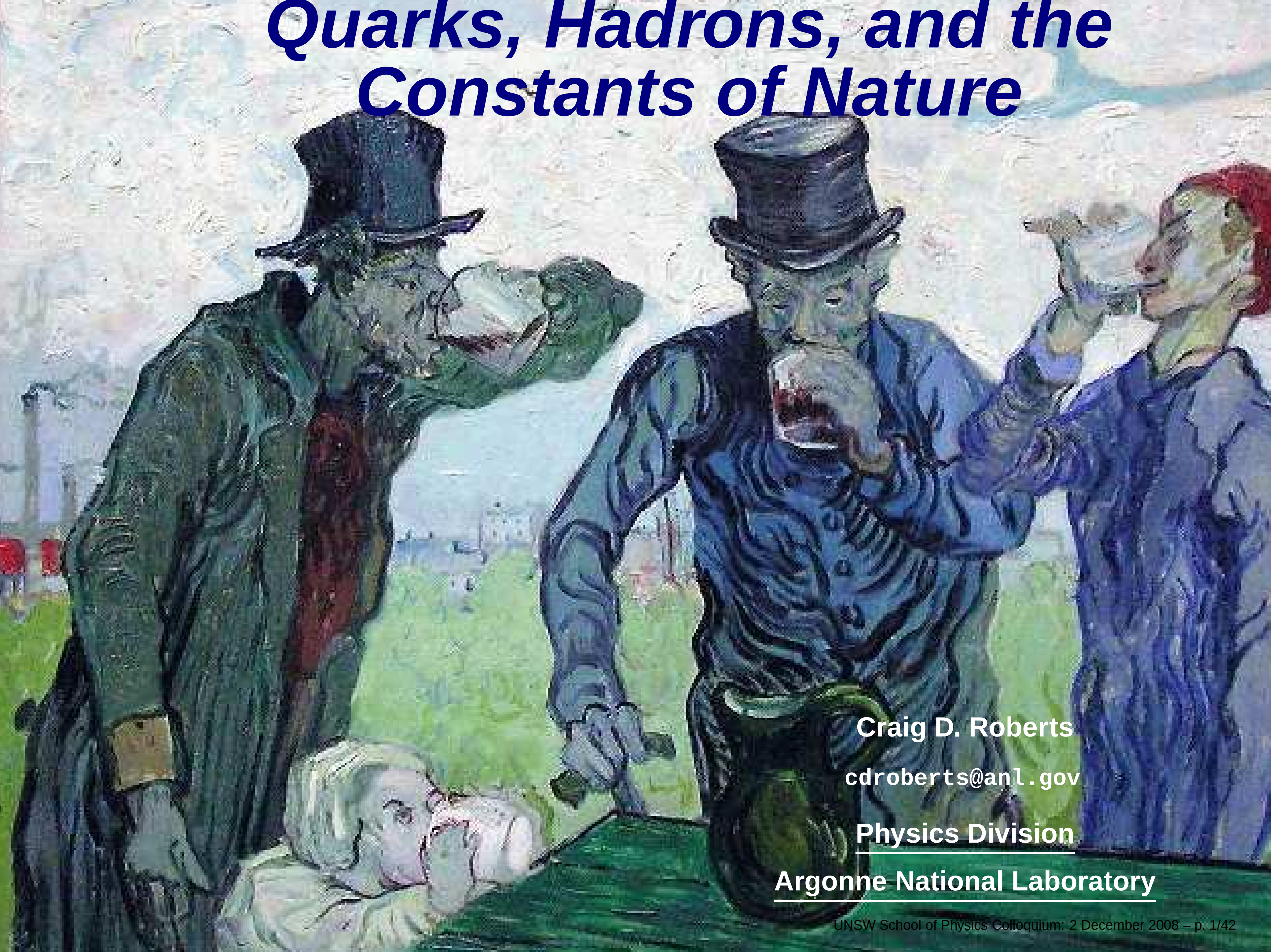
Quarks, Hadrons, and the Constants of Nature



Craig D. Roberts

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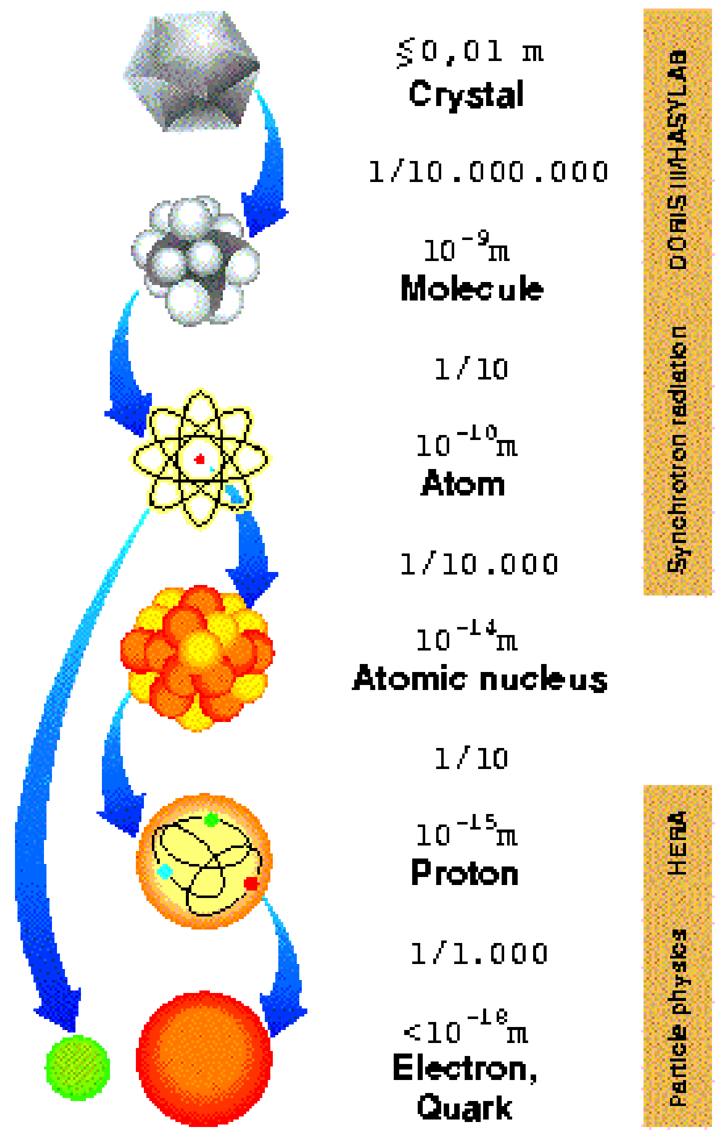
Physics Division

Argonne National Laboratory

Hadron Physics



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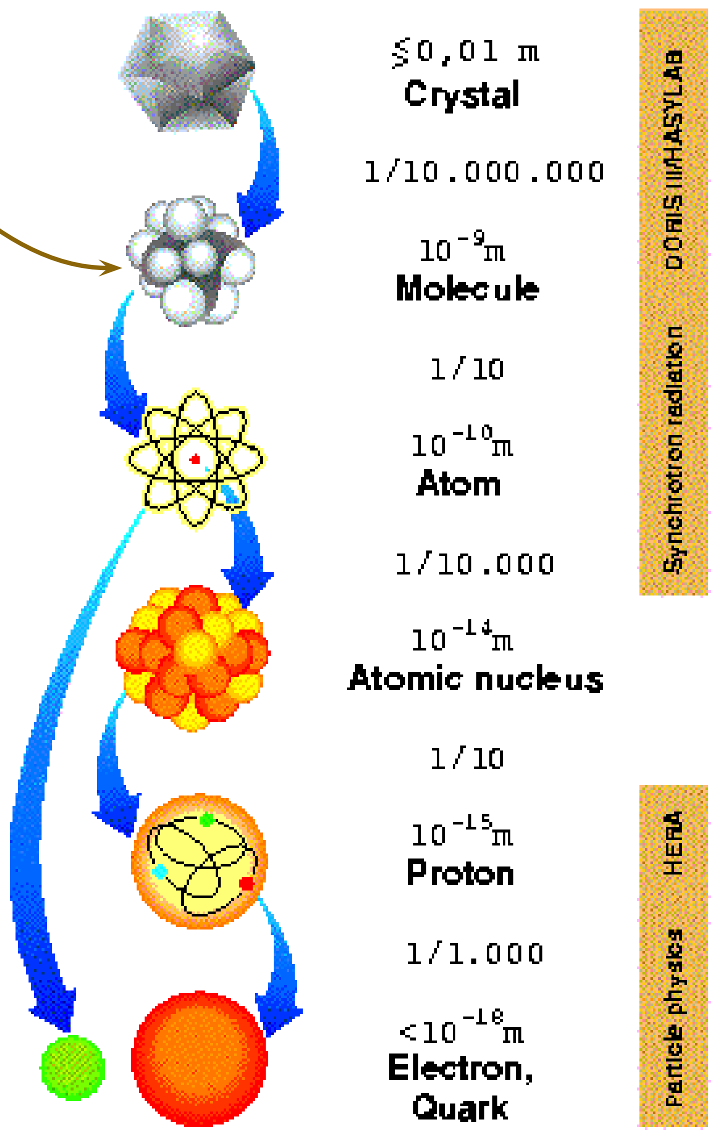
Hadron Physics

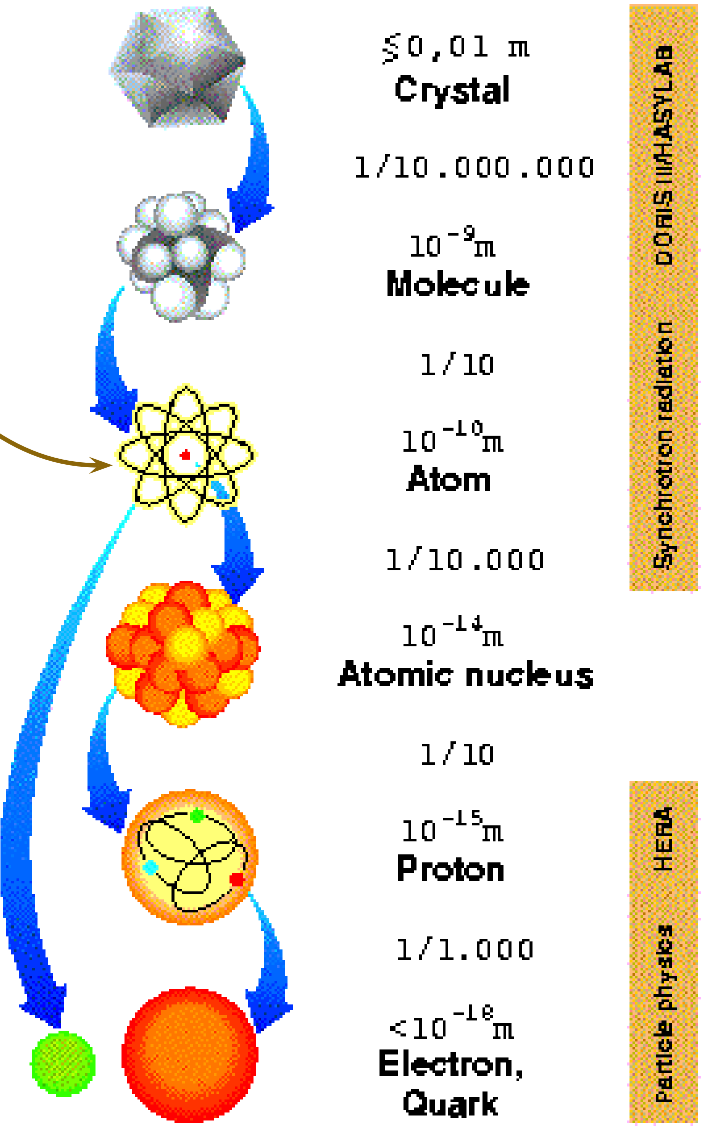
Molecular Physics
Scale = nm



Hadron Physics

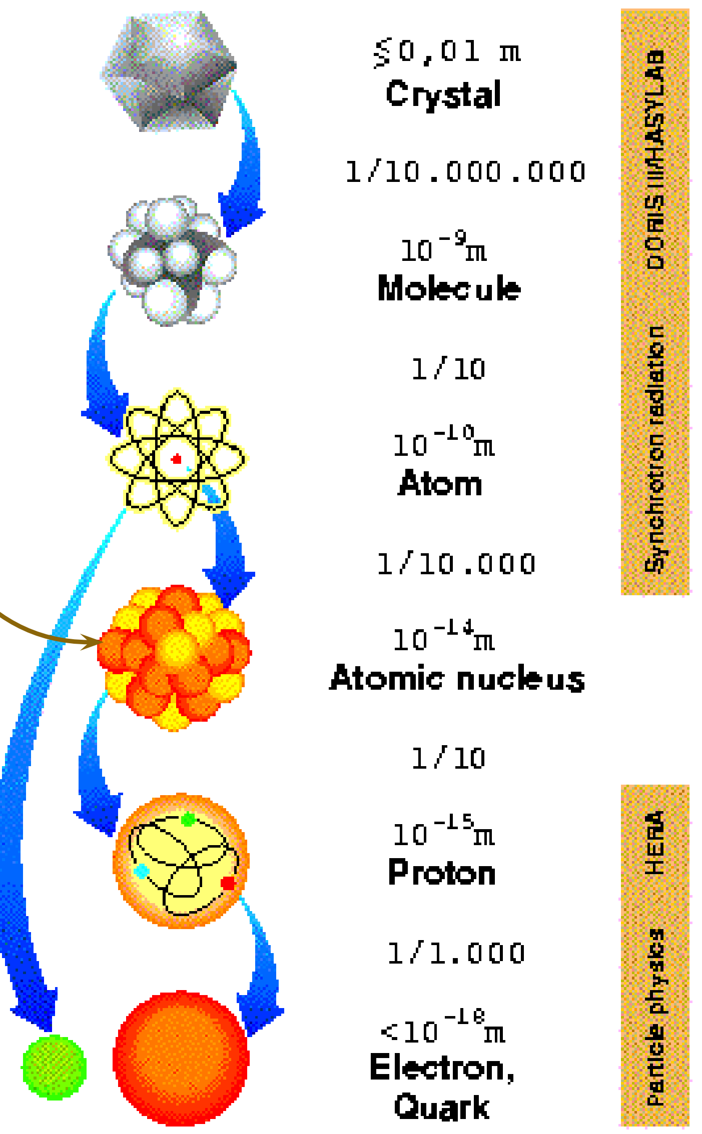
Atomic Physics

Scale = Å



Hadron Physics

Nuclear Physics
Scale = 10 fm



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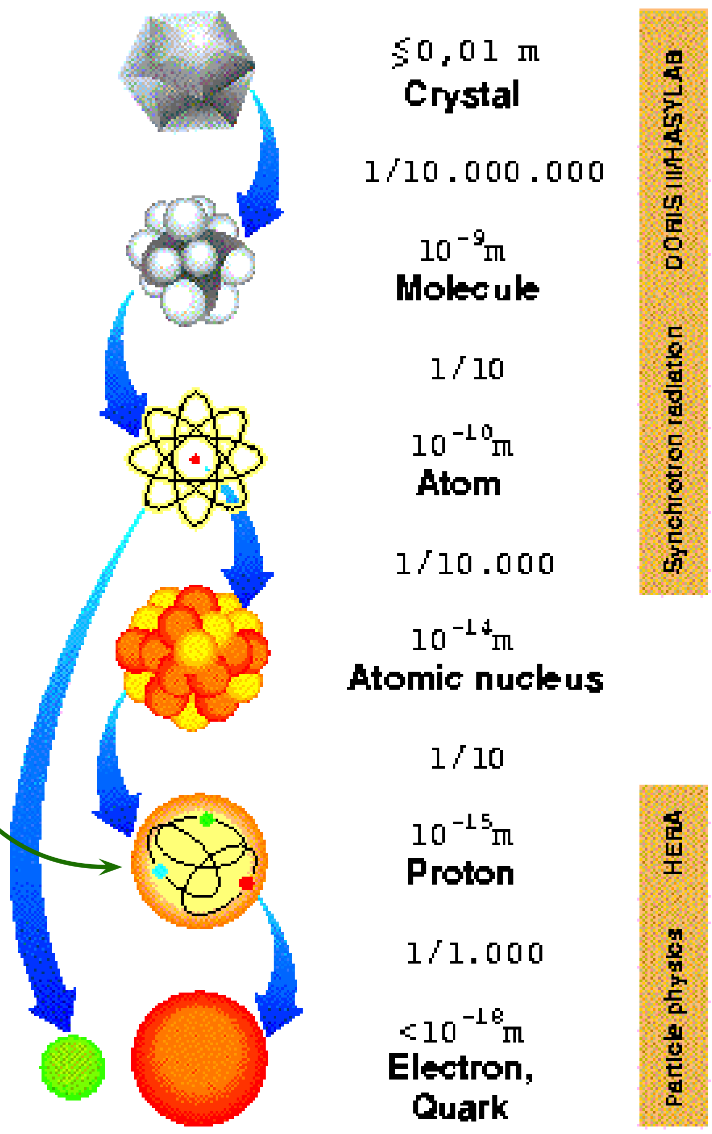
Hadron Physics
Scale = 1 fm



Hadron Physics

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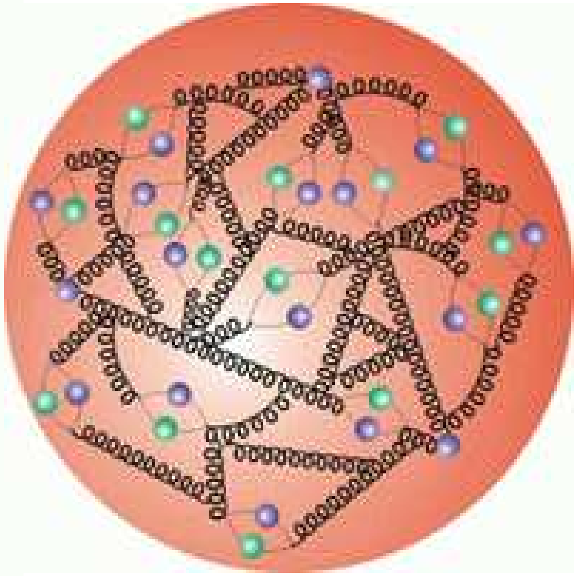
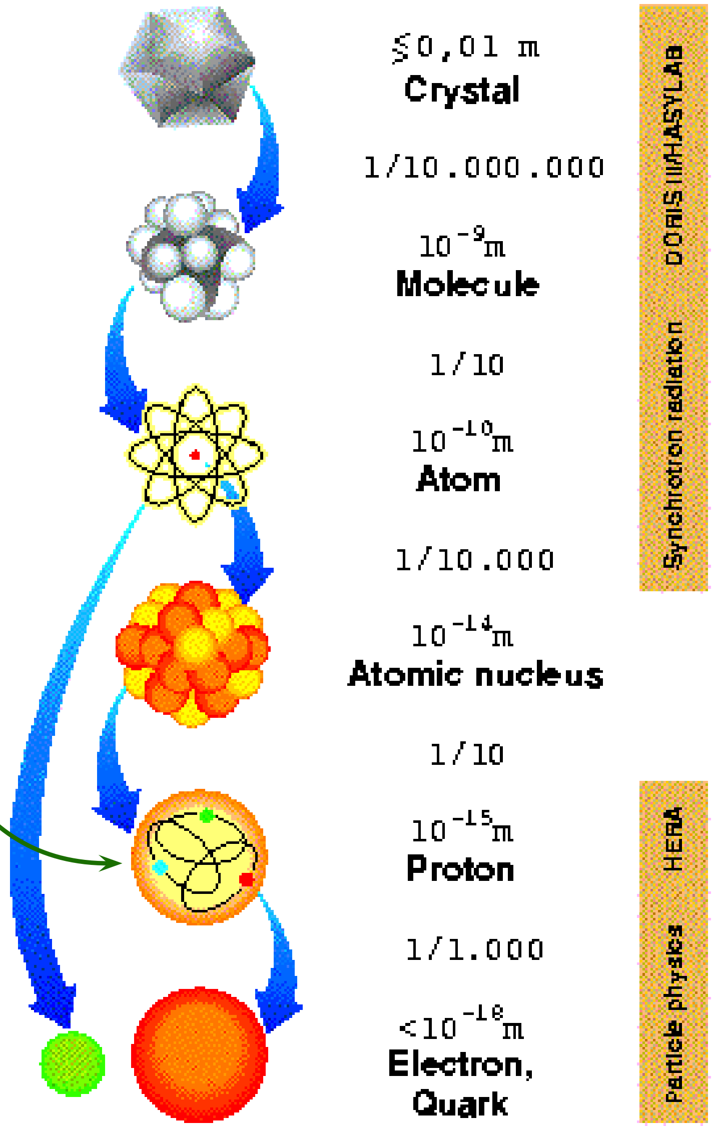
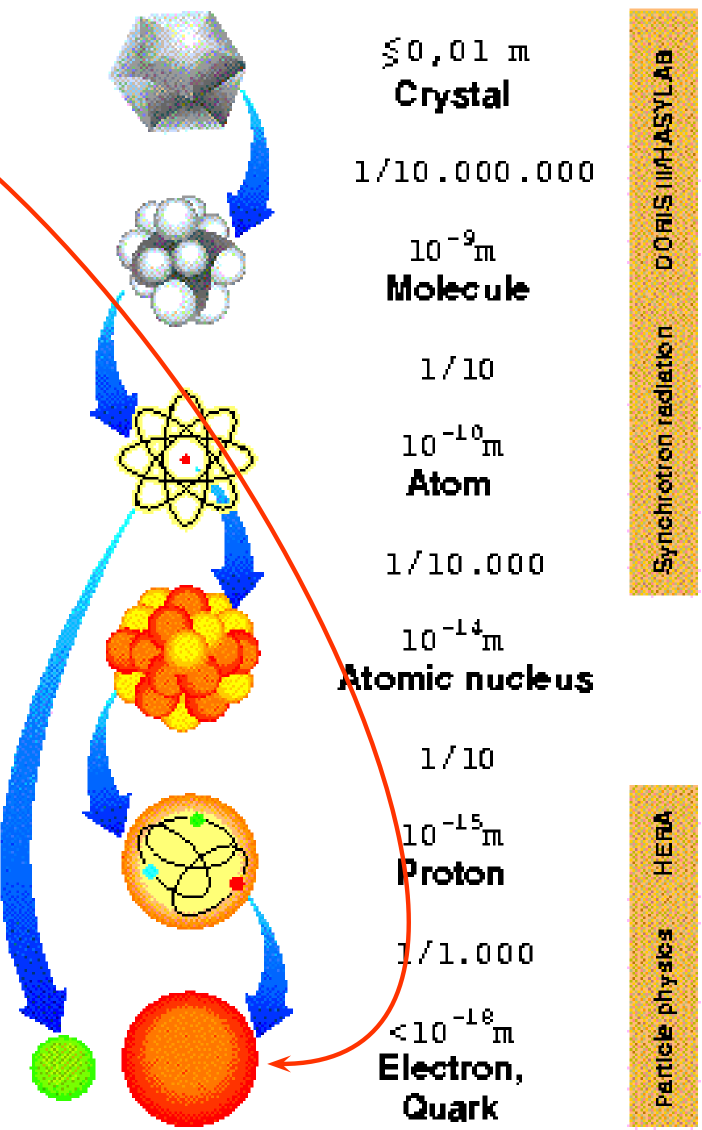




Hadron Physics

Meta-Physics
Scale = Limited only
by Theorists
Imagination



Nucleon . . . 2 Key Hadrons
= Proton and Neutron

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  proton electric charge $= +1$; and magnetic moment, μ_p

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- Stern (1933) – $\mu_p = (1 + 1.79)\dfrac{e\hbar}{2M}$
  - Big Hint that Proton is not a point particle
  - Proton has constituents
  - These are Quarks and Gluons
    Quark discovery via $e^- p$-scattering at SLAC in 1968
    – the elementary quanta of Quantum Chromo-dynamics

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What is QCD?

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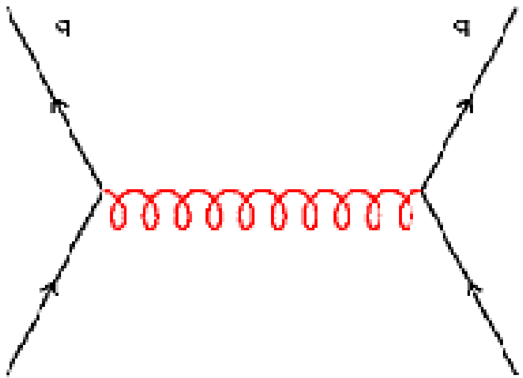
What is QCD?

- Gauge Theory:

Interactions Mediated by massless vector bosons

Feynman Diagram of Quark–Quark Scattering



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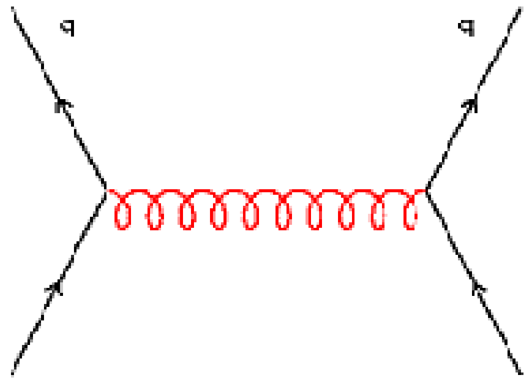

- Similar interaction in QED

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What is QCD?

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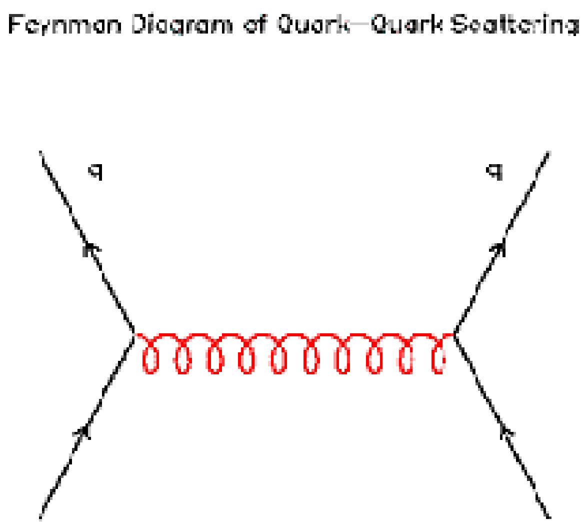

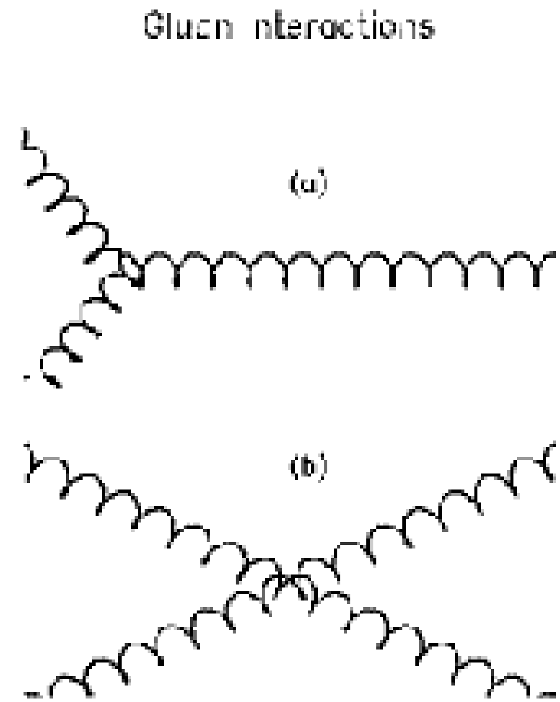

- Similar interaction in QED

- Special Feature of QCD – gluon self-interactions

  Completely Change the Character of the Theory

QED cf. QCD

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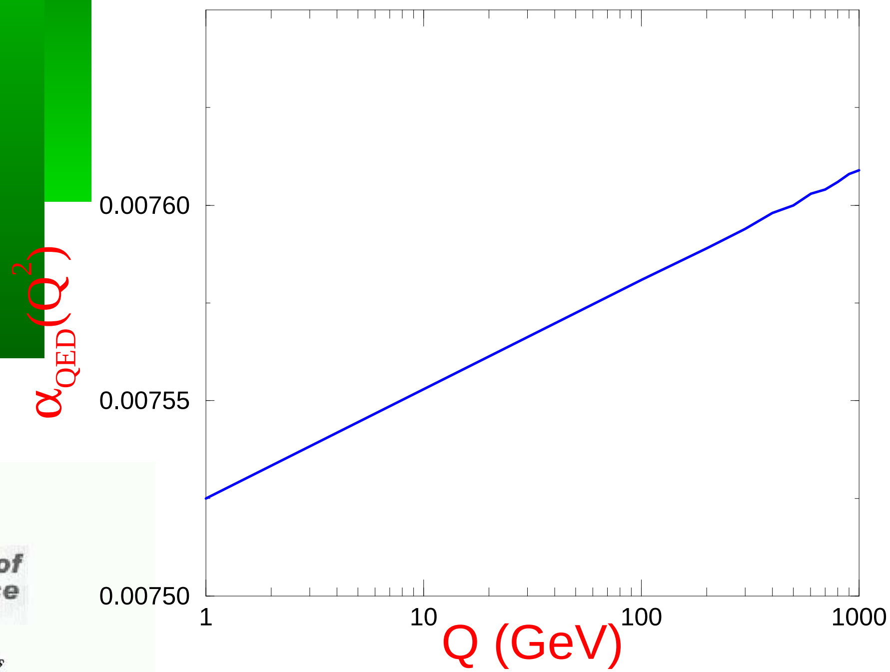
QED cf. QCD



$$\alpha_{\mathrm{QED}} = \frac{\alpha}{1 - \alpha/3\pi \ln\left(Q^2/m_e^2\right)}$$

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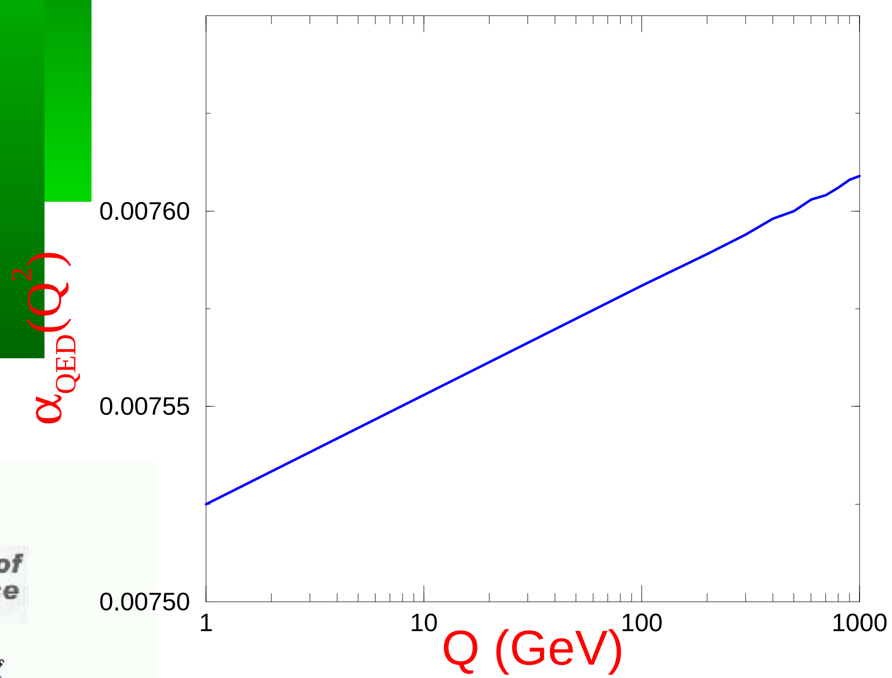
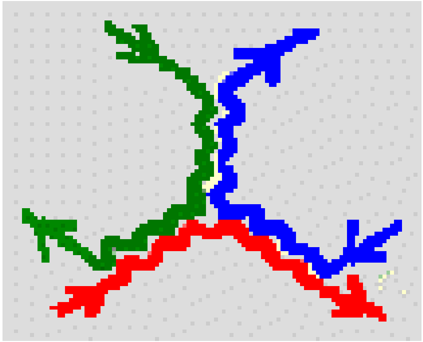
Add three-gluon interaction





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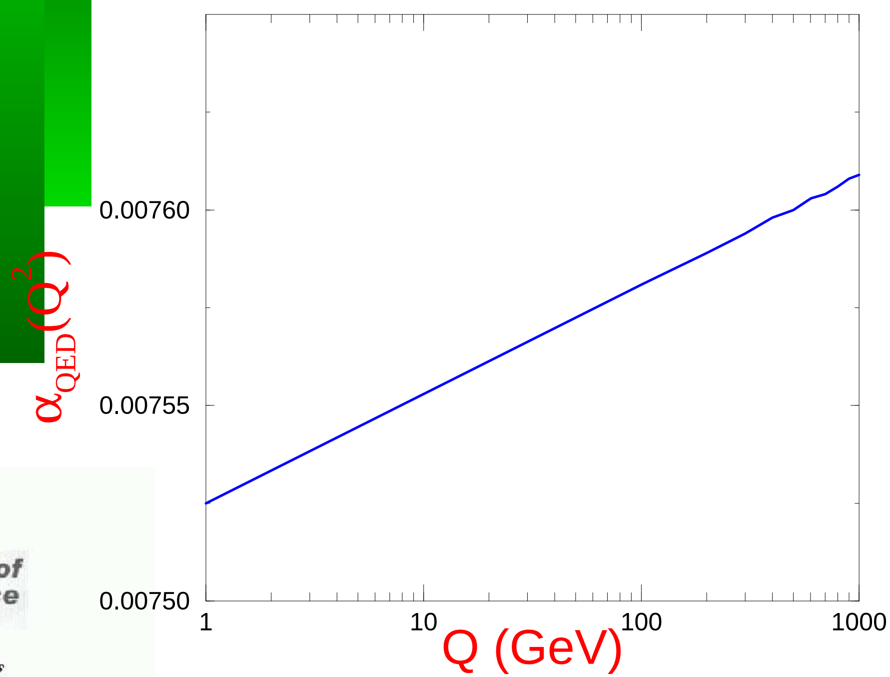

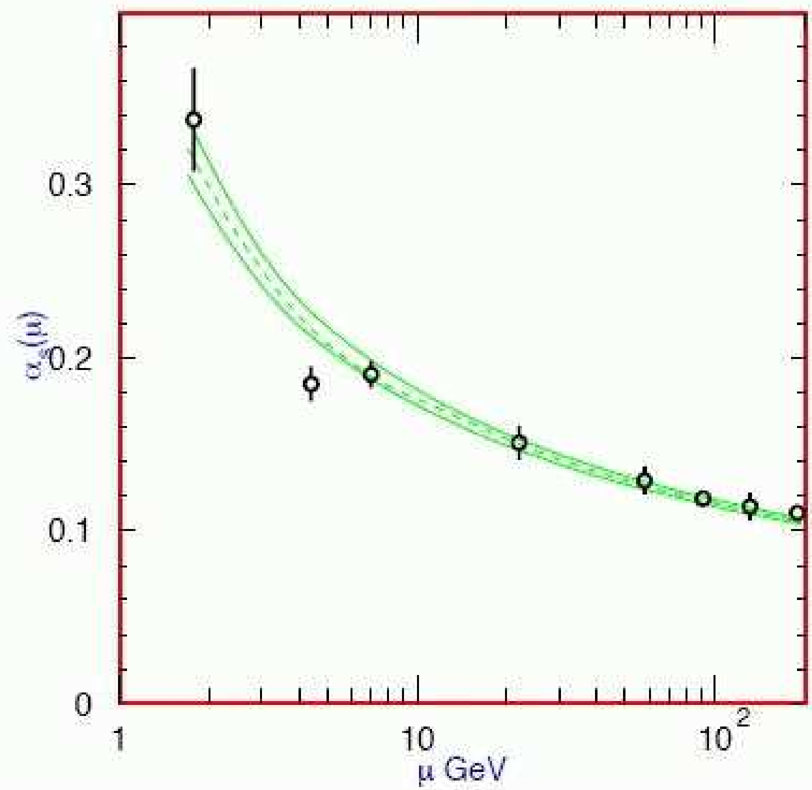

Figure 9.2: Summary of the values of $\alpha_s(\mu)$ at the values of μ where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average. The figure clearly shows the decrease in $\alpha_s(\mu)$ with increasing μ. The data are, in increasing order of μ, τ width, Υ decays, deep inelastic scattering, e^+e^- event shapes at 22 GeV from the JADE data, shapes at TRISTAN at 58 GeV, Z width, and e^+e^- event shapes at 135 and 189 GeV.

$$\alpha_{\rm QED} = \frac{\alpha}{1 - \alpha/3\pi \ln\left(Q^2/m_e^2\right)}$$

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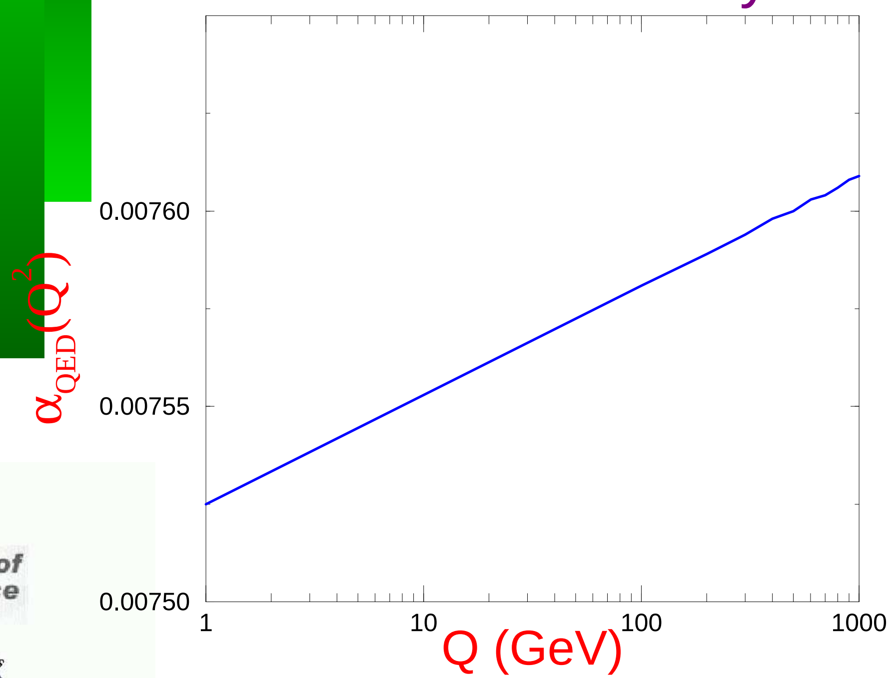
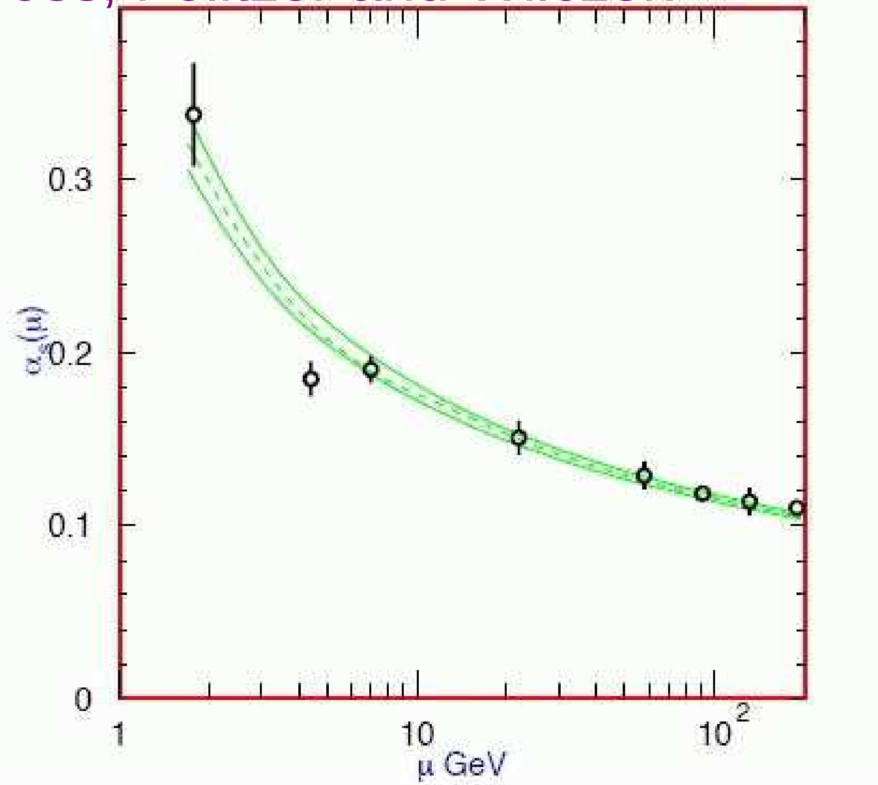
2004 Nobel Prize in Physics: Gross, Politzer and Wilczek





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  Nobody knows! A bit exaggerated. The mathematical origin is plain. The classical theory has no scale but in defining the quantum field theory associated with it, this mass scale arises through regularisation of ultraviolet divergences and subsequent renormalisation.

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  Such effects occur often in passing from classical to quantum theory. In this instance one says that QCD has a conformal anomaly: quantisation destroys a symmetry of the classical theory.

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Fundamental Parameter of QCD

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- In contemporary models for a Grand Unification of all forces, $\Lambda_{\rm QCD}$ can be time-dependent and even location dependent.

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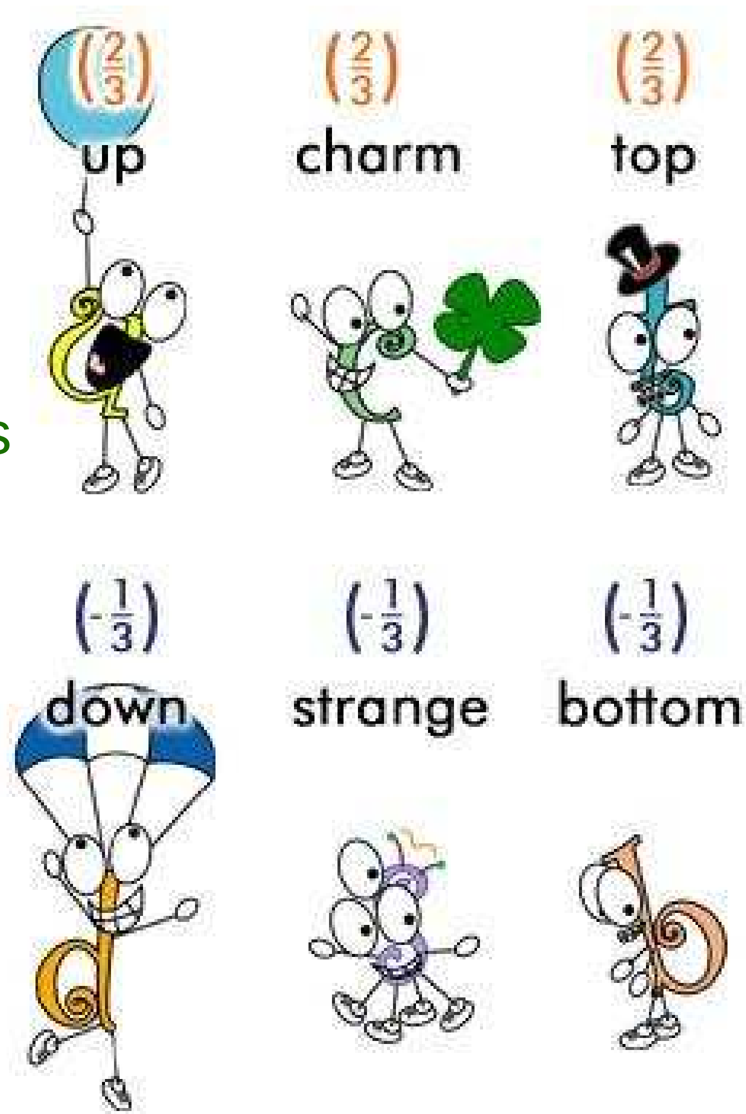
Quarks and Nuclear Physics

Quarks and Nuclear Physics

Standard Model

of Particle Physics

Six Flavours

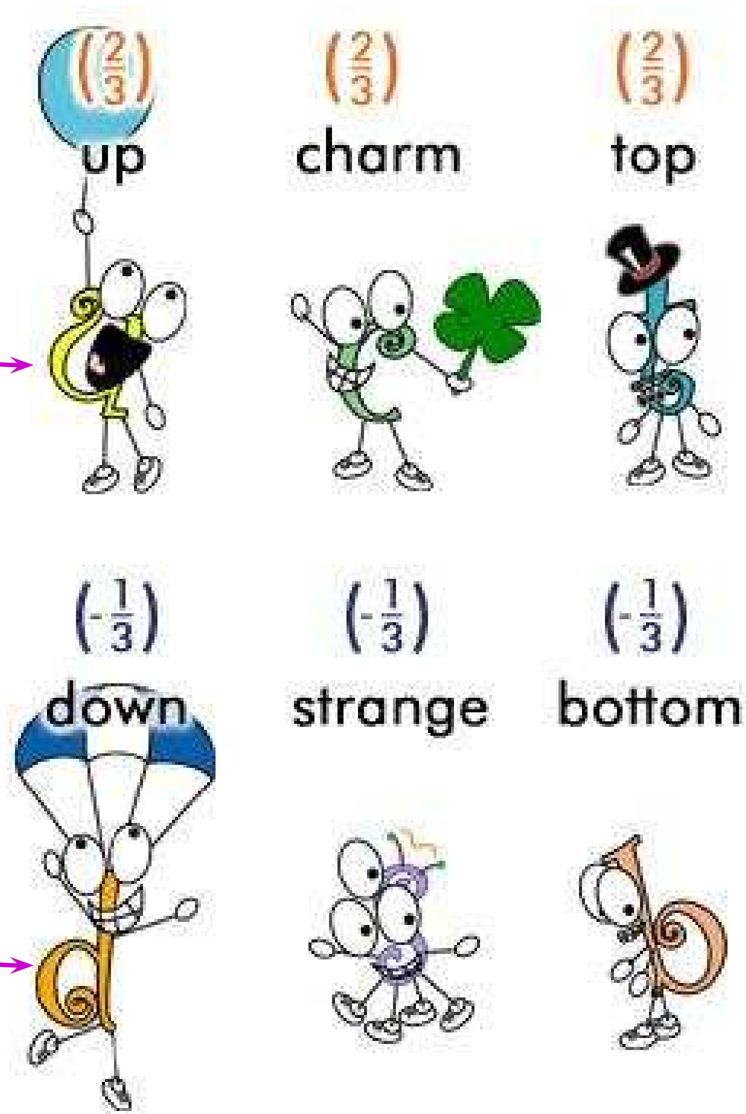


Real World

Normal Matter . . .

Only Two Light

Flavours Active



Quarks and Nuclear Physics

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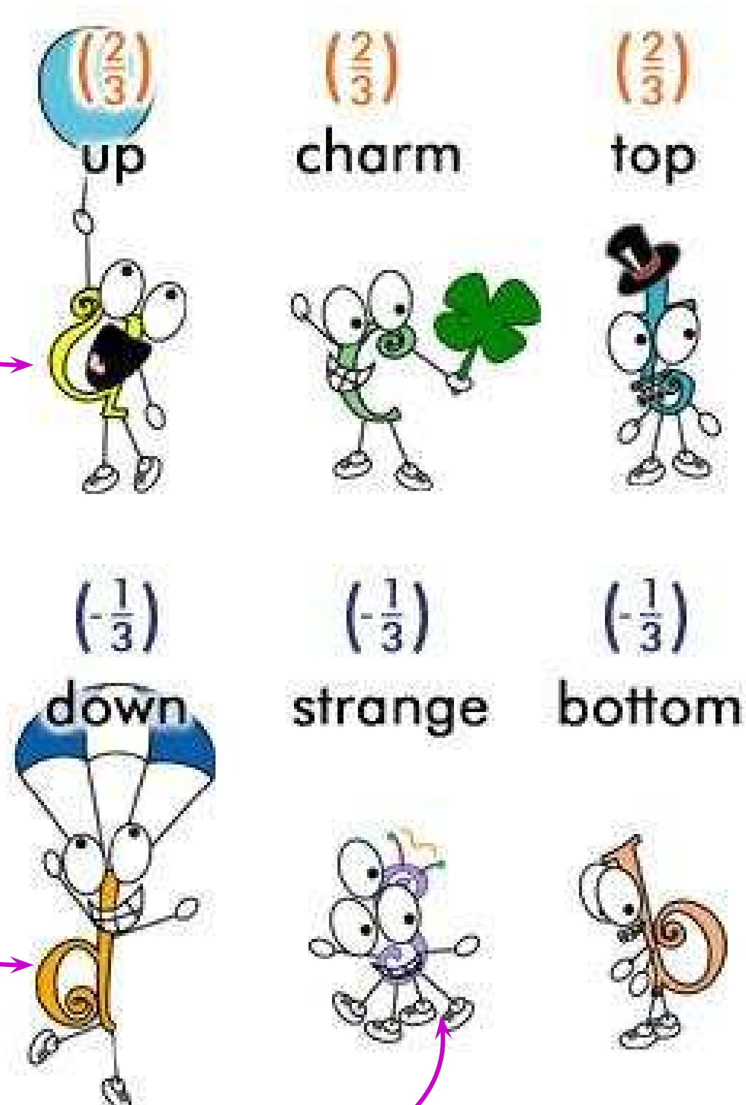
or, perhaps, three



Quarks and Nuclear Physics

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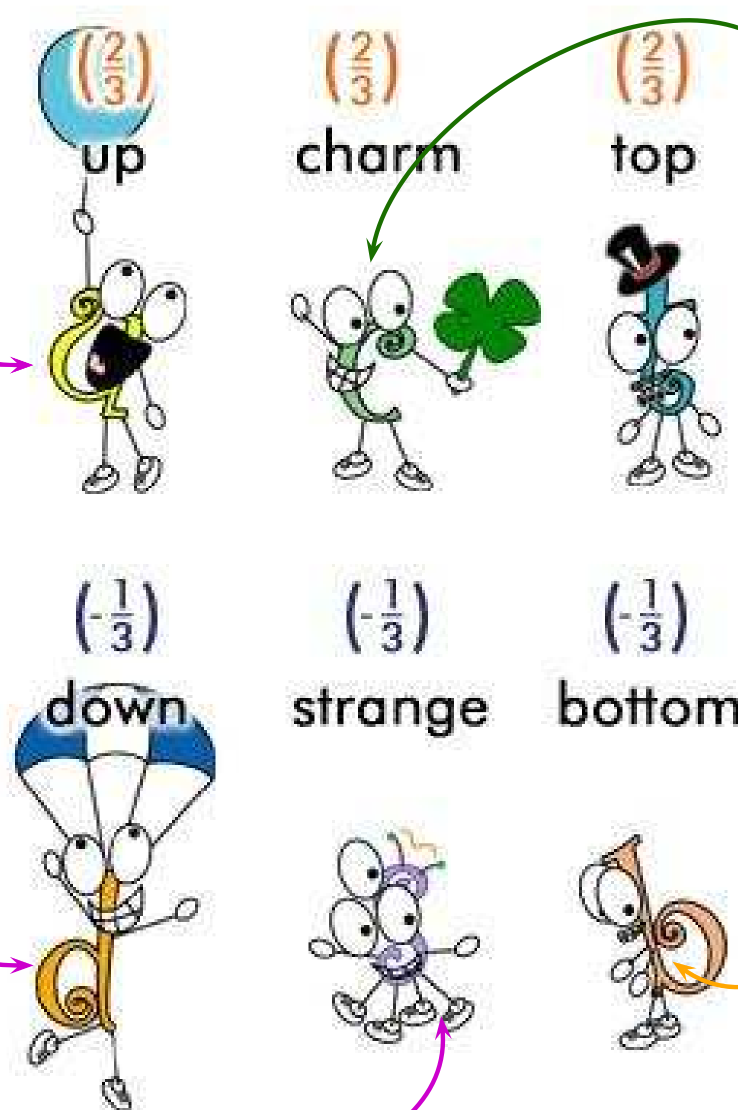


For numerous good reasons, much research also focuses on accessible heavy-quarks

or, perhaps, three

Quarks and Nuclear Physics

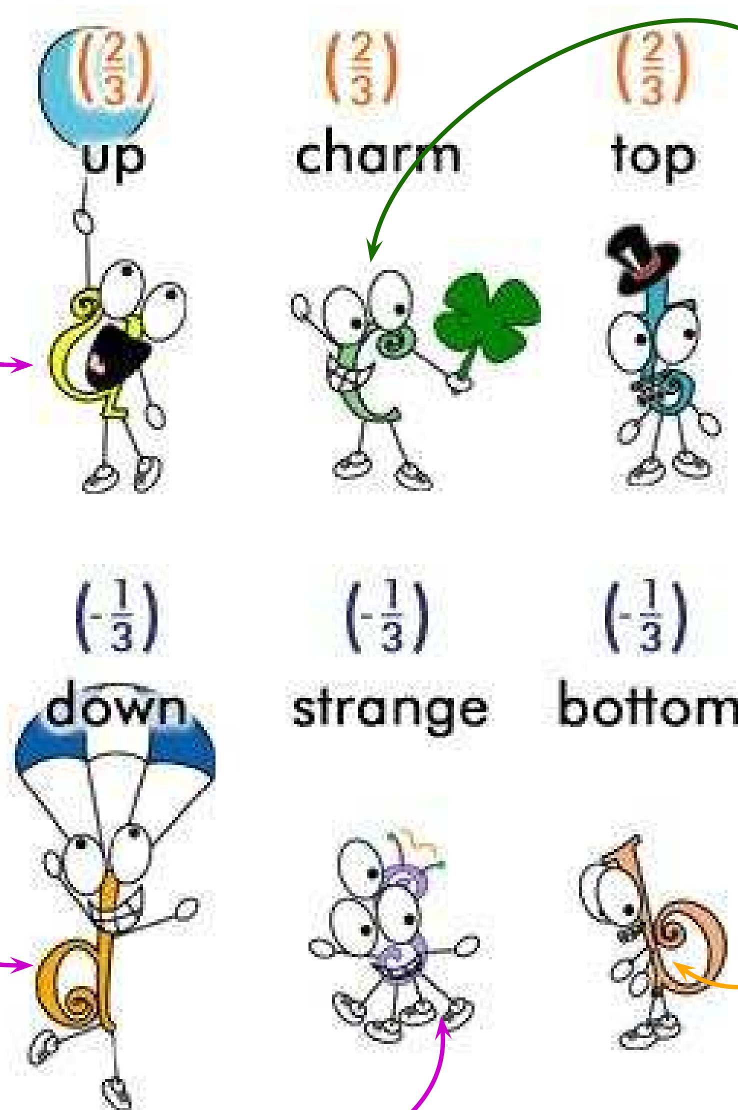
Nevertheless, I will focus primarily on the light-quarks.

Real World

Normal Matter . . .

Only Two Light Flavours Active

or, perhaps, three



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Simple Picture

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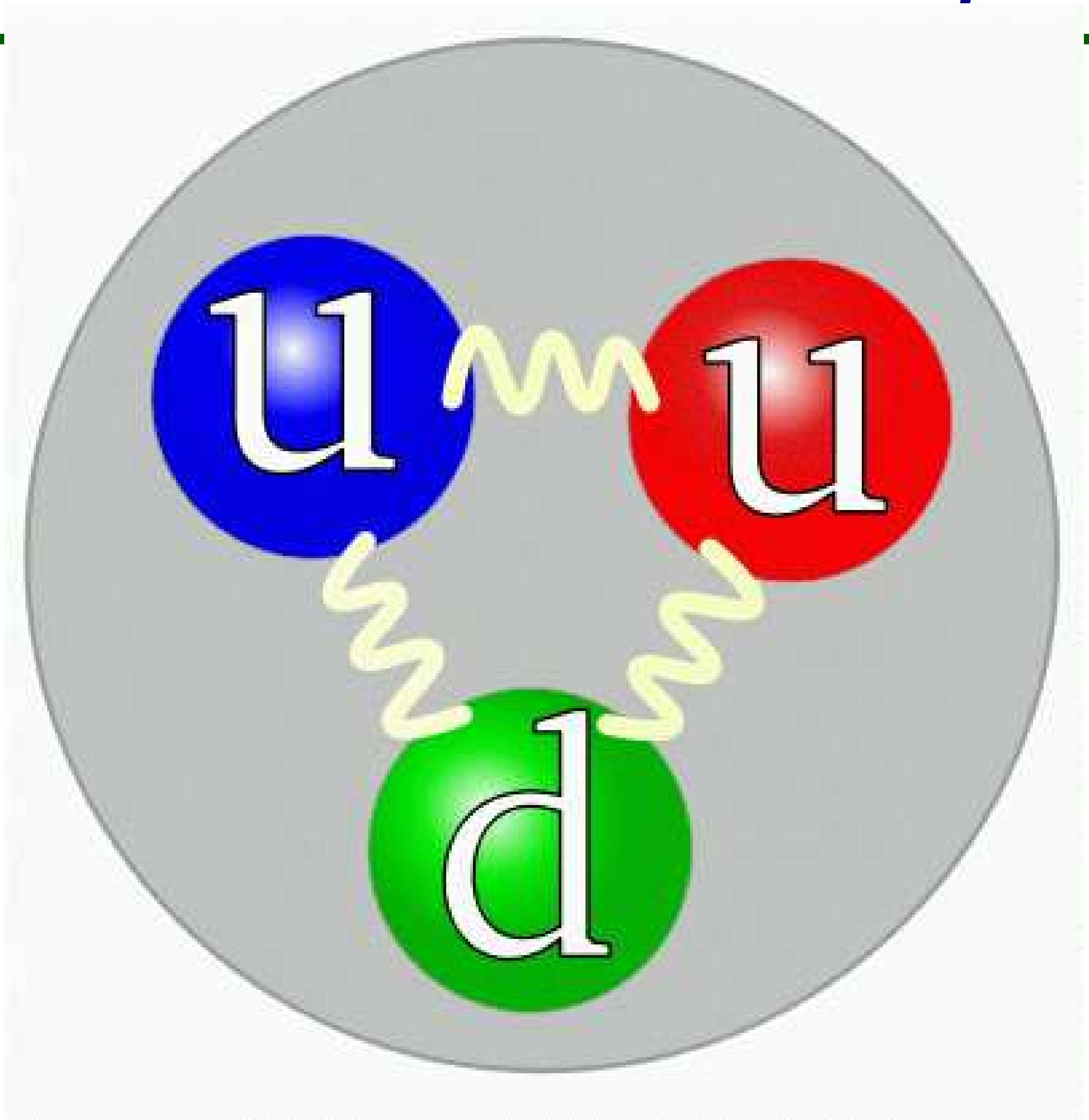

PROTON

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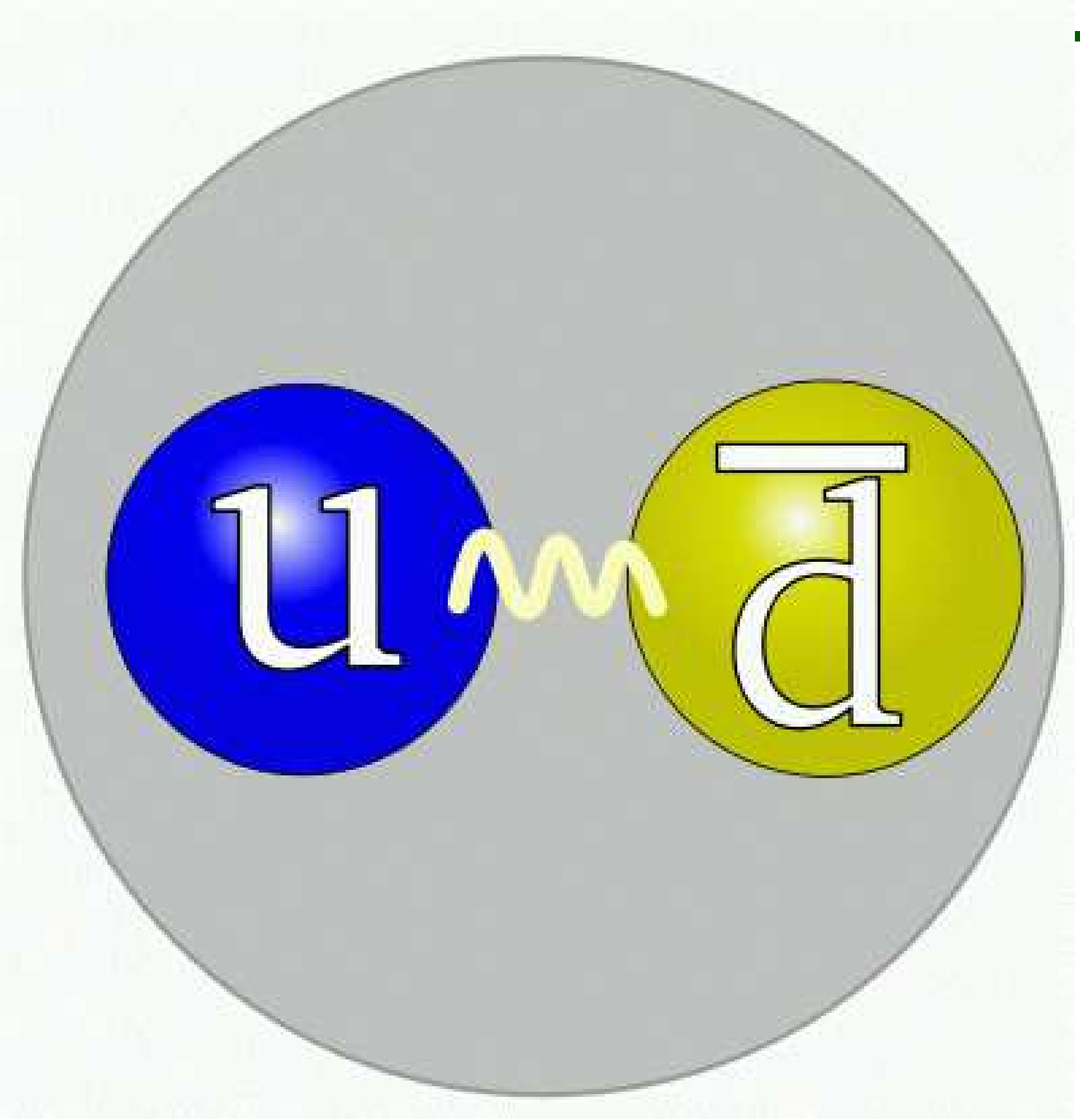
Simple Picture



PION

Meson Spectrum

LIGHT UNFLAVORED ($S = C = B = 0$)		$I^G(J^{PC})$		$I^G(J^{PC})$	STRANGE ($S = \pm 1$, $C = B = 0$)	$I(J^P)$
• $\pi^\pm$		$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$	• $K^\pm$	$1/2(0^-)$
• π^0	140 MeV	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$	• K^0	$1/2(0^-)$
• η		$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$	• K^0_S	$1/2(0^-)$
• $f_0(600)$		$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$	• K^0_L	$1/2(0^-)$
• $\rho(770)$	770	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$	$K^*_0(800)$	$1/2(0^+)$
• $\omega(782)$		$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$	• $K^*(892)$	$1/2(1^-)$
• $\eta'(958)$		$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$	• $K_1(1270)$	$1/2(1^+)$
• $f_0(980)$		$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$	• $K_1(1400)$	$1/2(1^+)$
• $a_0(980)$		$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$	• $K^*(1410)$	$1/2(1^-)$
• $\phi(1020)$		$0^-(1^{--})$	$X(1835)$	$?^?(?^{-+})$	• $K^*_0(1430)$	$1/2(0^+)$
• $h_1(1170)$		$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$	• $K^*_2(1430)$	$1/2(2^+)$
• $b_1(1235)$		$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$	$K(1460)$	$1/2(0^-)$
• $a_1(1260)$		$1^-(1^{++})$	$\rho(1900)$	$1^+(1^{--})$	$K_2(1580)$	$1/2(2^-)$
• $f_2(1270)$		$0^+(2^{++})$	$f_2(1910)$	$0^+(2^{++})$	$K(1630)$	$1/2(?^?)$
• $f_1(1285)$		$0^+(1^{++})$	• $f_2(1950)$	$0^+(2^{++})$	$K_1(1650)$	$1/2(1^+)$
• $\eta(1295)$		$0^+(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$	• $K^*(1680)$	$1/2(1^-)$
• $\pi(1300)$		$1^-(0^{-+})$	• $f_2(2010)$	$0^+(2^{++})$	• $K_2(1770)$	$1/2(2^-)$

Modern Miracles in Hadron Physics

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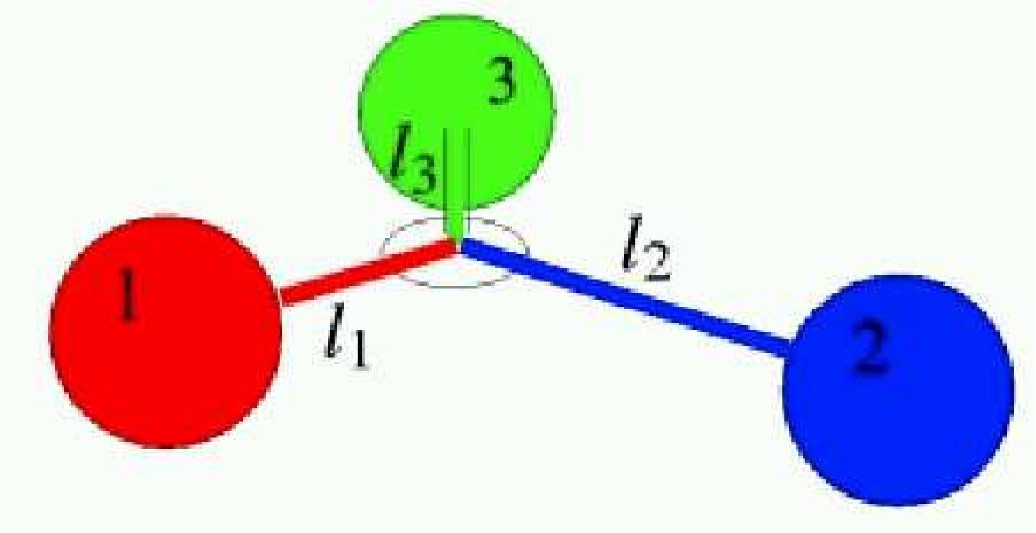
- proton = three constituent quarks



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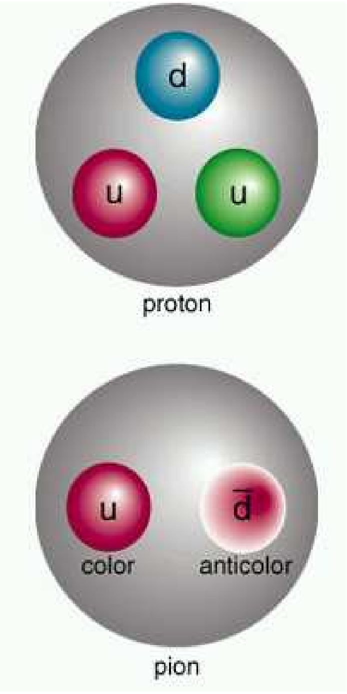

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- Another meson:
  .... $M_\rho = 770\,$MeV .... No Surprises Here

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- pion $=$
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- guess $M_{\text{pion}} \approx 2 \times \dfrac{M_{\text{proton}}}{3} \approx 700\,\mathbf{MeV}$
- WRONG . . . . $\boxed{M_{\text{pion}} = 140\,\mathbf{MeV}}$
- What is “wrong” with the pion?

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Dichotomy of Pion
– *Goldstone Mode* and *Bound state*

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- Not Allowed to do it by fine-tuning a potential

  Must exhibit $\boxed{m_\pi^2 \propto m_q}$

Current Algebra . . . 1968

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Current Algebra . . . 1968

The correct understanding of pion observables;
e.g. mass, decay constant and form factors,
requires an approach to contain a

- well-defined and valid chiral limit;
- and an accurate realisation of
  dynamical chiral symmetry breaking.

Dichotomy of Pion
– Goldstone Mode and *Bound state*

- How does one make an almost massless particle
 . . . . from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

Must exhibit $\boxed{m_\pi^2 \propto m_q}$

Current Algebra . . . 1968

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Highly Nontrivial

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  - Interaction between quarks – the Interquark "Potential" – unknown throughout $>$ 98% of a hadron's volume

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Intranucleon Interaction

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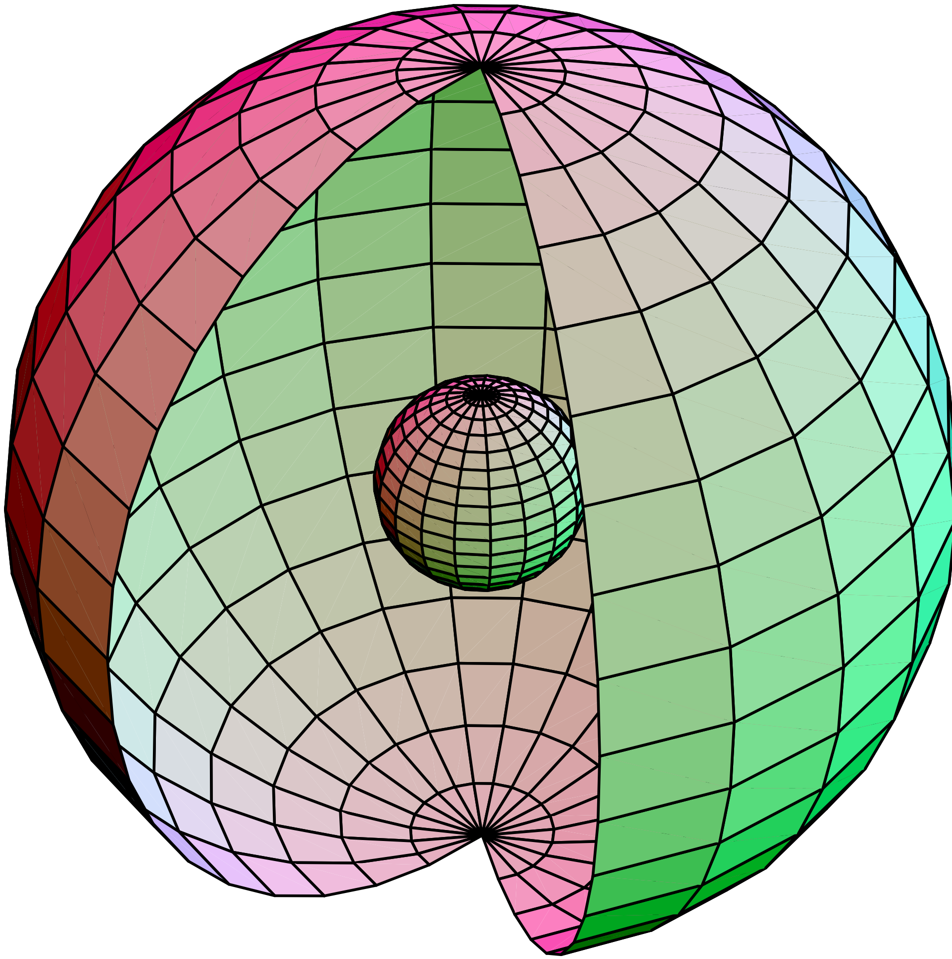
Intranucleon Interaction



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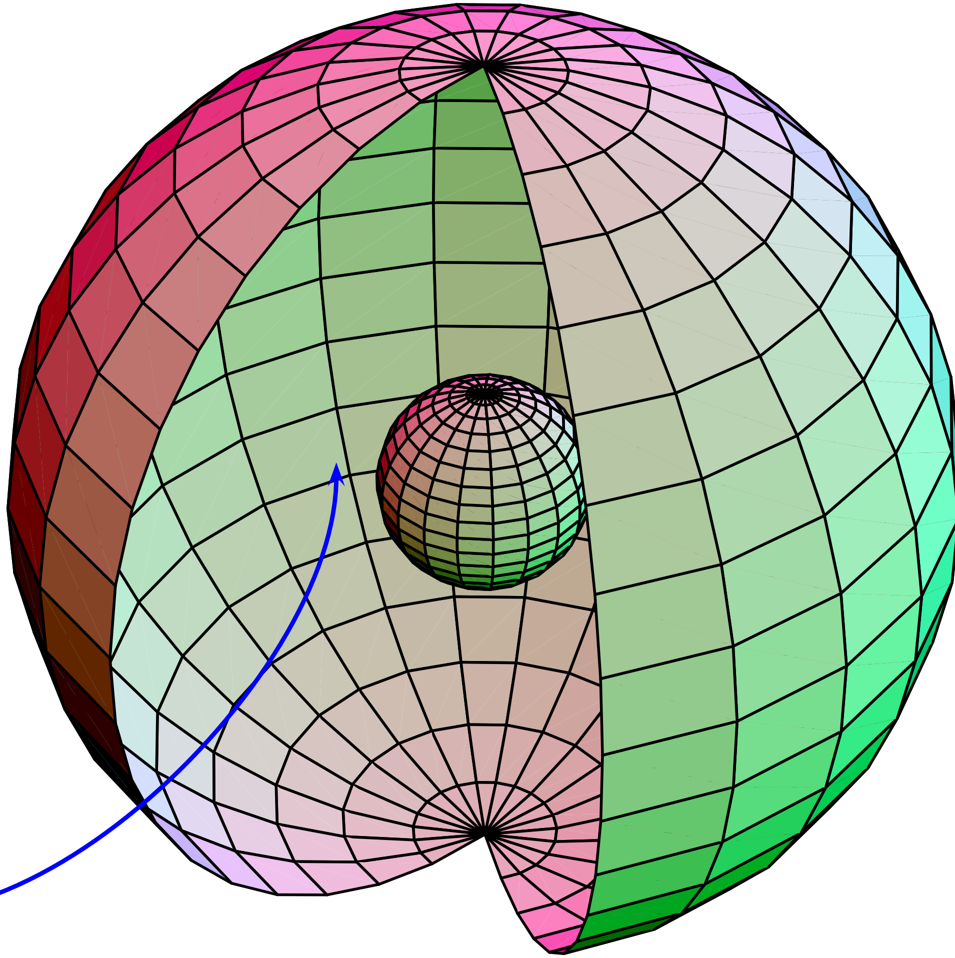

98% of the volume

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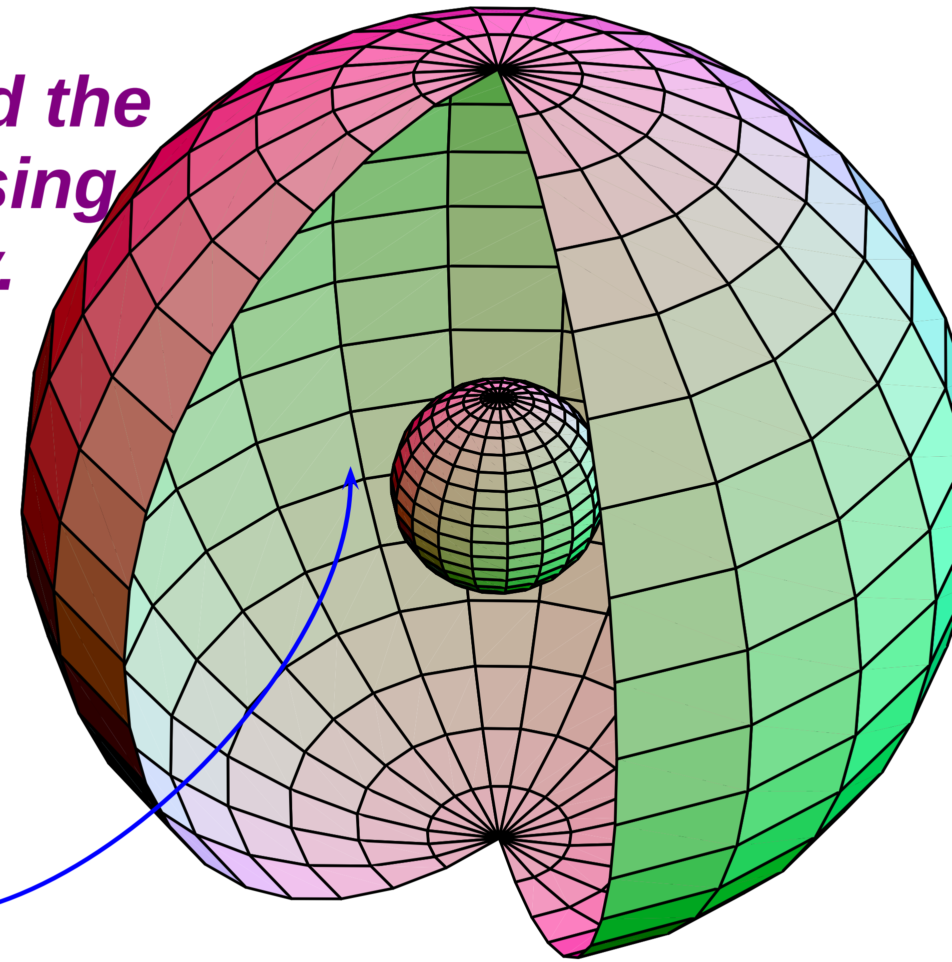
What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



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QCD's Challenges

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QCD's Challenges

Understand Emergent Phenomena

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- QCD – Complex behaviour
  arises from apparently simple rules

Why should You care?

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Absent DCSB: $m_\pi = m_\rho \;\Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!

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  - Probably not, if range range $\sim \dfrac{1}{2\,M_Q}$

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- How do such changes affect Big Bang Nucleosynthesis?

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Is a unique long-range interaction between light-quarks responsible for all this or are there an uncountable infinity of qualitatively equivalent interactions?

Model QCD

Traditional approach to strong force problem

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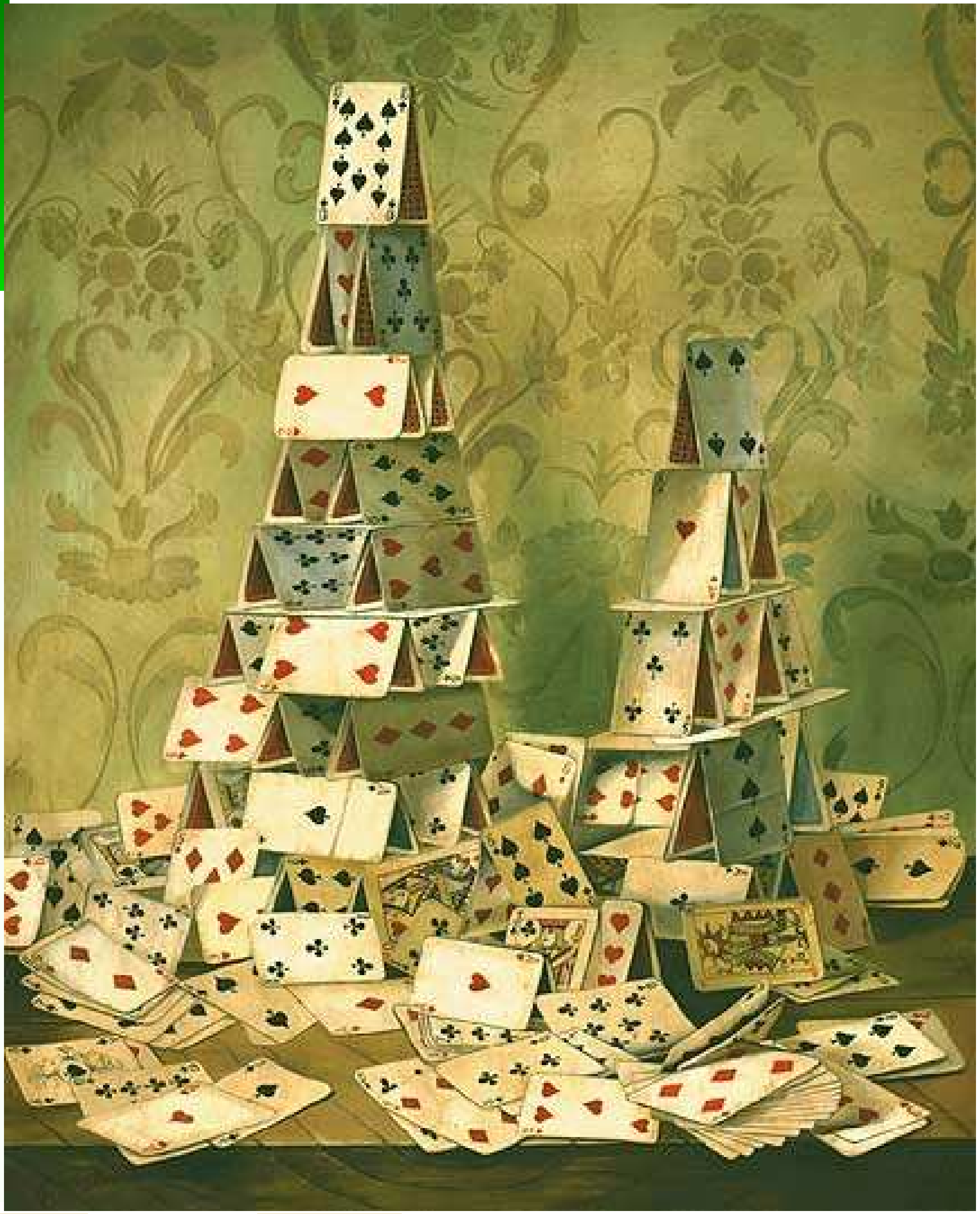
Model QCD



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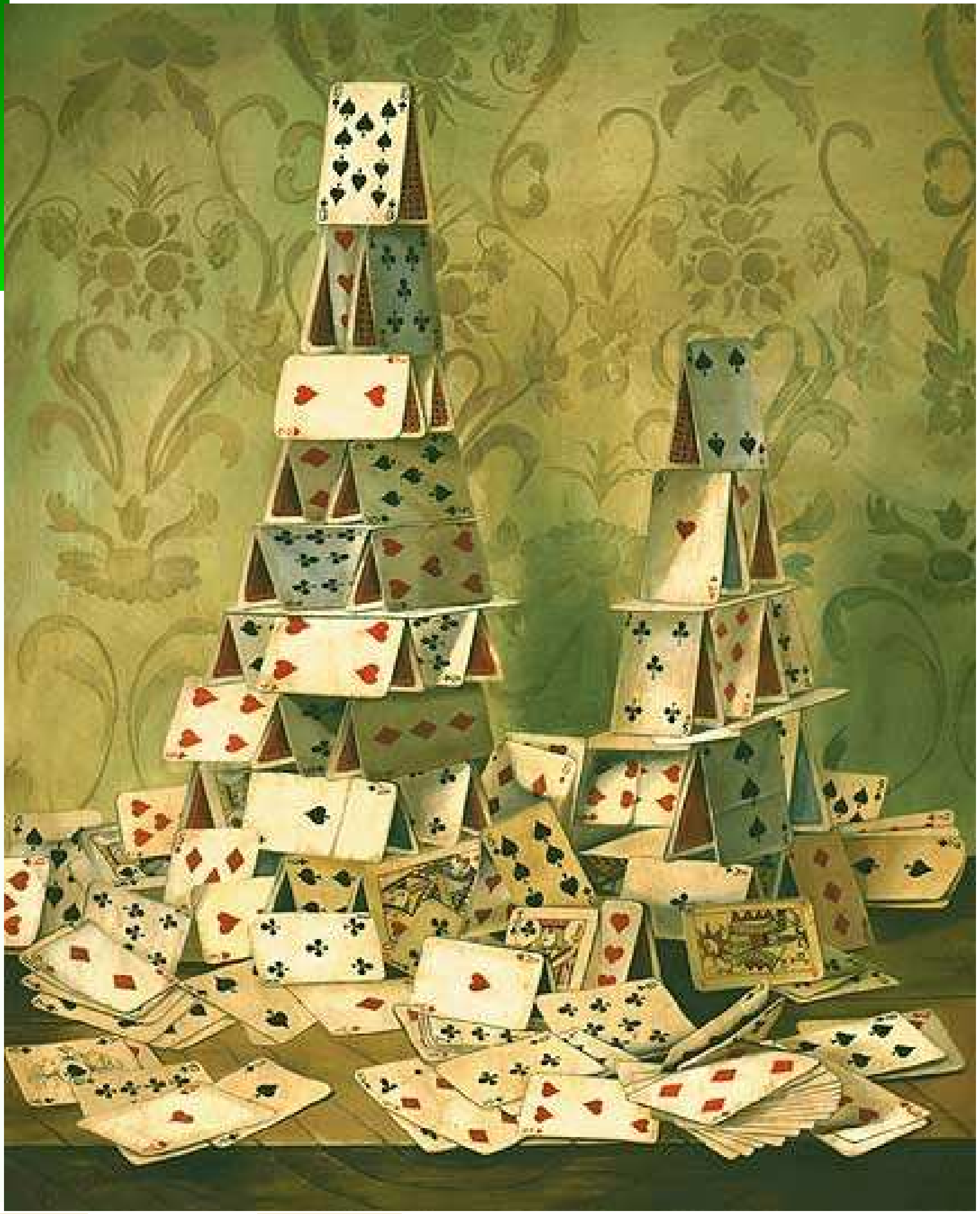
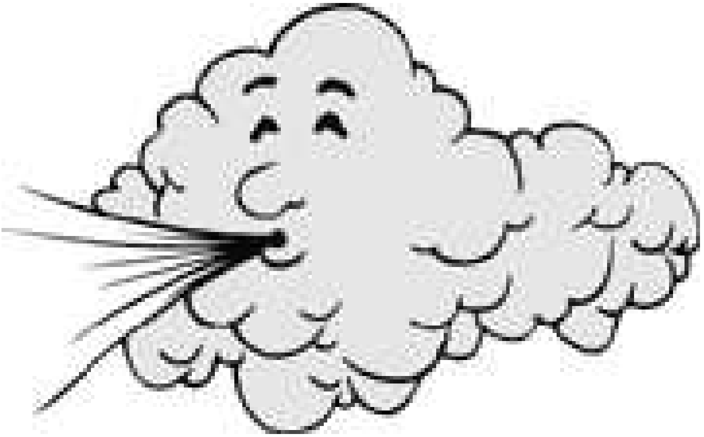
Model QCD





Lattice QCD

One modern nonperturbative approach *Lattice QCD*

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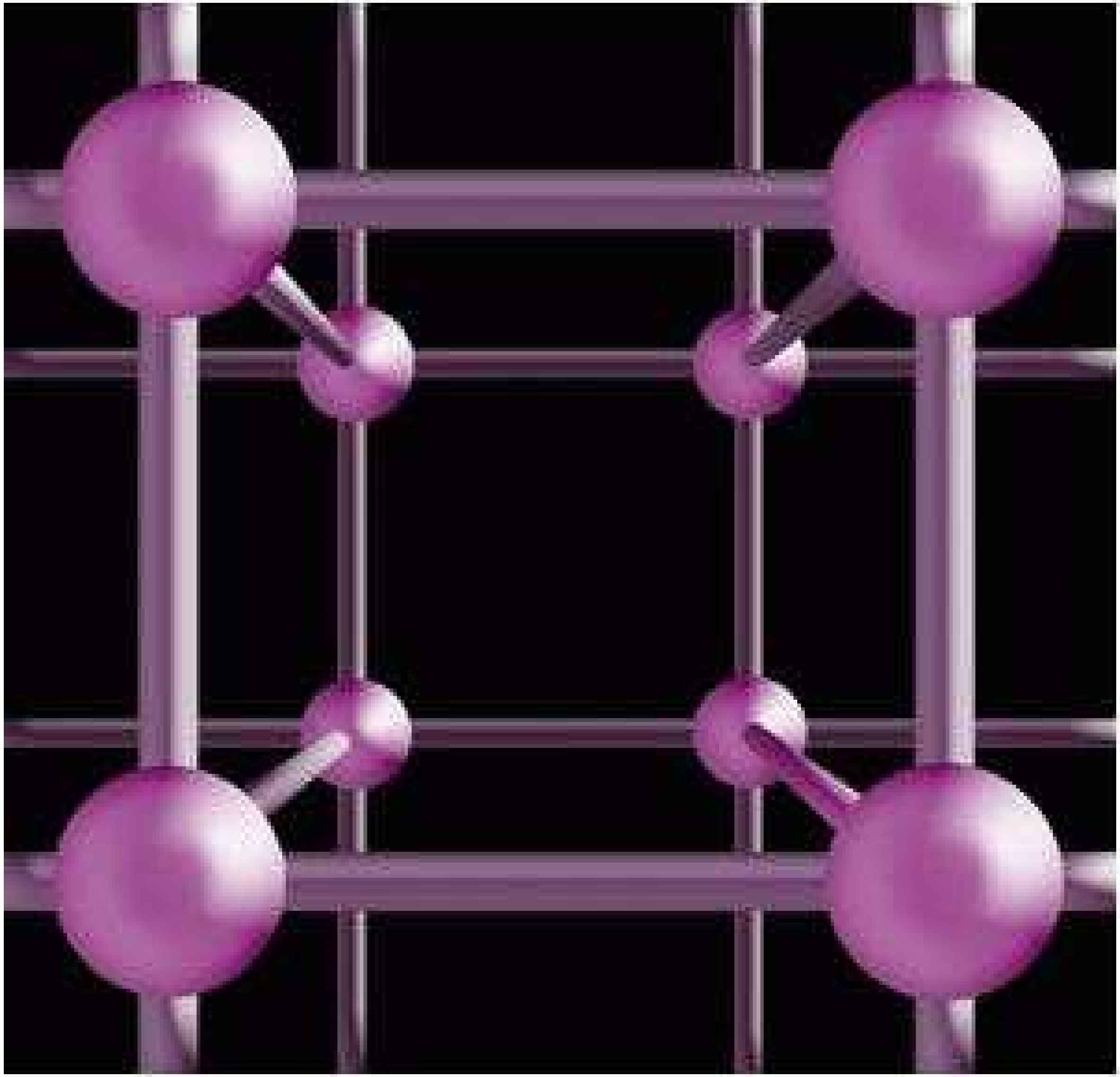

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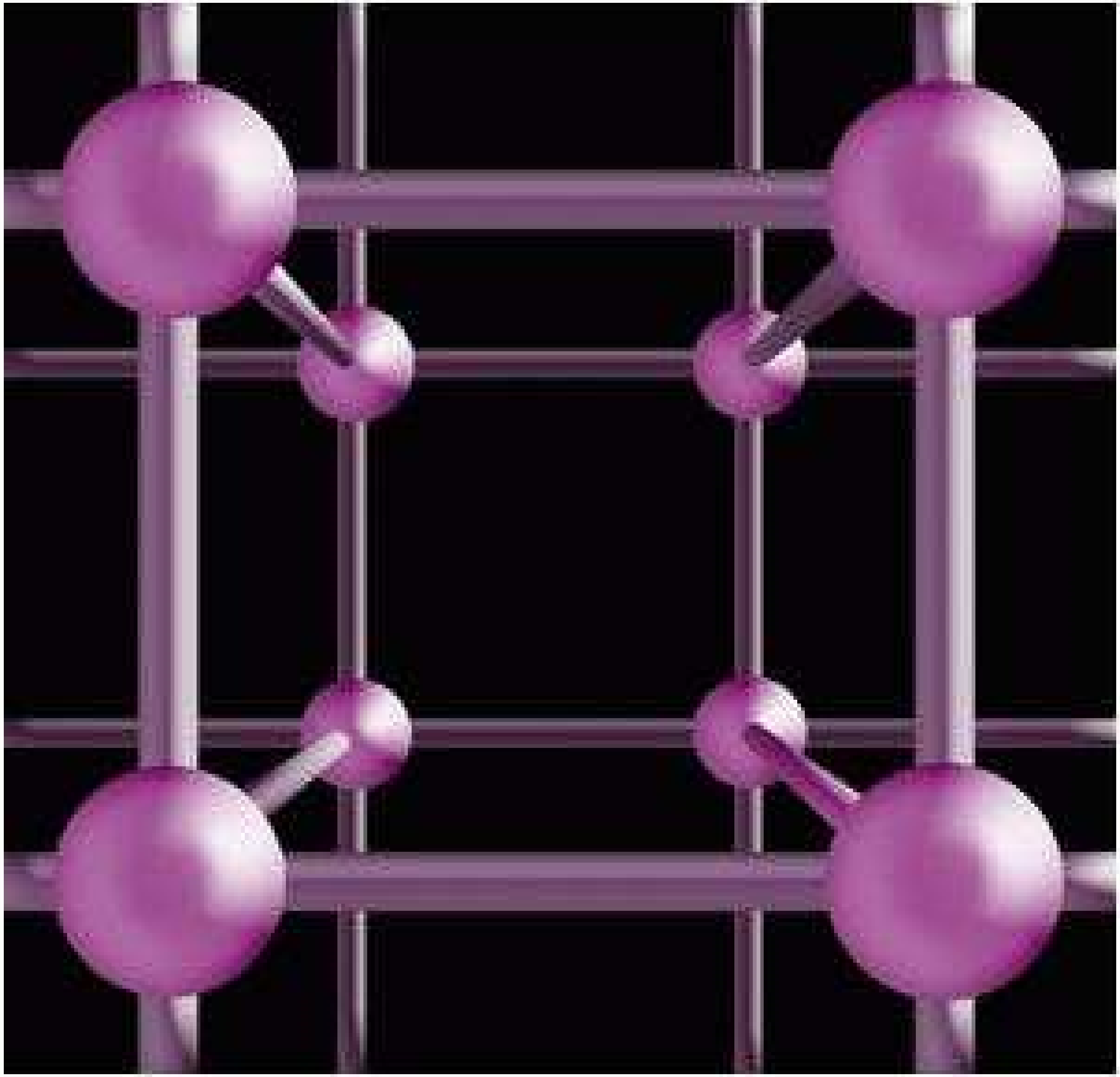
One modern nonperturbative approach *Lattice QCD*



$\sim$ 500 people worldwide.

Collaborations $\sim$ 20 people

One modern nonperturbative approach *Lattice QCD*

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A Compromise?

Dyson-Schwinger Equations

A Compromise?

Dyson-Schwinger Equations

- 1994 . . . “*As computer technology continues to improve, lattice gauge theory [LGT] will become an increasingly useful means of studying hadronic physics through investigations of discretised quantum chromodynamics [QCD]. . . .*”

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Dyson-Schwinger Equations

- 1994 . . . "*However, it is equally important to develop other complementary nonperturbative methods based on continuum descriptions. In particular, with the advent of new accelerators such as CEBAF and RHIC, there is a need for the development of approximation techniques and models which bridge the gap between short-distance, perturbative QCD and the extensive amount of low- and intermediate-energy phenomenology in a single covariant framework. . . .*"

A Compromise?
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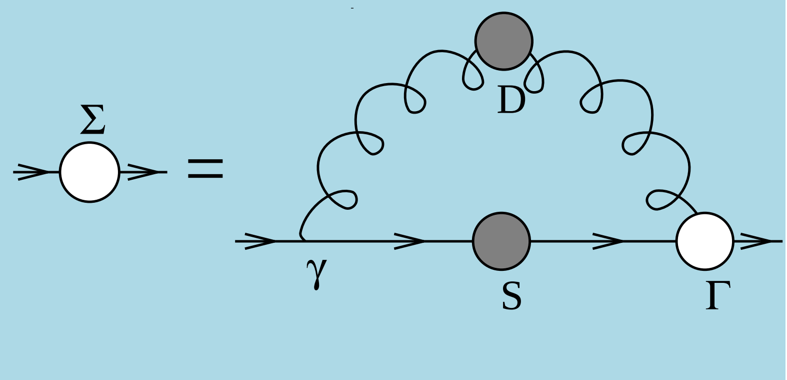

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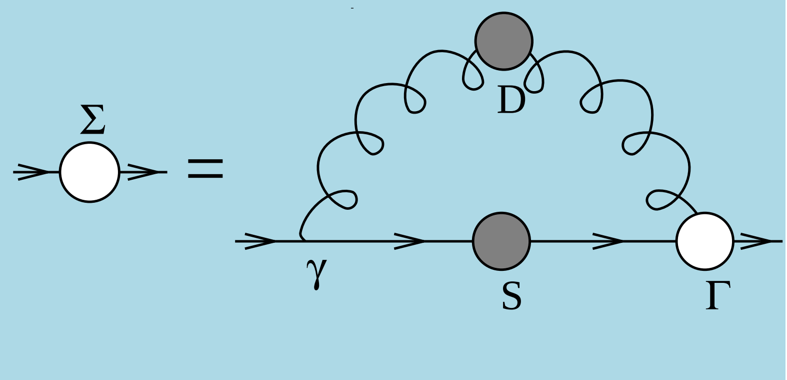

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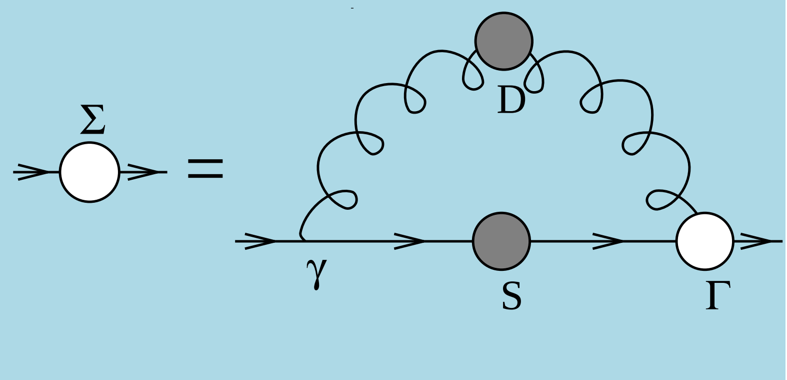

- These are nonperturbative equivalents in quantum field theory to the Lagrange equations of motion.
- Essential in simplifying the general proof of renormalisability of gauge field theories.

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Dyson-Schwinger Equations

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- Well suited to Relativistic Quantum Field Theory

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Cross-Sections built from Schwinger Functions

Perturbative Dressed-quark Propagator

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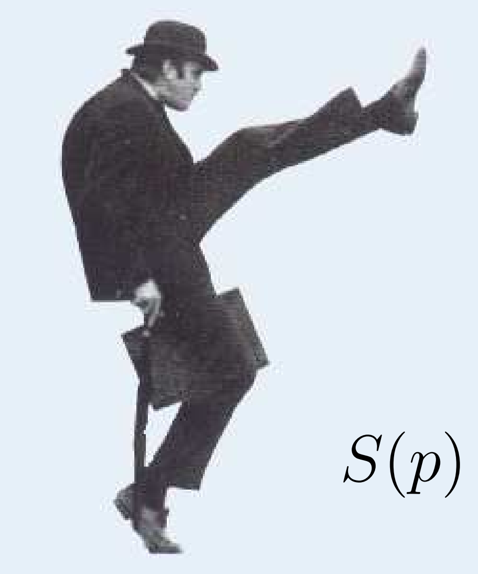
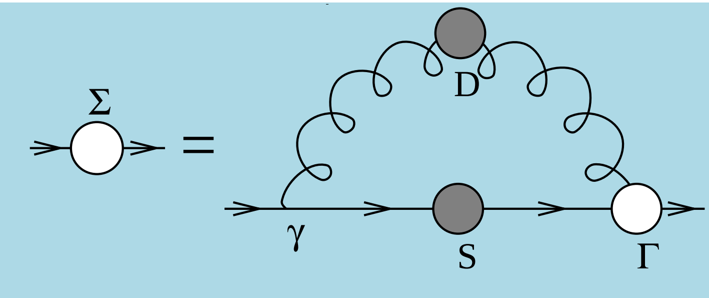
Perturbative Dressed-quark Propagator



$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$



Gap Equation

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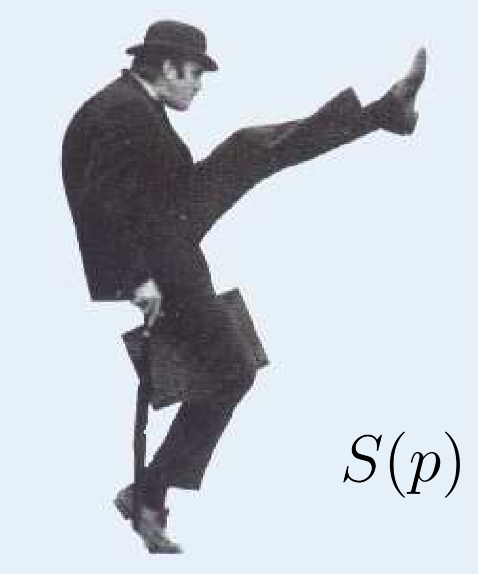

Perturbative Dressed-quark Propagator

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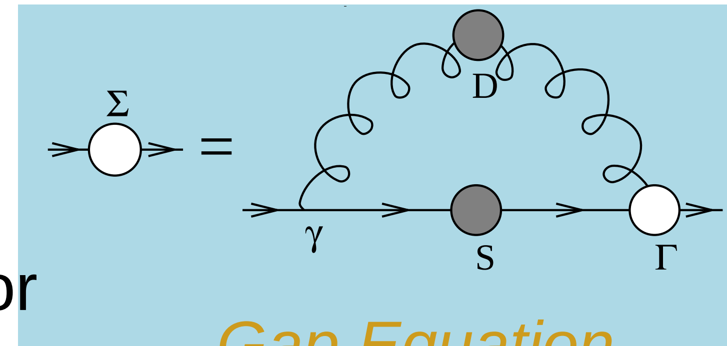

Gap Equation

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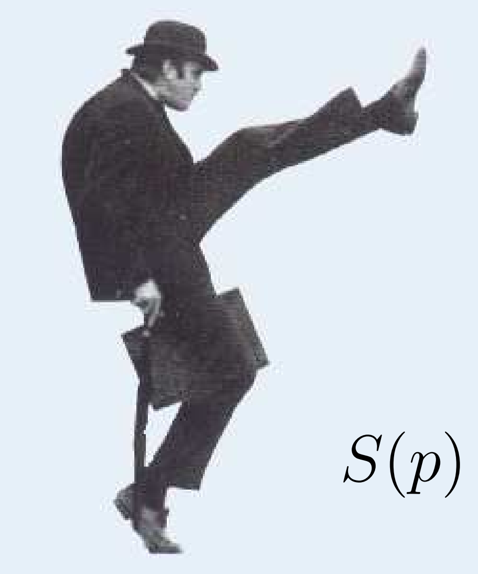

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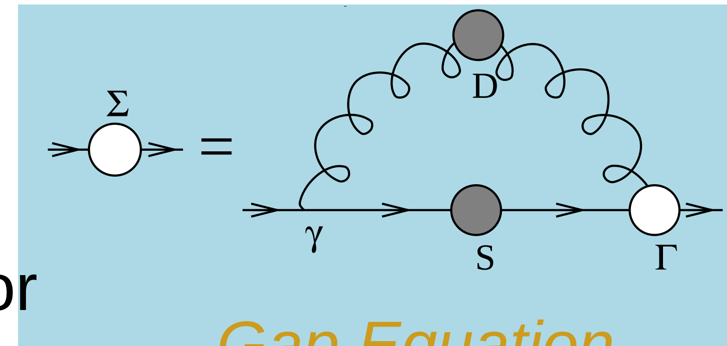

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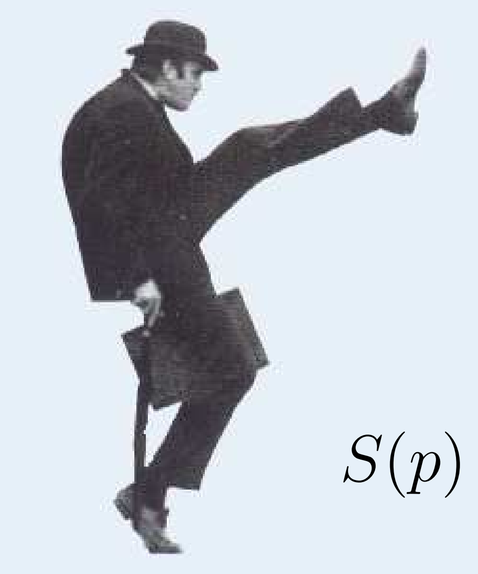
Reproduces Every Diagram in Perturbation Theory



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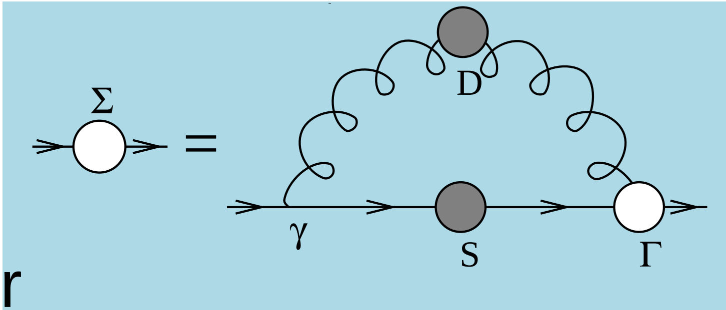

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$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\longrightarrow} 0$$

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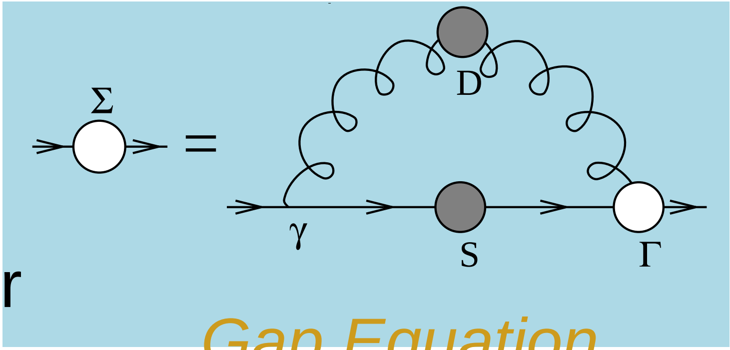

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QCD & Interaction Between Light-Quarks

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- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
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- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

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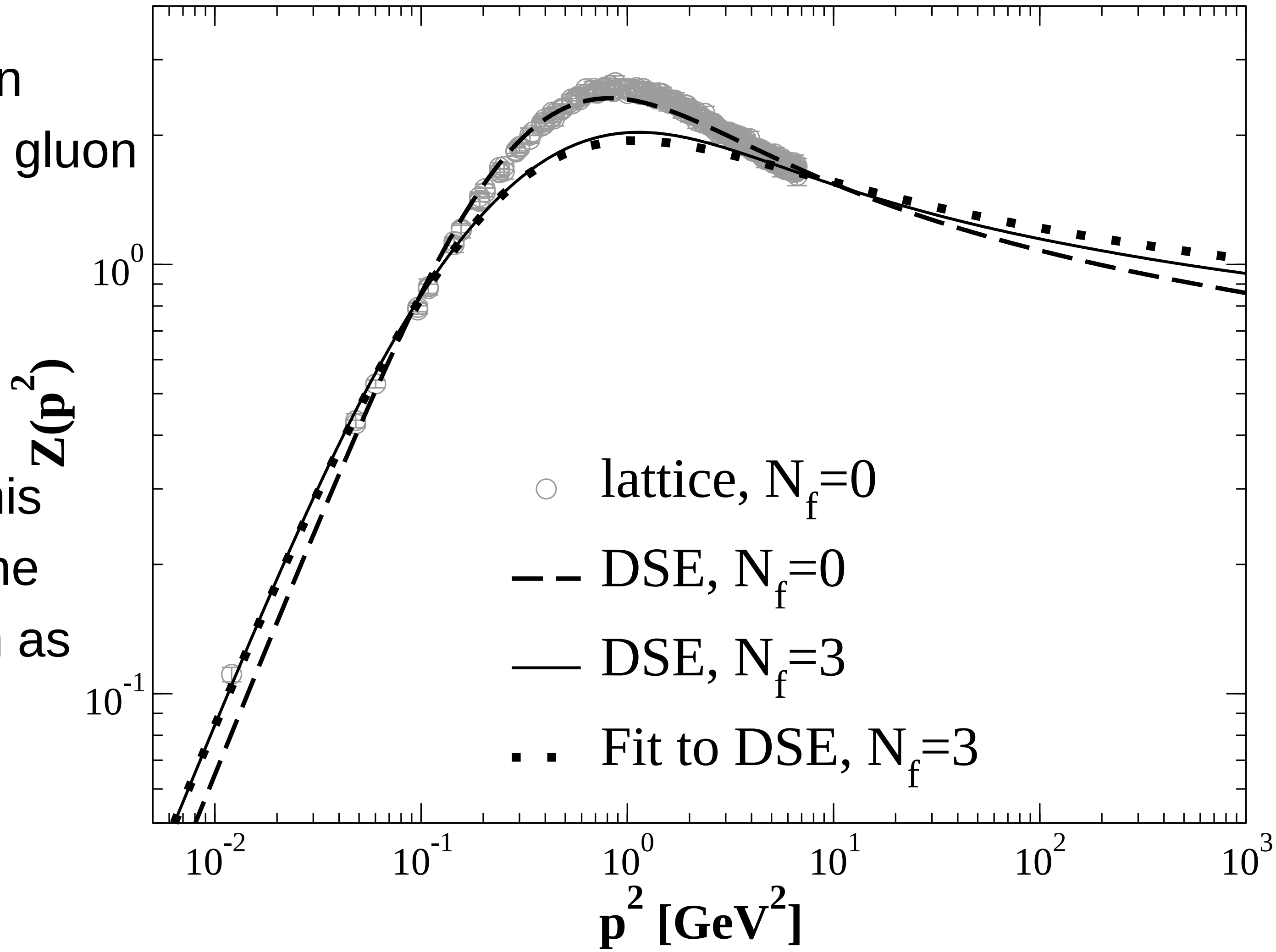
- Naturally, this scale has the same origin as $\Lambda_{\rm QCD}$



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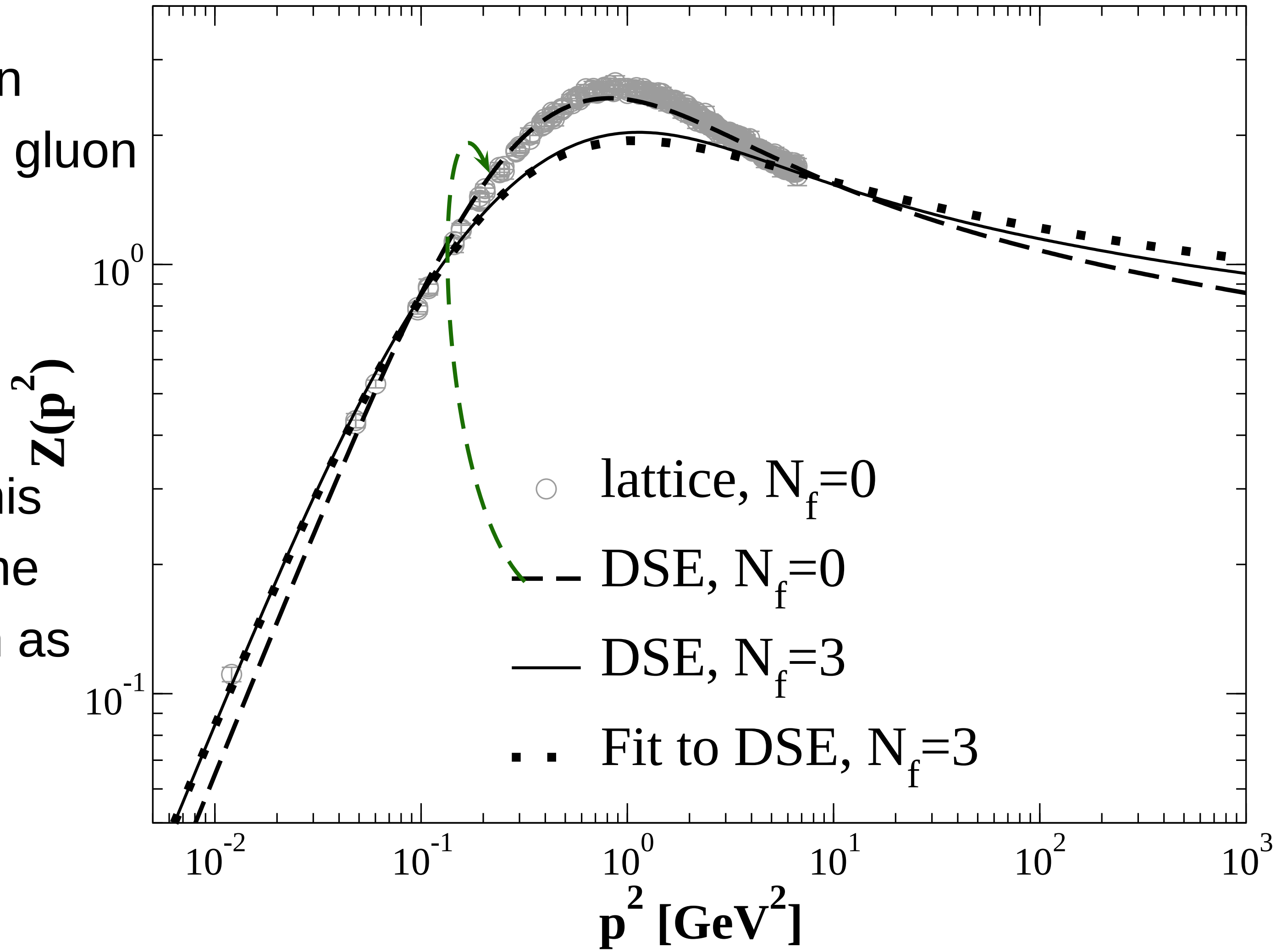

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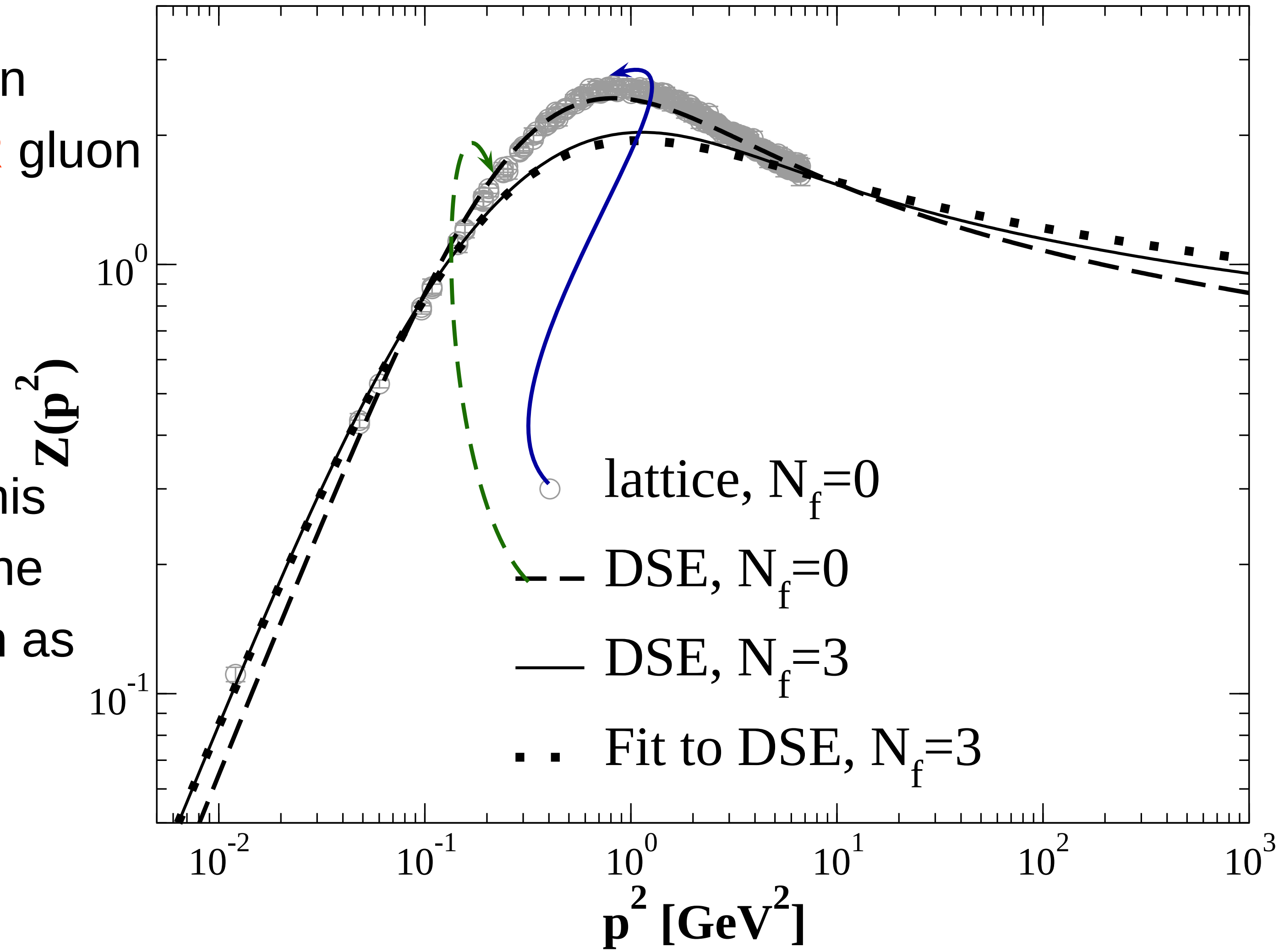

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Dyson-Schwinger Equations

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Dressed-Quark Propagator

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Dyson-Schwinger Equations
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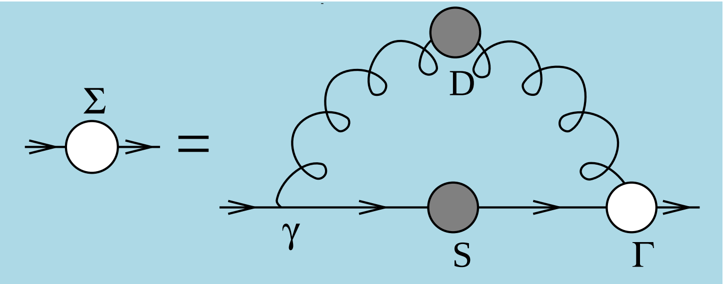
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



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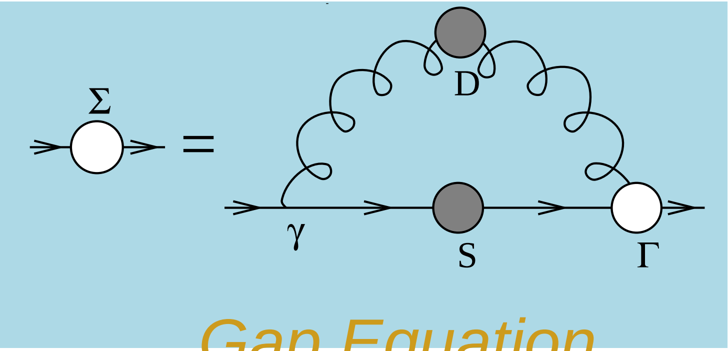

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- Gap Equation's Kernel Enhanced on IR domain

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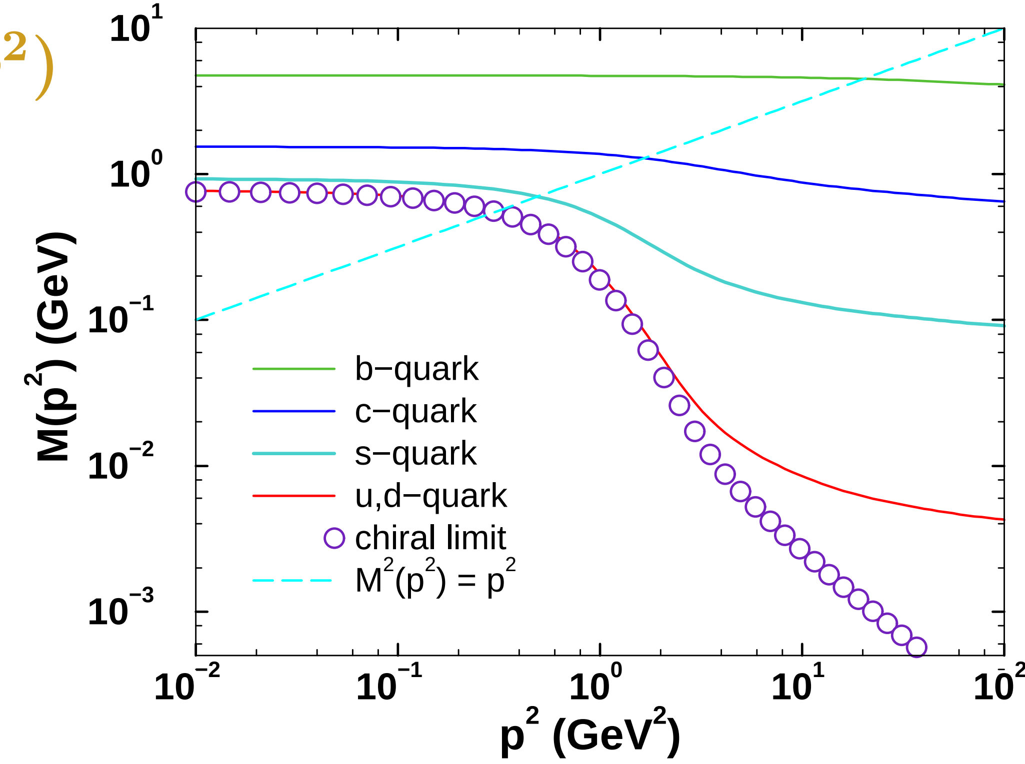

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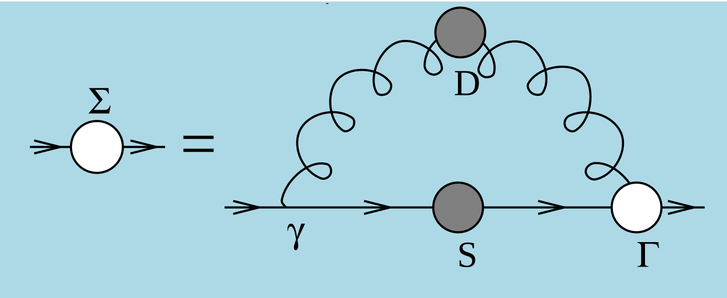

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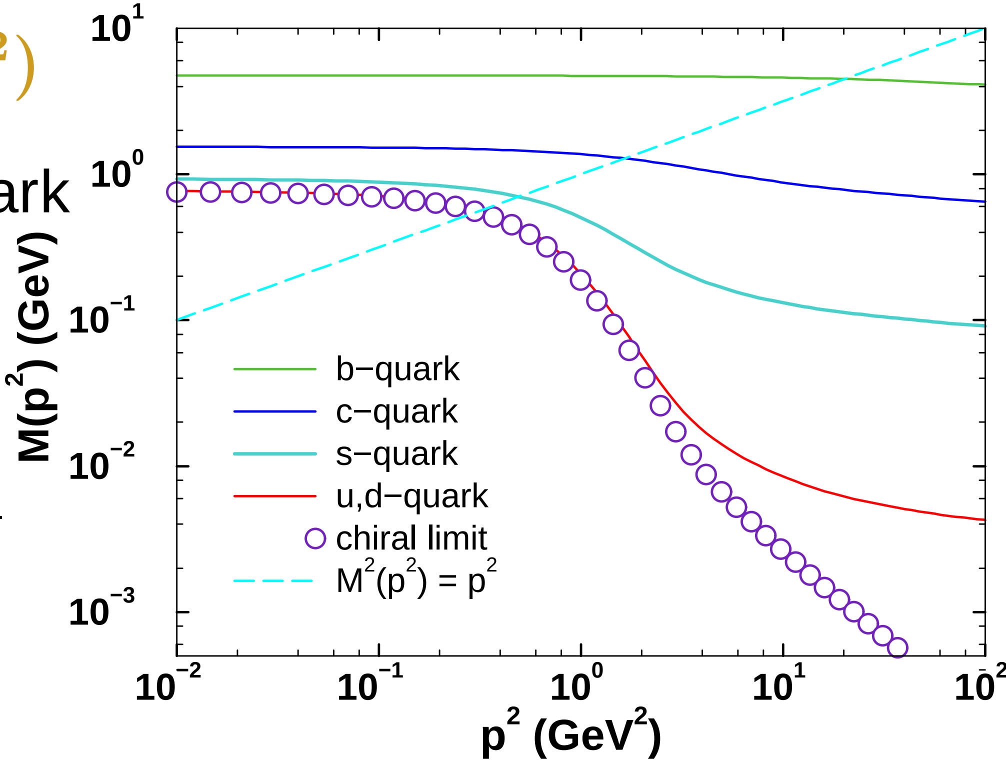
flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



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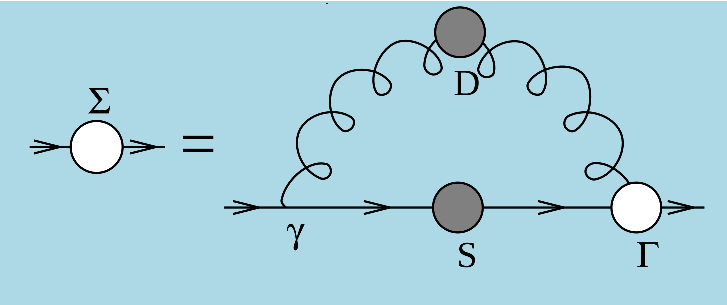

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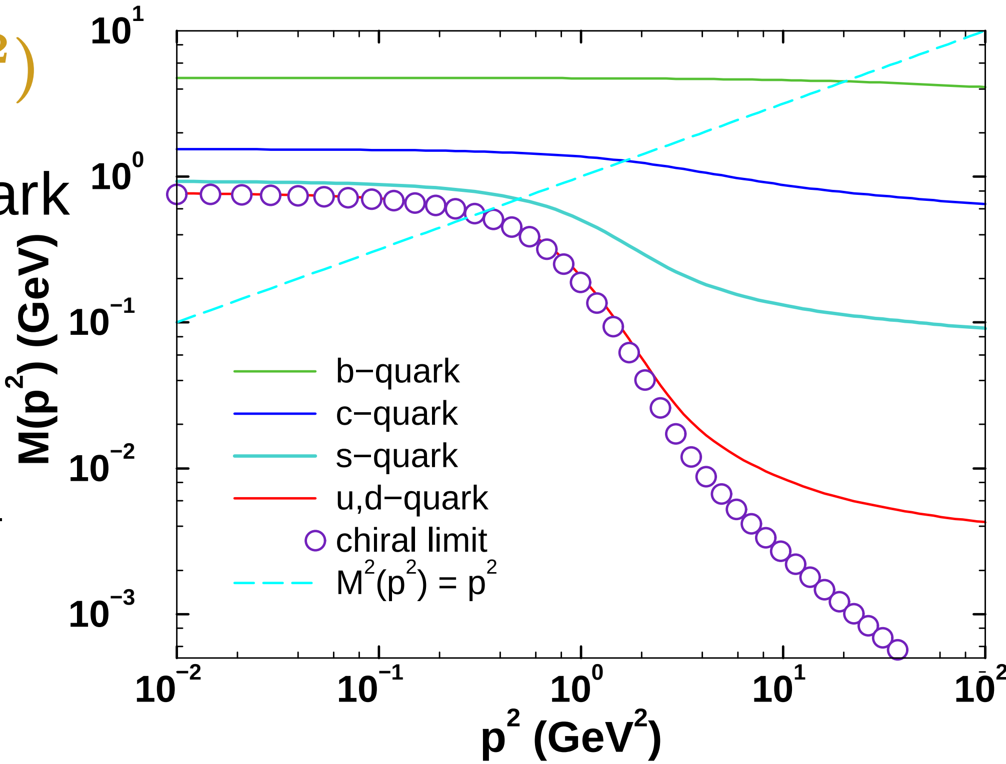
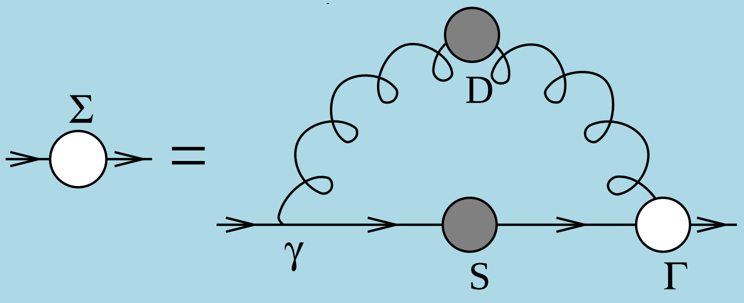


Predictions confirmed in numerical simulations of lattice-QCD

Frontiers of Nuclear Science:
A Long Range Plan (2007)



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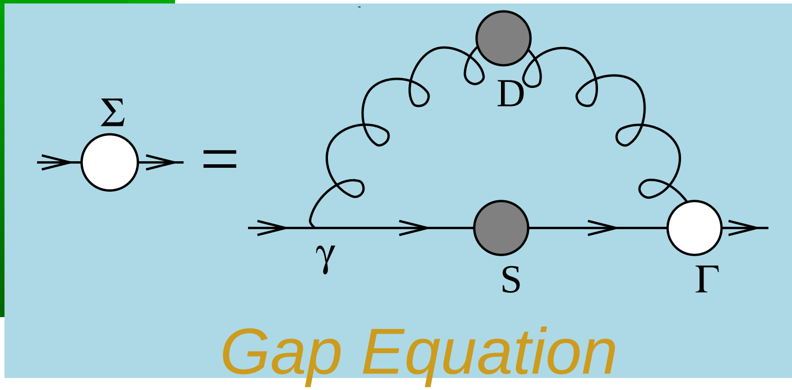

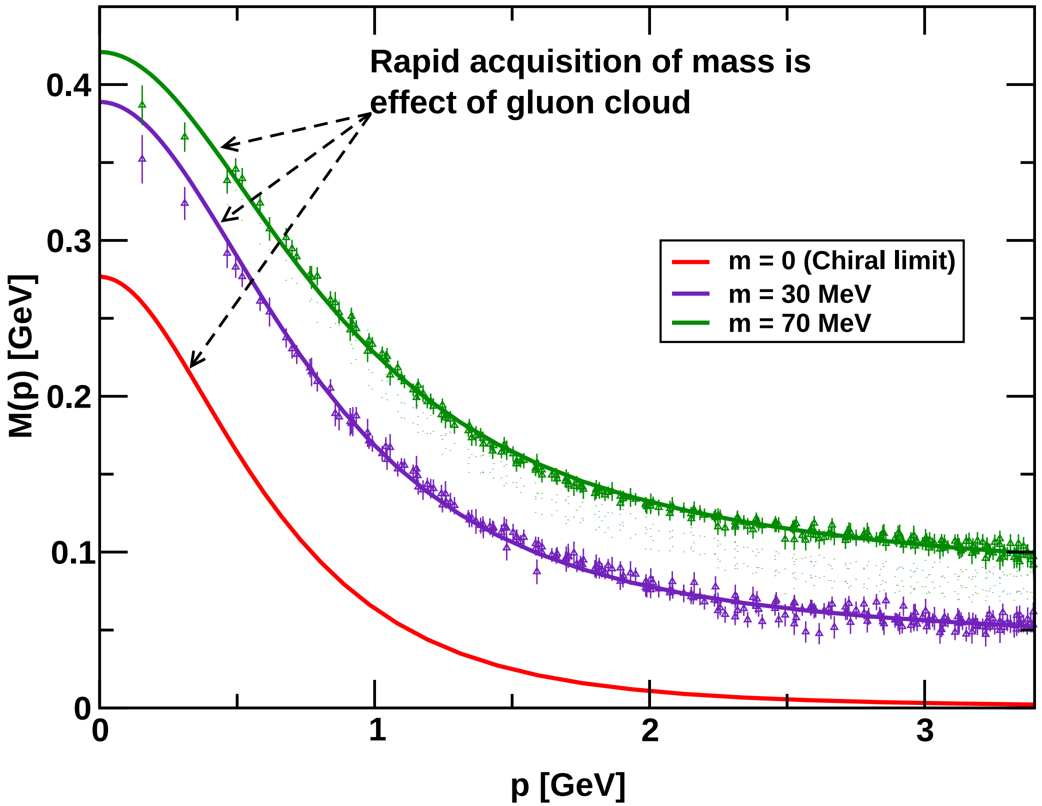

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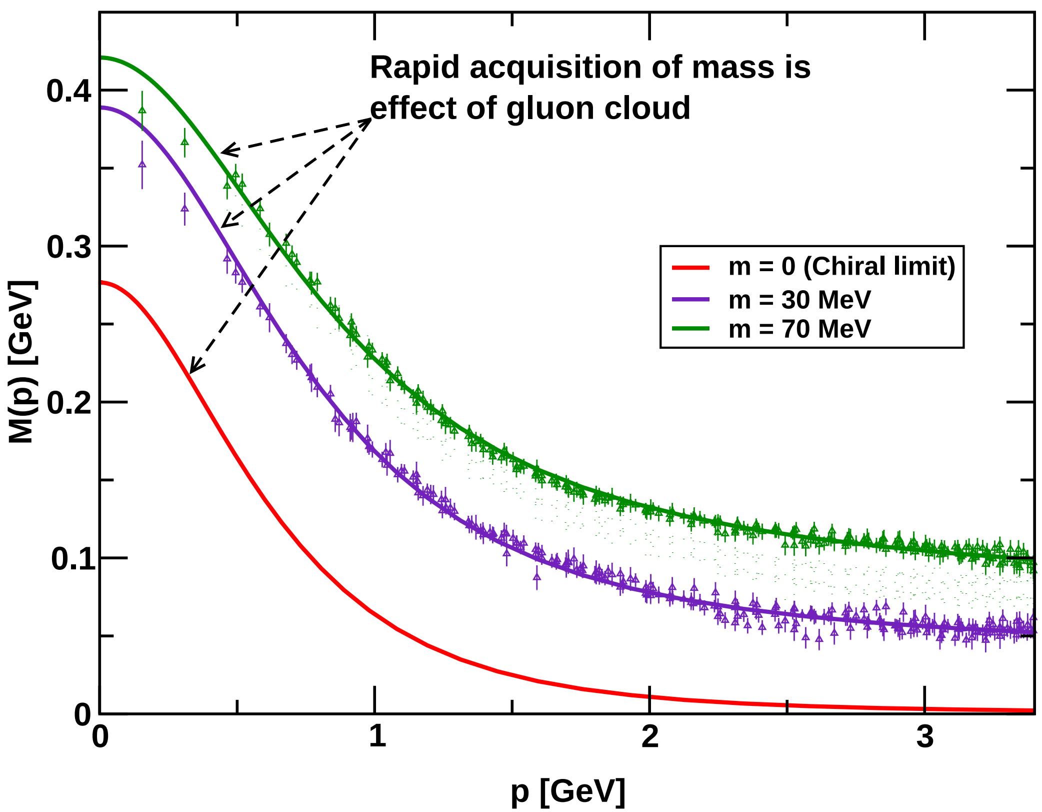
Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.



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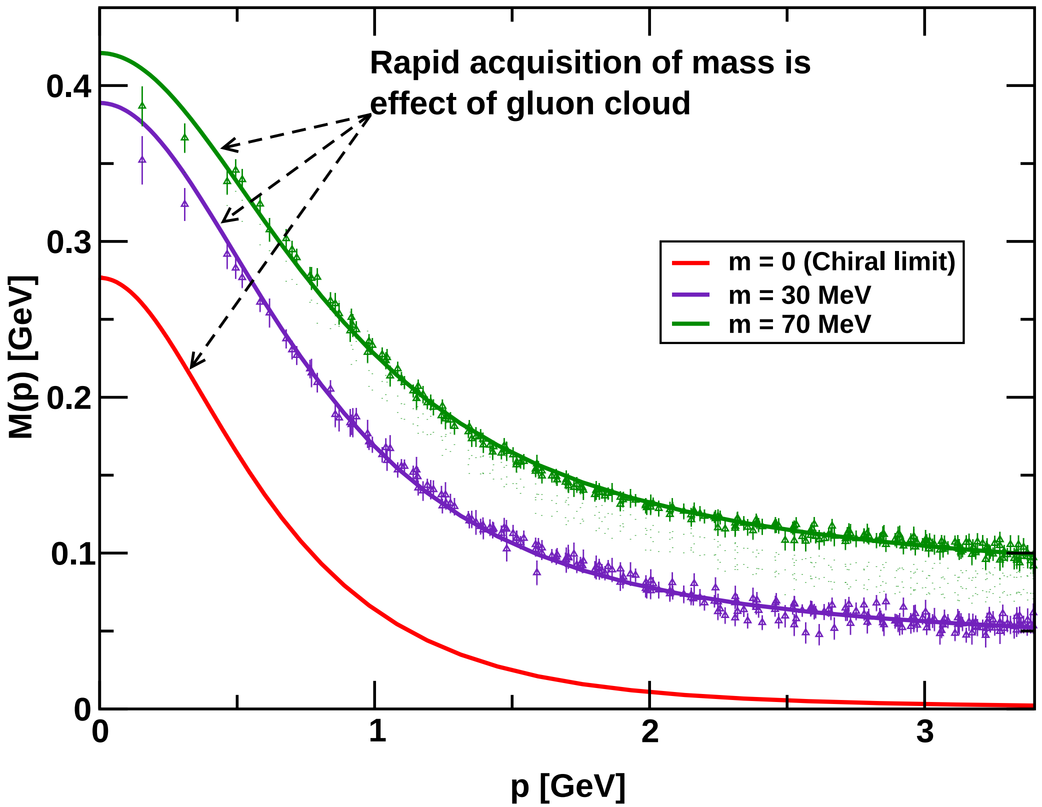


ALWAYS LOOK
ON THE
BRIGHT SIDE

- Established understanding of two- and three-point functions

Hadrons

- Established understanding of two- and three-point functions
- What about bound states?

Hadrons

- Without bound states, Comparison with experiment is impossible

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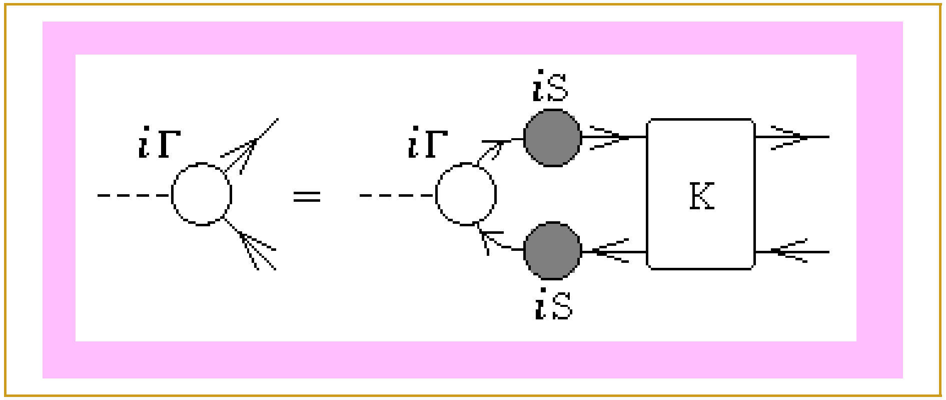

QFT Generalisation of Lippmann-Schwinger Equation.

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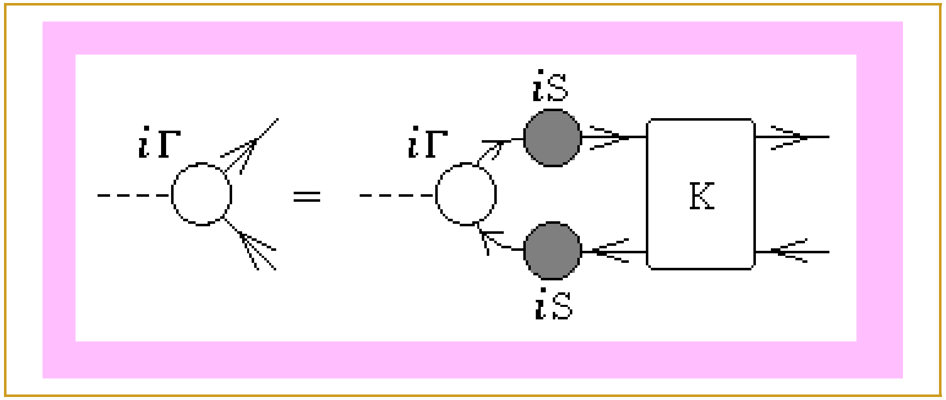

QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

or What is the long-range potential in QCD?

Bethe-Salpeter Kernel

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QFT Statement of Chiral Symmetry

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Radial Excitations & Chiral Symmetry

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(Maris, Roberts, Tandy nu-th/9707003)

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

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$$\boxed{f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H}$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

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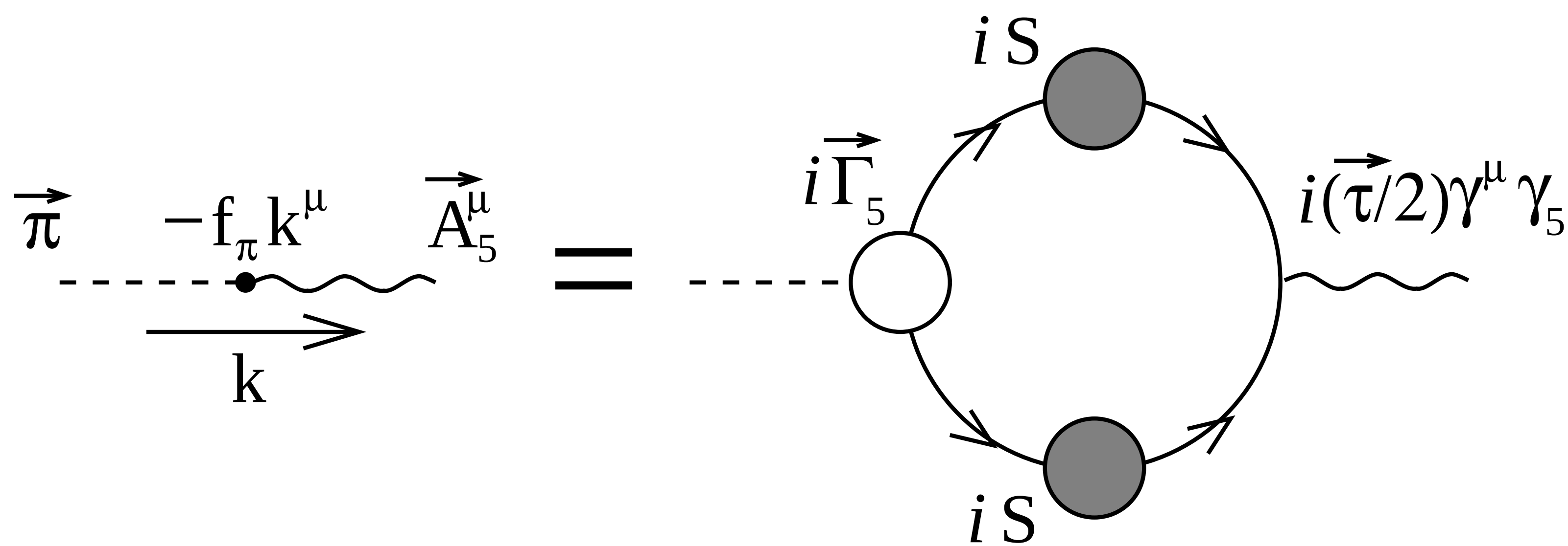
- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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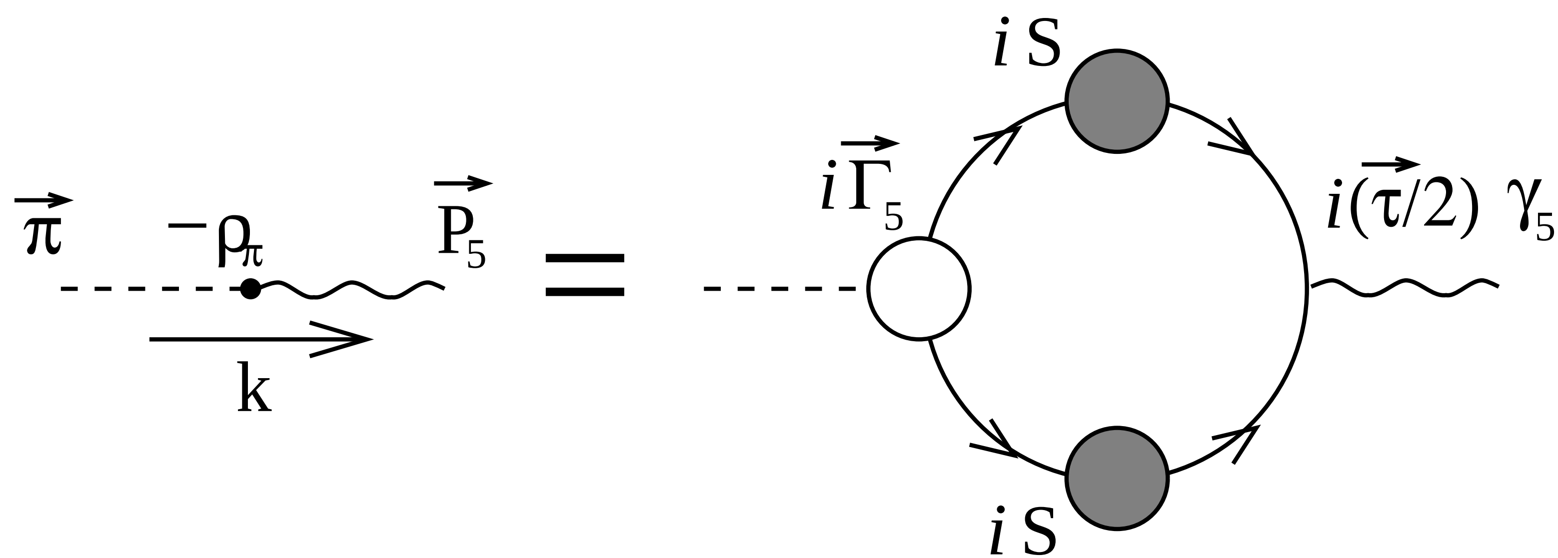

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  Hence $\boxed{m_H^2 = \dfrac{-\langle \bar{q}q\rangle_\zeta^0}{(f_H^0)^2}\, m_q}$ ... GMOR relation, a corollary
- Heavy-quark + light-quark

  $\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

  Hence, $\boxed{m_H \propto m_q}$

  ... QCD Proof of Potential Model result

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Explicit Chiral Symmetry Breaking

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- Chiral symmetry is explicitly broken in QCD by the current-quark mass term, which for the u- and d-quark sector is expressed in the action as

$$\begin{aligned}\int d^4z\, \bar{Q}(z)\, \mathcal{M}\, Q(z) &= \int d^4z\, (\bar{u}(z)\, \bar{d}(z)) \left(\begin{array}{cc} m_u & 0 \\ 0 & m_d \end{array}\right) \left(\begin{array}{c} u(z) \\ d(z) \end{array}\right) \\ &= \int d^4z\, \{\bar{m}\, \bar{Q}(z)\tau^0 Q(z) + \bar{Q}(z)\check{m}\tau^3 Q(z)\}\,,\end{aligned}$$

where: $(\tau^0)_{ij} = \delta_{ij}$ and $\{\tau^k; k = 1, 2, 3\}$ are Pauli matrices; and $\bar{m} = (m_u + m_d)/2$ and $\check{m} = (m_u - m_d)/2$.

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- Empirical success with the application of chiral effective theories to low-energy phenomena in QCD indicates that this term can often be treated as a perturbation.

Dynamical Chiral Symmetry Breaking

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  - K. D. Lane, "*Asymptotic Freedom And Goldstone Realization Of Chiral Symmetry*," Phys. Rev. **D 10**, 2605 (1974).
  - H. D. Politzer, "*Effective Quark Masses In The Chiral Limit*," Nucl. Phys. **B 117**, 397 (1976).
  - C. D. Roberts and A. G. Williams, "*Dyson-Schwinger equations and their application to hadronic physics*," Prog. Part. Nucl. Phys. **33**, 477 (1994).

Sigma Term

Höll, *et al.*, nu-th/0510075

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- Important for numerous reasons, some of longstanding.

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  - Hence, nature's "constants" may vary.

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- NB. Higher dimensional theories do not necessarily require varying "constants", but they provide a framework for describing the variations, if they exist.

Impact of Variation? . . . Example

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- *Calculation of current-quark mass dependence of hadron properties necessary* to enable use of observational data to *place constraints on variation of nature's* "constants".

Evidence of Variation?

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- Oklo
  - Uranium mine in Gabon, West Africa

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Evidence of Variation?

- Oklo
  - Evidence suggesting that a natural fission reactor was in operation there about 1.8 Gy ago

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$$\frac{\alpha_{\mathrm{em}}^{\mathrm{past}} - \alpha_{\mathrm{em}}^{\mathrm{now}}}{\alpha_{\mathrm{em}}} \gtrsim 4.5 \times 10^{-8}, \quad (6\,\sigma \text{ confidence})$$

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- "*Finally, these results might be interpreted more efficiently in terms of* $\frac{m}{\Lambda_{\text{QCD}}}$*, for which the sensitivity is about two orders of magnitude higher than the sensitivity to a variation in* α_{em}."

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- Interpretation of
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- Plus . . . a large number of new and more accurate measurements expected to appear soon
- Emphasise the important role that the *hadron physics calculations* can play in the *interpretation* of many *measurements performed in several areas of physics and astronomy*

Pion Sigma Term

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- Useful illustrative example . . . σ-term for π . . . begin with scalar form factor:

$$s_\pi(Q^2) = \langle \pi(P') | \bar{m}\, J_\sigma(Q) | \pi(P) \rangle \,,\; Q_\mu = (P' - P)_\mu \,.$$

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- In rainbow-ladder truncation of QCD's Dyson-Schwinger Equation: $(\ell_{\alpha,\beta} = \ell + \alpha P + \beta Q)$

$$\begin{aligned} s_\pi(Q^2) \;&=\; \mathrm{tr}_{CDF} \int \frac{d^4\ell}{(2\pi)^4}\, \mathcal{S}(\ell_{-1,\frac{1}{2}})\, \bar{m}\, \Gamma_{\tau^0}(\ell_{-1,0};Q)\, \mathcal{S}(\ell_{-1,-\frac{1}{2}}) \\ &\times \Gamma_\pi(\ell_{-\frac{1}{2},0};P')\, \mathcal{S}(\ell_{0,\frac{1}{2}})\, \Gamma_\pi(\ell_{-\frac{1}{2},\frac{1}{2}};P) \end{aligned}$$

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- rainbow-ladder ... first term in a nonperturbative, systematic and symmetry preserving truncation scheme $\Rightarrow$ triangle diagram
  - $\boldsymbol{\mathcal{S}(\ell)}$ ... two-flavour dressed-quark propagator
  - $\boldsymbol{\Gamma_\pi(\ell;P)}$ ... pion's Bethe-Salpeter amplitude
  - $\boldsymbol{\Gamma_{\tau^0}(\ell;Q)}$ ... two-flavour inhomogeneous isoscalar scalar vertex

Return to Pion Sigma Term

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$$2P_\mu = \mathrm{tr}_{CDF}\int_q^\Lambda \Gamma_\pi(q;-P)\frac{\partial}{\partial P_\mu}\,\mathcal{S}(q+Q/2)\Gamma_\pi(q;P)\mathcal{S}(q-Q/2)+\text{sym}$$

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Feynman-Hellmann Theorem

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- Concrete illustration of a general result that may be viewed as a consequence of the Feynman-Hellmann theorem.
  - Present case: theorem states that response of an eigenvalue of the QCD mass^2-operator to a change in a parameter in that operator is given by expectation value of the derivative of the mass^2-operator operator with respect to the parameter.

$$\boxed{\frac{\partial E_{M^2}}{\partial \lambda} = \left\langle \frac{\partial M^2}{\partial \lambda} \right\rangle}$$

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  Derived by Feynman, when 21, in his final year as an undergraduate. Has played an important role in theoretical chemistry and condensed matter physics.

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$$\boxed{\frac{\partial E_{M^2}}{\partial \lambda} = \left\langle \frac{\partial M^2}{\partial \lambda} \right\rangle}$$

- The result is valid in this form for all mesons; i.e.,

$$\boxed{2\, m_M \sigma_M := s_M(0) = \bar{m}(\zeta)\frac{\partial m_M^2}{\partial \bar{m}(\zeta)} \;\Rightarrow\; \sigma_M = \bar{m}(\zeta)\frac{\partial m_M}{\partial \bar{m}(\zeta)}}$$

NB. The σ-term is a renormalisation point invariant, in general and also in the explicit calculation, so long as a RGI rainbow-ladder truncation is used.

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$$\boxed{2\,m_\pi\,\sigma_\pi \stackrel{\bar{m}\simeq 0}{=} -2\,\bar{m}(\zeta)\,\frac{\langle \bar{q}q\rangle_\zeta^0}{(f_\pi^0)^2} \;\Rightarrow\; \sigma_\pi \stackrel{\bar{m}\simeq 0}{=} \frac{1}{2}\,m_\pi}$$

  - Model-independent result.
  - Essential consequence of DCSB.

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Example: BBN

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  - Standard Model of Nuclear Physics: Hamiltonian capable of decimal point accuracy in calculation of light-element binding energies.
  - DSE result for hadron mass dependence on $X_q := m_q/\Lambda_{\rm QCD}$
- 1% increase in X_q is sufficient by itself to resolve existing discrepancies between theoretical and measured abundances of ^{2}H, ^{4}He, ^{7}Li
- Since the BBN epoch, $\dfrac{\delta X_q}{X_q} = 0.007 - 0.026$.


I remember when the
only job a black man could
get in America was cleaning up
the mess that white folks made
PRESIDENT OBAMA
HISTORY MADE

Epilogue

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  - Manifest in dressed propagators and vertices
  - It predicts, amongst other things, that
    - light current-quarks become heavy constituent-quarks: $4 \to 400\,$MeV
    - pseudoscalar mesons are unnaturally light: $m_\rho = 770$ cf. $m_\pi = 140\,$MeV
    - pseudoscalar mesons couple unnaturally strongly to light-quarks: $g_{\pi\bar{q}q} \approx 4.3$
    - pseudscalar mesons couple unnaturally strongly to the lightest baryons $g_{\pi\bar{N}N} \approx 12.8 \approx 3 g_{\pi\bar{q}q}$

Epilogue

- DCSB exists in QCD: mass from *nothing*
  - Manifest in dressed propagators and vertices
  - Impacts enormously upon observables.

Epilogue

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- *The only thing constant in life is change*

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