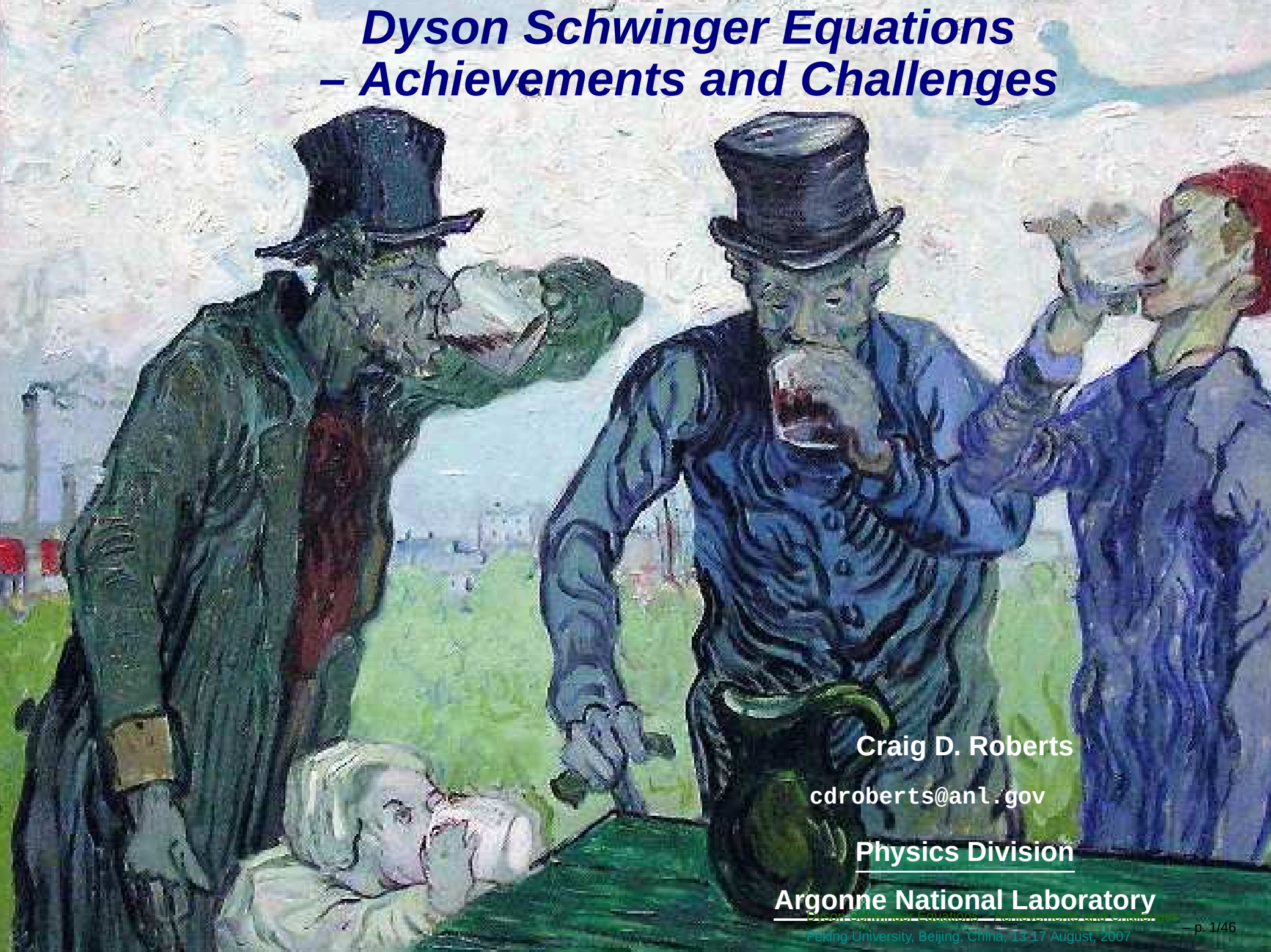
Dyson Schwinger Equations – Achievements and Challenges



Craig D. Roberts

cdroberts@anl.gov

Physics Division

Argonne National Laboratory

Study Structure via Nucleon Form Factors

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- Electron's relativistic electromagnetic current:

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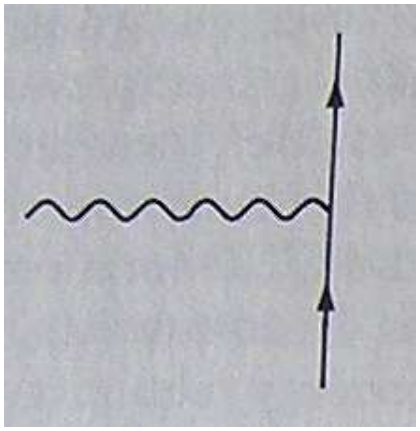

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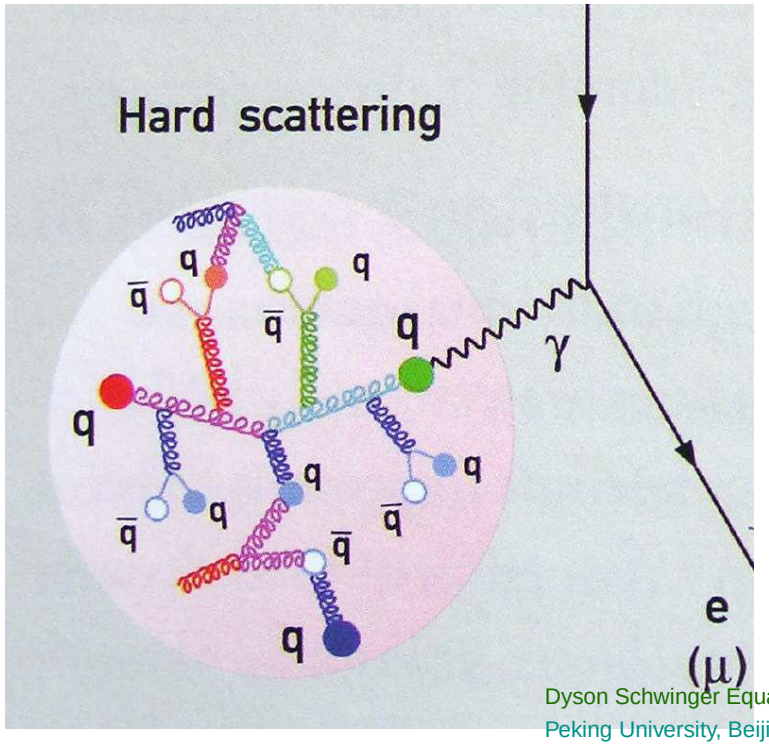

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Point-particle: $\boldsymbol{F_2 \equiv 0 \Rightarrow G_E \equiv G_M}$

Proton Form Factors, Reprise

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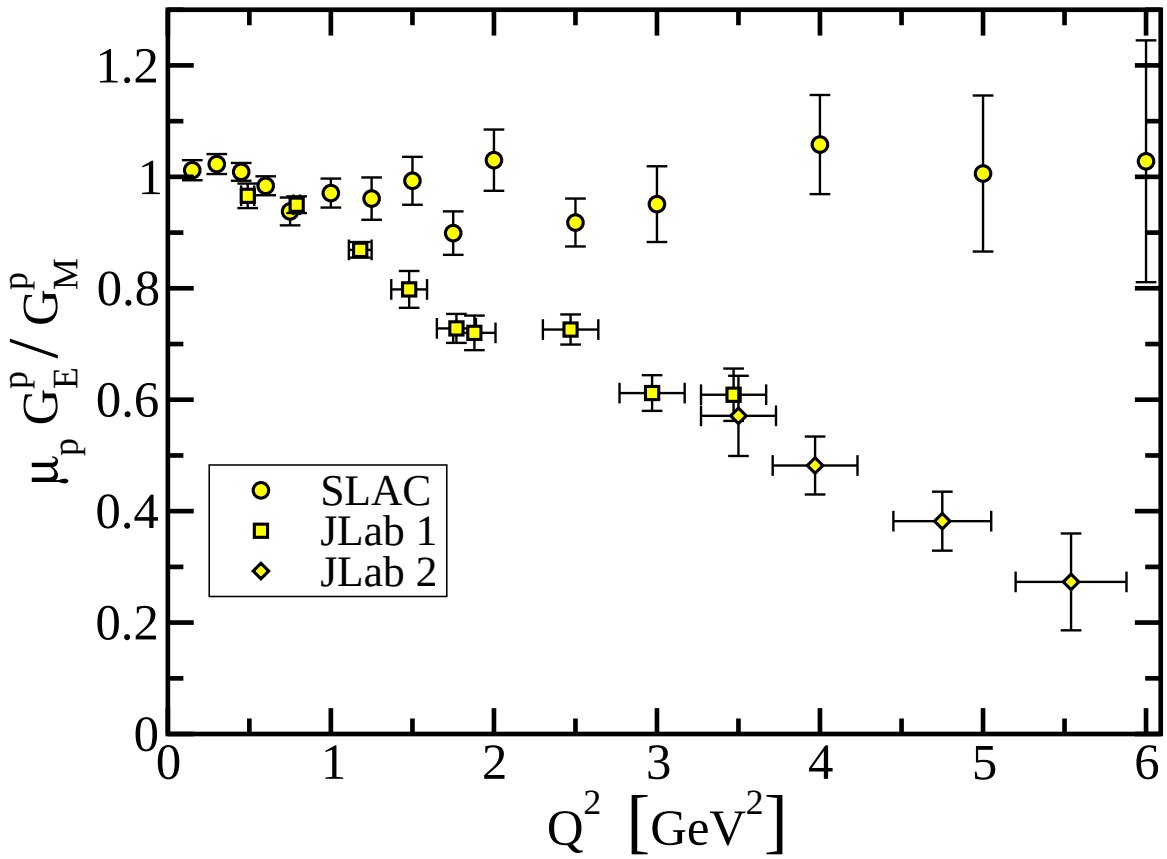
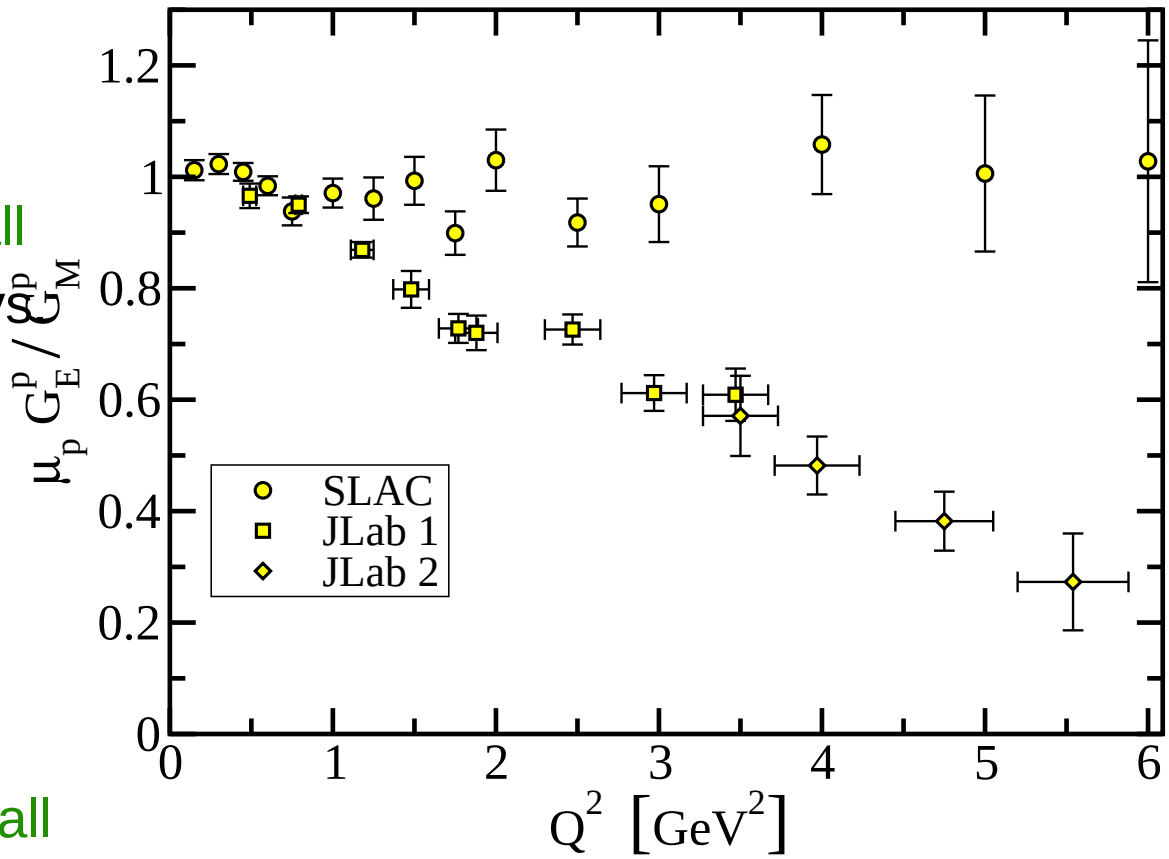
- SLAC and JLab have Measured Ratio of Proton's Electric and Magnetic Form Factors

Proton Form Factors, Reprise



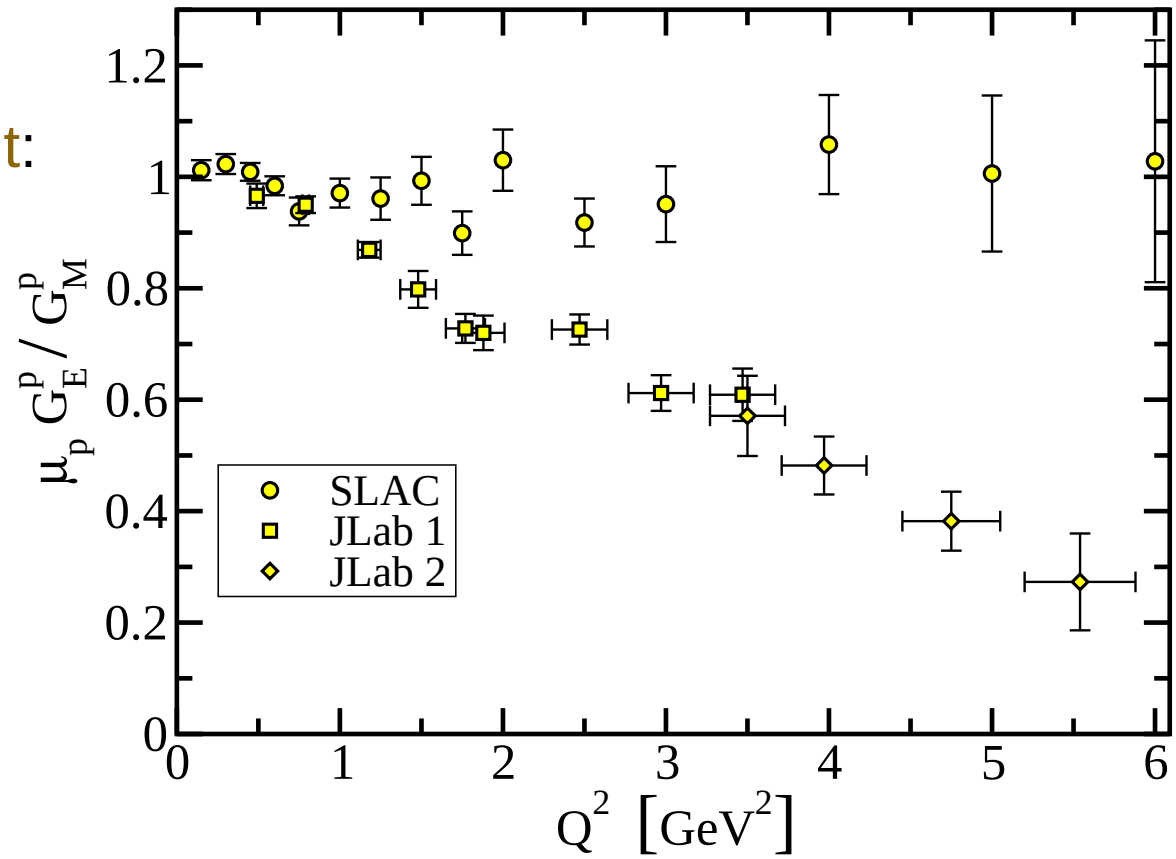
Proton Form Factors, Reprise

- Walker *et al.*, Phys. Rev. **D 49**, 5671 (1994). (SLAC)
- Jones *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **84**, 1398 (2000)
- Gayou, *et al.*, Phys. Rev. **C 64**, 038202 (2001)
- Gayou, *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **88** 092301 (2002)



Proton Form Factors, Reprise

- If JLab Correct, then Completely Unexpected Result: In the Proton
  – On Relativistic Domain
  – Distribution of Quark-Charge Not Equal Distribution of Quark-Current!



Modern Miracles in Hadron Physics

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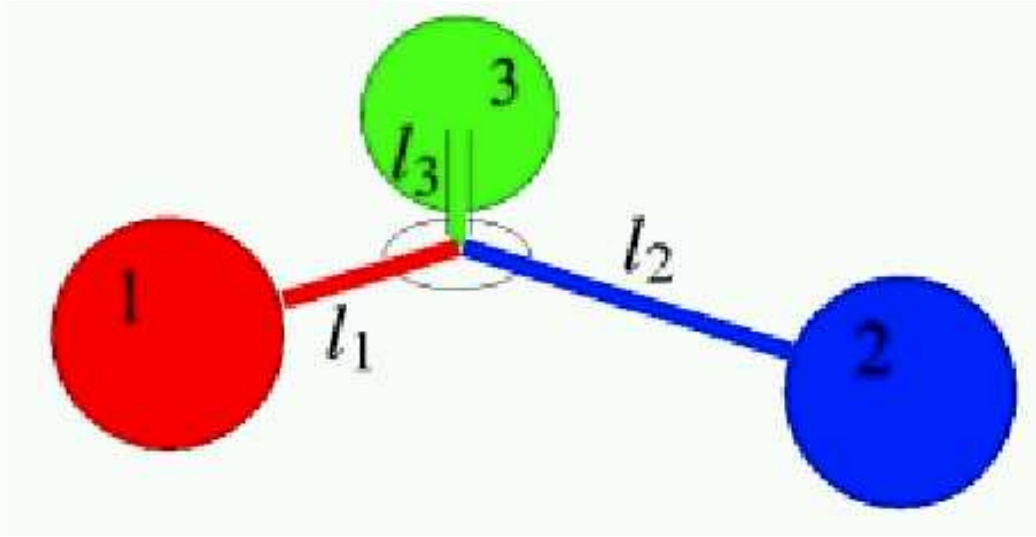

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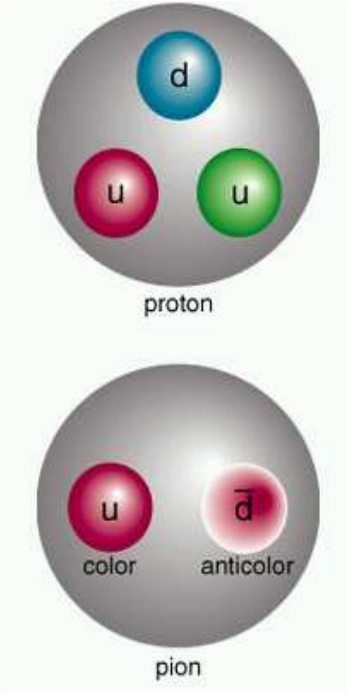

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- Another meson:
  ..... $M_\rho = \mathbf{770}$ MeV ..... No Surprises Here

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- What is "wrong" with the pion?

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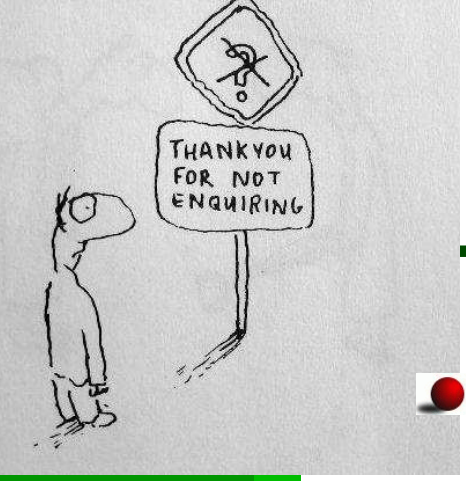
Using DSEs, we've provided this.

QCD's Emergent Phenomena



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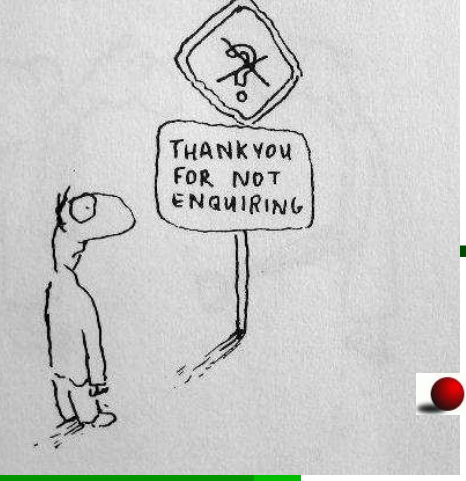

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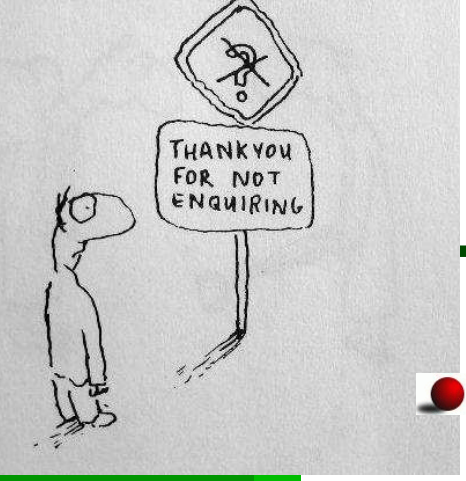

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- NSAC – Understanding these phenomena is one of the greatest intellectual challenges in physics

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- Modern Physics & Mathematics
  – Still quite some way from being able to do that

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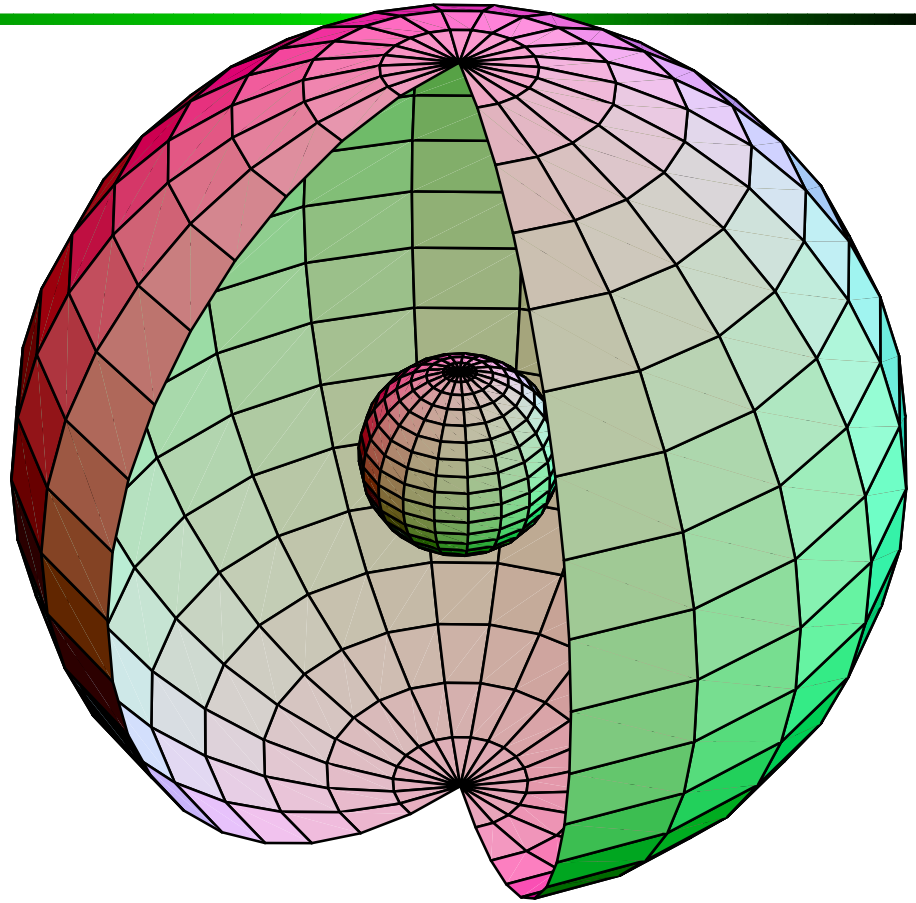

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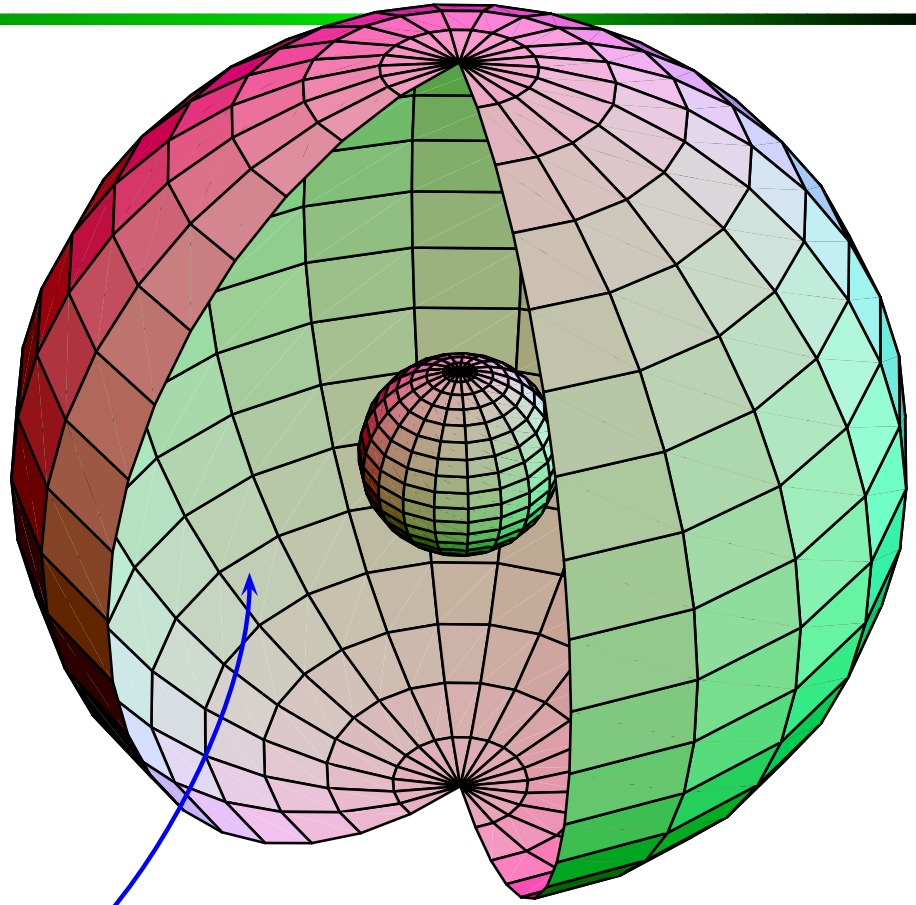

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Dyson-Schwinger Equations

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- $\Rightarrow$ Understanding InfraRed (long-range)
  ..... behaviour of $\alpha_s(Q^2)$

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- Method yields Schwinger Functions $\equiv$ Propagators

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Cross-Sections built from Schwinger Functions

Persistent Challenge

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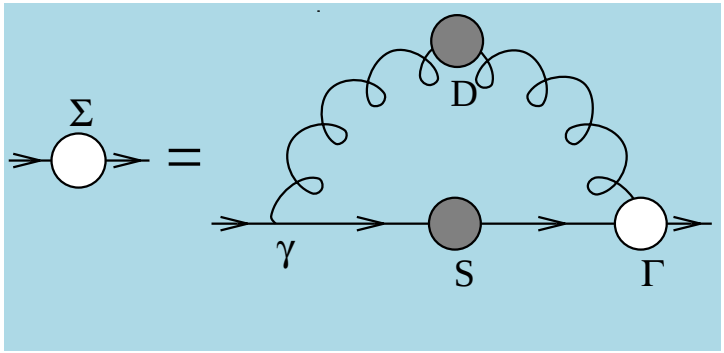

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  - Not all are Schwinger functions are experimentally observable but all are same VEVs measured in Lattice-QCD simulations . . . opportunity for comparisons at pre-experimental level . . . cross-fertilisation

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    Not useful for the nonperturbative problems in which we're interested

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  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
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Dressed-Quark Propagator

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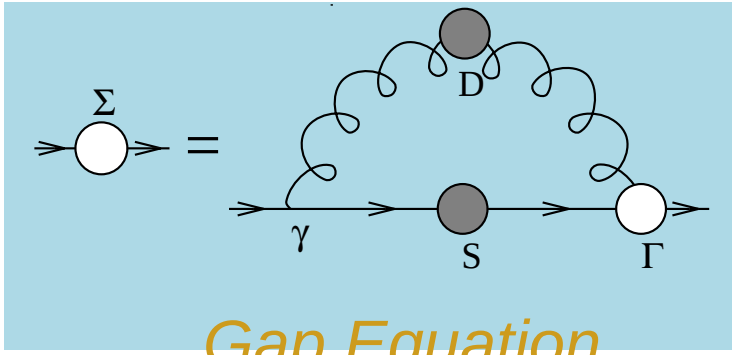
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

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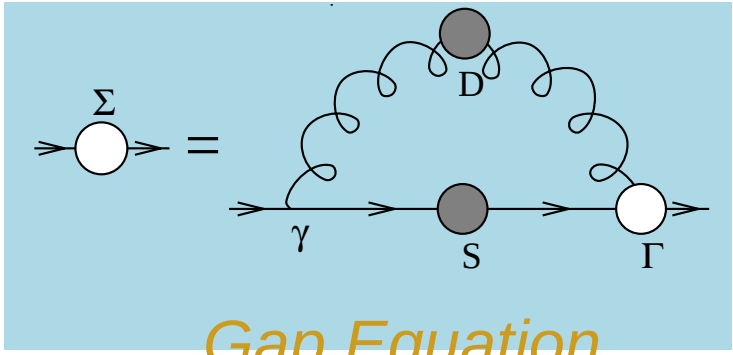
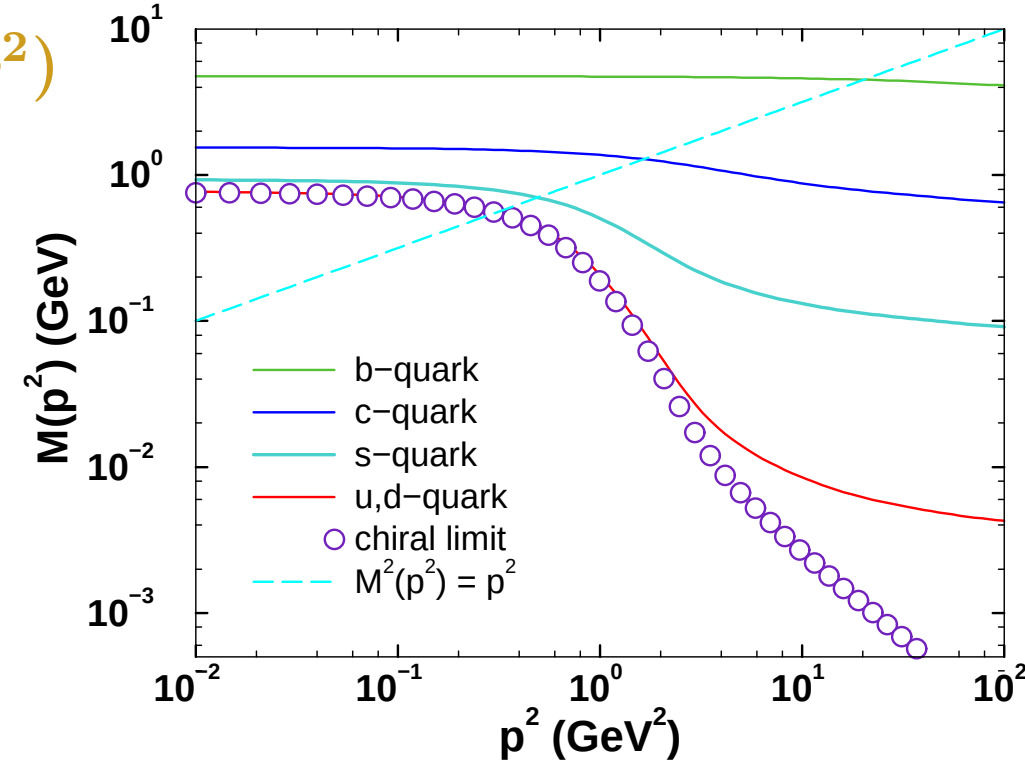


Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



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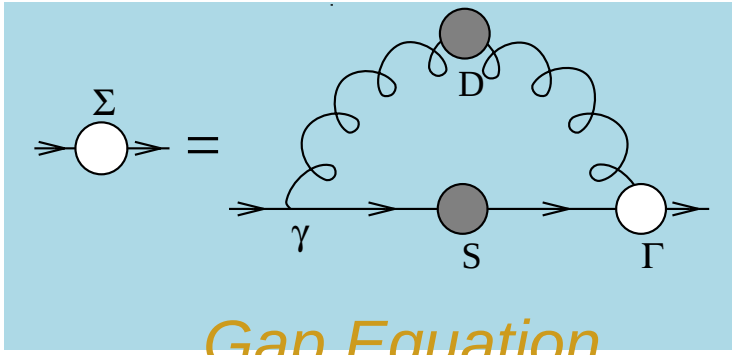

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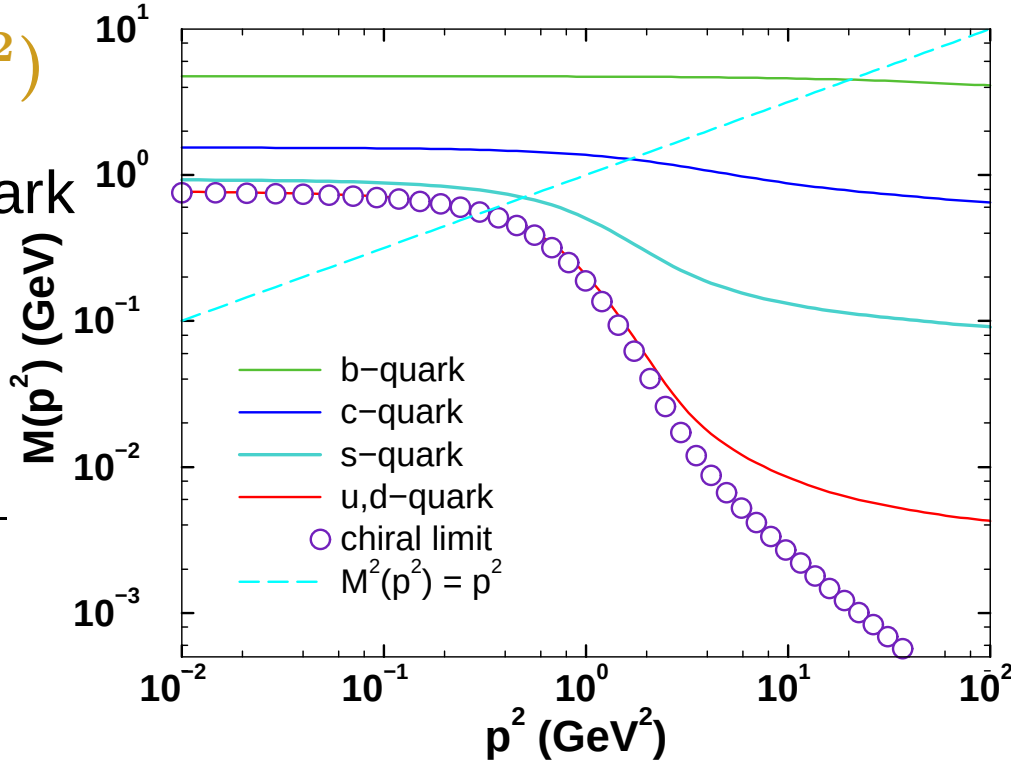
- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



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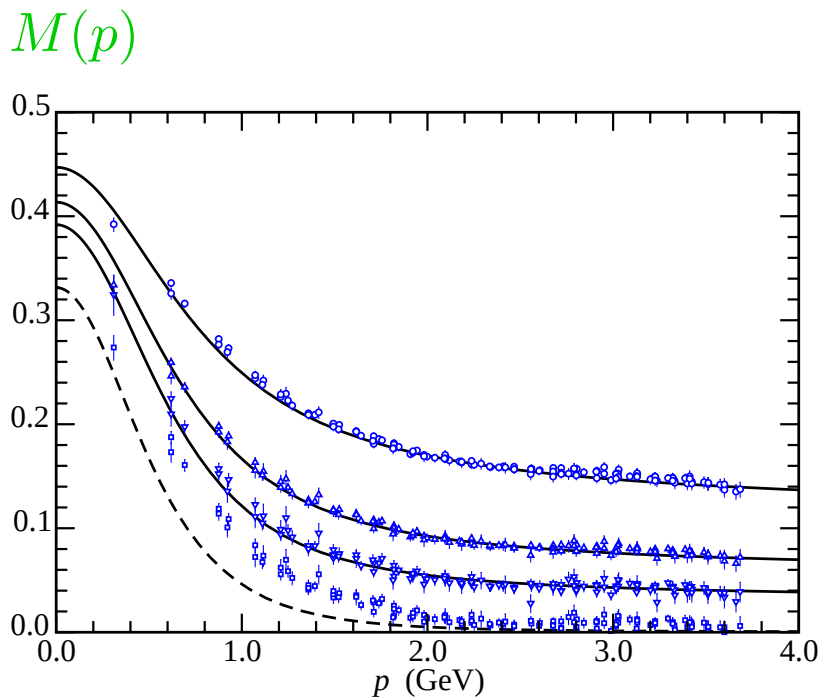
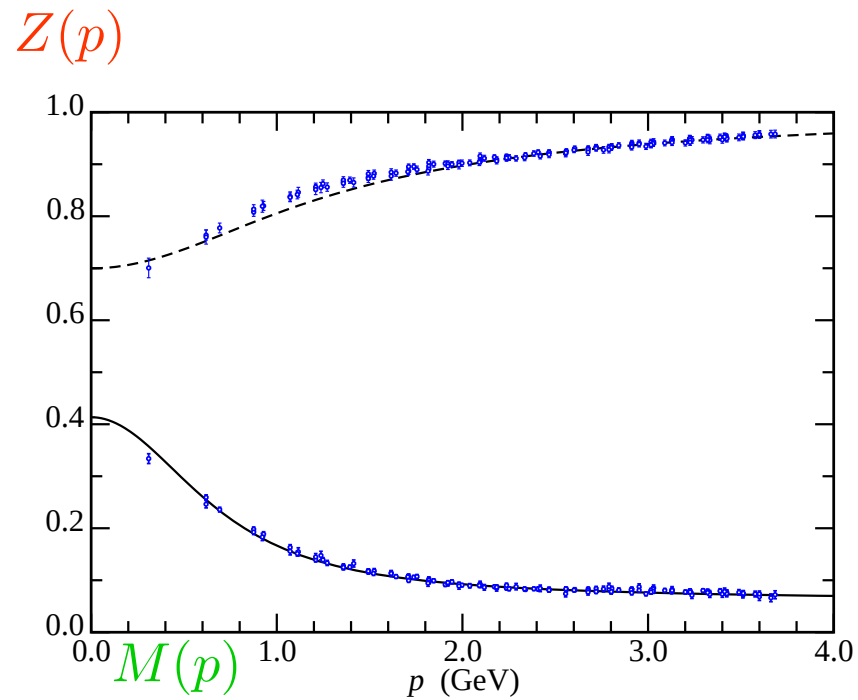
Quenched-QCD Dressed-Quark Propagator





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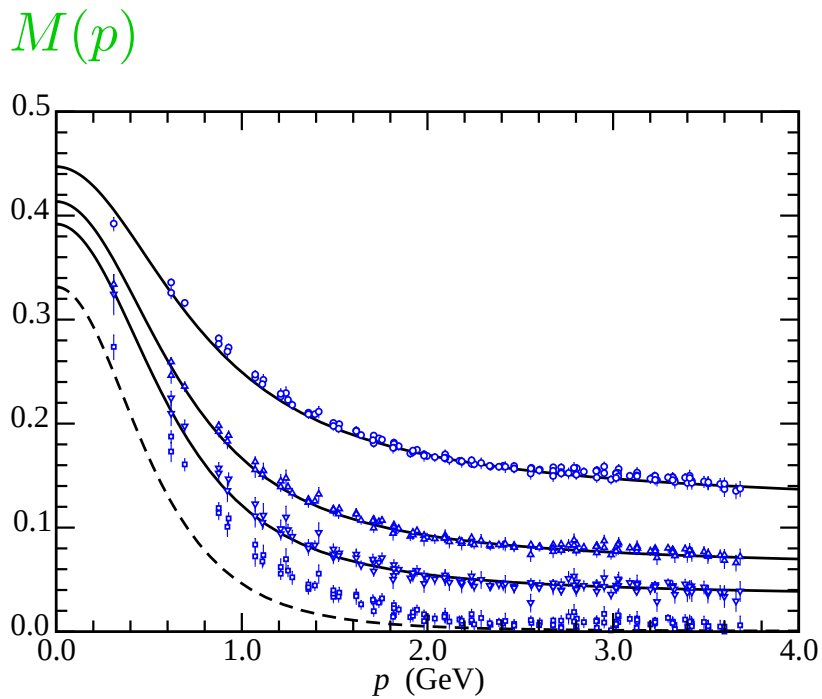
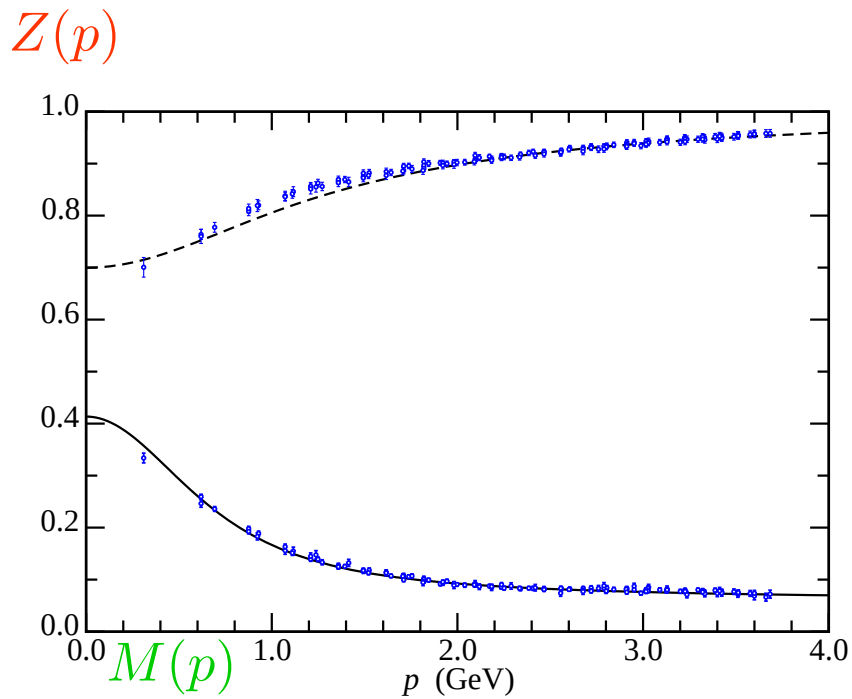
2002





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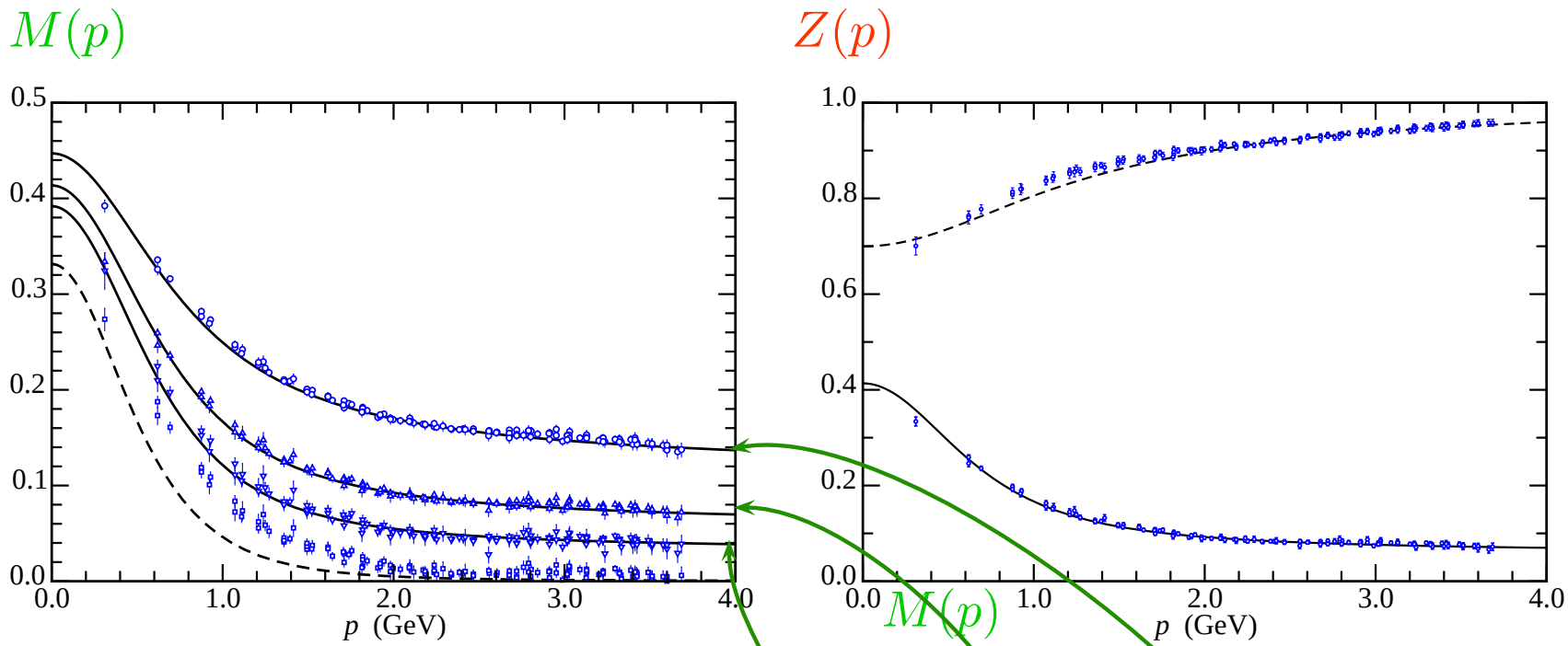

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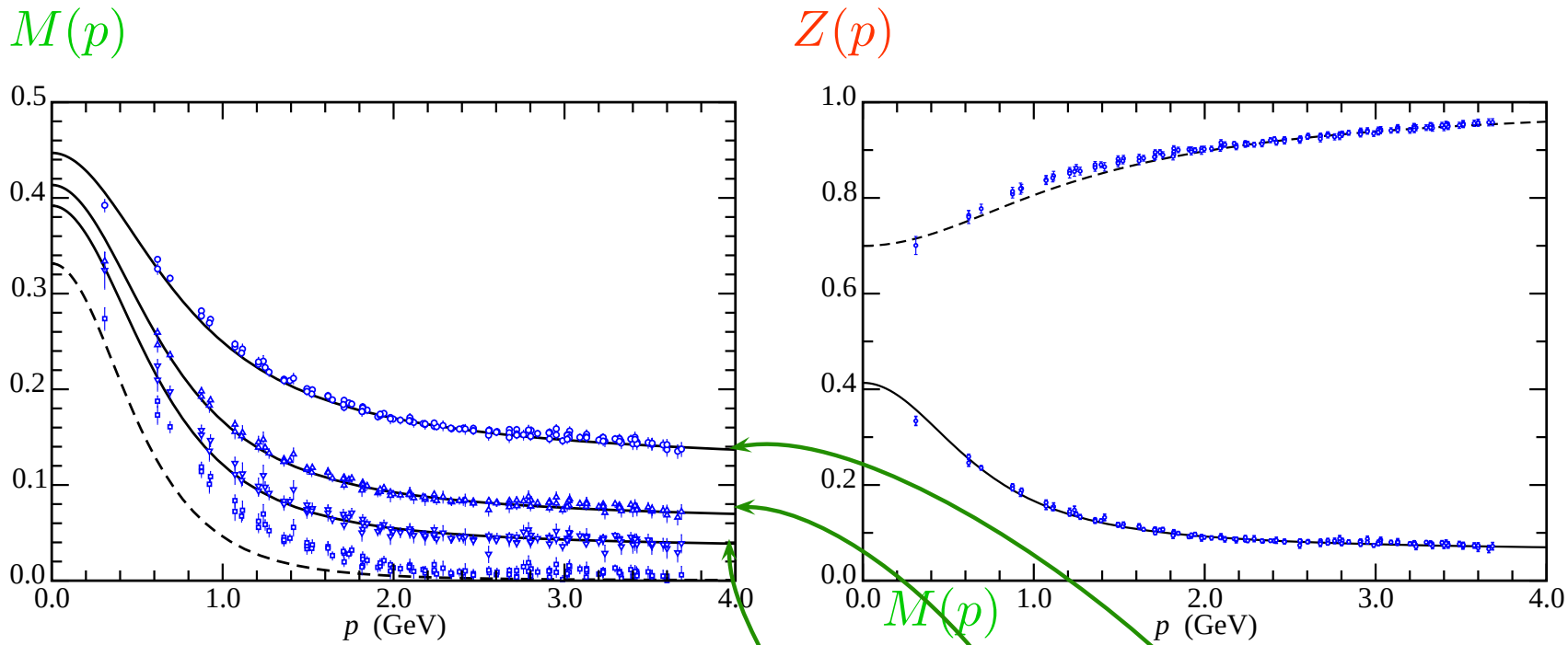

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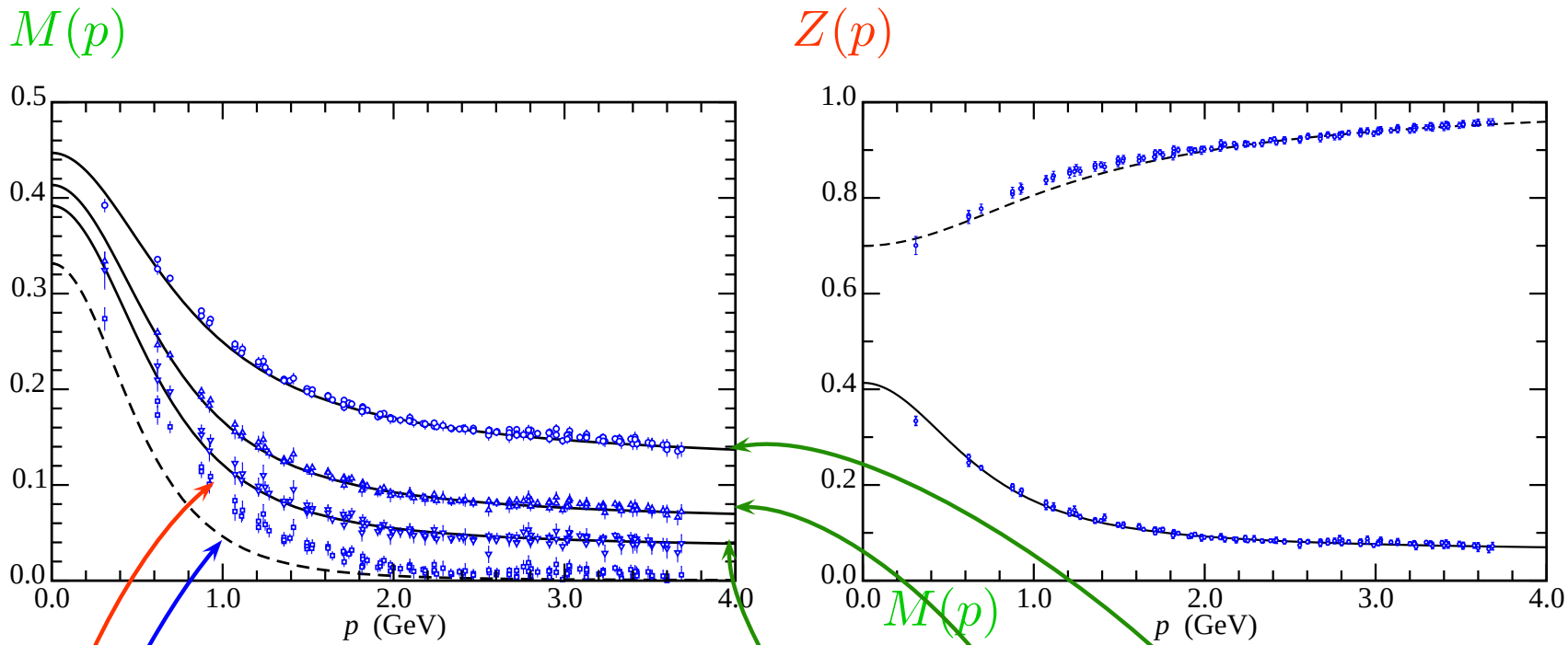

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- Linear extrapolation of lattice data to chiral limit is inaccurate

QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
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- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
  - D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD Collaboration], *Asymptotic scaling and infrared behavior of the gluon propagator*, Phys. Rev. D **60**, 094507 (1999) [Erratum-ibid. D **61**, 079901 (2000)].
- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

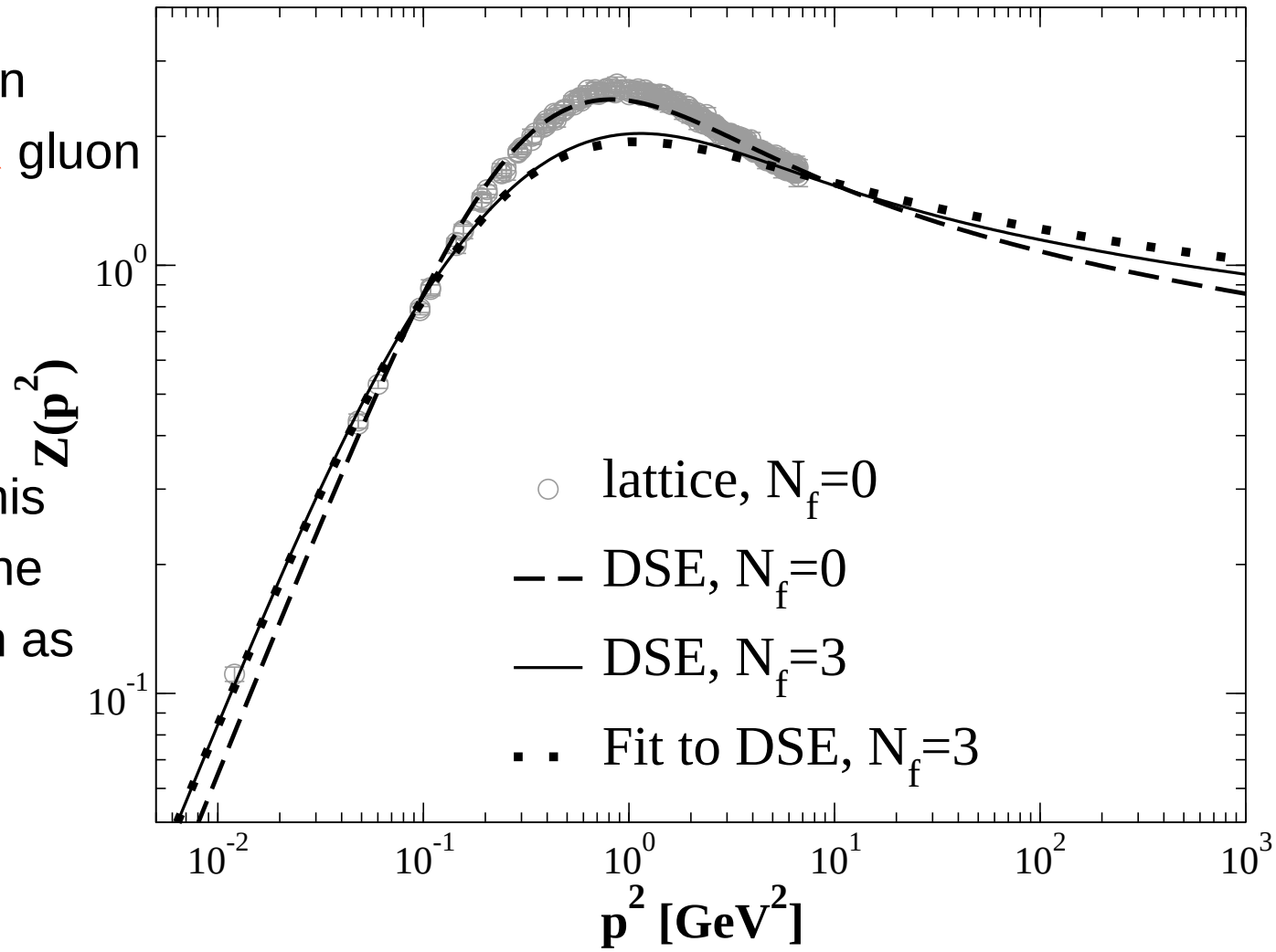
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



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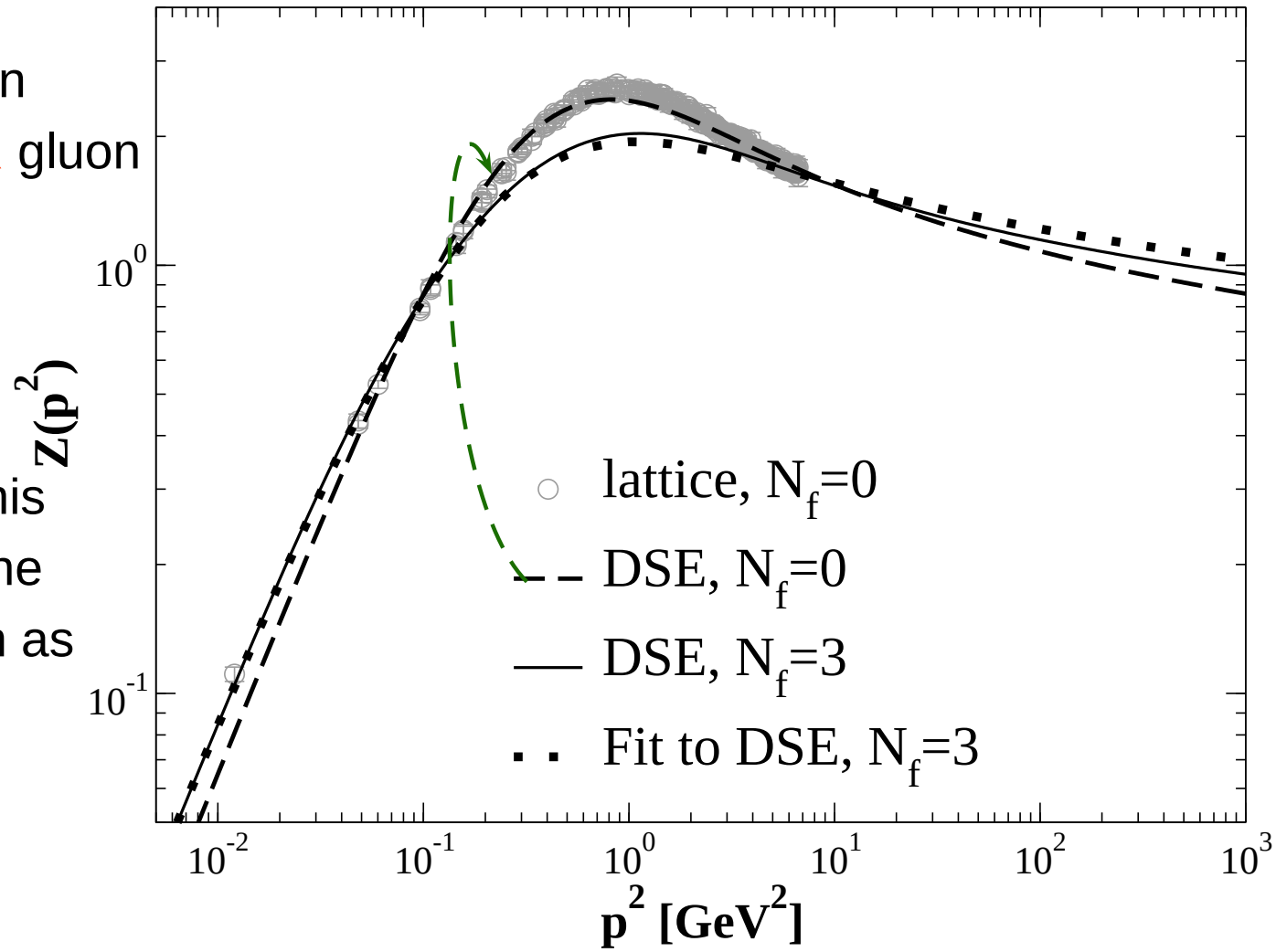

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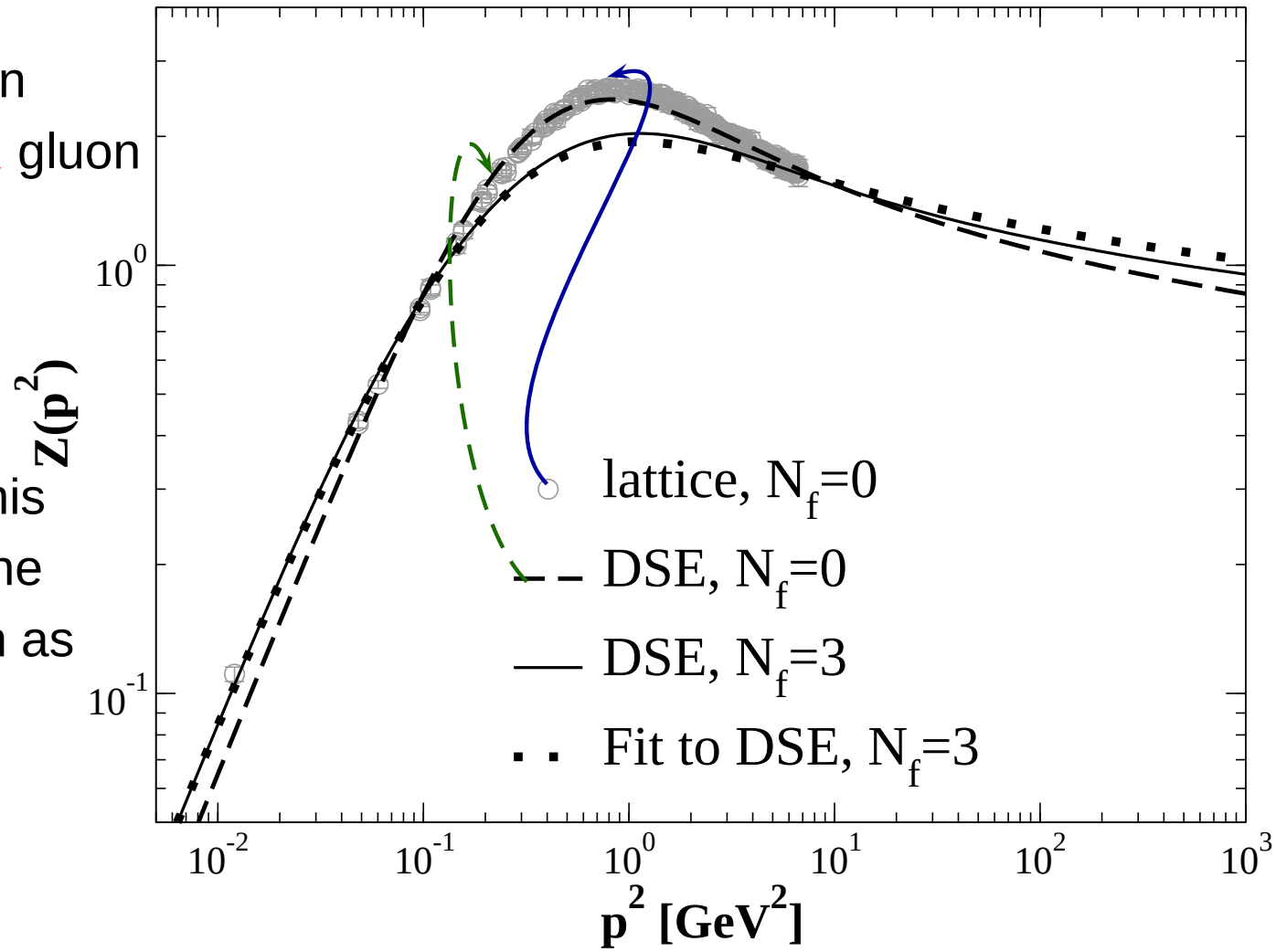

Critical Mass for Chiral Expansion

Dynamical chiral symmetry breaking and a critical mass

Lei Chang, Yu-Xin Liu, Mandar S. Bhagwat, Craig D. Roberts and Stewart V. Wright . . . nucl-th/0605058

Phys. Rev. C 75 (2007) 015201 (8 pages)

Critical Mass for Chiral Expansion

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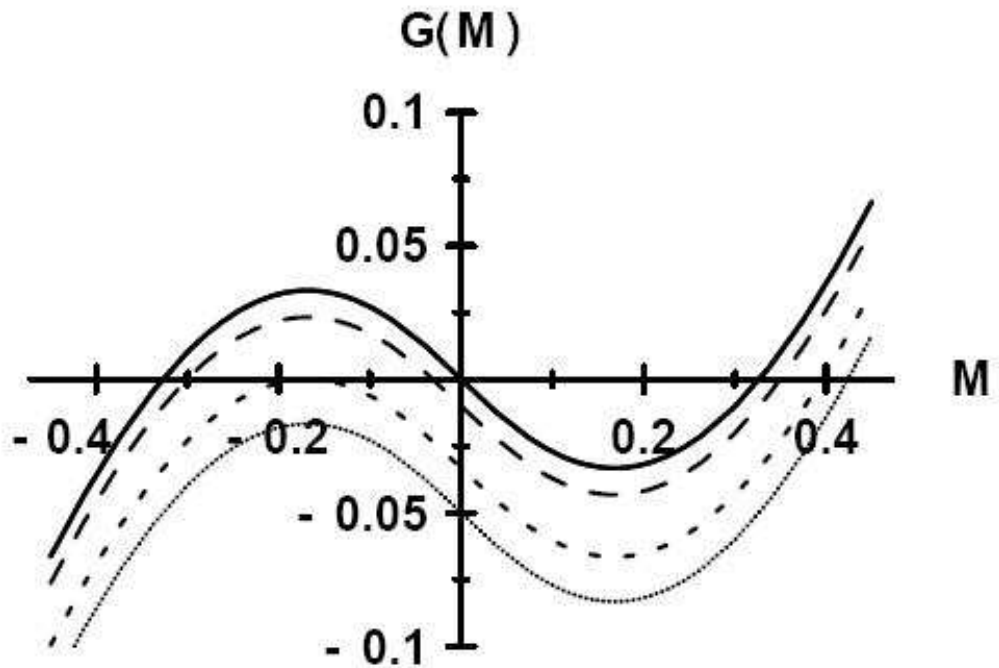
Critical Mass for Chiral Expansion

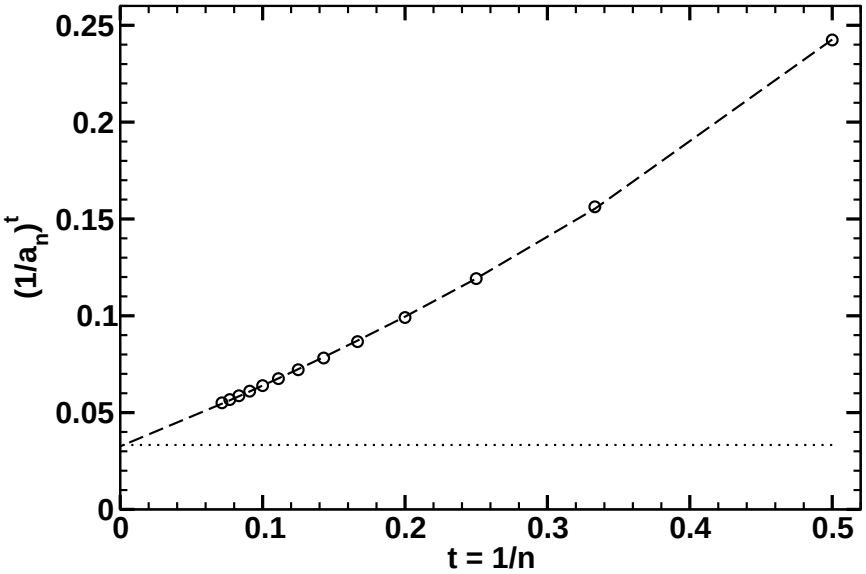
- Realistic models of QCD's gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.
- Wigner solution and one DCSB solution are destabilised by current-quark mass & both disappear when that mass exceeds a critical value, $m_{\rm cr}$



The zeros of $G(M)$ characterise the different solutions of the gap equation. *Solid curve*: obtained with $m^{\rm bm} = 0$, in which case $G(M)$ is odd under $M \to -M$; *long-dashed curve*: $m^{\rm bm} = 0.01$; *short-dashed curve*: $m^{\rm bm} = m^{\rm bm}_{\rm cr} = 0.033$; *dotted curve*: $m^{\rm bm} = 0.05$.

Critical Mass for Chiral Expansion

- Realistic models of QCD's gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.
- $m_{\rm cr}$ also bounds domain on which surviving DCSB solution possesses a chiral expansion: $m_{\rm cr} = \lim_{n\to\infty} \left(\frac{1}{|a_n|}\right)^{1/n}$



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- For a pseudoscalar meson constituted of equal mass current-quarks, it corresponds to a mass $m_{0^-}^{\rm cr} \sim 0.45$ GeV.
- Entails lattice-QCD simulations *must* have many results at $m_\pi < m_{0^-}^{\rm cr} \sim 0.45$ GeV for reliable extrapolation via EFT.

Critical Mass for Chiral Expansion

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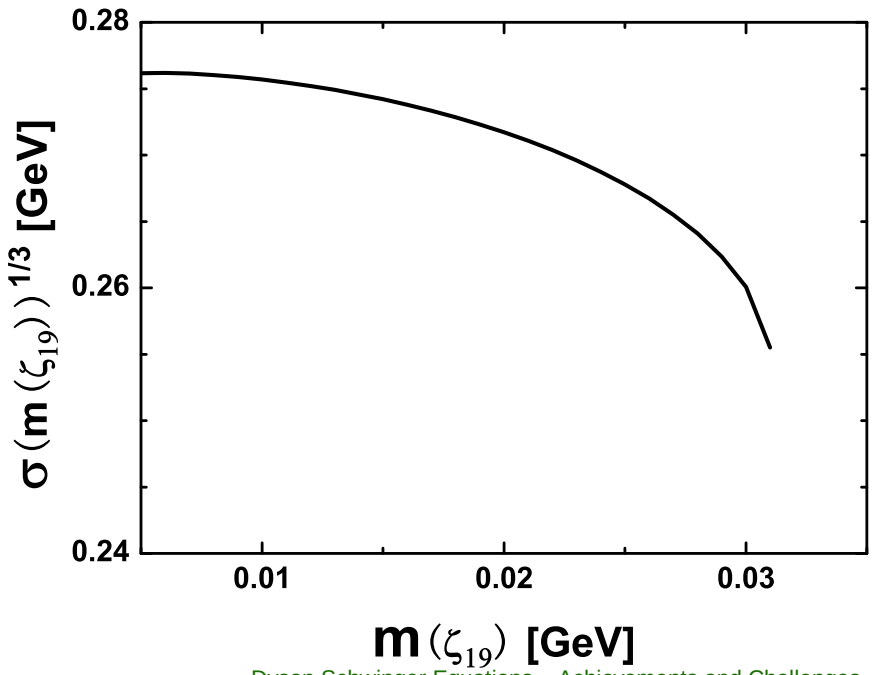
- The two DCSB solutions of the gap equation enable a valid definition of $\langle \bar{q}q \rangle$ in the presence of a nonzero current-mass.

Critical Mass for Chiral Expansion

- Realistic models of QCD's gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.
- The two DCSB solutions of the gap equation enable a valid definition of $\langle \bar{q}q \rangle$ in the presence of a nonzero current-mass.
- The behaviour of this condensate indicates that the essentially dynamical component of chiral symmetry breaking decreases with increasing current-quark mass.



Consituent-quark σ-term

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark σ-term

$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\; (M^E)^2 := s\,|\,s = M(s)^2\,.$$

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- Renormalisation-group-invariant and determined from solutions of the gap equation

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- Unambiguous probe of impact of explicit chiral symmetry breaking on the mass function

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- Ratio $\boxed{\dfrac{\sigma_f}{M_f^E} = \dfrac{\text{EXPLICIT}}{\text{EXPLICIT} + \text{DYNAMICAL}}}$

measures effect of *EXPLICIT* chiral symmetry breaking on dressed-quark mass-function

cf. *SUM* of effects of *EXPLICIT* AND *DYNAMICAL CHIRAL SYMMETRY BREAKING*

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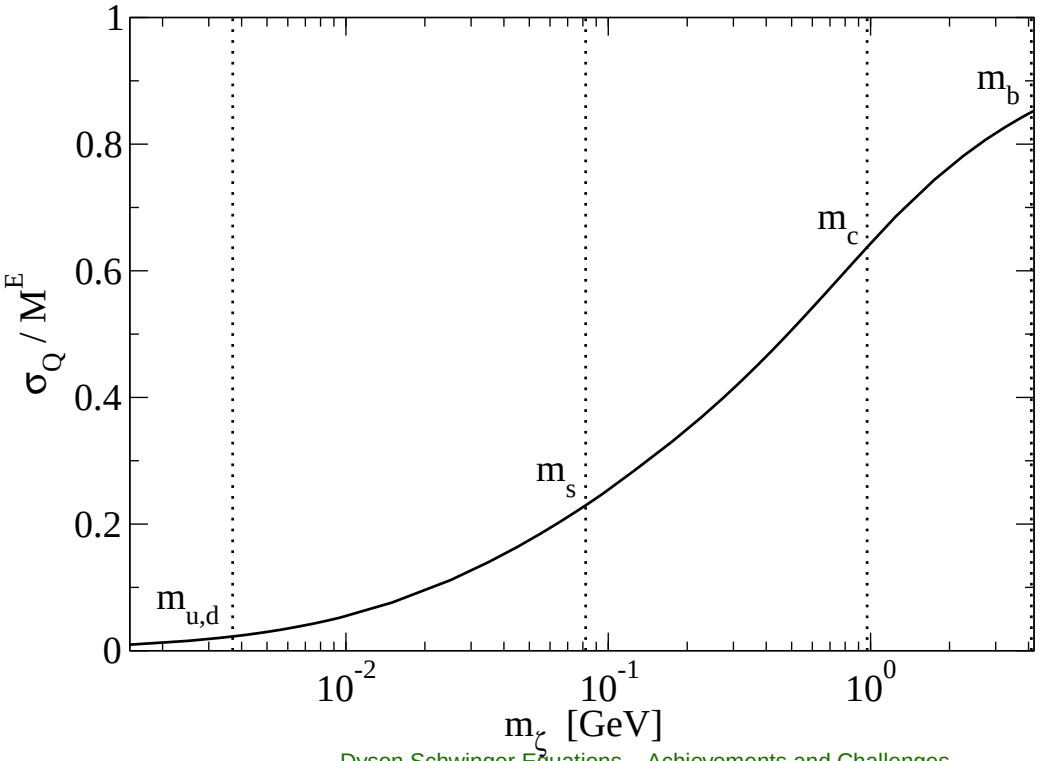

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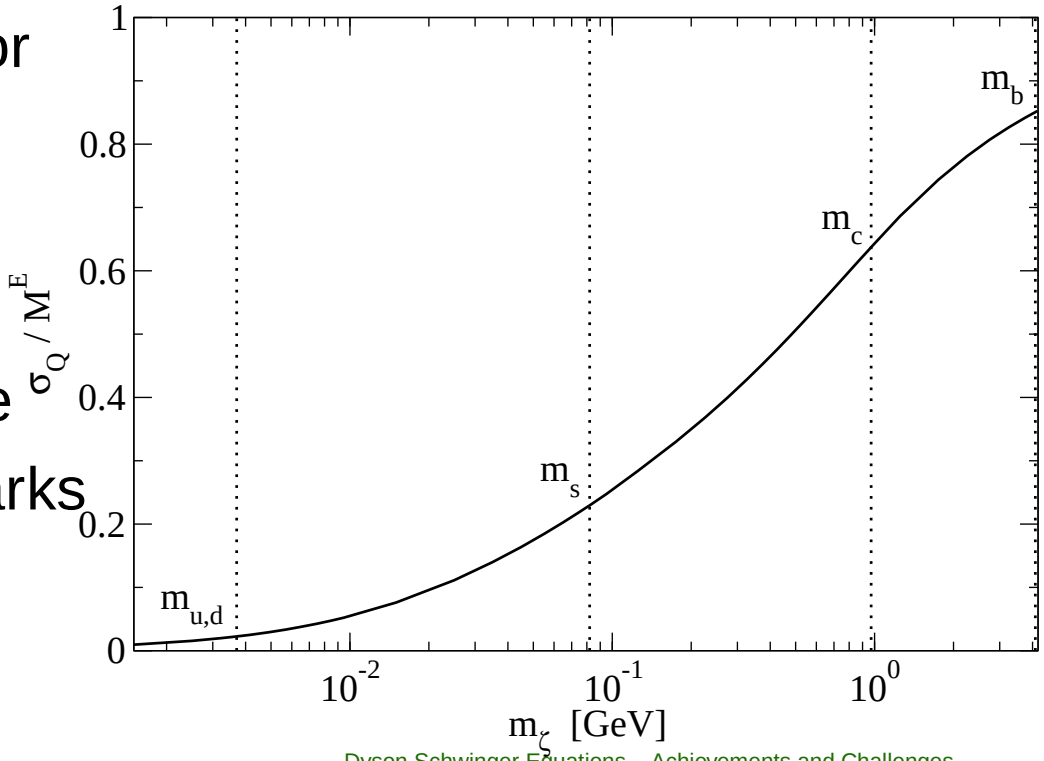
Obvious: ratio vanishes for light-quarks because magnitude of their constituent-mass owes primarily to DCSB. On the other hand, for heavy-quarks it approaches one.



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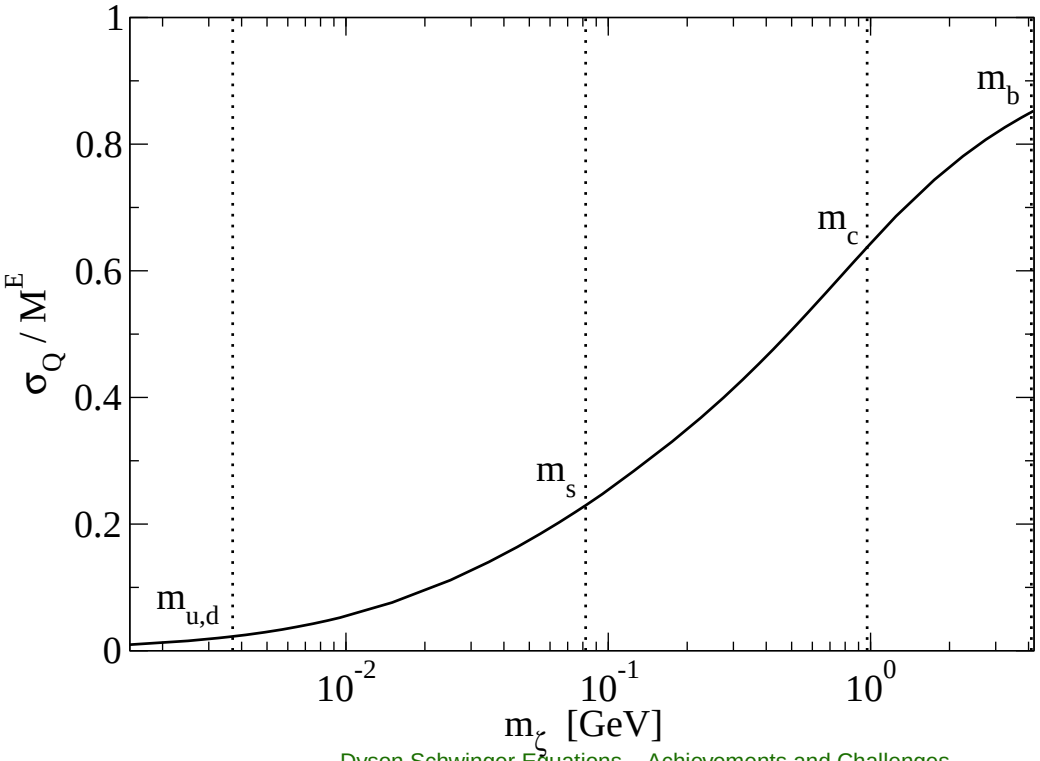
Essentially dynamical component of chiral symmetry breaking, and manifestation in all its order parameters, vanishes with increasing current-quark mass



Hadrons

- Established understanding of two- and three-point functions

Hadrons

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- What about bound states?

Hadrons

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- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

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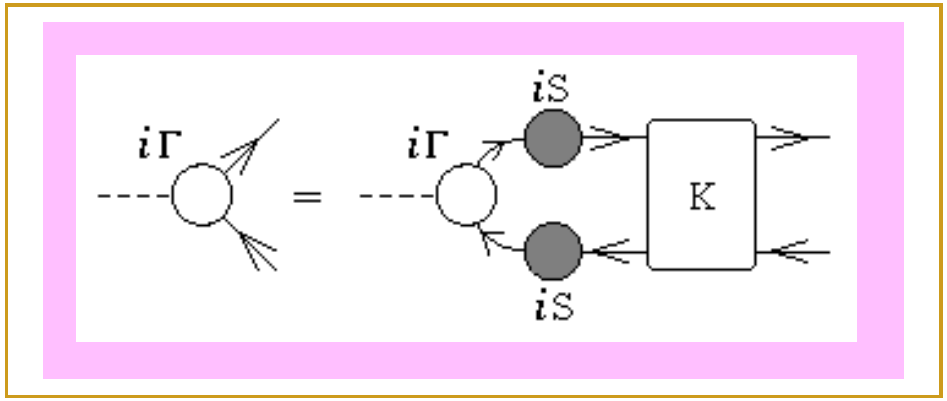
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

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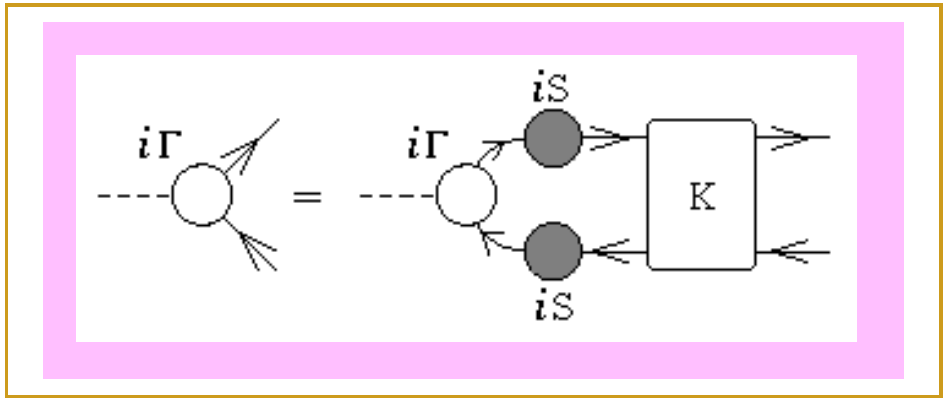

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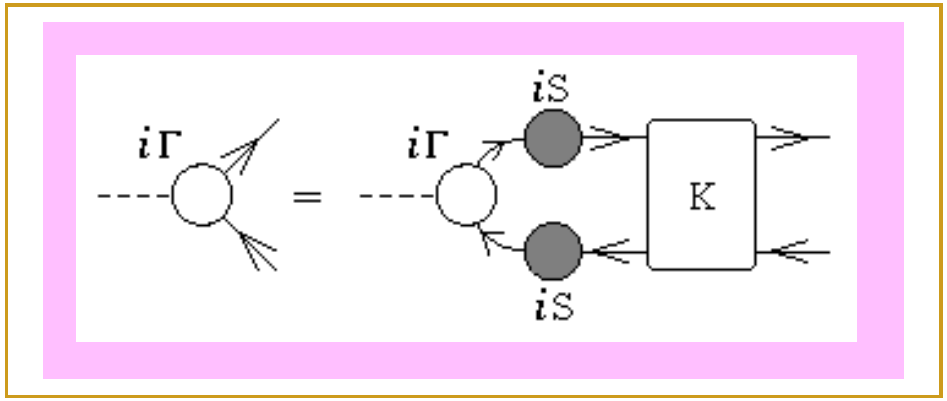

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or

What is the Long-Range Potential?

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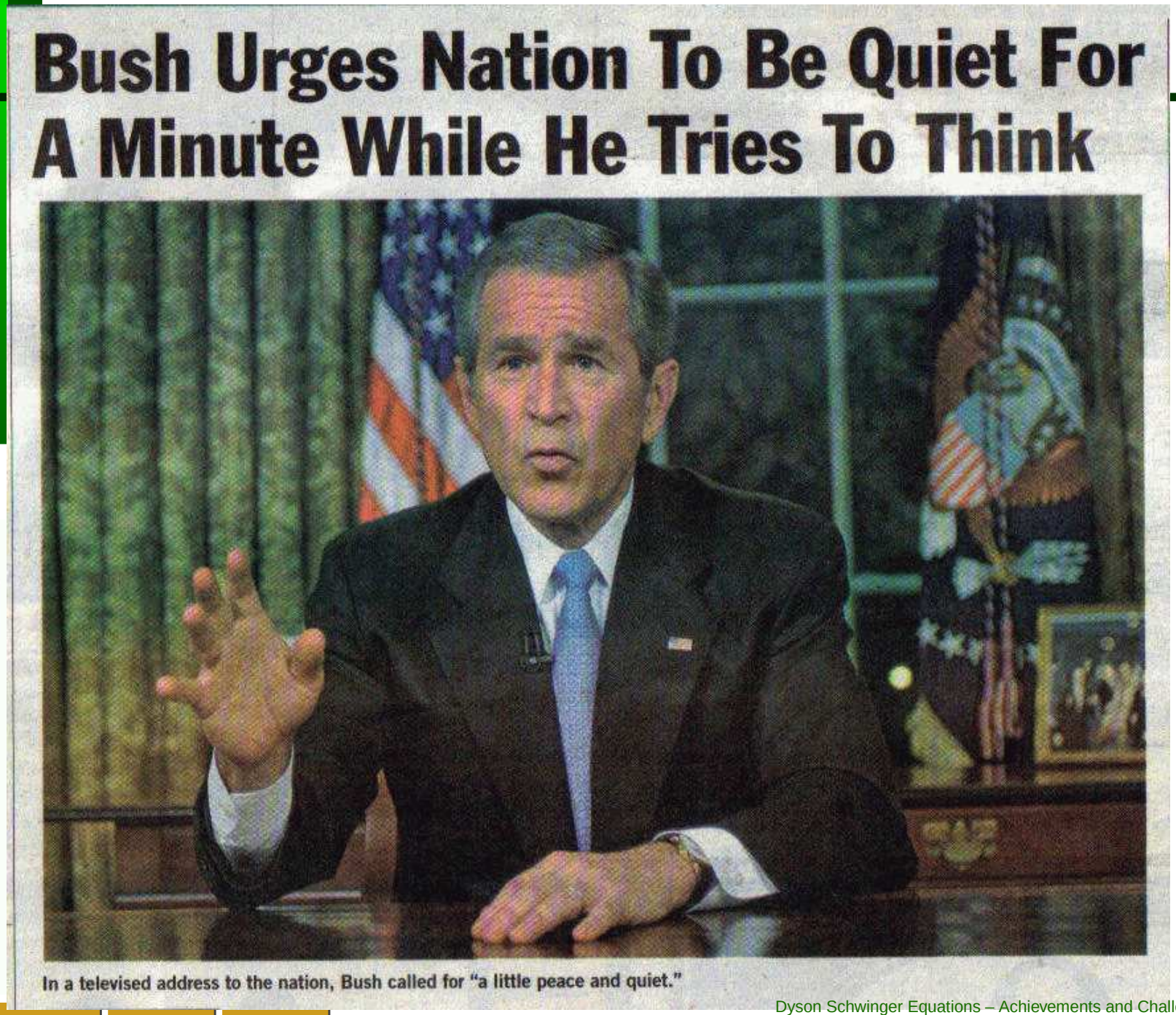

Bush Urges Nation To Be Quiet For A Minute While He Tries To Think

In a televised address to the nation, Bush called for "a little peace and quiet."

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

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- Nontrivial constraint

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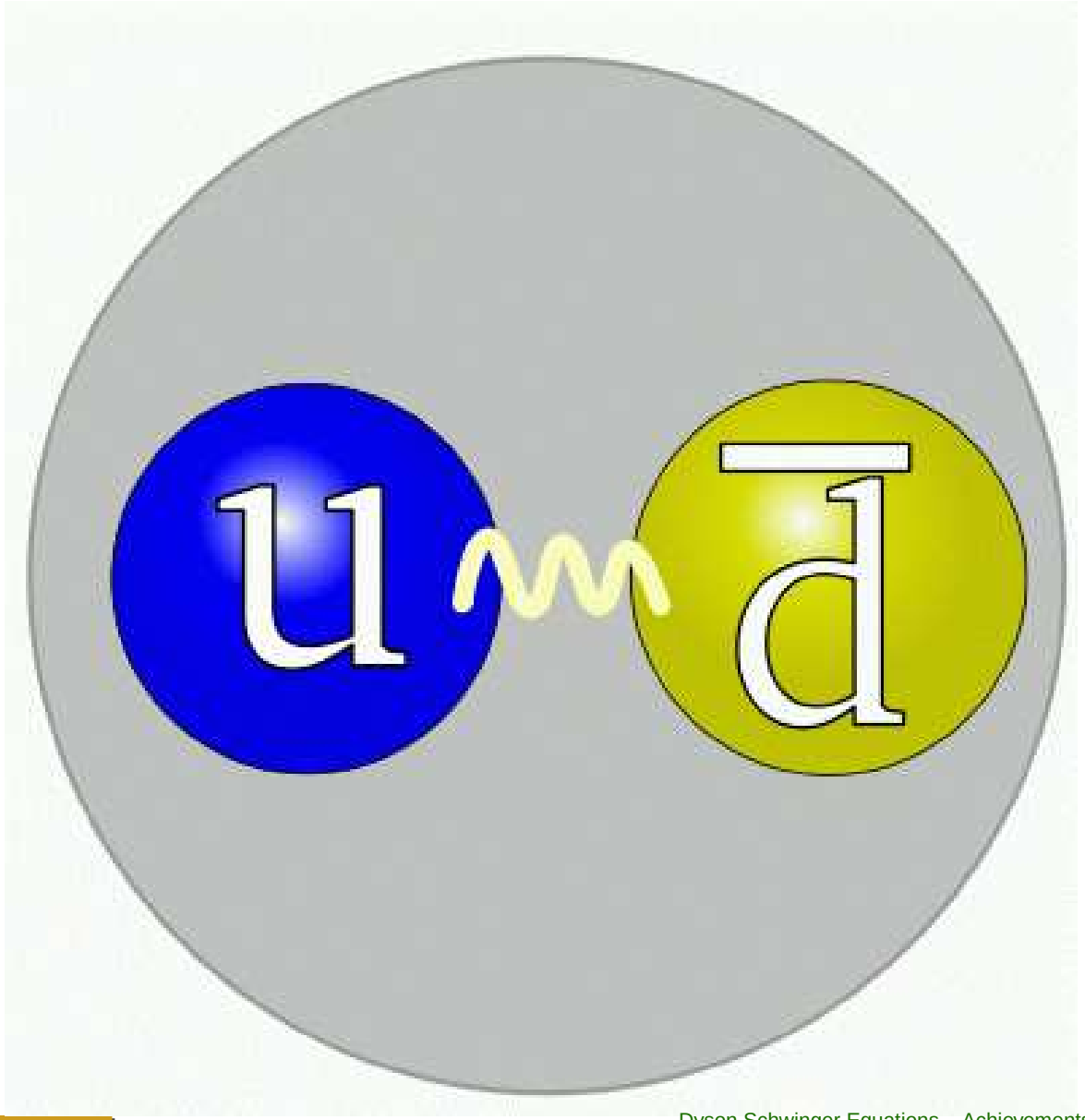
Satisfies BSE Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

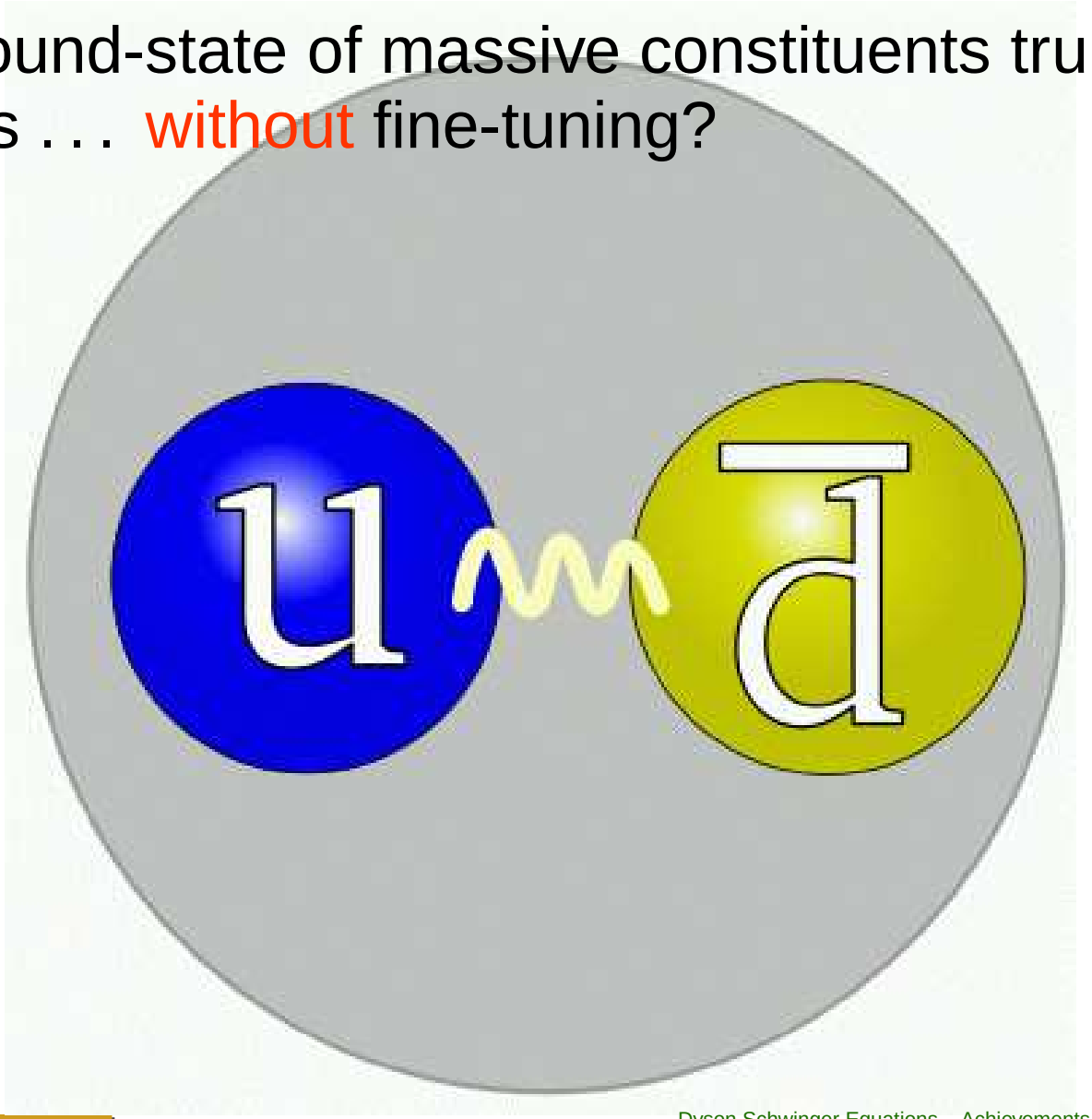
Pion and . . .

Pseudoscalar Mesons?



Pion and . . .
Pseudoscalar Mesons?

Can a bound-state of massive constituents truly be massless . . . without fine-tuning?



Radial Excitations & Chiral Symmetry

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(Maris, Roberts, Tandy nu-th/9707003)

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- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

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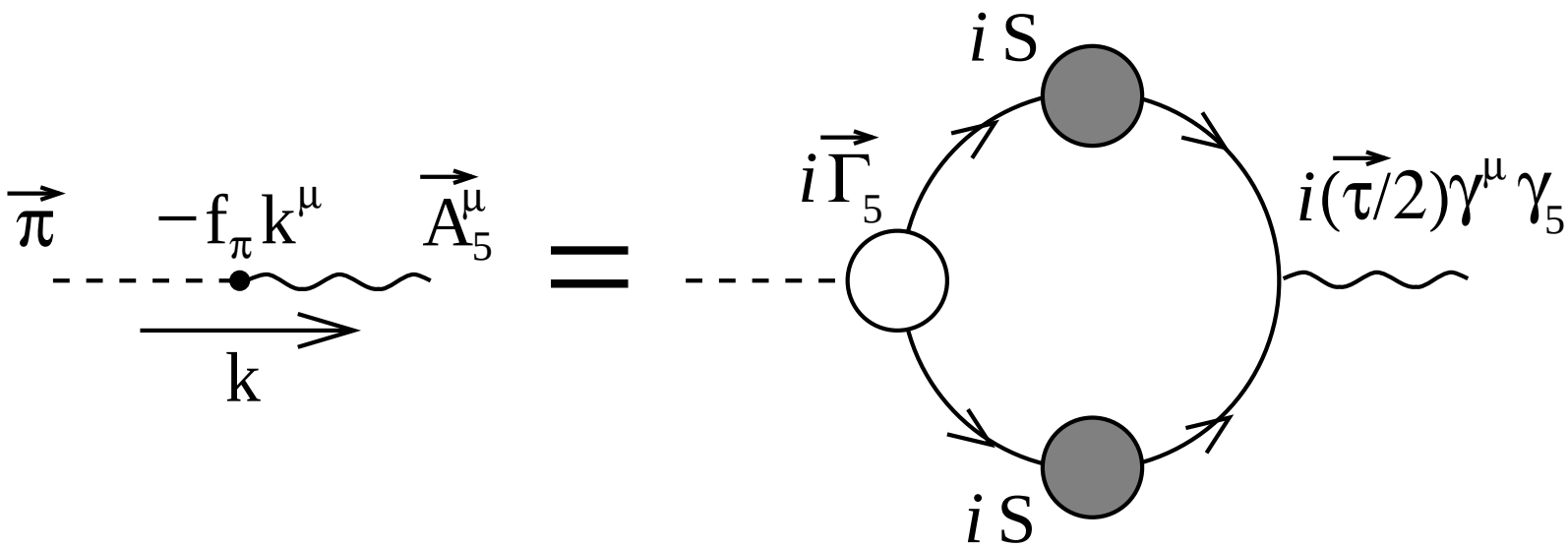
$$f_H\, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5\gamma_\mu\, \mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-) \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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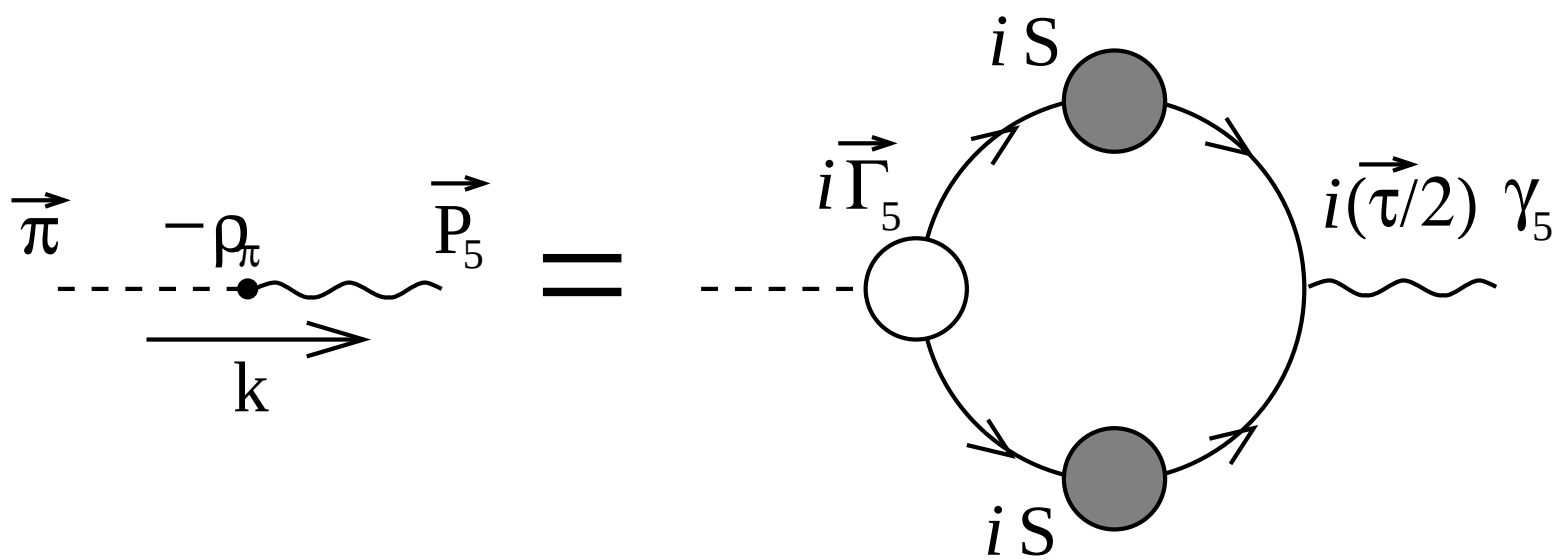

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$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ ...GMOR relation, a corollary

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- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $m_H \propto m_q$

... QCD Proof of Potential Model result

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Höll, Krassnigg, Roberts
nu-th/0406030

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Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

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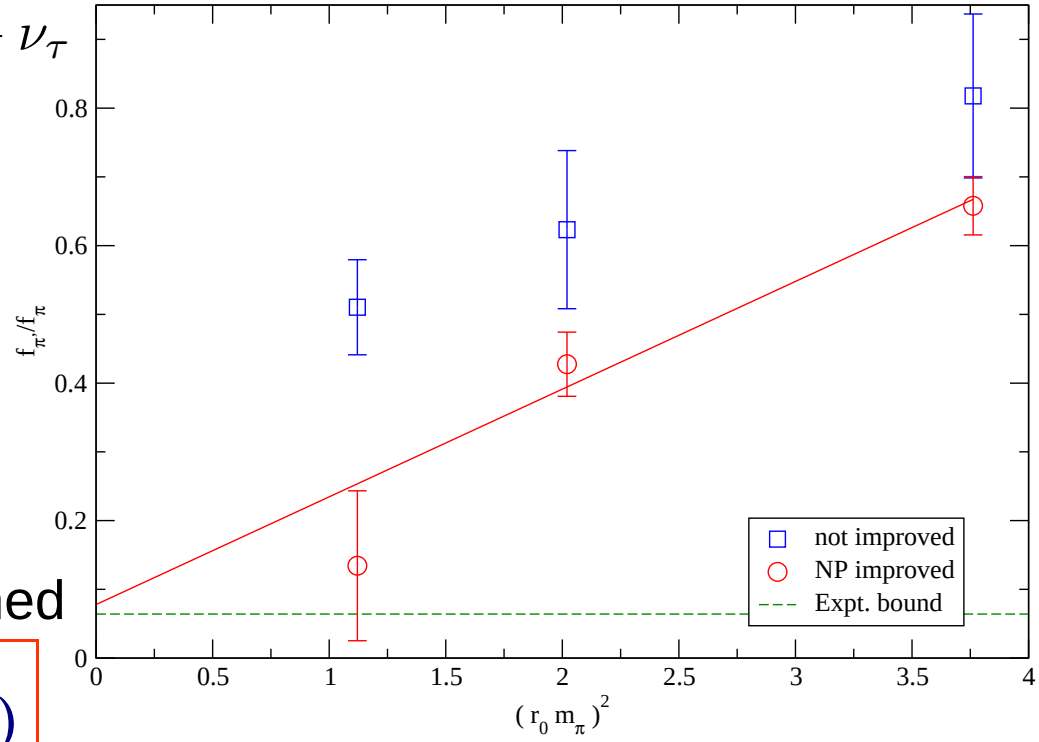
$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$



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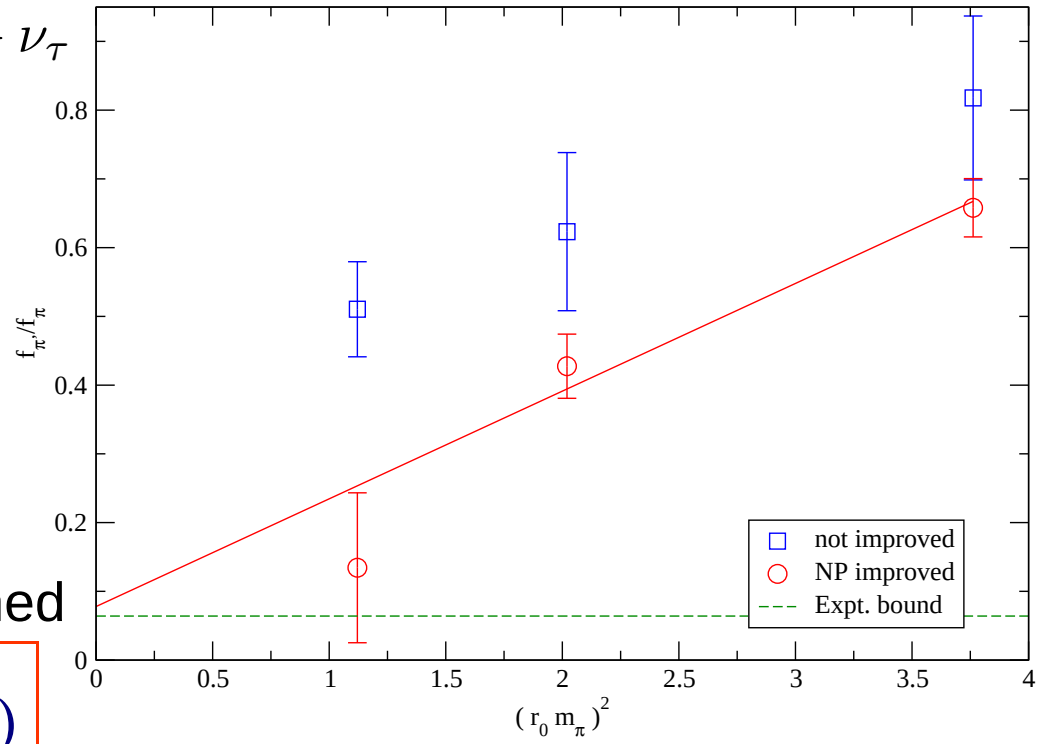

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- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

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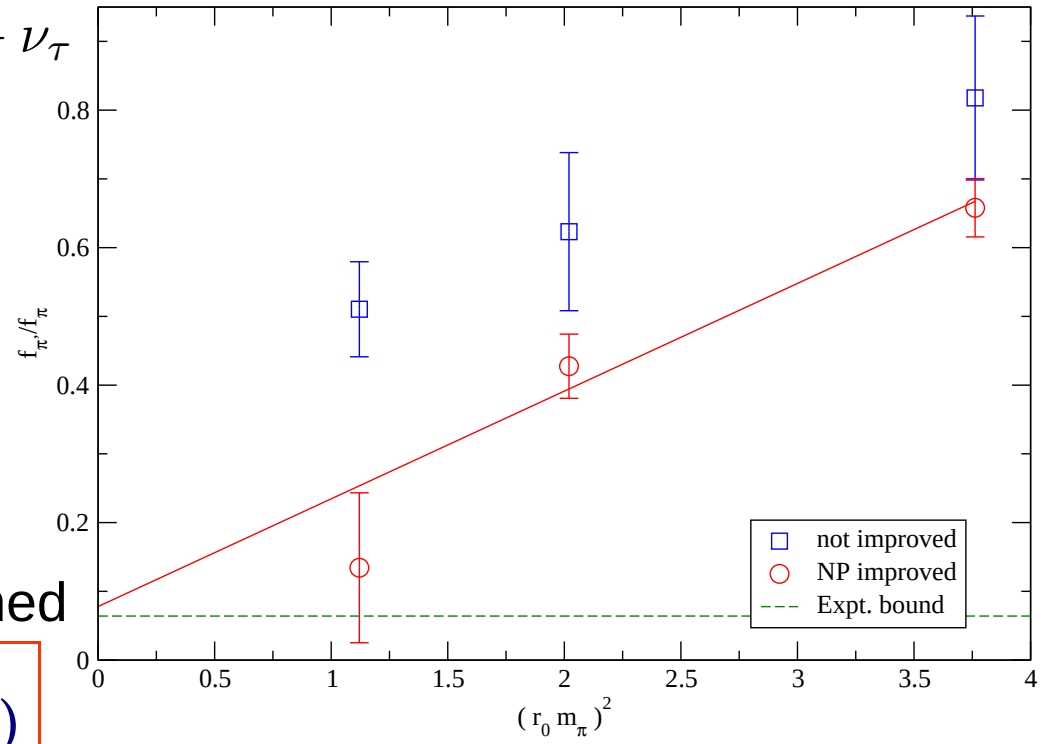

- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Radial Excitations

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- Spectrum contains 3 pseudoscalars [$I^G(J^P)L = 1^-(0^-)S$]

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

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- But $\pi(1800)$ is narrow ($\Gamma = 207 \pm 13$) & decay pattern might indicate some "flux tube angular momentum" content:
  $S_{\bar{Q}Q} = 1 \oplus L_F = 1 \Rightarrow J = 0$
  & $L_F = 1 \Rightarrow\ ^3S_1 \oplus\ ^3S_1$ ($\bar{Q}Q$) decays suppressed?

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- Radial excitations & Hybrids & Exotics $\Rightarrow$ Long-range radial wave functions $\Rightarrow$ sensitive to confinement
- NSAC Long-Range Plan, 2002: . . . an understanding of confinement "remains one of the greatest intellectual challenges in physics"

Pion ... $J = 0$ but ...

- Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.

Pion ... $J = 0$ but ...

- Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.

Pion ... $J = 0$ but ...

- Pseudoscalar meson Bethe-Salpeter amplitude

$$\chi_\pi(k;P) = \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P\mathcal{F}_{\pi_n}(k;P) \right.$$
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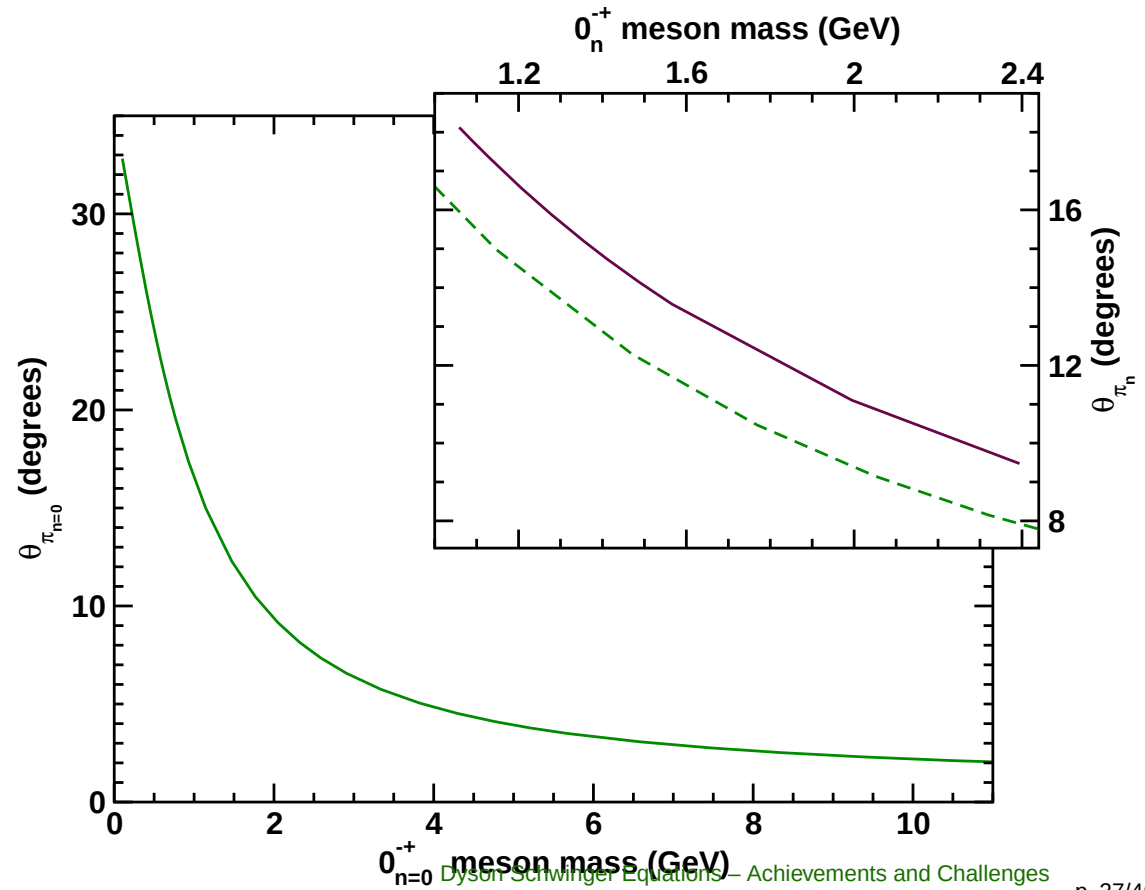

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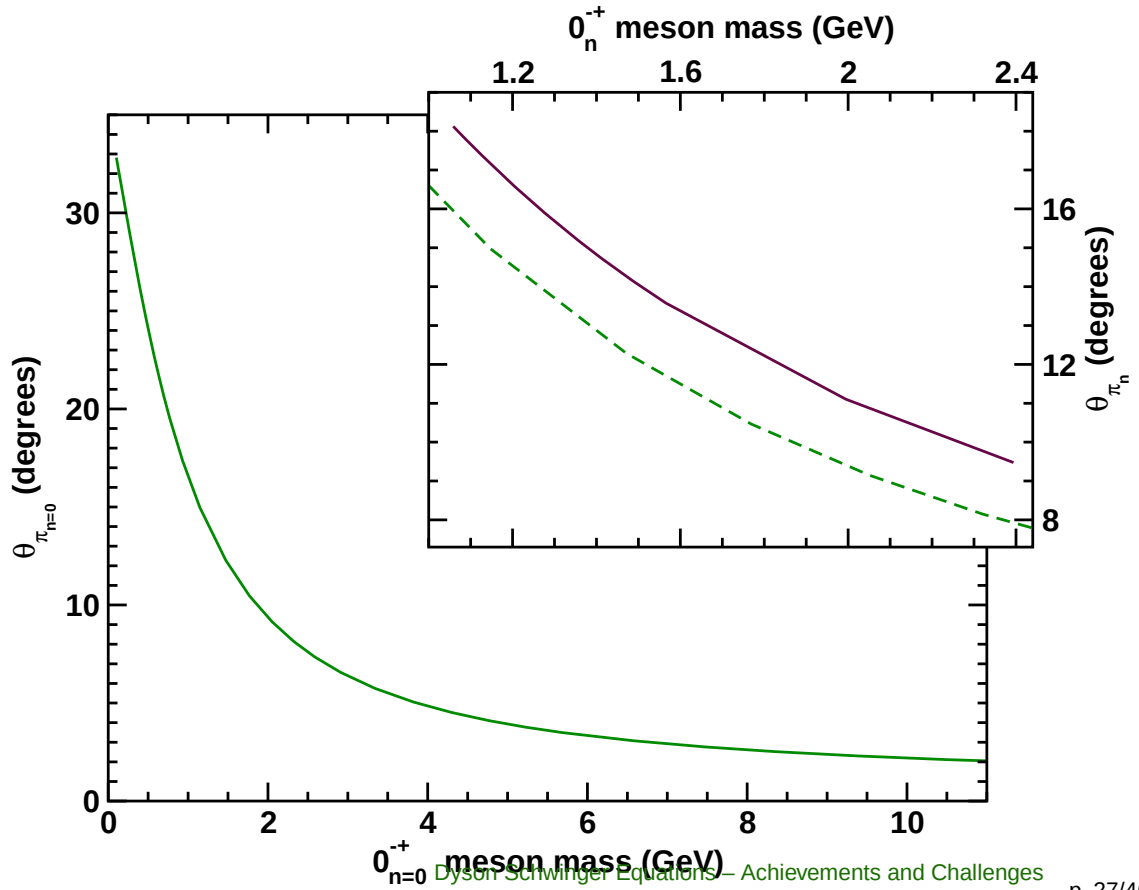
L is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.



Charge Neutral Pseudoscalar Mesons

Flavour symmetry breaking and meson masses

Mandar S. Bhagwat, Lei Chang, Yu-Xin Liu, Craig D. Roberts and Peter C. Tandy . . . nucl-th/arXiv:0708.1118

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- Determine effect of these differences in current-quark mass throughout the hadron spectrum, which leads one to consider the difference in mass between charged and neutral hadrons.
- Part of that splitting is electromagnetic, but constraining the strong component is necessary before one can know just how large that electromagnetic contribution might be.

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- Determine effect of these differences in current-quark mass throughout the hadron spectrum, which leads one to consider the difference in mass between charged and neutral hadrons.
- In considering neutral pseudoscalars, the $U(N_f)$ axial-vector Ward-Takahashi identity becomes relevant.
  - It necessarily includes a contribution from the non-Abelian anomaly.
  - A discussion of η–η' splitting is therefore unavoidable.

Charge Neutral Pseudoscalar Mesons

$$
\begin{aligned}
P_\mu \Gamma^a_{5\mu}(k;P) &= \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\
&\quad -2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)
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- The final term in the last line expresses the non-Abelian axial anomaly.

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- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$

  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \left\langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y) \right\rangle$

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  … The topological charge density operator.

  (Trace is over colour indices & $F_{\mu\nu} = \frac{1}{2}\lambda^a F^a_{\mu\nu}$.)

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  … The topological charge density operator.
- Important that only $\mathcal{A}^{a=0}$ is nonzero.
- NB. While $\mathcal{Q}(x)$ is gauge invariant, the associated Chern-Simons current, K_μ, is not.
  Means that *no physical* boson can couple to K_μ.

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- Many new results
  - Here, focus on $N_f = 3$
  - Will only describe those results which are exact in QCD
  - Omit numerous illustrations and calculations

Charge Neutral Pseudoscalar Mesons

- Consider $\mathcal{A}^0 \not\equiv 0$

Charge Neutral Pseudoscalar Mesons

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  … otherwise all pseudoscalar mesons are Goldstone Modes!

Charge Neutral Pseudoscalar Mesons

- Consider $\mathcal{A}^0 \not\equiv 0$
- Allow that $\Gamma^0_{5\mu}$ might possess a longitudinal massless bound-state pole:

$$\Gamma^0_{5\mu}(k;P)\big|_{P^2\approx 0} = r^0_A \frac{P_\mu}{P^2}\Gamma_{BS}(k;P)$$

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- F^0_R, G^0_R, H^0_R, $\Gamma^0_{5\mu}(k;P)$ are regular as $P^2 \to 0$
- $P_\mu\tilde{\Gamma}^0_{5\mu} \sim$O$(P^2)$
- $\Gamma_{BS}(k;P)$ is the possible bound-state's Bethe-Salpeter amplitude & r^0_A is its residue

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- Structure of anomaly contribution is similar

$$\mathcal{A}^0(k;P) = \mathcal{F}^0\gamma_5\left[i\mathcal{E}_{\mathcal{A}}(k;P) + \gamma\cdot P\mathcal{F}_{\mathcal{A}}(k;P)\right.$$
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- Consider $\mathcal{A}^0 \not\equiv 0$
- Leads to generalised Goldberger-Treiman relations

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\begin{aligned}
2r_A^0 E_{BS}(k;0) &= 2B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0), \\
F_R^0(k;0) + 2r_A^0 F_{BS}(k;0) &= A_0(k^2) - \mathcal{F}_{\mathcal{A}}(k;0), \\
G_R^0(k;0) + 2r_A^0 G_{BS}(k;0) &= 2A_0'(k^2) - \mathcal{G}_{\mathcal{A}}(k;0), \\
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A_0, B_0 characterise gap equation's chiral limit solution.

- Follows that $\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)$ is necessary and sufficient condition for absence of massless bound-state pole in $\Gamma_{5\mu}^0$.

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  Discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
  - Hence, absence of massless η' bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.

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- Further highlighted . . . we also proved

$$\begin{aligned}\langle \bar{q}q\rangle^0_\zeta &= -\lim_{\Lambda\to\infty} Z_4(\zeta^2,\Lambda^2)\,\mathrm{tr}_{\mathrm{CD}}\int_q^\Lambda S^0(q,\zeta)\\ &= N_f\int d^4x\,\langle \bar{q}(x)i\gamma_5 q(x)\mathcal{Q}(0)\rangle^0,\end{aligned}$$

Charge Neutral Pseudoscalar Mesons

- Mass Formulae for neutral pseudoscalar mesons

$$m_{\pi_i}^2 f_{\pi_i}^a = 2\,\mathcal{M}^{ab}\rho_{\pi_i}^b + \delta^{a0} n_{\pi_i}$$

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$$\boxed{m_{\pi_i}^2 f_{\pi_i}^a = 2\,\mathcal{M}^{ab}\rho_{\pi_i}^b + \delta^{a0} n_{\pi_i}}$$

- Contribution from the meson's anomaly residue:

$$n_{\pi_i} = \sqrt{\frac{N_f}{2}}\,\nu_{\pi_i}\,,\ \nu_{\pi_i} = \langle 0|\mathcal{Q}|\pi_i\rangle$$

Connects bound-state to vacuum via topological charge operator

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- Mass matrix

$$\Big[M_{3\times 3}\Big] = \begin{bmatrix} m_{110} & \sqrt{\frac{1}{3}}\, m_{1-10} & \sqrt{\frac{2}{3}}\, m_{1-10} \\ \sqrt{\frac{1}{3}}\, m_{1-10} & \frac{1}{3}\, m_{114} & \sqrt{\frac{2}{9}}\, m_{11-2} \\ \sqrt{\frac{2}{3}}\, m_{1-10} & \sqrt{\frac{2}{9}}\, m_{11-2} & \frac{2}{3}\, m_{111} \end{bmatrix},$$

$m_{\alpha\beta\gamma} = \alpha\, m_u + \beta\, m_d + \gamma\, m_s$;

e.g., $m_{110} = m_u + m_d$

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- residue projections

$$f^a_{\pi_i} P_\mu = Z_2 \,\mathrm{tr} \int_q^\Lambda \mathcal{F}^a \gamma_5 \gamma_\mu \,\chi_{\pi_i}(q;P)\,,$$

$$i\rho^a_{\pi_i}(\zeta) = Z_4 \,\mathrm{tr} \int_q^\Lambda \mathcal{F}^a \gamma_5 \,\chi_{\pi_i}(q;P)\,,$$

$$\chi_{\pi_i}(k;P) = \mathcal{S}(k_+)\Gamma_{\pi_i}(k;P)\mathcal{S}(k_-),$$

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  Complete content of above equation is statement

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$$m_{\eta'}^2 f_{\eta'}^0 = n_{\eta'} + 2m(\zeta)\rho_{\eta'}^0(\zeta)$$

- It is argued that in QCD $n_{\eta'} \sim \dfrac{1}{\sqrt{N_c}}$

  And it can be shown that $f_{\eta'}^0 \sim \sqrt{N_c} \sim \rho_{\eta'}^0(\zeta)$

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- One thus obtains $$m_{\eta'}^2 = \frac{n_{\eta'}}{f_{\eta'}^0} + 2m(\zeta)\frac{\rho_{\eta'}^0(\zeta)}{f_{\eta'}^0}$$
  - First term vanishes as $N_c \to \infty$ but second remains finite
  - Subsequently taking $\hat{m} \to 0$, η' mass approaches zero in the manner characteristic of all Goldstone modes.

NB. One must take $N_c \to \infty$ before $\hat{m} \to 0$ because procedures do not commute

Charge Neutral Pseudoscalar Mesons

Flavour symmetry breaking and meson masses

Mandar S. Bhagwat, Lei Chang, Yu-Xin Liu, Craig D. Roberts and Peter C. Tandy . . . nucl-th/arXiv:0708.1118

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- Employed in an analysis of pseudoscalar- and vector-meson bound-states

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- The model involves two parameters, one of which is a mass-scale.
- Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts
  - η–η' mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)
  - π^0–η angles of $\sim 1.2^\circ$ (Expt. $p\,d \to {}^3$He π^0: $0.6^\circ \pm 0.3^\circ$)
  - Strong neutron-proton mass difference of 5.3 MeV (Lattice-QCD: 2.26 ± 0.72)

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- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

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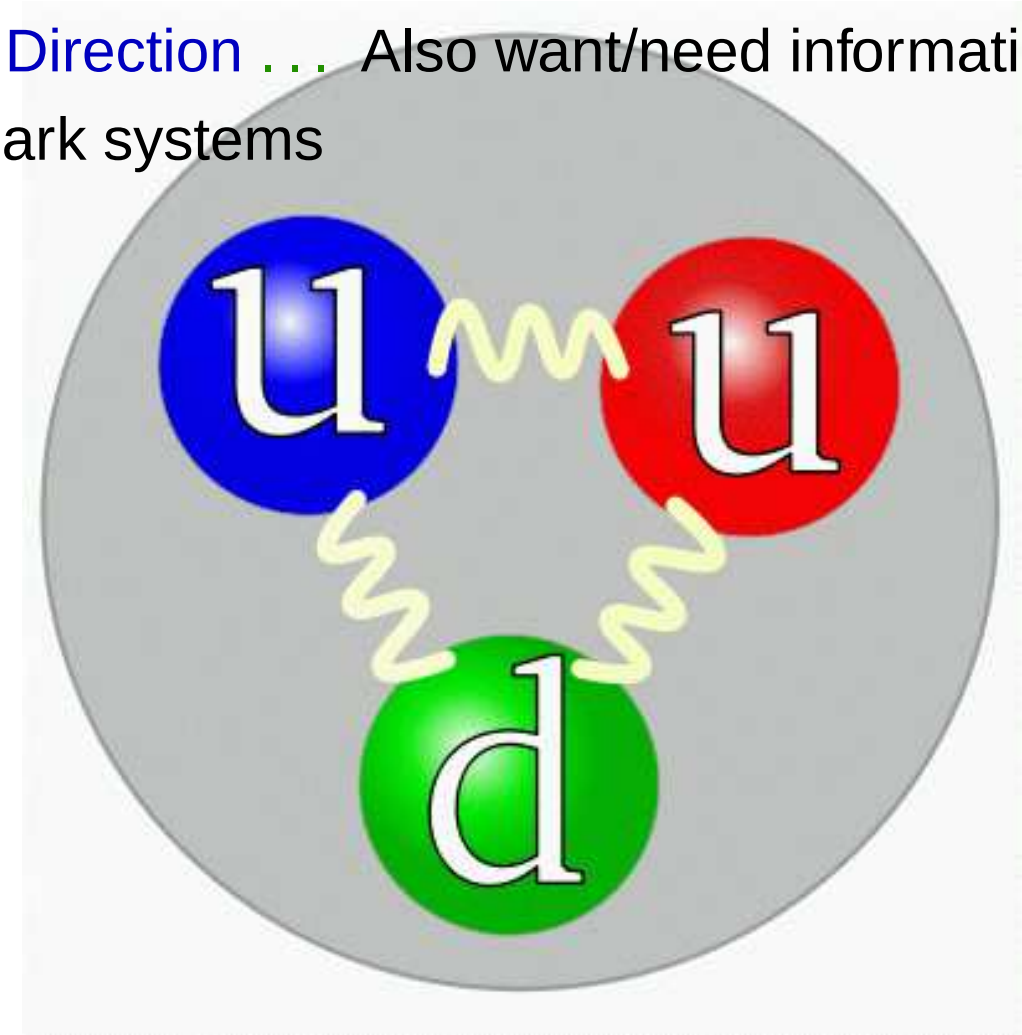
- Another Direction . . . Also want/need information about three-quark systems



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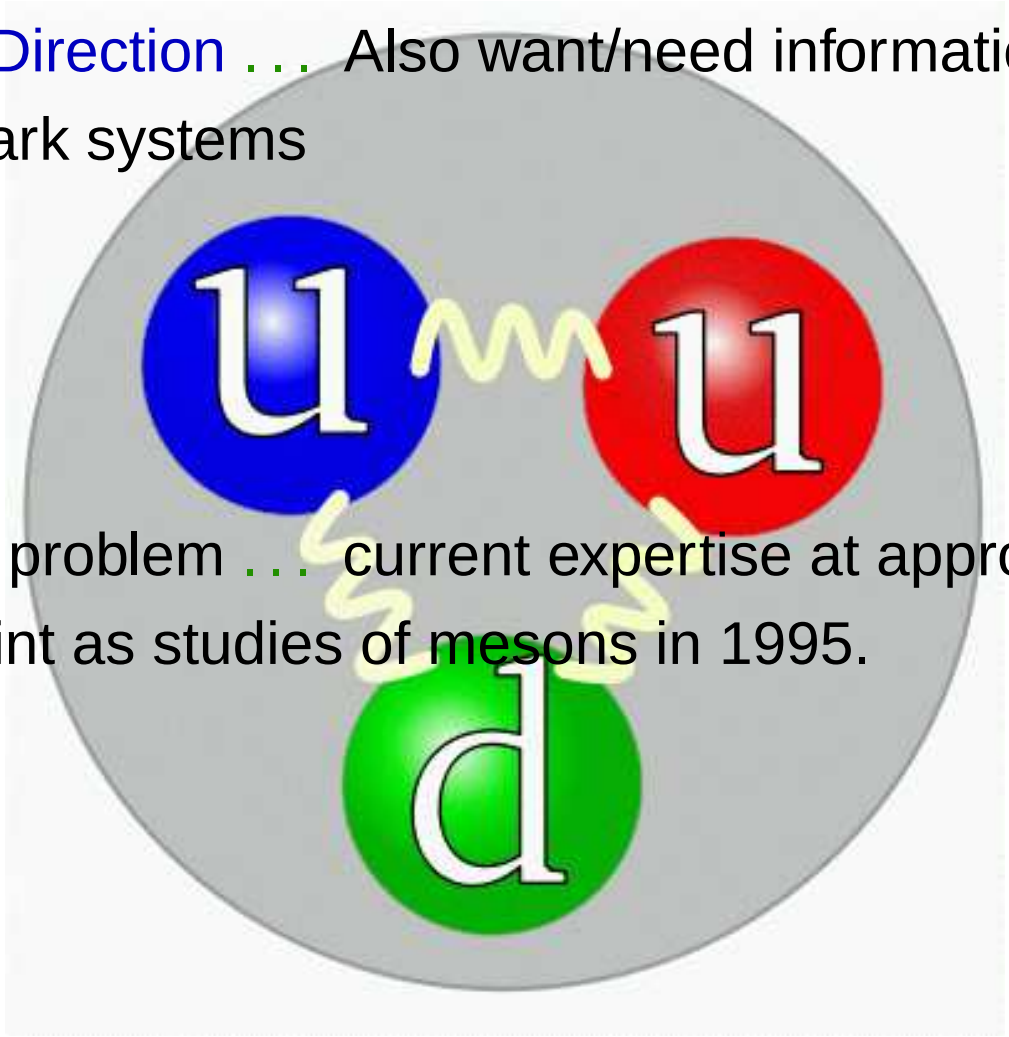

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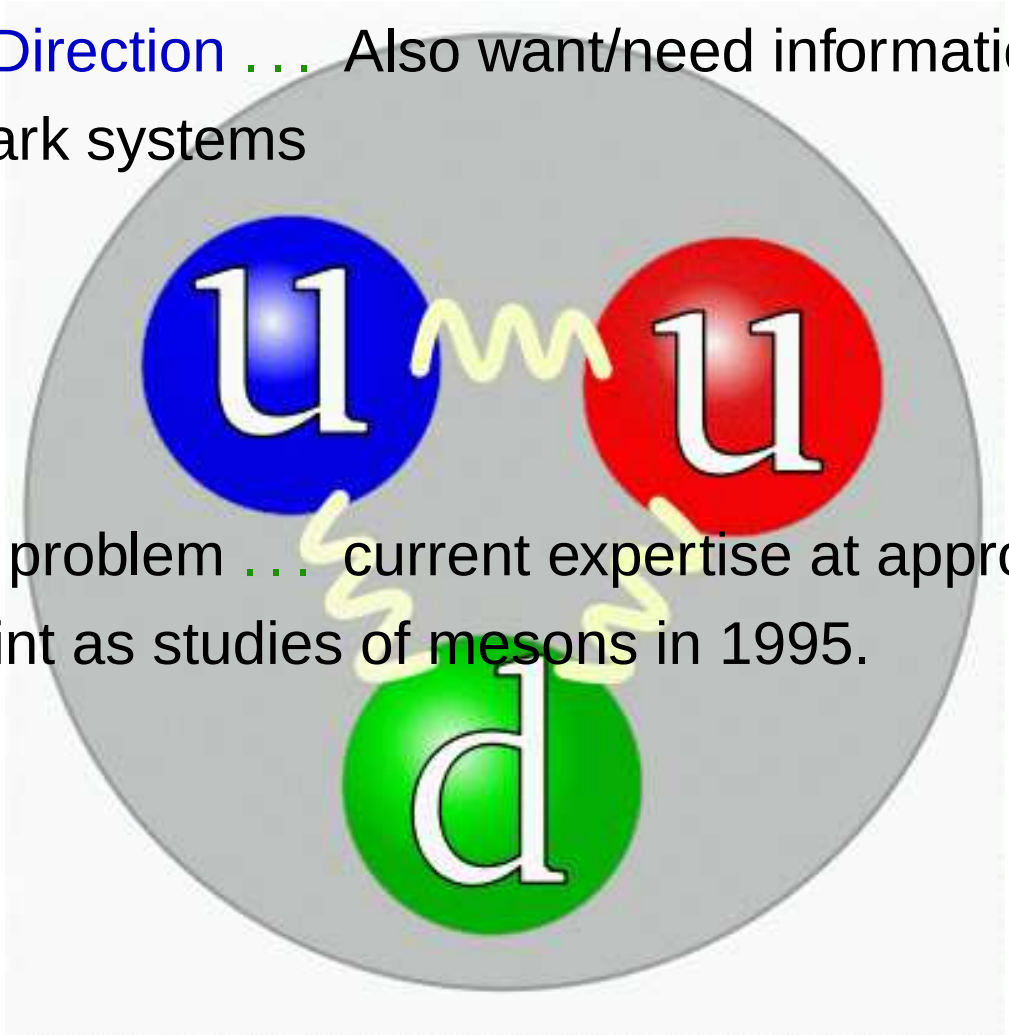

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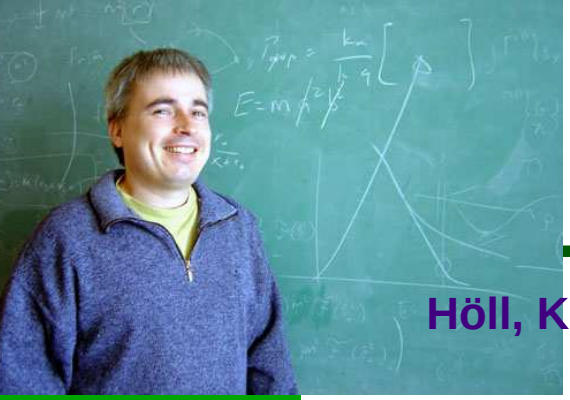
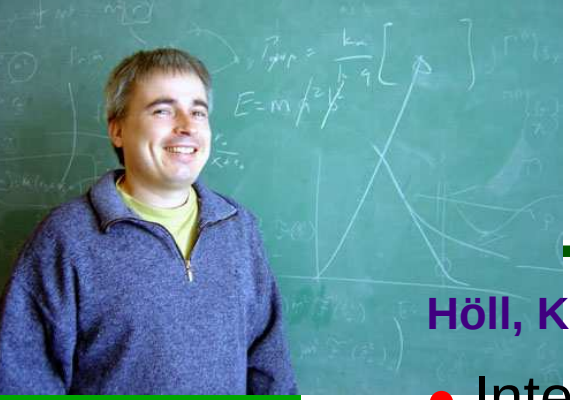
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.



Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033



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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

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- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

Faddeev equation

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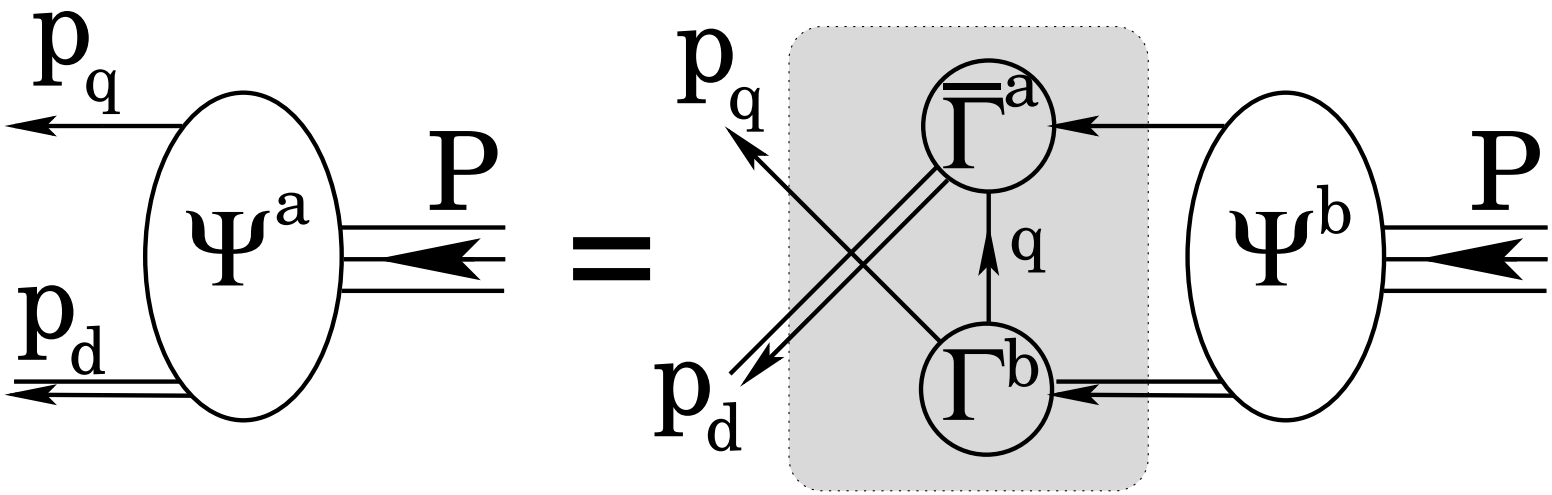

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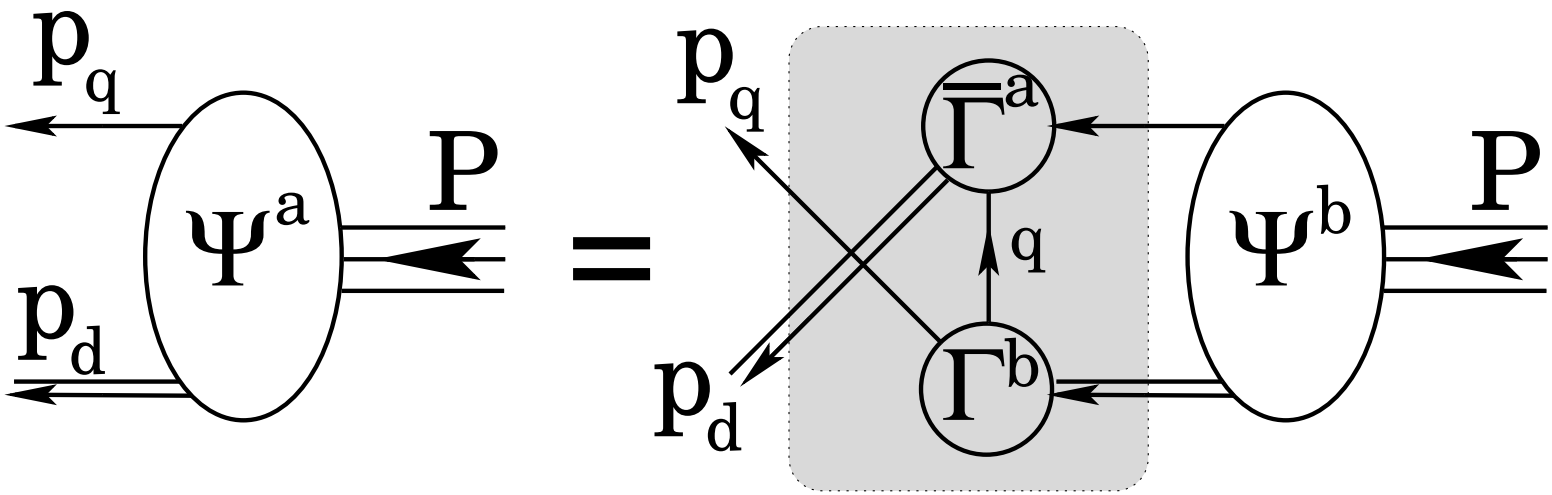
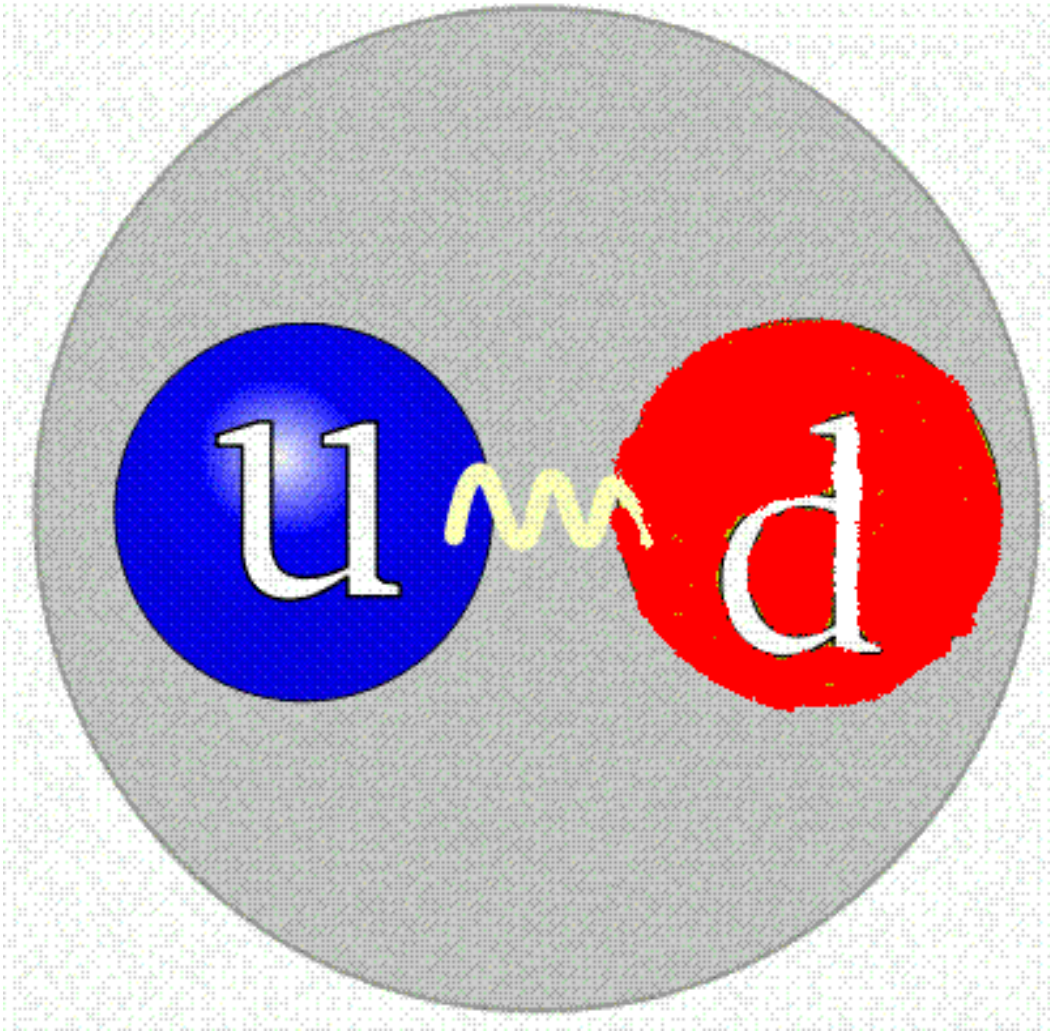


- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Diquark correlations



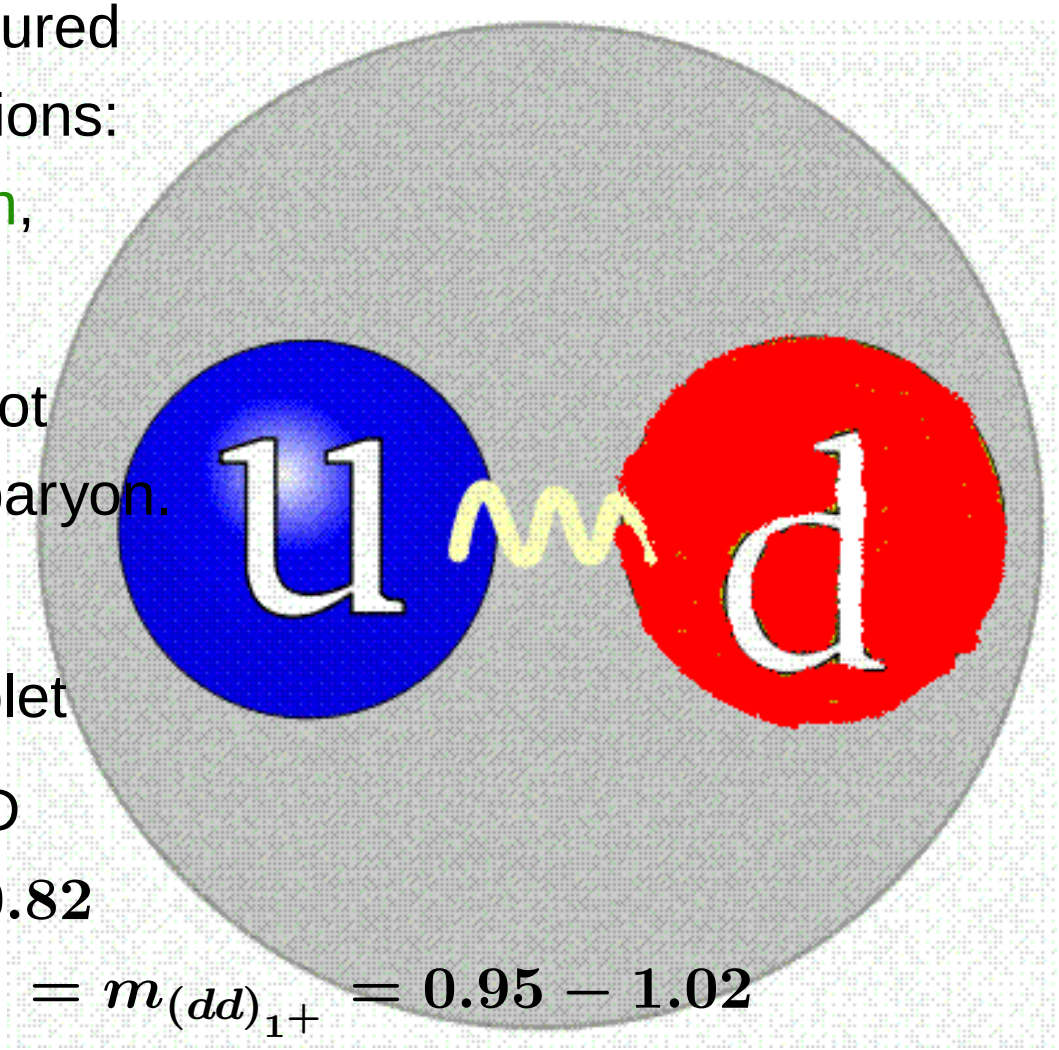
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined . . . Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

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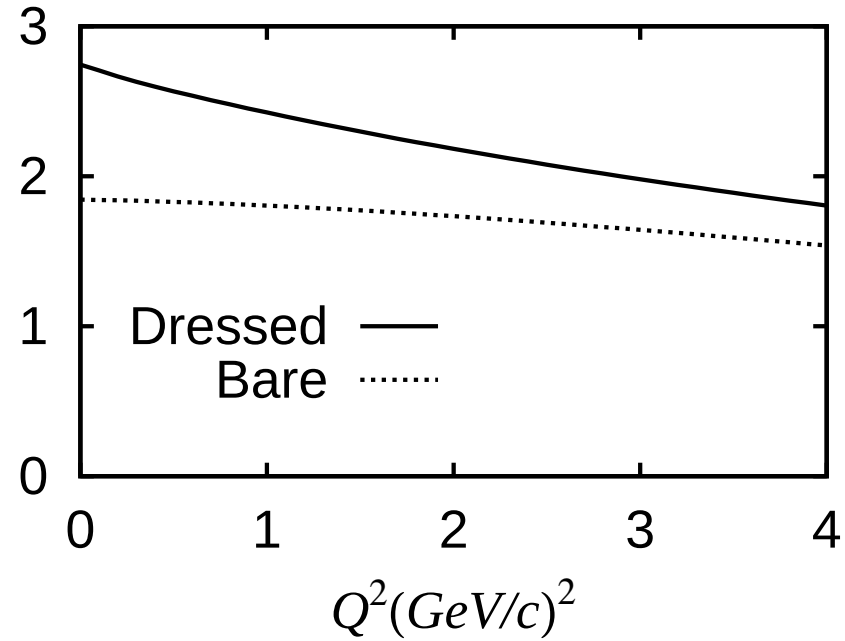
- Pion cloud effects are large in the low Q^2 region.

Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
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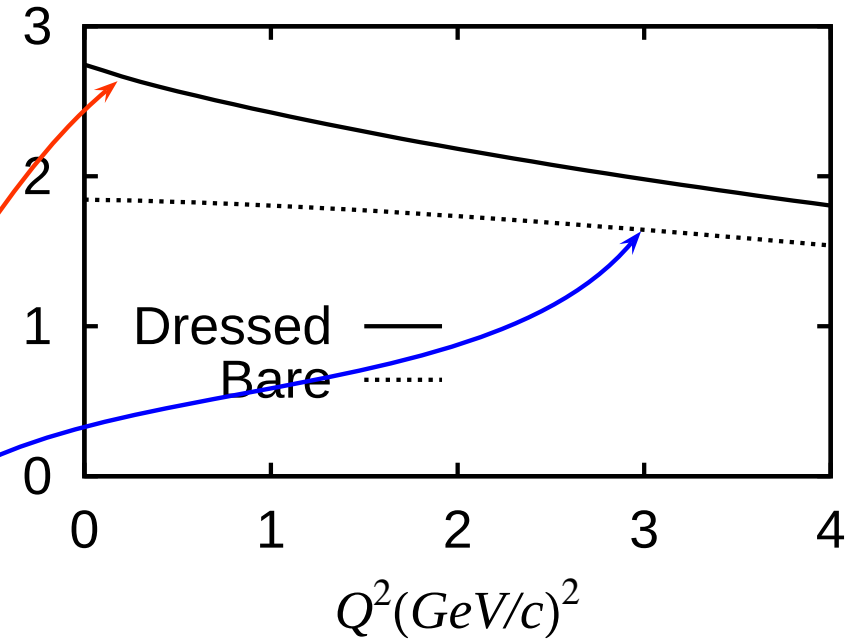

Quark Core

- Responsible for only 2/3 of result at small Q^2
- Dominant for $Q^2 >$2 – 3 GeV^2

Results: Nucleon and Δ Masses

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Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

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- Axial-vector diquark provides significant attraction

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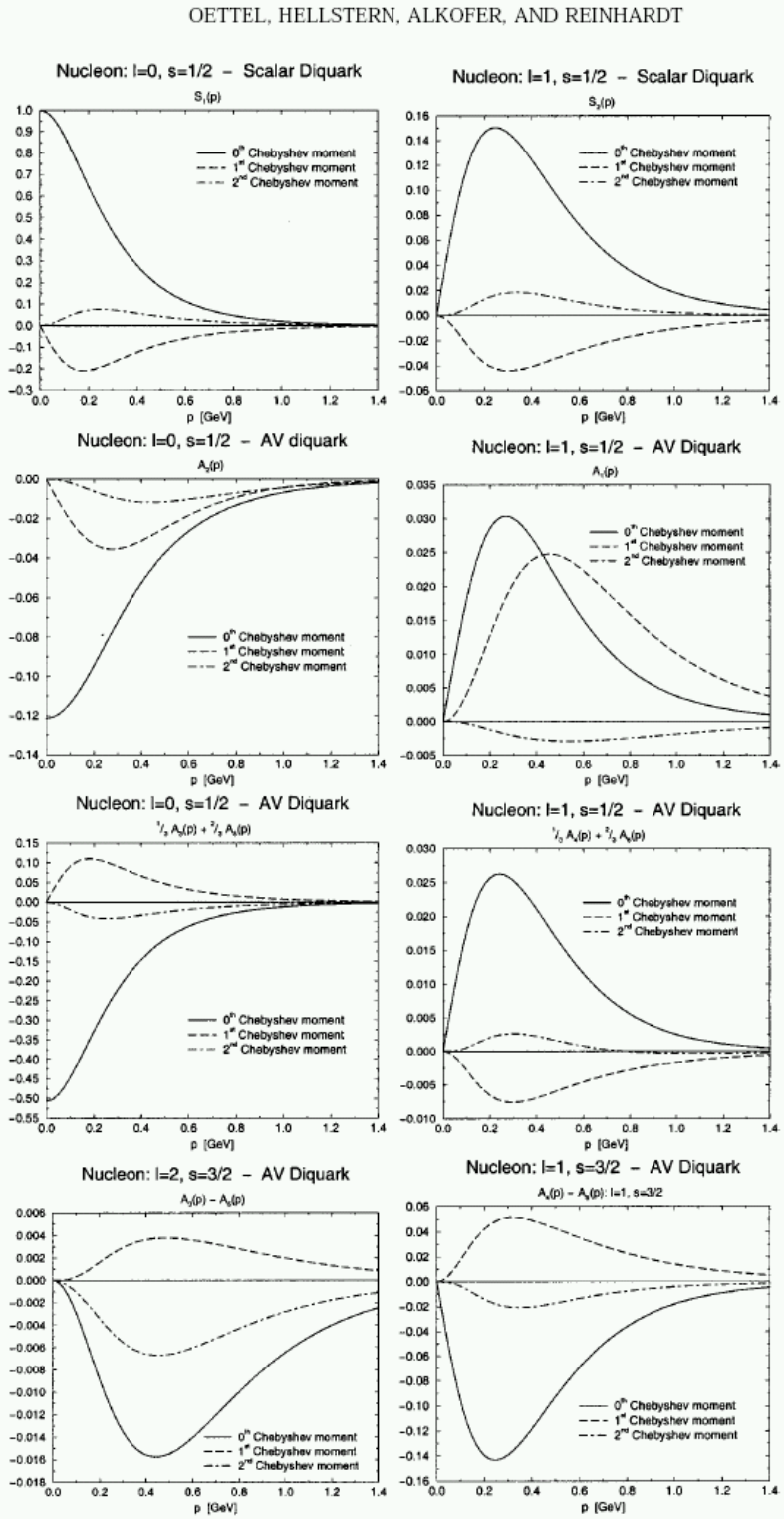
- Constructive Interference: 1^{++}-diquark $+ \partial_\mu \pi$

Angular Momentum Rest Frame



M. Oettel, *et al.*
nucl-th/9805054

Crude estimate based on magnitudes $\Rightarrow$ probability for a u-quark to carry the proton's spin is $P_{u\uparrow} \sim 80\,\%$, with $P_{u\downarrow} \sim 5\,\%$, $P_{d\uparrow} \sim 5\,\%$, $P_{d\downarrow} \sim 10\,\%$.

Hence, by this reckoning $\sim 30\%$ of proton's rest-frame spin is located in dressed-quark angular momentum.

Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

for on-shell nucleons described by Faddeev Amplitude

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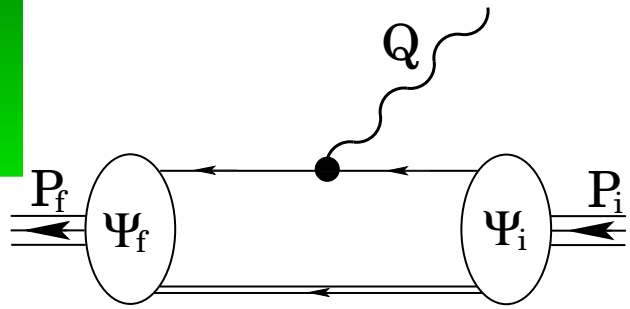
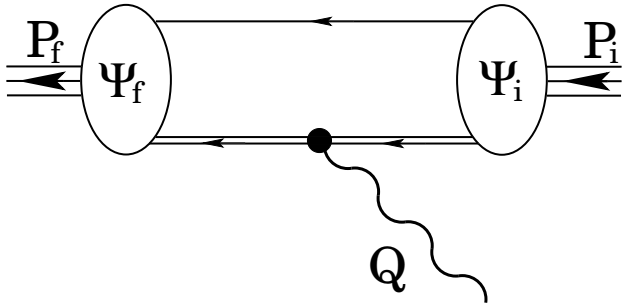
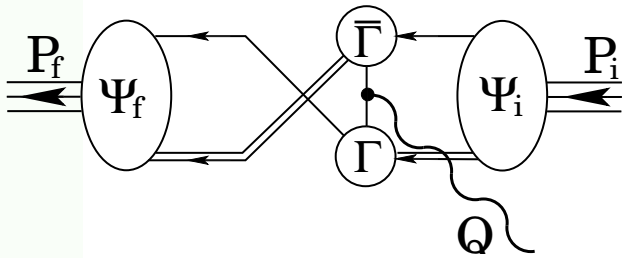
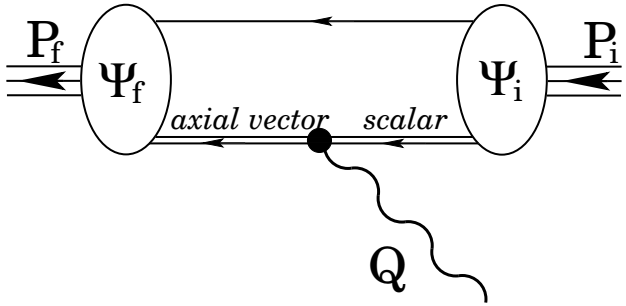
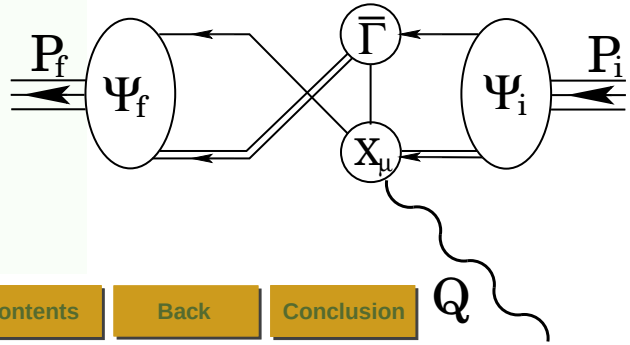
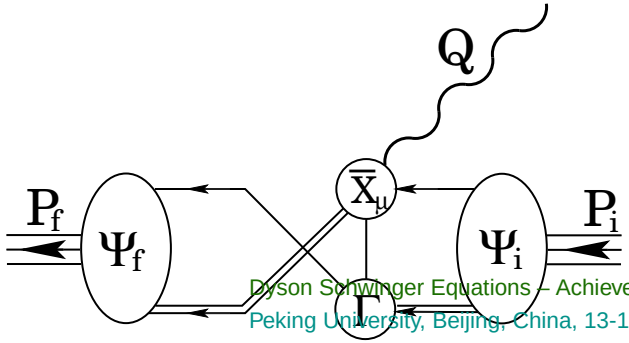




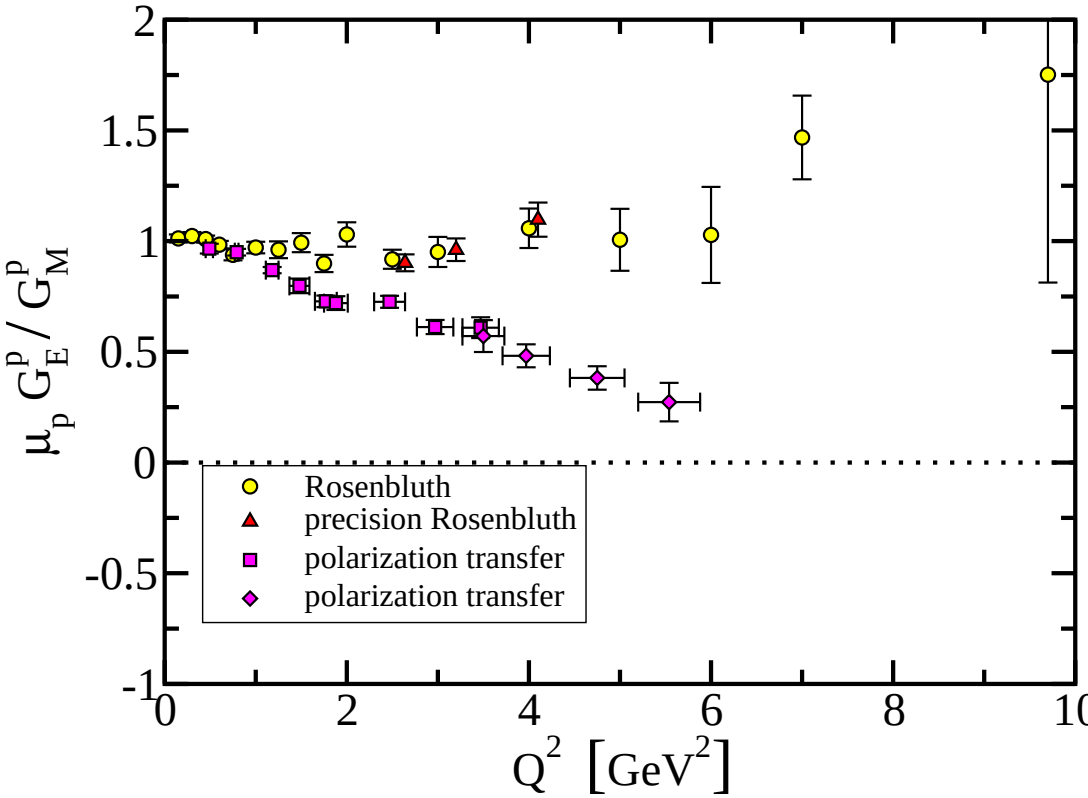








Form Factor Ratio: GE/GM



Form Factor Ratio: GE/GM

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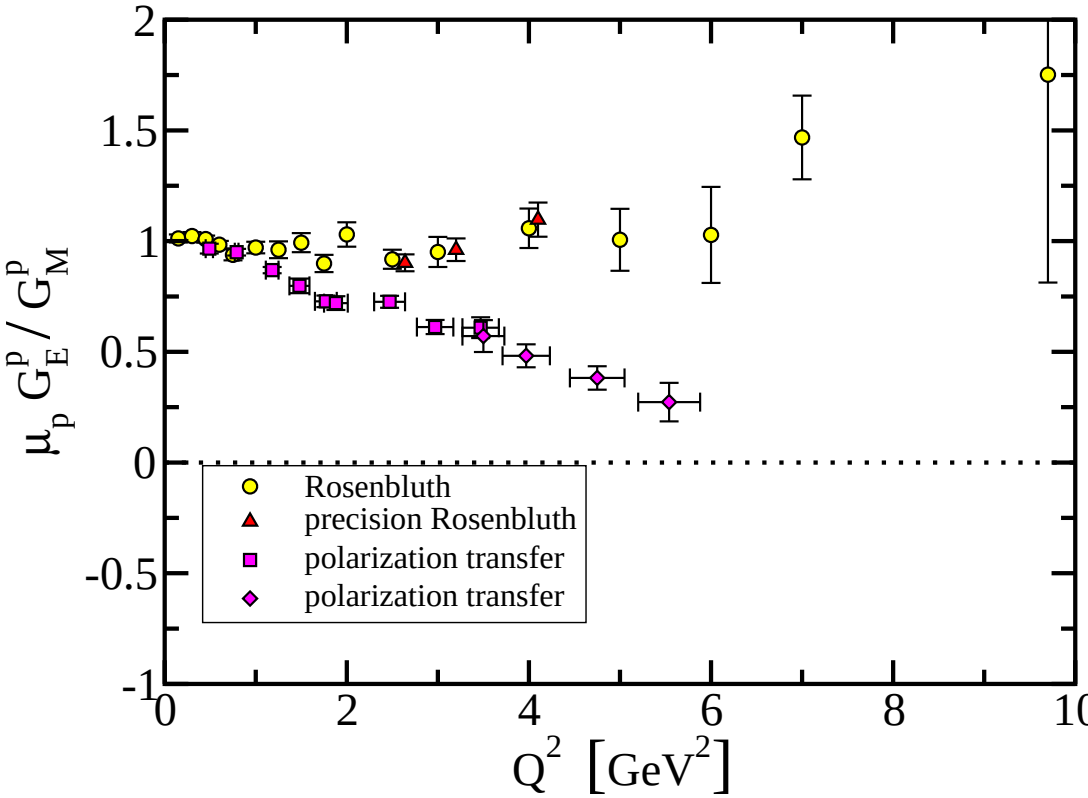

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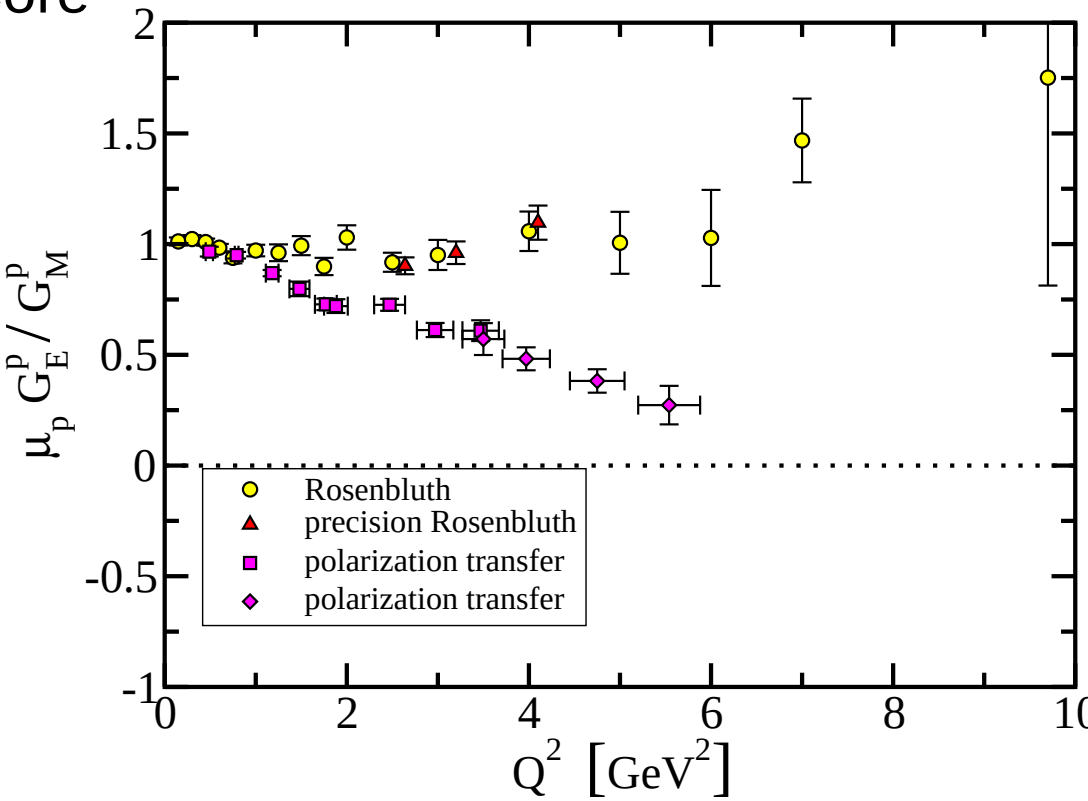

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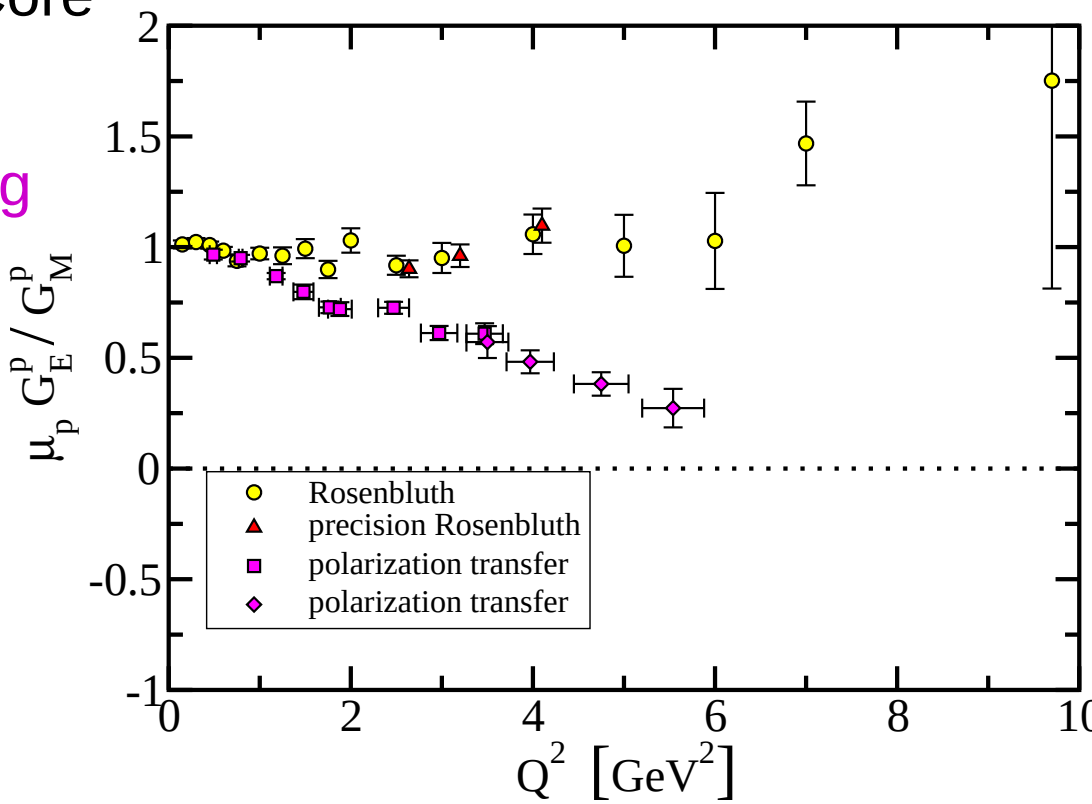

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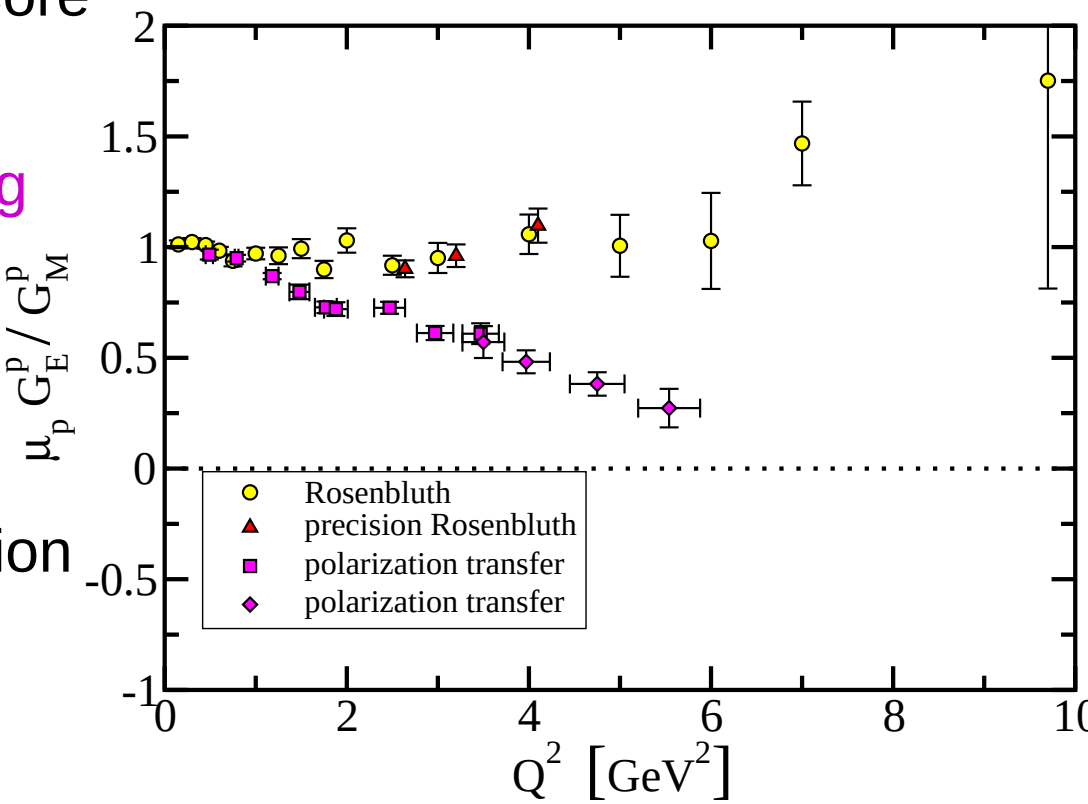
  - Anticipate and Estimate Pion Cloud's Contribution



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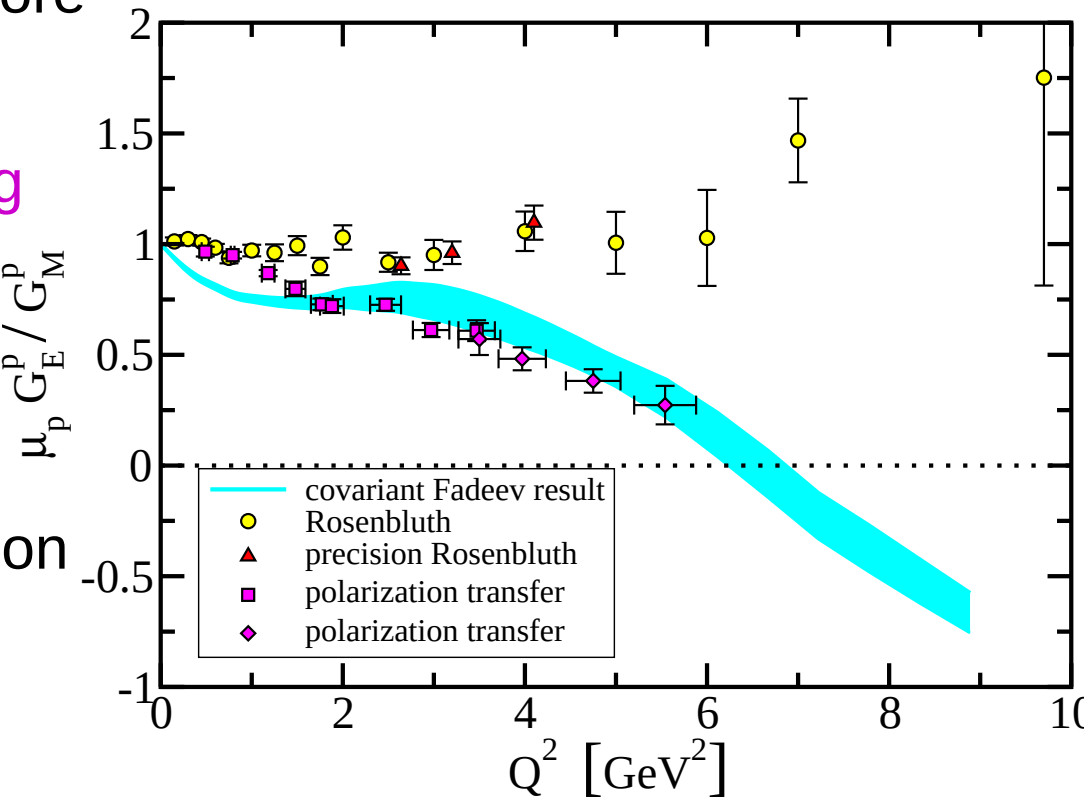

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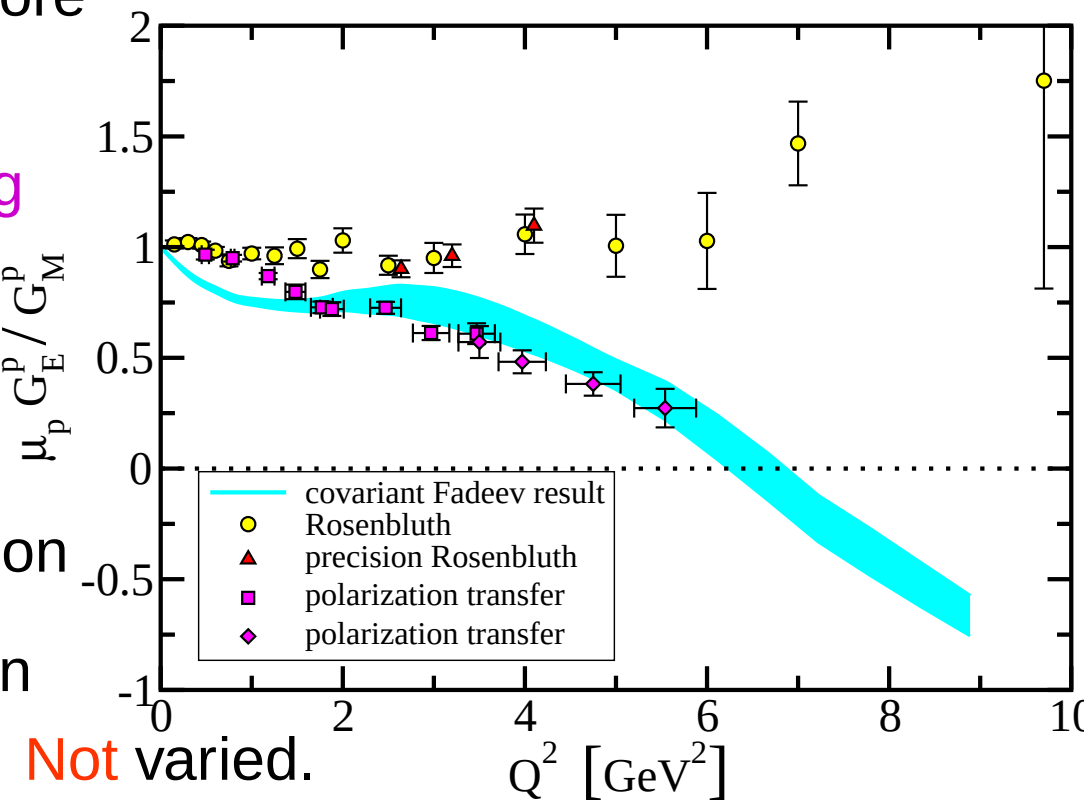

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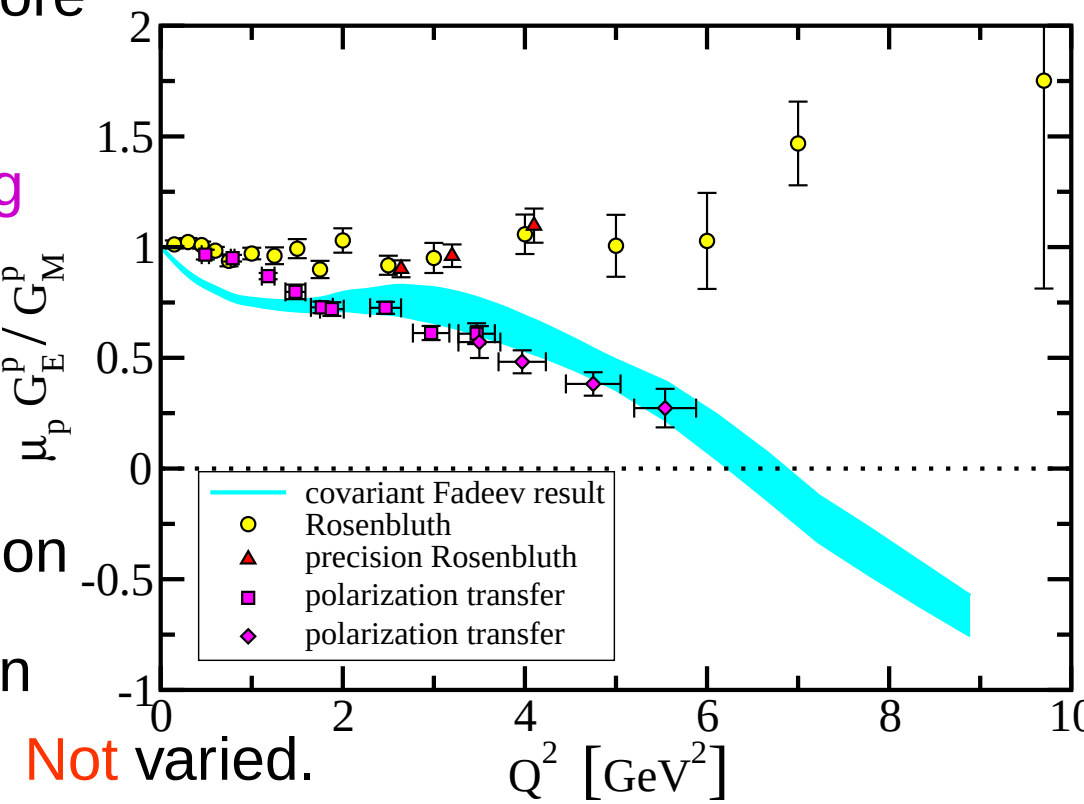

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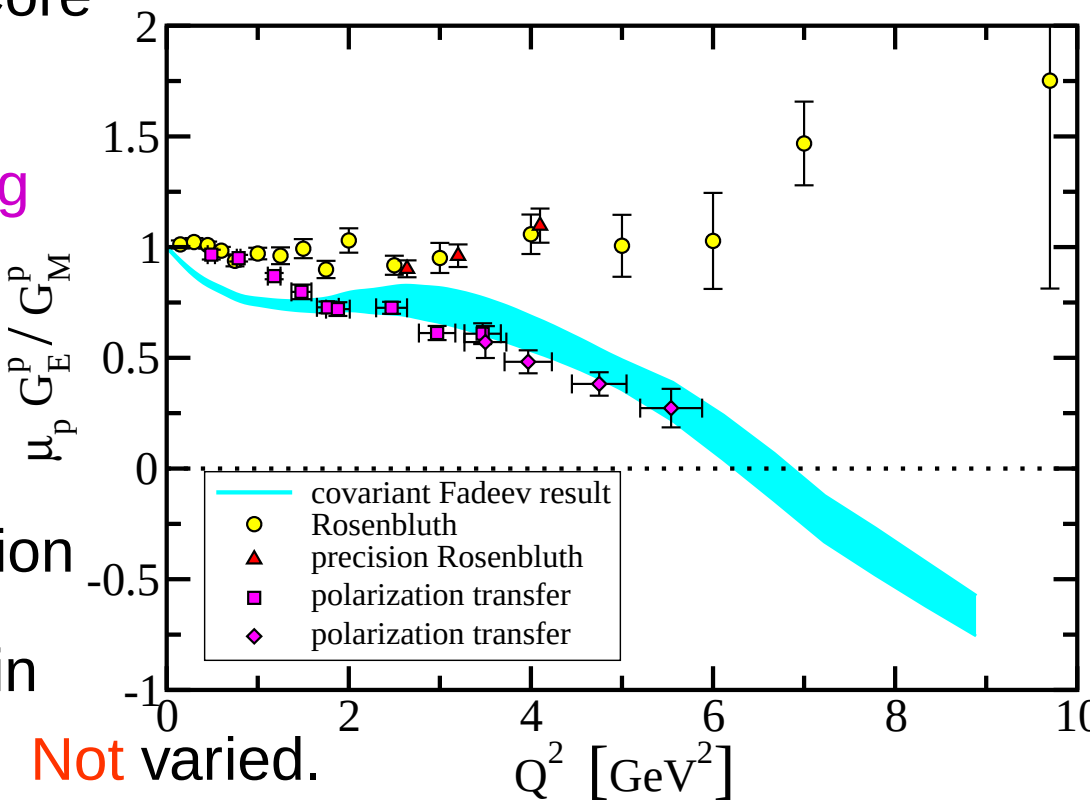

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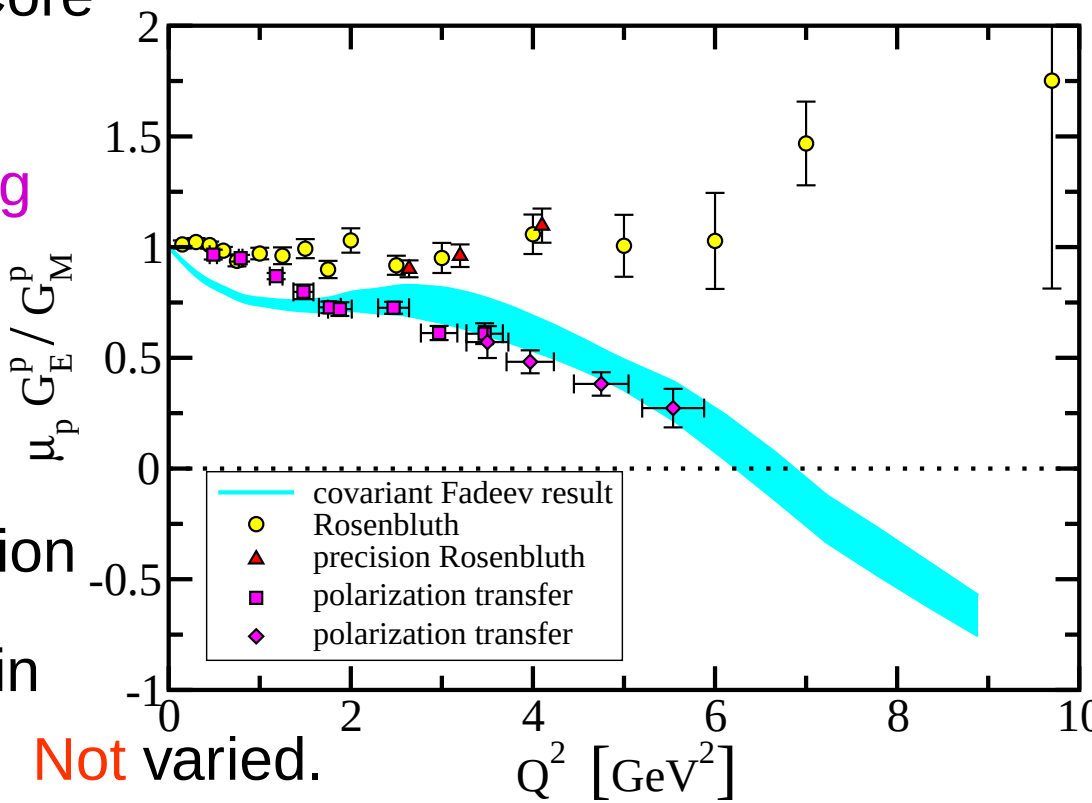

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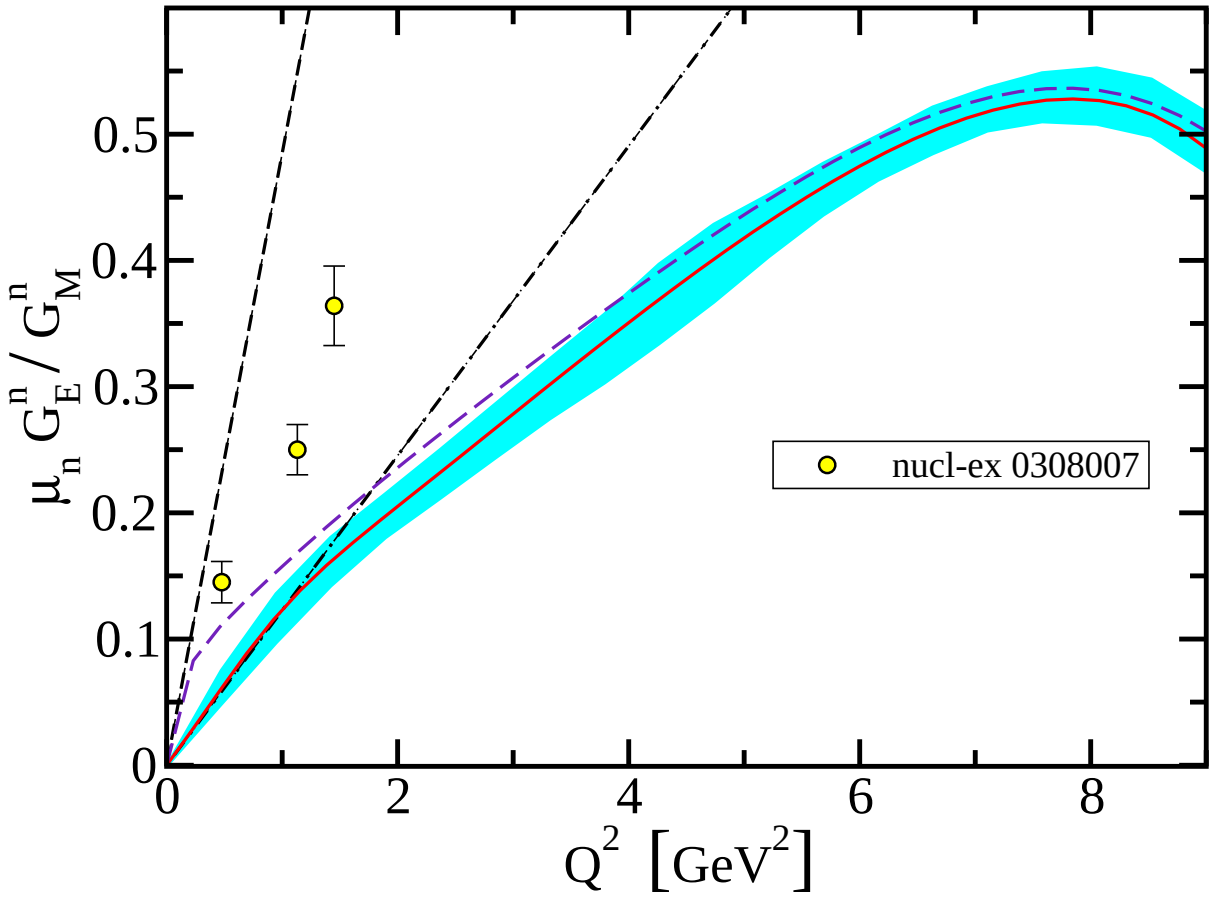
  - Predict Zero at $Q^2 \approx 6.5\mathrm{GeV}^2$

Neutron Form Factors



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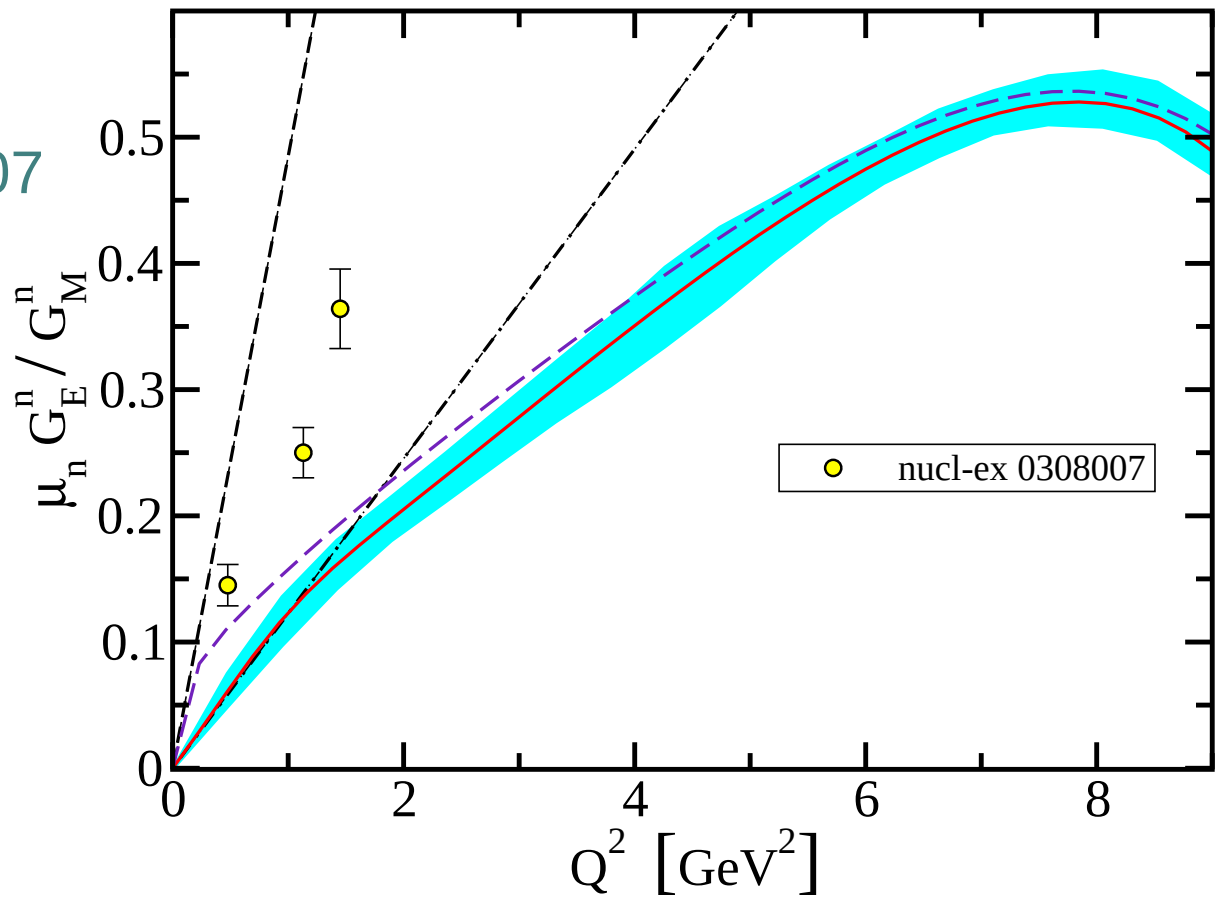

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- Calc. Bhagwat, *et al.* nu-th/0610080
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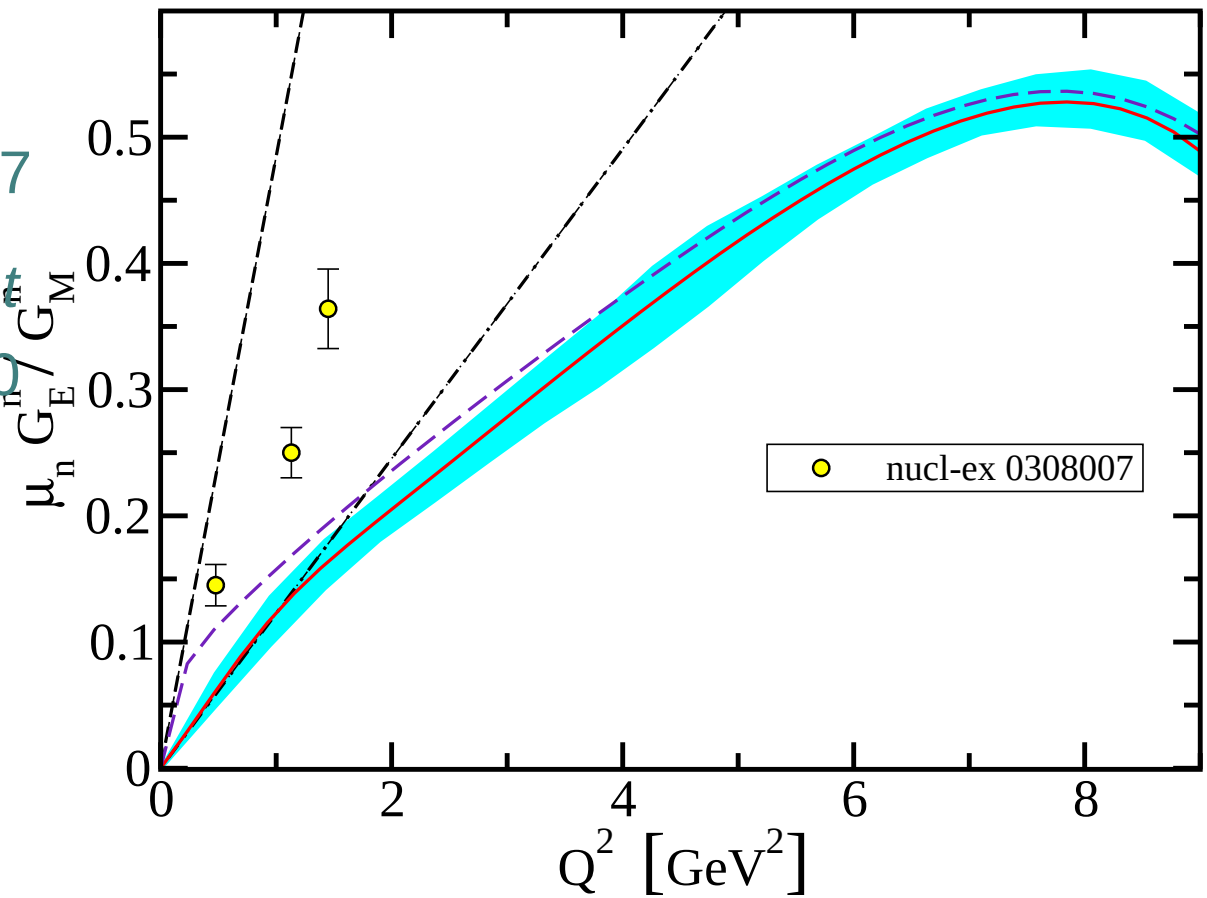
Valid for $r_n^2 Q^2 \lesssim 1$



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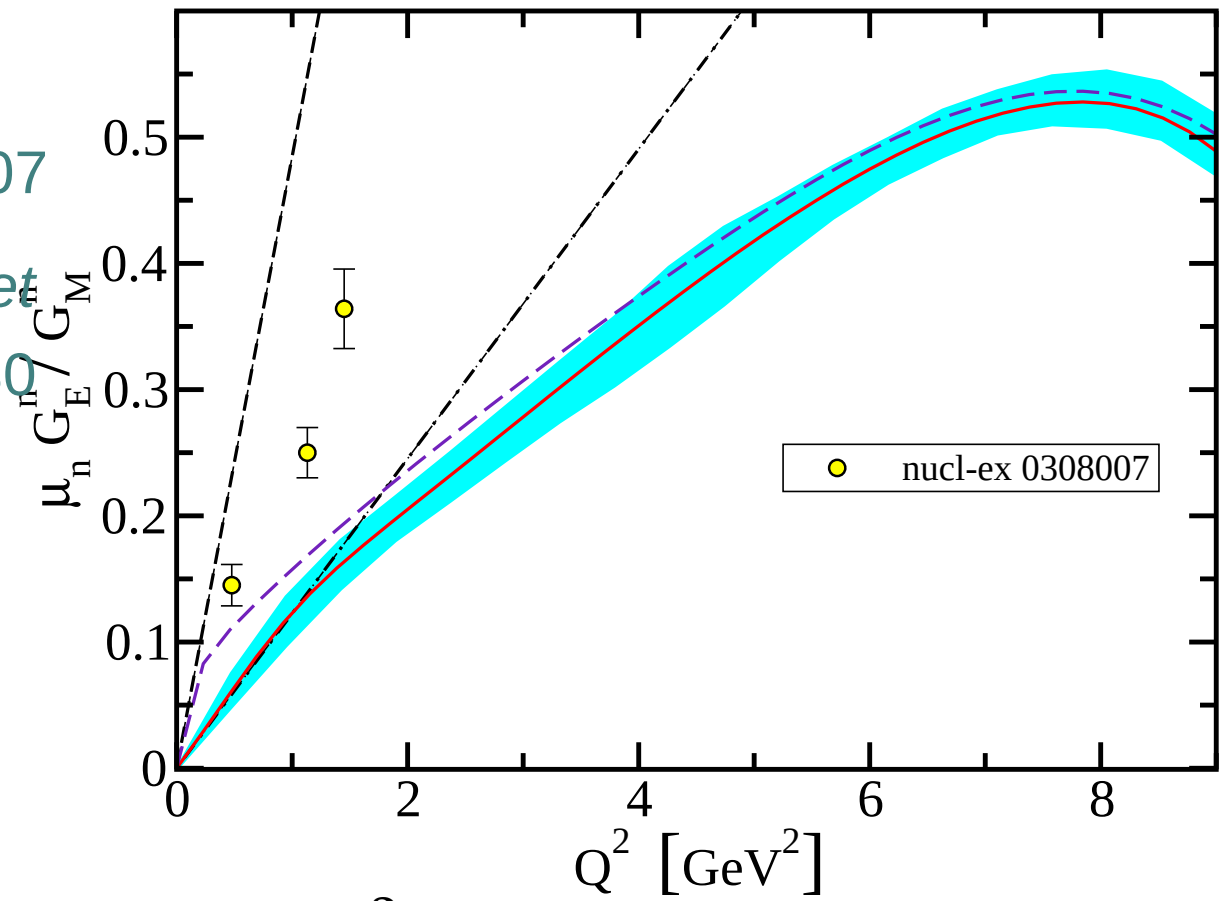

- No sign yet of a zero in $G_E^n(Q^2)$, even though calculation predicts $G_E^p(Q^2 \approx 6.5\,\mathrm{GeV}^2) = 0$
- Data to $Q^2 = 3.4\,\mathrm{GeV}^2$ is being analysed (JLab E02-013)

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- An excellent way to test conjectures and constrain the possibilities
- Physics is an Experimental science

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30. Contemporary Reviews

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Widths fixed by "asymptotic freedom" condition –

$$\frac{d}{dK^2}\left(\frac{1}{m_{J^P}^2}\,\mathcal{F}(K^2/\omega_{J^P}^2)\right)^{-1}\Bigg|_{K^2=0} = 1 \;\Rightarrow\; \omega_{J^P}^2 = \frac{1}{2}\,m_{J^P}^2\,,$$

Only two parameters; viz., diquark "masses": m_{J^P}

Contemporary Reviews

- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD
  C.D. Roberts and S.M. Schmidt, nu-th/0005064,
  Prog. Part. Nucl. Phys. **45** (2000) S1
- The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .
  R. Alkofer and L. von Smekal, he-ph/0007355,
  Phys. Rept. **353** (2001) 281
- Dyson-Schwinger equations: A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049,
  Int. J. Mod. Phys. **E 12** (2003) pp. 297-365
- Infrared properties of QCD from Dyson-Schwinger equations.
  C. S. Fischer, he-ph/0605173,
  J. Phys. **G 32** (2006) pp. R253-R291
- Nucleon electromagnetic form factors
  J. Arrington, C.D. Roberts and J.M. Zanotti, nucl-th/0611050,
  J. Phys. **G 34** (2007) pp. S23-S52.