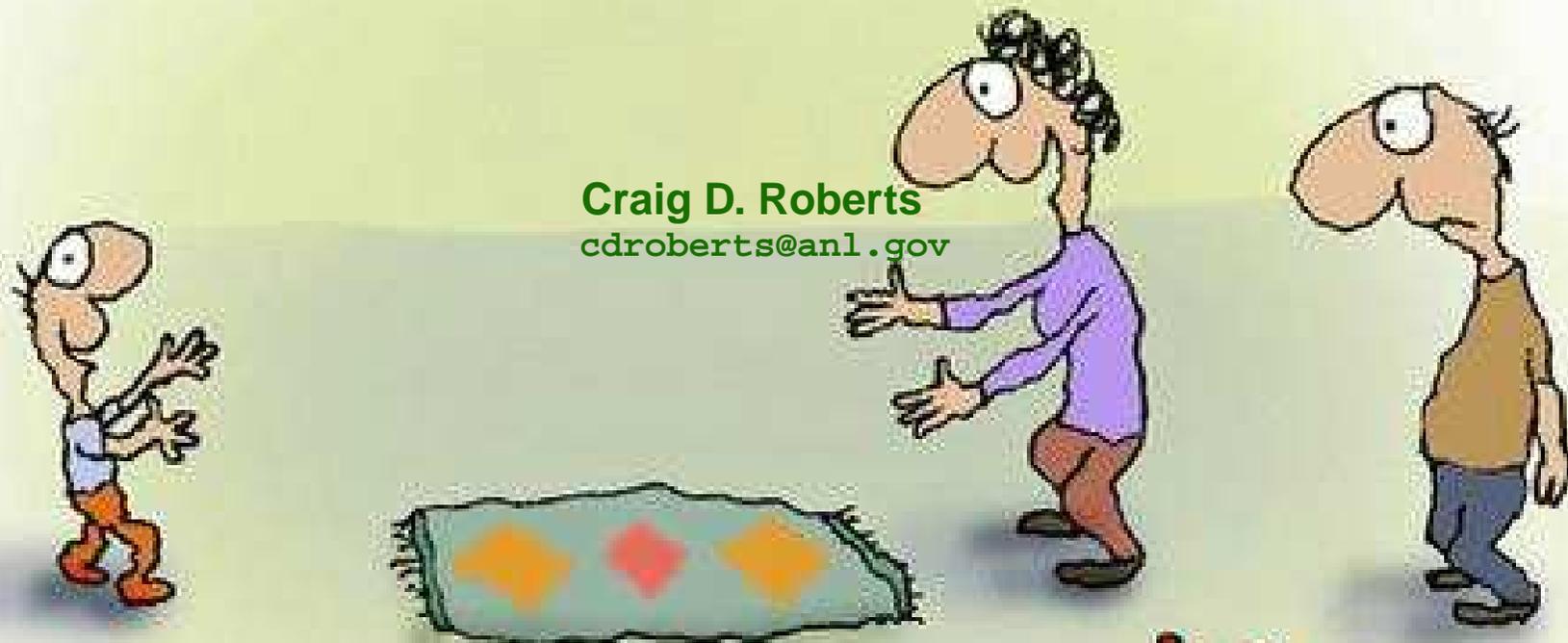


# Dynamics and Expression of Chiral Symmetry Breaking

How wonderful. He's moving  
with the times. He just took  
his first step backwards ...



Craig D. Roberts  
cdroberts@anl.gov

Physics Division & School of Physics  
Argonne National Laboratory Peking University

<http://www.phy.anl.gov/theory/staff/cdr.html>

# Universal Truths

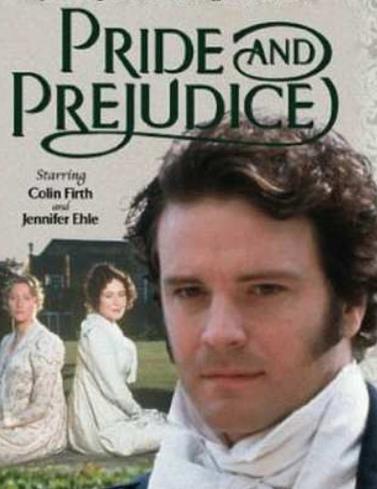


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# Universal Truths



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# PRIDE AND PREJUDICE

Starring  
Colin Firth  
and  
Jennifer Ehle



## Universal Truths

- Spectrum of excited states, and elastic and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.



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- Running of quark mass entails that calculations at even modest  $Q^2$  require a Poincaré-covariant approach. **Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.**



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# PRIDE AND PREJUDICE

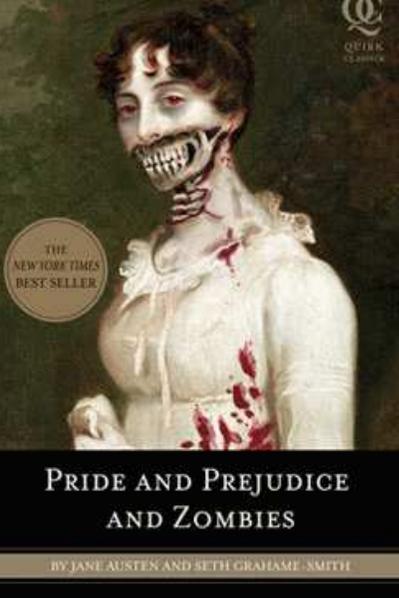
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- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe. **Higgs mechanism is irrelevant to light-quarks.**
- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. **Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.**



# QCD's Challenges

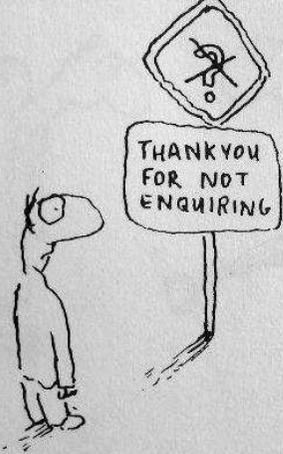


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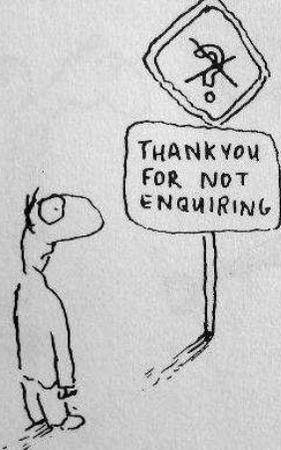
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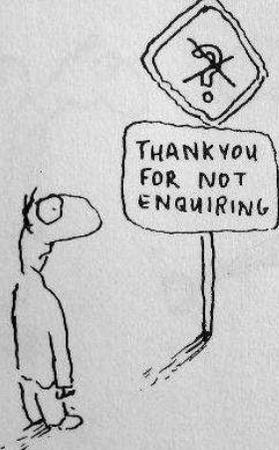
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  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon





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## Understand Emergent Phenomena

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- QCD – Complex behaviour  
arises from apparently simple rules



# Confinement



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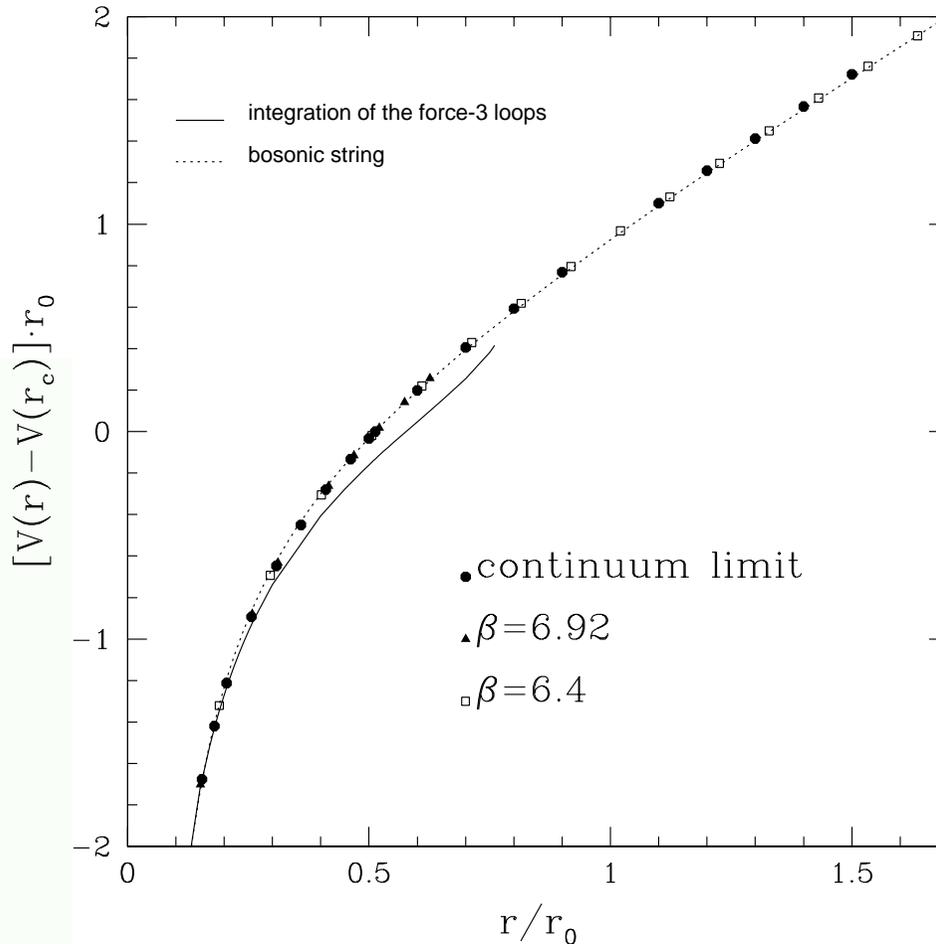
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# Confinement

## ● Infinitely Heavy Quarks ... Picture in Quantum Mechanics



$$V(r) = \sigma r - \frac{\pi}{12} \frac{1}{r}$$

$$\sqrt{\sigma} \sim 470 \text{ MeV}$$

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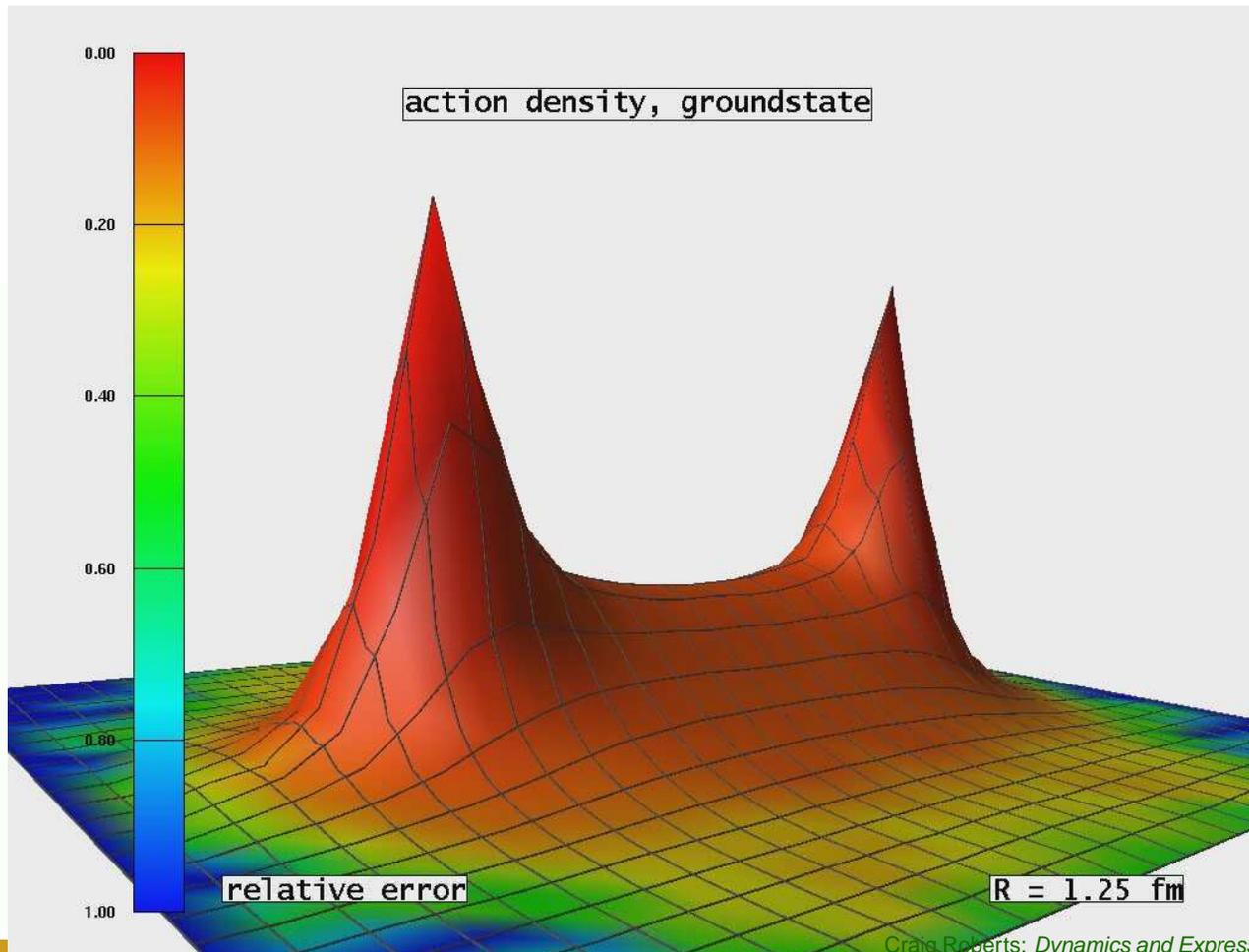
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# Confinement

- Illustrate this in terms of the action density ... analogous to plotting the Force =  $F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12} \frac{1}{r^2}$



# Confinement

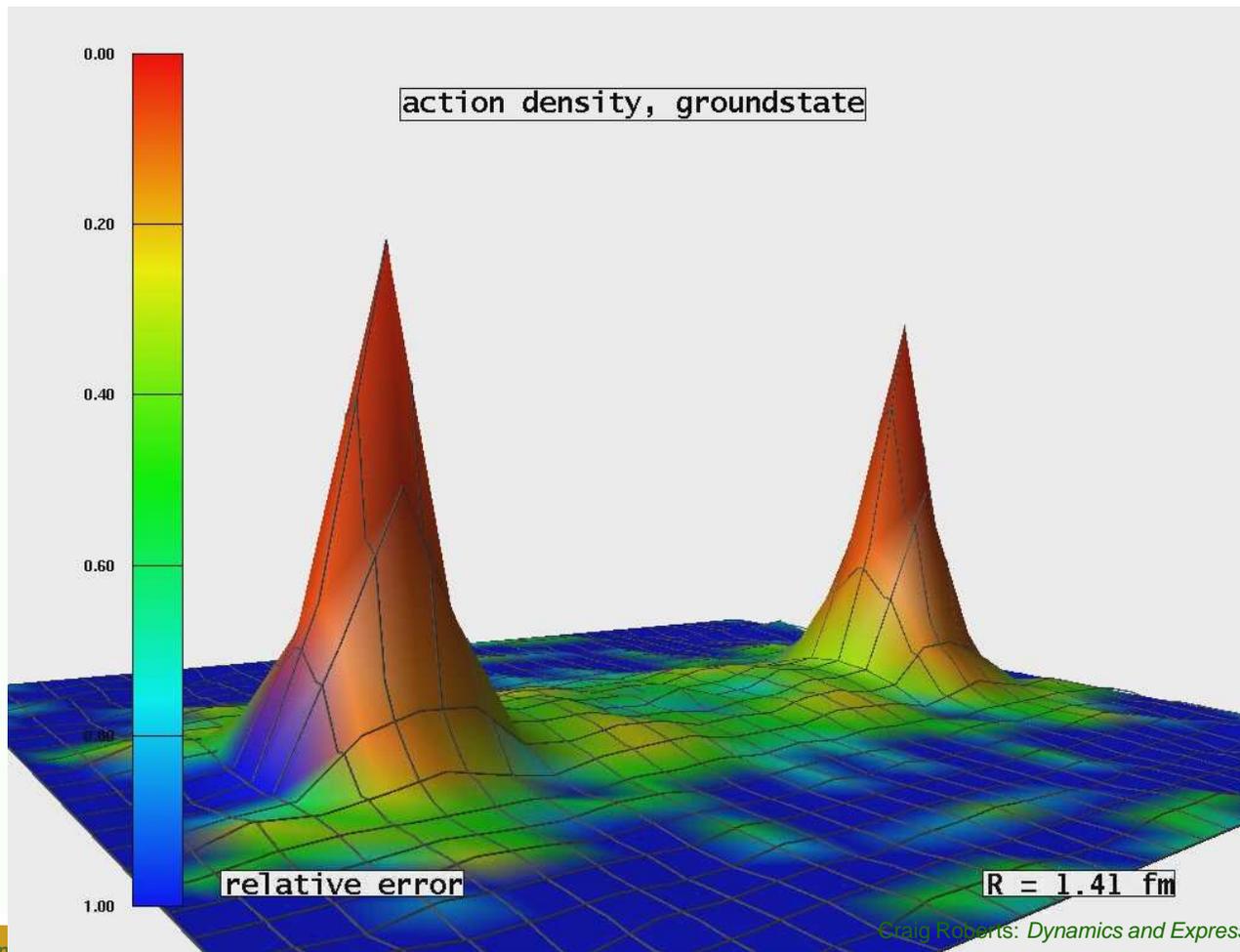
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Bali, *et al.*  
he-lq/0512018



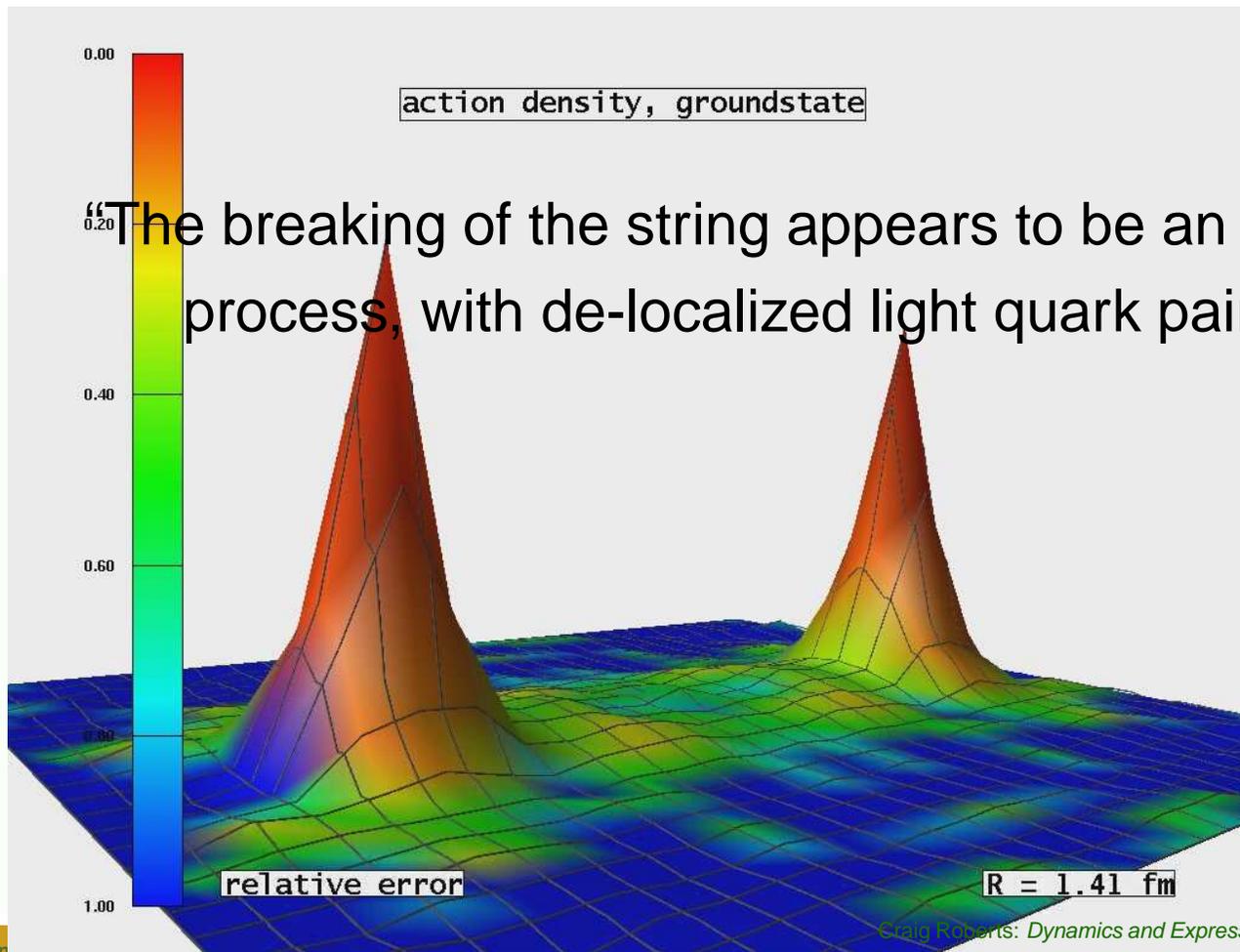
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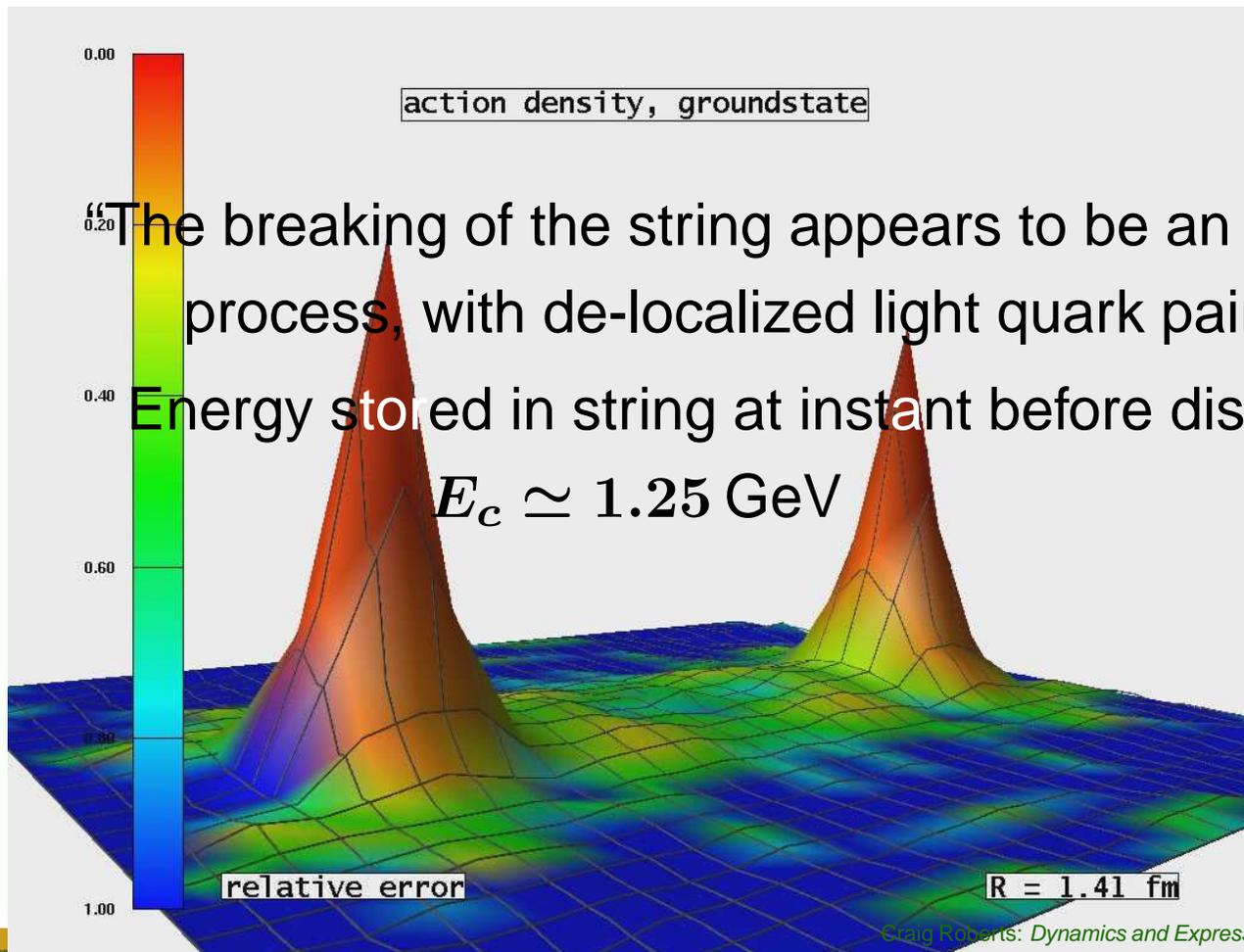


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“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”

Energy stored in string at instant before disappearance:

$$E_c \simeq 1.25 \text{ GeV}$$

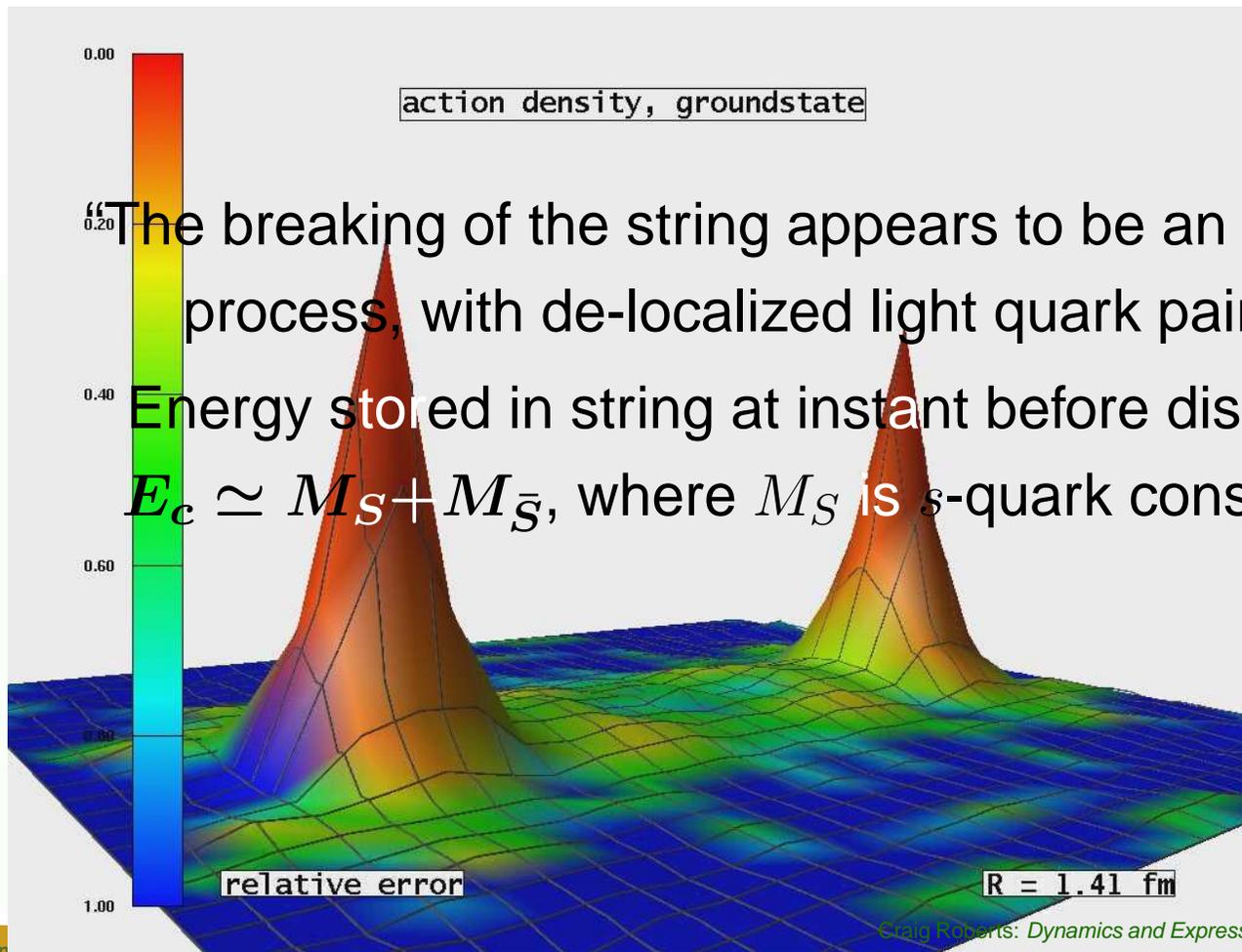


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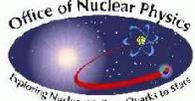
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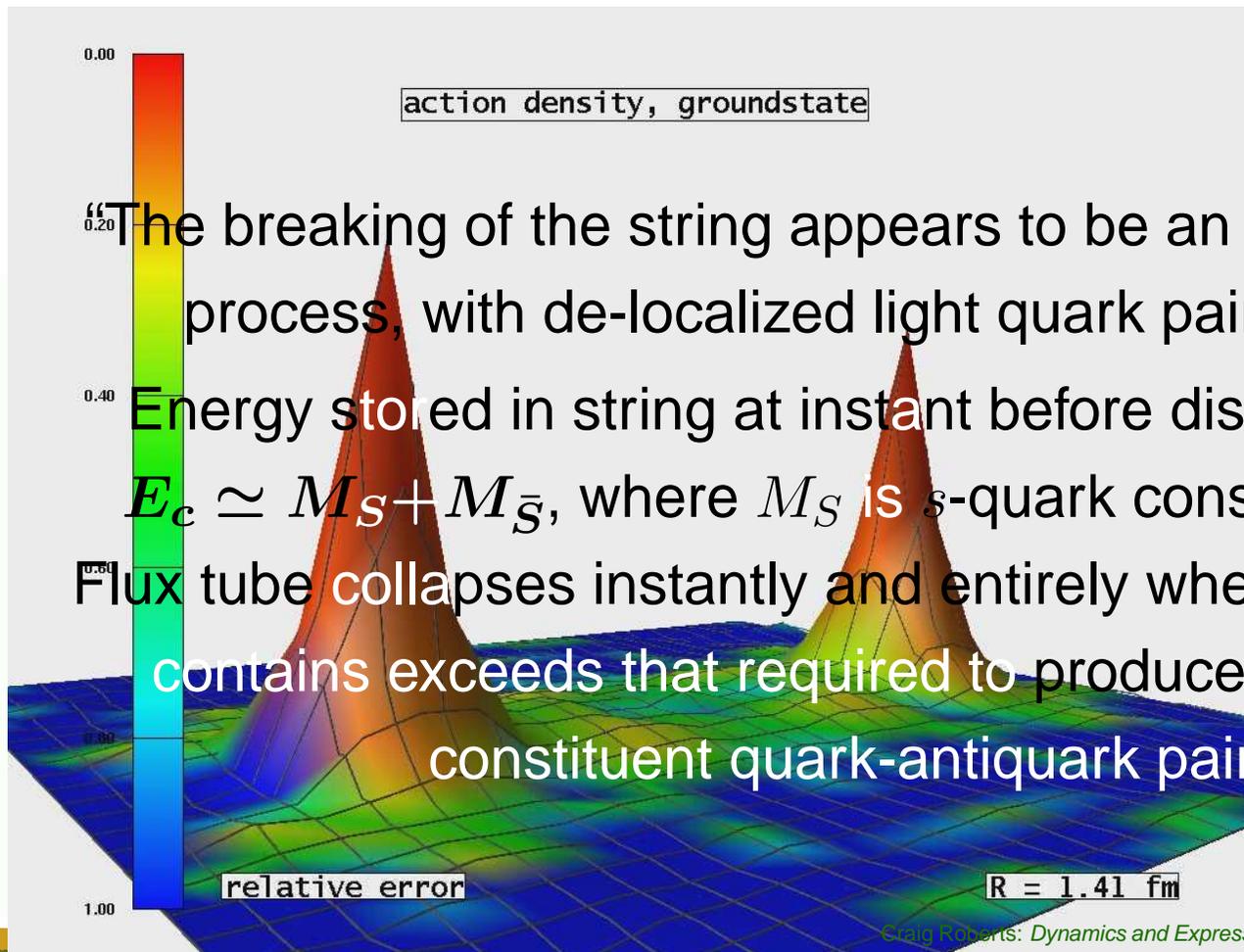


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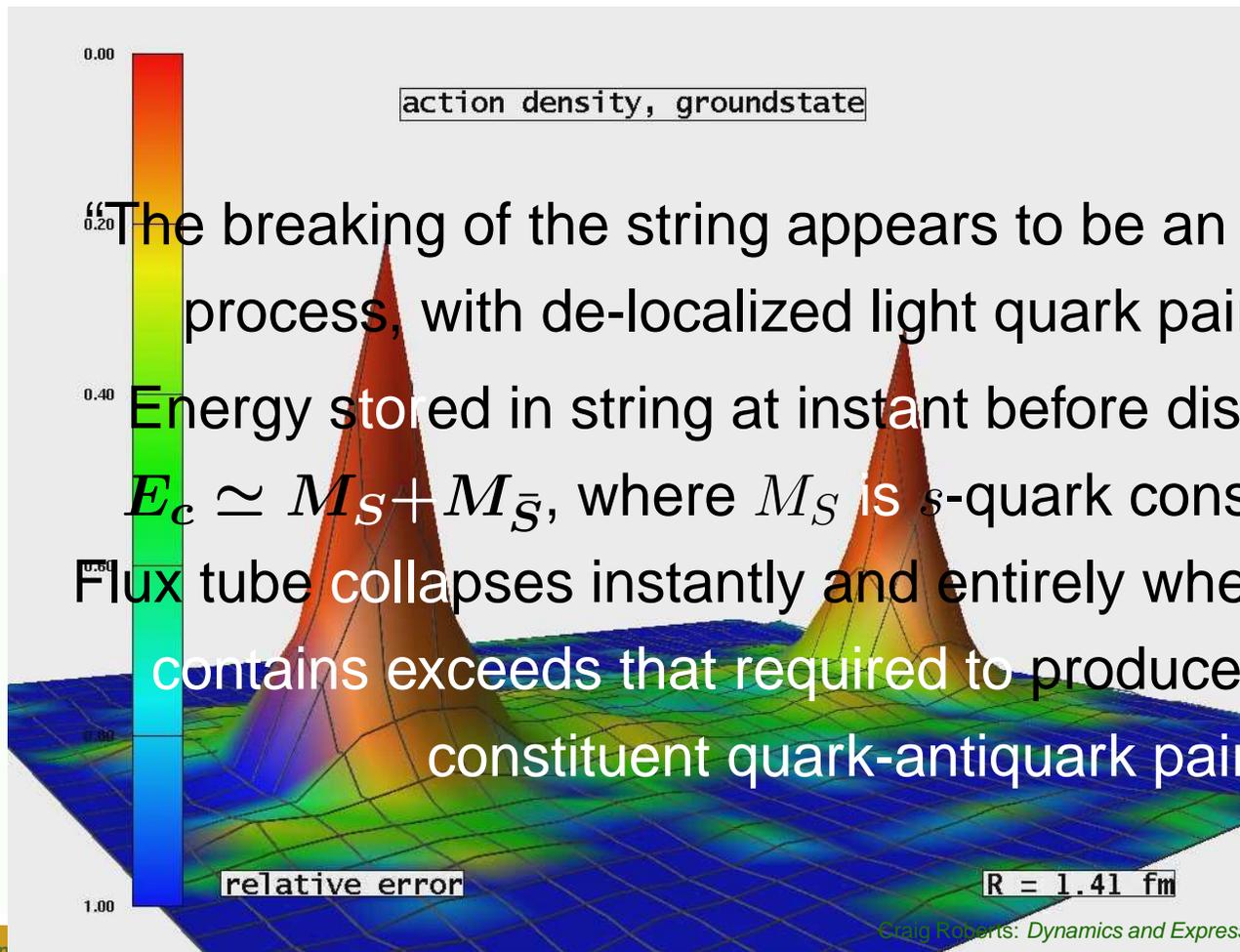


# Therefore ... No information on *potential* between light-quarks. **Confinement**

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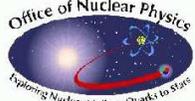


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# What is the light-quark Long-Range Potential?



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Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD **is not related** in any simple way to the light-quark interaction.



# Dyson-Schwinger Equations

- Euler-Lagrange equations for quantum field theory



# Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory



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..... behaviour of  $\alpha_s(Q^2)$



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- Method yields Schwinger Functions  $\equiv$  Propagators



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**Cross-Sections built from Schwinger Functions**



# Frontiers of Nuclear Science: A Long Range Plan (2007)



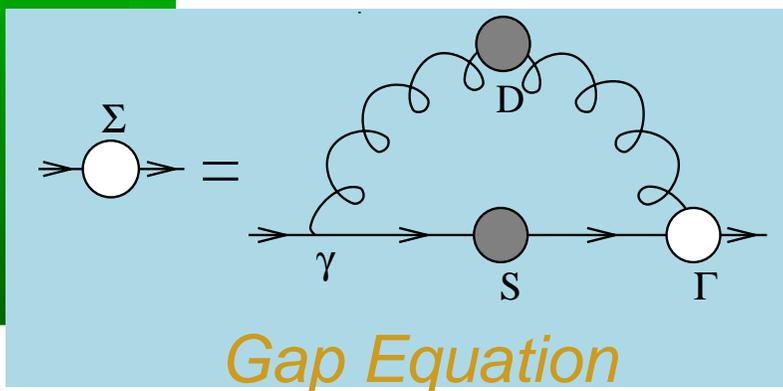
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# Frontiers of Nuclear Science: Theoretical Advances



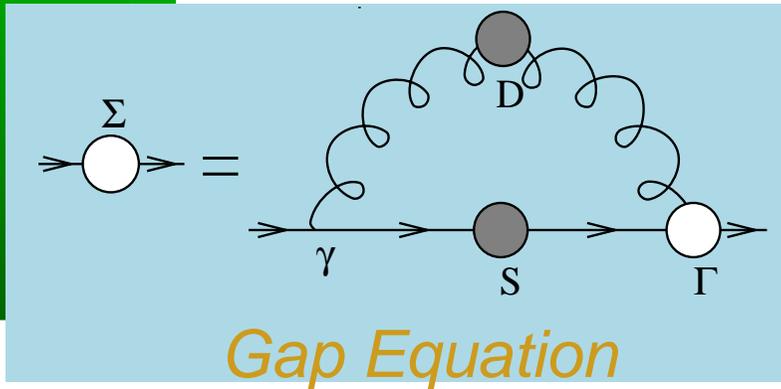
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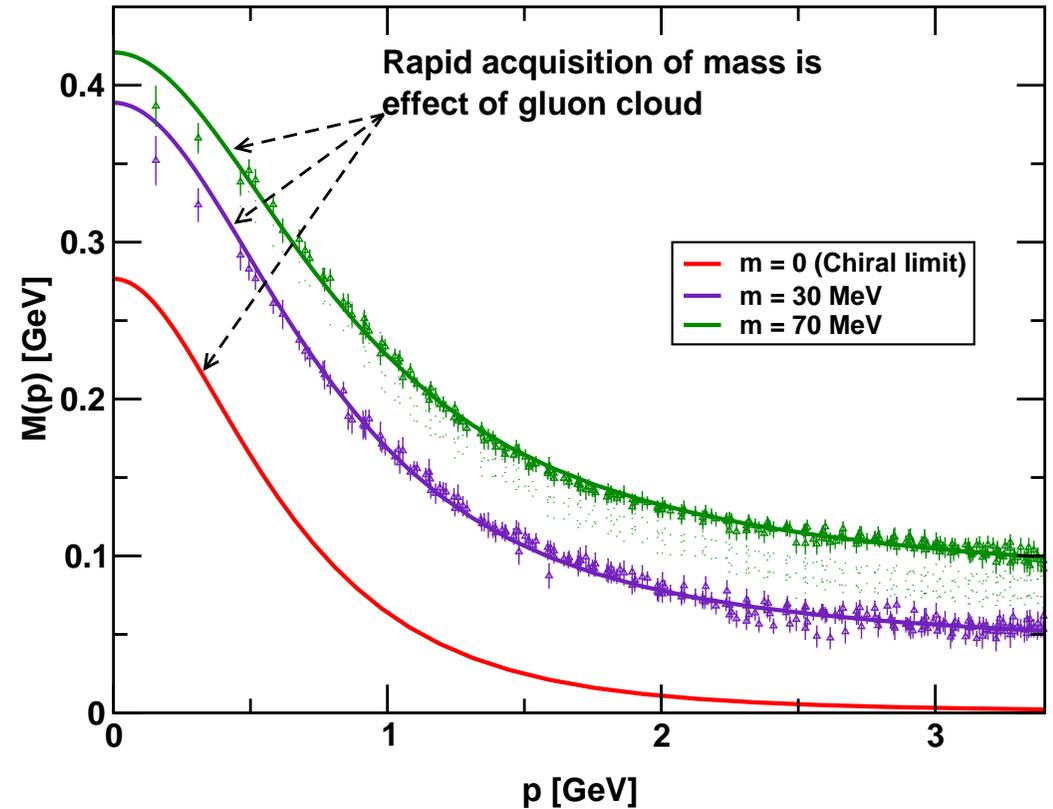
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# Frontiers of Nuclear Science: Theoretical Advances



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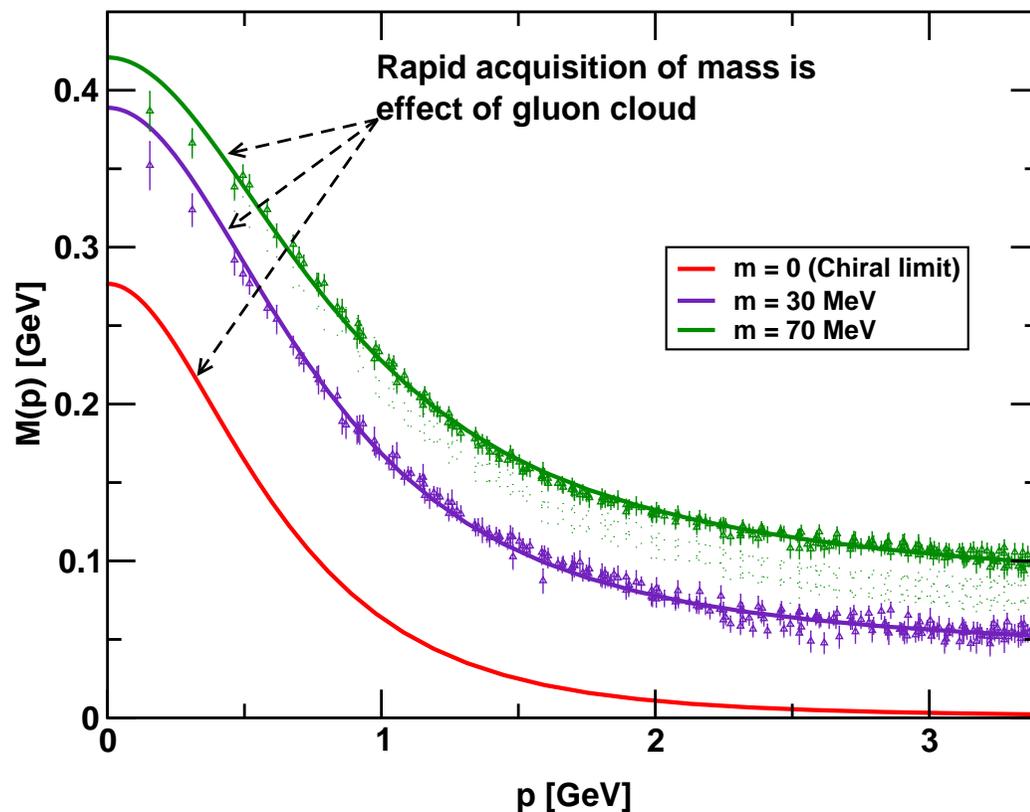


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In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ( $m = 0$ , red curve) acquires a large constituent mass at low energies.

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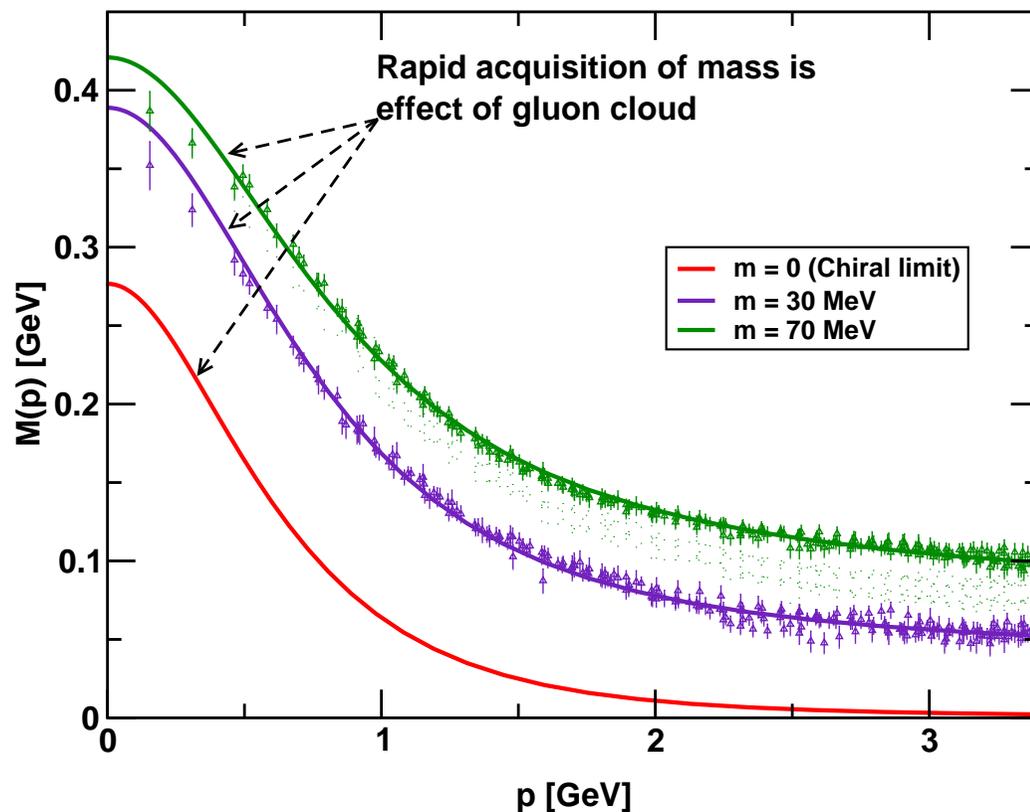


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# Charting the Interaction between light-quarks



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# Charting the Interaction between light-quarks

- Confinement can be related to the analytic properties of QCD's Schwinger functions



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  - Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's *universal*  $\beta$ -function
    - This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.
- Of course, the behaviour of the  $\beta$ -function on the perturbative domain is well known.
- This is a well-posed problem whose solution is an elemental goal of modern hadron physics.



# Charting the Interaction between light-quarks

- Through DSEs the pointwise behaviour of the  $\beta$ -function determines pattern of chiral symmetry breaking



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- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
  - Steady quantitative progress is being made with a scheme that is systematically improvable



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- DSEs connect  $\beta$ -function to experimental observables. Hence, comparison between computations and observations of, e.g., hadron mass spectrum can be used to chart  $\beta$ -function's long-range behaviour
- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
  - On other hand, at present significant qualitative advances possible with symmetry-preserving kernel *Ansätze* that express important additional nonperturbative effects –  $M(p^2)$  – difficult/impossible to capture in any finite sum of contributions





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- Established understanding of two- and three-point functions



# Hadrons



- Established understanding of two- and three-point functions
- What about bound states?



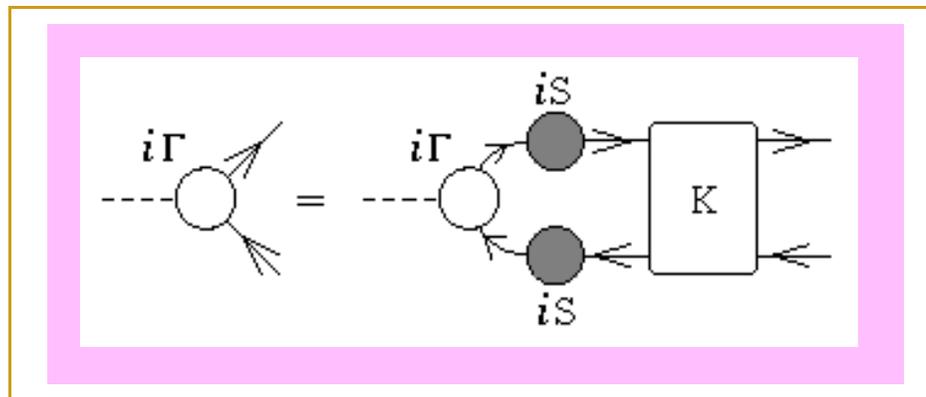
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- They appear as pole contributions to  $n \geq 3$ -point colour-singlet Schwinger functions

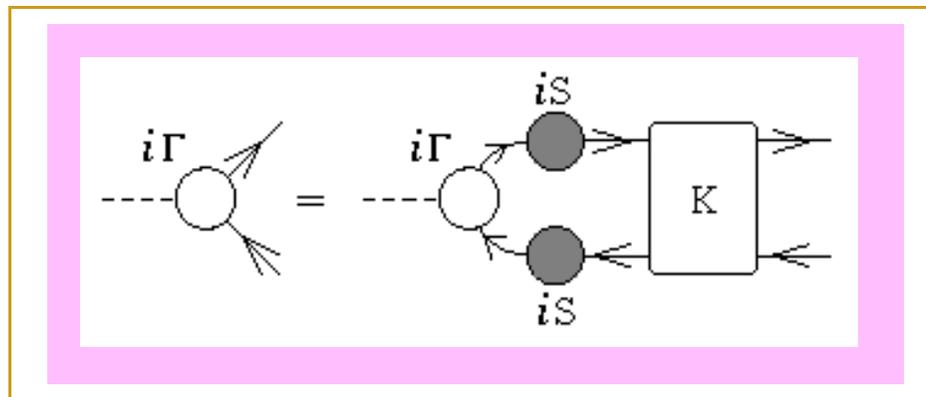


- Without bound states, Comparison with experiment is **impossible**
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QFT Generalisation of Lippmann-Schwinger Equation.

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QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel,  $K$ ?
- or What is the **long-range** potential in QCD?

# Bethe-Salpeter Kernel



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# Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^l(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f^l i\gamma_5 + \frac{1}{2} \lambda_f^l i\gamma_5 \mathcal{S}^{-1}(k_-)$$
$$- M_\zeta i\Gamma_5^l(k; P) - i\Gamma_5^l(k; P) M_\zeta$$

QFT Statement of Chiral Symmetry



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Satisfies BSE

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- **Nontrivial** constraint





# Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^l(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f^l i\gamma_5 + \frac{1}{2} \lambda_f^l i\gamma_5 \mathcal{S}^{-1}(k_-) - M_\zeta i\Gamma_5^l(k; P) - i\Gamma_5^l(k; P) M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels very different

but must be *intimately* related

- Relation **must** be preserved by truncation
- **Failure**  $\Rightarrow$  Explicit Violation of QCD's Chiral Symmetry



# Persistent Challenge



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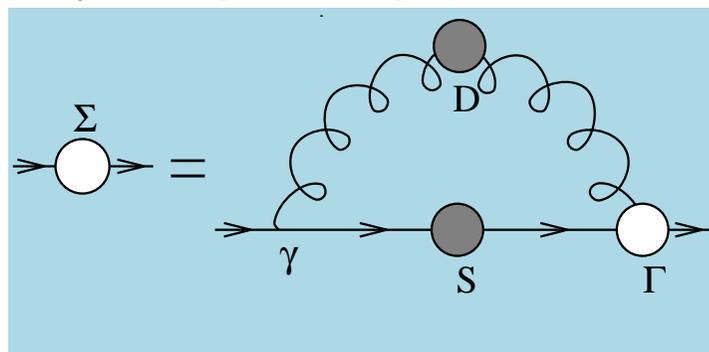
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# Persistent Challenge

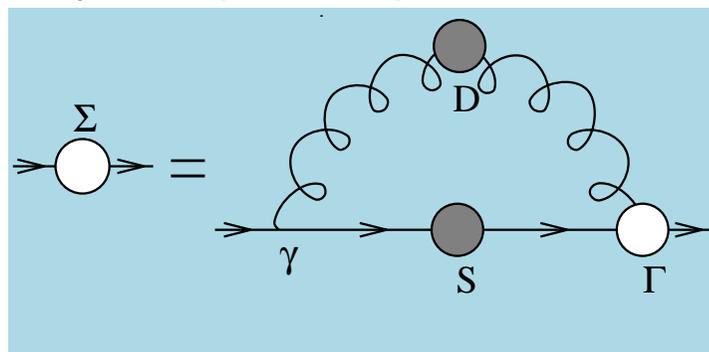
- Infinitely Many Coupled Equations





# Persistent Challenge

- Infinitely Many Coupled Equations



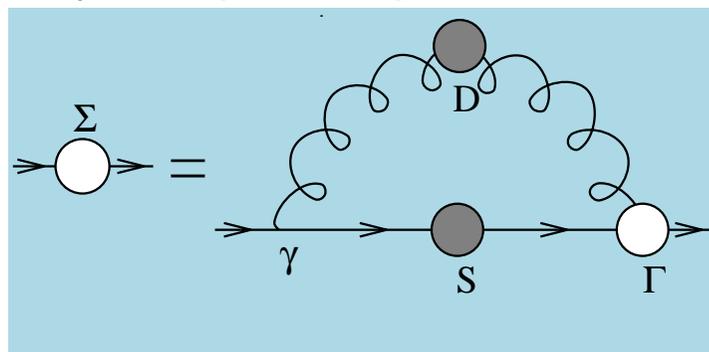
- Coupling between equations **necessitates** truncation





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- Infinitely Many Coupled Equations



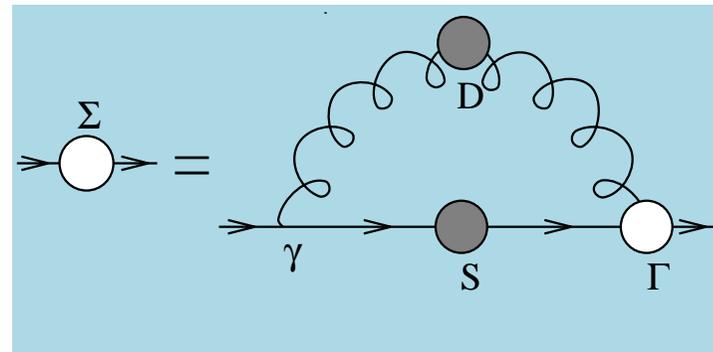
- Coupling between equations **necessitates** truncation
  - Weak coupling expansion  $\Rightarrow$  Perturbation Theory





# Persistent Challenge

- Infinitely Many Coupled Equations



- Coupling between equations **necessitates** truncation

- Weak coupling expansion  $\Rightarrow$  Perturbation Theory  
**Not useful** for the nonperturbative problems  
in which we're interested





# Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme

H.J. Munczek Phys. Rev. D **52** (1995) 4736

*Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*

A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7

*Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*





# Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme
- Has Enabled Proof of **EXACT** Results in QCD



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# Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme
- Has Enabled Proof of **EXACT** Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results



# Persistent Challenge



- Infinitely Many Coupled Equations
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  - Make Predictions with Readily Quantifiable Errors





# Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme
- Has Enabled Proof of **EXACT** Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results
  - Make Predictions with Readily Quantifiable Errors
- Examples:

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Review Articles and Tutorials in an Encyclopædic Format

[web.mit.edu/readingtn/www/netadv/Xdysonschw.html](http://web.mit.edu/readingtn/www/netadv/Xdysonschw.html)



# Gap Equation

## General Form



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# Gap Equation

## General Form

- Return to general bound-state problem . . .



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- Return to general bound-state problem . . .
- To study the Poincaré covariant bound-state problem for mesons, one must first solve the gap equation

$$S_f(p)^{-1} = Z_2 (i\gamma \cdot p + m_f^{\text{bm}}) + \Sigma_f(p),$$

$$\Sigma_f(p) = Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S_f(q) \frac{\lambda^a}{2} \Gamma_\nu^f(q,p),$$



- Return to general bound-state problem . . .
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- $D_{\mu\nu}(k)$  is the dressed-gluon propagator;
- $\Gamma_\nu^f(q,p)$  is the dressed-quark-gluon vertex;
- $m^{\text{bm}}(\Lambda)$  is the Lagrangian current-quark bare mass;
- $Z_{1,2}(\zeta^2, \Lambda^2)$  are respectively the vertex and quark wave function renormalisation constants, with  $\zeta$  the renormalisation point.



# Bethe-Salpeter Equation

## General Form



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# Bethe-Salpeter Equation

## General Form

- Pseudoscalar and axial-vector mesons appear as poles in the inhomogeneous Bethe-Salpeter equation.



- Pseudoscalar and axial-vector mesons appear as poles in the inhomogeneous Bethe-Salpeter equation.
- Exact form:

$$\begin{aligned}\Gamma_{5\mu}^{fg}(k; P) &= Z_2 \gamma_5 \gamma_\mu - \int_q g^2 D_{\alpha\beta}(k - q) \\ &\quad \times \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \Gamma_{5\mu}^{fg}(q; P) S_g(q_-) \frac{\lambda^a}{2} \Gamma_\beta^g(q_-, k_-) \\ &\quad + \int_q g^2 D_{\alpha\beta}(k - q) \frac{\lambda^a}{2} \gamma_\alpha S_f(q_+) \frac{\lambda^a}{2} \Lambda_{5\mu\beta}^{fg}(k, q; P),\end{aligned}$$



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- $\Lambda_{5\mu\beta}^{fg}$  is defined completely via the dressed-quark self-energy and, owing to Poincaré covariance, one can employ, e.g.,  $q_\pm = q \pm P/2$ , etc., without loss of generality



# *Ward-Takahashi Identity*

## *Bethe-Salpeter Kernel*



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# Ward-Takahashi Identity

## Bethe-Salpeter Kernel

- In any reliable study of light-quark hadrons, axial-vector vertex must satisfy

$$P_\mu \Gamma_{5\mu}^{fg}(k; P) = S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-) - i [m_f(\zeta) + m_g(\zeta)] \Gamma_5^{fg}(k; P),$$

expresses chiral symmetry & pattern by which it's broken



# Ward-Takahashi Identity

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expresses chiral symmetry & pattern by which it's broken

- The condition ( $\Lambda_{5\beta}^{fg}$  pseudoscalar analogue of  $\Lambda_{5\mu\beta}^{fg}$ )

$$P_\mu \Lambda_{5\mu\beta}^{fg}(k, q; P) = \Gamma_\beta^f(q_+, k_+) i\gamma_5 + i\gamma_5 \Gamma_\beta^g(q_-, k_-) - i [m_f(\zeta) + m_g(\zeta)] \Lambda_{5\mu\beta}^{fg}(k, q; P),$$

**NECESSARY & SUFFICIENT**

to ensure Ward-Takahashi identity satisfied.



# Ward-Takahashi Identity

## Bethe-Salpeter Kernel

- The condition ( $\Lambda_{5\beta}^{fg}$  pseudoscalar analogue of  $\Lambda_{5\mu\beta}^{fg}$ )

$$P_\mu \Lambda_{5\mu\beta}^{fg}(k, q; P) = \Gamma_\beta^f(q_+, k_+) i\gamma_5 + i\gamma_5 \Gamma_\beta^g(q_-, k_-) - i[m_f(\zeta) + m_g(\zeta)] \Lambda_{5\beta}^{fg}(k, q; P),$$

### NECESSARY & SUFFICIENT

to ensure Ward-Takahashi identity satisfied.

- Rainbow-ladder ...

- $\Gamma_\beta^f(q, k) = \gamma_\mu$

$$\Rightarrow \Lambda_{5\mu\beta}^{fg}(k, q; P) = 0 = \Lambda_{5\beta}^{fg}(k, q; P)$$



# Bethe-Salpeter Kernel



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# Bethe-Salpeter Kernel

## 60 year problem

- Bethe-Salpeter equation introduced in 1951



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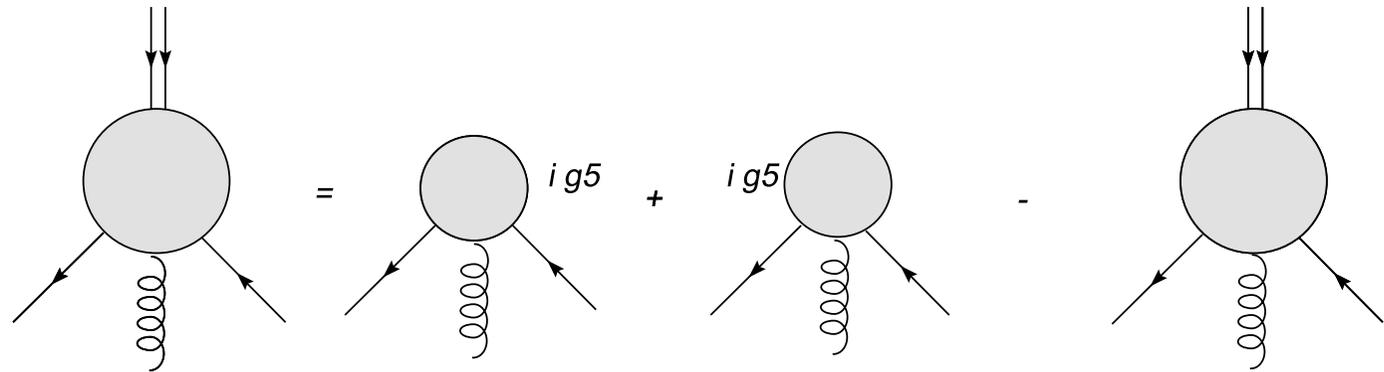
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# Bethe-Salpeter Kernel

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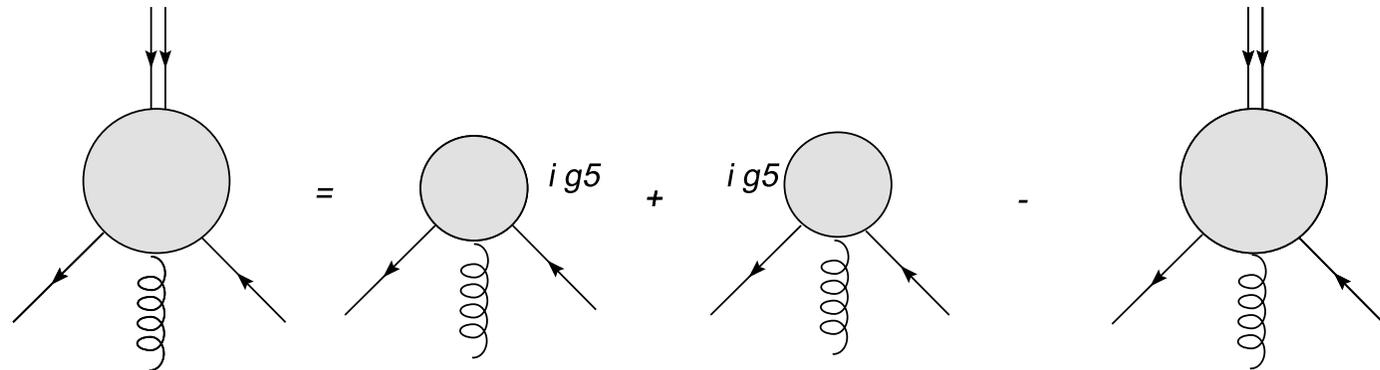


$$P_\mu \Lambda_{5\mu\beta}^{fg}(k, q; P) = \Gamma_\beta^f(q_+, k_+) i\gamma_5 + i\gamma_5 \Gamma_\beta^g(q_-, k_-) - i[m_f(\zeta) + m_g(\zeta)] \Lambda_{5\beta}^{fg}(k, q; P),$$



Chang Lei (IAPCM, Beijing) & CDR  
arXiv:0903.5461 [nucl-th] – in Phys.Rev.Lett.

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- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
  - Consistent means - all symmetries preserved!

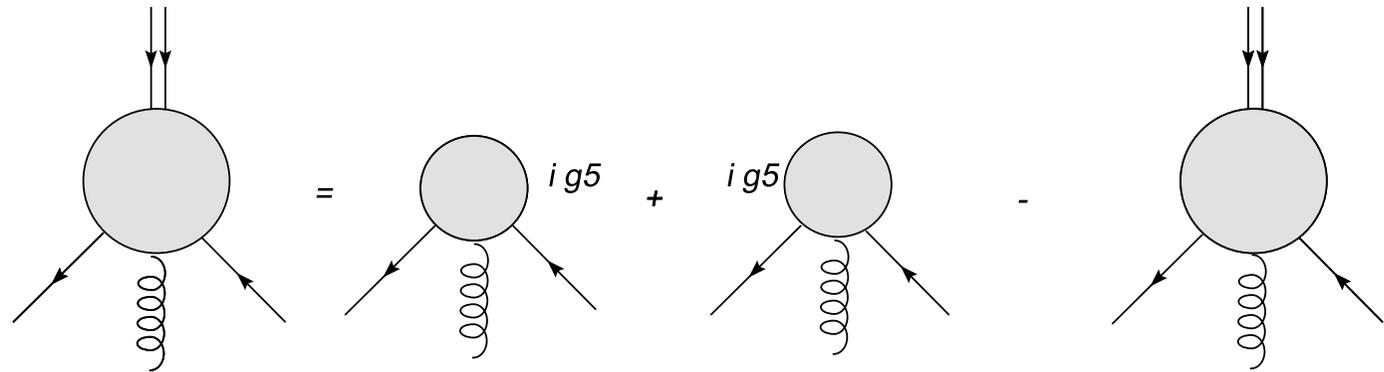


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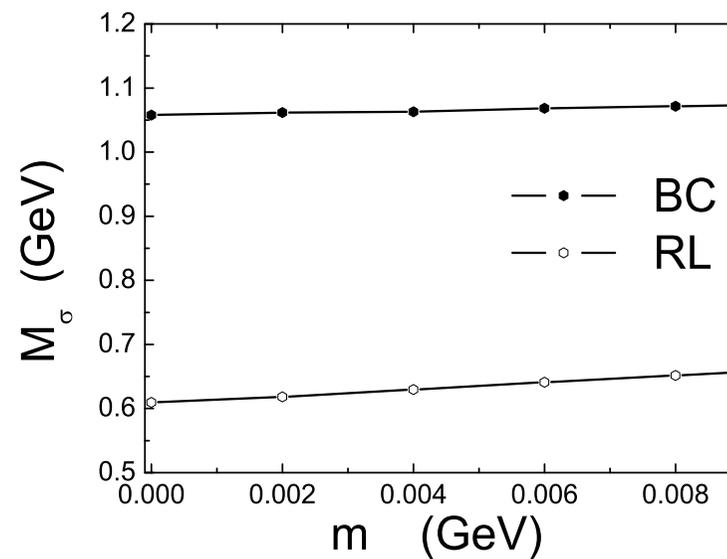
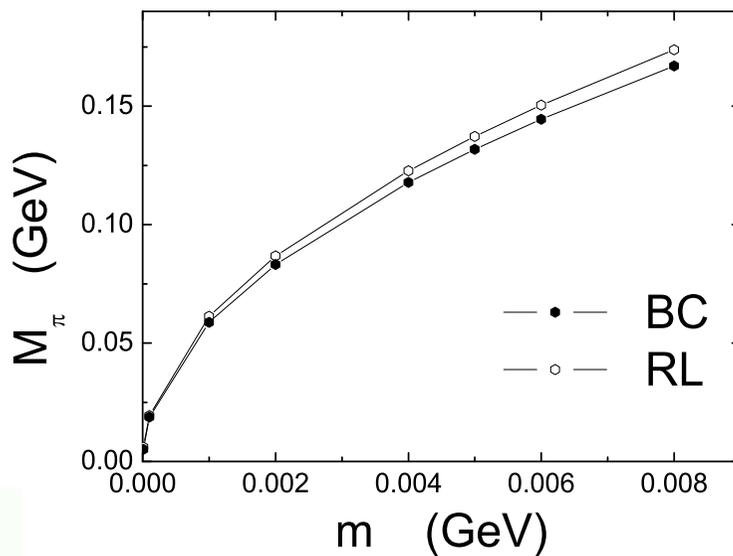
- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
- Exemplified the procedure and results to expect ...



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# Numerical Illustration

Chang Lei & CDR  
arXiv:0903.5461 [nucl-th], in Phys.Rev.Lett.



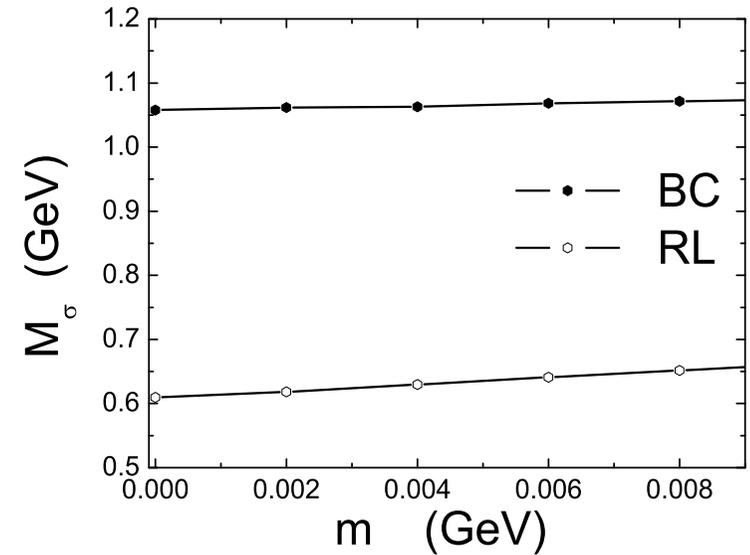
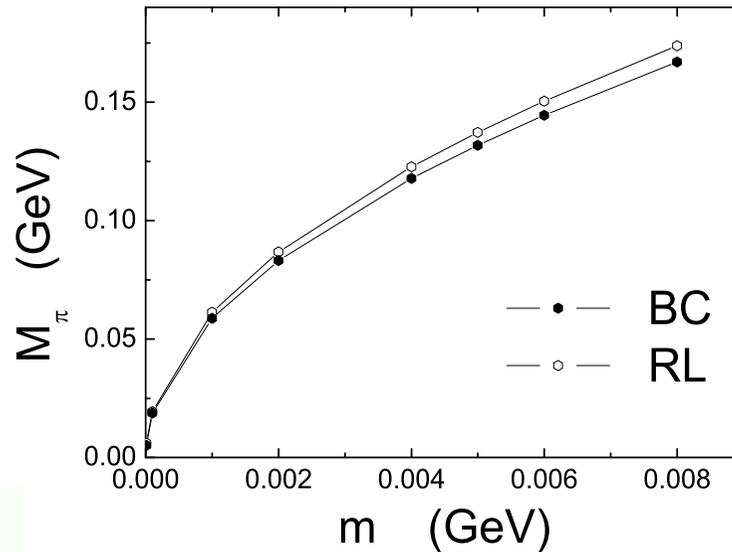
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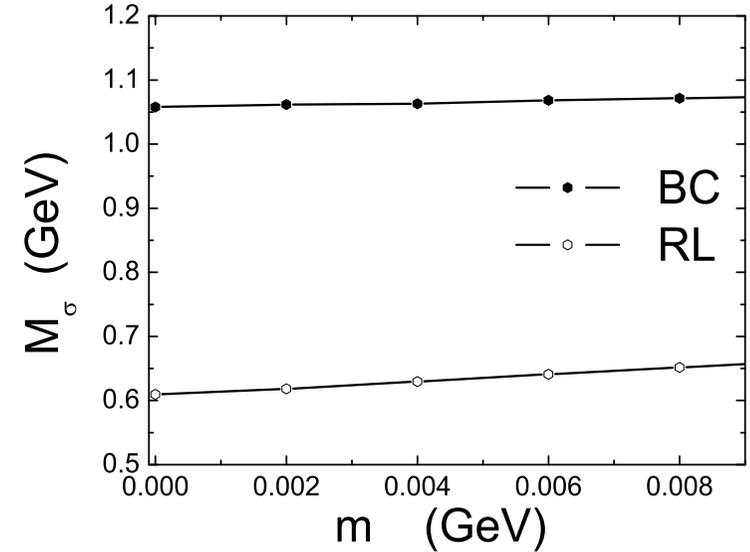
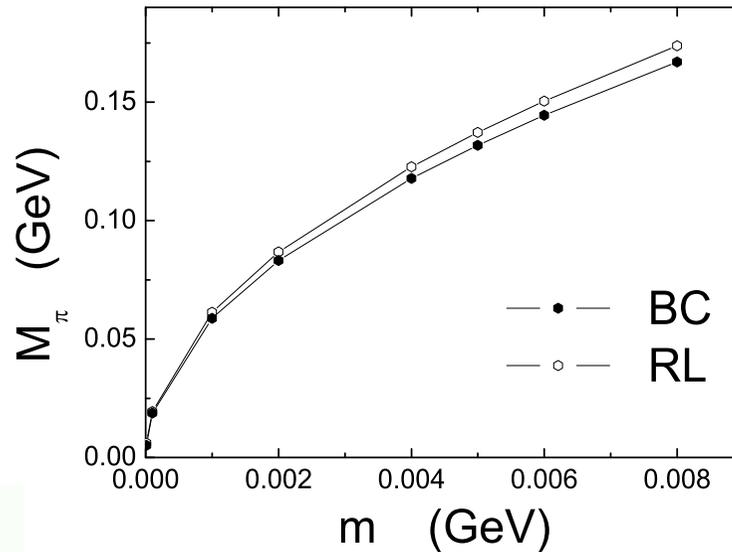
Chang Lei & CDR  
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- Single interaction, common mass scale:  
rainbow-ladder cf. BC-consistent truncation



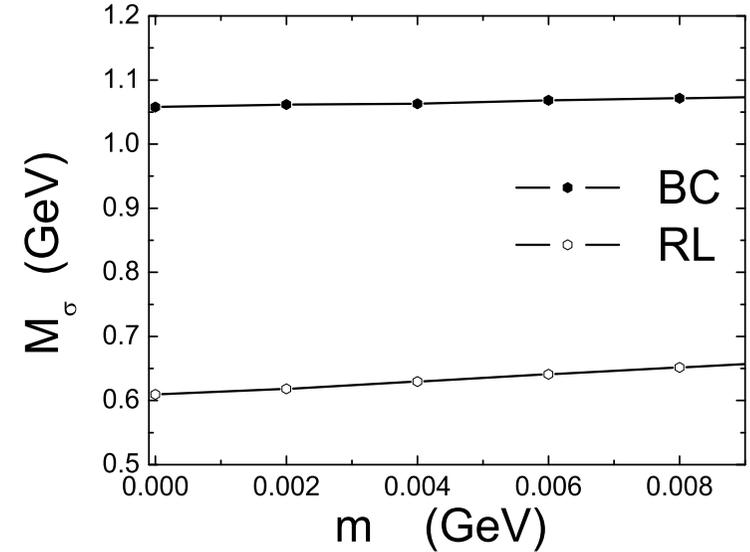
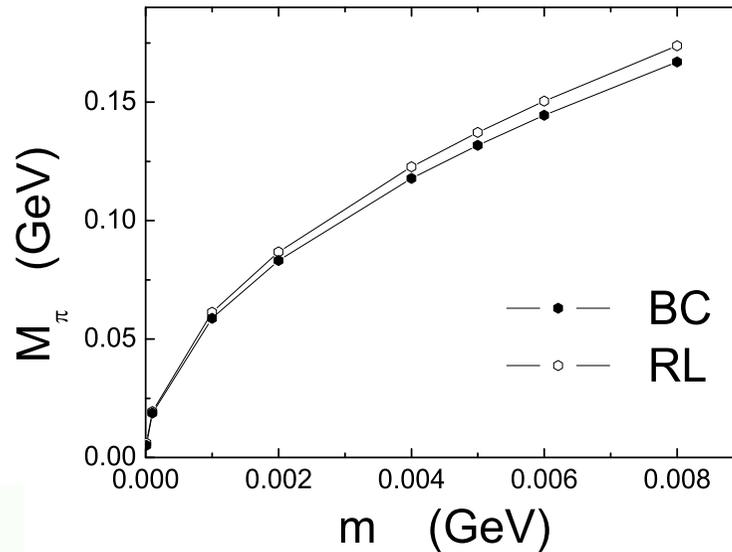
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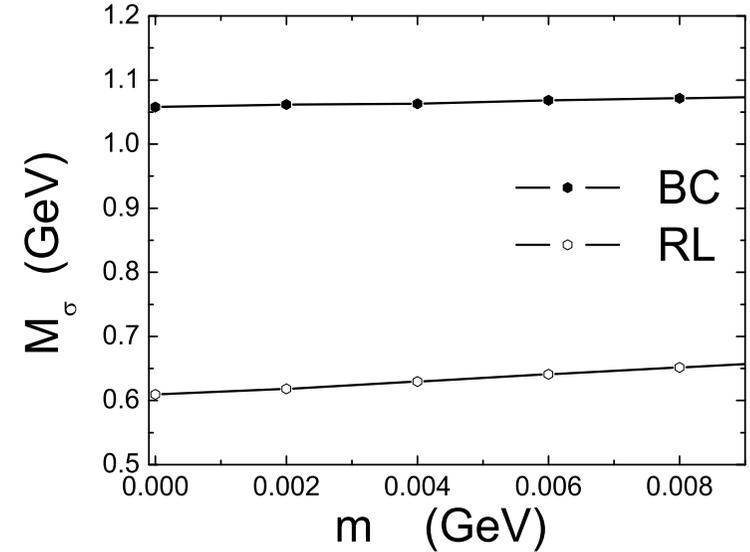
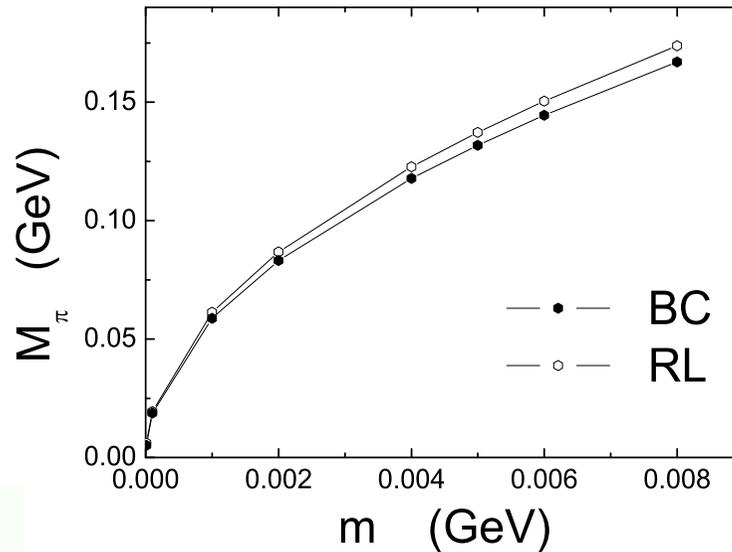


Chang Lei & CDR  
arXiv:0903.5461 [nucl-th], in Phys.Rev.Lett.



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- Single interaction, common mass scale:  
rainbow-ladder cf. BC-consistent truncation
- GMOR ... plainly satisfied by both truncations
- Added **attraction** in pseudoscalar channel
- Added **repulsion** in scalar channel



Chang Lei & CDR  
arXiv:0903.5461 [nucl-th], in Phys.Rev.Lett.

- Rainbow-ladder DSE truncation,

$$\epsilon_{\sigma}^{\text{RL}} := \frac{2M(0) - m_{\sigma}}{2M(0)} \Big|_{\text{RL}} = (0.3 \pm 0.1) .$$

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- Scalar mesons =  ${}^3P_0$  states: Constituents' spins aligned and one unit of constituent orbital angular momentum
- From this viewpoint scalar is a spin and orbital excitation of a pseudoscalar meson



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- Scalar mesons =  ${}^3P_0$  states: Constituents' spins aligned and one unit of constituent orbital angular momentum
- Extant studies of realistic corrections to the rainbow-ladder truncation show that they reduce hyperfine splitting



- Rainbow-ladder DSE truncation,

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- Scalar mesons =  ${}^3P_0$  states: Constituents' spins aligned and one unit of constituent orbital angular momentum
- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit splitting. Effect owes to influence of quark's dynamically-enhanced scalar self-energy in the Bethe-Salpeter kernel.

Impossible to demonstrate effect without our new procedure



- Rainbow-ladder DSE truncation,

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- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit splitting.
- Expect this feature to have material impact on mesons with mass greater than 1 GeV.  
*prima facie* . . . can overcome longstanding shortcoming of RL truncation; viz., splitting between vector & axial-vector mesons is too small



- Rainbow-ladder DSE truncation,

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- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit splitting.
- Expect this feature to have material impact on mesons with mass greater than 1 GeV.
- Promise of **realistic** meson spectroscopy  
First time, also for mass  $> 1$  GeV



Chang Lei & CDR, in-preparation

# $a_1 - \rho$ mass splitting



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- Solve inhomogeneous vector and axial-vector Bethe-Salpeter equation at spacelike total momentum  
 $\Rightarrow \Gamma_{qq}(k = 0, P^2)$
- $\frac{1}{\Gamma_{qq}(k = 0, P^2)}$  exhibits a zero at ground-state mass-squared
- Padé approximant extrapolation to locate zero
  - Almost precisely method used for ground-state masses in lattice-QCD
  - Intelligent use gives dependable results  
 “Schwinger functions and light-quark bound states”  
 S.V. Wright, *et al.*, *Few Body Syst.* **40** (2007) 209, nucl-th/0701009

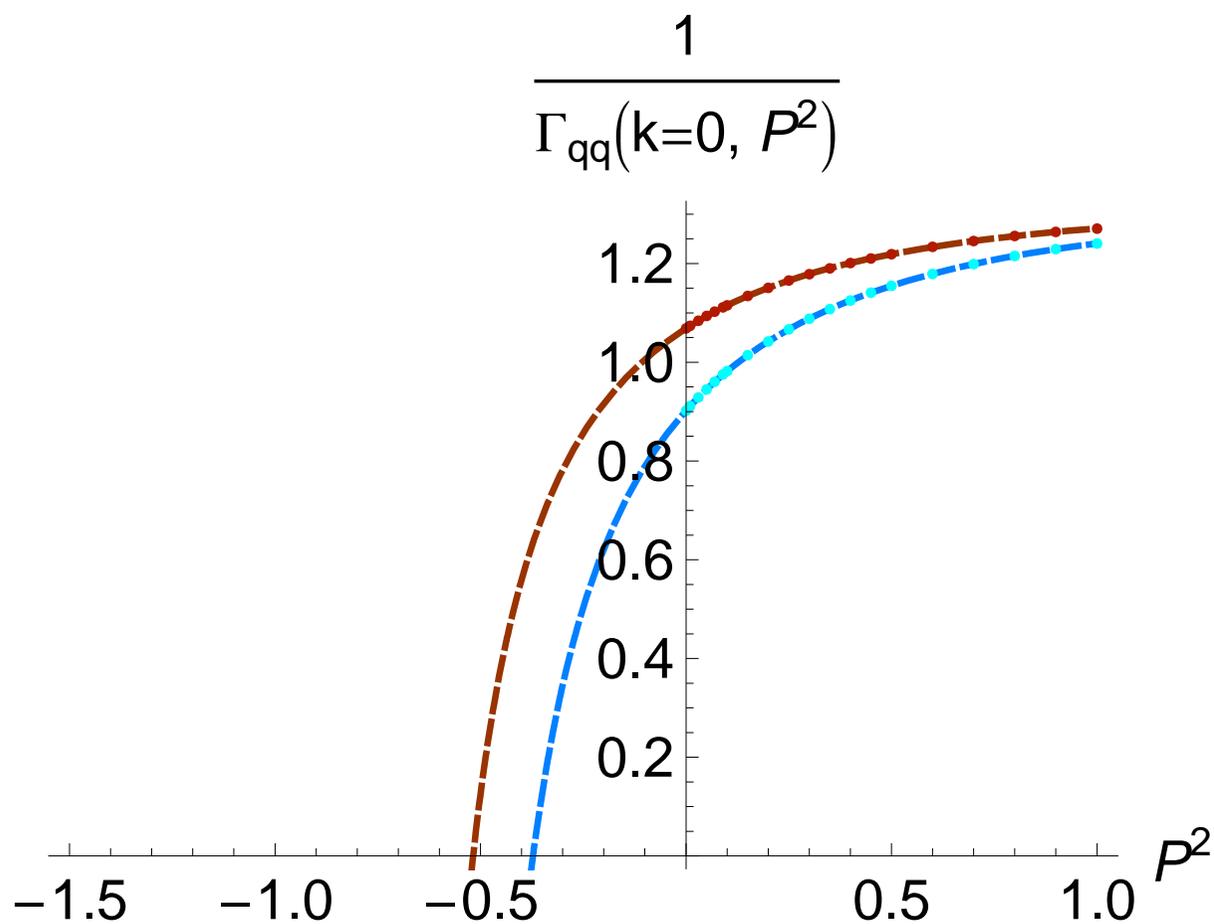


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# $a_1 - \rho$ mass splitting

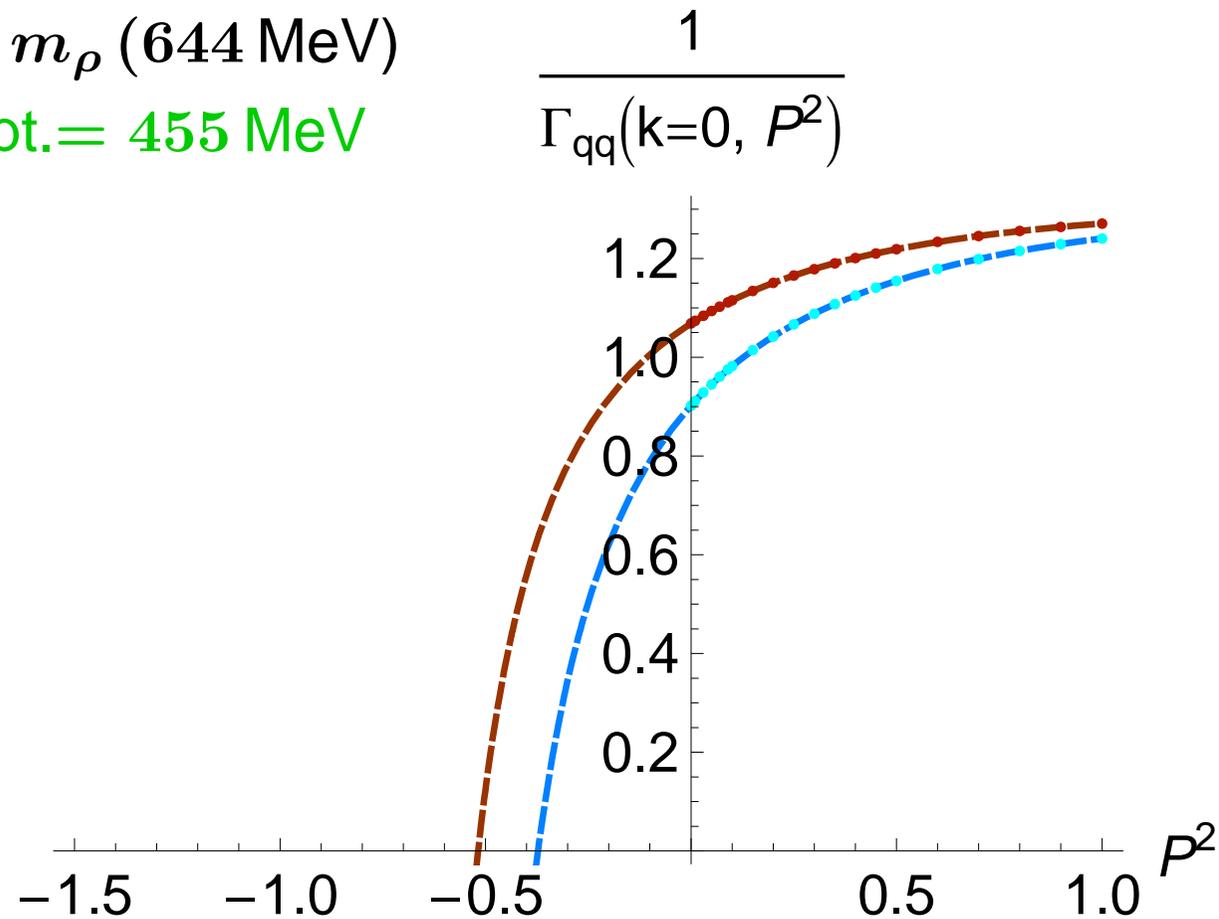


# $a_1 - \rho$ mass splitting

$$m_{a_1} (759 \text{ MeV}) - m_{\rho} (644 \text{ MeV}) = 115 \text{ MeV} \dots \text{expt.} = 455 \text{ MeV}$$

Rainbow-Ladder

$$\Gamma_{\mu}(q, k) = \gamma_{\mu}$$



# $a_1 - \rho$ mass splitting

$$m_{a_1} (1066 \text{ MeV}) - m_\rho (924 \text{ MeV})$$

$$= 142 \text{ MeV} \dots \text{expt.} = 455 \text{ MeV}$$

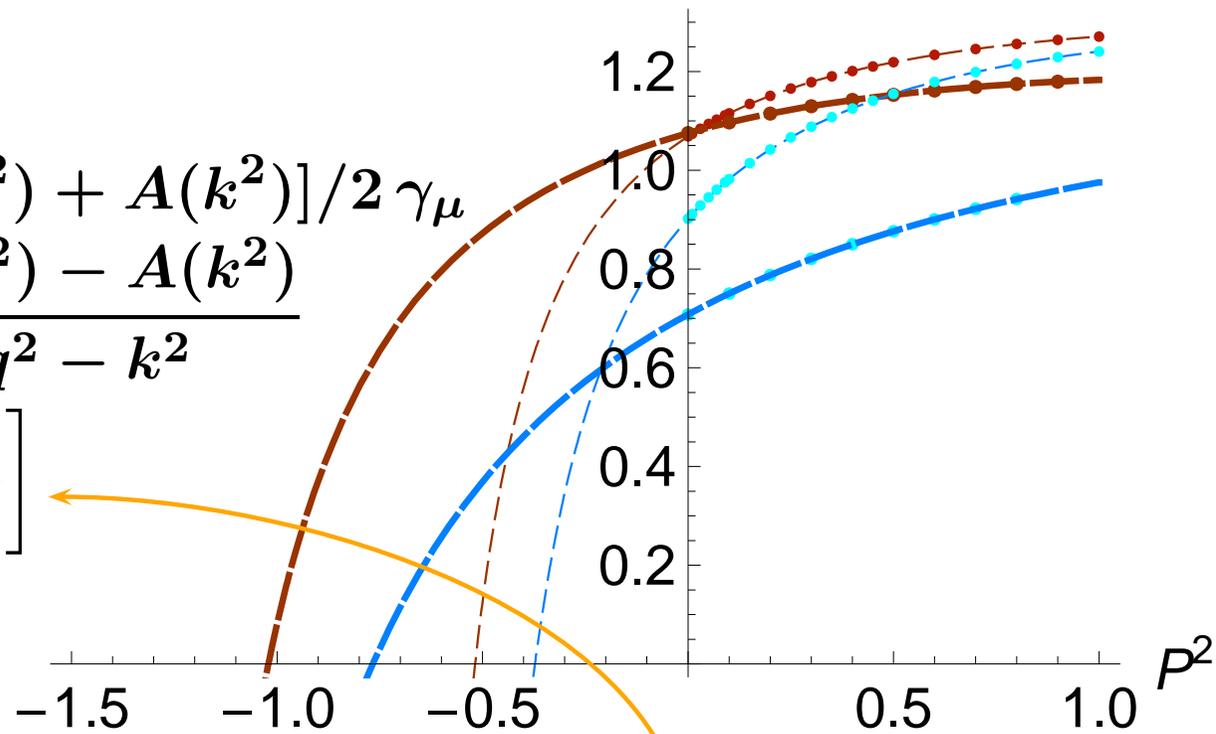
$$\frac{1}{\Gamma_{qq}(k=0, P^2)}$$

## Ball-Chiu

$$\Gamma_\mu(q, k) = i[A(q^2) + A(k^2)]/2 \gamma_\mu$$

$$+ 2k_\mu \left[ i\gamma \cdot k \frac{A(q^2) - A(k^2)}{q^2 - k^2} \right.$$

$$\left. + \frac{B(q^2) - B(k^2)}{q^2 - k^2} \right]$$



DCSB enhanced spin-orbit interaction



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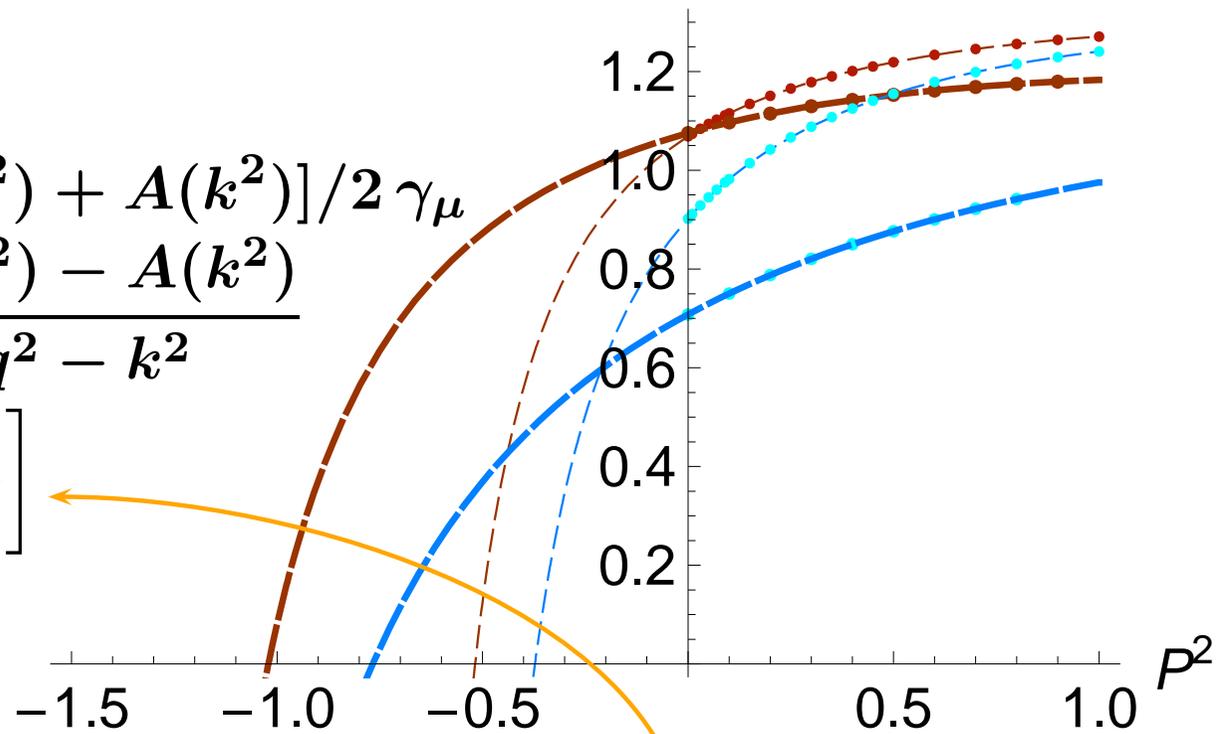
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What's missing?

DCSB enhanced spin-orbit interaction



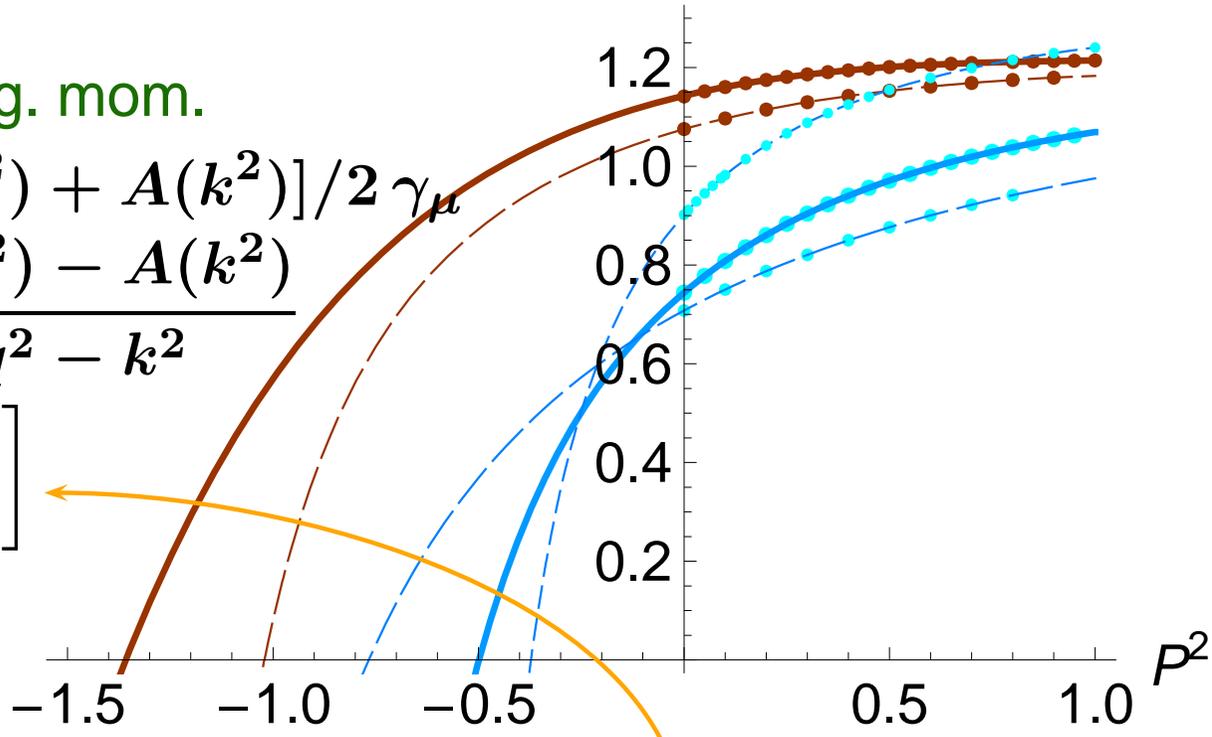
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$m_{a_1}$  (1230 MeV) –  $m_\rho$  (745 MeV)  
 = 485 MeV ... expt. = 455 MeV

$$\frac{1}{\Gamma_{qq}(k=0, P^2)}$$

Ball-Chiu + an. mag. mom.

$$\Gamma_\mu(q, k) = i[A(q^2) + A(k^2)]/2 \gamma_\mu + 2k_\mu \left[ i\gamma \cdot k \frac{A(q^2) - A(k^2)}{q^2 - k^2} + \frac{B(q^2) - B(k^2)}{q^2 - k^2} \right] + \sigma_{\mu\nu}(q - k)_\nu \frac{B(q^2) - B(k^2)}{q^2 - k^2}$$



DCSB enhanced spin-orbit interaction

DCSB enhanced anomalous magnetic moment



# Unifying Study of Mesons and Baryons



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# Unifying Study of Mesons and Baryons

- How does one incorporate dressed-quark mass function,  $M(p^2)$ , in study of baryons?



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- In quantum field theory a nucleon appears as a pole in a six-point quark Green function.
  - Residue is proportional to nucleon's Faddeev amplitude
  - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks



# Unifying Study of Mesons and Baryons

- How does one incorporate dressed-quark mass function,  $M(p^2)$ , in study of baryons? Behaviour of  $M(p^2)$  is essentially a quantum field theoretical effect.
- In quantum field theory a nucleon appears as a pole in a six-point quark Green function.
  - Residue is proportional to nucleon's Faddeev amplitude
  - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
  - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour- $\bar{3}$  (antitriplet) channel



# *Faddeev equation*

R. T. Cahill *et al.* Austral. J. Phys. **42** (1989) 129



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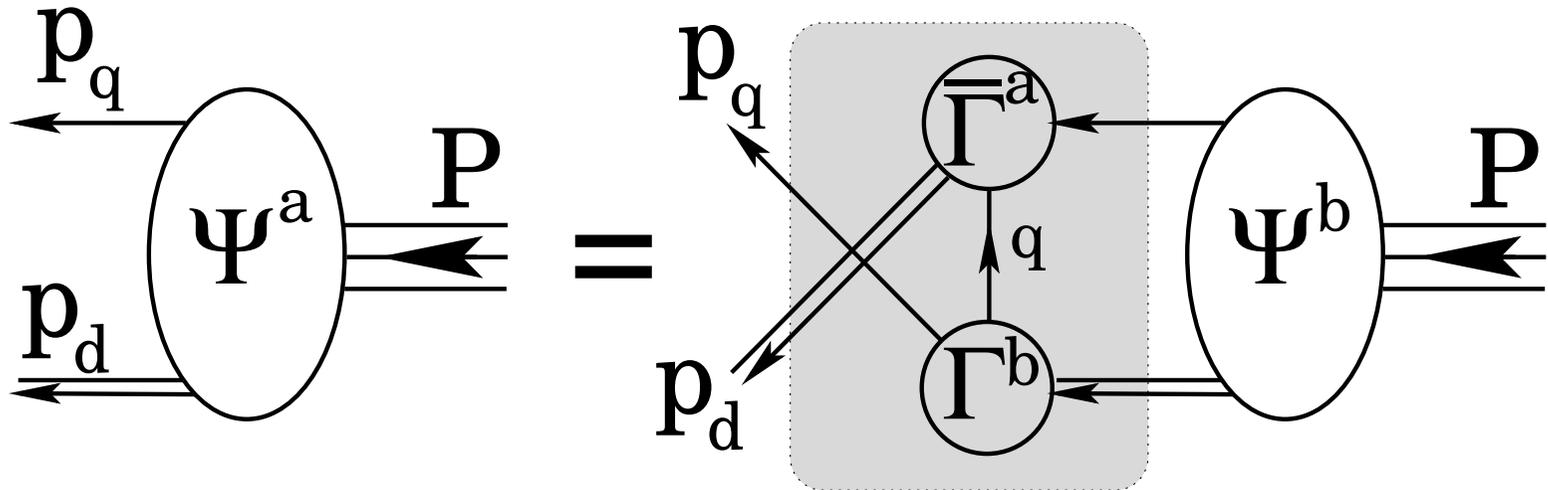
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# Faddeev equation

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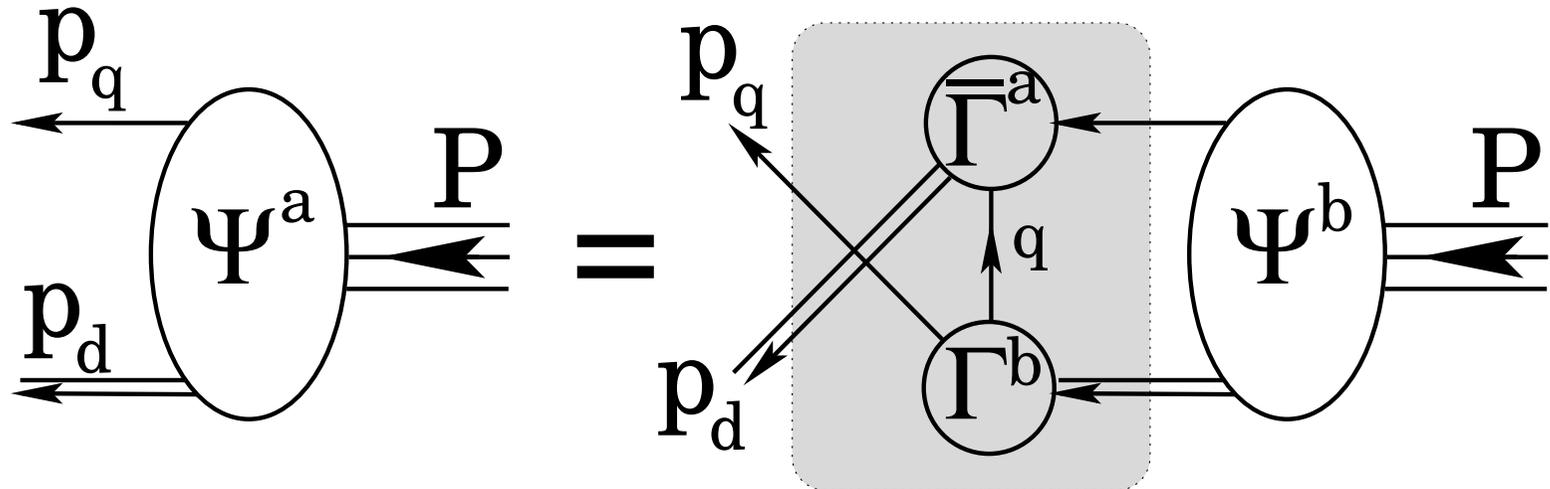
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# Faddeev equation

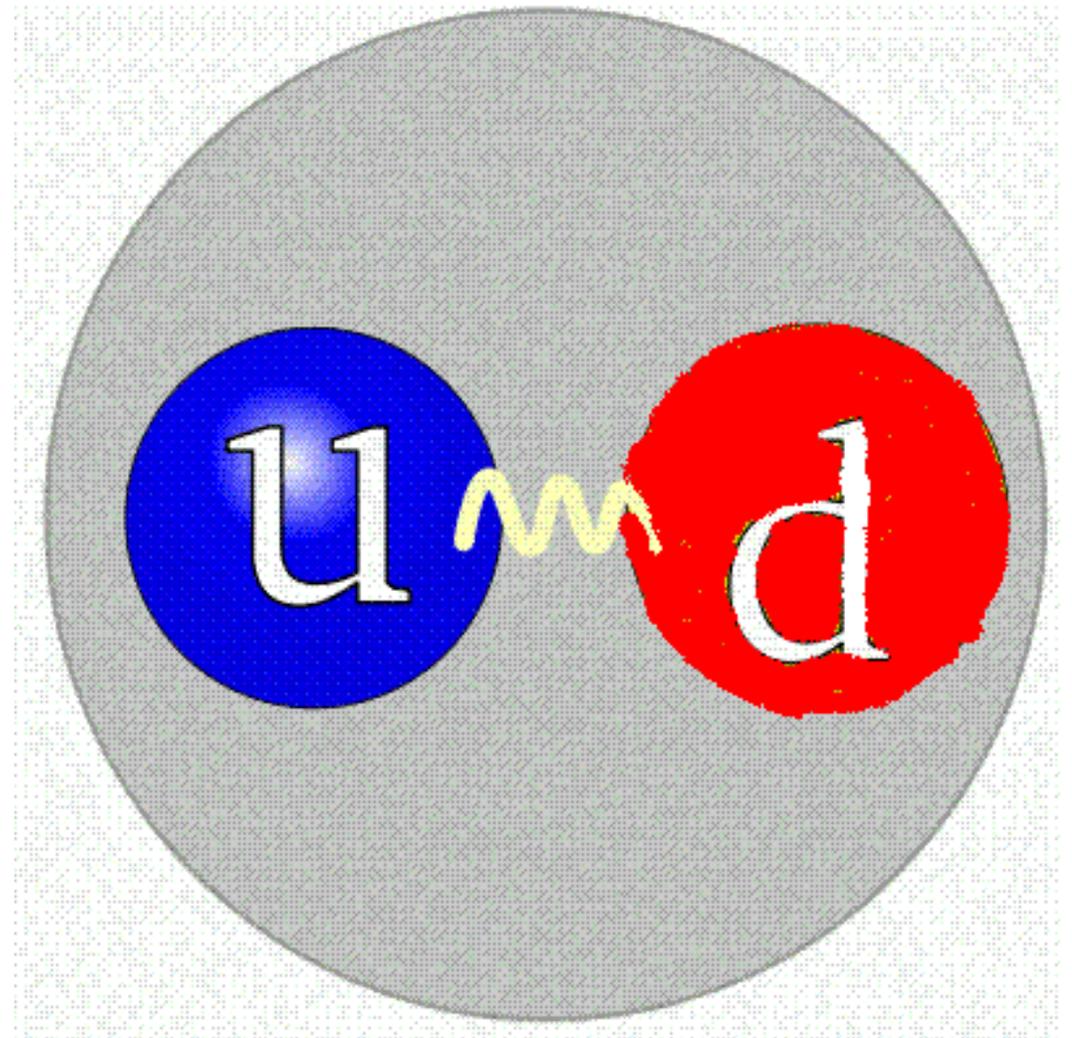
R. T. Cahill *et al.* Austral. J. Phys. **42** (1989) 129



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (**Poincaré Covariant Faddeev Amplitude**) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame **Amplitude** has ... *s*-, *p*- & *d*-wave correlations



# Diquark correlations



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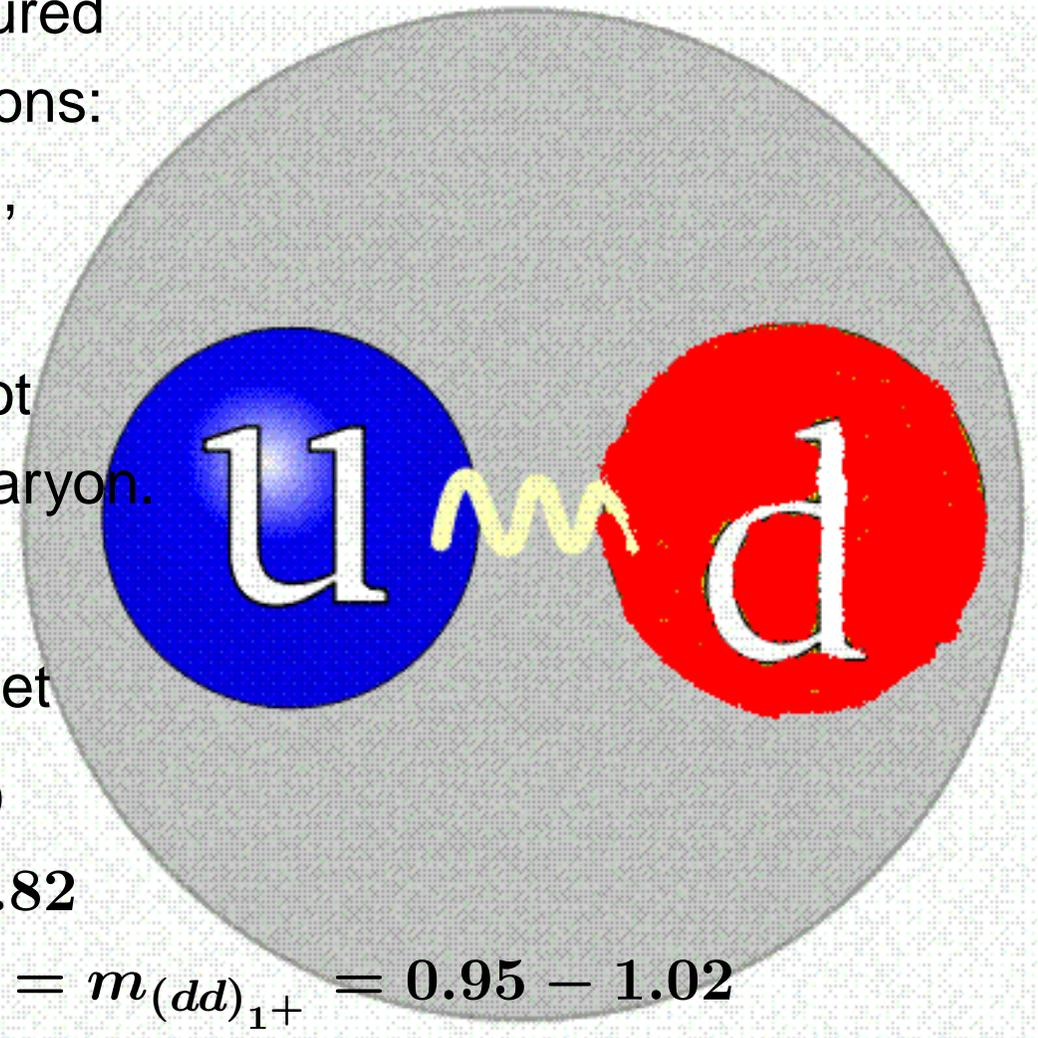
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## QUARK-QUARK

# Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue-red, blue-green, green-red
- Confined ... Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD
$$m_{[ud]_{0+}} = 0.74 - 0.82$$
$$m_{(uu)_{1+}} = m_{(ud)_{1+}} = m_{(dd)_{1+}} = 0.95 - 1.02$$



# Nucleon-Photon Vertex



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M. Oettel, M. Pichowsky  
and L. von Smekal, nu-th/9909082

6 terms ...

# ***Nucleon-Photon Vertex***

constructed systematically ... current conserved automatically  
for on-shell nucleons described by Faddeev Amplitude



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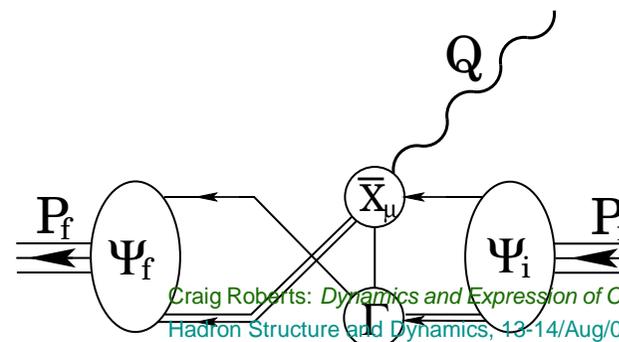
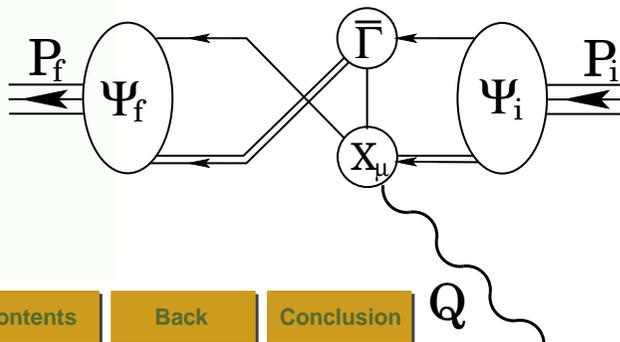
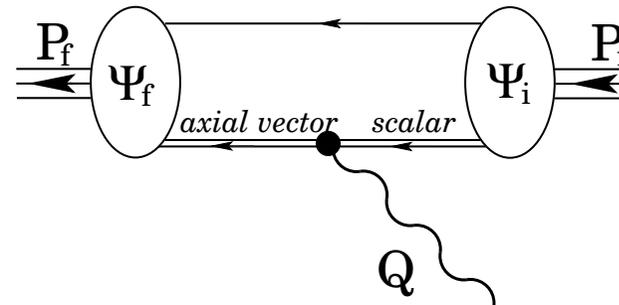
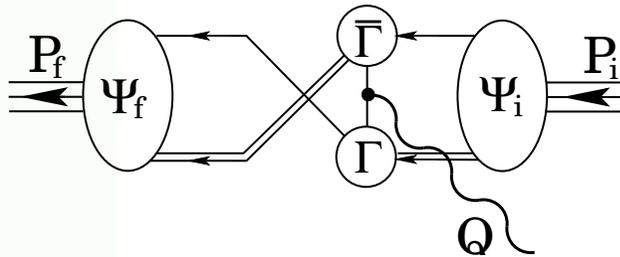
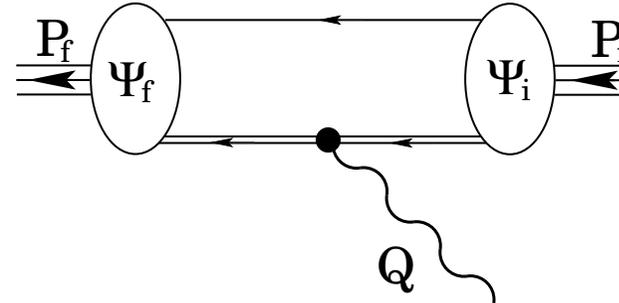
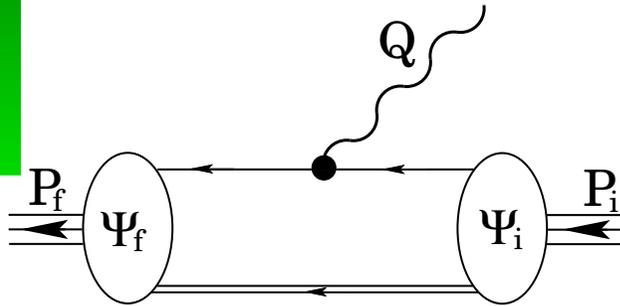
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# DSE-based Faddeev Equation

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Cloët *et al.*

– arXiv:0710.2059 [nucl-th]

– arXiv:0710.5746 [nucl-th]

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- Faddeev equation input – algebraic parametrisations of DSE results, constrained by  $\pi$  and  $K$  observables



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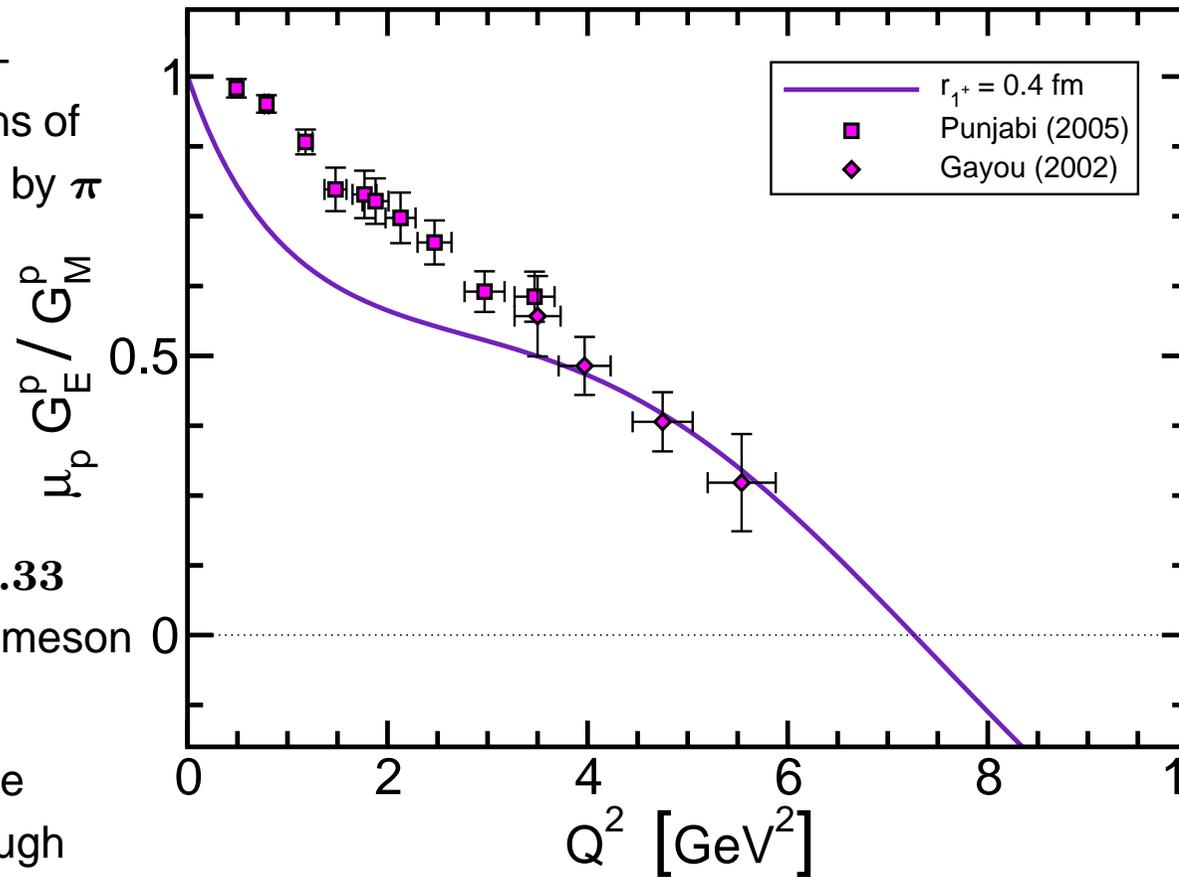
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- Faddeev equation input – algebraic parametrisations of DSE results, constrained by  $\pi$  and  $K$  observables
- Two parameters
  - $M_{0+} = 0.8$  GeV,
  - $M_{1+} = 0.9$  GeV
  - chosen to give  $M_N = 1.18$ ,  $M_\Delta = 1.33$
  - allow for pseudoscalar meson contributions



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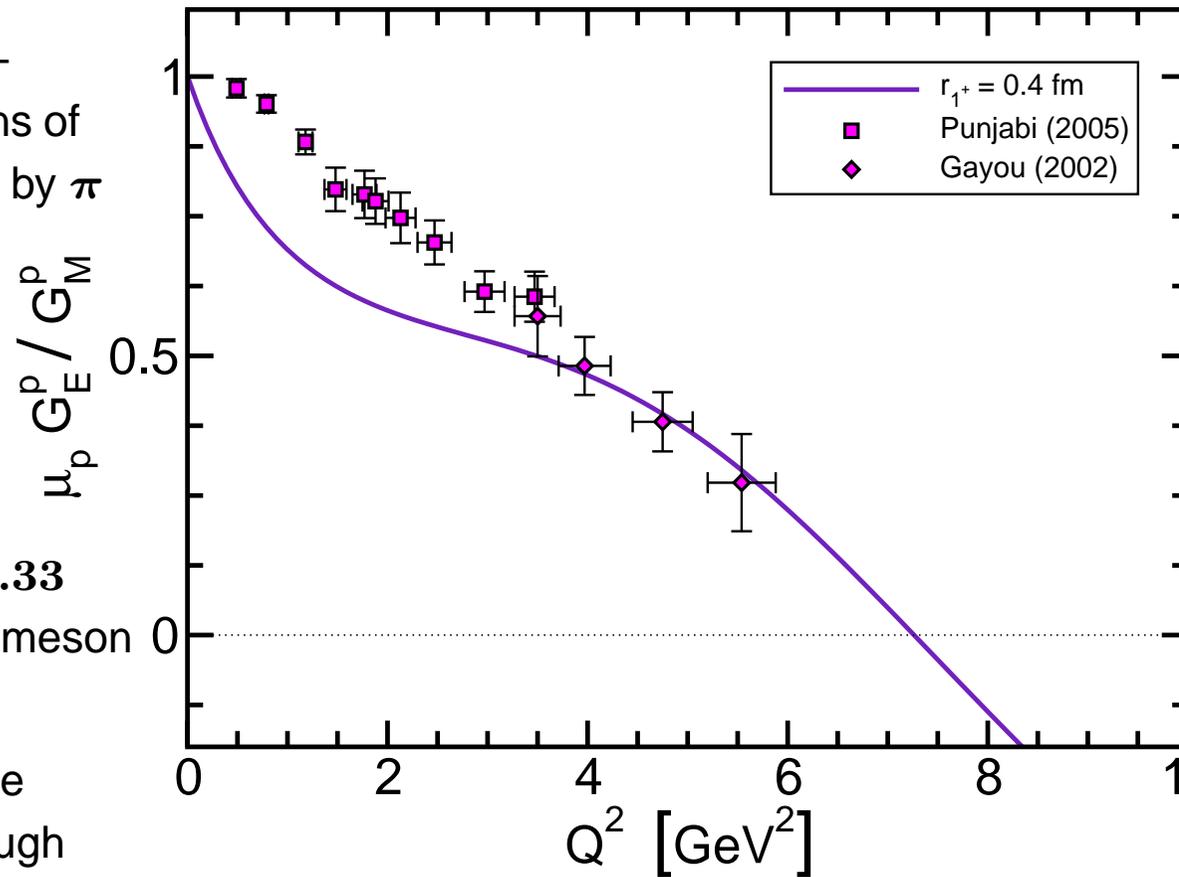
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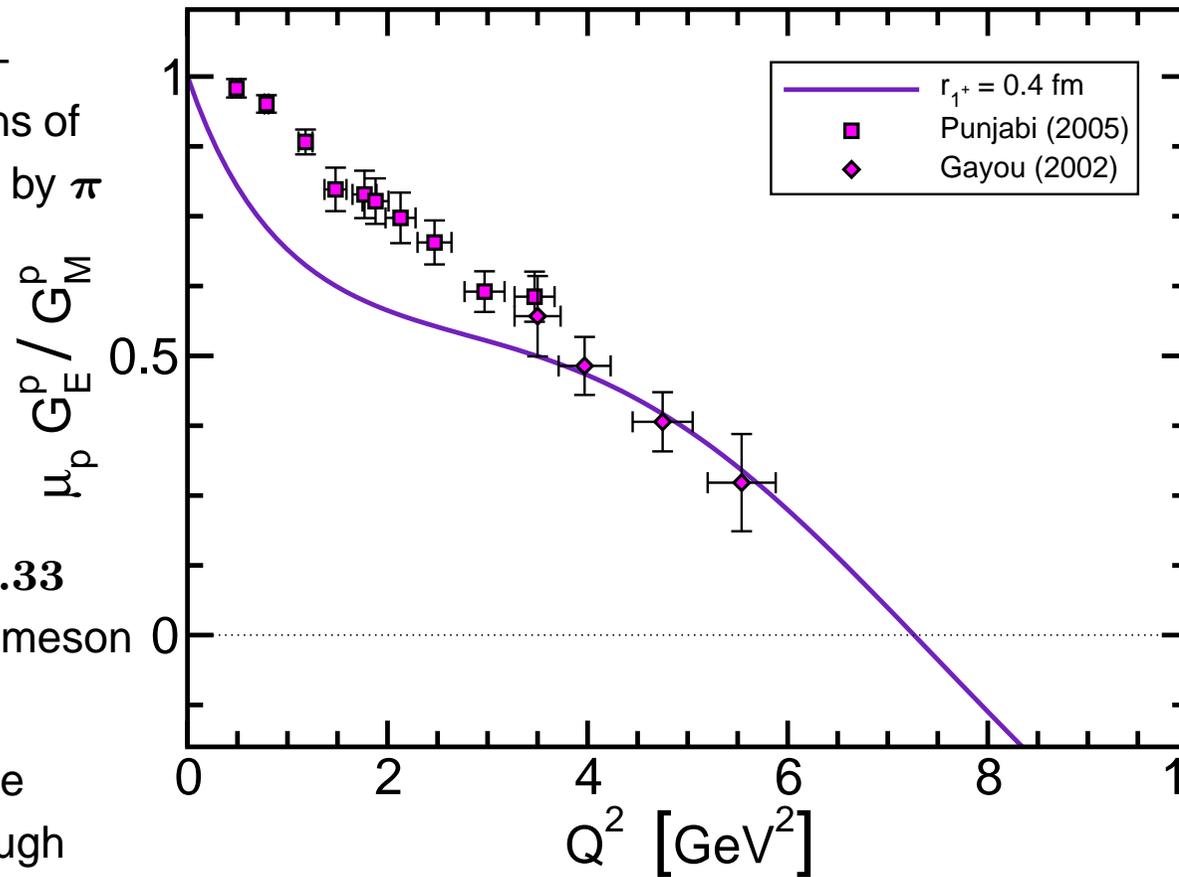
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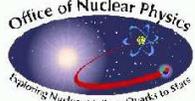
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- Sensitivity to details of the current – expressed through diquark radius
- On  $Q^2 \lesssim 4$  GeV<sup>2</sup> result lies below experiment. This can be attributed to omission of pseudoscalar-meson-cloud contributions
- Always a zero but position depends on details of current





$$\frac{\mu_n G_E(Q^2)}{G_M(Q^2)}$$



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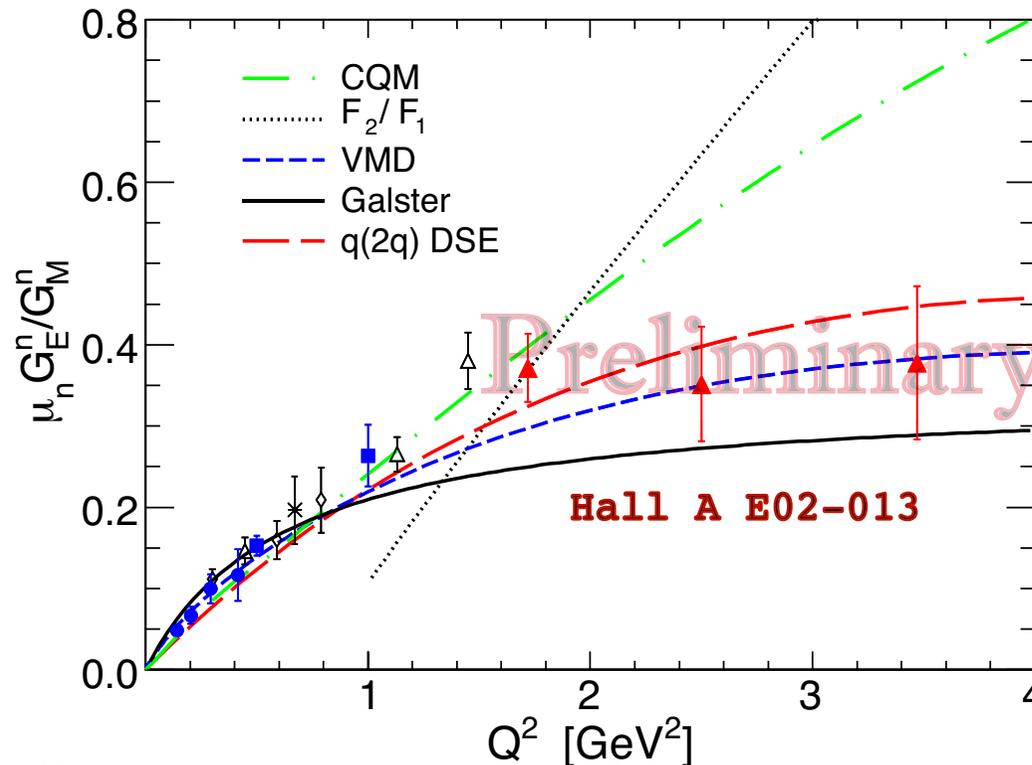


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Jefferson Lab E02-013 Collaboration, *in preparation.*



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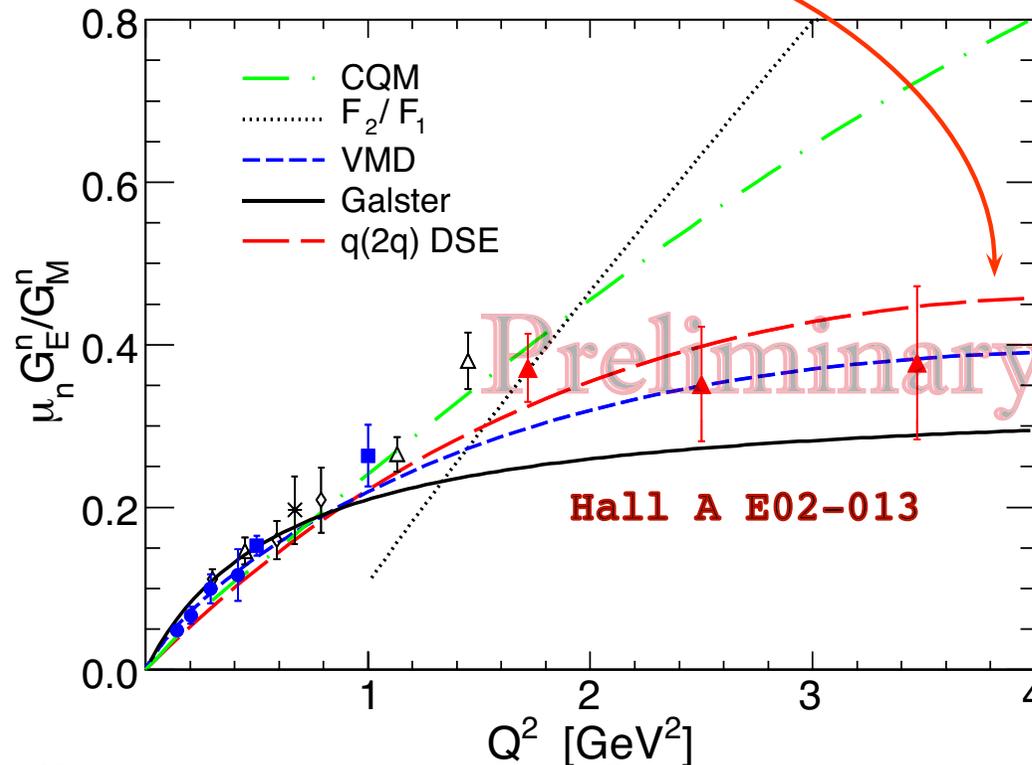
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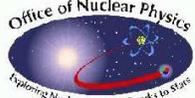
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● DSE-Faddeev Equation prediction

Red long-dashed curve



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# *Pion Cloud*

## *F2 – neutron*

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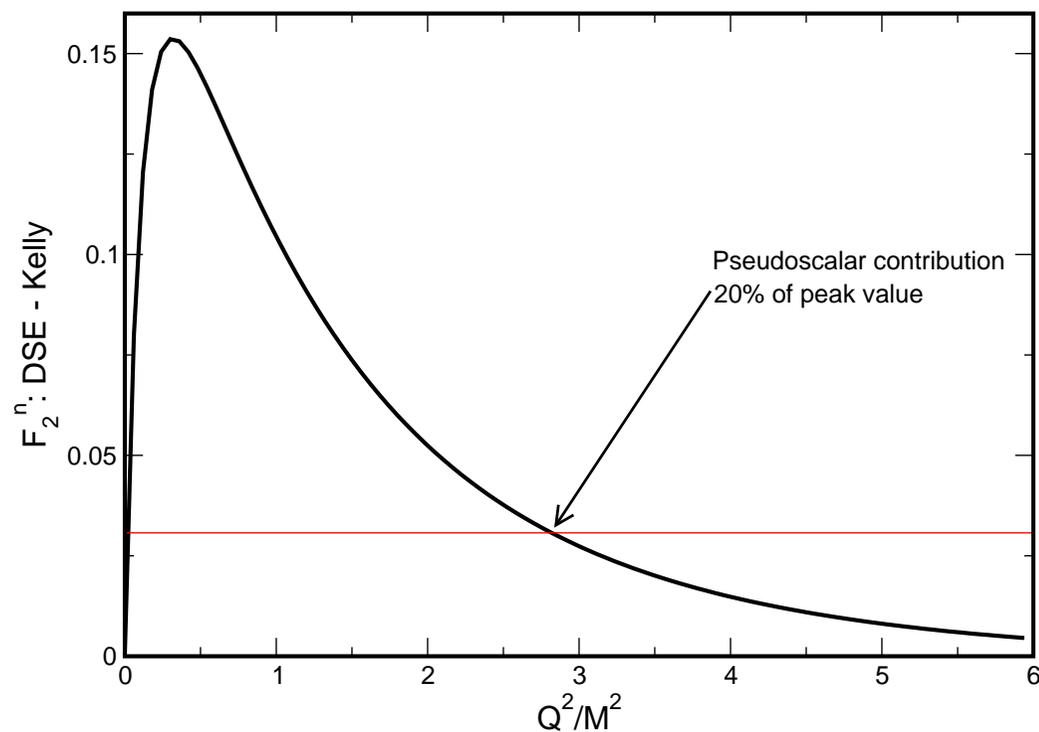
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# Pion Cloud

## F2 – neutron

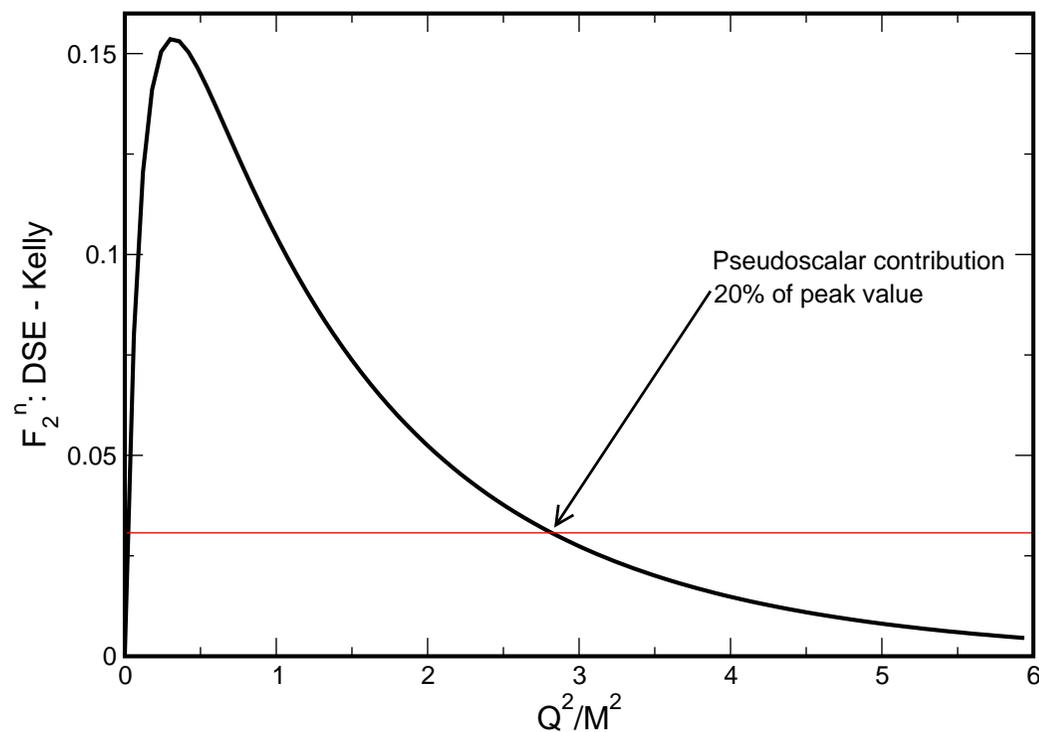
- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core



# Pion Cloud

## F2 – neutron

- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core
- Pseudoscalar meson cloud (and related effects) significant for  $Q^2 \lesssim 3 - 4 M_N^2$



# Epilogue

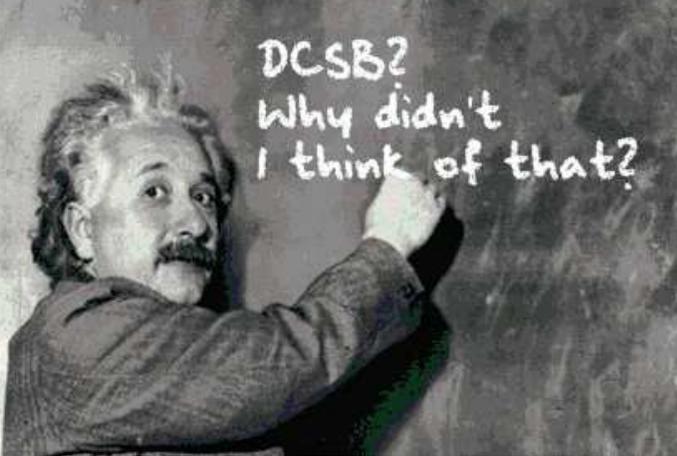


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- DCSB exists in QCD.

## Epilogue

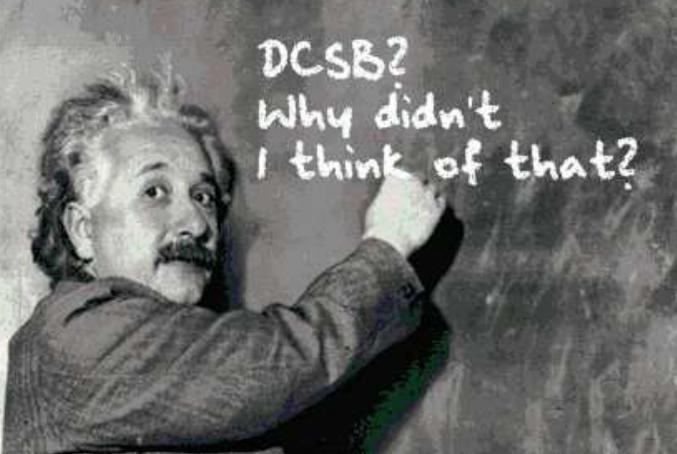


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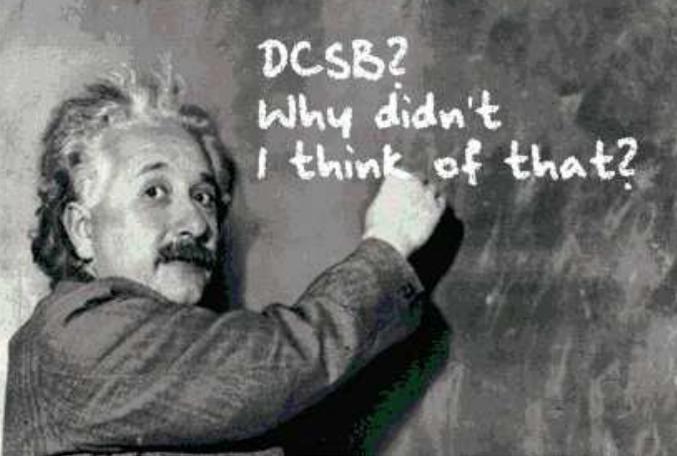
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## Epilogue

- DCSB exists in QCD.
- It is manifest in dressed propagators and vertices





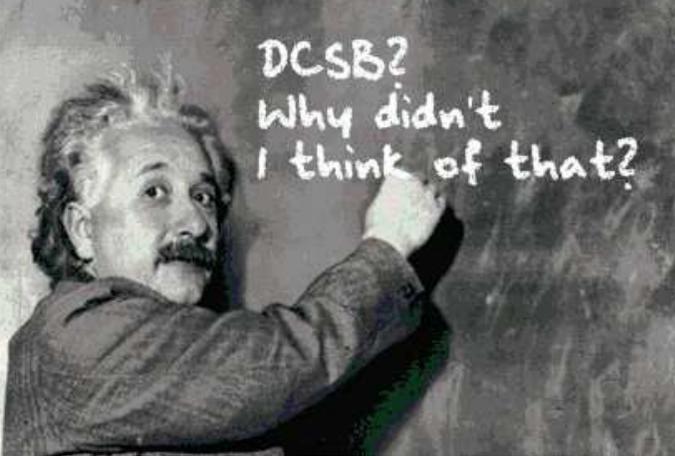
## Epilogue

### ● DCSB exists in QCD.

- It is manifest in dressed propagators and vertices
- It predicts, amongst other things, that
  - light current-quarks become heavy constituent-quarks:  $4 \rightarrow 400 \text{ MeV}$
  - pseudoscalar mesons are unnaturally light:  $m_\rho = 770$  cf.  $m_\pi = 140 \text{ MeV}$
  - pseudoscalar mesons couple unnaturally strongly to light-quarks:  $g_{\pi\bar{q}q} \approx 4.3$
  - pseudoscalar mesons couple unnaturally strongly to the lightest baryons

$$g_{\pi\bar{N}N} \approx 12.8 \approx 3g_{\pi\bar{q}q}$$



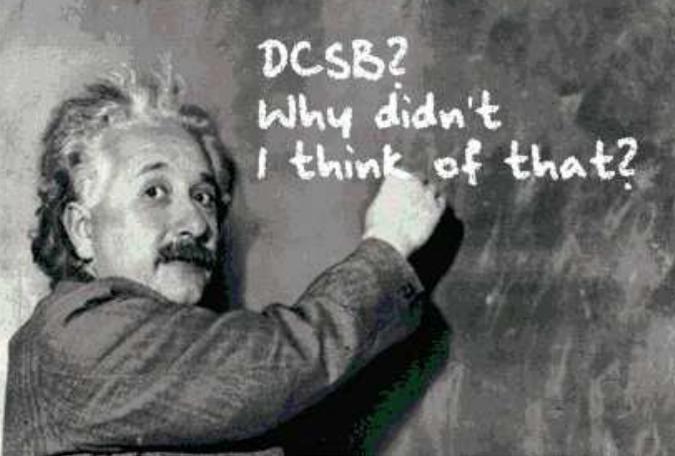


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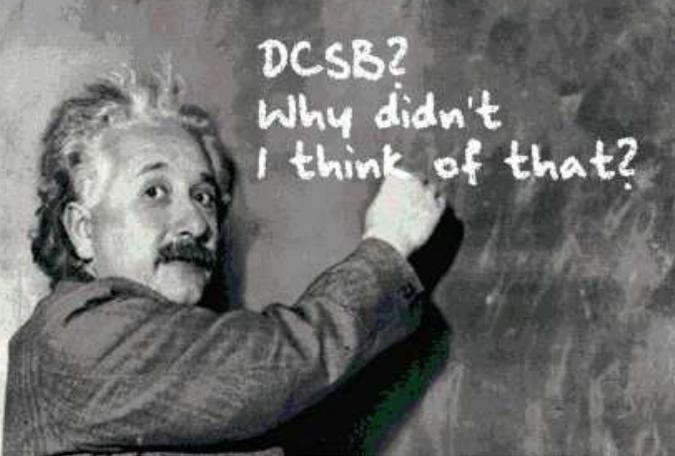
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- $$g_{\pi\bar{N}N} \approx 12.8 \approx 3g_{\pi\bar{q}q}$$
- **It impacts dramatically upon observables.**





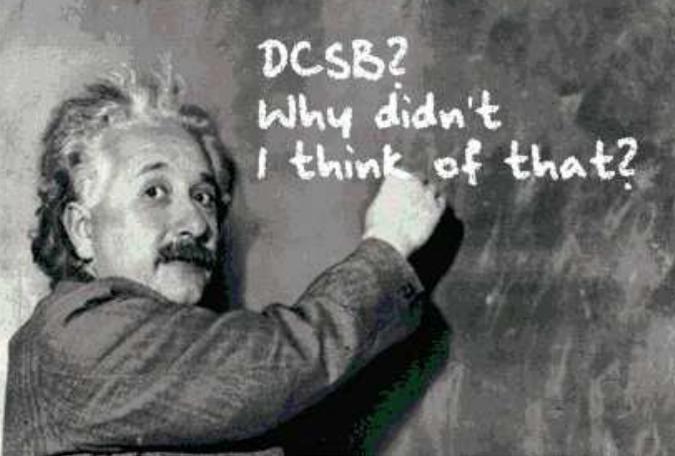
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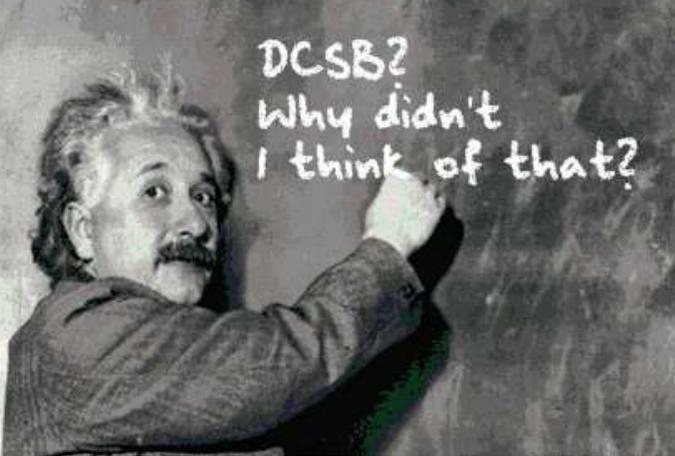
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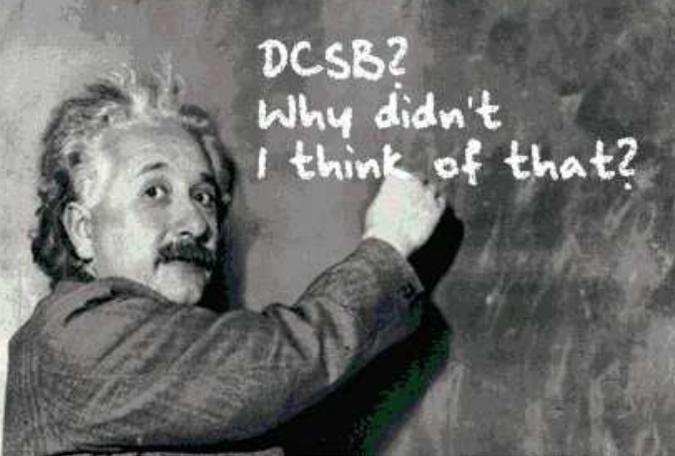
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  - Solved  $a_1$ - $\rho$  problem. Now positioned to provide first unified and reliable symmetry-preserving prediction of complete meson spectrum





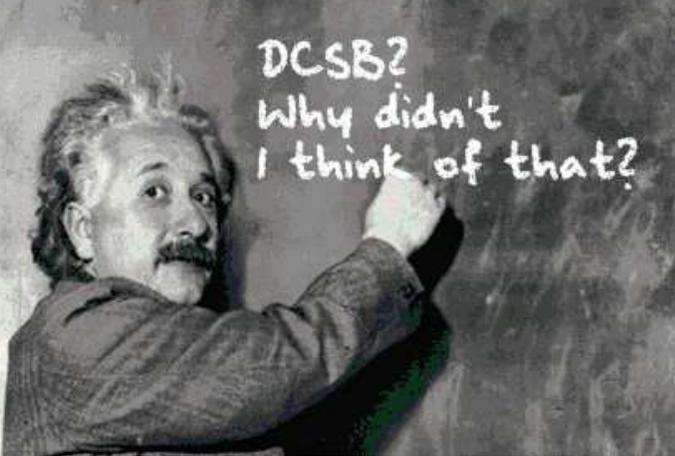
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    - Baryons are within practical reach





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- DSEs: Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks



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