Quarks, Hadrons, and the Constants of Nature . . . I

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http://www.phy.anl.gov/theory/staff/cdr.html

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= Proton and Neutron

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  - Big Hint that Proton is not a point particle
  - Proton has constituents
  - These are Quarks and Gluons
    Quark discovery via $e^- p$-scattering at SLAC in 1968
    – the elementary quanta of Quantum Chromo-dynamics

Study Structure via Nucleon Form Factors

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- Electron's relativistic electromagnetic current:

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\begin{aligned}
j_\mu(P',P) &= ie\,\bar{u}_e(P')\,\Lambda_\mu(Q,P)\,u_e(P)\,,\; Q = P' - P \\
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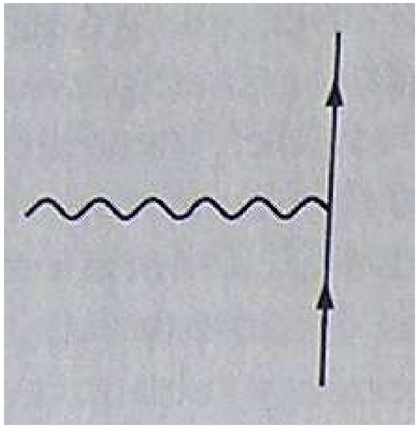

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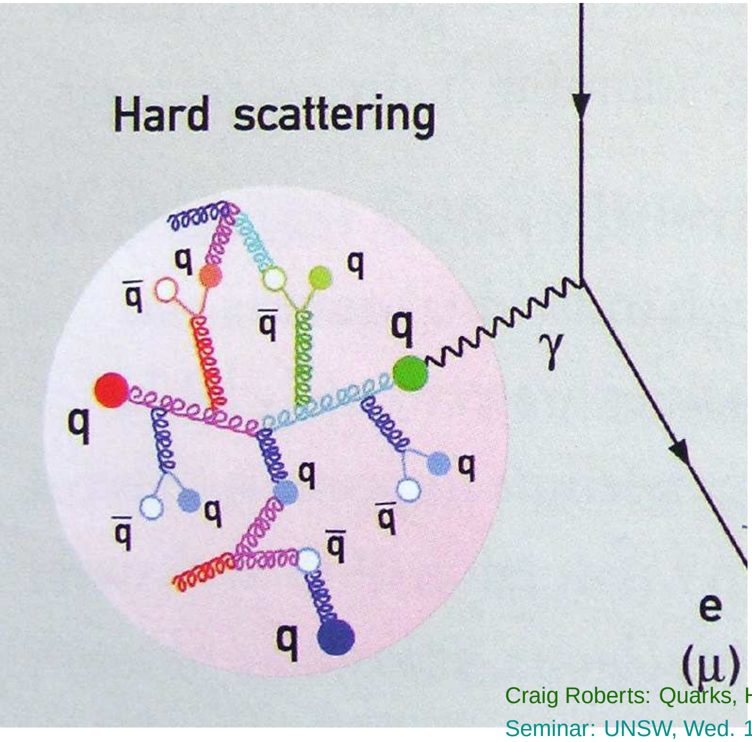

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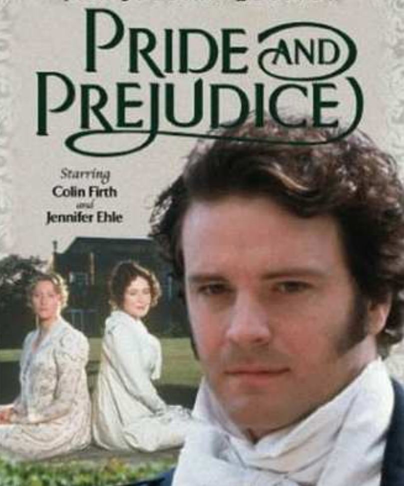
Point-particle: $\boldsymbol{F_2 \equiv 0 \Rightarrow G_E \equiv G_M}$

Universal Truths



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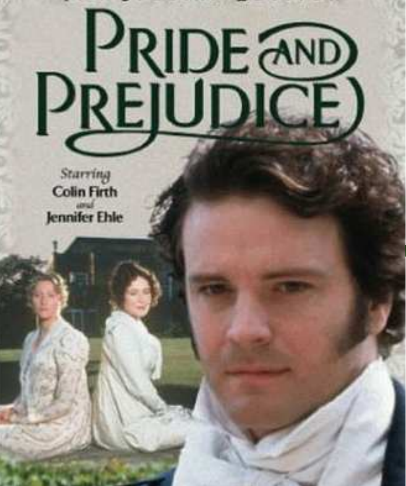

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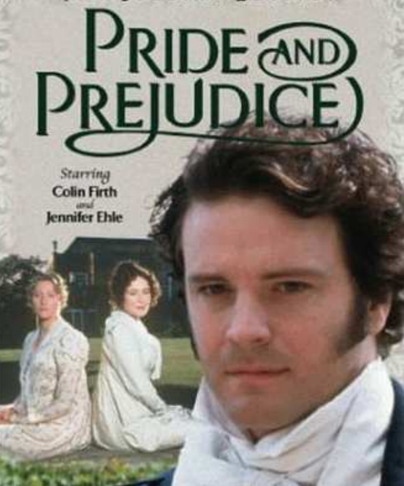

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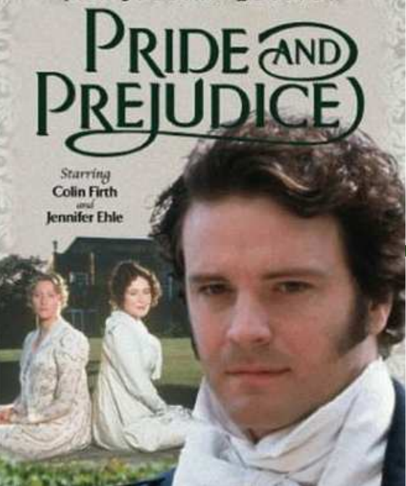

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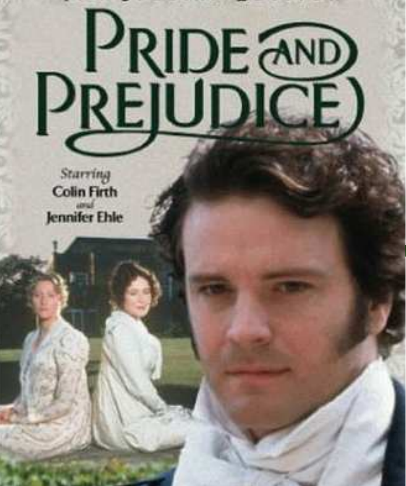

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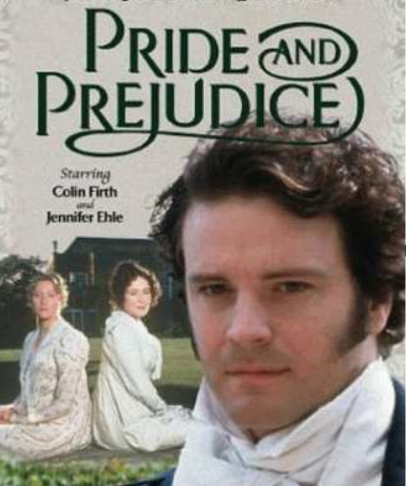

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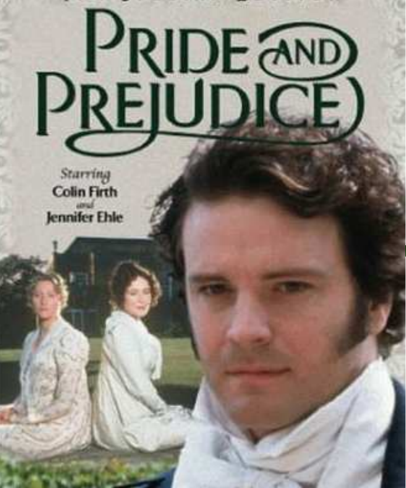
- Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.



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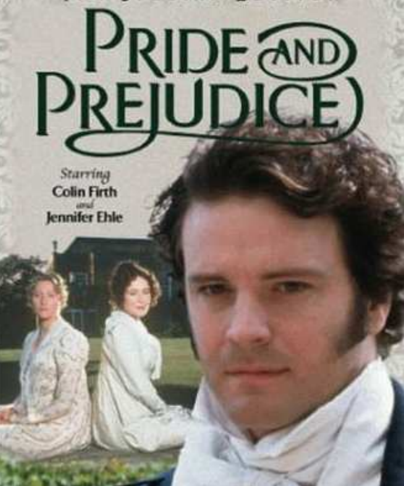

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- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe. Higgs mechanism is irrelevant to light-quarks.
- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.

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Understand Emergent Phenomena

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- QCD – Complex behaviour

  arises from apparently simple rules

Dichotomy of Pion

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Highly Nontrivial

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  - Interaction between quarks – the Interquark "Potential" – unknown throughout $>$ 98% of a hadron's volume

Intranucleon Interaction

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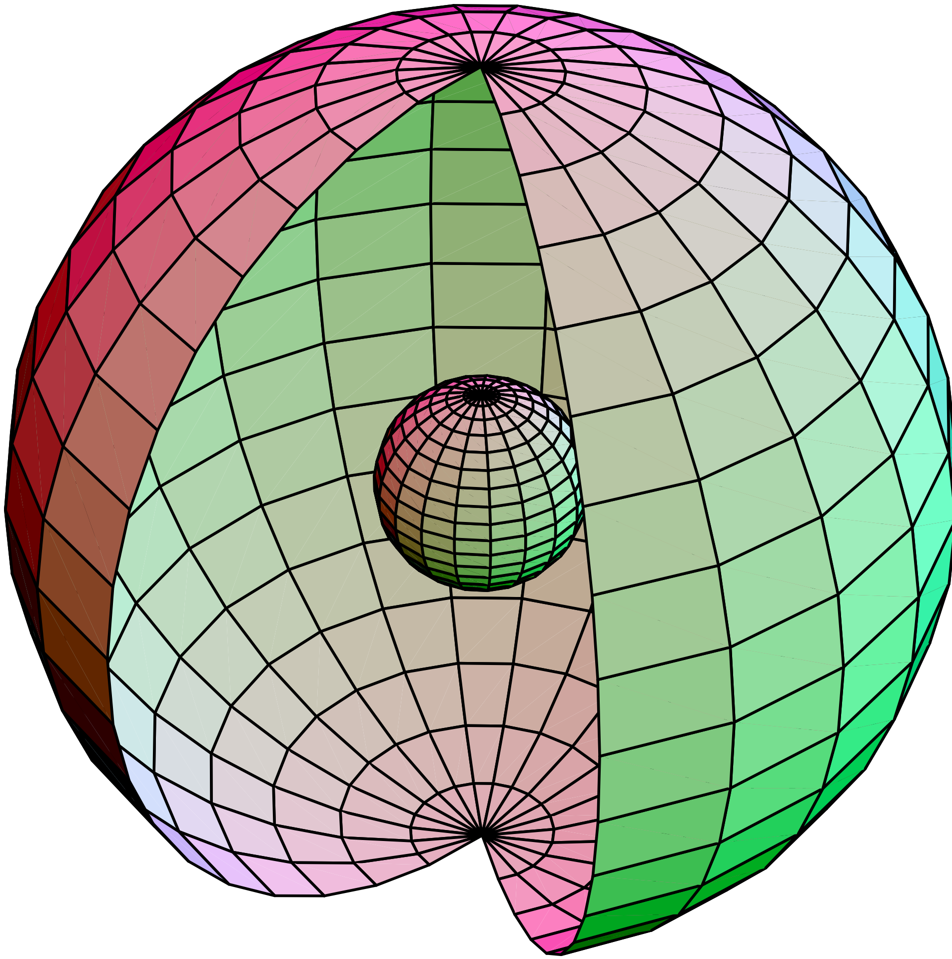

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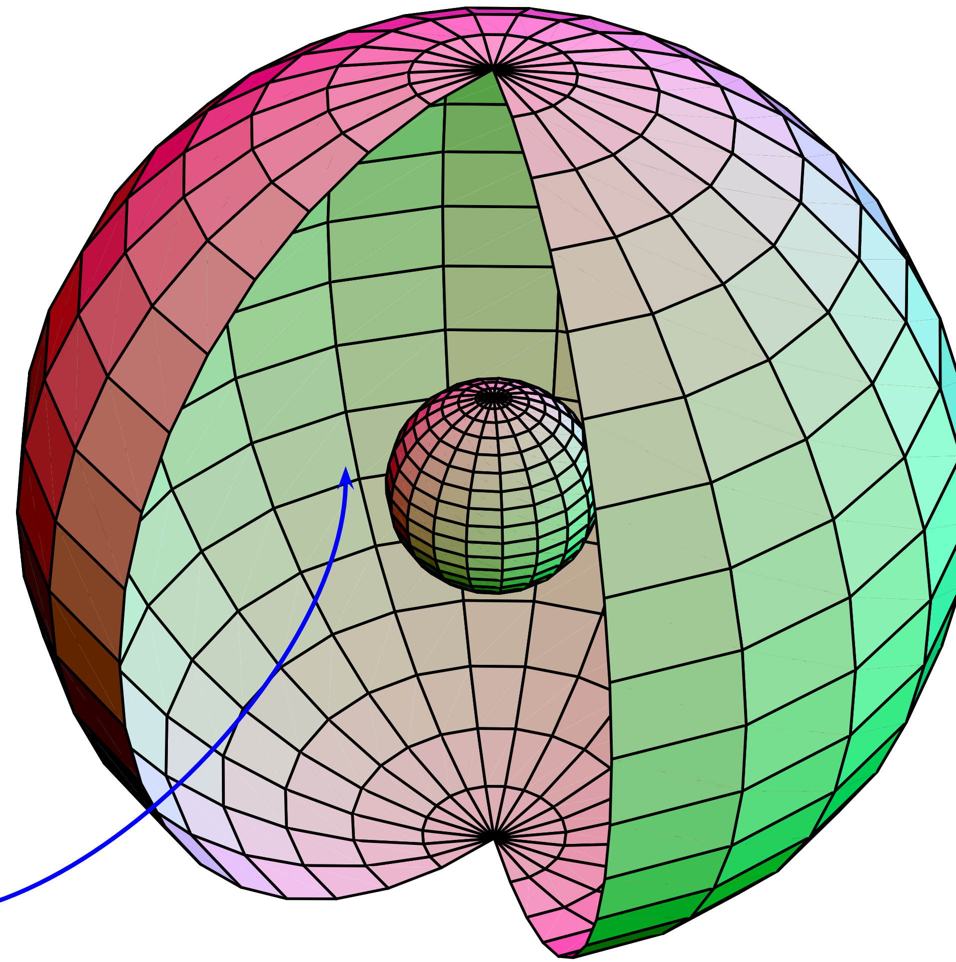
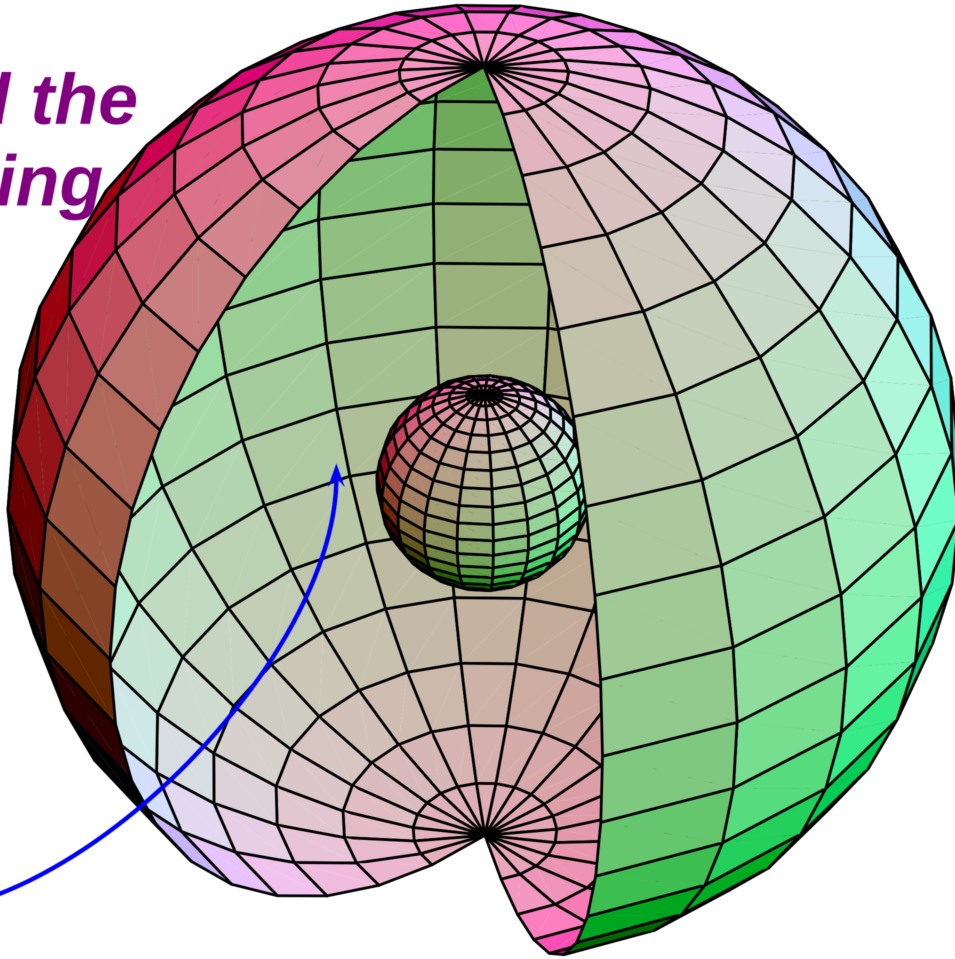


98% of the volume

What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



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Dyson-Schwinger Equations

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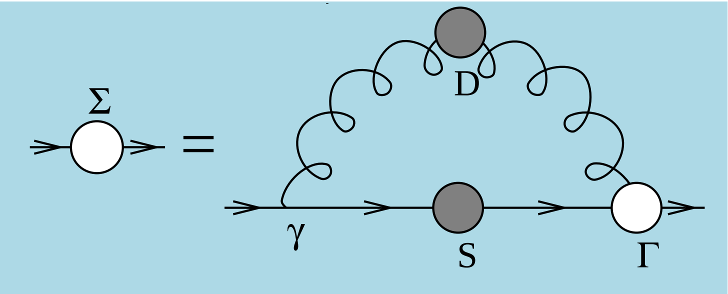

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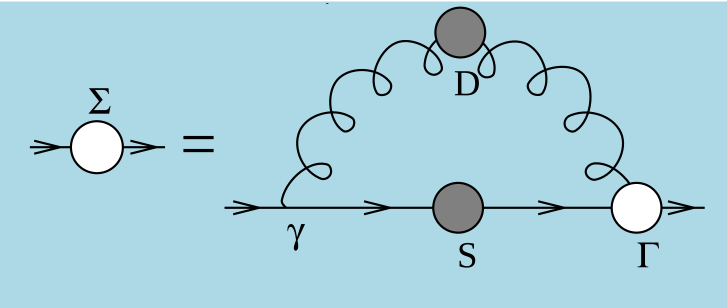
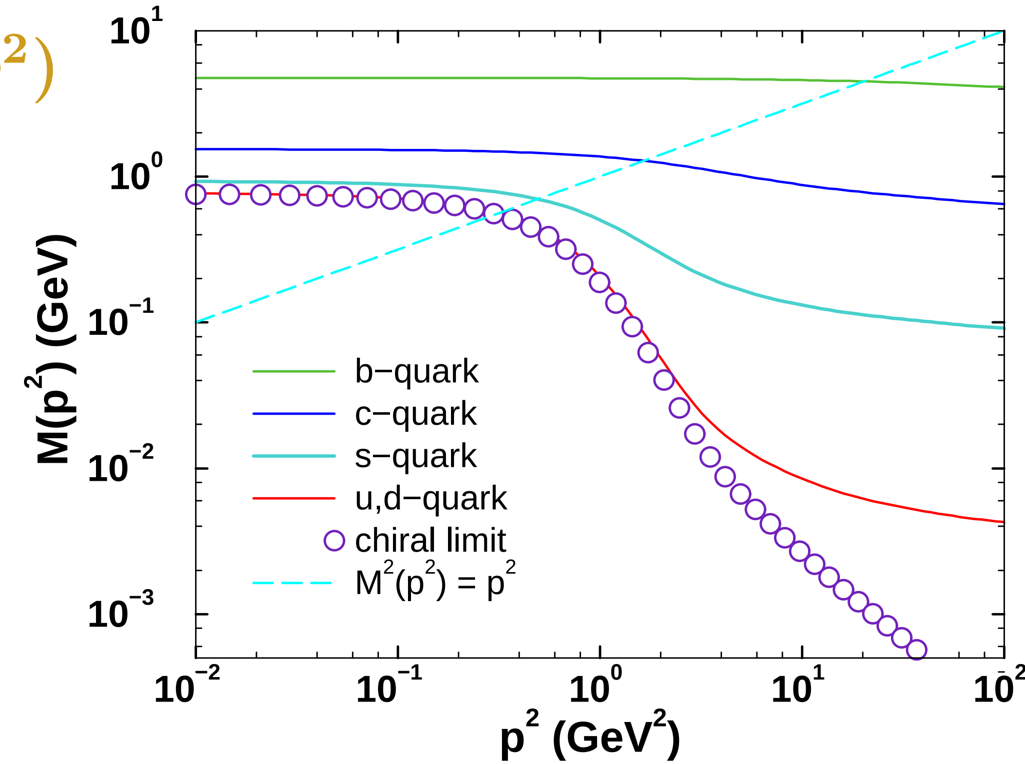


Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



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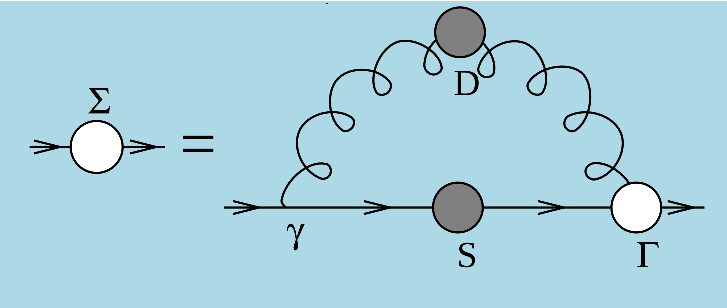

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flavour	u/d	s	c	b
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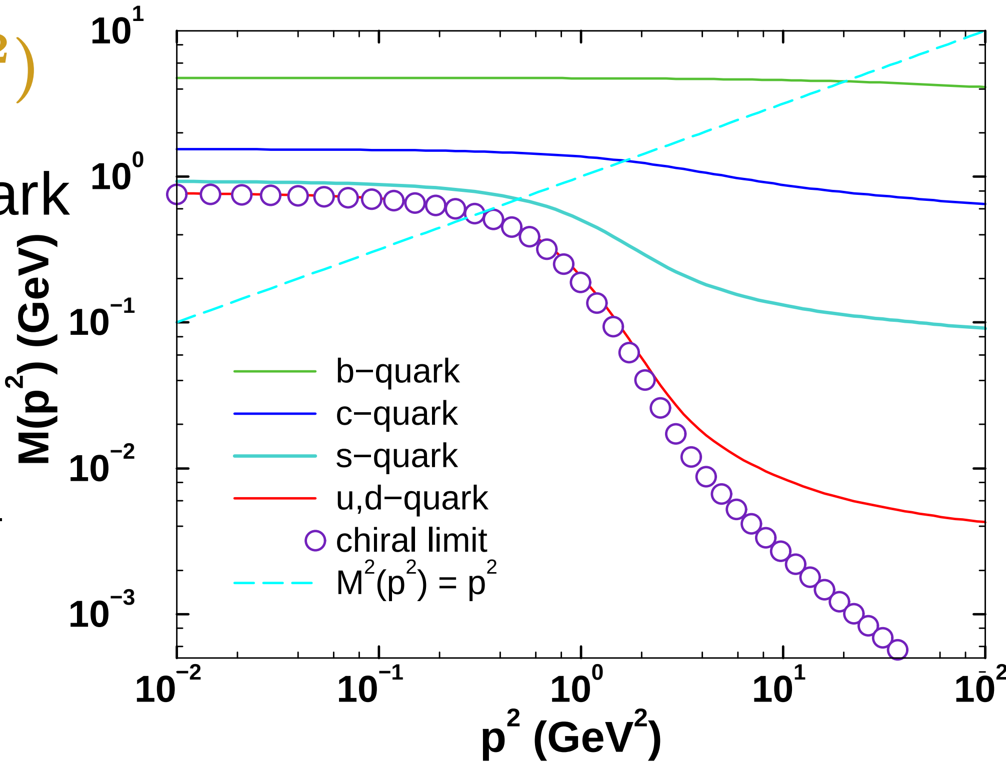

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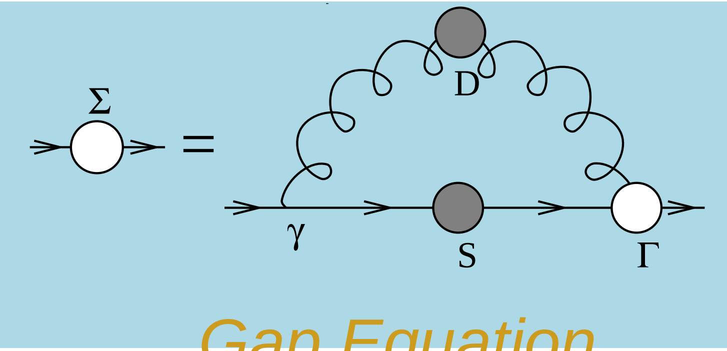

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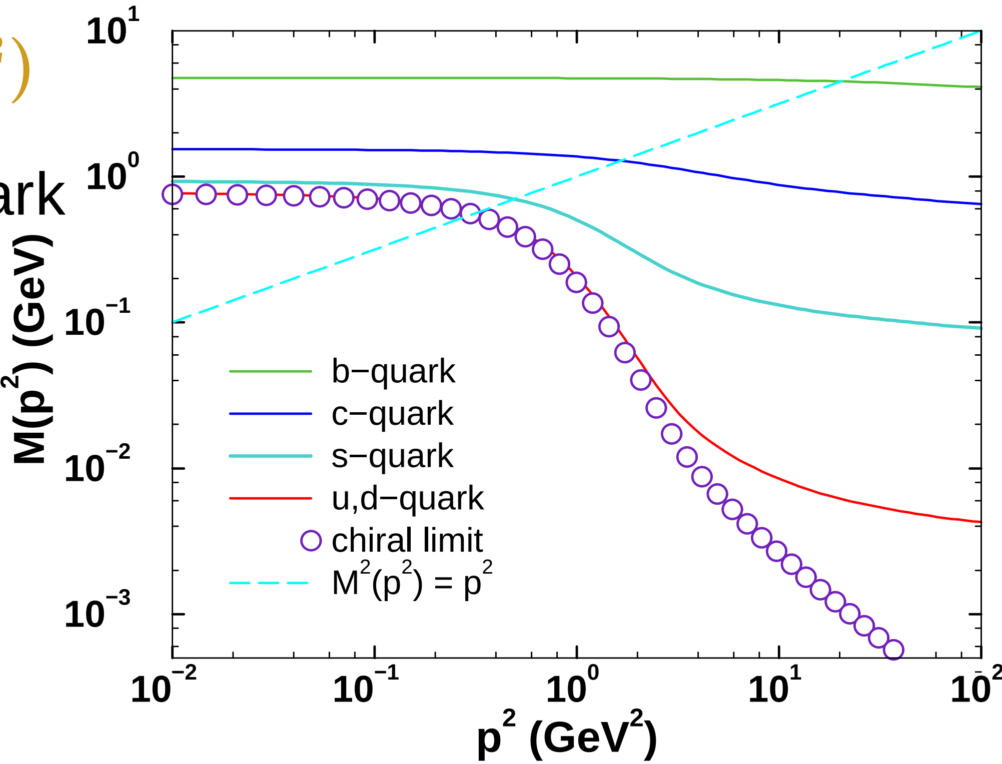

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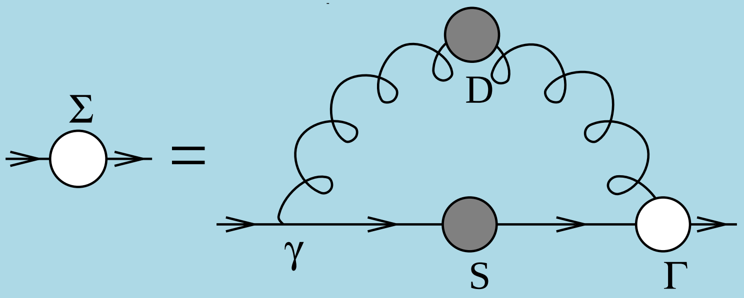
Predictions confirmed in numerical simulations of lattice-QCD

Frontiers of Nuclear Science: A Long Range Plan (2007)



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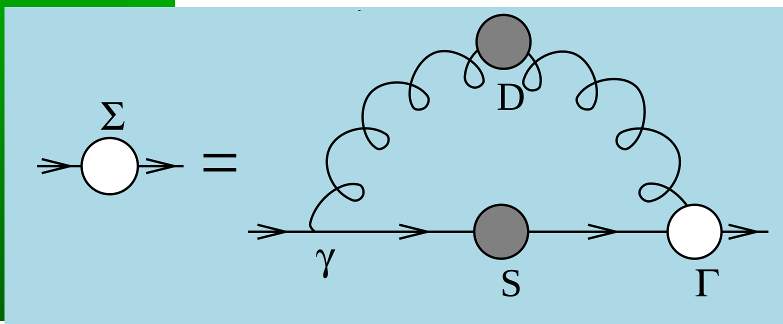
Frontiers of Nuclear Science: Theoretical Advances



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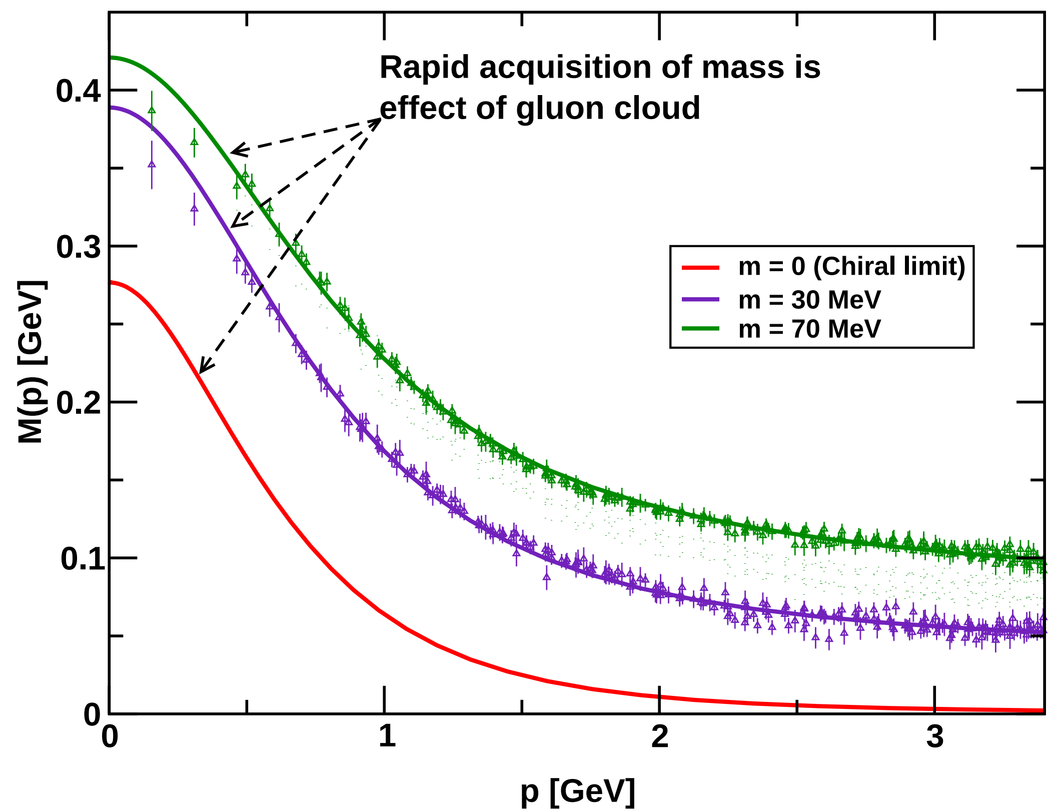
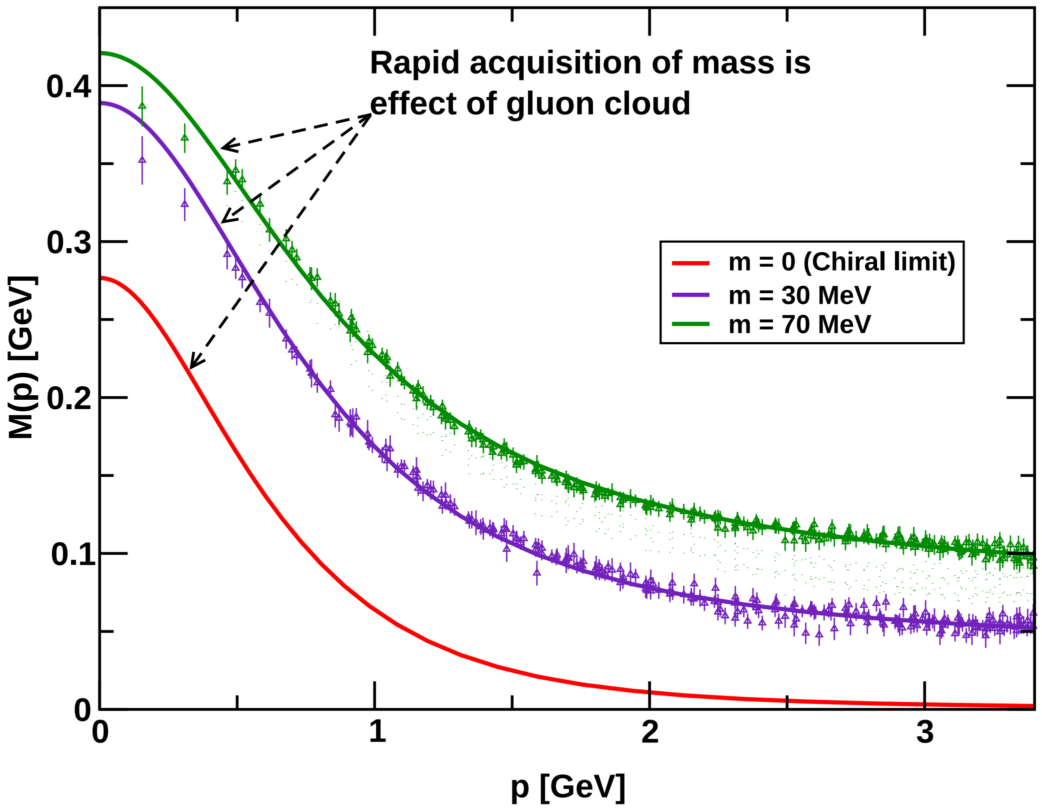


Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.



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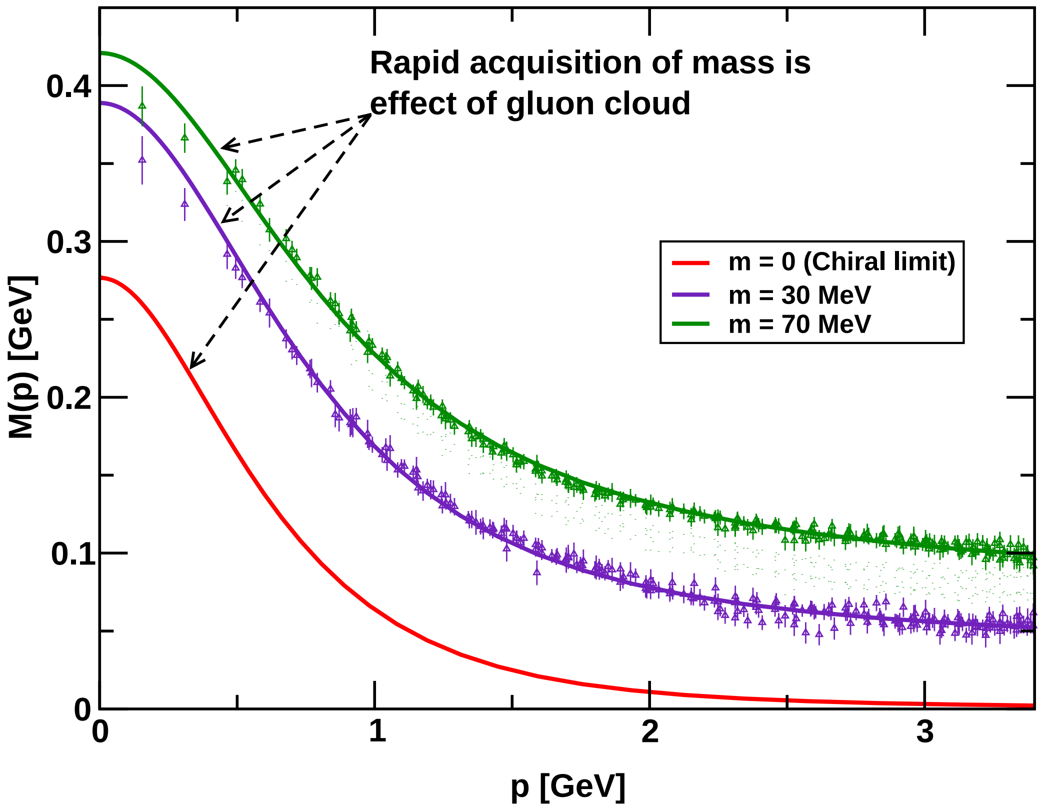


ALWAYS LOOK
ON THE
BRIGHT SIDE

- Established understanding of two- and three-point functions

Hadrons

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- What about bound states?

Hadrons

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- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

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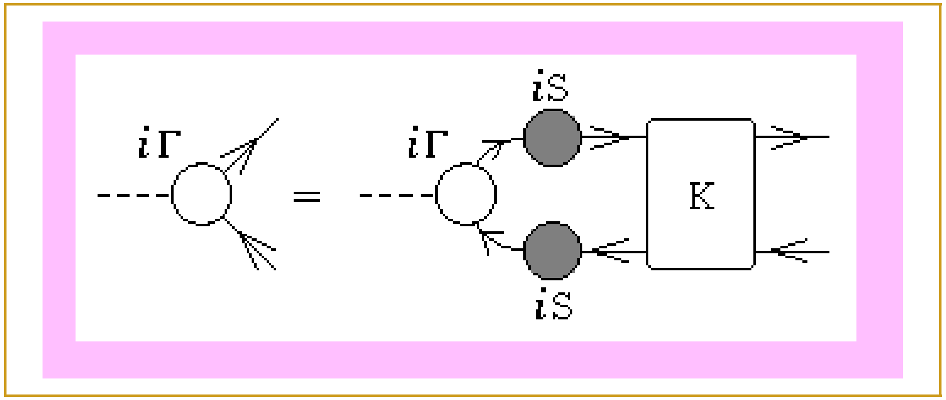

QFT Generalisation of Lippmann-Schwinger Equation.

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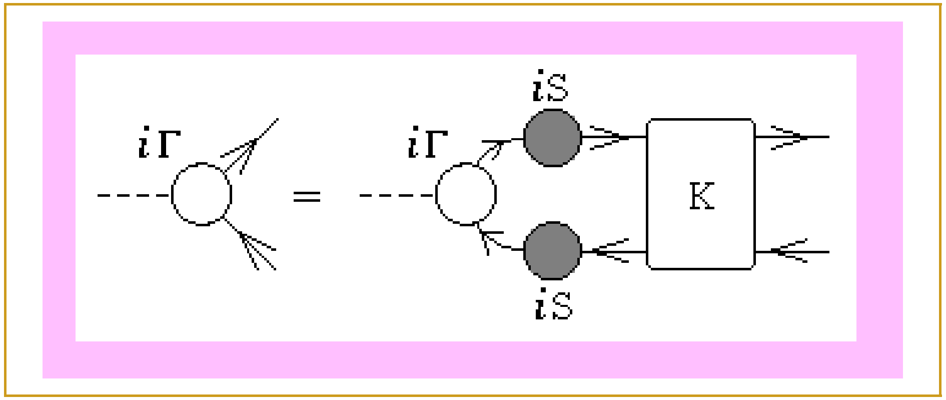


QFT Generalisation of Lippmann-Schwinger Equation.

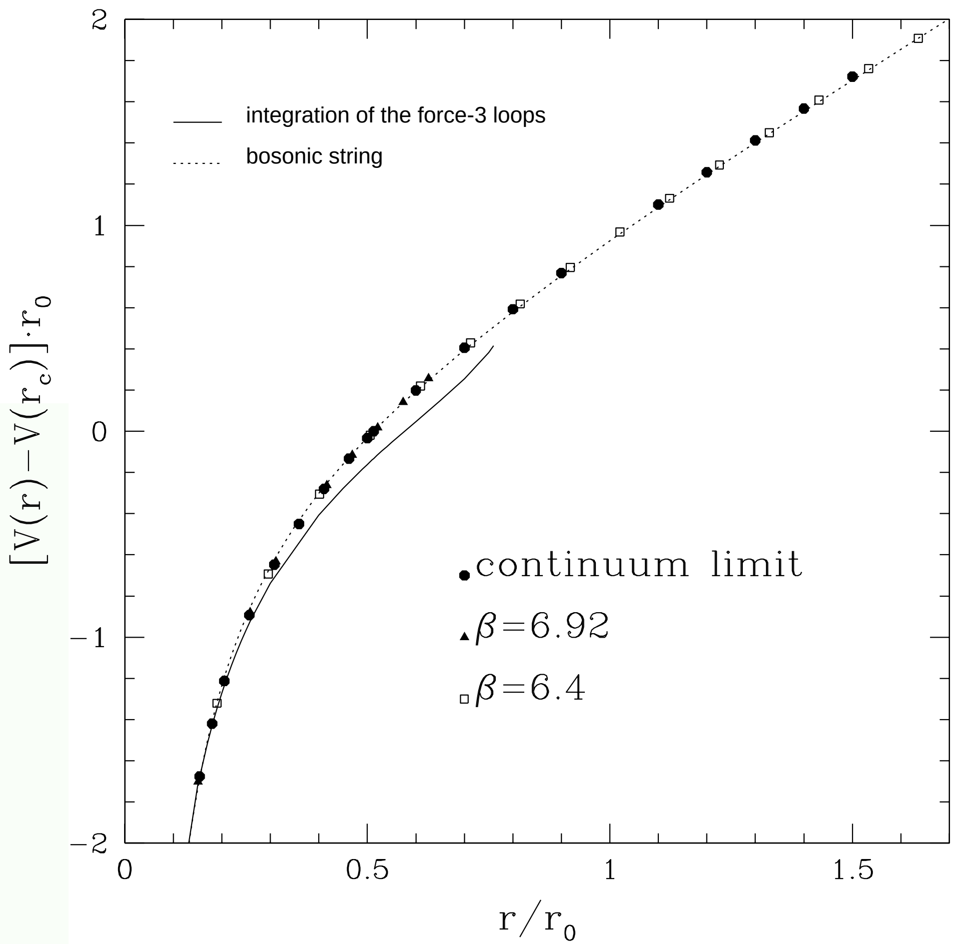
- What is the kernel, K?

or

Confinement

Confinement

- Infinitely Heavy Quarks . . . Picture in Quantum Mechanics



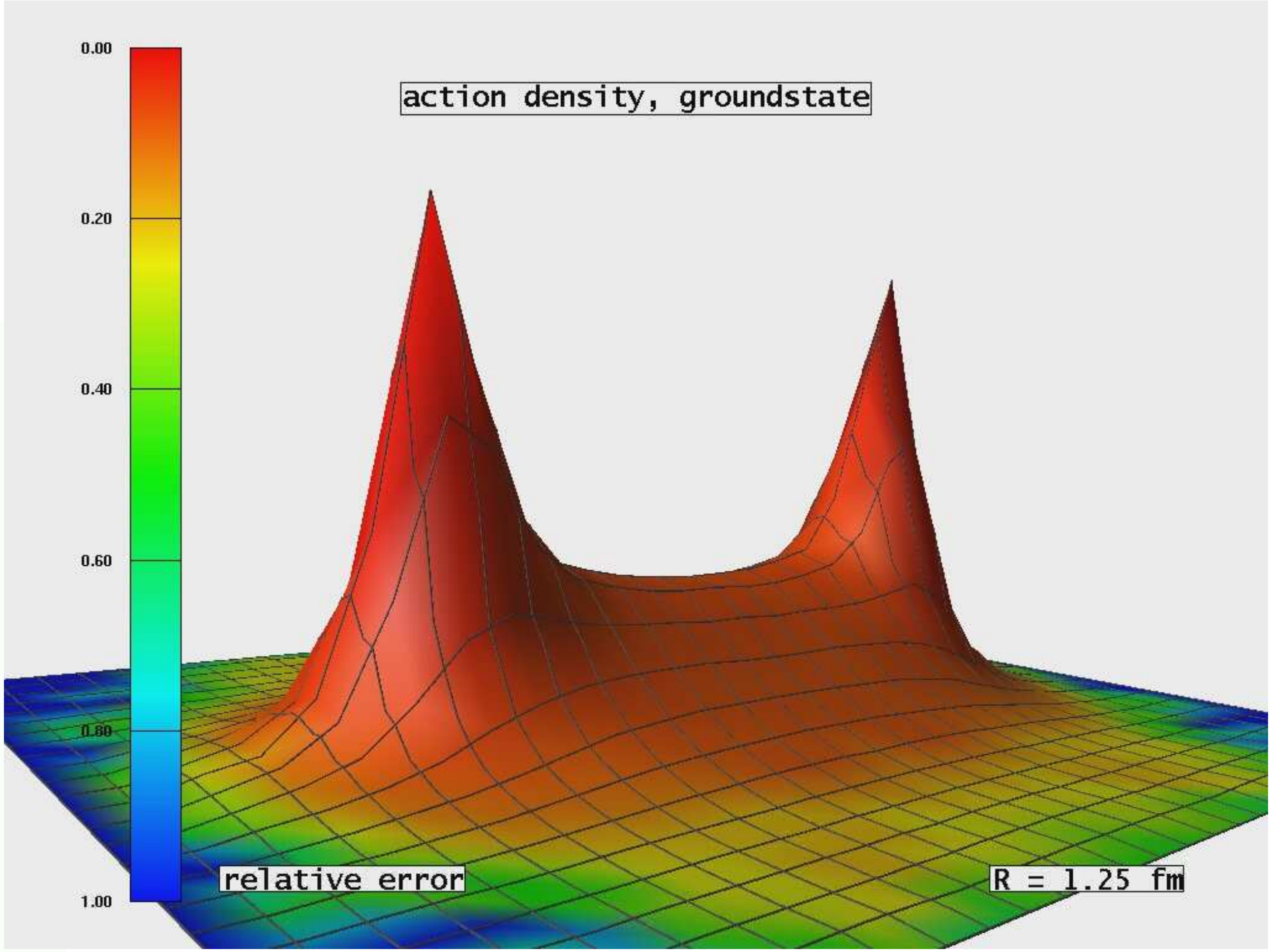
$$V(r) = \sigma\, r - \frac{\pi}{12}\,\frac{1}{r}$$

$\sigma \sim 470\,$MeV

Necco & Sommer
he-la/0108008

Confinement

- Illustrate this in terms of the action density … analogous to plotting the Force $= F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12}\frac{1}{r^2}$



Bali, *et al*.
he-la/0512018

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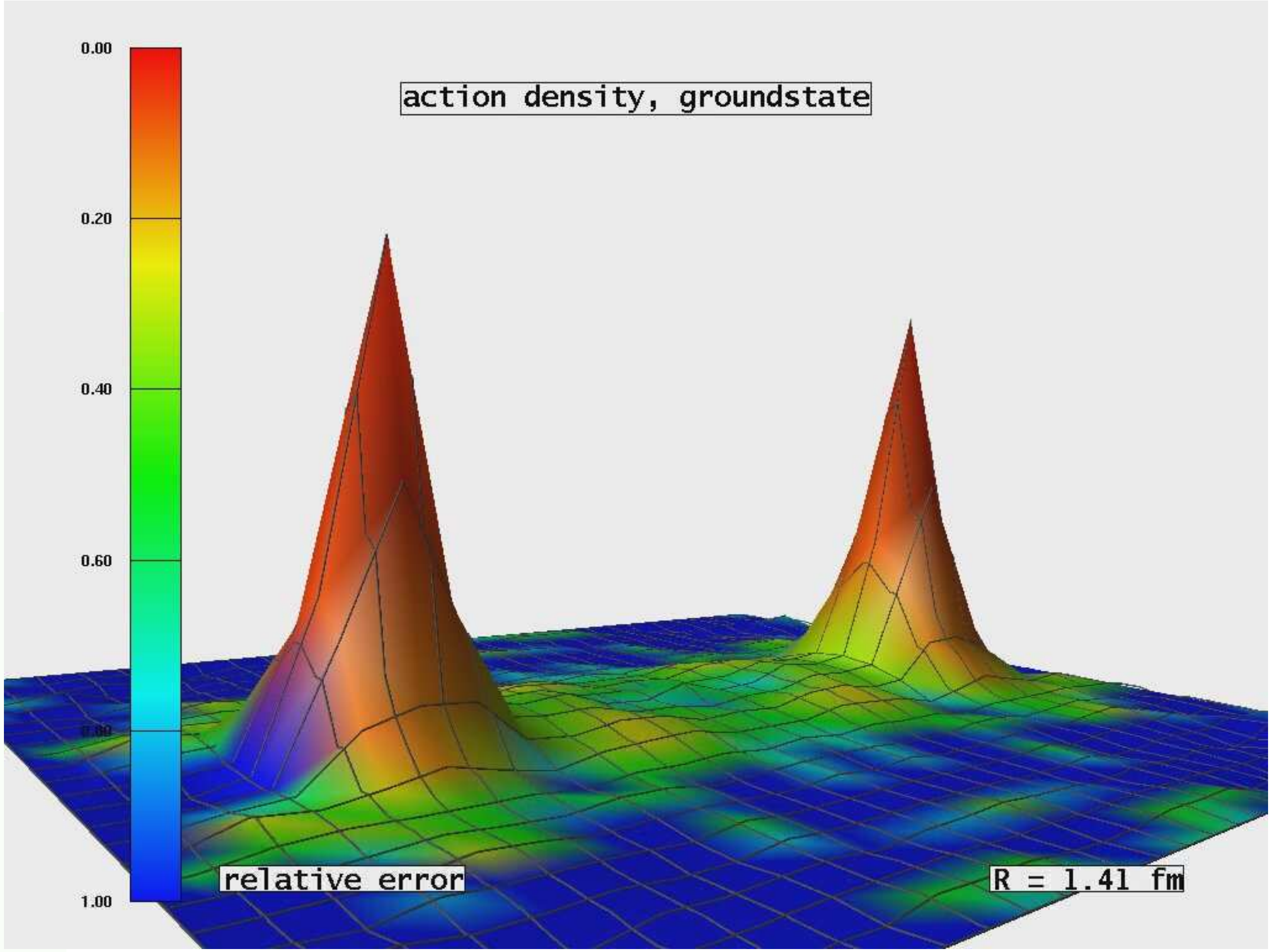

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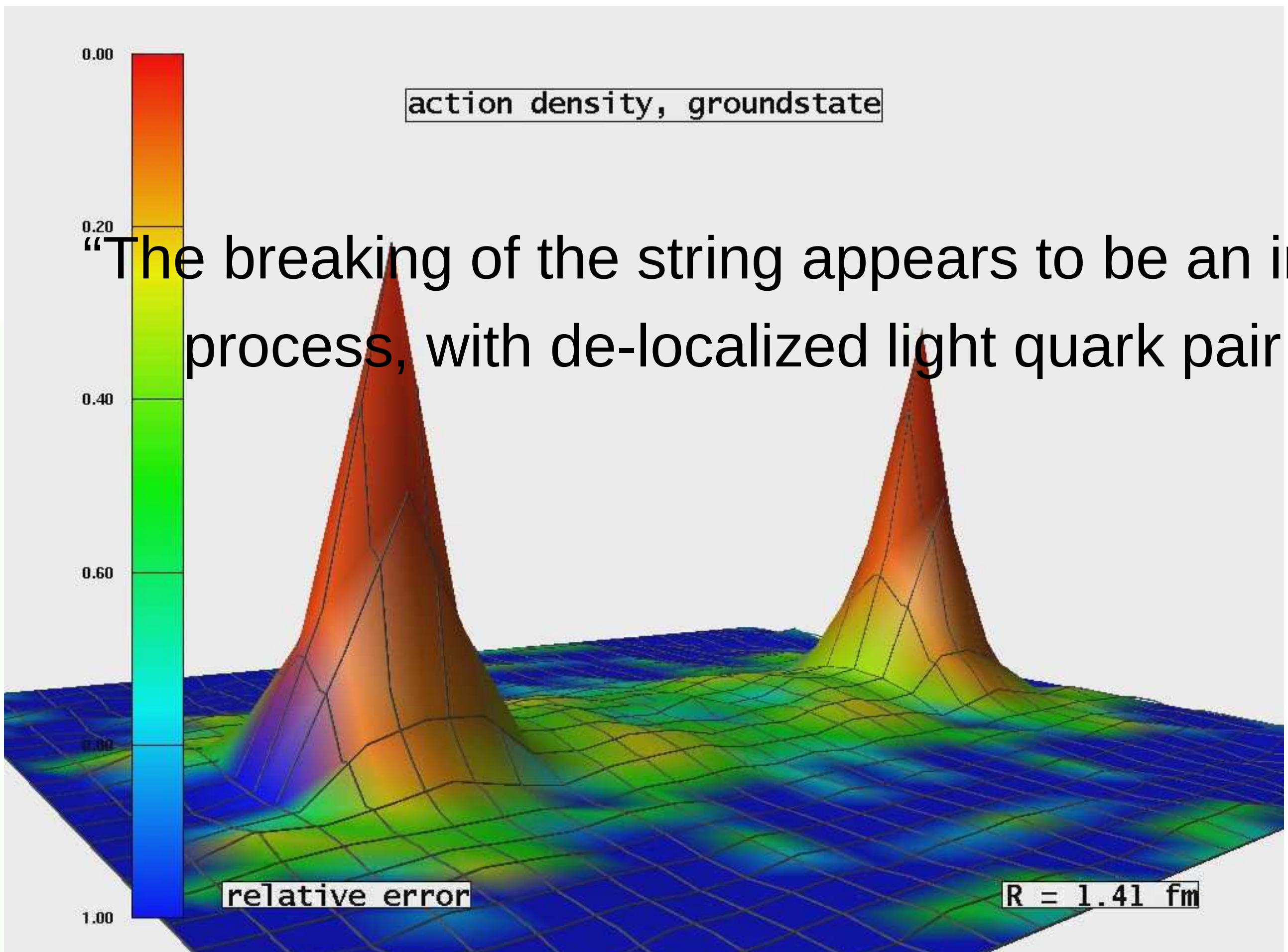
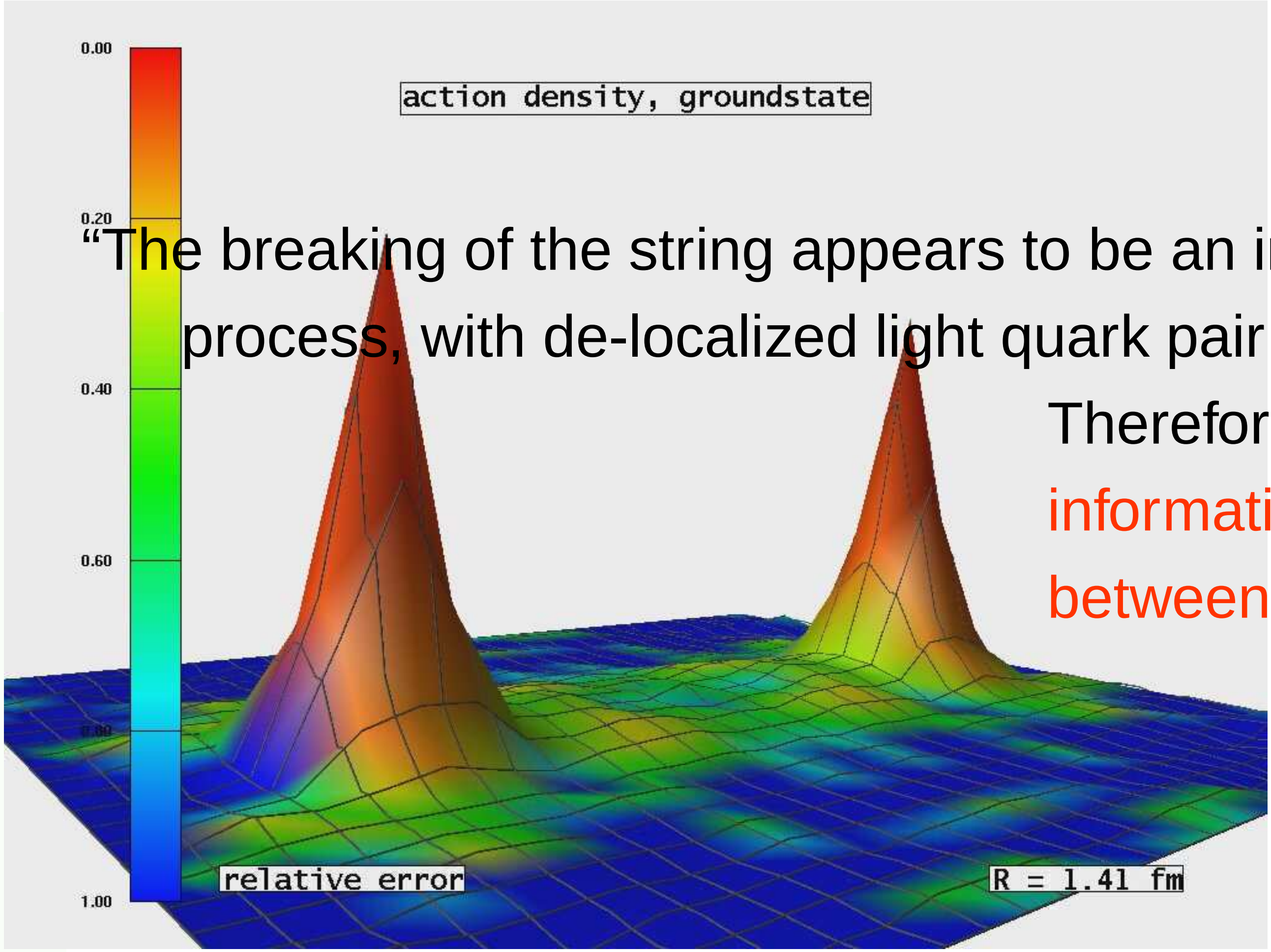


"The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation."

Confinement

- What happens in the real world; namely, in the presence of light-quarks? No one knows . . . but $\bar{Q}Q + \mathbf{2} \times \bar{q}q$

Bali, *et al*.
he-la/0512018



"The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation."

Therefore . . . No information on *potential* between light-quarks.

What is the light-quark Long-Range Potential?

What is the light-quark Long-Range Potential?

Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

Bethe-Salpeter Kernel

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Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

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- Nontrivial constraint

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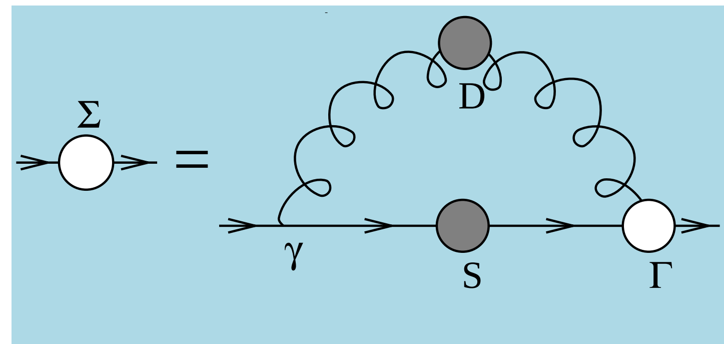
- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Persistent Challenge

Persistent Challenge

- Infinitely Many Coupled Equations



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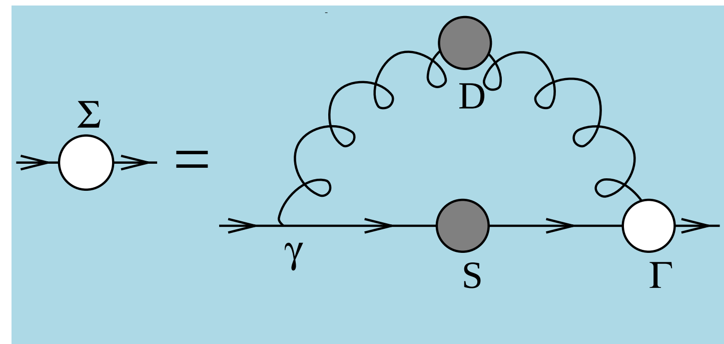

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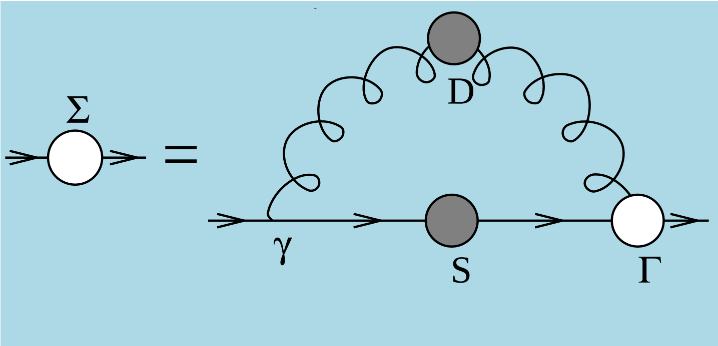

- Coupling between equations necessitates truncation
  - Weak coupling expansion ⇒ Perturbation Theory

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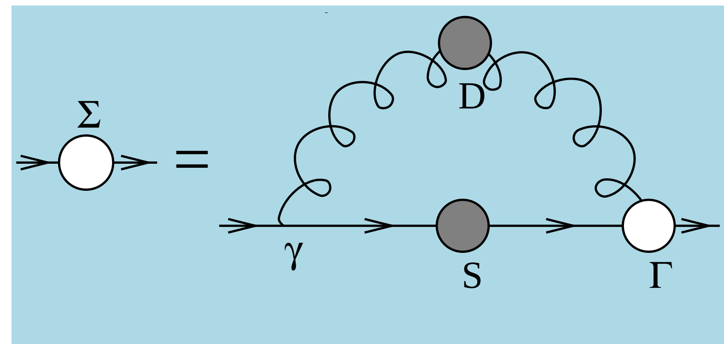

- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
    Not useful for the nonperturbative problems
    in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme

  H.J. Munczek Phys. Rev. D **52** (1995) 4736

  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations

  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7

  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

Persistent Challenge

- Infinitely Many Coupled Equations
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- Has Enabled Proof of EXACT Results in QCD

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Office of Nuclear Physics Exploring Nuclear Matter - Quarks to Stars

Argonne NATIONAL LABORATORY

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Persistent Challenge

- Infinitely Many Coupled Equations
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Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
nu-th/9707003)

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

Radial Excitations & Chiral Symmetry

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- Mass^2 of pseudoscalar hadron

(Maris, Roberts, Tandy nu-th/9707003)

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

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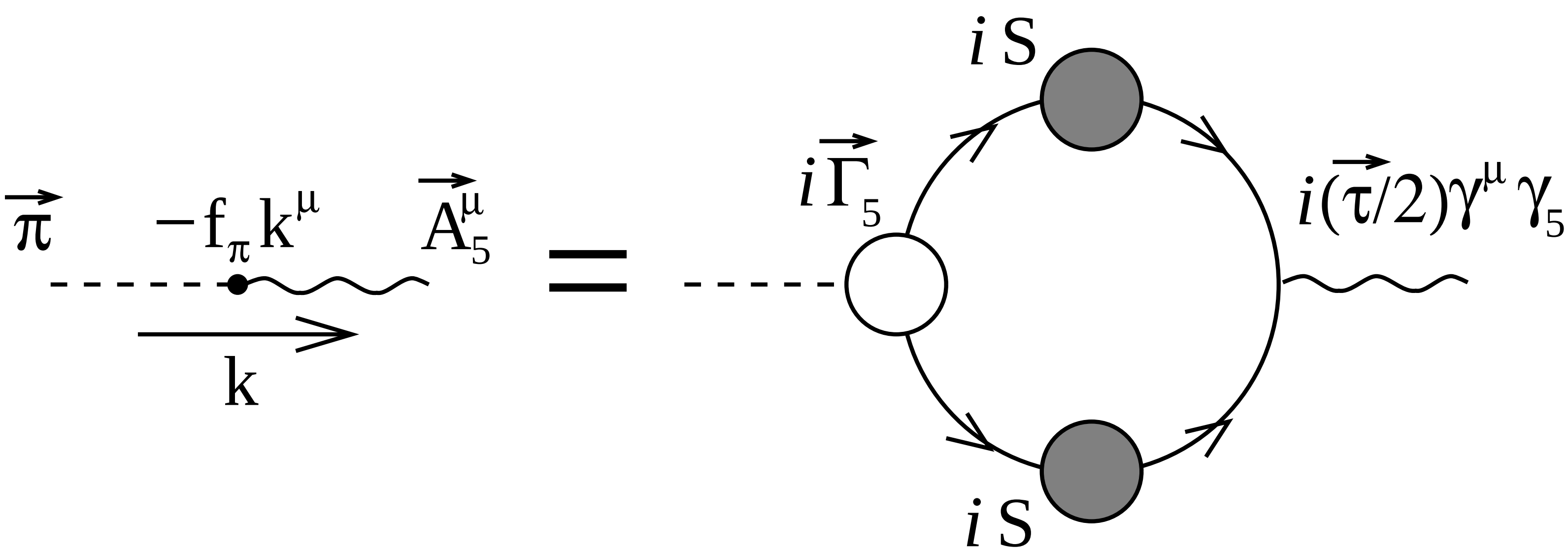
- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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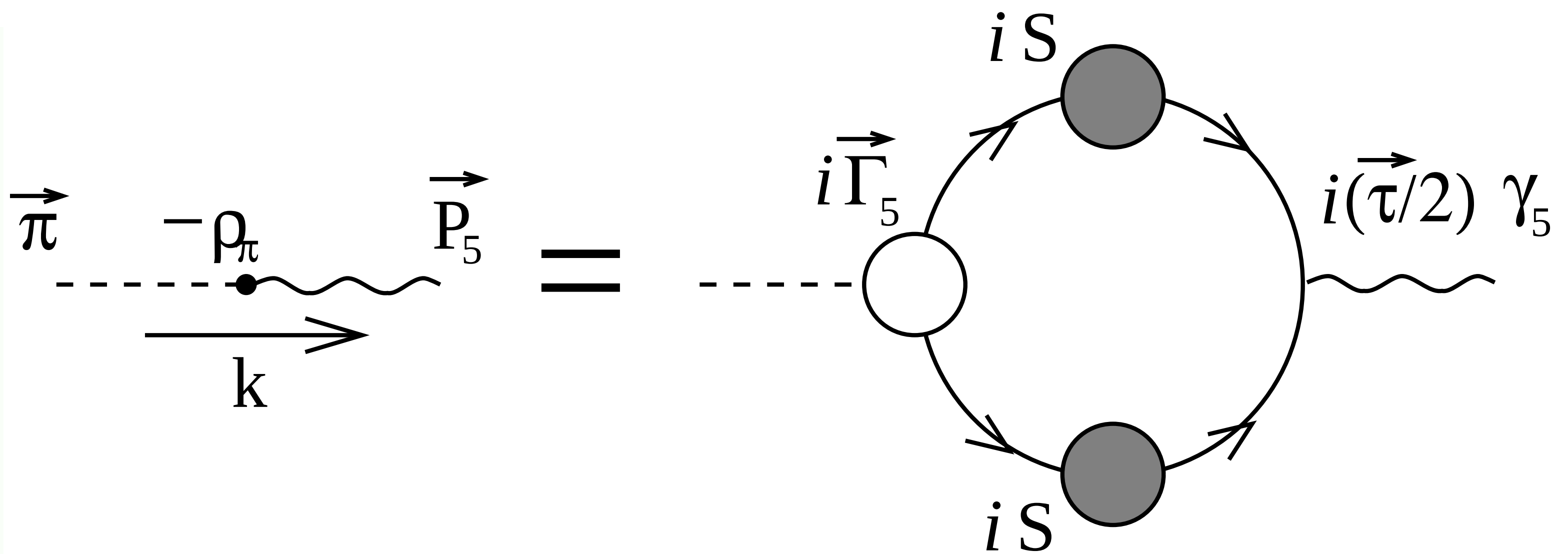

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- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $\boxed{m_H \propto m_q}$

. . . QCD Proof of Potential Model result

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

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- Dynamical Chiral Symmetry Breaking

  – Goldstone's Theorem –

  impacts upon *every* pseudoscalar meson

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McNeile and Michael
he-la/0607032

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  $16^3 \times 32$,
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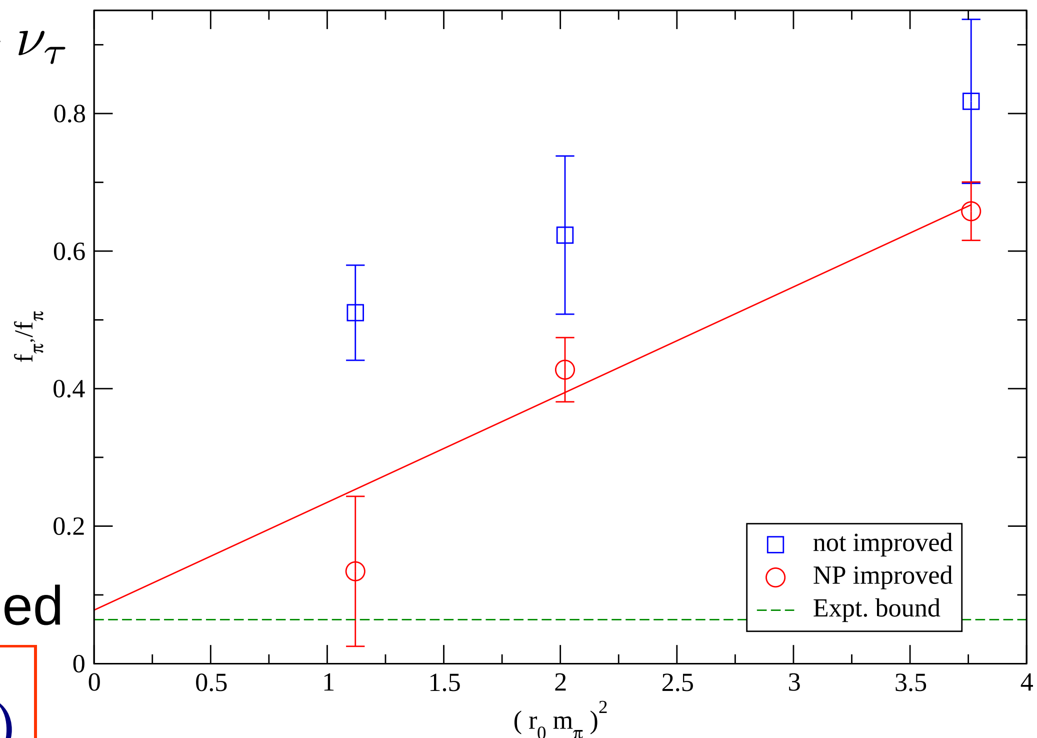
$$\boxed{\Rightarrow \frac{f_{\pi_1}}{f_\pi} = \mathbf{0.078\,(93)}}$$



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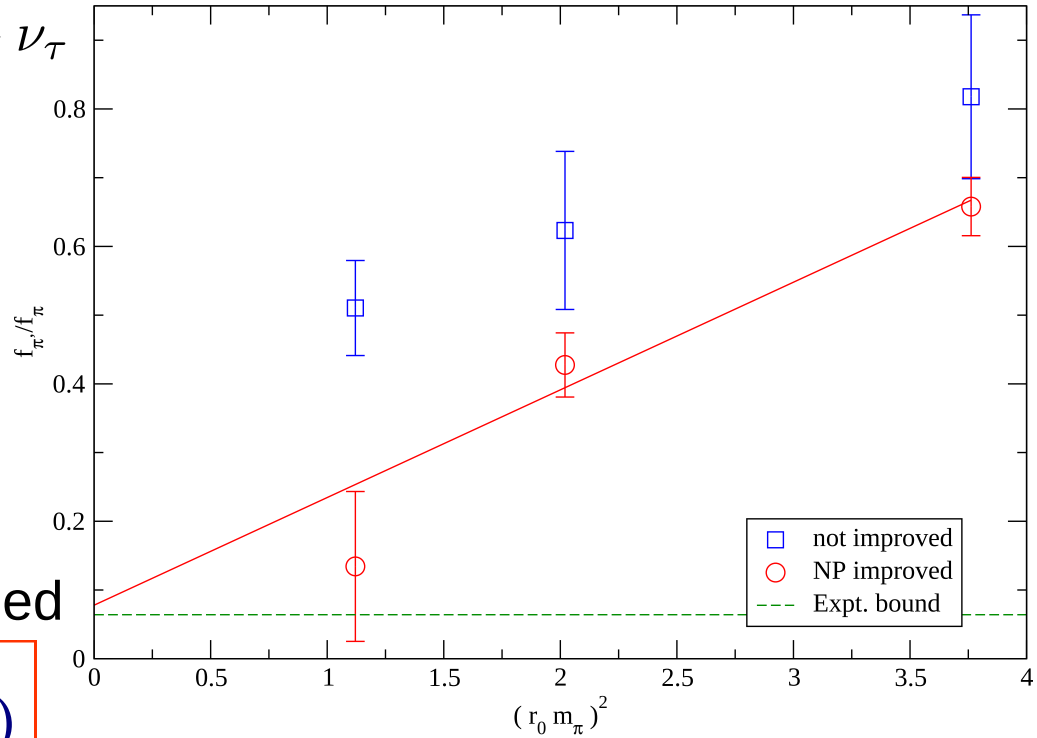

- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

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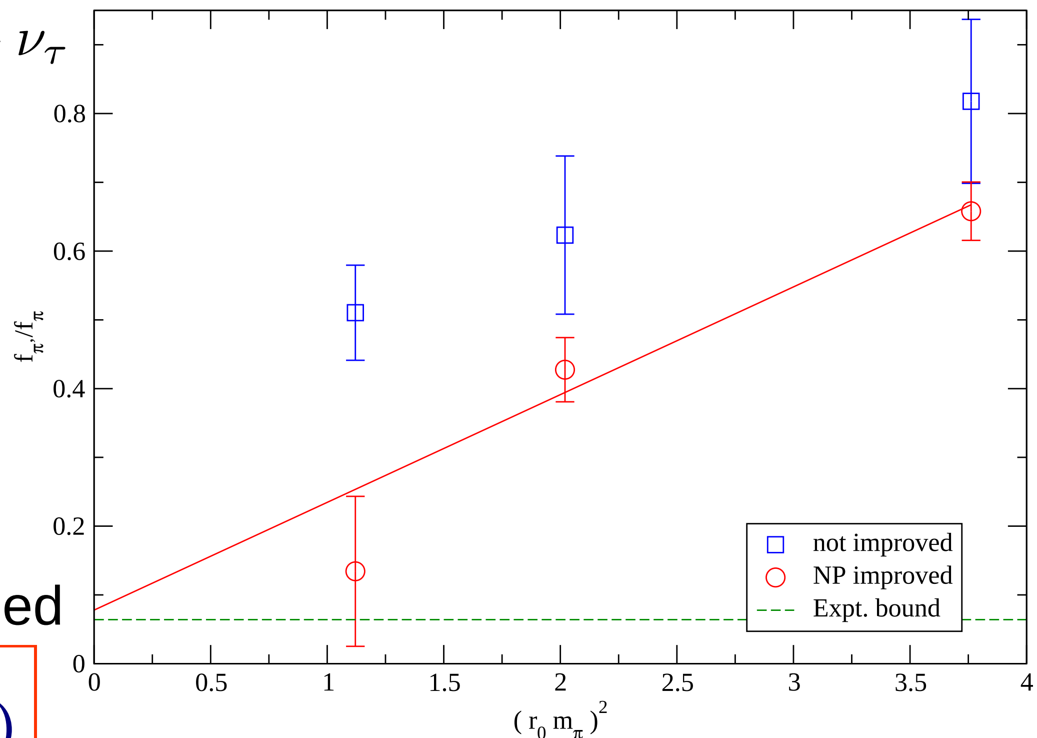
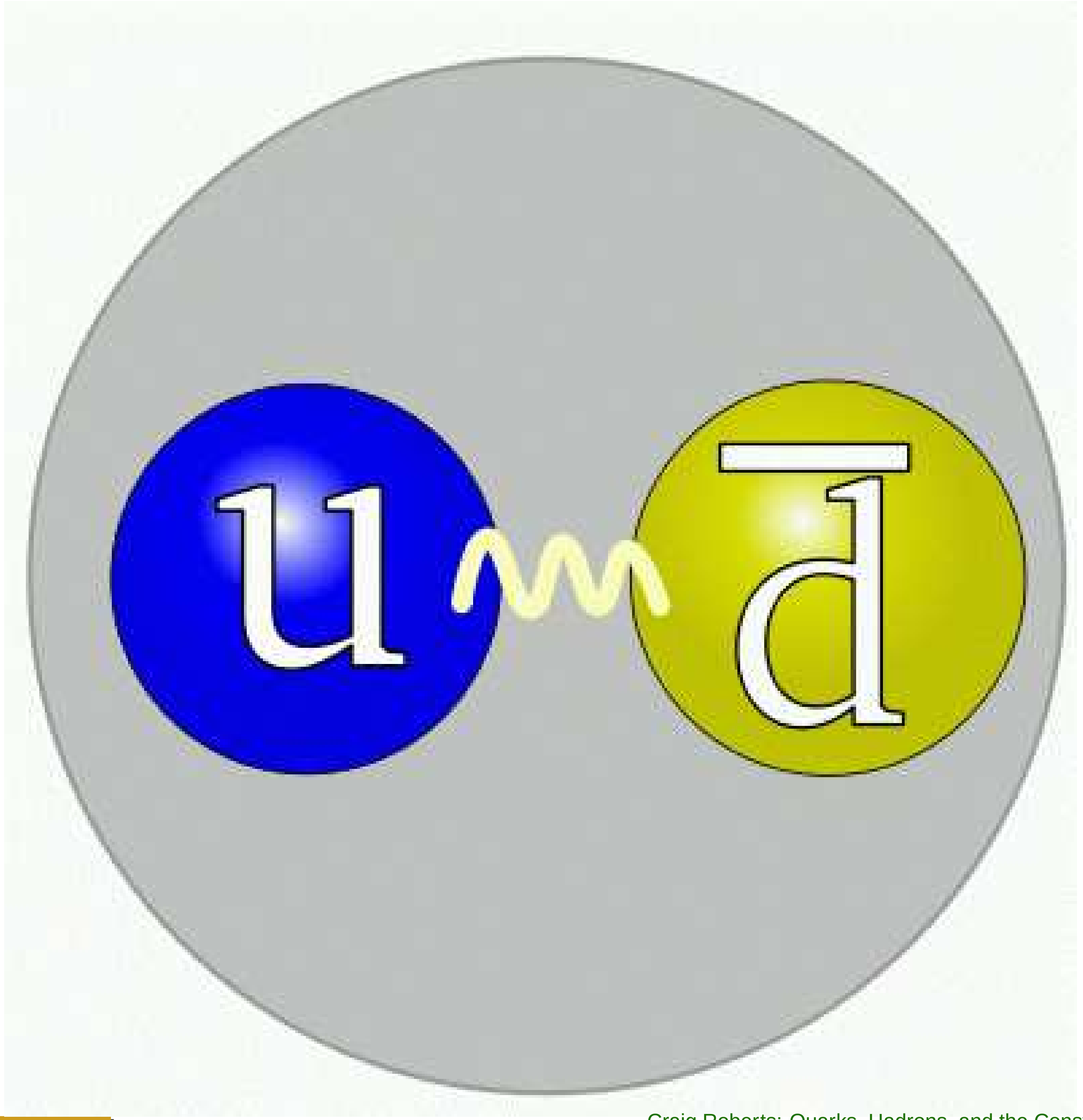


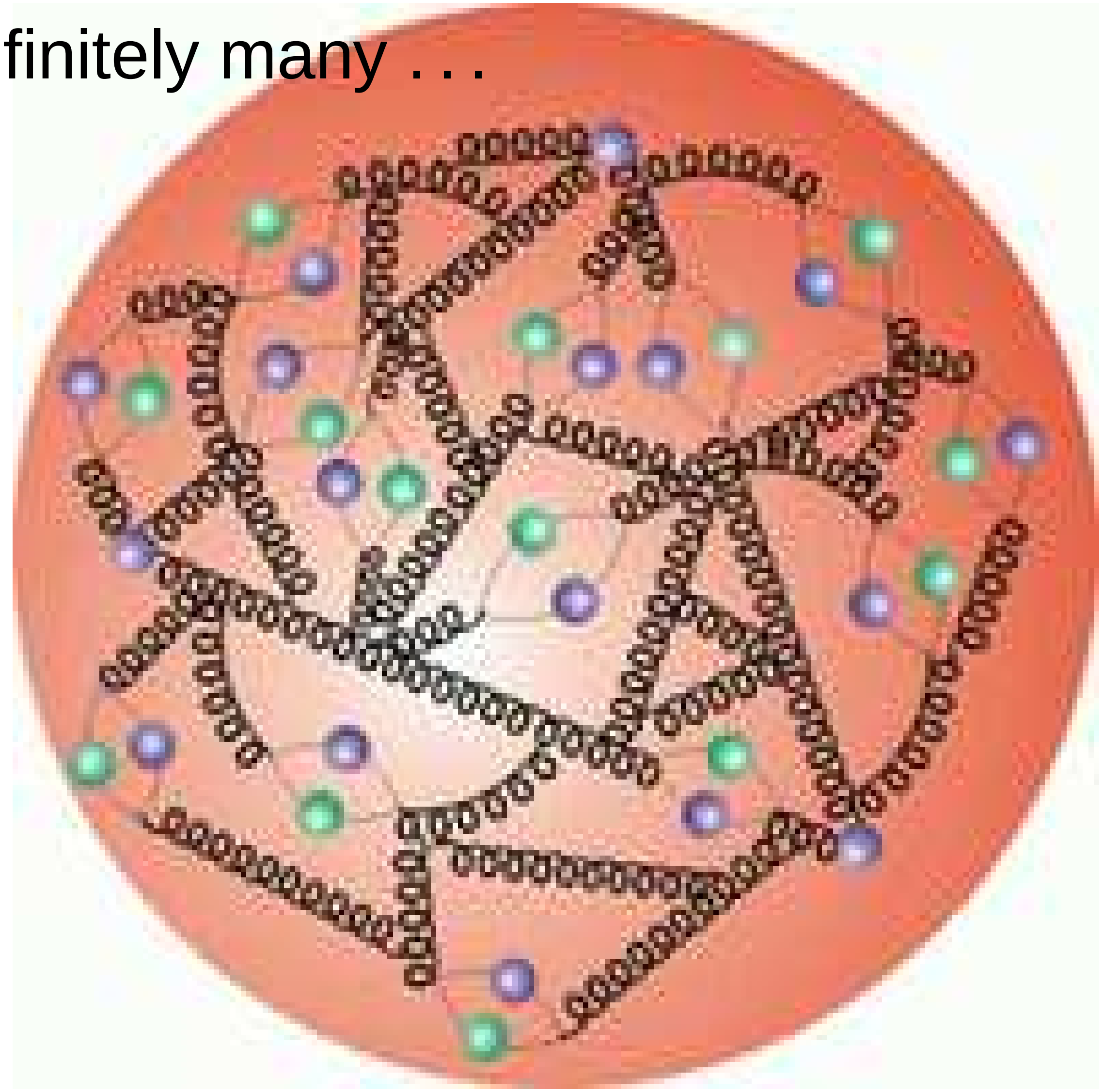
- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Answer for the pion

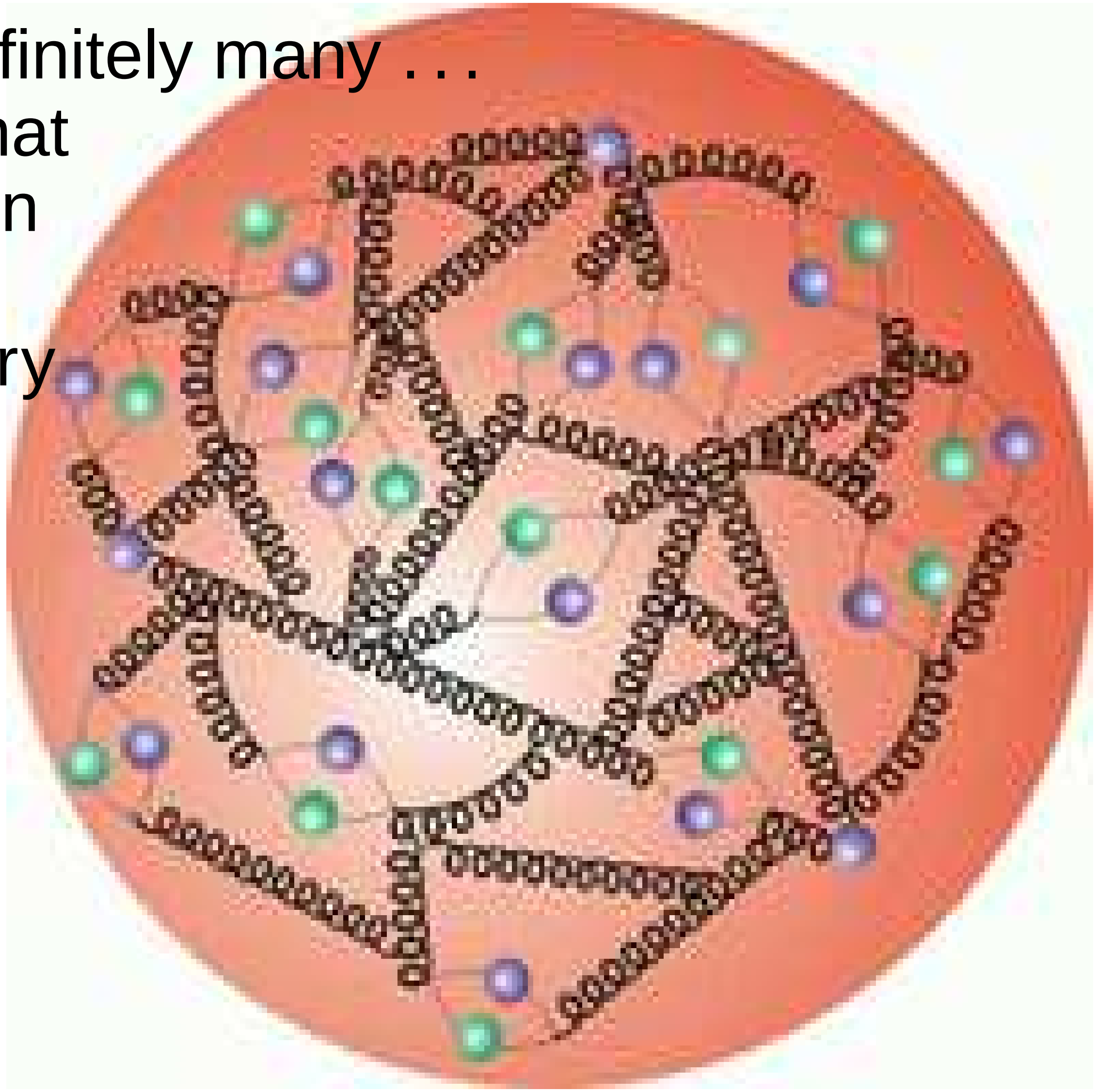


Answer for the pion

Two → Infinitely many . . .



Two $\rightarrow$ Infinitely many . . .
Handle that properly in quantum field theory



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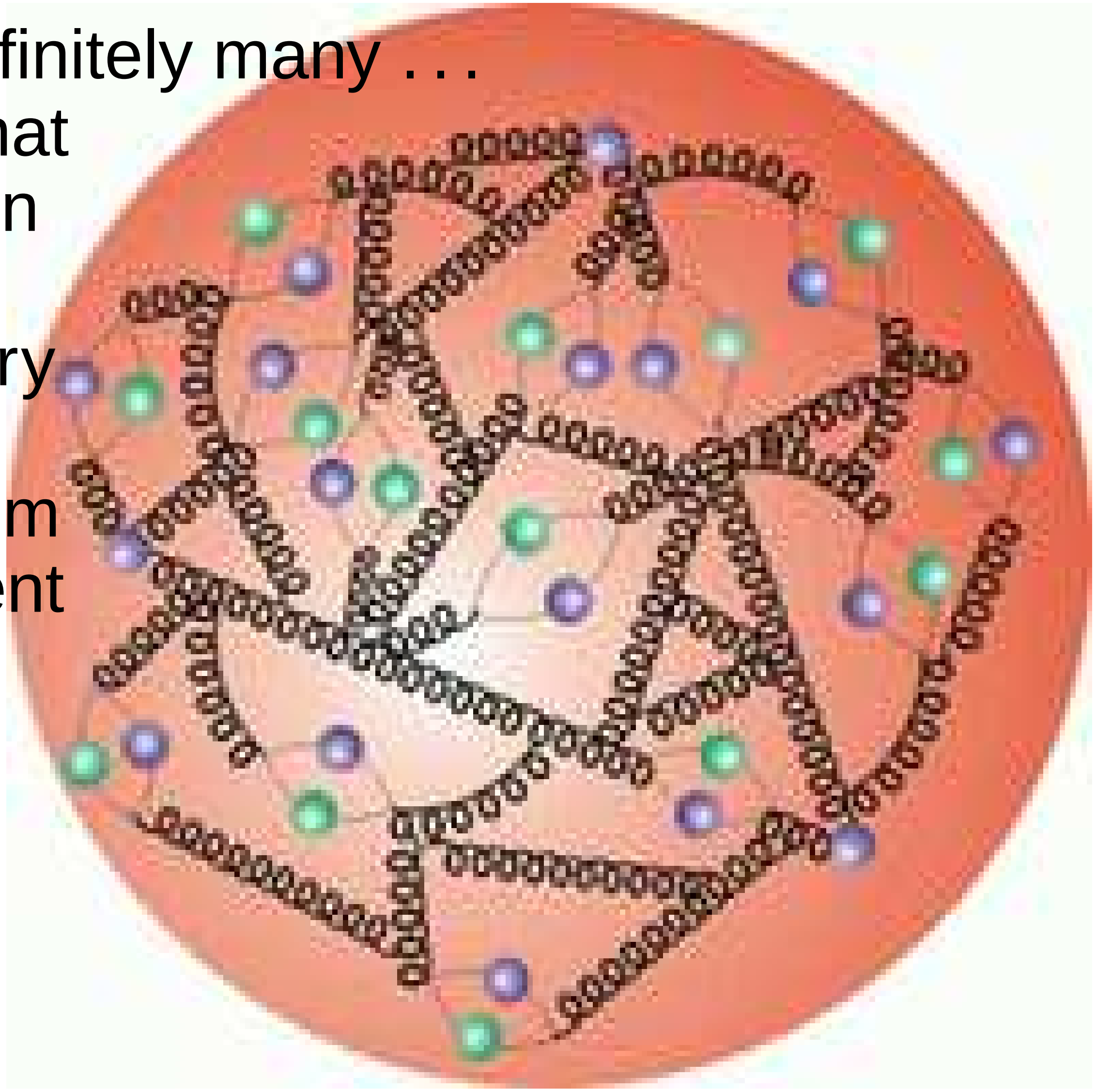

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Answer for the pion

Two → Infinitely many . . .
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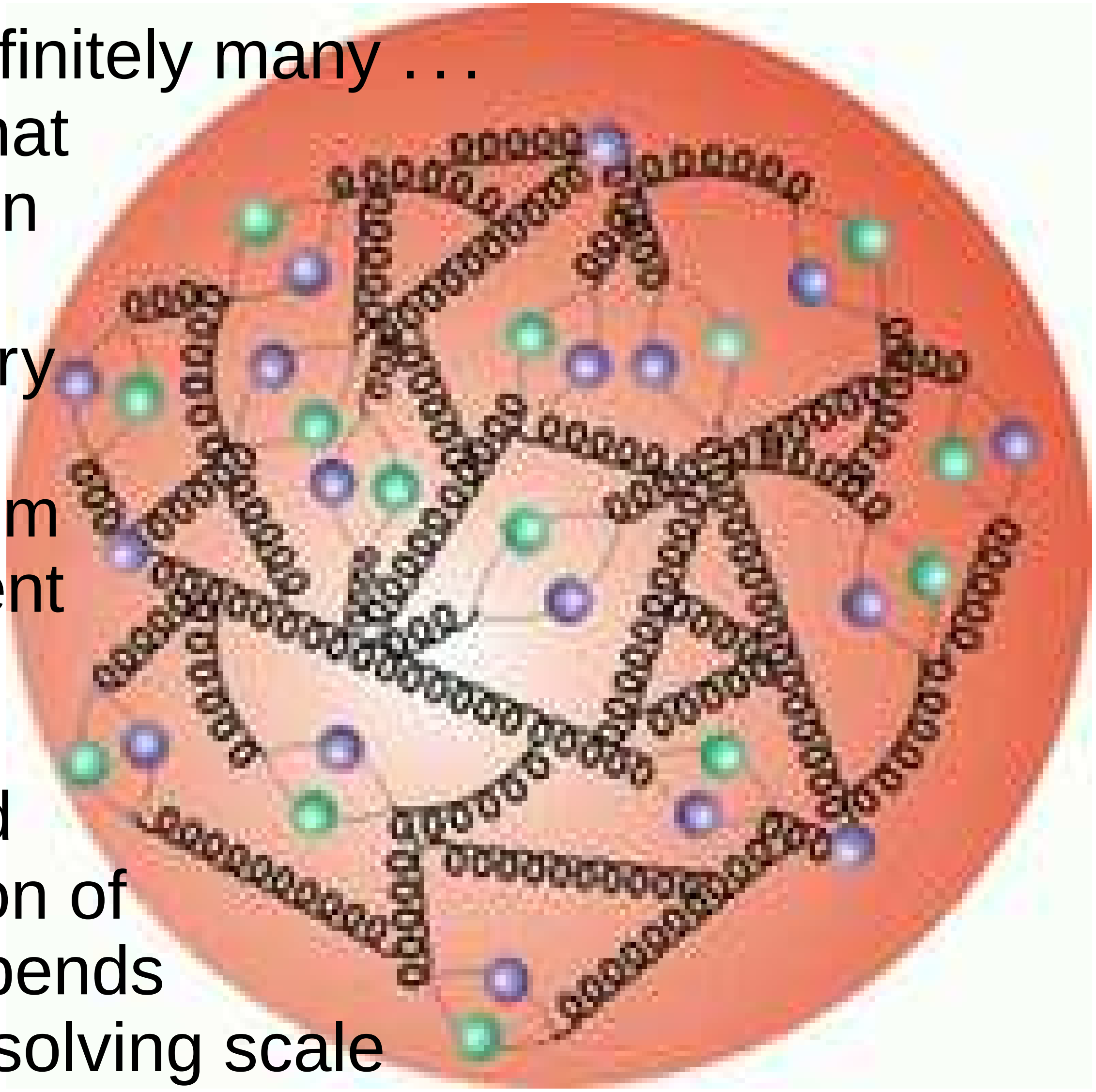
. . .
momentum-dependent dressing
. . .
perceived distribution of mass depends on the resolving scale



Explicit Chiral Symmetry Breaking

Explicit Chiral Symmetry Breaking

- Chiral symmetry is explicitly broken in QCD by the current-quark mass term, which for the u- and d-quark sector is expressed in the action as

$$\begin{aligned}\int d^4z\, \bar{Q}(z)\, \mathcal{M}\, Q(z) &= \int d^4z\, (\bar{u}(z)\, \bar{d}(z)) \left(\begin{array}{cc} m_u & 0 \\ 0 & m_d \end{array} \right) \left(\begin{array}{c} u(z) \\ d(z) \end{array} \right) \\ &= \int d^4z\, \{\bar{m}\, \bar{Q}(z)\tau^0 Q(z) + \bar{Q}(z)\check{m}\tau^3 Q(z)\}\,,\end{aligned}$$

where: $(\tau^0)_{ij} = \delta_{ij}$ and $\{\tau^k; k = 1, 2, 3\}$ are Pauli matrices; and $\bar{m} = (m_u + m_d)/2$ and $\check{m} = (m_u - m_d)/2$.

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- Empirical success with the application of chiral effective theories to low-energy phenomena in QCD indicates that this term can often be treated as a perturbation.

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Dynamical Chiral Symmetry Breaking

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- Success of Chiral Effective Theory owes fundamentally to the phenomenon of dynamical chiral symmetry breaking (DCSB) in QCD

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  - K. D. Lane, "*Asymptotic Freedom And Goldstone Realization Of Chiral Symmetry*," Phys. Rev. **D 10**, 2605 (1974).
  - H. D. Politzer, "*Effective Quark Masses In The Chiral Limit*," Nucl. Phys. **B 117**, 397 (1976).
  - C. D. Roberts and A. G. Williams, "*Dyson-Schwinger equations and their application to hadronic physics*," Prog. Part. Nucl. Phys. **33**, 477 (1994).

Höll, *et al.*, nu-th/0510075

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- Important for numerous reasons, some of longstanding.

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  - Hence, nature’s “constants” may vary.

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- *Interpretation* of some of these measurements *requires* calculations of the *current-quark mass dependence* of the parameters characterising nuclear systems.
- NB. Higher dimensional theories do not necessarily require varying “constants”, but they provide a framework for describing the variations, if they exist.

Pion Sigma Term

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- Useful illustrative example . . . $\boldsymbol{\sigma}$-term for $\boldsymbol{\pi}$ . . . begin with scalar form factor:

$$s_\pi(Q^2) = \langle \pi(P')|\bar{m}\, J_\sigma(Q)|\pi(P)\rangle\,,\; Q_\mu = (P'-P)_\mu\,.$$

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- rainbow-ladder … first term in a nonperturbative, systematic and symmetry preserving truncation scheme $\Rightarrow$ triangle diagram
  - $\mathcal{S}(\ell)$ … two-flavour dressed-quark propagator
  - $\Gamma_\pi(\ell; P)$ … pion's Bethe-Salpeter amplitude
  - $\Gamma_{\tau^0}(\ell; Q)$ … two-flavour inhomogeneous isoscalar scalar vertex

Return to Pion Sigma Term

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- Concrete illustration of a general result that may be viewed as a consequence of the Feynman-Hellmann theorem.
  - Present case: theorem states that response of an eigenvalue of the QCD mass^2-operator to a change in a parameter in that operator is given by expectation value of the derivative of the mass^2-operator operator with respect to the parameter.

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Derived by Feynman, when 21, in his final year as an undergraduate. Has played an important role in theoretical chemistry and condensed matter physics.

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$$\boxed{\frac{\partial E_{M^2}}{\partial \lambda} = \left\langle \frac{\partial M^2}{\partial \lambda} \right\rangle}$$

- The result is valid in this form for all mesons; i.e.,

$$\boxed{2\, m_M \sigma_M := s_M(0) = \bar{m}(\zeta)\frac{\partial m_M^2}{\partial \bar{m}(\zeta)} \;\Rightarrow\; \sigma_M = \bar{m}(\zeta)\frac{\partial m_M}{\partial \bar{m}(\zeta)}}$$

NB. The σ-term is a renormalisation point invariant, in general and also in the explicit calculation, so long as a RGI rainbow-ladder truncation is used.

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  - Model-independent result.
  - Essential consequence of DCSB.

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- Typical, calculated chiral limit values of:
  - $f_\pi^0 = 0.088$ GeV
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- Same value is obtained in one-loop chiral perturbation theory

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ARE WE THERE YET?

New Challenges

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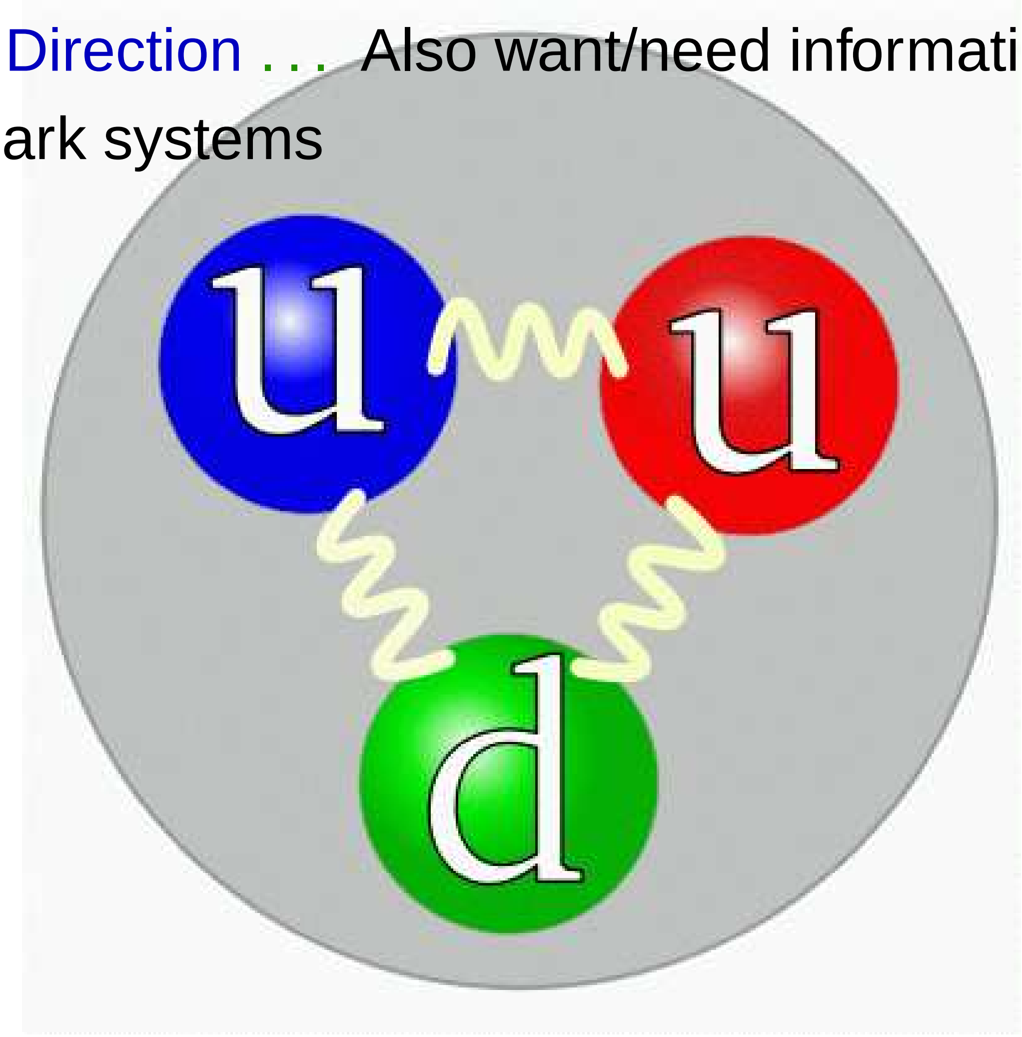
- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

New Challenges

- Another Direction . . . Also want/need information about three-quark systems



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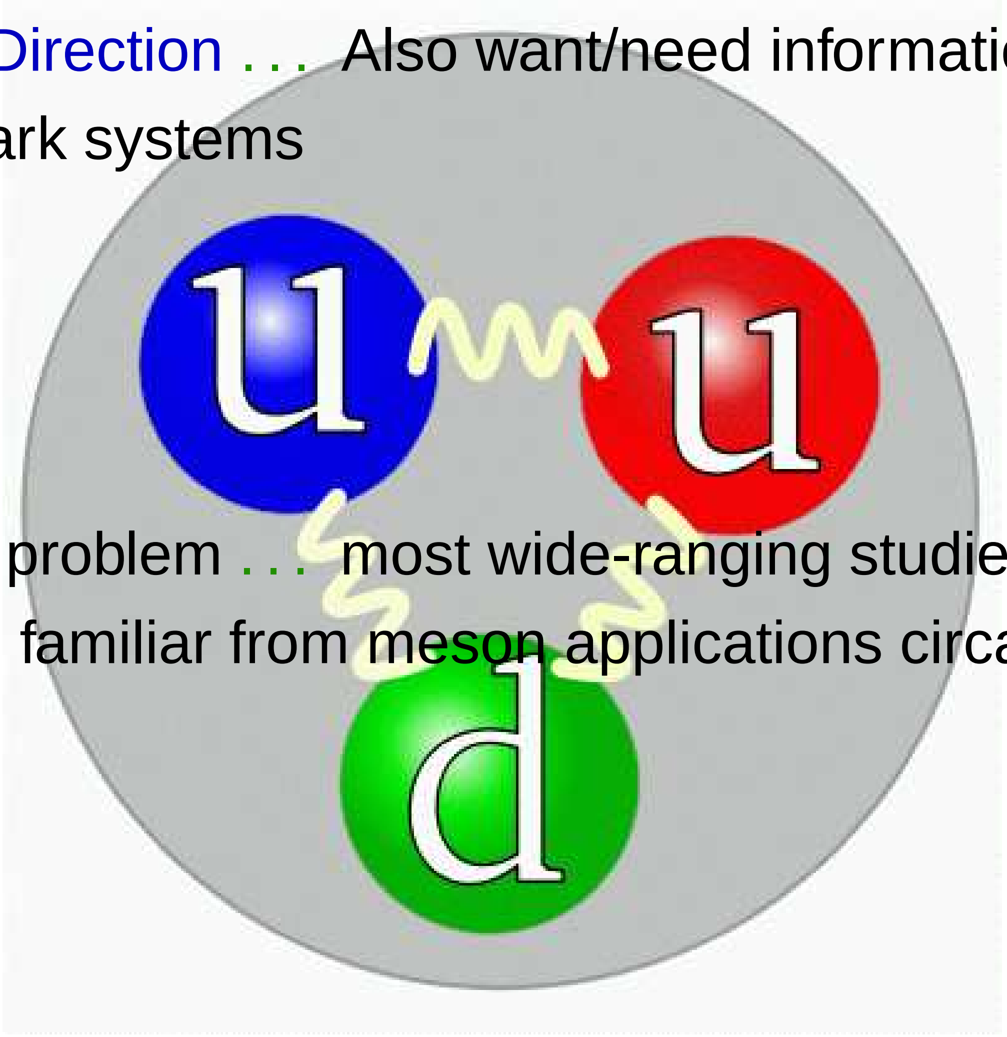

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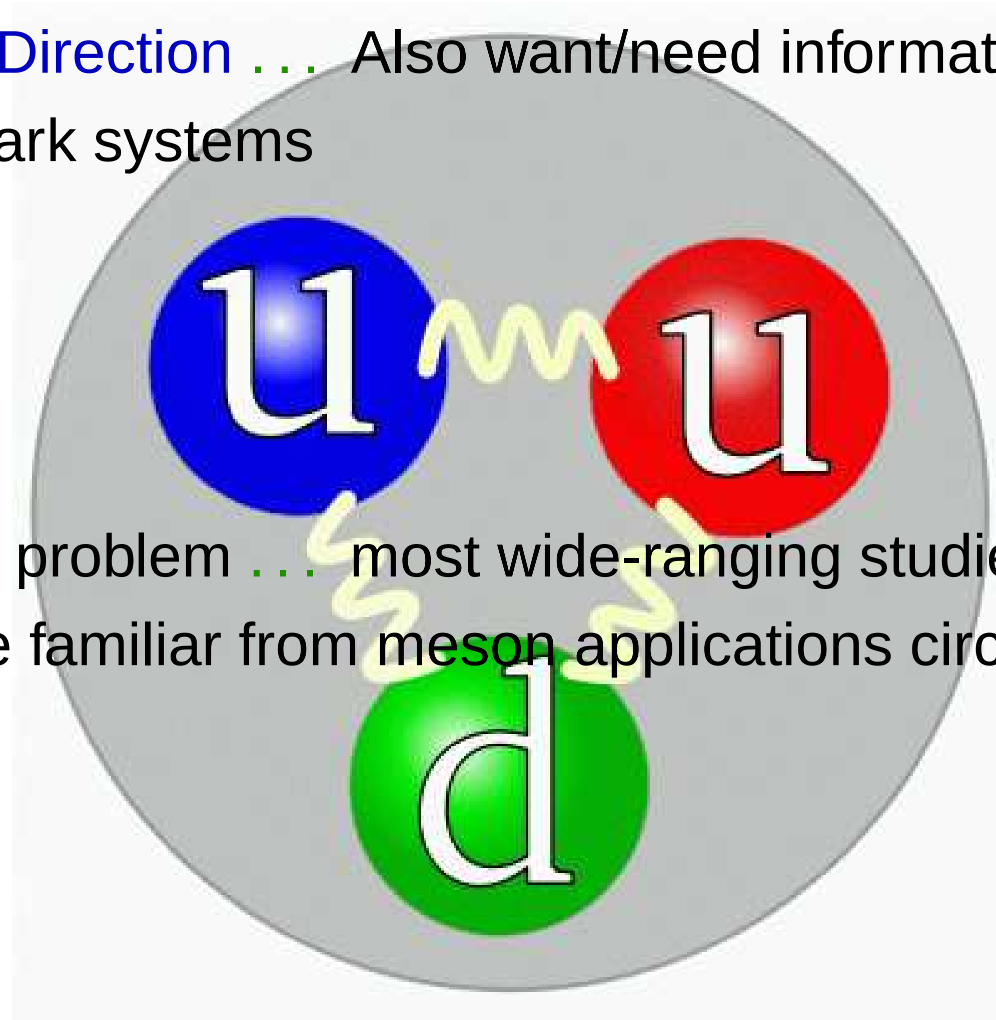

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- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.

New Challenges

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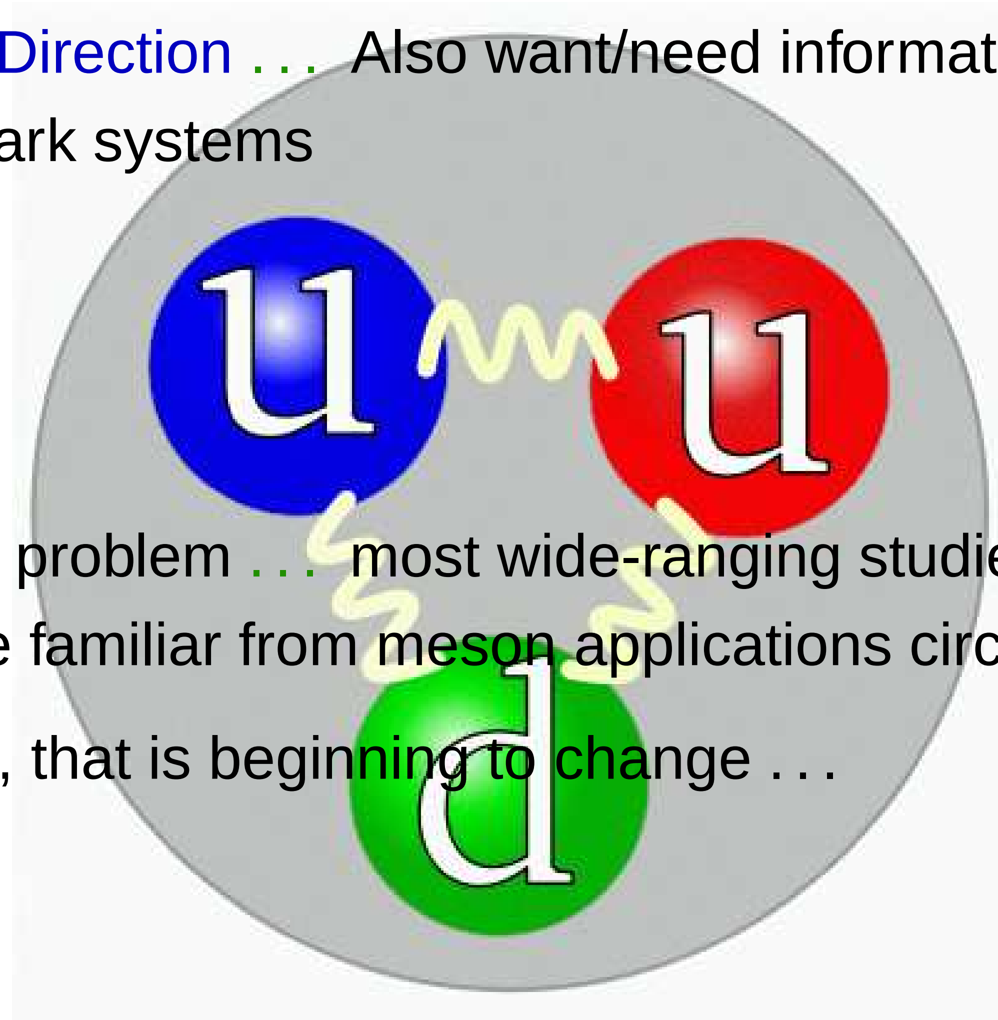

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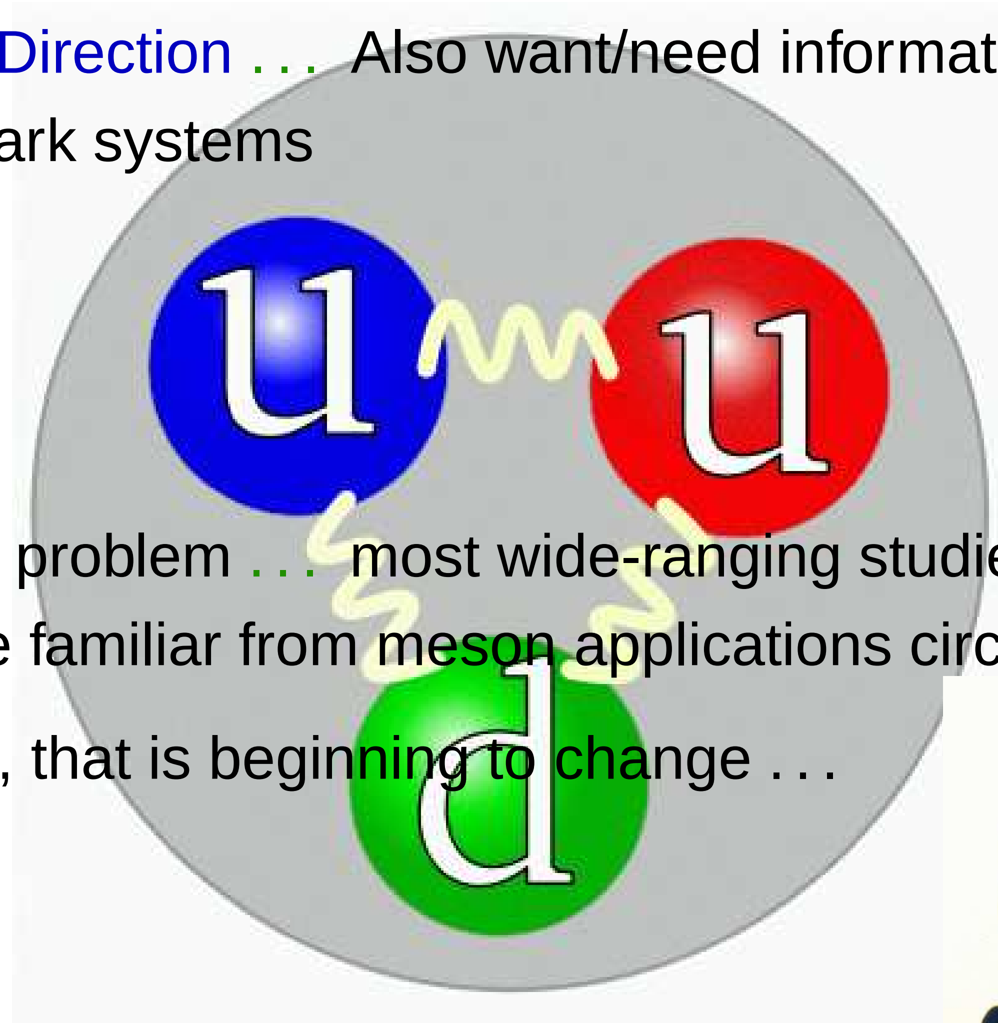

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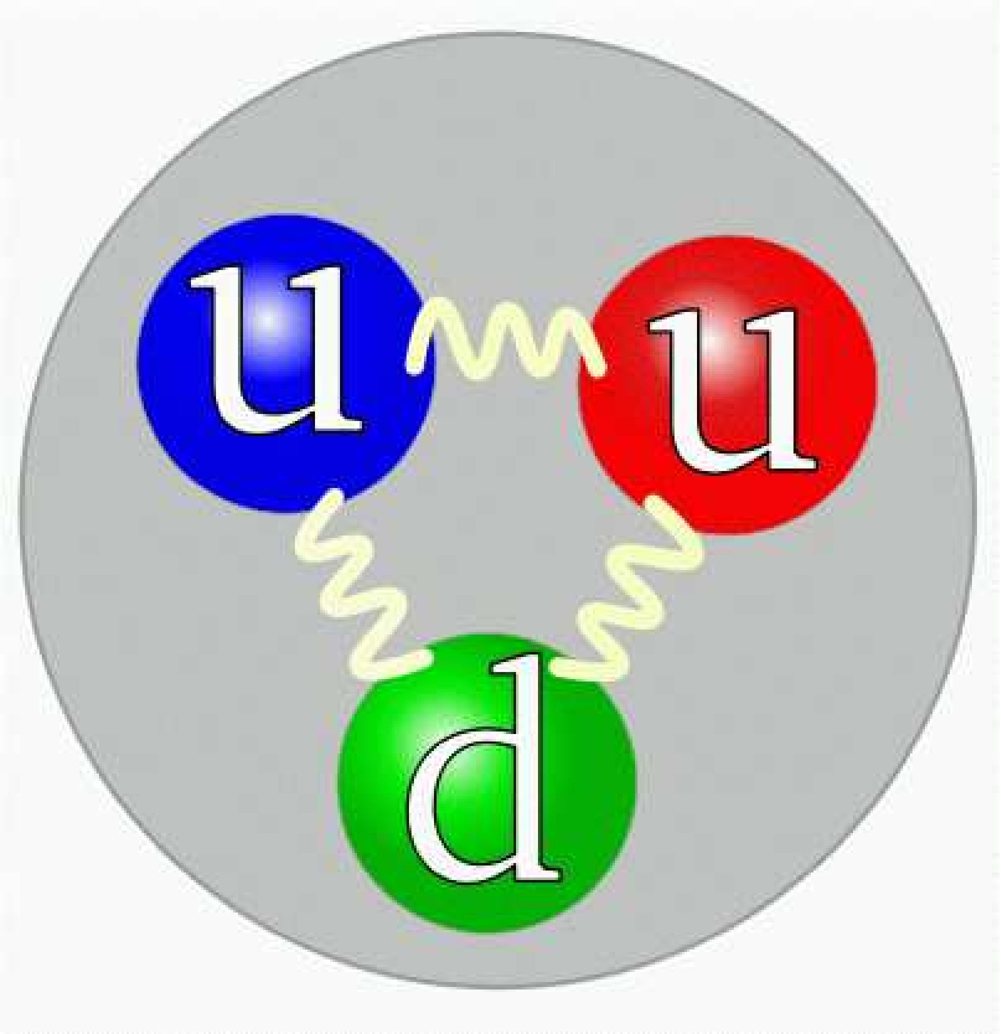
Nucleon . . .
Three-body Problem?



Nucleon . . .
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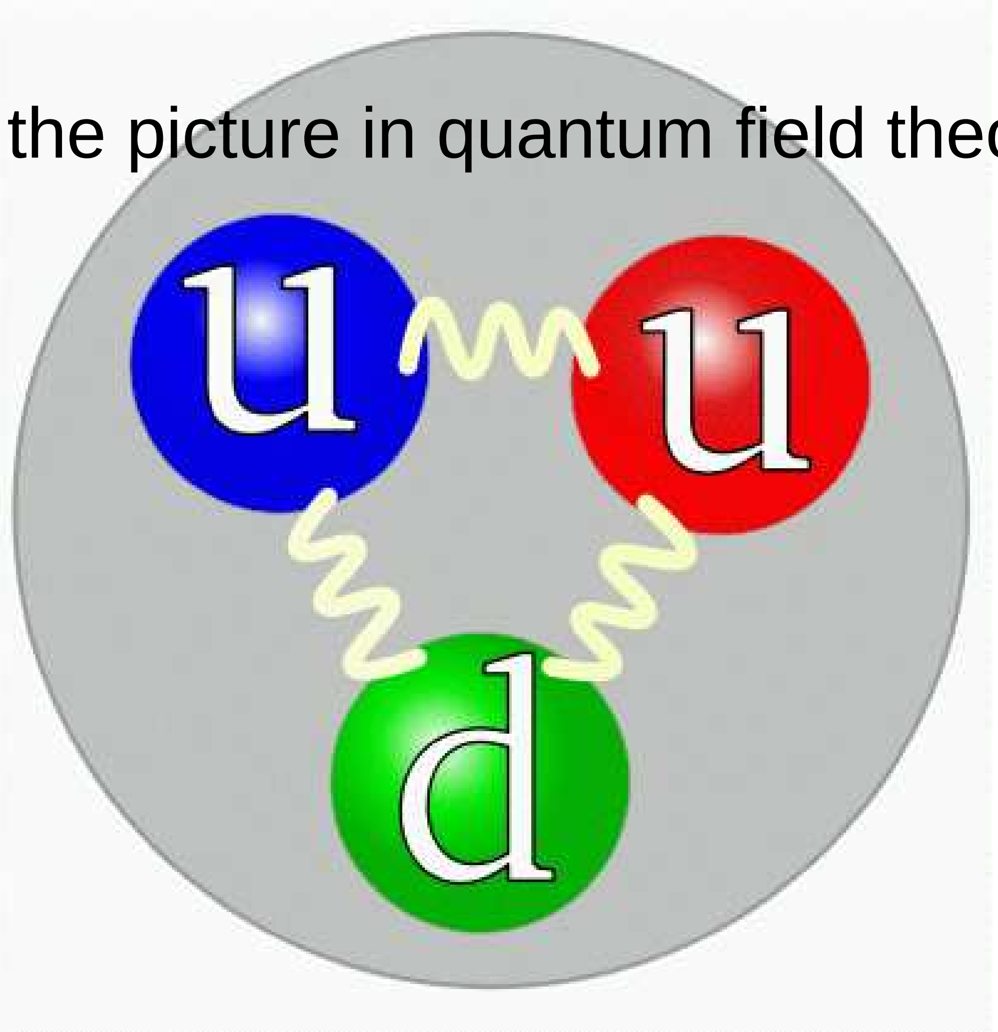

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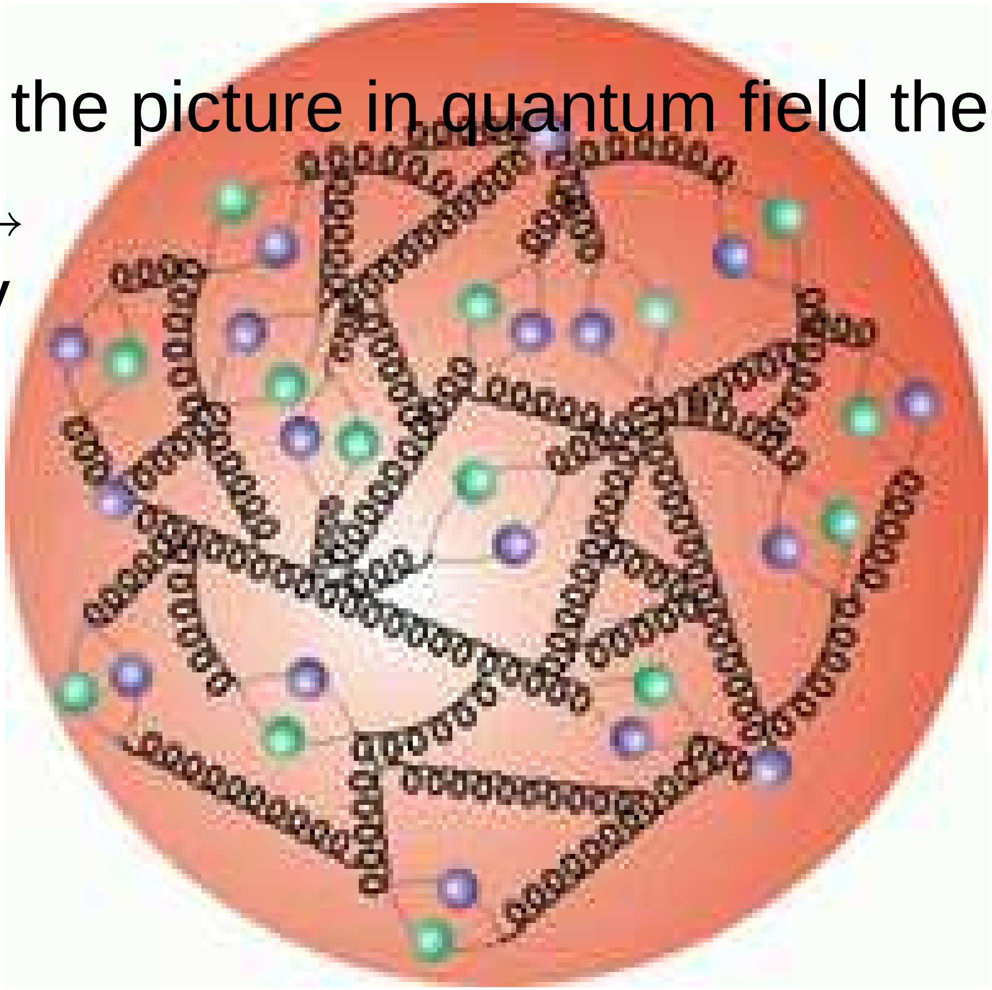
Three-body Problem?

- What is the picture in quantum field theory?
- Three → infinitely many!



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Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars
Argonne NATIONAL LABORATORY

Unifying Study of Mesons and Baryons

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  - Residue is proportional to nucleon's Faddeev amplitude
  - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
  - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour-$\bar{3}$ (antitriplet) channel

Faddeev equation

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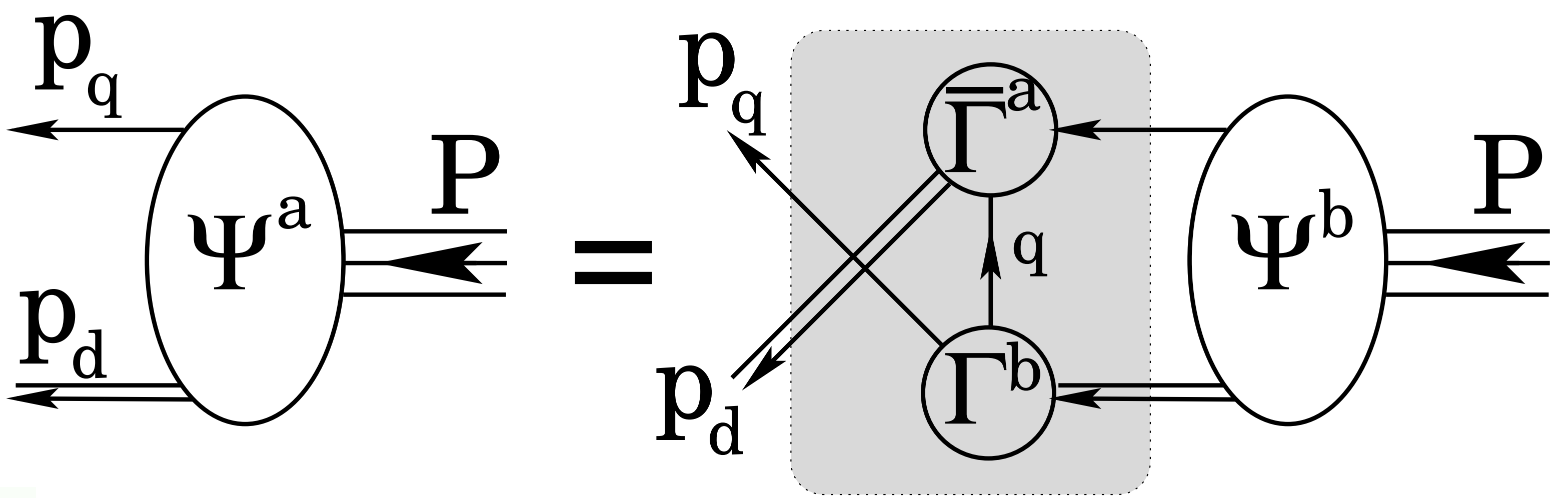

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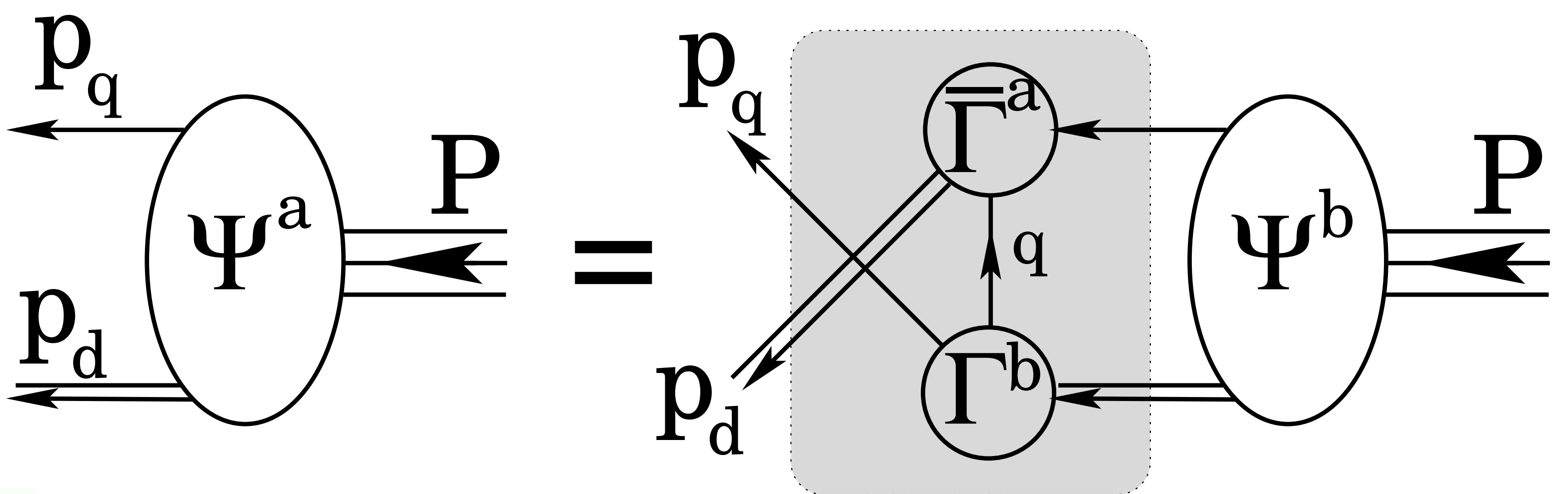
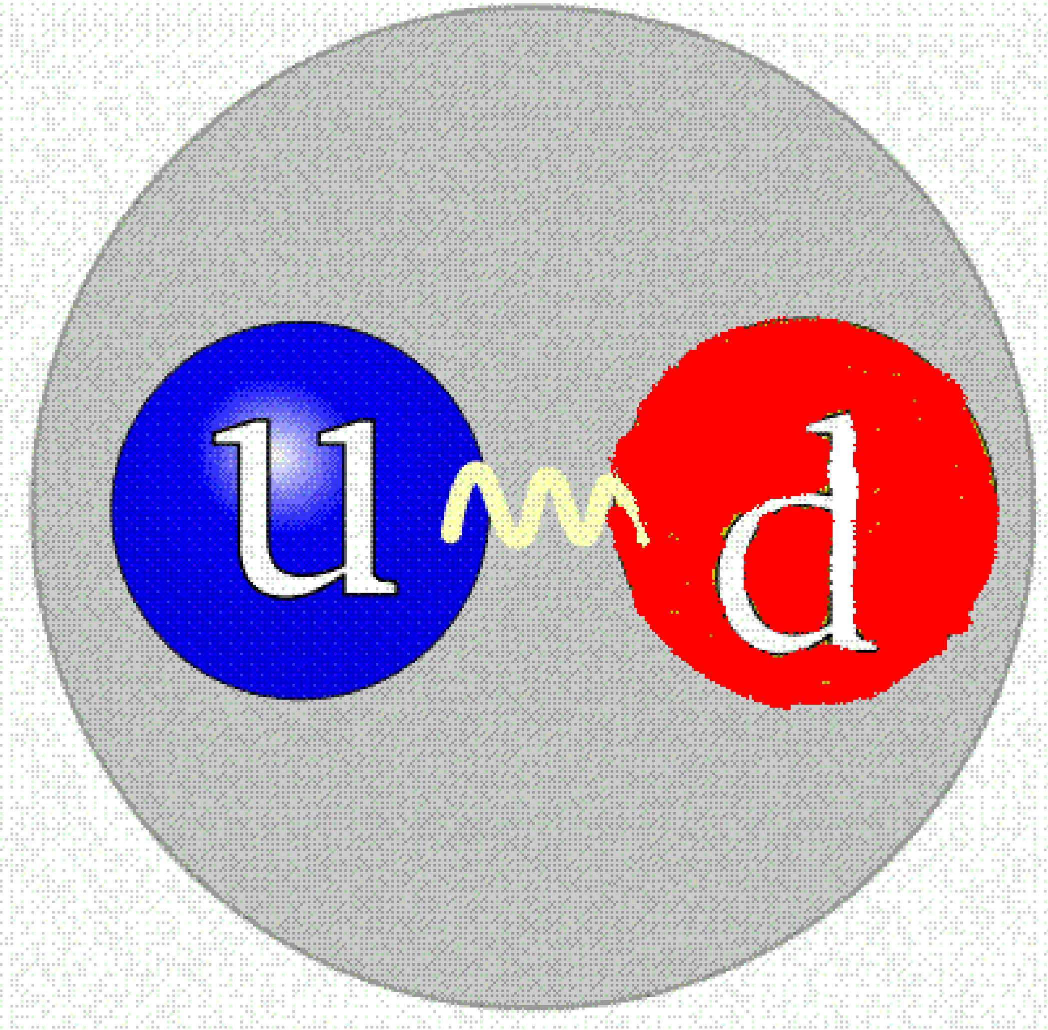


- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame Amplitude has ... $s-$, $p-$ & $d-$wave correlations

Diquark correlations



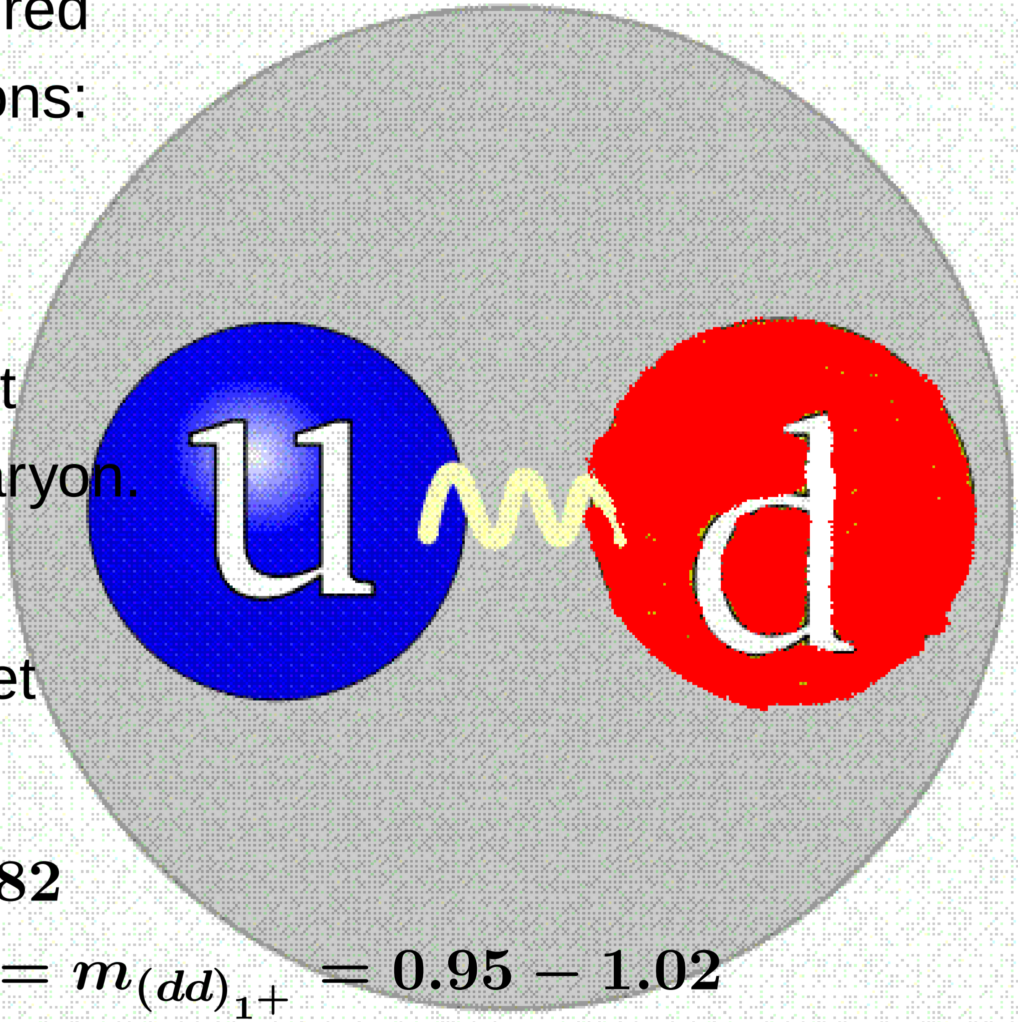
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined ... Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

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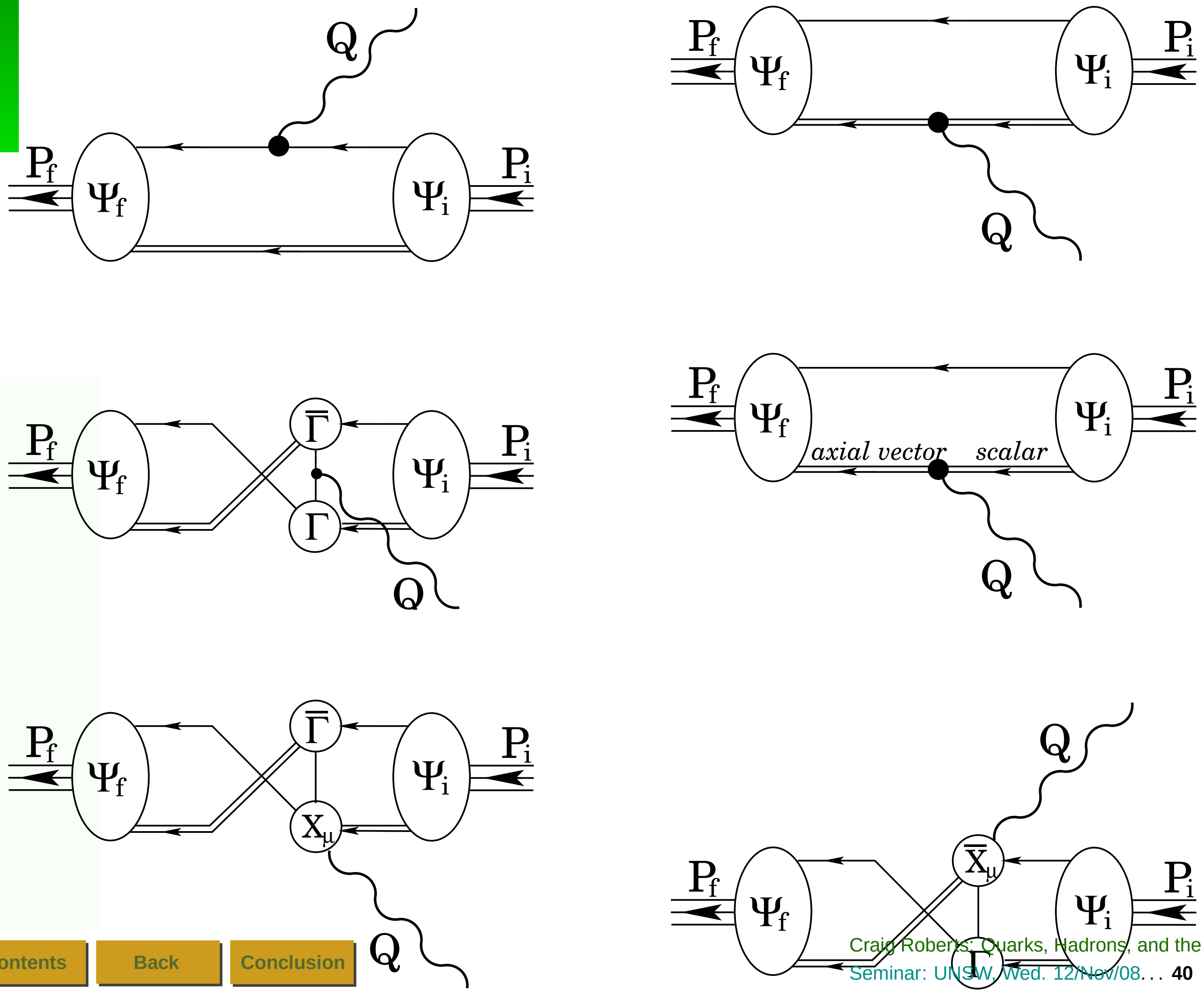

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Cloët *et al.*

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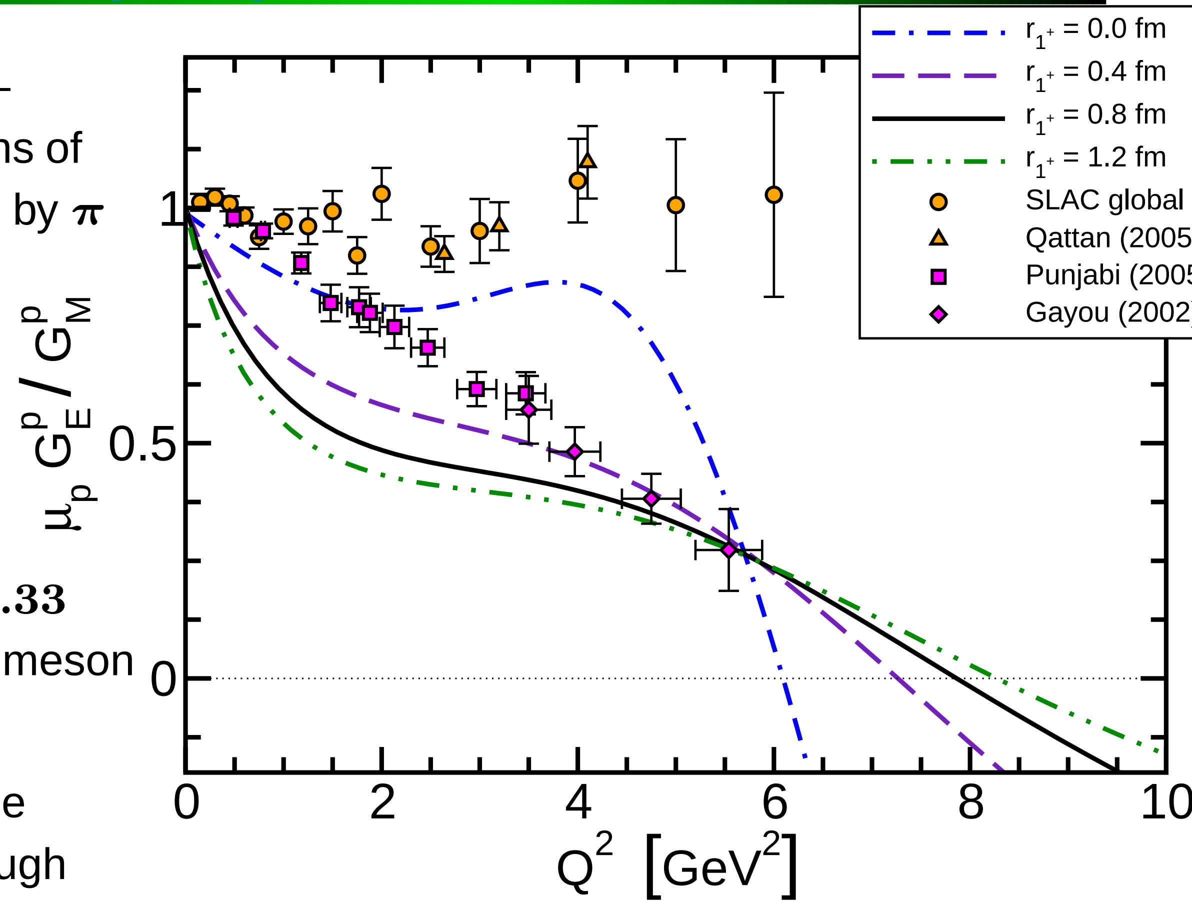

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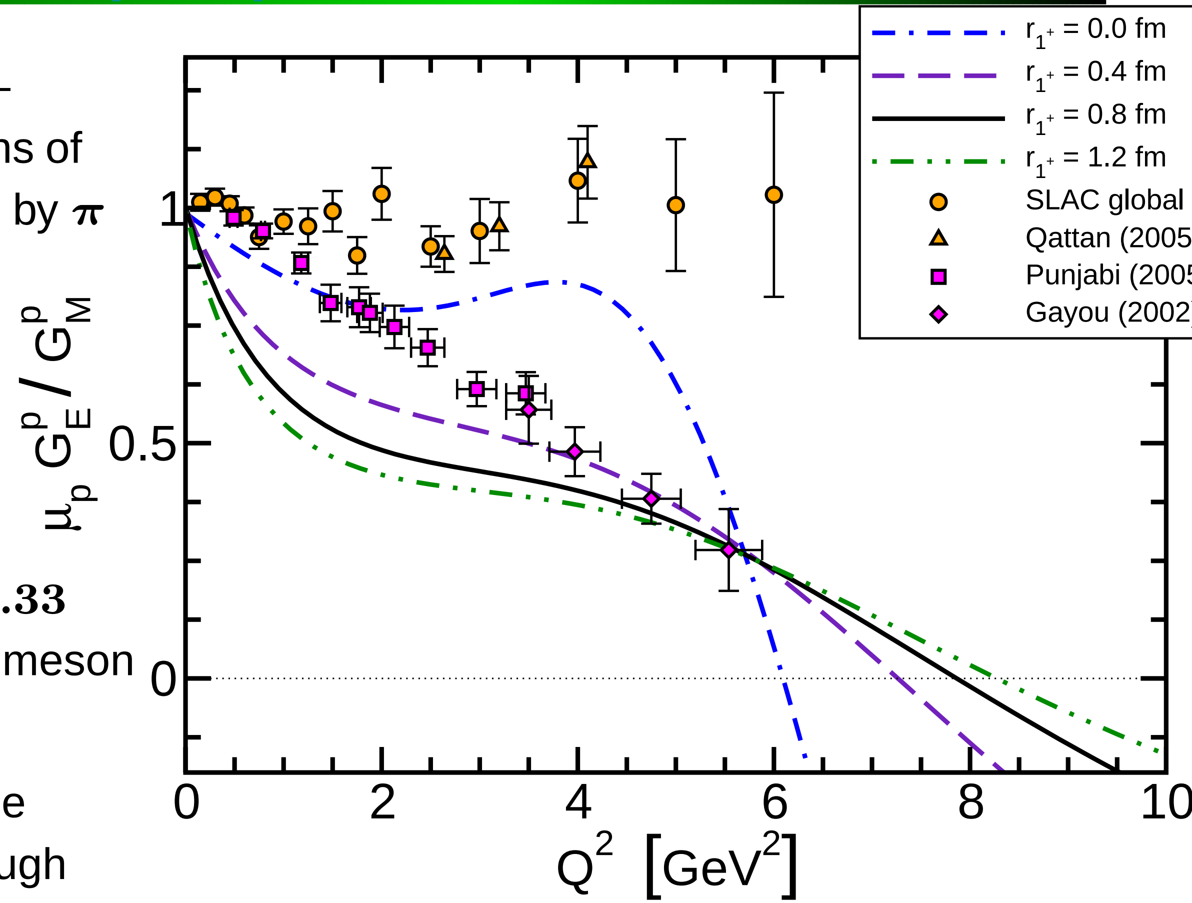

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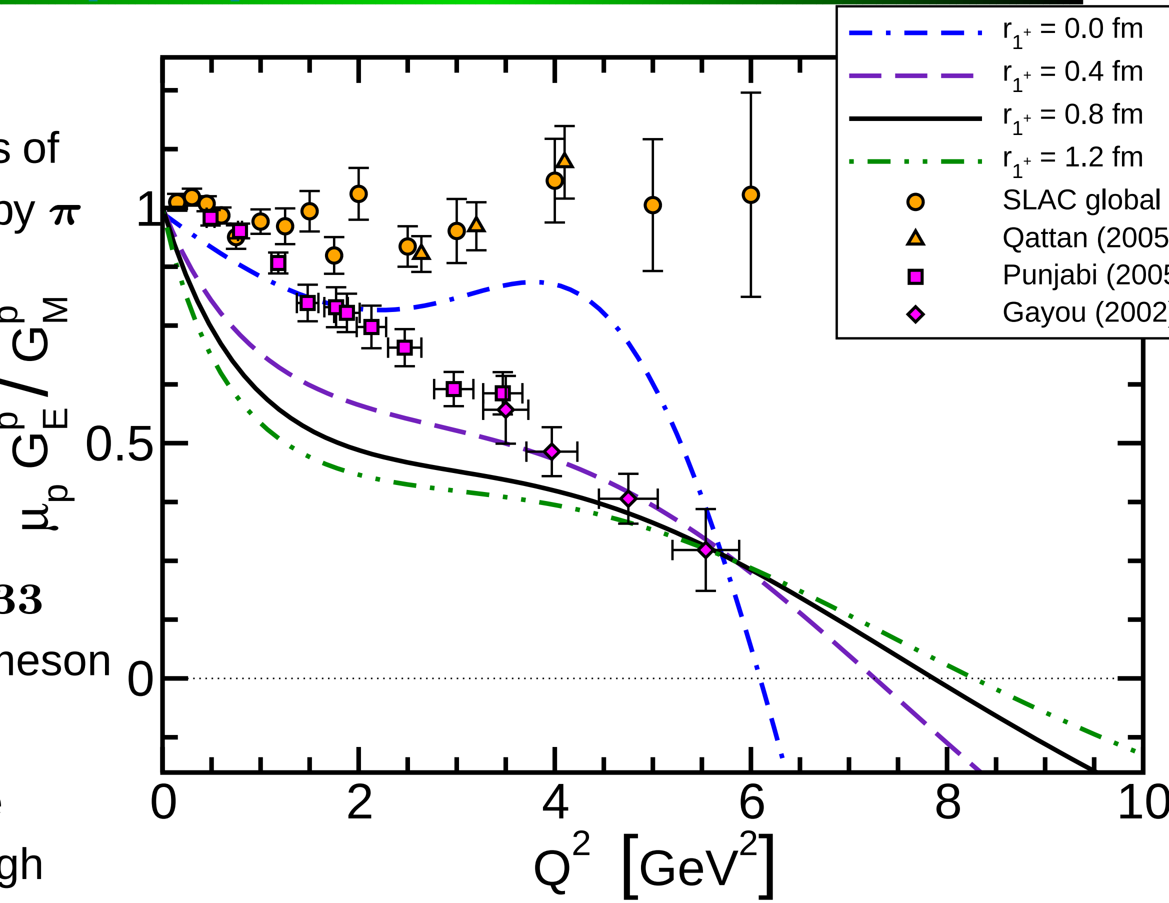

- On $Q^2 \lesssim 4\,\text{GeV}^2$ result lies below experiment. This can be attributed to omission of pseudoscalar-meson-cloud contributions
- Always a zero but position depends on details of current

Ratio of Neutron Pauli & Dirac Form Factors

- $$\frac{\hat{Q}^2}{(\ln \hat{Q}^2/\hat{\Lambda})^2} \frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)}$$

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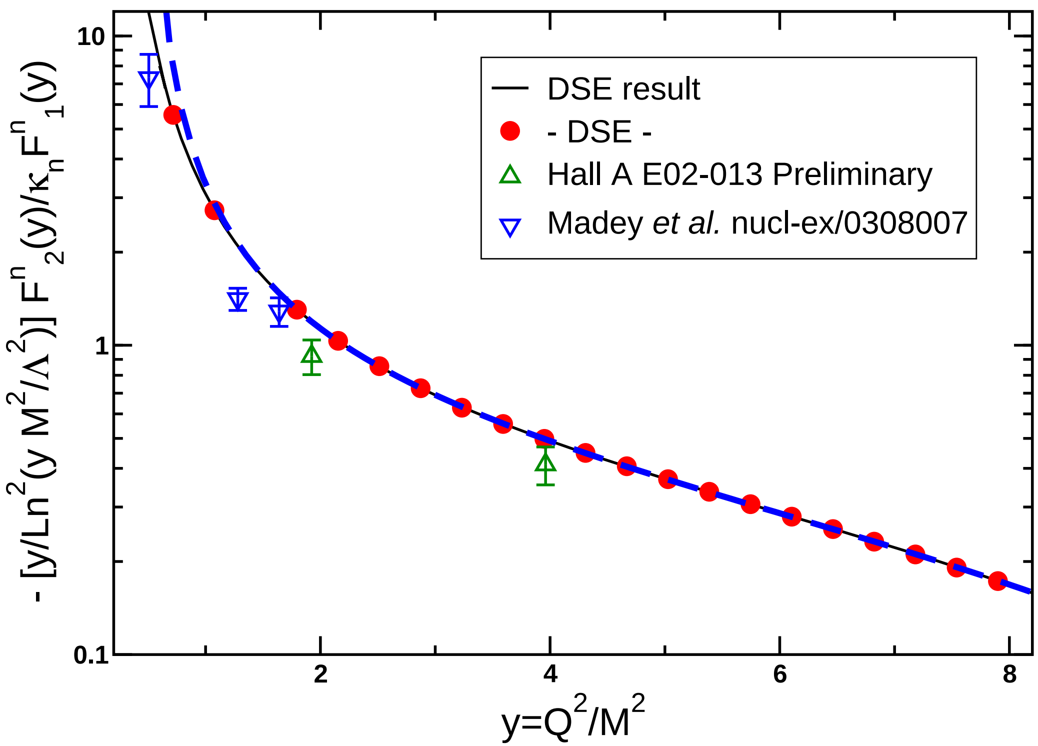

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Pion Cloud

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F2 – neutron

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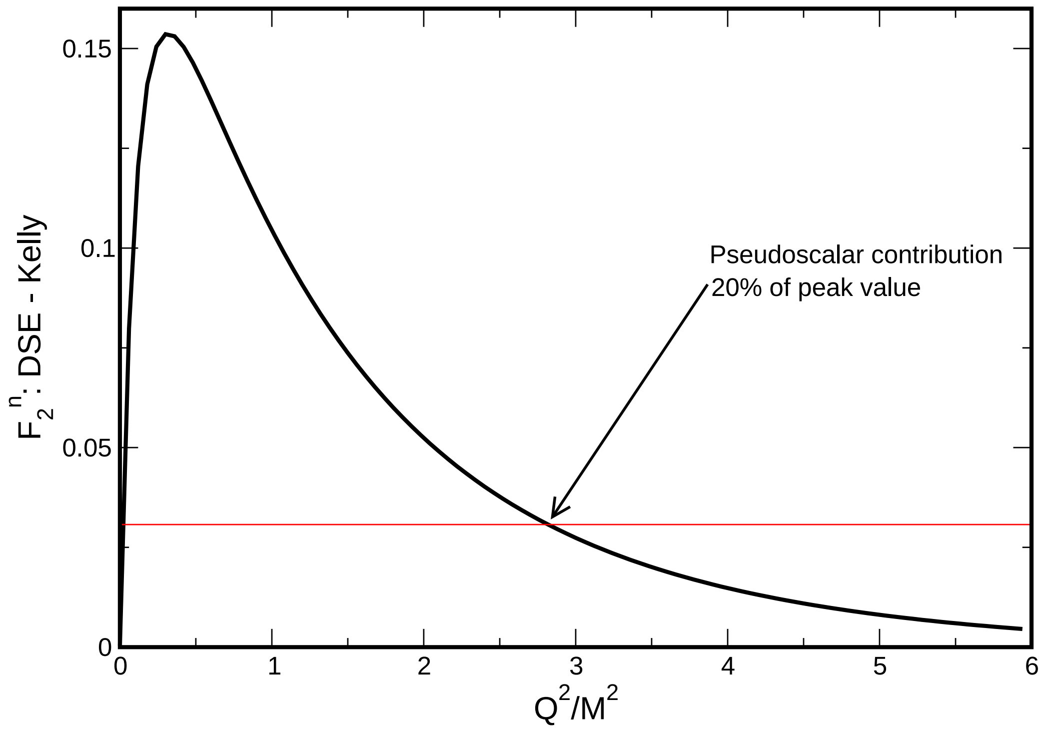

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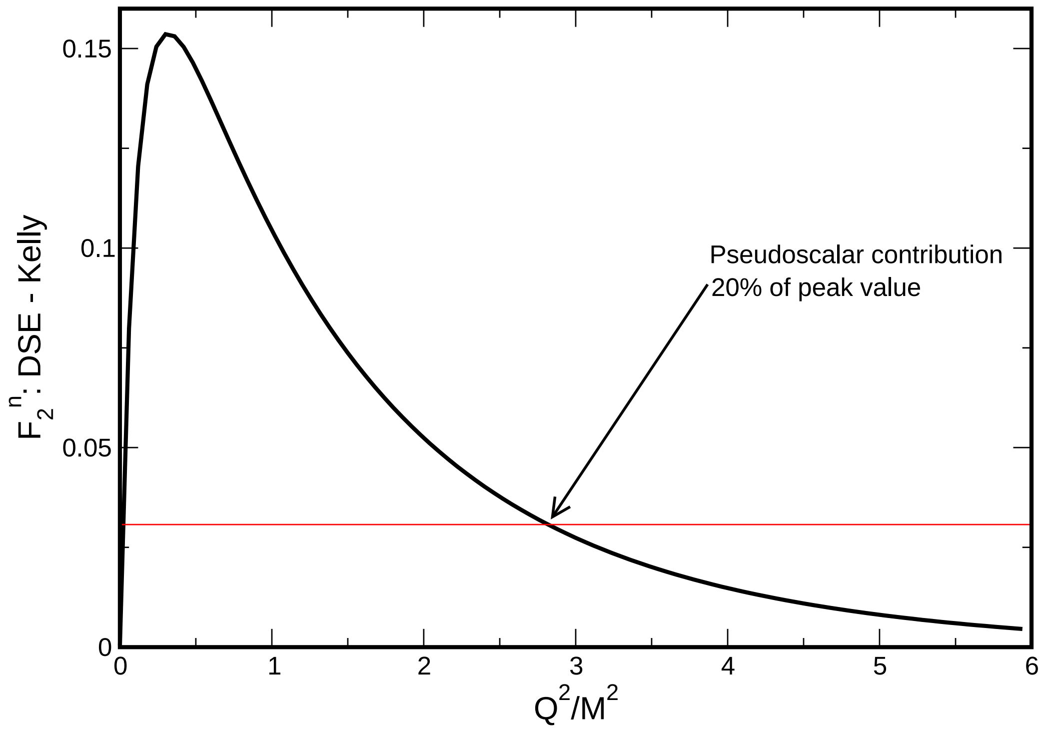

- Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4\, M_N^2$

Epilogue

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Epilogue

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Epilogue

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  - DCSB means that a small change in current-quark mass is amplified in the response of hadron masses
  - But DCSB suppresses the δm-response of quark-core radii and magnetic moments.
    The rapid change arises from the pseudoscalar meson cloud owing to the pion σ-term

Contents