

# *Hadron Form Factors*

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# Universal Truths



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# PRIDE AND PREJUDICE

Starring  
Colin Firth  
and  
Jennifer Ehle



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- Form factors give information about distribution of hadron's characterising properties amongst its QCD constituents.



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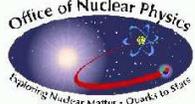
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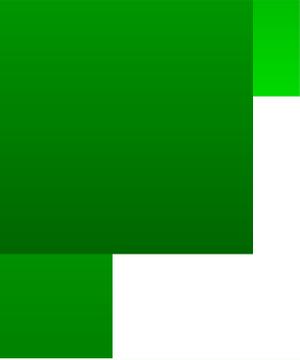


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- DCSB is most important mass generating mechanism for matter in the Universe. Higgs mechanism is irrelevant to light-quarks.
- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.



# QCD's Challenges

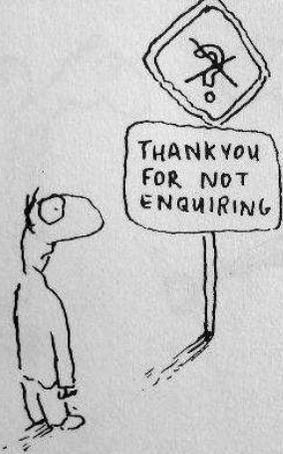


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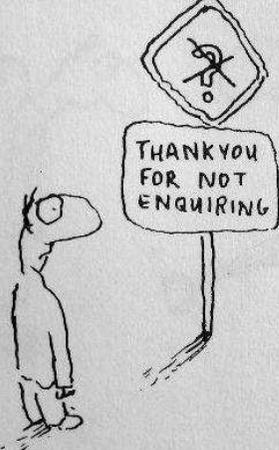
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  - Very unnatural pattern of bound state masses
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## Understand Emergent Phenomena

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- Neither of these phenomena is apparent in QCD's Lagrangian **yet** they are the dominant determining characteristics of real-world QCD.
- QCD – Complex behaviour  
arises from apparently simple rules



# Why?



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# Why?

- The nucleon and pion hold special places in non-perturbative studies of QCD.



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- Experimental and theoretical studies of nucleon electromagnetic form factors have made rapid and significant progress during the last several years, including new data in the time like region, and material gains have been made in studying the pion form factor.
- Despite this, many urgent questions remain unanswered.



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# Some Questions

- What is the role of pion cloud in nucleon electromagnetic structure?
- Can we understand the pion cloud in a more quantitative and, perhaps, model-independent way?



# Some Questions

- Where is the transition from non-pQCD to pQCD in the pion and nucleon electromagnetic form factors?



# Some Questions

- Do we understand the high  $Q^2$  behavior of the proton form factor ratio in the space-like region?
- Can we make model-independent statements about the role of relativity or orbital angular momentum in the nucleon?



# Some Questions

- Can we understand the rich structure of the time-like proton form factors in terms of resonances?
- What do we expect for the proton form factor ratio in the time-like region?
- What is the relation between proton and neutron form factor in the time-like region?
- How do we understand the ratio between time-like and space-like form factors?



# Some Questions

- What is the role of two-photon exchange contributions in understanding the discrepancy between the polarization and Rosenbluth measurements of the proton form factor ratio?
- What is the impact of these contributions on other form factor measurements?



# Some Questions

- How accurately can the pion form factor be extracted from the  $ep \rightarrow e'n\pi^+$  reaction?





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- Current status is described in
  - J. Arrington, C. D. Roberts and J. M. Zanotti  
“Nucleon electromagnetic form factors,”  
J. Phys. G **34**, S23 (2007); [arXiv:nucl-th/0611050].
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“Nucleon electromagnetic form factors,”  
Prog. Part. Nucl. Phys. **59**, 694 (2007);  
[arXiv:hep-ph/0612014].
- Most recently:  
“ECT\* Workshop on Hadron Electromagnetic Form Factors”  
Organisers: Alexandrou, Arrington, Friedrich, Maas, Roberts  
Presentations, etc., available on-line  
<http://ect08.phy.anl.gov/>



# *Dichotomy of Pion*

## *– Goldstone Mode and Bound state*

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# *Dichotomy of Pion*

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How does one make an almost massless particle  
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# Dichotomy of Pion

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- **Not Allowed** to do it by fine-tuning a potential

Must exhibit  $m_\pi^2 \propto m_q$

Current Algebra ... 1968



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- well-defined and valid chiral limit;
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**Highly Nontrivial**



# Pion Form Factors



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- Must similarly require prediction of  $\gamma^* \pi \rightarrow \pi\pi$  and all other anomalous processes



# What's the Problem?



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# What's the Problem?

- Minimal requirements
  - detailed understanding of connection between **Current-quark** and **Constituent-quark** masses;
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  - Interaction between quarks – the **Interquark “Potential”** – **unknown** throughout **> 98%** of a hadron's volume



# *Intranucleon Interaction*



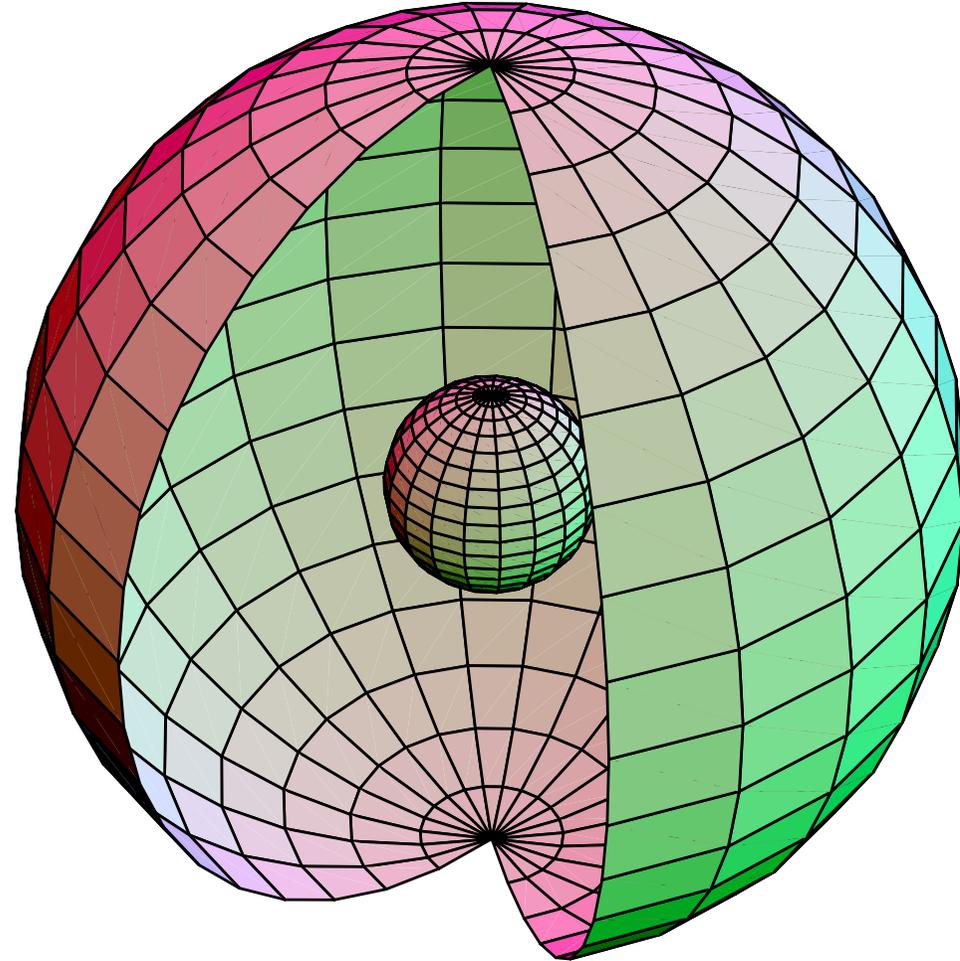
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# Intranucleon Interaction



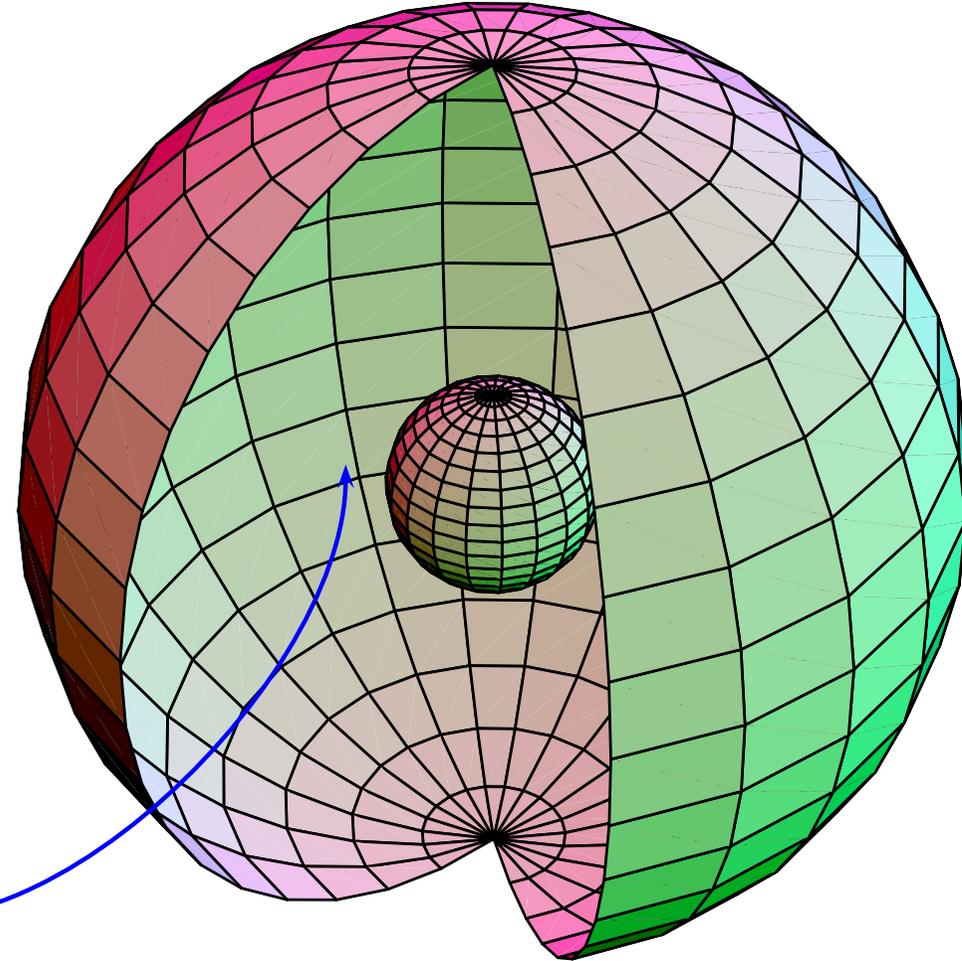
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# Intranucleon Interaction



98% of the volume



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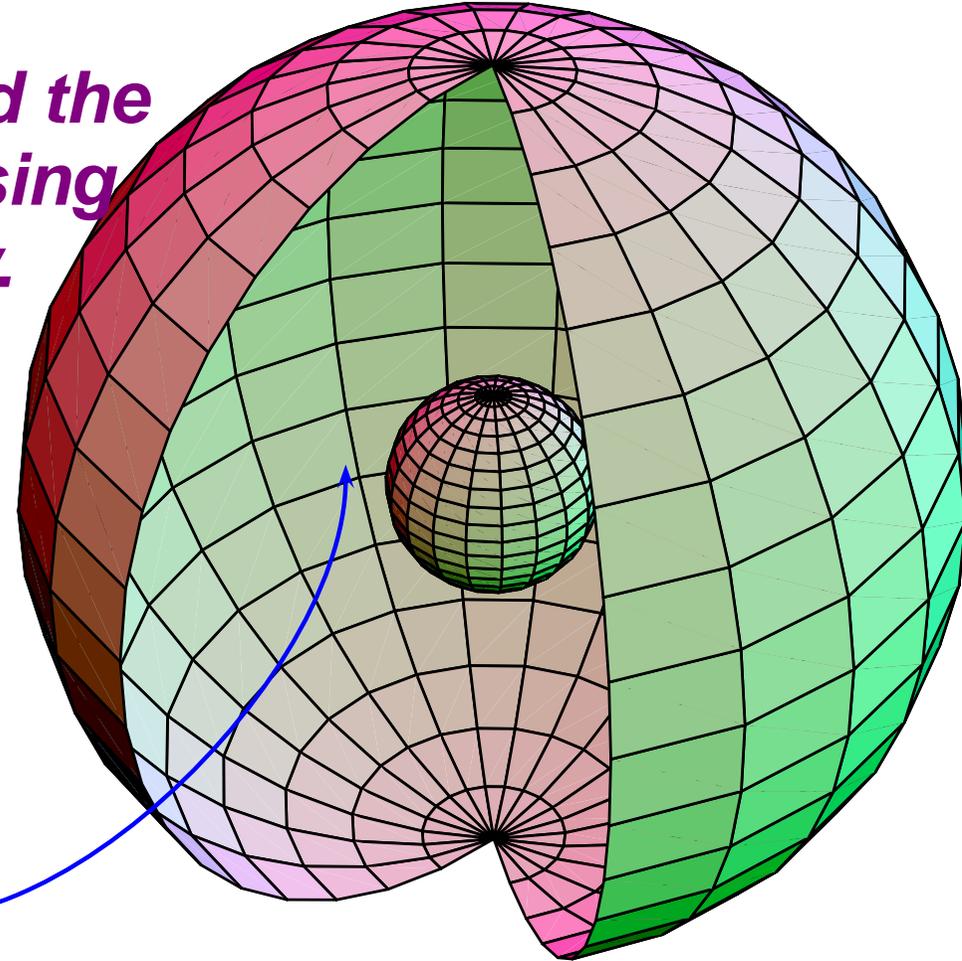
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# What is the Intranucleon Interaction?

*The question must be rigorously defined, and the answer mapped out using experiment and theory.*



98% of the volume



# Dyson-Schwinger Equations



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# *Dyson-Schwinger Equations*

## *Dressed-Quark Propagator*

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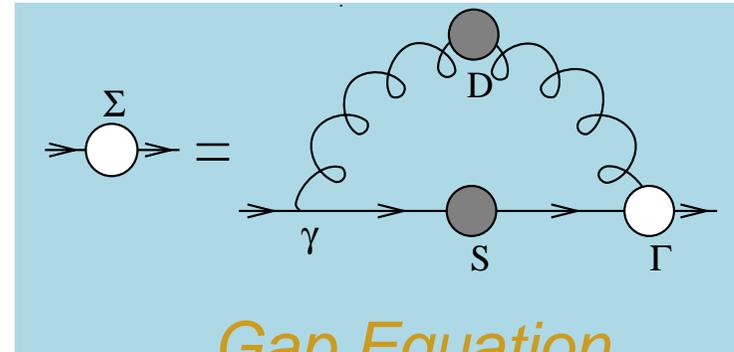
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# Dyson-Schwinger Equations

## Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



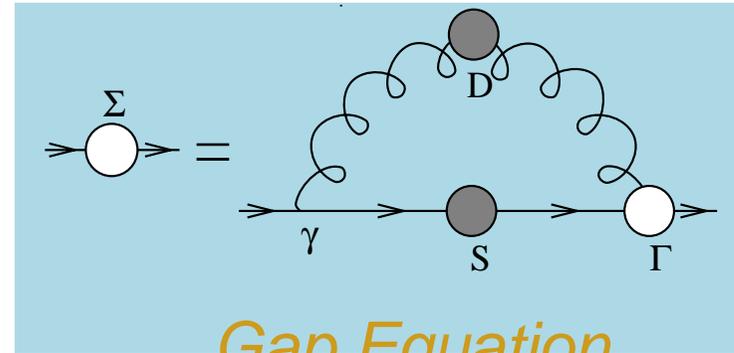
Gap Equation



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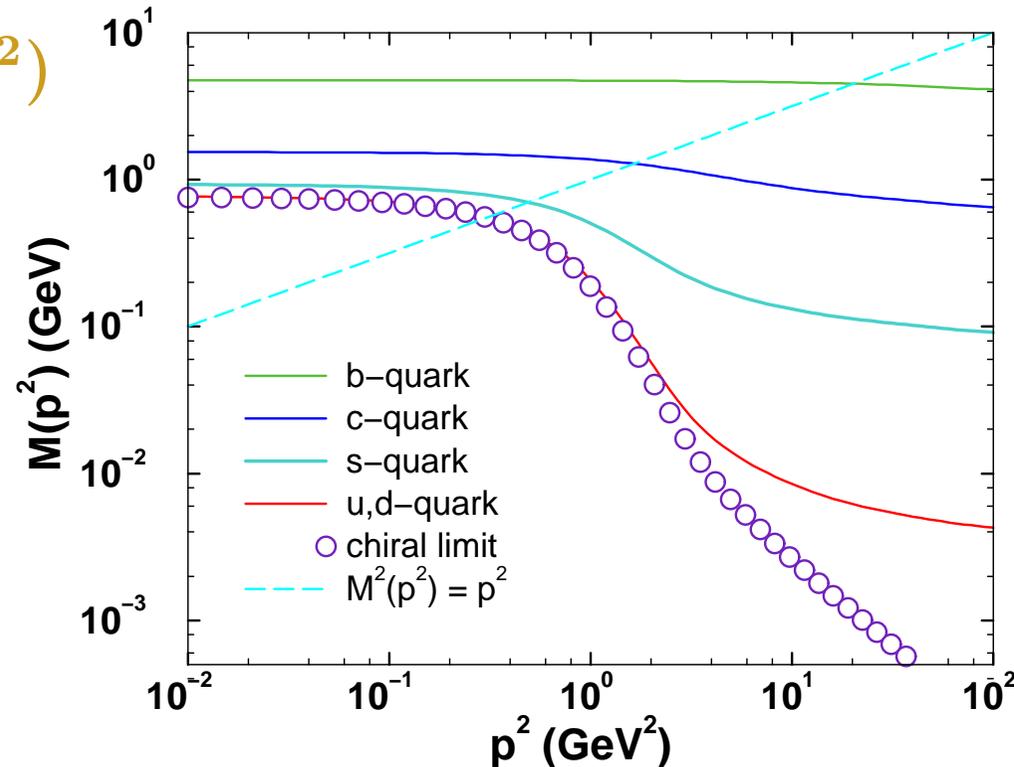
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Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

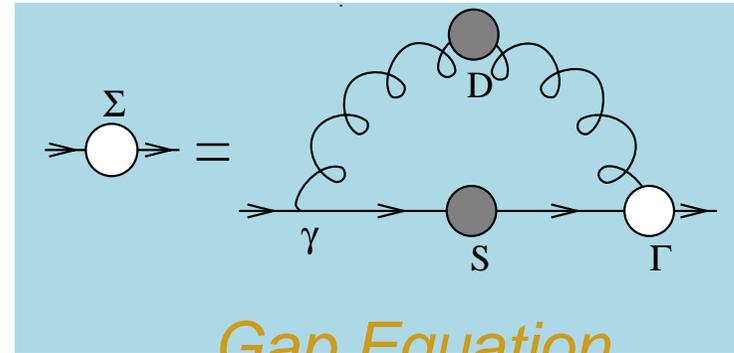
⇒ IR Enhancement of  $M(p^2)$



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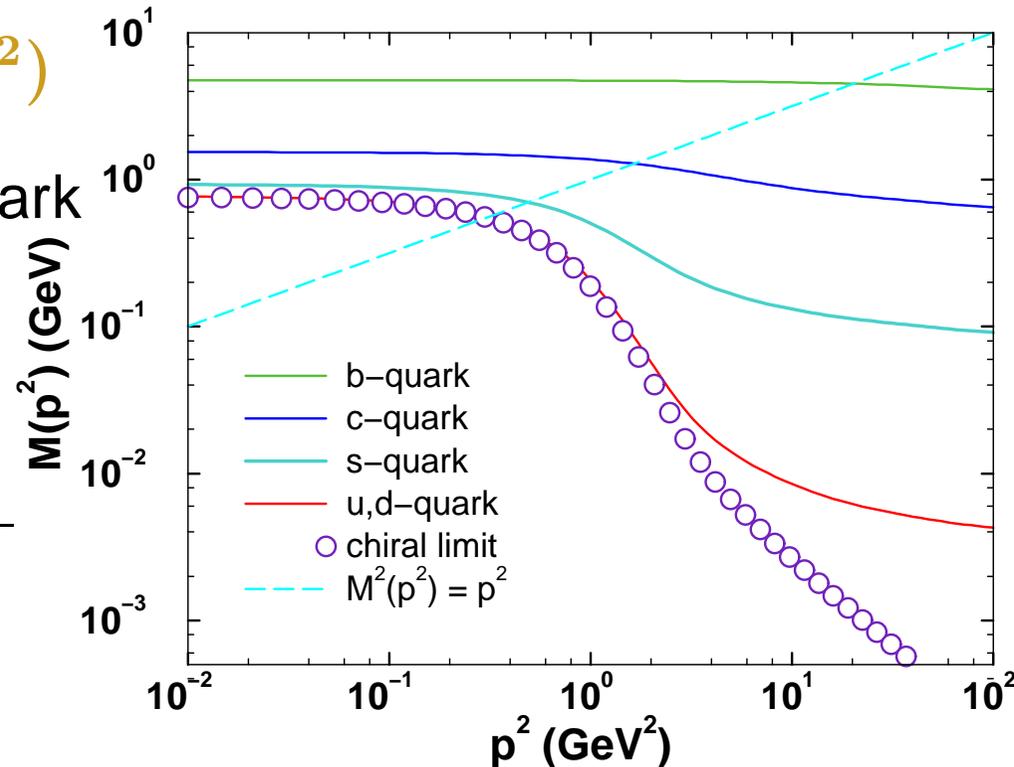
- Gap Equation's Kernel Enhanced on IR domain

⇒ IR Enhancement of  $M(p^2)$

● Euclidean Constituent-Quark

Mass:  $M_f^E: p^2 = M(p^2)^2$

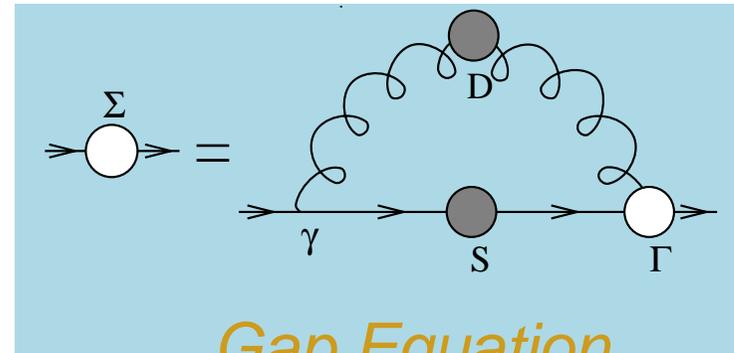
flavour	$u/d$	$s$	$c$	$b$
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	$\sim 10$	$\sim 1.5$	$\sim 1.1$



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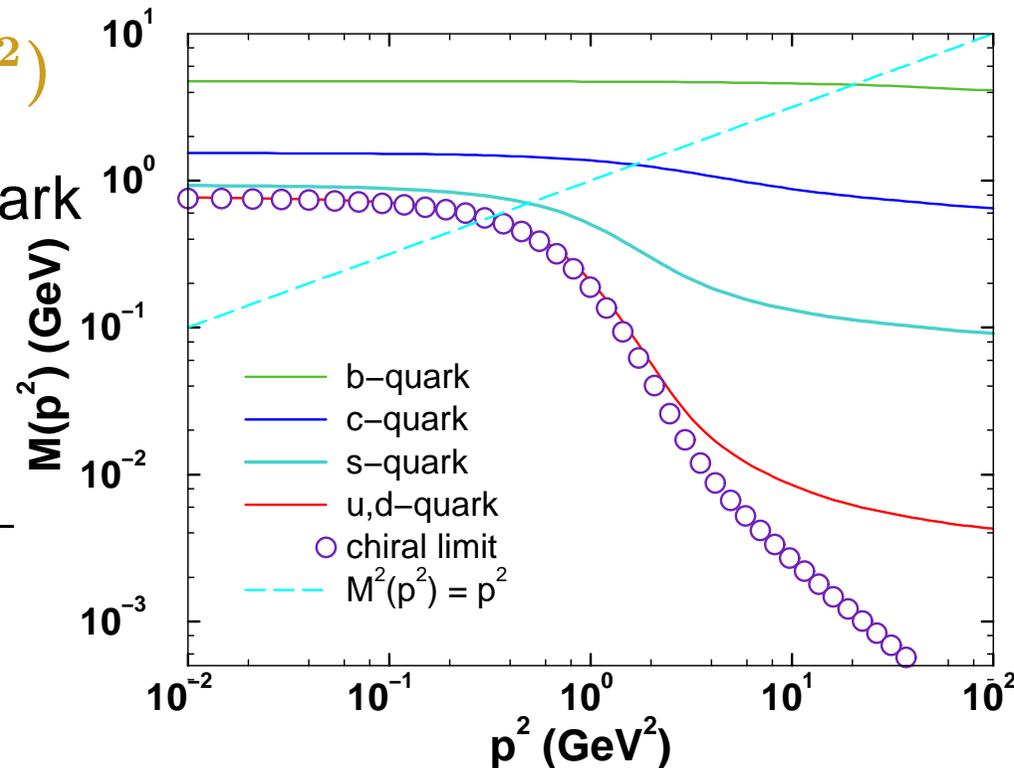
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Predictions confirmed in numerical simulations of lattice-QCD



# Hadrons



- Established understanding of two- and three-point functions



# Hadrons



- Established understanding of two- and three-point functions
- What about bound states?



# Hadrons



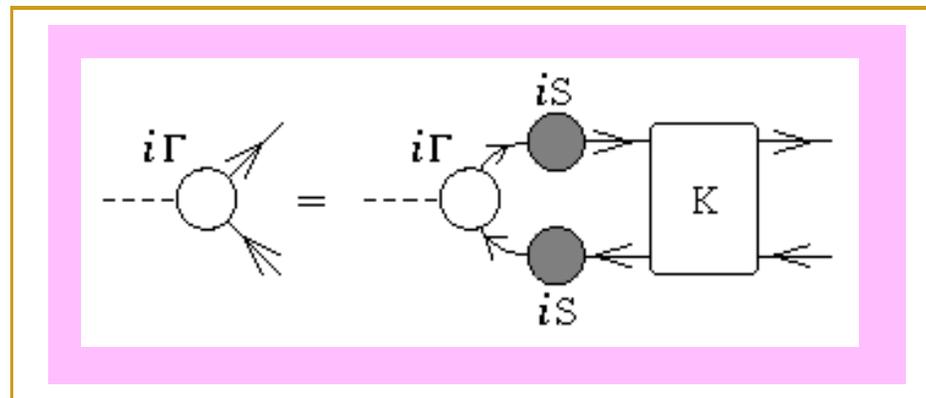
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Comparison with experiment is  
**impossible**
- They appear as pole contributions  
to  $n \geq 3$ -point colour-singlet  
Schwinger functions

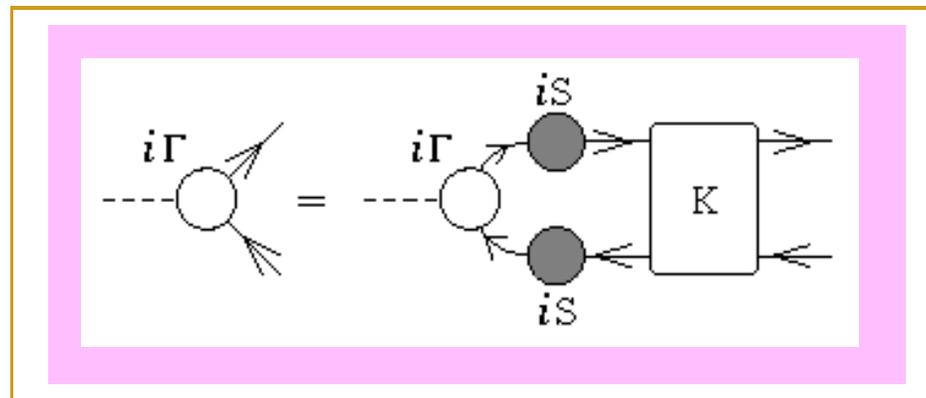


- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

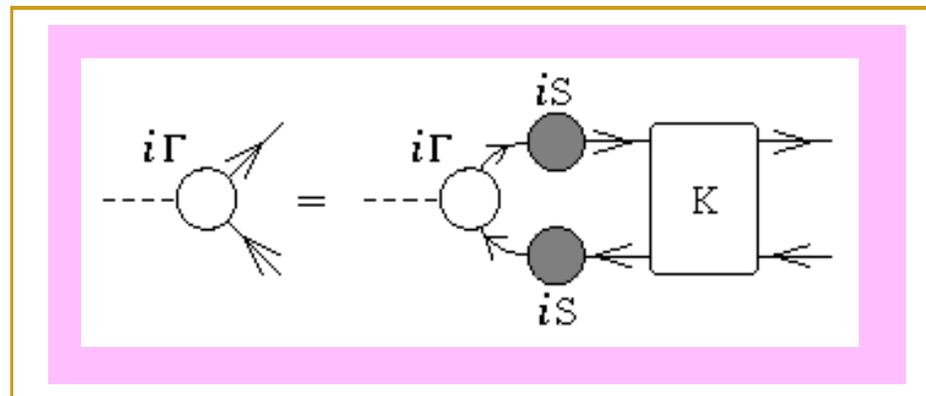
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QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel,  $K$ ?

- Without bound states, Comparison with experiment is impossible
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QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel,  $K$ ?

or

# What is the light-quark Long-Range Potential?



# What is the light-quark Long-Range Potential?



Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD **is not related** in any simple way to the light-quark interaction.



# Bethe-Salpeter Kernel



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Exploring Nuclear Matter - Quarks to Stars



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# Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^l(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f^l i\gamma_5 + \frac{1}{2} \lambda_f^l i\gamma_5 \mathcal{S}^{-1}(k_-)$$
$$- M_\zeta i\Gamma_5^l(k; P) - i\Gamma_5^l(k; P) M_\zeta$$

QFT Statement of Chiral Symmetry



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Satisfies BSE

Satisfies DSE



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Kernels very different

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- Relation **must** be preserved by truncation
- **Failure**  $\Rightarrow$  Explicit Violation of QCD's Chiral Symmetry



# Pion Form Factor

Procedure Now Straightforward



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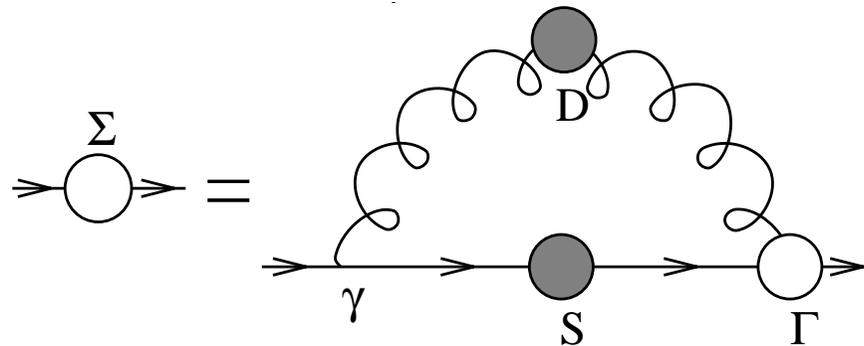
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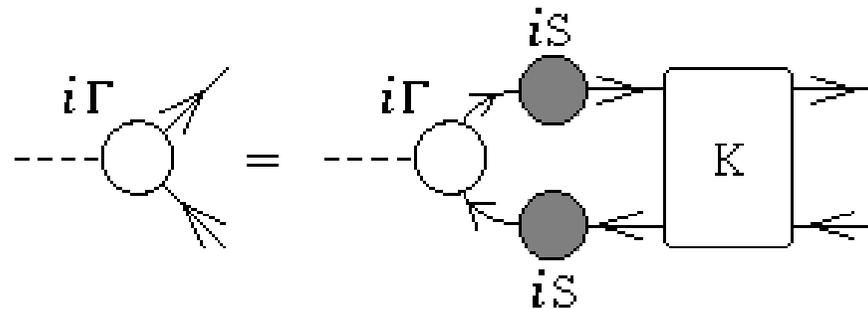
# Pion Form Factor

- Solve Gap Equation
  - ⇒ Dressed-Quark Propagator,  $S(p)$



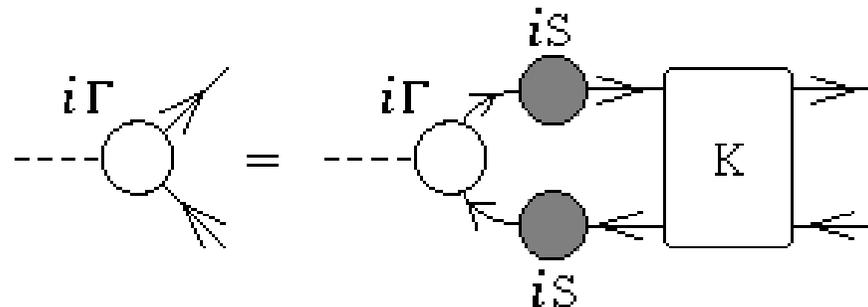
# Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel,  $K$
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude,  $\Gamma_\pi$



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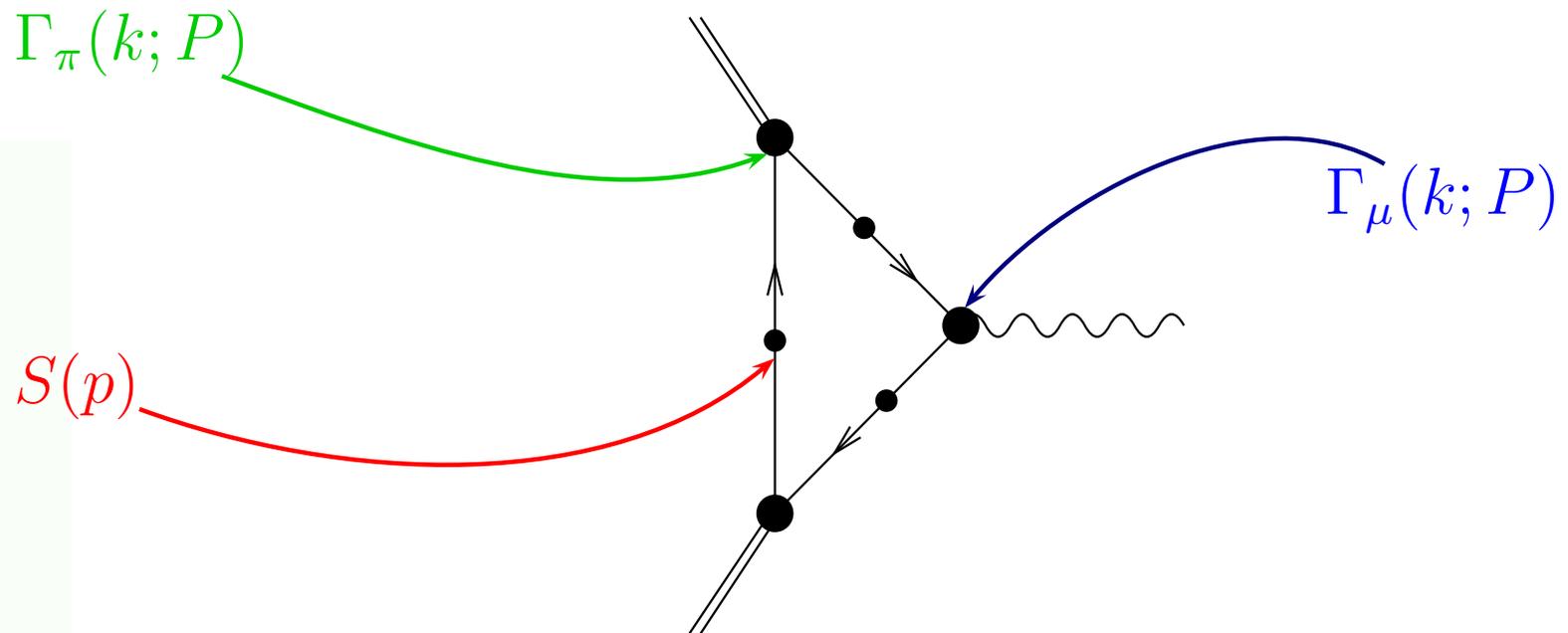


- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Gluon Vertex,  $\Gamma_\mu$



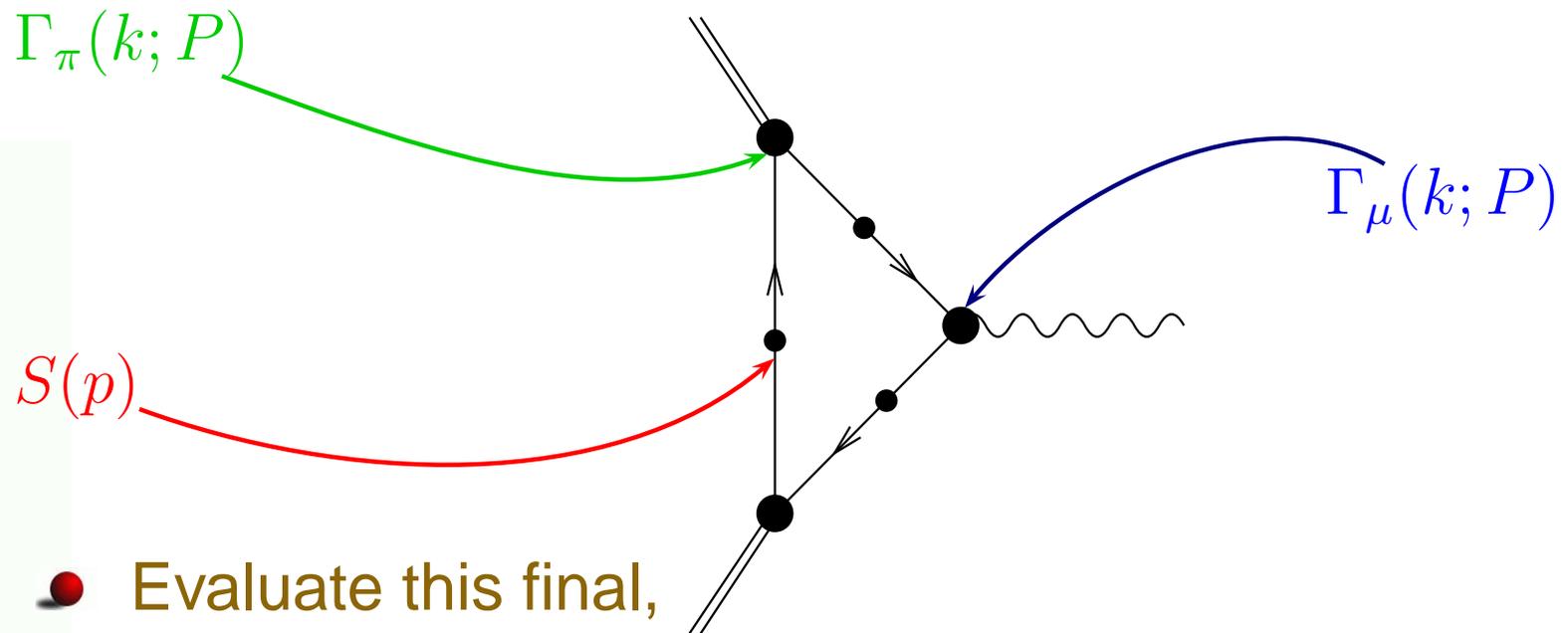
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- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



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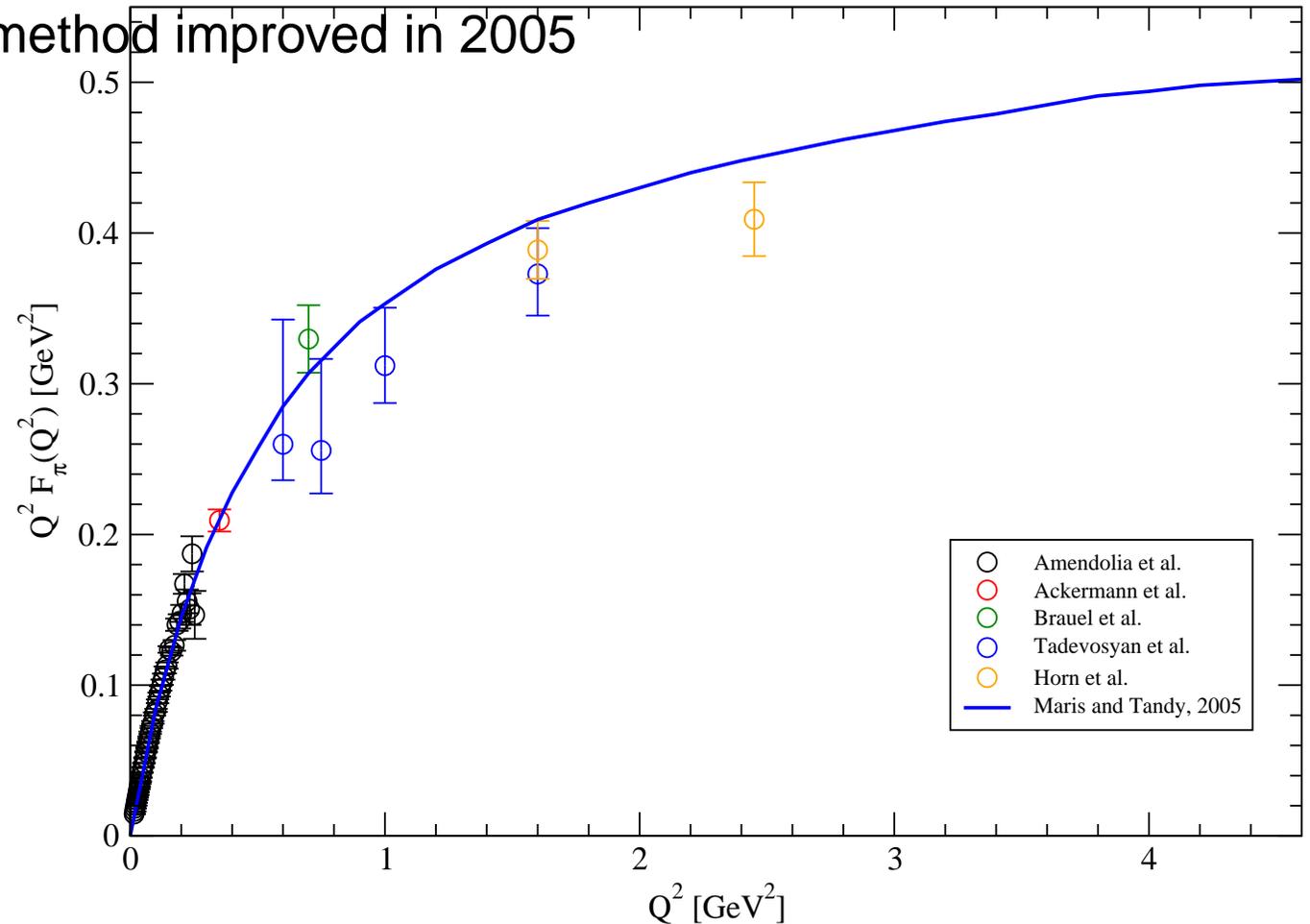
- Evaluate this final, three-dimensional integral



# Calculated Pion Form Factor

Calculation first published in 1999; No Parameters Varied

Numerical method improved in 2005

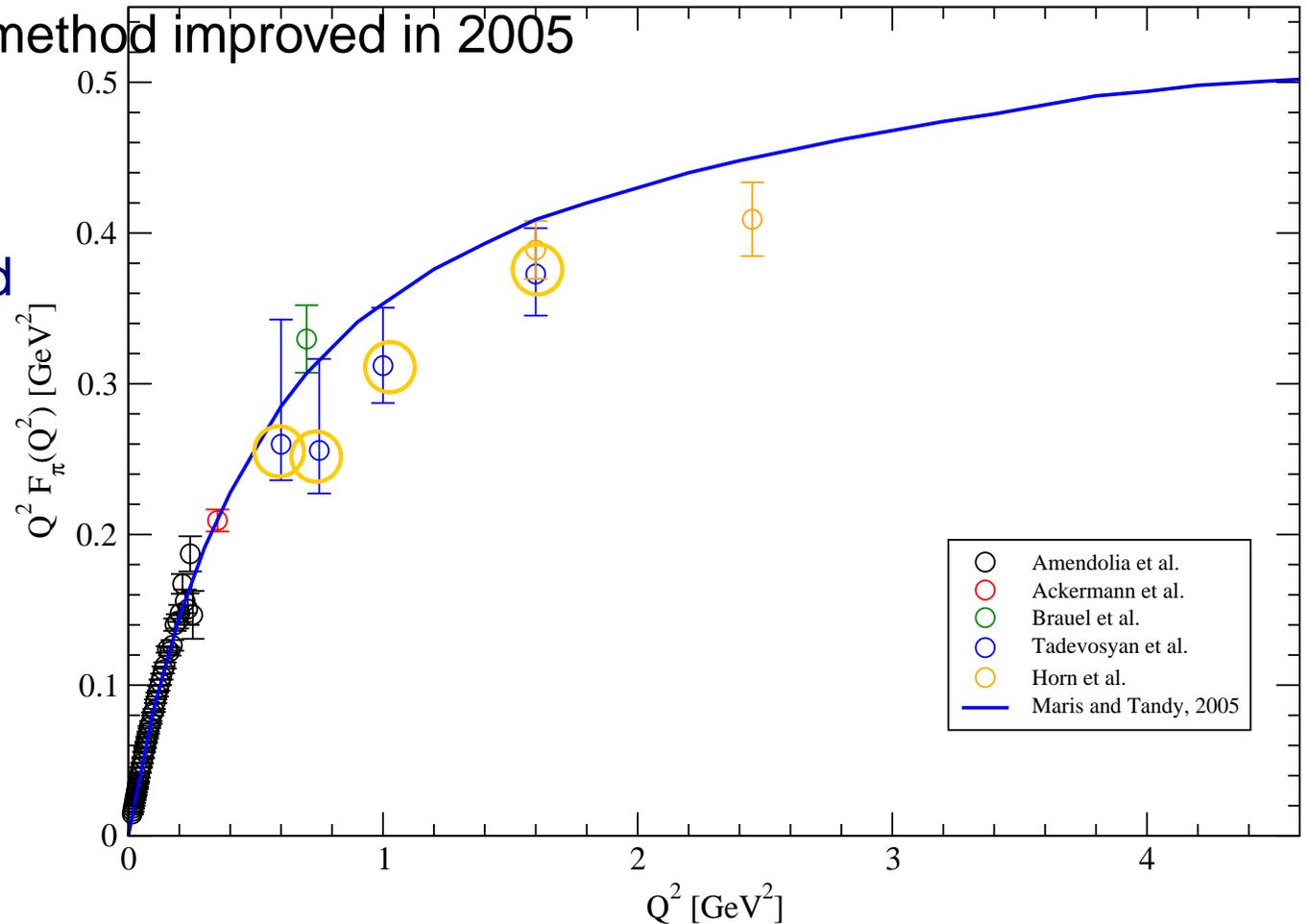


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Numerical method improved in 2005

Data published  
in 2001.  
Subsequently  
revised





# Timelike Pion Form Factor



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# Timelike Pion Form Factor

*Ab initio* calculation into timelike region  
Deeper than ground-state  $\rho$ -meson pole



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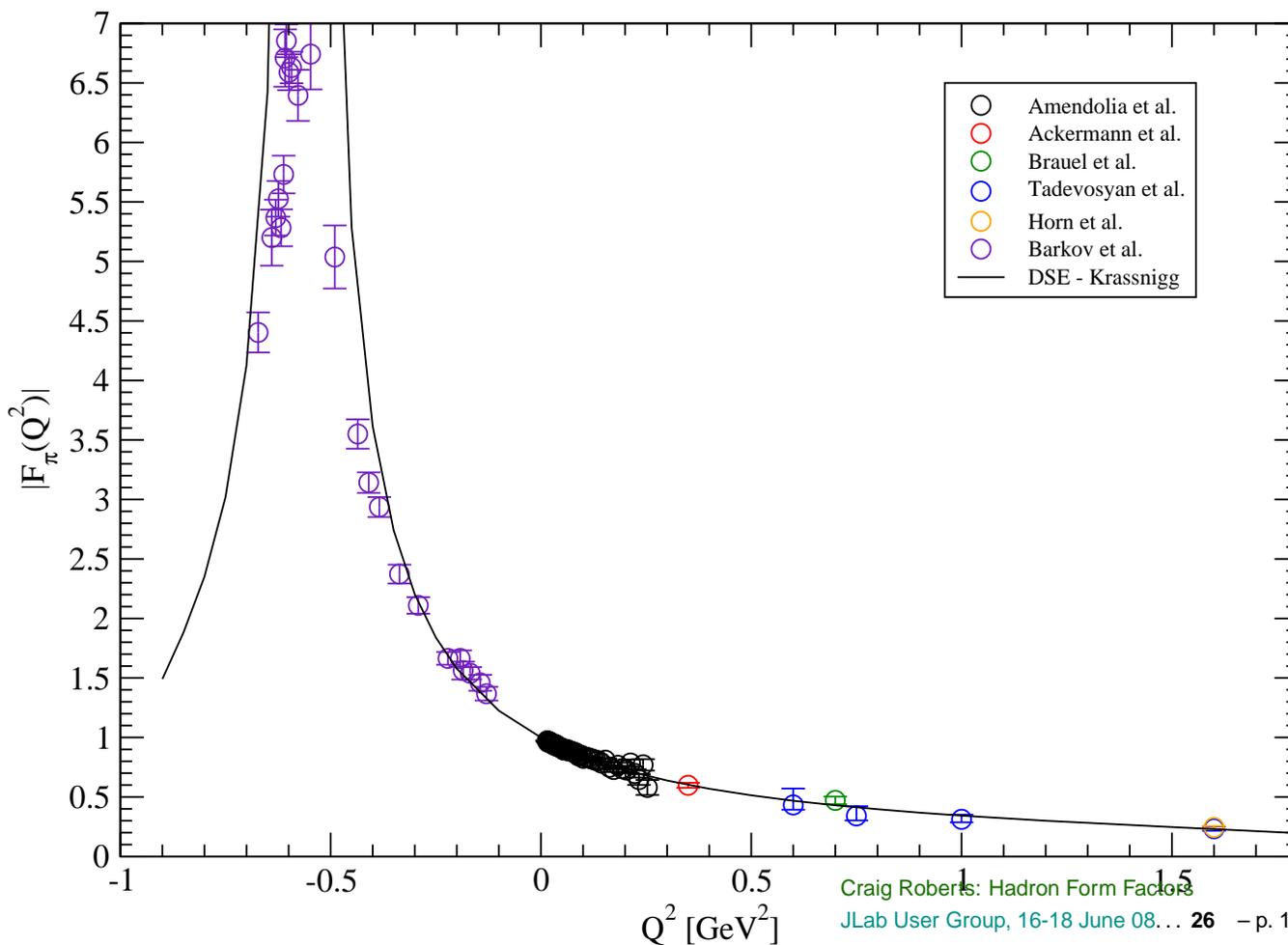
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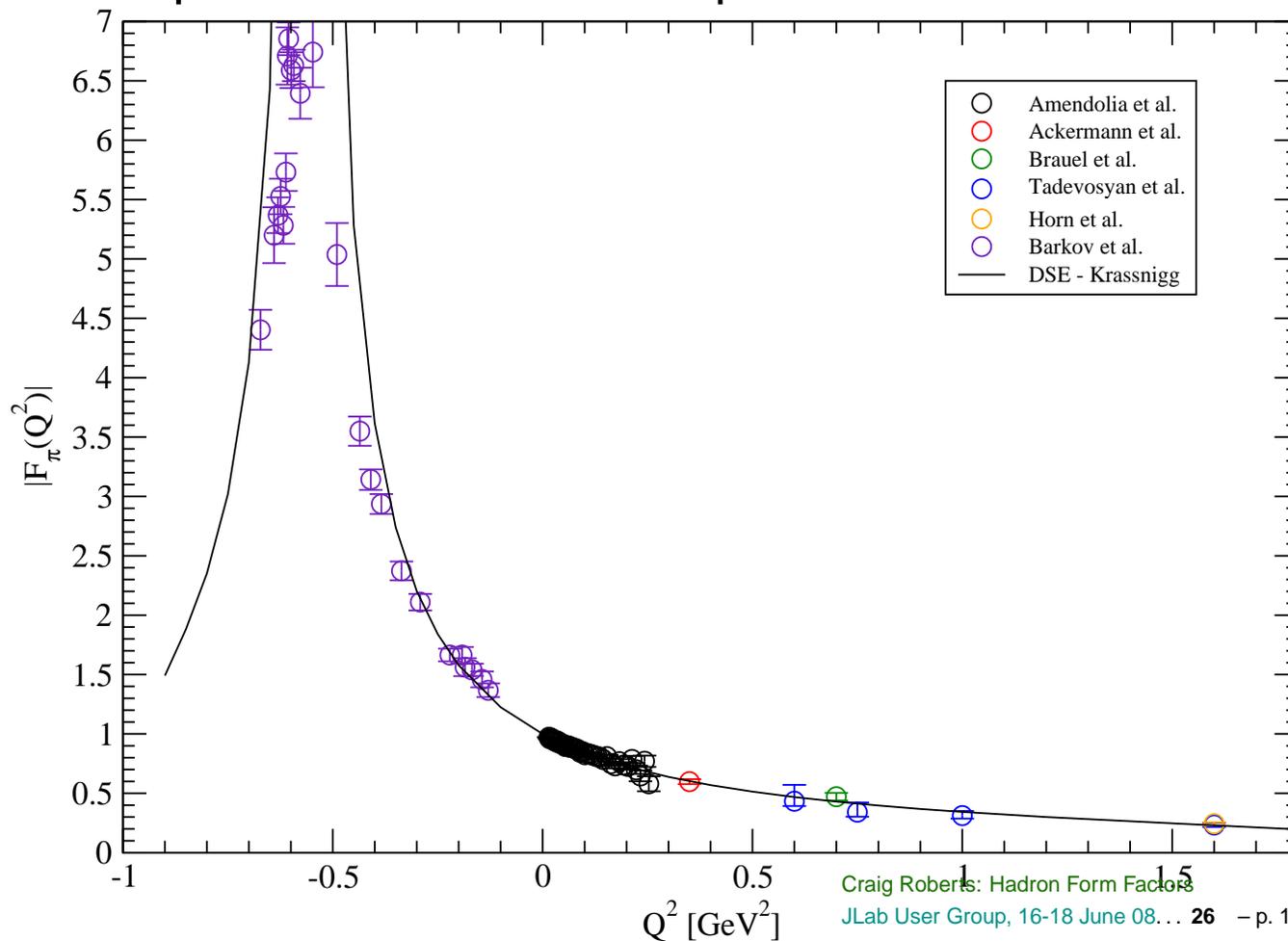


# Timelike Pion Form Factor

*Ab initio* calculation into timelike region

Deeper than ground-state  $\rho$ -meson pole

$\rho$ -meson not put in “by hand” – generated dynamically as a bound-state of dressed-quark and dressed-antiquark



# *Nucleon Challenge*



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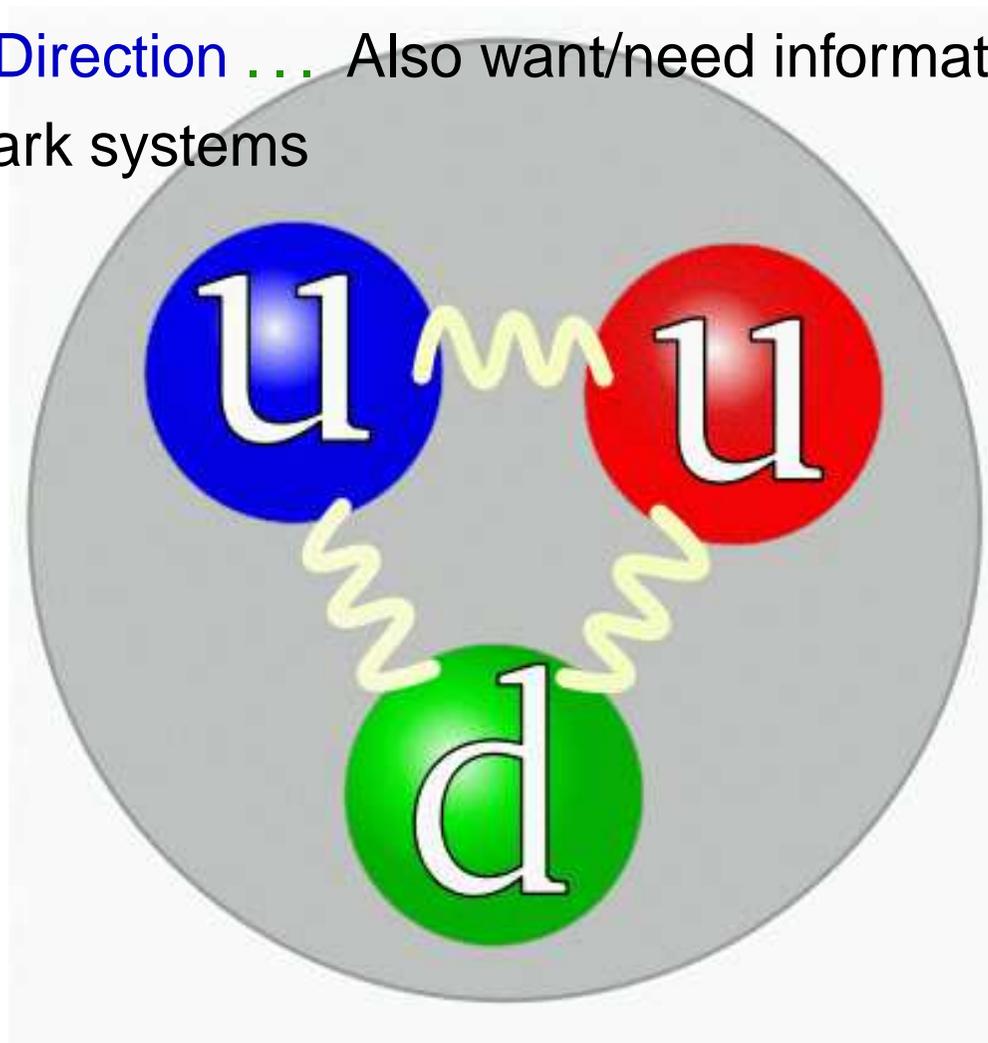
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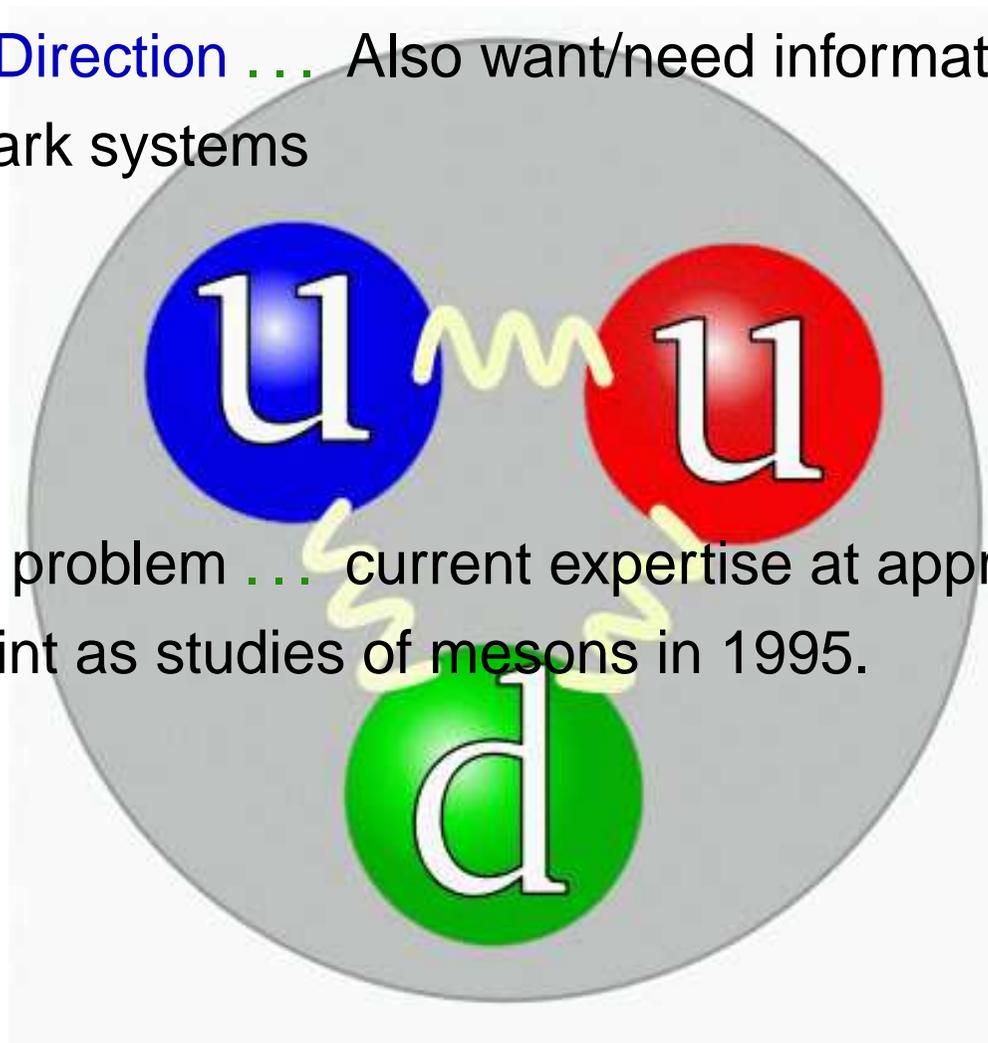
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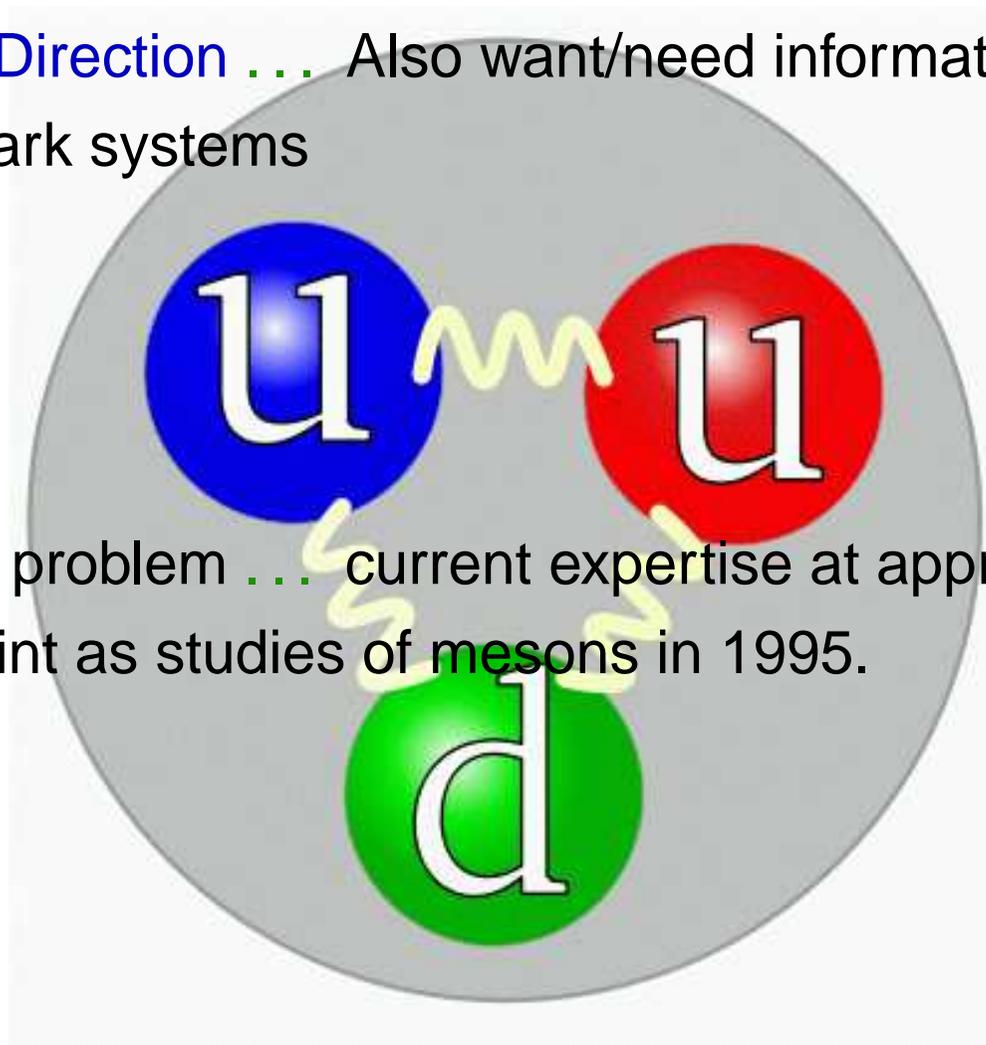


- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.



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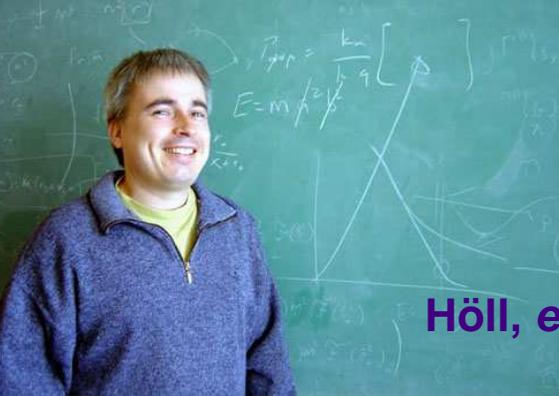
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- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.





# Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033

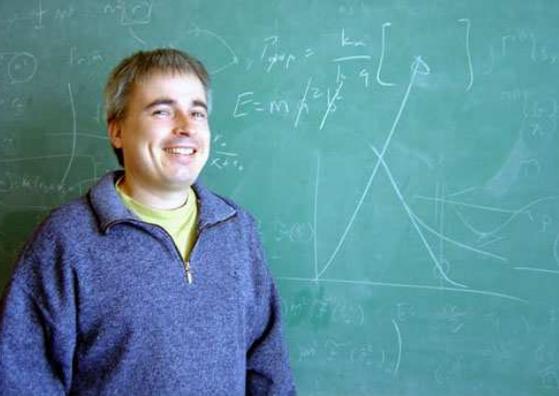


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# Nucleon EM Form Factors: A Précis

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# *Nucleon EM Form Factors: A Précis*

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# Nucleon EM Form Factors: A Précis

Cloët, *et al.*:  
arXiv:0710.2059, arXiv:0710.5746 & arXiv:0804.3118



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$$\left( \frac{1}{N_H} \sum_H \frac{[M_H^{\text{exp}} - M_H^{\text{calc}}]^2}{[M_H^{\text{exp}}]^2} \right)^{1/2} = 2\%$$



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(Oettel, Hellstern, Alkofer, Reinhardt: [nucl-th/9805054](#))



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- **But** is that good?
  - Cloudy Bag:  $\delta M_+^{\pi\text{-loop}} = -300$  to  $-400$  MeV!
- **Critical** to anticipate pion cloud effects

Roberts, Tandy, Thomas, *et al.*, nu-th/02010084



# *Faddeev equation*



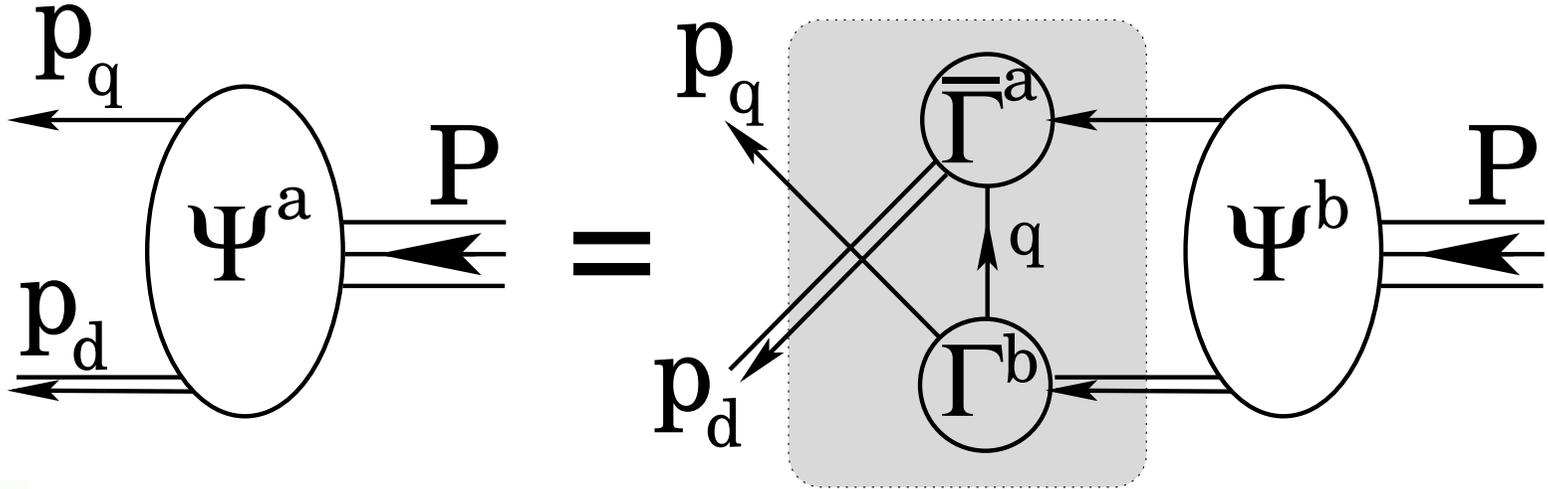
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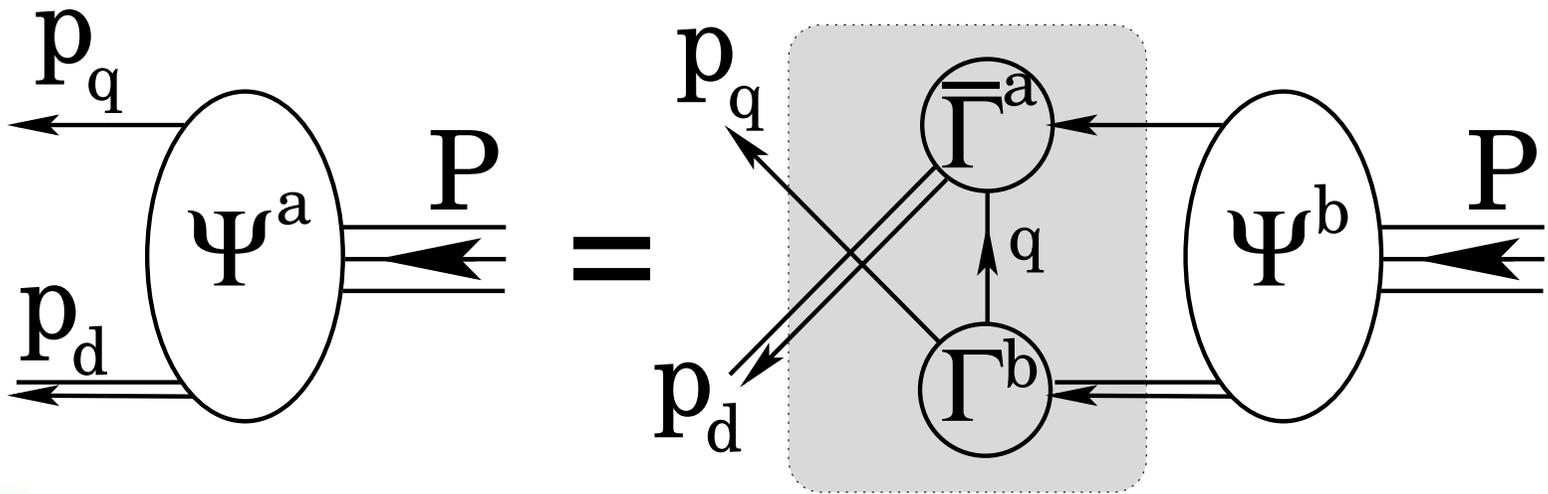
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# Faddeev equation



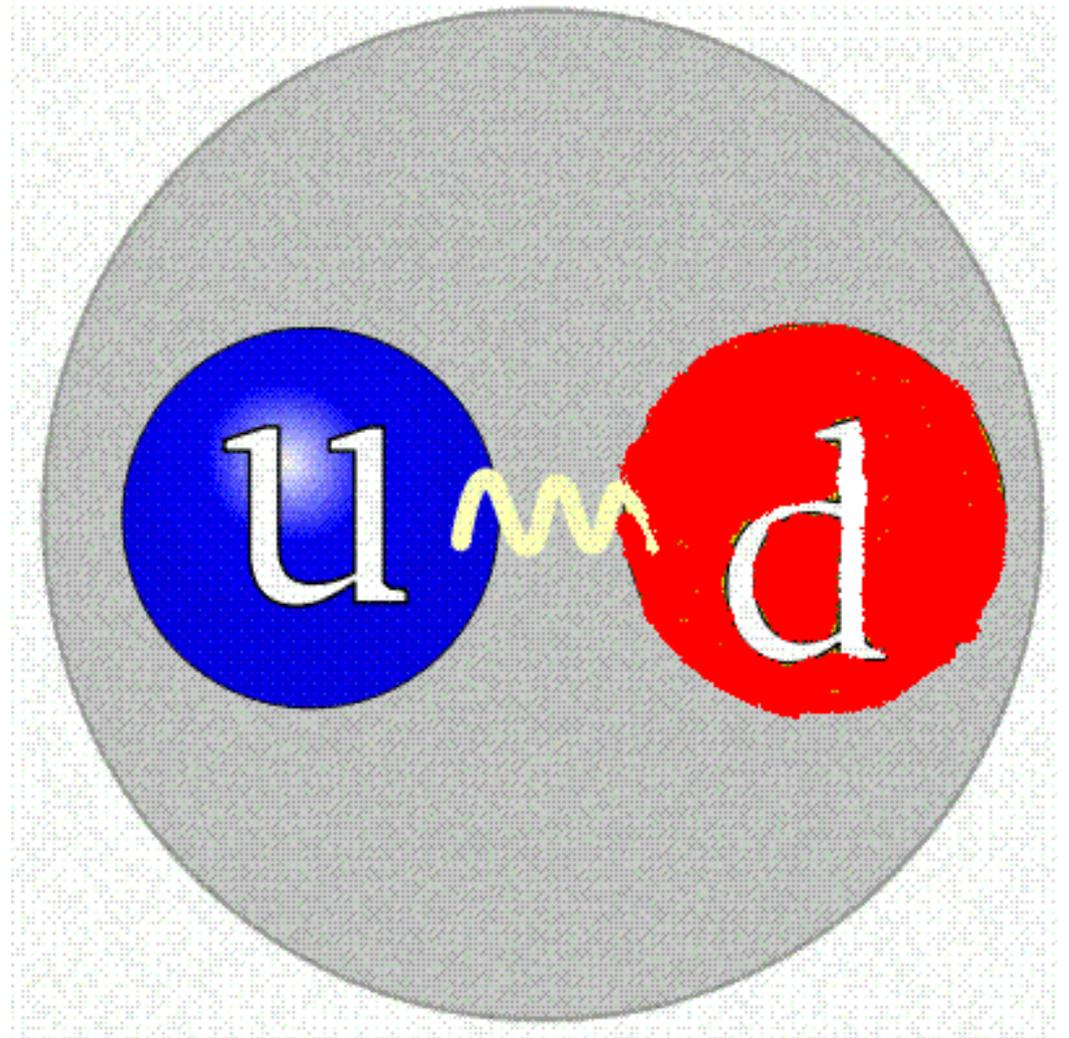
# Faddeev equation



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame Amplitude has ... *s*- , *p*- & *d*-wave correlations



# Diquark correlations



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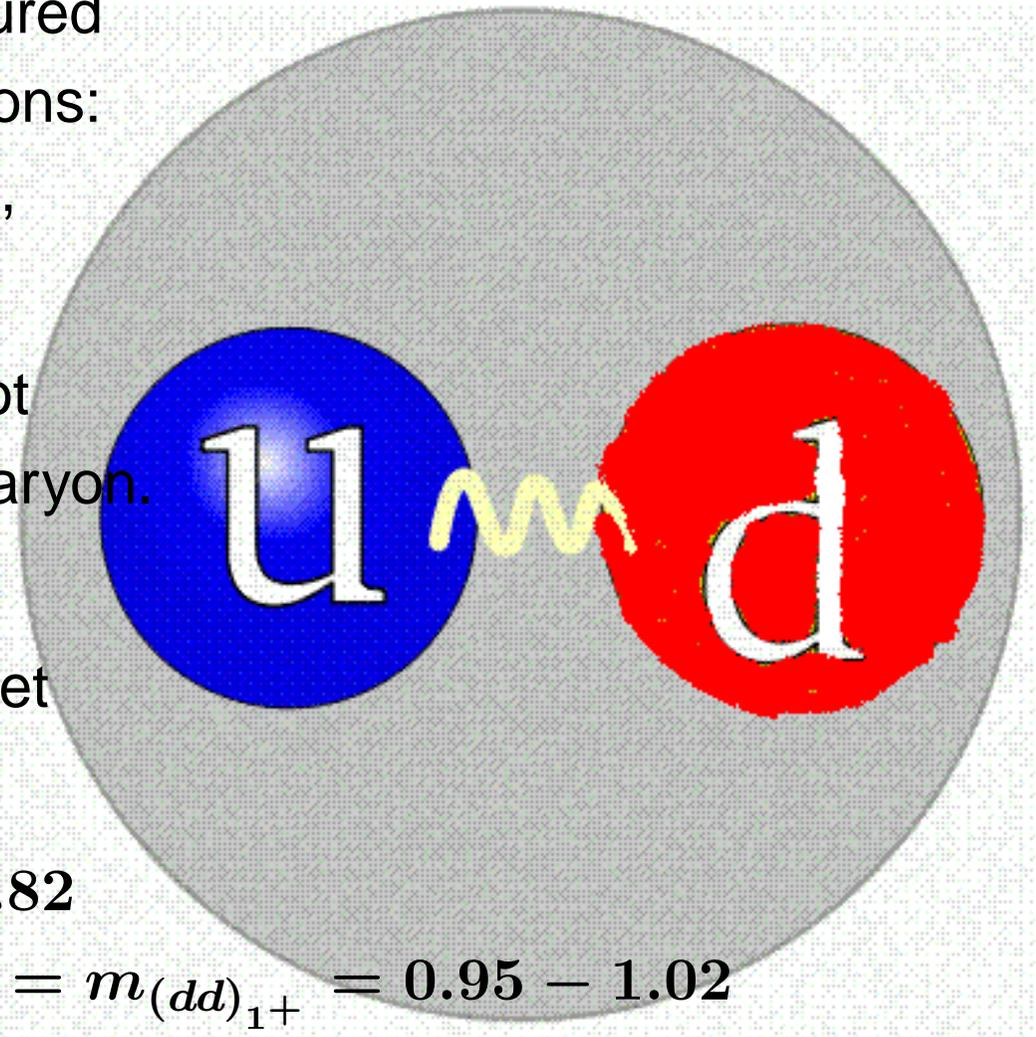
## QUARK-QUARK

Craig Roberts: Hadron Form Factors

JLab User Group, 16-18 June 08. . . 26 - p. 22/45

# Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue-red, blue-green, green-red
- Confined ... Does not escape from within baryon.



- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0+}} = 0.74 - 0.82$$

$$m_{(uu)_{1+}} = m_{(ud)_{1+}} = m_{(dd)_{1+}} = 0.95 - 1.02$$



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# *Nucleon-Photon Vertex*



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M. Oettel, M. Pichowsky  
and L. von Smekal, nu-th/9909082

6 terms ...

# ***Nucleon-Photon Vertex***

constructed systematically ... current conserved automatically  
for on-shell nucleons described by Faddeev Amplitude



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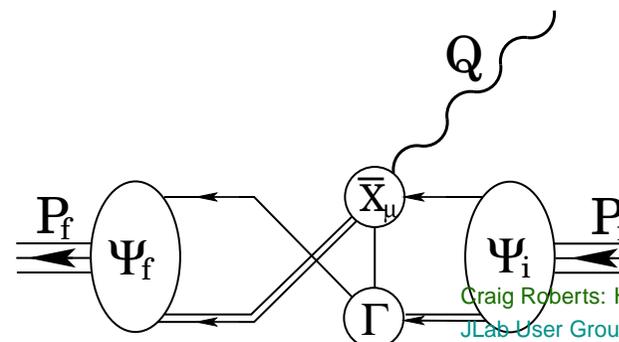
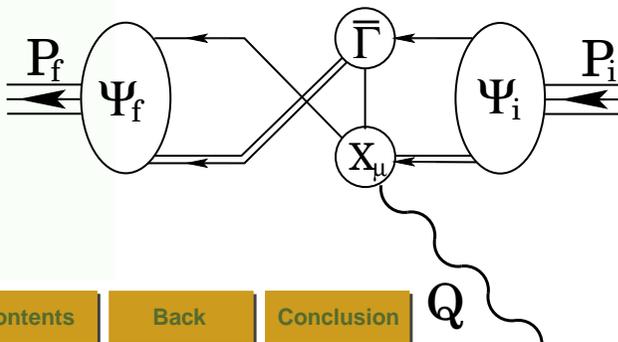
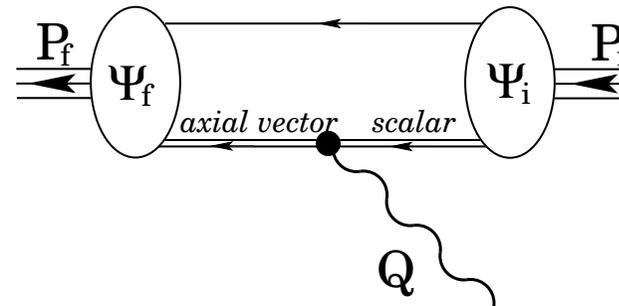
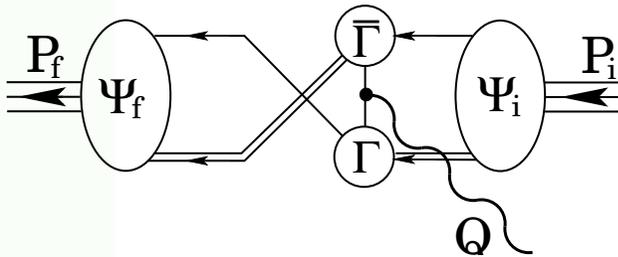
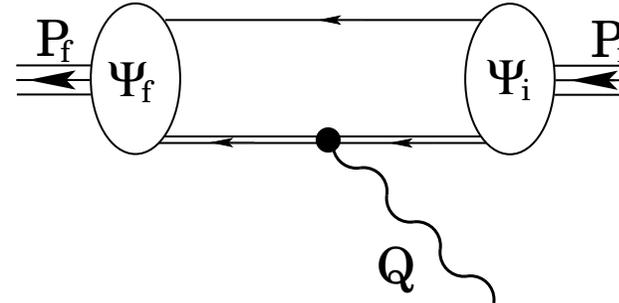
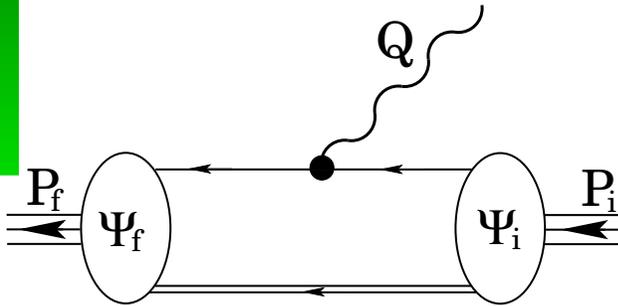
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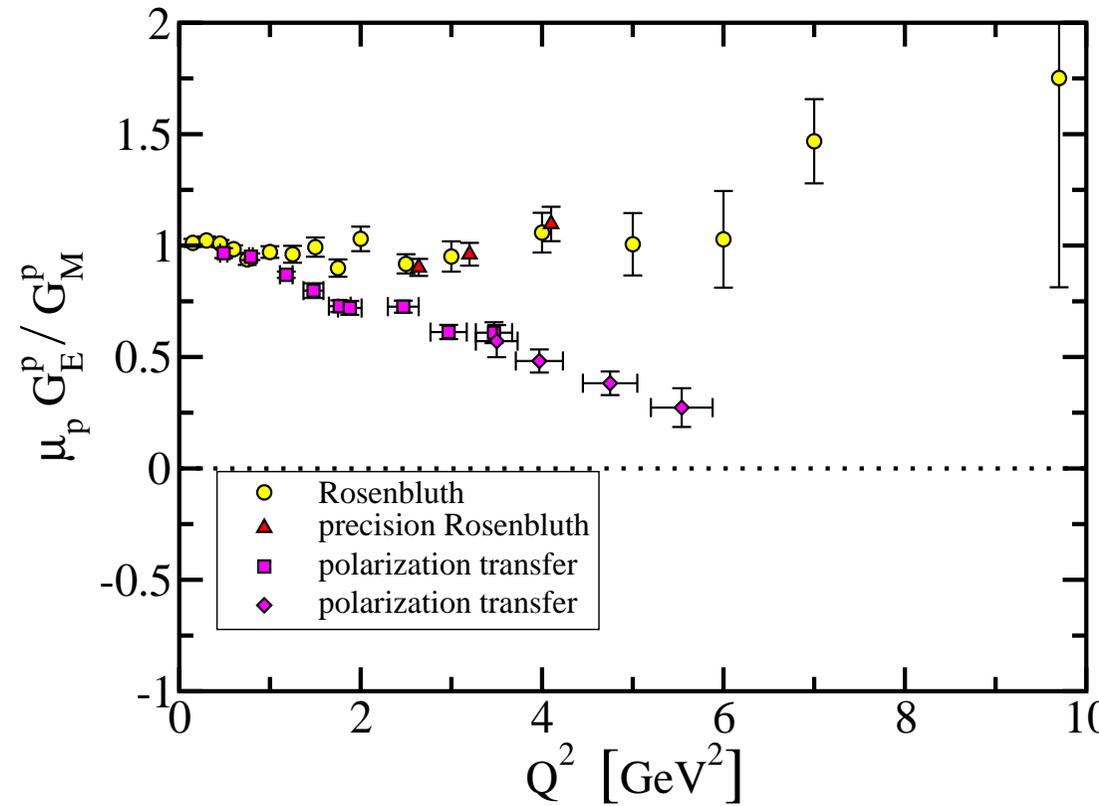
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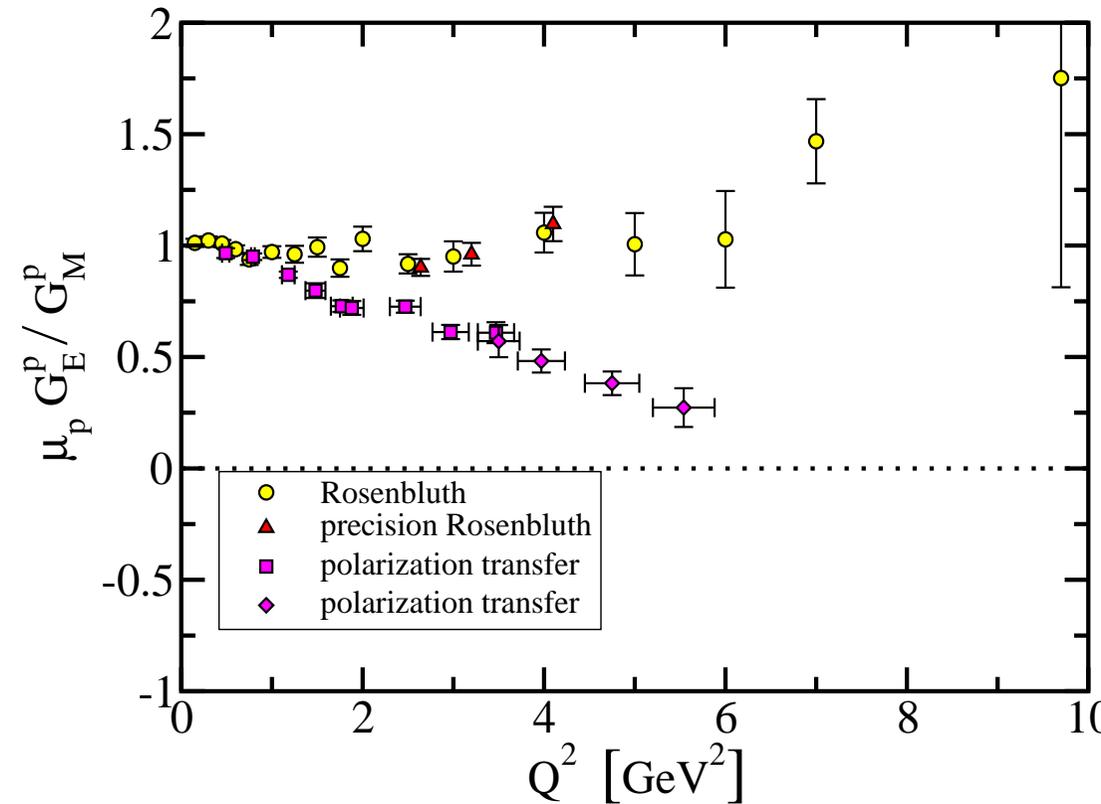
## $G_E/G_M$



# Form Factor Ratio:

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● Combine these elements ...

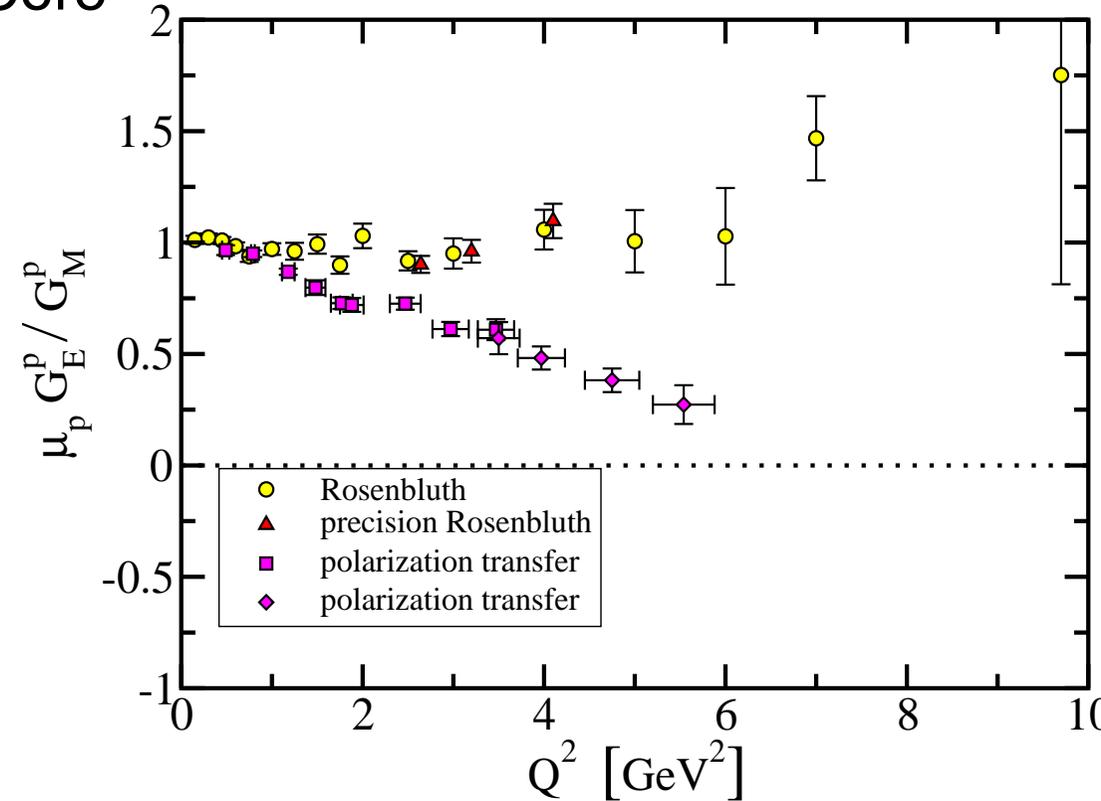


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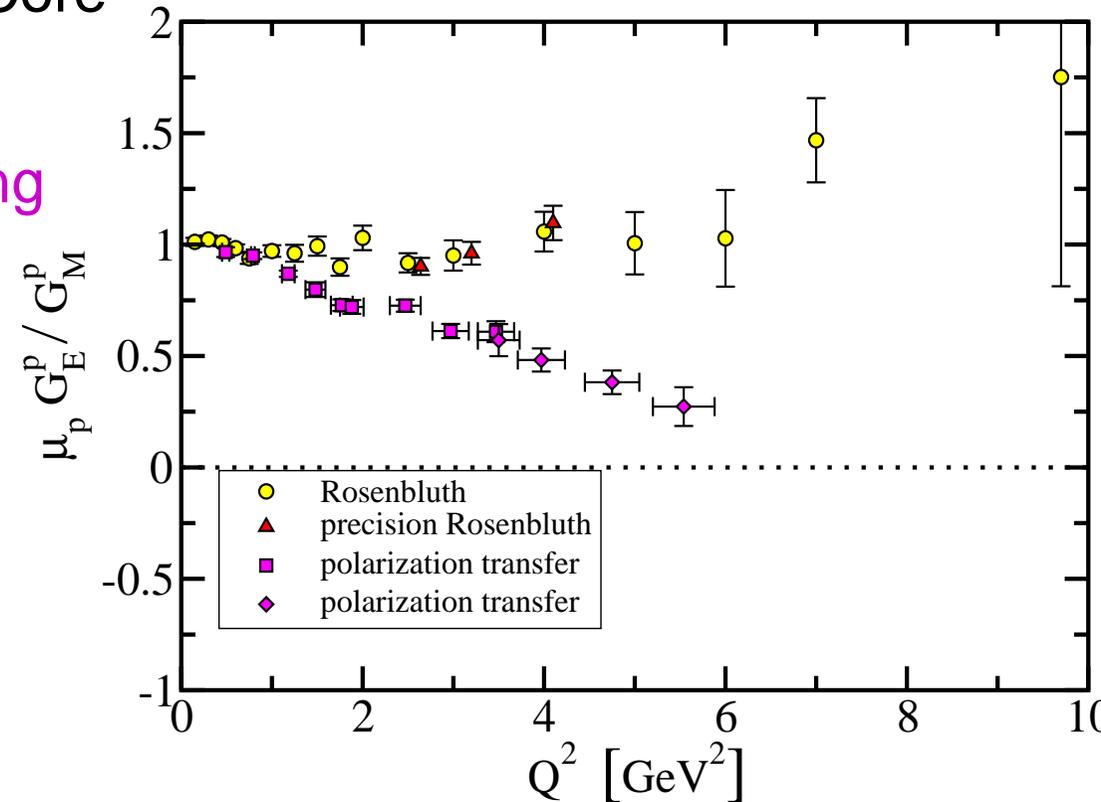


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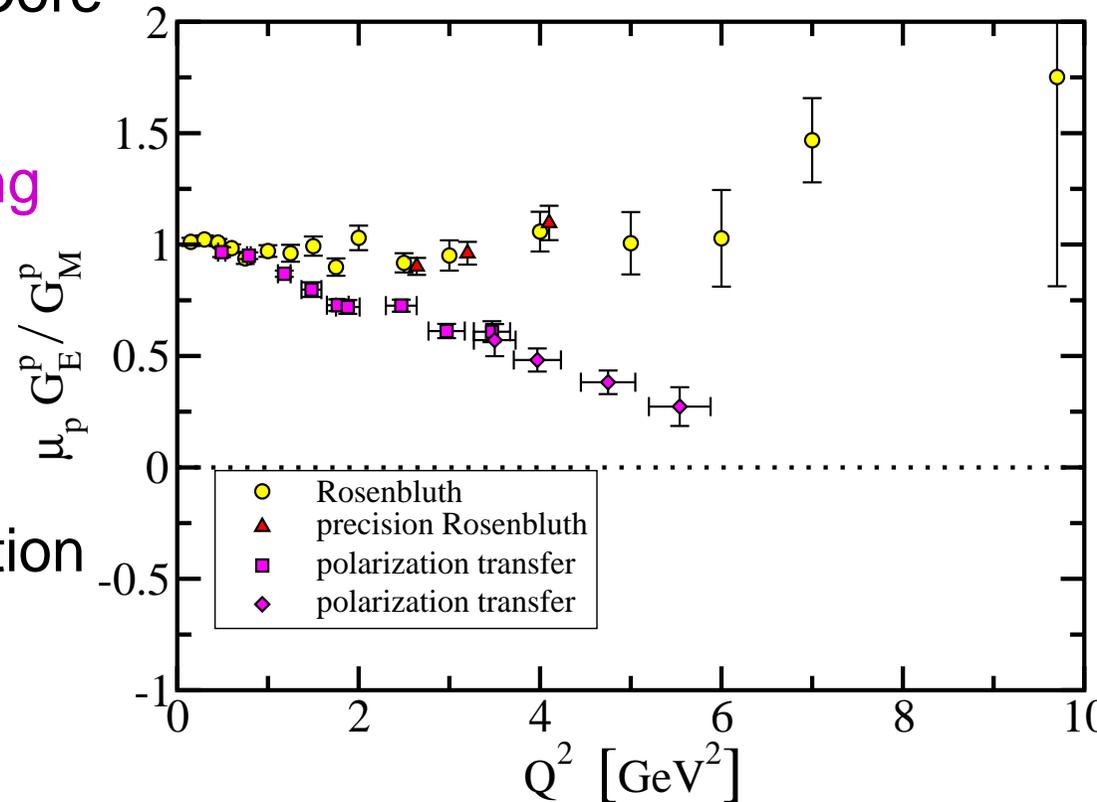
● Combine these elements ...

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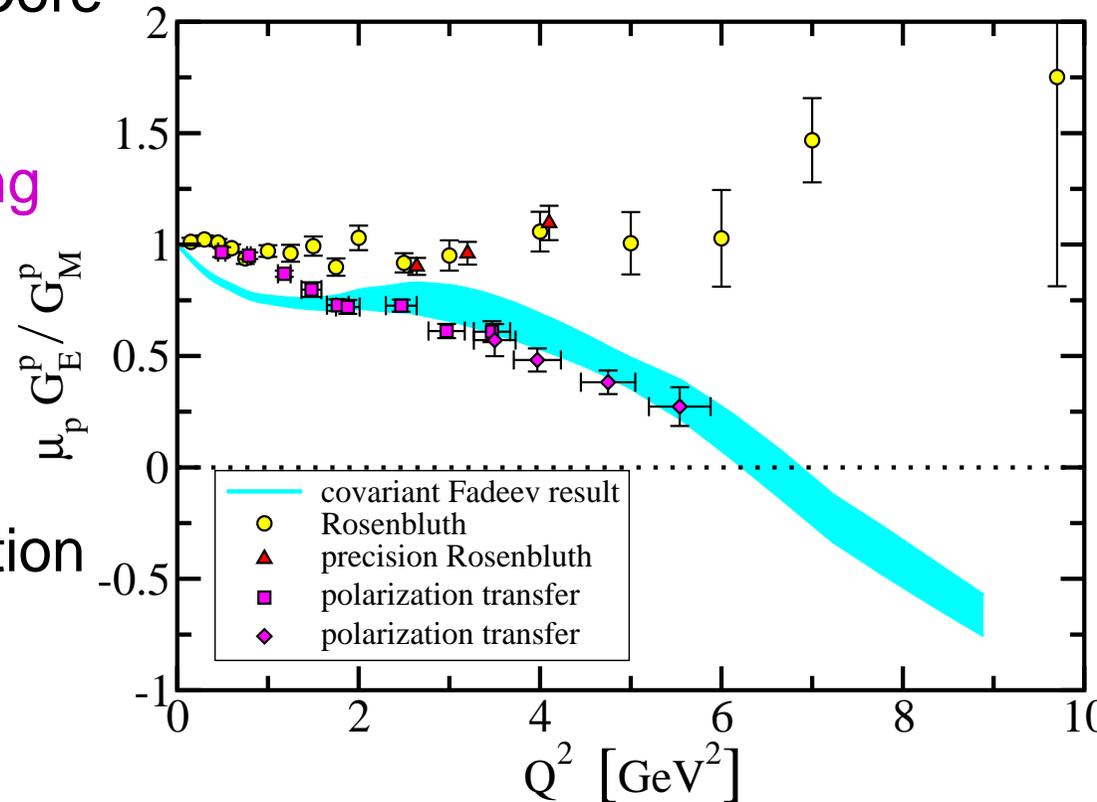
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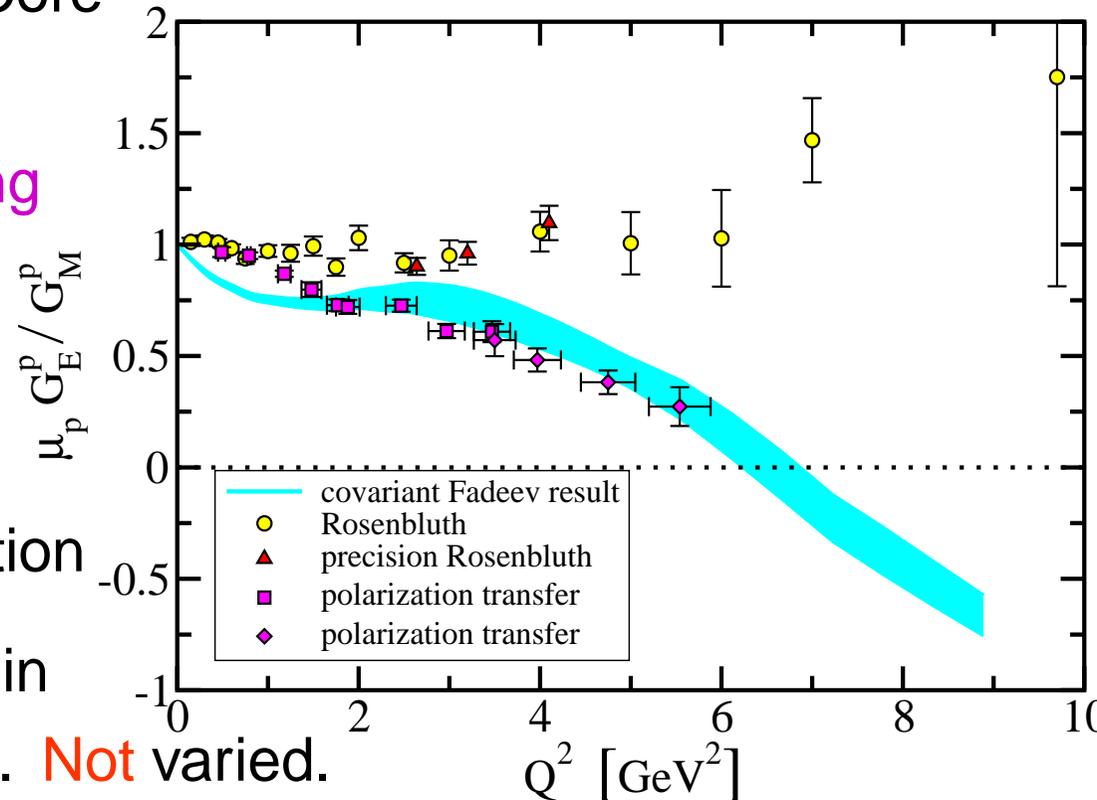
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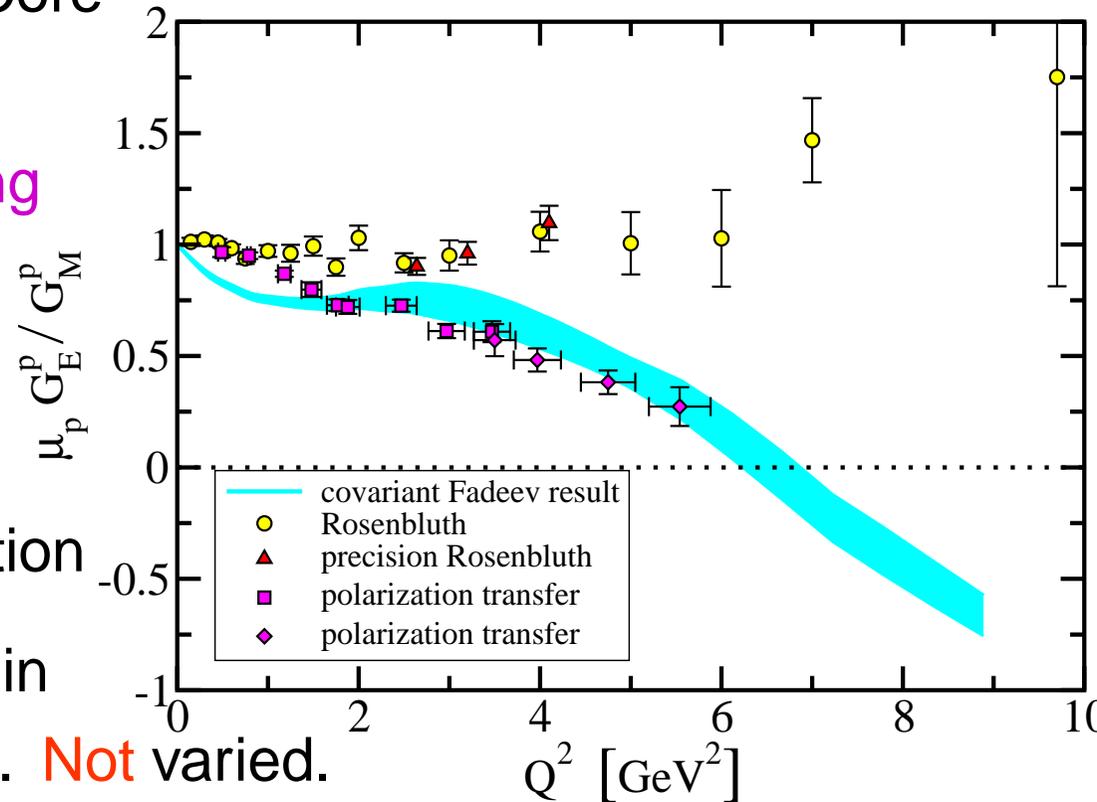
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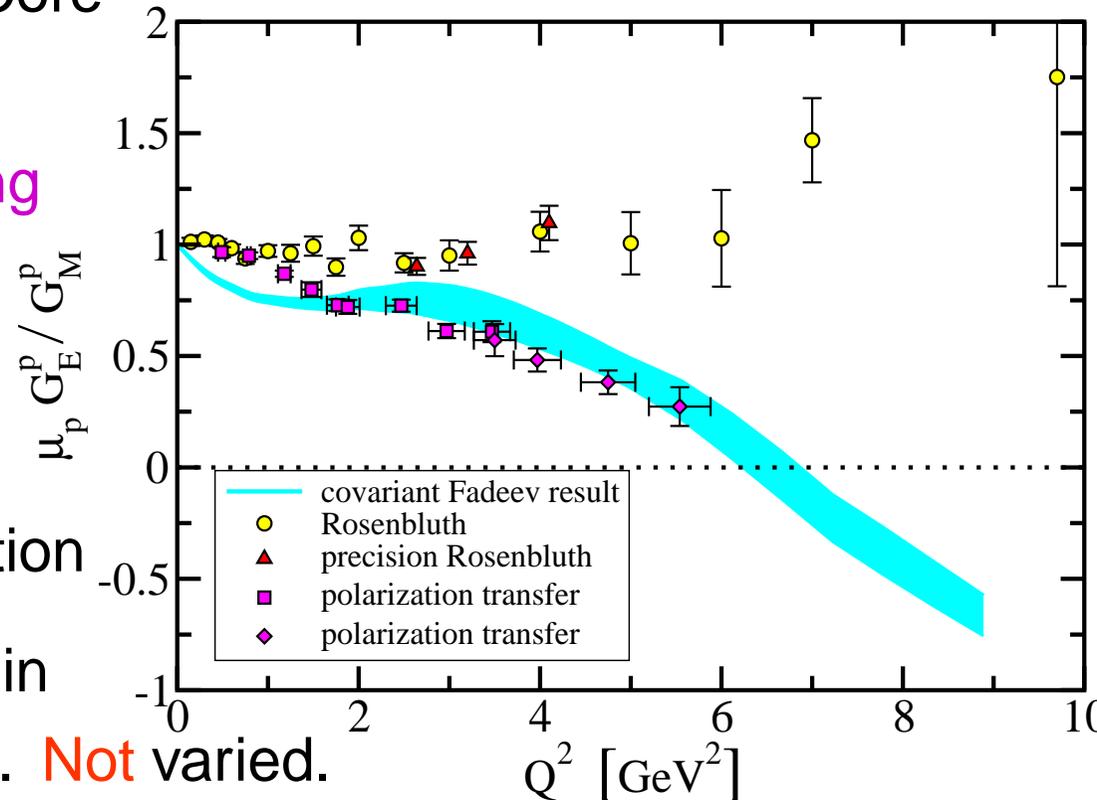
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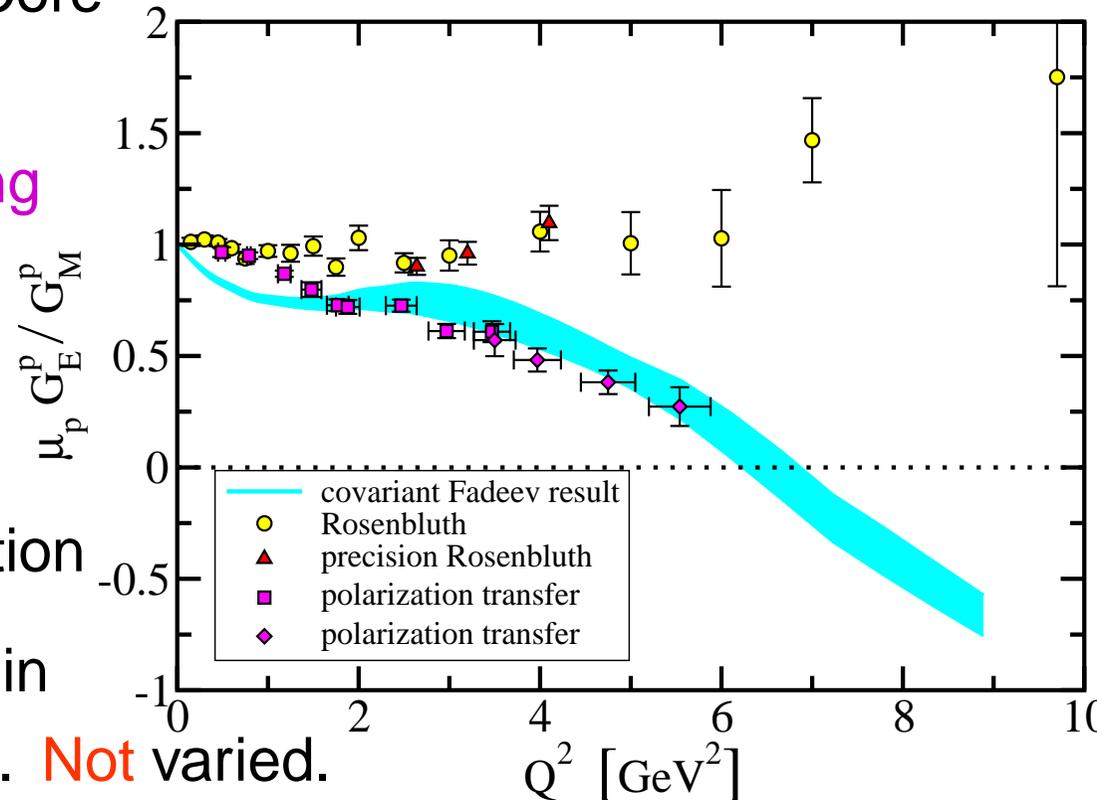
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- Predict Zero at  $Q^2 \approx 6.5 \text{ GeV}^2$



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# *Improved current*



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# Improved current

- Composite axial-vector diquark correlation
  - Electromagnetic current can be complicated
  - Limited constraints on large- $Q^2$  behaviour



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  - Implemented corrections so that large- $Q^2$  behaviour of form factors could be reliably calculated
  - Exposed two weaknesses in rudimentary *Ansatz*



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  - Implemented corrections so that large- $Q^2$  behaviour of form factors could be reliably calculated
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    - Diquark effectively pointlike to hard probe
    - Didn't account for diquark being off-shell in recoil



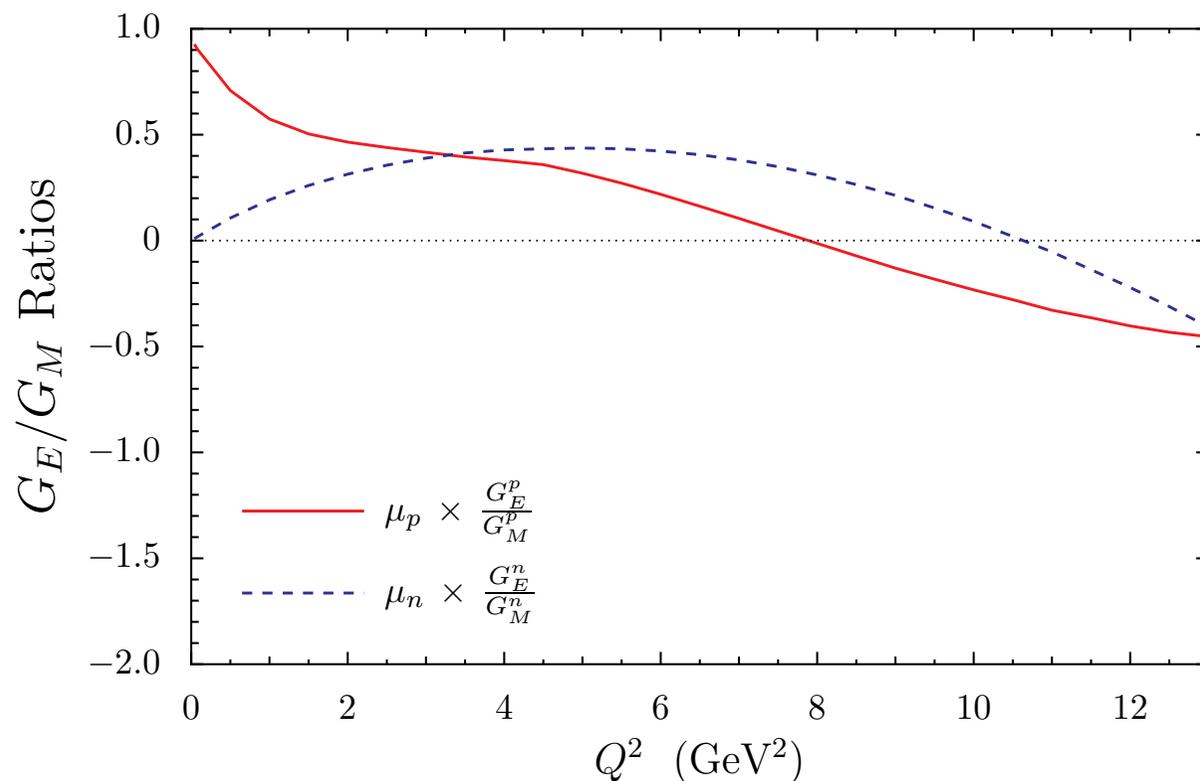
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  - Introduce form factor: radius 0.8 fm
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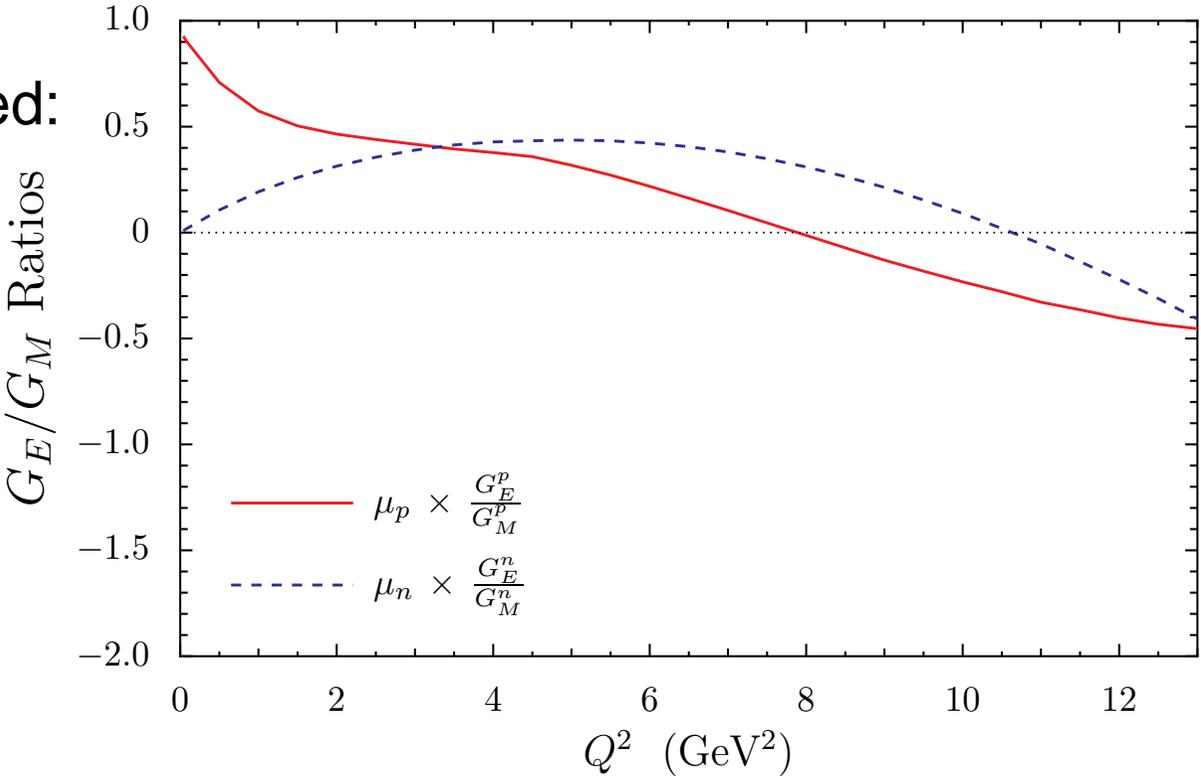
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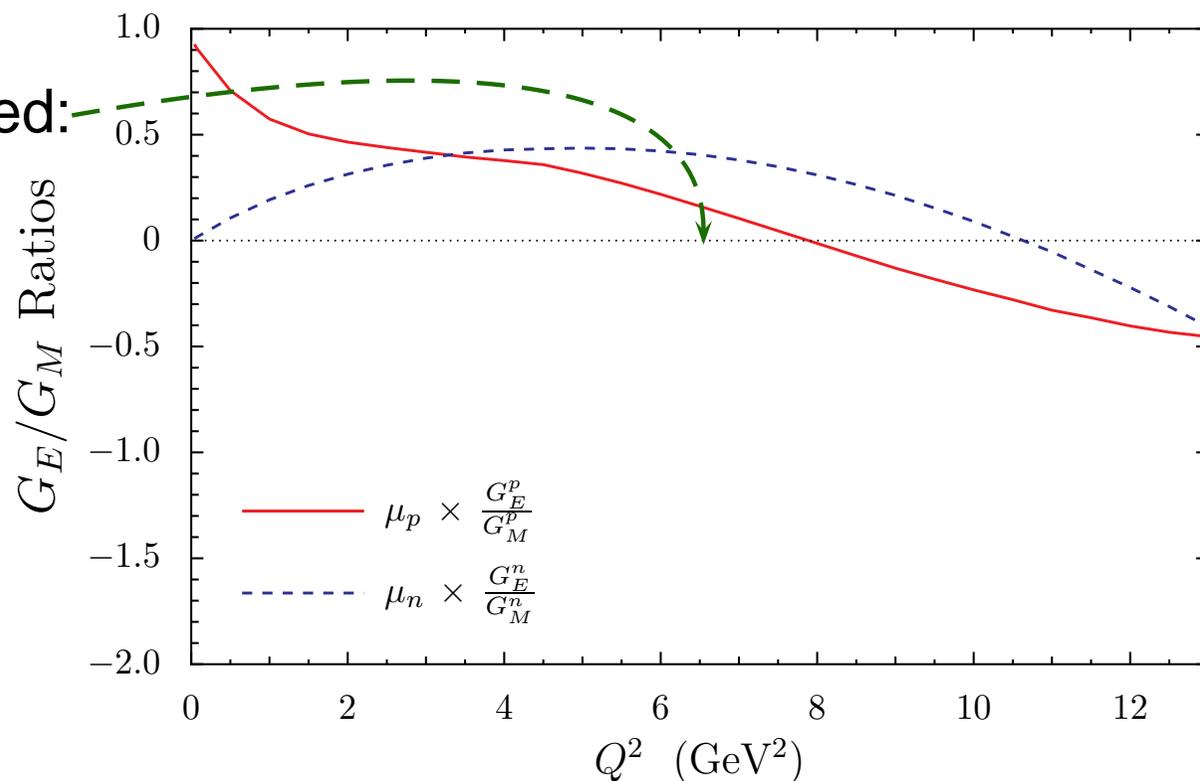
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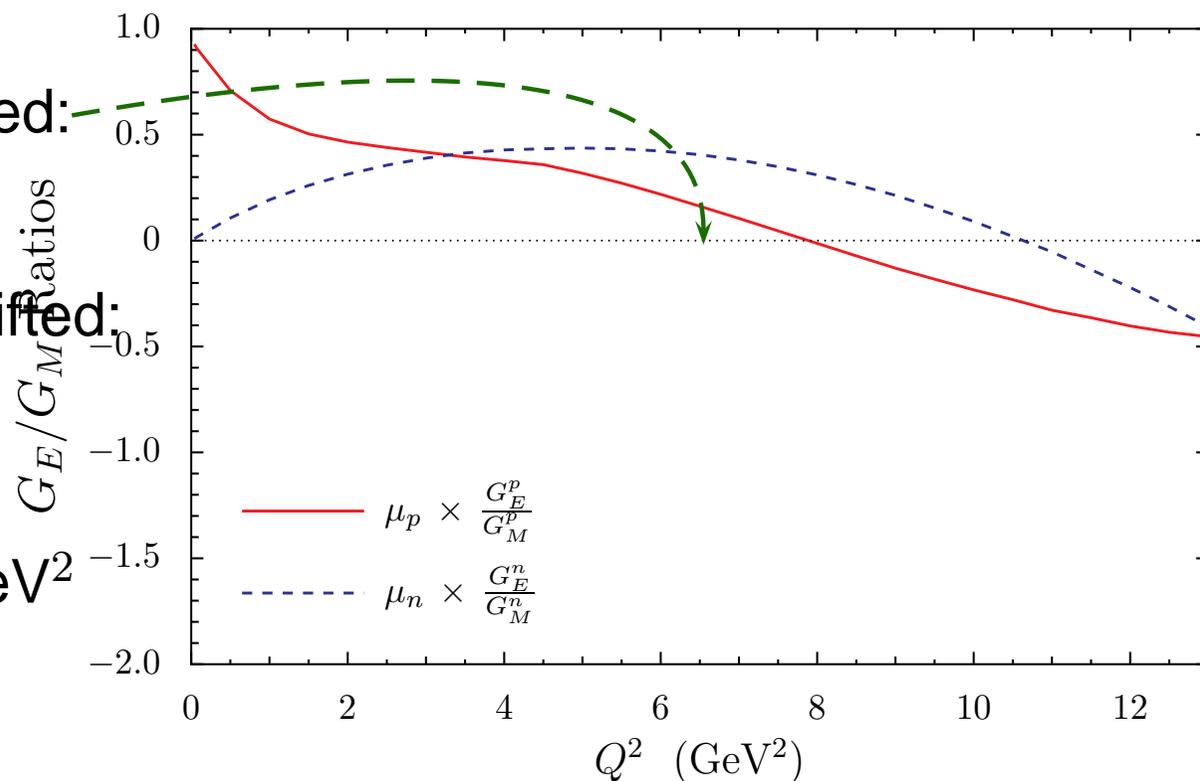
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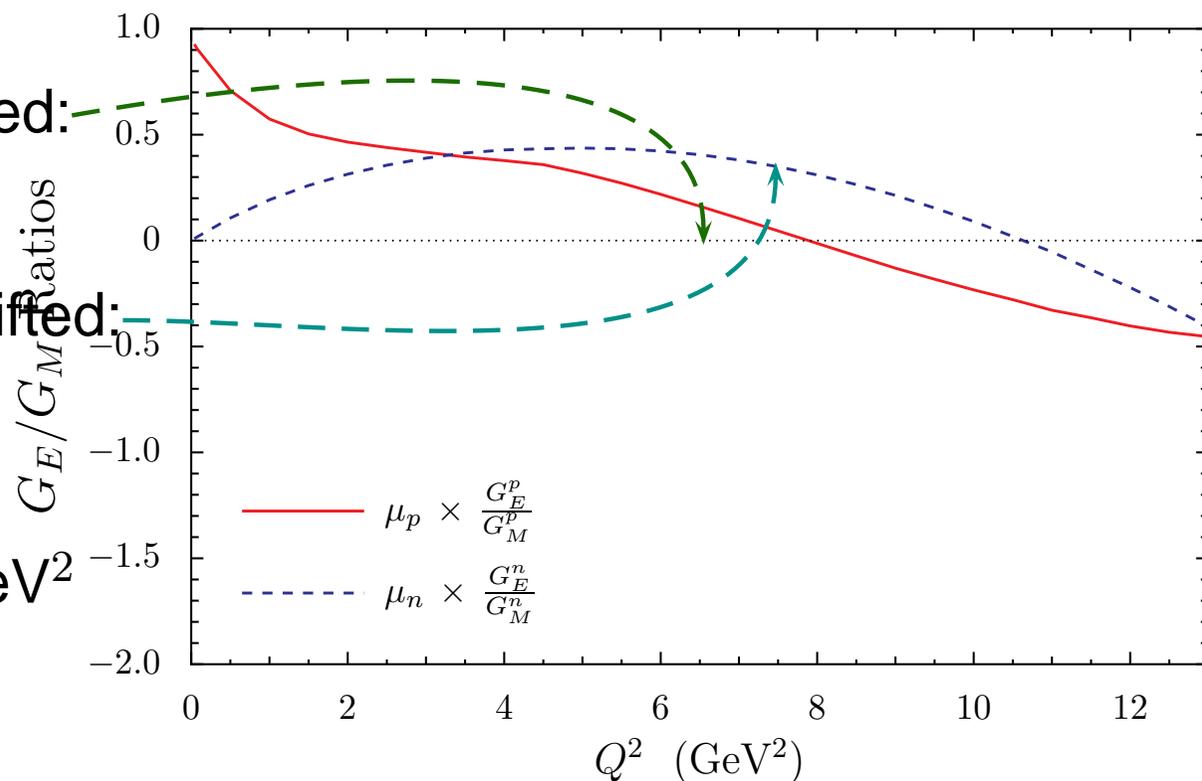
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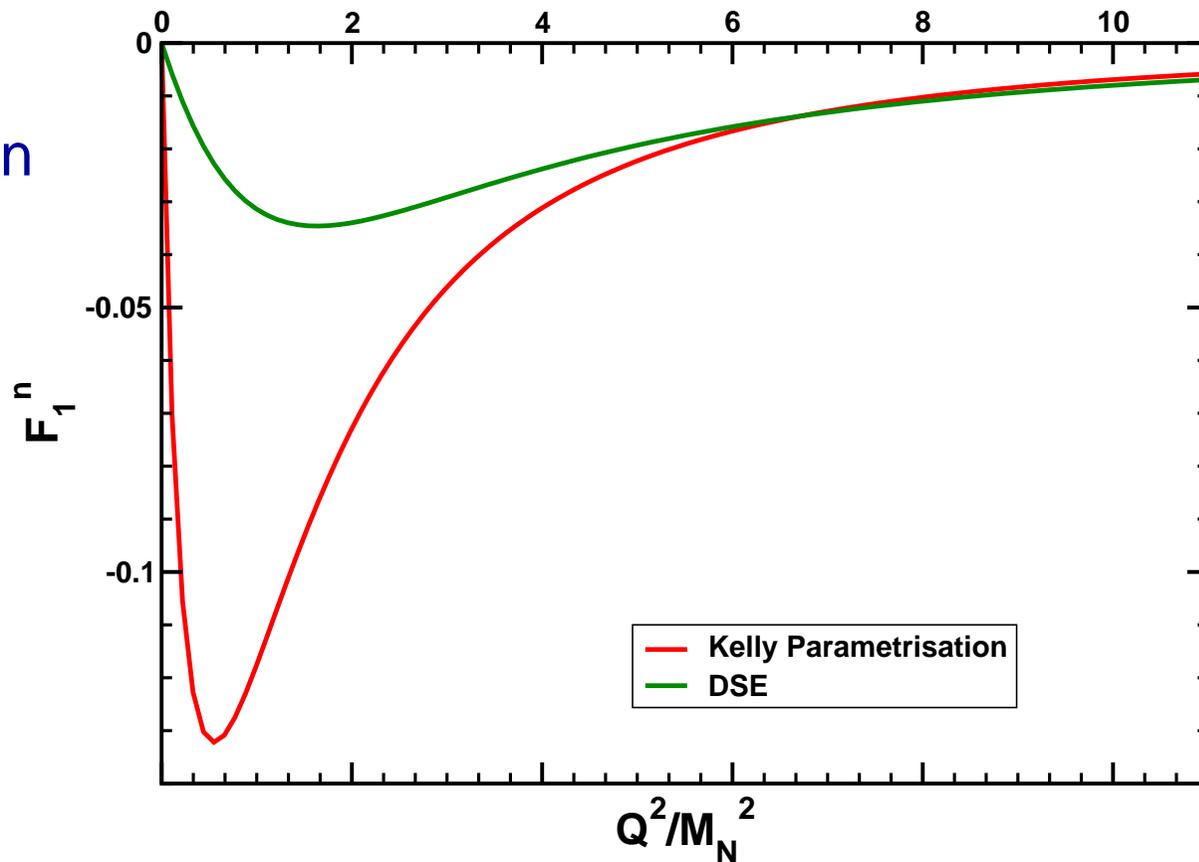
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# Pion Cloud

## $F_1$ – neutron

- Comparison between Faddeev equation result and Kelly's parametrisation



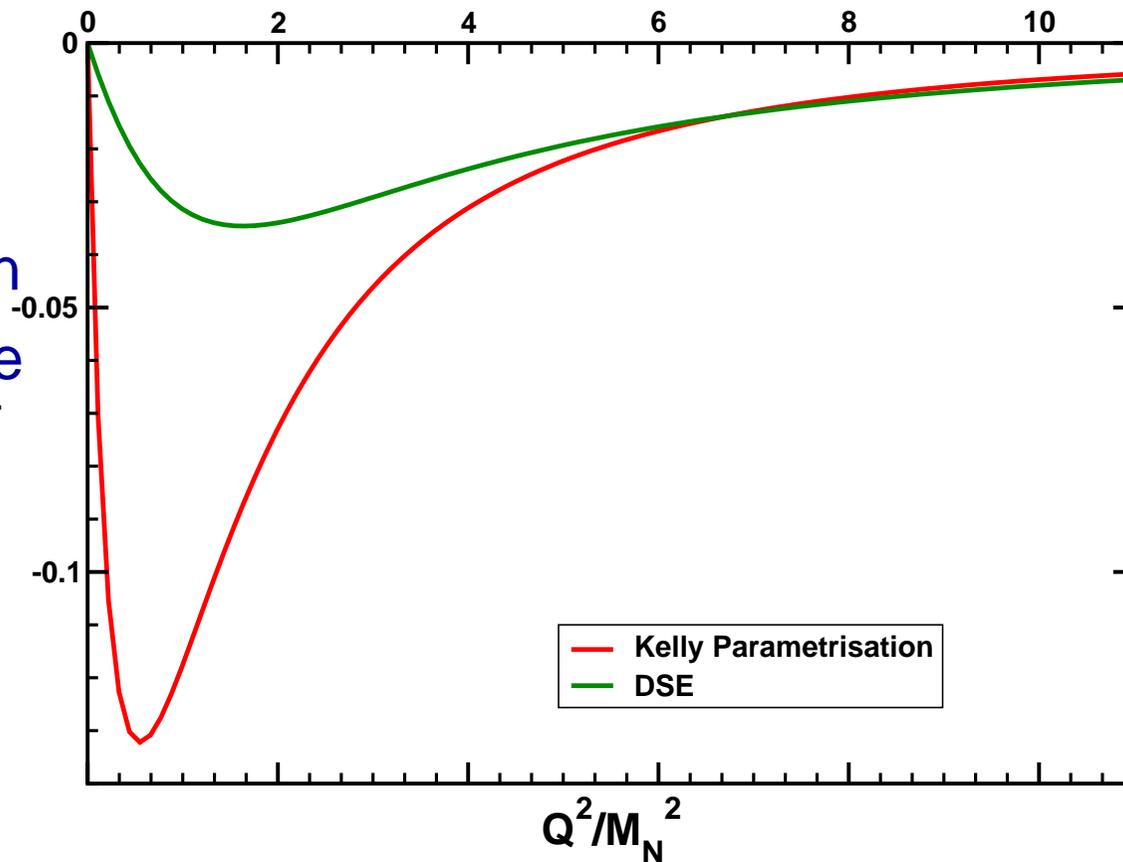
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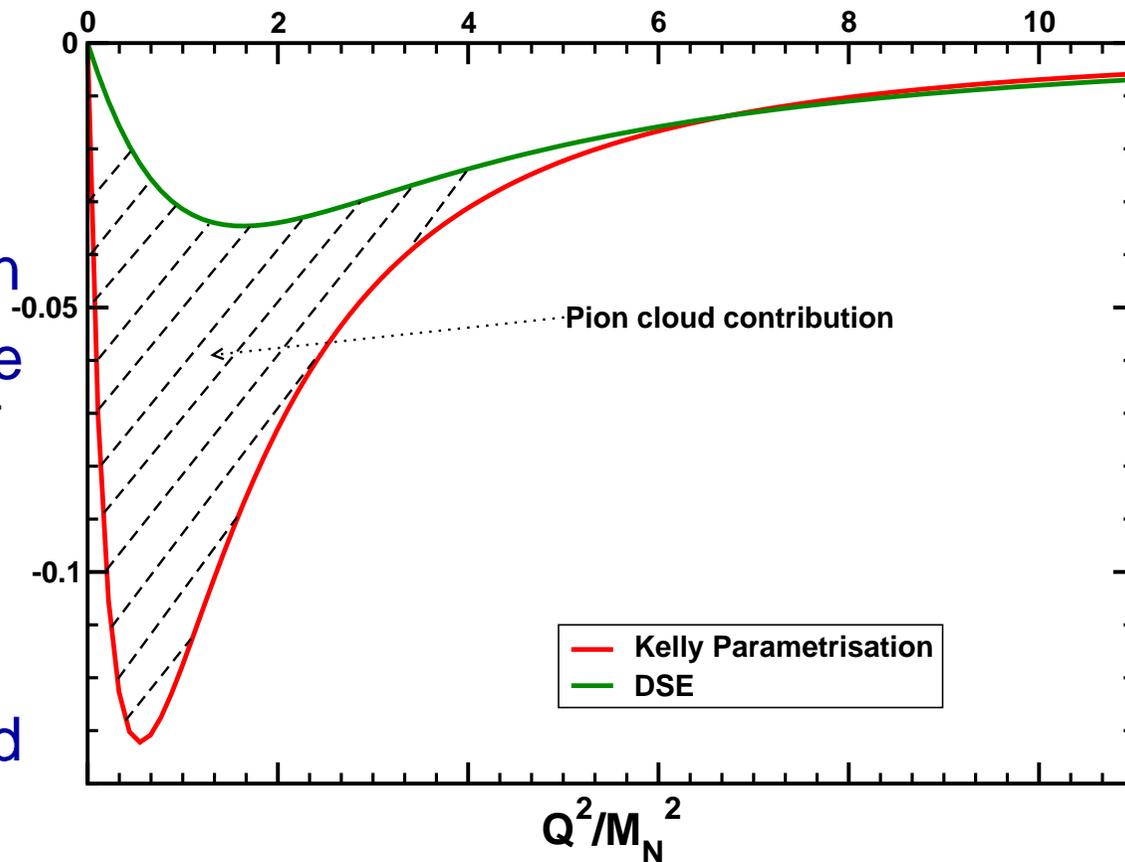
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- Faddeev equation set-up to describe dressed-quark core

- Pseudoscalar meson cloud (and related effects) significant for  $Q^2 \lesssim 3 - 4 M_N^2$





# Epilogue



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# Epilogue



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# Epilogue

- DCSB exists in QCD.



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# Epilogue



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  - It predicts, amongst other things, that
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# Epilogue

- Form Factors - progress anticipated in near- to medium-term
  - Quantifying pseudoscalar meson “cloud” effects





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  - Detailing broadly the role of two-photon exchange contributions





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  - Detailing broadly the role of two-photon exchange contributions
  - Explaining relationship between parton properties on the light-front and rest frame structure of hadrons



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2. QCD's Challenges
3. Why?
4. Some Questions
5. Status
6. Dichotomy of the Pion
7. Pion Form Factors
8. Dressed-Quark Propagator
9. Hadrons
10. Bethe-Salpeter Kernel
11. Pion FF
12. Calculated Pion FF
13. Nucleon Challenge
14. Nucleon EM Form Factors
15. Faddeev equation
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19. Improved current
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# *Dyson-Schwinger Equations*



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# Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory



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- Simplest level: **Generating Tool for Perturbation Theory**  
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..... behaviour of  $\alpha_s(Q^2)$



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- Method yields Schwinger Functions  $\equiv$  Propagators



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**Cross-Sections built from Schwinger Functions**



# Schwinger Functions



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# *Schwinger Functions*

- Solutions are Schwinger Functions  
(Euclidean **Green** Functions)



# Schwinger Functions

- Solutions are Schwinger Functions (Euclidean **Green** Functions)
- Not all are Schwinger functions are experimentally observable



# Schwinger Functions

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  - **all are** same VEVs measured in numerical simulations of lattice-regularised QCD
  - opportunity for comparisons at pre-experimental level ... cross-fertilisation



# Schwinger Functions

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- Not all are Schwinger functions are experimentally observable but ...
  - **all are** same VEVs measured in numerical simulations of lattice-regularised QCD
  - opportunity for comparisons at pre-experimental level ... cross-fertilisation
- Proving fruitful.



# Persistent Challenge



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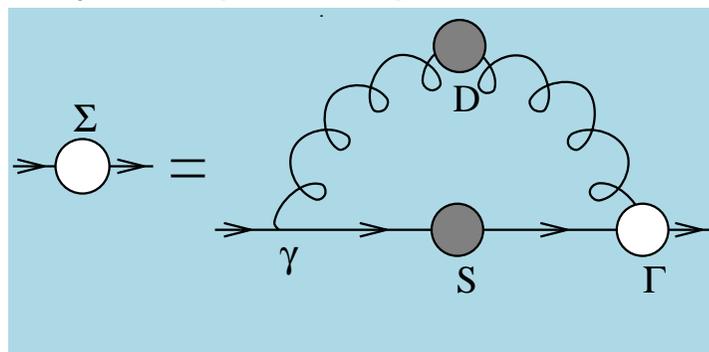
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# Persistent Challenge

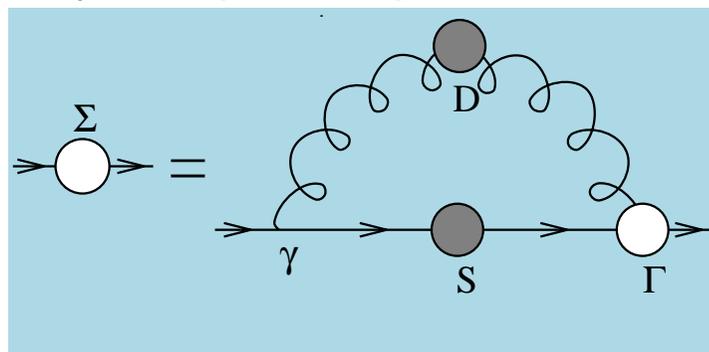
- Infinitely Many Coupled Equations





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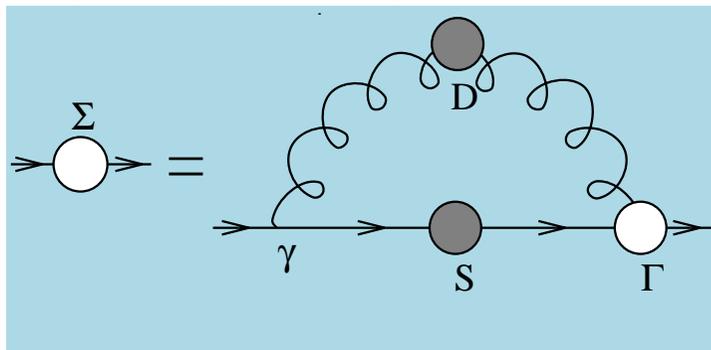
- Coupling between equations **necessitates** truncation





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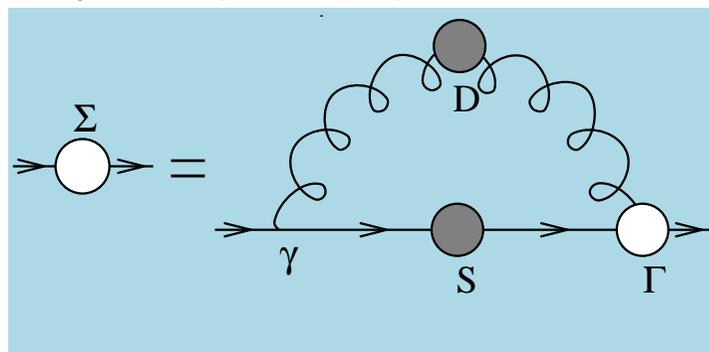
- Coupling between equations **necessitates** truncation
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# Persistent Challenge

- Infinitely Many Coupled Equations



- Coupling between equations **necessitates** truncation

- Weak coupling expansion  $\Rightarrow$  Perturbation Theory  
**Not useful** for the nonperturbative problems  
in which we're interested





# Persistent Challenge

- Infinitely Many Coupled Equations
  - There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme
- H.J. Munczek Phys. Rev. D **52** (1995) 4736  
*Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*
- A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7  
*Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*





## Persistent Challenge

- Infinitely Many Coupled Equations
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## Persistent Challenge

- Infinitely Many Coupled Equations
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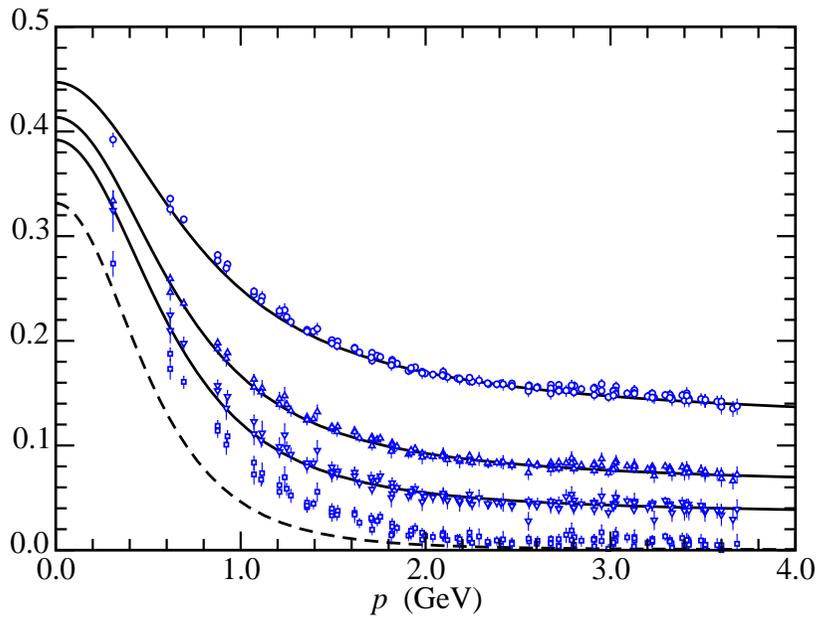
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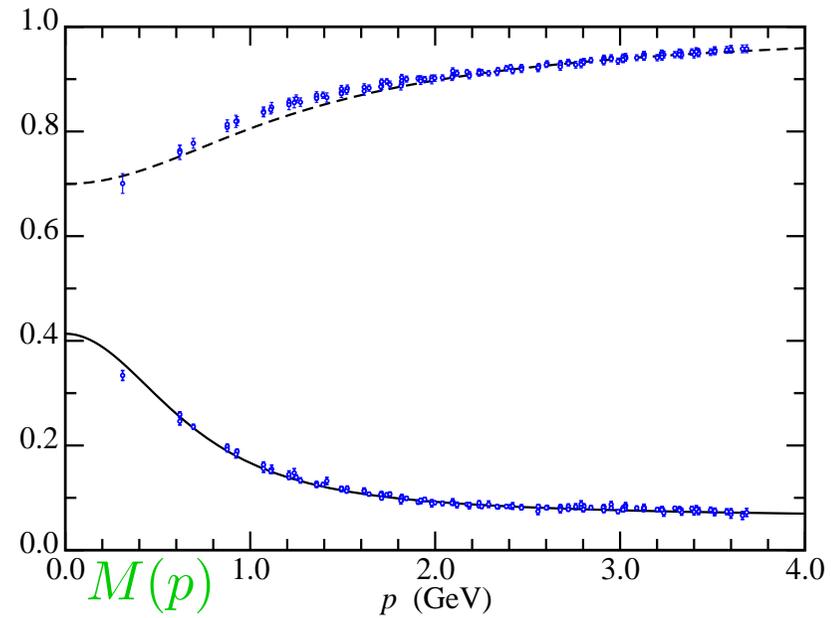
# Quenched-QCD

## Dressed-Quark Propagator

$M(p)$

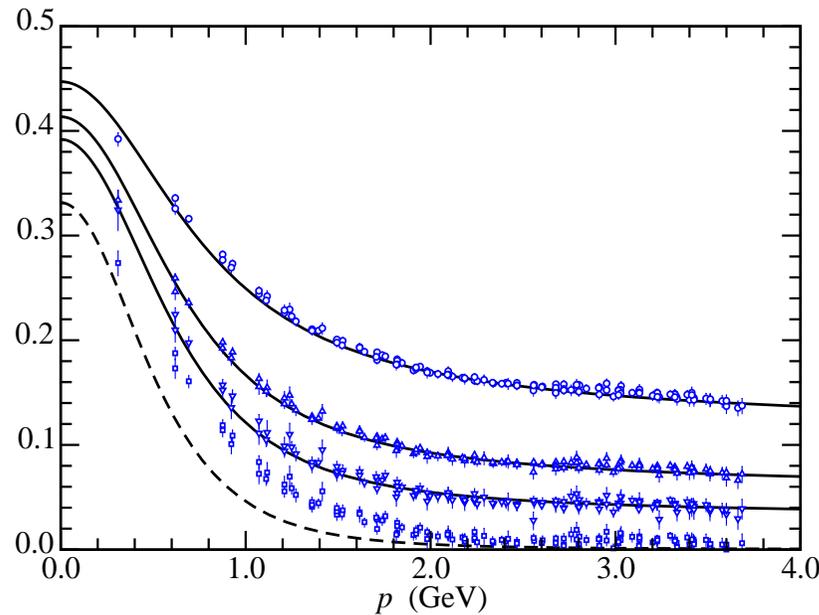


$Z(p)$

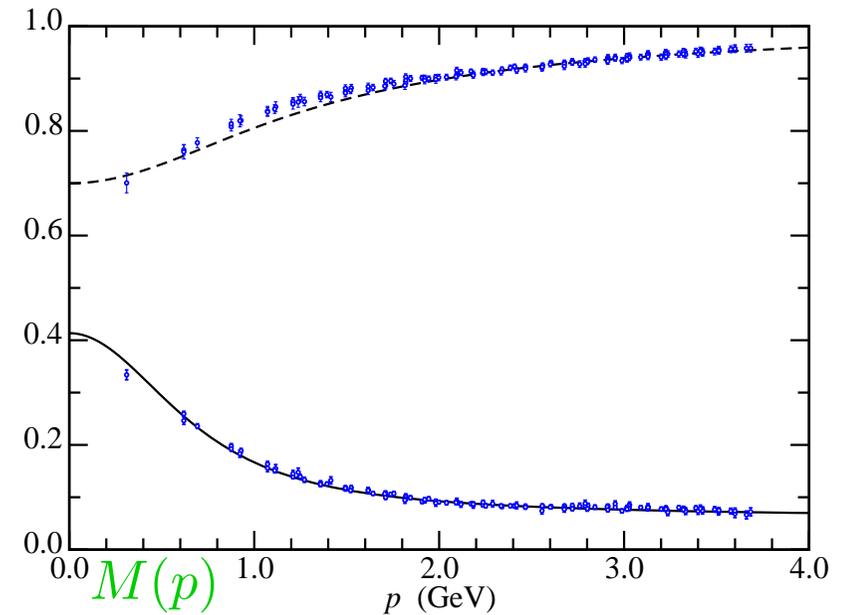


2002

$M(p)$



$Z(p)$



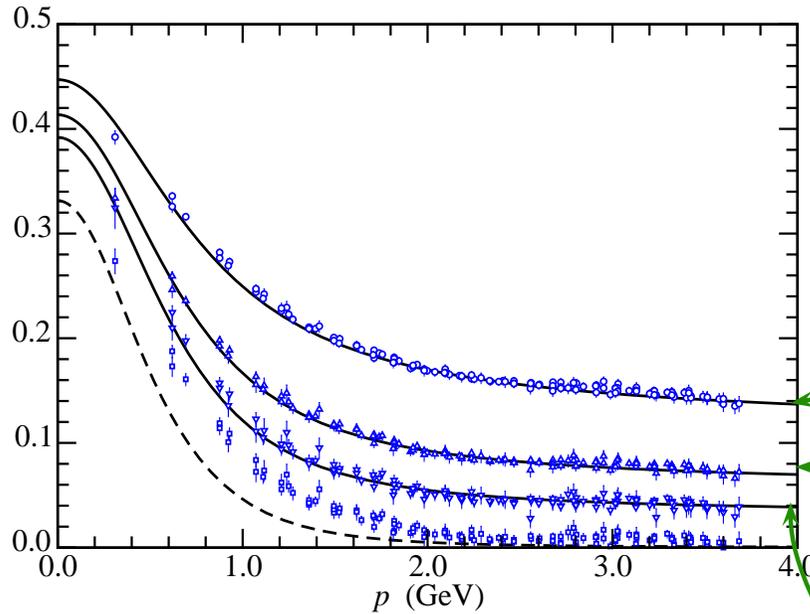
“*data*:” Quenched Lattice Meas.

– Bowman, Heller, Leinweber, Williams: [he-lat/0209129](https://arxiv.org/abs/he-lat/0209129)

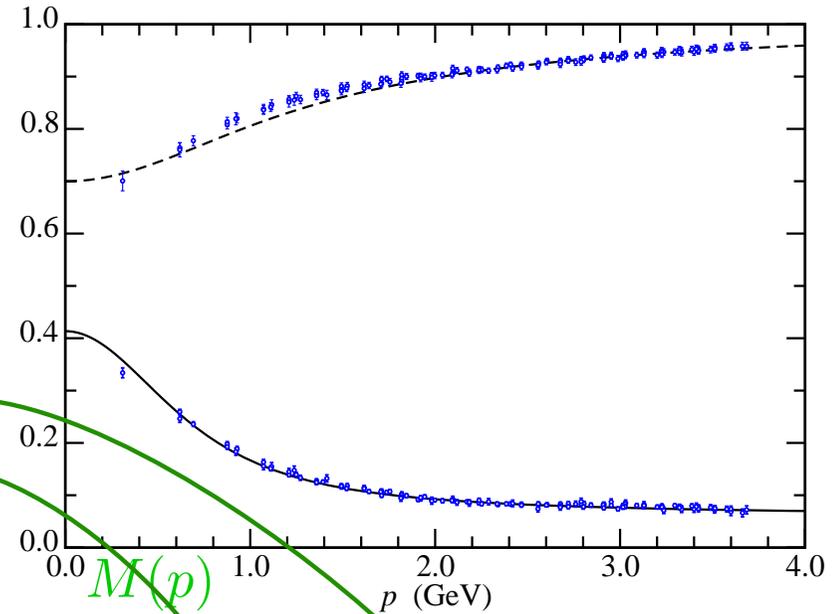


2002

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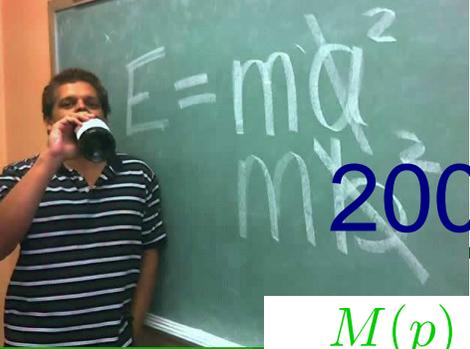


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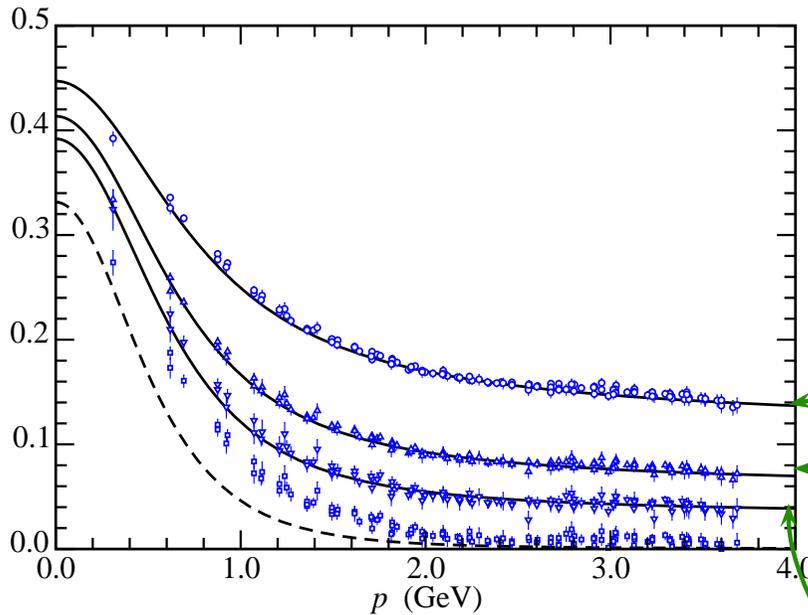
– Bowman, Heller, Leinweber, Williams: [he-lat/0209129](https://arxiv.org/abs/he-lat/0209129)  
 current-quark masses: 30 MeV, 50 MeV, 100 MeV



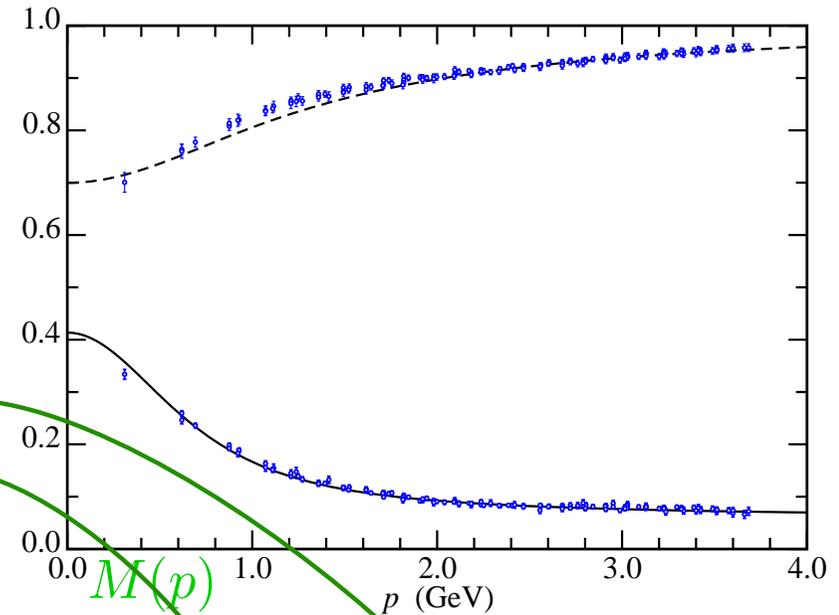
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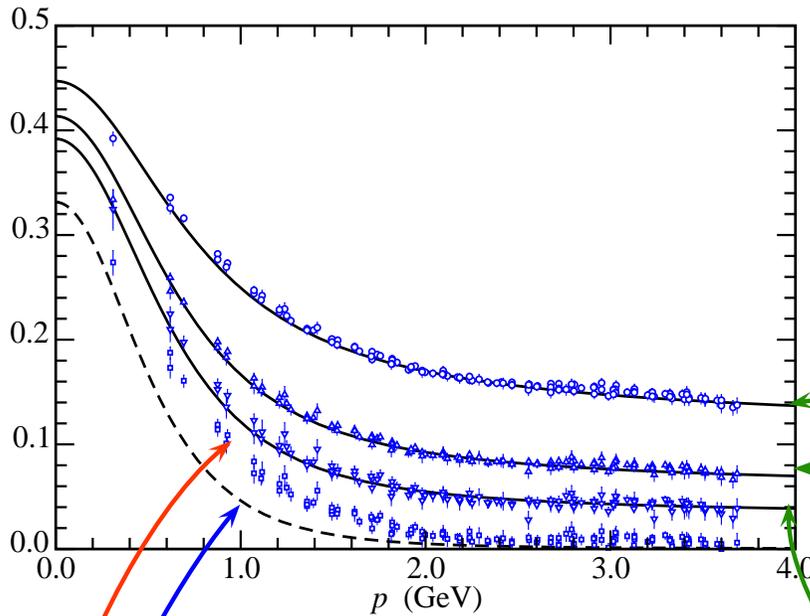
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- **Curves:** Quenched DSE Cal.
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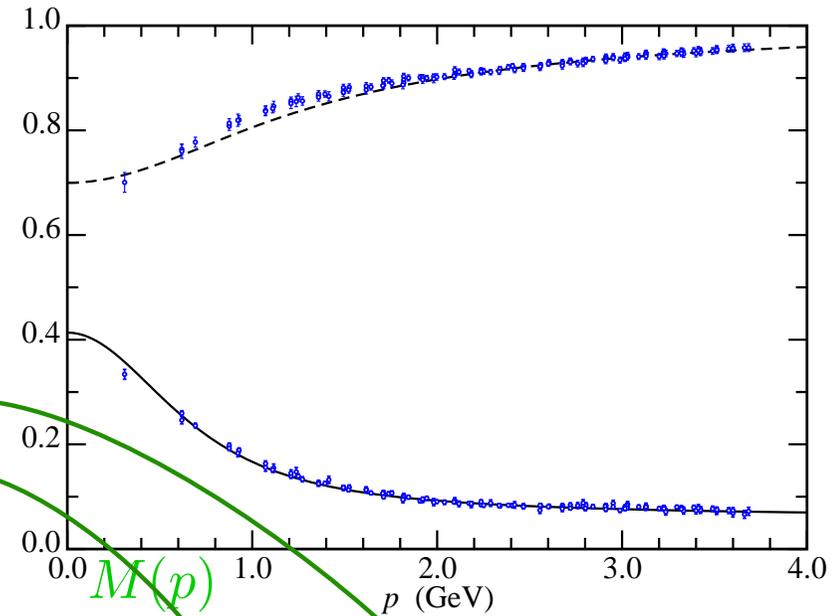
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Linear extrapolation of lattice data to chiral limit is inaccurate

# Frontiers of Nuclear Science: A Long Range Plan (2007)



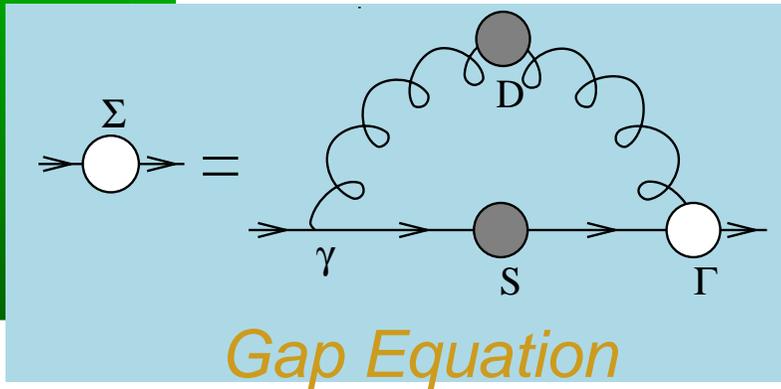
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# Frontiers of Nuclear Science: Theoretical Advances



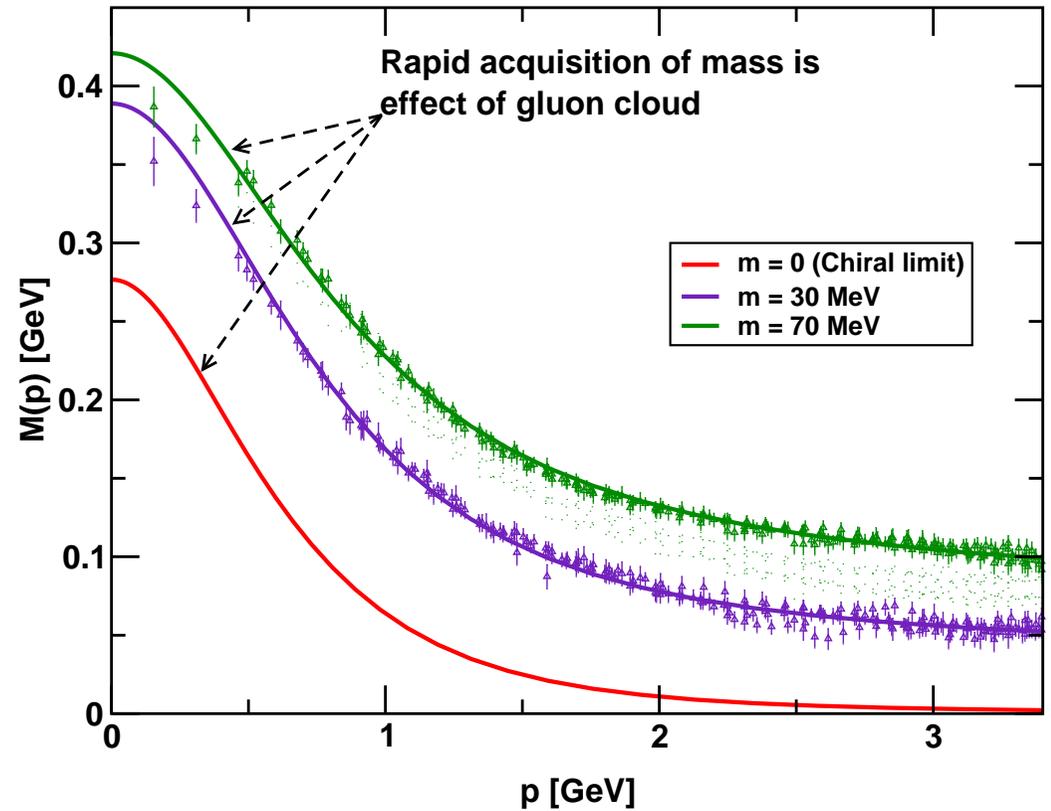
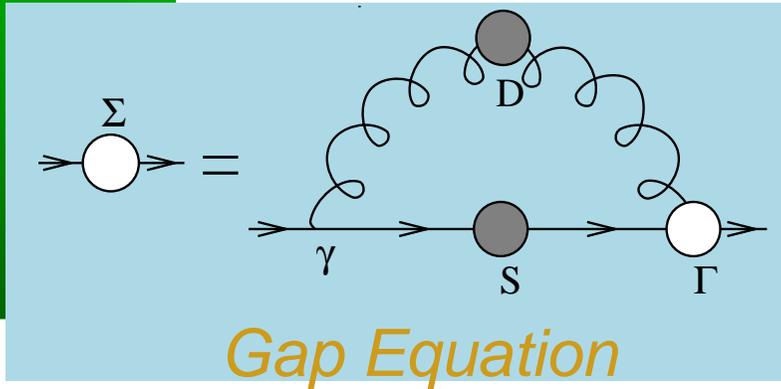
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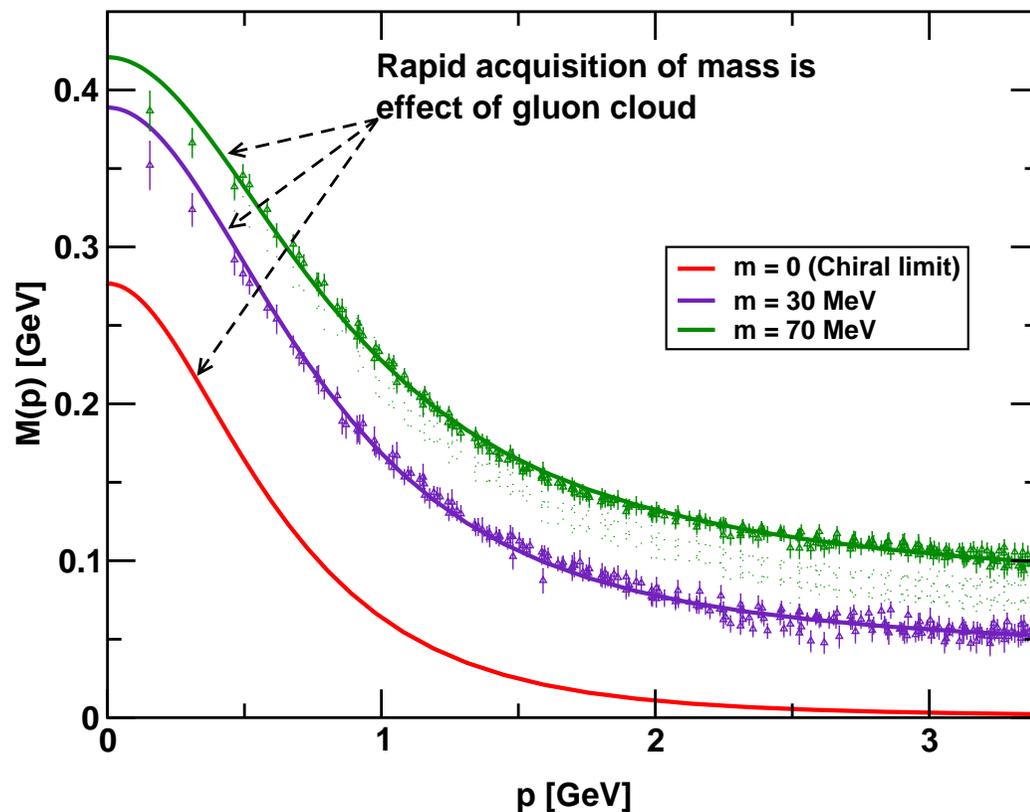
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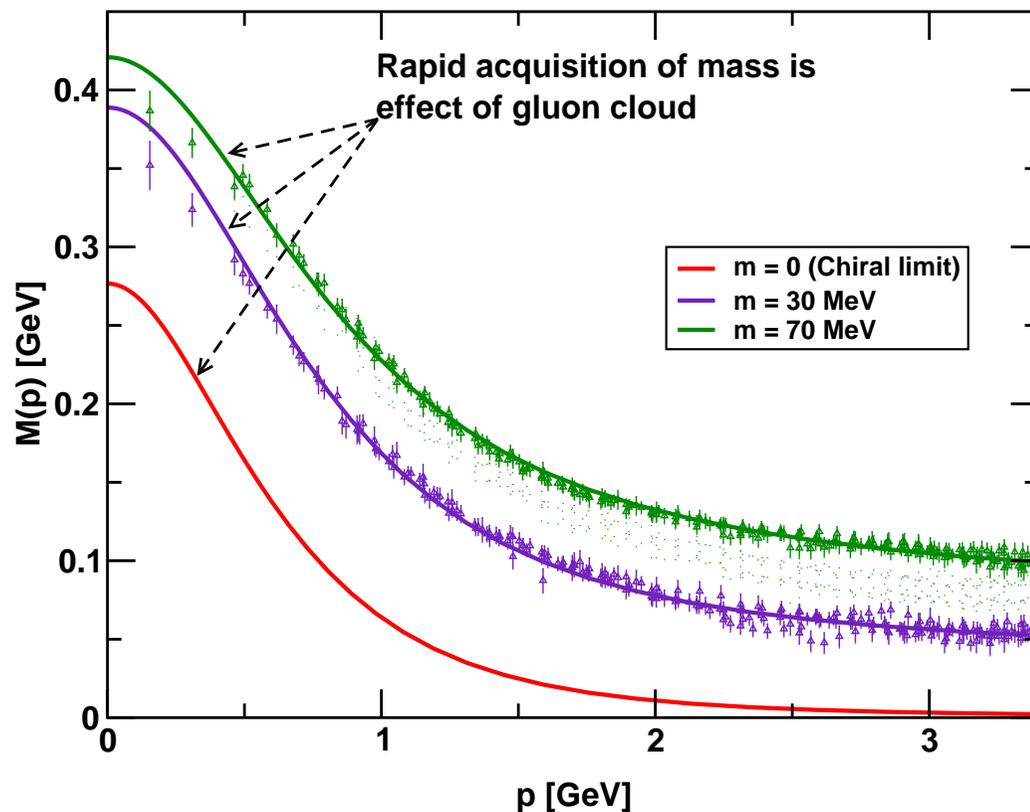
## Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ( $m = 0$ , red curve) acquires a large constituent mass at low energies.



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# *Dimensionless product: $r_\pi f_\pi$*



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## *Dimensionless product: $r_\pi f_\pi$*

- Improved rainbow-ladder interaction





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## *Dimensionless product: $r_\pi f_\pi$*

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- Great strides towards placing nucleon studies on same footing as mesons



# Dimensionless product: $r_\pi f_\pi$

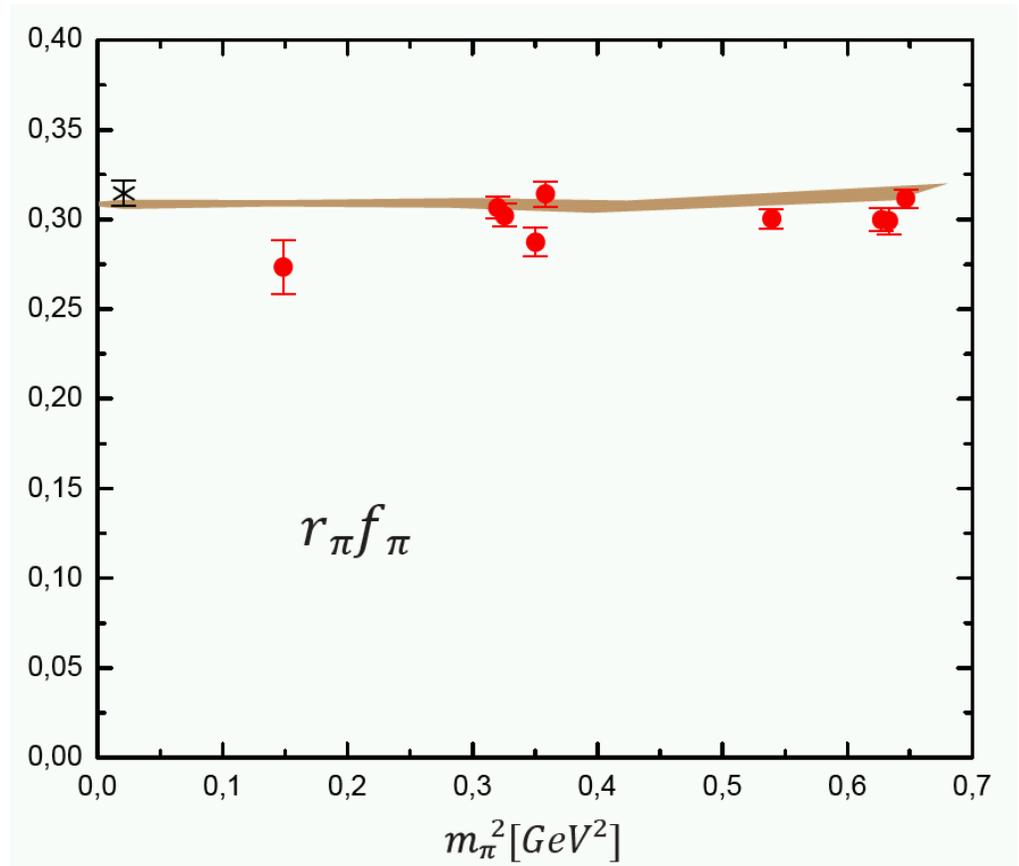
- Improved rainbow-ladder interaction
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- Experimentally:  $r_\pi f_\pi = 0.315 \pm 0.005$



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● DSE prediction



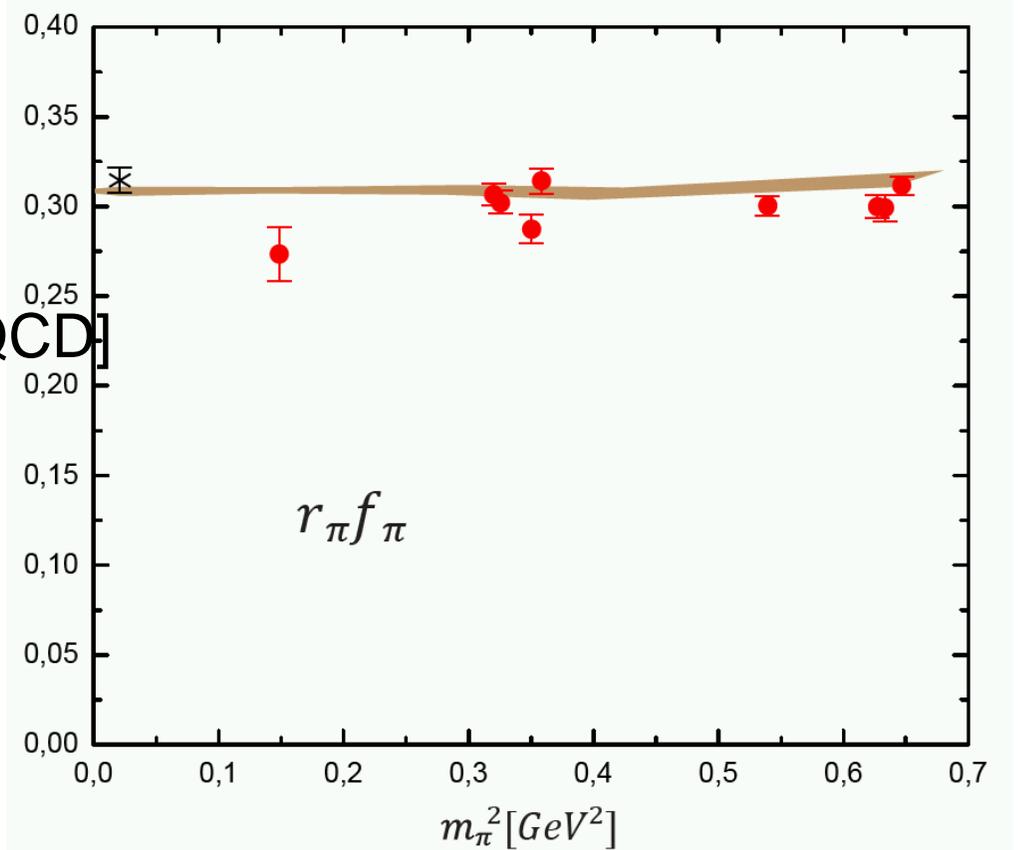
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● DSE prediction

● Lattice results

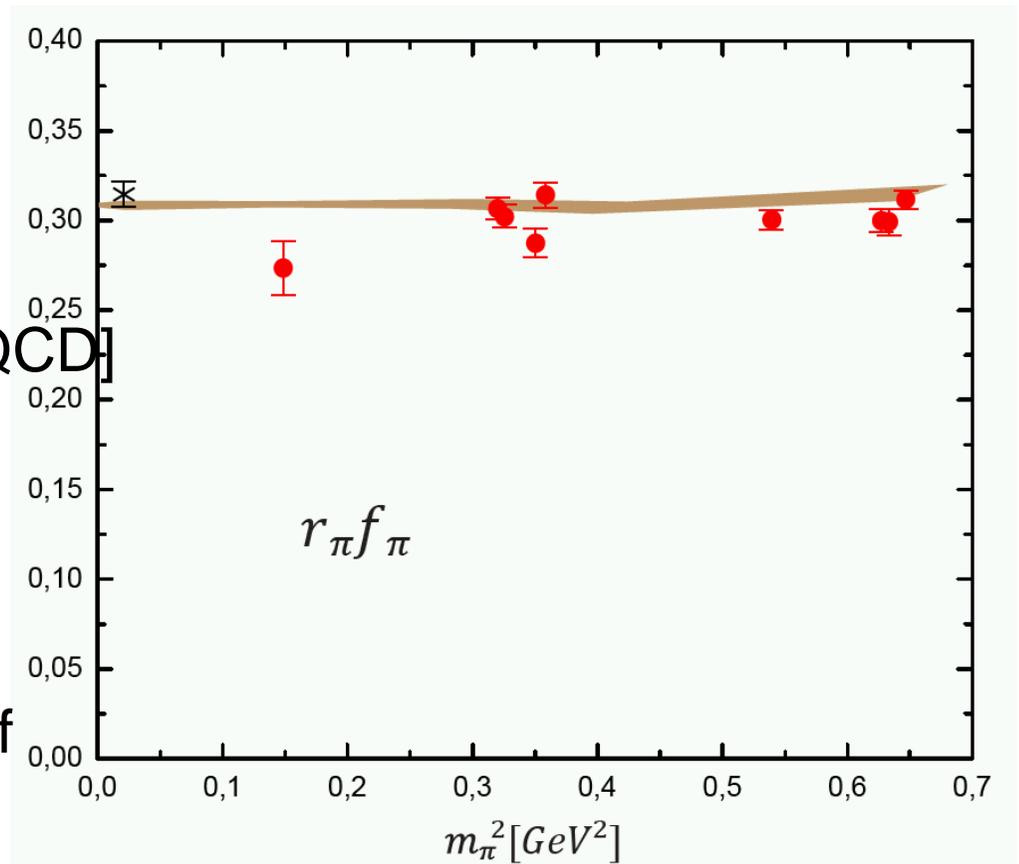
– James Zanotti [UK QCD]



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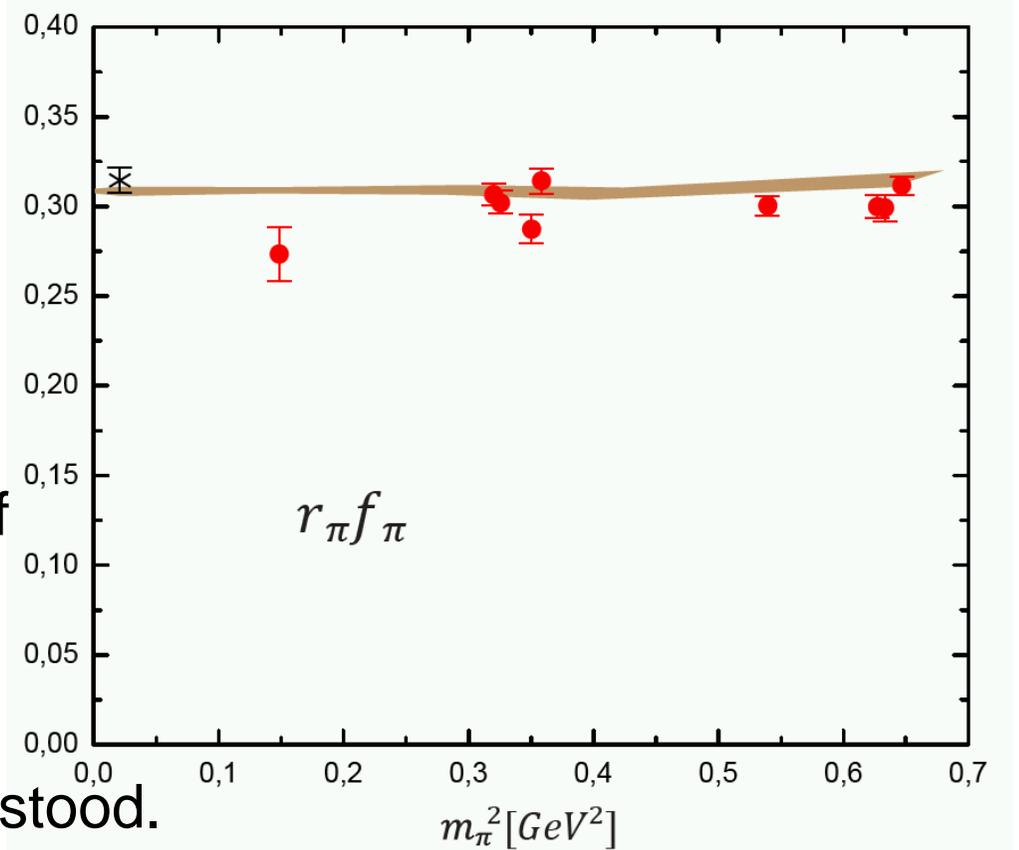
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DSE and Lattice
  - Experimental value obtains independent of current-quark mass.



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# Two-photon Couplings of

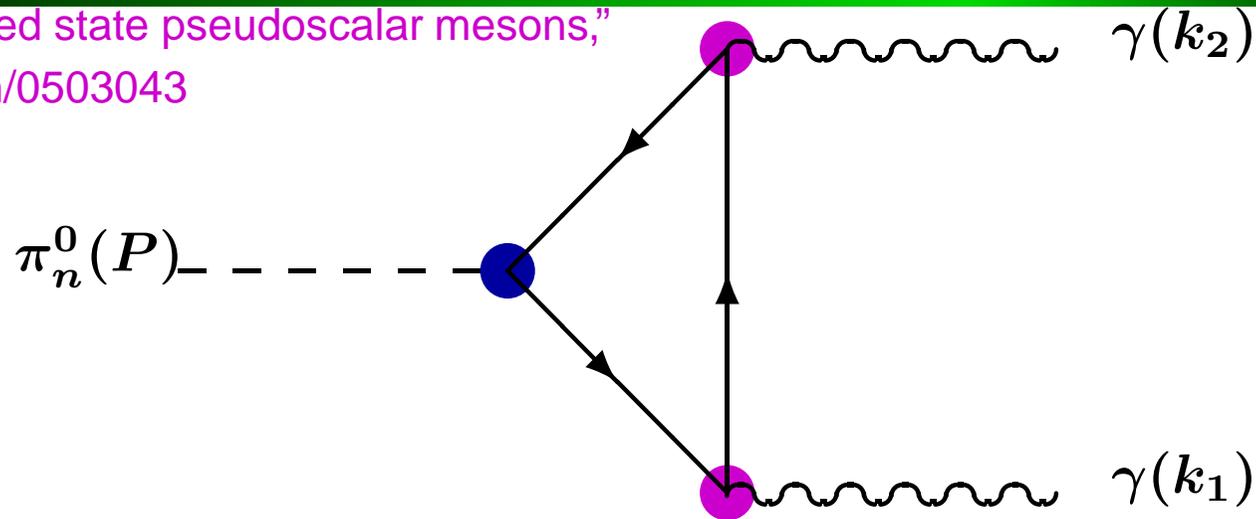
# Pseudoscalar Mesons

Höll, Krassnigg, Maris, *et al.*,

“Electromagnetic properties of ground and

excited state pseudoscalar mesons,”

nu-th/0503043



$$T_{\mu\nu}^{\pi_n^0}(k_1, k_2) = \frac{\alpha}{\pi} i \varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma} G^{\pi_n^0}(k_1, k_2)$$

- Define:  $\mathcal{T}_{\pi_n^0}(P^2, Q^2) = G^{\pi_n^0}(k_1, k_2) \Big|_{k_1^2=Q^2=k_2^2}$

This is a transition form factor.

- Physical Processes described by couplings:

$$g_{\pi_n^0\gamma\gamma} := \mathcal{T}_{\pi_n^0}(-m_{\pi_n^0}^2, 0)$$

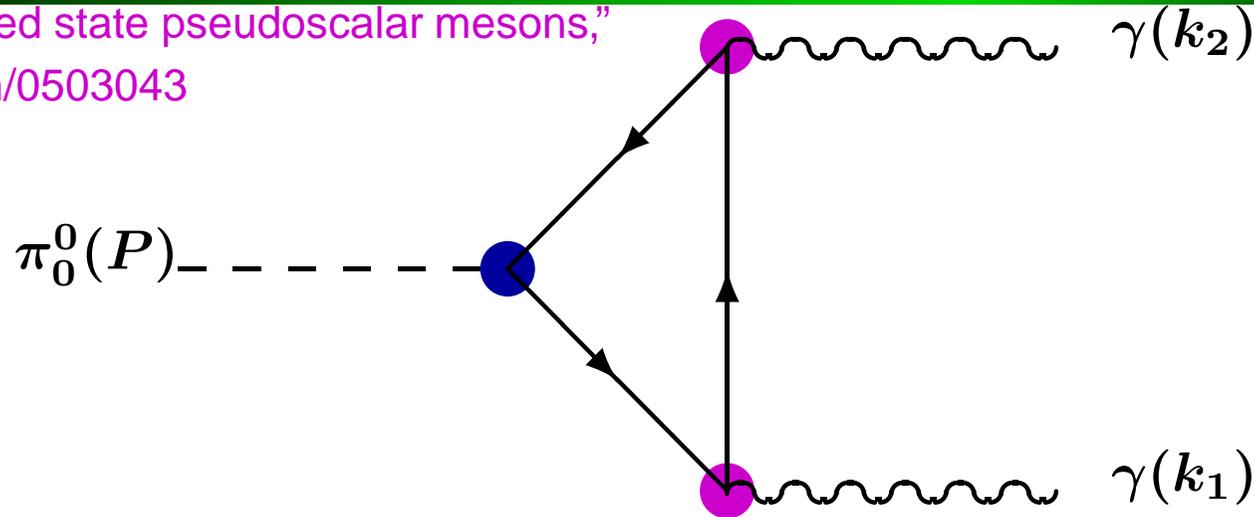
$$\text{Width: } \Gamma_{\pi_n^0\gamma\gamma} = \alpha_{\text{em}}^2 \frac{m_{\pi_n}^3}{16\pi^3} g_{\pi_n\gamma\gamma}^2$$



# Two-photon Couplings:

## Goldstone Mode

Höll, Krassnigg, Maris, *et al.*,  
 “Electromagnetic properties of ground and  
 excited state pseudoscalar mesons,”  
 nu-th/0503043



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Chiral limit, model-independent and algebraic result

$$g_{\pi_0^0\gamma\gamma} := \mathcal{T}_{\pi_0^0}(-m_{\pi_0^0}^2 = 0, 0) = \frac{1}{2} \frac{1}{f_{\pi_0}}$$

So long as truncation veraciously preserves chiral symmetry  
 and the pattern of its dynamical breakdown

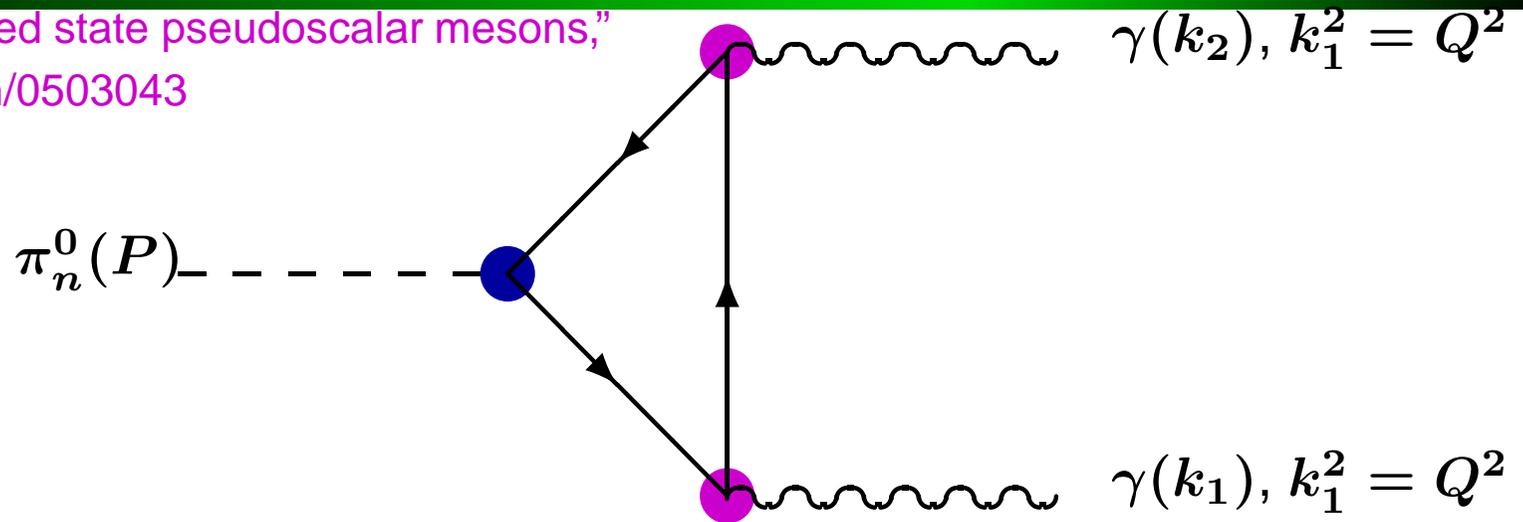
The most widely known consequence of the **Abelian anomaly**



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# Two-photon Couplings: Transition Form Factor

Höll, Krassnigg, Maris, *et al.*,  
“Electromagnetic properties of ground and  
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nu-th/0503043



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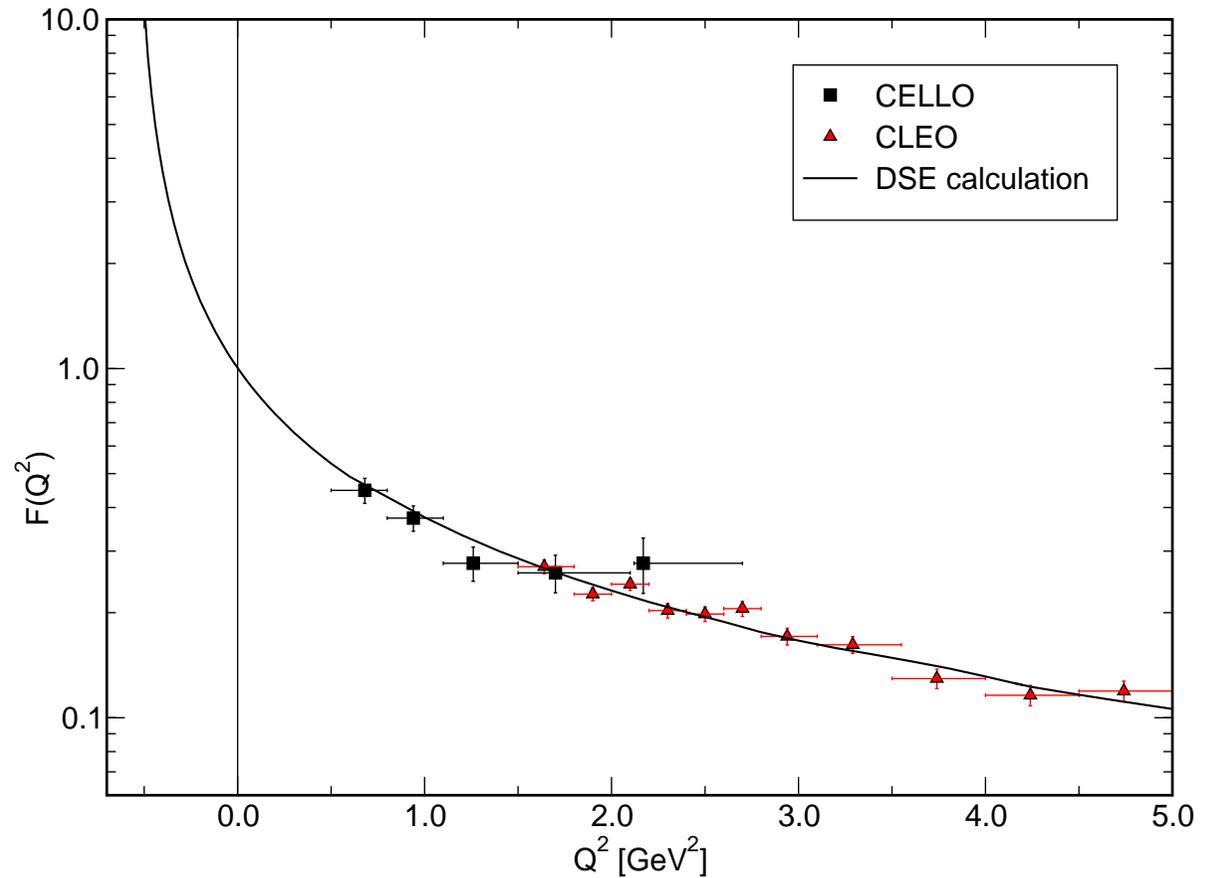
So long as truncation preserves chiral symmetry and the pattern of its dynamical breakdown, and the one-loop renormalisation group properties of QCD: model-independent result –  $\forall n$ :

$$T_{\pi_n^0}(P^2, Q^2) = G^{\pi_n^0}(k_1, k_2) \Big|_{k_1^2=Q^2=k_2^2} \stackrel{Q^2 \gg \Lambda_{\text{QCD}}^2}{=} \frac{4\pi^2}{3} \frac{f_{\pi_n}}{Q^2}$$



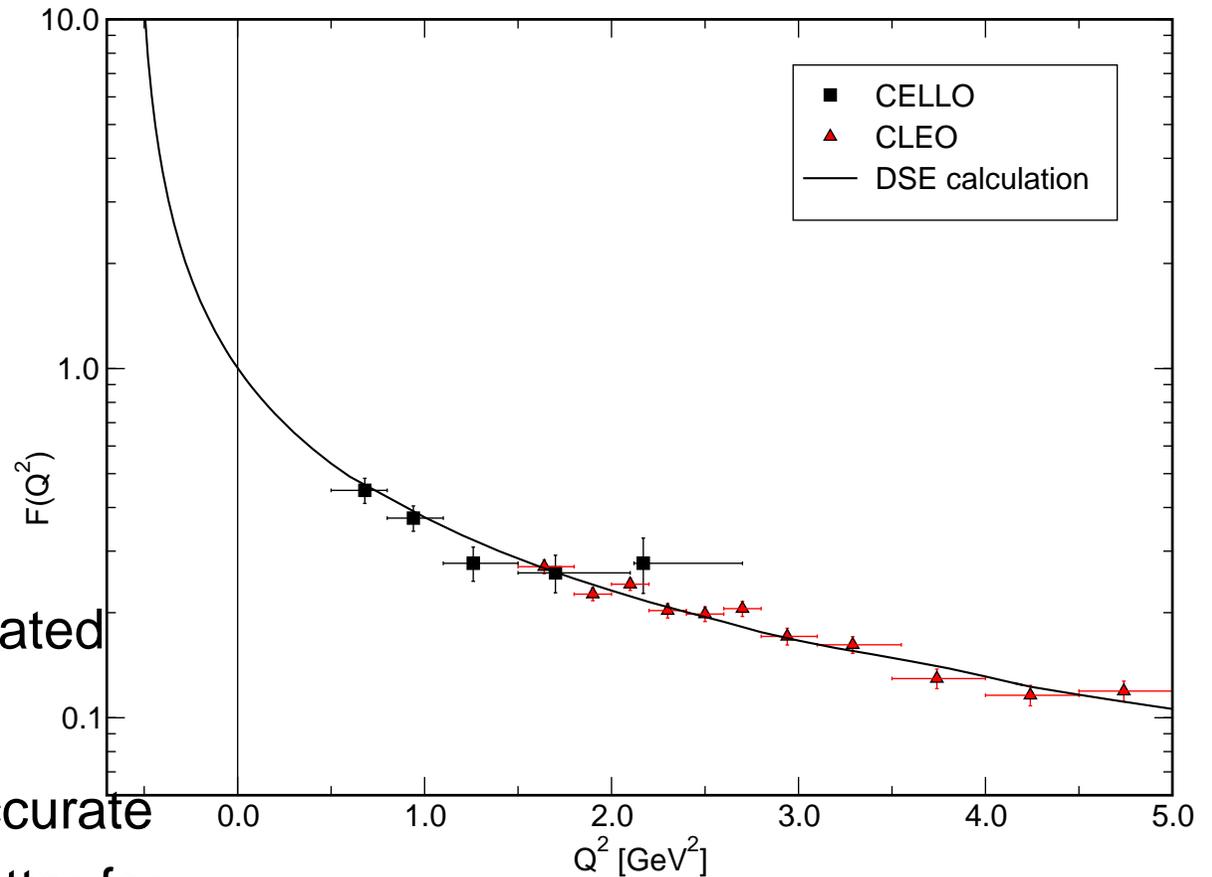
# Two-photon Couplings: Transition Form Factor

Maris and Tandy, " Electromagnetic  
transition form-factors of light mesons,"  
nucl-th/0201017



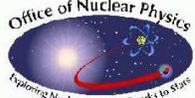
# Two-photon Couplings: Transition Form Factor

Maris and Tandy, " Electromagnetic transition form-factors of light mesons,"  
nucl-th/0201017



DSE result:

- normalisation calculated
- $\rho$ -meson generated dynamically
- pQCD result accurate to  $\sim 20\%$  or better for  $Q^2 \geq 3 \text{ GeV}^2$



*Harry Lee*

# *Pions and Form Factors*



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# Pions and Form Factors

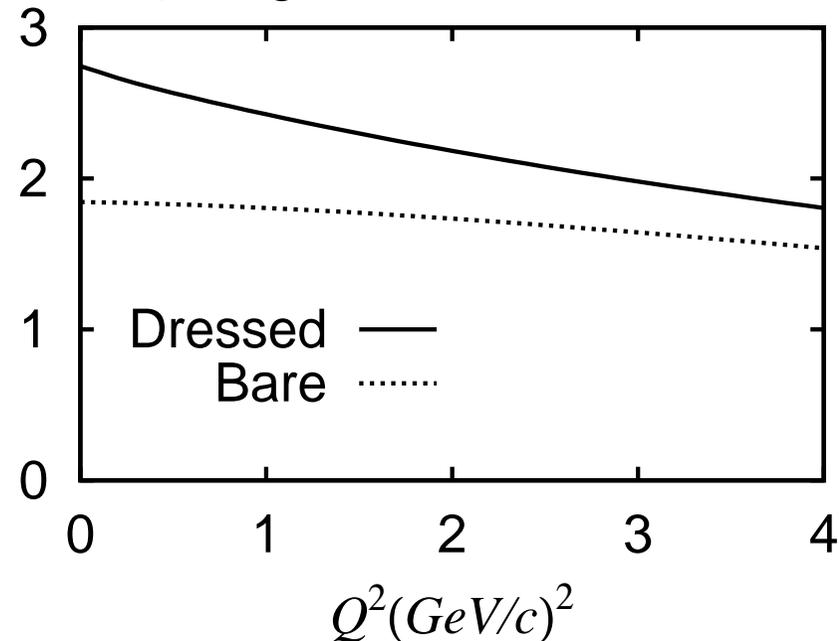
- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the  $\Delta(1236)$ 
  - *Meson Exchange Model for  $\pi N$  Scattering and  $\gamma N \rightarrow \pi N$  Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the  $\Delta$  Excitation in  $N(e, e'\pi)$  Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C **63**, 055201/1-13 (2001)



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Ratio of the M1 form factor in  $\gamma N \rightarrow \Delta$  transition and proton dipole form factor  $G_D$ . Solid curve is  $G_M^*(Q^2)/G_D(Q^2)$  including pions; Dotted curve is  $G_M(Q^2)/G_D(Q^2)$  without pions.



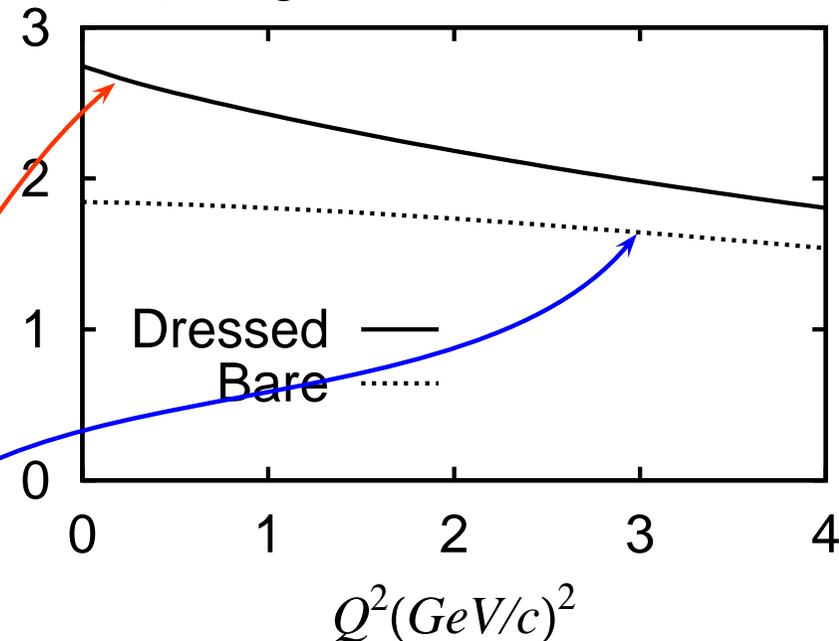
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### Quark Core

- Responsible for only 2/3 of result at small  $Q^2$
- Dominant for  $Q^2 > 2 - 3 \text{ GeV}^2$



# Results: Nucleon and $\Delta$ Masses



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# Results: Nucleon and $\Delta$ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and  $\Delta$  masses



**Set A** – fit to the actual masses was required; whereas for **Set B** – fitted mass was offset to allow for “ $\pi$ -cloud” contributions

set	$M_N$	$M_\Delta$	$m_{0+}$	$m_{1+}$	$\omega_{0+}$	$\omega_{1+}$
<b>A</b>	0.94	1.23	0.63	0.84	$0.44=1/(0.45 \text{ fm})$	$0.59=1/(0.33 \text{ fm})$
<b>B</b>	1.18	1.33	0.80	0.89	$0.56=1/(0.35 \text{ fm})$	$0.63=1/(0.31 \text{ fm})$

$m_{1+} \rightarrow \infty$ :  $M_N^A = 1.15 \text{ GeV}$ ;  $M_N^B = 1.46 \text{ GeV}$



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# Results: Nucleon and $\Delta$ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and  $\Delta$  masses



**Set A** – fit to the actual masses was required; whereas for **Set B** – fitted mass was offset to allow for “ $\pi$ -cloud” contributions

set	$M_N$	$M_\Delta$	$m_{0+}$	$m_{1+}$	$\omega_{0+}$	$\omega_{1+}$
<b>A</b>	0.94	1.23	0.63	0.84	$0.44=1/(0.45 \text{ fm})$	$0.59=1/(0.33 \text{ fm})$
<b>B</b>	1.18	1.33	0.80	0.89	$0.56=1/(0.35 \text{ fm})$	$0.63=1/(0.31 \text{ fm})$

●  $m_{1+} \rightarrow \infty$ :  $M_N^A = 1.15 \text{ GeV}$ ;  $M_N^B = 1.46 \text{ GeV}$

● Axial-vector diquark provides significant attraction



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● **Constructive Interference:**  $1^{++}$ -diquark +  $\partial_\mu \pi$



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# Deep-inelastic scattering



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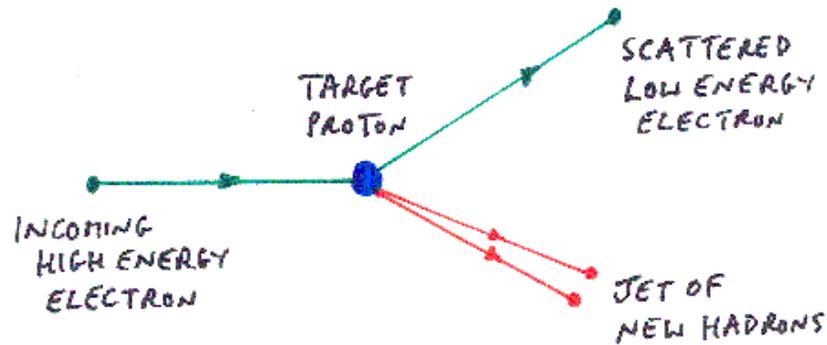
# Deep-inelastic scattering



- Looking for Quarks



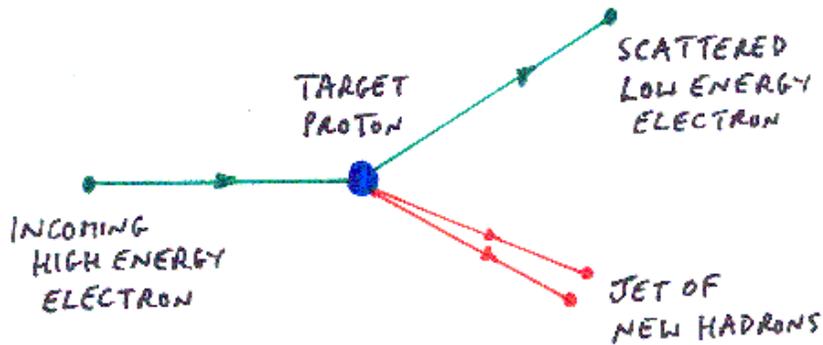
# Deep-inelastic scattering



## ● Looking for Quarks



# Deep-inelastic scattering



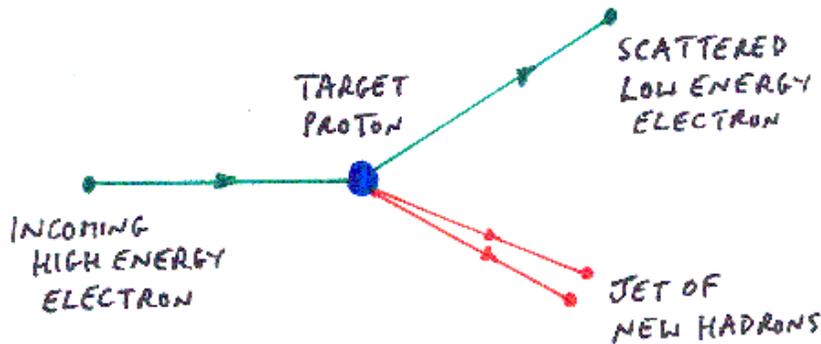
- Looking for Quarks

● **Signature Experiment** for QCD:

Discovery of Quarks at SLAC



# Deep-inelastic scattering



- Looking for Quarks



**Signature Experiment** for QCD:

Discovery of Quarks at SLAC



Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution:  $q(x)$ ,  $g(x)$



# *Pion's valence quark distn*



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# Pion's valence quark distn

- $\pi$  is Two-Body System: "Easiest" Bound State in QCD
- However, NO  $\pi$  Targets!



# Pion's valence quark distn

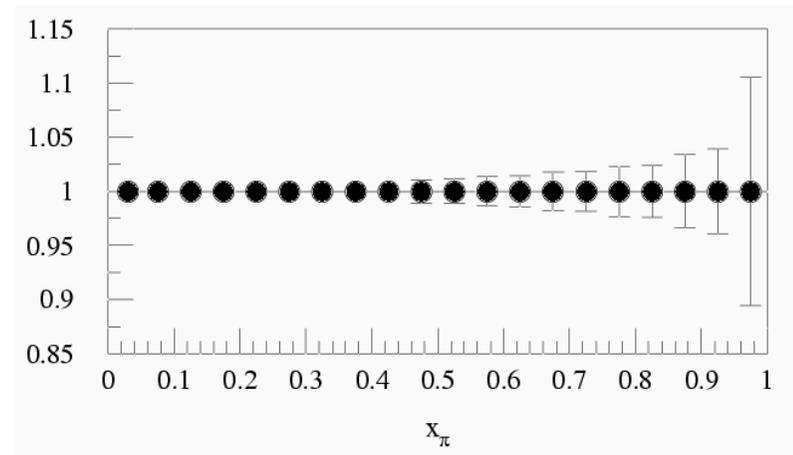
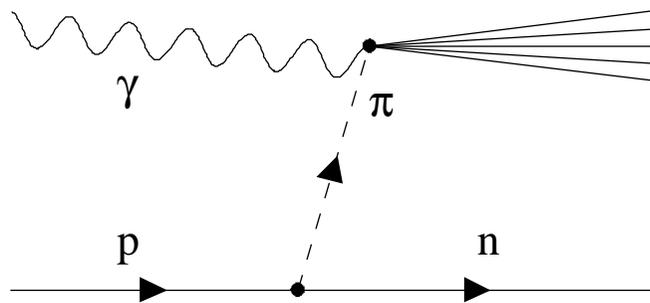
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$$\pi N \rightarrow \mu^+ \mu^- X$$



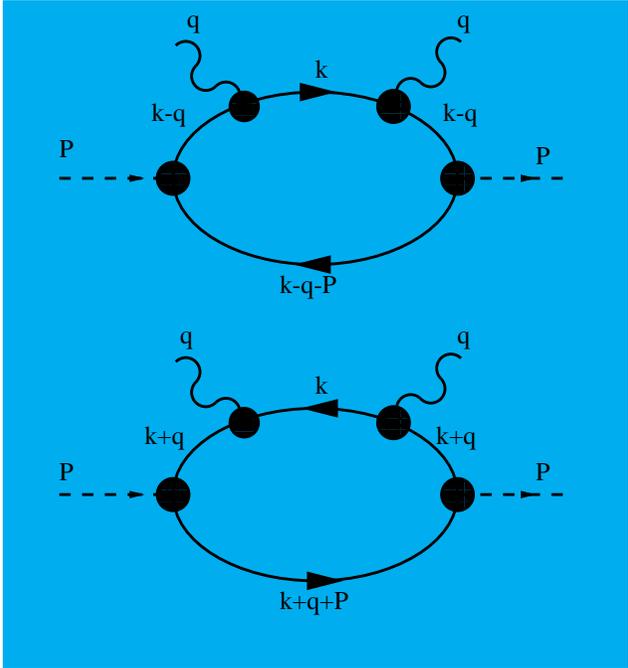
# Pion's valence quark distn

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- However, NO  $\pi$  Targets!
- Existing Measurement Inferred from Drell-Yan:  
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- Proposal (Holt & Reimer, ANL, nu-ex/0010004)

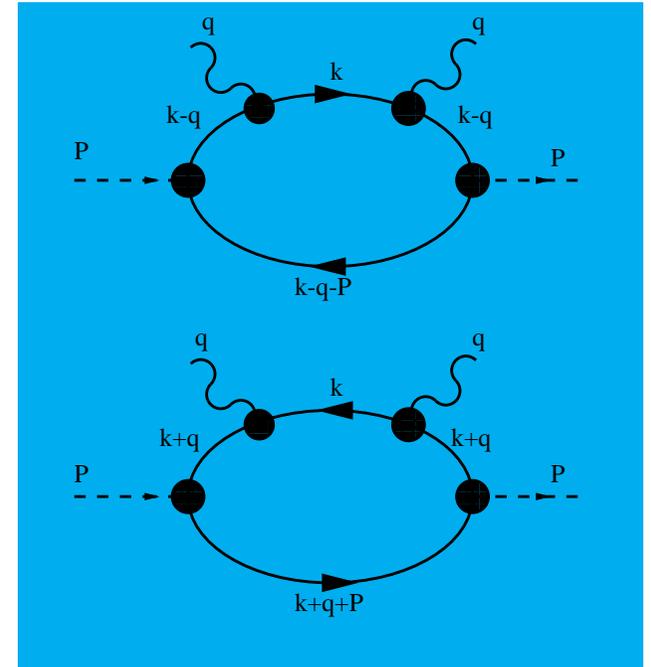
$e_{5\text{GeV}}^- - p_{25\text{GeV}}$  Collider  $\rightarrow$  Accurate "Measurement"



# Handbag diagrams



# Handbag diagrams



$$W_{\mu\nu}(q; P) = \frac{1}{2\pi} \text{Im} [T_{\mu\nu}^+(q; P) + T_{\mu\nu}^-(q; P)]$$

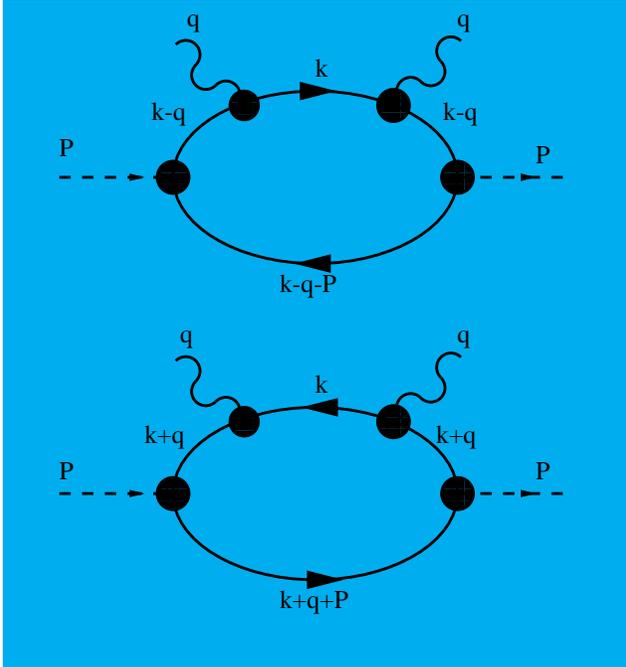
$$T_{\mu\nu}^+(q, P) = \text{tr} \int \frac{d^4 k}{(2\pi)^4} \tau_- \bar{\Gamma}_\pi(k_{-\frac{1}{2}}; -P) S(k_{-0}) ieQ\Gamma_\nu(k_{-0}, k) \\ \times S(k) ieQ\Gamma_\mu(k, k_{-0}) S(k_{-0}) \tau_+ \Gamma_\pi(k_{-\frac{1}{2}}; P) S(k_{--})$$



# Handbag diagrams

Bjorken Limit:  $q^2 \rightarrow \infty$ ,  $P \cdot q \rightarrow -\infty$   
 but  $x := -\frac{q^2}{2P \cdot q}$  fixed.

Numerous algebraic simplifications



$$W_{\mu\nu}(q; P) = \frac{1}{2\pi} \text{Im} [T_{\mu\nu}^+(q; P) + T_{\mu\nu}^-(q; P)]$$

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# Extant theory vs. experiment

K. Wijersooriya, P. Reimer and R. Holt,  
nu-ex/0509012 ... Phys. Rev. C (Rapid)

