Dynamics, Symmetries, and Hadron Properties

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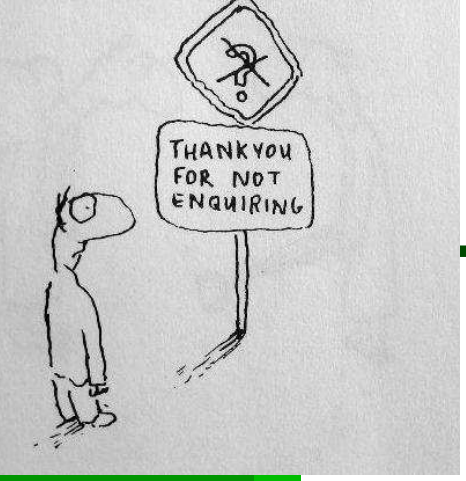
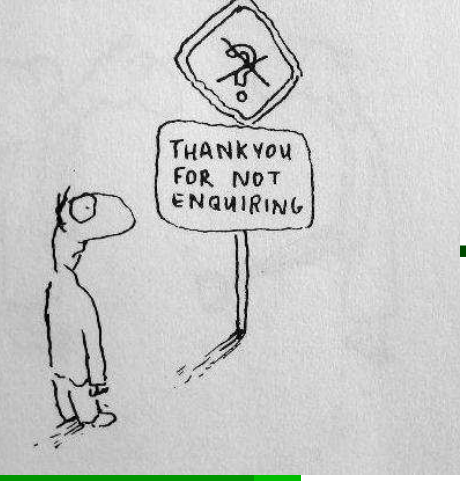
http://www.phy.anl.gov/theory/staff/cdr.html



QCD's Challenges



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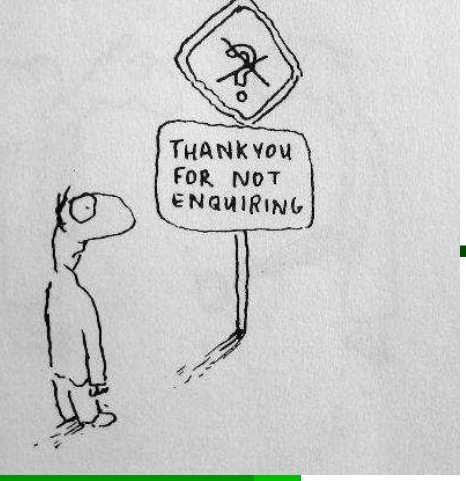
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon



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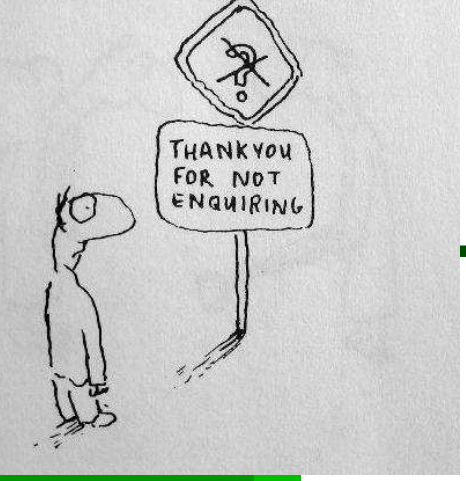

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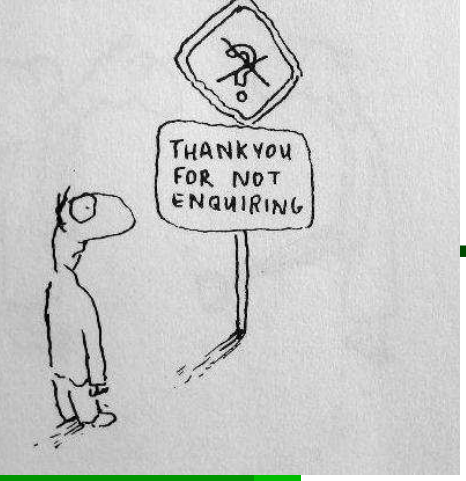

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Understand Emergent Phenomena

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- QCD – Complex behaviour

  arises from apparently simple rules

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Highly Nontrivial

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  - Interaction between quarks – the Interquark Potential – unknown throughout $> 98\%$ of an hadron's volume

Intranucleon Interaction

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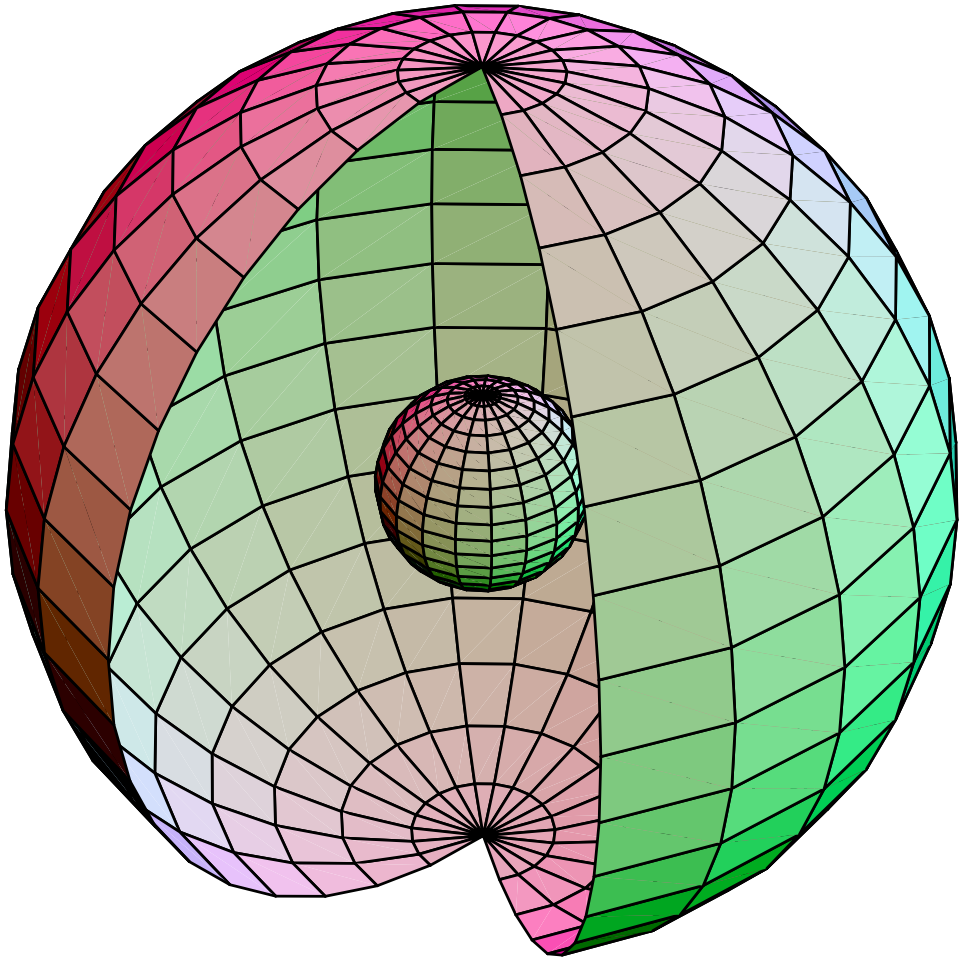

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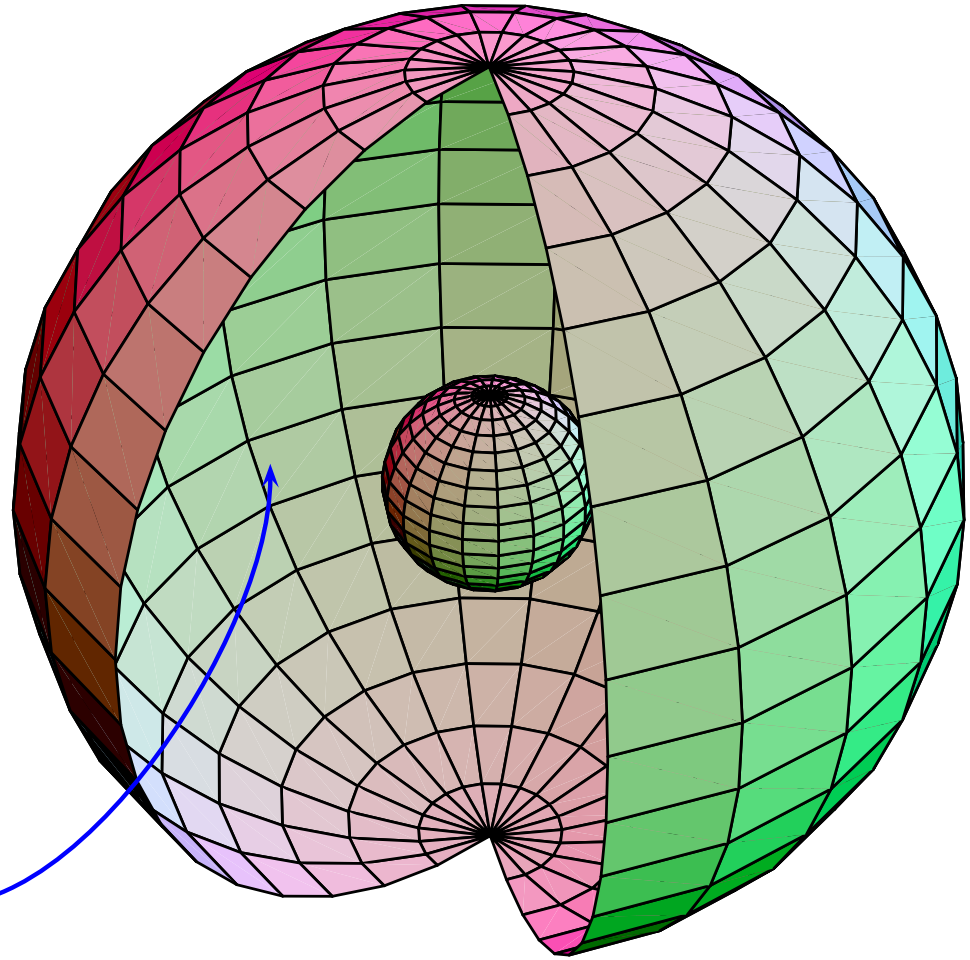
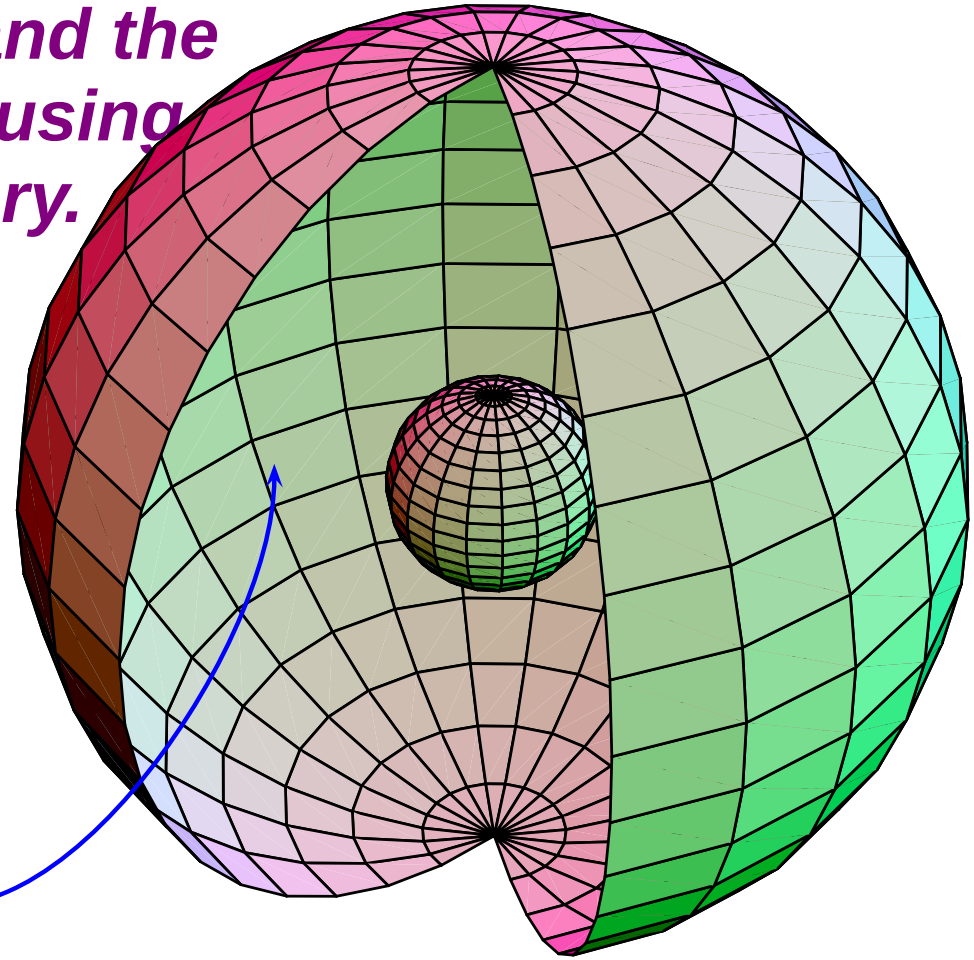


98% of the volume

Intranucleon Interaction

The question must be rigorously defined, and the answer mapped out using experiment and theory.



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- $\Rightarrow$ Understanding InfraRed (long-range)

  ..... behaviour of $\alpha_s(Q^2)$

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- Method yields Schwinger Functions $\equiv$ Propagators

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Cross-Sections built from Schwinger Functions

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- Proving fruitful.

World . . .

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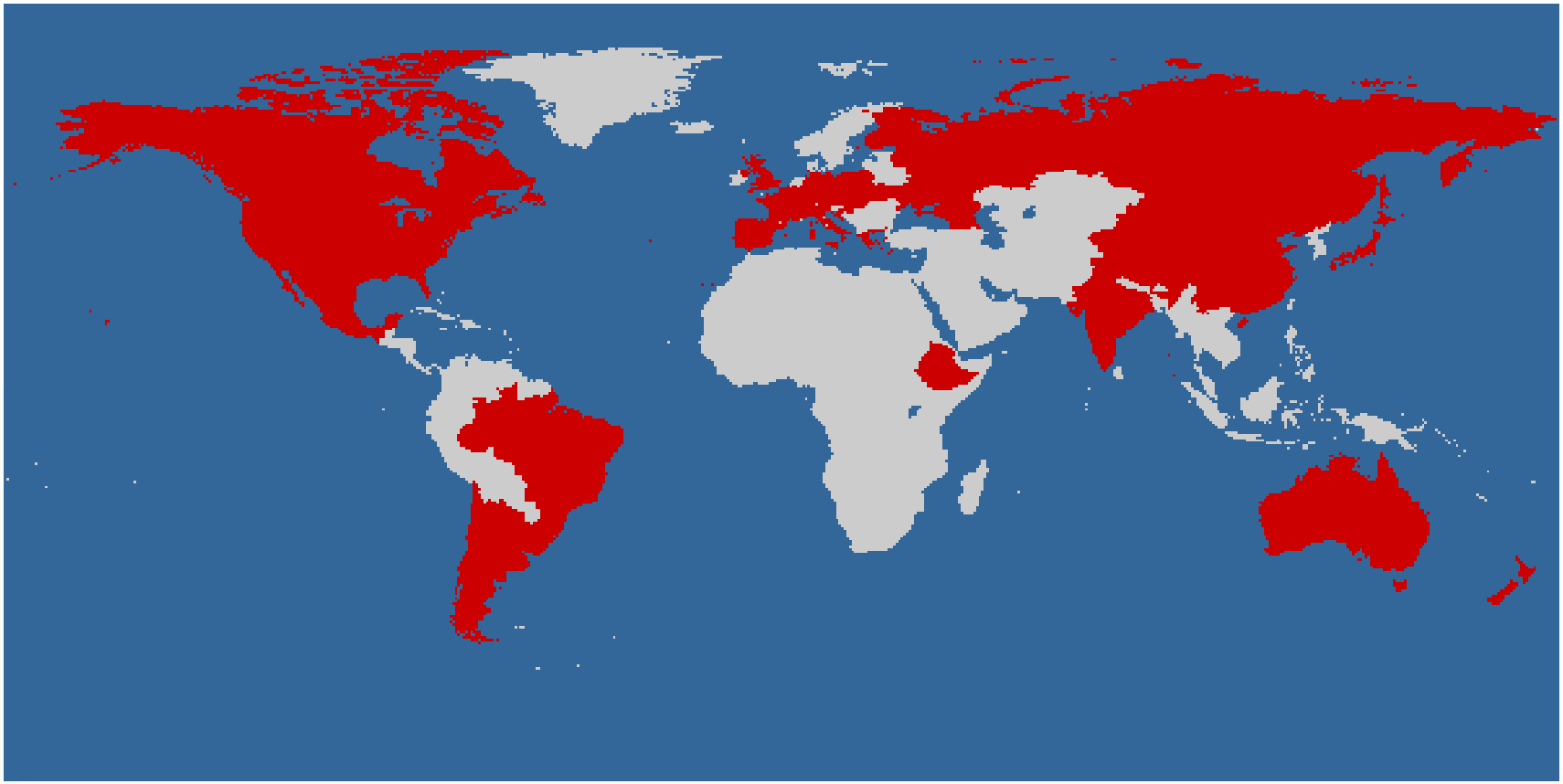
DSE Perspective



Persistent Challenge

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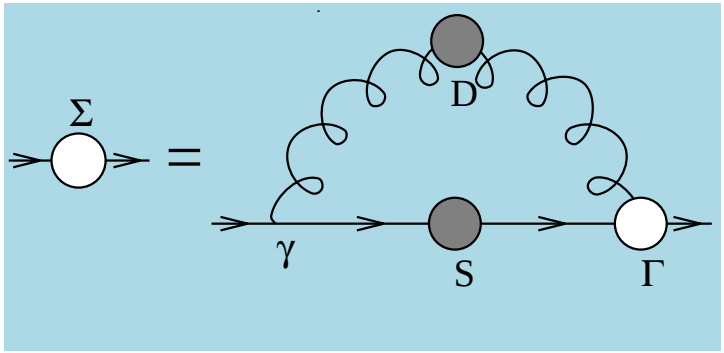

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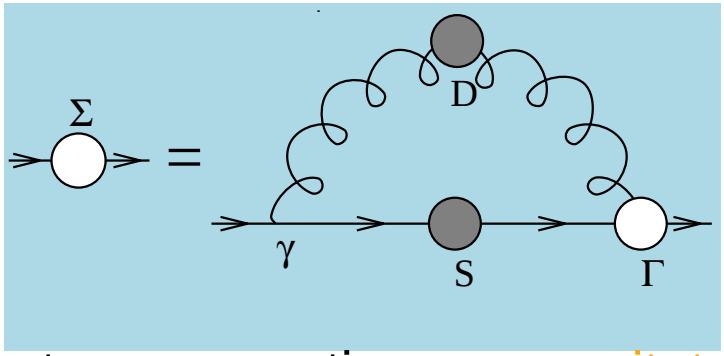

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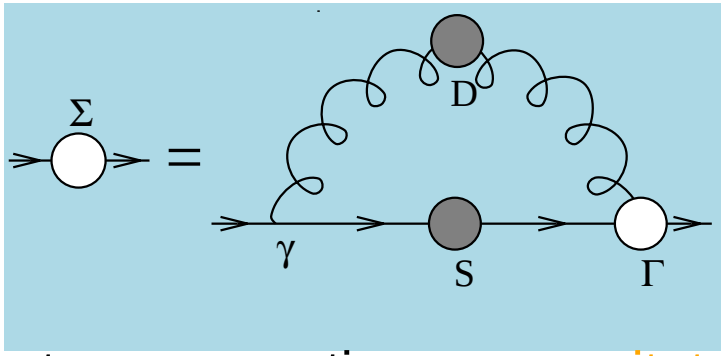

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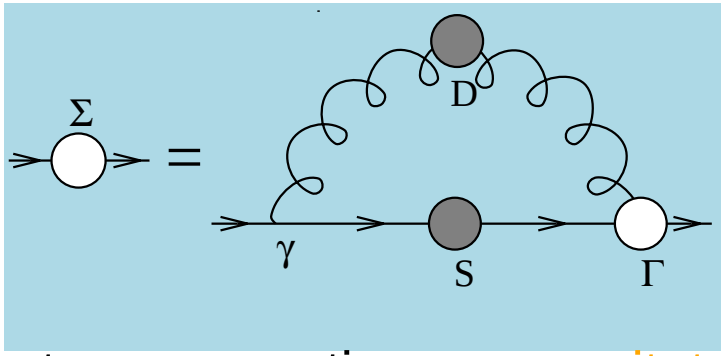

- Coupling between equations necessitates truncation
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    Not useful for the nonperturbative problems
    in which we're interested

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- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

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  - Make Predictions with Readily Quantifiable Errors

Dressed-Quark Propagator

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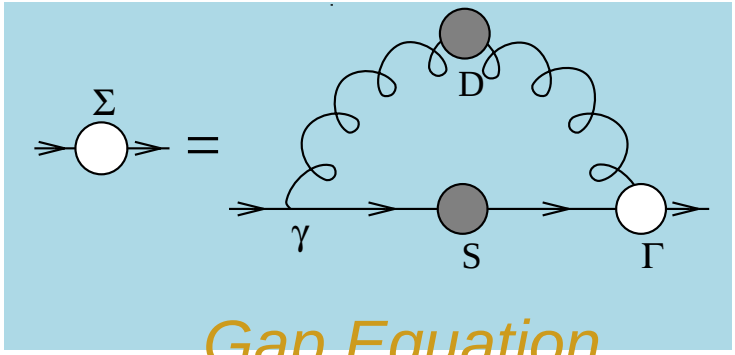
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



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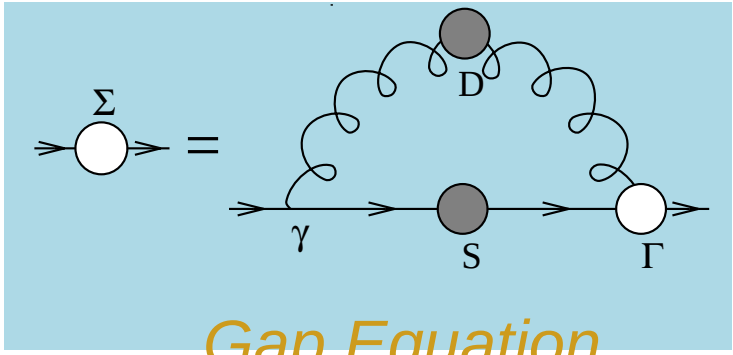
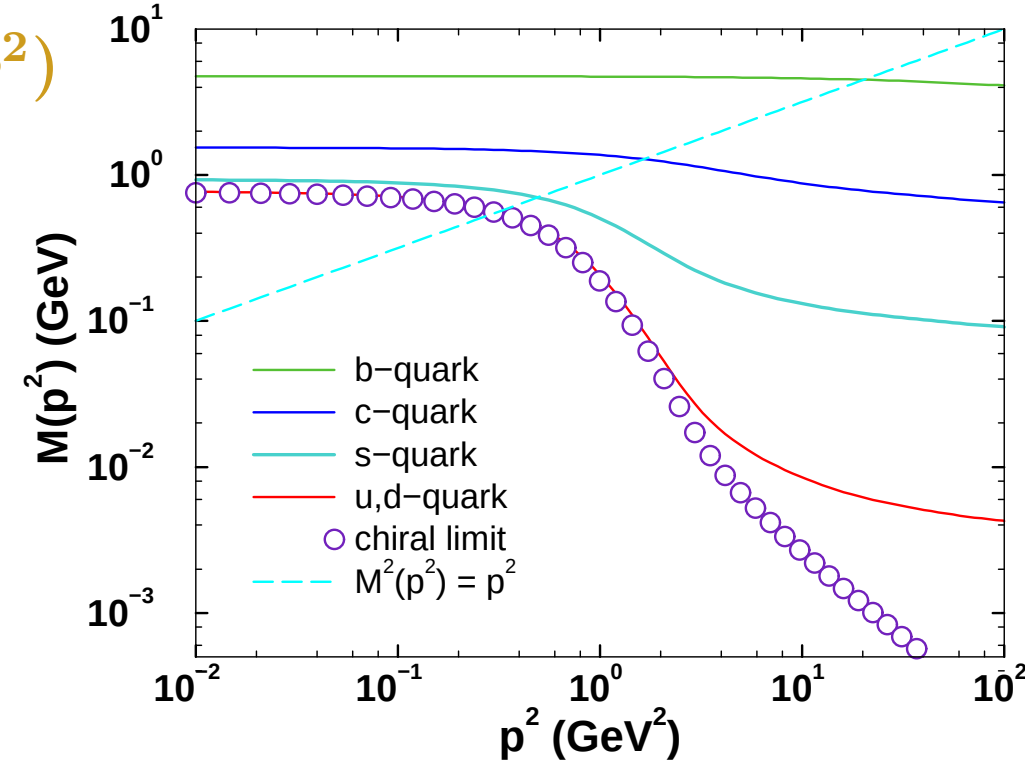


Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



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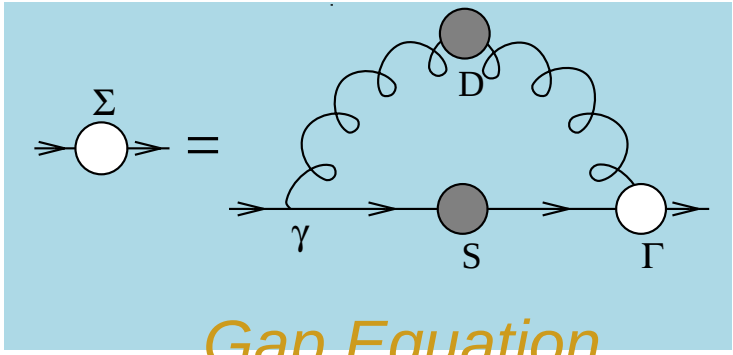

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flavour	u/d	s	c	b
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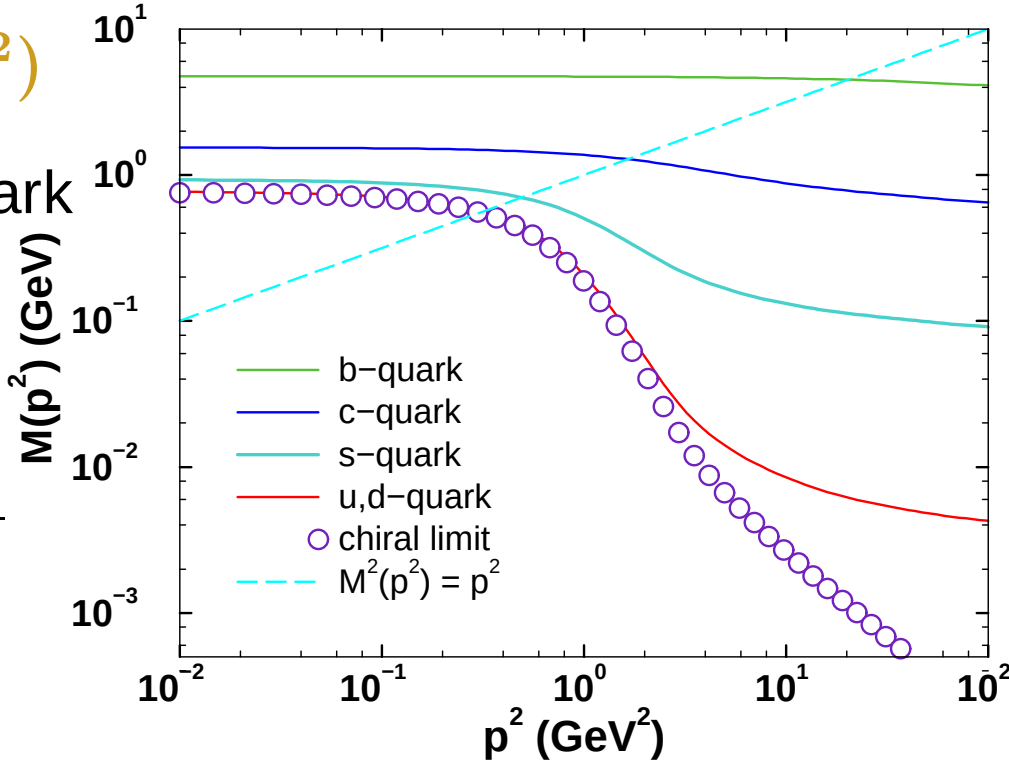

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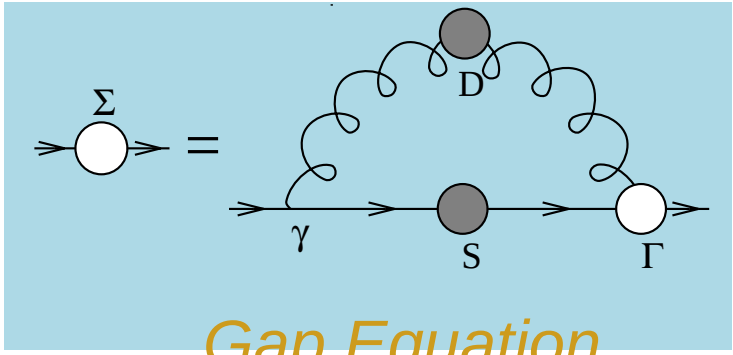

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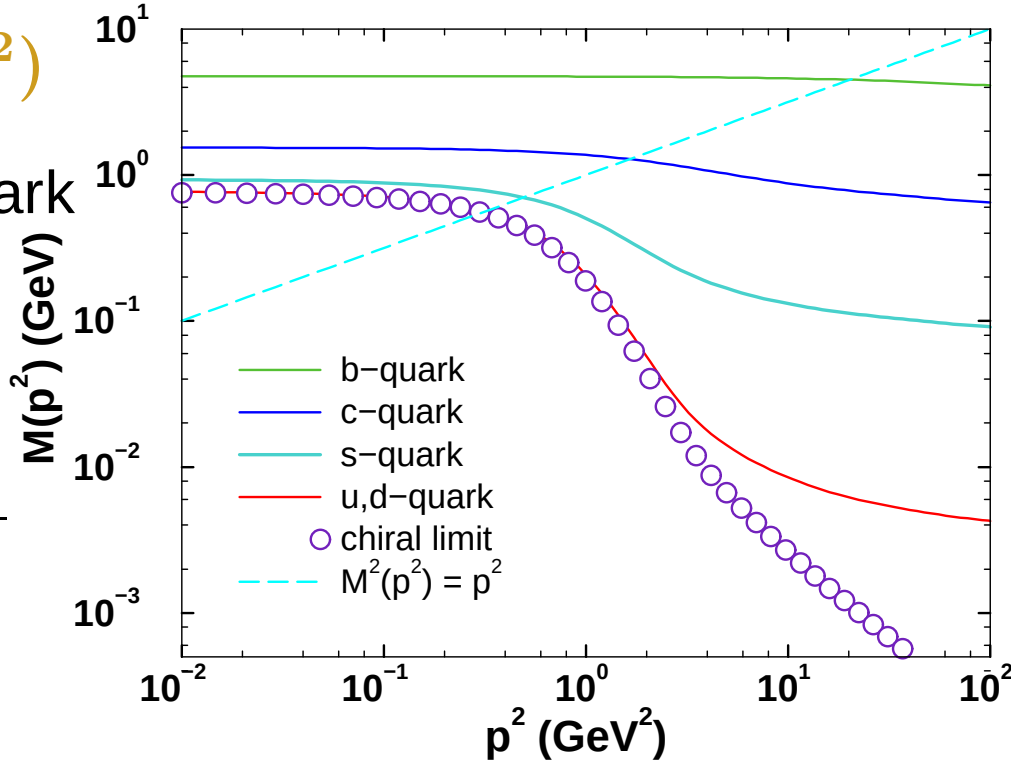

Predictions confirmed in numerical simulations of lattice-QCD

Hadrons

- Established understanding of two- and three-point functions

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- What about bound states?

Hadrons

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- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

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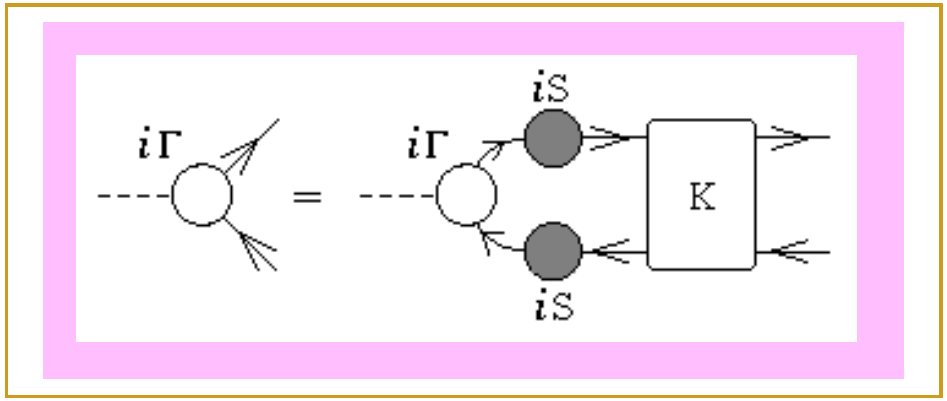
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

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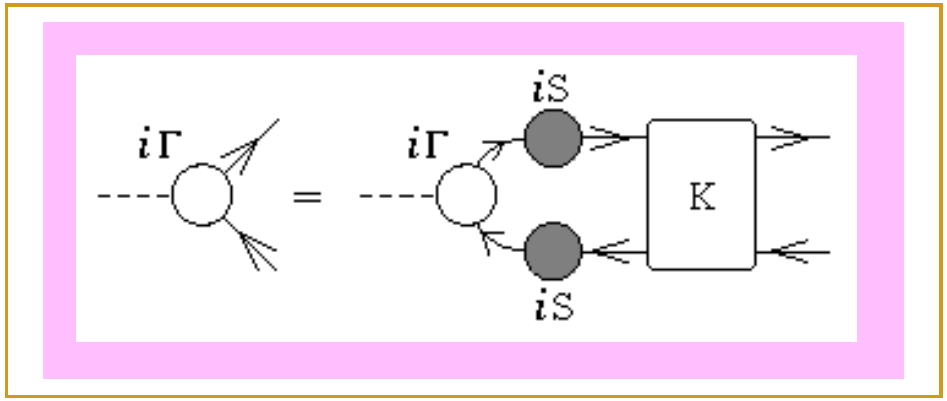

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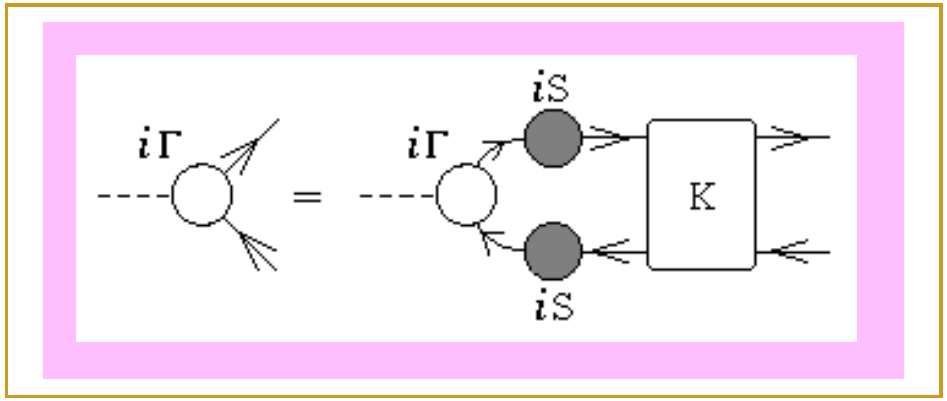

QFT Generalisation of Lippmann-Schwinger Equation.

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What is the light-quark Long-Range Potential?

Bethe-Salpeter Kernel

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QFT Statement of Chiral Symmetry

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- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Radial Excitations & Chiral Symmetry

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(Maris, Roberts, Tandy nu-th/9707003)

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- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

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$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

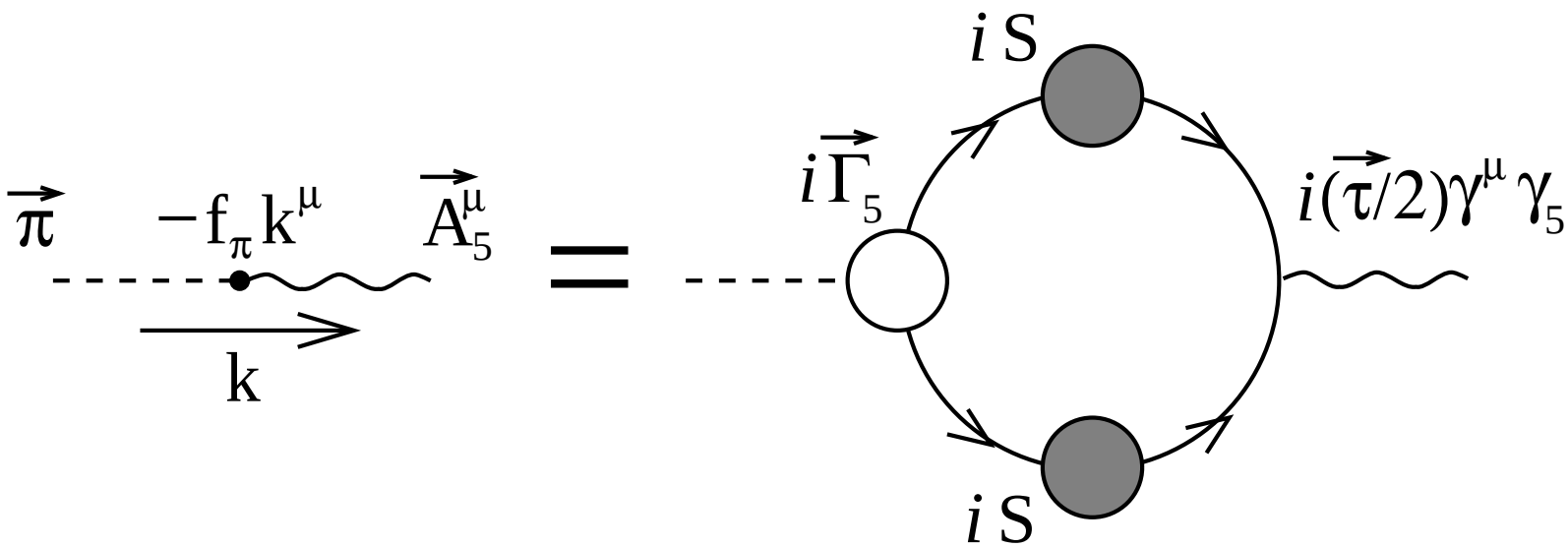
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H\, m_H^2 = -\ \rho_\zeta^H\ \mathcal{M}_H$$

$$f_H\, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5\gamma_\mu\, \mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-) \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



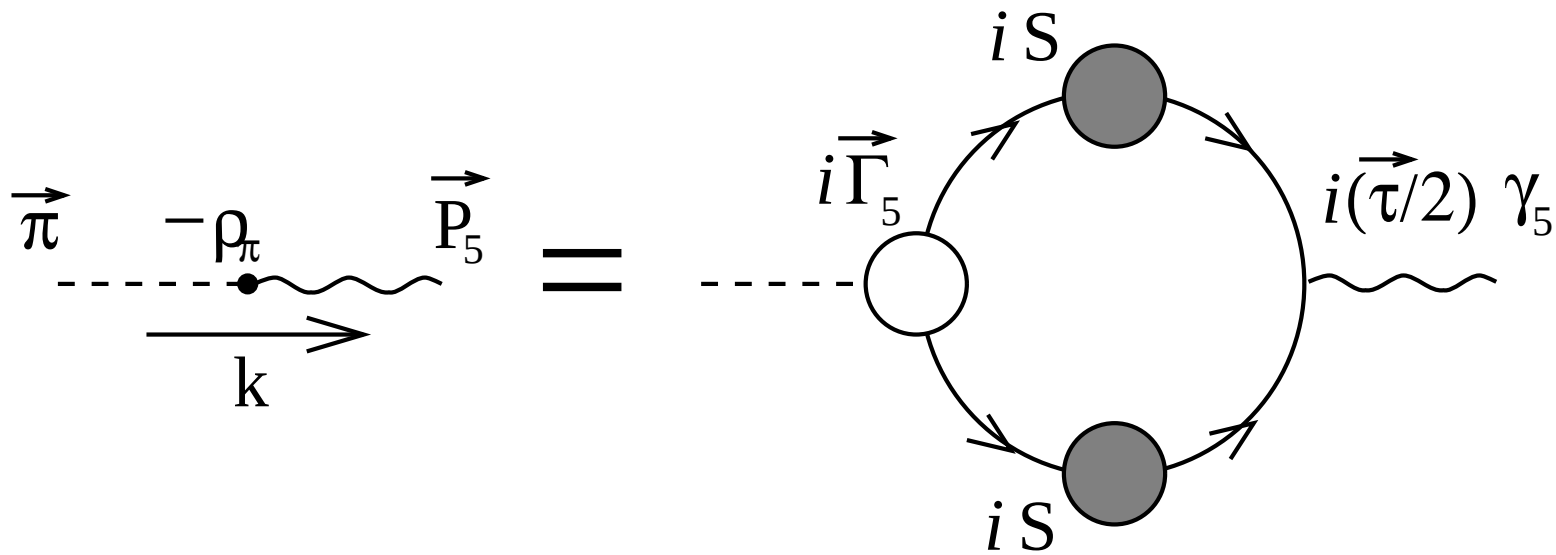
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = -\rho_\zeta^H \mathcal{M}_H$$

$$i\rho_\zeta^H = Z_4 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5 \mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-) \right\}$$

- Pseudoscalar projection of BS wave function at $x = 0$



Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ ...GMOR relation, a corollary

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

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  ALL pseudoscalar mesons except $\boldsymbol{\pi(140)}$ in chiral limit
- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

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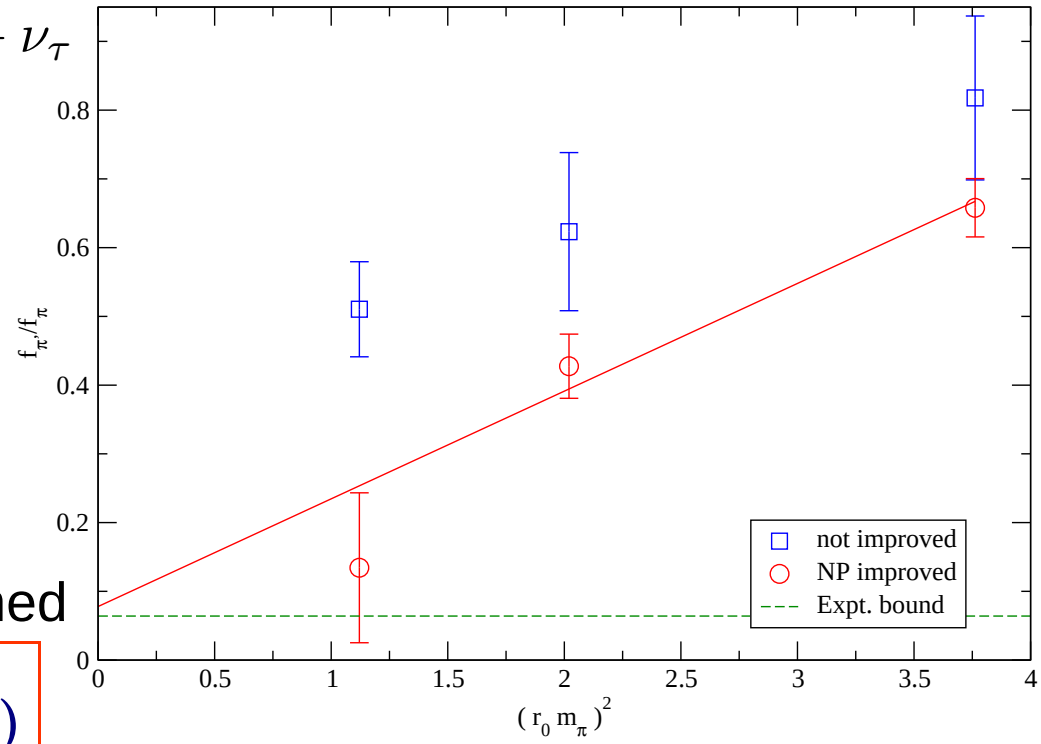
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$



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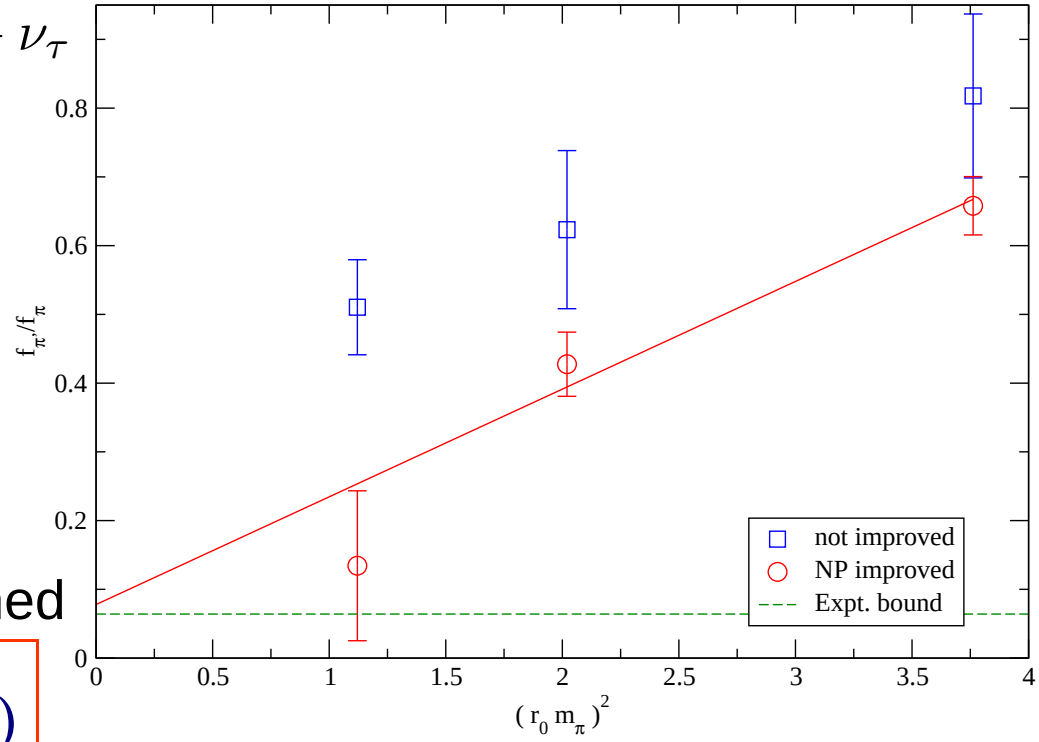

- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

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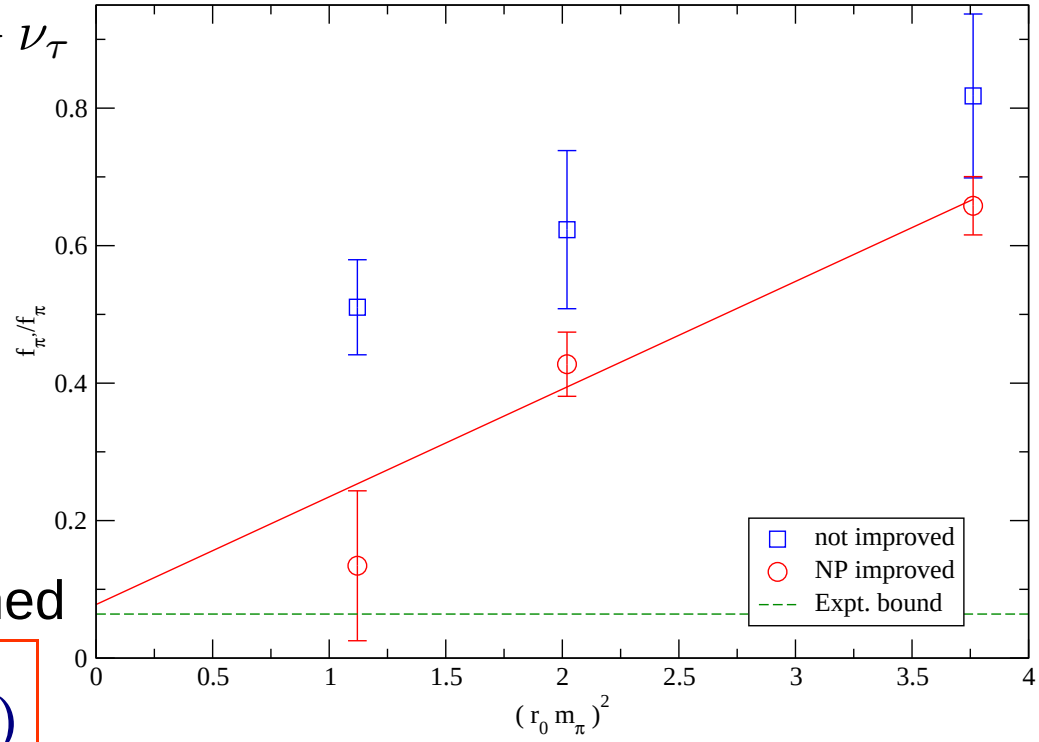

- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Pion … $J = 0$ but …

- Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.

Pion … $J = 0$ but …

- Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.

Pion ... $J = 0$ but ...

- Pseudoscalar meson Bethe-Salpeter amplitude

$$\chi_\pi(k;P) = \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P\mathcal{F}_{\pi_n}(k;P) \right.$$
$$\left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P) \right]$$

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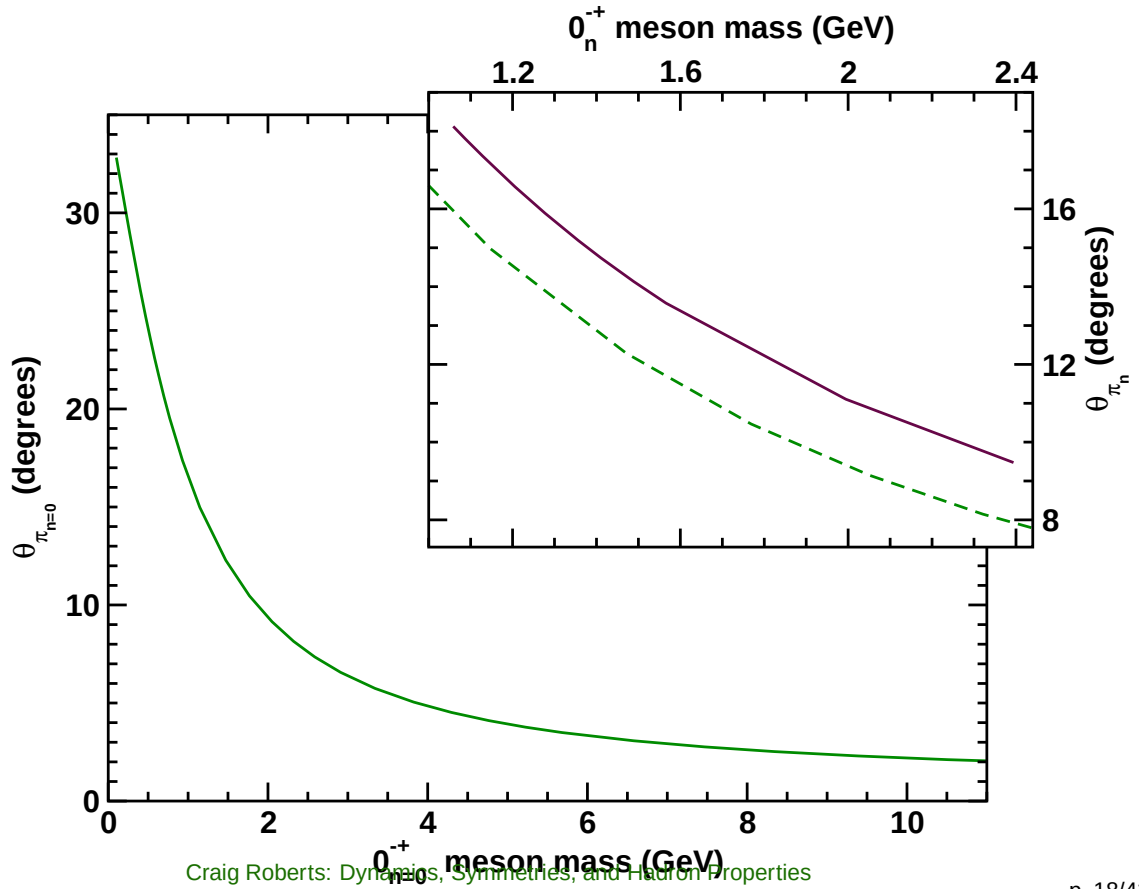

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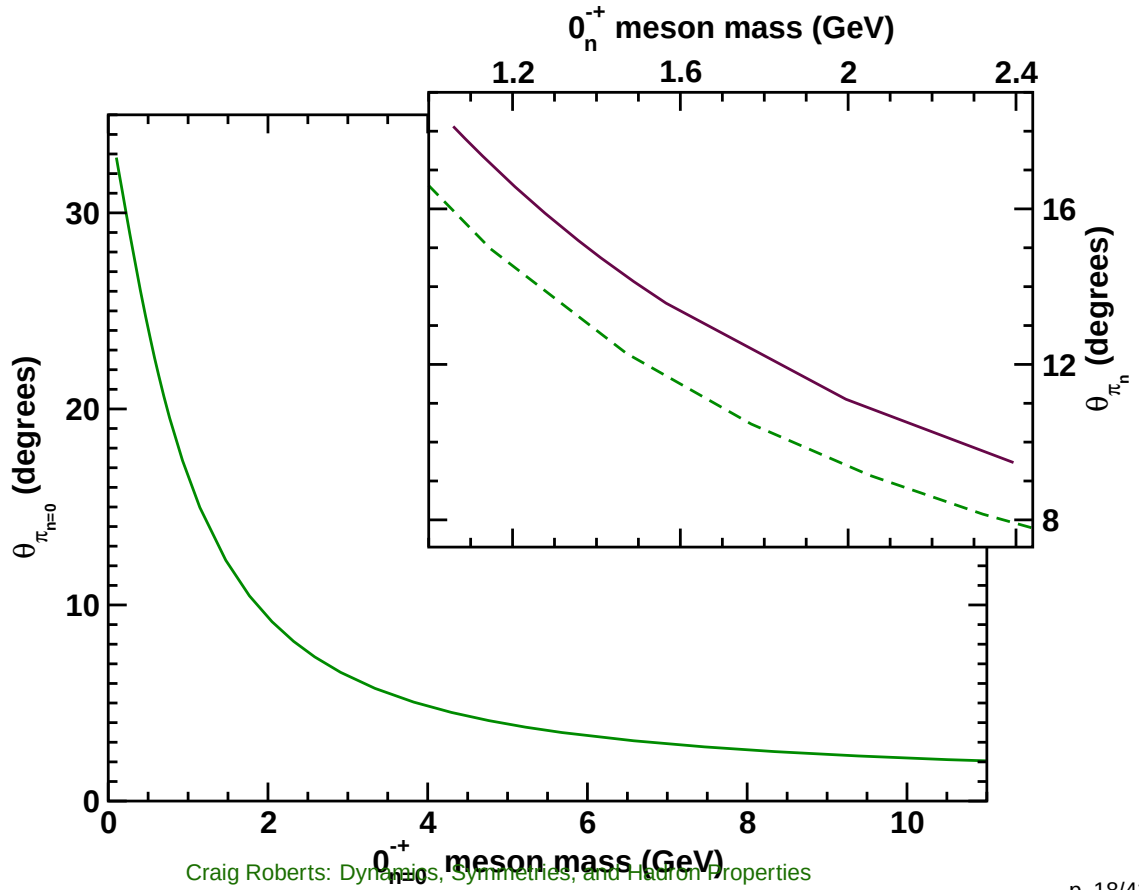
L is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.



Charge Neutral Pseudoscalar Mesons

$$
\begin{aligned}
P_\mu \Gamma^a_{5\mu}(k;P) &= \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\
&\quad -2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)
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- The final term in the second line expresses the non-Abelian axial anomaly.

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  … The topological charge density operator.

  (Trace is over colour indices & $F_{\mu\nu} = \frac{1}{2}\lambda^a F^a_{\mu\nu}$.)

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- Important that only $\mathcal{A}^{a=0}$ is nonzero.

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  … The topological charge density operator.
- NB. While $\mathcal{Q}(x)$ is gauge invariant, the associated Chern-Simons current, K_μ, is not $\Rightarrow$ in QCD *no physical* boson can couple to K_μ and hence *no physical* states can contribute to resolution of U$_A$(1) problem.

Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

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Bhagwat, Chang, Liu, Roberts, Tandy
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Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting ... otherwise all pseudoscalar mesons are Goldstone Modes!

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- Anomaly term has structure

$$\mathcal{A}^0(k;P) = \mathcal{F}^0\gamma_5\left[i\mathcal{E}_{\mathcal{A}}(k;P) + \gamma\cdot P\mathcal{F}_{\mathcal{A}}(k;P)\right.$$
$$\left.+\gamma\cdot k k\cdot P\mathcal{G}_{\mathcal{A}}(k;P) + \sigma_{\mu\nu}k_\mu P_\nu\mathcal{H}_{\mathcal{A}}(k;P)\right]$$

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- AVWTI gives generalised Goldberger-Treiman relations

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2 f^0_{\eta'} E_{BS}(k;0) &= 2B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0), \\
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A_0, B_0 characterise gap equation's chiral limit solution.

- Follows that $\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)$ is necessary and sufficient condition for absence of massless η' bound-state.

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Charge Neutral Pseudoscalar Mesons

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  Discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
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- Further highlighted . . . proved

$$\begin{aligned}\langle \bar{q}q\rangle_\zeta^0 &= -\lim_{\Lambda\to\infty} Z_4(\zeta^2,\Lambda^2)\,\mathrm{tr}_{\mathrm{CD}}\int_q^\Lambda S^0(q,\zeta)\\ &= N_f\int d^4x\,\langle \bar{q}(x)i\gamma_5 q(x)\mathcal{Q}(0)\rangle^0\,.\end{aligned}$$

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Charge Neutral Pseudoscalar Mesons

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- Employed in an analysis of pseudoscalar- and vector-meson bound-states

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- Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts
  - η–η' mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)
  - π^0–η angles of $\sim 1.2^\circ$ (Expt. $p\,d \to {}^3$He π^0: $0.6^\circ \pm 0.3^\circ$)
  - Strong neutron-proton mass difference . . .
    $\lesssim 75\,\%$ current-quark mass-difference

New Challenges

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- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.

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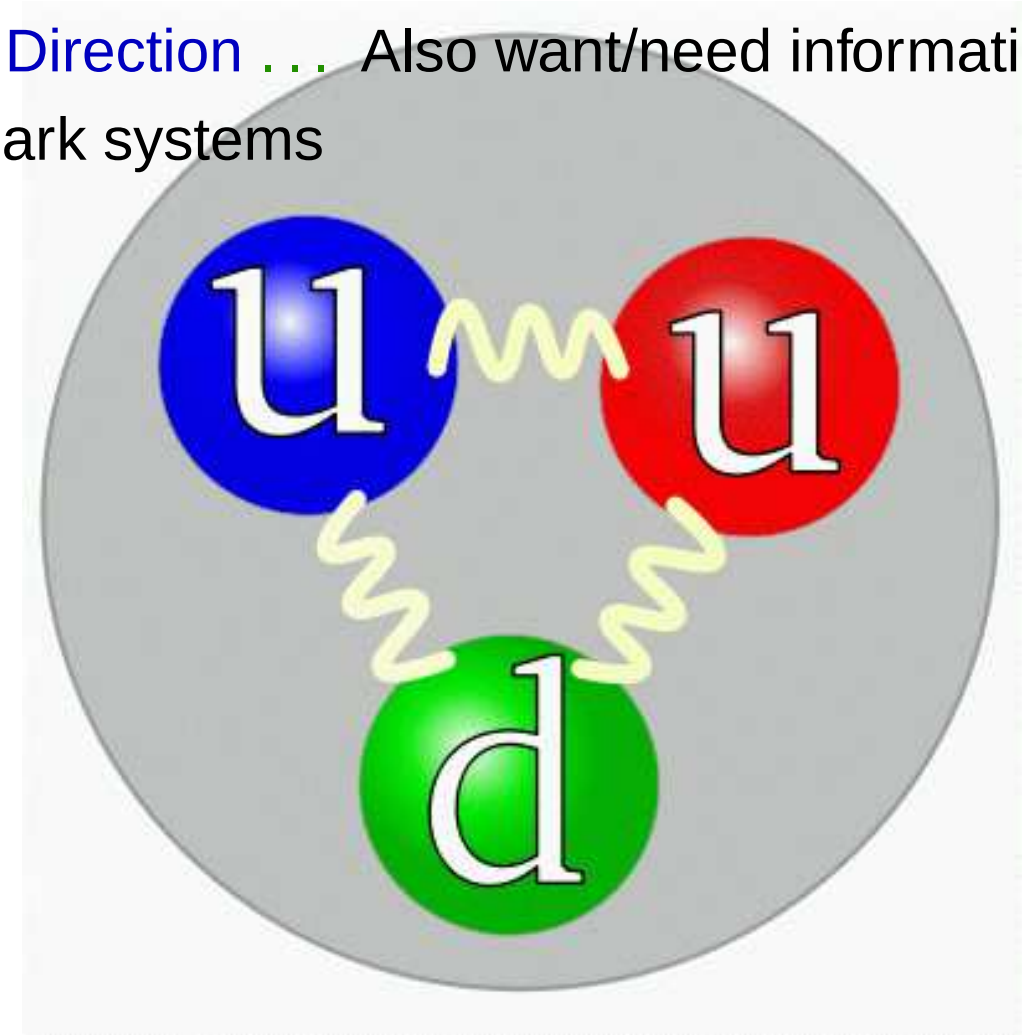
- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

New Challenges

- Another Direction . . . Also want/need information about three-quark systems



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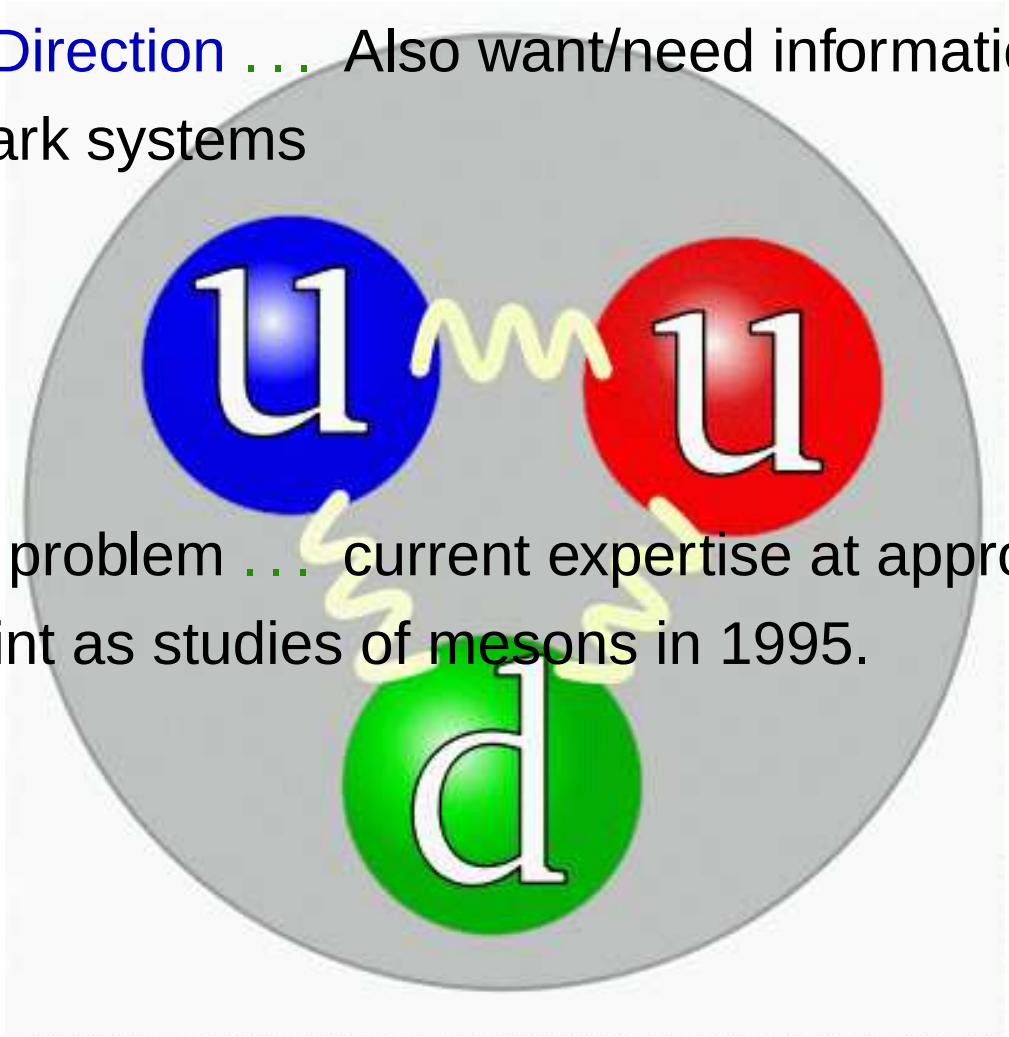

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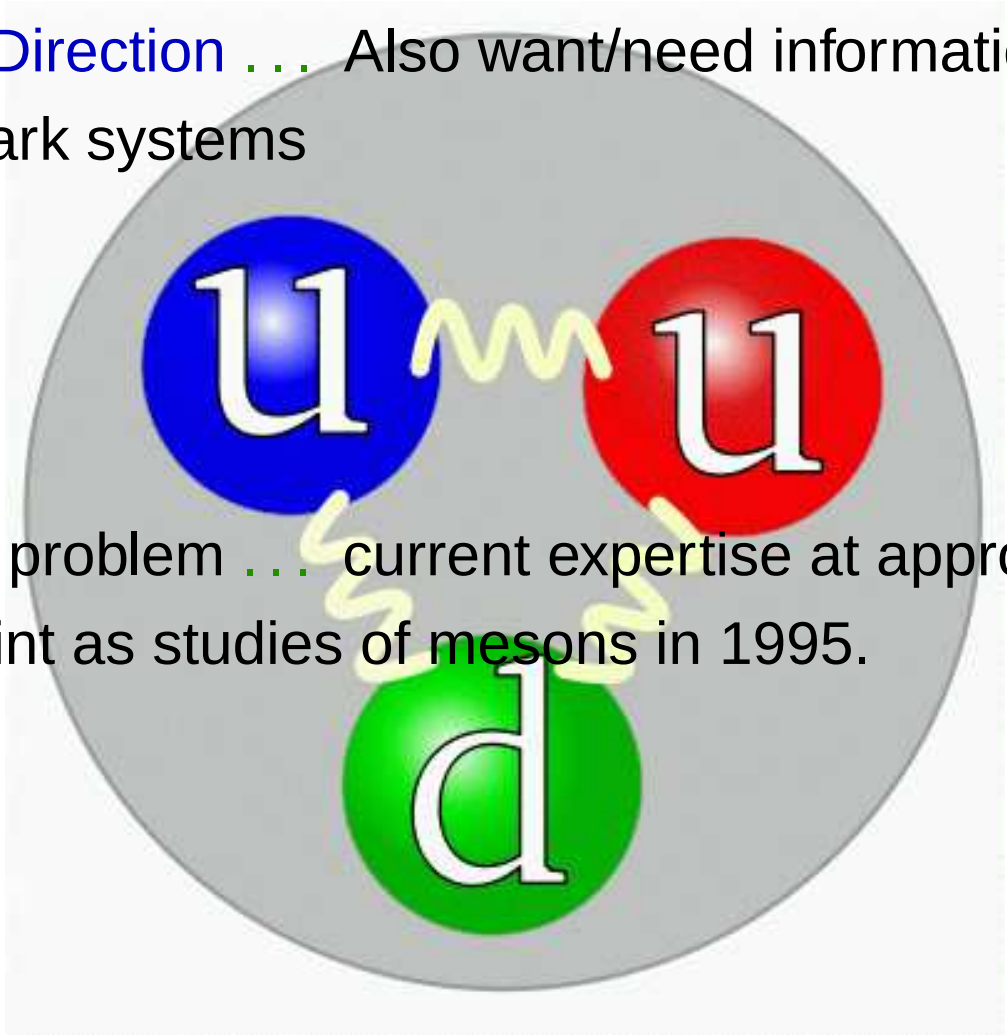

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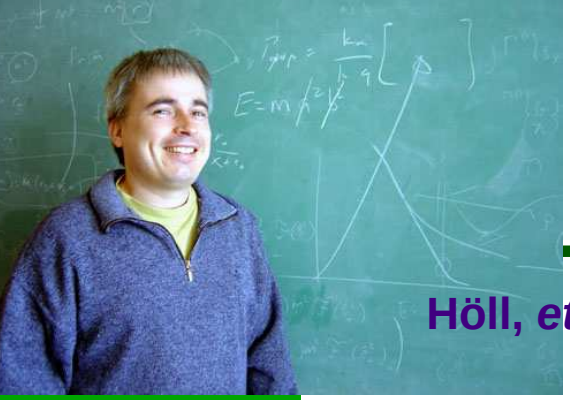
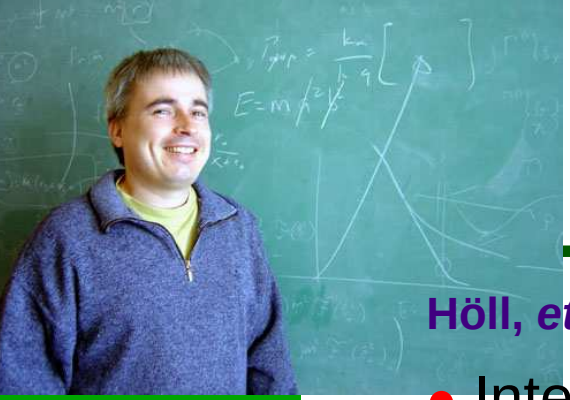
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.



Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033



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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

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- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

Faddeev equation

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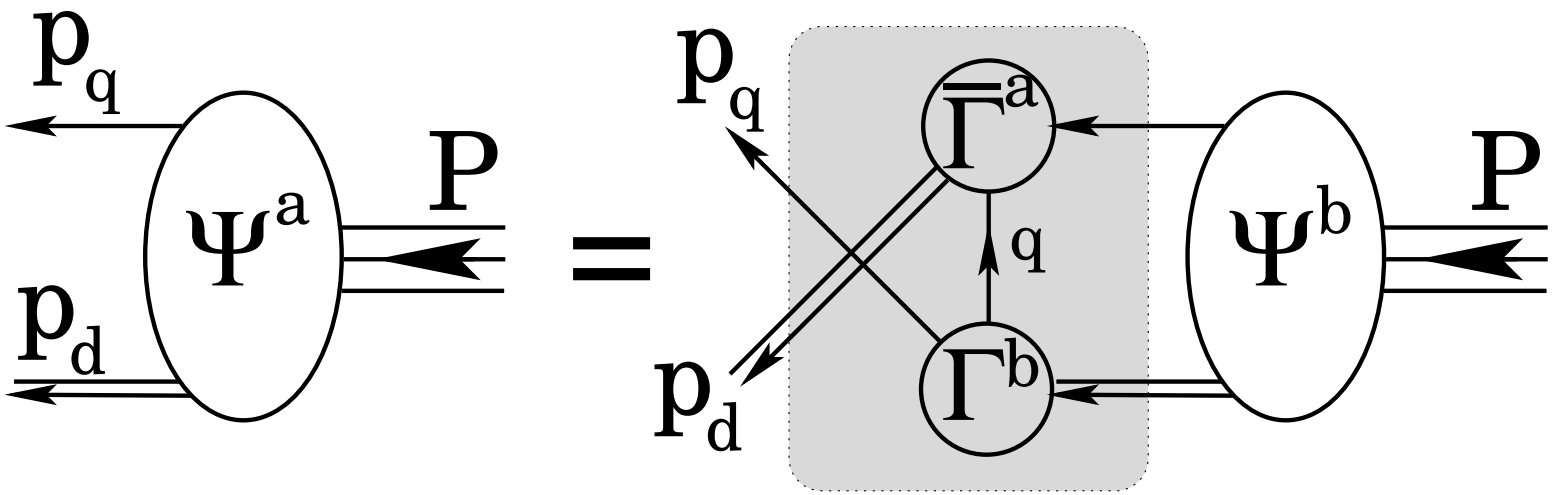

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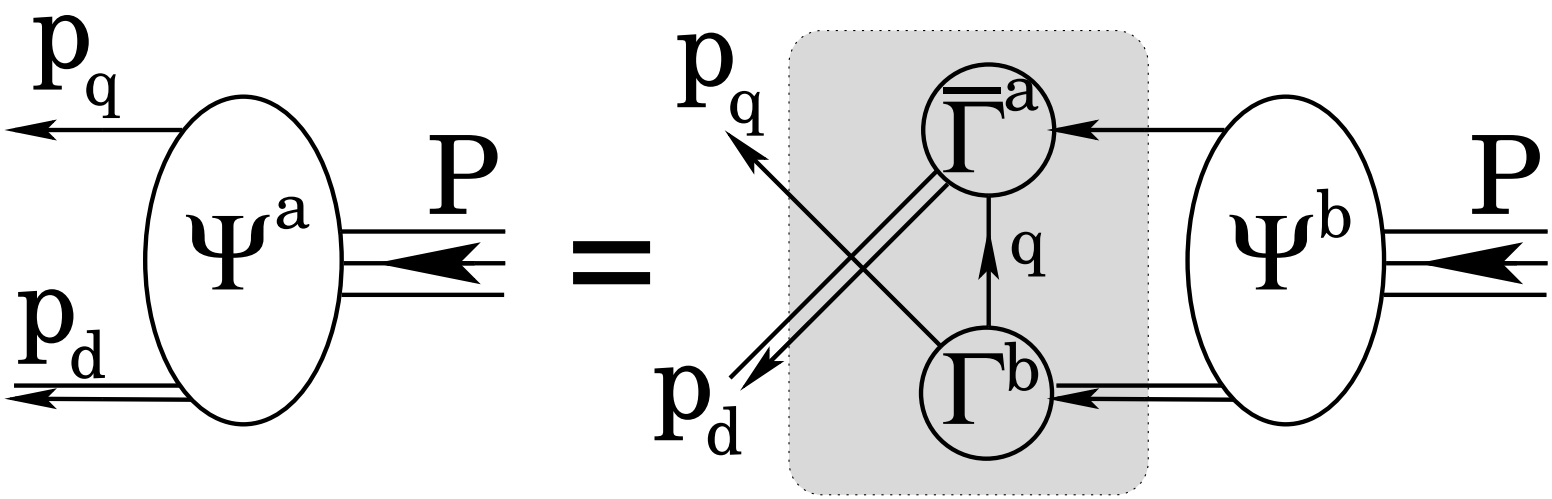
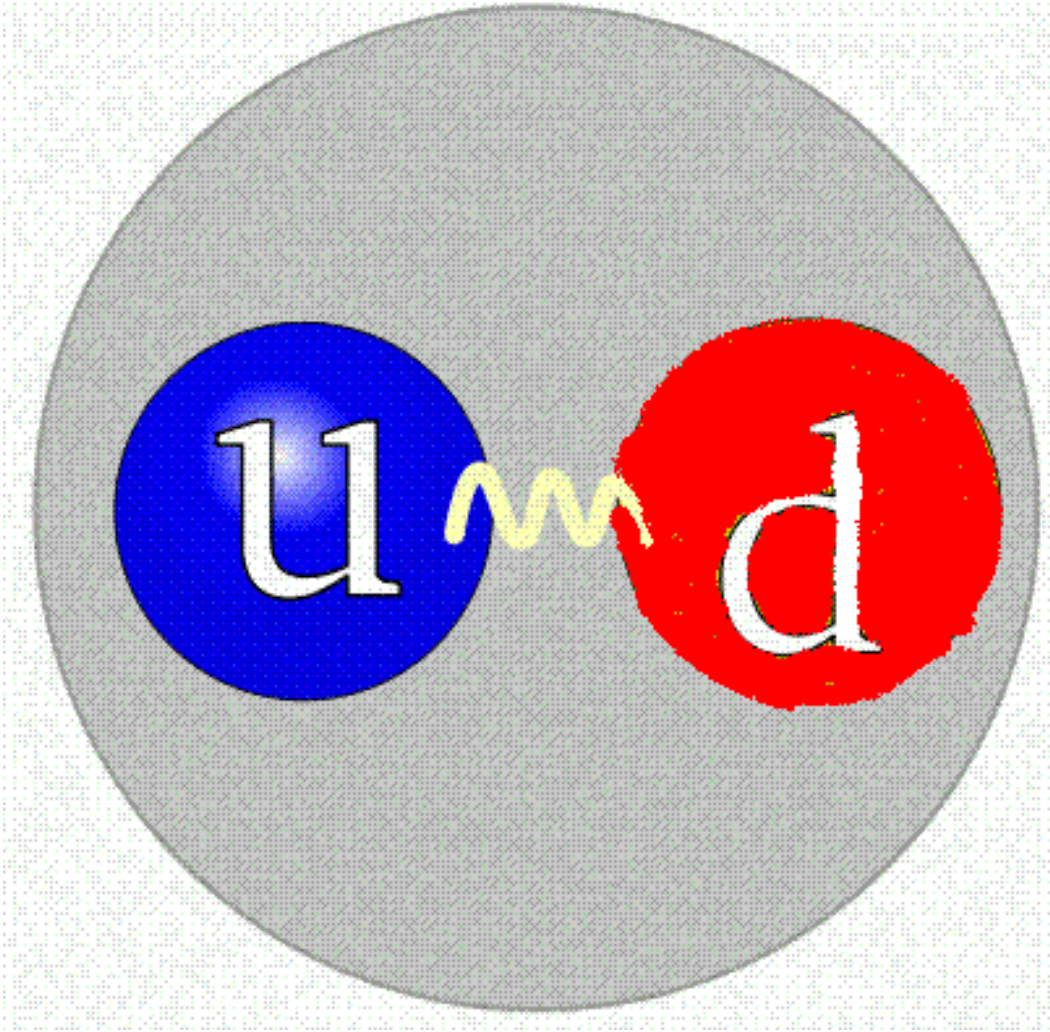


- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Diquark correlations



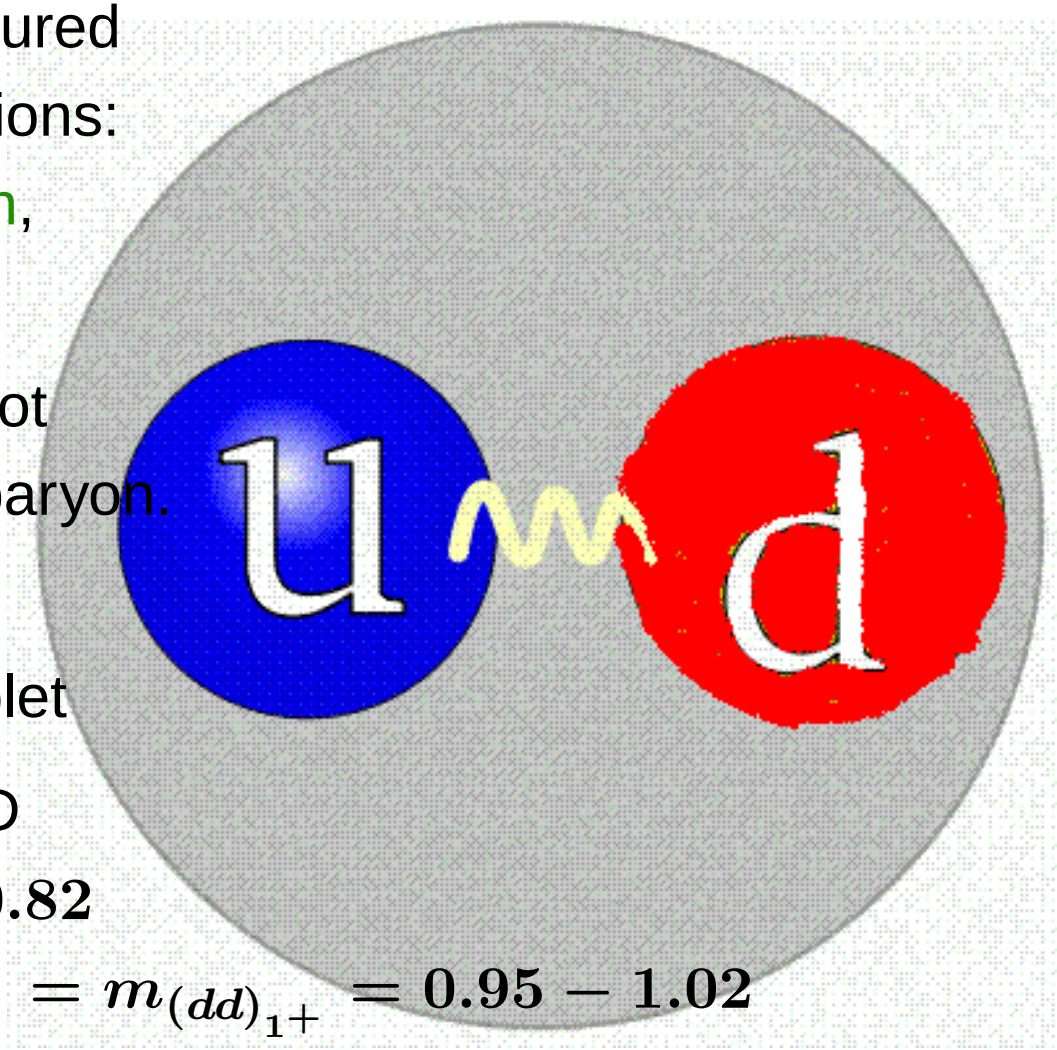
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined . . . Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

Results: Nucleon and Δ Masses

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Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

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- Axial-vector diquark provides significant attraction

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- Constructive Interference: 1^{++}-diquark $+ \partial_\mu \pi$

Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

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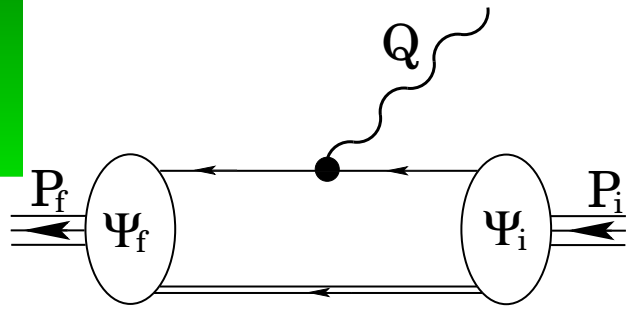
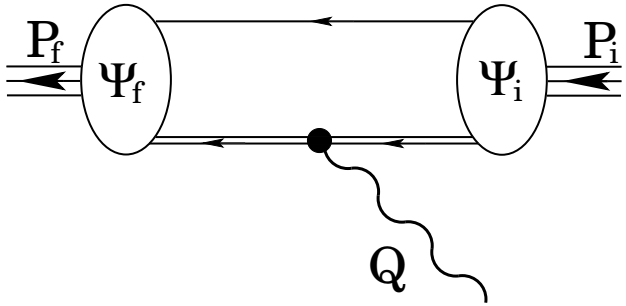
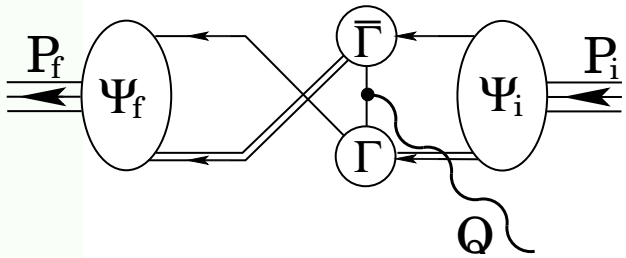
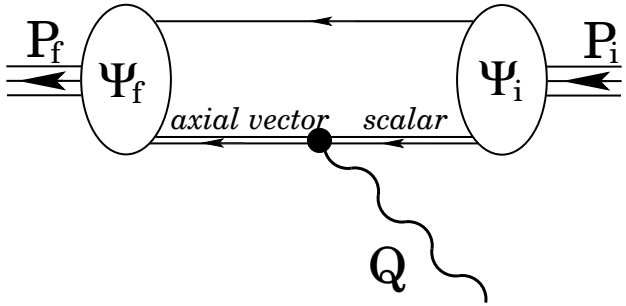
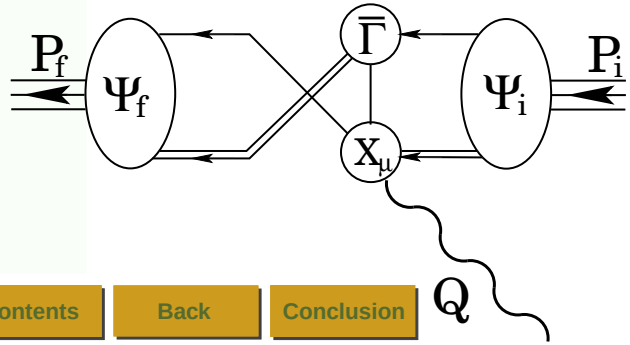
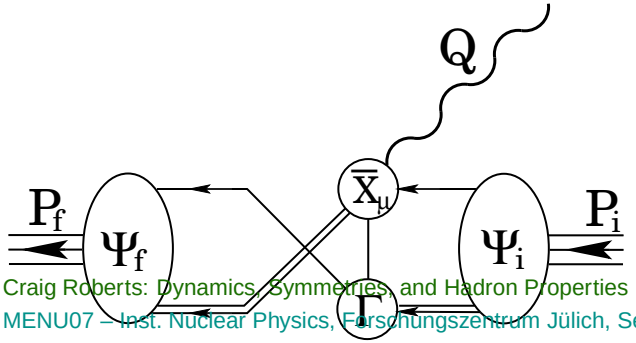




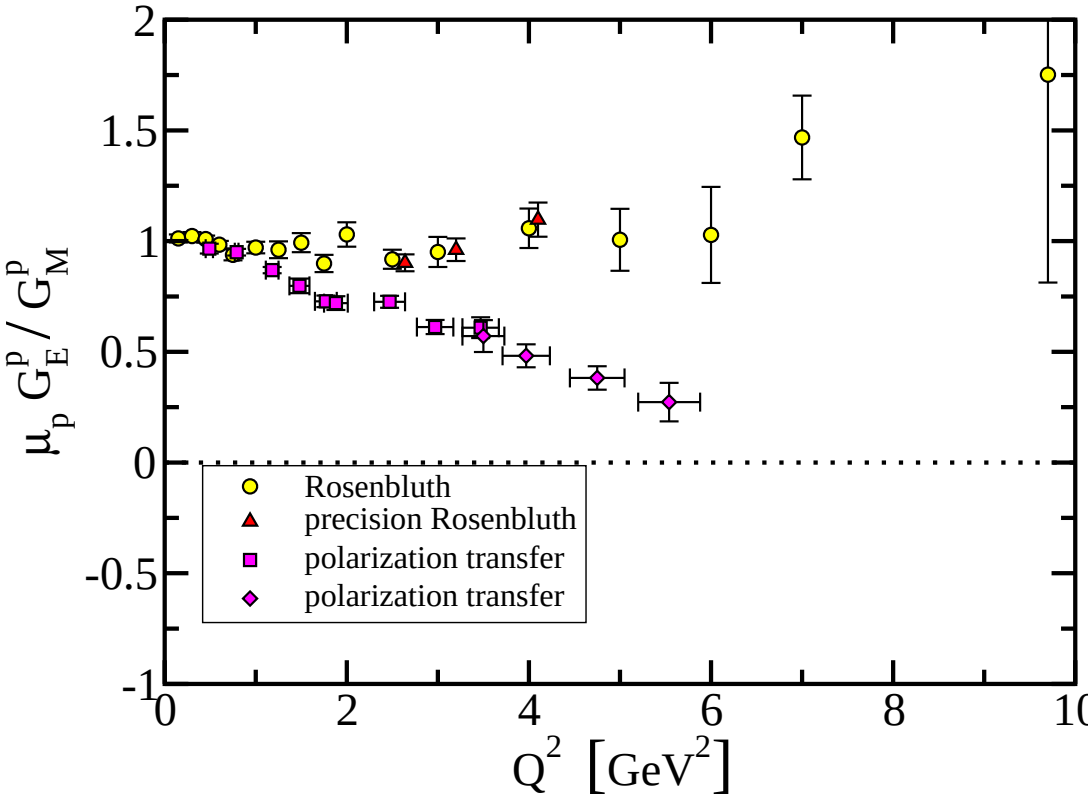








Form Factor Ratio: GE/GM



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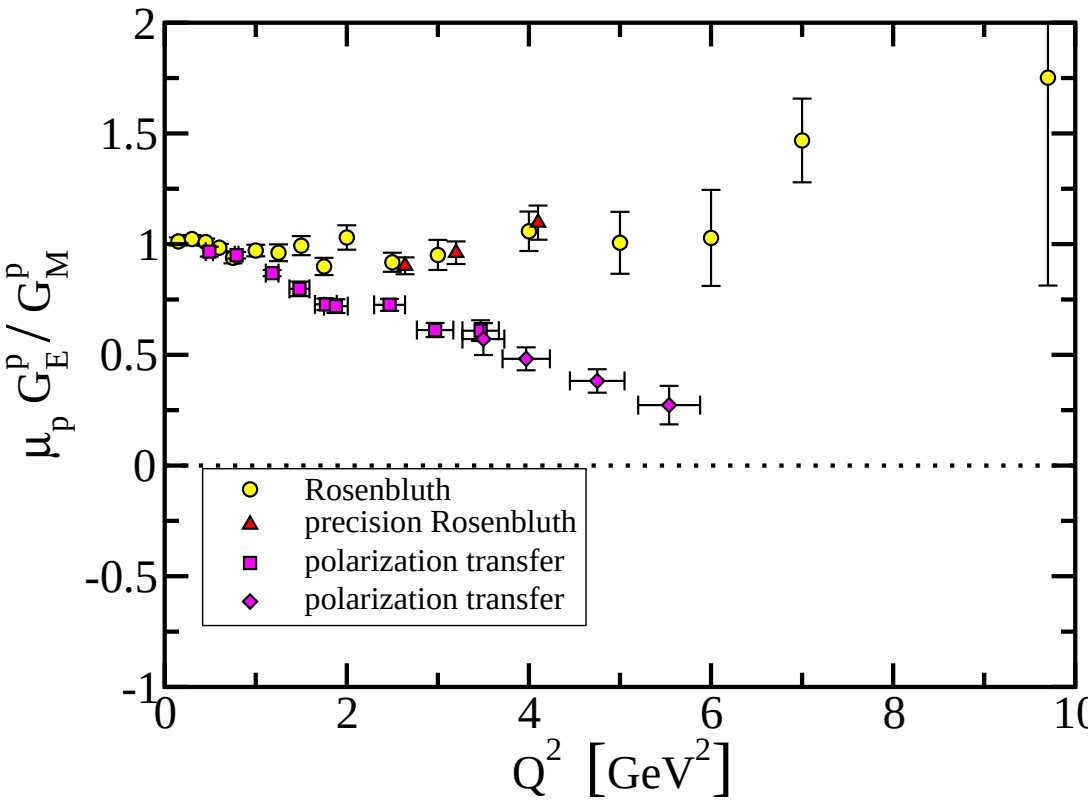

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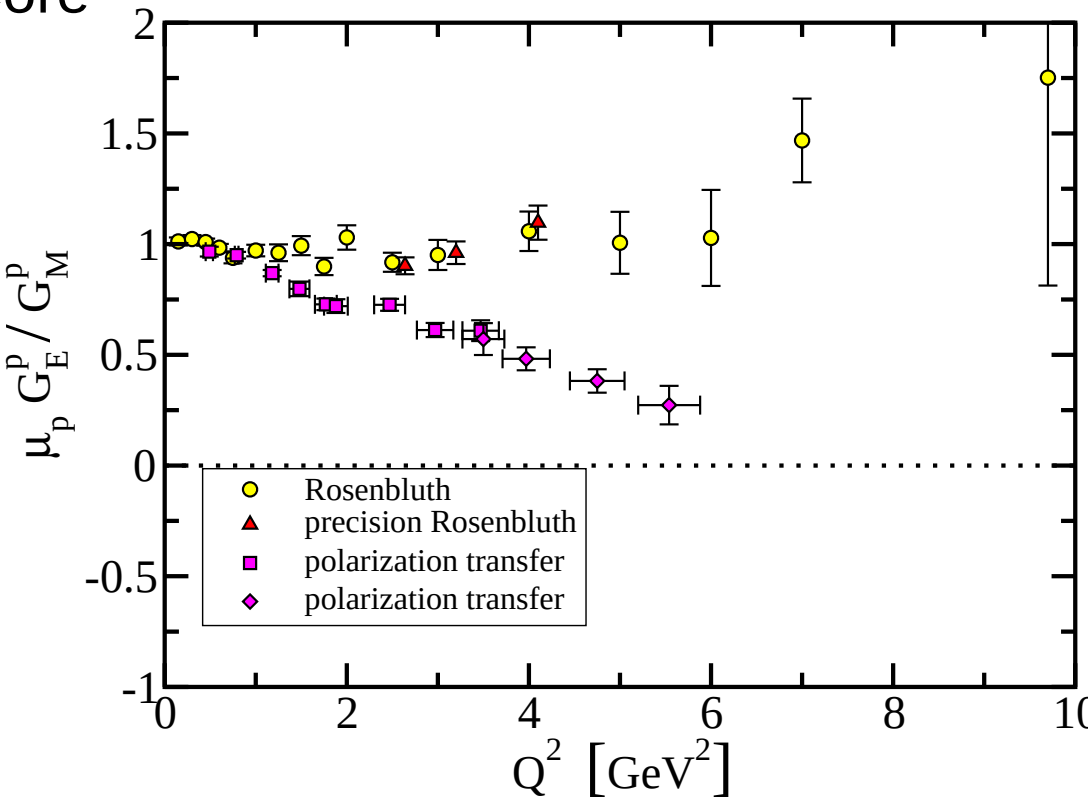

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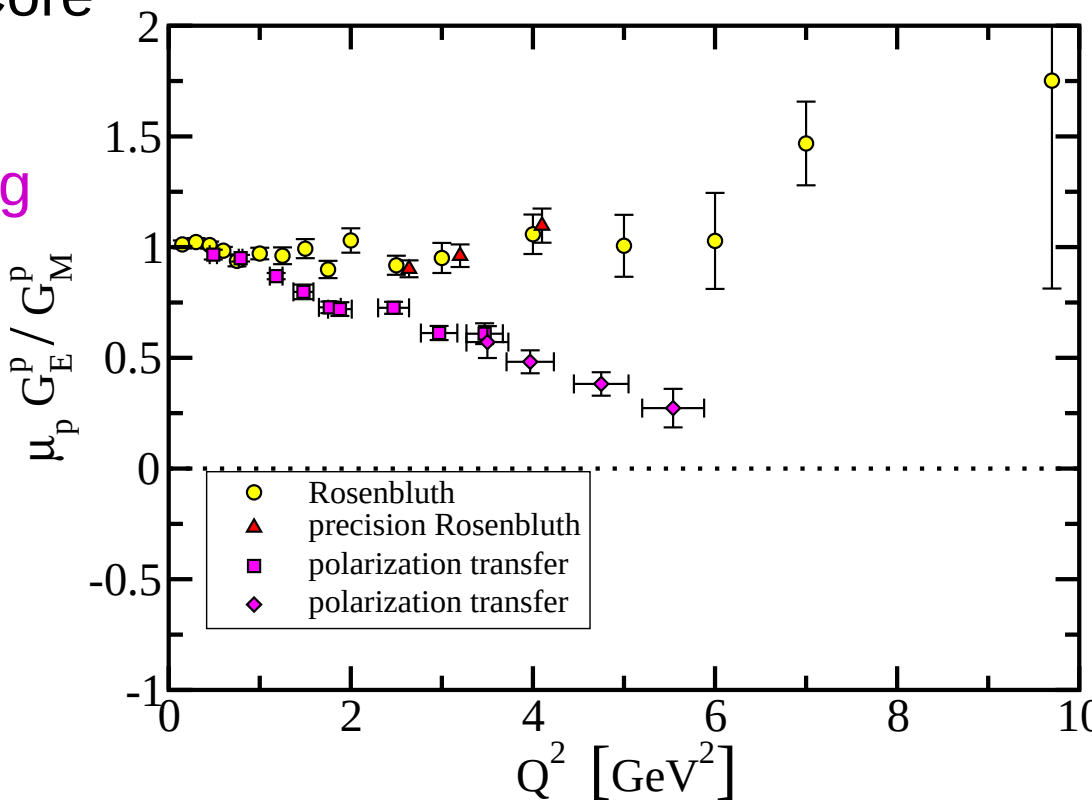

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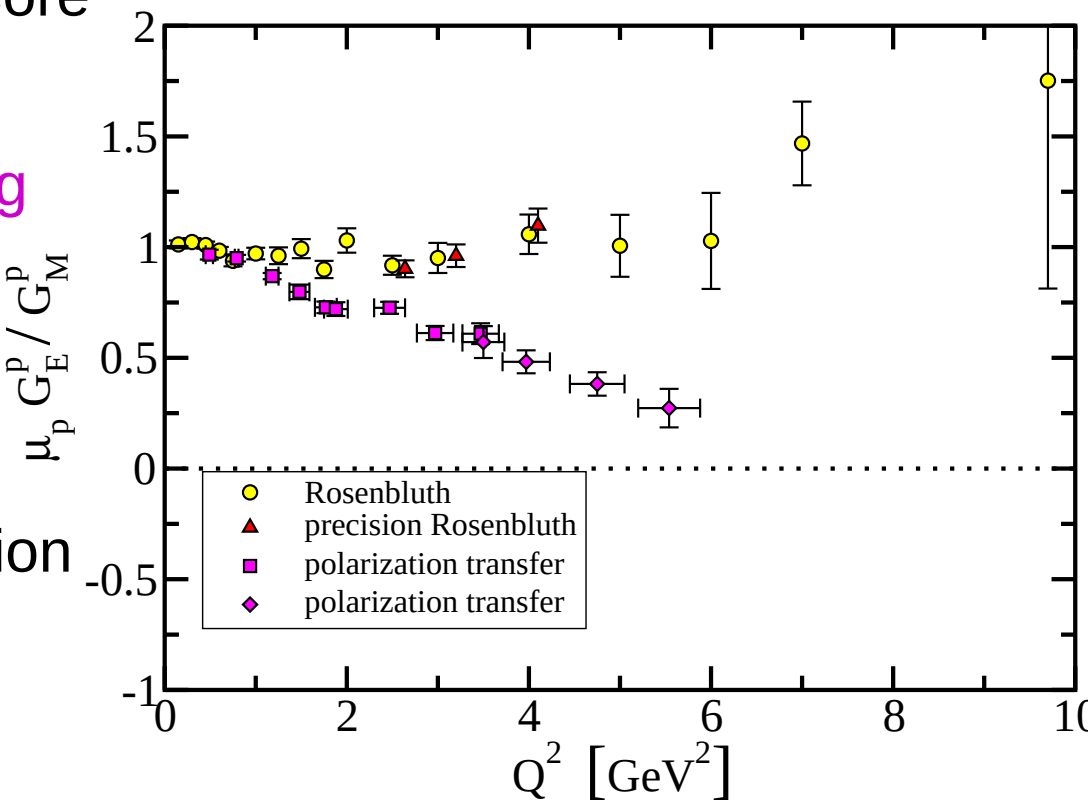

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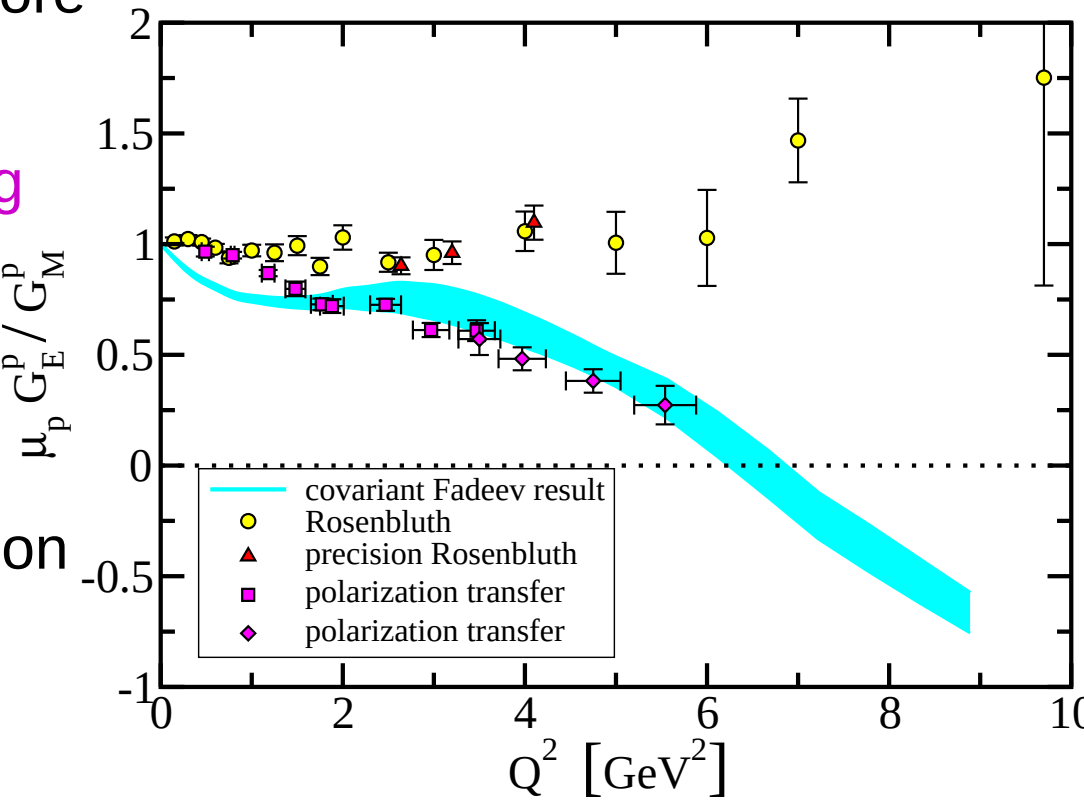

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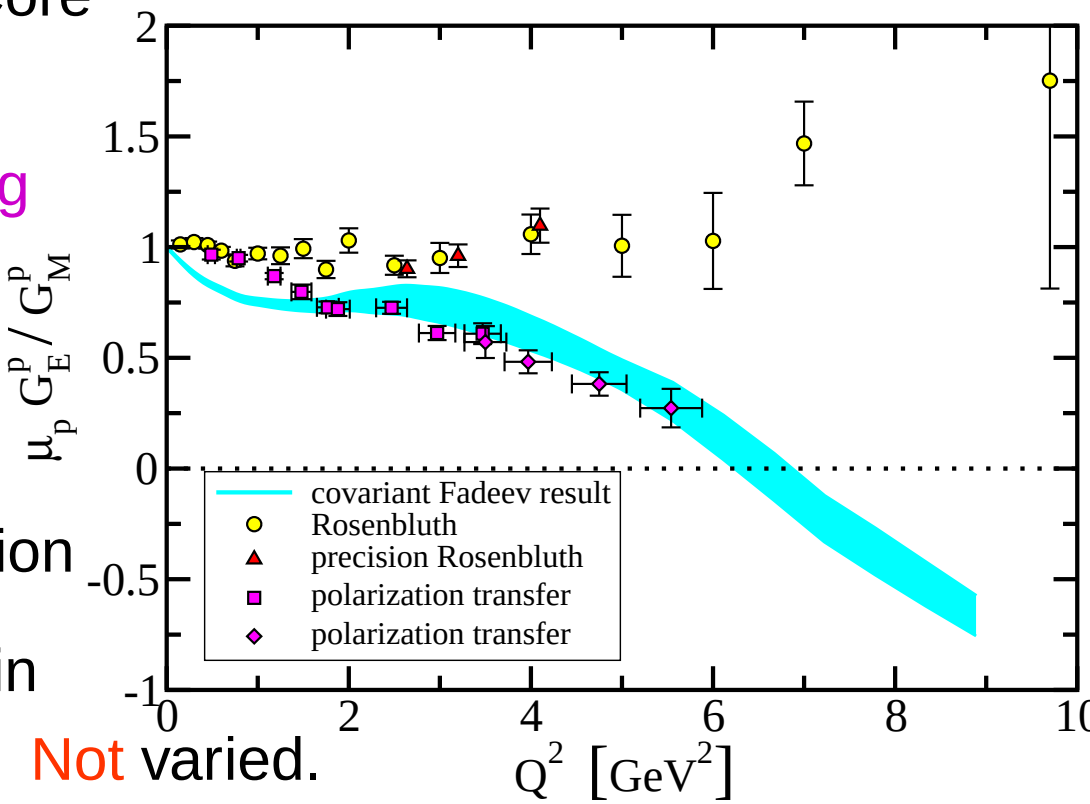

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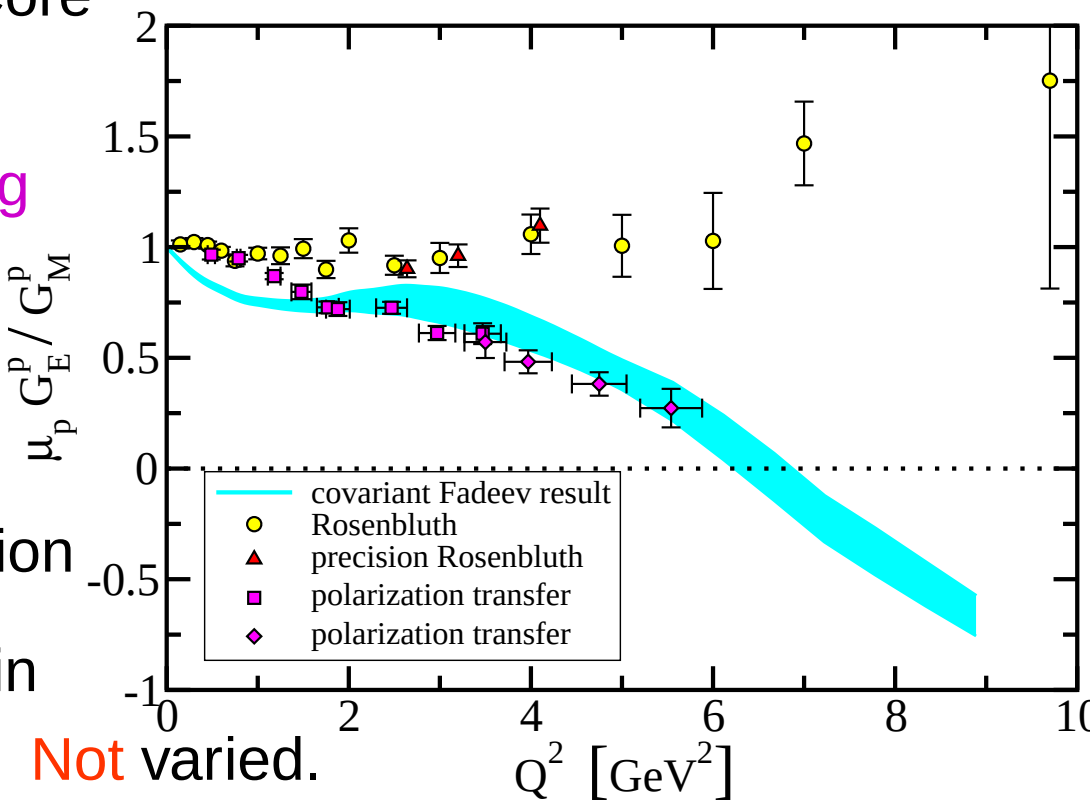

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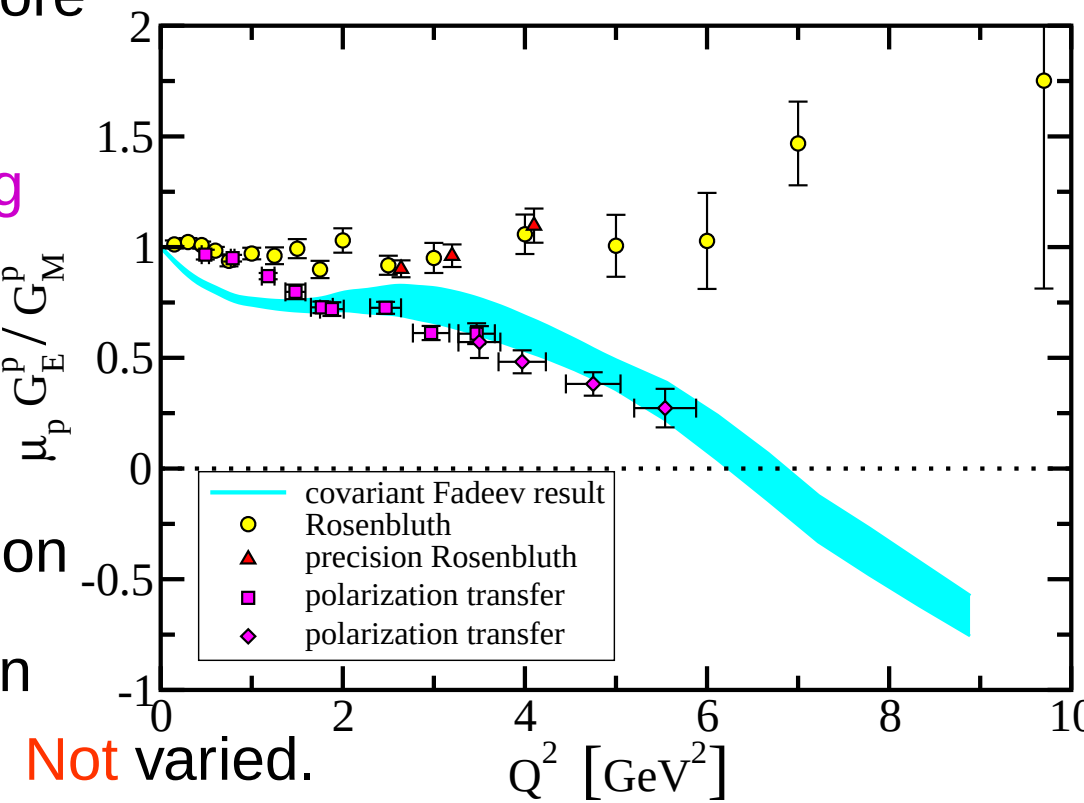

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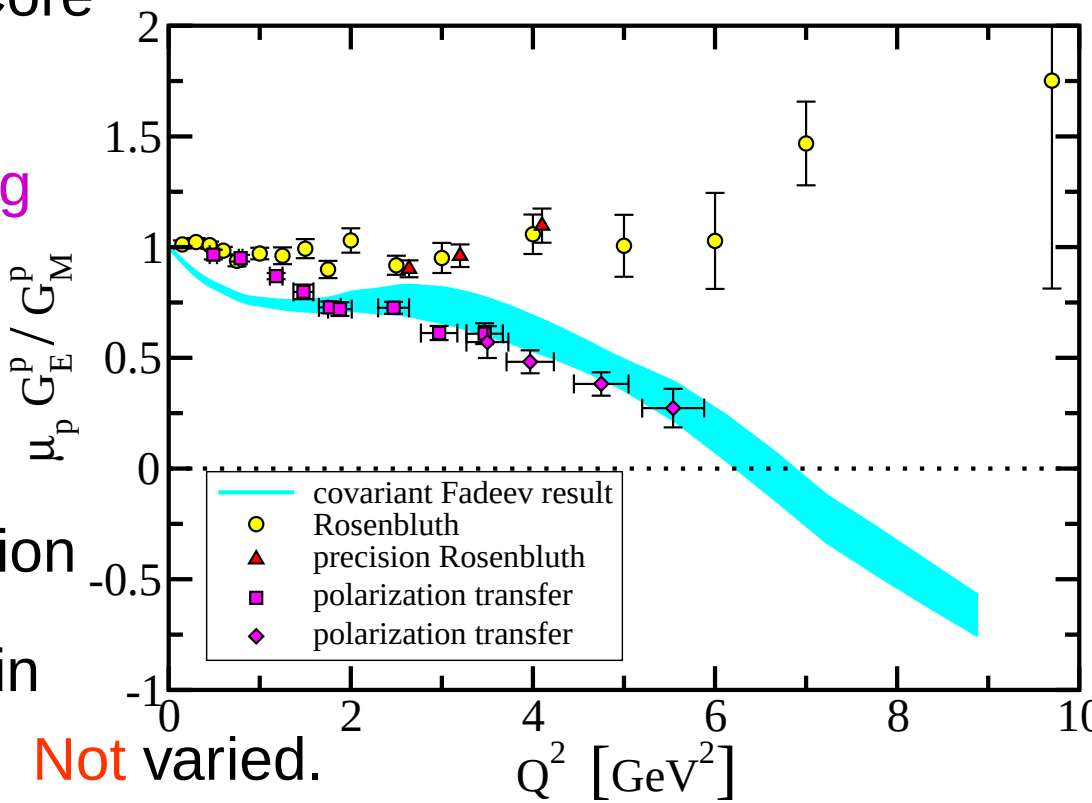

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Quark Distribution Functions

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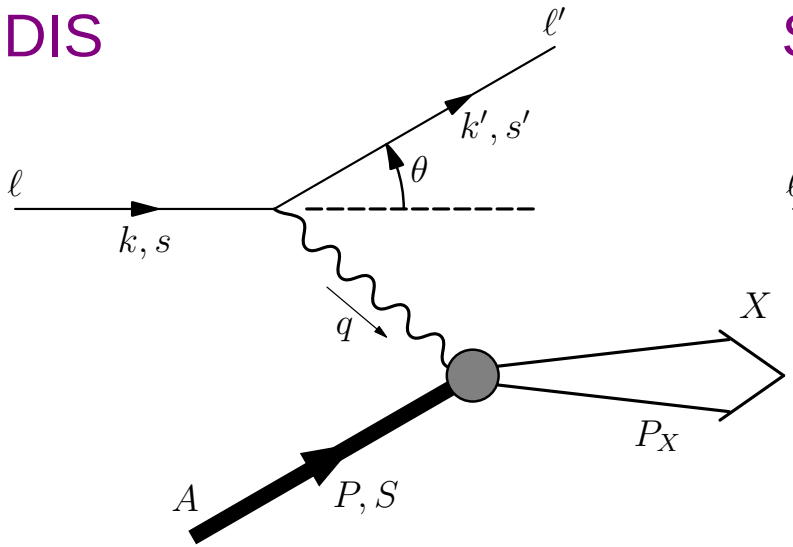
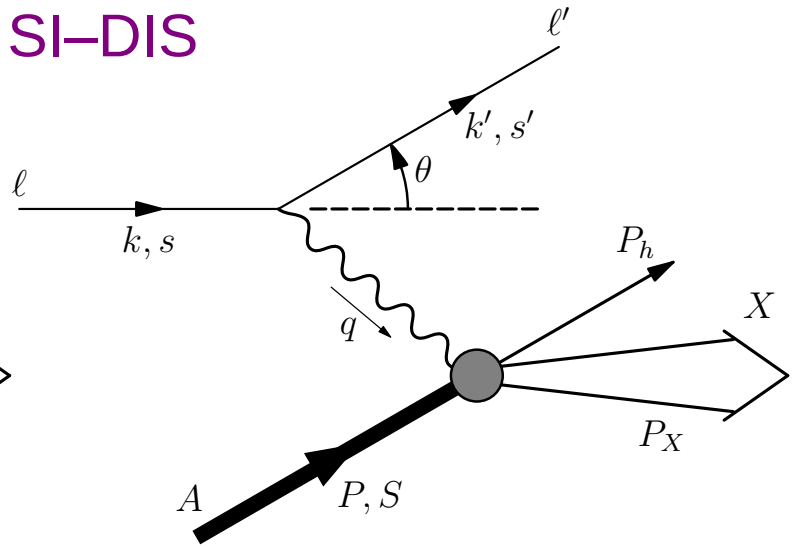
DIS



SI–DIS



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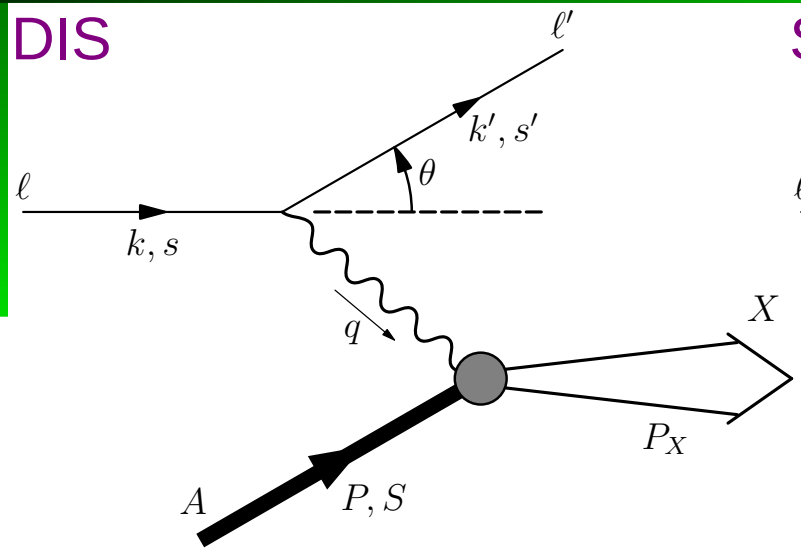

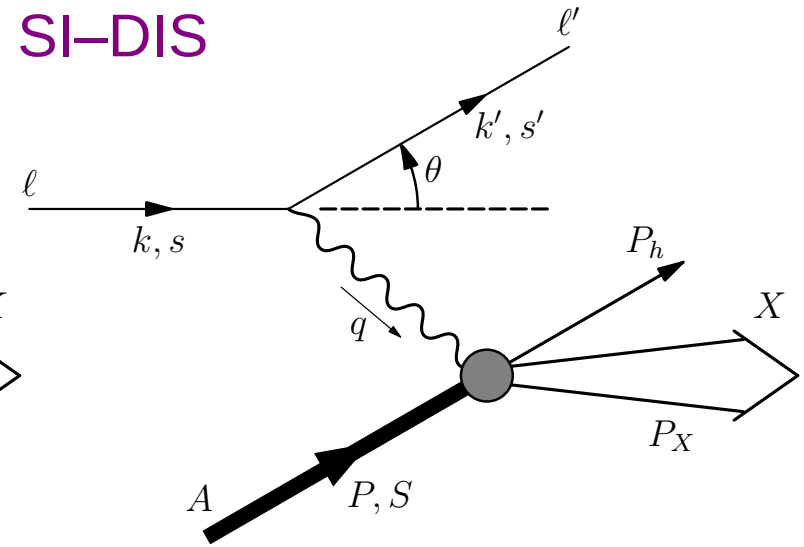

- Three twist-2 parton distributions $(k_\perp = 0)$:
  - Spin-Independent: $q(x)$
  - Helicity: $\Delta q(x)$
  - Transversity: $\Delta_T q(x)$
- All distributions have probability interpretation.
- By definition, contain essentially non-perturbative information about a given process.

Definition and Sum Rules

Definition and Sum Rules

- Light-cone Fourier transforms :

$$\Delta_T q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\xi^-} \langle p,s|\overline{\psi}_q(0)\gamma^+\gamma^1\gamma_5\psi_q(\xi^-)|p,s\rangle_c$$

$$q(x) = \langle\gamma^+\rangle, \qquad \Delta q(x) = \langle\gamma^+\gamma_5\rangle$$

- Related to the nucleon axial & tensor charges via

$$g_A = \int dx[\Delta u(x) - \Delta d(x)], \quad g_T = \int dx[\Delta_T u(x) - \Delta_T d(x)],$$

- Must satisfy: positivity constraints and Soffer bound

$$\Delta q(x), \Delta_T q(x) \leqslant q(x), \quad q(x) + \Delta q(x) \geqslant 2\,|\Delta_T q(x)|$$

Ian Cloët

JLab, now ANL

Ian Cloët

JLab, now ANL

Once more on the one that got away.

Cloët, Bentz, Thomas

arXiv:0708.3246 [hep-ph]

Model predictions

Cloët, Bentz, Thomas

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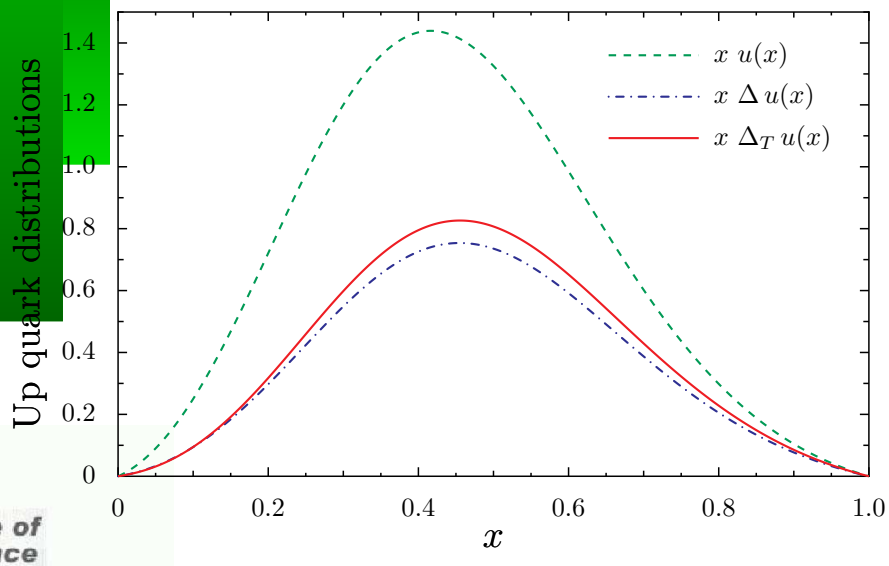
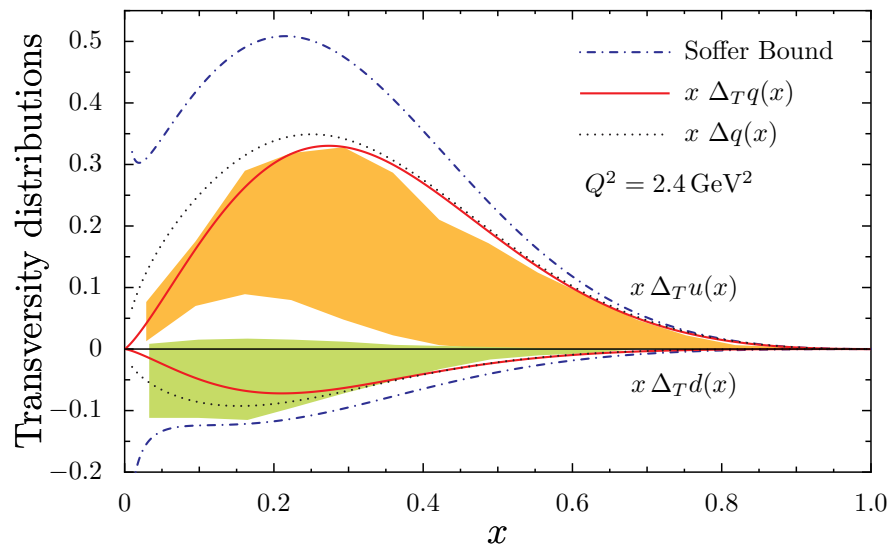
Model predictions

- Simplified Faddeev equation





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Cloët, Bentz, Thomas

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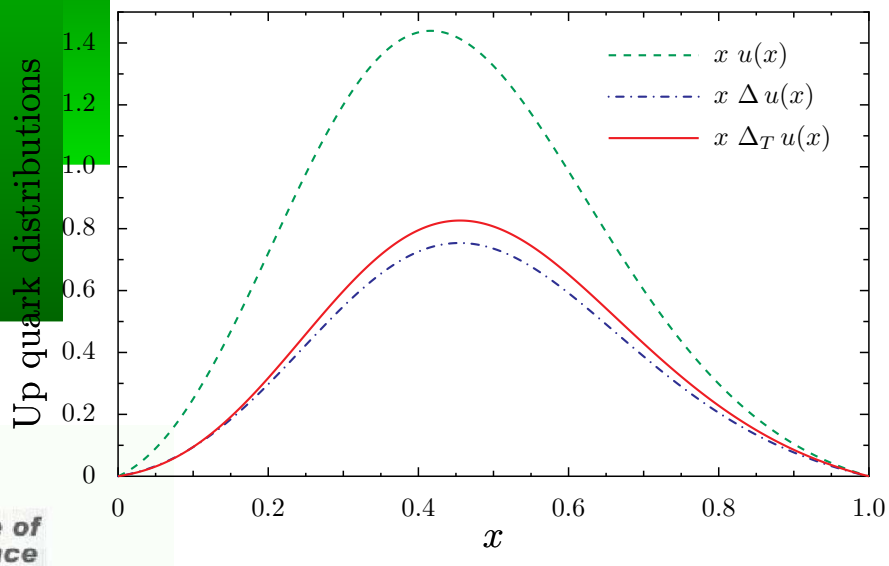

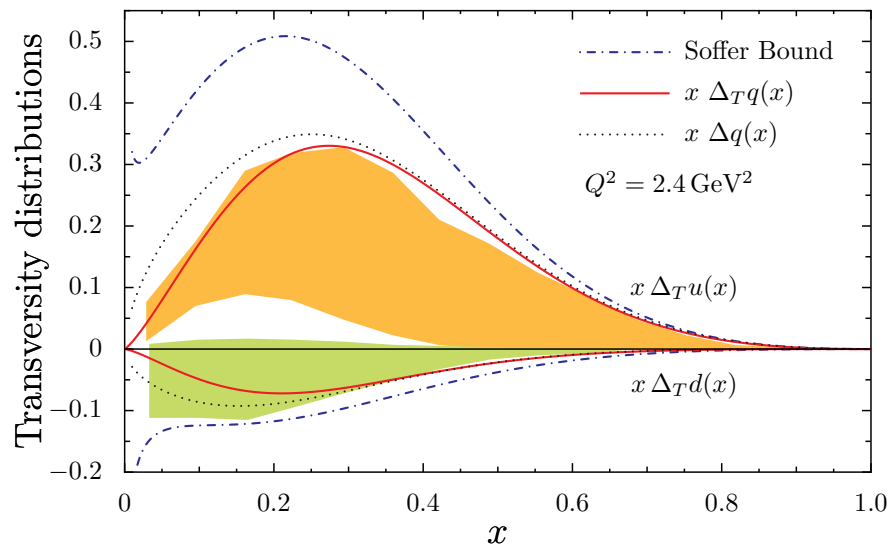

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- Moments at $Q^2 = 0.16\,\mathrm{GeV}^2$:

$$\Delta u = 0.97, \quad \Delta d = -0.30 \quad \Longrightarrow \quad g_A = 1.267$$

$$\Delta_T u = 1.04, \; \Delta_T d = -0.24 \quad \Longrightarrow \quad g_T = 1.28$$

Model constraint

Cloët, Bentz, Thomas

arXiv:0708.3246 [hep-ph]

Model predictions

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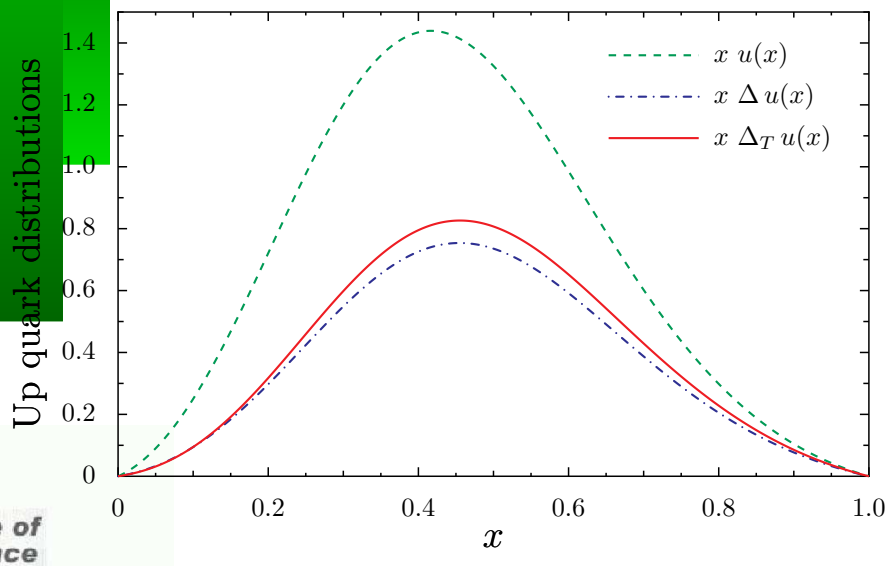

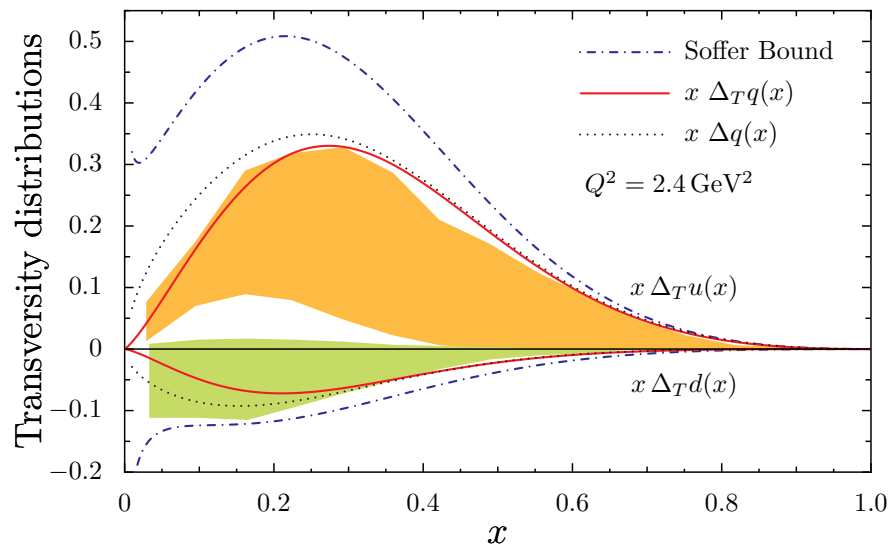

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$$\Delta_T u = 1.04, \; \Delta_T d = -0.24 \quad \Longrightarrow \quad g_T = 1.28$$

- $\Delta q(x) \sim \Delta_T q(x)$ in valence region for $Q^2 \lesssim 10\,\mathrm{GeV}^2$

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  - Observables can be used to explore model realisations
- DSEs . . . contemporary tool that describes and explains these phenomena, and connects them with prediction of observables

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4. Dyson-Schwinger Equations
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9. Hadrons
10. Bethe-Salpeter Kernel
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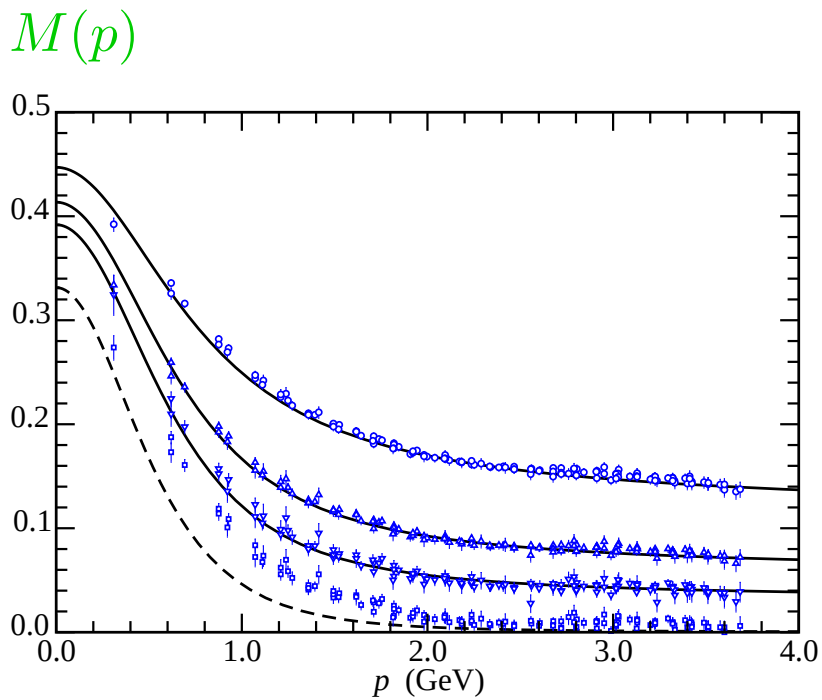
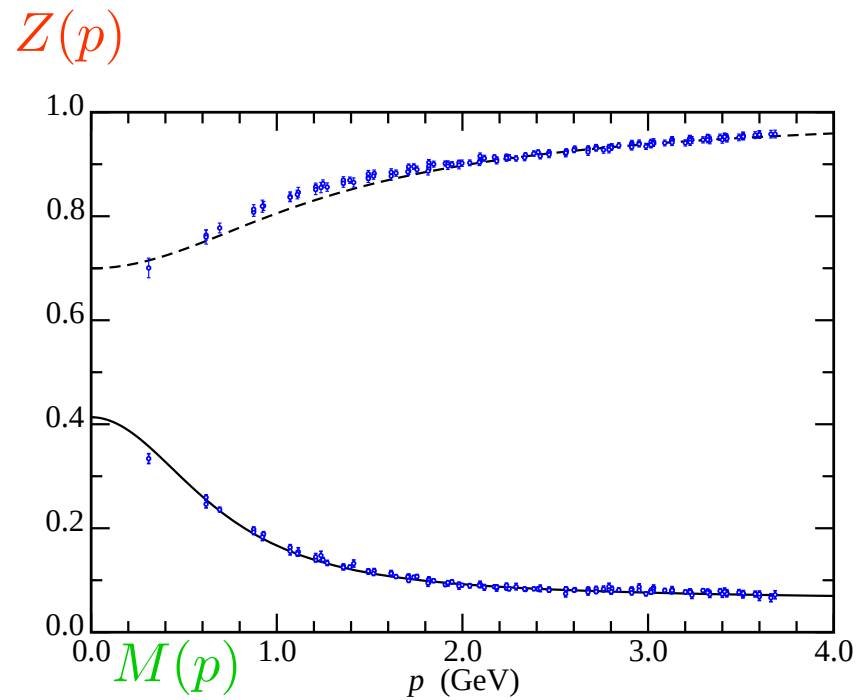
27. σ-term
28. Angular Momentum
29. Neutron Form Factors
30. Contemporary Reviews

Quenched-QCD Dressed-Quark Propagator





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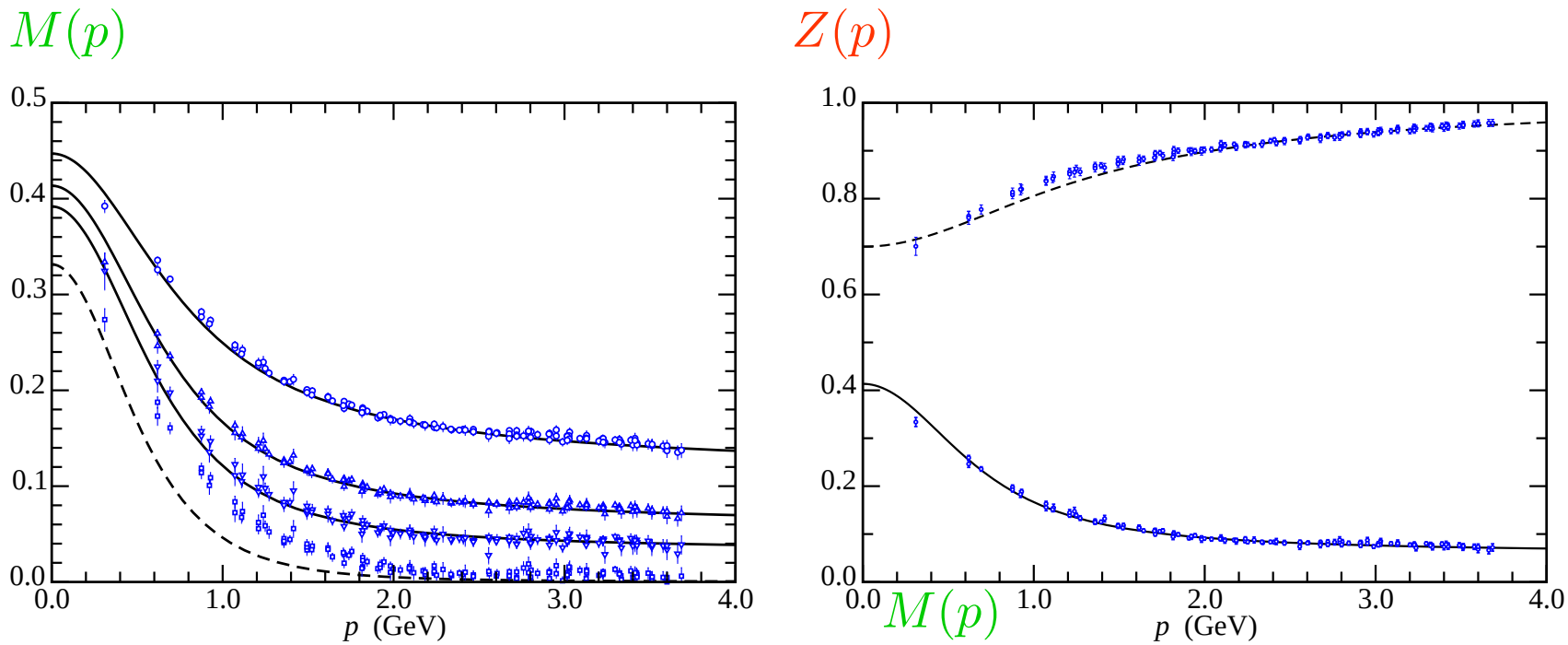
2002



- "*data*:" Quenched Lattice Meas.
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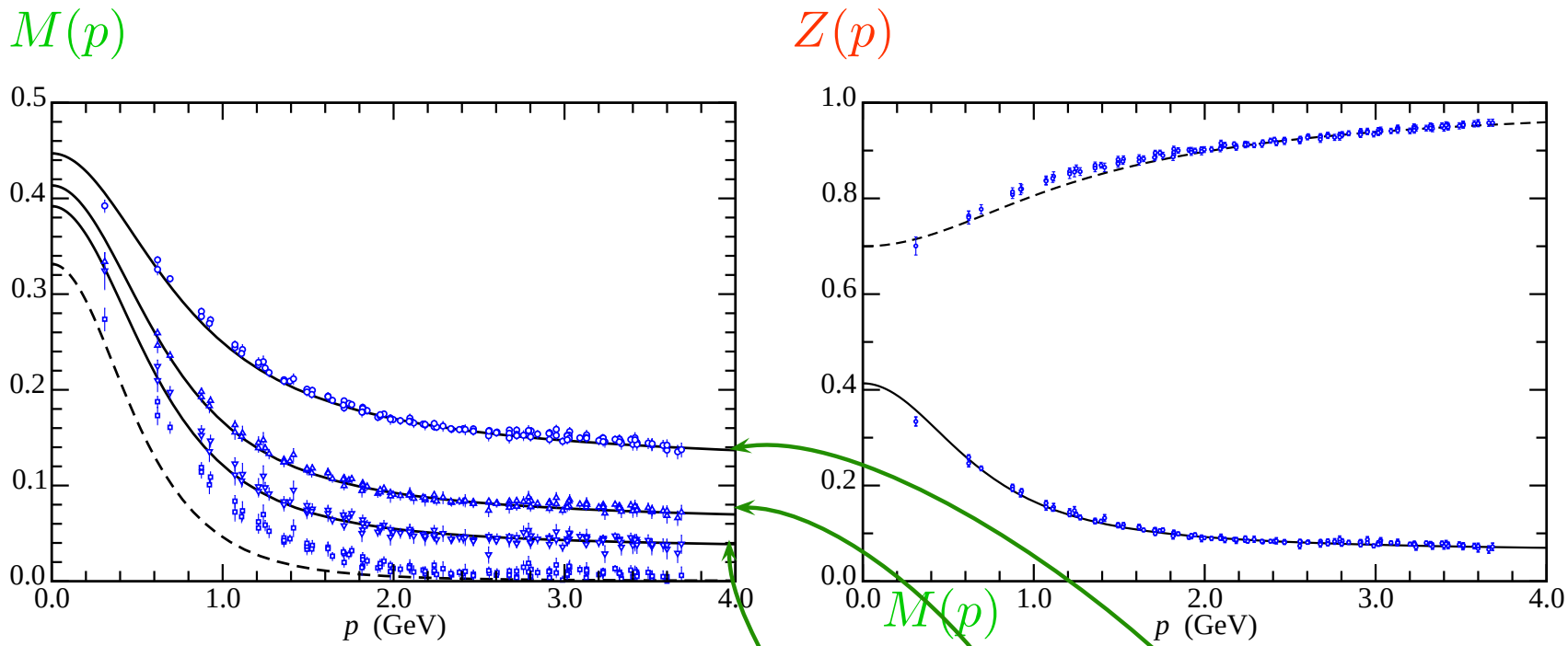

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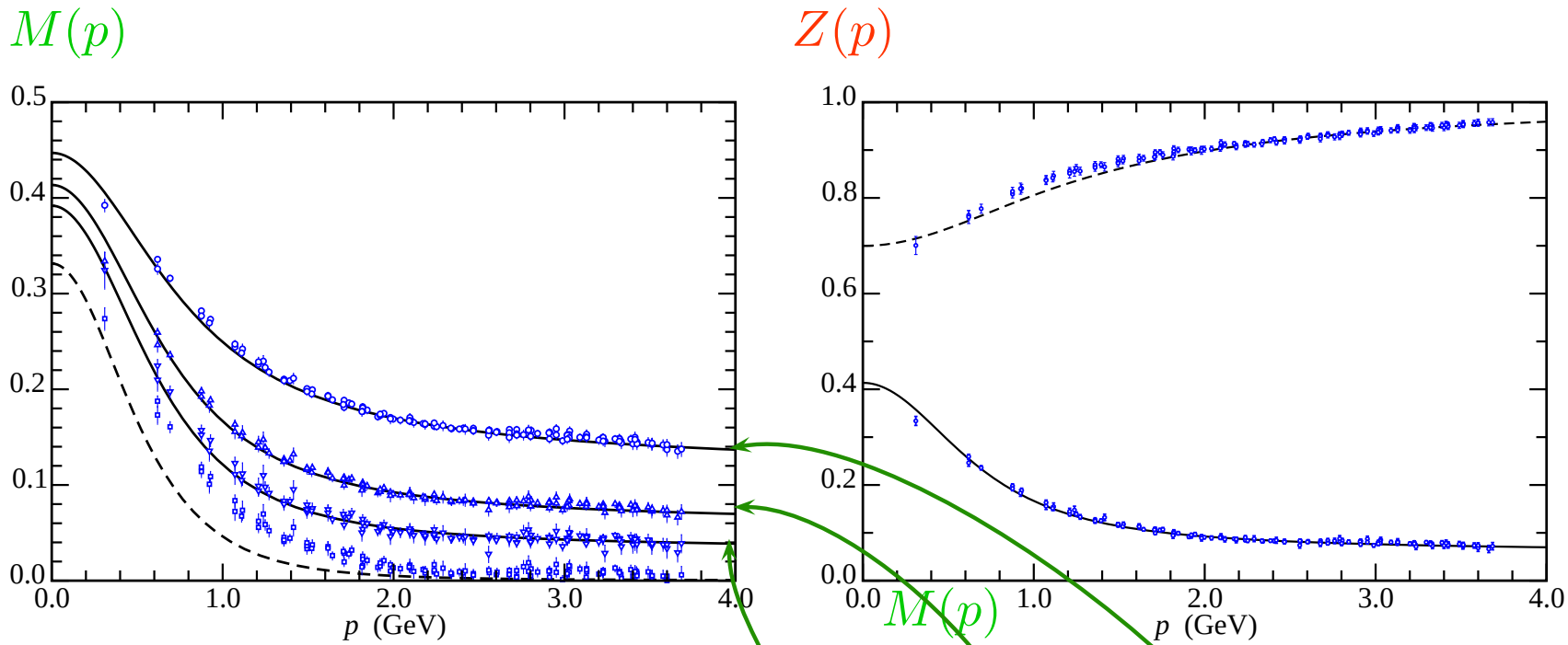

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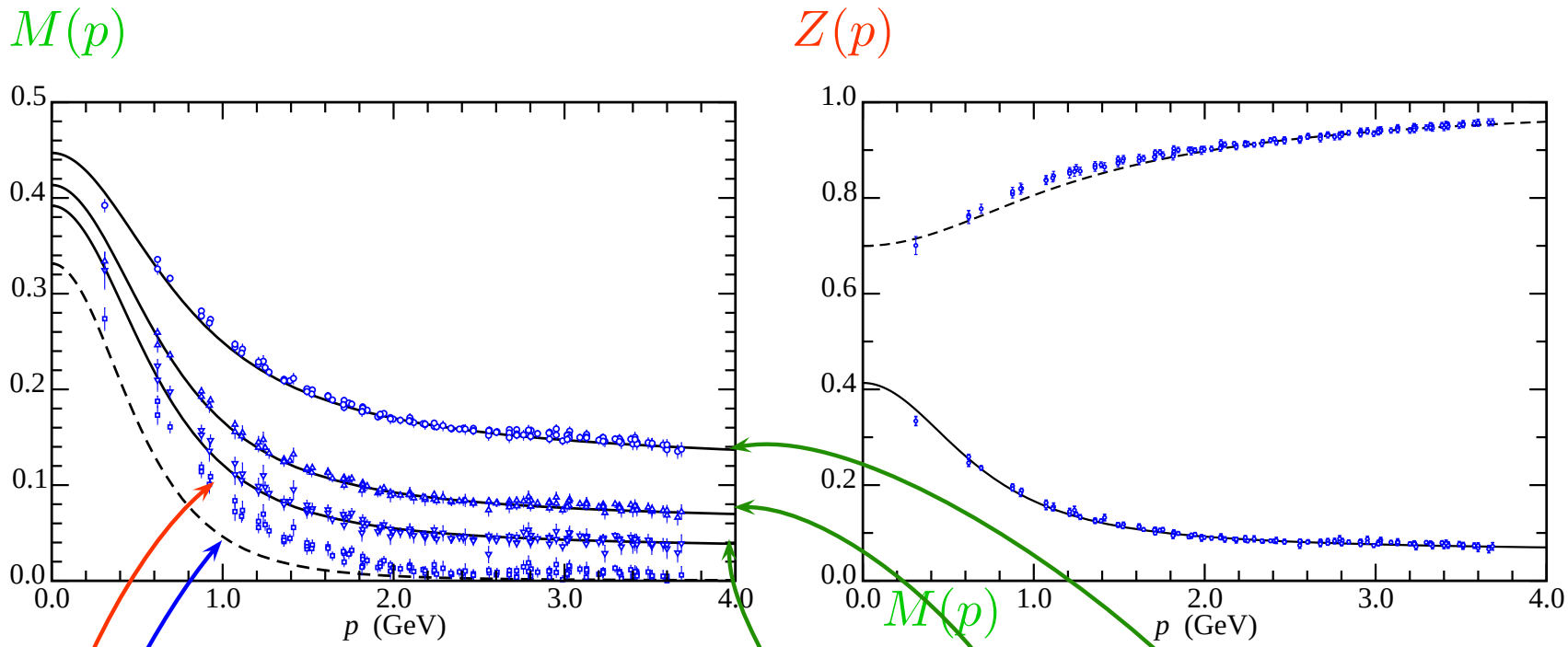

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- Linear extrapolation of lattice data to chiral limit is inaccurate

QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
  - R. Alkofer and L. von Smekal, *The infrared behavior of QCD Green's functions* . . . , Phys. Rept. **353**, 281 (2001).

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- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
  - D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD Collaboration], *Asymptotic scaling and infrared behavior of the gluon propagator*, Phys. Rev. D **60**, 094507 (1999) [Erratum-ibid. D **61**, 079901 (2000)].
- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

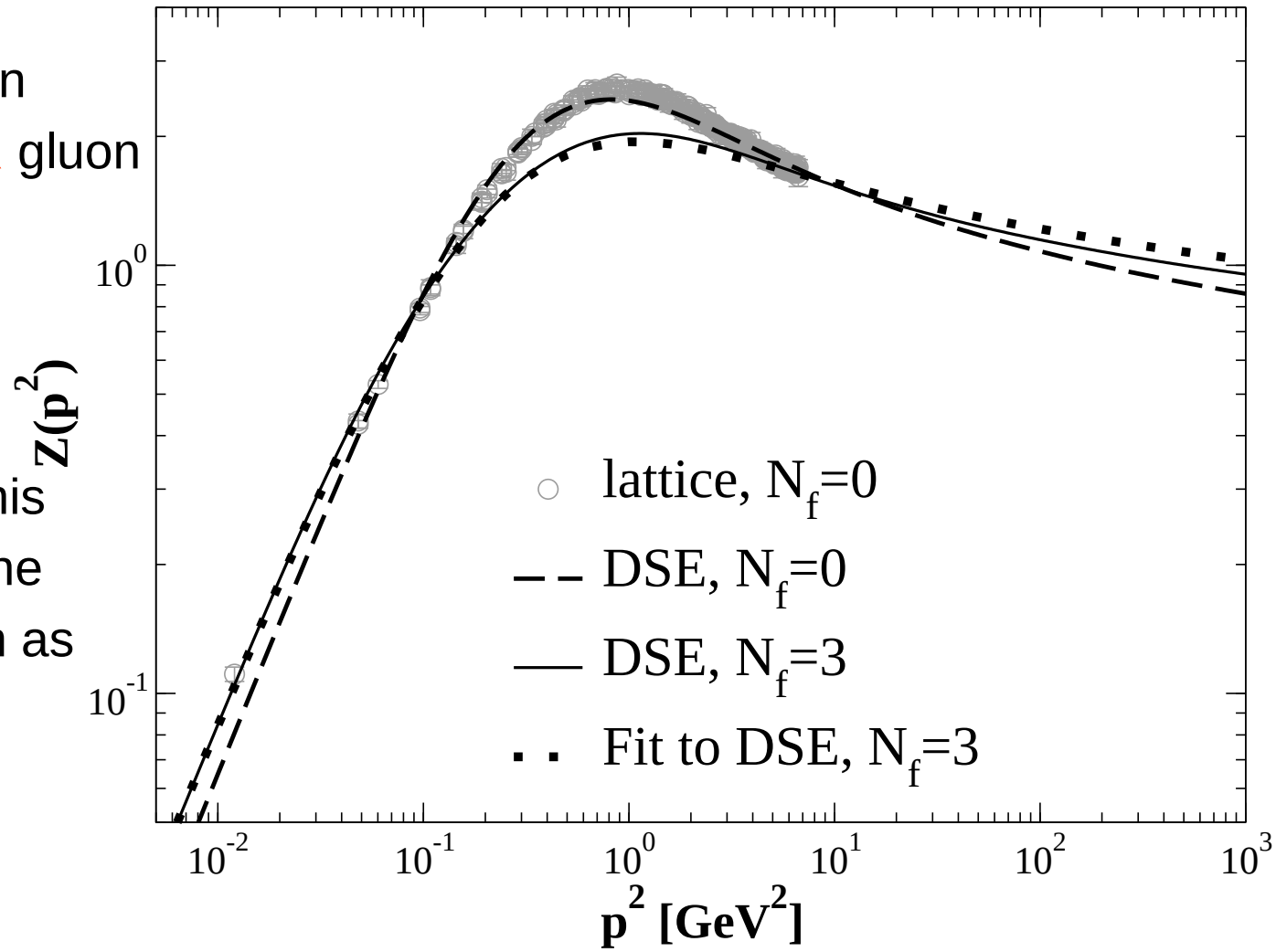
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



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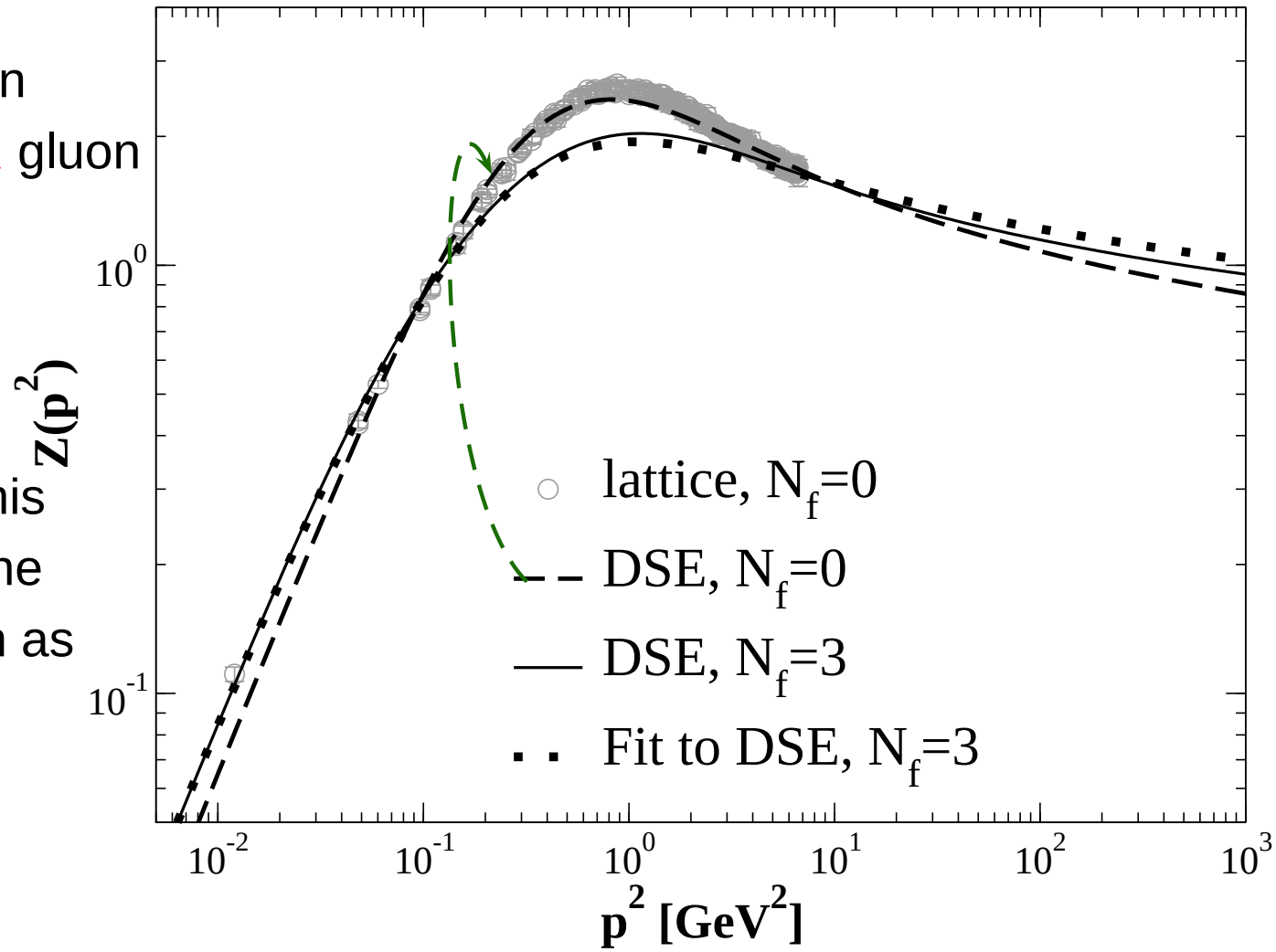

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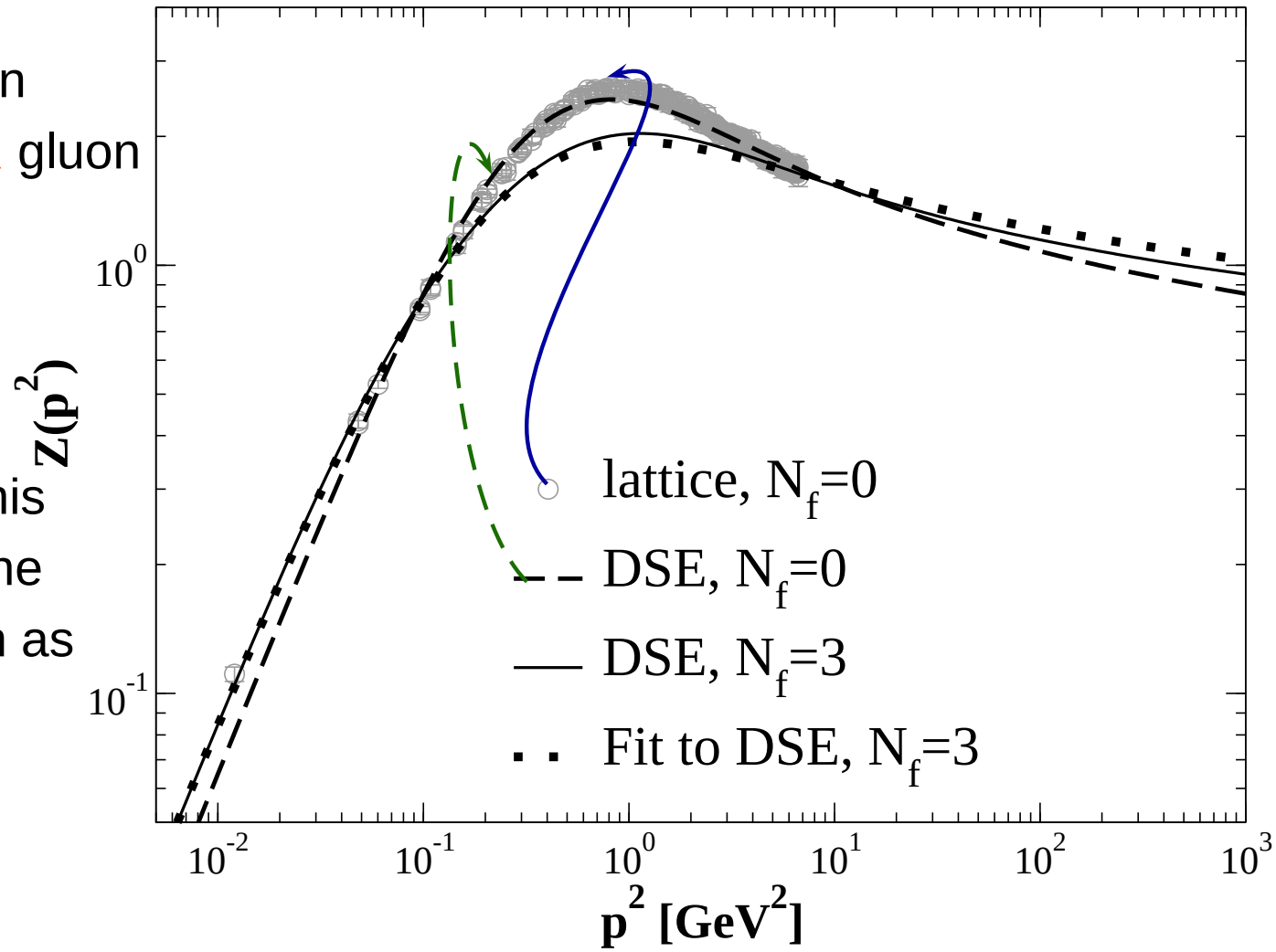

Critical Mass for Chiral Expansion

Dynamical chiral symmetry breaking and a critical mass

Lei Chang, Yu-Xin Liu, Mandar S. Bhagwat, Craig D. Roberts and Stewart V. Wright . . . nucl-th/0605058

Phys. Rev. C 75 (2007) 015201 (8 pages)

Critical Mass for Chiral Expansion

- Realistic models of QCD's gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.

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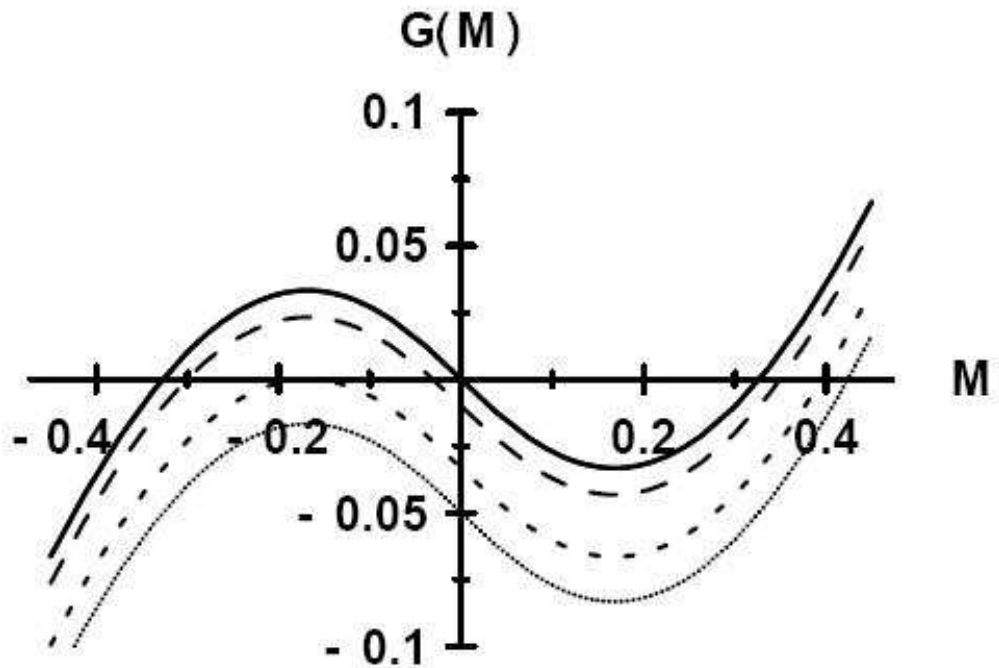
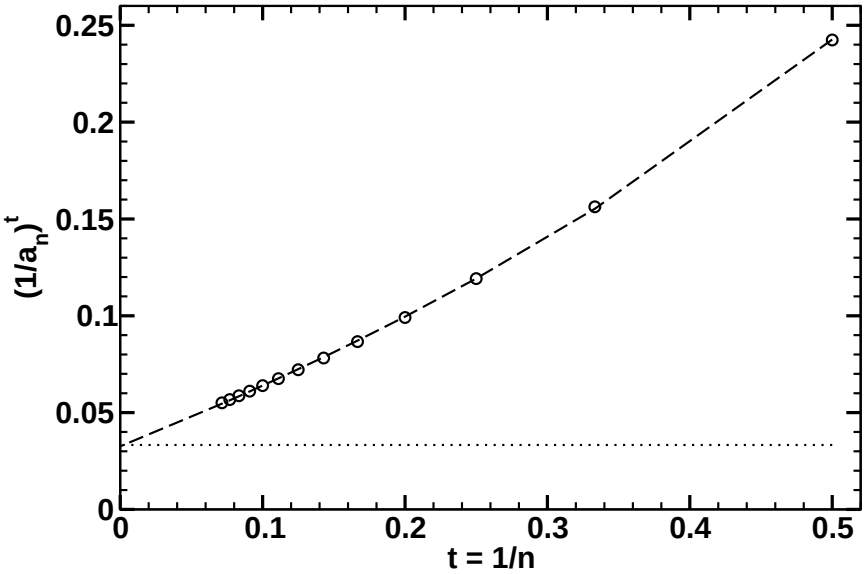
- Realistic models of QCD's gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.
- Wigner solution and one DCSB solution are destabilised by current-quark mass & both disappear when that mass exceeds a critical value, $m_{\rm cr}$



The zeros of $G(M)$ characterise the different solutions of the gap equation. *Solid curve*: obtained with $m^{\rm bm} = 0$, in which case $G(M)$ is odd under $M \to -M$; *long-dashed curve*: $m^{\rm bm} = 0.01$; *short-dashed curve*: $m^{\rm bm} = m^{\rm bm}_{\rm cr} = 0.033$; *dotted curve*: $m^{\rm bm} = 0.05$.

Critical Mass for Chiral Expansion

- Realistic models of QCD's gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.
- $m_{\rm cr}$ also bounds domain on which surviving DCSB solution possesses a chiral expansion: $m_{\rm cr} = \lim_{n\to\infty} \left(\frac{1}{|a_n|}\right)^{1/n}$



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- Entails lattice-QCD simulations *must* have many results at $m_\pi < m_{0^-}^{\rm cr} \sim 0.45$ GeV for reliable extrapolation via EFT.

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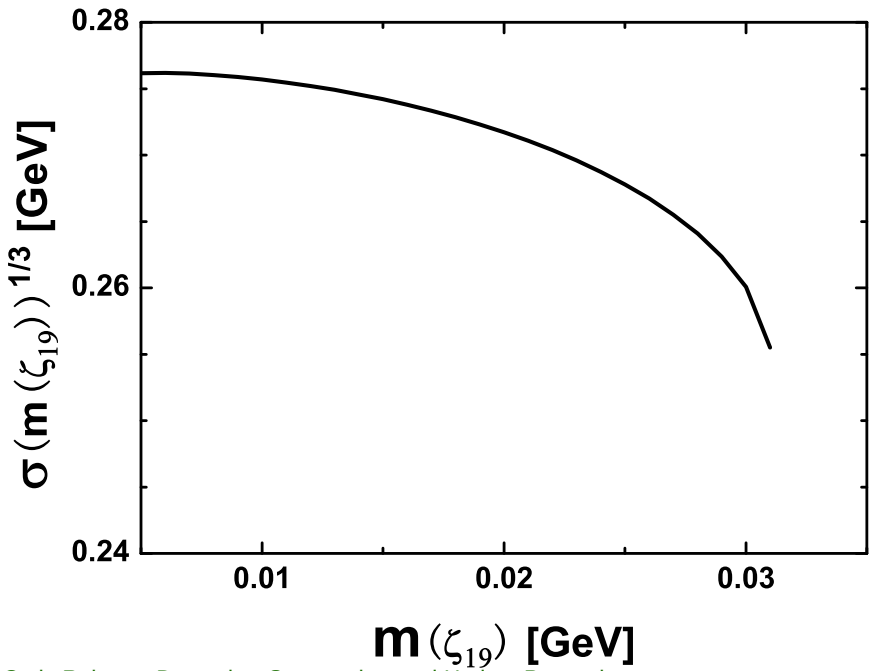
- Realistic models of QCD's gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.
- The two DCSB solutions of the gap equation enable a valid definition of $\langle \bar{q} q \rangle$ in the presence of a nonzero current-mass.

Critical Mass for Chiral Expansion

- Realistic models of QCD's gap equation simultaneously admit two inequivalent DCSB solutions & solution connected with realisation of chiral symmetry in Wigner mode.
- The two DCSB solutions of the gap equation enable a valid definition of $\langle \bar{q}q \rangle$ in the presence of a nonzero current-mass.
- The behaviour of this condensate indicates that the essentially dynamical component of chiral symmetry breaking decreases with increasing current-quark mass.



Consituent-quark σ-term

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark σ-term

$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\; (M^E)^2 := s\,|\,s = M(s)^2\,.$$

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- Renormalisation-group-invariant and determined from solutions of the gap equation

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- Unambiguous probe of impact of explicit chiral symmetry breaking on the mass function

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- Ratio $\boxed{\dfrac{\sigma_f}{M_f^E} = \dfrac{\text{EXPLICIT}}{\text{EXPLICIT} + \text{DYNAMICAL}}}$

measures effect of *EXPLICIT* chiral symmetry breaking on dressed-quark mass-function

cf. *SUM* of effects of *EXPLICIT* AND *DYNAMICAL CHIRAL SYMMETRY BREAKING*

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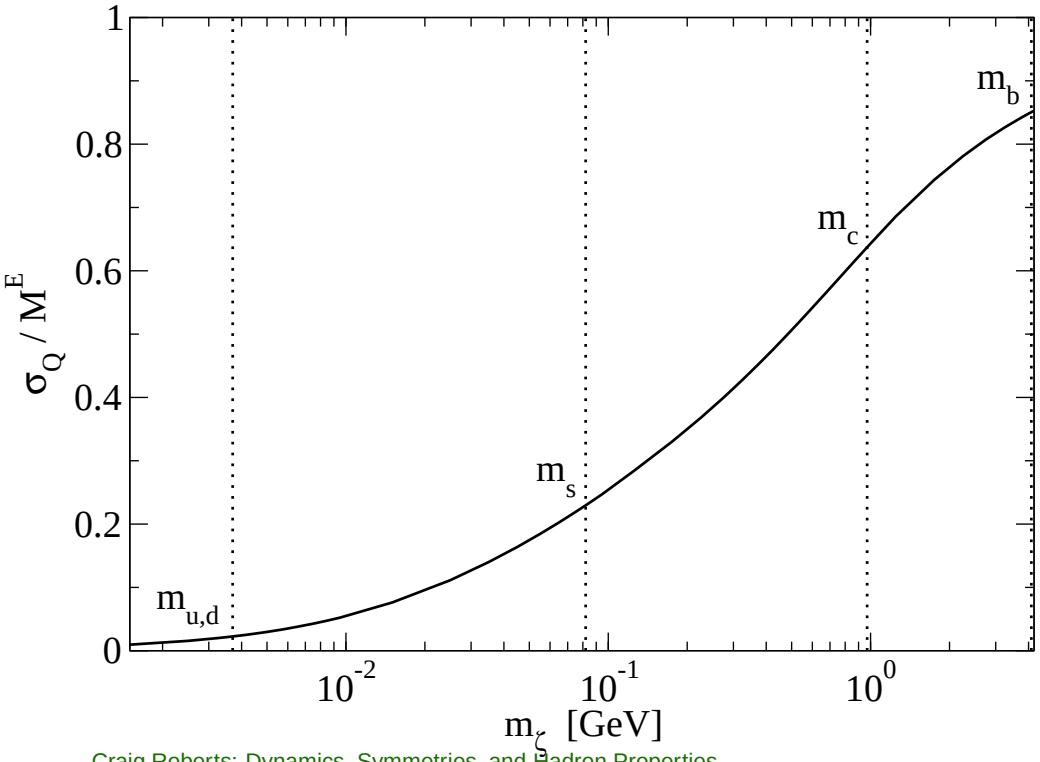

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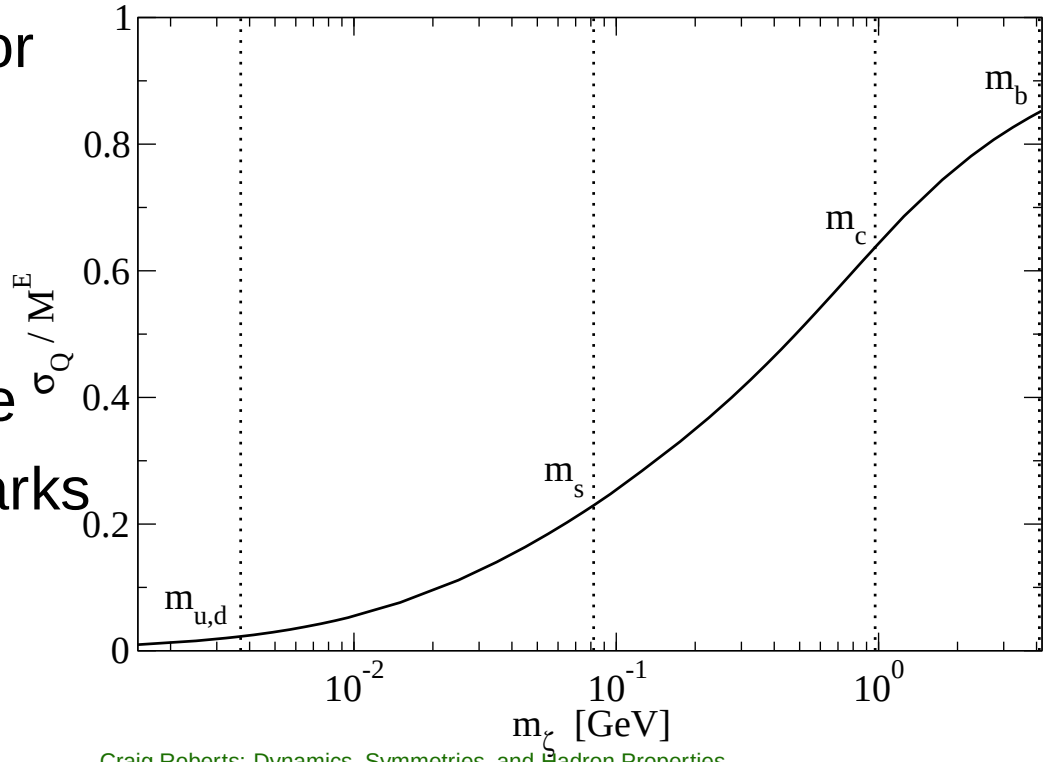
Obvious: ratio vanishes for light-quarks because magnitude of their constituent-mass owes primarily to DCSB. On the other hand, for heavy-quarks it approaches one.



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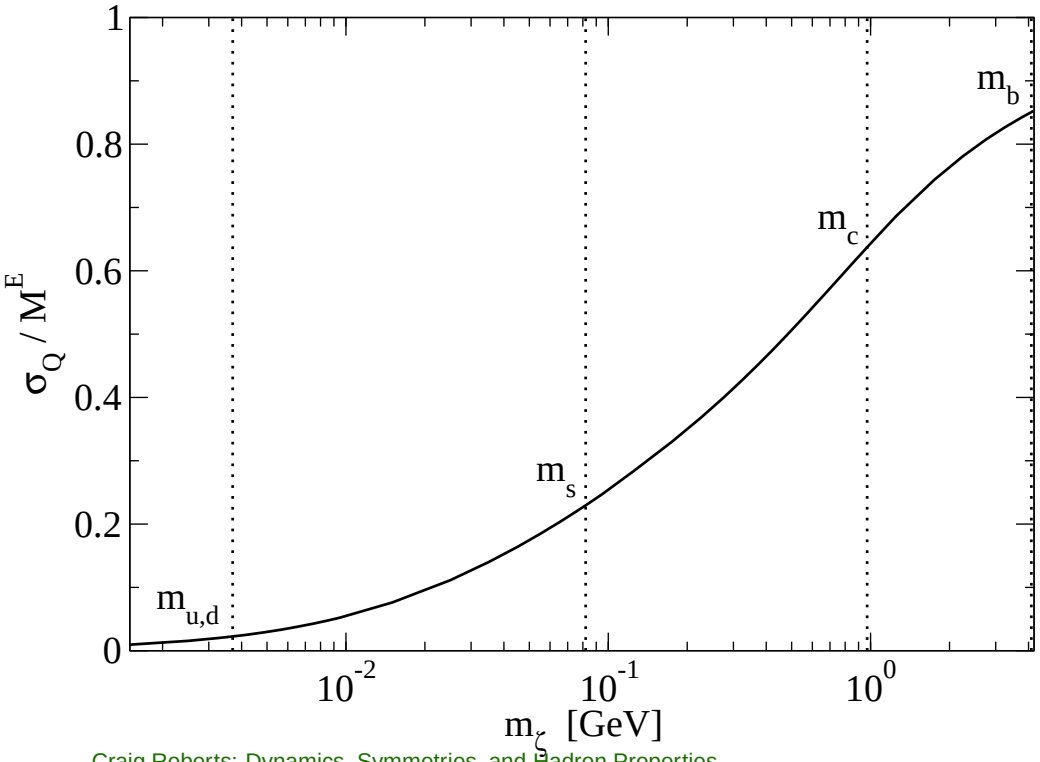
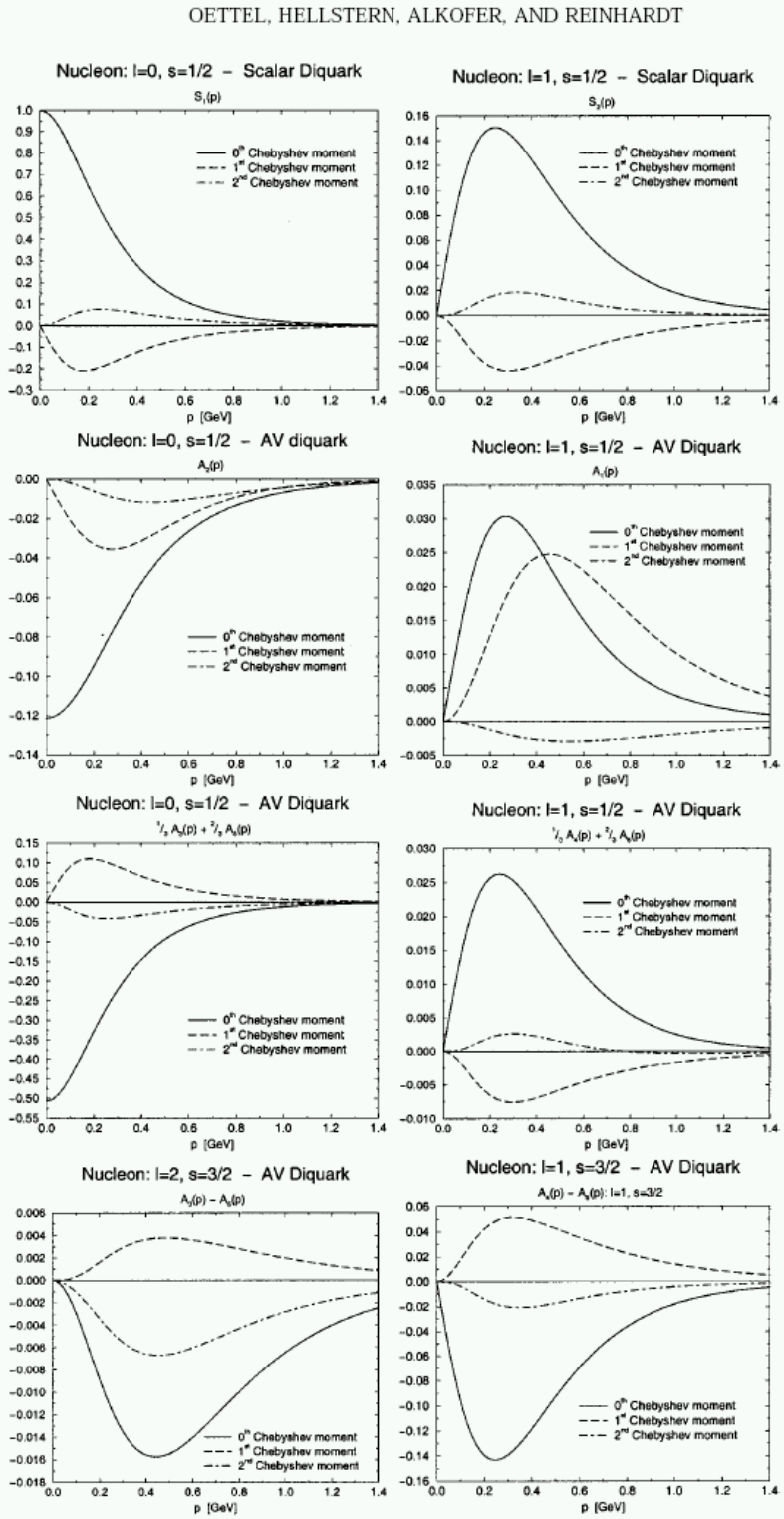
Essentially dynamical component of chiral symmetry breaking, and manifestation in all its order parameters, vanishes with increasing current-quark mass



Angular Momentum Rest Frame

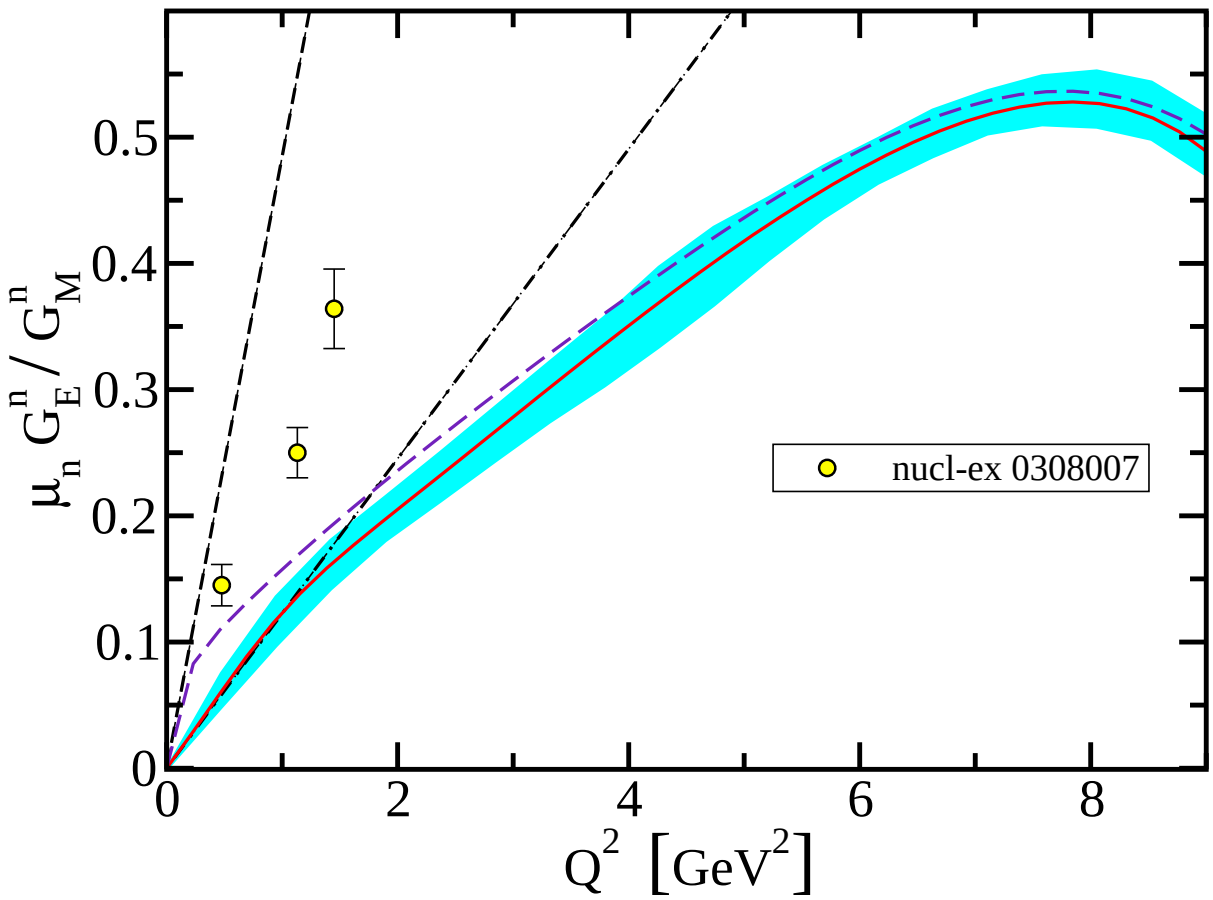


M. Oettel, *et al.*
nucl-th/9805054

Crude estimate based on magnitudes $\Rightarrow$ probability for a u-quark to carry the proton's spin is $P_{u\uparrow} \sim 80\,\%$, with $P_{u\downarrow} \sim 5\,\%$, $P_{d\uparrow} \sim 5\,\%$, $P_{d\downarrow} \sim 10\,\%$.

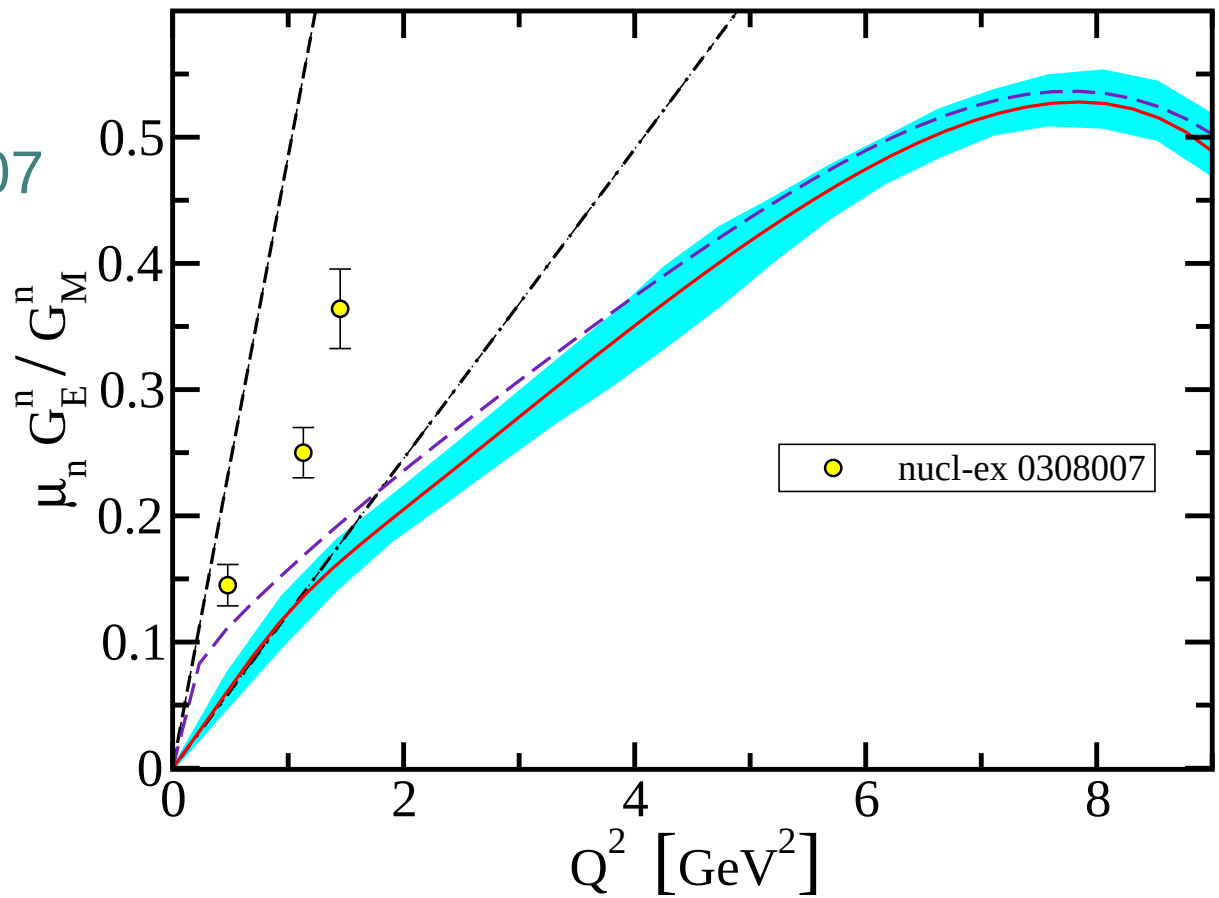
Hence, by this reckoning $\sim 30\%$ of proton's rest-frame spin is located in dressed-quark angular momentum.

Neutron Form Factors



Neutron Form Factors

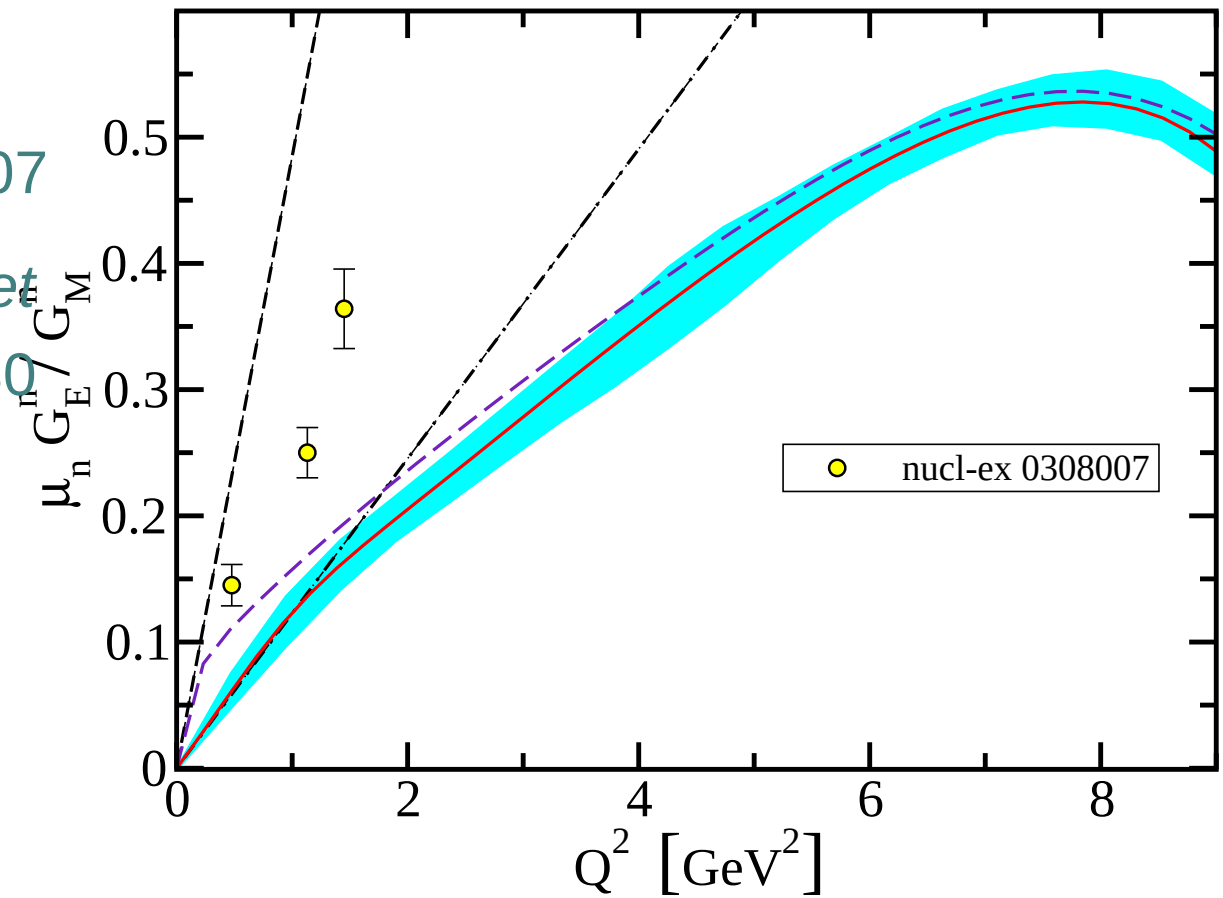
- Expt. Madey, *et al.* nu-ex/0308007



Neutron Form Factors

- Expt. Madey, *et al.* nu-ex/0308007
- Calc. Bhagwat, *et al.* nu-th/0610080
- $\mu_p \dfrac{G_E^n(Q^2)}{G_M^n(Q^2)} = -\dfrac{r_n^2}{6} Q^2$

Valid for $r_n^2 Q^2 \lesssim 1$



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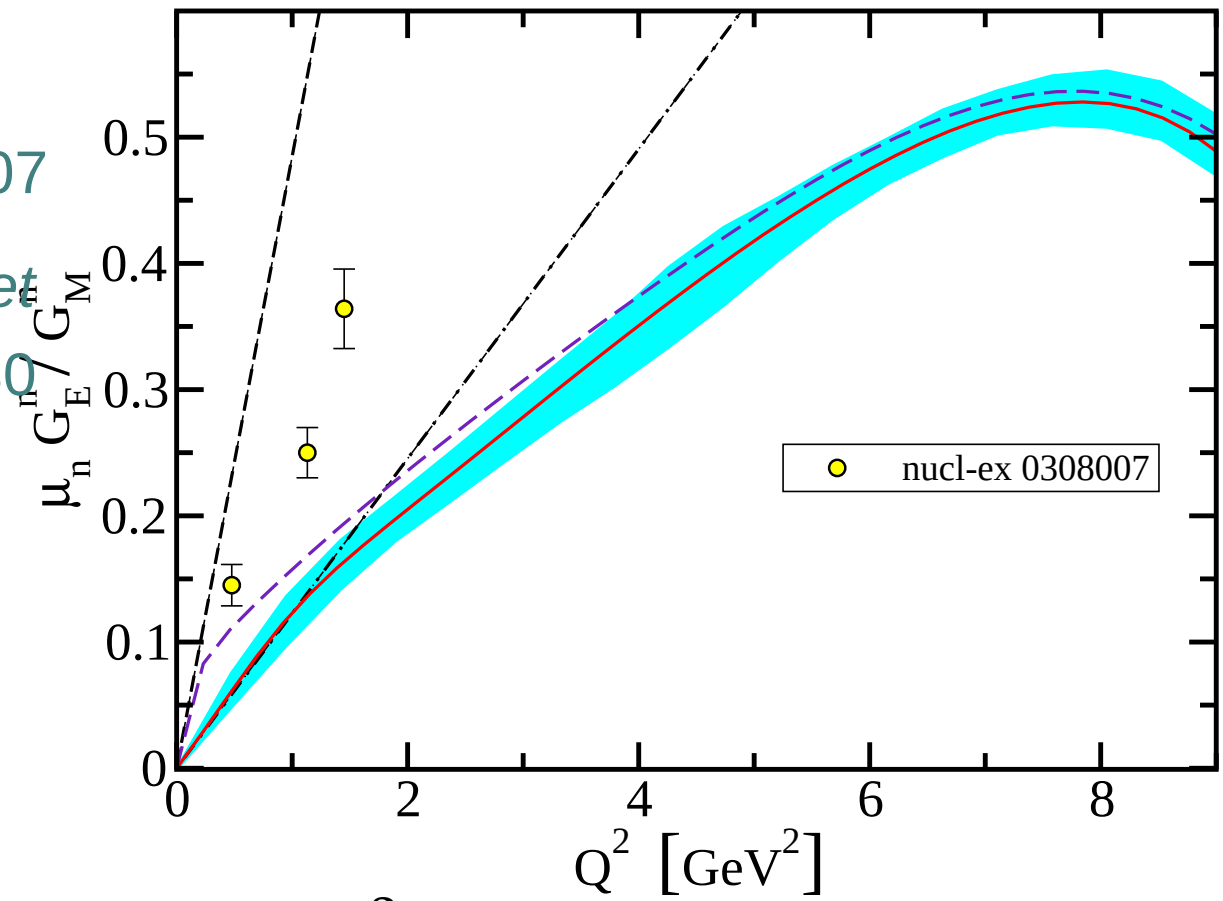

- No sign yet of a zero in $G_E^n(Q^2)$, even though calculation predicts $G_E^p(Q^2 \approx 6.5\,\mathrm{GeV}^2) = 0$
- Data to $Q^2 = 3.4\,\mathrm{GeV}^2$ is being analysed (JLab E02-013)

Contemporary Reviews

- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD
  C.D. Roberts and S.M. Schmidt, nu-th/0005064,
  Prog. Part. Nucl. Phys. **45** (2000) S1
- The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .
  R. Alkofer and L. von Smekal, he-ph/0007355,
  Phys. Rept. **353** (2001) 281
- Dyson-Schwinger equations: A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049,
  Int. J. Mod. Phys. **E 12** (2003) pp. 297-365
- Infrared properties of QCD from Dyson-Schwinger equations.
  C. S. Fischer, he-ph/0605173,
  J. Phys. **G 32** (2006) pp. R253-R291
- Nucleon electromagnetic form factors
  J. Arrington, C.D. Roberts and J.M. Zanotti, nucl-th/0611050,
  J. Phys. **G 34** (2007) pp. S23-S52.