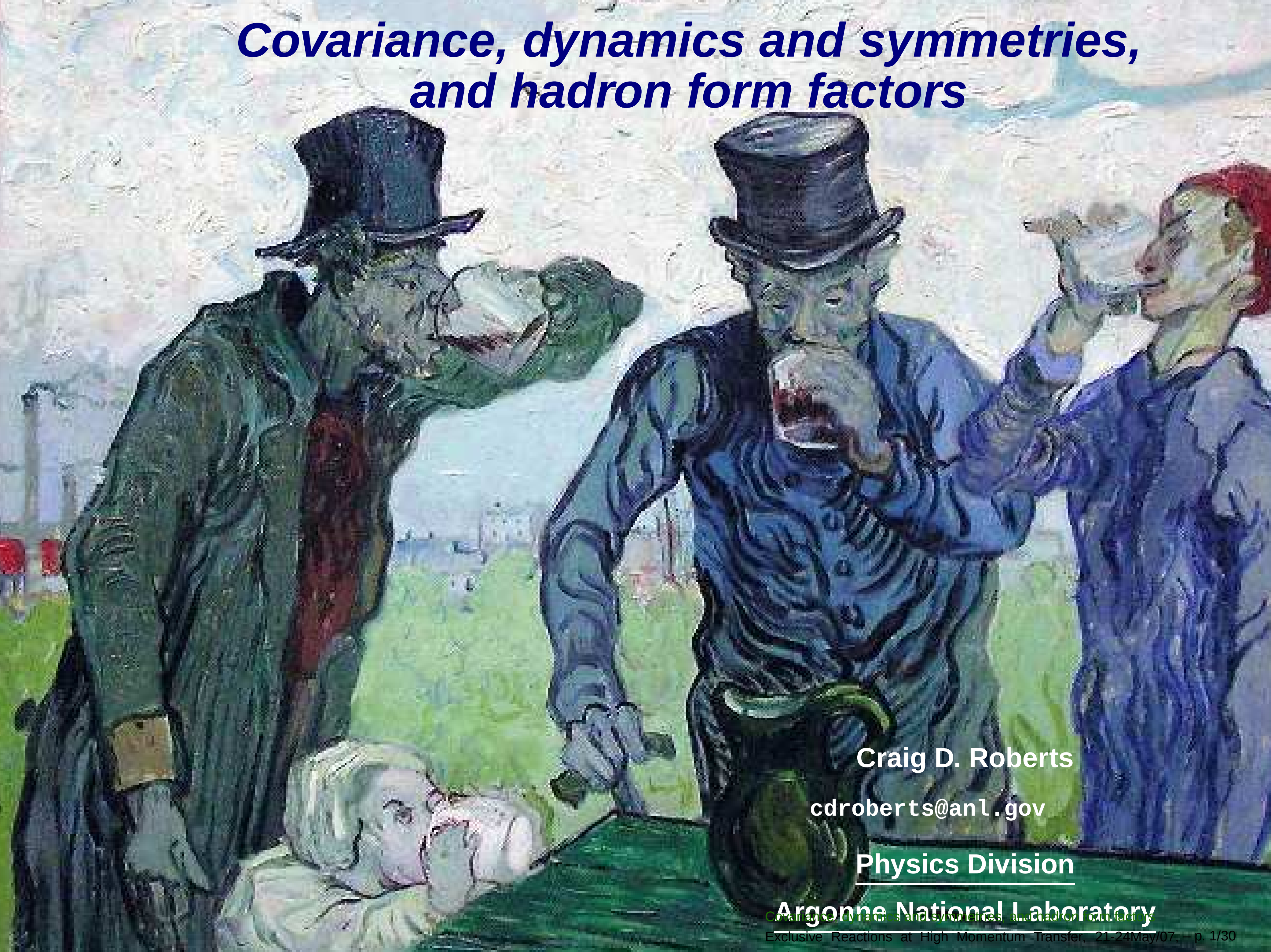

Covariance, dynamics and symmetries, and hadron form factors

Craig D. Roberts

`cdroberts@anl.gov`

Physics Division

Argonne National Laboratory

Dichotomy of the Pion

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Dichotomy of the Pion

- How does one make an almost massless particle . . . . from two massive constituent-quarks?

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Dichotomy of the Pion

- How does one make an almost massless particle
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Must exhibit $\boxed{m_\pi^2 \propto m_q}$

Current Algebra ... 1968

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Using DSEs,
we've provided this.

What's the Problem?

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    throughout $>$ 98% of the pion's/proton's volume

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  of fully-fledged relativistic quantum field theory
- Modern Physics & Mathematics
  – Still quite some way from being able to do that

Dyson-Schwinger Equations

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Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory

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- ⇒ Understanding InfraRed (long-range)
  .... behaviour of $\alpha_s(Q^2)$

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- Method yields Schwinger Functions $\equiv$ Propagators

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Cross-Sections built from Schwinger Functions

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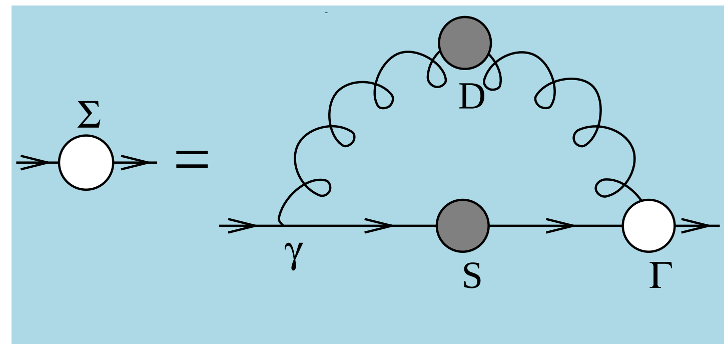
Persistent Challenge

Persistent Challenge

- Infinitely Many Coupled Equations



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- Infinitely Many Coupled Equations
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  - Not all are Schwinger functions are experimentally observable but all are same VEVs measured in Lattice-QCD simulations . . . opportunity for comparisons at pre-experimental level . . . cross-fertilisation

Persistent Challenge

- Infinitely Many Coupled Equations
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- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory

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Persistent Challenge

- Infinitely Many Coupled Equations
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    Not useful for the nonperturbative problems in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

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Dressed-Quark Propagator

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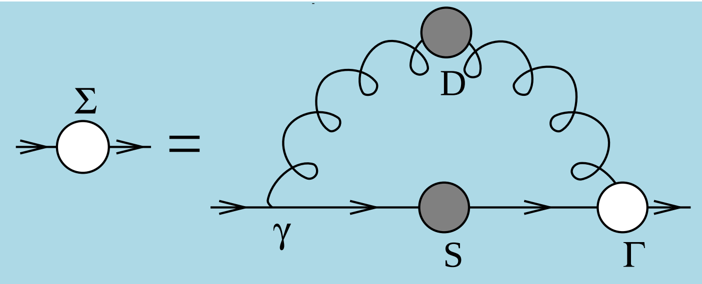
Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

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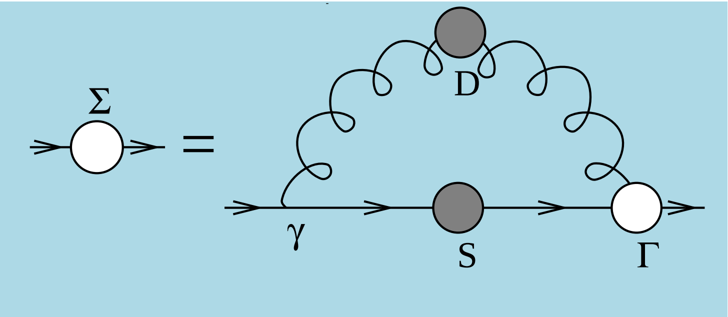
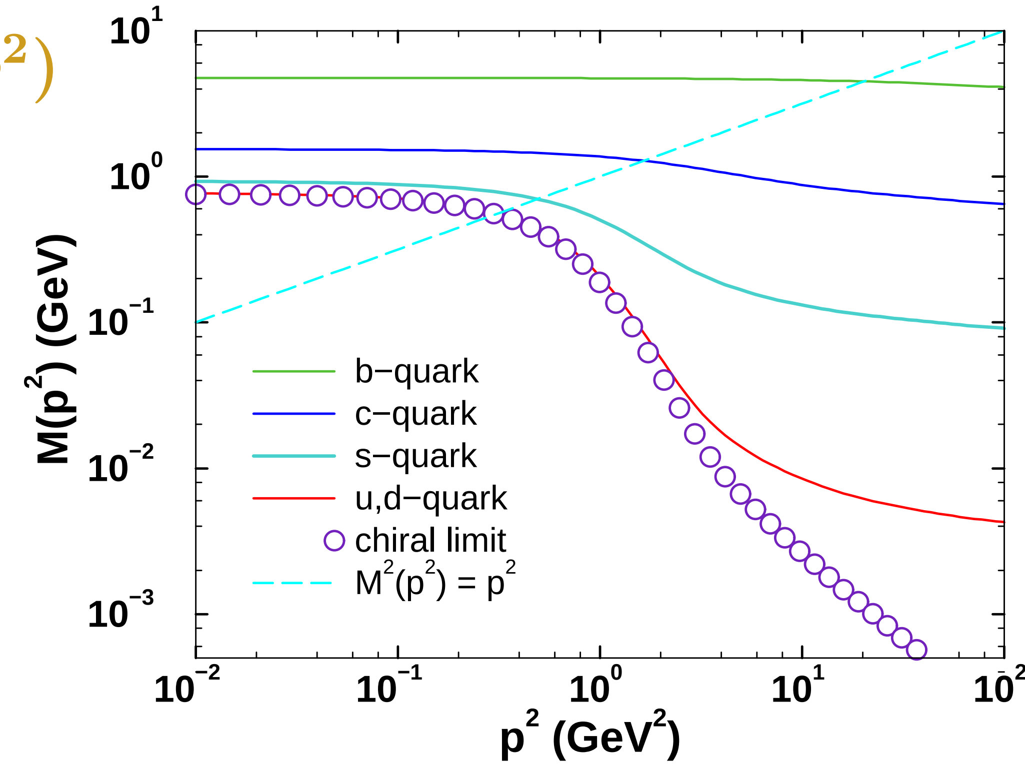


Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



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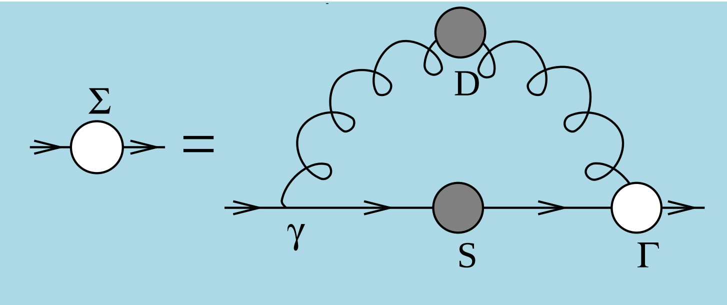

Gap Equation

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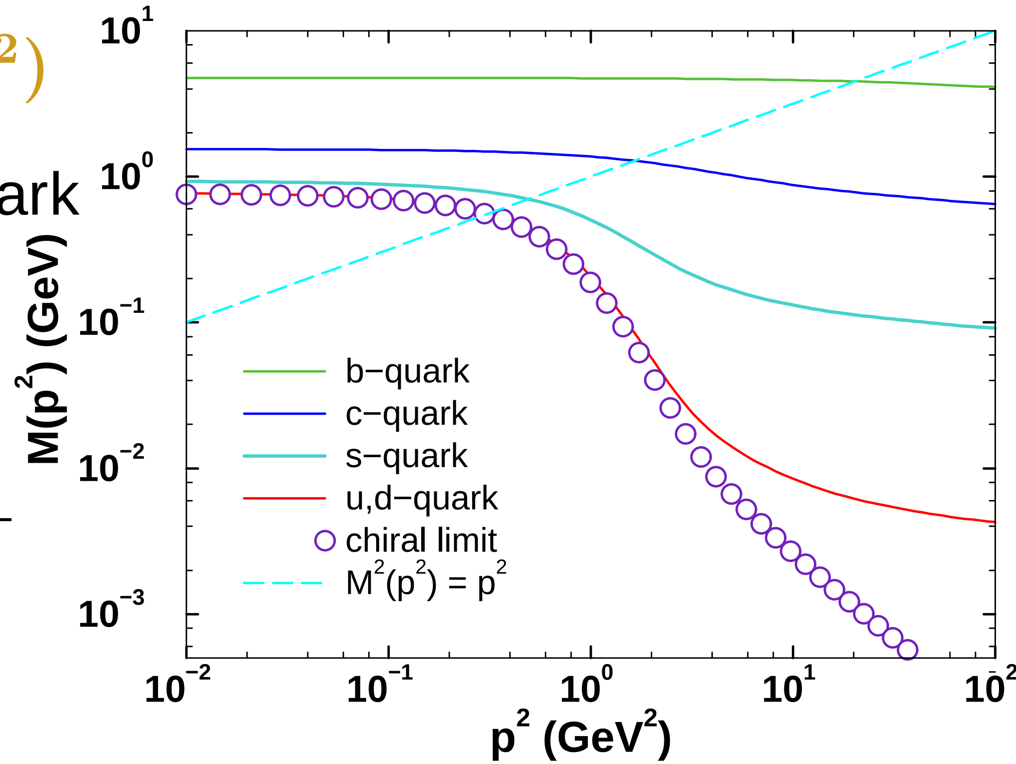
- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



Dressed-Quark Propagator

- Longstanding Prediction of Dyson-Schwinger Equation Studies

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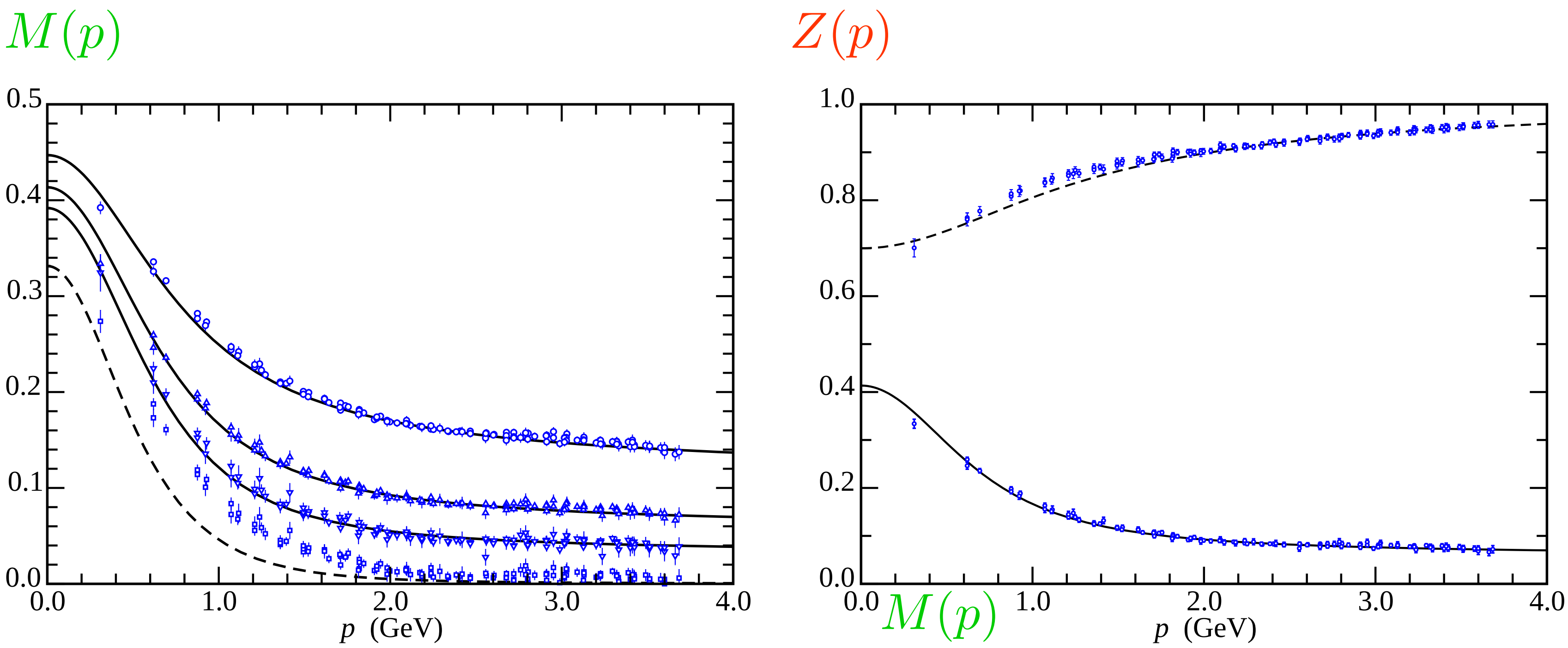
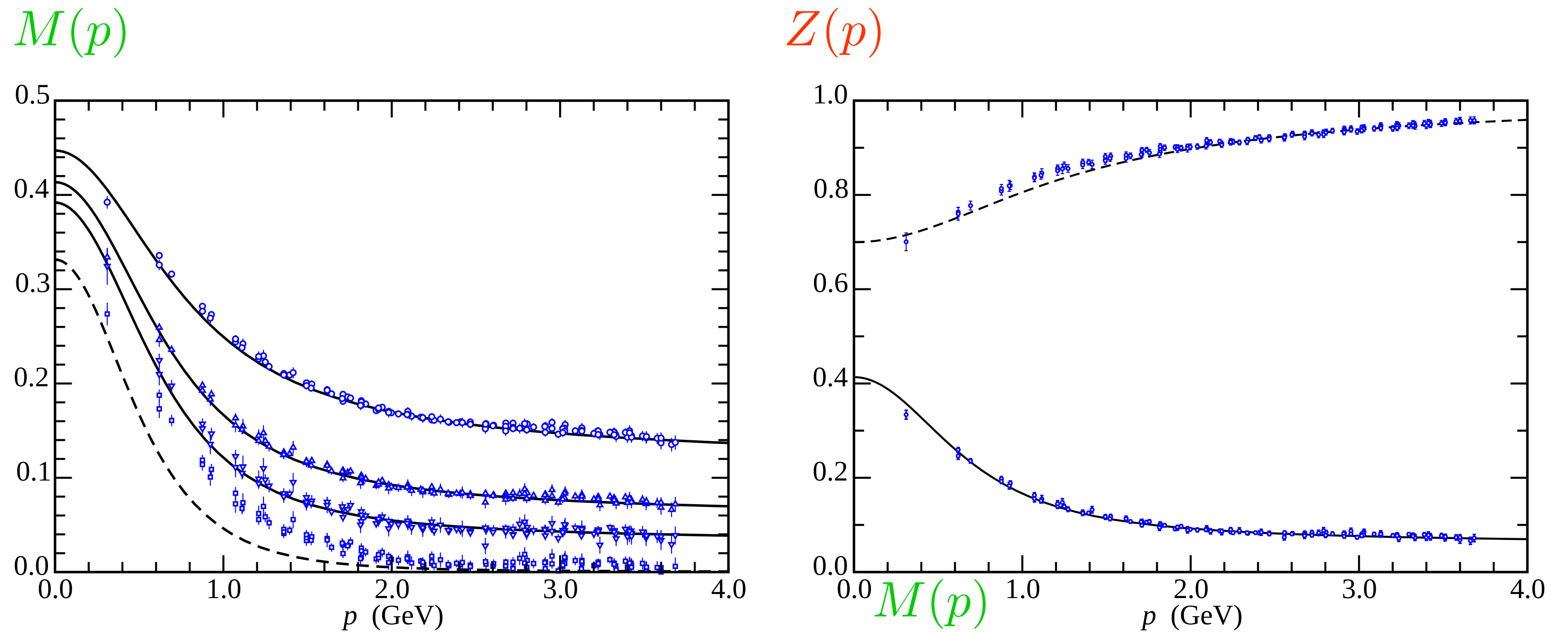
Quenched-QCD Dressed-Quark Propagator



Quenched-QCD Dressed-Quark Propagator

2002



- “*data*:” Quenched Lattice Meas.
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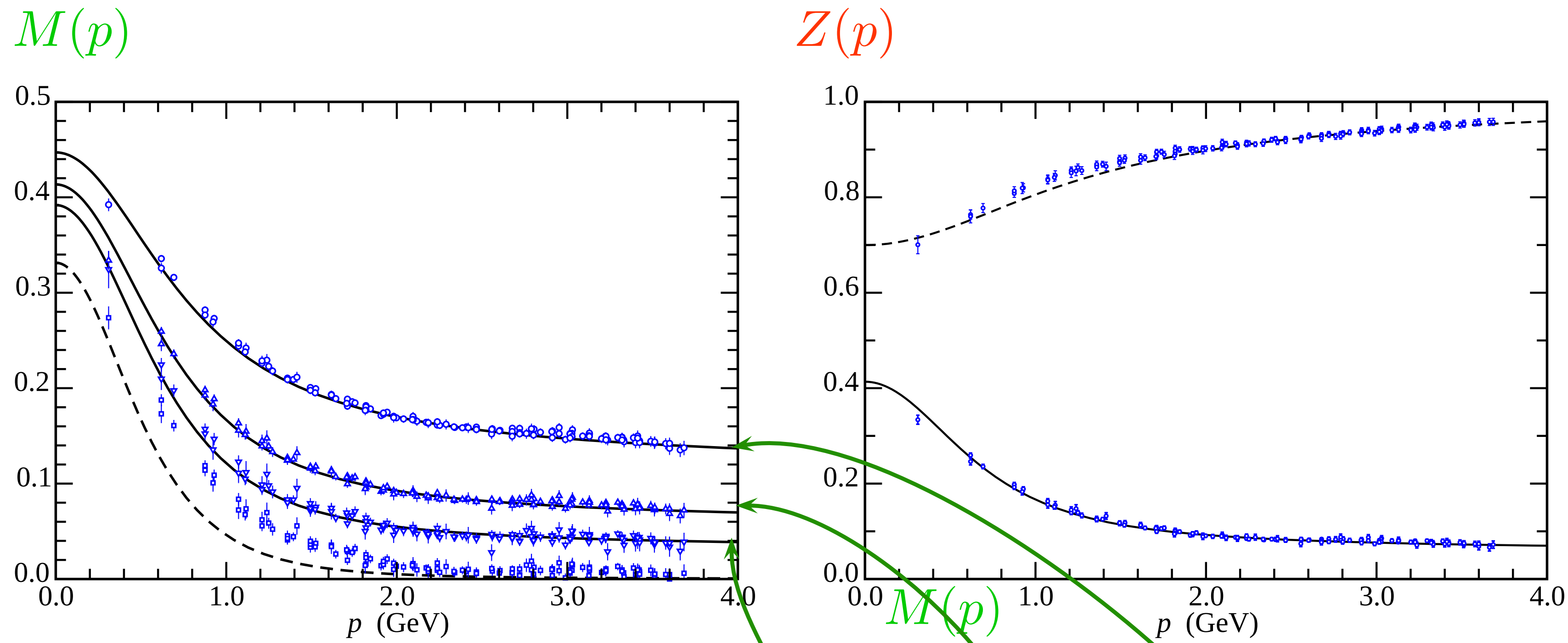

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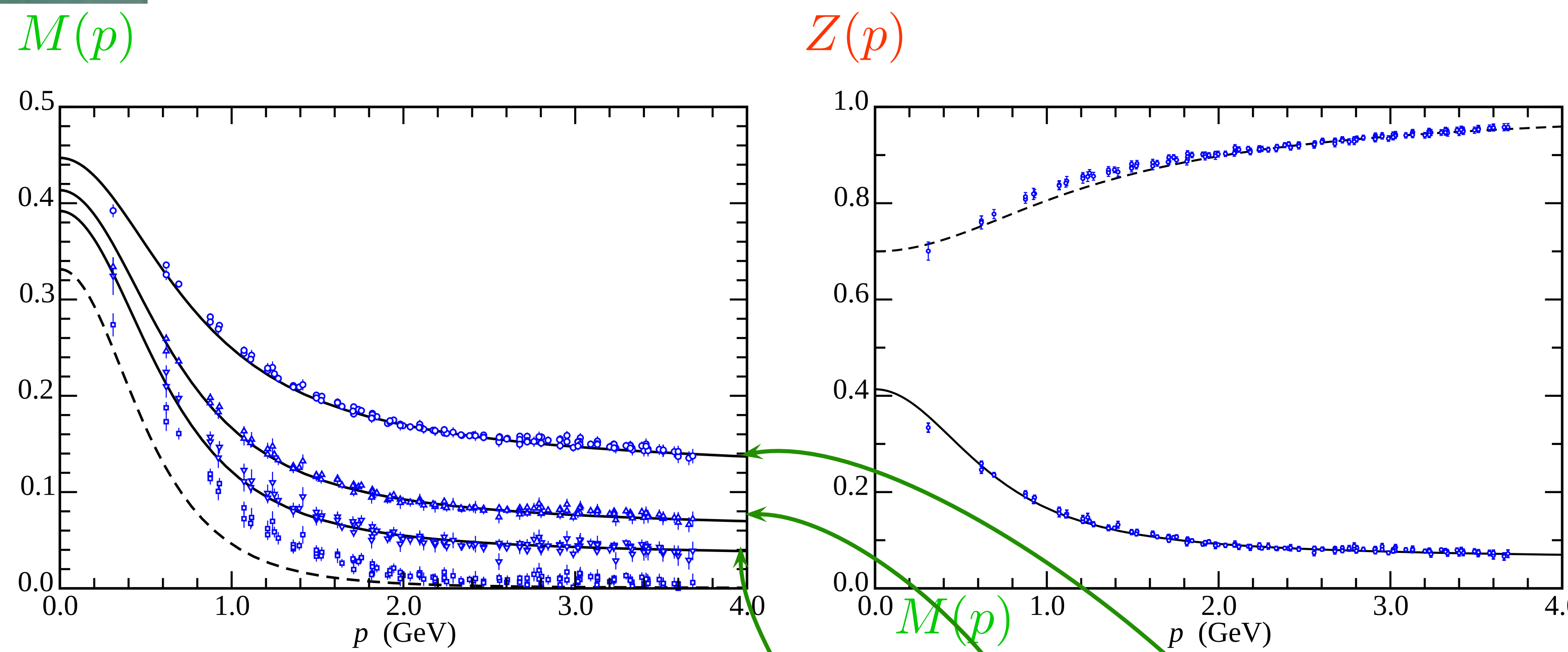

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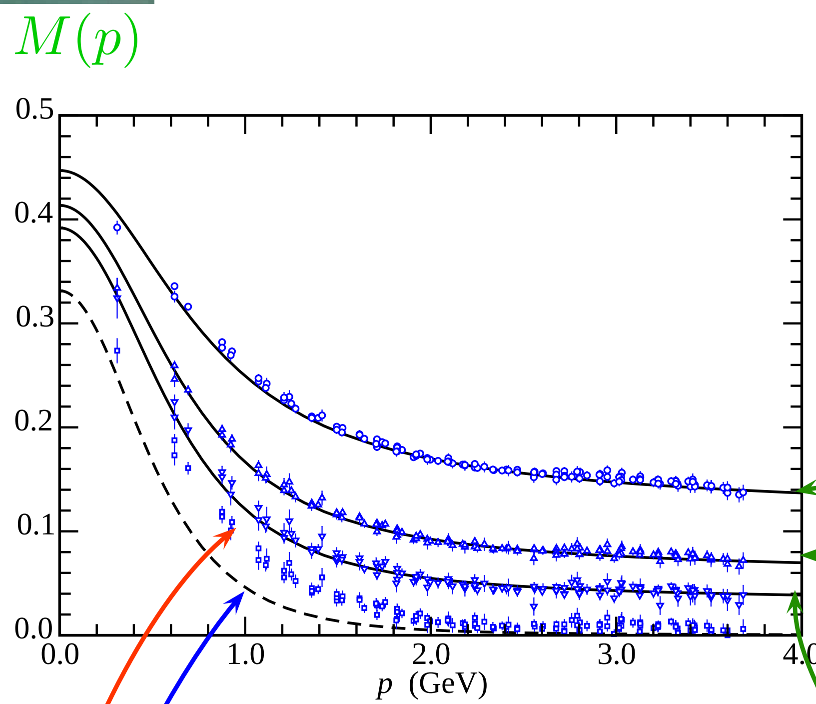

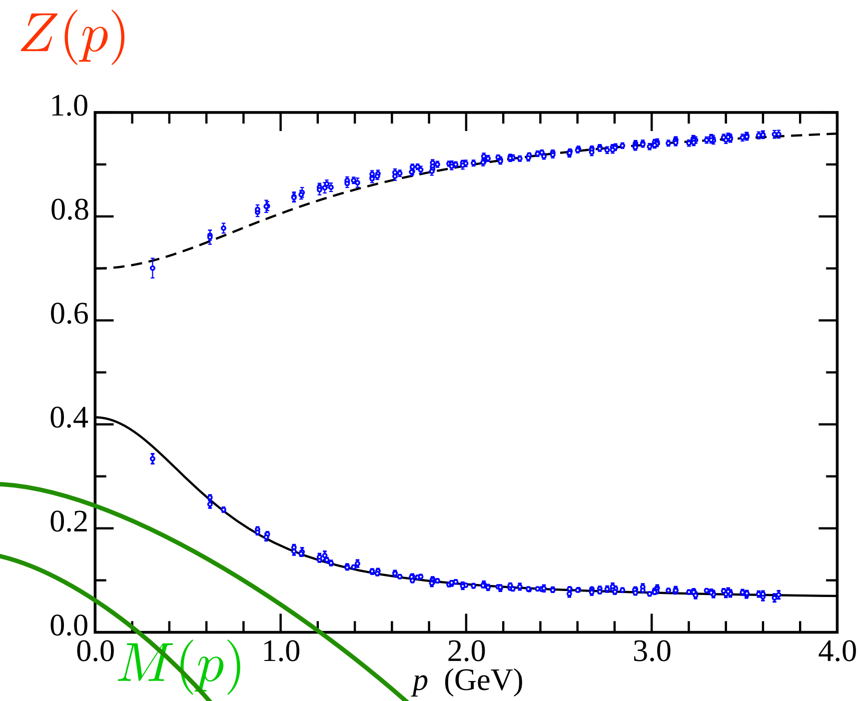

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- Linear extrapolation of lattice data to chiral limit is inaccurate

Hadrons

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Hadrons

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- What about bound states?

Hadrons

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- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

- Without bound states, Comparison with experiment is impossible
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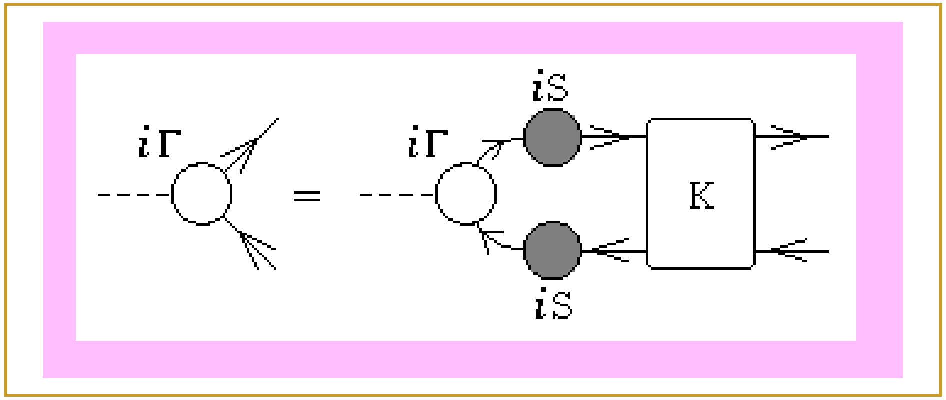
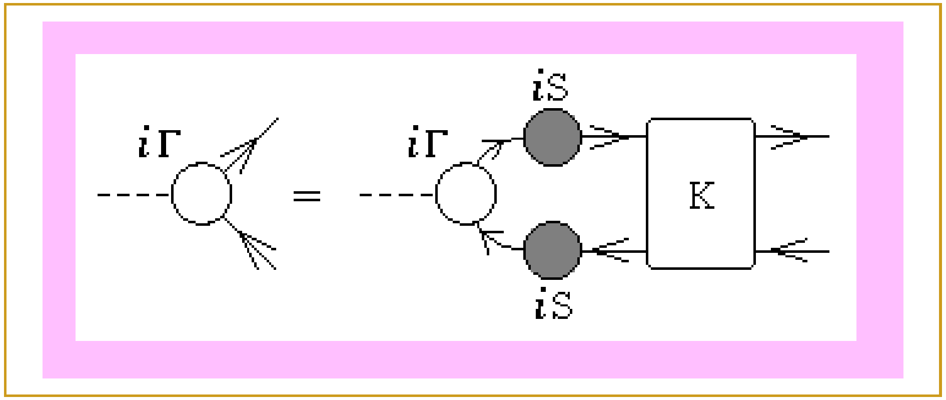


QFT Generalisation of Lippmann-Schwinger Equation.

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



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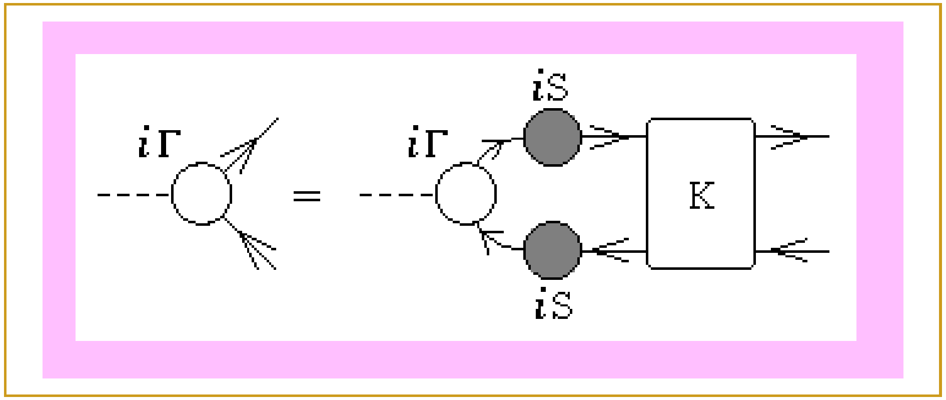

QFT Generalisation of Lippmann-Schwinger Equation.

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or

What is the Long-Range Potential?

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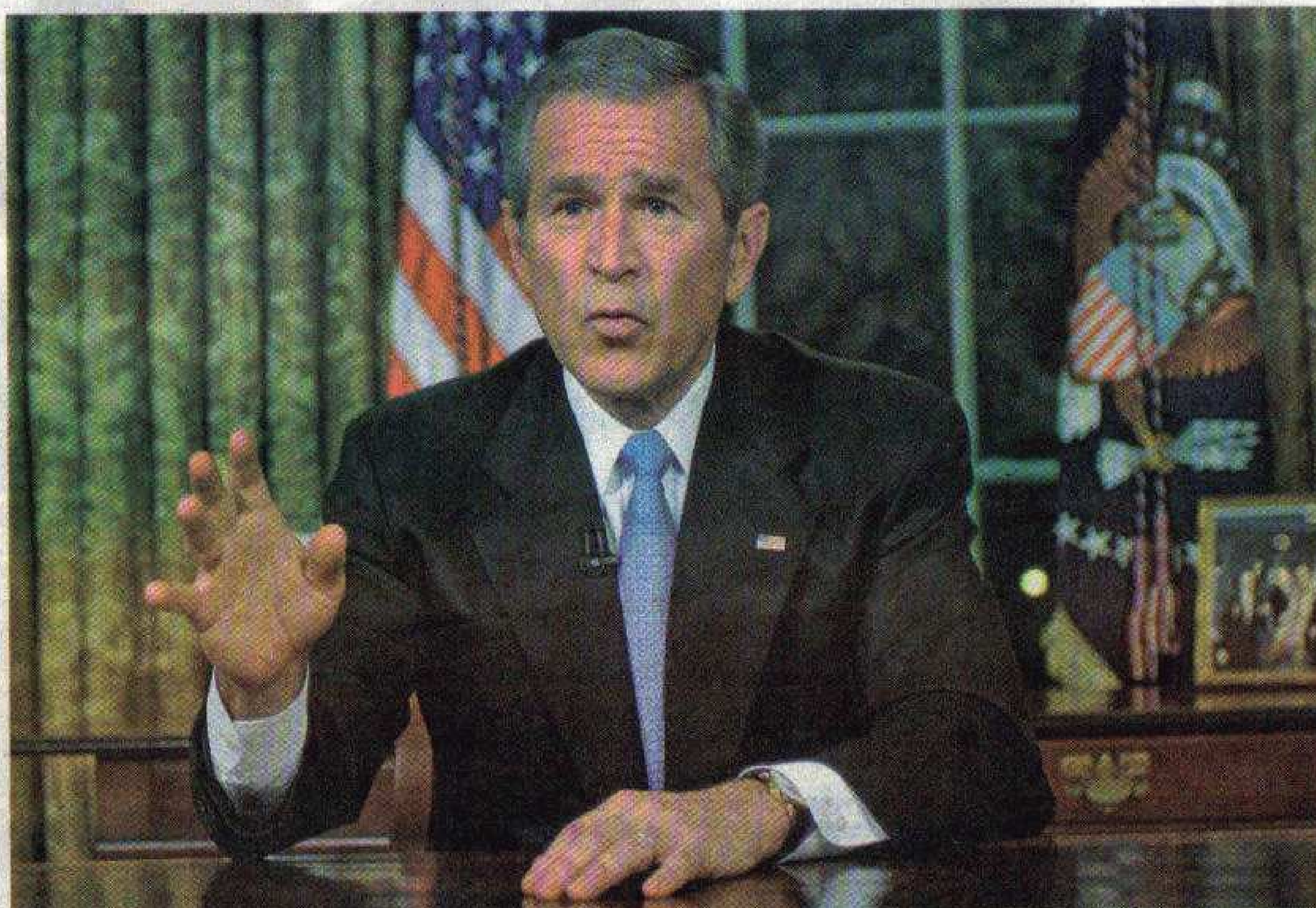
What is the Long-Range Potential?

Bush Urges Nation To Be Quiet For A Minute While He Tries To Think



In a televised address to the nation, Bush called for "a little peace and quiet."

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \ \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \ \ \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

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Satisfies BSE

Satisfies DSE

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Kernels must be intimately related

Bethe-Salpeter Kernel

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Satisfies BSE — Satisfies DSE

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Satisfies BSE — Satisfies DSE

Kernels must be intimately related

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- Nontrivial constraint

- Axial-vector Ward-Takahashi identity

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Satisfies BSE — Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation
- Failure ⇒ Explicit Violation of QCD's Chiral Symmetry

Radial Excitations
& Chiral Symmetry

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H$$

Radial Excitations & Chiral Symmetry

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Mass^2 of pseudoscalar hadron

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2} \left(\lambda^4 + i\lambda^5 \right)$

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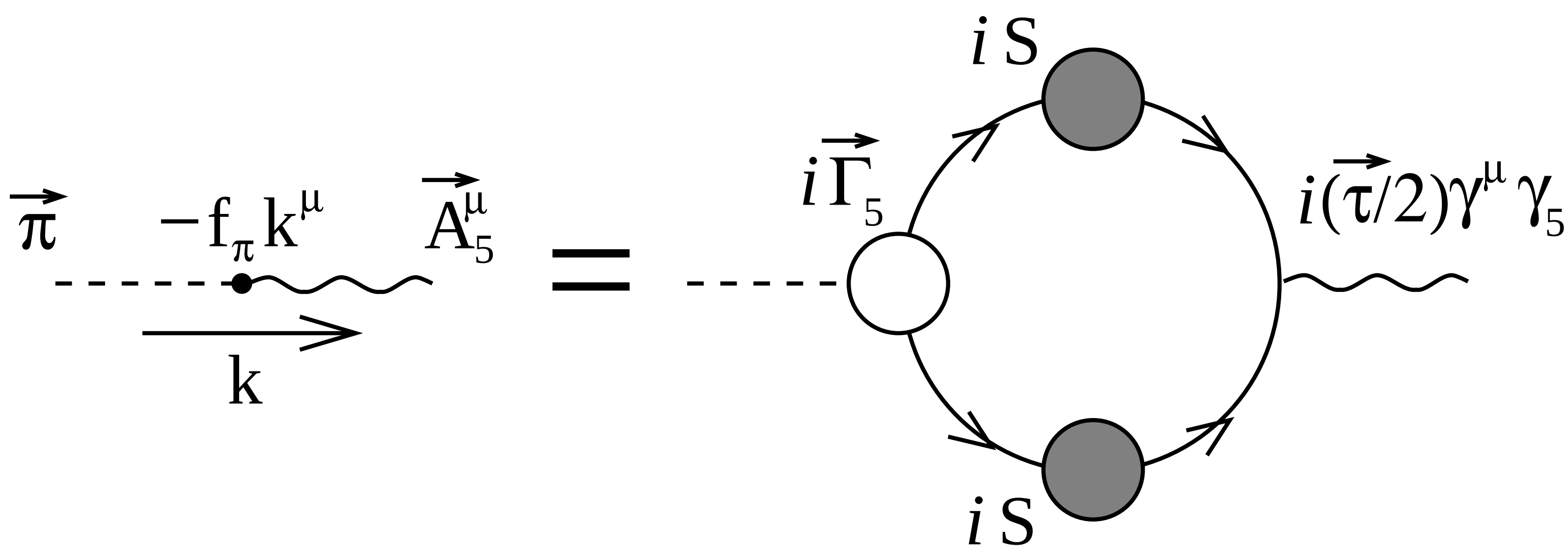
Radial Excitations & Chiral Symmetry

$$f_H \, m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$f_H \, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2} \mathrm{tr} \left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5 \gamma_\mu \, \mathcal{S}(q_+) \Gamma_H(q;P) \mathcal{S}(q_-) \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

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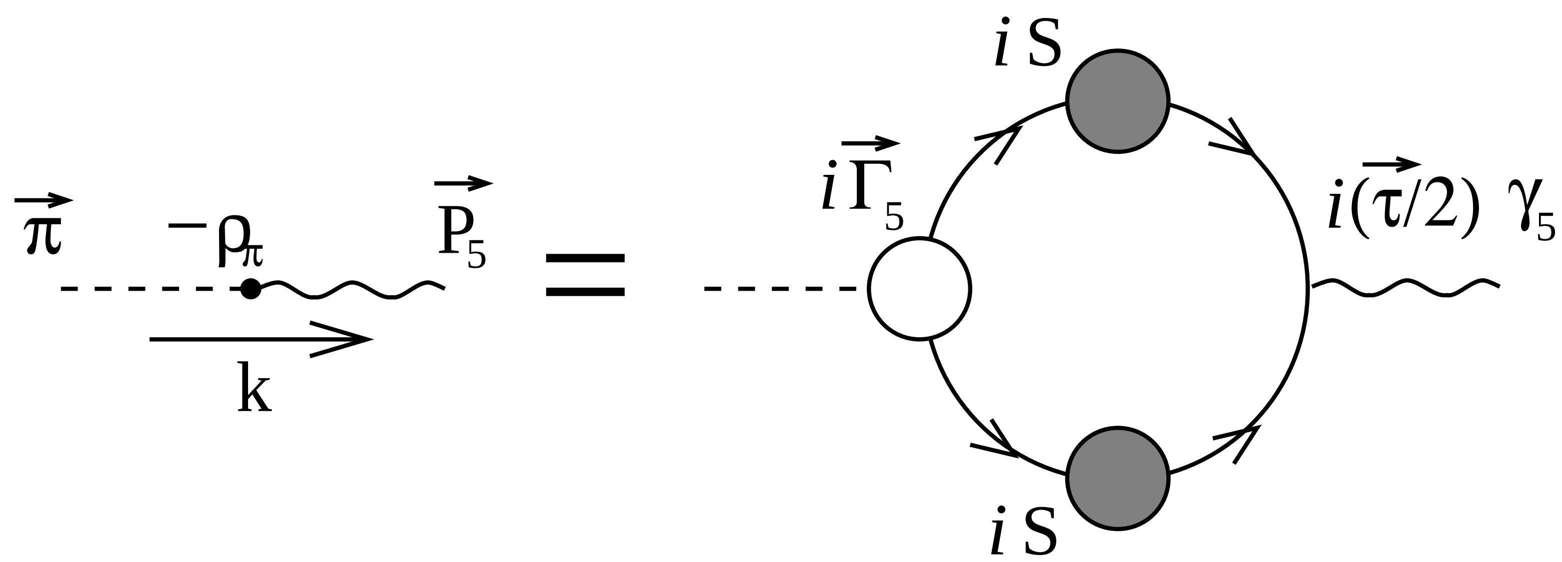
- Pseudoscalar projection of BS wave function at $x = 0$



Radial Excitations & Chiral Symmetry

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \frac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $$m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} \, m_q$$ . . . GMOR relation, a corollary

Radial Excitations & Chiral Symmetry

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$

Höll, Krassnigg, Roberts
nu-th/0406030

Radial Excitations & Chiral Symmetry

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson,
  $\boldsymbol{m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0}$, in chiral limit

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Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson,
  $\boldsymbol{m^2_{\pi_{n\neq0}} > m^2_{\pi_{n=0}} = 0}$, in chiral limit
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- Dynamical Chiral Symmetry Breaking

  – Goldstone's Theorem –

  impacts upon *every* pseudoscalar meson

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

Radial Excitations & Lattice-QCD

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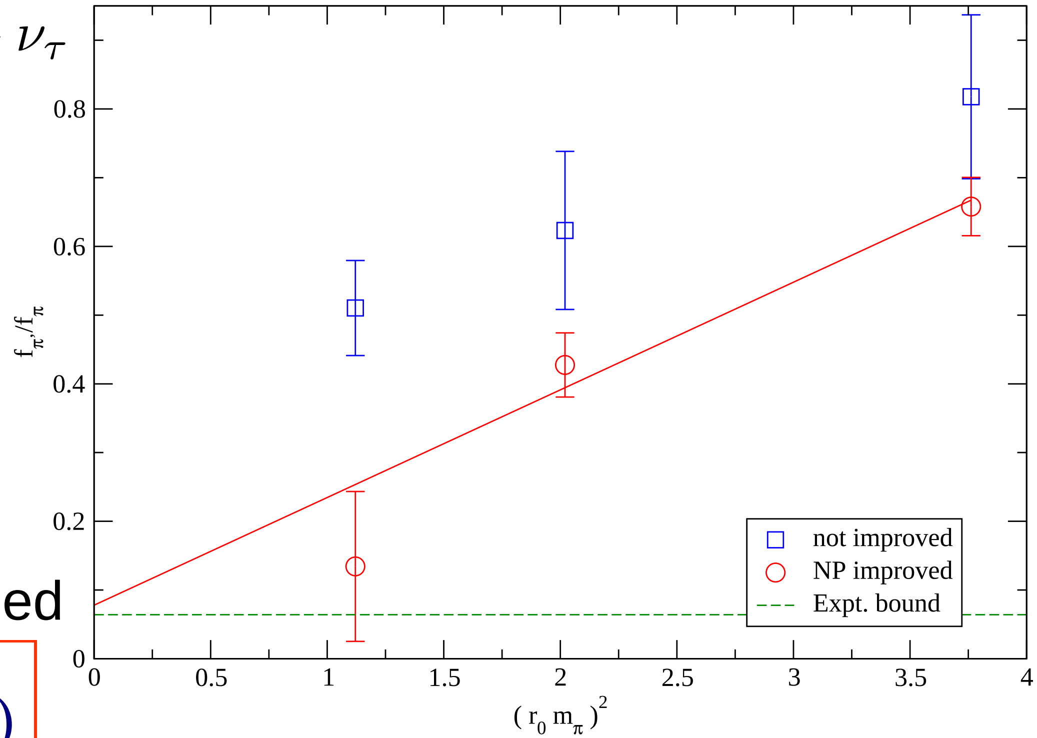
$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = \mathbf{0.078\,(93)}$$



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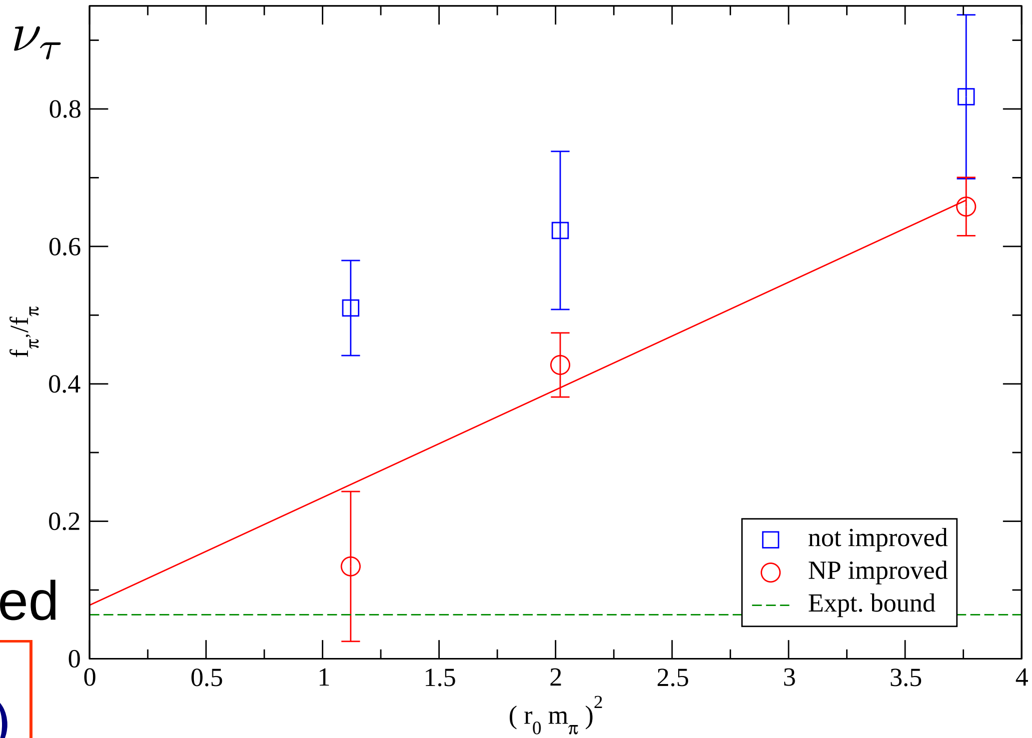

- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

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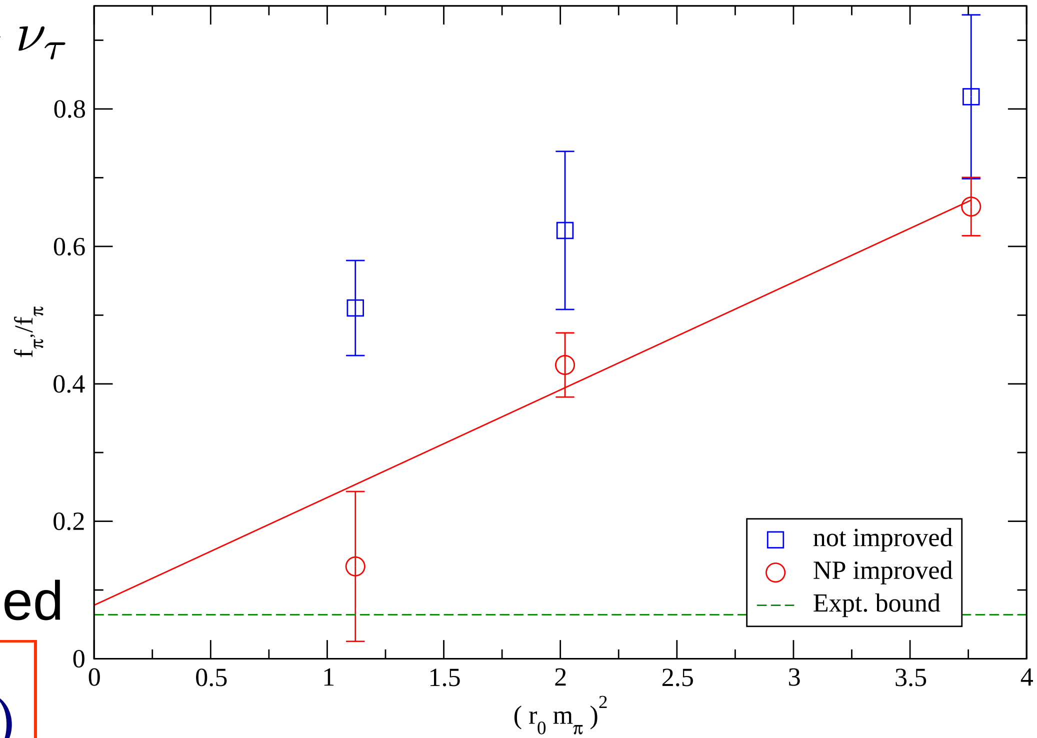

- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Pion . . . $J = 0$
but . . .

- Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.

Pion . . . $J = 0$ but . . .

- Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.

Pion . . . $J = 0$
but . . .

- Pseudoscalar meson Bethe-Salpeter amplitude

$$\chi_\pi(k;P) = \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P \mathcal{F}_{\pi_n}(k;P) \right.$$
$$\left. \gamma\cdot k \, k\cdot P \, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu} \, k_\mu P_\nu \, \mathcal{H}_{\pi_n}(k;P) \right]$$

Office of Science
U.S. DEPARTMENT OF ENERGY
Office of Nuclear Physics
Exploring Nuclear Matter - Quarks to Stars
Argonne
NATIONAL LABORATORY

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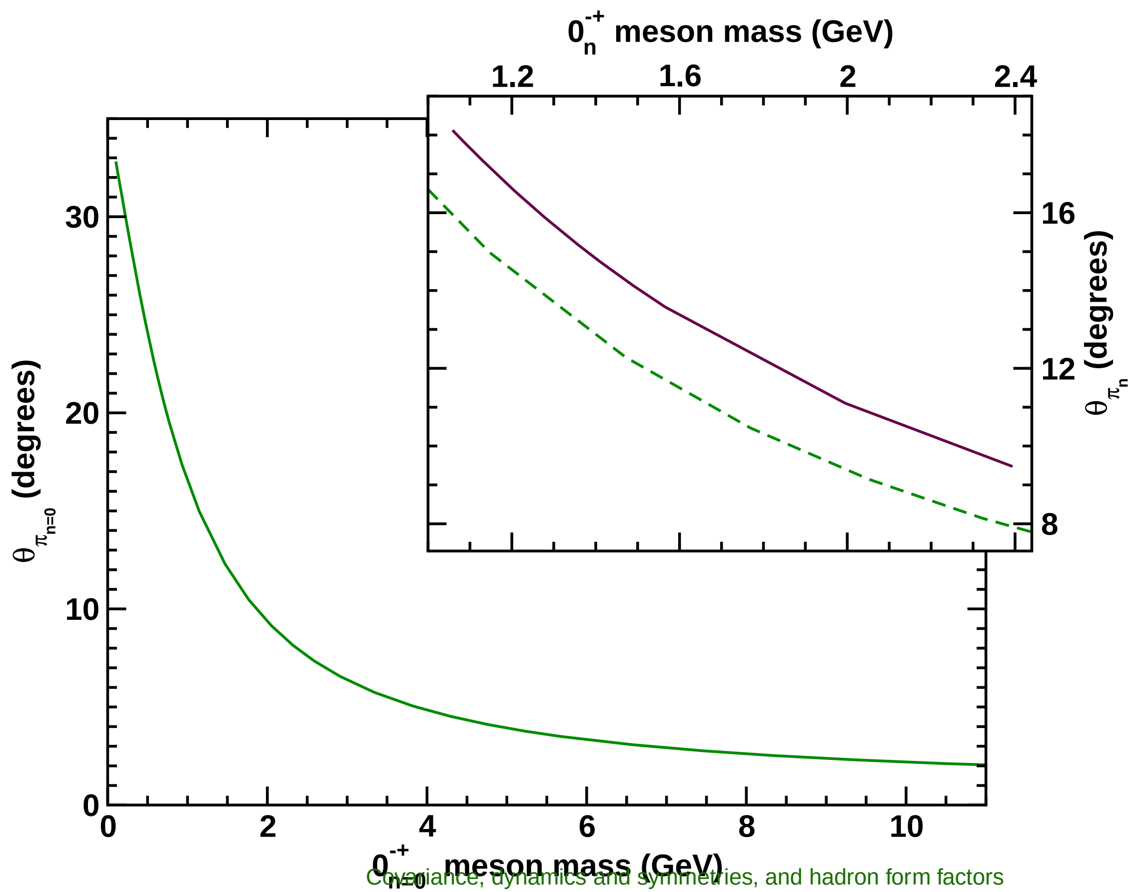

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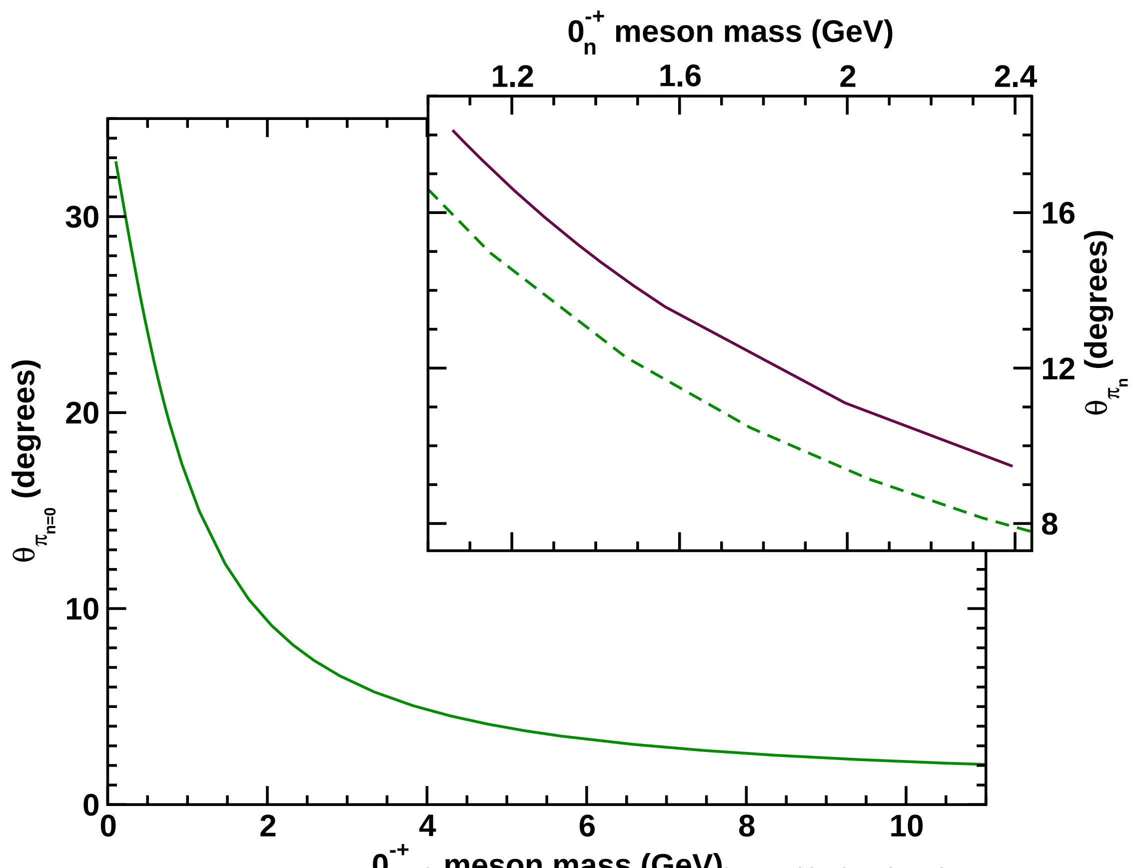
L is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.



New Challenges

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- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.

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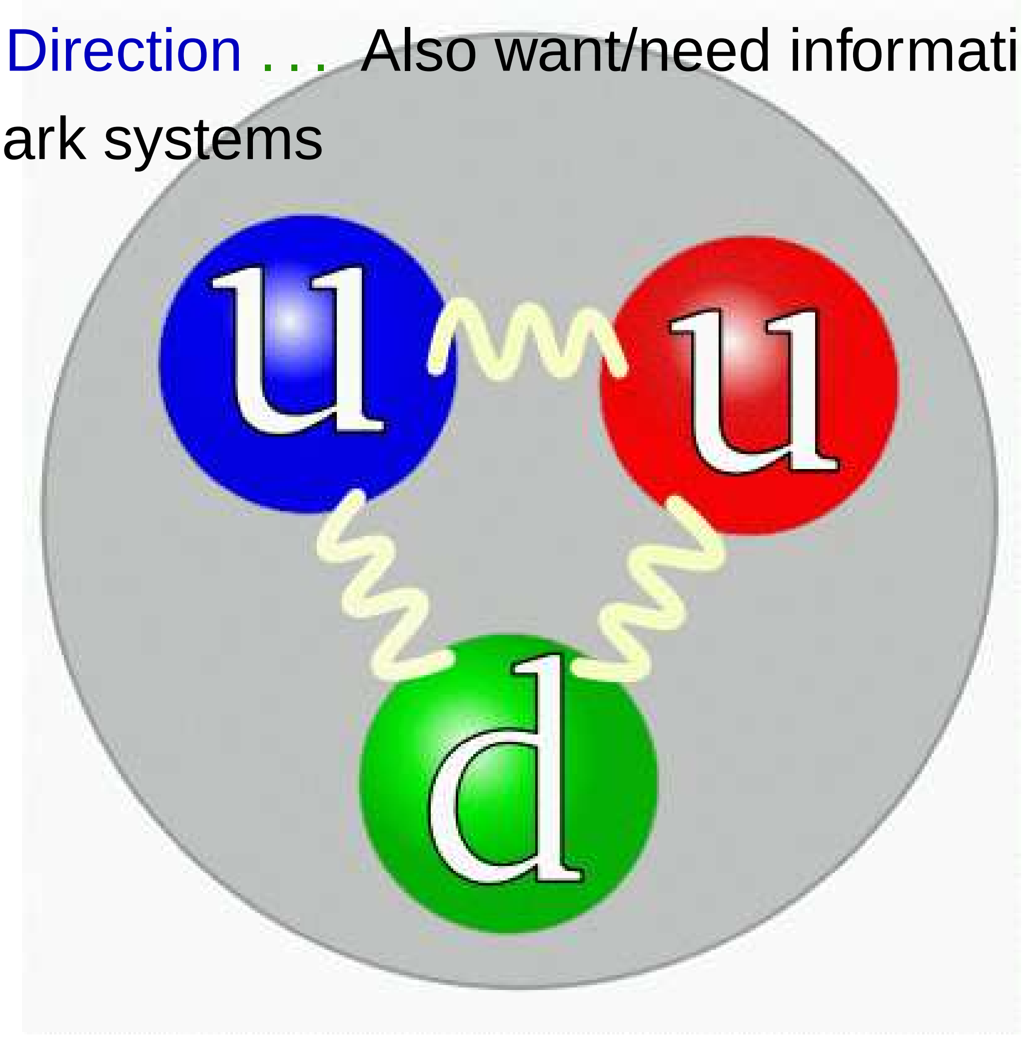
New Challenges

- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

New Challenges

- Another Direction . . . Also want/need information about three-quark systems



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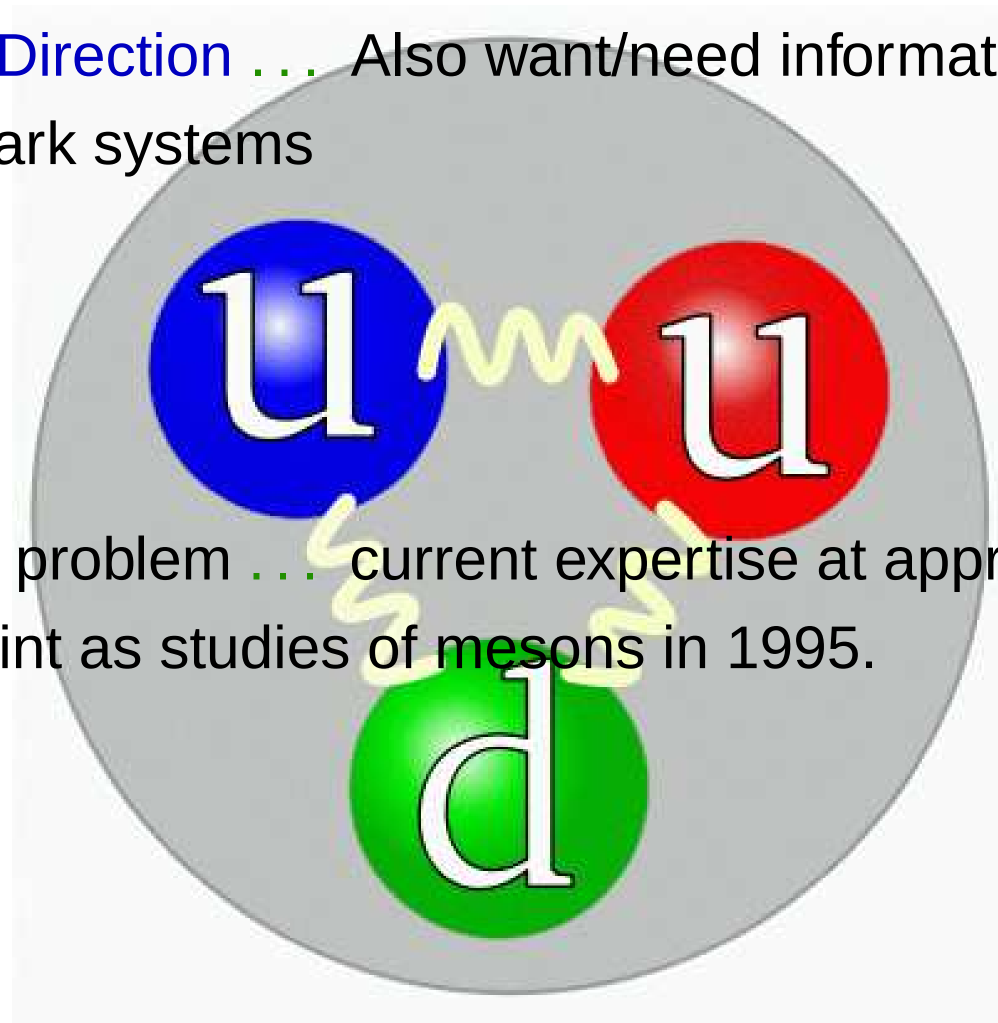

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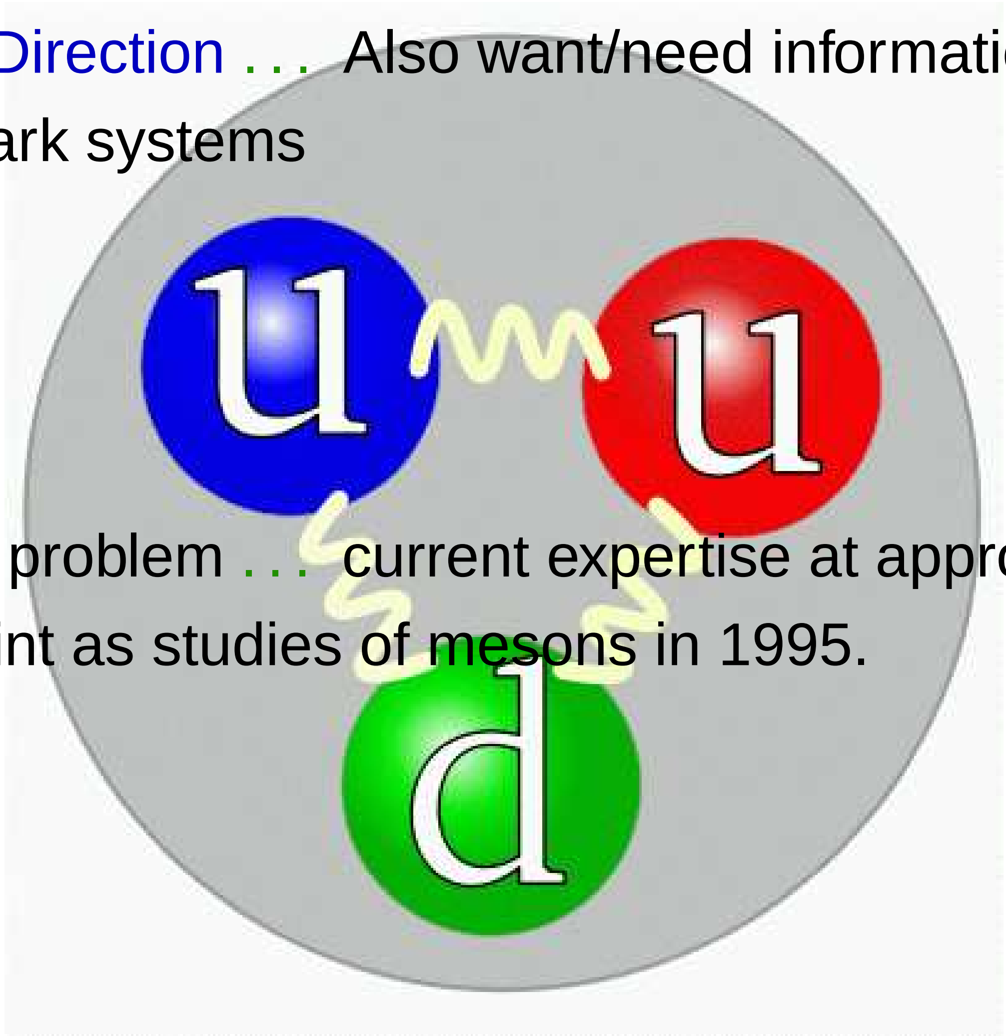

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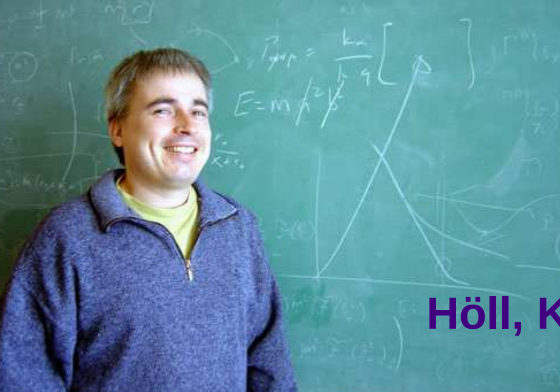
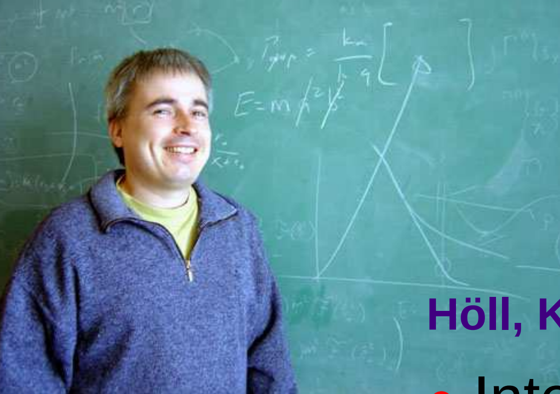
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.



Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033



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$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

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- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

Faddeev equation

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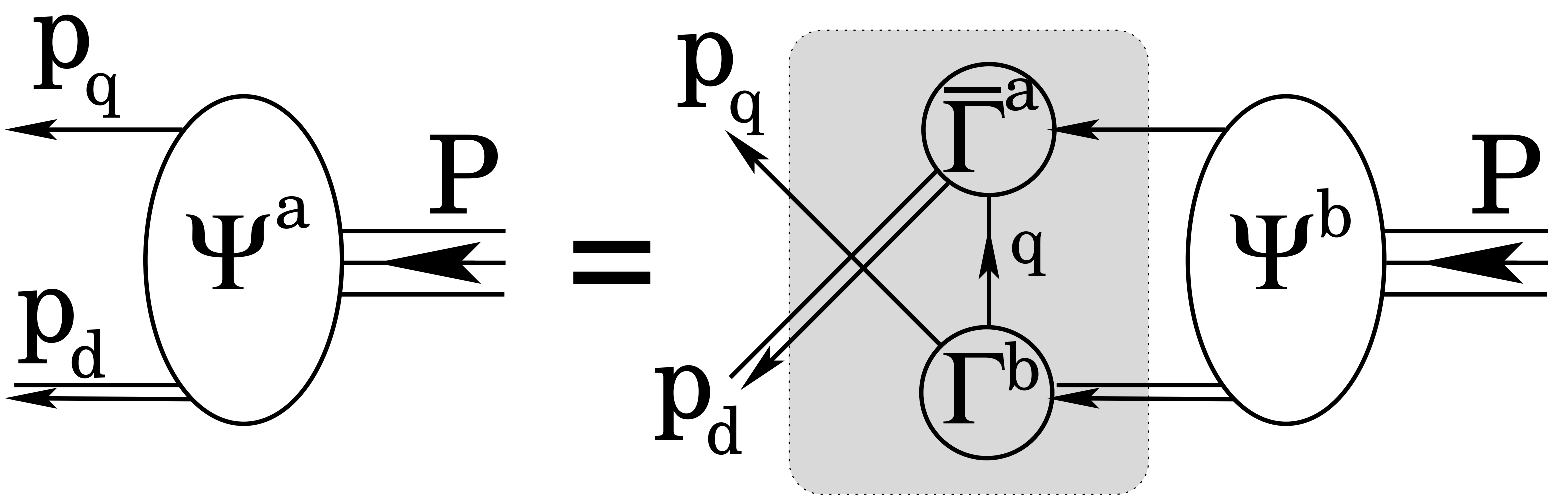

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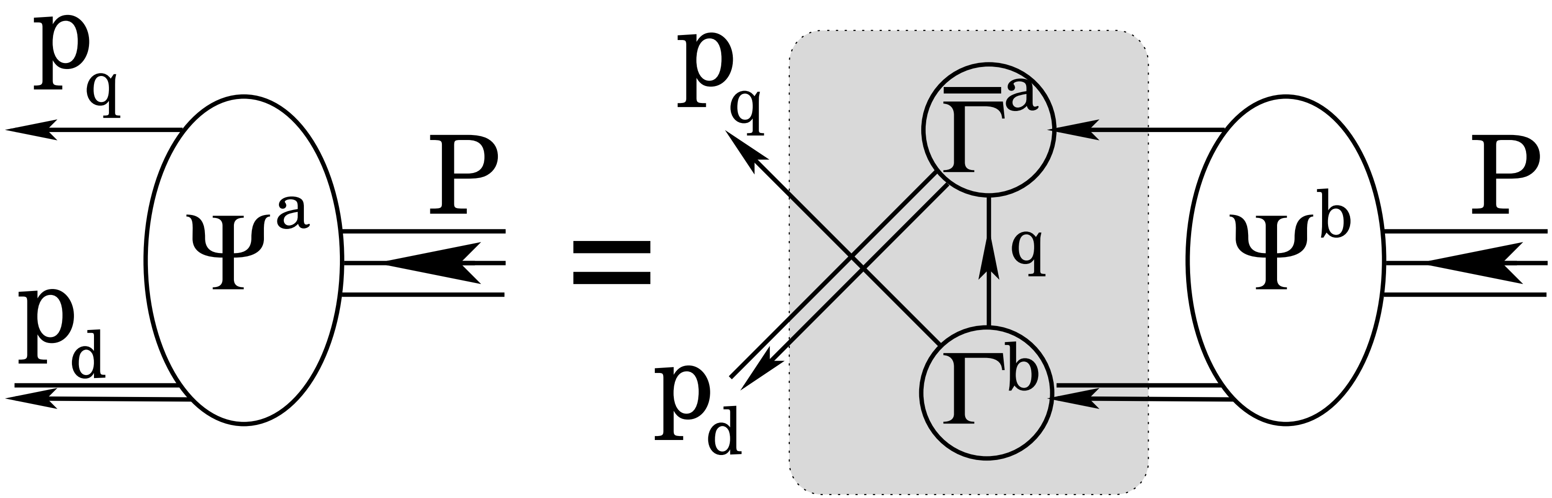
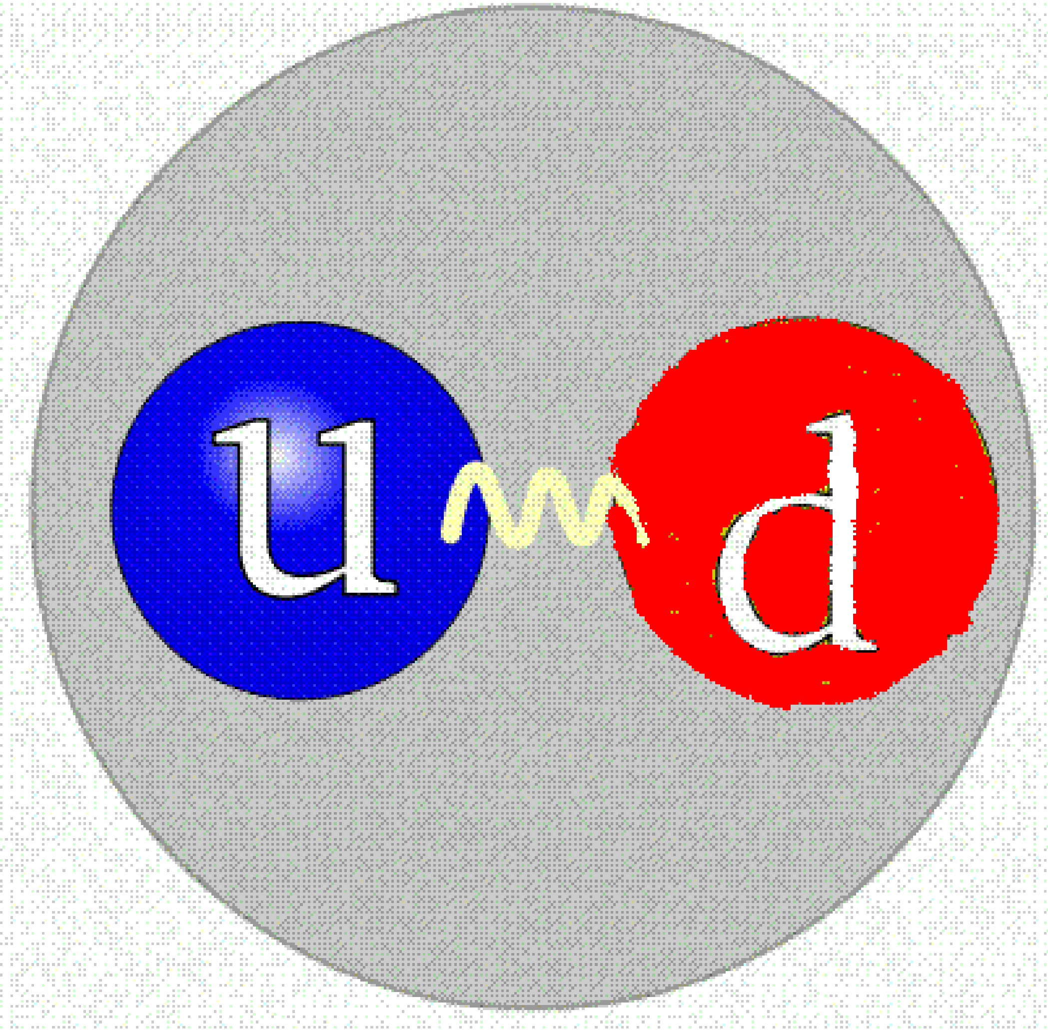


- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Diquark correlations



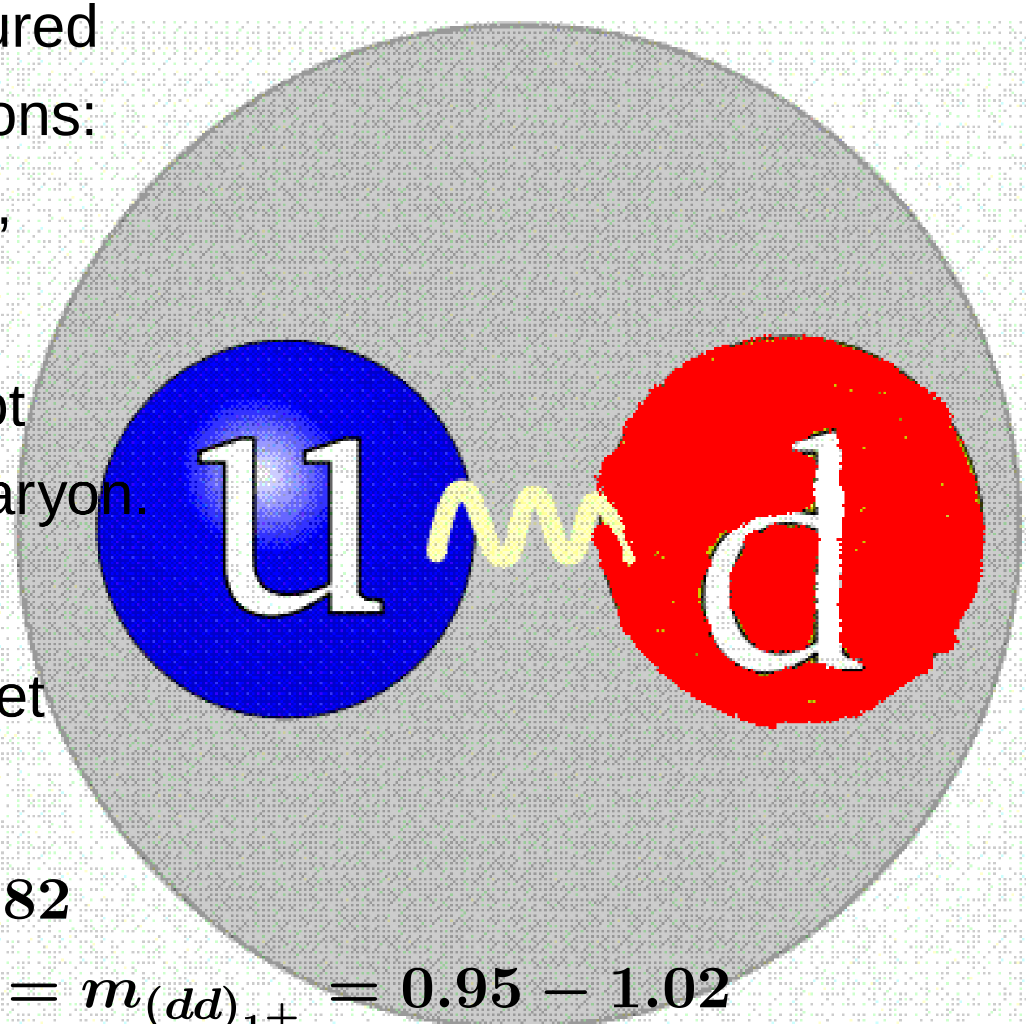
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined ... Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

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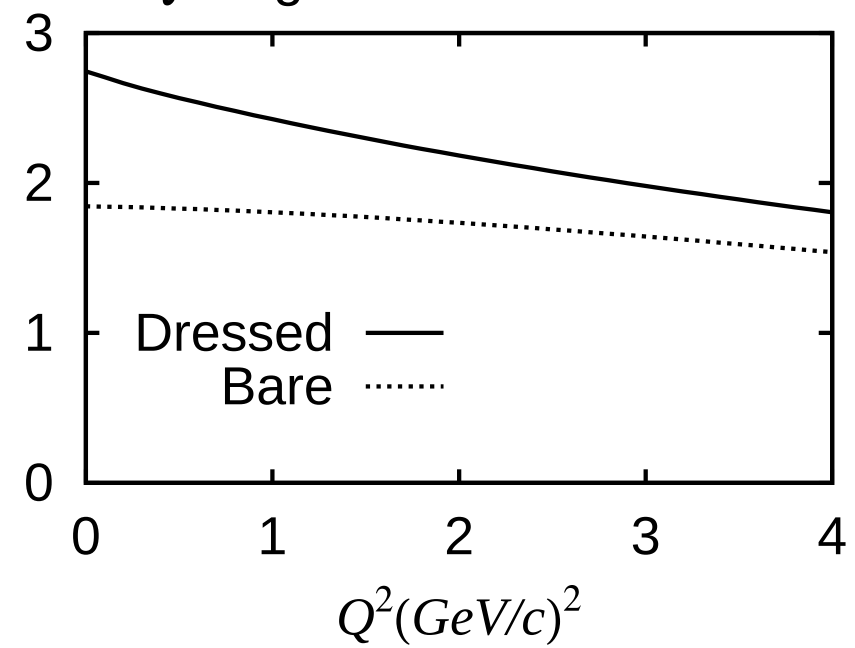
Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



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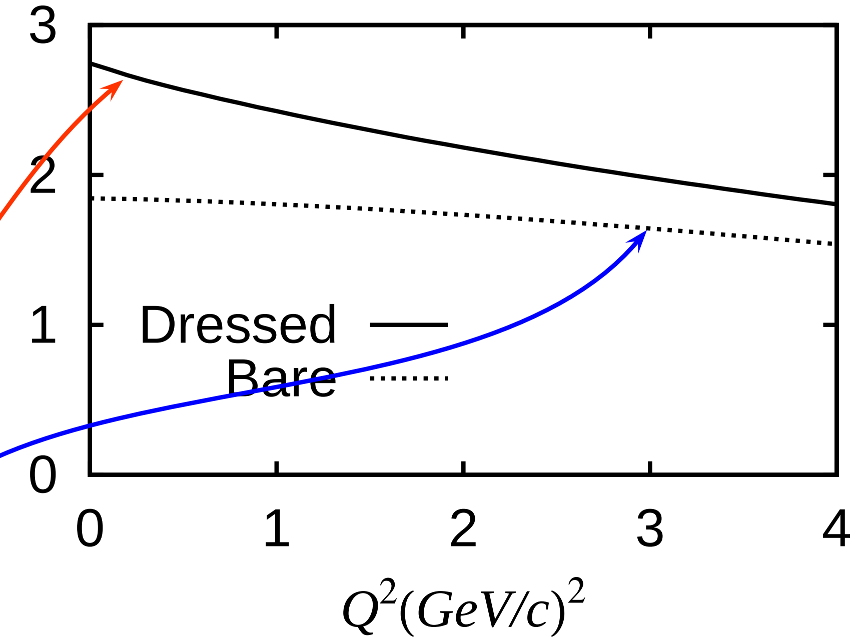

Quark Core

- Responsible for only 2/3 of result at small Q^2
- Dominant for $Q^2 > 2 - 3\,\text{GeV}^2$

Results: Nucleon and Δ Masses

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Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0^+}	m_{1^+}	ω_{0^+}	ω_{1^+}
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- $m_{1^+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

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- Axial-vector diquark provides significant attraction

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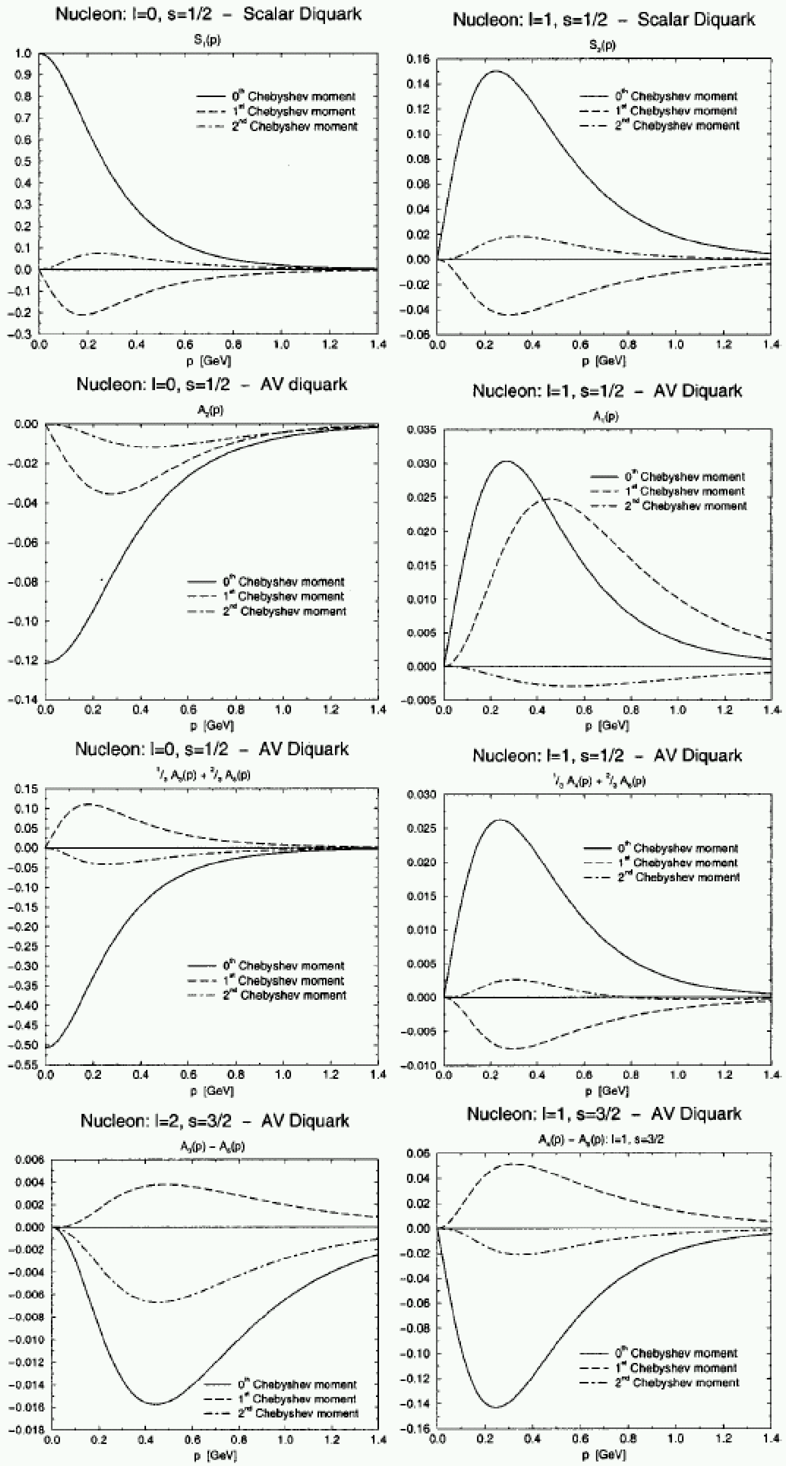
- Constructive Interference: 1^{++}-diquark $+ \partial_\mu \pi$



Angular Momentum Rest Frame

M. Oettel, *et al.*
nucl-th/9805054

Crude estimate based on magnitudes $\Rightarrow$ probability for a u-quark to carry the proton's spin is $P_{u\uparrow} \sim 80\,\%$, with $P_{u\downarrow} \sim 5\,\%$, $P_{d\uparrow} \sim 5\,\%$, $P_{d\downarrow} \sim 10\,\%$.

Hence, by this reckoning $\sim 30\%$ of proton's rest-frame spin is located in dressed-quark angular momentum.

Nucleon-Photon Vertex

M. Oettel, M. Pichowsky and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

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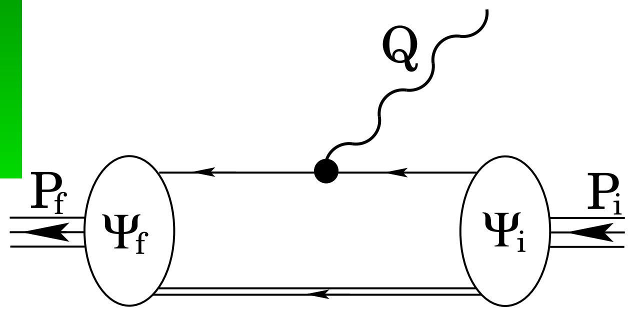
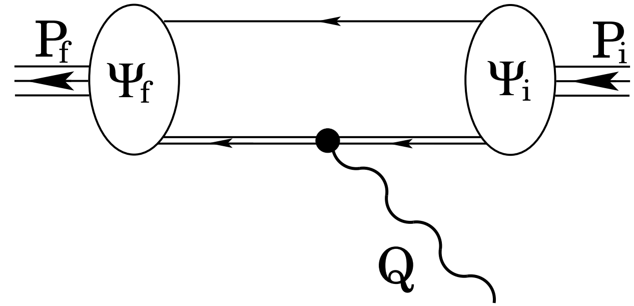
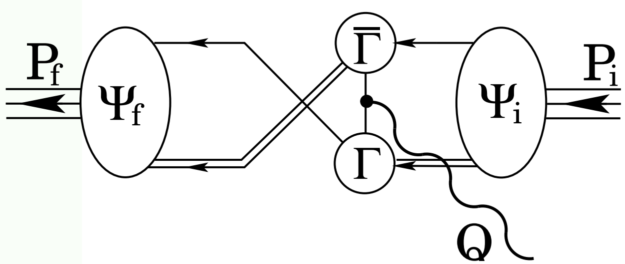
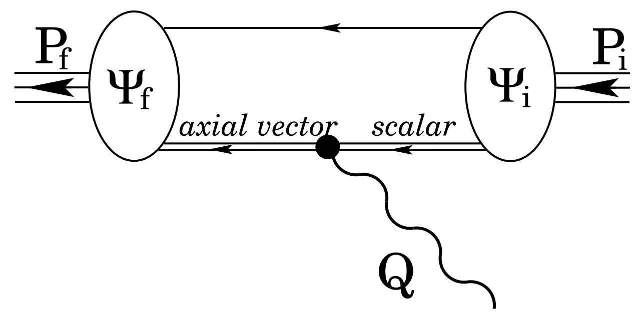
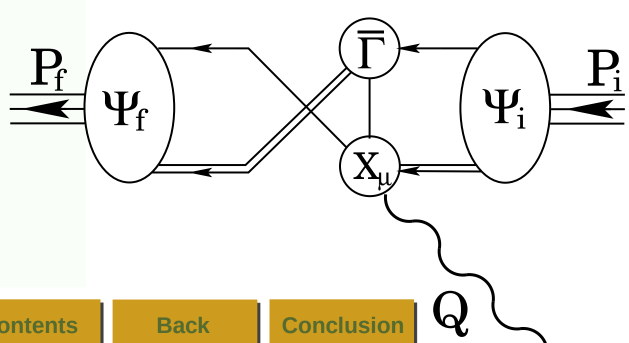
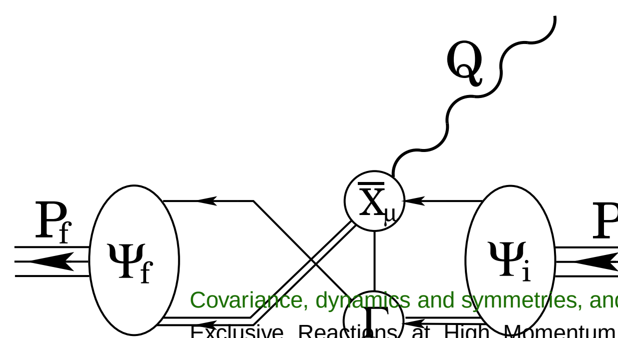




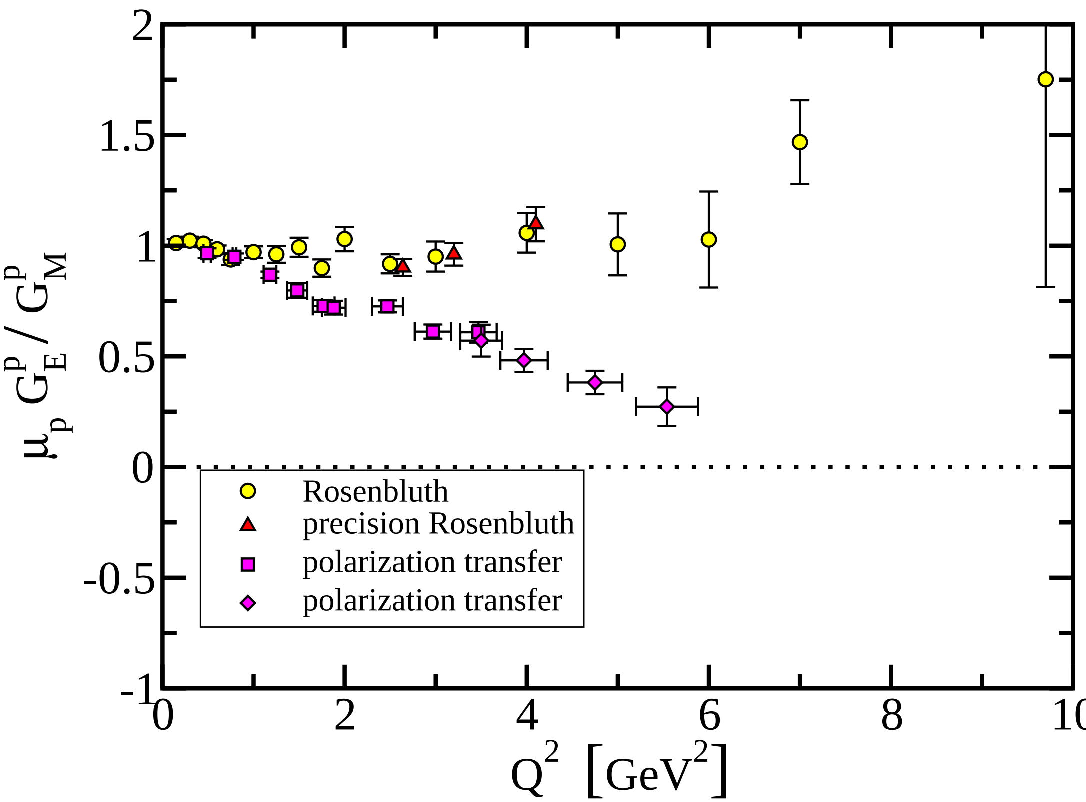








Form Factor Ratio: GE/GM



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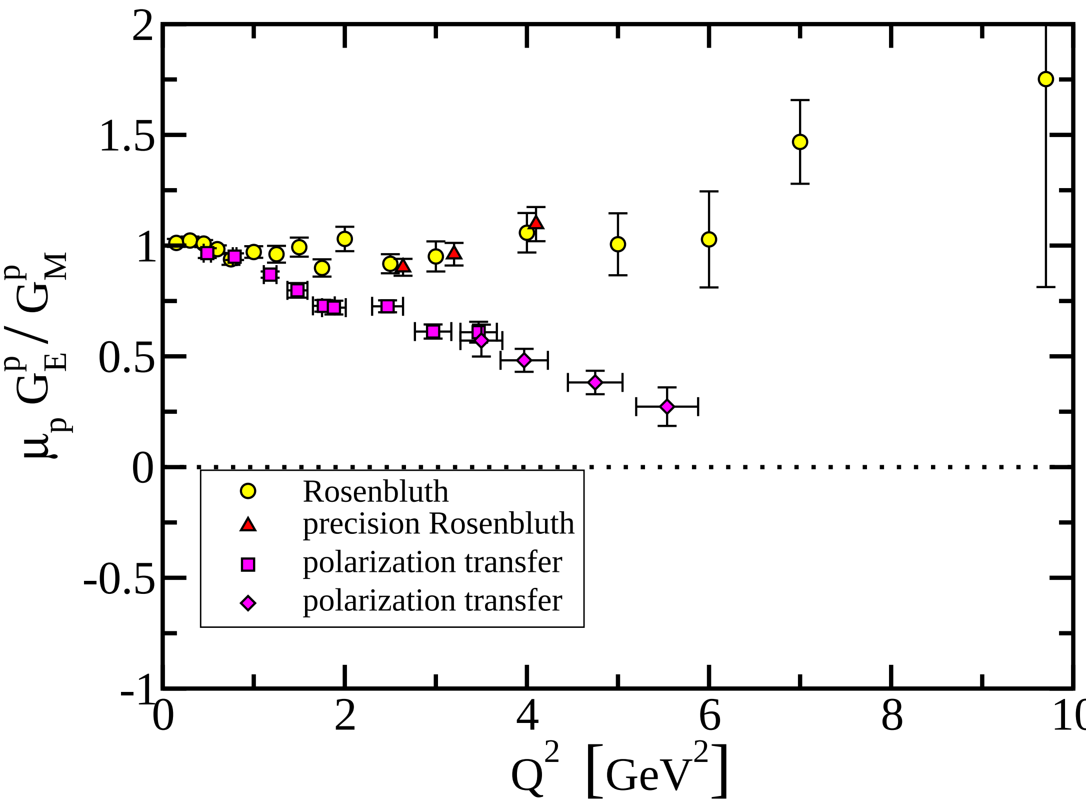
Form Factor Ratio: GE/GM

- Combine these elements . . .



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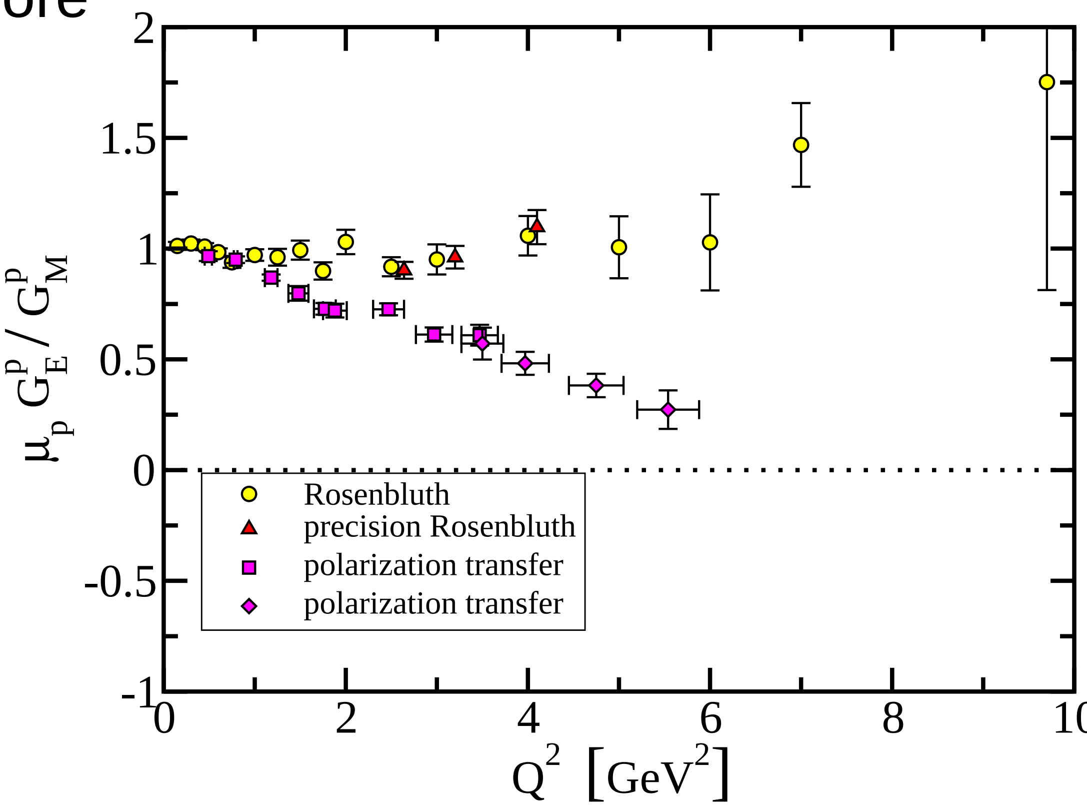

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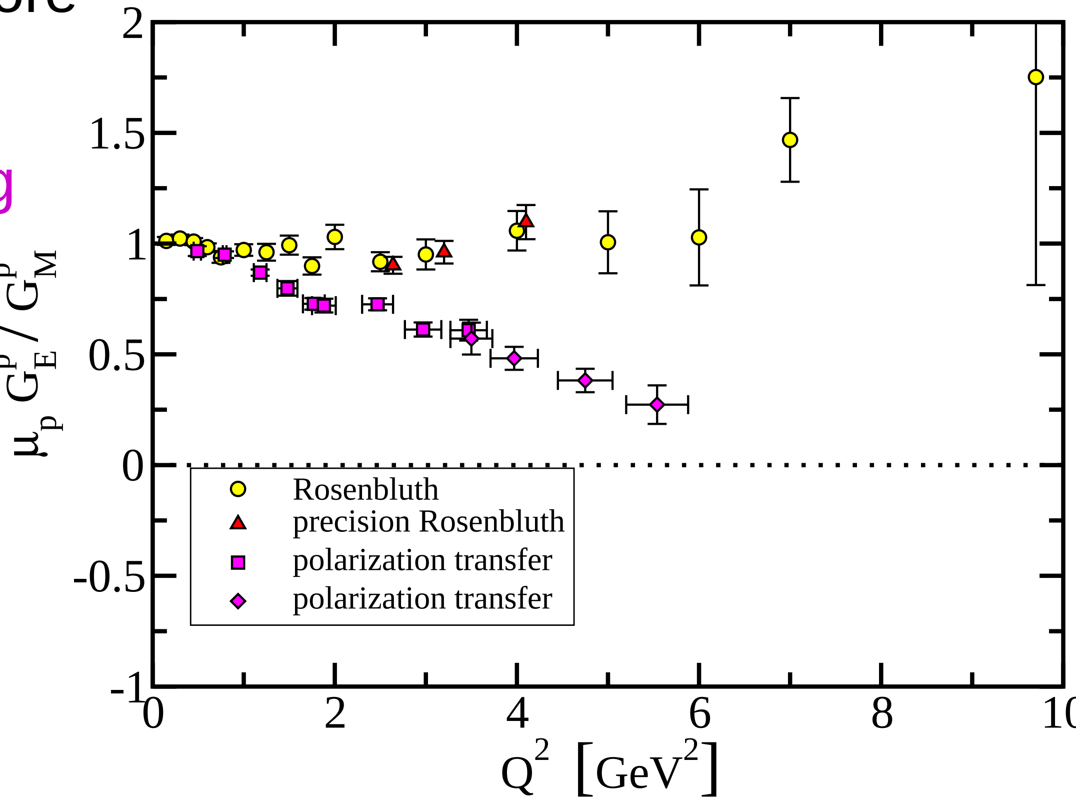

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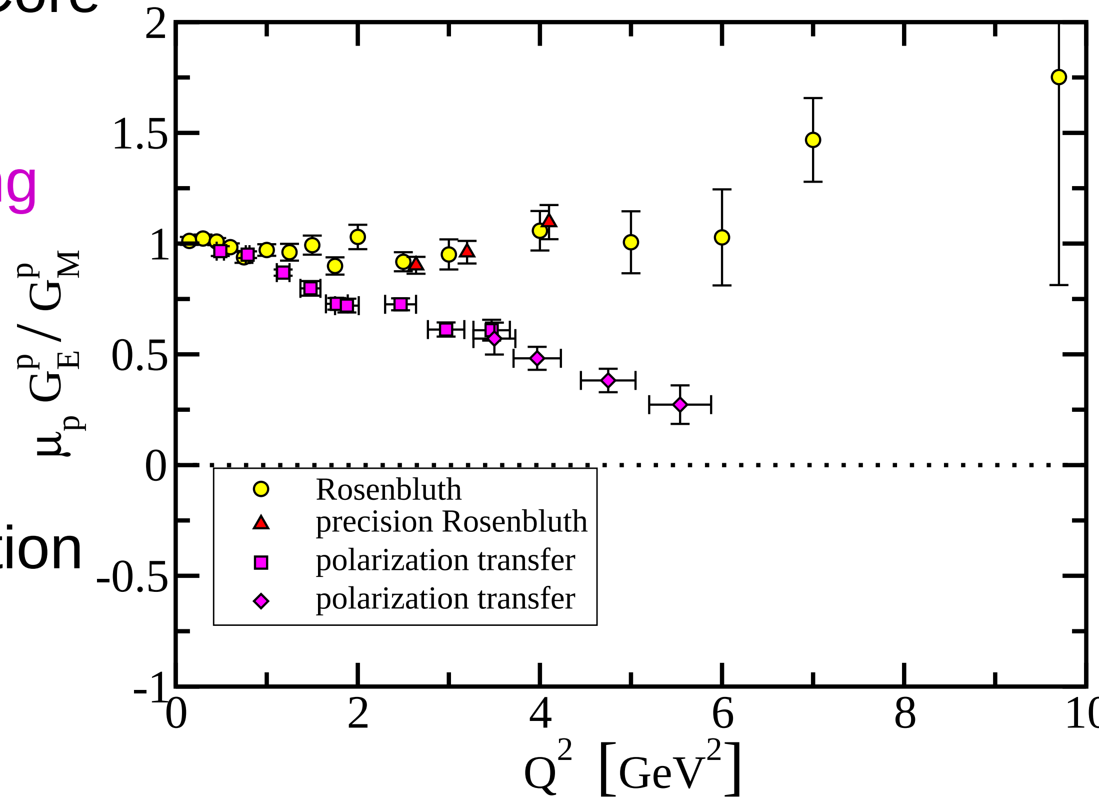

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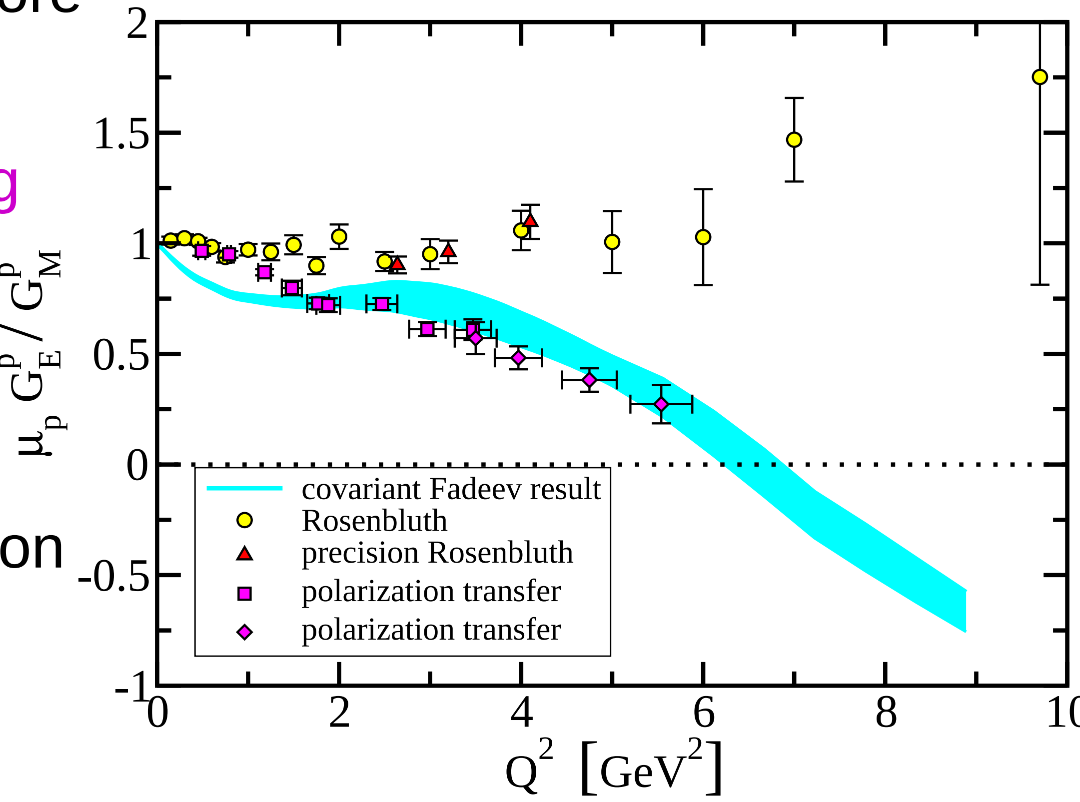

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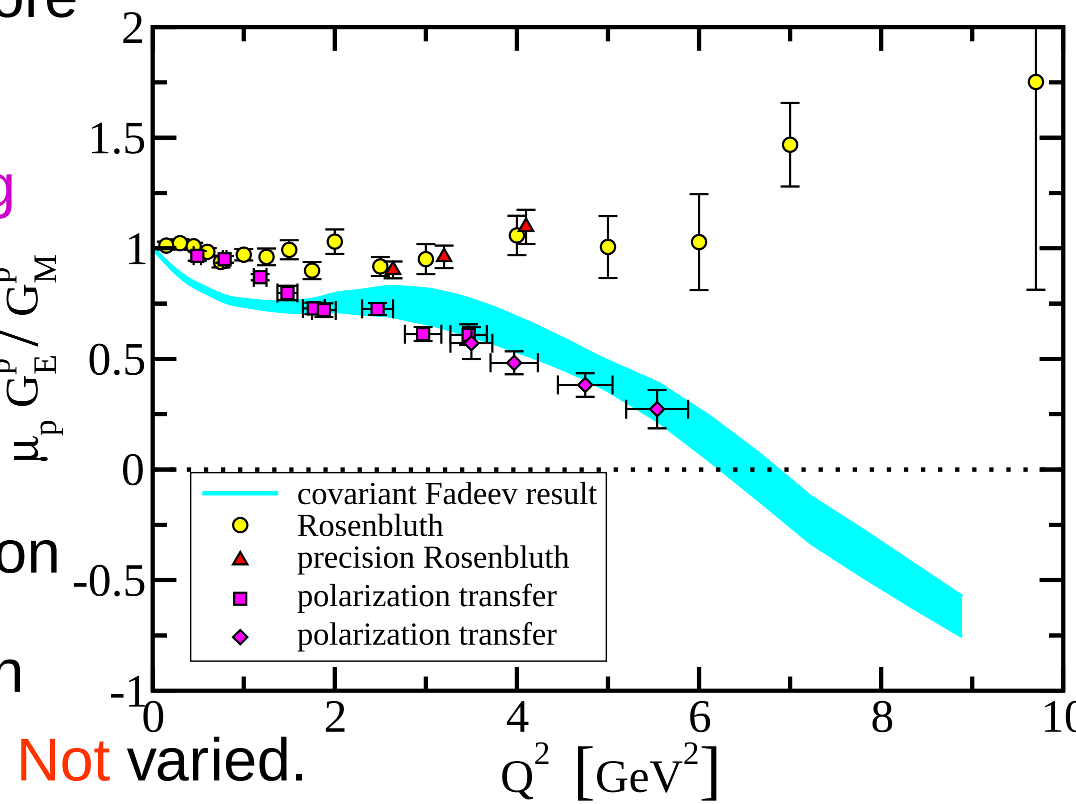

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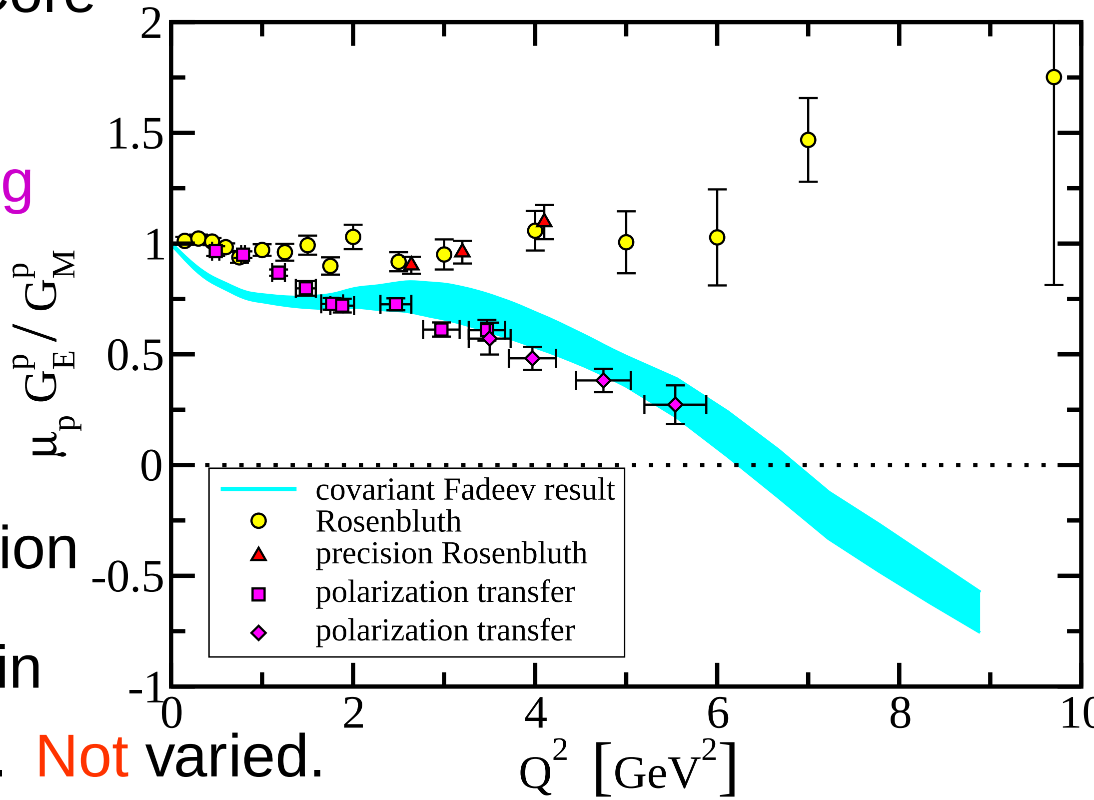

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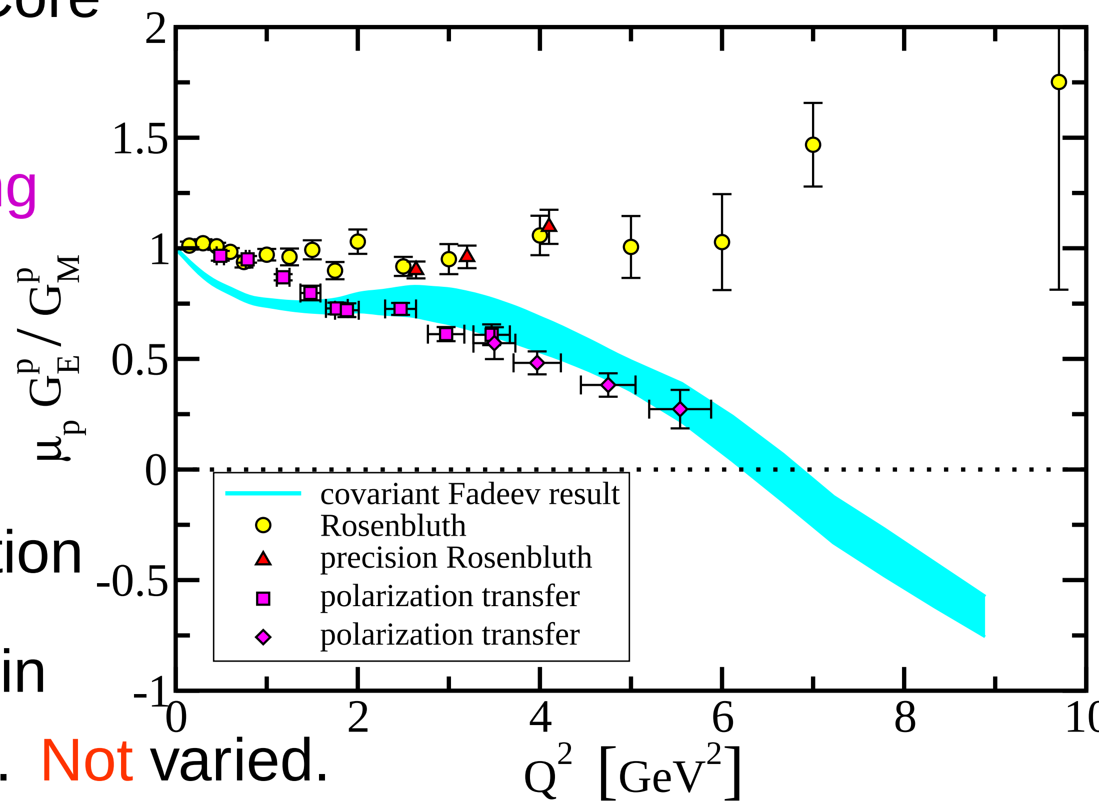

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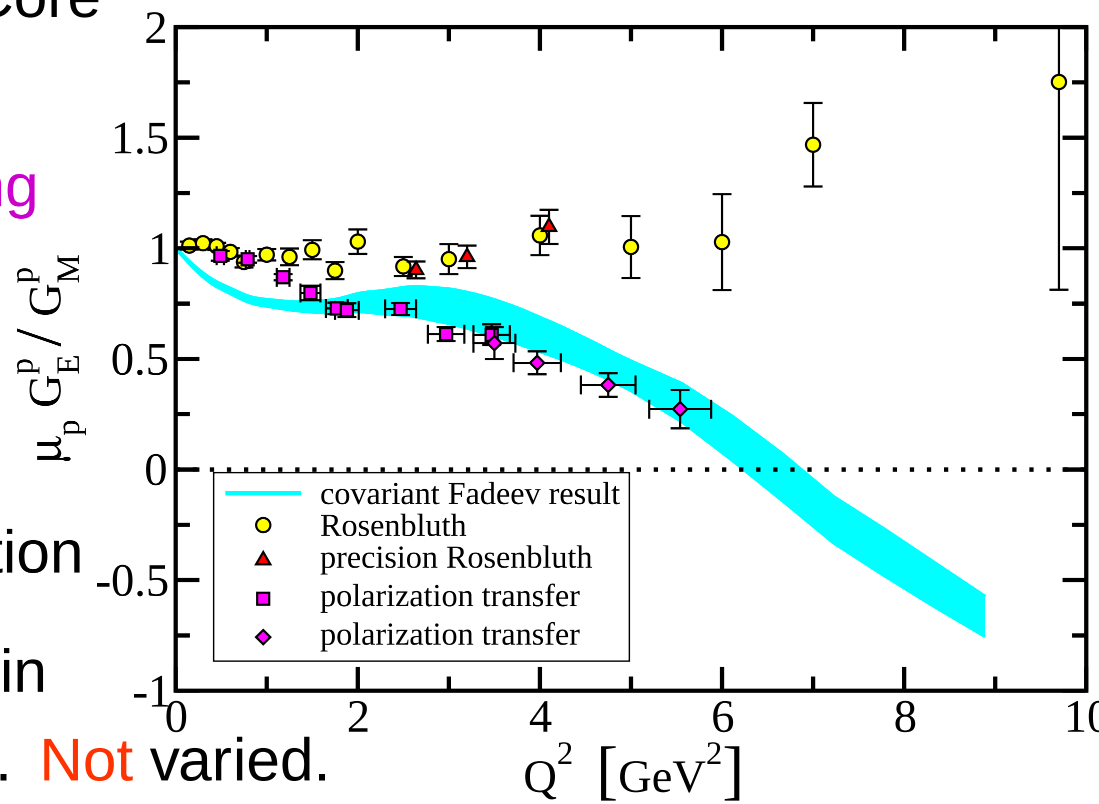

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  - Predict Zero at $Q^2 \approx 6.5$GeV^2

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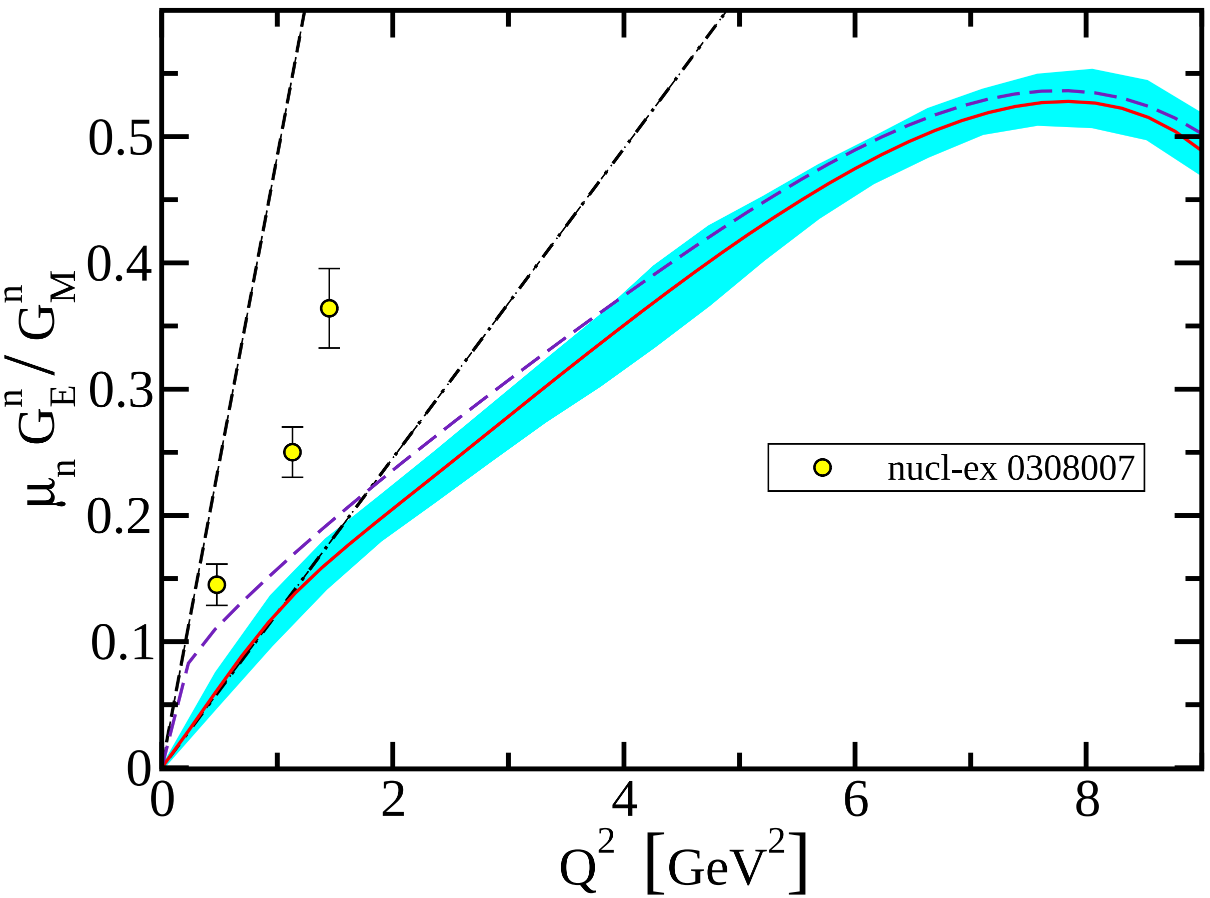
Neutron Form Factors



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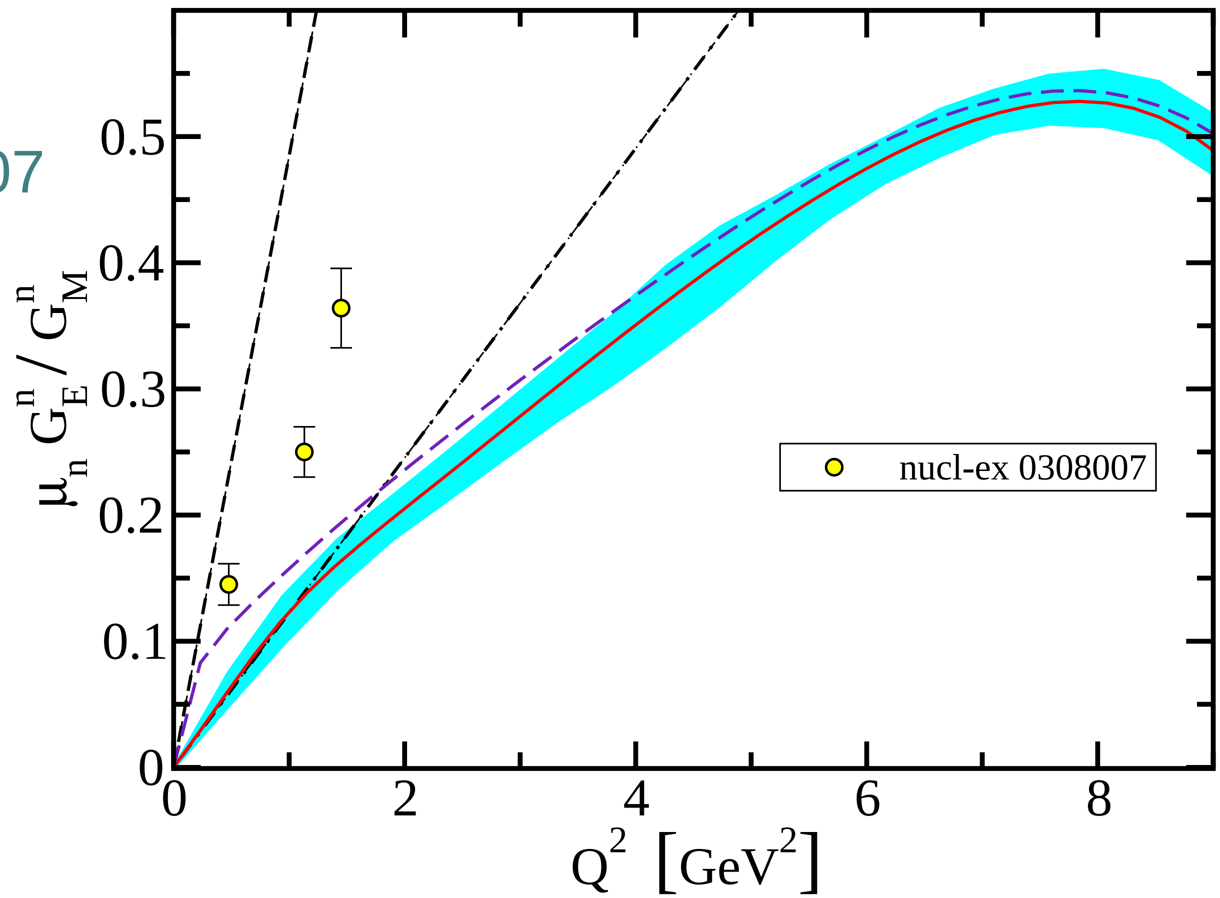

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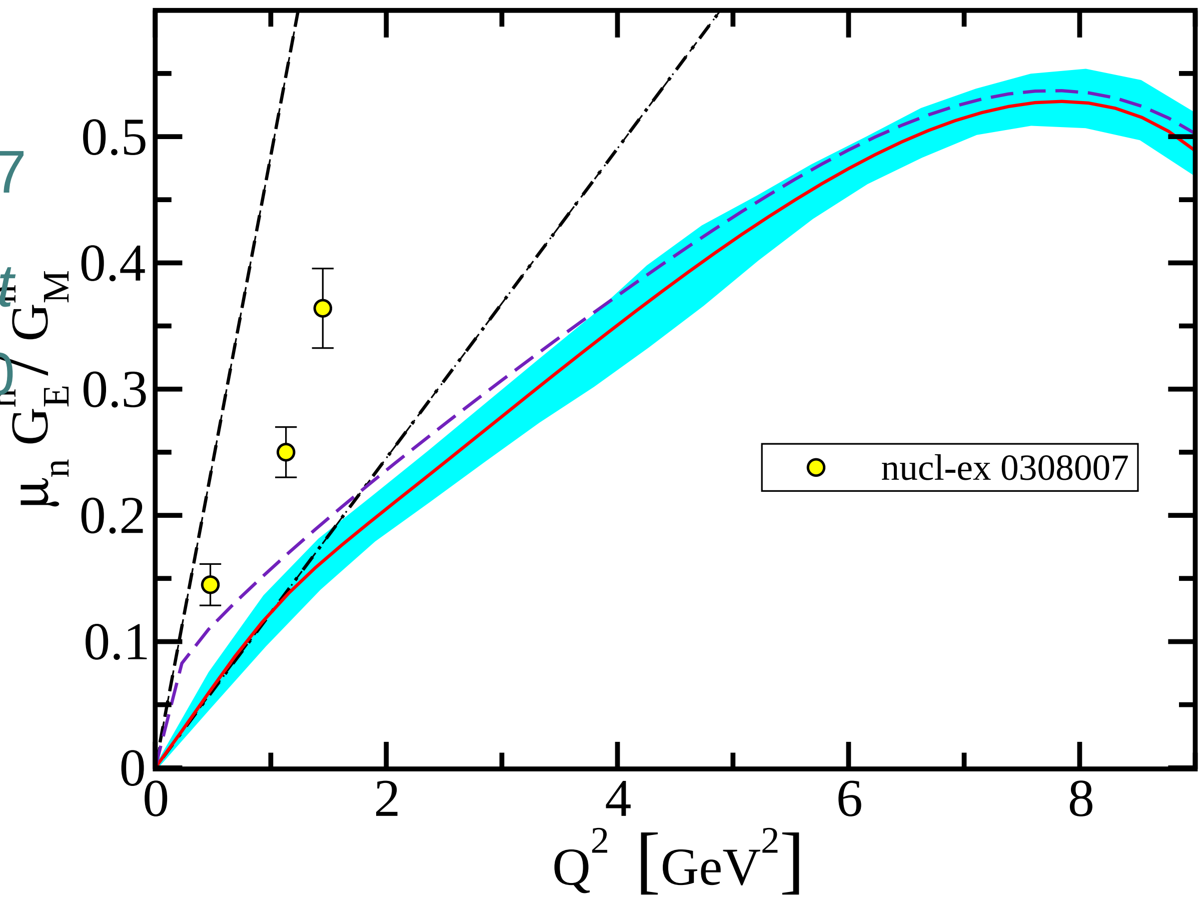

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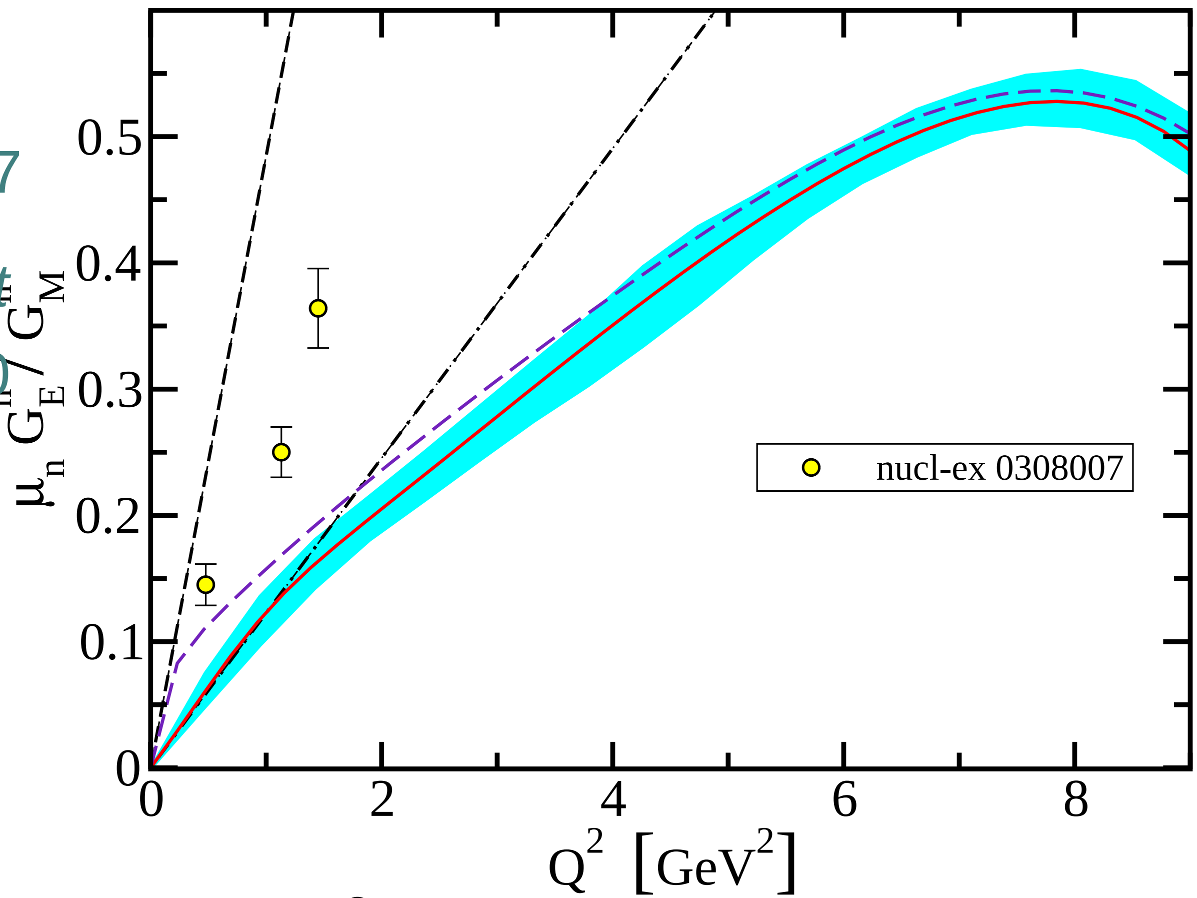

- No sign yet of a zero in $G_E^n(Q^2)$, even though calculation predicts $G_E^p(Q^2 \approx 6.5\,\mathrm{GeV}^2) = 0$
- Data to $Q^2 = 3.4$ GeV^2 is being analysed (JLab E02-013)

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- An excellent way to test conjectures and constrain the possibilities

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  Widths fixed by “asymptotic freedom” condition –

$$\frac{d}{dK^2}\left(\frac{1}{m_{J^P}^2}\mathcal{F}(K^2/\omega_{J^P}^2)\right)^{-1}\Bigg|_{K^2=0} = 1 \;\Rightarrow\; \omega_{J^P}^2 = \frac{1}{2}\,m_{J^P}^2\,,$$

Only two parameters; viz., diquark “masses”: m_{J^P}

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