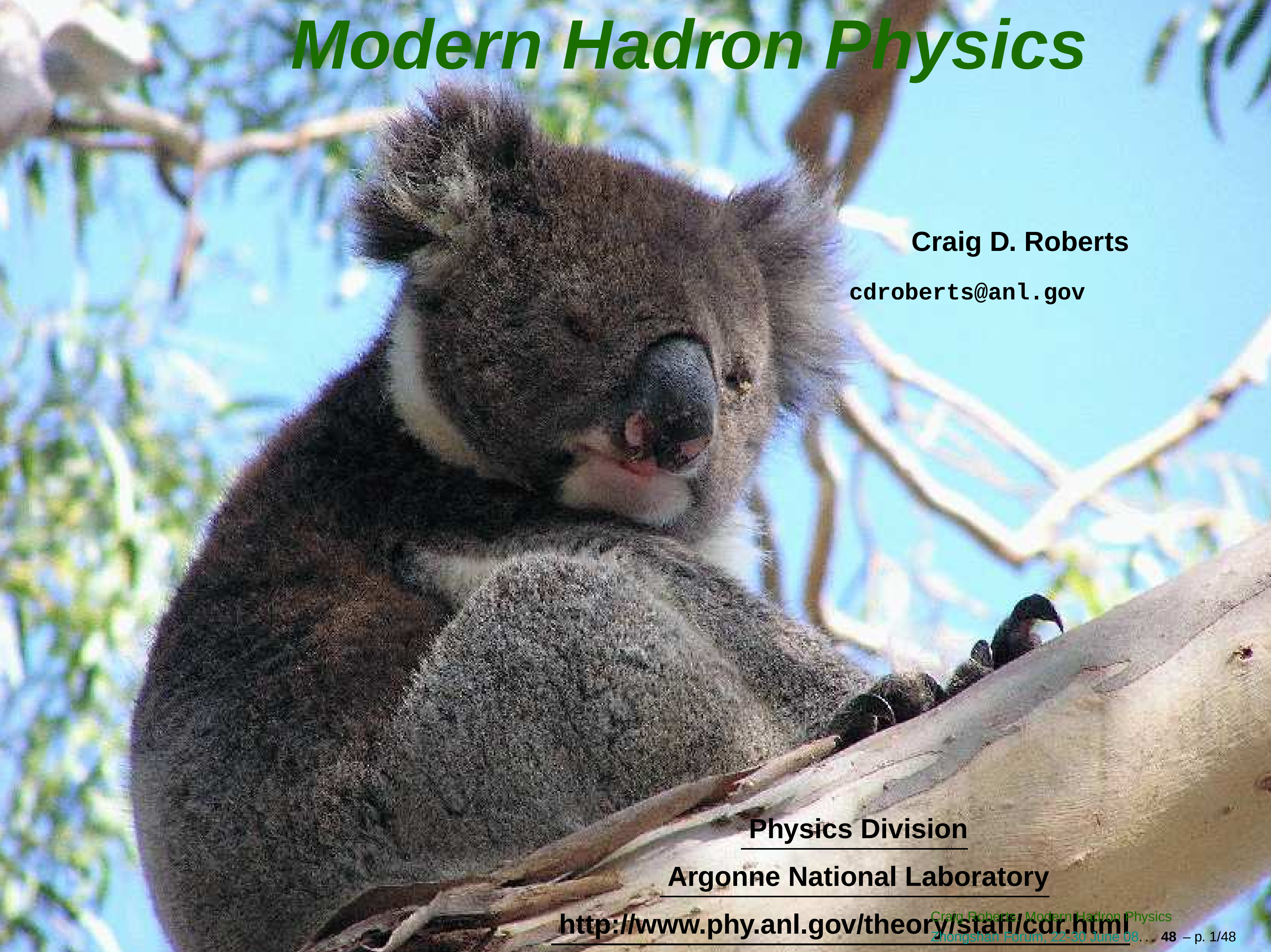


Modern Hadron Physics

Craig D. Roberts

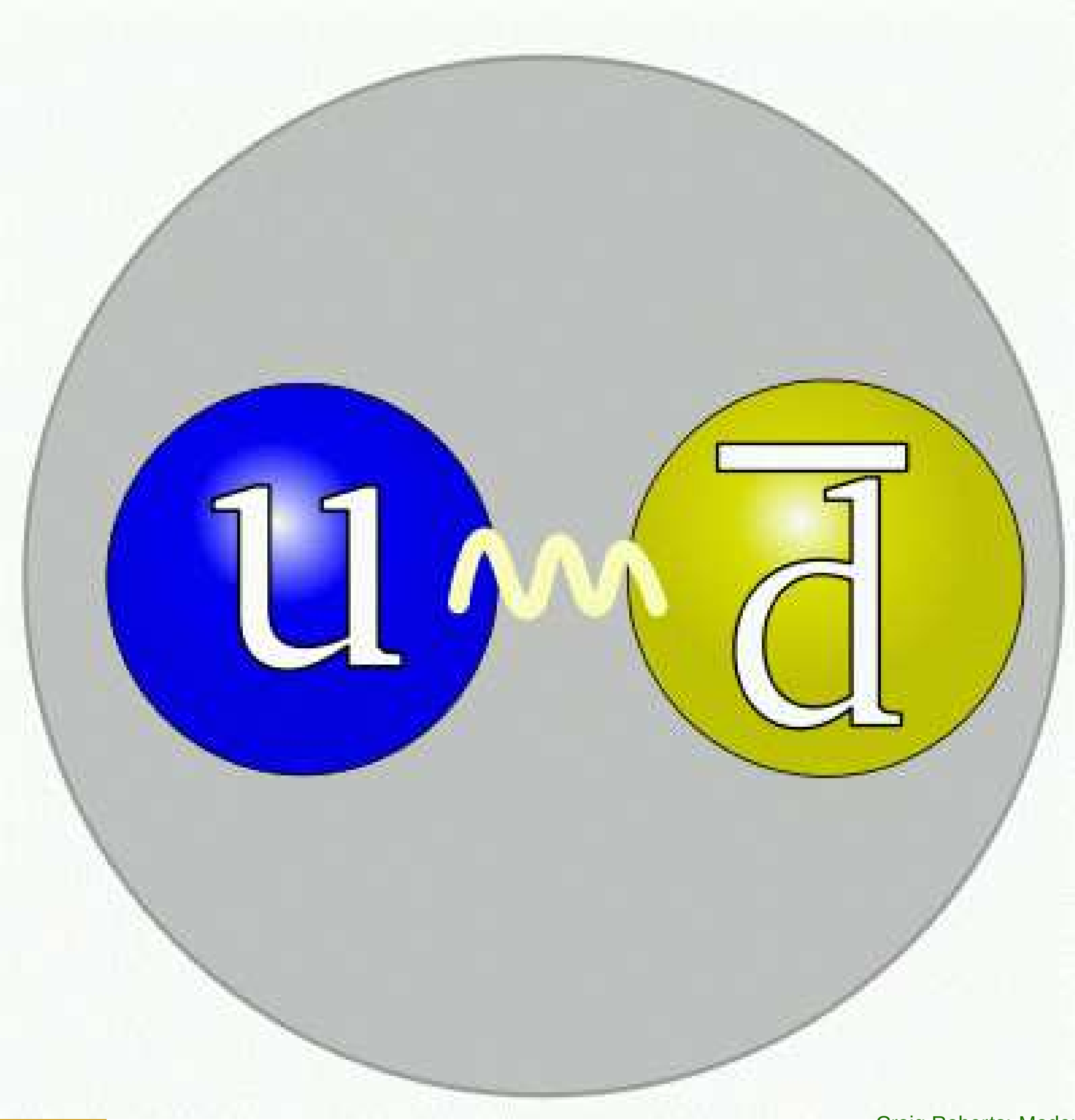
`cdroberts@anl.gov`

Physics Division

Argonne National Laboratory

http://www.phy.anl.gov/theory/staff/cdr.html

Answer for the pion



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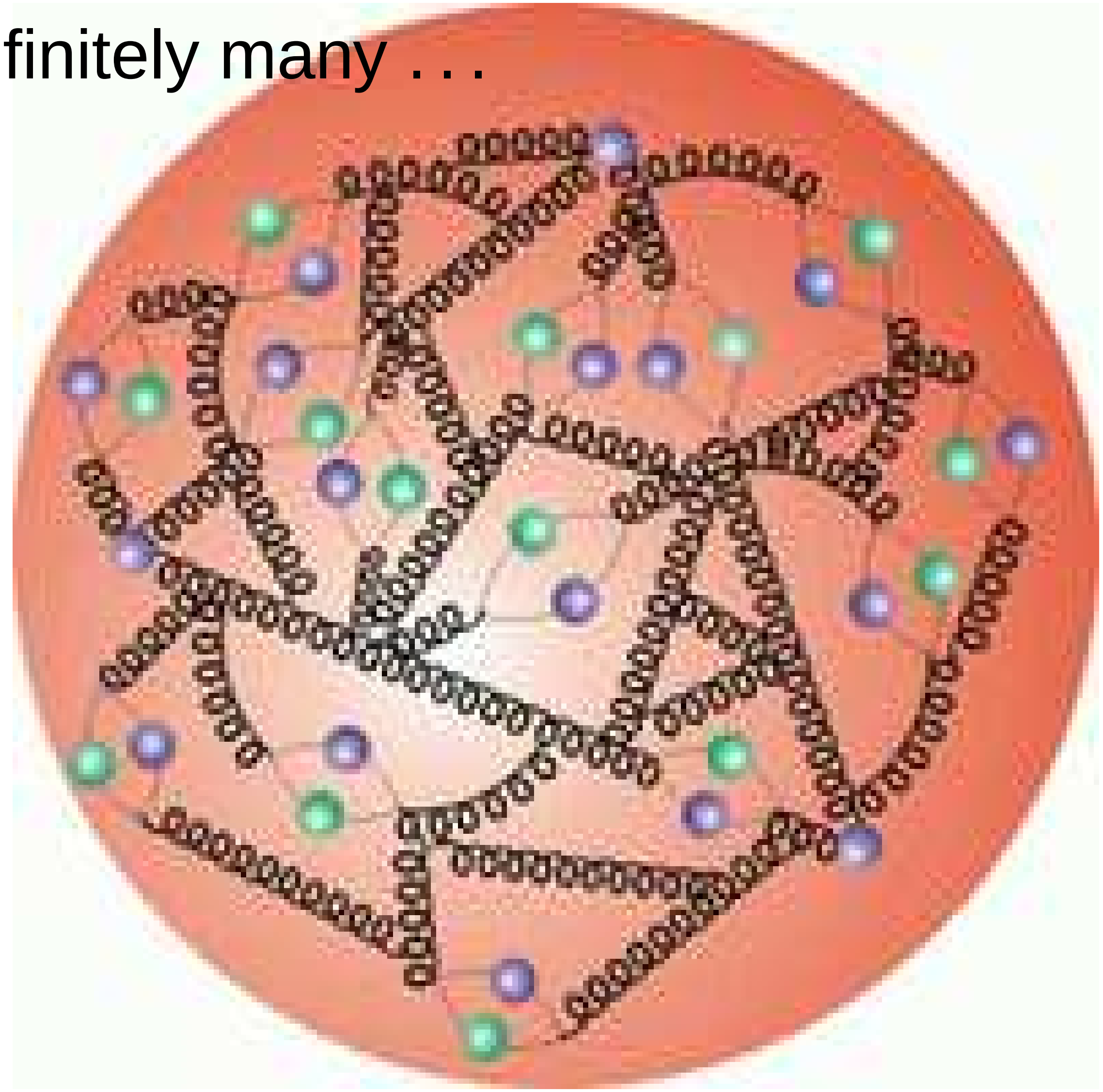
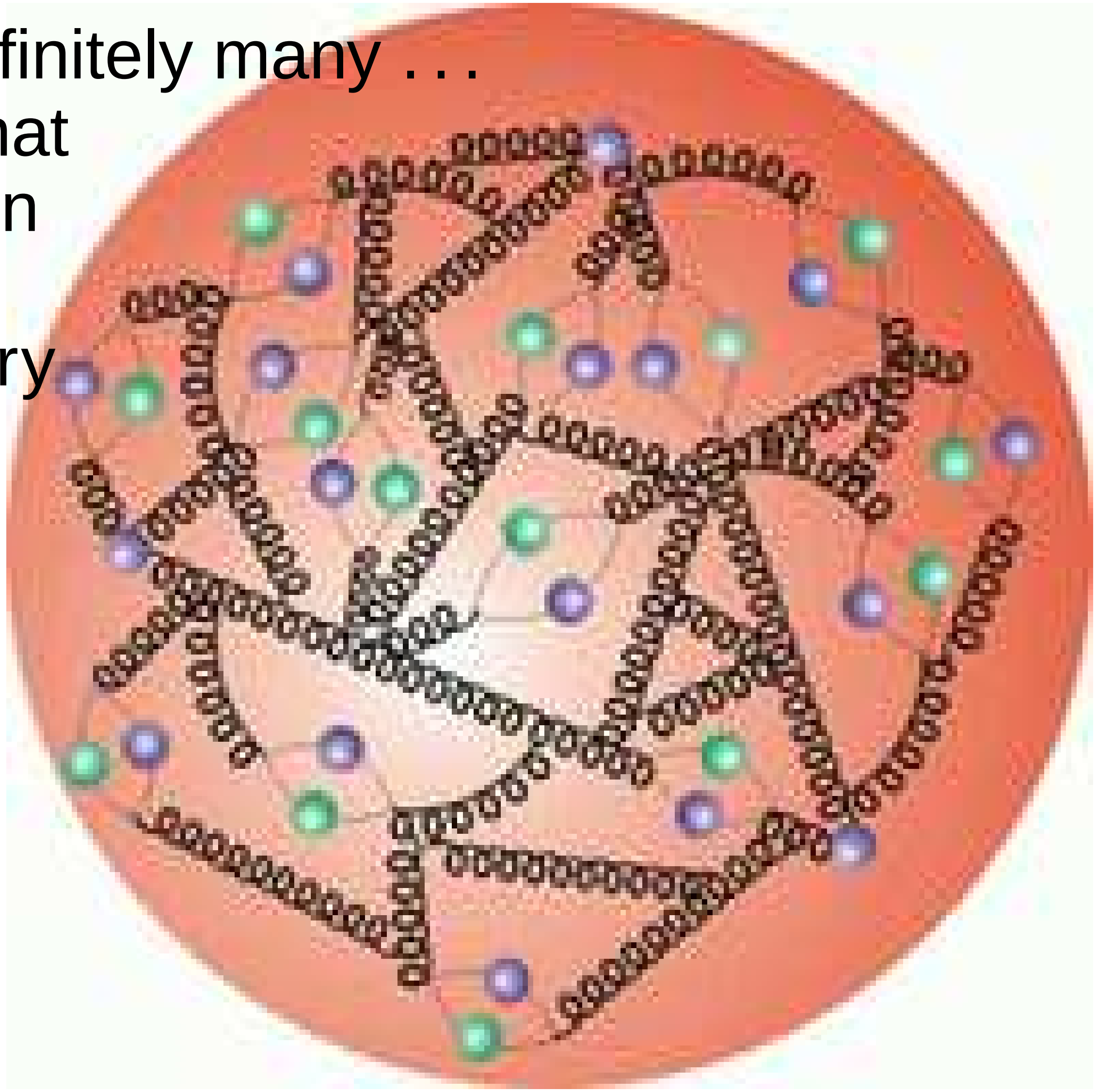
Answer for the pion

Two → Infinitely many . . .



Answer for the pion

Two → Infinitely many . . .
Handle that properly in quantum field theory



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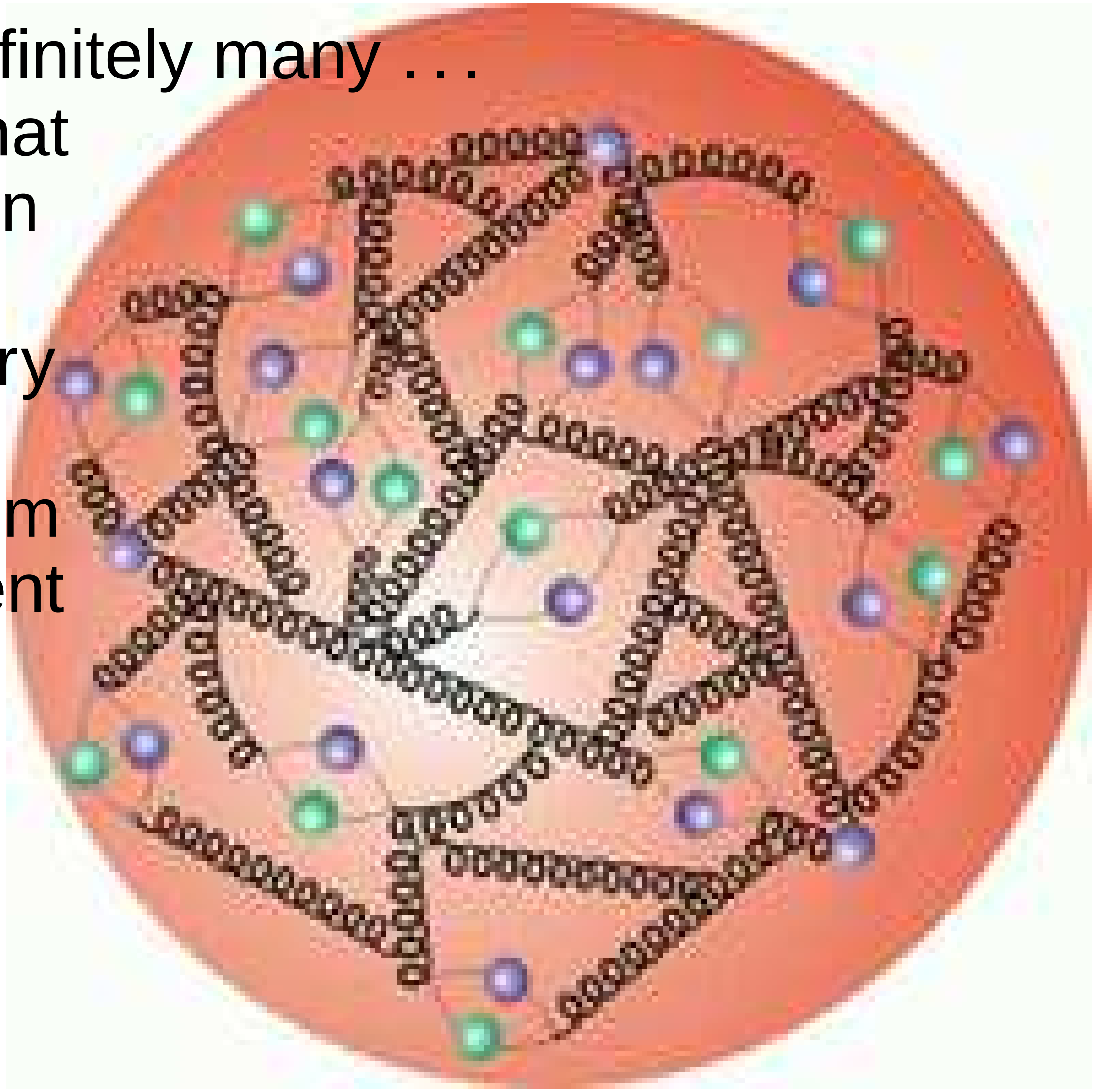
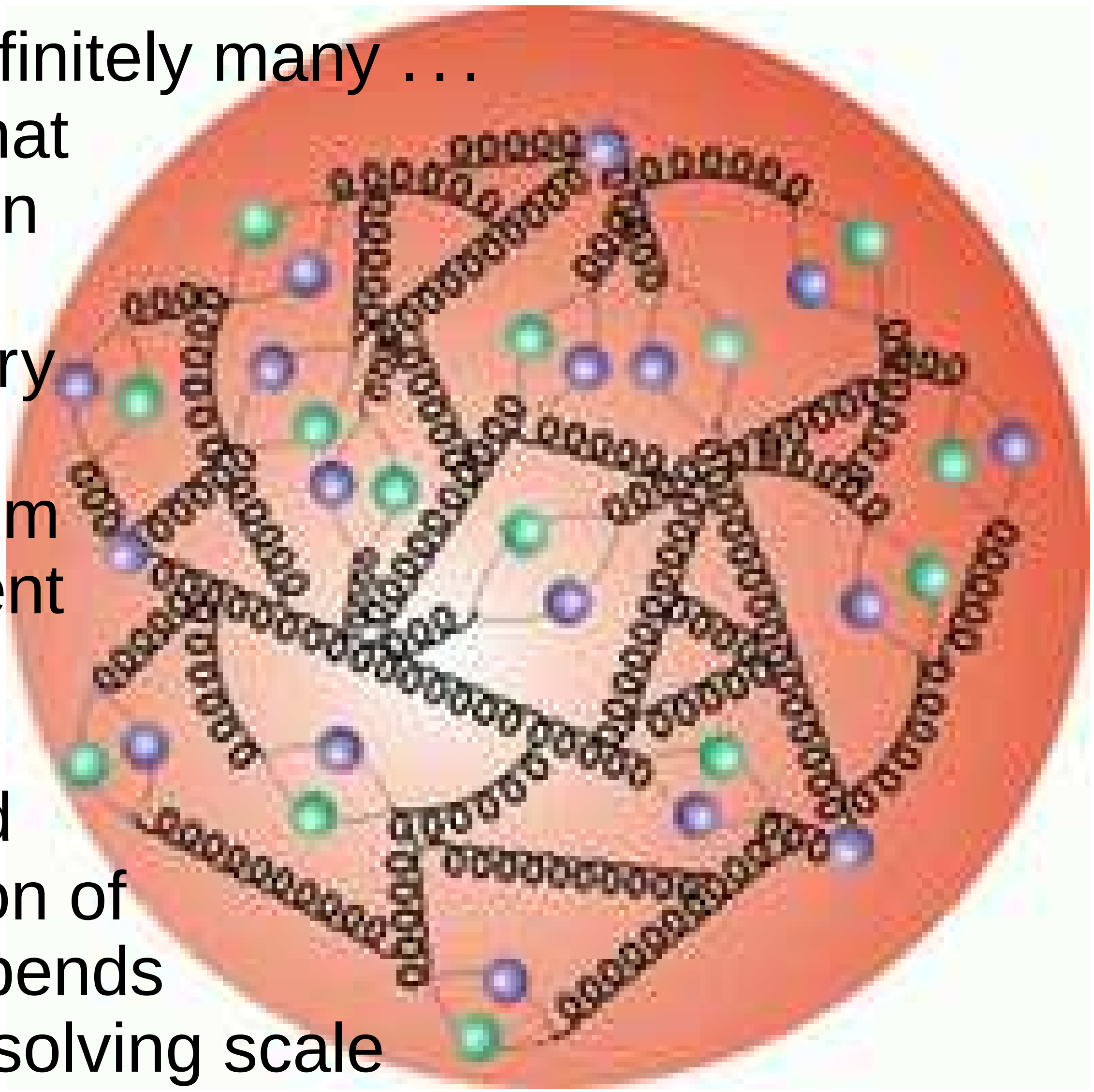
. . .
momentum -dependent dressing



Answer for the pion

Two $\rightarrow$ Infinitely many . . .
Handle that properly in quantum field theory
. . .
momentum-dependent dressing
. . .
perceived distribution of mass depends on the resolving scale

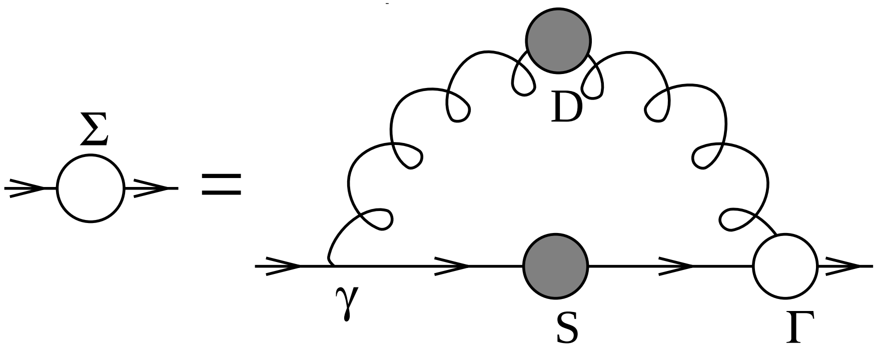


Procedure Now Straightforward

Pion Form Factor

- Solve Gap Equation

  $\Rightarrow$ Dressed-Quark Propagator, $S(p)$



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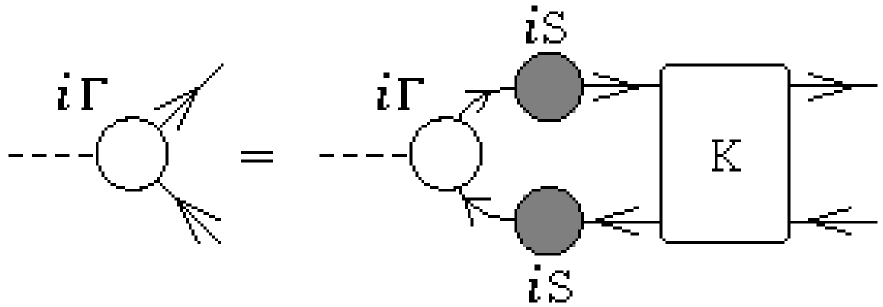
Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, K
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π



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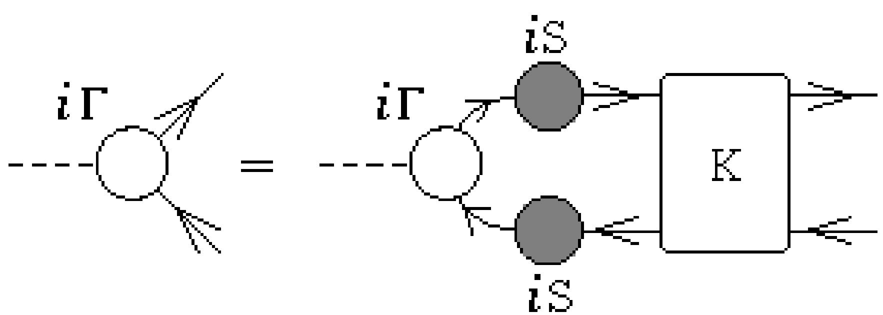

- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Gluon Vertex, Γ_μ

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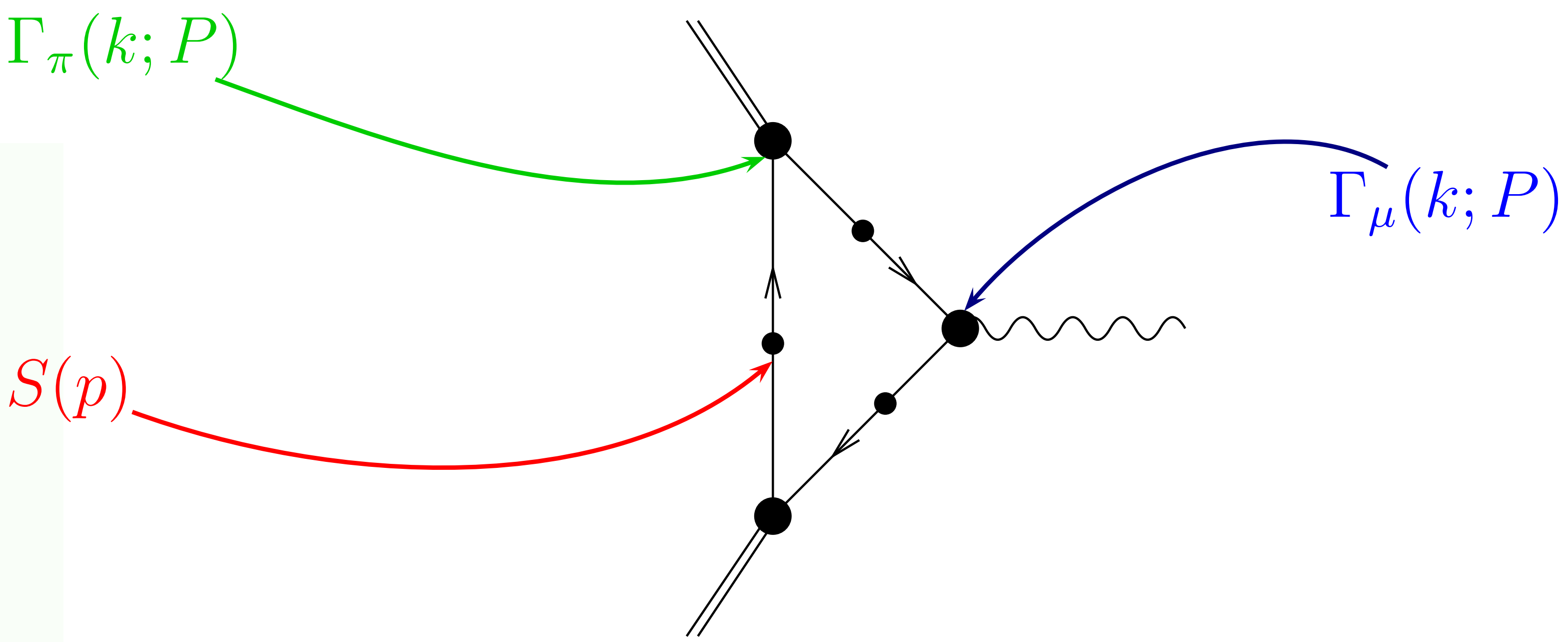
Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



Pion Form Factor

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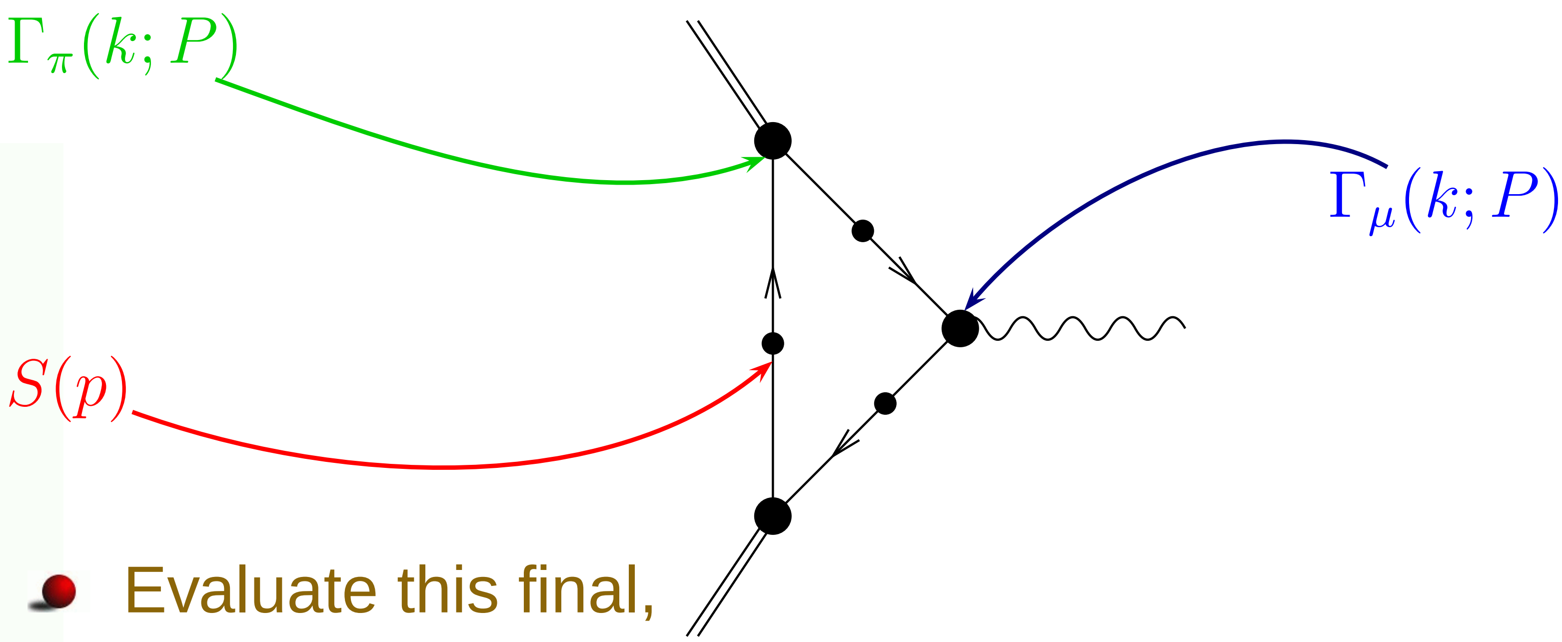
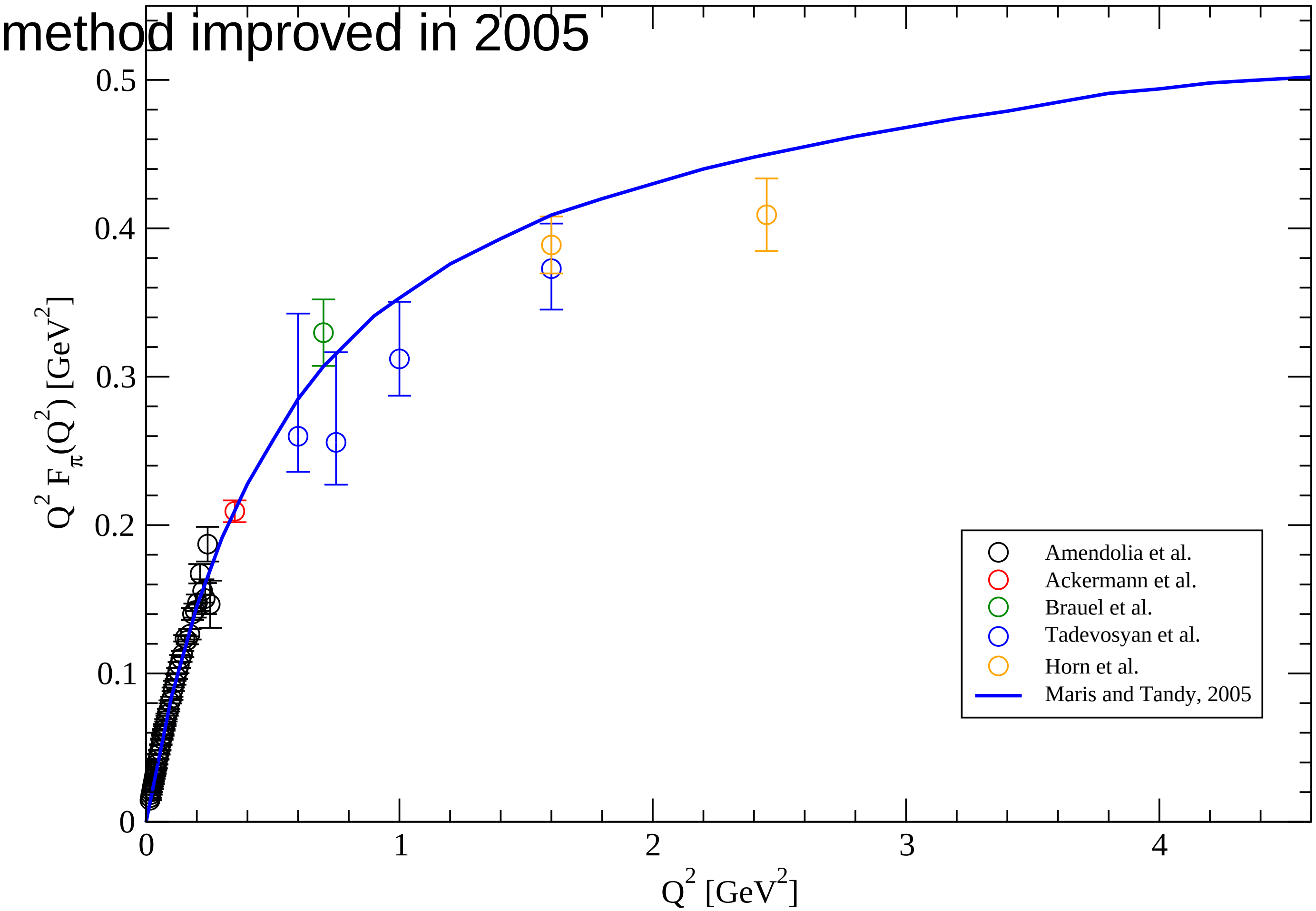


- Evaluate this final, three-dimensional integral

Calculated Pion Form Factor

Calculation first published in 1999; No Parameters Varied

Numerical method improved in 2005



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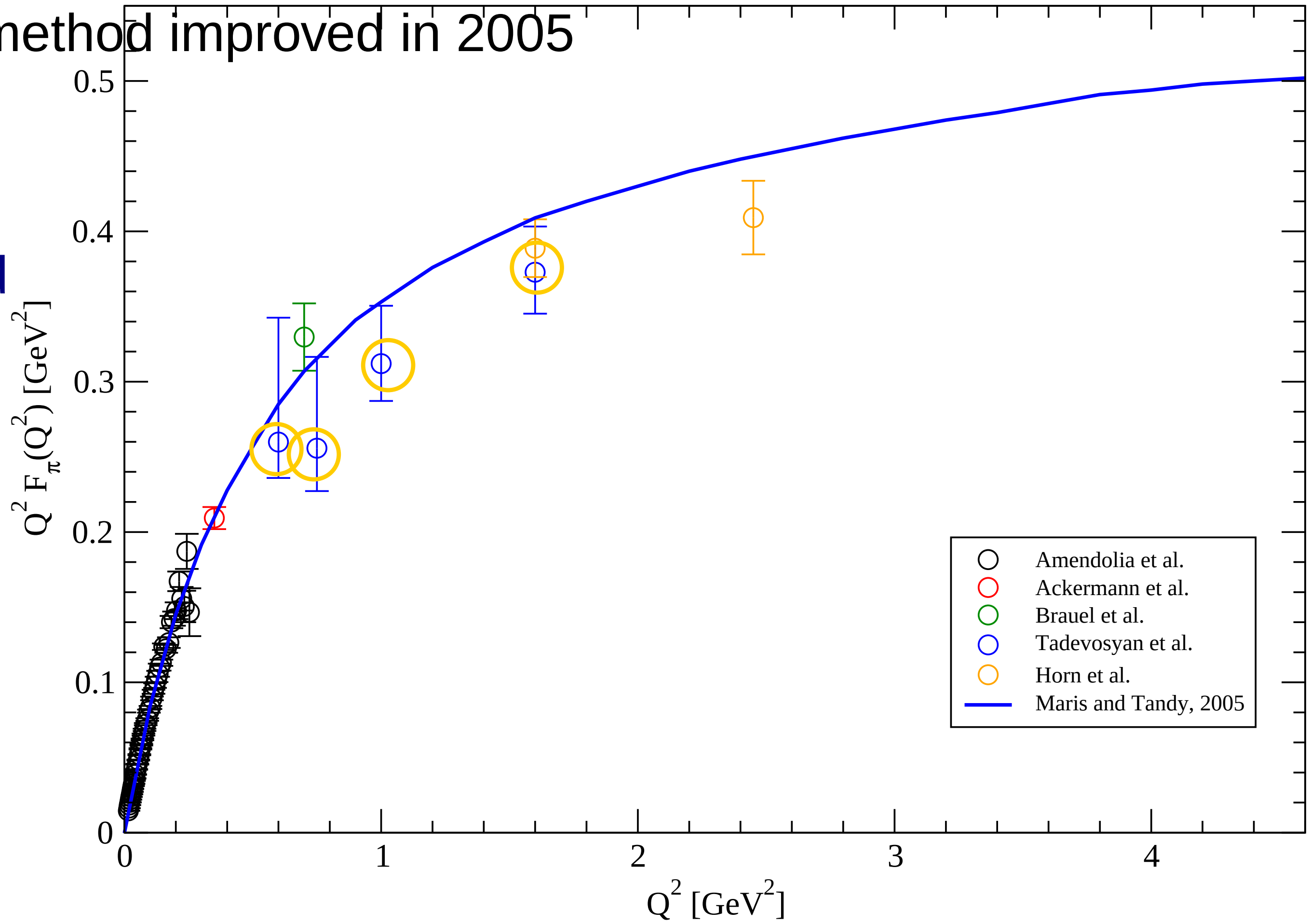

Timelike Pion Form Factor

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Ab initio calculation into timelike region

Deeper than ground-state ρ-meson pole

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Ab initio calculation into timelike region

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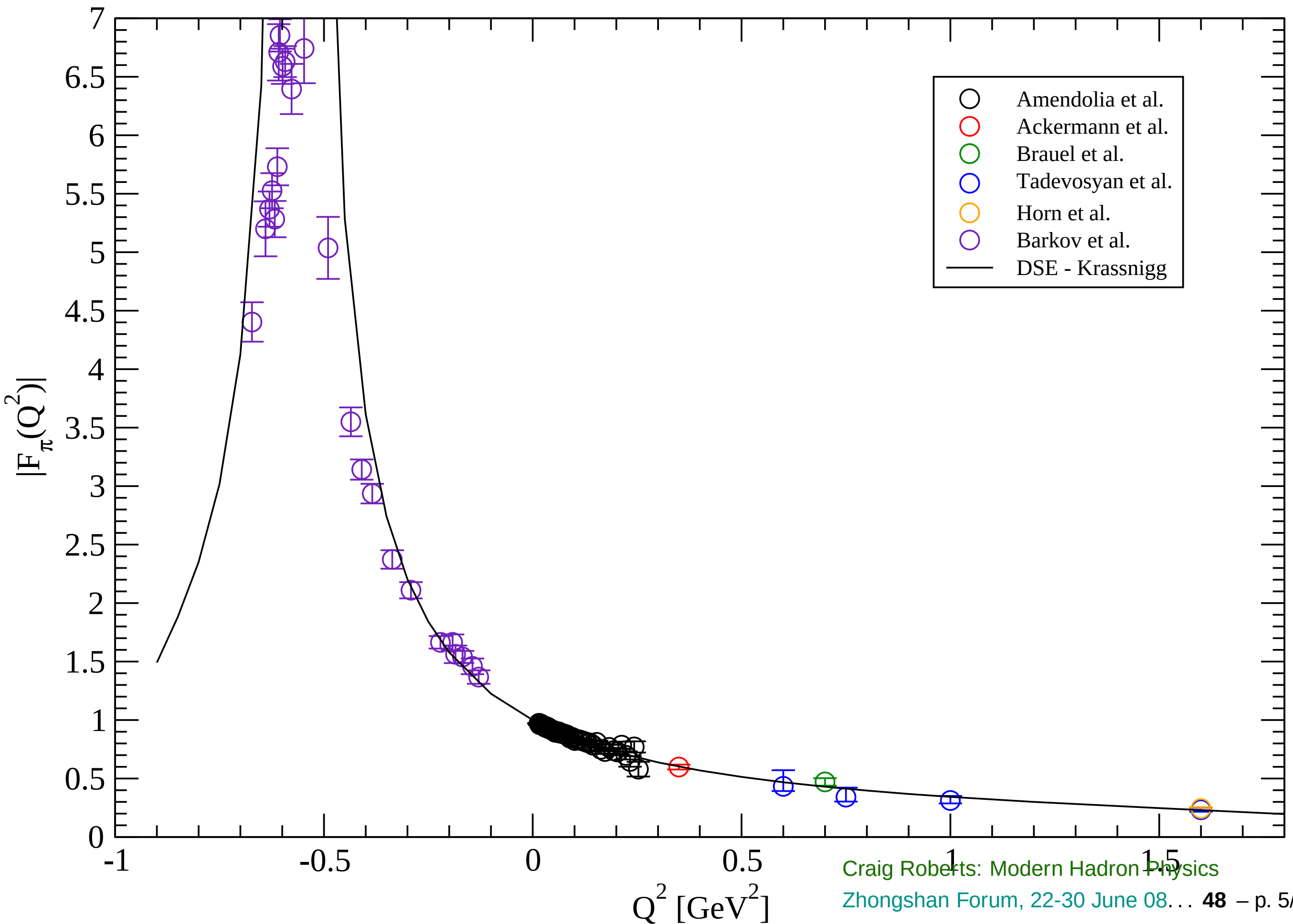
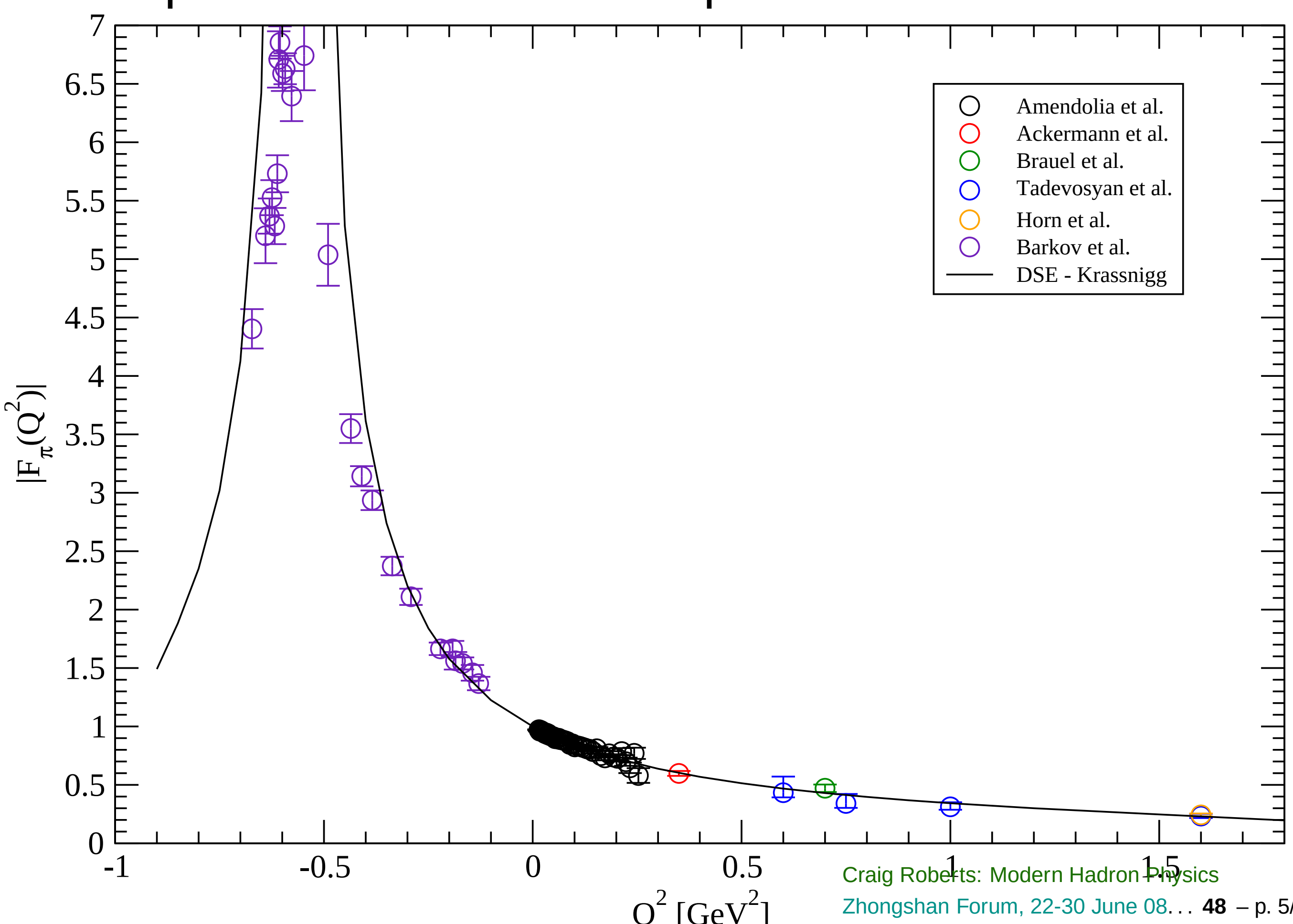


Timelike Pion Form Factor

- *Ab initio* calculation into timelike region
- Deeper than ground-state ρ-meson pole
- ρ-meson not put in "by hand" – generated dynamically as a bound-state of dressed-quark and dressed-antiquark



Dimensionless product: $r_\pi \, f_\pi$

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- Improved rainbow-ladder interaction

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Dimensionless product: $r_\pi\, f_\pi$

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- Great strides towards placing nucleon studies on same footing as mesons

Dimensionless product: $r_\pi \, f_\pi$

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- Experimentally: $r_\pi f_\pi = 0.315 \pm 0.005$

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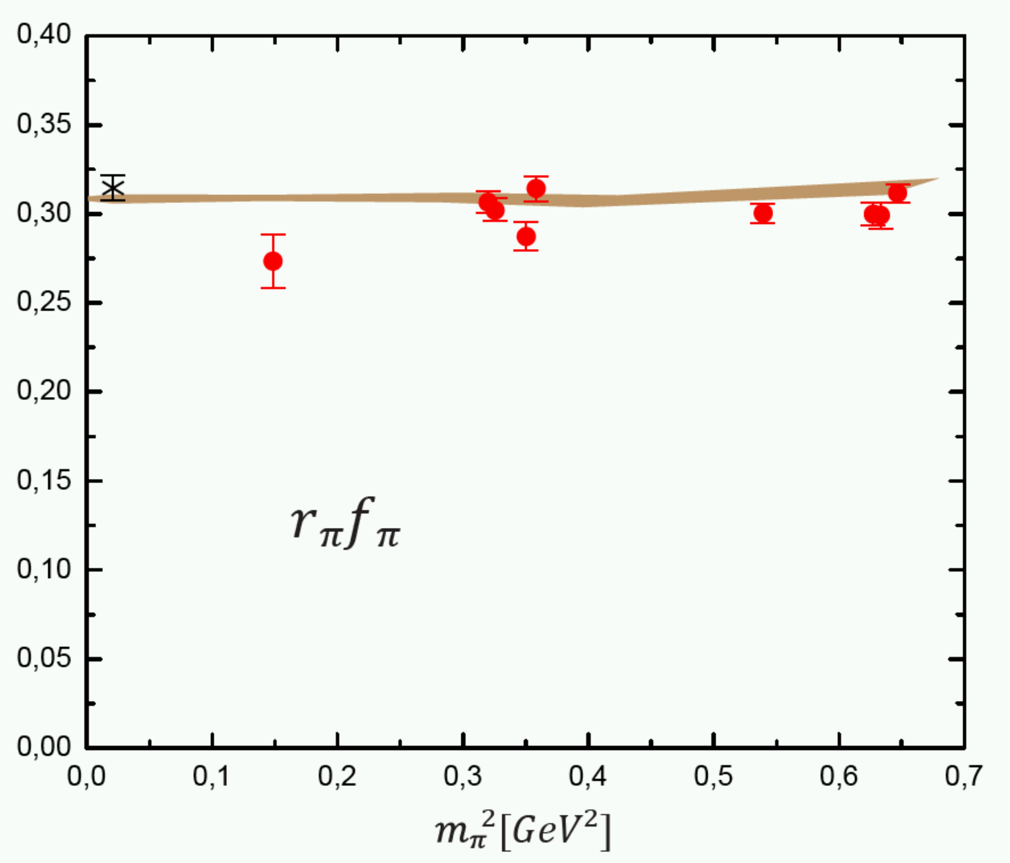
- DSE prediction



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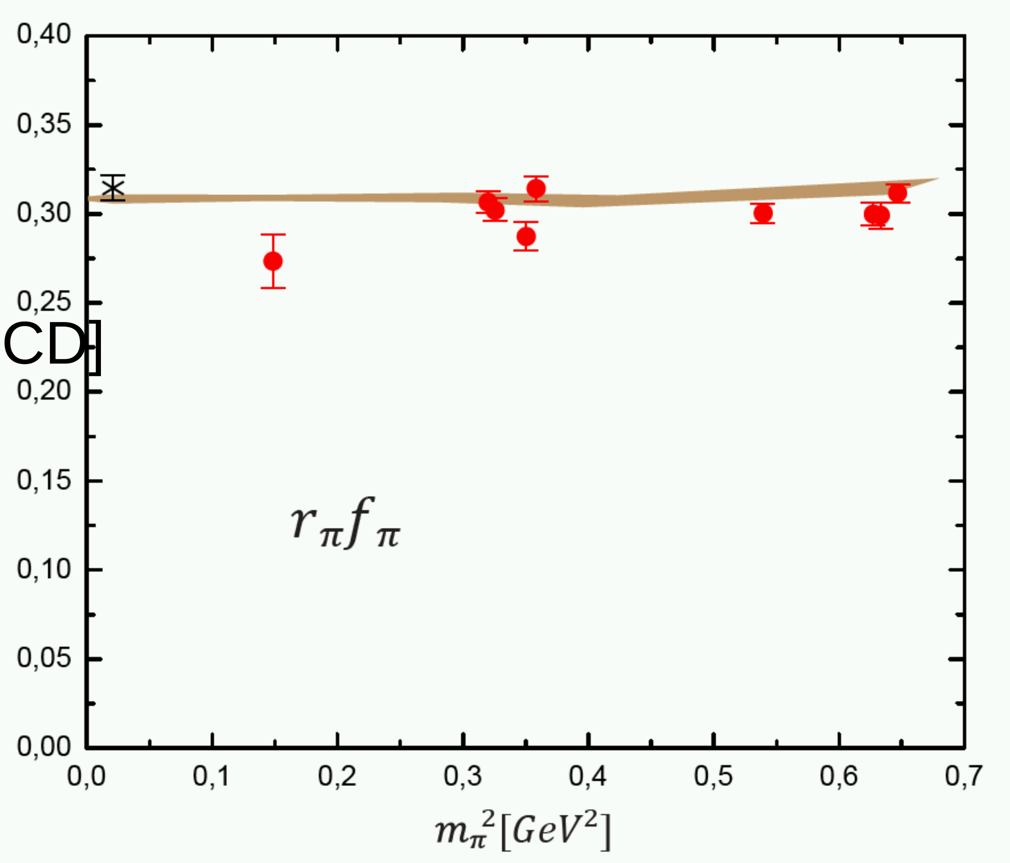

- DSE prediction
- Lattice results
  – James Zanotti [UK QCD]

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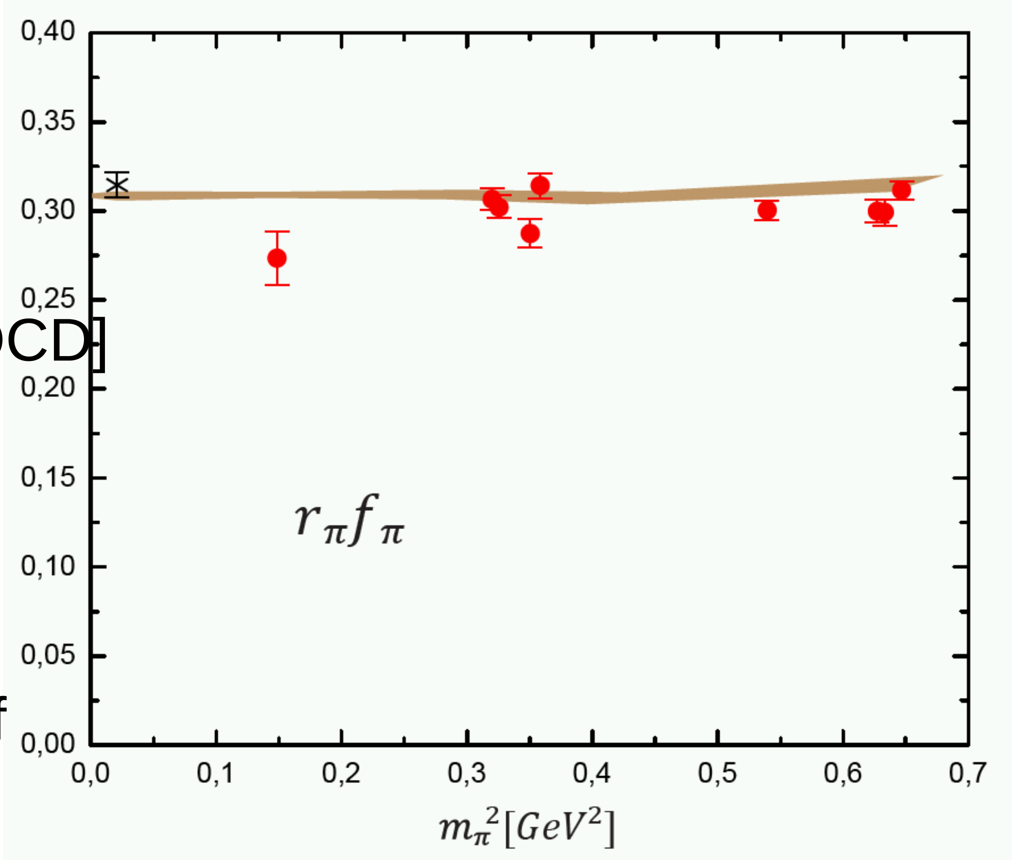

- DSE prediction
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  – James Zanotti [UK QCD]
- Fascinating result: DSE and Lattice
  – Experimental value obtains independent of current-quark mass.

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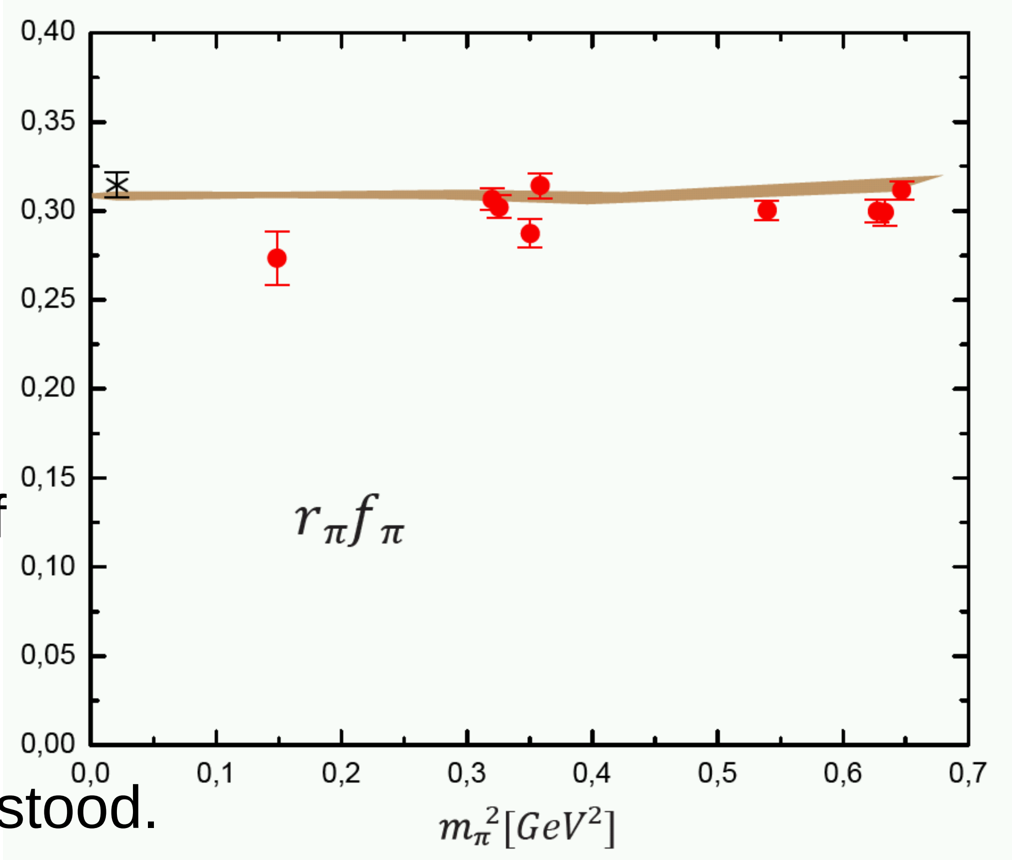

- DSE prediction
- Fascinating result: DSE and Lattice – Experimental value obtains independent of current-quark mass.
- Potentially useful but must first be understood.


ARE WE THERE YET?

Deep-inelastic scattering

Deep-inelastic scattering

- Looking for Quarks

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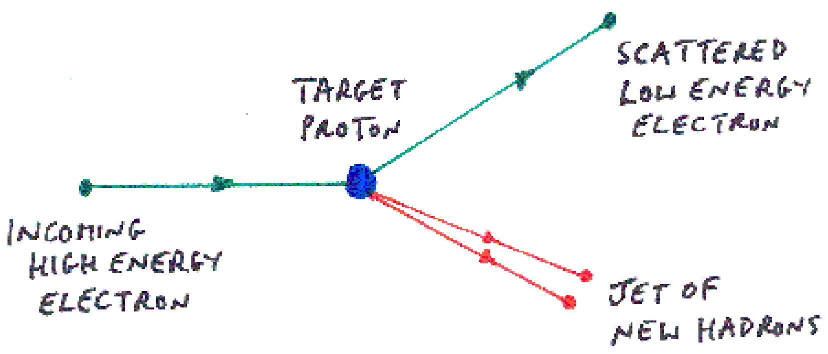
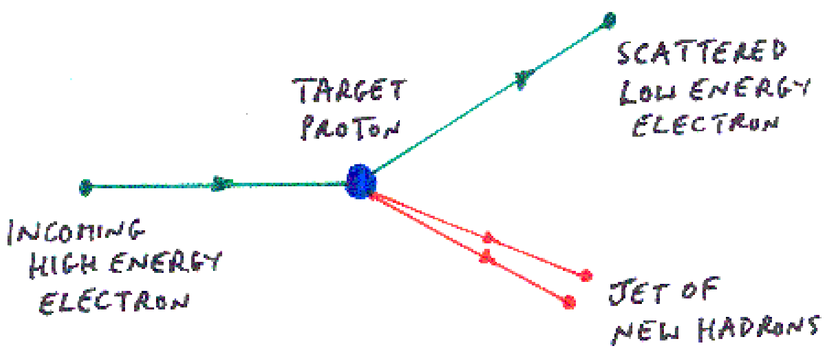
Deep-inelastic scattering



- Looking for Quarks

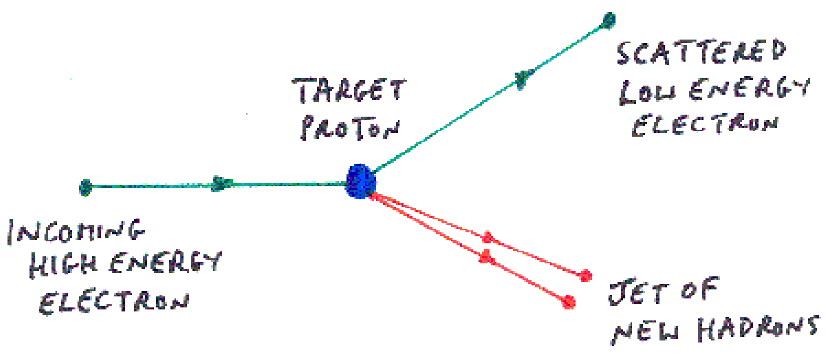
Deep-inelastic scattering



- Looking for Quarks

- Signature Experiment for QCD:
  Discovery of Quarks at SLAC

Deep-inelastic scattering



- Looking for Quarks

- **Signature Experiment** for QCD:
  Discovery of Quarks at SLAC
- Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x)$, $g(x)$

Pion’s valence quark distn

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- However, NO π Targets!

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Office of Nuclear Physics Exploring Nuclear Matter - Quarks to Stars

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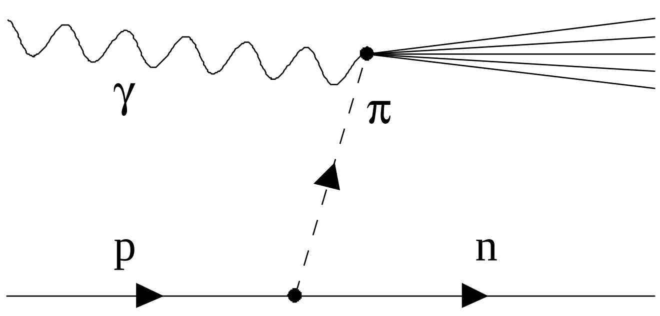
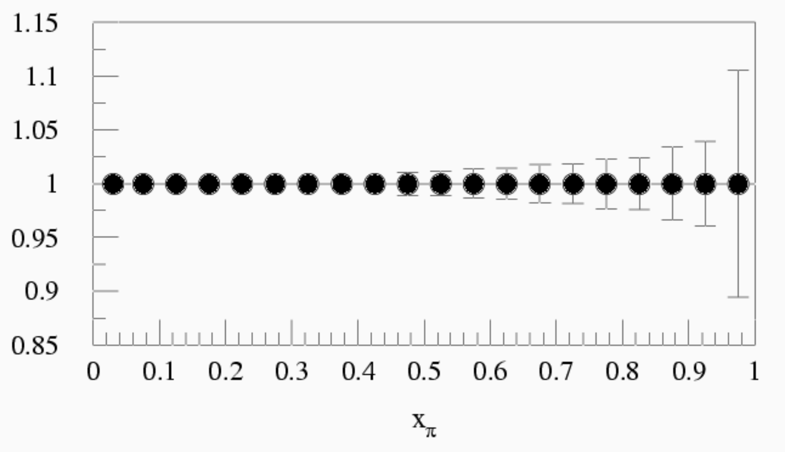
Pion's valence quark distn

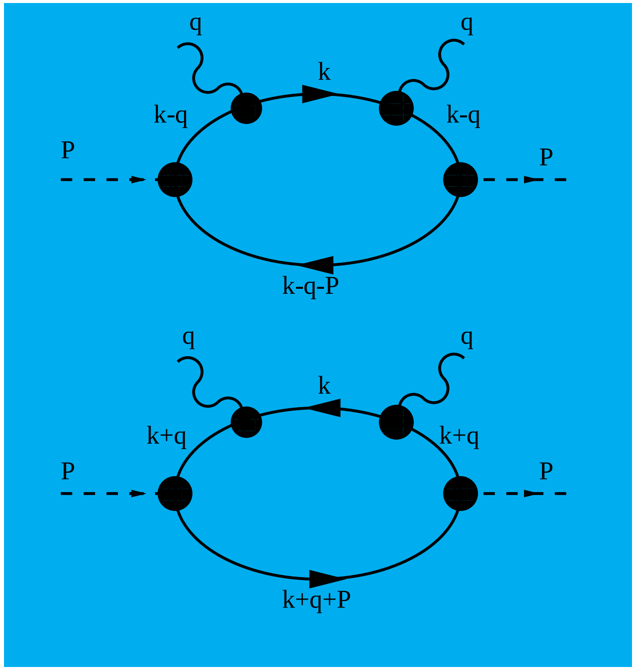
- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!
- Existing Measurement Inferred from Drell-Yan:
  $\pi N \to \mu^+ \mu^- X$
- Proposal (Holt & Reimer, ANL, nu-ex/0010004)

$e^-_{5\text{GeV}} - p_{25\,\text{GeV}}$ Collider $\to$ Accurate "Measurement"





Handbag diagrams



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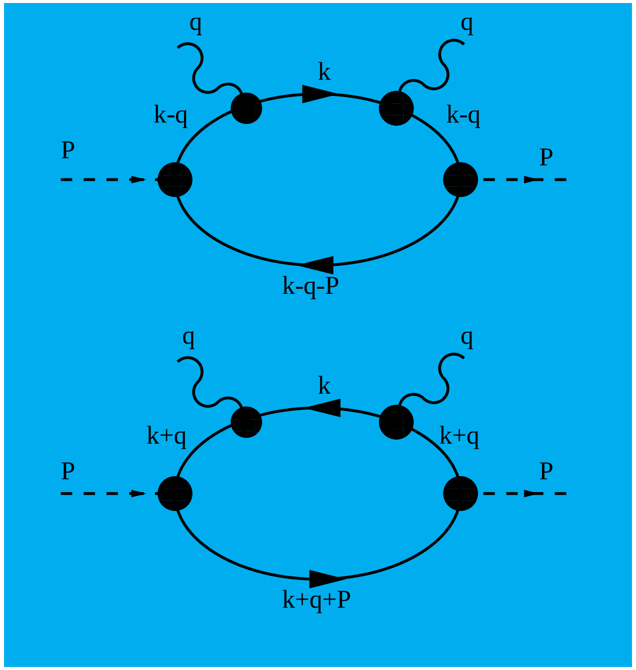


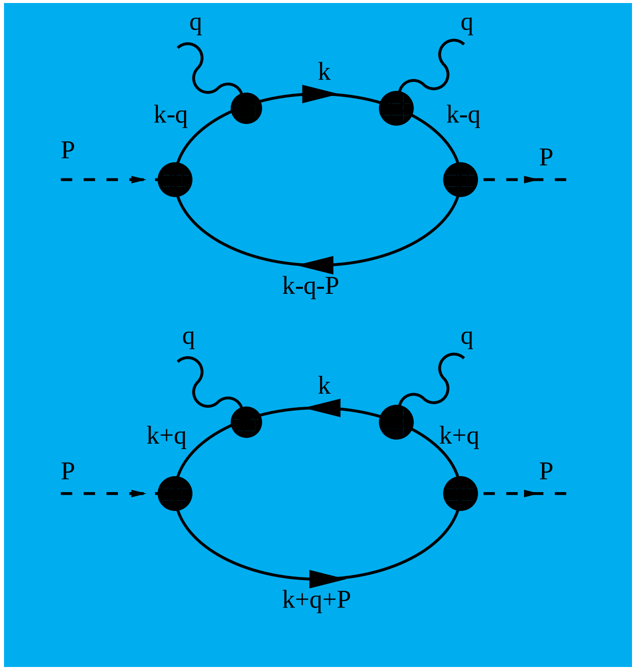
$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

$$T^{+}_{\mu\nu}(q,P) = \mathrm{tr}\int\frac{d^4k}{(2\pi)^4}\,\tau_-\bar{\Gamma}_\pi(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_\nu(k_{-0},k)$$
$$\times S(k)\,ieQ\Gamma_\mu(k,k_{-0})\,S(k_{-0})\tau_+\Gamma_\pi(k_{-\frac{1}{2}};P)\,S(k_{--})$$

Handbag diagrams

Bjorken Limit: $q^2 \to \infty\,, \quad P\cdot q \to -\infty$

but $x := -\dfrac{q^2}{2P\cdot q}$ fixed.



Numerous algebraic simplifications

$$W_{\mu\nu}(q;P) \quad = \quad \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

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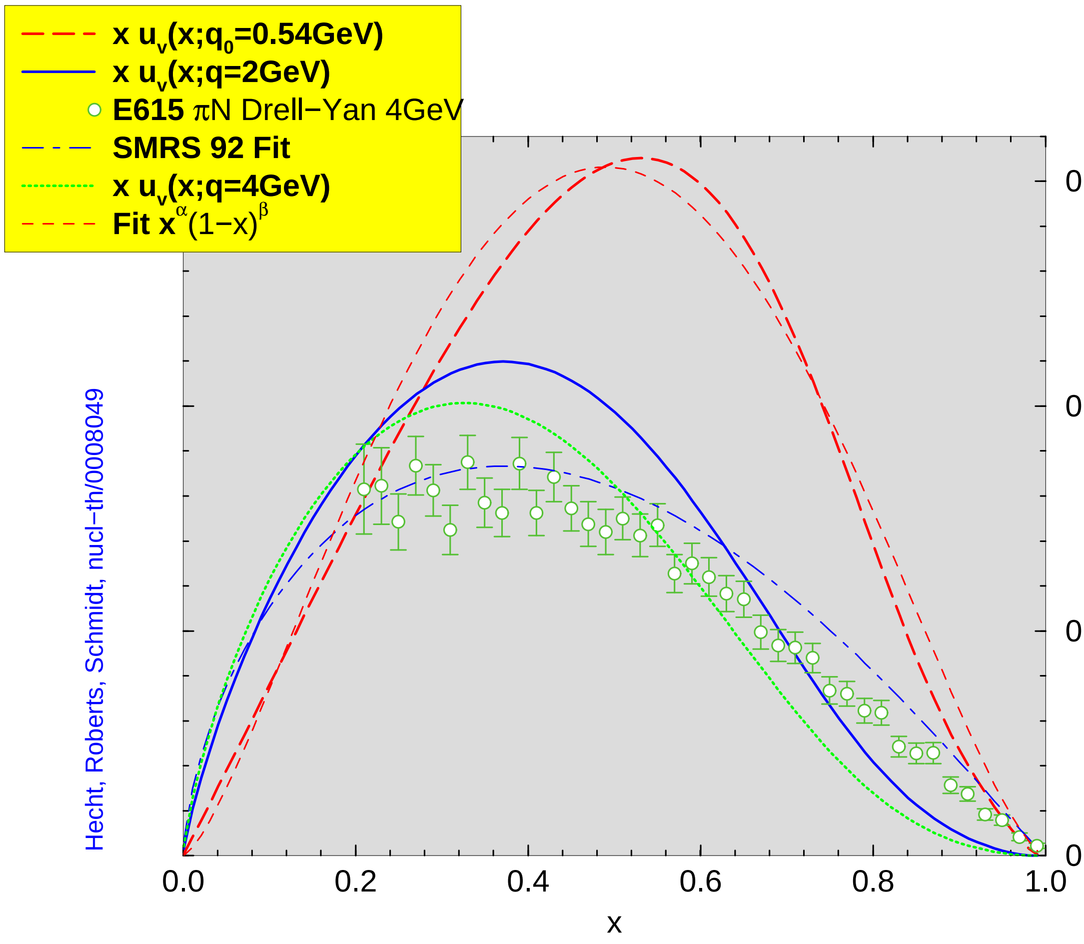
Hecht, Roberts, Schmidt
nucl-th/0008049

Calc. $u_V(x)$ cf. Drell-Yan data

Hecht, Roberts, Schmidt
nucl-th/0008049

Calc. $u_V(x)$ *cf. Drell-Yan data*



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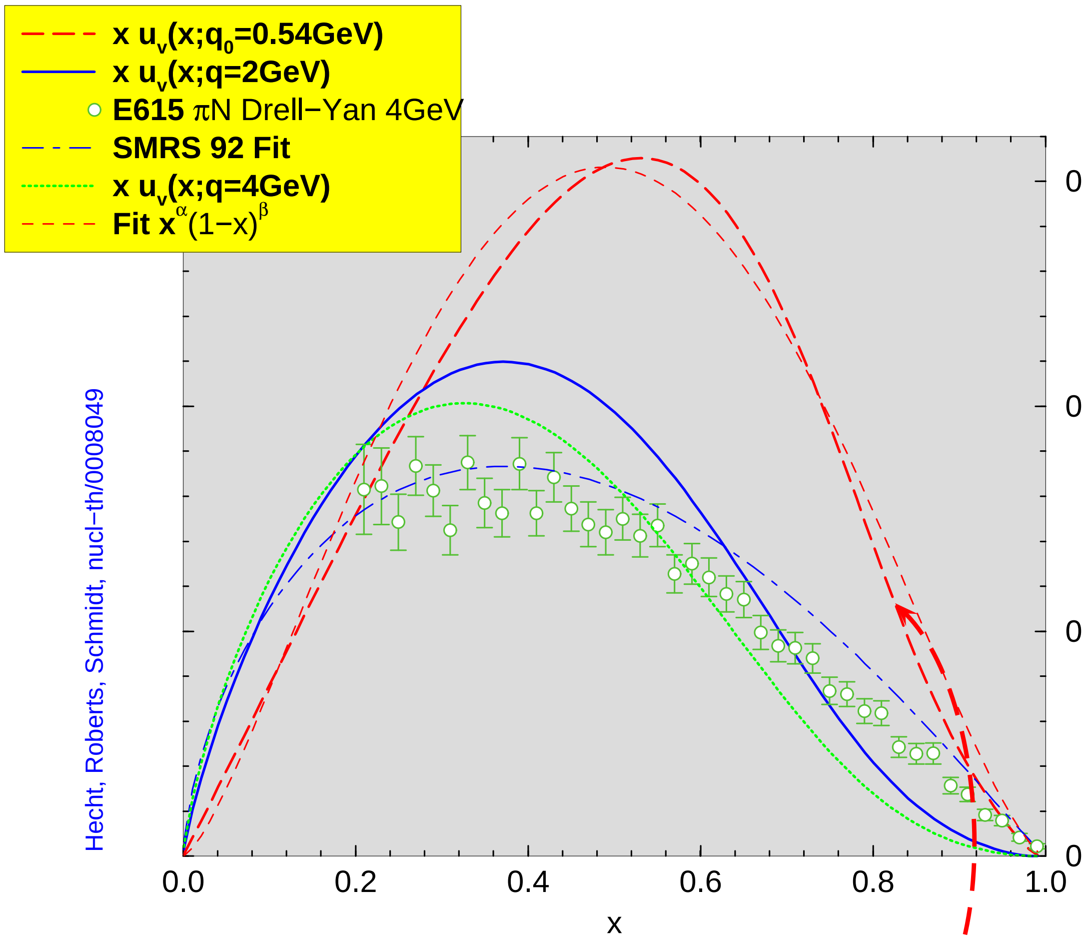
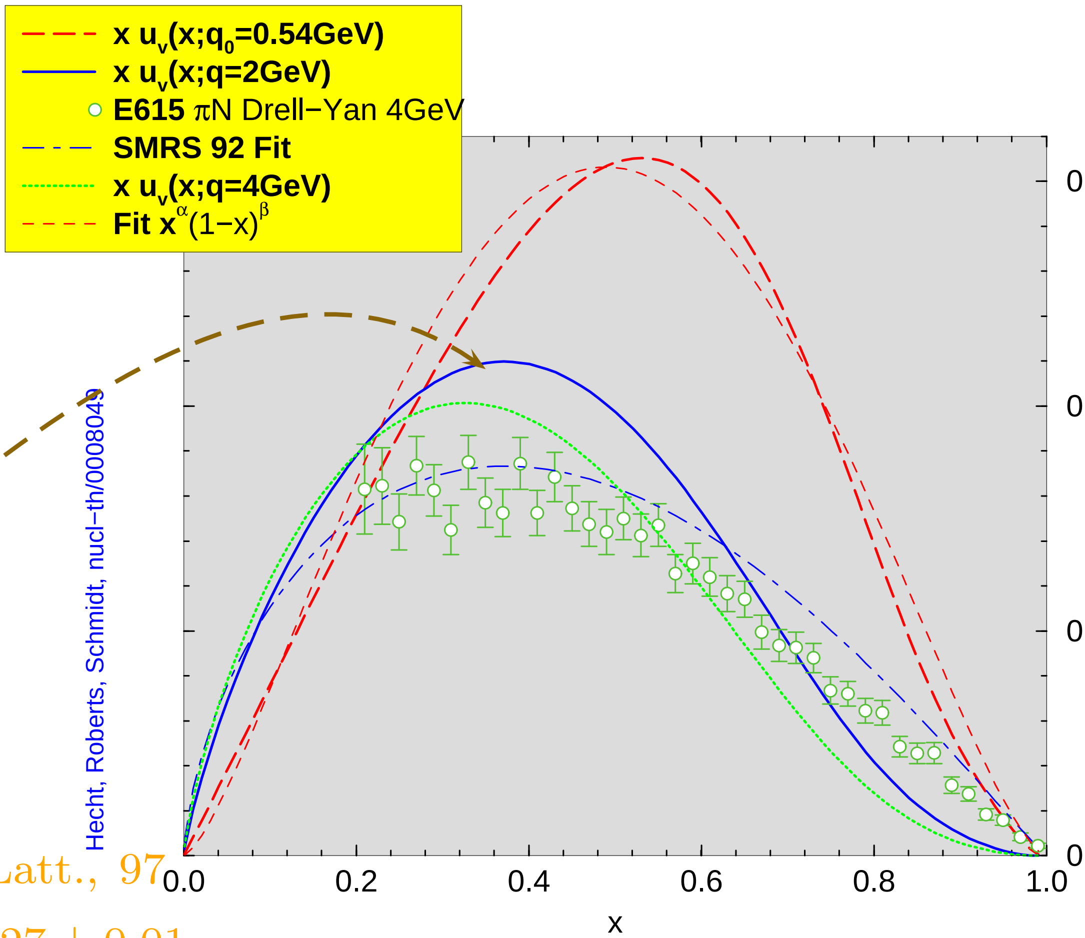


Resolving Scale: $q_0 = 0.54\,\mathrm{GeV} = 1/(0.37\,\mathrm{fm})$

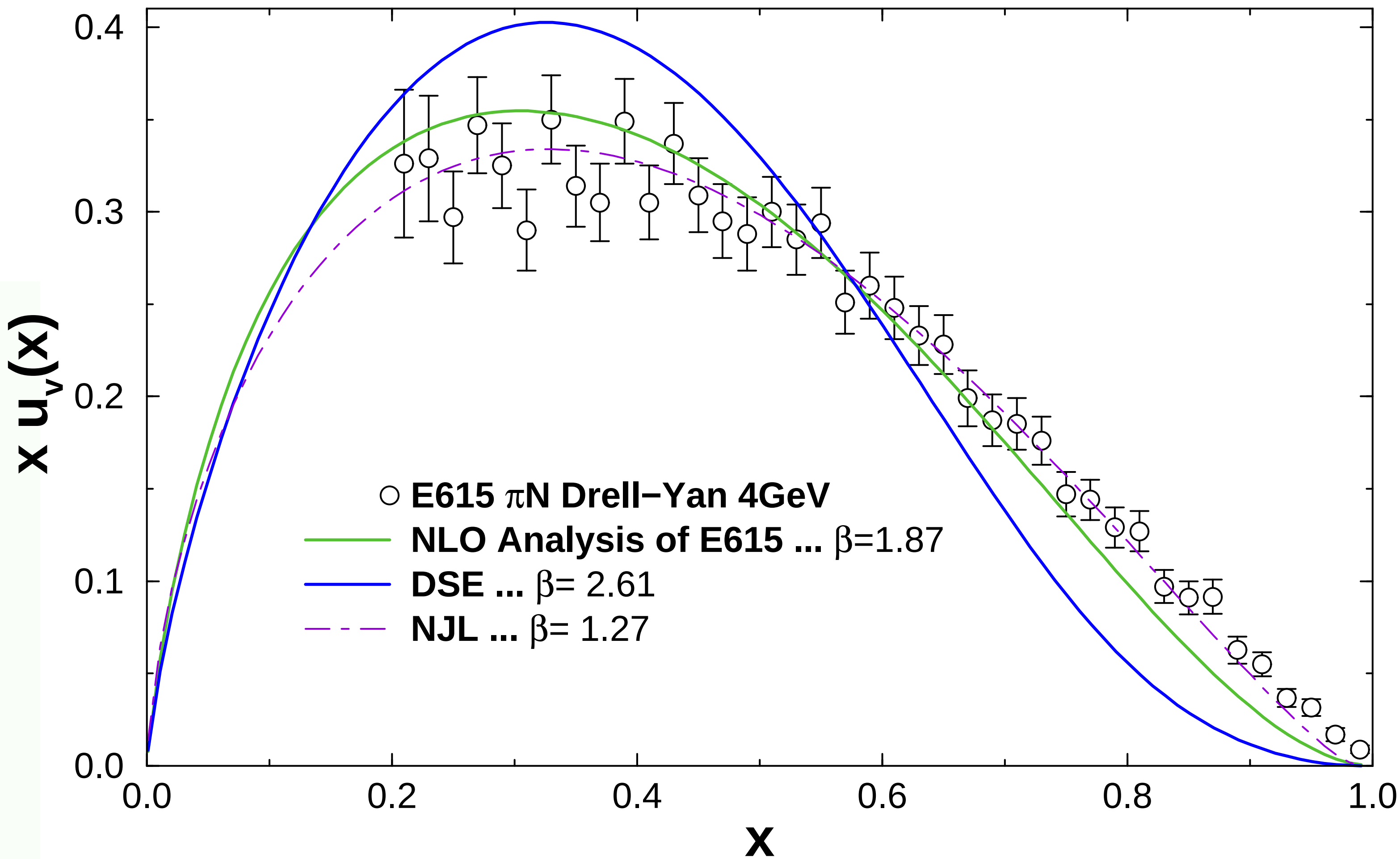
Calc. $u_V(x)$ cf. Drell-Yan data



$q =$ 2 GeV	Calc.	Fit, 92	Latt., 97
$\langle x \rangle_q$	0.24	0.24 ± 0.01	0.27 ± 0.01
$\langle x^2 \rangle_q$	0.10	0.10 ± 0.01	0.11 ± 0.3
$\langle x^3 \rangle_q$	0.050	0.058 ± 0.004	0.048 ± 0.020

Extant theory vs. experiment

K. Wijersooriya, P. Reimer and R. Holt,
nu-ex/0509012 ... Phys. Rev. C (Rapid)



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New Challenges

New Challenges

- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.

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New Challenges

- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

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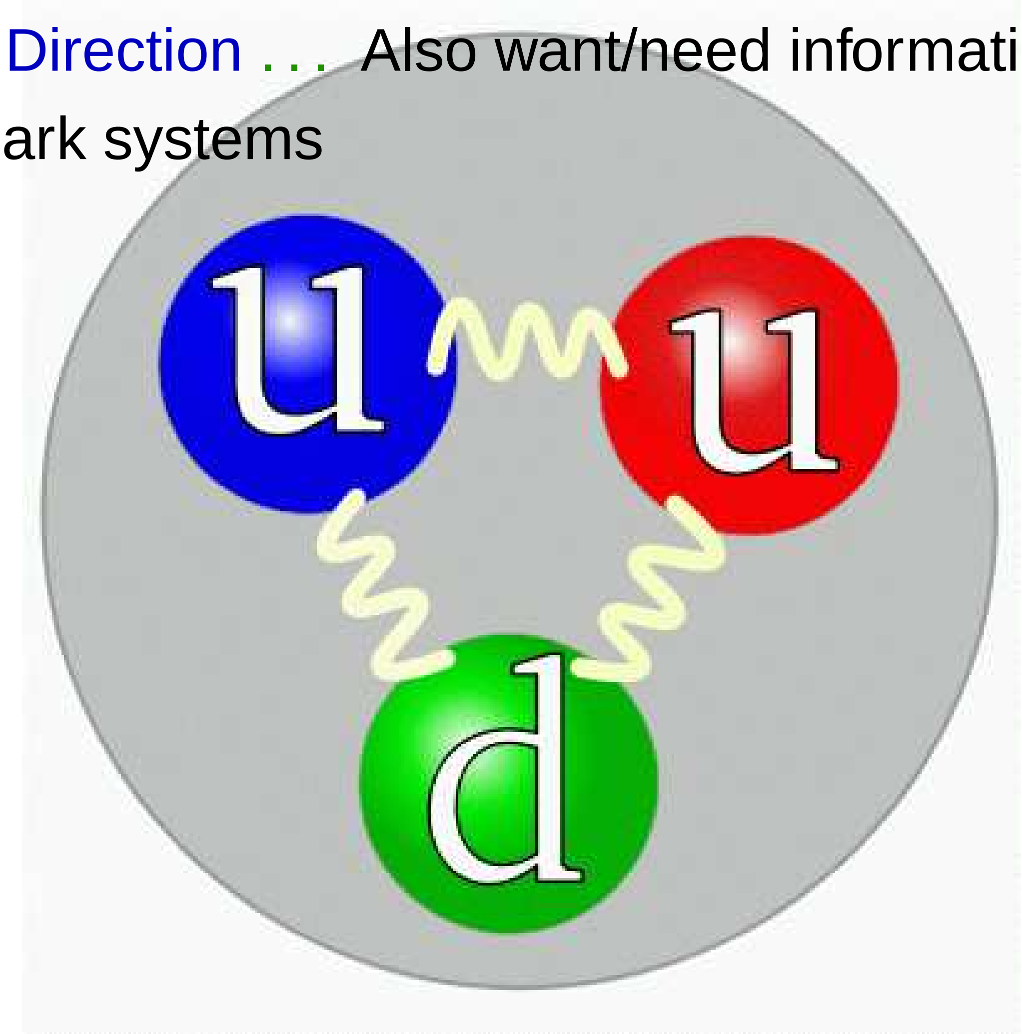
New Challenges

- Another Direction . . . Also want/need information about three-quark systems



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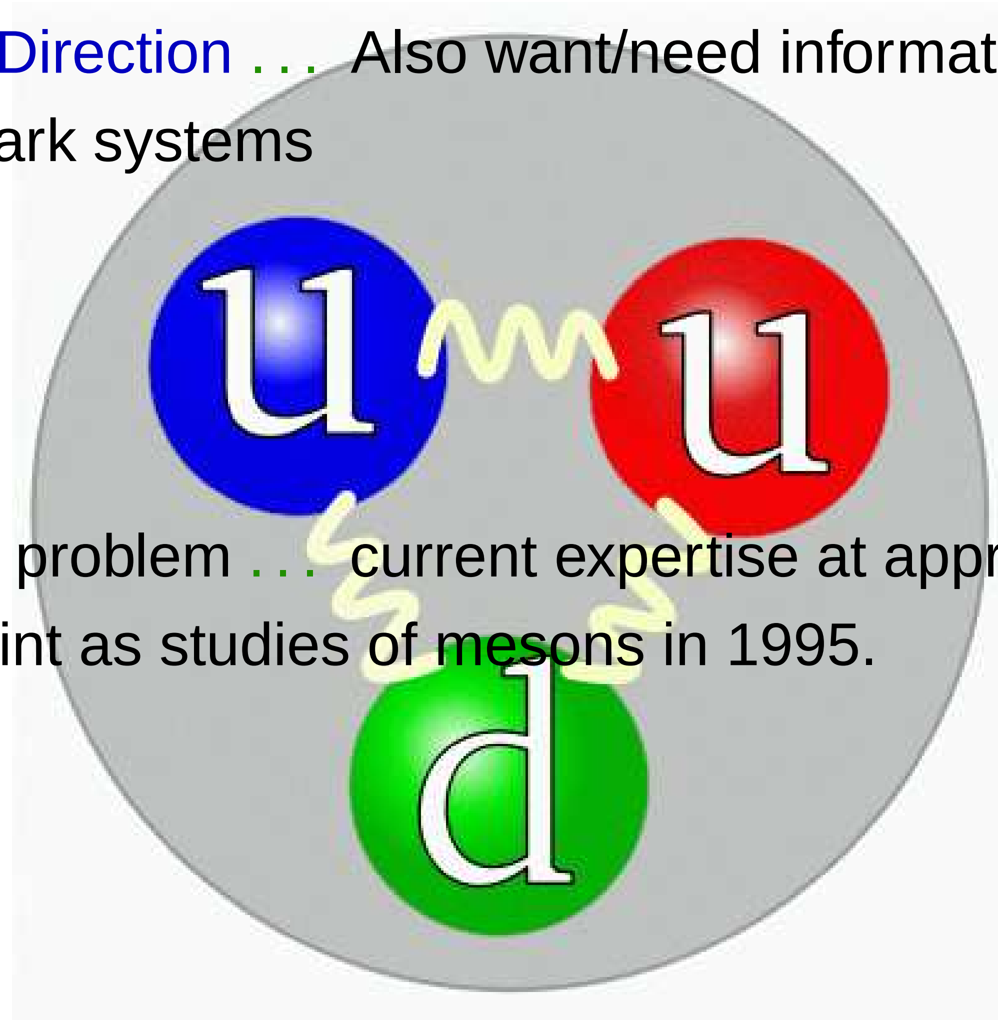
- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.



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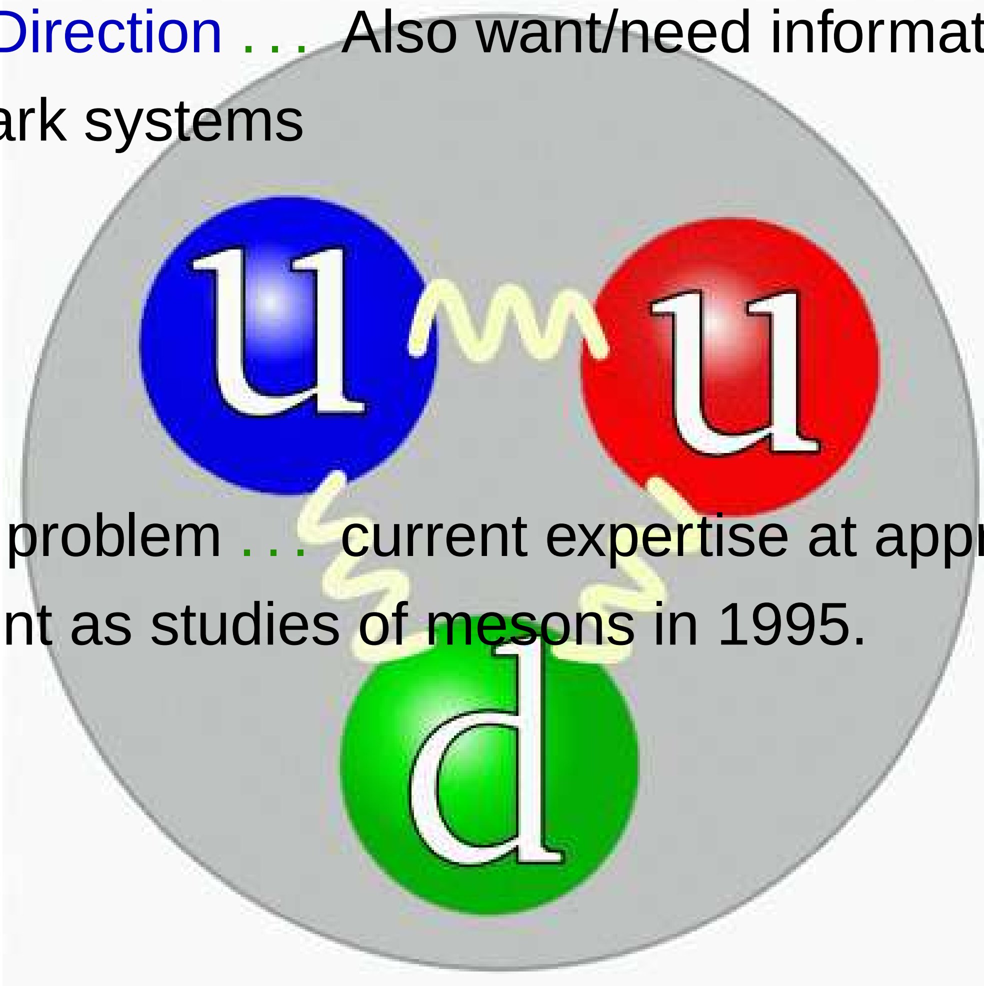
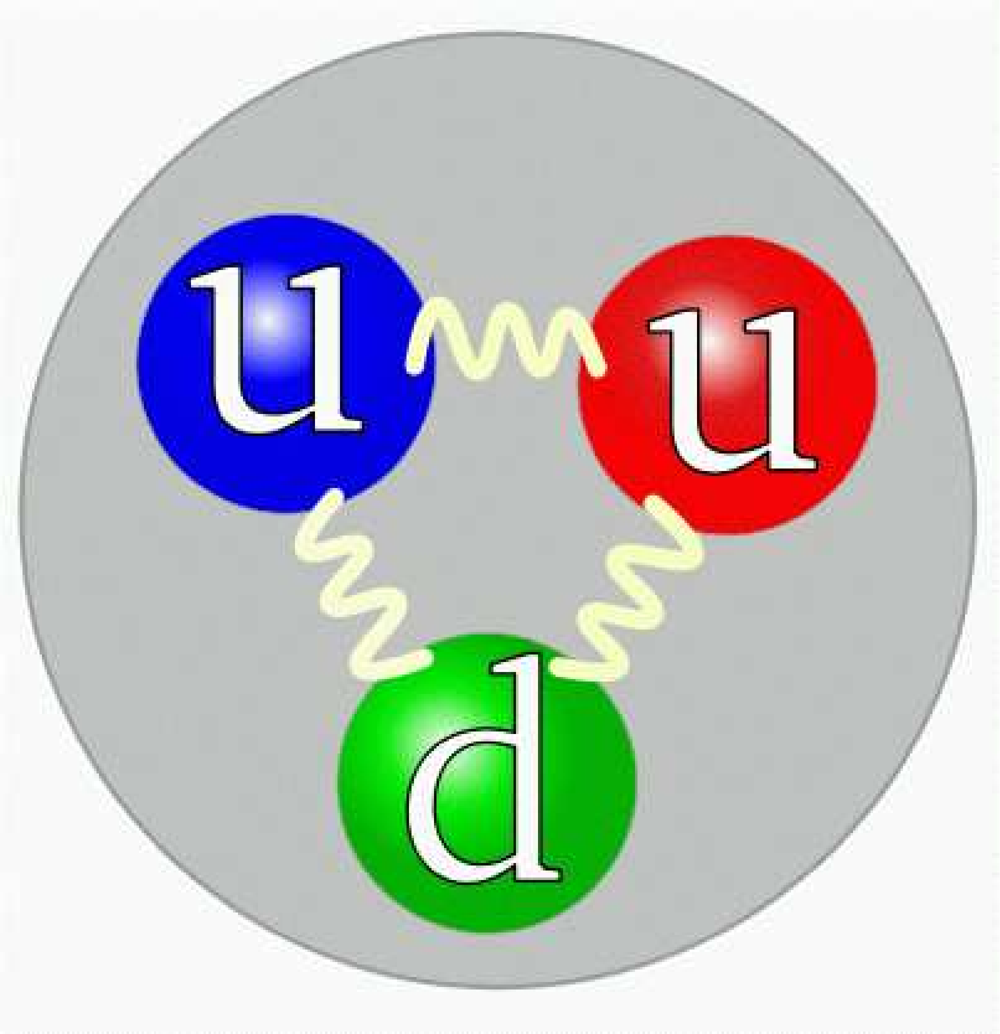


- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

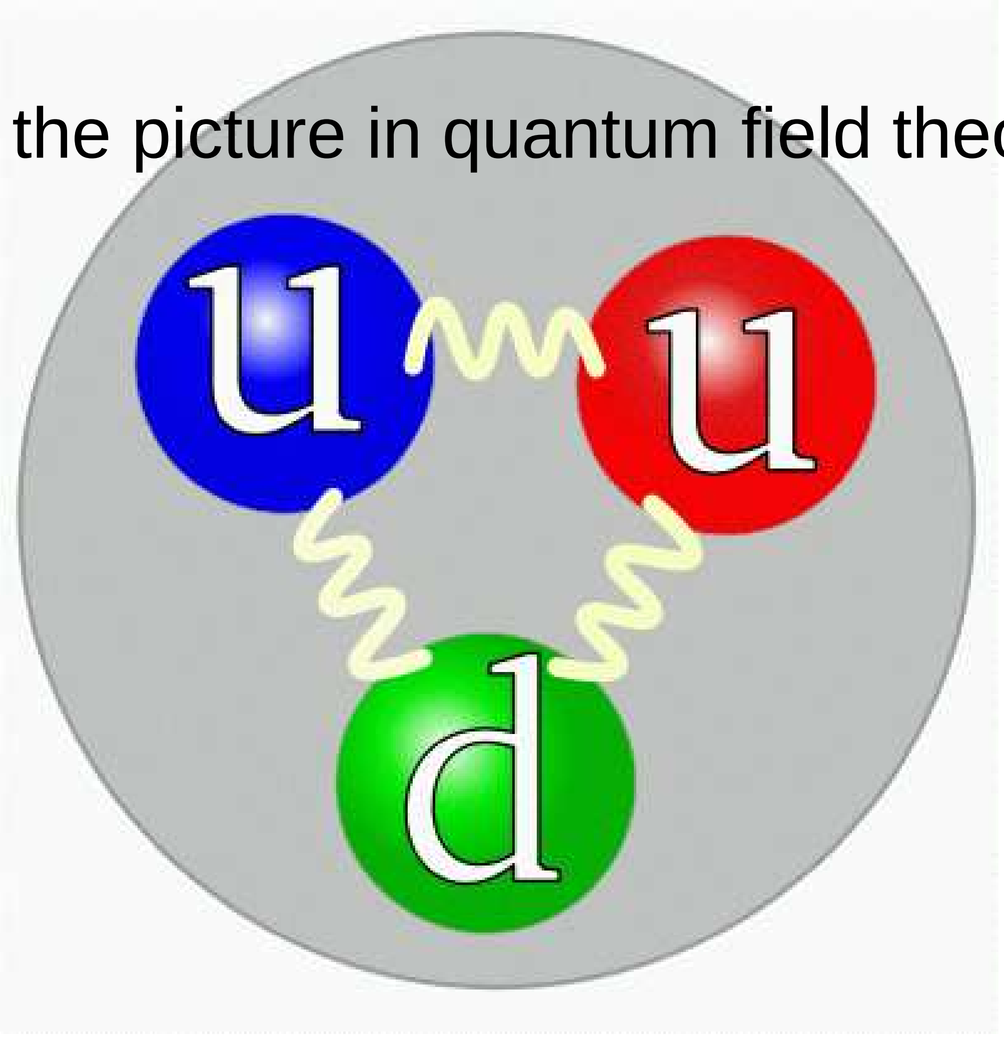
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.

Nucleon . . .

Three-body Problem?

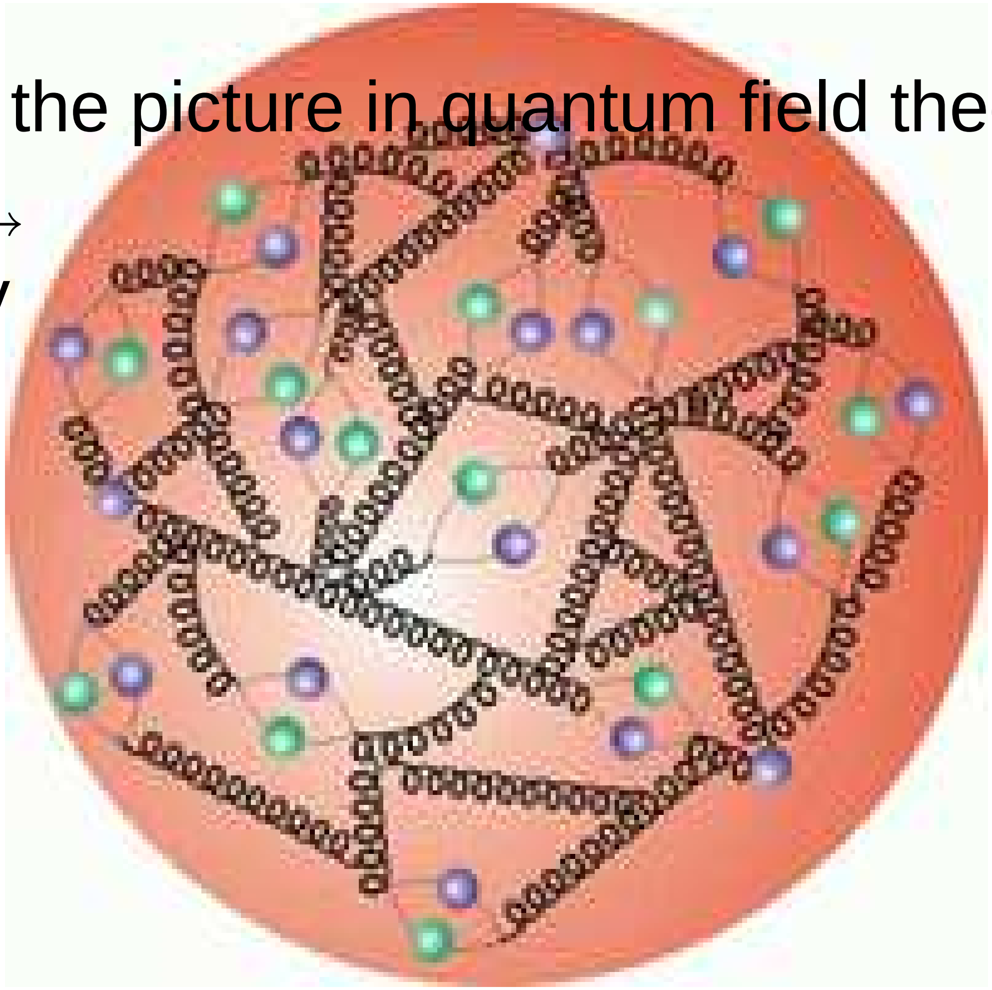


- What is the picture in quantum field theory?



Three-body Problem?

- What is the picture in quantum field theory?
- Three $\rightarrow$ infinitely many!



Faddeev equation

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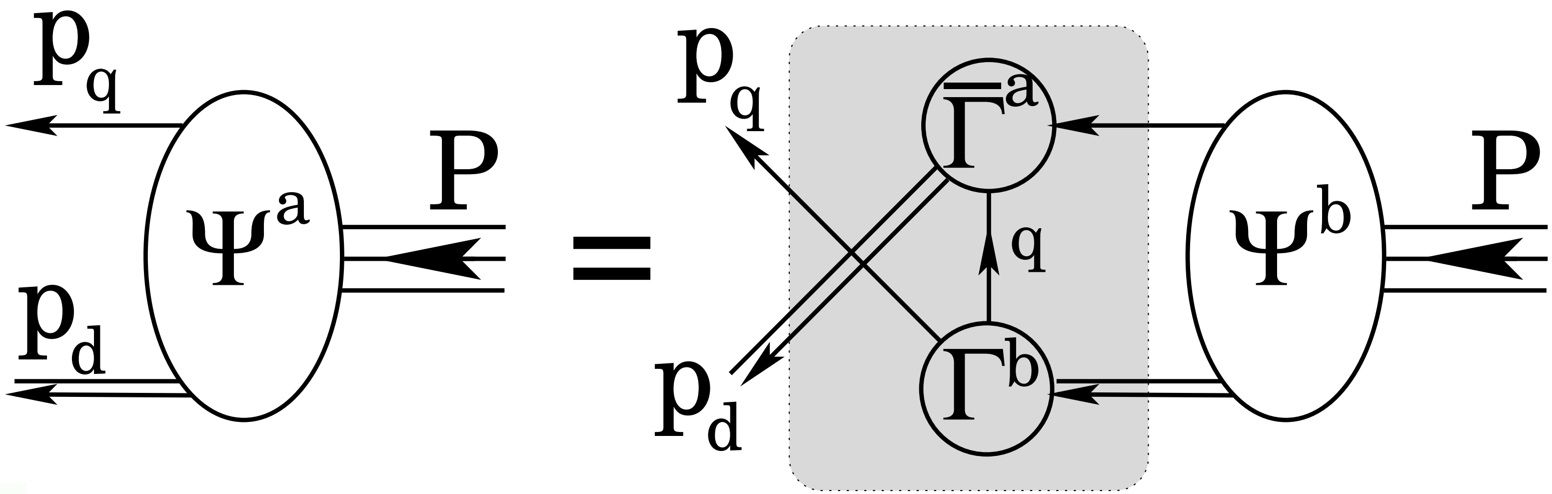
Faddeev equation



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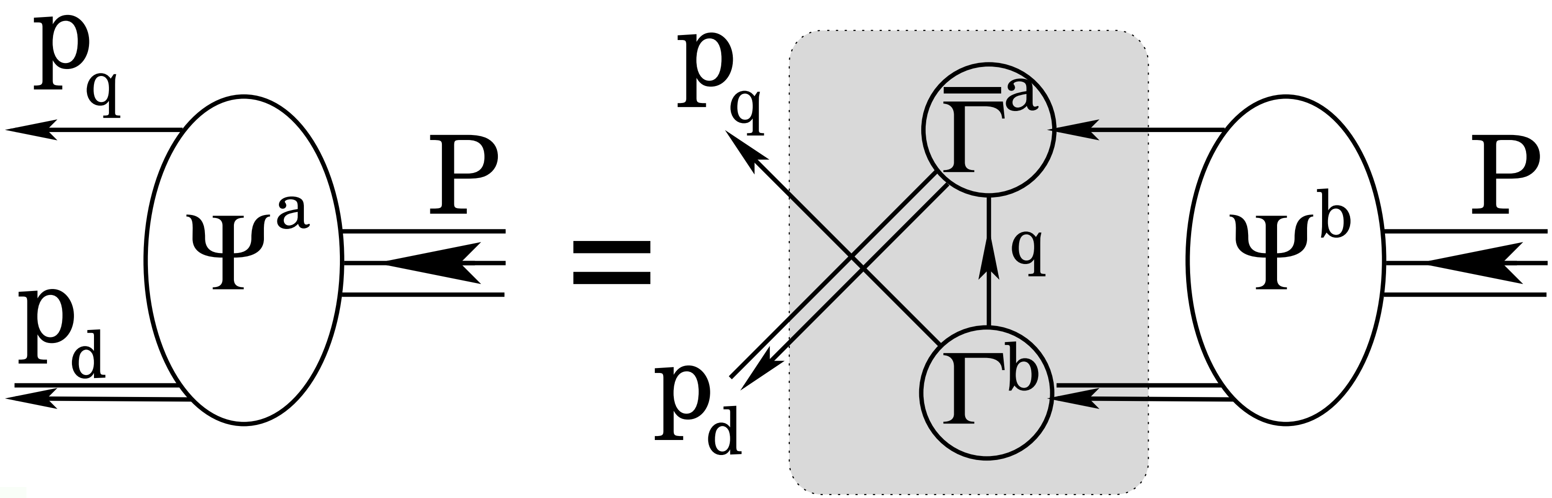

- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame Amplitude has ... $s-$, $p-$ & $d-$wave correlations

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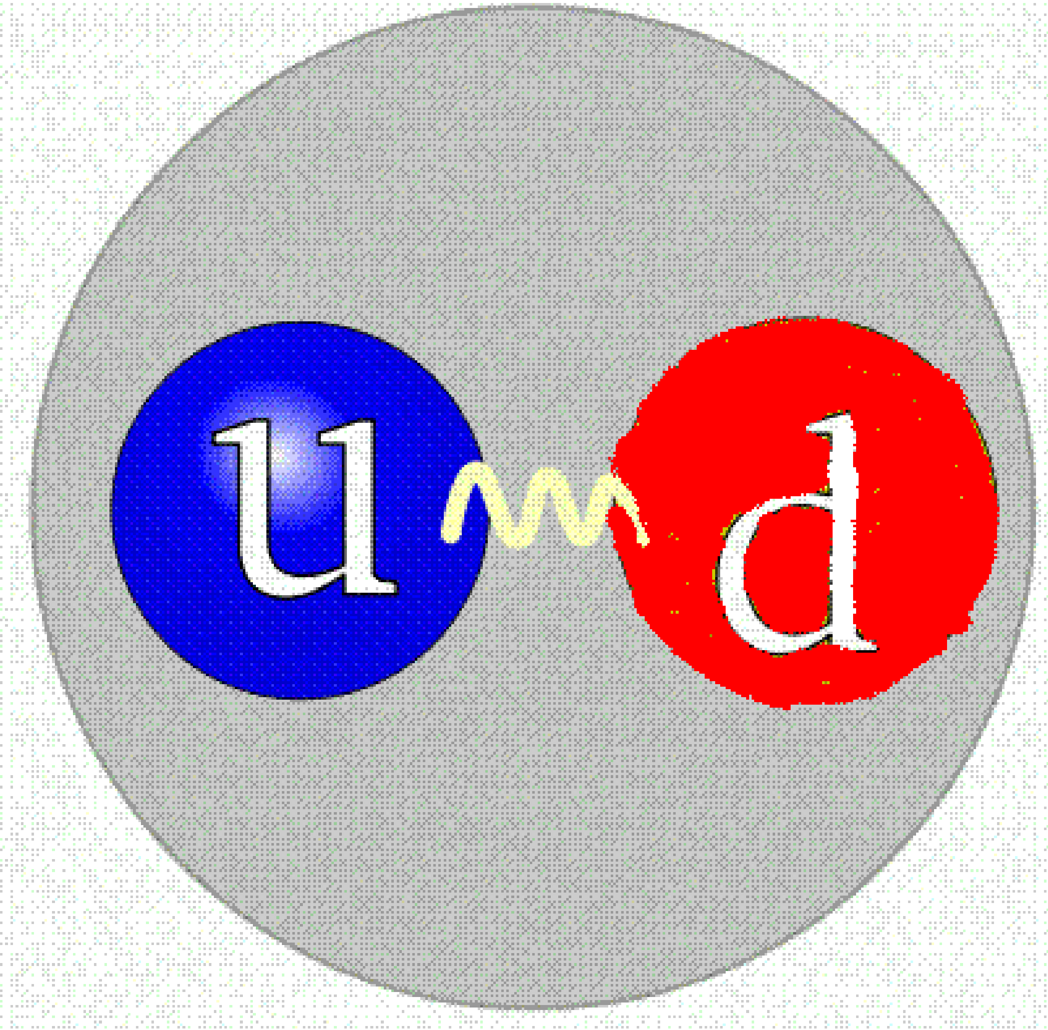
Diquark correlations



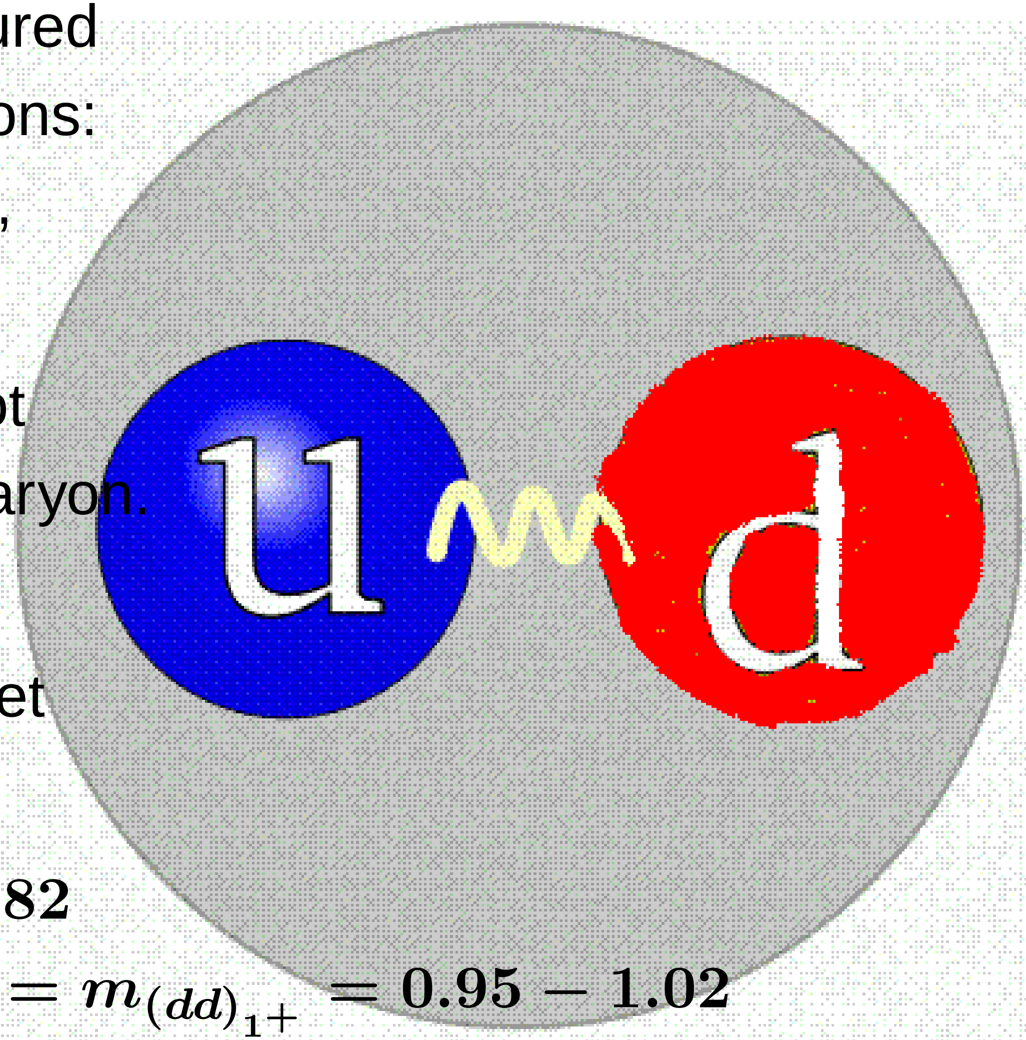
QUARK-QUARK

Diquark correlations

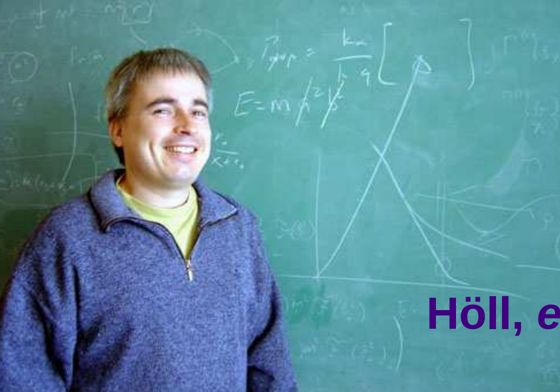
- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined … Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$

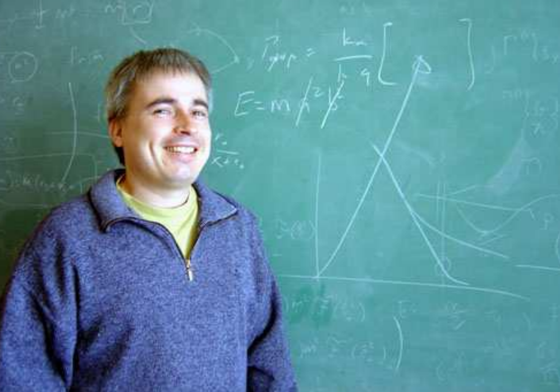


QUARK-QUARK



Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033



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  Easily obtained:

$$\left(\frac{1}{N_H}\sum_H \frac{[M_H^{\rm exp}-M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2}\right)^{1/2} = 2\%$$

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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

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Nucleon EM Form Factors: A Précis

Cloët, *et al.*:
arXiv:0710.2059, arXiv:0710.5746 & arXiv:0804.3118

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

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- But is that good?

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  - Cloudy Bag: $\delta M_+^{\pi-\rm loop} = -300$ to -400 MeV!

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- But is that good?
  - Cloudy Bag: $\delta M_+^{\pi-\rm loop} = -300$ to -400 MeV!
- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

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Nucleon's self-energy - pion loop

Nucleon's self-energy - pion loop

$$\begin{aligned}\Sigma(P) &= 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2(P,k)\,\Delta_\pi((P-k)^2) \\ &\quad \times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5} \\ &= i\gamma\cdot k\,[\mathcal{A}(k^2)-1]+\mathcal{B}(k^2)\end{aligned}$$

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- Pseudovector coupling

Nucleon's self-energy - pion loop

$$\Sigma(P) \;=\; 3\int \frac{d^4k}{(2\pi)^4}\, g^2_{PV}(P,k)\,\Delta_\pi((P-k)^2)$$
$$\times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5}$$
$$= \; i\gamma\cdot k\,[\mathcal{A}(k^2)-1] + \mathcal{B}(k^2)$$

- Pseudovector coupling
- Completely equivalent to pseudoscalar coupling
  IF that is treated completely
  - Tadpole contribution can't be neglected

(Hecht, Oettel, Roberts, Schmidt, Tandy, Thomas: nucl-th/0201084)

Nucleon's self-energy - pion loop

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- $g_{PV}(P,k)$, πN vertex function
- Calculated using Γ_π and Ψ_N
  - Always soft: Monopole $\lambda \sim 0.6$ GeV

Nucleon's self-energy - pion loop

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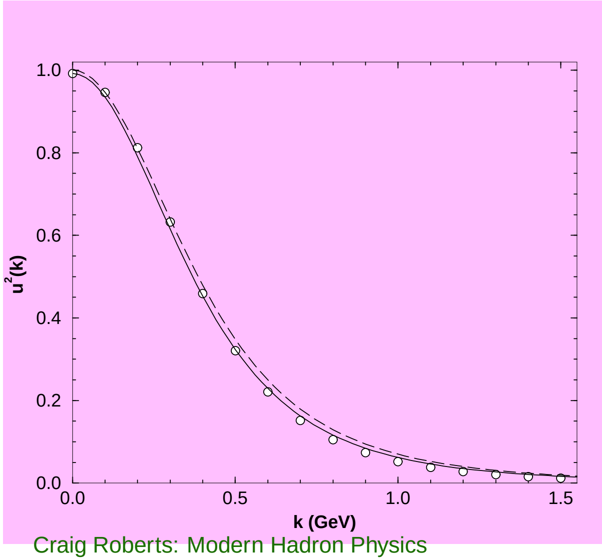
- $g_{PV}(P,k)$, πN vertex function
- Calculated using Γ_π and Ψ_N
  - Always soft: Monopole $\lambda \sim 0.6$ GeV
    - Corresponds to range $r_\lambda \sim 0.8$ fm

      ... pion cloud does not penetrate deeply within nucleon.



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$$\times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5}$$
$$= i\gamma\cdot k\,[\mathcal{A}(k^2)-1] + \mathcal{B}(k^2)$$

- $$G(k) = 1/[i\gamma\cdot k + M + \Sigma(P)]$$
$$= -i\gamma\cdot k\,\sigma_V(k^2) + \sigma_S(k^2)$$
Pole Position Not Known *a priori*
- Mass shift calculated via self-consistent solution

Nucleon's self-energy - pion loop

$$\Sigma(P) = 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2 (\text{const.})\, \Delta_\pi((P-k)^2)$$

$$\times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5}$$

$$= i\gamma\cdot k\,[\mathcal{A}(k^2)-1] + \mathcal{B}(k^2)$$

- Obtain Integral Equation Kernels

$$\int d\Omega_k\, f((P-k)^2) = \frac{2}{\pi}\int_{-1}^{1} dz\sqrt{1-z^2}\, f(P^2+k^2-2Pkz)$$

E.g.

$$\omega_{\mathcal{B}}(P^2,k^2) = \int d\Omega_k\, \frac{(P-k)^2}{(P-k)^2+m_\pi^2} = 1 - \frac{2\,m_\pi^2}{a+\sqrt{a^2-b^2}}\,,$$

$a = P^2+k^2+m_\pi^2$, b $= 2Pk$

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$$\begin{aligned}\Sigma(P) &= 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2(P,k)\,\Delta_\pi((P-k)^2) \\ &\quad \times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5} \\ &= i\gamma\cdot k\,[\mathcal{A}(k^2)-1]+\mathcal{B}(k^2)\end{aligned}$$

- But $g_{PV} = g_{PV}(P^2, k^2, P\cdot k)$
  Therefore, In General, Kernel only known Numerically
- Complicates analysis . . .
  locating, incorporating poles in integrand

Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

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Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

- Let's look what happens when $m_\pi \longrightarrow 0$
  - Minkowski Space
  - Pseudovector Coupling

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Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

- Let's look what happens when $m_\pi \to 0$
  - Minkowski Space
  - Pseudovector Coupling
- One-loop nucleon self energy

$$\boxed{\Sigma(P) = 3i\frac{g^2}{4M^2}\int\frac{d^4k}{(2\pi)^4}\,\Delta(k^2,m_\pi^2)\,\not k\,\gamma_5\,G_0(P-k)\,\not k\,\gamma_5\,.}$$

This integral is divergent. Assume a Poincaré covariant regularisation, characterised by a mass-scale λ

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This integral is divergent. Assume a Poincaré covariant regularisation, characterised by a mass-scale λ

- Decompose nucleon propagator into positive and negative energy components

$$\begin{aligned} G_0(P) &= \frac{1}{\not P - M_0} = G_0^+(P) + G_0^-(P) \\ &= \frac{M}{\omega_N(\vec P)}\left[\Lambda_+(\vec P)\frac{1}{P_0-\omega_N(\vec P)+i\varepsilon} + \Lambda_-(\vec P)\frac{1}{P_0+\omega_N(\vec P)-i\varepsilon}\right] \end{aligned} \quad (4)$$

$\omega_N^2(\vec P) = \vec P^2 + M^2$, and $\Lambda_\pm(\vec P) = (\tilde{\not P} \pm M)/(2M)$, $\tilde P = (\omega(\vec P), \vec P)$

Nucleon Self Energy: Chiral Limit

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- Shift in the mass of a positive energy nucleon nucleon:

$$\delta M_+ = \tfrac{1}{2}\mathrm{tr}_D\left[\Lambda_+(\vec{P}=0)\,\Sigma(P_0=M,\vec{P}=0)\right]$$

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- Focus on positive energy nucleon's contribution to the loop integral; i.e., $\Delta(k)\,G^+(P-k)$, which we denote: $\delta_F M_+^+$

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$\delta M_+ = \frac{1}{2}\mathrm{tr}_D\left[\Lambda_+(\vec{P}=0)\,\Sigma(P_0=M,\vec{P}=0)\right]$

- Focus on positive energy nucleon's contribution to the loop integral; i.e., $\Delta(k)\,G^+(P-k)$, which we denote: $\delta_F M_+^+$
  - To evaluate k_0 integral, close contour in lower half-plane, thereby encircling only the positive-energy pion pole.

$$\delta_F M_+^+ = -3g^2\int\frac{d^3k}{(2\pi)^3}\,\frac{\omega_N(\vec{k}^2)-M_0}{4\,\omega_N(\vec{k}^2)}\,\frac{1}{\omega_\pi(\vec{k}^2)\,[\omega_\pi(\vec{k}^2)+\omega_N(\vec{k}^2)-M_0]} \quad (9)$$

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Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

$$\delta_F M_+^+ = -3g^2 \int \frac{d^3k}{(2\pi)^3} \, \frac{\omega_N(\vec{k}^2) - M_0}{4\,\omega_N(\vec{k}^2)} \, \frac{1}{\omega_\pi(\vec{k}^2)\,[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_0]} \tag{10}$$

Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

$$\delta_F M_+^+ = -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\omega_N(\vec{k}^2) - M_0}{4\,\omega_N(\vec{k}^2)} \frac{1}{\omega_\pi(\vec{k}^2)\,[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_0]} \quad (14)$$

- On the domain for which the regularised integral has significant support, assume that M_0 is very much greater than all other mass scales.

$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \quad (15)$$

Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

$$\delta_F M_+^+ = -3g^2 \int \frac{d^3 k}{(2\pi)^3} \, \frac{\omega_N(\vec{k}^2) - M_0}{4\,\omega_N(\vec{k}^2)} \, \frac{1}{\omega_\pi(\vec{k}^2)\,[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_0]} \tag{18}$$

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$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \tag{19}$$

- Then

$$\delta_F M_+^+ \approx -3g^2 \int \frac{d^3 k}{(2\pi)^3} \, \frac{\vec{k}^2}{8\,M^2} \, \frac{1}{\omega_{\lambda_i}^2(\vec{k}^2)} \tag{20}$$

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- On the domain for which the regularised integral has significant support, assume that M_0 is very much greater than all other mass scales.

$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \tag{23}$$

- Then

$$\delta_F M_+^+ \approx -3g^2 \int \frac{d^3k}{(2\pi)^3} \, \frac{\vec{k}^2}{8\,M^2} \, \frac{1}{\omega^2_{\lambda_i}(\vec{k}^2)} \tag{24}$$

- So that

$$\frac{d^2\,\delta_F M_+^+}{(dm_\pi^2)^2} \approx -\frac{3g^2}{4M^2} \int \frac{d^3k}{(2\pi)^3} \, \frac{\vec{k}^2}{\omega_\pi^6(\vec{k}^2)} = -\frac{9}{128\pi} \, \frac{g^2}{M^2} \, \frac{1}{m_\pi} \,. \tag{25}$$

Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

$$\delta_F M_+^+ = -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\omega_N(\vec{k}^2) - M_0}{4\,\omega_N(\vec{k}^2)} \frac{1}{\omega_\pi(\vec{k}^2)\,[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_0]} \tag{26}$$

- On the domain for which the regularised integral has significant support, assume that M_0 is very much greater than all other mass scales.

$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \tag{27}$$

- Then

$$\delta_F M_+^+ \approx -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{8\,M^2} \frac{1}{\omega_{\lambda_i}^2(\vec{k}^2)} \tag{28}$$

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- Namely $\delta_F M_+^+ = -\frac{3}{32\pi}\frac{g^2}{M^2} m_\pi^3 + f_{(1)}^+(\lambda_1,\lambda_2)\,m_\pi^2 + f_{(0)}^+(\lambda_1,\lambda_2)$
where the last two terms express the necessary contribution from the regulator.

Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

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Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

- Nucleon's self energy

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Office of Nuclear Physics

Argonne National Laboratory

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Nucleon Self Energy: Chiral Limit

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- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.

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- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.
  - This is the Leading Nonanalytic Contribution much touted in effective field theory.
  - Its form is completely fixed by chiral symmetry and the pattern of its dynamical breaking.

NB. Contribution from negative energy nucleon is $\propto \dfrac{1}{M^3}$.

Nucleon Self Energy: Chiral Limit

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  NB. Contribution from negative energy nucleon is $\propto \dfrac{1}{M^3}$.

- The remaining terms are regular in the current-quark mass. Their exact nature depends on the explicit form of regularisation procedure.

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Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

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- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.
- The Leading Nonanalytic Contribution is a somewhat magical model-independent result.

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- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.
- The Leading Nonanalytic Contribution is a somewhat magical model-independent result.
- Unfortunately, it is not of much relevance in the real world. The actual value of the pion loop contribution to the nucleon's mass is completely determined by the regularisation dependent terms.
  - It is essential for a framework to veraciously express the leading nonanalytic contribution . . . it serves as a check that DCSB is truly described.
  - However, beyond that, one must accept that the world is messy.
    - The pion has a finite size. So does the nucleon.
    - These sizes set the mass-scale which determines the nucleon's mass shift.

Model pion-nucleon coupling

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$$g_{PV}(P,k) = \frac{g}{2M}\,\exp(-(P-k)^2/\Lambda^2)$$

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Model pion-nucleon coupling

$$g_{PV}(P,k) = \frac{g}{2M}\, \exp(-(P-k)^2/\Lambda^2)$$

- $\mathcal{B}$-Kernel

$$\int d\Omega_k\, g_{PV}^2((P-k)^2)\, \left[1 - \frac{2m_\pi^2}{(P-k)^2 + m_\pi^2}\right]$$

Clearly the sum of two independent terms.

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Model pion-nucleon coupling

$$g_{PV}(P,k) = \frac{g}{2M}\,\exp(-(P-k)^2/\Lambda^2)$$

- $\mathcal{B}$-Kernel

$$\int d\Omega_k\, g_{PV}^2((P-k)^2)\left[1 - \frac{2m_\pi^2}{(P-k)^2+m_\pi^2}\right]$$

- First term can be evaluated exactly

$$\begin{aligned}\bar{g}_{PV}^2(P^2,k^2) &= \int d\Omega_k\, g_{PV}^2((P-k)^2) \\ &= \frac{g^2}{4M^2}\,\mathrm{e}^{-2(P^2+k^2)/\Lambda^2}\,\frac{\Lambda^2}{2Pk}\,\mathrm{I}_1(4Pk/\Lambda^2)\,,\end{aligned}$$

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Model pion-nucleon coupling

$$g_{PV}(P,k) = \frac{g}{2M}\, \exp(-(P-k)^2/\Lambda^2)$$

- $\mathcal{B}$-Kernel

$$\int d\Omega_k\, g_{PV}^2((P-k)^2)\, \left[1 - \frac{2m_\pi^2}{(P-k)^2+m_\pi^2}\right]$$

- Second term can be approximated

$$\begin{aligned}\omega_{g^2}(P^2,k^2) &= 2\,m_\pi^2 \int d\Omega_k\, \frac{g_{PV}^2((P-k)^2)}{(P-k)^2+m_\pi^2} \\ &\approx g_{PV}^2(|P-k|^2)\frac{2\,m_\pi^2}{a+\sqrt{a^2-b^2}}\end{aligned}$$

- Reliable when analytic

structure of g_{PV} is not key to that of solution

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Model pion-nucleon coupling

$$g_{PV}(P,k) = \frac{g}{2M}\,\exp(-(P-k)^2/\Lambda^2)$$

- $\mathcal{B}$-Kernel

$$\int d\Omega_k\, g_{PV}^2((P-k)^2)\,\left[1 - \frac{2m_\pi^2}{(P-k)^2+m_\pi^2}\right]$$

- Total Kernel:

$$\begin{aligned} &\approx \quad \bar{g}_{PV}^2(P^2,k^2) - g_{PV}^2(|P^2-k^2|)\frac{2\,m_\pi^2}{a+\sqrt{a^2-b^2}}\,, \\ &=: \quad \bar{g}_{PV}^2(P^2,k^2) - \tilde{g}_{PV}^2(P^2,k^2))\frac{2\,m_\pi^2}{a+\sqrt{a^2-b^2}}\,, \end{aligned}$$

- Analytic structure is transparent

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Nucleon's self energy and mass shift

Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$M_D^2 \mathcal{A}^2(-M_D^2) = [M + \mathcal{B}(-M_D^2)]^2$$
$$\delta M = M_D - M$$

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Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$
\begin{aligned}
M_D^2 \mathcal{A}^2(-M_D^2) &= [M + \mathcal{B}(-M_D^2)]^2 \\
\delta M &= M_D - M
\end{aligned}
$$

- Vector self energy

Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$M_D^2 \mathcal{A}^2(-M_D^2) = [M + \mathcal{B}(-M_D^2)]^2$$
$$\delta M = M_D - M$$

- Scalar self energy

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Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$\begin{aligned} M_D^2 \mathcal{A}^2(-M_D^2) &= [M + \mathcal{B}(-M_D^2)]^2 \\ \delta M &= M_D - M \end{aligned}$$

	(Λ, Λ_N)	(Λ, Λ_N)	(Λ, Λ_N)
	$(0.9, \infty)$	$(0.9, 1.5)$	$(0.9, 2.0)$
$-\delta M$ (MeV)	222	61	99

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	$(0.9, \infty)$	$(0.9, 1.5)$	$(0.9, 2.0)$
$-\delta M$ (MeV)	222	61	99

- No suppression for nucleon off-shell in self-energy loop; i.e, $g_{PV}((P-k^2), P^2, k^2)$

Neglected this dependence

Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$\begin{aligned} M_D^2 \mathcal{A}^2(-M_D^2) &= [M + \mathcal{B}(-M_D^2)]^2 \\ \delta M &= M_D - M \end{aligned}$$

	(Λ, Λ_N)	(Λ, Λ_N)	(Λ, Λ_N)
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$$g_{PV}(P^2, k^2, P\cdot k) = \frac{g}{2M}\,\mathrm{e}^{-(P-k)^2/\Lambda^2}\,\mathrm{e}^{-(P^2+M^2+k^2+M^2)/\Lambda_N^2}$$

- Correct on-shell limit:

$$g_{PV}(P^2 = -M^2, k^2 = -M^2, (P-k)^2 = 0) = \frac{g}{2M}$$

Nucleon's self energy and mass shift

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Range from meson exchange model phen.

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- $\Lambda_N \to \infty \Rightarrow$ pointlike nucleon

Pion loop's effect

- Nonpointlike πN-loop

  . . . reduces nucleon's mass by $\sim 100\,$MeV

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- $$\boxed{-\delta M_N \sim 200\,\text{MeV}}$$

- Qualitative effect of this?

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Too much of a good thing

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Too much of a good thing

- Refit Faddeev model parameters,

  allowing for heavier “quark-core” mass

Too much of a good thing

	ω_{0^+}	ω_{1^+}	M_N	M_Δ	ω_{f_1}	ω_{f_2}	R
0^+	0.45	-	1.44	-	0.36	0.35	2.32
0^+ & 1^+	0.45	1.36	1.14	1.33	0.44	0.36	0.54
0^+	0.64	-	1.59	-	0.39	0.41	1.28
0^+ & 1^+	0.64	1.19	0.94	1.23	0.49	0.44	0.25

- 50% reduction in role of axial-vector diquark

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- 50% reduction in role of axial-vector diquark
- 10% increase in role of scalar diquark

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Unsurprisingly:

Requiring Exact Fit to N, Δ masses

with only q, $(qq)_{J^P}$ Degrees of Freedom

$\Rightarrow$ Forces 1^+ to mimic, in part, effect of π

Pseudoscalar mesons and Form Factors

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- Another example … pseudoscalar mesons also contribute materially to form factors.
- Illustrate with $\gamma N \to \boldsymbol{\Delta}$ transition form factor. Focus on the M1 (spin-flip) form factor, $G_M^*(Q^2)$.

Pions and Form Factors

Pions and Form Factors

- Dynamical coupled-channels model ... Analyzed extensive JLab data ... Completed a study of the $\Delta(1236)$
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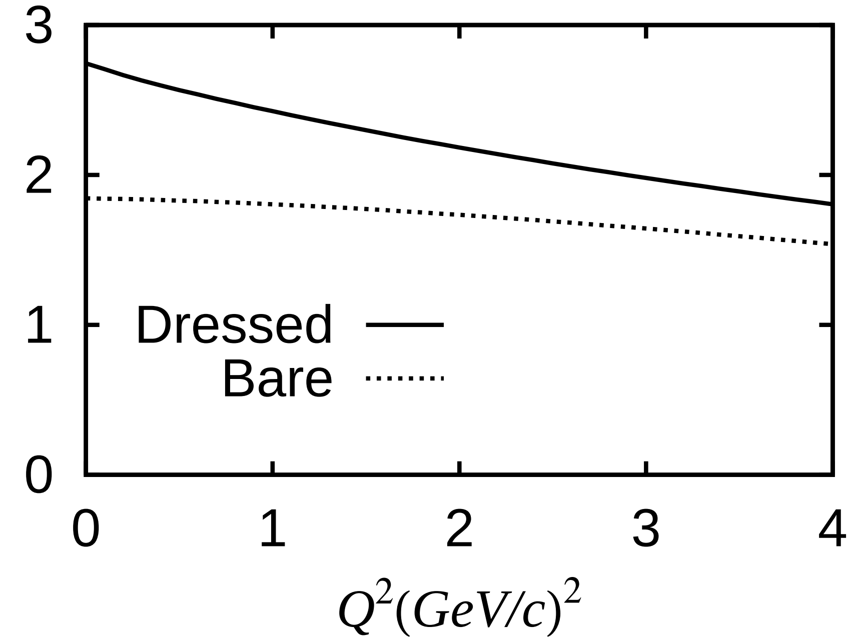
Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



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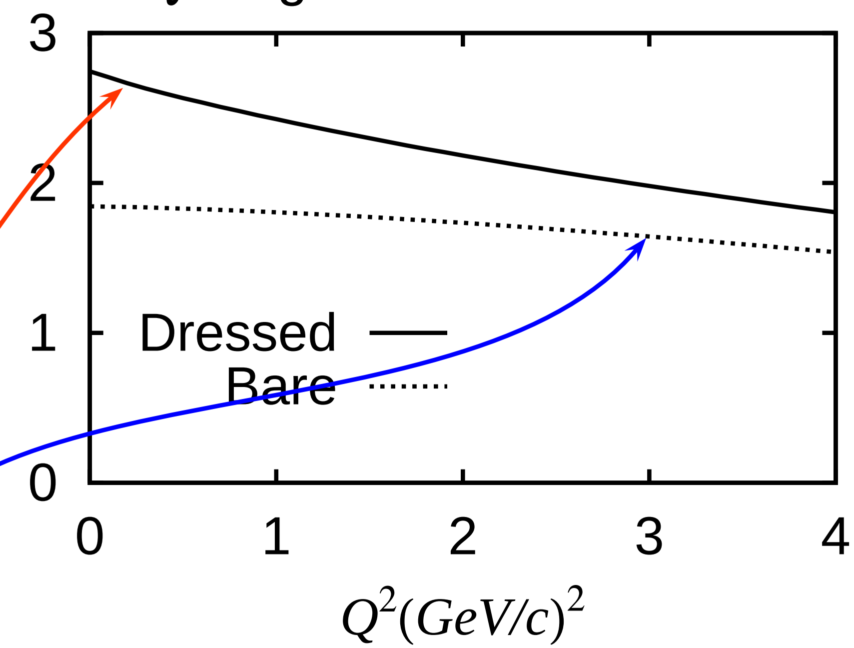

Quark Core

- Responsible for only 2/3 of result at small Q^2
- Dominant for Q^2 >2 – 3 GeV^2

Results: Nucleon and Δ Masses

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Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for
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A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
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Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically
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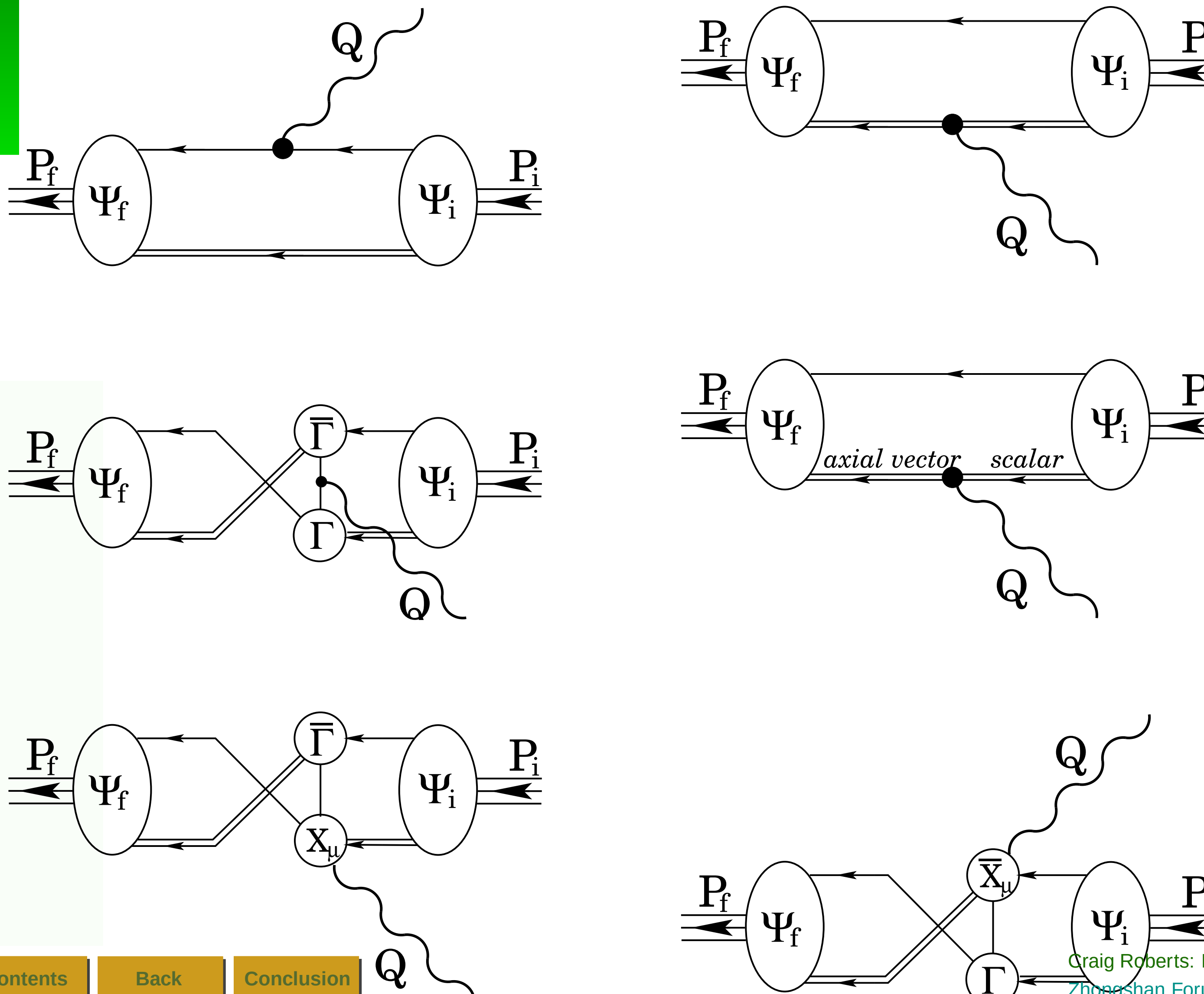
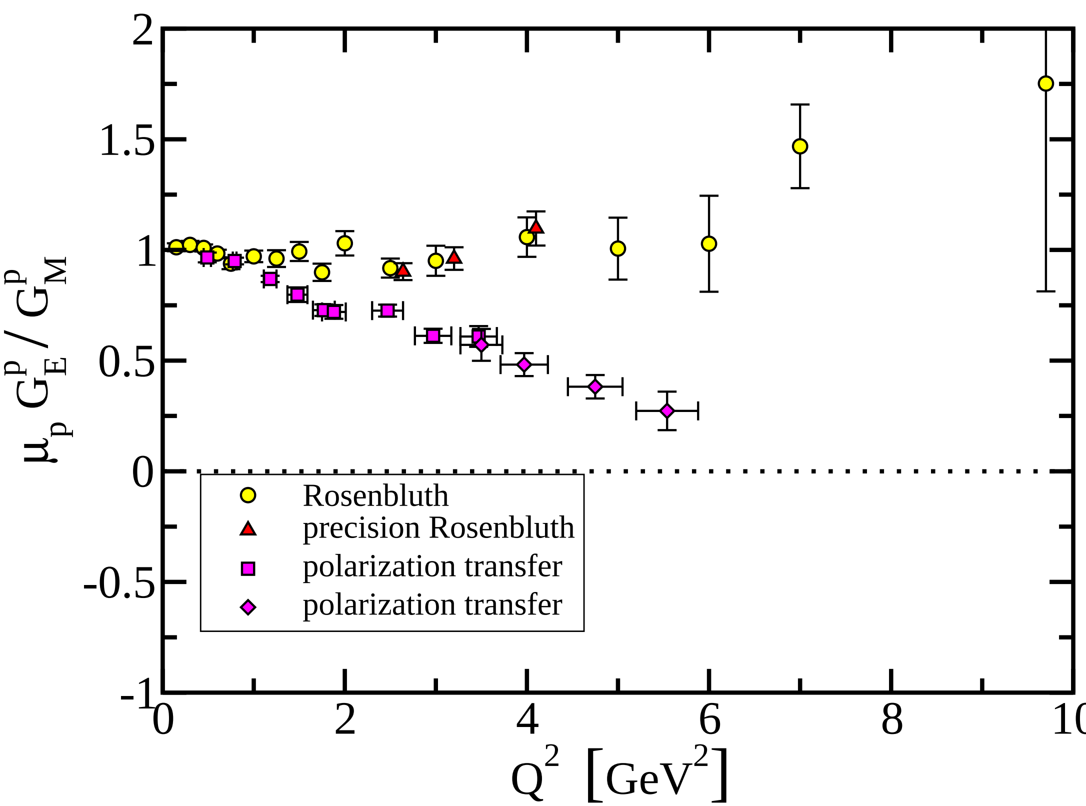


Form Factor Ratio: GE/GM



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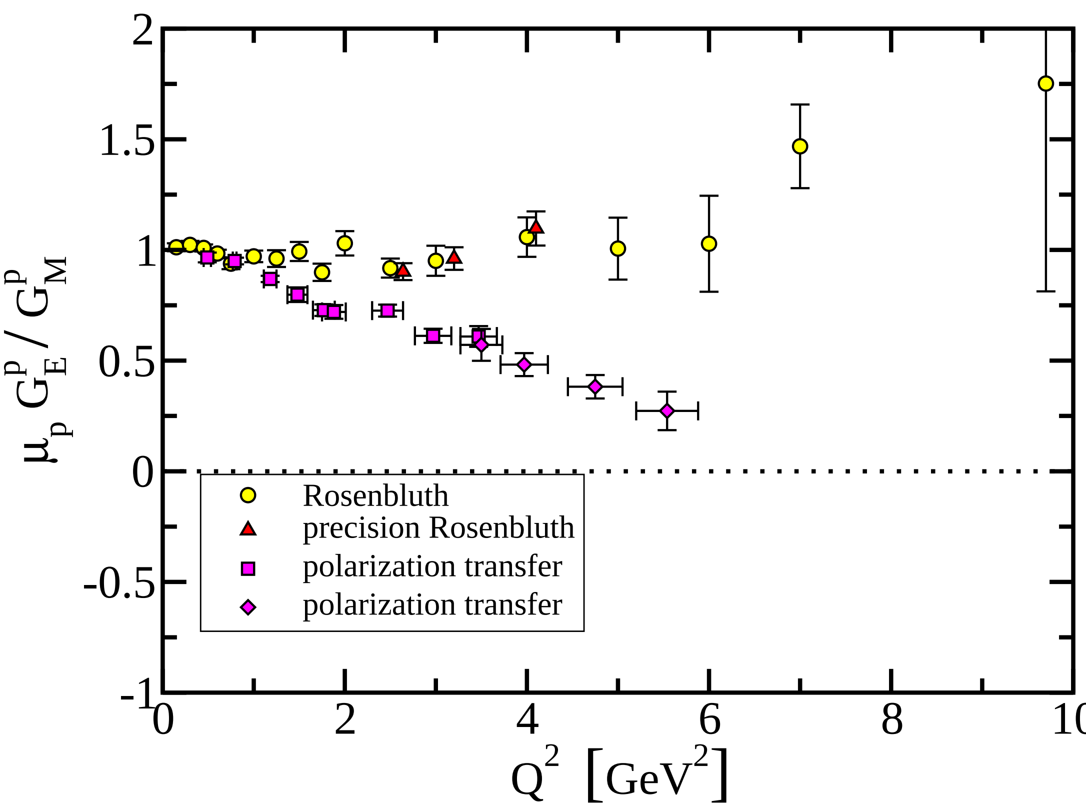

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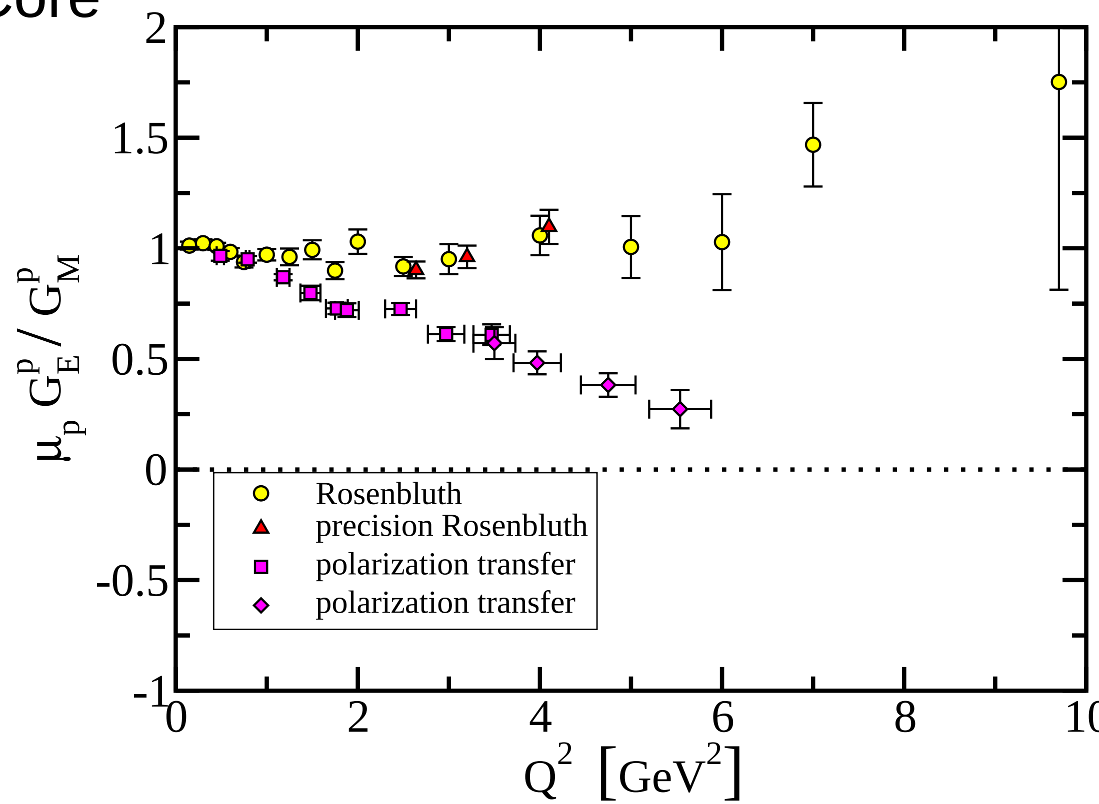

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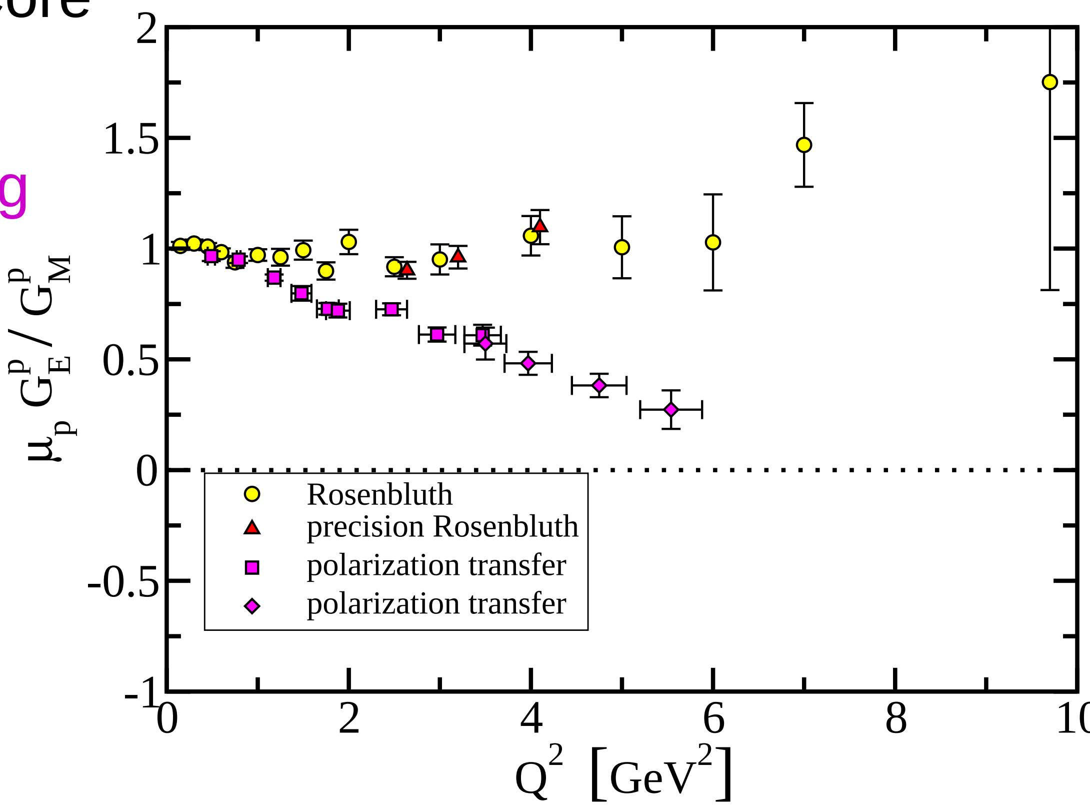

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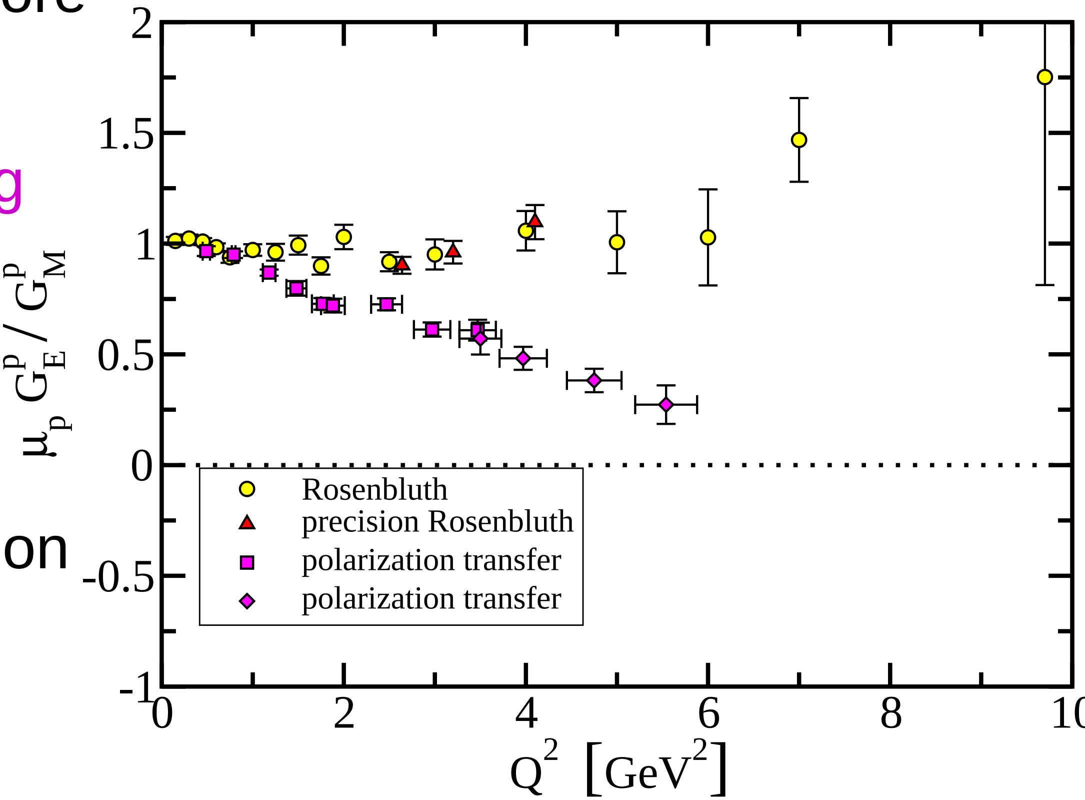

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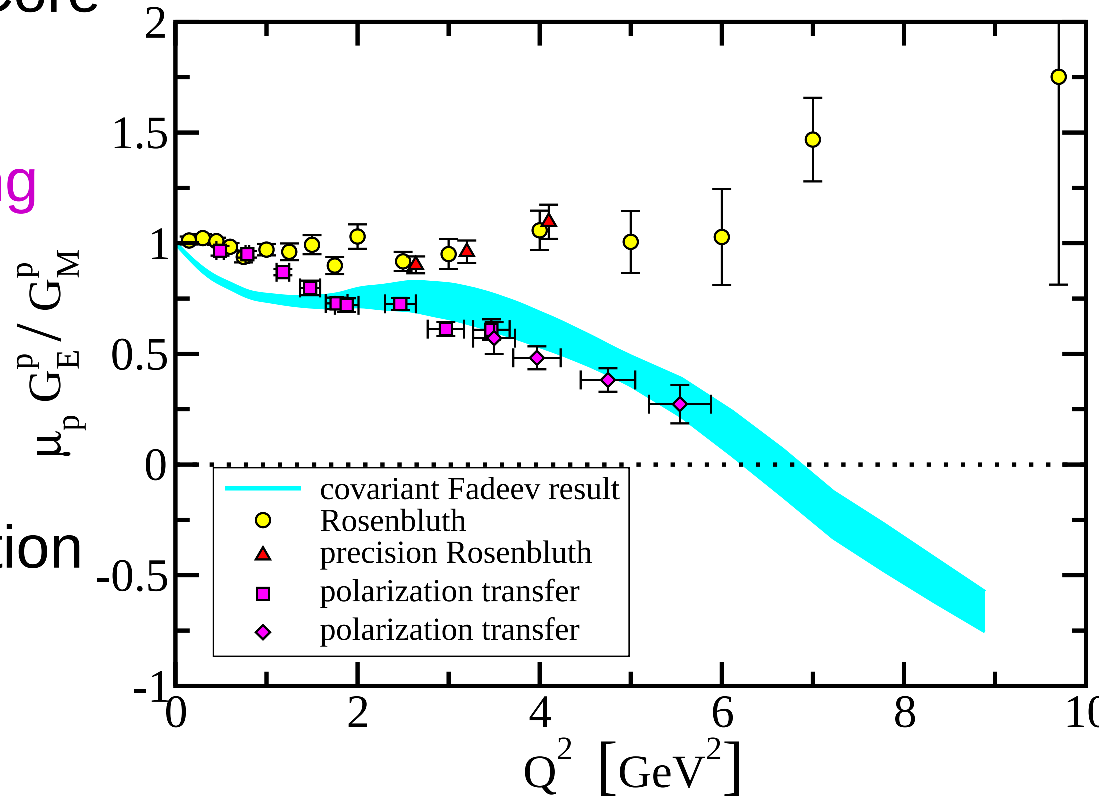

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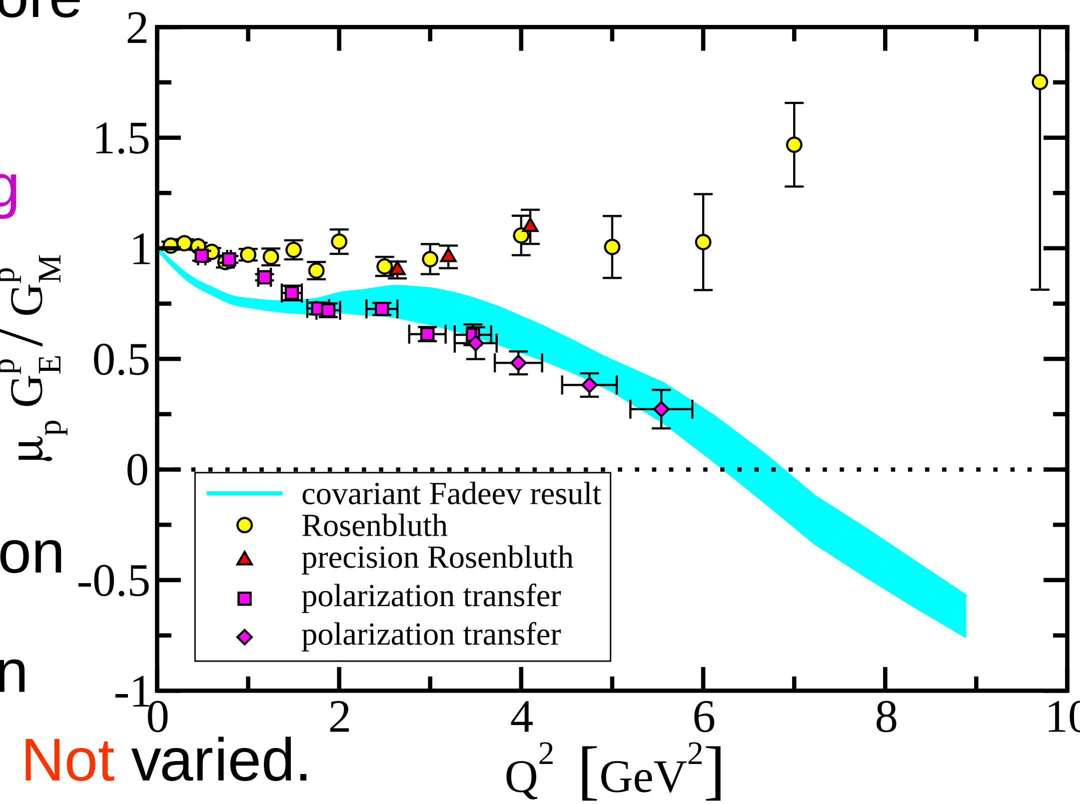

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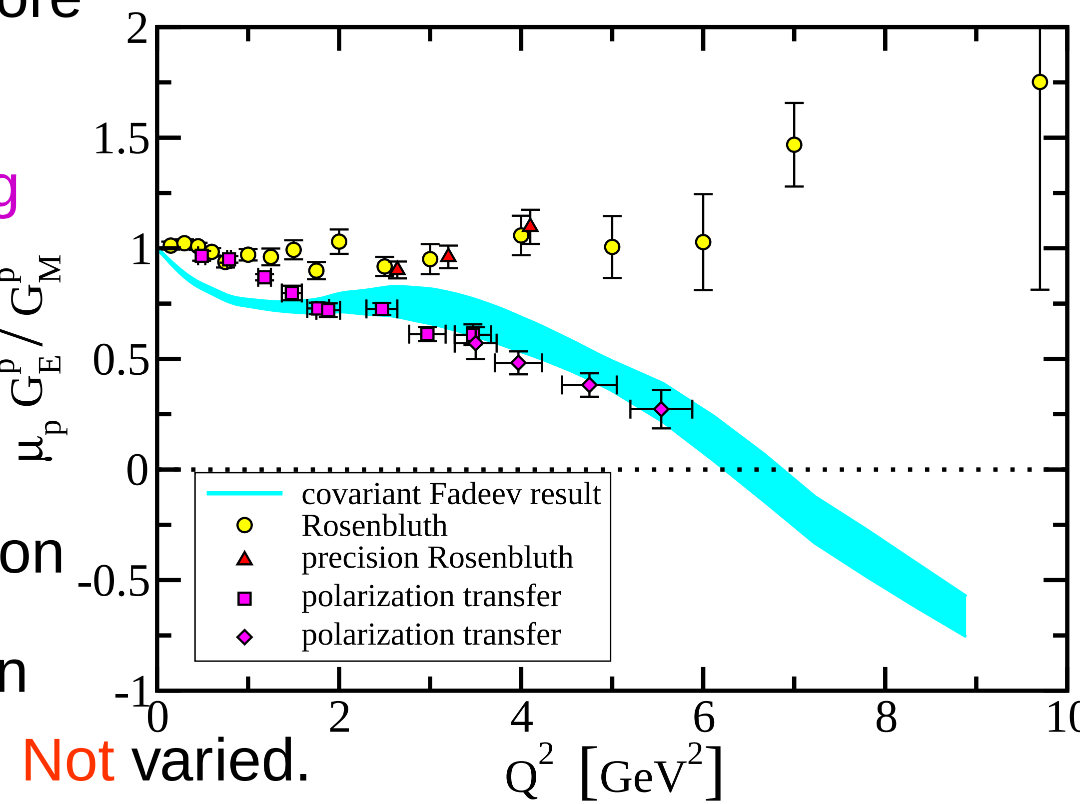

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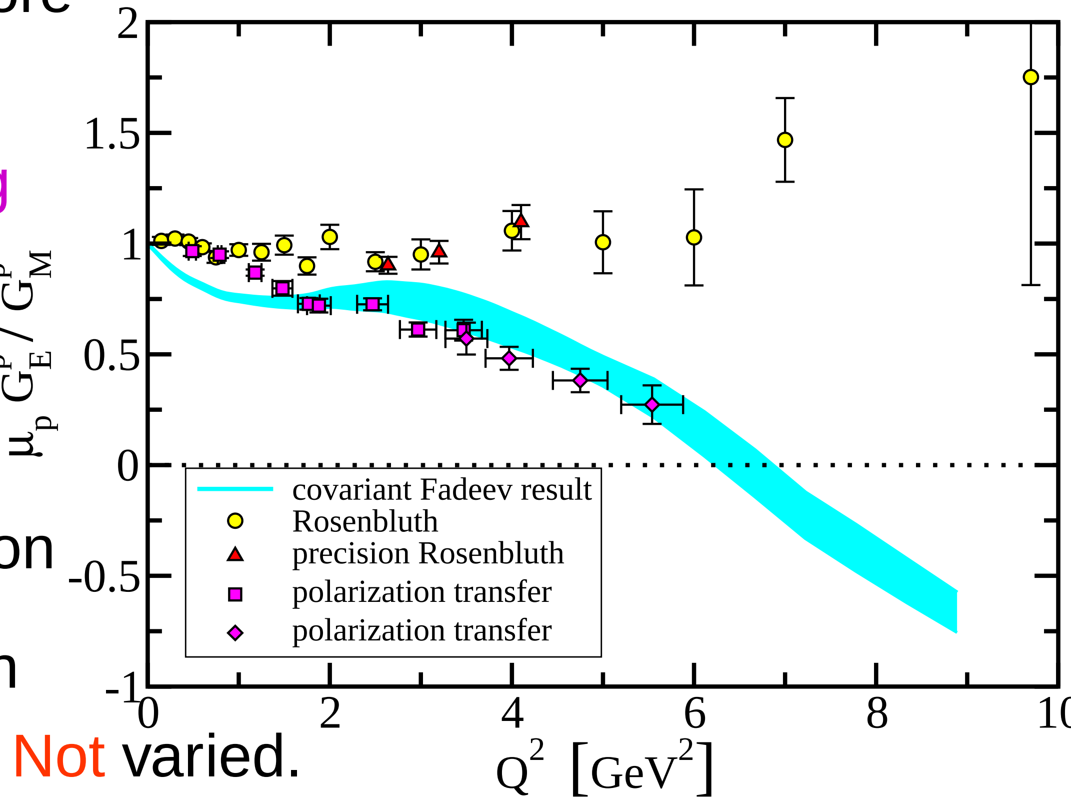

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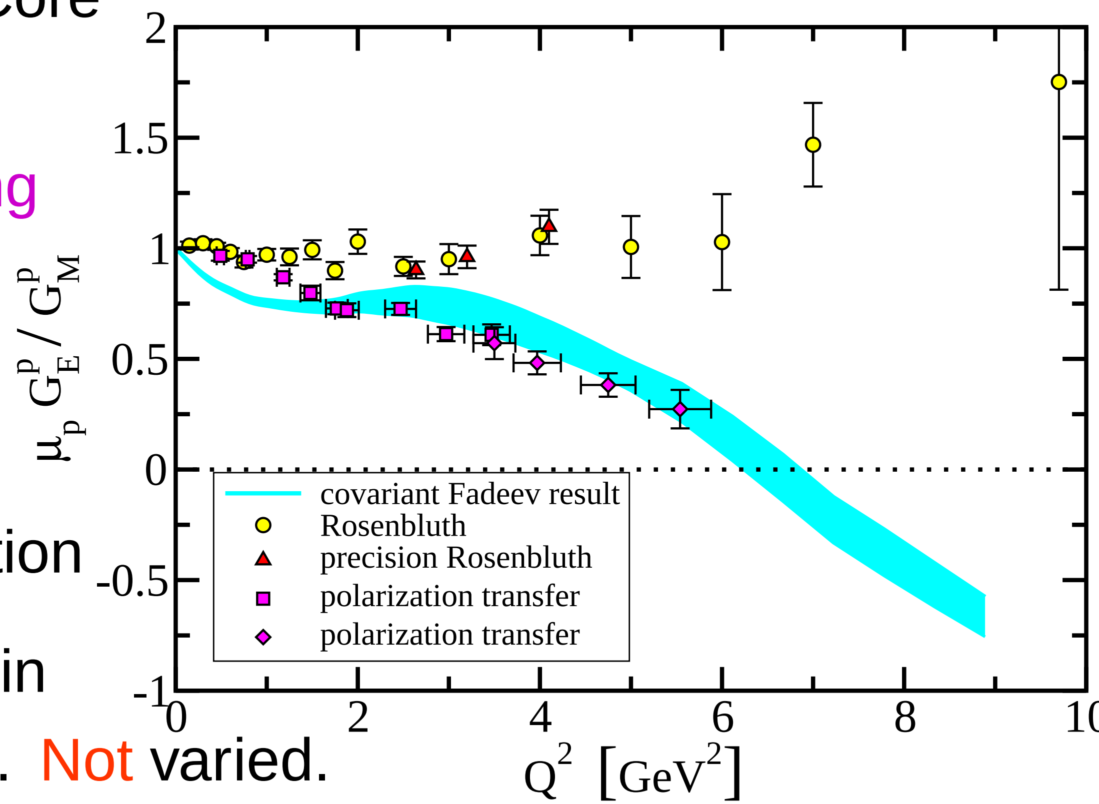

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  - Predict Zero at $Q^2 \approx 6.5\text{GeV}^2$

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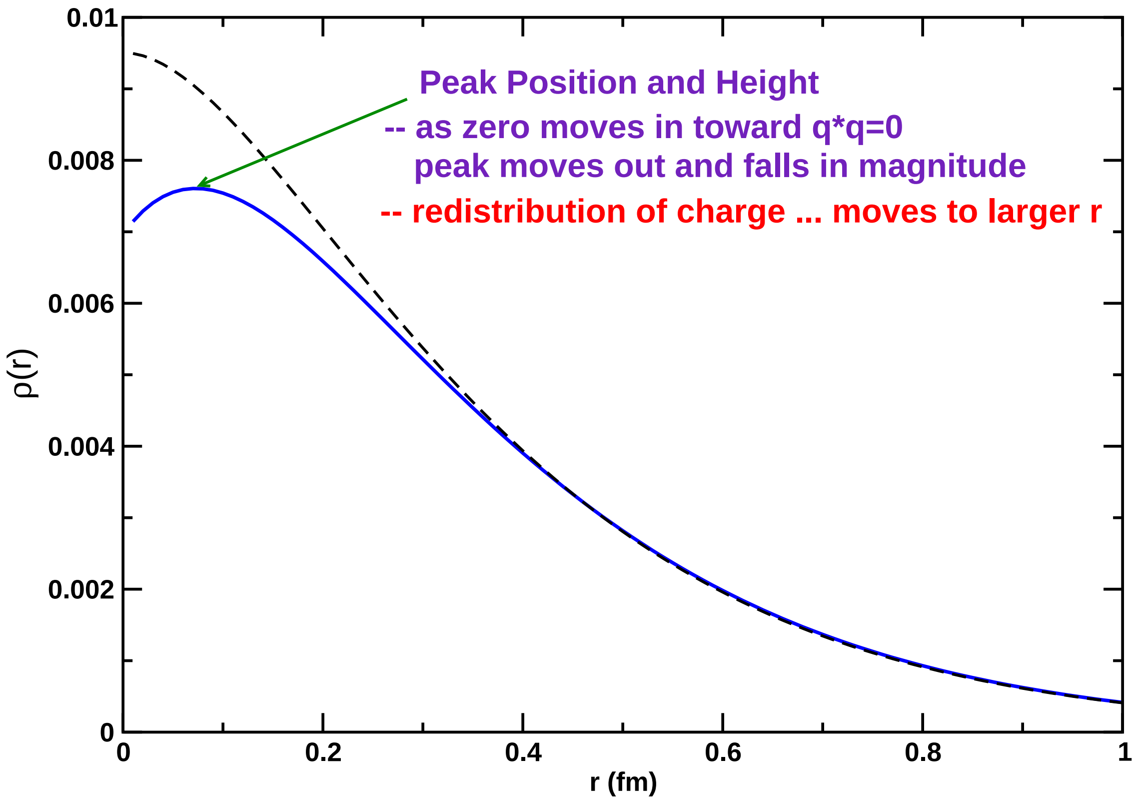

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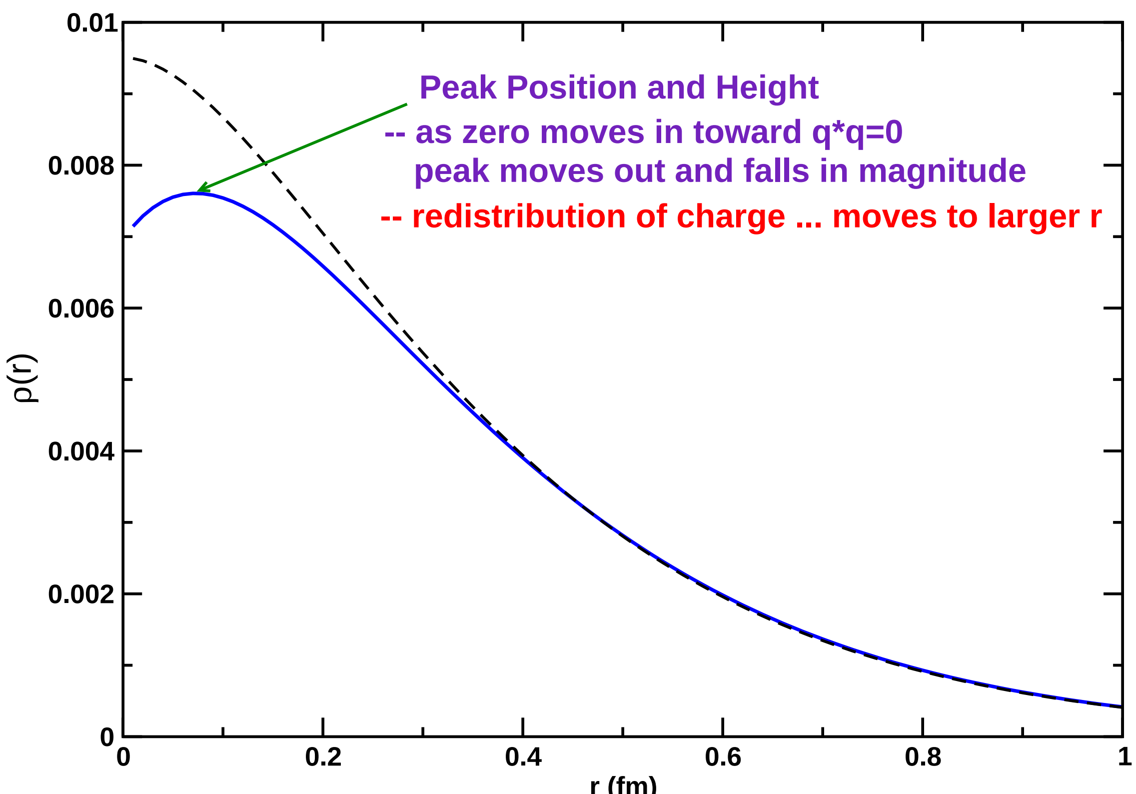

- However, Current Density remains peaked at $r=0$!

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U.S. DEPARTMENT OF ENERGY
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Exploring Nuclear Matter - Quarks to Stars
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- *Simple independent-particle three-quark bag-model picture is profoundly incorrect*

Improved current

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    - Diquark effectively pointlike to hard probe
    - Didn't account for diquark being off-shell in recoil

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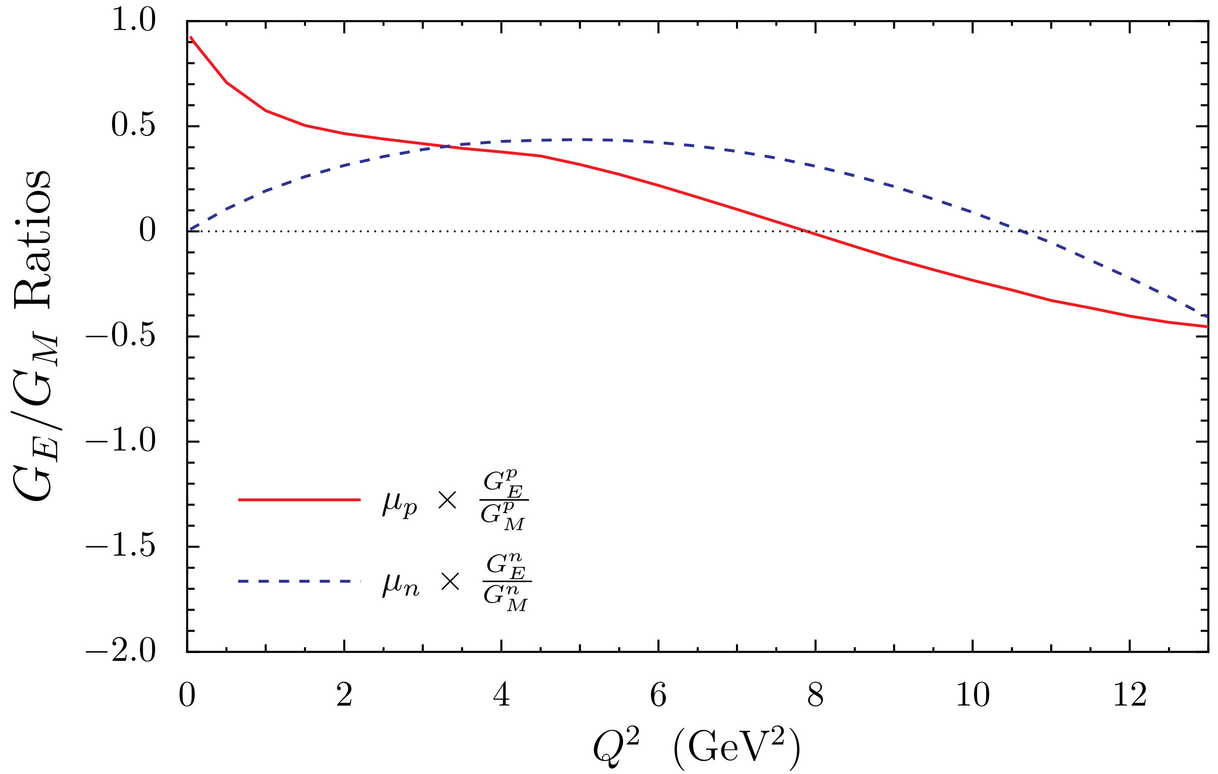

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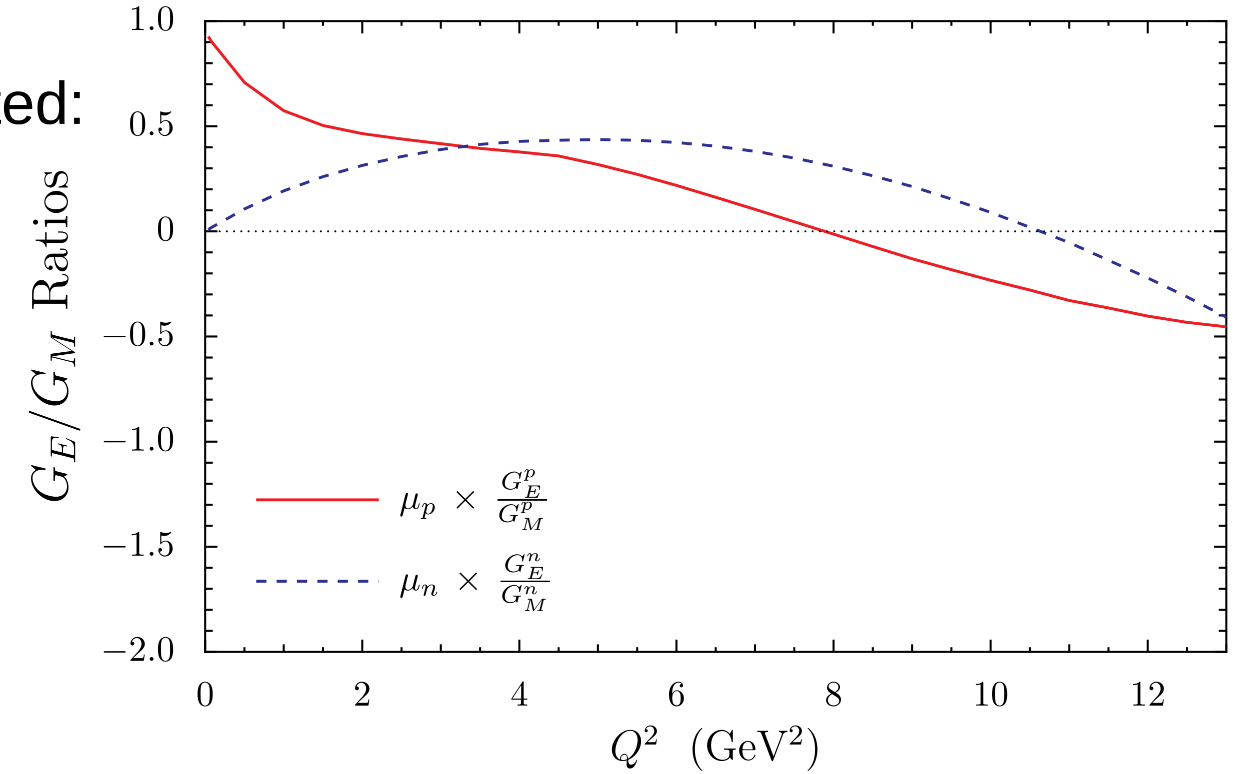

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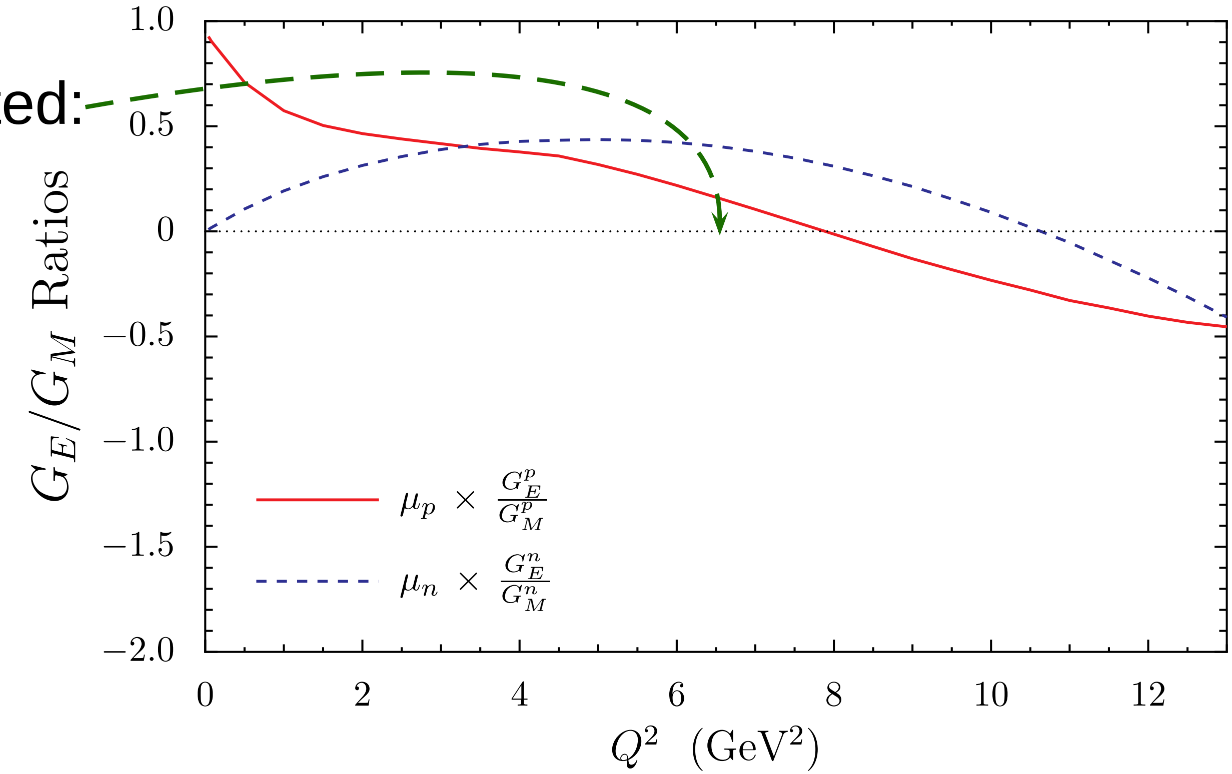

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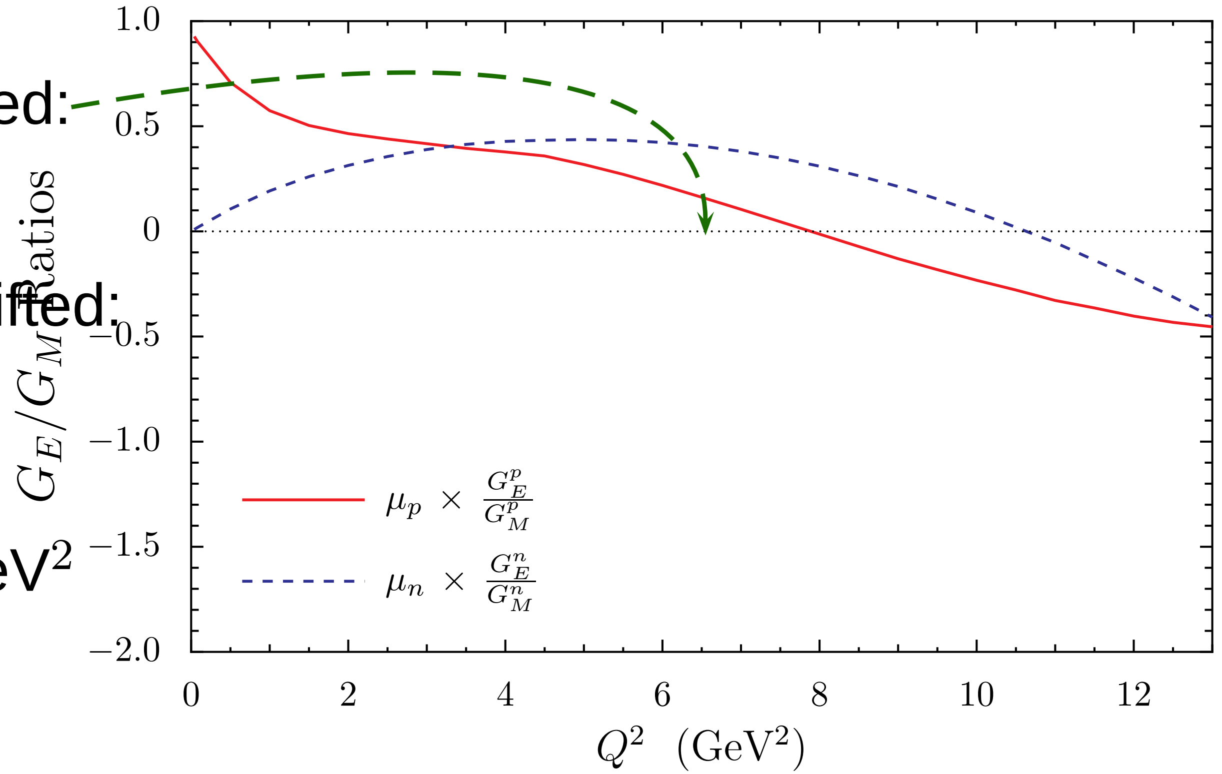

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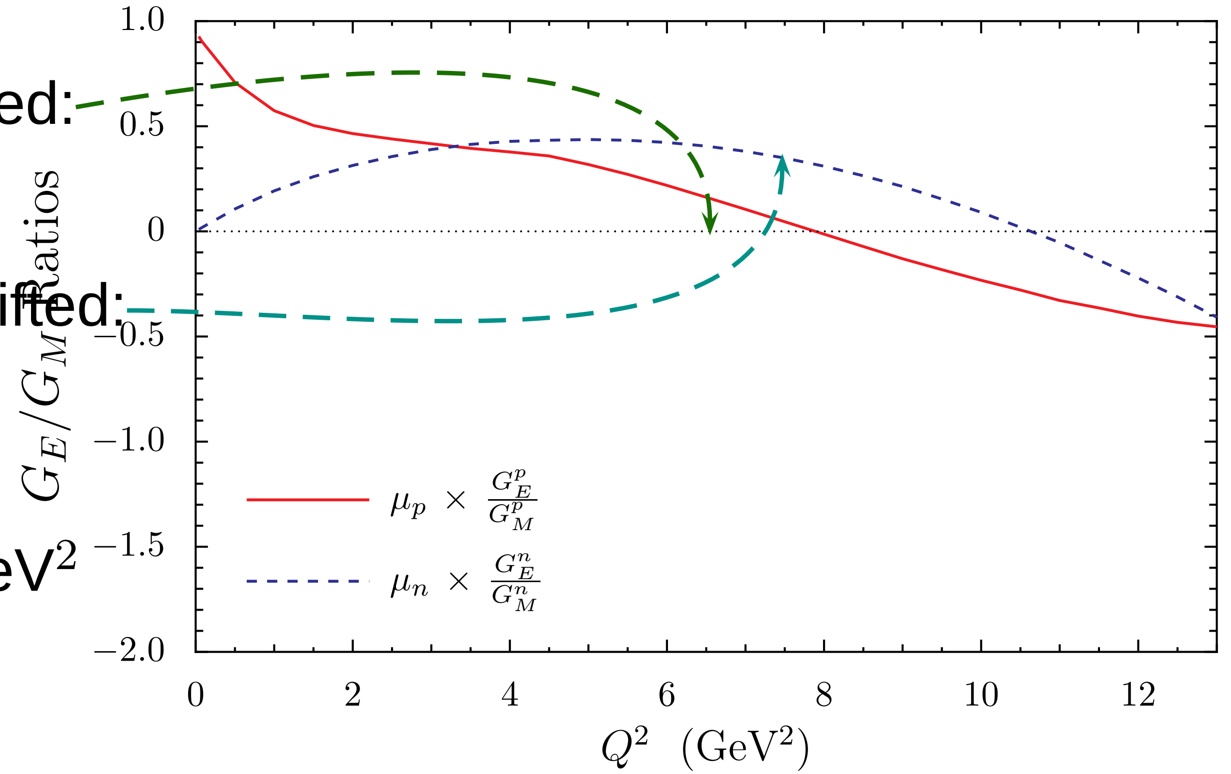

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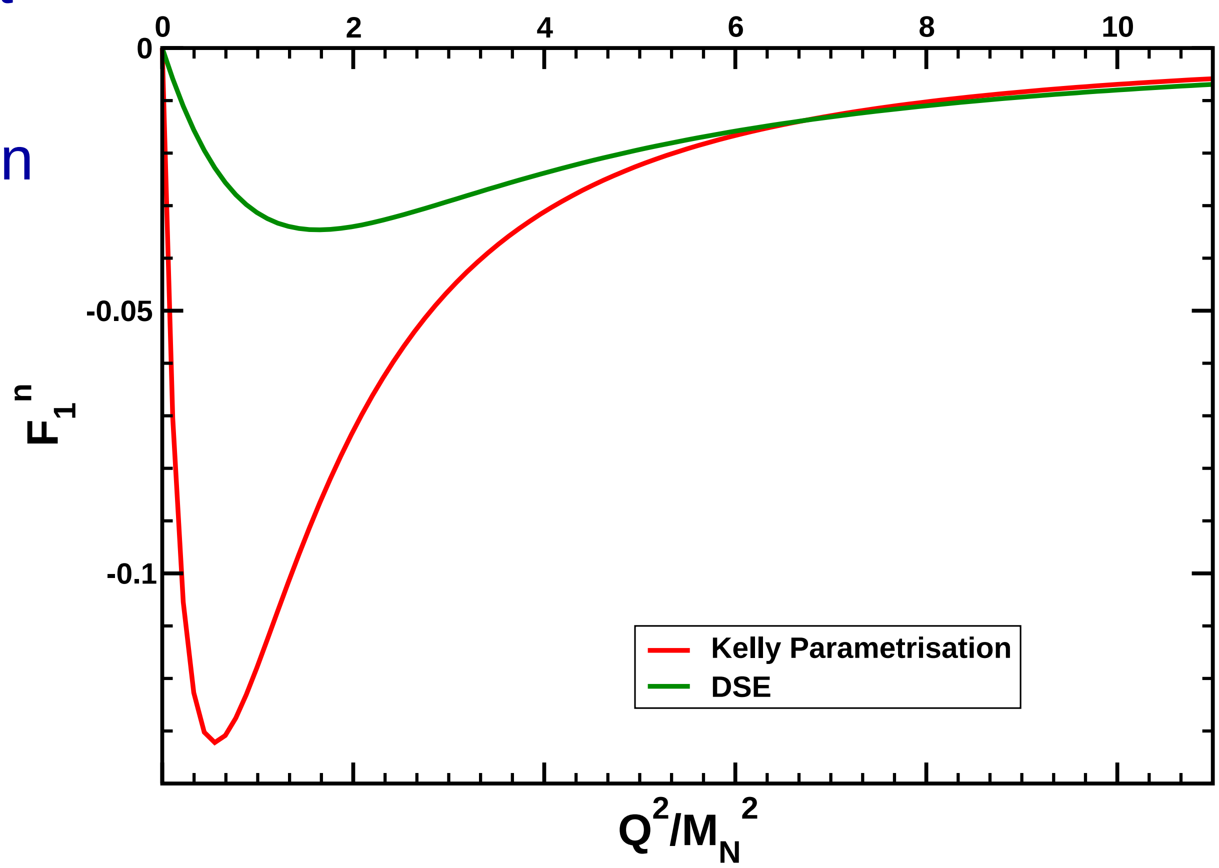
Pion Cloud

- Comparison between Faddeev equation result and Kelly's parametrisation



Pion Cloud
F1 – neutron

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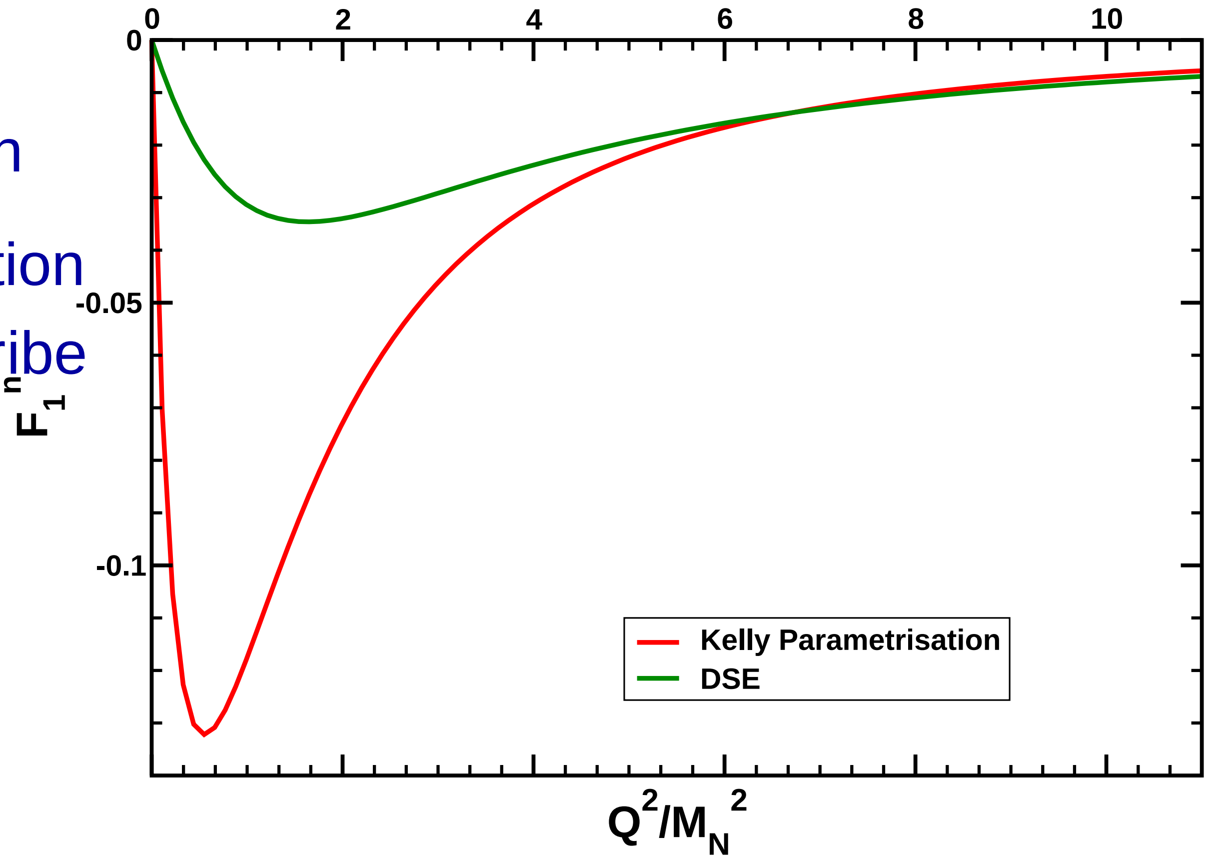
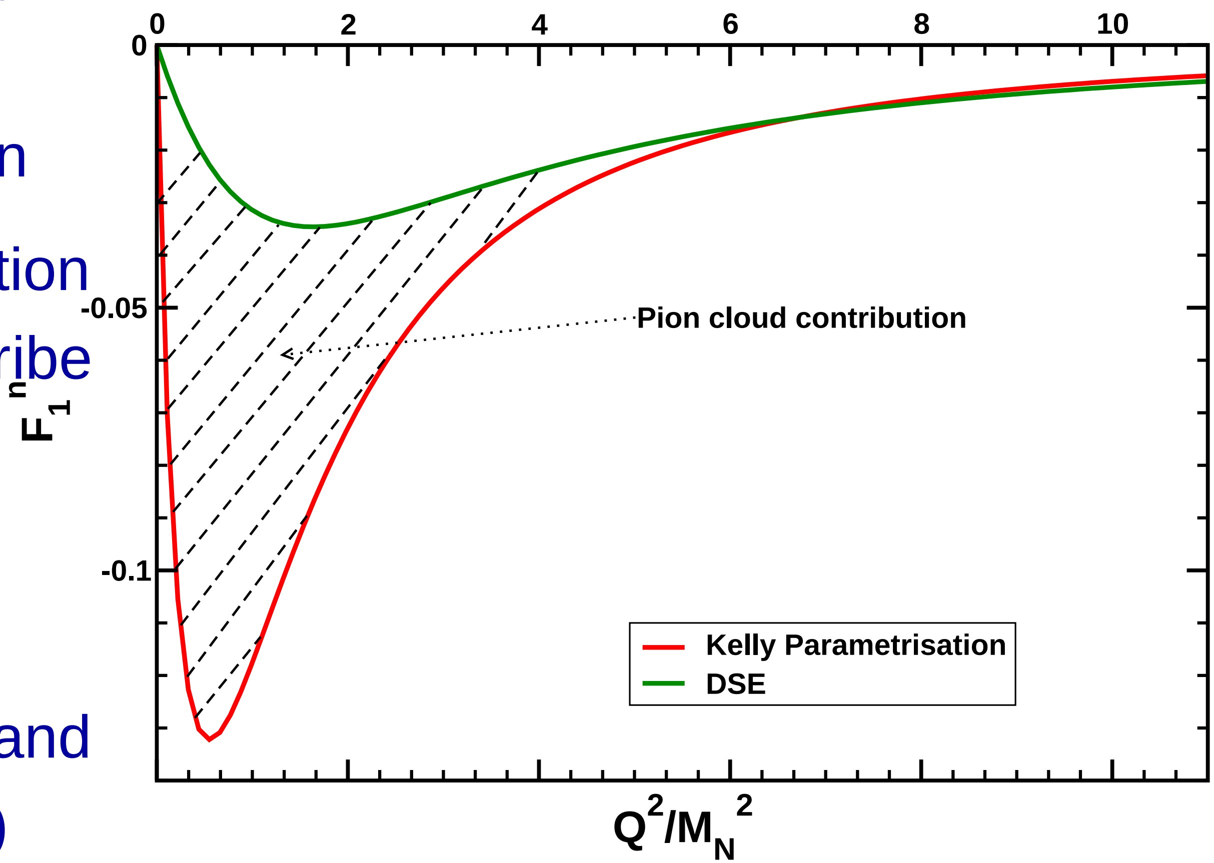


- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core
- Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4\, M_N^2$



Next Five Years

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- What is the internal landscape of the nucleons? . . . “For many years, we have known that the nucleons are composite particles made up of quarks and gluons, and we have partial answers concerning the internal structure of protons and neutrons from years of measurements with high-energy probes. New experiments will provide an unprecedented, tomographic view of the quarks and their motion inside the nucleons, and map the distributions of quarks and gluons in space, momentum, type of quark, and spin orientation.”

Next Five Years

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- What does QCD predict for the properties of strongly interacting matter? . . . "A critical step in the quest to understand strongly interacting matter is to confront the results of experiments with the quantitative implications of QCD. Doing so is exceedingly challenging because the strong force cannot be accurately described at the relevant scales by means of analytical calculations. Future progress will require extensive numerical simulations on a scale that has never before been undertaken."

Next Five Years

- Long Range Plan: http://www.sc.doe.gov/np/nsac/nsac.html
- What governs the transition of quarks and gluons into pions and nucleons? . . . "Nucleons, and the pions that bind them together into atomic nuclei, must emerge from nearly massless quarks and gluons. The process that transforms deconfined matter into hadrons and nuclei remains poorly understood. Dedicated measurements are needed to launch a new stage in understanding both how quarks accrete partners from the vacuum or debris of high-energy collisions to form hadrons, and how the interaction among protons and neutrons arises from QCD."

Next Five Years

- Long Range Plan: http://www.sc.doe.gov/np/nsac/nsac.html
- What are the phases of strongly interacting matter and what roles do they play in the Cosmos? . . . "QCD predicts that soon after the birth of the universe, a sea of quarks and gluons – the quark-gluon plasma – coalesced into protons and neutrons. Can we replicate that transition in the laboratory by creating a high-temperature, high-density environment that temporarily frees quarks from their normal confinement within protons and neutrons? Experiments to address this most fundamental question have produced tantalizing indications of just such a transition. Further studies will lead to an understanding of matter in the early universe and provide important clues on matter as it now exists in the interior of compact stars."

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- Au + Au collisions create conditions kindred to those shortly after the "Big Bang" . . . looking for a quark-gluon plasma . . . deconfinement and chiral symmetry restoration

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- That's an enormous challenge
  . . . Requires vast diversity of theory
  - From relativistic hydrodynamics . . .
  - . . . to equilibrium thermal quantum field theory

Thermal Field Theory

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- Necessary precursor to exploring the response of hadron properties to chemical potential and temperature.

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- First steps toward elucidating the Phase Diagram of QCD
- Particularly noteworthy that, while numerical simulations of lattice-regularised QCD are contributing toward our understanding of $T \neq 0$, contemporary algorithms are inapplicable at $\mu \neq 0$.
- Moreover, lattice methods have hitherto provided little in connection with the evolution of hadron properties.

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  - fermions . . . $\boldsymbol{ip_0 \to \omega_n = (2n+1)\pi T\,,\ n \in \mathcal{Z}}$
  - bosons . . . $\boldsymbol{ip_0 \to \Omega_n = 2n\pi T\,,\ n \in \mathcal{Z}}$

NB. For fermions, $\boldsymbol{\omega_0 = \pi T}$ . . . this is a *crucial* difference between fermions and bosons ($\Omega_0 = 0$)

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- Within the Matsubara formalism, time is "lost" . . . the system is in equilibrium. However, it can be recovered by summing over all Matsubara frequencies and subsequently performing an analytic continuation.

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- In practical terms this means $\boldsymbol{ip_0 \to ip_0 + i\mu}$

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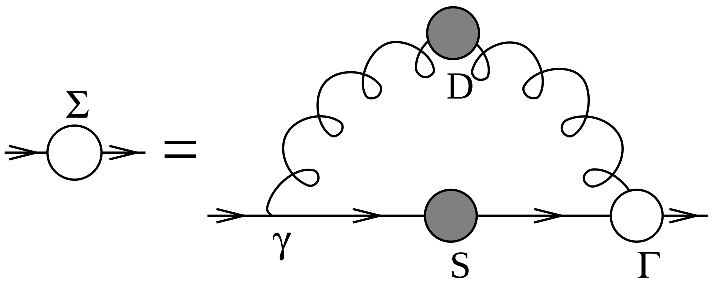

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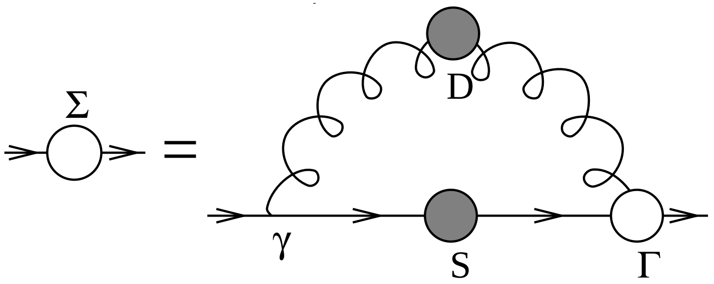

$$D_{\mu\nu}(p) = \left[\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right] 2\pi^2\eta^2 \, \frac{2\pi}{T}\delta_{0\,k}\,\delta^3(\vec{p})\,, \quad \Gamma_\nu = \gamma_\nu$$

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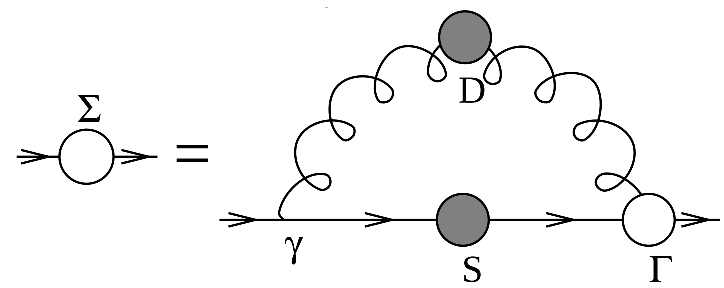

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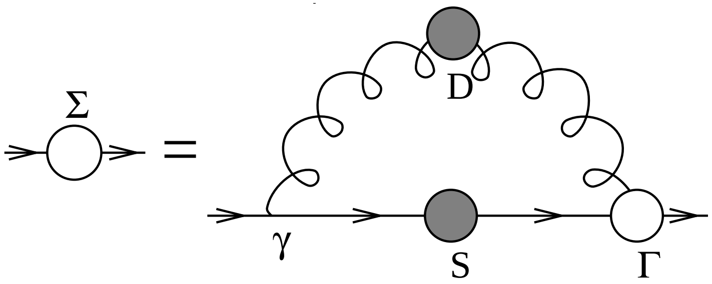

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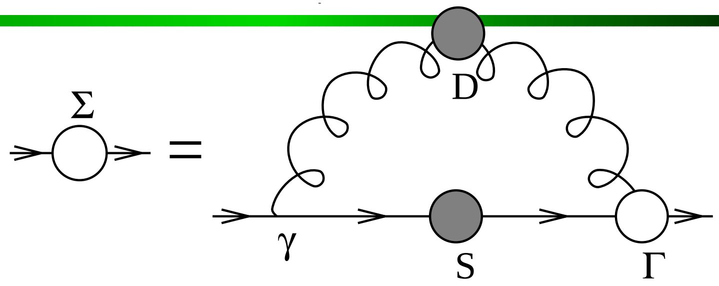
- Simplicity now apparent: model allows reduction of an integral equation to an algebraic equation. Extremely useful step toward developing an intuitive understanding of complicated phenomena.

Model Gap Equation



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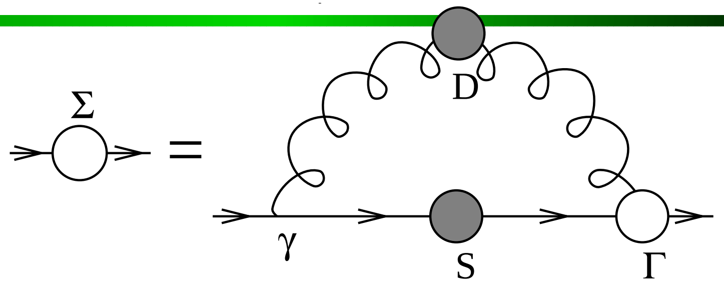

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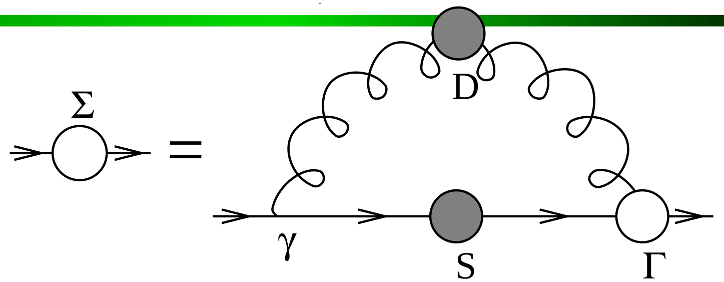

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- Three scalar functions: two vector self-energies: A, C, and one scalar self-energy, B.
  In solution of gap equation they express dressing owing to emission and reabsorption of hot gluons

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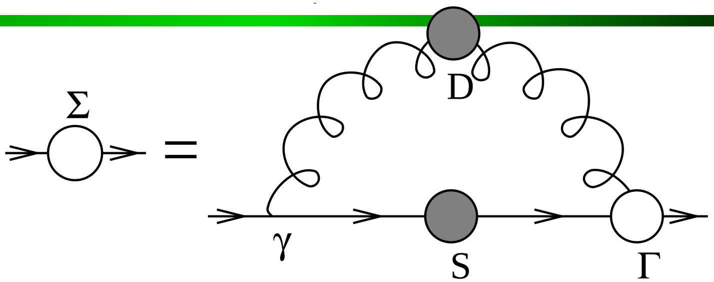

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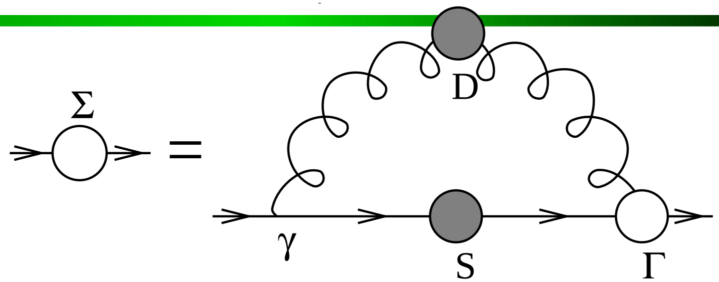

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- Structure is typical; i.e., nonlinear, coupled equations

Phases of QCD

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if $\pi T > \dfrac{\eta}{2}$; namely, for $T > 169$ MeV
- Critical temperature:

$$\boxed{T_c \approx 170\,\text{MeV}}$$

. . . For temperatures greater than this only the Wigner phase can be realised

Phase Transition

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$$
\begin{aligned}
\mathcal{B}(\mu,T) &:= p_{\Sigma_{\rm NG}}(\mu,T) - p_{\Sigma_{\rm W}}(\mu,T)\,, \\
&= 4N_c \int_{l,q}^{\bar\Lambda} \left\{ \ln\left[\frac{|\vec{p}|^2 A^2 + \tilde\omega_k^2 C^2 + B^2}{|\vec{p}|^2 \hat{A}^2 + \tilde\omega_k^2 \hat{C}^2}\right] \right. \\
&\quad \left. + |\vec{p}|^2\left(\sigma_A - \hat\sigma_A\right) + \tilde\omega_k^2\left(\sigma_C - \hat\sigma_C\right) \right\}
\end{aligned}
$$

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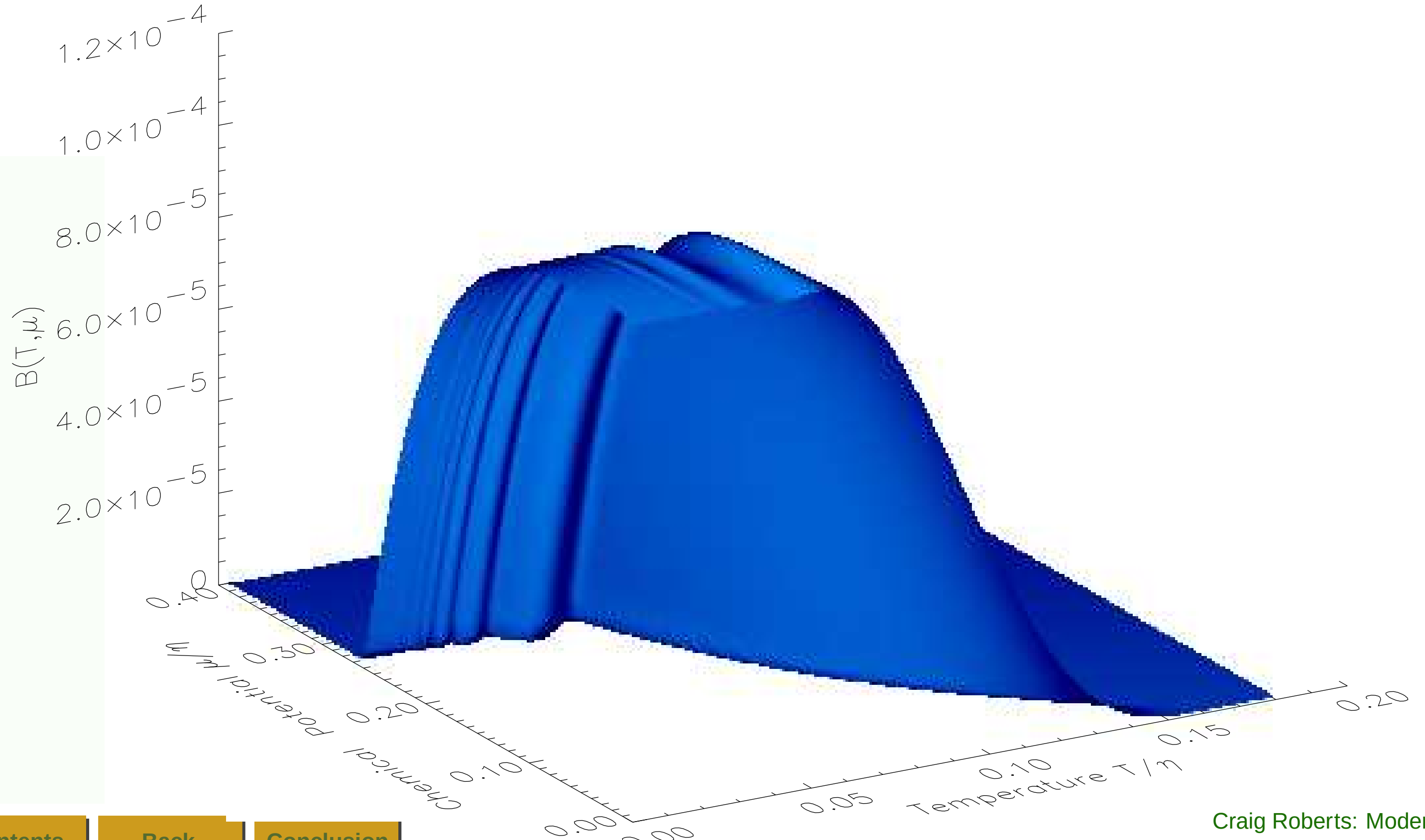

Phase Transition

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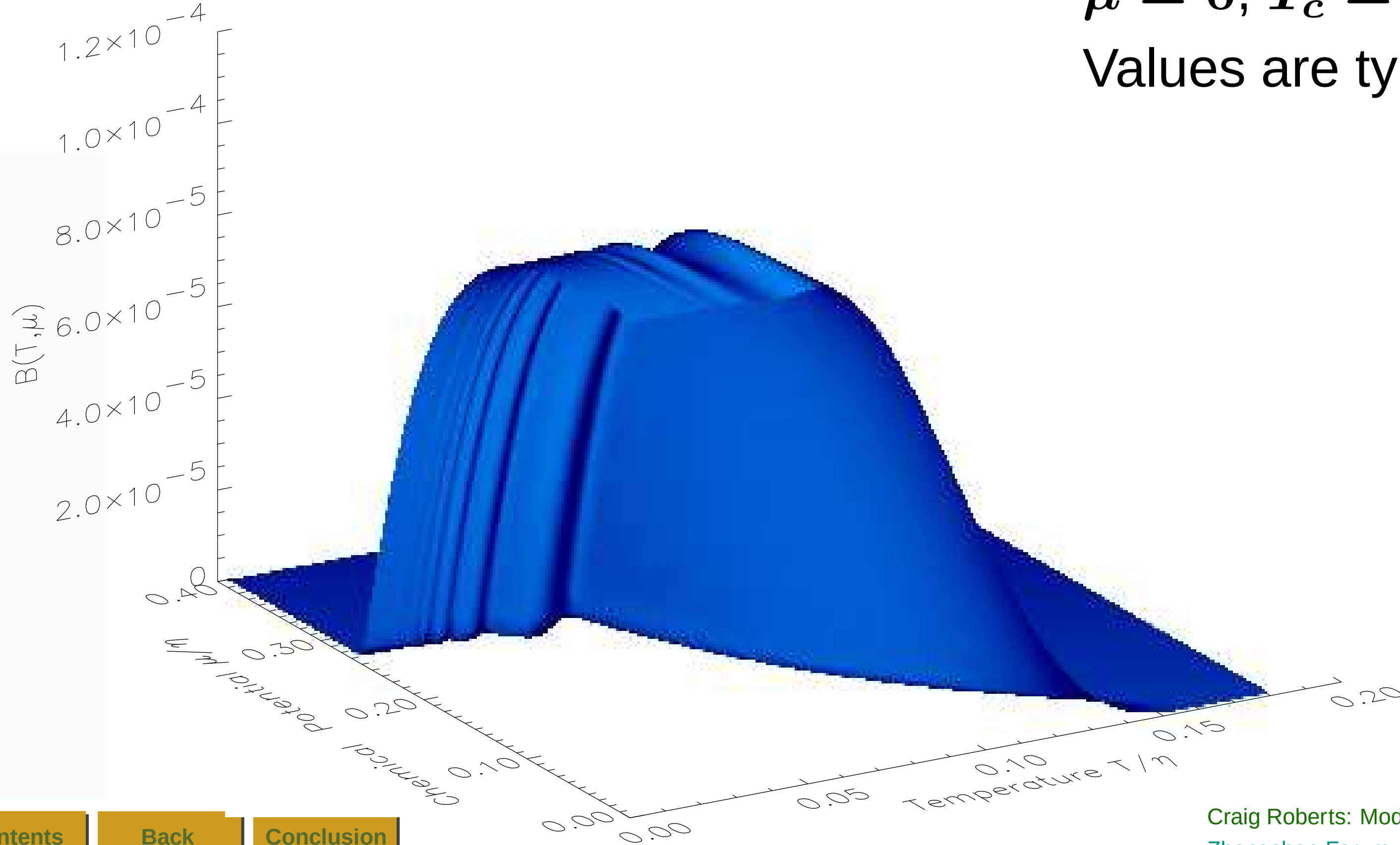
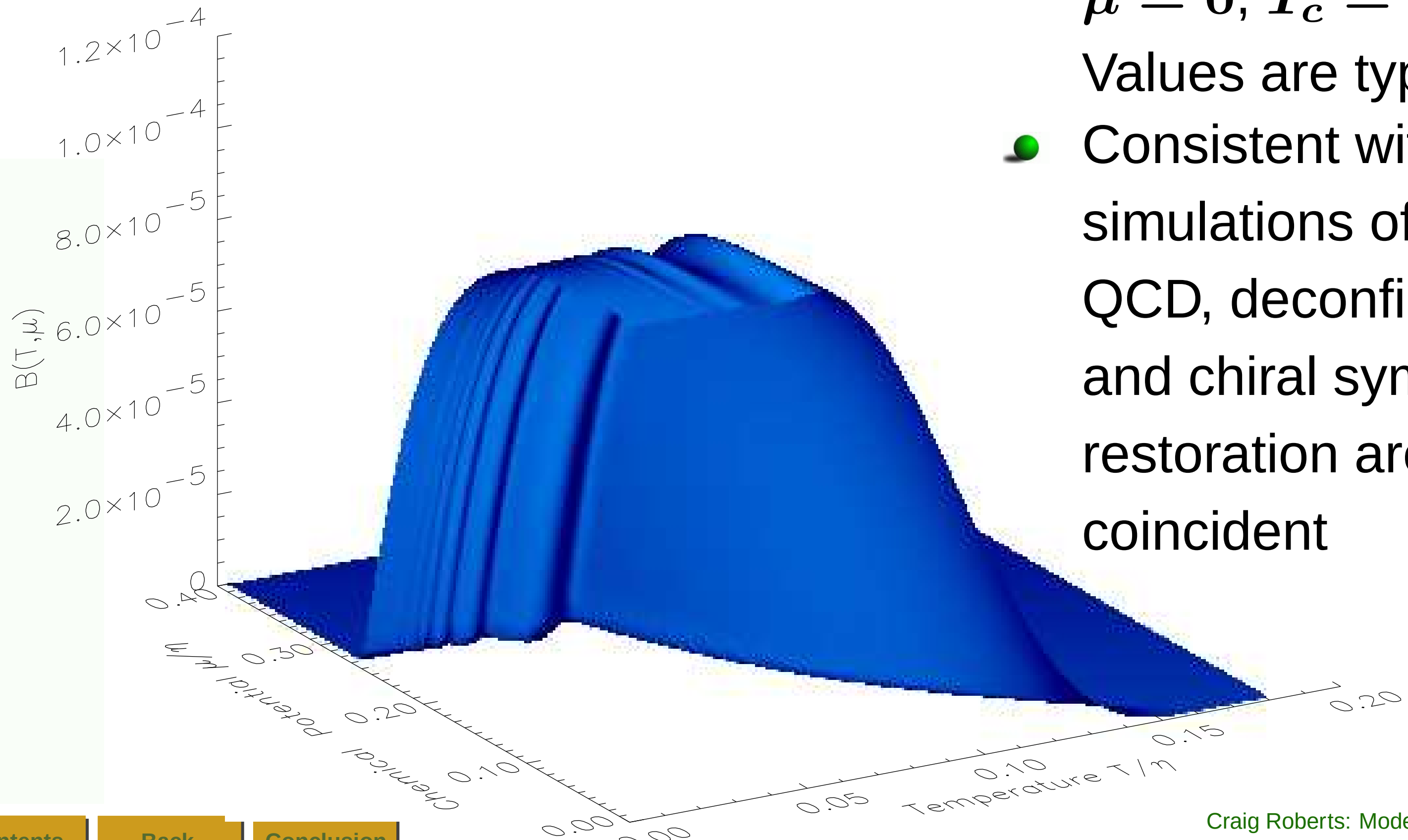
  - $T = 0$, $\mu_c = 300$ MeV
    $\mu = 0$, $T_c = 170$ MeV
    Values are typical



- In general a theory realises the phase with greatest pressure
- Must calculate the pressure difference between Wigner and Nambu phases



- $T = 0$, $\mu_c = 300$ MeV
  $\mu = 0$, $T_c = 170$ MeV
  Values are typical
- Consistent with $\mu = 0$ simulations of lattice – QCD, deconfinement and chiral symmetry restoration are coincident

Quark Pressure

First Contents Back Conclusion

Quark Pressure

- Ultrarelativistic gas of fermions

$$\boxed{p_{\rm UR}(\mu,T) = N_c N_f \frac{1}{12\pi^2}\left(\mu^4 + 2\pi^2\mu^2T^2 + \frac{7}{15}\pi^4T^4\right)}$$

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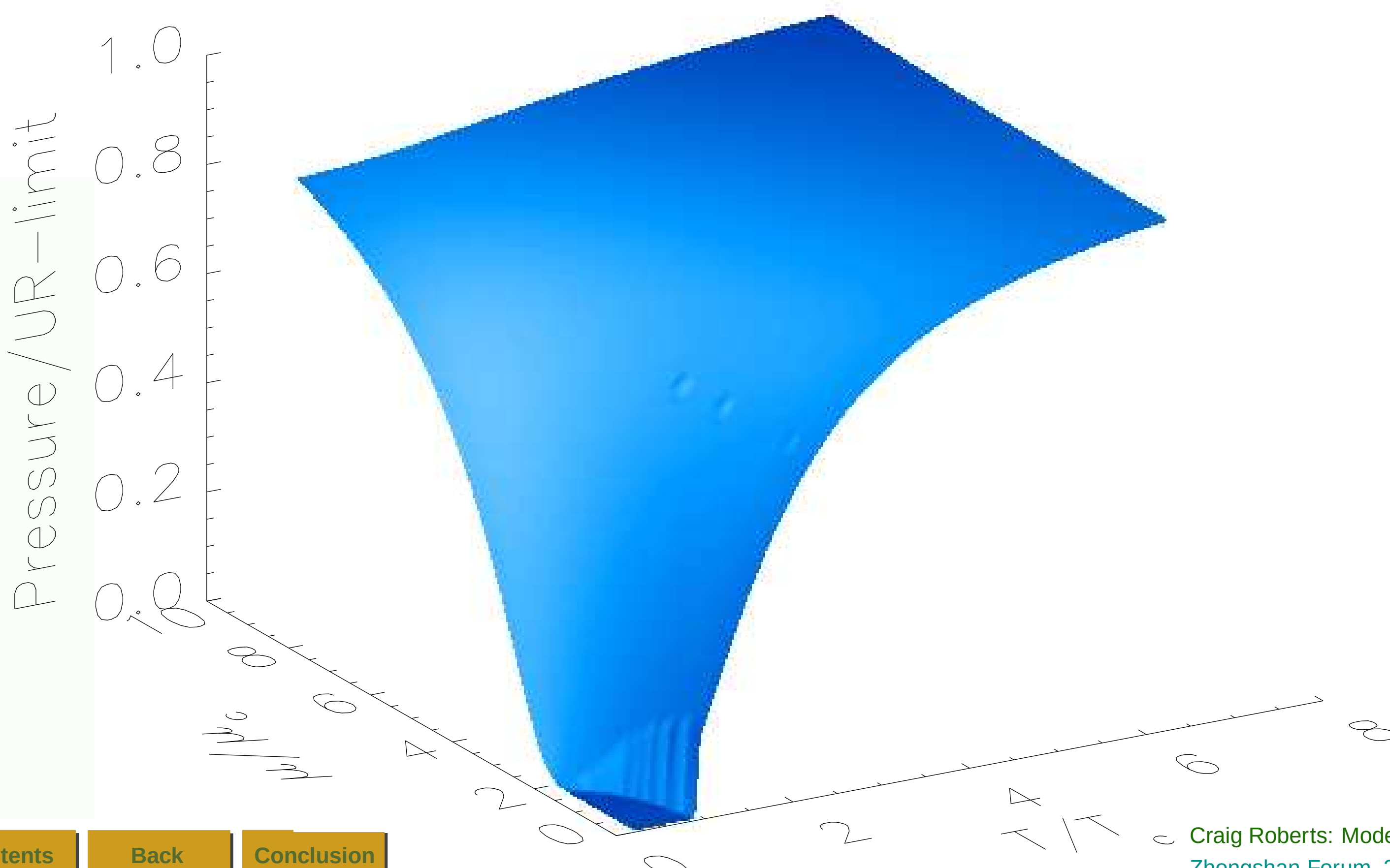

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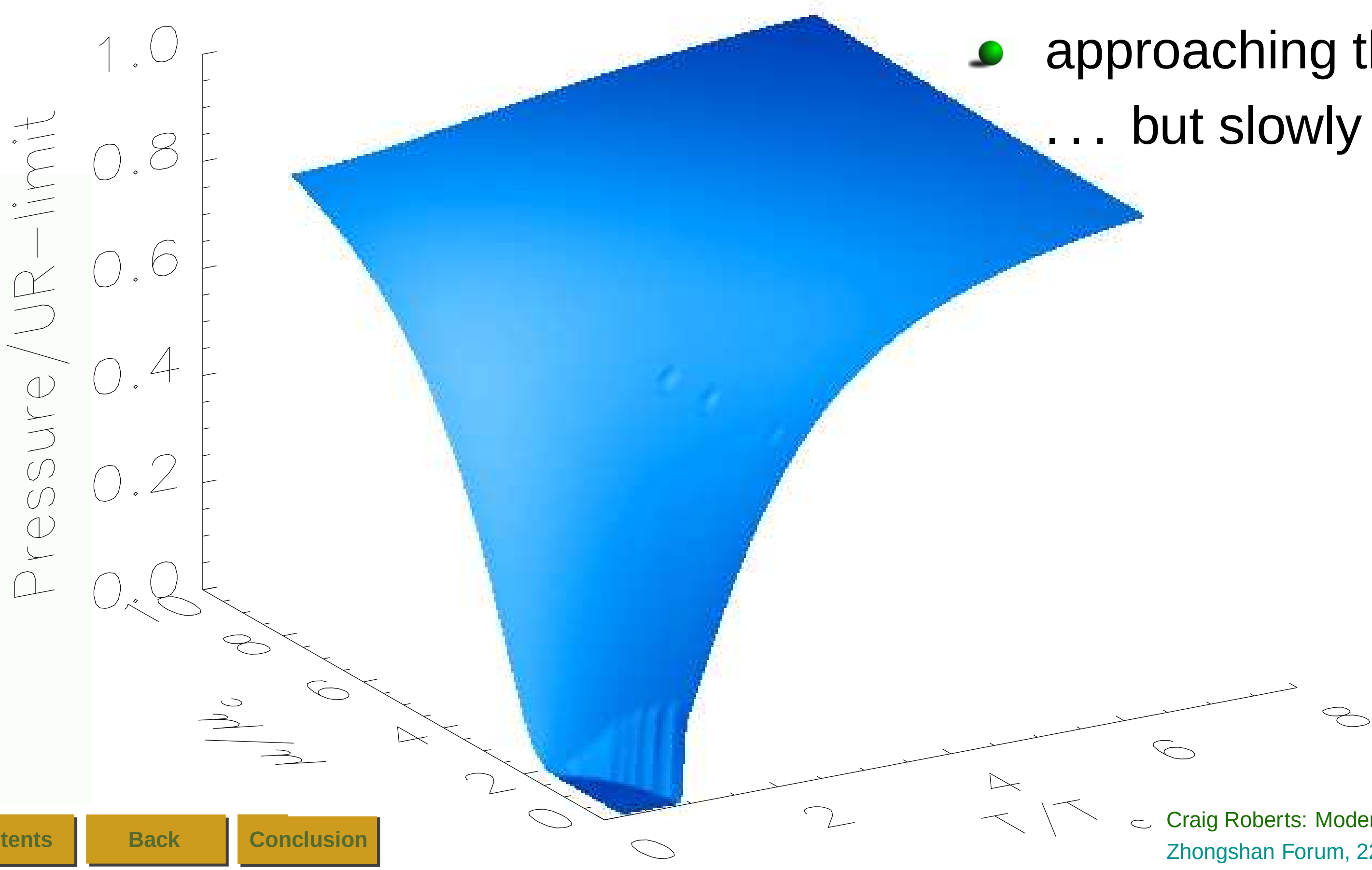

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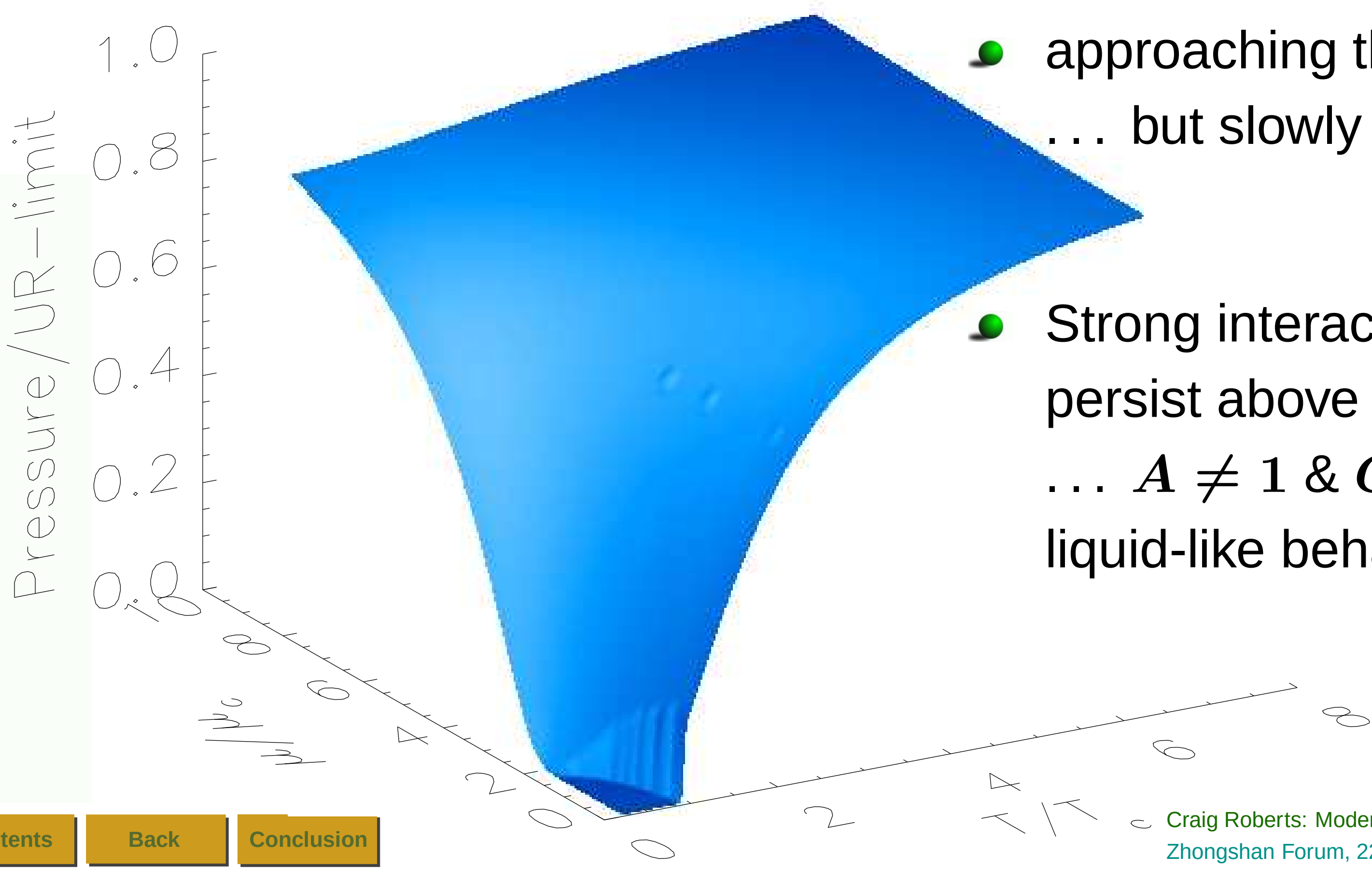

- approaching the limit
  . . . but slowly
- Strong interactions
  persist above transition
  . . . $A \neq 1$ & $C \neq 1$ . . .
  liquid-like behaviour

Epilogue

Epilogue

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- Hadron Physics is a world-wide endeavour
  - Nonperturbative quantum field theory – truly frontier physics
  - Emergent phenomena

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Epilogue

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- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
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- Confinement
  - Expressed and realised in dressed propagators and vertices associated with elementary excitations
  - Observables can be used to explore model realisations

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nothing!

Epilogue

- Hadron Physics is a world-wide endeavour
  - Nonperturbative quantum field theory – truly frontier physics
  - Emergent phenomena
- Dyson-Schwinger Equations
  - Contemporary tool
  - Describes and explains these phenomena – in-vacuum and in-medium
  - Connects them with prediction of observables