# Form Factors: A DSE Perspective

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http://www.phy.anl.gov/theory/staff/cdr.html

# Universal Truths

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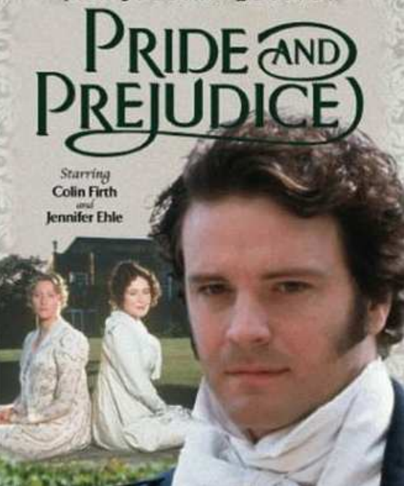


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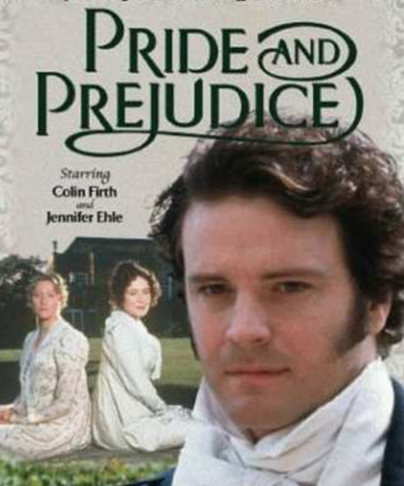


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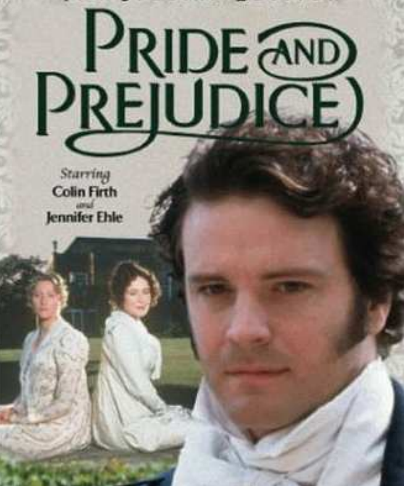


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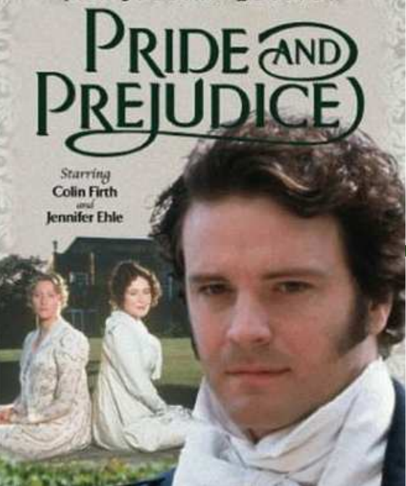


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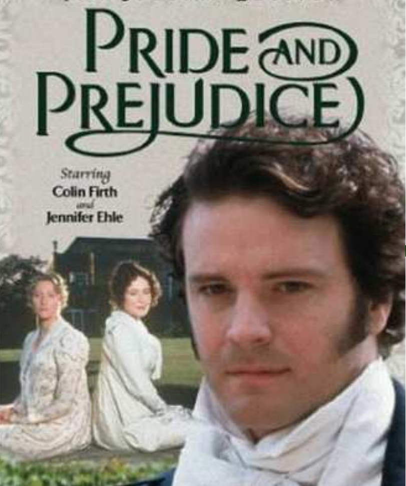


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- DCSB is most important mass generating mechanism for matter in the Universe. Higgs mechanism is irrelevant to light-quarks.
- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.


# QCD’s Challenges

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# QCD’s Challenges

- Quark and Gluon Confinement
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## Understand Emergent Phenomena

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- QCD – Complex behaviour

  arises from apparently simple rules

# *Dichotomy of Pion*

# *– Goldstone Mode* and *Bound state*

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Current Algebra . . . 1968

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  - Interaction between quarks – the Interquark "Potential" – unknown throughout $>$ 98% of a hadron's volume

# Intranucleon Interaction

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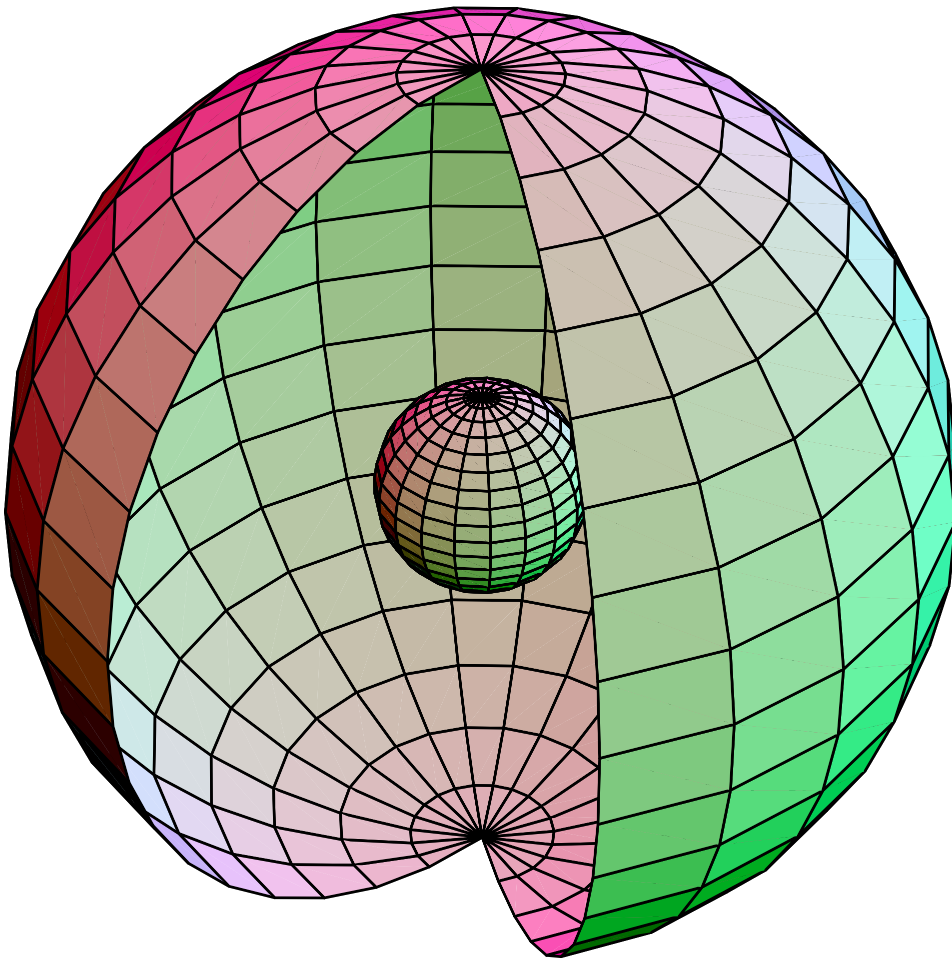
# Intranucleon Interaction



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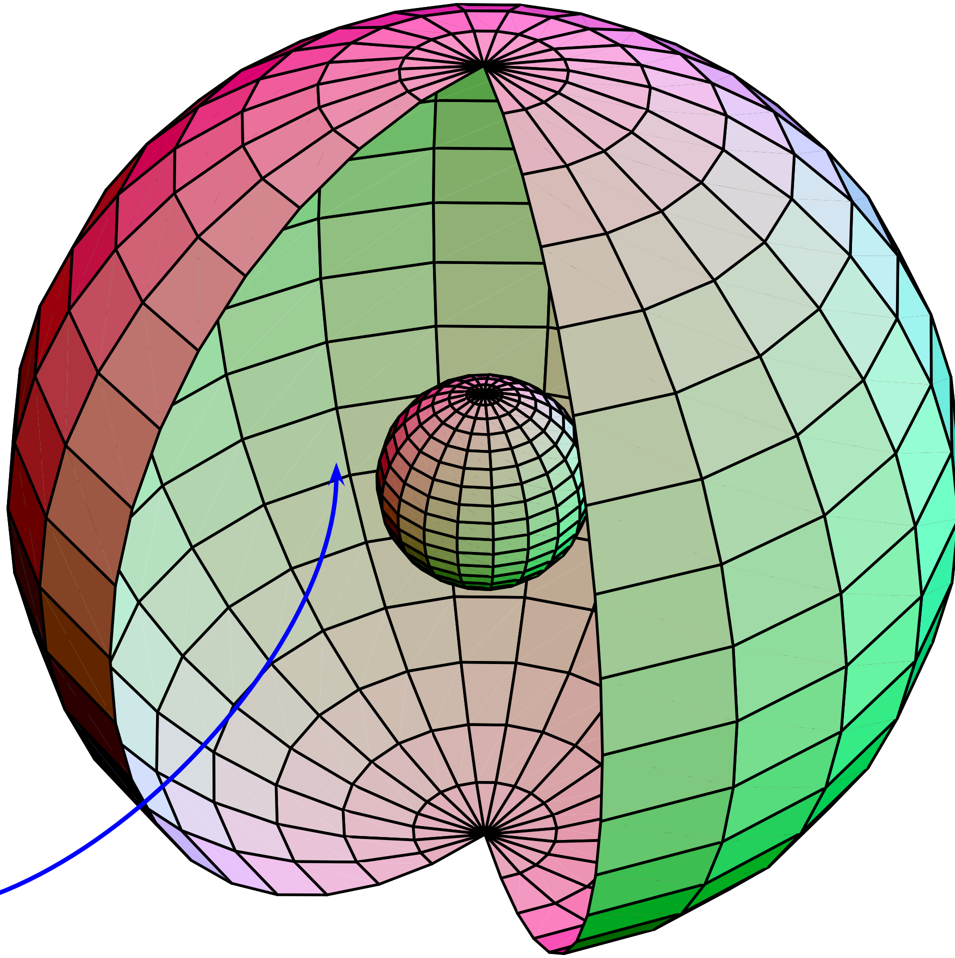
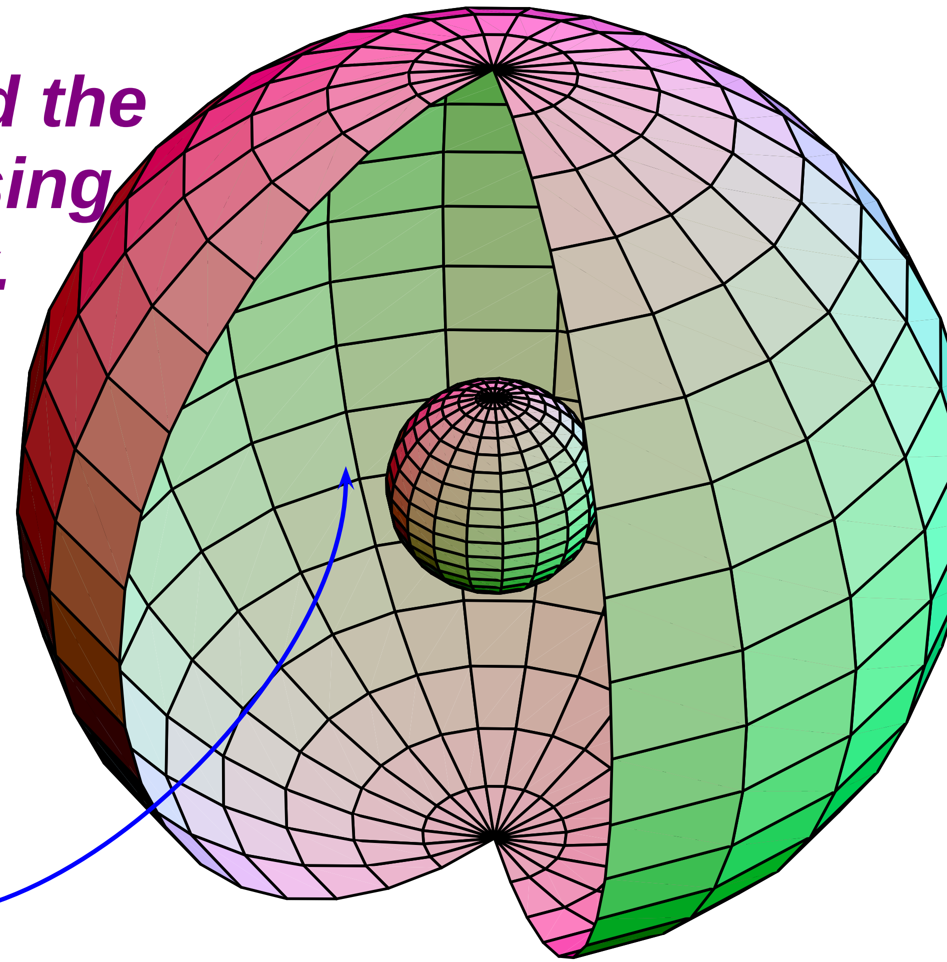


98% of the volume

# What is the Intranucleon Interaction?

***The question must be rigorously defined, and the answer mapped out using experiment and theory.***

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# Dyson-Schwinger Equations

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- $\Rightarrow$ Understanding InfraRed (long-range)

  . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . behaviour of $\alpha_s(Q^2)$

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- Method yields Schwinger Functions $\equiv$ Propagators

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Cross-Sections built from Schwinger Functions

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- Proving fruitful.

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# World . . .

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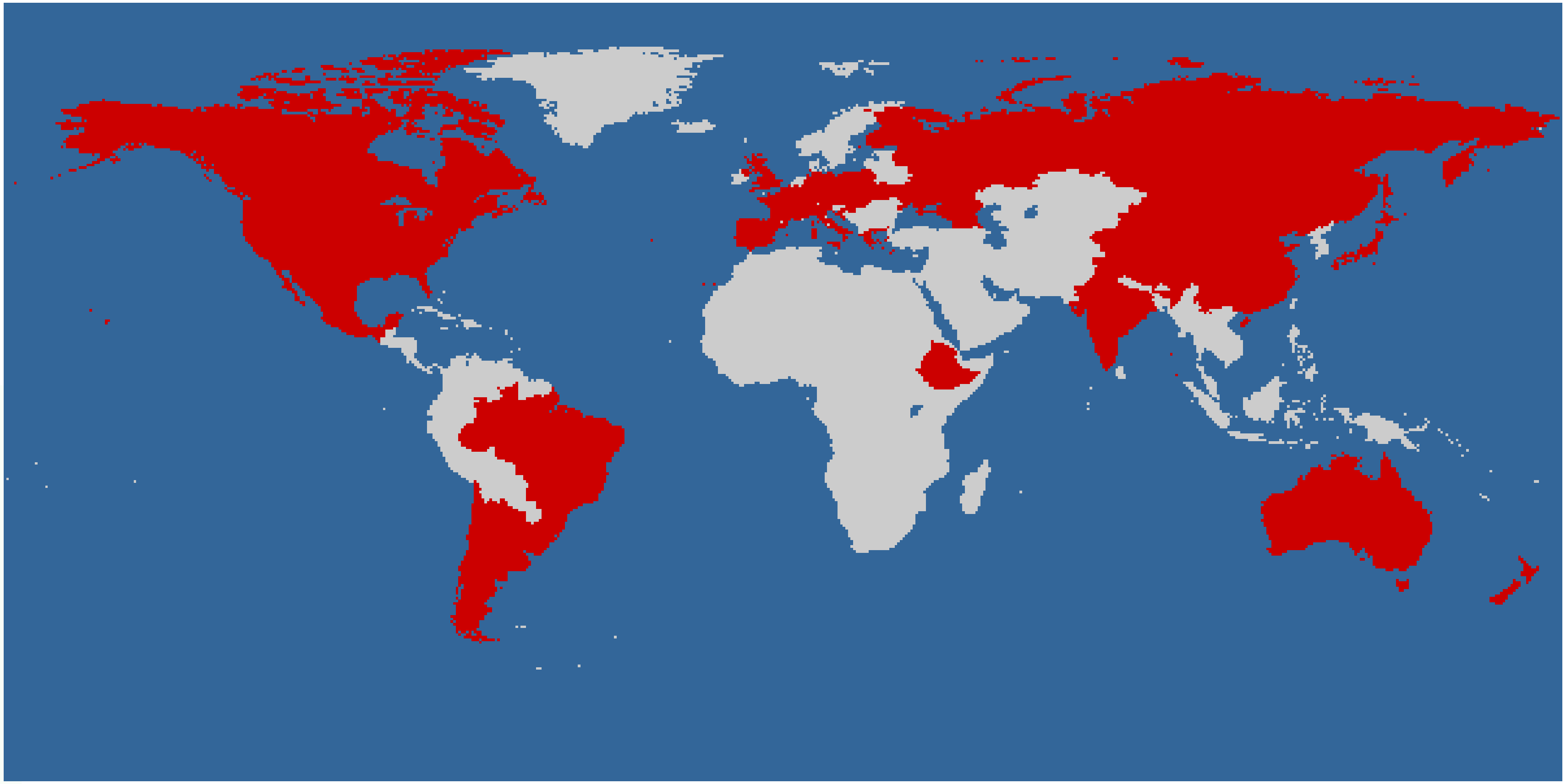
# DSE Perspective



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# Persistent Challenge


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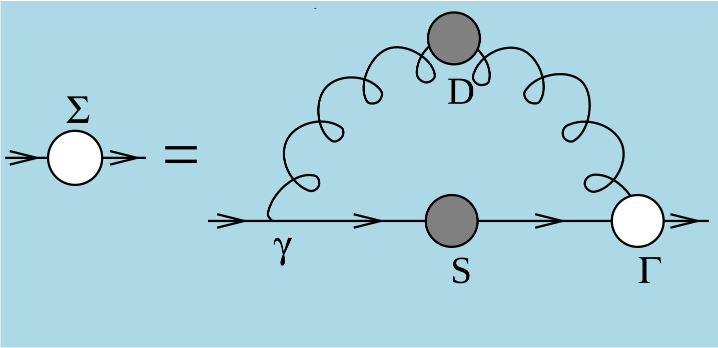


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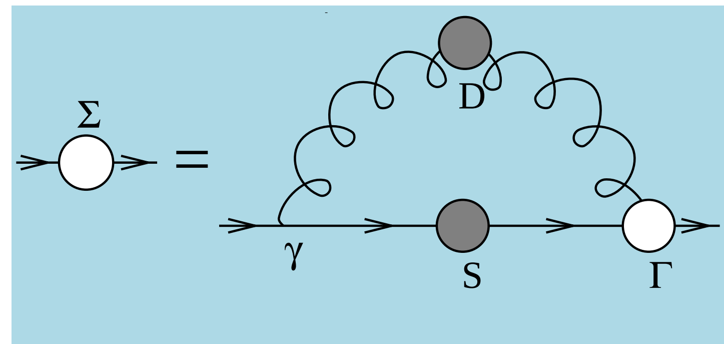


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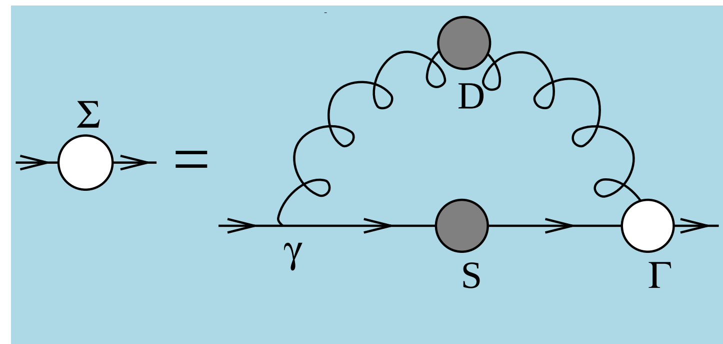


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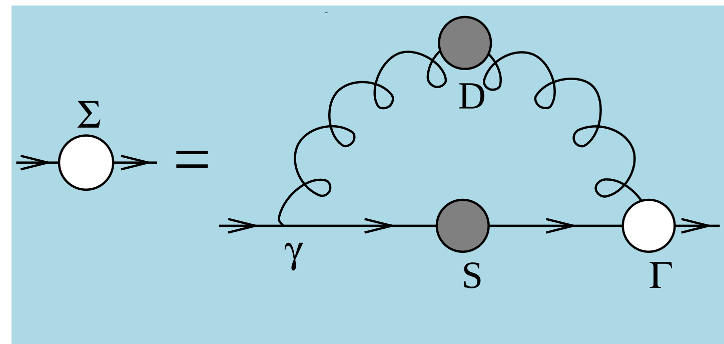


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    Not useful for the nonperturbative problems
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  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  *Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  *Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*

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  - Make Predictions with Readily Quantifiable Errors

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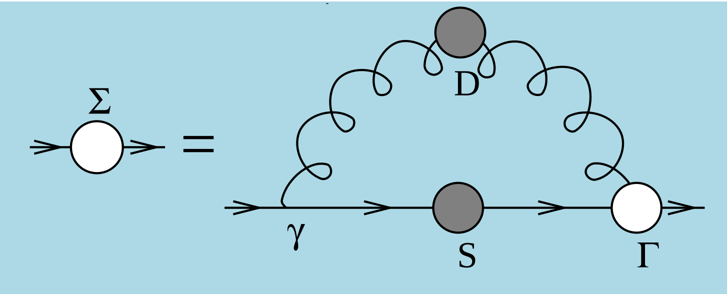


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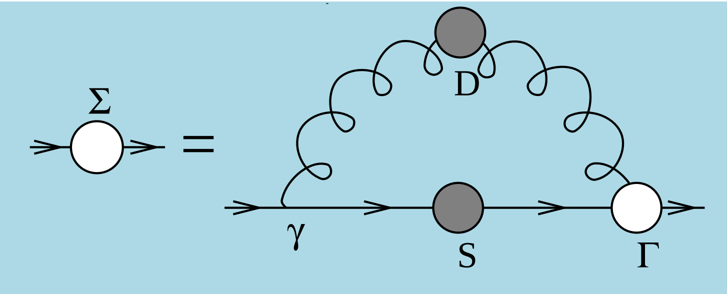
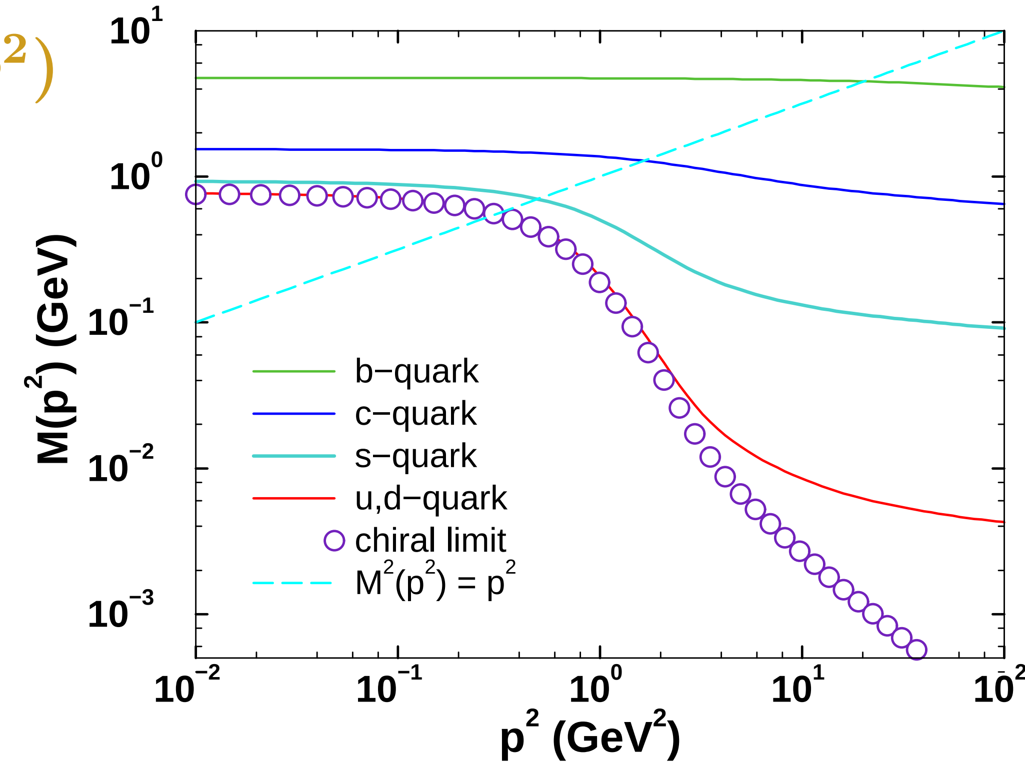



Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



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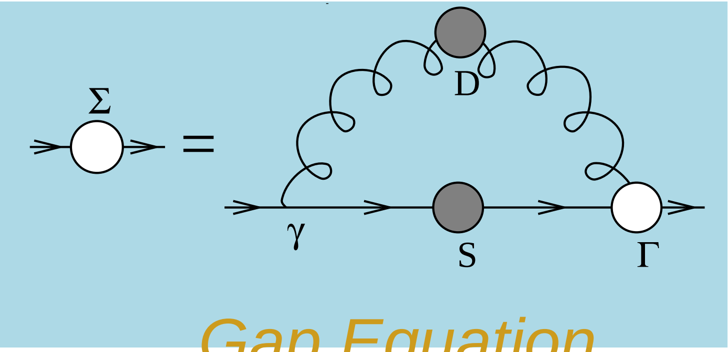


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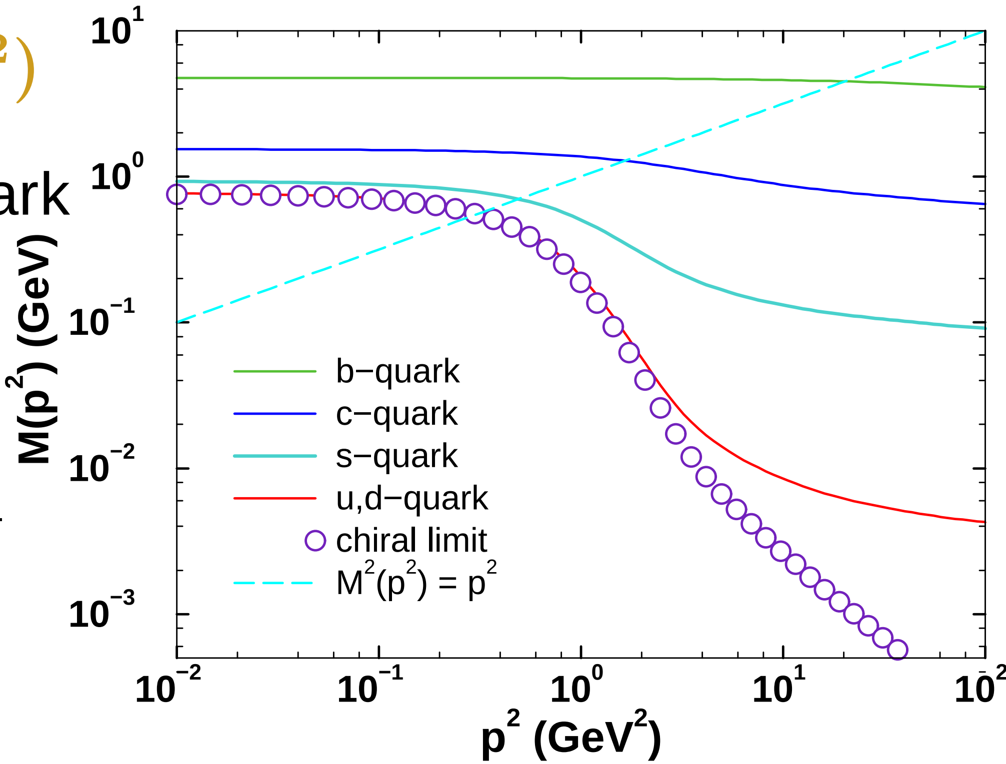
- Euclidean Constituent–Quark Mass: $M_f^E$: $p^2 = M(p^2)^2$

| flavour | $u/d$ | $s$ | $c$ | $b$ |
|---|---|---|---|---|
| $\frac{M^E}{m_\zeta}$ | $\sim 10^2$ | $\sim 10$ | $\sim 1.5$ | $\sim 1.1$ |



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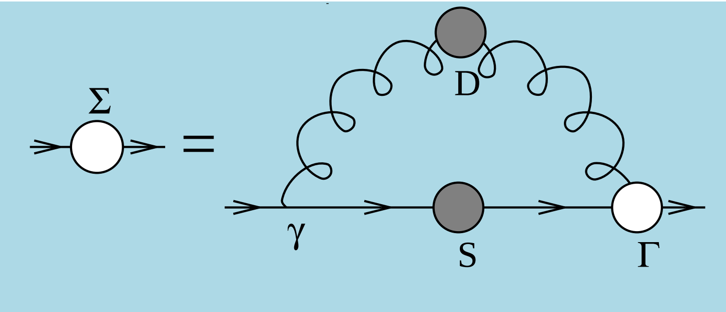


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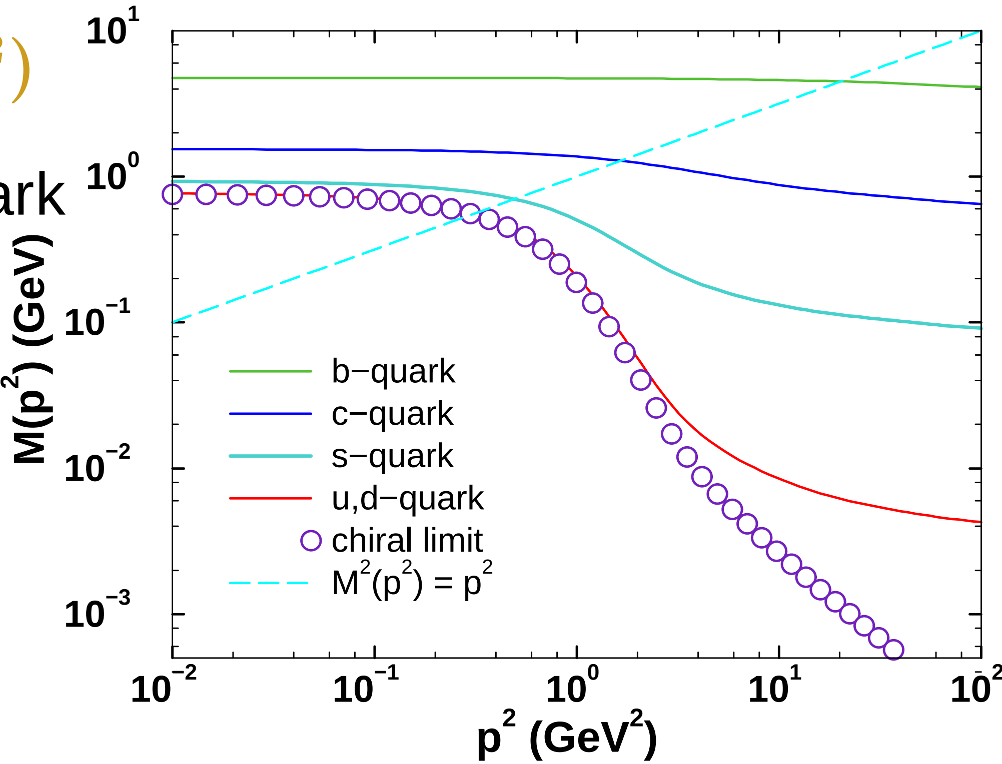


Predictions confirmed in numerical simulations of lattice-QCD

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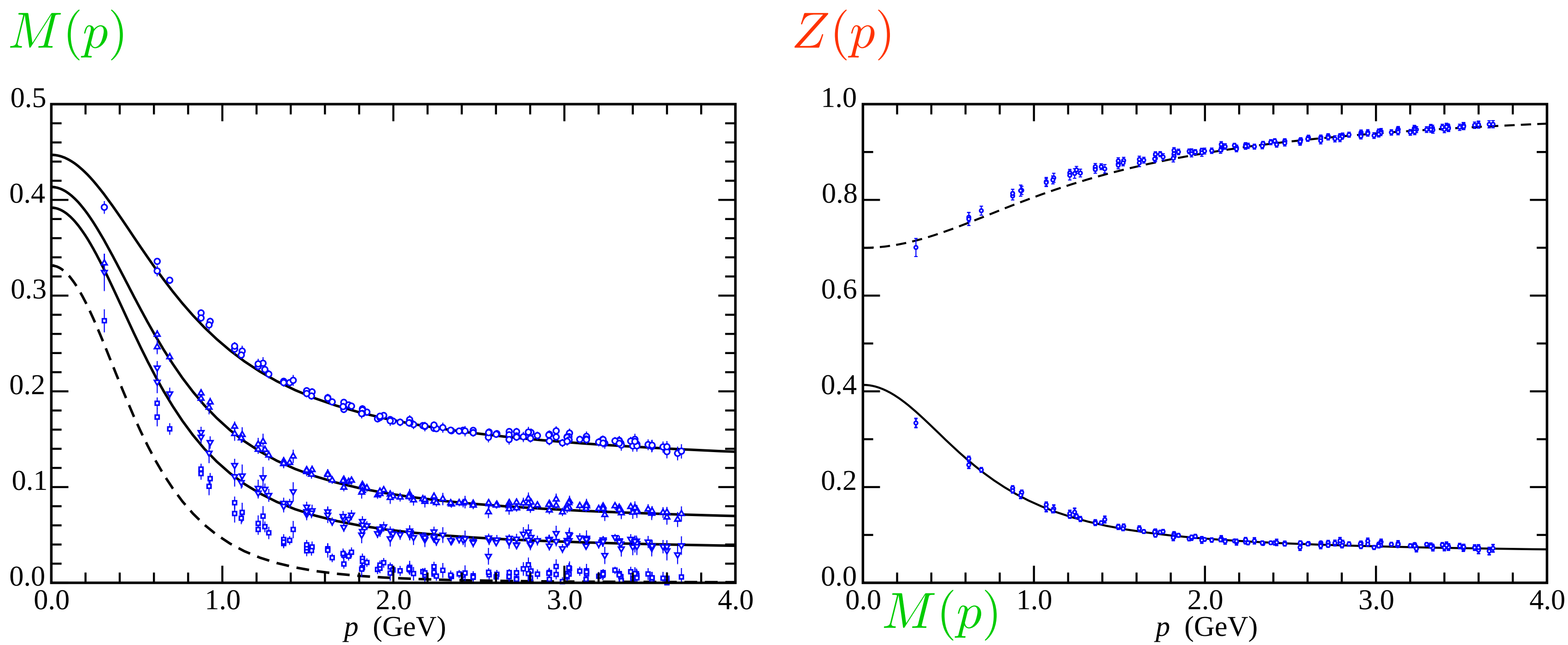


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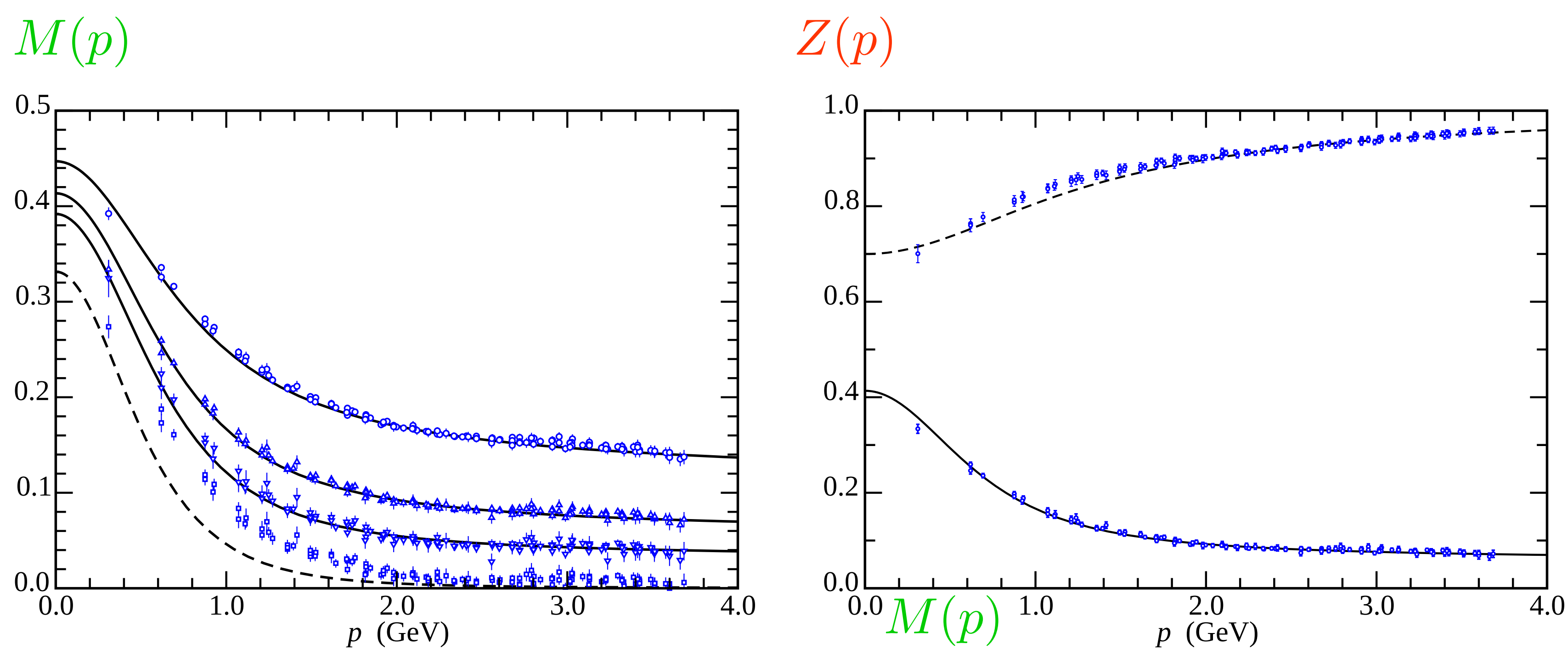
# 2002

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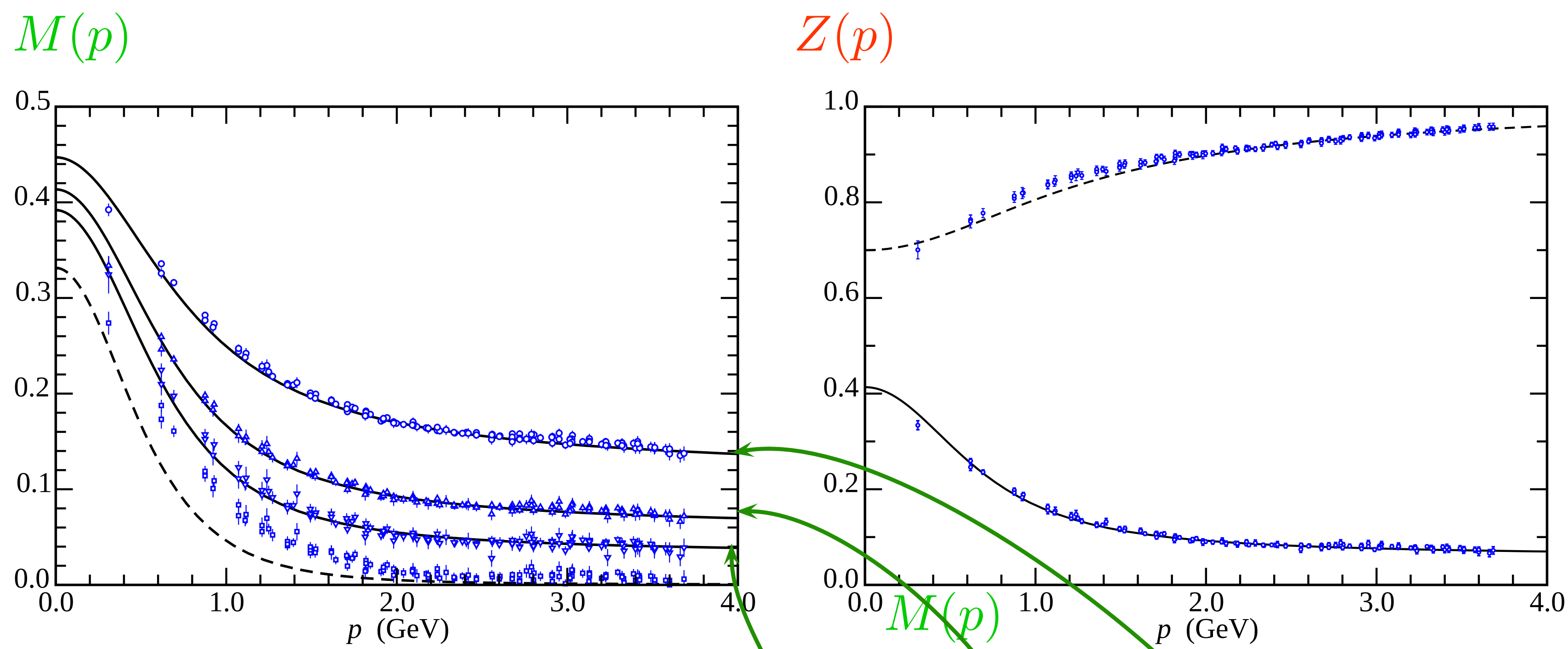


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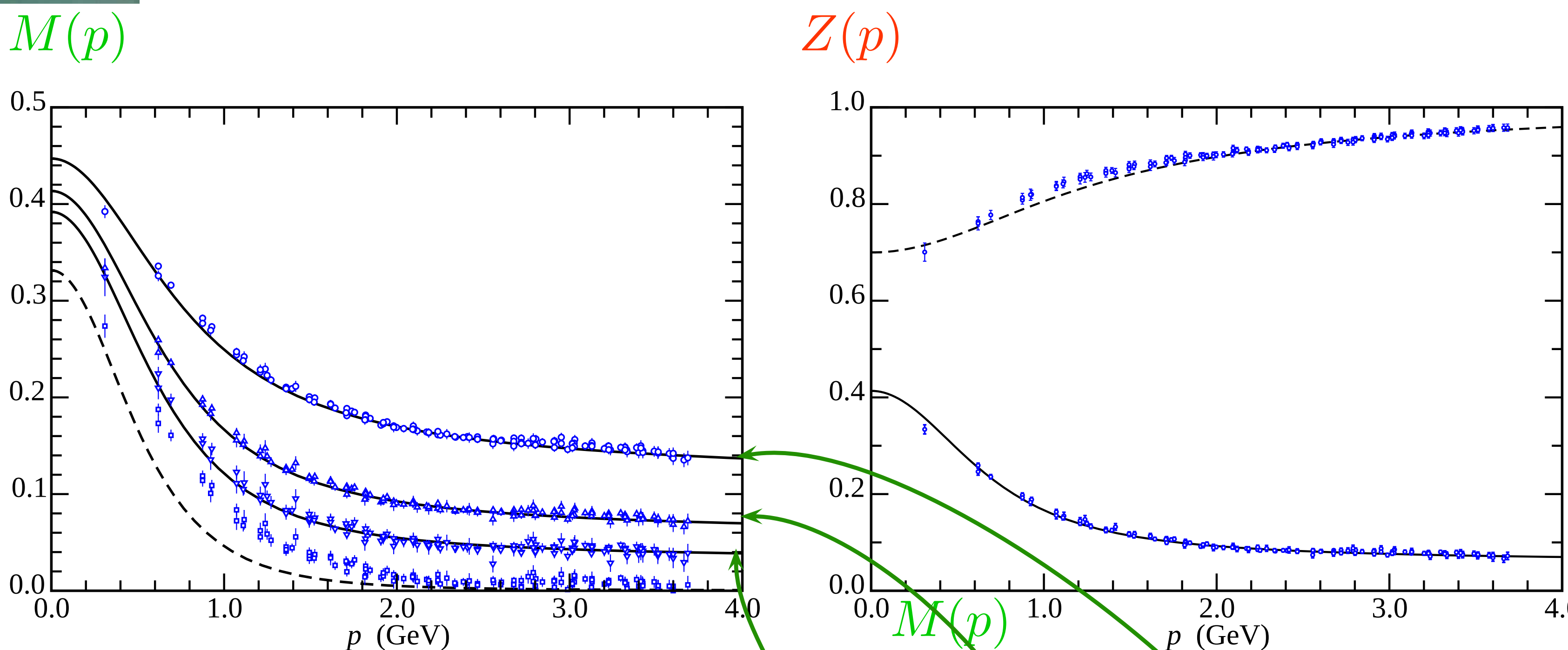


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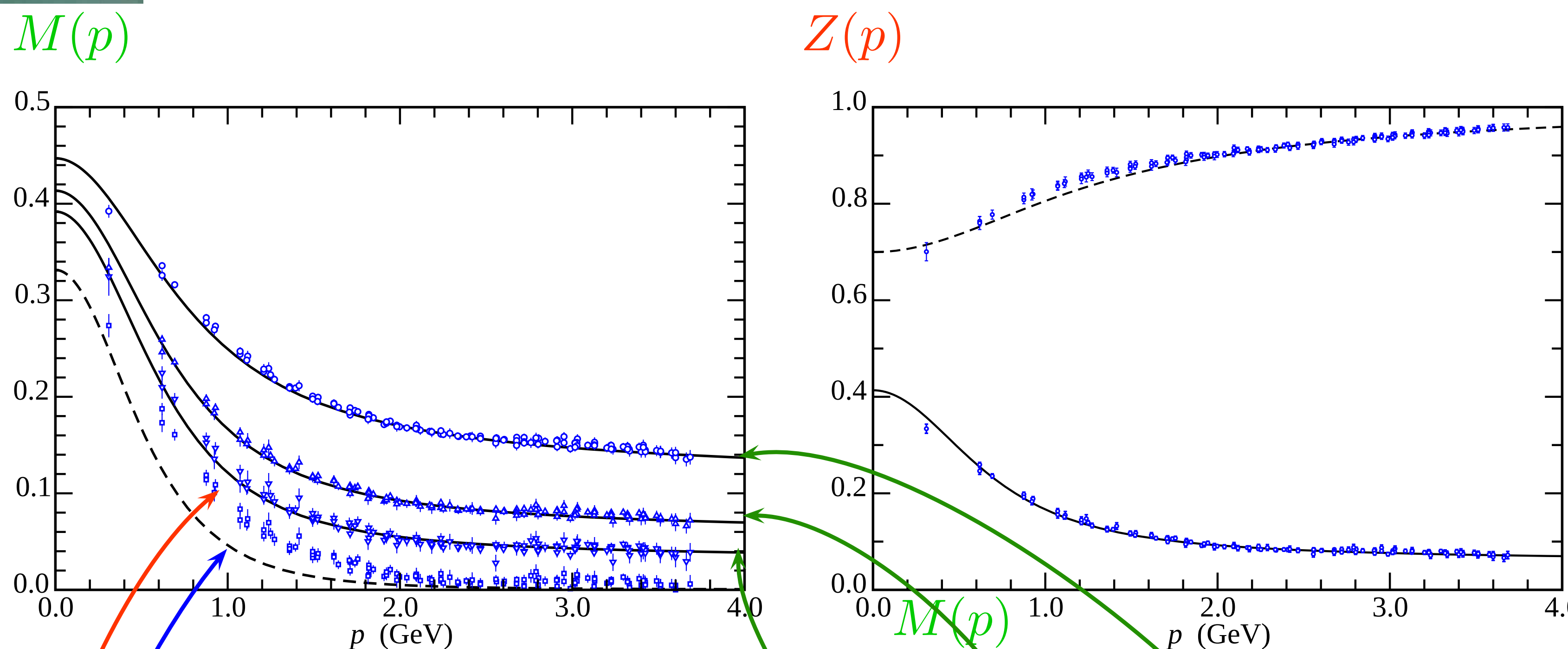


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- Linear extrapolation of lattice data to chiral limit is inaccurate

# Frontiers of Nuclear Science:
# A Long Range Plan (2007)

# Frontiers of Nuclear Science:
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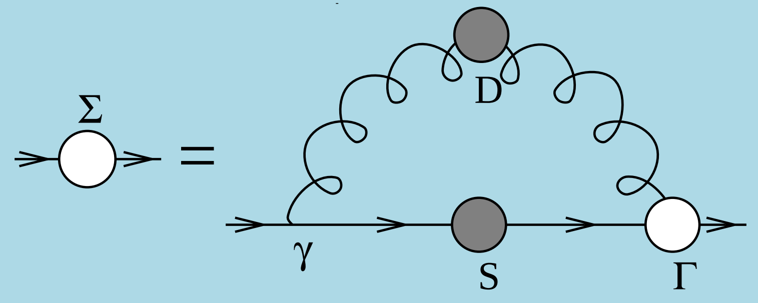

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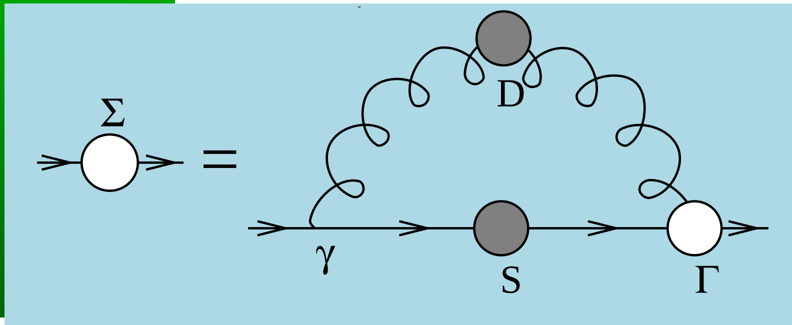
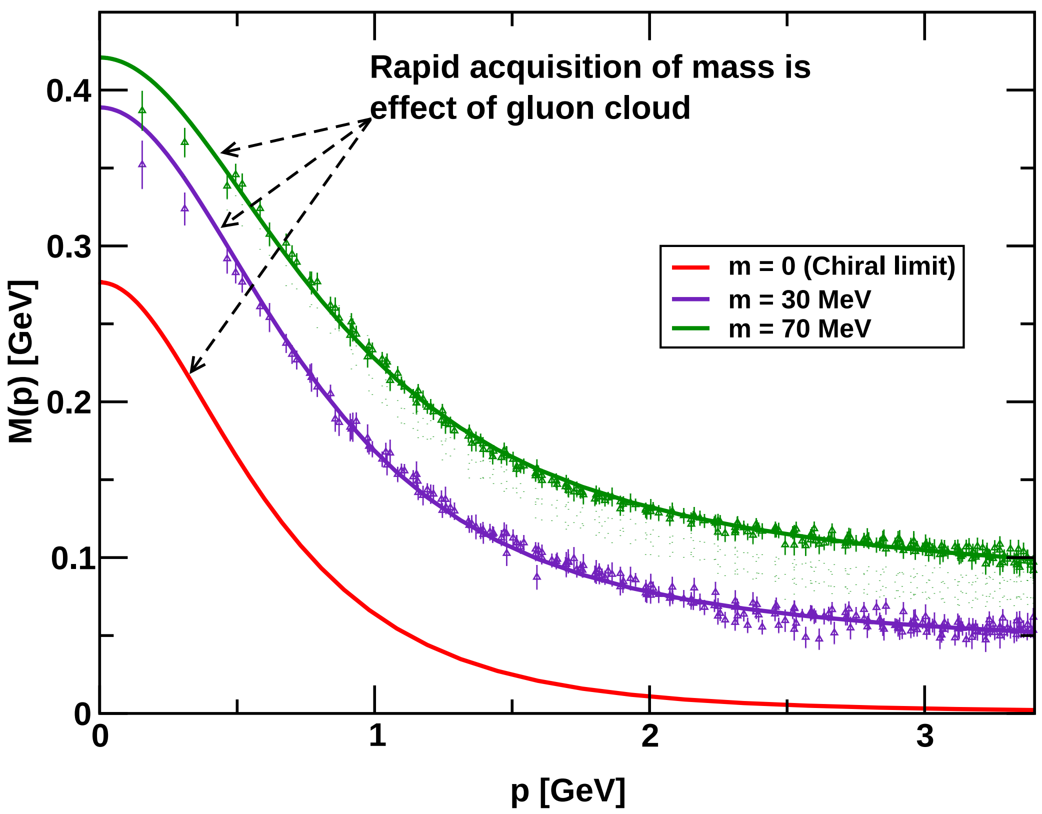
# *Frontiers of Nuclear Science: Theoretical Advances*

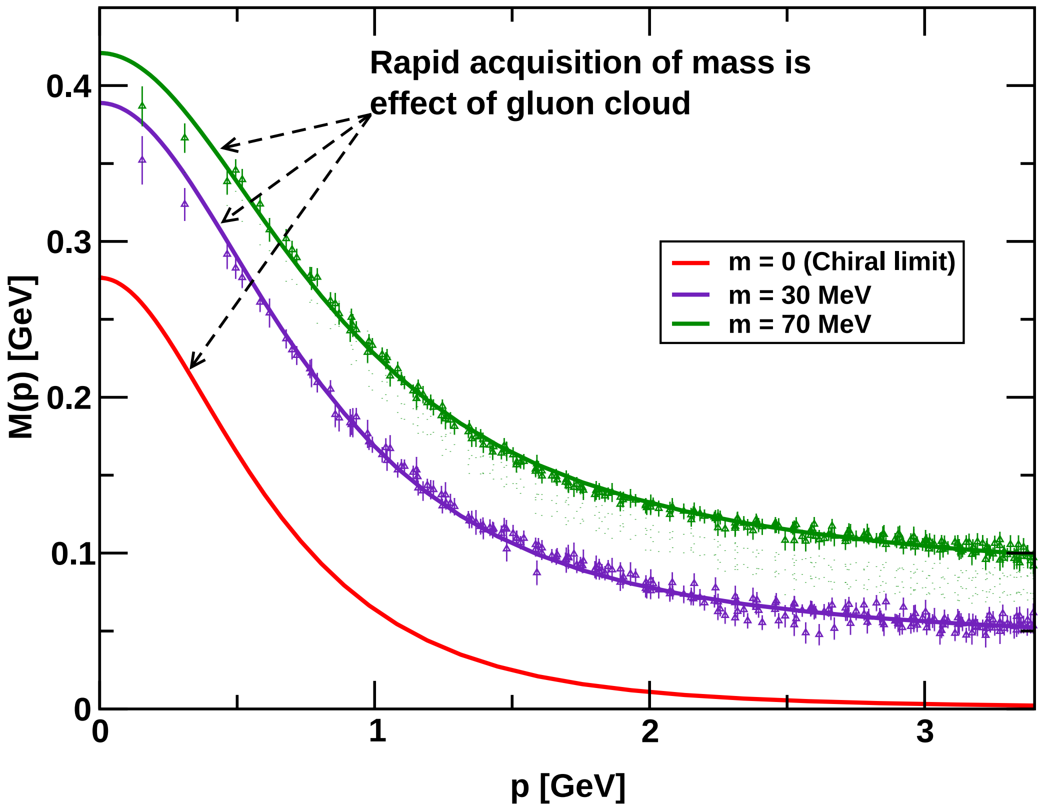



*Gap Equation*



# Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.



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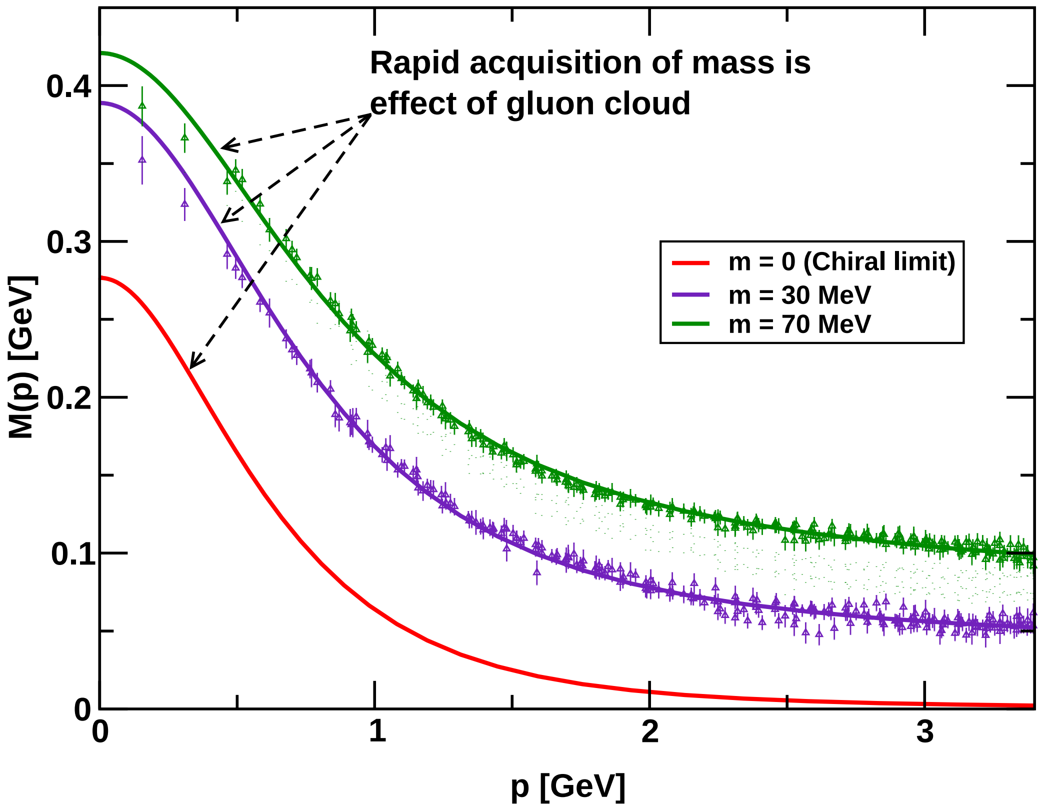

# *Consituent-quark $\sigma$-term*

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark $\sigma$-term

$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\;(M^E)^2 := s\,|\,s = M(s)^2\,.$$

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$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;(M^E)^2 := s\,|\, s = M(s)^2\,.$$

- Renormalisation-group-invariant and determined from solutions of the gap equation

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- Unambiguous probe of impact of explicit chiral symmetry breaking on the mass function

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$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\; (M^E)^2 := s\,|\,s = M(s)^2\,.$$

- Ratio $\boxed{\dfrac{\sigma_f}{M_f^E} = \dfrac{\text{EXPLICIT}}{\text{EXPLICIT} + \text{DYNAMICAL}}}$

measures effect of *EXPLICIT* chiral symmetry breaking on dressed-quark mass-function

cf. *SUM* of effects of *EXPLICIT* AND *DYNAMICAL CHIRAL SYMMETRY BREAKING*

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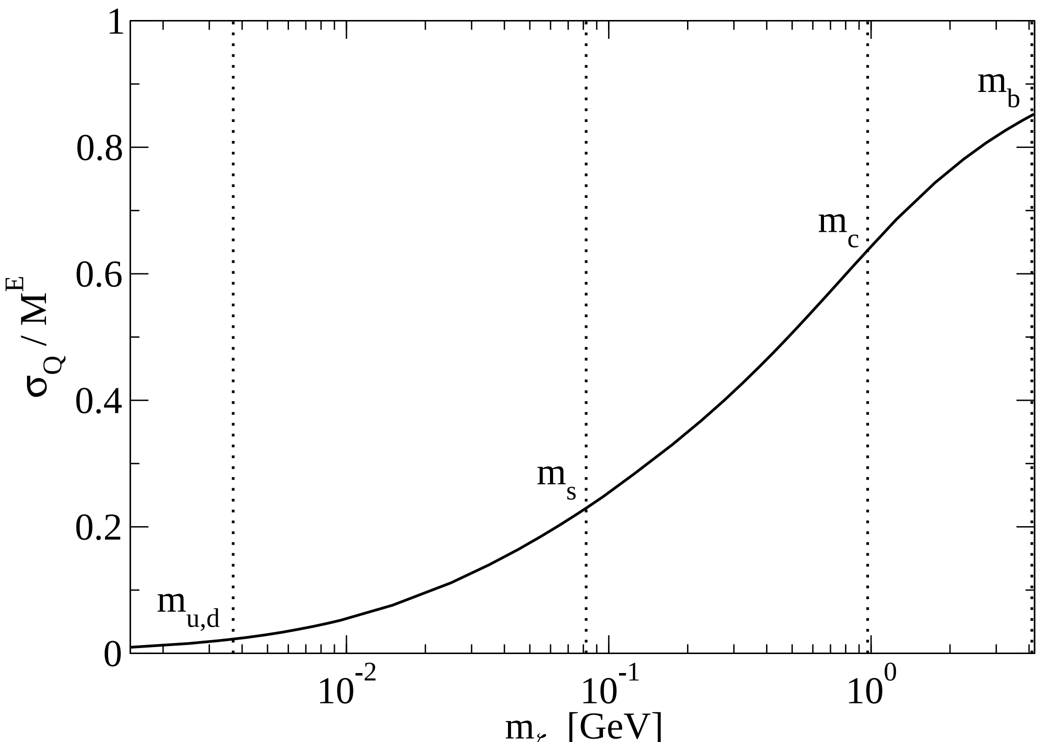


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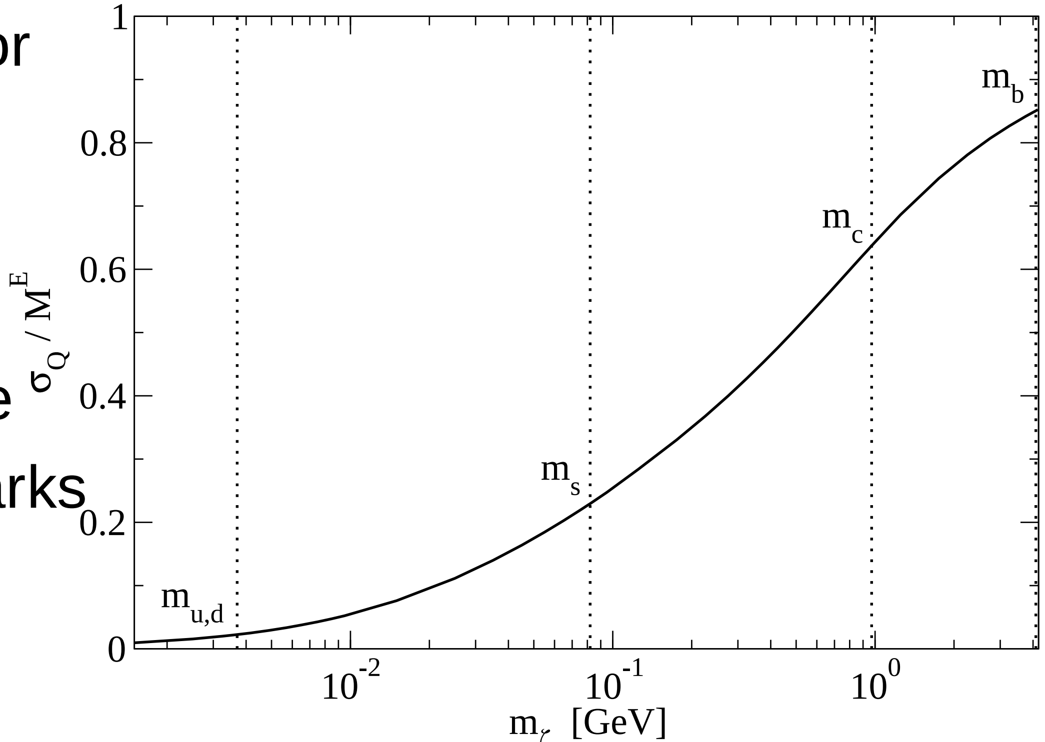
Obvious: ratio vanishes for light-quarks because magnitude of their constituent-mass owes primarily to DCSB. On the other hand, for heavy-quarks it approaches one.



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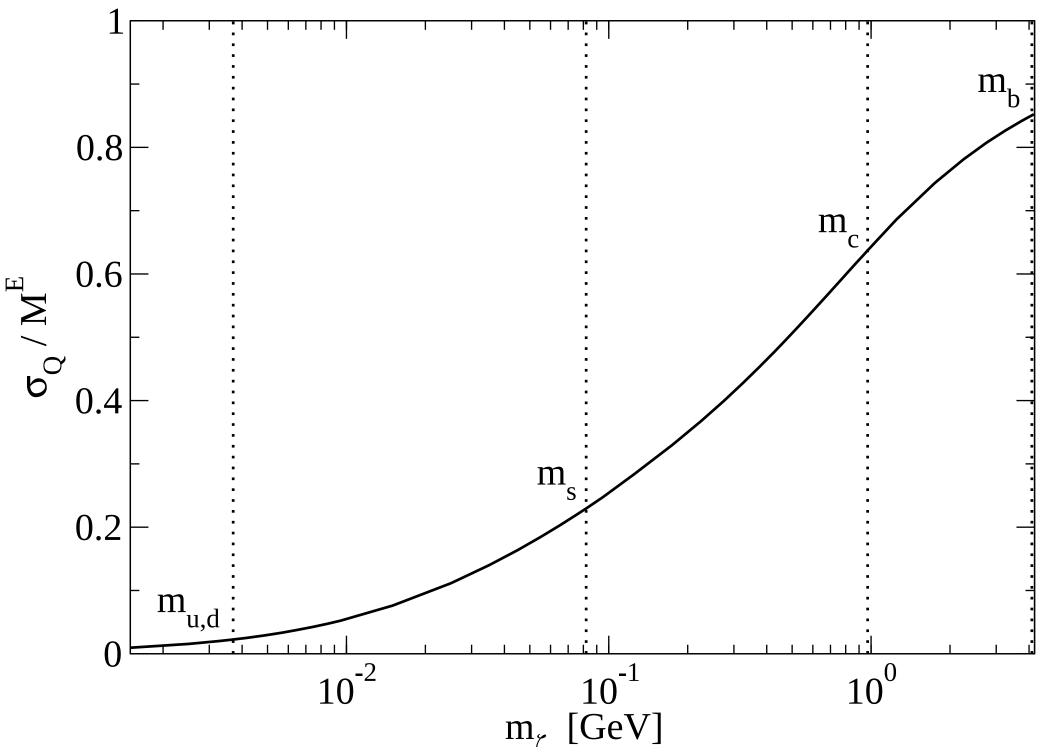
$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\; (M^E)^2 := s\,|\, s = M(s)^2\,.$$

Essentially dynamical component of chiral symmetry breaking, and manifestation in all its order parameters, vanishes with increasing current-quark mass

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# Critical Mass for Chiral Expansion

Lei Chang, *et al*., nucl-th/0605058

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$$\boxed{M(p^2; m = 0) \neq 0}.$$

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- Does this mass function have a ***convergent*** expansion in current-quark mass about its nonzero chiral-limit value:

$$M(0;m) = M(0,0) + m\, \frac{\partial}{\partial m} M(0;m)\Big|_{m=0} + \ldots\ ?$$

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- $$M(0;m) = M(0,0) + \sum_{n=1}^{\infty} m^n \, a_n$$

  Radius of convergence: $m_{\rm rc} = \lim_{n\to\infty} \left( \frac{1}{|a_n|} \right)^{1/n}$

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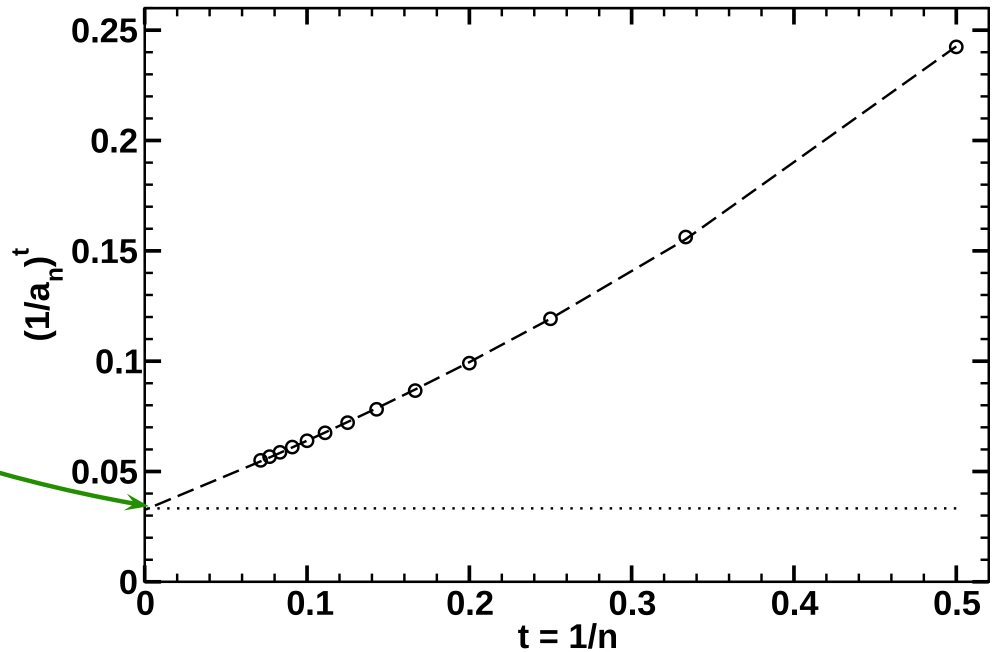
$$m_{\rm rc} = 0.034 \pm 0.001$$



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- Entails, e.g., lattice-QCD simulations *must* have results at $m_\pi^2 < [m_{0^-}^{\rm cr}]^2 \sim 0.2\,$GeV$^2$ for reasonable extrapolation via EFT.

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# Hadrons

- Established understanding
  of two- and three-point functions


# Hadrons

- Established understanding
  of two- and three-point functions
- What about bound states?


# Hadrons

- Without bound states,
Comparison with experiment is
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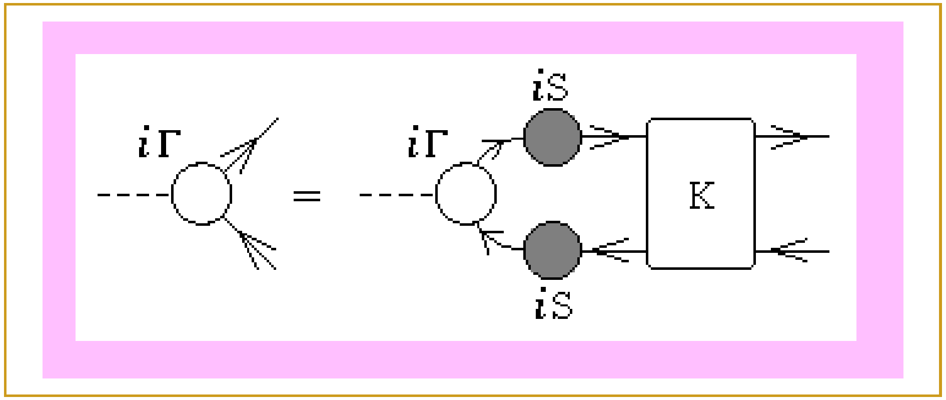
## *Hadrons*

- Without bound states, Comparison with experiment is impossible
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

- Without bound states, Comparison with experiment is impossible

- Bethe-Salpeter Equation




QFT Generalisation of Lippmann-Schwinger Equation.

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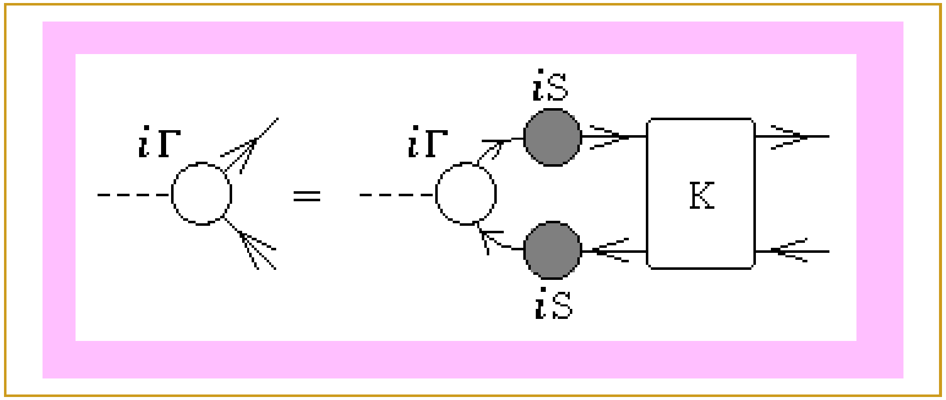


QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, $K$?

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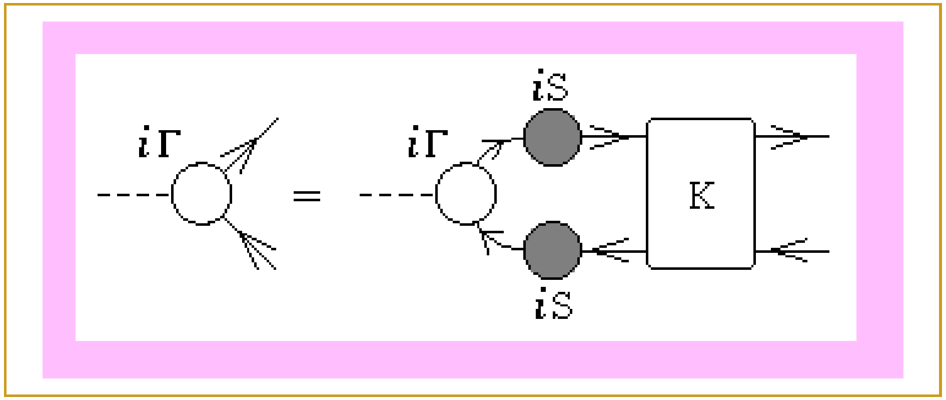


QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, $K$?

or

# What is the light-quark Long-Range Potential?


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# What is the light-quark Long-Range Potential?


Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

# Bethe-Salpeter Kernel

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- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

# Bethe-Salpeter Kernel

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- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

# Radial Excitations & Chiral Symmetry

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(Maris, Roberts, Tandy nu-th/9707003 )

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

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- Mass$^2$ of pseudoscalar hadron

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$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[ M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2} \left( \lambda^4 + i\lambda^5 \right)$

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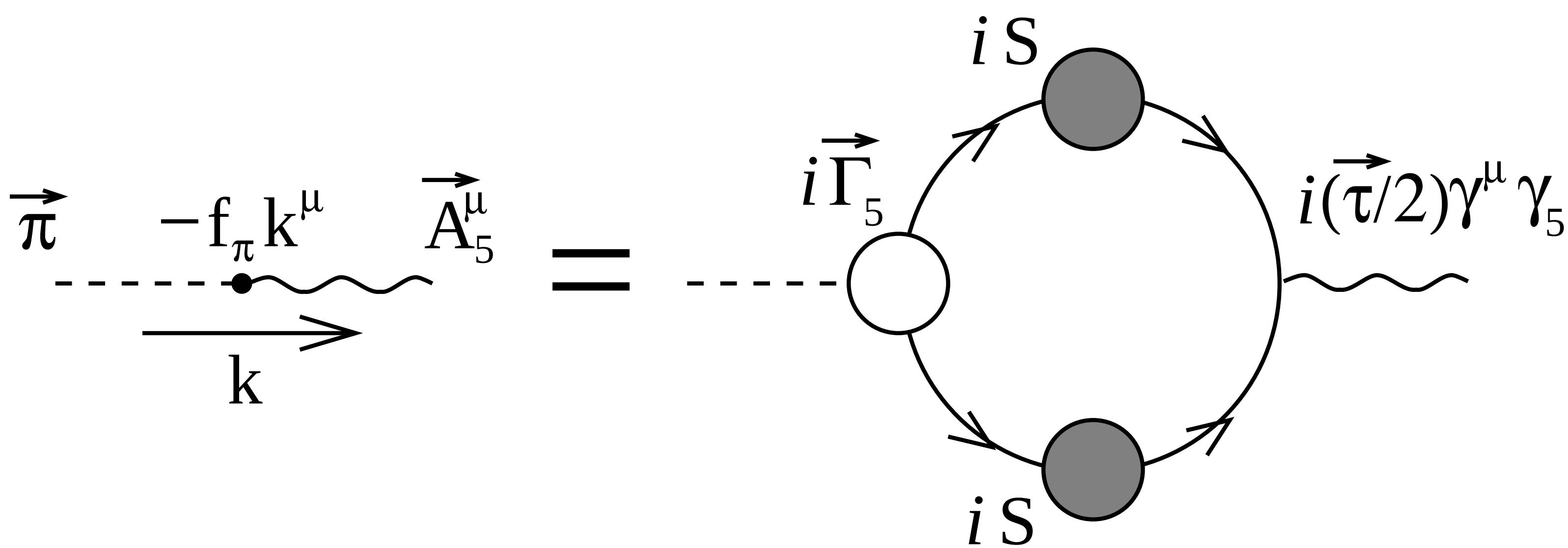
$$f_H \, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5 \gamma_\mu \boxed{\mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-)} \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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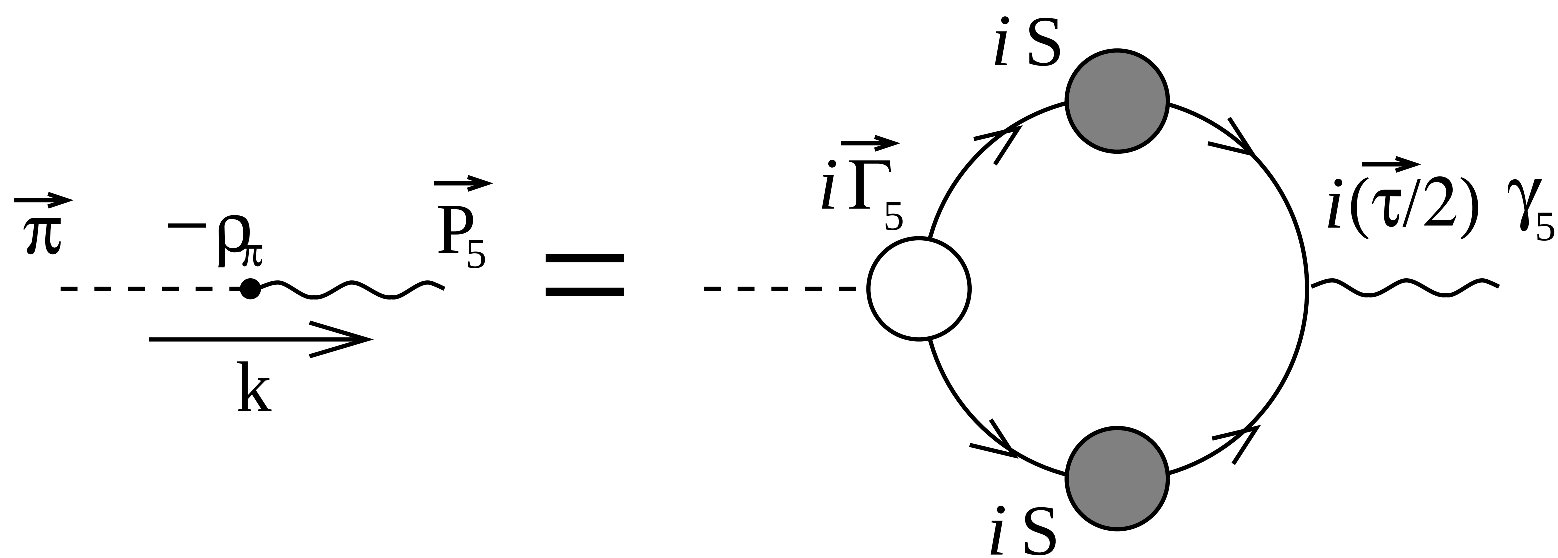

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- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of $m_q$

Hence $\boxed{m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2}\, m_q}$ . . . GMOR relation, a corollary

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- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $\boxed{m_H \propto m_q}$

. . . QCD Proof of Potential Model result

Höll, Krassnigg, Roberts
nu-th/0406030

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  ALL pseudoscalar mesons except $\pi(140)$ in chiral limit

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  ALL pseudoscalar mesons except $\pi(140)$ in chiral limit
- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

McNeile and Michael
he-la/0607032

# Radial Excitations & Lattice-QCD

McNeile and Michael
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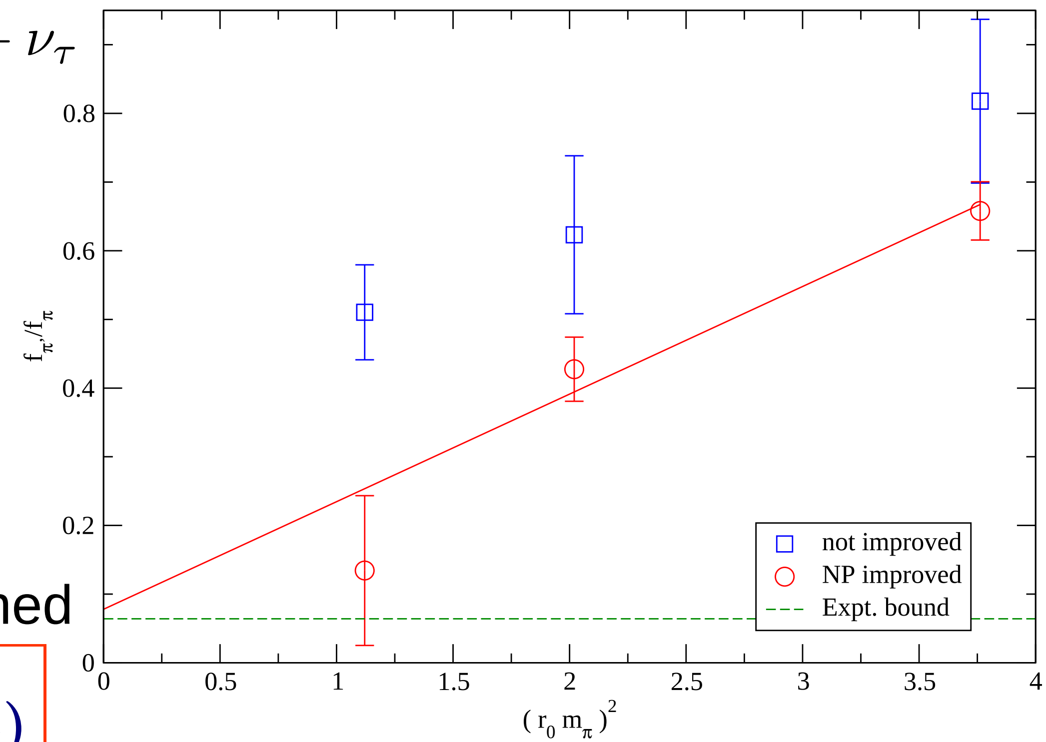

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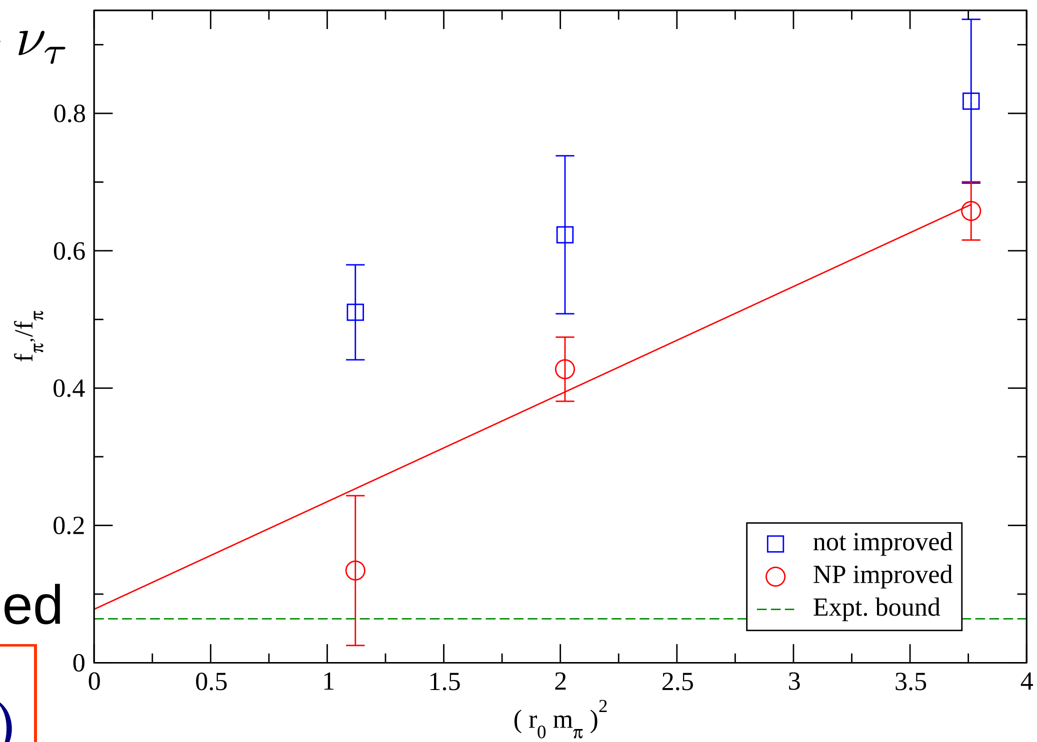


- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

# Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

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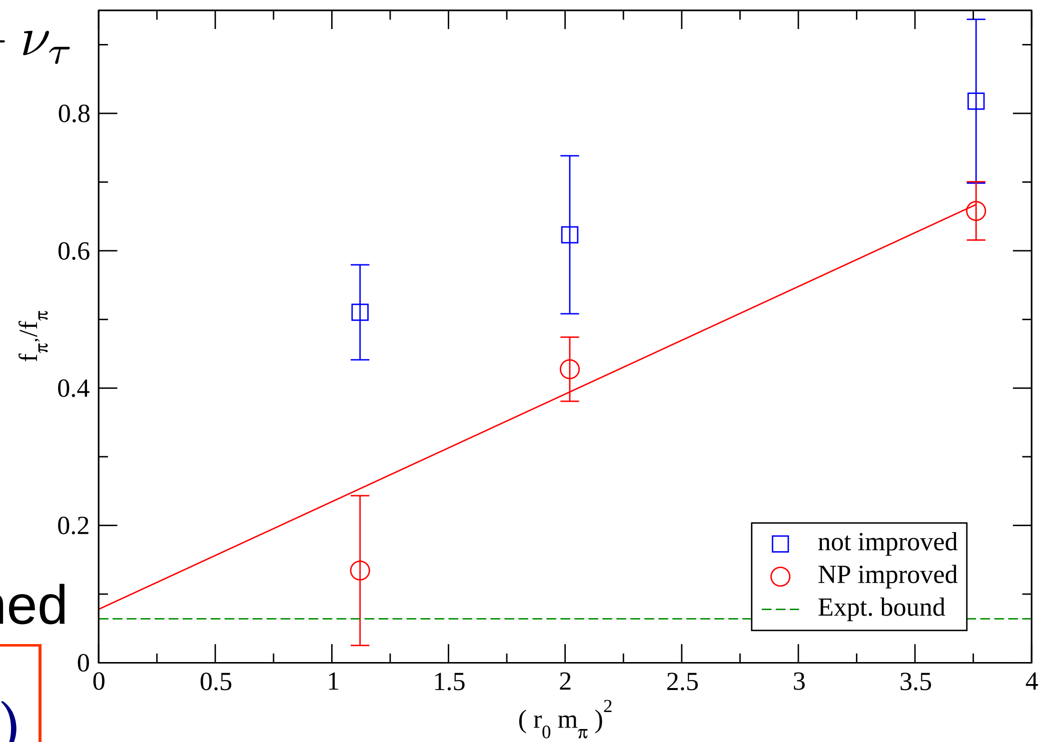


- *The suppression of $f_{\pi_1}$ is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

# Pion . . . $J = 0$
# but . . .

- Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.

# Pion . . . $J = 0$
# but . . .

- Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.

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# Pion . . . $J = 0$
# but . . .

- Pseudoscalar meson Bethe-Salpeter amplitude

$$\begin{aligned}\chi_\pi(k;P) &= \gamma_5 \left[ i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P\mathcal{F}_{\pi_n}(k;P) \right. \\ &\quad \left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P) \right]\end{aligned}$$

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# Pion ... $J = 0$ but ...

- Pseudoscalar meson Bethe-Salpeter amplitude

$$\begin{aligned}\chi_\pi(k;P) &= \gamma_5 \left[ i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P \mathcal{F}_{\pi_n}(k;P) \right.\\ &\left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P)\right]\end{aligned}$$

- $J = 0$ ... *but* while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both $S$- and $P$-wave components.

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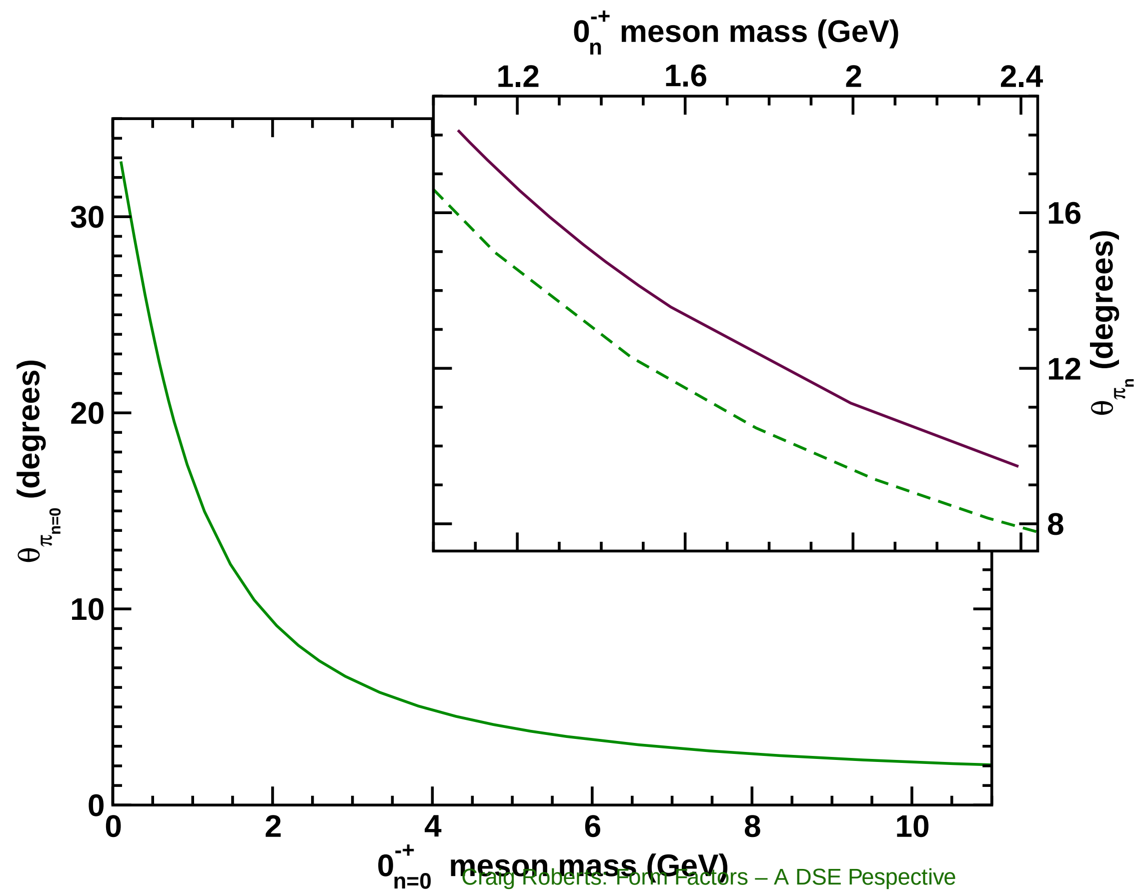


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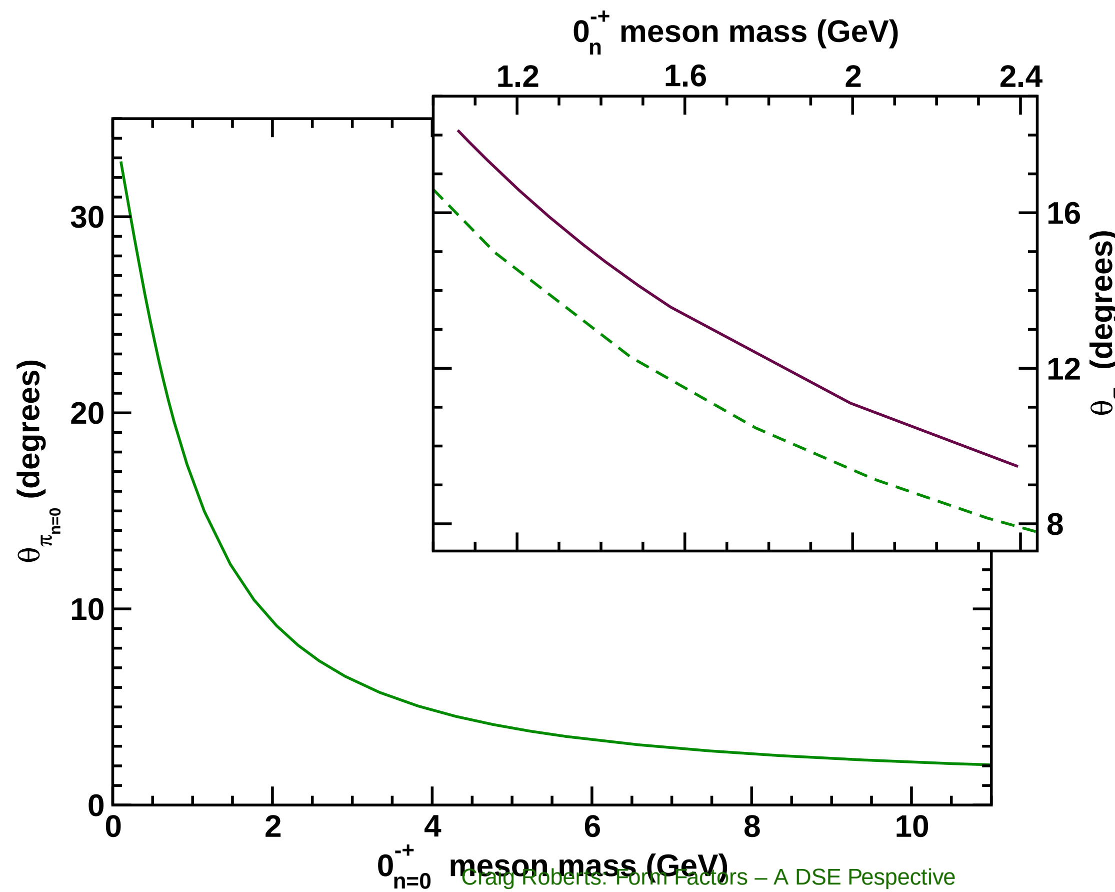
$L$ is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.



# Pion Form Factor

Procedure Now Straightforward

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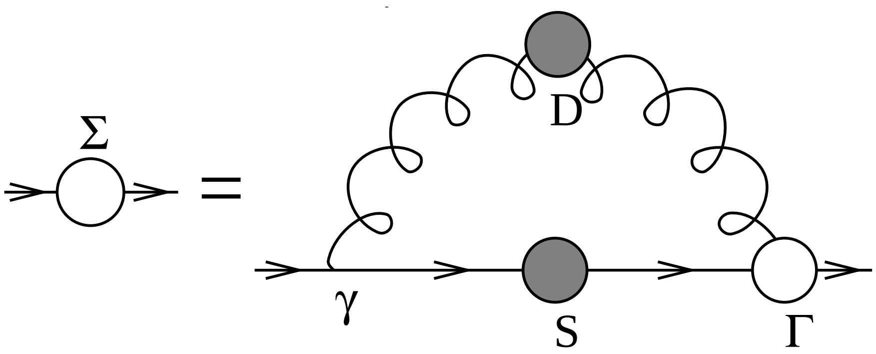
# Pion Form Factor

- Solve Gap Equation
  $\Rightarrow$ Dressed-Quark Propagator, $S(p)$



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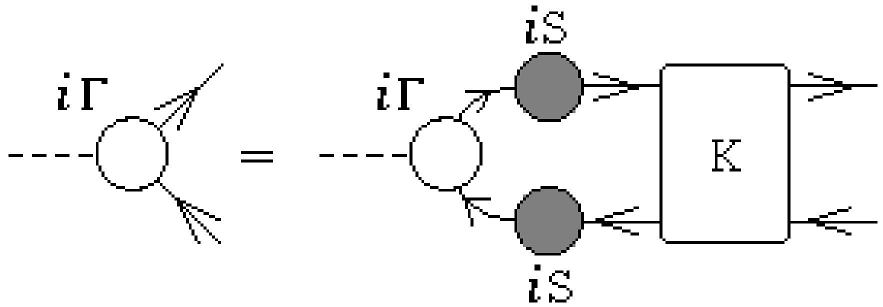
- Use that to Complete Bethe Salpeter Kernel, $K$
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, $\Gamma_\pi$

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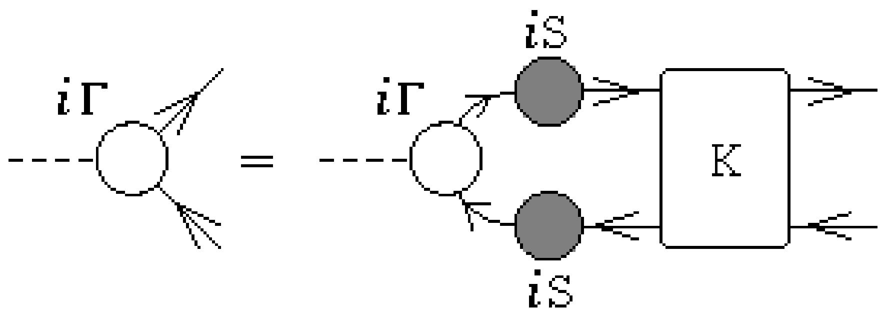
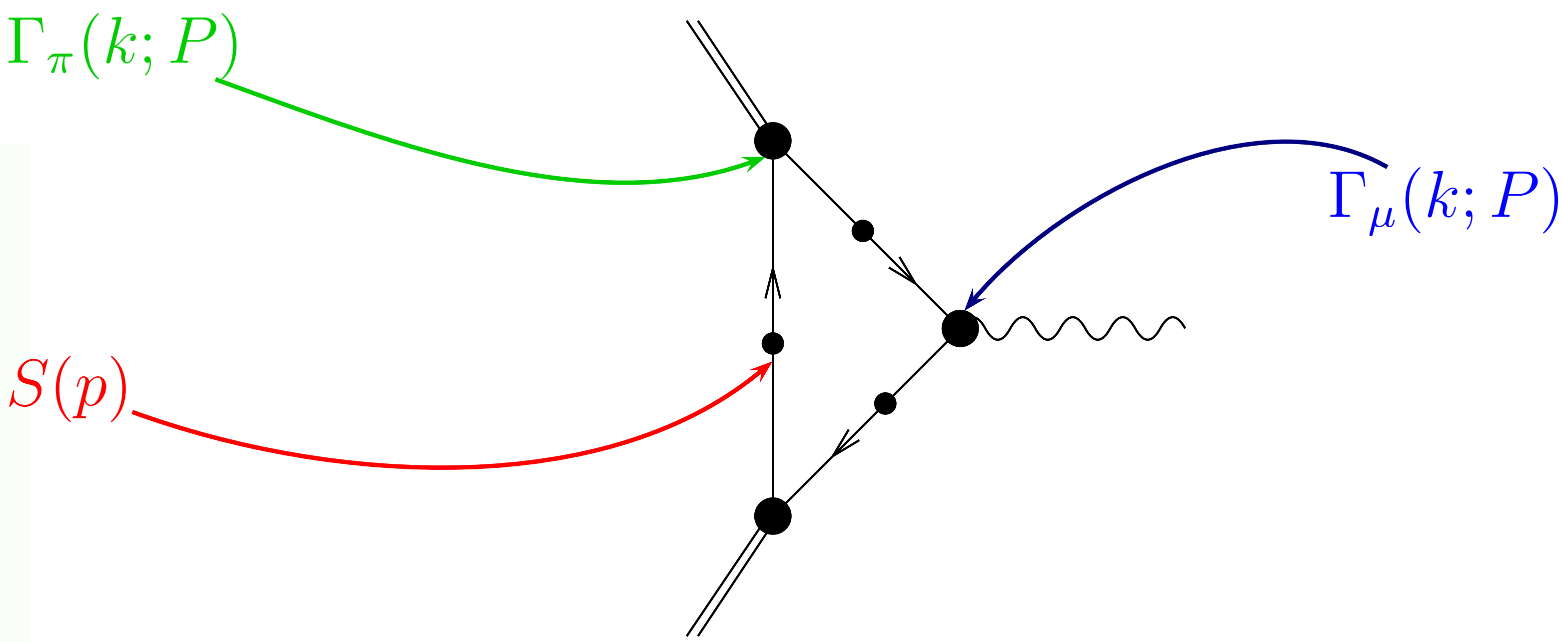



- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Gluon Vertex, $\Gamma_\mu$

## Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor

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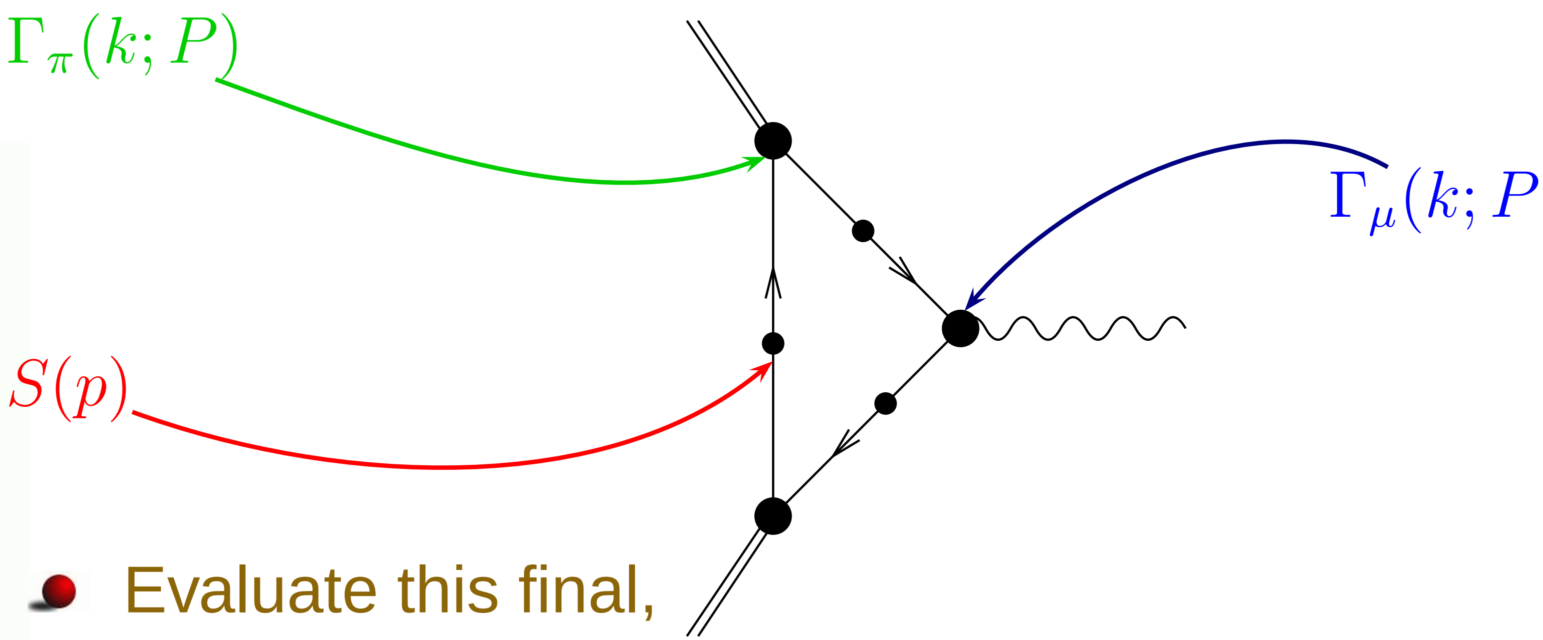


- Evaluate this final, three-dimensional integral

# Dimensionless product: $r_\pi \, f_\pi$

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- Great strides towards placing nucleon studies on same footing as mesons

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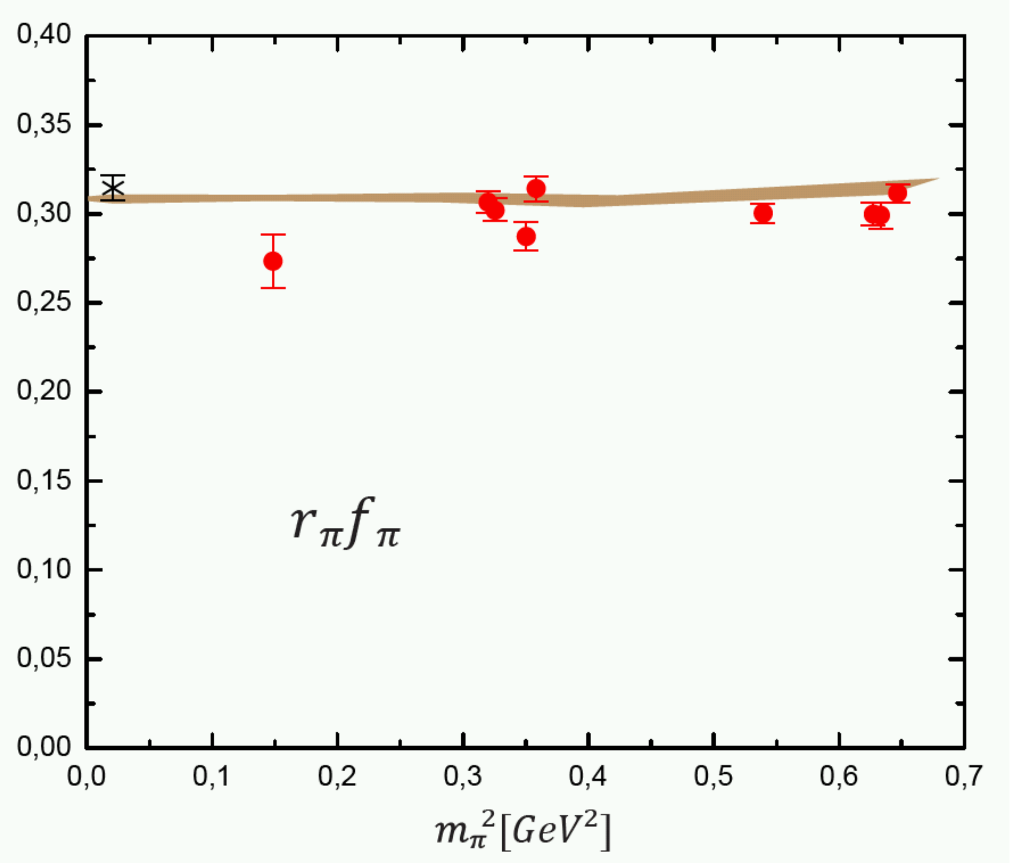


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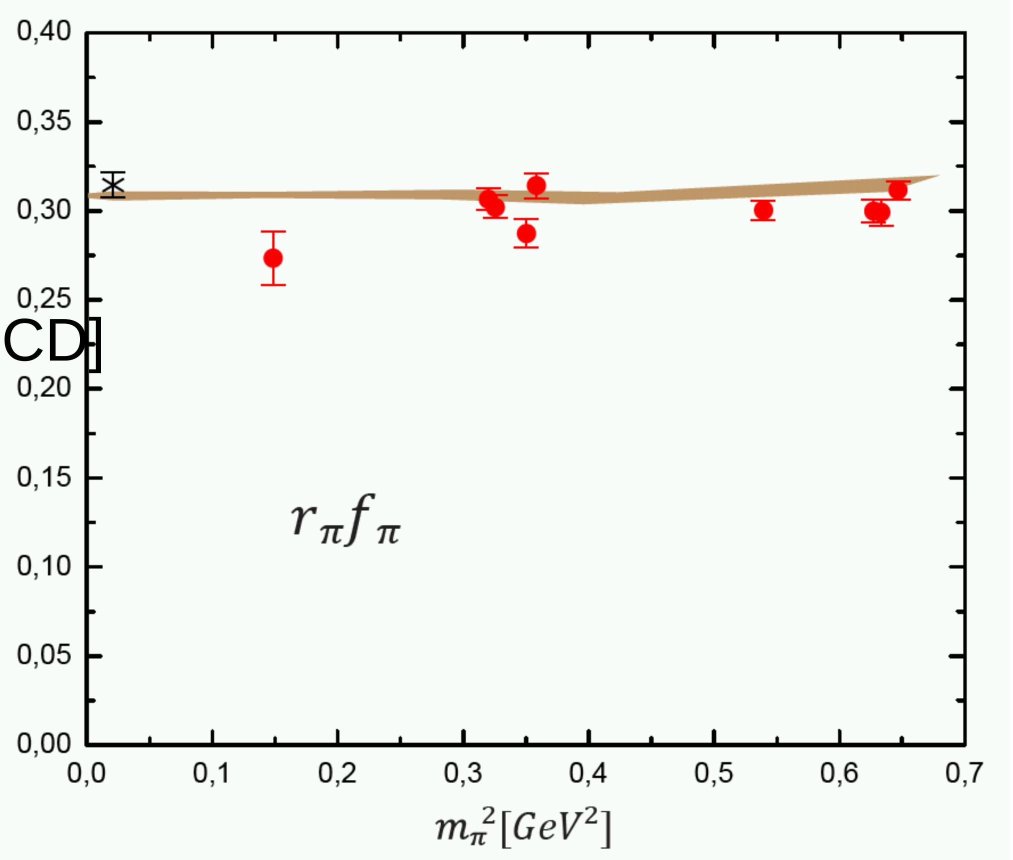


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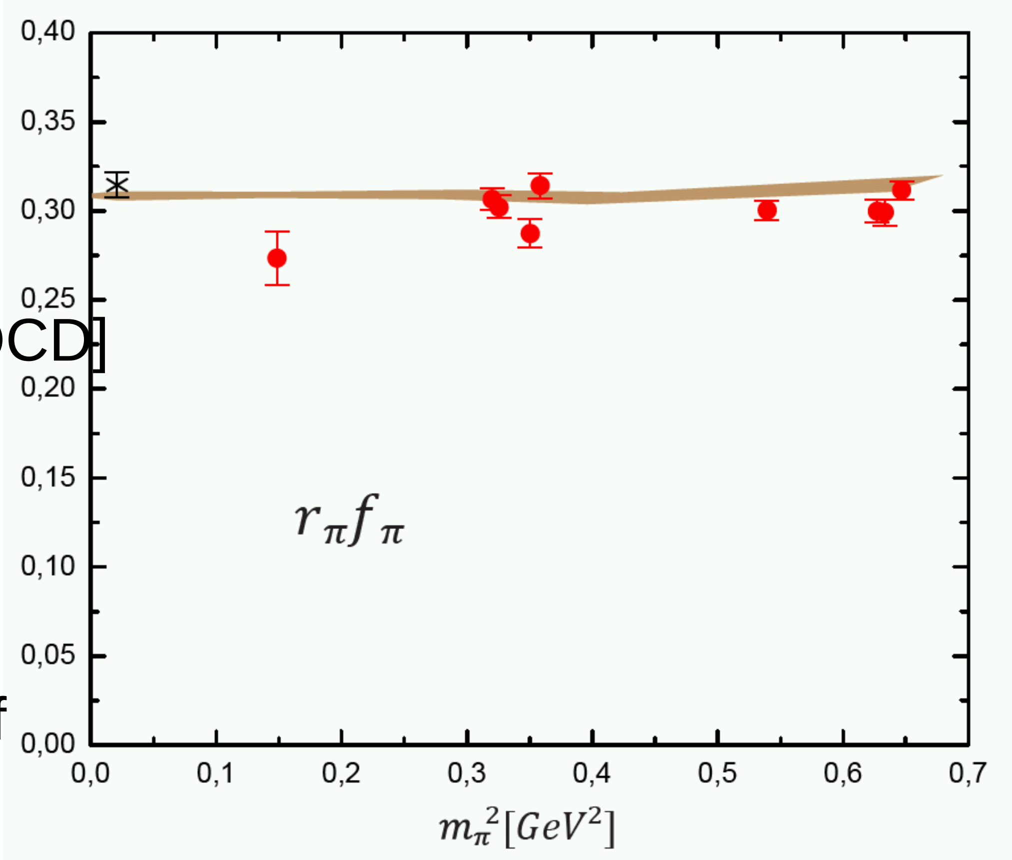


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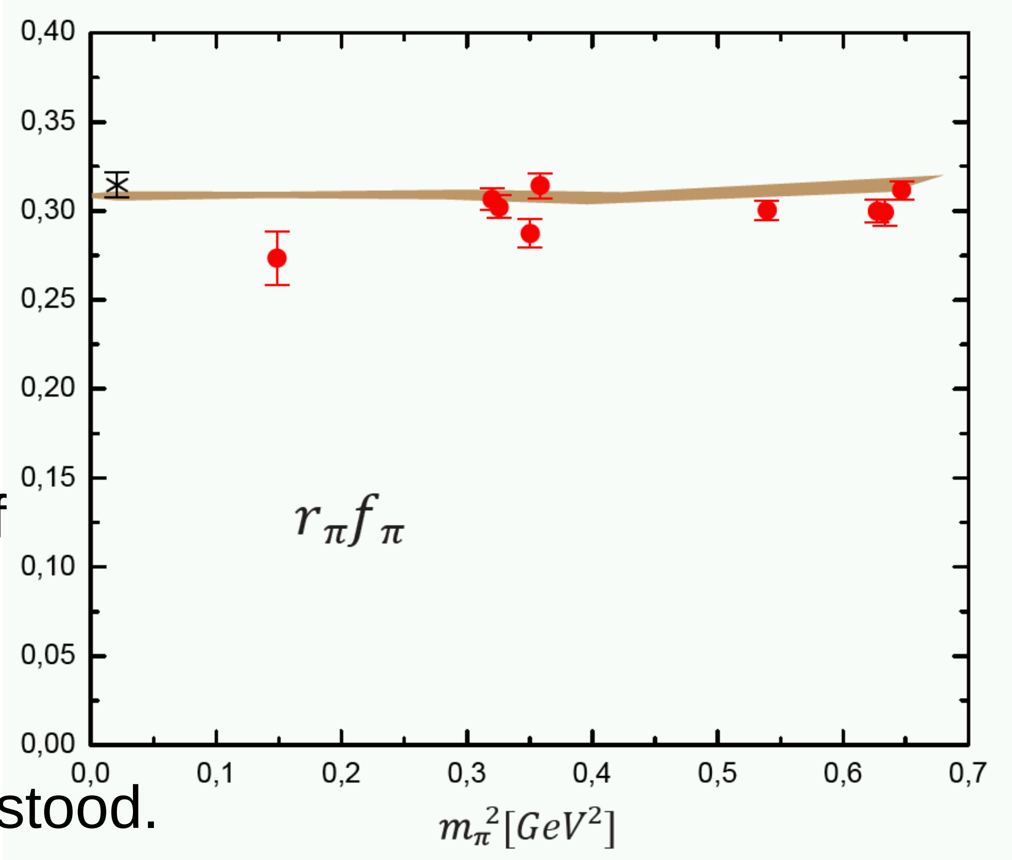


- DSE prediction
- Fascinating result:
  DSE and Lattice
  – Experimental value
  obtains independent of
  current-quark mass.
- Potentially useful
  but must first be understood.

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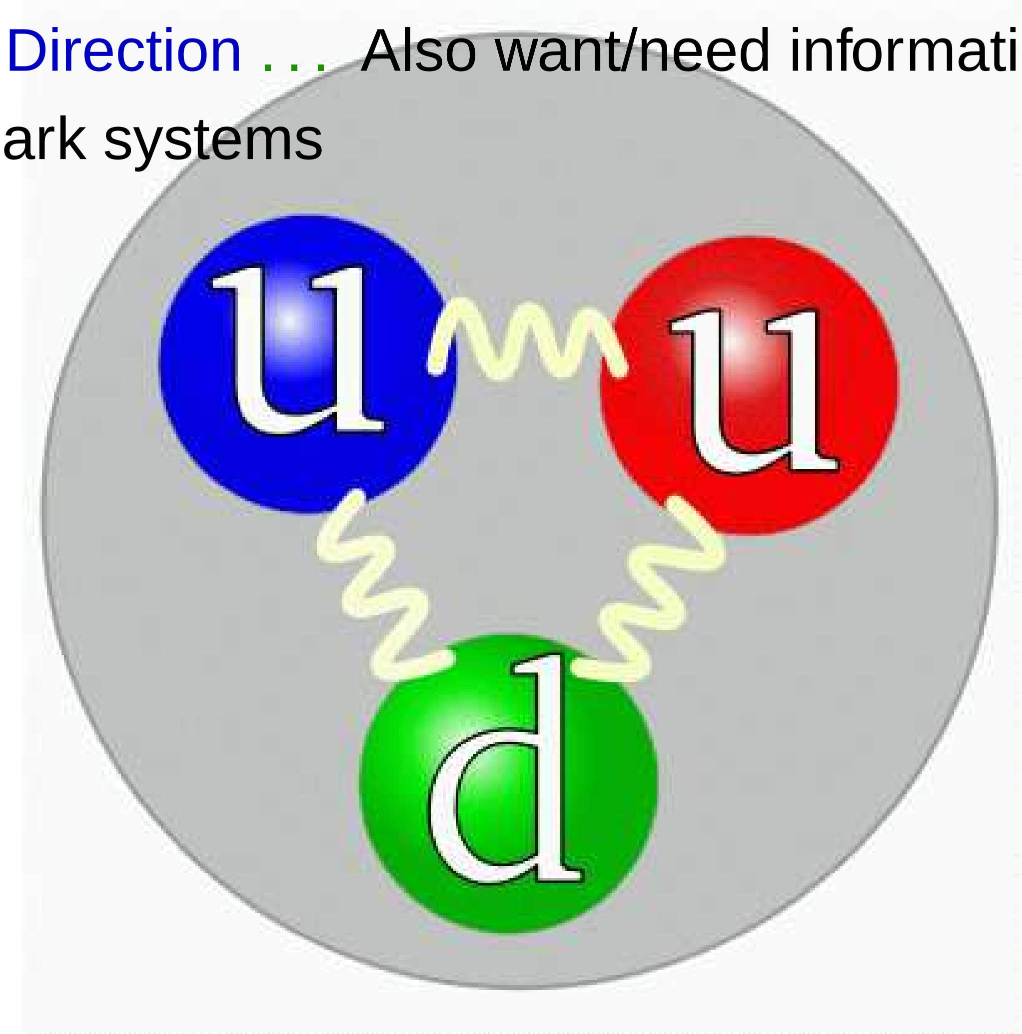
- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

- Another Direction . . . Also want/need information about three-quark systems



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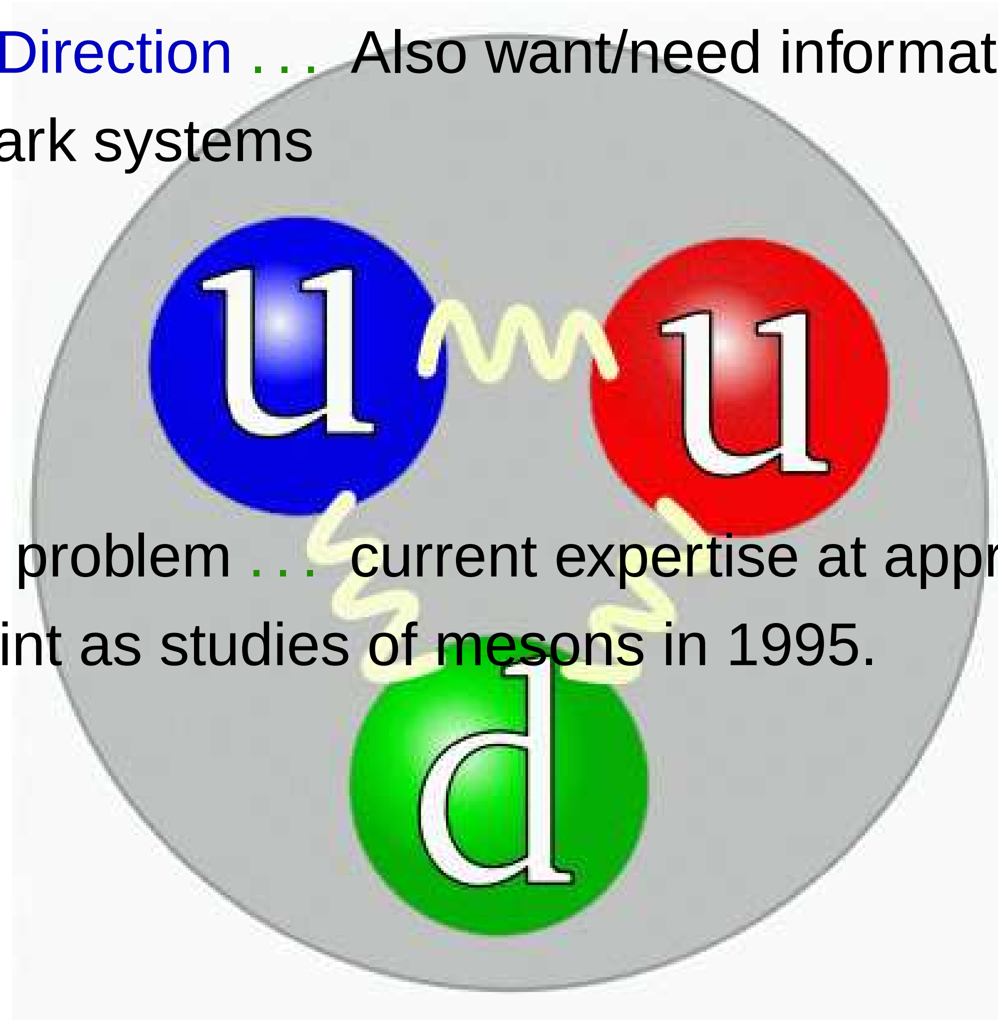


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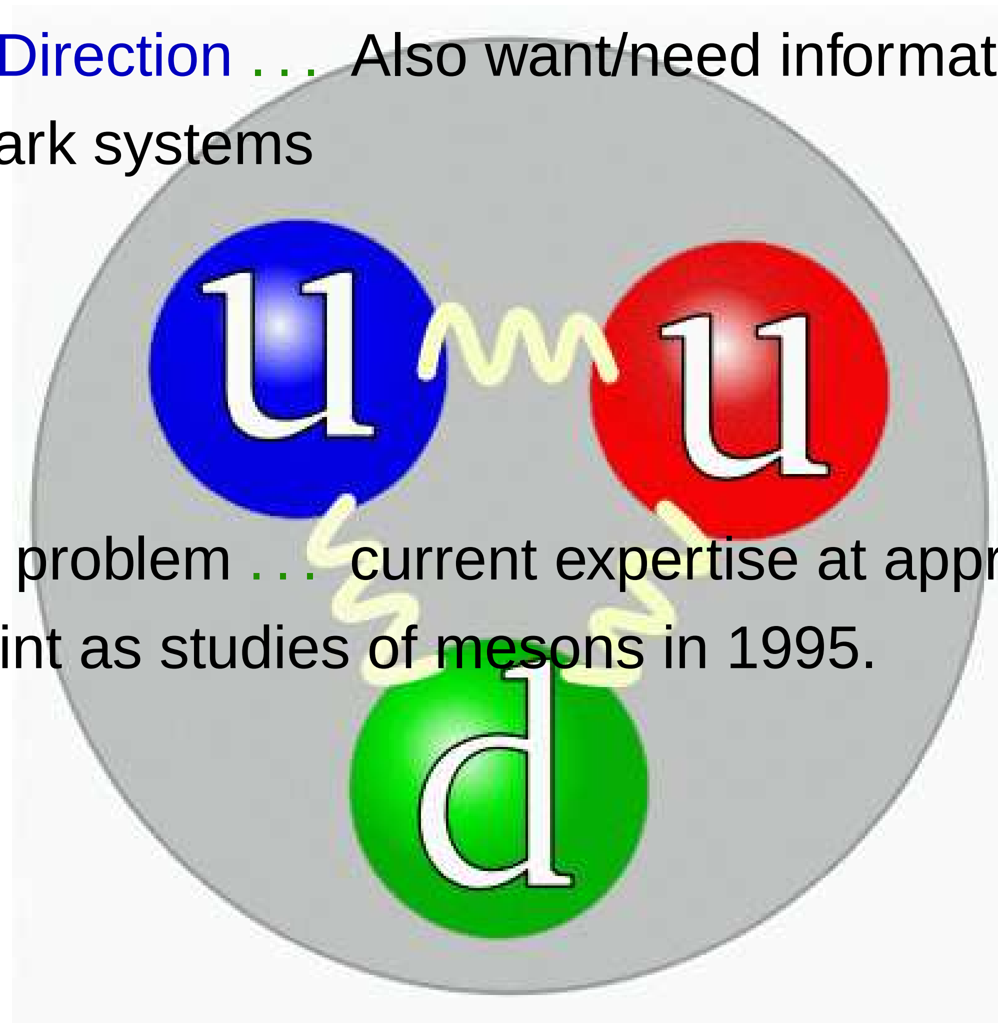


- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.

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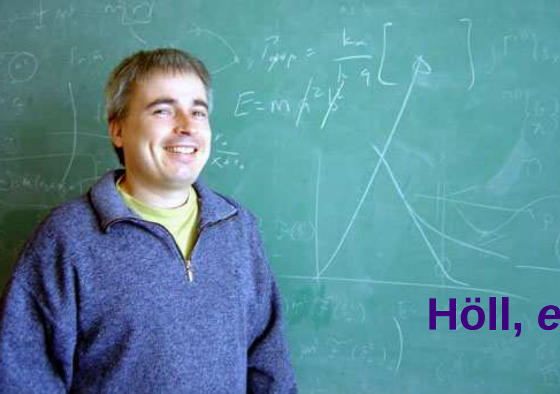
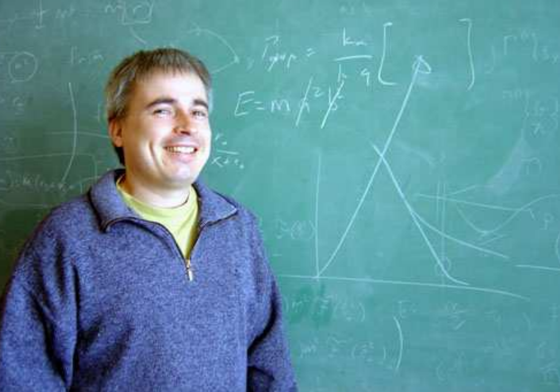


# *Nucleon EM Form Factors: A Précis*

**Höll, *et al.*: nu-th/0412046 & nu-th/0501033**



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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

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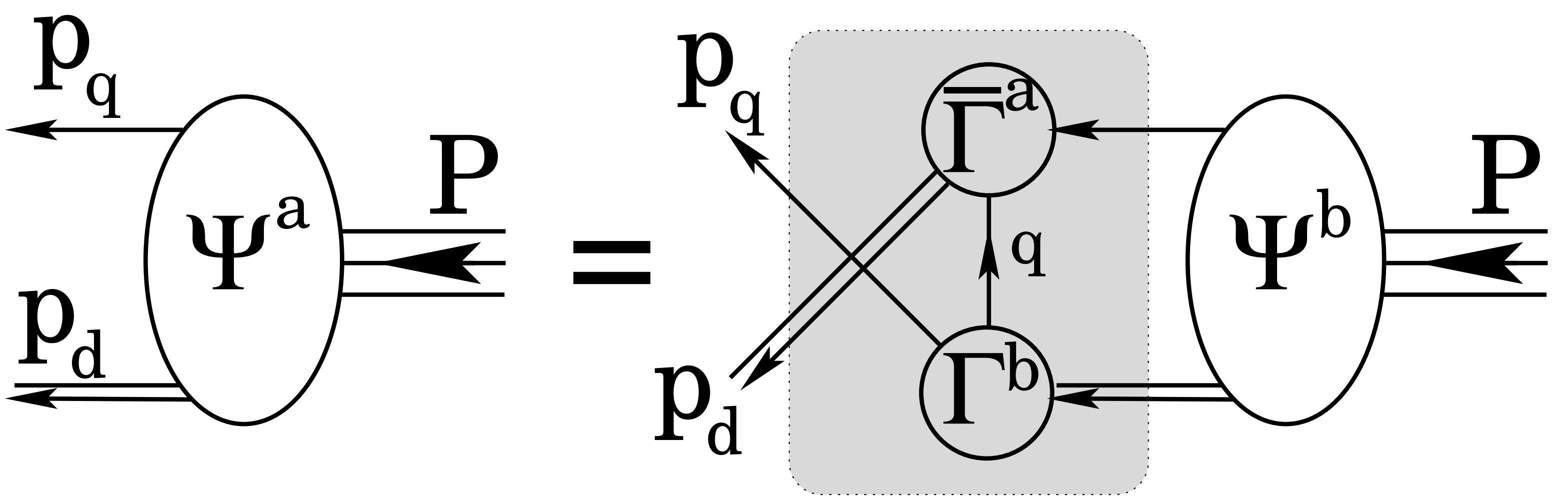
  - Cloudy Bag: $\delta M_+^{\pi-\rm loop} = -300$ to $-400$ MeV!
- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

# Faddeev equation

# *Faddeev equation*



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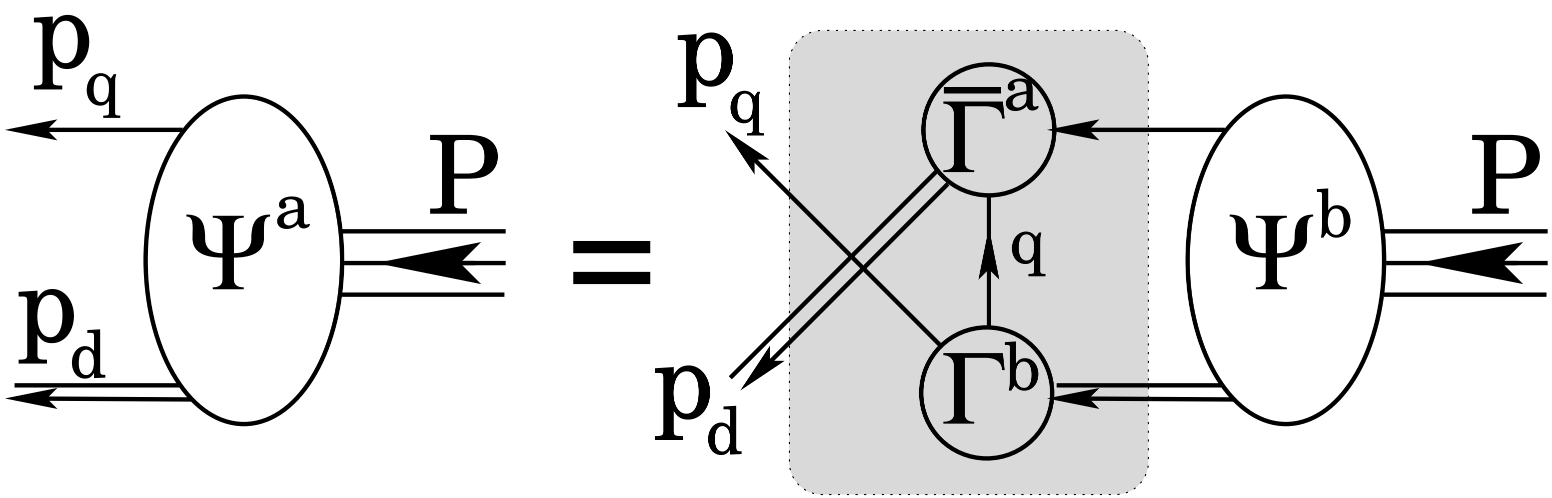
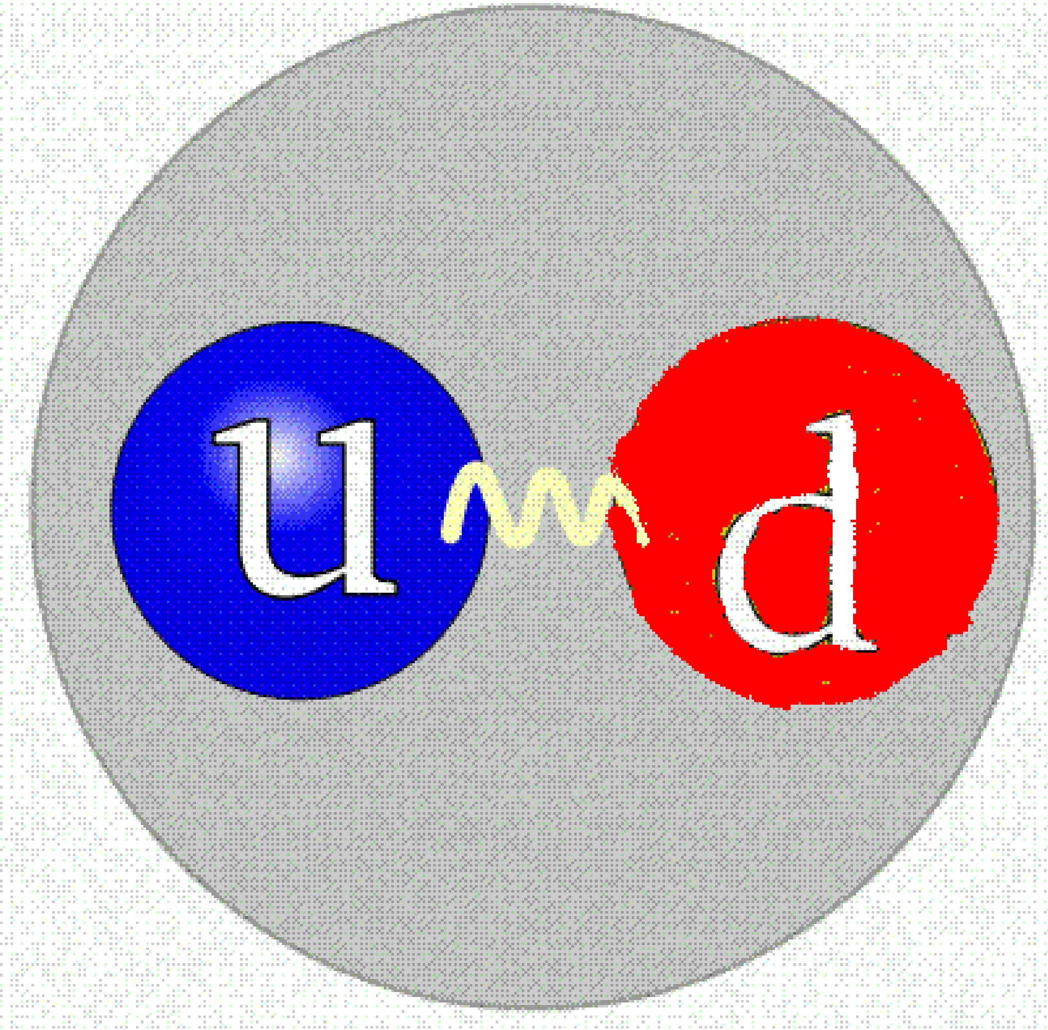



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame Amplitude has ... $s-$, $p-$ & $d-$wave correlations

# *Diquark correlations*




QUARK-QUARK

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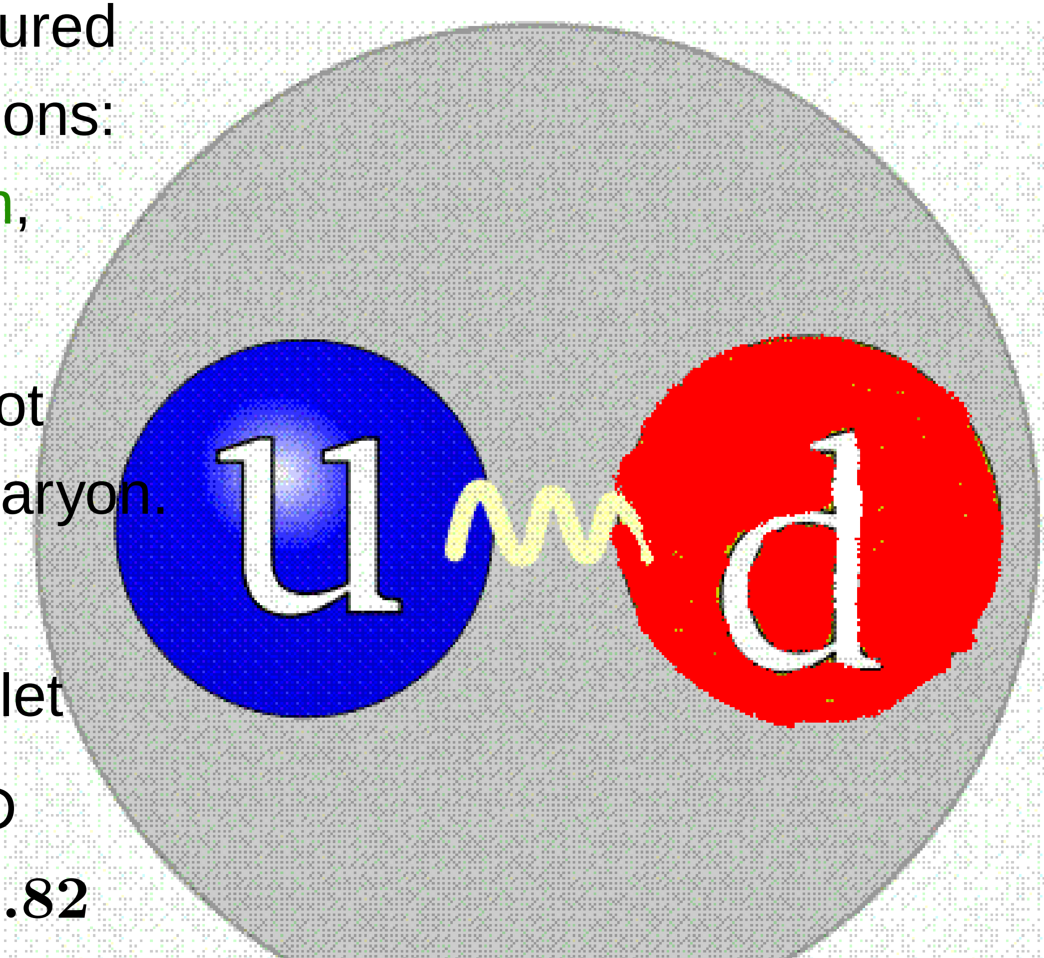
# *Diquark correlations*

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined ... Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$




QUARK-QUARK

# Pions and Form Factors

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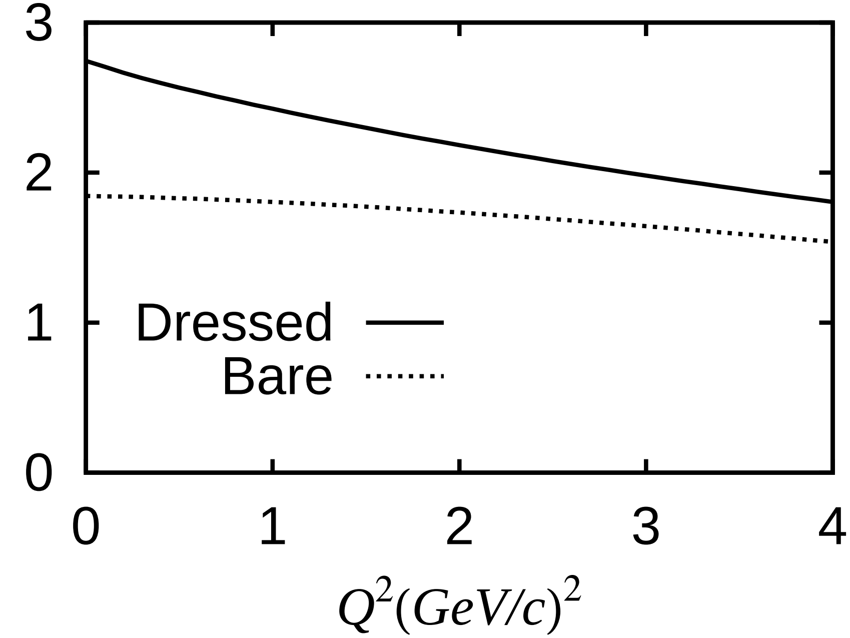
*Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor $G_D$. Solid curve is $G^*_M(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.*



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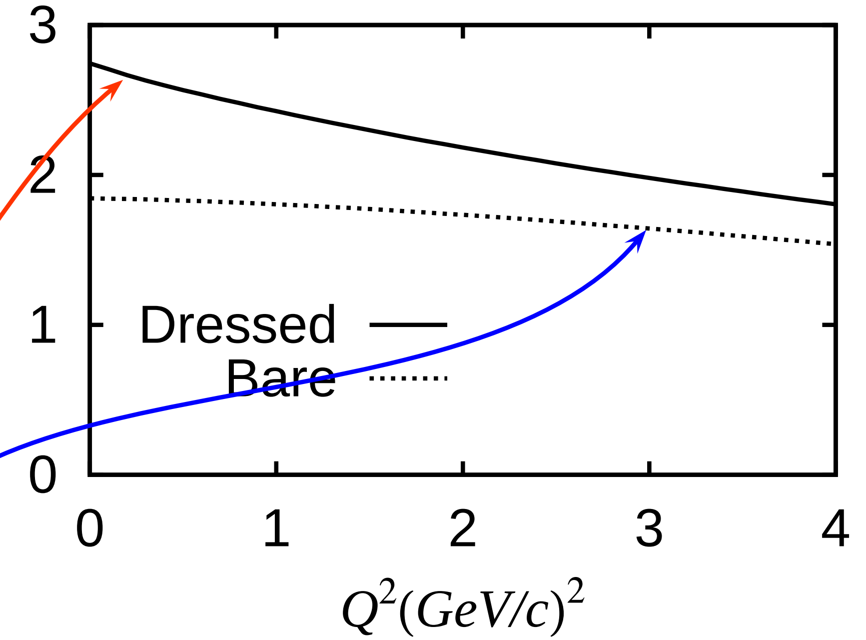


Quark Core

- Responsible for only 2/3 of result at small $Q^2$
- Dominant for $Q^2$ >2 – 3 GeV$^2$

# Results: Nucleon and $\Delta$ Masses


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# Results: Nucleon and $\Delta$ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and $\Delta$ masses

Set A – fit to the actual masses was required; whereas for
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| set | $M_N$ | $M_\Delta$ | $m_{0^+}$ | $m_{1^+}$ | $\omega_{0^+}$ | $\omega_{1^+}$ |
|---|---|---|---|---|---|---|
| A | 0.94 | 1.23 | 0.63 | 0.84 | 0.44=1/(0.45 fm) | 0.59=1/(0.33 fm) |
| B | 1.18 | 1.33 | 0.80 | 0.89 | 0.56=1/(0.35 fm) | 0.63=1/(0.31 fm) |

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- Constructive Interference: $1^{++}$-diquark $+ \partial_\mu \pi$

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# Nucleon-Photon Vertex

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M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

# Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

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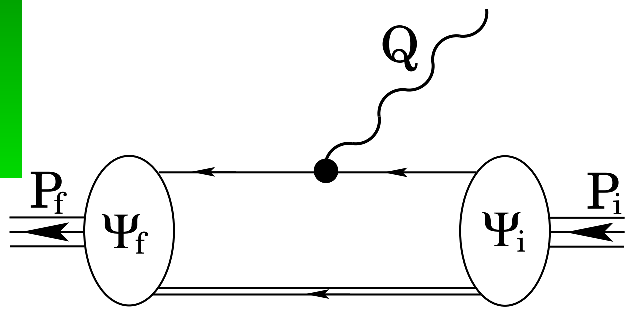


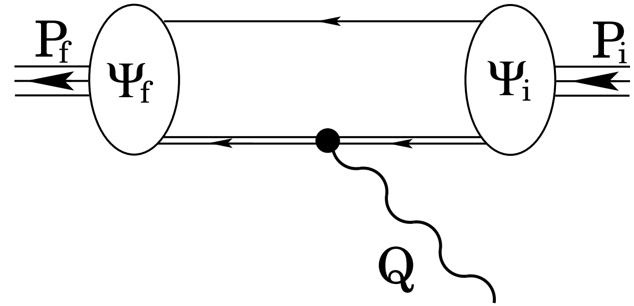


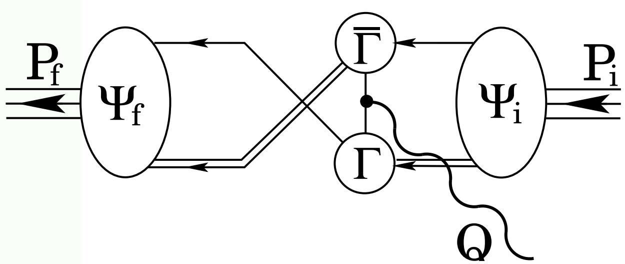


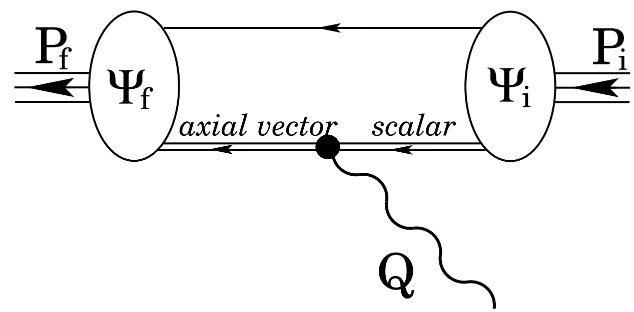


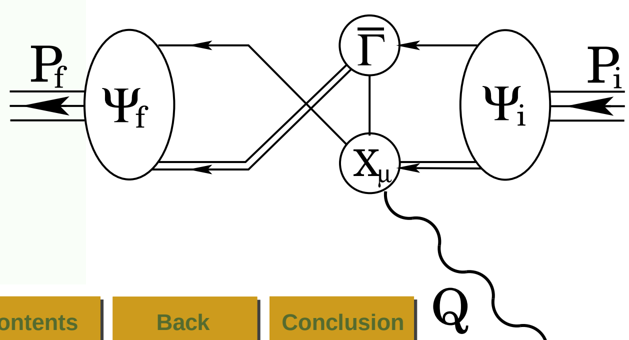


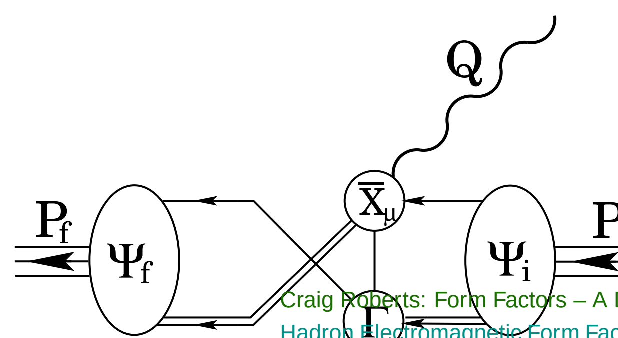


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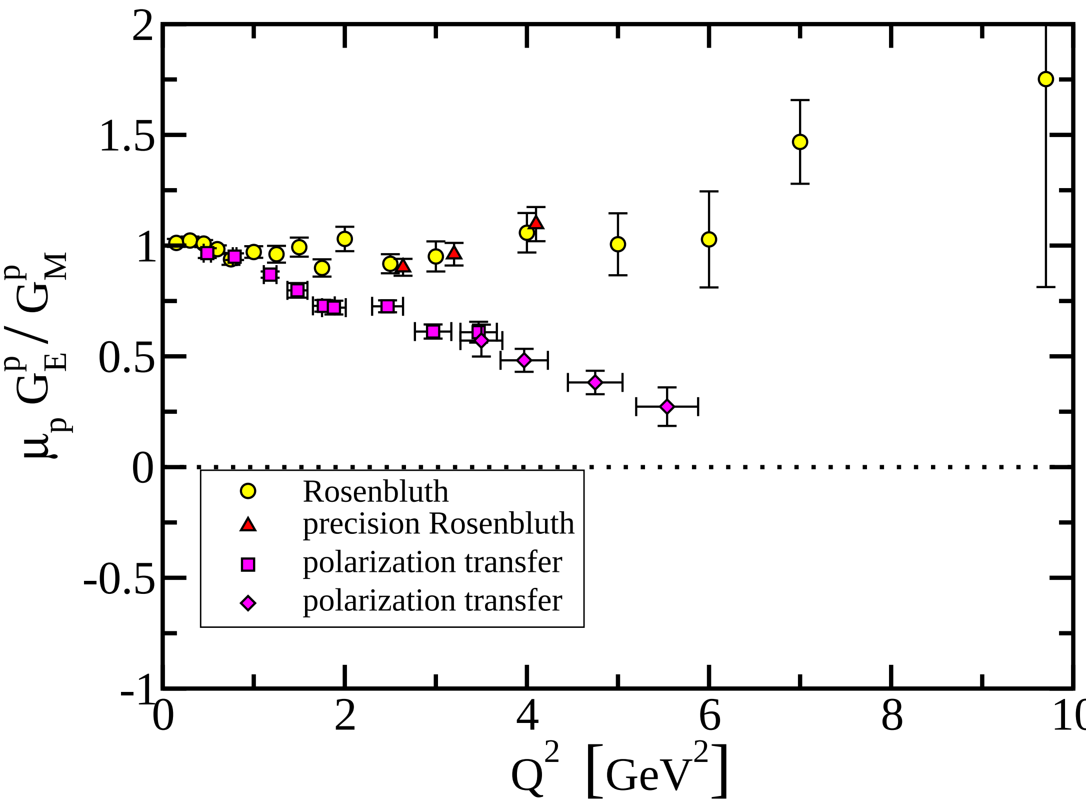
# Form Factor Ratio: GE/GM



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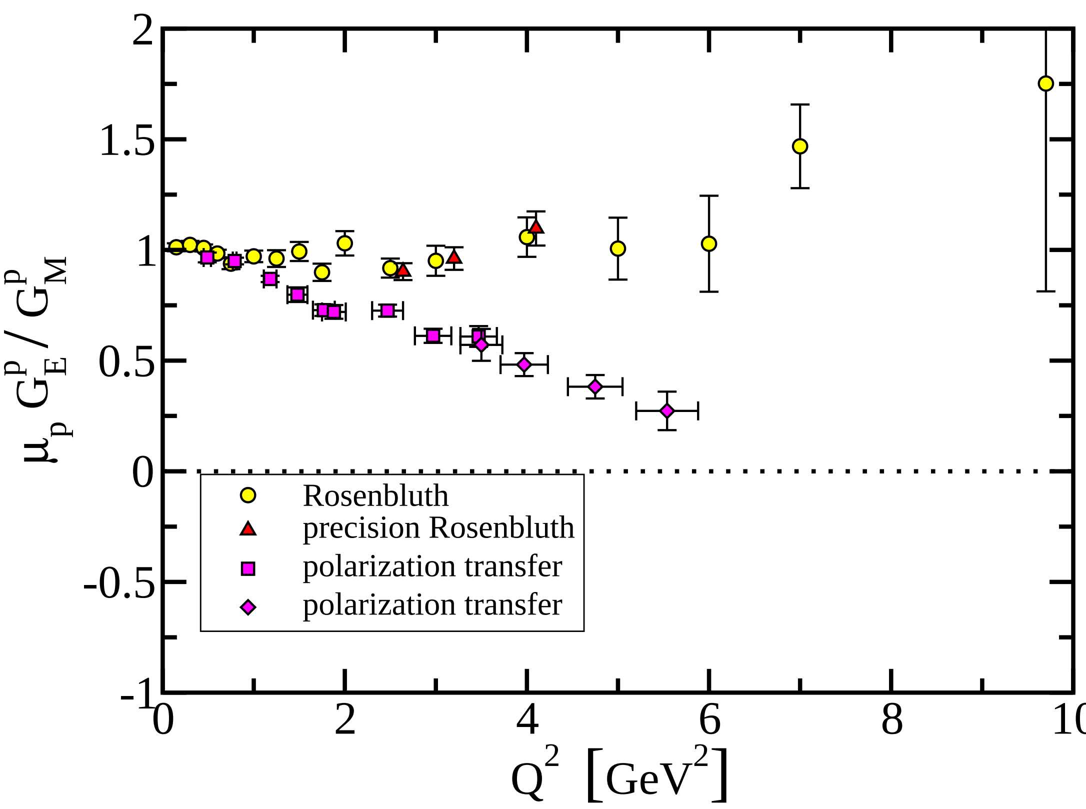


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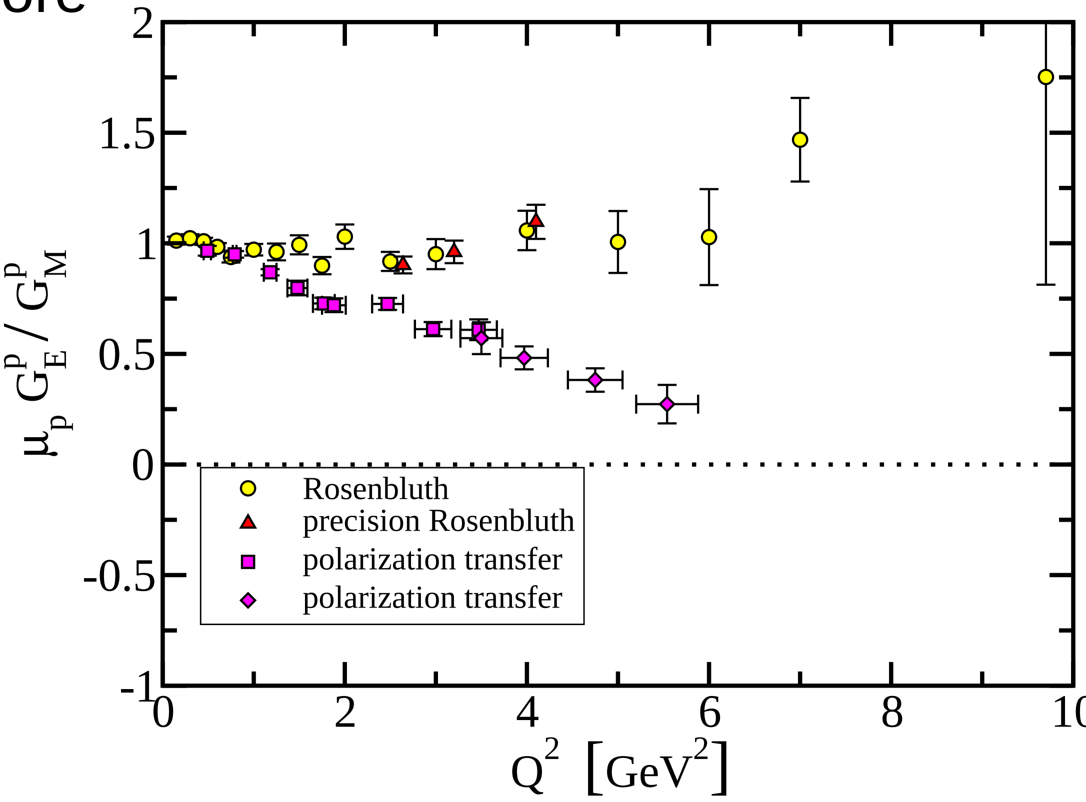

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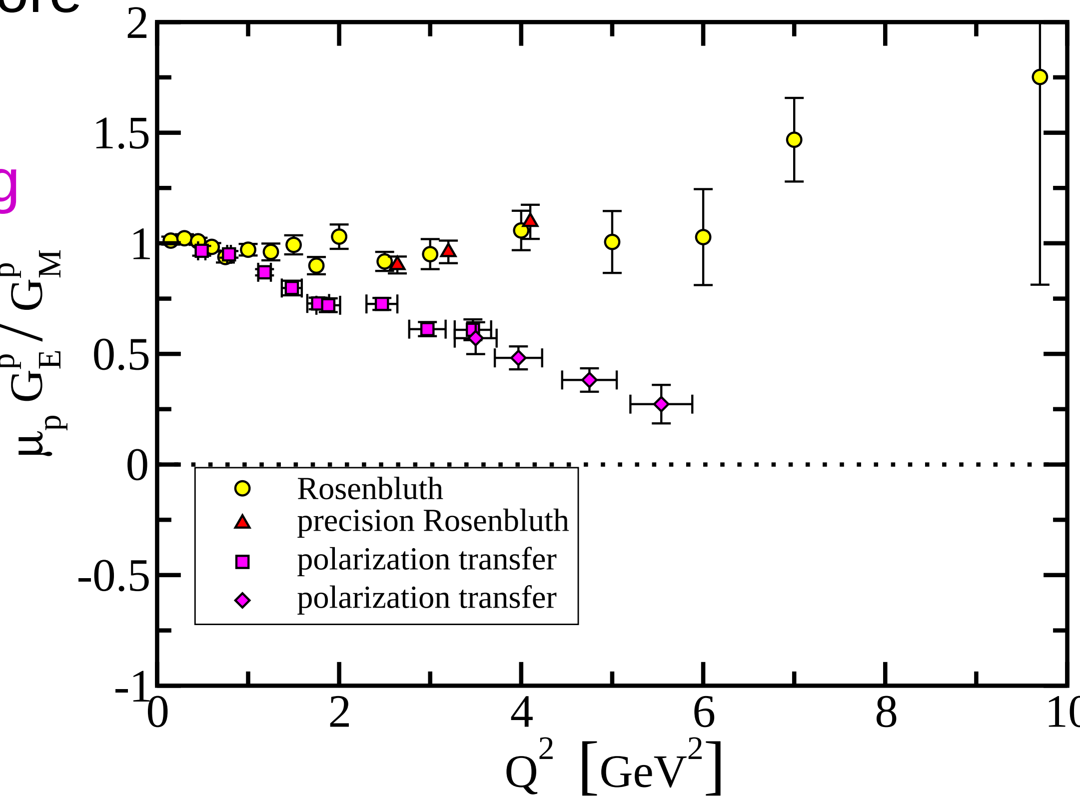


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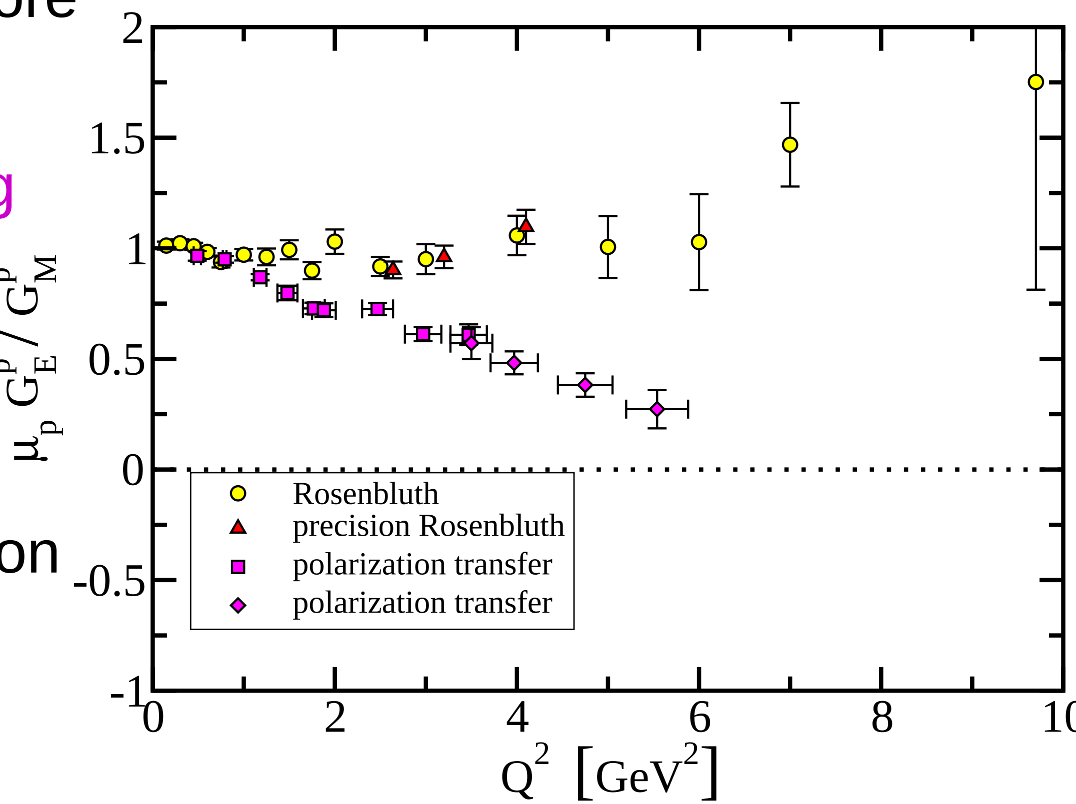

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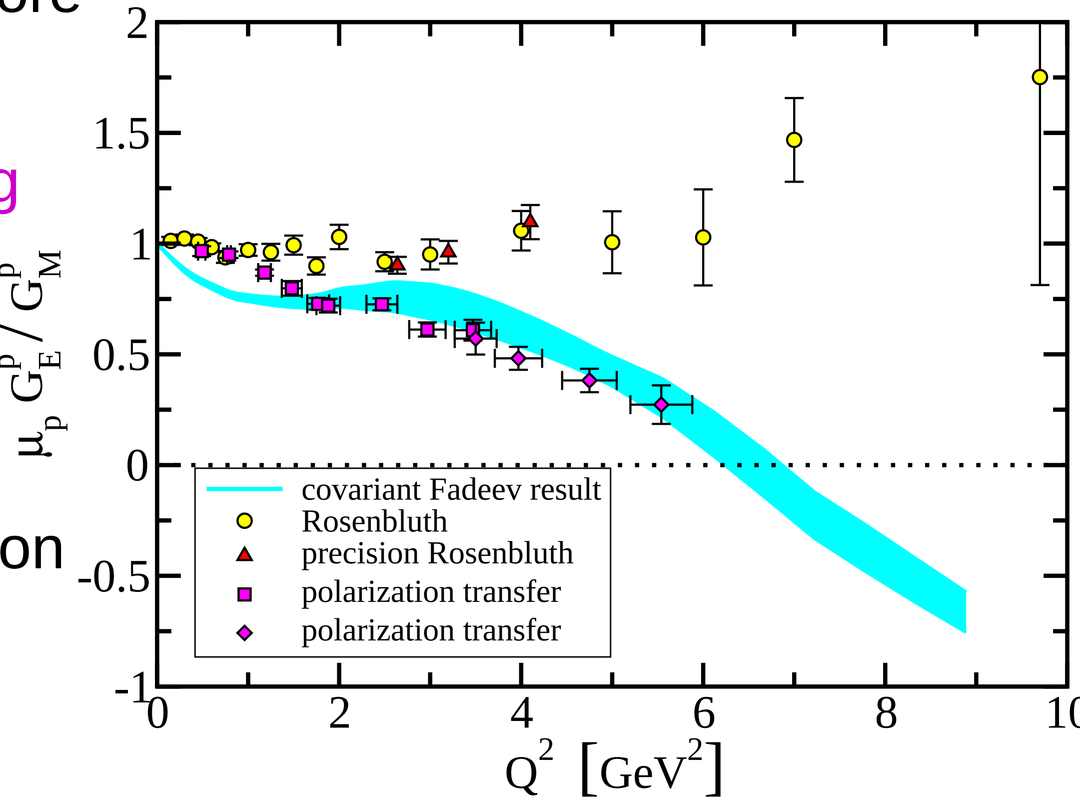


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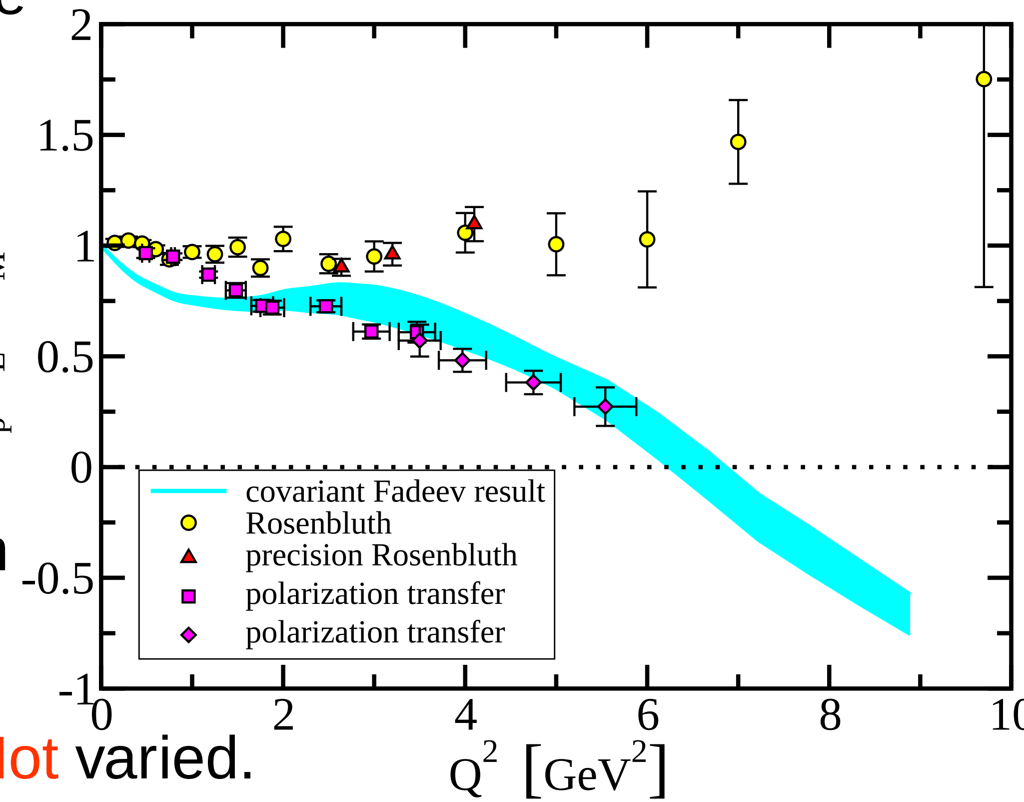

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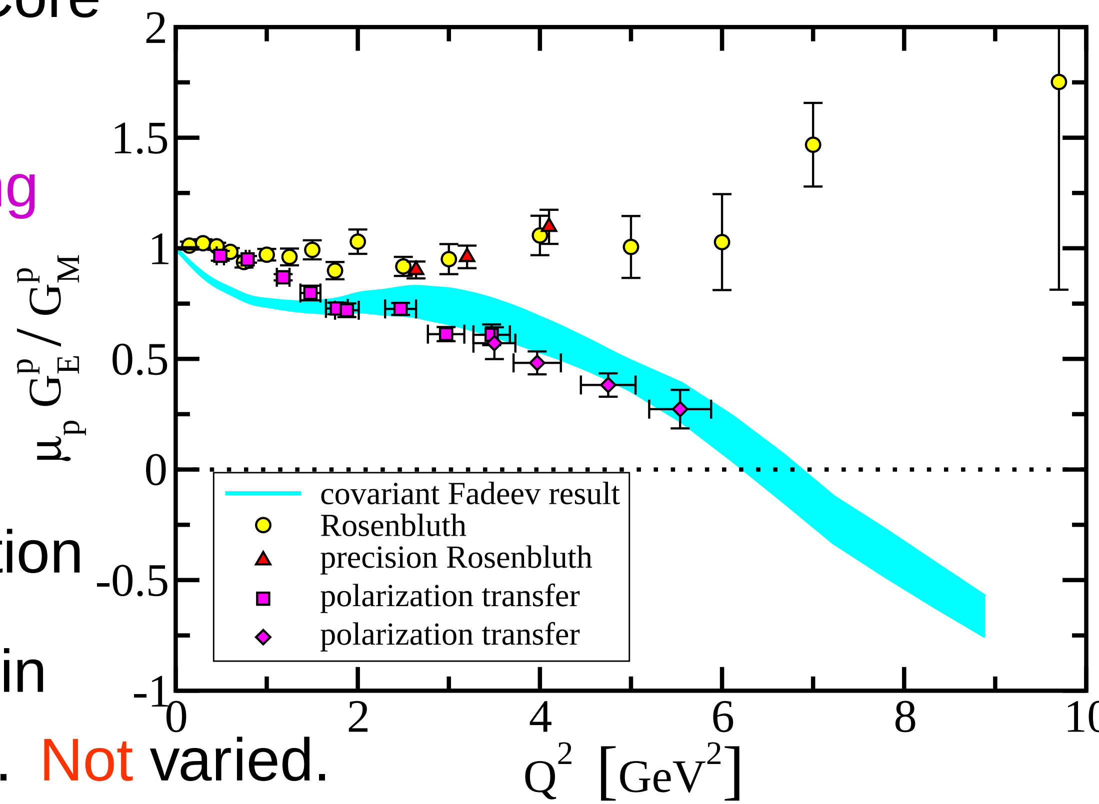


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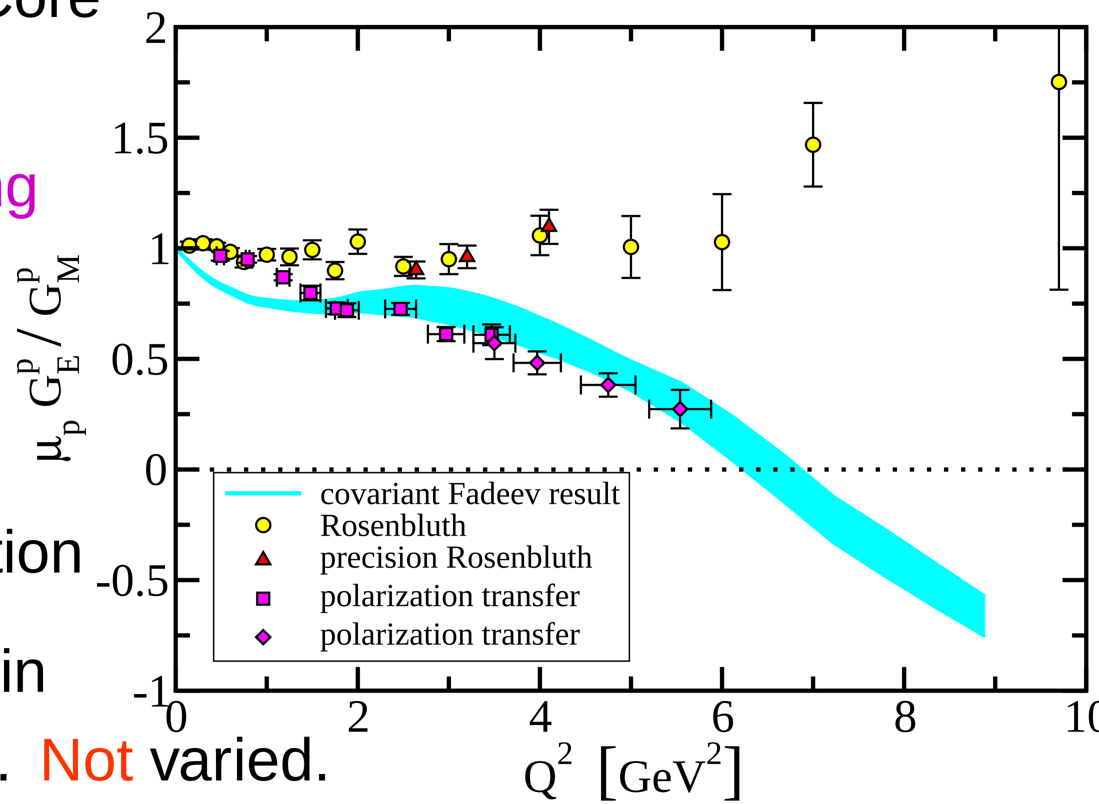


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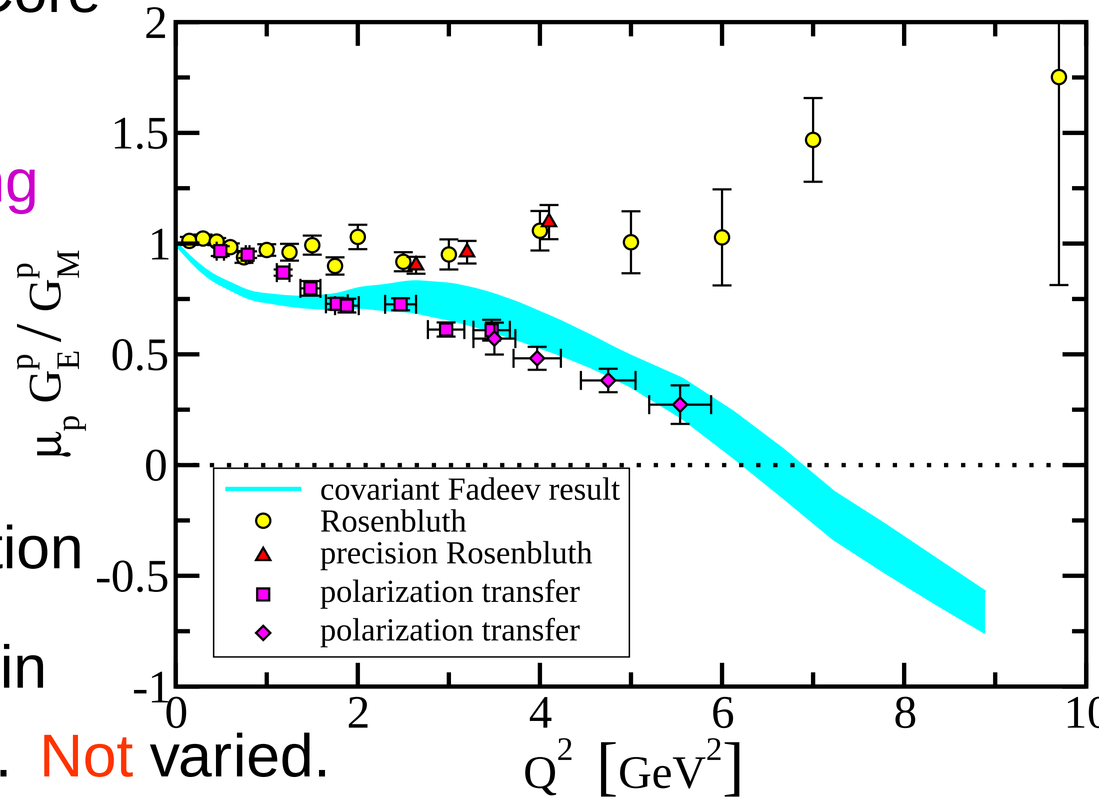


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  - Predict Zero at $Q^2 \approx 6.5$GeV$^2$

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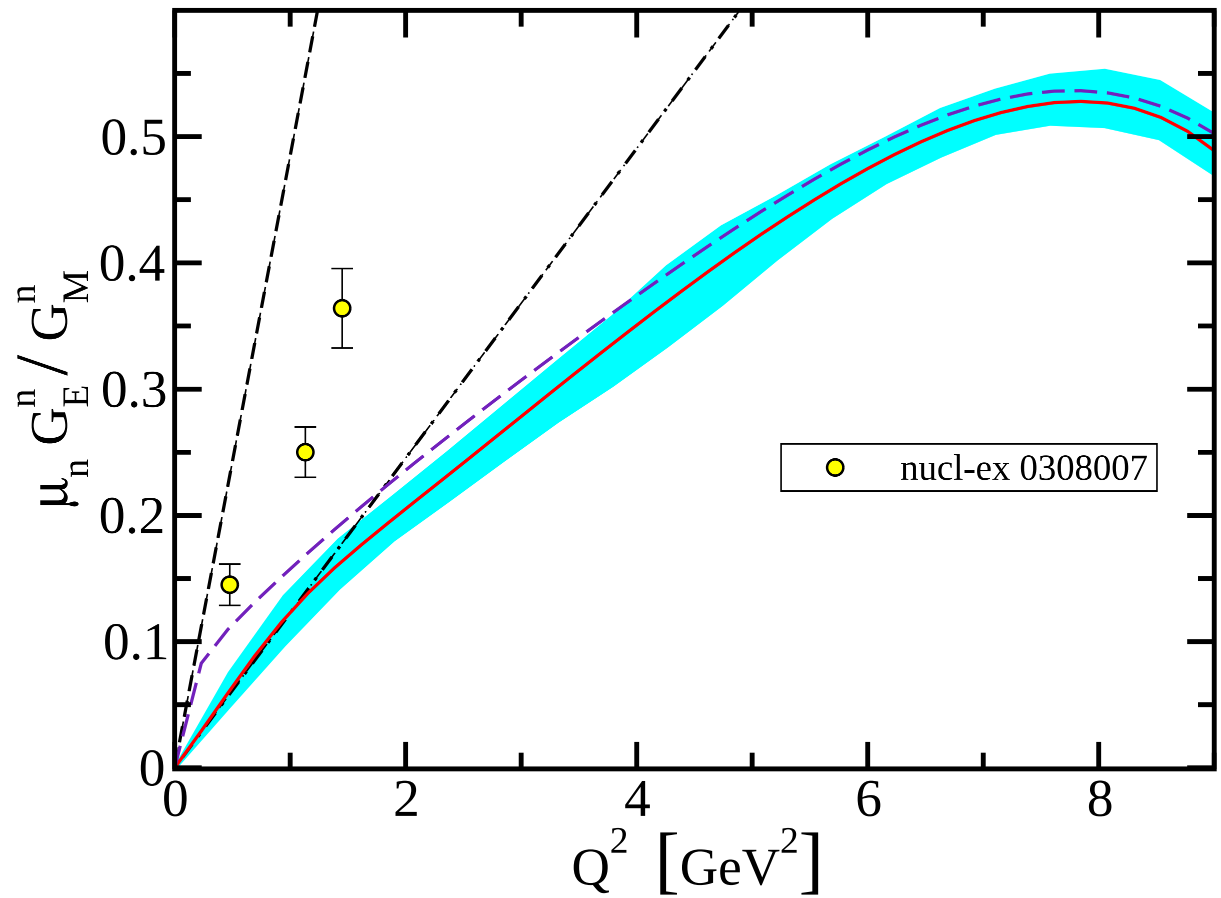
# Neutron Form Factors

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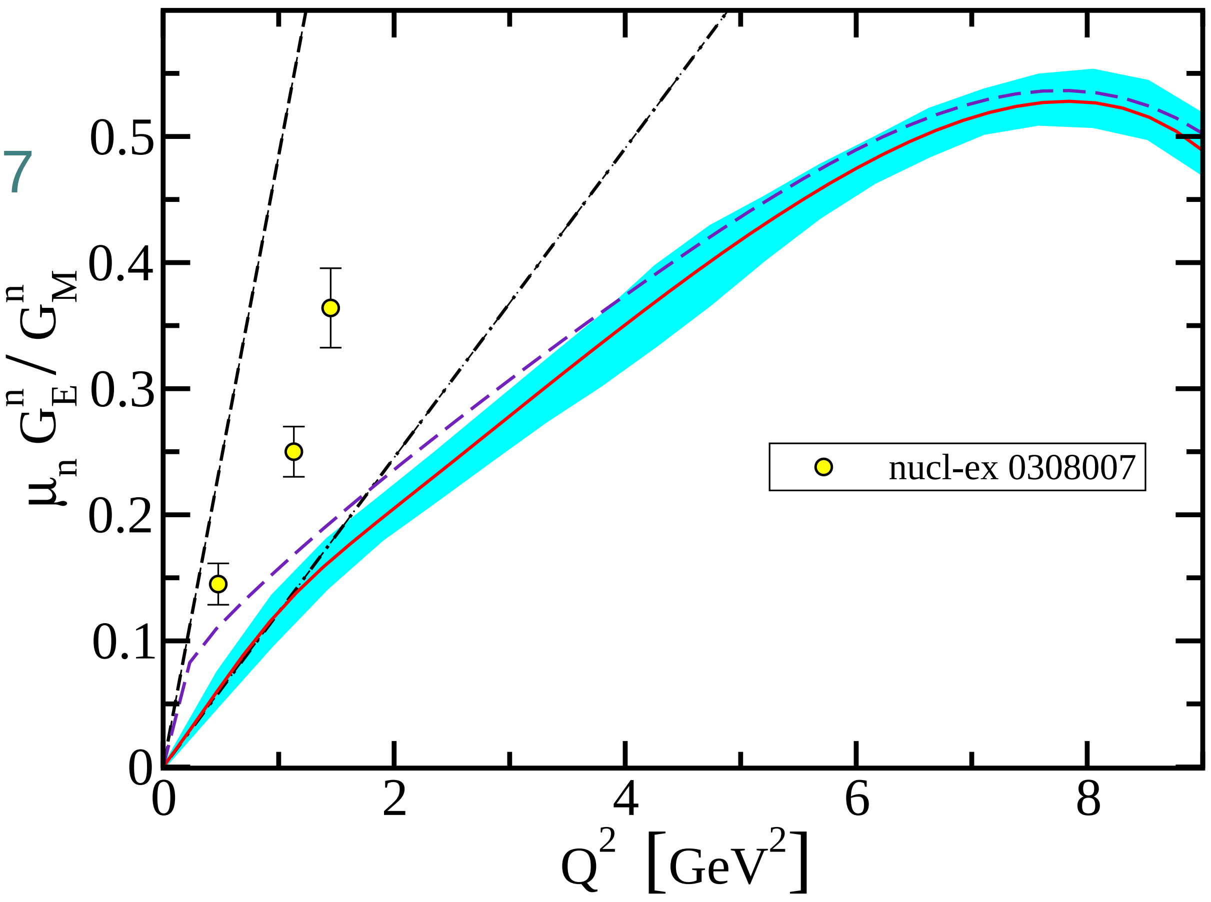

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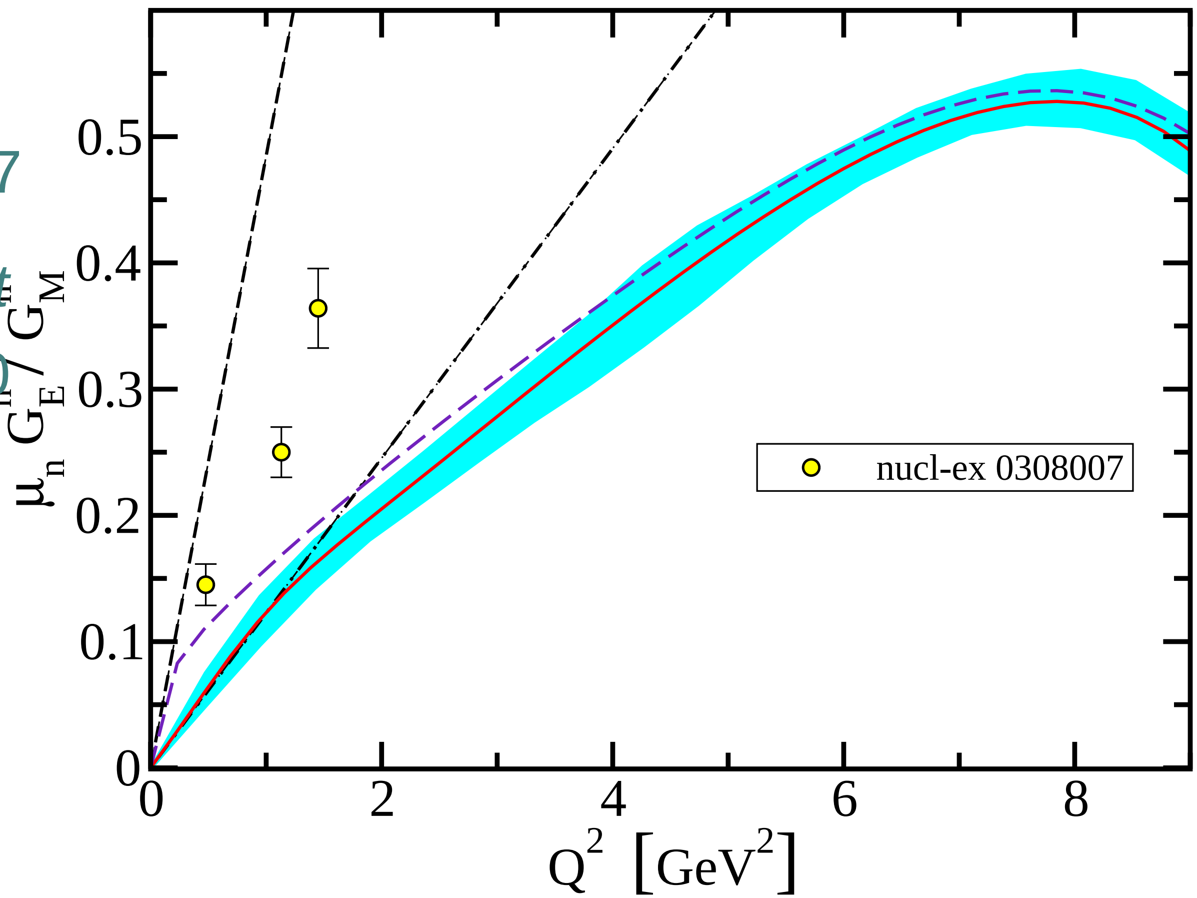

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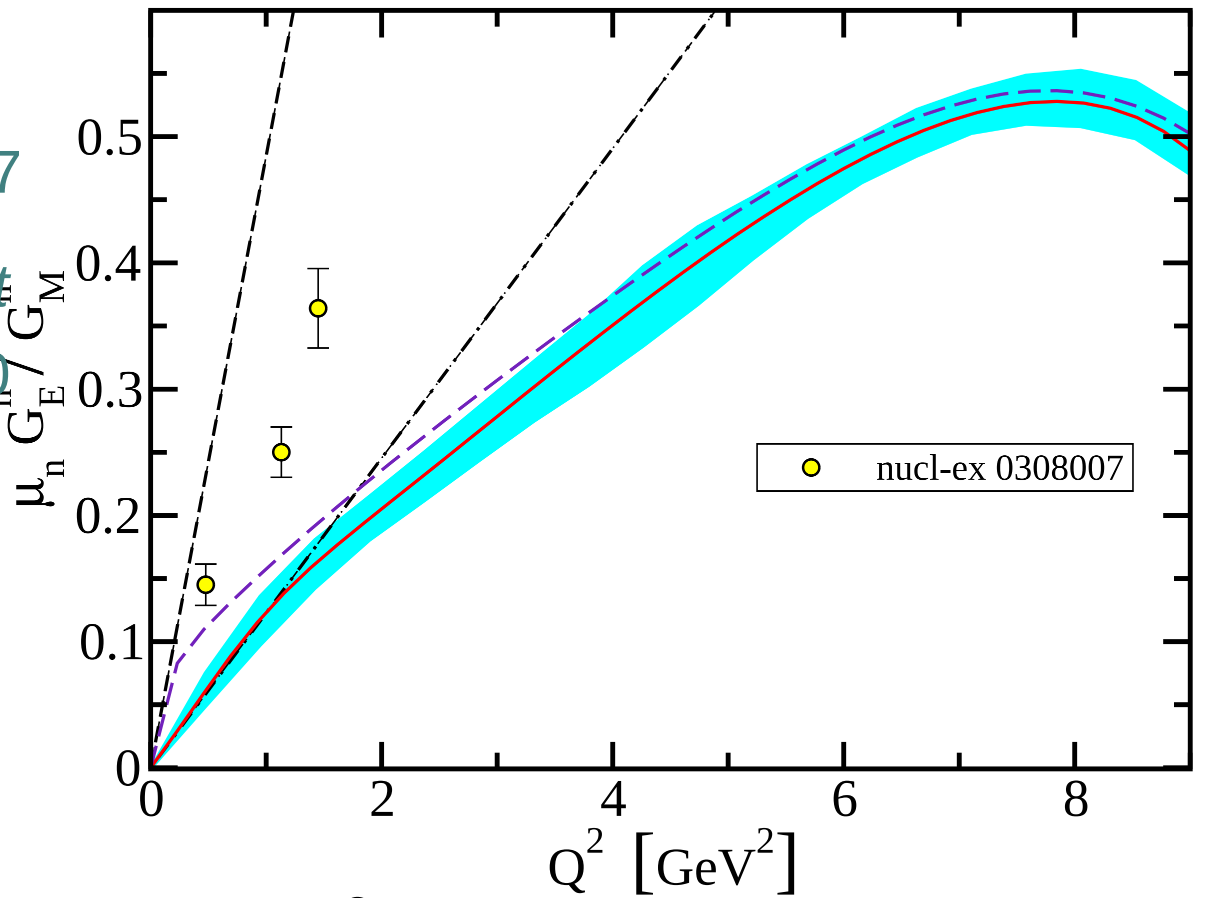


- No sign yet of a zero in $G_E^n(Q^2)$, even though calculation predicts $G_E^p(Q^2 \approx 6.5\,\mathrm{GeV}^2) = 0$
- Data to $Q^2 = 3.4$ GeV$^2$ is being analysed (JLab E02-013)

# Improved current

# *Improved current*

- Composite axial-vector diquark correlation
  - Electromagnetic current can be complicated
  - Limited constraints on large-$Q^2$ behaviour

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    - Diquark effectively pointlike to hard probe
    - Didn't account for diquark being off-shell in recoil

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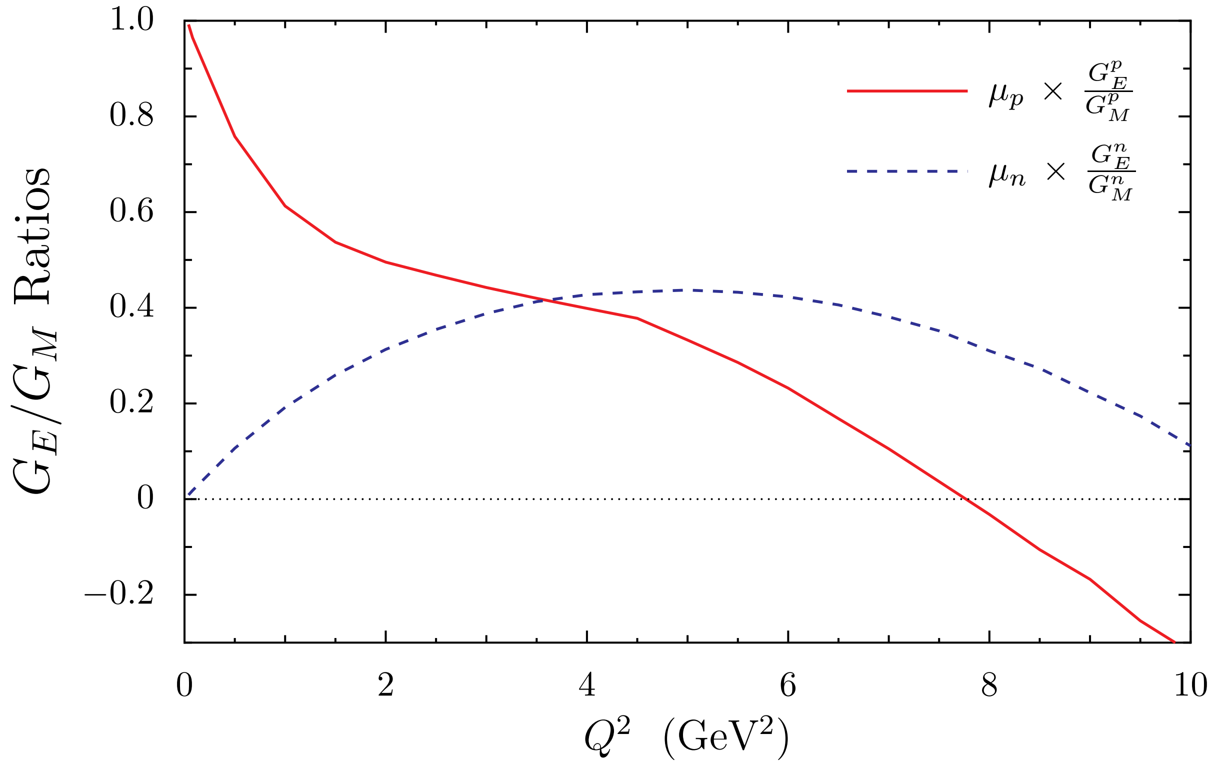


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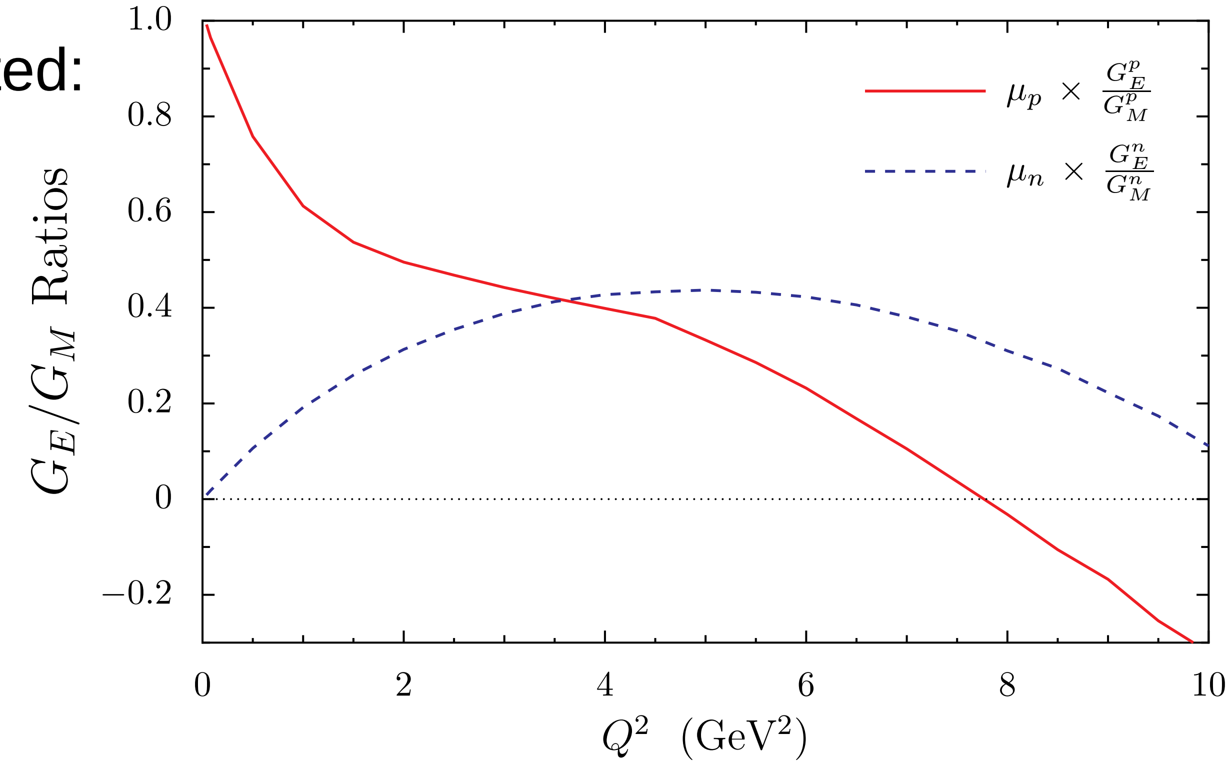


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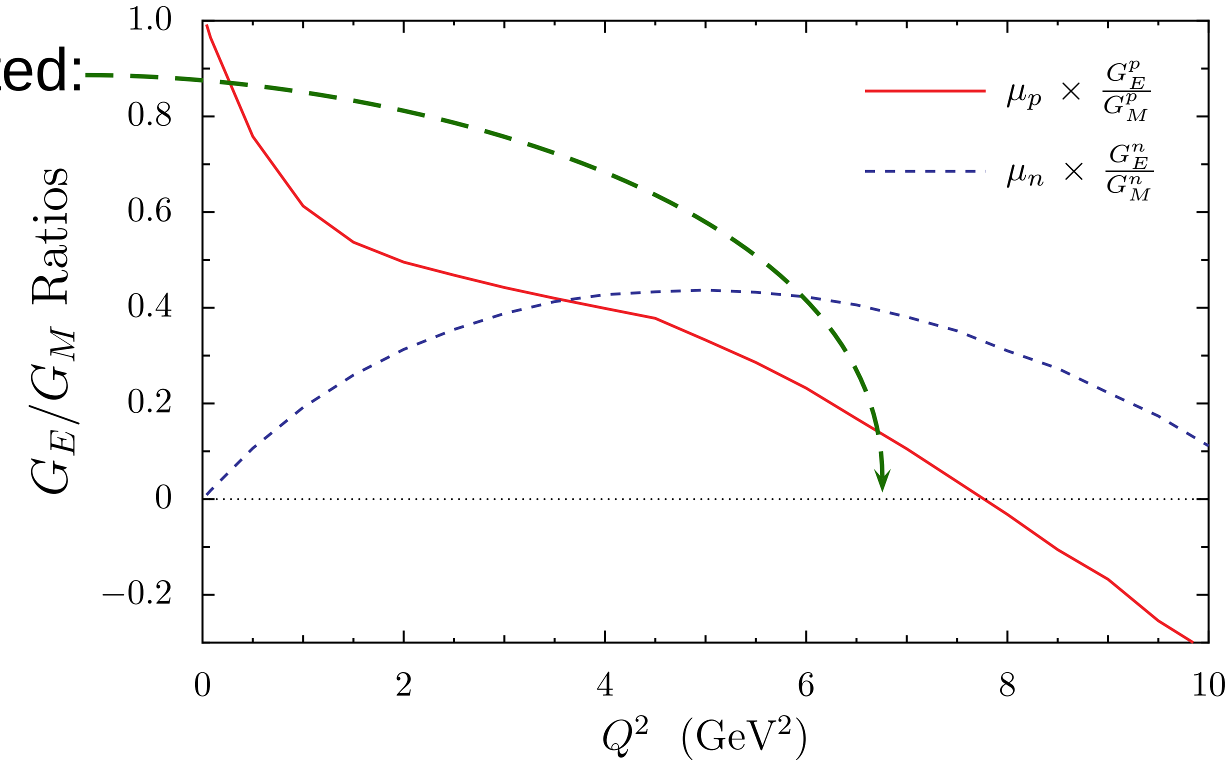

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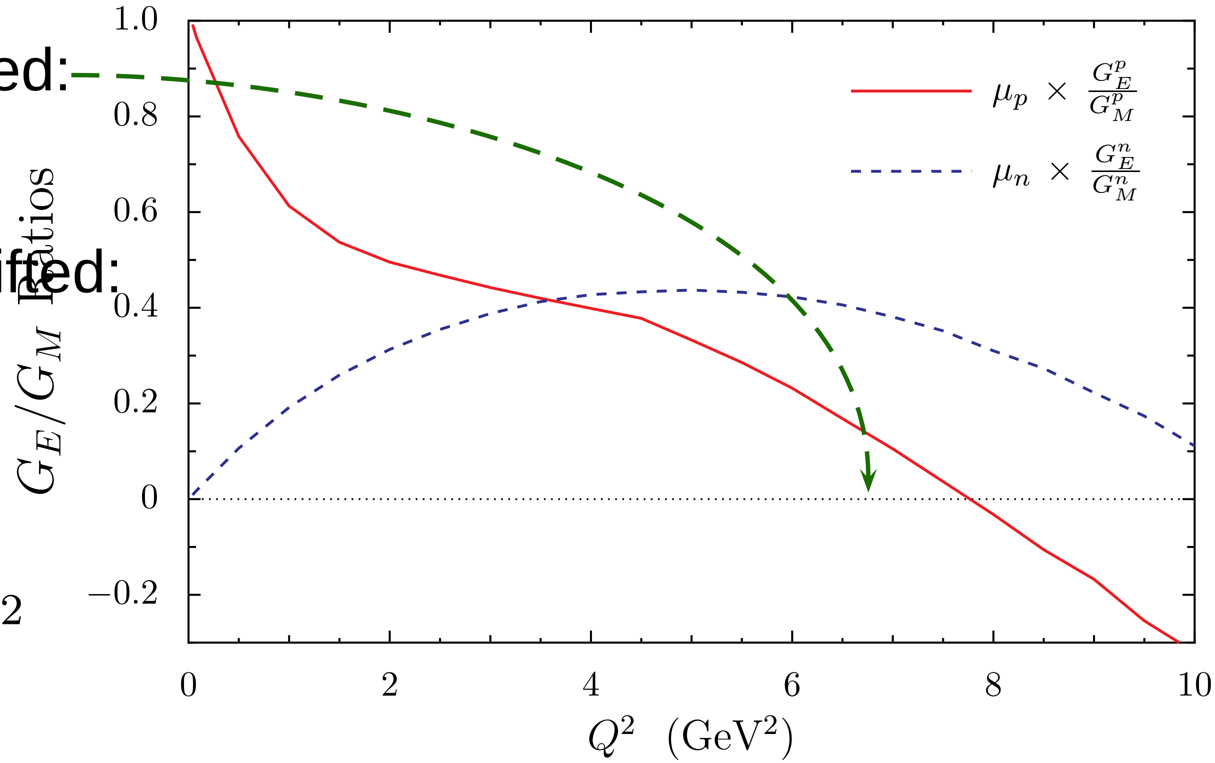

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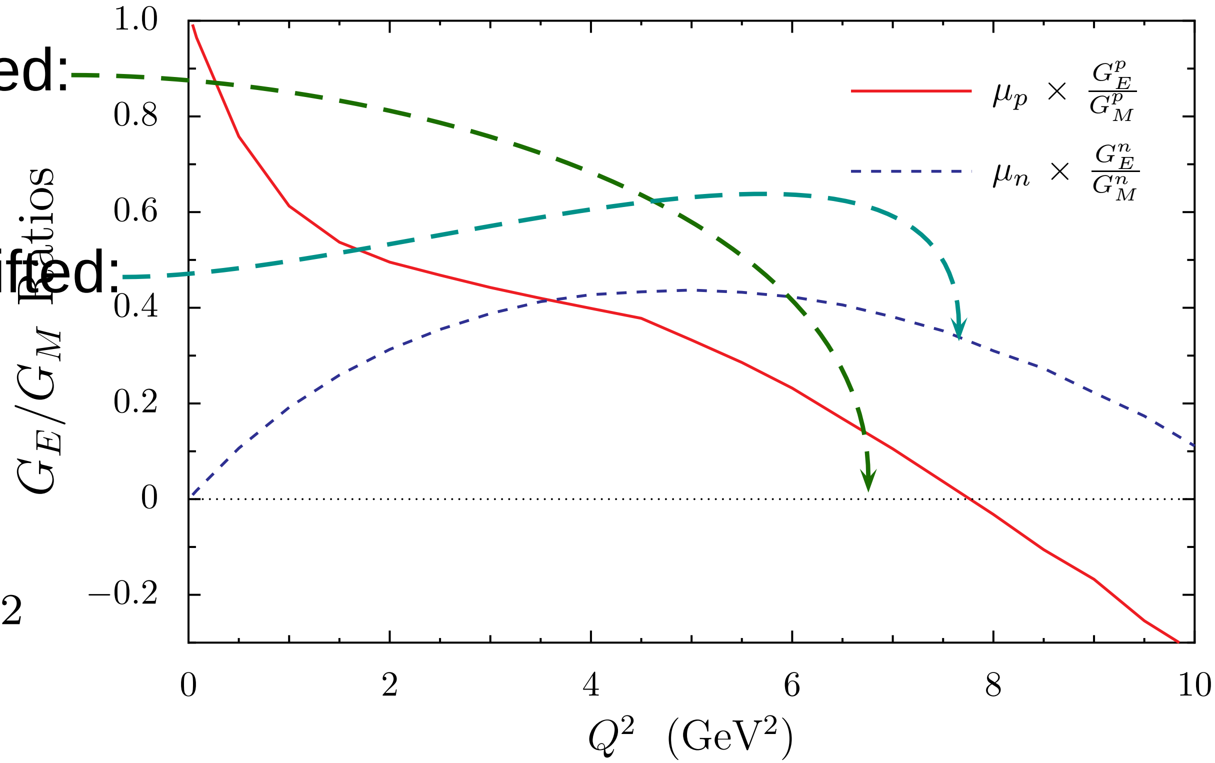


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# Epilogue

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# Epilogue

- DCSB exists in QCD.
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  - Observables can be used to explore model realisations

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- DSEs . . . contemporary tool that describes and explains these phenomena, and connects them with prediction of observables

# Contents

# Contemporary Reviews

- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD
  C.D. Roberts and S.M. Schmidt, nu-th/0005064,
  Prog. Part. Nucl. Phys. **45** (2000) S1
- The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .
  R. Alkofer and L. von Smekal, he-ph/0007355,
  Phys. Rept. **353** (2001) 281
- Dyson-Schwinger equations: A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049,
  Int. J. Mod. Phys. **E 12** (2003) pp. 297-365
- Infrared properties of QCD from Dyson-Schwinger equations.
  C. S. Fischer, he-ph/0605173,
  J. Phys. **G 32** (2006) pp. R253-R291
- Nucleon electromagnetic form factors
  J. Arrington, C.D. Roberts and J.M. Zanotti, nucl-th/0611050,
  J. Phys. **G 34** (2007) pp. S23-S52.

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# Perturbative Dressed-quark Propagator

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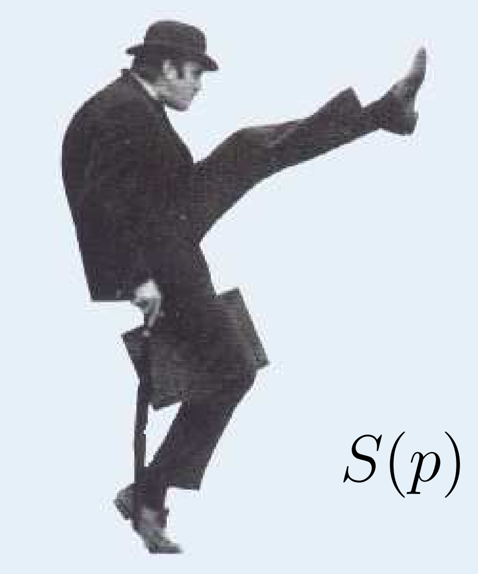
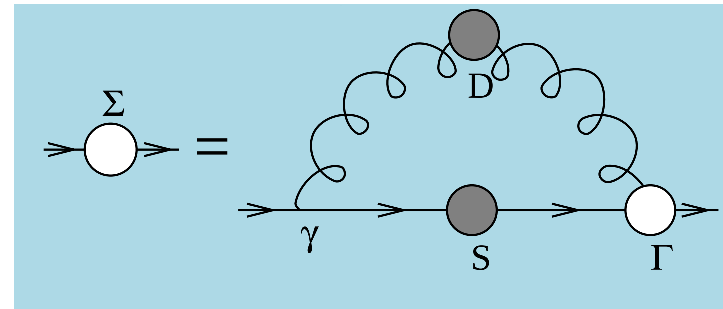


# *Perturbative Dressed-quark Propagator*

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$




*Gap Equation*

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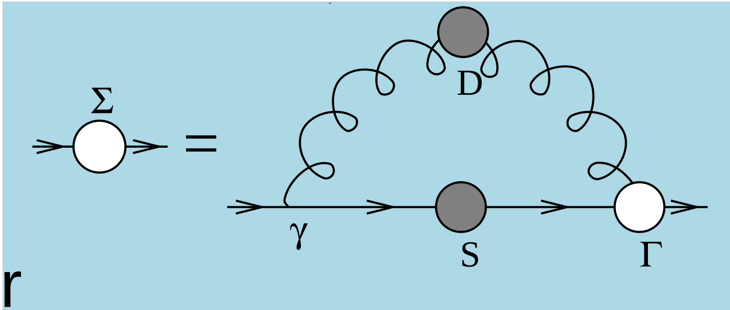


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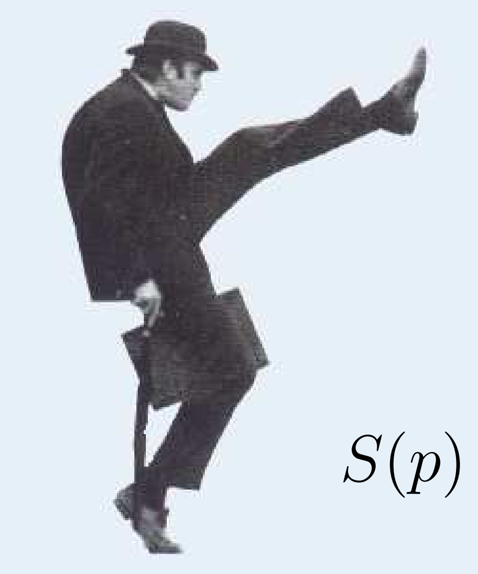
$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$



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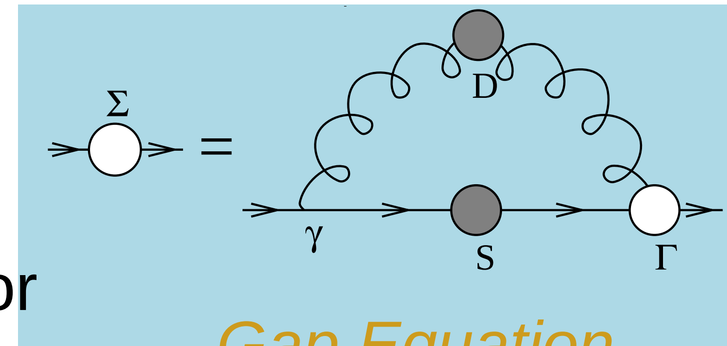


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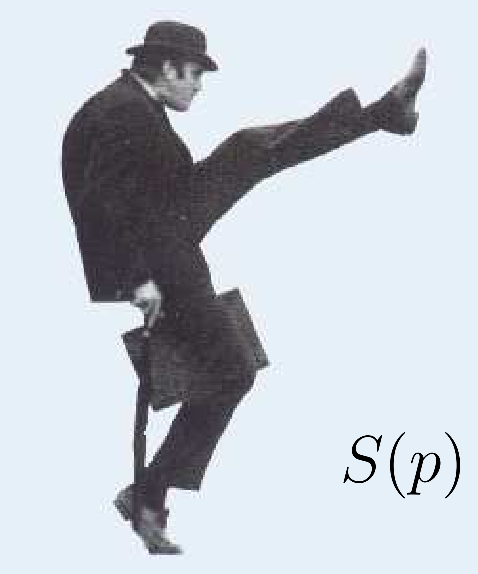

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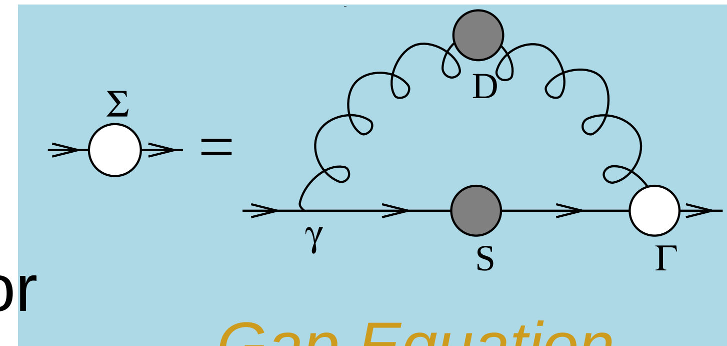


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$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\longrightarrow} 0$$

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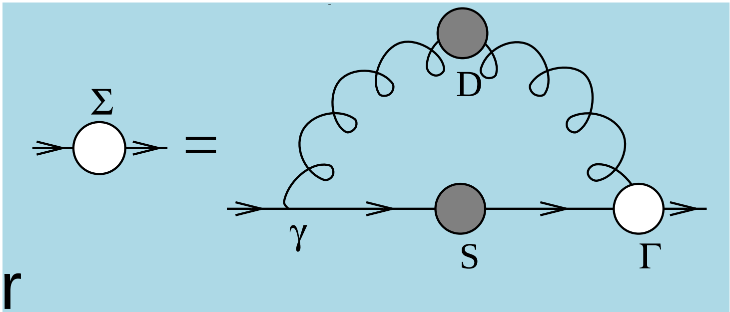


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# QCD & Interaction Between Light-Quarks

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- Reliable DSE studies of Dressed-gluon propagator:
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- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
  - D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD Collaboration], *Asymptotic scaling and infrared behavior of the gluon propagator*, Phys. Rev. D **60**, 094507 (1999) [Erratum-ibid. D **61**, 079901 (2000)].
- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

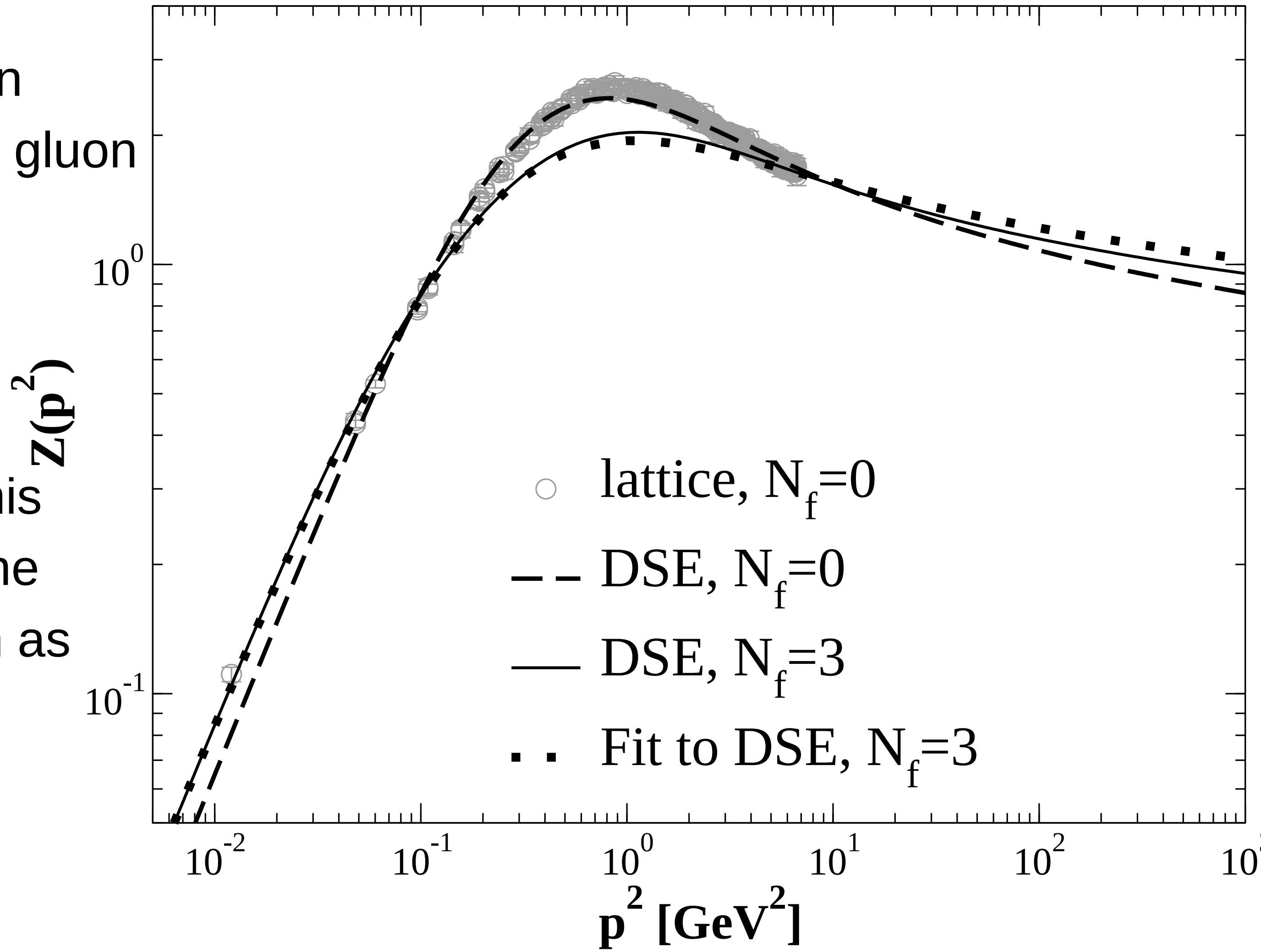
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

# Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as $\Lambda_{\mathrm{QCD}}$

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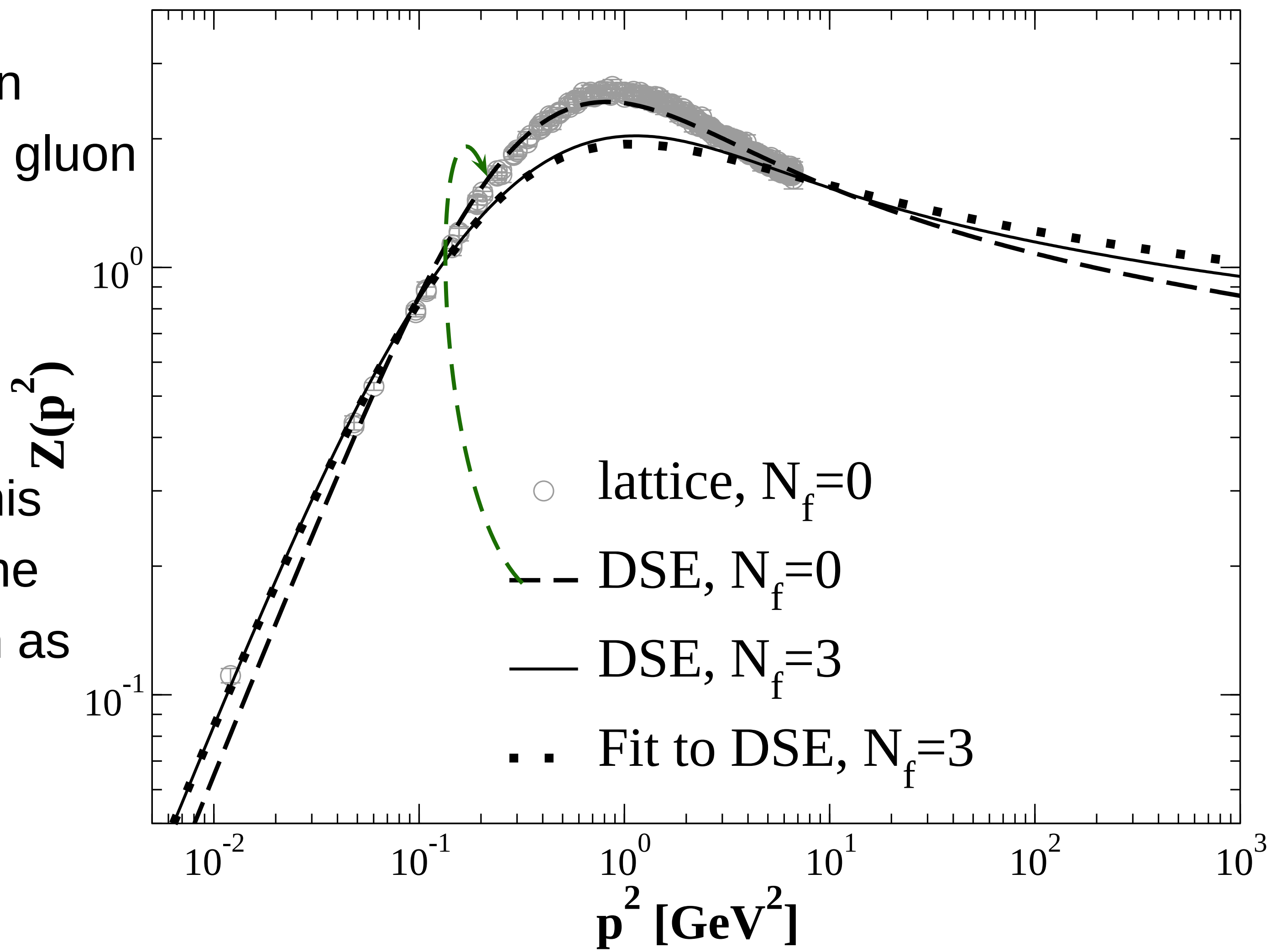

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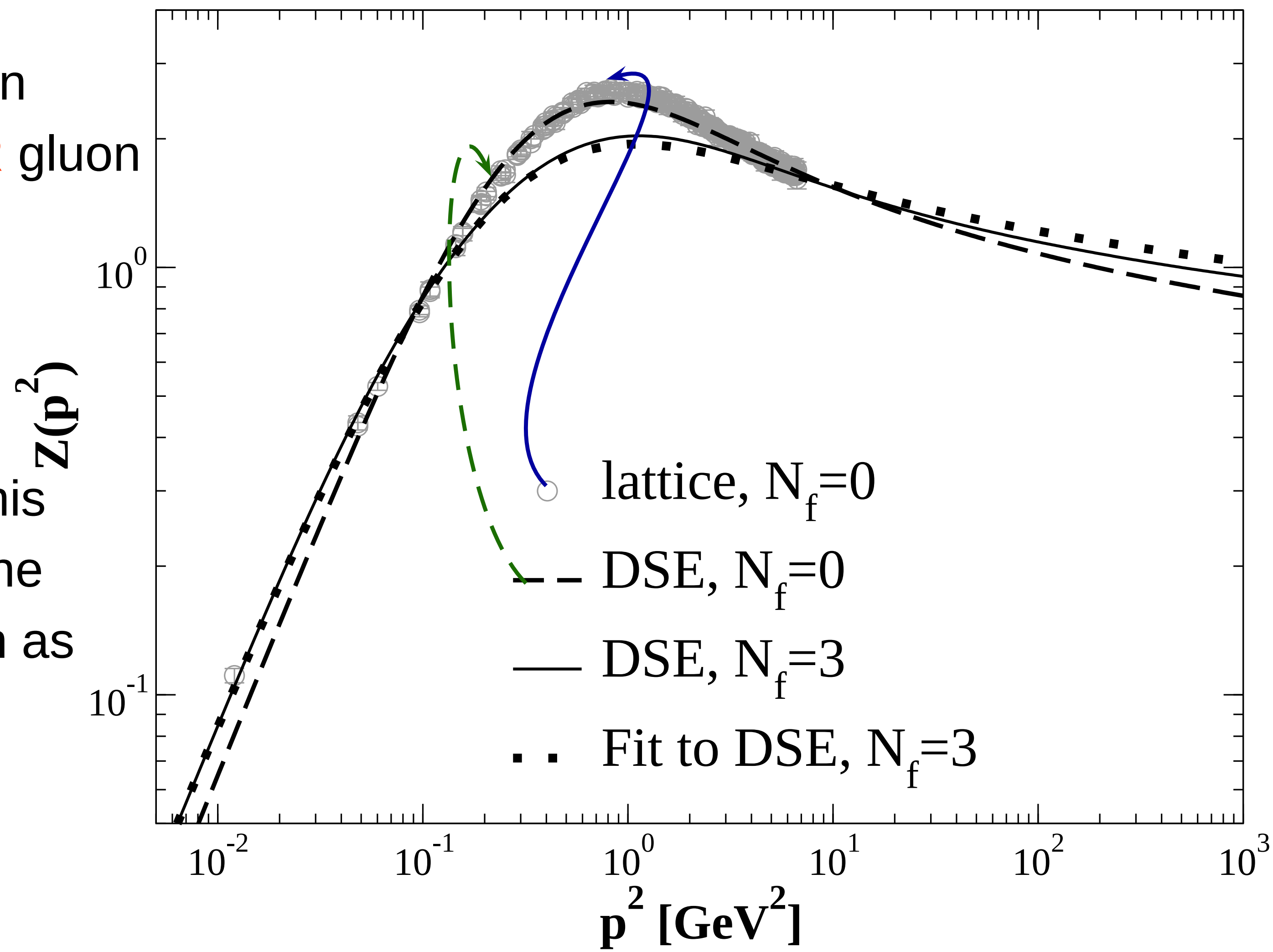


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# Colour-singlet Bethe-Salpeter equation

Detmold *et al*., nu-th/0202082

Bhagwat, *et al*., nu-th/0403012

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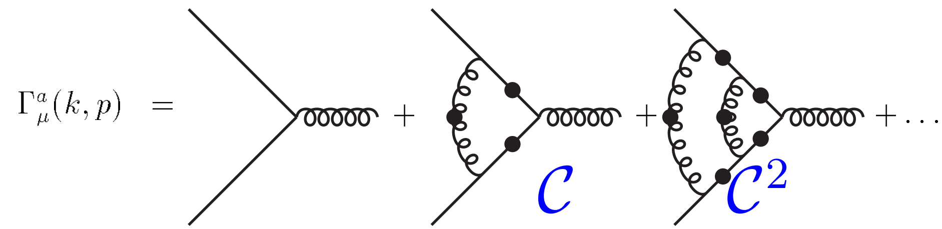
- Coupling-modified dressed-ladder vertex



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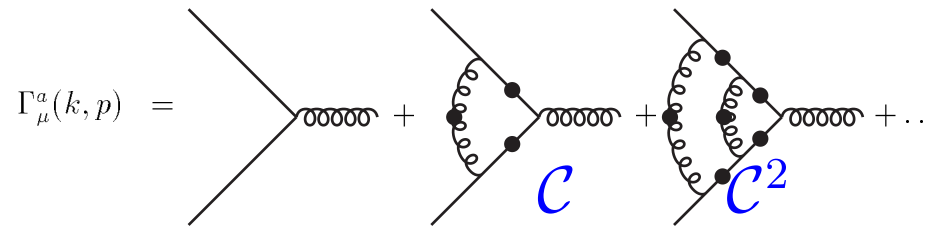


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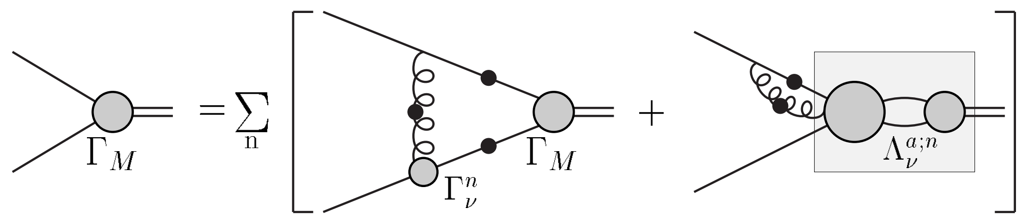

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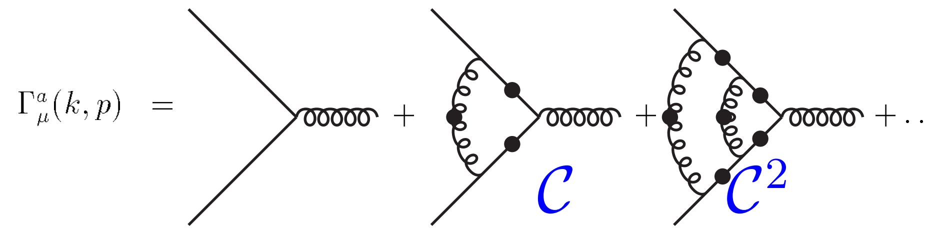


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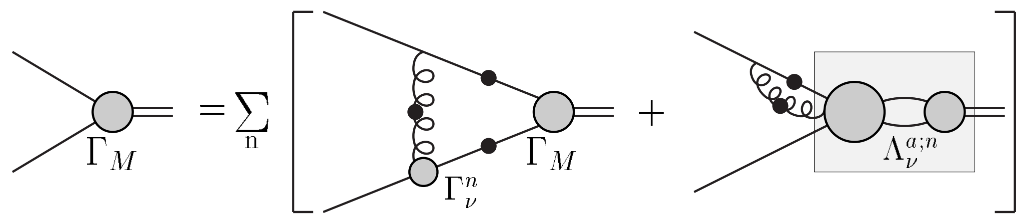
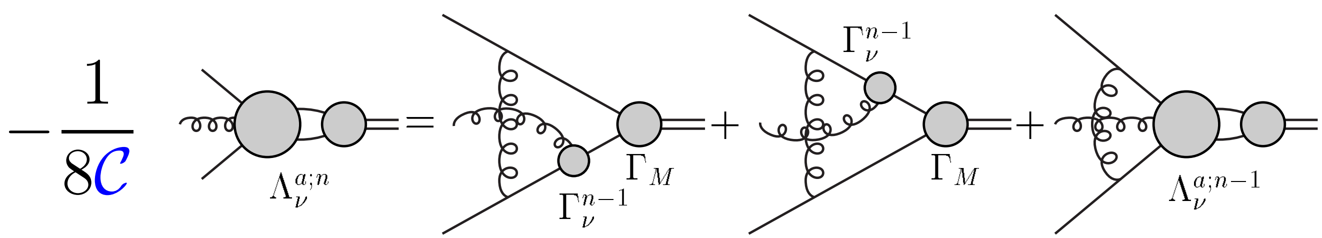



- Bethe-Salpeter kernel . . . recursion relation

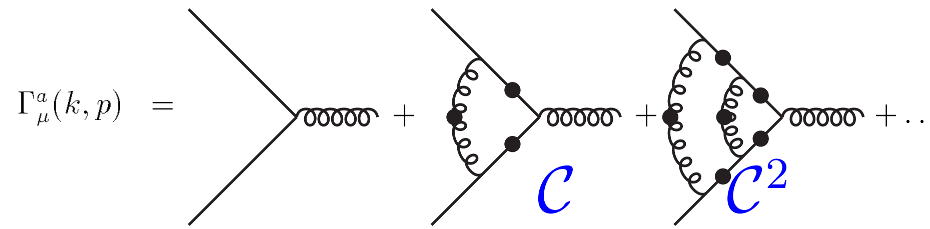


# Colour-singlet Bethe-Salpeter equation

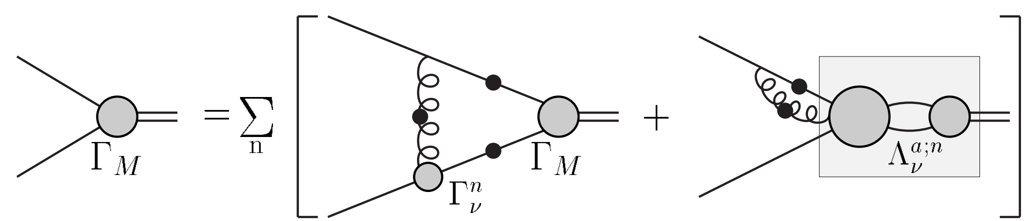
Detmold *et al.*, nu-th/0202082

Bhagwat, *et al.*, nu-th/0403012

- Coupling-modified dressed-ladder vertex




- BSE consistent with vertex

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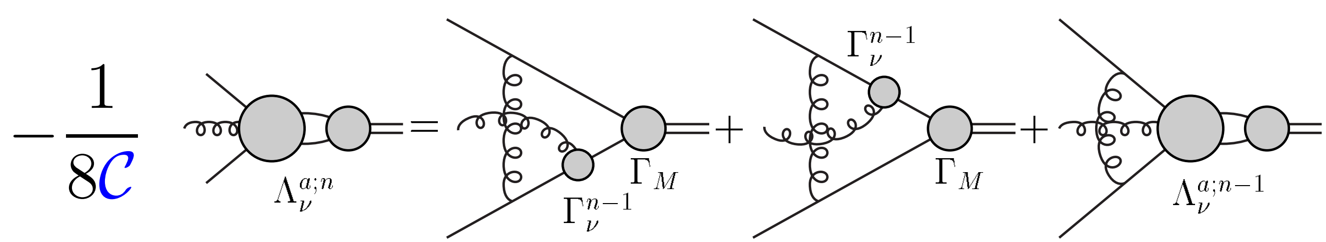


- Kernel necessarily non-planar,

even with planar vertex

# $\pi$ and $\rho$ mesons

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## $\pi$ and $\rho$ mesons

| | $M_H^{n=0}$ | $M_H^{n=1}$ | $M_H^{n=2}$ | $M_H^{n=\infty}$ |
|---|---|---|---|---|
| $\pi$, $m = 0$ | 0 | 0 | 0 | 0 |
| $\pi$, $m = 0.011$ | 0.147 | 0.135 | 0.139 | 0.138 |
| $\rho$, $m = 0$ | 0.920 | 0.648 | 0.782 | 0.754 |
| $\rho$, $m = 0.011$ | 0.936 | 0.667 | 0.798 | 0.770 |

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- $\pi$ massless in chiral limit . . . **NO** Fine Tuning

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  Not constituent-quark-model-like hyperfine splitting

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  For $m_\rho$ – zeroth order, accurate to 20%

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  For $m_\rho$ – zeroth order, accurate to 20%
  – one loop, accurate to 13%

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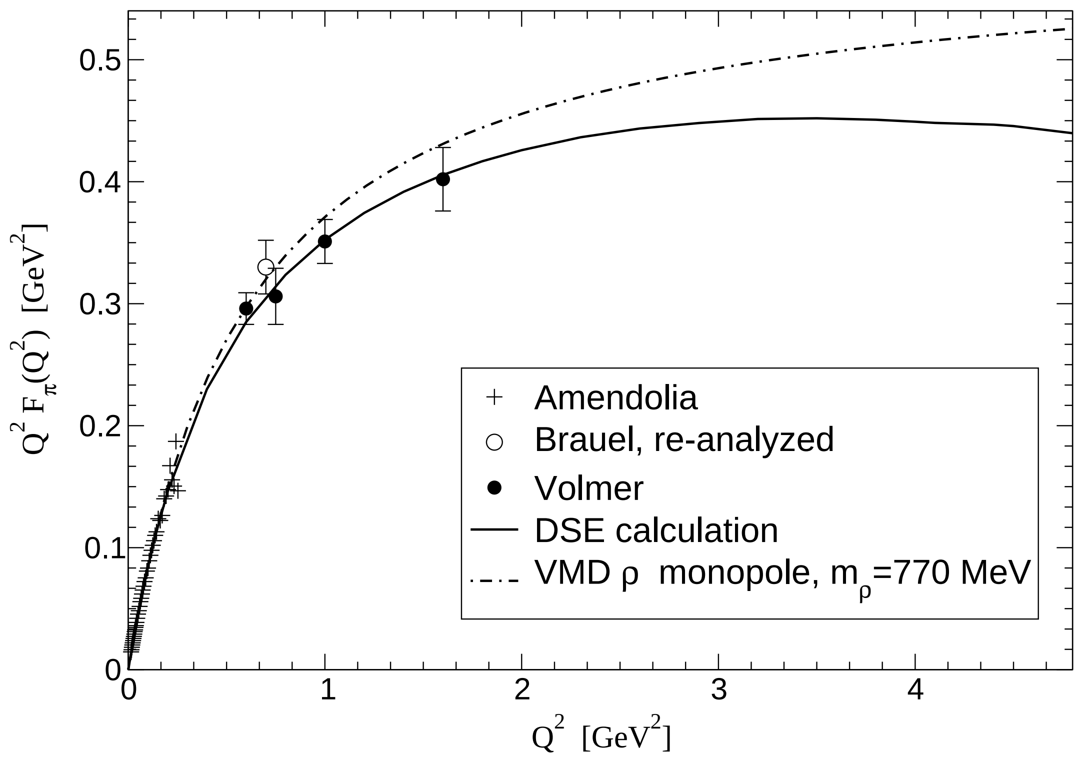
- Extending kernel: NO effect on $m_\pi$
  For $m_\rho$ – zeroth order, accurate to 20%
  – one loop, accurate to 13%
  – two loop, accurate to 4%

# Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

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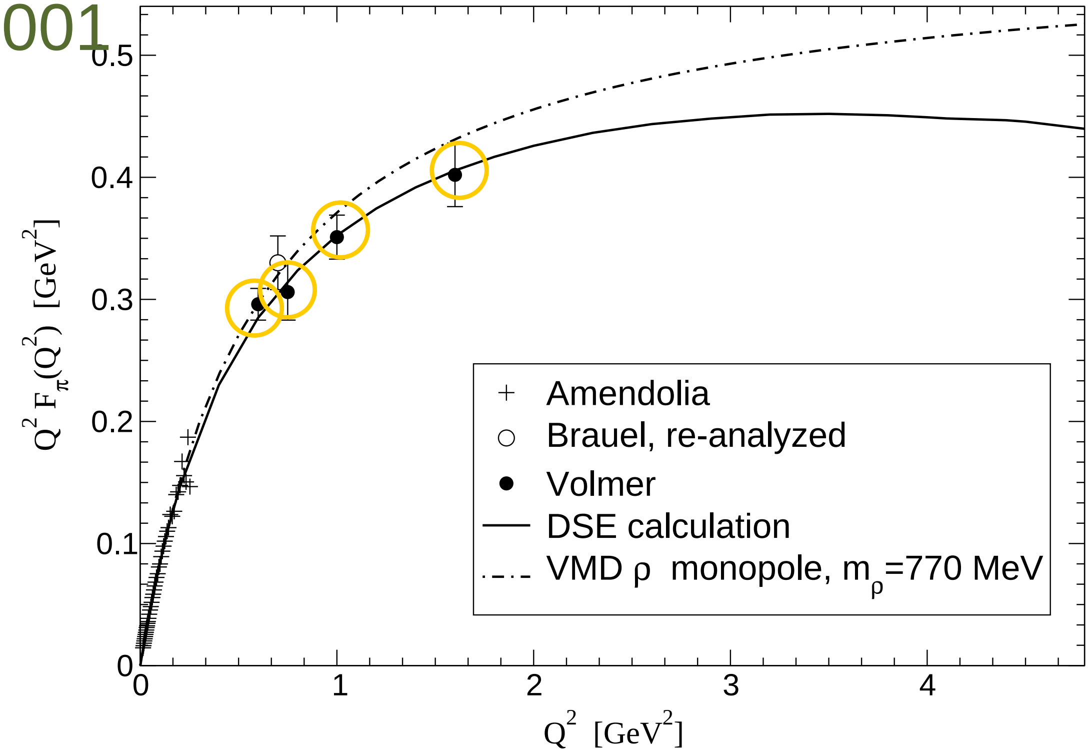
# Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

Data published in 2001

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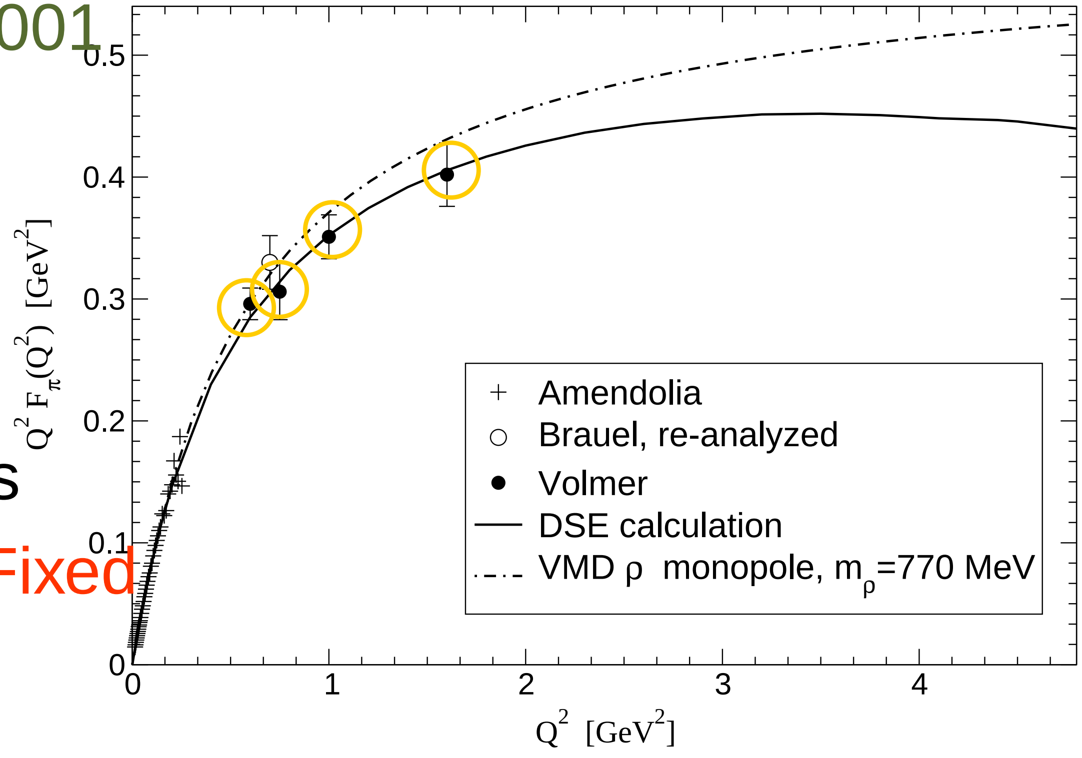
# Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

Data published in 2001




Many subsequent successful applications

. . Again, parameters Fixed

Notably $\pi\ \pi$ Scattering

Maris, et al., Phys. Rev. D 65, 076008

Bicudo, Phys. Rev. C 67, 035201

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# Charge Neutral Pseudoscalar Mesons

$$
\begin{aligned}
P_\mu \Gamma^a_{5\mu}(k;P) &= \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\
&\quad - 2i \mathcal{M}^{ab} \Gamma^b_5(k;P) - \mathcal{A}^a(k;P)
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$$

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# Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) \;=\; \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-)$$
$$-2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\{\mathcal{F}^a \,|\, a = 0,\ldots,N_f^2-1\}$ are the generators of $U(N_f)$
- $\mathcal{S} = \mathrm{diag}[S_u, S_d, S_s, S_c, S_b, \ldots]$
- $\mathcal{M}^{ab} = \mathrm{tr}_F \left[ \{\mathcal{F}^a, \mathcal{M}\}\mathcal{F}^b \right]$,
  $\mathcal{M} = \mathrm{diag}[m_u, m_d, m_s, m_c, m_b, \ldots]$ = matrix of current-quark bare masses

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# Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) \;=\; \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-)$$
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  $\mathcal{M} = \mathrm{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] =$ matrix of current-quark bare masses
- The final term in the second line expresses the non-Abelian axial anomaly.

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# Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$
  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \langle \mathcal{F}^0 q(x)\,\mathcal{Q}(0)\,\bar{q}(y)\rangle$

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- $\mathcal{Q}(x) = i\dfrac{\alpha_s}{4\pi}\mathrm{tr}_C\left[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}(x)\right] = \partial_\mu K_\mu(x)$

  … The topological charge density operator.

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  … The topological charge density operator.
  (Trace is over colour indices & $F_{\mu\nu} = \frac{1}{2}\lambda^a F^a_{\mu\nu}$.)

# Charge Neutral Pseudoscalar Mesons

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  … The topological charge density operator.
- Important that only $\mathcal{A}^{a=0}$ is nonzero.

# Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\ -2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

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  … The topological charge density operator.
- NB. While $\mathcal{Q}(x)$ is gauge invariant, the associated Chern-Simons current, $K_\mu$, is not $\Rightarrow$ in QCD *no physical* boson can couple to $K_\mu$ and hence *no physical* states can contribute to resolution of $U_A(1)$ problem.

# Charge Neutral Pseudoscalar Mesons

*Bhagwat, Chang, Liu, Roberts, Tandy*
nucl-th/arXiv:0708.1118

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# Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting

*Bhagwat, Chang, Liu, Roberts, Tandy*
nucl-th/arXiv:0708.1118

# Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting ... otherwise all pseudoscalar mesons are Goldstone Modes!

# Charge Neutral Pseudoscalar Mesons

- Anomaly term has structure

$$\mathcal{A}^0(k;P) = \mathcal{F}^0\gamma_5\left[i\mathcal{E}_{\mathcal{A}}(k;P) + \gamma\cdot P\mathcal{F}_{\mathcal{A}}(k;P)\right.$$
$$\left.+\gamma\cdot k k\cdot P\mathcal{G}_{\mathcal{A}}(k;P) + \sigma_{\mu\nu}k_\mu P_\nu\mathcal{H}_{\mathcal{A}}(k;P)\right]$$

# Charge Neutral Pseudoscalar Mesons

- AVWTI gives generalised Goldberger-Treiman relations

$$\begin{aligned}
2f^0_{\eta'}E_{BS}(k;0) &= 2B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0),\\
F^0_R(k;0) + 2f^0_{\eta'}F_{BS}(k;0) &= A_0(k^2) - \mathcal{F}_{\mathcal{A}}(k;0),\\
G^0_R(k;0) + 2f^0_{\eta'}G_{BS}(k;0) &= 2A_0'(k^2) - \mathcal{G}_{\mathcal{A}}(k;0),\\
H^0_R(k;0) + 2f^0_{\eta'}H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0),
\end{aligned}$$

$A_0$, $B_0$ characterise gap equation's chiral limit solution.

# Charge Neutral Pseudoscalar Mesons

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G^0_R(k;0) + 2f^0_{\eta'} G_{BS}(k;0) &= 2A'_0(k^2) - \mathcal{G}_{\mathcal{A}}(k;0), \\
H^0_R(k;0) + 2f^0_{\eta'} H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0),
\end{aligned}
$$

  $A_0$, $B_0$ characterise gap equation's chiral limit solution.

- Follows that $\boldsymbol{\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)}$ is necessary and sufficient condition for absence of massless $\boldsymbol{\eta'}$ bound-state.

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# Charge Neutral Pseudoscalar Mesons

- $\boldsymbol{\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)}$
  Discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
  - Hence, absence of massless $\eta'$ bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.

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- Further highlighted … proved

$$\begin{aligned} \langle \bar{q}q \rangle_\zeta^0 &= -\lim_{\Lambda\to\infty} Z_4(\zeta^2,\Lambda^2)\,\mathrm{tr}_{\rm CD}\int_q^\Lambda S^0(q,\zeta) \\ &= N_f \int d^4x\,\langle \bar{q}(x) i\gamma_5 q(x) \mathcal{Q}(0)\rangle^0 \,. \end{aligned}$$

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# *Charge Neutral Pseudoscalar Mesons*

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons

# Charge Neutral Pseudoscalar Mesons

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- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly

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# *Charge Neutral Pseudoscalar Mesons*

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
- Employed in an analysis of pseudoscalar- and vector-meson bound-states

*Bhagwat, Chang, Liu, Roberts, Tandy*
nucl-th/arXiv:0708.1118

# *Charge Neutral Pseudoscalar Mesons*

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
- Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts
  - $\eta$–$\eta'$ mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)
  - $\pi^0$–$\eta$ angles of $\sim 1.2^\circ$ (Expt. $p\,d \rightarrow {}^3$He $\pi^0$: $0.6^\circ \pm 0.3^\circ$)
  - Strong neutron-proton mass difference . . .
    $\lesssim 75\,$% current-quark mass-difference

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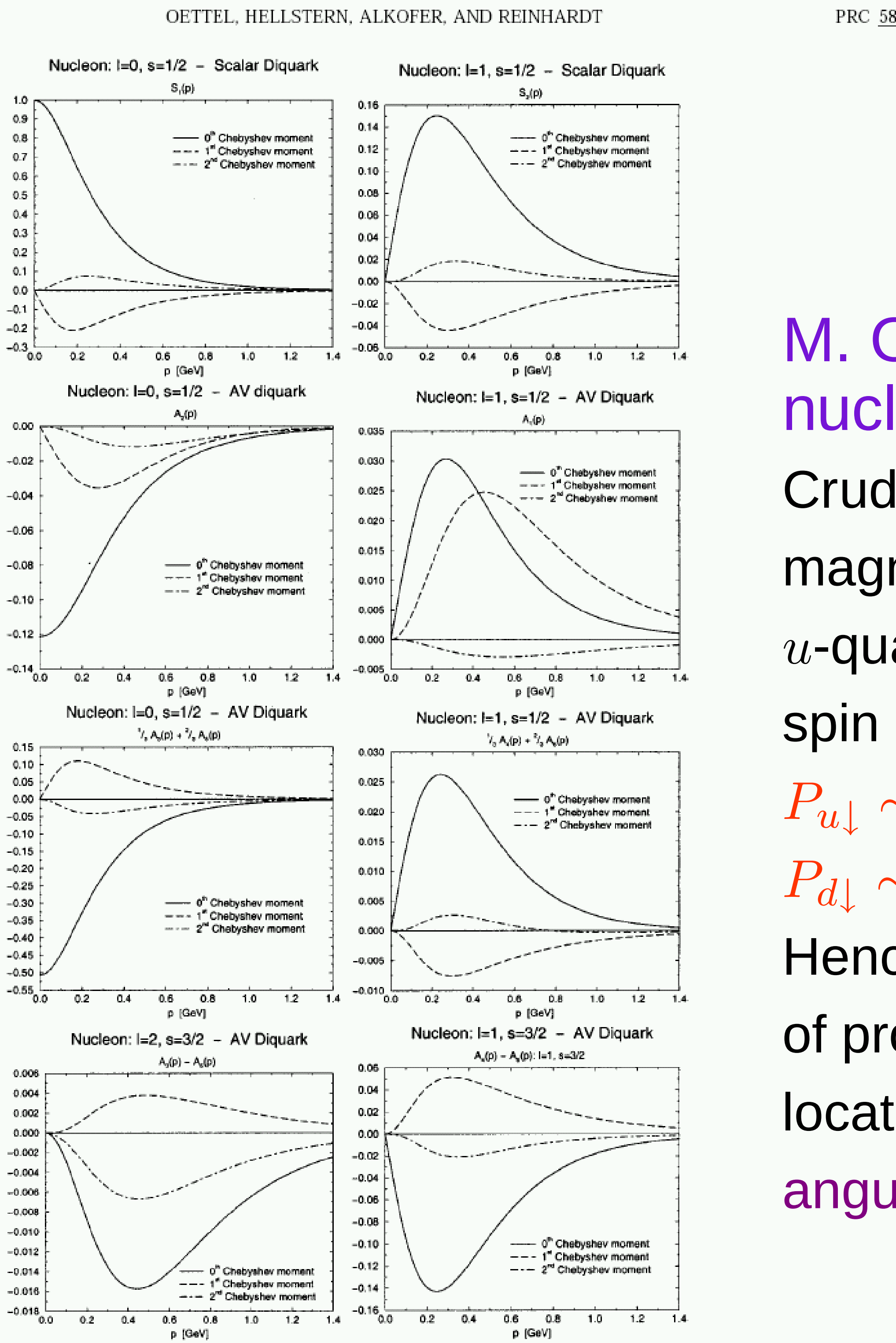
# Angular Momentum
# Rest Frame




M. Oettel, *et al.*
nucl-th/9805054

Crude estimate based on magnitudes $\Rightarrow$ probability for a $u$-quark to carry the proton's spin is $P_{u\uparrow} \sim 80\,\%$, with $P_{u\downarrow} \sim 5\,\%$, $P_{d\uparrow} \sim 5\,\%$, $P_{d\downarrow} \sim 10\,\%$.

Hence, by this reckoning $\sim 30\%$ of proton's rest-frame spin is located in dressed-quark angular momentum.

# Deep-inelastic scattering

# *Deep-inelastic scattering*

- Looking for Quarks

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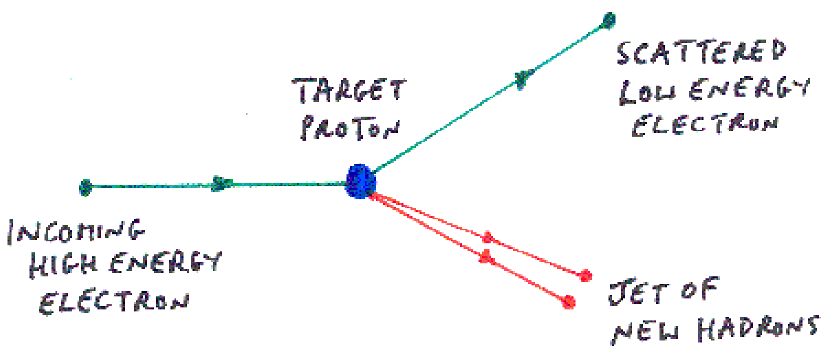
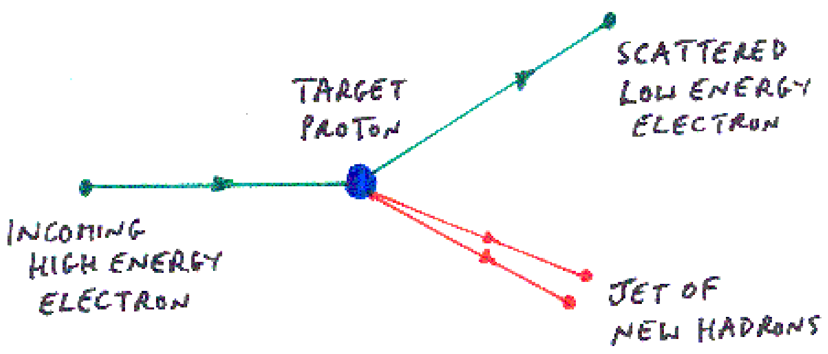
# Deep-inelastic scattering




- Looking for Quarks

# *Deep-inelastic scattering*

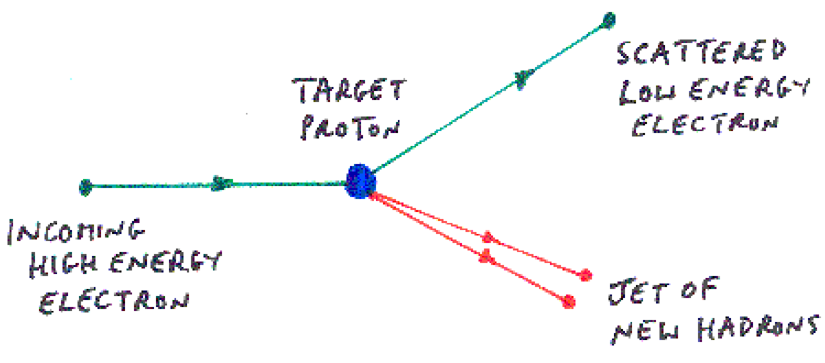



- Looking for Quarks

- Signature Experiment for QCD:

  Discovery of Quarks at SLAC

# Deep-inelastic scattering




- Looking for Quarks

- $\boxed{\text{Signature Experiment}}$ for QCD:

  Discovery of Quarks at SLAC

- Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x)$, $g(x)$

# Pion's valence quark distn

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# Pion's valence quark distn

- $\pi$ is Two-Body System: "Easiest" Bound State in QCD
- However, NO $\pi$ Targets!

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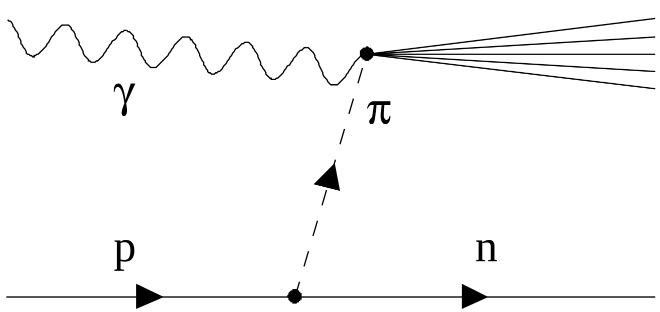
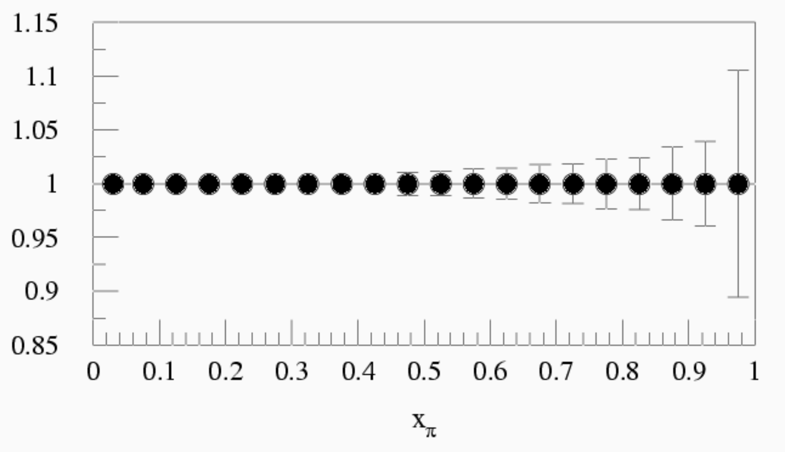
  $\pi N \to \mu^+ \mu^- X$

# Pion's valence quark distn

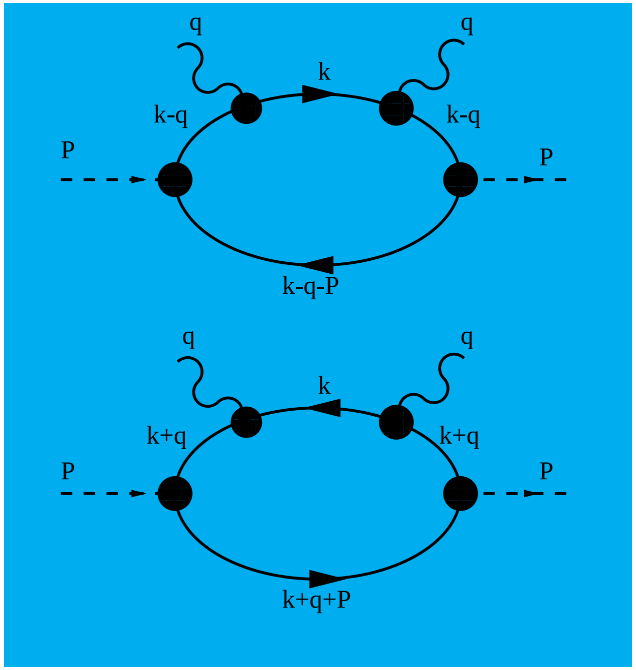
- $\pi$ is Two-Body System: "Easiest" Bound State in QCD
- However, NO $\pi$ Targets!
- Existing Measurement Inferred from Drell-Yan:
  $\pi N \to \mu^+ \mu^- X$
- Proposal (Holt & Reimer, ANL, nu-ex/0010004)

  $e^-_{5\mathrm{GeV}} - p_{25\,\mathrm{GeV}}$ Collider $\to$ Accurate "Measurement"



# Handbag diagrams

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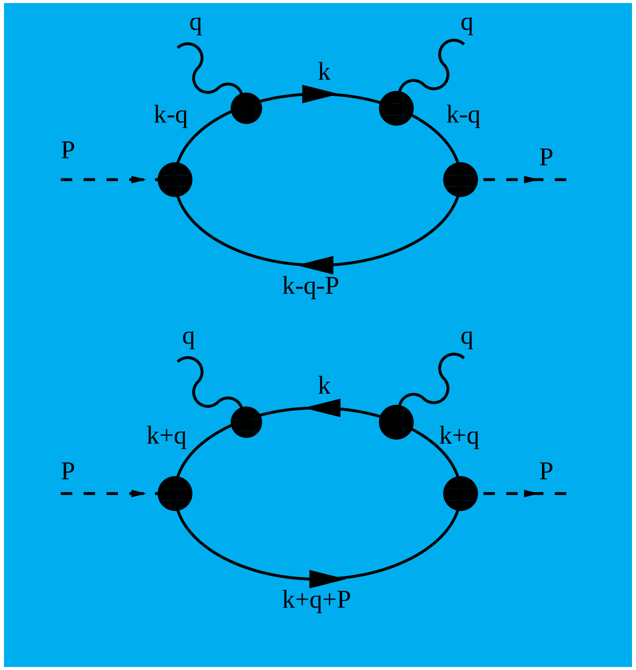
# Handbag diagrams




$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

$$T^{+}_{\mu\nu}(q,P) = \mathrm{tr}\int\frac{d^4k}{(2\pi)^4}\,\tau_-\bar{\Gamma}_\pi(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_\nu(k_{-0},k)$$
$$\times\, S(k)\,ieQ\Gamma_\mu(k,k_{-0})\,S(k_{-0})\tau_+\Gamma_\pi(k_{-\frac{1}{2}};P)\,S(k_{--})$$

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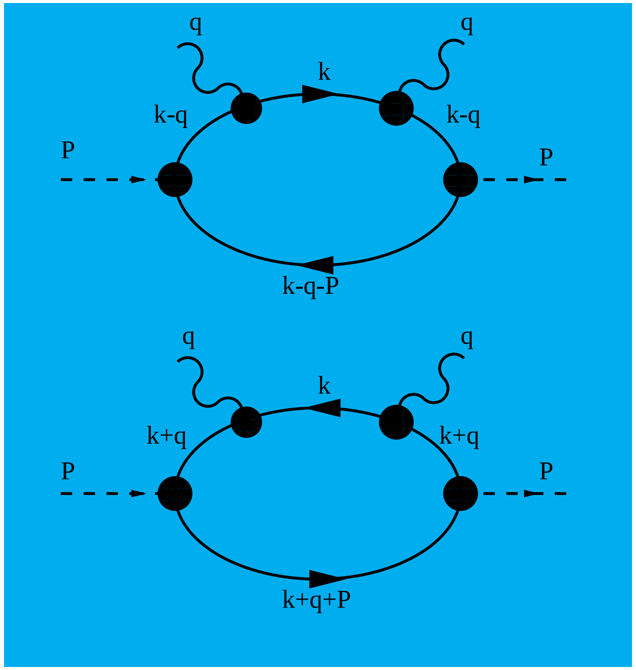
# Handbag diagrams

Bjorken Limit: $q^2 \to \infty\,,\quad P\cdot q \to -\infty$

but $x := -\dfrac{q^2}{2P\cdot q}$ fixed.




Numerous algebraic simplifications

$$W_{\mu\nu}(q;P) \;=\; \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

$$T^{+}_{\mu\nu}(q,P) \;=\; \mathrm{tr}\int\frac{d^4k}{(2\pi)^4}\,\tau_-\bar{\Gamma}_\pi(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_\nu(k_{-0},k)$$
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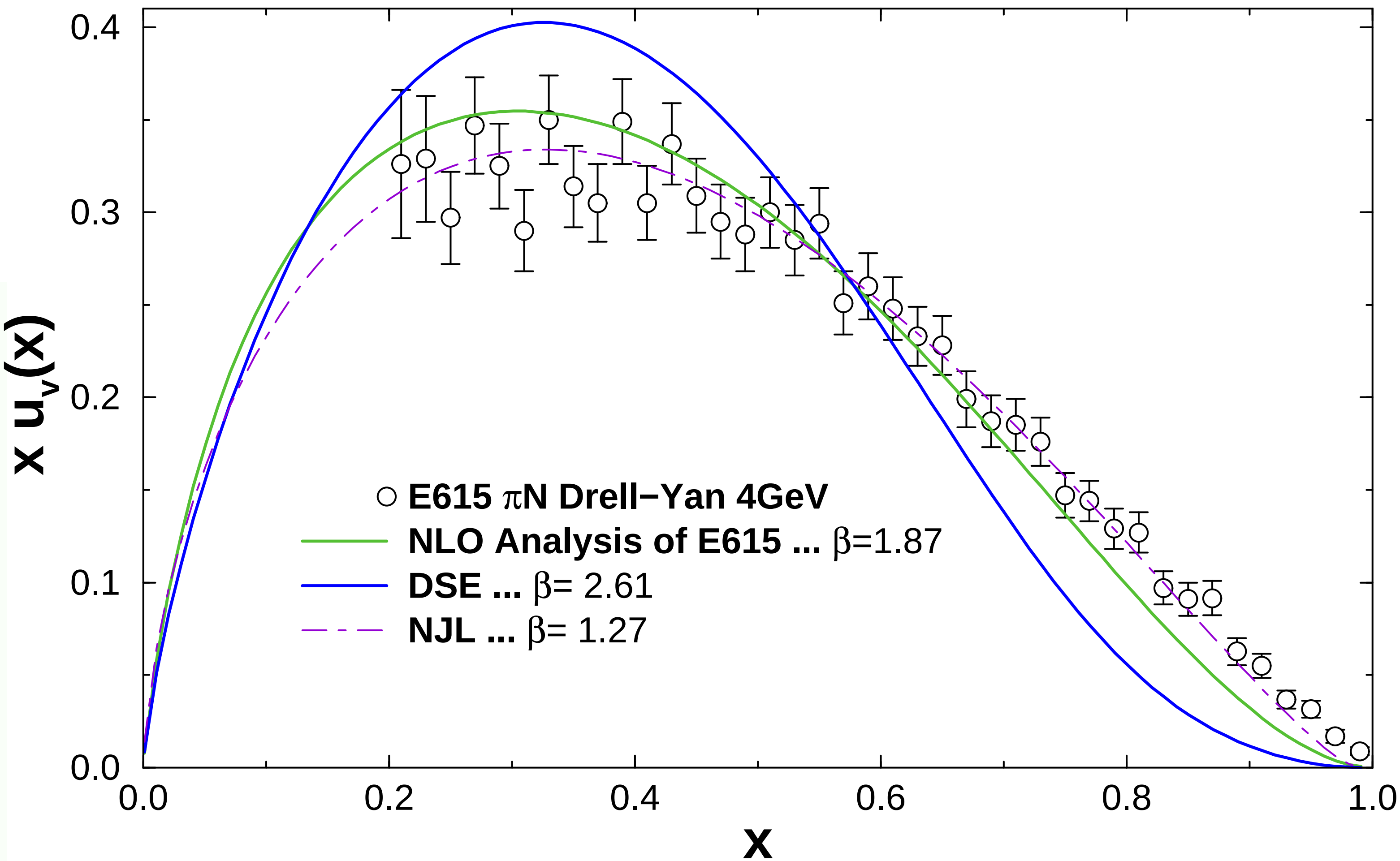
# *Extant theory vs. experiment*

K. Wijersooriya, P. Reimer and R. Holt,
nu-ex/0509012 ... Phys. Rev. C (Rapid)



# Nucleon's Quark Distribution Functions

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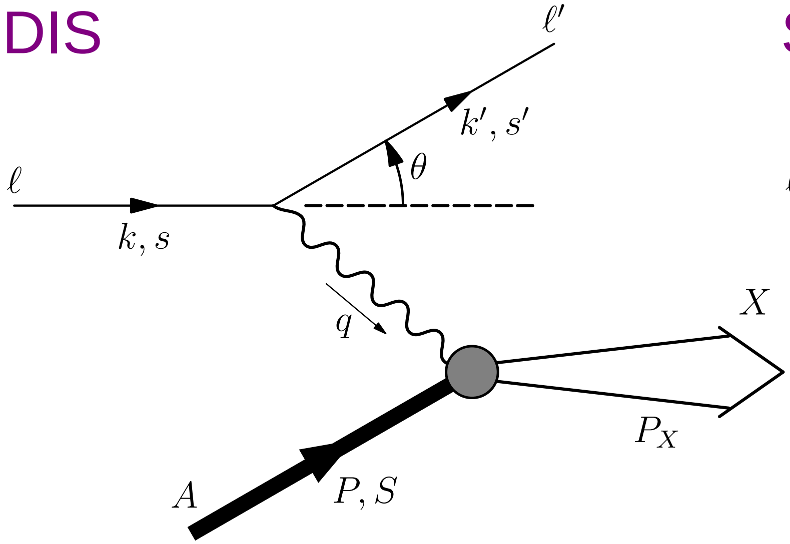
DIS
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k, s
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A
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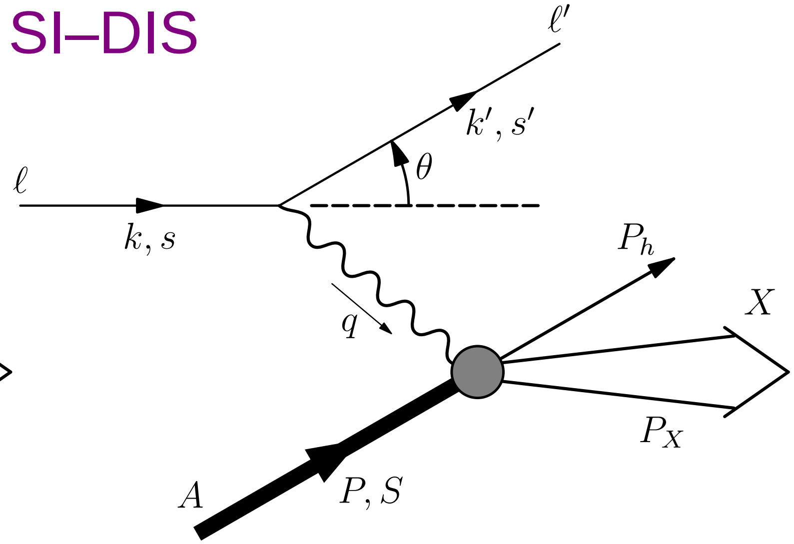
SI–DIS
ℓ
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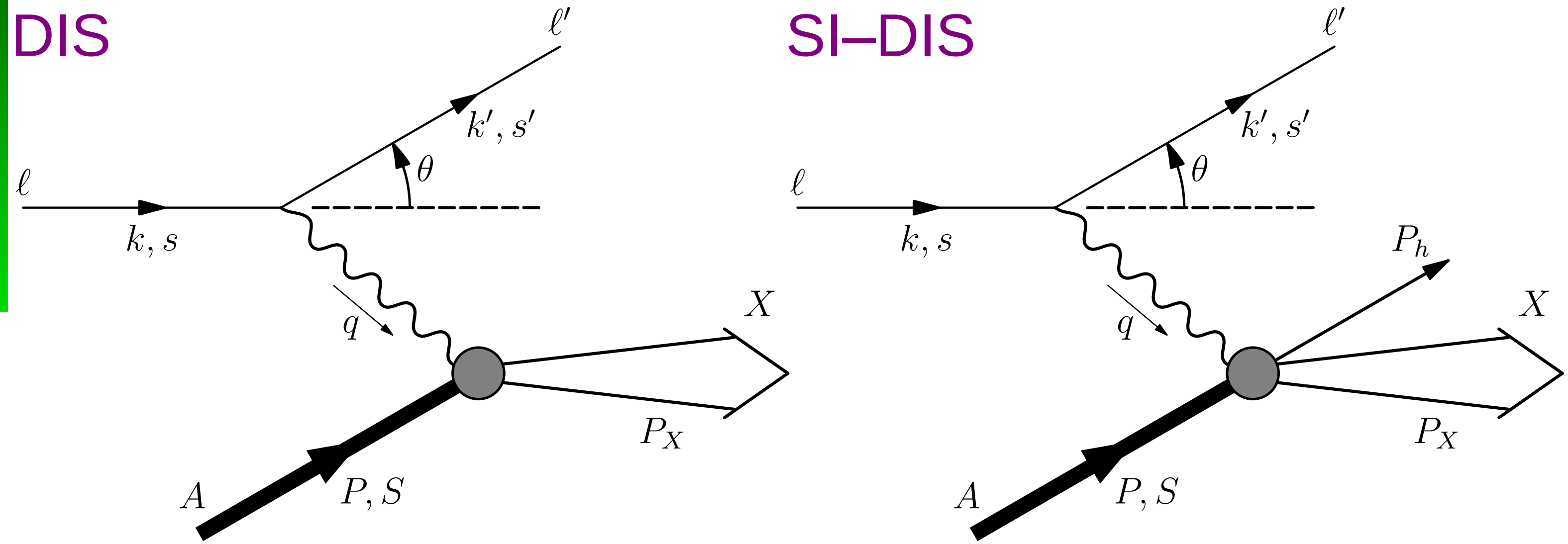


- Three twist-2 parton distributions $(k_\perp = 0)$:
  - Spin-Independent: $q(x)$
  - Helicity: $\Delta q(x)$
  - Transversity: $\Delta_T q(x)$
- All distributions have probability interpretation.
- By definition, contain essentially non-perturbative information about a given process.

# Definition and Sum Rules

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# Definition and Sum Rules

- Light-cone Fourier transforms :

$$\Delta_T q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\xi^-} \langle p,s|\overline{\psi}_q(0)\gamma^+\gamma^1\gamma_5\psi_q(\xi^-)|p,s\rangle_c$$

$$q(x) = \langle\gamma^+\rangle, \qquad \Delta q(x) = \langle\gamma^+\gamma_5\rangle$$

- Related to the nucleon axial & tensor charges via

$$g_A = \int dx[\Delta u(x) - \Delta d(x)], \quad g_T = \int dx[\Delta_T u(x) - \Delta_T d(x)],$$

- Must satisfy: positivity constraints and Soffer bound

$$\Delta q(x), \Delta_T q(x) \leqslant q(x), \quad q(x) + \Delta q(x) \geqslant 2\,|\Delta_T q(x)|$$

# Ian Cloët

# JLab, now ANL

# Ian Cloët

# JLab, now ANL

Once more on the one that got away.

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Cloët, Bentz, Thomas

arXiv:0708.3246 [hep-ph]

# *Model predictions*

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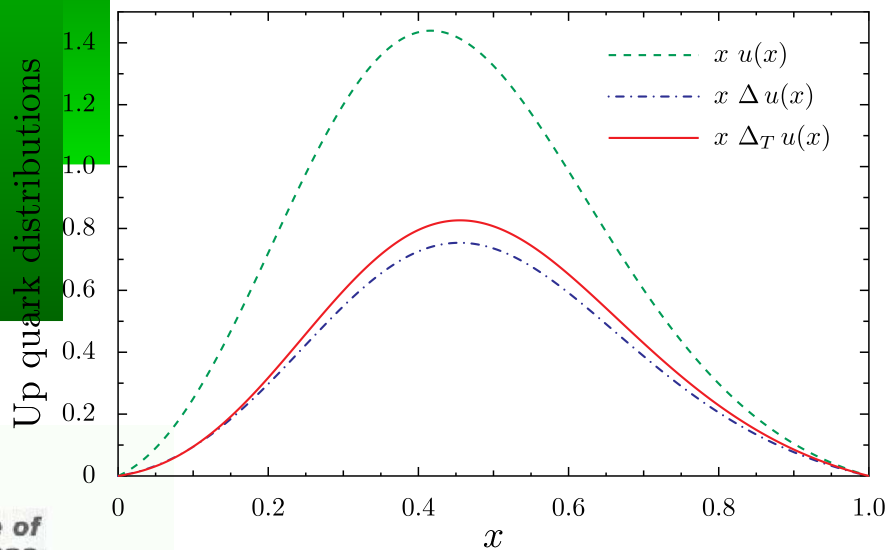
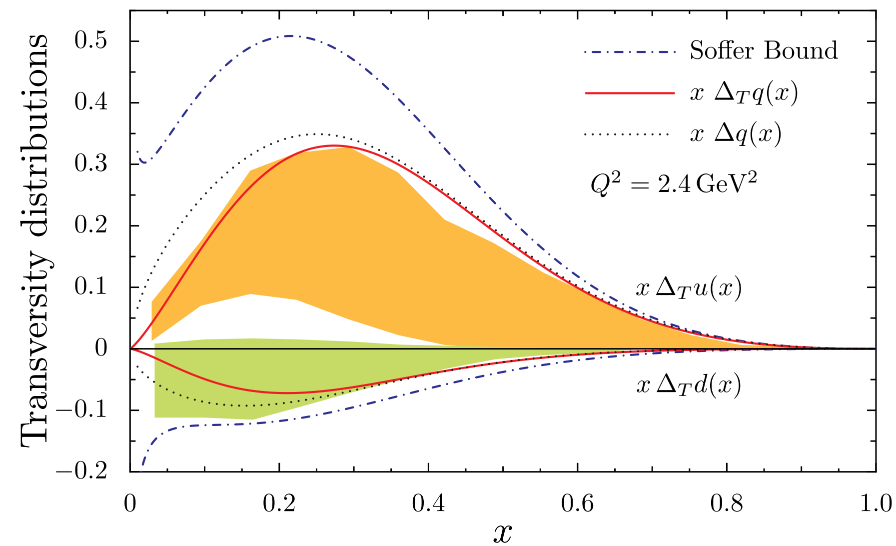
# Model predictions

- Simplified Faddeev equation

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Cloët, Bentz, Thomas

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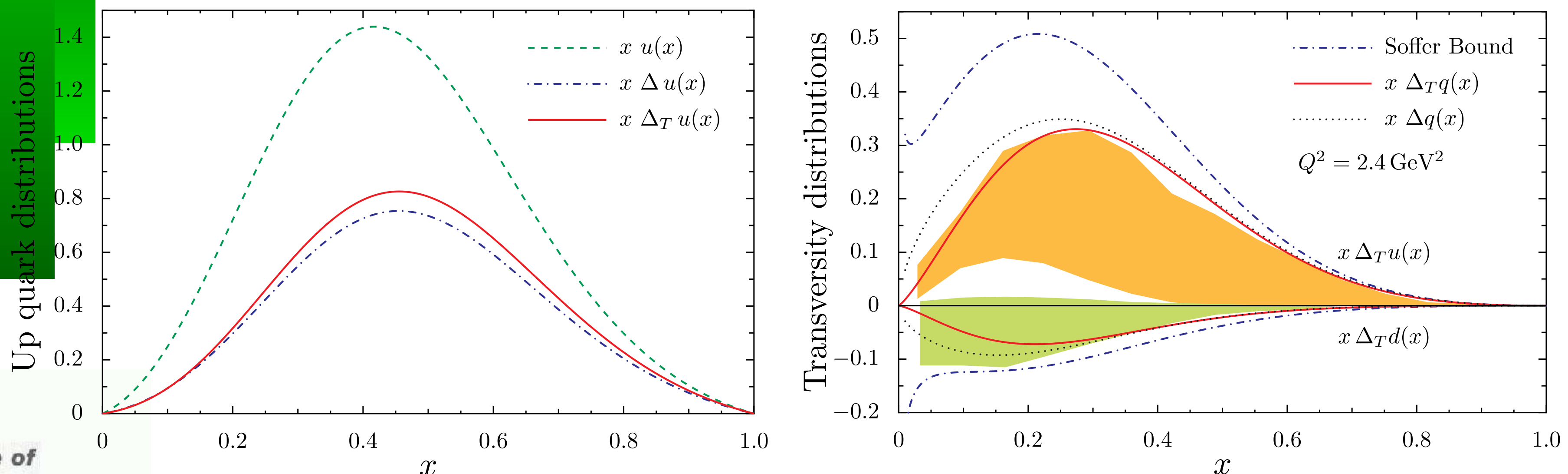


- Satisfy: Soffer bound, baryon & momentum SRs.

- Moments at $Q^2 = 0.16\,$GeV$^2$:

$$\Delta u = 0.97, \quad \Delta d = -0.30 \quad \Longrightarrow \quad g_A = 1.267$$
$$\Delta_T u = 1.04, \, \Delta_T d = -0.24 \quad \Longrightarrow \quad g_T = 1.28$$

Model constraint

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# Model predictions

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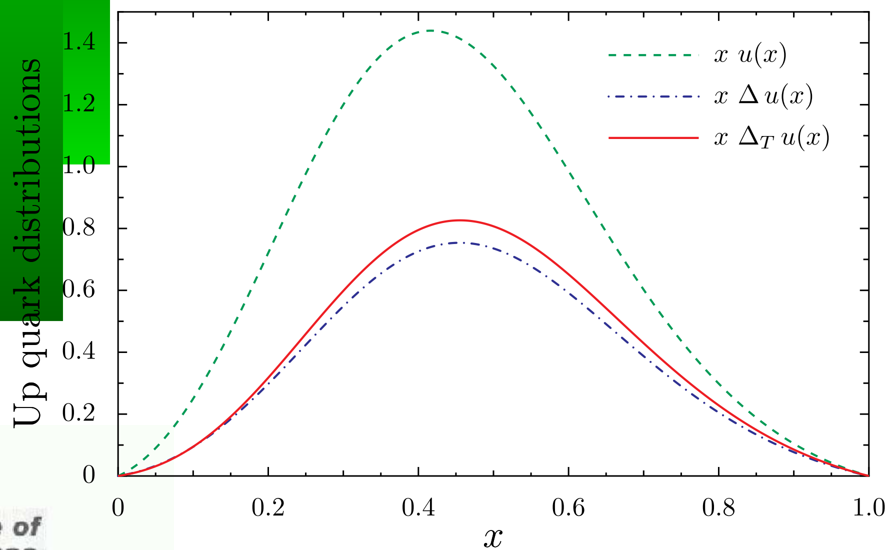


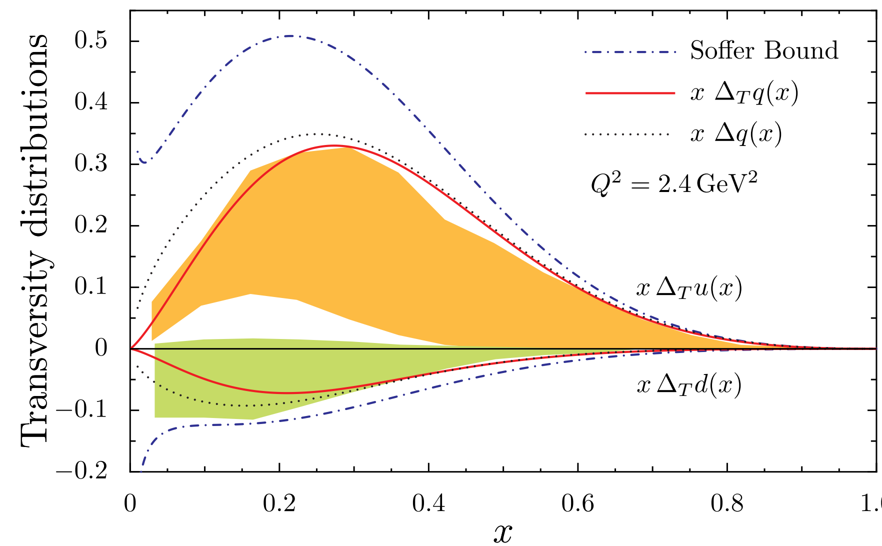


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- $\Delta q(x) \sim \Delta_T q(x)$ in valence region for $Q^2 \lesssim 10\,$GeV$^2$