

Unified description of mesons and baryons

Craig D. Roberts

cdroberts@anl.gov

Physics Division

Argonne National Laboratory

<http://www.phy.anl.gov/theory/staff/cdr.html>

Universal Truths

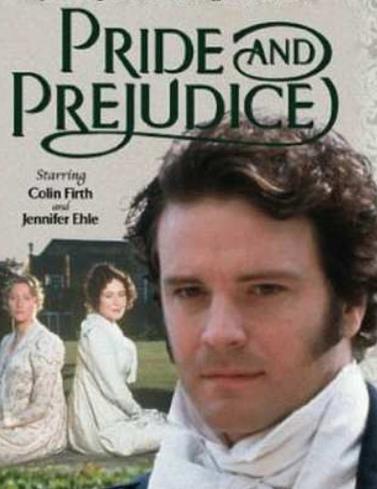


[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Universal Truths



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

PRIDE AND PREJUDICE

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Colin Firth
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Universal Truths

- Spectrum of excited states and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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- Spectrum of excited states and transition form factors provide unique information about long-range interaction between light-quarks and distribution of hadron's characterising properties amongst its QCD constituents.
- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe.



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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- Running of quark mass entails that calculations at even modest Q^2 require a Poincaré-covariant approach. **Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.**



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- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.



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- Dynamical Chiral Symmetry Breaking (DCSB) is most important mass generating mechanism for visible matter in the Universe. **Higgs mechanism is irrelevant to light-quarks.**
- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. **Problem** because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.



QCD's Challenges

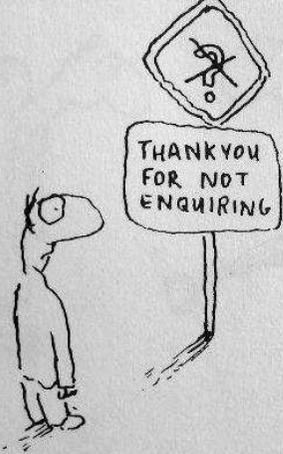


[First](#)

[Contents](#)

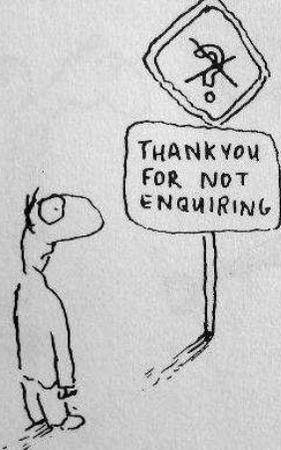
[Back](#)

[Conclusion](#)



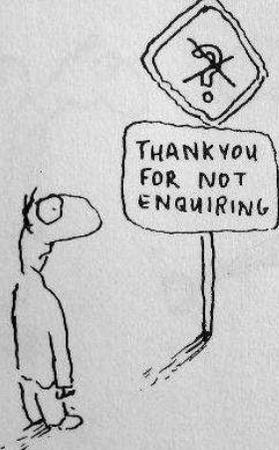
- Quark and Gluon Confinement
 - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon





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 - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
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 - Very unnatural pattern of bound state masses
 - e.g., Lagrangian (pQCD) quark mass is small but ... no degeneracy between $J^{P=+}$ and $J^{P=-}$





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Understand Emergent Phenomena

- Quark and Gluon Confinement
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- Neither of these phenomena is apparent in QCD's Lagrangian **yet** they are the dominant determining characteristics of real-world QCD.
- QCD – Complex behaviour
arises from apparently simple rules



Dichotomy of Pion

– Goldstone Mode and Bound state



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Dichotomy of Pion

– Goldstone Mode and Bound state

- How does one make an **almost massless** particle from two **massive** constituent-quarks?





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Must exhibit $m_{\pi}^2 \propto m_q$

Current Algebra ... 1968





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The **correct understanding** of pion observables; e.g. **mass**, **decay constant** and **form factors**, **requires** an approach to contain a

- **well-defined** and **valid chiral limit**;
- and an **accurate realisation** of **dynamical chiral symmetry breaking**.





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Highly Nontrivial



What's the Problem?



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

What's the Problem?

- Minimal requirements
 - detailed understanding of connection between **Current-quark** and **Constituent-quark** masses;
 - and systematic, symmetry preserving means of realising this connection in bound-states.



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- Differences!



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Relativistic QFT!

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 - Here relativistic effects are crucial – *virtual particles*, quintessence of **Relativistic Quantum Field Theory** – must be included



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- Differences!
 - Here relativistic effects are crucial – *virtual particles*, quintessence of **Relativistic Quantum Field Theory** – must be included
 - Interaction between quarks – the **Interquark “Potential”** – **unknown** throughout **> 98%** of a hadron's volume



Intranucleon Interaction



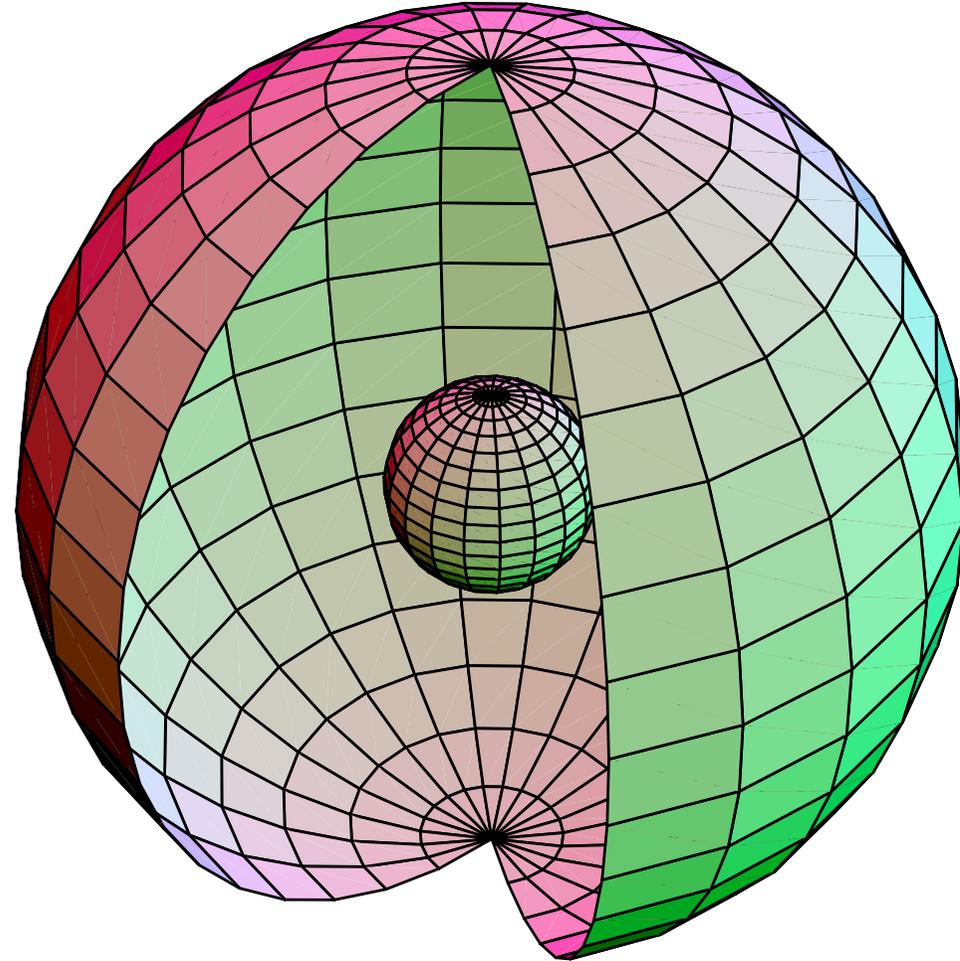
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Intranucleon Interaction



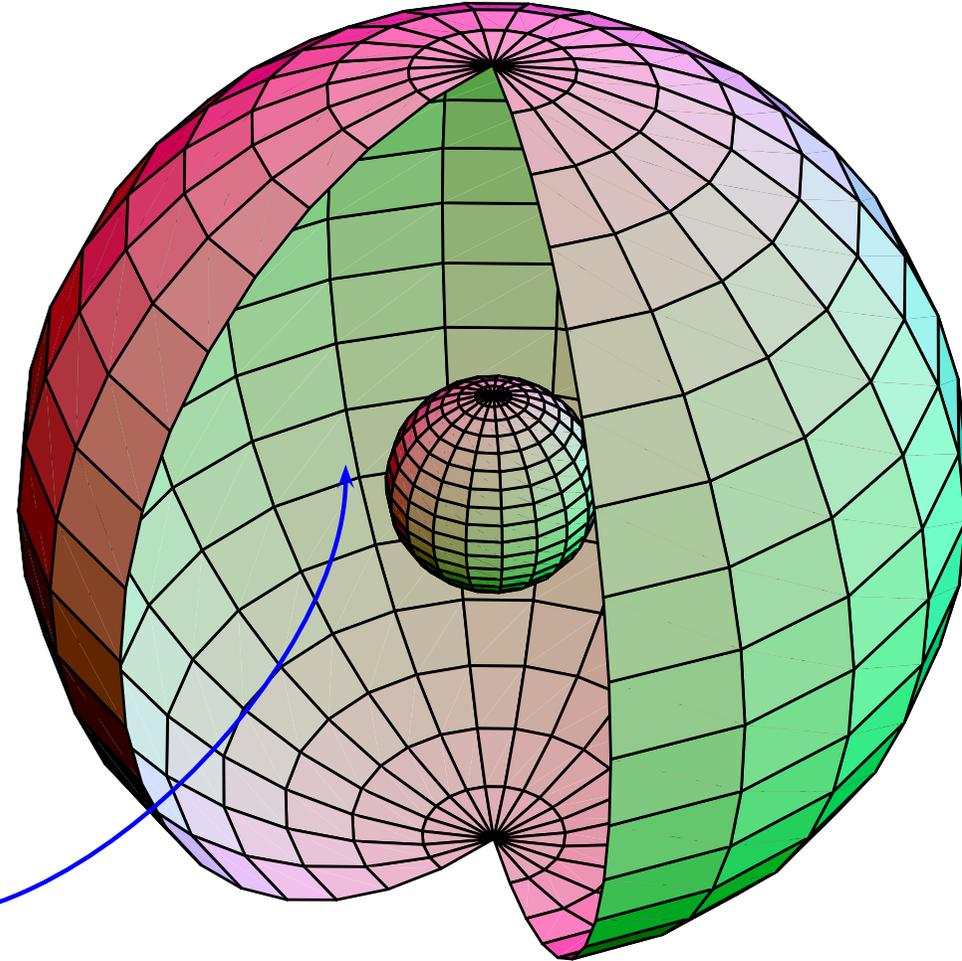
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Intranucleon Interaction



98% of the volume



[First](#)

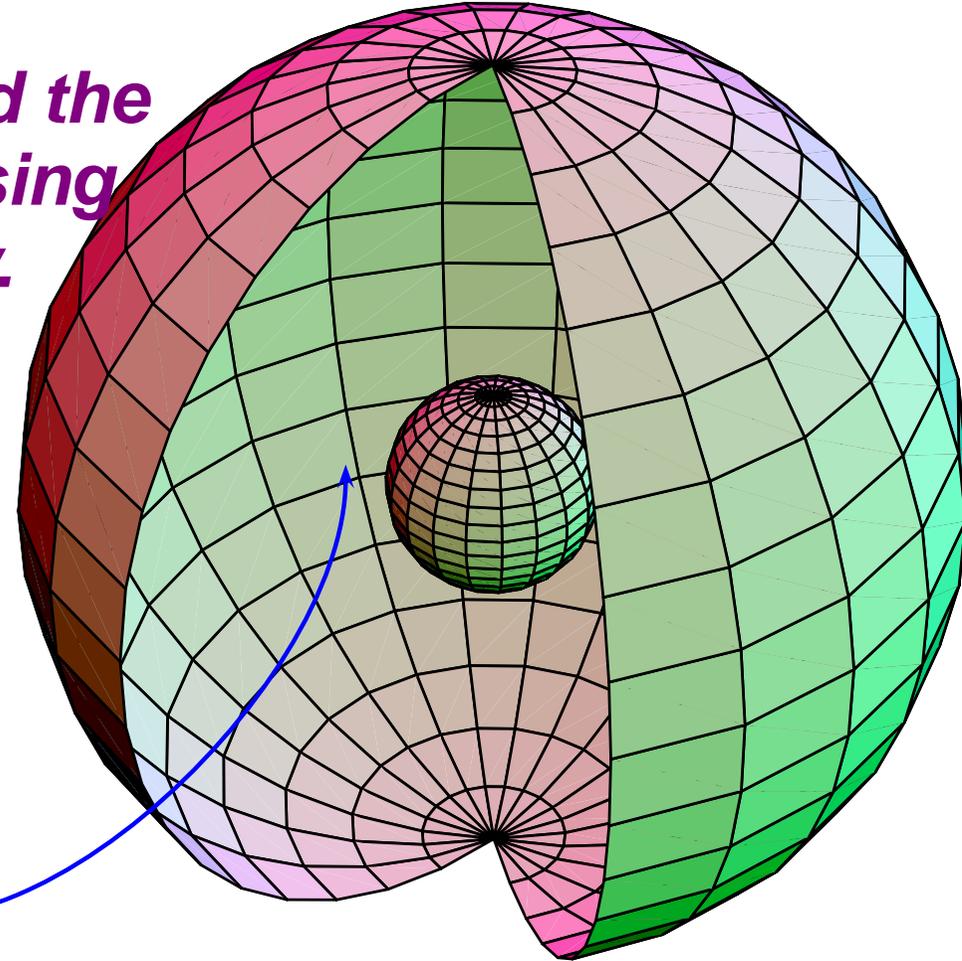
[Contents](#)

[Back](#)

[Conclusion](#)

What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



98% of the volume



Dyson-Schwinger Equations



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Dyson-Schwinger Equations

Dressed-Quark Propagator



[First](#)

[Contents](#)

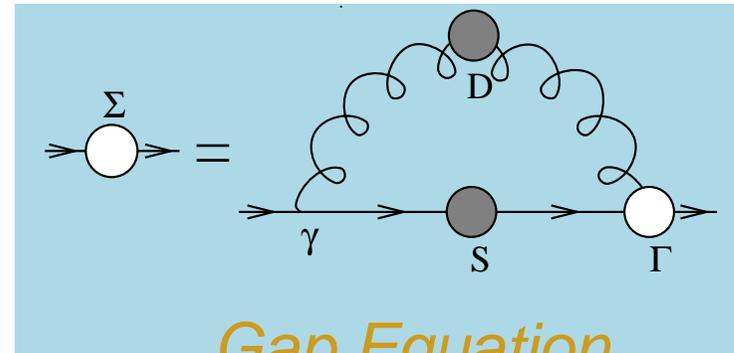
[Back](#)

[Conclusion](#)

Dyson-Schwinger Equations

Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



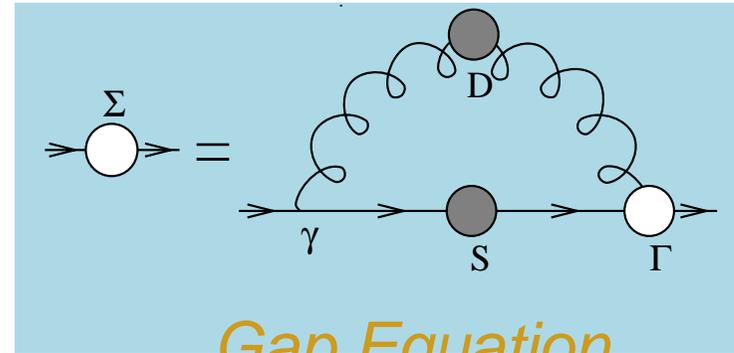
Gap Equation



Dyson-Schwinger Equations

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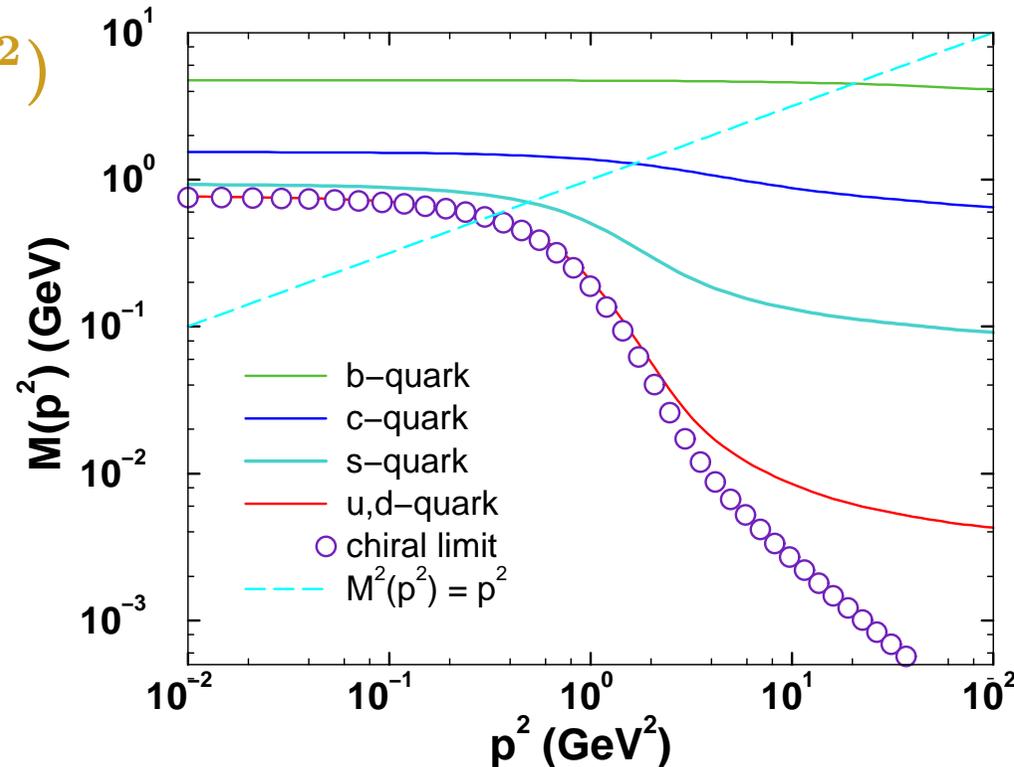
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Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

⇒ IR Enhancement of $M(p^2)$

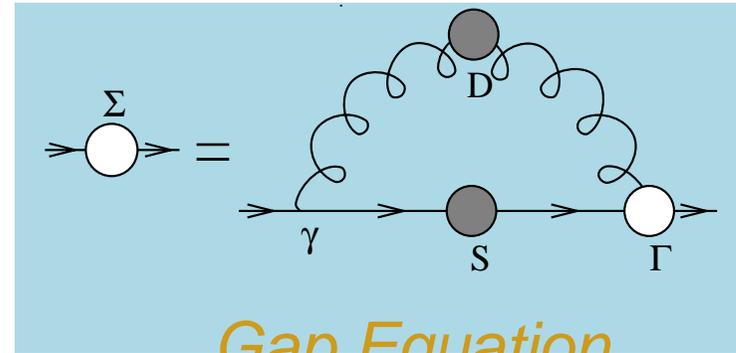


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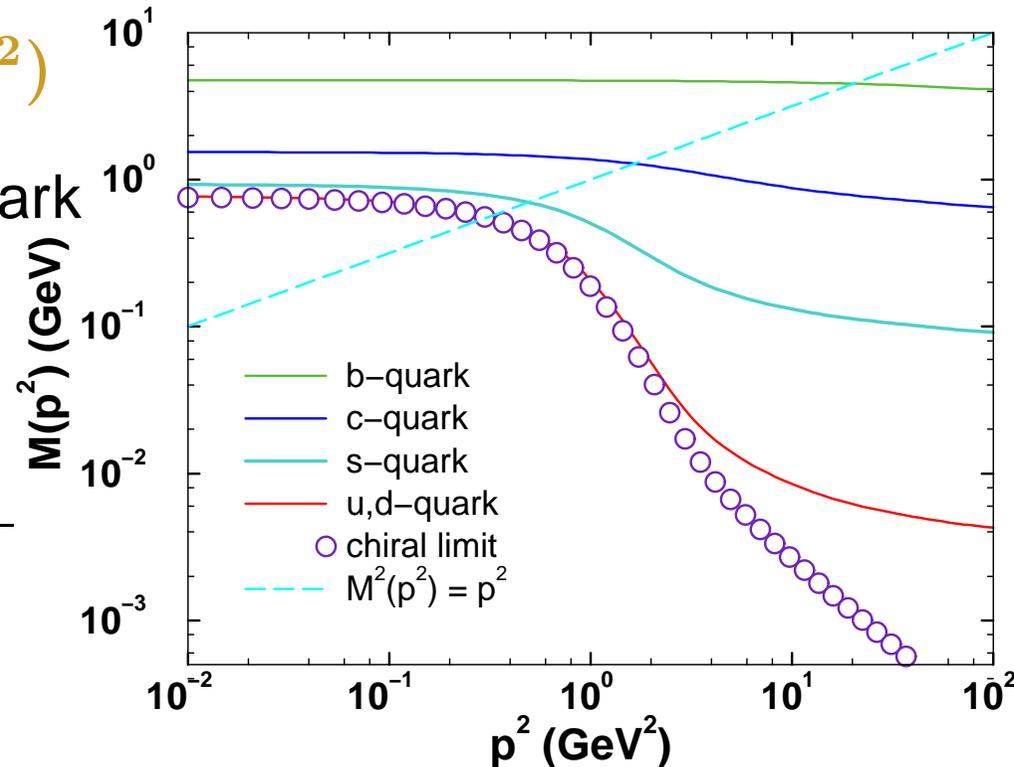
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⇒ IR Enhancement of $M(p^2)$

Euclidean Constituent-Quark

Mass: $M_f^E: p^2 = M(p^2)^2$

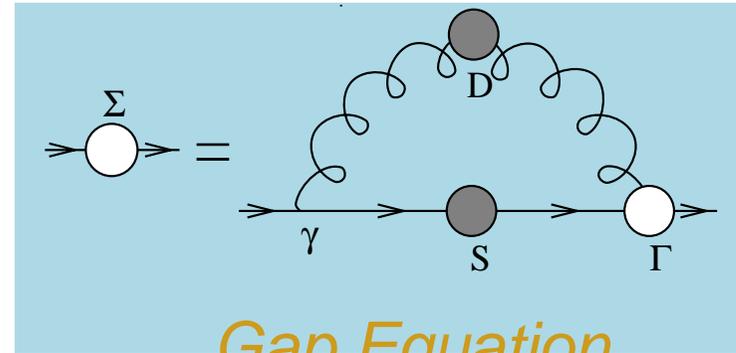
flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



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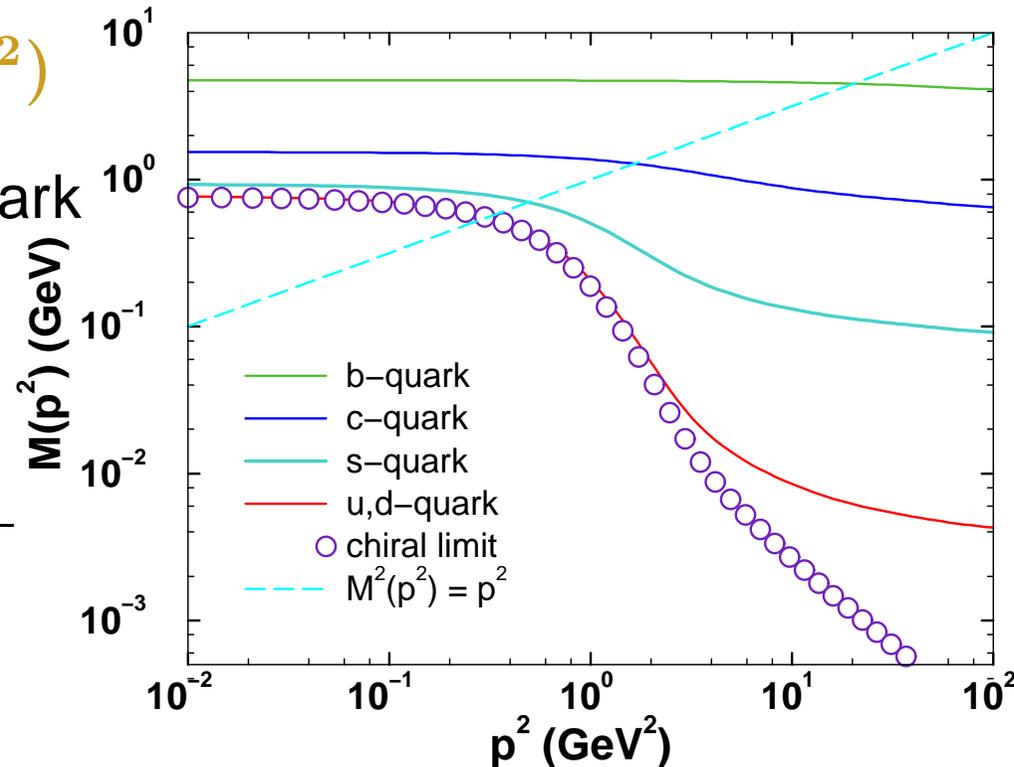
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Predictions confirmed in numerical simulations of lattice-QCD



Frontiers of Nuclear Science: A Long Range Plan (2007)



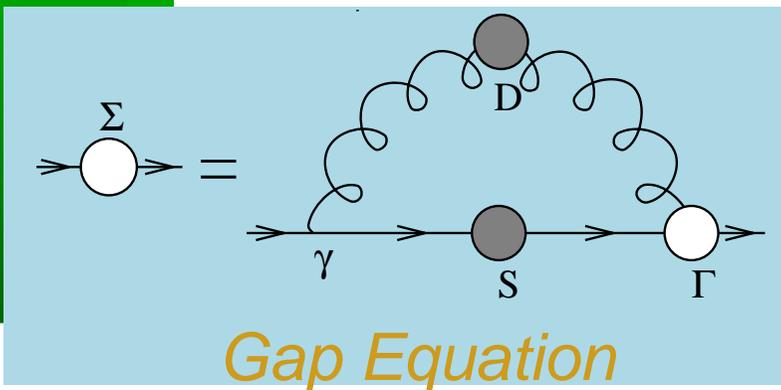
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Frontiers of Nuclear Science: Theoretical Advances



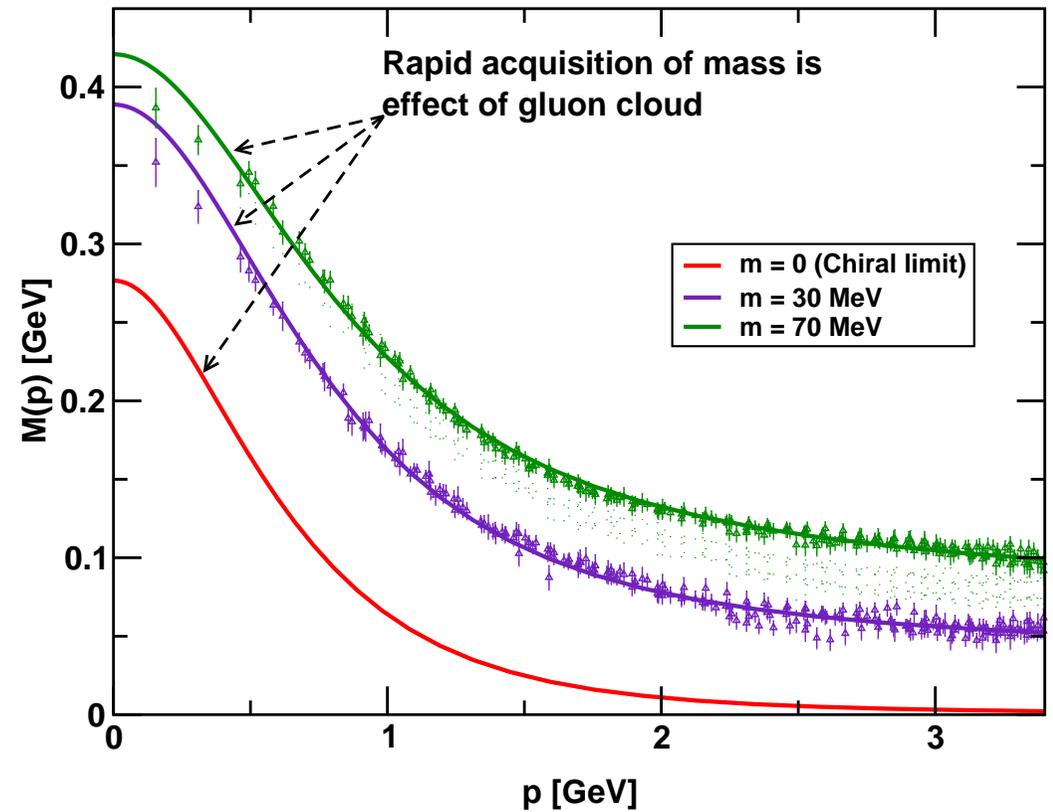
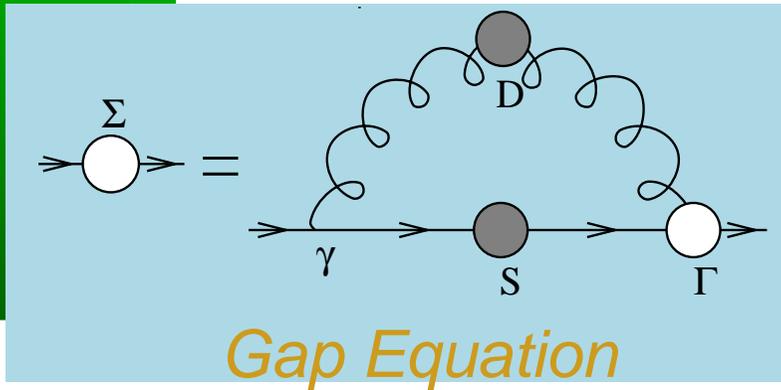
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

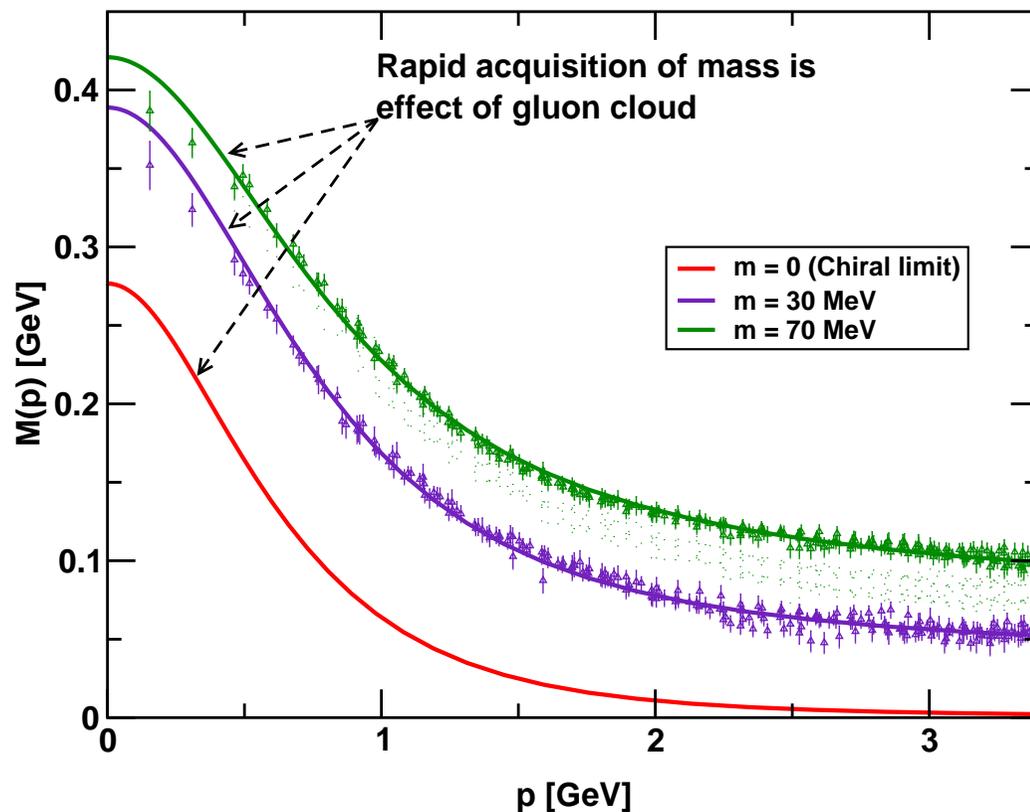
Frontiers of Nuclear Science: Theoretical Advances



Frontiers of Nuclear Science: Theoretical Advances

Mass from nothing.

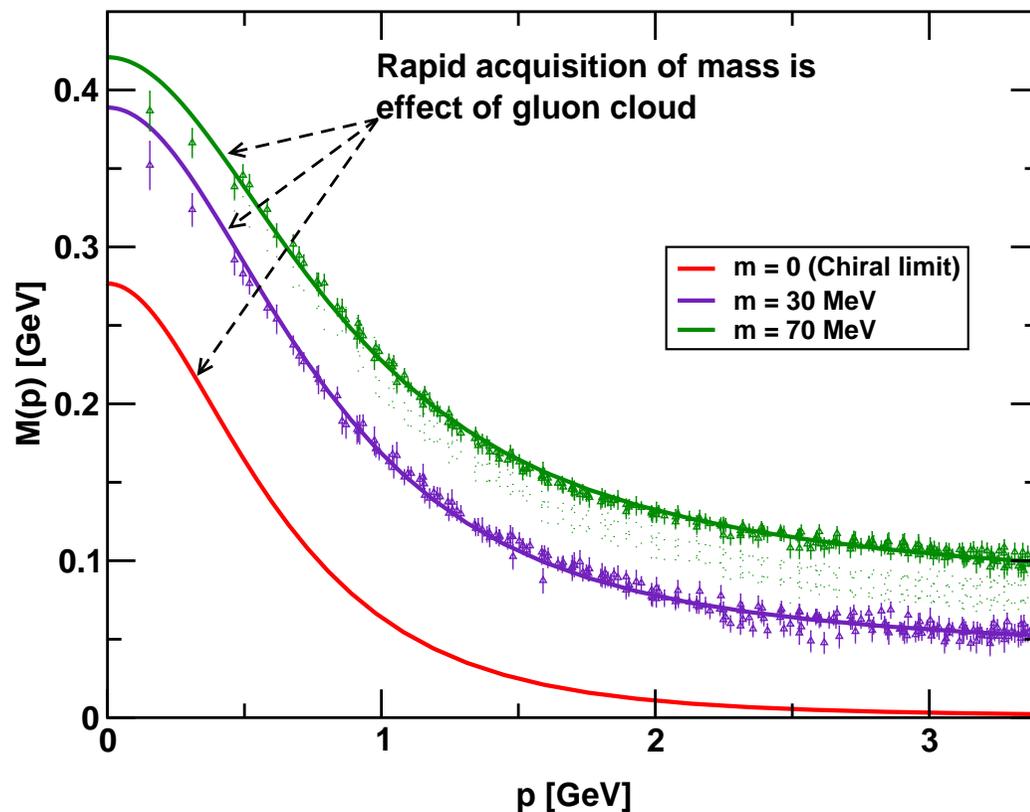
In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.



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[First](#)[Contents](#)[Back](#)[Conclusion](#)



- Established understanding of two- and three-point functions



Hadrons



- Established understanding of two- and three-point functions
- What about bound states?



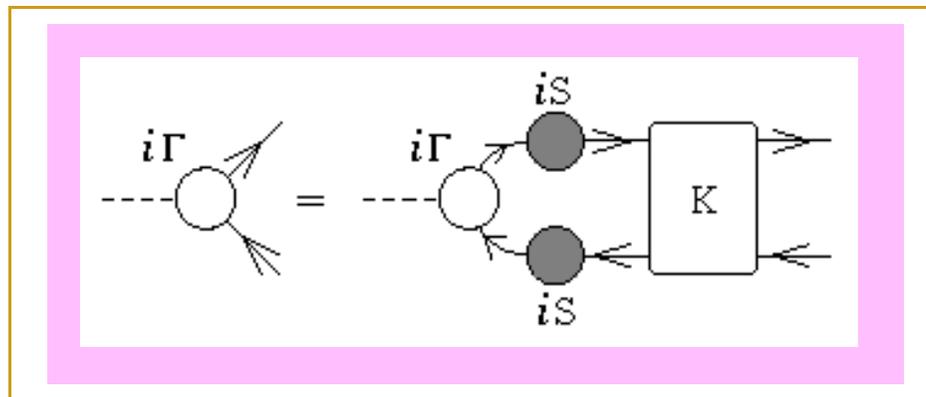
- Without bound states, Comparison with experiment is **impossible**



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- They appear as pole contributions to $n \geq 3$ -point colour-singlet Schwinger functions

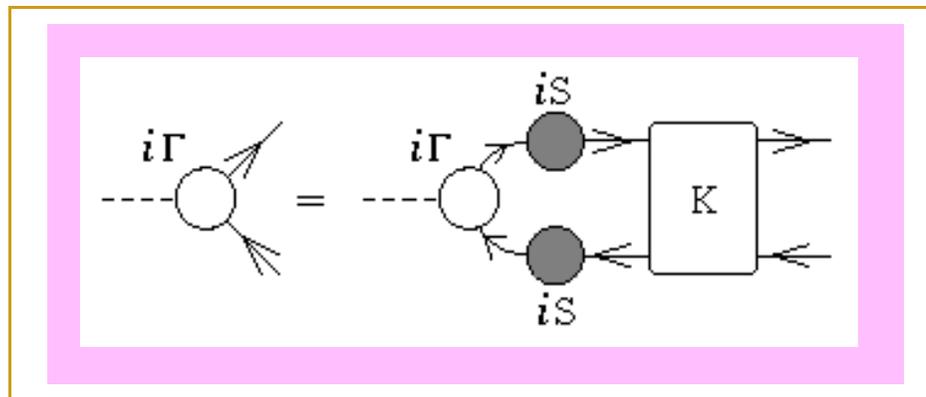


- Without bound states, Comparison with experiment is **impossible**
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

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QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K ?

or



Confinement



[First](#)

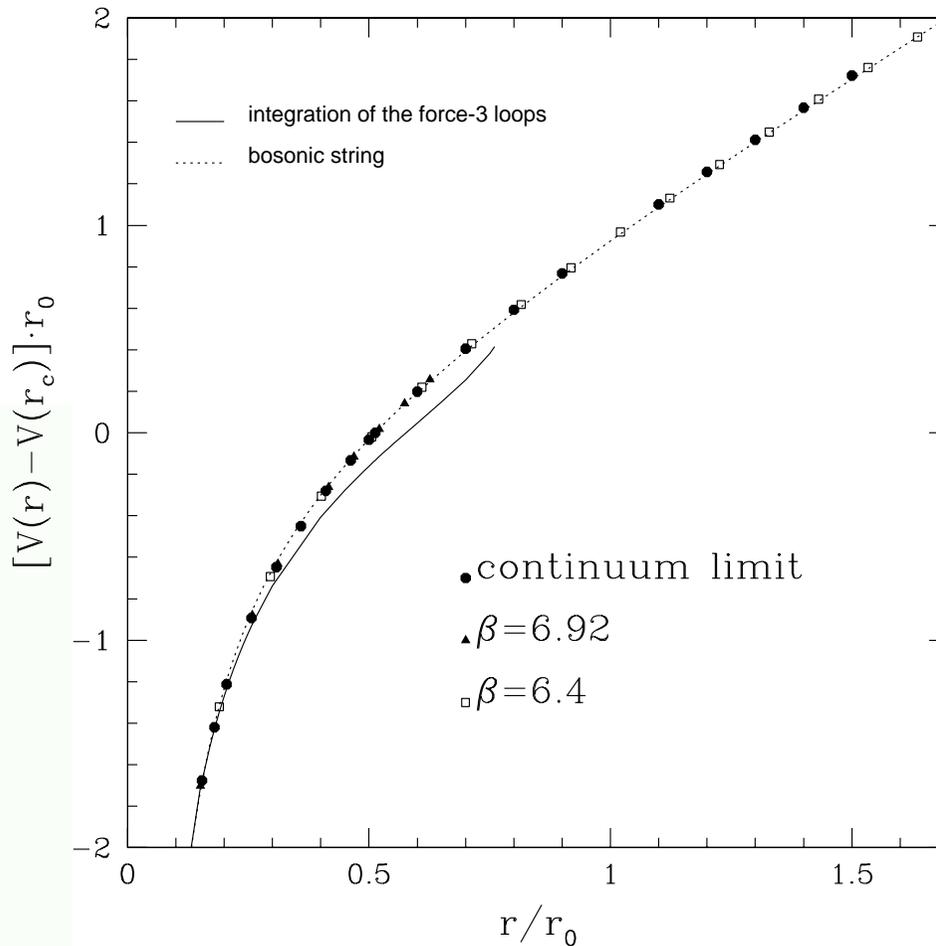
[Contents](#)

[Back](#)

[Conclusion](#)

Confinement

● Infinitely Heavy Quarks ... Picture in Quantum Mechanics



$$V(r) = \sigma r - \frac{\pi}{12} \frac{1}{r}$$

$$\sigma \sim 470 \text{ MeV}$$

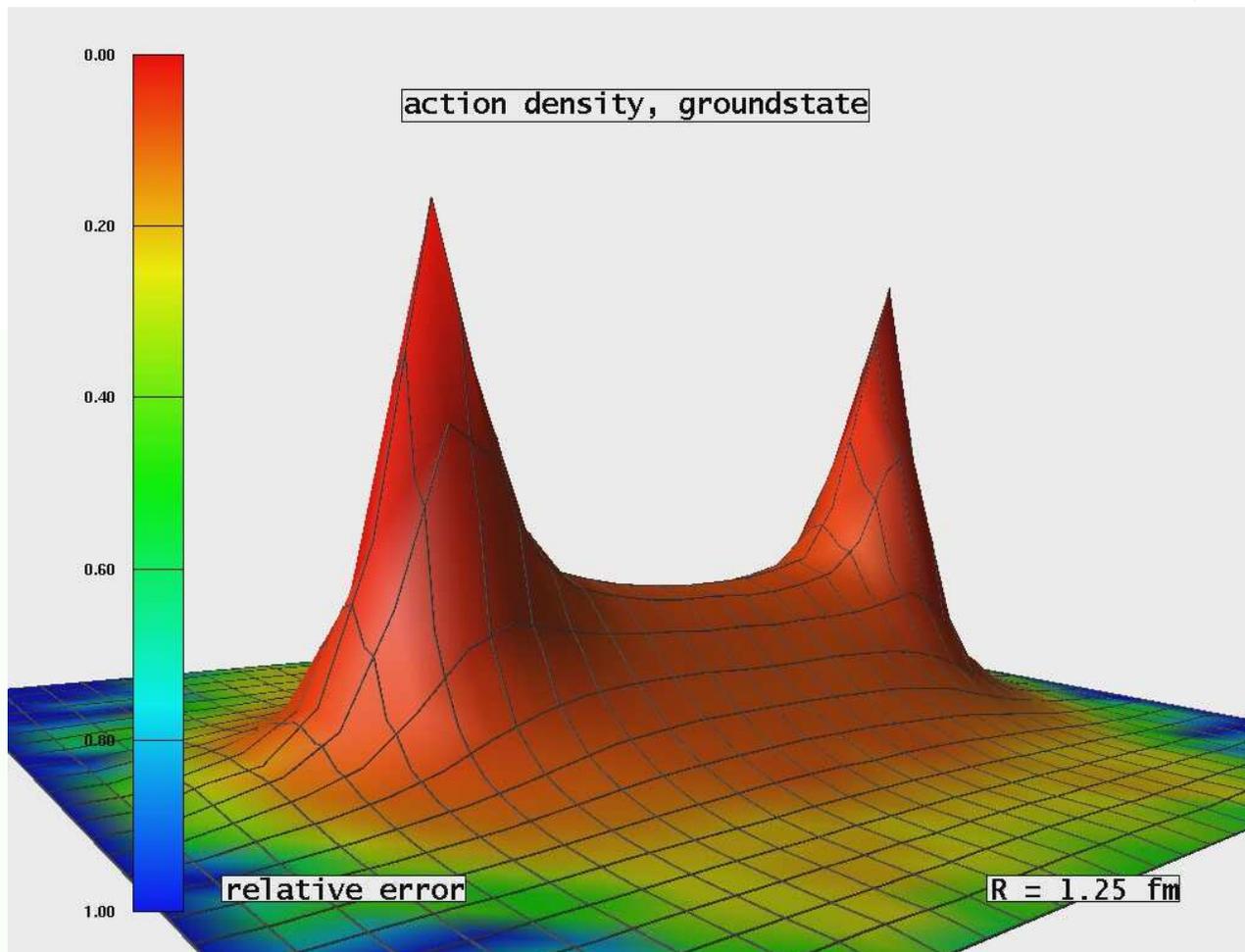
Necco & Sommer
 he-lq/0108008



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Confinement

- Illustrate this in terms of the action density ... analogous to plotting the Force = $F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12} \frac{1}{r^2}$



Bali, *et al.*
he-lq/0512018



Confinement

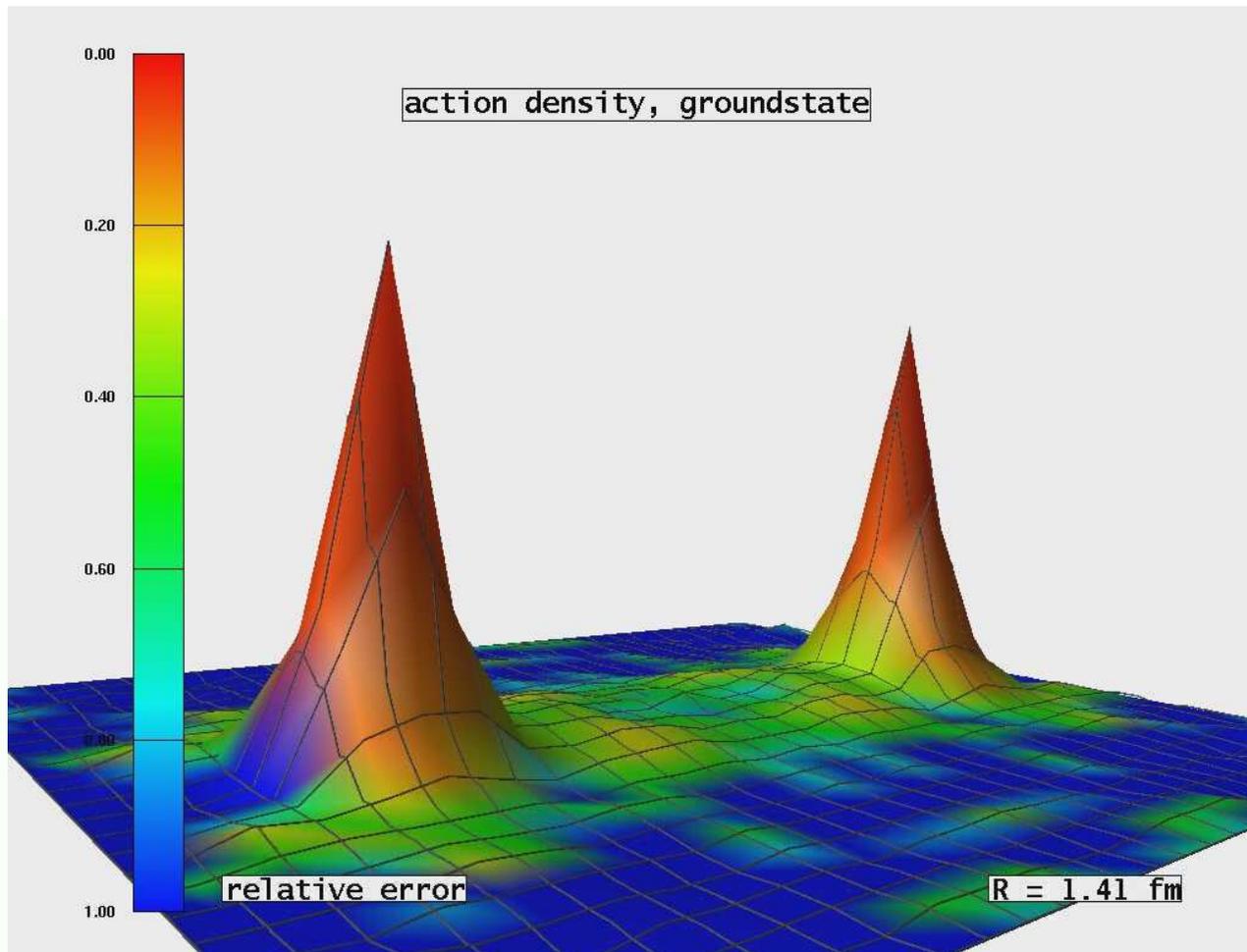
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Confinement

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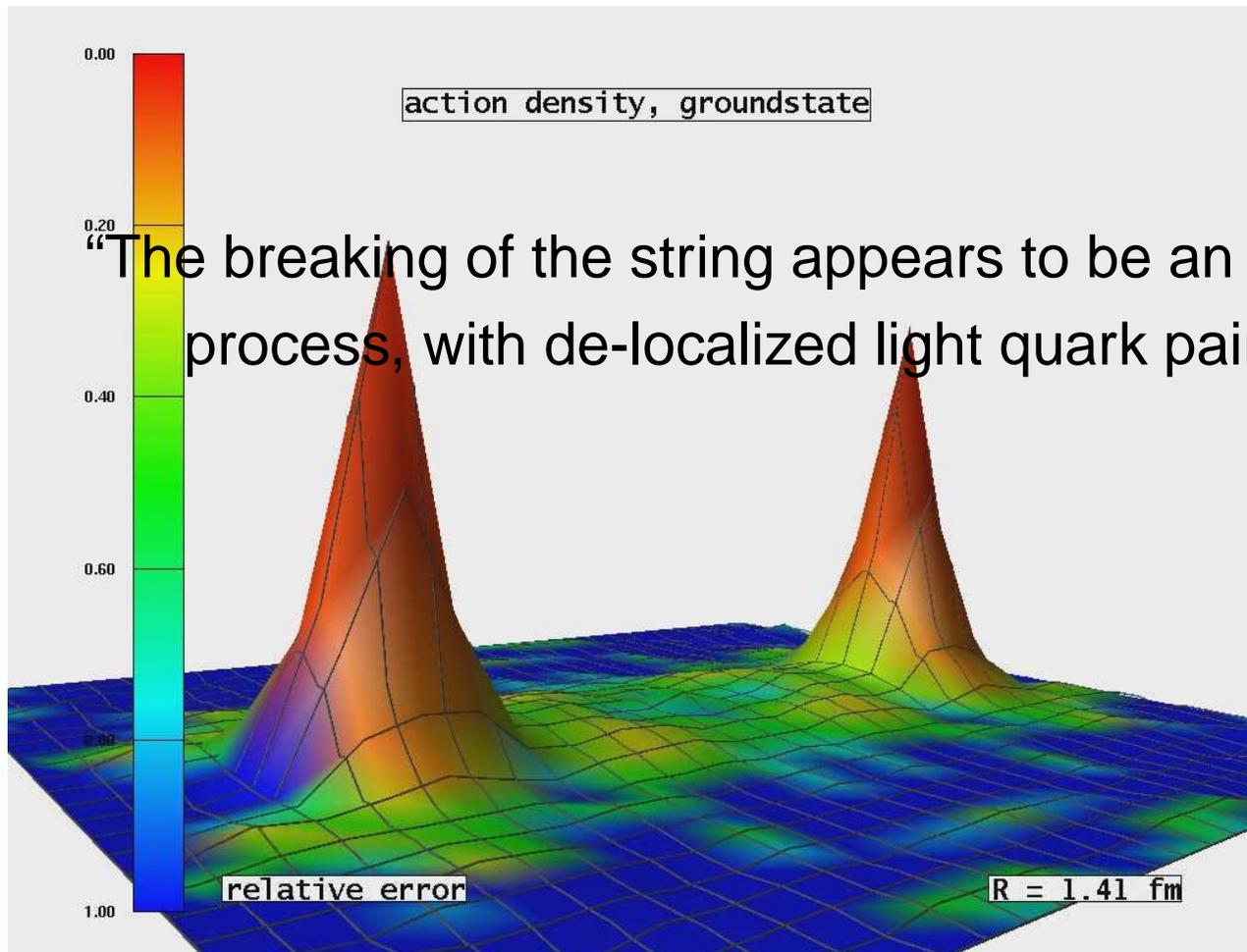
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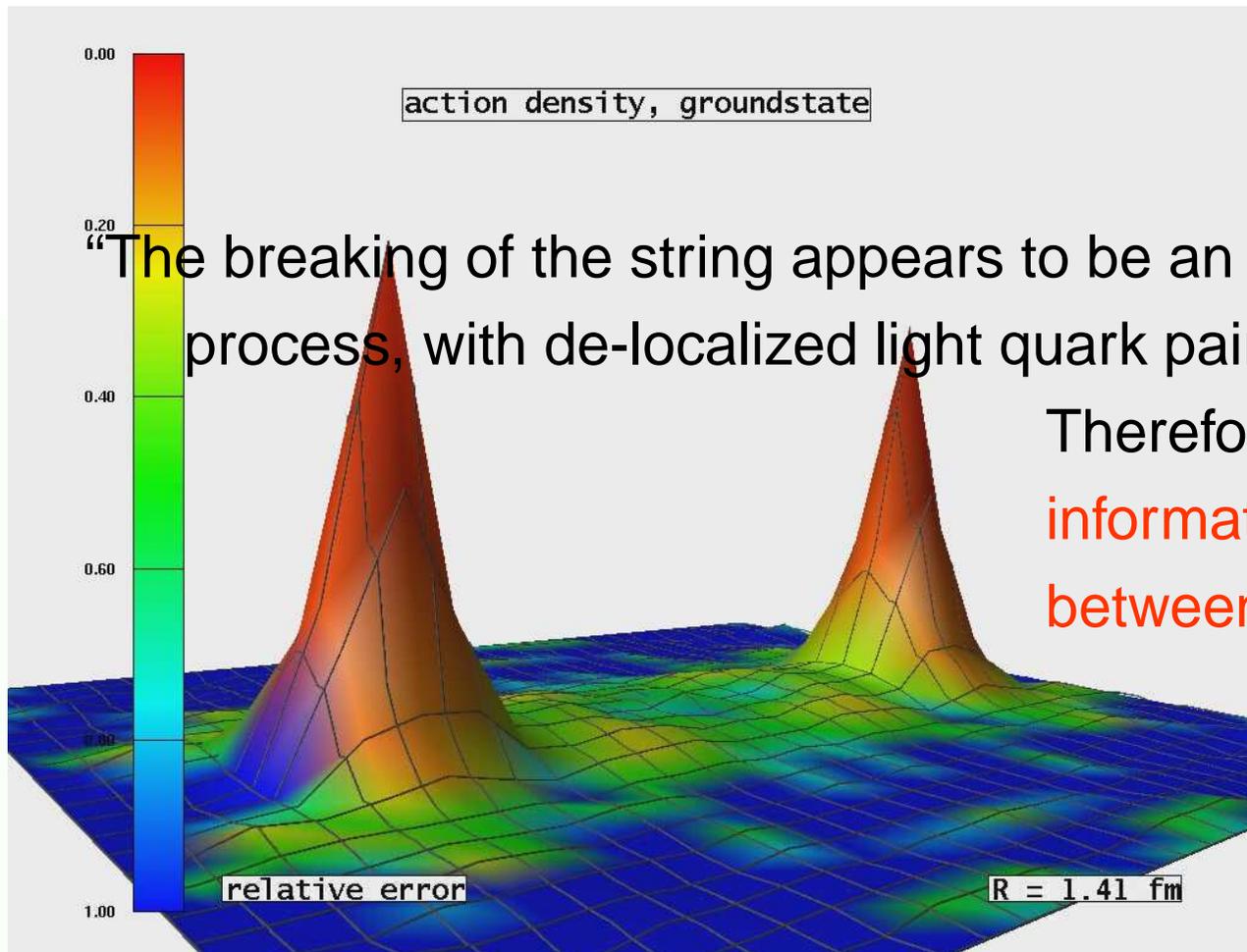
“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”



Confinement

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“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”

Therefore ... **No**
information on *potential*
between light-quarks.



What is the light-quark Long-Range Potential?



What is the light-quark Long-Range Potential?



Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD *is not related* in any simple way to the light-quark interaction.



Bethe-Salpeter Kernel



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^l(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f^l i\gamma_5 + \frac{1}{2} \lambda_f^l i\gamma_5 \mathcal{S}^{-1}(k_-) \\ - M_\zeta i\Gamma_5^l(k; P) - i\Gamma_5^l(k; P) M_\zeta$$

QFT Statement of Chiral Symmetry



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Satisfies BSE

Satisfies DSE



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- **Nontrivial** constraint





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Satisfies BSE

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Kernels very different

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- Relation **must** be preserved by truncation
- **Failure** \Rightarrow Explicit Violation of QCD's Chiral Symmetry



Persistent Challenge



[First](#)

[Contents](#)

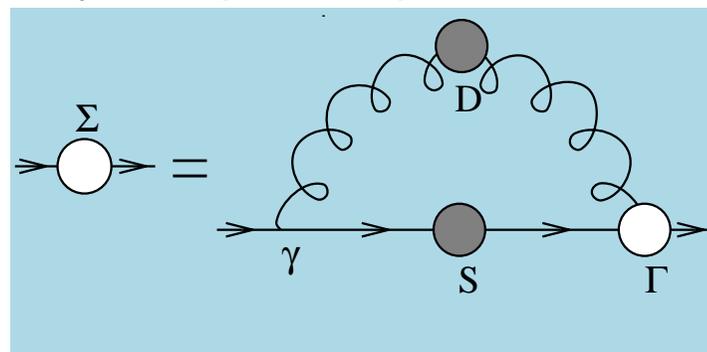
[Back](#)

[Conclusion](#)



Persistent Challenge

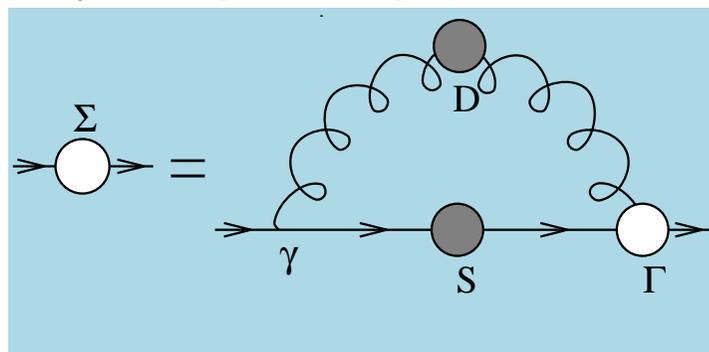
- Infinitely Many Coupled Equations





Persistent Challenge

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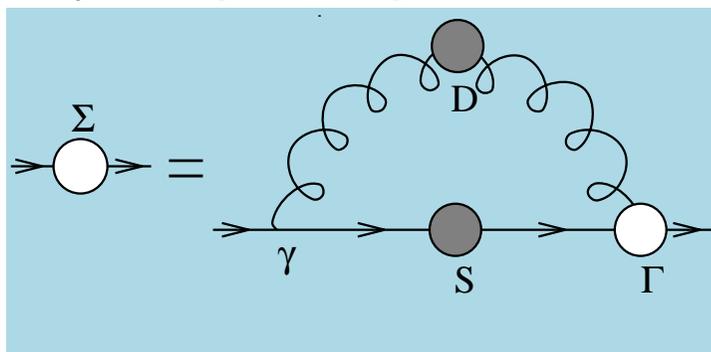
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Persistent Challenge

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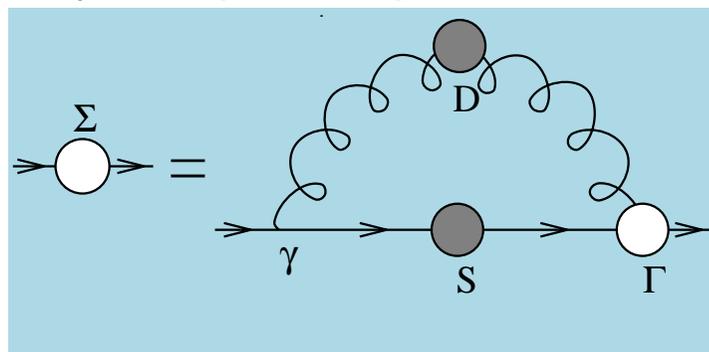
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 - Weak coupling expansion \Rightarrow Perturbation Theory





Persistent Challenge

- Infinitely Many Coupled Equations



- Coupling between equations **necessitates** truncation

- Weak coupling expansion \Rightarrow Perturbation Theory
Not useful for the nonperturbative problems
in which we're interested





Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme

H.J. Munczek Phys. Rev. D **52** (1995) 4736

Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations

A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7

Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation





Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme
- Has Enabled Proof of **EXACT** Results in QCD



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Persistent Challenge

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Radial Excitations & Chiral Symmetry



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
nu-th/9707003)

$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Radial Excitations & Chiral Symmetry

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$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

- Mass² of pseudoscalar hadron



Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
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$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

$$\mathcal{M}_H := \text{tr}_{\text{flavour}} \left[M_{(\mu)} \left\{ T^H, (T^H)^t \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2} (\lambda^4 + i\lambda^5)$



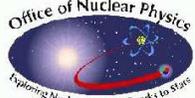
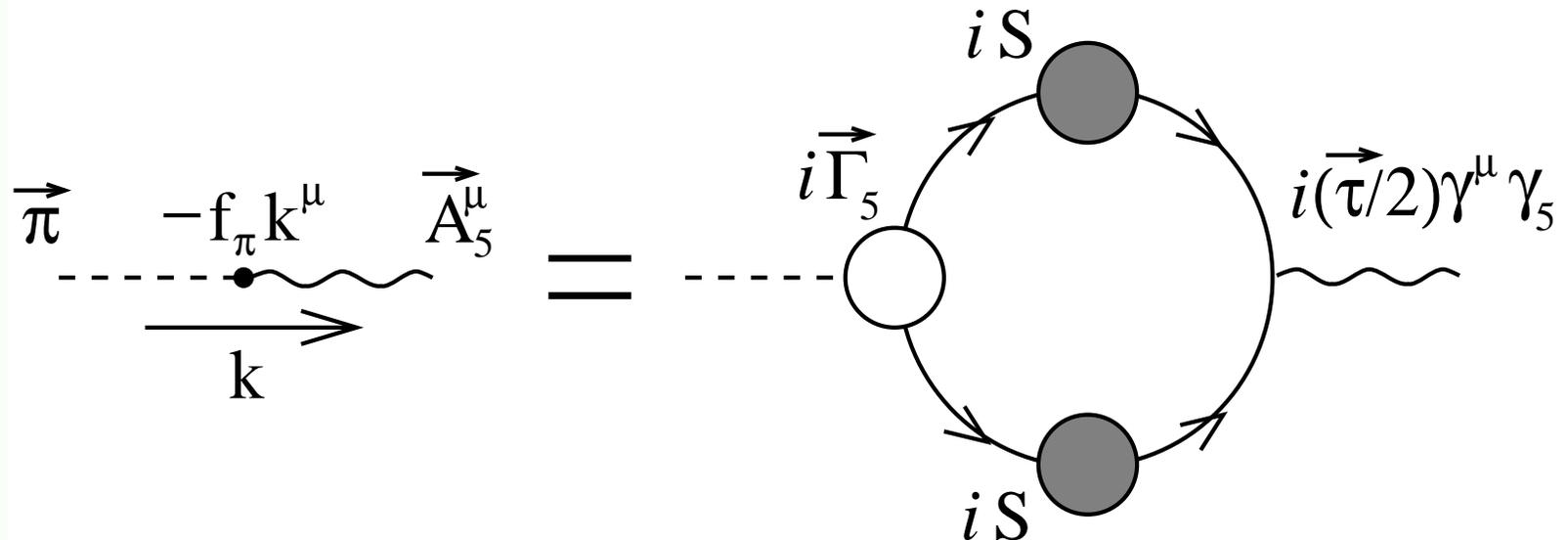
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- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



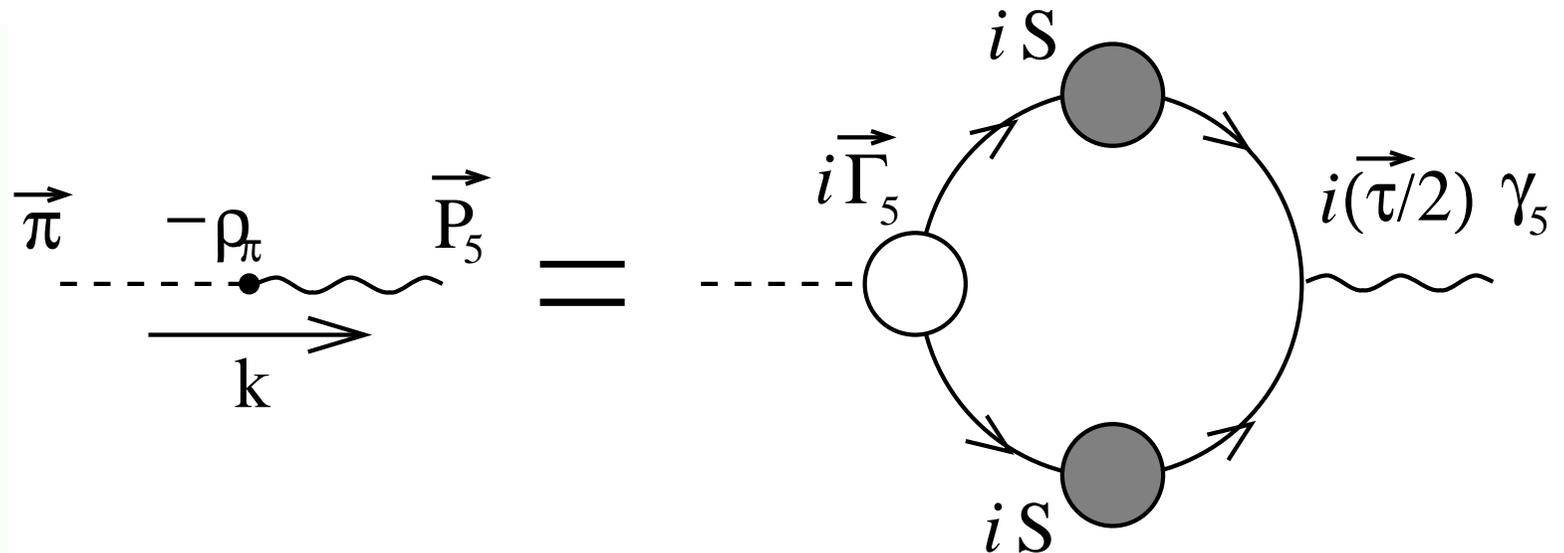
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- $f_H \rightarrow f_H^0$ & $\rho_\zeta^H \rightarrow \frac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q \dots$ GMOR relation, a corollary



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- Heavy-quark + light-quark

$\Rightarrow f_H \propto \frac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $m_H \propto m_q$

\dots QCD Proof of Potential Model result

Craig Roberts: Unifying description of mesons and baryons

Electromagnetic N-N* Transition Form Factors Workshop ... 40 - p. 15/42



Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

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[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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ALL pseudoscalar mesons **except $\pi(140)$** in **chiral limit**
- **Dynamical Chiral Symmetry Breaking**
– Goldstone’s Theorem –
impacts upon **every pseudoscalar meson**



Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Radial Excitations & Lattice-QCD

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- *When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.*



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Radial Excitations & Lattice-QCD

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- When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.
- CLEO: $\tau \rightarrow \pi(1300) + \nu_\tau$
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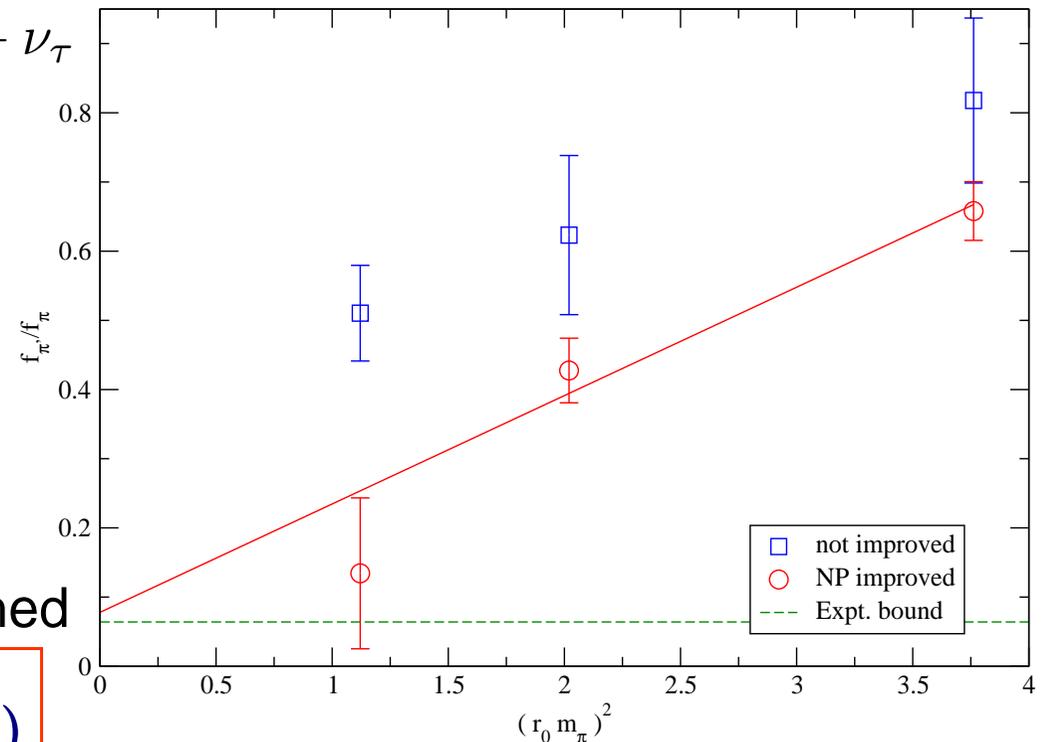
- Lattice-QCD check:

$$16^3 \times 32,$$

$$a \sim 0.1 \text{ fm},$$

two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078 (93)$$



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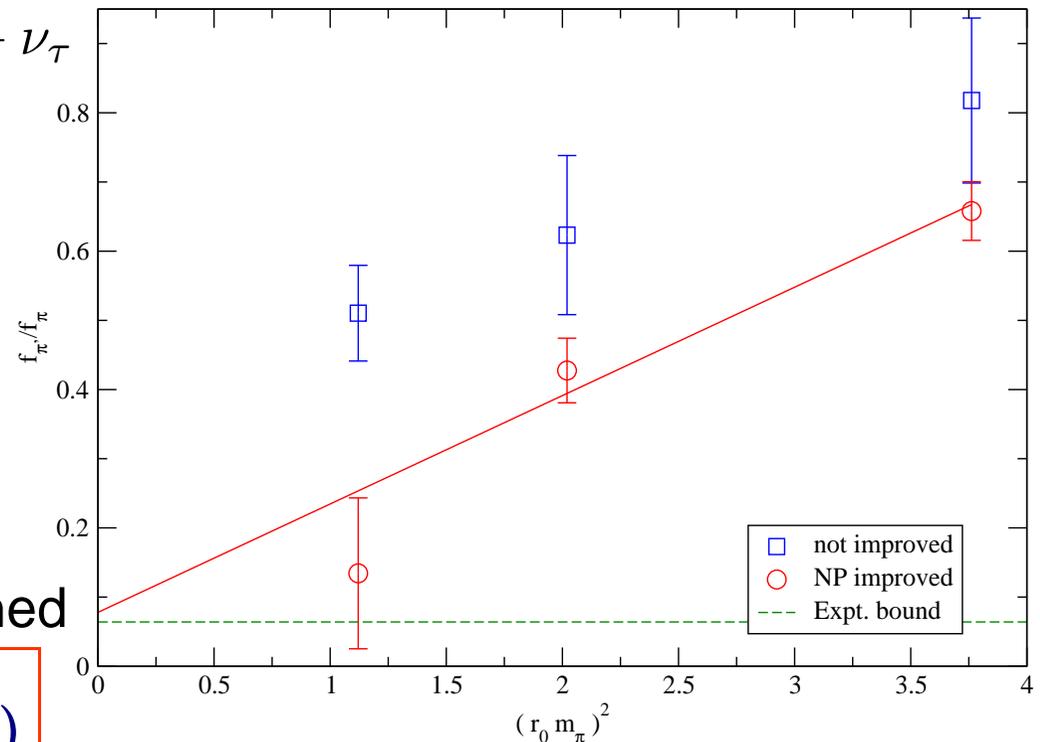
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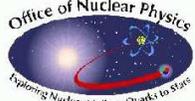
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- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)



Radial Excitations & Lattice-QCD

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he-la/0607032

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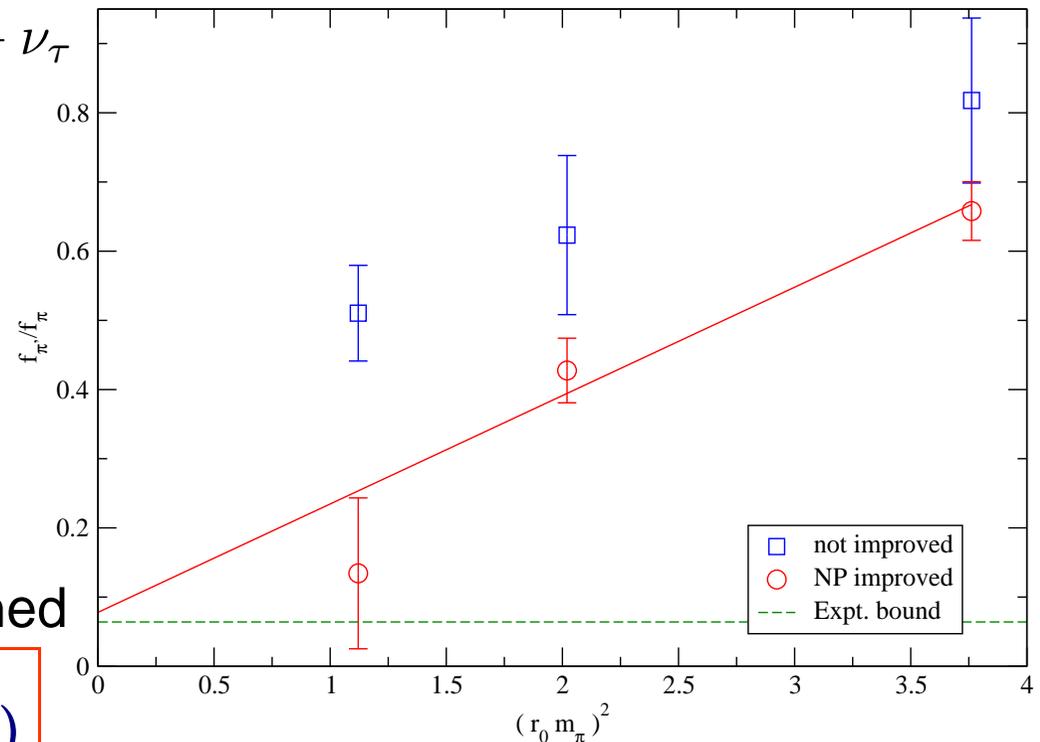
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- The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.



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Pion Form Factor

Procedure Now Straightforward



[First](#)

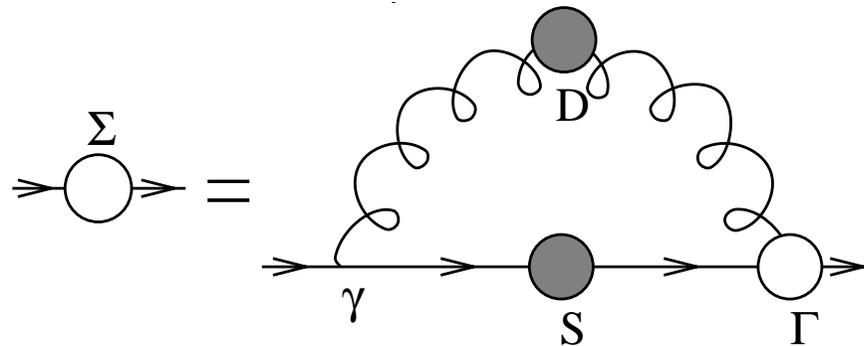
[Contents](#)

[Back](#)

[Conclusion](#)

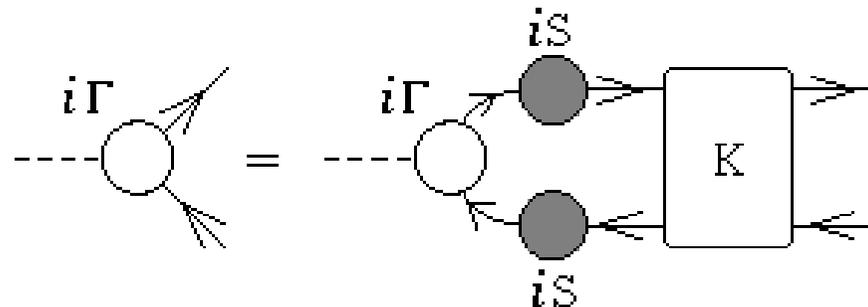
Pion Form Factor

- Solve Gap Equation
⇒ Dressed-Quark Propagator, $S(p)$



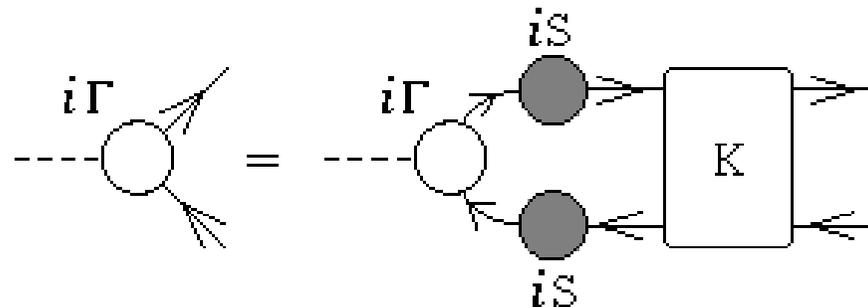
Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, K
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π



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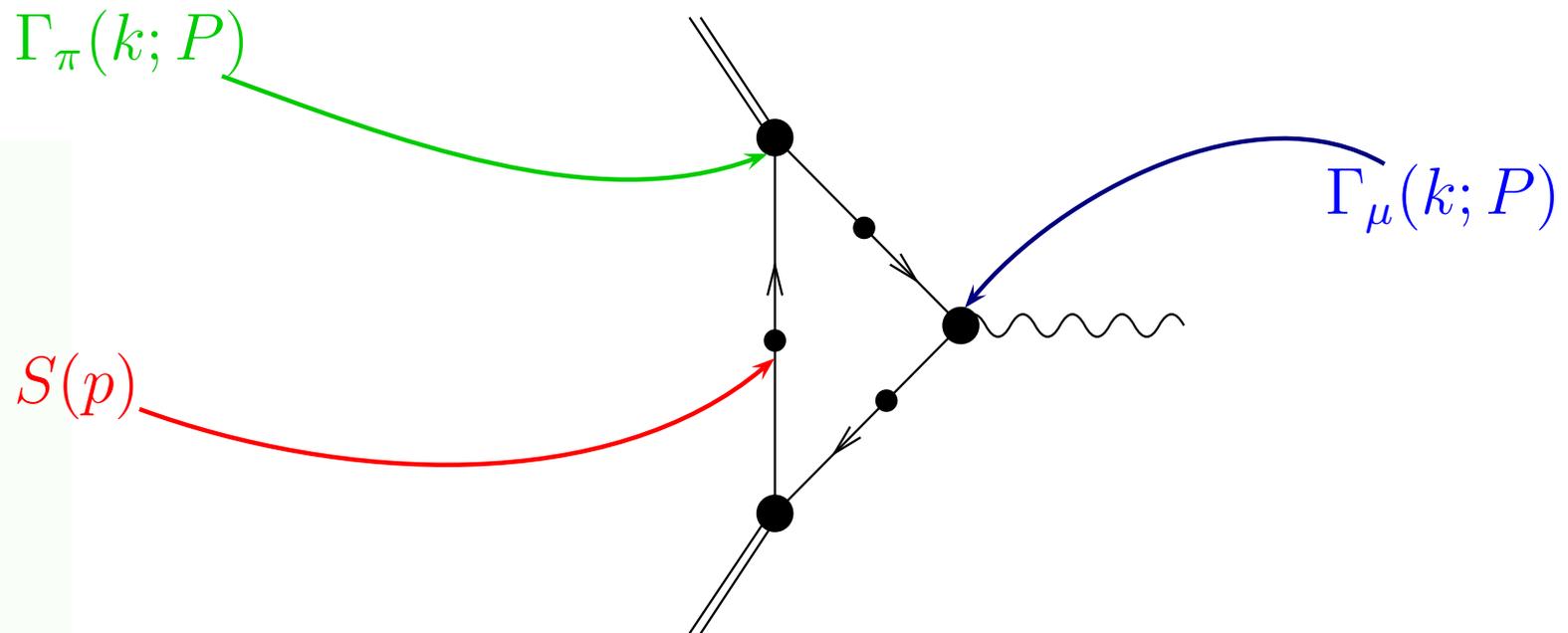


- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Photon Vertex, Γ_μ



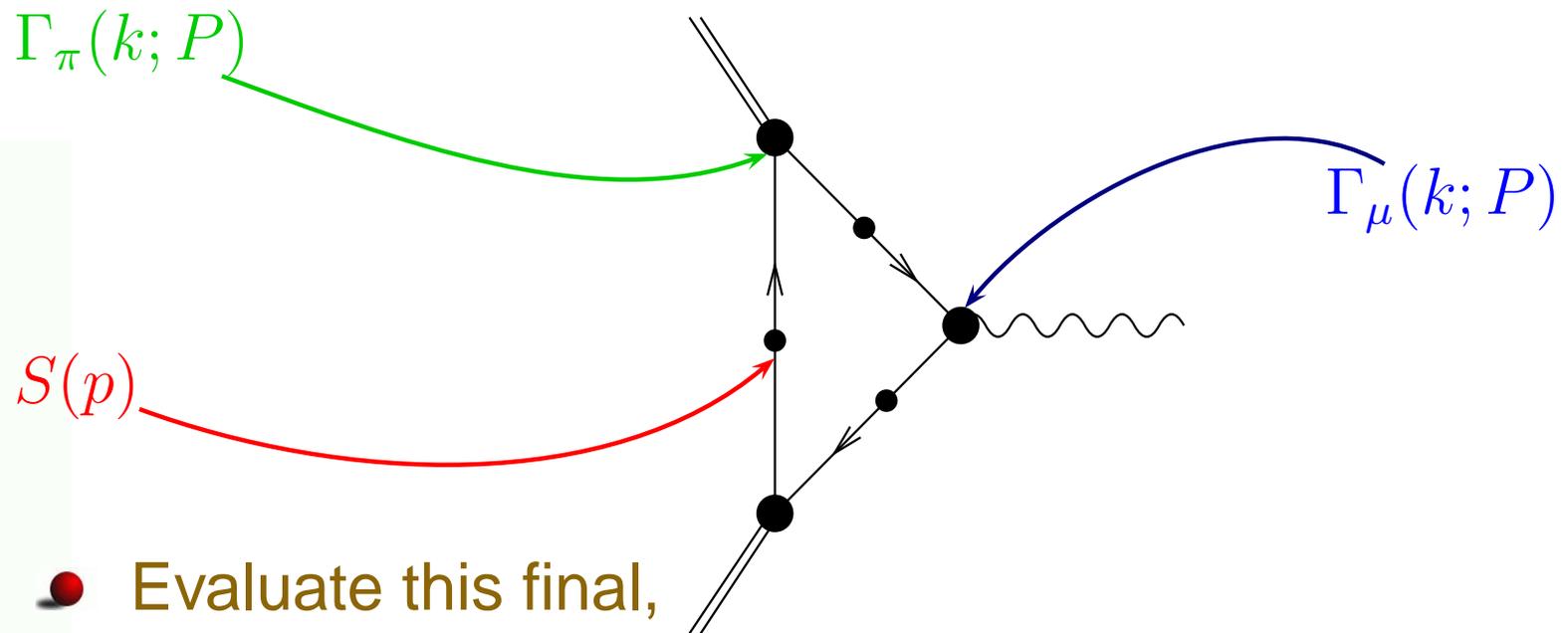
Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



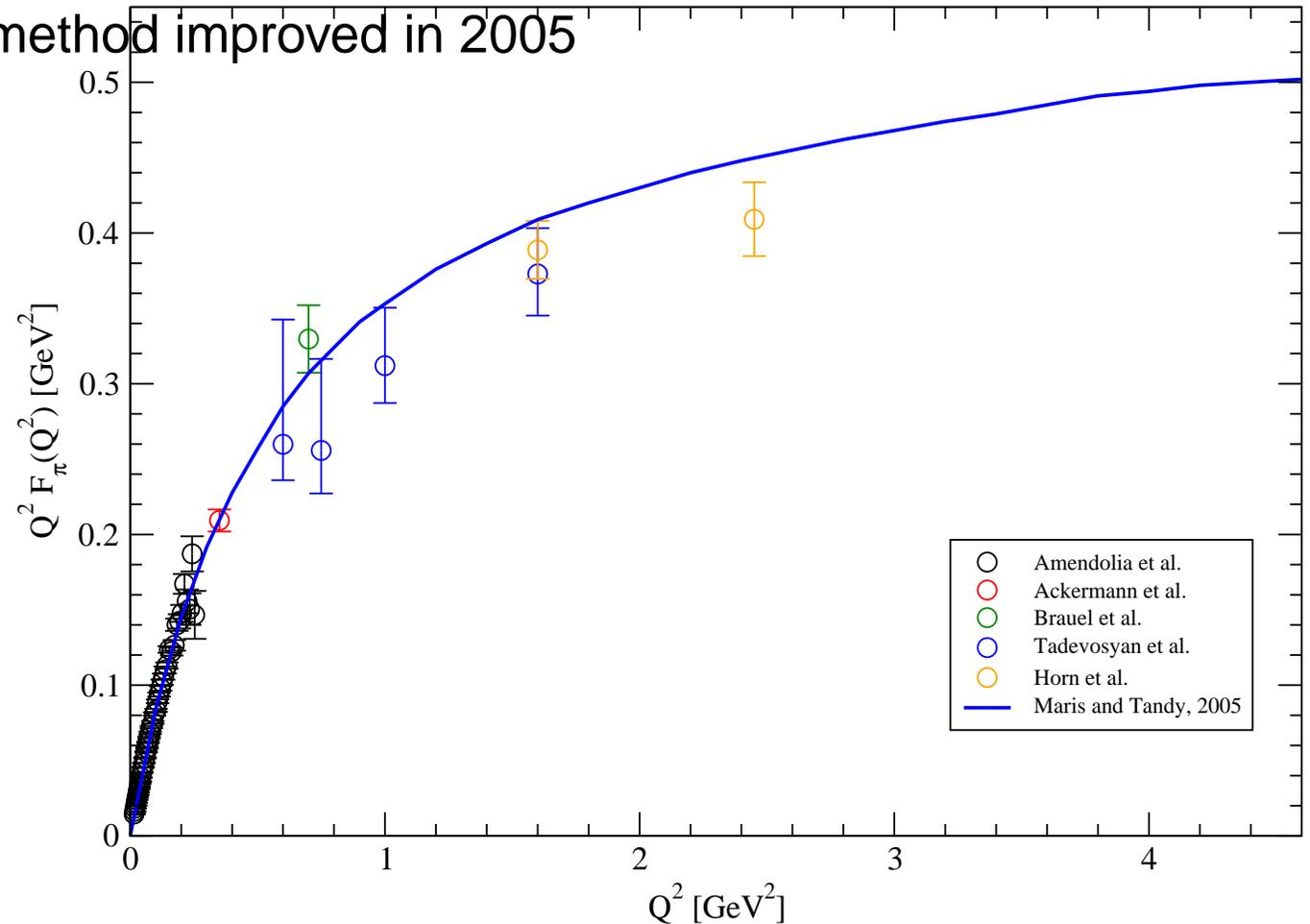
- Evaluate this final, three-dimensional integral



Calculated Pion Form Factor

Calculation first published in 1999; No Parameters Varied

Numerical method improved in 2005

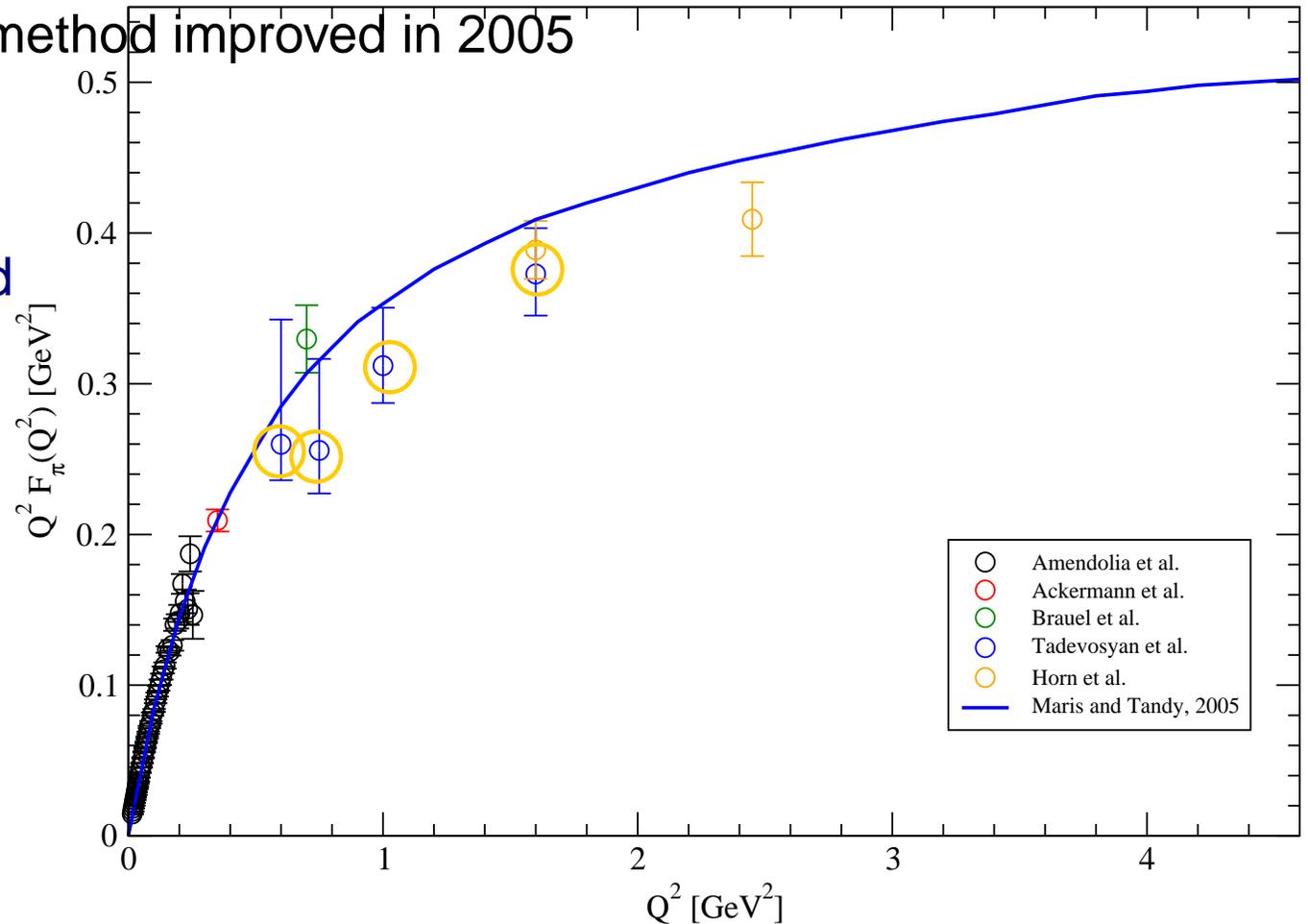


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Calculation first published in 1999; No Parameters Varied

Numerical method improved in 2005

Data published
in 2001.
Subsequently
revised





Timelike Pion Form Factor



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[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Ab initio calculation into
timelike region. Deeper than
ground-state ρ -meson pole

Timelike Pion Form Factor



[First](#)

[Contents](#)

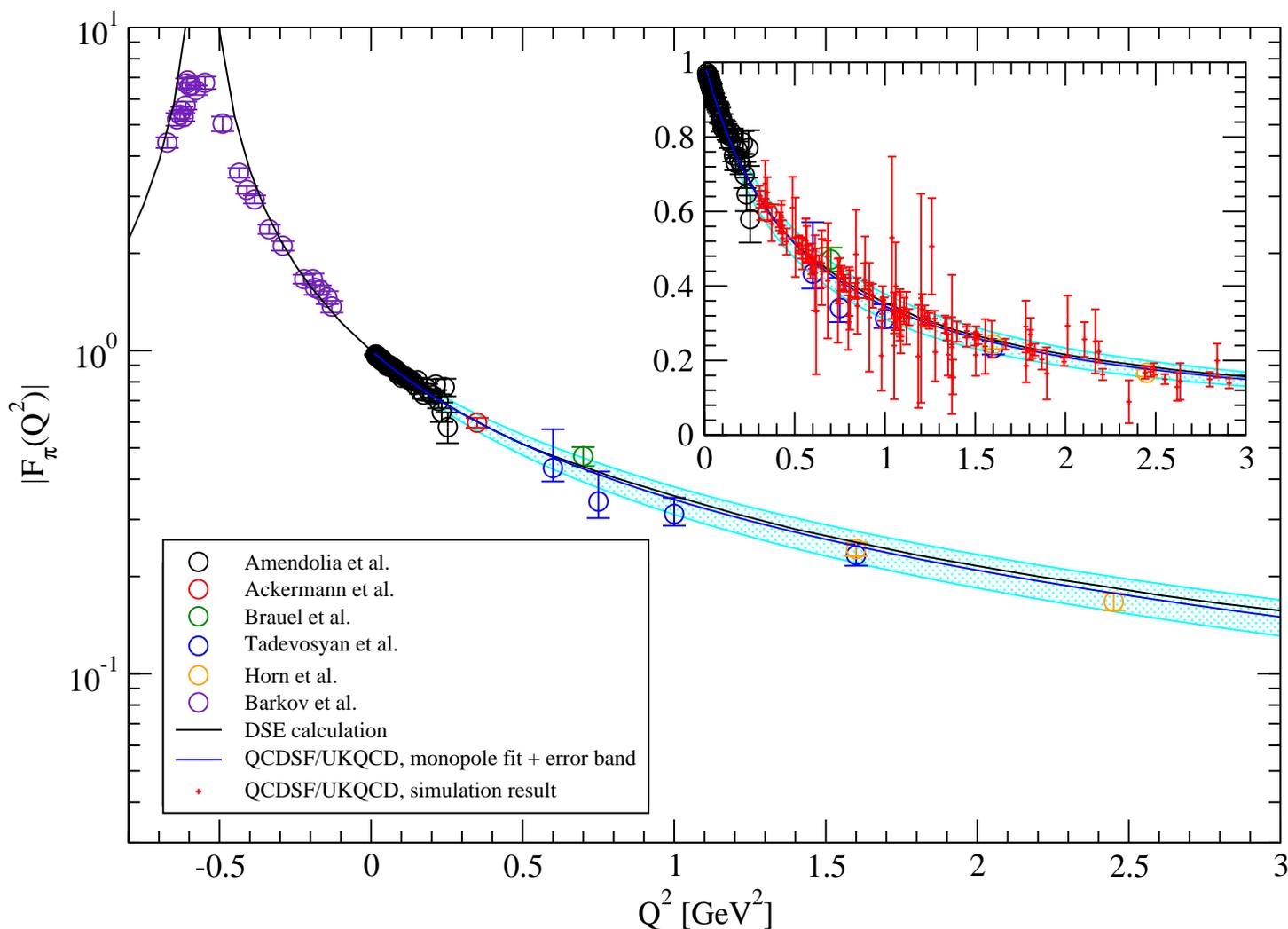
[Back](#)

[Conclusion](#)



Ab initio calculation into timelike region. Deeper than ground-state ρ -meson pole

Timelike Pion Form Factor

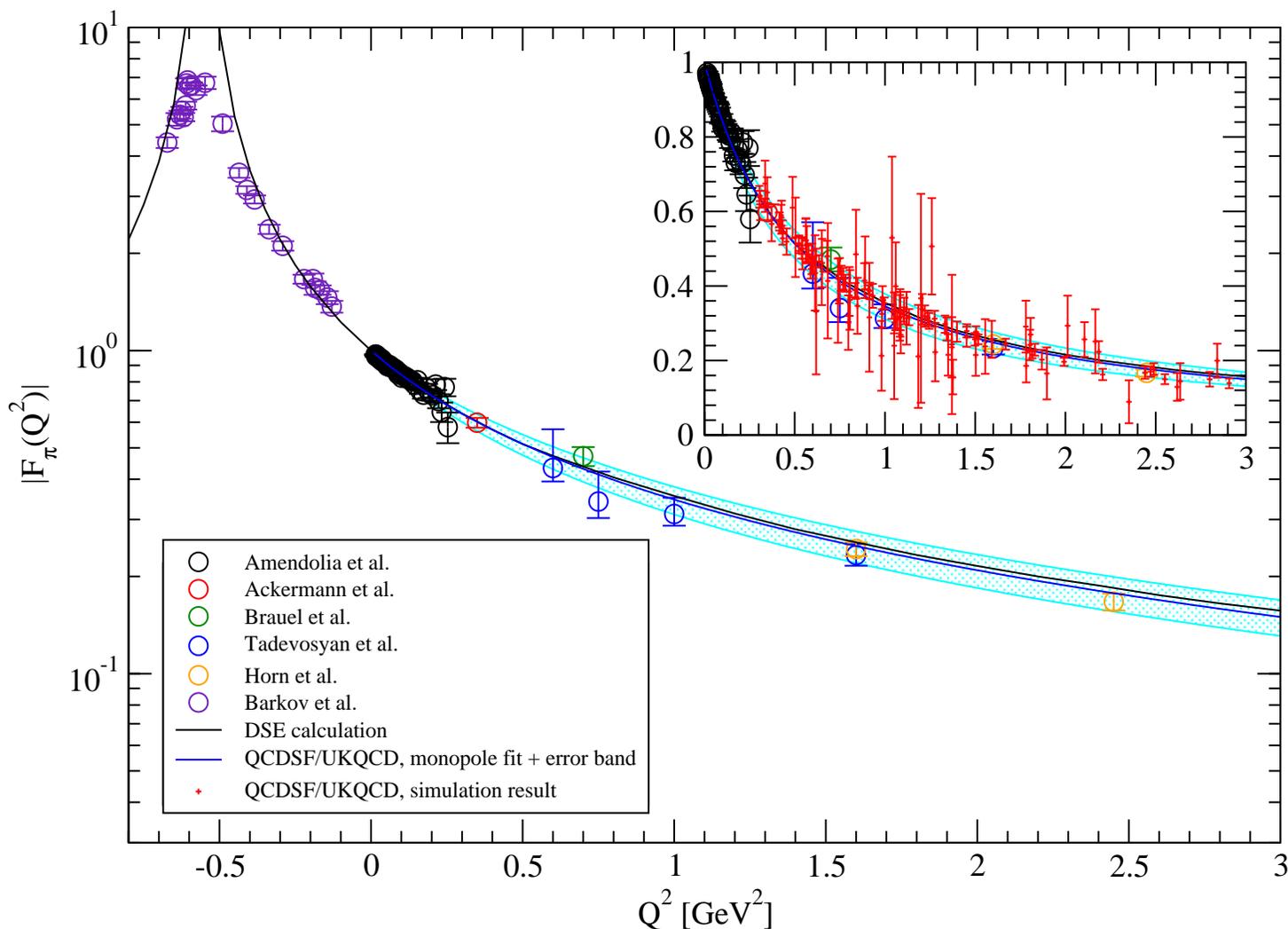




Ab initio calculation into timelike region. Deeper than ground-state ρ -meson pole

Timelike Pion Form Factor

ρ -meson not put in “by hand” – generated dynamically as a bound-state of dressed-quark and dressed-antiquark



Pion Form Factors



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

- There is a sense in which it is easy to fabricate a model that can *reproduce* the elastic electromagnetic pion form factor



- There is a sense in which it is easy to fabricate a model that can *reproduce* the elastic electromagnetic pion form factor
- However, a **veracious description** of the pion will **simultaneously predict** the elastic electromagnetic form factor, $F_\pi(Q^2)$ AND the $\gamma^*\pi \rightarrow \gamma$ transition form factor



Infidelity without simultaneity

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- The latter is connected with the **Abelian anomaly** – therefore **fundamentally connected** with **chiral symmetry and its dynamical breaking** – no mere model can successfully describe this without fine tuning

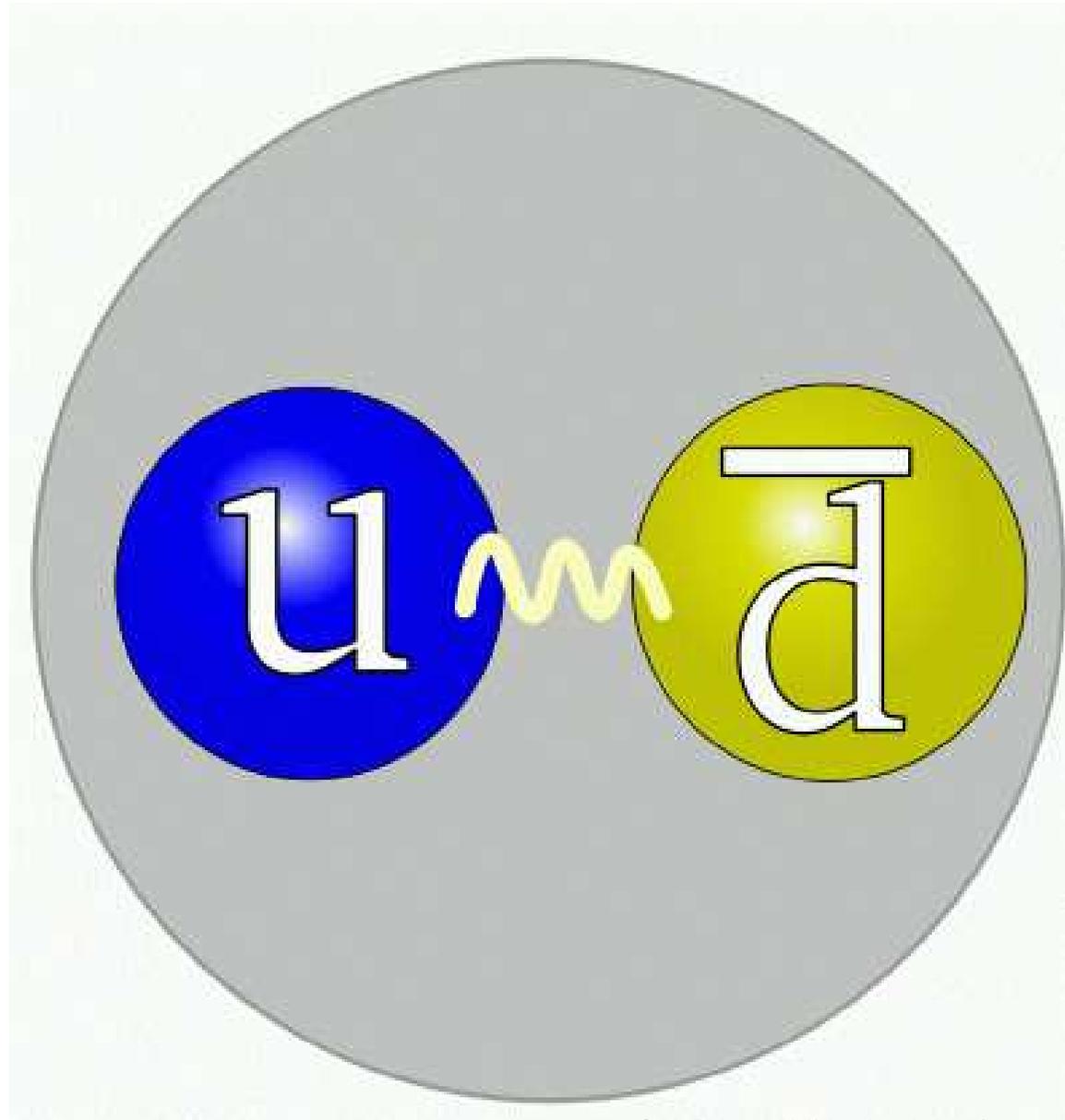


Infidelity without simultaneity

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- The latter is connected with the **Abelian anomaly** – therefore **fundamentally connected** with **chiral symmetry and its dynamical breaking** – no mere model can successfully describe this without fine tuning
- Must similarly require prediction of $\gamma^* \pi \rightarrow \pi\pi$ and all other anomalous processes

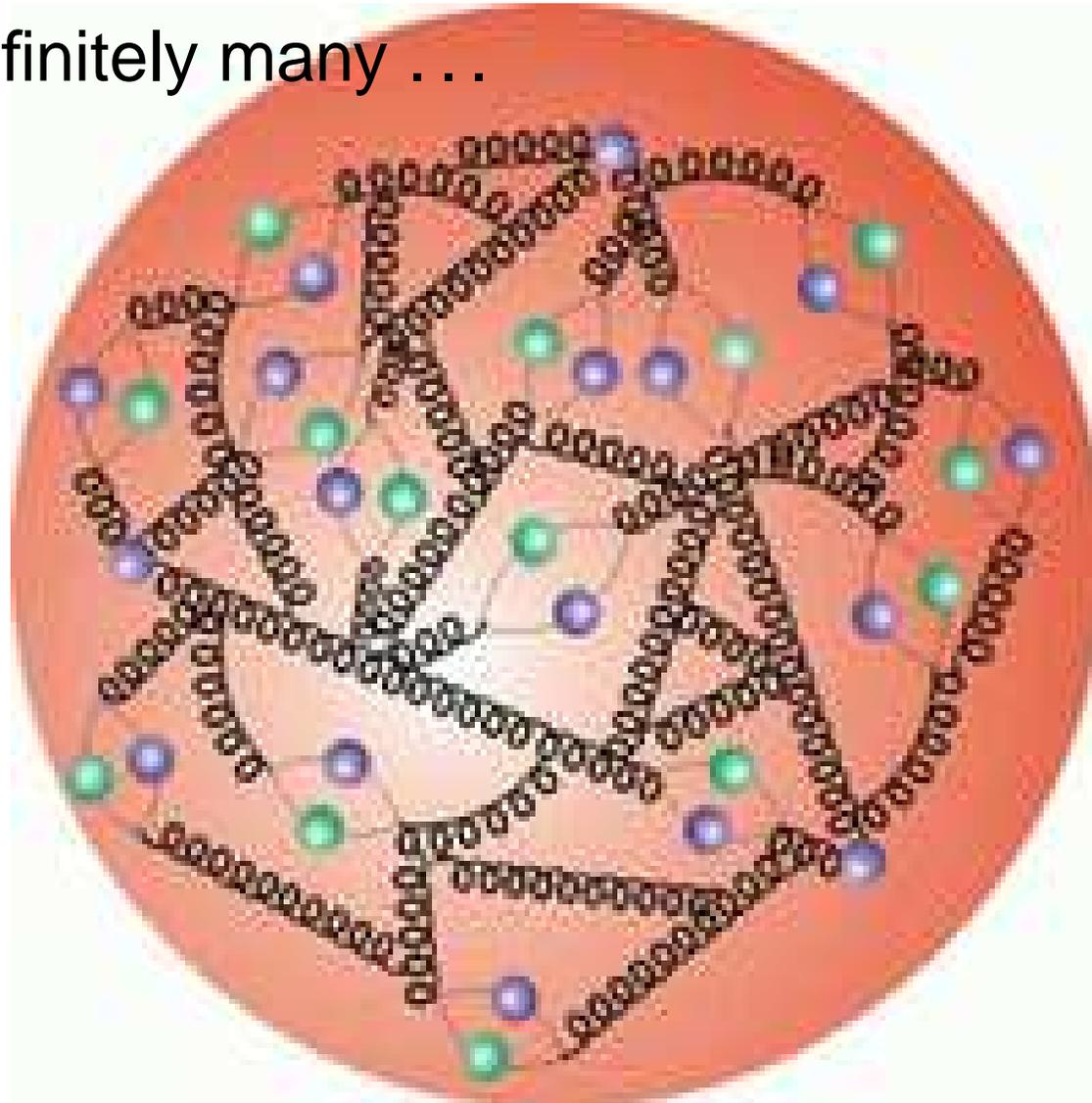


Answer for the pion



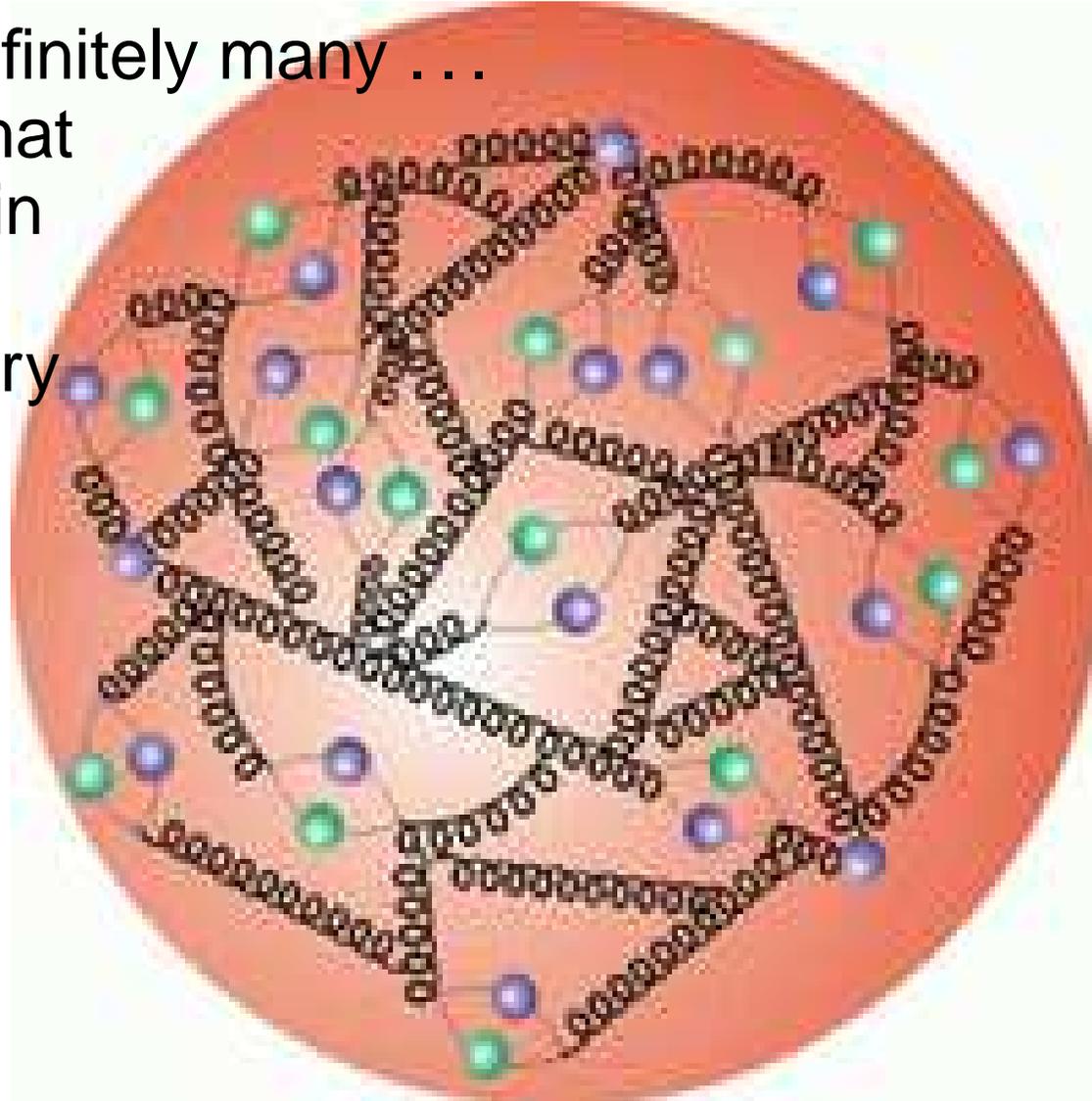
Answer for the pion

Two \rightarrow Infinitely many ...



Answer for the pion

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Handle that
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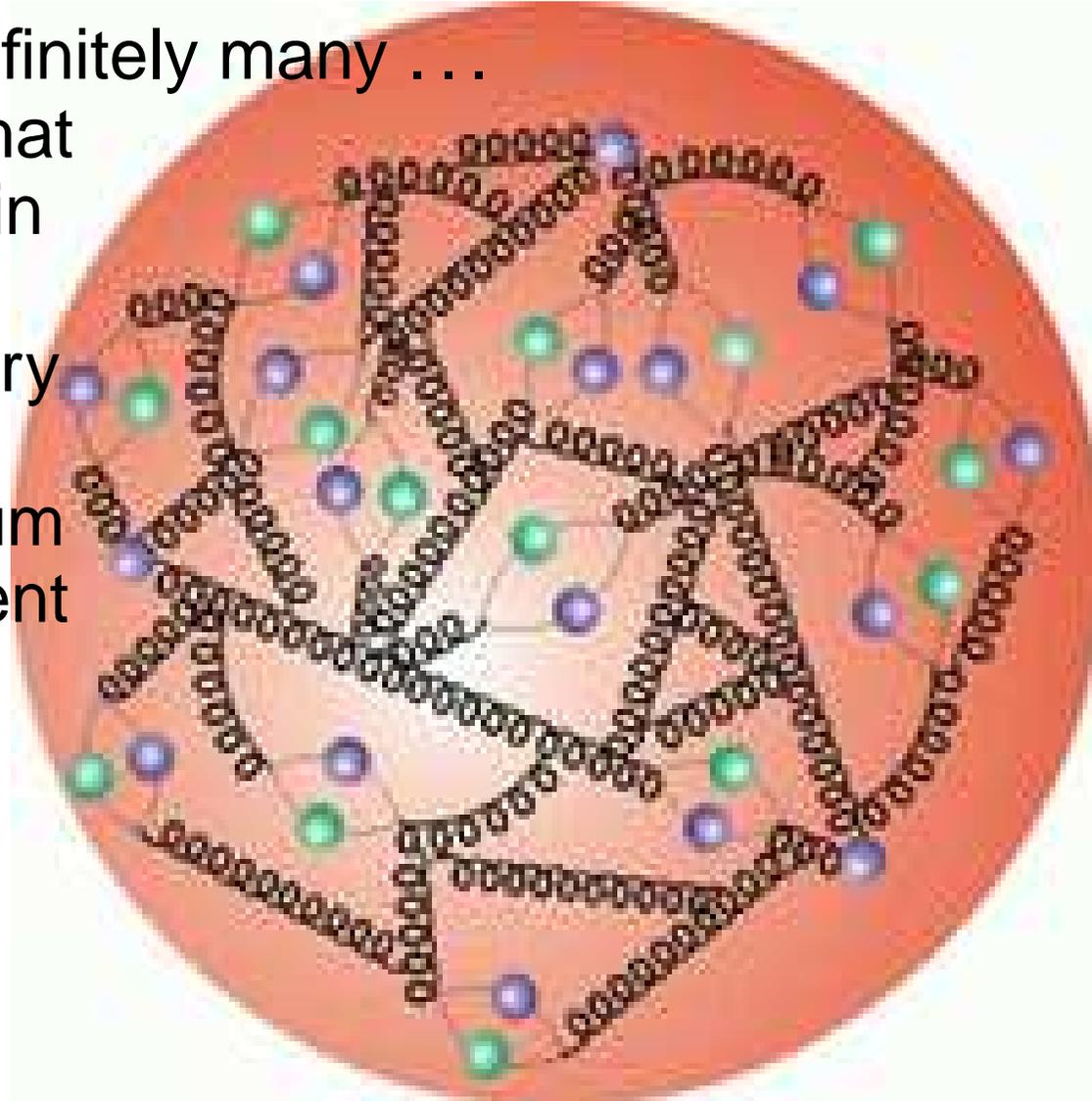


Answer for the pion

Two \rightarrow Infinitely many ...

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...
momentum-dependent dressing



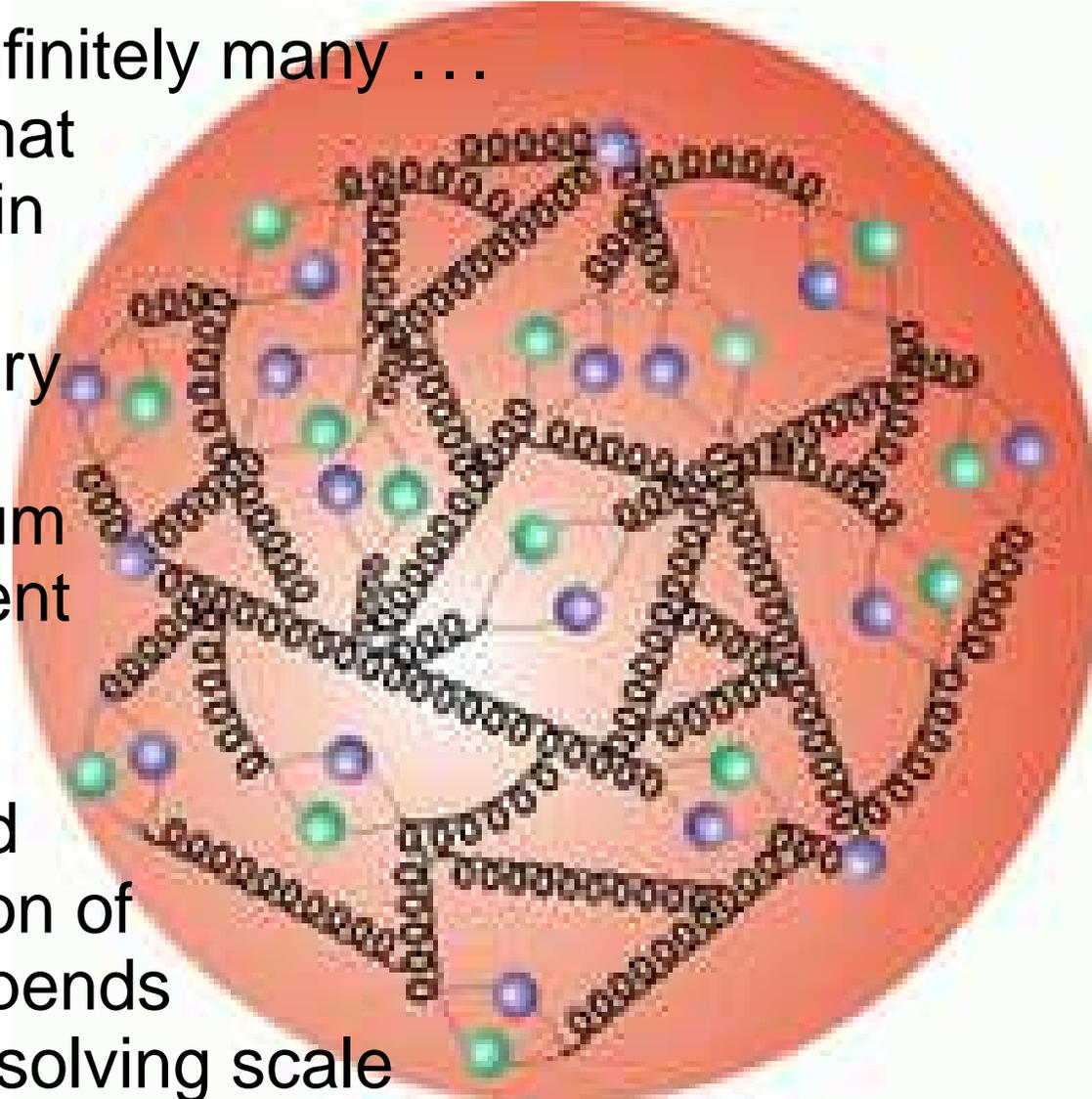
Answer for the pion

Two \rightarrow Infinitely many ...

Handle that properly in quantum field theory

...
momentum-dependent dressing

...
perceived distribution of mass depends on the resolving scale





[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

New Challenges



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

New Challenges

- **Next Steps . . .** Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.



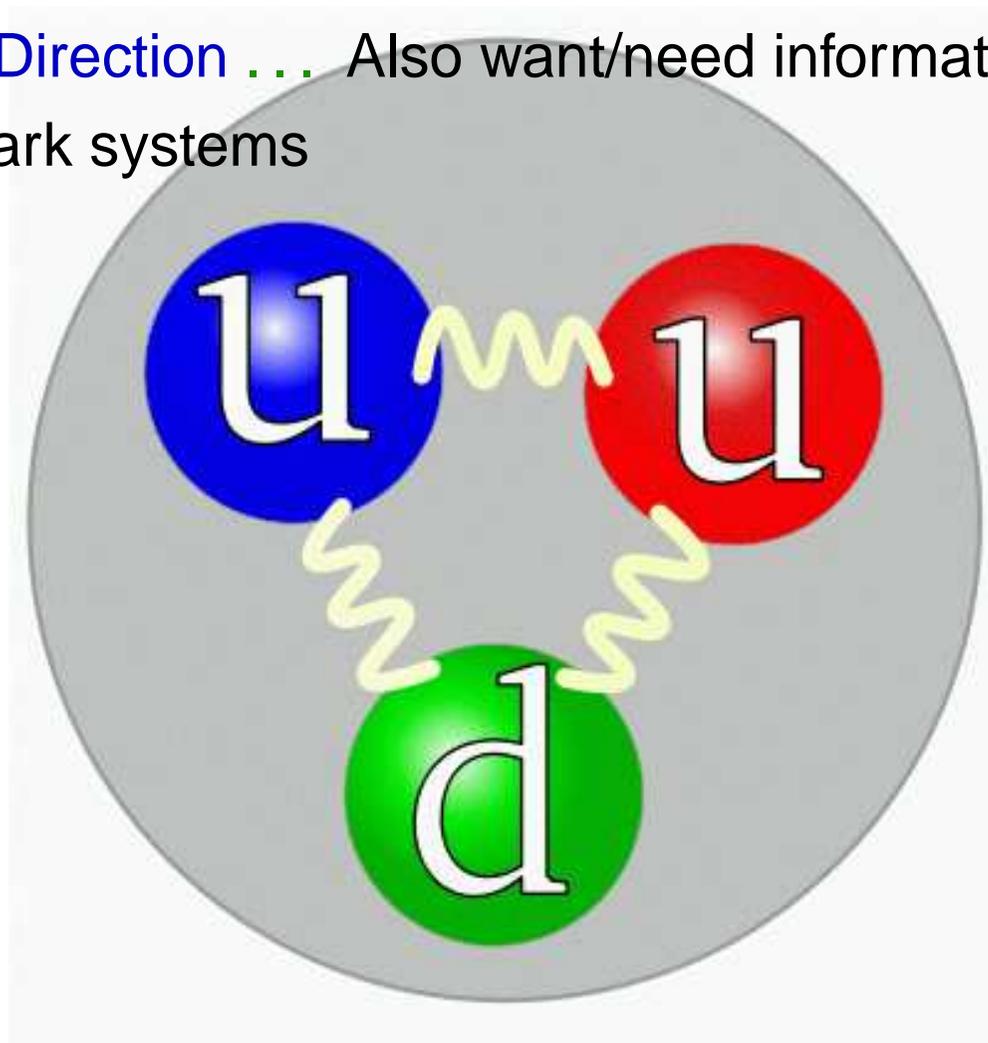
New Challenges

- **Next Steps** . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
- Move on to the problem of a **symmetry preserving** treatment of hybrids and exotics.



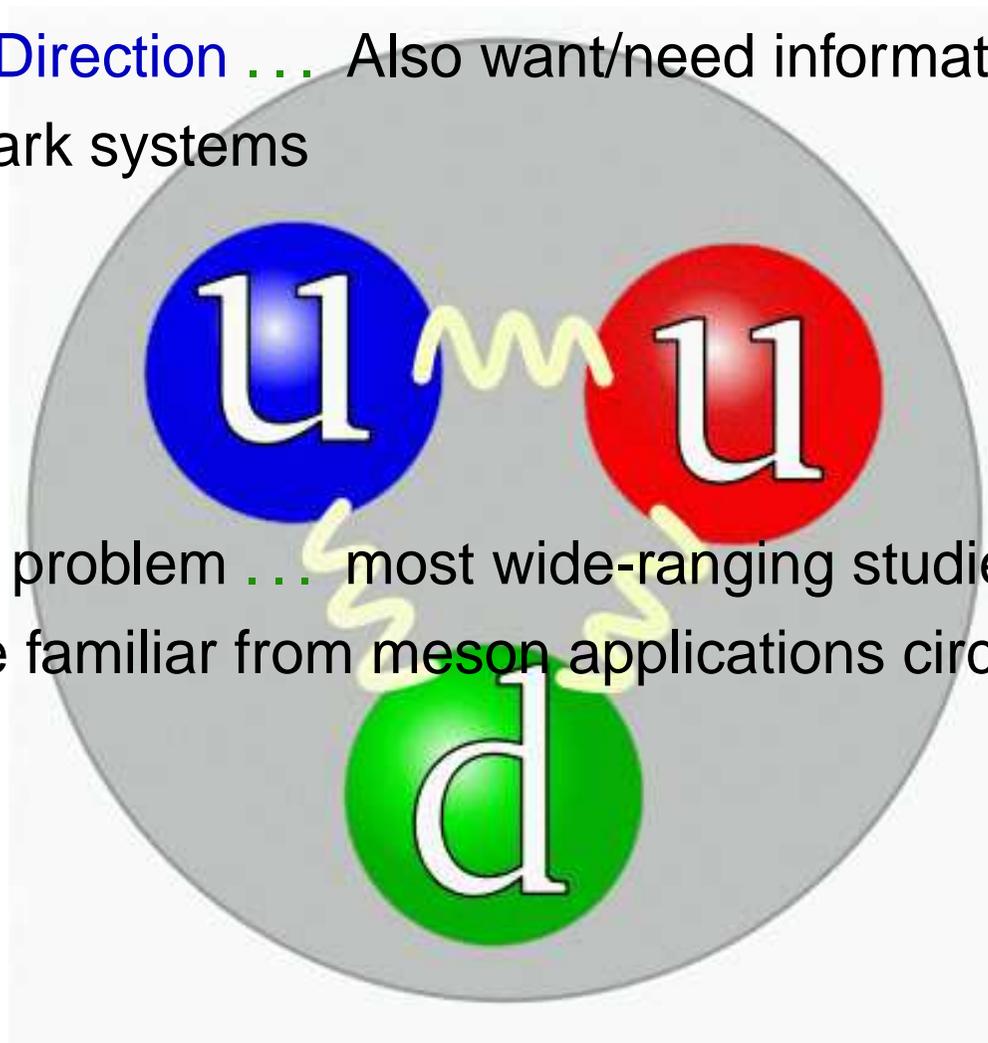
New Challenges

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New Challenges

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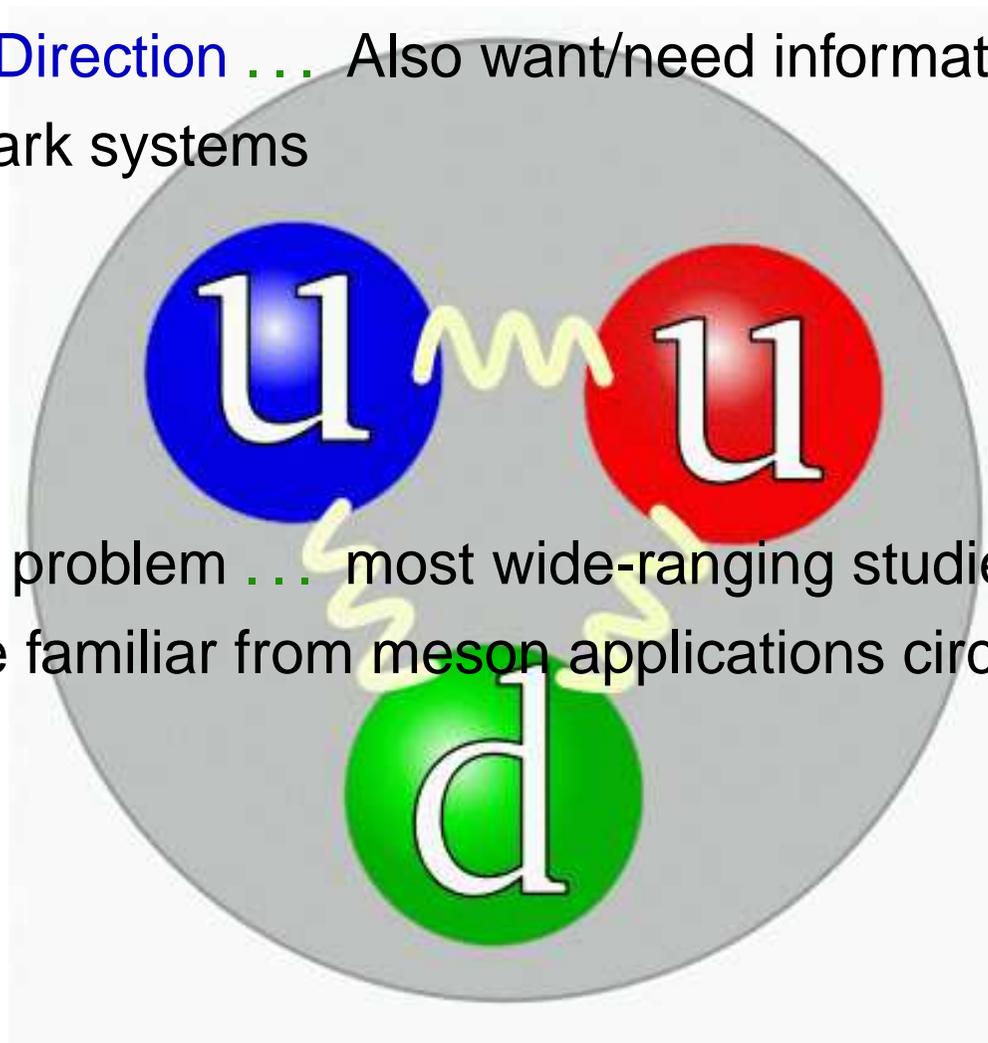


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New Challenges

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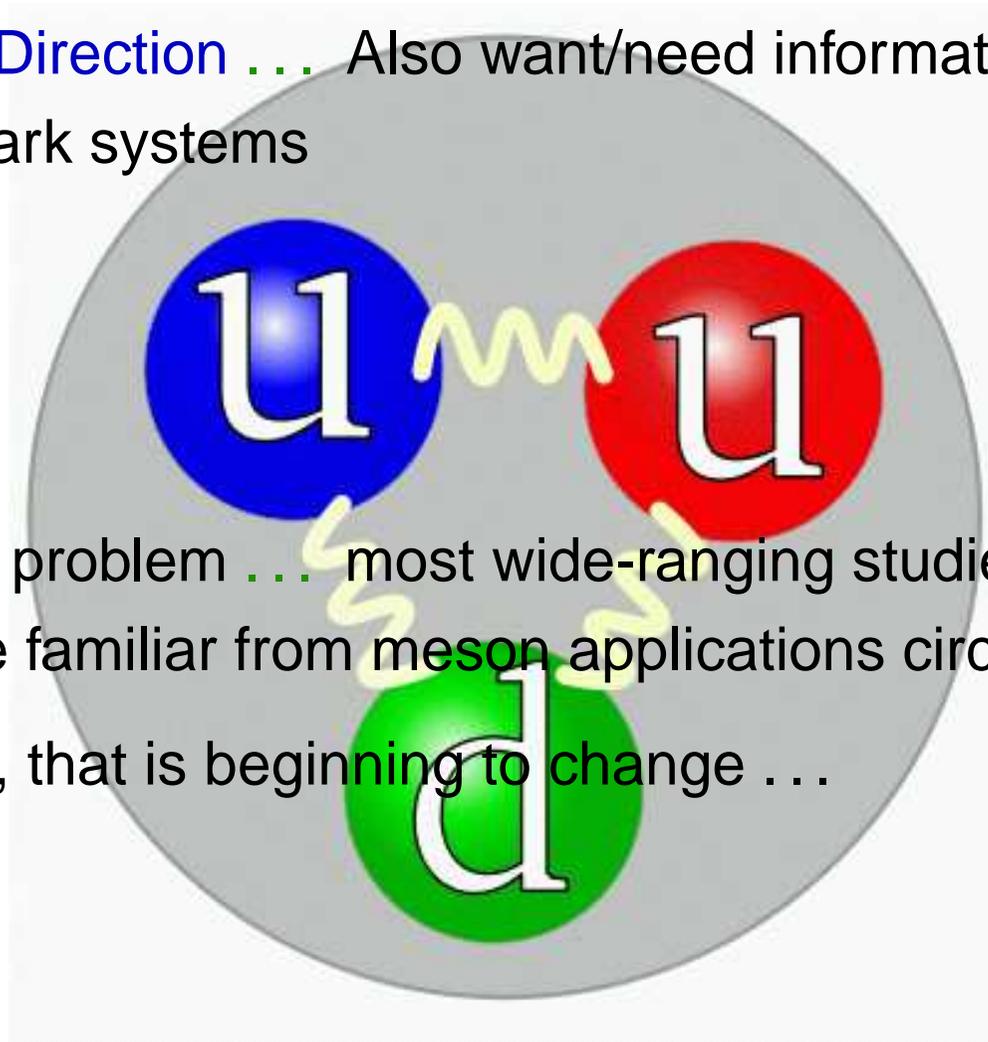
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- **Namely . . . Model-building and Phenomenology, constrained** by the **DSE results** outlined already.



New Challenges

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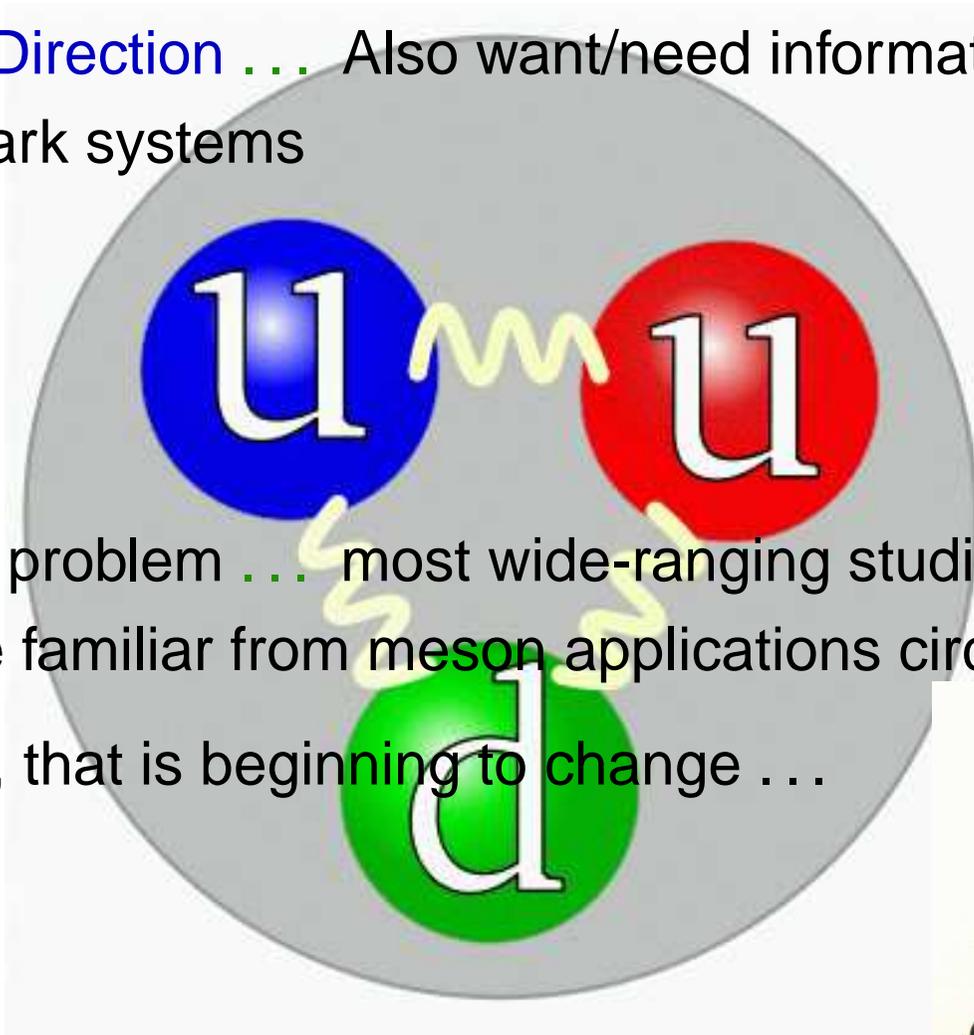


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New Challenges

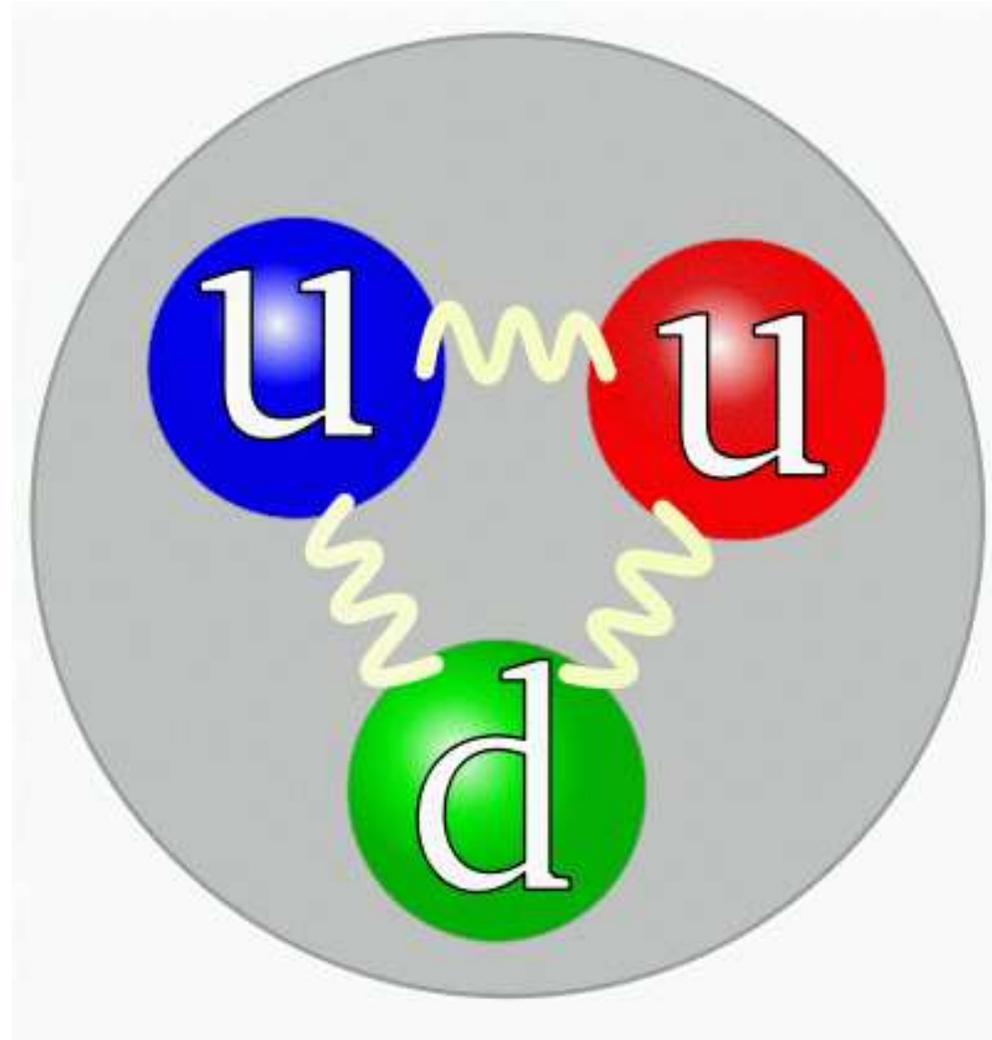
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Nucleon ... Three-body Problem?



[First](#)

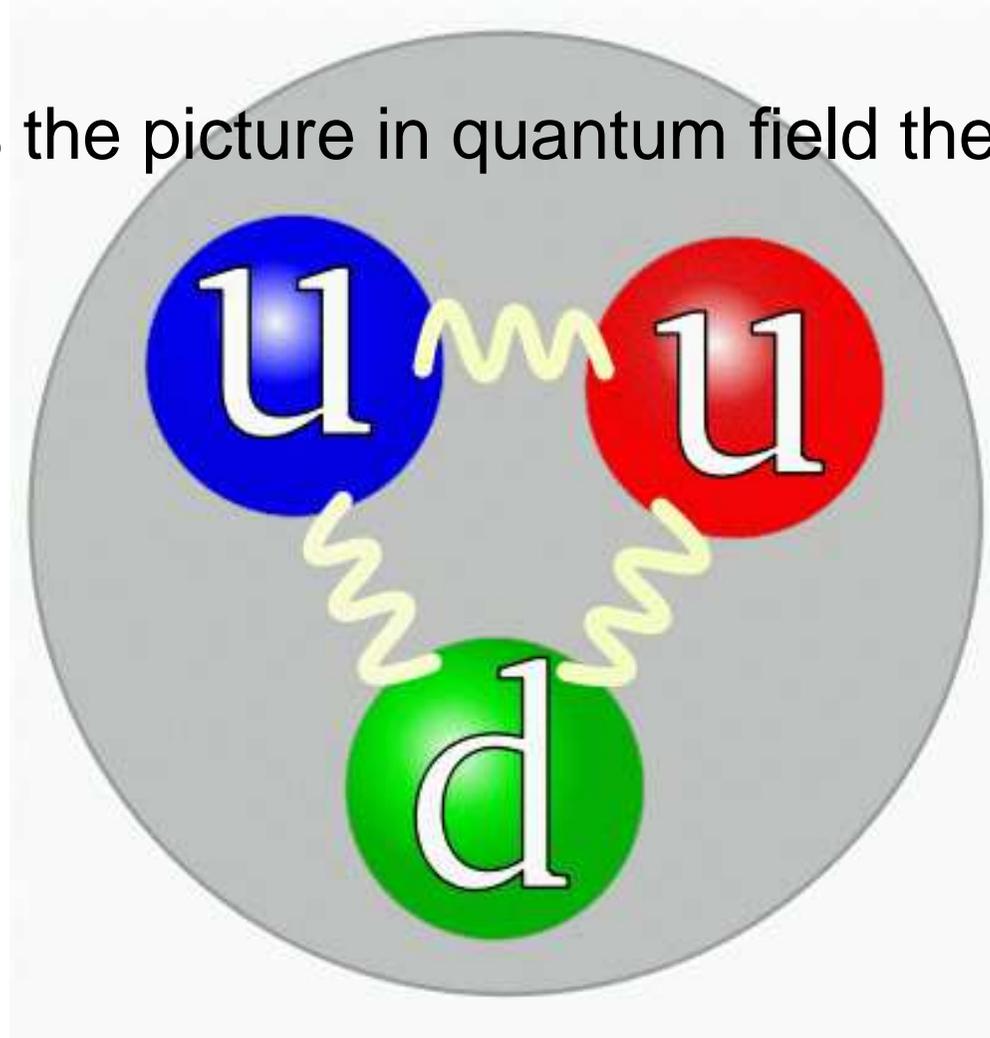
[Contents](#)

[Back](#)

[Conclusion](#)

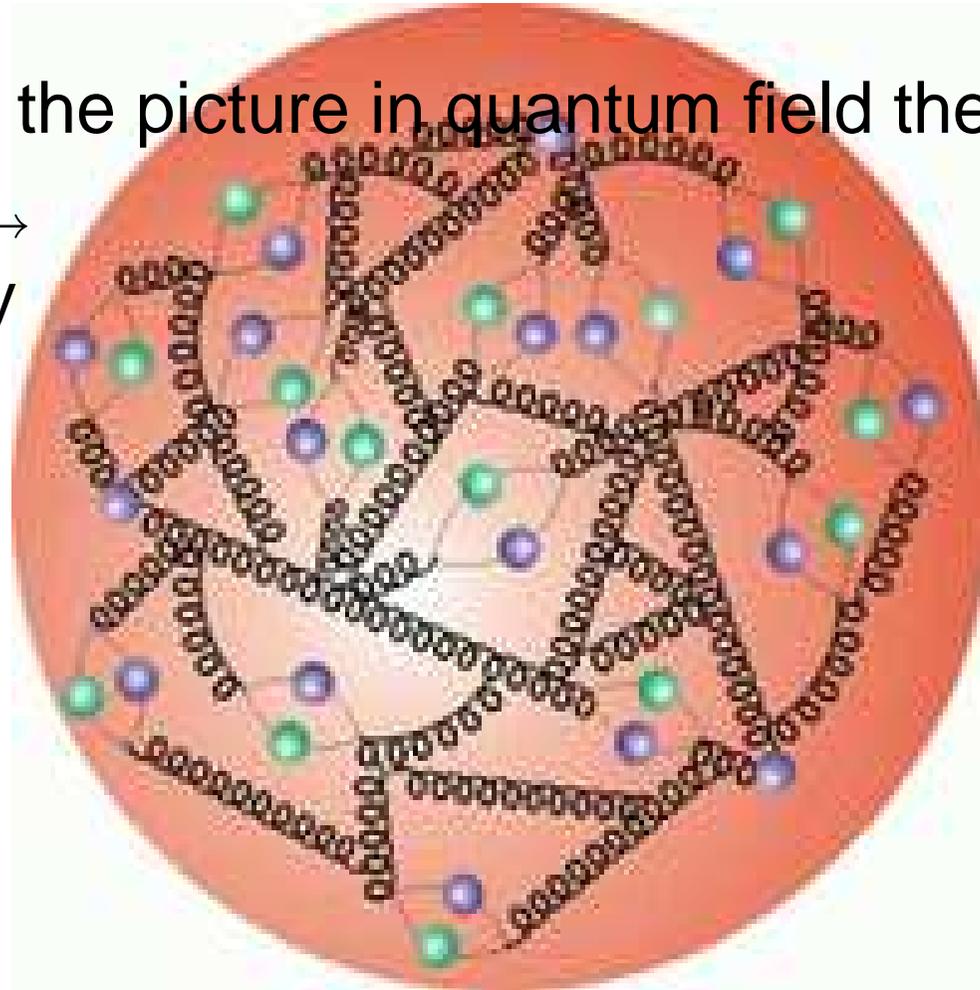
Nucleon ... Three-body Problem?

- What is the picture in quantum field theory?



Nucleon ... Three-body Problem?

- What is the picture in quantum field theory?
- Three → infinitely many!



Unifying Study of Mesons and Baryons



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Unifying Study of Mesons and Baryons

- How does one incorporate dressed-quark mass function, $M(p^2)$, in study of baryons?



Unifying Study of Mesons and Baryons

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Unifying Study of Mesons and Baryons

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 - Residue is proportional to nucleon's Faddeev amplitude
 - Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
 - Tractable equation is founded on observation that an interaction which describes colour-singlet mesons also generates quark-quark (diquark) correlations in the colour- $\bar{3}$ (antitriplet) channel



Faddeev equation



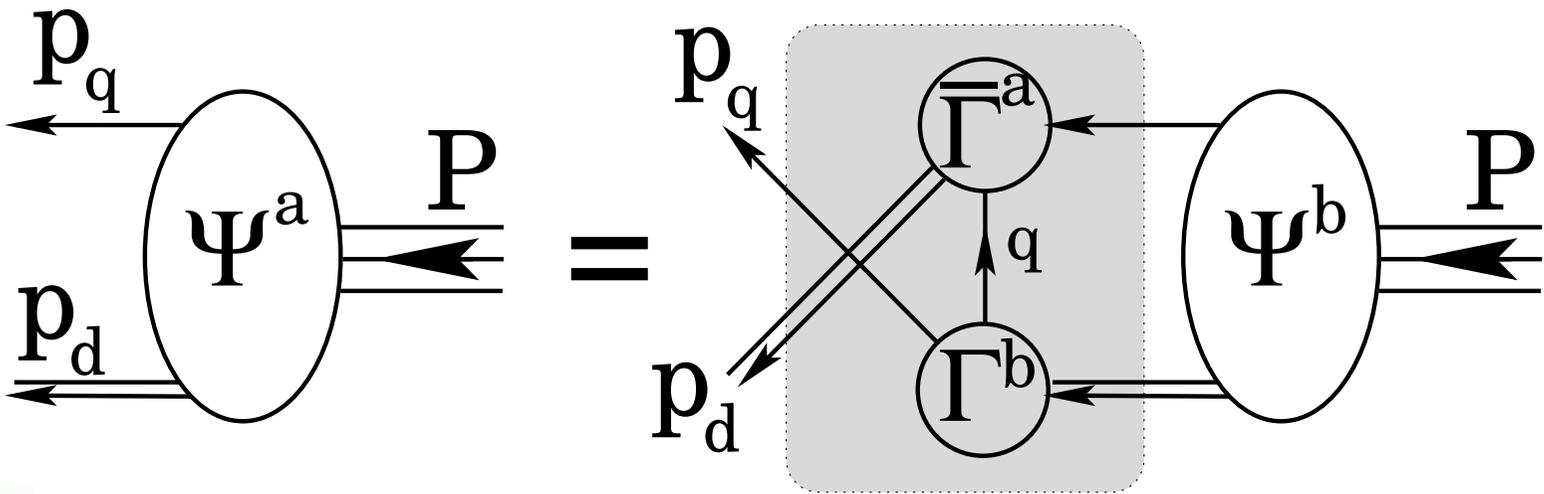
[First](#)

[Contents](#)

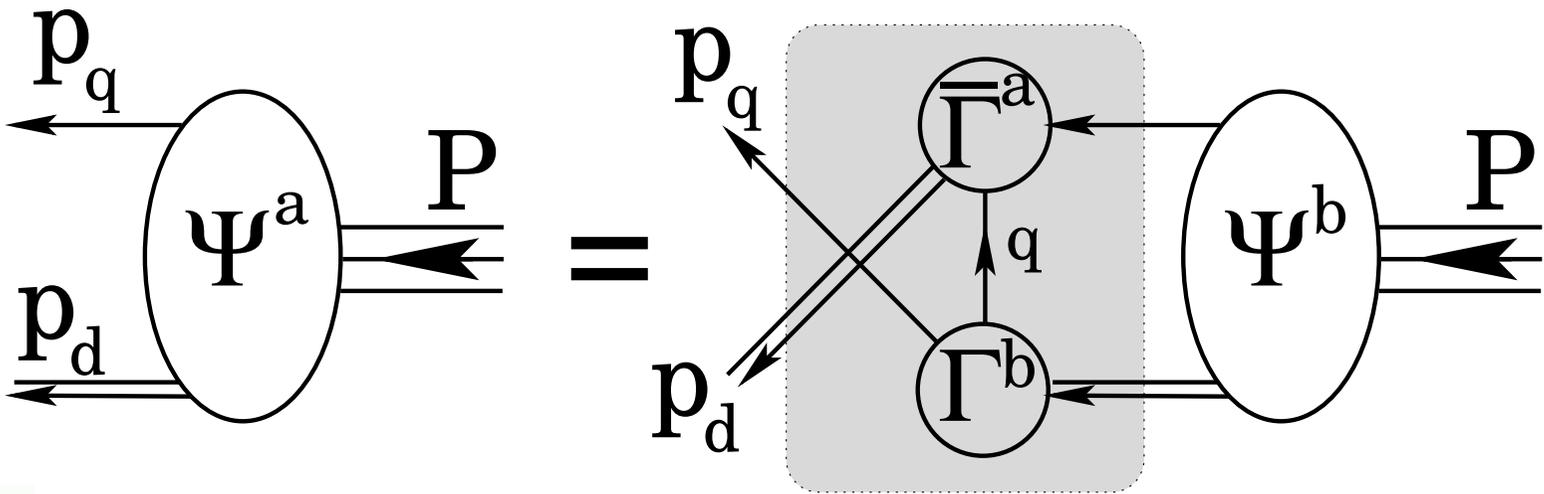
[Back](#)

[Conclusion](#)

Faddeev equation

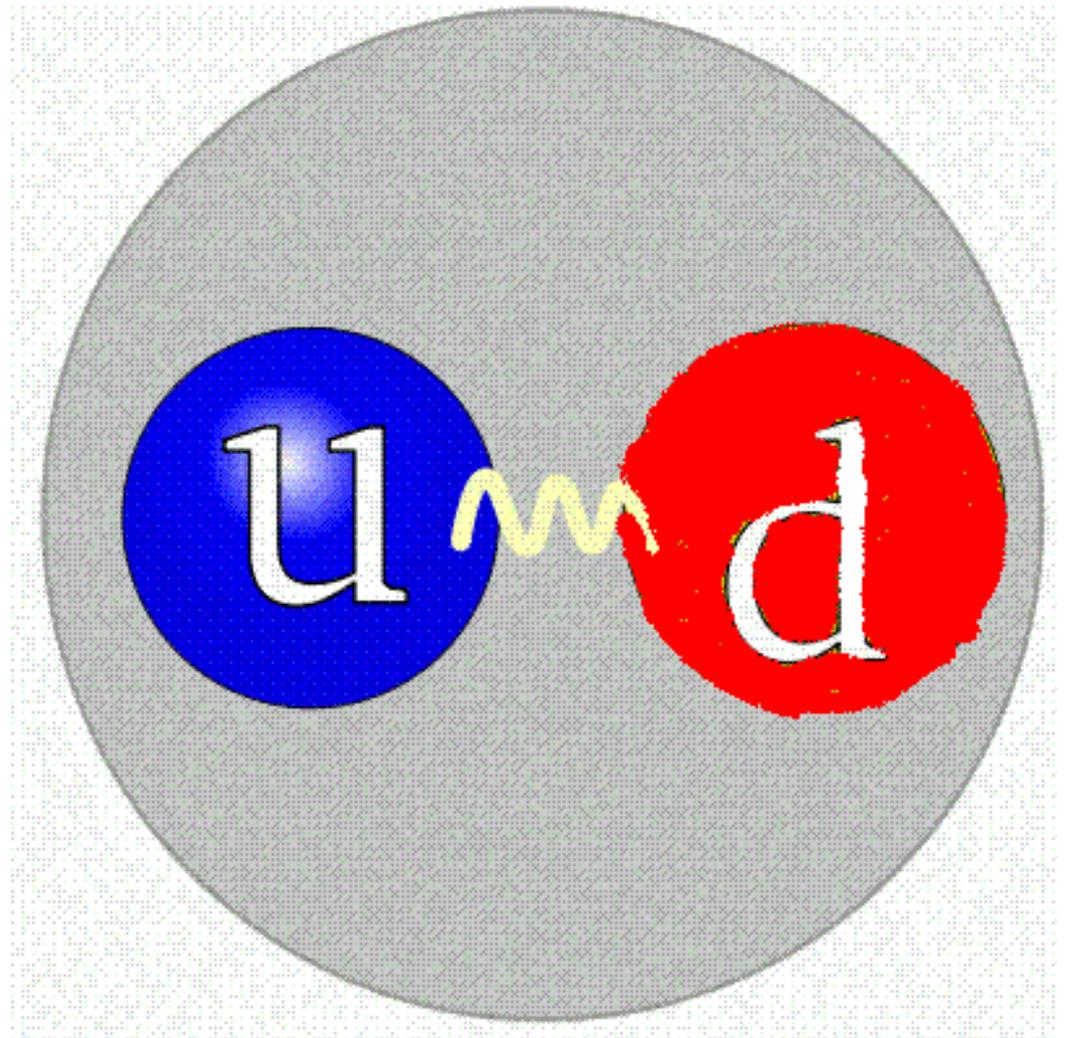


Faddeev equation



- Linear, Homogeneous Matrix equation
 - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame Amplitude has ... *s*- , *p*- & *d*-wave correlations

Diquark correlations



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

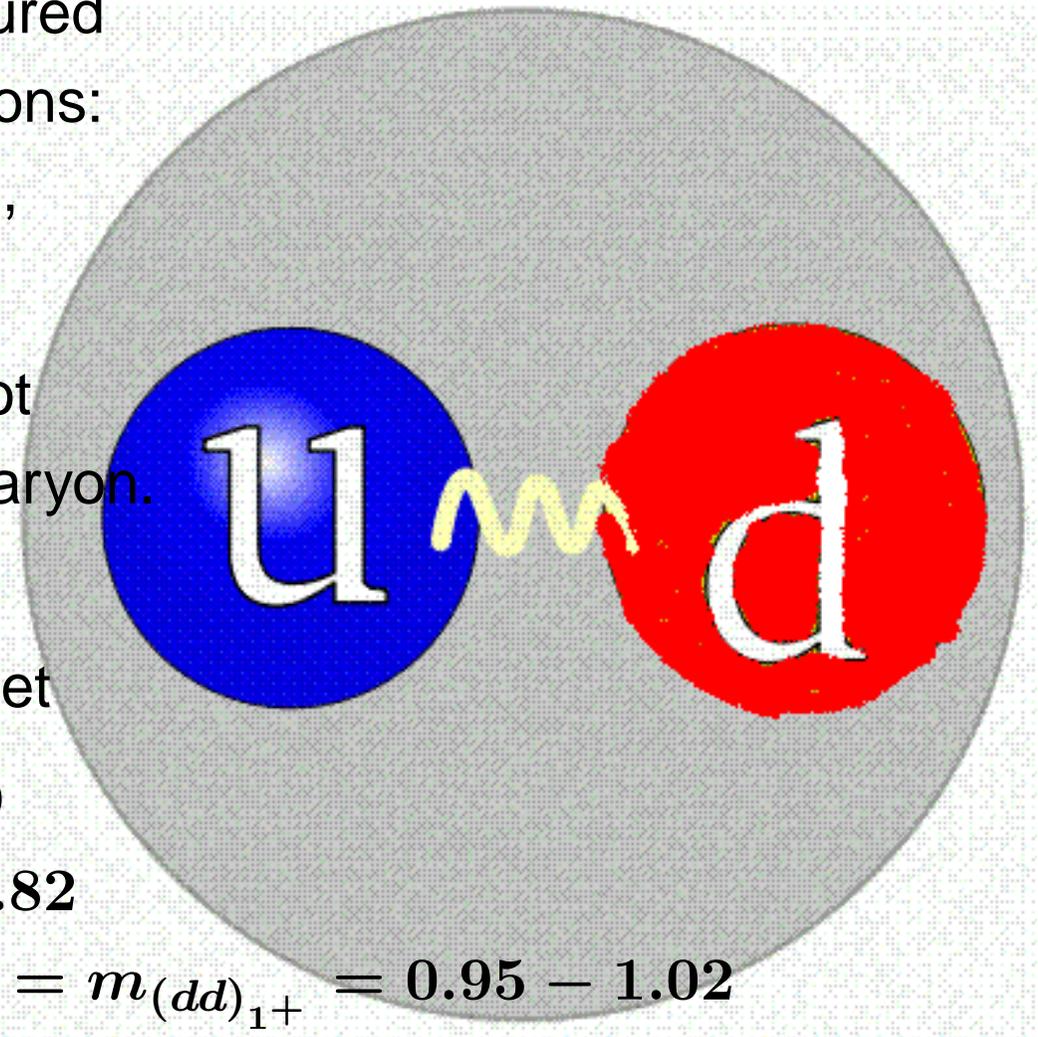
QUARK-QUARK

Craig Roberts: Unifying description of mesons and baryons

Electromagnetic N-N* Transition Form Factors Workshop ... 40 - p. 28/42

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue-red, blue-green, green-red
- Confined ... Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD
$$m_{[ud]_{0+}} = 0.74 - 0.82$$
$$m_{(uu)_{1+}} = m_{(ud)_{1+}} = m_{(dd)_{1+}} = 0.95 - 1.02$$





Ab-initio study of mesons & nucleons



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Eichmann *et al.*

- arXiv:0802.1948 [nucl-th]
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Ab-initio study of mesons & nucleons



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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Ab-initio study of mesons & nucleons

- Leading-order truncation of DSEs – rainbow-ladder



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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- Symmetry preserving and systematic approach can elucidate and account for these effects
 - Use this knowledge to constrain interaction in infrared
 - Interaction in ultraviolet predicted by perturbative expansion of DSEs

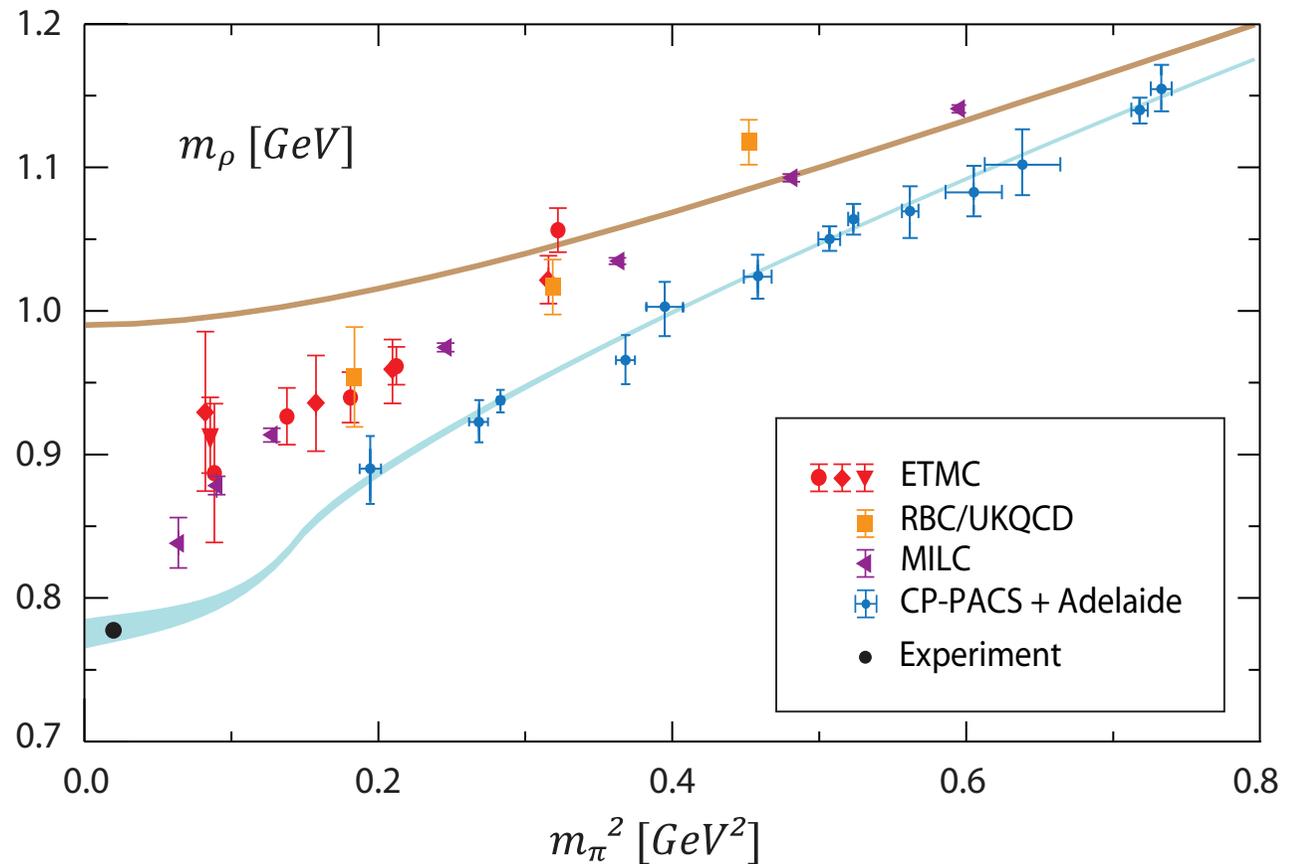


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First

Contents

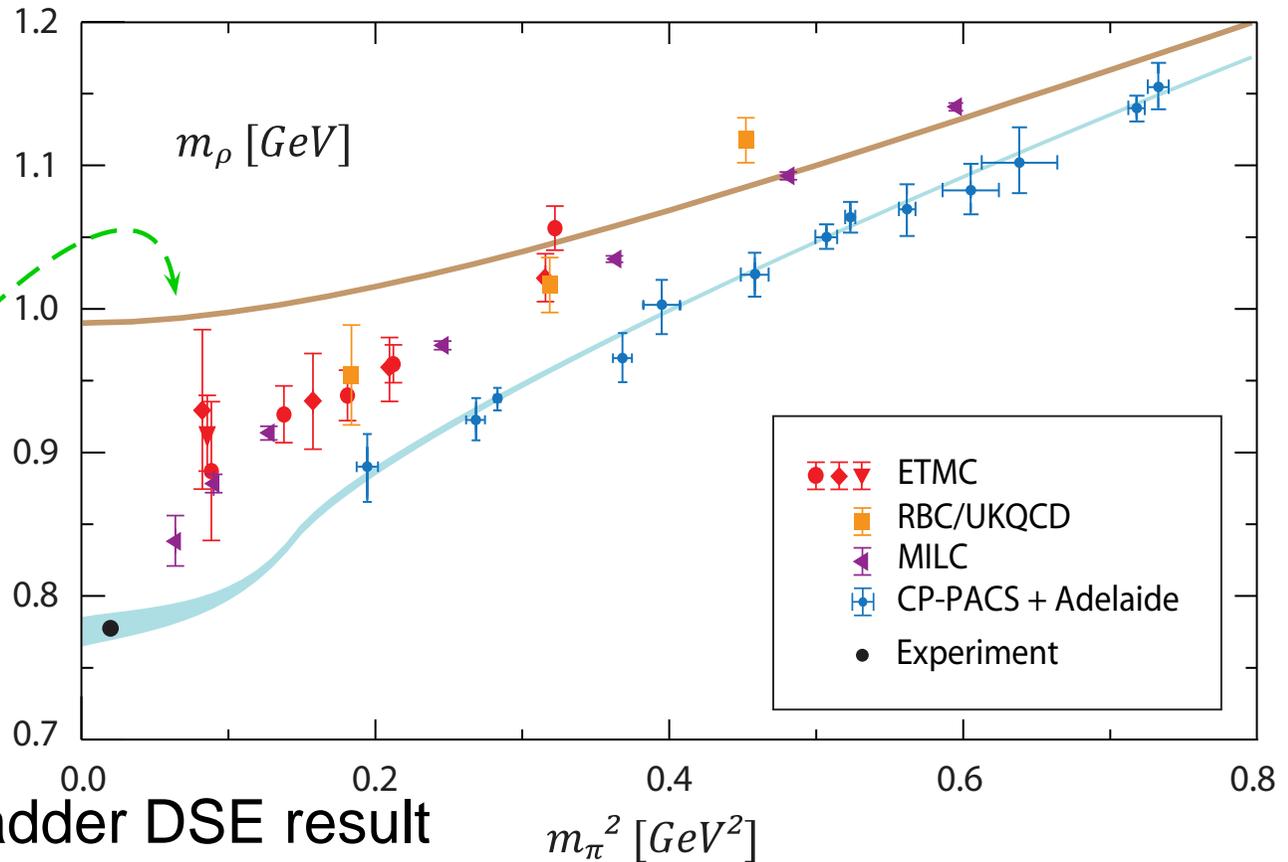
Back

Conclusion

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Rainbow-Ladder DSE result

one parameter for IR – “confinement radius”

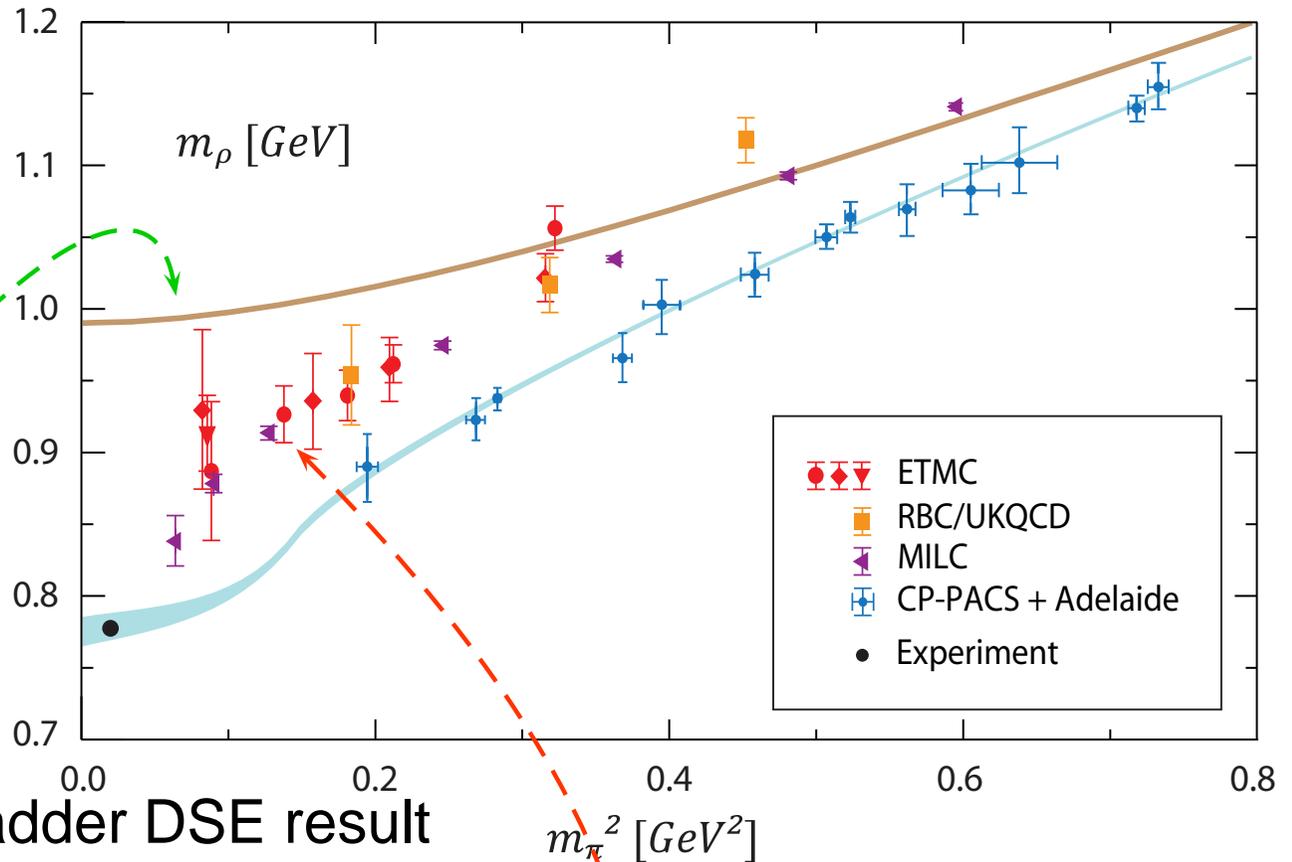
Results insensitive to value on material domain



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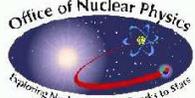


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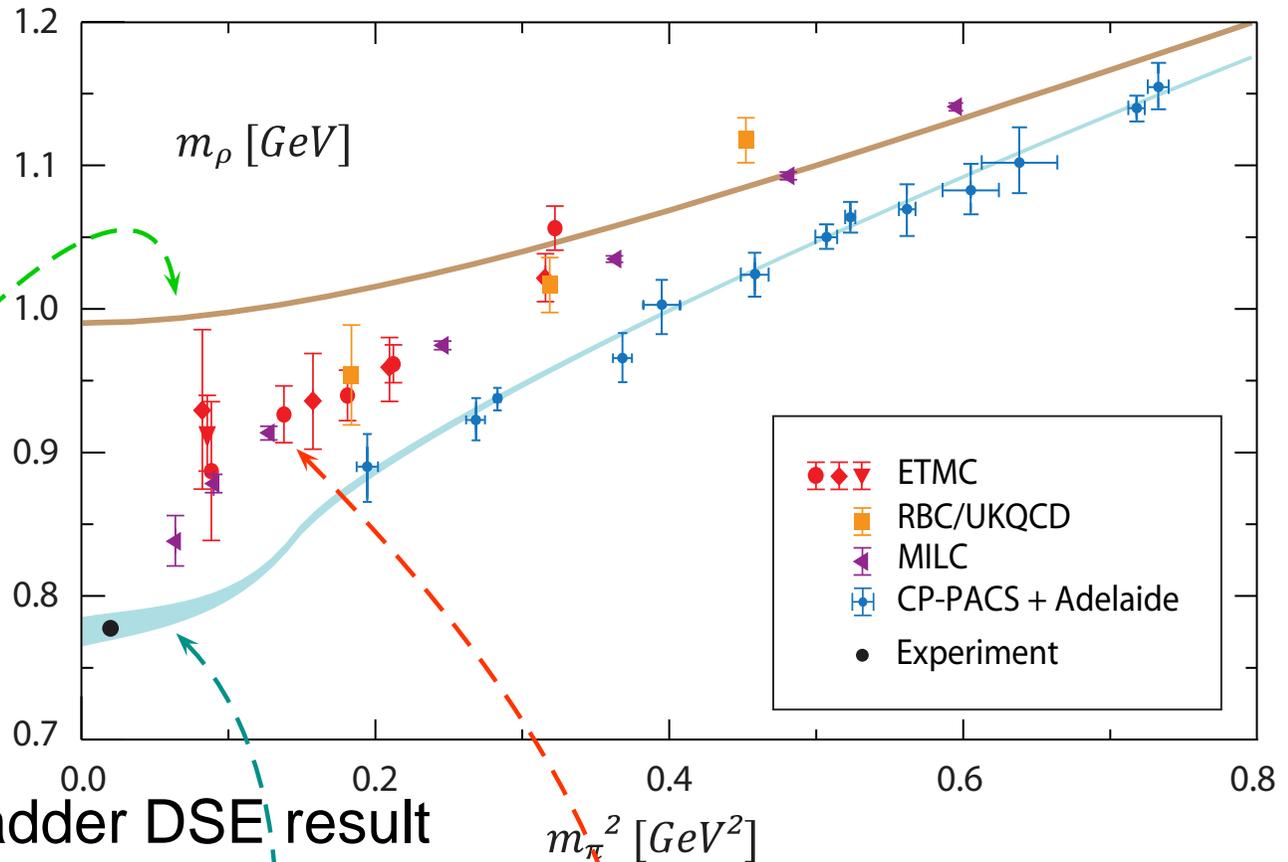
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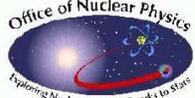
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● FRR extrapolation of lattice CP-PACS result



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- Precisely the same interaction



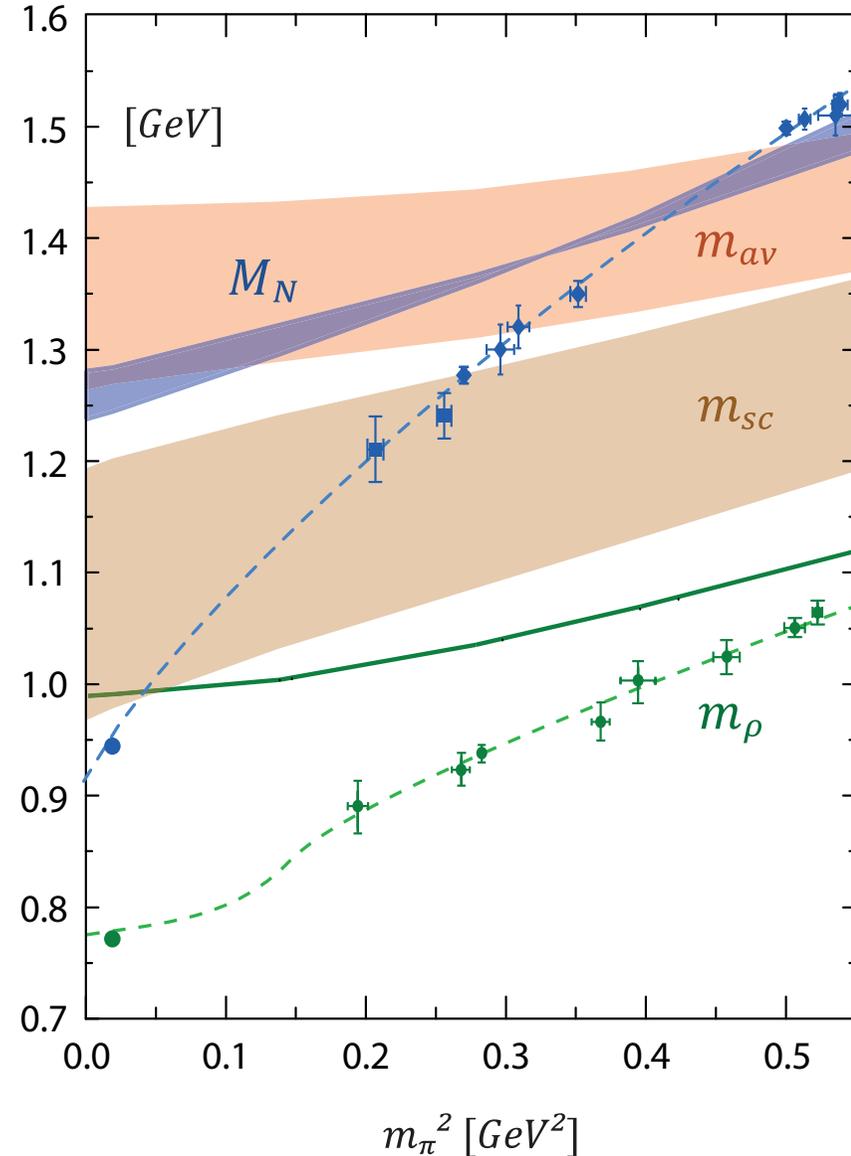
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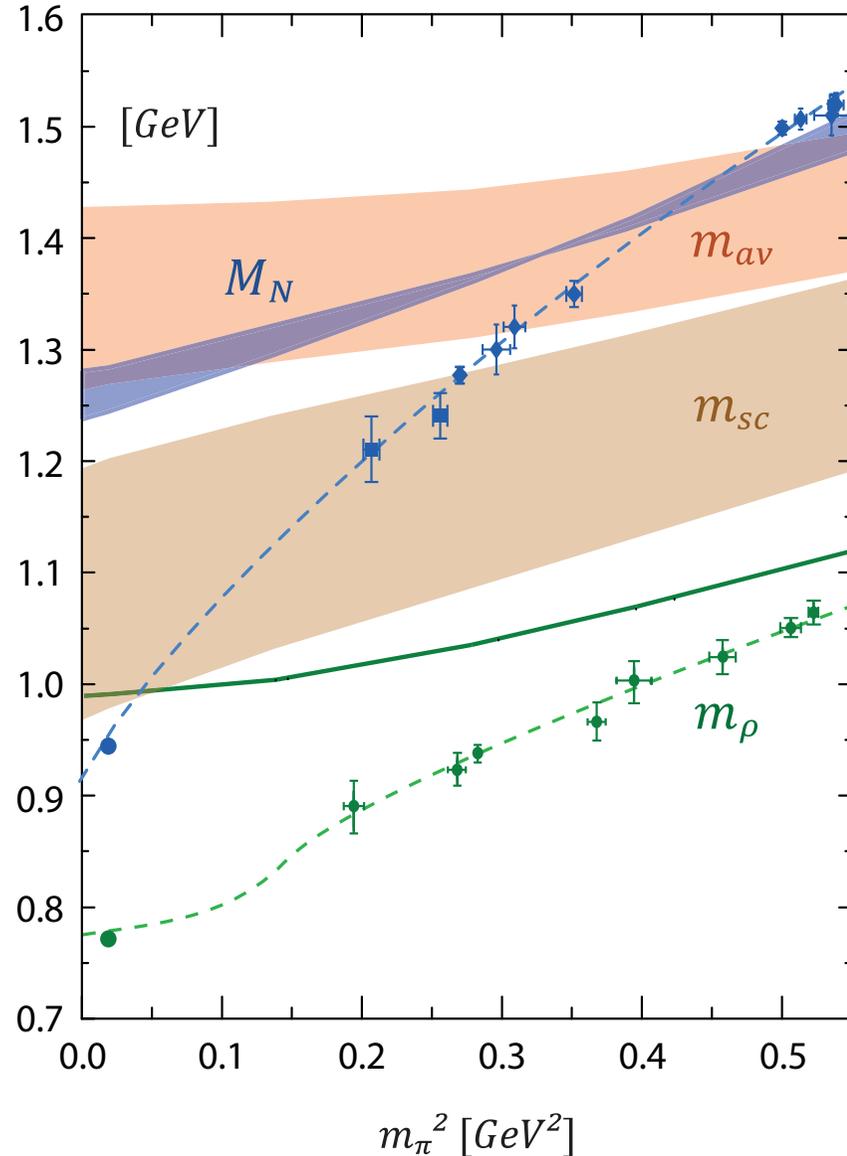
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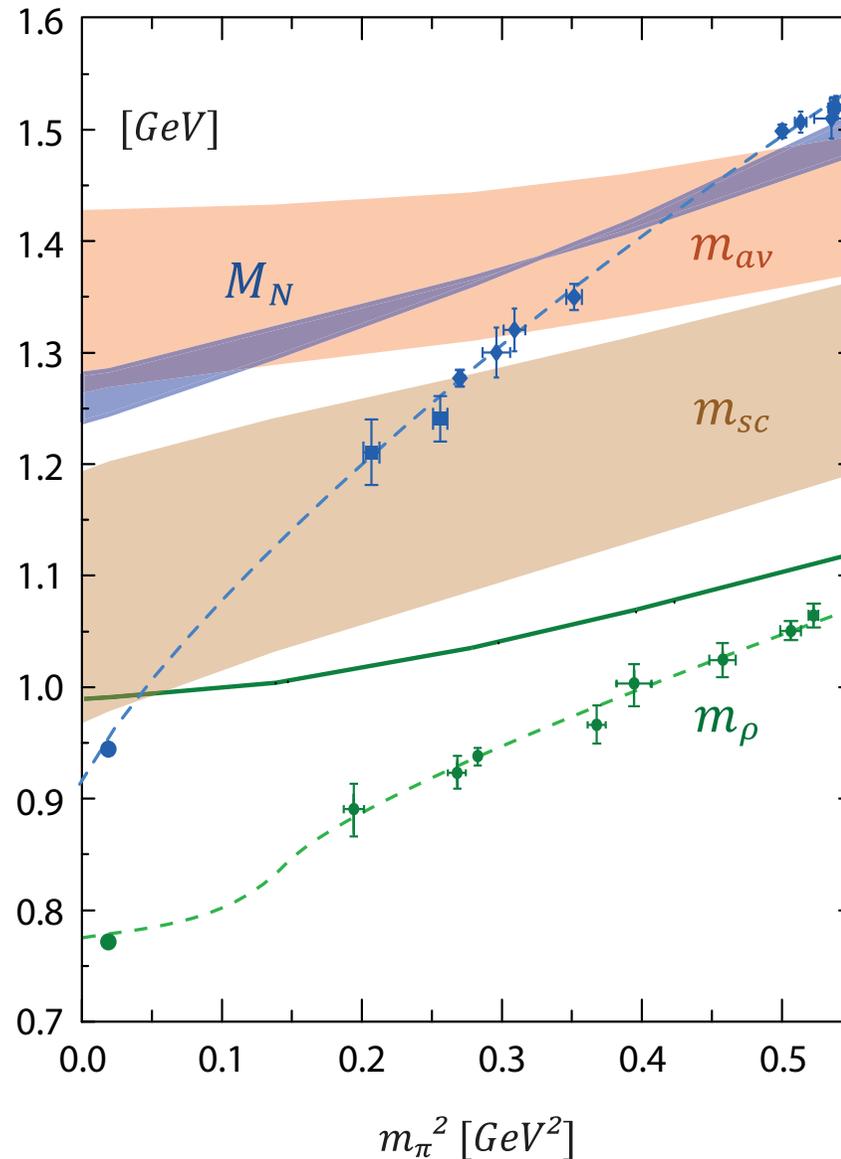
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- But ... $[m_{av} - m_{sc}], m_\rho$ & M_N ... are *independent* of that parameter



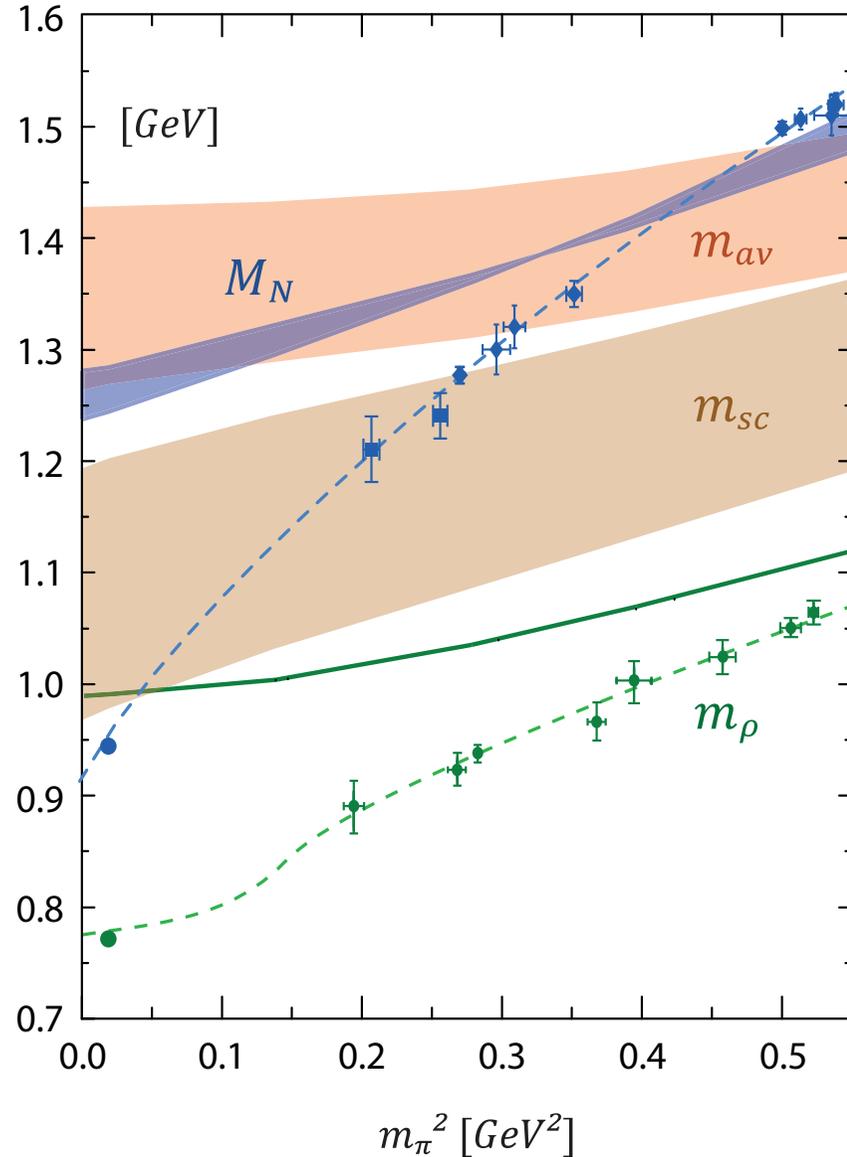
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Ab-initio study of mesons & nucleons

- Parameter-independent RL-DSE predictions, with veracious description of Goldstone mode



First

Contents

Back

Conclusion

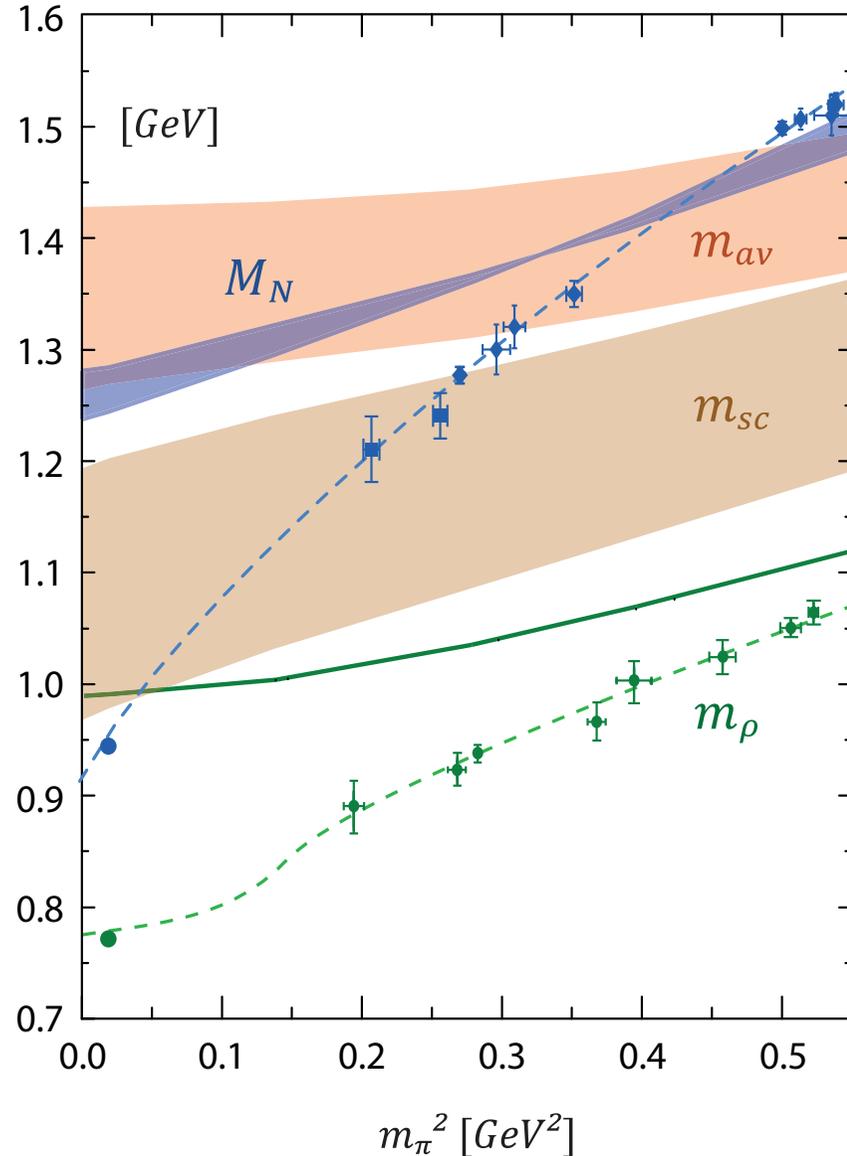
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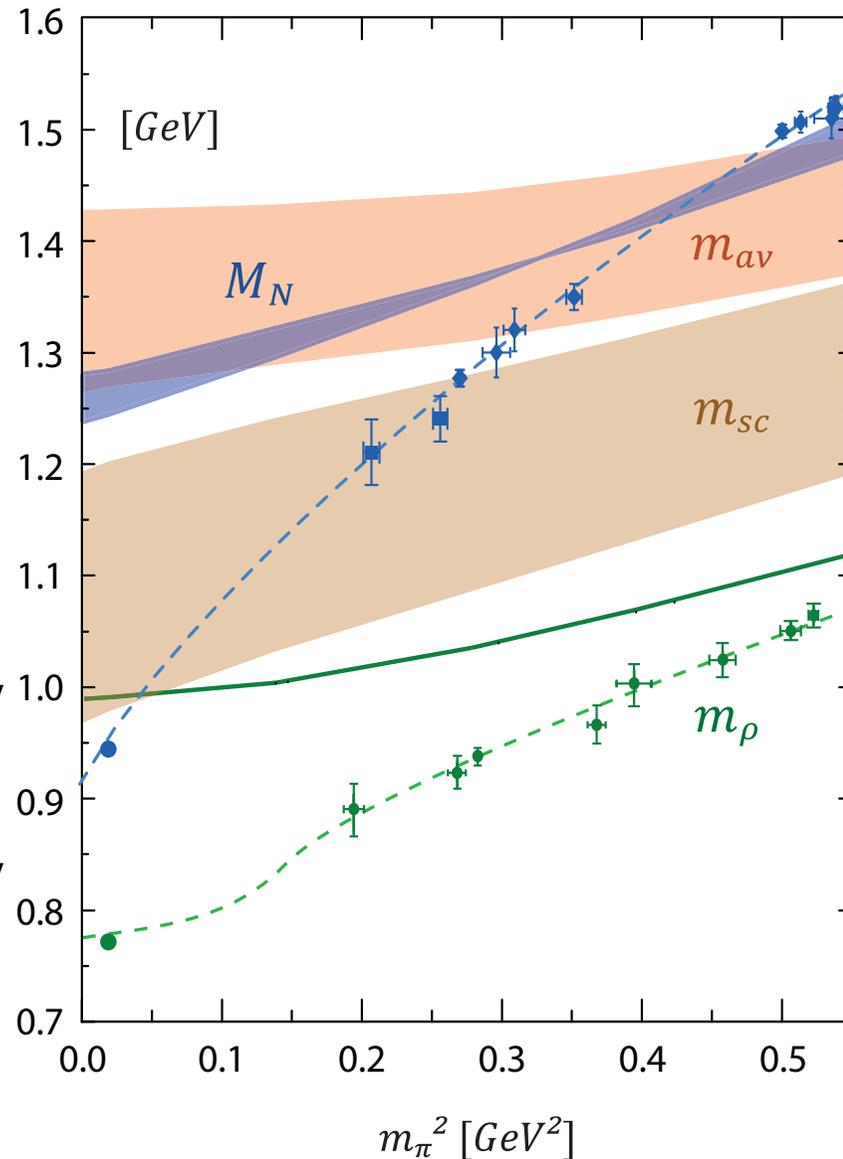
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Ab-initio study of mesons & nucleons

- Parameter-independent RL-DSE predictions, with veracious description of Goldstone mode
- DSE and lattice agree on heavy-quark domain
- Prediction: at physical m_π^2 ,
 $M_N^{\text{quark-core}} = 1.26(2) \text{ GeV}$
 cf. FRR+lattice-QCD,
 $M_N^{\text{quark-core}} = 1.27(2) \text{ GeV}$
 \Rightarrow subleading corrections,
 including 0^- -meson loops,
 $\delta M_N = -320 \text{ MeV}$,



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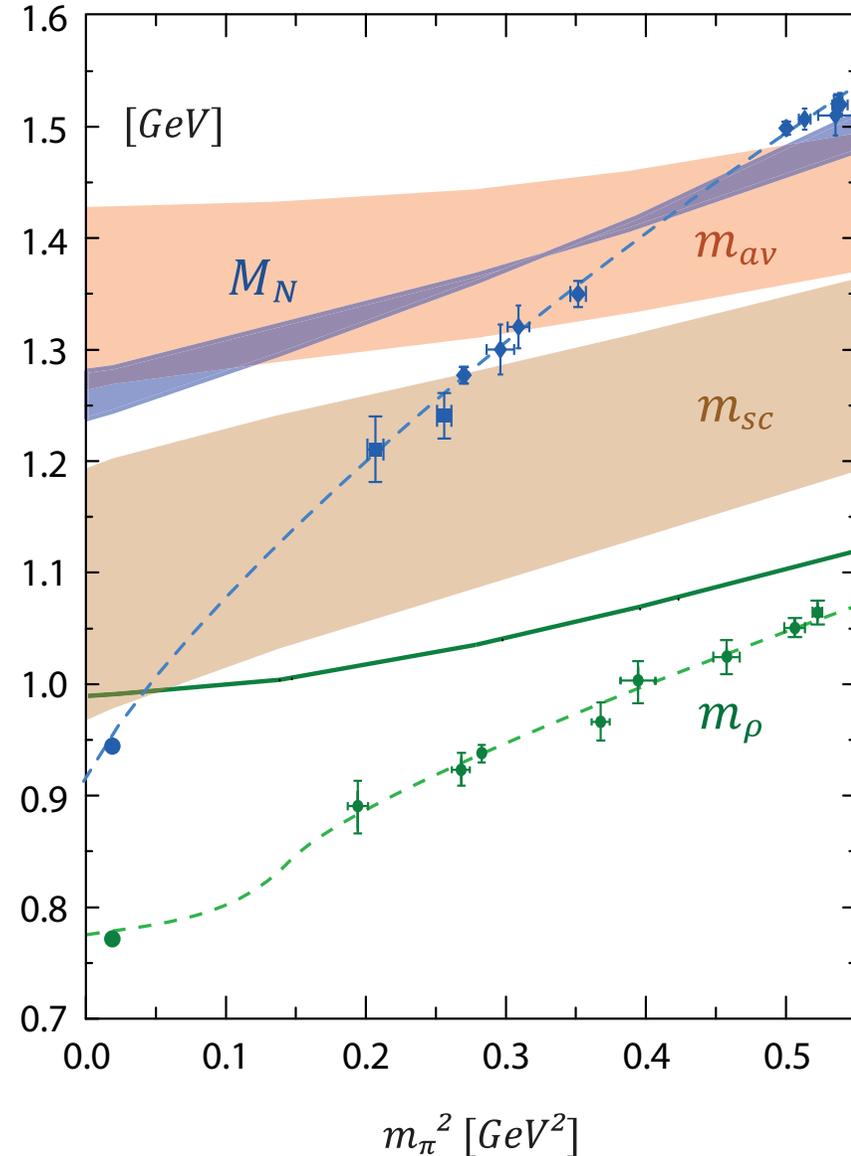
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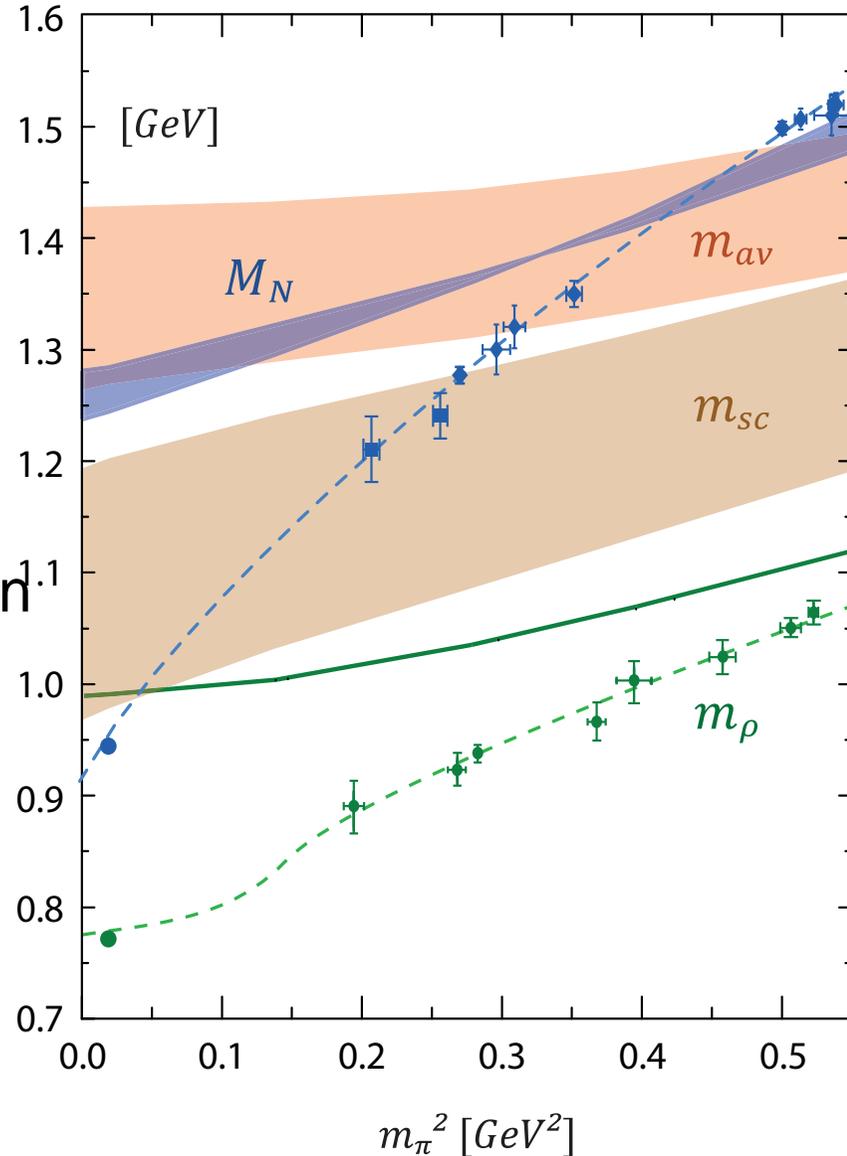
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First

Contents

Back

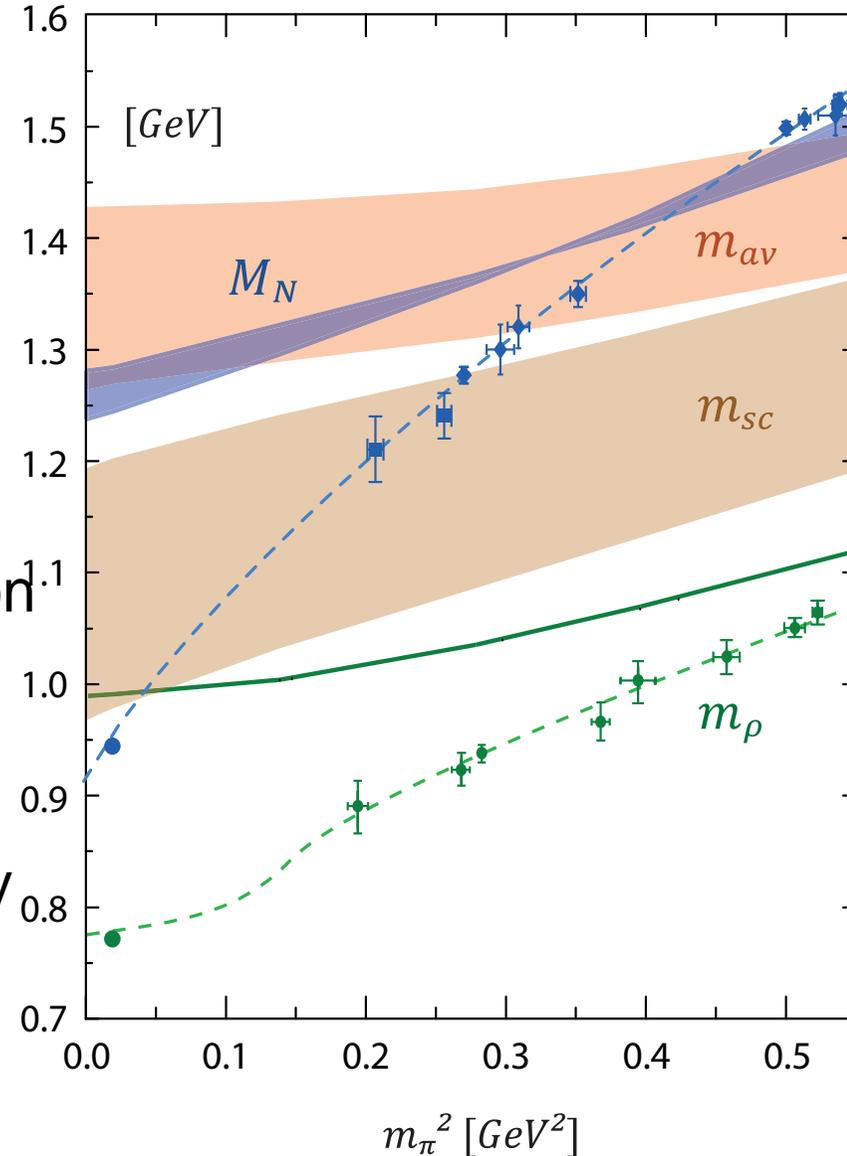
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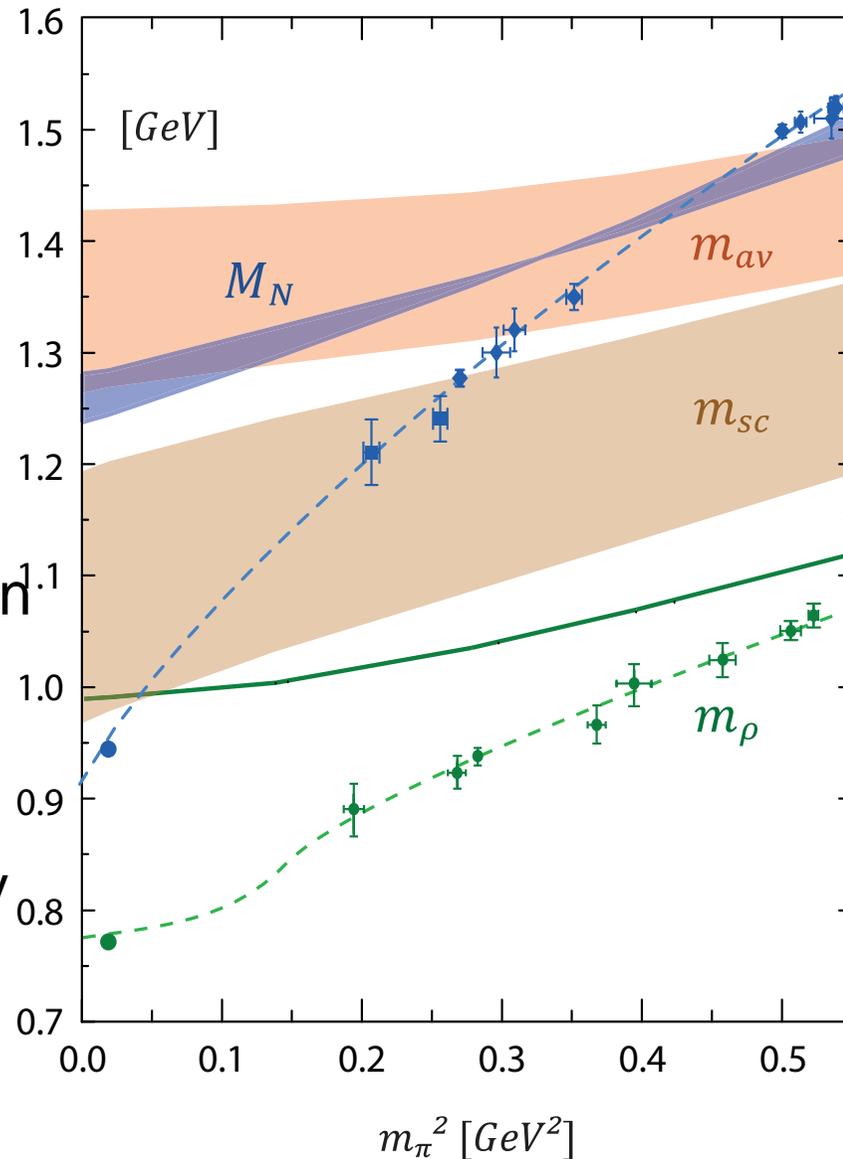
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● *Systematically improvable*



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First

Contents

Back

Conclusion

Dimensionless product: $r_\pi f_\pi$



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Dimensionless product: $r_\pi f_\pi$



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Dimensionless product: $r_\pi f_\pi$

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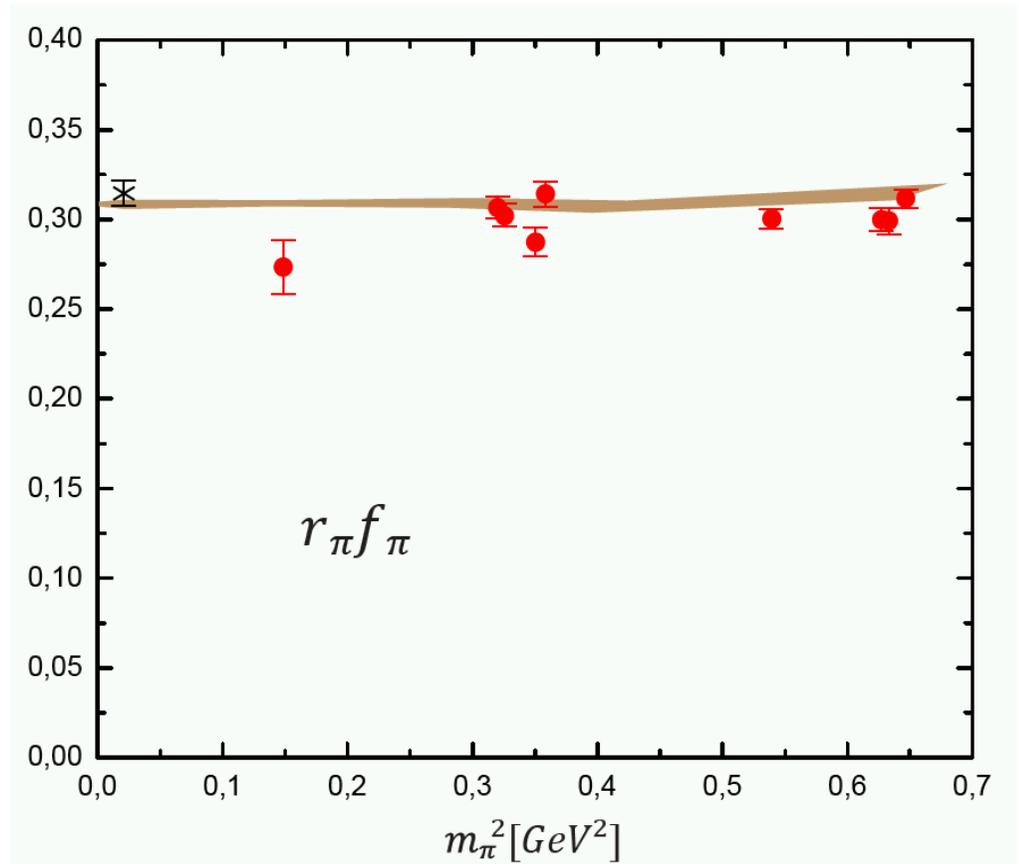
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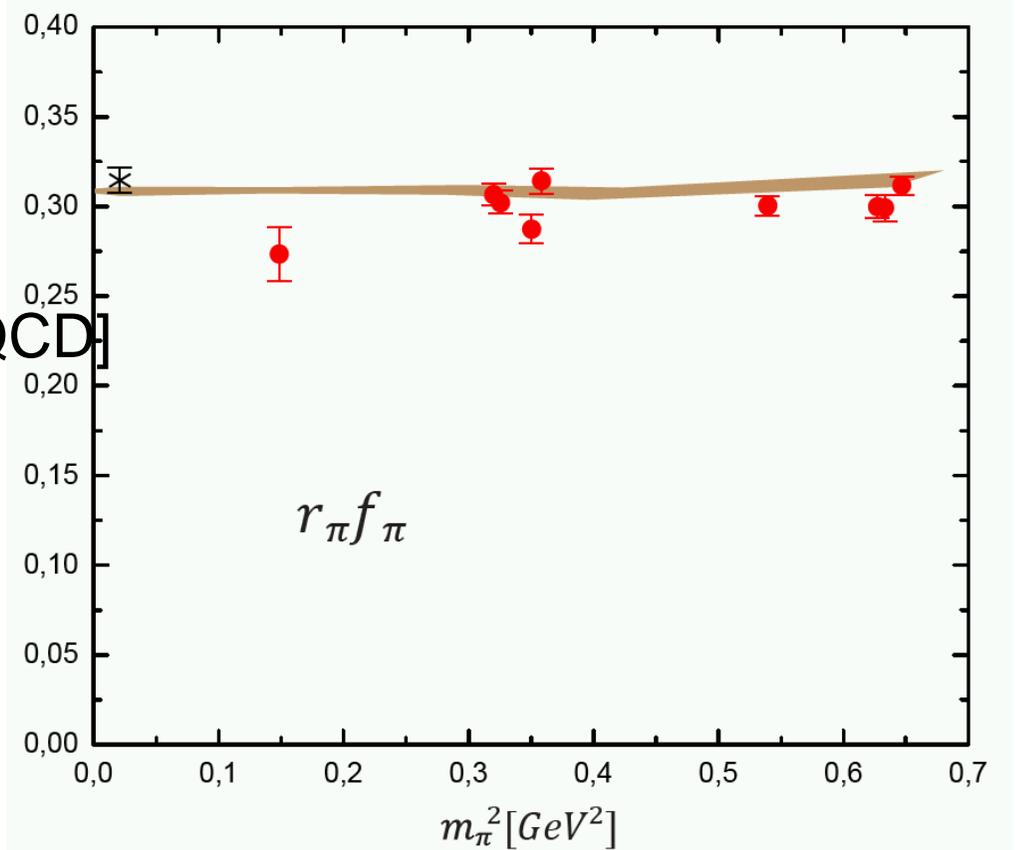
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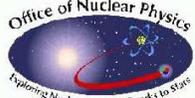
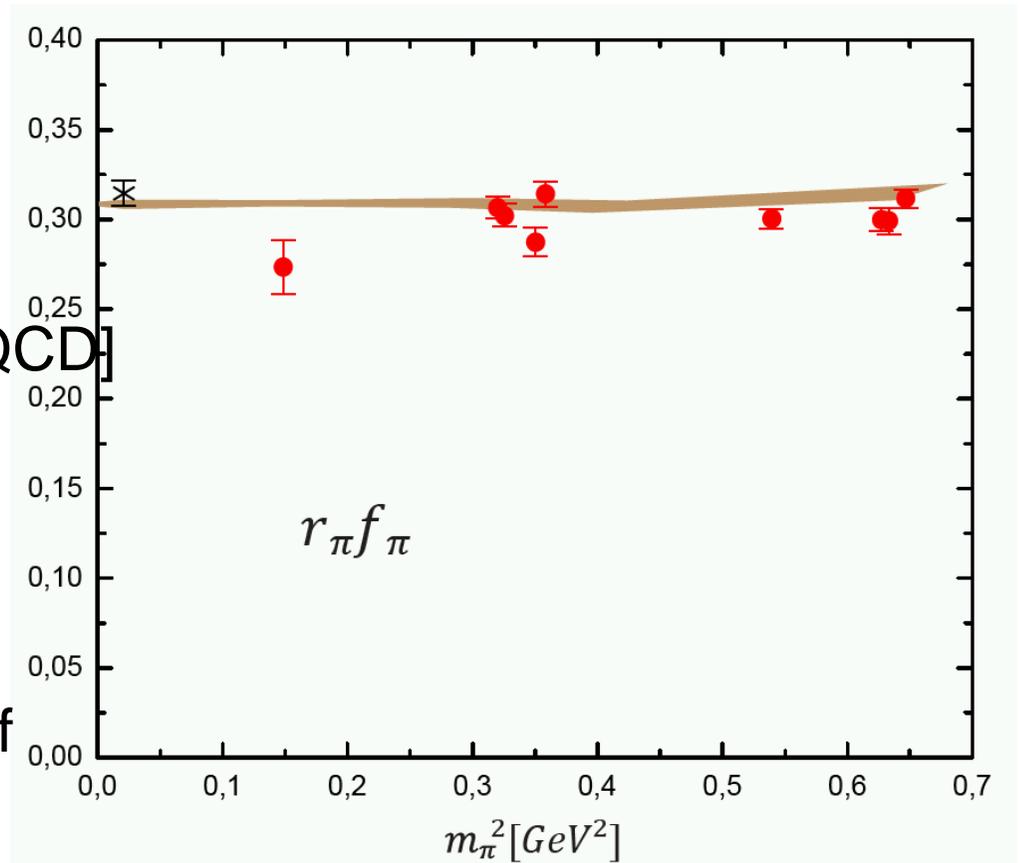
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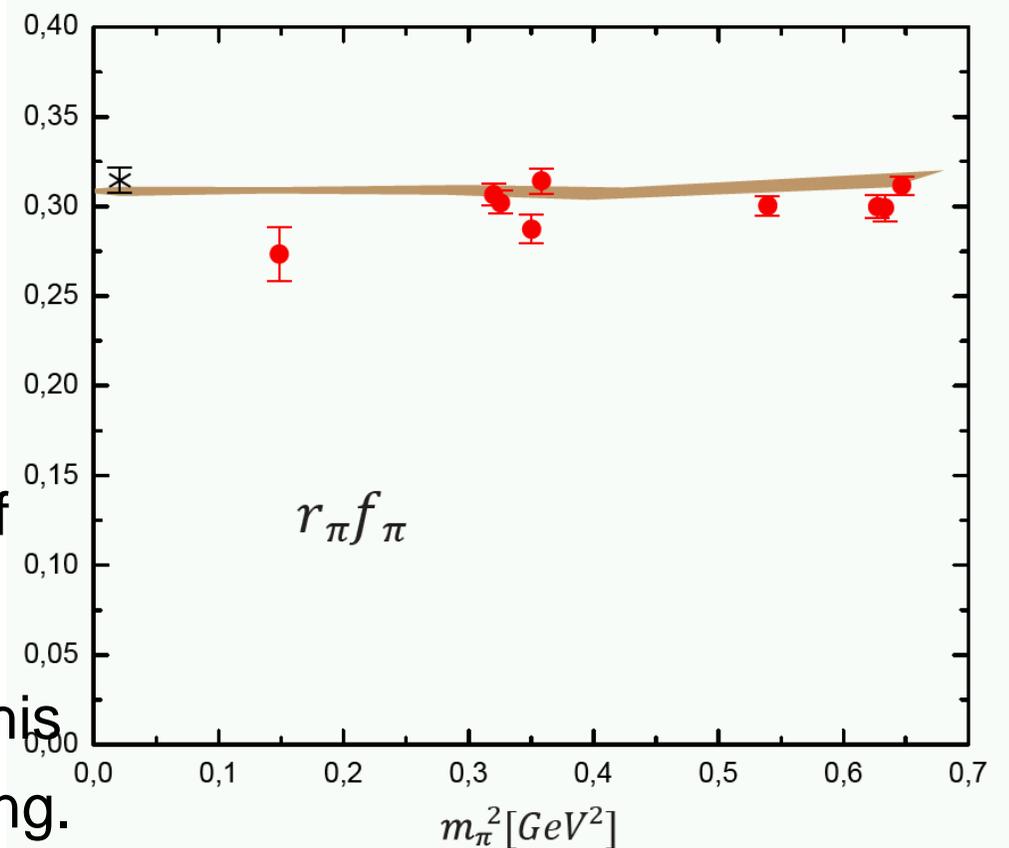
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DSE and Lattice
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- Fascinating result:
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obtains independent of
current-quark mass.
- We have understood this
Implications far-reaching.



Nucleon-Photon Vertex



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

6 terms . . .

Nucleon-Photon Vertex

constructed systematically . . . current conserved automatically
for on-shell nucleons described by Faddeev Amplitude



[First](#)

[Contents](#)

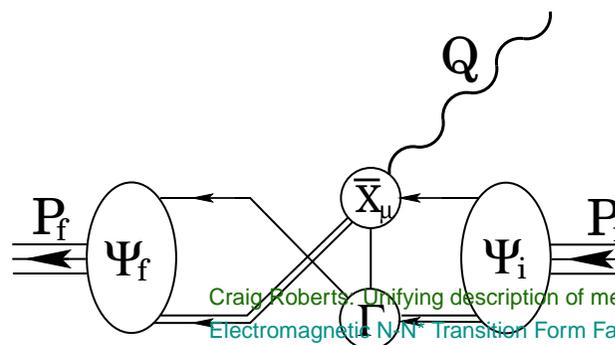
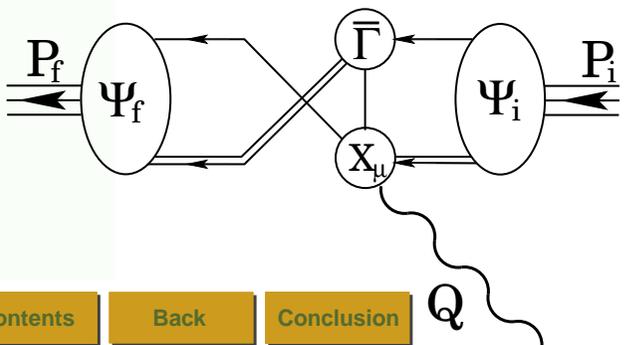
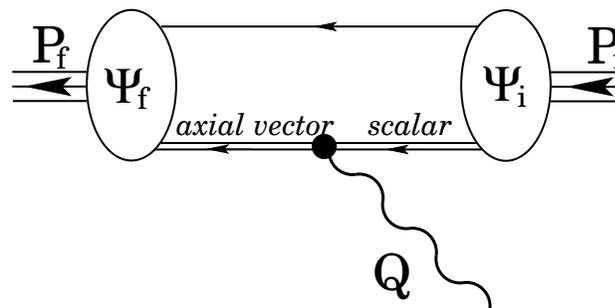
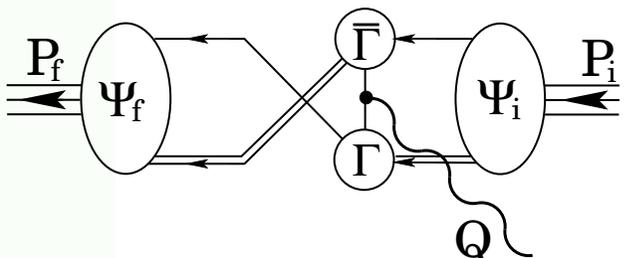
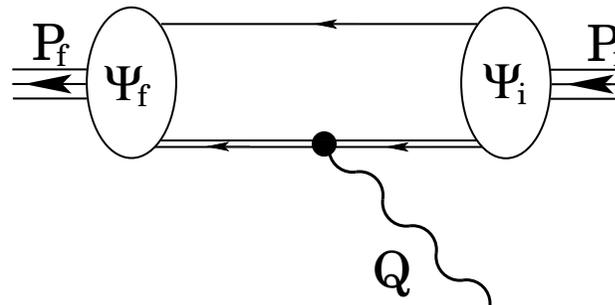
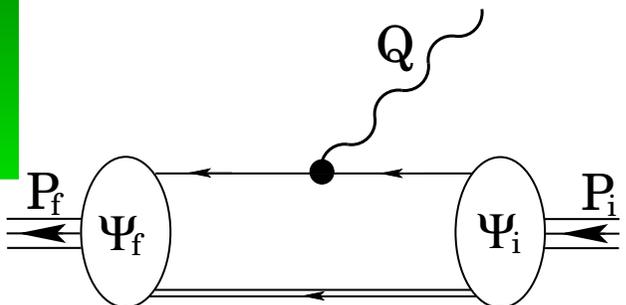
[Back](#)

[Conclusion](#)

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DSE-based Faddeev Equation



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Cloët *et al.*

- arXiv:0710.2059 [nucl-th]
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DSE-based Faddeev Equation



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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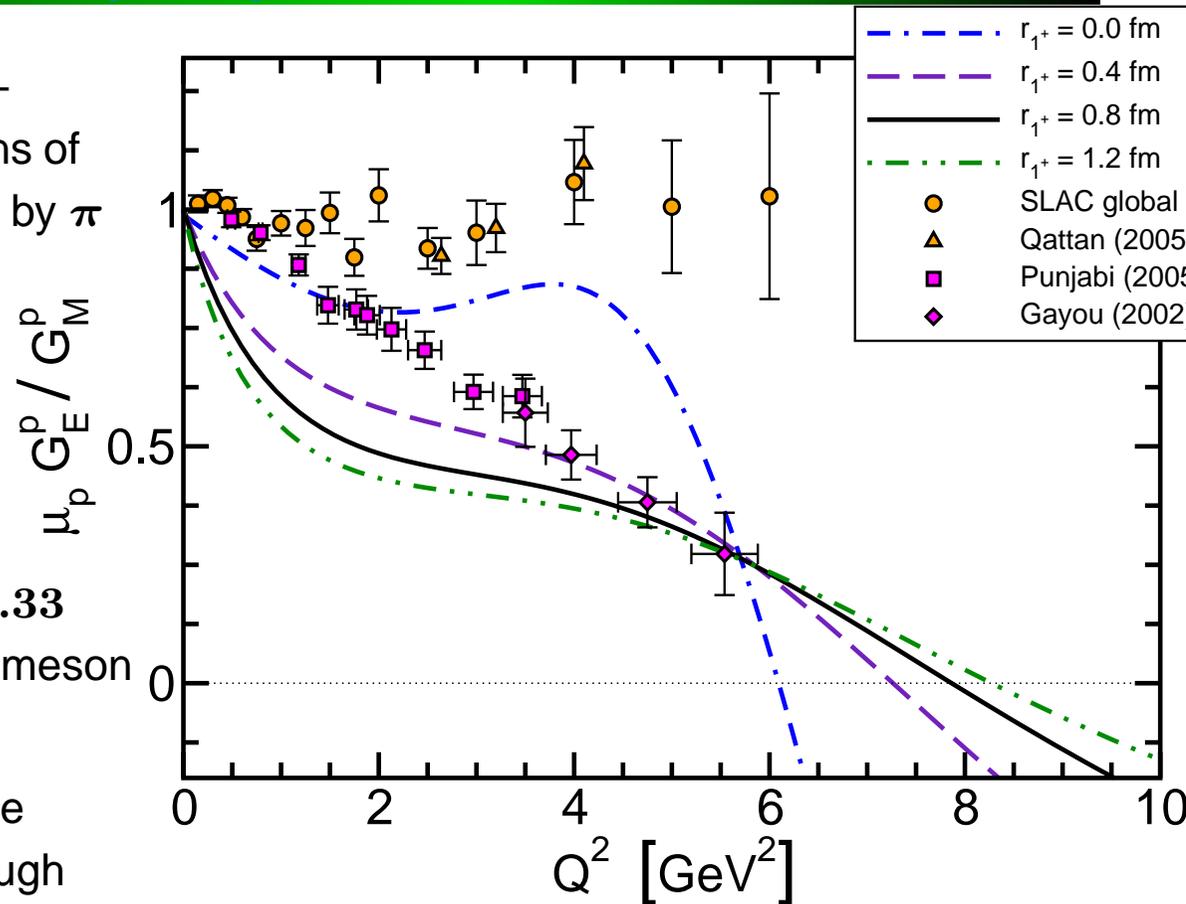
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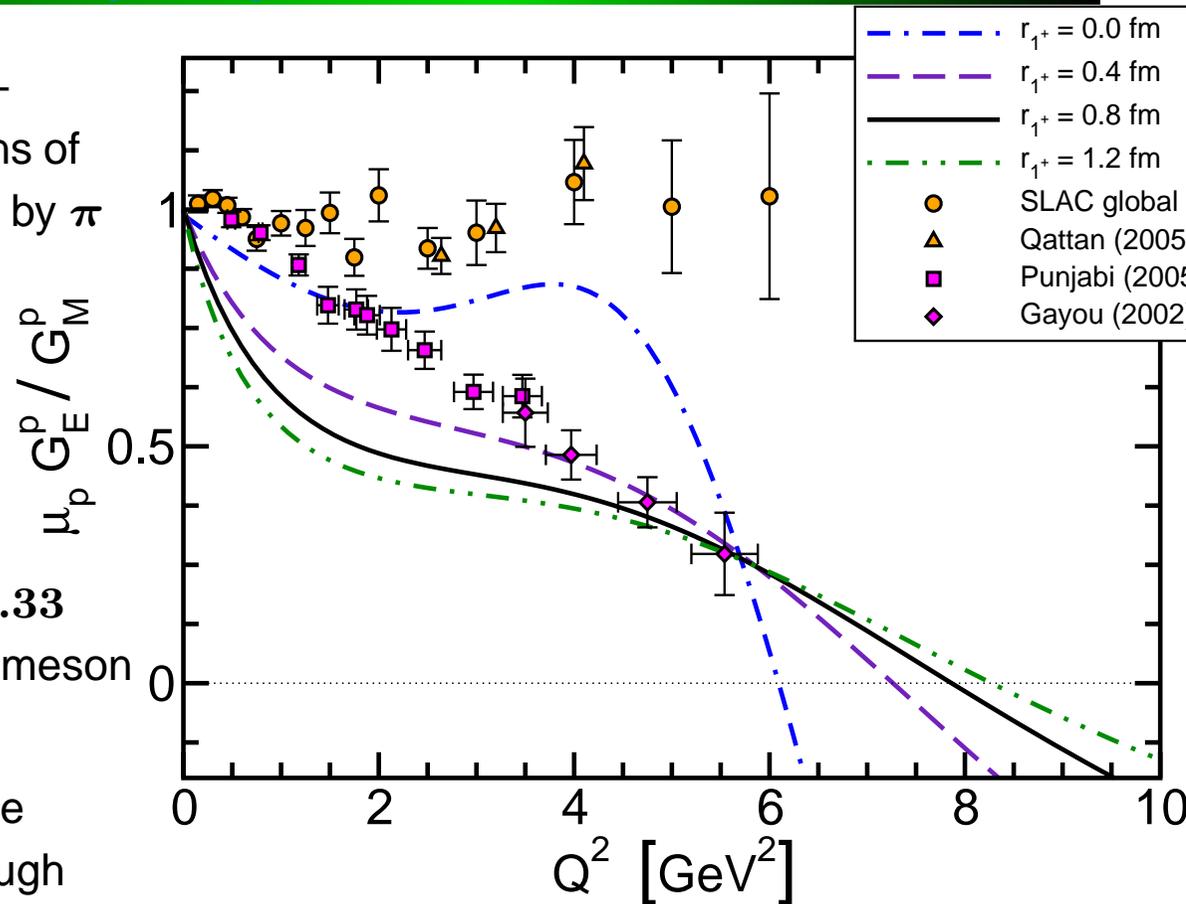
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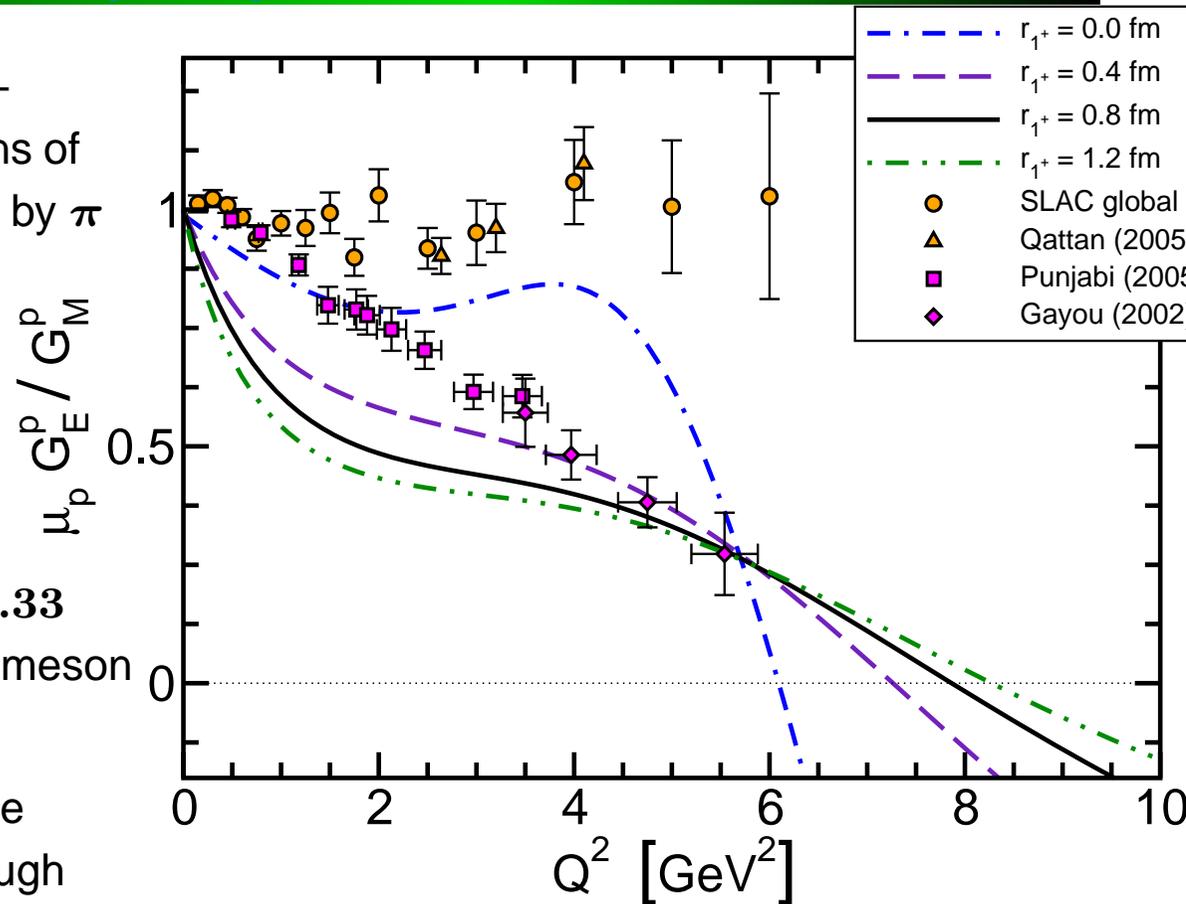


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- Always a zero but position depends on details of current



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ab initio *Faddeev Equation*



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

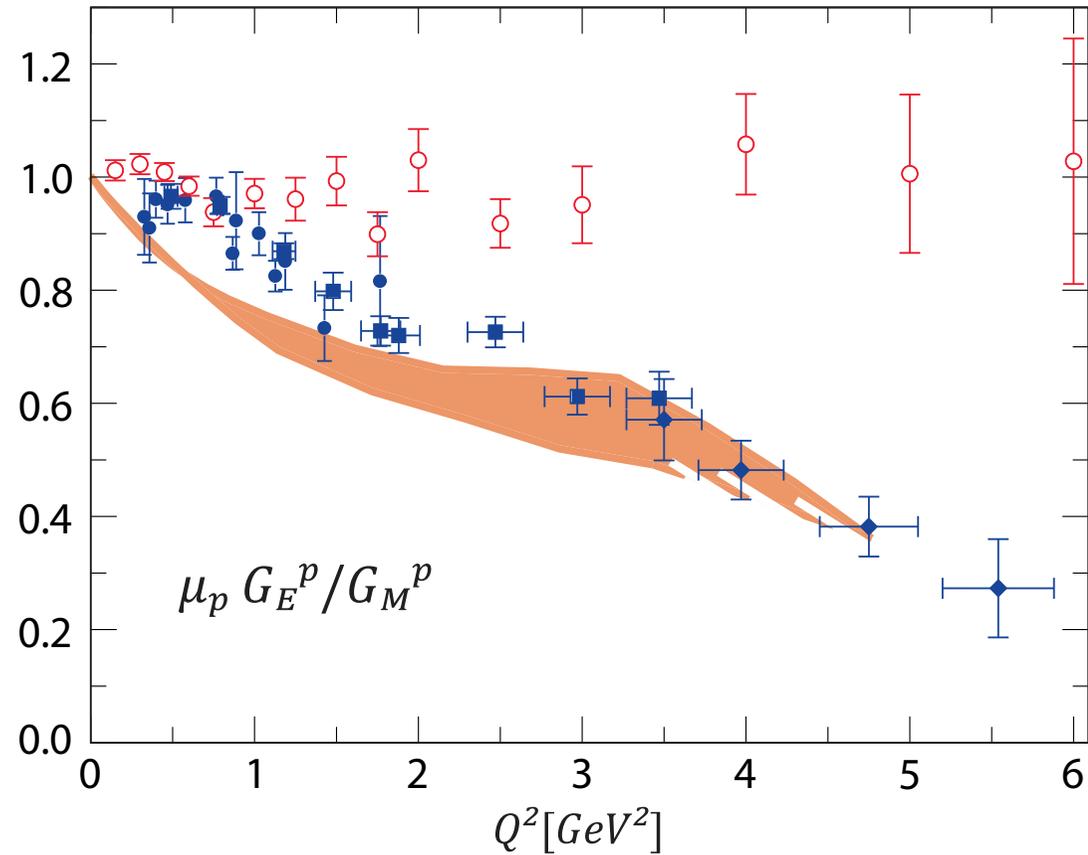


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ab initio

Faddeev Equation



First

Contents

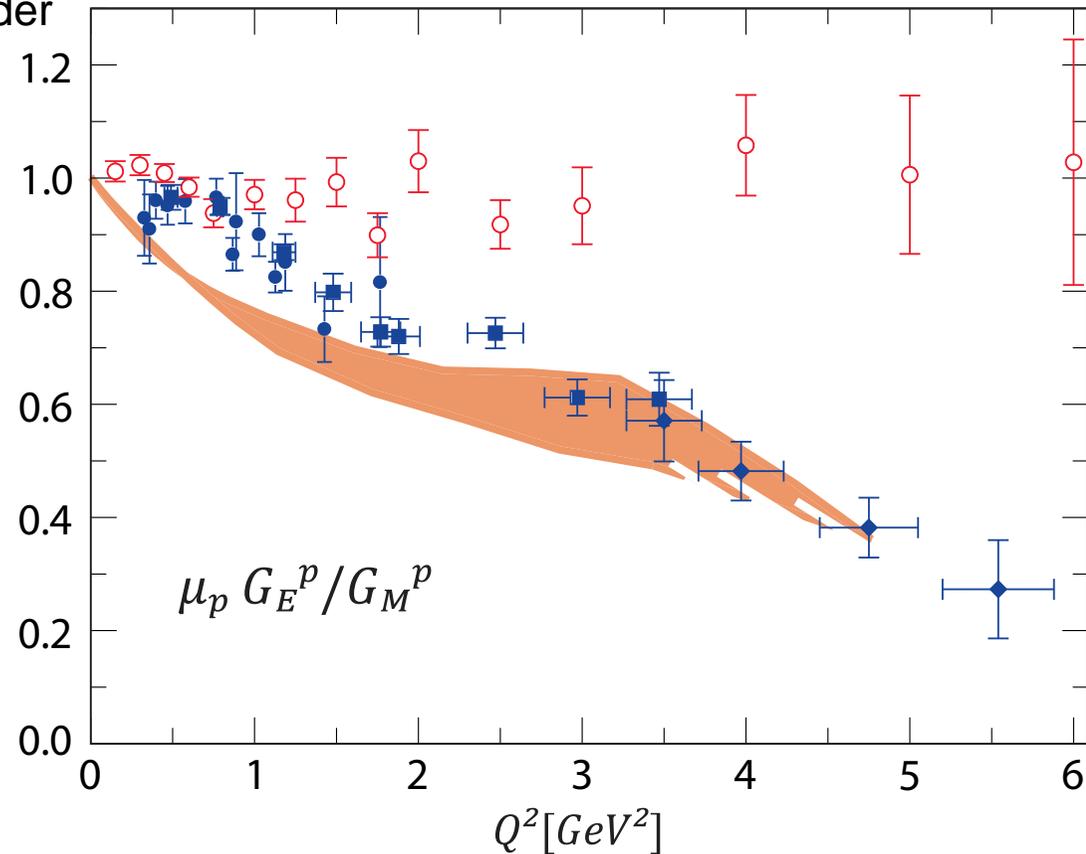
Back

Conclusion

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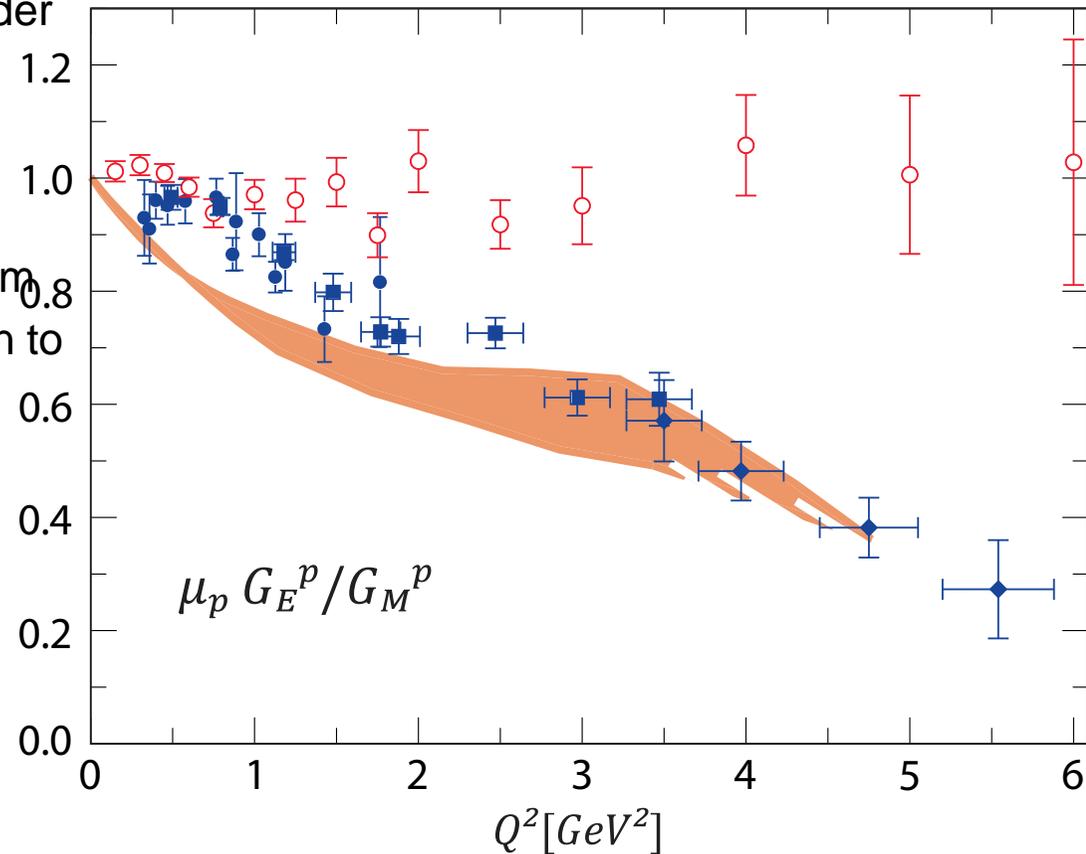
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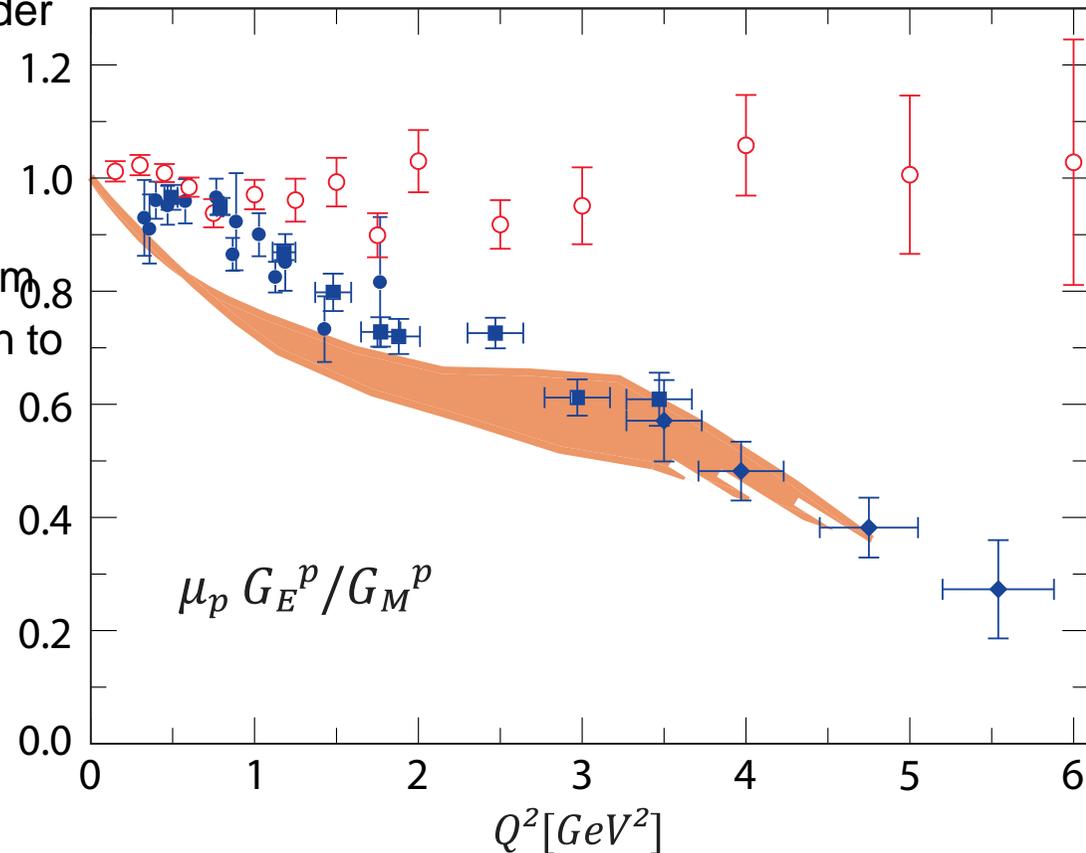
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- Calculation unifies π , ρ and nucleon properties – keystone is behaviour of dressed-quark mass function and hence veracious description of QCD's Goldstone mode



Ratio of Neutron Pauli & Dirac Form Factors

$$\frac{\hat{Q}^2}{(\ln \hat{Q}^2 / \hat{\Lambda})^2} \frac{F_2^n(\hat{Q}^2)}{F_1^n(\hat{Q}^2)}$$

$$\hat{\Lambda} = \Lambda / M_N = 0.44$$

Ensures proton ratio
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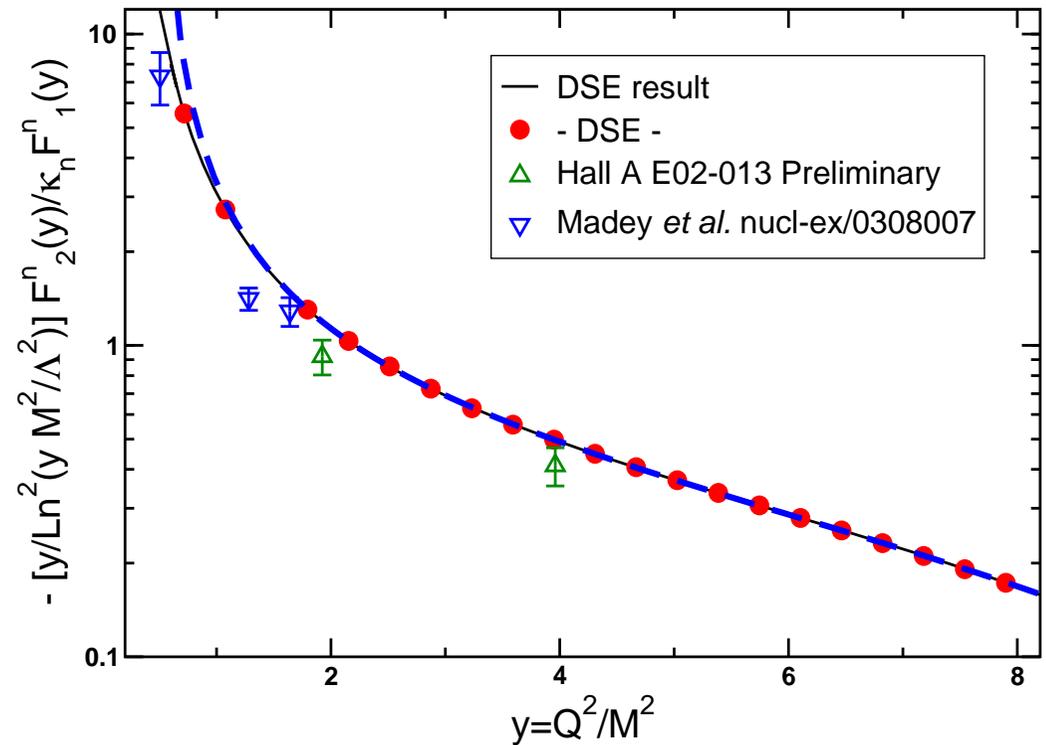


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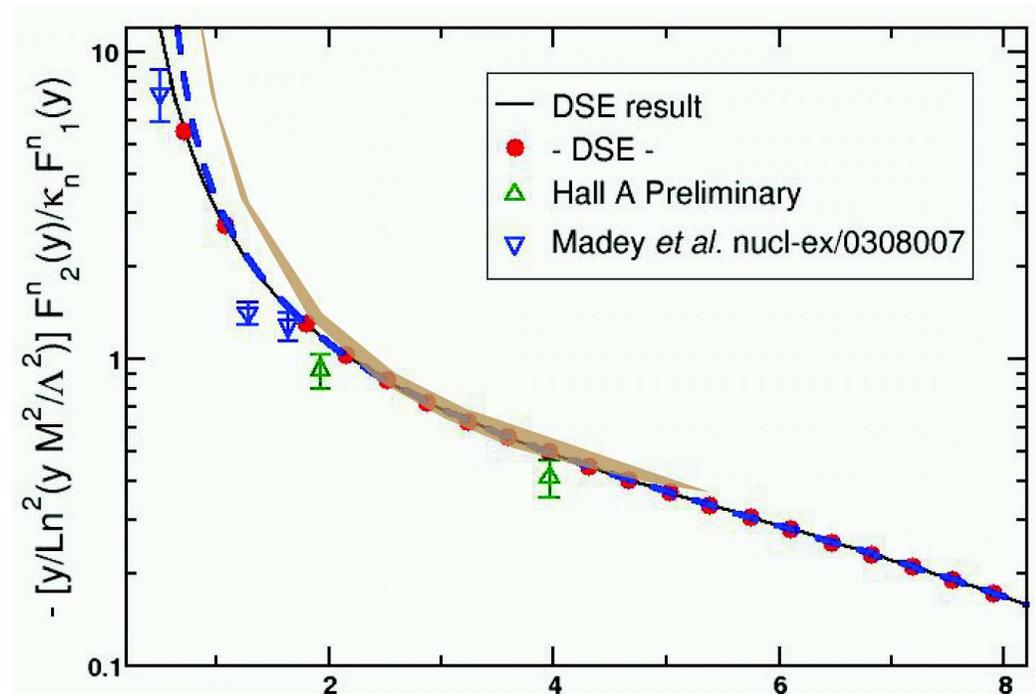
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Brown band
– *ab initio* RL result





[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Pion Cloud

F2 – neutron



[First](#)

[Contents](#)

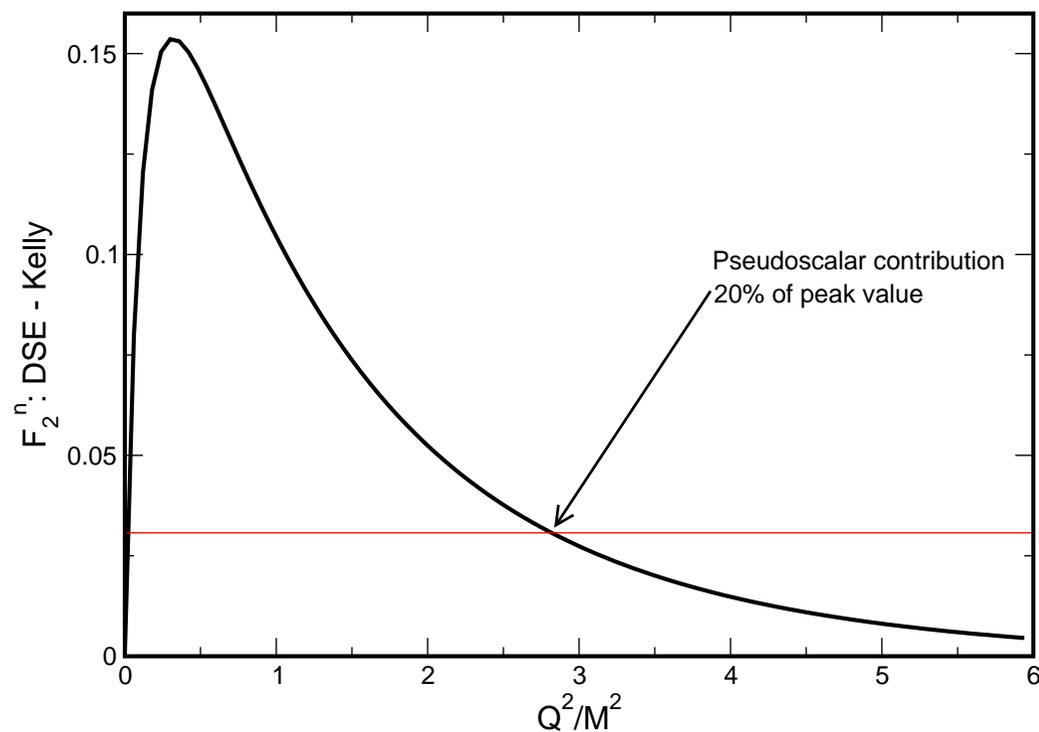
[Back](#)

[Conclusion](#)

Pion Cloud

F2 – neutron

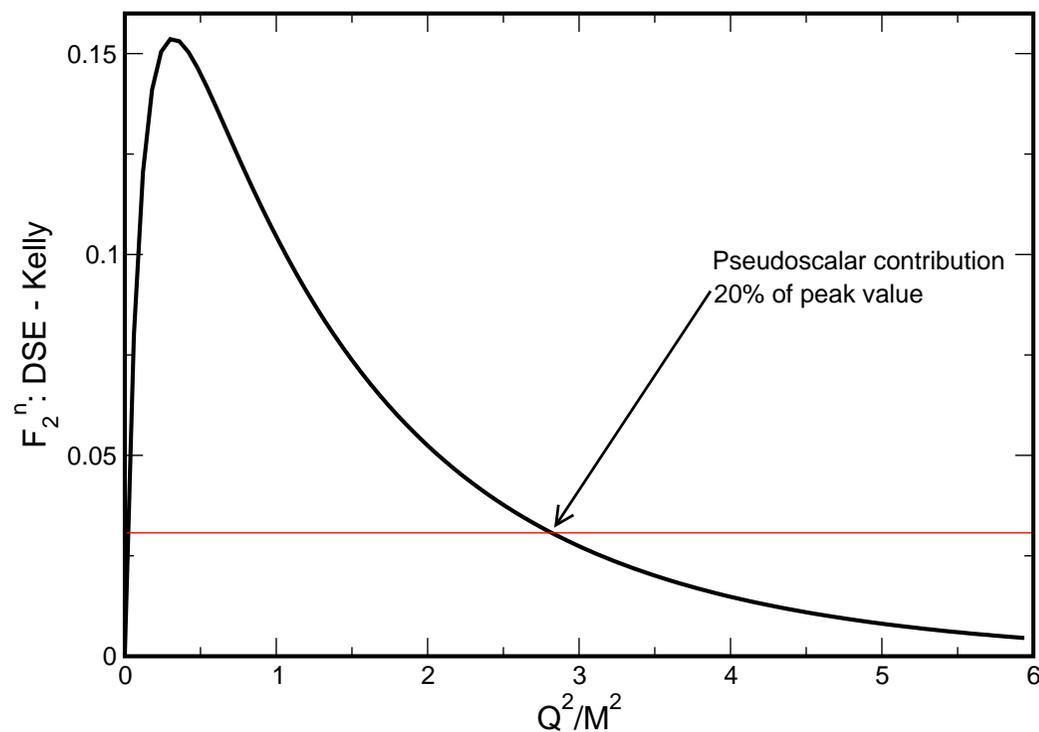
- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core



Pion Cloud

F2 – neutron

- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core
- Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4 M_N^2$





Epilogue



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Epilogue



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Epilogue

- DCSB exists in QCD.



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Epilogue

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 - It predicts, amongst other things, that
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Epilogue

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nothing!

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- Tool enabling insight to be drawn from experiment into long-range piece of interaction between light-quarks



Contents

1. Universal Truths
2. QCD's Challenges
3. Dichotomy of the Pion
4. Dressed-Quark Propagator
5. Frontiers of Nuclear Science
6. Hadrons
7. Confinement
8. Bethe-Salpeter Kernel
9. Persistent Challenge
10. Radial Excitations
11. Radial Excitations & Lattice-QCD
12. Pion FF
13. Calculated Pion FF
14. All Pion Form Factors
15. Nucleon Challenge
16. Unifying Meson & Nucleon
17. Faddeev equation
18. Diquark correlations
19. Ab-initio study of mesons & nucleons
20. $r_\pi f_\pi$
21. Nucleon-Photon Vertex
22. DSE-based Faddeev Equation
23. Ratio of Neutron FFs
24. Pion Cloud

