Dyson-Schwinger Equations and Quantum Chromodynamics

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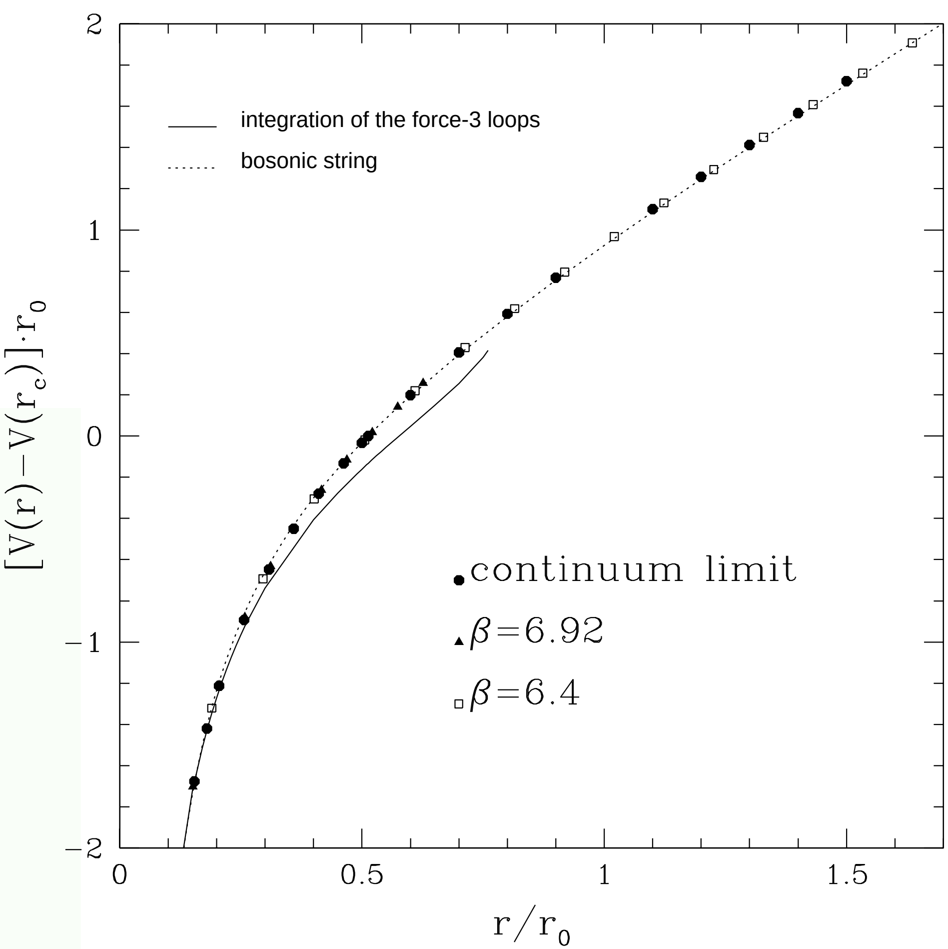
http://www.phy.anl.gov/theory/staff/cdr.html

Confinement

Confinement

- Infinitely Heavy Quarks . . . Picture in Quantum Mechanics



$$V(r) = \sigma\, r - \frac{\pi}{12}\,\frac{1}{r}$$

$\sigma \sim 470$ MeV

Necco & Sommer
he-la/0108008

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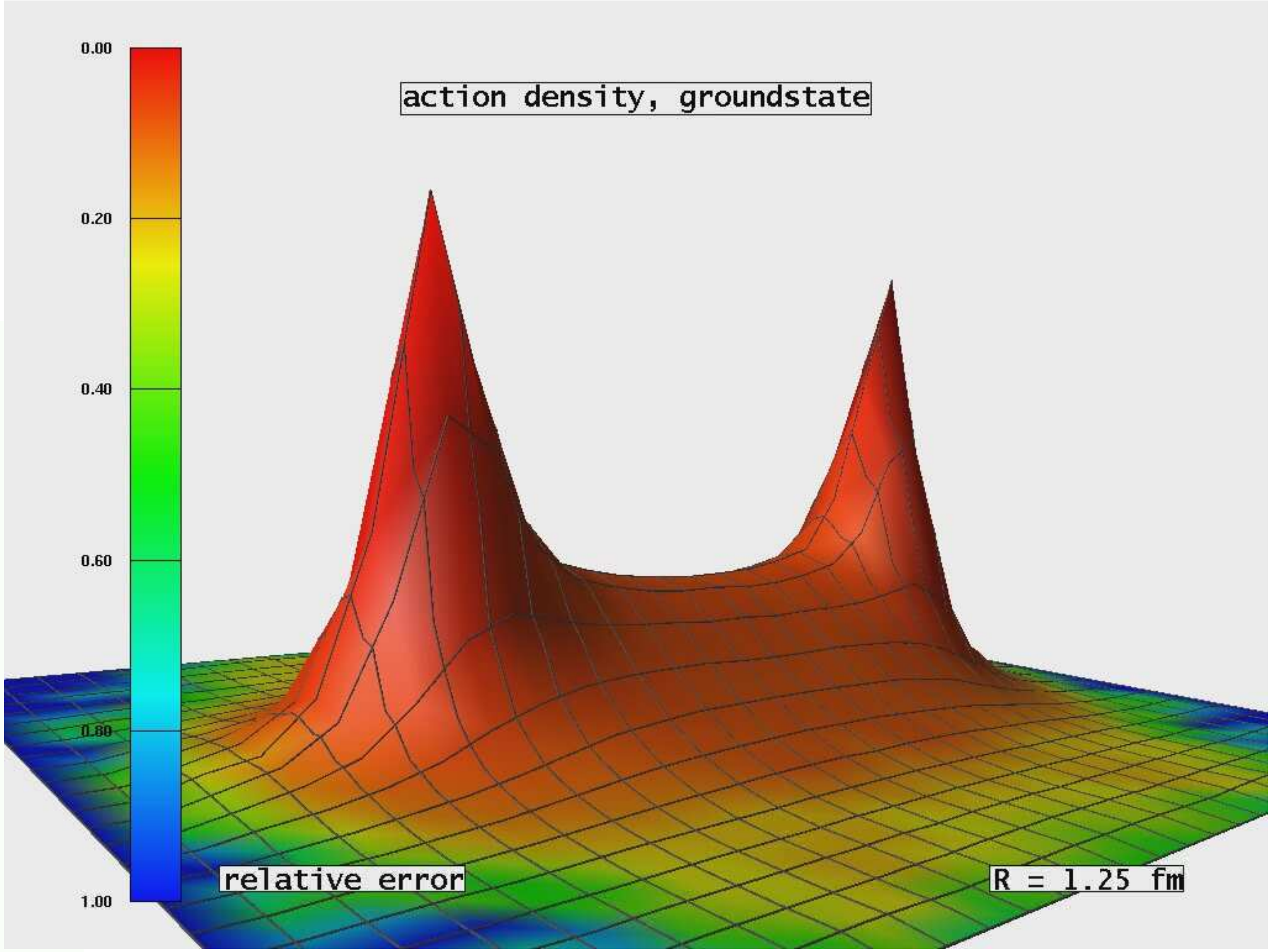
Confinement

- Illustrate this in terms of the action density … analogous to plotting the Force $= F_{\bar{Q}Q}(r) = \sigma + \dfrac{\pi}{12}\,\dfrac{1}{r^2}$



Bali, *et al*.
he-la/0512018

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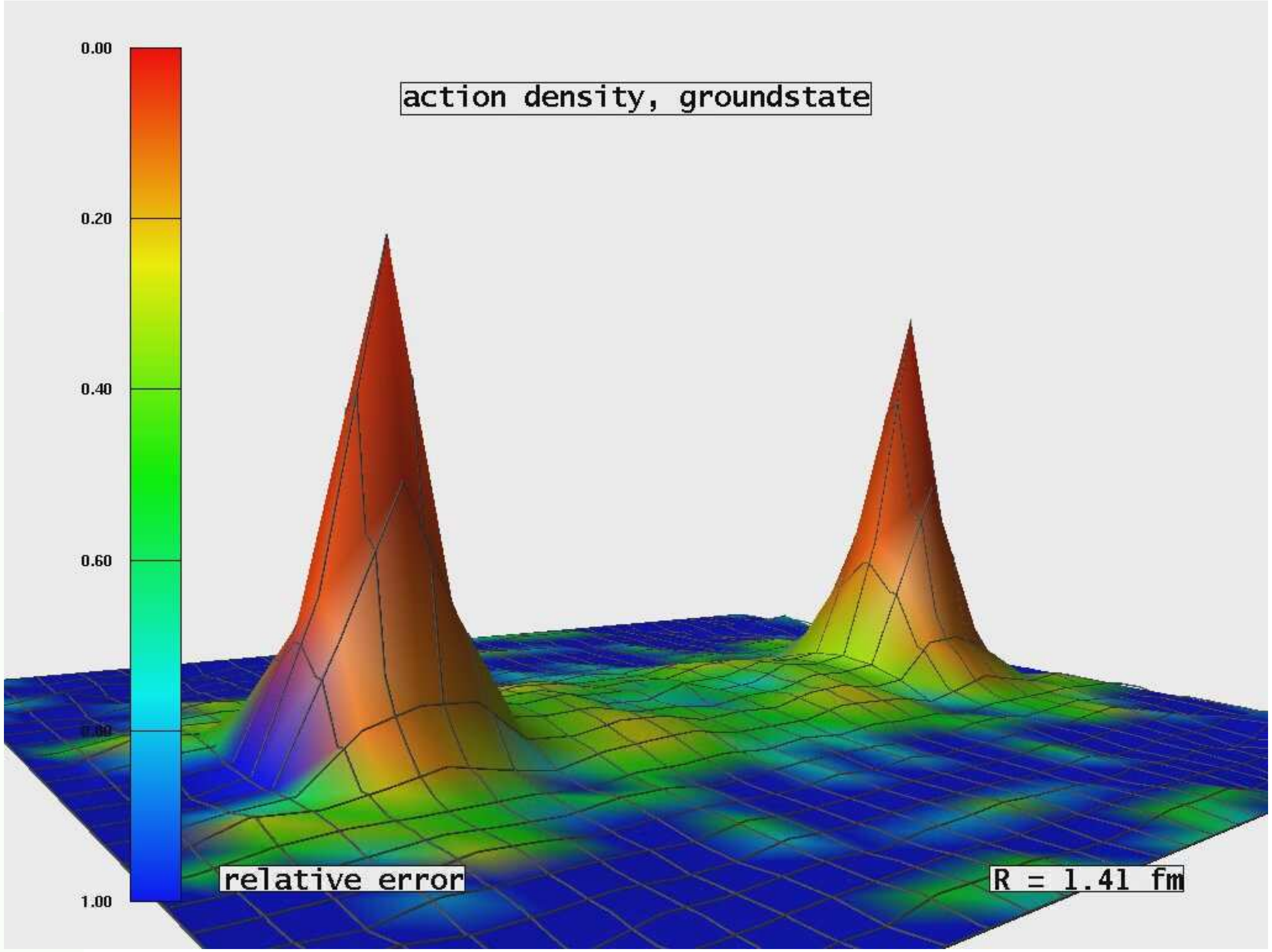

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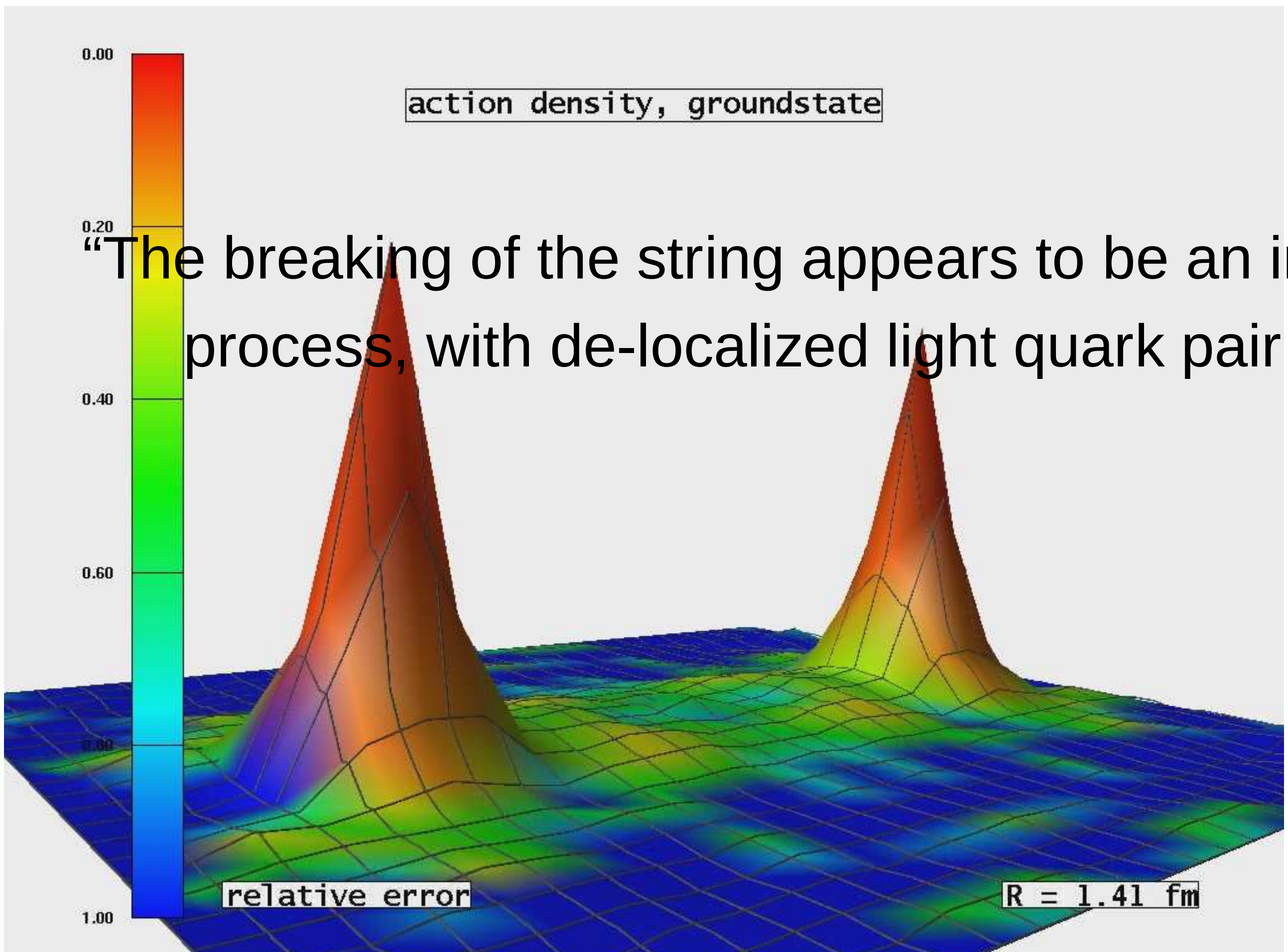

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"The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation."

Confinement

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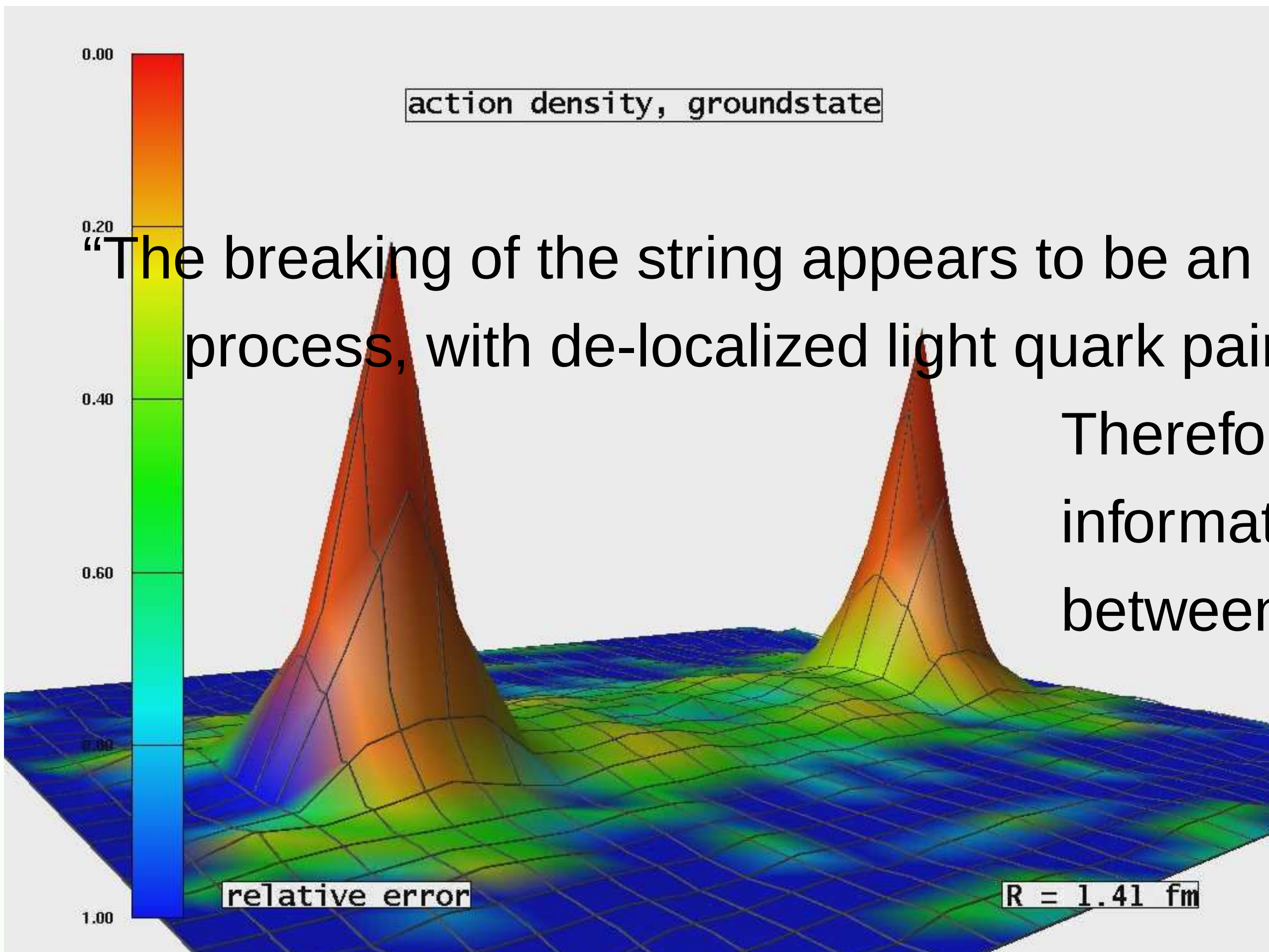

"The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation."

Therefore … No information on *potential* between light-quarks.

Dyson-Schwinger Equations . . .
Continuum Nonperturbative Method

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Continuum Nonperturbative Method

- 1994 . . . “*As computer technology continues to improve, lattice gauge theory [LGT] will become an increasingly useful means of studying hadronic physics through investigations of discretised quantum chromodynamics [QCD]. . . . .*”

- 1994 . . . “*However, it is equally important to develop other complementary nonperturbative methods based on continuum descriptions. In particular, with the advent of new accelerators such as CEBAF and RHIC, there is a need for the development of approximation techniques and models which bridge the gap between short-distance, perturbative QCD and the extensive amount of low- and intermediate-energy phenomenology in a single covariant framework. . . .*”

- 1994 . . . “*Cross-fertilisation between LGT studies and continuum techniques provides a particularly useful means of developing a detailed understanding of nonperturbative QCD.*”

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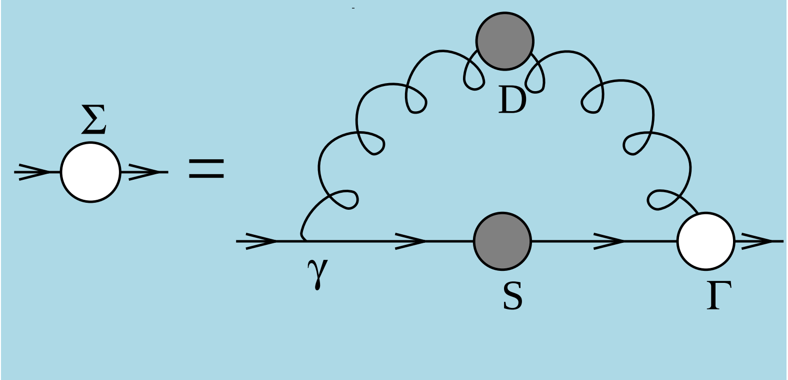

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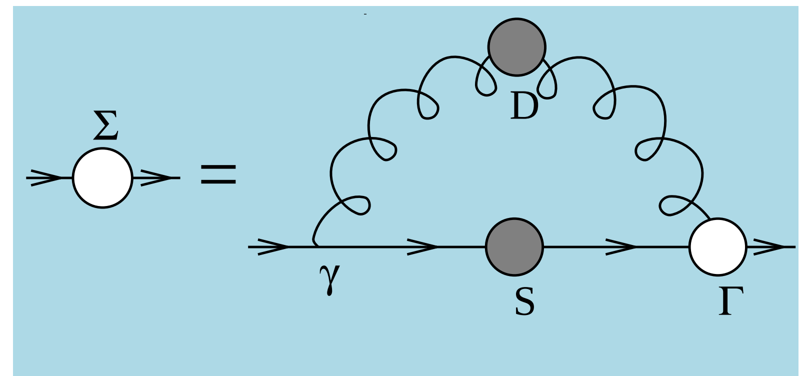

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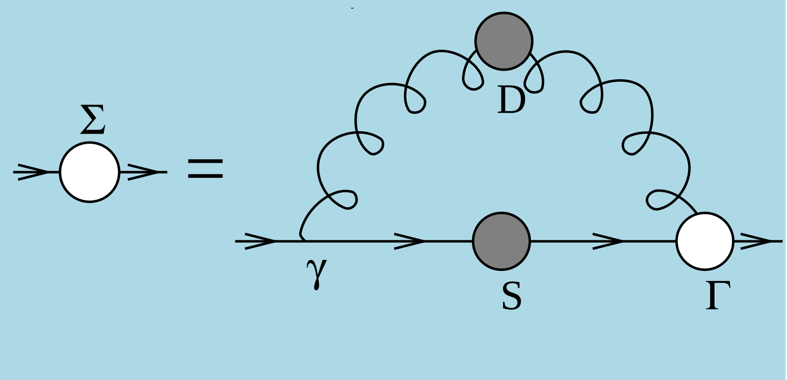

- These are nonperturbative equivalents in quantum field theory to the Lagrange equations of motion.
- Essential in simplifying the general proof of renormalisability of gauge field theories.

Dyson-Schwinger Equations

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- Well suited to Relativistic Quantum Field Theory

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- Simplest level: Generating Tool for Perturbation Theory
  .... Materially Reduces Model Dependence

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      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?

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- $\Rightarrow$ Understanding InfraRed (long-range)

  .... behaviour of $\alpha_s(Q^2)$

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- Method yields Schwinger Functions $\equiv$ Propagators

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Cross-Sections built from Schwinger Functions

Schwinger Functions

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- Proving fruitful.

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World . . .

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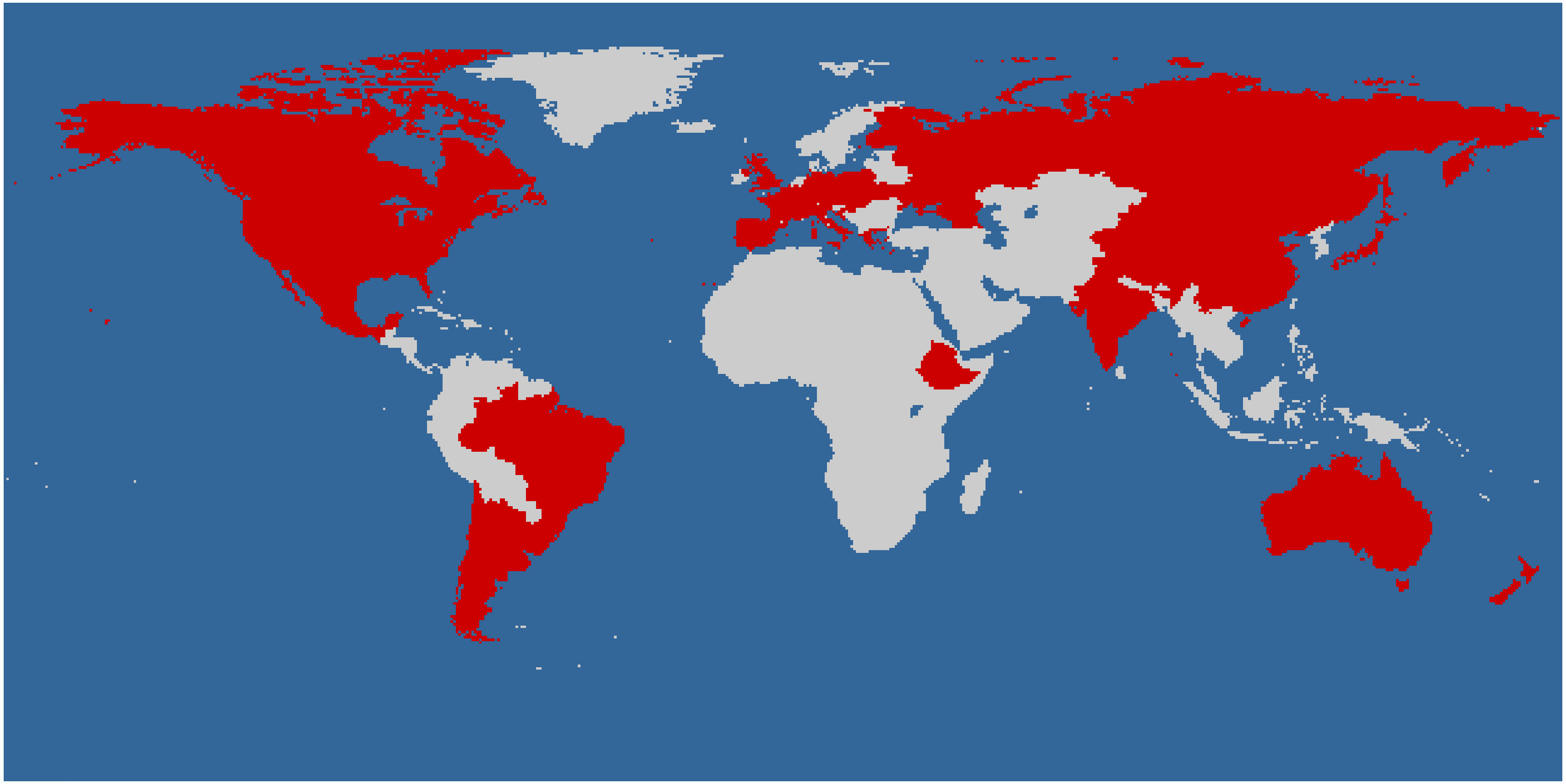
World . . .
DSE Perspective



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Persistent Challenge

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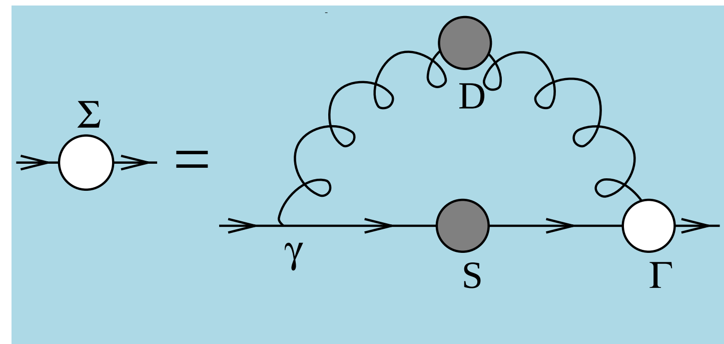
- Infinitely Many Coupled Equations



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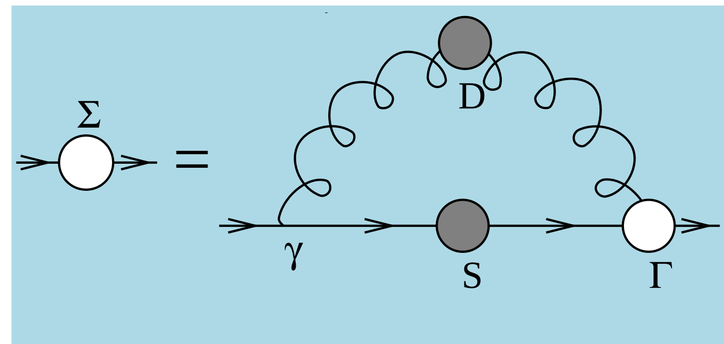

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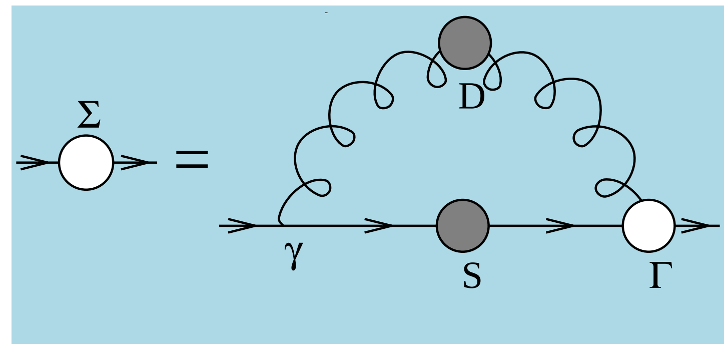

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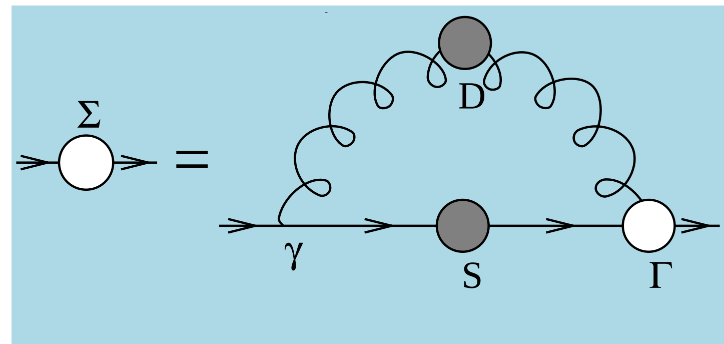

- Coupling between equations necessitates truncation
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    Not useful for the nonperturbative problems
    in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

Persistent Challenge

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- Has Enabled Proof of EXACT Results in QCD

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  - Make Predictions with Readily Quantifiable Errors

Perturbative Dressed-quark Propagator

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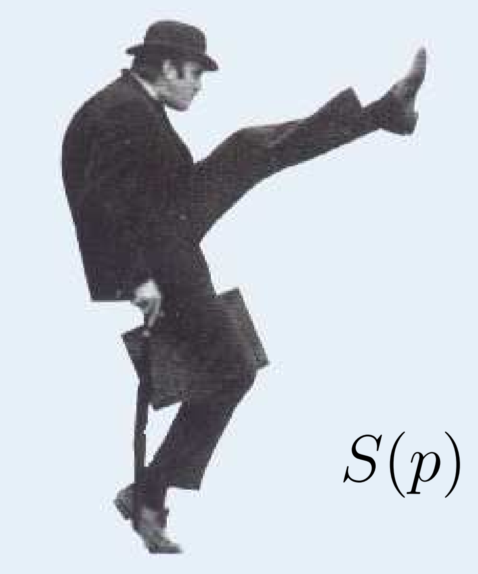
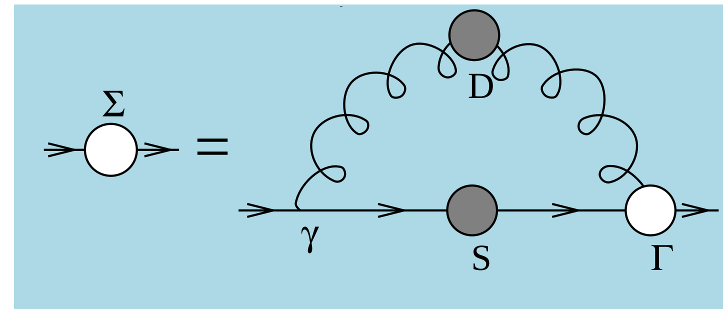
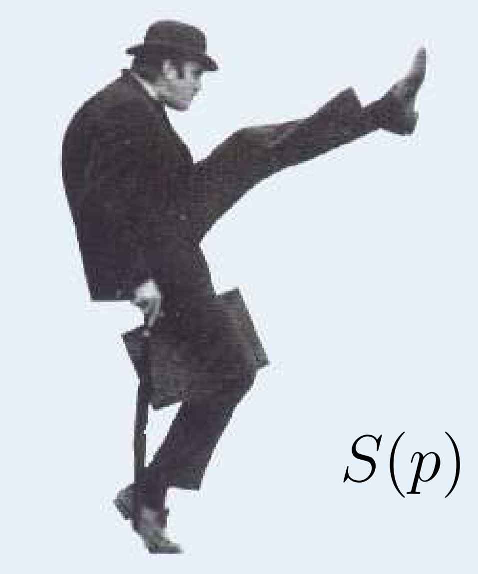


Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$



Gap Equation



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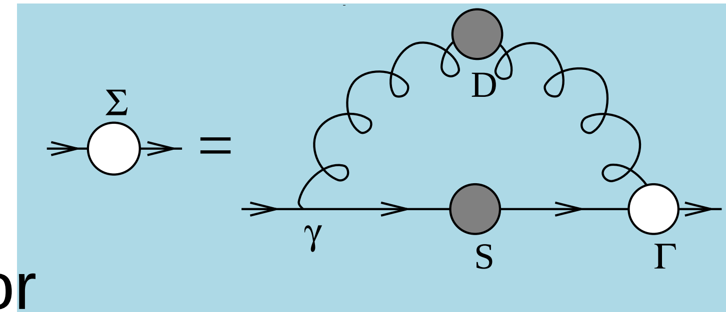

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$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

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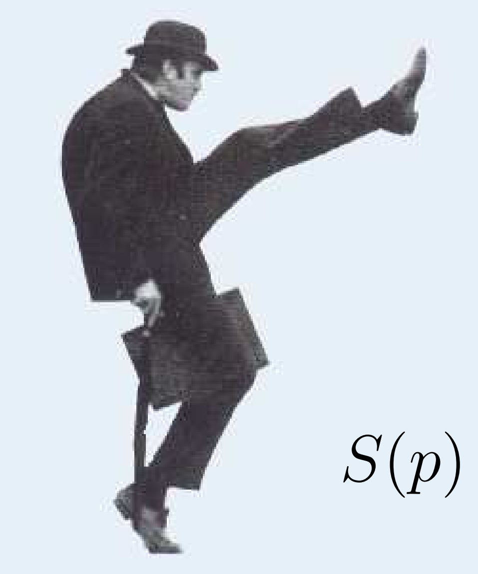

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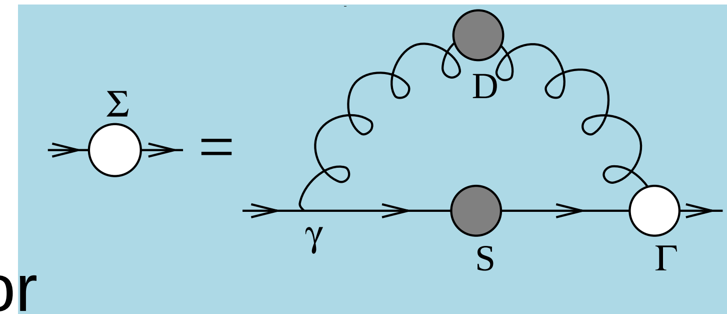

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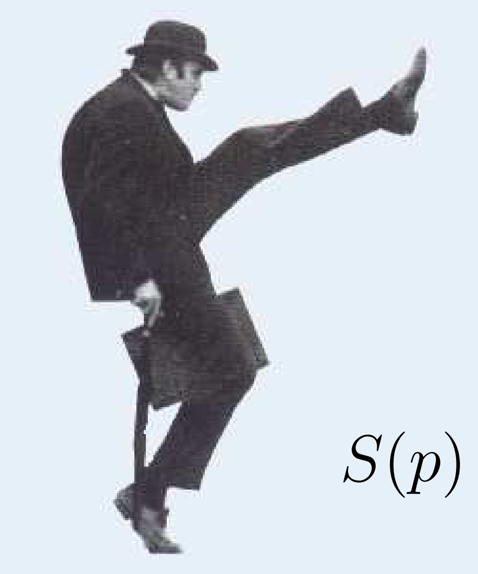

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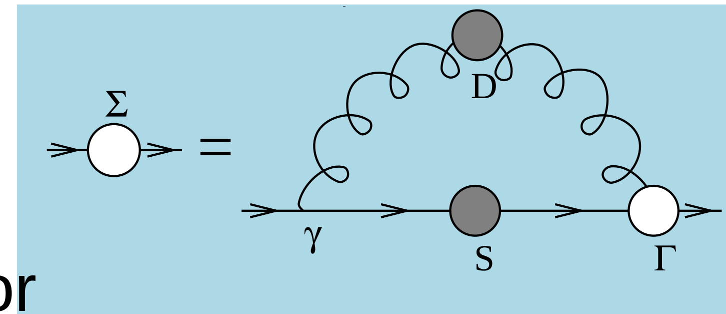

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$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\to} 0$$

Perturbative
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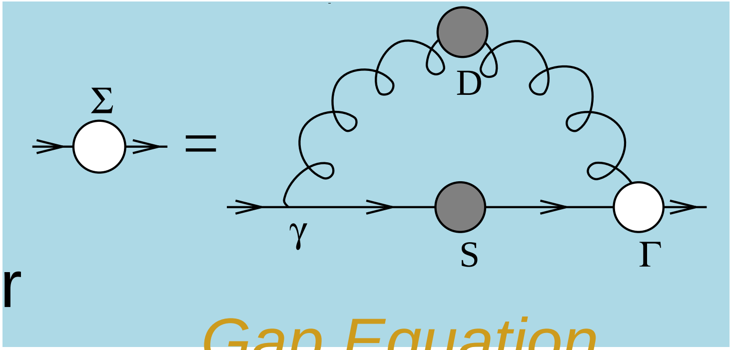

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No DCSB Here!

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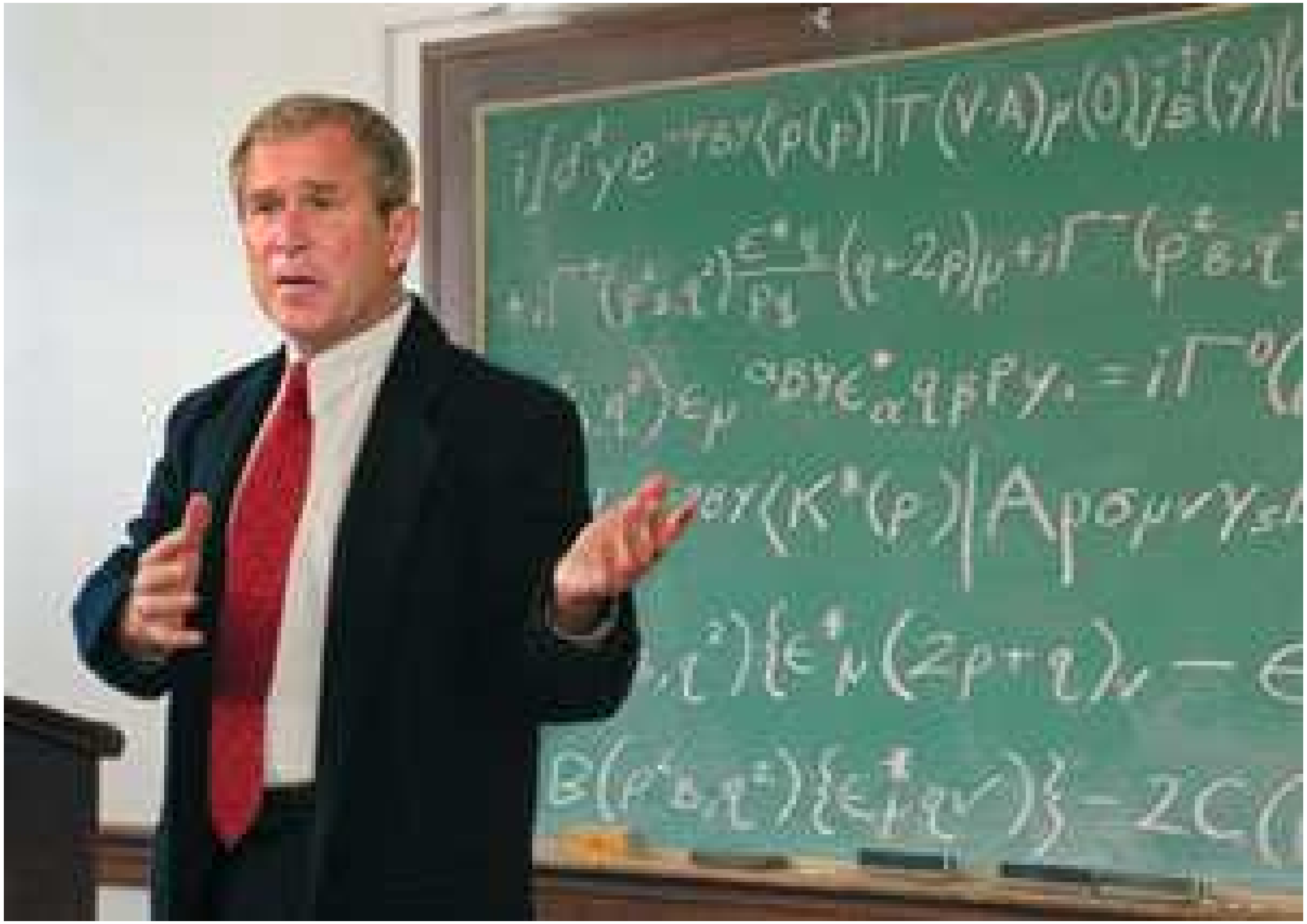
Explanation?



QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
  - R. Alkofer and L. von Smekal, *The infrared behavior of QCD Green's functions* . . . , Phys. Rept. **353**, 281 (2001).

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- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
  - D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD Collaboration], *Asymptotic scaling and infrared behavior of the gluon propagator*, Phys. Rev. D **60**, 094507 (1999) [Erratum-ibid. D **61**, 079901 (2000)].
- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

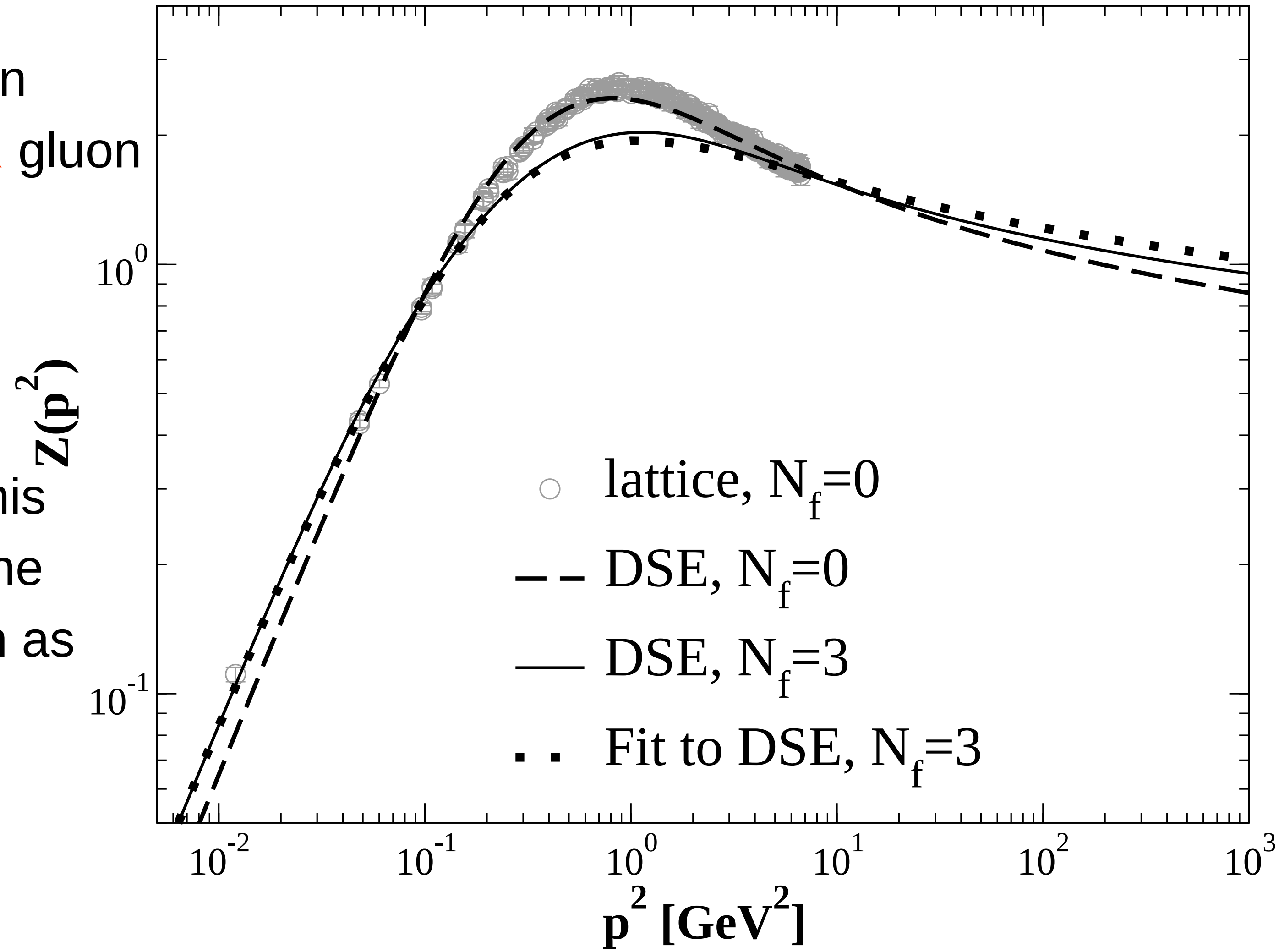
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx$ 1 GeV

- Naturally, this scale has the same origin as $\Lambda_{\rm QCD}$



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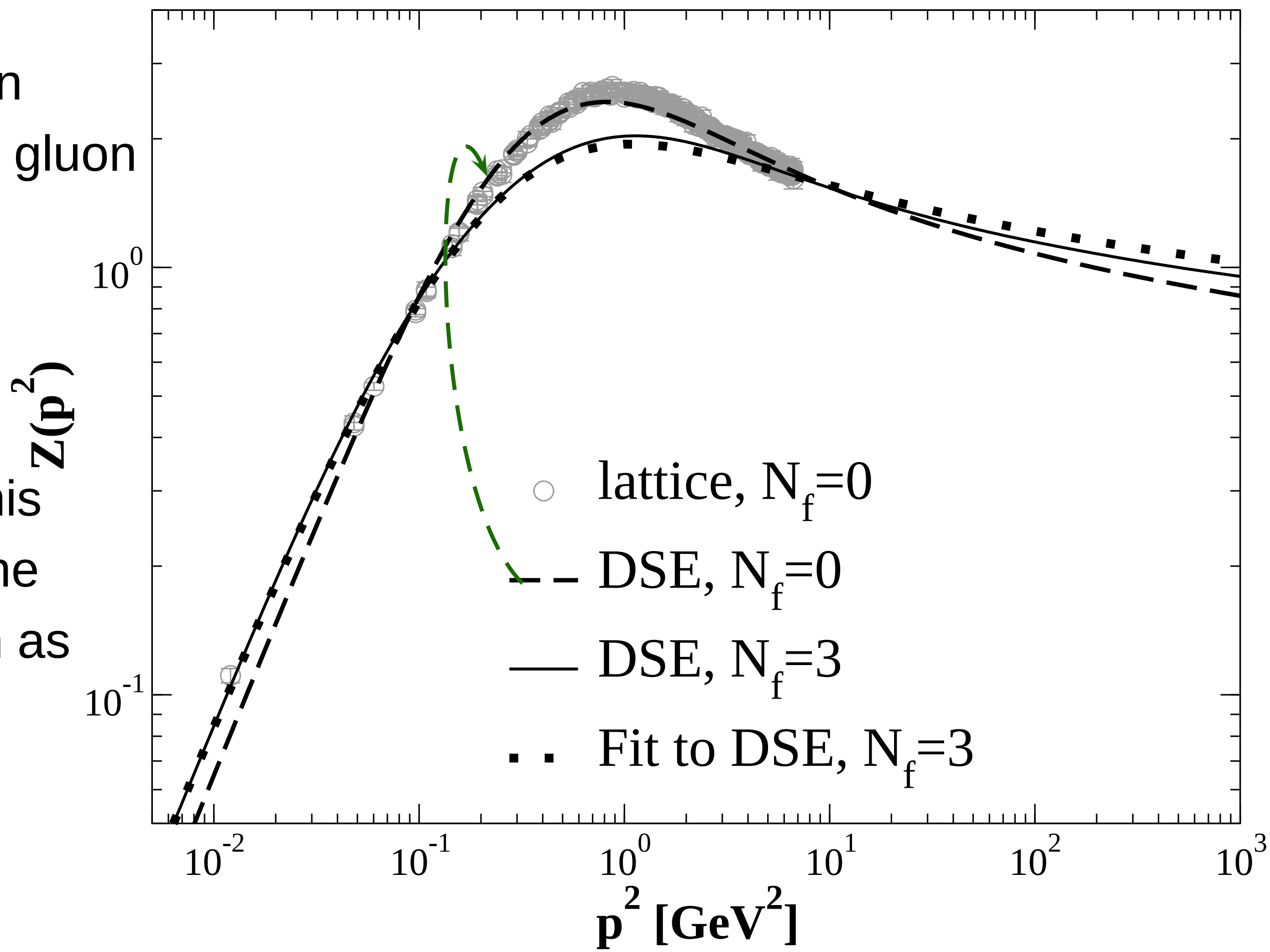

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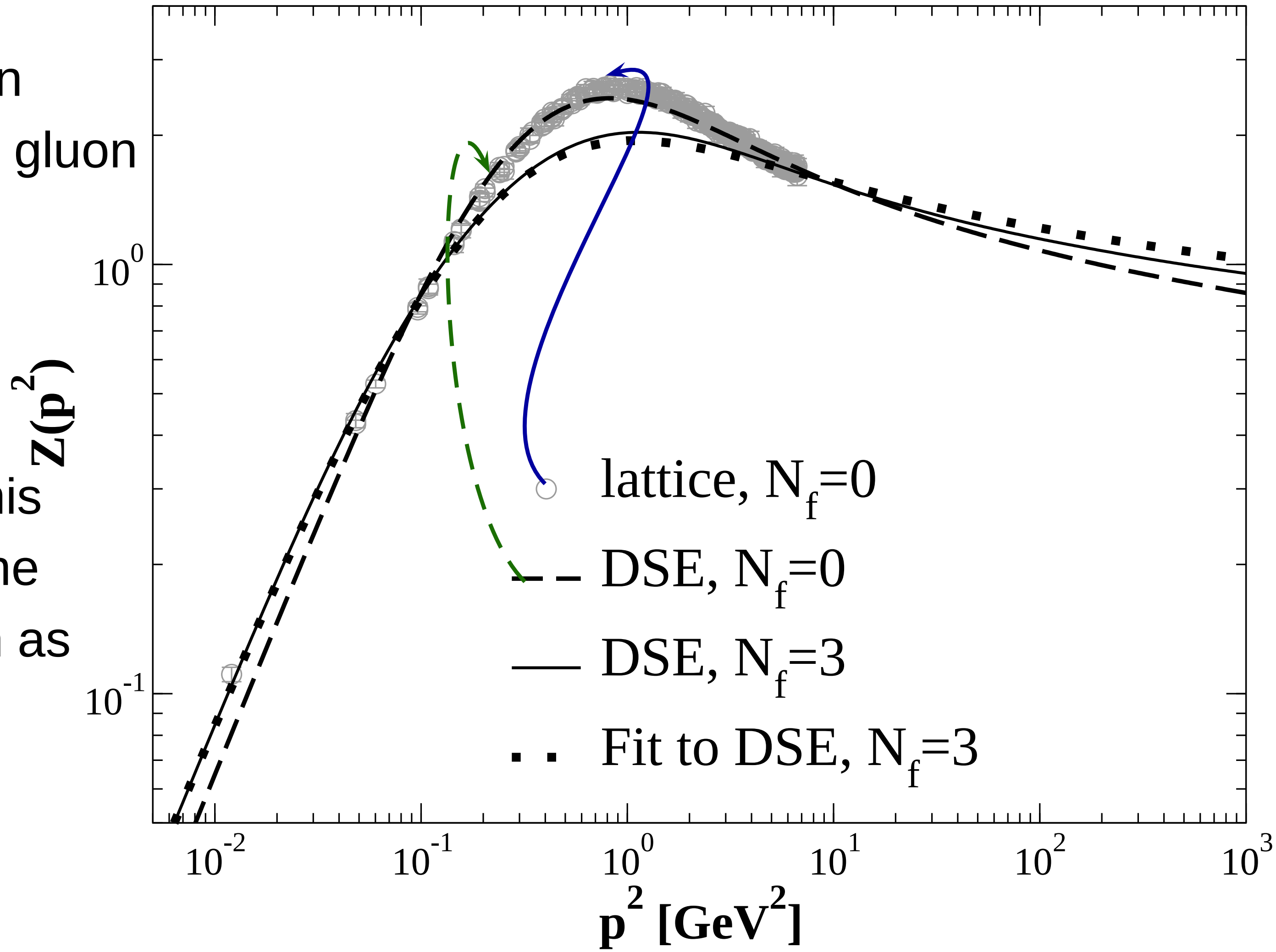

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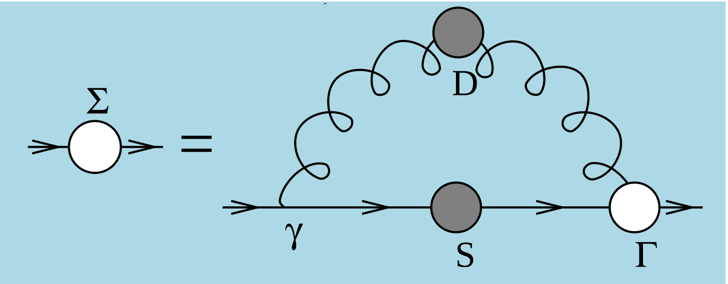

Gap Equation

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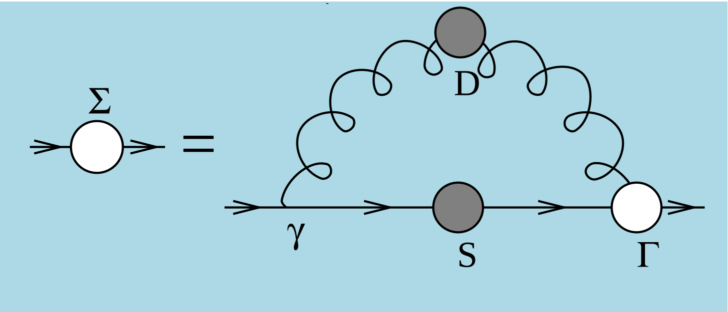
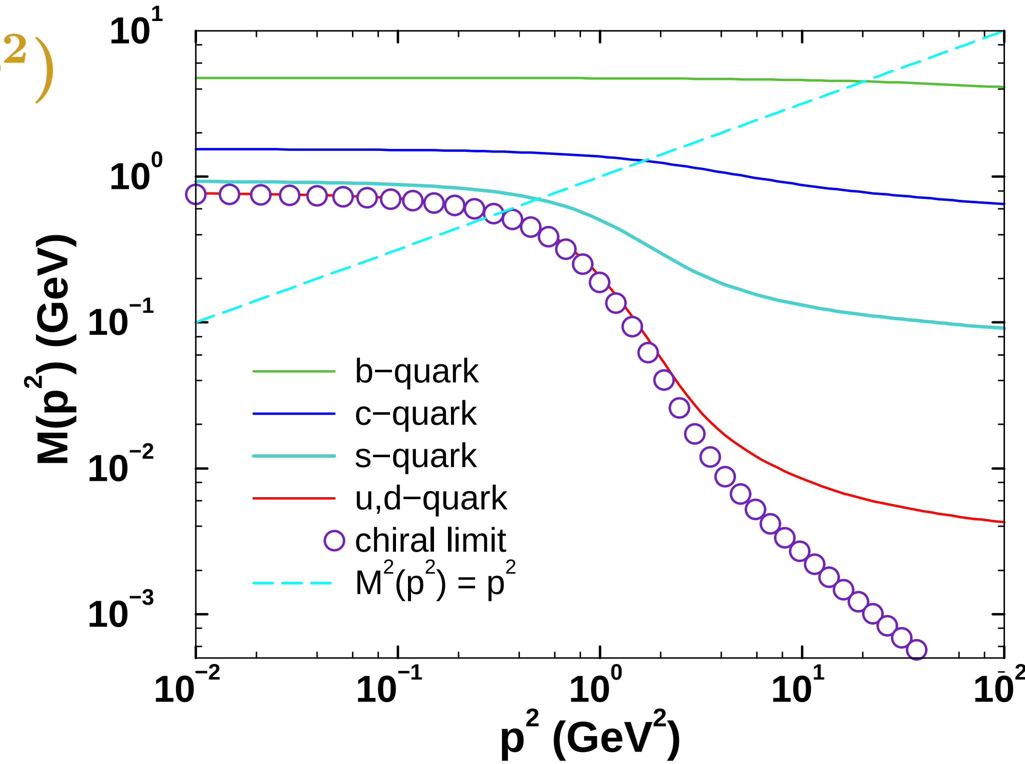


Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



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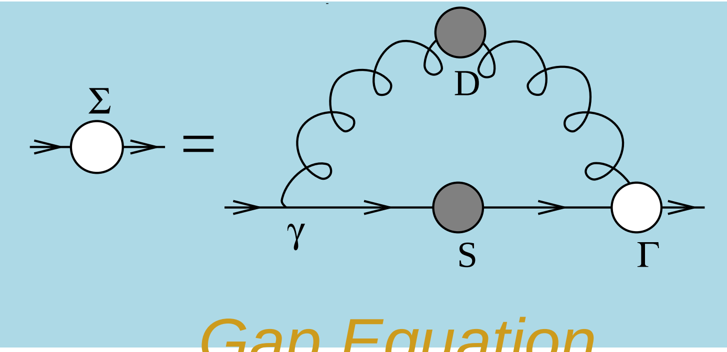

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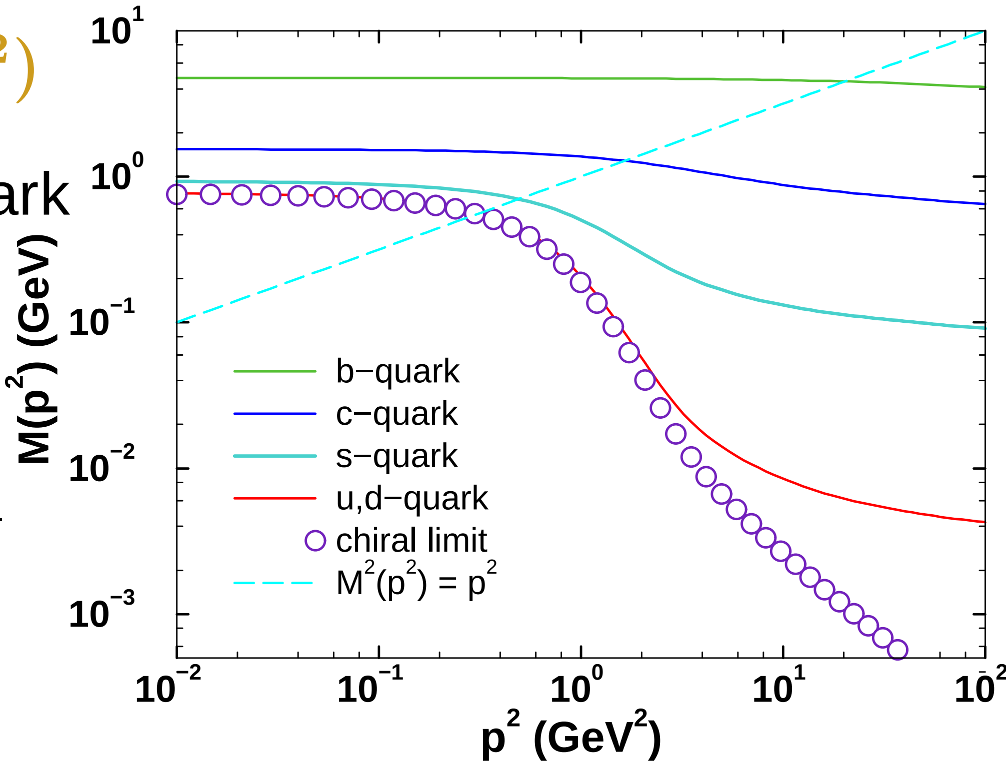
flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



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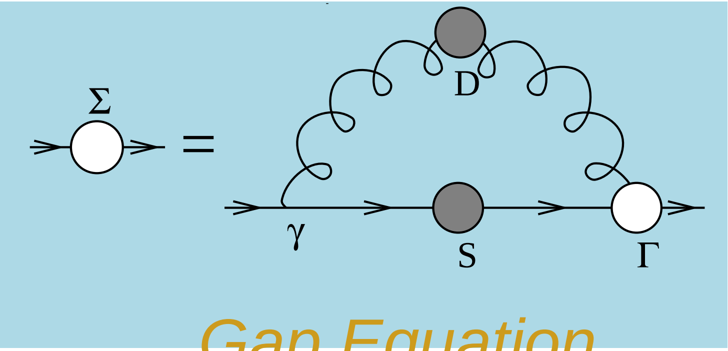

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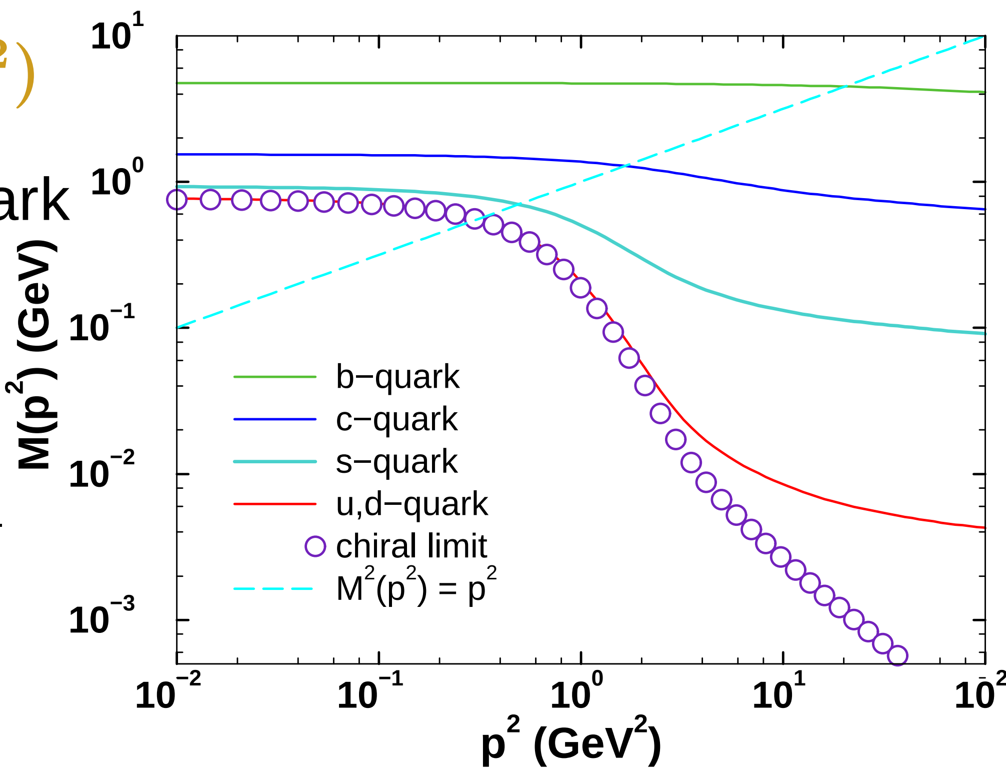

Predictions confirmed in numerical simulations of lattice-QCD

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Dressed-Quark Propagator

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- Longstanding Prediction of Dyson-Schwinger Equation Studies

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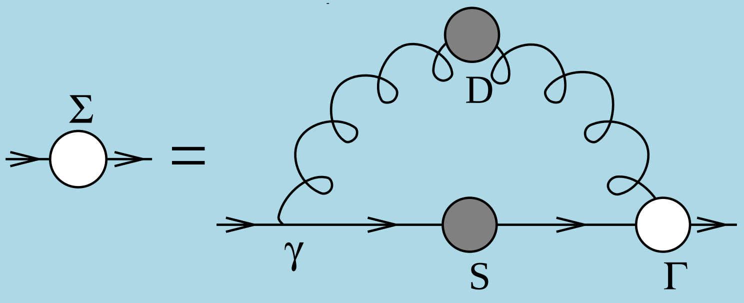
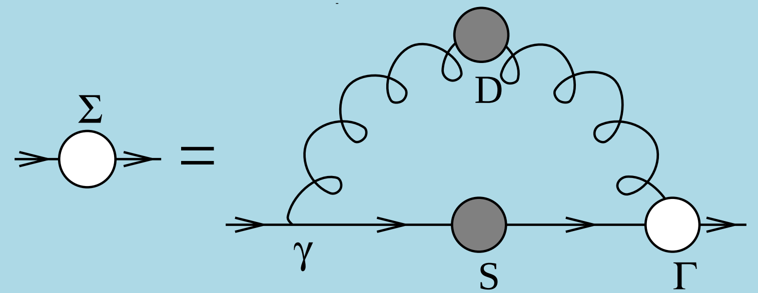
Frontiers of Nuclear Science: A Long Range Plan (2007)



Gap Equation



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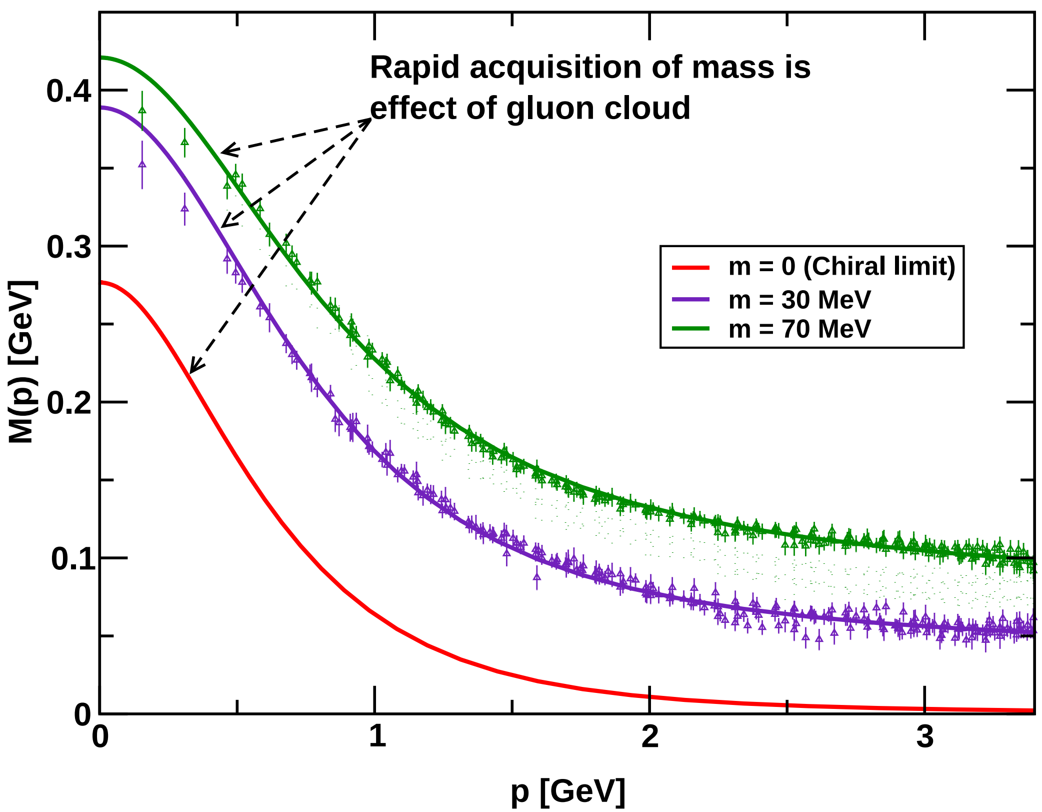

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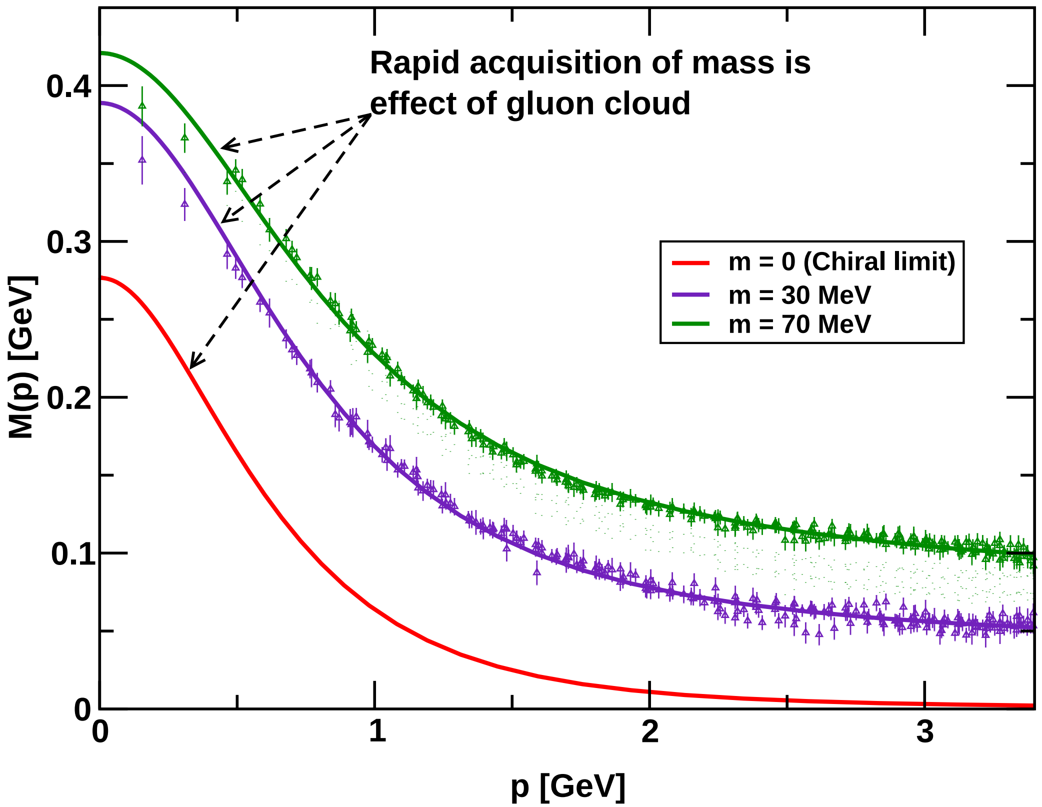
Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.



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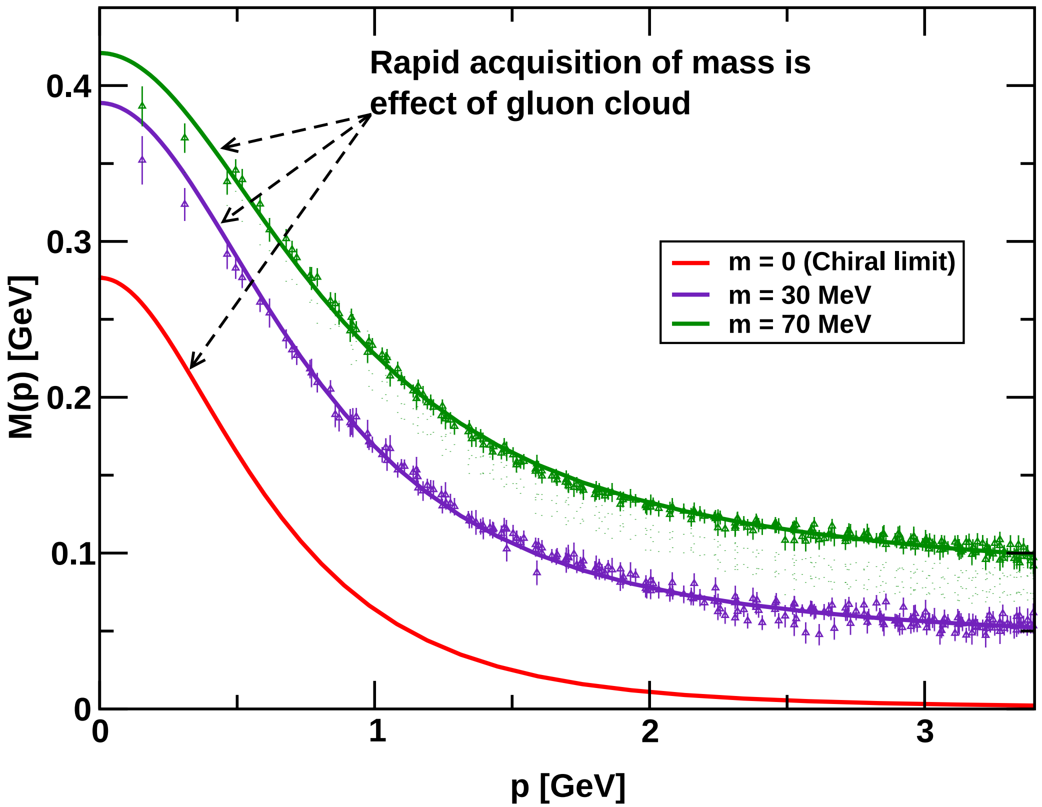


ALWAYS LOOK ON THE BRIGHT SIDE

- Established understanding of two- and three-point functions

Hadrons

- Established understanding of two- and three-point functions
- What about bound states?

Hadrons

- Without bound states, Comparison with experiment is impossible

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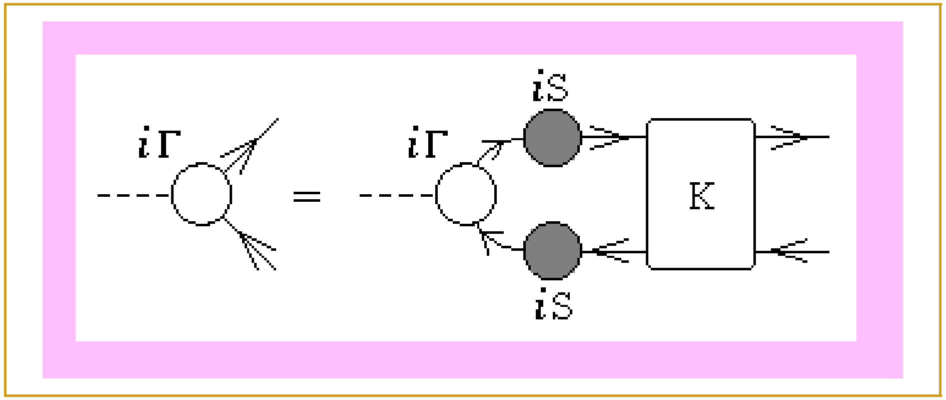
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

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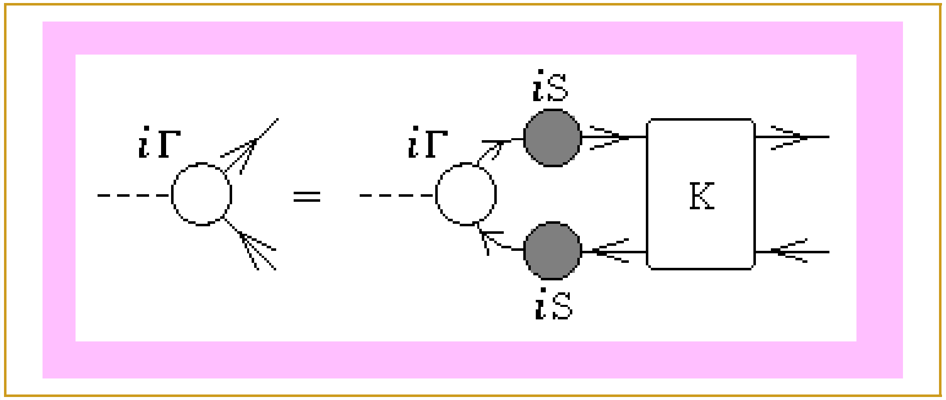

QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

or

What is the light-quark Long-Range Potential?

What is the light-quark Long-Range Potential?

Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

Bethe-Salpeter Kernel

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- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

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Kernels very different

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- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Goldstone's Theorem

- In the chiral limit the QCD Action possesses chiral symmetry
- The chiral limit is a good approximation in QCD for u- and d-quarks
- If this $SU(N_f = 2)$ chiral symmetry is dynamically broken, then there is a massless composite particle associated with each generator of chiral transformations; i.e., three Goldstone Bosons
- These three Goldstone Bosons have long been identified with the pions: $\boldsymbol{\pi}^+, \boldsymbol{\pi}^0, \boldsymbol{\pi}^-$

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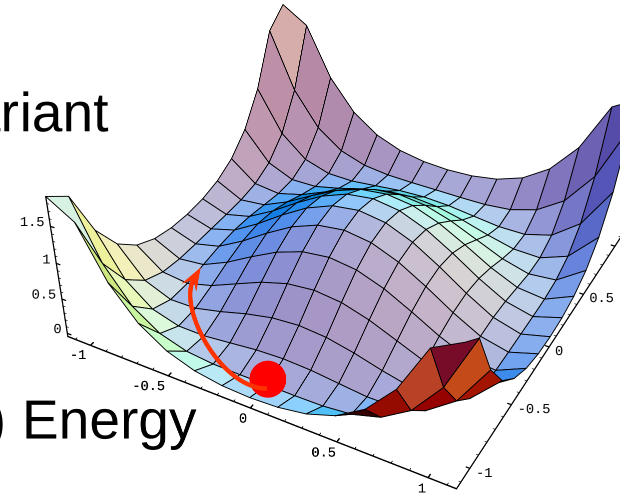
  E.g., $V(x,y) = (\sigma^2 + \pi^2 - 1)^2$

  – Hamiltonian: $T + V$, is Rotationally Invariant

  Ground State

  - Ball at any (σ, π) for which $\sigma^2 + \pi^2 = 1$
    - All Positions have Same (Minimum) Energy
    - But not invariant under rotations



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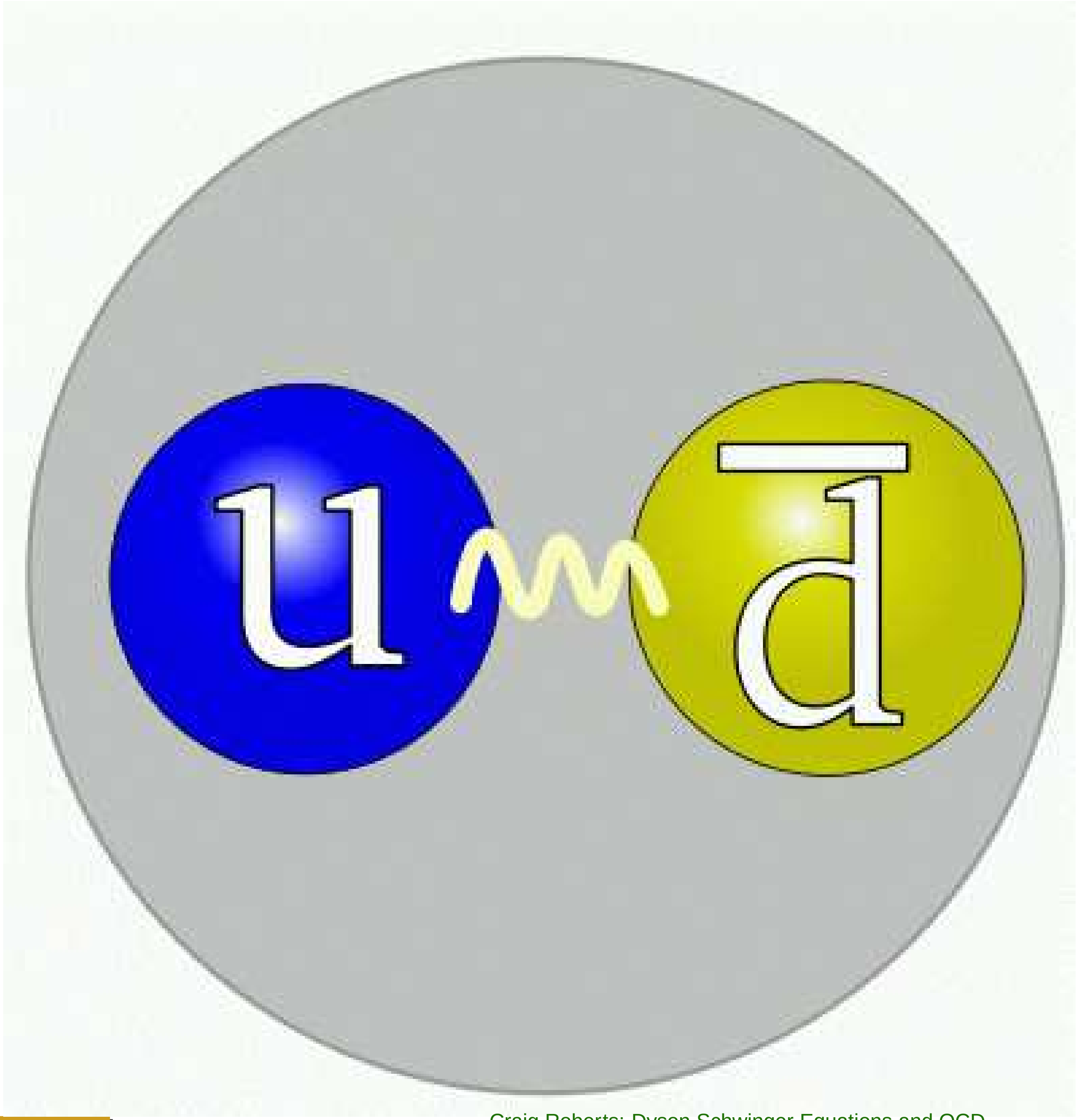
- If one assumes the s-quark is also light; namely, assumes that $SU(N_f = 3)$ chiral symmetry is a good approximation, then the kaons are four more Goldstone Bosons

Pion and . . .
Pseudoscalar Mesons?



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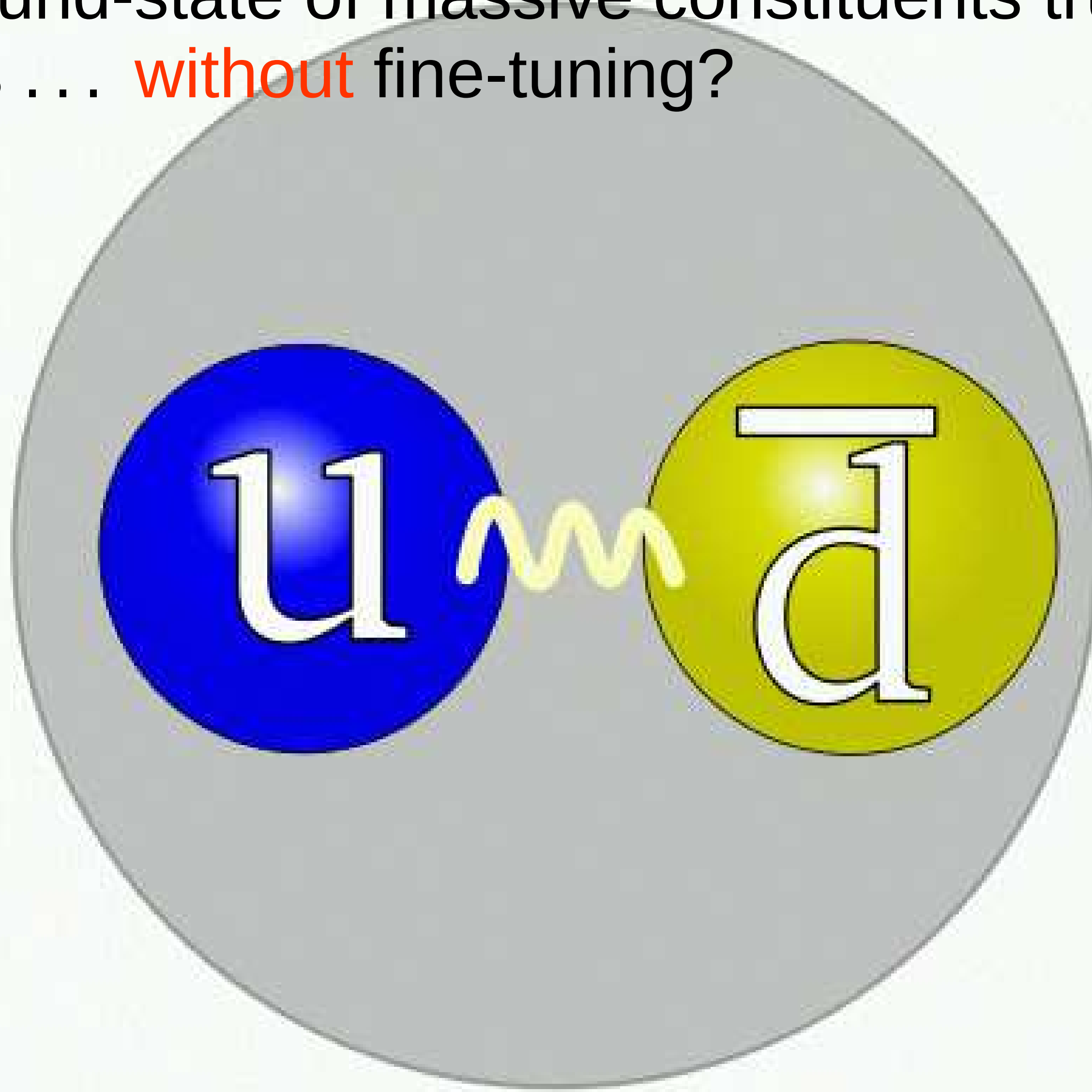
Can a bound-state of massive constituents truly be massless . . . without fine-tuning?



Dichotomy of Pion
– *Goldstone Mode* and *Bound state*

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– Goldstone Mode and Bound state

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Must exhibit $\boxed{m_\pi^2 \propto m_q}$

Current Algebra . . . 1968

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requires an approach to contain a

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Highly Nontrivial

Resolving the Dichotomy

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.

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- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
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- Satisfying these requirements enables
  - Proof of numerous exact results for pseudoscalar mesons
  - Formulation of reliable models
    - To illustrate those results
    - Make predictions of observables with quantifiable errors

Goldberger-Treiman for pion

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\begin{aligned}\Gamma_{\pi^j}(k;P) &= \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P)\right.\\ &\left.+ \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]\end{aligned}$$

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Pseudovector components necessarily nonzero

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$\Rightarrow$ Exact in Chiral QCD

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Radial Excitations & Chiral Symmetry

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(Maris, Roberts, Tandy nu-th/9707003)

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Radial Excitations & Chiral Symmetry

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- Mass^2 of pseudoscalar hadron

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Radial Excitations & Chiral Symmetry

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

$$\mathcal{M}_H := \mathrm{tr}_{\text{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

(Maris, Roberts, Tandy nu-th/9707003)

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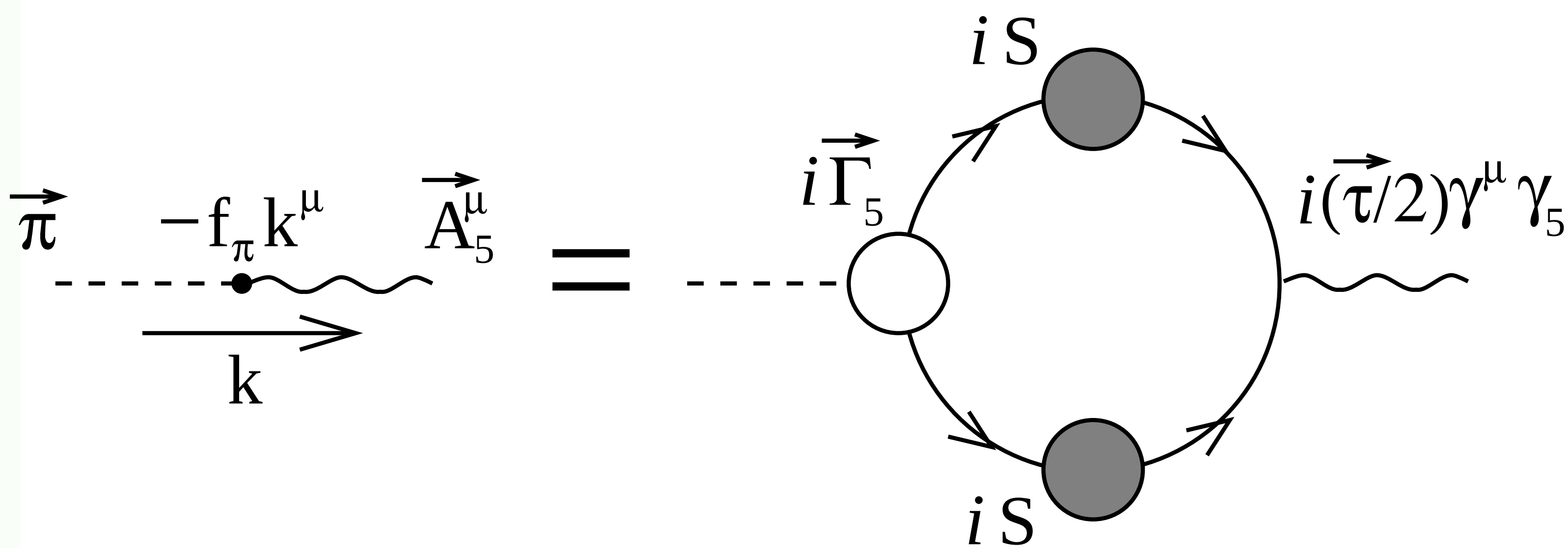
$$f_H \, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2} \mathrm{tr} \left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5 \gamma_\mu \boxed{\mathcal{S}(q_+) \Gamma_H(q;P) \mathcal{S}(q_-)} \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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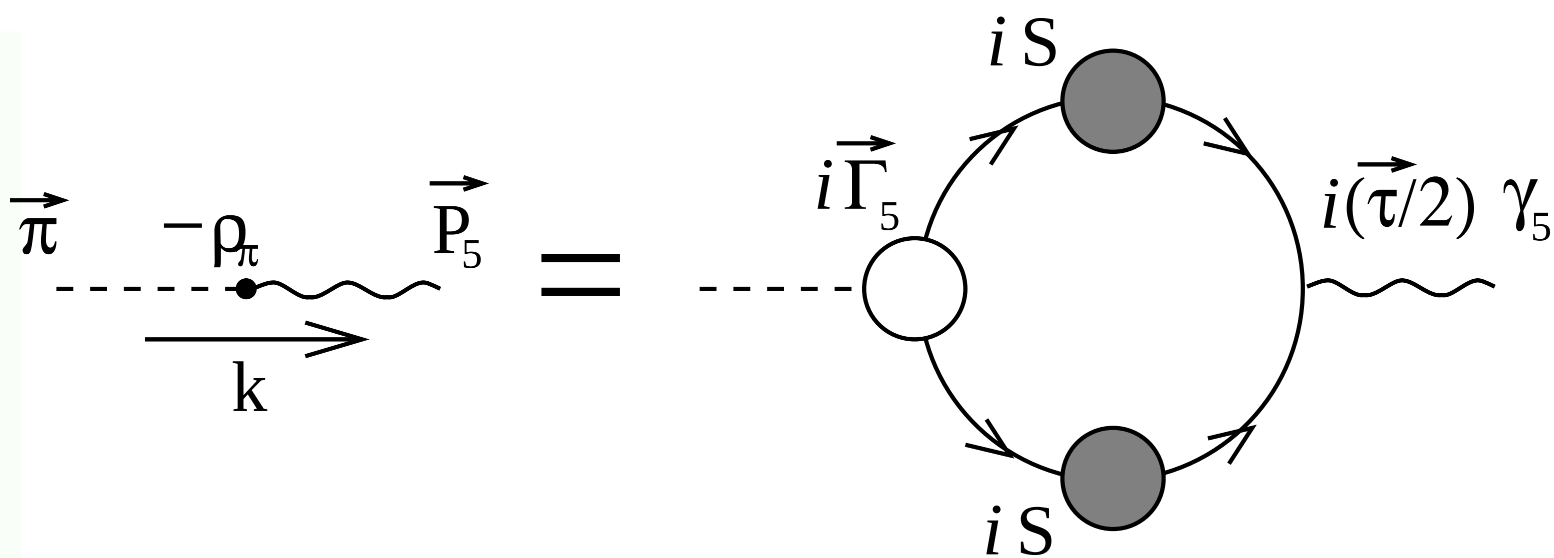

Radial Excitations & Chiral Symmetry

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- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $\boxed{m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q}$ ...GMOR relation, a corollary

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- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $\boxed{m_H \propto m_q}$

. . . QCD Proof of Potential Model result

Radial Excitations

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Radial Excitations

- Spectrum contains 3 pseudoscalars $[I^G(J^P)L = 1^-(0^-)S]$

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

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- But $\pi(1800)$ is narrow ($\Gamma = 207 \pm 13$) & decay pattern might indicate some "flux tube angular momentum" content:
  $S_{\bar{Q}Q} = 1 \oplus L_F = 1 \Rightarrow J = 0$
  & $L_F = 1 \Rightarrow\ {}^3S_1 \oplus\ {}^3S_1$ ($\bar{Q}Q$) decays suppressed?

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Höll, Krassnigg, Roberts
nu-th/0406030

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- $\Rightarrow \boldsymbol{f_H = 0}$

  ALL pseudoscalar mesons except $\boldsymbol{\pi(140)}$ in chiral limit

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- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson, not the ground state, so $\boldsymbol{m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0}$, in chiral limit
- $\Rightarrow \boldsymbol{f_H = 0}$
  ALL pseudoscalar mesons except $\boldsymbol{\pi(140)}$ in chiral limit
- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

McNeile and Michael
he-la/0607032

Radial Excitations & Lattice-QCD

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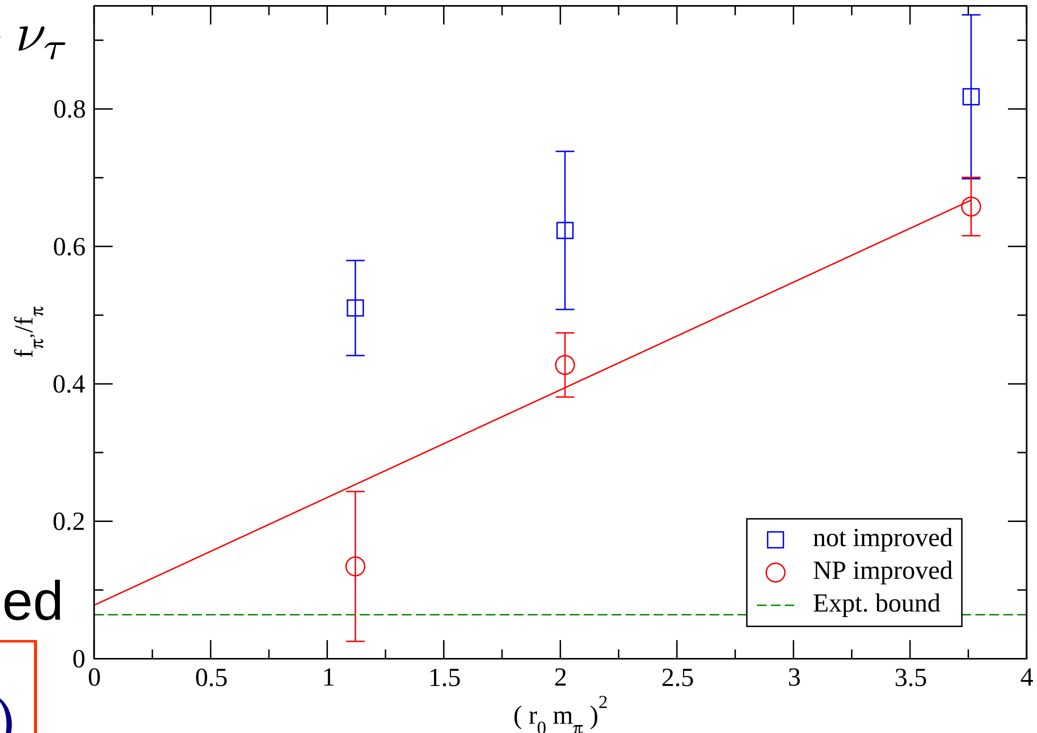
$$\Rightarrow \boxed{\frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)}$$



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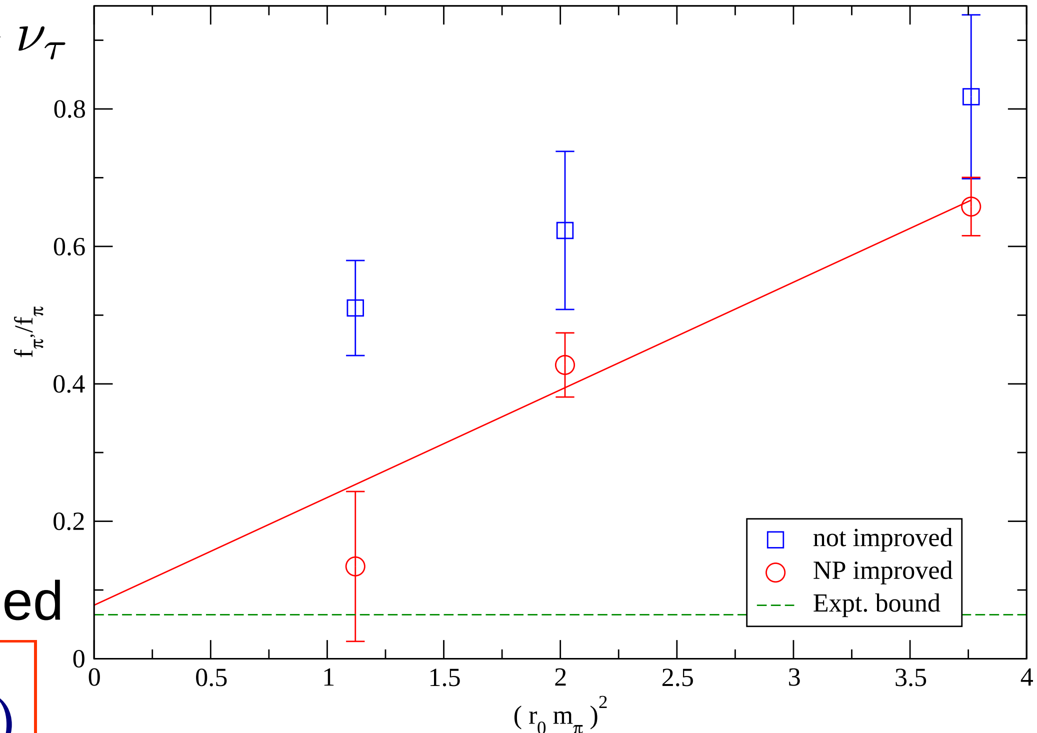

- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

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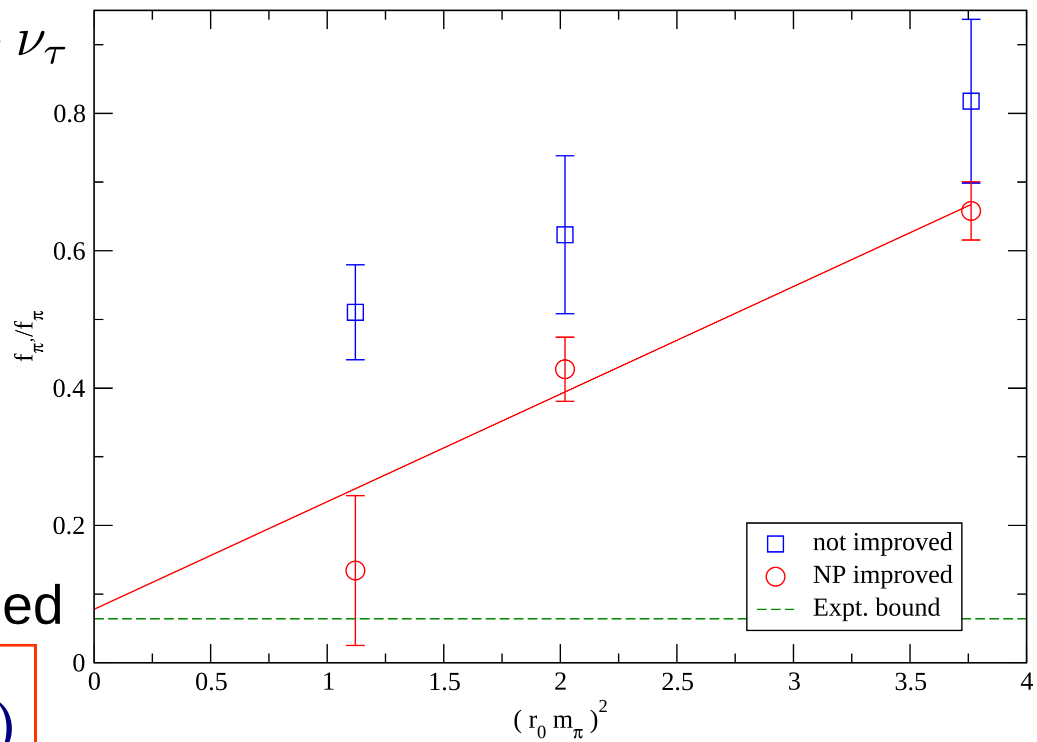

- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Pion . . . $J = 0$
but . . .

Pseudoscalar meson Bethe-Salpeter amplitude

$$\begin{aligned}\chi_\pi(k;P) &= \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P \mathcal{F}_{\pi_n}(k;P) \right. \\ &\quad \left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P) \right]\end{aligned}$$

First Contents Back Conclusion

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- Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.

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- Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.

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- In QCD, a Poincaré invariant theory with interactions, $\mathcal{E} \neq 0$ forces nonzero results for $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$
- $J = 0 \ldots$ *but* while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both S- and P-wave components.

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Introduce mixing angle θ_π such that

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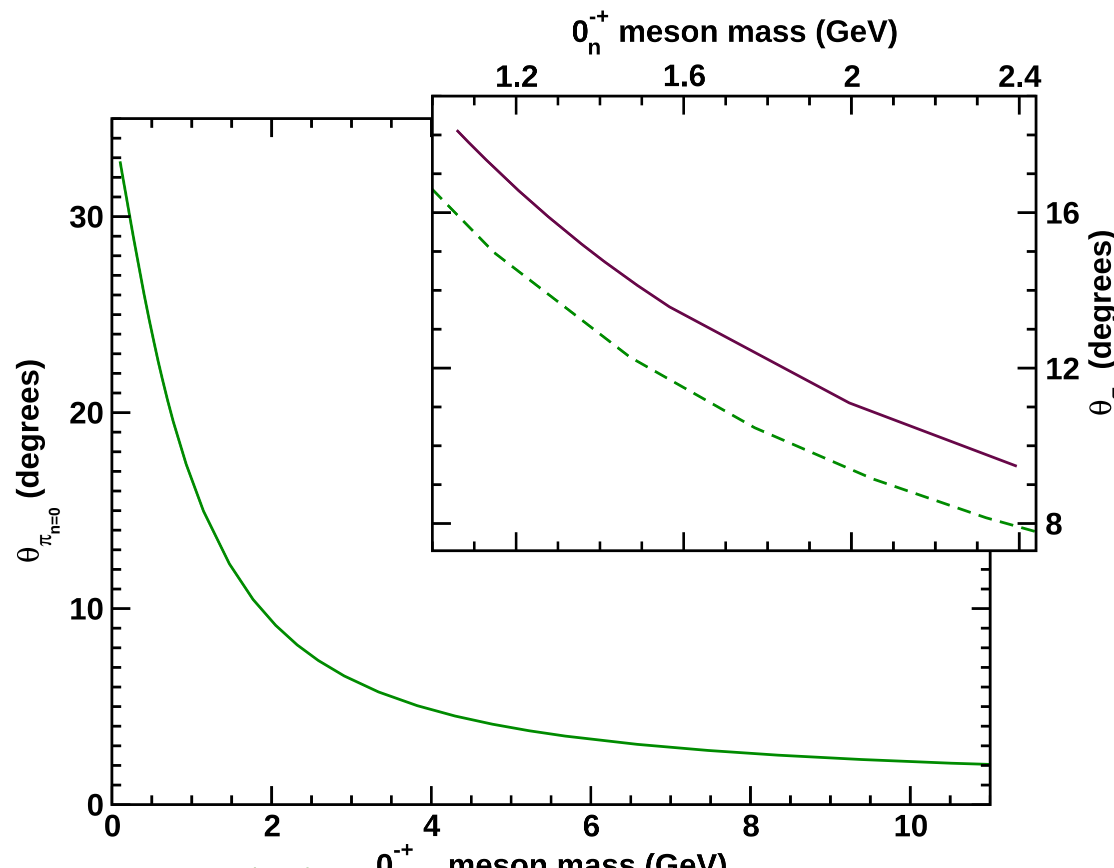

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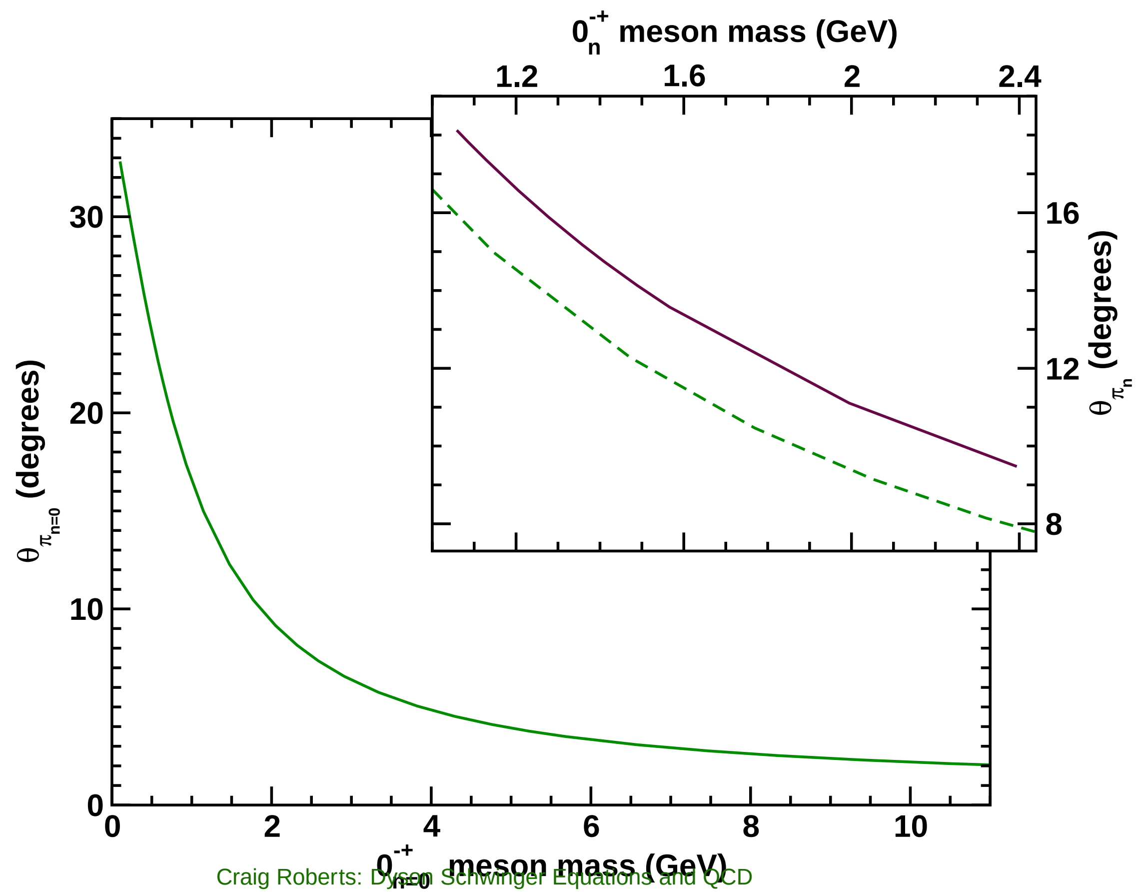
L is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.



Charge Neutral Pseudoscalar Mesons

Charge Neutral Pseudoscalar Mesons

non-Abelian Anomaly and η-η' mixing

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non-Abelian Anomaly and η-η' mixing

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non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
  Problem: η' is a pseudoscalar meson but it's much more massive than the other eight constituted from light-quarks.
  Origin: While the classical action associated with QCD is invariant under $U_A(1)$ (Abelian axial transformations generated by $\lambda^0\gamma_5$), the quantum field theory is not!

non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
- Flavour mixing takes place in singlet channel: $\lambda^0 \Leftrightarrow \lambda^8$

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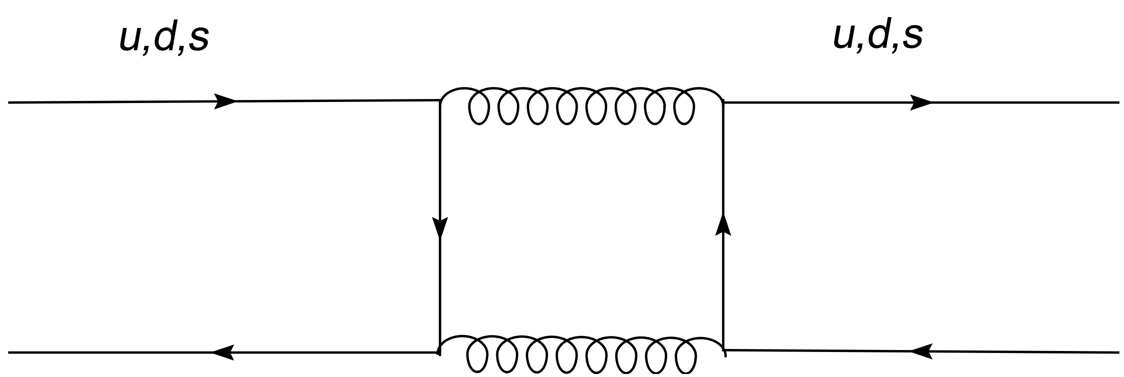

non-Abelian Anomaly and η-η' mixing

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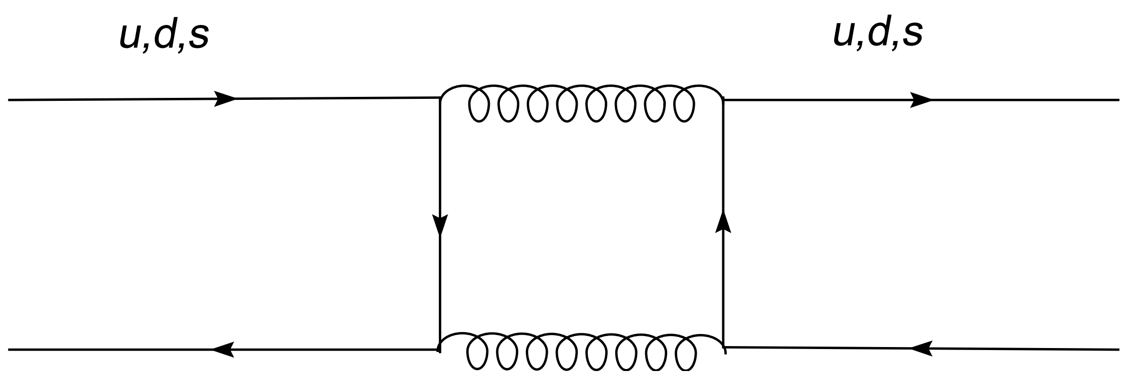

- This is a perturbative diagram.

  It has almost nothing to do with $\eta \Leftrightarrow \eta'$ mixing.

non-Abelian Anomaly and η-η' mixing

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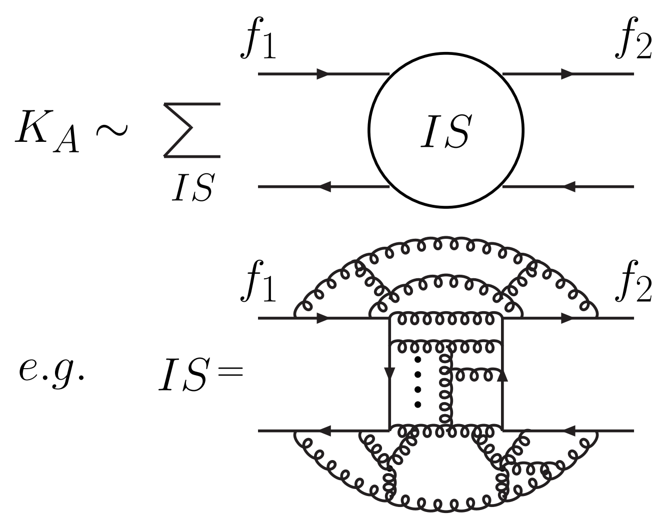
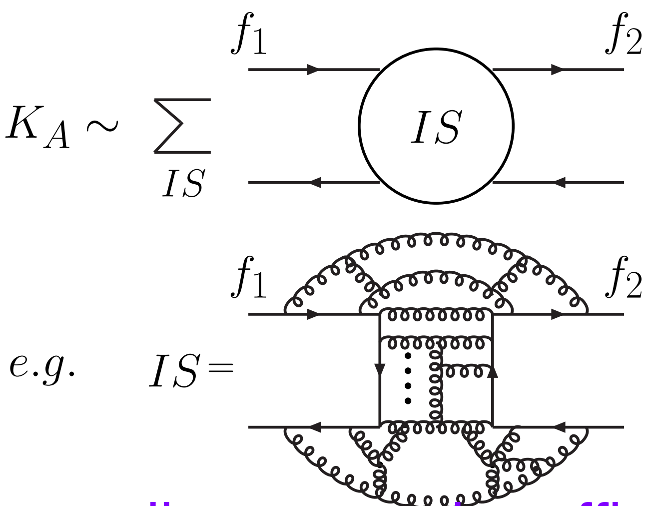


non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
- Driver is the non-Abelian anomaly
- Contribution to the Bethe-Salpeter kernel associated with the non-Abelian anomaly. All terms have the "hairpin" structure.



No finite sum of such intermediate states is sufficient to veraciously represent the anomaly.

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

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Charge Neutral Pseudoscalar Mesons

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$$

- $\{\mathcal{F}^a \,|\, a = 0, \ldots, N_f^2 - 1\}$ are the generators of $U(N_f)$
- $\mathcal{S} = \text{diag}[S_u, S_d, S_s, S_c, S_b, \ldots]$
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Charge Neutral Pseudoscalar Mesons

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- The final term in the second line expresses the non-Abelian axial anomaly.

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- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$
  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y)\rangle$

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... The topological charge density operator.

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(Trace is over colour indices & $F_{\mu\nu} = \frac{1}{2}\lambda^a F^a_{\mu\nu}$.)

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  . . . The topological charge density operator.
- Important that only $\mathcal{A}^{a=0}$ is nonzero.

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  … The topological charge density operator.
- NB. While $\mathcal{Q}(x)$ is gauge invariant, the associated Chern-Simons current, K_μ, is not $\Rightarrow$ in QCD *no physical* boson can couple to K_μ and hence *no physical* states can contribute to resolution of U$_A$(1) problem.

Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting ... otherwise all pseudoscalar mesons are Goldstone Modes!

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Charge Neutral Pseudoscalar Mesons

- Anomaly term has structure

$$\mathcal{A}^0(k;P) = \mathcal{F}^0\gamma_5\left[i\mathcal{E}_{\mathcal{A}}(k;P) + \gamma\cdot P\mathcal{F}_{\mathcal{A}}(k;P)\right.$$
$$\left. +\gamma\cdot k k\cdot P\mathcal{G}_{\mathcal{A}}(k;P) + \sigma_{\mu\nu}k_\mu P_\nu\mathcal{H}_{\mathcal{A}}(k;P)\right]$$

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Charge Neutral Pseudoscalar Mesons

- AVWTI gives generalised Goldberger-Treiman relations

$$
\begin{aligned}
2 f_{\eta'}^0 E_{BS}(k;0) &= 2B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0), \\
F_R^0(k;0) + 2 f_{\eta'}^0 F_{BS}(k;0) &= A_0(k^2) - \mathcal{F}_{\mathcal{A}}(k;0), \\
G_R^0(k;0) + 2 f_{\eta'}^0 G_{BS}(k;0) &= 2A_0'(k^2) - \mathcal{G}_{\mathcal{A}}(k;0), \\
H_R^0(k;0) + 2 f_{\eta'}^0 H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0),
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$$

A_0, B_0 characterise gap equation's chiral limit solution.

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G^0_R(k;0) + 2f^0_{\eta'}G_{BS}(k;0) &= 2A'_0(k^2) - \mathcal{G}_{\mathcal{A}}(k;0),\\
H^0_R(k;0) + 2f^0_{\eta'}H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0),
\end{aligned}$$

A_0, B_0 characterise gap equation's chiral limit solution.

- Follows that $\boldsymbol{\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)}$ is necessary and sufficient condition for absence of massless $\boldsymbol{\eta'}$ bound-state.

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Charge Neutral Pseudoscalar Mesons

- $\boldsymbol{\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)}$
  Discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
  - Hence, absence of massless η' bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.

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- Further highlighted ... proved

$$\begin{aligned} \langle \bar{q}q \rangle_\zeta^0 &= -\lim_{\Lambda\to\infty} Z_4(\zeta^2,\Lambda^2)\,\mathrm{tr}_{\mathrm{CD}} \int_q^\Lambda S^0(q,\zeta) \\ &= N_f \int d^4x\, \langle \bar{q}(x) i\gamma_5 q(x) \mathcal{Q}(0) \rangle^0 . \end{aligned}$$

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Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons

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- Employed in an analysis of pseudoscalar- and vector-meson bound-states

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Charge Neutral Pseudoscalar Mesons

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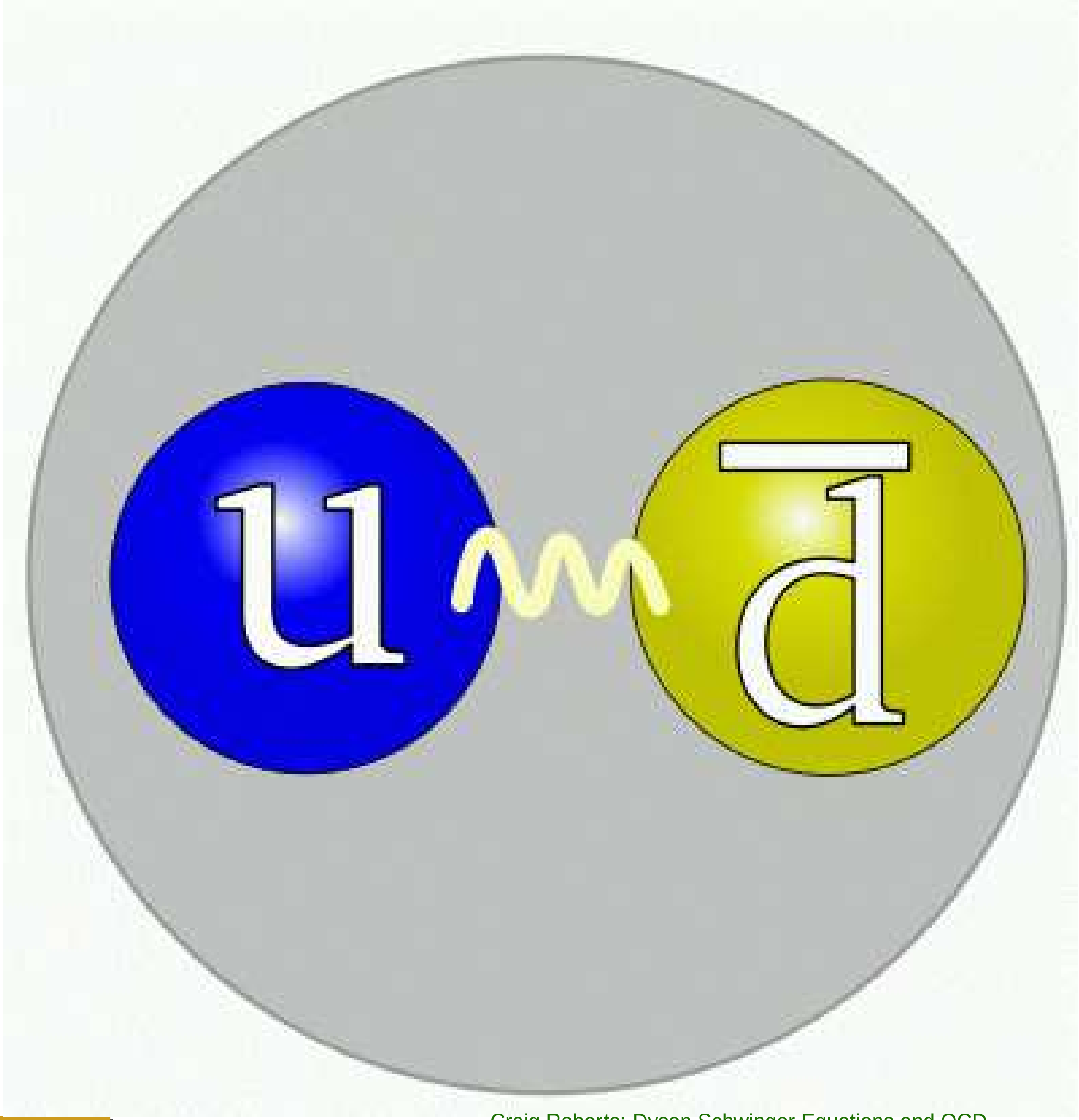
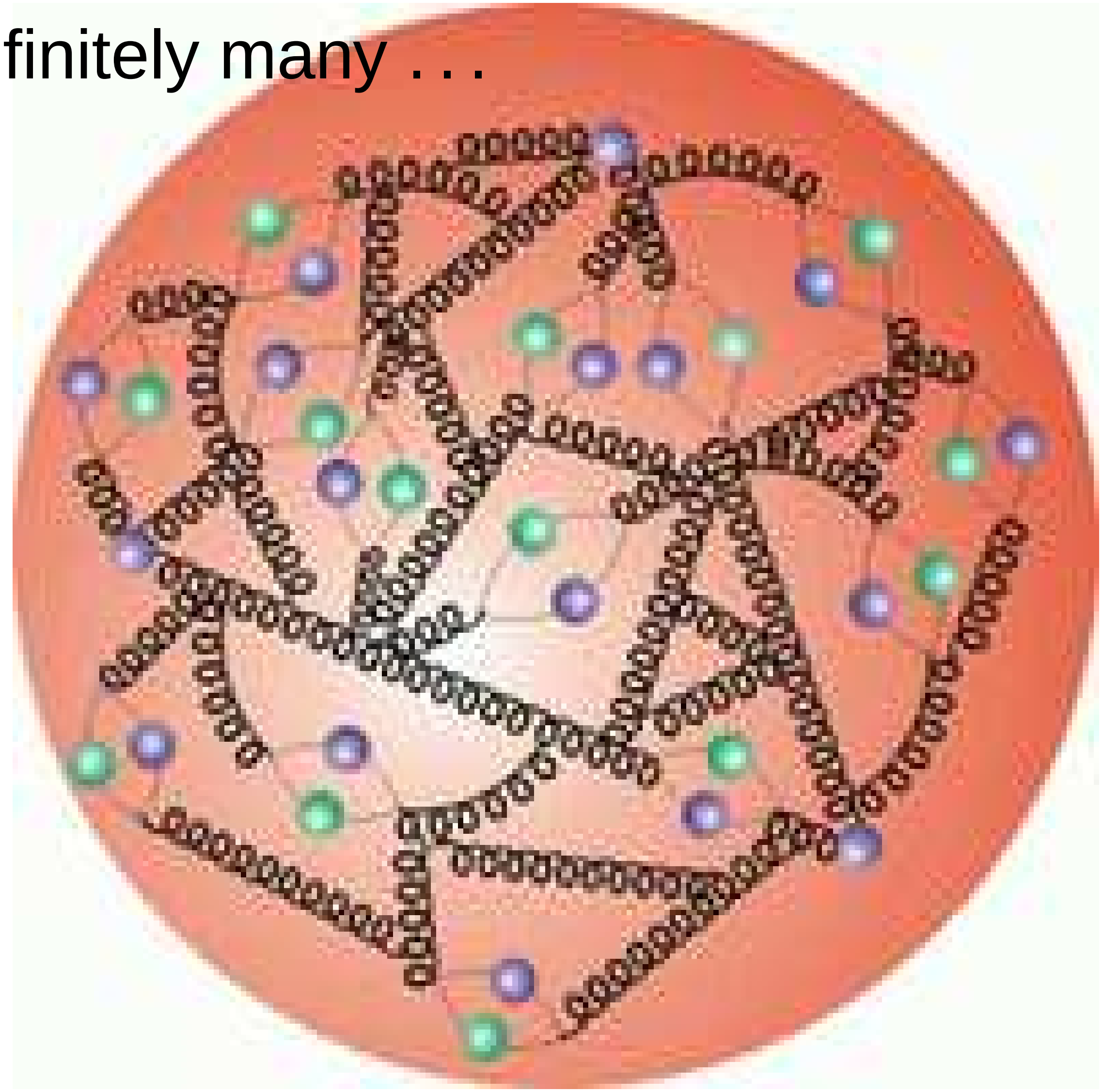
- Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts
  - η–η' mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)
  - π^0–η angles of $\sim 1.2^\circ$ (Expt. $p\, d \to {}^3$He π^0: $0.6^\circ \pm 0.3^\circ$)
  - Strong neutron-proton mass difference . . .

    $\lesssim 75$ % current-quark mass-difference

Answer for the pion



Two → Infinitely many . . .



Answer for the pion

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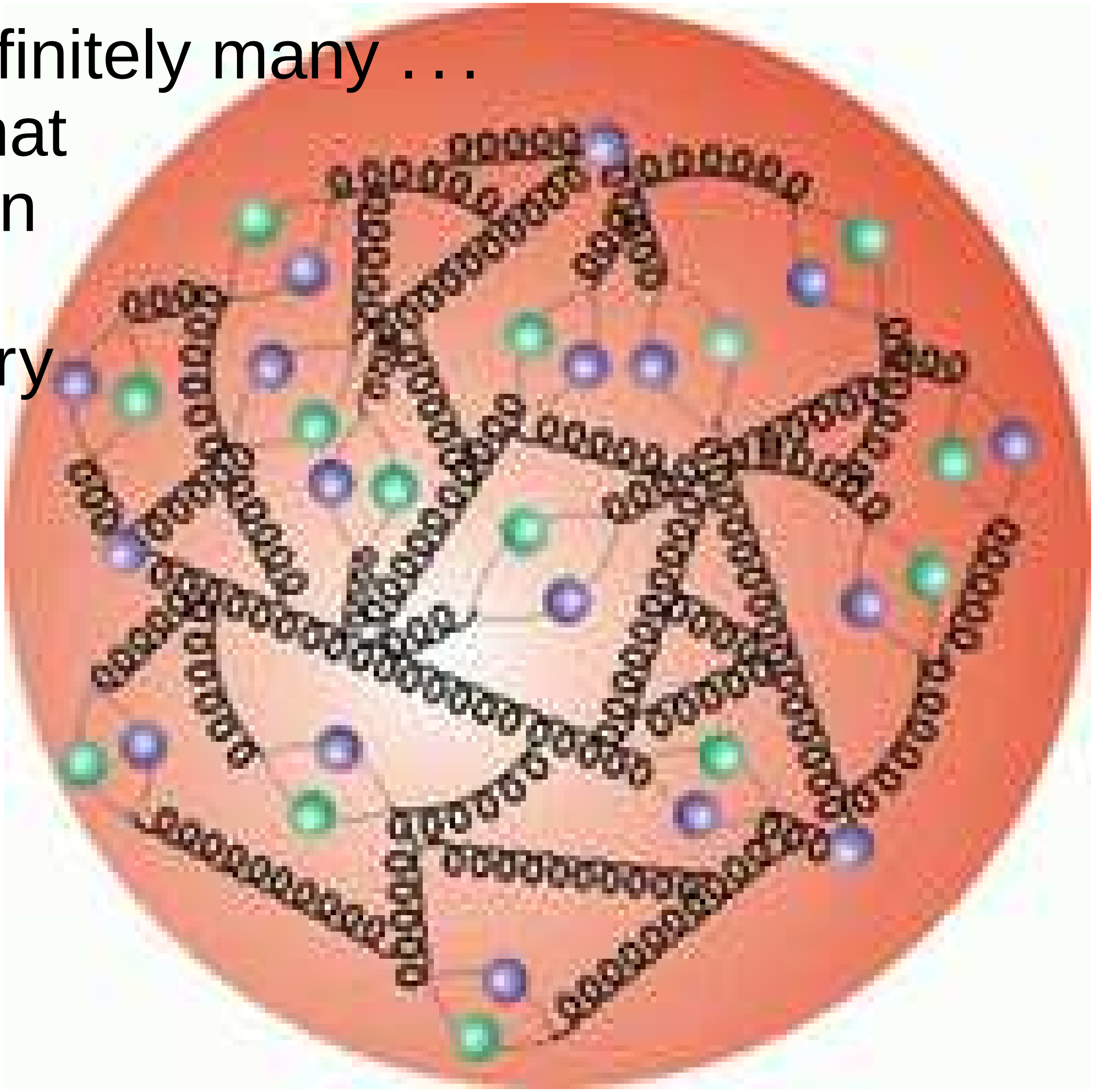

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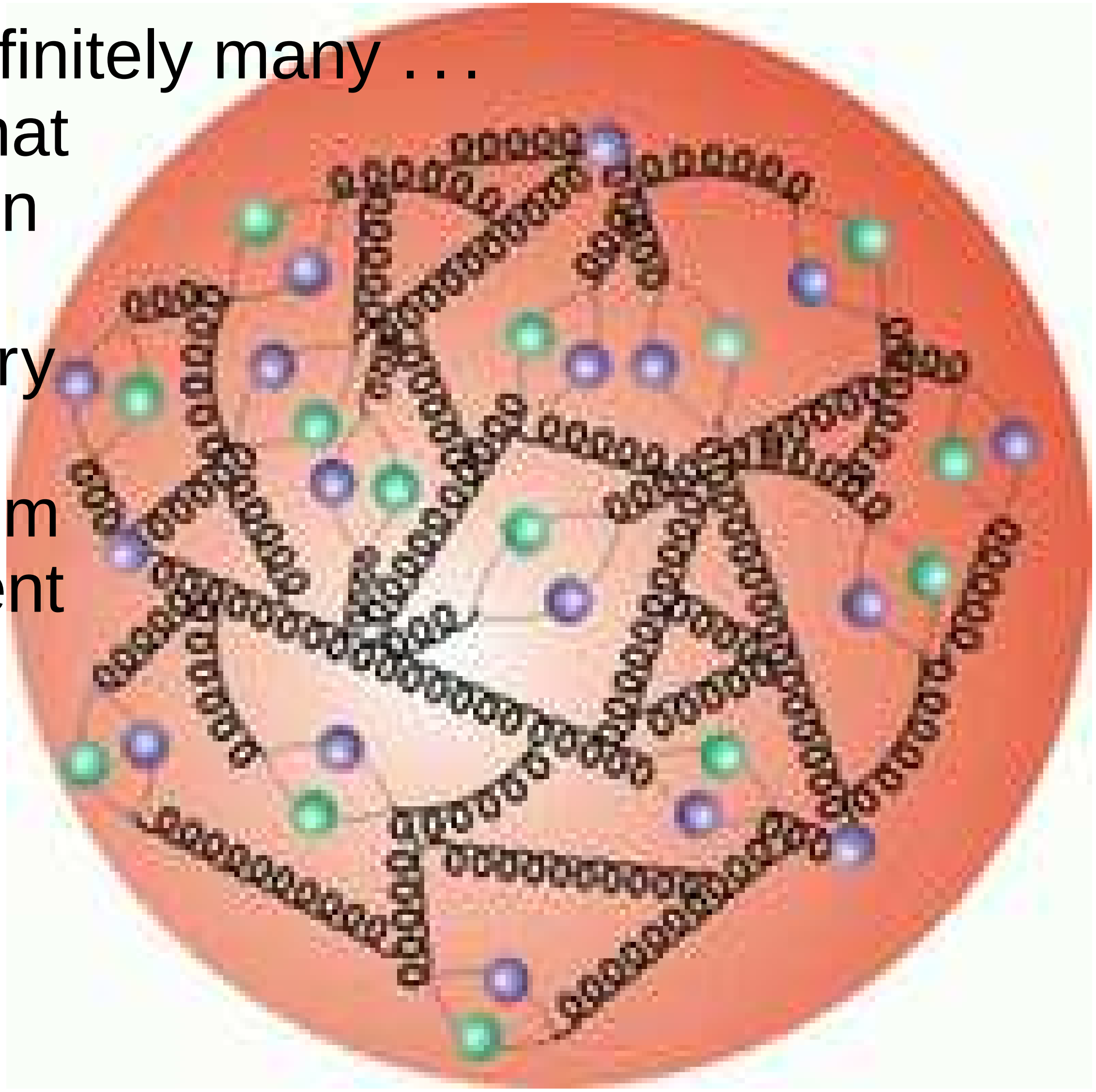
momentum-dependent dressing

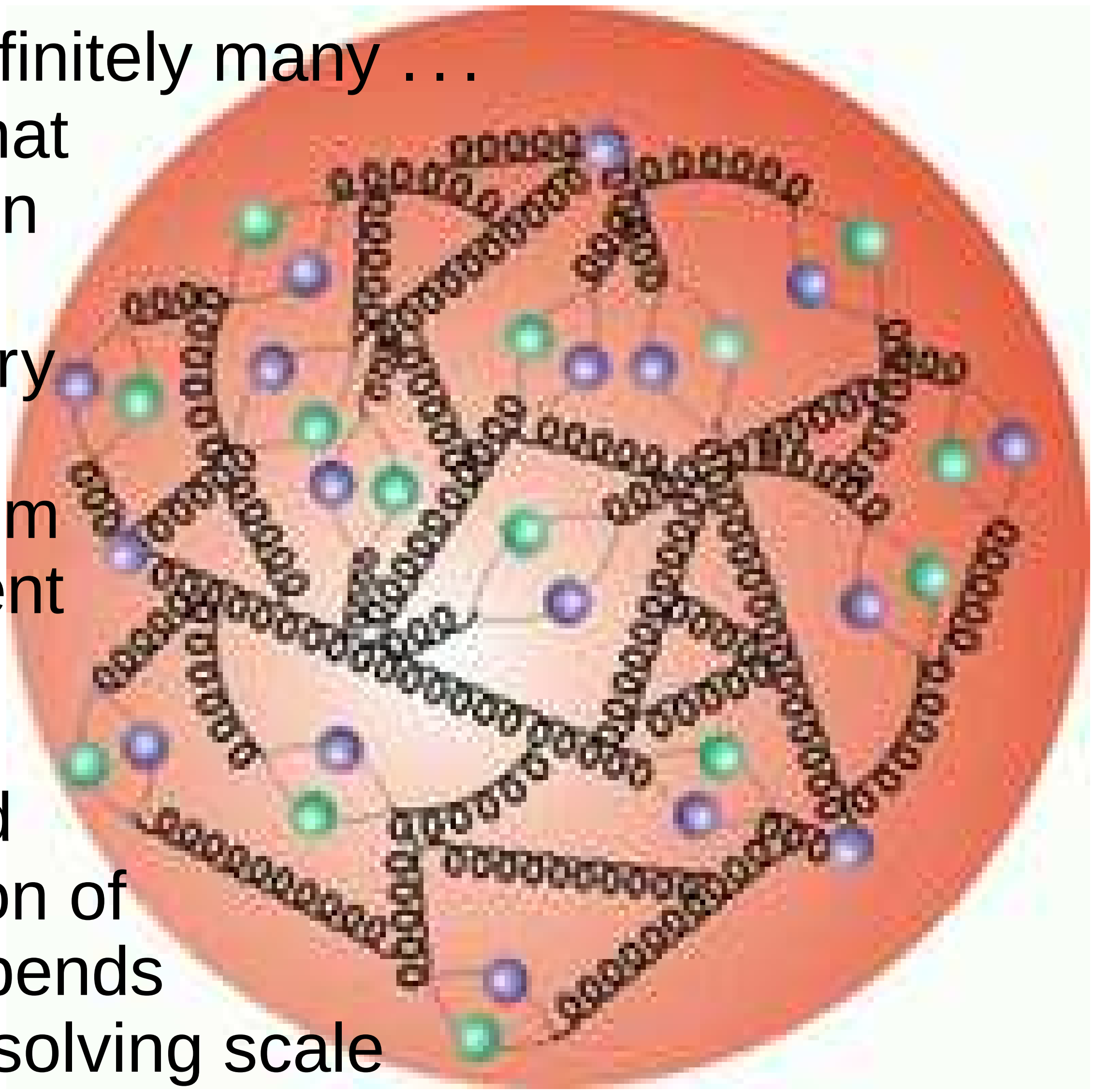


Answer for the pion

Two $\rightarrow$ Infinitely many ...
Handle that properly in quantum field theory

...
momentum -dependent dressing

...
perceived distribution of mass depends on the resolving scale



Exegesis

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  - Nature of Baryons