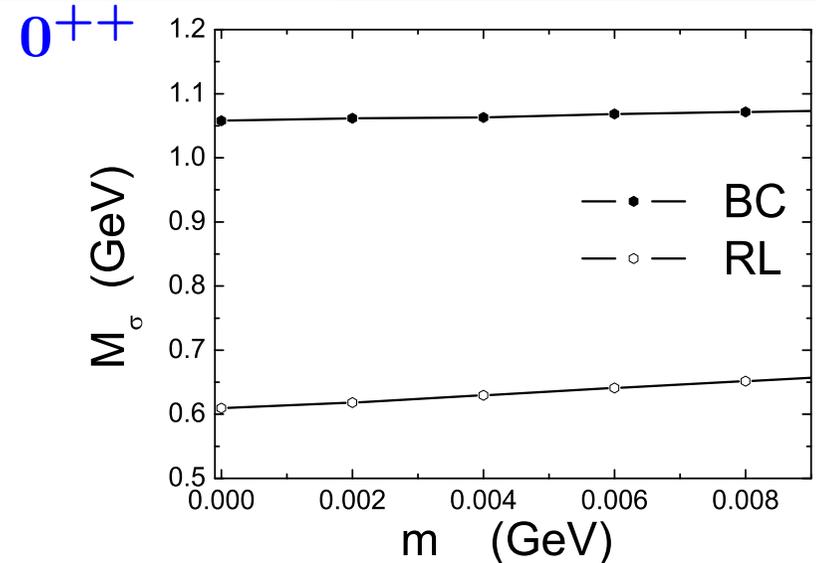
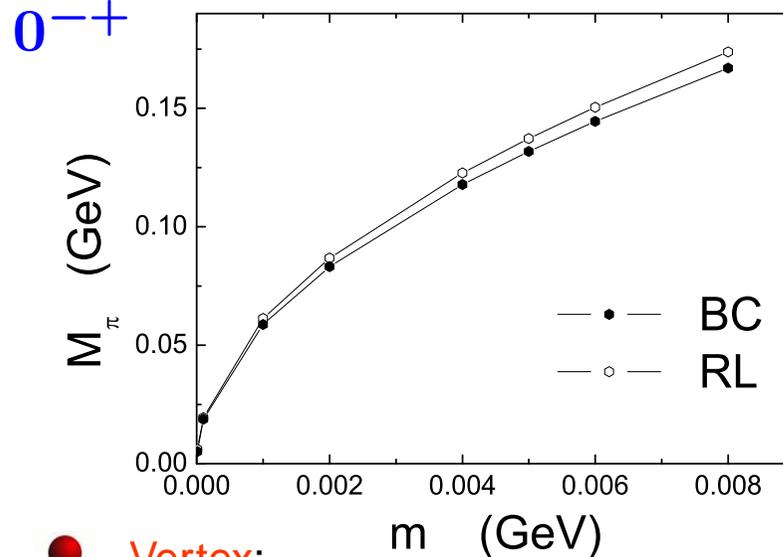
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Same interaction. One mass-scale in both truncations:  $1/\omega = 0.4$  fm, defining border between **IR** & **UV**.

- GMOR ... plainly satisfied by both truncations
- A little **attraction** introduced in pseudoscalar channel
- **Enormous repulsion** introduced in scalar channel



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# Spin-orbit Interaction

L. Chang and C. D. Roberts  
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- Rainbow-ladder DSE truncation,  $\epsilon_{\sigma}^{\text{RL}} := \frac{2M(0) - m_{\sigma}}{2M(0)} \Big|_{\text{RL}} = (0.3 \pm 0.1)$ .
- BC-consistent Bethe-Salpeter kernel; viz.,  $\epsilon_{\sigma}^{\text{BC}} \lesssim 0.1$ .



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- Scalar mesons =  ${}^3P_0$  states: Constituents' spins aligned and one unit of constituent orbital angular momentum
  - From this viewpoint,  
scalar is a spin and orbital excitation of a pseudoscalar meson



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- Extant studies of realistic corrections to the rainbow-ladder truncation show that they reduce hyperfine splitting in the absence of orbital angular momentum
- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit interaction.
  - Effect owes to influence of quark's dynamically-enhanced scalar self-energy in the Bethe-Salpeter kernel.

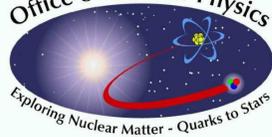
Impossible to demonstrate effect without our new procedure



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- Impossible to demonstrate effect without our new procedure
- Expect this feature to have material impact
    - Especially on mesons with mass greater than 1 GeV.
- prima facie* ... can overcome longstanding shortcoming of systematic, symmetry-preserving truncations;  
viz., splitting between vector & axial-vector mesons is too small



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- Expect this feature to have material impact
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- *prima facie* ... can overcome longstanding shortcoming of systematic, symmetry-preserving truncations; viz., splitting between vector & axial-vector mesons is too small
- Promise of realistic meson spectroscopy ... First time, also for mass  $> 1$  GeV



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$$[m_{a_1} - m_\rho] / \dots$$

- That was where things stood in March/09

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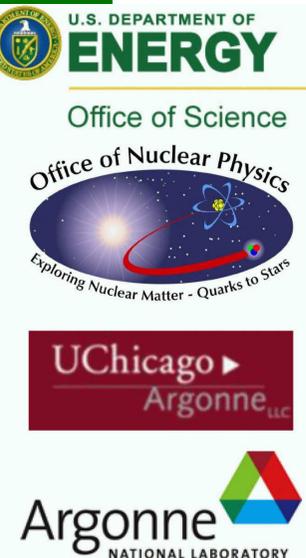
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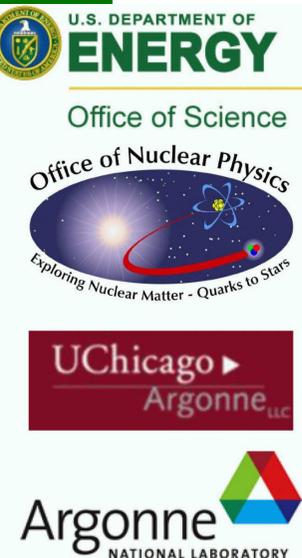
Chang Lei & CDR, in-preparation

- That was where things stood in March/09
- Now, we've solved inhomogeneous vector and axial-vector Bethe-Salpeter equation at spacelike total momentum  
 $\Rightarrow \Gamma_{qq}(k = 0, P^2)$
- $\frac{1}{\Gamma_{qq}(k = 0, P^2)}$  Exhibits a zero at ground-state mass-squared
- Padé approximant extrapolation to locate zero



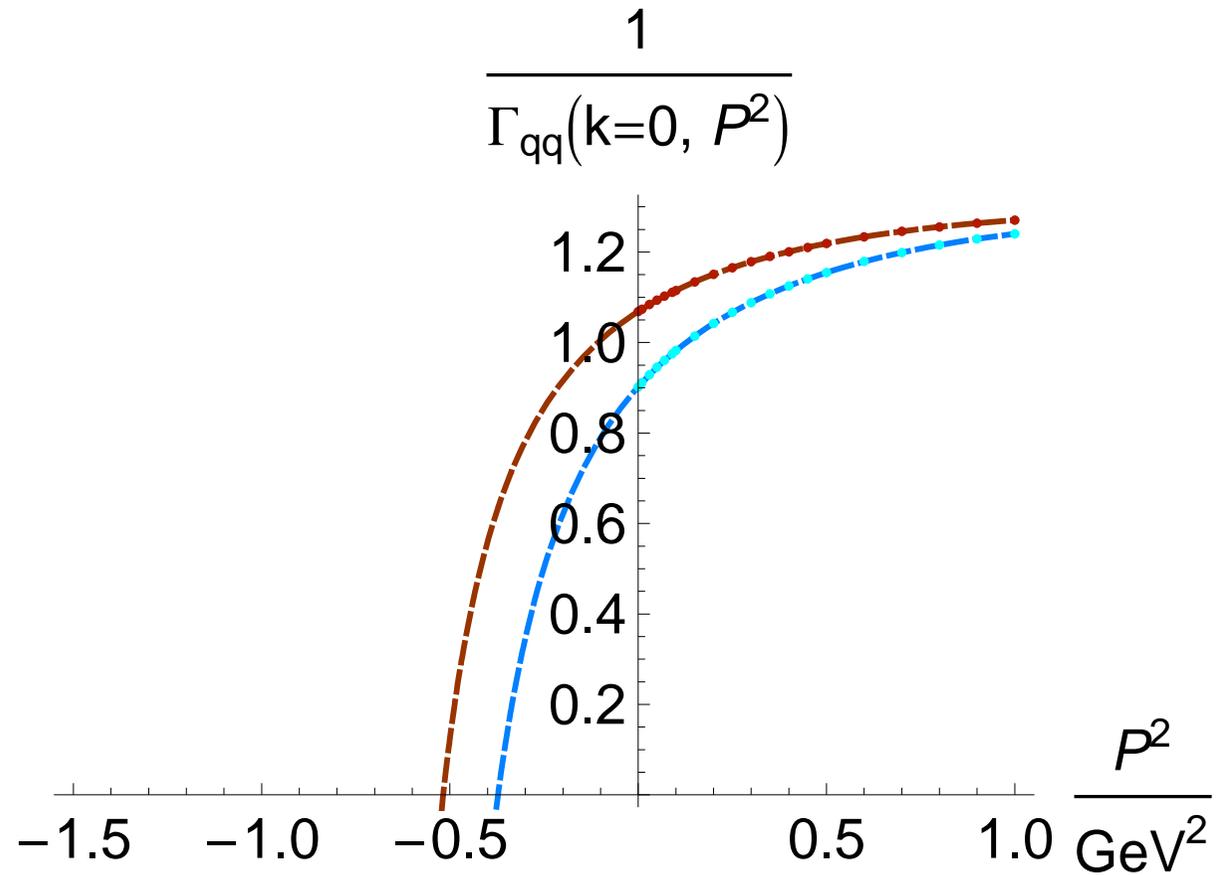
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- |                                     |
|-------------------------------------|
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|-------------------------------------|

 Exhibits a zero at ground-state mass-squared
- Padé approximant extrapolation to locate zero
  - Almost precisely method used for ground-state masses in lattice-QCD
  - Intelligent use gives dependable results  
 “Schwinger functions and light-quark bound states”  
[Bhagwat](#), [Höll](#), [Krassnigg](#), [Roberts](#) & [Wright](#),  
 Few Body Syst. **40** (2007) 209, nucl-th/0701009



Rainbow-Ladder

$$\Gamma_\mu(q, k) = \gamma_\mu$$



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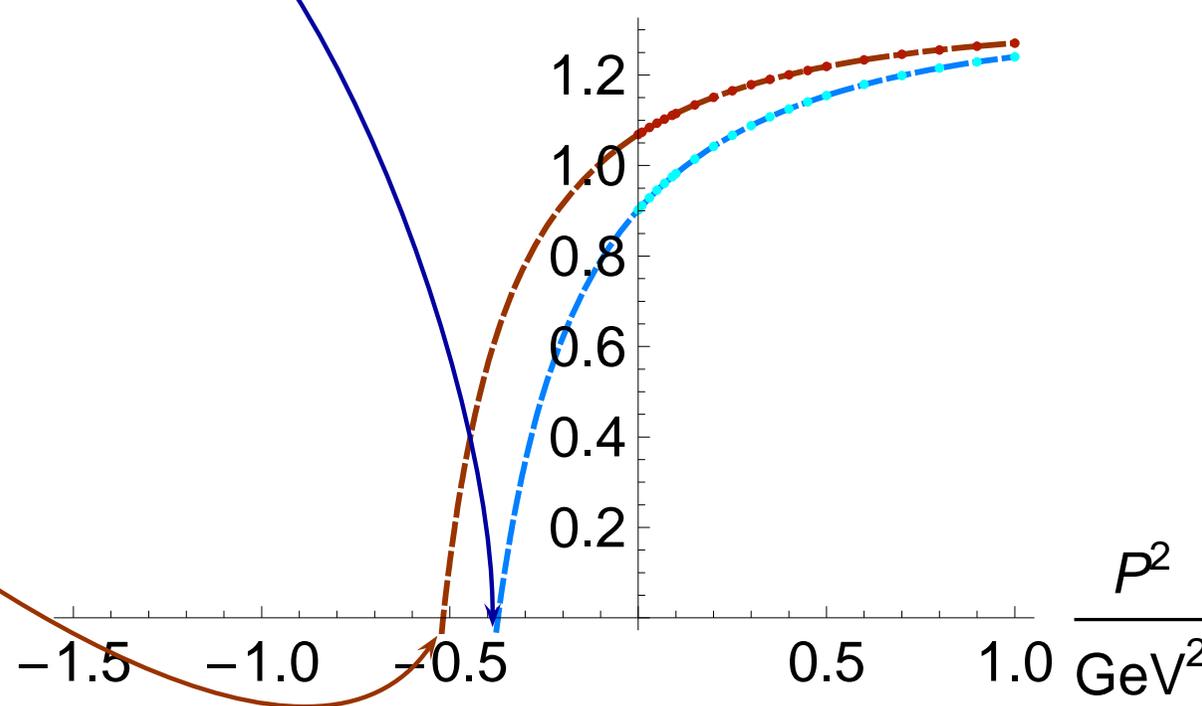


$m_{a_1} (759 \text{ MeV}) - m_\rho (644 \text{ MeV})$   
 $= 115 \text{ MeV} \dots \text{expt.} = 455 \text{ MeV}$

Rainbow-Ladder

$\Gamma_\mu(q, k) = \gamma_\mu$

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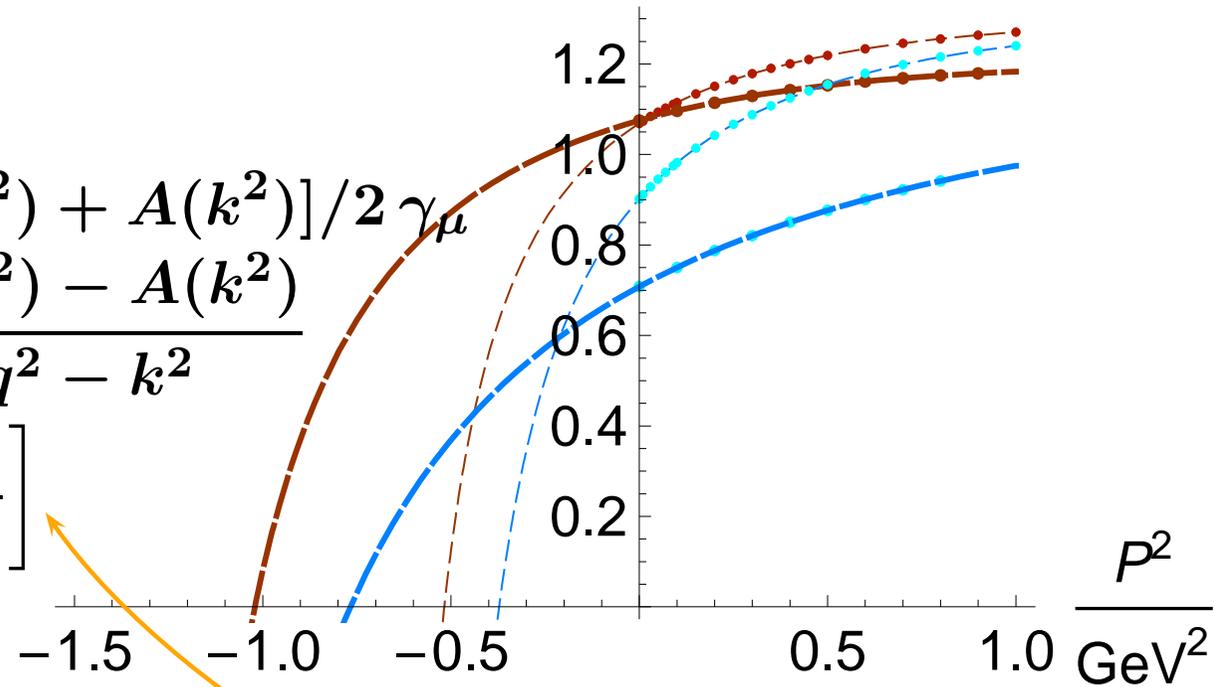

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$$m_{a_1} (1066 \text{ MeV}) - m_\rho (924 \text{ MeV}) = 142 \text{ MeV} \dots \text{expt.} = 455 \text{ MeV}$$

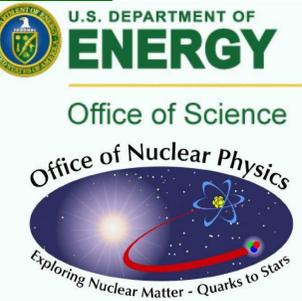
$$\frac{1}{\Gamma_{qq}(k=0, P^2)}$$

Ball-Chiu

$$\Gamma_\mu(q, k) = i[A(q^2) + A(k^2)]/2 \gamma_\mu + 2k_\mu \left[ i\gamma \cdot k \frac{A(q^2) - A(k^2)}{q^2 - k^2} + \frac{B(q^2) - B(k^2)}{q^2 - k^2} \right]$$



DCSB enhanced spin-orbit interaction

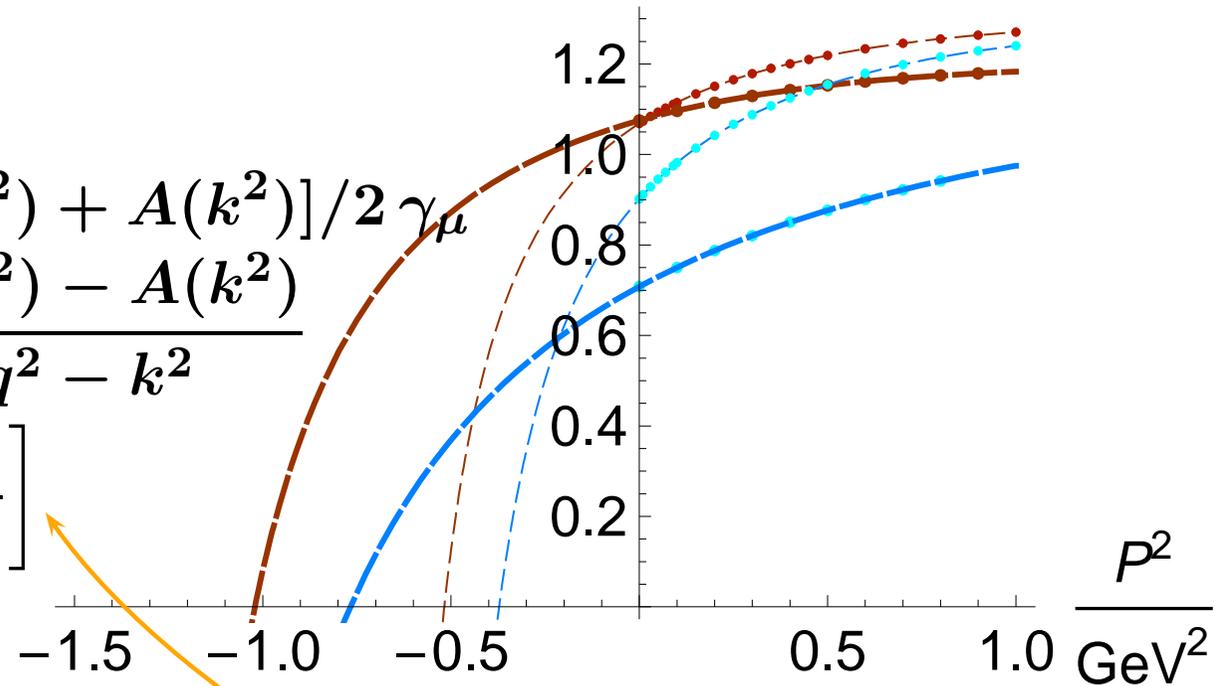


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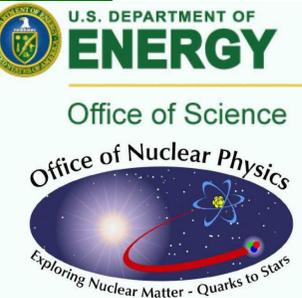
### Ball-Chiu

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DCSB enhanced spin-orbit interaction

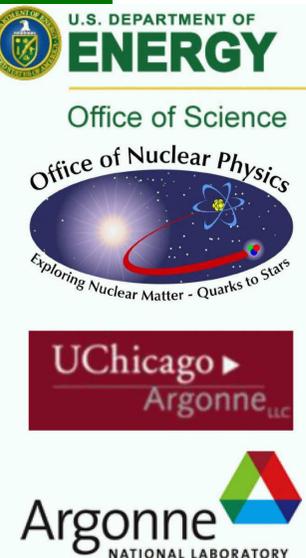
What's missing?



# Missing Link

$$[m_{a_1} - m_\rho]$$

Chang Lei & CDR, in-preparation



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$$[m_{a_1} - m_\rho]$$

Chang Lei & CDR, in-preparation

$$m_{a_1} (1230 \text{ MeV}) - m_\rho (745 \text{ MeV})$$

$$= 485 \text{ MeV} \dots \text{expt.} = 455 \text{ MeV}$$

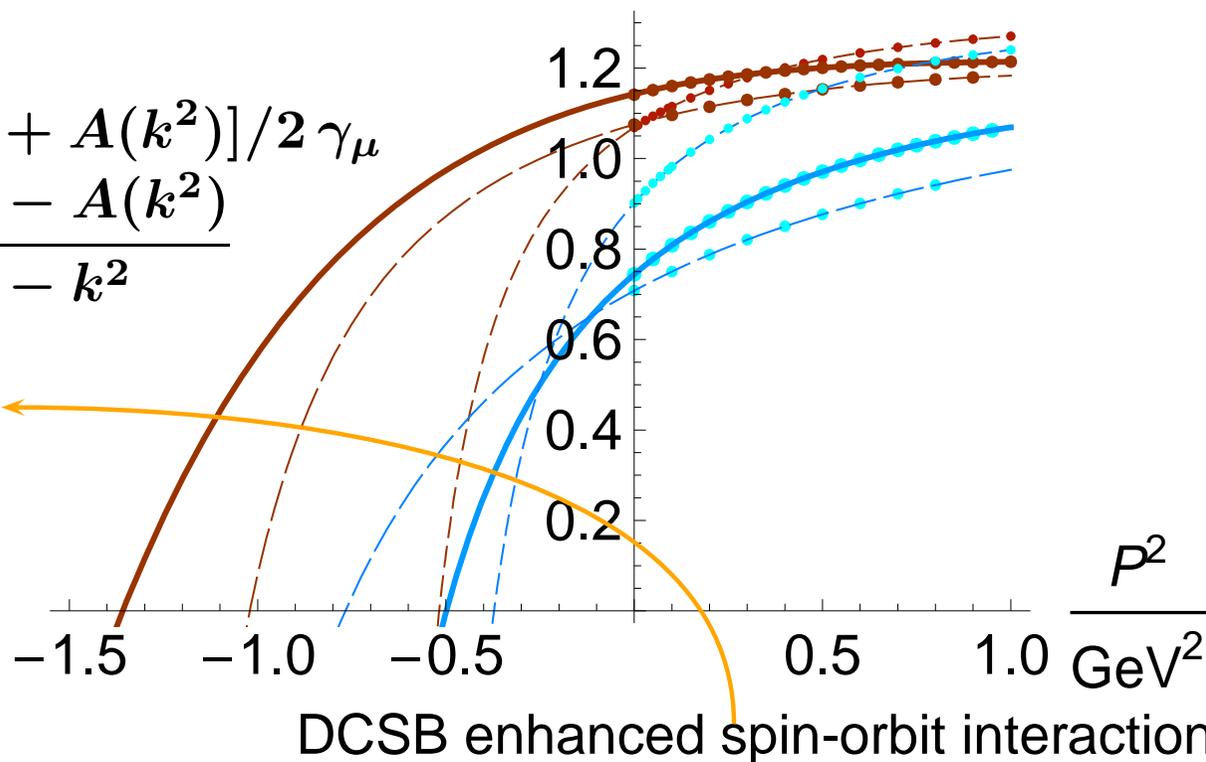
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$$\frac{1}{\Gamma_{qq}(k=0, P^2)}$$



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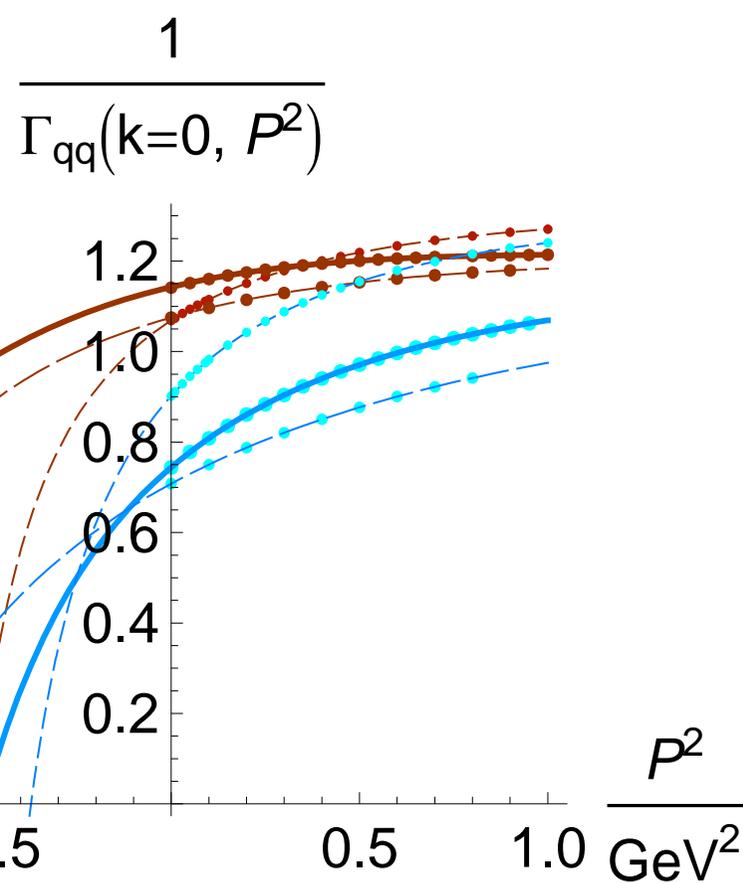
$$[m_{a_1} - m_\rho]$$

Chang Lei & CDR, in-preparation

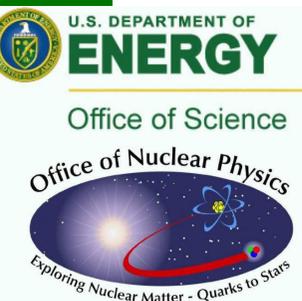
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Ball-Chiu + an. mag. mom.

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DCSB enhanced anomalous chromomagnetic moment



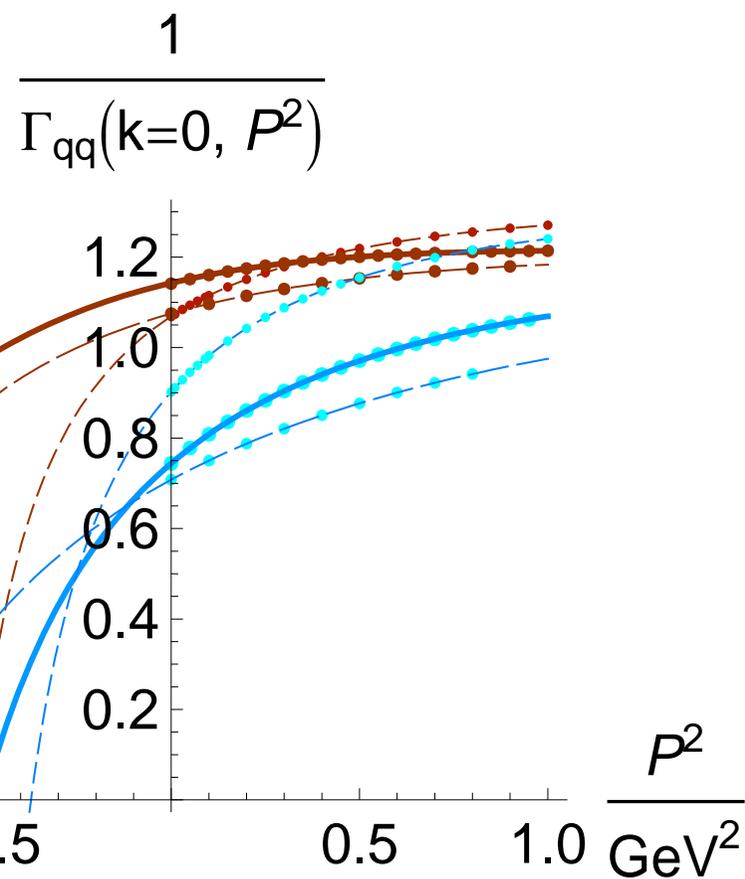
$$[m_{a_1} - m_\rho]$$

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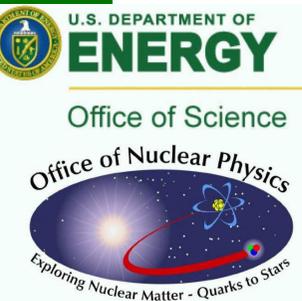
Inextricably connected

with DCSB.

DCSB enhanced anomalous chromomagnetic moment

Can't appear in chirally symmetric theory

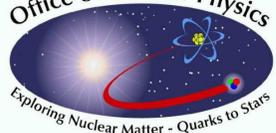
Paves way for truly reliable light-quark meson spectrum



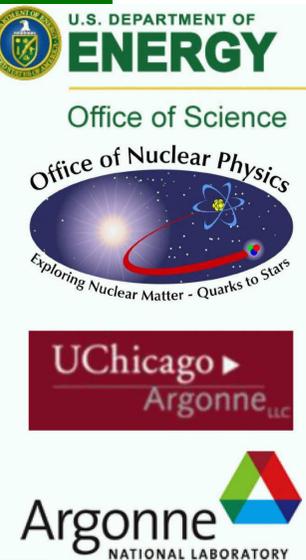


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# *Unifying Study of Mesons and Baryons*



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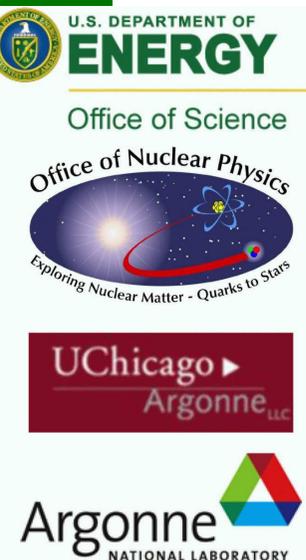
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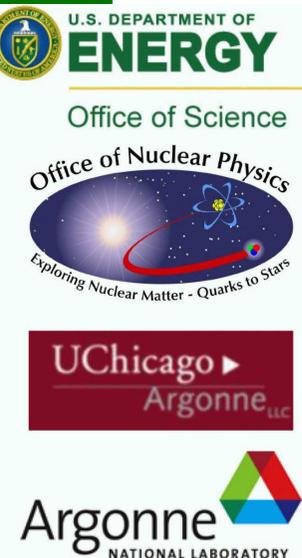
# Unifying Study of Mesons and Baryons

- How does one incorporate dressed-quark mass function,  $M(p^2)$ , in study of baryons? Behaviour of  $M(p^2)$  is essentially a quantum field theoretical effect.



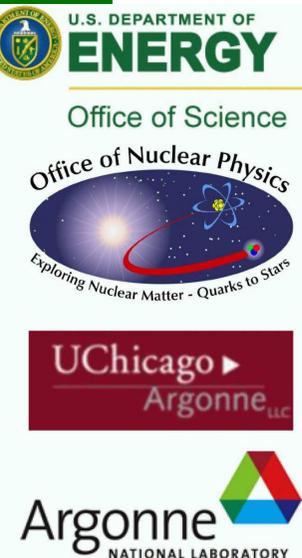
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- In quantum field theory a nucleon appears as a pole in a six-point quark Green function.
  - Residue is proportional to nucleon's Faddeev amplitude
  - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks



# Unifying Study of Mesons and Baryons

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  - Residue is proportional to nucleon's Faddeev amplitude
  - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
  - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour- $\bar{3}$  (antitriplet) channel



# *Faddeev equation*

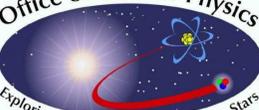
R. T. Cahill *et al.* Austral. J. Phys. **42** (1989) 129



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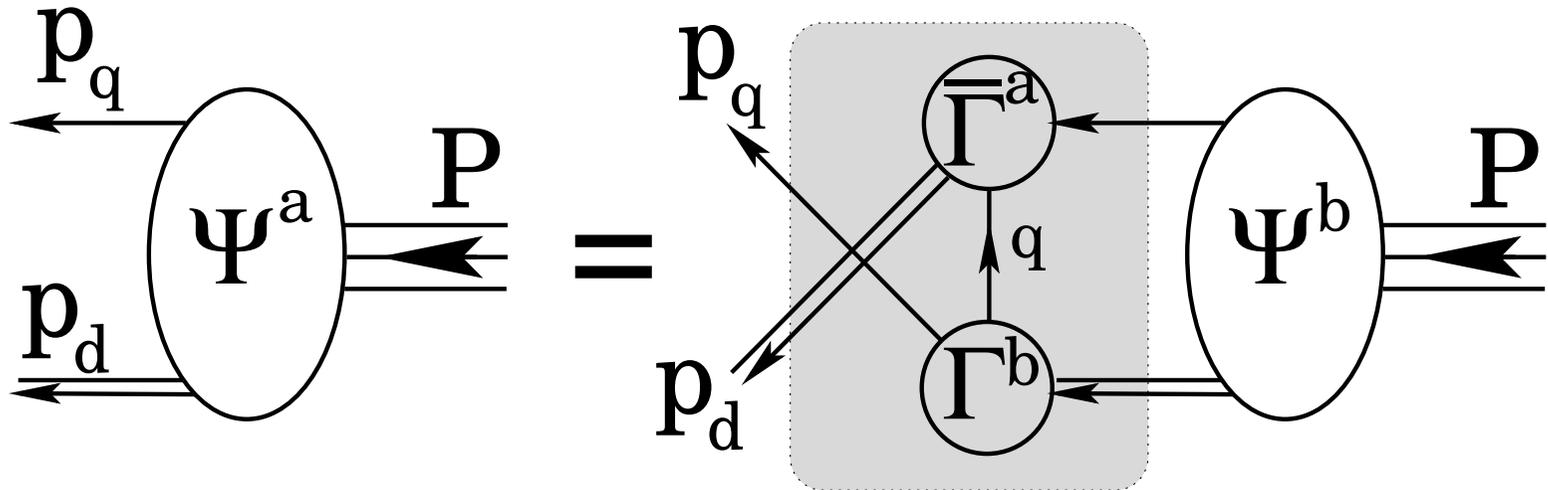
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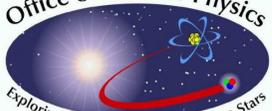
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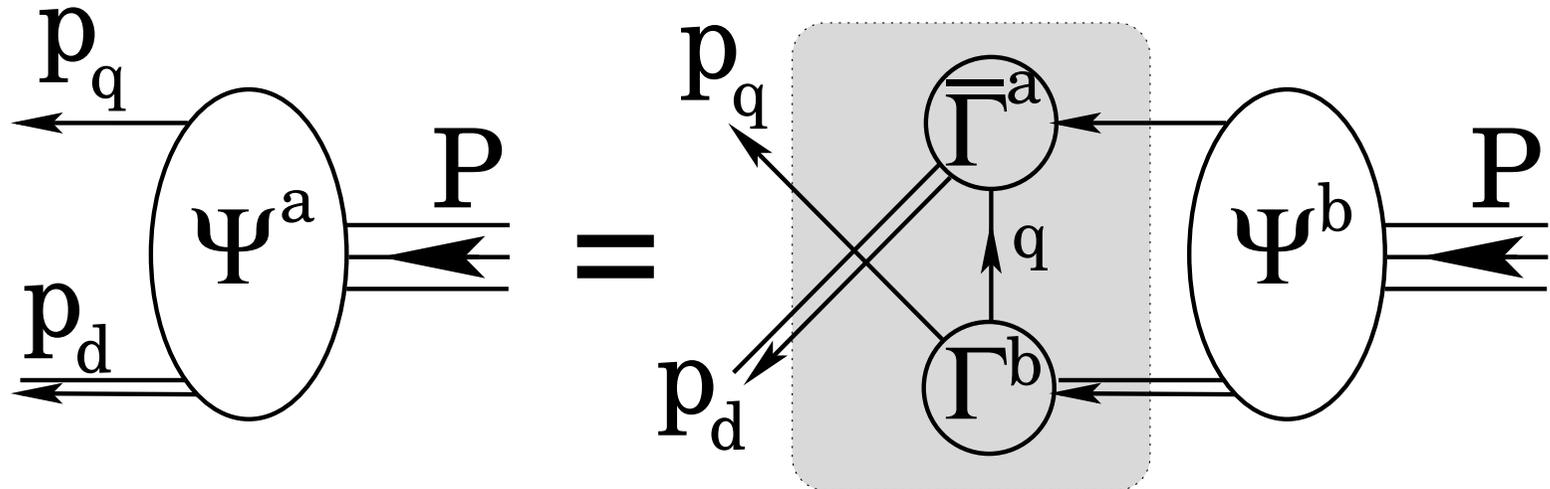
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# Faddeev equation

R. T. Cahill *et al.* Austral. J. Phys. **42** (1989) 129



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (**Poincaré Covariant Faddeev Amplitude**) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame **Amplitude** has ... *s*-, *p*- & *d*-wave correlations



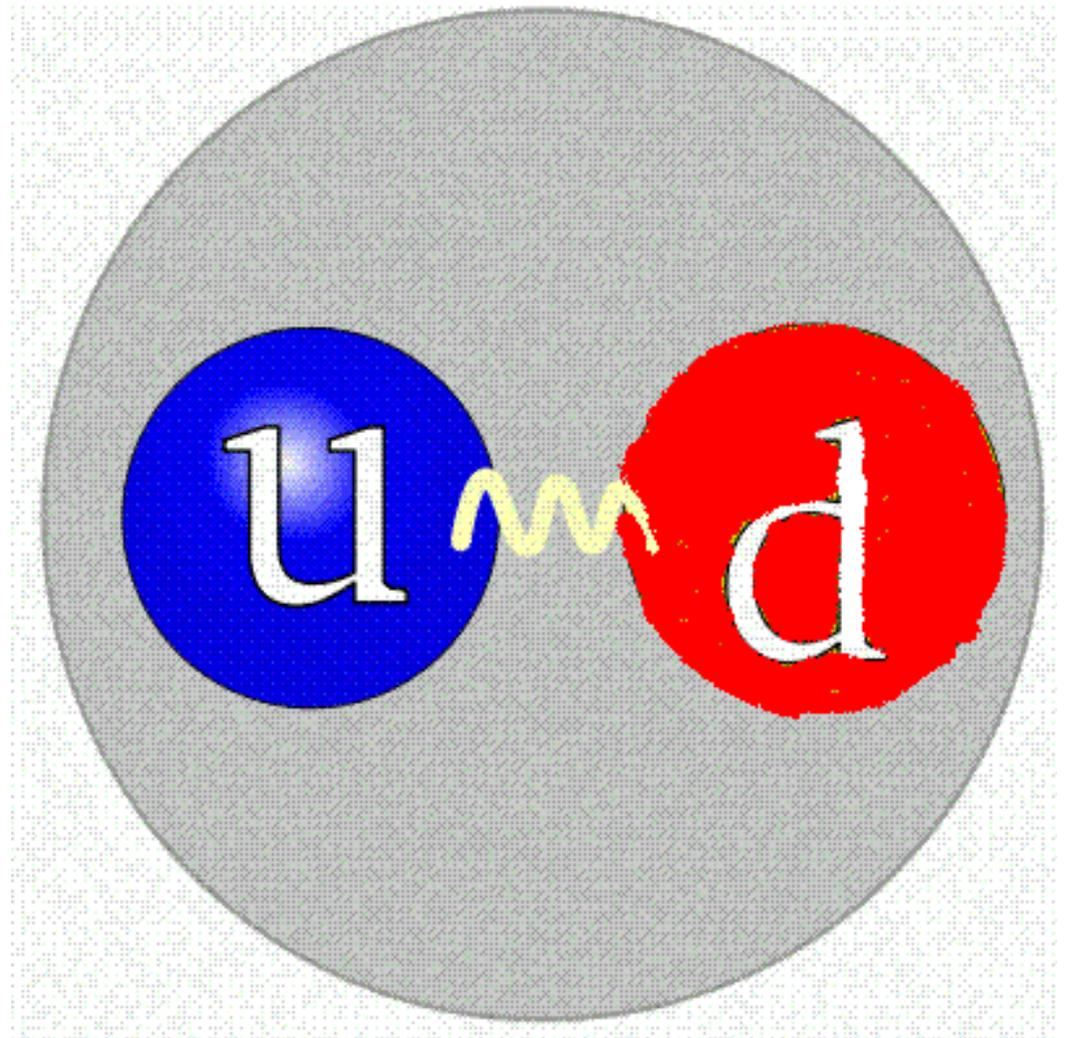
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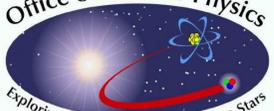
# Diquark correlations



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## QUARK-QUARK

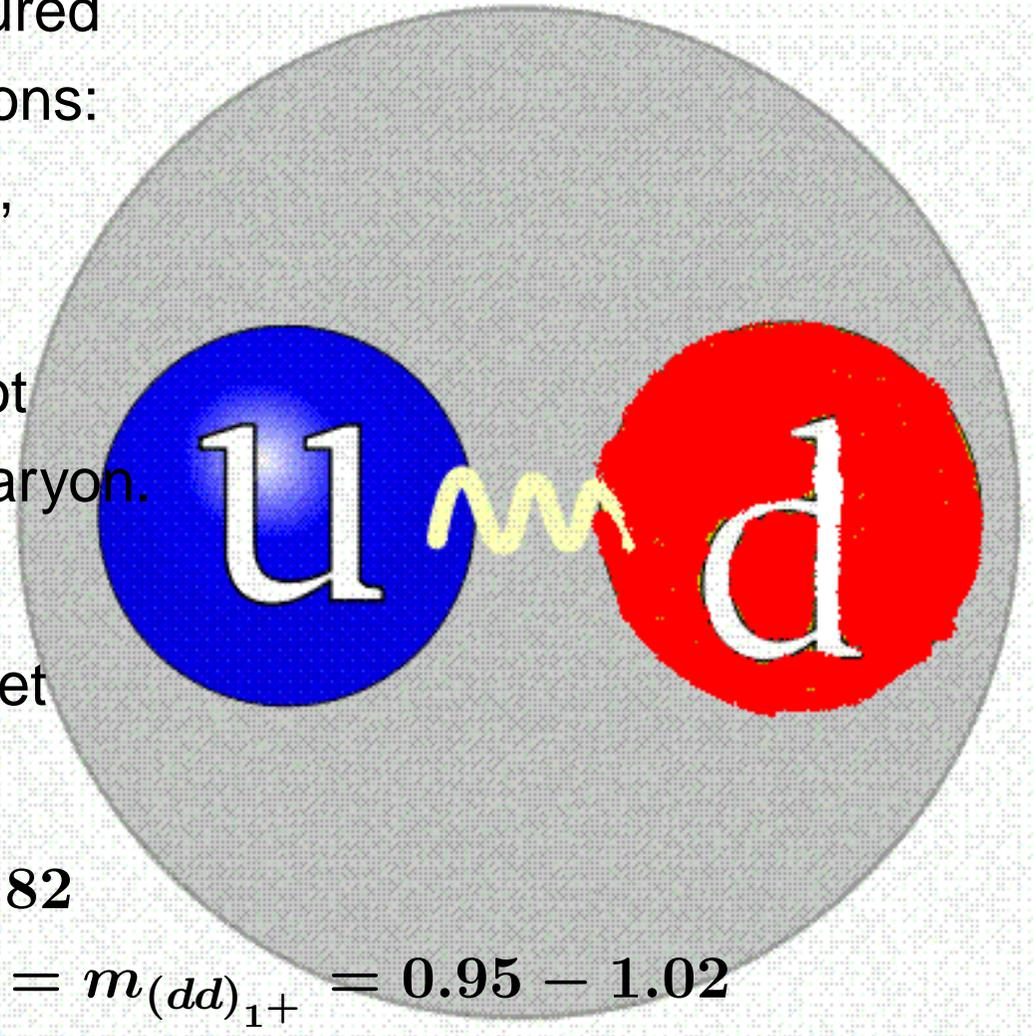
Craig Roberts – Hadron Physics: The Essence of Matter

XII Mexican Workshop on Particles and Fields, 9-14 Nov. 2009... 45 – p. 37/50

# Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: **blue-red**, **blue-green**, **green-red**
- Confined ... Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD
 
$$m_{[ud]_{0+}} = 0.74 - 0.82$$

$$m_{(uu)_{1+}} = m_{(ud)_{1+}} = m_{(dd)_{1+}} = 0.95 - 1.02$$



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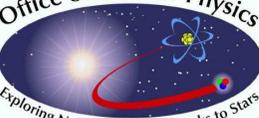
# Nucleon-Photon Vertex



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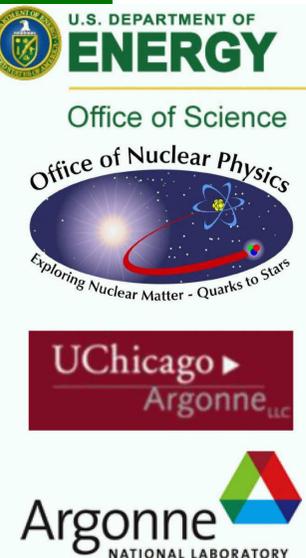
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M. Oettel, M. Pichowsky  
and L. von Smekal, nu-th/9909082

6 terms ...

# ***Nucleon-Photon Vertex***

constructed systematically ... current conserved automatically  
for on-shell nucleons described by Faddeev Amplitude



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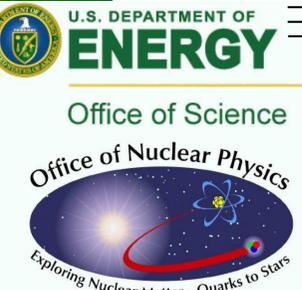
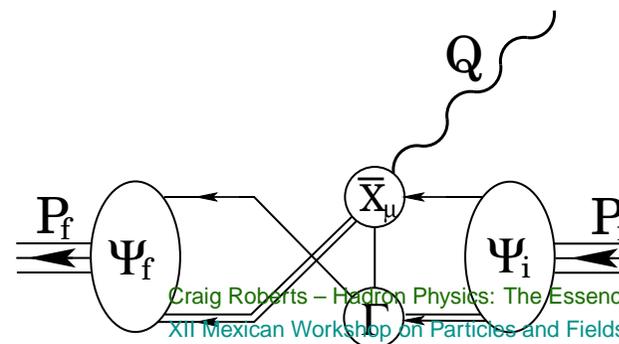
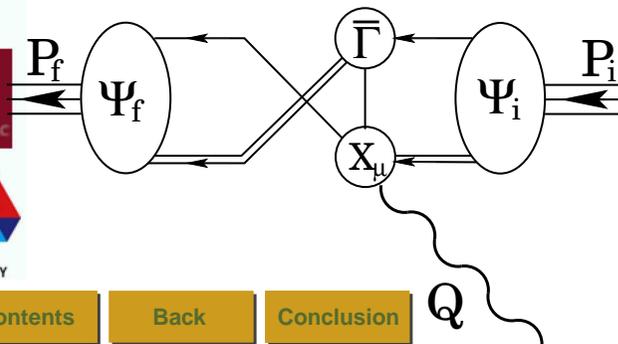
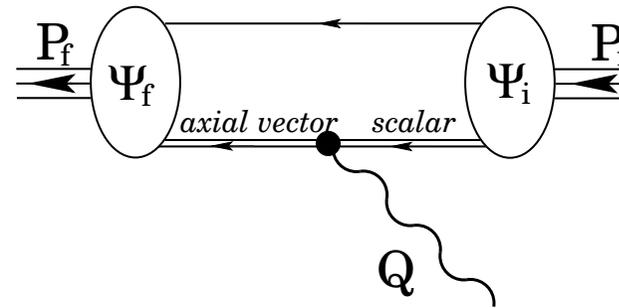
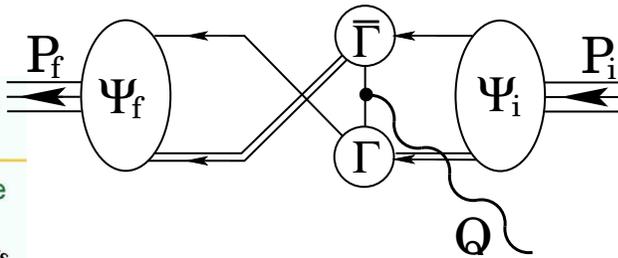
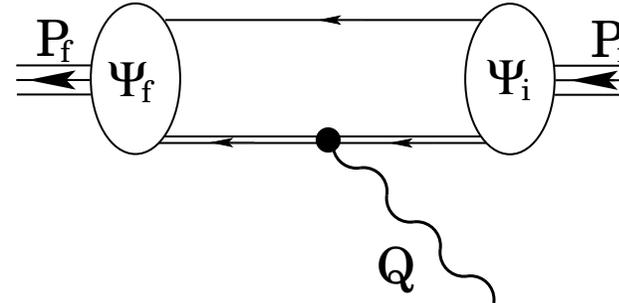
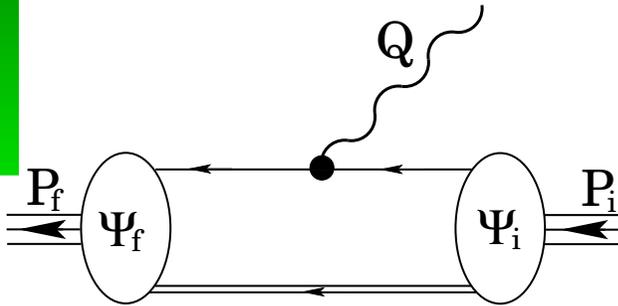
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# Nucleon-Photon Vertex

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Cloët, Roberts *et al.*

- arXiv:0710.2059 [nucl-th]
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$$\frac{\mu_n G_E(Q^2)}{G_M(Q^2)}$$

- arXiv:0812.0416 [nucl-th] – *Survey of nucleon EM form factors*



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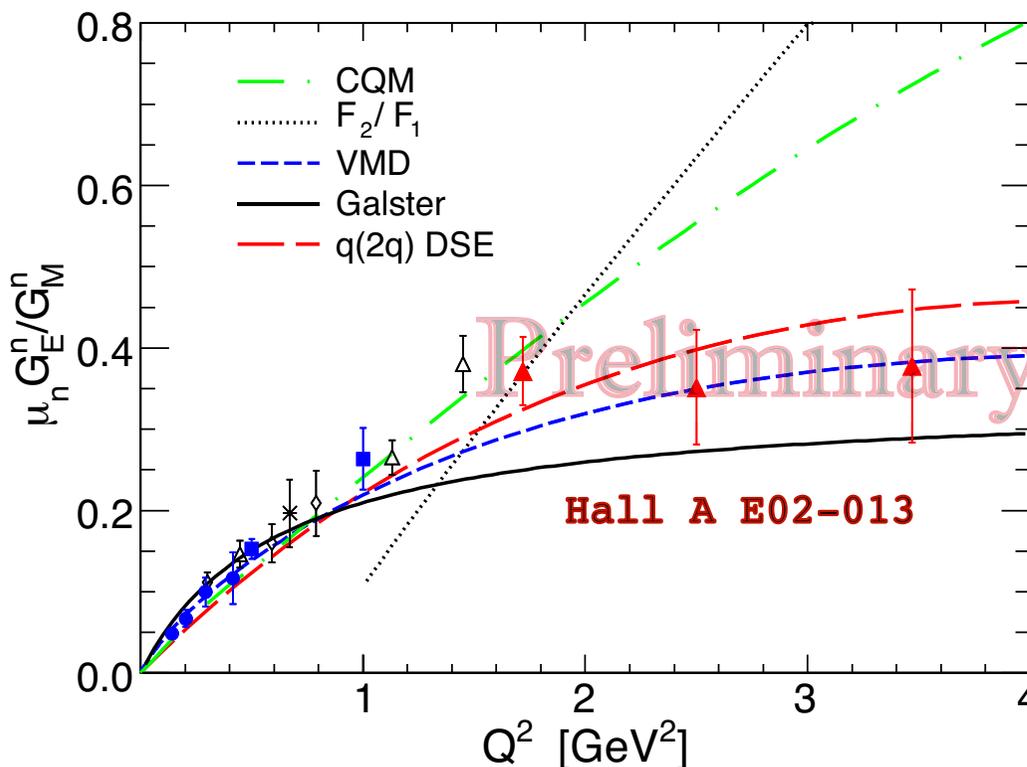
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● DSE-Faddeev Equation prediction



Jefferson Lab E02-013 Collaboration, *in preparation.*

Bogdan Wojtsekhowski



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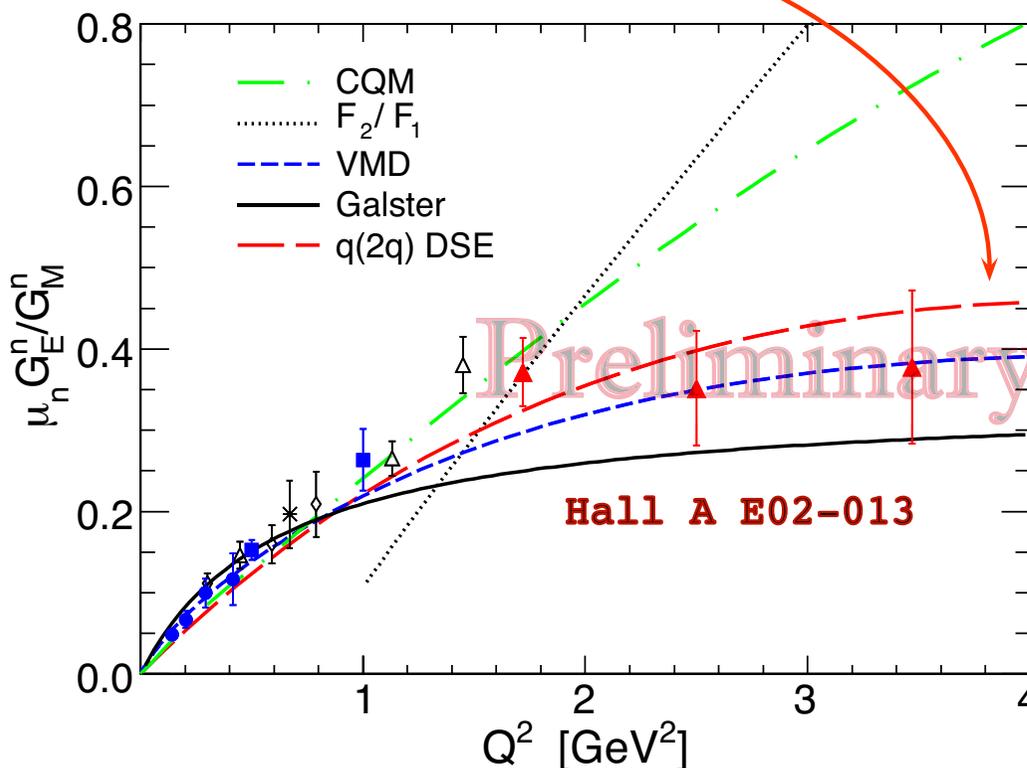
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Red long-dashed curve



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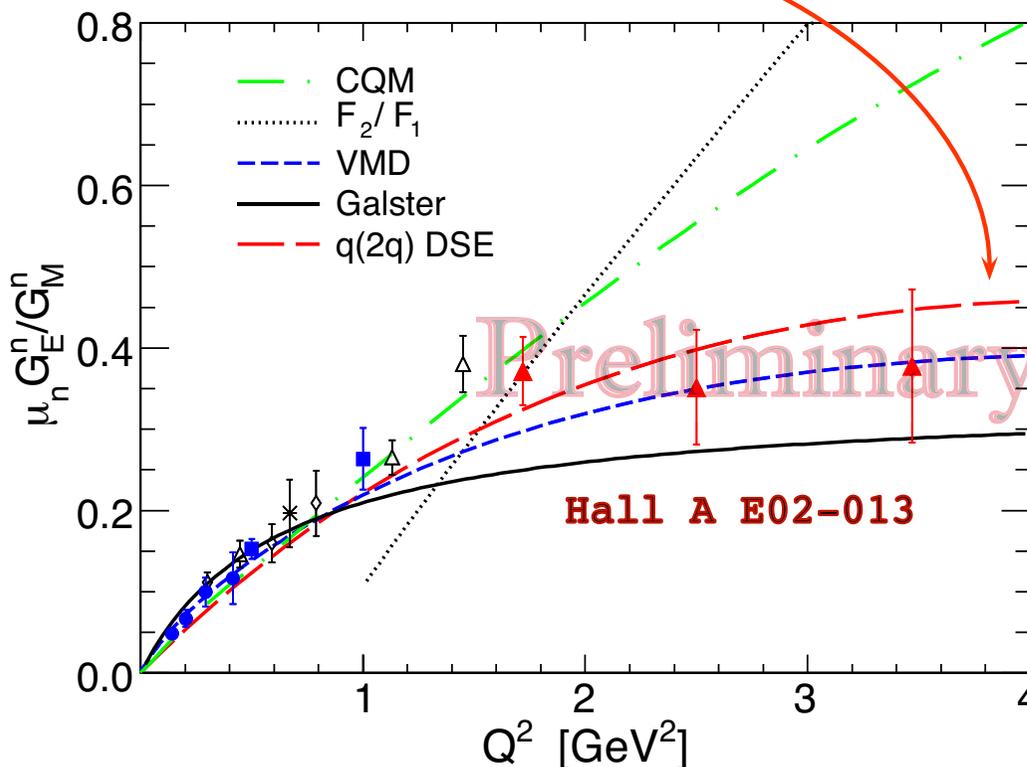
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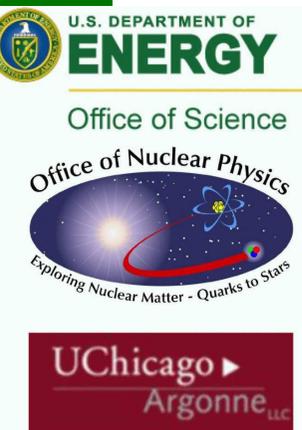
Red long-dashed curve



This evolution very sensitive to momentum dependence dressed-quark propagator

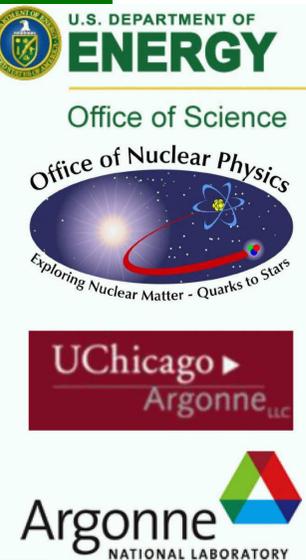
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Bogdan Wojtsekhowski



Maris, Roberts, Tandy  
nucl-th/9707003

# Goldberger-Treiman for pion



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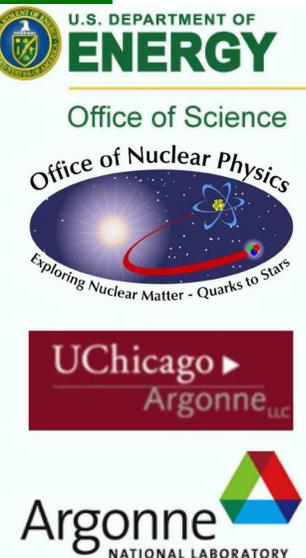
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# Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi j}(k; P) = \tau^{\pi j} \gamma_5 \left[ iE_{\pi}(k; P) + \gamma \cdot P F_{\pi}(k; P) \right. \\ \left. + \gamma \cdot k k \cdot P G_{\pi}(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} H_{\pi}(k; P) \right]$$

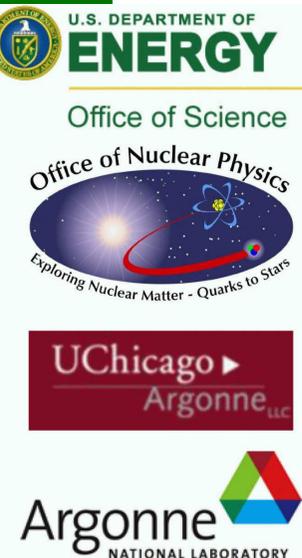


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- Dressed-quark Propagator:  $S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$



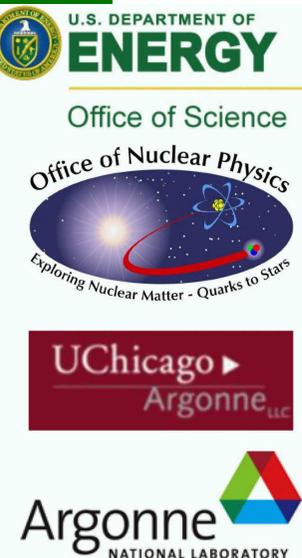
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$$\Rightarrow f_{\pi} E_{\pi}(k; P = 0) = B(p^2)$$



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$$F_R(k; 0) + 2 f_{\pi} F_{\pi}(k; 0) = A(k^2)$$

$$G_R(k; 0) + 2 f_{\pi} G_{\pi}(k; 0) = 2A'(k^2)$$

$$H_R(k; 0) + 2 f_{\pi} H_{\pi}(k; 0) = 0$$

# Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

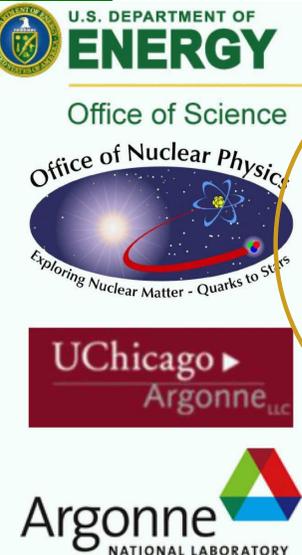
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Pseudovector components necessarily nonzero

- Dressed-quark Propagator:  $S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$
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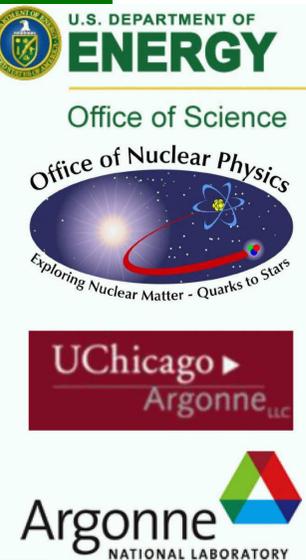
Exact in Chiral QCD

$$\begin{aligned} f_{\pi} E_{\pi}(k; P = 0) &= B(p^2) \\ F_R(k; 0) + 2 f_{\pi} F_{\pi}(k; 0) &= A(k^2) \\ G_R(k; 0) + 2 f_{\pi} G_{\pi}(k; 0) &= 2A'(k^2) \\ H_R(k; 0) + 2 f_{\pi} H_{\pi}(k; 0) &= 0 \end{aligned}$$



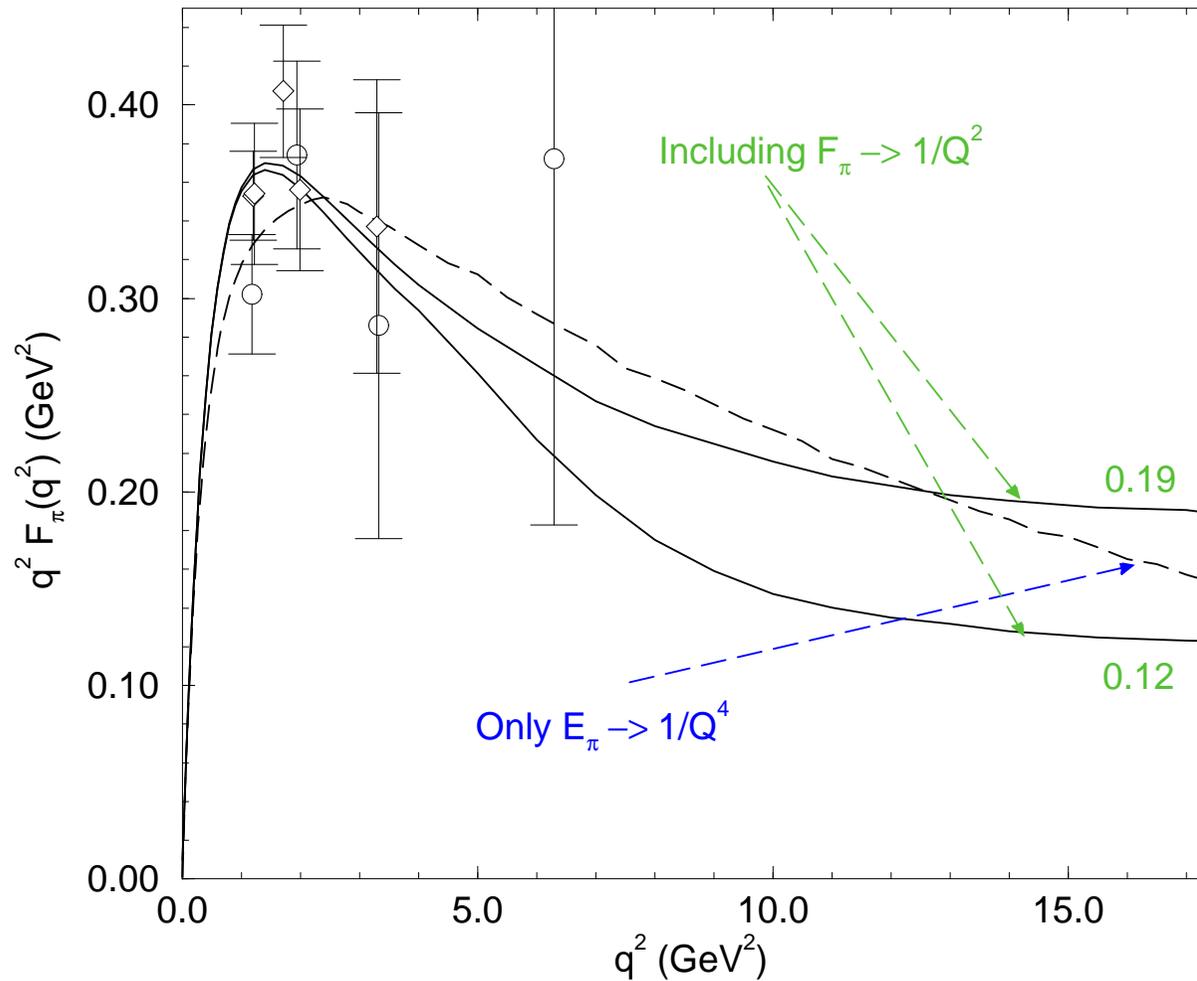
Maris, Roberts  
nucl-th/9804062

- What does this mean for observables?



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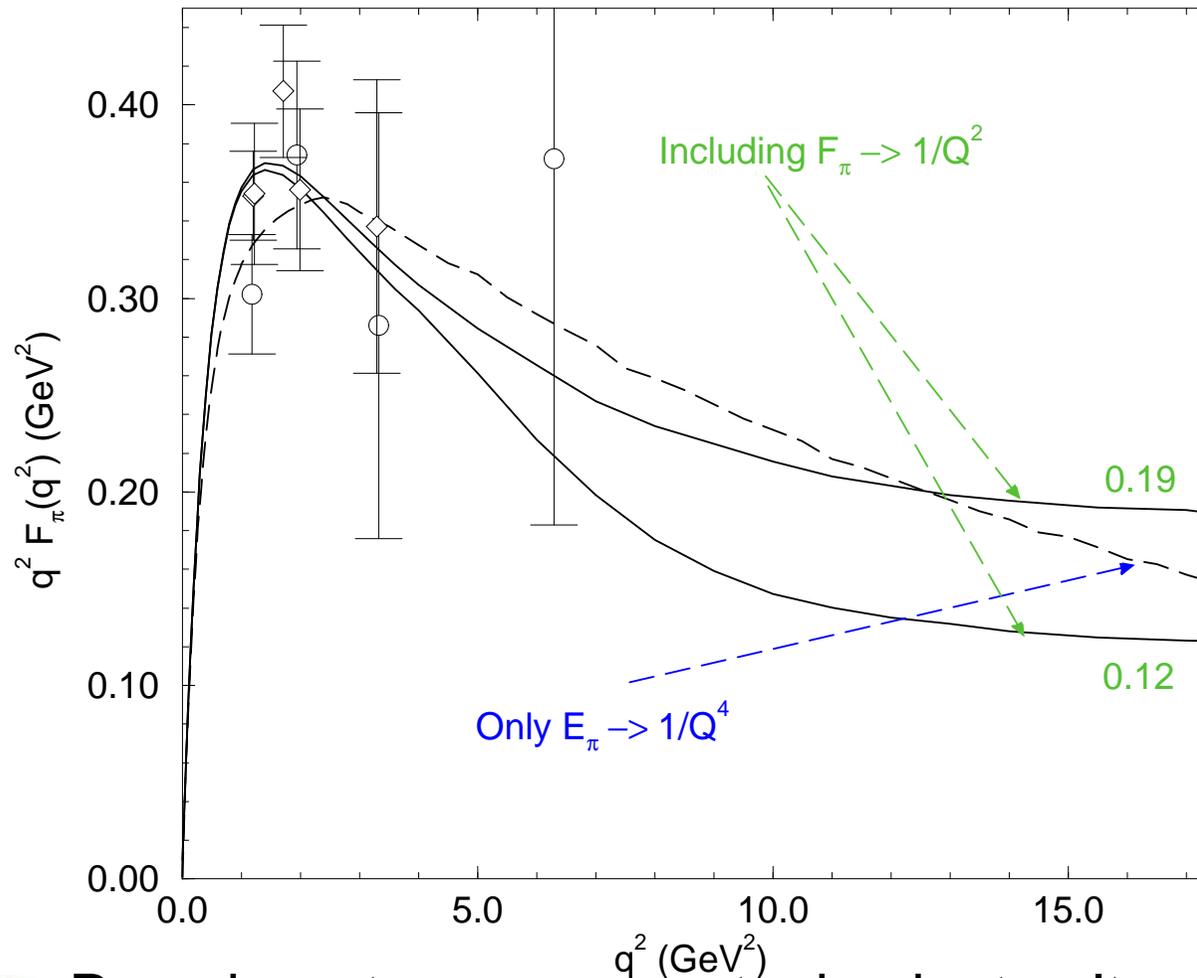
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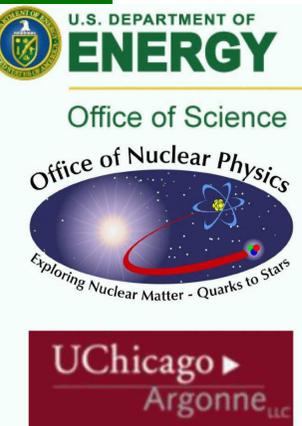
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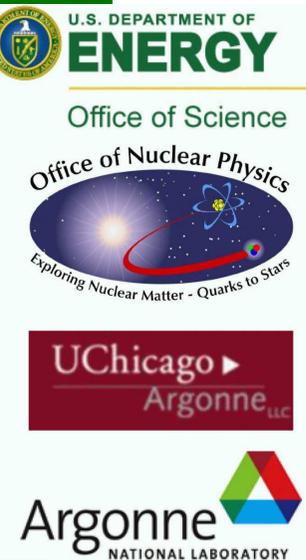


Pseudovector components dominate ultraviolet behaviour of electromagnetic form factor



Bashir, Gutierrez, Roberts:  
*in progress*

# *GT for pion – NJL*



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- Bethe-Salpeter amplitude can't depend on relative momentum

⇒ General Form 
$$\Gamma_{\pi}(P) = i\gamma_5 E_{\pi}(P) + \frac{1}{M_Q} \gamma \cdot P F_{\pi}(P)$$

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$$P^2 = 0 : M_Q = 0.40, E_\pi = 0.98, \frac{F_\pi}{M_Q} = 0.50$$



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- RHS Bethe-Salpeter equation:

$$\gamma_\mu S(k + P/2) i\gamma_5 E_\pi S(k - P/2) \gamma_\mu$$



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- Has pseudovector component

$$\sim E_\pi [\sigma_S(k_+) \sigma_V(k_-) + \sigma_S(k_-) \sigma_V(k_+)]$$

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- Hence  $F_\pi$  on LHS is forced to be nonzero because  $E_\pi$  on RHS is nonzero owing to DCSB



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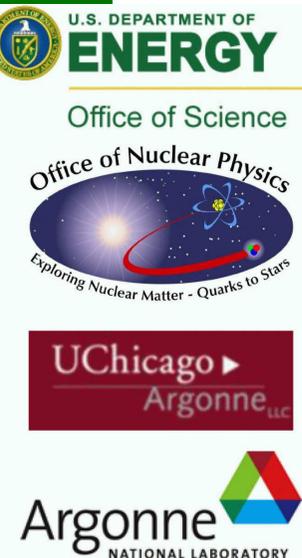
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- Asymptotic form of electromagnetic pion form factor



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- Asymptotic form of electromagnetic pion form factor

- $E_{\pi}^2$ -term  $\Rightarrow F_{\pi E}^{\text{em}}(Q^2) \sim \frac{M^2}{Q^2}$ , photon( $Q$ )



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- $E_{\pi} F_{\pi}$ -term.



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pion( $P = (0, 0, -Q/2, iQ/2)$ )



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$$\text{pion}(P = (0, 0, -Q/2, iQ/2))$$

$$F_{\pi EF}^{\text{em}}(Q^2) \sim 2 S \gamma \cdot (P + Q) F_{\pi} S \gamma_4 S E_{\pi}$$

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- Asymptotic form of electromagnetic pion form factor

- $E_{\pi}^2$ -term  $\Rightarrow F_{\pi E}^{\text{em}}(Q^2) \sim \frac{M^2}{Q^2}$ , photon( $Q$ )

- $E_{\pi} F_{\pi}$ -term. Breit Frame:

pion( $P = (0, 0, -Q/2, iQ/2)$ )

$$F_{\pi EF}^{\text{em}}(Q^2) \sim 2 S \gamma \cdot (P + Q) F_{\pi} S \gamma_4 S E_{\pi}$$

$$\Rightarrow F_{\pi EF}^{\text{em}}(Q^2) \propto \frac{Q^2}{M_Q^2} \frac{F_{\pi}}{E_{\pi}} \times E_{\pi}^2\text{-term} = \text{constant!}$$

- Bethe-Salpeter amplitude: General Form

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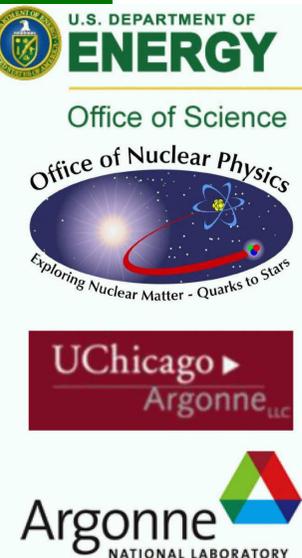
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- This behaviour dominates for  $Q^2 \gtrsim M_Q^2 \frac{E_{\pi}}{F_{\pi}} > 0.8 \text{ GeV}^2$

# Computation: Elastic Pion Form Factor

Bashir, Gutierrez, Roberts:  
*in progress*

- DSE prediction:  $M(p^2)$ ; i.e., interaction  $\frac{1}{|x - y|^2}$
- cf.  $M(p^2) = \text{Constant}$ ; i.e., interaction  $\delta^4(x - y)$

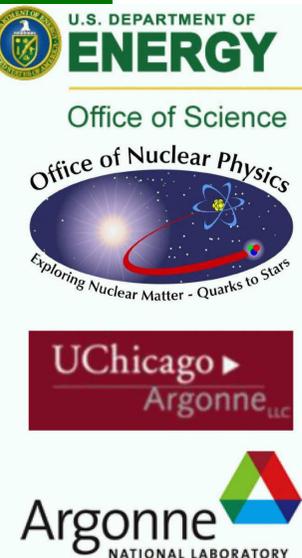


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Single mass-scale parameter  
in both studies



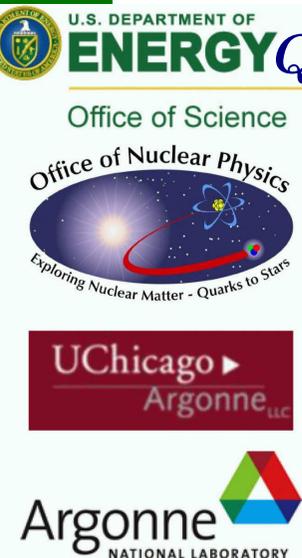
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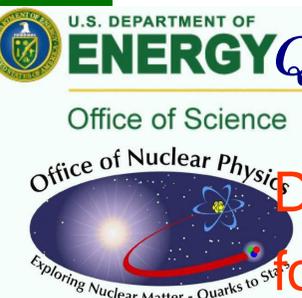
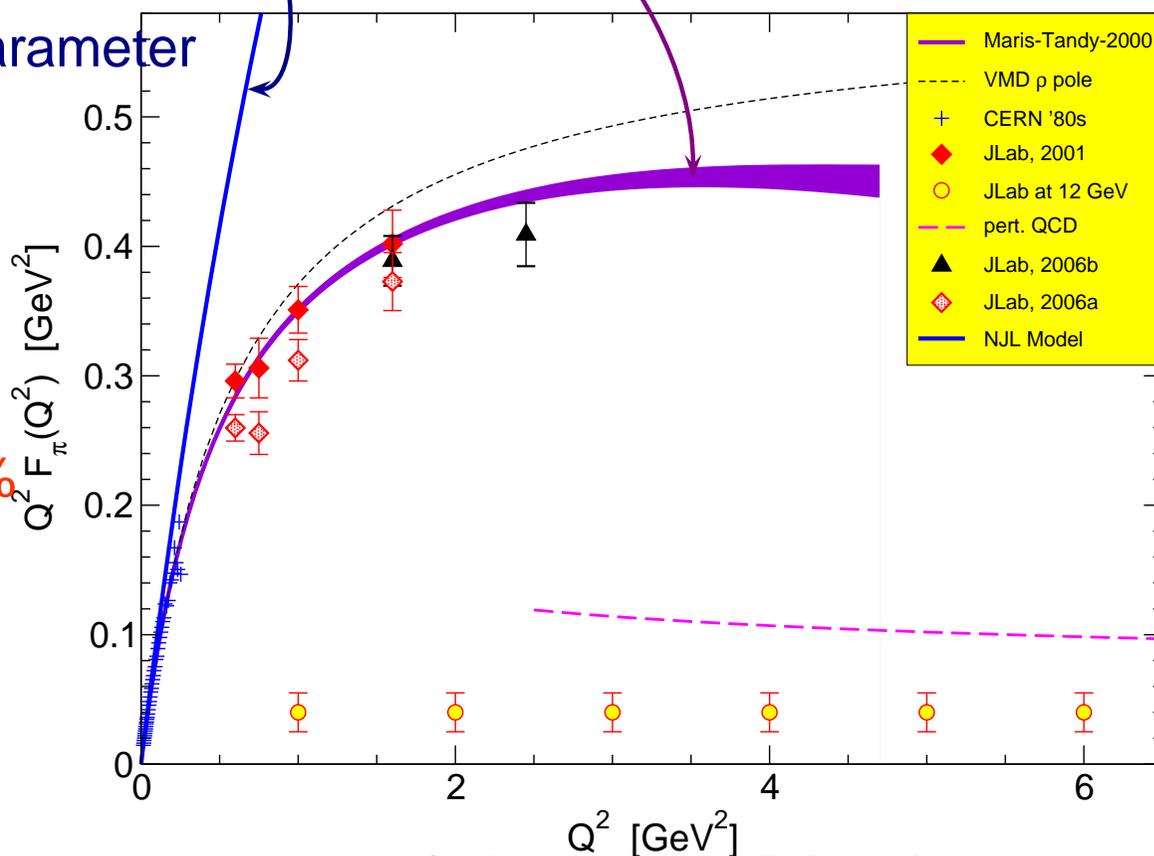
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Single mass-scale parameter  
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Same predictions for  
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Disagreement > 20%  
for  $Q^2 > 0.2 \text{ GeV}^2$



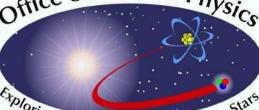
# Epilogue



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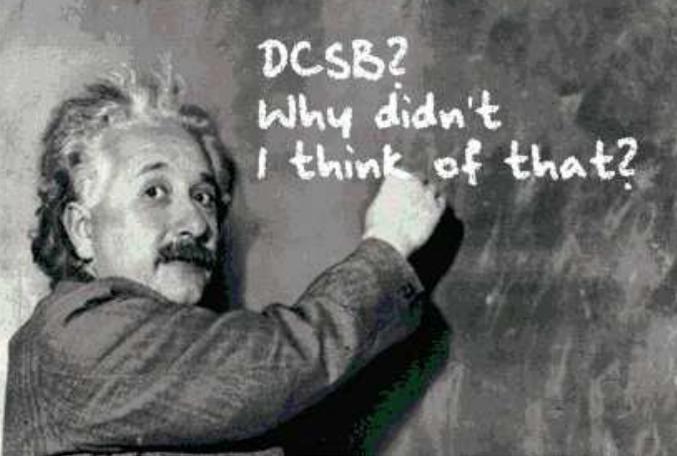
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- DCSB exists in QCD.

## *Epilogue*

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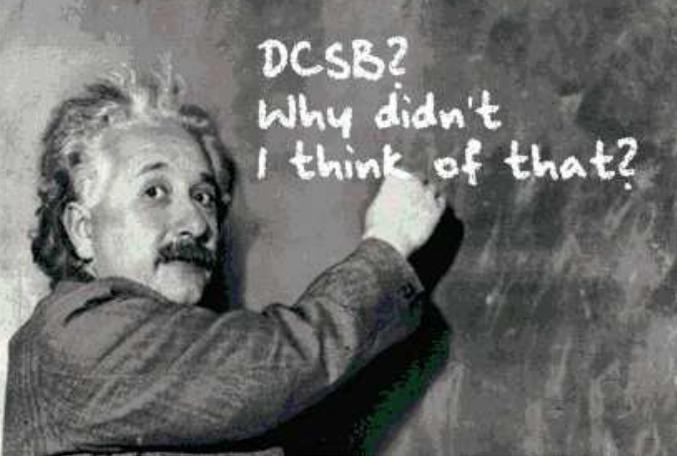
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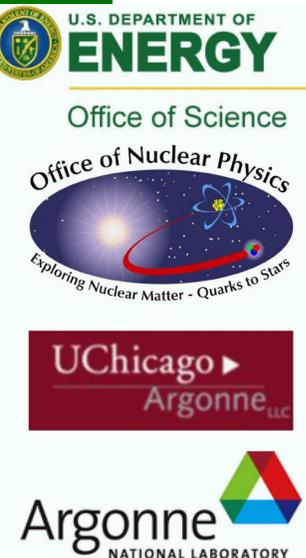
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- DCSB exists in QCD.
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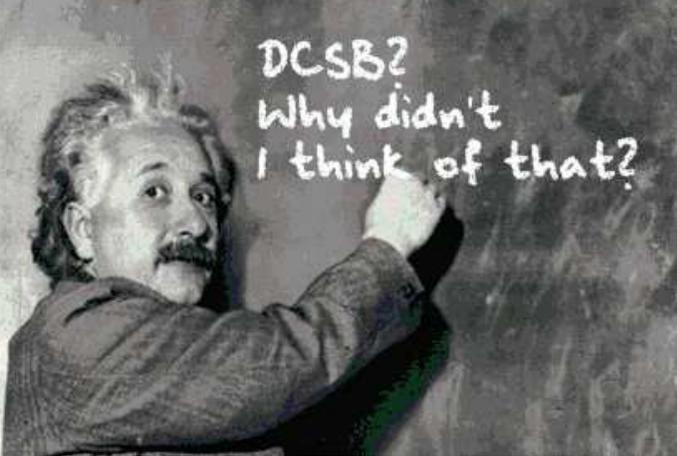


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## Epilogue

### ● DCSB exists in QCD.

- It is manifest in dressed propagators and vertices
- It predicts, amongst other things, that
  - light current-quarks become heavy constituent-quarks:  $4 \rightarrow 400 \text{ MeV}$
  - pseudoscalar mesons are unnaturally light:  $m_\rho = 770$  cf.  $m_\pi = 140 \text{ MeV}$
  - pseudoscalar mesons couple unnaturally strongly to light-quarks:  $g_{\pi\bar{q}q} \approx 4.3$
  - pseudoscalar mesons couple unnaturally strongly to the lightest baryons

$$g_{\pi\bar{N}N} \approx 12.8 \approx 3g_{\pi\bar{q}q}$$



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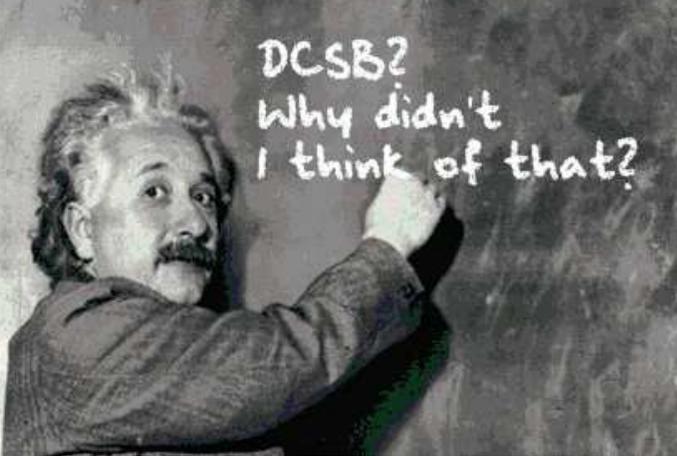


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# Epilogue

- DCSB impacts dramatically upon observables

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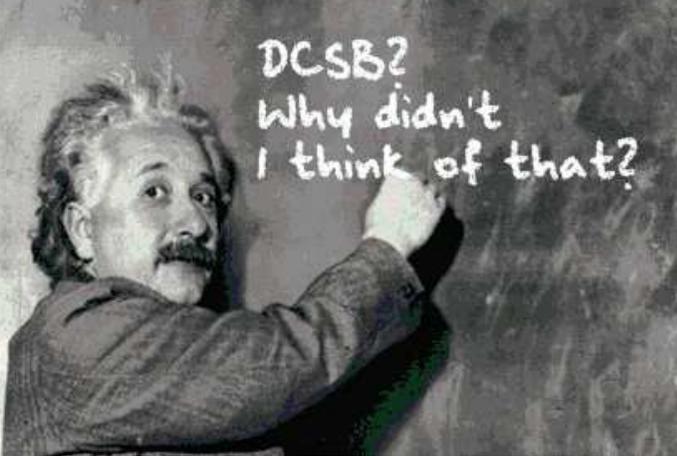
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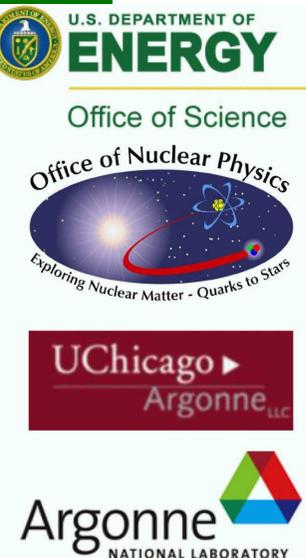
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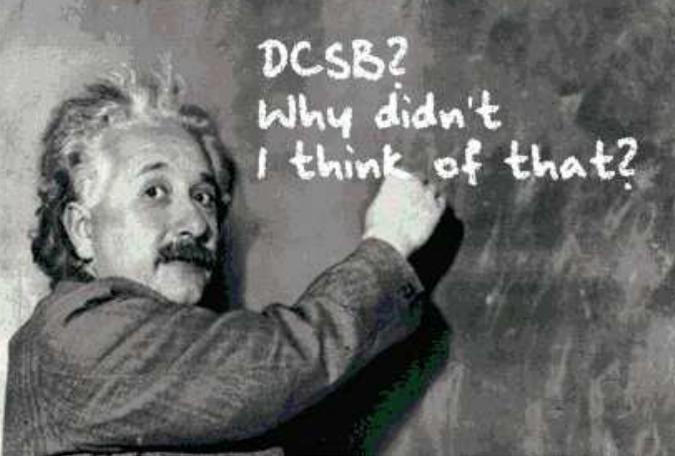


## Epilogue

- DCSB impacts dramatically upon observables
  - Spectrum; e.g., splittings:  $\sigma-\pi$  &  $a_1-\rho$
  - Form Factors; e.g., e.m. pion and nucleon

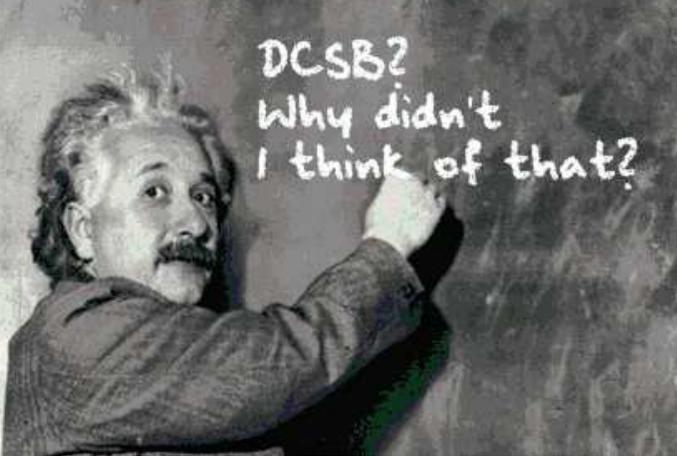


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  - Exposing & elucidating its effect in hadron physics requires nonperturbative, symmetry preserving framework; i.e., Poincaré covariance, chiral and e.m. current conservation, etc.





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- DSEs provide such a framework.
  - Studies underway will identify observable signals of  $M(p^2)$ , the most important mass-generating mechanism for visible matter in the Universe



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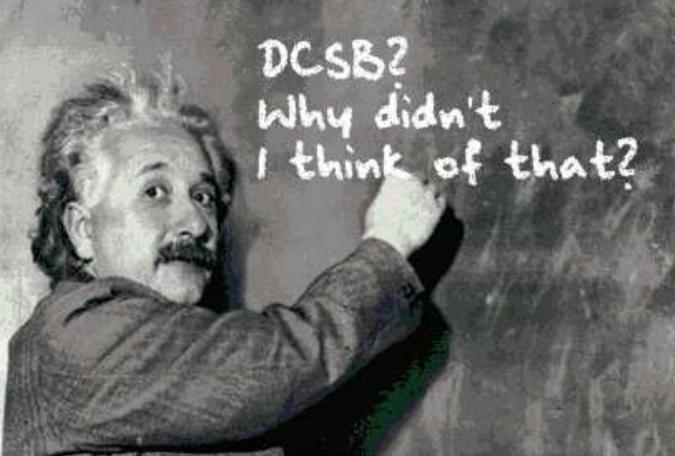


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- DSEs: Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks

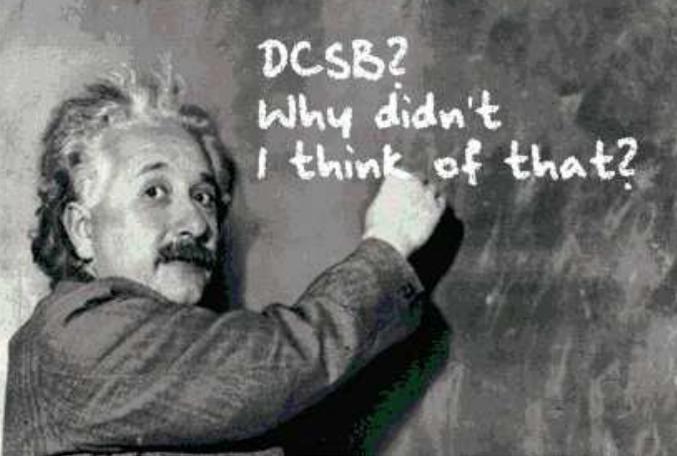


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# Epilogue

Now is an exciting time . . .

Positioned to unify phenomena as apparently disparate as

- Hadron spectrum
- Elastic and transition form factors, from small- to large- $Q^2$
- Parton structure functions



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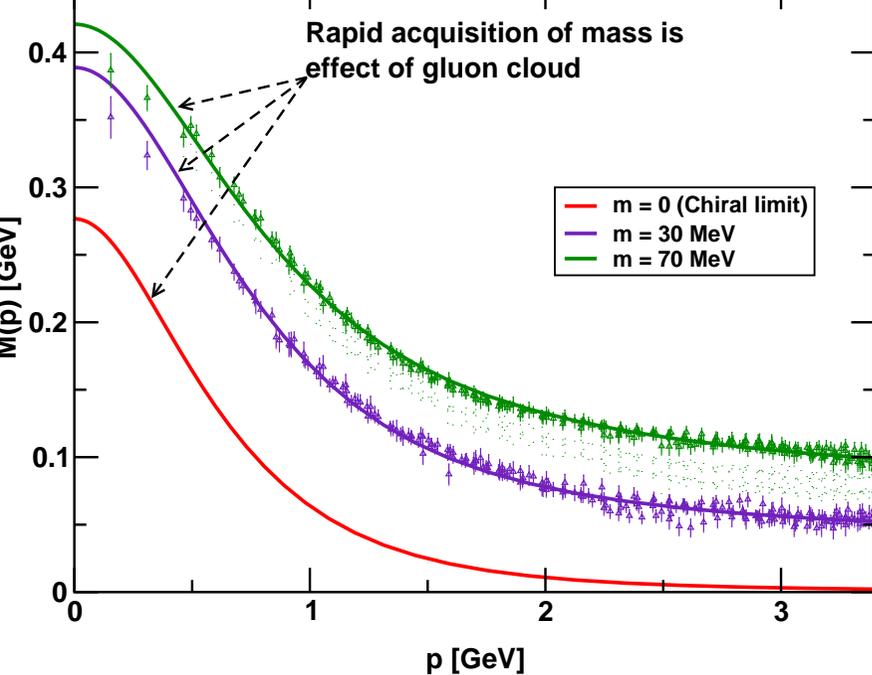


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Now is an exciting time . . .

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Key: an understanding of both the fundamental origin of nuclear mass and the far-reaching consequences of the mechanism responsible; namely, **Dynamical Chiral Symmetry Breaking**



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26. Pion Cloud



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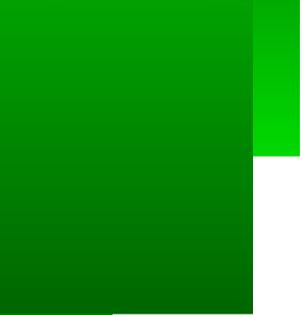
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# *DSE-based Faddeev Equation*



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Cloët *et al.*

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# DSE-based Faddeev Equation



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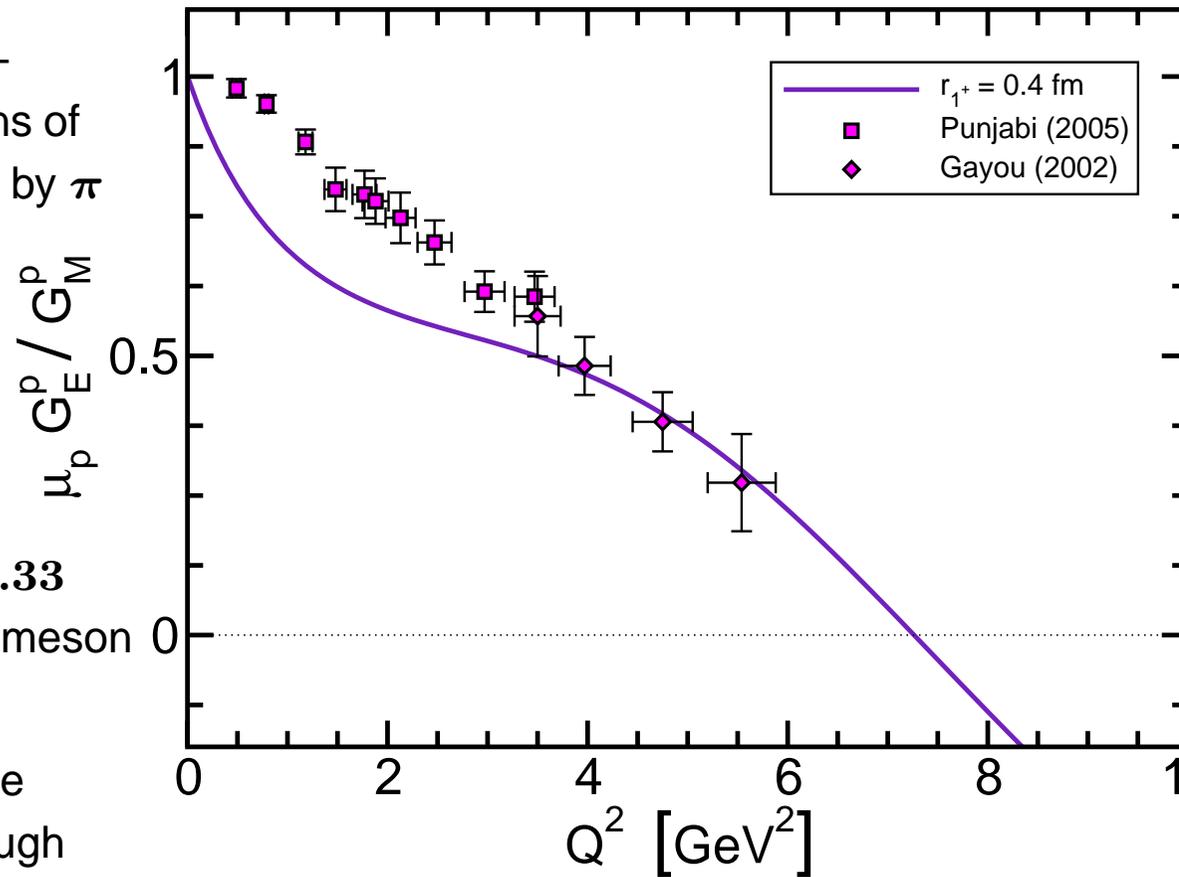
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  - chosen to give
  - $M_N = 1.18$ ,  $M_\Delta = 1.33$
  - allow for pseudoscalar meson contributions



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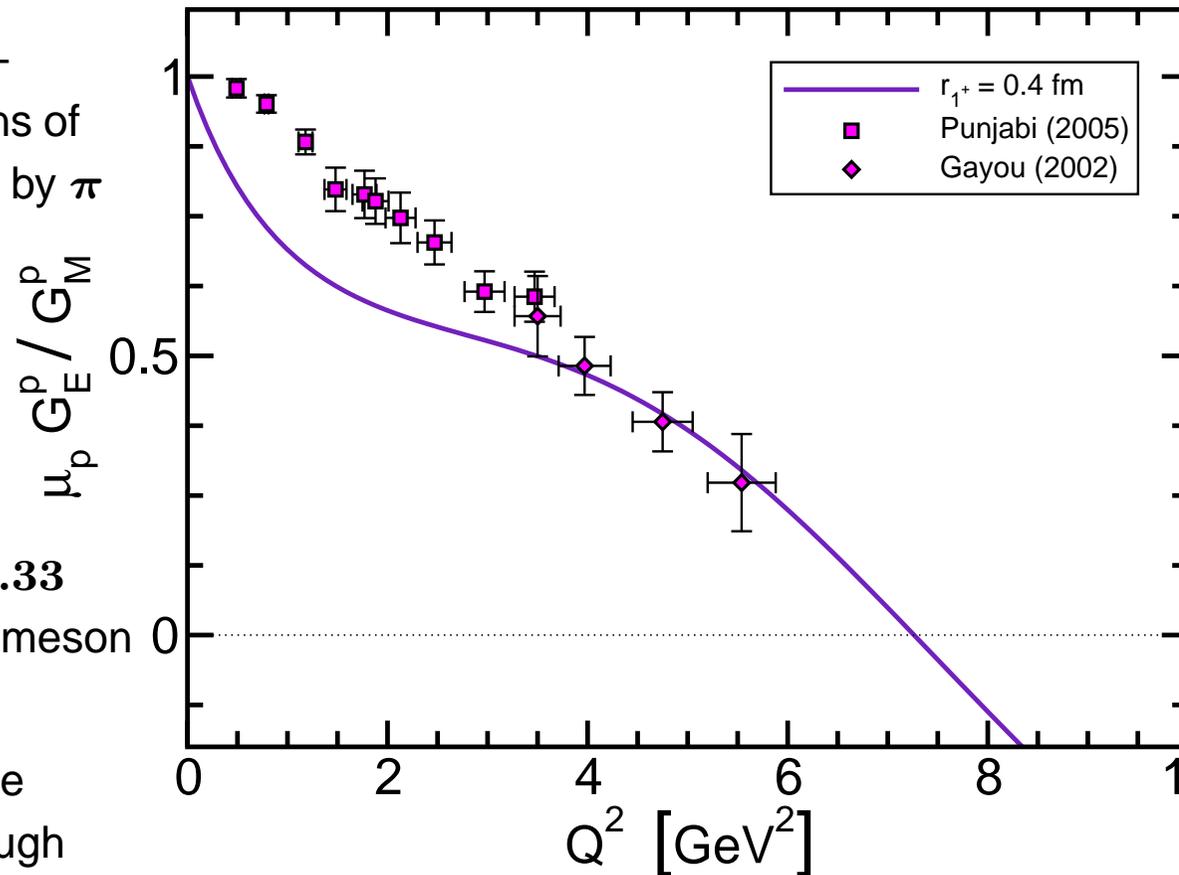
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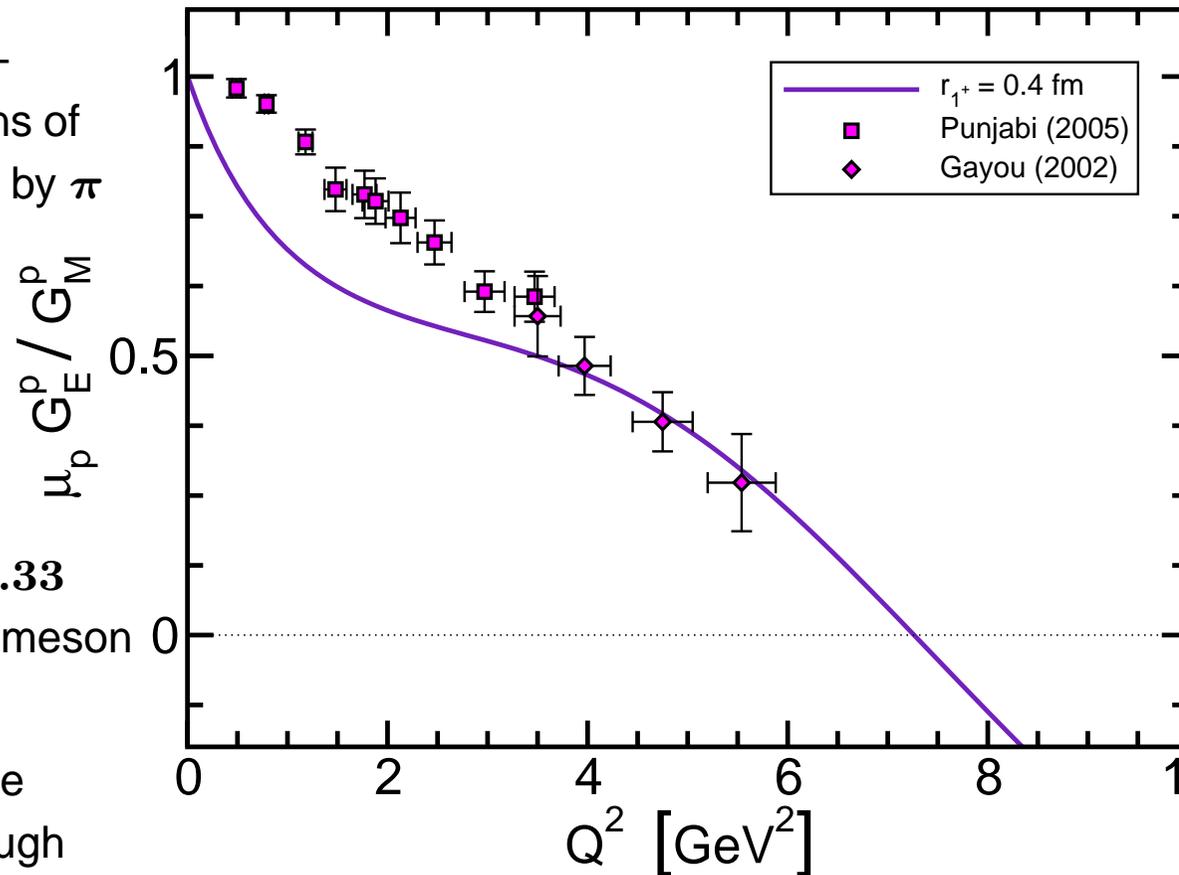
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- Always a zero but position depends on details of current

*Harry Lee*

# *Pions and Form Factors*

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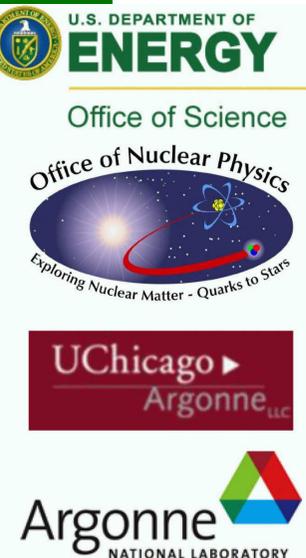
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# Pions and Form Factors

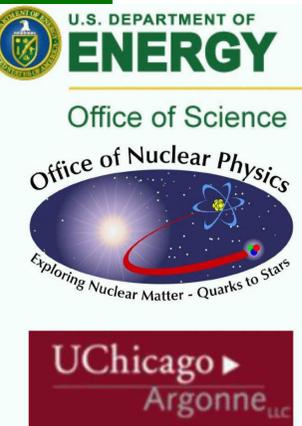
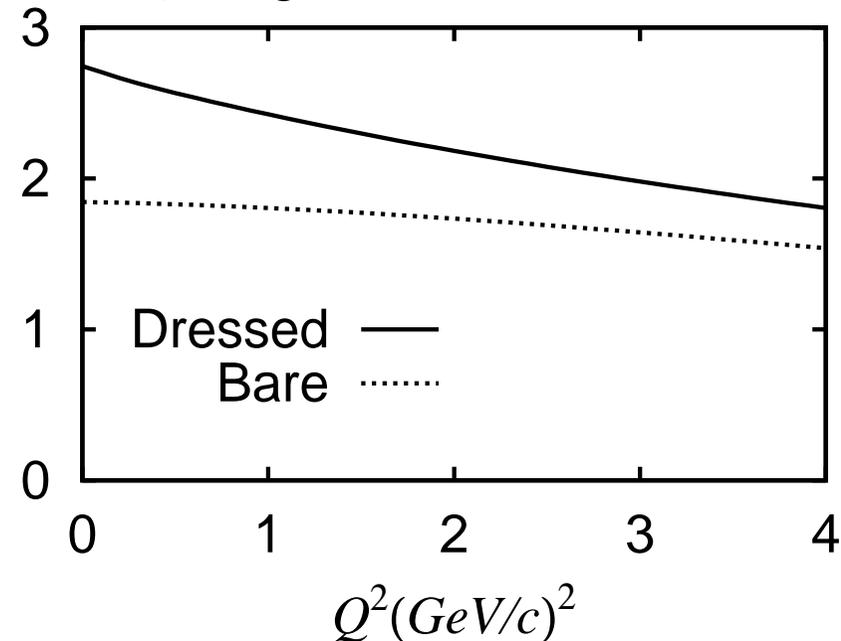
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*Ratio of the M1 form factor in  $\gamma N \rightarrow \Delta$  transition and proton dipole form factor  $G_D$ . Solid curve is  $G_M^*(Q^2)/G_D(Q^2)$  including pions; Dotted curve is  $G_M(Q^2)/G_D(Q^2)$  without pions.*



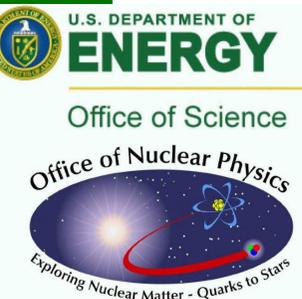
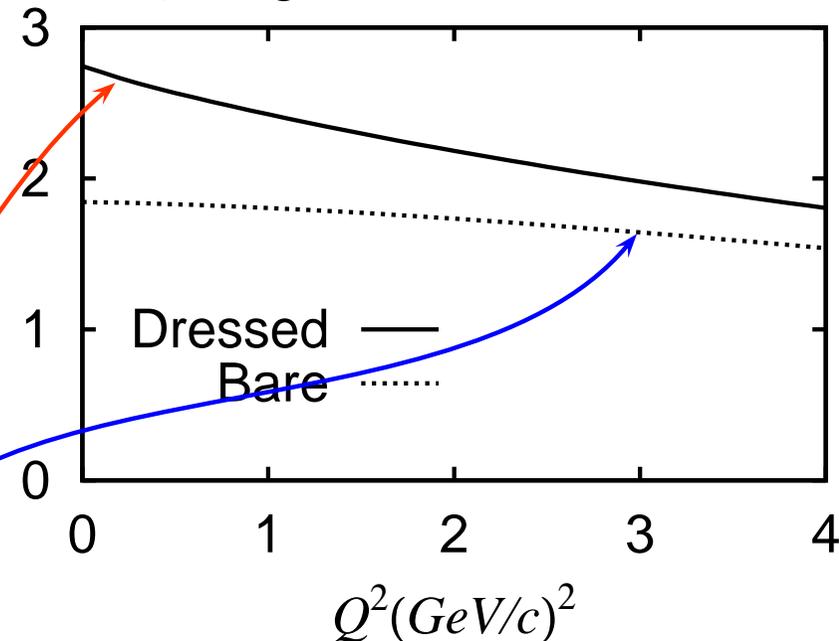
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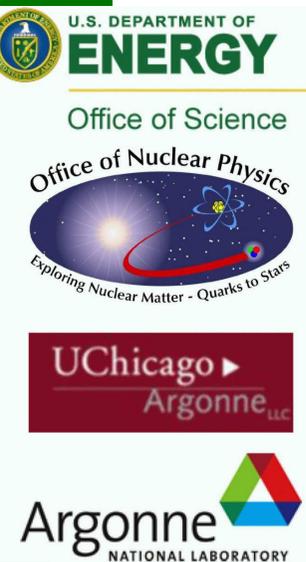
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Quark Core

- Responsible for only 2/3 of result at small  $Q^2$
- Dominant for  $Q^2 > 2 - 3 \text{ GeV}^2$





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# *Pion Cloud*

## *F2 – neutron*

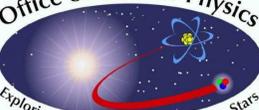
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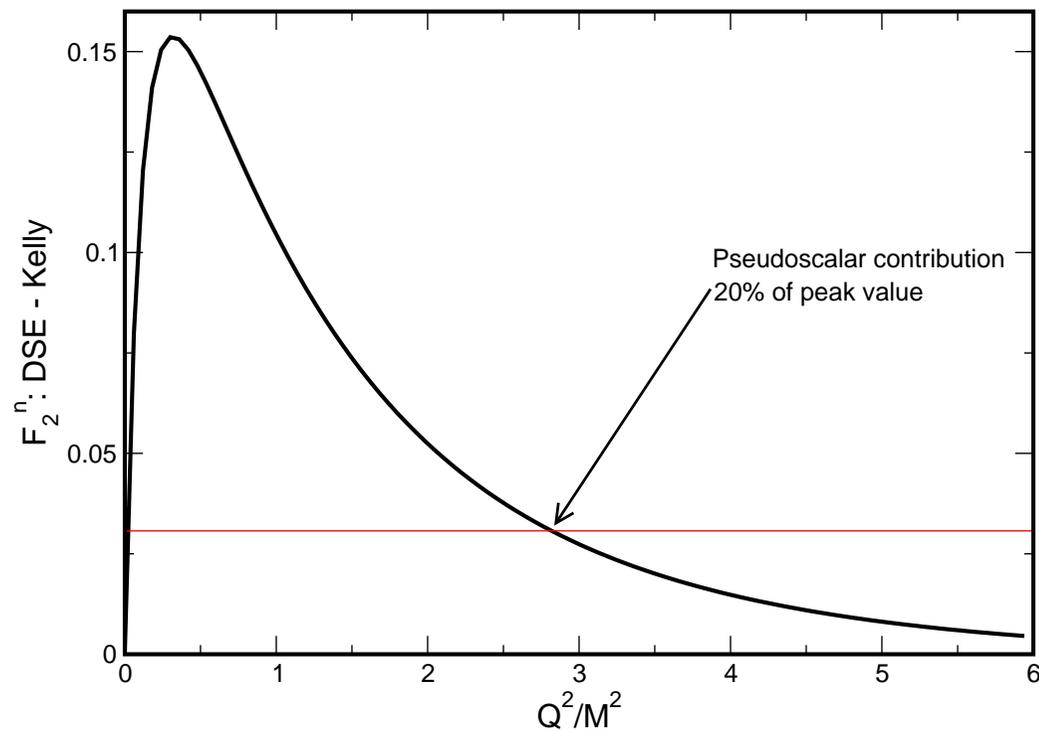
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# Pion Cloud

## F2 – neutron

- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core



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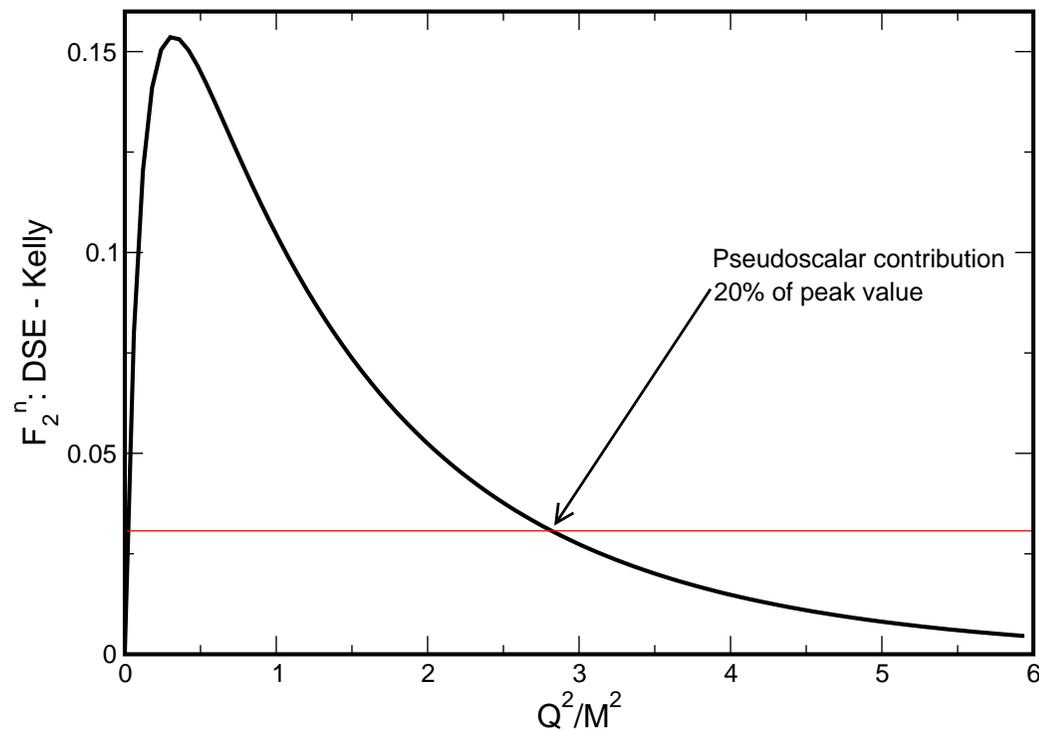
# Pion Cloud

## $F_2$ – neutron

- Comparison between Faddeev equation result and Kelly's parametrisation

- Faddeev equation set-up to describe dressed-quark core

- Pseudoscalar meson cloud (and related effects) significant for  $Q^2 \lesssim 3 - 4 M_N^2$



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