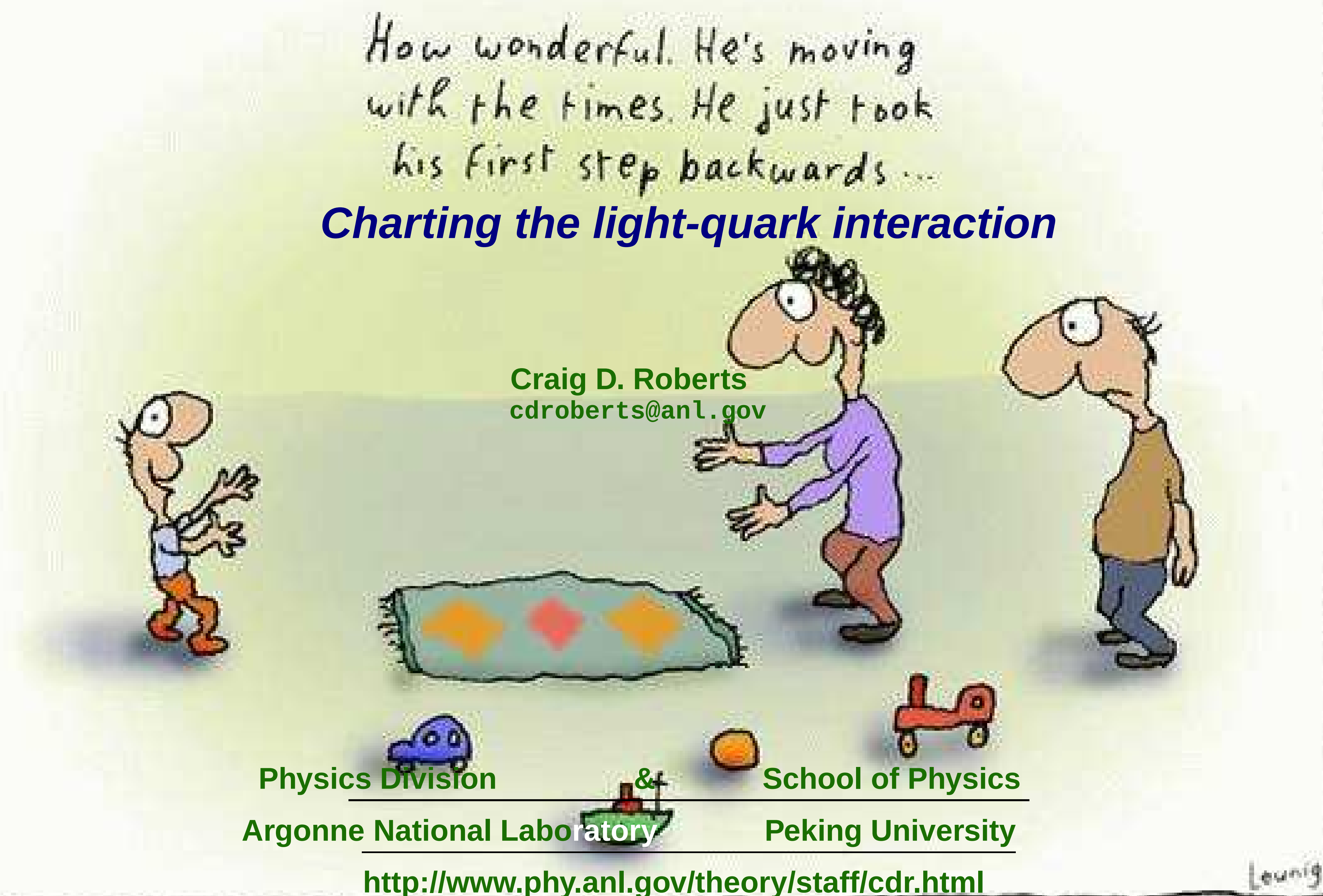

Charting the light-quark interaction

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Physics Division & School of Physics

Argonne National Laboratory Peking University

http://www.phy.anl.gov/theory/staff/cdr.html

Universal Truths

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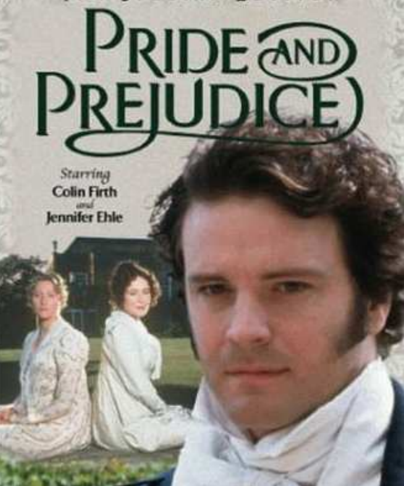

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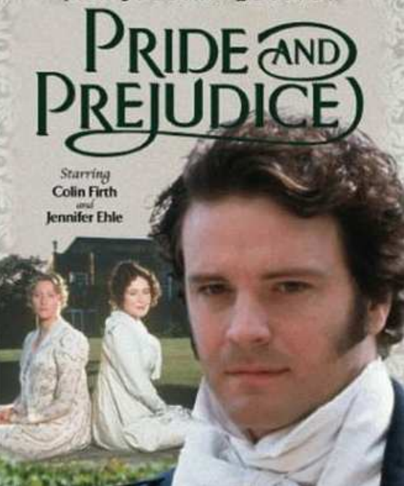

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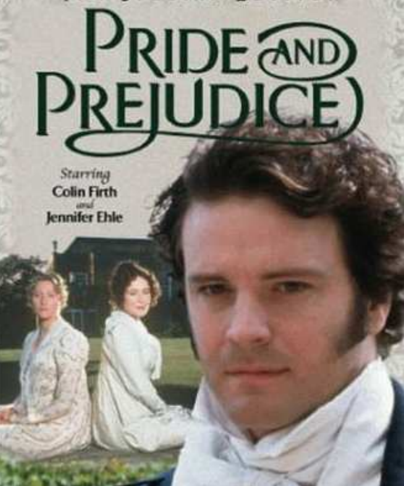

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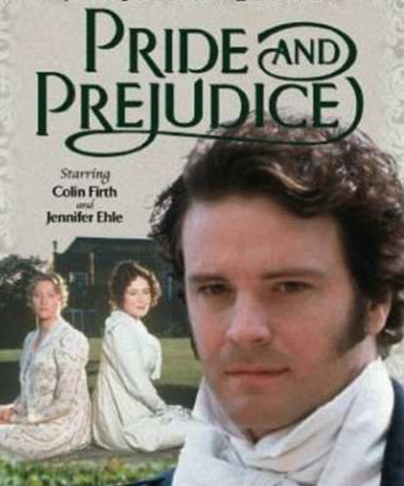

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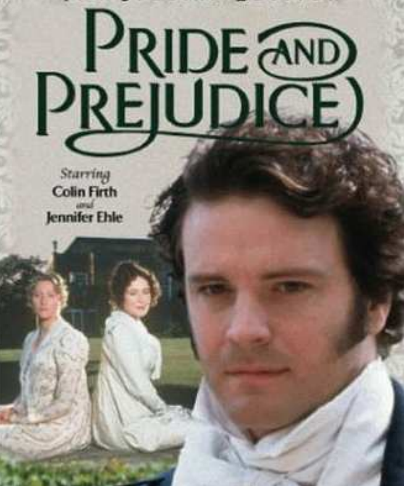

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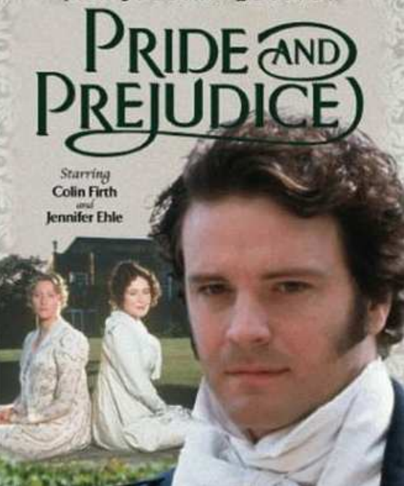

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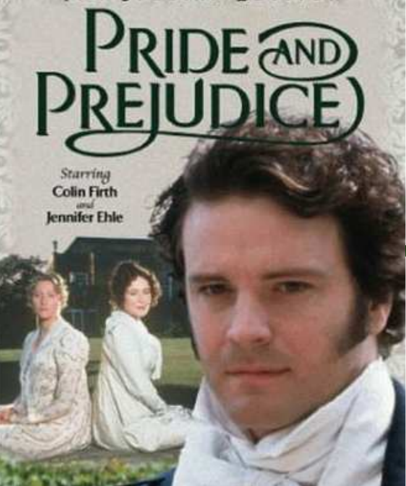

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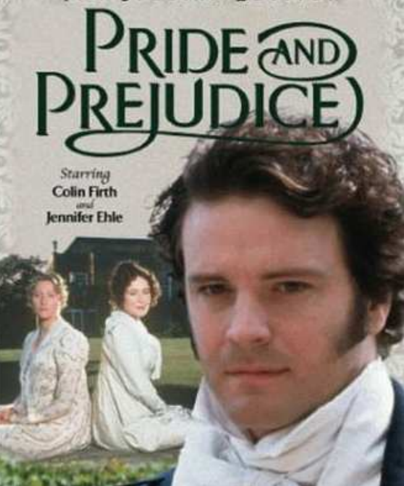

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- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.

QCD’s Challenges

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- QCD – Complex behaviour arises from apparently simple rules

Dichotomy of Pion
– Goldstone Mode and *Bound state*

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Highly Nontrivial

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Office of Nuclear Physics Exploring Nuclear Matter - Quarks to Stars

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  - Interaction between quarks – the Interquark "Potential" – unknown throughout $> 98\%$ of a hadron's volume

Intranucleon Interaction

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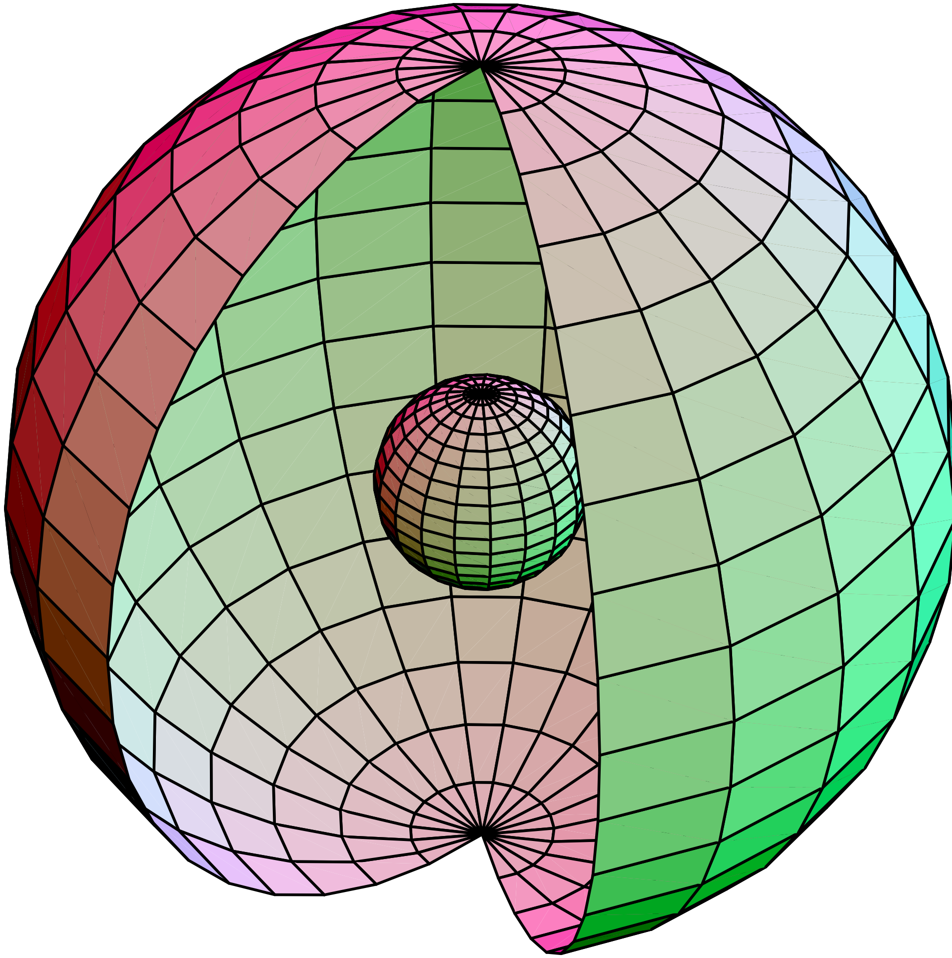
Intranucleon Interaction



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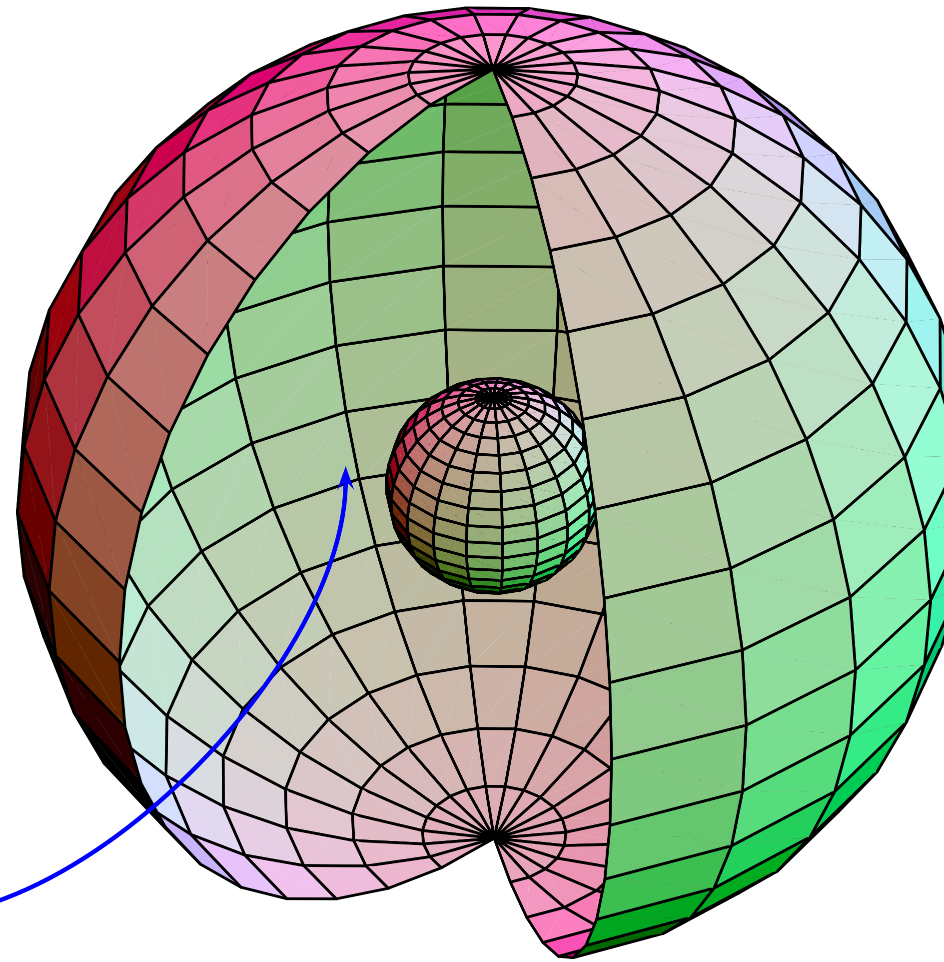
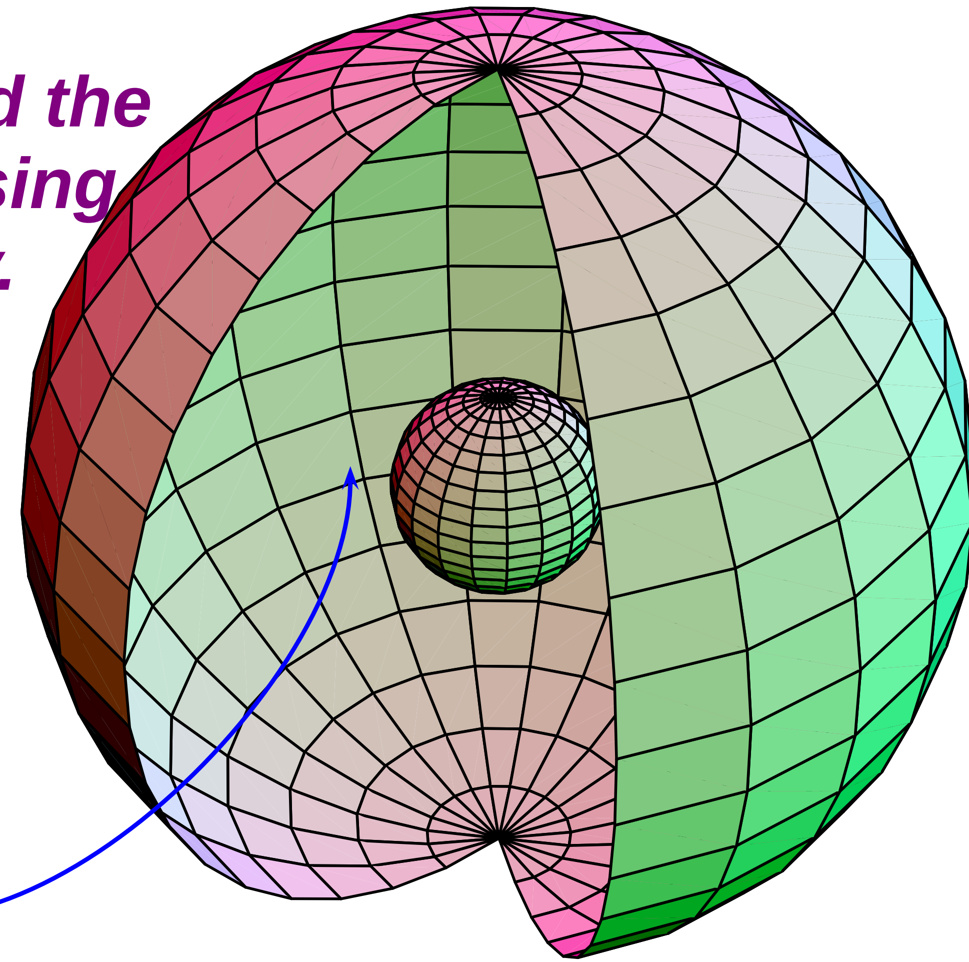


98% of the volume

What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



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Confinement

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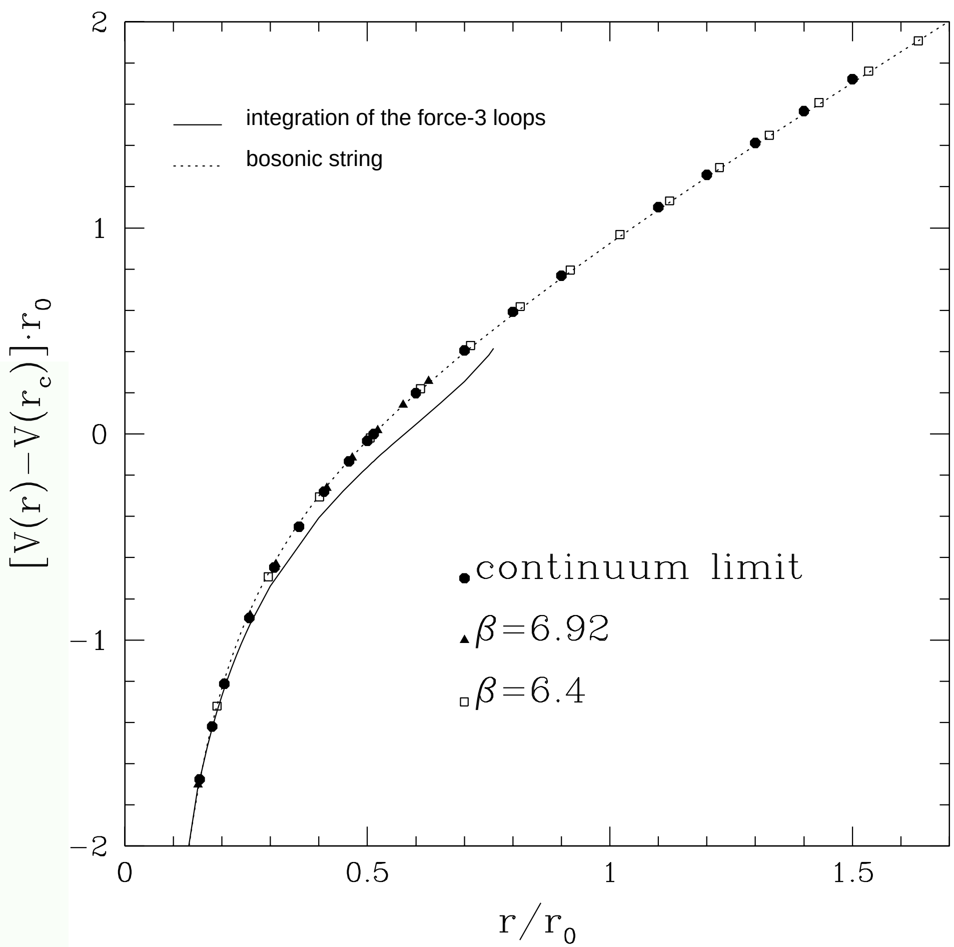
Confinement

- Infinitely Heavy Quarks . . . Picture in Quantum Mechanics



$$V(r) = \sigma\, r - \frac{\pi}{12}\frac{1}{r}$$

$$\sqrt{\sigma} \sim 470\,\text{MeV}$$

Necco & Sommer
he-la/0108008

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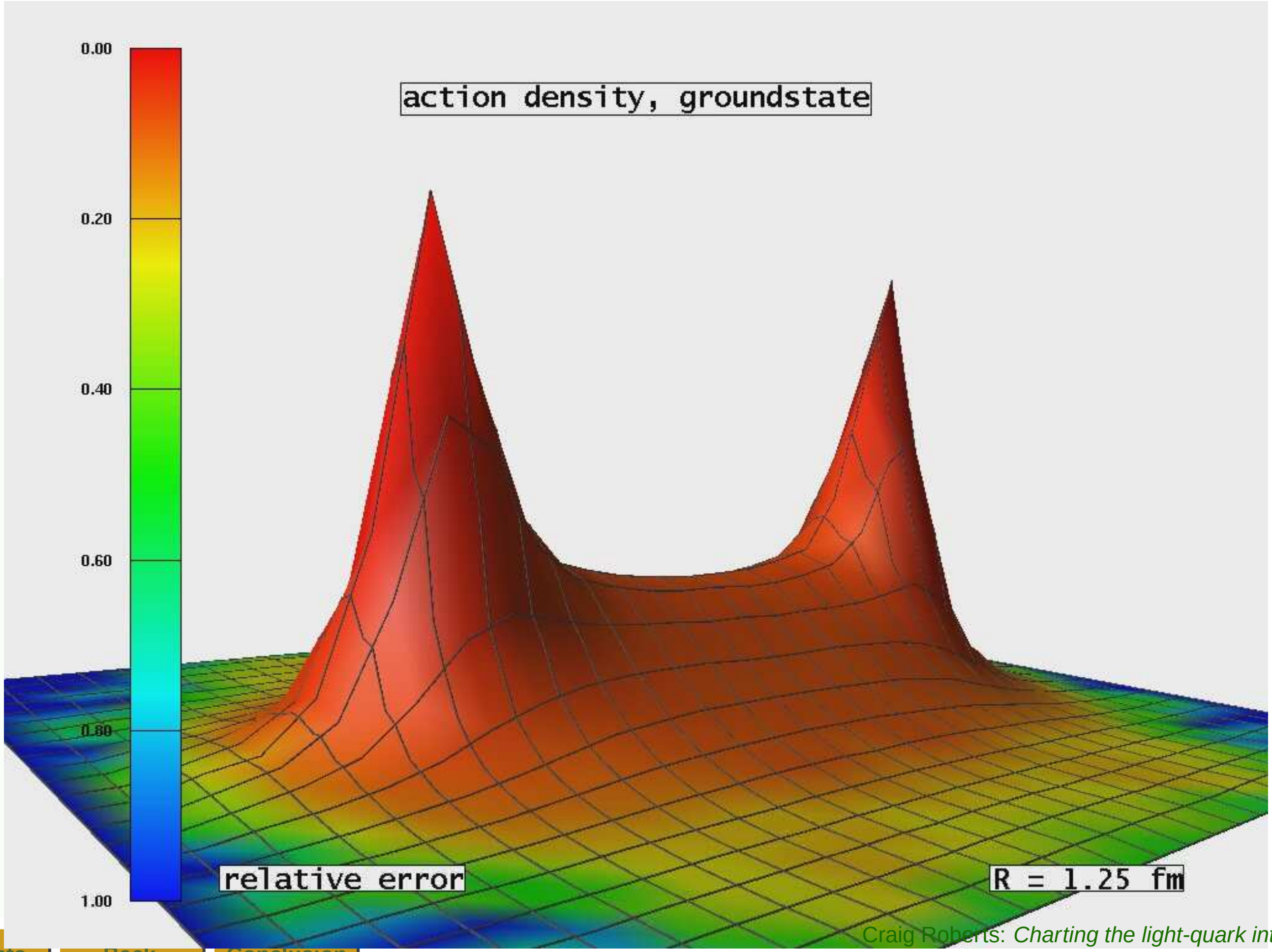
Confinement

- Illustrate this in terms of the action density ... analogous to plotting the Force $= F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12}\frac{1}{r^2}$



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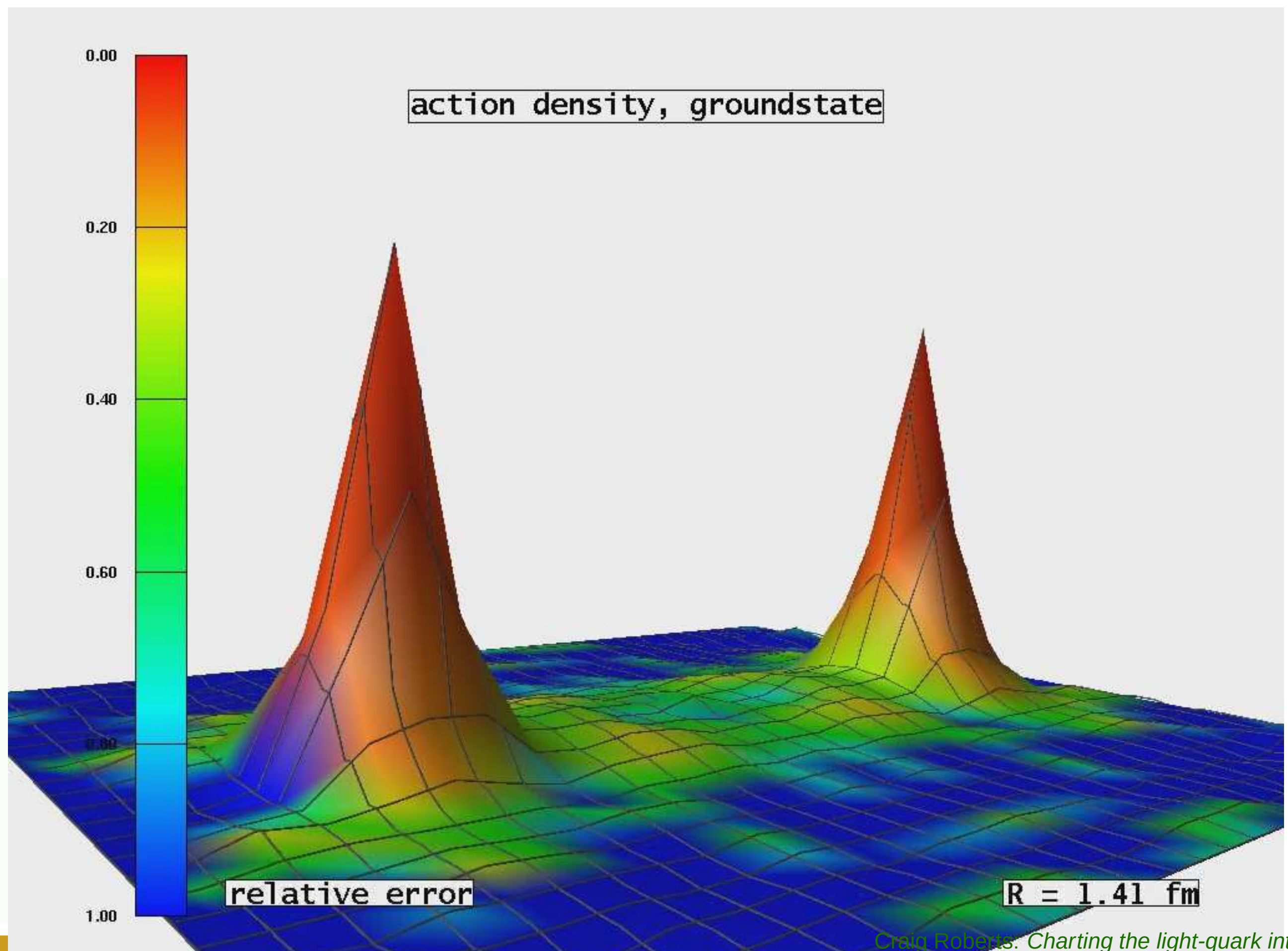

Bali, *et al*.
he-la/0512018

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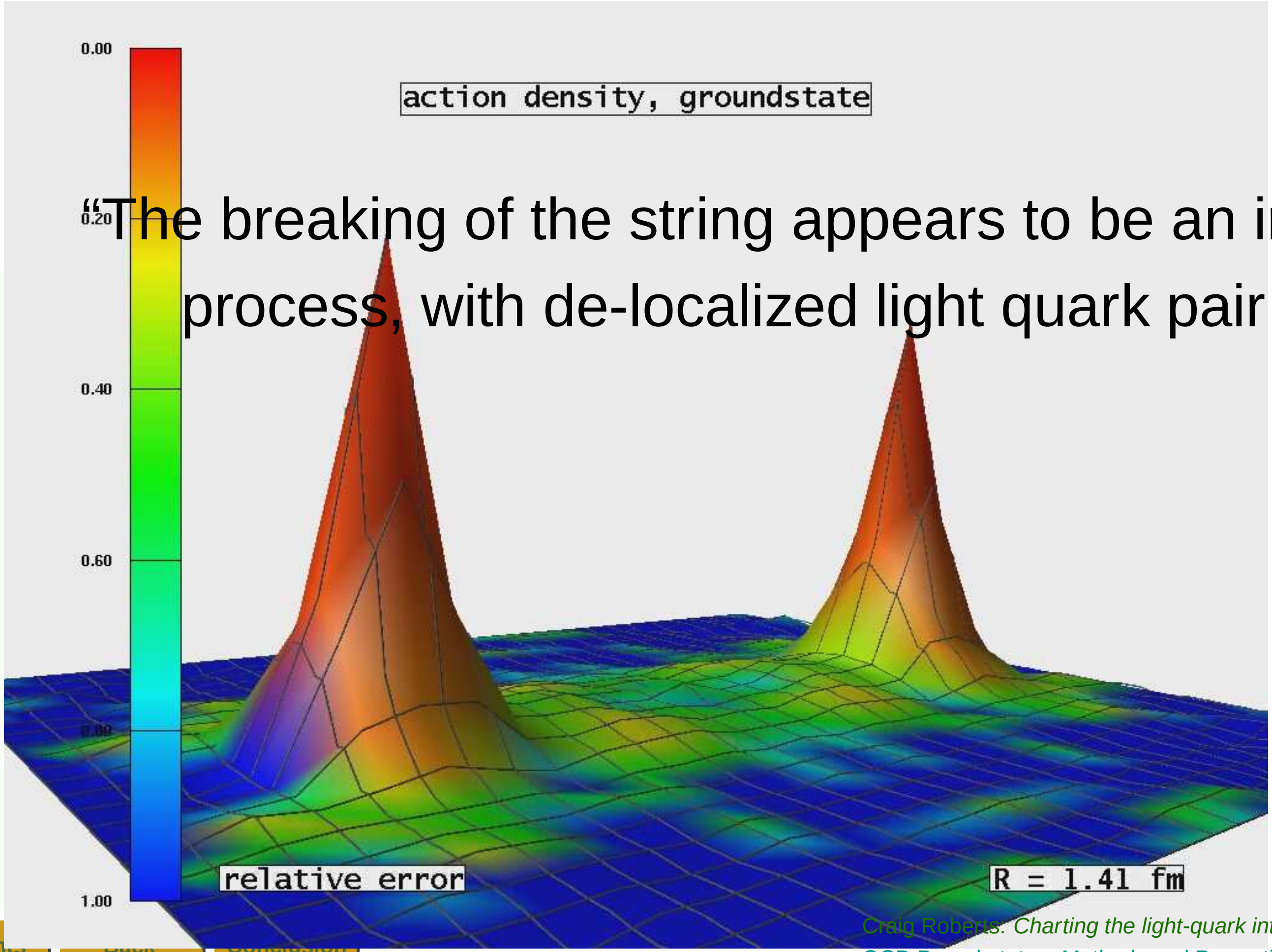

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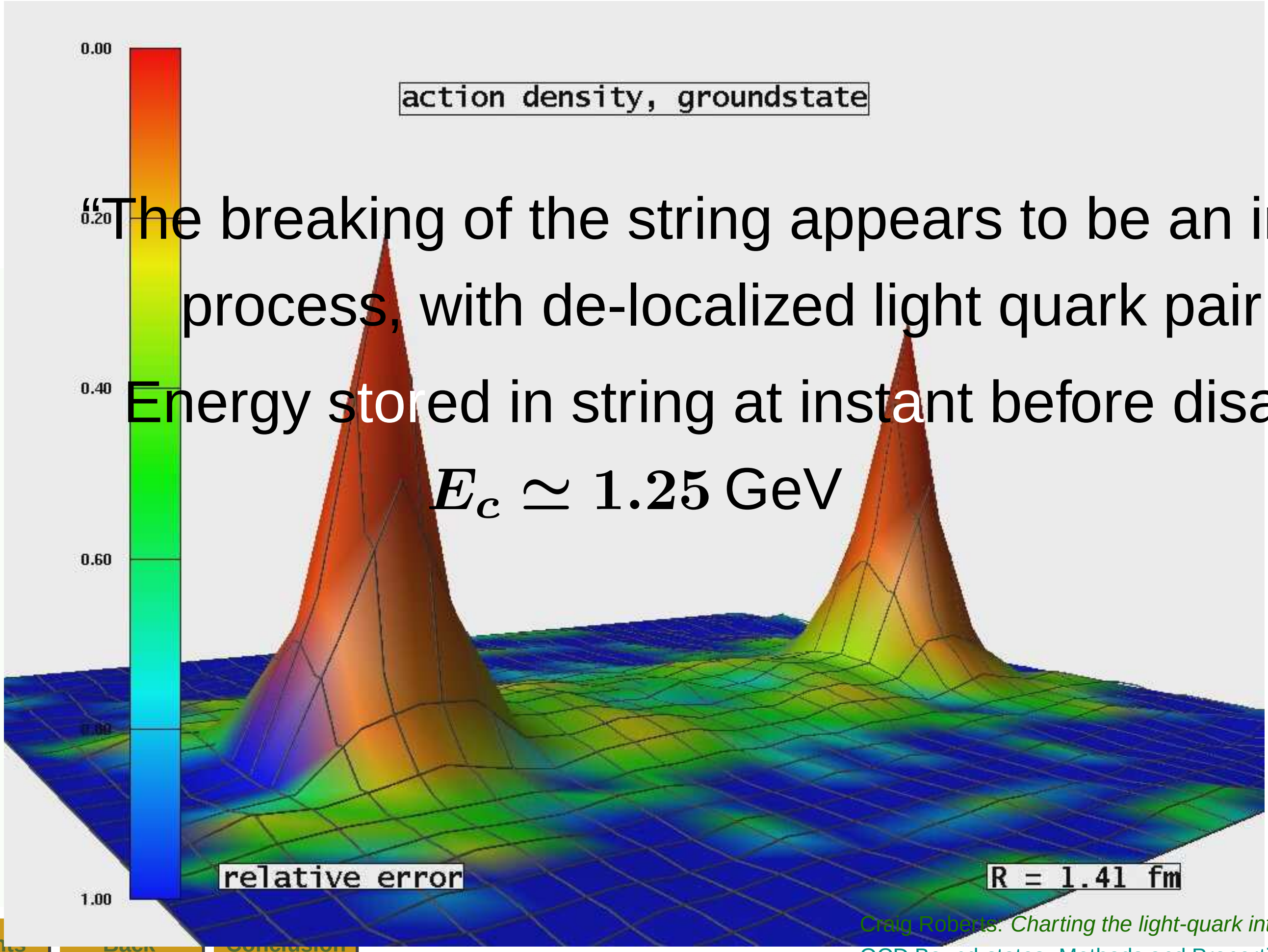

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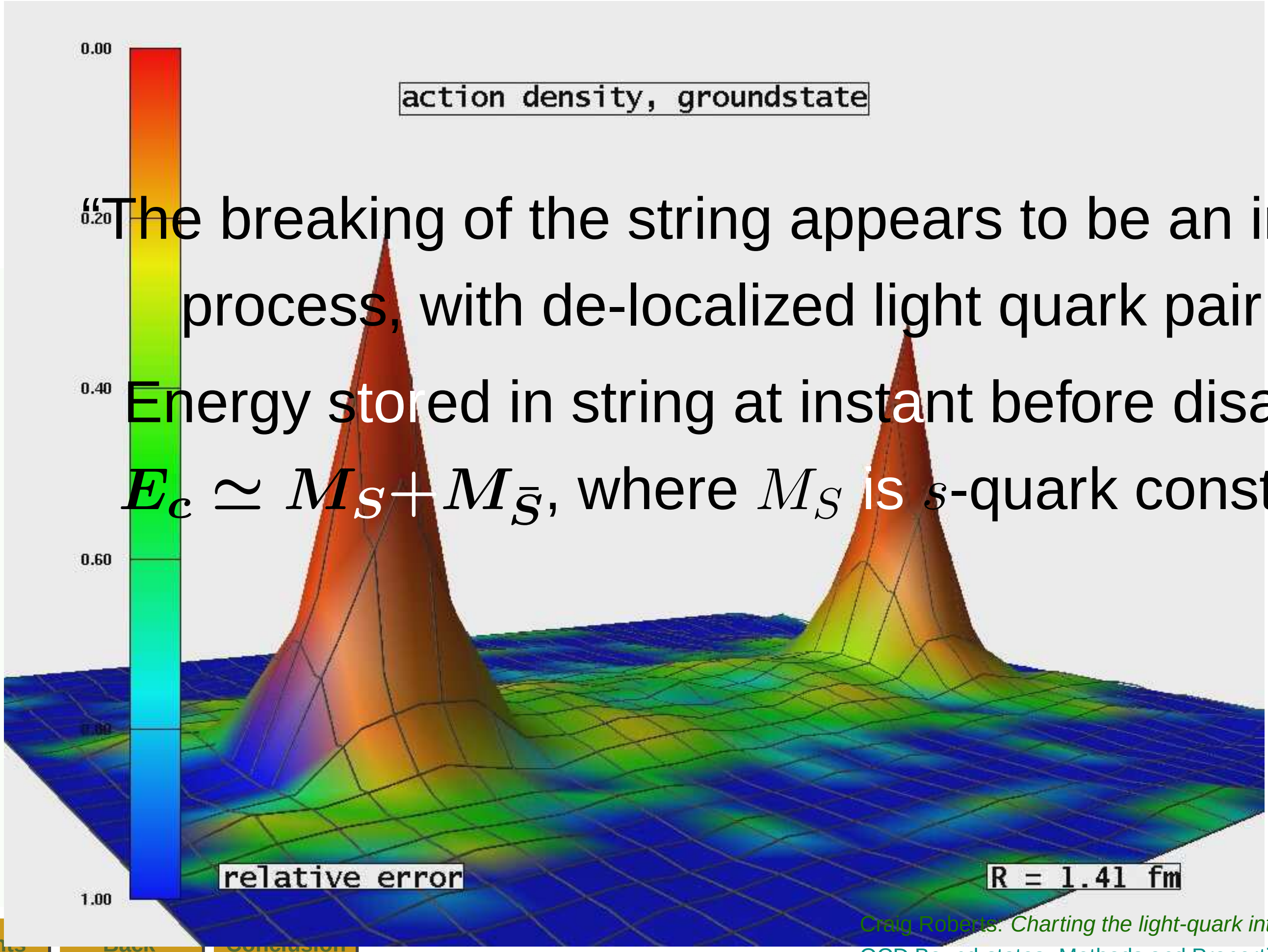

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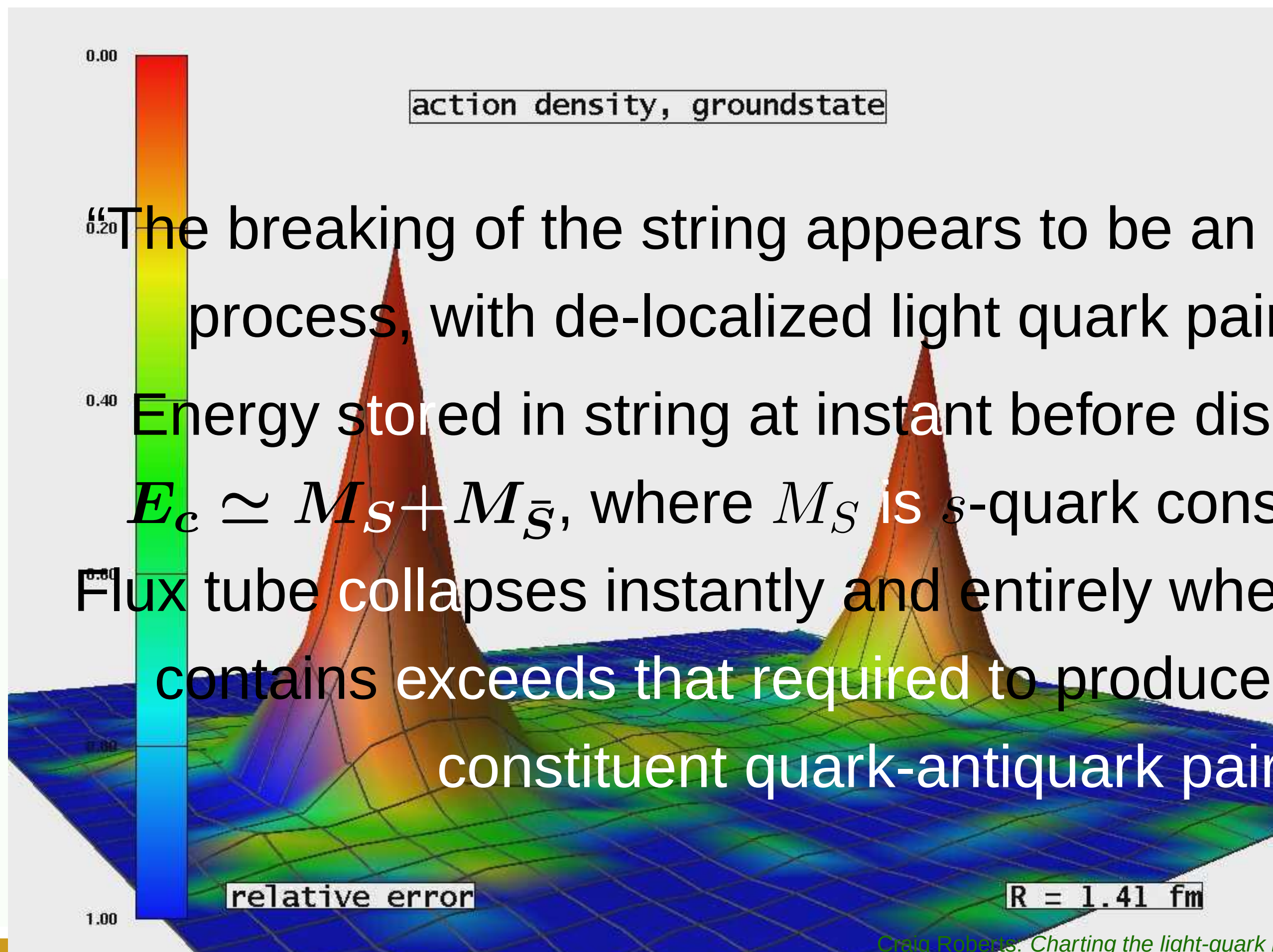

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Therefore ... No information on *potential* between light-quarks. **Confinement**

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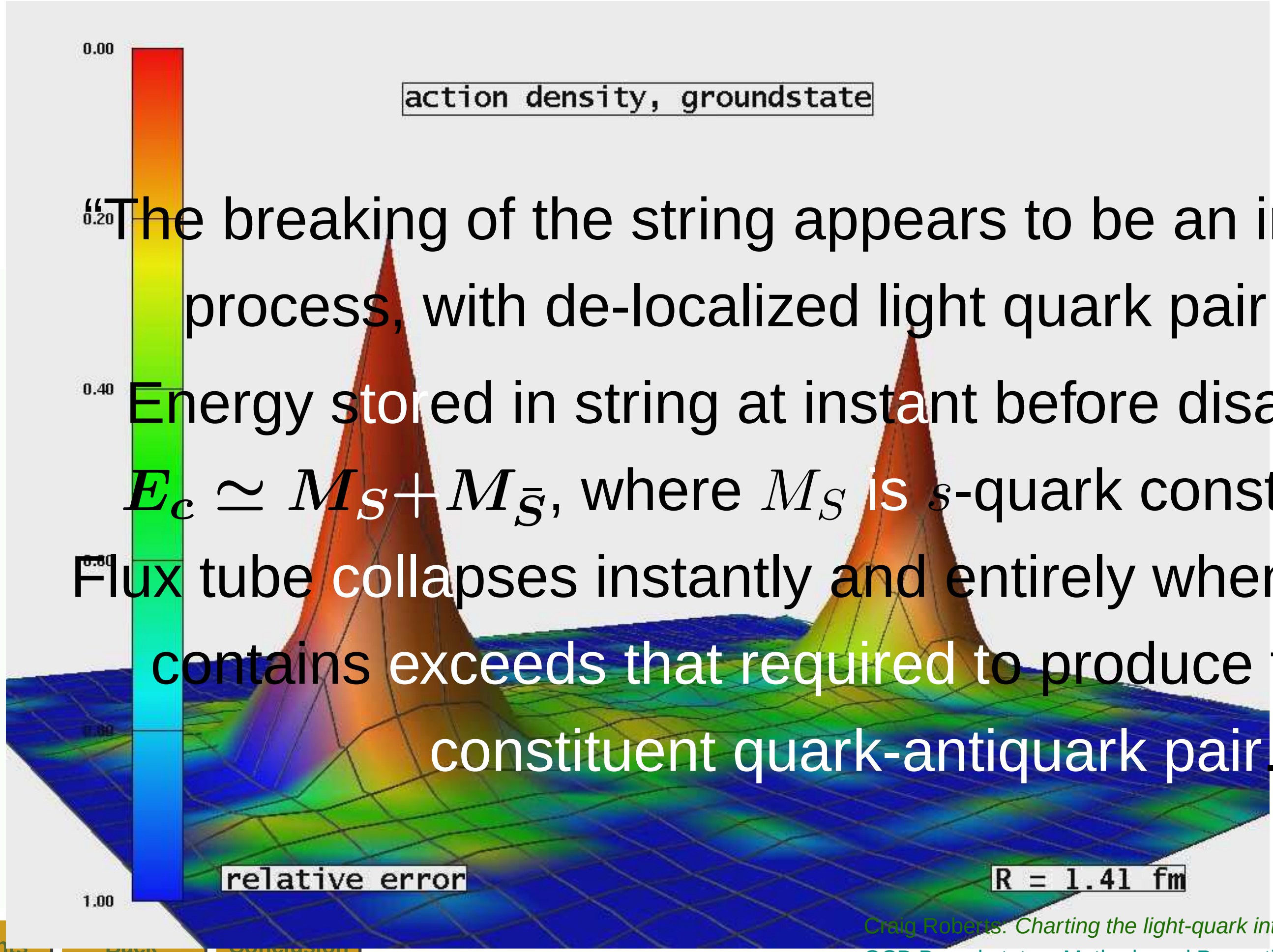

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What is the light-quark Long-Range Potential?

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Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

Dyson-Schwinger Equations

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- $\Rightarrow$ Understanding InfraRed (long-range)

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Cross-Sections built from Schwinger Functions

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- This is a well-posed problem whose solution is an elemental goal of modern hadron physics.

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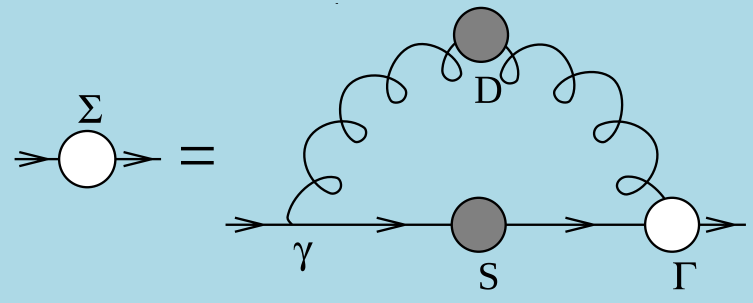
- To realise this goal, a nonperturbative symmetry-preserving DSE truncation is necessary
  - On other hand, at present significant qualitative advances possible with symmetry-preserving kernel *Ansätze* that express important additional nonperturbative effects, difficult to capture in any finite sum of contributions

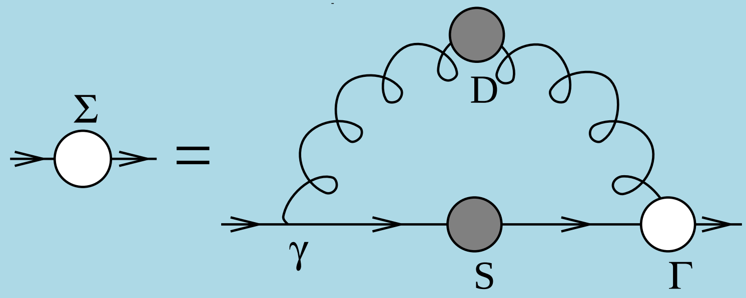
Frontiers of Nuclear Science: A Long Range Plan (2007)

Frontiers of Nuclear Science: Theoretical Advances



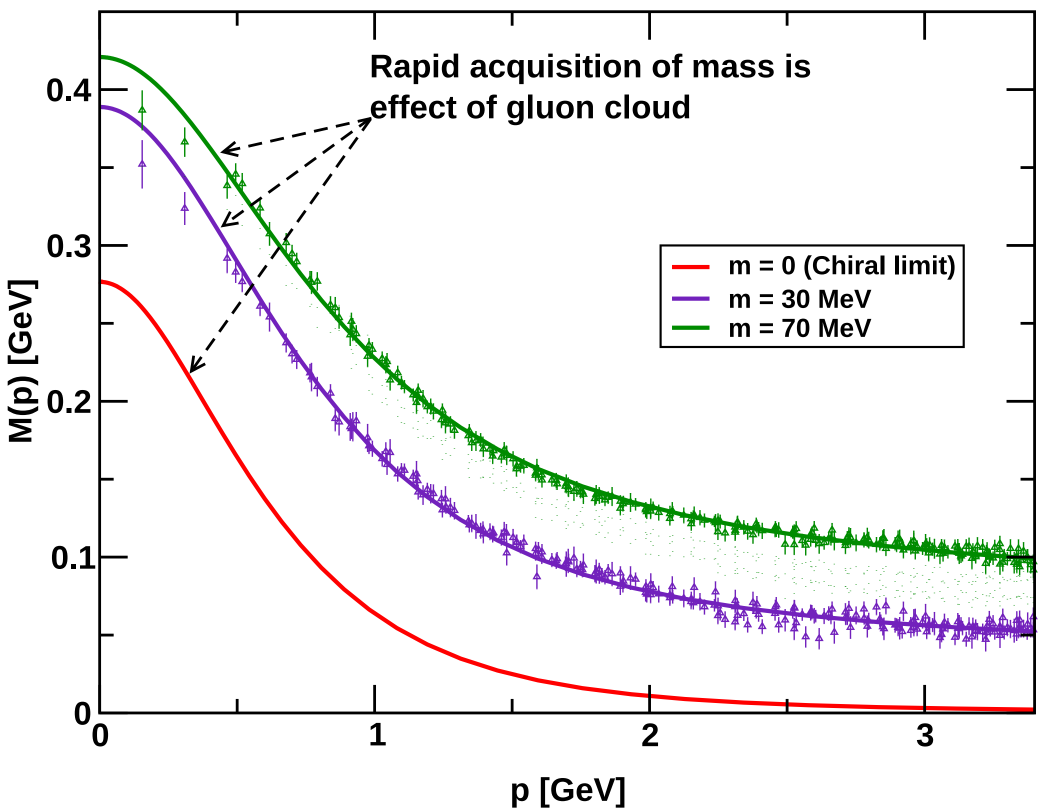
Gap Equation

Frontiers of Nuclear Science: Theoretical Advances



Gap Equation

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Mass from nothing.

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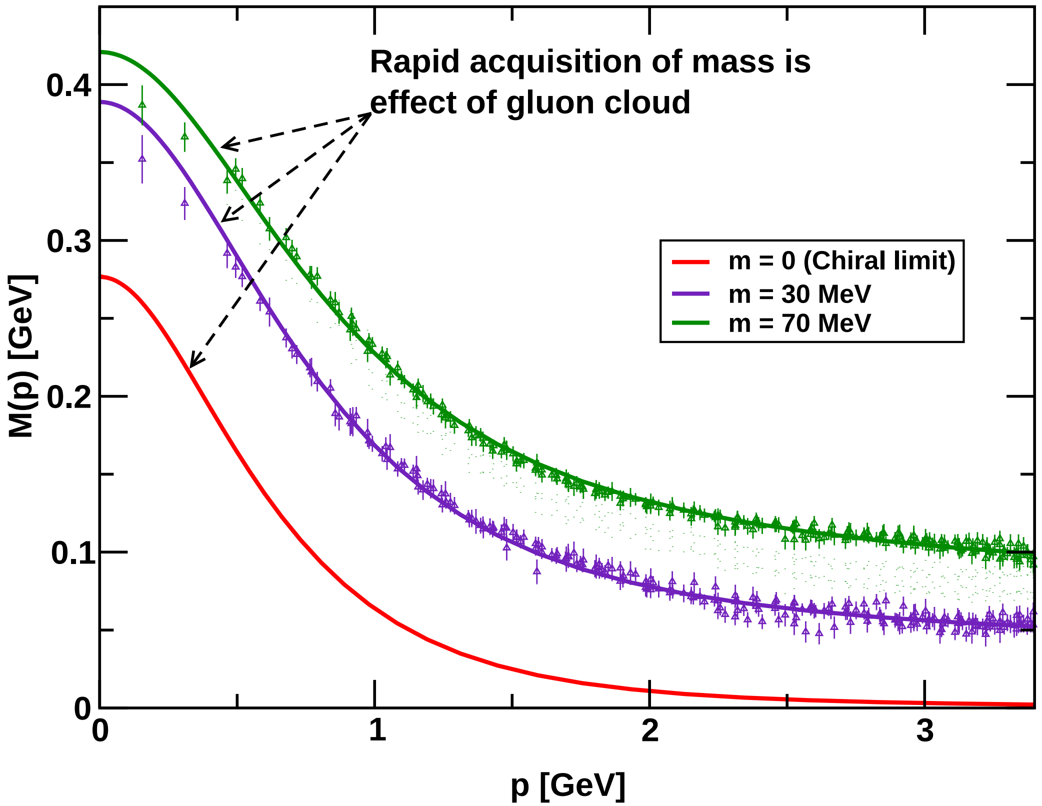
In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.



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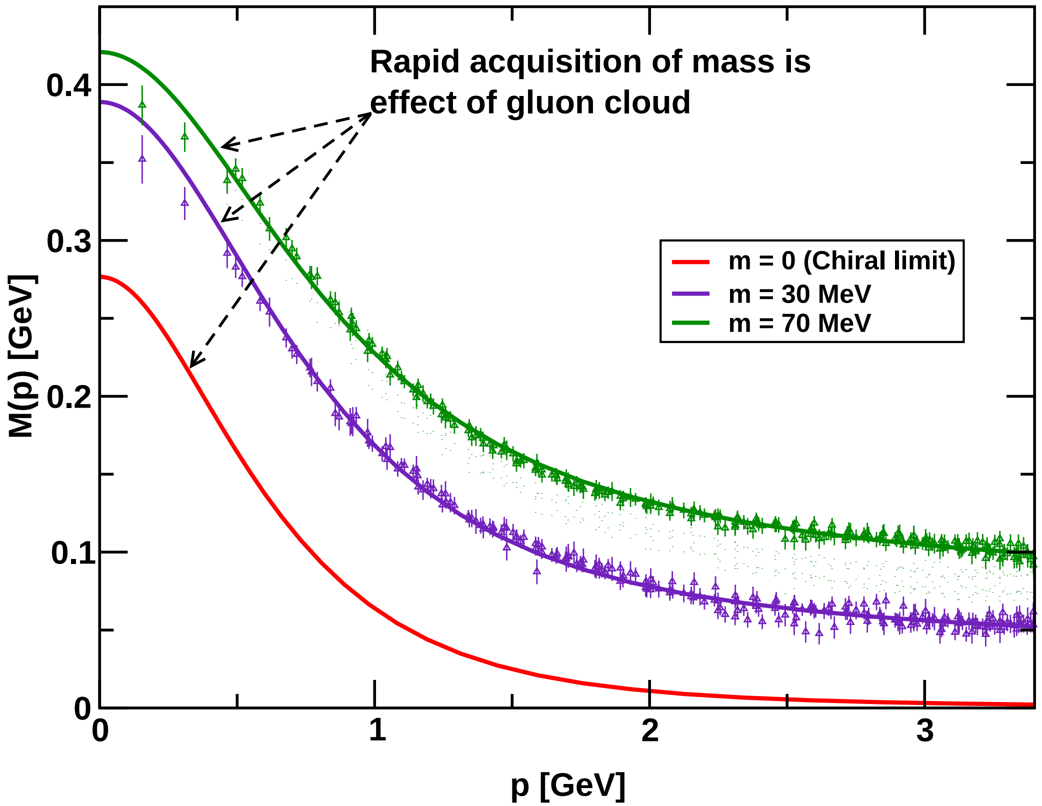
In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.




ALWAYS LOOK
ON THE
BRIGHT SIDE

- Established understanding of
  two- and three-point functions

Hadrons

- Established understanding of two- and three-point functions
- What about bound states?

Hadrons

- Without bound states, Comparison with experiment is impossible

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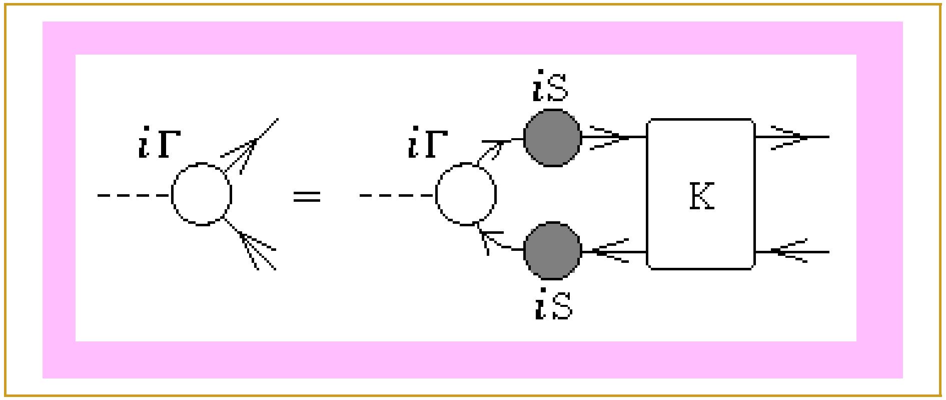
- Without bound states, Comparison with experiment is impossible
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

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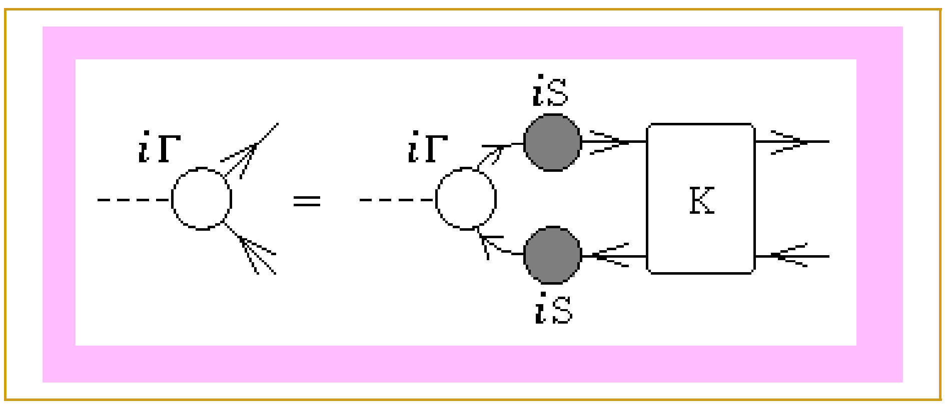

QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

or What is the long-range potential in QCD?

Bethe-Salpeter Kernel

First Contents Back Conclusion

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

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- Nontrivial constraint

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Satisfies BSE — Satisfies DSE

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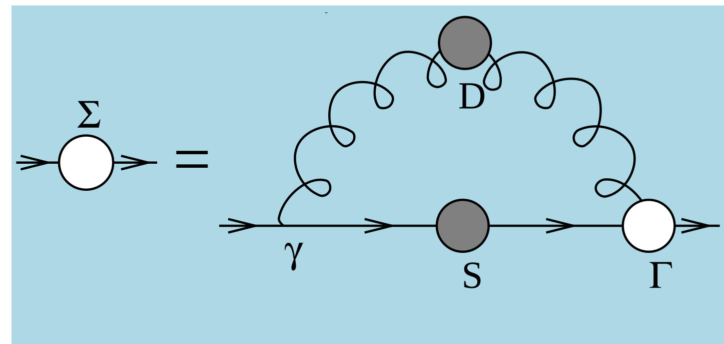
- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Persistent Challenge

Persistent Challenge

- Infinitely Many Coupled Equations



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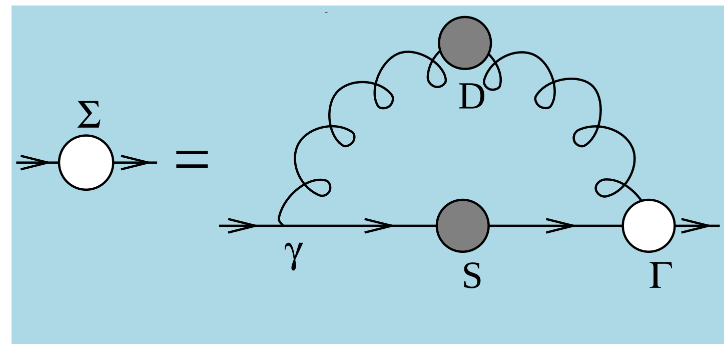

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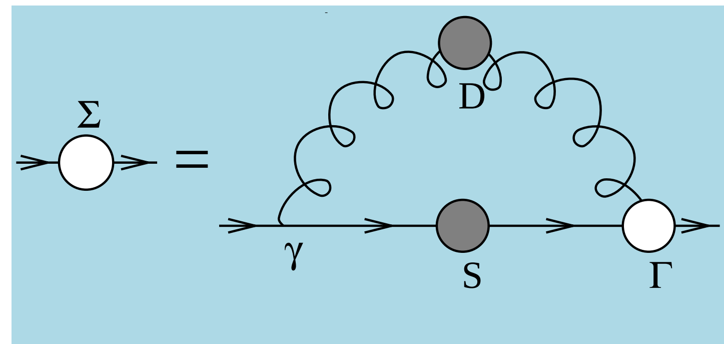

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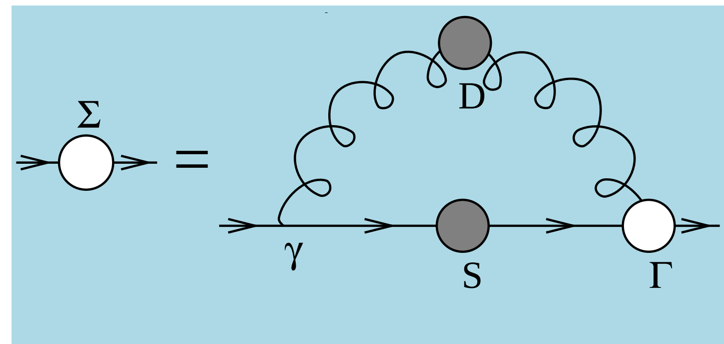

- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
    Not useful for the nonperturbative problems
    in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

Persistent Challenge

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Office of Nuclear Physics
Argonne NATIONAL LABORATORY

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Radial Excitations & Chiral Symmetry

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Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H$$

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- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

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$$f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

First Contents Back Conclusion

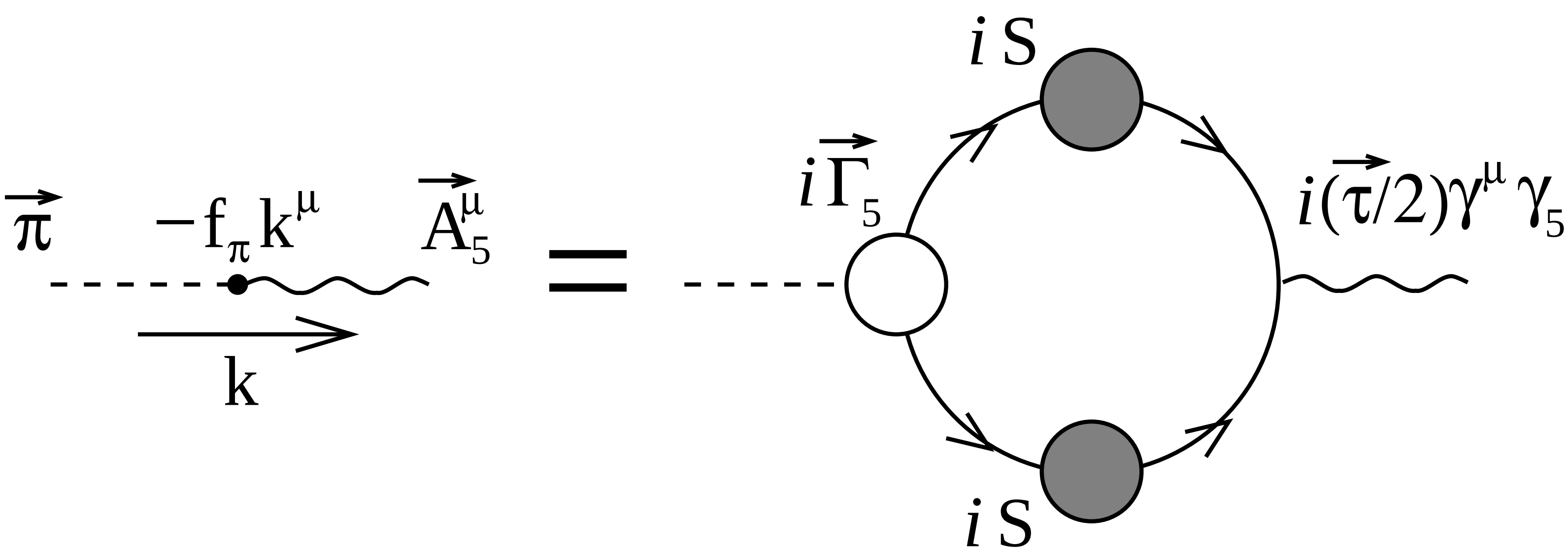
(Maris, Roberts, Tandy nu-th/9707003)

$\langle 0|\bar{q}\gamma_5\gamma_\mu q|\pi\rangle$

$$f_H \, m_H^2 = - \, \rho_\zeta^H \, \mathcal{M}_H$$

$$f_H \, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5\gamma_\mu \mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-) \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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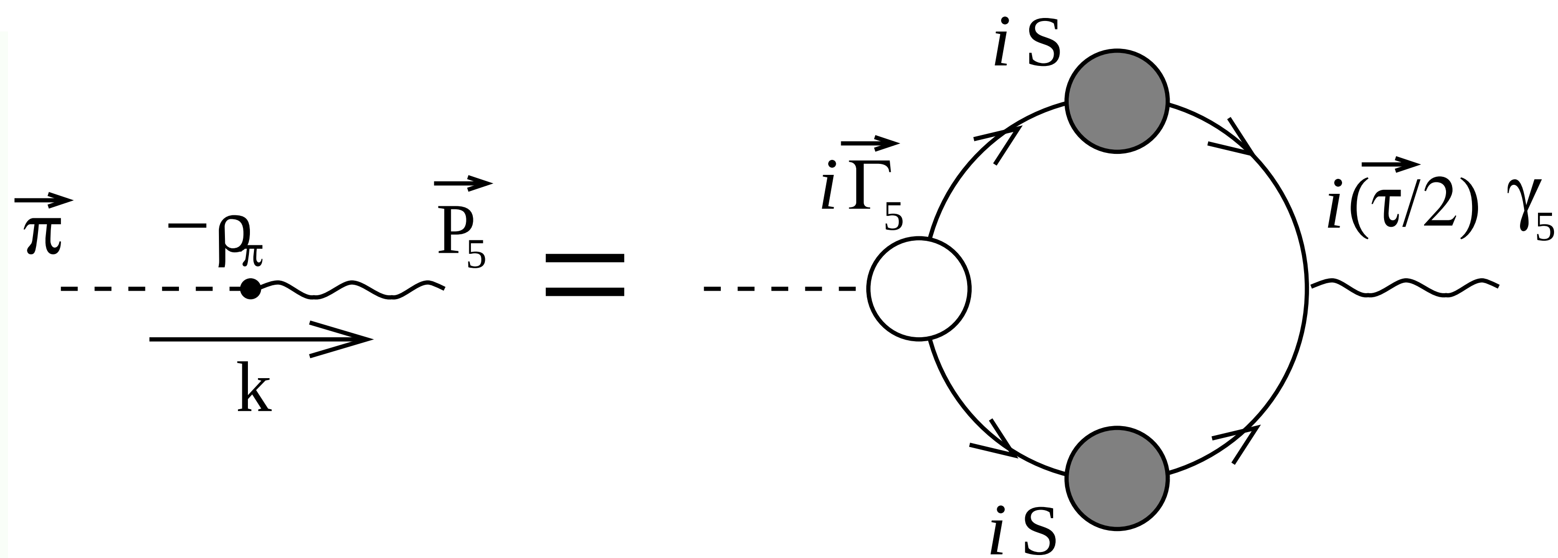

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Radial Excitations & Chiral Symmetry

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  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $\boxed{m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2}\, m_q}$ . . . GMOR relation, a corollary

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  Hence $m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ ...GMOR relation, a corollary
- Heavy-quark + light-quark

  $\Rightarrow f_H \propto \frac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

  Hence, $m_H \propto m_q$

  ... QCD Proof of Potential Model result

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Höll, Krassnigg, Roberts
nu-th/0406030

Radial Excitations & Chiral Symmetry

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- Dynamical Chiral Symmetry Breaking

  – Goldstone's Theorem –

impacts upon *every* pseudoscalar meson

Radial Excitations & Lattice-QCD

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McNeile and Michael
he-la/0607032

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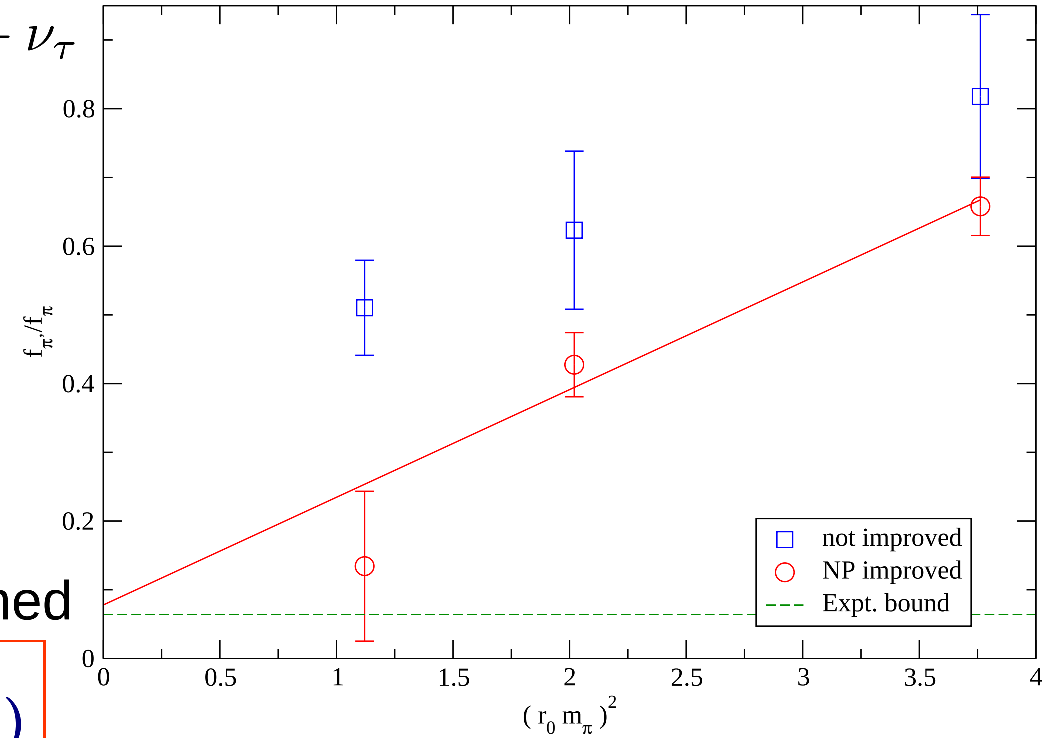

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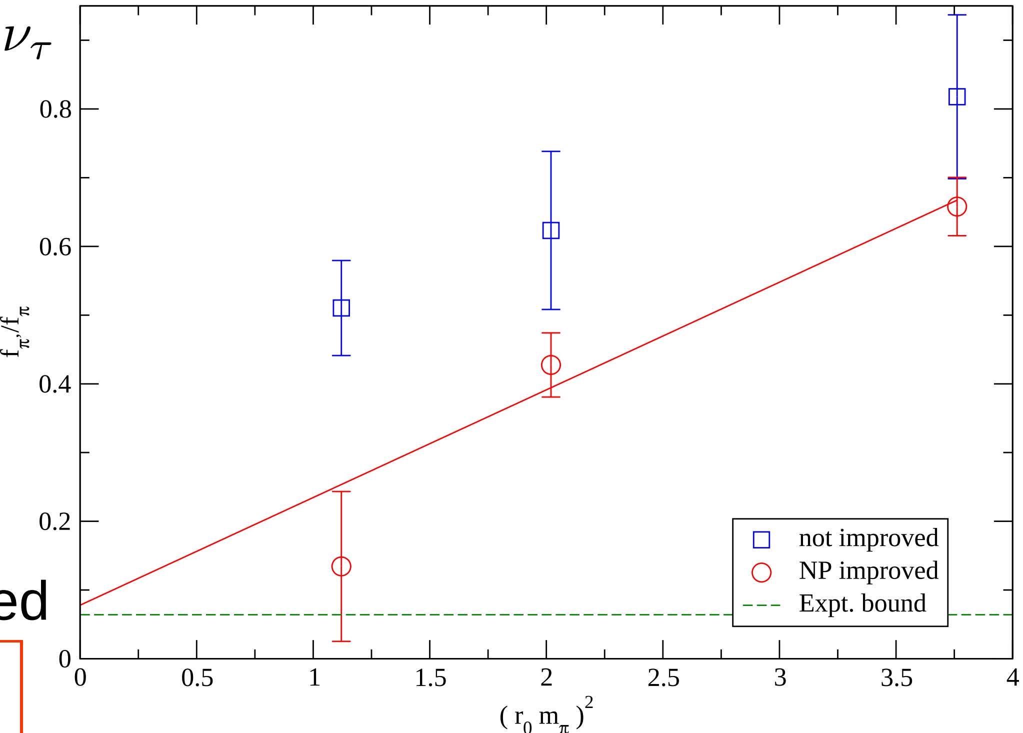

- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

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Radial Excitations & Lattice-QCD

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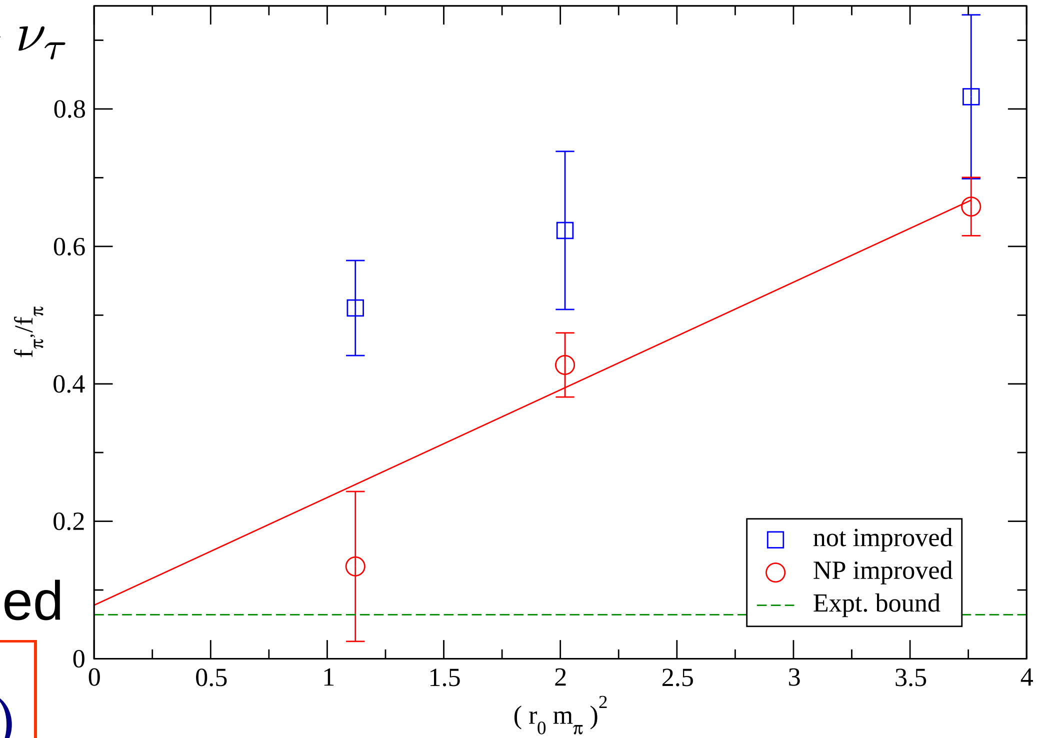

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- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Gap Equation
General Form

- Return to general bound-state problem . . .

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- To study the Poincaré covariant bound-state problem for mesons, one must first solve the gap equation

$$
\begin{aligned}
S_f(p)^{-1} &= Z_2\,(i\gamma\cdot p + m_f^{\rm bm}) + \Sigma_f(p)\,,\\
\Sigma_f(p) &= Z_1 \int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S_f(q)\frac{\lambda^a}{2}\Gamma^f_\nu(q,p),
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$$

- $D_{\mu\nu}(k)$ is the dressed-gluon propagator;
- $\Gamma^f_\nu(q,p)$ is the dressed-quark-gluon vertex;
- $m^{\rm bm}(\Lambda)$ is the Lagrangian current-quark bare mass;
- $Z_{1,2}(\zeta^2,\Lambda^2)$ are respectively the vertex and quark wave function renormalisation constants, with ζ the renormalisation point.

Bethe-Salpeter Equation
General Form

Bethe-Salpeter Equation
General Form

- Pseudoscalar and axial-vector mesons appear as poles in the inhomogeneous Bethe-Salpeter equation.

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- Exact form:

$$\Gamma^{fg}_{5\mu}(k;P) = Z_2\gamma_5\gamma_\mu - \int_q g^2 D_{\alpha\beta}(k-q) \times \frac{\lambda^a}{2}\gamma_\alpha S_f(q_+)\Gamma^{fg}_{5\mu}(q;P)S_g(q_-)\frac{\lambda^a}{2}\,\Gamma^g_\beta(q_-,k_-) + \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

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$$\times \frac{\lambda^a}{2}\gamma_\alpha S_f(q_+)\Gamma^{fg}_{5\mu}(q;P)S_g(q_-)\frac{\lambda^a}{2}\,\Gamma^g_\beta(q_-,k_-)$$
$$+ \int_q g^2 D_{\alpha\beta}(k-q)\,\frac{\lambda^a}{2}\gamma_\alpha S_f(q_+)\frac{\lambda^a}{2}\Lambda^{fg}_{5\mu\beta}(k,q;P),$$

  - $\Lambda^{fg}_{5\mu\beta}$ is defined completely via the dressed-quark self-energy and, owing to Poincaré covariance, one can employ, e.g., $q_\pm = q \pm P/2$, etc., without loss of generality

Ward-Takahashi Identity
Bethe-Salpeter Kernel

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- In any reliable study of light-quark hadrons, axial-vector vertex must satisfy

$$P_\mu \Gamma^{fg}_{5\mu}(k;P) = S_f^{-1}(k_+) i\gamma_5 + i\gamma_5 S_g^{-1}(k_-) - i\left[m_f(\zeta) + m_g(\zeta)\right] \Gamma_5^{fg}(k;P)\,,$$

expresses chiral symmetry & pattern by which it's broken

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expresses chiral symmetry & pattern by which it's broken

- The condition ($\Lambda^{fg}_{5\beta}$ pseudoscalar analogue of $\Lambda^{fg}_{5\mu\beta}$)

$$P_\mu \Lambda^{fg}_{5\mu\beta}(k,q;P) = \Gamma^f_\beta(q_+,k_+)\, i\gamma_5 + i\gamma_5\, \Gamma^g_\beta(q_-,k_-) - i[m_f(\zeta) + m_g(\zeta)]\Lambda^{fg}_{5\beta}(k,q;P),$$

NECESSARY & SUFFICIENT

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NECESSARY & SUFFICIENT

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- Rainbow-ladder . . .
  - $\Gamma^f_\beta(q,k) = \gamma_\mu$

    $\Rightarrow \Lambda^{fg}_{5\mu\beta}(k,q;P) = 0 = \Lambda^{fg}_{5\beta}(k,q;P)$

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel
60 year problem

- Bethe-Salpeter equation introduced in 1951

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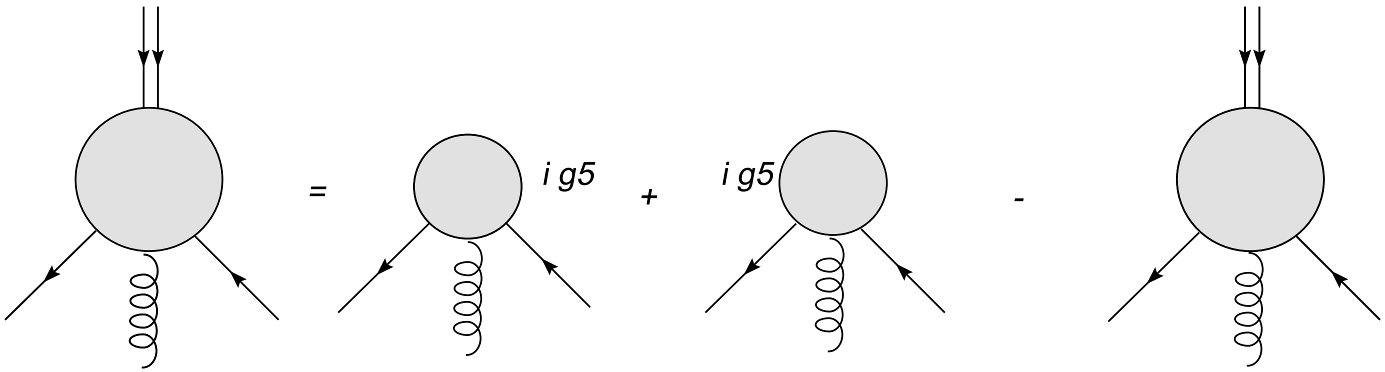

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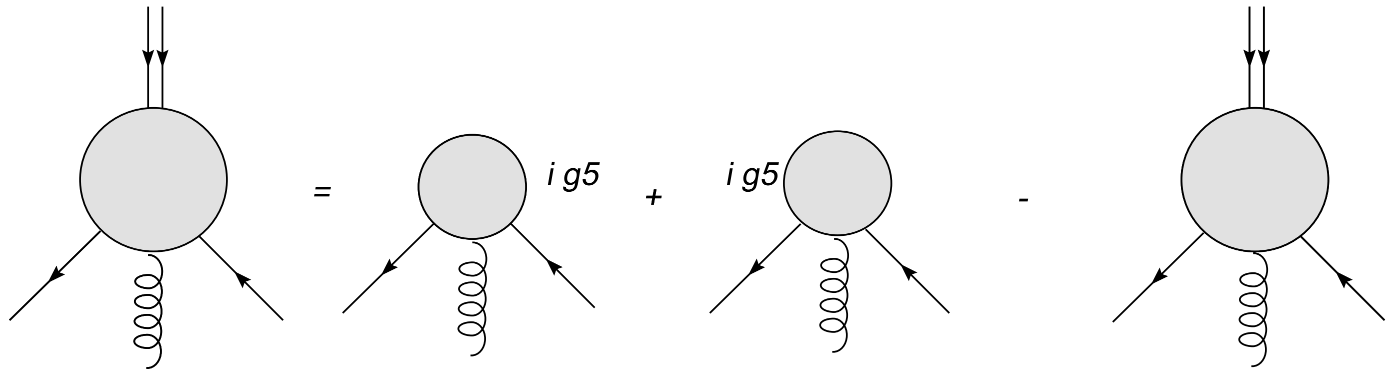

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- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
  - Consistent means - all symmetries preserved!

Chang Lei (IAPCM, Beijing) & CDR
arXiv:0903.5461 [nucl-th]

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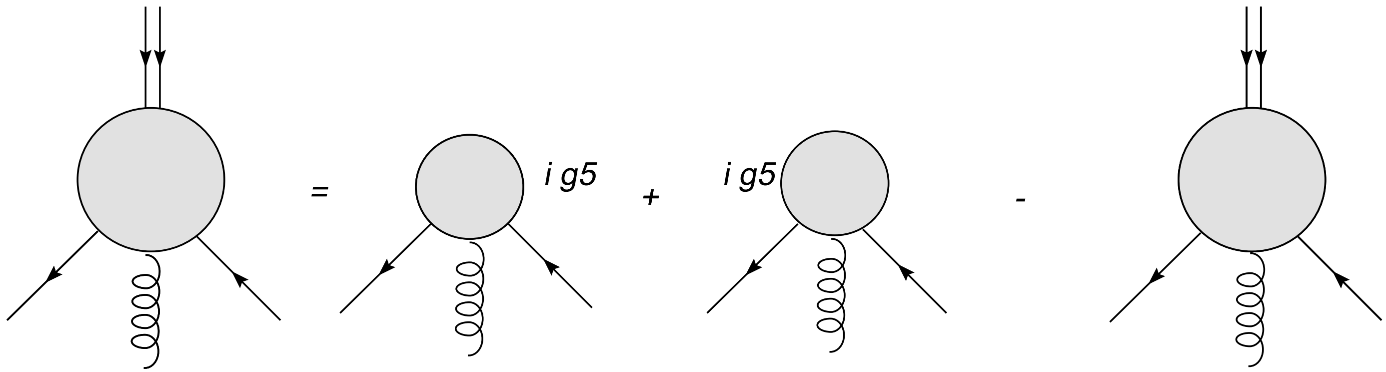
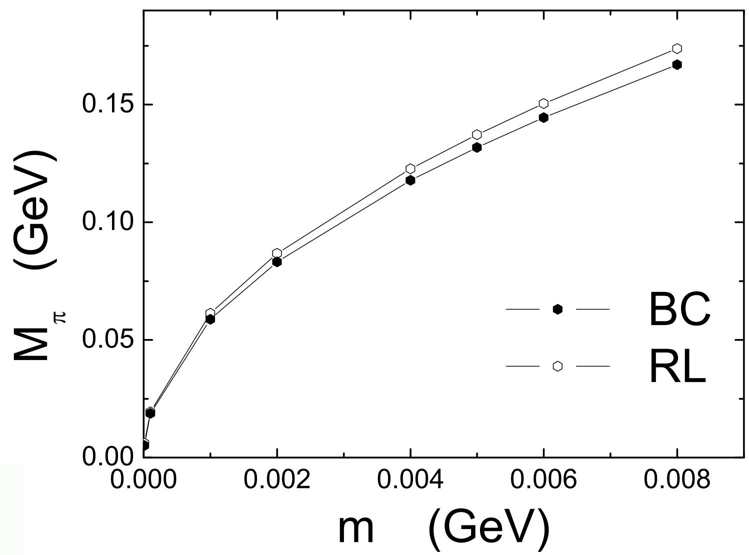


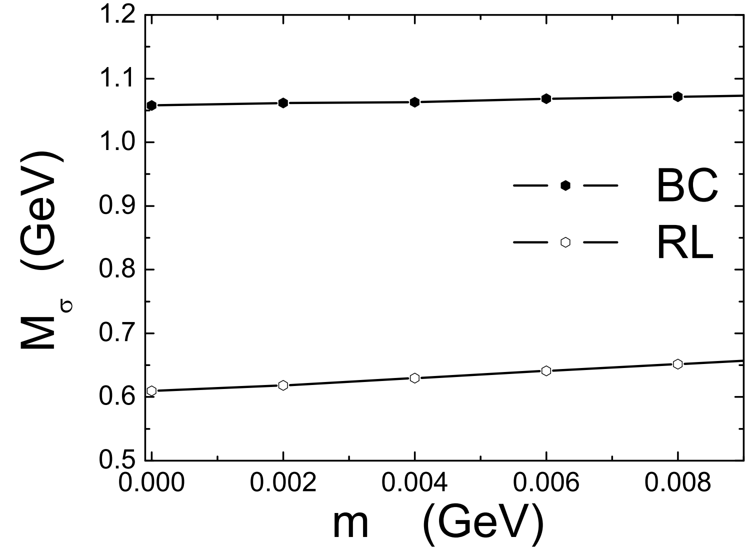
$$P_\mu \Lambda^{fg}_{5\mu\beta}(k,q;P) = \Gamma^f_\beta(q_+,k_+)\, i\gamma_5 + i\gamma_5\, \Gamma^g_\beta(q_-,k_-) - i[m_f(\zeta)+m_g(\zeta)]\Lambda^{fg}_{5\beta}(k,q;P),$$

- For first time: can construct *Ansatz* for Bethe-Salpeter kernel consistent with any reasonable quark-gluon vertex
- Exemplified the procedure and results to expect . . .

Numerical Illustration

Chang Lei & CDR, arXiv:0903.5461 [nucl-th]





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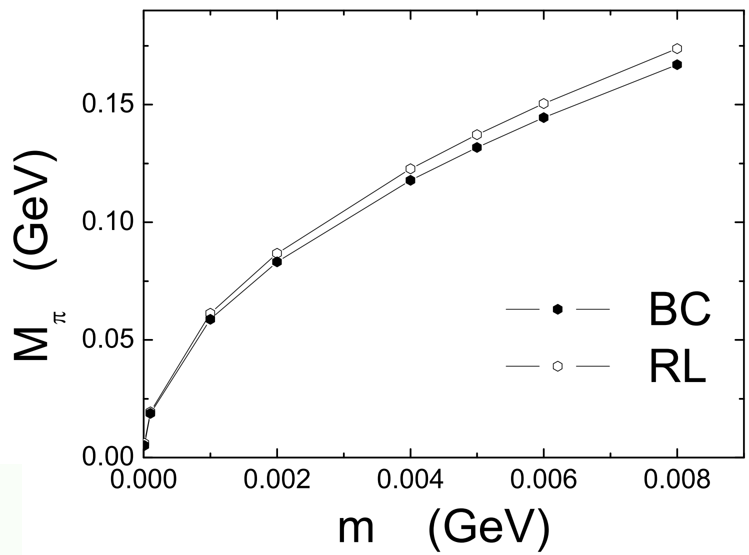
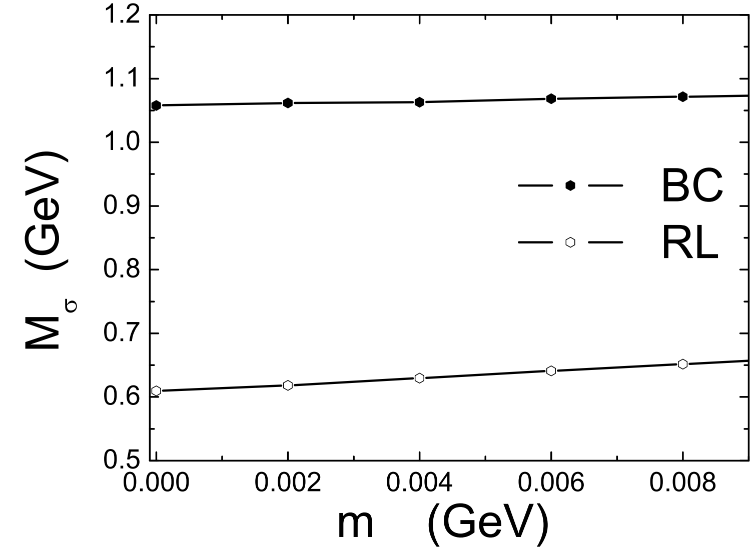
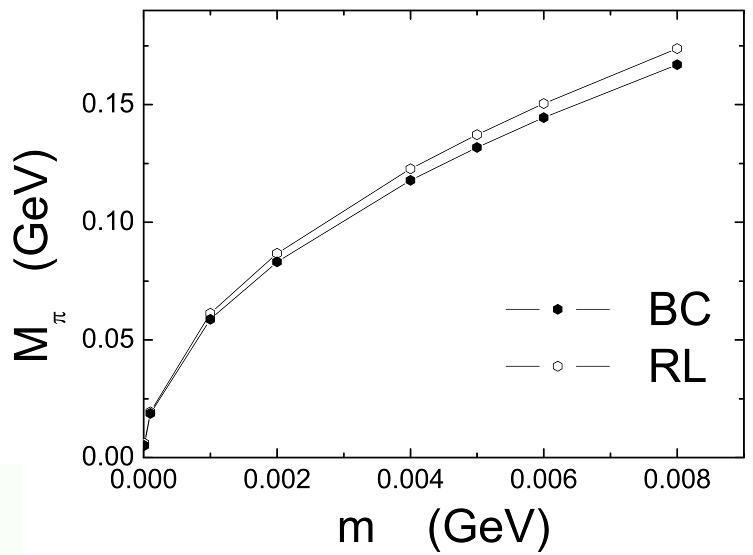
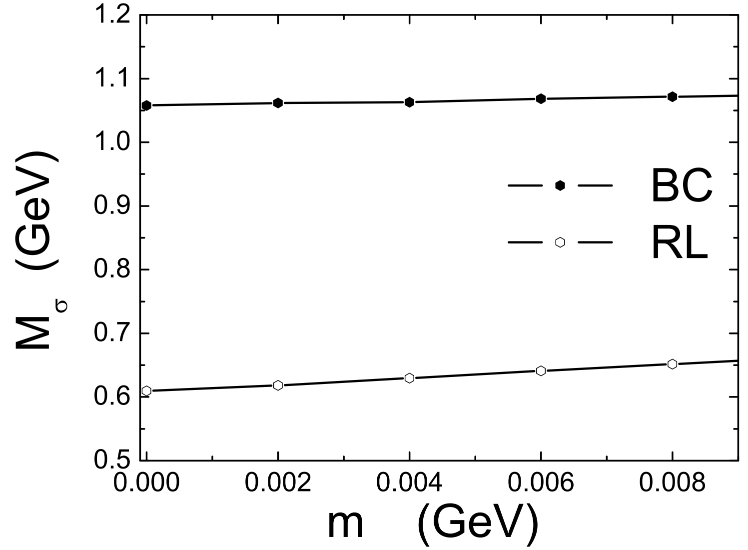




- Single interaction, common mass scale: rainbow-ladder cf. BC-consistent truncation

Numerical Illustration





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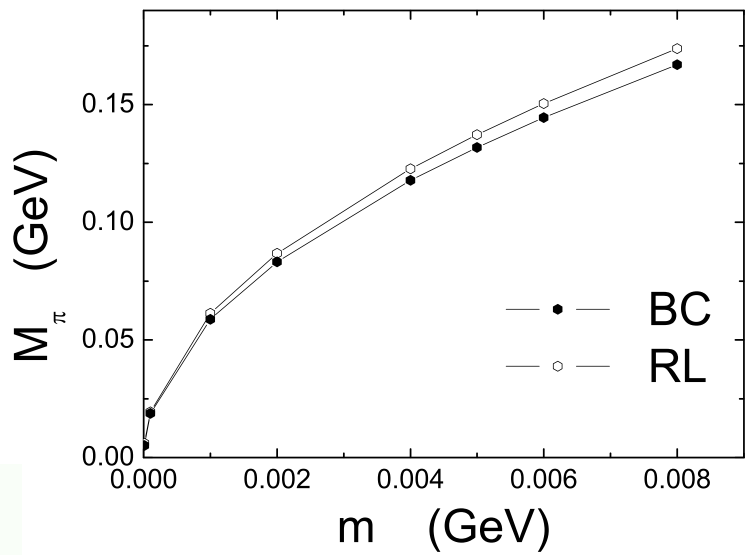

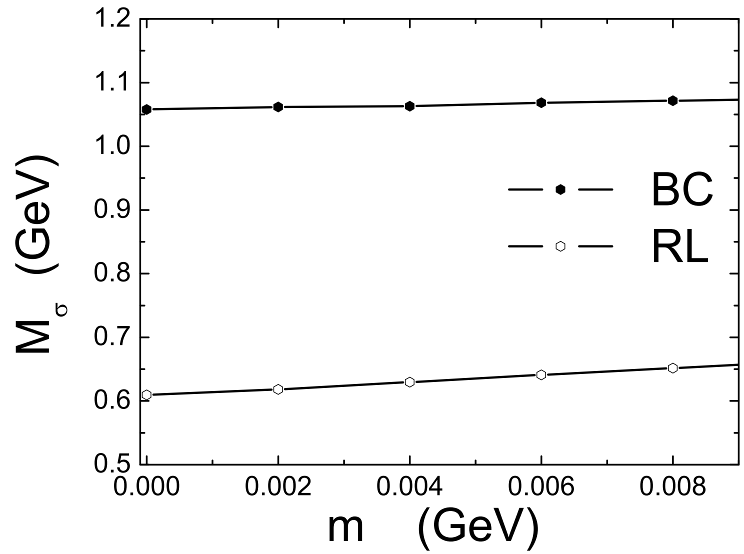

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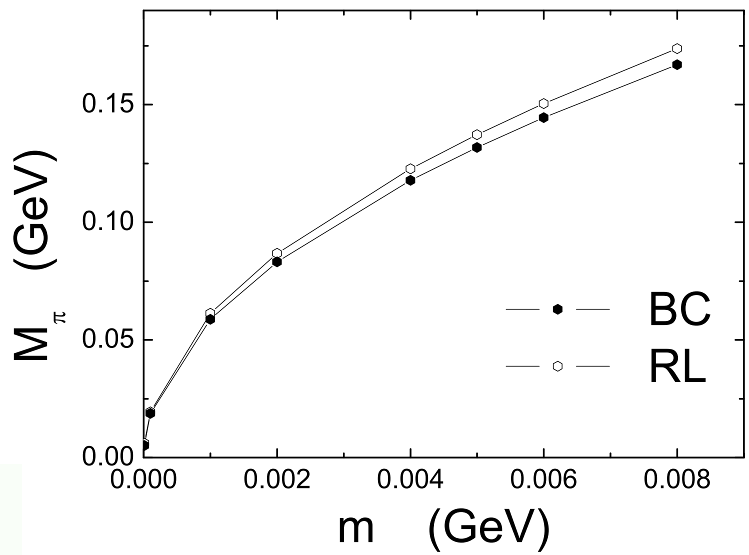

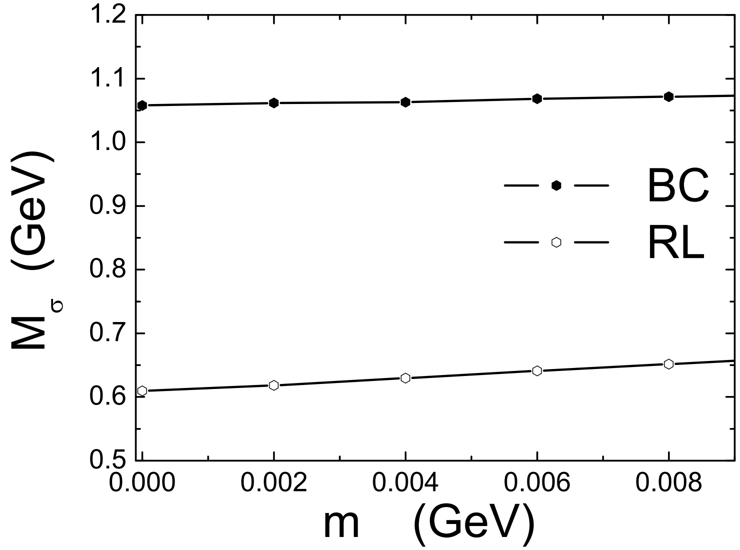

- Single interaction, common mass scale: rainbow-ladder cf. BC-consistent truncation
  - GMOR . . . plainly satisfied by both truncations
  - Added attraction in pseudoscalar channel
  - Added repulsion in scalar channel

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Spin-orbit Interaction

Chang Lei & CDR, arXiv:0903.5461 [nucl-th]

- Rainbow-ladder DSE truncation,

$$\varepsilon_\sigma^{\rm RL} := \left. \frac{2M(0) - m_\sigma}{2M(0)} \right|_{\rm RL} = (0.3 \pm 0.1)\,.$$

- BC-consistent Bethe-Salpeter kernel; viz., $\varepsilon_\sigma^{\rm BC} \lesssim 0.1\,.$

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- From this viewpoint scalar is a spin and orbital excitation of a pseudoscalar meson

Spin-orbit Interaction

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- Scalar mesons $=$ ${}^3P_0$ states: Constituents' spins aligned and one unit of constituent orbital angular momentum
- Extant studies of realistic corrections to the rainbow-ladder truncation show that they reduce hyperfine splitting

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Spin-orbit Interaction

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- Scalar mesons $=\ {}^3P_0$ states: Constituents' spins aligned and one unit of constituent orbital angular momentum
- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit splitting. Effect owes to influence of quark's dynamically-enhanced scalar self-energy in the Bethe-Salpeter kernel.
  Impossible to demonstrate effect without our new procedure

Spin-orbit Interaction

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- Expect this feature to have material impact on mesons with mass greater than 1 GeV.

  prima facie . . . can overcome longstanding shortcoming of RL truncation; viz., splitting between vector & axial-vector mesons is too small

Spin-orbit Interaction

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- Clear sign that in a Poincaré covariant treatment the BC-consistent truncation magnifies spin-orbit splitting.
- Expect this feature to have material impact on mesons with mass greater than 1 GeV.
- Promise of realistic meson spectroscopy
  First time, also for mass > 1 GeV

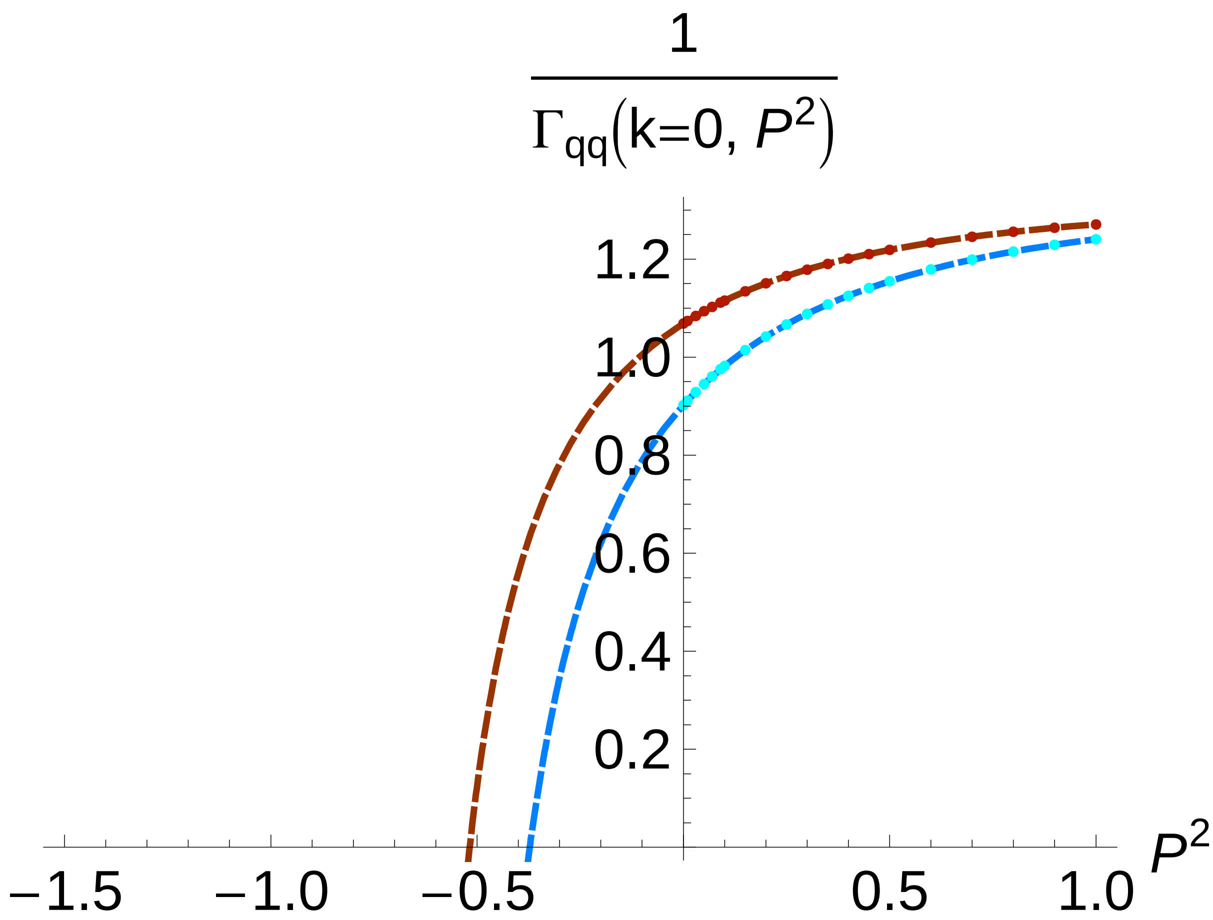
$a_1 - \rho$ *mass splitting*

$a_1 - \rho$ mass splitting

- Solve inhomogeneous vector and axial-vector Bethe-Salpeter equation at spacelike total momentum $\Rightarrow \Gamma_{qq}(k=0,P^2)$
- $$\frac{1}{\Gamma_{qq}(k=0,P^2)}$$ exhibits a zero at ground-state mass-squared
- Padé approximant extrapolation to locate zero
  - Almost precisely method used for ground-state masses in lattice-QCD
  - Intelligent use gives dependable results
    "Schwinger functions and light-quark bound states"
    S.V. Wright, *et al*., Few Body Syst. **40** (2007) 209, nucl-th/0701009

Chang Lei & CDR, in-preparation

$a_1 - \rho$ *mass splitting*



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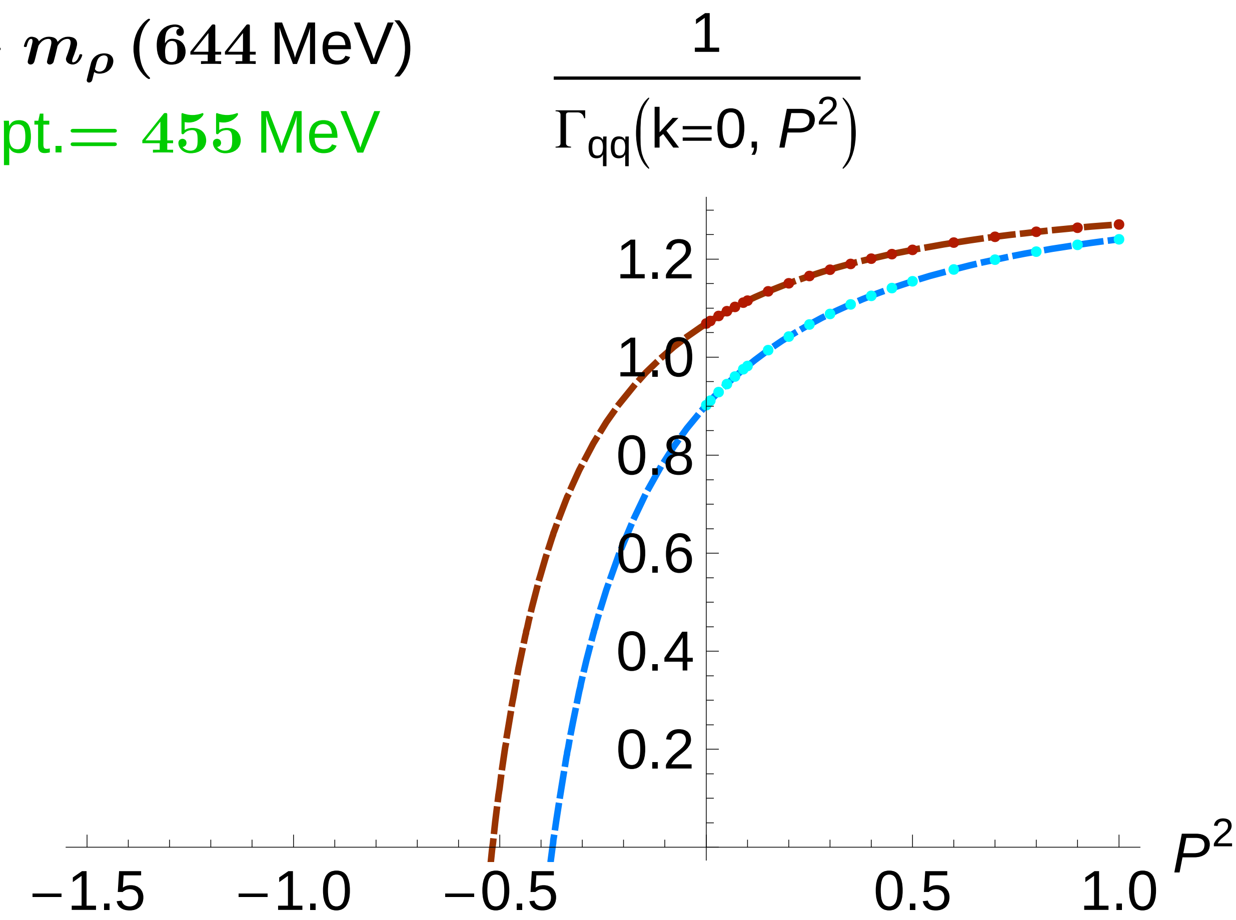
Chang Lei & CDR, in-preparation

$a_1 - \rho$ mass splitting

$m_{a_1}\,(759\,\text{MeV}) - m_\rho\,(644\,\text{MeV})$
$= 115\,\text{MeV}$ ... expt.$=455\,\text{MeV}$

Rainbow-Ladder

$\Gamma_\mu(q,k) = \gamma_\mu$



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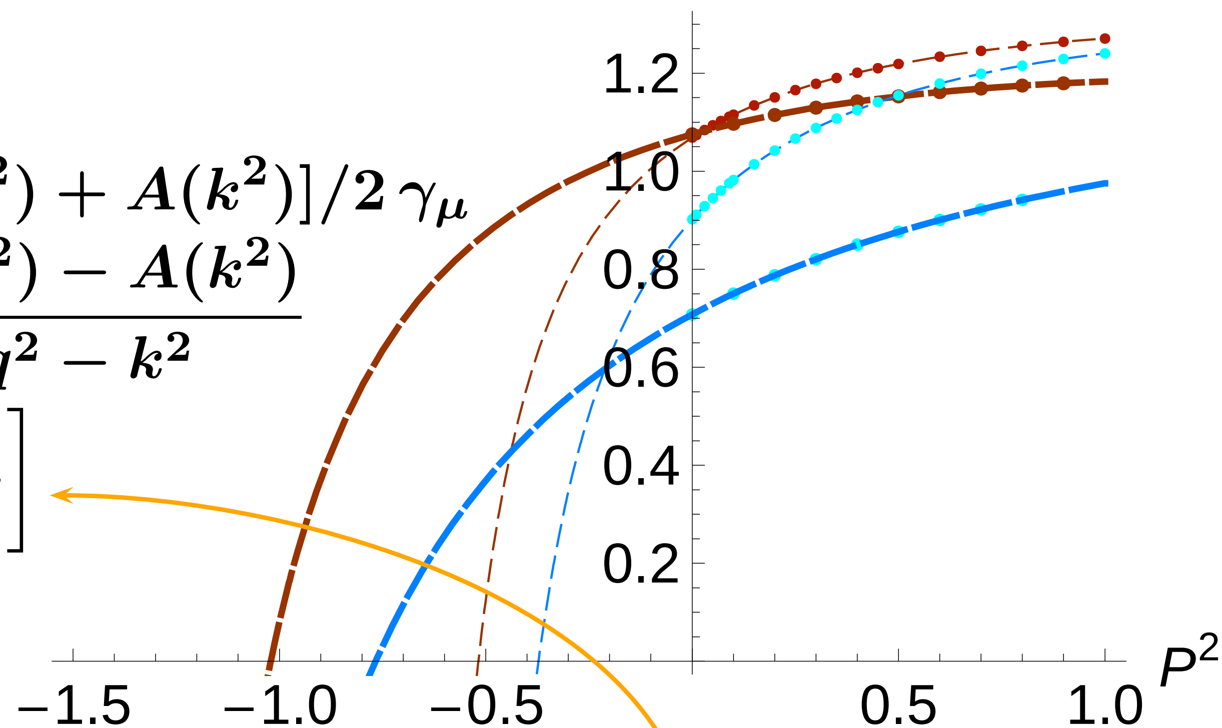
Chang Lei & CDR, in-preparation

$a_1 - \rho$ mass splitting

m_{a_1} (1066 MeV) $-\ m_\rho$ (924 MeV)
$= 142$ MeV . . . expt.$= 455$ MeV



Ball-Chiu

$$\Gamma_\mu(q,k) = i[A(q^2) + A(k^2)]/2\,\gamma_\mu$$
$$+2k_\mu \left[i\gamma \cdot k\, \frac{A(q^2) - A(k^2)}{q^2 - k^2} + \frac{B(q^2) - B(k^2)}{q^2 - k^2} \right]$$

DCSB enhanced spin-orbit interaction

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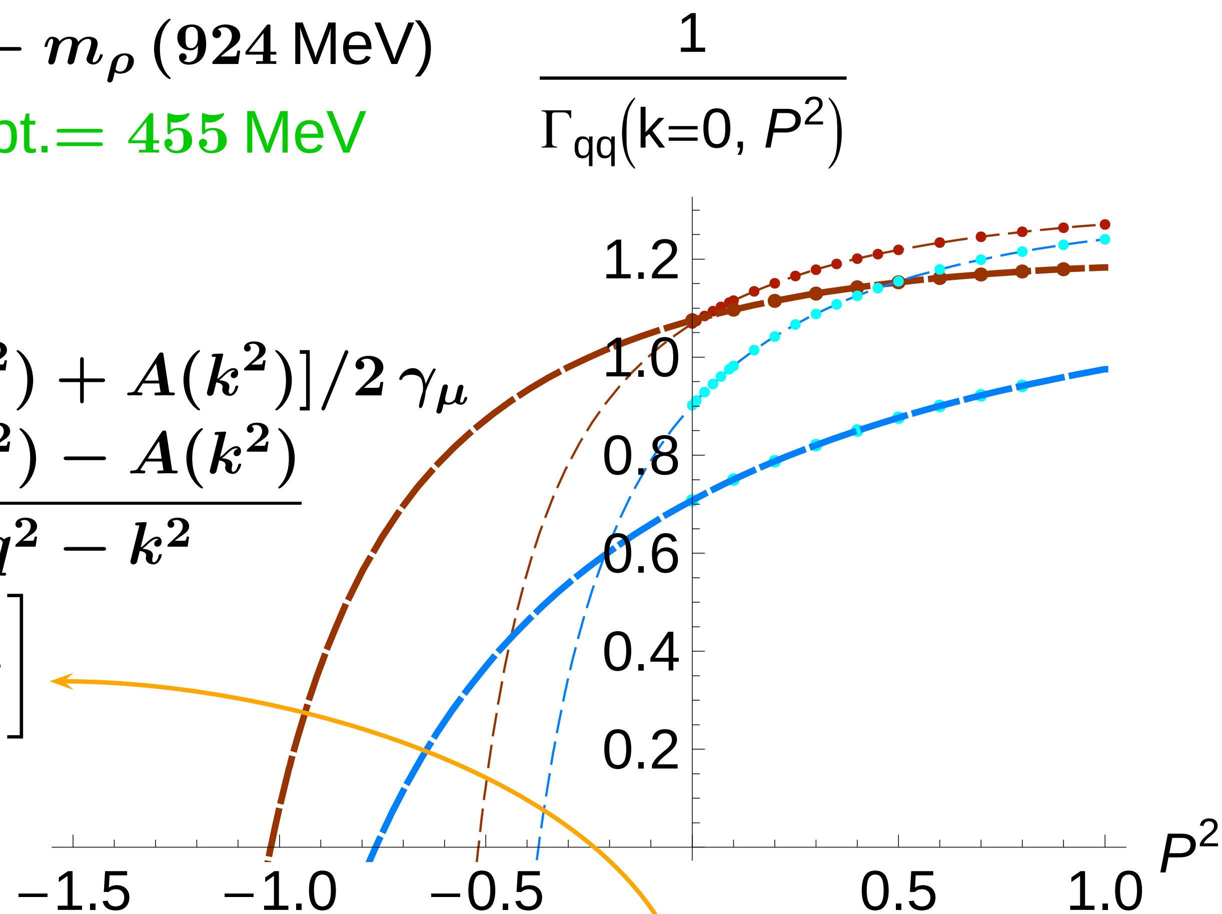


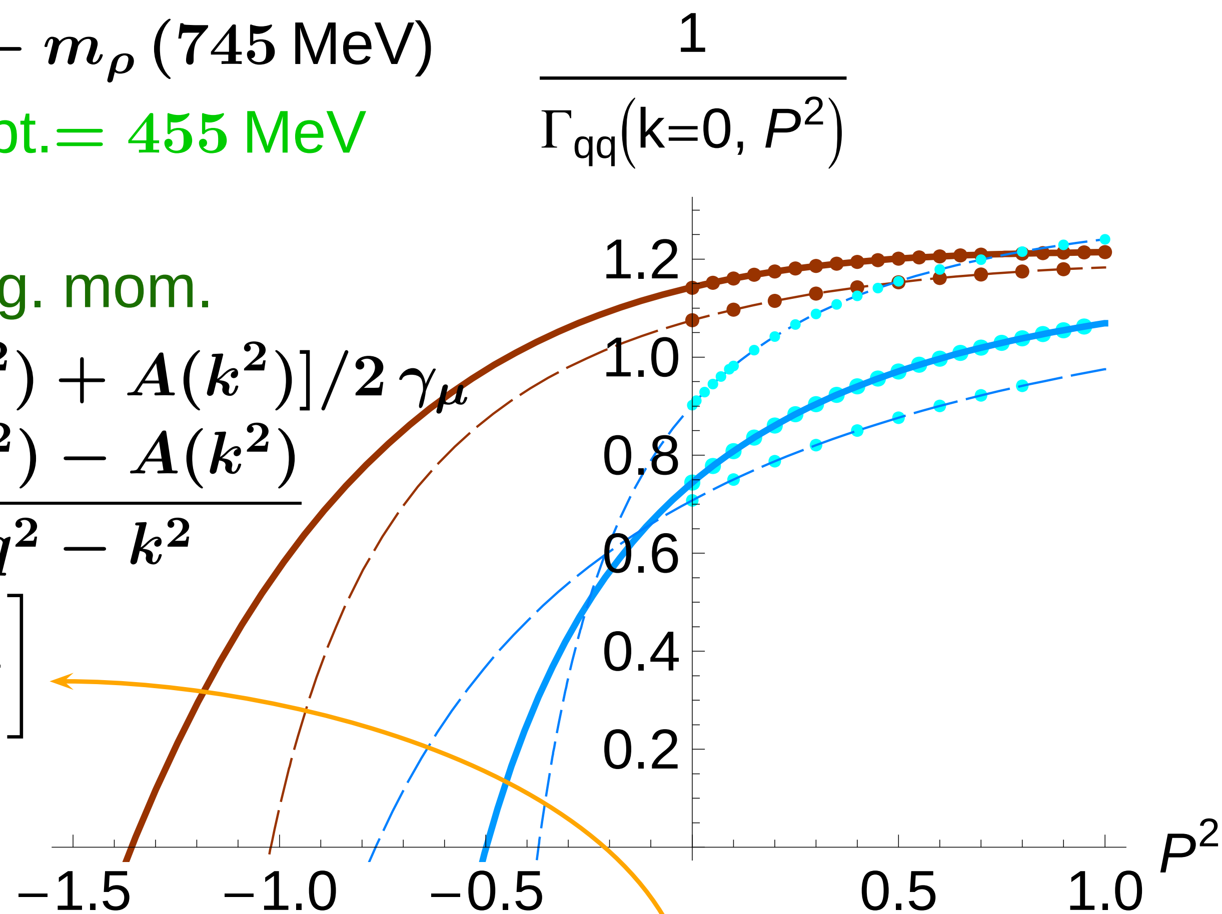
What's missing?

DCSB enhanced spin-orbit interaction

$a_1 - \rho$ mass splitting

m_{a_1} (1230 MeV) $- m_\rho$ (745 MeV)

$= 485$ MeV … expt.$= 455$ MeV



Ball-Chiu + an. mag. mom.

$$\Gamma_\mu(q,k) = i[A(q^2)+A(k^2)]/2\,\gamma_\mu$$
$$+2k_\mu\left[i\gamma\cdot k\,\frac{A(q^2)-A(k^2)}{q^2-k^2} + \frac{B(q^2)-B(k^2)}{q^2-k^2}\right]$$
$$+\sigma_{\mu\nu}(q-k)_\nu\,\frac{B(q^2)-B(k^2)}{q^2-k^2}\Bigg]$$

DCSB enhanced spin-orbit interaction

DCSB enhanced anomalous magnetic moment


ARE WE THERE YET?

Unifying Study of Mesons and Baryons

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- How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons?

Unifying Study of Mesons and Baryons

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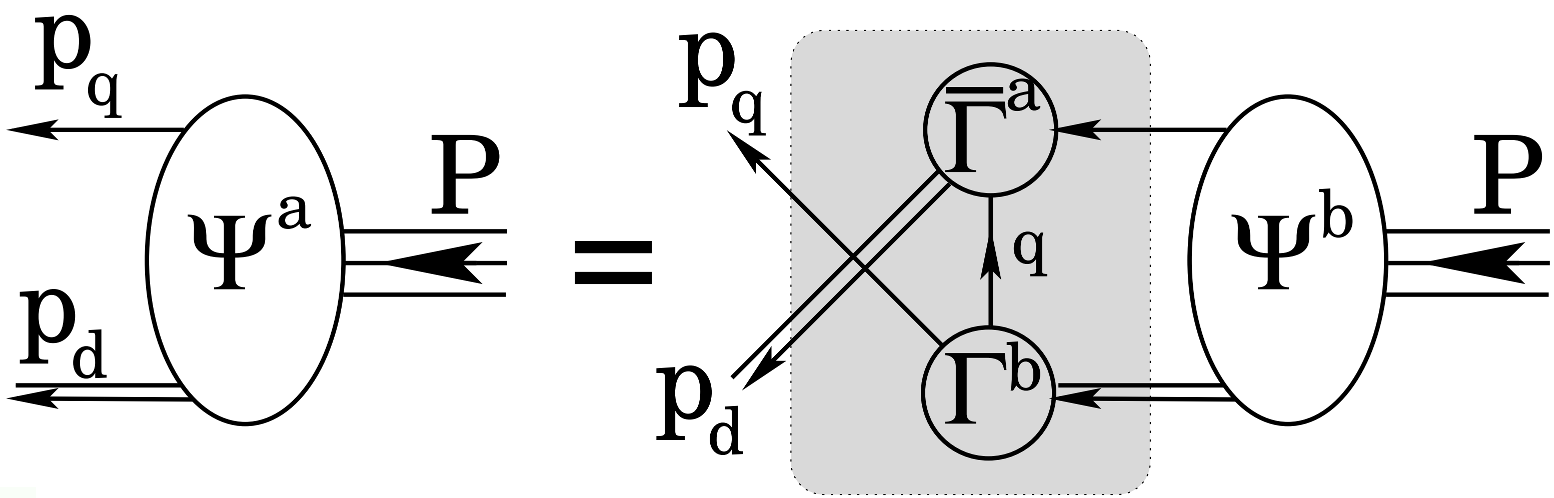
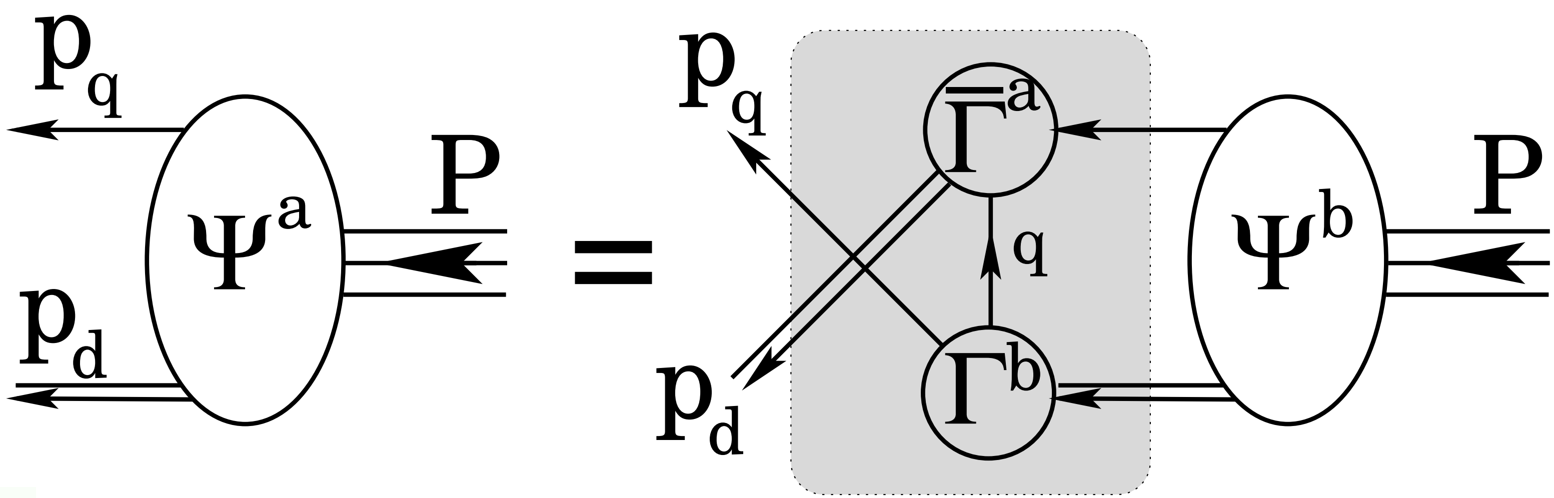
  - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
  - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour-$\bar{3}$ (antitriplet) channel

Faddeev equation

Faddeev equation

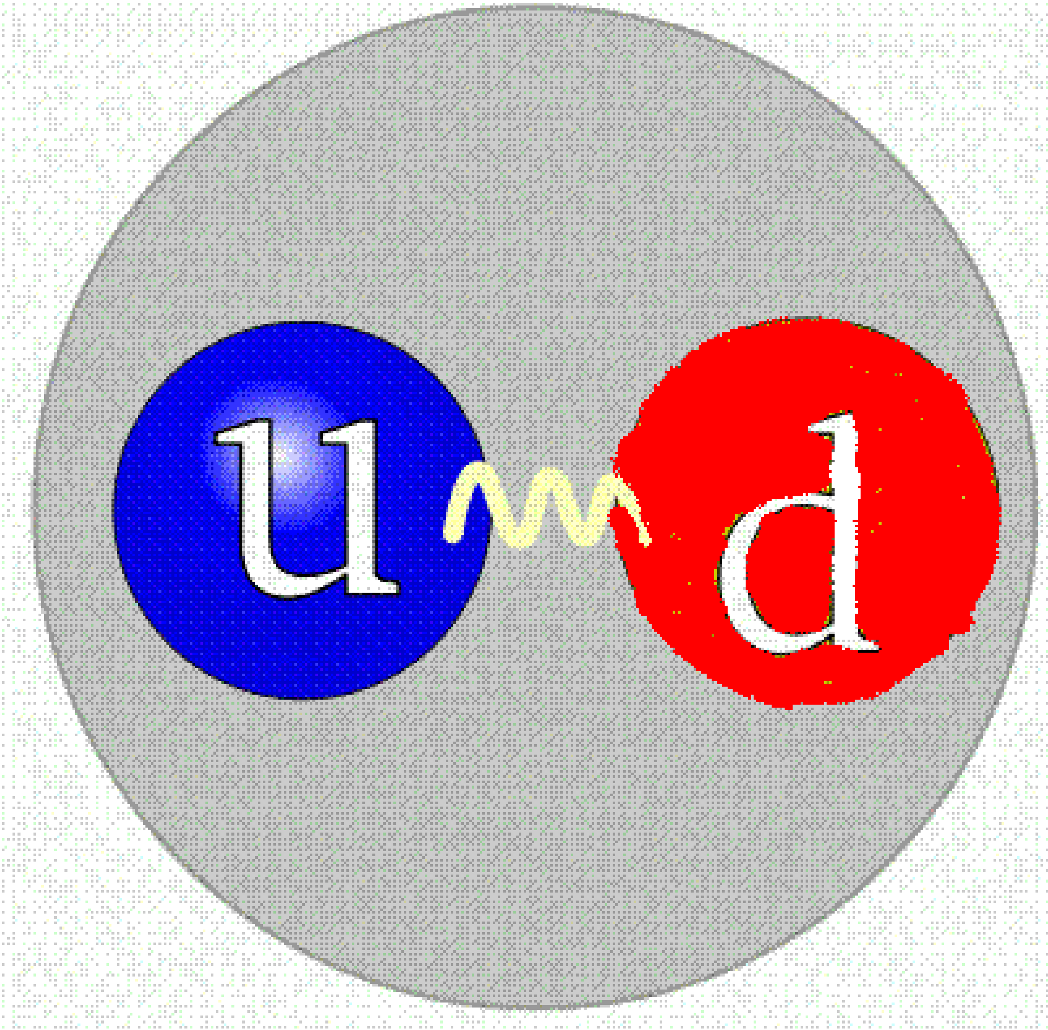


Faddeev equation



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Diquark correlations



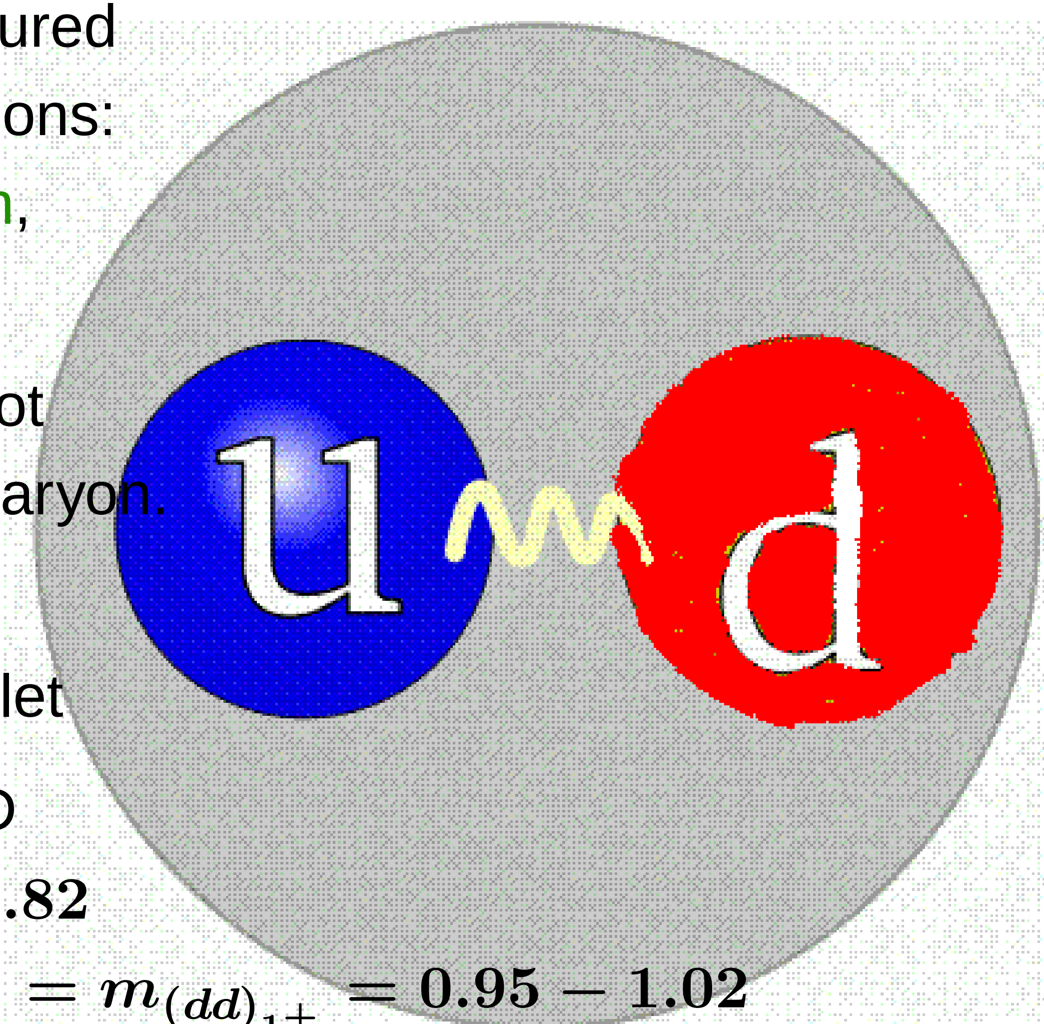
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined . . . Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

Nucleon-Photon Vertex

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M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

6 terms . . .

Nucleon-Photon Vertex

constructed systematically . . . current conserved automatically
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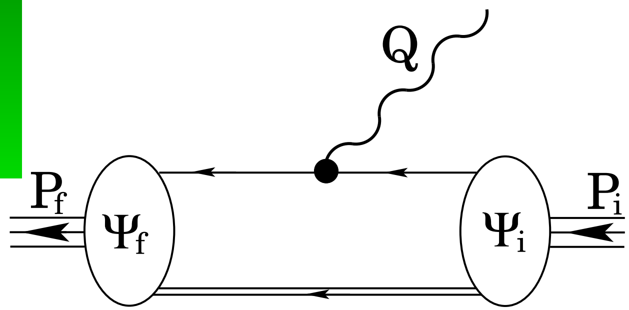

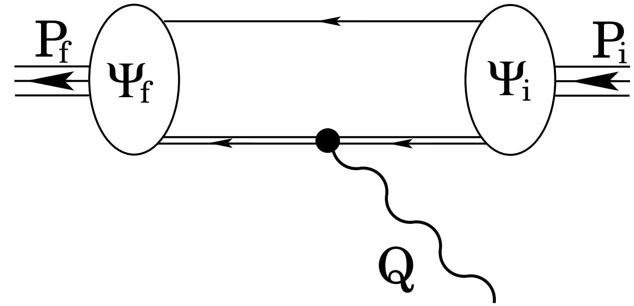

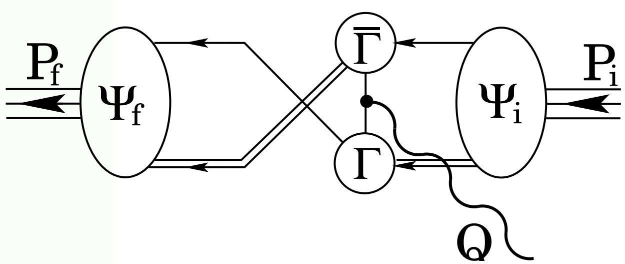

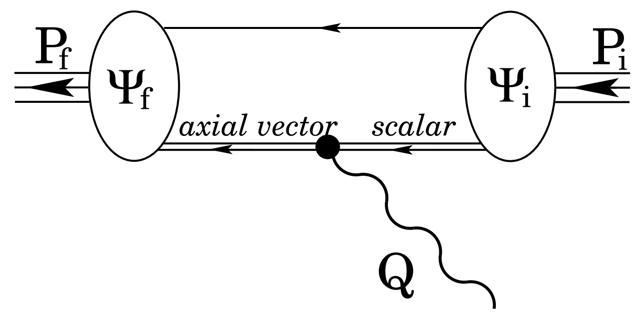

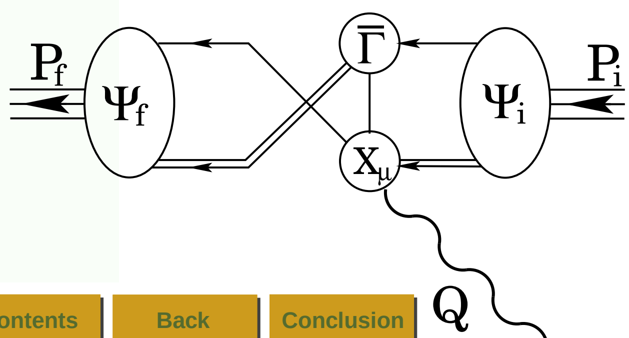

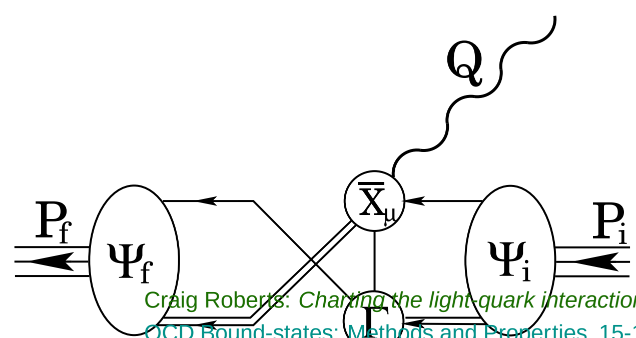

$$\frac{\mu_n G_E(Q^2)}{G_M(Q^2)}$$

Cloët *et al.*

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– arXiv:0710.5746 [nucl-th]

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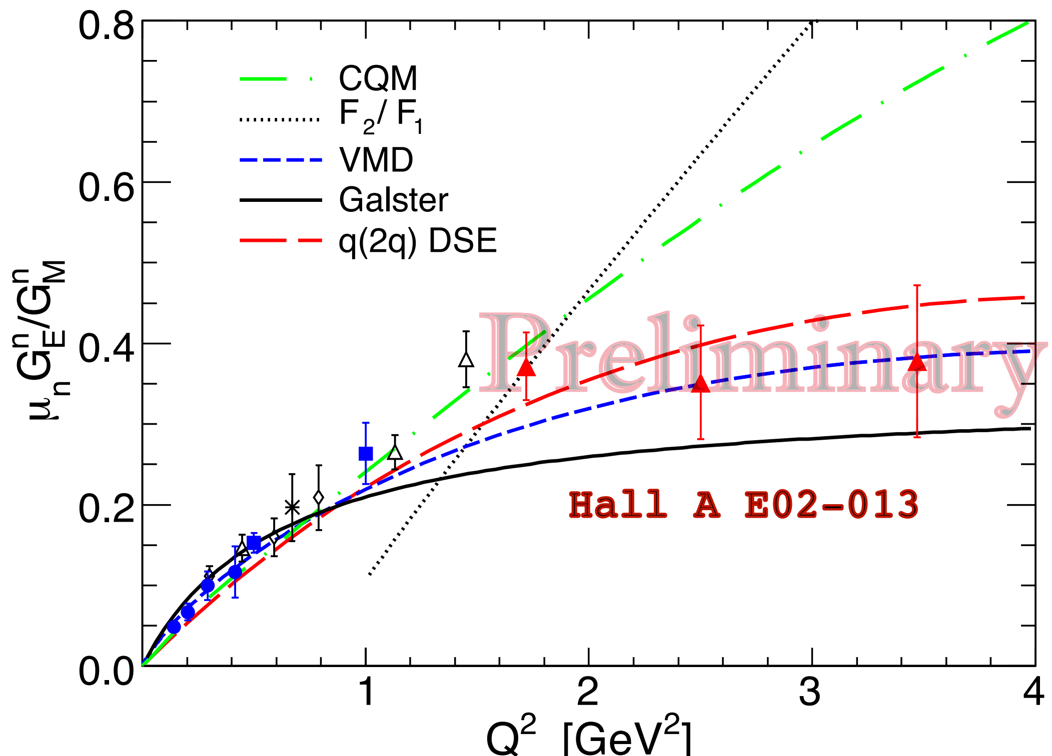

Jefferson Lab E02-013 Collaboration, *in preparation*.

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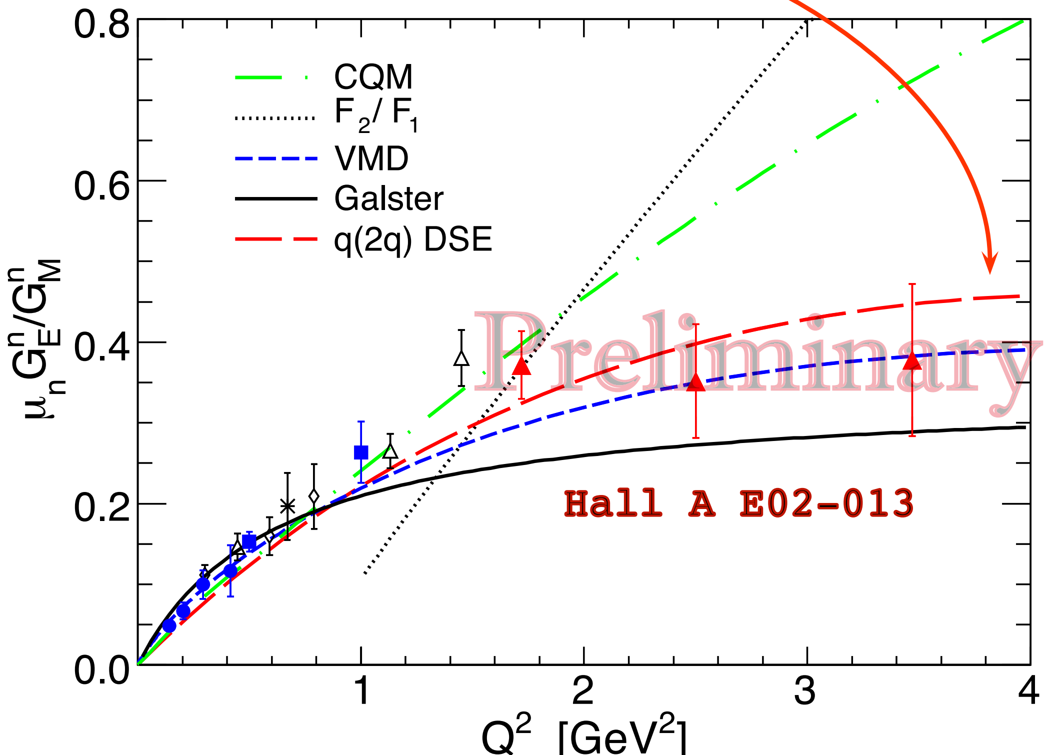
Red long-dashed curve



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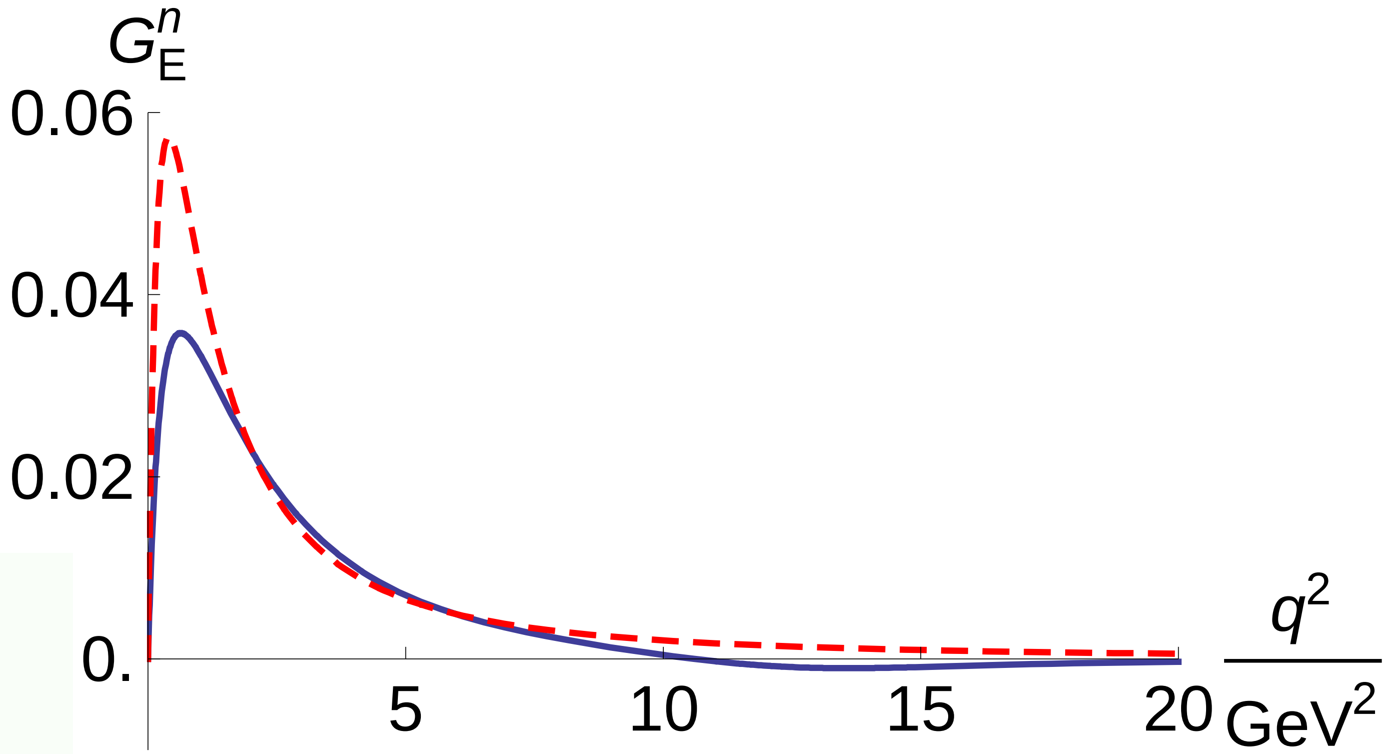
Neutron Charge Density



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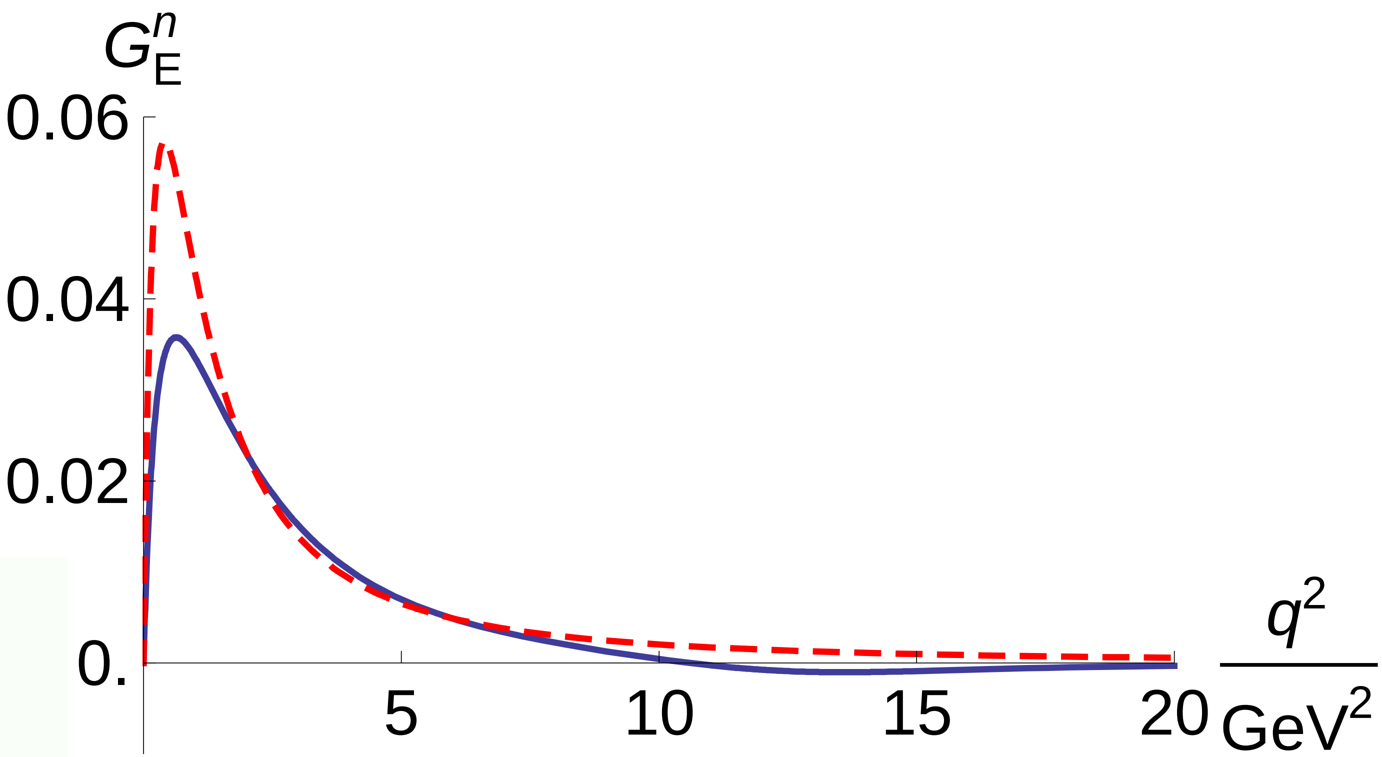

$$\rho_n(r) = \frac{1}{2\pi^2 r}\int_0^\infty dq\, q\, \sin(qr)\, G_E^n(q^2)$$

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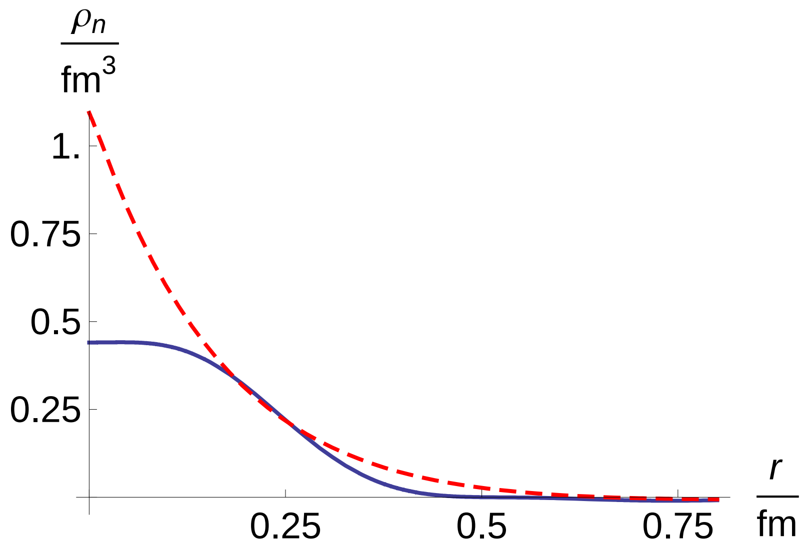

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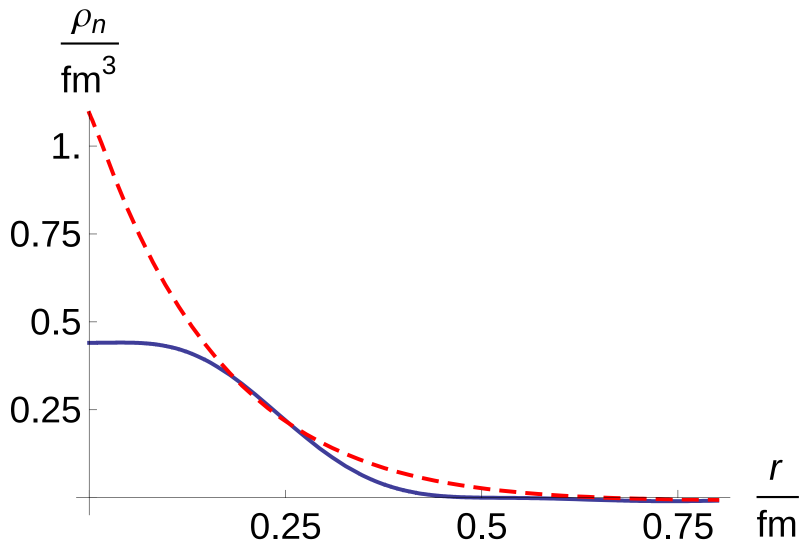

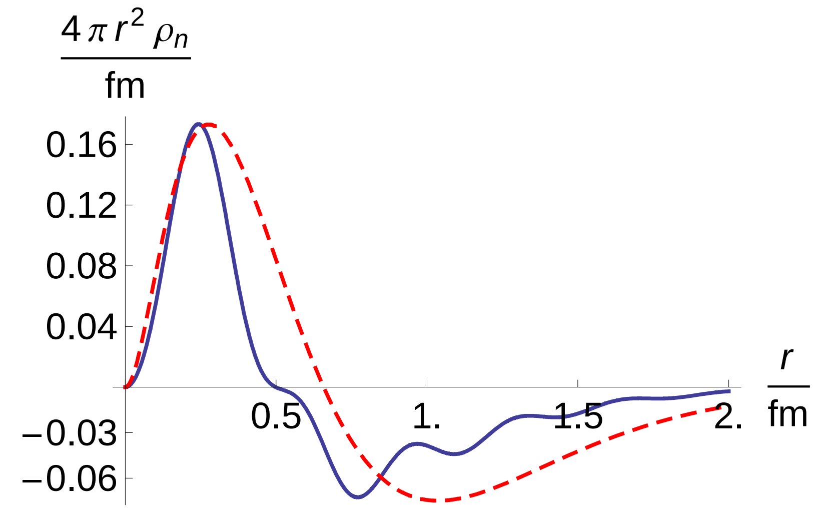

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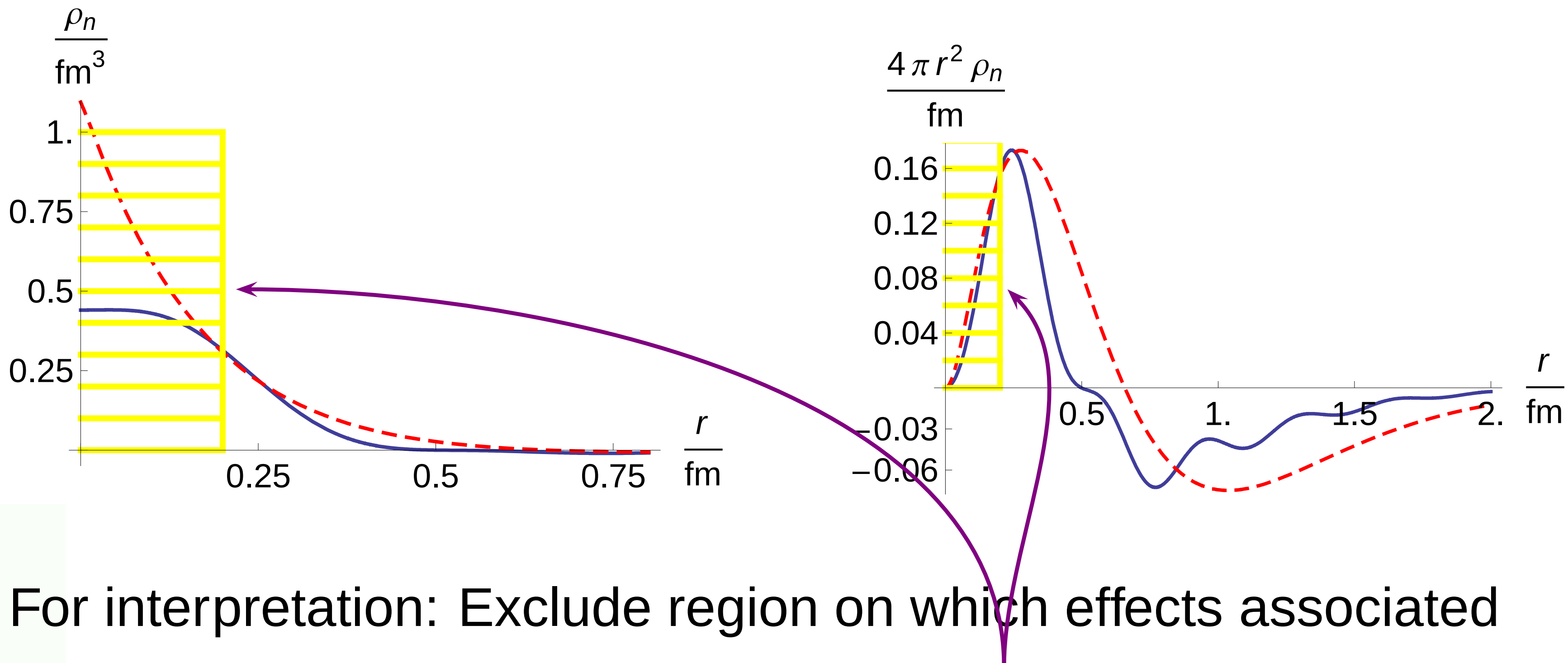

For interpretation: Exclude region on which effects associated with recoil of a DCSB dressed-quark are large

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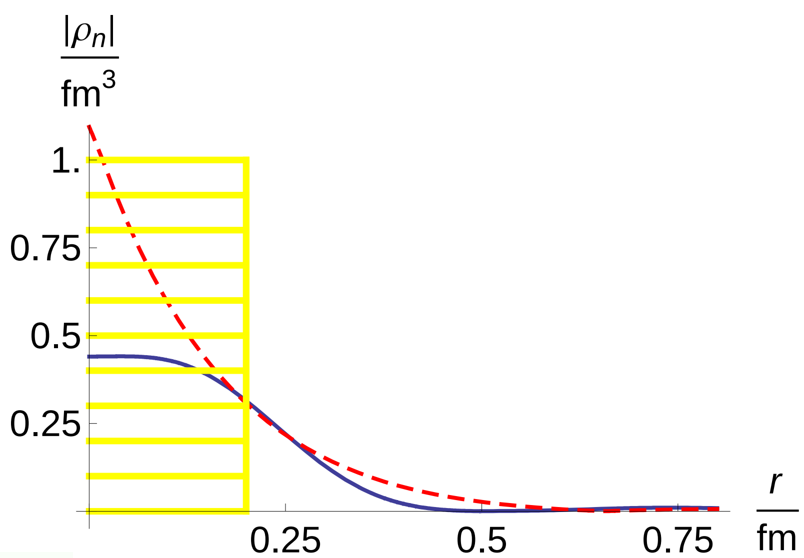
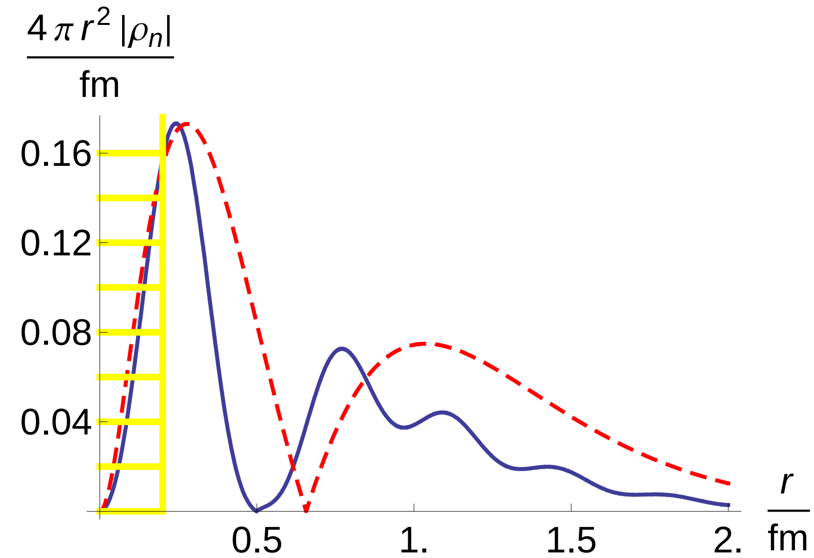
Neutron Charge Density



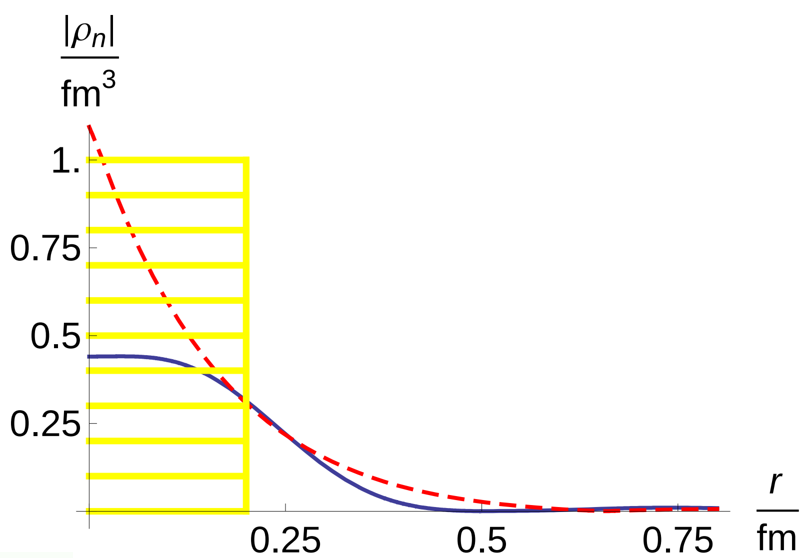


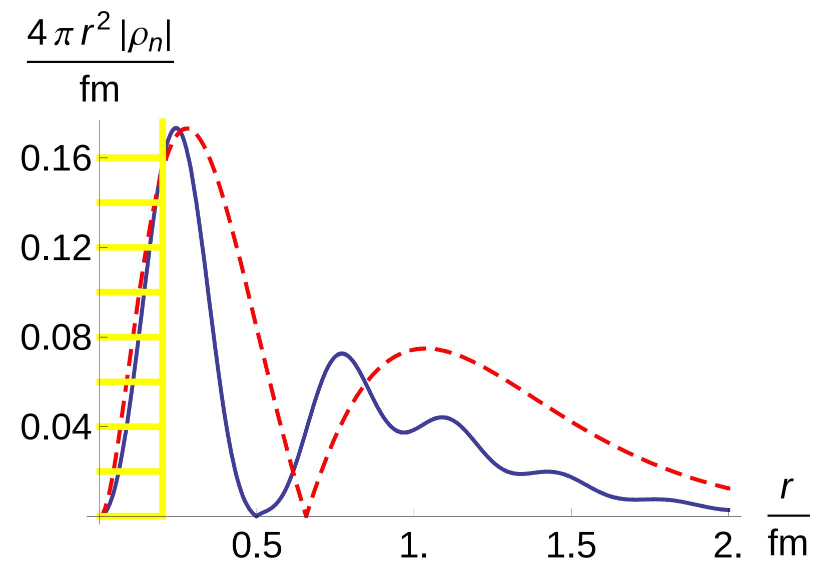
- Sharper localisation of charge
  - Interior – contraction of domain of positive charge
  - Exterior – greater concentration of negative charge toward surface

Cloët *et al.*

– arXiv:0812.0416 [nucl-th] – *Survey of nucleon EM form factors*

Neutron Charge Density





- Oscillations at $r \gtrsim 0.5$ fm – connected with shape of $G_E^n(q^2)$ on its domain of positive support; viz.,
  - $G_E^n(0) = 0$;
  - Location and magnitude of the maximum of $G_E^n(q^2)$
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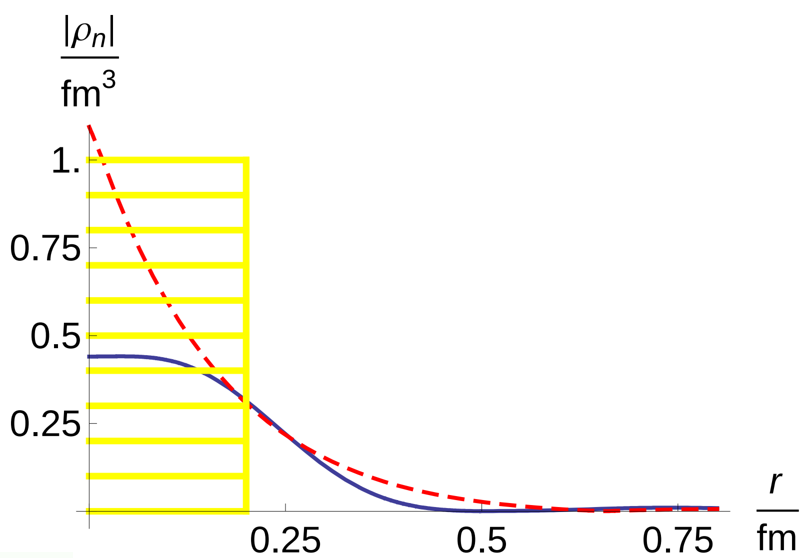

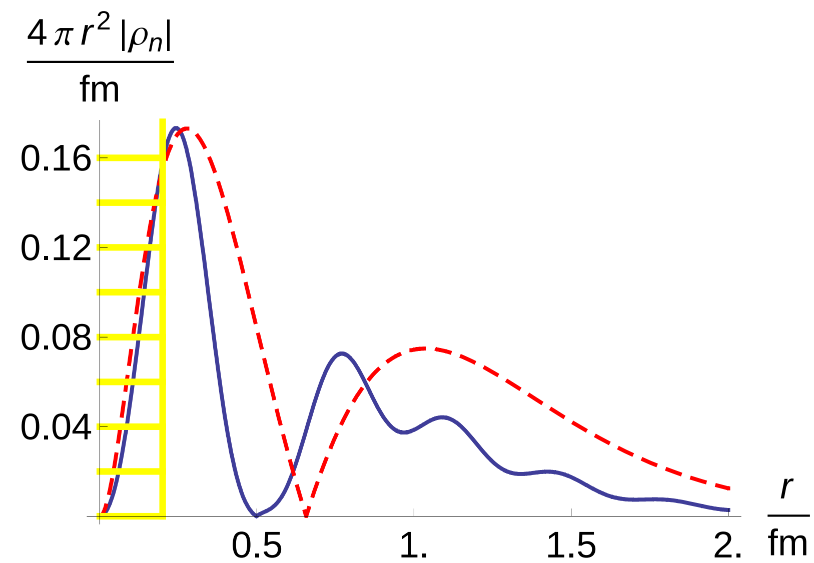

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  - Fact that the domain of positive support is bounded.
- Apparent in F&W parametrisation of data and in Hammer *et al.* dispersion relation fit

Pion Cloud

Pion Cloud
F2 – neutron

Pion Cloud
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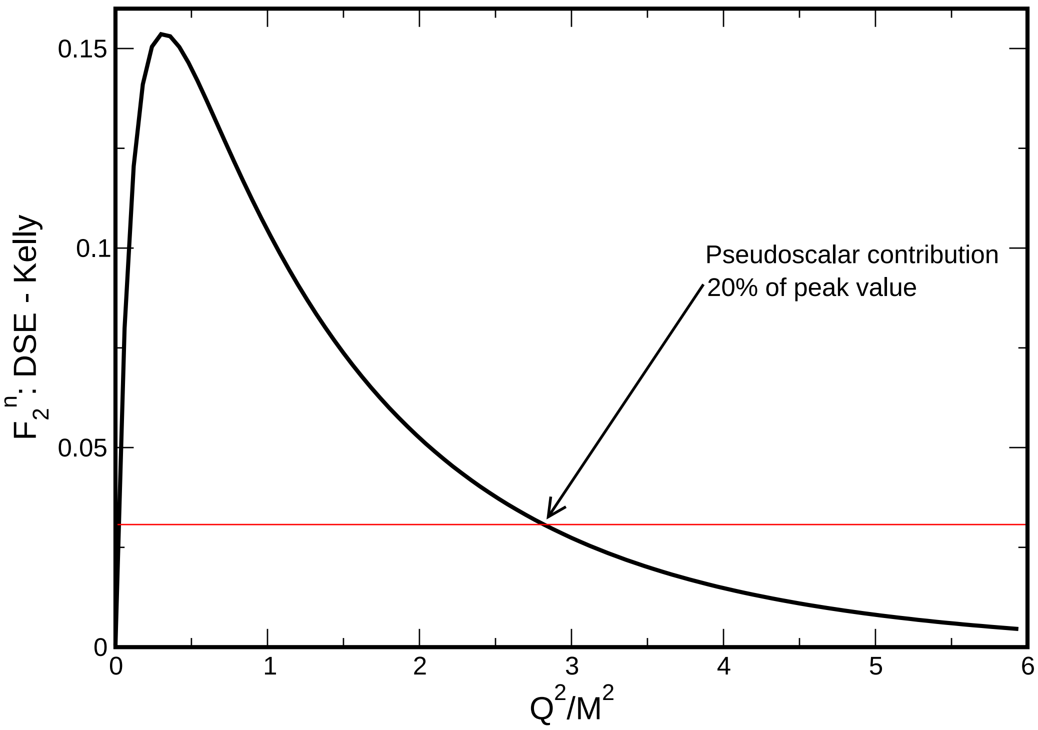
- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core



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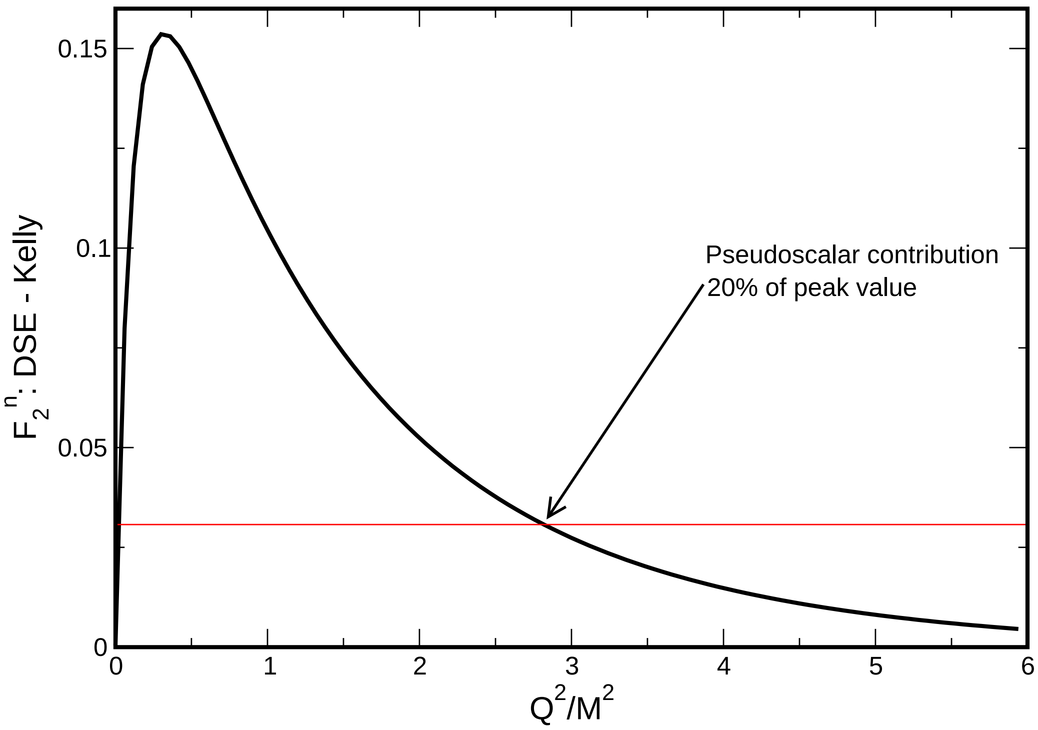

- Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4\, M_N^2$

Epilogue

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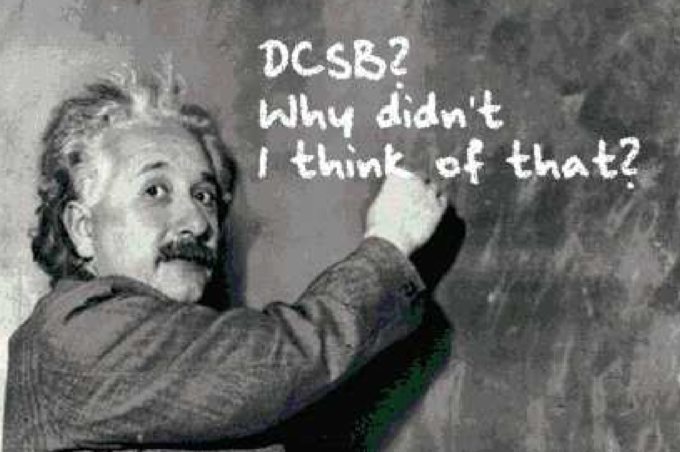
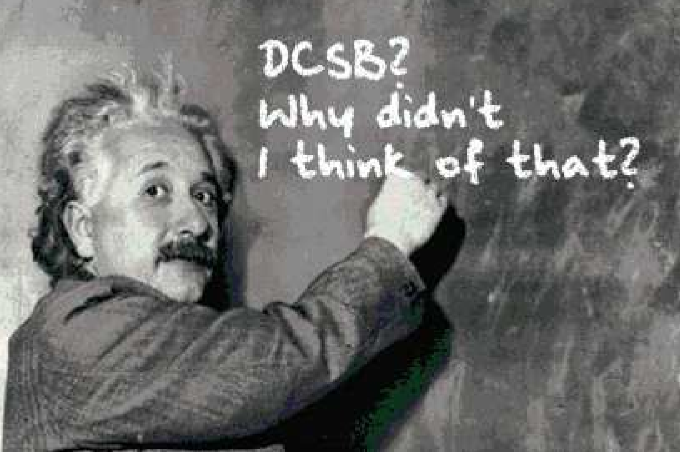


Epilogue

- DCSB exists in QCD.



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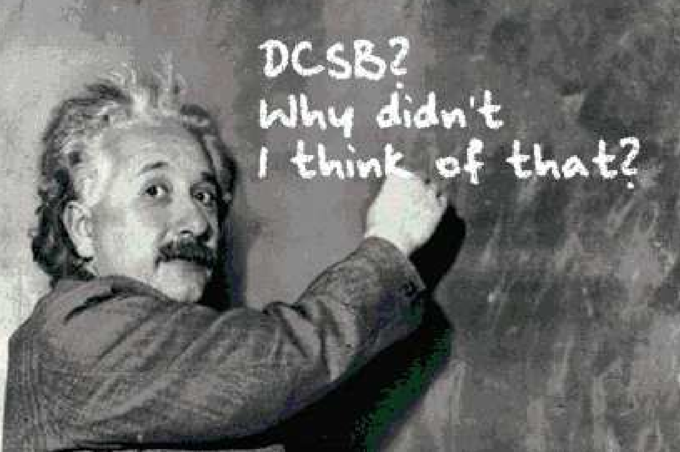

Epilogue

- DCSB exists in QCD.
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  - It predicts, amongst other things, that
    - light current-quarks become heavy constituent-quarks: $4 \to 400\,$MeV
    - pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140\,$MeV
    - pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi\bar{q}q} \approx 4.3$
    - pseudscalar mesons couple unnaturally strongly to the lightest baryons $g_{\pi\bar{N}N} \approx 12.8 \approx 3 g_{\pi\bar{q}q}$

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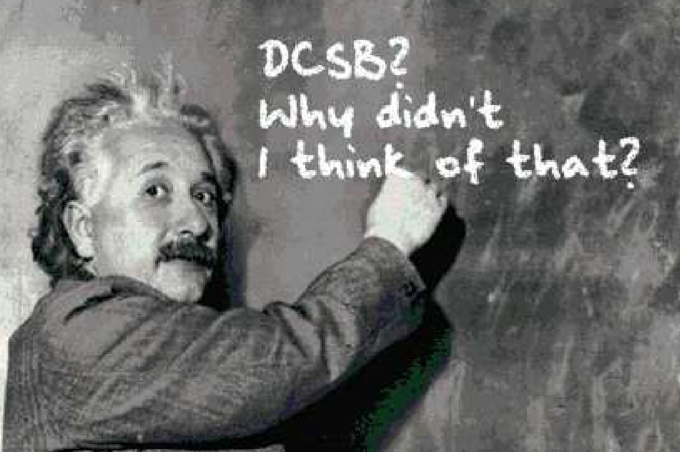

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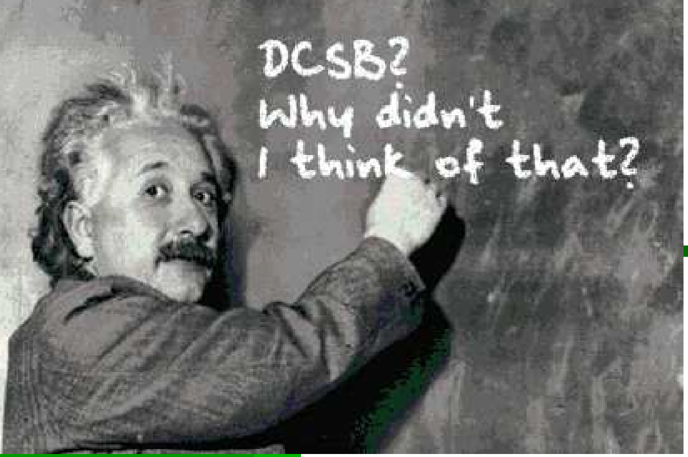
  - It impacts dramatically upon observables.



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- Dyson-Schwinger Equations
  - Poincaré covariant unification of meson and baryon observables

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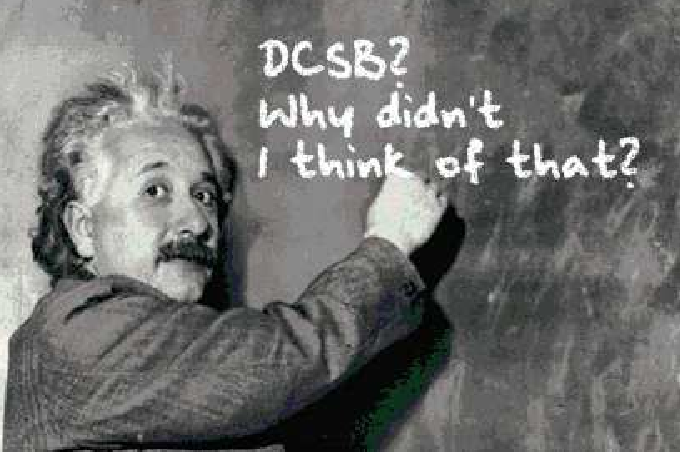

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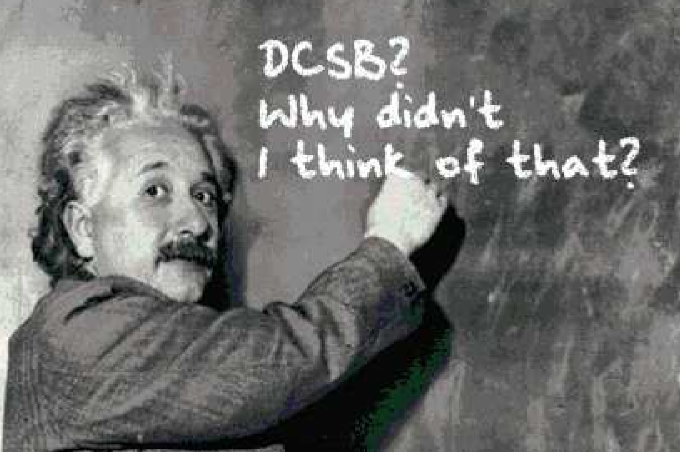

Epilogue

- Dyson-Schwinger Equations
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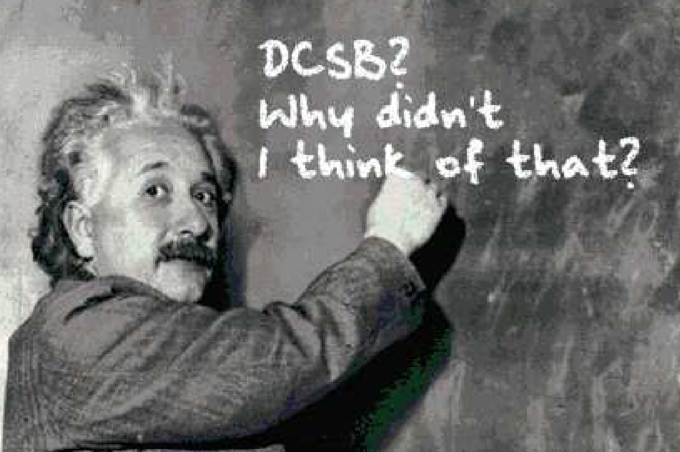
  - Solved a_1-ρ problem. Now positioned to provide first unified and reliable symmetry-preserving prediction of complete meson spectrum



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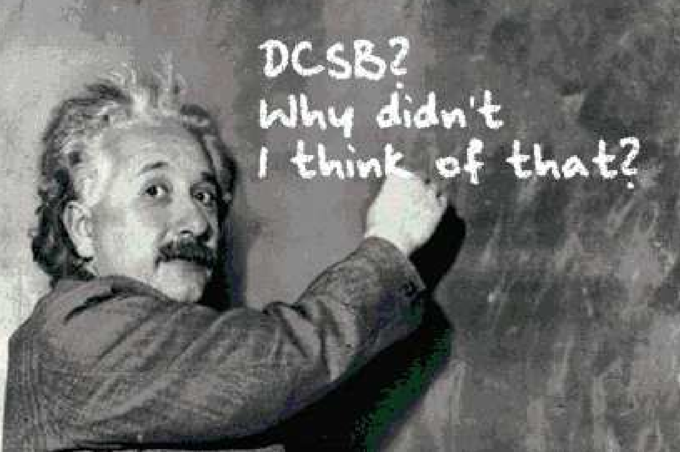

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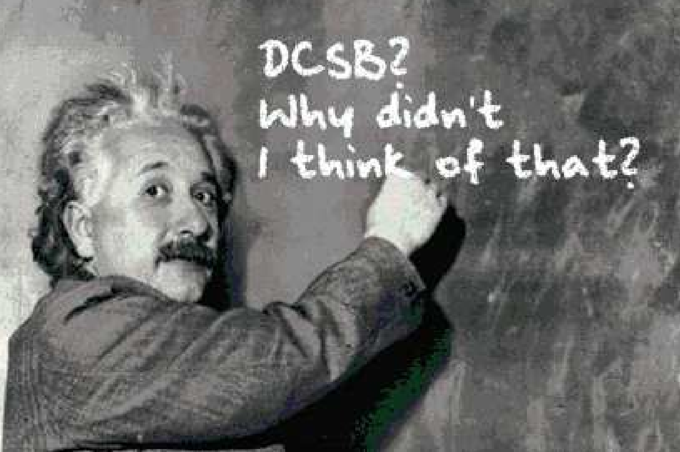

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- DSEs: Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks

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DSE-based Faddeev Equation

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Cloët *et al.*

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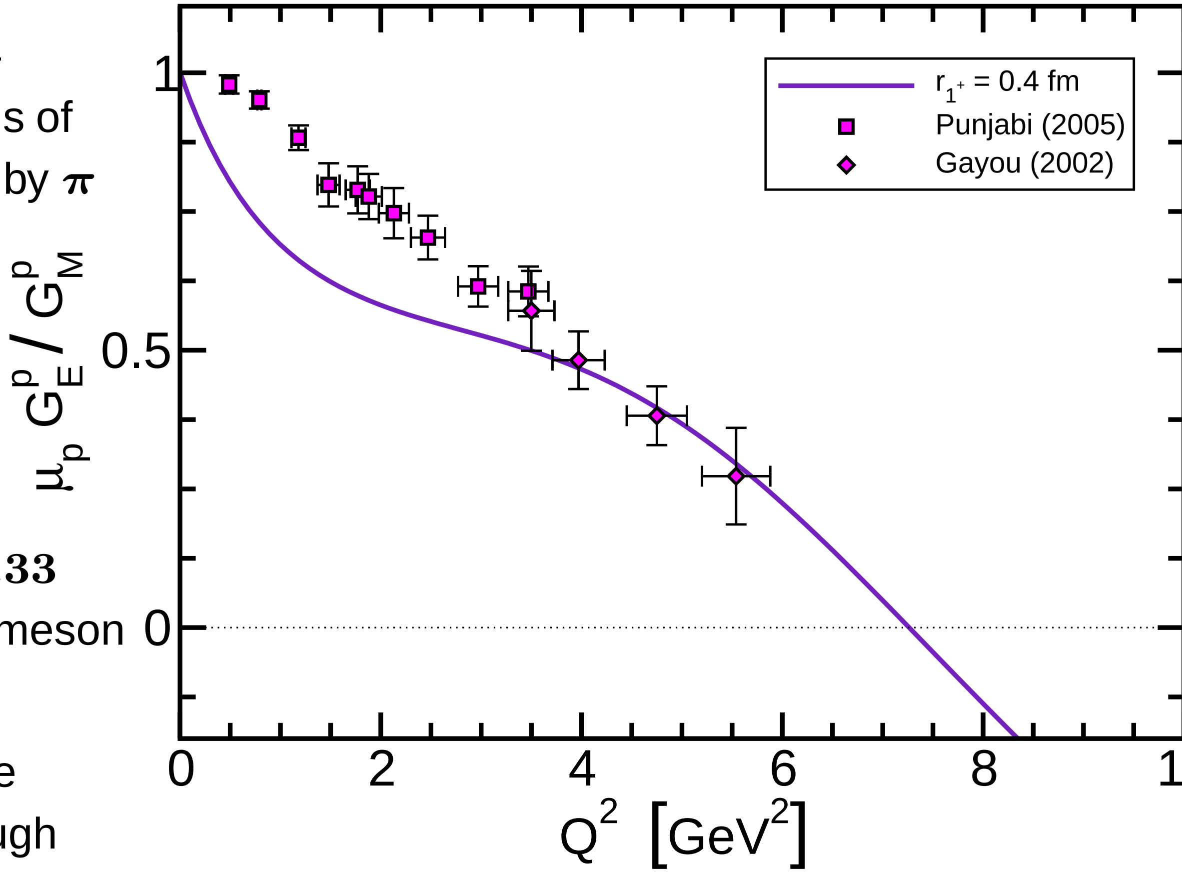

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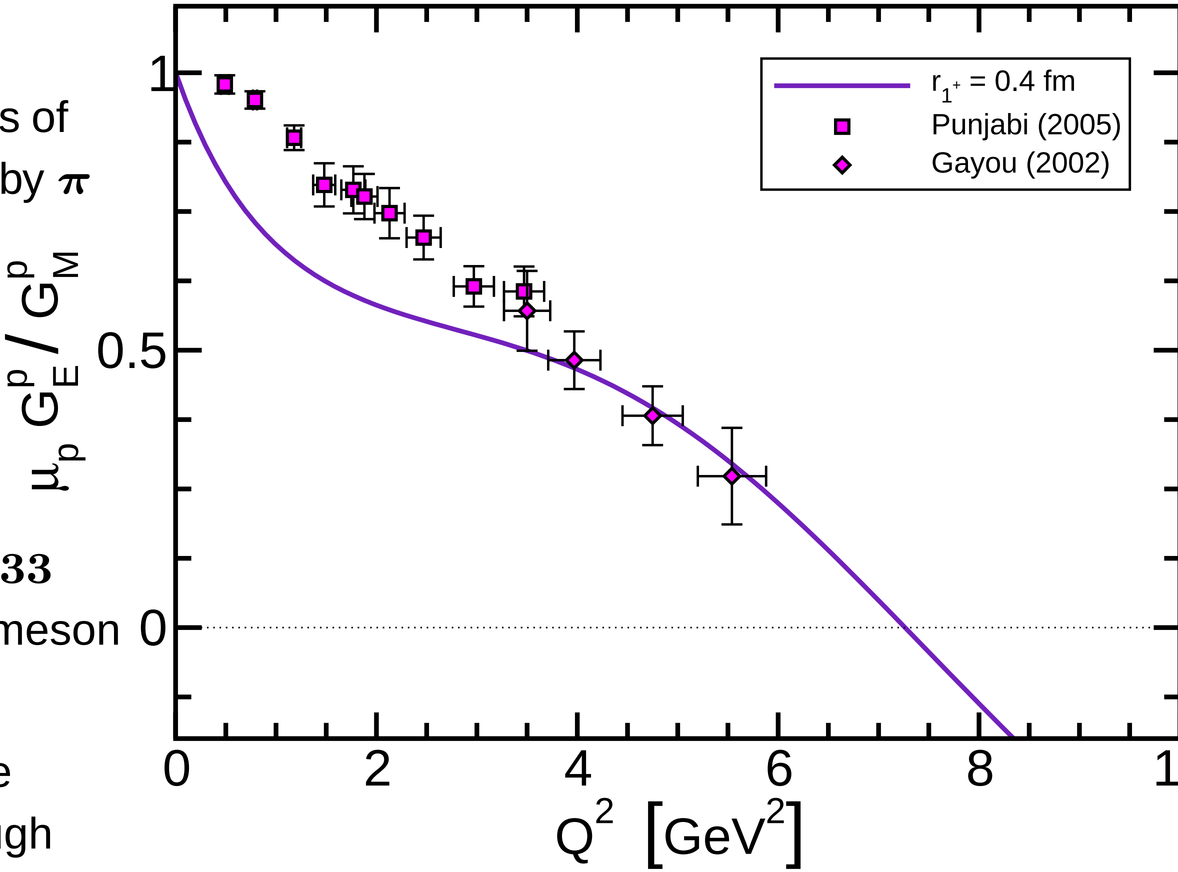

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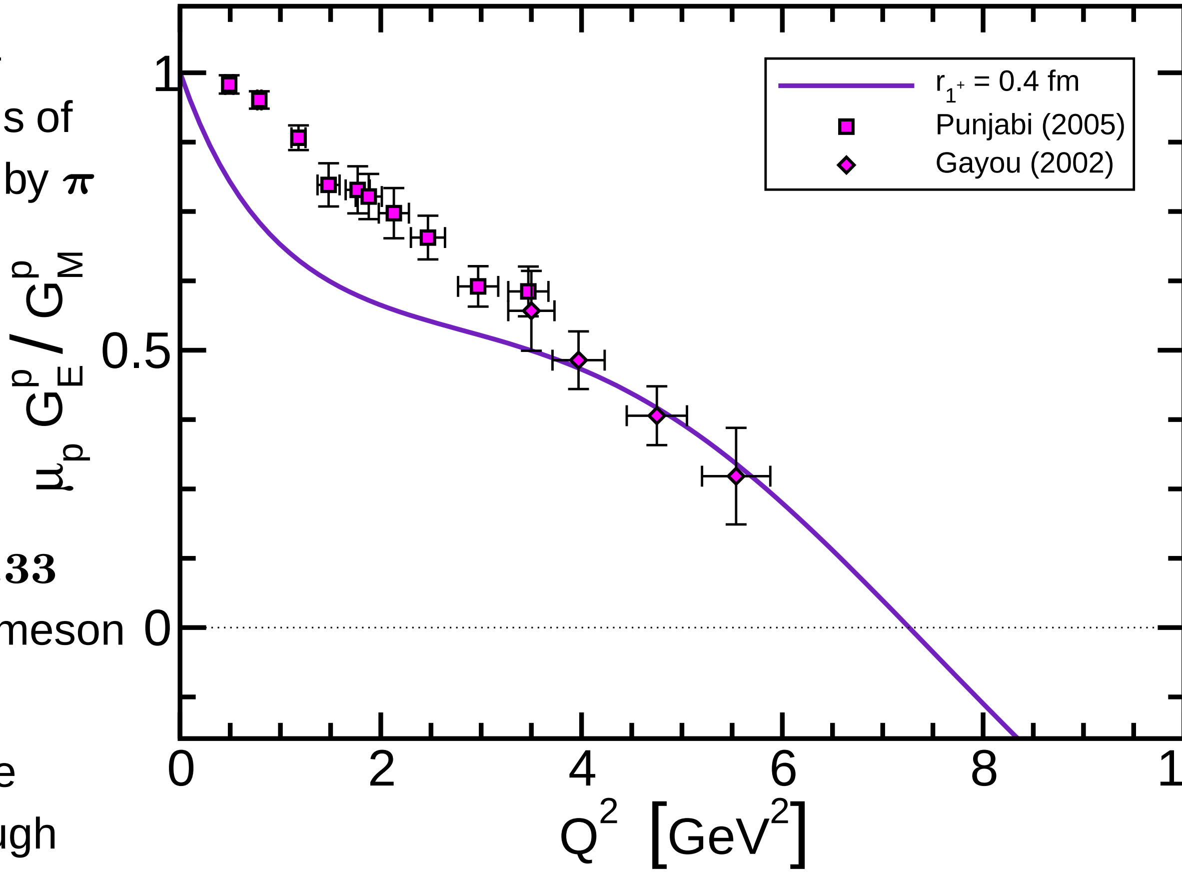

- On $Q^2 \lesssim 4$ GeV^2 result lies below experiment. This can be attributed to omission of pseudoscalar-meson-cloud contributions
- Always a zero but position depends on details of current

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Ratio of Neutron Pauli & Dirac Form Factors

- $$\frac{\hat{Q}^2}{(\ln \hat{Q}^2/\hat{\Lambda})^2} \frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)}$$

  $\hat{\Lambda} = \Lambda/M_N = 0.44$

  Ensures proton ratio constant for $\hat{Q}^2 \geq 4$

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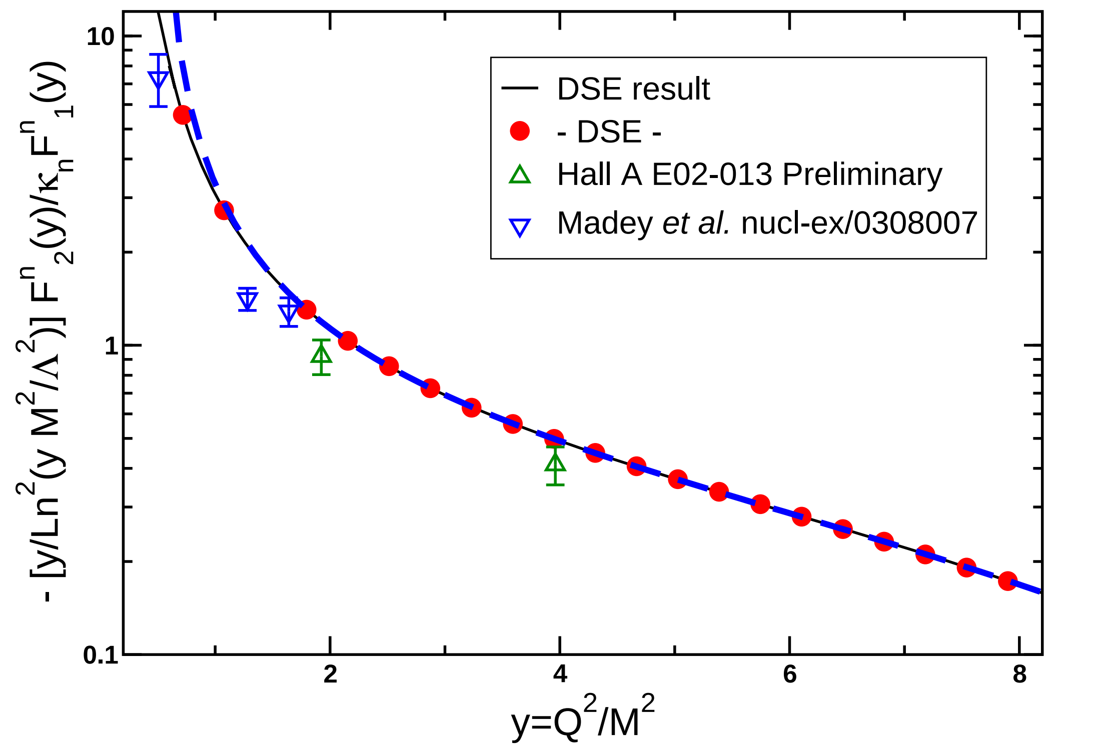

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