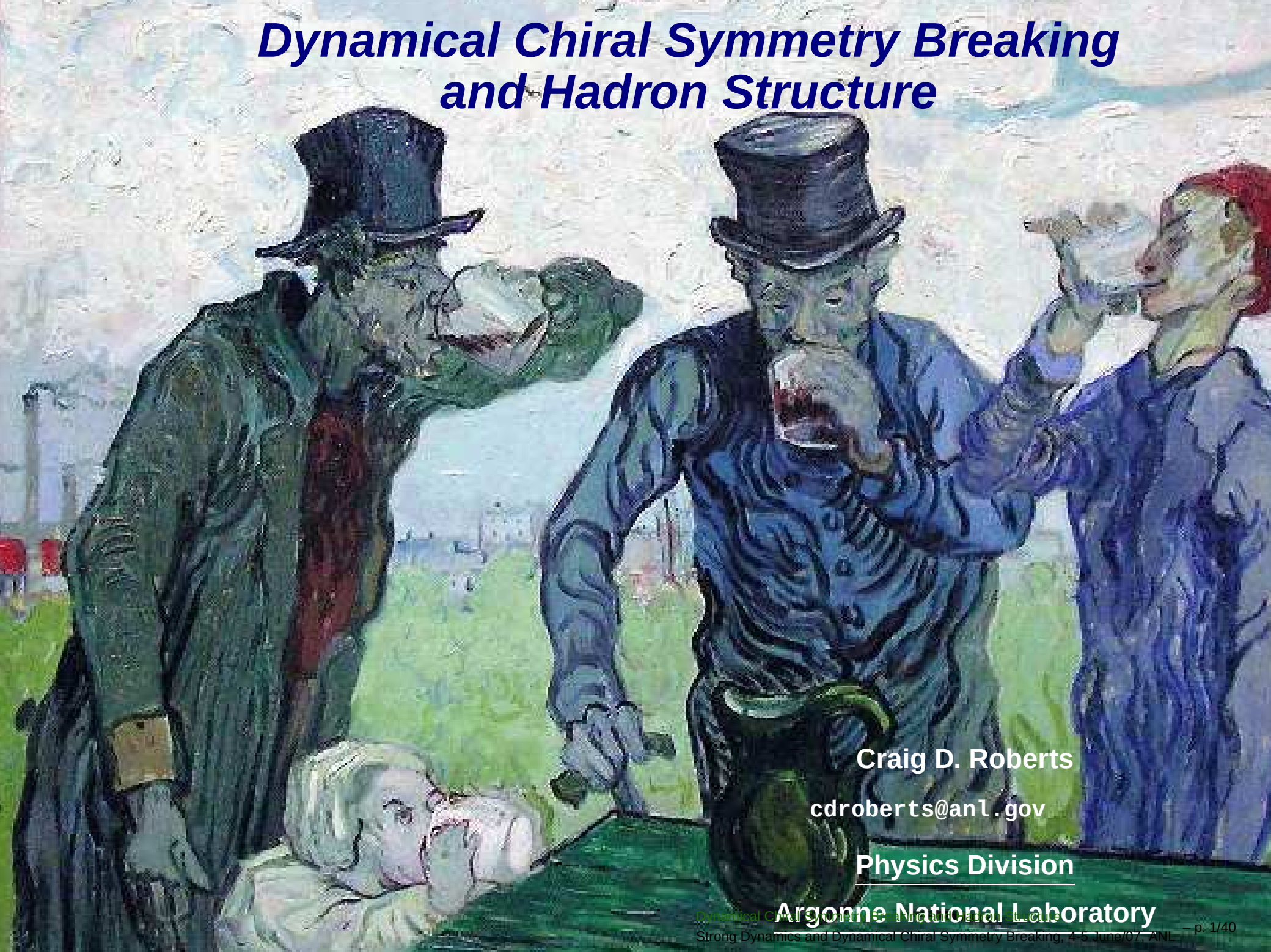
Dynamical Chiral Symmetry Breaking and Hadron Structure



Craig D. Roberts

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Physics Division

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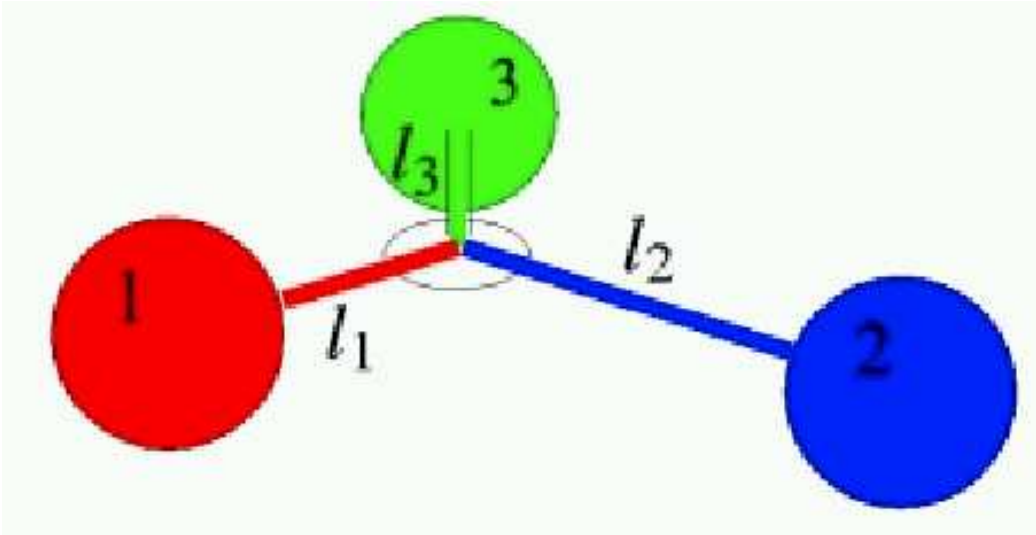
Modern Miracles in Hadron Physics

- proton = three constituent quarks



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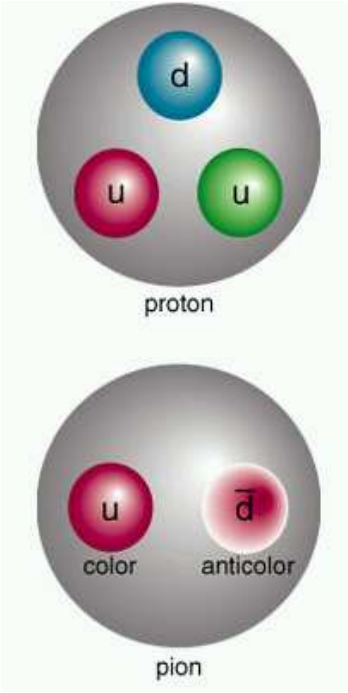

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- Another meson:
  ..... $M_\rho = \mathbf{770}$ MeV ..... No Surprises Here

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- What is "wrong" with the pion?

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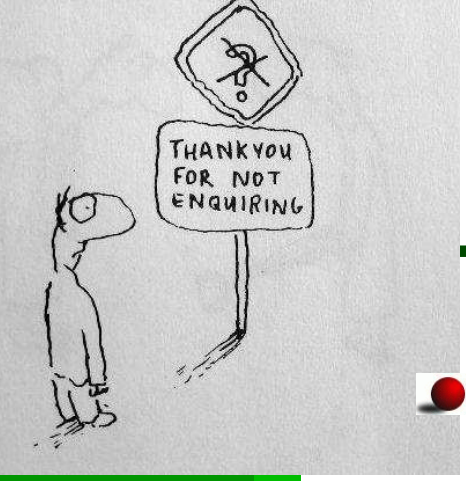
Using DSEs, we've provided this.

QCD's Emergent Phenomena



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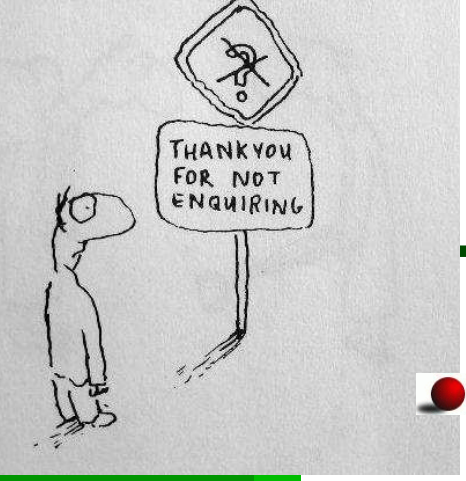

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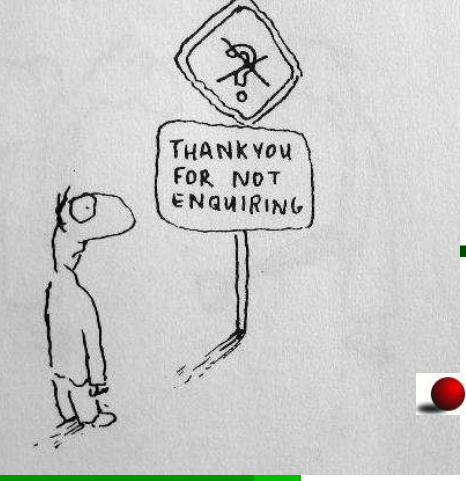

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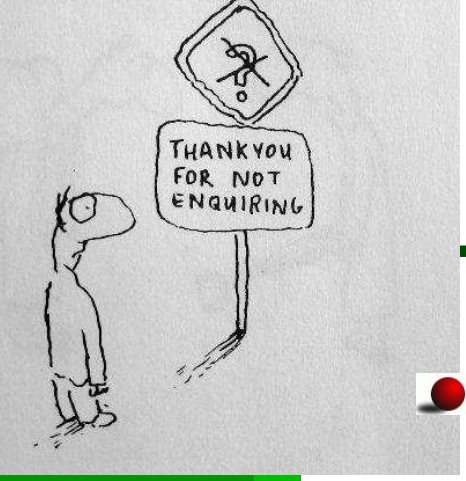

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- NSAC – Understanding these phenomena is one of the greatest intellectual challenges in physics

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- Modern Physics & Mathematics
  – Still quite some way from being able to do that

Intranucleon Interaction

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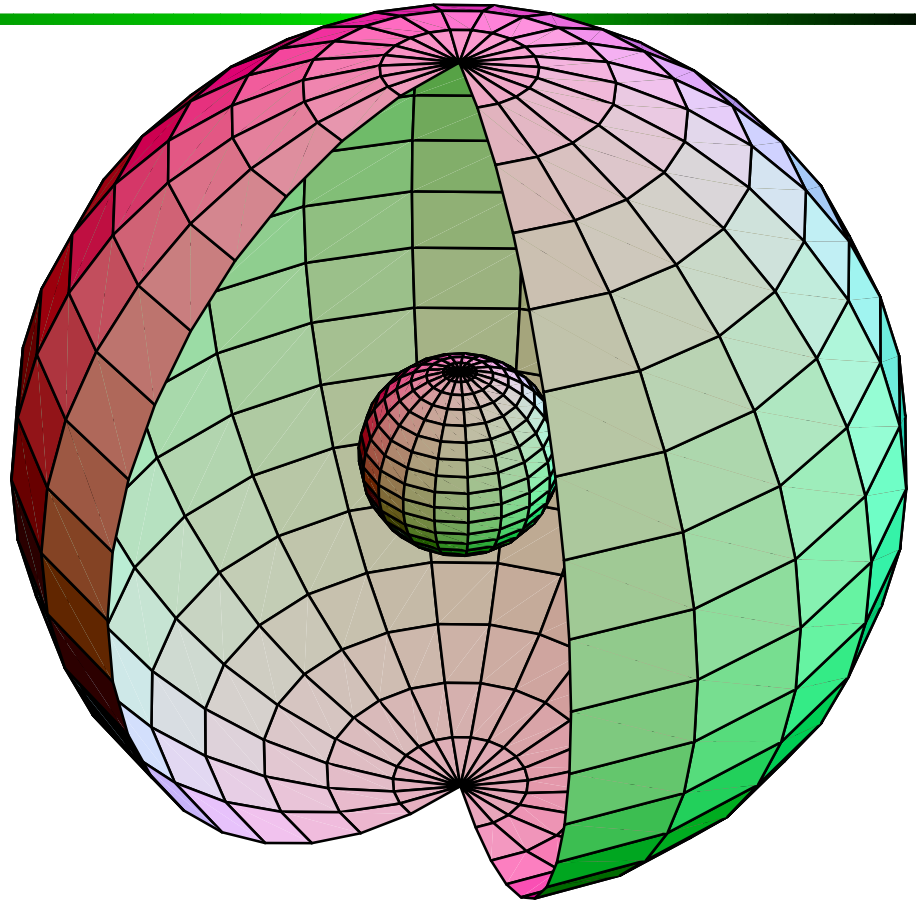

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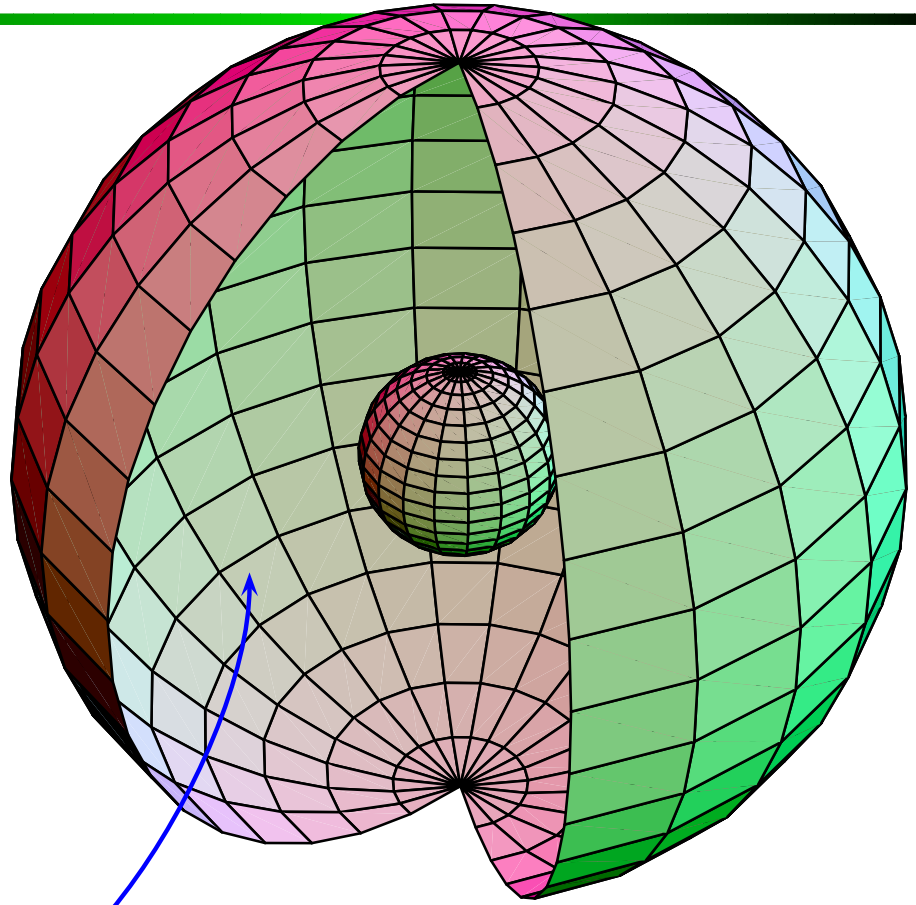

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- $\Rightarrow$ Understanding InfraRed (long-range)
  ..... behaviour of $\alpha_s(Q^2)$

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- Method yields Schwinger Functions $\equiv$ Propagators

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Cross-Sections built from Schwinger Functions

Persistent Challenge

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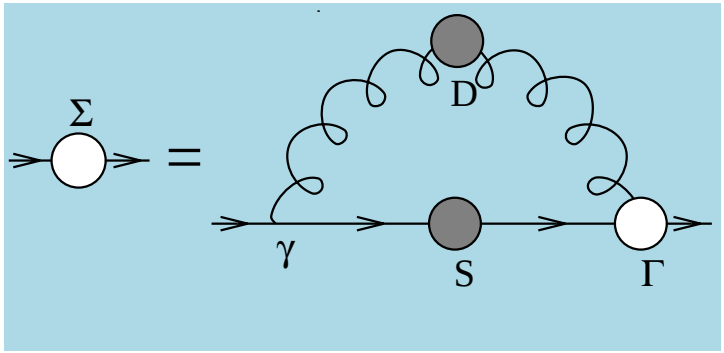

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  - Not all are Schwinger functions are experimentally observable but all are same VEVs measured in Lattice-QCD simulations . . . opportunity for comparisons at pre-experimental level . . . cross-fertilisation

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    Not useful for the nonperturbative problems in which we're interested

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  H.J. Munczek Phys. Rev. D **52** (1995) 4736
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  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
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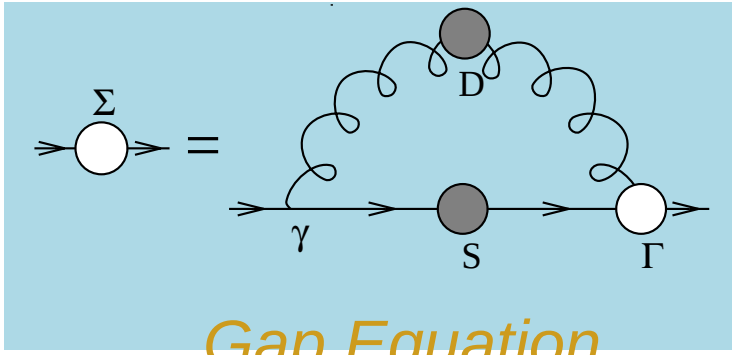

Gap Equation

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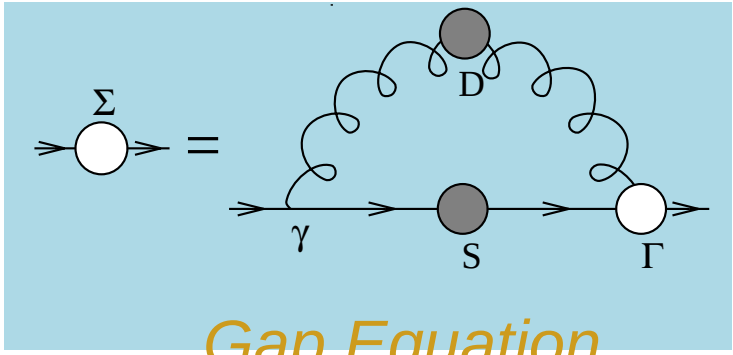
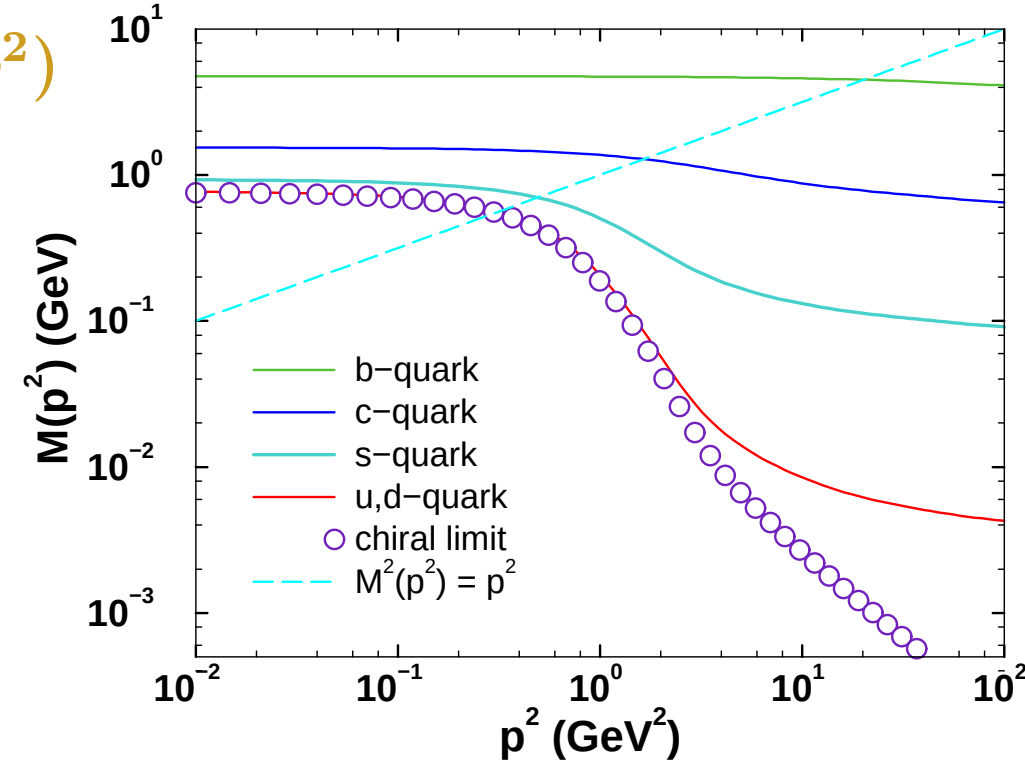


Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



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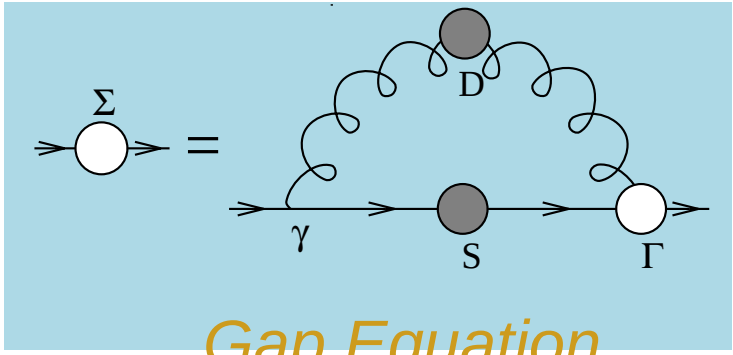

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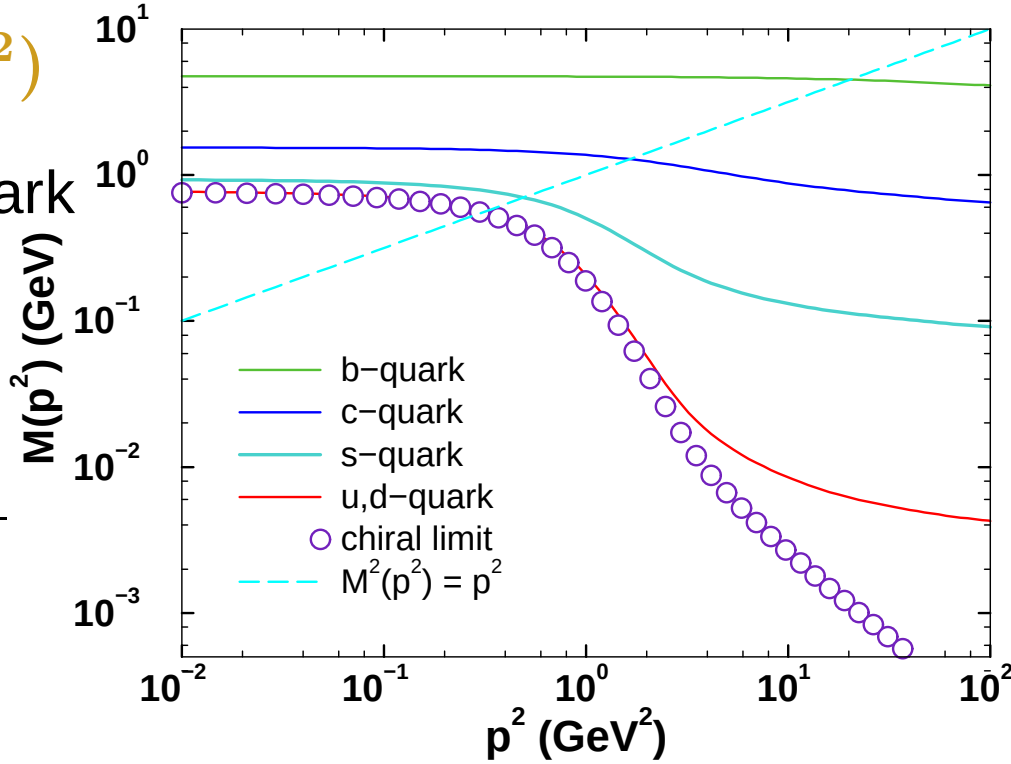
- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



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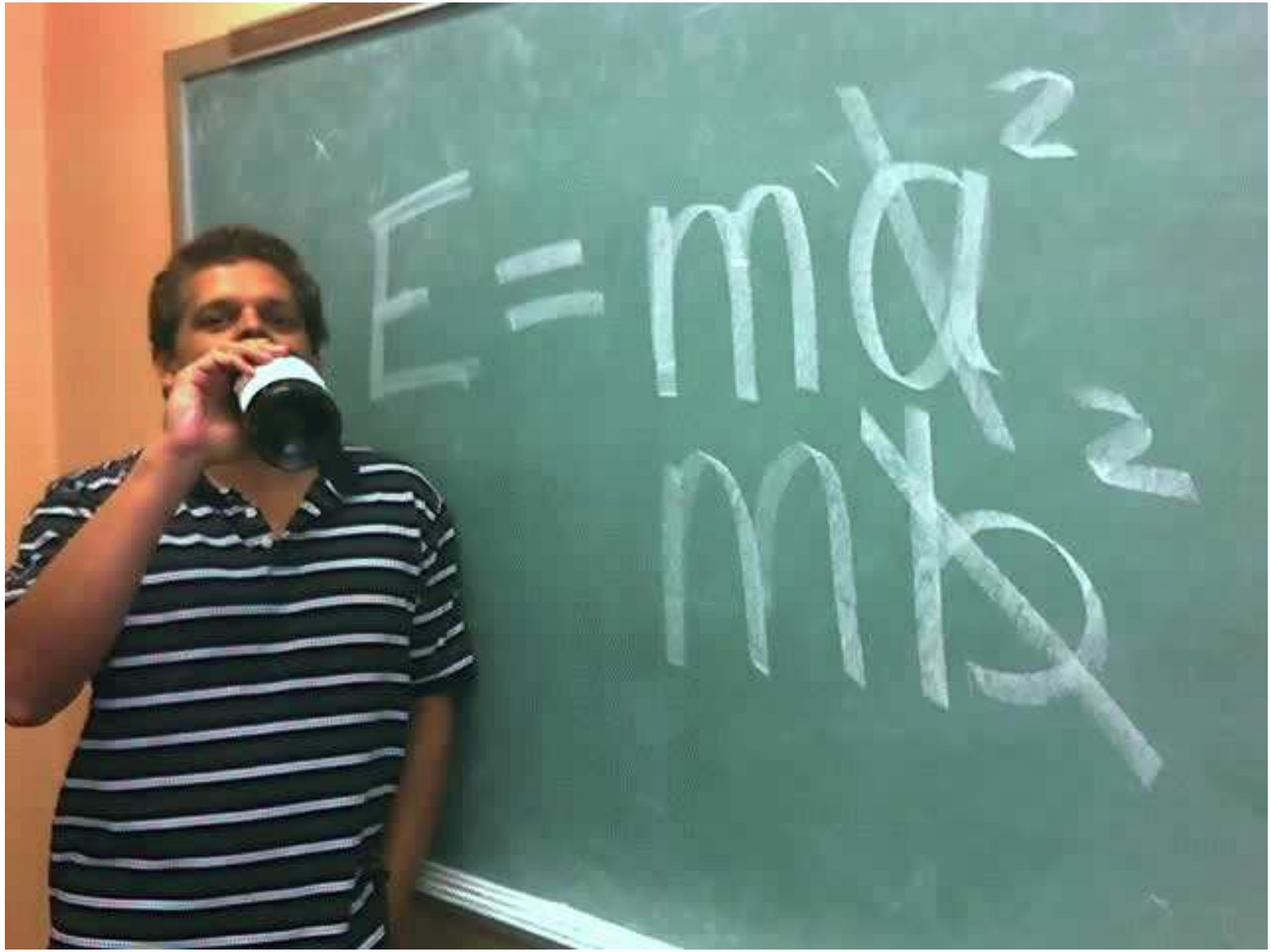
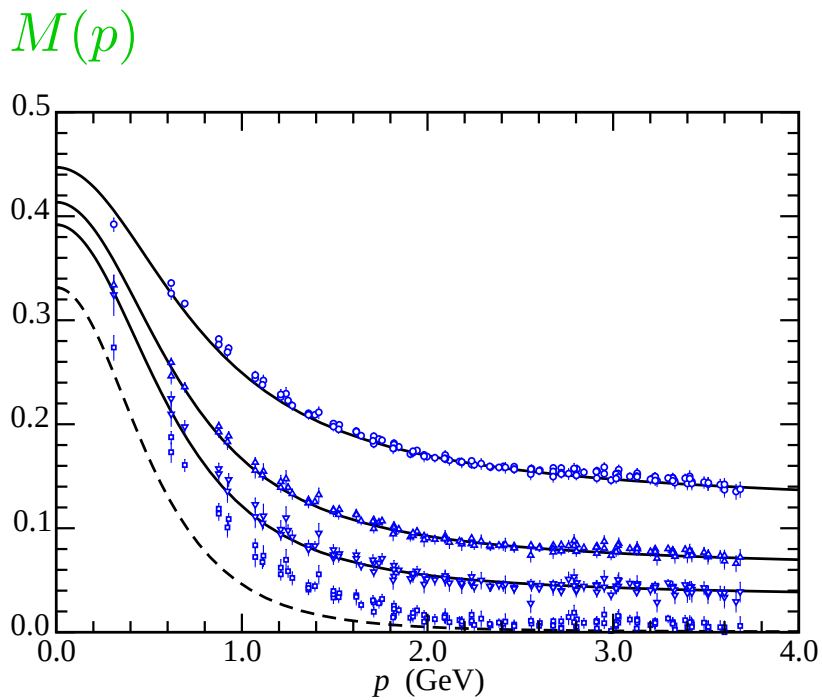
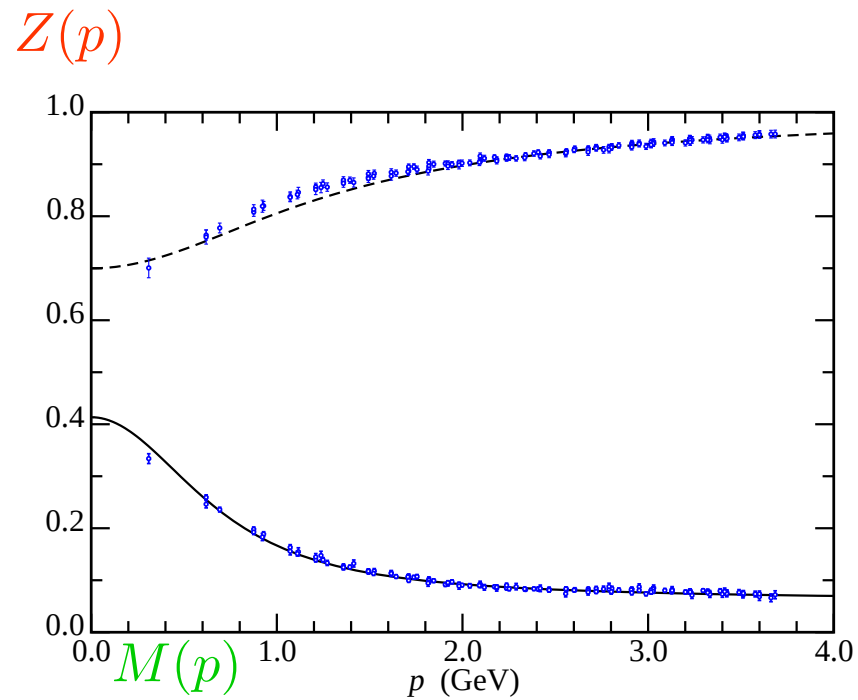


Mandar Bhagwat

Emulating supervisor's approach to physics

Quenched-QCD Dressed-Quark Propagator





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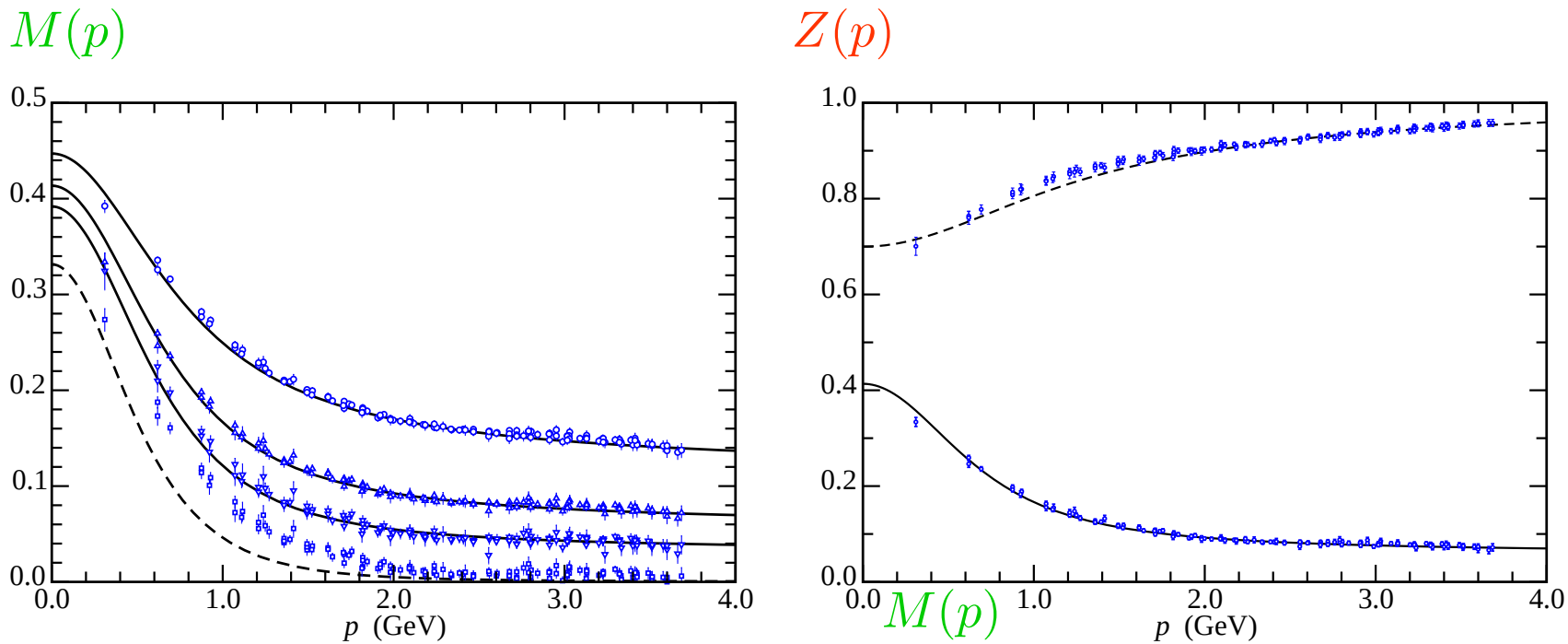
2002



- "*data*:" Quenched Lattice Meas.
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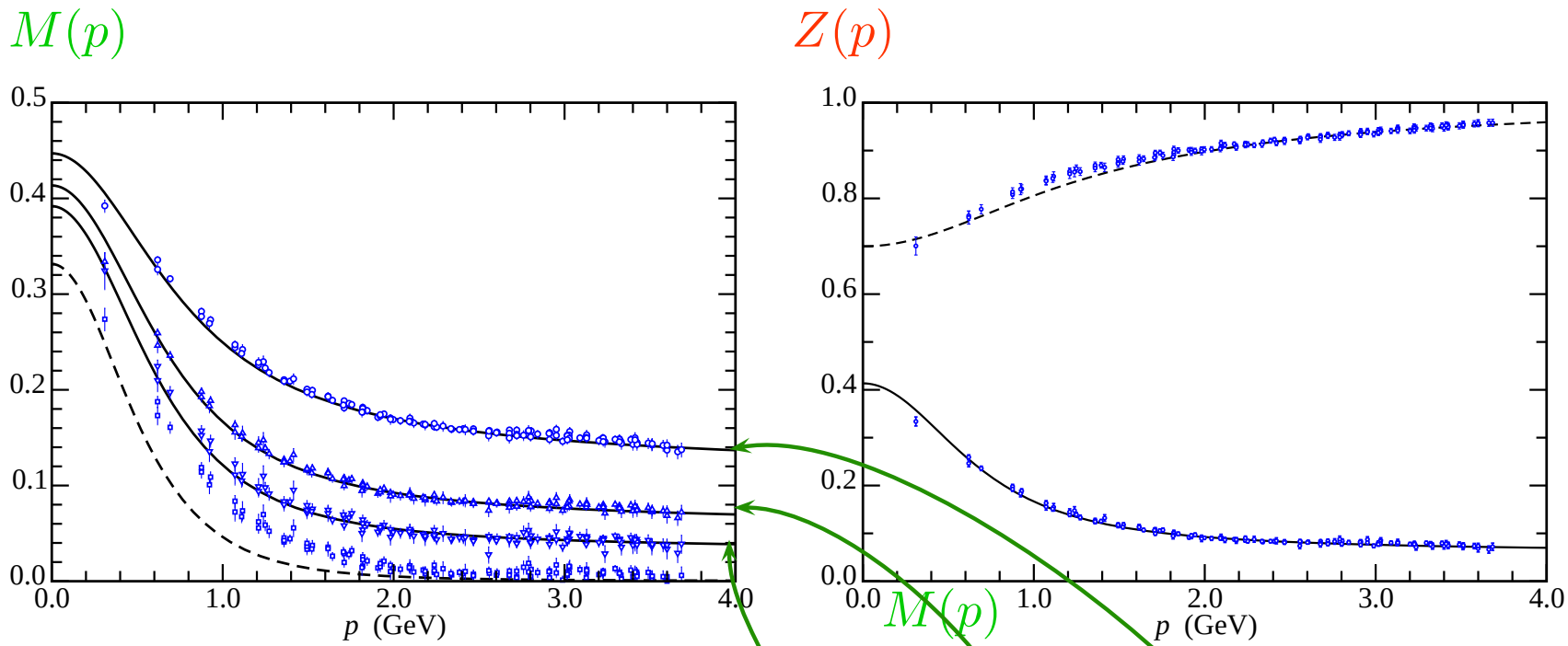

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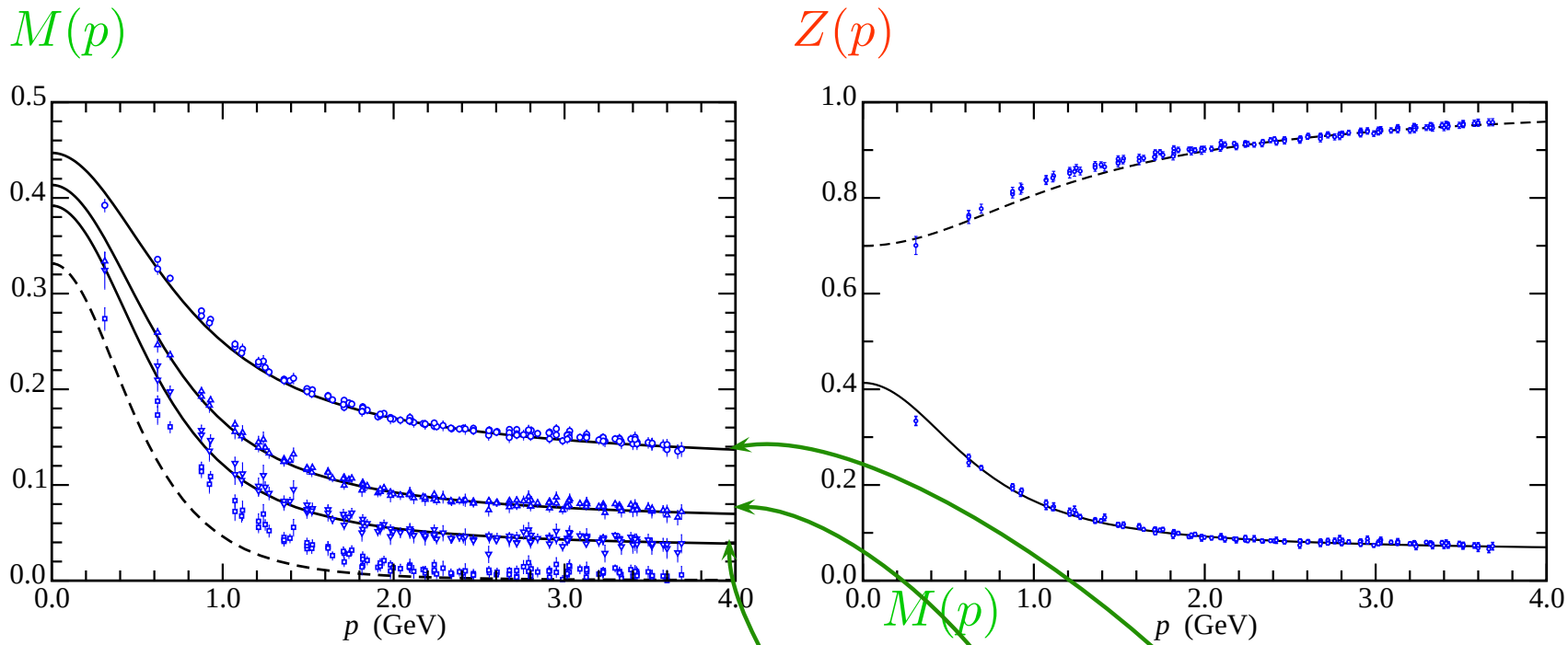

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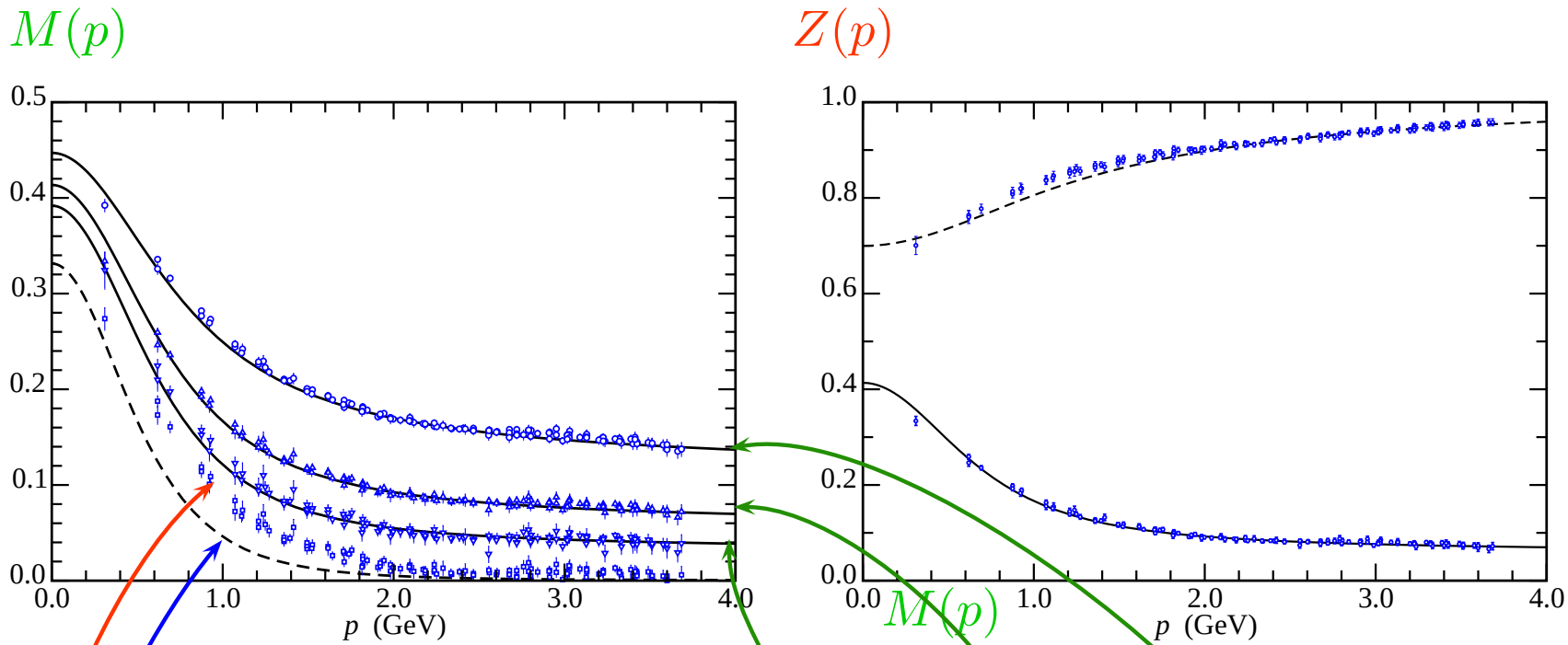

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- Linear extrapolation of lattice data to chiral limit is inaccurate

QCD & Interaction Between Light-Quarks

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- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

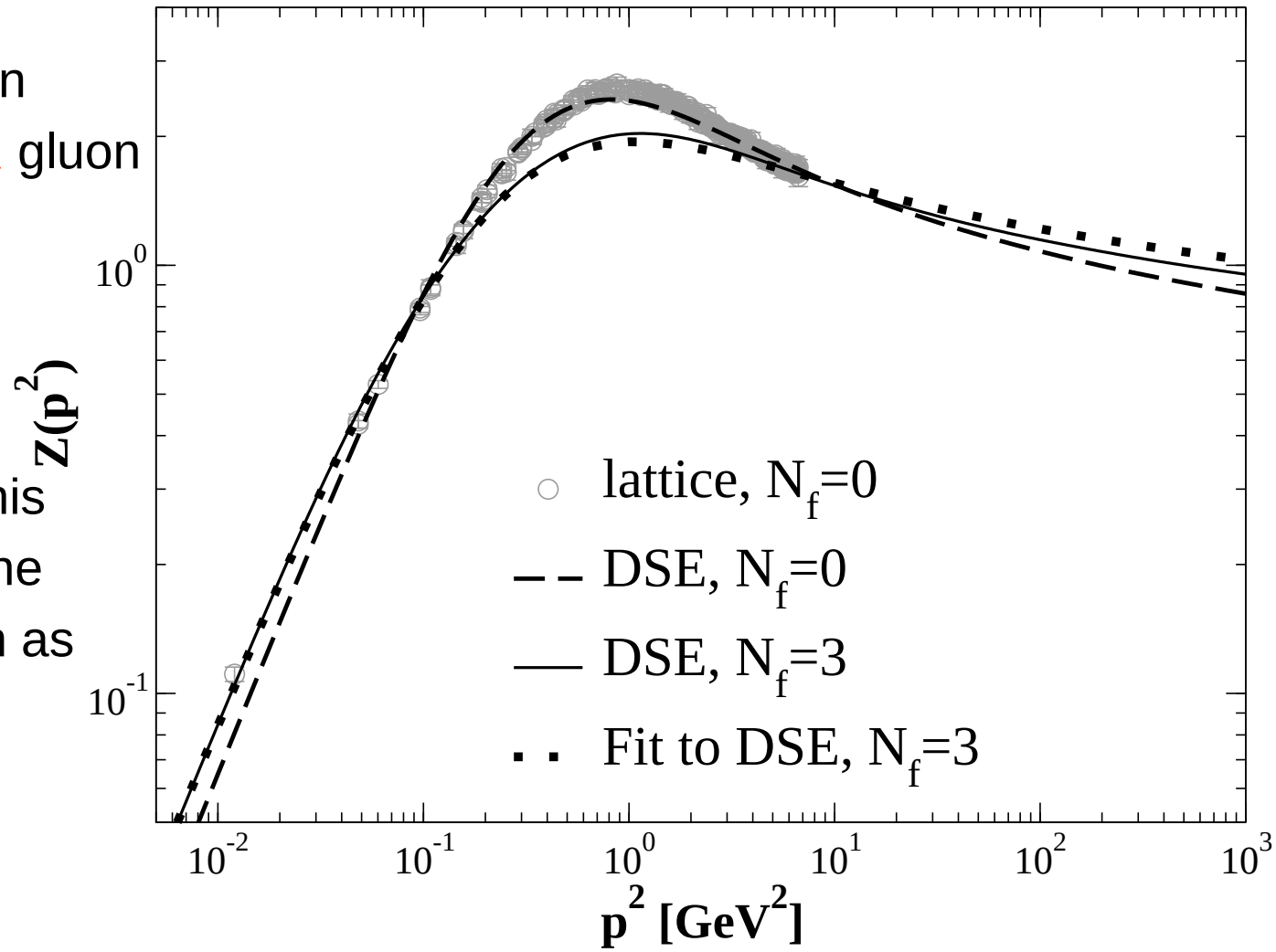
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



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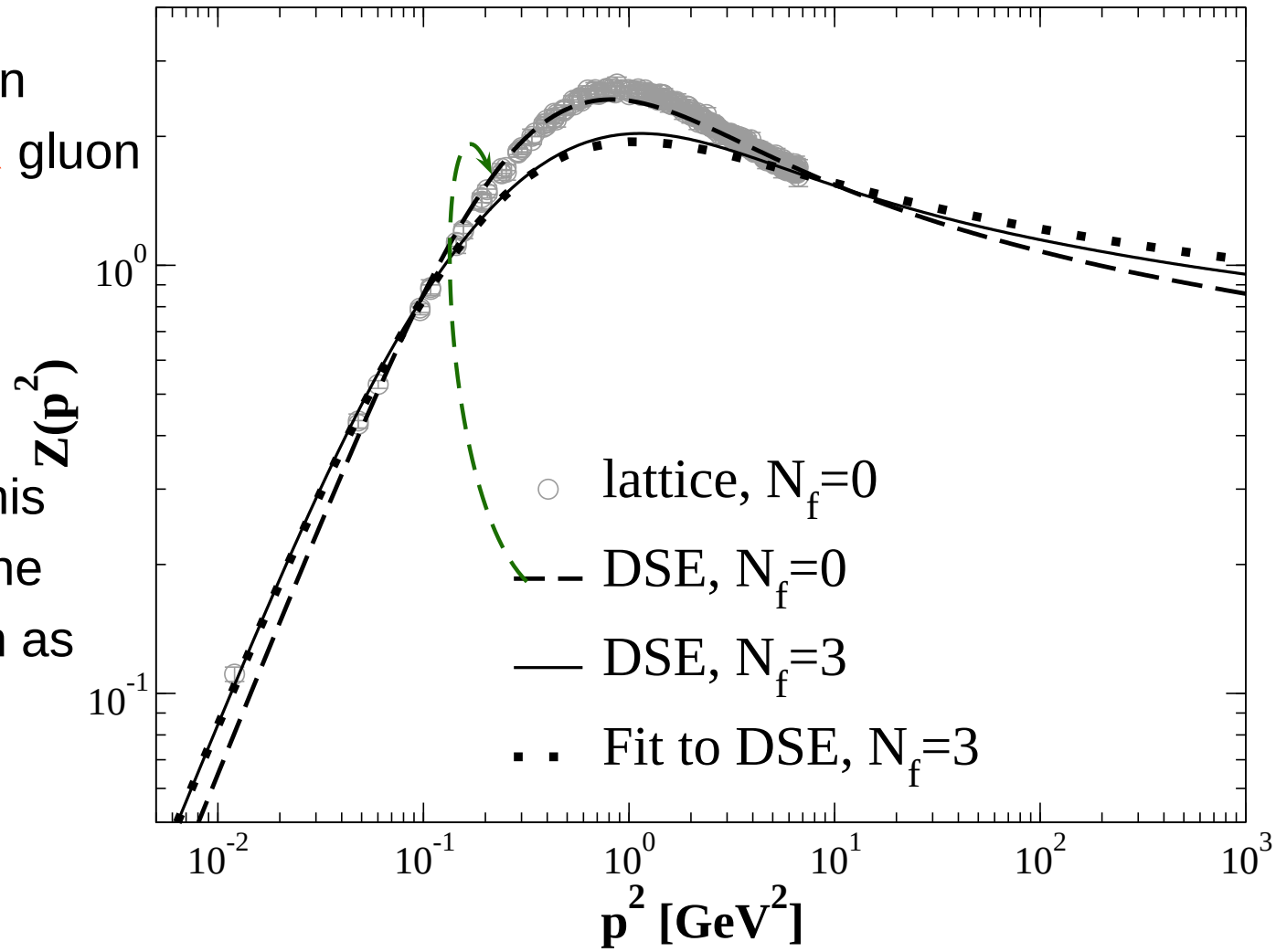

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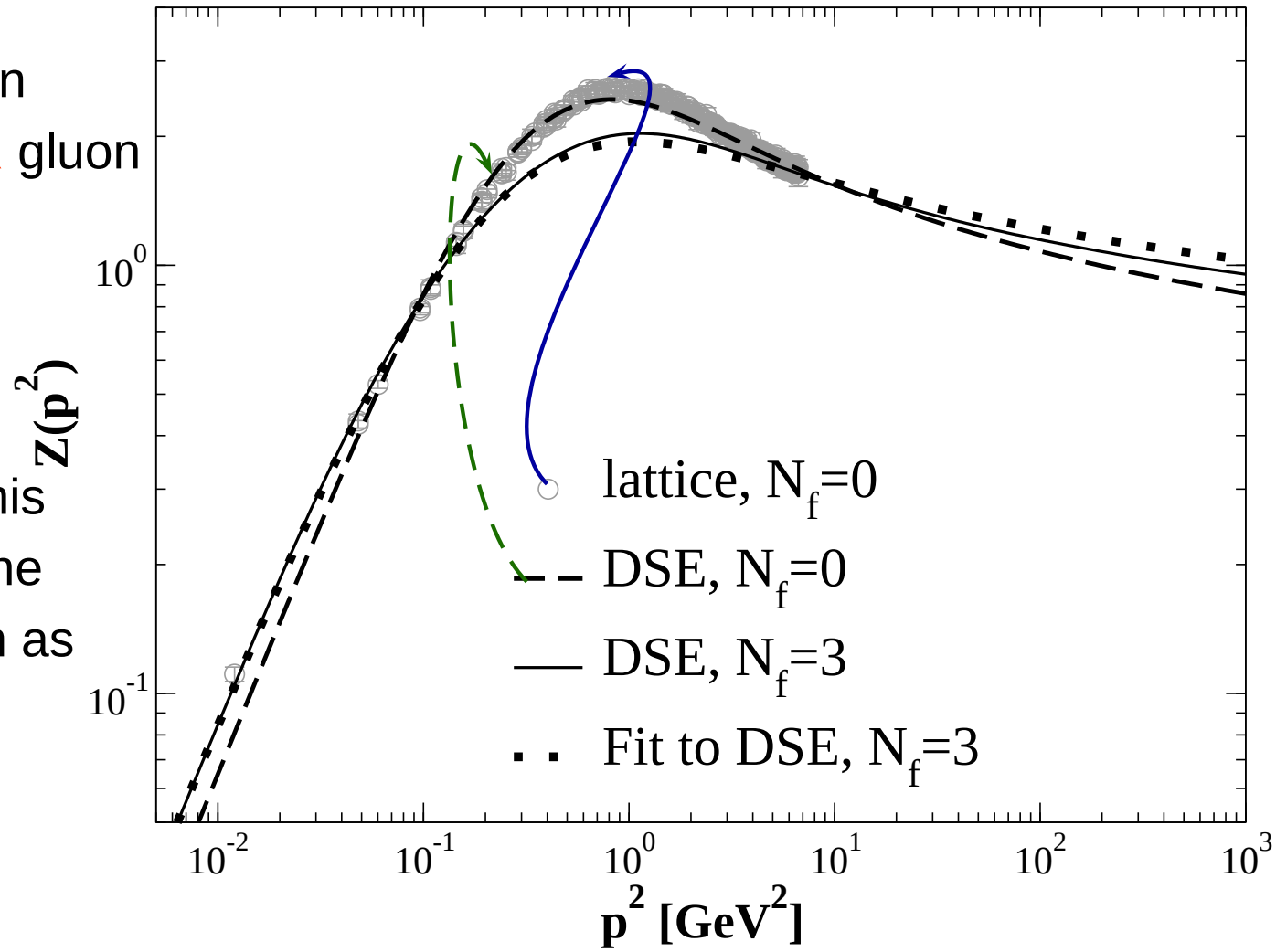

Consituent-quark σ-term

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark σ-term

$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\; (M^E)^2 := s\,|\,s = M(s)^2\,.$$

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- Renormalisation-group-invariant and determined from solutions of the gap equation

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- Unambiguous probe of impact of explicit chiral symmetry breaking on the mass function

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- Ratio $\boxed{\dfrac{\sigma_f}{M_f^E} = \dfrac{\text{EXPLICIT}}{\text{EXPLICIT} + \text{DYNAMICAL}}}$

measures effect of *EXPLICIT* chiral symmetry breaking on dressed-quark mass-function

cf. *SUM* of effects of *EXPLICIT* AND *DYNAMICAL CHIRAL SYMMETRY BREAKING*

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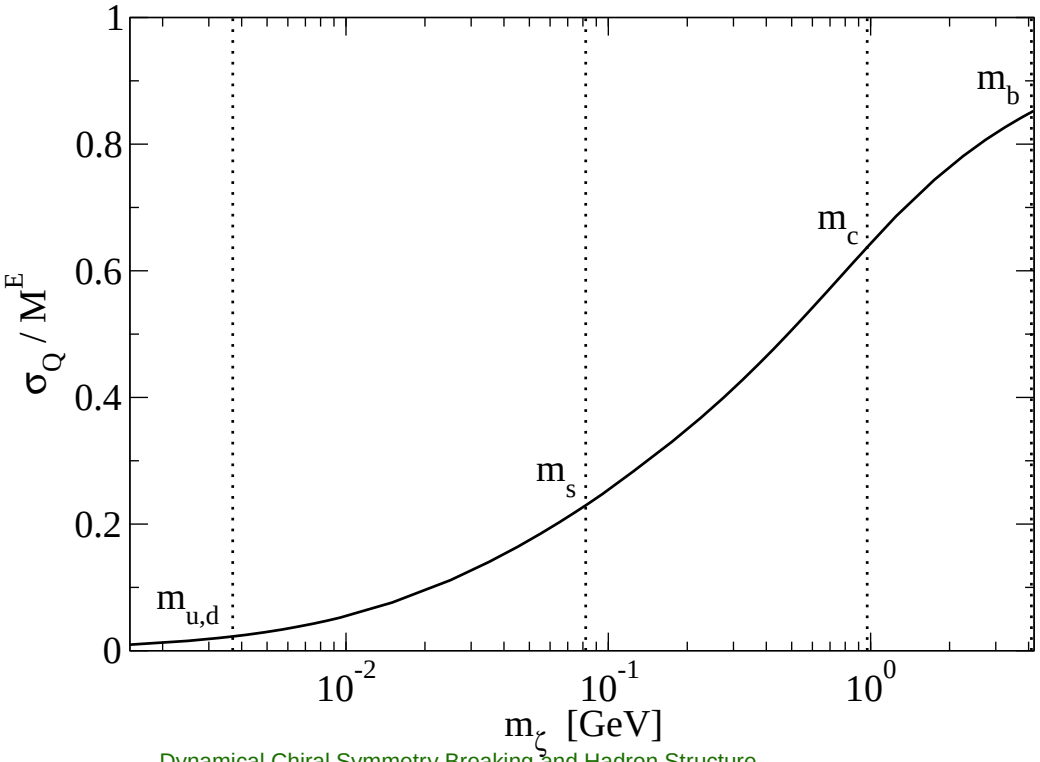

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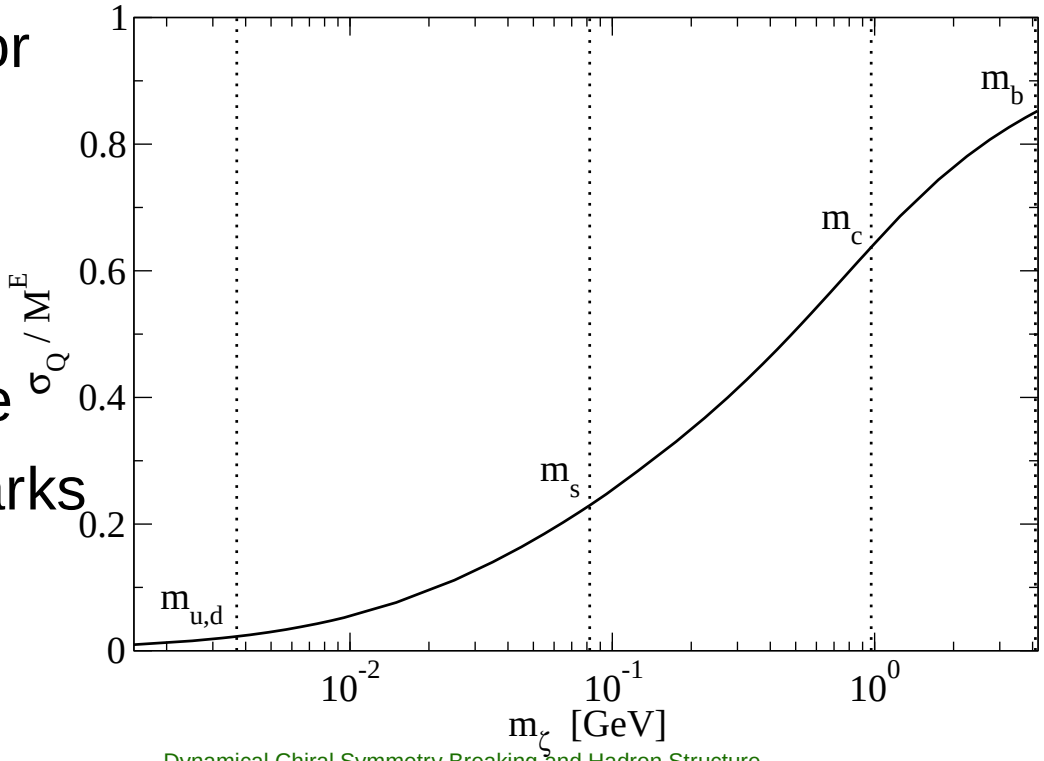
Obvious: ratio vanishes for light-quarks because magnitude of their constituent-mass owes primarily to DCSB. On the other hand, for heavy-quarks it approaches one.



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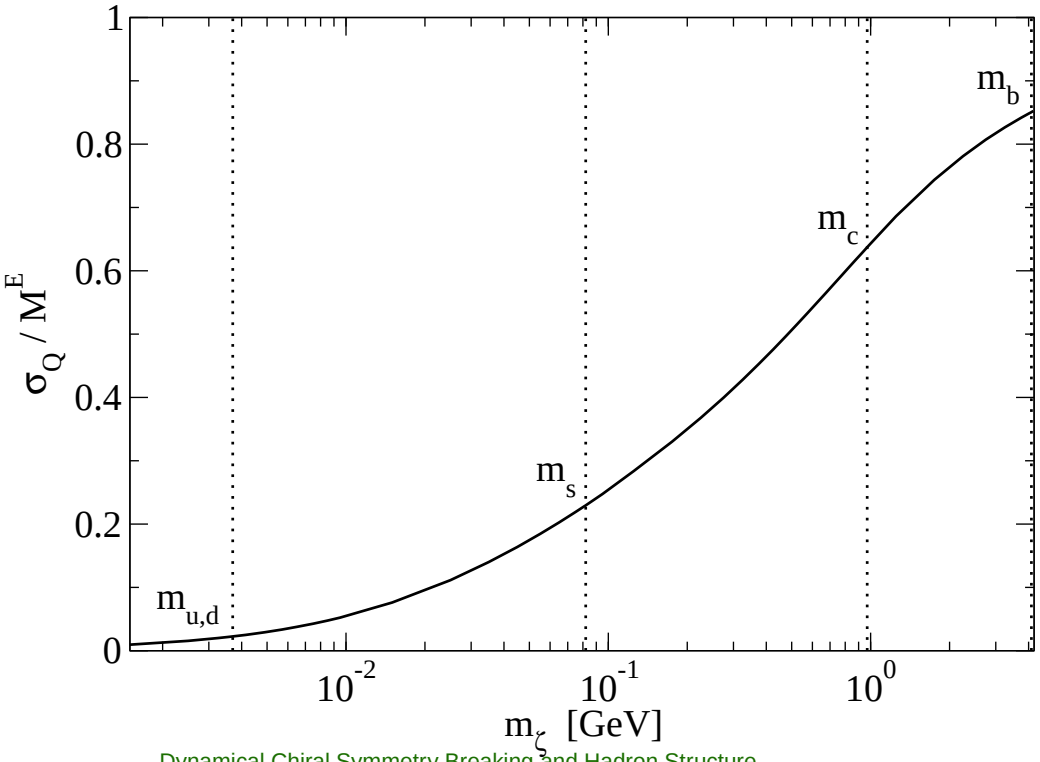
Essentially dynamical component of chiral symmetry breaking, and manifestation in all its order parameters, vanishes with increasing current-quark mass



Hadrons

- Established understanding of two- and three-point functions

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- What about bound states?

Hadrons

- Without bound states, Comparison with experiment is impossible

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- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

Hadrons

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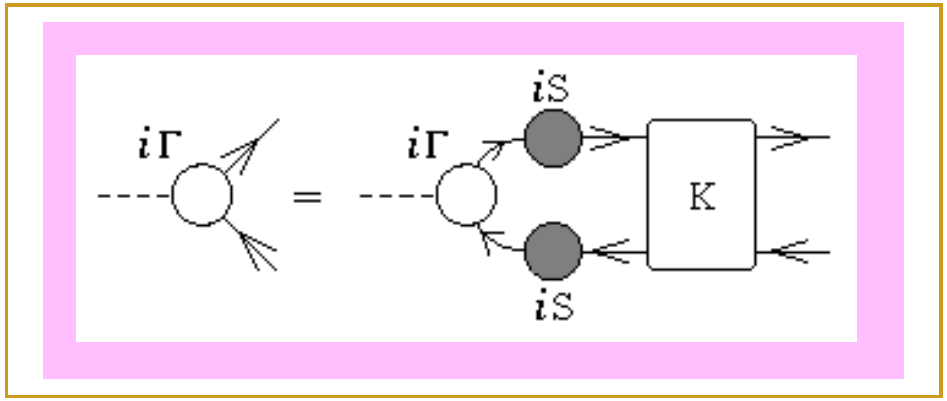

QFT Generalisation of Lippmann-Schwinger Equation.

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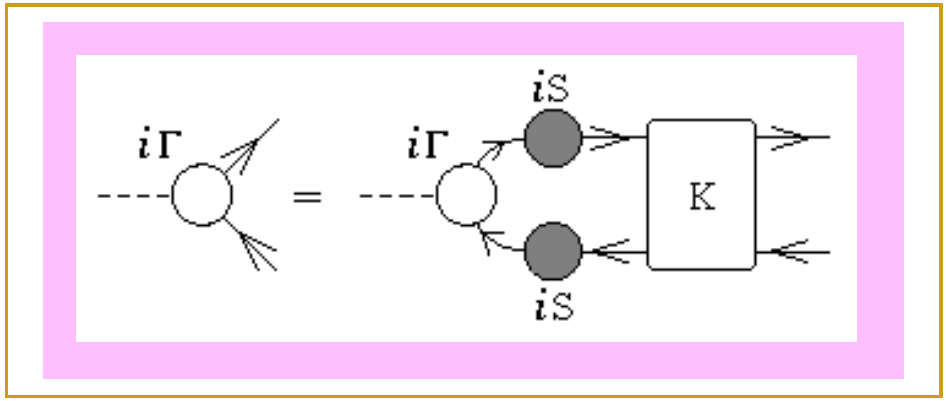

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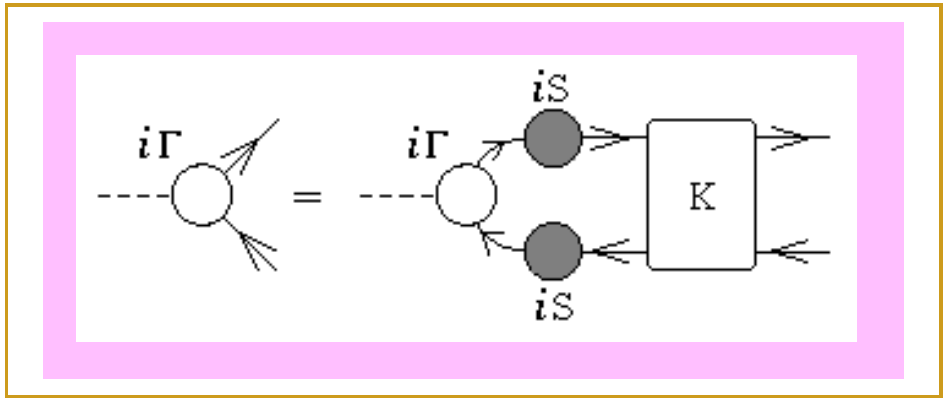

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or

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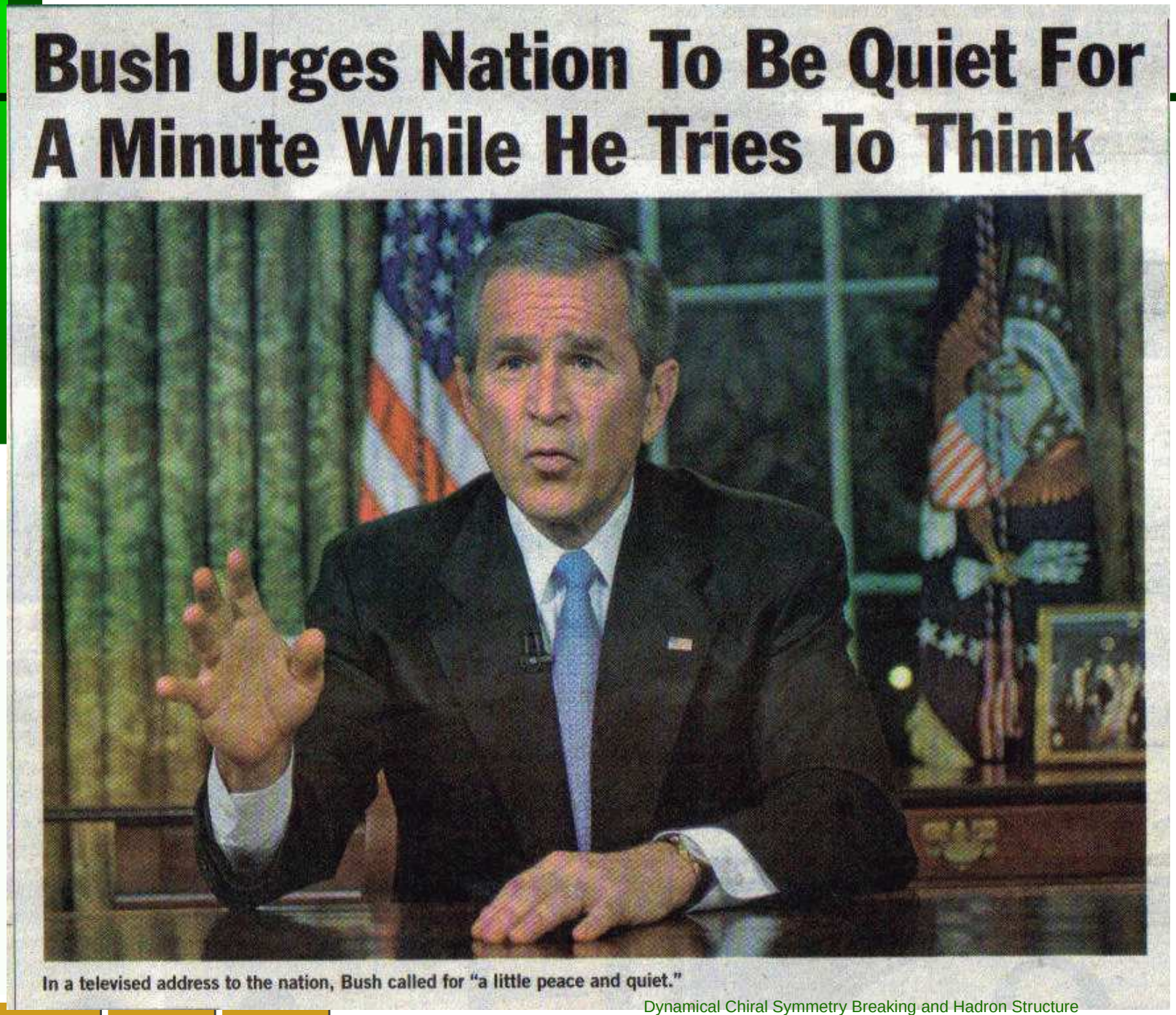

Bush Urges Nation To Be Quiet For A Minute While He Tries To Think

In a televised address to the nation, Bush called for "a little peace and quiet."

Bethe-Salpeter Kernel

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- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

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QFT Statement of Chiral Symmetry

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Satisfies BSE

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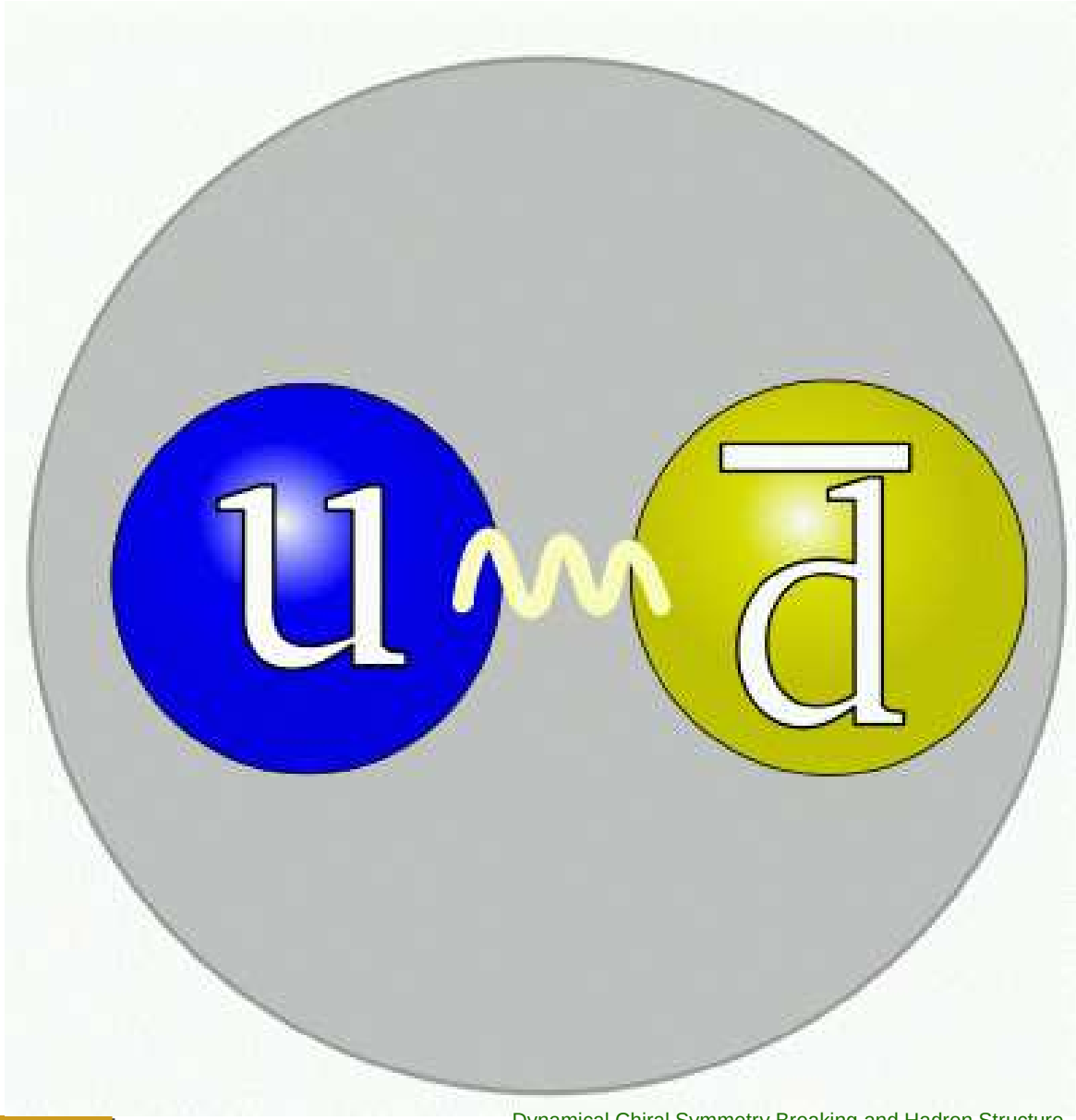
- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Pion and . . .

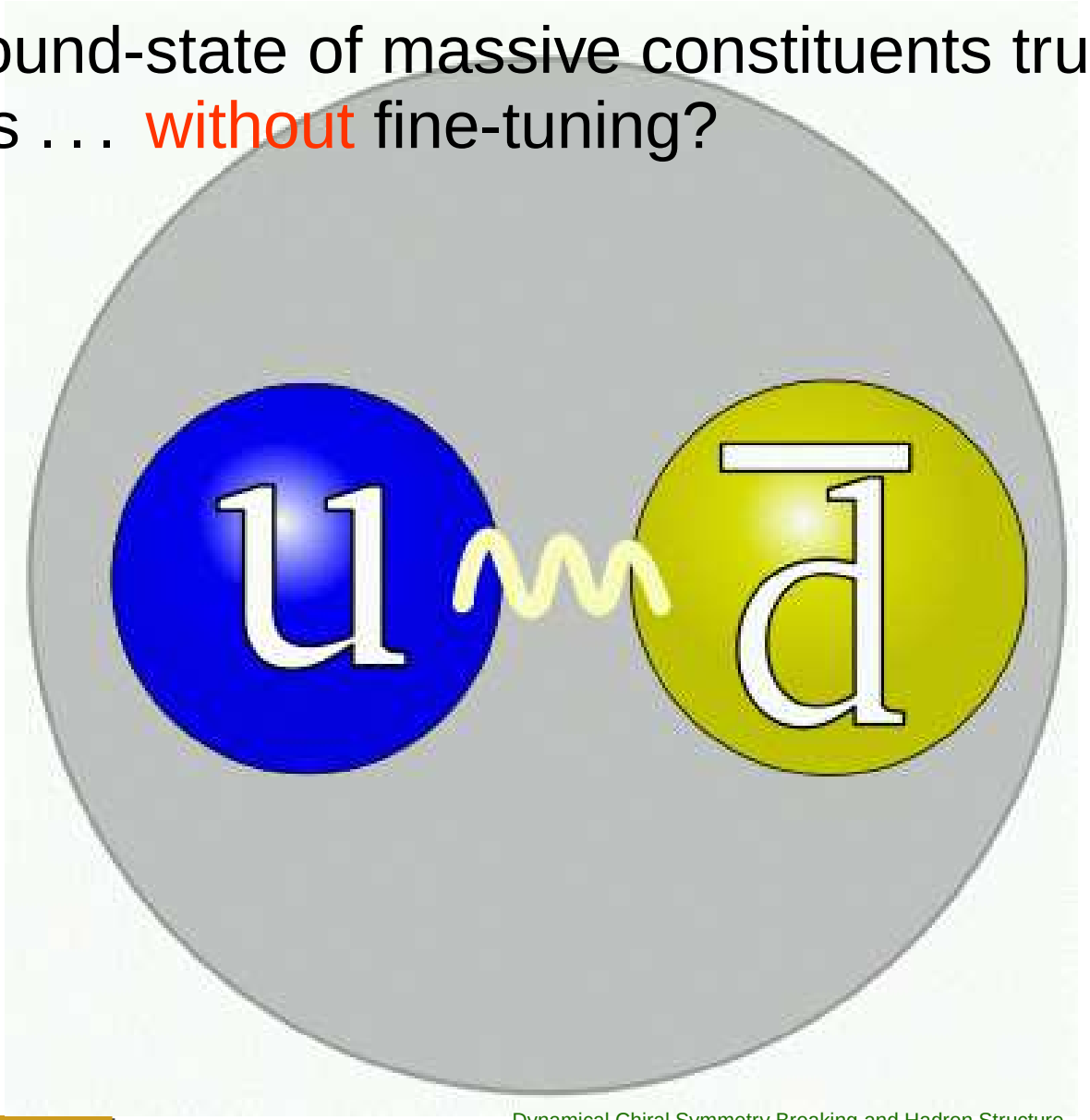
Pseudoscalar Mesons?



Pion and . . .

Pseudoscalar Mesons?

Can a bound-state of massive constituents truly be massless . . . without fine-tuning?



Radial Excitations & Chiral Symmetry

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(Maris, Roberts, Tandy nu-th/9707003)

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- Mass^2 of pseudoscalar hadron

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$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

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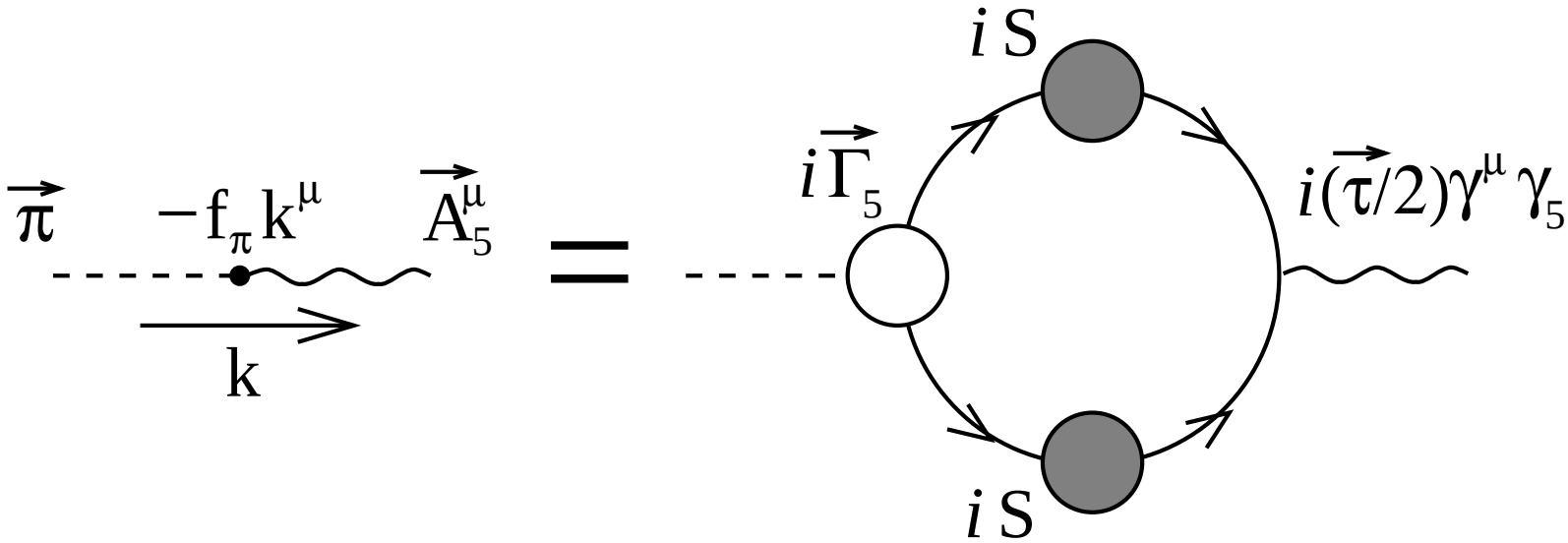
- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ ...GMOR relation, a corollary

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- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

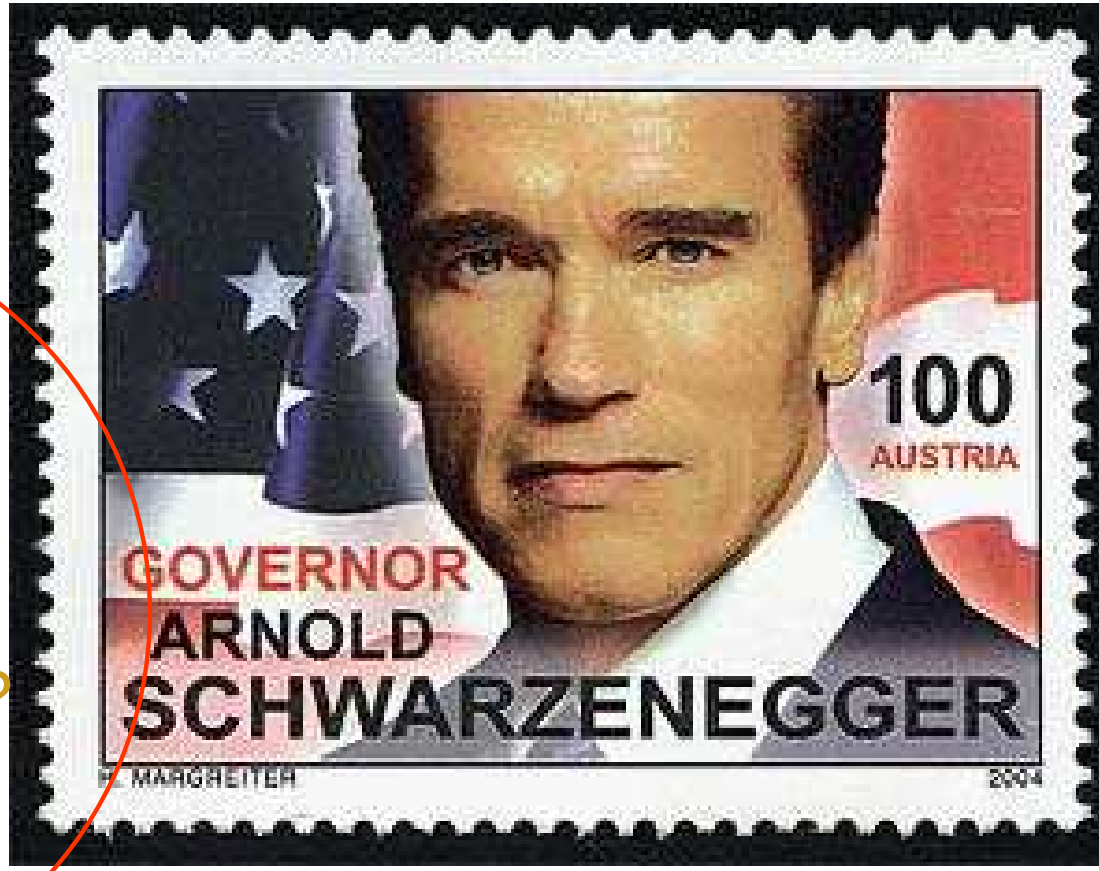
Hence, $m_H \propto m_q$

... QCD Proof of Potential Model result

Andreas Krassnigg

FWF “Erwin Schrödinger Fellow,” ANL 2003-2005

Almost Blood
Relative of Arnold
. . . Future
President?
. . . Executioner?



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- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

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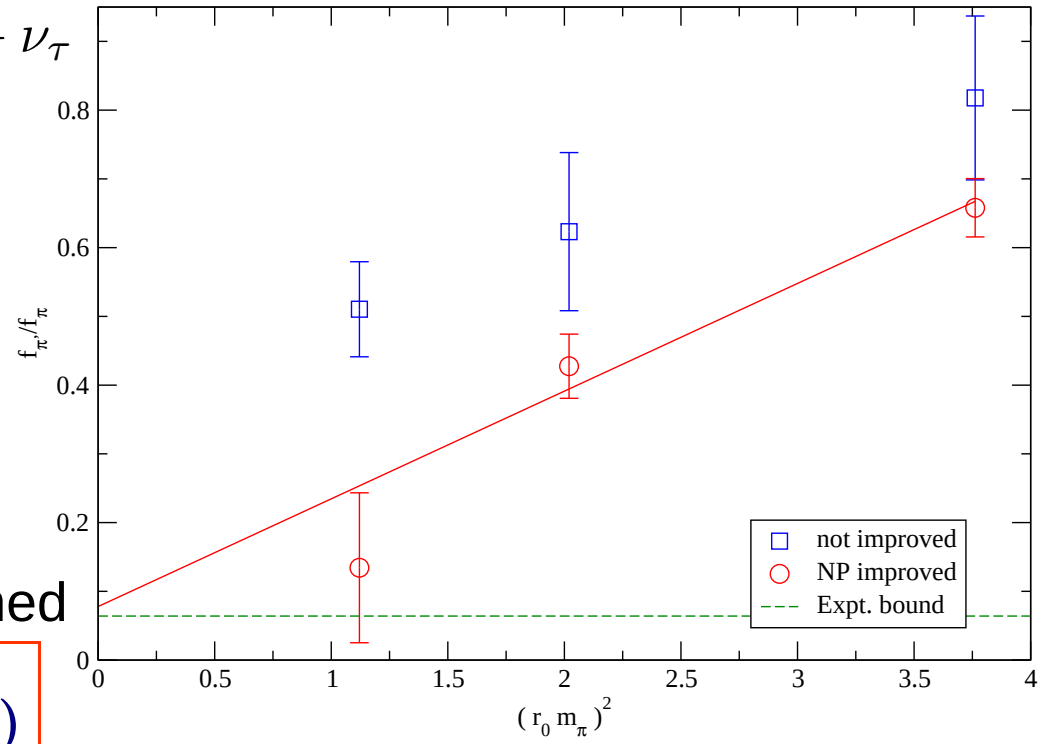
$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$



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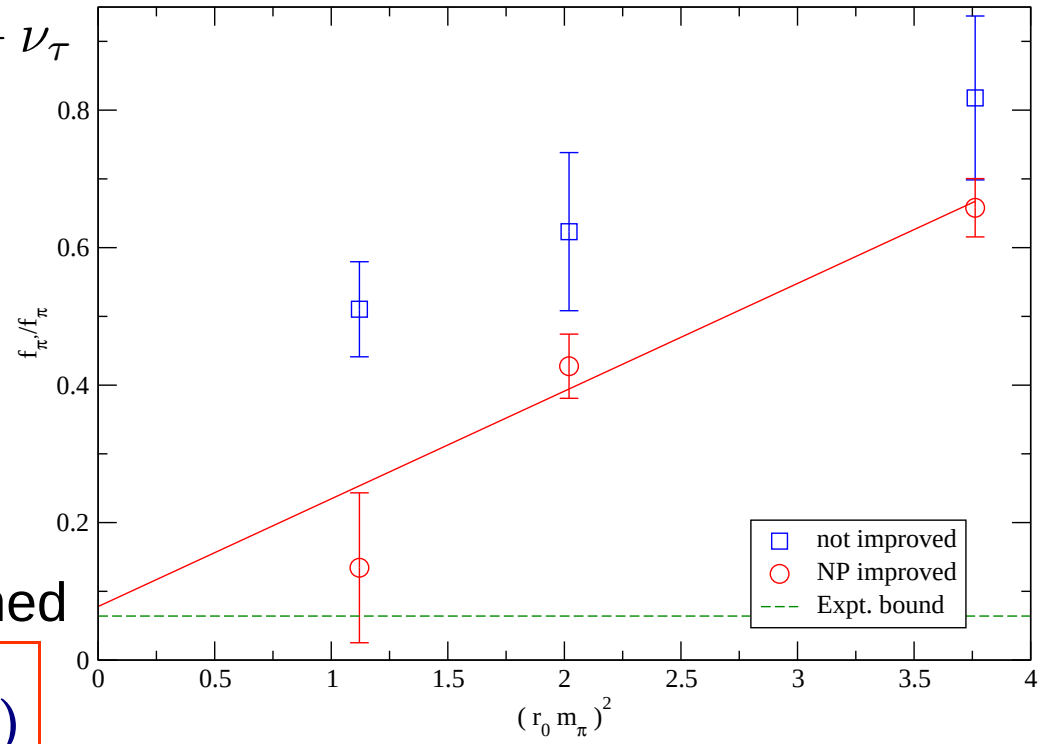

- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

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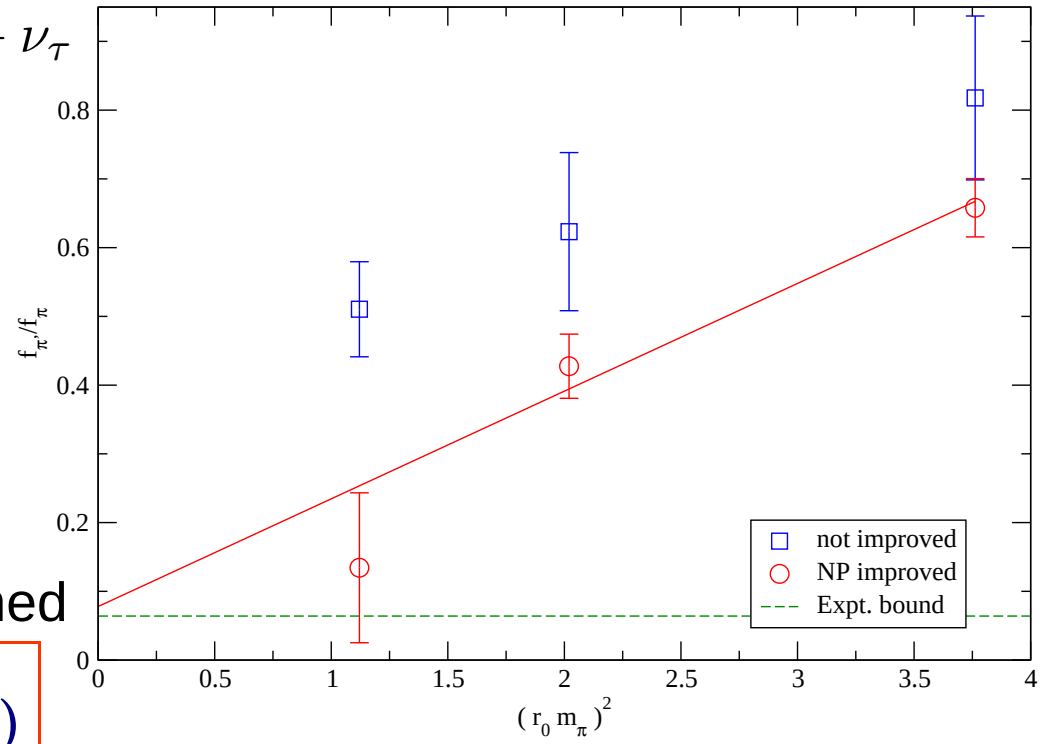

- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Radial Excitations

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- Spectrum contains 3 pseudoscalars $[I^G(J^P)L = 1^-(0^-)S]$

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

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- Consituent-Q Model: 1st three members of $\boldsymbol{n\,{}^1S_0}$ trajectory; i.e., ground state plus radial excitations?

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- But $\pi(1800)$ is narrow ($\Gamma = 207 \pm 13$) & decay pattern might indicate some "flux tube angular momentum" content:
  $S_{\bar{Q}Q} = 1 \oplus L_F = 1 \Rightarrow J = 0$
  & $L_F = 1 \Rightarrow\ ^3S_1 \oplus\ ^3S_1$ ($\bar{Q}Q$) decays suppressed?

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- NSAC Long-Range Plan, 2002: . . . an understanding of confinement "remains one of the greatest intellectual challenges in physics"

Pion ... $J = 0$ but ...

- Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.

Pion ... $J = 0$ but ...

- Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.

Pion ... $J = 0$ but ...

- Pseudoscalar meson Bethe-Salpeter amplitude

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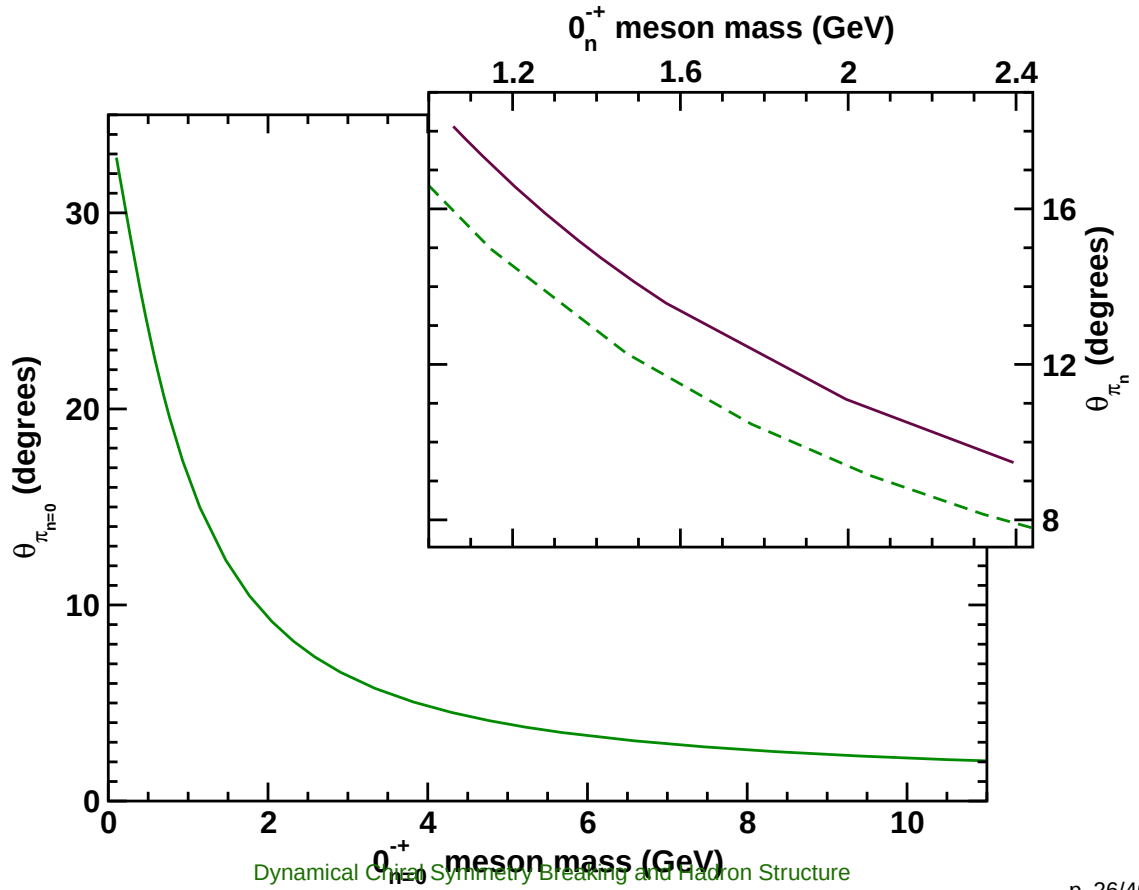

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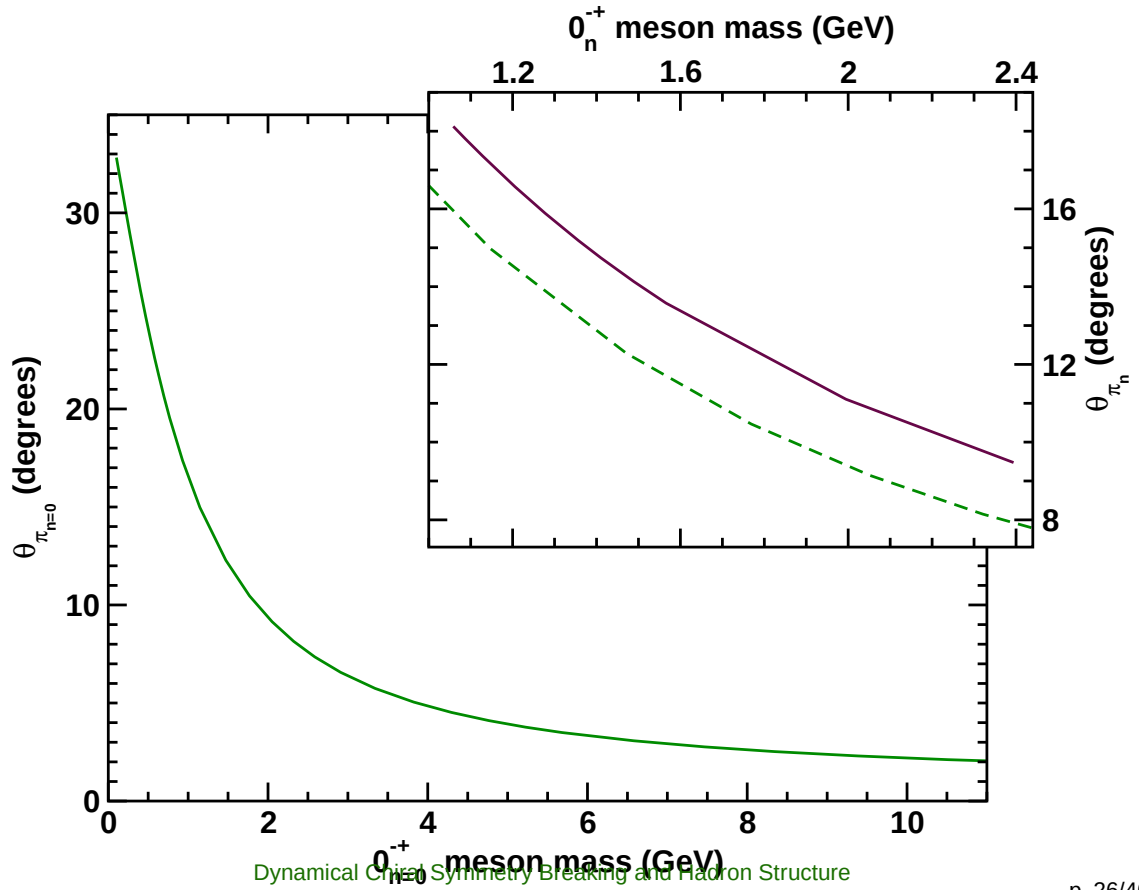
L is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.



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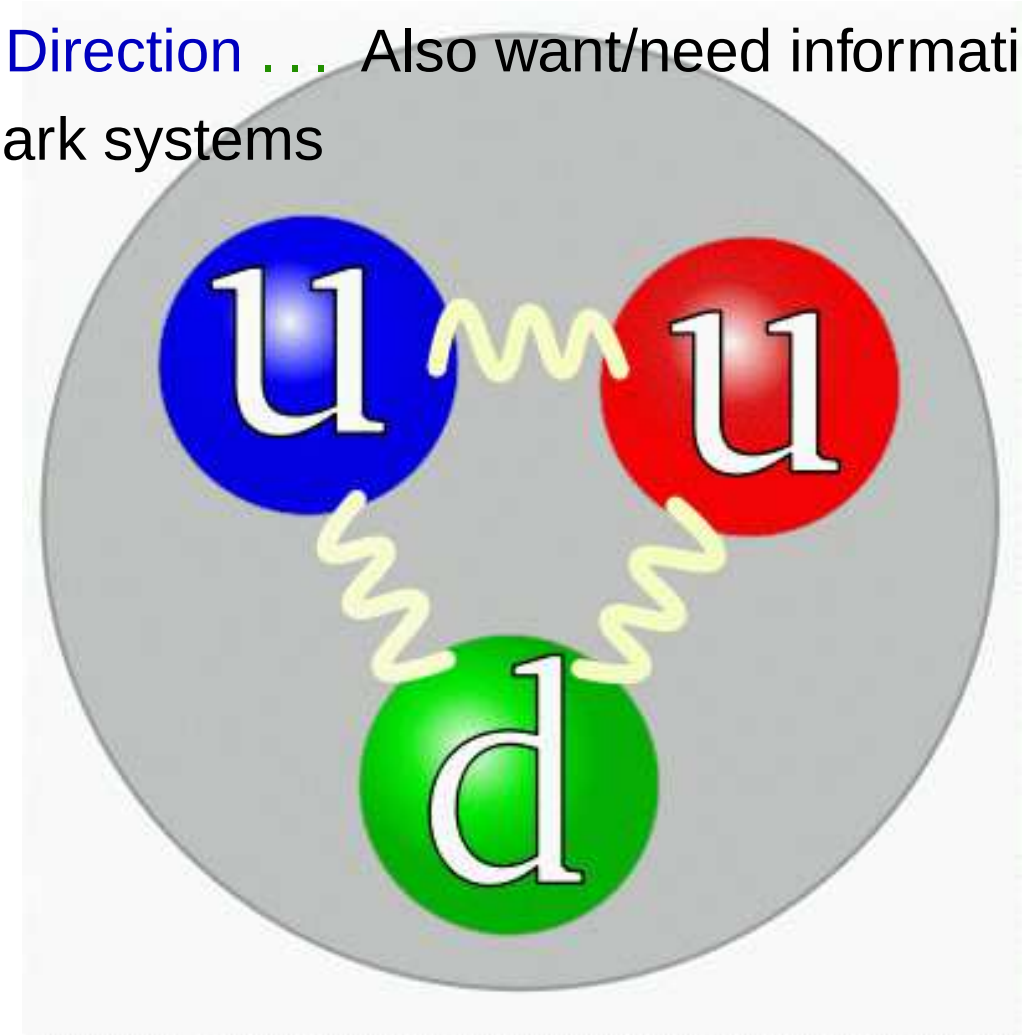
- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

New Challenges

- Another Direction . . . Also want/need information about three-quark systems



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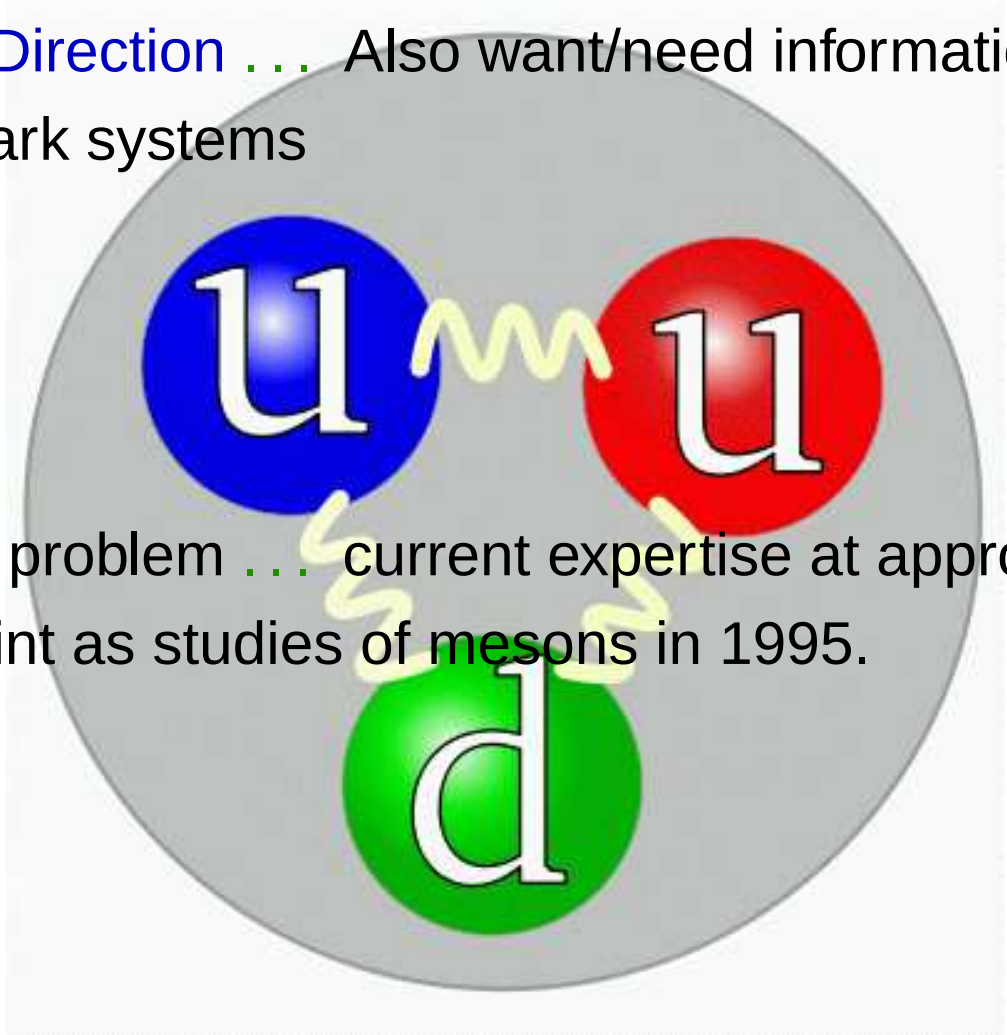

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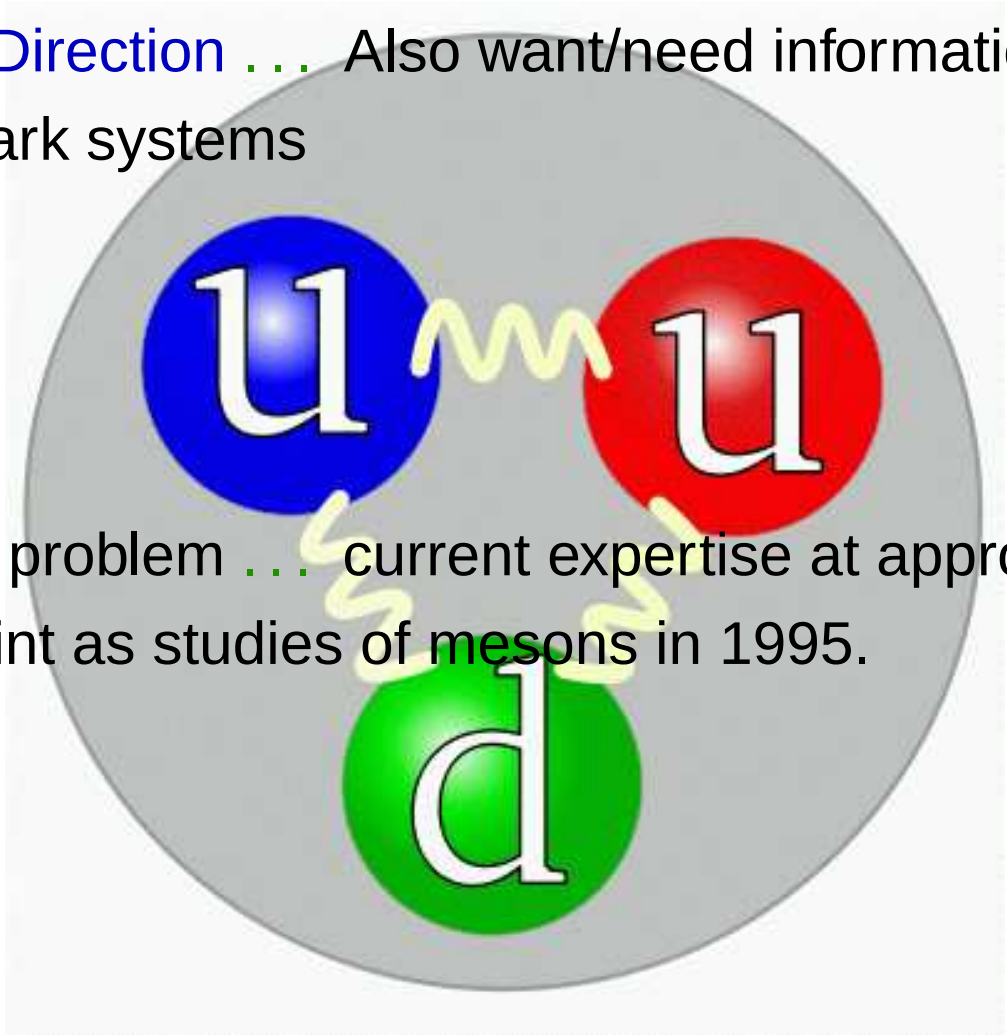

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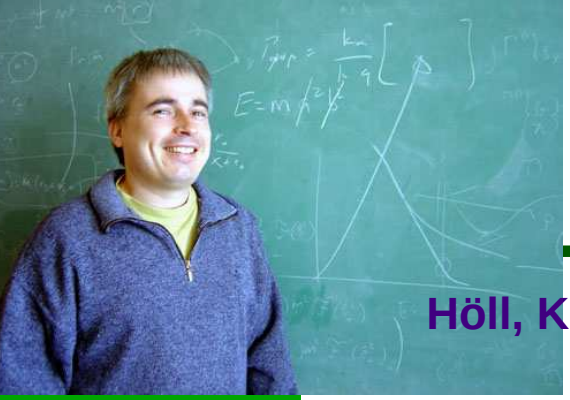
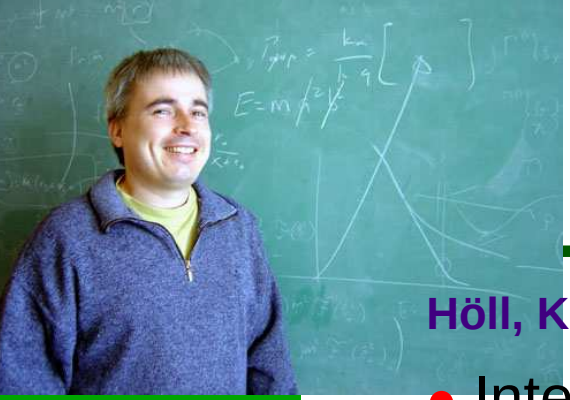
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.



Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033



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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

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- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

Faddeev equation

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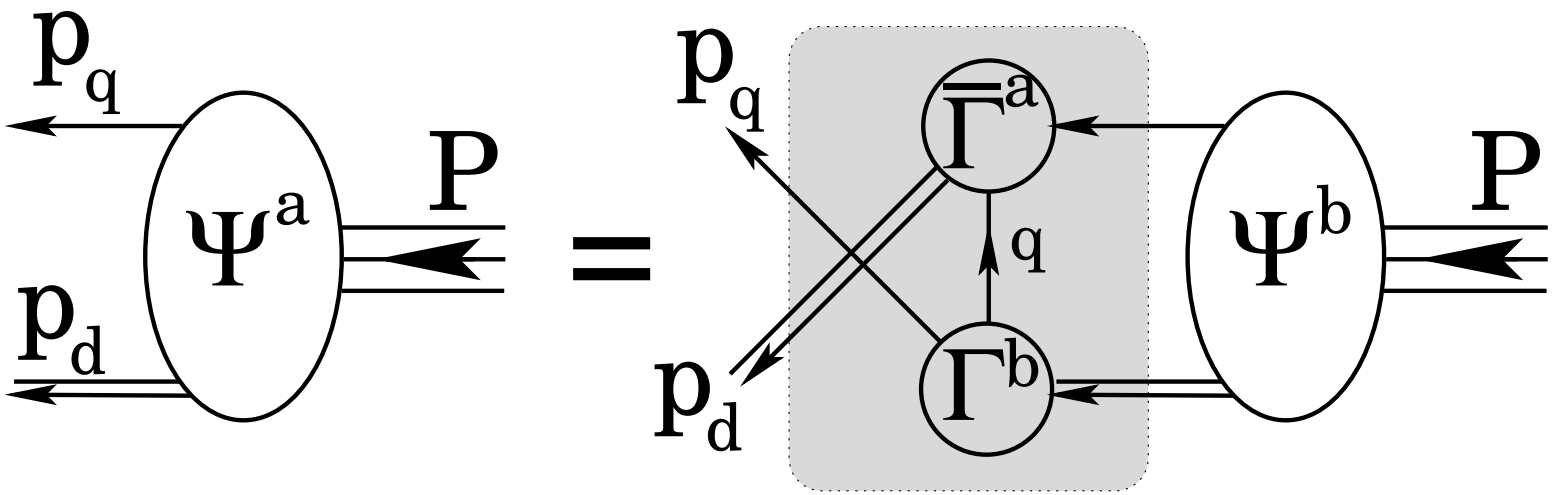

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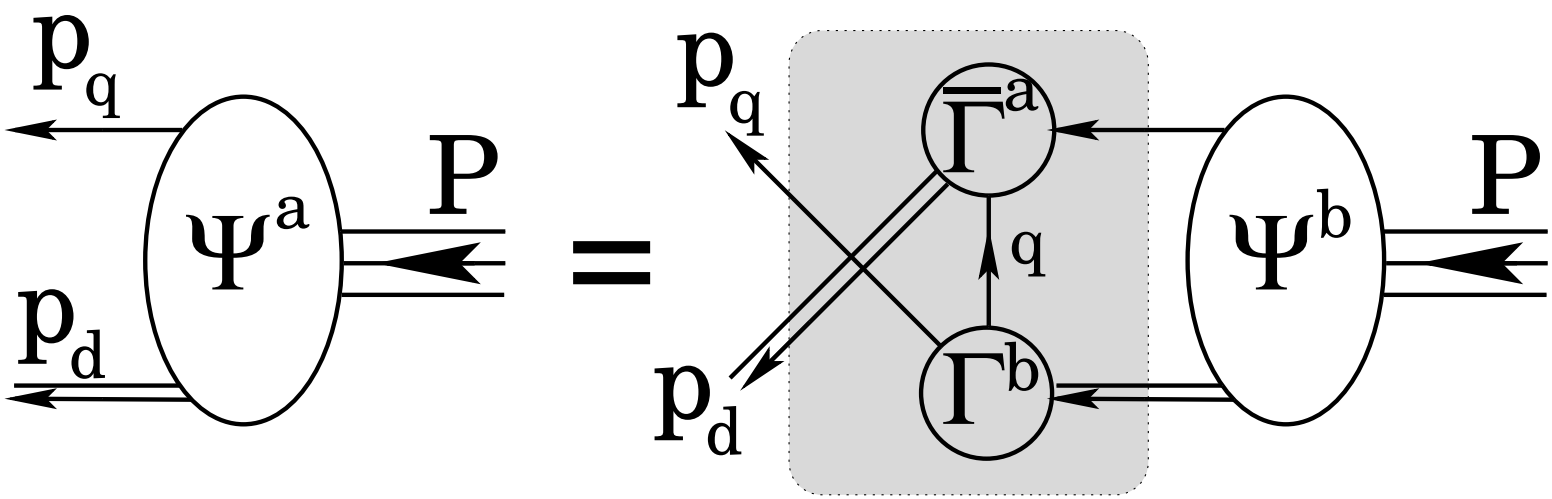
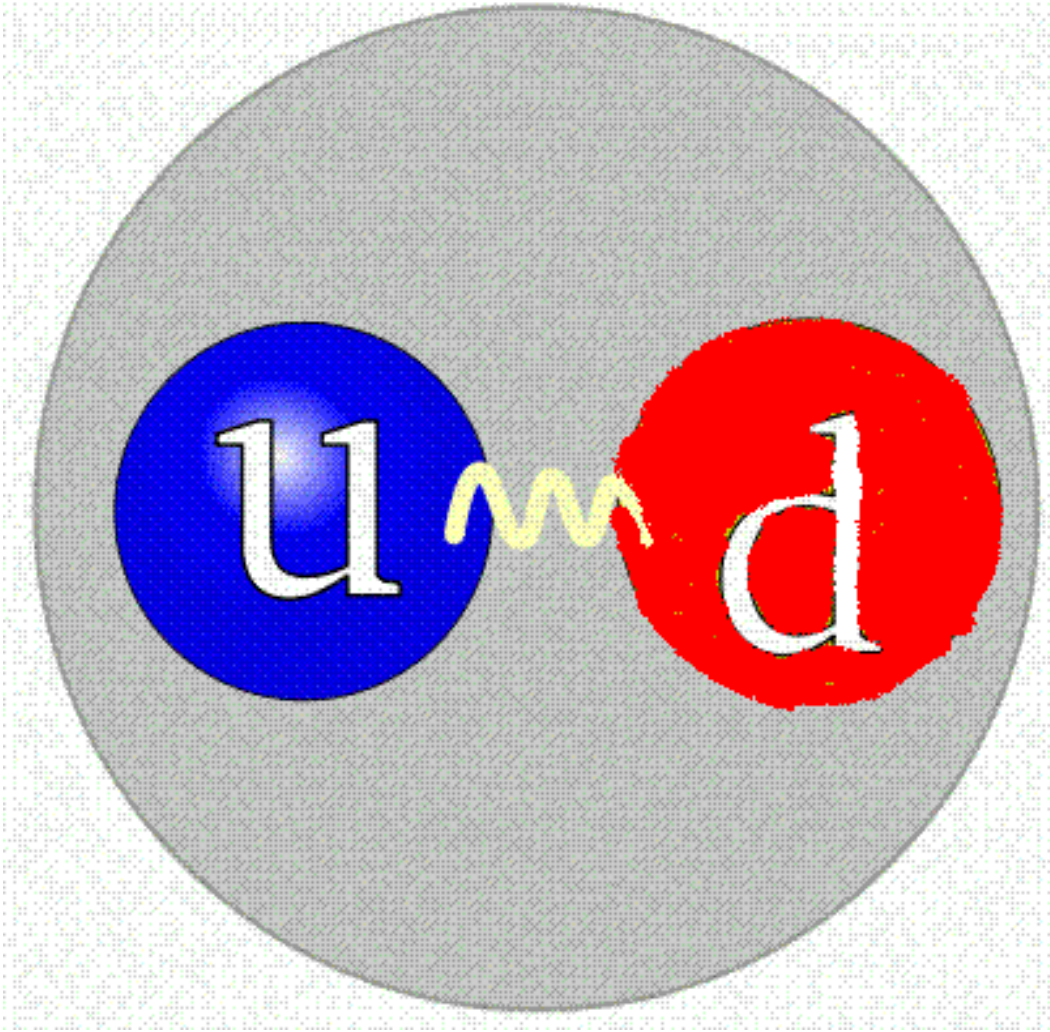


- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Diquark correlations



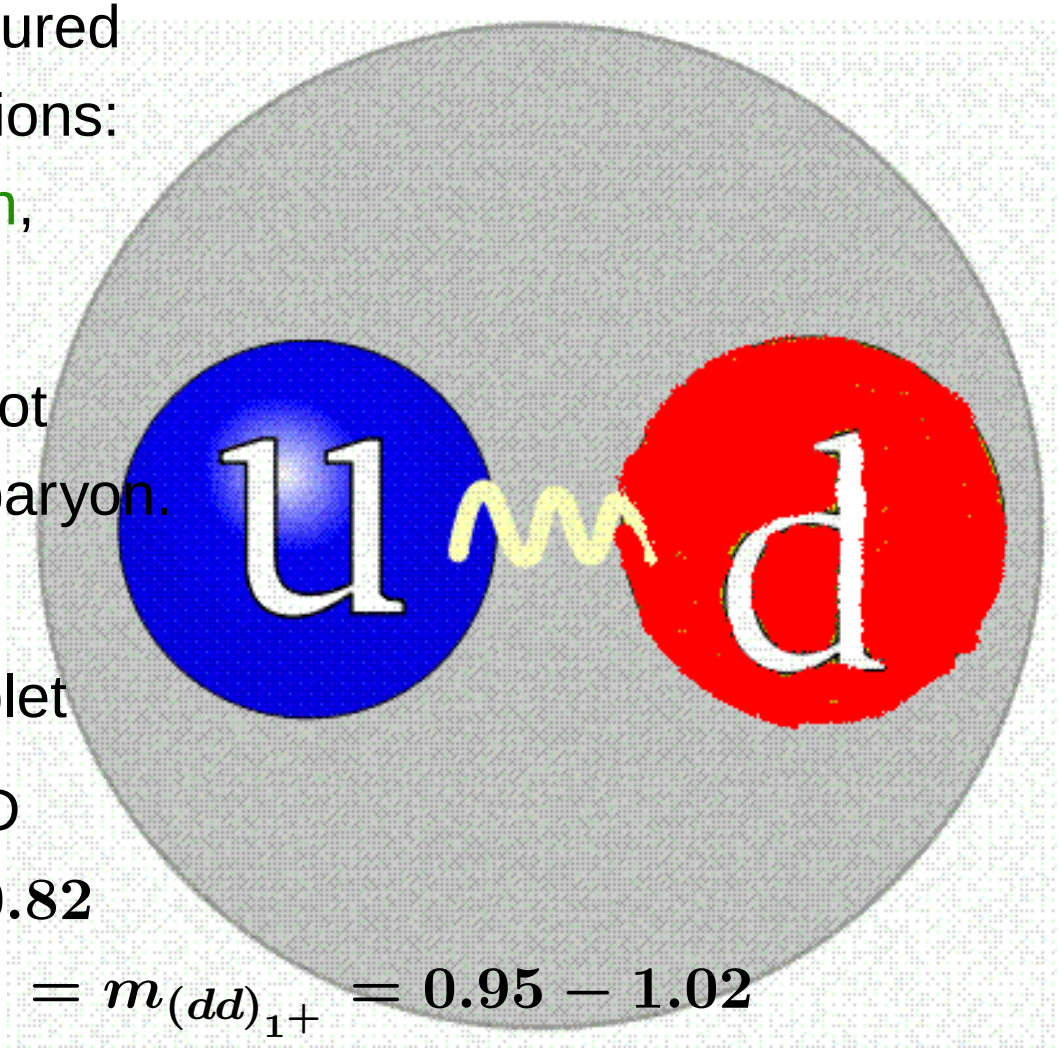
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined . . . Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

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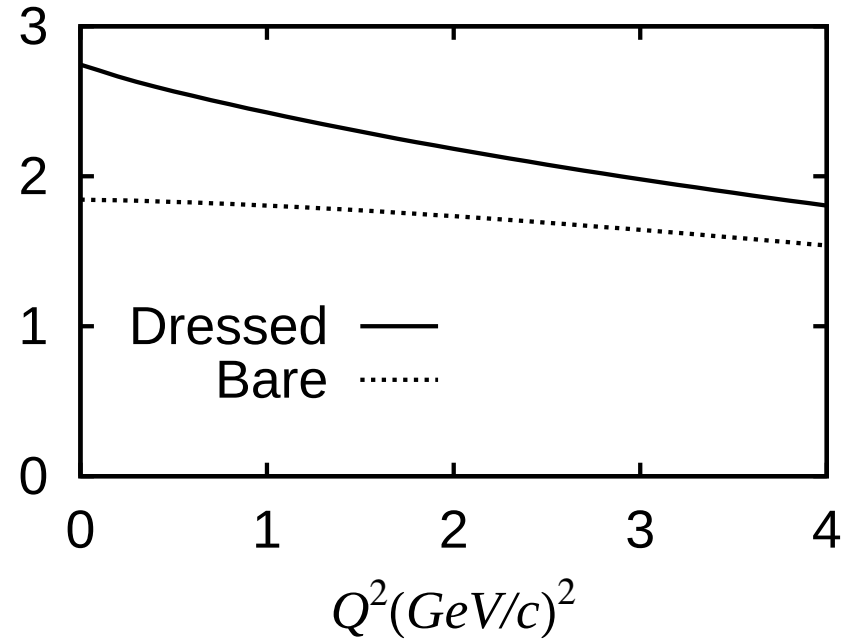
Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



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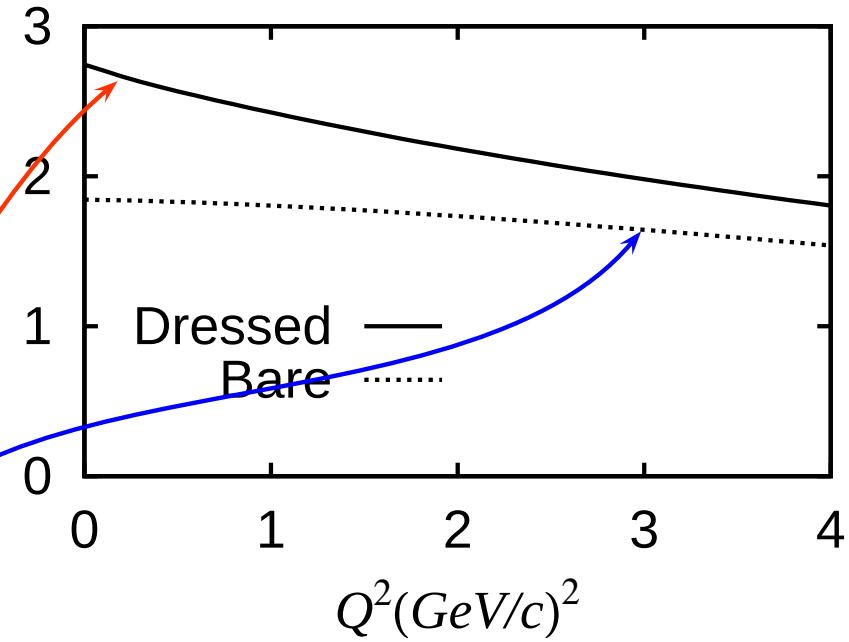

Quark Core

- Responsible for only 2/3 of result at small Q^2
- Dominant for $Q^2 >$2 – 3 GeV^2

Results: Nucleon and Δ Masses

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Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
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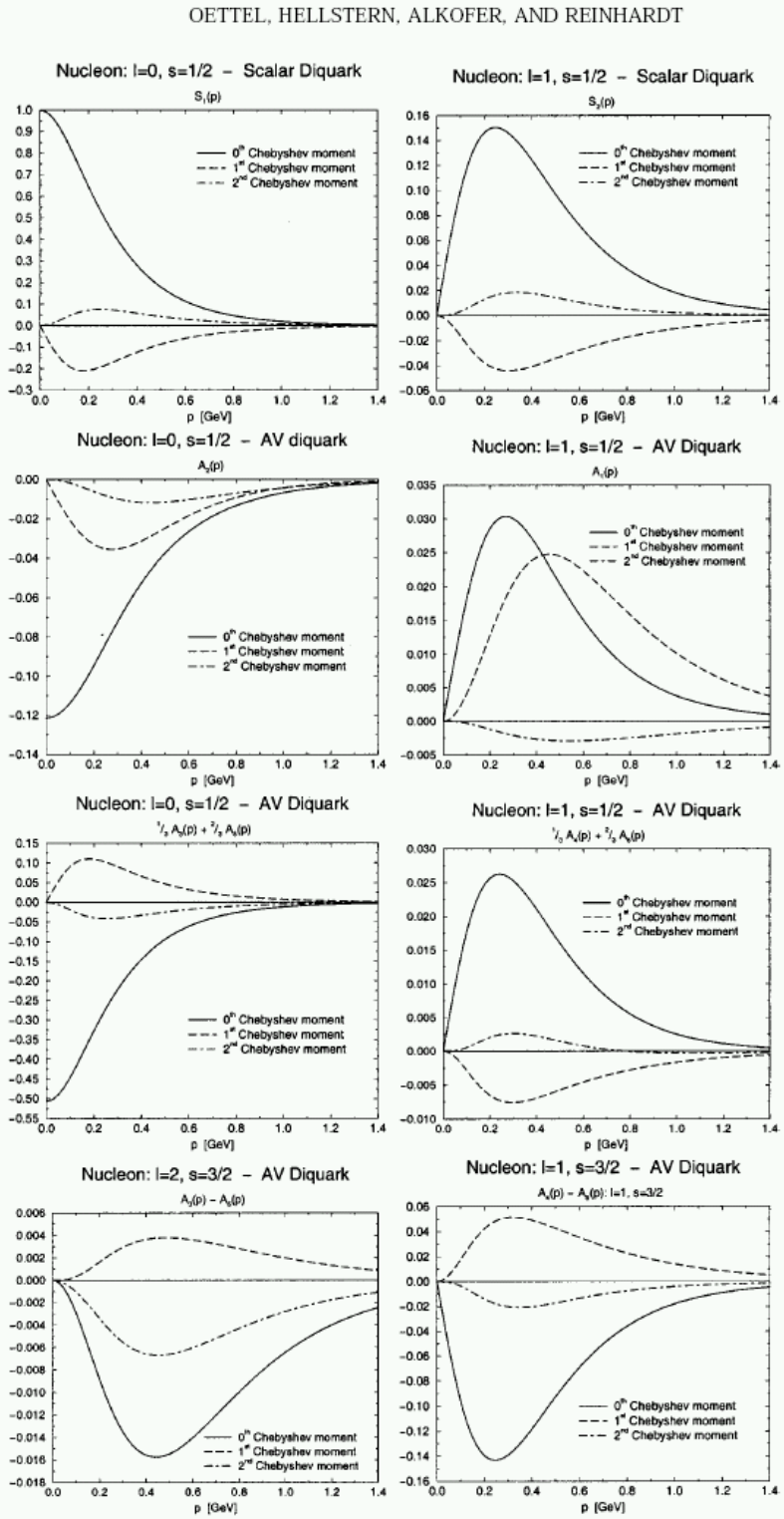
- Constructive Interference: 1^{++}-diquark $+ \partial_\mu \pi$

Angular Momentum Rest Frame



M. Oettel, *et al.*
nucl-th/9805054

Crude estimate based on magnitudes $\Rightarrow$ probability for a u-quark to carry the proton's spin is $P_{u\uparrow} \sim 80\,\%$, with $P_{u\downarrow} \sim 5\,\%$, $P_{d\uparrow} \sim 5\,\%$, $P_{d\downarrow} \sim 10\,\%$.

Hence, by this reckoning $\sim 30\%$ of proton's rest-frame spin is located in dressed-quark angular momentum.

Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

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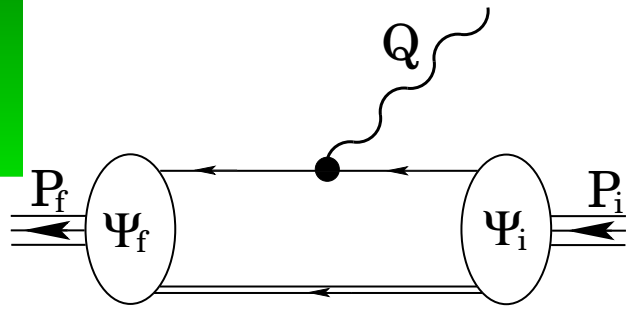
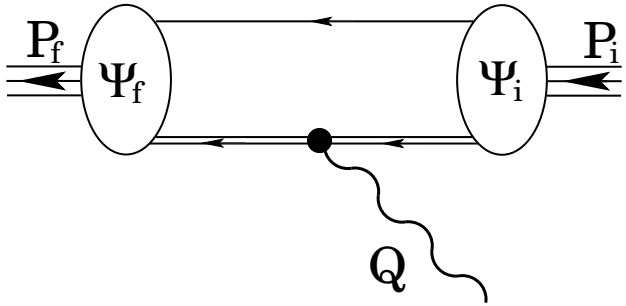
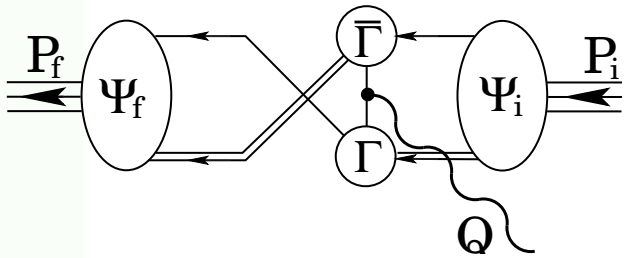
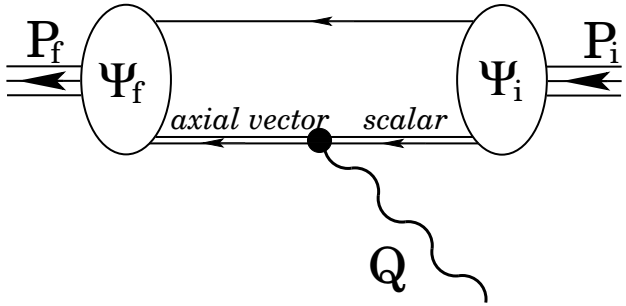
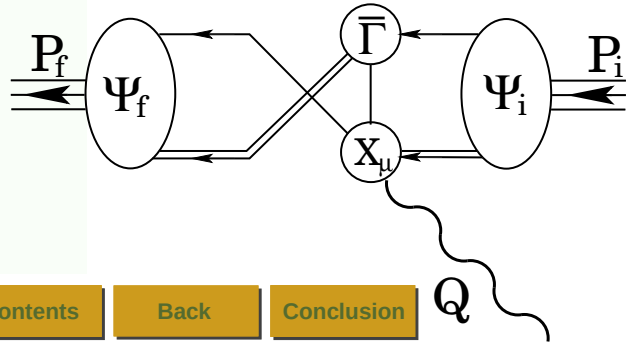
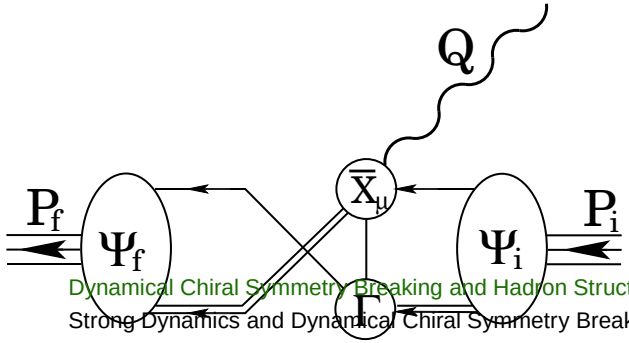




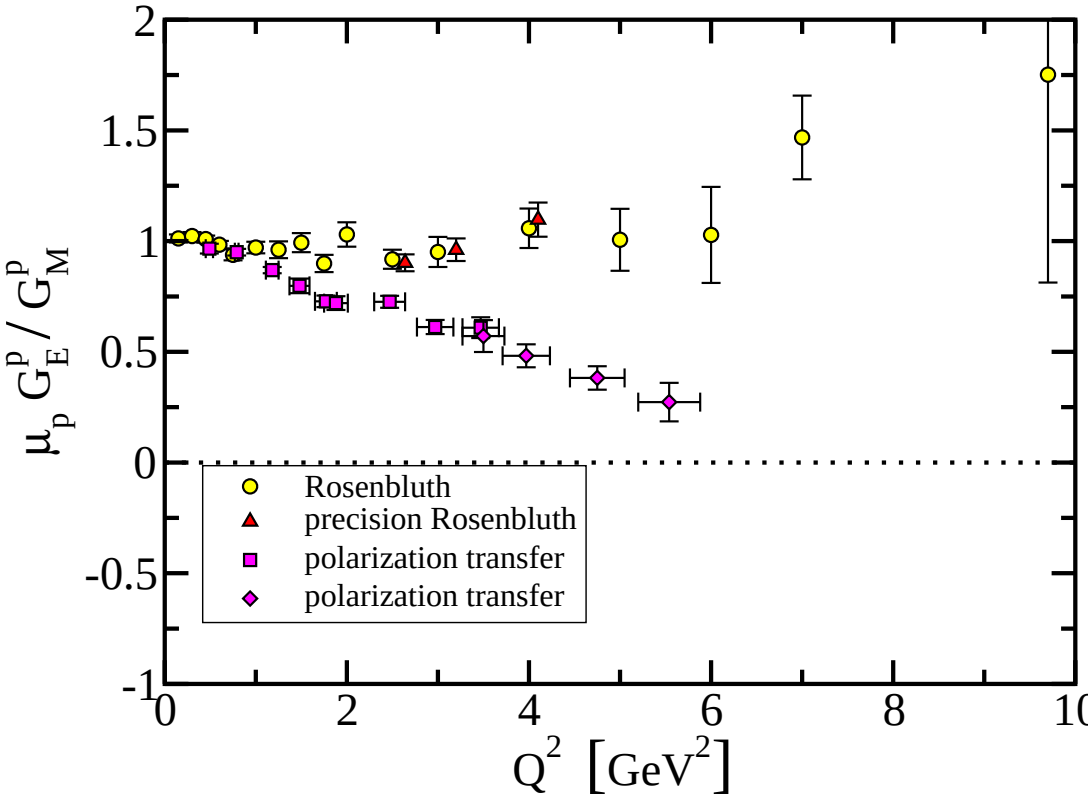








Form Factor Ratio: GE/GM



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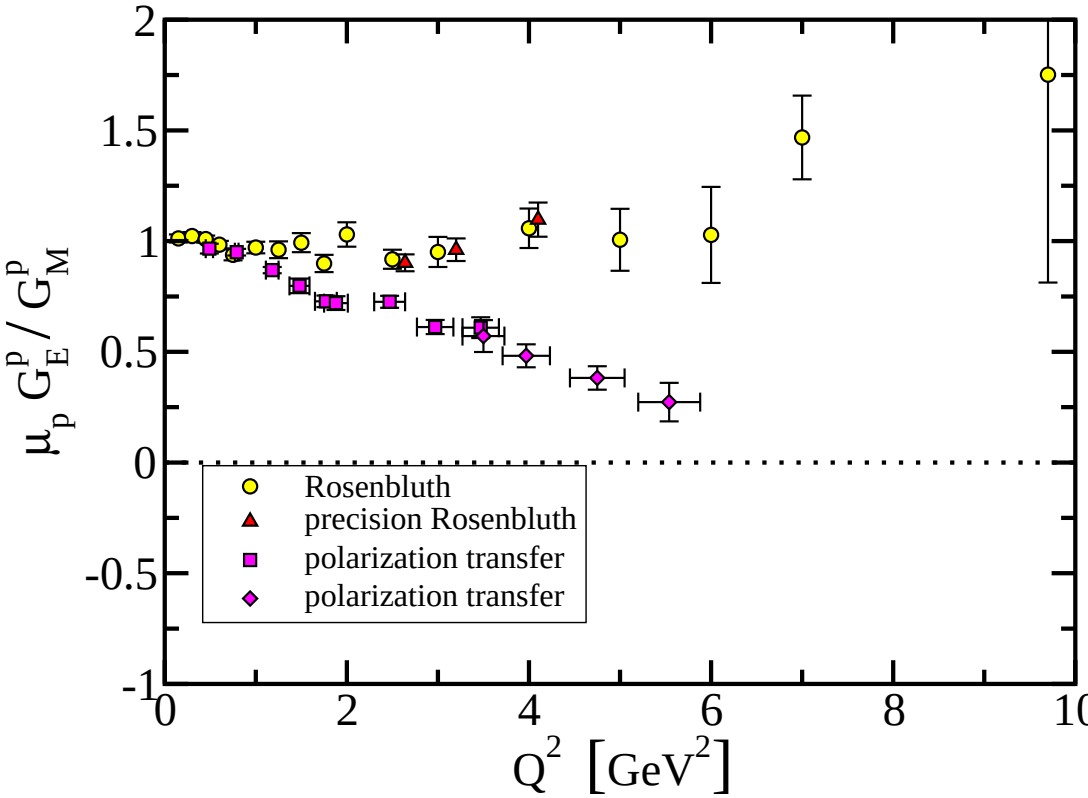

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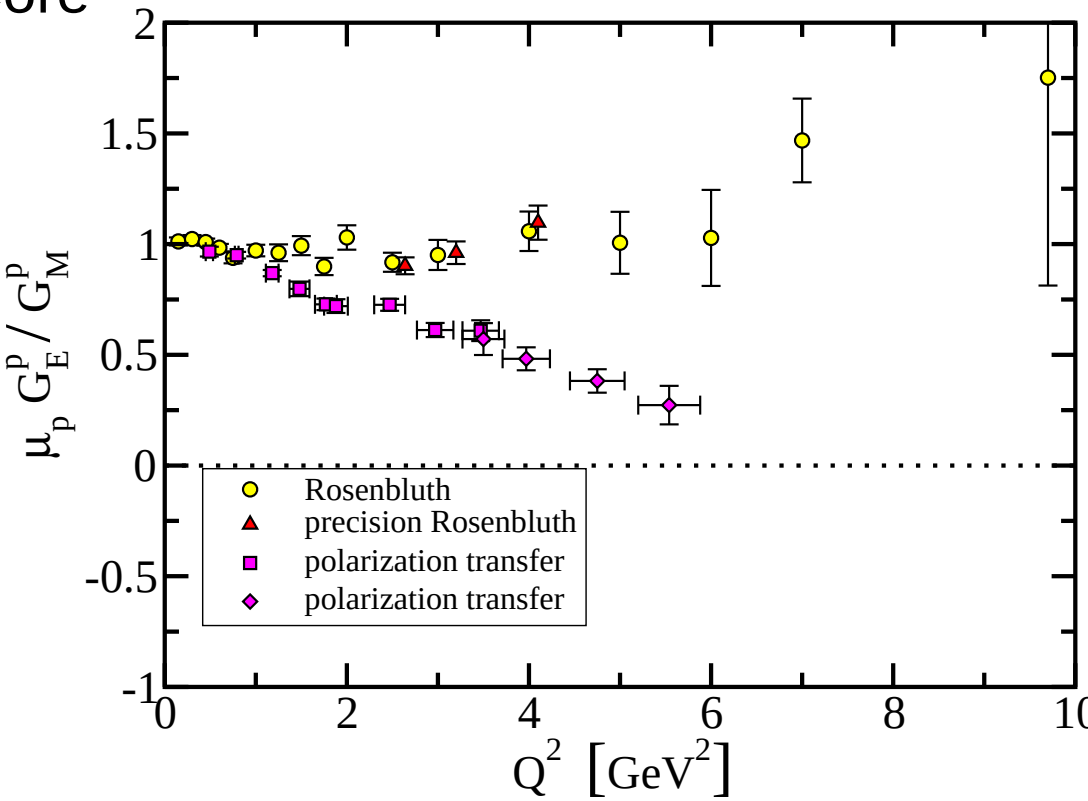

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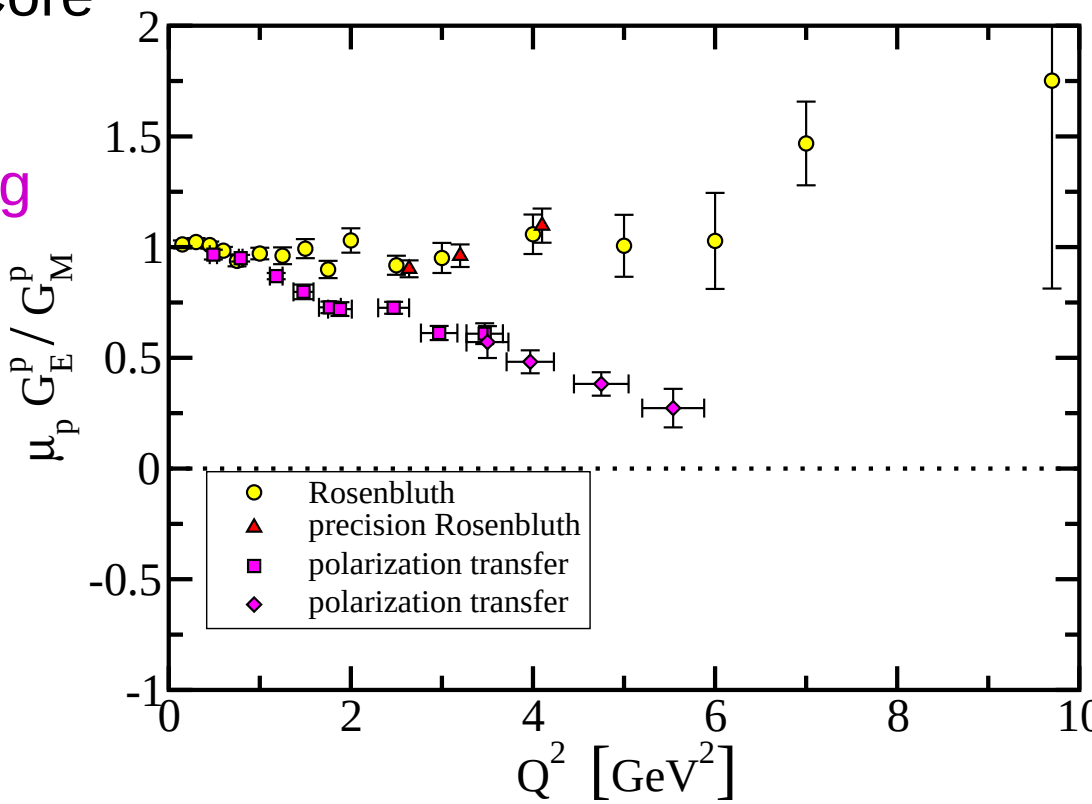

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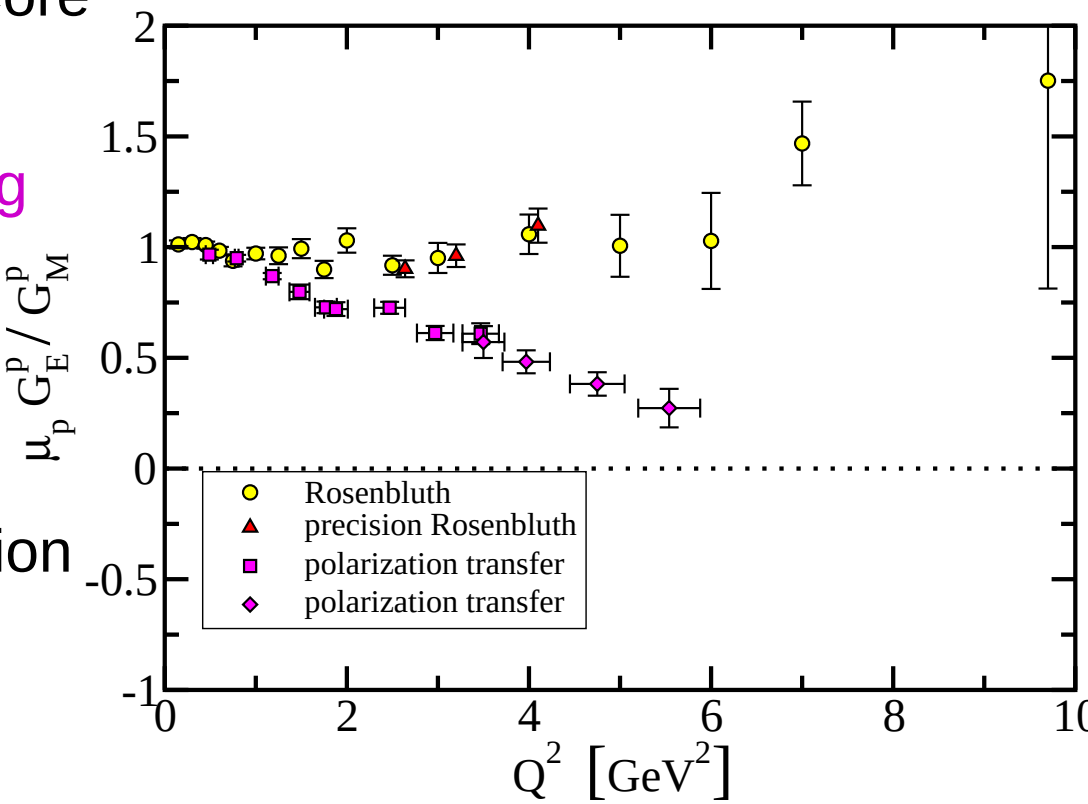

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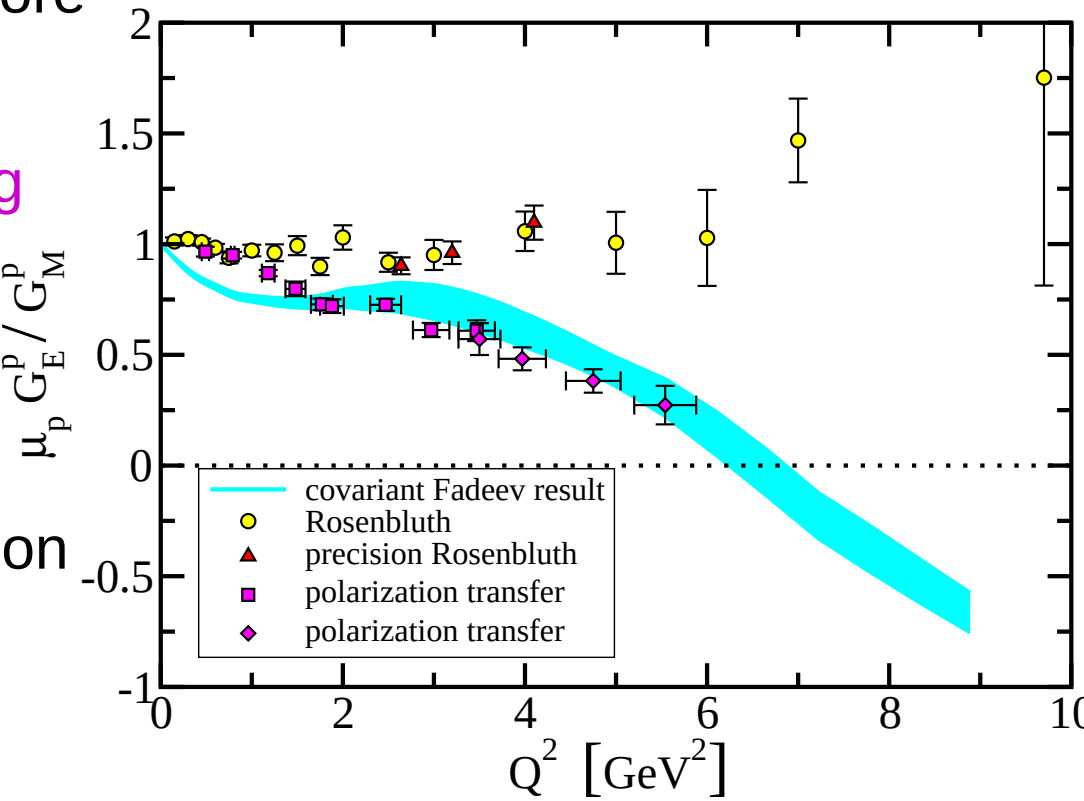

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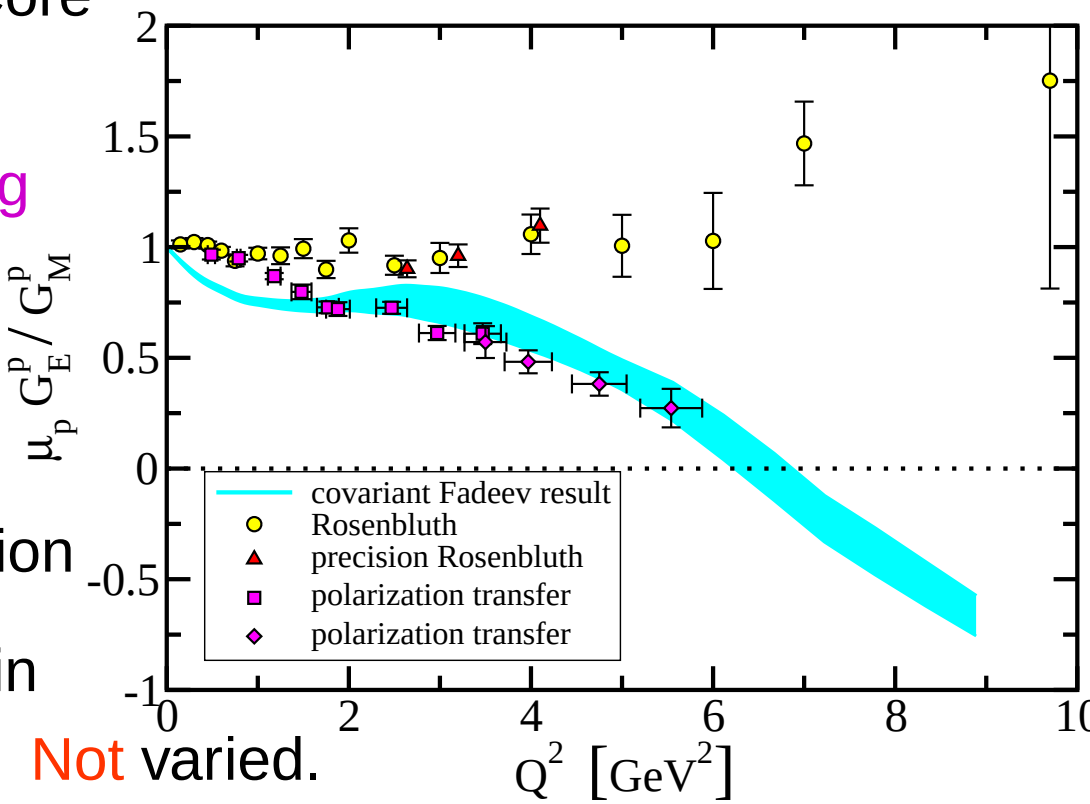
- All parameters fixed in other applications . . . Not varied.



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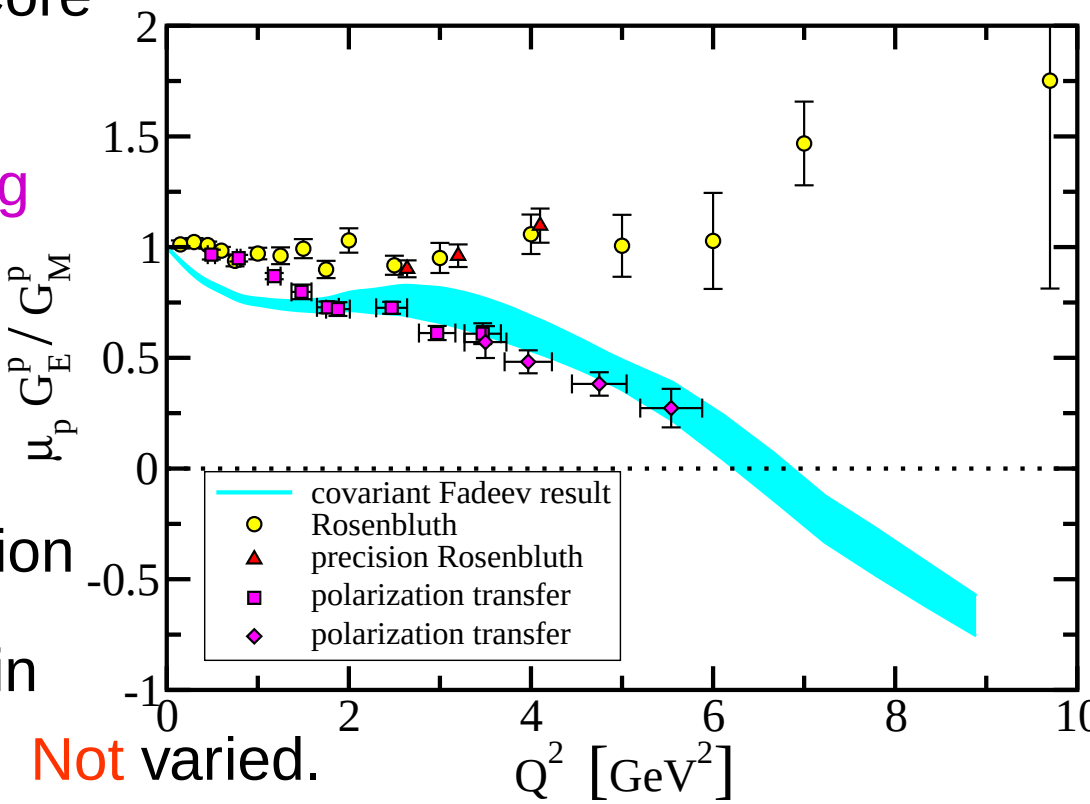

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Form Factor Ratio: GE/GM

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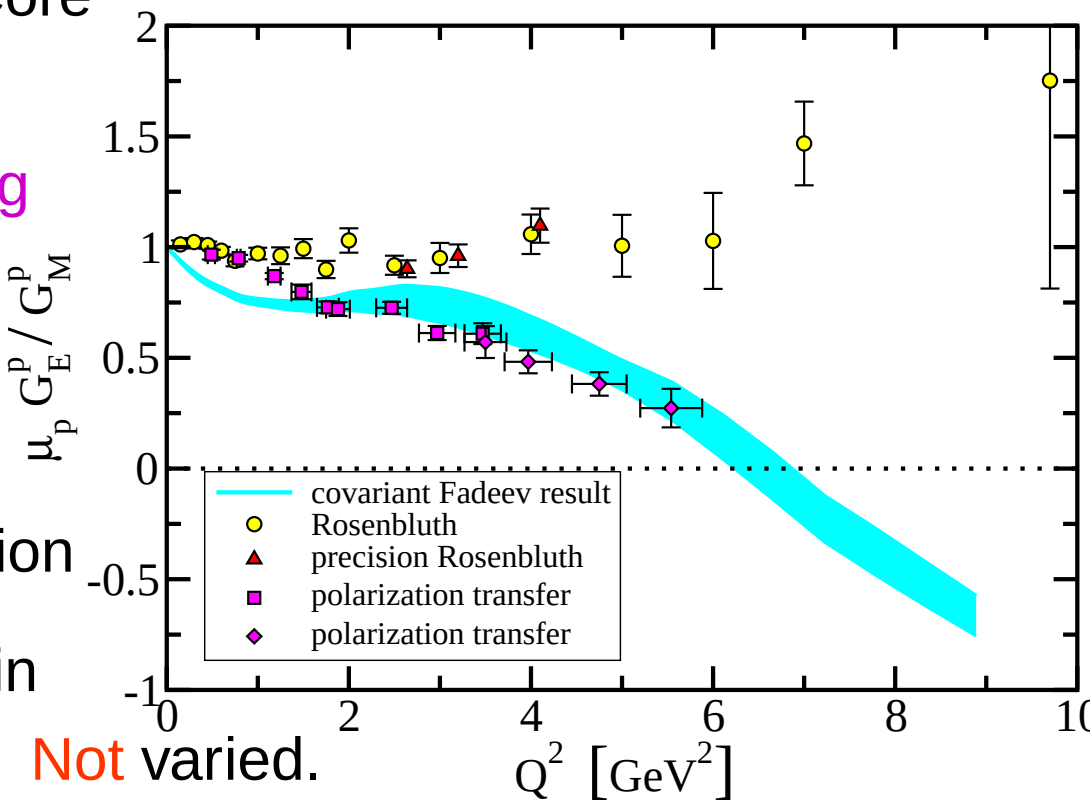

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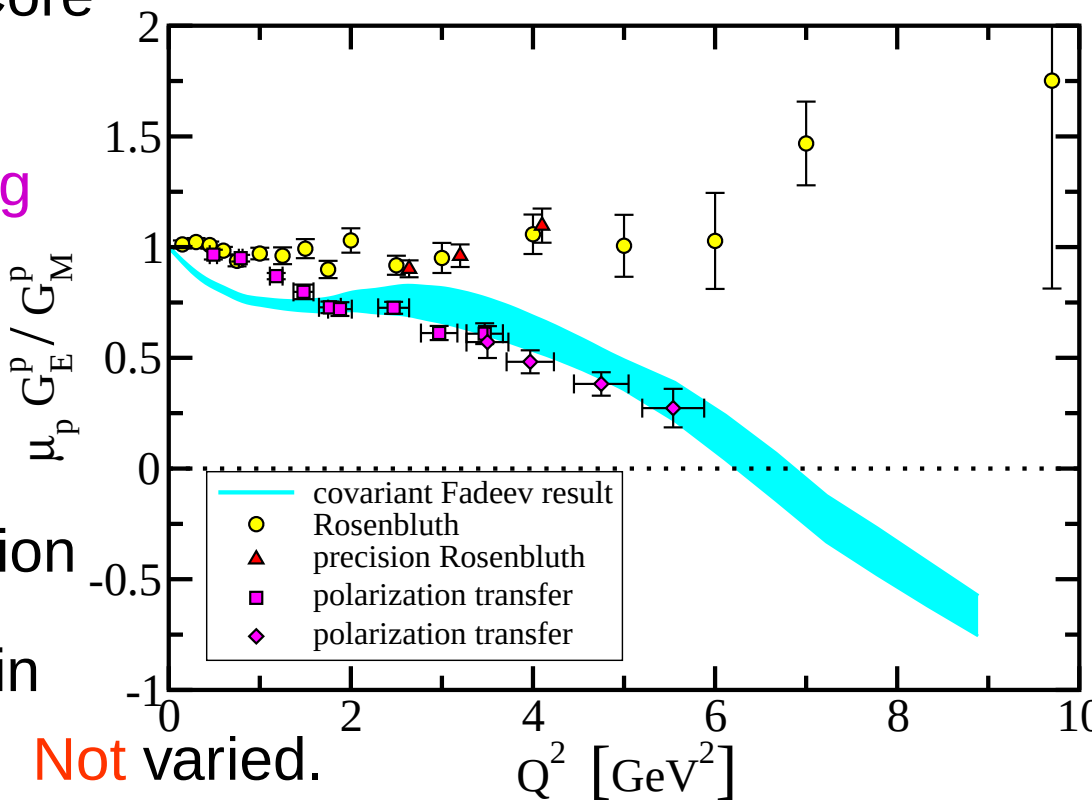

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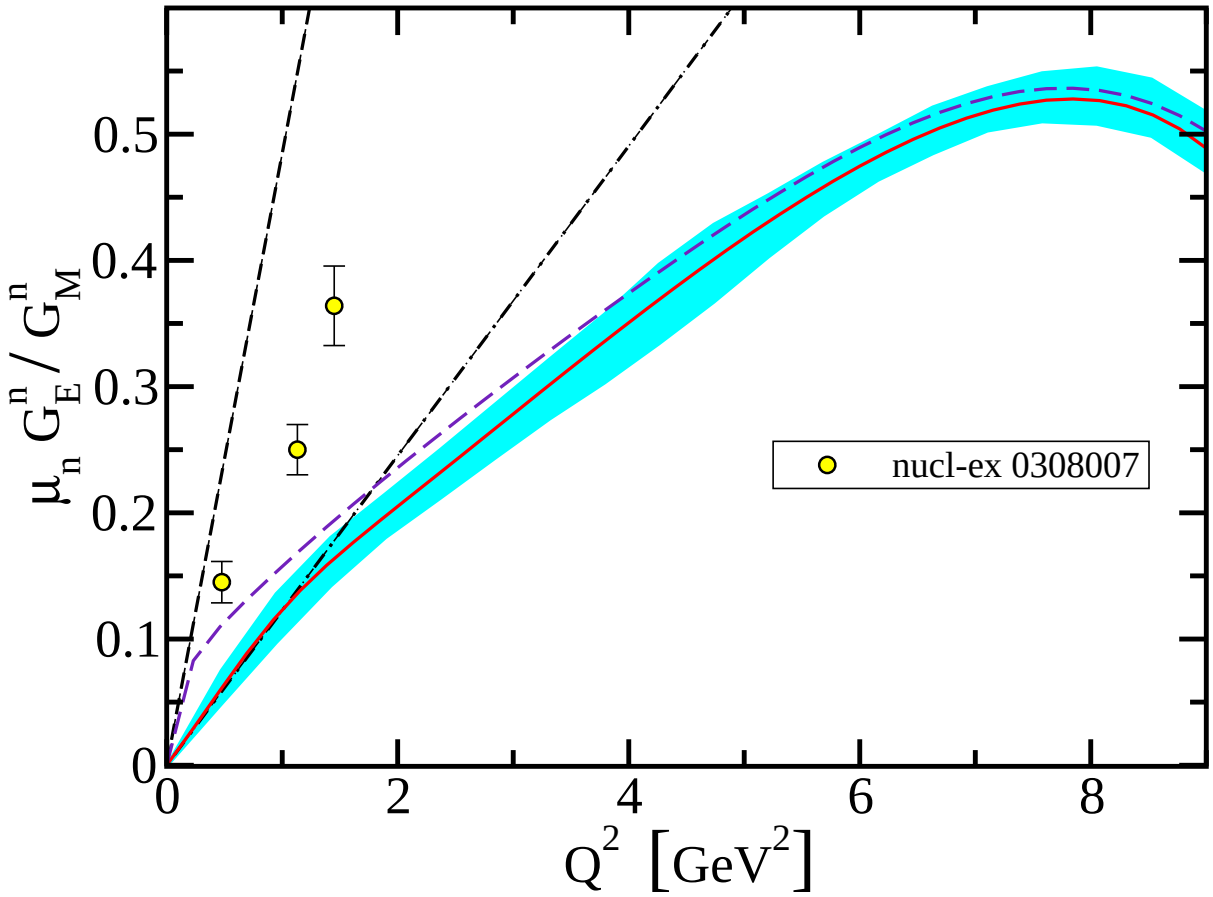
Neutron Form Factors



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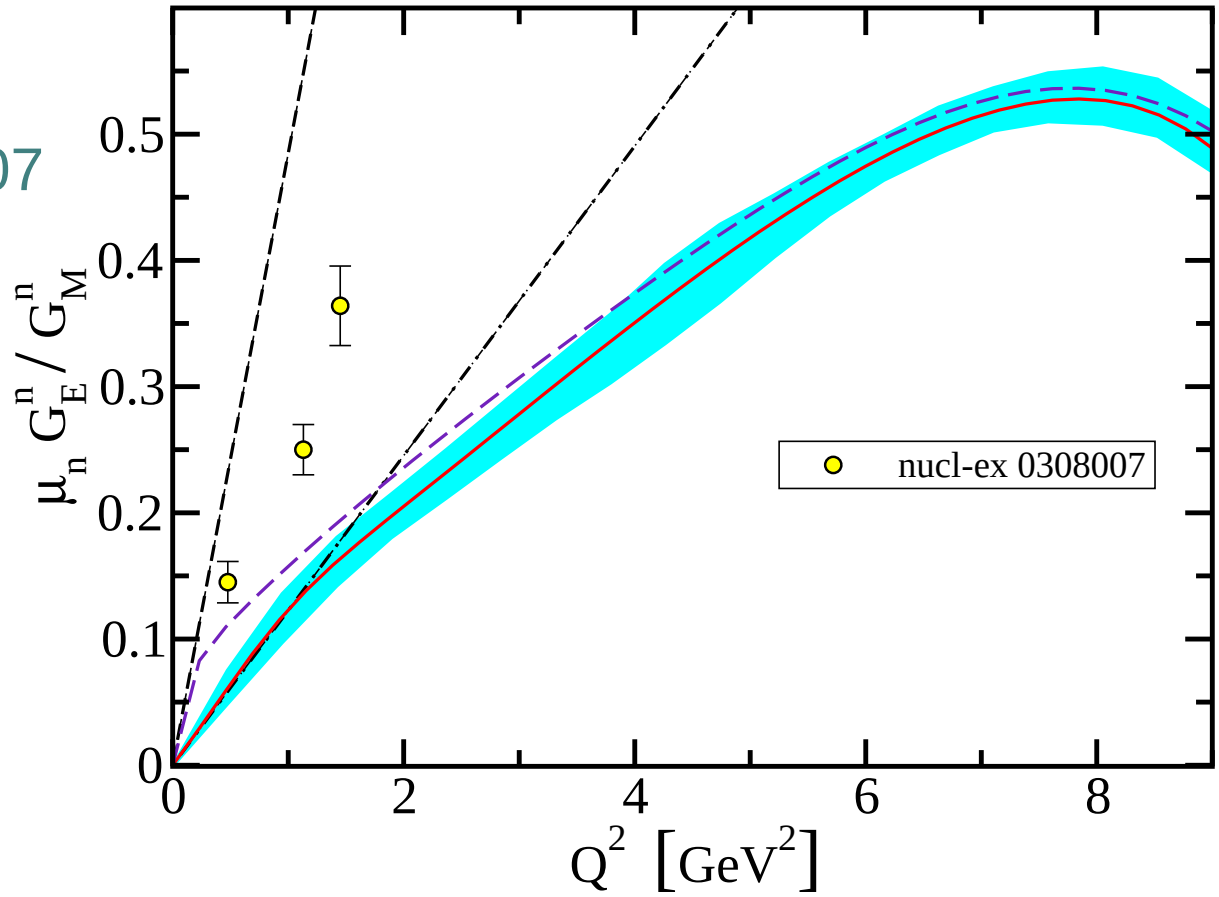

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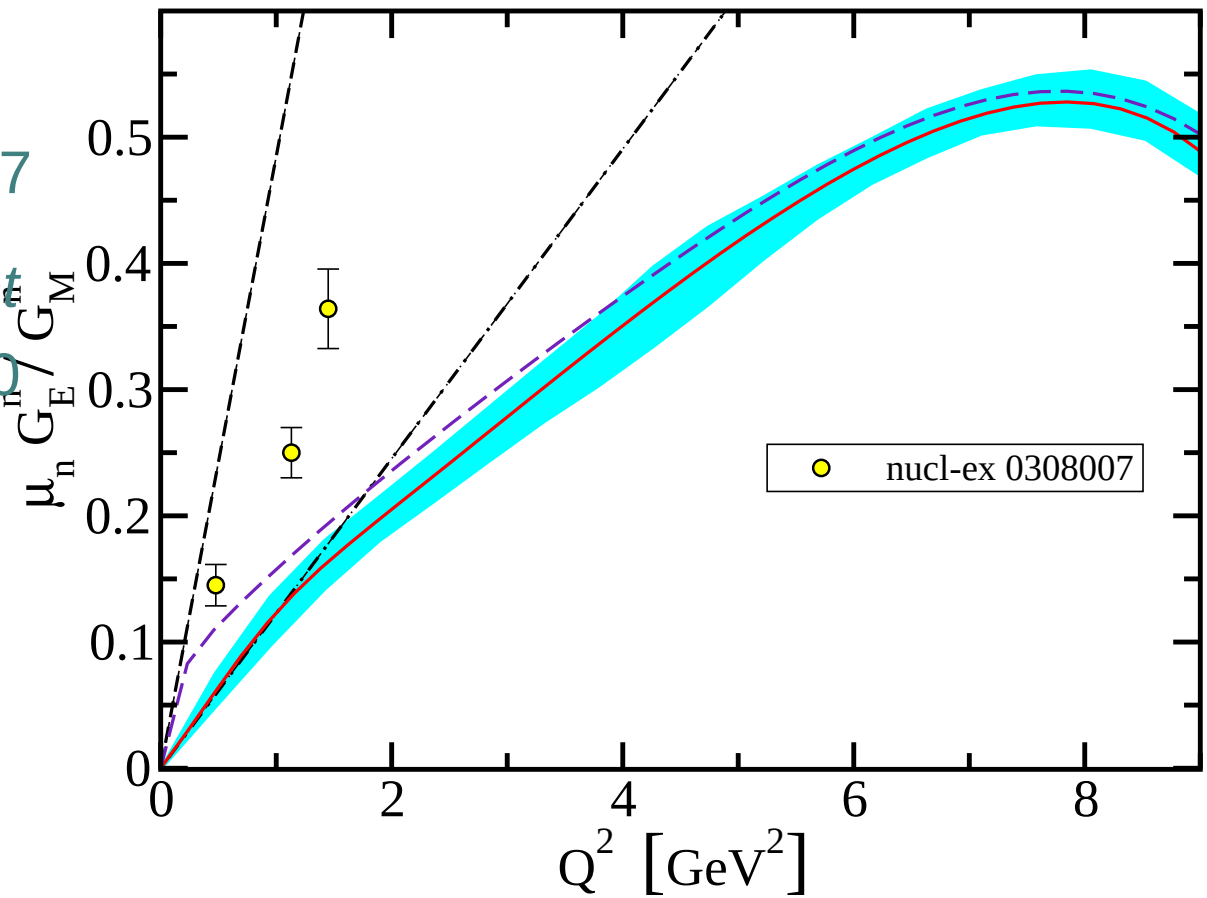
- $\mu_p \dfrac{G_E^n(Q^2)}{G_M^n(Q^2)} = -\dfrac{r_n^2}{6} Q^2$

Valid for $r_n^2 Q^2 \lesssim 1$



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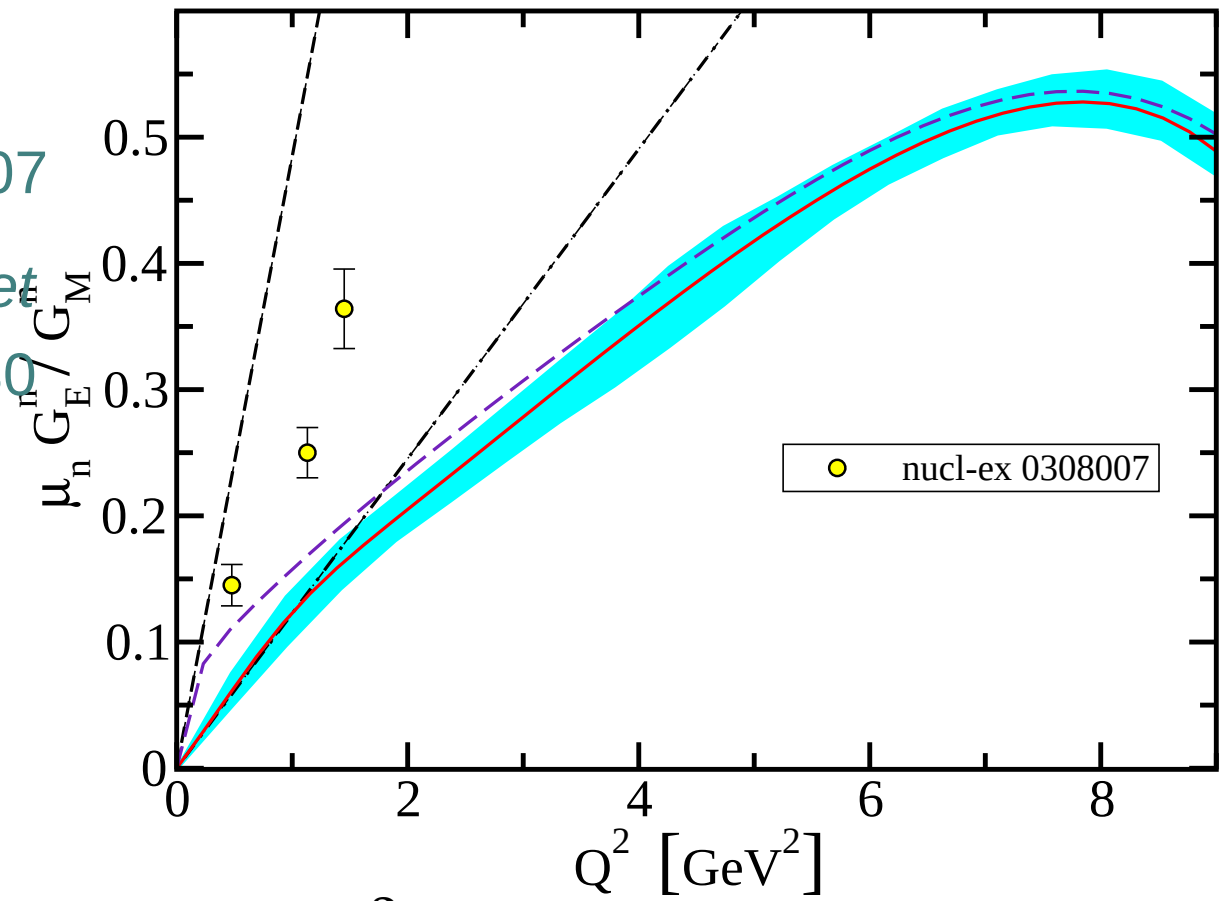

- No sign yet of a zero in $G_E^n(Q^2)$, even though calculation predicts $G_E^p(Q^2 \approx 6.5\,\mathrm{GeV}^2) = 0$
- Data to $Q^2 = 3.4\,\mathrm{GeV}^2$ is being analysed (JLab E02-013)

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Widths fixed by "asymptotic freedom" condition –

$$\frac{d}{dK^2}\left(\frac{1}{m_{J^P}^2}\,\mathcal{F}(K^2/\omega_{J^P}^2)\right)^{-1}\Bigg|_{K^2=0} = 1 \;\Rightarrow\; \omega_{J^P}^2 = \frac{1}{2}\,m_{J^P}^2\,,$$

Only two parameters; viz., diquark "masses": m_{J^P}

Contemporary Reviews

- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD
  C.D. Roberts and S.M. Schmidt, nu-th/0005064,
  Prog. Part. Nucl. Phys. **45** (2000) S1
- The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .
  R. Alkofer and L. von Smekal, he-ph/0007355,
  Phys. Rept. **353** (2001) 281
- Dyson-Schwinger equations: A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049,
  Int. J. Mod. Phys. **E 12** (2003) pp. 297-365
- Infrared properties of QCD from Dyson-Schwinger equations.
  C. S. Fischer, he-ph/0605173,
  J. Phys. **G 32** (2006) pp. R253-R291