Hadron Form Factors

Craig D. Roberts

`cdroberts@anl.gov`

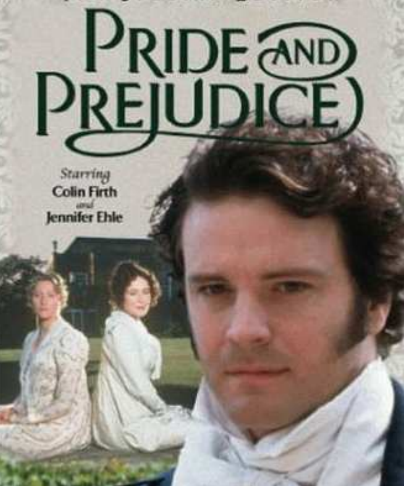
Physics Division

Argonne National Laboratory

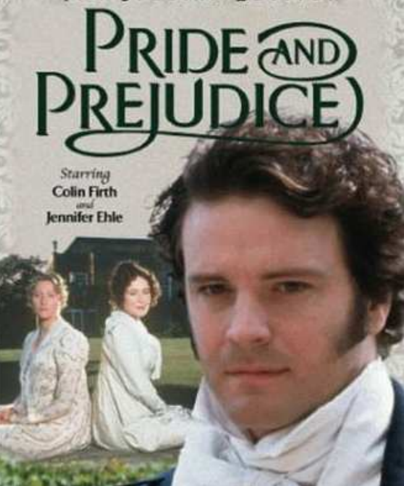
http://www.phy.anl.gov/theory/staff/cdr.html

Universal Truths

First Contents Back Conclusion

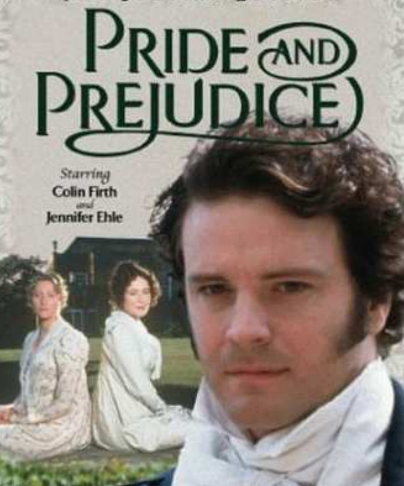


Universal Truths



Universal Truths

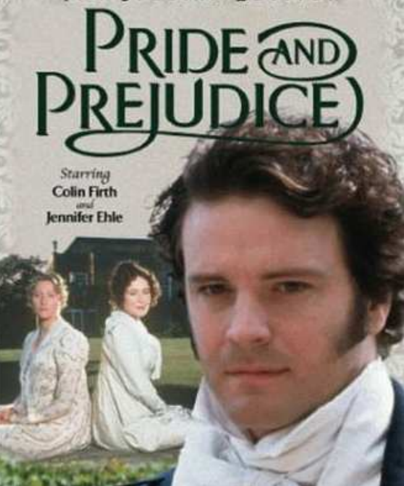
- Form factors give information about distribution of hadron's characterising properties amongst its QCD constituents.



Universal Truths

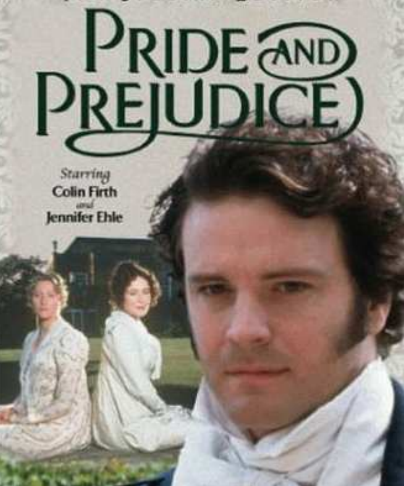
- Form factors give information about distribution of hadron's characterising properties amongst its QCD constituents.
- Calculations at $Q^2 > 1\,\mathrm{GeV}^2$ require a Poincaré-covariant approach.

Office of Science
U.S. DEPARTMENT OF ENERGY
Office of Nuclear Physics
Argonne NATIONAL LABORATORY



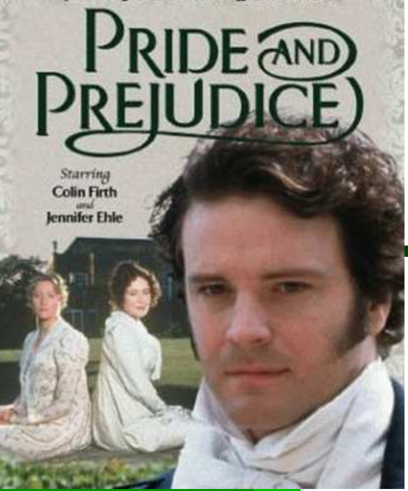
Universal Truths

- Form factors give information about distribution of hadron's characterising properties amongst its QCD constituents.
- Calculations at $Q^2 > 1\,\mathrm{GeV}^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.



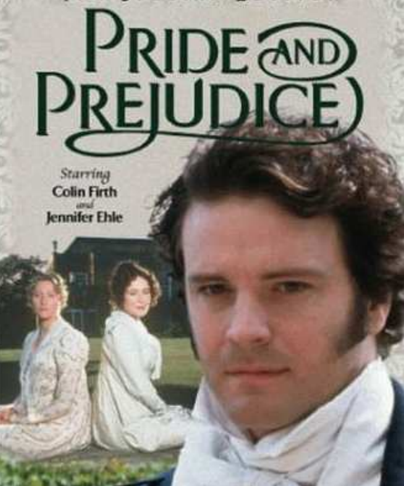
Universal Truths

- Form factors give information about distribution of hadron's characterising properties amongst its QCD constituents.
- Calculations at $Q^2 > 1\,\mathrm{GeV}^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.
- DCSB is most important mass generating mechanism for matter in the Universe.



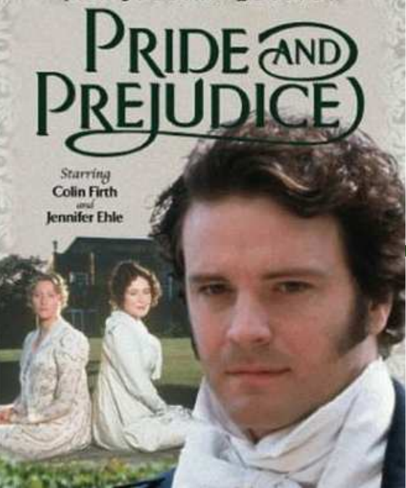
Universal Truths

- Form factors give information about distribution of hadron's characterising properties amongst its QCD constituents.
- Calculations at $Q^2 > 1\,\mathrm{GeV}^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.
- DCSB is most important mass generating mechanism for matter in the Universe. Higgs mechanism is irrelevant to light-quarks.



Universal Truths

- Form factors give information about distribution of hadron's characterising properties amongst its QCD constituents.
- Calculations at $Q^2 > 1\,\mathrm{GeV}^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.
- DCSB is most important mass generating mechanism for matter in the Universe. Higgs mechanism is irrelevant to light-quarks.
- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons.



Universal Truths

- Form factors give information about distribution of hadron's characterising properties amongst its QCD constituents.
- Calculations at $Q^2 > 1\,\mathrm{GeV}^2$ require a Poincaré-covariant approach. Covariance requires existence of quark orbital angular momentum in hadron's rest-frame wave function.
- DCSB is most important mass generating mechanism for matter in the Universe. Higgs mechanism is irrelevant to light-quarks.
- Challenge: understand relationship between parton properties on the light-front and rest frame structure of hadrons. Problem because, e.g., DCSB - an established keystone of low-energy QCD and the origin of constituent-quark masses - has not been realised in the light-front formulation.

QCD's Challenges

First Contents Back Conclusion

QCD's Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

QCD's Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^{P=+}$ and $J^{P=-}$

First Contents Back Conclusion

QCD’s Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^{P=+}$ and $J^{P=-}$
- Neither of these phenomena is apparent in QCD’s Lagrangian ***yet*** they are the dominant determining characteristics of real-world QCD.

Understand Emergent Phenomena

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^{P=+}$ and $J^{P=-}$
- Neither of these phenomena is apparent in QCD's Lagrangian ***yet*** they are the dominant determining characteristics of real-world QCD.
- QCD – Complex behaviour
  arises from apparently simple rules

Office of Science U.S. DEPARTMENT OF ENERGY

Office of Nuclear Physics

Argonne NATIONAL LABORATORY

First Contents Back Conclusion

Form Factors: Why?

First Contents Back Conclusion

Form Factors: Why?

- The nucleon and pion hold special places in non-perturbative studies of QCD.

First Contents Back Conclusion

Form Factors: Why?

- The nucleon and pion hold special places in non-perturbative studies of QCD.
- An explanation of nucleon and pion structure and interactions is central to hadron physics – they are respectively the archetypes for baryons and mesons.

First Contents Back Conclusion

Form Factors: Why?

- The nucleon and pion hold special places in non-perturbative studies of QCD.
- An explanation of nucleon and pion structure and interactions is central to hadron physics – they are respectively the archetypes for baryons and mesons.
- Form factors have long been recognized as a basic tool for elucidating bound state properties. They can be studied from very low momentum transfer, the region of non-perturbative QCD, up to a region where perturbative QCD predictions can be tested.

Form Factors: Why?

- The nucleon and pion hold special places in non-perturbative studies of QCD.
- An explanation of nucleon and pion structure and interactions is central to hadron physics – they are respectively the archetypes for baryons and mesons.
- Form factors have long been recognized as a basic tool for elucidating bound state properties. They can be studied from very low momentum transfer, the region of non-perturbative QCD, up to a region where perturbative QCD predictions can be tested.
- Experimental and theoretical studies of nucleon electromagnetic form factors have made rapid and significant progress during the last several years, including new data in the time like region, and material gains have been made in studying the pion form factor.

Form Factors: Why?

- The nucleon and pion hold special places in non-perturbative studies of QCD.
- An explanation of nucleon and pion structure and interactions is central to hadron physics – they are respectively the archetypes for baryons and mesons.
- Form factors have long been recognized as a basic tool for elucidating bound state properties. They can be studied from very low momentum transfer, the region of non-perturbative QCD, up to a region where perturbative QCD predictions can be tested.
- Experimental and theoretical studies of nucleon electromagnetic form factors have made rapid and significant progress during the last several years, including new data in the time like region, and material gains have been made in studying the pion form factor.
- Despite this, many urgent questions remain unanswered.

Some Questions

- What is the role of pion cloud in nucleon electromagnetic structure?
- Can we understand the pion cloud in a more quantitative and, perhaps, model-independent way?

First Contents Back Conclusion

Some Questions

- Where is the transition from non-pQCD to pQCD in the pion and nucleon electromagnetic form factors?

Some Questions

- Do we understand the high Q^2 behavior of the proton form factor ratio in the space-like region?
- Can we make model-independent statements about the role of relativity or orbital angular momentum in the nucleon?

First Contents Back Conclusion

Some Questions

- Can we understand the rich structure of the time-like proton form factors in terms of resonances?
- What do we expect for the proton form factor ratio in the time-like region?
- What is the relation between proton and neutron form factor in the time-like region?
- How do we understand the ratio between time-like and space-like form factors?

Some Questions

- What is the role of two-photon exchange contributions in understanding the discrepancy between the polarization and Rosenbluth measurements of the proton form factor ratio?
- What is the impact of these contributions on other form factor measurements?

First Contents Back Conclusion

Some Questions

- How accurately can the pion form factor be extracted from the $ep \to e' n \pi^+$ reaction?

First Contents Back Conclusion

Status

Status

- Current status is described in
  - J. Arrington, C. D. Roberts and J. M. Zanotti
    "Nucleon electromagnetic form factors,"
    J. Phys. G **34**, S23 (2007); [arXiv:nucl-th/0611050].
  - C. F. Perdrisat, V. Punjabi and M. Vanderhaeghen,
    "Nucleon electromagnetic form factors,"
    Prog. Part. Nucl. Phys. **59**, 694 (2007);
    [arXiv:hep-ph/0612014].

Status

- Current status is described in
  - J. Arrington, C. D. Roberts and J. M. Zanotti
    "Nucleon electromagnetic form factors,"
    J. Phys. G **34**, S23 (2007); [arXiv:nucl-th/0611050].
  - C. F. Perdrisat, V. Punjabi and M. Vanderhaeghen,
    "Nucleon electromagnetic form factors,"
    Prog. Part. Nucl. Phys. **59**, 694 (2007);
    [arXiv:hep-ph/0612014].
- Most recently:
  "ECT* Workshop on Hadron Electromagnetic Form Factors"
  Organisers: Alexandrou, Arrington, Friedrich, Maas, Roberts
  Presentations, etc., available on-line
  http://ect08.phy.anl.gov/

Dichotomy of Pion
– Goldstone Mode and *Bound state*

First Contents Back Conclusion

Dichotomy of Pion
– Goldstone Mode and *Bound state*

- How does one make an almost massless particle
  . . . . from two massive constituent-quarks?

First Contents Back Conclusion

Dichotomy of Pion – Goldstone Mode and Bound state

- How does one make an almost massless particle
  . . . . from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

  Must exhibit $m_\pi^2 \propto m_q$

  Current Algebra . . . 1968

Dichotomy of Pion
– *Goldstone Mode* and *Bound state*

- How does one make an almost massless particle
  . . . . from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

  Must exhibit $\boxed{m_\pi^2 \propto m_q}$

  Current Algebra . . . 1968

The correct understanding of pion observables;
e.g. mass, decay constant and form factors,
requires an approach to contain a

- well-defined and valid chiral limit;
- and an accurate realisation of
  dynamical chiral symmetry breaking.

Dichotomy of Pion
– Goldstone Mode and Bound state

- How does one make an almost massless particle
  . . . . from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

  Must exhibit $m_\pi^2 \propto m_q$

  Current Algebra . . . 1968

The correct understanding of pion observables;
e.g. mass, decay constant and form factors,
requires an approach to contain a

- well-defined and valid chiral limit;
- and an accurate realisation of
  dynamical chiral symmetry breaking.

Highly Nontrivial

First | Contents | Back | Conclusion

Pion Form Factors

- There is a sense in which it is easy to fabricate a model that can *reproduce* the elastic electromagnetic pion form factor

Pion Form Factors

- There is a sense in which it is easy to fabricate a model that can *reproduce* the elastic electromagnetic pion form factor
- However, a veracious description of the pion will simultaneously predict the elastic electromagnetic form factor, $F_\pi(Q^2)$ AND the $\gamma^*\pi \to \gamma$ transition form factor

First Contents Back Conclusion

Infidelity without simultaneity

- There is a sense in which it is easy to fabricate a model that can *reproduce* the elastic electromagnetic pion form factor
- However, a veracious description of the pion will simultaneously predict the elastic electromagnetic form factor, $F_\pi(Q^2)$ AND the $\gamma^*\pi \to \gamma$ transition form factor
- The latter is connected with the Abelian anomaly – therefore fundamentally connected with chiral symmetry and its dynamical breaking – no mere model can successfully describe this without fine tuning

Infidelity without simultaneity

- There is a sense in which it is easy to fabricate a model that can *reproduce* the elastic electromagnetic pion form factor
- However, a veracious description of the pion will simultaneously predict the elastic electromagnetic form factor, $F_\pi(Q^2)$ AND the $\gamma^*\pi \to \gamma$ transition form factor
- The latter is connected with the Abelian anomaly – therefore fundamentally connected with chiral symmetry and its dynamical breaking – no mere model can successfully describe this without fine tuning
- Must similarly require prediction of $\gamma^*\pi \to \pi\pi$ and all other anomalous processes

What's the Problem?

What's the Problem?

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.

First Contents Back Conclusion

What's the Problem?

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
- Means … must calculate hadron *wave functions* – Can't be done using perturbation theory

First Contents Back Conclusion

What's the Problem?

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
- Means . . . must calculate hadron *wave functions* – Can't be done using perturbation theory
- Why problematic? Isn't same true in quantum mechanics?

First Contents Back Conclusion

What's the Problem?

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
- Means . . . must calculate hadron *wave functions*
  – Can't be done using perturbation theory
- Why problematic? Isn't same true in quantum mechanics?
- Differences!

What's the Problem? Relativistic QFT!

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
- Differences!
  - Here relativistic effects are crucial – *virtual particles*, quintessence of Relativistic Quantum Field Theory – must be included

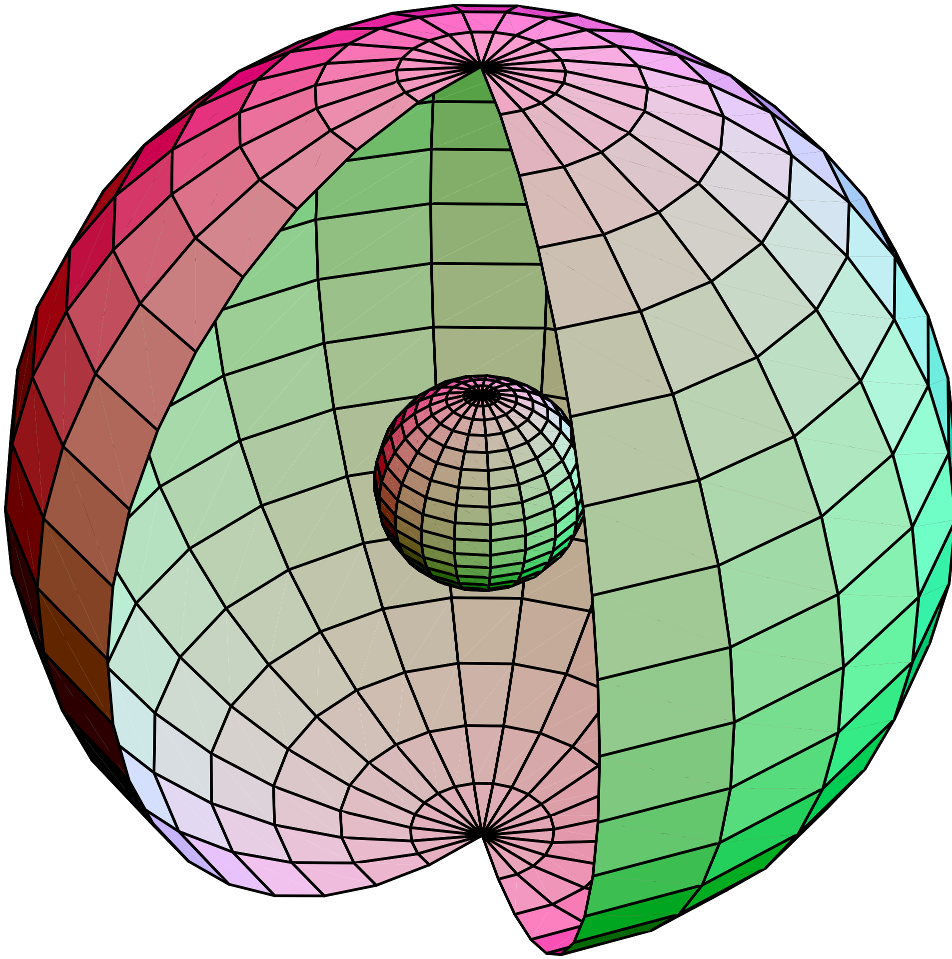
What's the Problem? Relativistic QFT!

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
- Differences!
  - Here relativistic effects are crucial – *virtual particles*, quintessence of Relativistic Quantum Field Theory – must be included
  - Interaction between quarks – the Interquark "Potential" – unknown throughout $>$ 98% of a hadron's volume

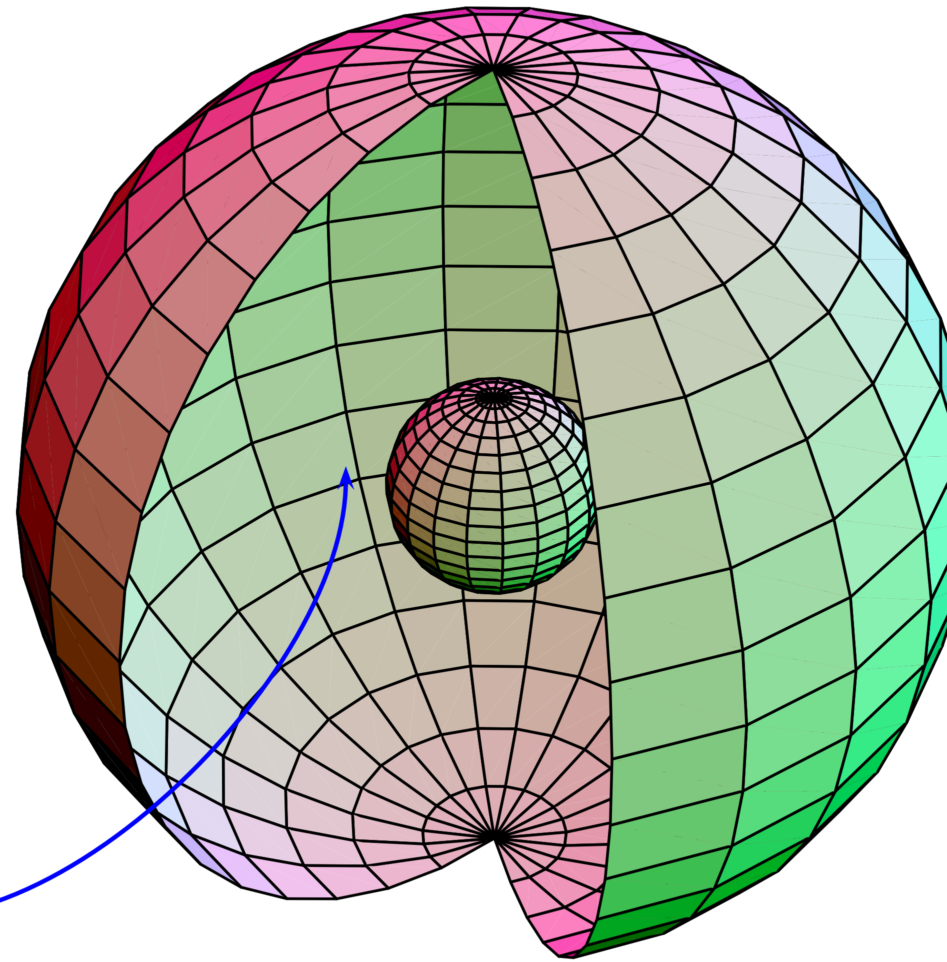
Intranucleon Interaction

First Contents Back Conclusion

Intranucleon Interaction



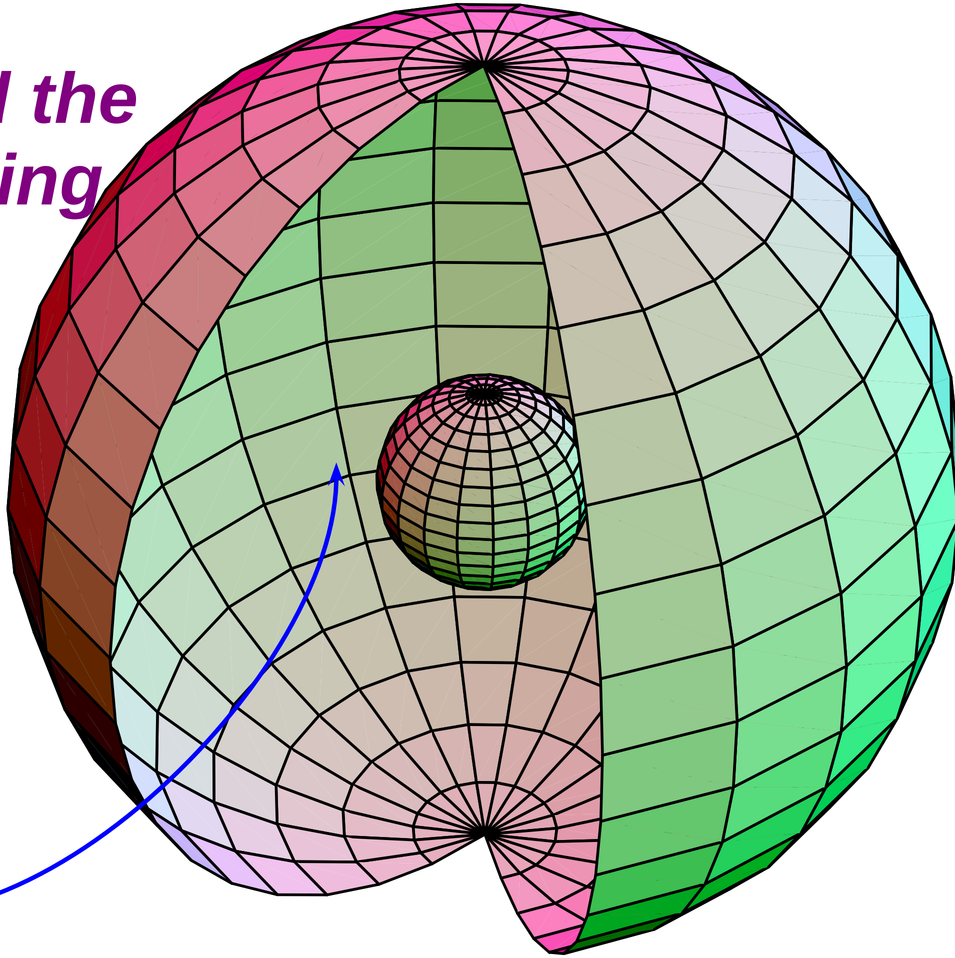
Intranucleon Interaction



98% of the volume

What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



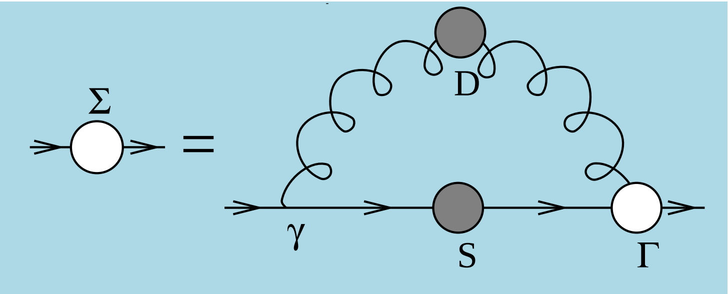
98% of the volume

Dyson-Schwinger Equations
Dressed-Quark Propagator

First Contents Back Conclusion

Dyson-Schwinger Equations
Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

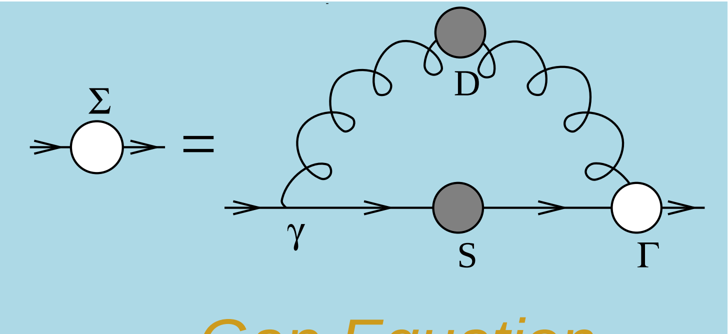


Gap Equation

First | Contents | Back | Conclusion

Dyson-Schwinger Equations
Dressed-Quark Propagator

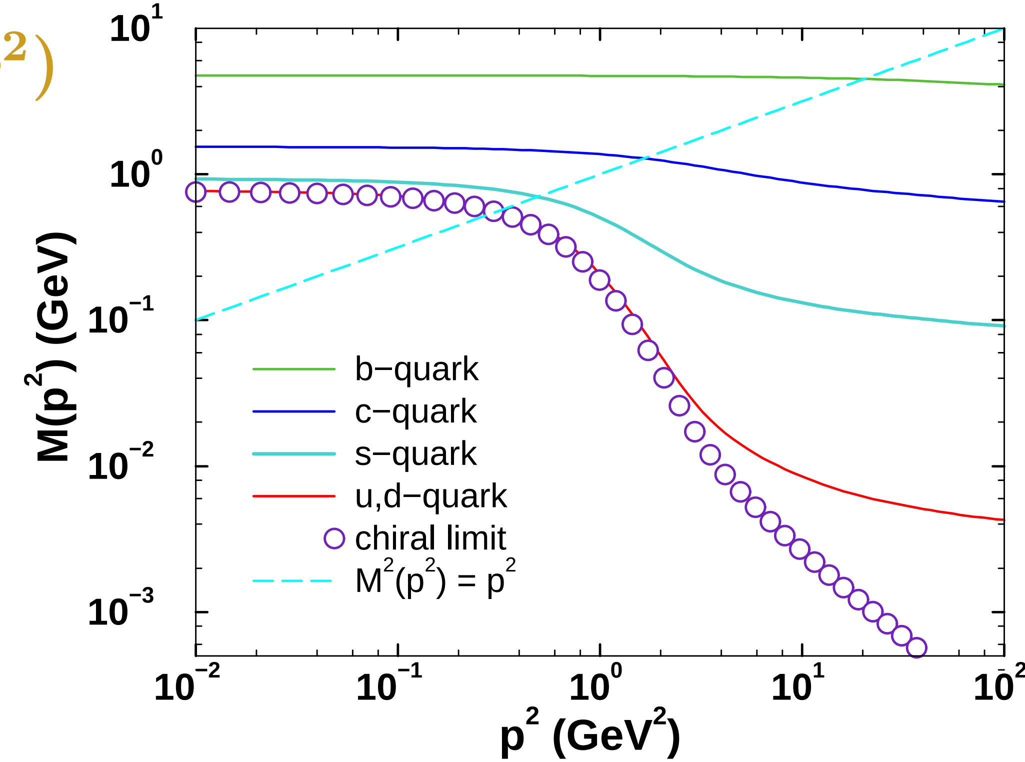
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$

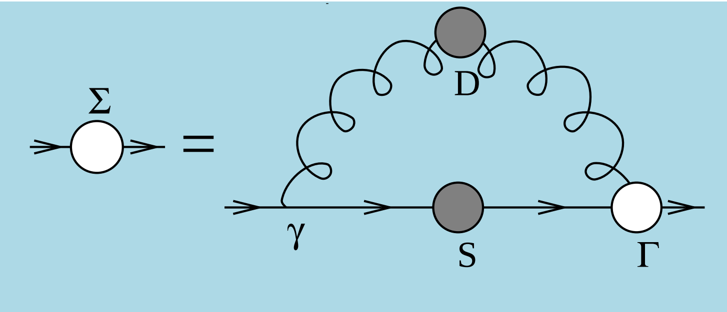


Office of Science U.S. DEPARTMENT OF ENERGY

Office of Nuclear Physics

Argonne NATIONAL LABORATORY

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$



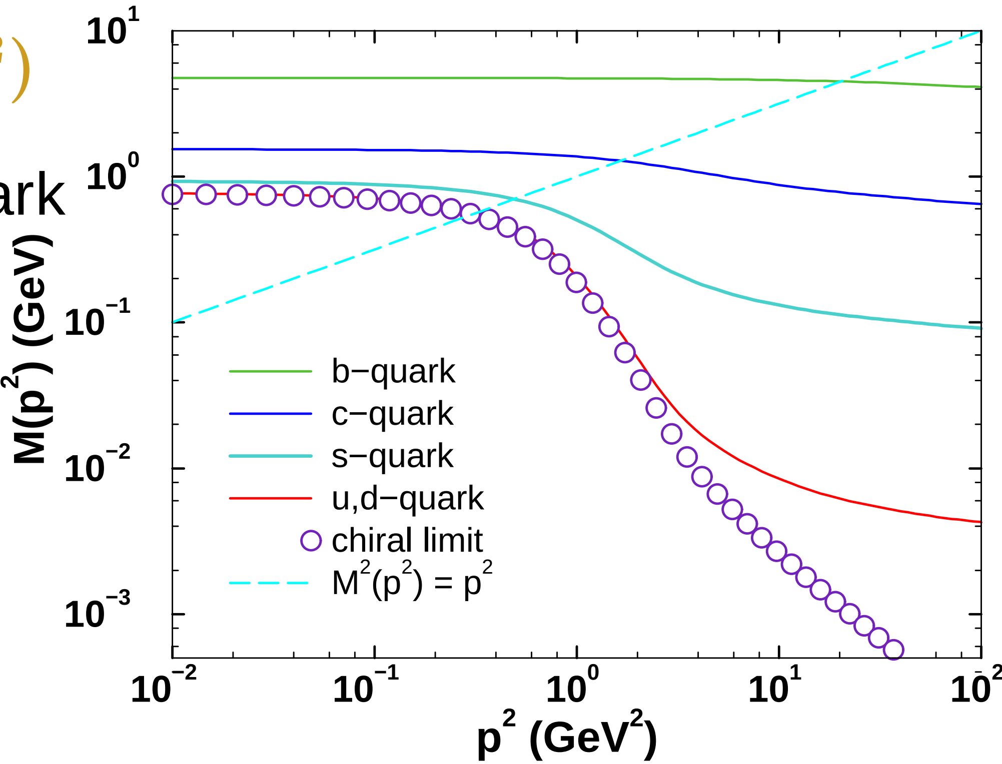
Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

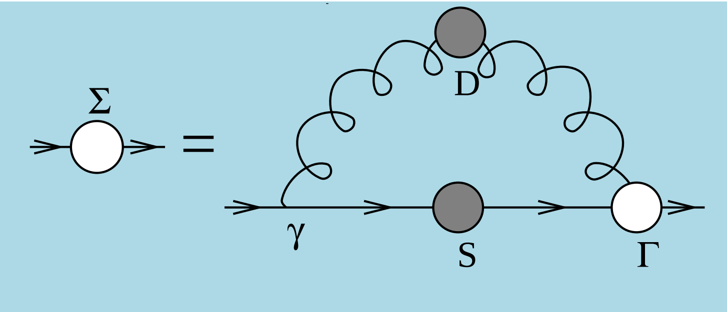
$\Rightarrow$ IR Enhancement of $M(p^2)$

- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



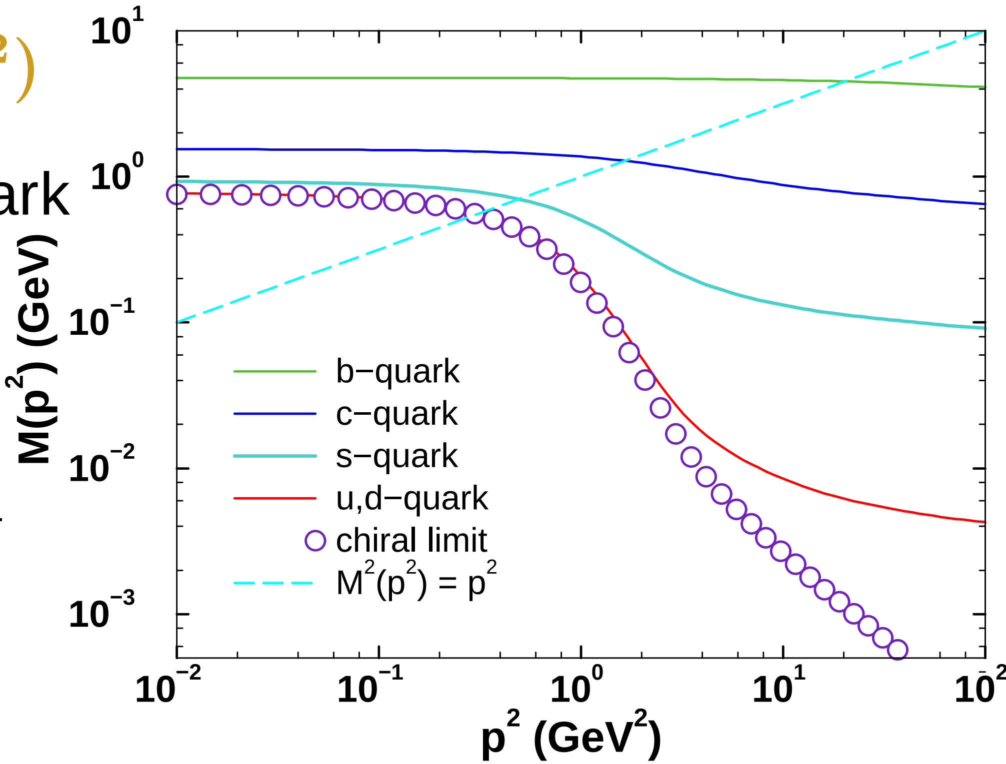
Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$

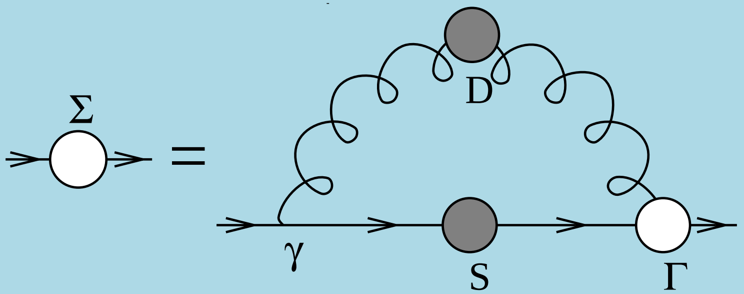
- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



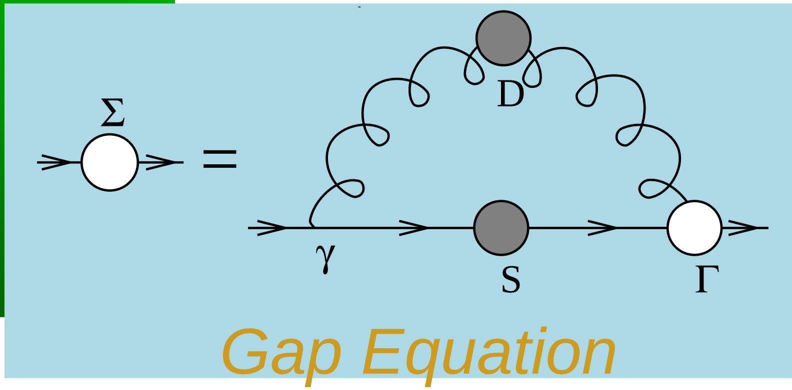
Predictions confirmed in numerical simulations of lattice-QCD

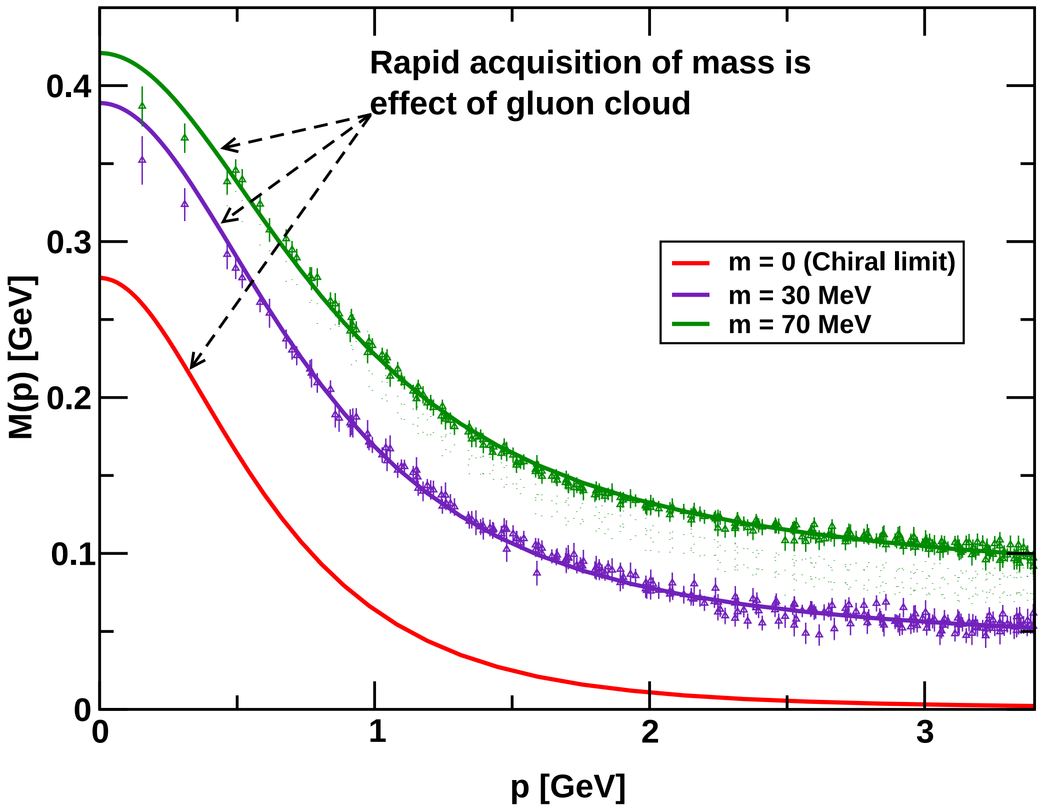
Frontiers of Nuclear Science:
A Long Range Plan (2007)



Gap Equation

Frontiers of Nuclear Science: Theoretical Advances

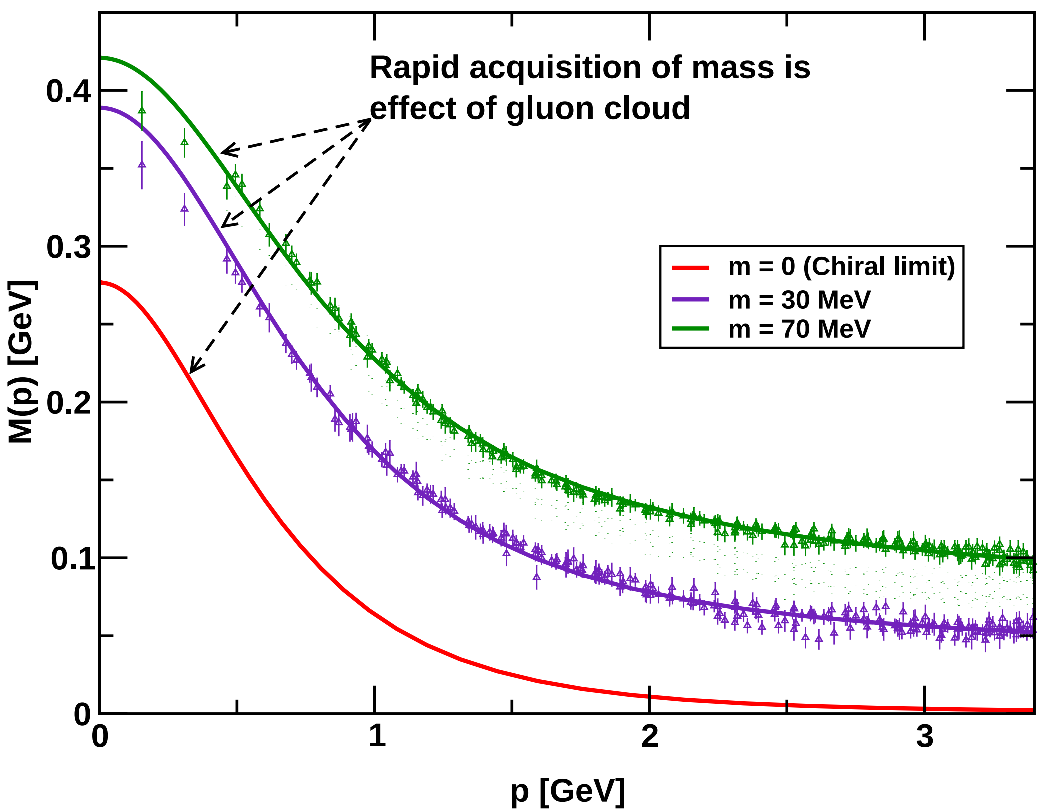




First | Contents | Back | Conclusion

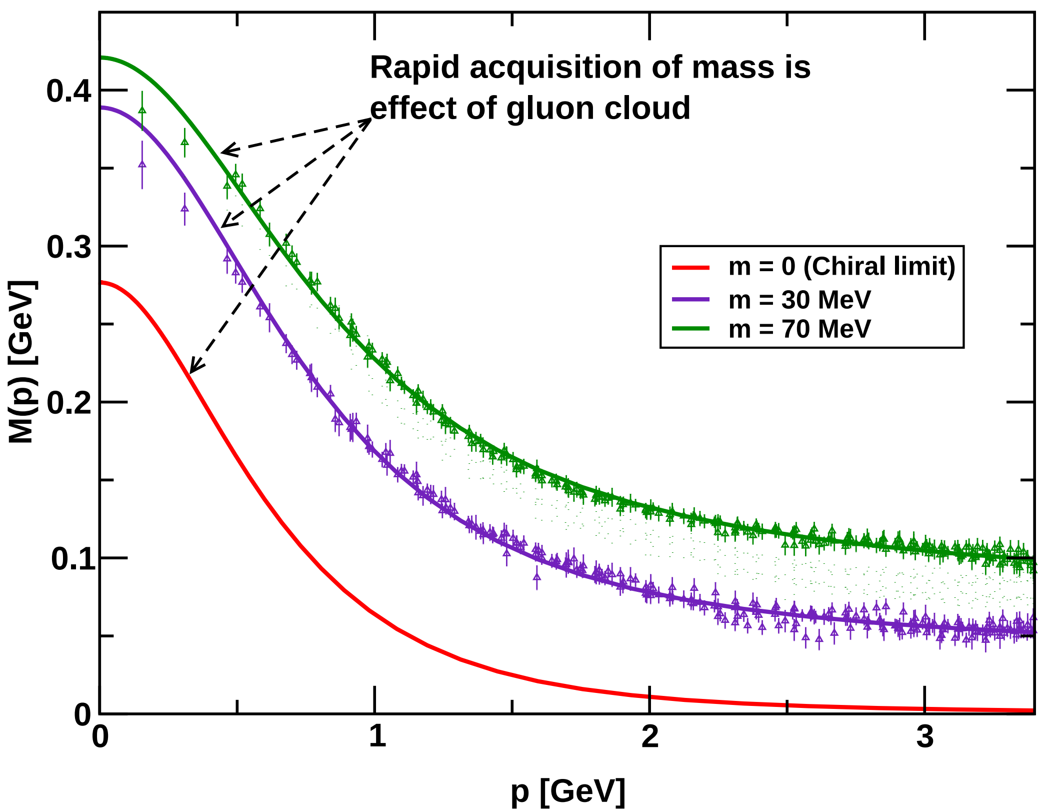
Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.



Mass from nothing.

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies ($m = 0$, red curve) acquires a large constituent mass at low energies.




ALWAYS LOOK ON THE BRIGHT SIDE

- Established understanding of two- and three-point functions

First Contents Back Conclusion

Hadrons

- Established understanding of two- and three-point functions
- What about bound states?

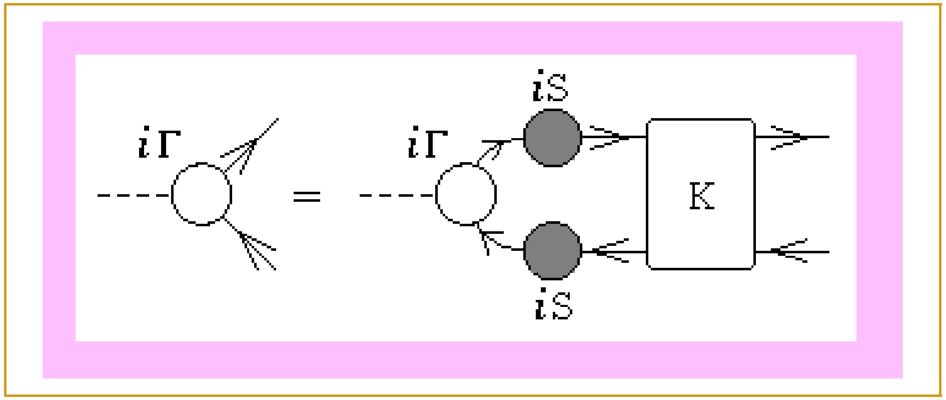
- Without bound states, Comparison with experiment is impossible

Hadrons

- Without bound states, Comparison with experiment is impossible
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

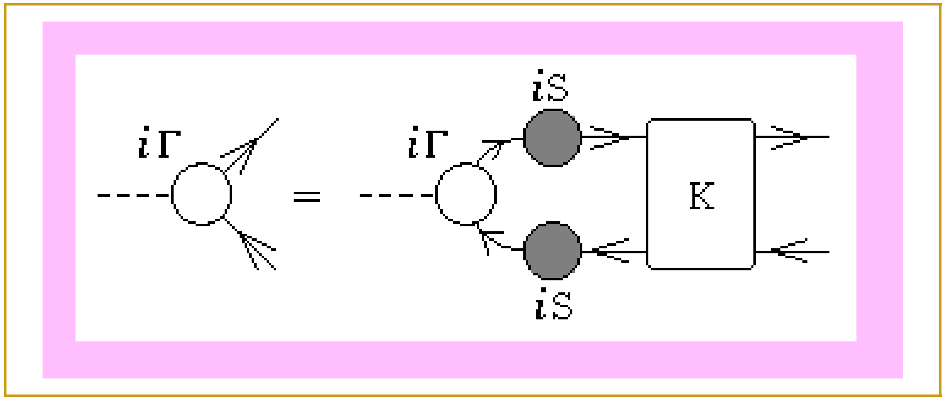
Hadrons

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

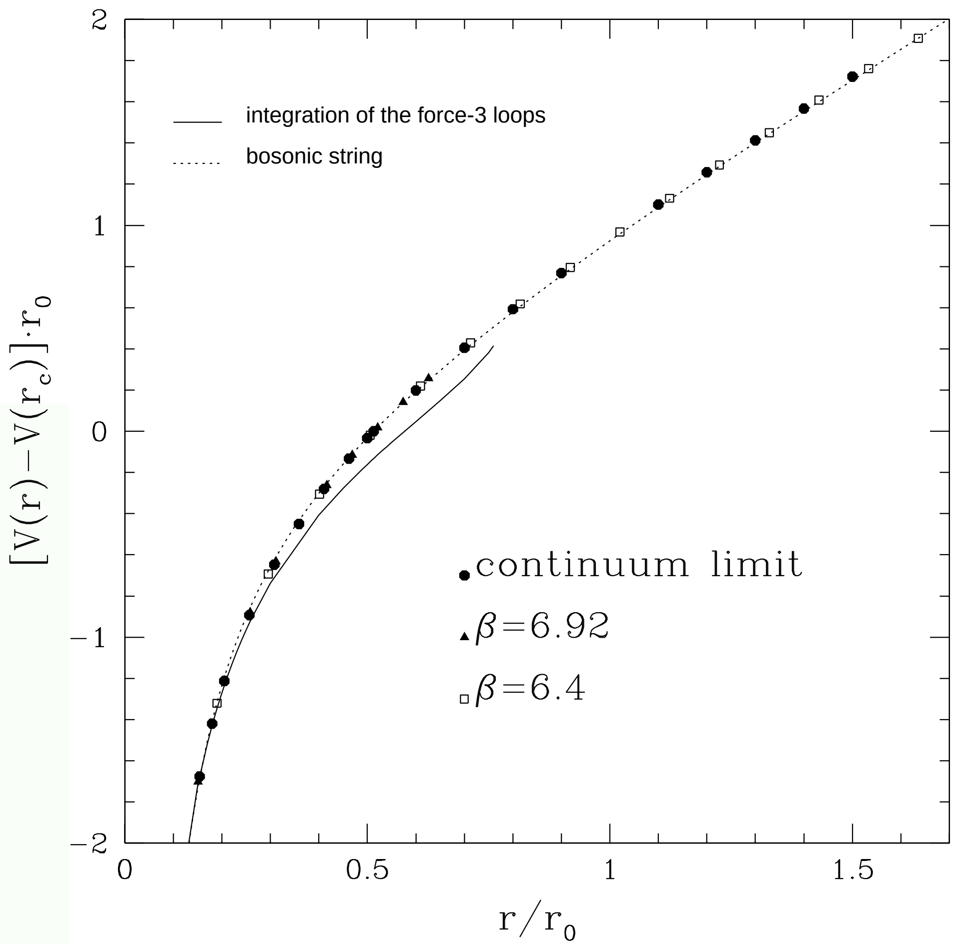
- What is the kernel, K?

or

Confinement

Confinement

- Infinitely Heavy Quarks … Picture in Quantum Mechanics



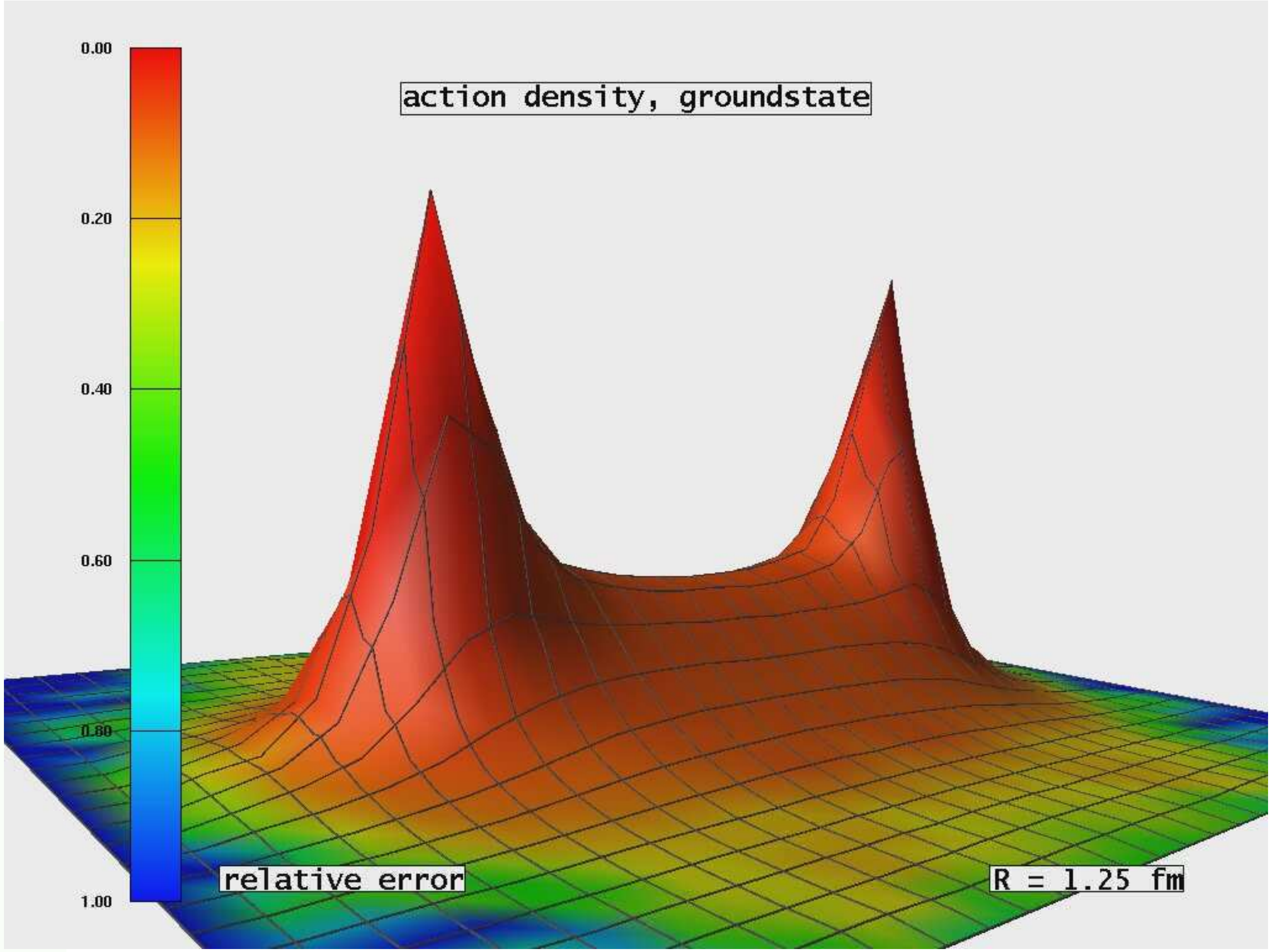
$$V(r) = \sigma\, r - \frac{\pi}{12}\,\frac{1}{r}$$

$\sigma \sim 470$ MeV

Necco & Sommer
he-la/0108008

Confinement

- Illustrate this in terms of the action density ... analogous to plotting the Force $= F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12}\frac{1}{r^2}$



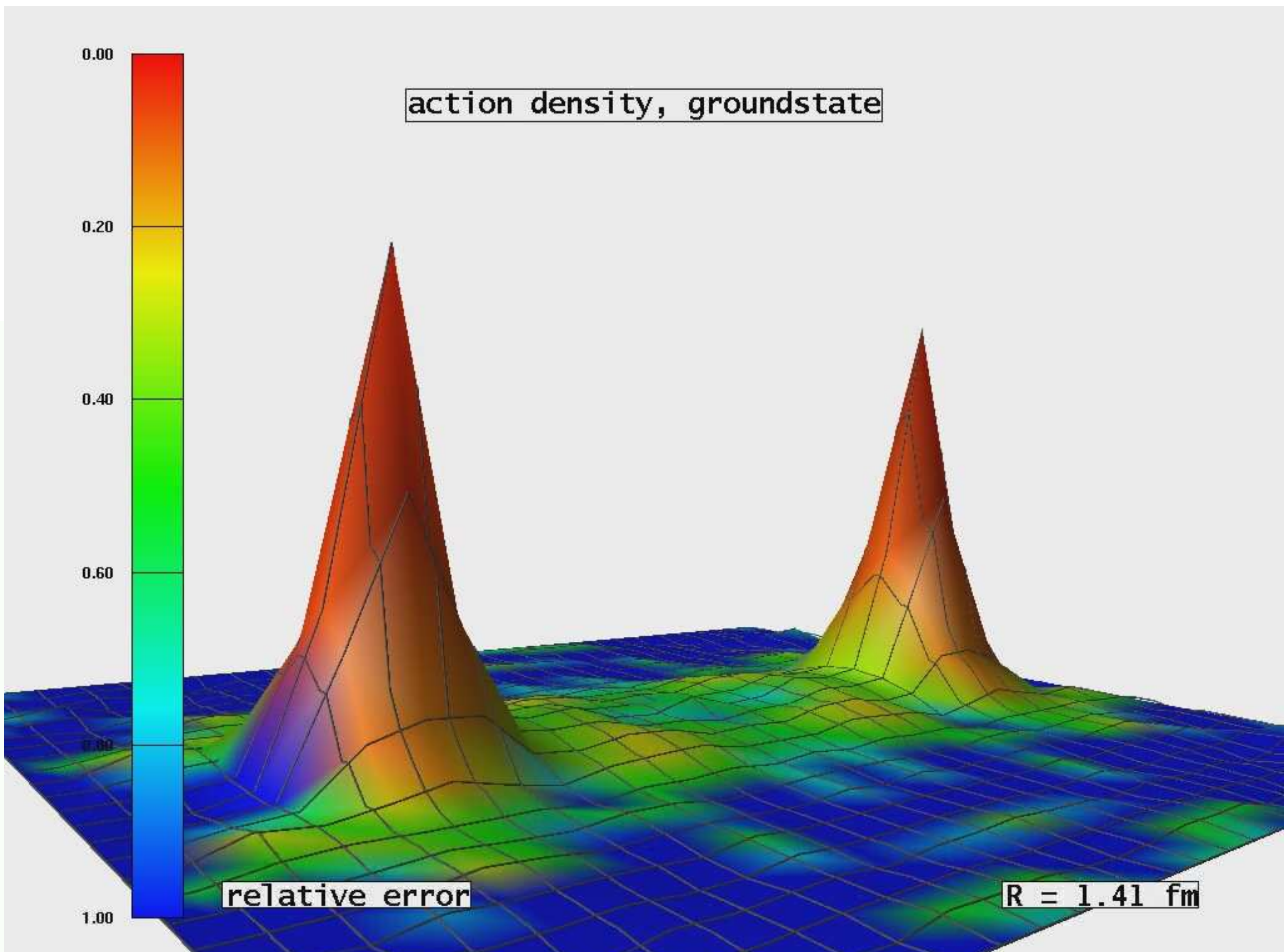
Bali, *et al*.
he-la/0512018

Confinement

- What happens in the real world; namely, in the presence of light-quarks?

Confinement

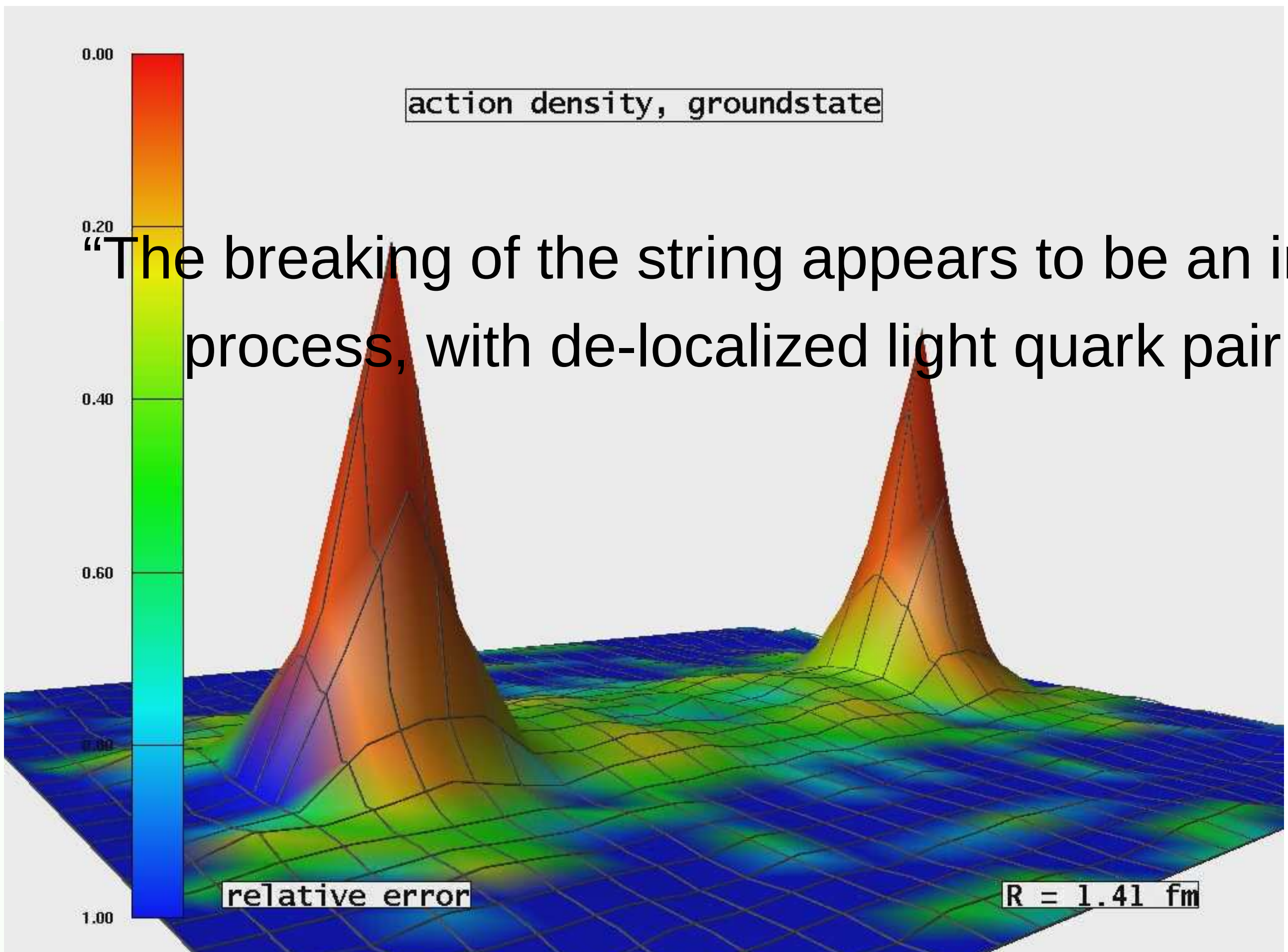
- What happens in the real world; namely, in the presence of light-quarks? No one knows ... but $\bar{Q}Q + 2 \times \bar{q}q$



Bali, *et al*.
he-la/0512018

Confinement

- What happens in the real world; namely, in the presence of light-quarks? No one knows . . . but $\bar{Q}Q + \mathbf{2} \times \bar{q}q$



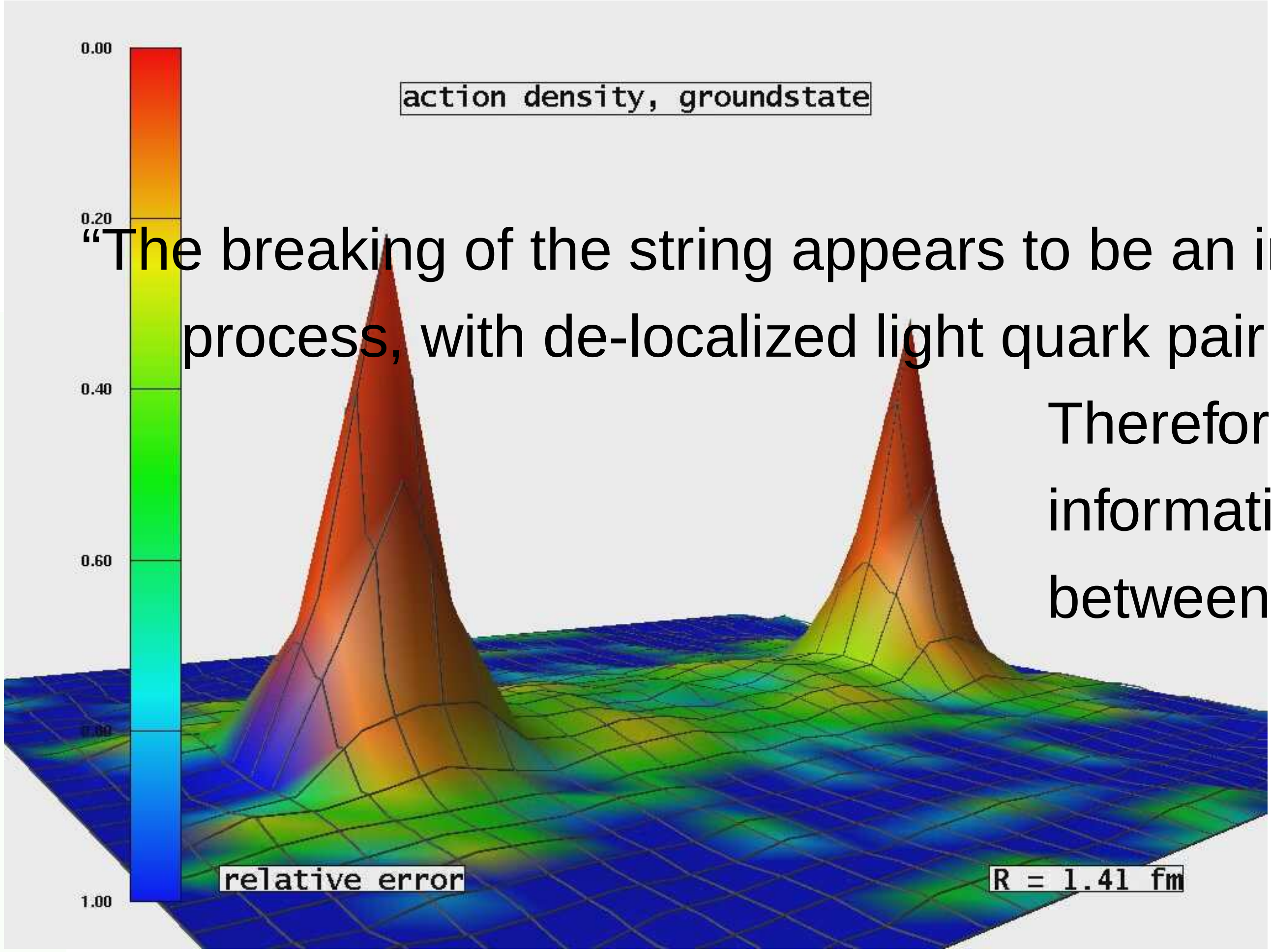
Bali, *et al*.
he-la/0512018

"The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation."

Confinement

- What happens in the real world; namely, in the presence of light-quarks? No one knows . . . but $\bar{Q}Q + 2 \times \bar{q}q$

Bali, *et al*.
he-la/0512018



"The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation."

Therefore . . . No information on *potential* between light-quarks.

What is the light-quark Long-Range Potential?

First Contents Back Conclusion

What is the light-quark Long-Range Potential?

Potential between static (infinitely heavy) quarks measured in simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

First Contents Back Conclusion

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \, \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\, \mathcal{S}^{-1}(k_-)$$

$$- M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels very different

but must be intimately related

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \, \Gamma^l_{5\mu}(k;P) \;=\; \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\,M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels very different

but must be intimately related

- Relation must be preserved by truncation

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE　　　　Satisfies DSE

Kernels very different

but must be intimately related

- Relation must be preserved by truncation
- Nontrivial constraint

First　Contents　Back　Conclusion

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \mathcal{S}^{-1}(k_-)$$

$$- M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE — Satisfies DSE

Kernels very different

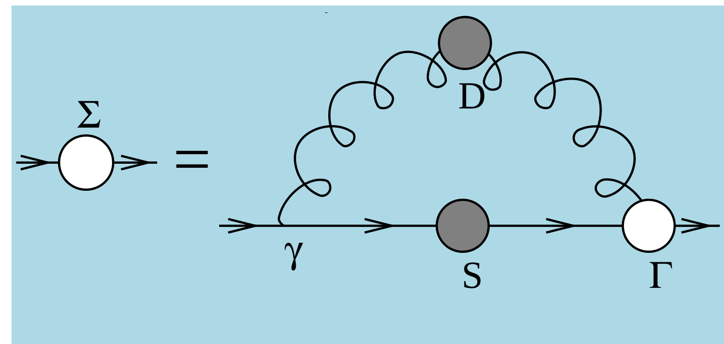
but must be intimately related

- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Persistent Challenge

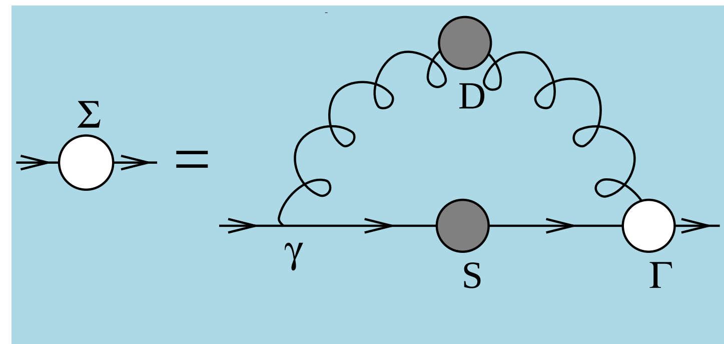
Persistent Challenge

- Infinitely Many Coupled Equations



Persistent Challenge

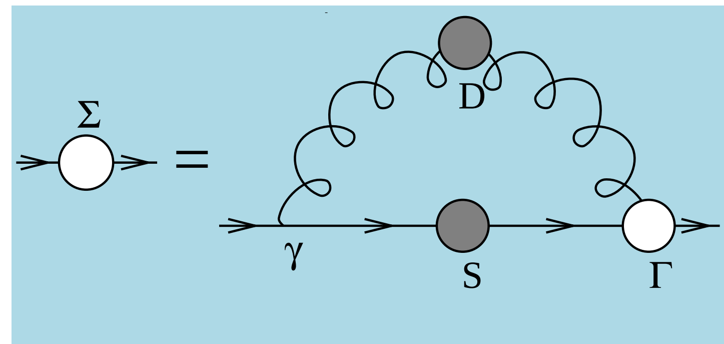
- Infinitely Many Coupled Equations



- Coupling between equations necessitates truncation

Persistent Challenge

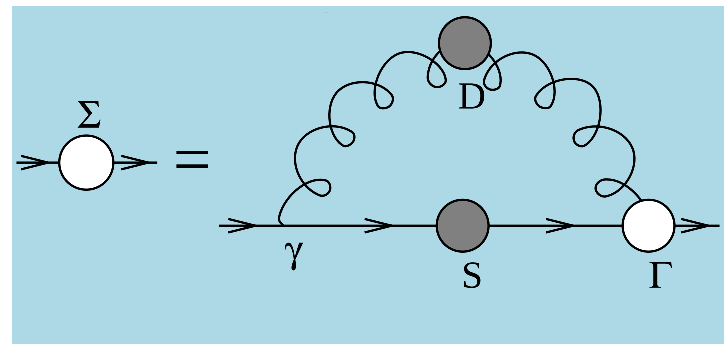
- Infinitely Many Coupled Equations



- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory

Persistent Challenge

- Infinitely Many Coupled Equations



- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
    Not useful for the nonperturbative problems
    in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of EXACT Results in QCD

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of EXACT Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of EXACT Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results
  - Make Predictions with Readily Quantifiable Errors

Radial Excitations
& Chiral Symmetry

First Contents Back Conclusion

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H$$

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

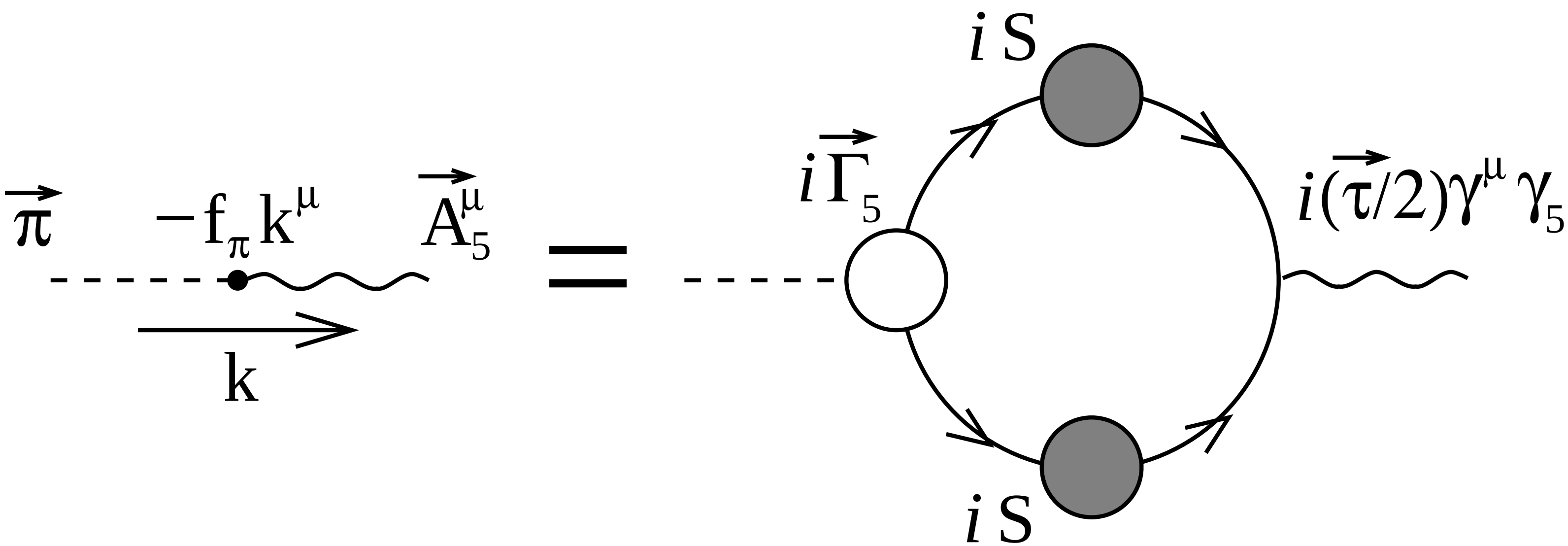
First Contents Back Conclusion

Radial Excitations & Chiral Symmetry

$$\boxed{f_H\, m_H^2 = -\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

$$f_H\, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5 \gamma_\mu\, \boxed{\mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-)} \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



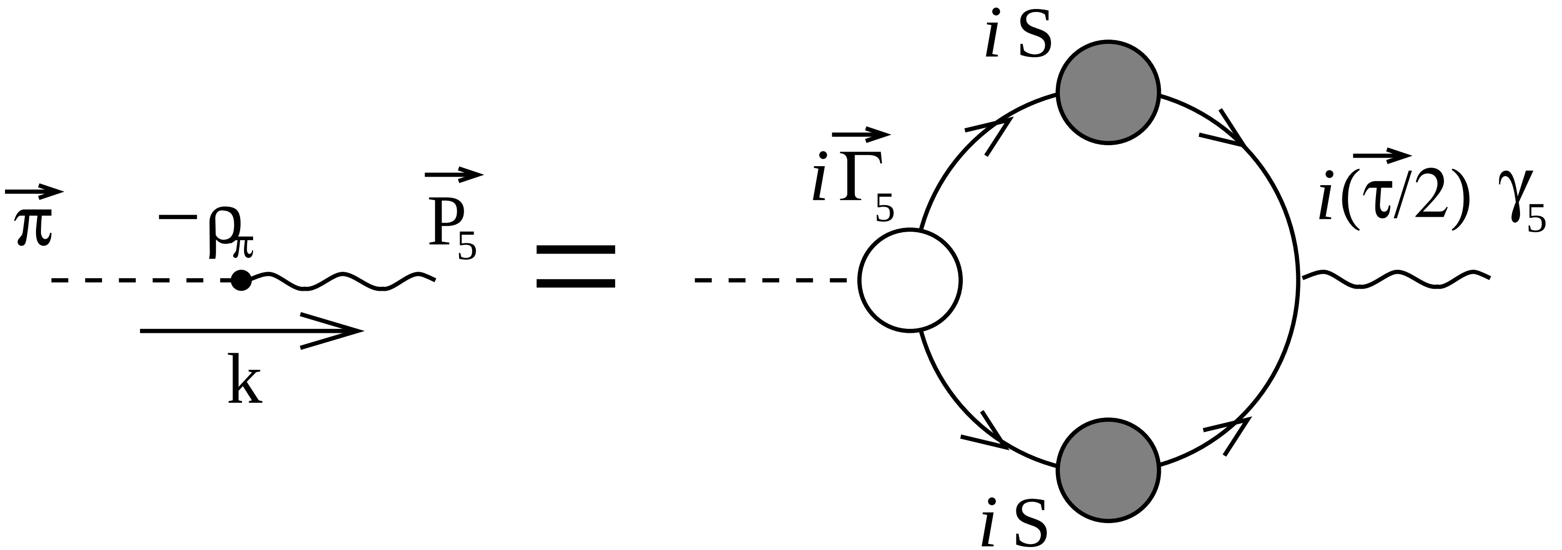
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \rho_\zeta^H \, \mathcal{M}_H$$

$$i\rho_\zeta^H = Z_4 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5 \, \mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-) \right\}$$

- Pseudoscalar projection of BS wave function at $x = 0$



Radial Excitations & Chiral Symmetry

$$f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \frac{-\langle \bar{q} q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \frac{-\langle \bar{q} q \rangle_\zeta^0}{(f_H^0)^2} m_q$ . . . GMOR relation, a corollary

Radial Excitations & Chiral Symmetry

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \frac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ . . . GMOR relation, a corollary

- Heavy-quark + light-quark

$\Rightarrow f_H \propto \frac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $m_H \propto m_q$

. . . QCD Proof of Potential Model result

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$

Höll, Krassnigg, Roberts
nu-th/0406030

Radial Excitations & Chiral Symmetry

$$f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson, not the ground state, so $m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0$, in chiral limit

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson, not the ground state, so $\boldsymbol{m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0}$, in chiral limit
- $\Rightarrow \boldsymbol{f_H = 0}$

ALL pseudoscalar mesons except $\boldsymbol{\pi(140)}$ in chiral limit

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson, not the ground state, so $\boldsymbol{m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0}$, in chiral limit
- $\Rightarrow \boldsymbol{f_H = 0}$

  ALL pseudoscalar mesons except $\boldsymbol{\pi(140)}$ in chiral limit
- Dynamical Chiral Symmetry Breaking

  – Goldstone's Theorem –

impacts upon *every* pseudoscalar meson

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”*.

First Contents Back Conclusion

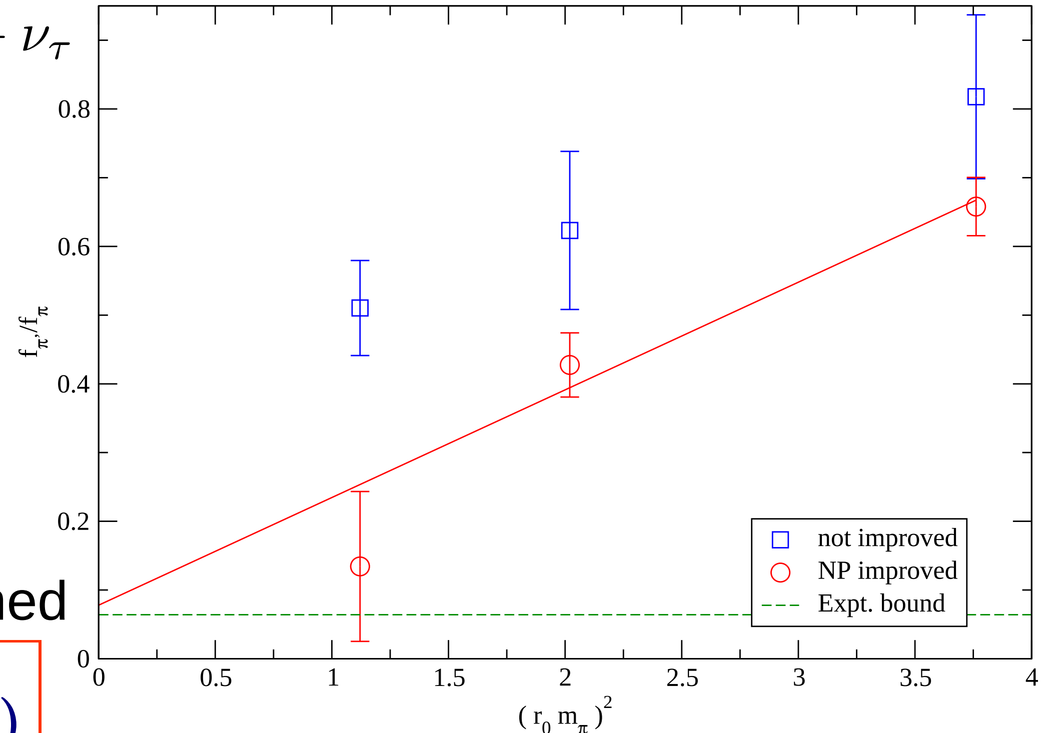
Radial Excitations & Lattice-QCD

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable".*
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4$ MeV
  Diehl & Hiller
  he-ph/0105194

Radial Excitations & Lattice-QCD

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable".*
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4$ MeV
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1$ fm,
  two-flavour, unquenched

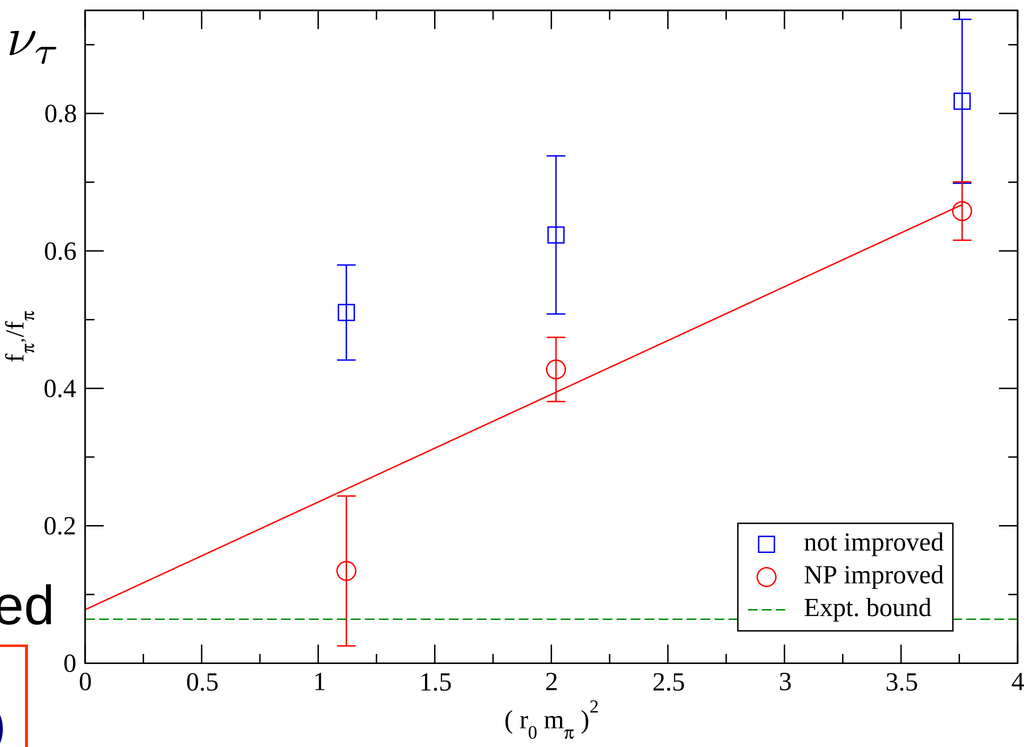
$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = \mathbf{0.078\,(93)}$$



Radial Excitations & Lattice-QCD

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable".*
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4$ MeV
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1$ fm,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = \mathbf{0.078\,(93)}$$



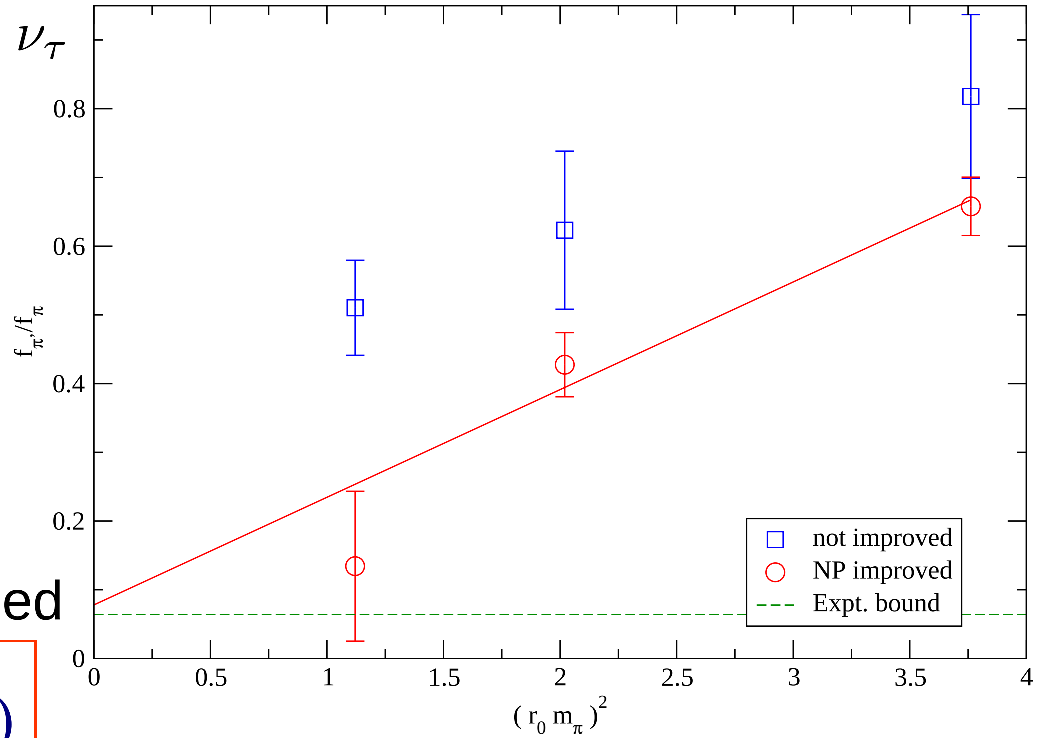
- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.*
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4$ MeV
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1$ fm,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$



- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Pseudoscalar meson Bethe-Salpeter amplitude

$$\begin{aligned}\chi_\pi(k;P) &= \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P \mathcal{F}_{\pi_n}(k;P) \right. \\ &\left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P) \right]\end{aligned}$$

Pion . . . $J = 0$ but . . .

Pseudoscalar meson Bethe-Salpeter amplitude

$$\chi_\pi(k;P) = \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P \mathcal{F}_{\pi_n}(k;P) \right.$$
$$\left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P) \right]$$

- Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.

First Contents Back Conclusion

Pion . . . $J = 0$ but . . .

Pseudoscalar meson Bethe-Salpeter amplitude

$$\begin{aligned}\chi_\pi(k;P) &= \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P \mathcal{F}_{\pi_n}(k;P) \right. \\ &\left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P) \right]\end{aligned}$$

- Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.

First Contents Back Conclusion

Pion ... $J = 0$ but ...

Pseudoscalar meson Bethe-Salpeter amplitude

$$\begin{aligned}\chi_\pi(k;P) &= \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P\mathcal{F}_{\pi_n}(k;P) \right. \\ & \left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P)\right]\end{aligned}$$

- In QCD, a Poincaré invariant theory with interactions, $\mathcal{E} \neq 0$ forces nonzero results for $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$

First Contents Back Conclusion

Pion . . . $J = 0$
but . . .

Pseudoscalar meson Bethe-Salpeter amplitude

$$\begin{aligned}\chi_\pi(k;P) &= \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P\mathcal{F}_{\pi_n}(k;P) \right. \\ & \left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P) \right]\end{aligned}$$

- In QCD, a Poincaré invariant theory with interactions, $\mathcal{E} \neq 0$ forces nonzero results for $\mathcal{F}$, $\mathcal{G}$ and $\mathcal{H}$
- $J = 0 \ldots$ *but* while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both S- and P-wave components.

Pion . . . $J = 0$ *but . . .*

- $J = 0 \ldots$ *but* while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both S- and P-wave components.

Introduce mixing angle θ_π such that

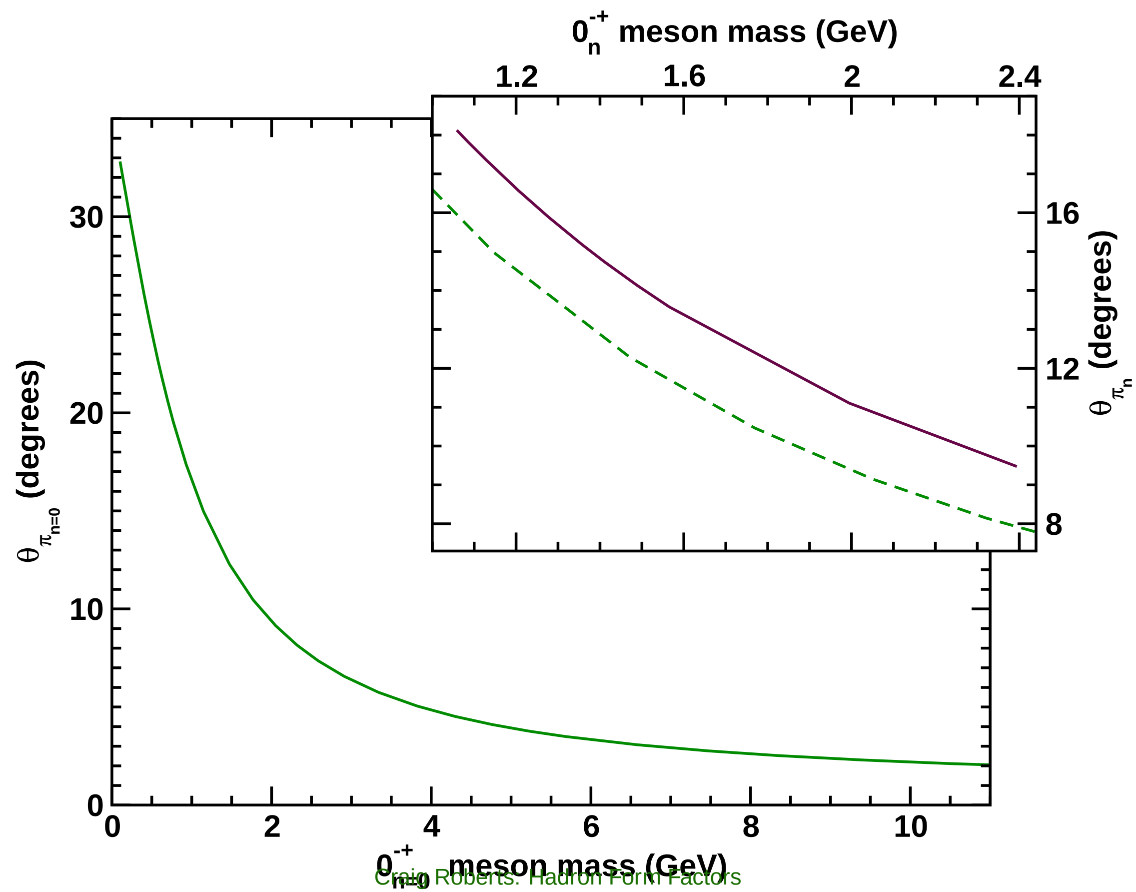
$$\chi_\pi \sim \cos\theta_\pi |L = 0\rangle + \sin\theta_\pi |L = 1\rangle$$

Pion . . . $J = 0$ but . . .

- $J = 0$ . . . *but* while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both S- and P-wave components.

Introduce mixing angle θ_π such that

$$\chi_\pi \sim \cos\theta_\pi |L=0\rangle + \sin\theta_\pi |L=1\rangle$$



First Contents Back Conclusion

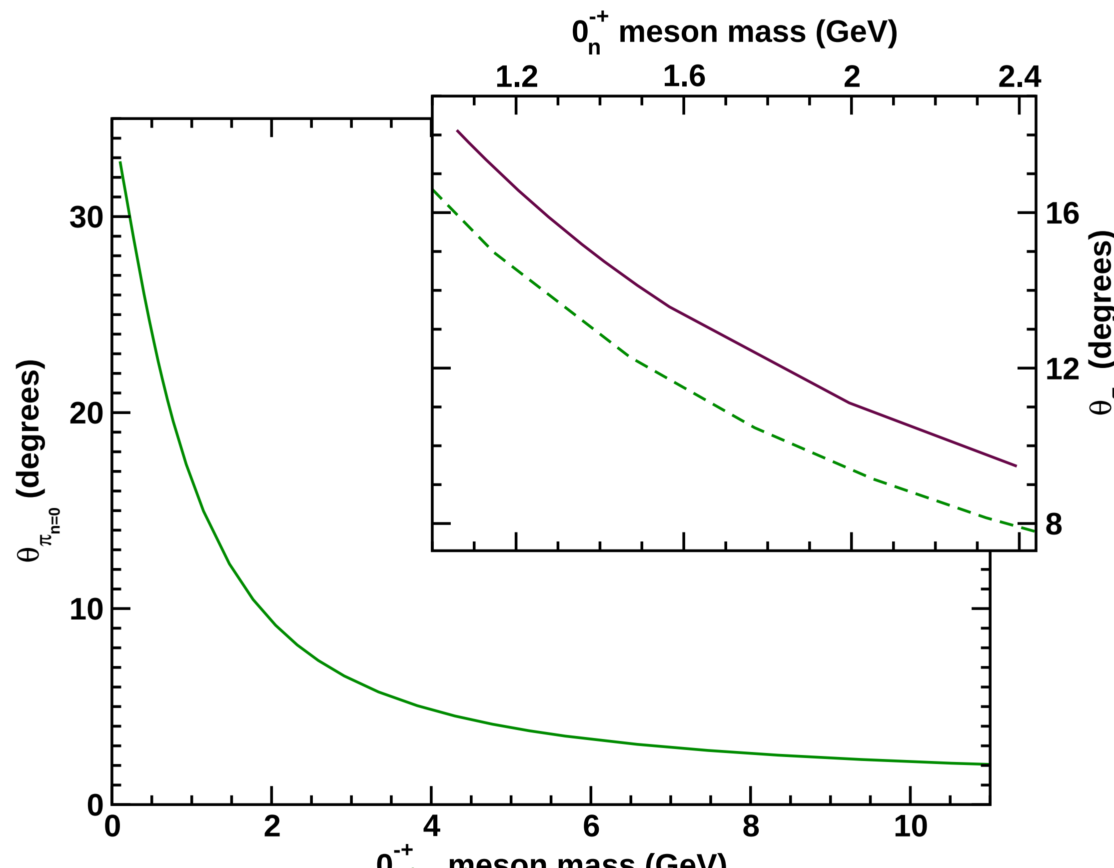
Pion . . . $J = 0$
but . . .

- $J = 0 \ldots$ *but* while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both S- and P-wave components.

Introduce mixing angle θ_π such that

$$\chi_\pi \sim \cos\theta_\pi |L=0\rangle + \sin\theta_\pi |L=1\rangle$$

L is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.



Charge Neutral Pseudoscalar Mesons

non-Abelian Anomaly and η-η' mixing

non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'

non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
  Problem: η' is a pseudoscalar meson but it's much more massive than the other eight constituted from light-quarks.

non-Abelian Anomaly and η-η' mixing

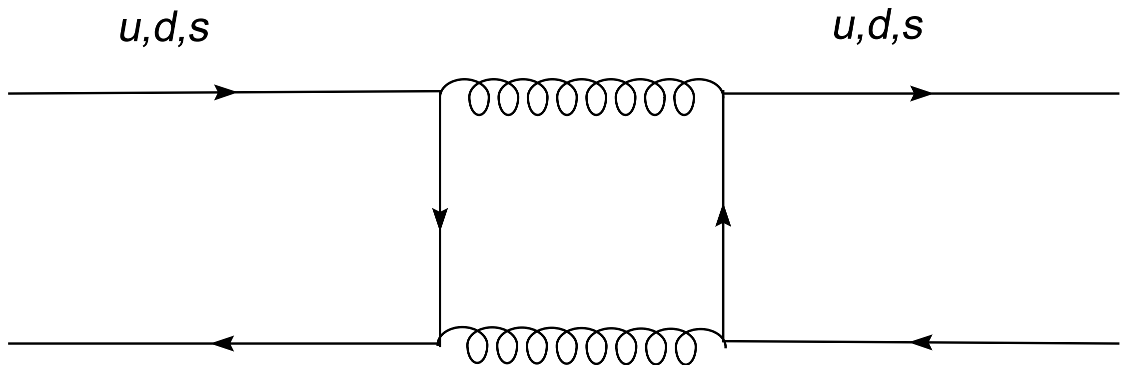
- Mesons containing $\bar{s}$-s are special: η & η'
  Problem: η' is a pseudoscalar meson but it's much more massive than the other eight constituted from light-quarks.
  Origin: While the classical action associated with QCD is invariant under $U_A(1)$ (Abelian axial transformations generated by $\lambda^0\gamma_5$), the quantum field theory is not!

non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
- Flavour mixing takes place in singlet channel: $\lambda^0 \Leftrightarrow \lambda^8$

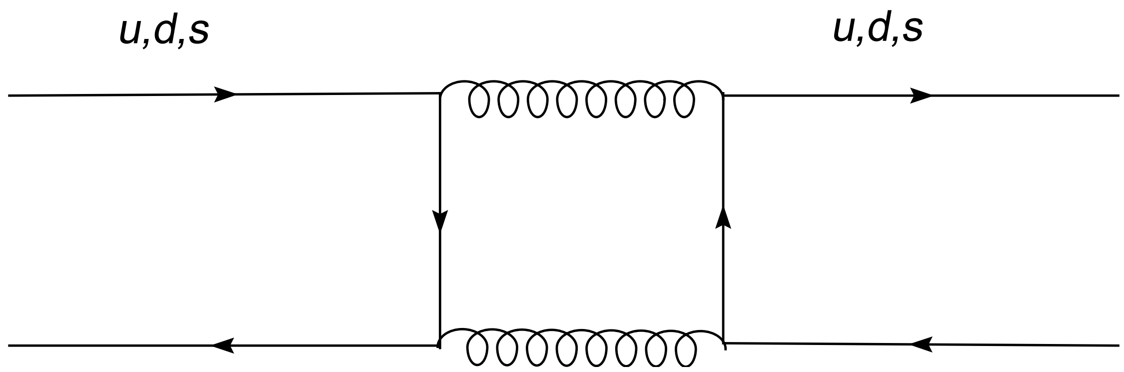
non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
- Flavour mixing takes place in singlet channel: $\lambda^0 \Leftrightarrow \lambda^8$



non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'



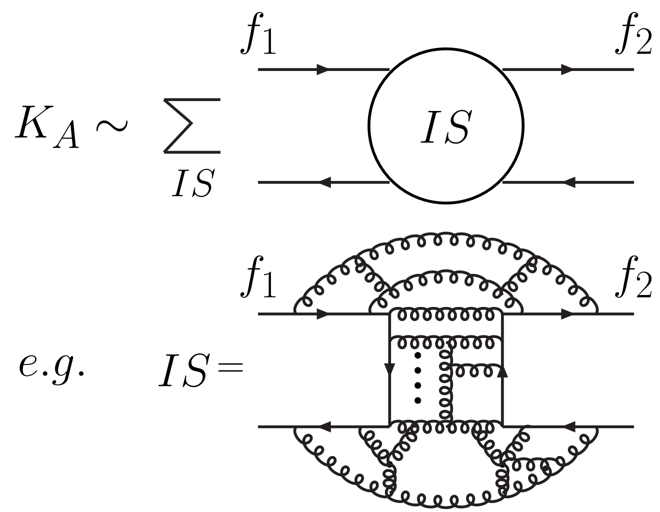
- This is a perturbative diagram.
  It has almost nothing to do with $\eta \Leftrightarrow \eta'$ mixing.

non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
- Driver is the non-Abelian anomaly

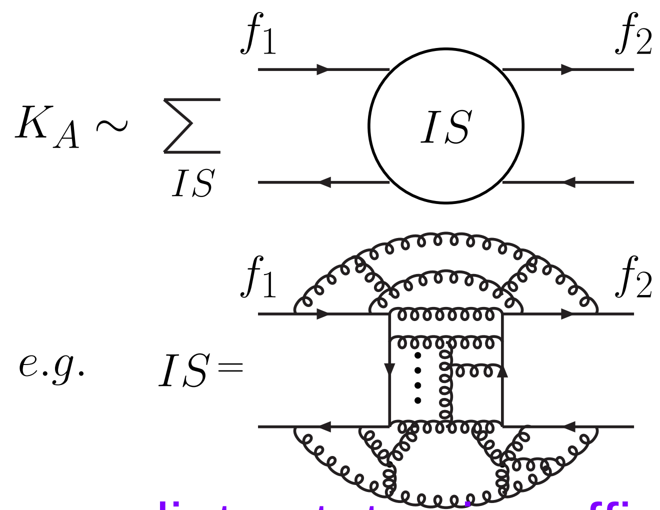
non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
- Driver is the non-Abelian anomaly



non-Abelian Anomaly and η-η' mixing

- Mesons containing $\bar{s}$-s are special: η & η'
- Driver is the non-Abelian anomaly
- Contribution to the Bethe-Salpeter kernel associated with the non-Abelian anomaly. All terms have the "hairpin" structure.



No finite sum of such intermediate states is sufficient to veraciously represent the anomaly.

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

First Contents Back Conclusion

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\{\mathcal{F}^a \,|\, a = 0,\ldots,N_f^2-1\}$ are the generators of $U(N_f)$
- $\mathcal{S} =$ diag$[S_u, S_d, S_s, S_c, S_b, \ldots]$
- $\mathcal{M}^{ab} = \mathrm{tr}_F\left[\{\mathcal{F}^a, \mathcal{M}\}\mathcal{F}^b\right]$,
  $\mathcal{M} = \mathrm{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] =$ matrix of current-quark bare masses

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\{\mathcal{F}^a \,|\, a = 0, \ldots, N_f^2 - 1\}$ are the generators of $U(N_f)$
- $\mathcal{S} =$ diag$[S_u, S_d, S_s, S_c, S_b, \ldots]$
- $\mathcal{M}^{ab} = \mathrm{tr}_F \left[\{\mathcal{F}^a, \mathcal{M}\} \mathcal{F}^b \right]$,
  $\mathcal{M} = \mathrm{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] =$ matrix of current-quark bare masses
- The final term in the second line expresses the non-Abelian axial anomaly.

Charge Neutral Pseudoscalar Mesons

$$\begin{aligned} P_\mu \Gamma^a_{5\mu}(k;P) &= \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\ &\quad -2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P) \end{aligned}$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\, \delta^{a0}\, \mathcal{A}_U(k;P) \mathcal{S}^{-1}(k_-)$
  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y) \rangle$

Charge Neutral Pseudoscalar Mesons

$$
\begin{aligned}
P_\mu \Gamma^a_{5\mu}(k;P) &= \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\
&\quad -2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)
\end{aligned}
$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$
  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y)\rangle$
- $\mathcal{Q}(x) = i\dfrac{\alpha_s}{4\pi} \mathrm{tr}_C\left[\epsilon_{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}(x)\right] = \partial_\mu K_\mu(x)$

… The topological charge density operator.

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-)$$
$$- 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$
  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y)\rangle$
- $\mathcal{Q}(x) = i\frac{\alpha_s}{4\pi}\mathrm{tr}_C\left[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}(x)\right] = \partial_\mu K_\mu(x)$

  . . . The topological charge density operator.
  (Trace is over colour indices & $F_{\mu\nu} = \frac{1}{2}\lambda^a F^a_{\mu\nu}$.)

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-)$$
$$-2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$
  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y)\rangle$
- $\mathcal{Q}(x) = i\frac{\alpha_s}{4\pi}\mathrm{tr}_C\left[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}(x)\right] = \partial_\mu K_\mu(x)$

  … The topological charge density operator.
- Important that only $\mathcal{A}^{a=0}$ is nonzero.

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\, \delta^{a0}\, \mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$
  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y)\rangle$
- $\mathcal{Q}(x) = i\frac{\alpha_s}{4\pi}\mathrm{tr}_C\left[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}(x)\right] = \partial_\mu K_\mu(x)$

  … The topological charge density operator.
- NB. While $\mathcal{Q}(x)$ is gauge invariant, the associated Chern-Simons current, K_μ, is not $\Rightarrow$ in QCD *no physical* boson can couple to K_μ and hence *no physical* states can contribute to resolution of U$_A$(1) problem.

Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

First Contents Back Conclusion

Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting

Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting ... otherwise all pseudoscalar mesons are Goldstone Modes!

Charge Neutral Pseudoscalar Mesons

- Anomaly term has structure

$$\mathcal{A}^0(k;P) = \mathcal{F}^0\gamma_5\left[i\mathcal{E}_\mathcal{A}(k;P) + \gamma\cdot P\mathcal{F}_\mathcal{A}(k;P)\right.$$
$$\left. +\gamma\cdot k k\cdot P\mathcal{G}_\mathcal{A}(k;P) + \sigma_{\mu\nu}k_\mu P_\nu \mathcal{H}_\mathcal{A}(k;P)\right]$$

Charge Neutral Pseudoscalar Mesons

- AVWTI gives generalised Goldberger-Treiman relations

$$
\begin{aligned}
2 f_{\eta'}^{0} E_{BS}(k;0) &= 2 B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0), \\
F_R^0(k;0) + 2 f_{\eta'}^{0} F_{BS}(k;0) &= A_0(k^2) - \mathcal{F}_{\mathcal{A}}(k;0), \\
G_R^0(k;0) + 2 f_{\eta'}^{0} G_{BS}(k;0) &= 2 A_0'(k^2) - \mathcal{G}_{\mathcal{A}}(k;0), \\
H_R^0(k;0) + 2 f_{\eta'}^{0} H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0),
\end{aligned}
$$

A_0, B_0 characterise gap equation's chiral limit solution.

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- AVWTI gives generalised Goldberger-Treiman relations

$$
\begin{aligned}
2 f_{\eta'}^0 E_{BS}(k;0) &= 2B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0), \\
F_R^0(k;0) + 2 f_{\eta'}^0 F_{BS}(k;0) &= A_0(k^2) - \mathcal{F}_{\mathcal{A}}(k;0), \\
G_R^0(k;0) + 2 f_{\eta'}^0 G_{BS}(k;0) &= 2A_0'(k^2) - \mathcal{G}_{\mathcal{A}}(k;0), \\
H_R^0(k;0) + 2 f_{\eta'}^0 H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0),
\end{aligned}
$$

A_0, B_0 characterise gap equation's chiral limit solution.

- Follows that $\boldsymbol{\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)}$ is necessary and sufficient condition for absence of massless $\boldsymbol{\eta'}$ bound-state.

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- $\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)$
  Discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
  - Hence, absence of massless η' bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.

First | Contents | Back | Conclusion

Charge Neutral Pseudoscalar Mesons

- $\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)$
  Discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
  - Hence, absence of massless η' bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.
- Further highlighted ... proved

$$\begin{aligned} \langle \bar{q}q \rangle_\zeta^0 &= -\lim_{\Lambda\to\infty} Z_4(\zeta^2,\Lambda^2)\, \mathrm{tr}_{\mathrm{CD}} \int_q^\Lambda S^0(q,\zeta) \\ &= N_f \int d^4x\, \langle \bar{q}(x) i\gamma_5 q(x) \mathcal{Q}(0) \rangle^0 . \end{aligned}$$

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons

First Contents Back Conclusion

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly

First Contents Back Conclusion

Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
- Employed in an analysis of pseudoscalar- and vector-meson bound-states

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
- Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts
  - η–η' mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)
  - π^0–η angles of $\sim 1.2^\circ$ (Expt. $p\,d \to {}^3$He π^0: $0.6^\circ \pm 0.3^\circ$)
  - Strong neutron-proton mass difference . . .

    $\lesssim 75\,$% current-quark mass-difference

First | Contents | Back | Conclusion

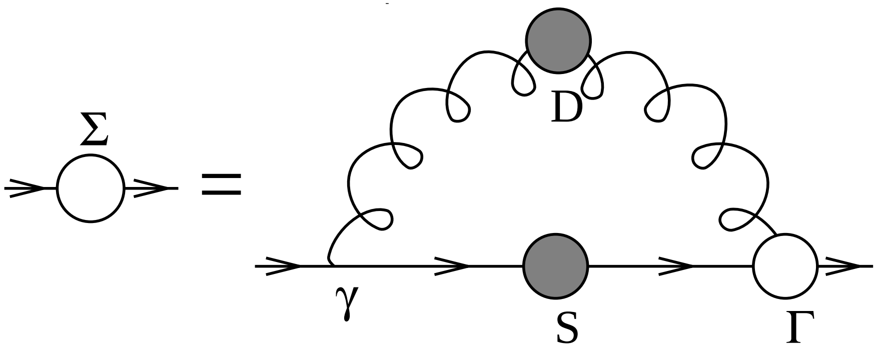
Pion Form Factor

Procedure Now Straightforward

First | Contents | Back | Conclusion

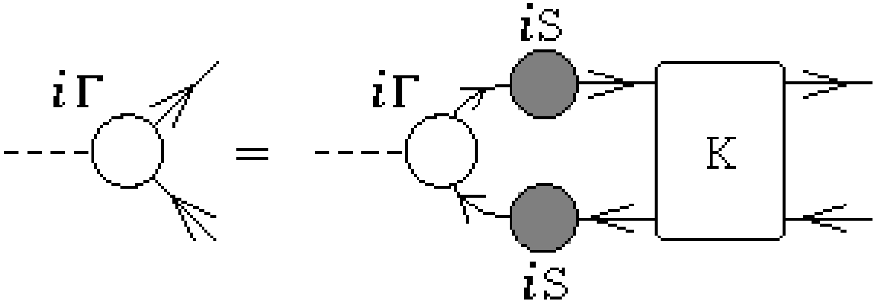
- Solve Gap Equation

  $\Rightarrow$ Dressed-Quark Propagator, $S(p)$



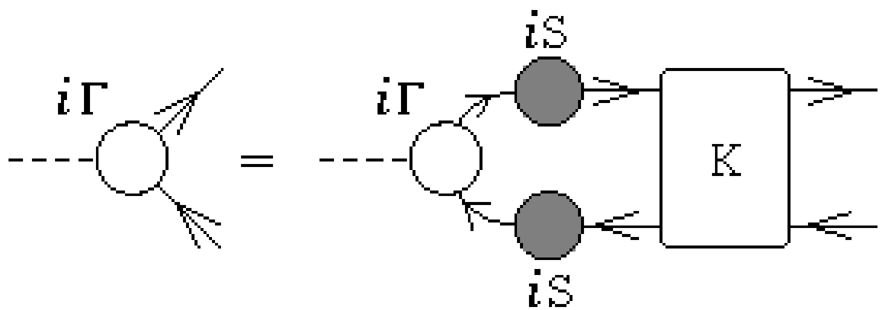
Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, K
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π



Pion Form Factor

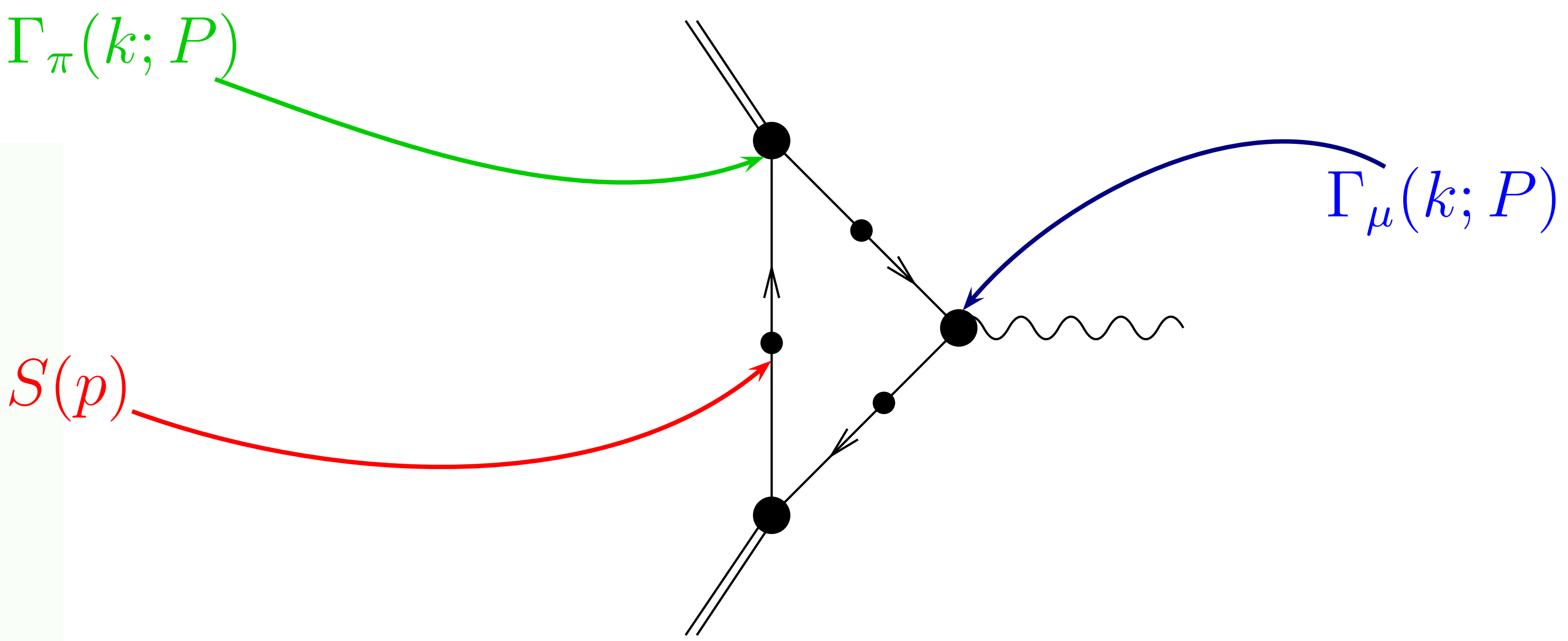
- Use that to Complete Bethe Salpeter Kernel, K
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π



- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Photon Vertex, Γ_μ

Pion Form Factor

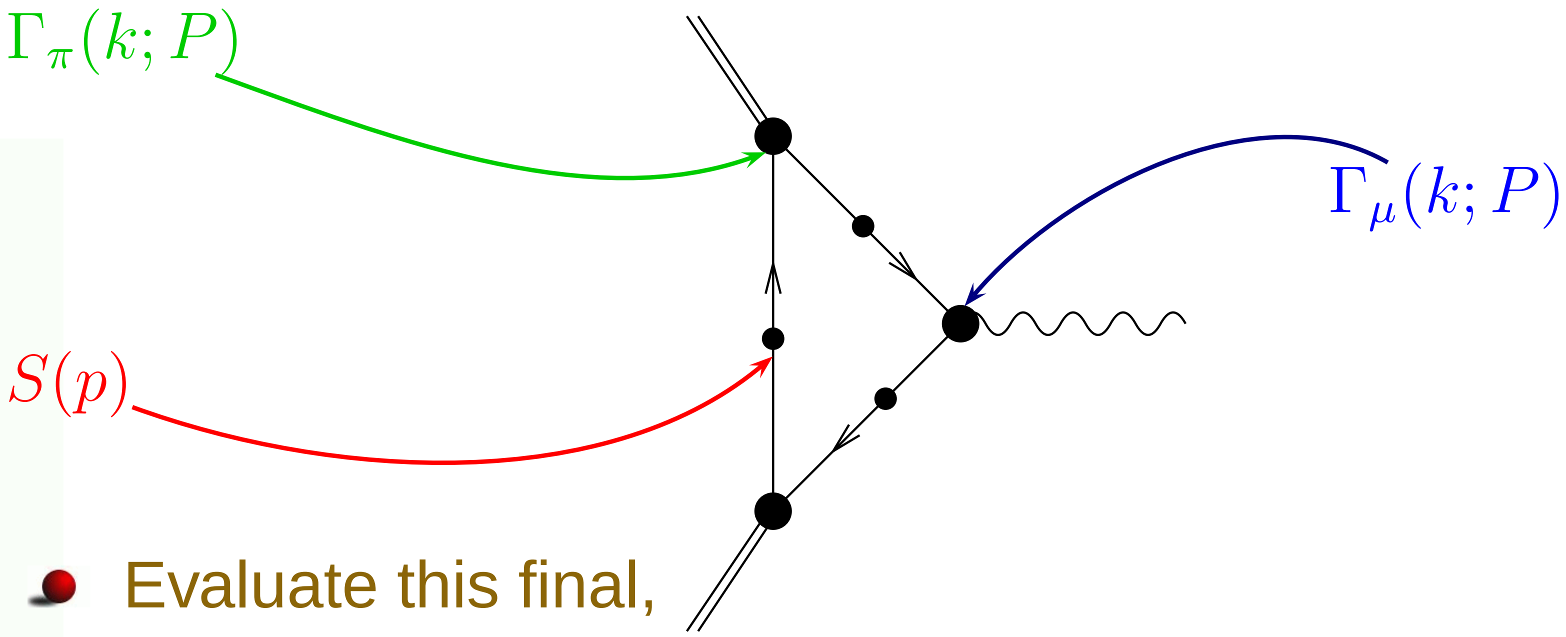
- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



First Contents Back Conclusion

Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor

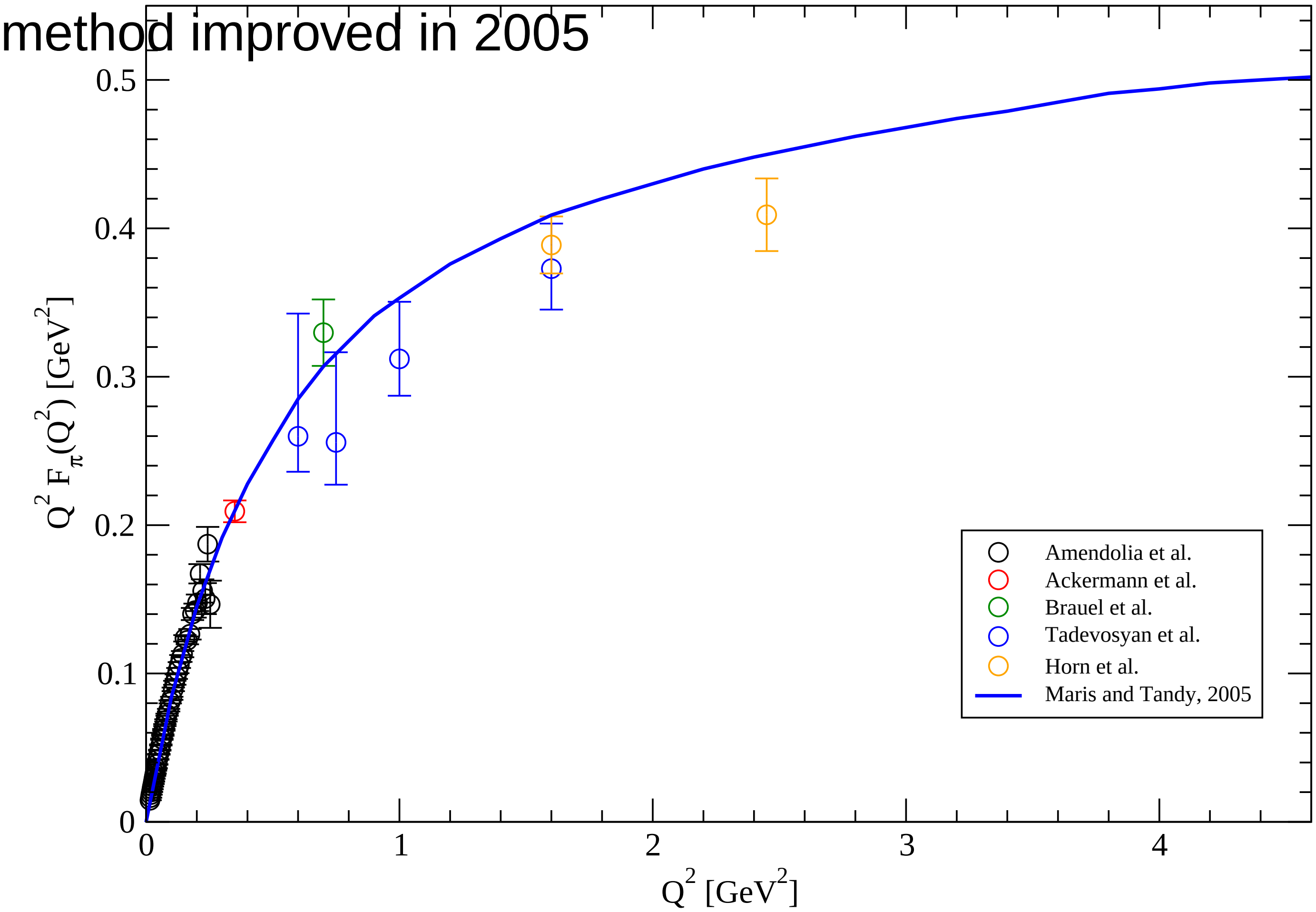


- Evaluate this final, three-dimensional integral

Calculated Pion Form Factor

Calculation first published in 1999; No Parameters Varied

Numerical method improved in 2005



Office of Science U.S. Department of Energy

Office of Nuclear Physics

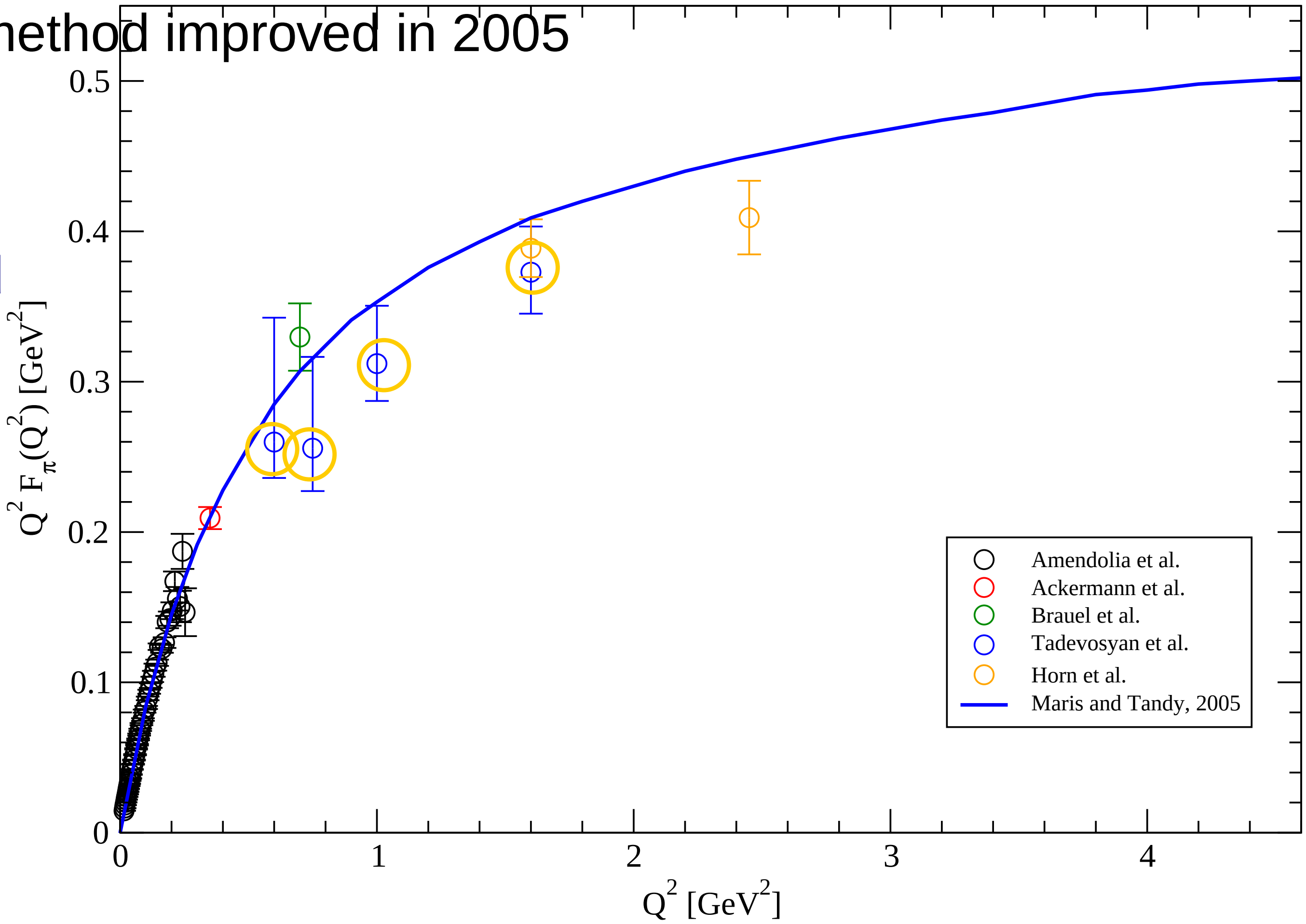
Argonne National Laboratory

Calculated Pion Form Factor

Calculation first published in 1999; No Parameters Varied

Numerical method improved in 2005

Data published in 2001. Subsequently revised



Office of Science U.S. DEPARTMENT OF ENERGY

Office of Nuclear Physics

Argonne NATIONAL LABORATORY

First Contents Back Conclusion

Timelike Pion Form Factor

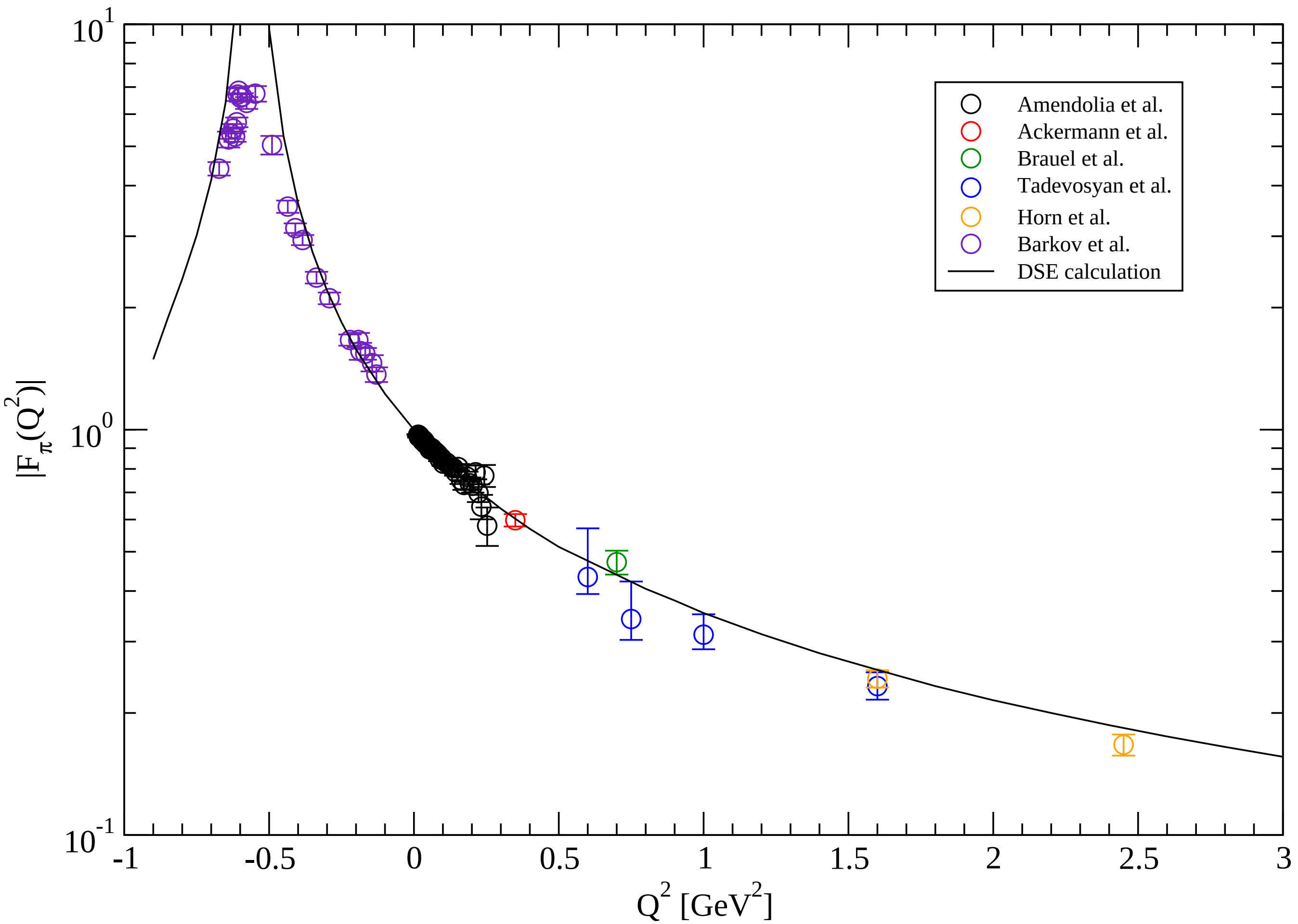
Timelike Pion Form Factor

- *Ab initio* calculation into timelike region
- Deeper than ground-state ρ-meson pole

Timelike Pion Form Factor

Ab initio calculation into timelike region

Deeper than ground-state ρ-meson pole

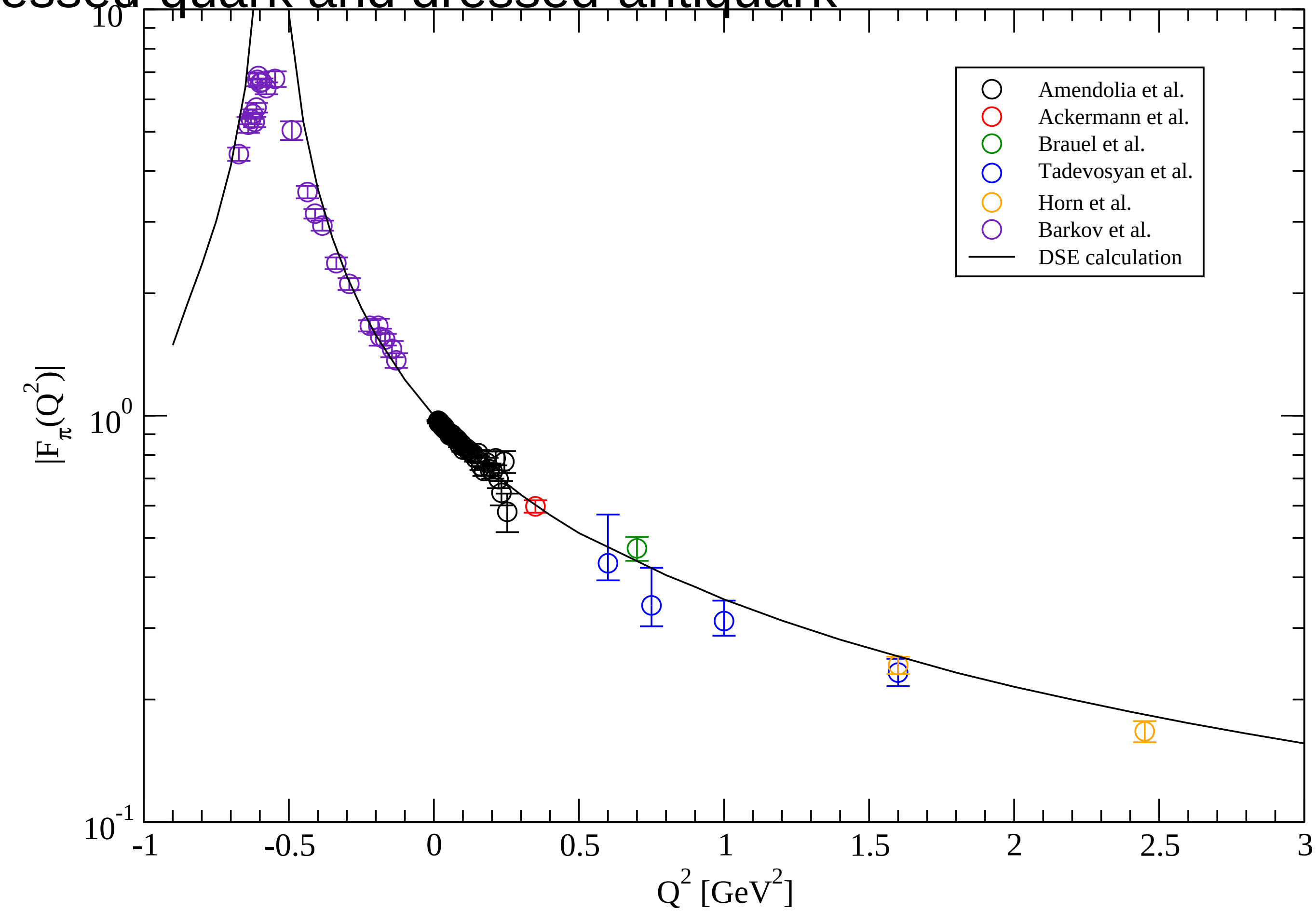


Timelike Pion Form Factor

Ab initio calculation into timelike region

Deeper than ground-state ρ-meson pole

ρ-meson not put in "by hand" – generated dynamically as a bound-state of dressed-quark and dressed-antiquark



First Contents Back Con

Dimensionless product: $r_\pi \, f_\pi$

First Contents Back Conclusion

Dimensionless product: $r_\pi \, f_\pi$

First Contents Back Conclusion

Dimensionless product: $r_\pi\, f_\pi$

- Improved rainbow-ladder interaction

Dimensionless product: $r_\pi \, f_\pi$

- Improved rainbow-ladder interaction
- Repeating $F_\pi(Q^2)$ calculation

Dimensionless product: $r_\pi\, f_\pi$

- Improved rainbow-ladder interaction
- Repeating $F_\pi(Q^2)$ calculation
- Great strides towards placing nucleon studies on same footing as mesons

Dimensionless product: $r_\pi \, f_\pi$

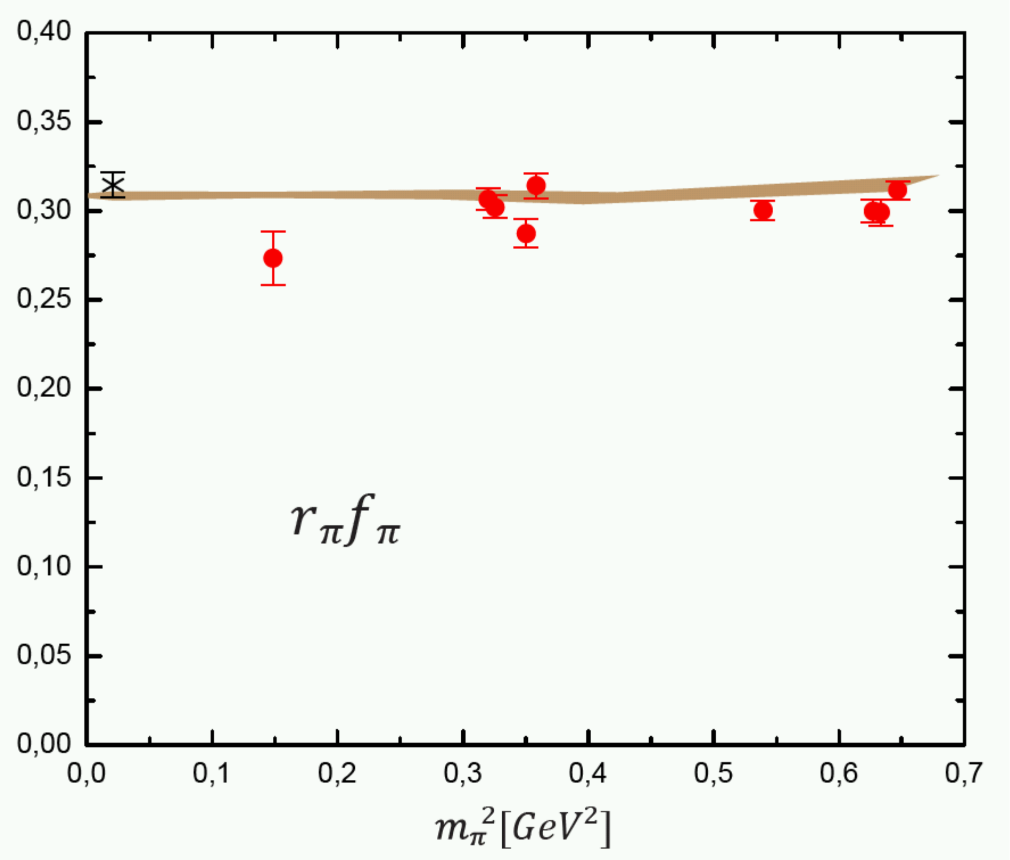
- Improved rainbow-ladder interaction
- Repeating $F_\pi(Q^2)$ calculation
- Experimentally: $r_\pi f_\pi = 0.315 \pm 0.005$

First Contents Back Conclusion

Dimensionless product: $r_\pi\, f_\pi$

- Improved rainbow-ladder interaction
- Repeating $F_\pi(Q^2)$ calculation
- Experimentally: $r_\pi f_\pi = 0.315 \pm 0.005$

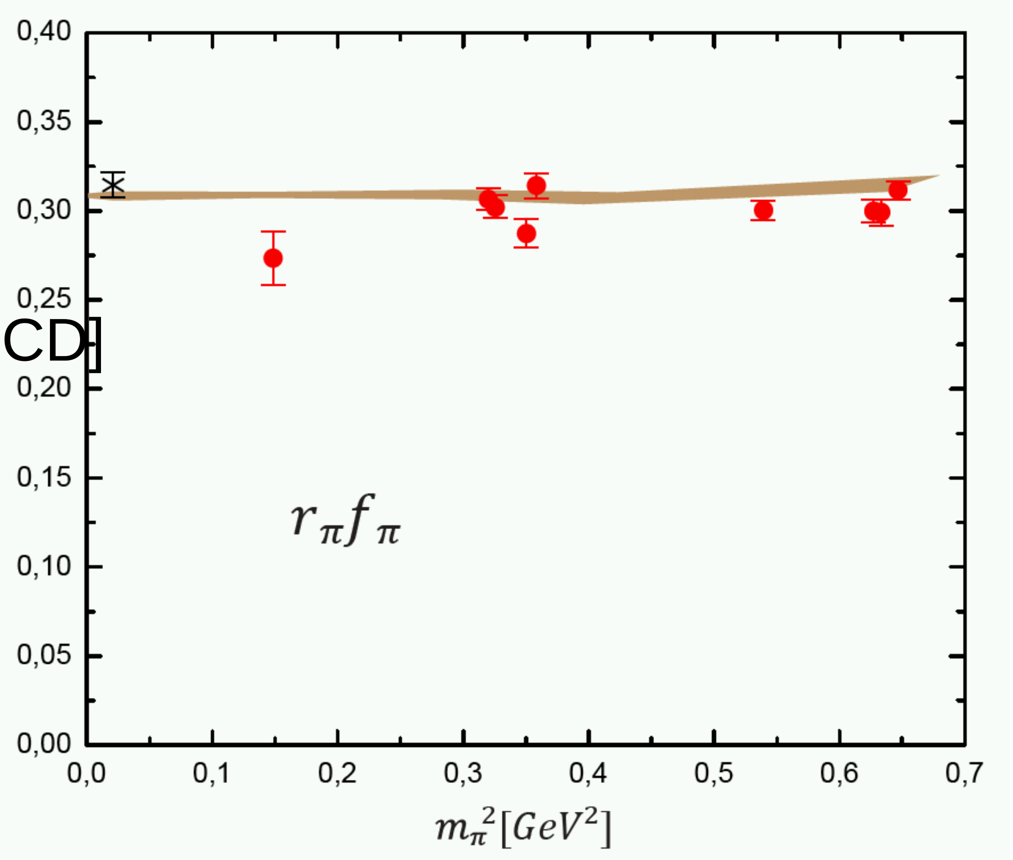
- DSE prediction



First Contents Back Conclusion

Dimensionless product: $r_\pi\, f_\pi$

- Improved rainbow-ladder interaction
- Repeating $F_\pi(Q^2)$ calculation
- Experimentally: $r_\pi f_\pi = 0.315 \pm 0.005$

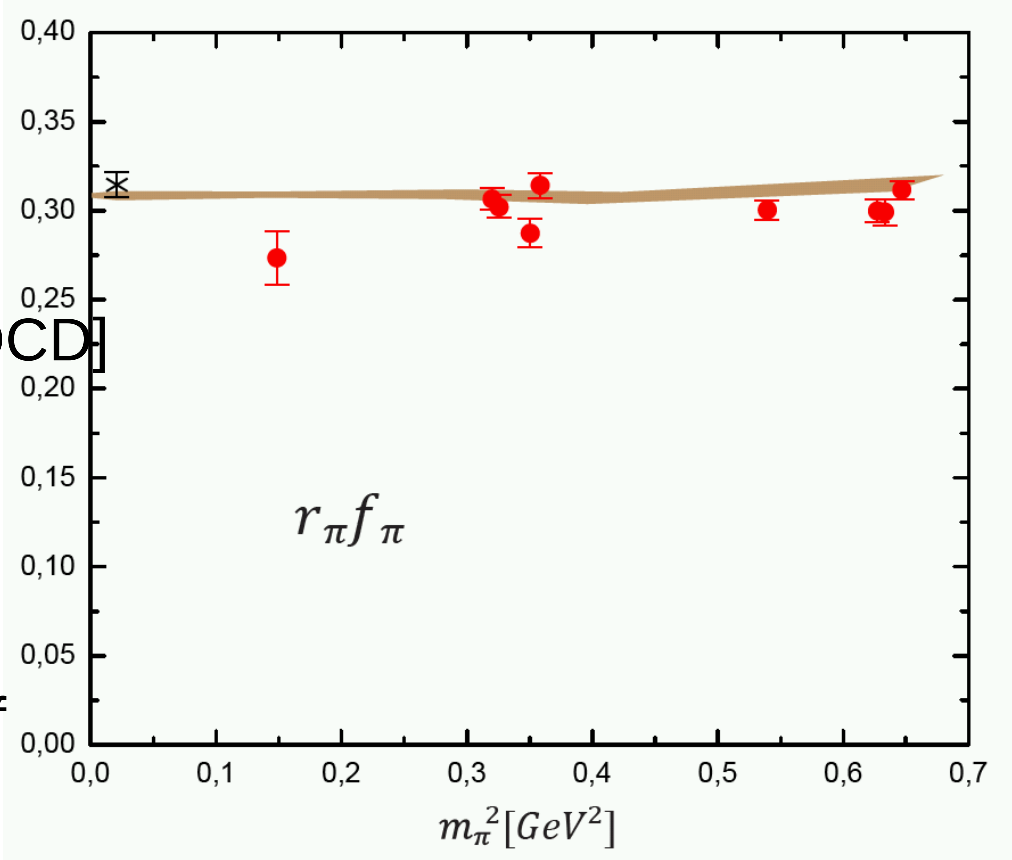


- DSE prediction
- Lattice results
  – James Zanotti [UK QCD]

First Contents Back Conclusion

Dimensionless product: $r_\pi\, f_\pi$

- Improved rainbow-ladder interaction
- Repeating $F_\pi(Q^2)$ calculation
- Experimentally: $r_\pi f_\pi = 0.315 \pm 0.005$



- DSE prediction
- Lattice results

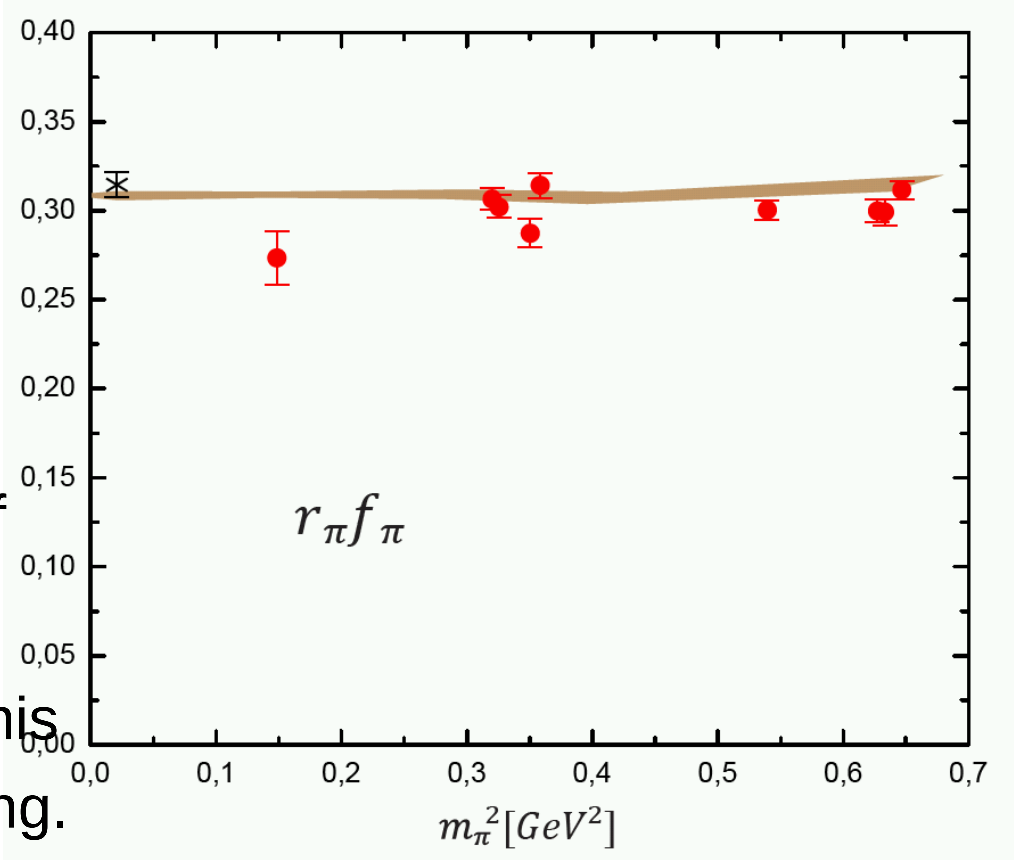
  – James Zanotti [UK QCD]

- Fascinating result: DSE and Lattice

  – Experimental value obtains independent of current-quark mass.

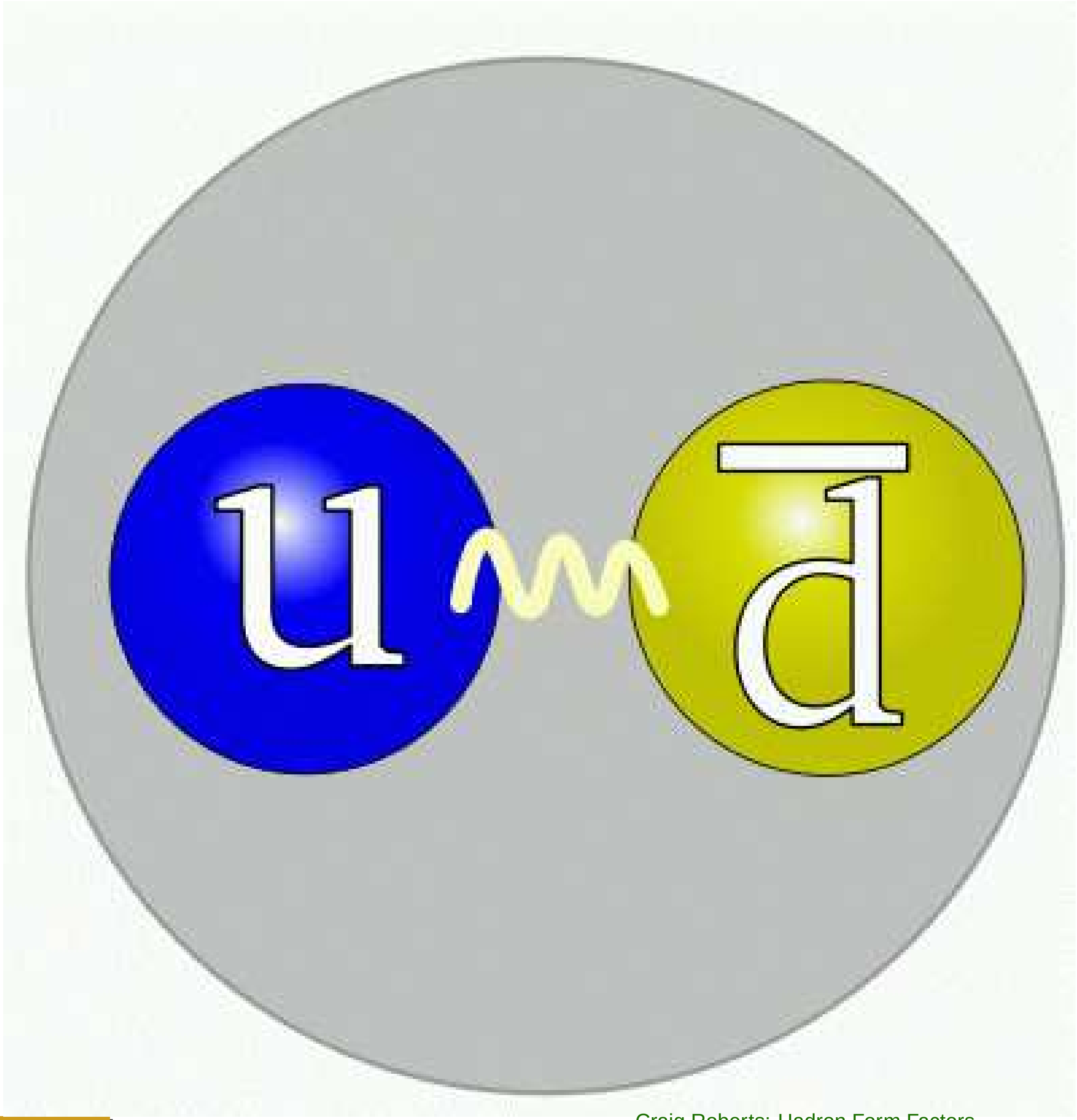
Dimensionless product: $r_\pi\, f_\pi$

- Improved rainbow-ladder interaction
- Repeating $F_\pi(Q^2)$ calculation
- Experimentally: $r_\pi f_\pi = 0.315 \pm 0.005$



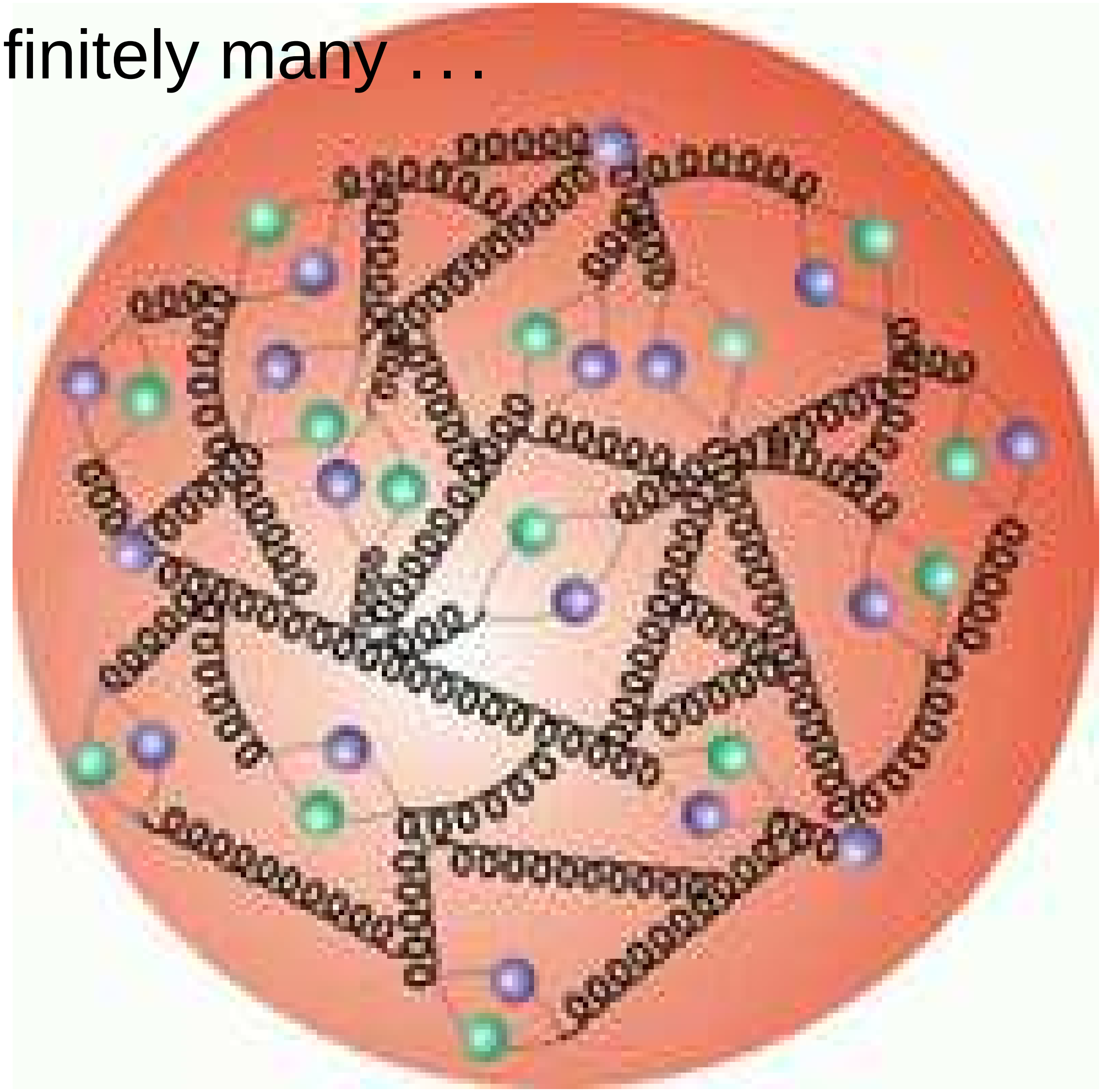
- DSE prediction
- Fascinating result: DSE and Lattice – Experimental value obtains independent of current-quark mass.
- We have understood this. Implications far-reaching.

Answer for the pion



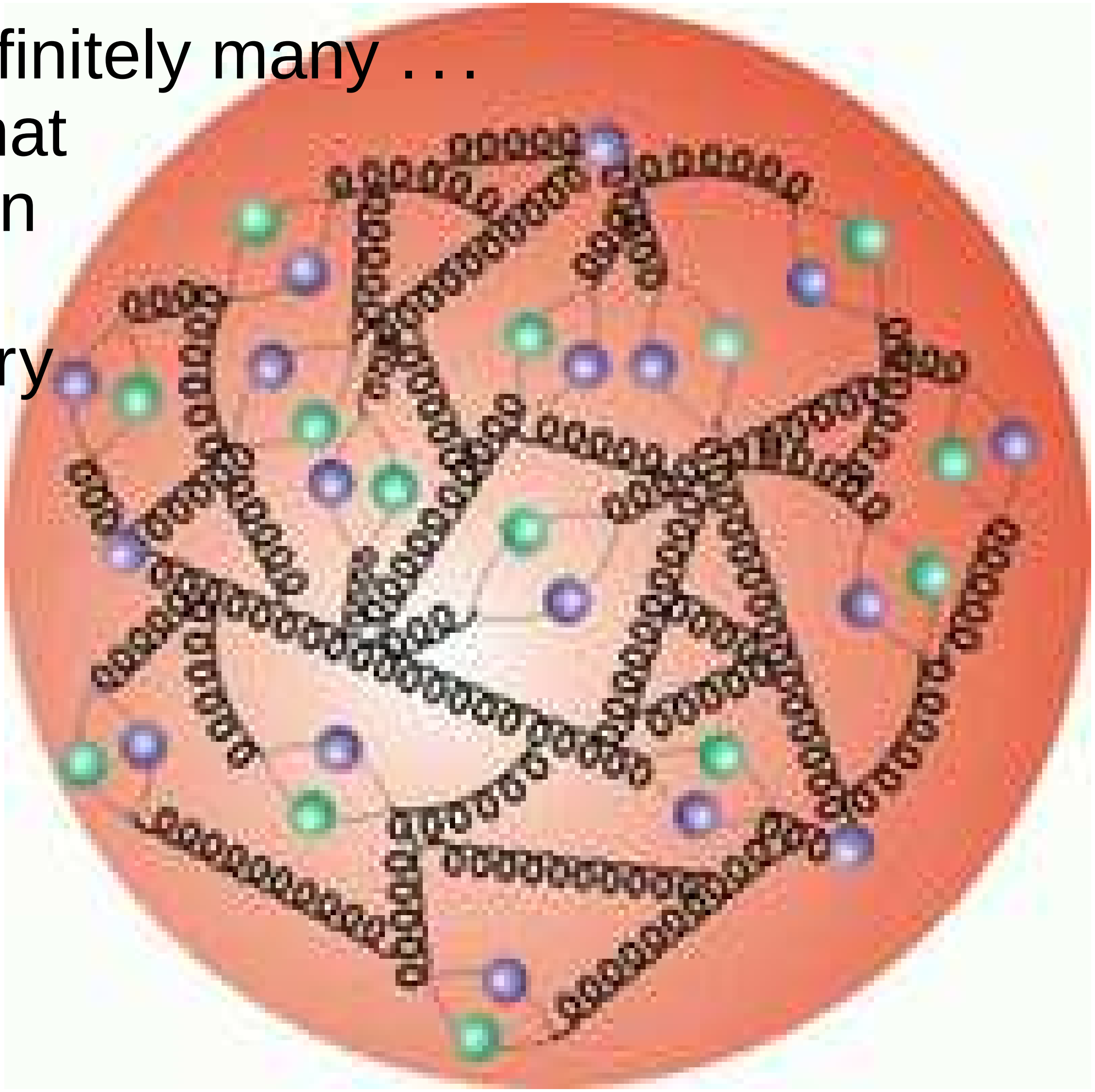
Answer for the pion

Two → Infinitely many . . .



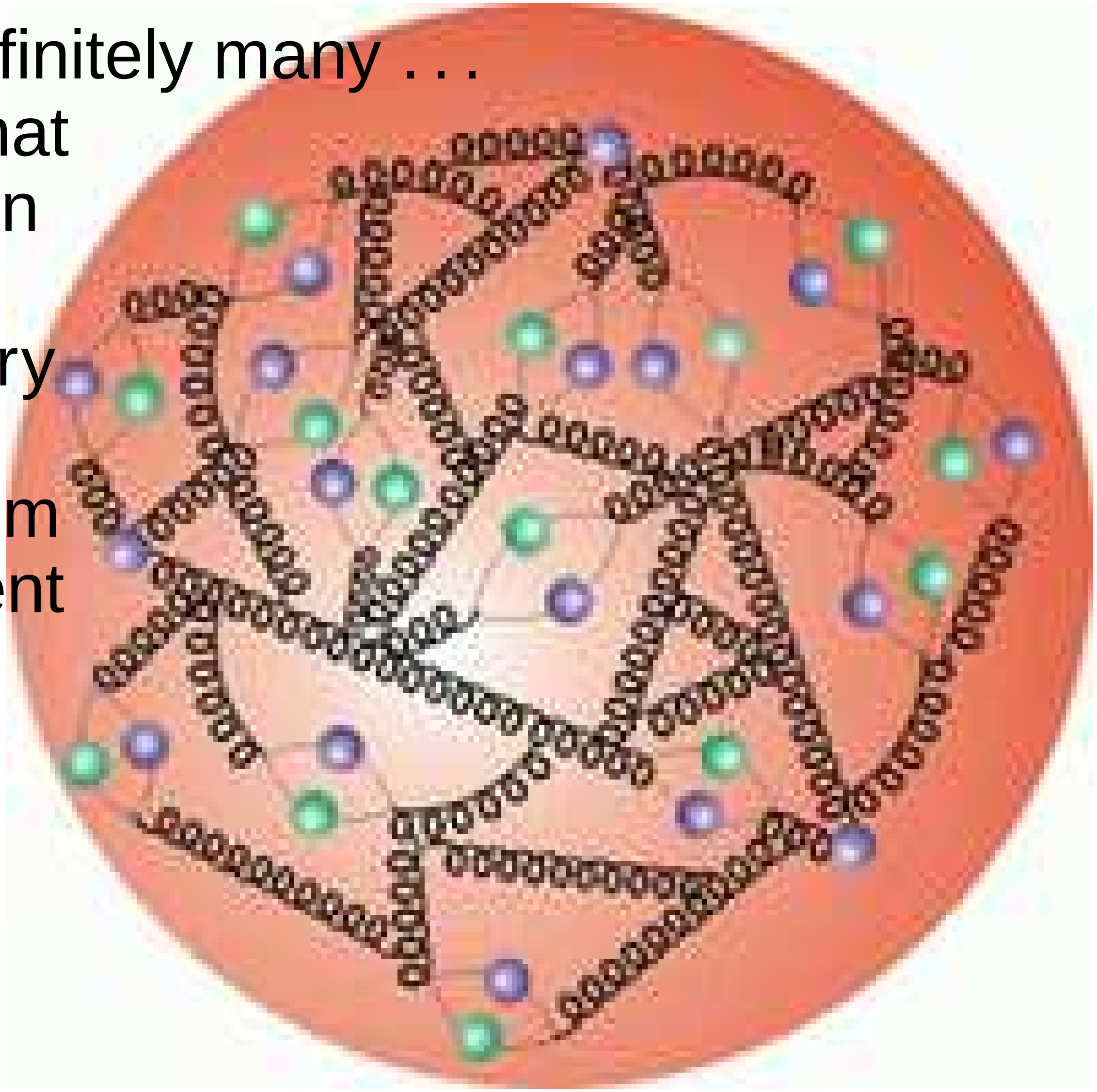
Answer for the pion

Two → Infinitely many . . .
Handle that properly in quantum field theory



Answer for the pion

Two → Infinitely many . . .
Handle that properly in quantum field theory
. . .
momentum-dependent dressing



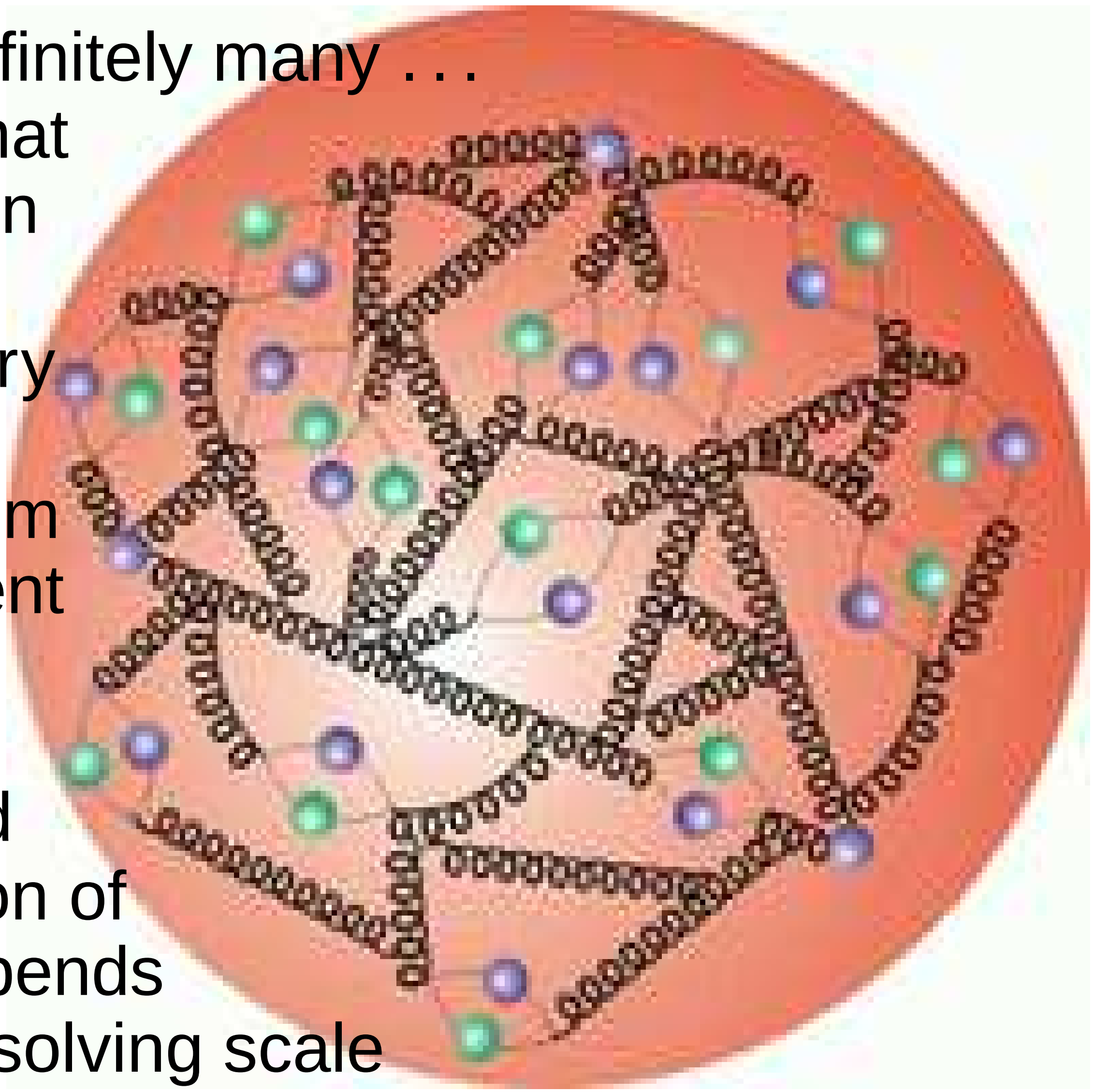
First Contents Back Conclusion

Answer for the pion

Two → Infinitely many . . .
Handle that properly in quantum field theory

. . .
momentum-dependent dressing

. . .
perceived distribution of mass depends on the resolving scale



First Contents Back Conclusion

New Challenges

First Contents Back Conclusion

New Challenges

- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.

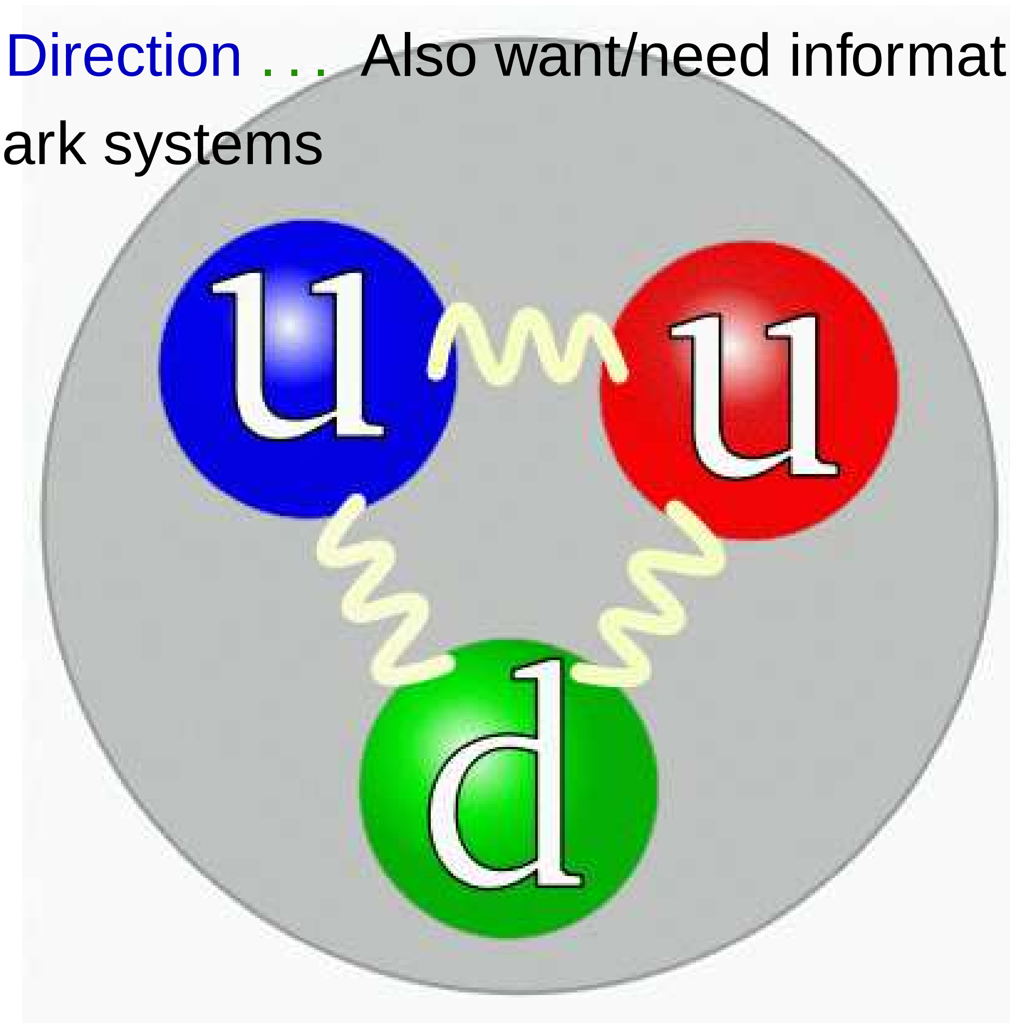
First Contents Back Conclusion

New Challenges

- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

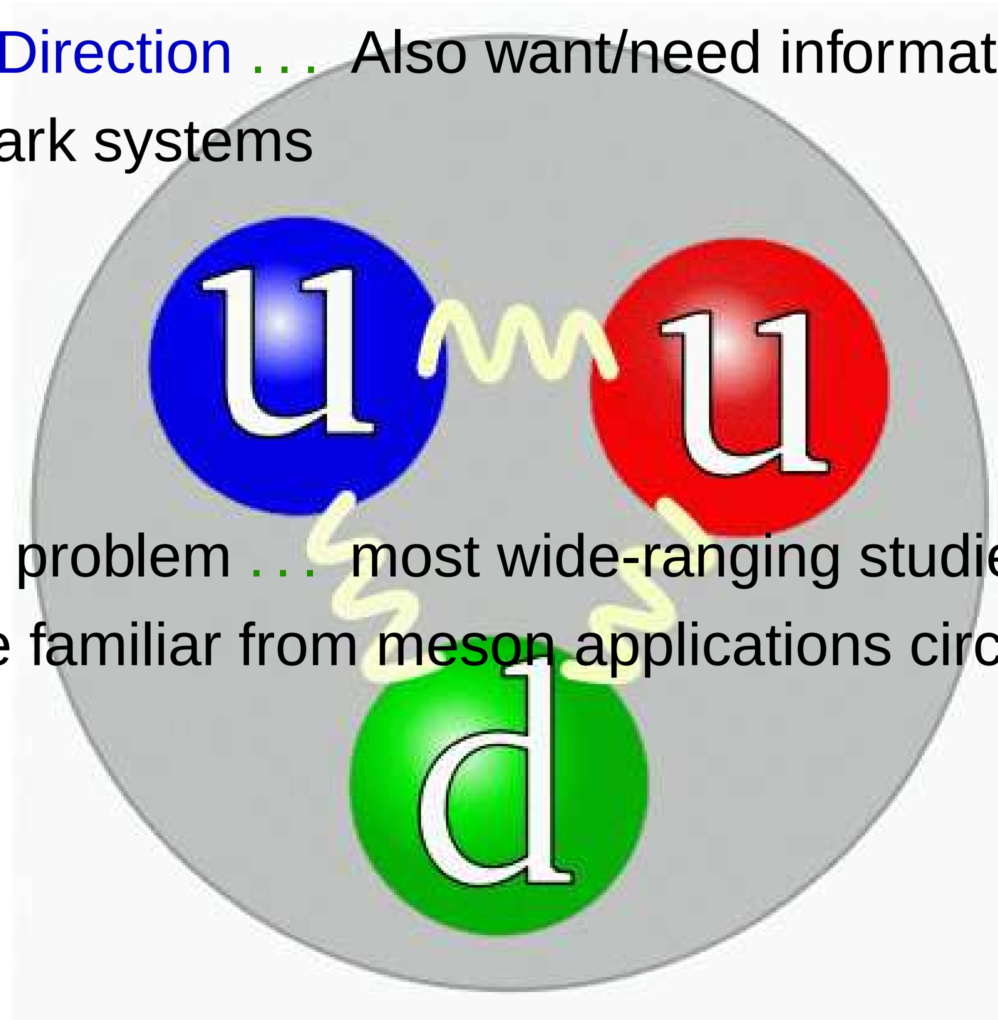
New Challenges

- Another Direction . . . Also want/need information about three-quark systems



New Challenges

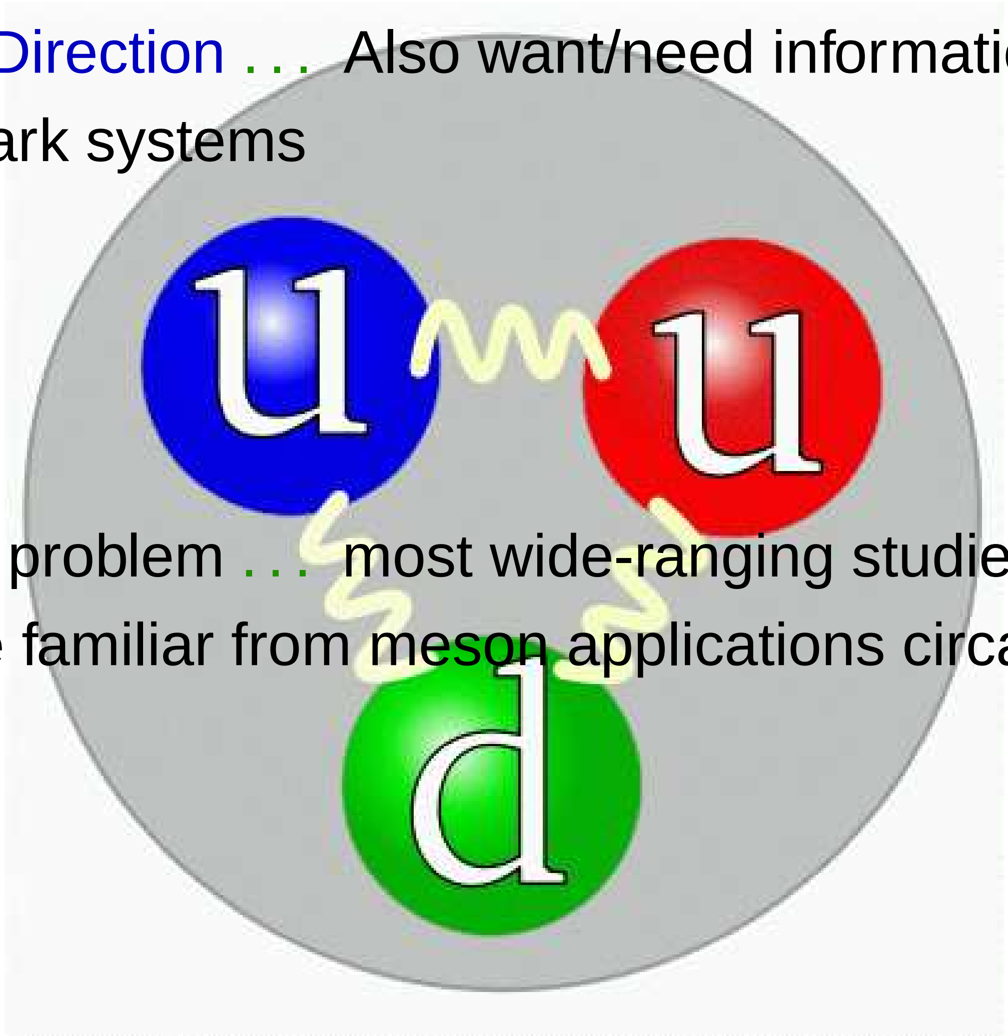
- Another Direction . . . Also want/need information about three-quark systems



- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa ~ 1995.

New Challenges

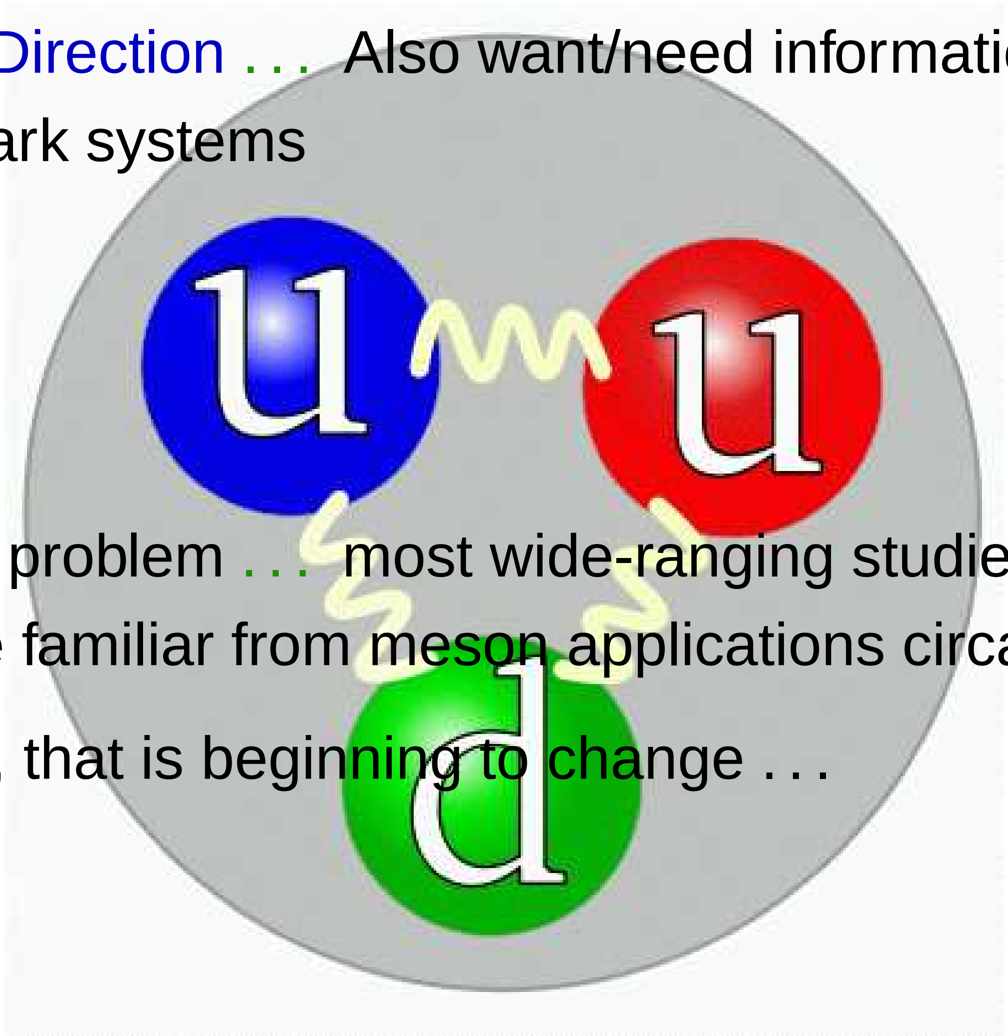
- Another Direction . . . Also want/need information about three-quark systems



- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa $\sim$1995.

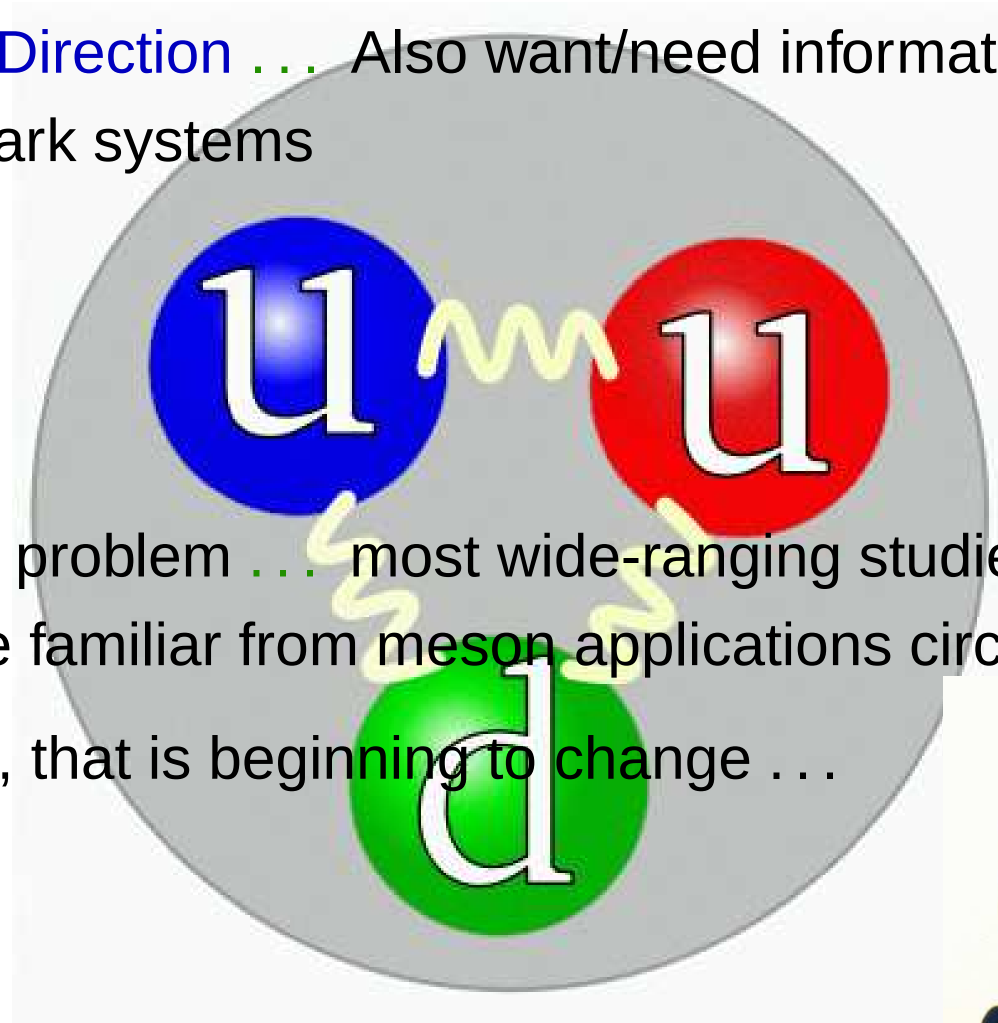
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.

New Challenges



- Another Direction . . . Also want/need information about three-quark systems
- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa $\sim$1995.
- However, that is beginning to change . . .

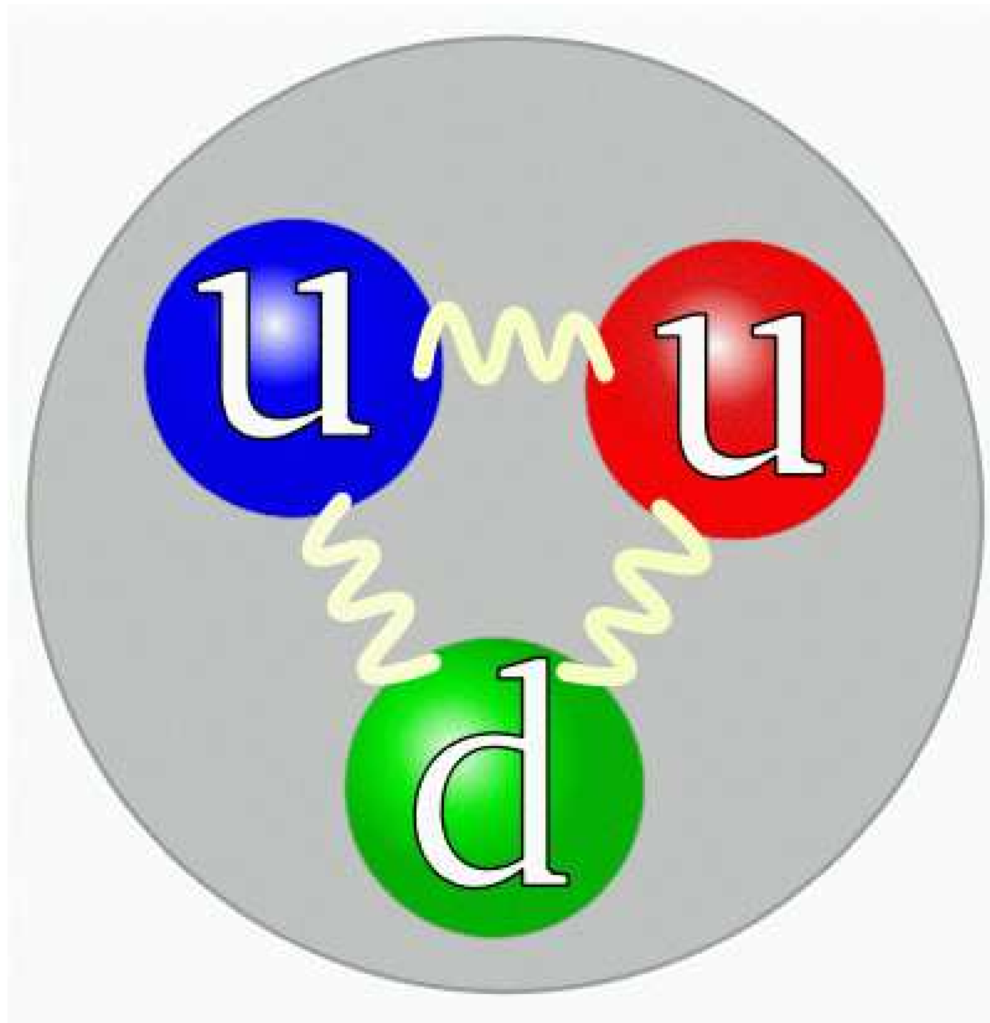
New Challenges



- Another Direction . . . Also want/need information about three-quark systems
- With this problem . . . most wide-ranging studies employ expertise familiar from meson applications circa $\sim$1995.
- However, that is beginning to change . . .

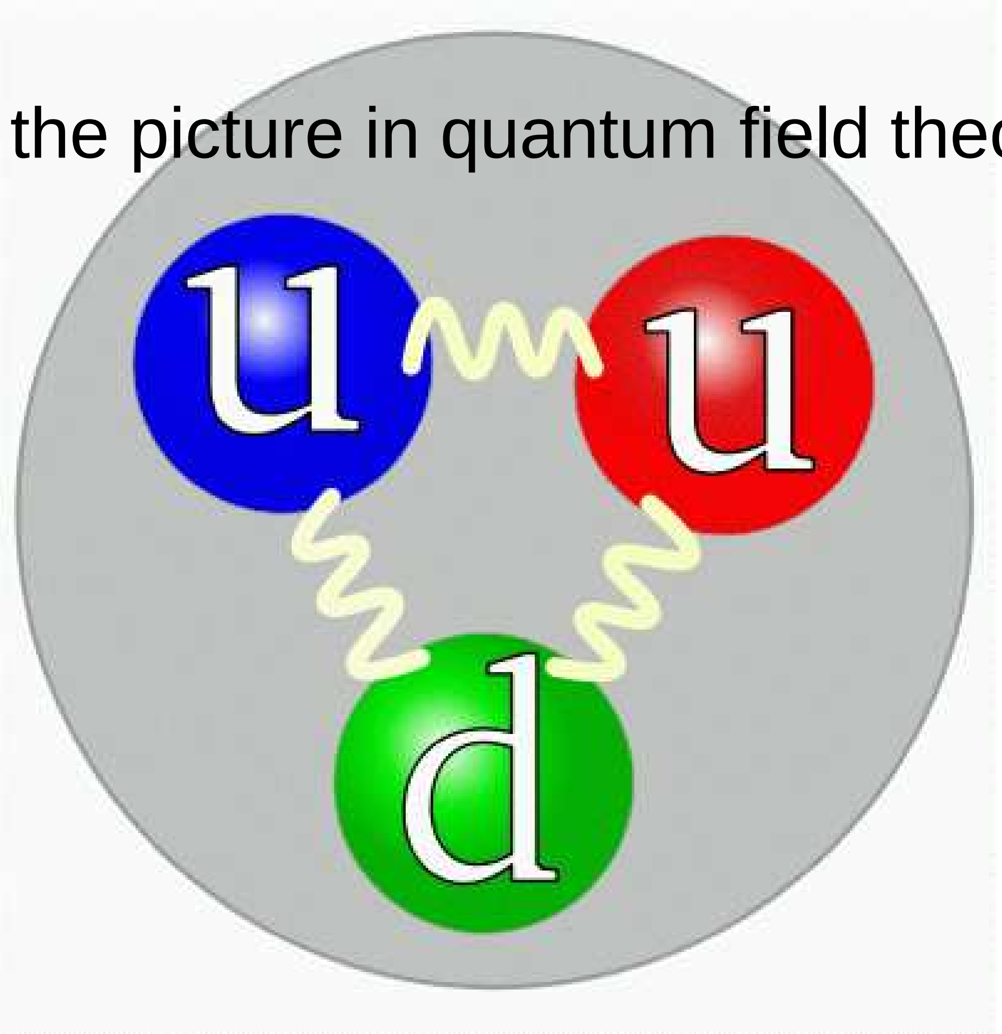
Nucleon . . .

Three-body Problem?



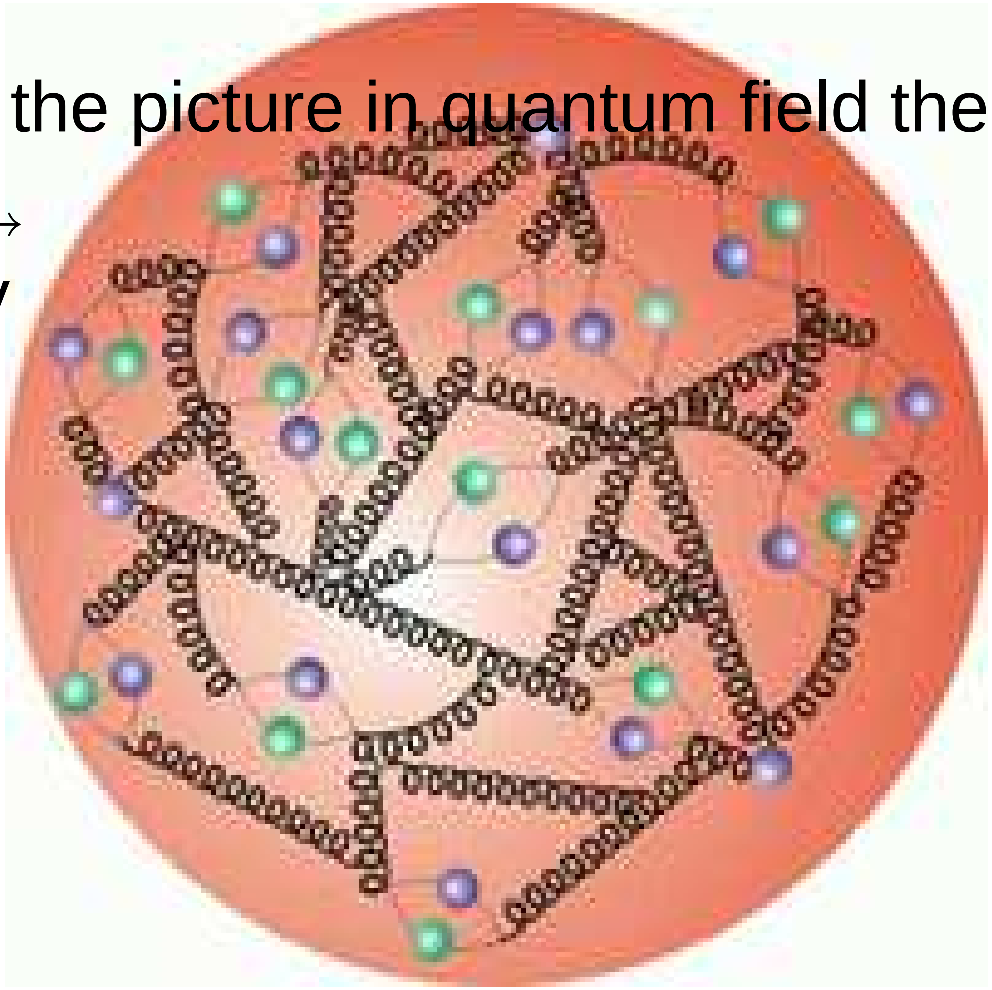
First Contents Back Conclusion

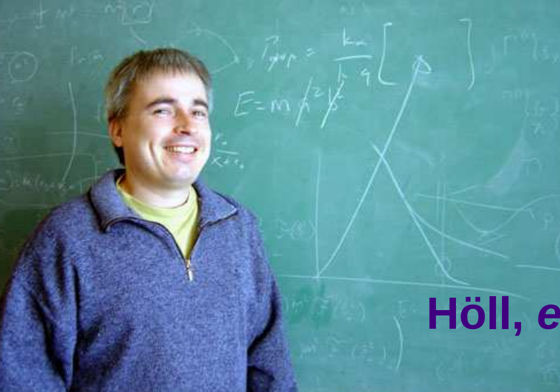
- What is the picture in quantum field theory?



Three-body Problem?

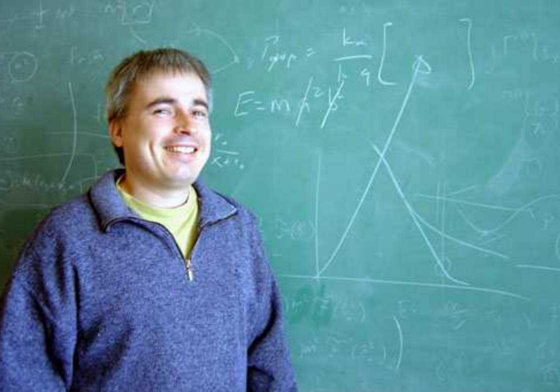
- What is the picture in quantum field theory?
- Three → infinitely many!





Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033



Nucleon EM Form Factors: A Précis

Nucleon EM Form Factors: A Précis

Nucleon EM Form Factors: A Précis

Cloët, *et al.*:
arXiv:0710.2059, arXiv:0710.5746 & arXiv:0804.3118

Nucleon EM Form Factors: A Précis

Cloët, *et al.*:
arXiv:0710.2059, arXiv:0710.5746 & arXiv:0804.3118

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons

Nucleon EM Form Factors: A Précis

Cloët, *et al.*:
arXiv:0710.2059, arXiv:0710.5746 & arXiv:0804.3118

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation

First Contents Back Conclusion

Nucleon EM Form Factors: A Précis

Cloët, *et al.*:
arXiv:0710.2059, arXiv:0710.5746 & arXiv:0804.3118

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)

  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

First Contents Back Conclusion

Nucleon EM Form Factors: A Précis

Cloët, *et al.*:
arXiv:0710.2059, arXiv:0710.5746 & arXiv:0804.3118

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

First Contents Back Conclusion

Nucleon EM Form Factors: A Précis

Cloët, *et al.*:
arXiv:0710.2059, arXiv:0710.5746 & arXiv:0804.3118

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\mathrm{exp}} - M_H^{\mathrm{calc}}]^2}{[M_H^{\mathrm{exp}}]^2} \right)^{1/2} = 2\%$$

- But is that good?

Nucleon EM Form Factors: A Précis

Cloët, *et al.*:
arXiv:0710.2059, arXiv:0710.5746 & arXiv:0804.3118

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H}\sum_H \frac{[M_H^{\rm exp}-M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2}\right)^{1/2} = 2\%$$

- But is that good?
  - Cloudy Bag: $\delta M_+^{\pi-\rm loop} = -300$ to -400 MeV!

First | Contents | Back | Conclusion

Nucleon EM Form Factors: A Précis

Cloët, *et al.*:
arXiv:0710.2059, arXiv:0710.5746 & arXiv:0804.3118

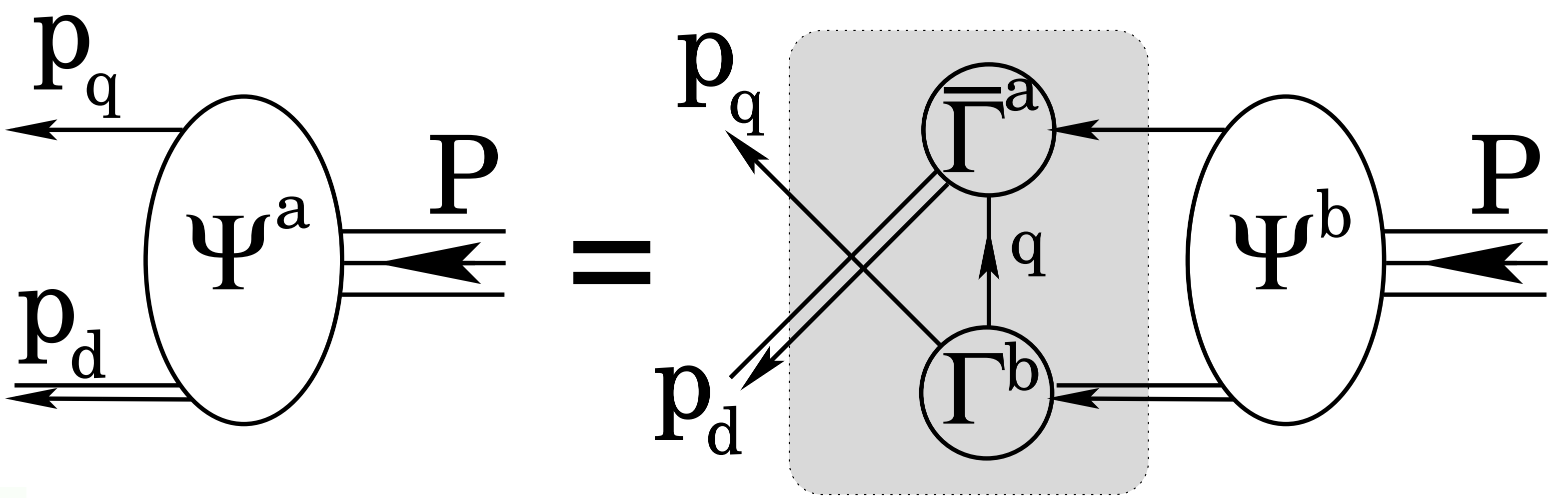
- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H}\sum_H \frac{[M_H^{\rm exp}-M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2}\right)^{1/2} = 2\%$$

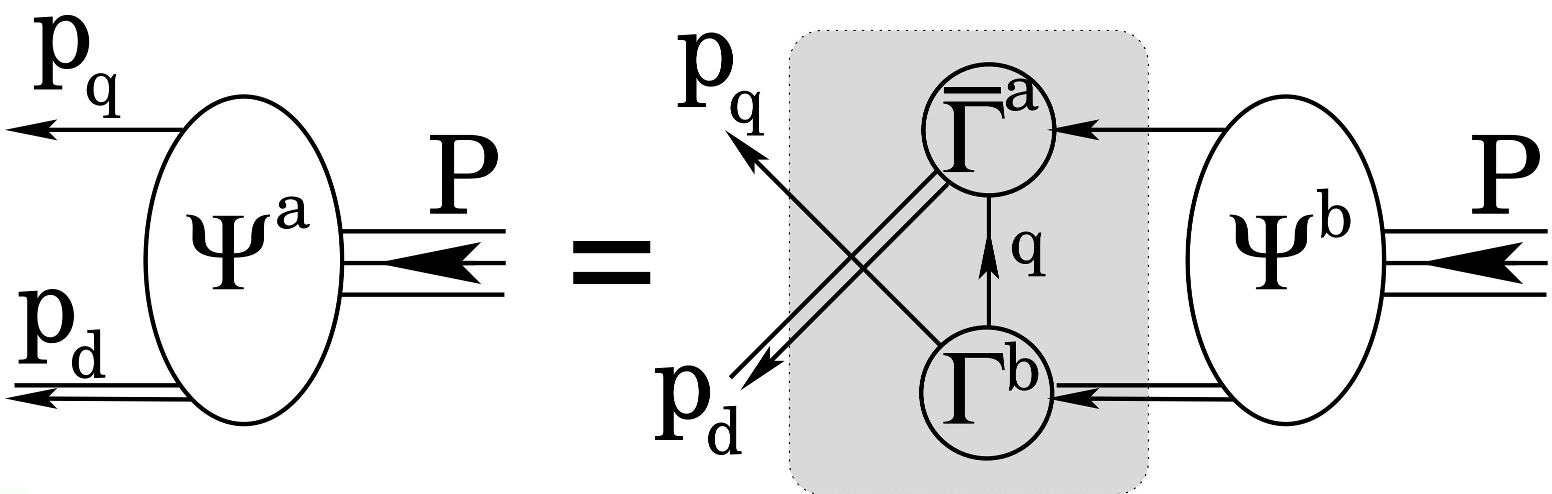
- But is that good?
  - Cloudy Bag: $\delta M_+^{\pi-\rm loop} = -300$ to -400 MeV!
- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

Faddeev equation

Faddeev equation

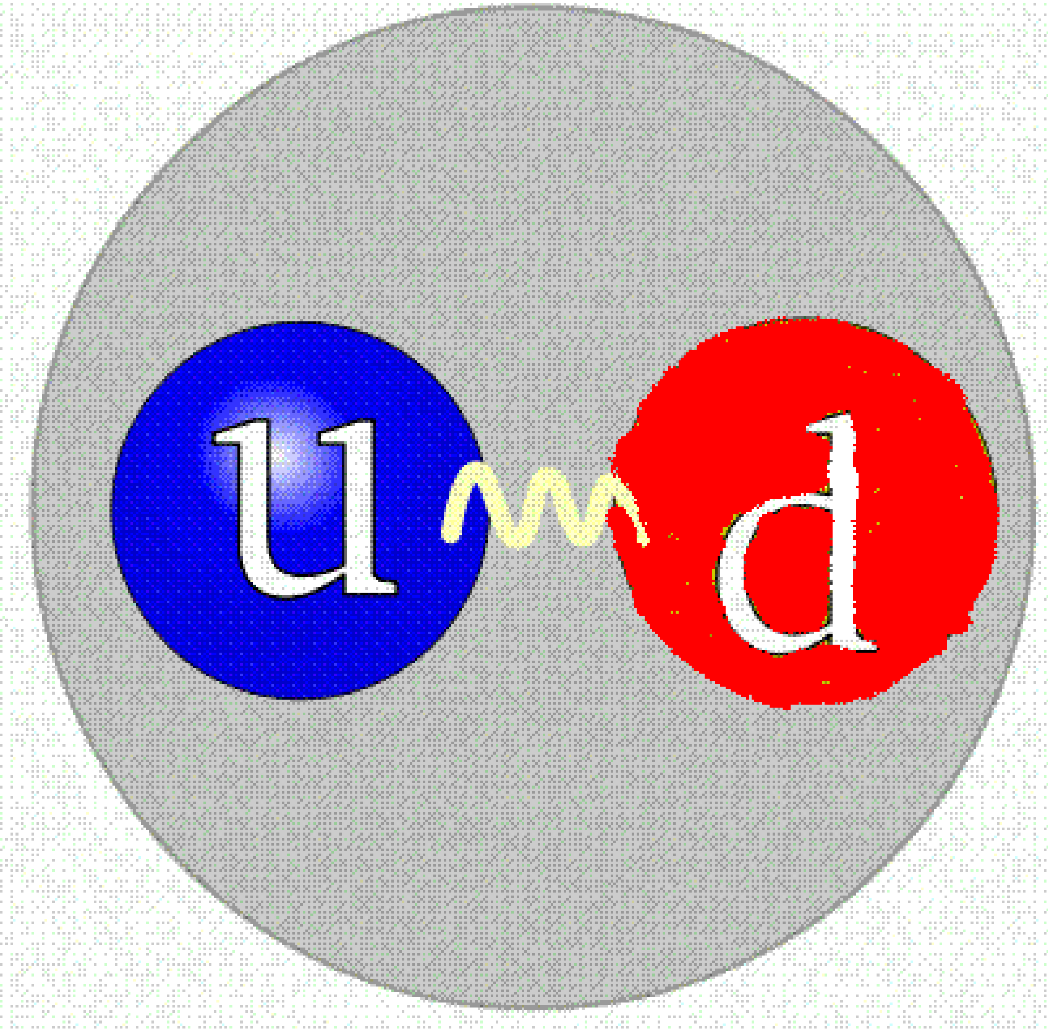


First Contents Back Conclusion



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Diquark correlations



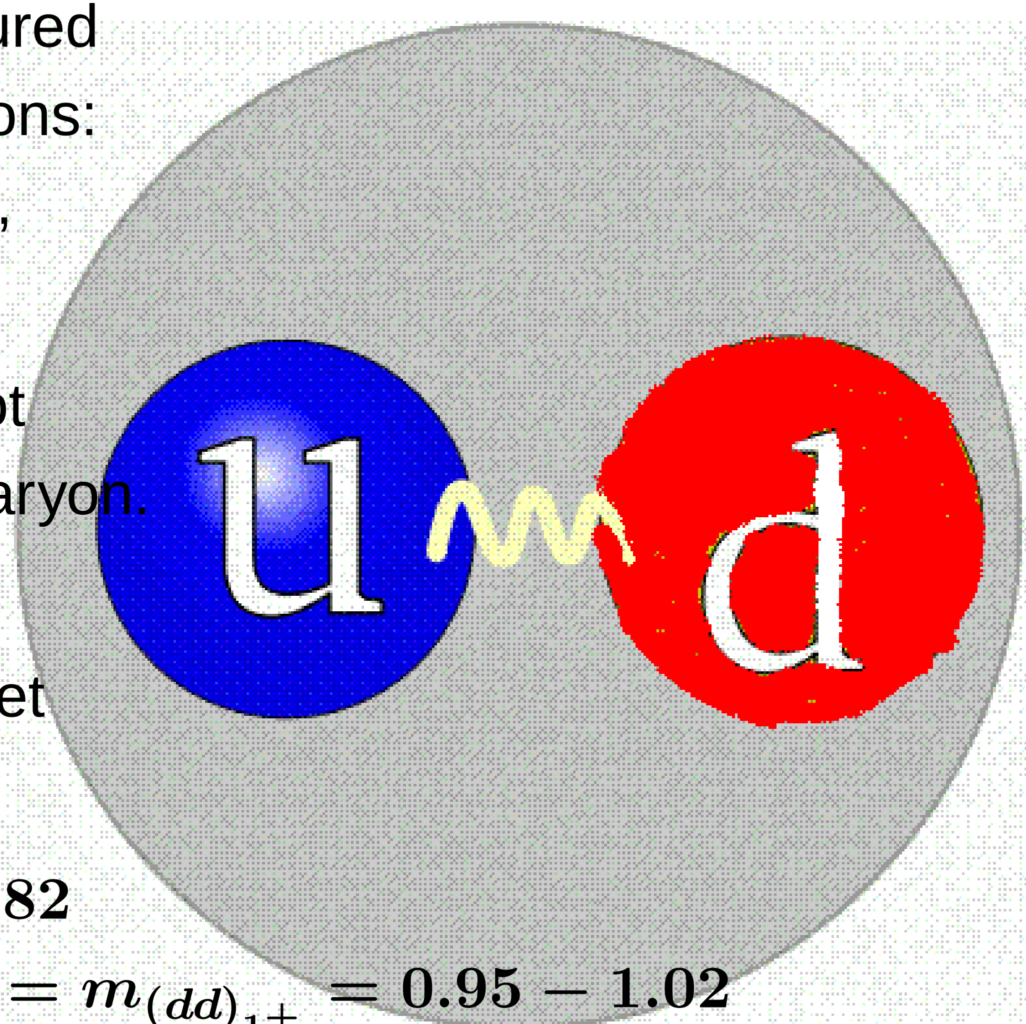
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined . . . Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

for on-shell nucleons described by Faddeev Amplitude

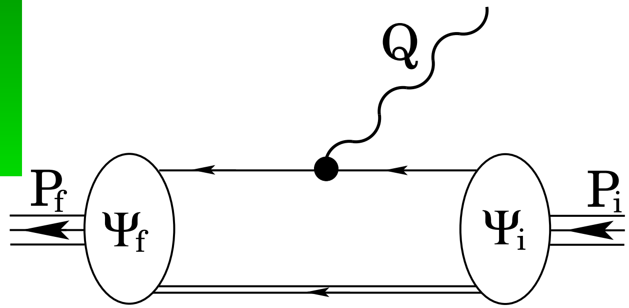
First Contents Back Conclusion

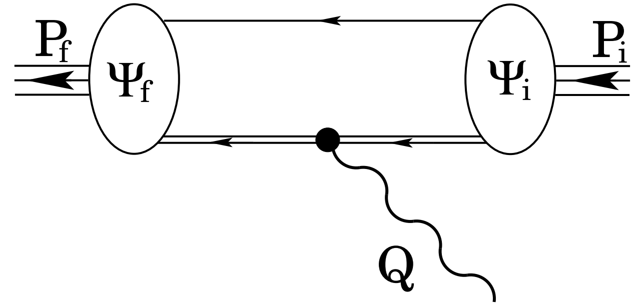
M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

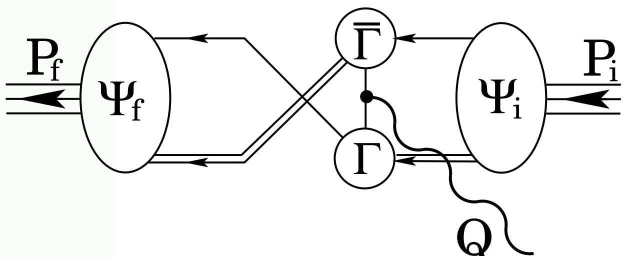
6 terms . . .

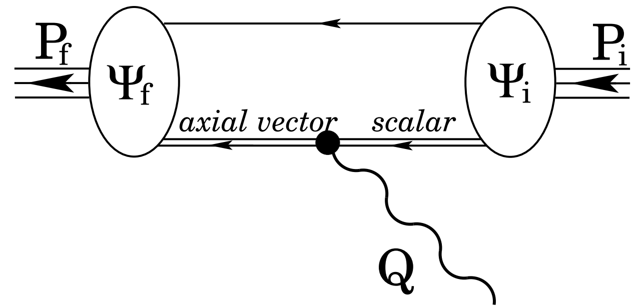
Nucleon-Photon Vertex

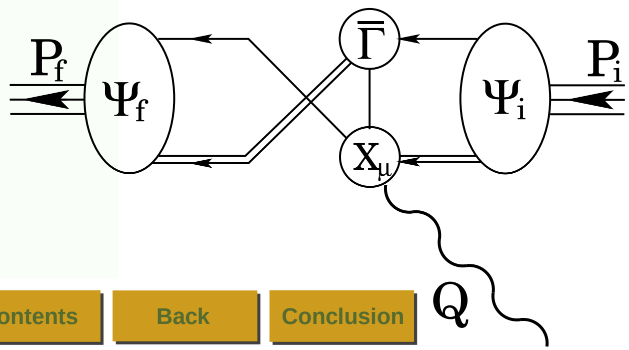
constructed systematically . . . current conserved automatically
for on-shell nucleons described by Faddeev Amplitude

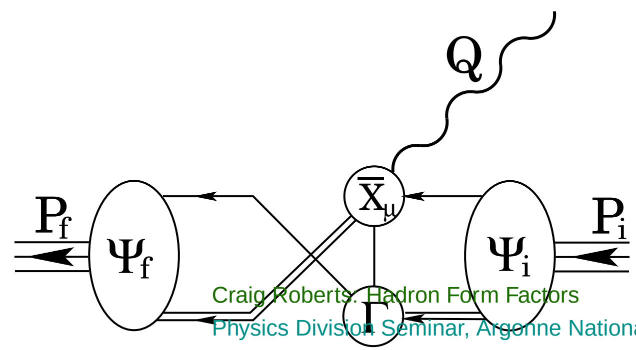






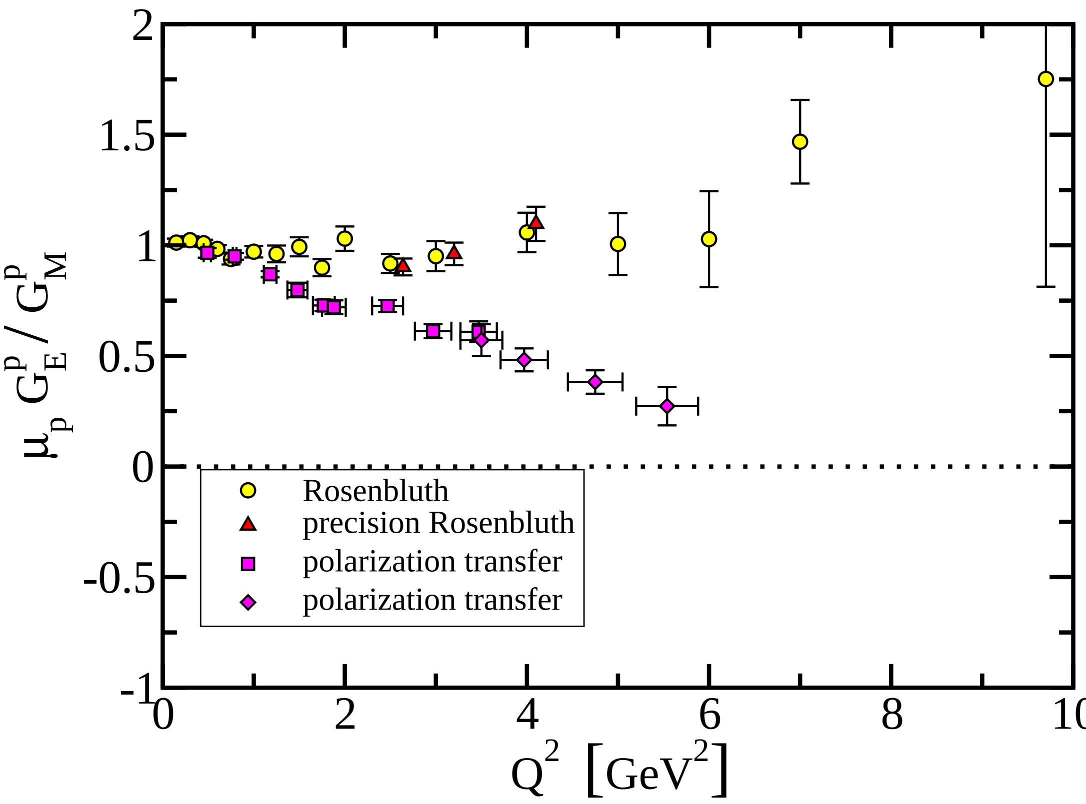






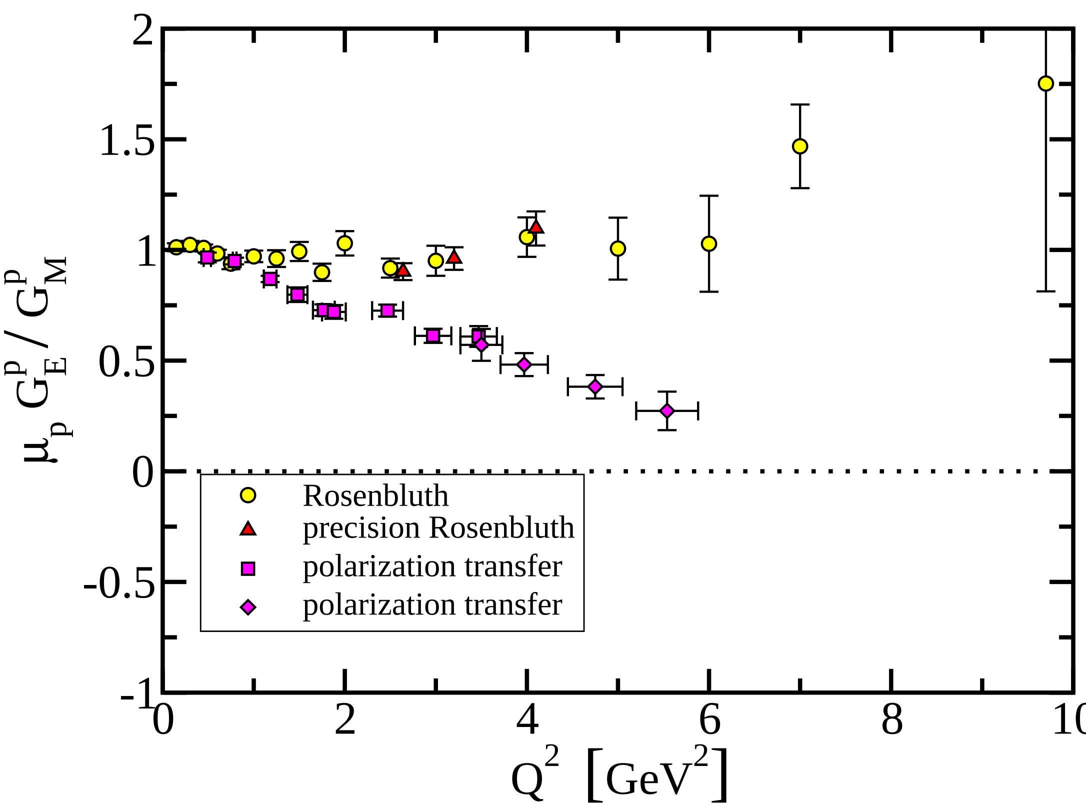
First Contents Back Conclusion

Form Factor Ratio: GE/GM



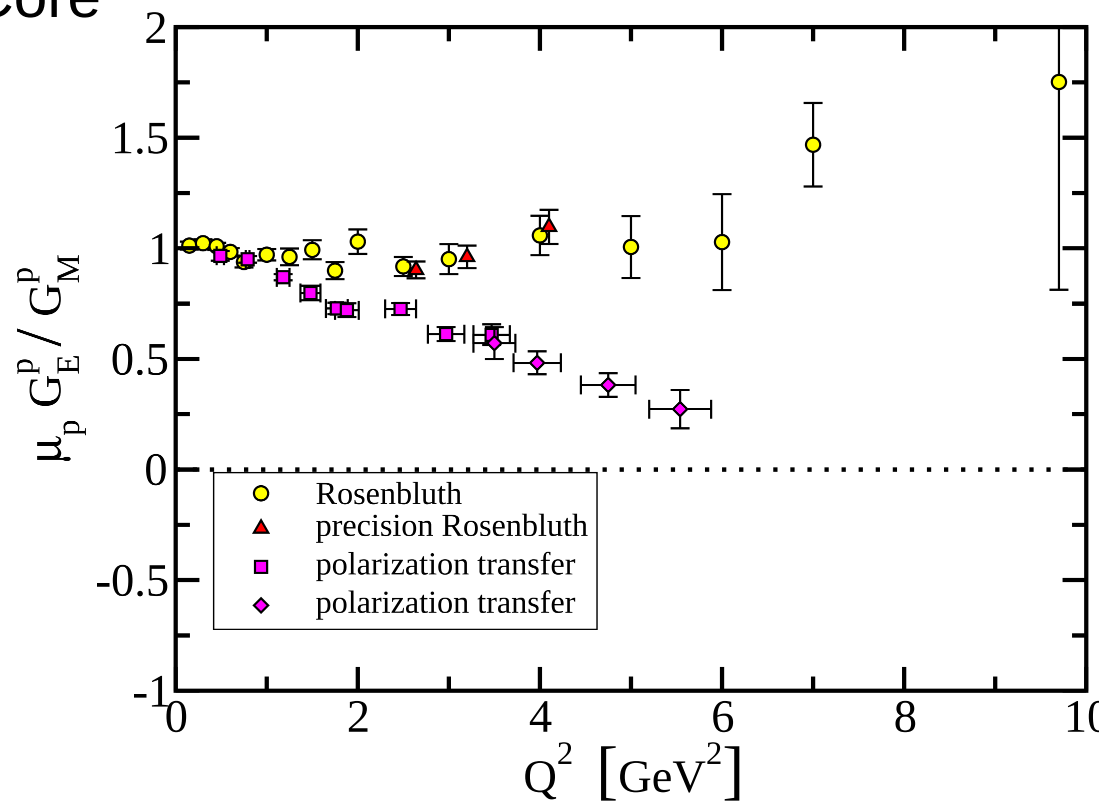
First Contents Back Conclusion

- Combine these elements . . .



Form Factor Ratio: GE/GM

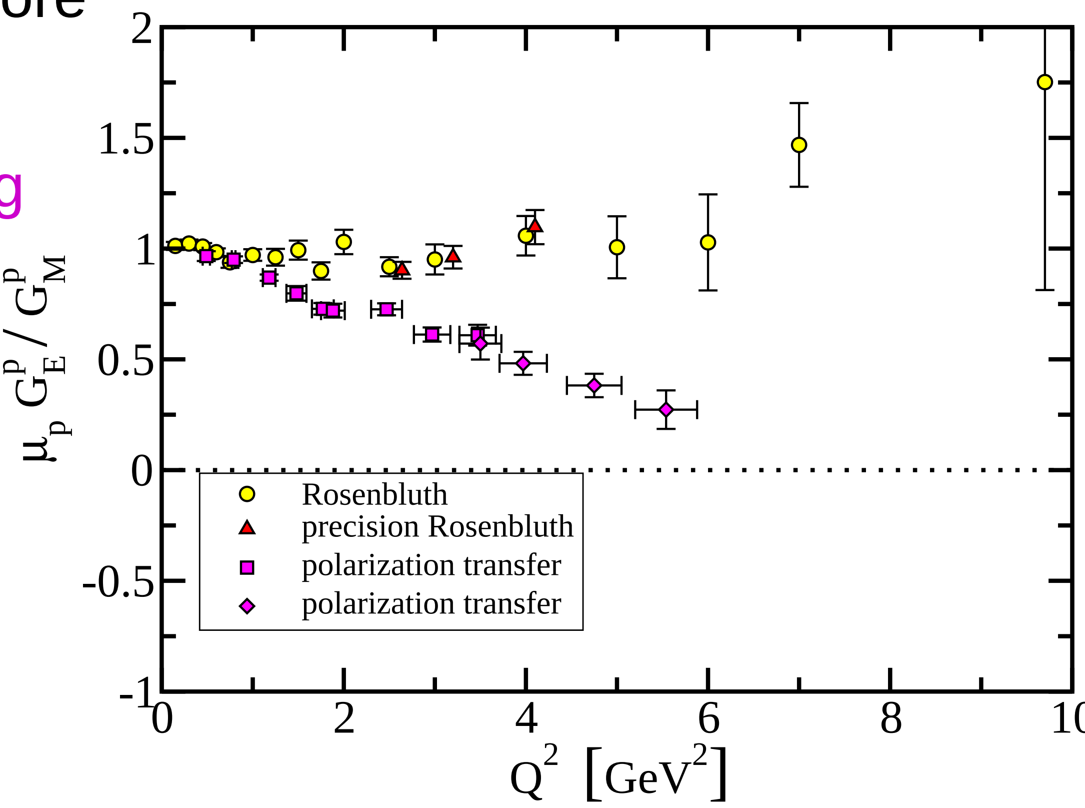
- Combine these elements . . .
  - Dressed-Quark Core



First | Contents | Back | Conclusion

Form Factor Ratio: GE/GM

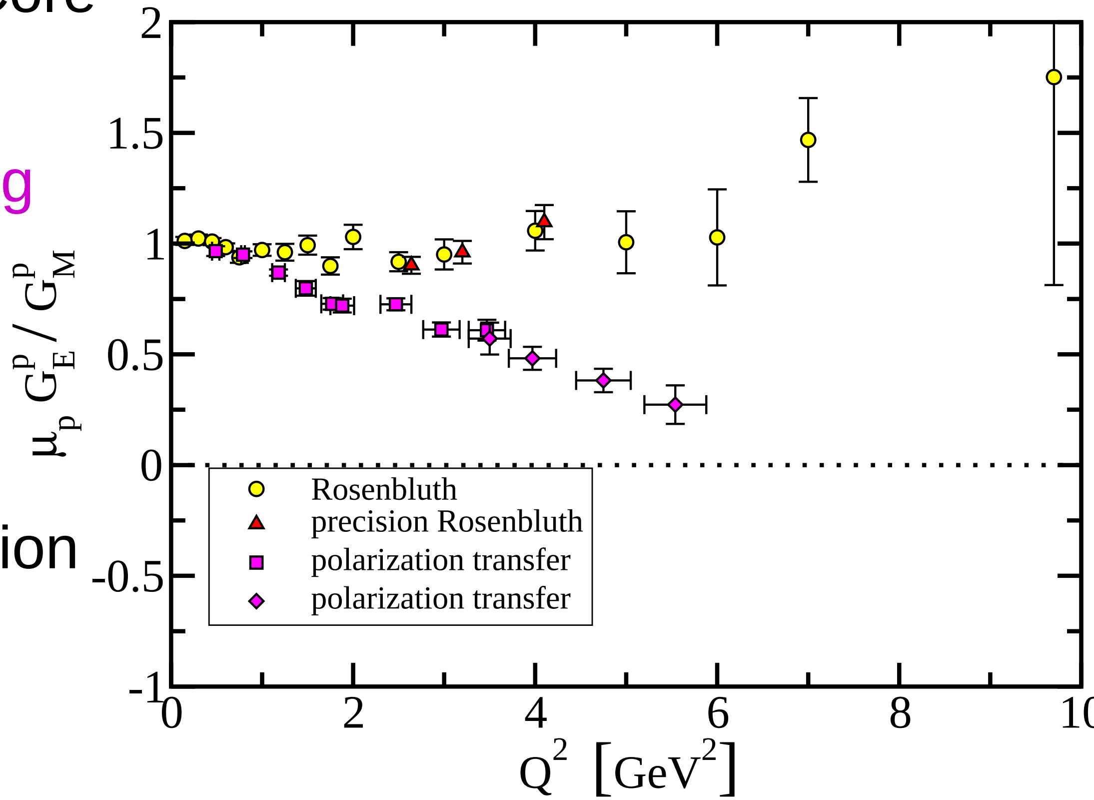
- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current



First Contents Back Conclusion

Form Factor Ratio: GE/GM

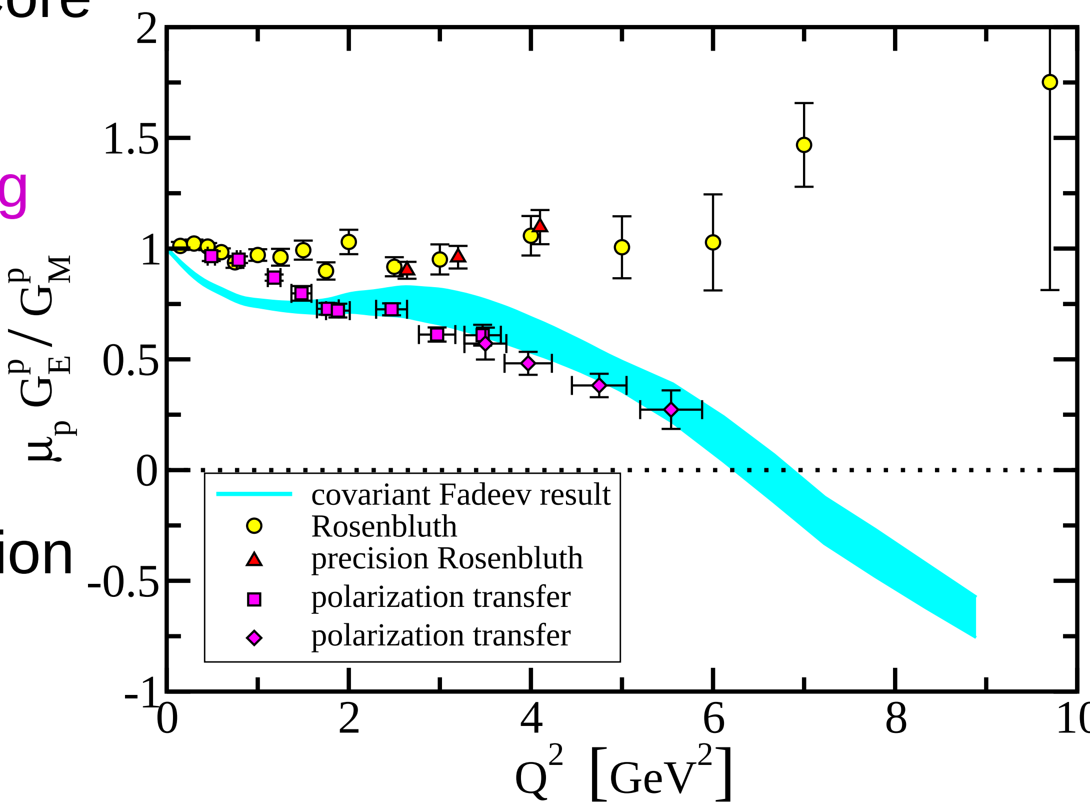
- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



First | Contents | Back | Conclusion

Form Factor Ratio: GE/GM

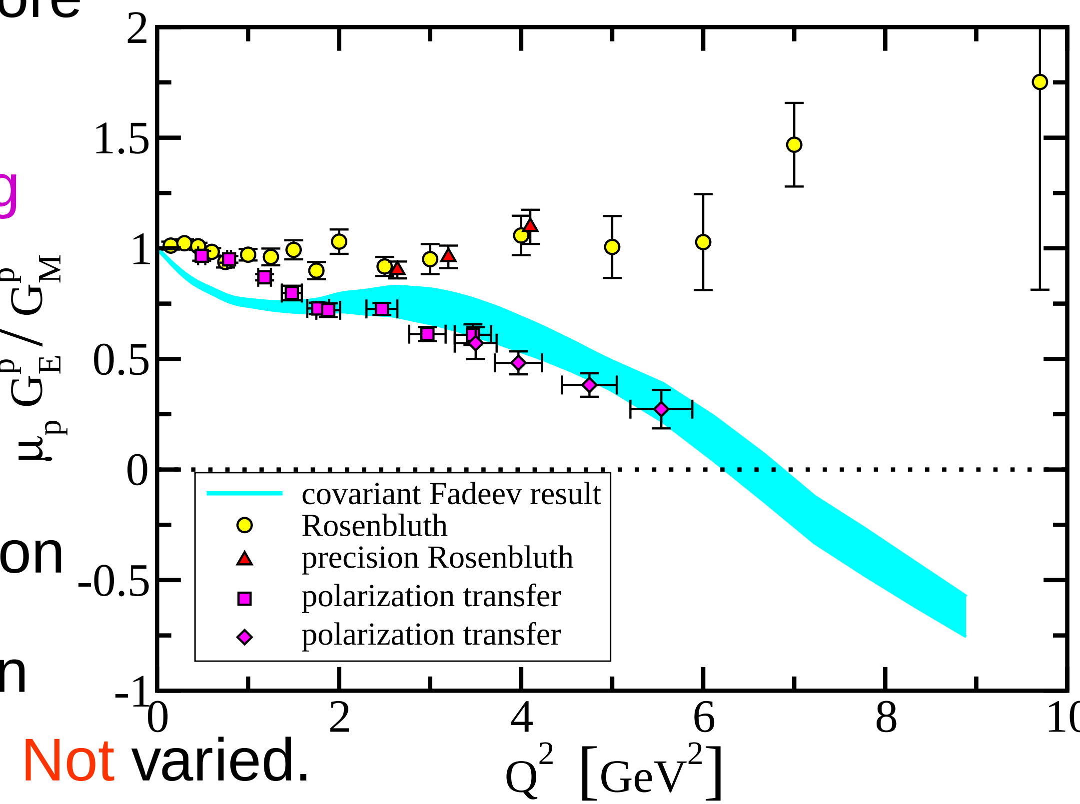
- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



First | Contents | Back | Conclusion

Form Factor Ratio: GE/GM

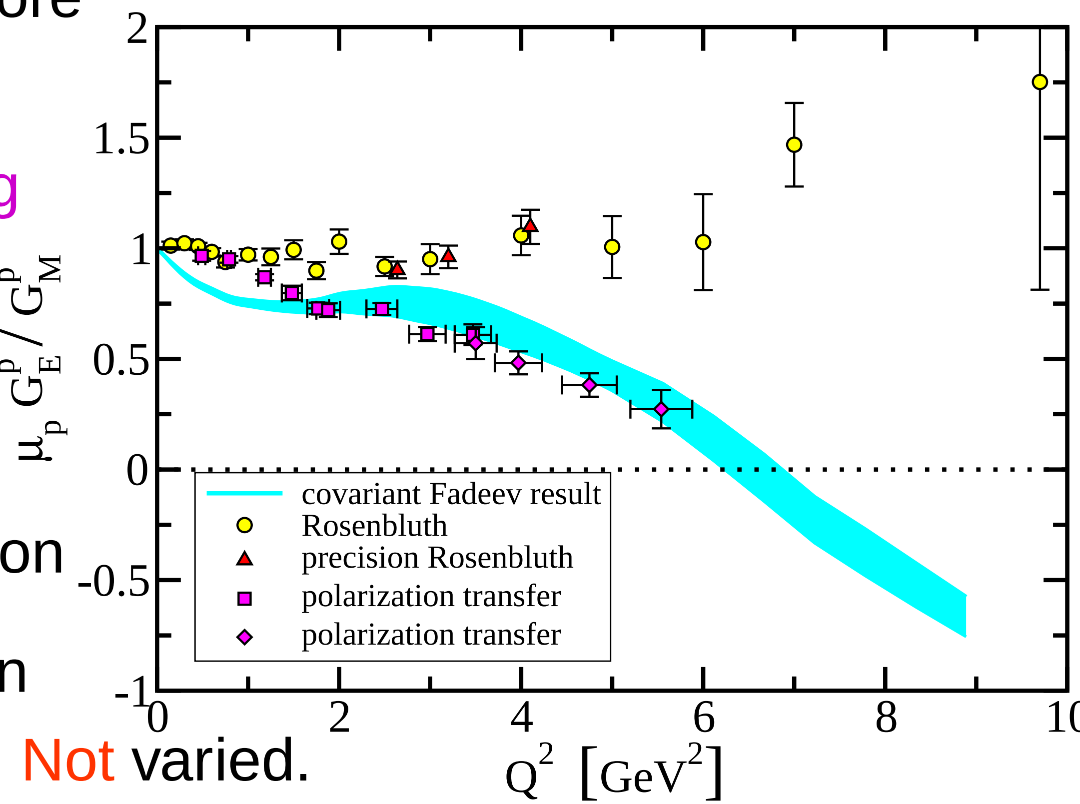
- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution
- All parameters fixed in other applications . . . Not varied.



First Contents Back Conclusion

Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution

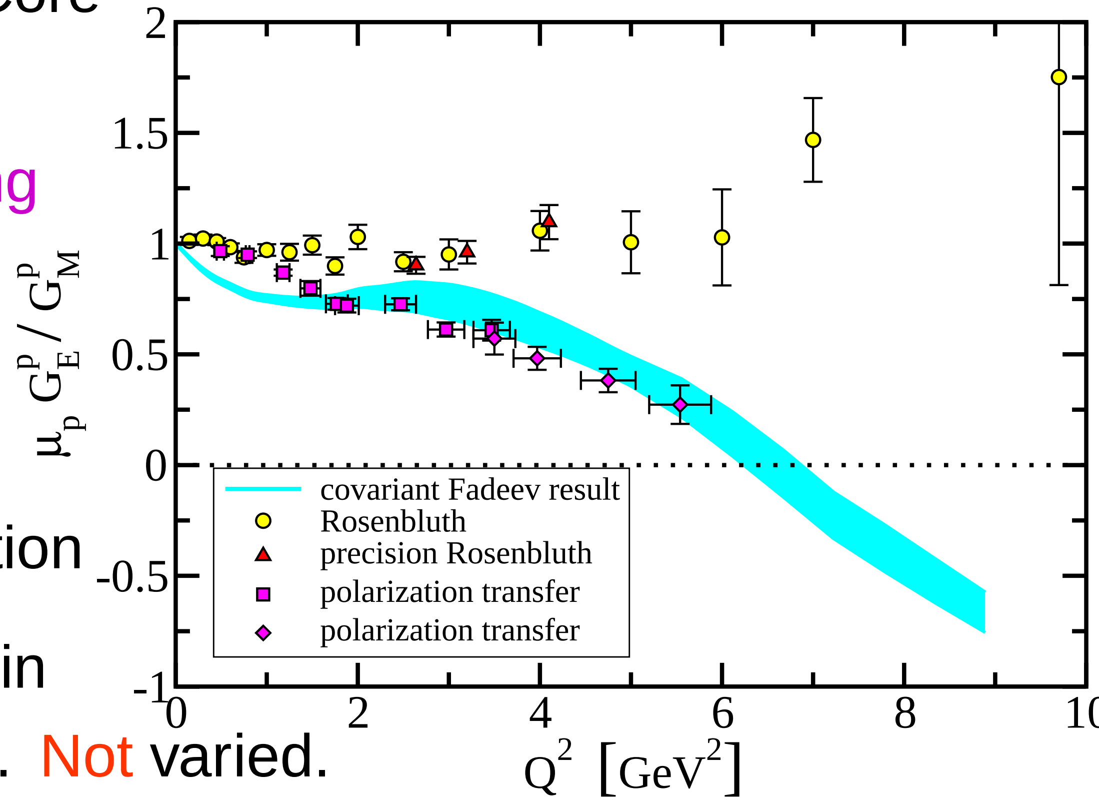


- All parameters fixed in other applications . . . Not varied.
  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2$ GeV^2

First Contents Back Conclusion

Form Factor Ratio: GE/GM

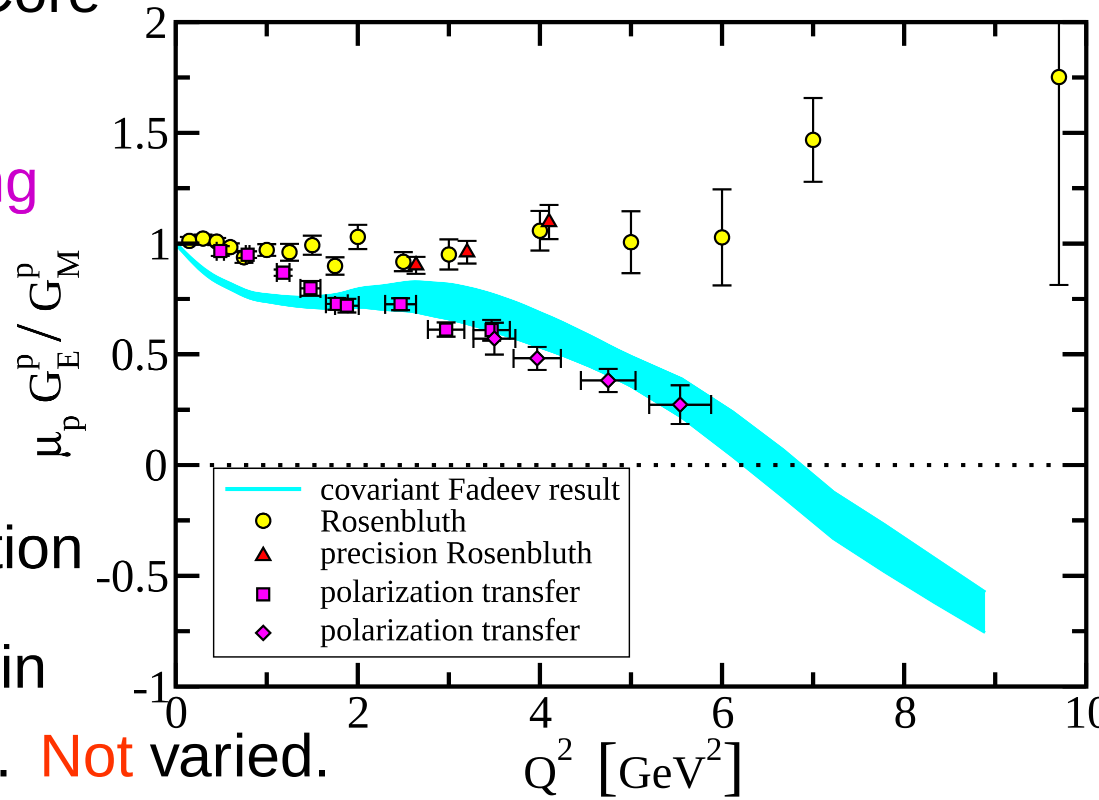
- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



- All parameters fixed in other applications . . . Not varied.
  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\,$GeV^2
  - Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement

Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



- All parameters fixed in other applications . . . Not varied.
  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2$ GeV^2
  - Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
  - Predict Zero at $Q^2 \approx 6.5$GeV^2

Hall-A Neutron FF

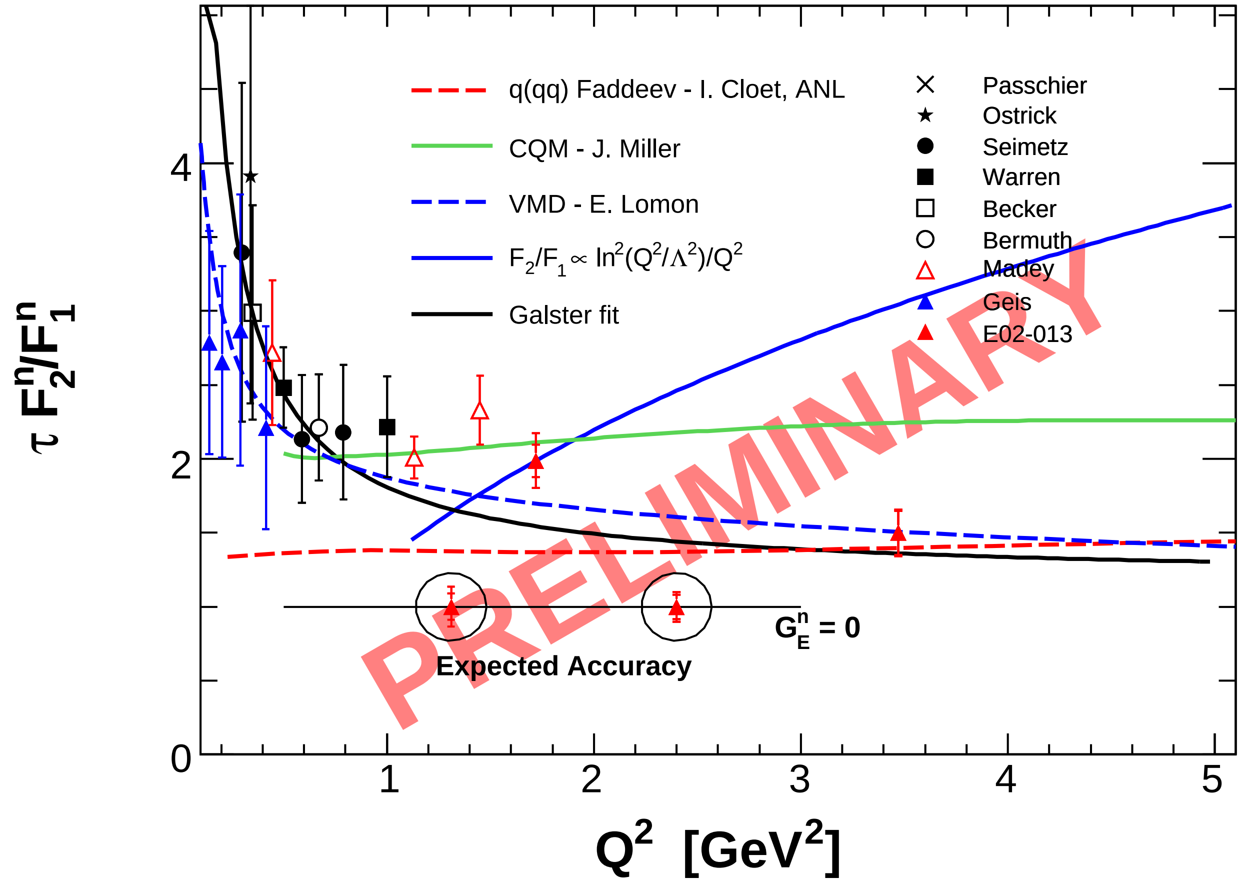
- Preliminary result

  E02-013 - Wojtsekhowski & Cates & Liyanage

Hall-A Neutron FF

- Preliminary result

E02-013 - Wojtsekhowski & Cates & Liyanage



First | Contents | Back | Conclusion

Improved current

First Contents Back Conclusion

Improved current

- Composite axial-vector diquark correlation
  - Electromagnetic current can be complicated
  - Limited constraints on large-Q^2 behaviour

First Contents Back Conclusion

Improved current

- Composite axial-vector diquark correlation
  - Electromagnetic current can be complicated
  - Limited constraints on large-Q^2 behaviour
  - Improved performance of code
  - Implemented corrections so that large-Q^2 behaviour of form factors could be reliably calculated
  - Exposed two weaknesses in rudimentary *Ansatz*

Improved current

- Composite axial-vector diquark correlation
  - Improved performance of code
  - Implemented corrections so that large-Q^2 behaviour of form factors could be reliably calculated
  - Exposed two weaknesses in rudimentary *Ansatz*
    - Diquark effectively pointlike to hard probe
    - Didn't account for diquark being off-shell in recoil

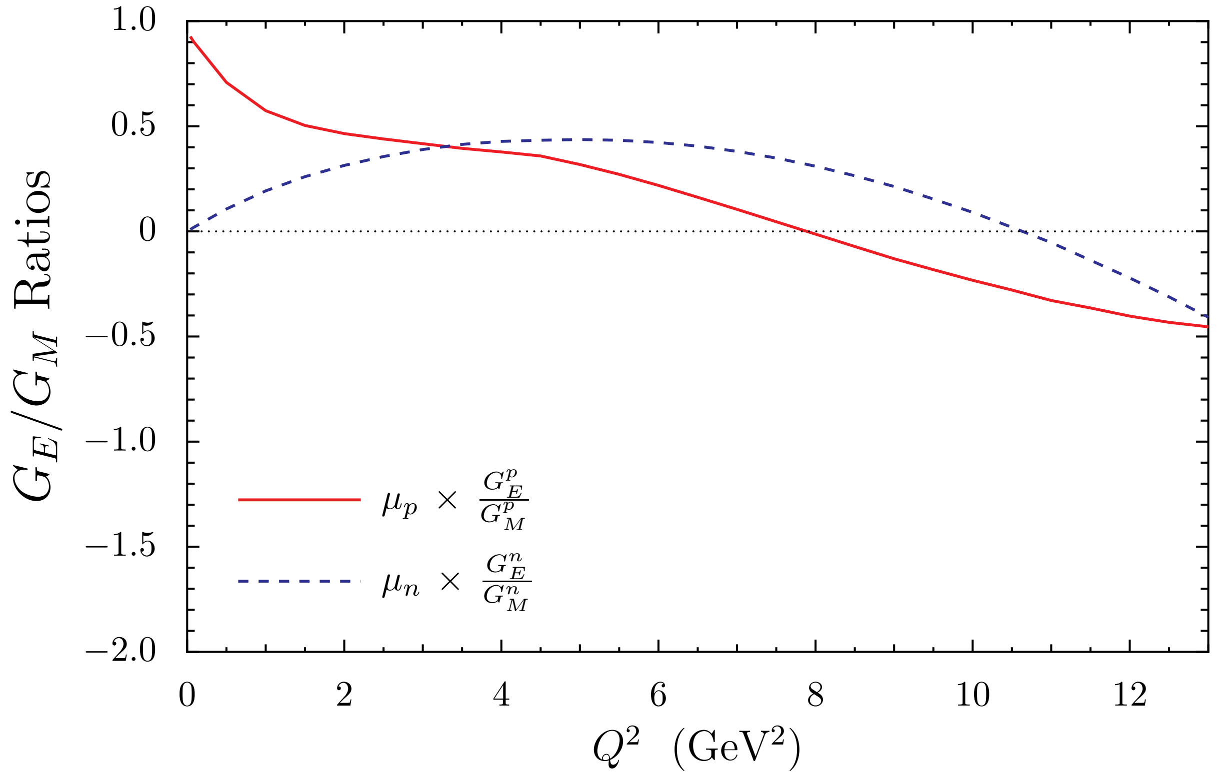
Improved current

- Composite axial-vector diquark correlation
- Minor but material improvements to current
  - Introduce form factor: radius 0.8 fm
  - Increase recoil mass by 10%

First Contents Back Conclusion

Improved current

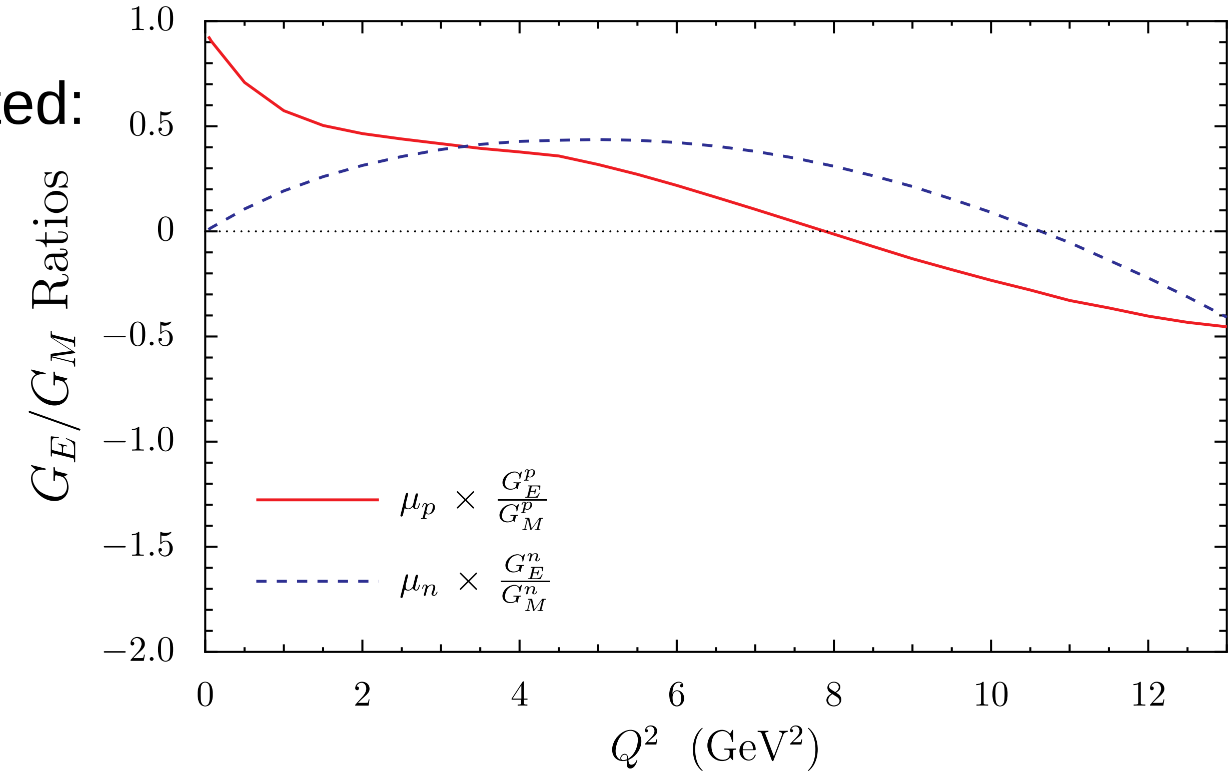
- Composite axial-vector diquark correlation
- Minor but material improvements to current
  - Introduce form factor: radius 0.8 fm
  - Increase recoil mass by 10%



First Contents Back Conclusion

Improved current

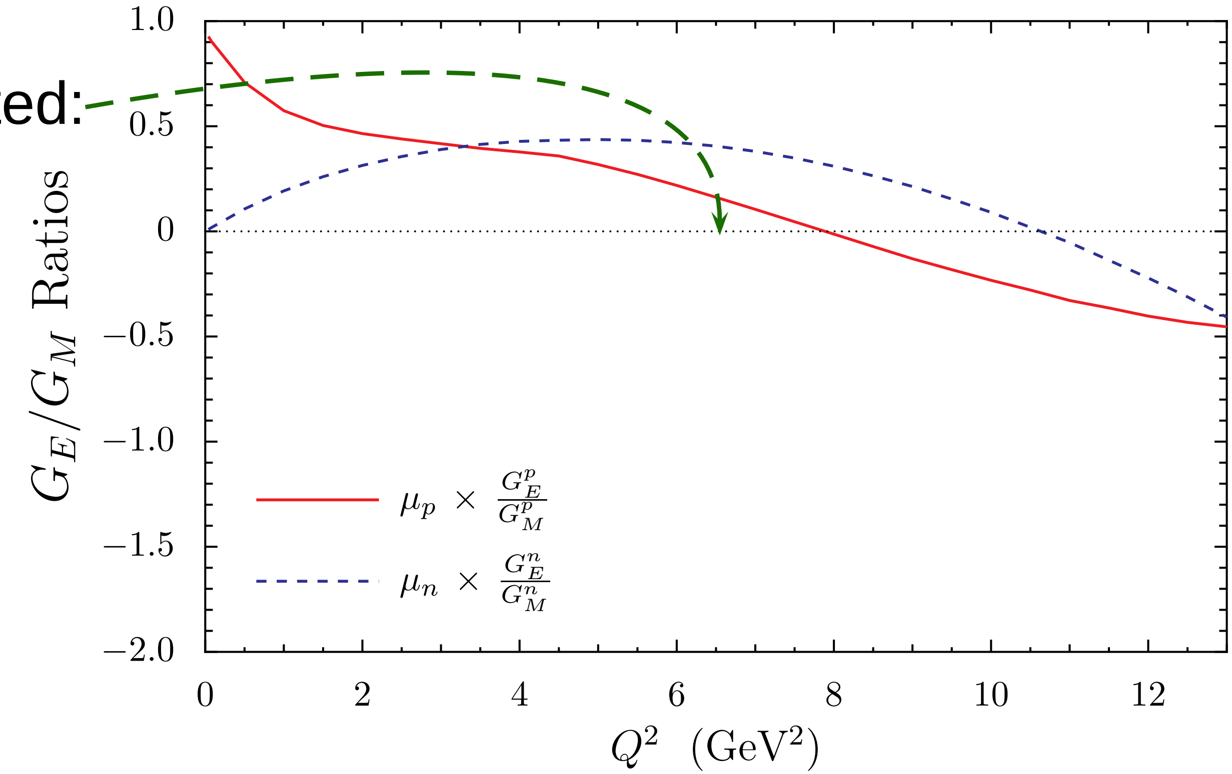
- Composite axial-vector diquark correlation
- Minor but material improvements to current
  - Introduce form factor: radius 0.8 fm
  - Increase recoil mass by 10%
- Proton – zero shifted:
  6.5 → 8.0 GeV^2



First Contents Back Conclusion

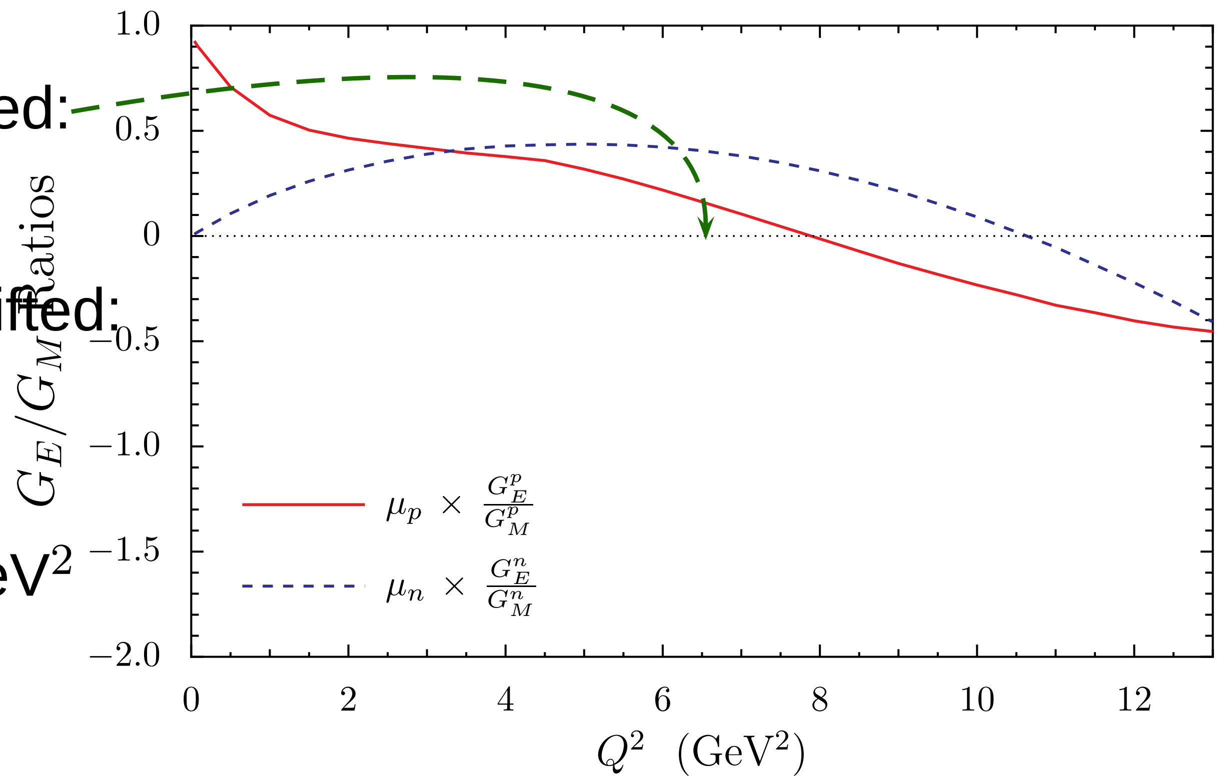
Improved current

- Composite axial-vector diquark correlation
- Minor but material improvements to current
  - Introduce form factor: radius 0.8 fm
  - Increase recoil mass by 10%
- Proton – zero shifted: $6.5 \rightarrow 8.0\,\mathrm{GeV}^2$



Improved current

- Composite axial-vector diquark correlation
- Minor but material improvements to current
  - Introduce form factor: radius 0.8 fm
  - Increase recoil mass by 10%
- Proton – zero shifted: 6.5 → 8.0 GeV^2
- Neutron – peak shifted: 7.5 → 5.0 GeV^2
  & now predict zero a little above 11 GeV^2

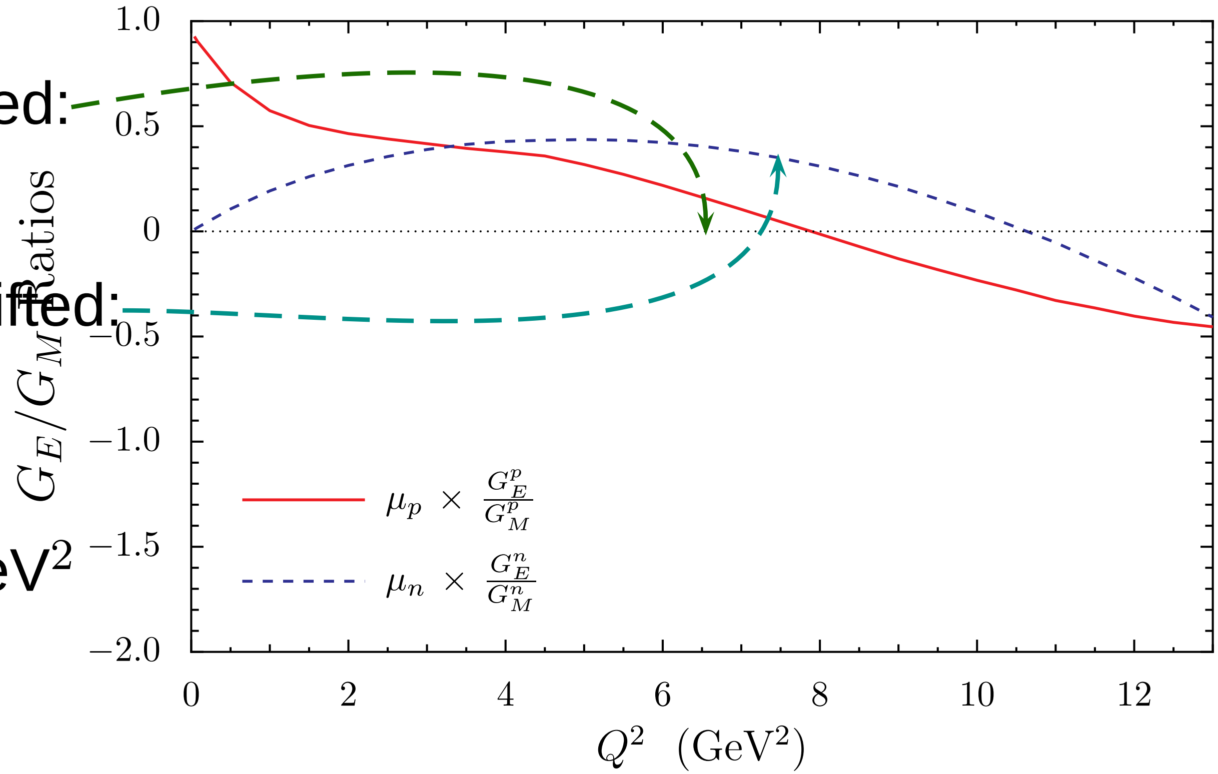


First Contents Back Conclusion

Improved current

- Composite axial-vector diquark correlation
- Minor but material improvements to current
  - Introduce form factor: radius 0.8 fm
  - Increase recoil mass by 10%

- Proton – zero shifted: 6.5 → 8.0 GeV^2
- Neutron – peak shifted: 7.5 → 5.0 GeV^2 & now predict zero a little above 11 GeV^2

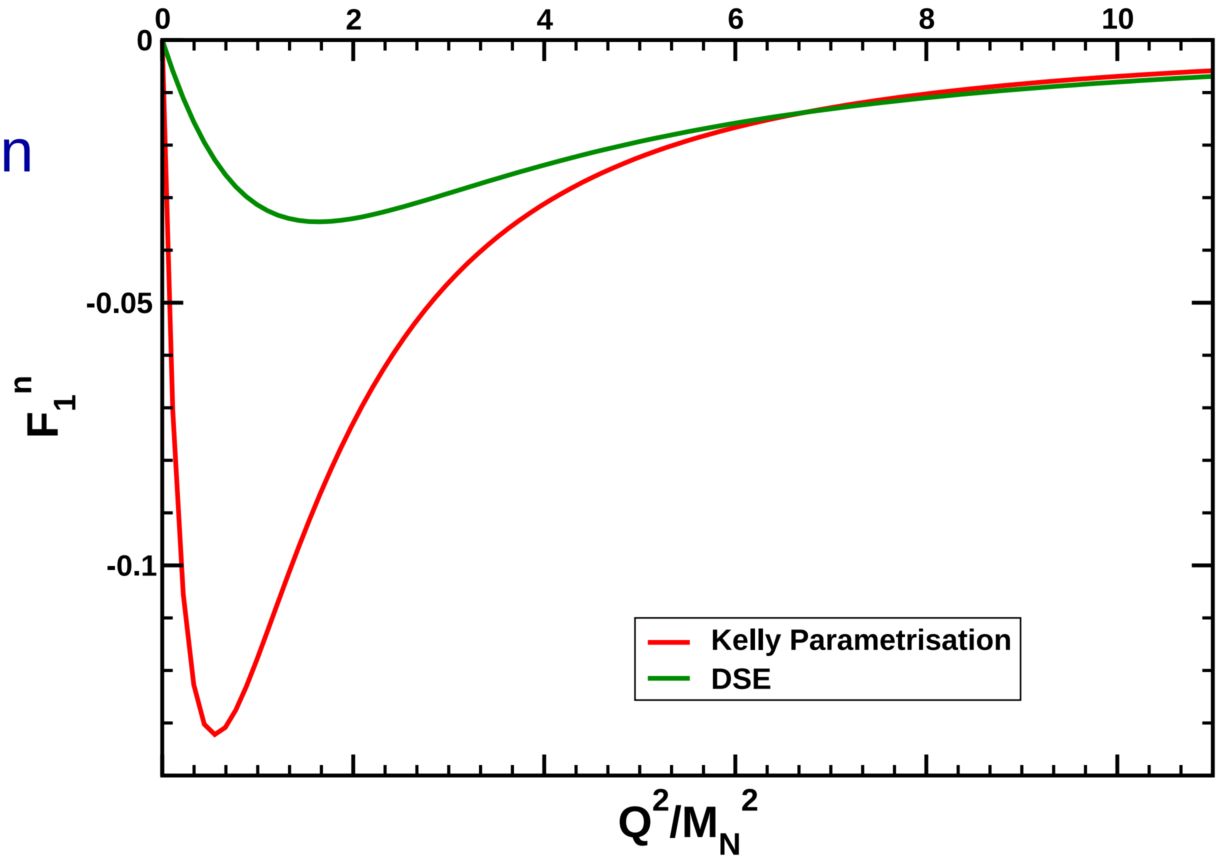


First Contents Back Conclusion

Pion Cloud

Pion Cloud
F1 – neutron

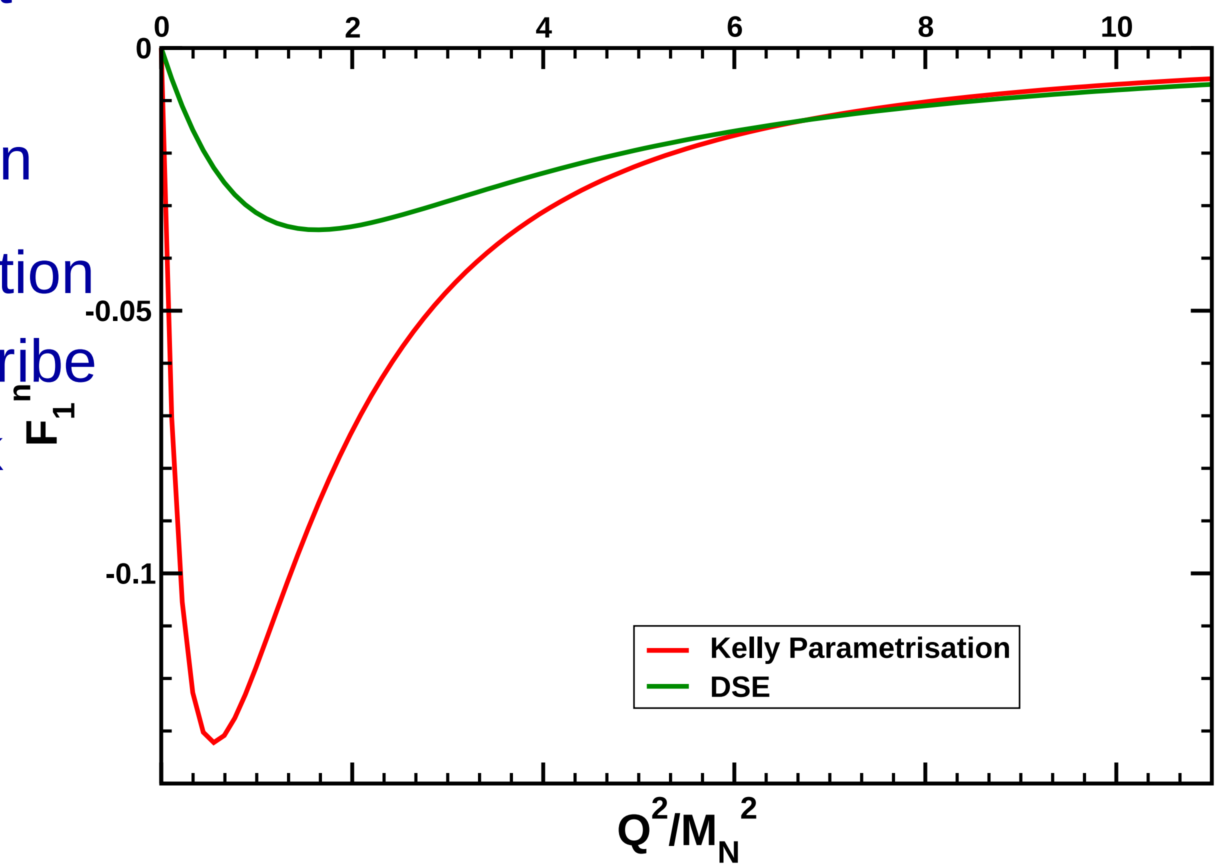
- Comparison between Faddeev equation result and Kelly's parametrisation



First Contents Back Conclusion

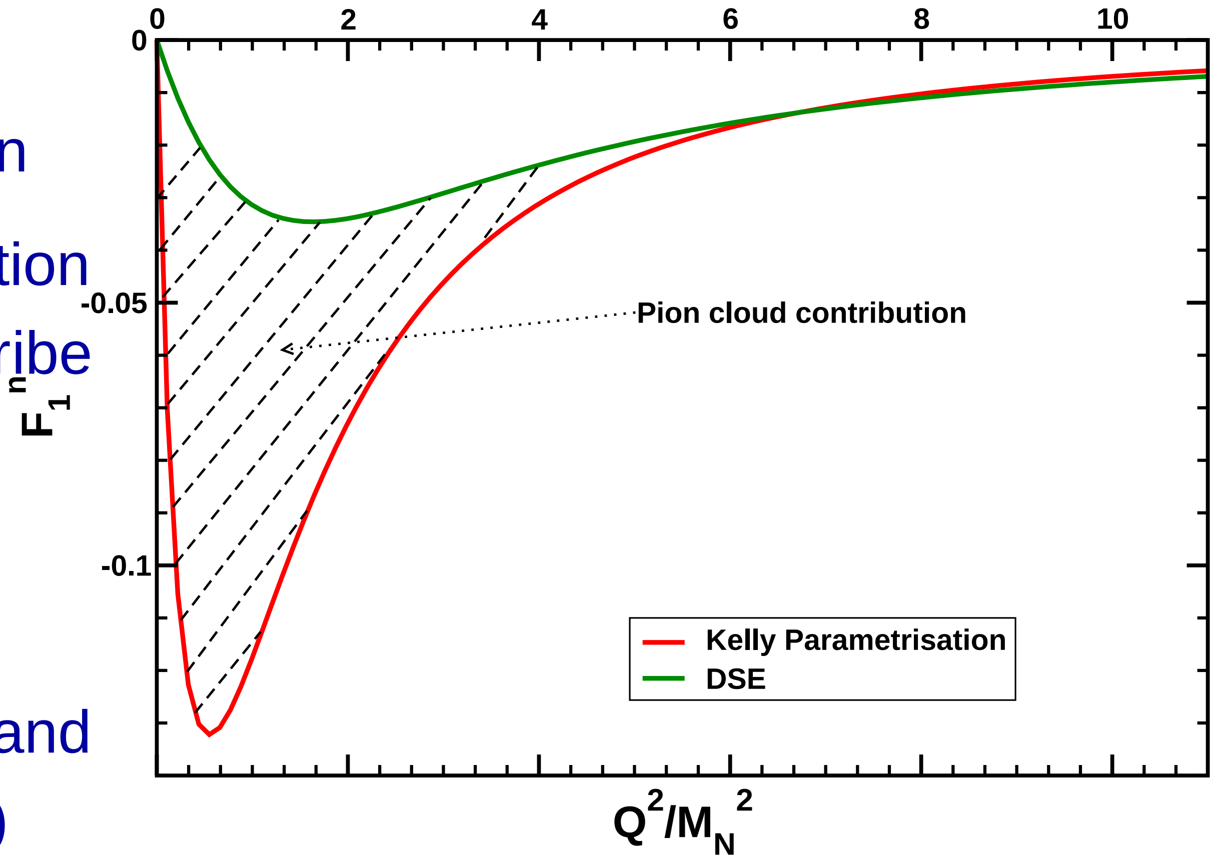
Pion Cloud
F1 – neutron

- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core



Pion Cloud
F1 – neutron

- Comparison between Faddeev equation result and Kelly's parametrisation
- Faddeev equation set-up to describe dressed-quark core
- Pseudoscalar meson cloud (and related effects) significant for $Q^2 \lesssim 3 - 4\, M_N^2$



Epilogue

Epilogue

Epilogue

- DCSB exists in QCD.

Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices

Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
  - It predicts, amongst other things, that
    - light current-quarks become heavy constituent-quarks
    - pseudoscalar mesons are unnaturally light
    - pseudoscalar mesons couple unnaturally strongly to light-quarks
    - pseudscalar mesons couple unnaturally strongly to the lightest baryons

Epilogue

- DCSB exists in QCD.
    - It is manifest in dressed propagators and vertices
    - It predicts, amongst other things, that
        - light current-quarks become heavy constituent-quarks
        - pseudoscalar mesons are unnaturally light
        - pseudoscalar mesons couple unnaturally strongly to light-quarks
        - pseudscalar mesons couple unnaturally strongly to the lightest baryons
    - It impacts dramatically upon observables.

Epilogue

- Form Factors - progress anticipated in near- to medium-term
  - Quantifying pseudoscalar meson “cloud” effects

Epilogue

- Form Factors - progress anticipated in near- to medium-term
  - Quantifying pseudoscalar meson "cloud" effects
  - Locating and explaining the transition from nonp-QCD to p-QCD in the pion and nucleon electromagnetic form factors

Epilogue

- Form Factors - progress anticipated in near- to medium-term
  - Quantifying pseudoscalar meson “cloud” effects
  - Locating and explaining the transition from nonp-QCD to p-QCD in the pion and nucleon electromagnetic form factors
  - Explaining the high Q^2 behavior of the proton form factor ratio in the space-like region

Epilogue

- Form Factors - progress anticipated in near- to medium-term
  - Quantifying pseudoscalar meson “cloud” effects
  - Locating and explaining the transition from nonp-QCD to p-QCD in the pion and nucleon electromagnetic form factors
  - Explaining the high Q^2 behavior of the proton form factor ratio in the space-like region
  - Detailing broadly the role of two-photon exchange contributions

nothing!

Epilogue

- Form Factors - progress anticipated in near- to medium-term
  - Quantifying pseudoscalar meson “cloud” effects
  - Locating and explaining the transition from nonp-QCD to p-QCD in the pion and nucleon electromagnetic form factors
  - Explaining the high Q^2 behavior of the proton form factor ratio in the space-like region
  - Detailing broadly the role of two-photon exchange contributions
  - Explaining relationship between parton properties on the light-front and rest frame structure of hadrons

Contents

First Contents Back Conclusion

Dyson-Schwinger Equations

First Contents Back Conclusion

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory

First Contents Back Conclusion

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  . . . . Materially Reduces Model Dependence

First Contents Back Conclusion

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  .... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD

Office of Science
U.S. DEPARTMENT OF ENERGY

Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars

Argonne NATIONAL LABORATORY

First Contents Back Conclusion

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  . . . . Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons

First Contents Back Conclusion

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  . . . . Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  .... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking

        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement

        – Coloured objects not detected, not detectable?
- $\Rightarrow$ Understanding InfraRed (long-range)

  .... behaviour of $\alpha_s(Q^2)$

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  .... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?
- Method yields Schwinger Functions $\equiv$ Propagators

First Contents Back Conclusion

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  .... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?

Cross-Sections built from Schwinger Functions

First | Contents | Back | Conclusion

Schwinger Functions

Schwinger Functions

- Solutions are Schwinger Functions (Euclidean Green Functions)

First Contents Back Conclusion

Schwinger Functions

- Solutions are Schwinger Functions (Euclidean Green Functions)
- Not all are Schwinger functions are experimentally observable

First | Contents | Back | Conclusion

Schwinger Functions

- Solutions are Schwinger Functions (Euclidean Green Functions)
- Not all are Schwinger functions are experimentally observable but . . .
  - all are same VEVs measured in numerical simulations of lattice-regularised QCD
  - opportunity for comparisons at pre-experimental level . . . cross-fertilisation

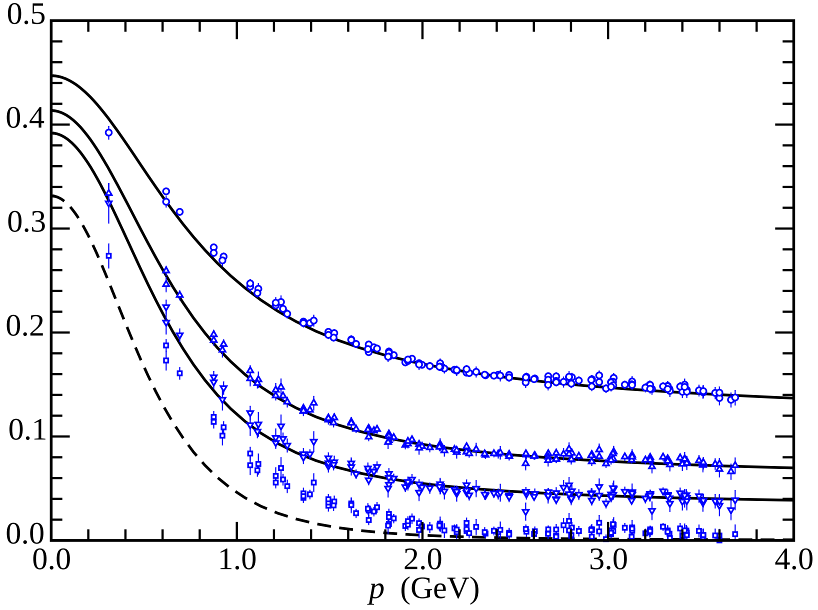
First Contents Back Conclusion

Schwinger Functions

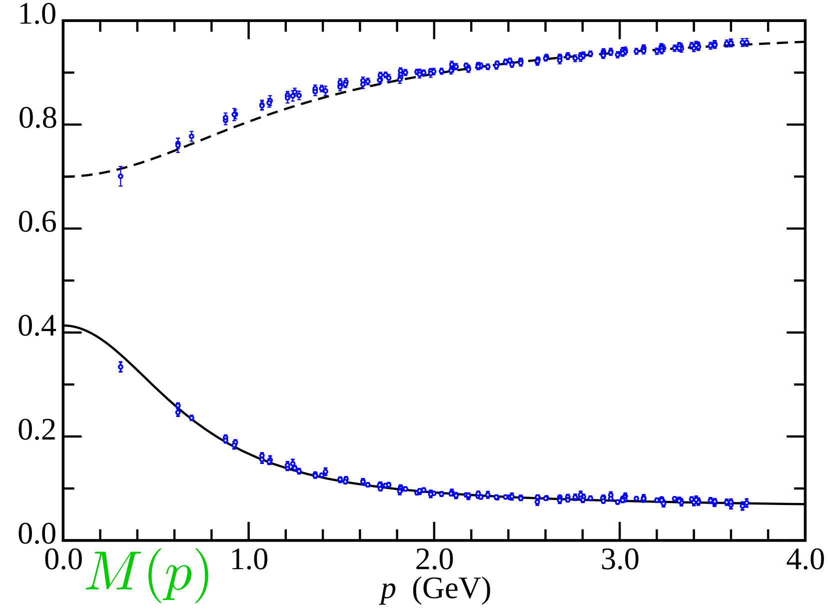
- Solutions are Schwinger Functions (Euclidean Green Functions)
- Not all are Schwinger functions are experimentally observable but . . .
  - all are same VEVs measured in numerical simulations of lattice-regularised QCD
  - opportunity for comparisons at pre-experimental level . . . cross-fertilisation
- Proving fruitful.

Quenched-QCD Dressed-Quark Propagator

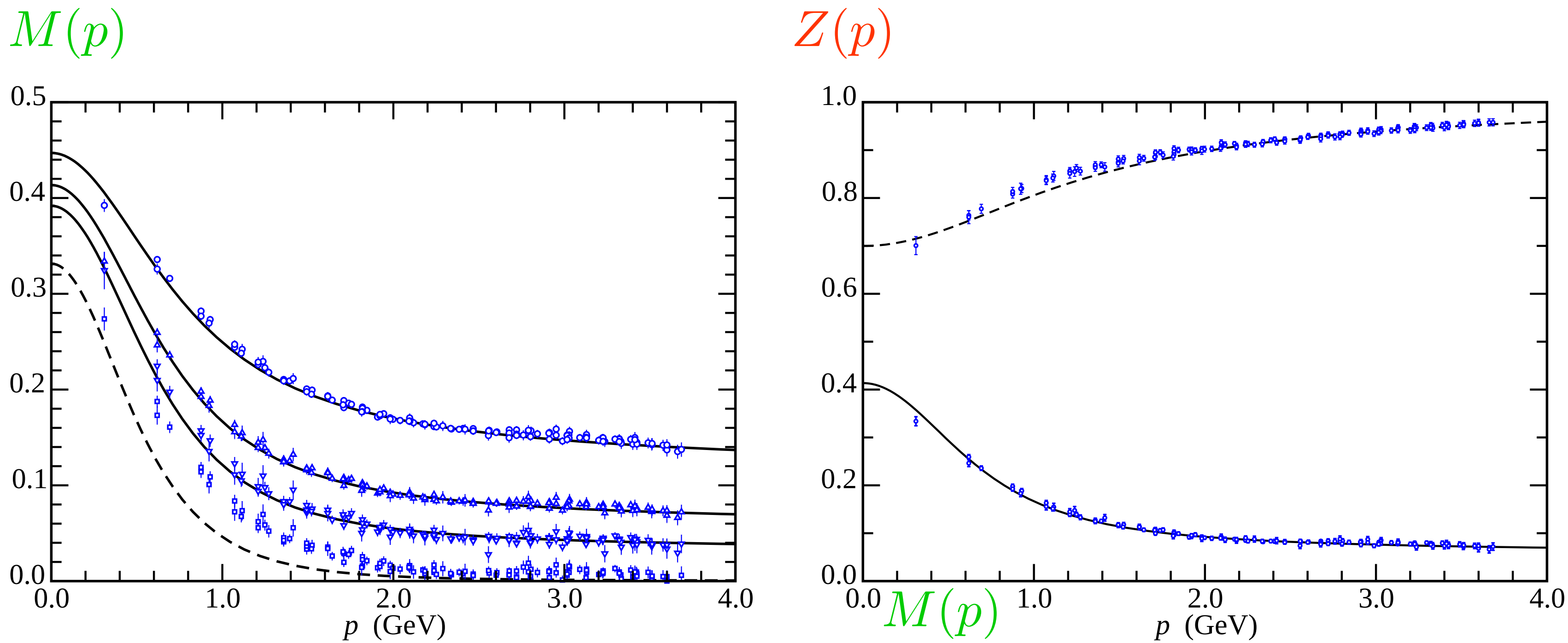
$M(p)$



$Z(p)$

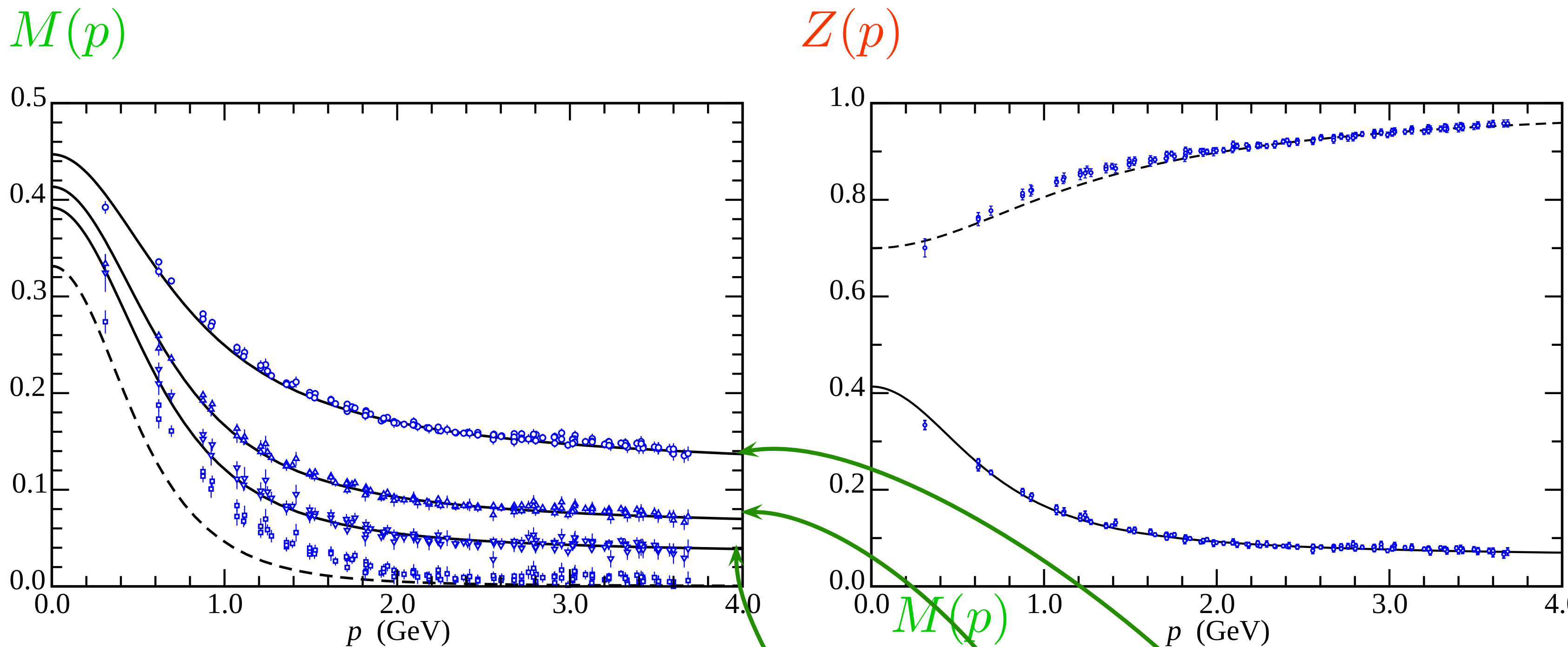


2002 Quenched-QCD Dressed-Quark Propagator



- "*data*:" Quenched Lattice Meas.
  – Bowman, Heller, Leinweber, Williams: he-lat/0209129

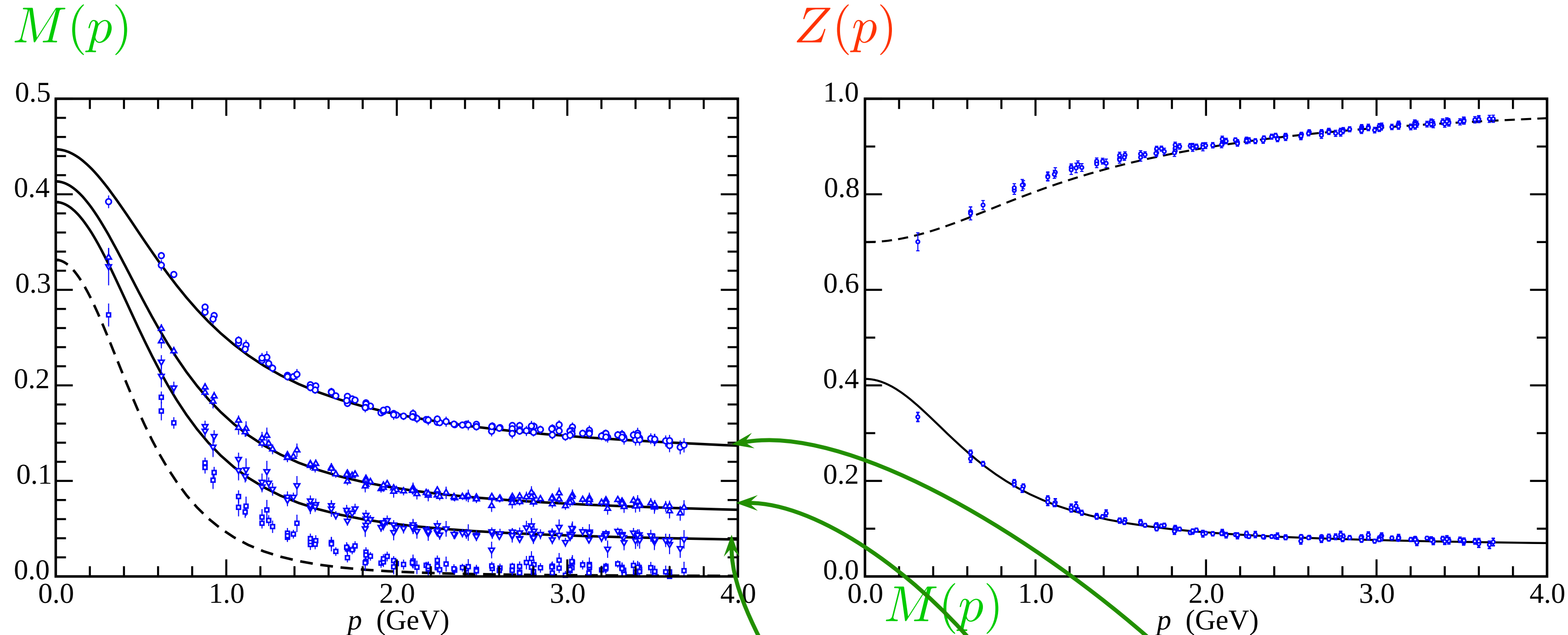
2002 Quenched-QCD Dressed-Quark Propagator



- "*data*:" Quenched Lattice Meas.

  – Bowman, Heller, Leinweber, Williams: he-lat/0209129
  current-quark masses: 30 MeV, 50 MeV, 100 MeV

2002 Quenched-QCD Dressed-Quark Propagator



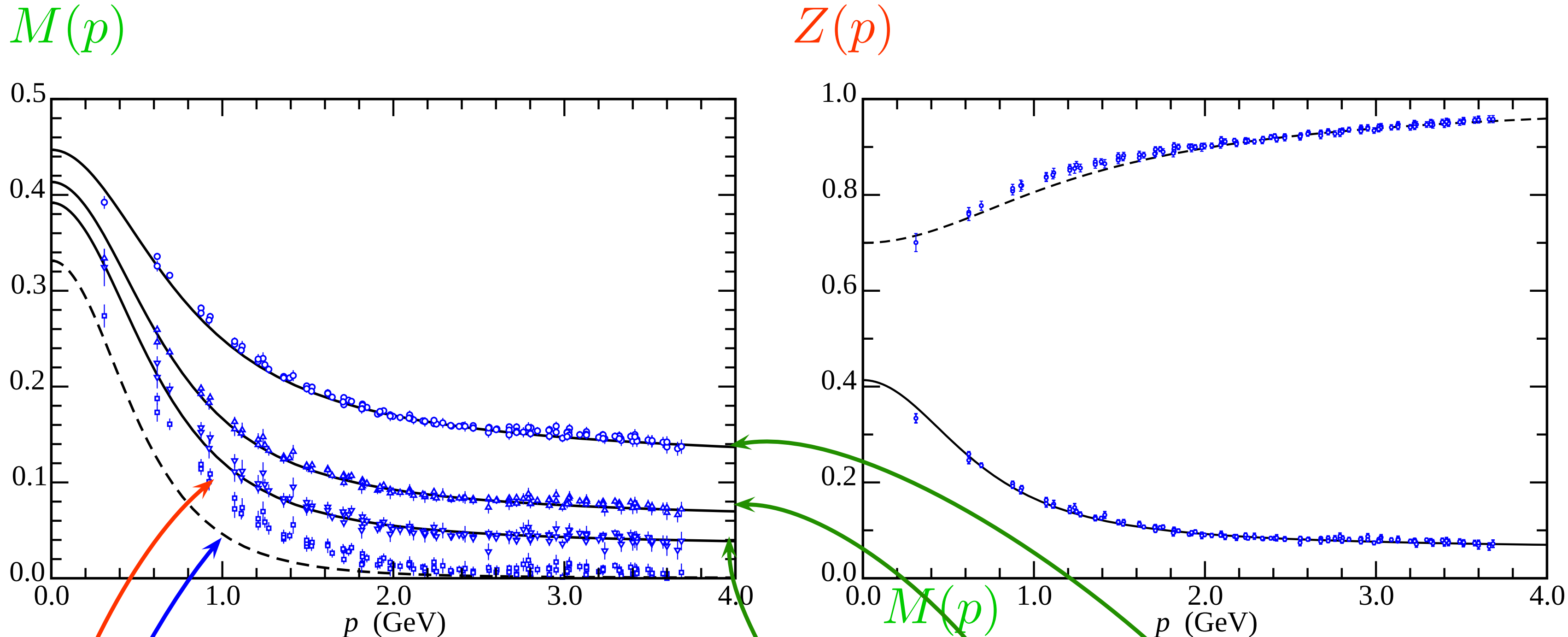
- "*data*:" Quenched Lattice Meas.

  – Bowman, Heller, Leinweber, Williams: he-lat/0209129
  current-quark masses: 30 MeV, 50 MeV, 100 MeV

- *Curves*: Quenched DSE Cal.

– Bhagwat, Pichowsky, Roberts, Tandy nu-th/0304003

2002 Quenched-QCD Dressed-Quark Propagator

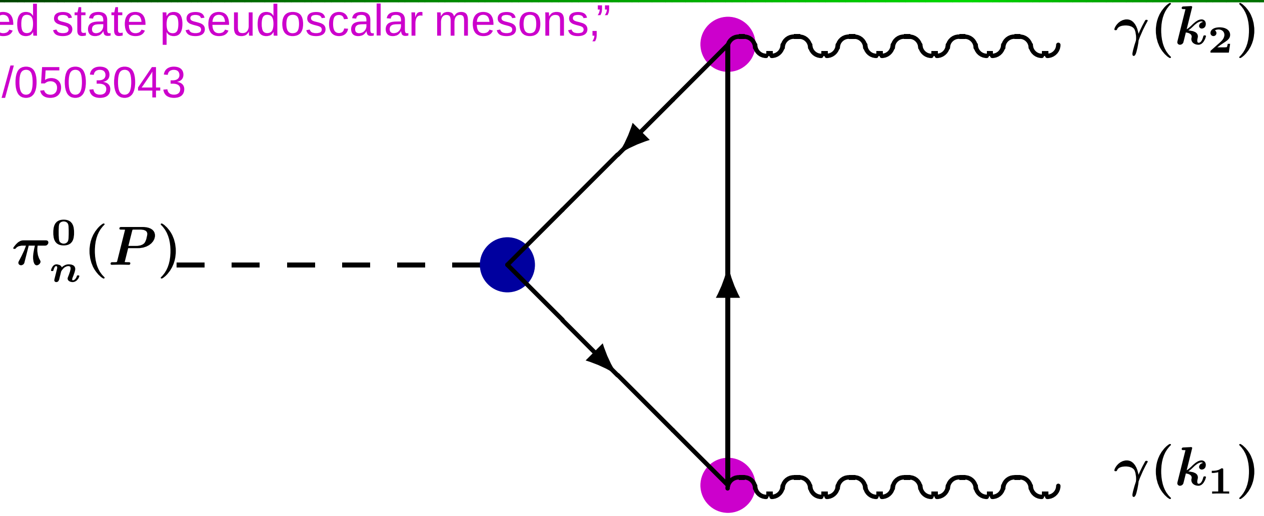


- "*data*:" Quenched Lattice Meas.
  – Bowman, Heller, Leinweber, Williams: he-lat/0209129
  current-quark masses: 30 MeV, 50 MeV, 100 MeV
- *Curves*: Quenched DSE Cal.
  – Bhagwat, Pichowsky, Roberts, Tandy nu-th/0304003
- Linear extrapolation of lattice data to chiral limit is inaccurate

First Contents Back Conclusion

Two-photon Couplings of Pseudoscalar Mesons

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



$$T_{\mu\nu}^{\pi_n^0}(k_1,k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}\, G^{\pi_n^0}(k_1,k_2)$$

- Define: $\mathcal{T}_{\pi_n^0}(P^2,Q^2) = G^{\pi_n^0}(k_1,k_2)\Big|_{k_1^2=Q^2=k_2^2}$

  This is a transition form factor.

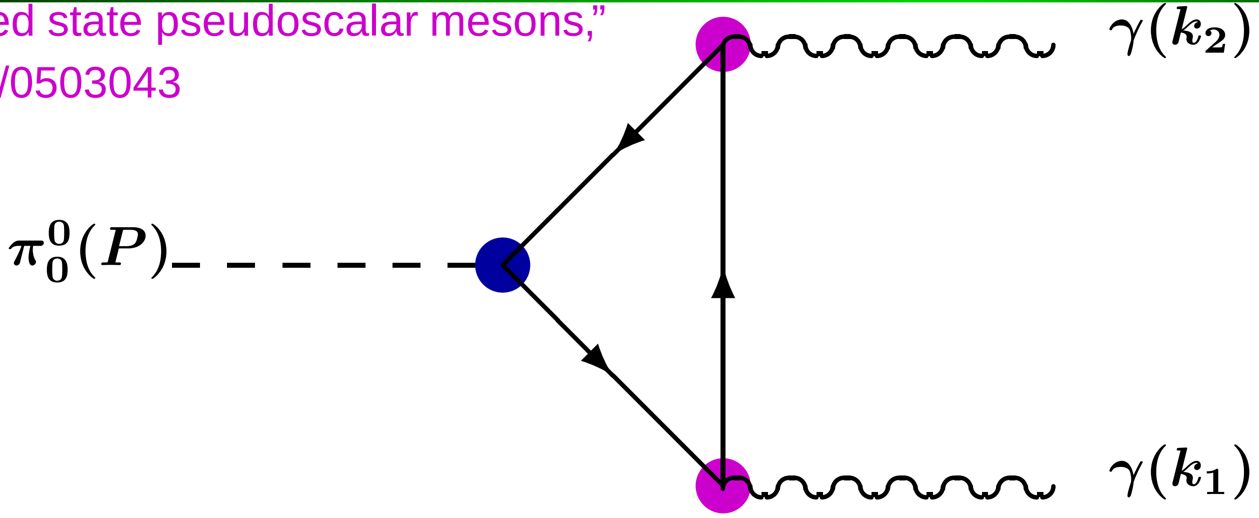
- Physical Processes described by couplings:

  $g_{\pi_0^0\gamma\gamma} := \mathcal{T}_{\pi_0^0}(-m_{\pi_0^0}^2, 0)$

  Width: $\Gamma_{\pi_n^0\gamma\gamma} = \alpha_{\rm em}^2\, \frac{m_{\pi_n}^3}{16\pi^3}\, g_{\pi_n\gamma\gamma}^2$

Two-photon Couplings: Goldstone Mode

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



$$T^{\pi^0_0}_{\mu\nu}(k_1,k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}\, G^{\pi^0_0}(k_1,k_2)$$

- Chiral limit, model-independent and algebraic result

  $$g_{\pi^0_0\gamma\gamma} := \mathcal{T}_{\pi^0_0}(-m^2_{\pi^0_0}=0,0) = \frac{1}{2}\frac{1}{f_{\pi_0}}$$

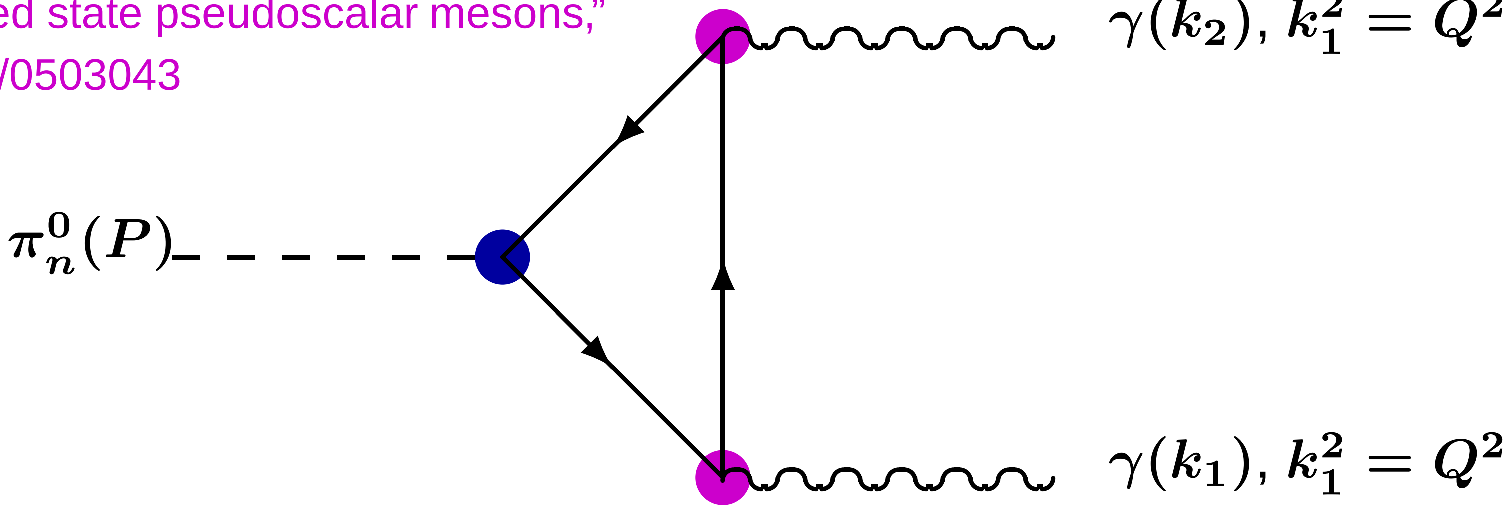
  So long as truncation veraciously preserves chiral symmetry and the pattern of its dynamical breakdown

- The most widely known consequence of the Abelian anomaly

First Contents Back Conclusion

Two-photon Couplings: Transition Form Factor

Höll, Krassnigg, Maris, *et al.*,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043



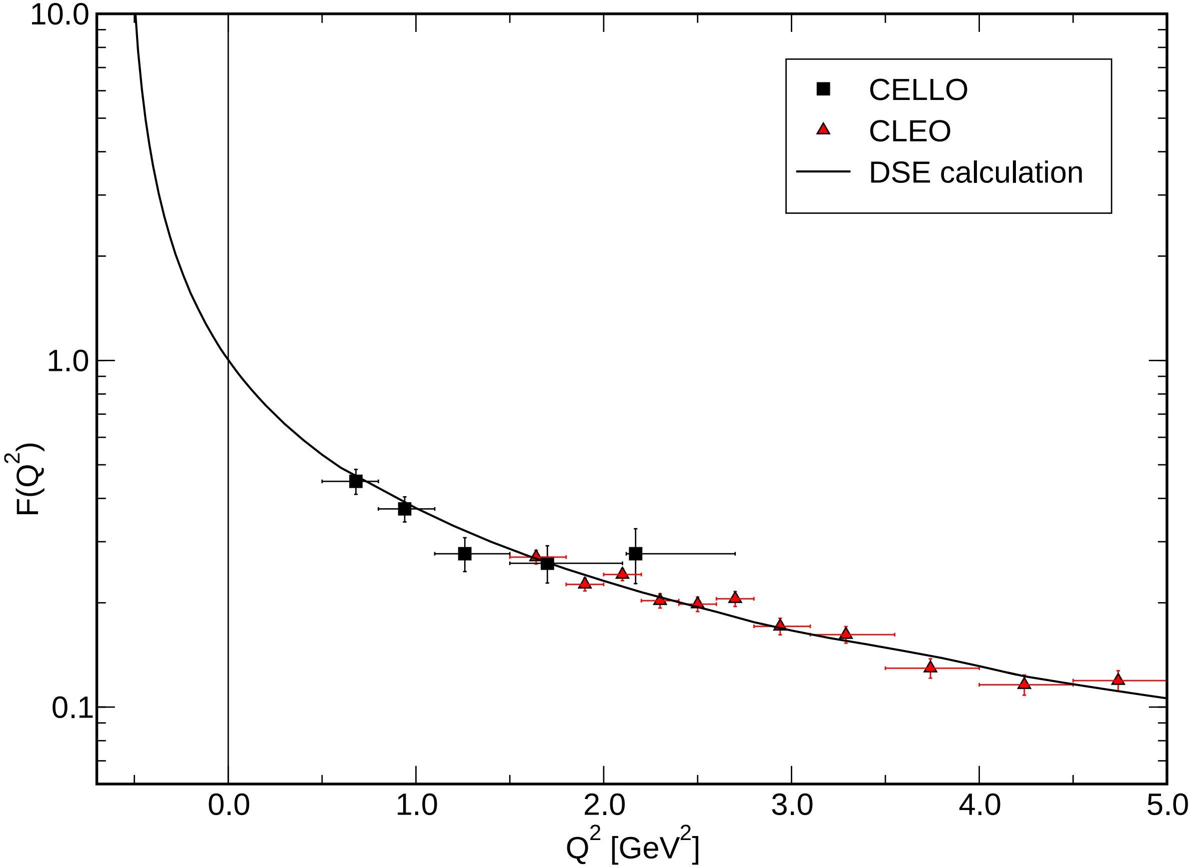
$$T^{\pi^0_n}_{\mu\nu}(k_1,k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}\, G^{\pi^0_n}(k_1,k_2)$$

So long as truncation preserves chiral symmetry and the pattern of its dynamical breakdown, and the one-loop renormalisation group properties of QCD: model-independent result – $\forall n$:

$$\mathcal{T}_{\pi^0_n}(P^2,Q^2) = G^{\pi^0_n}(k_1,k_2)\Big|_{k_1^2=Q^2=k_2^2} \stackrel{Q^2\gg\Lambda^2_{\rm QCD}}{=} \frac{4\pi^2}{3}\frac{f_{\pi_n}}{Q^2}$$

Two-photon Couplings: Transition Form Factor

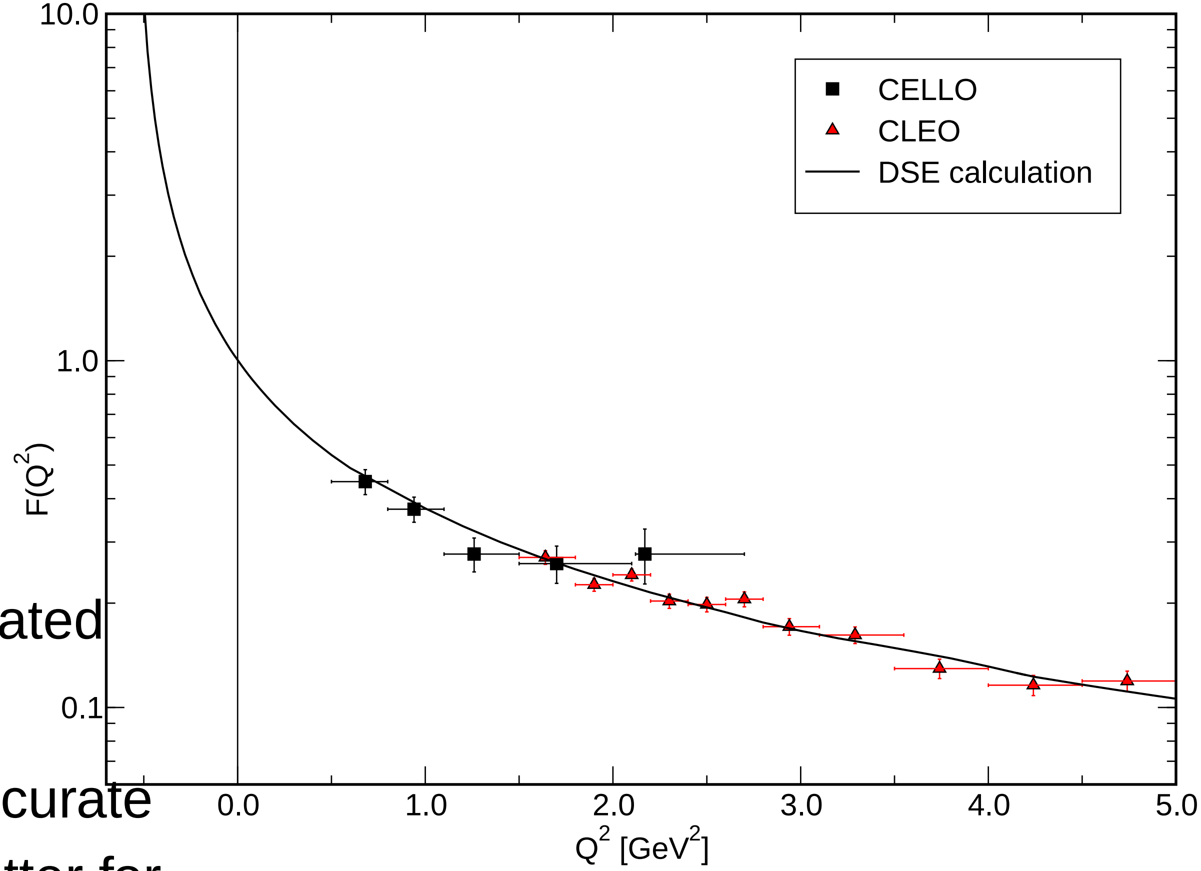
Maris and Tandy, “ Electromagnetic transition form-factors of light mesons,” nucl-th/0201017



First Contents Back Conclusion

Two-photon Couplings: Transition Form Factor

Maris and Tandy, " Electromagnetic transition form-factors of light mesons," nucl-th/0201017



DSE result:

- normalisation calculated
- ρ-meson generated dynamically
- pQCD result accurate to $\sim$ 20% or better for $Q^2 \geq 3$ GeV^2

First Contents Back Conclusion

Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al.*,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$

First Contents Back Conclusion

Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al*.,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$
- BUT, $f_{\pi_n} \equiv 0$, $\forall n$!

Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al*.,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$
- BUT, $f_{\pi_n} \equiv 0$, $\forall n$!
- Model-independent result, in chiral limit: $\forall n \geq 1$

$$\lim_{\hat{m}\to 0} \mathcal{T}_{\pi_n^0}(-m_{\pi_n}^2, Q^2)$$

$$\stackrel{Q^2 \gg \Lambda_{QCD}^2}{=} \frac{4\pi^2}{3} F_n^{(2)}(-m_{\pi_n}^2) \frac{\ln^\gamma Q^2/\omega_{\pi_n}^2}{Q^4}\Big|_{\hat{m}=0}$$

where:

- γ is an anomalous dimension
- ω_{π_n} is a width mass-scale

both determined, in part, by properties of the meson's Bethe-Salpeter wave function.

Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al*.,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$
- BUT, $f_{\pi_n} \equiv 0, \forall n$!
- Model-independent result, in chiral limit: $\forall n \geq 1$

$$\lim_{\hat{m}\to 0} \mathcal{T}_{\pi^0_n}(-m^2_{\pi_n}, Q^2) \overset{Q^2 \gg \Lambda^2_{\rm QCD}}{=} \frac{4\pi^2}{3} F_n^{(2)}(-m^2_{\pi_n}) \frac{\ln^\gamma Q^2/\omega^2_{\pi_n}}{Q^4}\Bigg|_{\hat{m}=0}$$

where:

  - γ is an anomalous dimension
  - ω_{π_n} is a width mass-scale

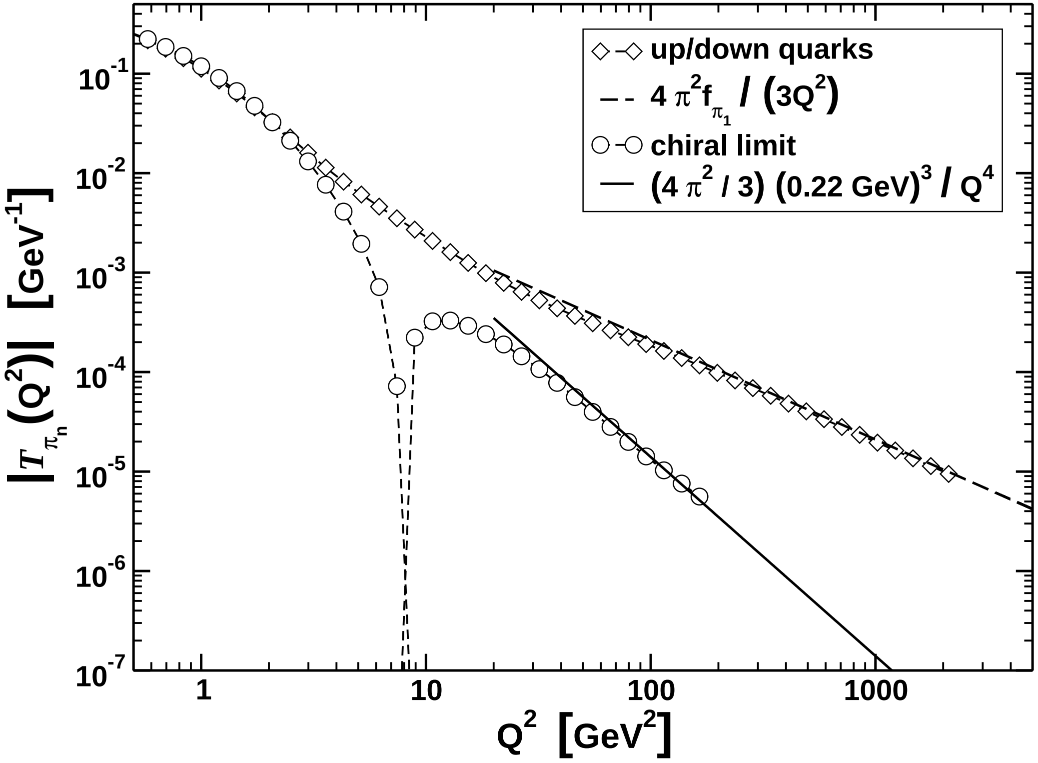
both determined, in part, by properties of the meson's Bethe-Salpeter wave function.

- Importantly, $F_n^{(2)}(-m^2_{\pi_n}) \not\propto f_{\pi_n}$. Instead, it is determined by DCSB mass-scales for π_n that do not vanish in the chiral limit.

Transition Form Factor (Chiral): RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al.*,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043

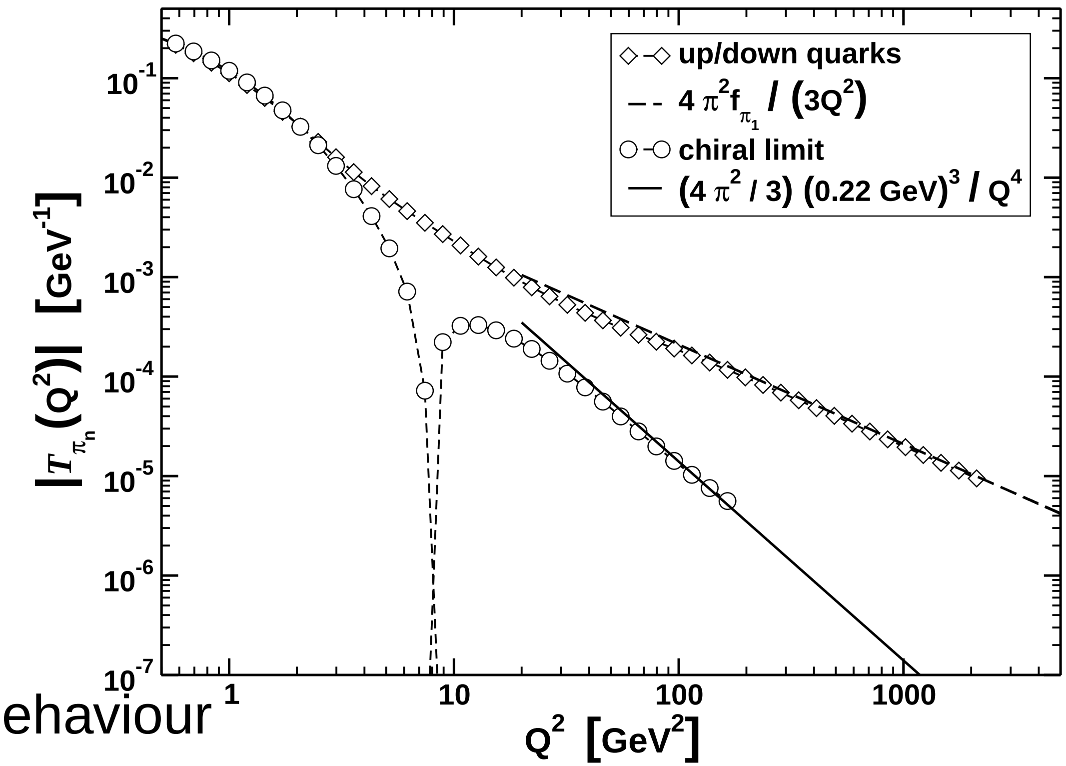
- $m_u(1\,\text{GeV}) = m_d(1\,\text{GeV}) = 0$



First Contents Back Conclusion

Transition Form Factor (Chiral): RGI Rainbow-Ladder

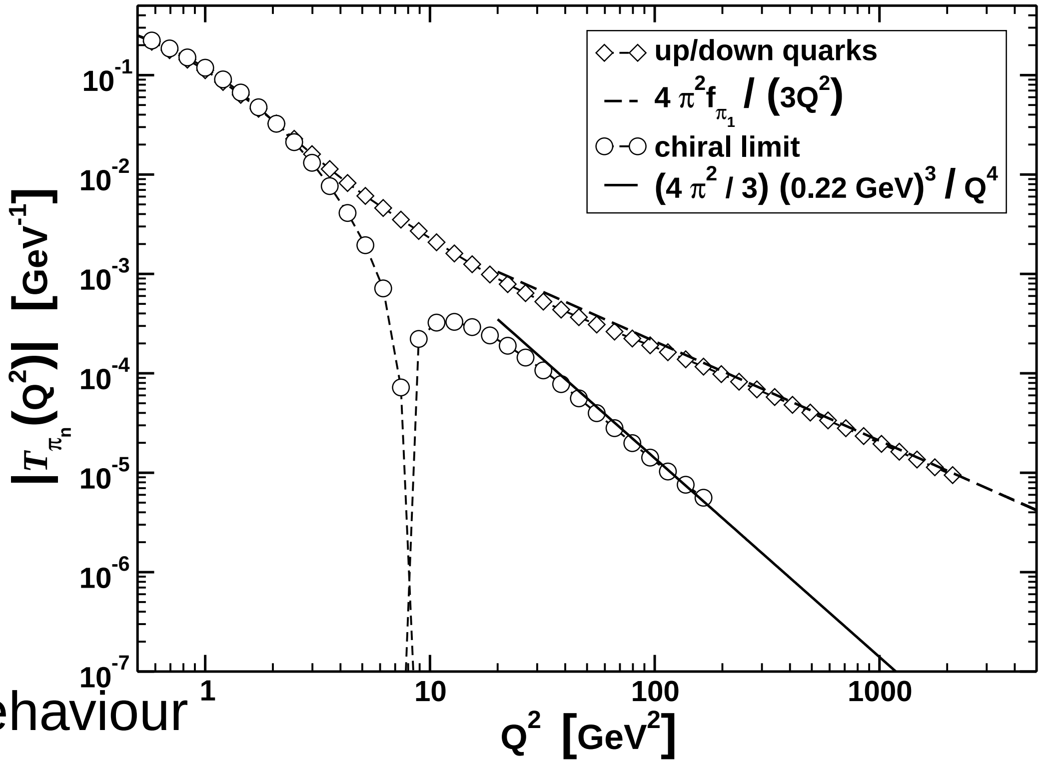
Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 0$
- Again, Predicted UV-behaviour is abundantly clear
  - precise for $Q^2 > 120\,\mathrm{GeV}^2$

Transition Form Factor (Chiral): RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043

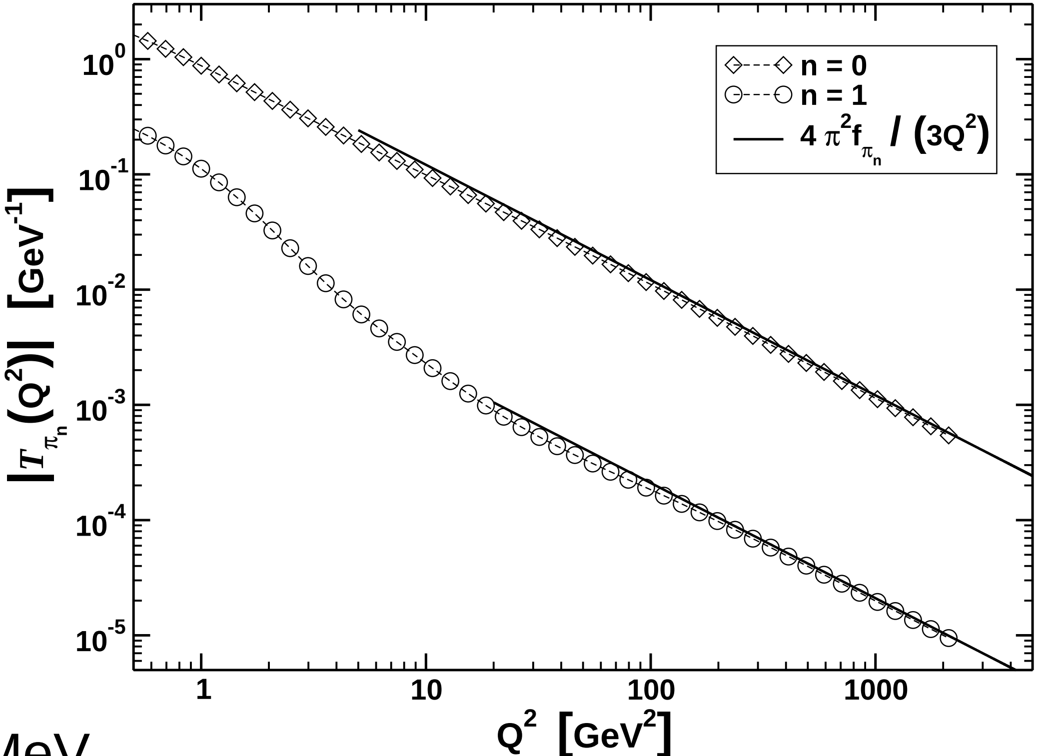


- $m_u(1\,\text{GeV}) = m_d(1\,\text{GeV}) = 0$
- Again, Predicted UV-behaviour is abundantly clear
  - precise for $Q^2 > 120\,\text{GeV}^2$
- $$F_1^{(2)}(-m_{\pi_1}^2)\ \ln^\gamma Q^2/\omega_{\pi_1}^2\Big|_{\hat m=0} \approx (0.22\,\text{GeV})^3 \simeq -\langle \bar q q\rangle^0 \quad (3)$$

First | Contents | Back | Conclusion

Calculated Transition Form Factor: RGI Rainbow-Ladder

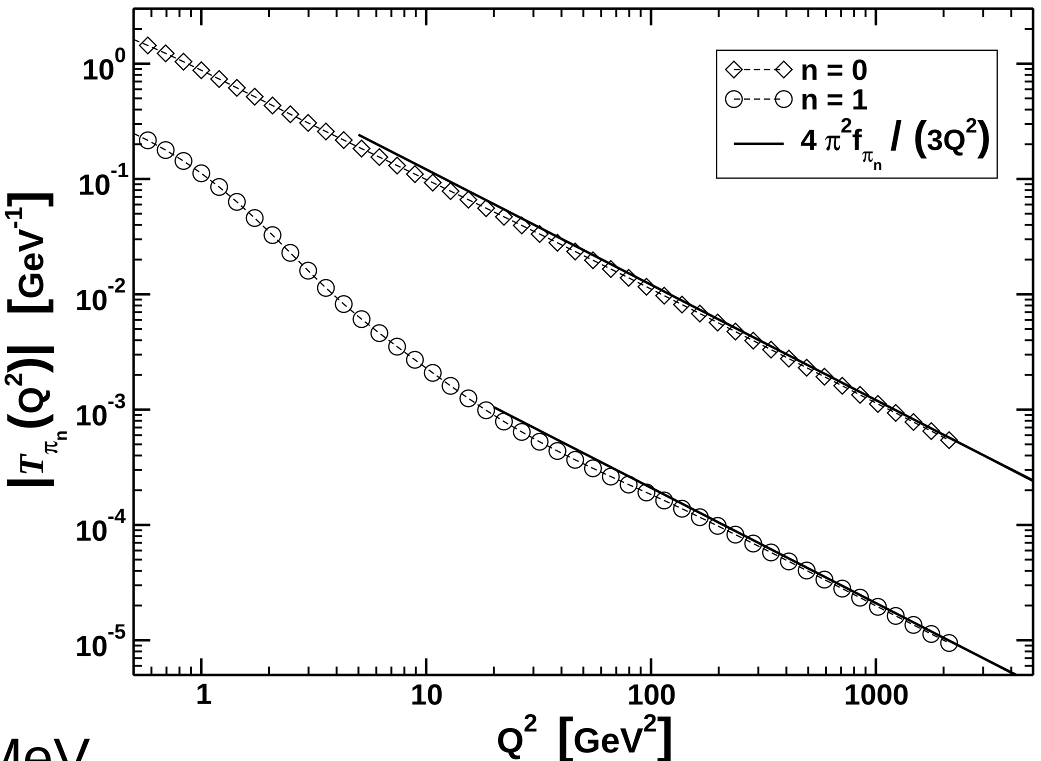
Höll, Krassnigg, Maris, *et al.*,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043



- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5$ MeV

Calculated Transition Form Factor: RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al.*,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043



- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5$ MeV
- Predicted UV-behaviour is abundantly clear
  - precise for $Q^2 > 120$ GeV^2

Results: Nucleon and Δ Masses

Office of Science
U.S. DEPARTMENT OF ENERGY
Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars
Argonne NATIONAL LABORATORY

First Contents Back Conclusion

Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0^+}	m_{1^+}	ω_{0^+}	ω_{1^+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.80	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1^+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for
Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0^+}	m_{1^+}	ω_{0^+}	ω_{1^+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.80	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1^+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV
- Axial-vector diquark provides significant attraction

First Contents Back Conclusion

Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0^+}	m_{1^+}	ω_{0^+}	ω_{1^+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.80	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1^+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV
- Constructive Interference: 1^{++}-diquark $+ \partial_\mu \pi$

First Contents Back Conclusion

Pions and Form Factors

Pions and Form Factors

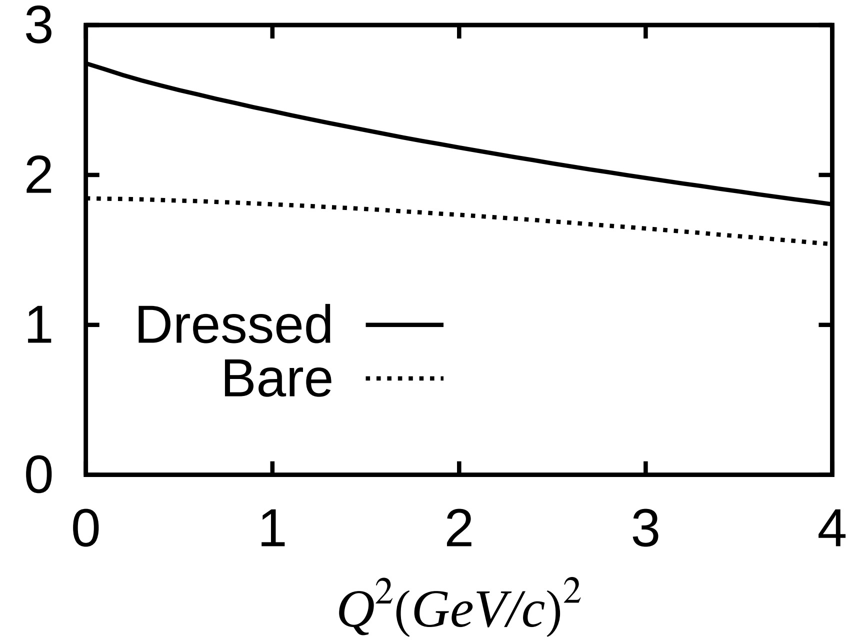
- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e, e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)

Harry Lee

Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e, e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)
- Pion cloud effects are large in the low Q^2 region.

Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G^*_M(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.

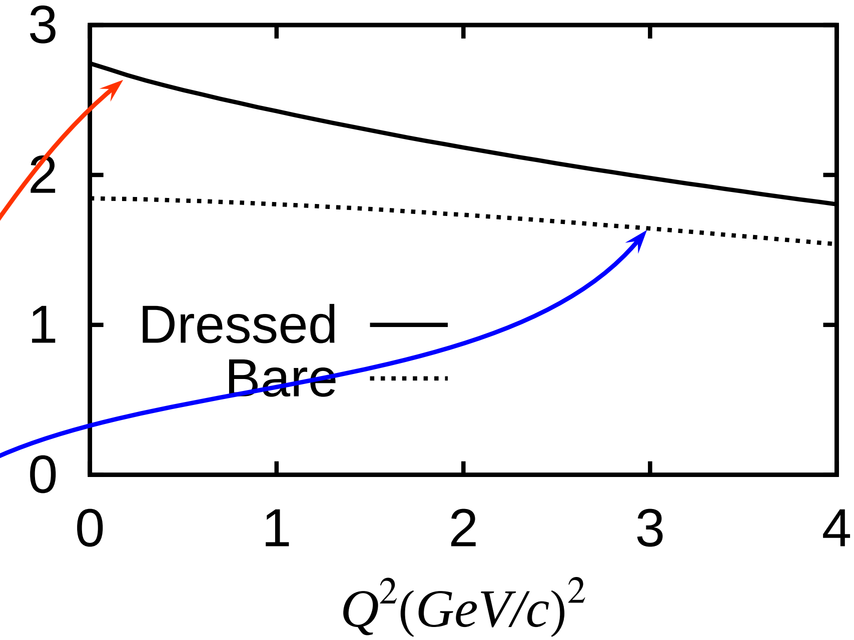


Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e, e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)
- Pion cloud effects are large in the low Q^2 region.

Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G^*_M(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.

Quark Core



- Responsible for only 2/3 of result at small Q^2
- Dominant for $Q^2 > 2 - 3\,\text{GeV}^2$