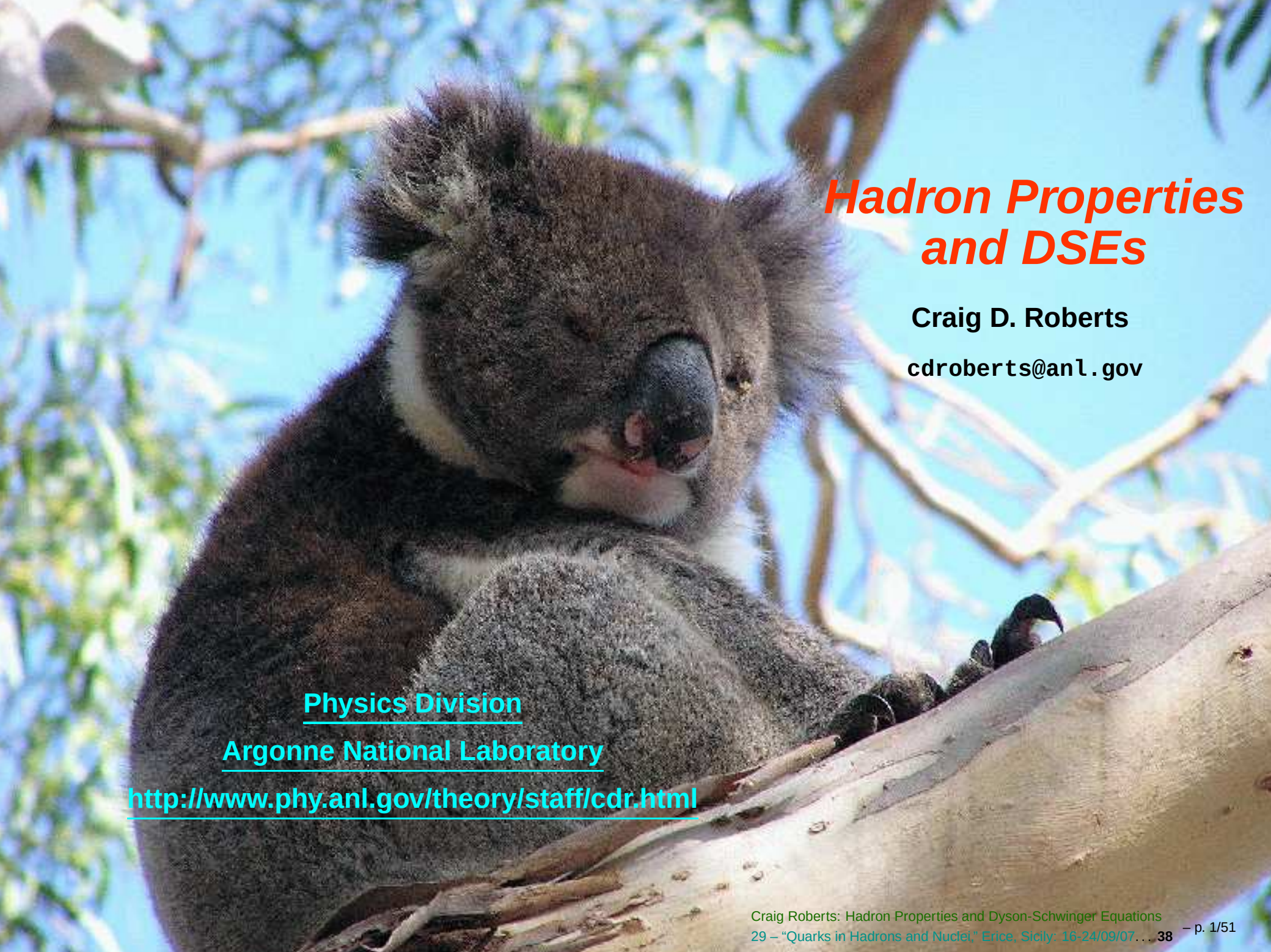


Hadron Properties and DSEs

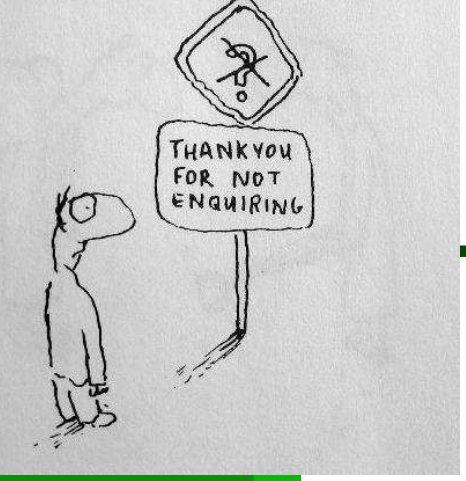
Craig D. Roberts

`cdroberts@anl.gov`

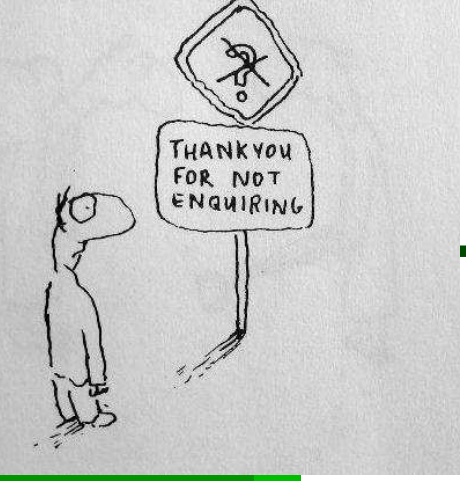
Physics Division

Argonne National Laboratory

http://www.phy.anl.gov/theory/staff/cdr.html

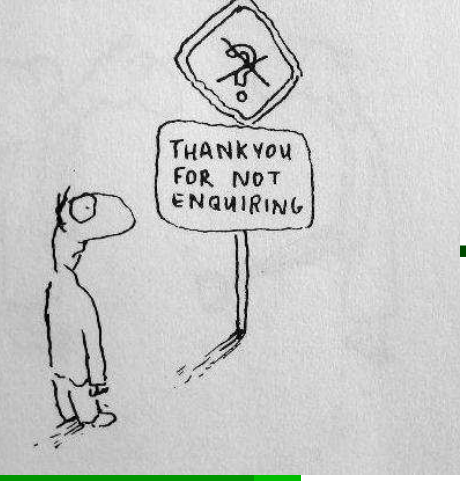


QCD's Challenges



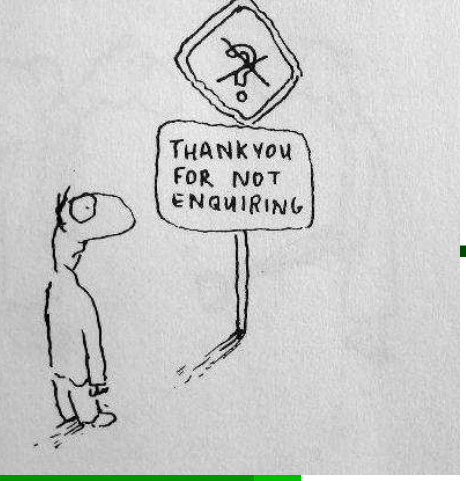
QCD's Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon



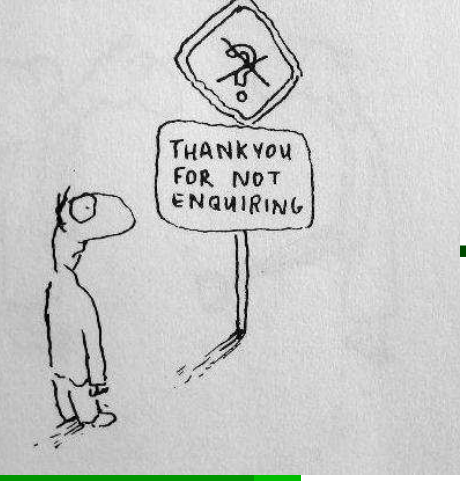
QCD's Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^{P=+}$ and $J^{P=-}$



QCD's Challenges

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^{P=+}$ and $J^{P=-}$
- Neither of these phenomena is apparent in QCD's Lagrangian ***yet*** they are the dominant determining characteristics of real-world QCD.



QCD's Challenges

Understand Emergent Phenomena

- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
    - e.g., Lagrangian (pQCD) quark mass is small but . . . no degeneracy between $J^{P=+}$ and $J^{P=-}$
- Neither of these phenomena is apparent in QCD's Lagrangian ***yet*** they are the dominant determining characteristics of real-world QCD.
- QCD – Complex behaviour

  arises from apparently simple rules

Dichotomy of Pion
– Goldstone Mode and *Bound state*

Dichotomy of Pion – Goldstone Mode and *Bound state*

- How does one make an almost massless particle
  . . . . . from two massive constituent-quarks?

Dichotomy of Pion
– Goldstone Mode and *Bound state*

- How does one make an almost massless particle
  . . . . . from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

  Must exhibit $\boxed{m_\pi^2 \propto m_q}$

Current Algebra . . . 1968

Dichotomy of Pion – Goldstone Mode and *Bound state*

- How does one make an almost massless particle ..... from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

  Must exhibit $\boxed{m_\pi^2 \propto m_q}$

  Current Algebra . . . 1968

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a

- well-defined and valid chiral limit;
- and an accurate realisation of dynamical chiral symmetry breaking.

Dichotomy of Pion
– Goldstone Mode and *Bound state*

- How does one make an almost massless particle
  . . . . . from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

  Must exhibit $\boxed{m_\pi^2 \propto m_q}$

  Current Algebra . . . 1968

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a

- well-defined and valid chiral limit;
- and an accurate realisation of dynamical chiral symmetry breaking.

Highly Nontrivial

What's the Problem?

What's the Problem?

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.

What's the Problem?

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
- Means . . . must calculate hadron *wave functions*

  – Can't be done using perturbation theory

What's the Problem?

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
- Means . . . must calculate hadron *wave functions*
  – Can't be done using perturbation theory
- Why problematic? Isn't same true in quantum mechanics?

What's the Problem?

- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
- Means . . . must calculate hadron *wave functions*
  – Can't be done using perturbation theory
- Why problematic? Isn't same true in quantum mechanics?
- Differences!

What's the Problem? Relativistic QFT!

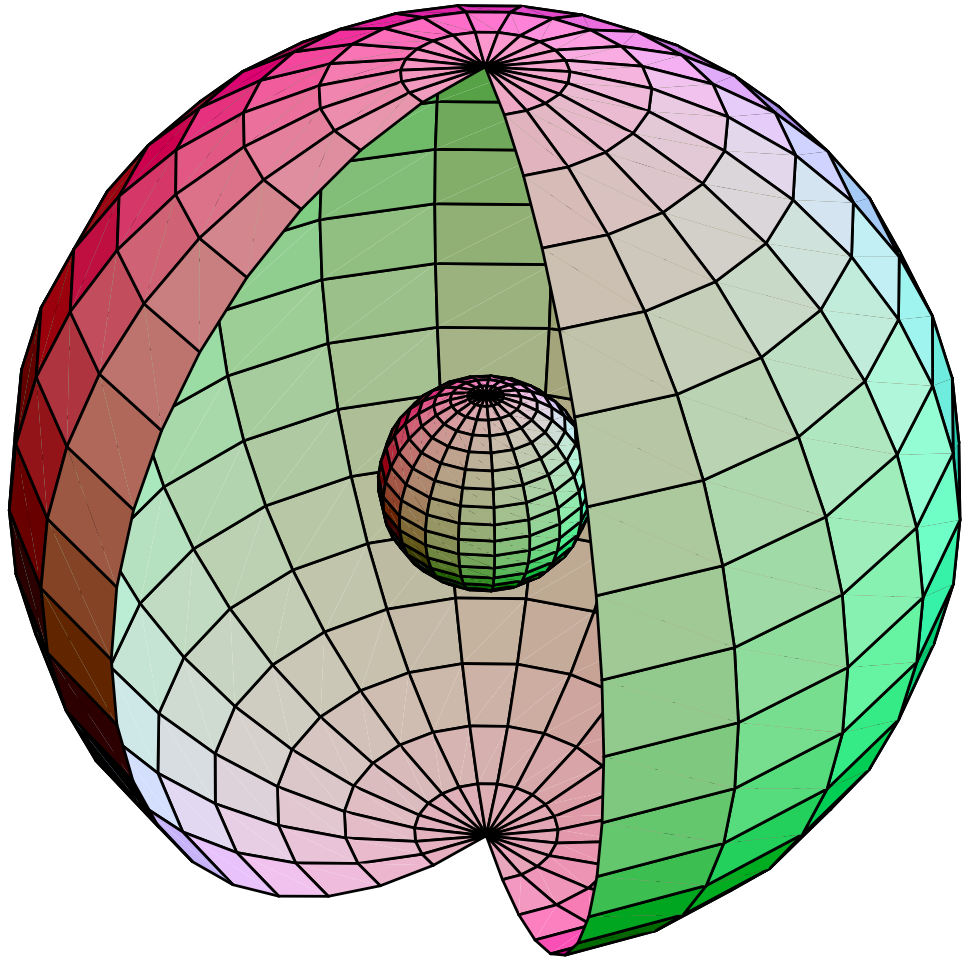
- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
- Differences!
  - Here relativistic effects are crucial – *virtual particles*, quintessence of Relativistic Quantum Field Theory – must be included

What's the Problem? Relativistic QFT!

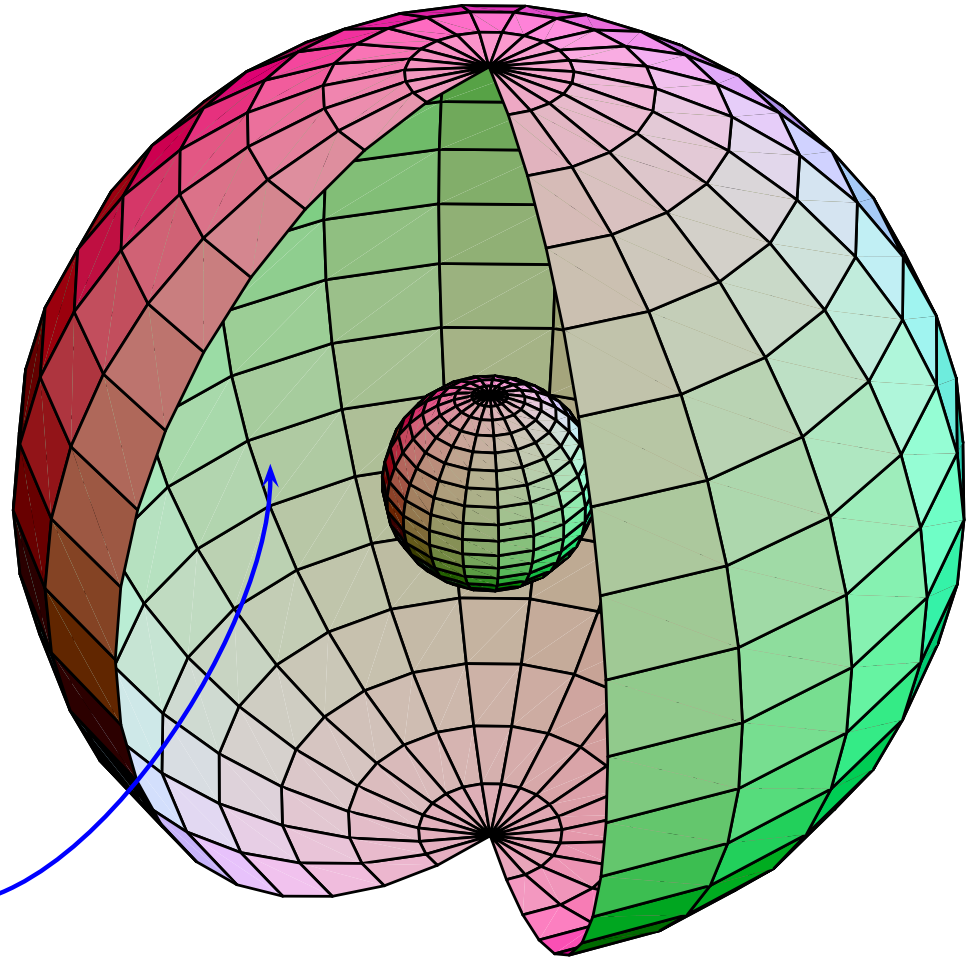
- Minimal requirements
  - detailed understanding of connection between Current-quark and Constituent-quark masses;
  - and systematic, symmetry preserving means of realising this connection in bound-states.
- Differences!
  - Here relativistic effects are crucial – *virtual particles*, quintessence of Relativistic Quantum Field Theory – must be included
  - Interaction between quarks – the Interquark "Potential" – unknown throughout $> 98\%$ of a hadron's volume

Intranucleon Interaction

Intranucleon Interaction



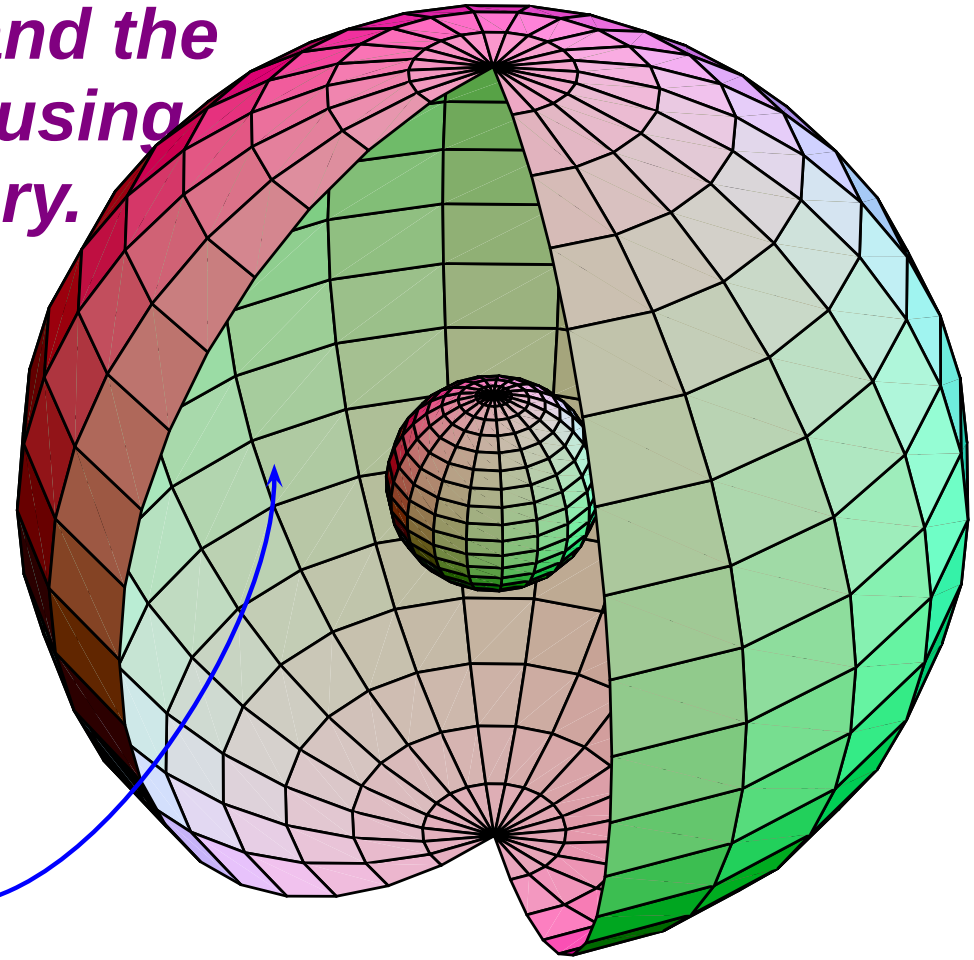
Intranucleon Interaction



98% of the volume

What is the Intranucleon Interaction?

The question must be rigorously defined, and the answer mapped out using experiment and theory.



98% of the volume

Dyson-Schwinger Equations

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking

        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement

        – Coloured objects not detected, not detectable?
- $\Rightarrow$ Understanding InfraRed (long-range)

  ..... behaviour of $\alpha_s(Q^2)$

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?
- Method yields Schwinger Functions $\equiv$ Propagators

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?

Cross-Sections built from Schwinger Functions

Schwinger Functions

Schwinger Functions

- Solutions are Schwinger Functions (Euclidean Green Functions)

Schwinger Functions

- Solutions are Schwinger Functions (Euclidean Green Functions)
- Not all are Schwinger functions are experimentally observable

Schwinger Functions

- Solutions are Schwinger Functions (Euclidean Green Functions)
- Not all are Schwinger functions are experimentally observable but . . .
  - all are same VEVs measured in numerical simulations of lattice-regularised QCD
  - opportunity for comparisons at pre-experimental level . . . cross-fertilisation

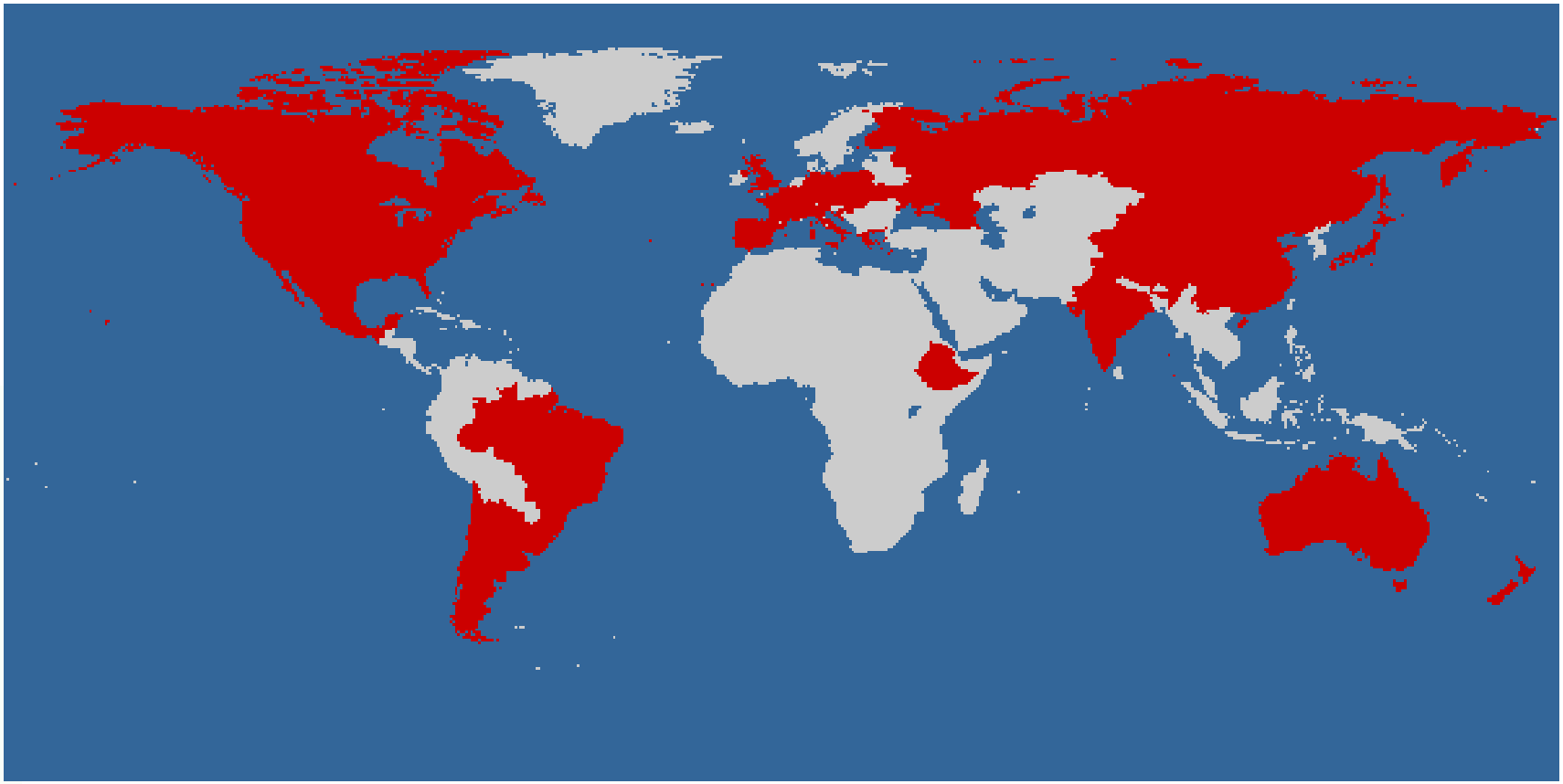
Schwinger Functions

- Solutions are Schwinger Functions (Euclidean Green Functions)
- Not all are Schwinger functions are experimentally observable but . . .
  - all are same VEVs measured in numerical simulations of lattice-regularised QCD
  - opportunity for comparisons at pre-experimental level . . . cross-fertilisation
- Proving fruitful.

World . . .

World . . .

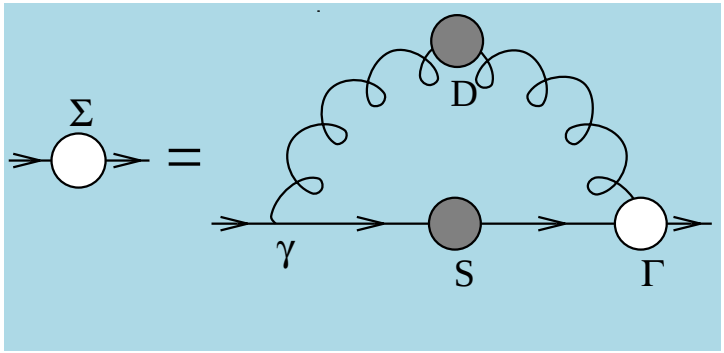
DSE Perspective



Persistent Challenge

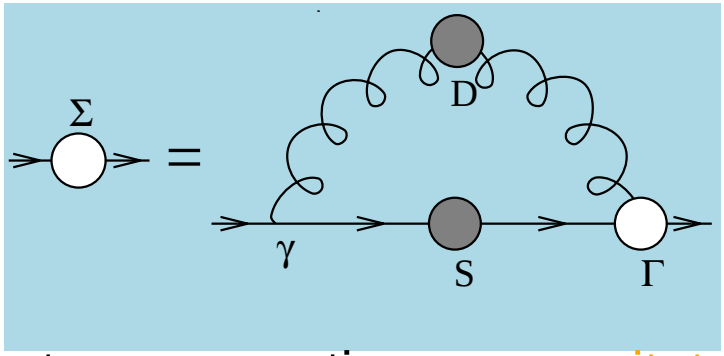
Persistent Challenge

- Infinitely Many Coupled Equations



Persistent Challenge

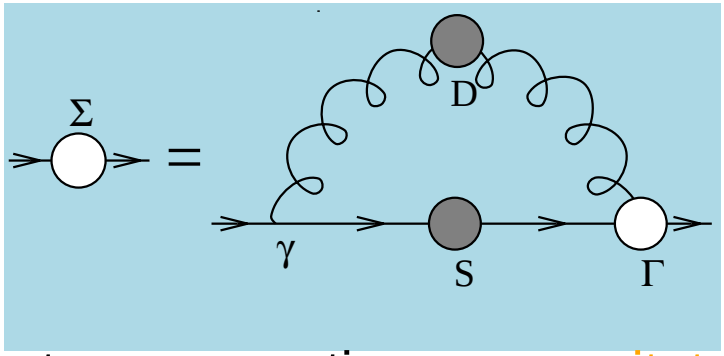
- Infinitely Many Coupled Equations



- Coupling between equations necessitates truncation

Persistent Challenge

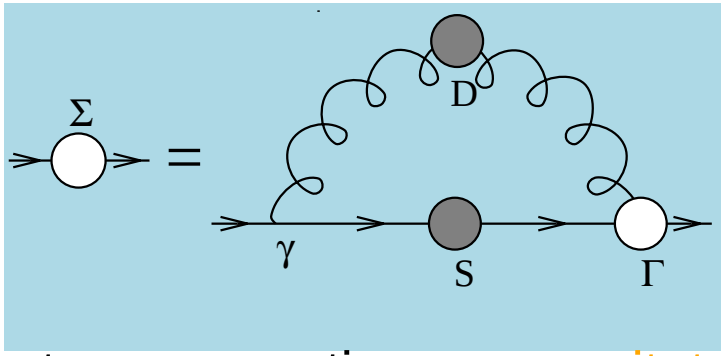
- Infinitely Many Coupled Equations



- Coupling between equations necessitates truncation
  - Weak coupling expansion ⇒ Perturbation Theory

Persistent Challenge

- Infinitely Many Coupled Equations



- Coupling between equations necessitates truncation
  - Weak coupling expansion ⇒ Perturbation Theory
    Not useful for the nonperturbative problems
    in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of EXACT Results in QCD

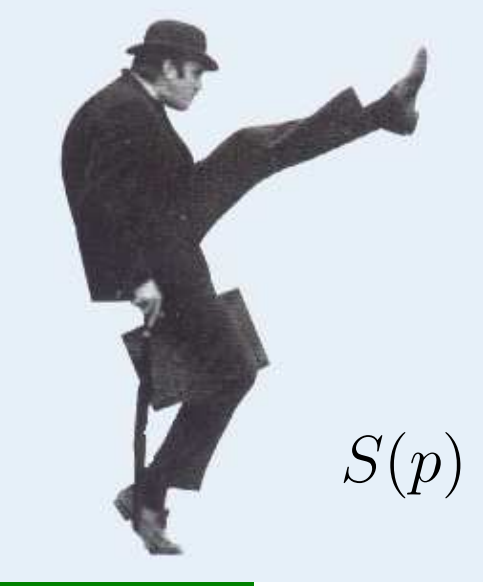
Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of EXACT Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results

Persistent Challenge

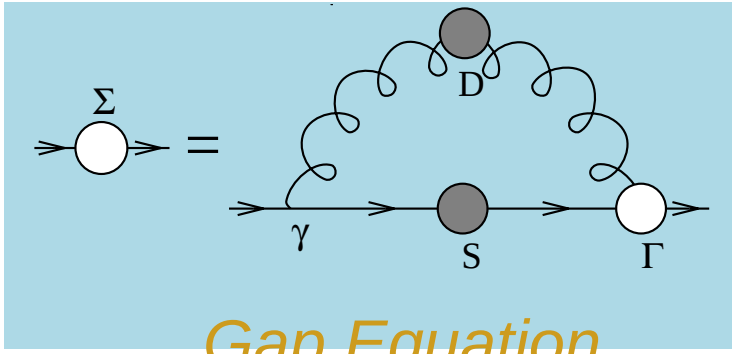
- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of EXACT Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results
  - Make Predictions with Readily Quantifiable Errors

Perturbative Dressed-quark Propagator

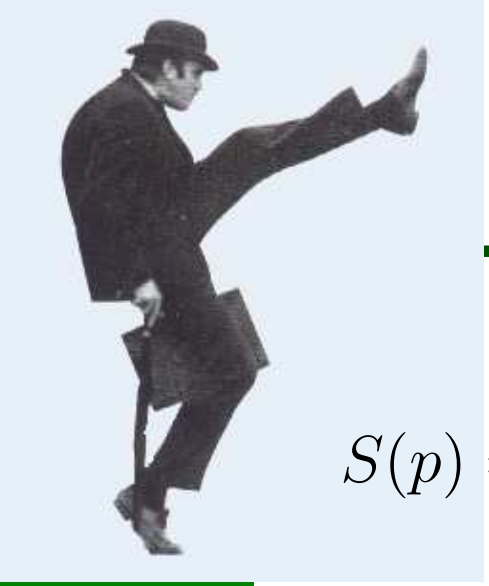


Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

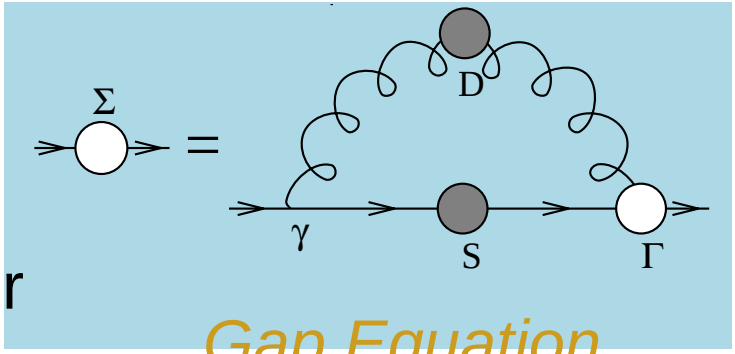


Gap Equation



Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$



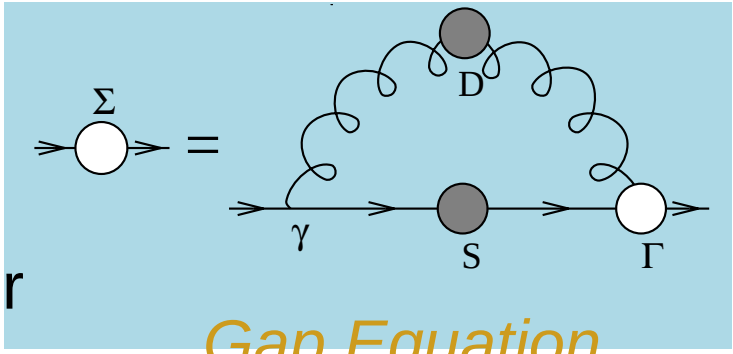
Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$



Gap Equation

- dressed-quark propagator

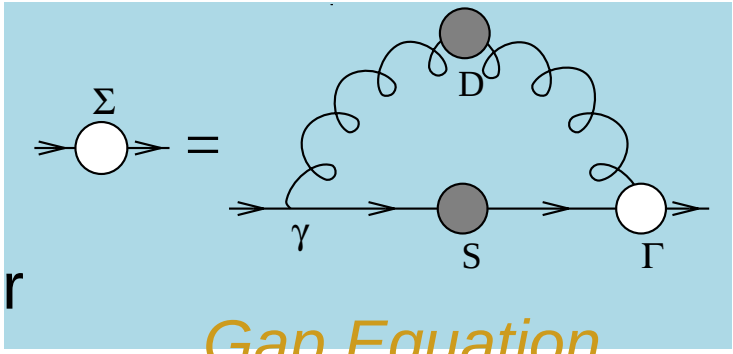
$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory

Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$



- dressed-quark propagator

Gap Equation

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

- Weak Coupling Expansion

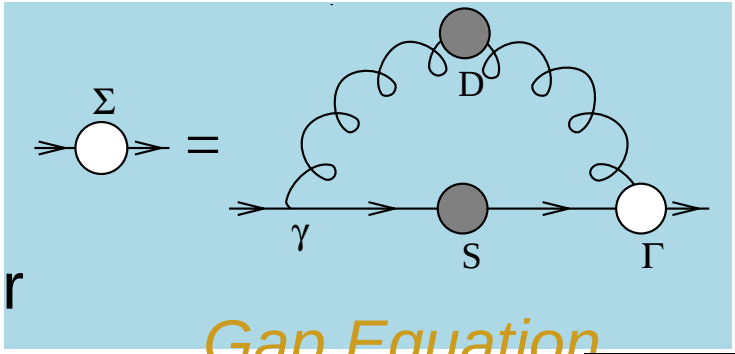
  Reproduces Every Diagram in Perturbation Theory

- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\longrightarrow} 0$$

Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$



Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

No DCSB Here!

- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory

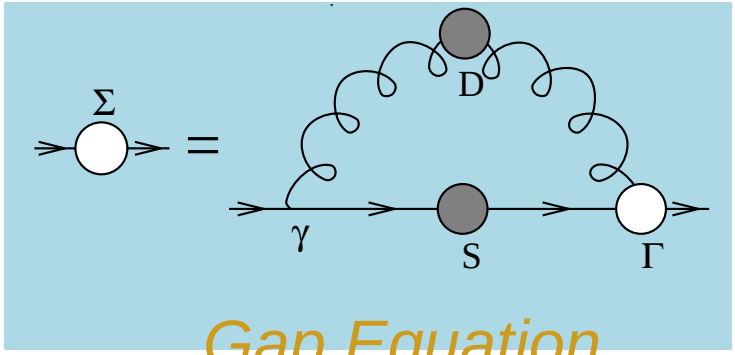
- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \xrightarrow{m\to 0} 0$$

Dressed-Quark Propagator

Dressed-Quark Propagator

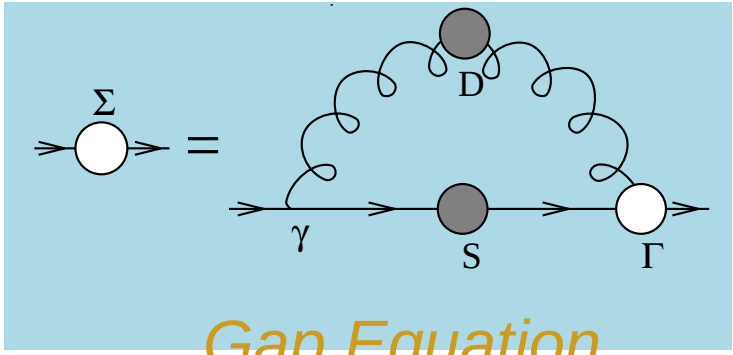
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

Dressed-Quark Propagator

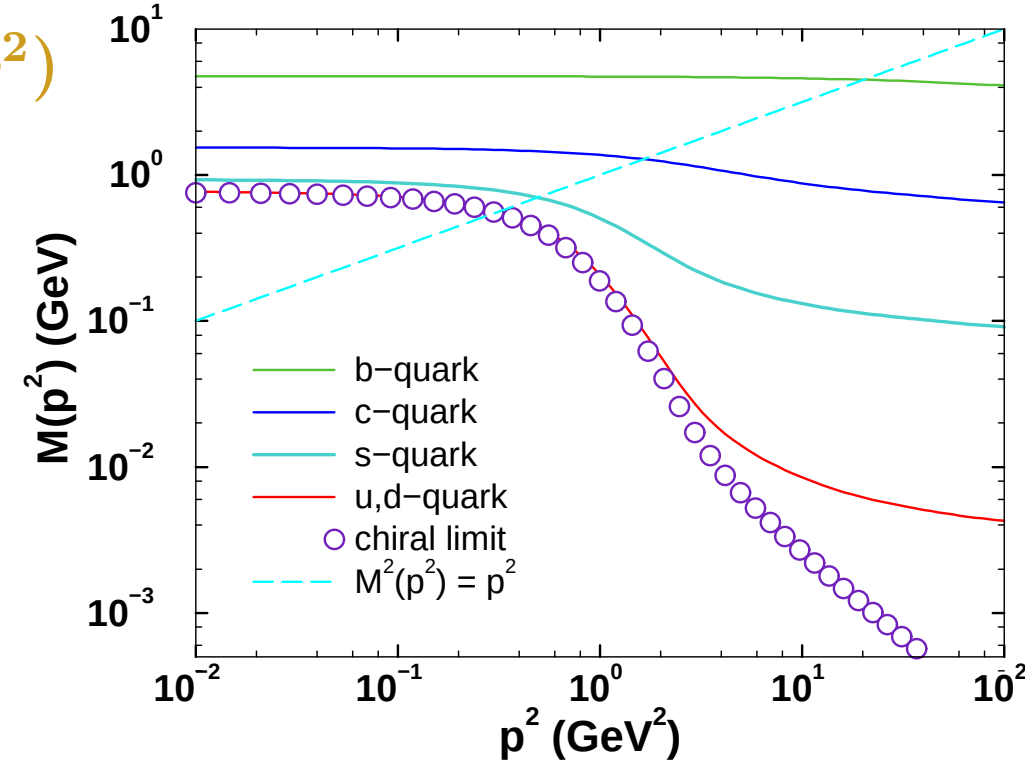
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

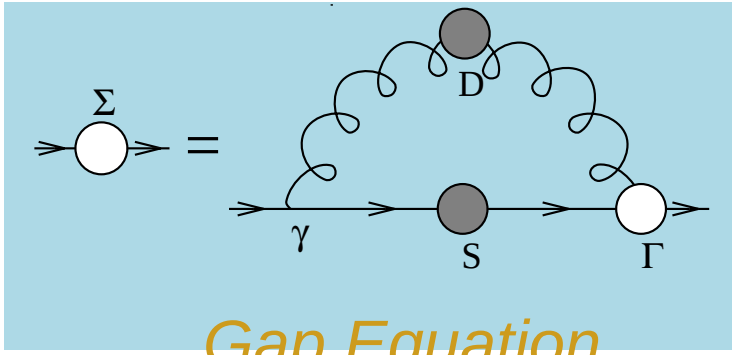
- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$



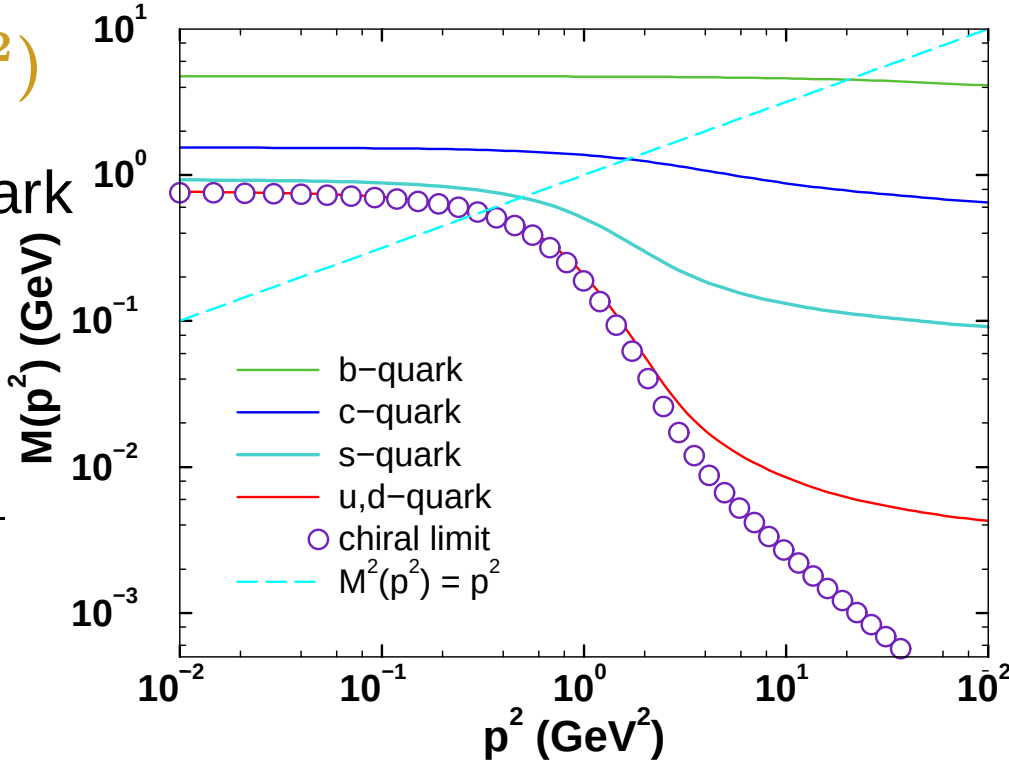
Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$

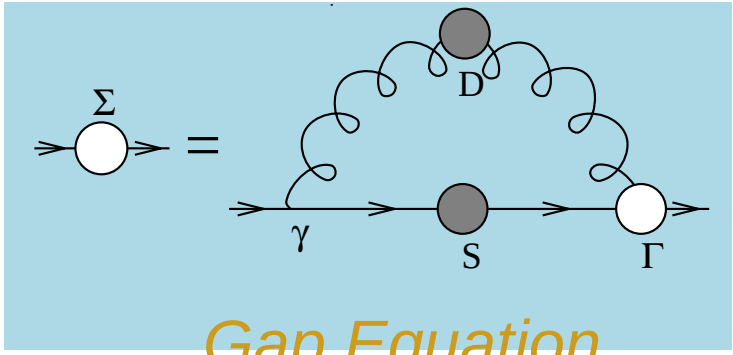
- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

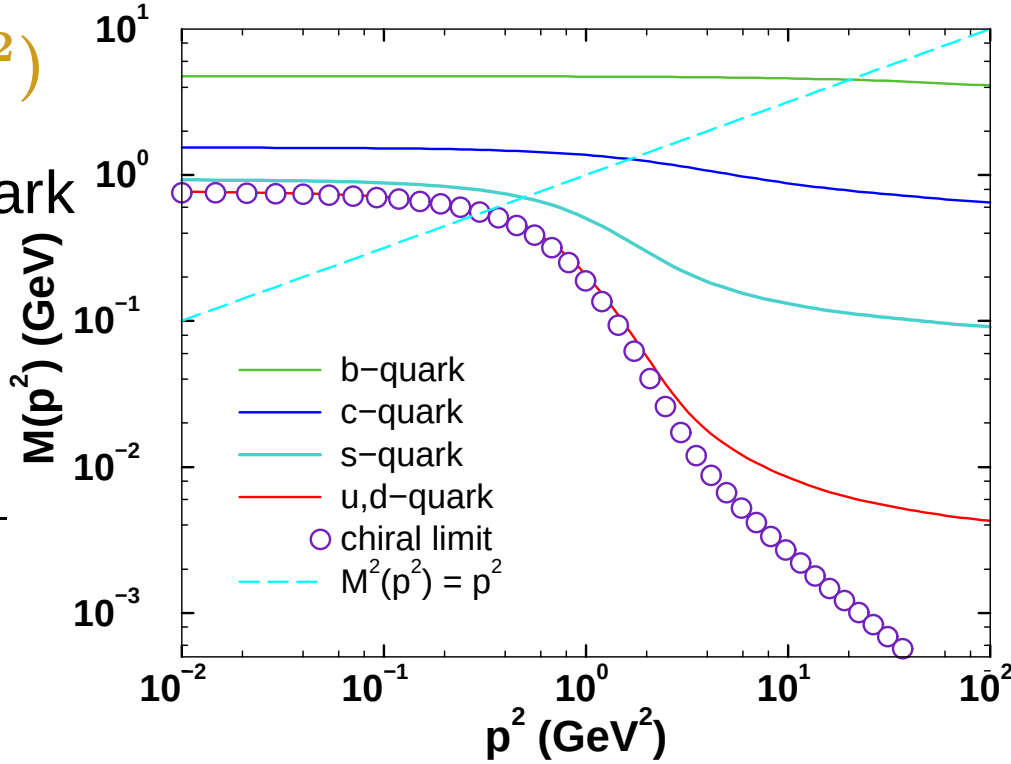
- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$

- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1

Predictions confirmed in numerical simulations of lattice-QCD



Dressed-Quark Propagator

- Longstanding Prediction of Dyson-Schwinger Equation Studies

Dressed-Quark Propagator

- Longstanding Prediction of Dyson-Schwinger Equation Studies
  - E.g., *Dyson-Schwinger equations and their application to hadronic physics*, C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477

Dressed-Quark Propagator

- Long used as basis for efficacious hadron physics phenomenology

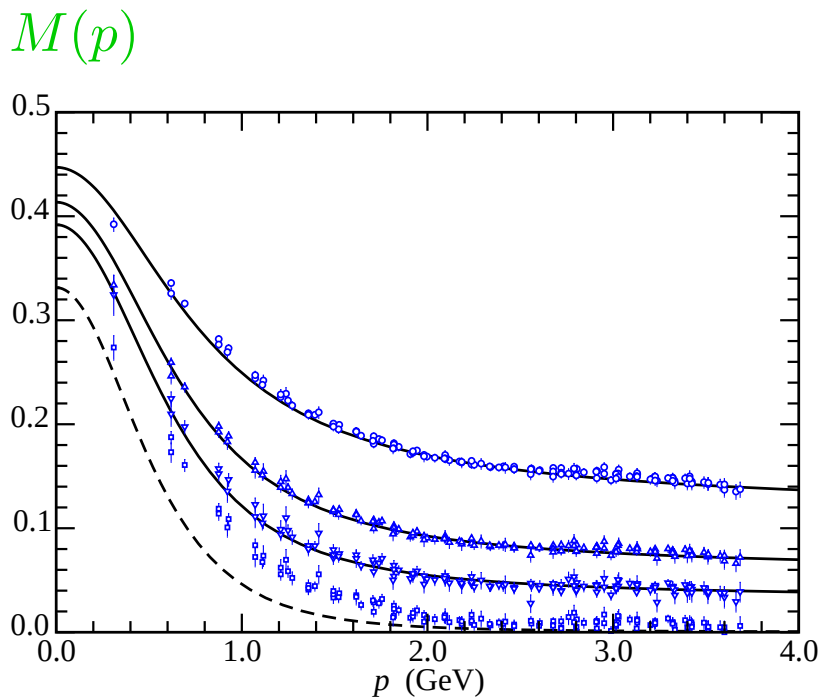
- Longstanding Prediction of Dyson-Schwinger Equation Studies
  - E.g., *Dyson-Schwinger equations and their application to hadronic physics*, C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477

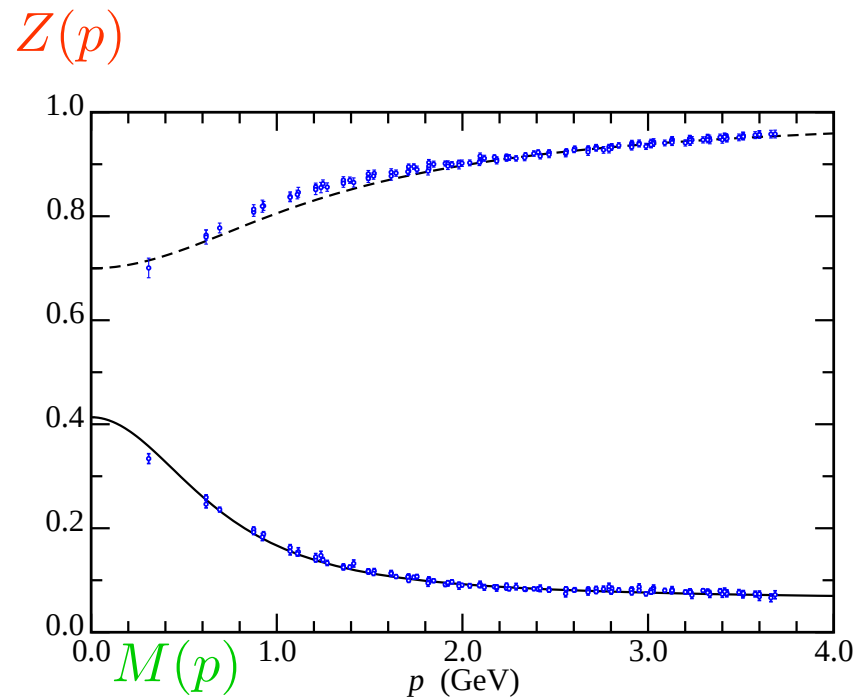
Dressed-Quark Propagator

- Long used as basis for efficacious hadron physics phenomenology
  - *Electromagnetic pion form-factor and neutral pion decay width*, C. D. Roberts, Nucl. Phys. A **605** (1996) 475

- Longstanding Prediction of Dyson-Schwinger Equation Studies
  - E.g., *Dyson-Schwinger equations and their application to hadronic physics*, C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477

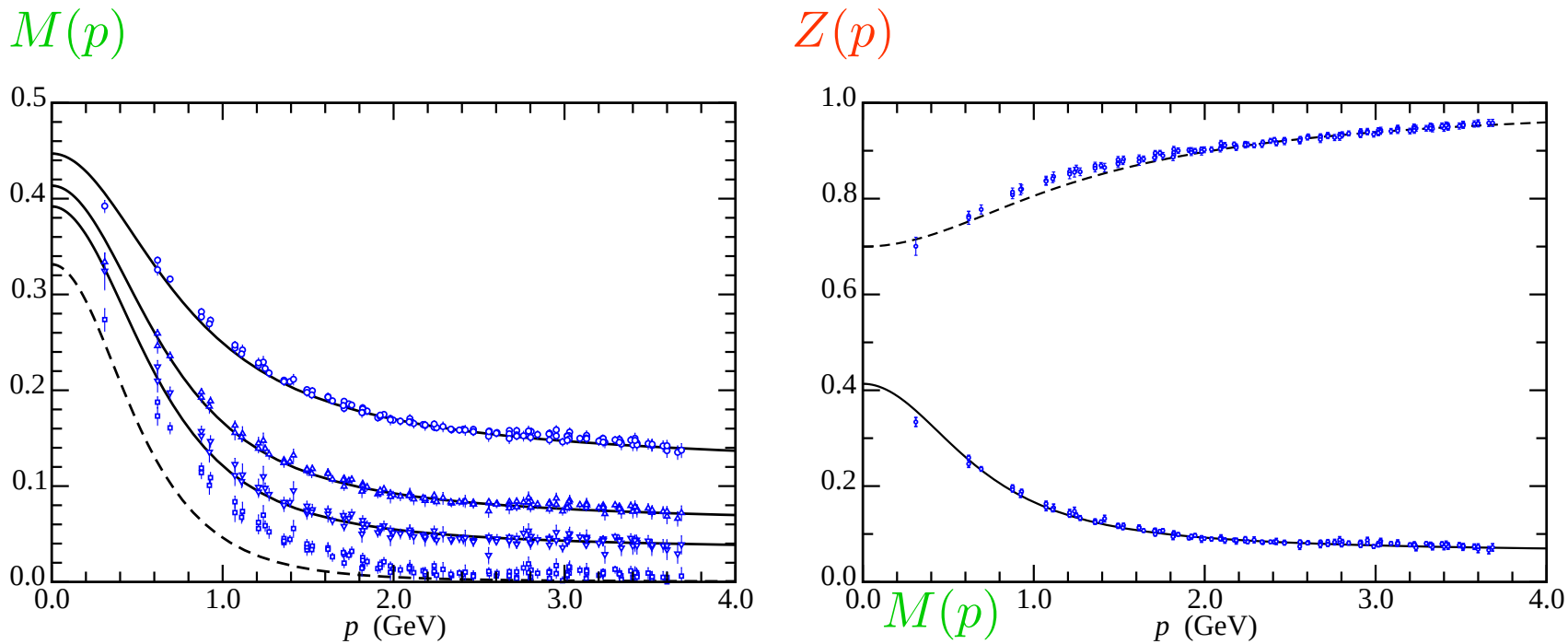
Quenched-QCD
Dressed-Quark Propagator





Quenched-QCD Dressed-Quark Propagator

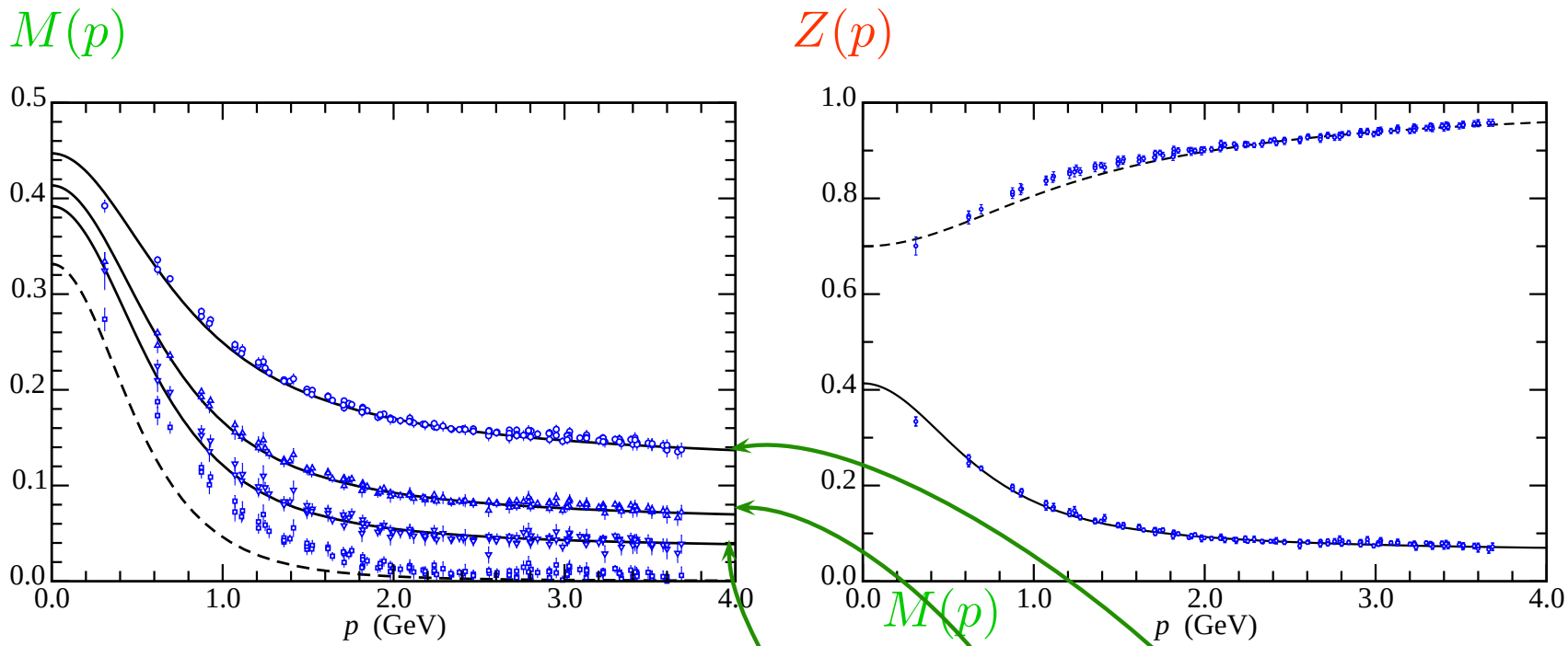
2002



- "*data*:" Quenched Lattice Meas.
  – Bowman, Heller, Leinweber, Williams: he-lat/0209129

Quenched-QCD Dressed-Quark Propagator

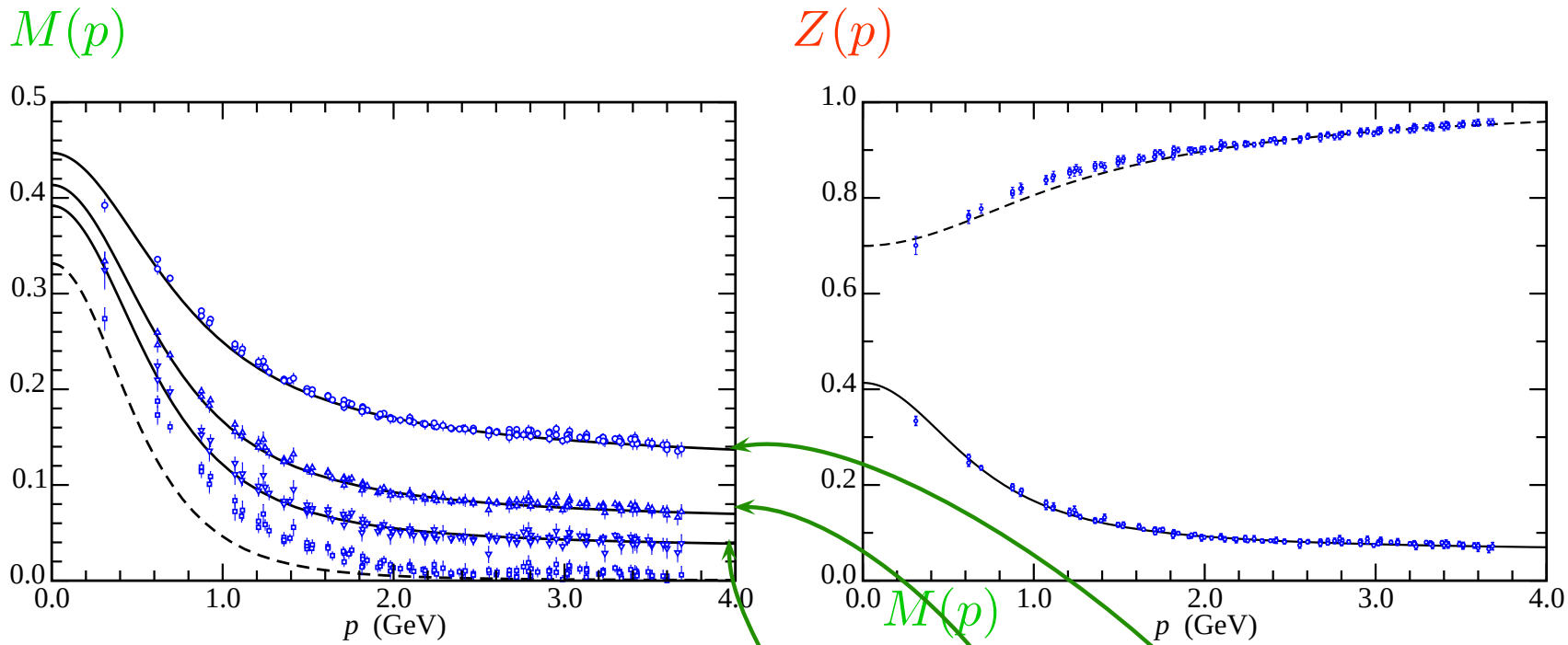
2002



- "*data*:" Quenched Lattice Meas.
  – Bowman, Heller, Leinweber, Williams: he-lat/0209129
  current-quark masses: 30 MeV, 50 MeV, 100 MeV

2002

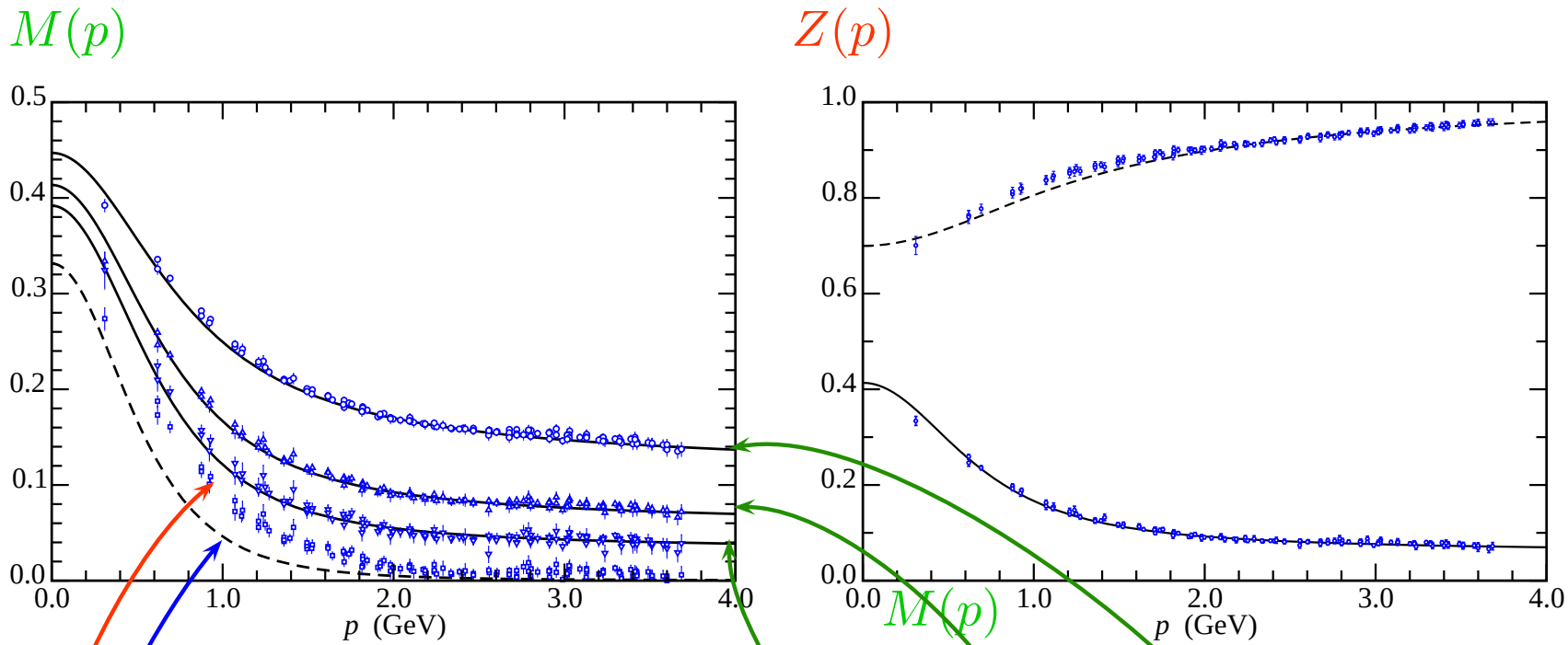
Quenched-QCD Dressed-Quark Propagator



- "*data*:" Quenched Lattice Meas.
  – Bowman, Heller, Leinweber, Williams: he-lat/0209129
  current-quark masses: 30 MeV, 50 MeV, 100 MeV
- *Curves*: Quenched DSE Cal.
  – Bhagwat, Pichowsky, Roberts, Tandy nu-th/0304003

2002

Quenched-QCD Dressed-Quark Propagator



- "*data*:" Quenched Lattice Meas.
  – Bowman, Heller, Leinweber, Williams: he-lat/0209129
  current-quark masses: 30 MeV, 50 MeV, 100 MeV
- *Curves*: Quenched DSE Cal.
  – Bhagwat, Pichowsky, Roberts, Tandy nu-th/0304003
- Linear extrapolation of lattice data to chiral limit is inaccurate

QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
  - R. Alkofer and L. von Smekal, *The infrared behavior of QCD Green's functions* . . . , Phys. Rept. **353**, 281 (2001).

QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
  - R. Alkofer and L. von Smekal, *The infrared behavior of QCD Green's functions* . . . , Phys. Rept. **353**, 281 (2001).
- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
  - D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD Collaboration], *Asymptotic scaling and infrared behavior of the gluon propagator*, Phys. Rev. D **60**, 094507 (1999) [Erratum-ibid. D **61**, 079901 (2000)].
- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

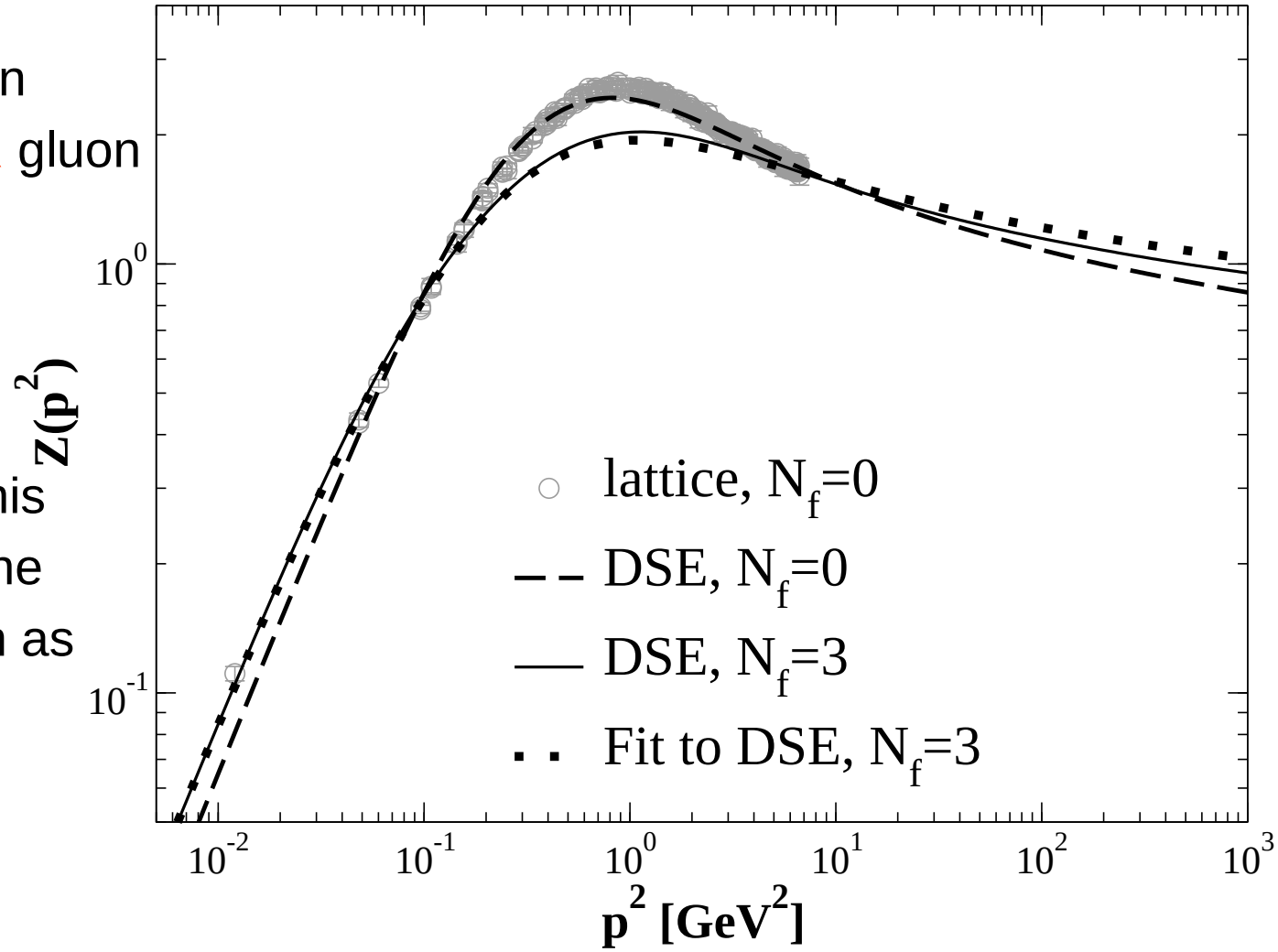
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



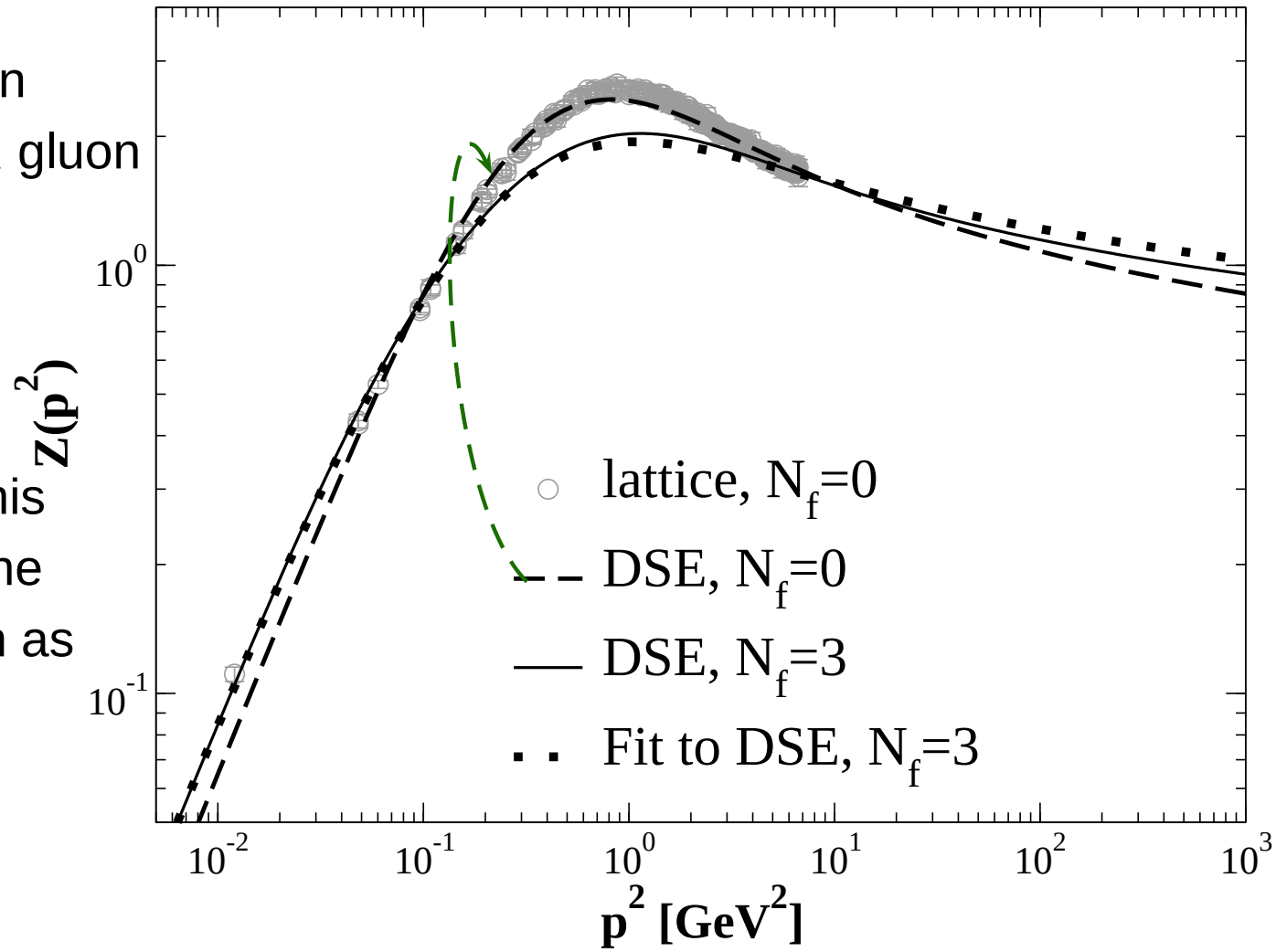
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



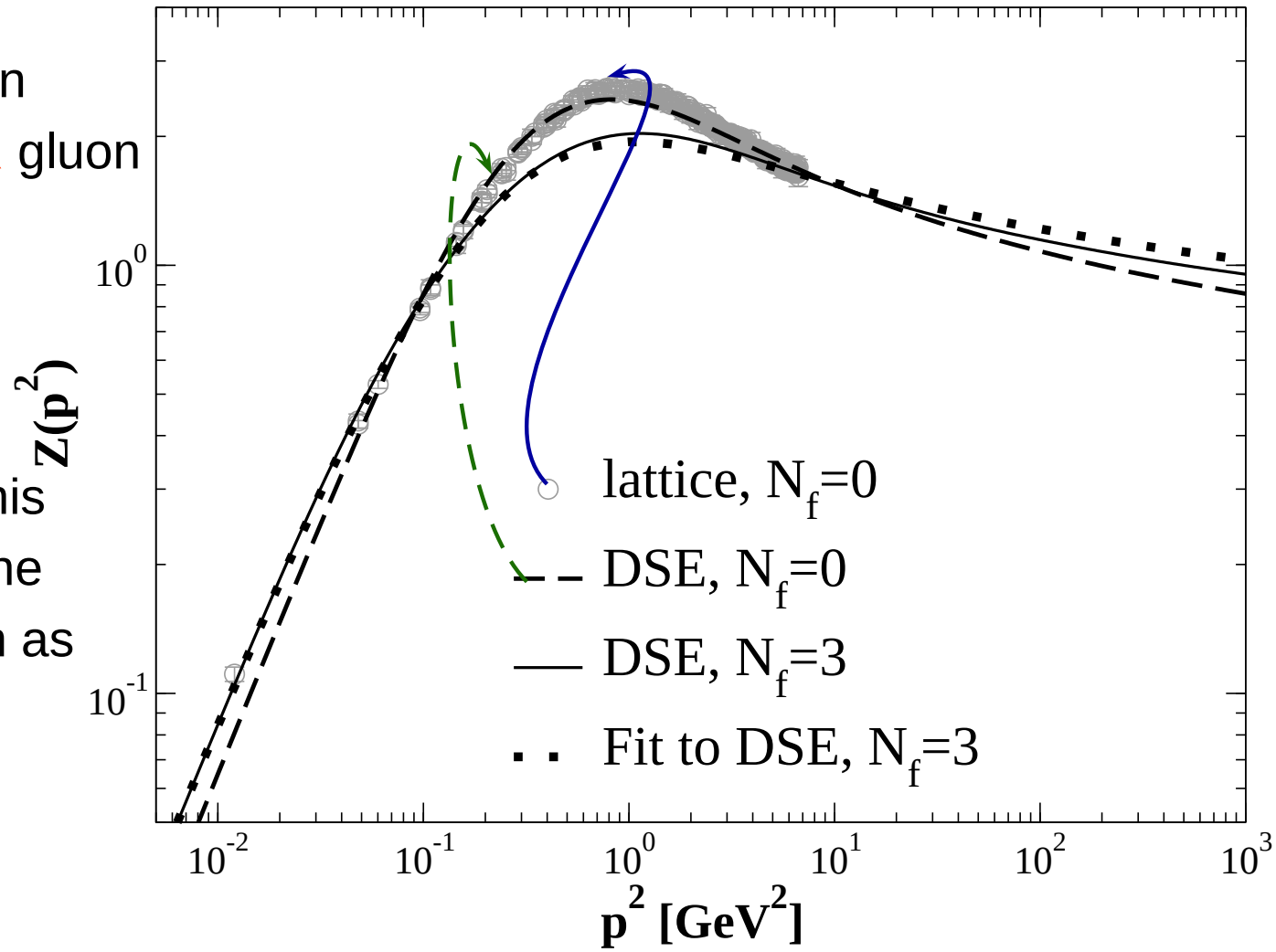
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



Critical Mass for Chiral Expansion

Dynamical chiral symmetry breaking and a critical mass

Lei Chang, *et al*., nucl-th/0605058

Critical Mass for Chiral Expansion

Dynamical chiral symmetry breaking and a critical mass

Lei Chang, *et al*., nucl-th/0605058

- Chiral symmetry realised in Nambu-Goldstone mode; i.e., Dynamical Chiral Symmetry Breaking – characterised by nonzero dressed-quark mass function in the chiral limit:

$$\boxed{M(p^2; m = 0) \neq 0}.$$

Critical Mass for Chiral Expansion

Dynamical chiral symmetry breaking and a critical mass

Lei Chang, *et al*., nucl-th/0605058

- Chiral symmetry realised in Nambu-Goldstone mode; i.e., Dynamical Chiral Symmetry Breaking – characterised by nonzero dressed-quark mass function in the chiral limit:

$$\boxed{M(p^2; m=0) \neq 0}.$$

- Does this mass function have a ***convergent*** expansion in current-quark mass about its nonzero chiral-limit value:

$$M(0;m) = M(0,0) + m \left.\frac{\partial}{\partial m} M(0;m)\right|_{m=0} + \ldots \ ?$$

Critical Mass for Chiral Expansion

Lei Chang, *et al*., nucl-th/0605058

- Chiral symmetry realised in Nambu-Goldstone mode; i.e., Dynamical Chiral Symmetry Breaking – characterised by nonzero dressed-quark mass function in the chiral limit:

$$\boxed{M(p^2; m=0) \neq 0}.$$

- $$M(0;m) = M(0,0) + \sum_{n=1}^{\infty} m^n a_n$$

Radius of convergence: $m_{\rm rc} = \lim_{n\to\infty} \left(\frac{1}{|a_n|}\right)^{1/n}$

Critical Mass for Chiral Expansion

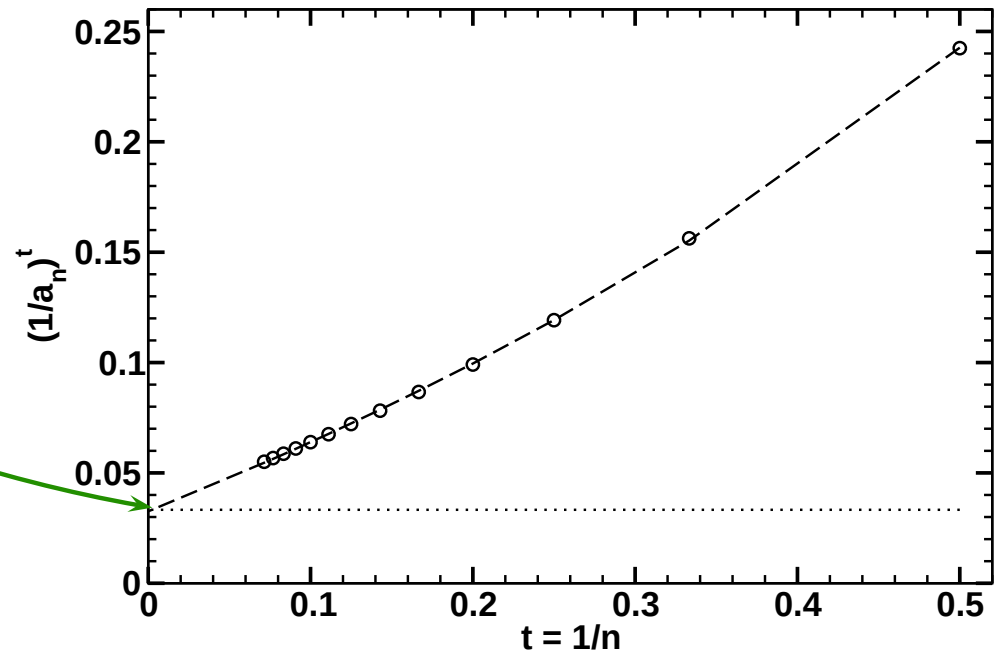
Dynamical chiral symmetry breaking and a critical mass

Lei Chang, *et al*., nucl-th/0605058

- Chiral symmetry realised in Nambu-Goldstone mode; i.e., Dynamical Chiral Symmetry Breaking – characterised by nonzero dressed-quark mass function in the chiral limit:

$$\boxed{M(p^2; m = 0) \neq 0}.$$

$$m_{\rm rc} = 0.034 \pm 0.001$$



Critical Mass for Chiral Expansion

Dynamical chiral symmetry breaking and a critical mass

Lei Chang, *et al*., nucl-th/0605058

- Chiral symmetry realised in Nambu-Goldstone mode; i.e., Dynamical Chiral Symmetry Breaking – characterised by nonzero dressed-quark mass function in the chiral limit:

$$\boxed{M(p^2; m = 0) \neq 0}.$$

- For a pseudoscalar meson constituted of equal mass current-quarks, it corresponds to a mass $m_{0^-}^{\rm cr} \sim 0.45\,$GeV, $[m_{0^-}^{\rm cr}]^2 \sim 0.2\,$GeV^2.

Critical Mass for Chiral Expansion

Dynamical chiral symmetry breaking and a critical mass

Lei Chang, *et al.*, nucl-th/0605058

- Chiral symmetry realised in Nambu-Goldstone mode; i.e., Dynamical Chiral Symmetry Breaking – characterised by nonzero dressed-quark mass function in the chiral limit:

$$\boxed{M(p^2; m = 0) \neq 0}.$$

- For a pseudoscalar meson constituted of equal mass current-quarks, it corresponds to a mass $m_{0^-}^{\rm cr} \sim 0.45$ GeV, $[m_{0^-}^{\rm cr}]^2 \sim 0.2$ GeV^2.
- Entails, e.g., lattice-QCD simulations *must* have results at $m_\pi^2 < [m_{0^-}^{\rm cr}]^2 \sim 0.2$ GeV^2 for reasonable extrapolation via EFT.

Consituent-quark σ-term

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark σ-term

$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\; (M^E)^2 := s\,|\,s = M(s)^2\,.$$

Consituent-quark σ-term

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark σ-term

$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\; (M^E)^2 := s\,|\,s = M(s)^2\,.$$

- Renormalisation-group-invariant and determined from solutions of the gap equation

Consituent-quark σ-term

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark σ-term

$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\; (M^E)^2 := s\,|\,s = M(s)^2\,.$$

- Unambiguous probe of impact of explicit chiral symmetry breaking on the mass function

Consituent-quark σ-term

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark σ-term

$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\; (M^E)^2 := s\,|\,s = M(s)^2\,.$$

- Ratio $\boxed{\dfrac{\sigma_f}{M_f^E} = \dfrac{\text{EXPLICIT}}{\text{EXPLICIT} + \text{DYNAMICAL}}}$

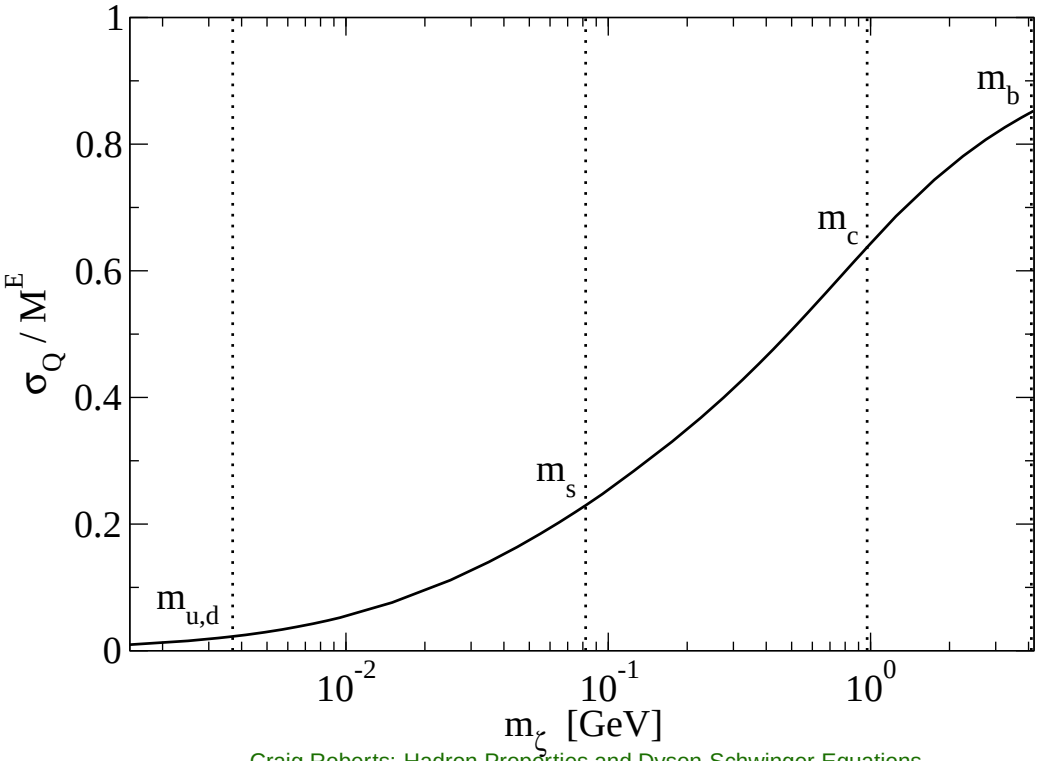
measures effect of *EXPLICIT* chiral symmetry breaking on dressed-quark mass-function

cf. *SUM* of effects of *EXPLICIT* AND *DYNAMICAL CHIRAL SYMMETRY BREAKING*

Consituent-quark σ-term

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark σ-term

$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\; (M^E)^2 := s\,|\,s = M(s)^2\,.$$

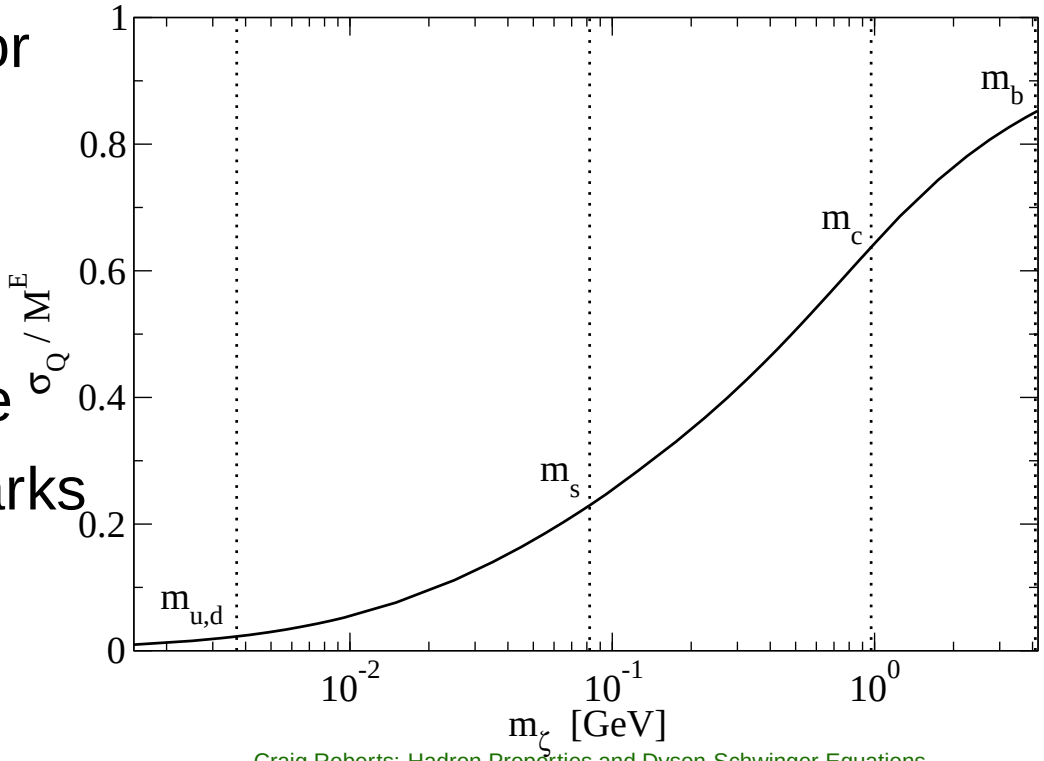


Consituent-quark σ-term

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark σ-term

$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\; (M^E)^2 := s\,|\,s = M(s)^2\,.$$

Obvious: ratio vanishes for light-quarks because magnitude of their constituent-mass owes primarily to DCSB. On the other hand, for heavy-quarks it approaches one.

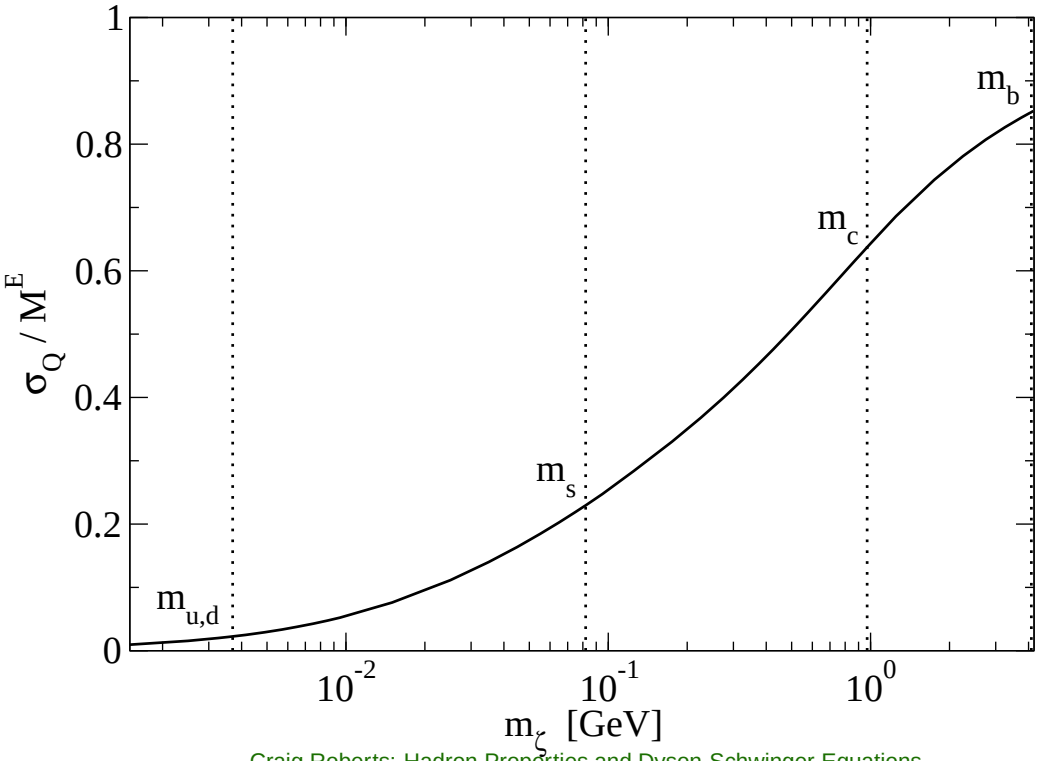


Consituent-quark σ-term

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark σ-term

$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;\; (M^E)^2 := s\,|\,s = M(s)^2\,.$$

Essentially dynamical component of chiral symmetry breaking, and manifestation in all its order parameters, vanishes with increasing current-quark mass



Hadrons

- Established understanding of two- and three-point functions

Hadrons

- Established understanding of two- and three-point functions
- What about bound states?

Hadrons

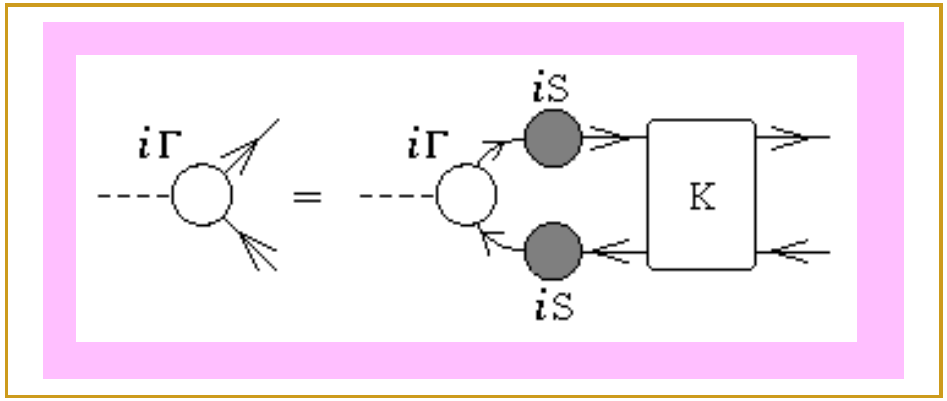
- Without bound states, Comparison with experiment is impossible

Hadrons

- Without bound states, Comparison with experiment is impossible
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

Hadrons

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation

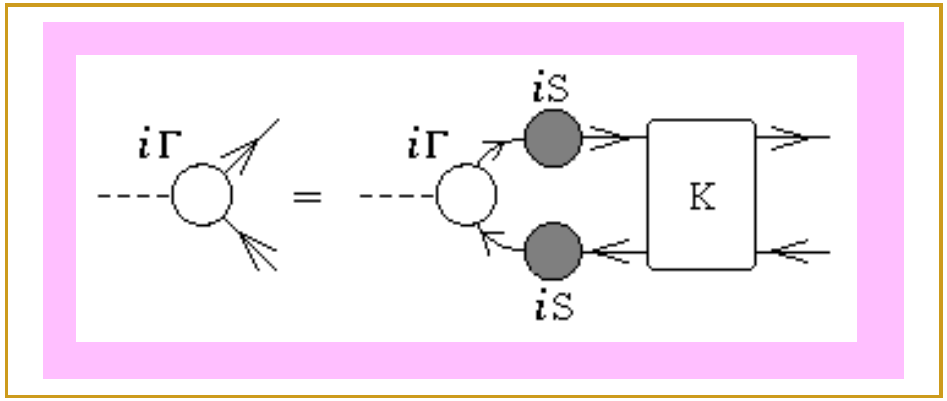


QFT Generalisation of Lippmann-Schwinger Equation.

Hadrons

- Without bound states, Comparison with experiment is impossible

- Bethe-Salpeter Equation

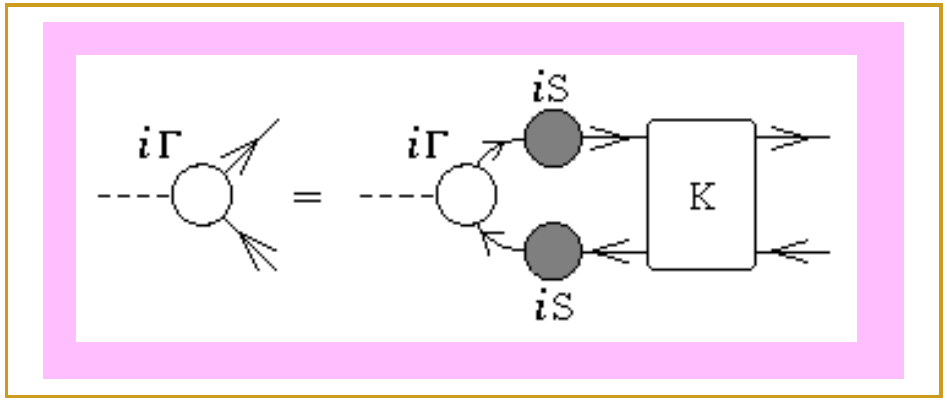


QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

Hadrons

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



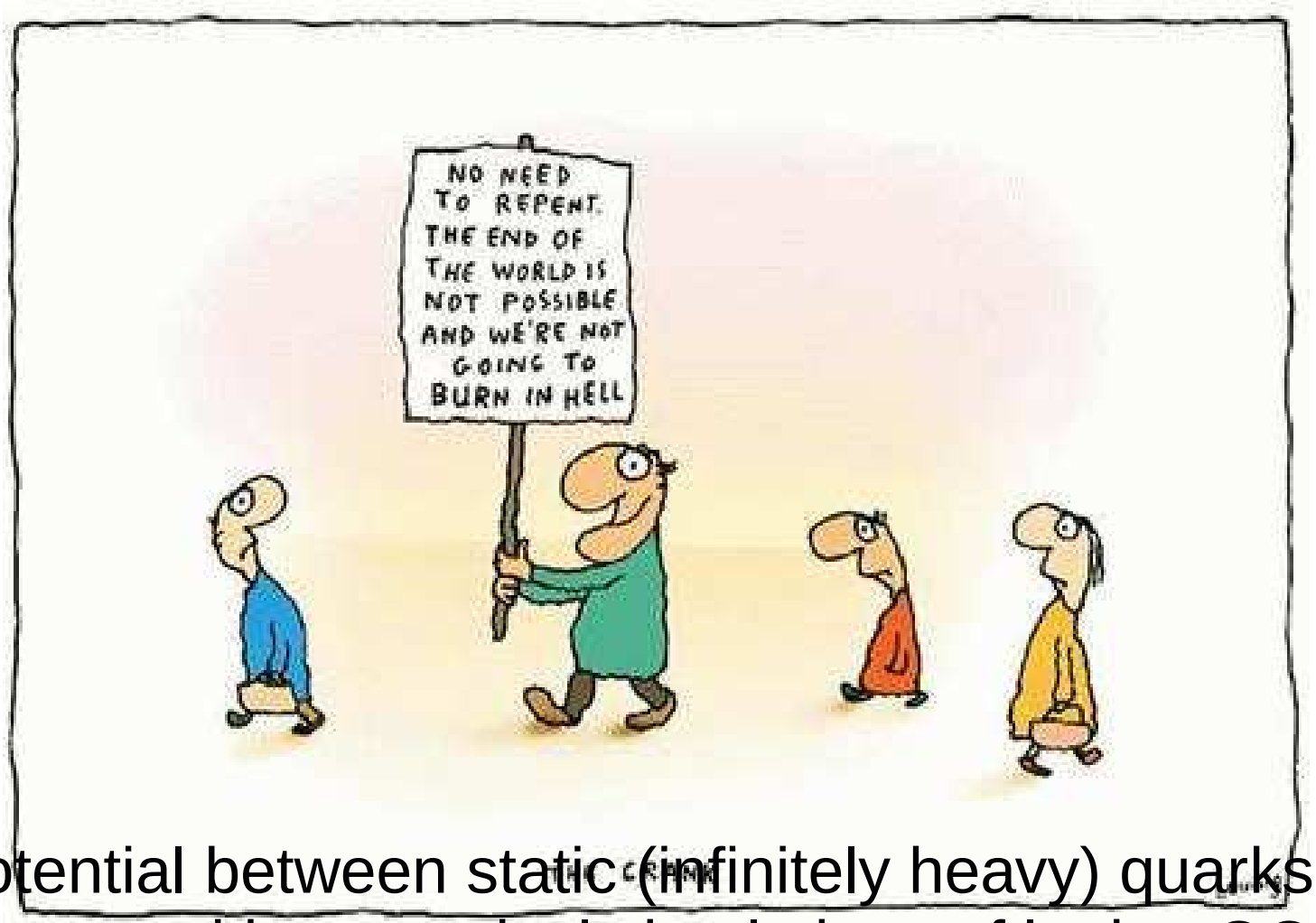
QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

or

What is the light-quark Long-Range Potential?

What is the light-quark Long-Range Potential?



Potential between static (infinitely heavy) quarks measured in numerical simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels very different

but must be intimately related

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels very different

but must be intimately related

- Relation must be preserved by truncation

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels very different

but must be intimately related

- Relation must be preserved by truncation
- Nontrivial constraint

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels very different

but must be intimately related

- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Radial Excitations & Chiral Symmetry

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

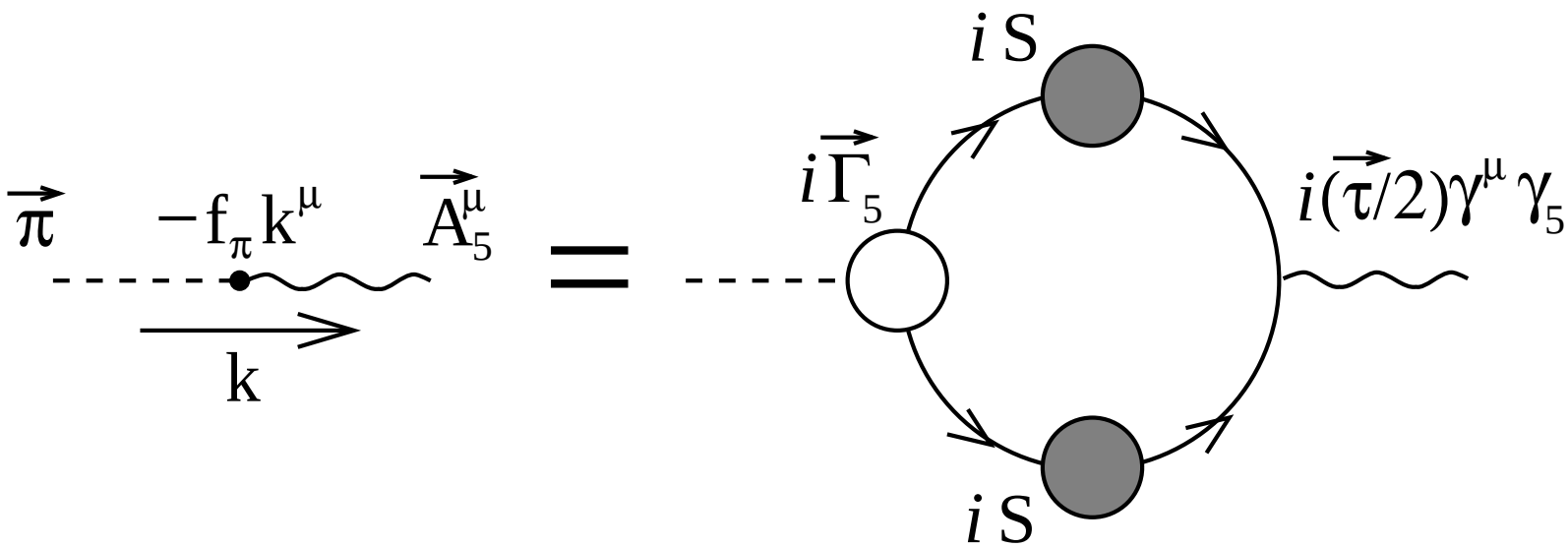
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H\, m_H^2 = -\ \rho_\zeta^H\ \mathcal{M}_H$$

$$f_H\, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5\gamma_\mu \boxed{\mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-)} \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



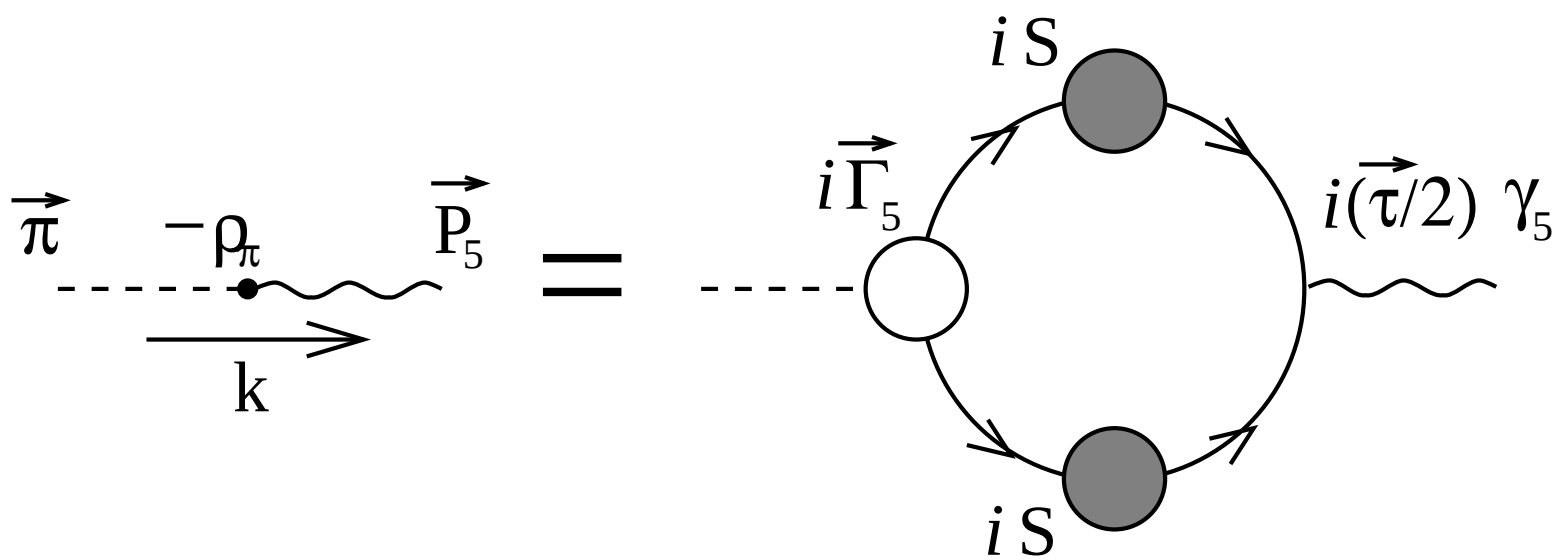
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = -\rho_\zeta^H \, \mathcal{M}_H$$

$$i\rho_\zeta^H = Z_4 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5 \, \mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-) \right\}$$

- Pseudoscalar projection of BS wave function at $x = 0$



Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} \, m_q$ ...GMOR relation, a corollary

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ ...GMOR relation, a corollary

- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $m_H \propto m_q$

... QCD Proof of Potential Model result

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson, not the ground state, so $\boldsymbol{m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0}$, in chiral limit

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson, not the ground state, so $\boldsymbol{m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0}$, in chiral limit
- $\Rightarrow \boldsymbol{f_H = 0}$
  ALL pseudoscalar mesons except $\boldsymbol{\pi(140)}$ in chiral limit

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson, not the ground state, so $m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0$, in chiral limit
- $\Rightarrow f_H = 0$
  ALL pseudoscalar mesons except $\pi(140)$ in chiral limit
- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

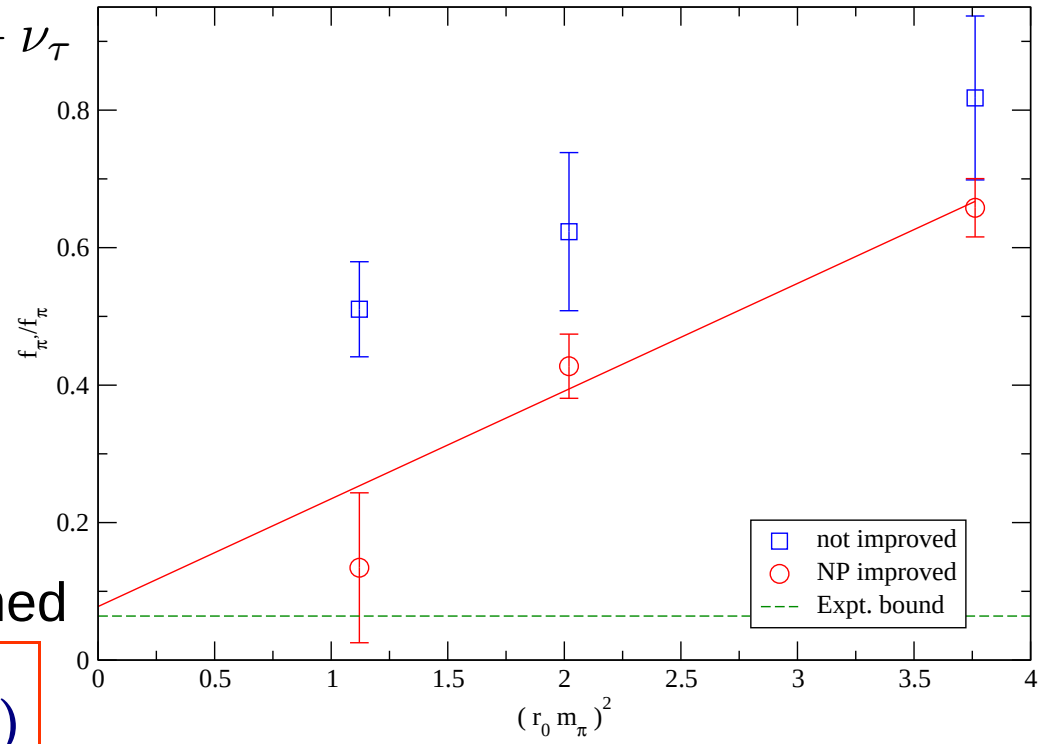
- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$

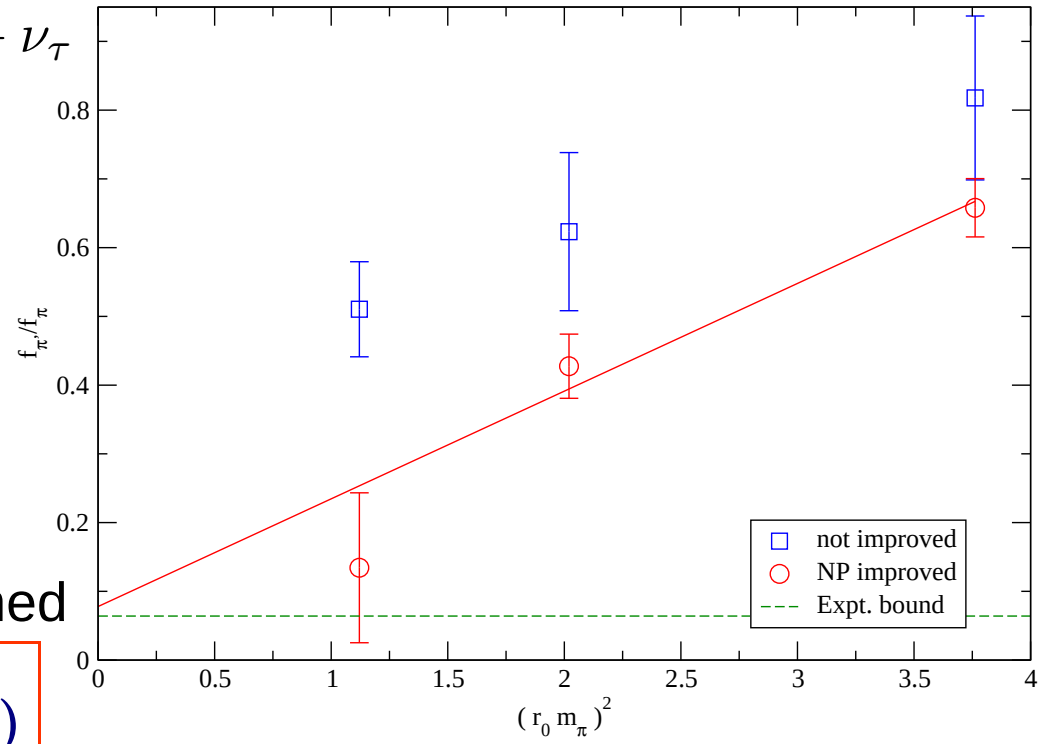


Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$



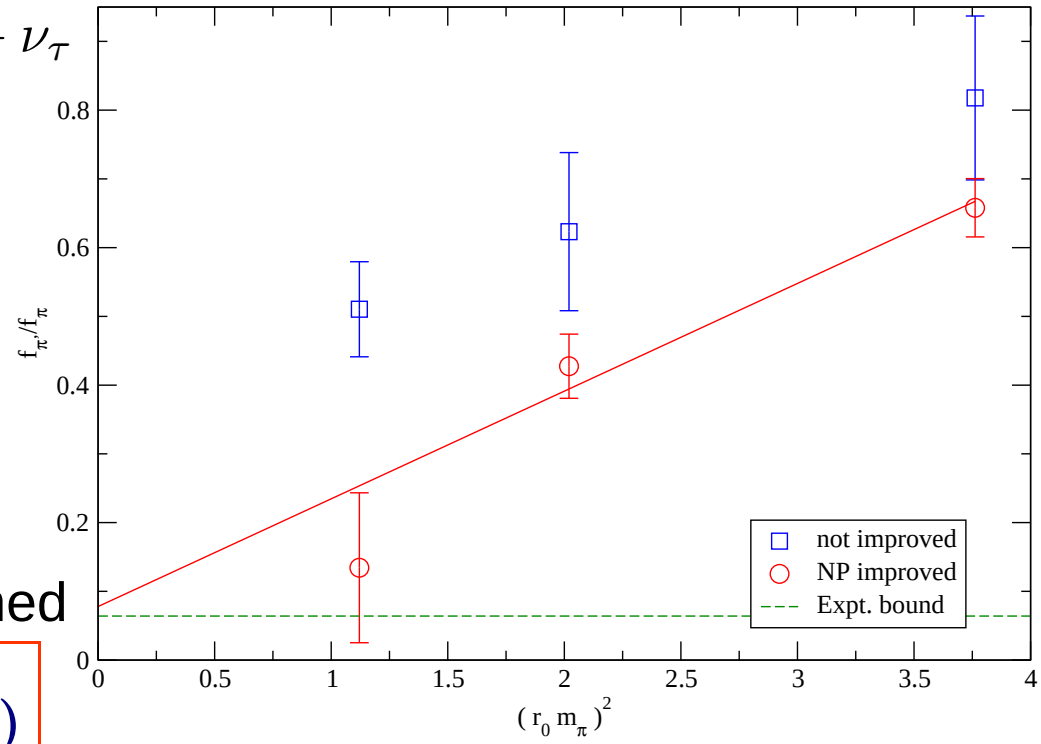
- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$



- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Pion ... $J = 0$ but ...

- Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.

Pion ... $J = 0$ but ...

- Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.

Pion ... $J = 0$ but ...

- Pseudoscalar meson Bethe-Salpeter amplitude

$$\chi_\pi(k;P) = \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P\mathcal{F}_{\pi_n}(k;P) \right.$$
$$\left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P) \right]$$

Pion ... $J = 0$ but ...

- Pseudoscalar meson Bethe-Salpeter amplitude

$$\chi_\pi(k;P) = \gamma_5 \left[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P\mathcal{F}_{\pi_n}(k;P) \right.$$
$$\left. \gamma\cdot k\, k\cdot P\, \mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, \mathcal{H}_{\pi_n}(k;P) \right]$$

- $J = 0$ ... *but* while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both S- and P-wave components.

Pion ... $J = 0$ but ...

- $J = 0$ ... *but* while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both S- and P-wave components.

Introduce mixing angle θ_π such that

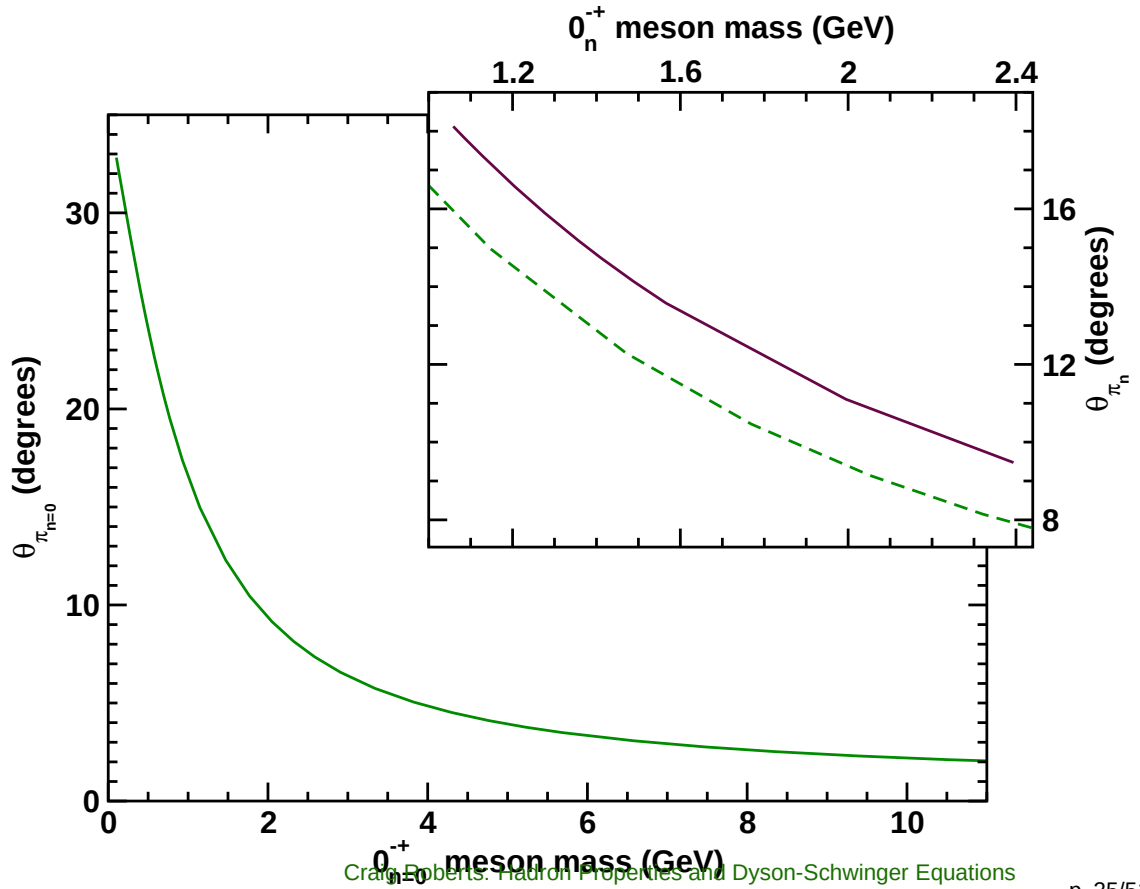
$$\chi_\pi \sim \cos\theta_\pi |L = 0\rangle + \sin\theta_\pi |L = 1\rangle$$

Pion … $J = 0$ but …

- $J = 0$ … *but* while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both S- and P-wave components.

Introduce mixing angle θ_π such that

$$\chi_\pi \sim \cos\theta_\pi |L = 0\rangle + \sin\theta_\pi |L = 1\rangle$$



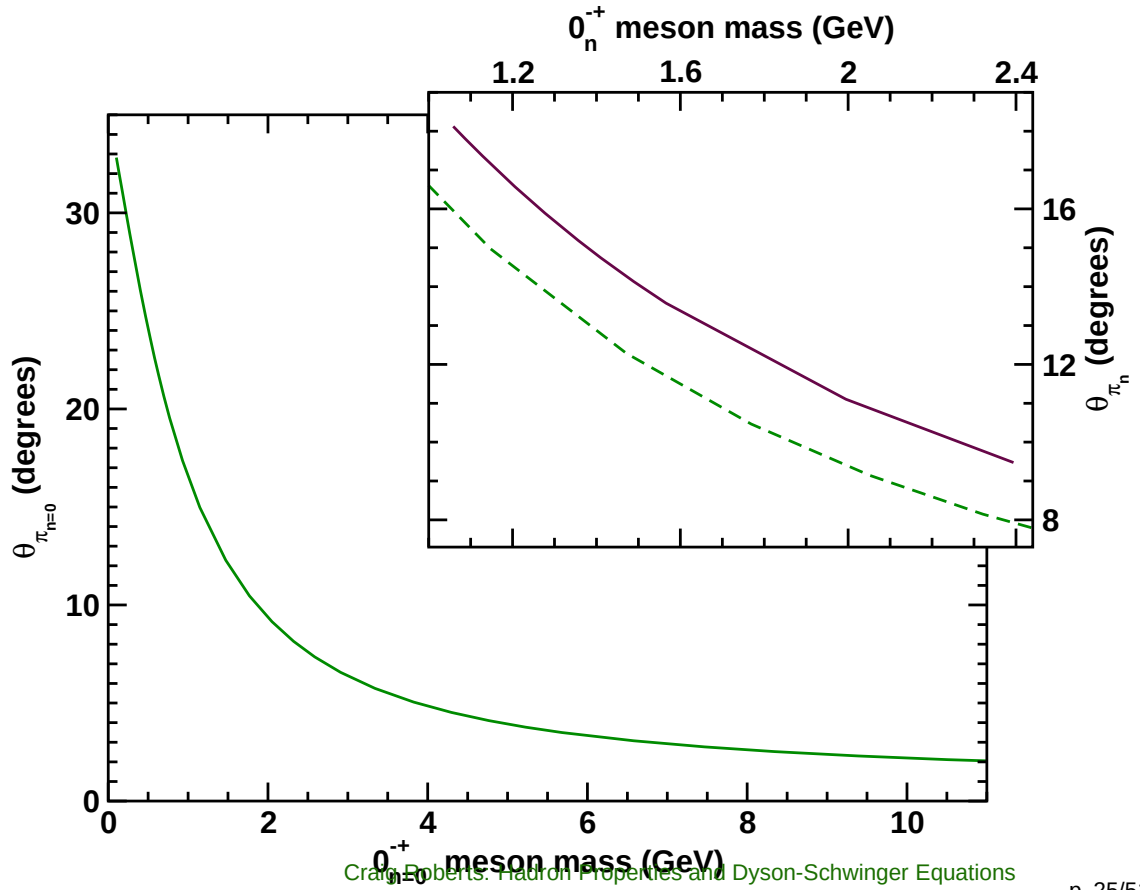
Pion … $J = 0$ but …

- $J = 0$ … *but* while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both S- and P-wave components.

Introduce mixing angle θ_π such that

$$\chi_\pi \sim \cos\theta_\pi |L = 0\rangle + \sin\theta_\pi |L = 1\rangle$$

L is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.



Charge Neutral Pseudoscalar Mesons

$$
\begin{aligned}
P_\mu \Gamma^a_{5\mu}(k;P) &= \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\
&\quad -2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)
\end{aligned}
$$

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\{\mathcal{F}^a | \, a = 0, \ldots, N_f^2 - 1\}$ are the generators of $U(N_f)$
- $\mathcal{S} =$ diag$[S_u, S_d, S_s, S_c, S_b, \ldots]$
- $\mathcal{M}^{ab} = \mathrm{tr}_F \left[\{\mathcal{F}^a, \mathcal{M}\} \mathcal{F}^b \right]$,
  $\mathcal{M} = \mathrm{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] =$ matrix of current-quark bare masses

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\{\mathcal{F}^a | \, a = 0, \ldots, N_f^2 - 1\}$ are the generators of $U(N_f)$
- $\mathcal{S} =$ diag$[S_u, S_d, S_s, S_c, S_b, \ldots]$
- $\mathcal{M}^{ab} = \mathrm{tr}_F \left[\{\mathcal{F}^a, \mathcal{M}\} \mathcal{F}^b \right]$,
  $\mathcal{M} = \mathrm{diag}[m_u, m_d, m_s, m_c, m_b, \ldots] =$ matrix of current-quark bare masses
- The final term in the second line expresses the non-Abelian axial anomaly.

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$

  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \left\langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y) \right\rangle$

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$

  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \left\langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y) \right\rangle$

- $\mathcal{Q}(x) = i\dfrac{\alpha_s}{4\pi}\mathrm{tr}_C\left[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}(x)\right] = \partial_\mu K_\mu(x)$

  … The topological charge density operator.

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$

  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \left\langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y) \right\rangle$

- $\mathcal{Q}(x) = i\dfrac{\alpha_s}{4\pi}\mathrm{tr}_C\left[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}(x)\right] = \partial_\mu K_\mu(x)$

  … The topological charge density operator.

  (Trace is over colour indices & $F_{\mu\nu} = \frac{1}{2}\lambda^a F^a_{\mu\nu}$.)

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$
  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \left\langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y)\right\rangle$
- $\mathcal{Q}(x) = i\frac{\alpha_s}{4\pi}\mathrm{tr}_C\left[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}(x)\right] = \partial_\mu K_\mu(x)$

  … The topological charge density operator.
- Important that only $\mathcal{A}^{a=0}$ is nonzero.

Charge Neutral Pseudoscalar Mesons

$$P_\mu \Gamma^a_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) - 2i\mathcal{M}^{ab}\Gamma^b_5(k;P) - \mathcal{A}^a(k;P)$$

- $\mathcal{A}^a(k;P) = \mathcal{S}^{-1}(k_+)\,\delta^{a0}\,\mathcal{A}_U(k;P)\mathcal{S}^{-1}(k_-)$
  $\mathcal{A}_U(k;P) = \int d^4x d^4y\, e^{i(k_+\cdot x - k_-\cdot y)} N_f \left\langle \mathcal{F}^0 q(x)\, \mathcal{Q}(0)\, \bar{q}(y)\right\rangle$
- $\mathcal{Q}(x) = i\frac{\alpha_s}{4\pi}\mathrm{tr}_C\left[\epsilon_{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma}(x)\right] = \partial_\mu K_\mu(x)$

  … The topological charge density operator.
- NB. While $\mathcal{Q}(x)$ is gauge invariant, the associated Chern-Simons current, K_μ, is not $\Rightarrow$ in QCD *no physical* boson can couple to K_μ and hence *no physical* states can contribute to resolution of U$_A$(1) problem.

Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting ... otherwise all pseudoscalar mesons are Goldstone Modes!

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- Anomaly term has structure

$$\mathcal{A}^0(k;P) = \mathcal{F}^0\gamma_5\left[i\mathcal{E}_{\mathcal{A}}(k;P) + \gamma\cdot P\mathcal{F}_{\mathcal{A}}(k;P)\right.$$
$$\left.+\gamma\cdot k k\cdot P\mathcal{G}_{\mathcal{A}}(k;P) + \sigma_{\mu\nu}k_\mu P_\nu\mathcal{H}_{\mathcal{A}}(k;P)\right]$$

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- AVWTI gives generalised Goldberger-Treiman relations

$$
\begin{aligned}
2 f^0_{\eta'} E_{BS}(k;0) &= 2B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0), \\
F^0_R(k;0) + 2 f^0_{\eta'} F_{BS}(k;0) &= A_0(k^2) - \mathcal{F}_{\mathcal{A}}(k;0), \\
G^0_R(k;0) + 2 f^0_{\eta'} G_{BS}(k;0) &= 2A'_0(k^2) - \mathcal{G}_{\mathcal{A}}(k;0), \\
H^0_R(k;0) + 2 f^0_{\eta'} H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0),
\end{aligned}
$$

A_0, B_0 characterise gap equation's chiral limit solution.

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- AVWTI gives generalised Goldberger-Treiman relations

$$
\begin{aligned}
2 f^0_{\eta'} E_{BS}(k;0) &= 2B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0), \\
F^0_R(k;0) + 2 f^0_{\eta'} F_{BS}(k;0) &= A_0(k^2) - \mathcal{F}_{\mathcal{A}}(k;0), \\
G^0_R(k;0) + 2 f^0_{\eta'} G_{BS}(k;0) &= 2A'_0(k^2) - \mathcal{G}_{\mathcal{A}}(k;0), \\
H^0_R(k;0) + 2 f^0_{\eta'} H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0),
\end{aligned}
$$

A_0, B_0 characterise gap equation's chiral limit solution.

- Follows that $\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)$ is necessary and sufficient condition for absence of massless η' bound-state.

Charge Neutral Pseudoscalar Mesons

- $\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)$
  Discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
  - Hence, absence of massless η' bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.

Charge Neutral Pseudoscalar Mesons

- $\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)$
  Discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
  - Hence, absence of massless η' bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.
- Further highlighted . . . proved

$$\begin{aligned}\langle \bar{q}q\rangle_\zeta^0 &= -\lim_{\Lambda\to\infty} Z_4(\zeta^2,\Lambda^2)\,\mathrm{tr}_{\mathrm{CD}}\int_q^\Lambda S^0(q,\zeta)\\ &= N_f\int d^4x\,\langle \bar{q}(x)i\gamma_5 q(x)\mathcal{Q}(0)\rangle^0\,.\end{aligned}$$

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
- Employed in an analysis of pseudoscalar- and vector-meson bound-states

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
- Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts
  - η–η' mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)
  - π^0–η angles of $\sim 1.2^\circ$ (Expt. $p\,d \to {}^3$He π^0: $0.6^\circ \pm 0.3^\circ$)
  - Strong neutron-proton mass difference . . .
    $\lesssim 75\,\%$ current-quark mass-difference

New Challenges

New Challenges

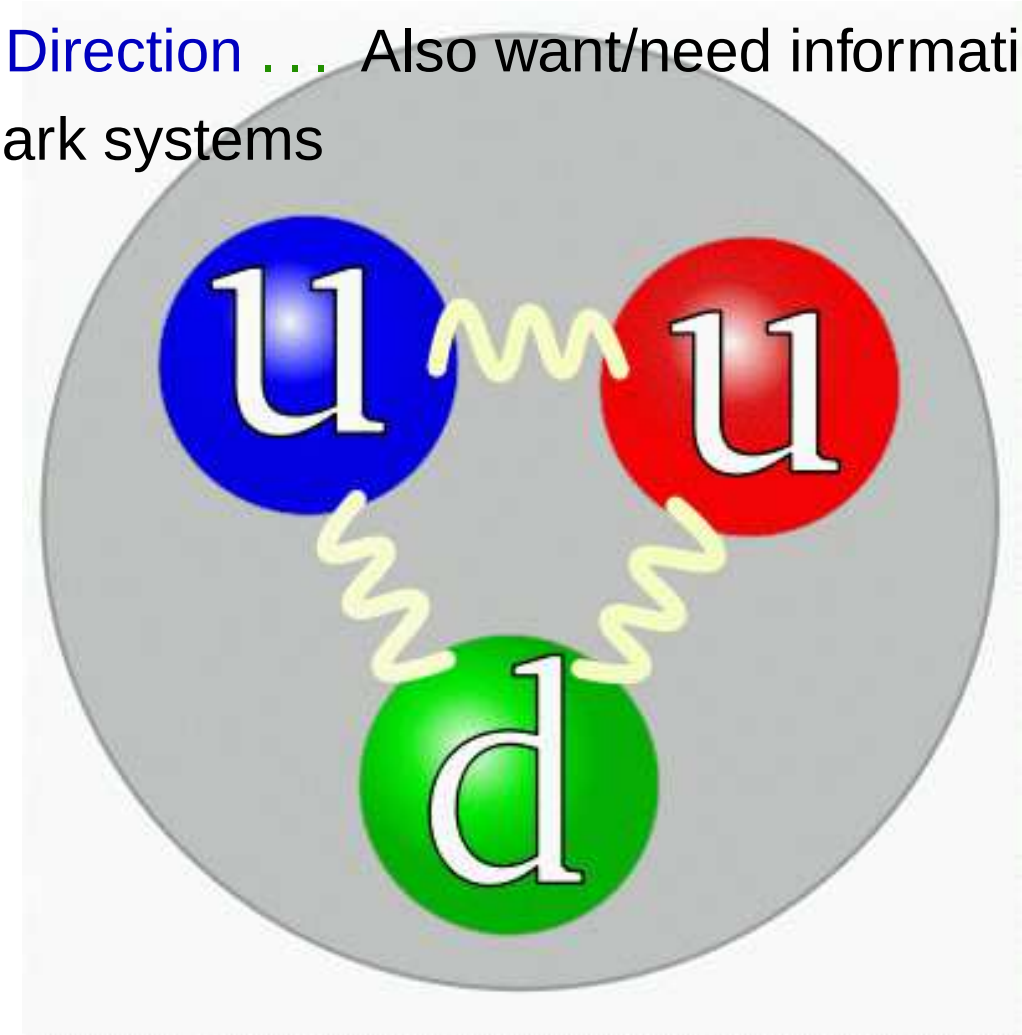
- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.

New Challenges

- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.
- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

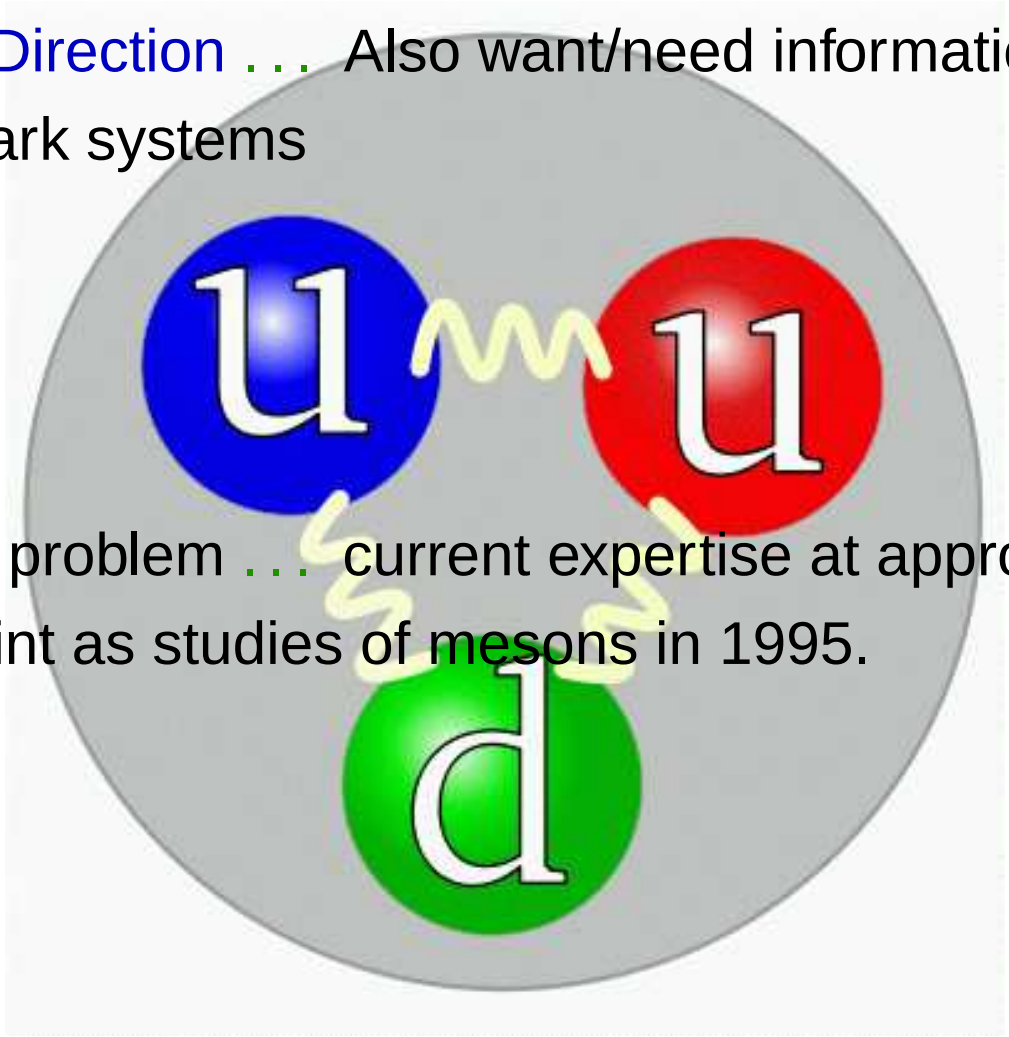
New Challenges

- Another Direction . . . Also want/need information about three-quark systems



New Challenges

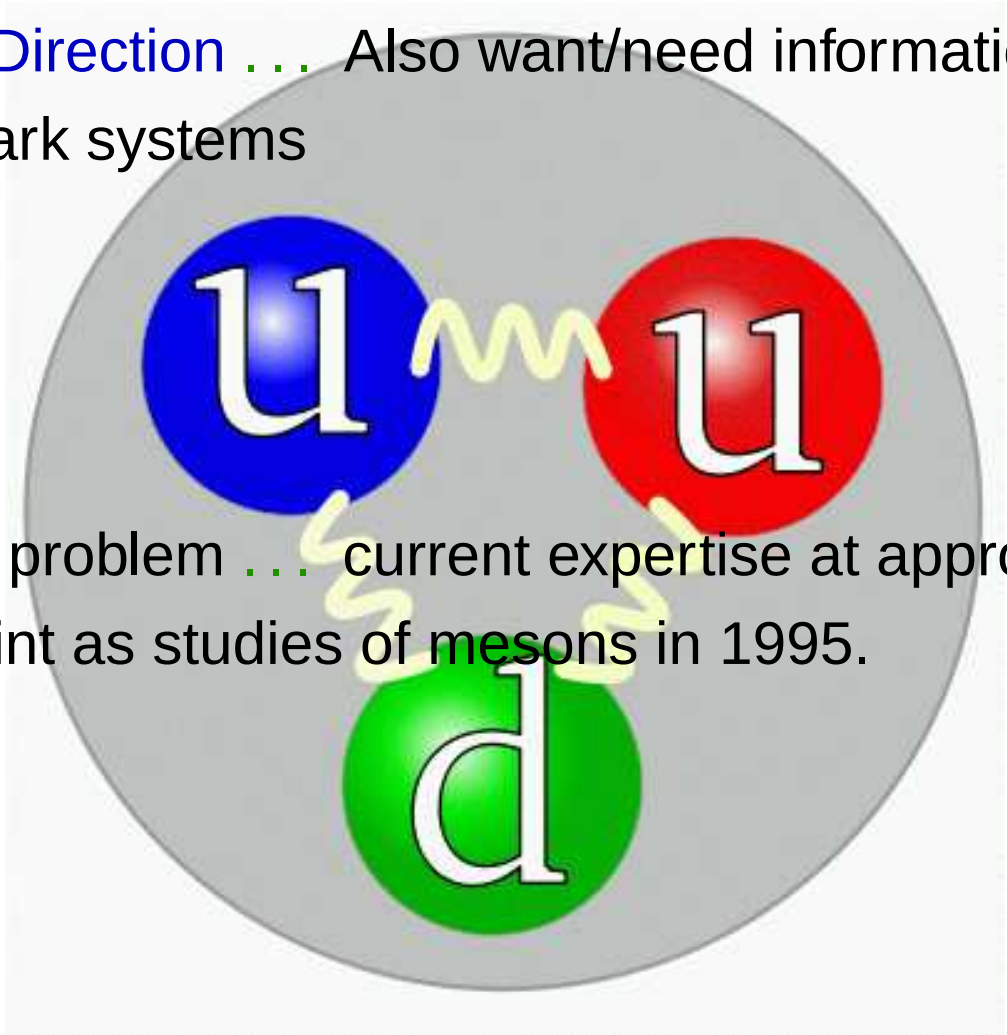
- Another Direction . . . Also want/need information about three-quark systems



- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

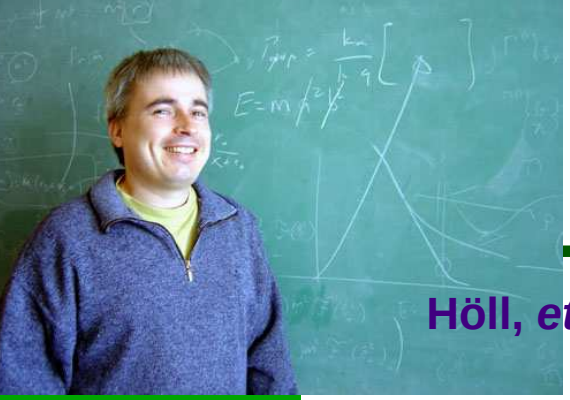
New Challenges

- Another Direction . . . Also want/need information about three-quark systems



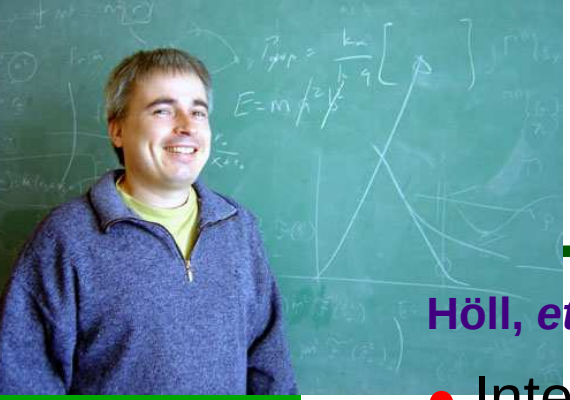
- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.



Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033



Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons

Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation

Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H}\sum_H \frac{[M_H^{\rm exp}-M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2}\right)^{1/2} = 2\%$$

- But is that good?

Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

- But is that good?
  - Cloudy Bag: $\delta M_+^{\pi-{\rm loop}} = -300$ to -400 MeV!

Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033

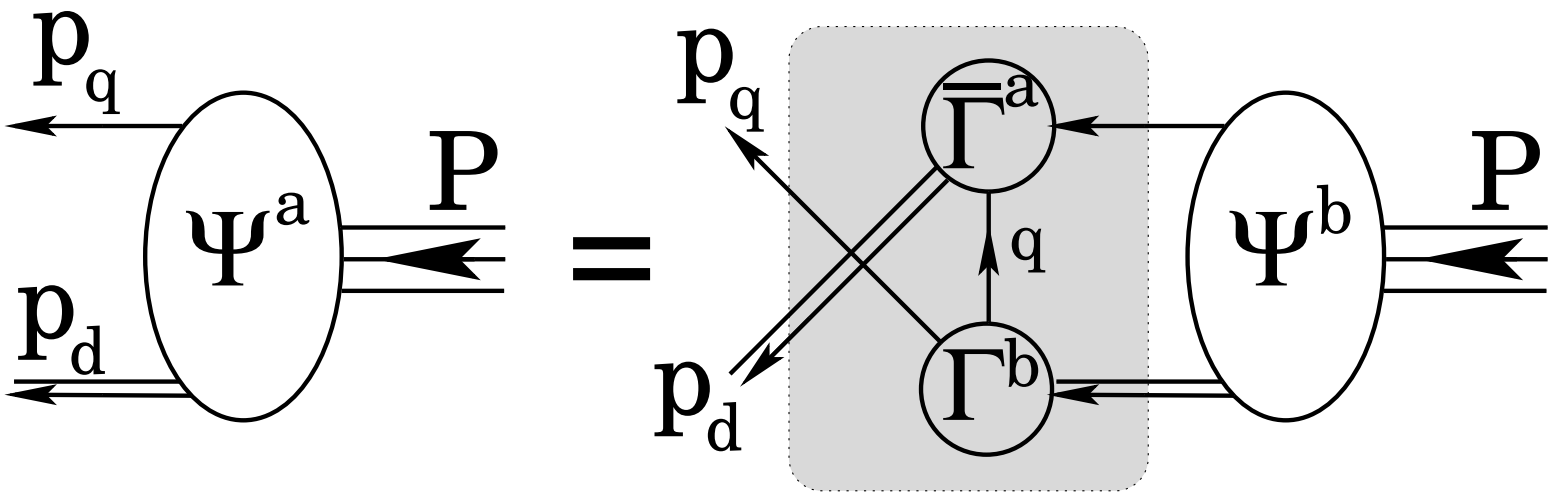
- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

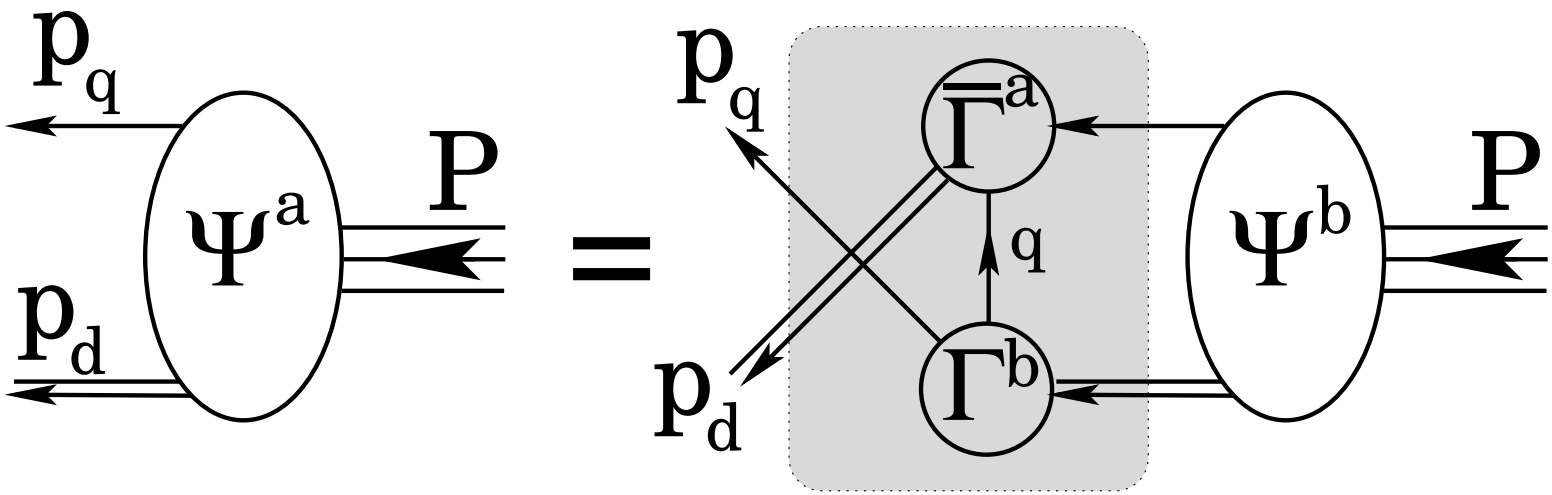
- But is that good?
  - Cloudy Bag: $\delta M_+^{\pi-\rm loop} = -300$ to -400 MeV!
- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

Faddeev equation

Faddeev equation

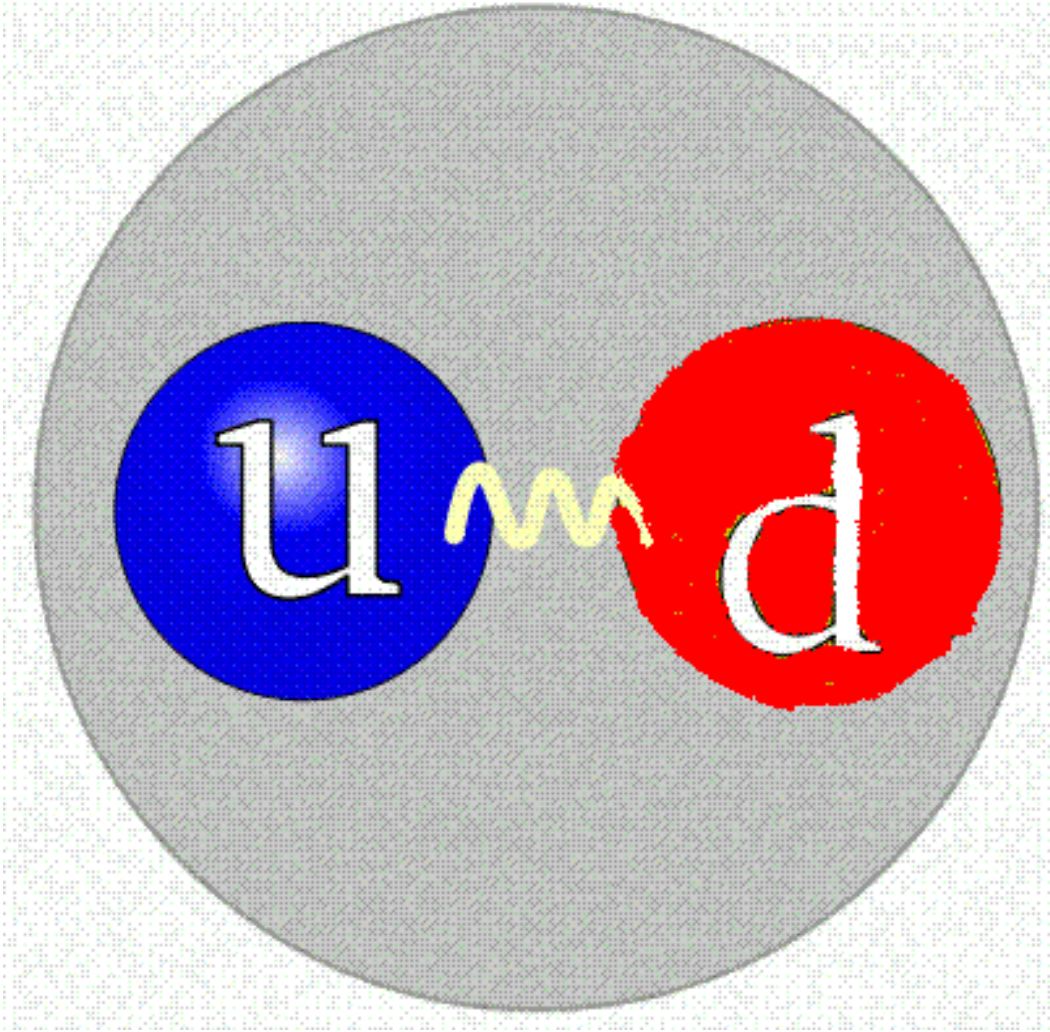


Faddeev equation



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Diquark correlations



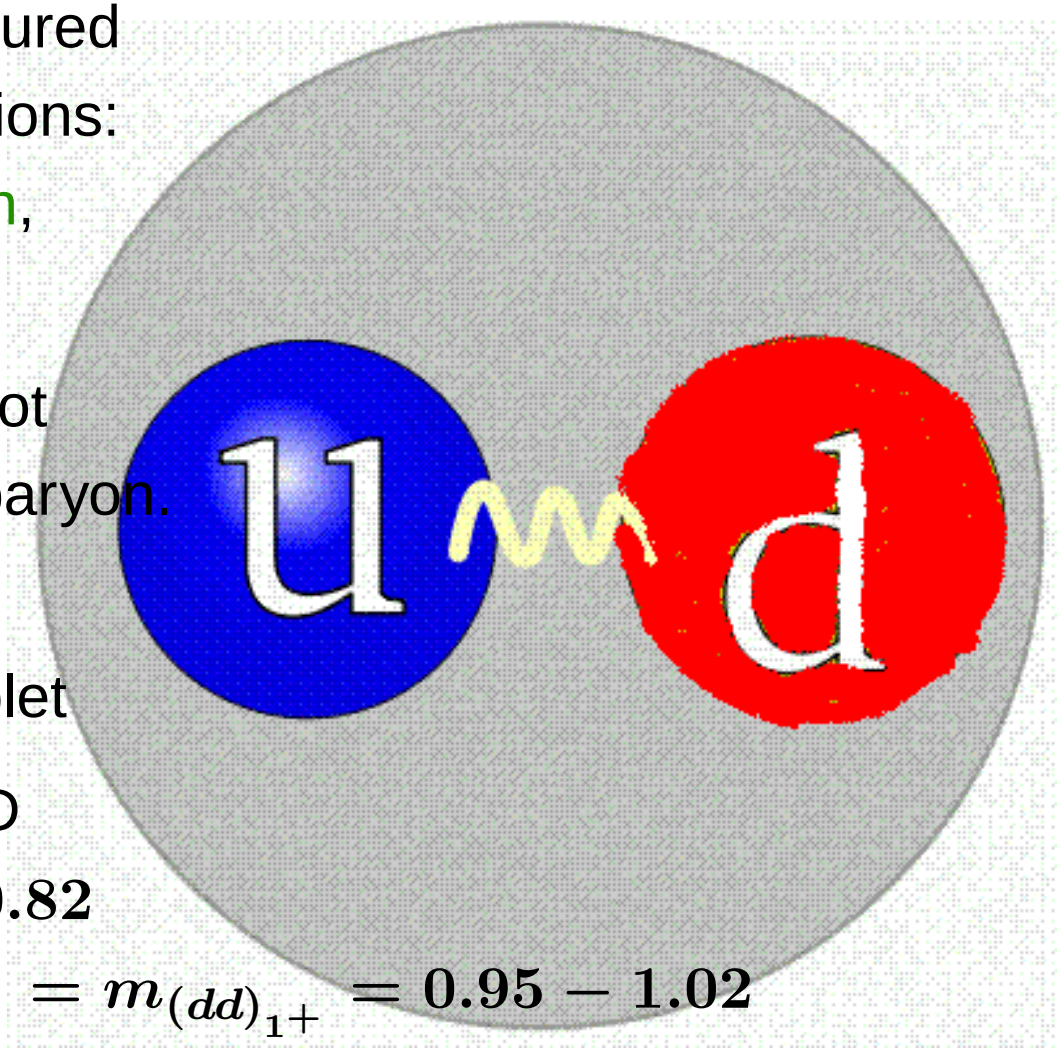
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined . . . Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

Harry Lee

Pions and Form Factors

Harry Lee

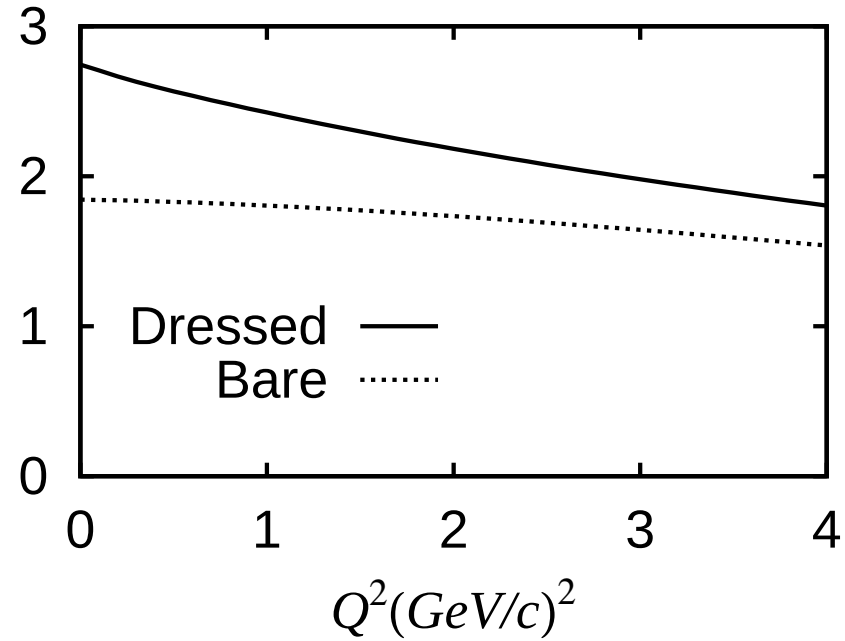
Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e, e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)

Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e, e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)
- Pion cloud effects are large in the low Q^2 region.

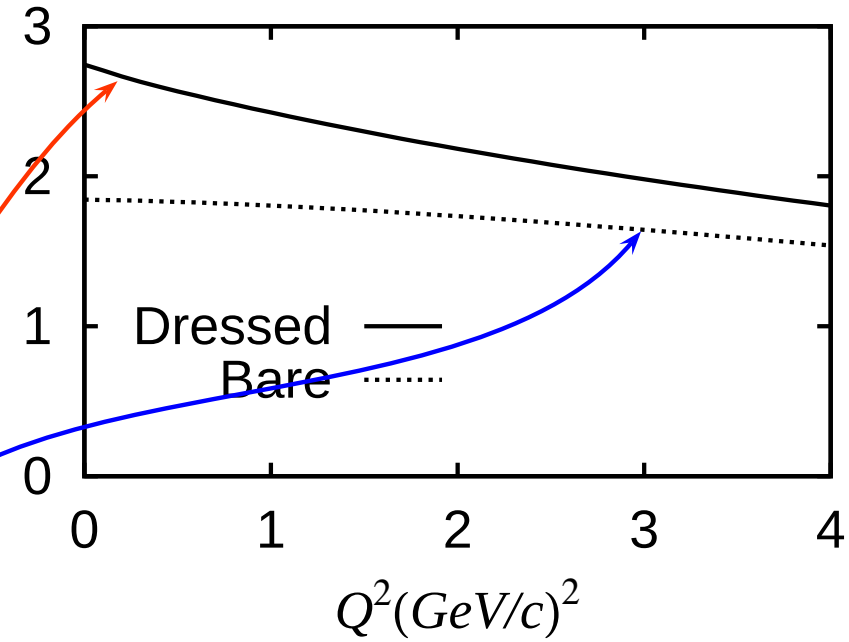
Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e, e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)
- Pion cloud effects are large in the low Q^2 region.

Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



Quark Core

- Responsible for only 2/3 of result at small Q^2
- Dominant for $Q^2 >$2 – 3 GeV^2

Results: Nucleon and Δ Masses

Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV
- Axial-vector diquark provides significant attraction

Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV
- Constructive Interference: 1^{++}-diquark $+ \partial_\mu \pi$

Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

for on-shell nucleons described by Faddeev Amplitude

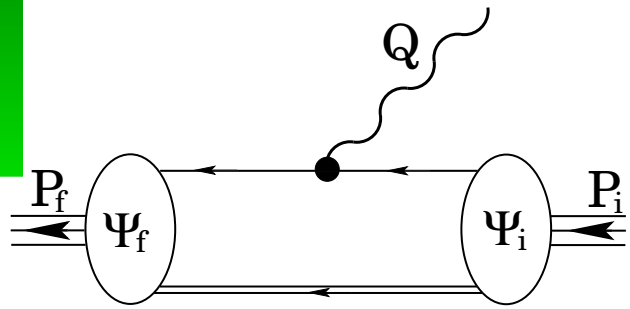
M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

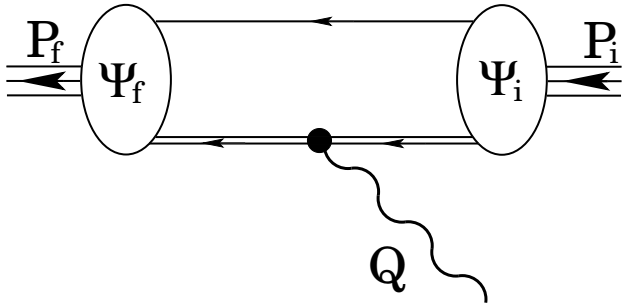
Nucleon-Photon Vertex

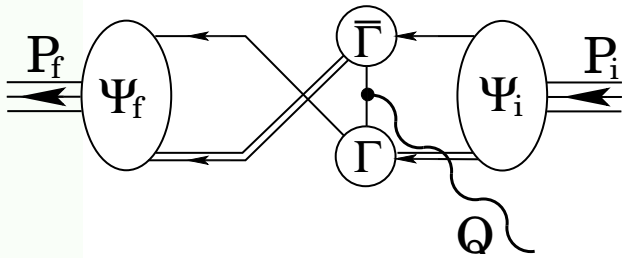
6 terms . . .

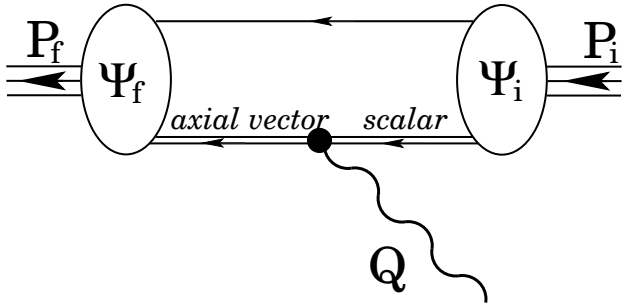
constructed systematically . . . current conserved automatically

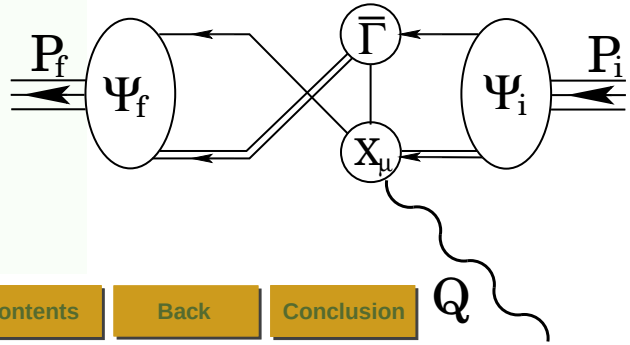
for on-shell nucleons described by Faddeev Amplitude

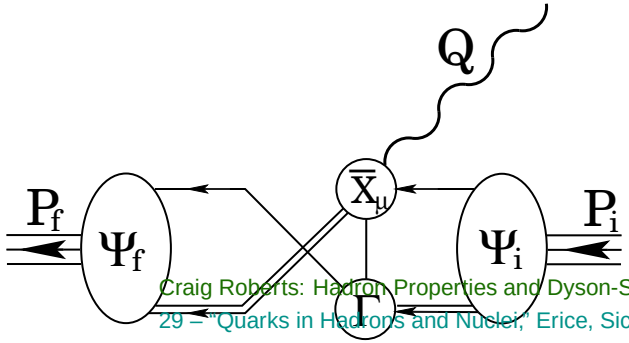




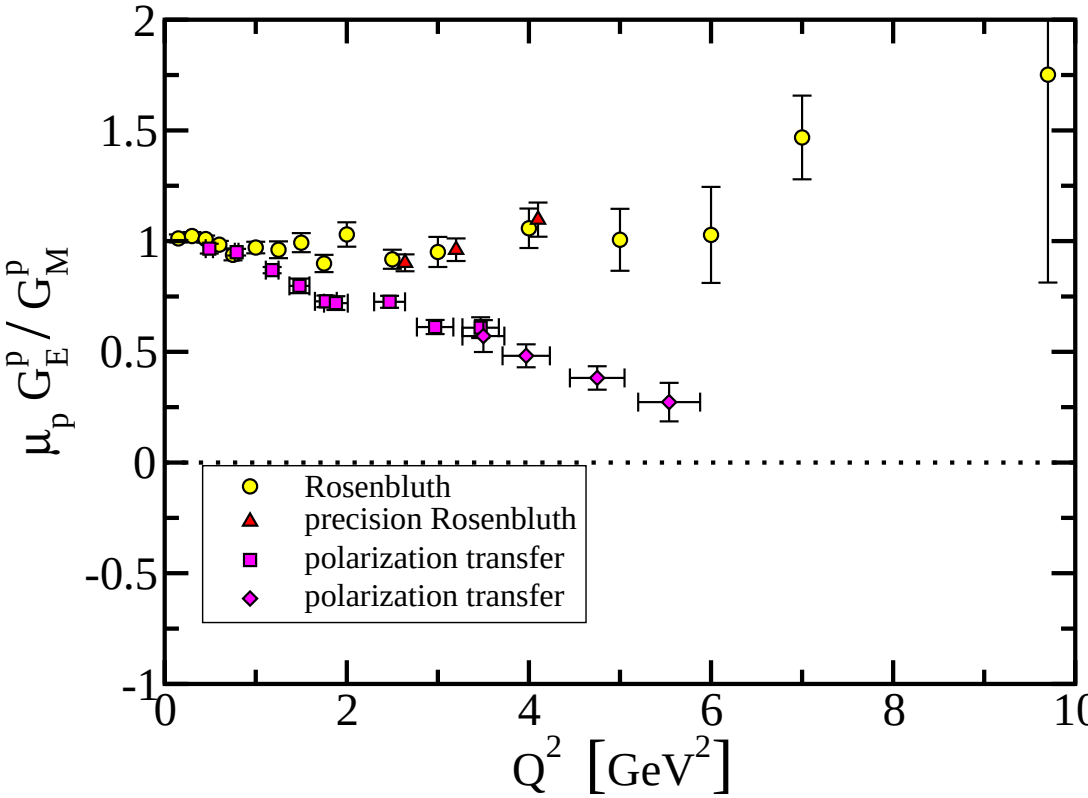






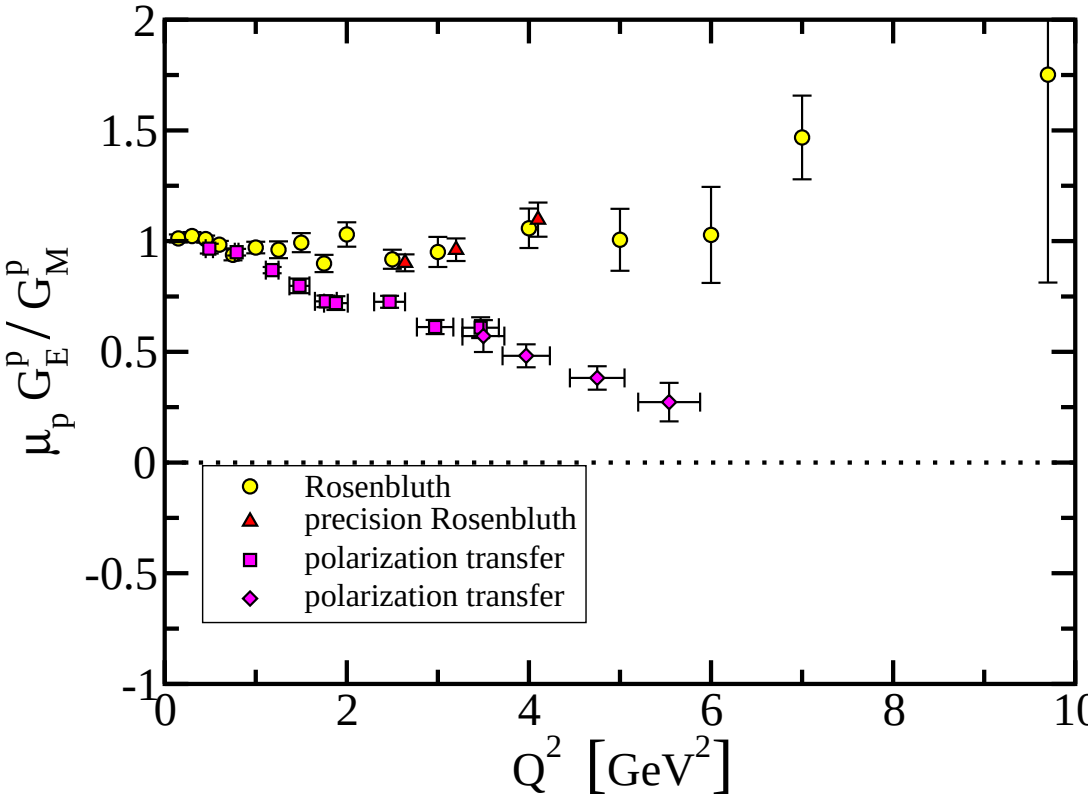


Form Factor Ratio: GE/GM



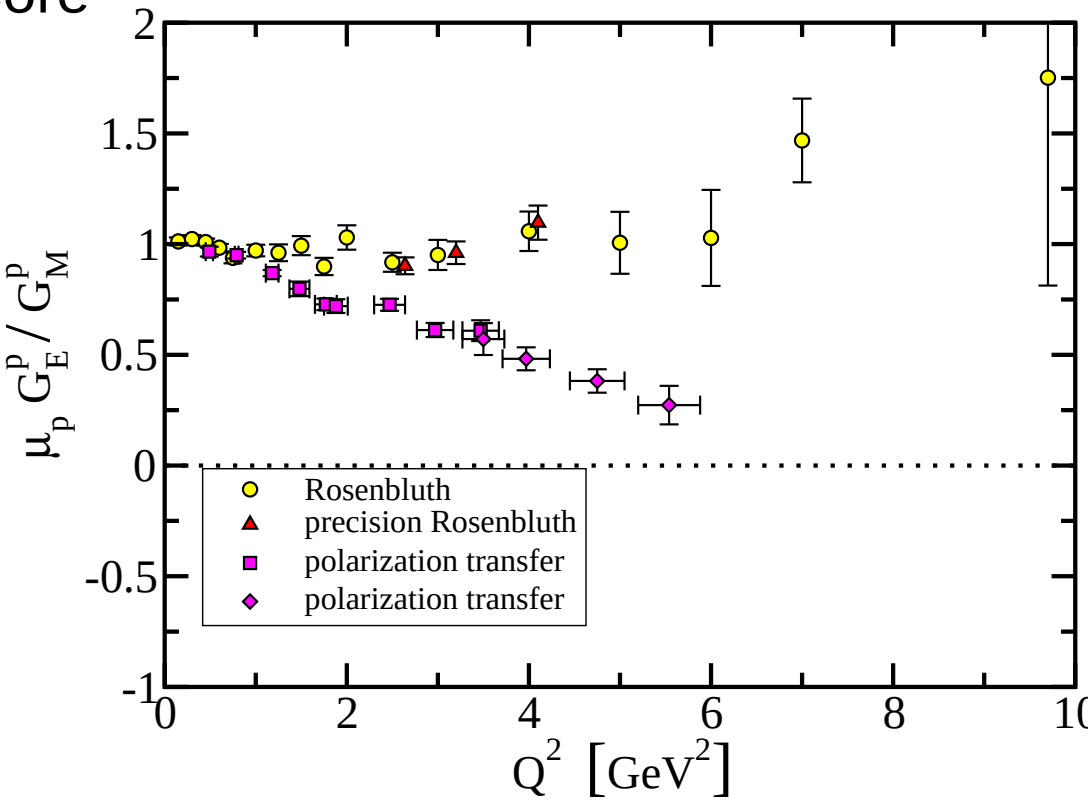
Form Factor Ratio: GE/GM

- Combine these elements . . .



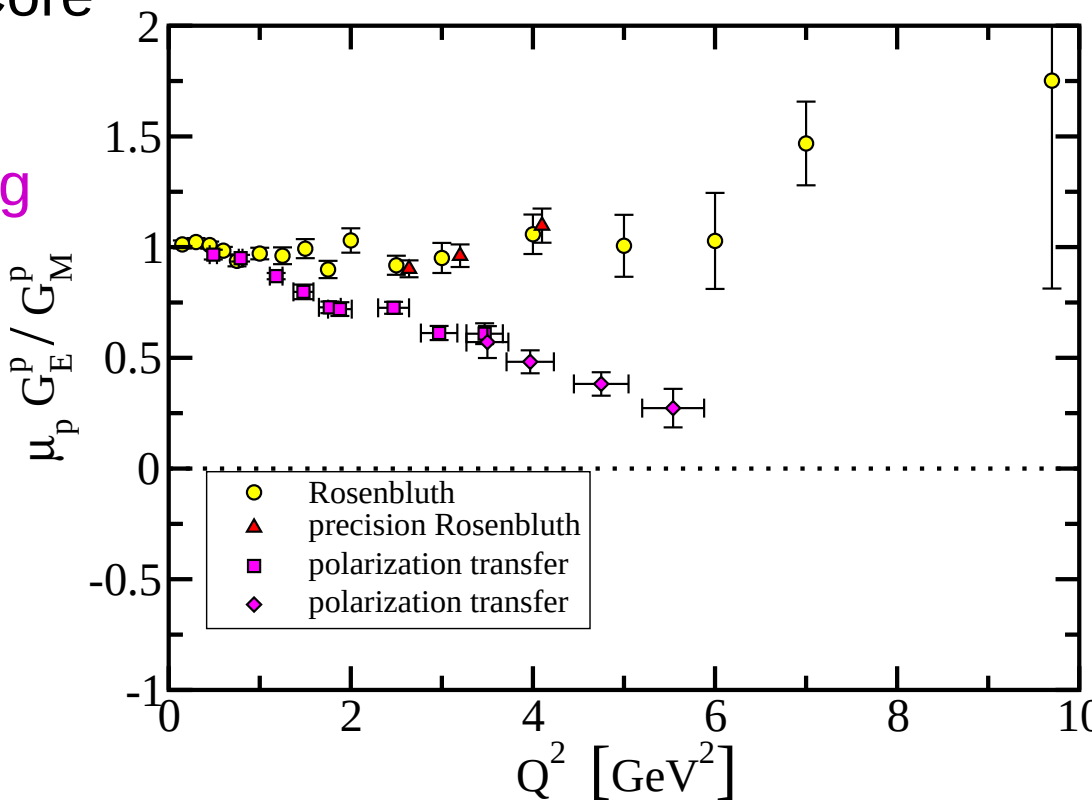
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core



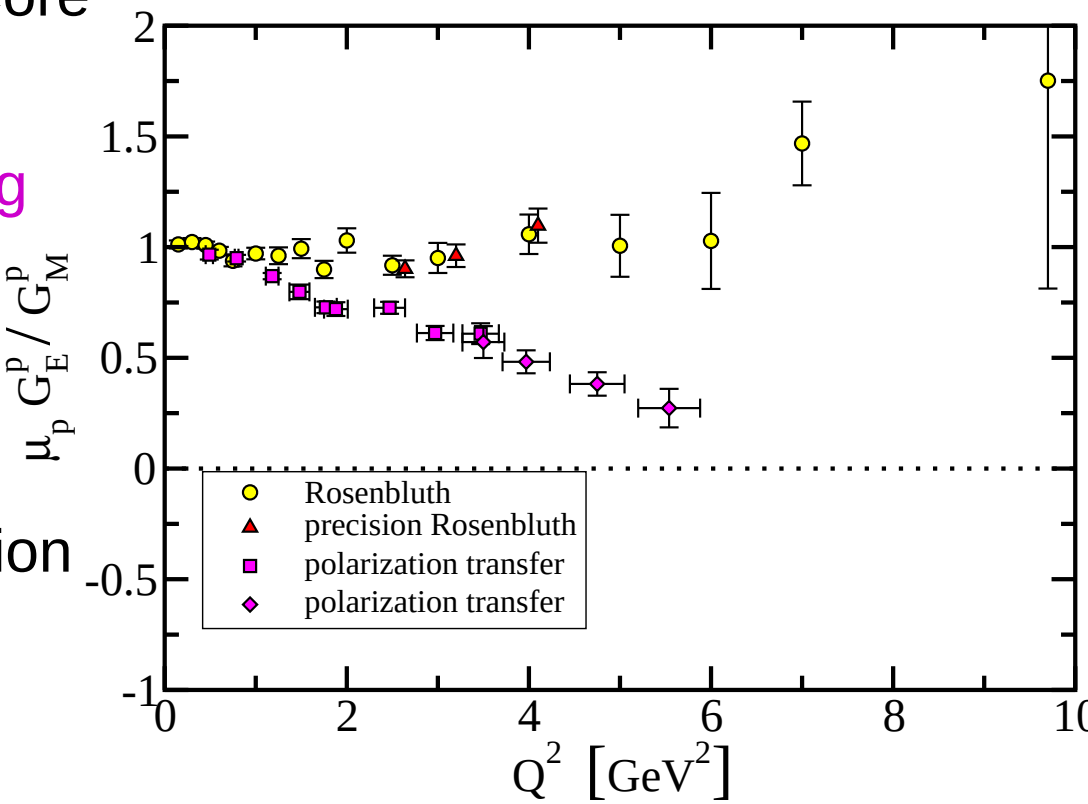
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current



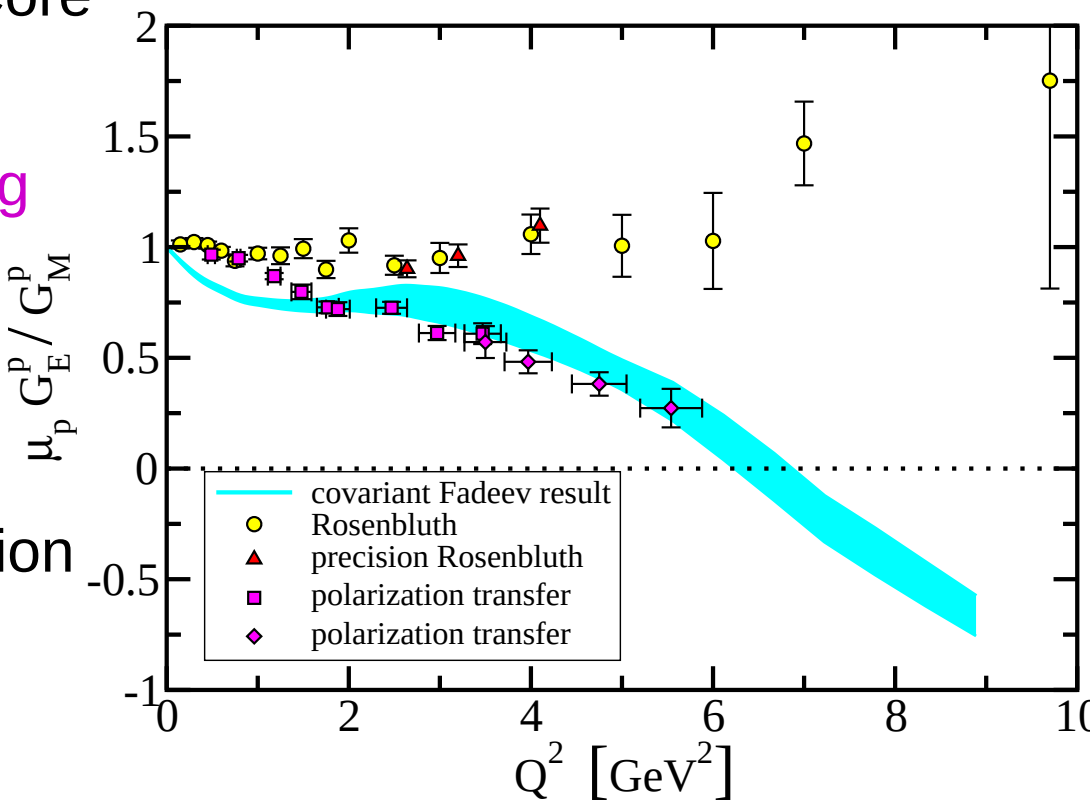
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



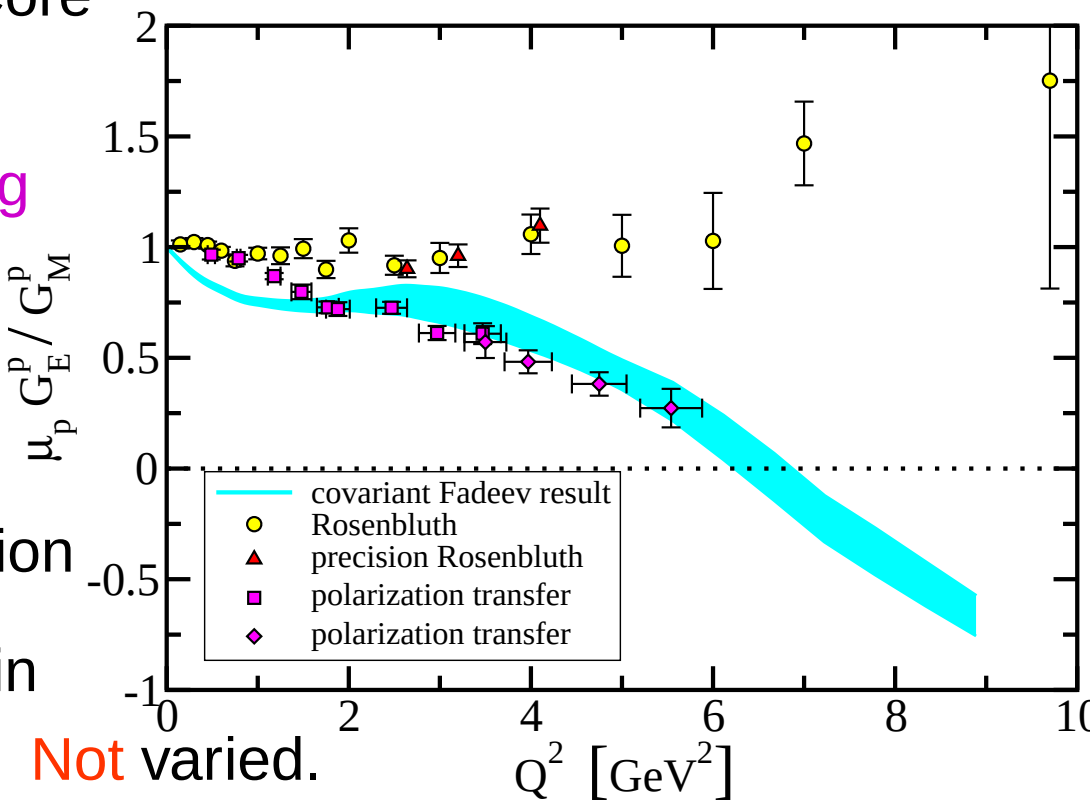
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



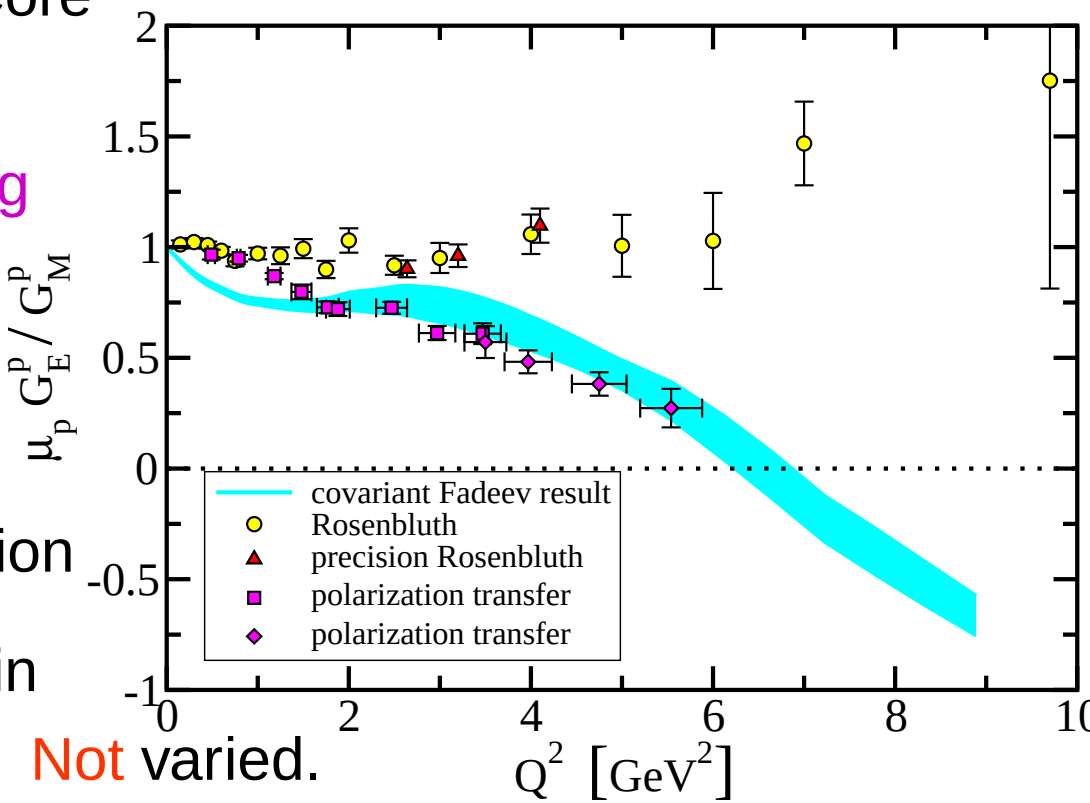
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution
- All parameters fixed in other applications . . . Not varied.



Form Factor Ratio: GE/GM

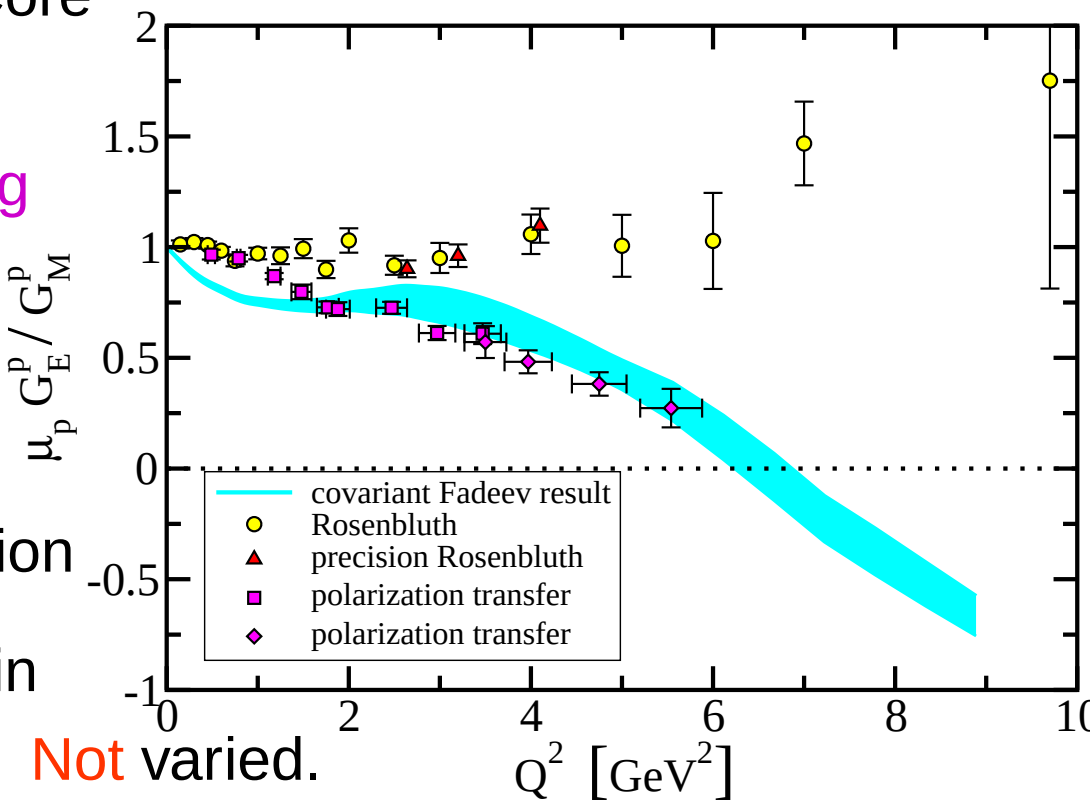
- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



- All parameters fixed in other applications . . . Not varied.
  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\,\mathrm{GeV}^2$

Form Factor Ratio: GE/GM

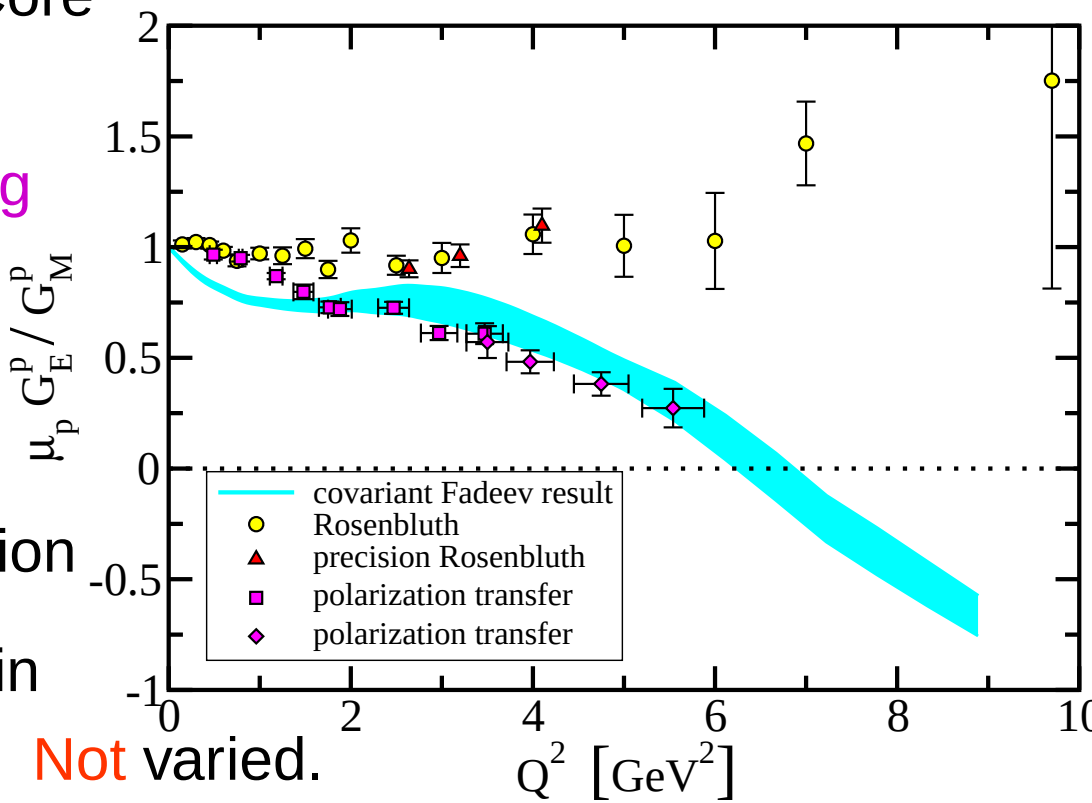
- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



- All parameters fixed in other applications . . . Not varied.
  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\,\mathrm{GeV}^2$
  - Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement

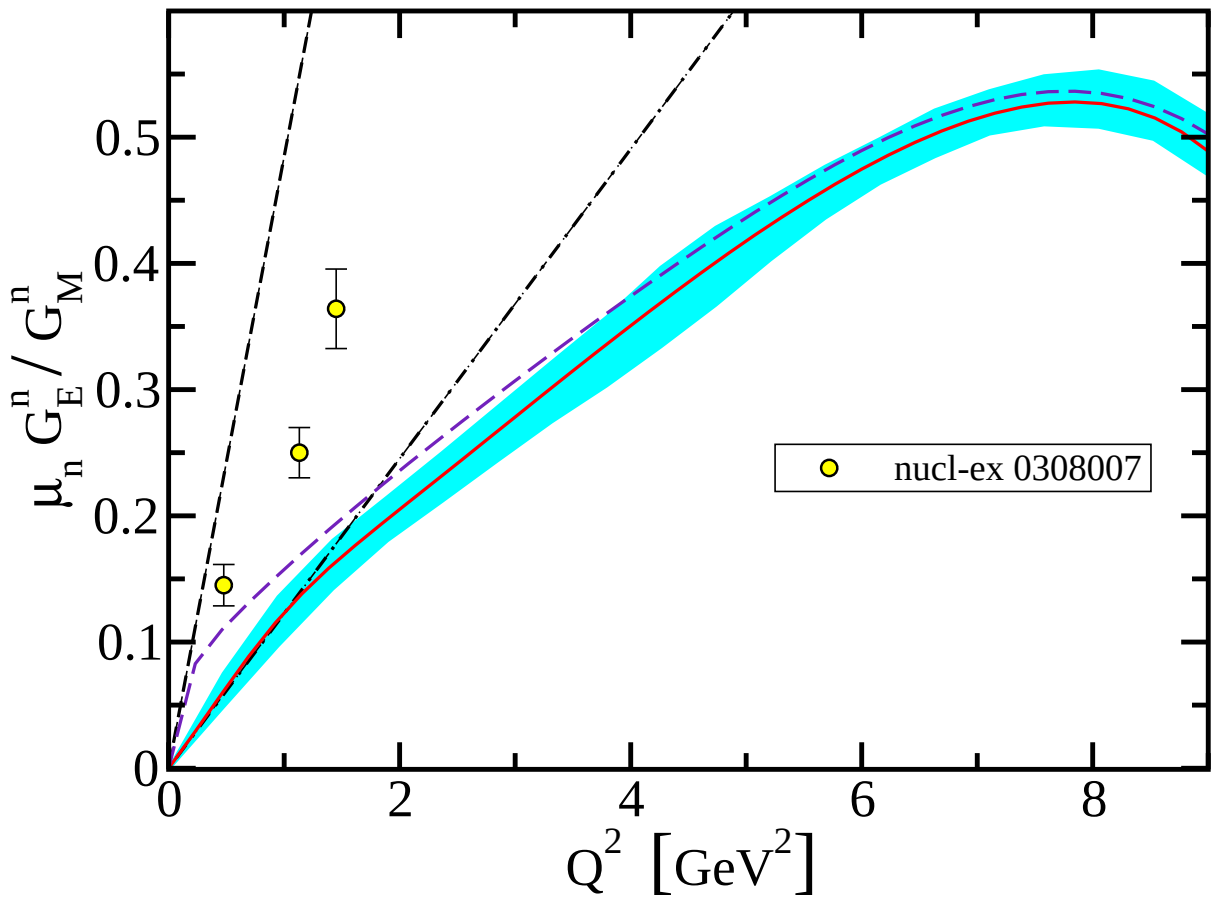
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



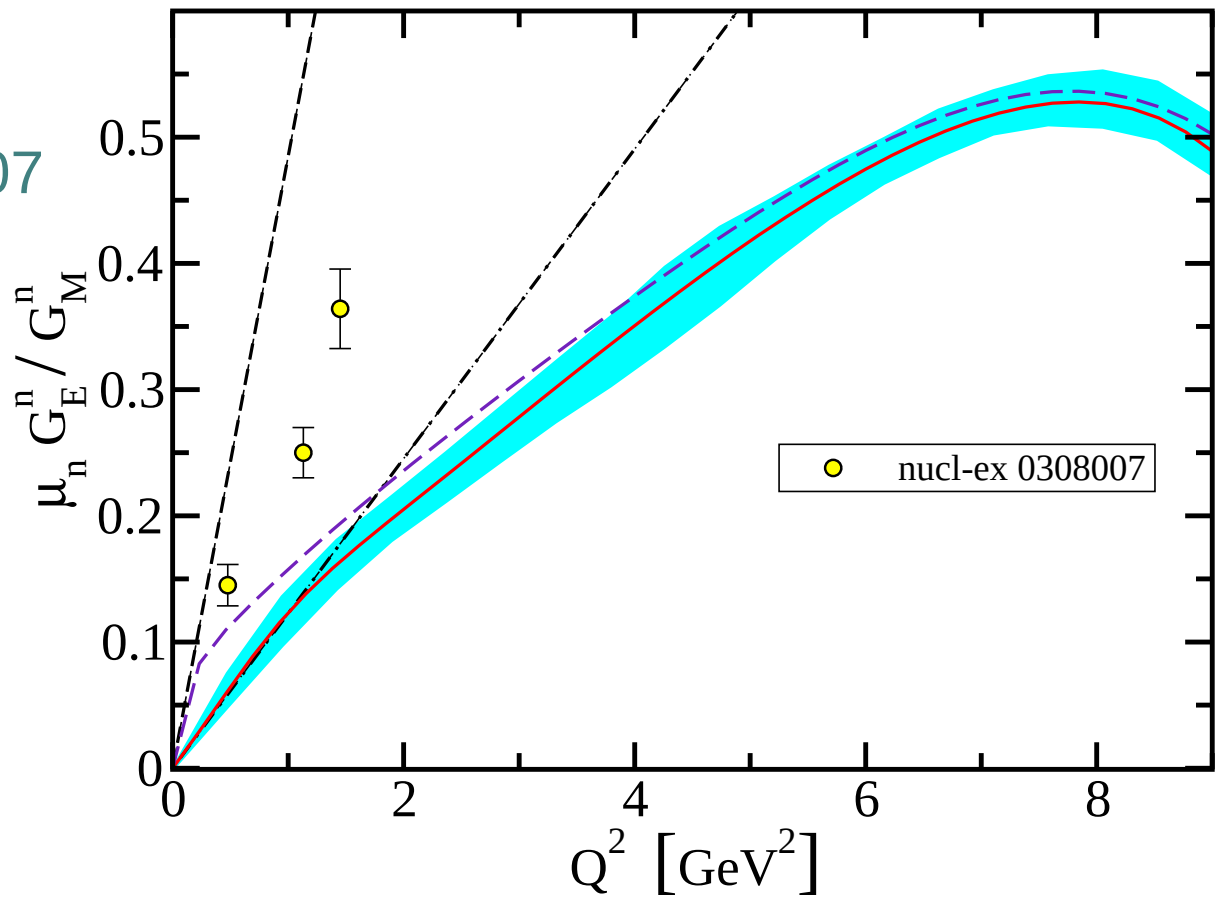
- All parameters fixed in other applications . . . Not varied.
  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\,\mathrm{GeV}^2$
  - Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
  - Predict Zero at $Q^2 \approx 6.5\mathrm{GeV}^2$

Neutron Form Factors



Neutron Form Factors

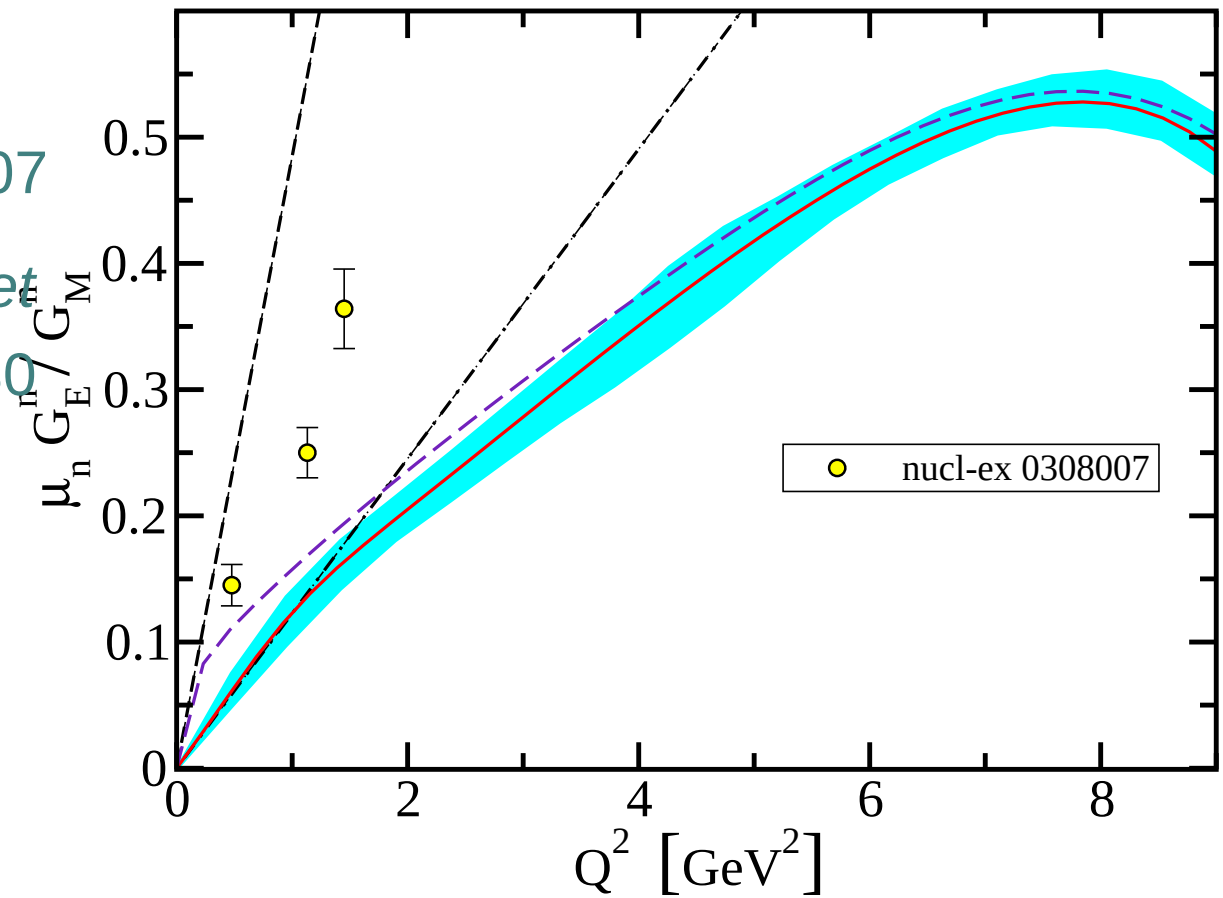
- Expt. Madey, *et al.* nu-ex/0308007



Neutron Form Factors

- Expt. Madey, *et al.* nu-ex/0308007
- Calc. Bhagwat, *et al.* nu-th/0610080
- $\mu_p \dfrac{G_E^n(Q^2)}{G_M^n(Q^2)} = -\dfrac{r_n^2}{6} Q^2$

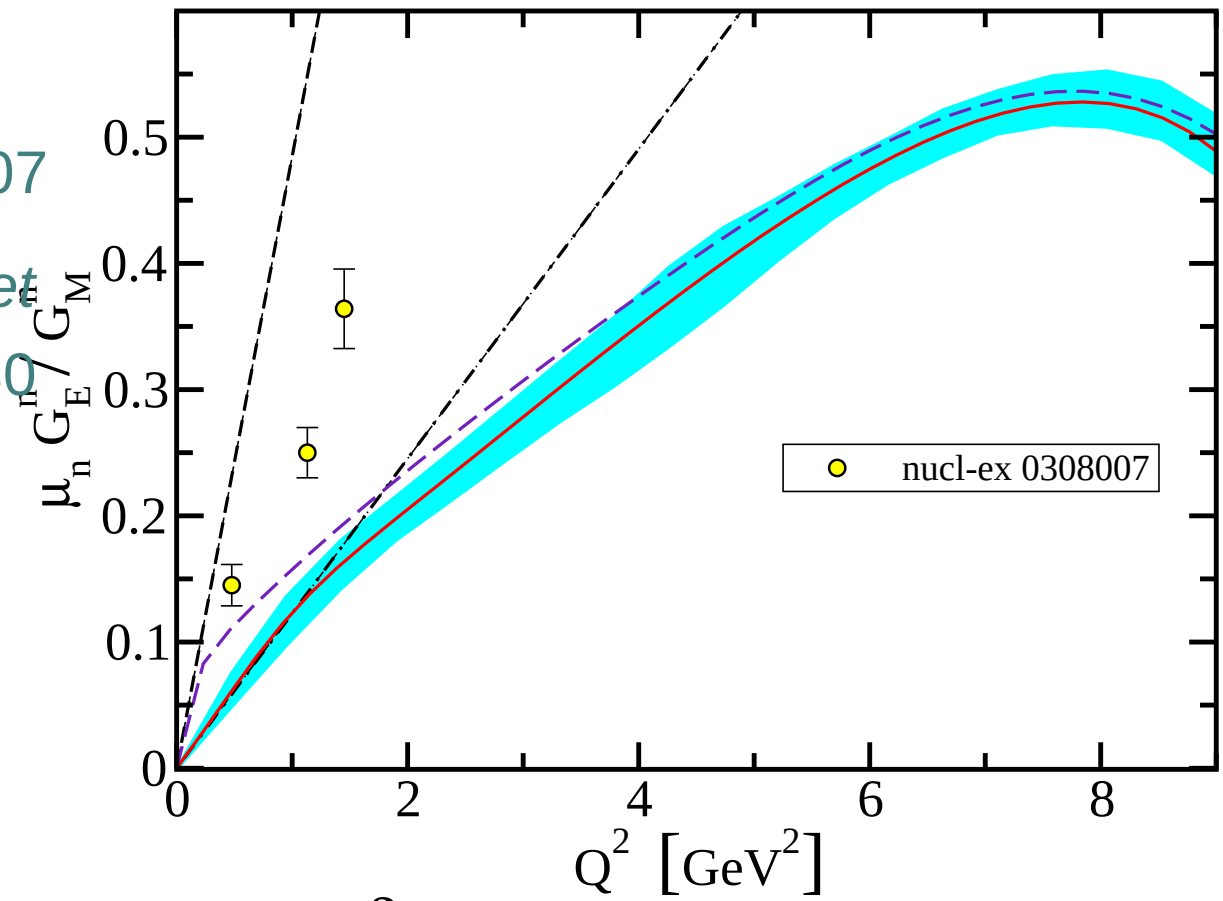
Valid for $r_n^2 Q^2 \lesssim 1$



Neutron Form Factors

- Expt. Madey, *et al.* nu-ex/0308007
- Calc. Bhagwat, *et al.* nu-th/0610080
- $\mu_p \dfrac{G_E^n(Q^2)}{G_M^n(Q^2)} = -\dfrac{r_n^2}{6} Q^2$

Valid for $r_n^2 Q^2 \lesssim 1$



- No sign yet of a zero in $G_E^n(Q^2)$, even though calculation predicts $G_E^p(Q^2 \approx 6.5\,\mathrm{GeV}^2) = 0$
- Data to $Q^2 = 3.4\,\mathrm{GeV}^2$ is being analysed (JLab E02-013)

Epilogue

Epilogue

Epilogue

- DCSB exists in QCD.

Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
  - It impacts dramatically upon observables.

Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
  - It impacts dramatically upon observables.
- Confinement

Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
  - It impacts dramatically upon observables.
- Confinement
  - Expressed and realised in dressed propagators and vertices associated with elementary excitations
  - Observables can be used to explore model realisations

Epilogue

- DCSB exists in QCD.
  - It is manifest in dressed propagators and vertices
  - It impacts dramatically upon observables.
- Confinement
  - Expressed and realised in dressed propagators and vertices associated with elementary excitations
  - Observables can be used to explore model realisations
- DSEs . . . contemporary tool that describes and explains these phenomena, and connects them with prediction of observables

Contents

1. QCD's Challenges
2. Dichotomy of the Pion
3. What's the Problem?
4. Dyson-Schwinger Equations
5. Schwinger Functions
6. Persistent Challenge
7. Truncation
8. Dressed-Quark Propagator
9. Quenched-QCD cf. Lattice
10. QCD & Interaction
11. Dressed-gluon
12. Critical Mass & Chiral Expansion
13. C-quark σ-term
14. Hadrons
15. Bethe-Salpeter Kernel
16. Radial Excitations
17. Radial Excitations (cont.)
18. Radial Excitations & Lattice-QCD
19. Pion OAM
20. Neutral Pseudoscalars
21. New Challenges
22. Nucleon EM Form Factors
23. Faddeev equation
24. Diquark correlations
25. Pions and Form Factors
26. Baryon Masses
27. Nucleon-Photon Vertex
28. Form Factor Ratio
29. Neutron Form Factors

---

30. Contemporary Reviews
31. Colour-singlet Kernel
32. π and ρ
33. Angular Momentum
34. Extant DIS π
35. Distribution function

Contemporary Reviews

- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD
  C.D. Roberts and S.M. Schmidt, nu-th/0005064,
  Prog. Part. Nucl. Phys. **45** (2000) S1
- The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .
  R. Alkofer and L. von Smekal, he-ph/0007355,
  Phys. Rept. **353** (2001) 281
- Dyson-Schwinger equations: A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049,
  Int. J. Mod. Phys. **E 12** (2003) pp. 297-365
- Infrared properties of QCD from Dyson-Schwinger equations.
  C. S. Fischer, he-ph/0605173,
  J. Phys. **G 32** (2006) pp. R253-R291
- Nucleon electromagnetic form factors
  J. Arrington, C.D. Roberts and J.M. Zanotti, nucl-th/0611050,
  J. Phys. **G 34** (2007) pp. S23-S52.

Colour-singlet Bethe-Salpeter equation

Detmold *et al*., nu-th/0202082

Bhagwat, *et al*., nu-th/0403012

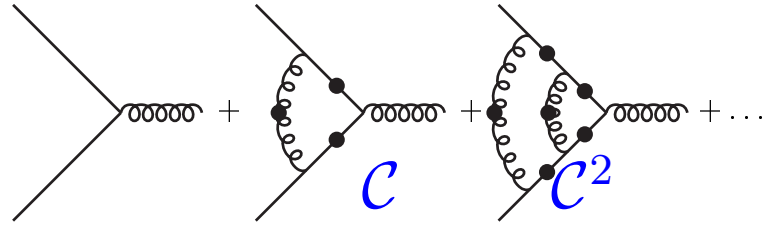
Colour-singlet Bethe-Salpeter equation

Detmold *et al*., nu-th/0202082

Bhagwat, *et al*., nu-th/0403012

- Coupling-modified dressed-ladder vertex

$\Gamma^a_\mu(k,p) \;=$

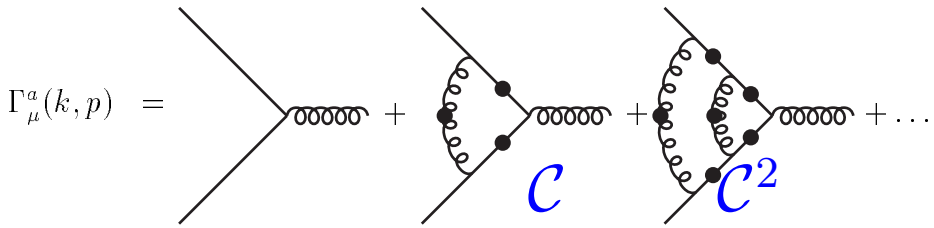


Colour-singlet Bethe-Salpeter equation

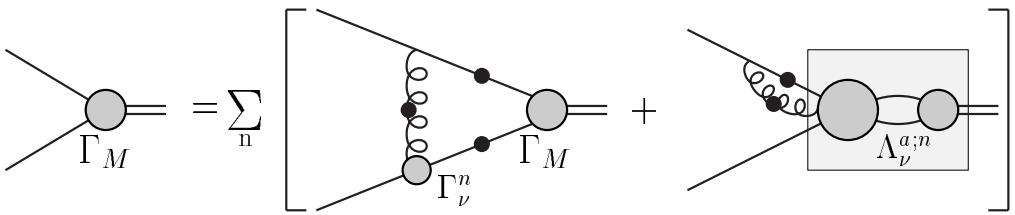
Detmold *et al*., nu-th/0202082

Bhagwat, *et al*., nu-th/0403012

- Coupling-modified dressed-ladder vertex



- BSE consistent with vertex

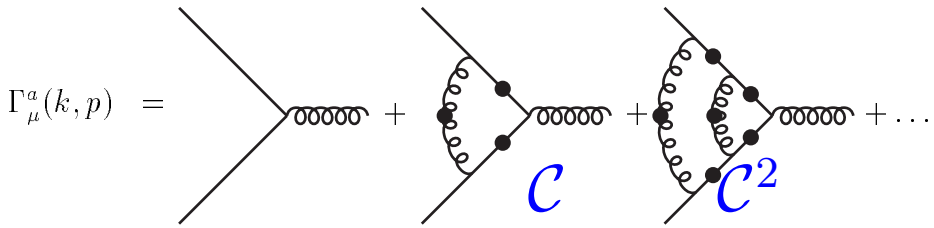


Colour-singlet Bethe-Salpeter equation

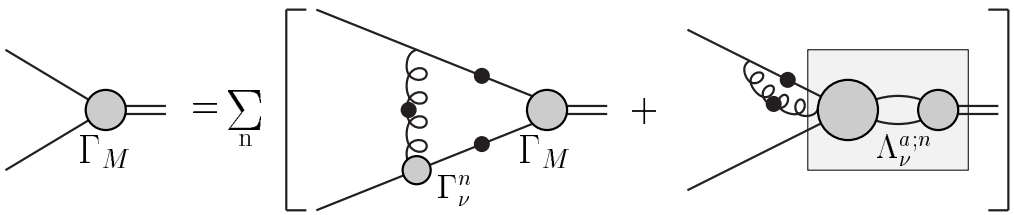
Detmold *et al*., nu-th/0202082

Bhagwat, *et al*., nu-th/0403012

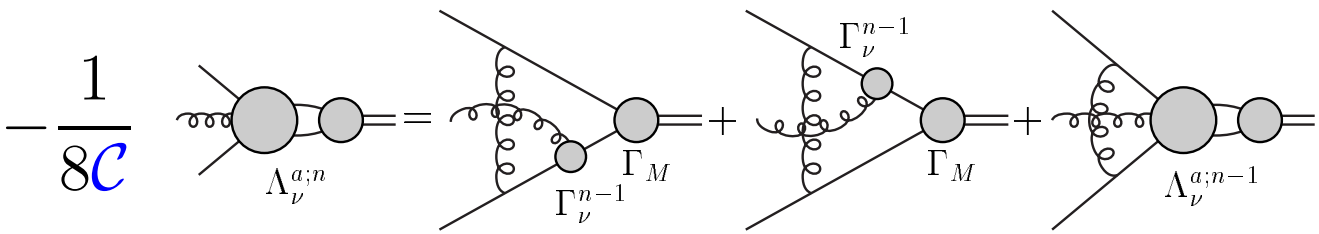
- Coupling-modified dressed-ladder vertex



- BSE consistent with vertex



- Bethe-Salpeter kernel . . . recursion relation

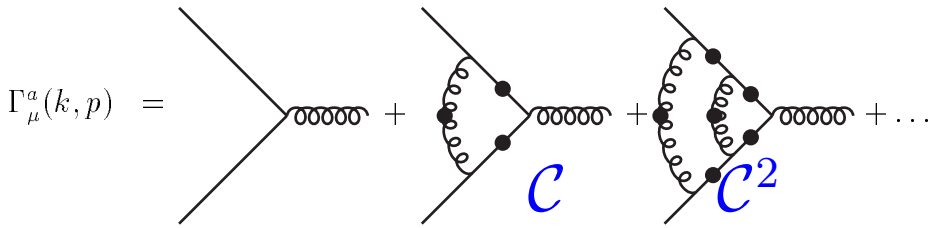


Colour-singlet Bethe-Salpeter equation

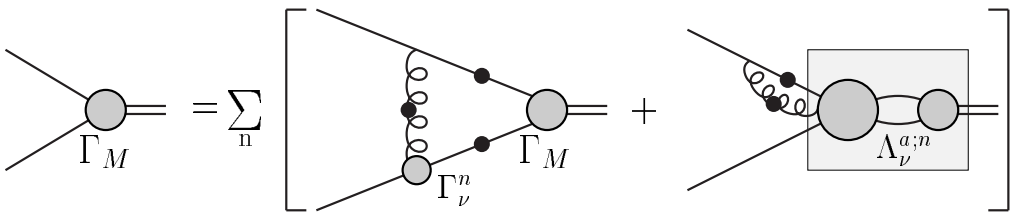
Detmold *et al*., nu-th/0202082

Bhagwat, *et al*., nu-th/0403012

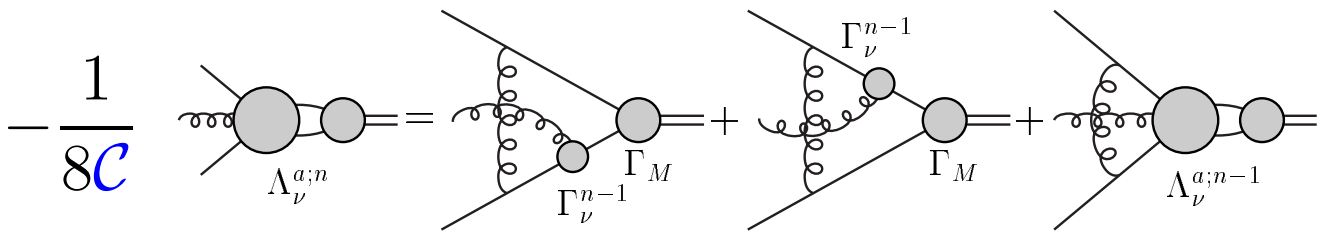
- Coupling-modified dressed-ladder vertex



- BSE consistent with vertex



- Bethe-Salpeter kernel . . . recursion relation



- Kernel necessarily non-planar,

  even with planar vertex

π and ρ mesons

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning
- ALL π-ρ mass splitting present in chiral limit

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning
- ALL π-ρ mass splitting present in chiral limit and with the Simplest kernel

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning
- π-ρ mass splitting driven by DχSB mechanism

  Not constituent-quark-model-like hyperfine splitting

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning
- π-ρ mass splitting driven by DχSB mechanism
  Not constituent-quark-model-like hyperfine splitting
- Extending kernel

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . **NO** Fine Tuning
- π-ρ mass splitting driven by DχSB mechanism
  Not constituent-quark-model-like hyperfine splitting
- Extending kernel: NO effect on m_π

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . **NO** Fine Tuning
- π-ρ mass splitting driven by DχSB mechanism

  Not constituent-quark-model-like hyperfine splitting
- Extending kernel: NO effect on m_π

  For m_ρ – zeroth order, accurate to 20%

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning
- π-ρ mass splitting driven by DχSB mechanism

  Not constituent-quark-model-like hyperfine splitting
- Extending kernel: NO effect on m_π

  For m_ρ – zeroth order, accurate to 20%

  – one loop, accurate to 13%

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . **NO** Fine Tuning
- π-ρ mass splitting driven by DχSB mechanism

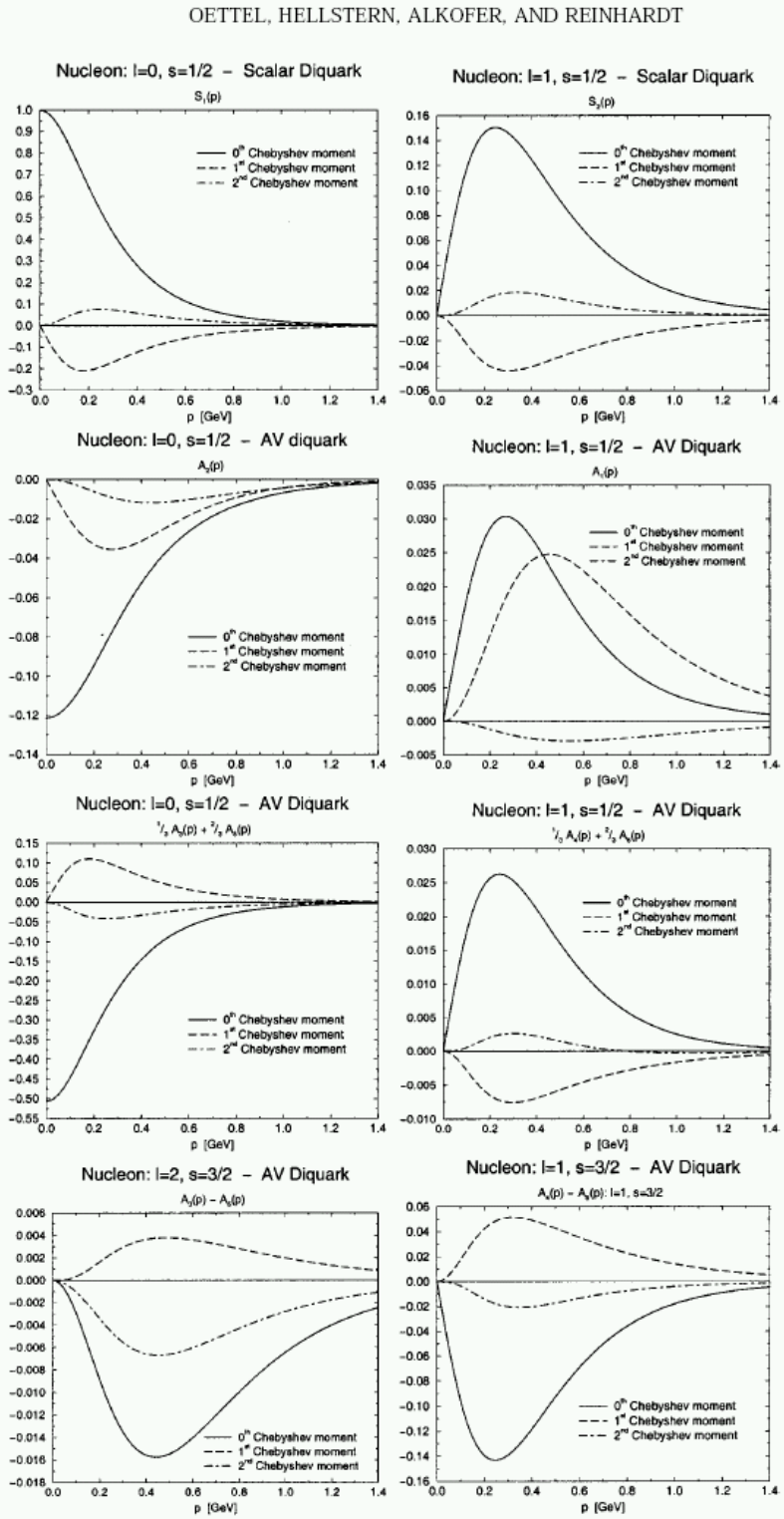
  Not constituent-quark-model-like hyperfine splitting
- Extending kernel: NO effect on m_π

  For m_ρ – zeroth order, accurate to 20%

  – one loop, accurate to 13%

  – two loop, accurate to 4%

Angular Momentum Rest Frame



M. Oettel, *et al.*
nucl-th/9805054

Crude estimate based on magnitudes $\Rightarrow$ probability for a u-quark to carry the proton's spin is $P_{u\uparrow} \sim 80\,\%$, with $P_{u\downarrow} \sim 5\,\%$, $P_{d\uparrow} \sim 5\,\%$, $P_{d\downarrow} \sim 10\,\%$.

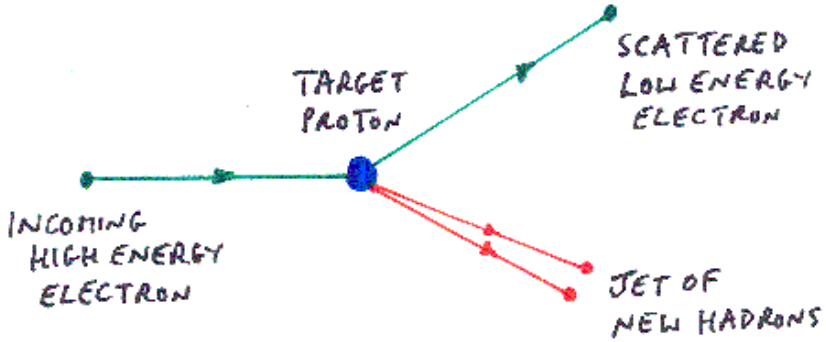
Hence, by this reckoning $\sim 30\%$ of proton's rest-frame spin is located in dressed-quark angular momentum.

Deep-inelastic scattering

Deep-inelastic scattering

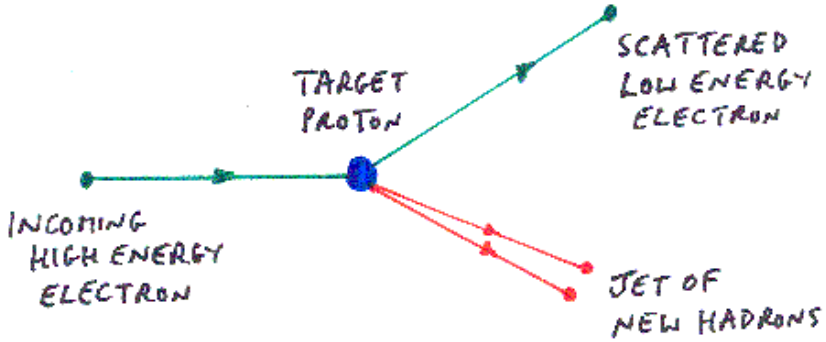
- Looking for Quarks

Deep-inelastic scattering



- Looking for Quarks

Deep-inelastic scattering

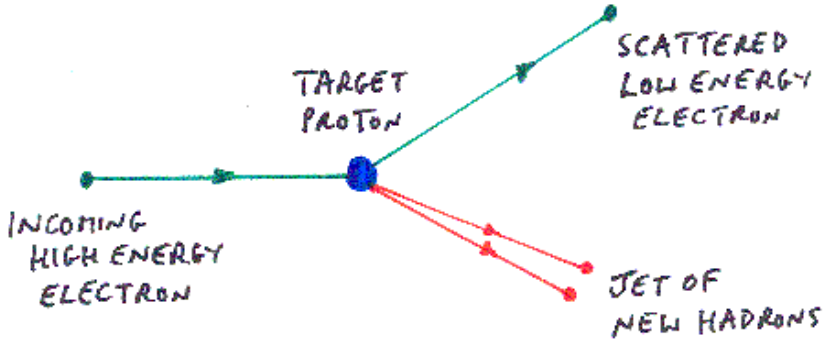


- Looking for Quarks

- Signature Experiment for QCD:

  Discovery of Quarks at SLAC

Deep-inelastic scattering



- Looking for Quarks

- Signature Experiment for QCD:

  Discovery of Quarks at SLAC

- Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x)$, $g(x)$

Pion's valence quark distn

Pion's valence quark distn

- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!

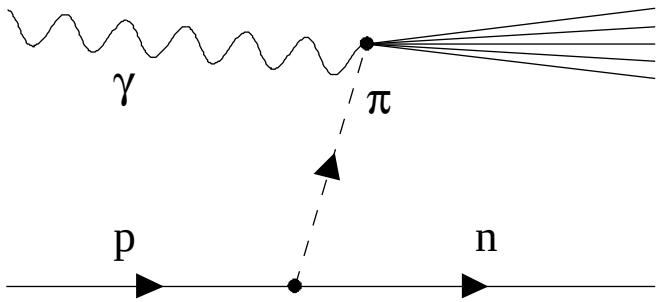
Pion's valence quark distn

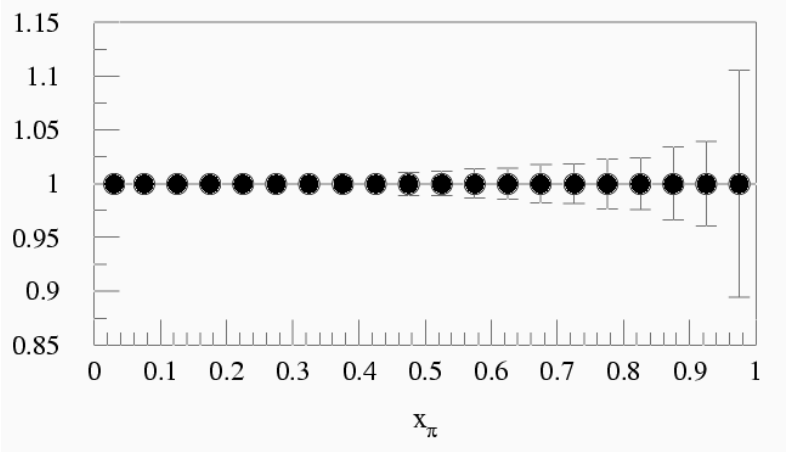
- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!
- Existing Measurement Inferred from Drell-Yan:
  $\pi N \to \mu^+ \mu^- X$

Pion's valence quark distn

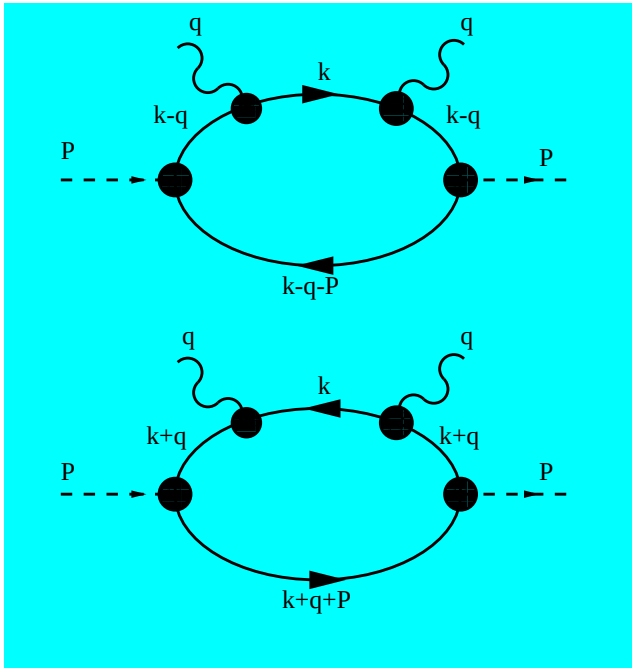
- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!
- Existing Measurement Inferred from Drell-Yan:
  $\pi N \to \mu^+\mu^- X$
- Proposal (Holt & Reimer, ANL, nu-ex/0010004)

$e^-_{5\mathrm{GeV}} - p_{25\,\mathrm{GeV}}$ Collider $\to$ Accurate "Measurement"

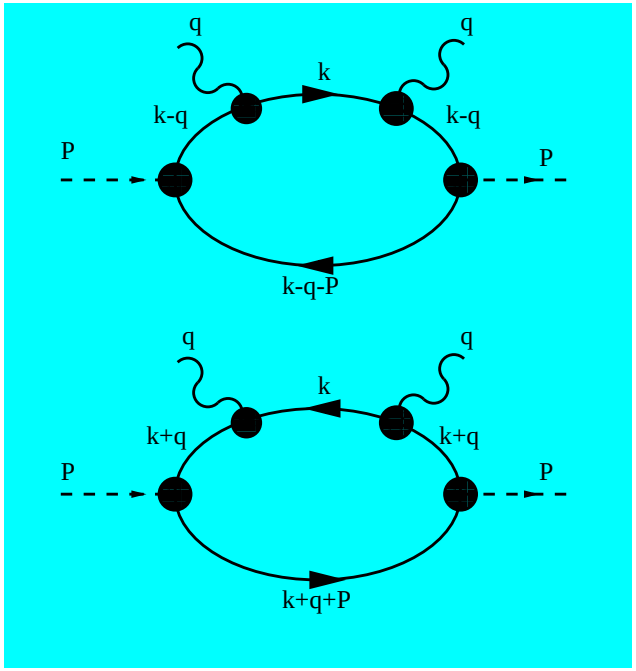




Handbag diagrams



Handbag diagrams



$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathsf{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

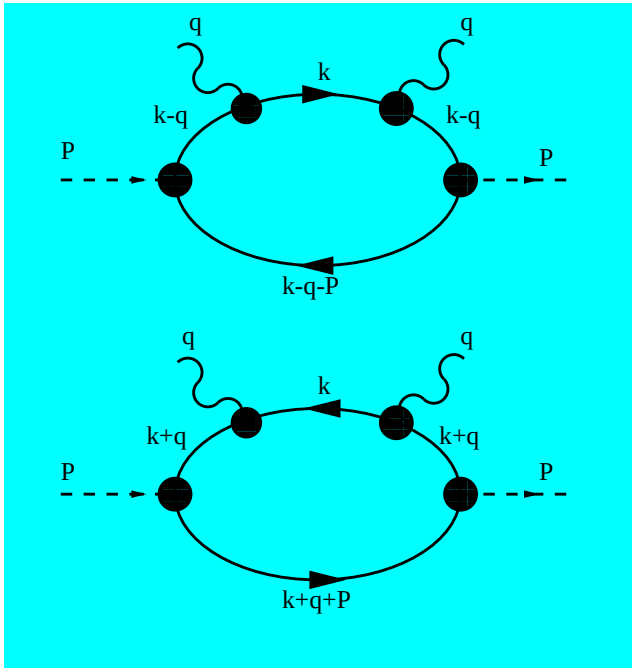
$$T^{+}_{\mu\nu}(q,P) = \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\,\tau_{-}\bar{\Gamma}_{\pi}(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_{\nu}(k_{-0},k)$$
$$\times\, S(k)\,ieQ\Gamma_{\mu}(k,k_{-0})\,S(k_{-0})\tau_{+}\Gamma_{\pi}(k_{-\frac{1}{2}};P)\,S(k_{--})$$

Handbag diagrams

Bjorken Limit: $q^2 \to \infty\,, \quad P\cdot q \to -\infty$

but $x := -\dfrac{q^2}{2P\cdot q}$ fixed.

Numerous algebraic simplifications

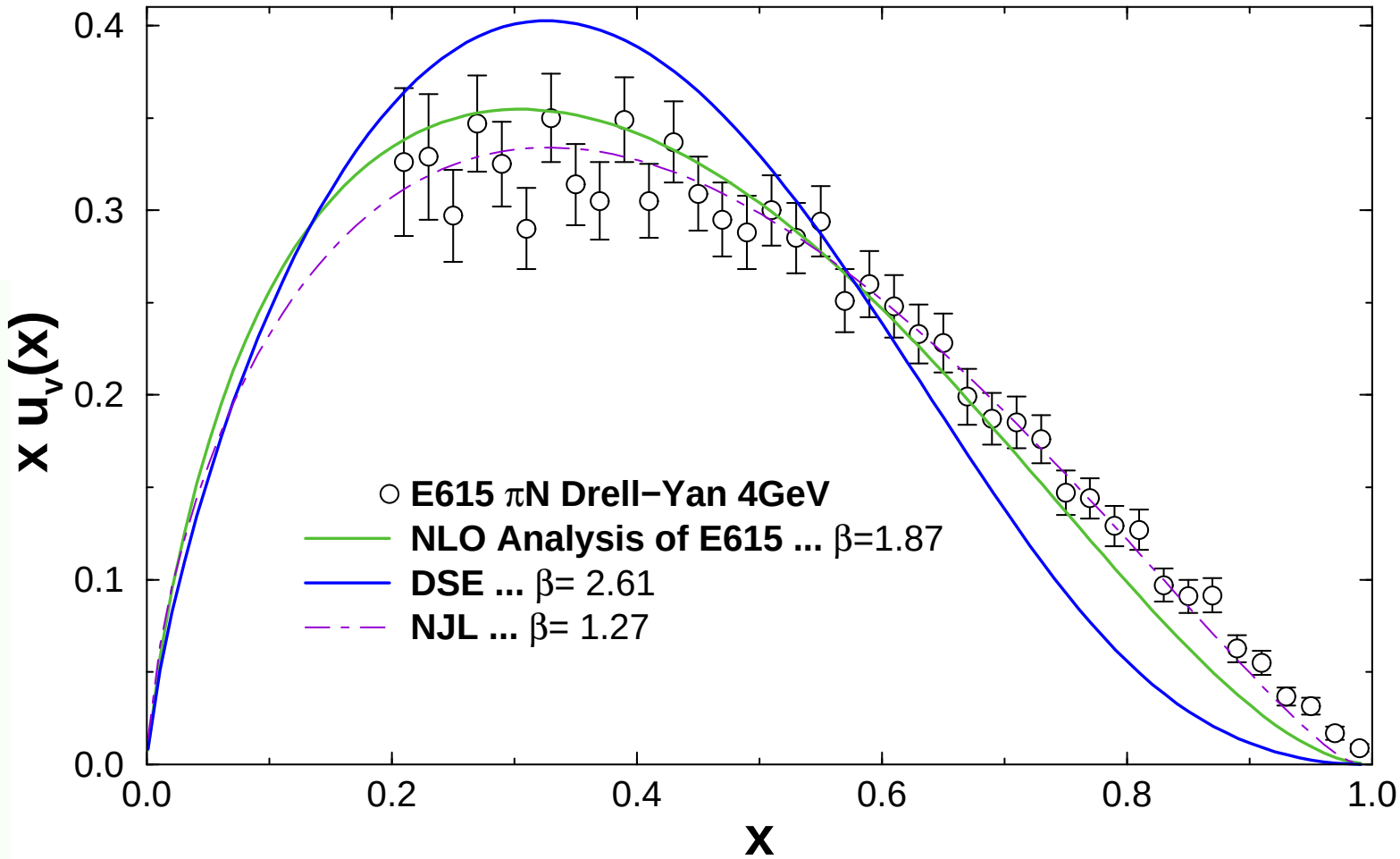


$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathrm{Im}\left[T^+_{\mu\nu}(q;P) + T^-_{\mu\nu}(q;P)\right]$$

$$T^+_{\mu\nu}(q,P) = \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\,\tau_-\bar{\Gamma}_\pi(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_\nu(k_{-0},k)$$
$$\times\, S(k)\,ieQ\Gamma_\mu(k,k_{-0})\,S(k_{-0})\tau_+\Gamma_\pi(k_{-\frac{1}{2}};P)\,S(k_{--})$$

Extant theory vs. experiment

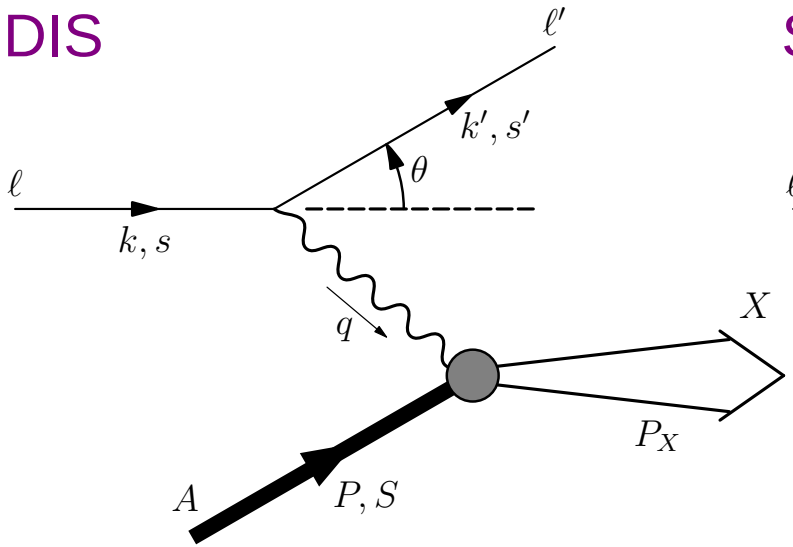
K. Wijersooriya, P. Reimer and R. Holt, nu-ex/0509012 ... Phys. Rev. C (Rapid)



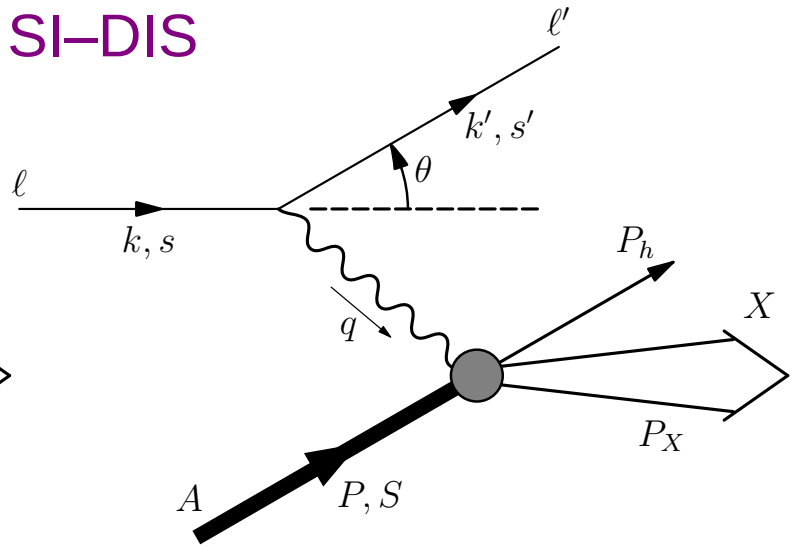
Quark Distribution Functions

Quark Distribution Functions

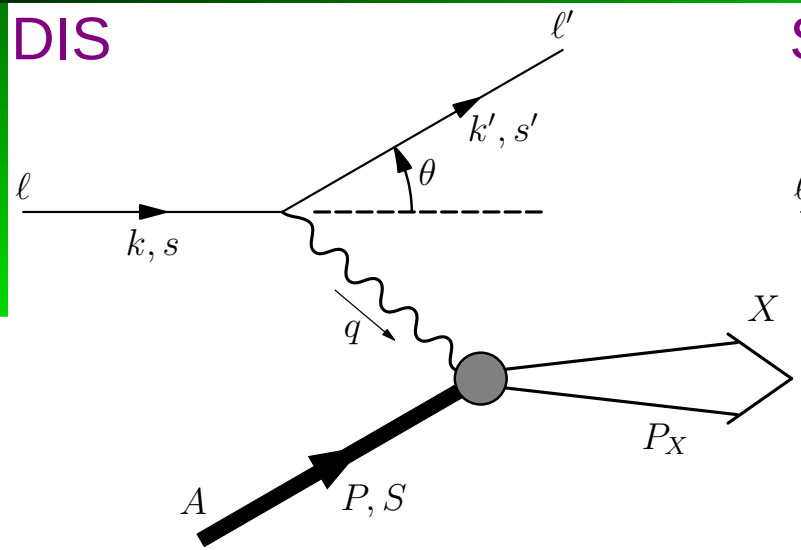
DIS

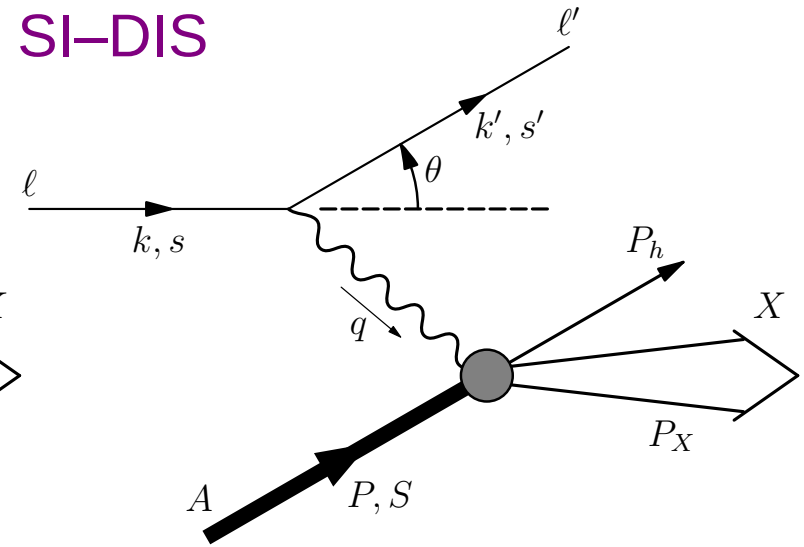


SI–DIS



Quark Distribution Functions





- Three twist-2 parton distributions $(k_\perp = 0)$:
  - Spin-Independent: $q(x)$
  - Helicity: $\Delta q(x)$
  - Transversity: $\Delta_T q(x)$
- All distributions have probability interpretation.
- By definition, contain essentially non-perturbative information about a given process.

Definition and Sum Rules

Definition and Sum Rules

- Light-cone Fourier transforms :

$$\Delta_T q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\xi^-} \langle p,s|\overline{\psi}_q(0)\gamma^+\gamma^1\gamma_5\psi_q(\xi^-)|p,s\rangle_c$$

$$q(x) = \langle\gamma^+\rangle, \qquad \Delta q(x) = \langle\gamma^+\gamma_5\rangle$$

- Related to the nucleon axial & tensor charges via

$$g_A = \int dx[\Delta u(x) - \Delta d(x)], \quad g_T = \int dx[\Delta_T u(x) - \Delta_T d(x)],$$

- Must satisfy: positivity constraints and Soffer bound

$$\Delta q(x), \Delta_T q(x) \leqslant q(x), \quad q(x) + \Delta q(x) \geqslant 2\,|\Delta_T q(x)|$$

Ian Cloët

JLab, now ANL

Ian Cloët

JLab, now ANL

Once more on the one that got away.

Cloët, Bentz, Thomas

arXiv:0708.3246 [hep-ph]

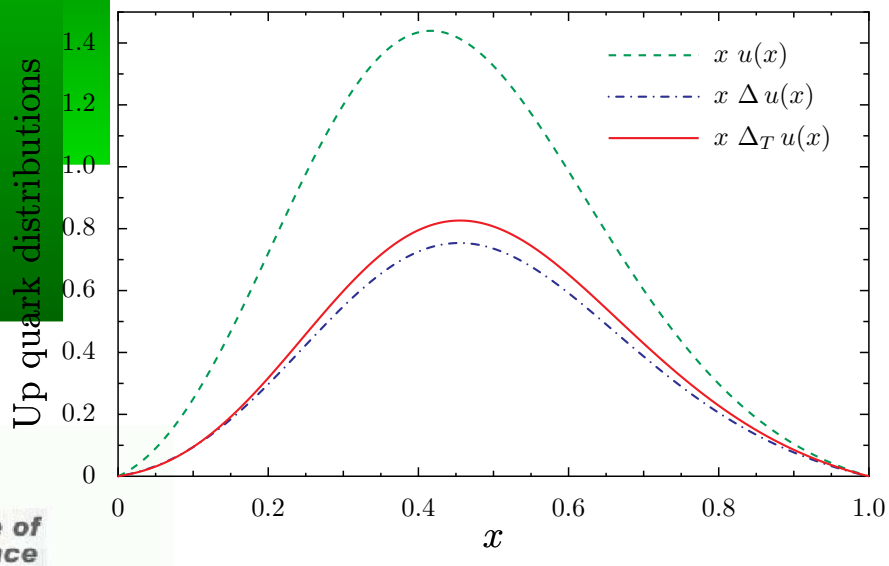
Model predictions

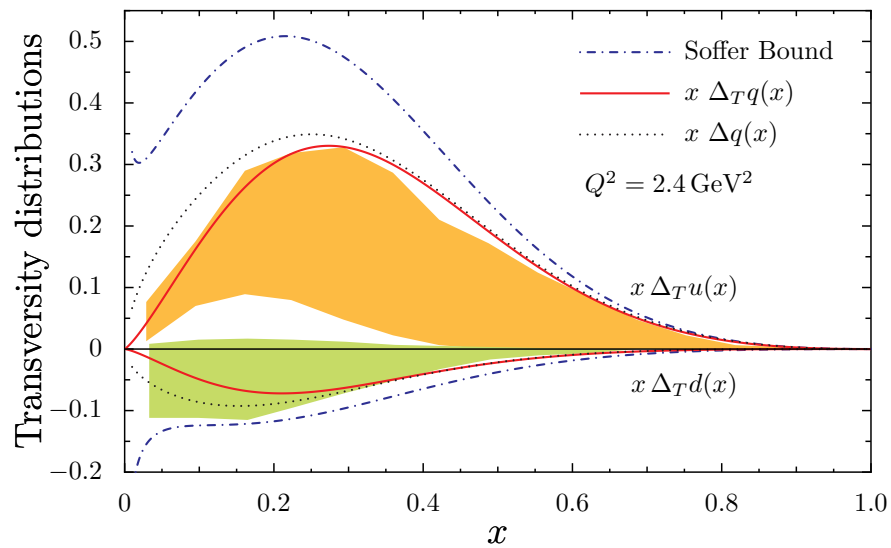
Cloët, Bentz, Thomas

arXiv:0708.3246 [hep-ph]

Model predictions

- Simplified Faddeev equation





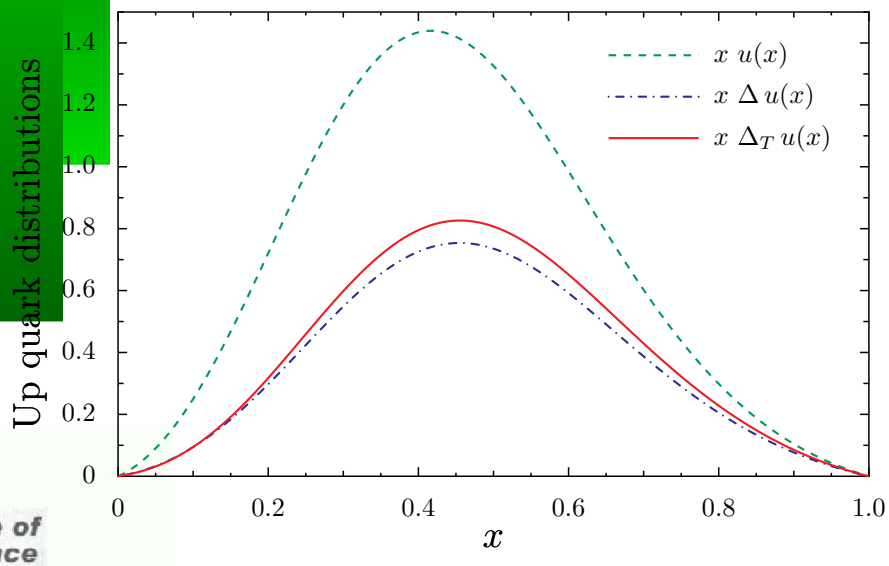
- Satisfy: Soffer bound, baryon & momentum SRs.

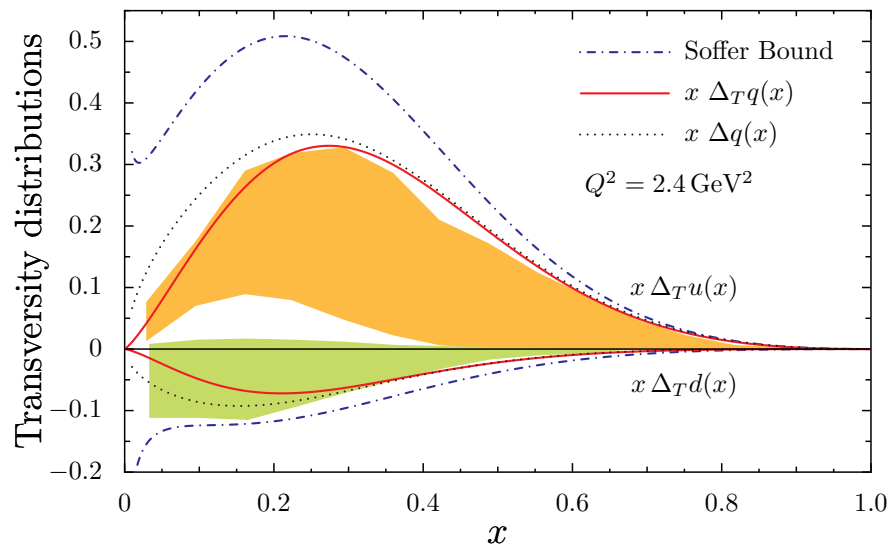
Cloët, Bentz, Thomas

arXiv:0708.3246 [hep-ph]

Model predictions

- Simplified Faddeev equation





- Satisfy: Soffer bound, baryon & momentum SRs.
- Moments at $Q^2 = 0.16\,\mathrm{GeV}^2$:

$$\Delta u = 0.97, \quad \Delta d = -0.30 \quad \Longrightarrow \quad g_A = 1.267$$

$$\Delta_T u = 1.04, \; \Delta_T d = -0.24 \quad \Longrightarrow \quad g_T = 1.28$$

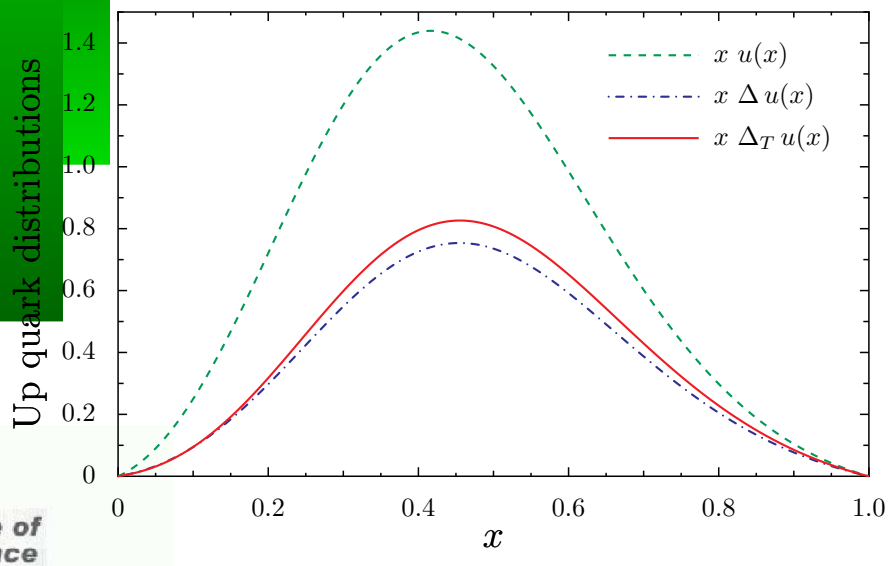
Model constraint

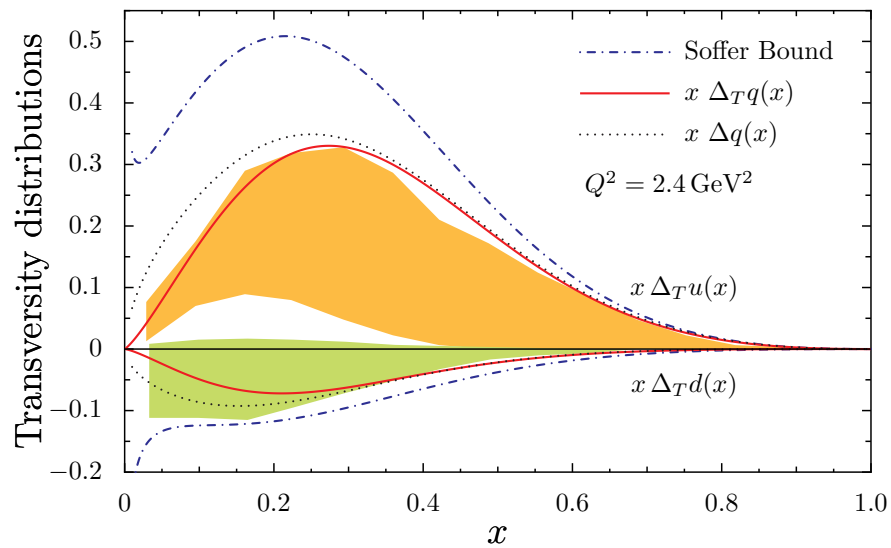
Cloët, Bentz, Thomas

arXiv:0708.3246 [hep-ph]

Model predictions

- Simplified Faddeev equation





- Satisfy: Soffer bound, baryon & momentum SRs.
- Moments at $Q^2 = 0.16\,\mathrm{GeV}^2$:

$$\Delta u = 0.97, \quad \Delta d = -0.30 \quad \Longrightarrow \quad g_A = 1.267$$

$$\Delta_T u = 1.04, \; \Delta_T d = -0.24 \quad \Longrightarrow \quad g_T = 1.28$$

- $\Delta q(x) \sim \Delta_T q(x)$ in valence region for $Q^2 \lesssim 10\,\mathrm{GeV}^2$