Exploring the Origin of Mass

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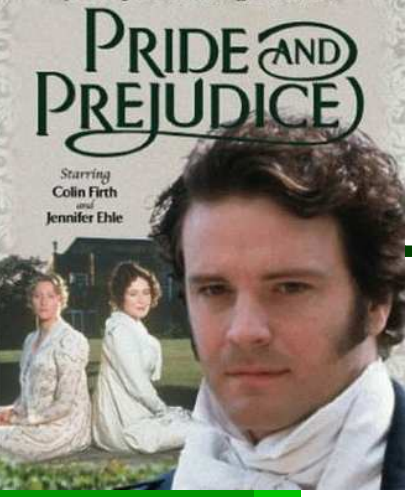
Argonne National Laboratory **Peking University**

http://www.phy.anl.gov/theory/staff/cdr.html

Universal Truths



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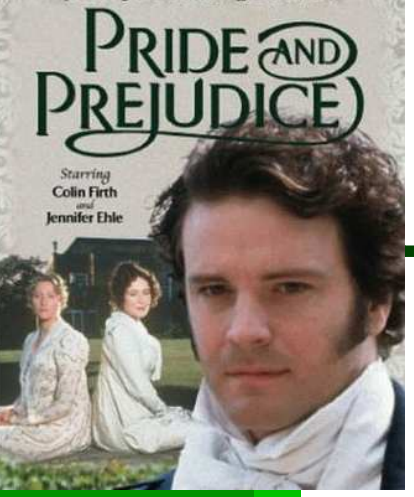

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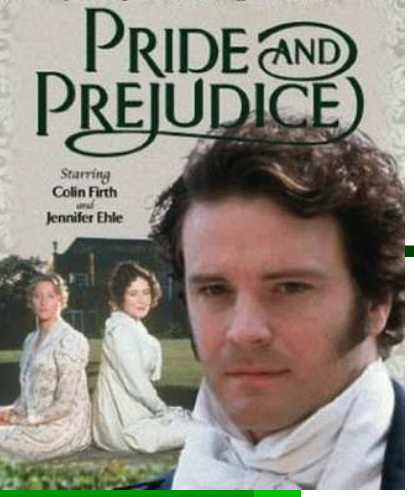
- Spectrum of excited states, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.



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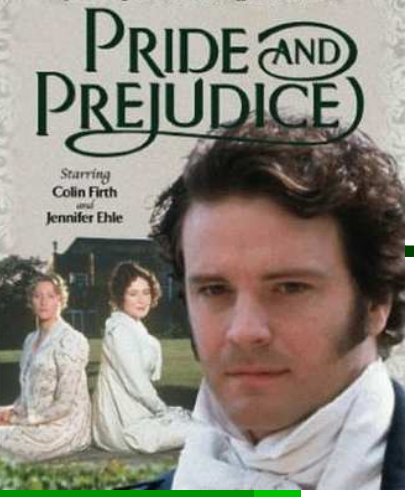

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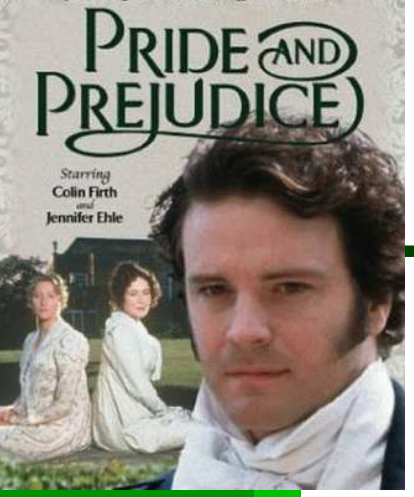

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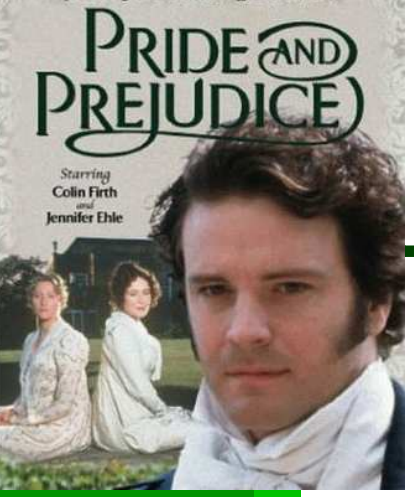

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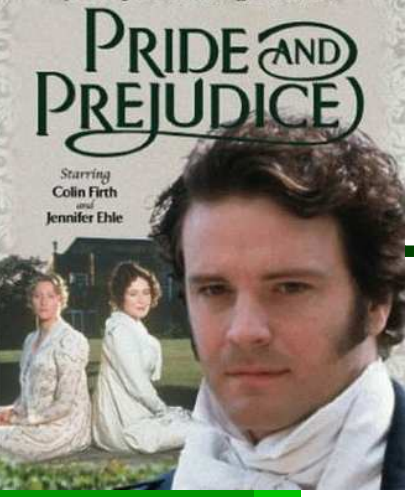

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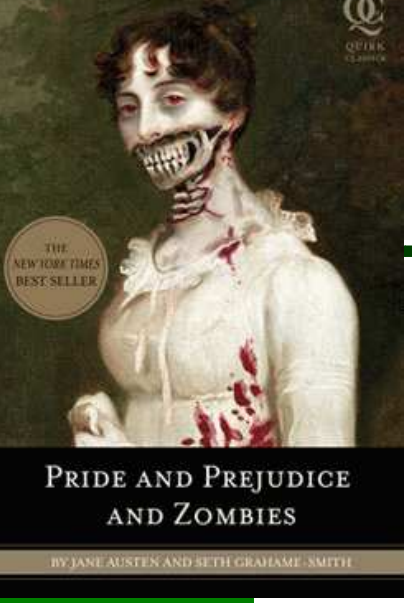

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QCD's Challenges

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- QCD – Complex behaviour arises from apparently simple rules

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Confinement

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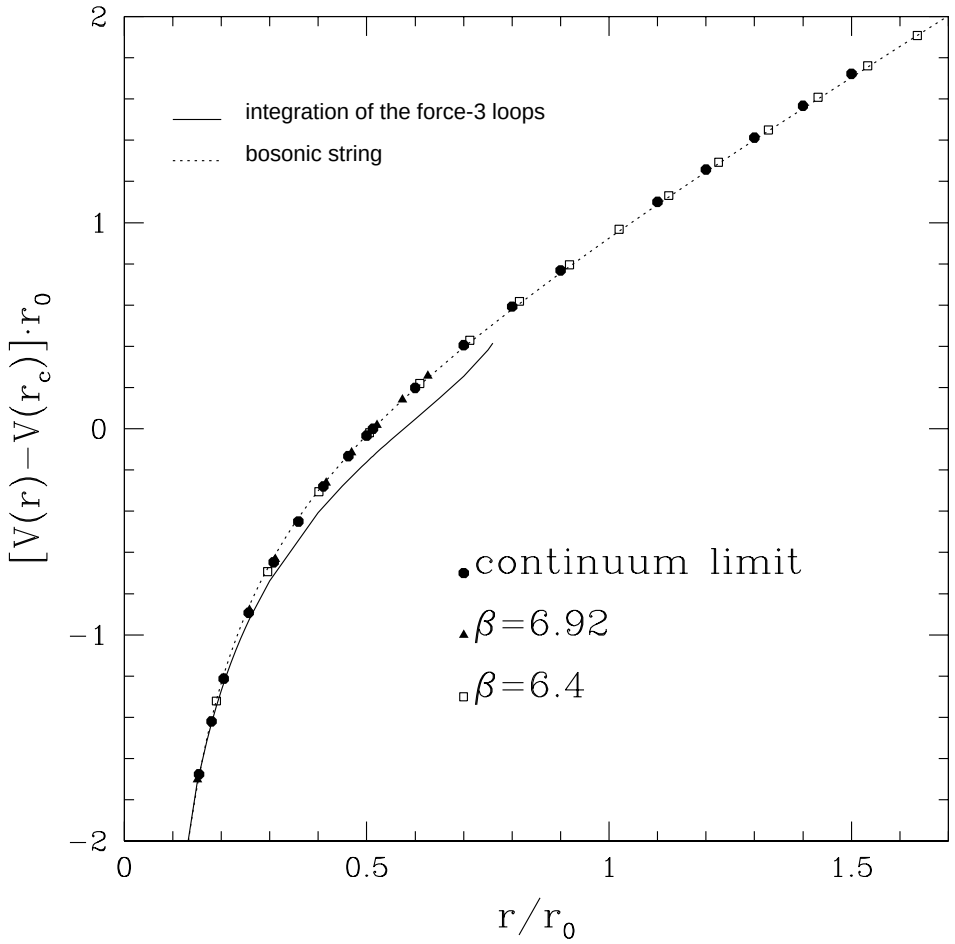
- Infinitely Heavy Quarks . . . Picture in Quantum Mechanics



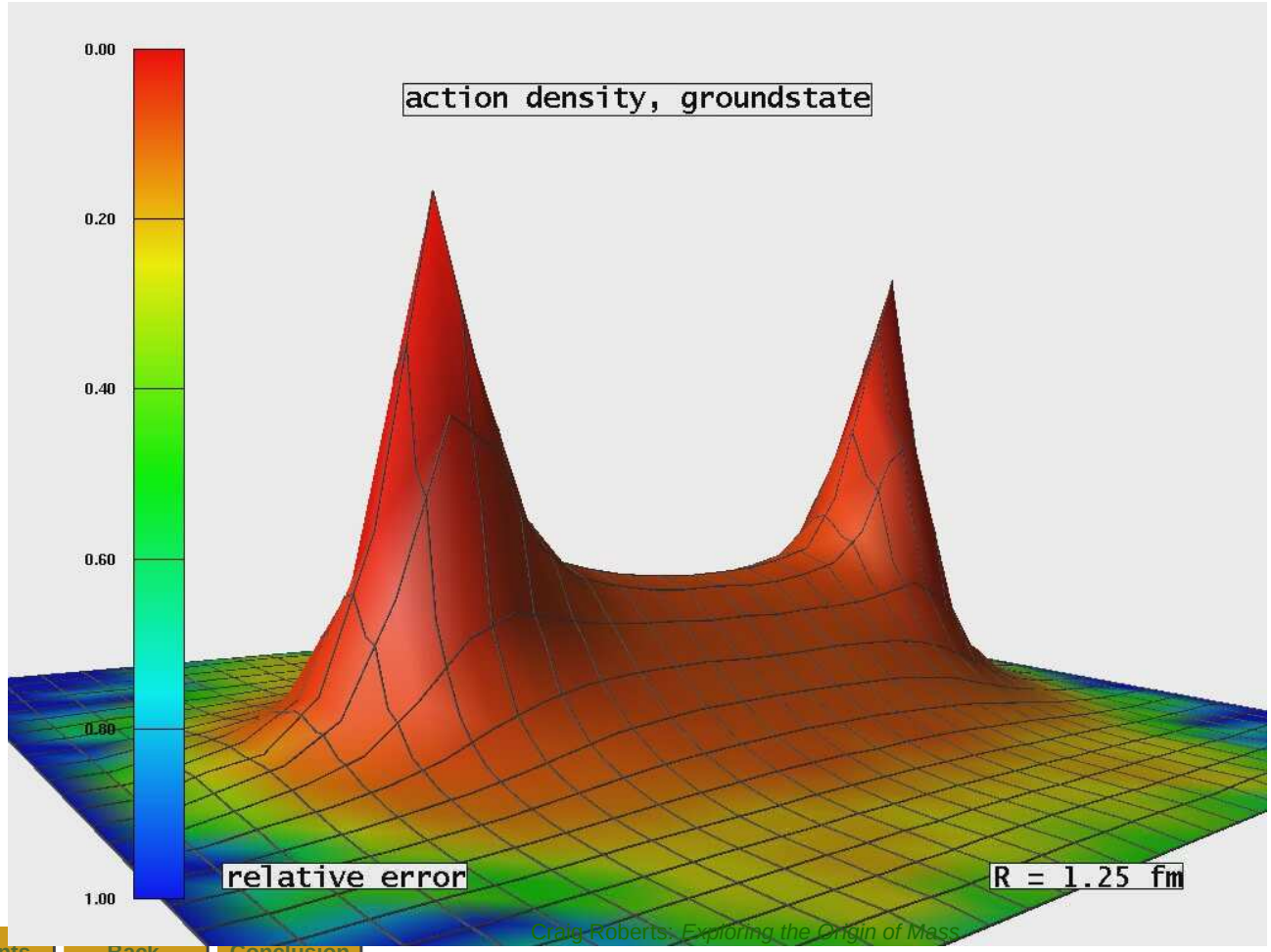
$$V(r) = \sigma\, r - \frac{\pi}{12}\,\frac{1}{r}$$

$$\sqrt{\sigma} \sim 470\,\text{MeV}$$

Necco & Sommer
he-la/0108008

Confinement

- Illustrate this in terms of the action density . . . analogous to plotting the Force $= F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12}\frac{1}{r^2}$



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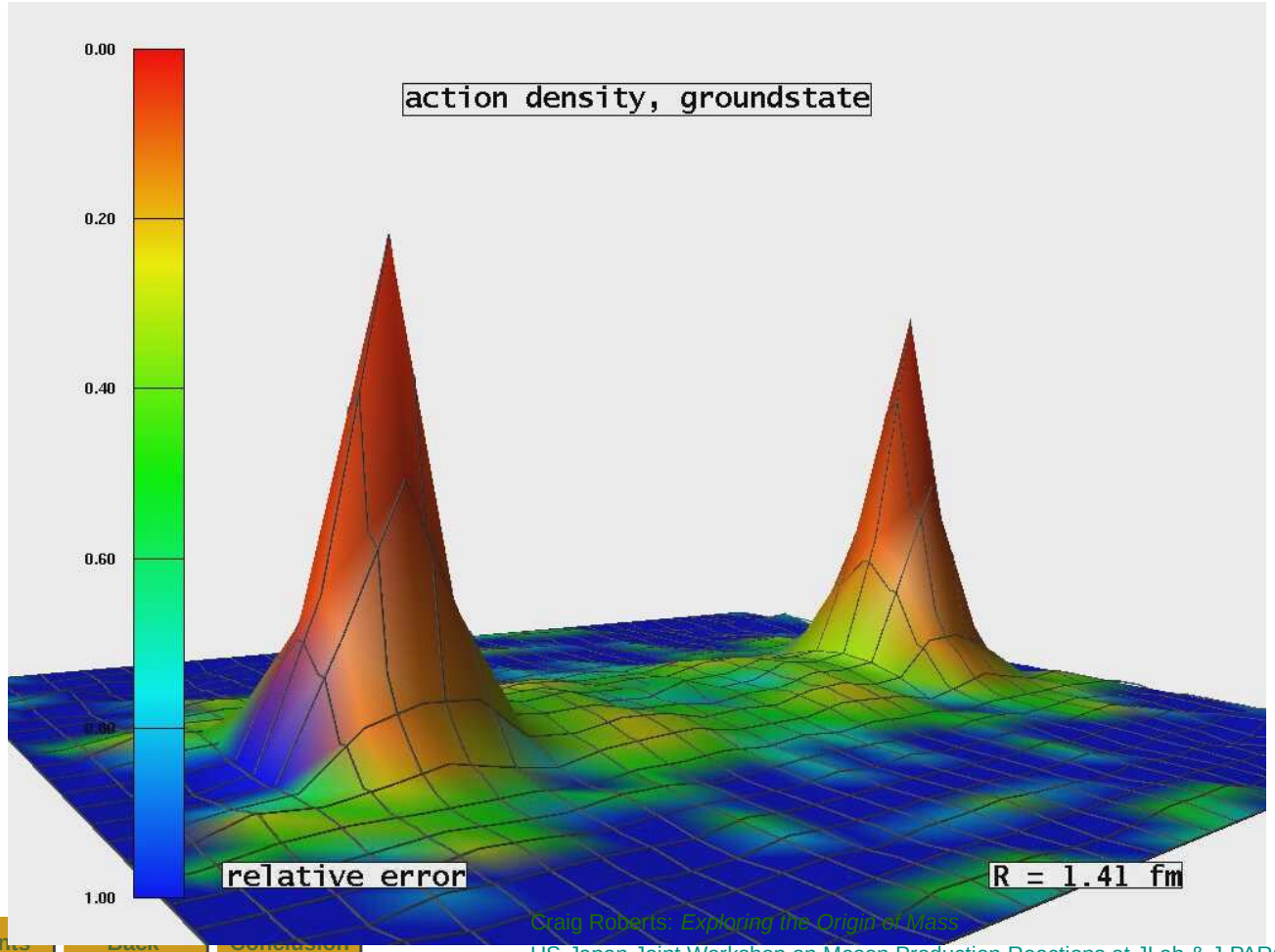

Bali, *et al.*
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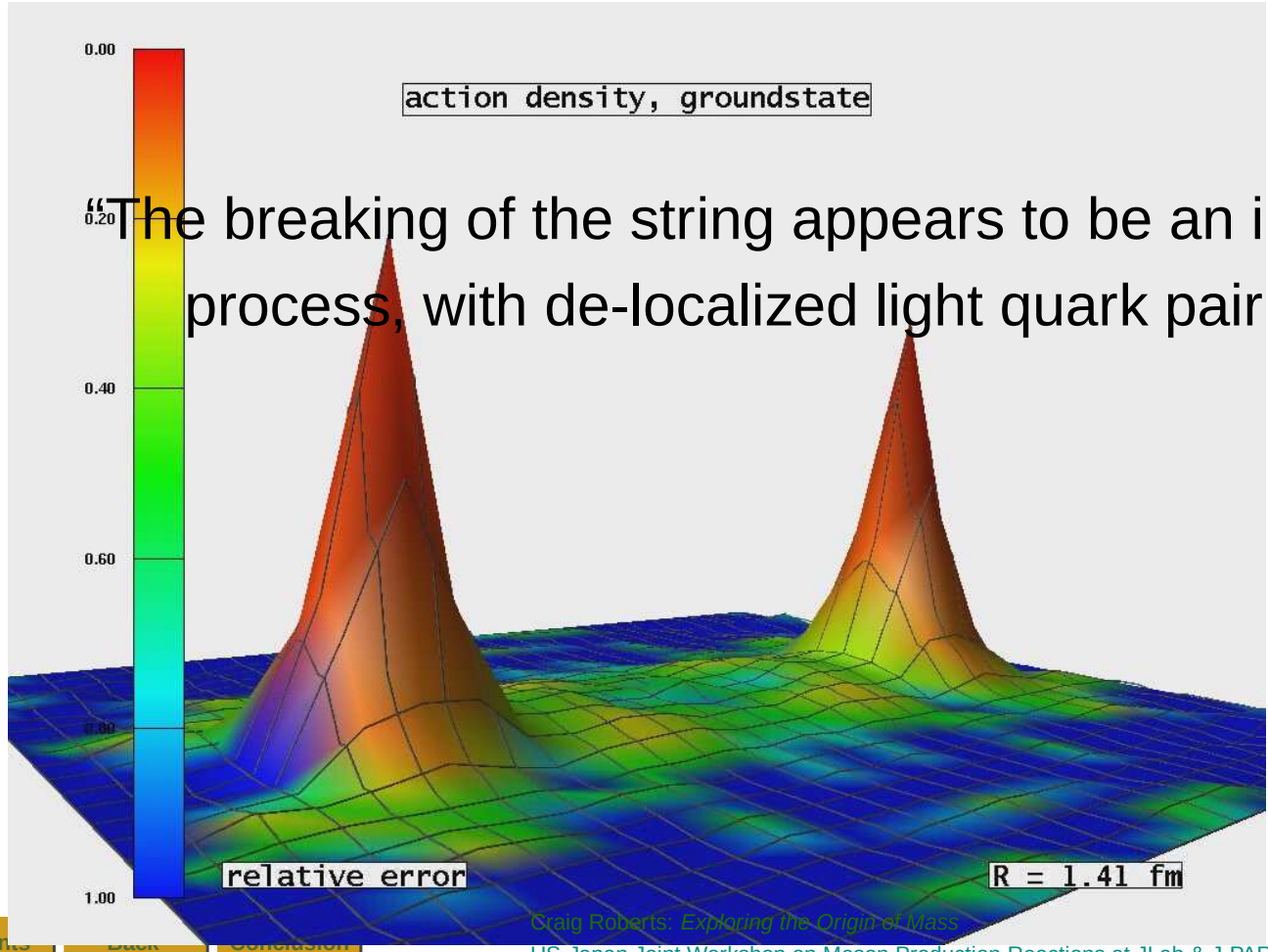

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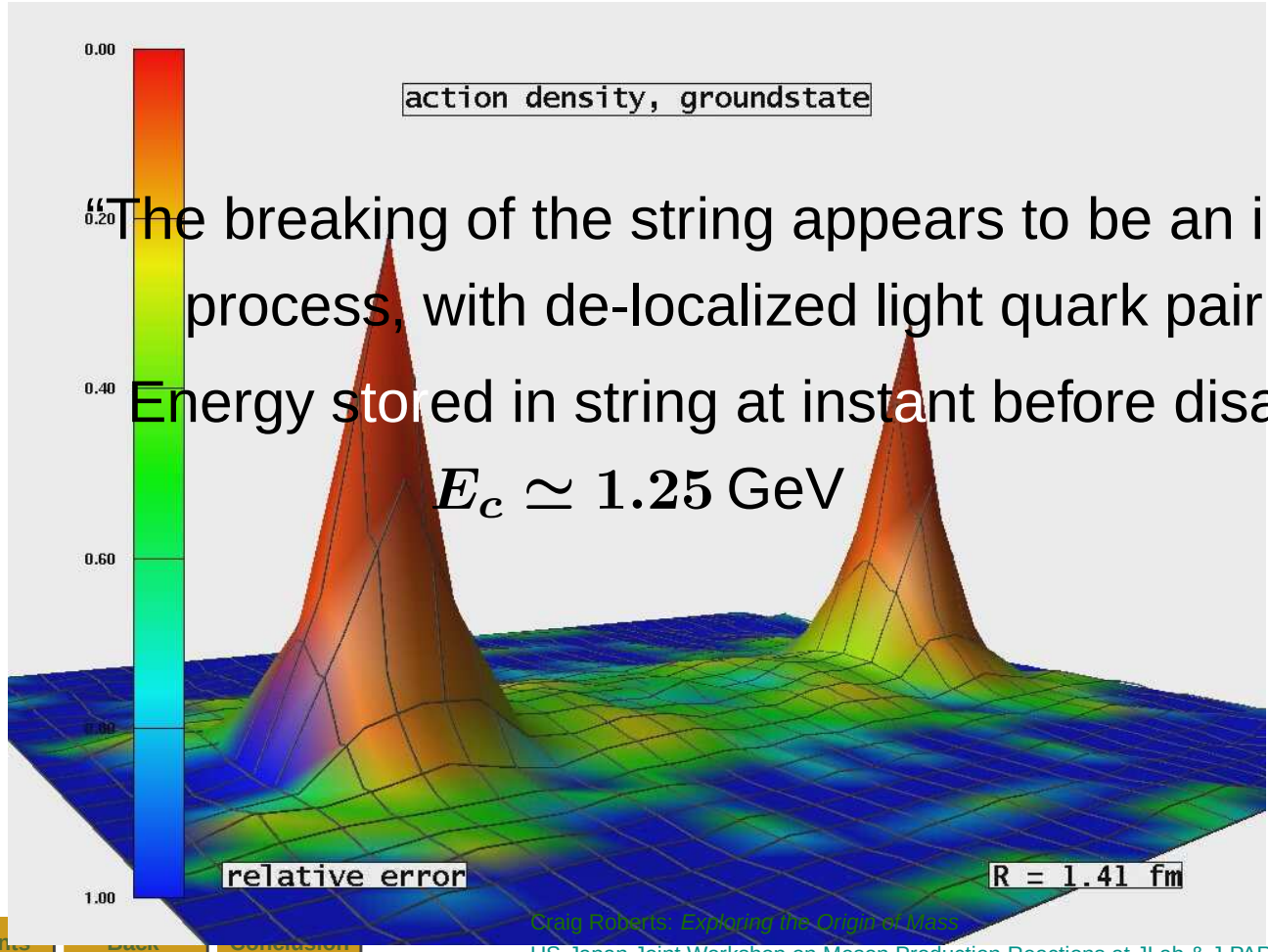

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Energy stored in string at instant before disappearance:

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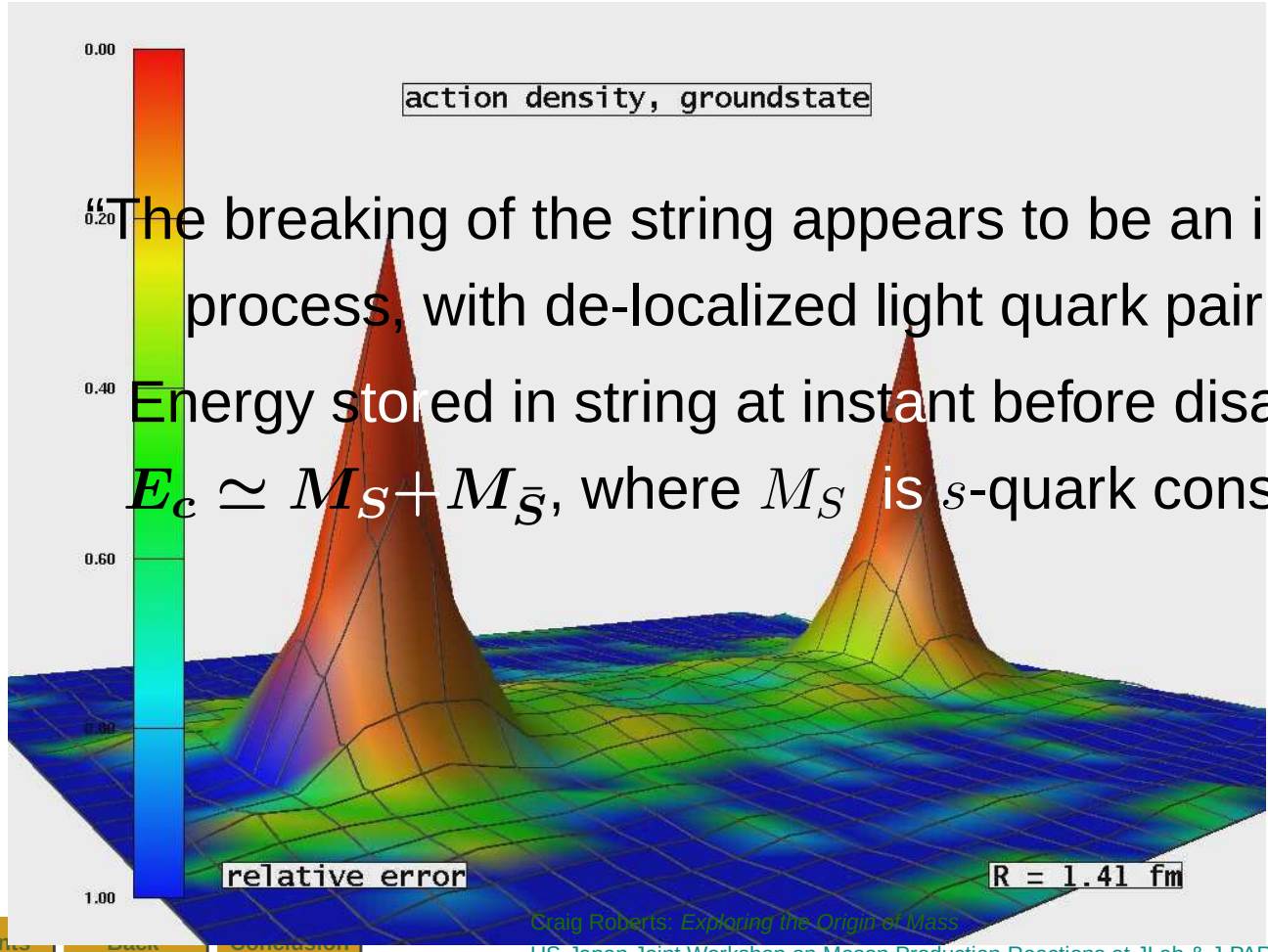

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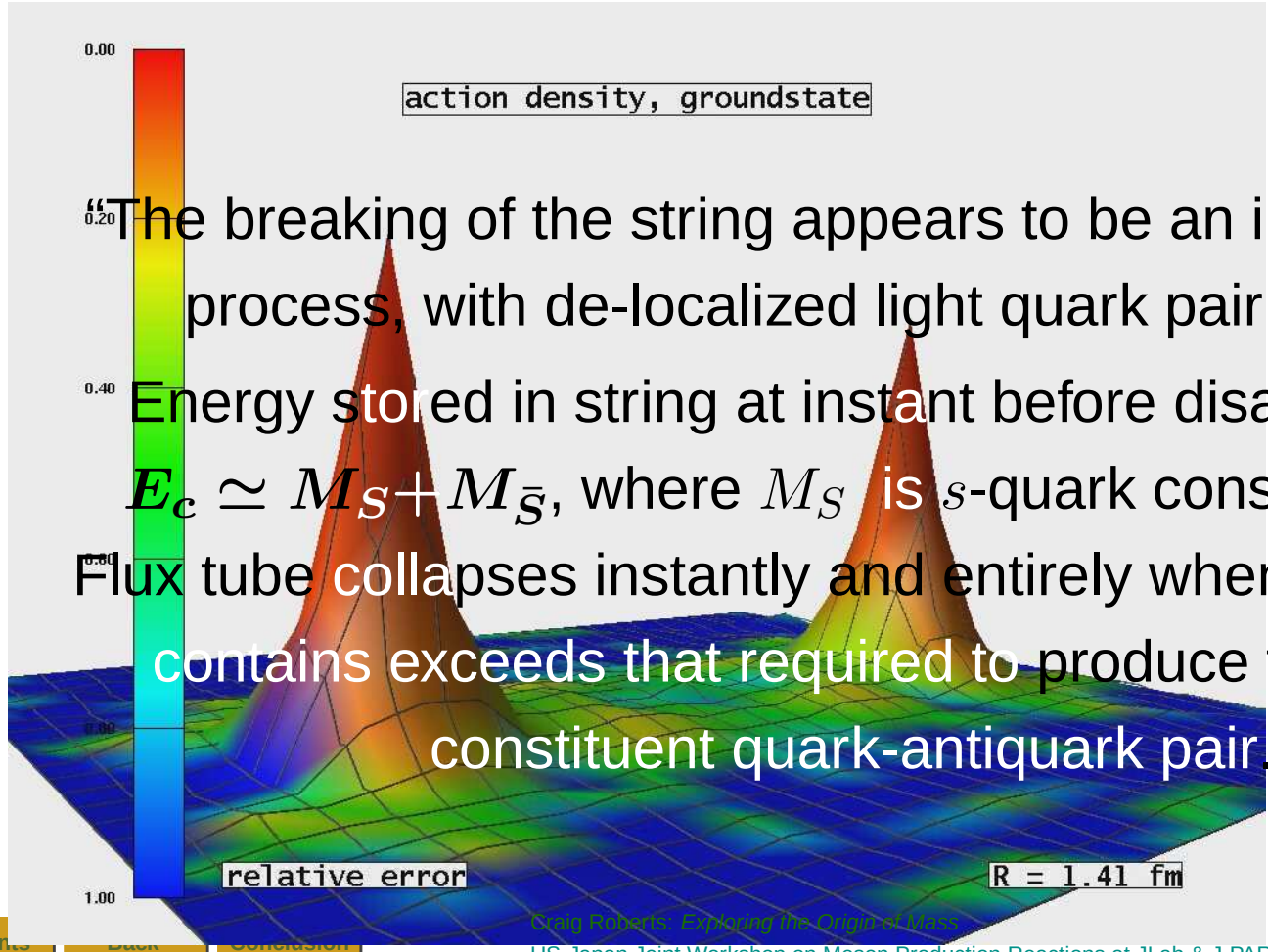

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Therefore . . . No information on *potential* between light-quarks. ***Confinement***

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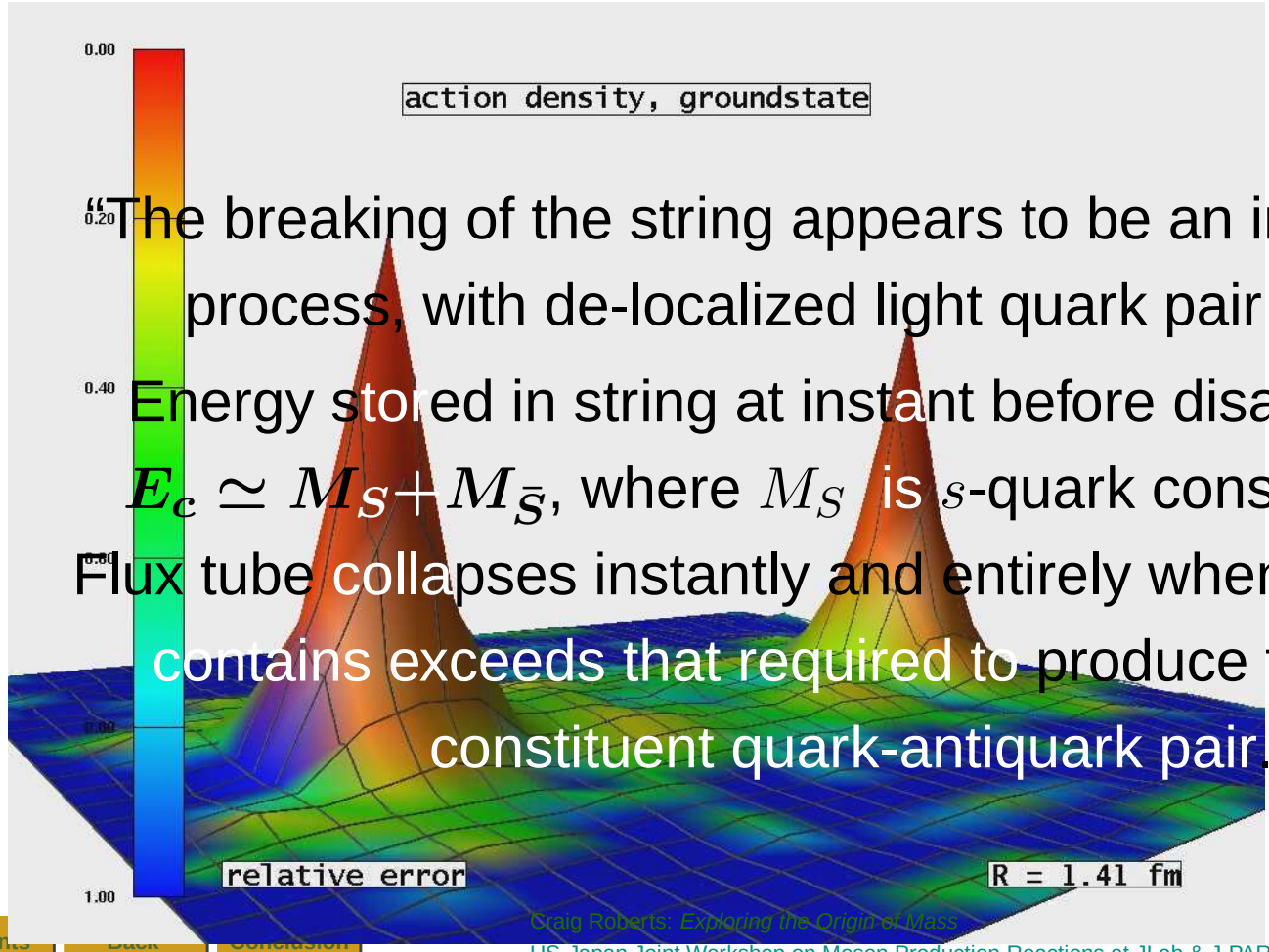

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Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

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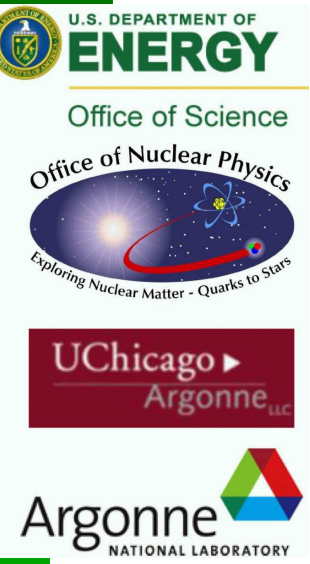

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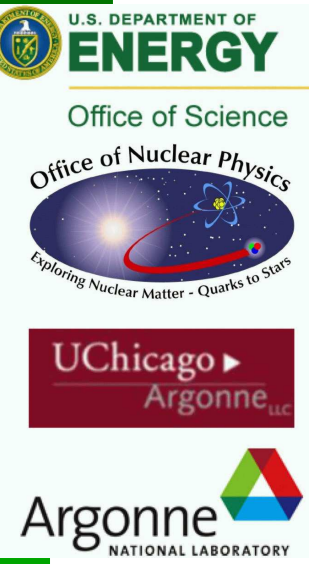

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- Method yields Schwinger Functions $\equiv$ Propagators

Cross-Sections built from Schwinger Functions

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- Proving fruitful.

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- This is a well-posed problem whose solution is an elemental goal of modern hadron physics.

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- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
  - Steady quantitative progress is being made with a scheme that is systematically improvable

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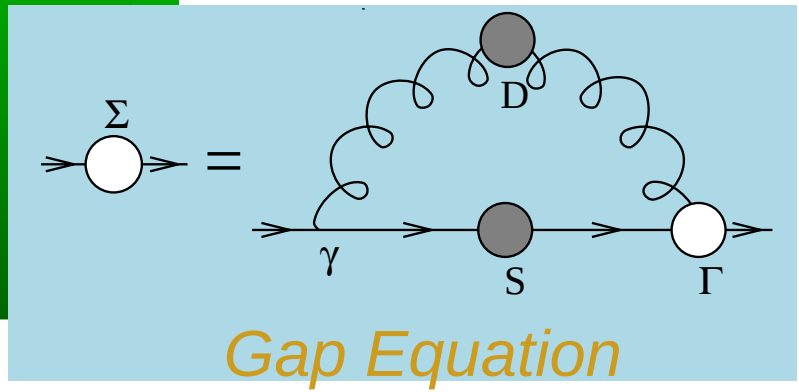
- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
  - On other hand, at present significant qualitative advances possible with symmetry-preserving kernel *Ansätze* that express important additional nonperturbative effects – $M(p^2)$ – difficult/impossible to capture in any finite sum of contributions

Frontiers of Nuclear Science: A Long Range Plan (2007)

Frontiers of Nuclear Science: Theoretical Advances



Gap Equation

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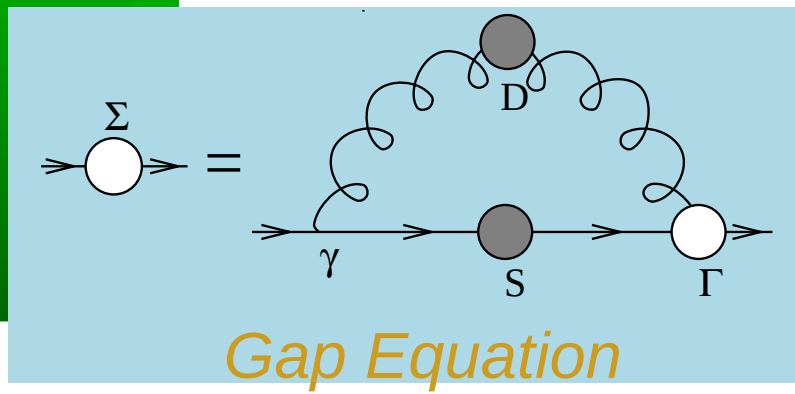
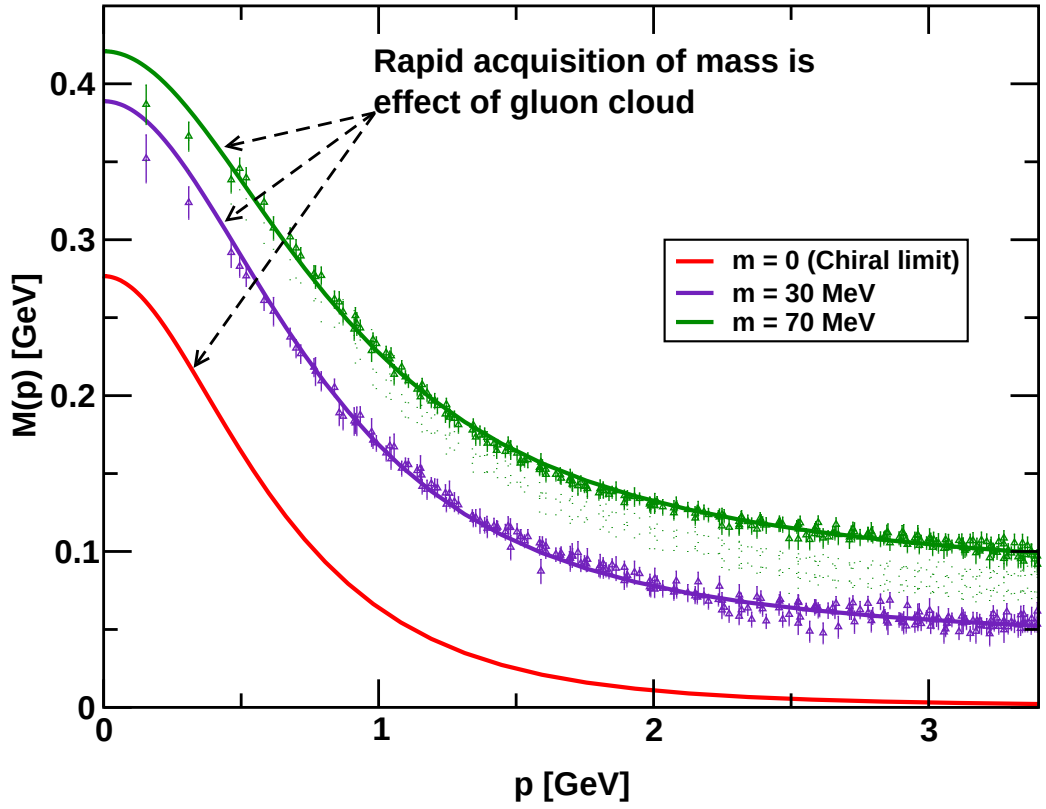


Gap Equation

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



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Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.

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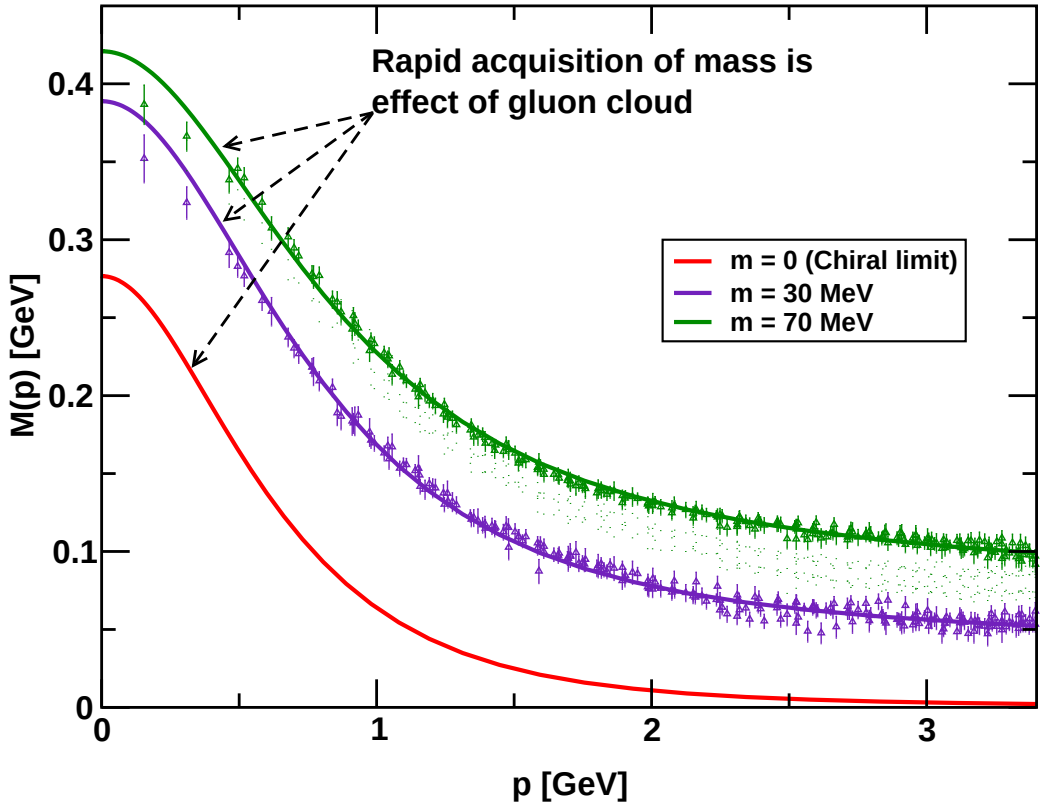

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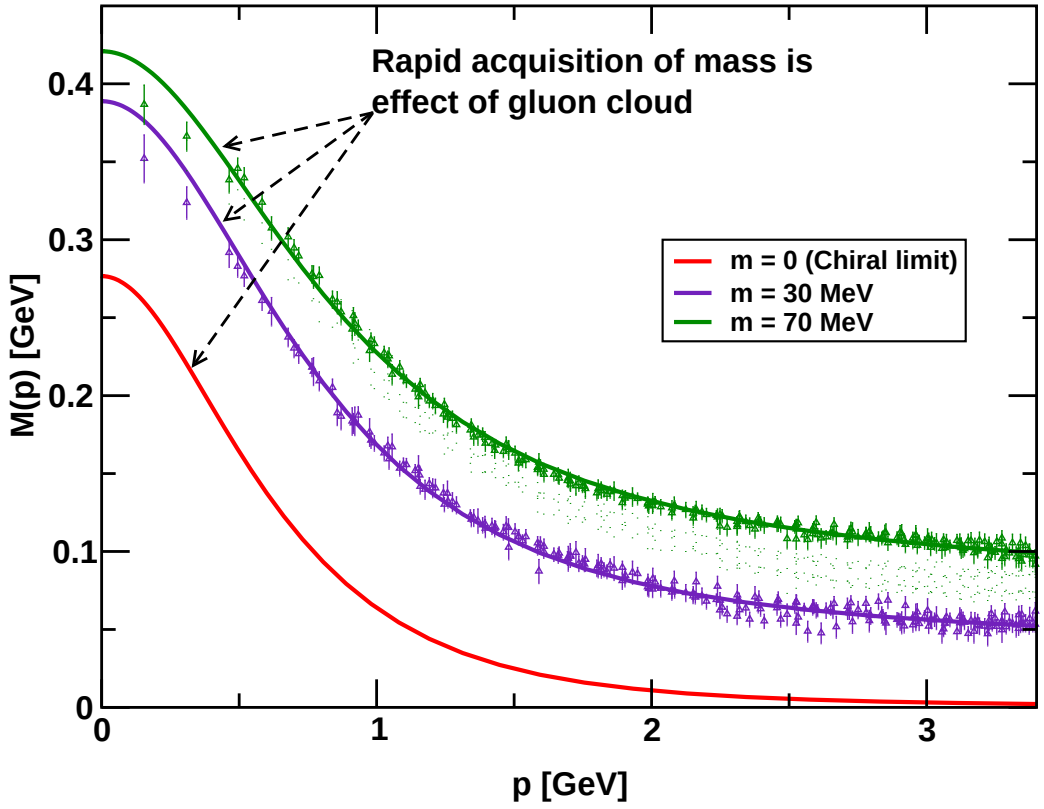

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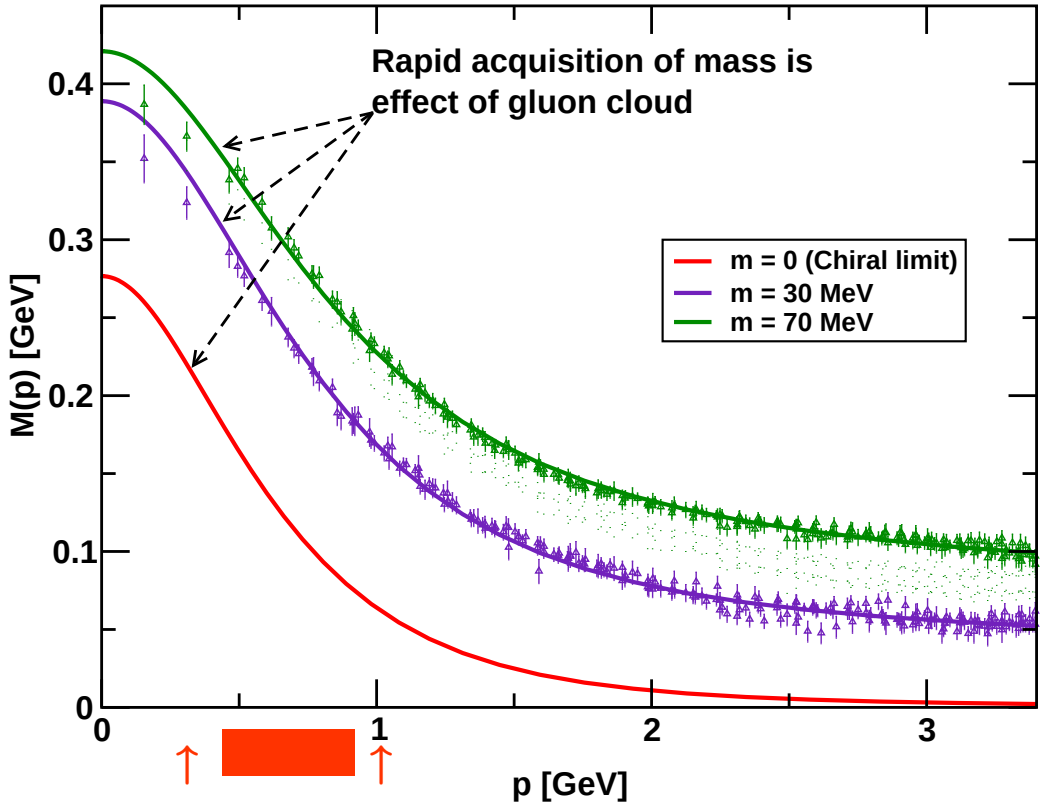

Scanned by $Q^2 \in [2, 9]\ \mathrm{GeV}^2$ Baryon Form Factors


ALWAYS LOOK
ON THE
BRIGHT SIDE

- Established understanding of two- and three-point functions

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Hadrons

- Established understanding of two- and three-point functions
- What about bound states?

Hadrons

- Without bound states, Comparison with experiment is impossible

Hadrons

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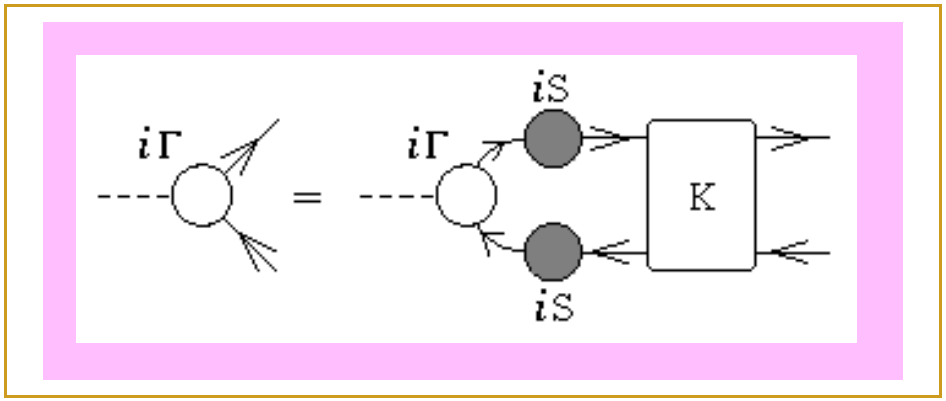
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

Hadrons

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

Hadrons

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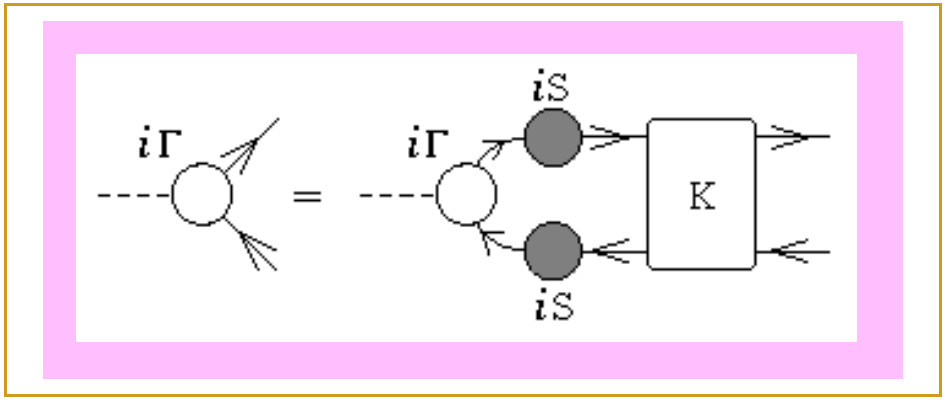
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

  or What is the long-range potential in QCD?

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

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Satisfies BSE

Satisfies DSE

Bethe-Salpeter Kernel

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Kernels very different

but must be intimately related

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- Nontrivial constraint

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Satisfies BSE — Satisfies DSE

Kernels very different

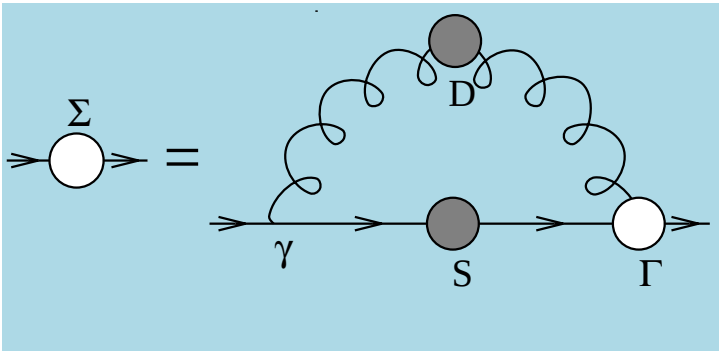
but must be intimately related

- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Persistent Challenge

Persistent Challenge

- Infinitely Many Coupled Equations



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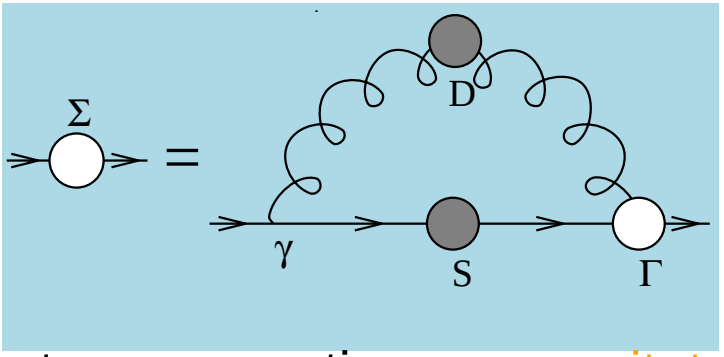

- Coupling between equations necessitates truncation

Persistent Challenge

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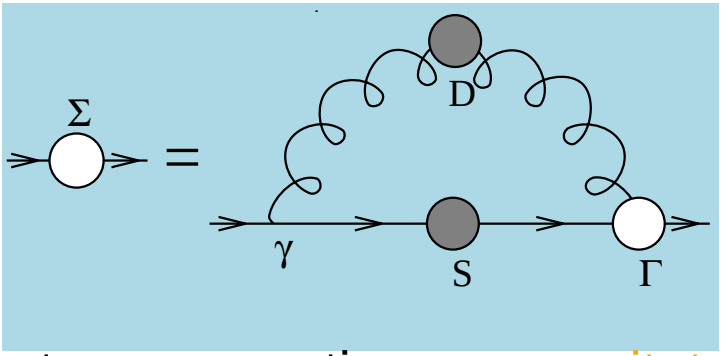

- Coupling between equations necessitates truncation
  - Weak coupling expansion ⇒ Perturbation Theory

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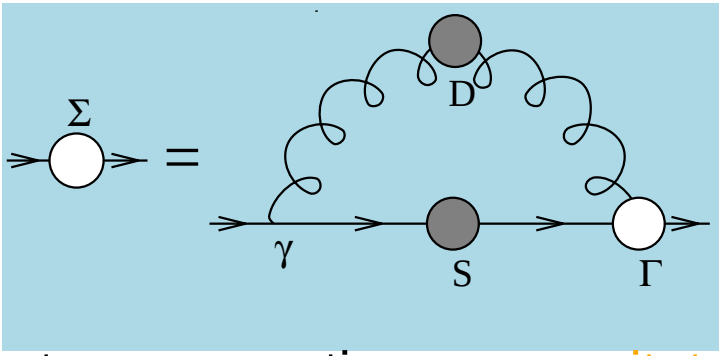

- Coupling between equations necessitates truncation
  - Weak coupling expansion ⇒ Perturbation Theory
    Not useful for the nonperturbative problems
    in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme

  H.J. Munczek Phys. Rev. D **52** (1995) 4736

  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations

  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7

  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of EXACT Results in QCD

Persistent Challenge

- Infinitely Many Coupled Equations
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- And Formulation of Practical Phenomenological Tool to
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- Examples:

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web.mit.edu/redingtn/www/netadv/Xdysonschw.html

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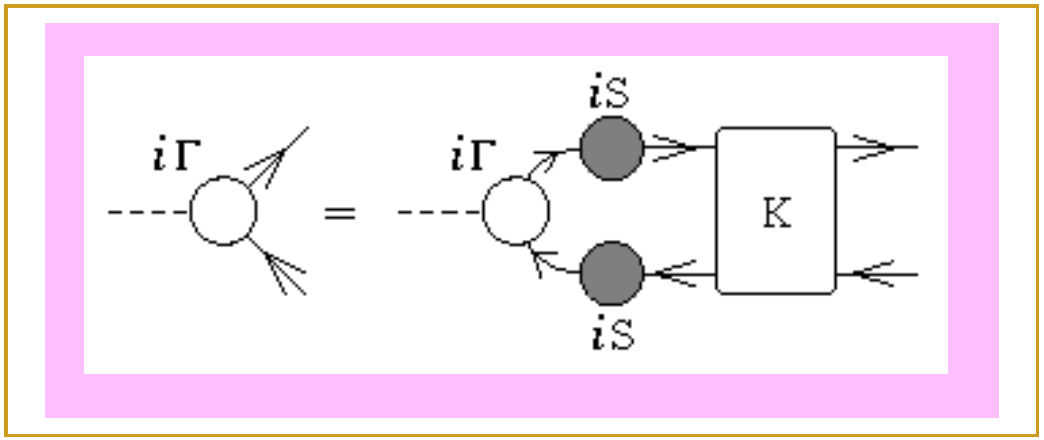
Bound-state DSE

Bound-state DSE
Bethe-Salpeter Equation

- Standard form, familiar from textbooks

$$\left[\Gamma^j_\pi(k;P)\right]_{tu} = \int_q^\Lambda \left[S(q+P/2)\Gamma^j_\pi(q;P)S(q-P/2)\right]_{sr} K^{rs}_{tu}(q,k;P)$$



$K(q,k;P)$: Fully-amputated, 2-particle-irreducible, quark-antiquark scattering kernel

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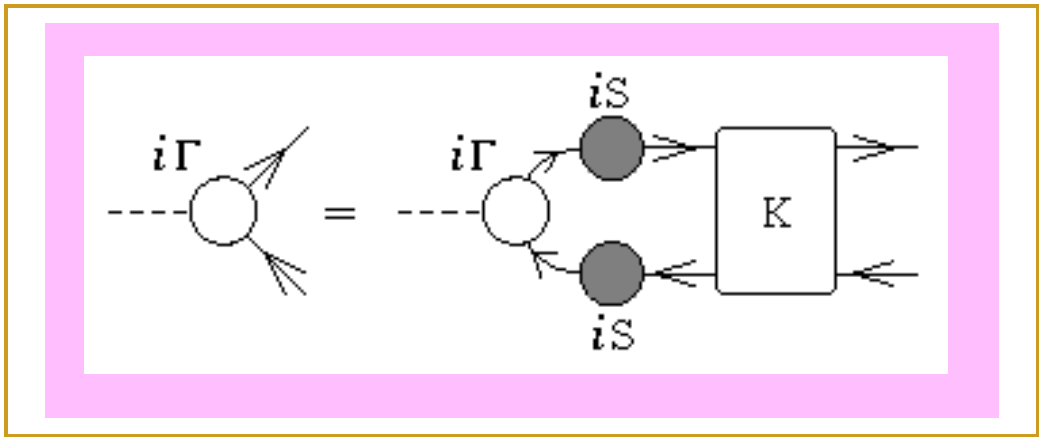

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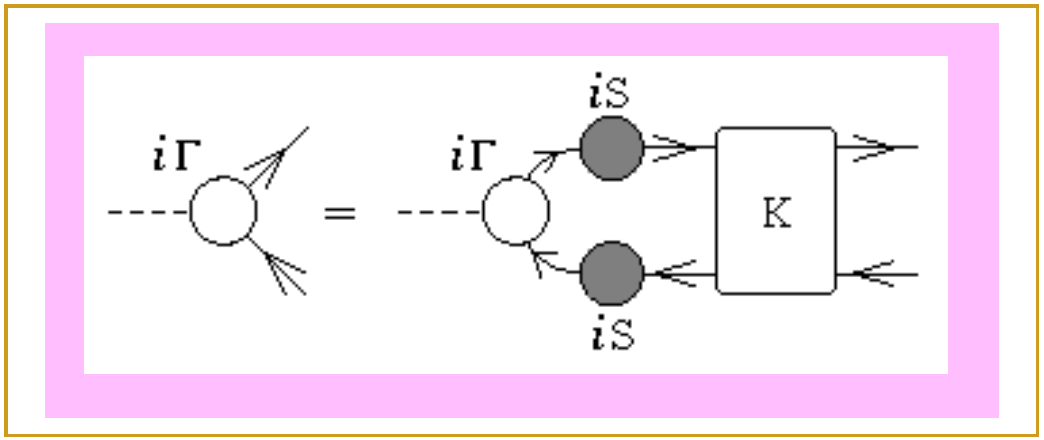
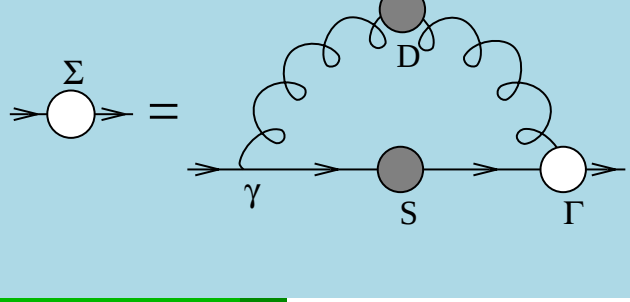


$K(q,k;P)$: Fully-amputated, 2-particle-irreducible, quark-antiquark scattering kernel

- Compact. Visually appealing. Correct.
- Blocked progress for more than 60 years.

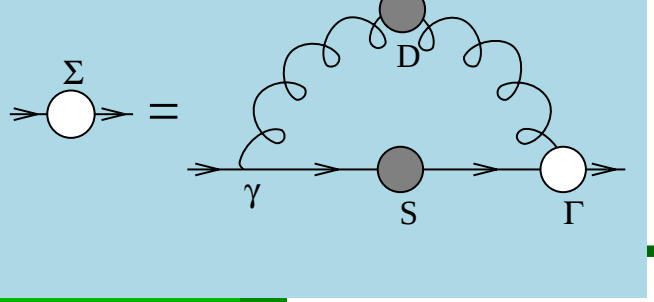
Gap Equation
General Form



Gap Equation
General Form

$$S_f(p)^{-1} = Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p),$$



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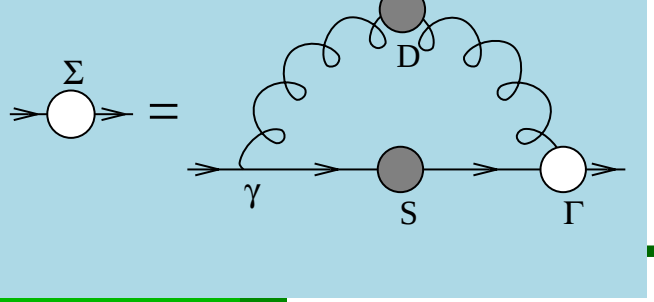
- $Z_{1,2}(\zeta^2,\Lambda^2)$ are respectively the vertex and quark wave function renormalisation constants, with ζ the renormalisation point
- $m^{\rm bm}(\Lambda)$ is the Lagrangian current-quark bare mass
- $D_{\mu\nu}(k)$ is the dressed-gluon propagator
- $\Gamma^f_\nu(q,p)$ is the dressed-quark-gluon vertex



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- $D_{\mu\nu}(k)$ is the dressed-gluon propagator
- $\Gamma^f_\nu(q,p)$ is the dressed-quark-gluon vertex
- Suppose one has in-hand the exact form of $\Gamma^f_\nu(q,p)$

  What is the associated

  Symmetry-preserving Bethe-Salpeter Kernel?

Bethe-Salpeter Equation

General Form

L. Chang and C. D. Roberts
0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

Bethe-Salpeter Equation
General Form

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- Equivalent exact form:

$$\Gamma^{fg}_{5\mu}(k;P) = Z_2\gamma_5\gamma_\mu$$
$$- \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\Gamma^{fg}_{5\mu}(q;P)S_g(q_-)\frac{\lambda^a}{2}\,\Gamma^g_\beta(q_-,k_-)$$
$$+ \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\,\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

(Poincaré covariance, hence $q_\pm = q \pm P/2$, etc., without loss of generality.)

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(Poincaré covariance, hence $q_\pm = q \pm P/2$, etc., without loss of generality.)

- In this form . . . $\Lambda^{fg}_{5\mu\beta}$
  is completely defined via the dressed-quark self-energy

Bethe-Salpeter Kernel & Symmetries

L. Chang and C. D. Roberts
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- E.g., in any reliable study of light-quark hadrons, axial-vector vertex must satisfy Ward-Takahashi identity

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Expresses chiral symmetry & pattern by which it's broken

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- The condition ($\Lambda^{fg}_{5\beta}$ pseudoscalar analogue of $\Lambda^{fg}_{5\mu\beta}$)

$$P_\mu \Lambda^{fg}_{5\mu\beta}(k,q;P) = \Gamma^f_\beta(q_+,k_+)\, i\gamma_5 + i\gamma_5\, \Gamma^g_\beta(q_-,k_-) - i[m_f(\zeta) + m_g(\zeta)]\Lambda^{fg}_{5\beta}(k,q;P),$$

a new Ward-Takahashi identity, is Necessary & Sufficient to ensure $\Gamma^{fg}_{5\mu}(k;P)$ Ward-Takahashi identity satisfied.

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- Rainbow-ladder . . .
  - $\Gamma^f_\beta(q,k) = \gamma_\mu$
    $\Rightarrow \Lambda^{fg}_{5\mu\beta}(k,q;P) = 0 = \Lambda^{fg}_{5\beta}(k,q;P)$

Bethe-Salpeter Kernel

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- Bethe-Salpeter equation introduced in 1951

Bethe-Salpeter Kernel

60 year problem

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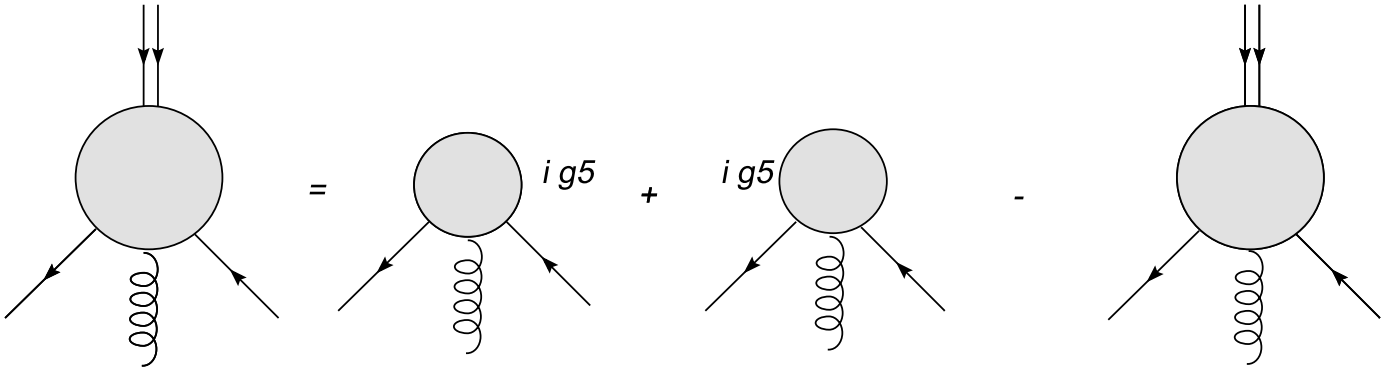

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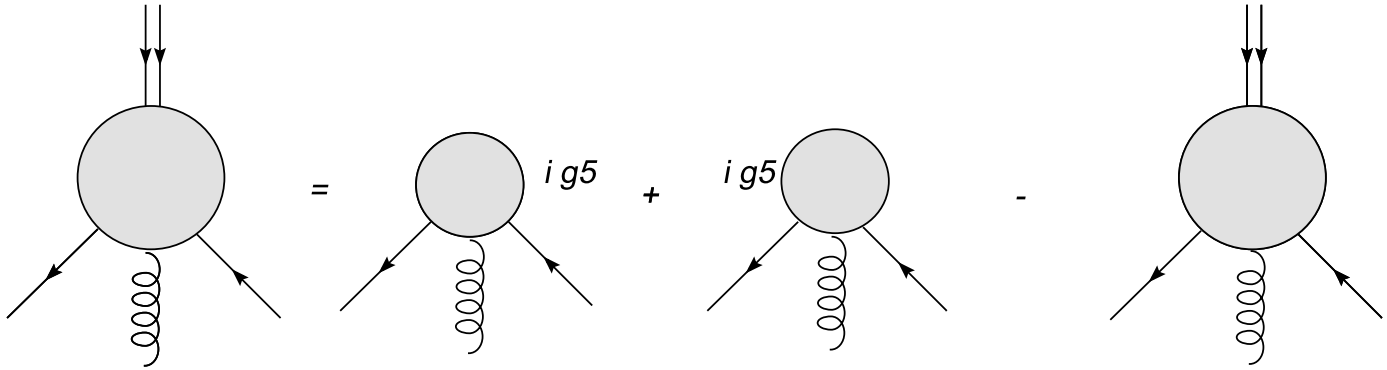

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- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
  - Consistent means - all symmetries preserved!

Bethe-Salpeter Kernel

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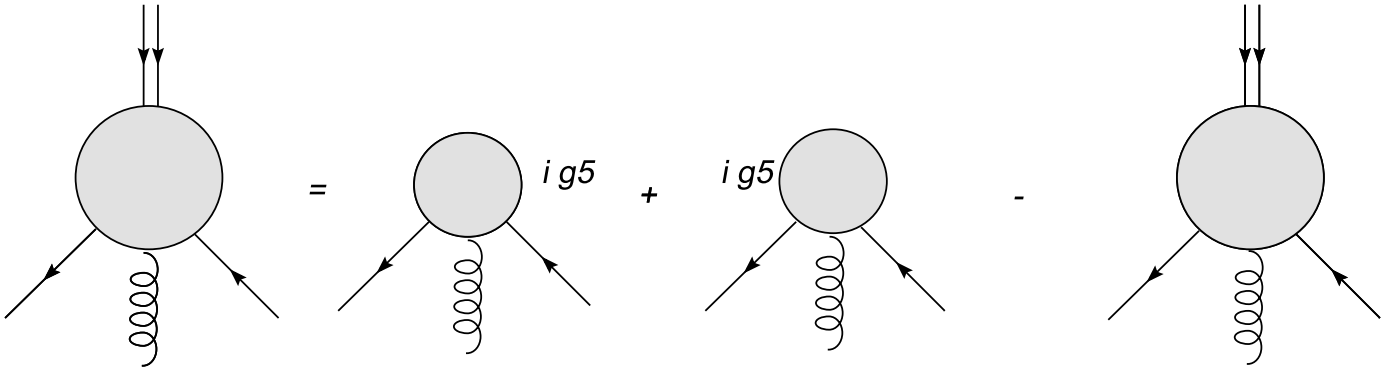

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- Exemplified the procedure and results to expect . . .

Numerical Illustration

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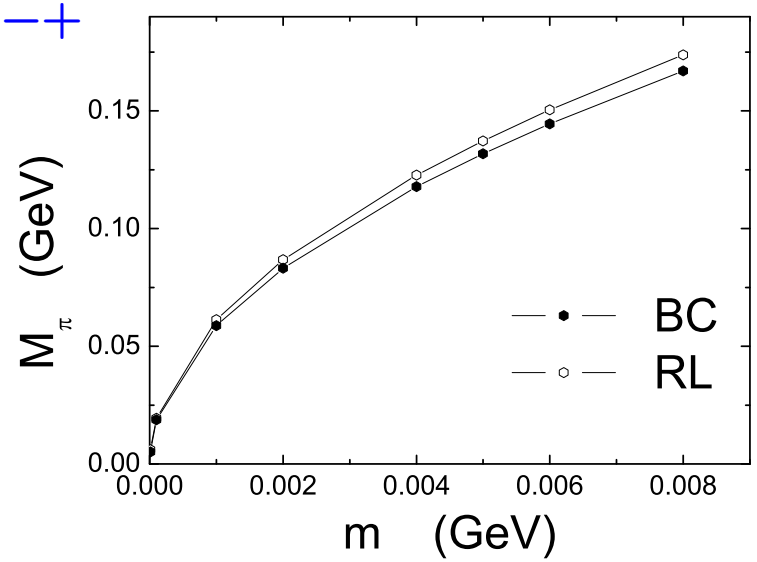
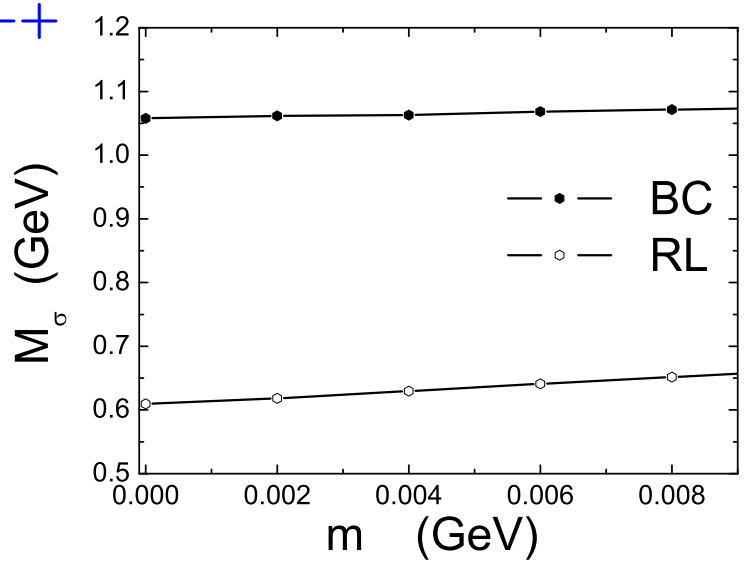
π cf. σ

0^{-+}



0^{++}



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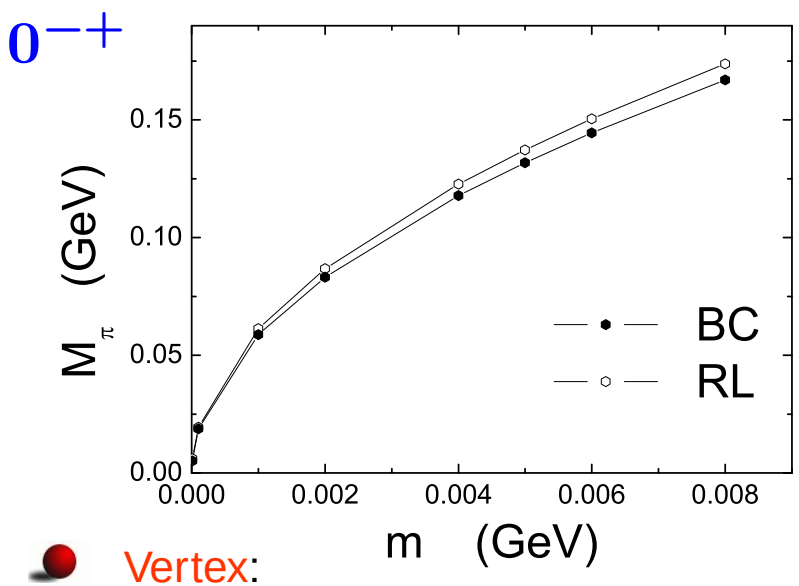

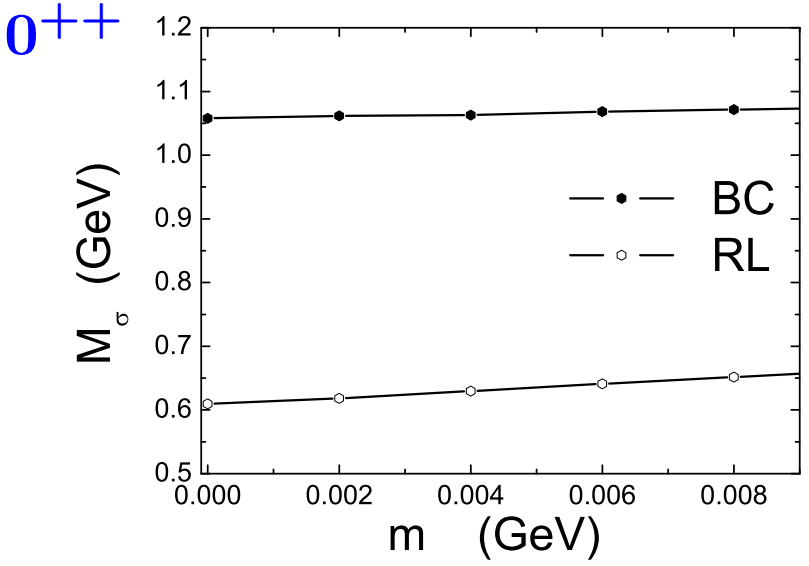

- Vertex:
  - leading-order rainbow-ladder truncation
  - cf. Ball-Chiu–consistent *Ansatz* – Essentially nonperturbative content; Expresses DCSB; Consistent with lattice-QCD simulations; Diagrammatic content unknown

Same interaction. One mass-scale in both truncations: $1/\omega = 0.4\,$fm, defining border between IR & UV.

Numerical Illustration

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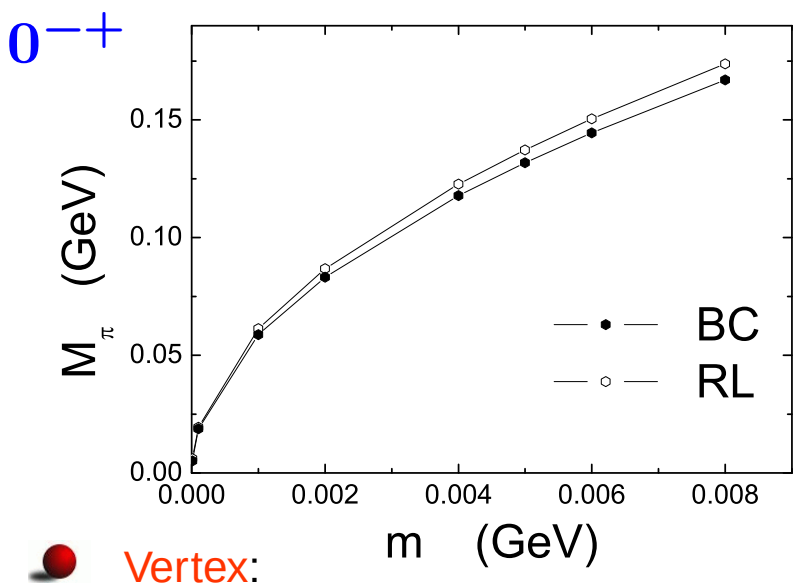

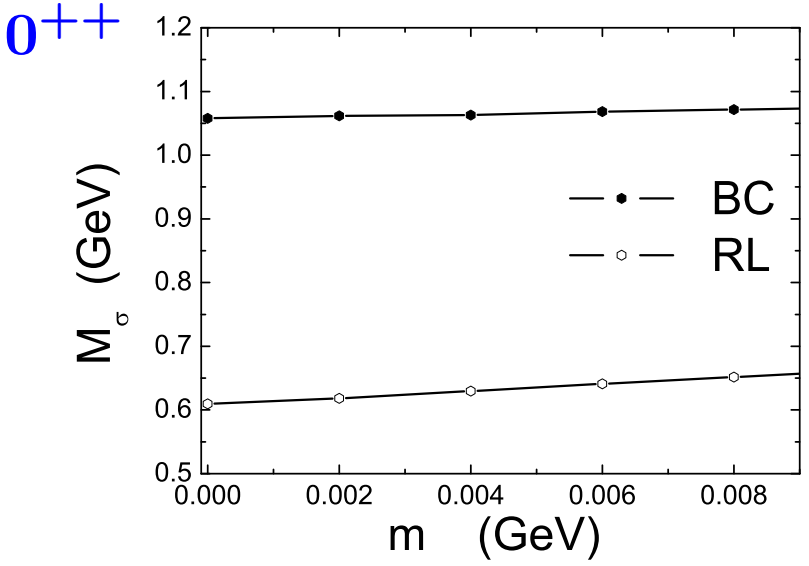

- Vertex:
  - leading-order rainbow-ladder truncation
  - cf. Ball-Chiu–consistent *Ansatz* – Essentially nonperturbative content; Expresses DCSB; Consistent with lattice-QCD simulations; Diagrammatic content unknown

  Same interaction. One mass-scale in both truncations: $1/\omega = 0.4\,$fm, defining border between IR & UV.
- GMOR . . . plainly satisfied by both truncations
- A little attraction introduced in pseudoscalar channel
- Enormous repulsion introduced in scalar channel

Spin-orbit Interaction

L. Chang and C. D. Roberts
0903.5461 [nucl-th], Phys. Rev. Lett. 103 (2009) 081601

- Rainbow-ladder DSE truncation, $\varepsilon_\sigma^{\rm RL} := \left. \frac{2M(0) - m_\sigma}{2M(0)} \right|_{\rm RL} = (0.3 \pm 0.1)\,.$
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- Scalar mesons $= {}^3P_0$ states: Constituents' spins aligned and one unit of constituent orbital angular momentum
  - From this viewpoint,
    scalar is a spin and orbital excitation of a pseudoscalar meson

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- Extant studies of realistic corrections to the rainbow-ladder truncation show that they reduce hyperfine splitting in the absence of orbital angular momentum
- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit interaction.
  - Effect owes to influence of quark's dynamically-enhanced scalar self-energy in the Bethe-Salpeter kernel.

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  Impossible to demonstrate effect without our new procedure
- Expect this feature to have material impact

  Especially on mesons with mass greater than 1 GeV.

prima facie . . . can overcome longstanding shortcoming of systematic, symmetry-preserving truncations;
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  viz., splitting between vector & axial-vector mesons is too small
- Promise of realistic meson spectroscopy . . . First time, also for mass > 1 GeV

$[m_{a_1} - m_\rho]$ *I* . . .

- That was where things stood in March/09

$[m_{a_1} - m_\rho]$ I . . .

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- Now, we've solved inhomogeneous vector and axial-vector Bethe-Salpeter equation at spacelike total momentum
  $\Rightarrow \Gamma_{qq}(k=0, P^2)$
- $$\boxed{\frac{1}{\Gamma_{qq}(k=0, P^2)}}$$ Exhibits a zero at ground-state mass-squared
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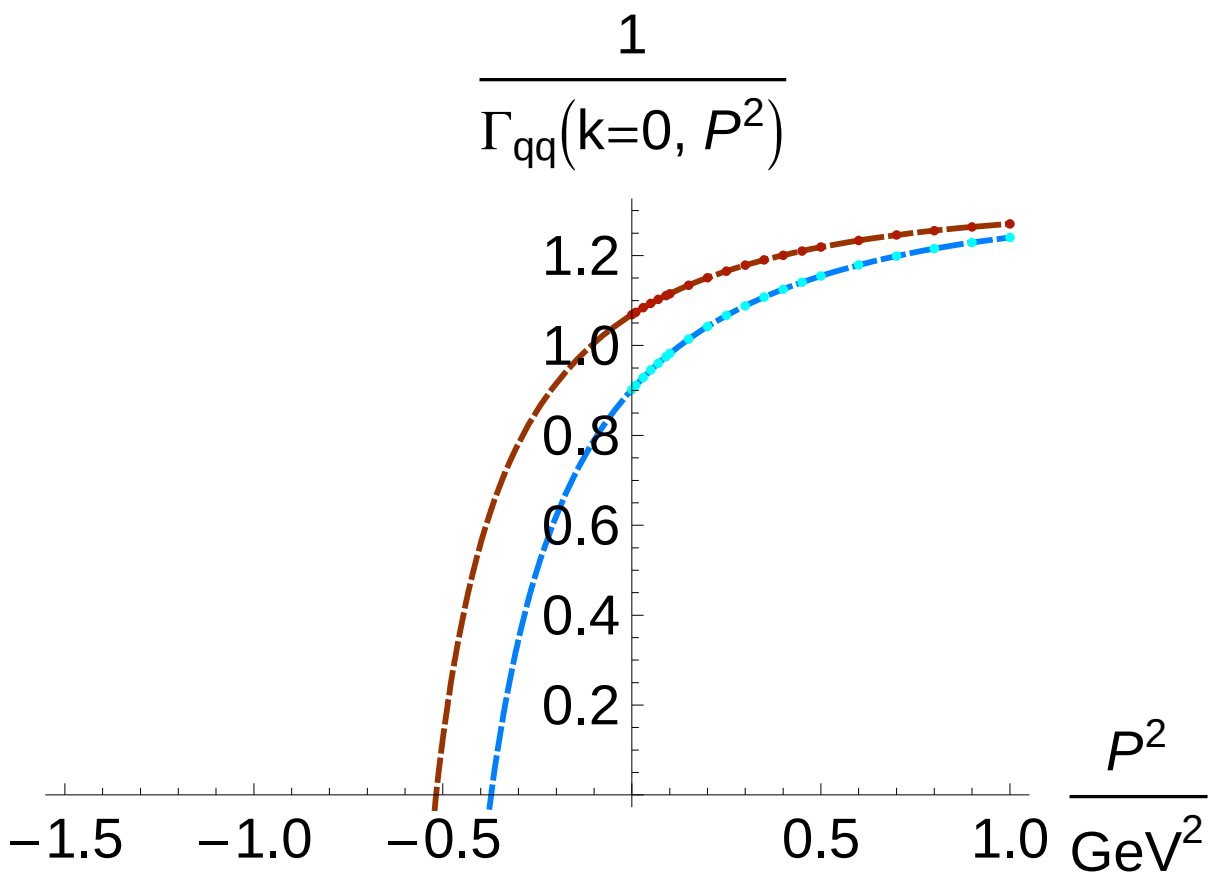
- Padé approximant extrapolation to locate zero
  - Almost precisely method used for ground-state masses in lattice-QCD
  - Intelligent use gives dependable results
    "Schwinger functions and light-quark bound states"
    Bhagwat, Höll, Krassnigg, Roberts & Wright,
    Few Body Syst. **40** (2007) 209, nucl-th/0701009

… $[m_{a_1} - m_\rho]$ II …

Rainbow-Ladder

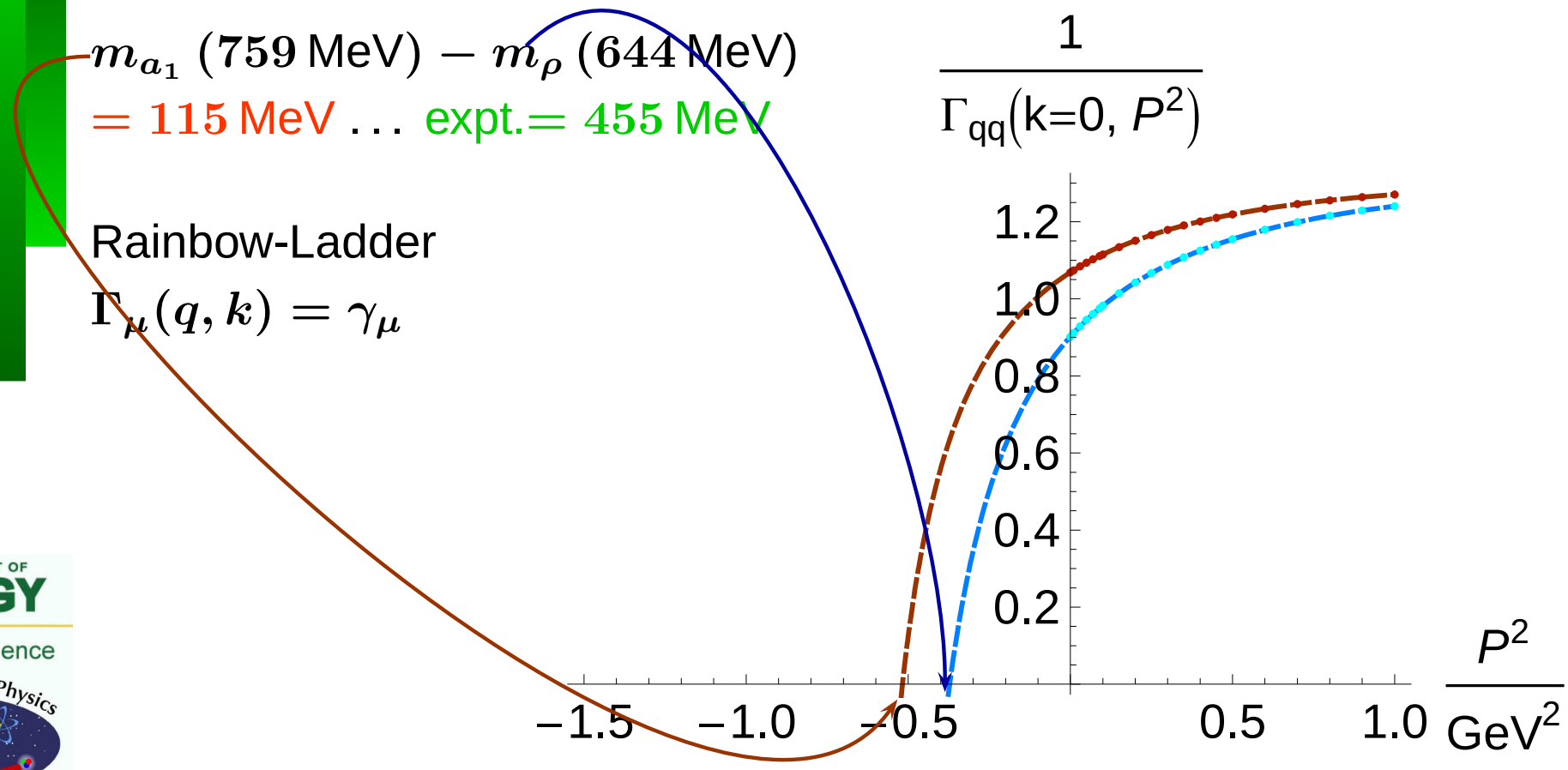
$\Gamma_\mu(q,k) = \gamma_\mu$



$\ldots\ [m_{a_1} - m_\rho]$ II $\ldots$

$m_{a_1}\,(759\,\text{MeV}) - m_\rho\,(644\,\text{MeV})$
$= 115\,\text{MeV}\ \ldots$ expt.$= 455\,\text{MeV}$

Rainbow-Ladder
$\Gamma_\mu(q,k) = \gamma_\mu$

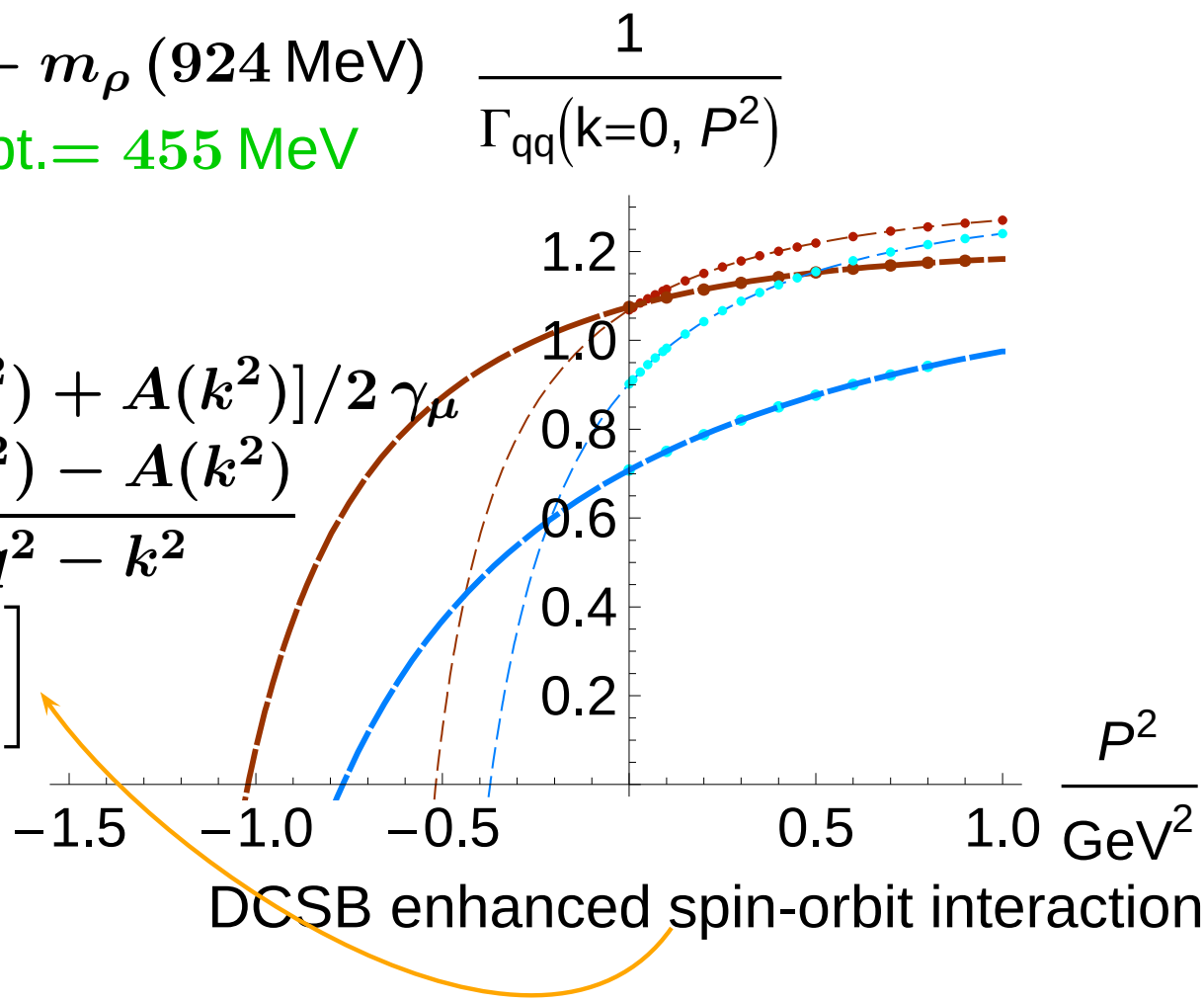


… $[m_{a_1} - m_\rho]$ III …

$m_{a_1}\,(1066\,\text{MeV}) - m_\rho\,(924\,\text{MeV})$
$= 142\,\text{MeV}$ … expt.$= 455\,\text{MeV}$

Ball-Chiu

$$\Gamma_\mu(q,k) = i[A(q^2)+A(k^2)]/2\,\gamma_\mu + 2k_\mu\left[i\gamma\cdot k\,\frac{A(q^2)-A(k^2)}{q^2-k^2} + \frac{B(q^2)-B(k^2)}{q^2-k^2}\right]$$



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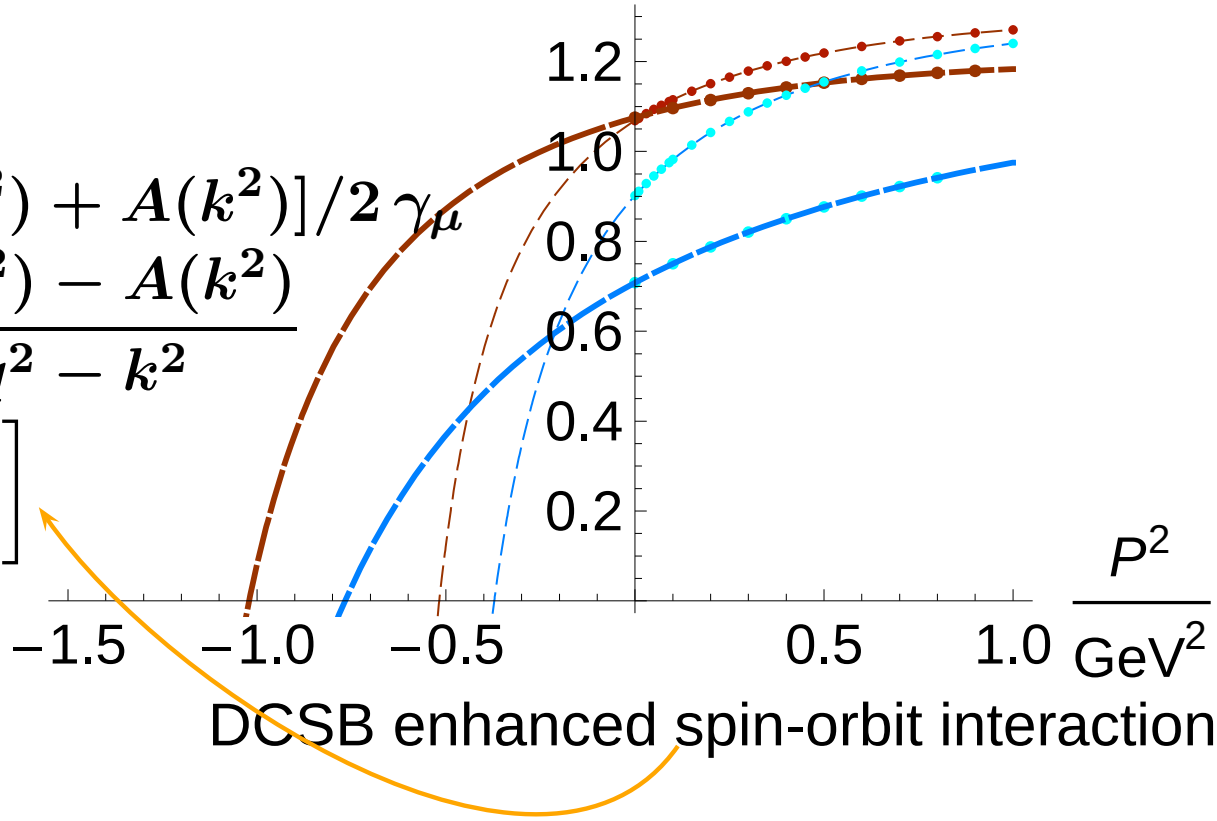
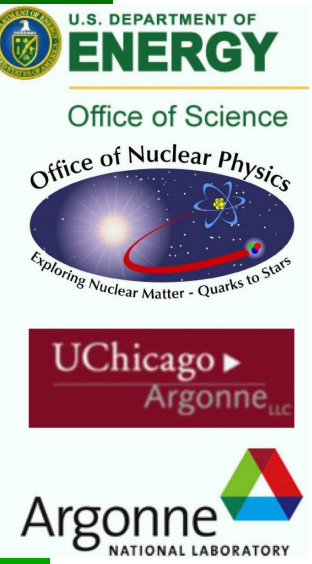


What's missing?

Missing Link

$[m_{a_1} - m_\rho]$

Chang Lei & CDR, in-preparation



Missing Link

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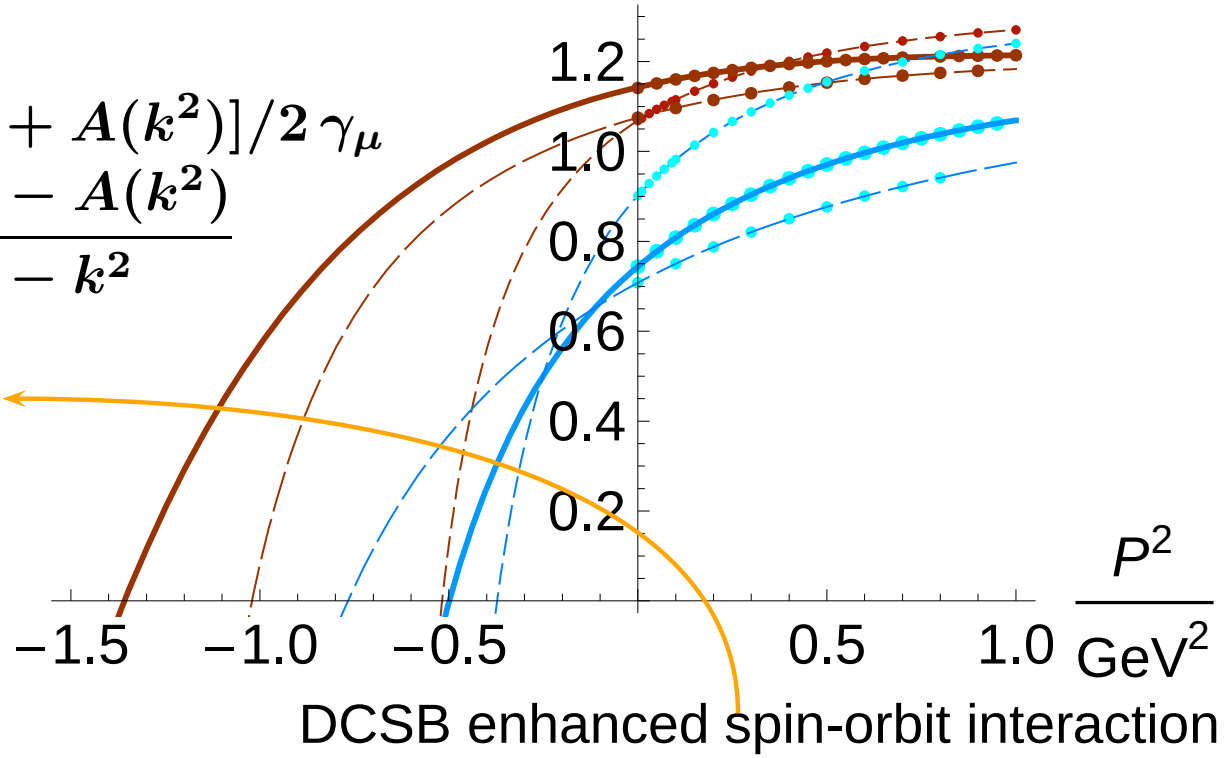
$$\frac{1}{\Gamma_{qq}(k=0,\,P^2)}$$



DCSB enhanced spin-orbit interaction

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Ball-Chiu + an. mag. mom.

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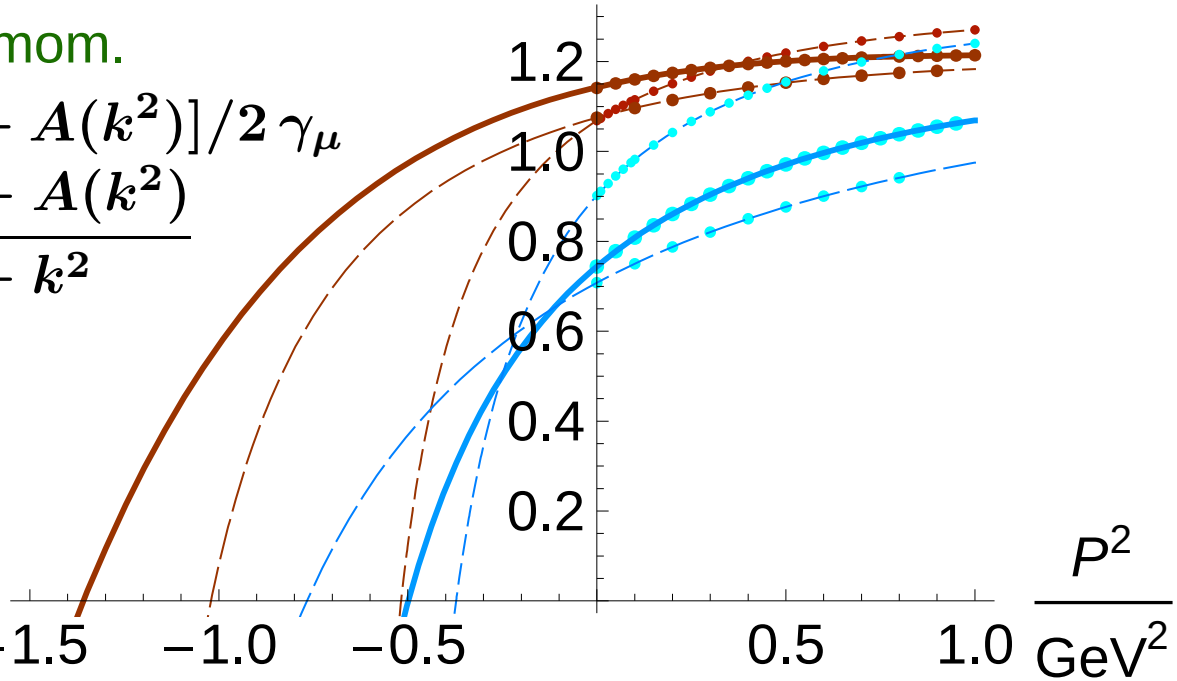

DCSB enhanced anomalous chromomagnetic moment

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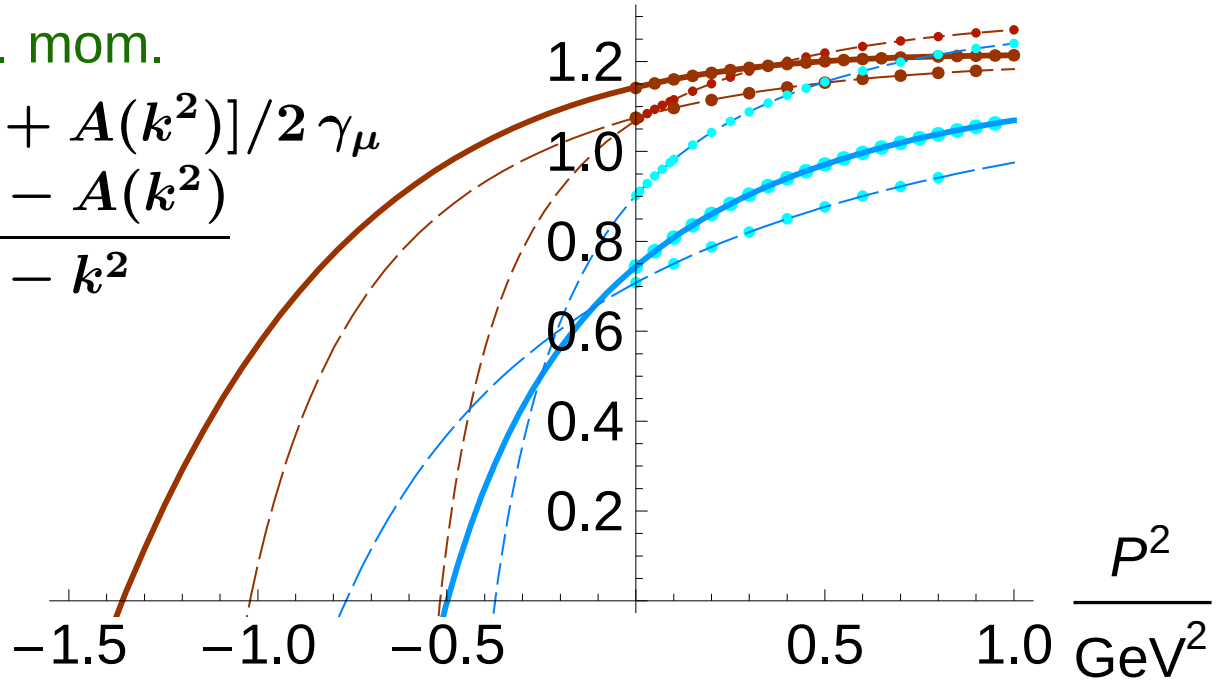

Inextricably connected with DCSB.

DCSB enhanced anomalous chromomagnetic moment

Can't appear in chirally symmetric theory

Paves way for truly reliable light-quark meson spectrum

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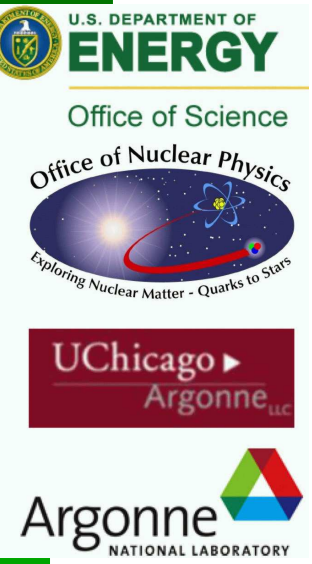
Unifying Study of Mesons and Baryons



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- How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons? Behaviour of $M(p^2)$ is essentially a quantum field theoretical effect.

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  - Residue is proportional to nucleon's Faddeev amplitude
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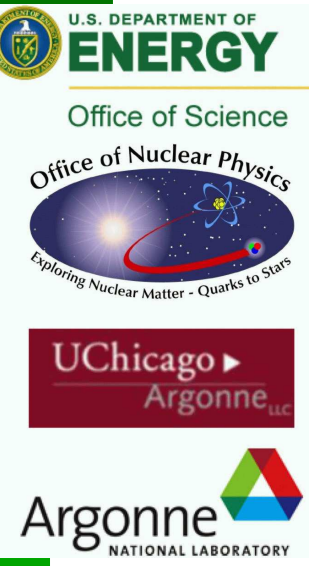
  - Residue is proportional to nucleon's Faddeev amplitude
  - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
  - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour-$\bar{3}$ (antitriplet) channel

Faddeev equation

R. T. Cahill *et al*. Austral. J. Phys. **42** (1989) 129



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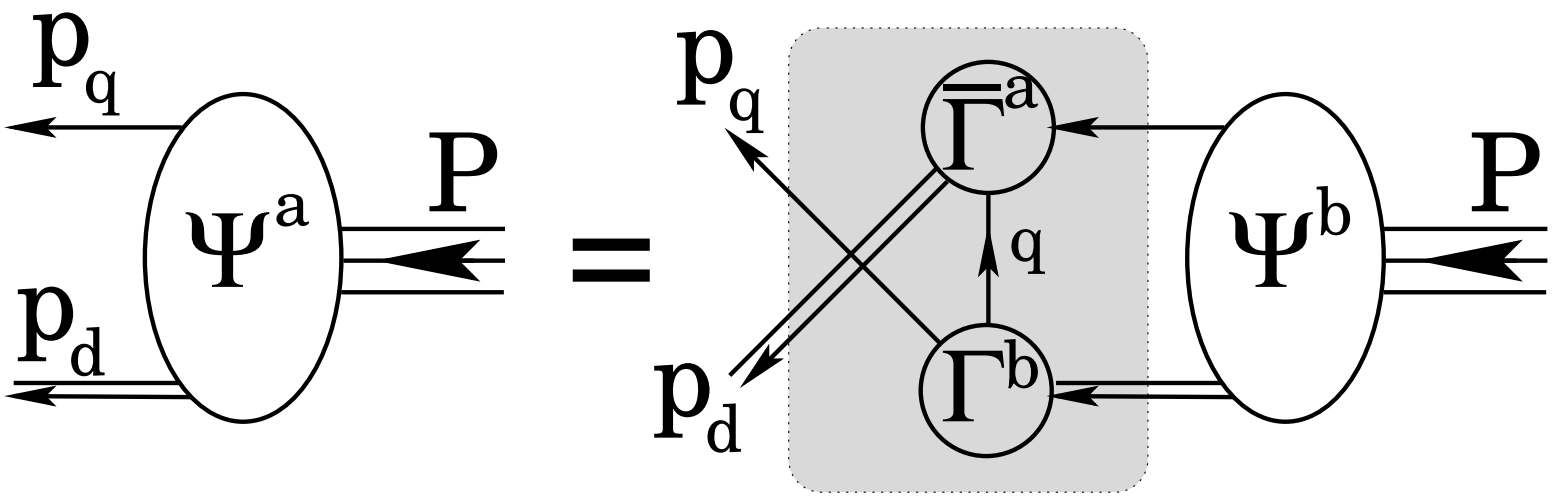

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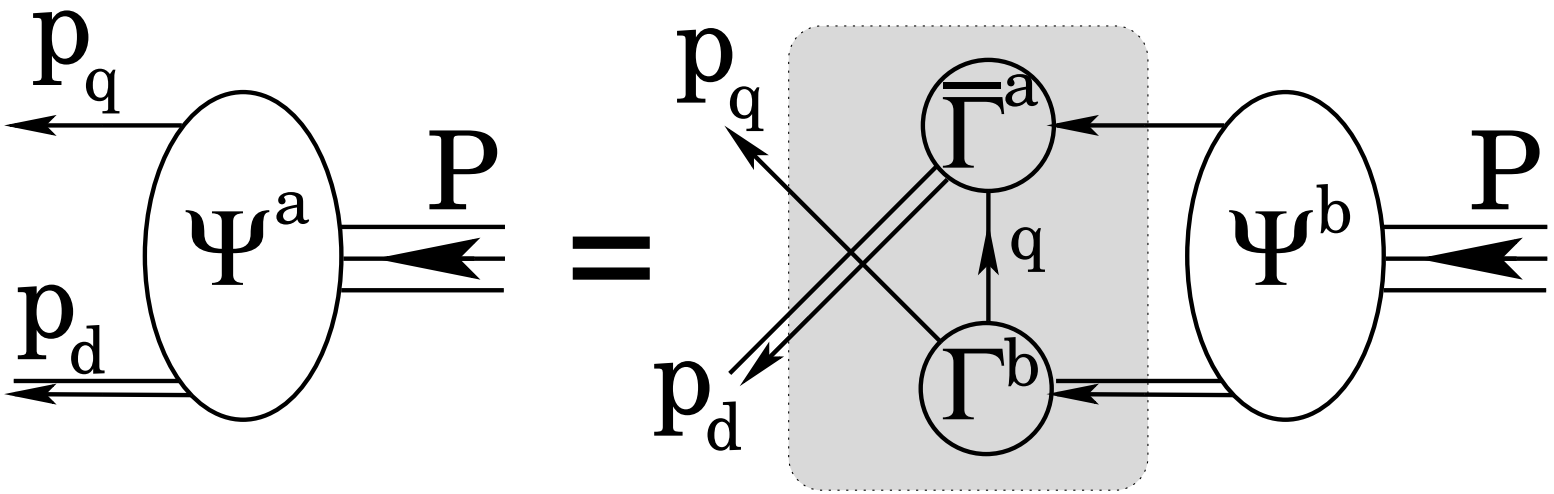
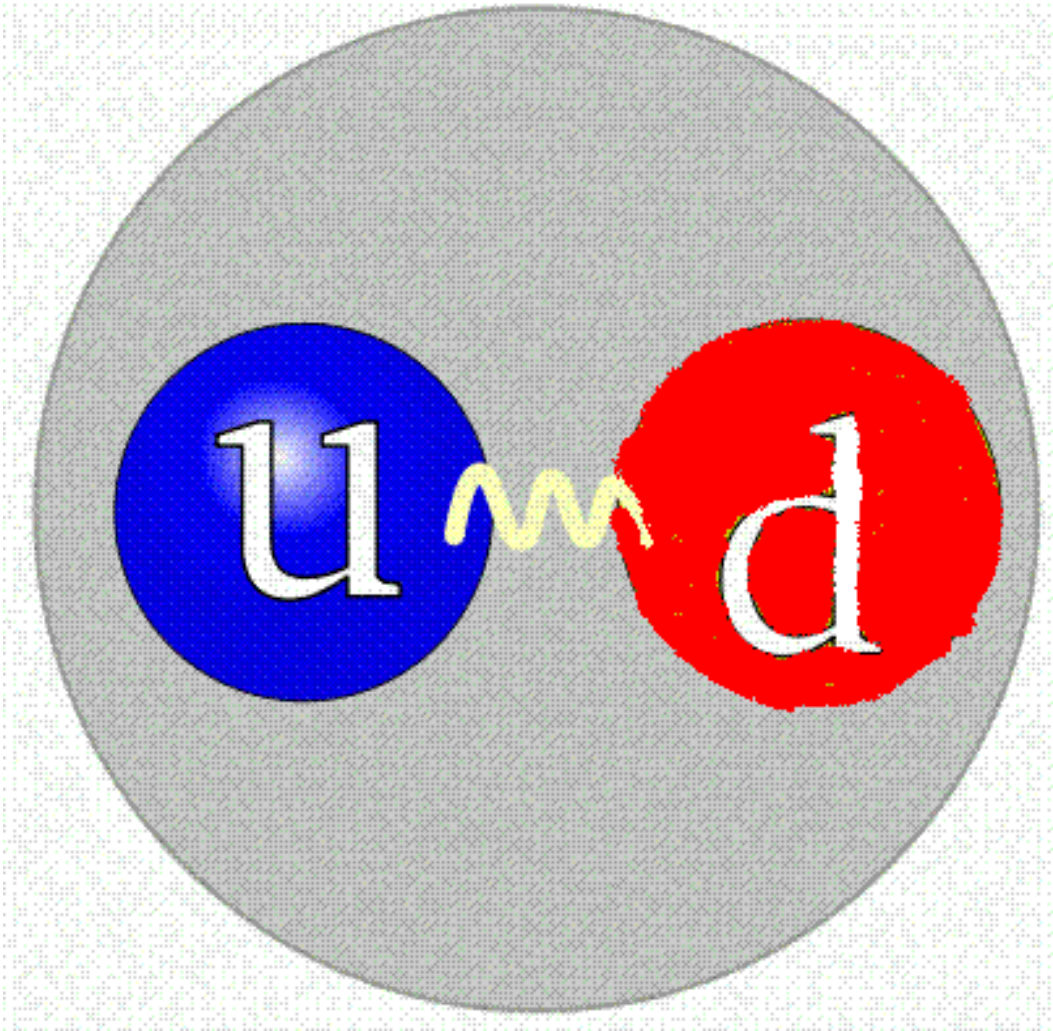


- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Diquark correlations



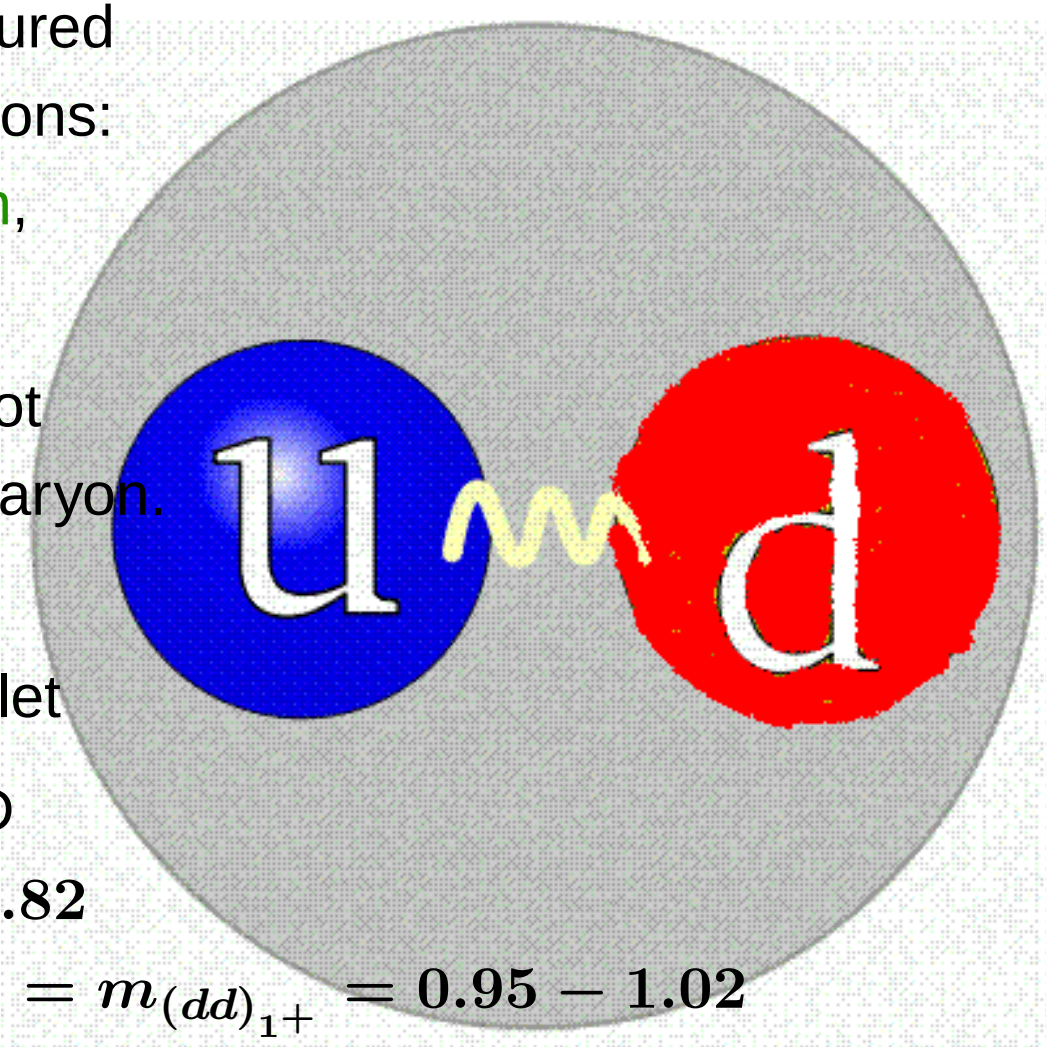
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined . . . Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

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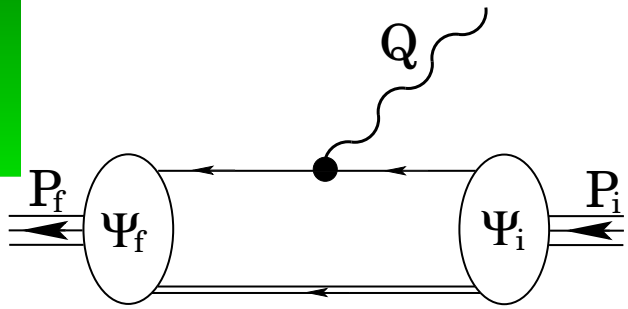

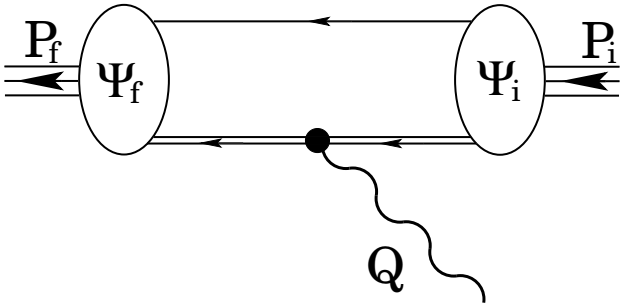

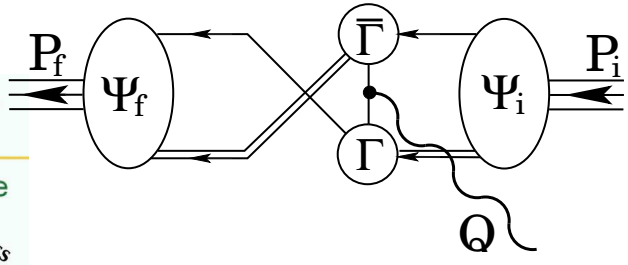

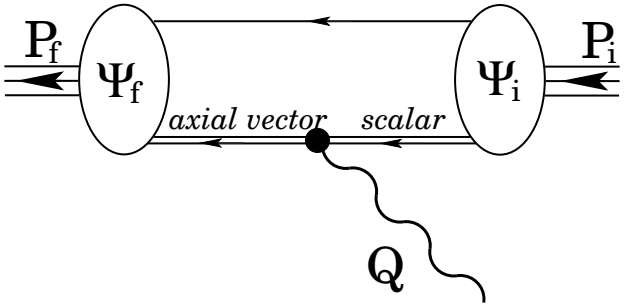

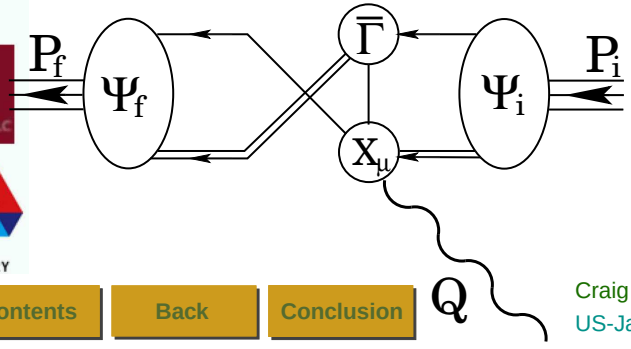

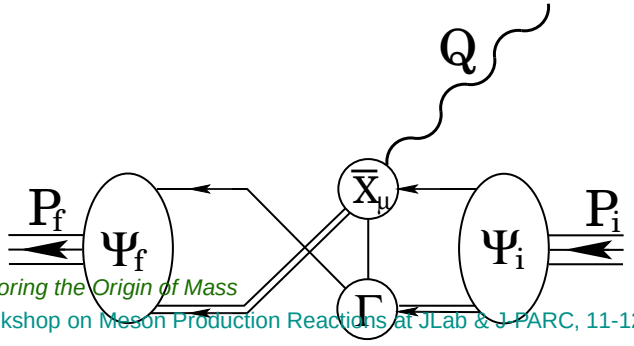

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$$\frac{\mu_n G_E(Q^2)}{G_M(Q^2)}$$

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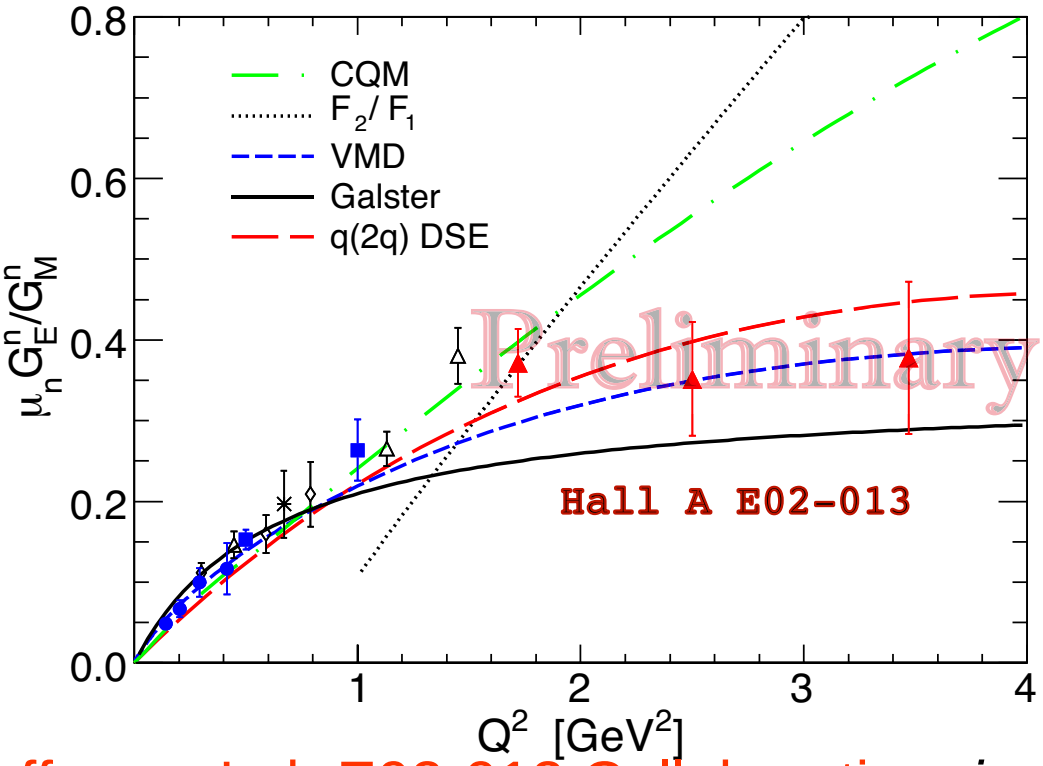

Jefferson Lab E02-013 Collaboration, *in preparation*.

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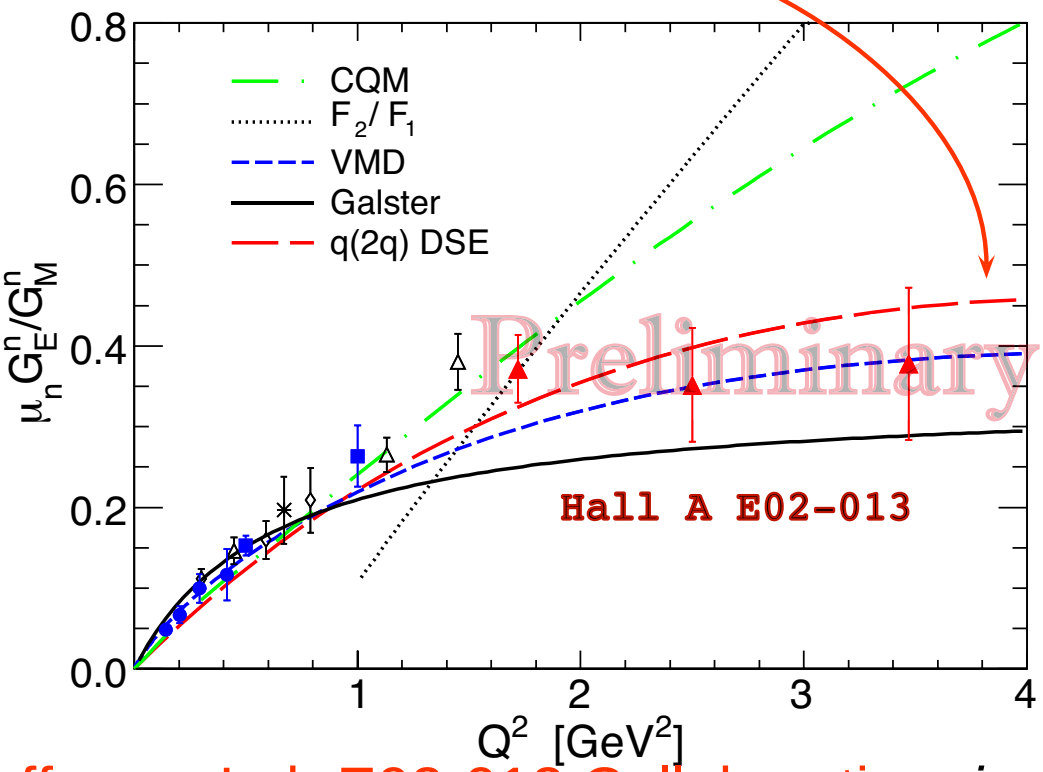
  Red long-dashed curve



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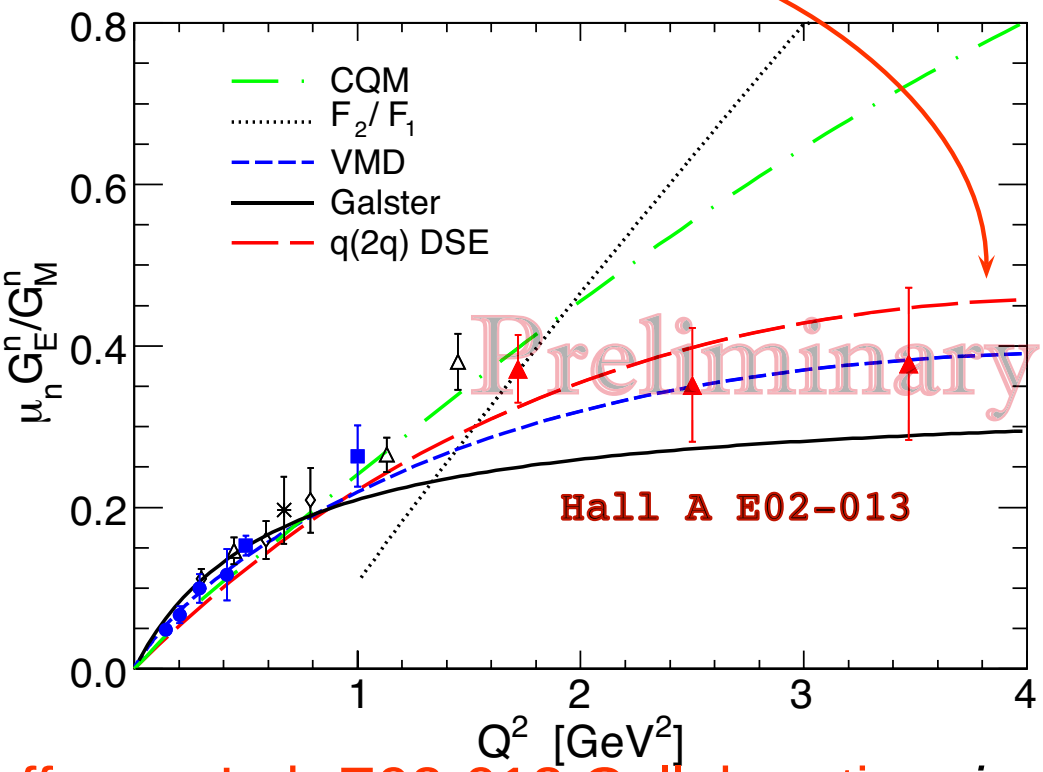

This evolution very sensitive to momentum -dependence dressed-quark propagator

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Exemplar: Elastic Pion Form Factor

Bashir, Guttierez, Roberts:
in progress

- DSE prediction: $M(p^2)$; i.e., interaction $\dfrac{1}{|x-y|^2}$
- cf. $M(p^2) =$ Constant; i.e., interaction $\delta^4(x-y)$

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Single mass-scale parameter
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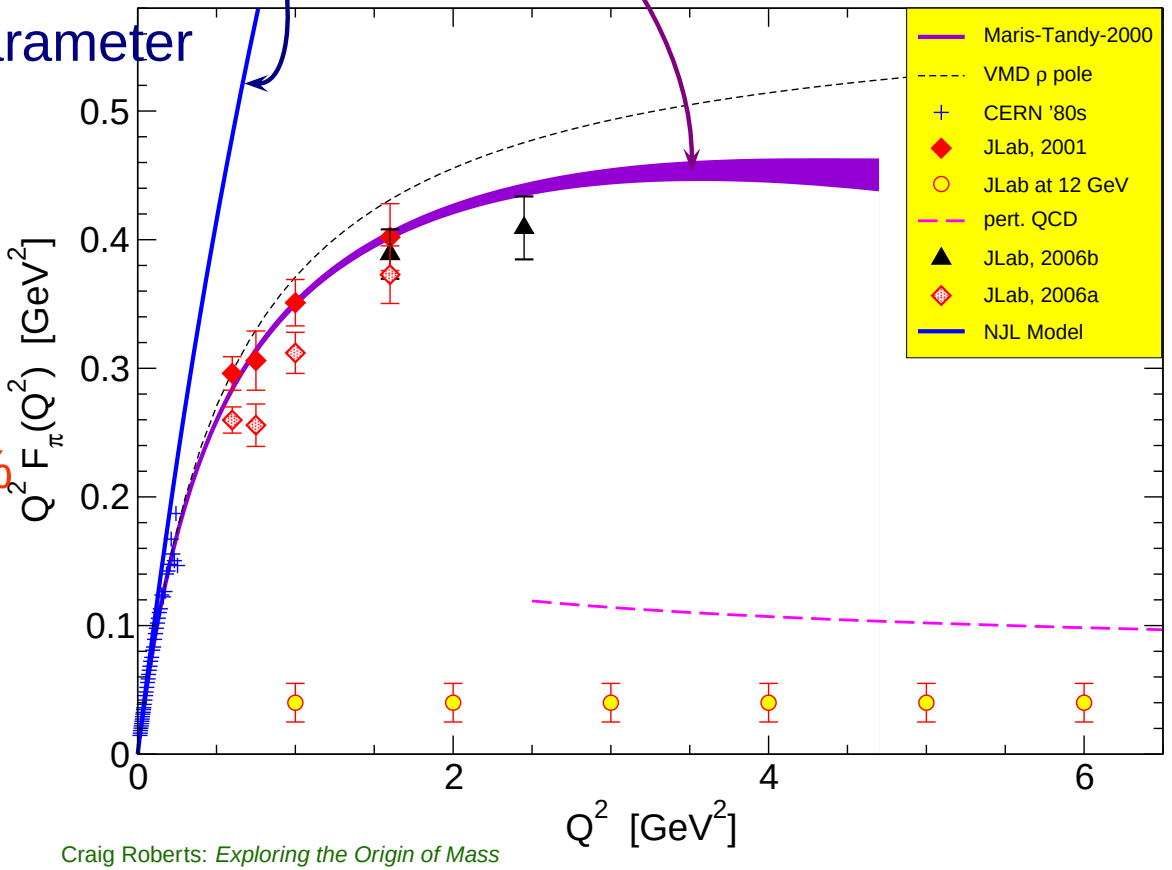
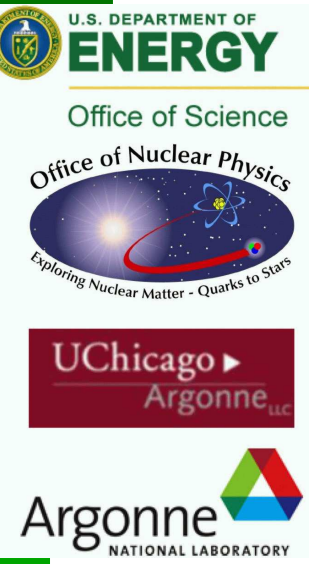
Single mass-scale parameter in both studies

Same predictions for $Q^2 = 0$ properties

Disagreement $> 20\%$ for $Q^2 > 0.2\,\text{GeV}^2$



Epilogue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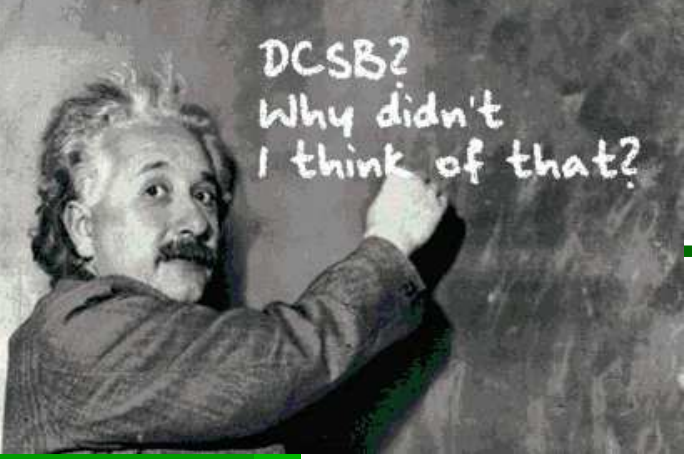

Epilogue

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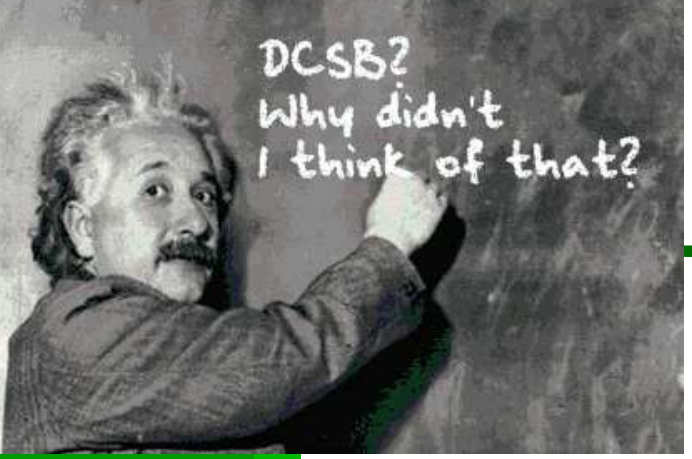

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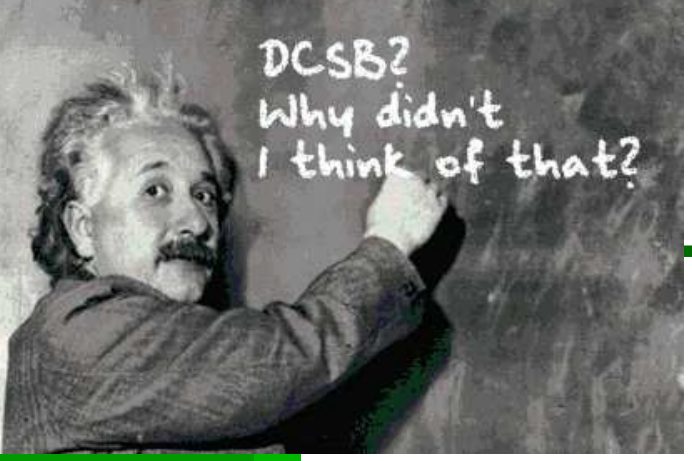
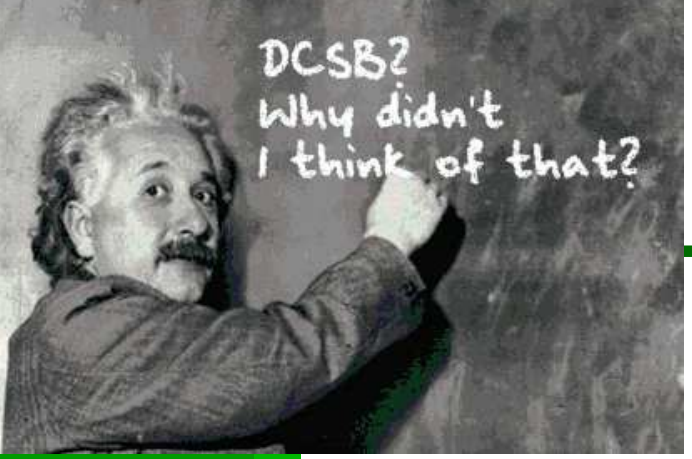


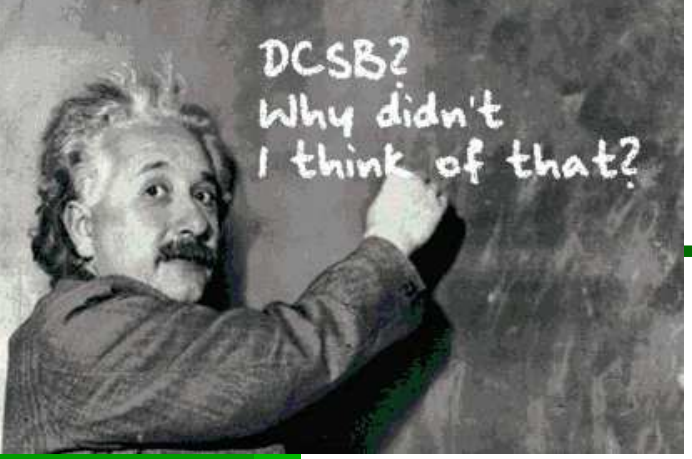
Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
  - It predicts, amongst other things, that
    - light current-quarks become heavy constituent-quarks: $4 \to 400\,$MeV
    - pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140\,$MeV
    - pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi\bar{q}q} \approx 4.3$
    - pseudscalar mesons couple unnaturally strongly to the lightest baryons $g_{\pi\bar{N}N} \approx 12.8 \approx 3 g_{\pi\bar{q}q}$



Epilogue

- DCSB impacts dramatically upon observables



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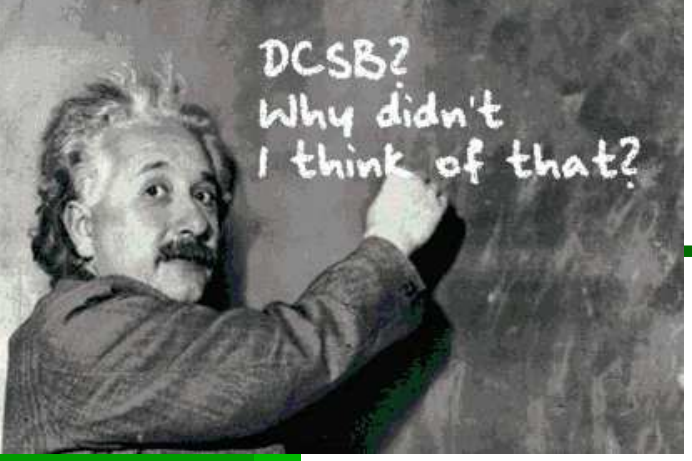
  - Spectrum; e.g., splittings: σ–π & a_1–ρ
  - Form Factors; e.g., e.m. pion and nucleon



Epilogue

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  - Exposing & elucidating its effect in hadron physics requires nonperturbative, symmetry preserving framework; i.e., Poincaré covariance, chiral and e.m. current conservation, etc.

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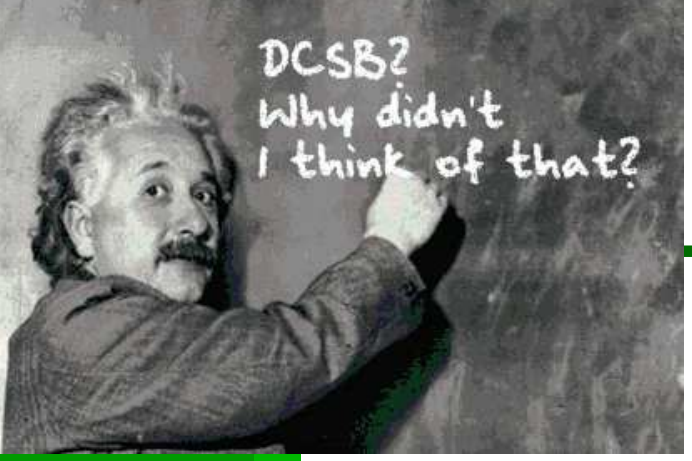

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- DSEs provide such a framework.
  - Studies underway will identify observable signals of $M(p^2)$, the most important mass-generating mechanism for visible matter in the Universe

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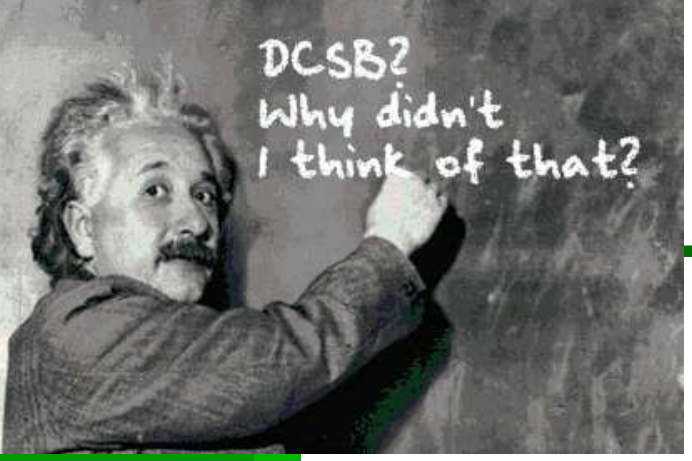

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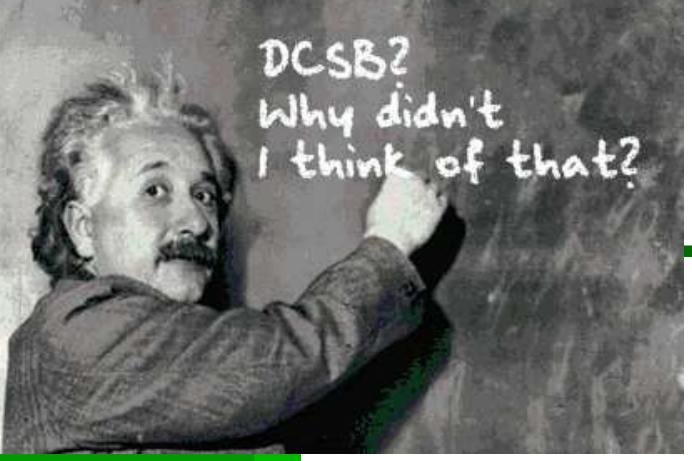

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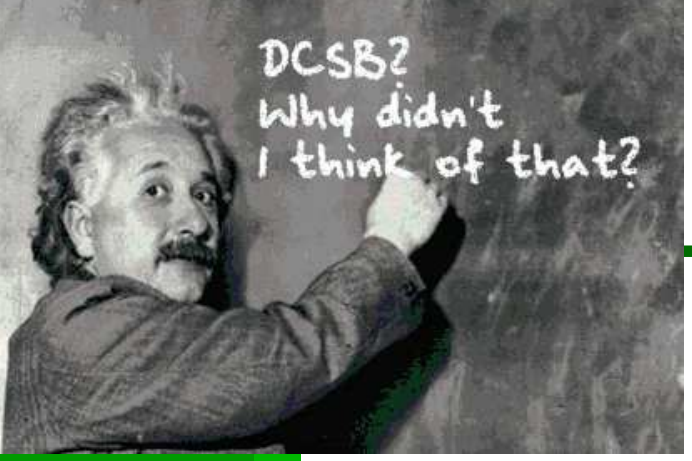

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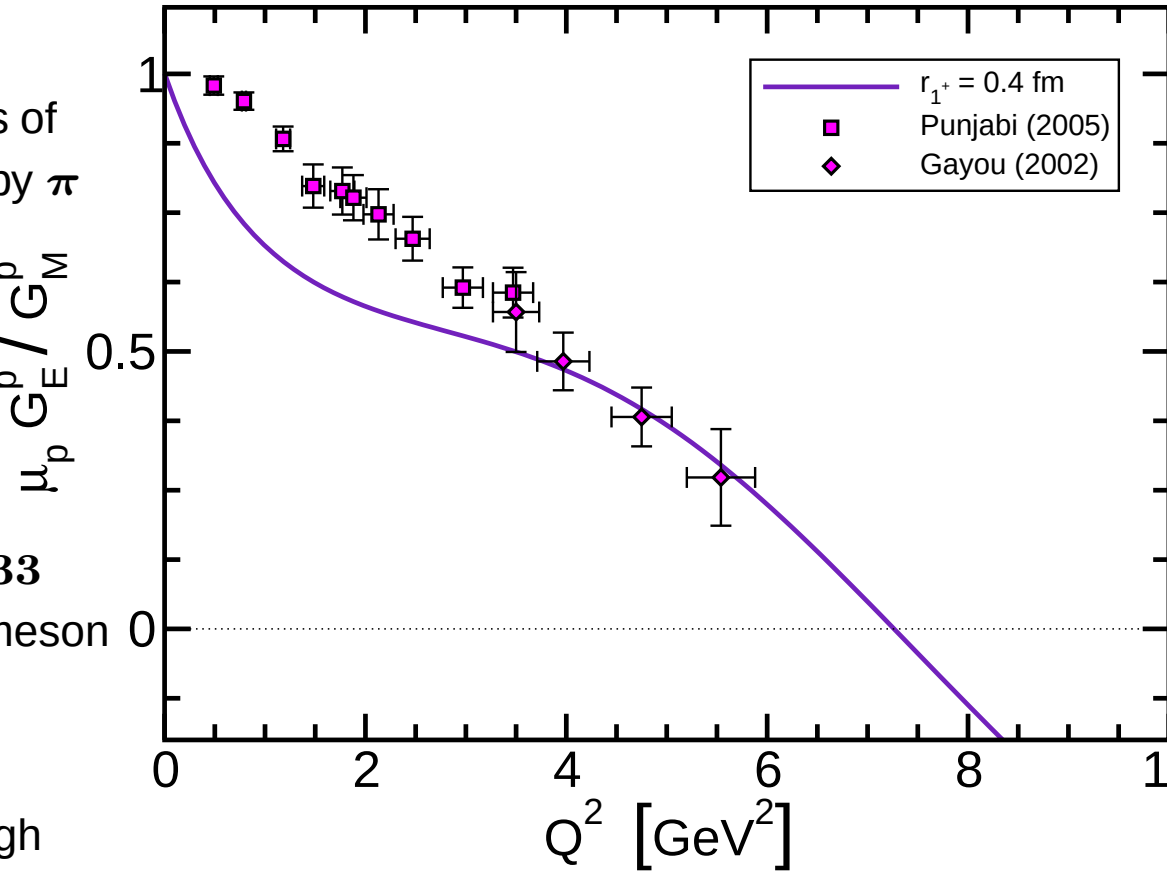

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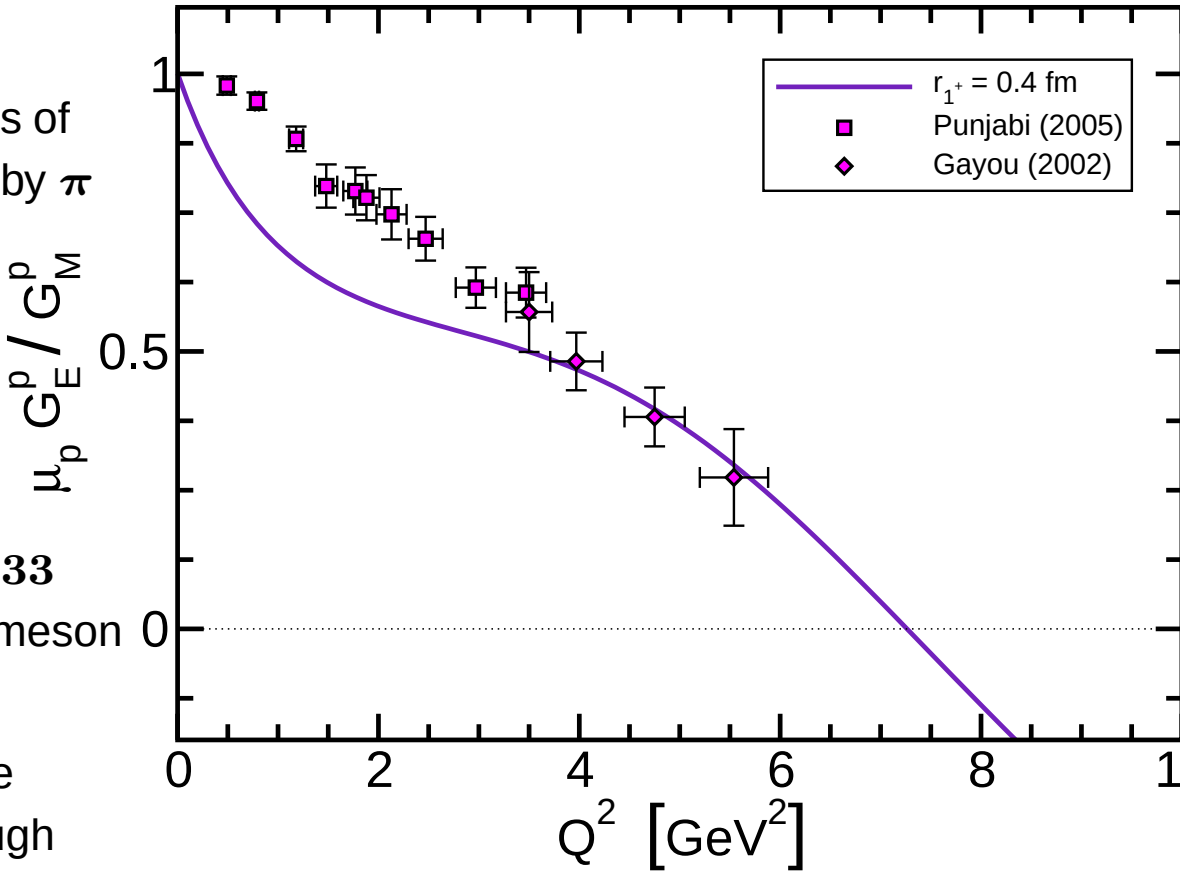

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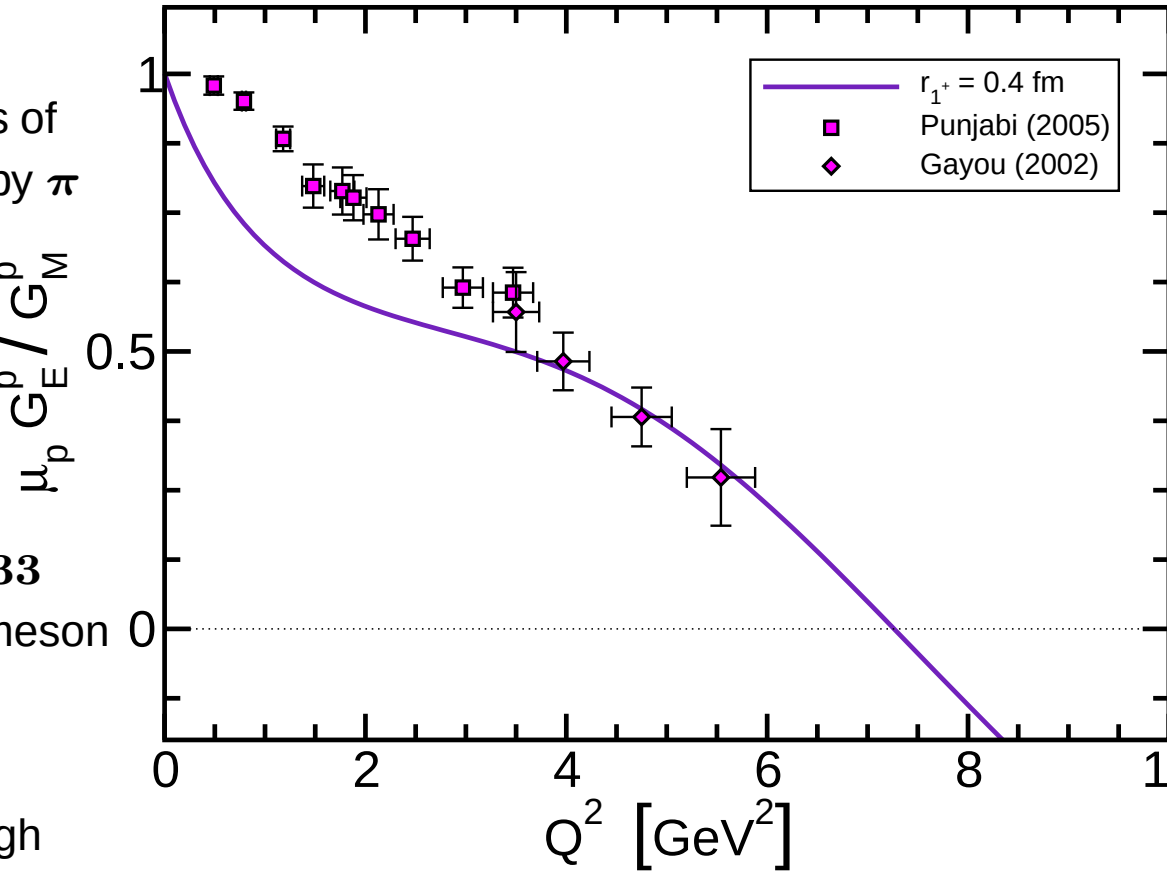

- On $Q^2 \lesssim 4\,\mathrm{GeV}^2$ result lies below experiment. This can be attributed to omission of pseudoscalar-meson-cloud contributions
- Always a zero but position depends on details of current

Pion Cloud

Pion Cloud

F2 – neutron

Pion Cloud
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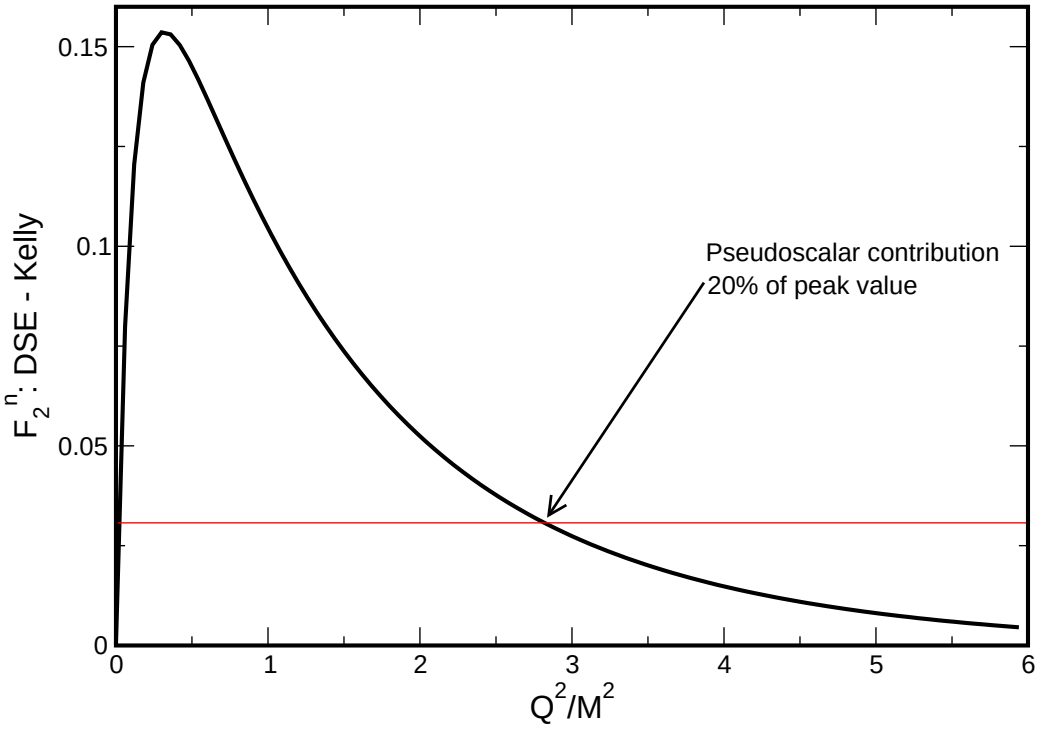
- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core



Pion Cloud
F2 – neutron

- Comparison between Faddeev equation result and Kelly's parametrisation
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- Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4\, M_N^2$