

# *Baryons through the DSQCD looking glass*

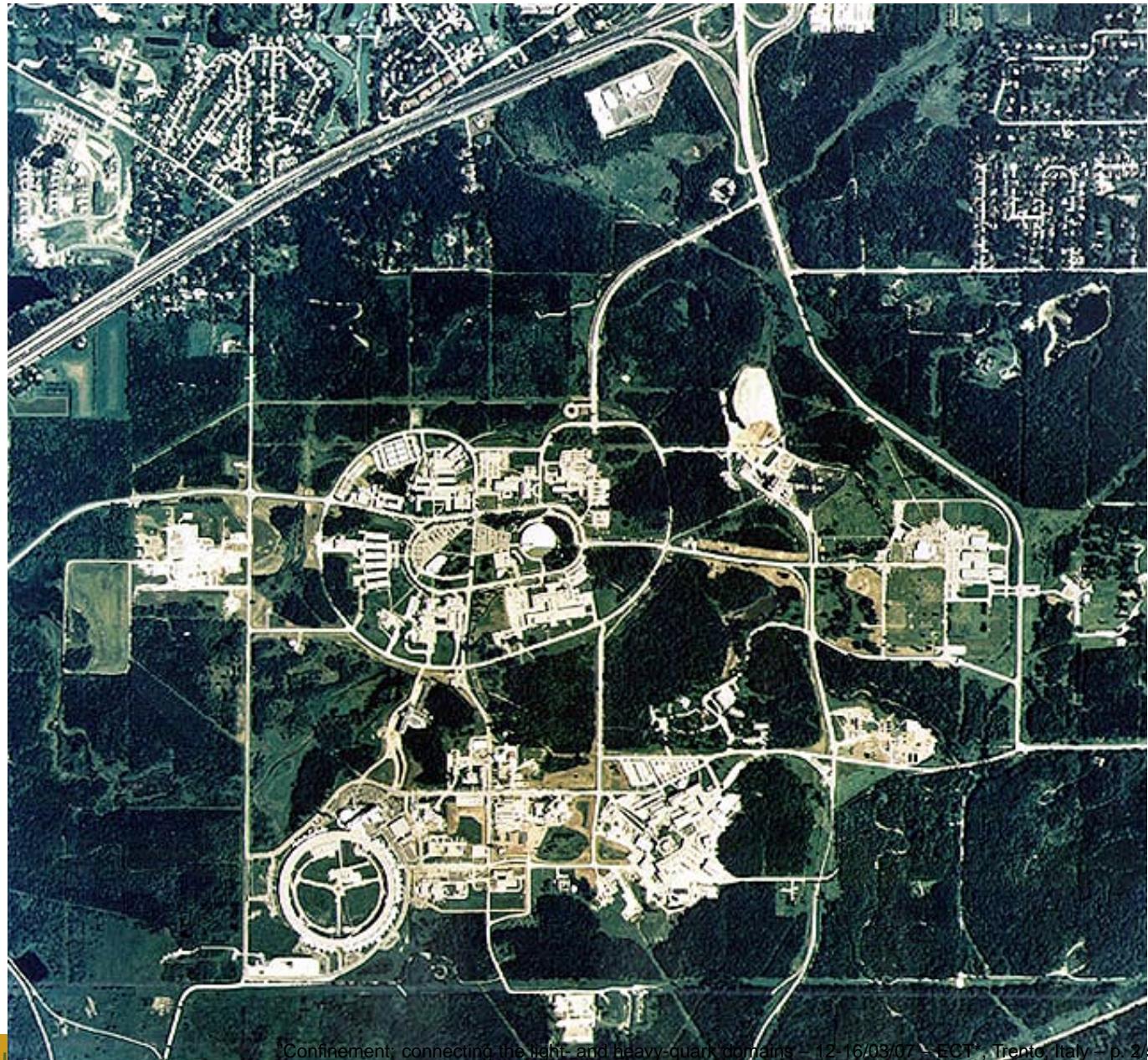
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Physics Division

Argonne National Laboratory

# Argonne National Laboratory



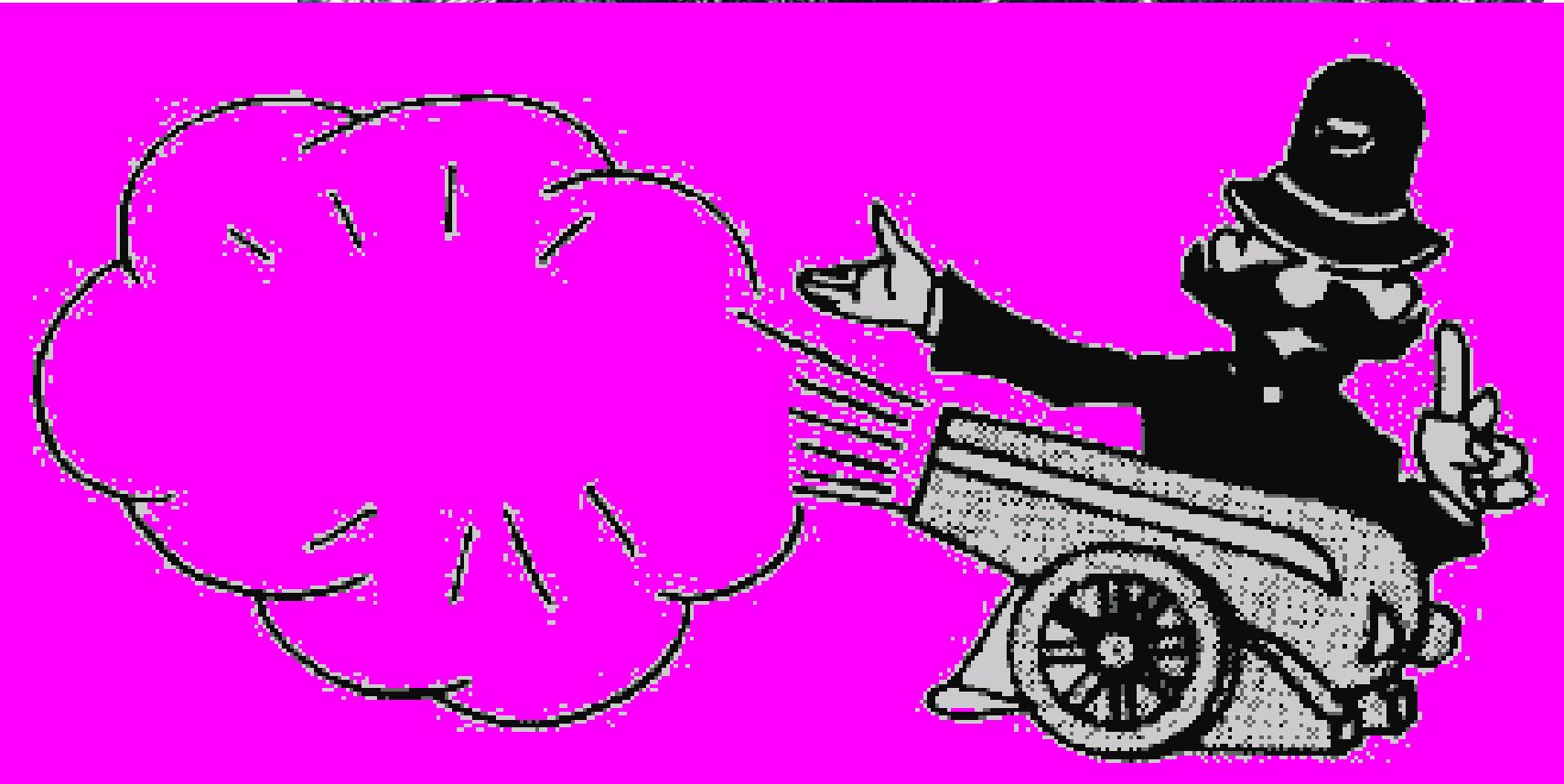
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# Argonne National Laboratory



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Conclu

# *And so ...*



# *And so . . . We gather in Trento*



# *And so . . . We gather in Trento*



*One thing was certain, that the WHITE kitten had had nothing to do with it:  
– it was the black kitten's fault entirely.*

# *Nucleon . . . 2 Key Hadrons*

## *= Proton and Neutron*

---



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- Stern (1933) –  $\mu_p = (1 + 1.79) \frac{e\hbar}{2M}$ 
  - Big Hint that Proton is not a point particle
  - Proton has constituents
  - These are Quarks and Gluons

Quark discovery via  $e^- p$ -scattering at SLAC in 1968  
– the elementary quanta of Quantum Chromo-dynamics



# ***QED cf. QCD***



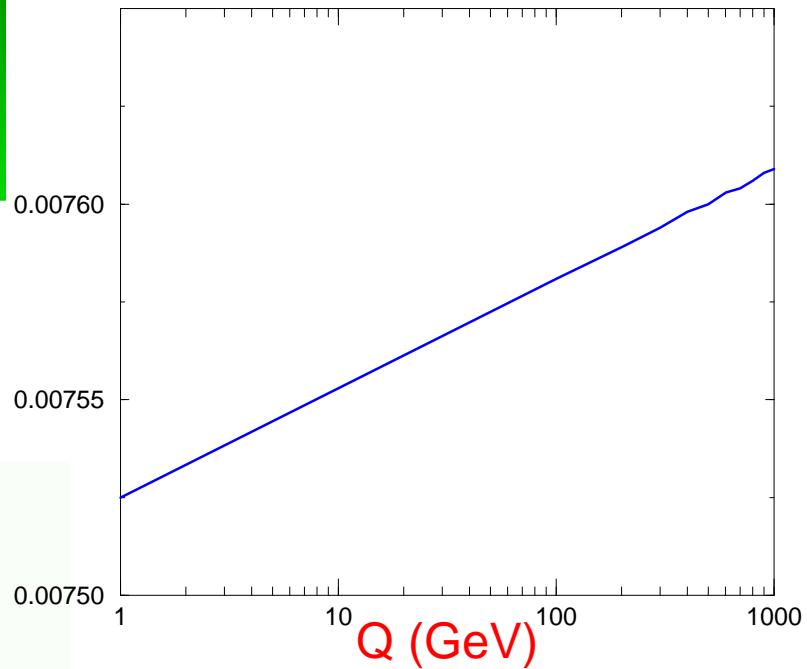
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# ***QED cf. QCD***



$\alpha_{\text{QED}}(Q^2)$

$$Q \text{ (GeV)}$$

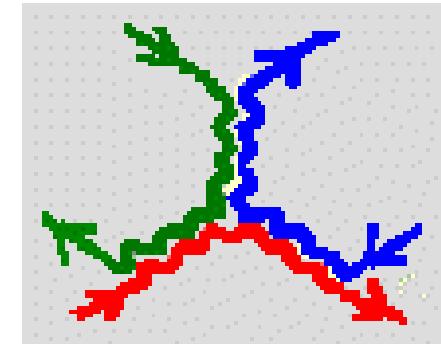
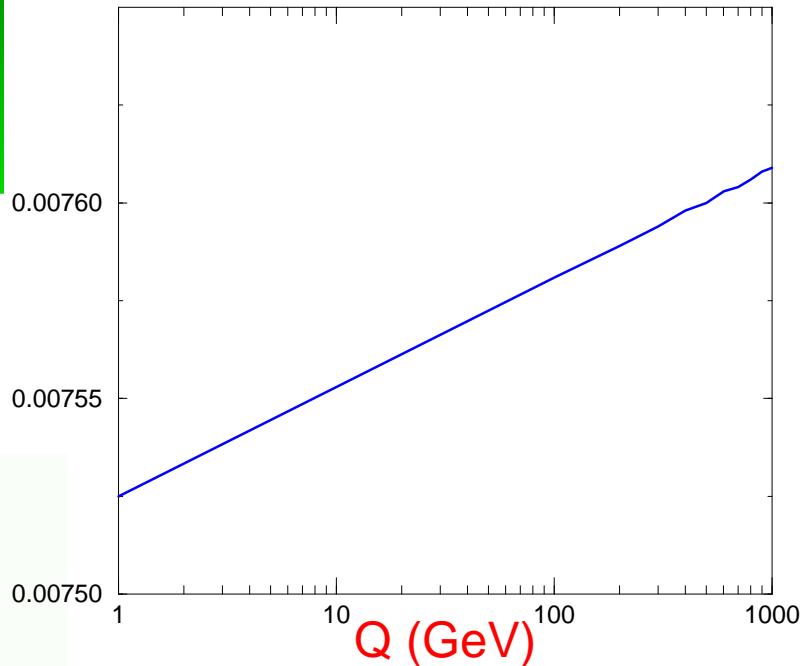


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$$\alpha_{\text{QED}} = \frac{\alpha}{1 - \alpha/3\pi \ln(Q^2/m_e^2)}$$

# ***QED cf. QCD***

Add three-gluon interaction

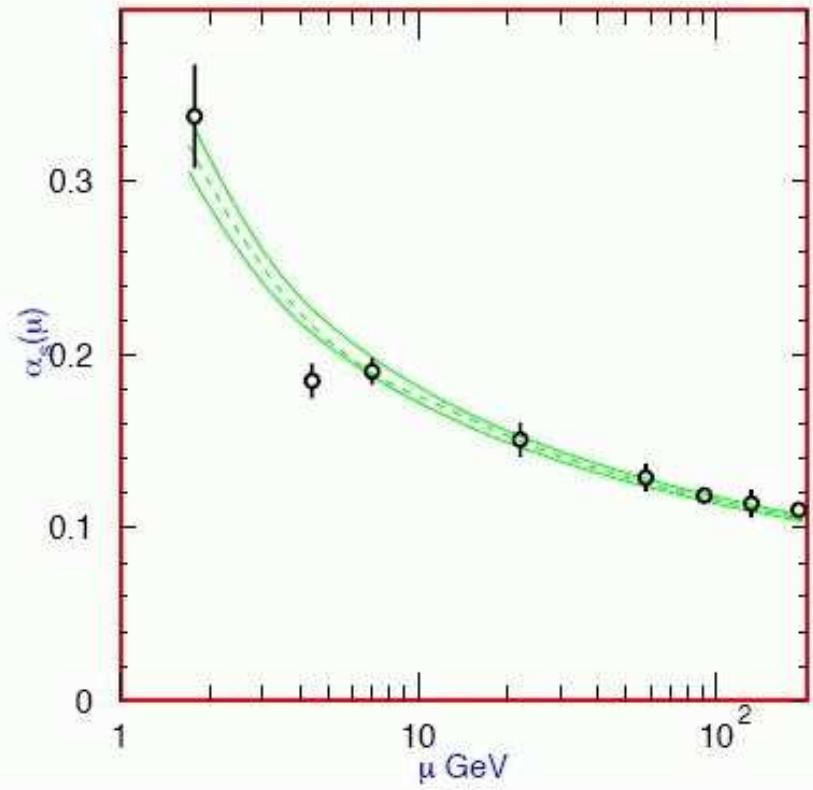
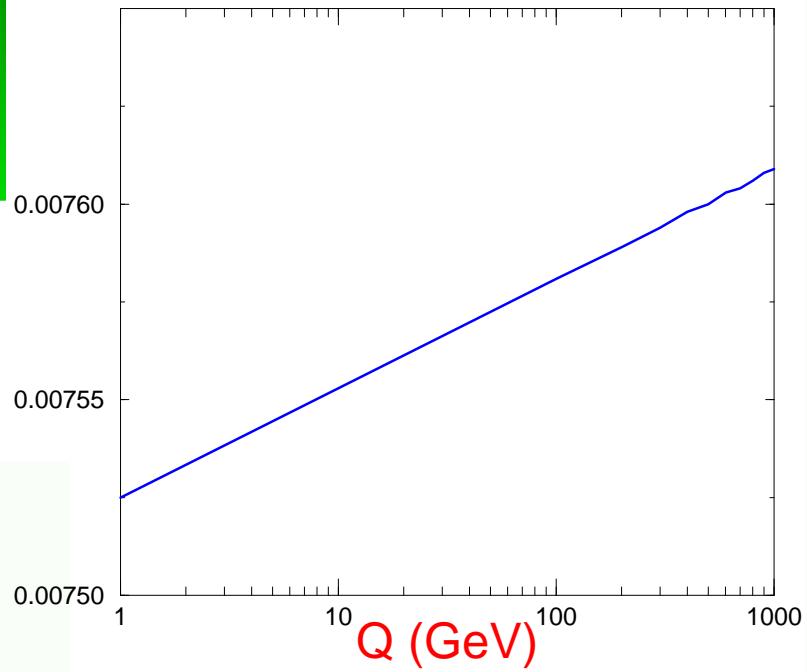


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**Figure 9.2:** Summary of the values of  $\alpha_s(\mu)$  at the values of  $\mu$  where they are measured. The lines show the central values and the  $\pm 1\sigma$  limits of our average. The figure clearly shows the decrease in  $\alpha_s(\mu)$  with increasing  $\mu$ . The data are, in increasing order of  $\mu$ ,  $\tau$  width,  $\Upsilon$  decays, deep inelastic scattering,  $e^+e^-$  event shapes at 22 GeV from the JADE data, shapes at TRISTAN at 58 GeV,  $Z$  width, and  $e^+e^-$  event shapes at 135 and 189 GeV.

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$$\alpha_{\text{QCD}} = \frac{12\pi}{(33 - 2N_f) \ln(Q^2/\Lambda^2)}$$



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## 2004 Nobel Prize in Physics: Gross, Politzer and Wilczek

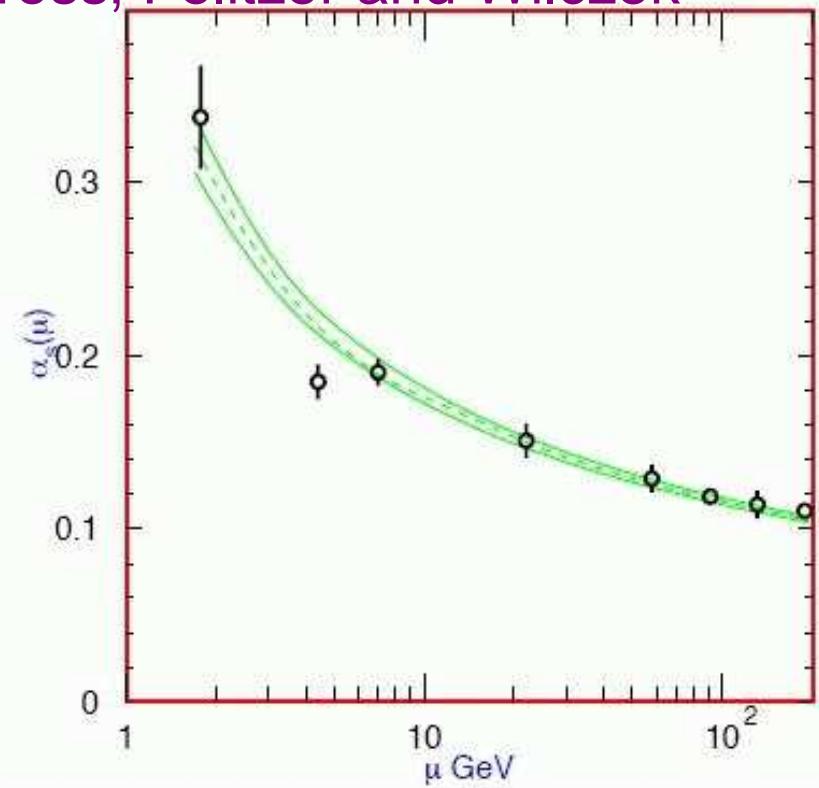
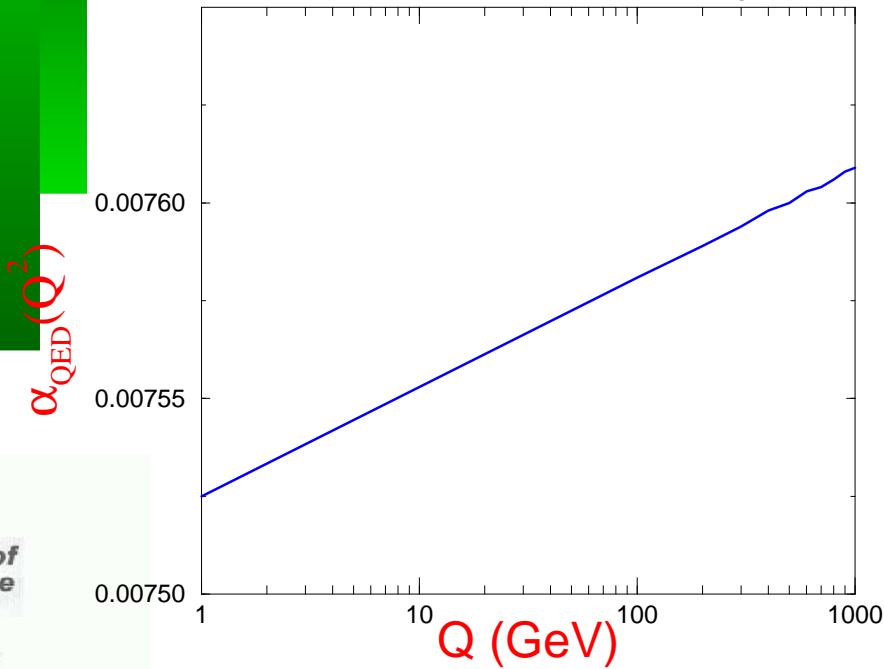


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# *Simple Picture*



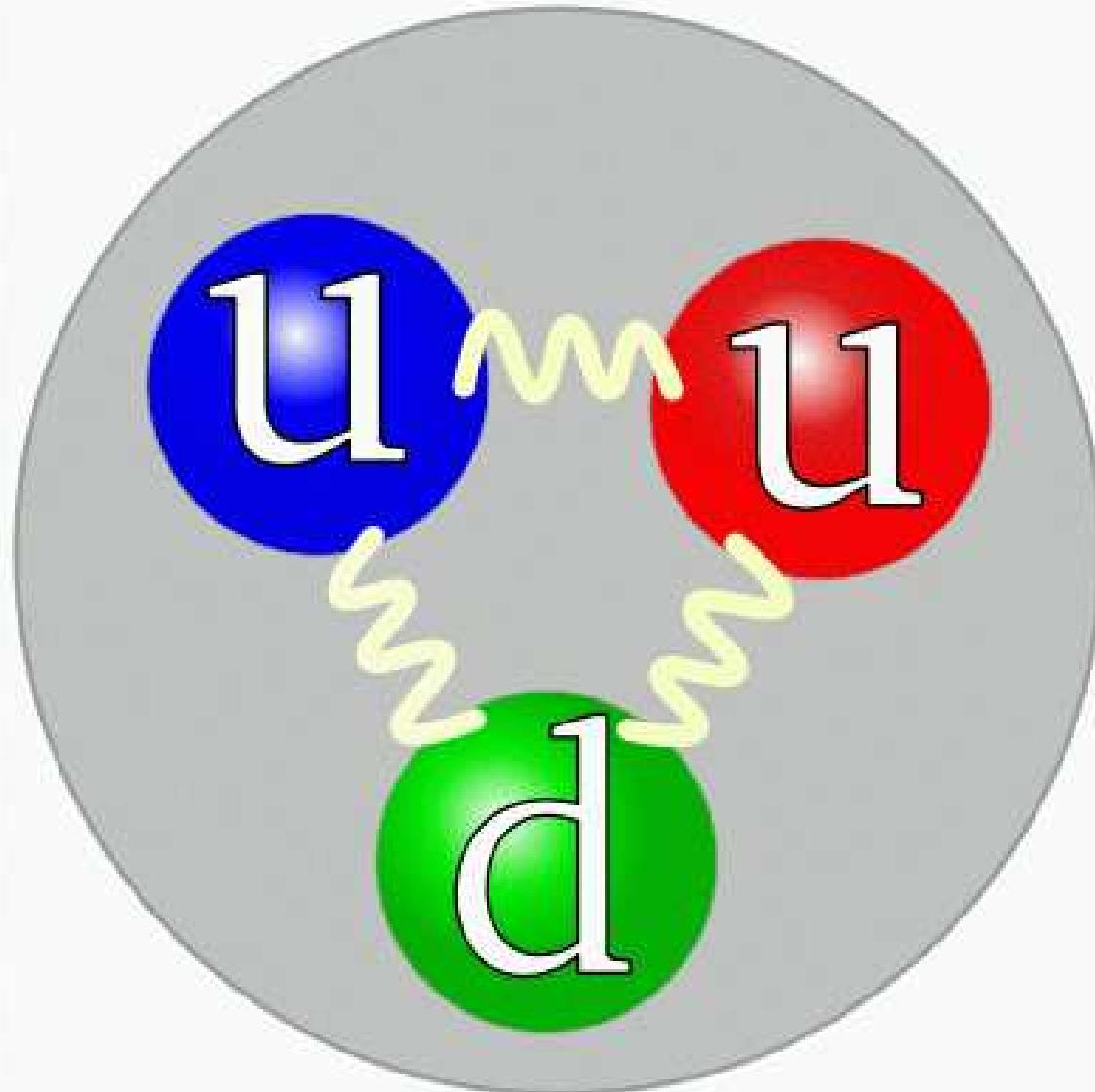
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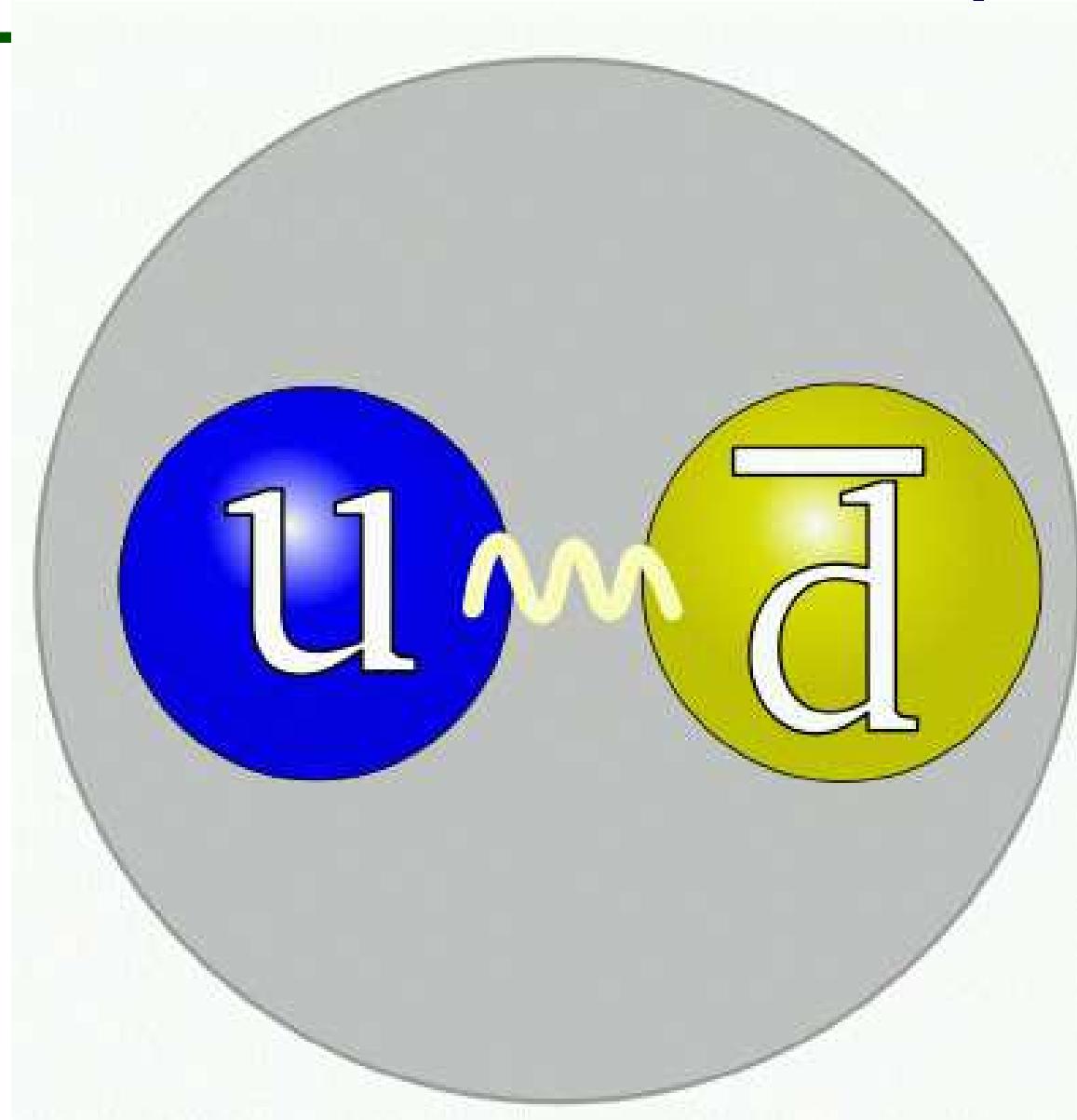
# *Simple Picture*



**PROTON**

"Confinement: connecting the light- and heavy-quark domains – 12-16/03/07 – ECT\*, Trento, Italy – p. 6/54

# *Simple Picture*



PION

"Confinement: connecting the light- and heavy-quark domains – 12-16/03/07 – ECT\*, Trento, Italy – p. 6/54

# *Study Structure via Nucleon Form Factors*

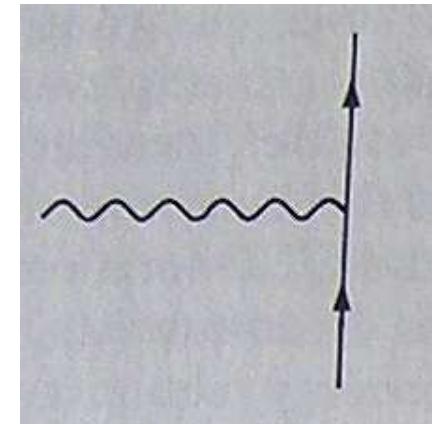
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# Study Structure via Nucleon Form Factors

- Electron's relativistic electromagnetic current:

$$\begin{aligned} j_\mu(P', P) &= ie \bar{u}_e(P') \Lambda_\mu(Q, P) u_e(P), \quad Q = P' - P \\ &= ie \bar{u}_e(P') \gamma_\mu(-1) u_e(P) \end{aligned}$$

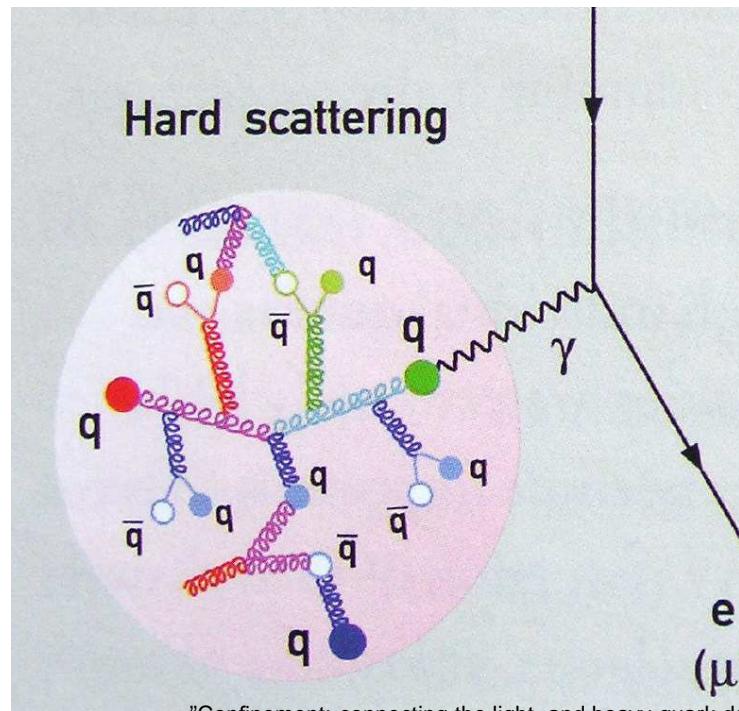


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$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$



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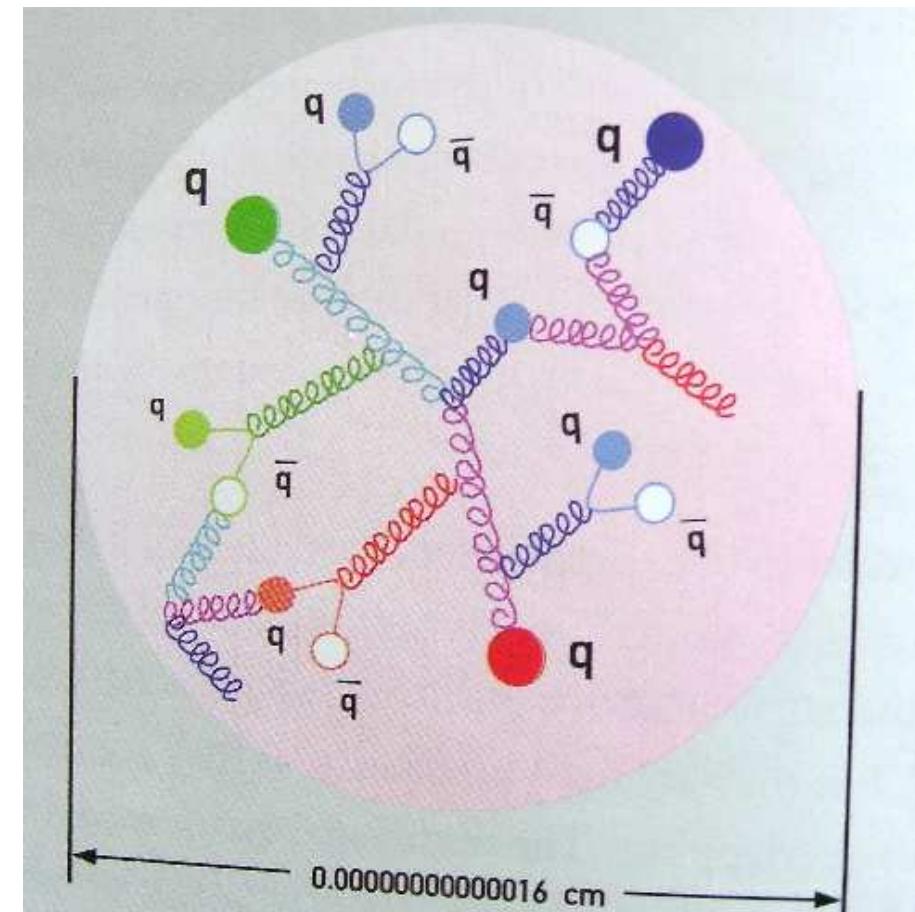
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Point-particle:  $F_2 \equiv 0 \Rightarrow G_E \equiv G_M$



# NSAC Long Range Plan

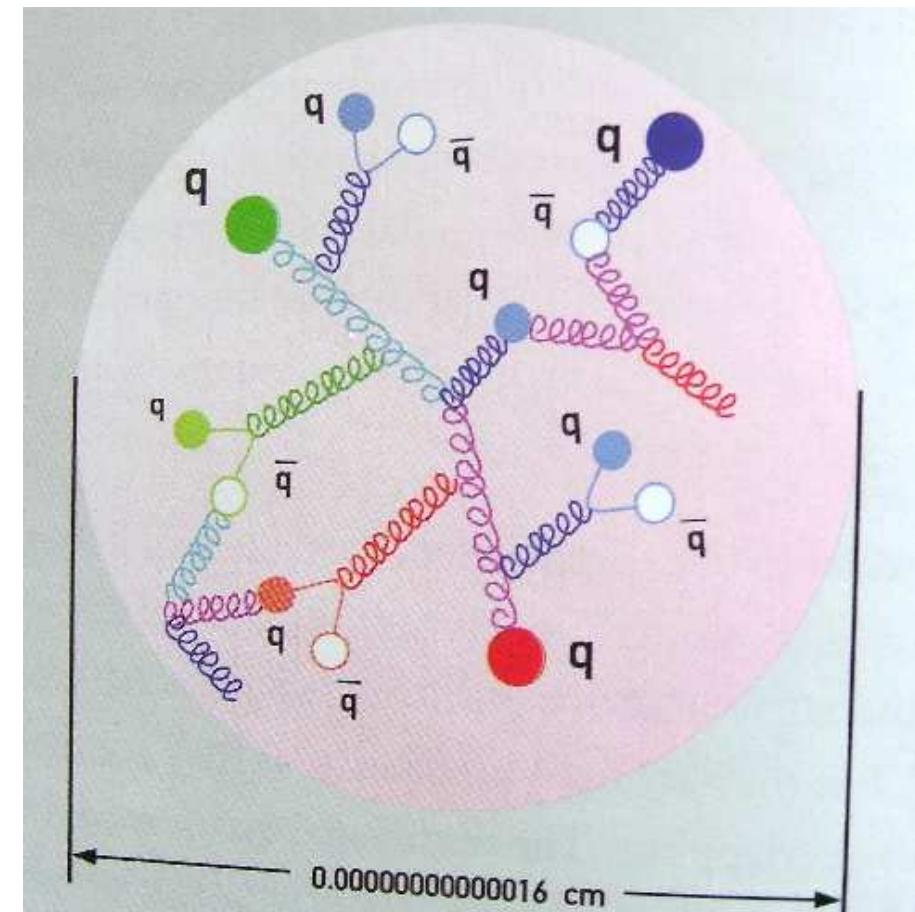
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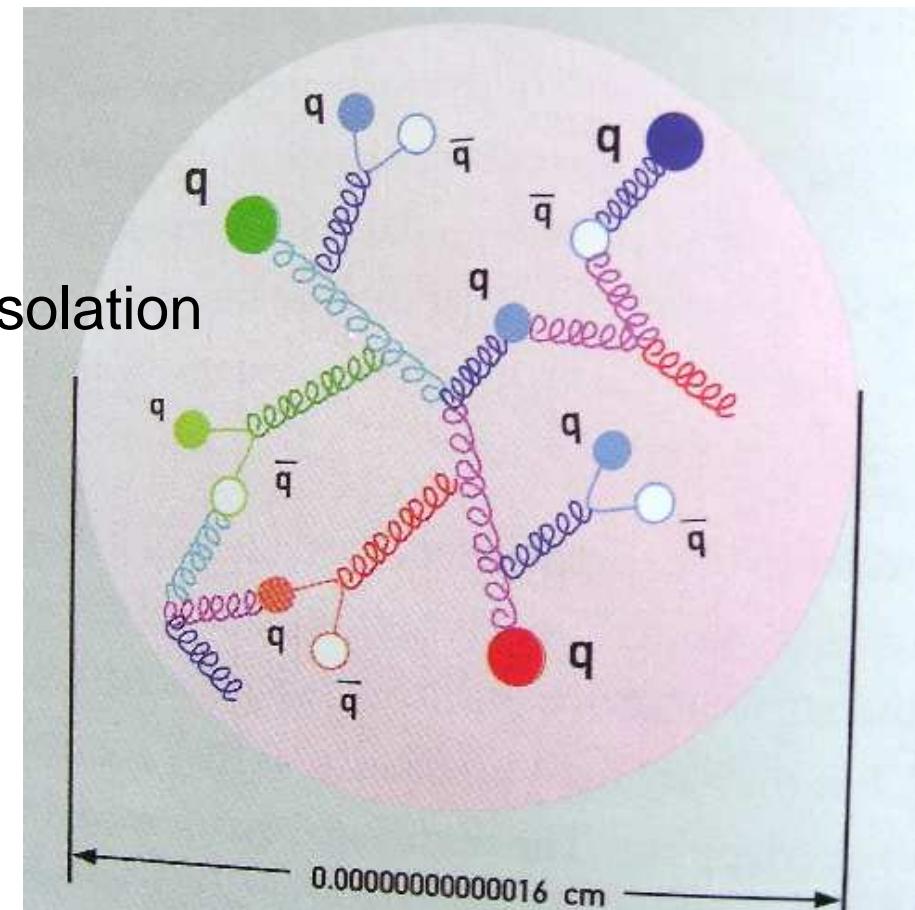


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- Confinement
  - No quark ever seen in isolation

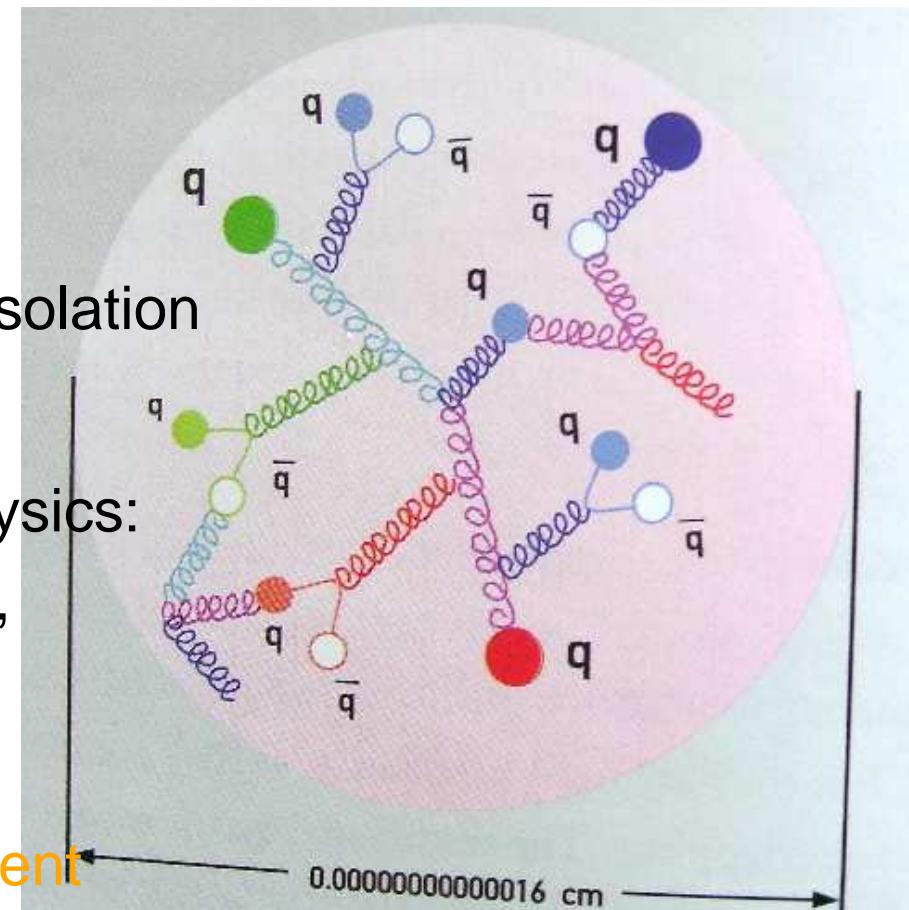


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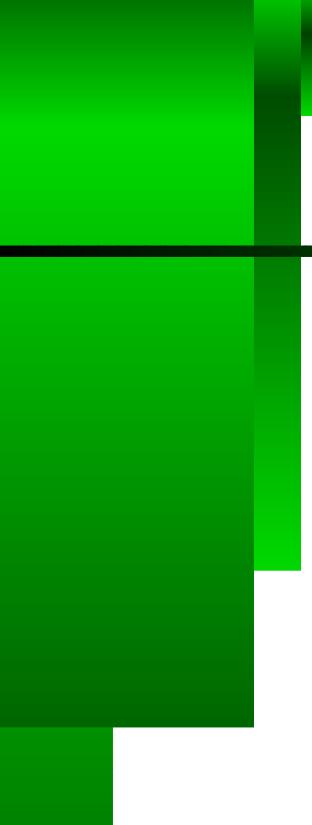
- Confinement
  - No quark ever seen in isolation
- Weightlessness
  - 2004 Nobel Prize in Physics:  
Mass of  $u-$  &  $d-$ quarks,  
each just 5 MeV;  
Proton Mass is 940 MeV  
 $\Rightarrow$  No Explanation Apparent



for 98.4% of Mass

"Confinement: connecting the light- and heavy-quark domains – 12-16/03/07 – ECT\*, Trento, Italy – p. 8/54"

# *Modern Miracles in Hadron Physics*



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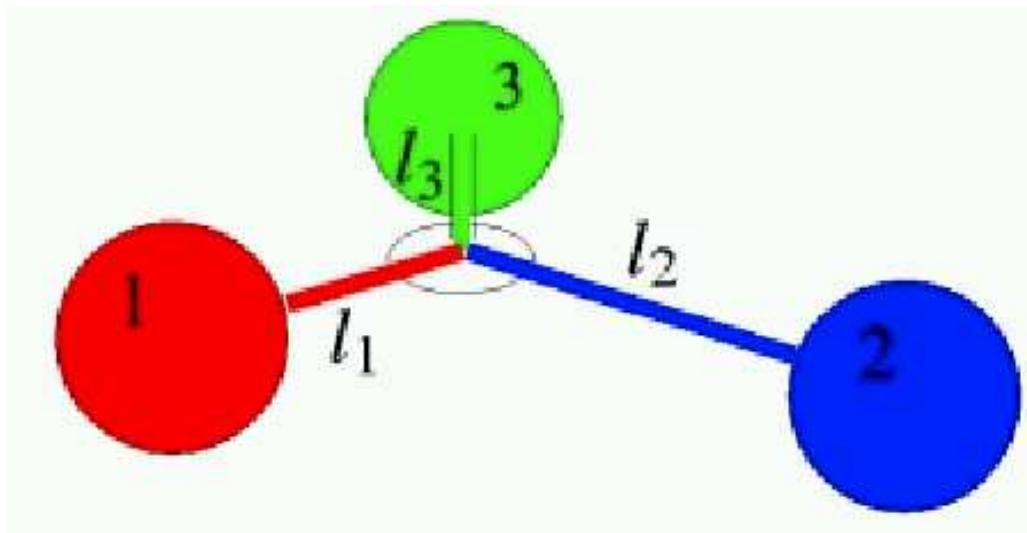
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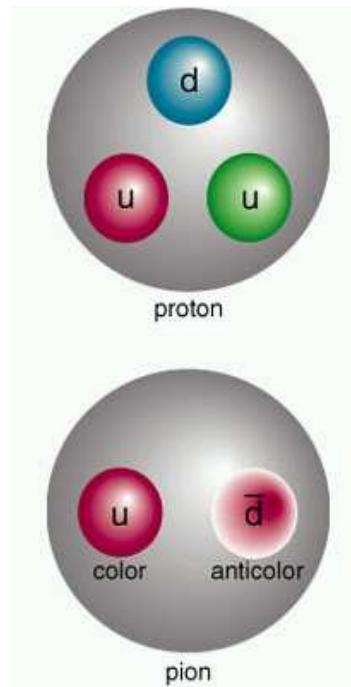
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- Another meson:  
.....  $M_{\rho} = 770 \text{ MeV}$  ..... No Surprises Here



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- What is “wrong” with the pion?



Mass Destruction? Is this



# *Dichotomy of the Pion*



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# Dichotomy of the Pion

- How does one make an **almost massless** particle  
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- How does one make an **almost massless** particle from two **massive** constituent-quarks?
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Using DSEs,  
we've provided this.



# *QCD's Emergent Phenomena*

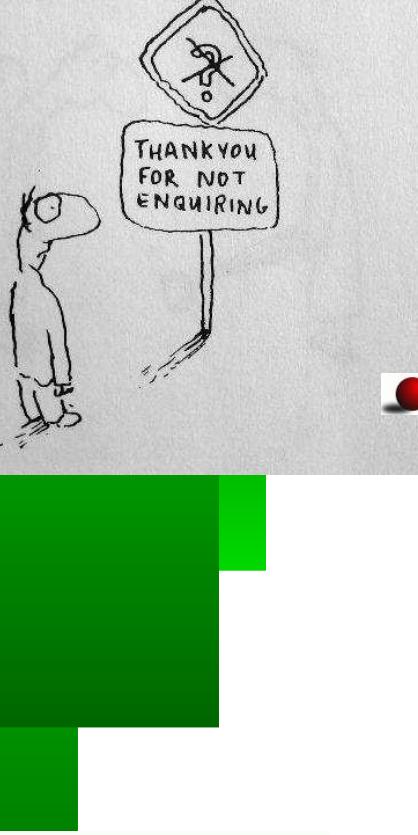


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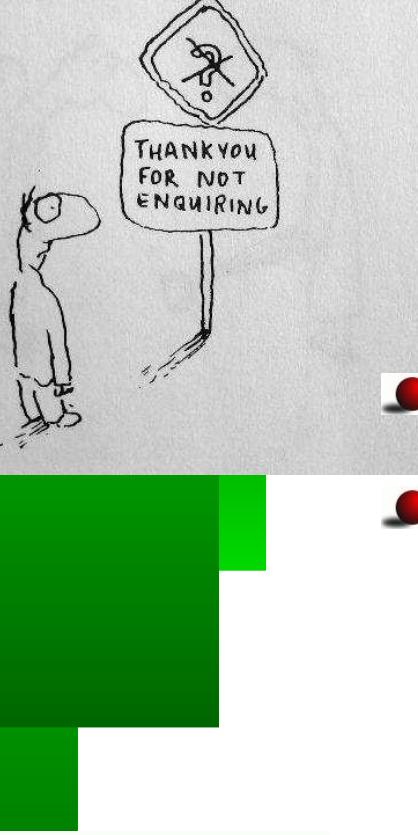


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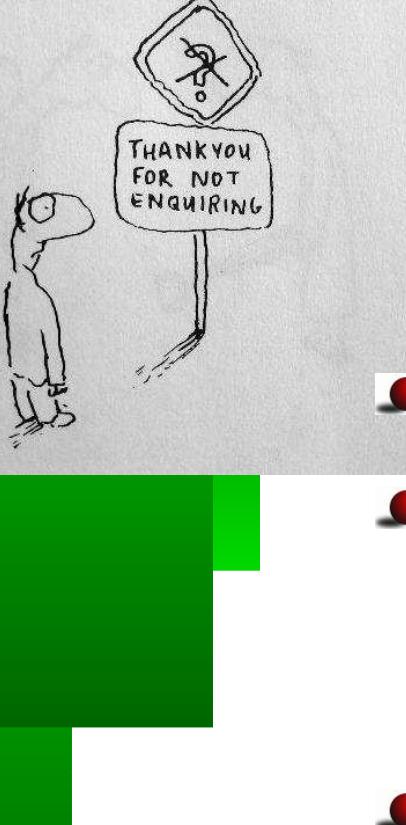


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- NSAC – Understanding these phenomena is one of the greatest intellectual challenges in physics



# *What's the Problem?*



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Quintessence of Relativistic Quantum Field Theory



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- Interaction between quarks – the *Interquark Potential* –
  - Unknown
  - throughout  $> 98\%$  of the pion's/proton's volume



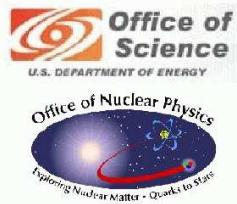
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- Determination of proton's wave function requires *ab initio* nonperturbative solution of fully-fledged relativistic quantum field theory
- Modern Physics & Mathematics
  - Still quite some way from being able to do that



# Intranucleon Interaction



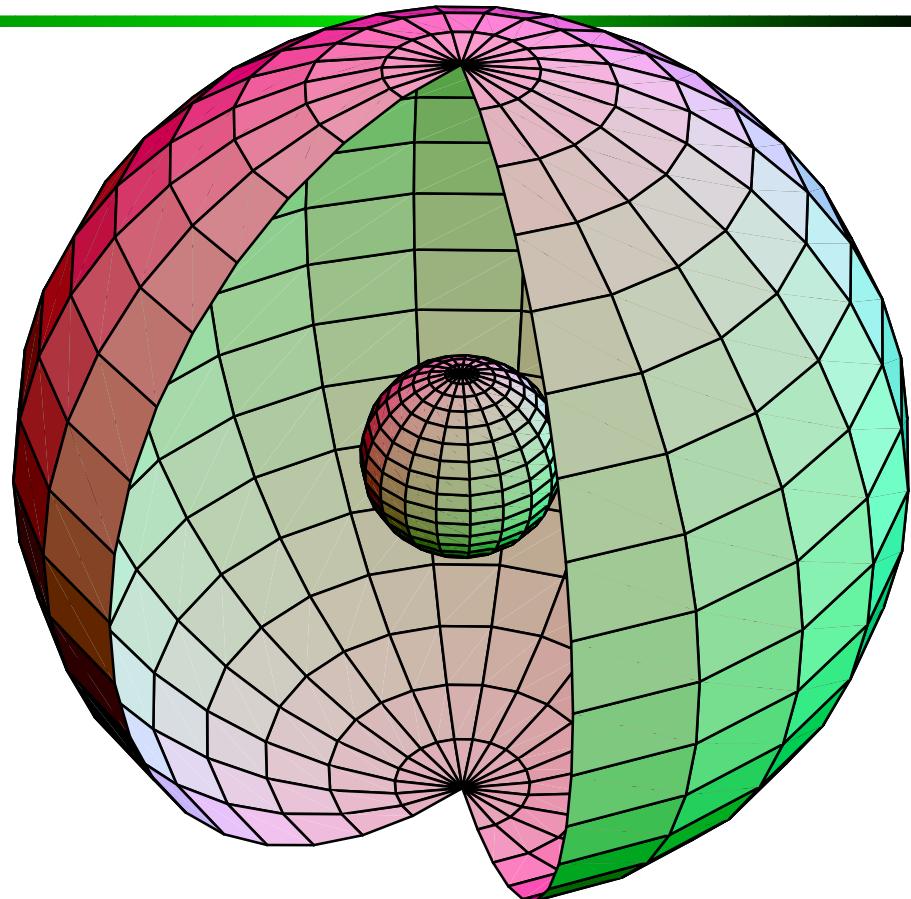
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# Intranucleon Interaction



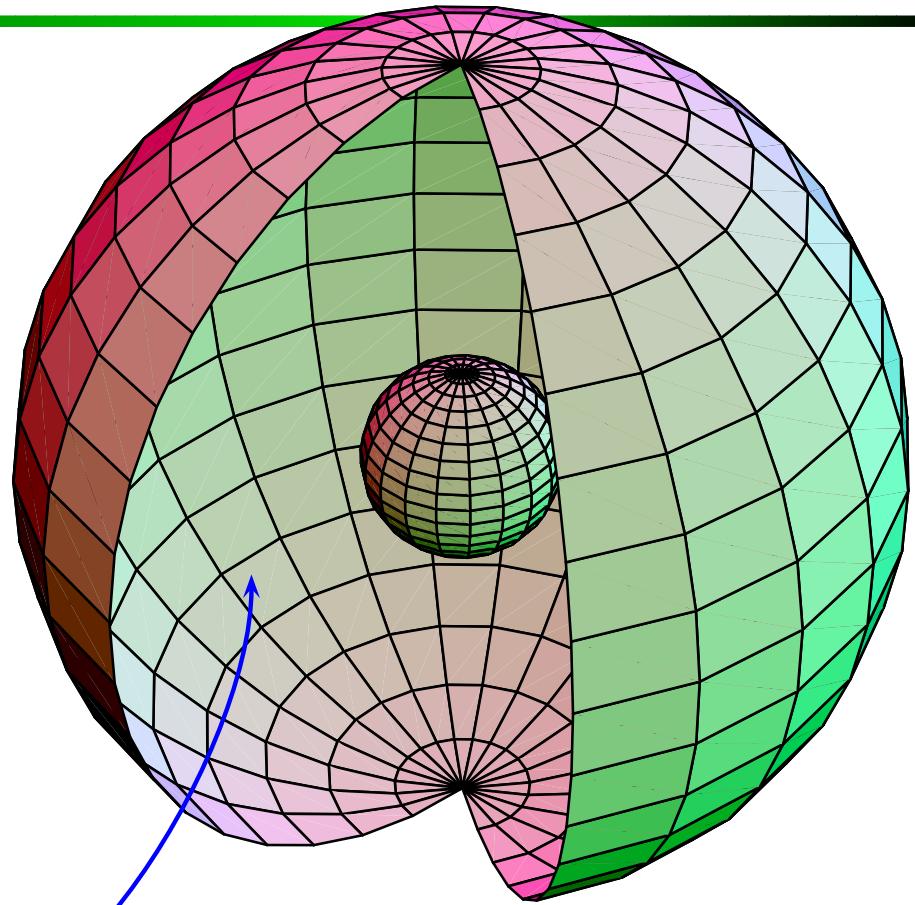
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# Intranucleon Interaction



98% of the volume



# Why should You care?



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Absent DCSB:  $m_\pi = m_\rho \Rightarrow$  repulsive and attractive forces in nucleon-nucleon interaction both have **SAME** range and there is **No** intermediate range attraction!



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- Is  $^{12}\text{C}$  stable?



# Why should You care?

Absent DCSB:  $m_\pi = m_\rho \Rightarrow$  repulsive and attractive forces in nucleon-nucleon interaction both have **SAME** range and there is **No** intermediate range attraction!  
Under these circumstances,

- What is the range:  $\frac{1}{2m_q} \sim 20 \text{ fm}$  or  $\frac{1}{2M_Q} \sim \frac{1}{3} \text{ fm}$ ?
- Is  $^{12}\text{C}$  stable?
  - Probably not, if range  $\text{range} \sim \frac{1}{2M_Q}$



# Why should You care?

Absent DCSB:  $m_\pi = m_\rho \Rightarrow$  repulsive and attractive forces in nucleon-nucleon interaction both have **SAME** range and there is **No** intermediate range attraction!  
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  - How does  $m_u - m_d$  relate to  $M_U - M_D$ ?



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  - Can one guarantee  $M_n > M_p$ ?
- How do such changes affect Big Bang Nucleosynthesis?



# Why should You care?

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  - How does  $m_u - m_d$  relate to  $M_U - M_D$ ?
  - Can one guarantee  $M_n > M_p$ ?

Is a unique long-range interaction between light-quarks responsible for all this or are there an uncountable infinity of qualitatively equivalent interactions?



# *Model QCD*



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# Traditional approach to strong force problem

***Model QCD***



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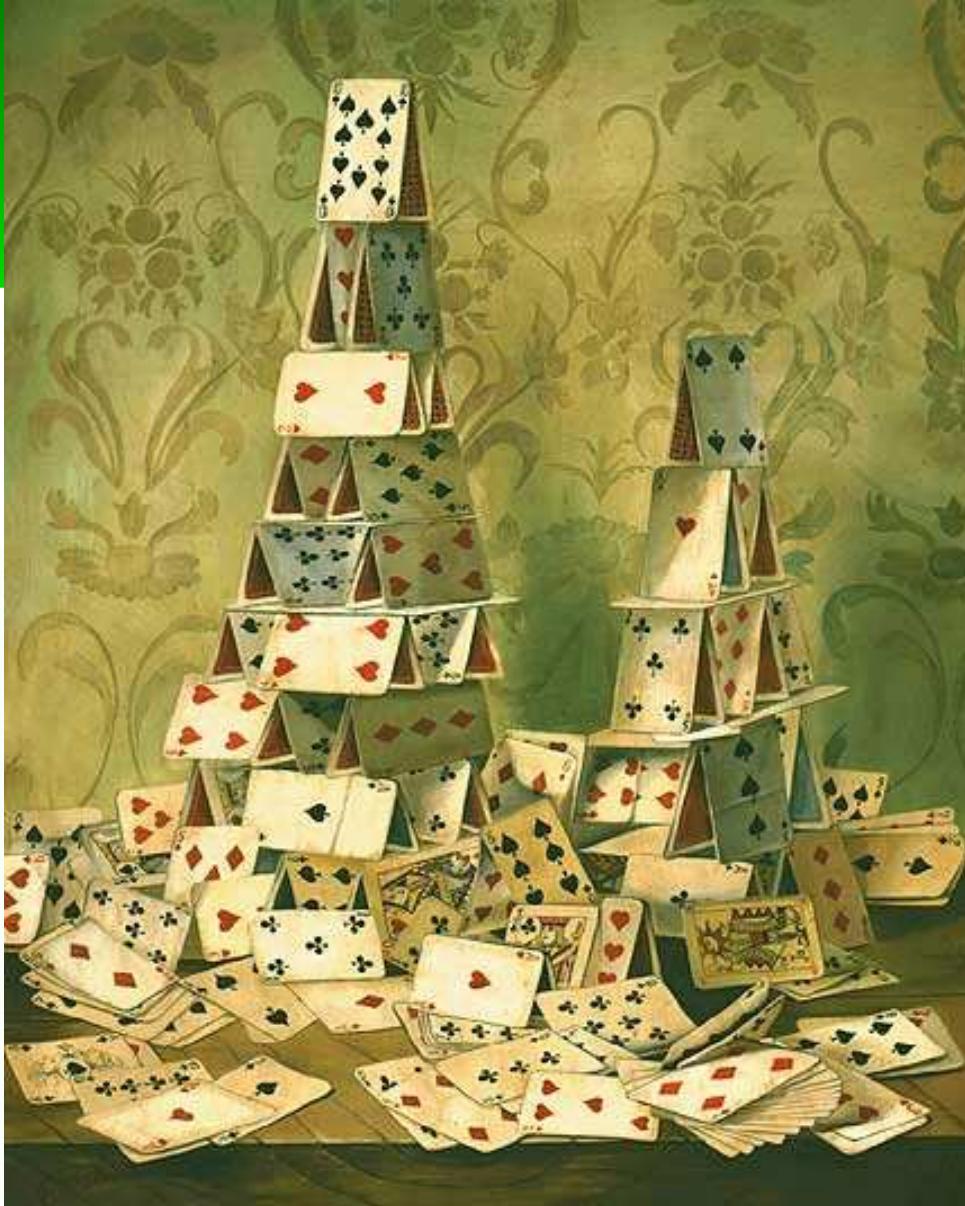
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# Traditional approach to strong force problem

**Model QCD**

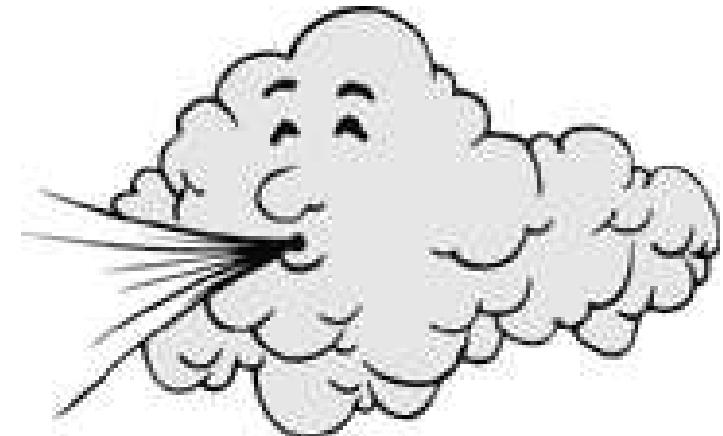
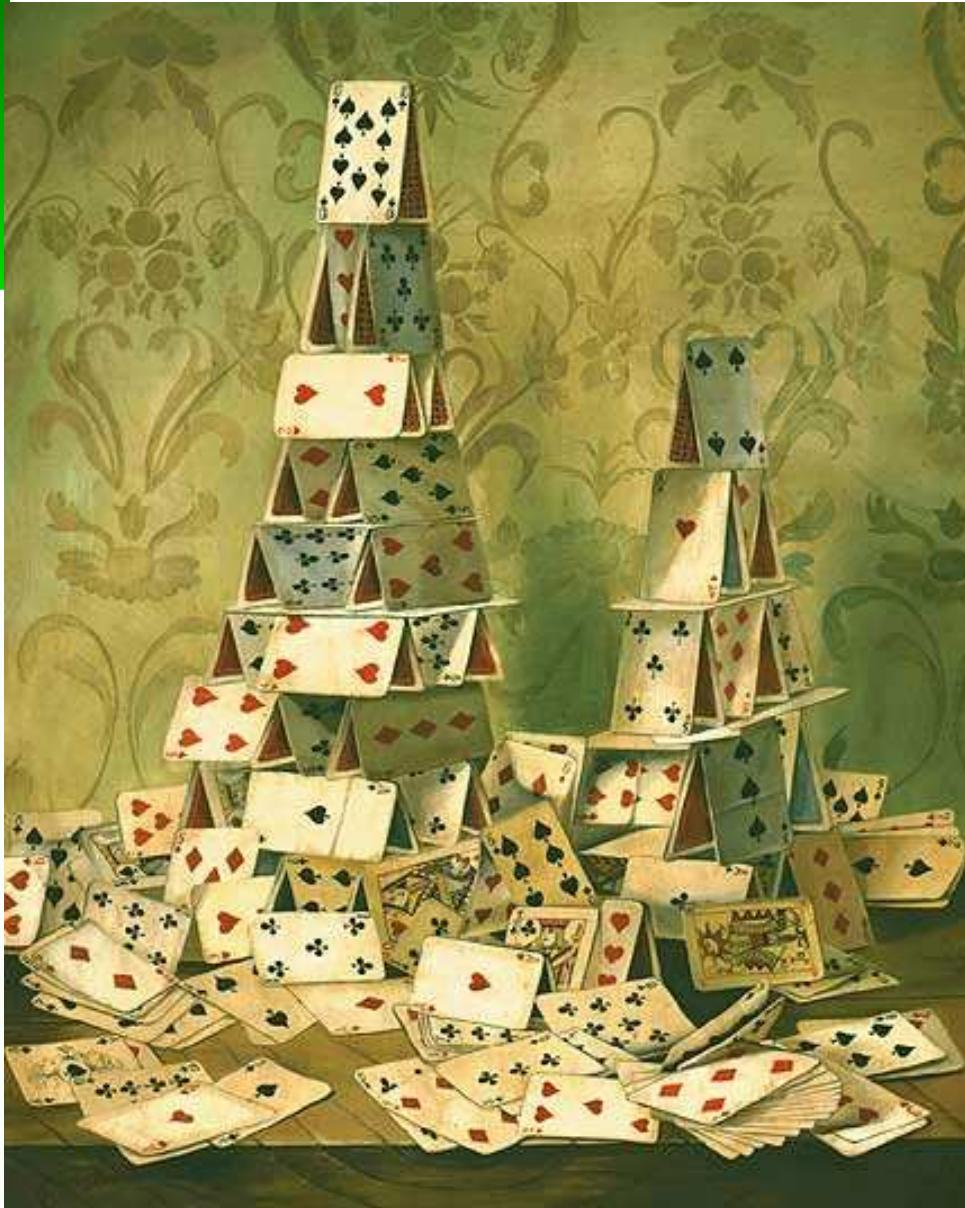


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# Traditional approach to strong force problem

**Model QCD**



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Office of Nuclear Physics  
Exploring Nuclear Matter - Quarks to Matter



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# Lattice QCD



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# One modern nonperturbative approach *Lattice QCD*



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# One modern nonperturbative approach *Lattice QCD*



# Dyson-Schwinger Equations



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# *Dyson-Schwinger Equations*

- Well suited to Relativistic Quantum Field Theory



# Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory  
..... Materially Reduces Model Dependence



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      - Generation of fermion mass from *nothing*
    - Quark & Gluon Confinement
      - Coloured objects not detected, not detectable?



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  - ⇒ Understanding InfraRed (long-range)
    - ..... behaviour of  $\alpha_s(Q^2)$



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  - Method yields Schwinger Functions  $\equiv$  Propagators



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Cross-Sections built from Schwinger Functions



# *Persistent Challenge*

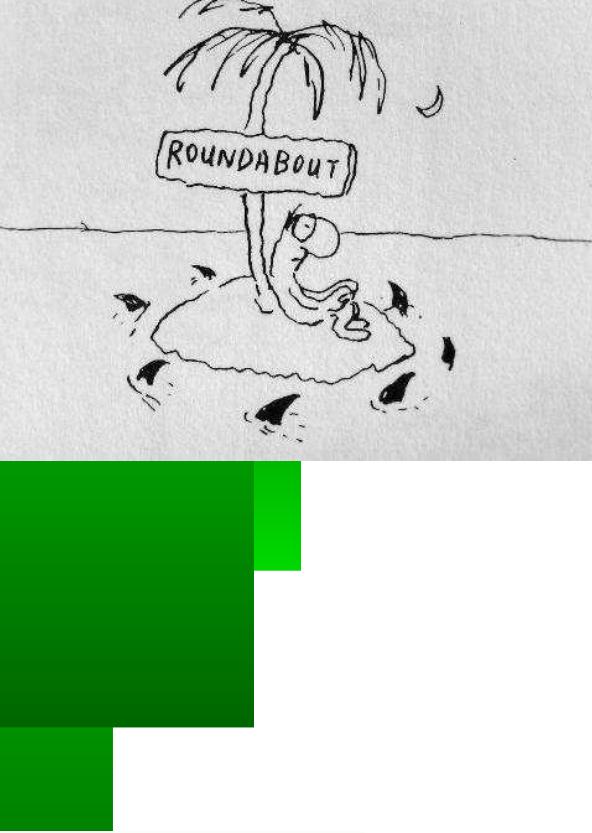


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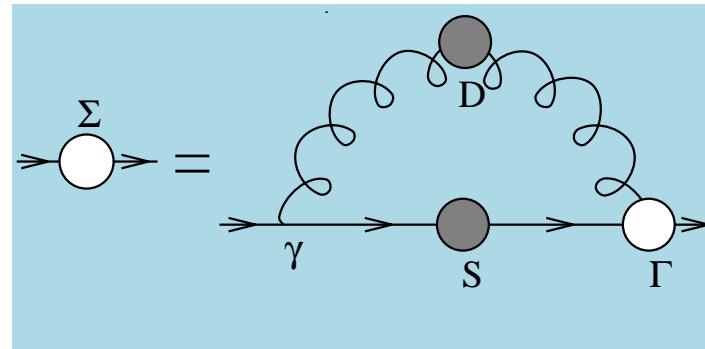
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# Persistent Challenge

---

- Infinitely Many Coupled Equations





# Persistent Challenge

---

- Infinitely Many Coupled Equations
  - Solutions are **Schwinger Functions**  
(Euclidean **Green Functions**)





# Persistent Challenge

---

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions  
(Euclidean **Green** Functions)
  - Not all are Schwinger functions are experimentally observable but **all are** same VEVs measured in Lattice-QCD simulations . . . opportunity for comparisons at pre-experimental level . . . cross-fertilisation





# Persistent Challenge

---

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions  
(Euclidean **Green** Functions)
- Coupling between equations **necessitates** truncation
  - Weak coupling expansion  $\Rightarrow$  Perturbation Theory





# Persistent Challenge

---

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions  
(Euclidean **Green** Functions)
- Coupling between equations **necessitates** truncation
  - Weak coupling expansion  $\Rightarrow$  Perturbation Theory  
**Not useful** for the nonperturbative problems  
in which we're interested





# Persistent Challenge

---

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions  
(Euclidean **Green** Functions)
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme  
H.J. Munczek Phys. Rev. D **52** (1995) 4736  
*Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations*  
A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7  
*Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*





# Persistent Challenge

---

- Infinitely Many Coupled Equations
  - Solutions are **Schwinger Functions**  
(Euclidean **Green Functions**)
- There is at least one **systematic nonperturbative, symmetry-preserving truncation scheme**
- Has Enabled Proof of **EXACT** Results in QCD





# Persistent Challenge

---

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions  
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- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme
- Has Enabled Proof of **EXACT** Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results





# Persistent Challenge

---

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions  
(Euclidean **Green** Functions)
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme
- Has Enabled Proof of **EXACT** Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Make Predictions with Readily Quantifiable Errors



# Perturbative Dressed-quark Propagator



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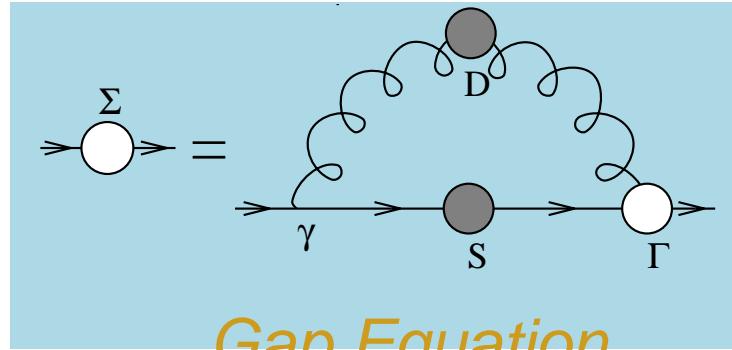
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# Perturbative Dressed-quark Propagator

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$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

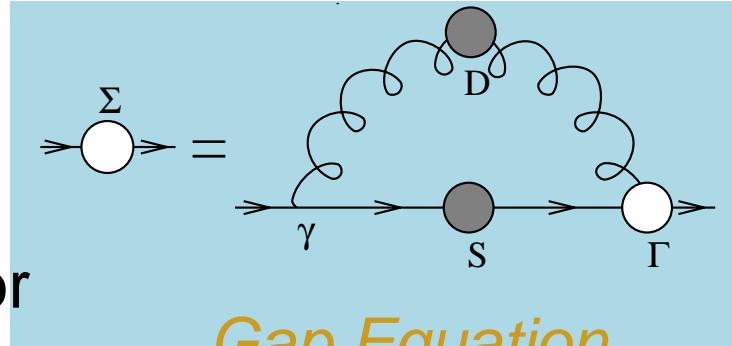


# Dressed-quark Propagator



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

- dressed-quark propagator



$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

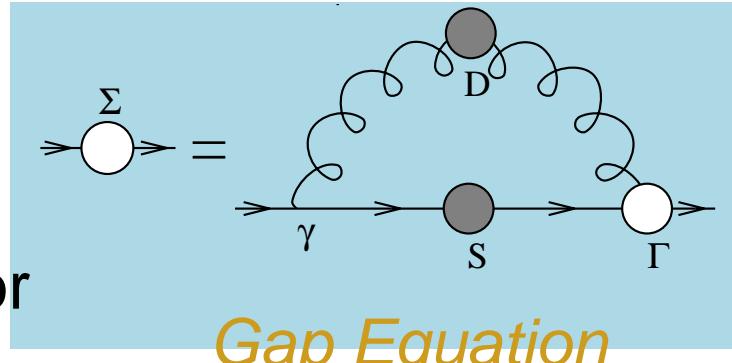




## Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

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$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

- Weak Coupling Expansion  
Reproduces Every Diagram in Perturbation Theory

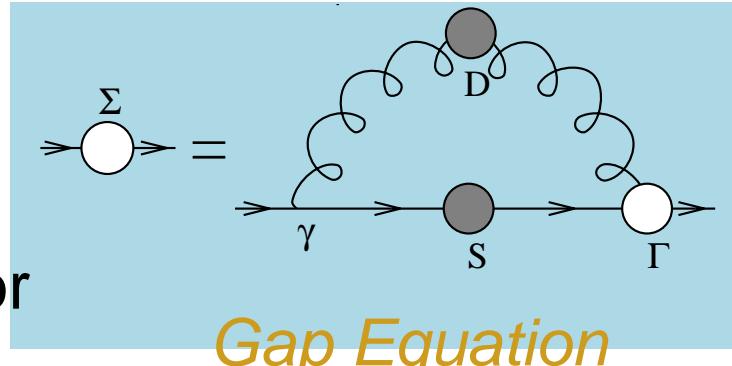




## Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

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$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

- Weak Coupling Expansion  
Reproduces Every Diagram in Perturbation Theory
- But in Perturbation Theory

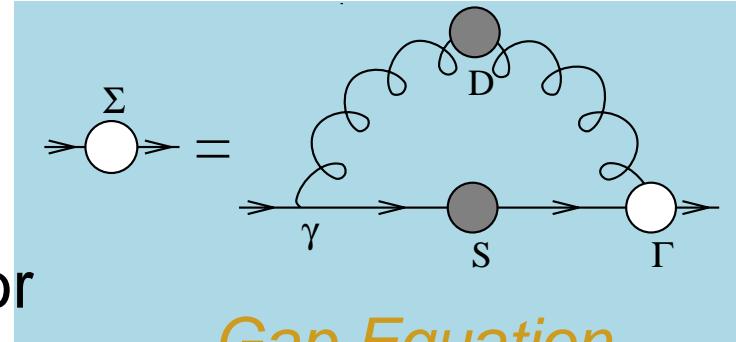
$$B(p^2) = m \left( 1 - \frac{\alpha}{\pi} \ln \left[ \frac{p^2}{m^2} \right] + \dots \right) \xrightarrow{m \rightarrow 0} 0$$



## Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

- dressed-quark propagator



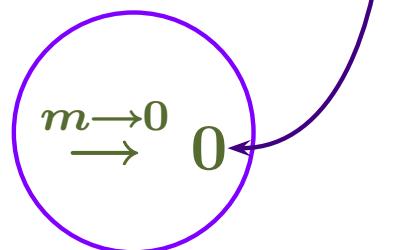
Gap Equation

$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

No DCSB  
Here!

- Weak Coupling Expansion  
Reproduces Every Diagram in Perturbation Theory
- But in Perturbation Theory

$$B(p^2) = m \left( 1 - \frac{\alpha}{\pi} \ln \left[ \frac{p^2}{m^2} \right] + \dots \right)$$



# Dressed-Quark Propagator



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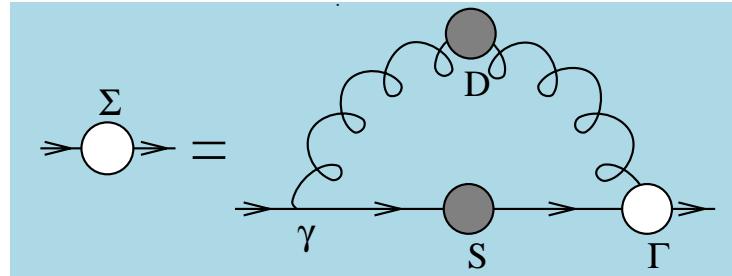
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# Dressed-Quark Propagator

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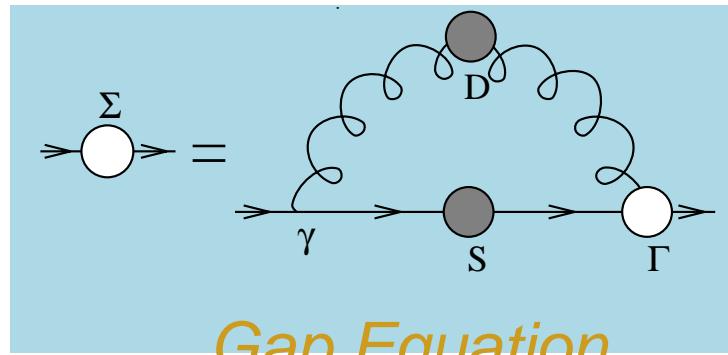


*Gap Equation*



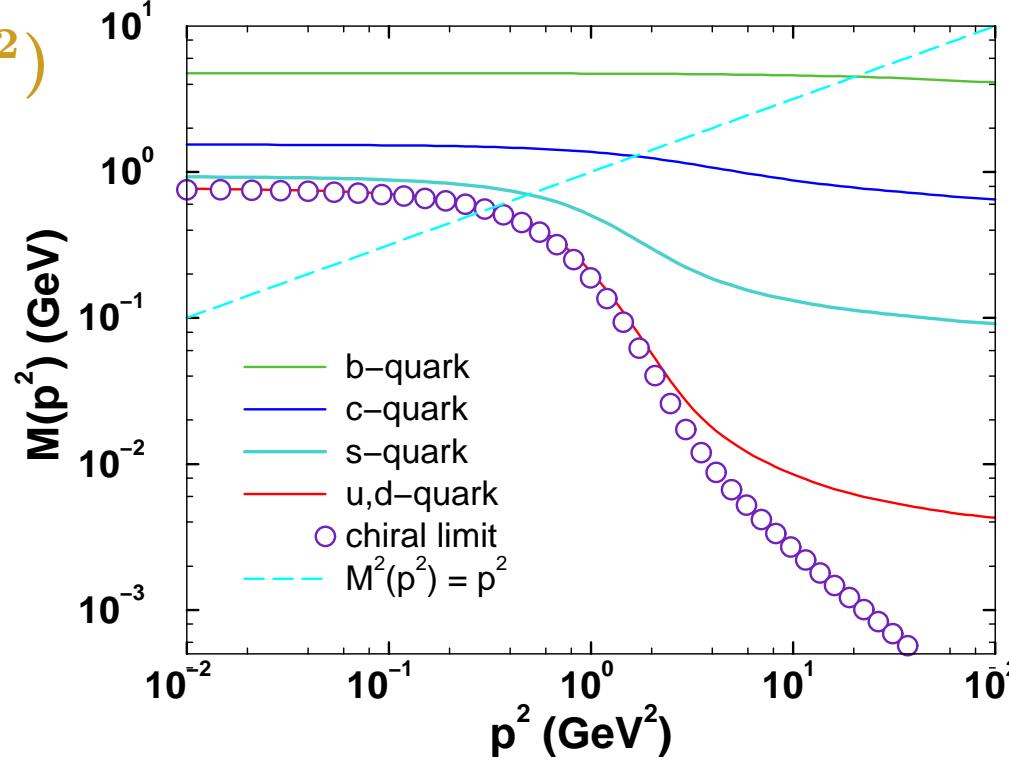
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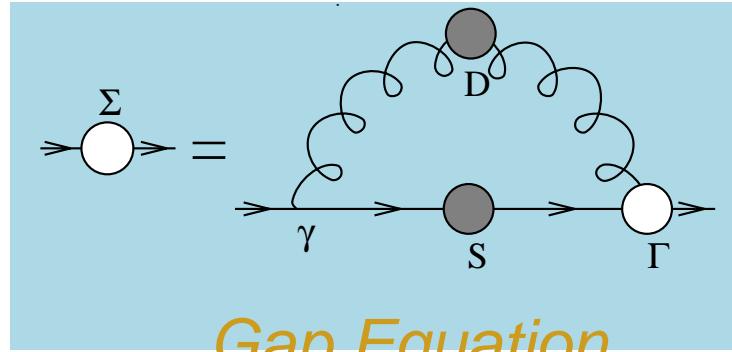
*Gap Equation*

- Gap Equation's Kernel Enhanced on **IR domain**  
⇒ **IR Enhancement of  $M(p^2)$**



# Dressed-Quark Propagator

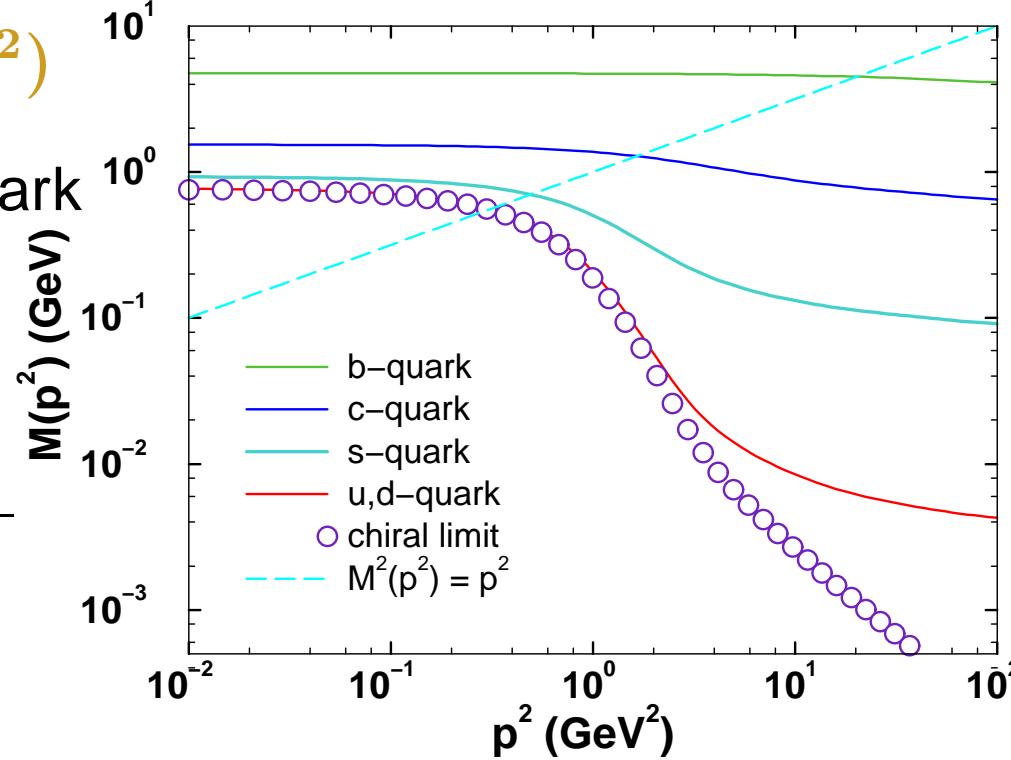
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



- Gap Equation's Kernel Enhanced on **IR domain**  
⇒ **IR Enhancement of  $M(p^2)$**

- Euclidean Constituent–Quark Mass:  $M_f^E$ :  $p^2 = M(p^2)^2$

flavour	$u/d$	$s$	$c$	$b$
$\frac{M_f^E}{m_\zeta}$	$\sim 10^2$	$\sim 10$	$\sim 1.5$	$\sim 1.1$

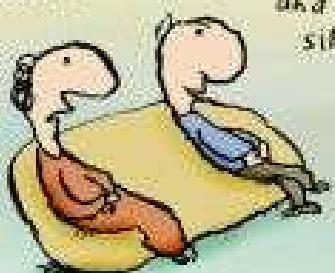


# Dressed-Quark Propagator

---

DO YOU  
THINK KEN'S  
CONSTIPATION  
WILL END  
HAPPILY?

The ending is  
unimportant; what  
matters most is  
the sheer drama  
of his difficult  
and lonely  
situation.



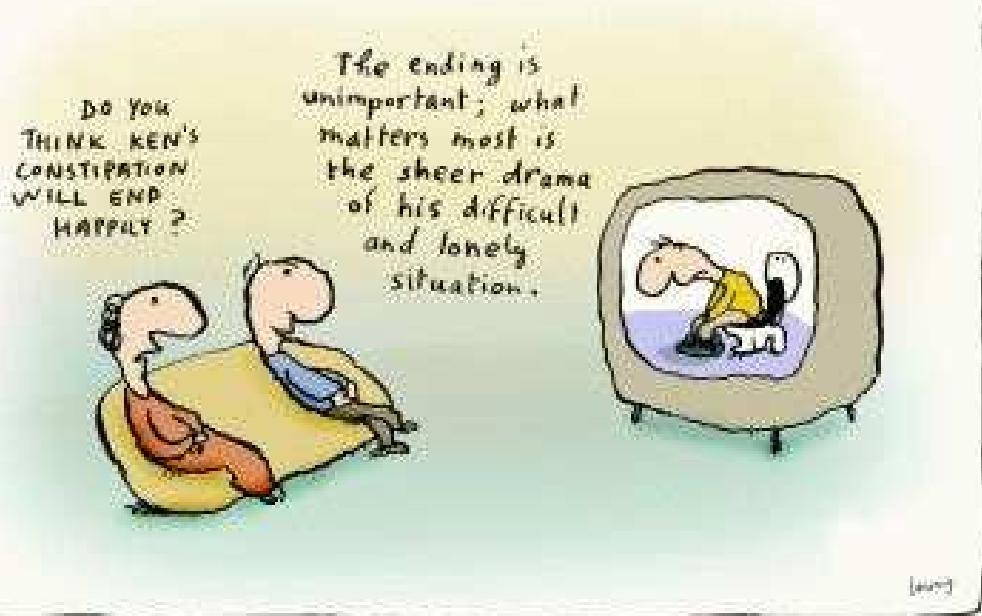
[407]

- Longstanding Prediction of Dyson-Schwinger Equation Studies



# Dressed-Quark Propagator

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- Longstanding Prediction of Dyson-Schwinger Equation Studies
  - E.g., *Dyson-Schwinger equations and their application to hadronic physics*,  
C. D. Roberts and  
A. G. Williams,  
Prog. Part. Nucl. Phys.  
**33** (1994) 477

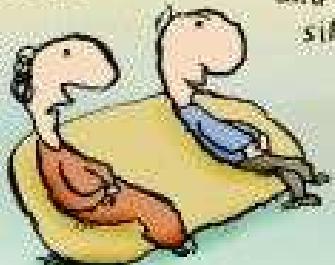


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[167]

- Long used as basis for efficacious hadron physics phenomenology

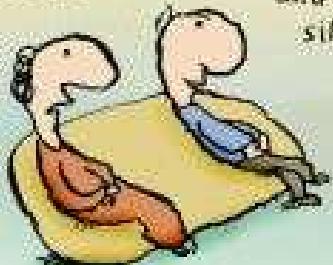


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  - *Electromagnetic pion form-factor and neutral pion decay width,*  
C. D. Roberts,  
Nucl. Phys. A **605**  
(1996) 475



# *Mandar Bhagwat*



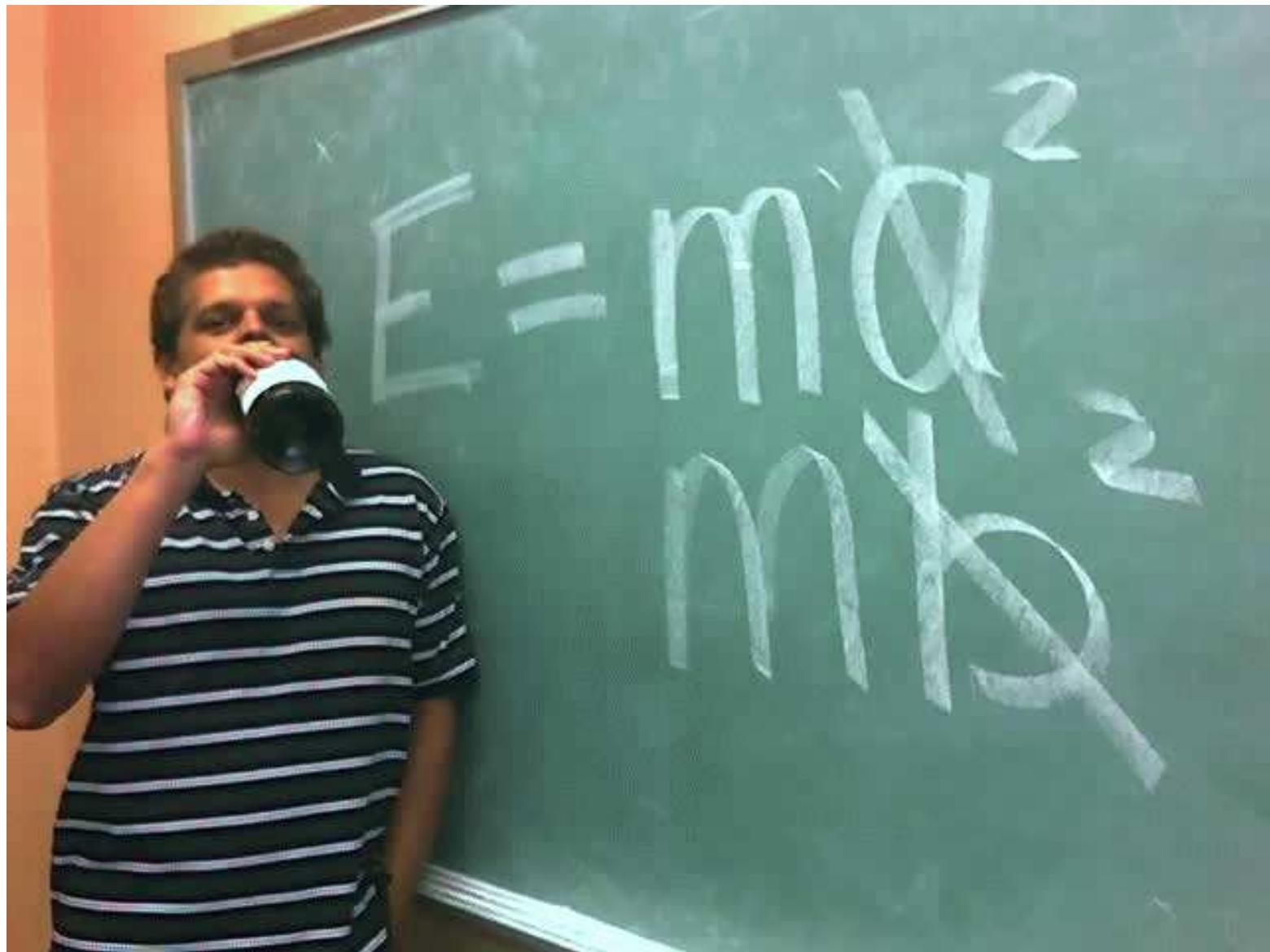
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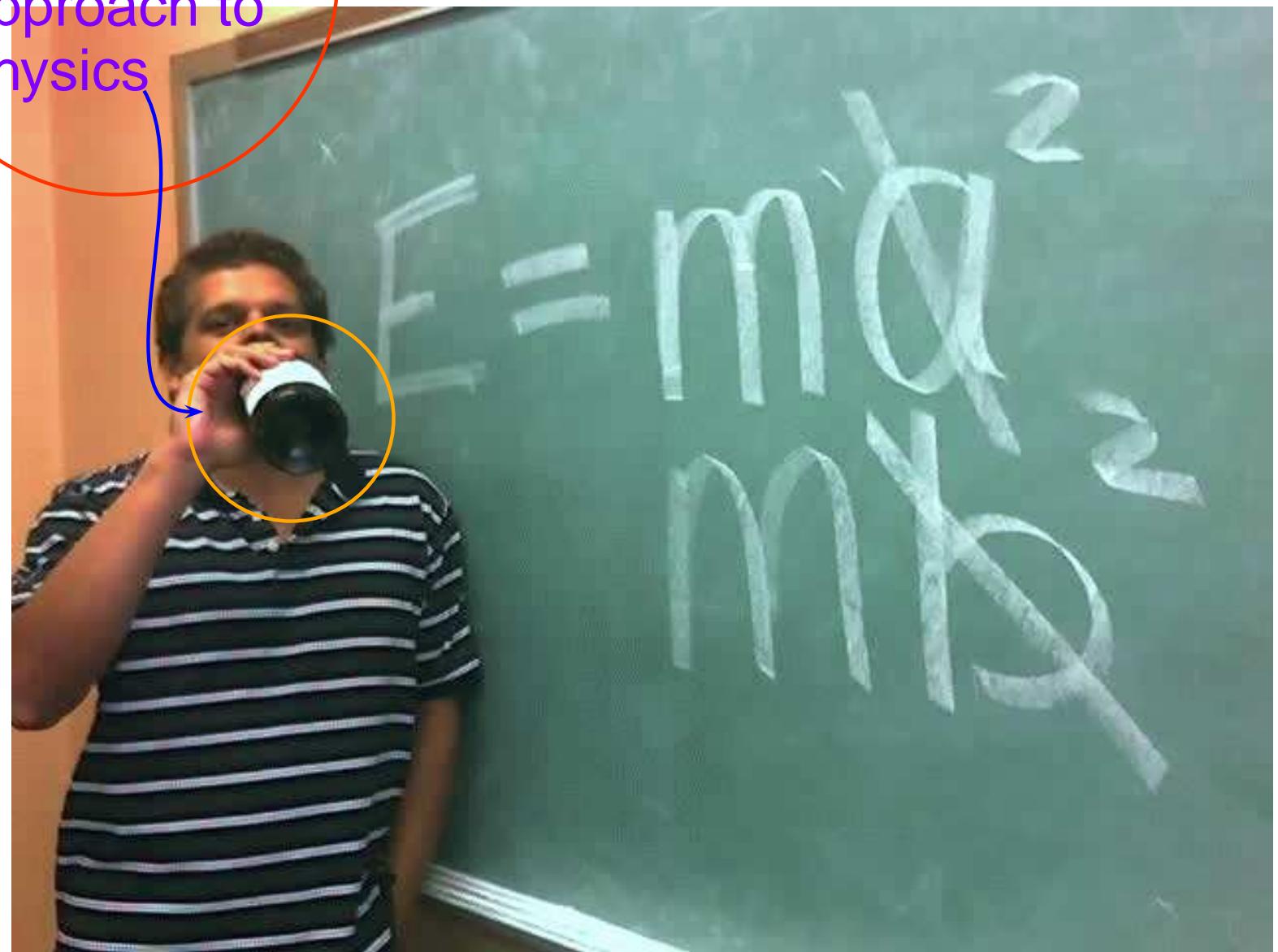
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# Mandar Bhagwat



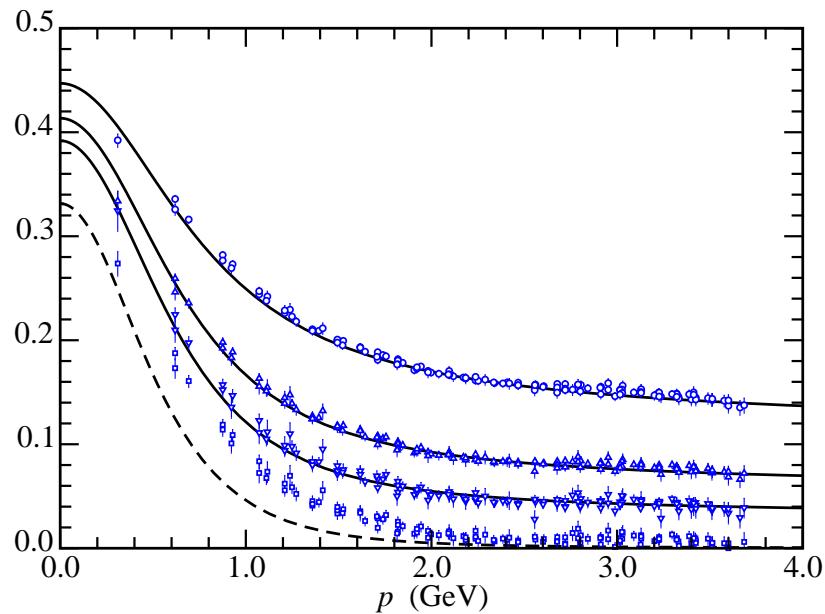
Emulating  
supervisor's  
approach to  
physics

*Mandar Bhagwat*

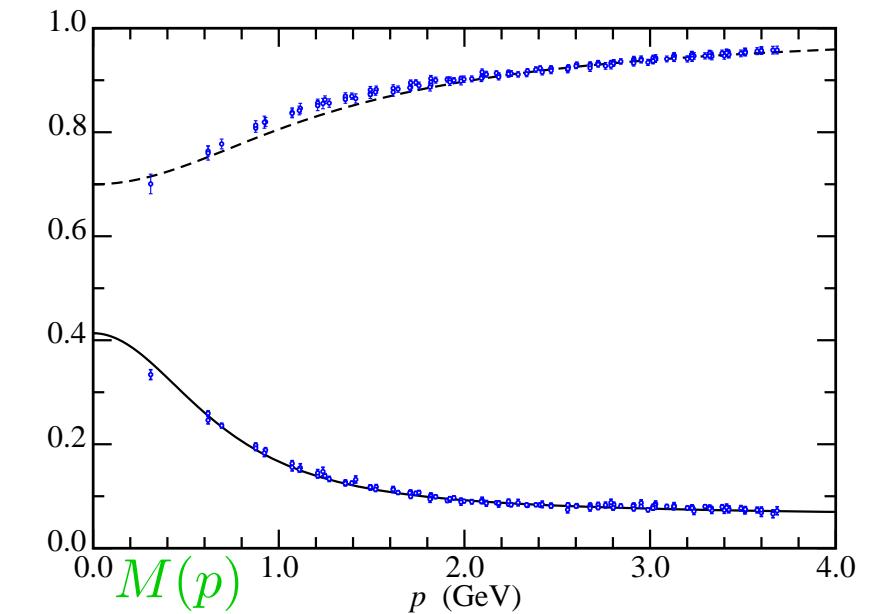


## Dressed-Quark Propagator

$M(p)$

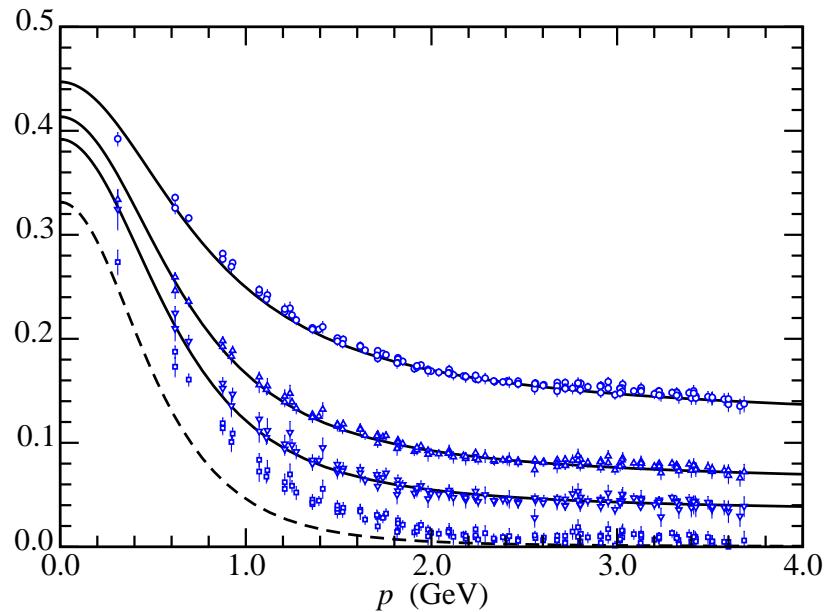
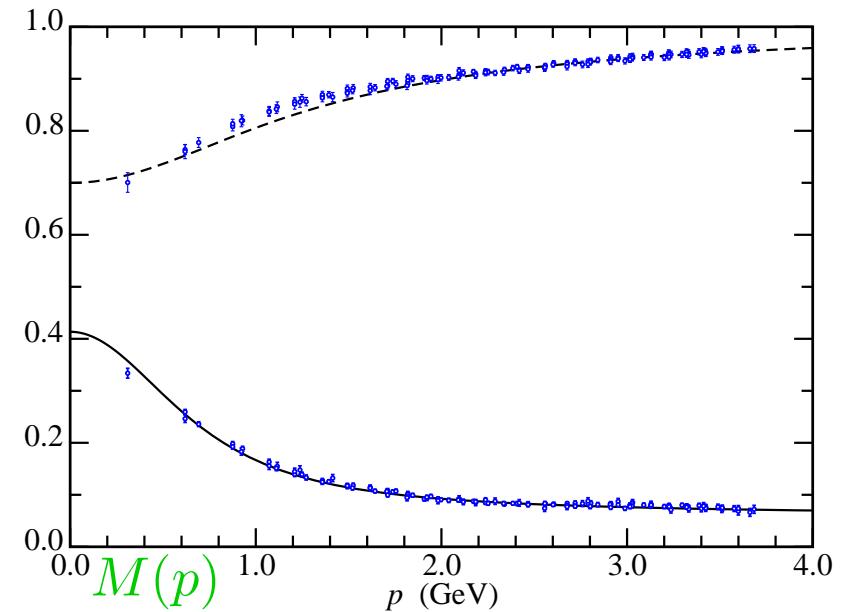


$Z(p)$



2002

## Dressed-Quark Propagator

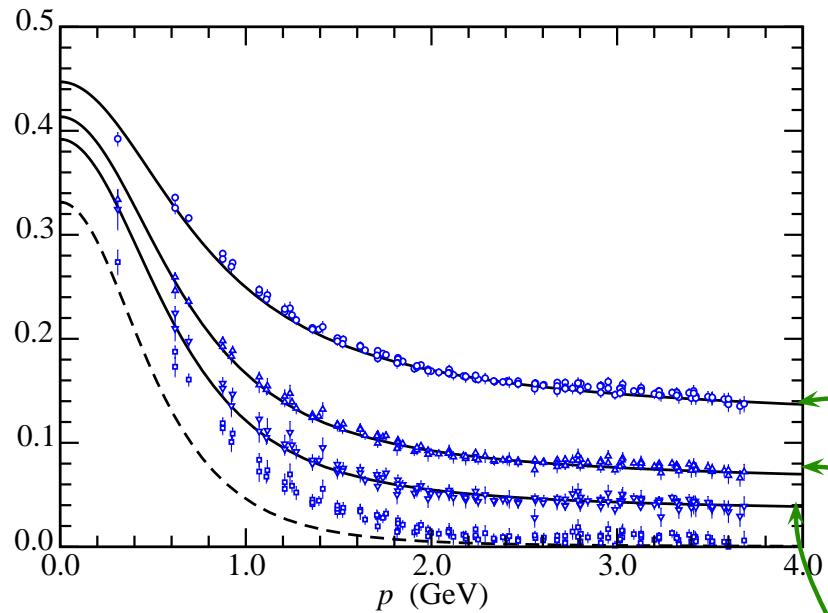
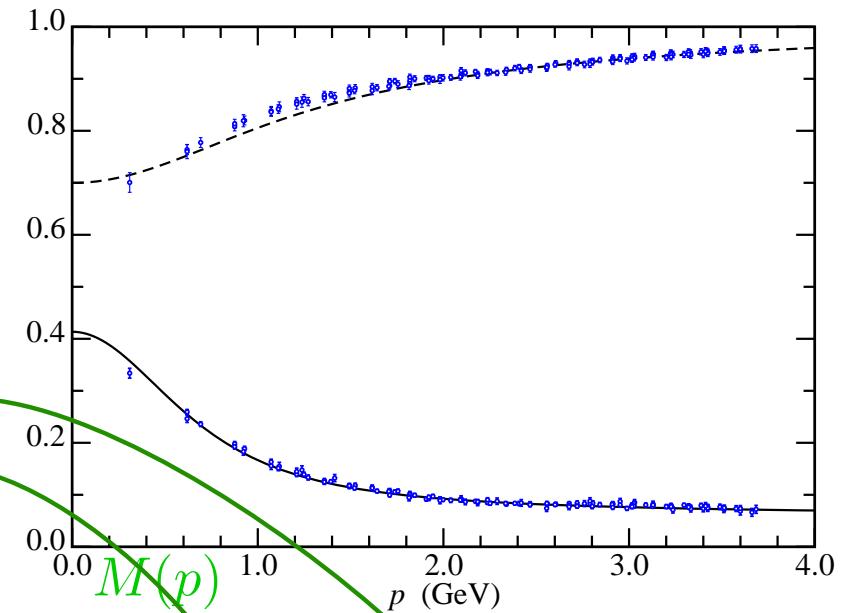
 $M(p)$  $Z(p)$ 

- “*data*:” Quenched Lattice Meas.
  - Bowman, Heller, Leinweber, Williams: [he-lat/0209129](https://arxiv.org/abs/hep-lat/0209129)

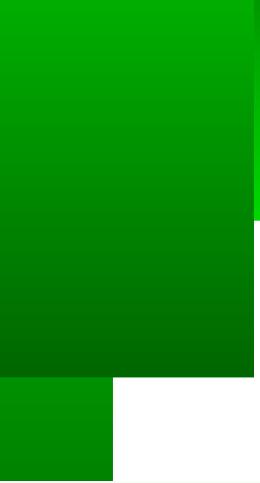


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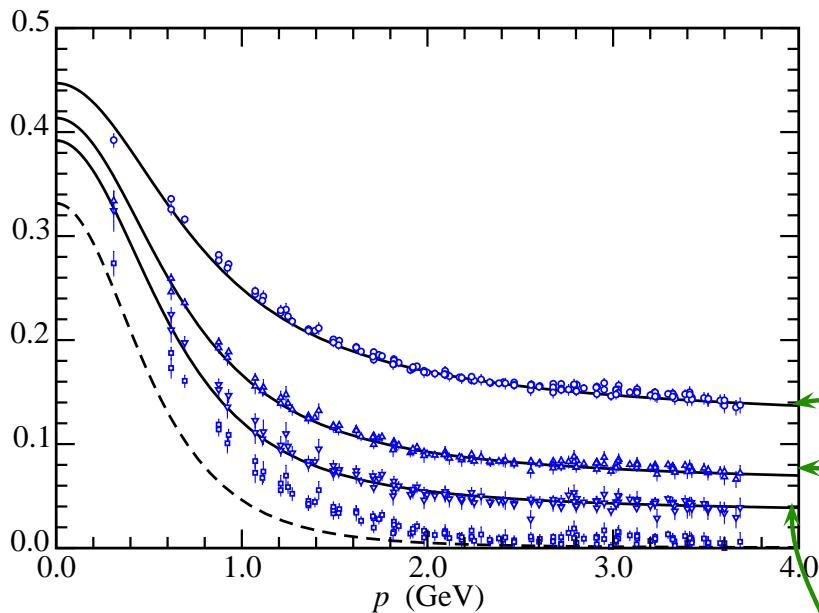
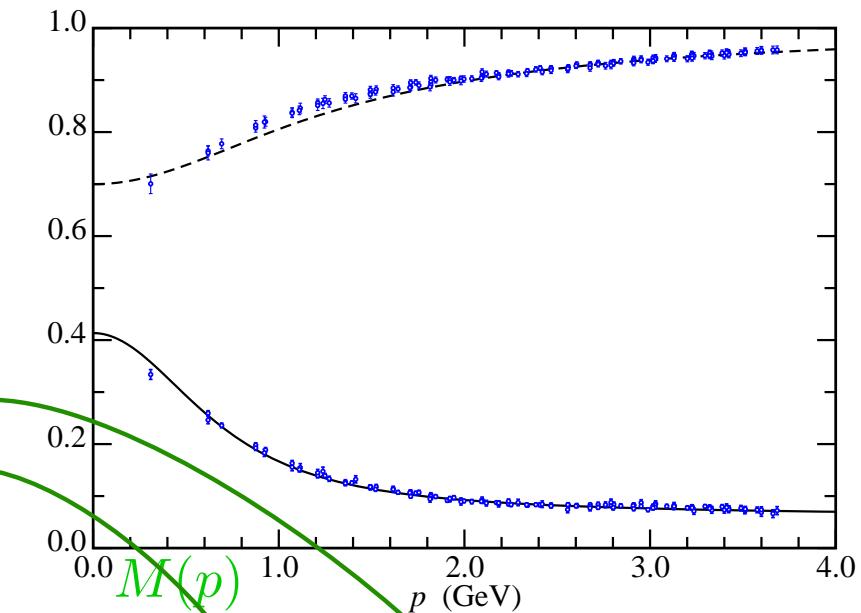
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current-quark masses: 30 MeV, 50 MeV, 100 MeV



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## Dressed-Quark Propagator

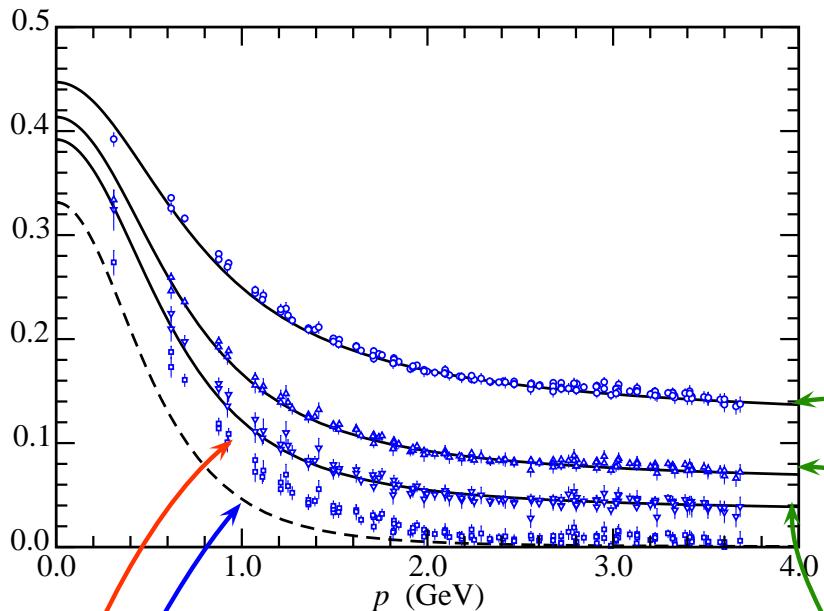
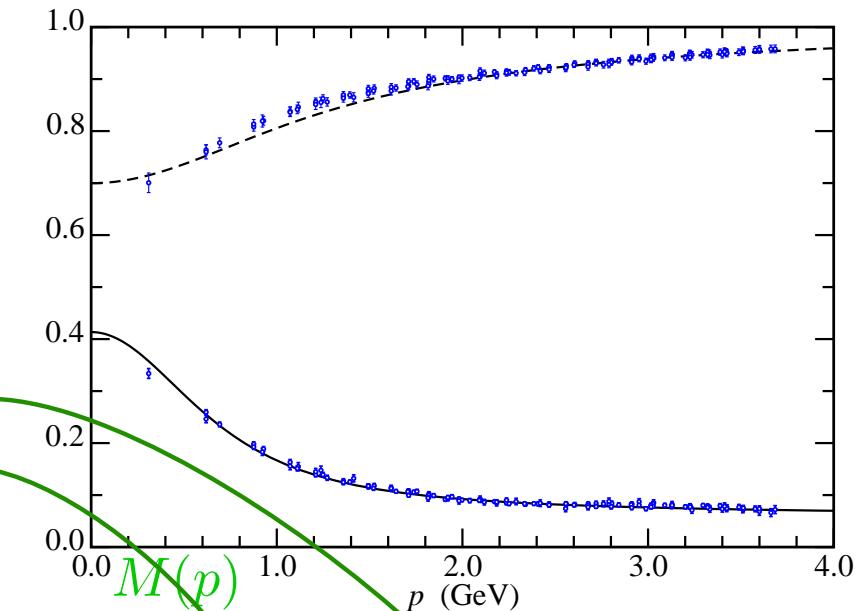
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- *Curves*: Quenched DSE Cal.
  - Bhagwat, Pichowsky, Roberts, Tandy [nu-th/0304003](#)



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## Dressed-Quark Propagator

 $M(p)$  $Z(p)$ 

**“data:” Quenched Lattice Meas.**

- Bowman, Heller, Leinweber, Williams: [he-lat/0209129](#)  
current-quark masses: 30 MeV, 50 MeV, 100 MeV

**Curves: Quenched DSE Cal.**

- Bhagwat, Pichowsky, Roberts, Tandy [nu-th/0304003](#)

**Linear extrapolation of lattice data to chiral limit is inaccurate**



# *Constituent-quark $\sigma$ -term*

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark  $\sigma$ -term

$$\sigma_f := m_f(\zeta) \frac{\partial M_f^E}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.$$



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- Renormalisation-group-invariant and determined from solutions of the gap equation



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- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark  $\sigma$ -term

$$\sigma_f := m_f(\zeta) \frac{\partial M_f^E}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.$$

- Unambiguous probe of impact of explicit chiral symmetry breaking on the mass function



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- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark  $\sigma$ -term

$$\sigma_f := m_f(\zeta) \frac{\partial M_f^E}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.$$

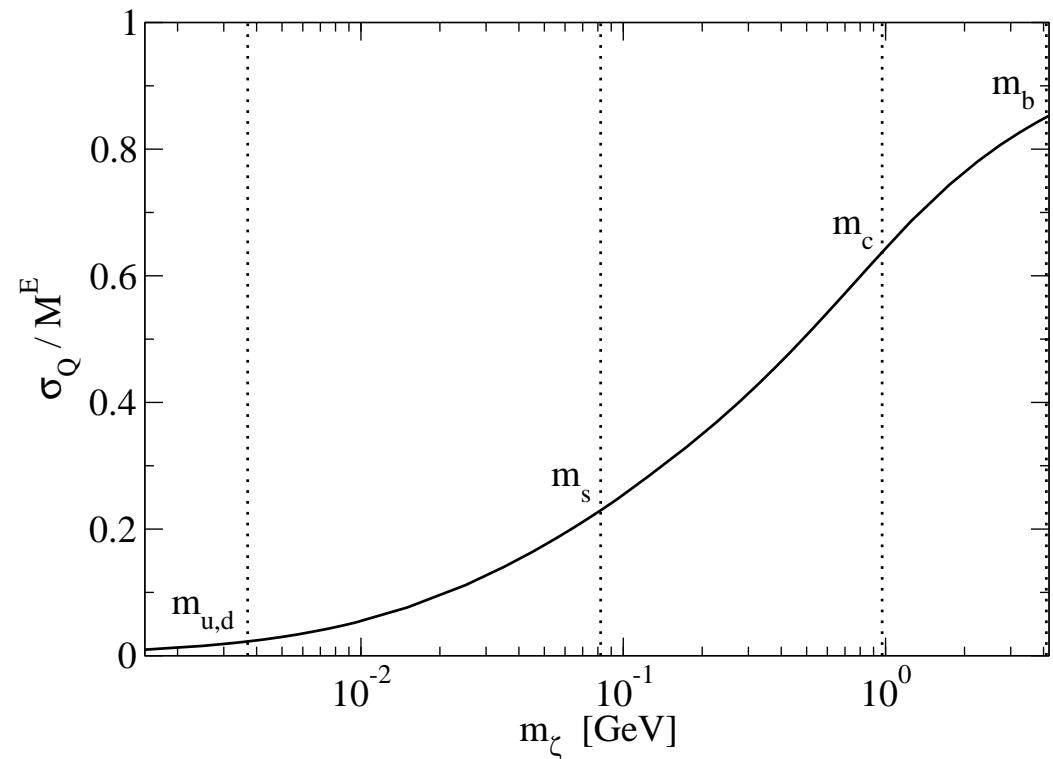
- Ratio 
$$\frac{\sigma_f}{M_f^E} = \frac{\text{EXPLICIT}}{\text{EXPLICIT} + \text{DYNAMICAL}}$$
measures effect of **EXPLICIT** chiral symmetry breaking on dressed-quark mass-function  
cf. **SUM** of effects of **EXPLICIT AND DYNAMICAL CHIRAL SYMMETRY BREAKING**



# Constituent-quark $\sigma$ -term

- Impact of Dynamical chiral symmetry breaking . . . exhibited via constituent-quark  $\sigma$ -term

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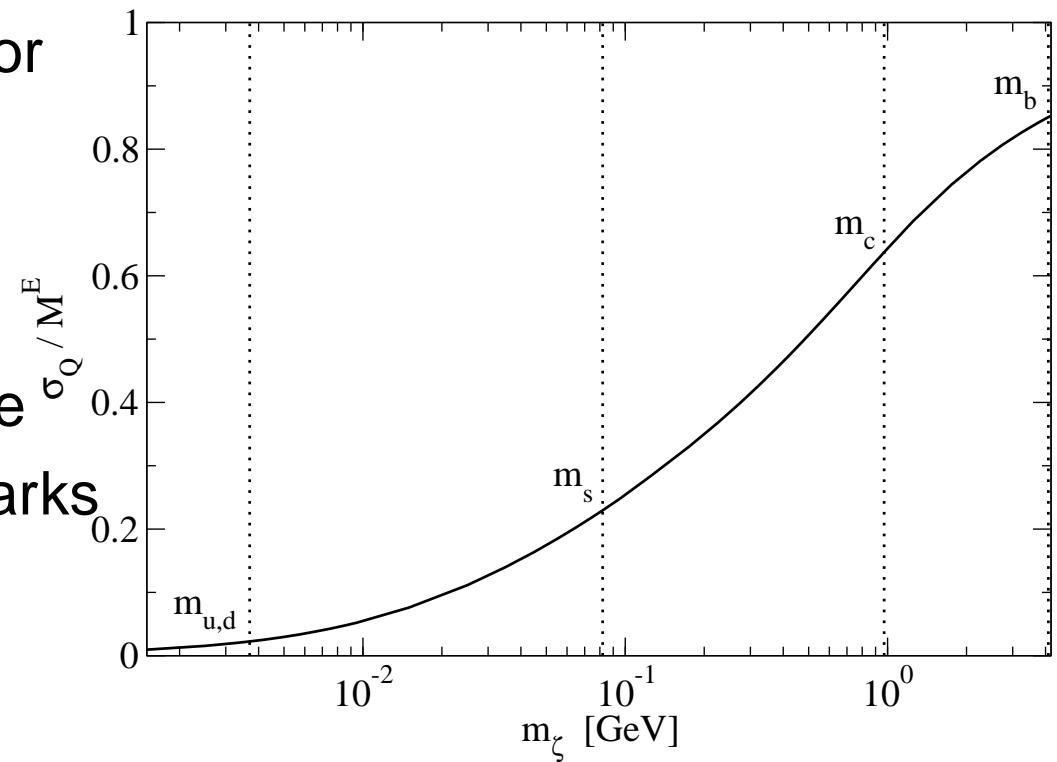
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$$\sigma_f := m_f(\zeta) \frac{\partial M_f^E}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.$$

Obvious: ratio vanishes for light-quarks because magnitude of their constituent-mass owes primarily to DCSB. On the other hand, for heavy-quarks it approaches one.

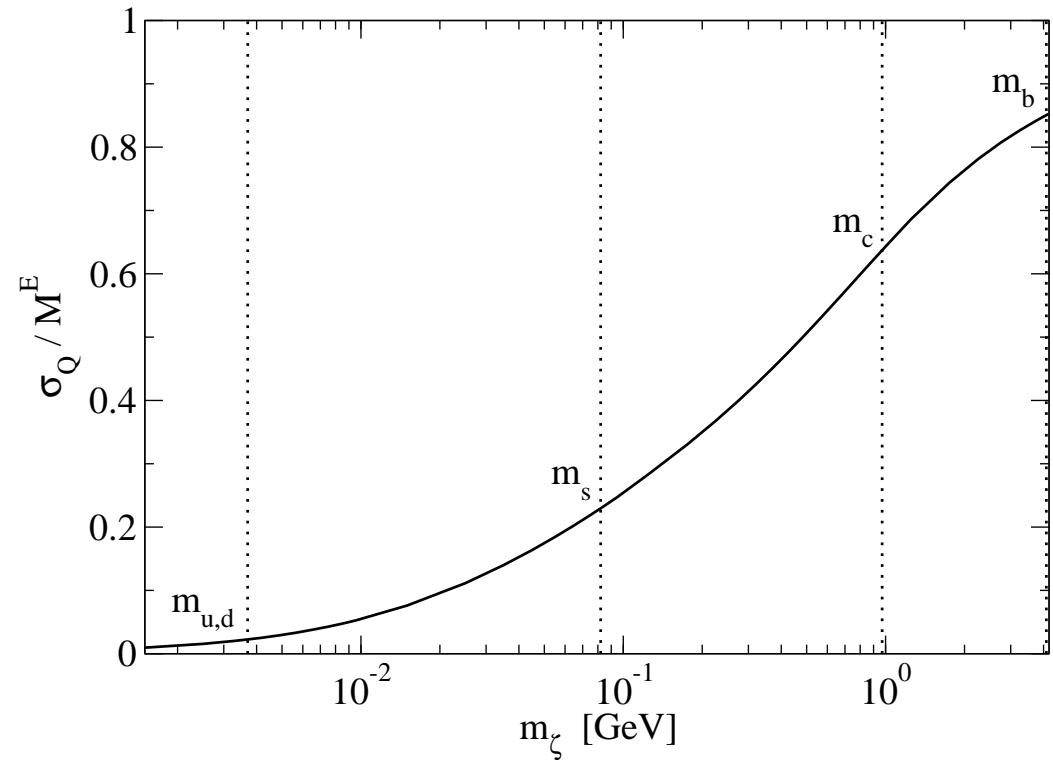


# Constituent-quark $\sigma$ -term

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$$\sigma_f := m_f(\zeta) \frac{\partial M_f^E}{\partial m_f(\zeta)}, \quad (M^E)^2 := s \mid s = M(s)^2.$$

Essentially dynamical component of chiral symmetry breaking, and manifestation in all its order parameters, vanishes with increasing current-quark mass





# Hadrons

---

- Established understanding of two- and three-point functions





# Hadrons

---

- Established understanding of two- and three-point functions
- What about bound states?





# Hadrons

---

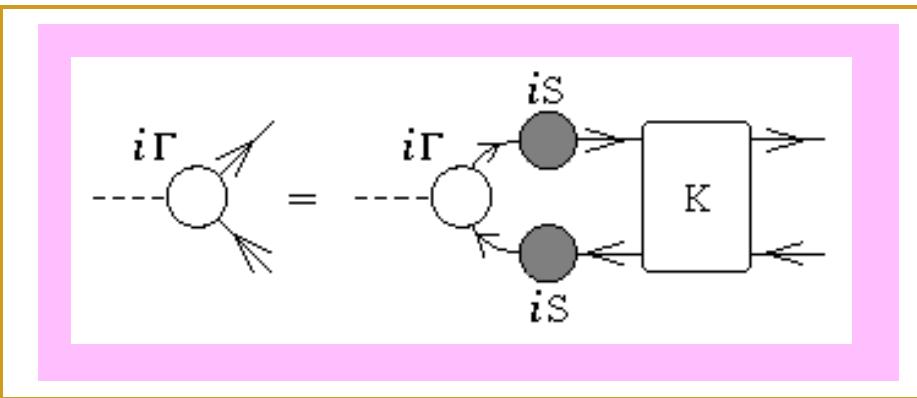
- Without bound states,  
Comparison with experiment is  
impossible



- Without bound states,  
Comparison with experiment is  
impossible
- They appear as pole contributions  
to  $n \geq 3$ -point colour-singlet  
Schwinger functions



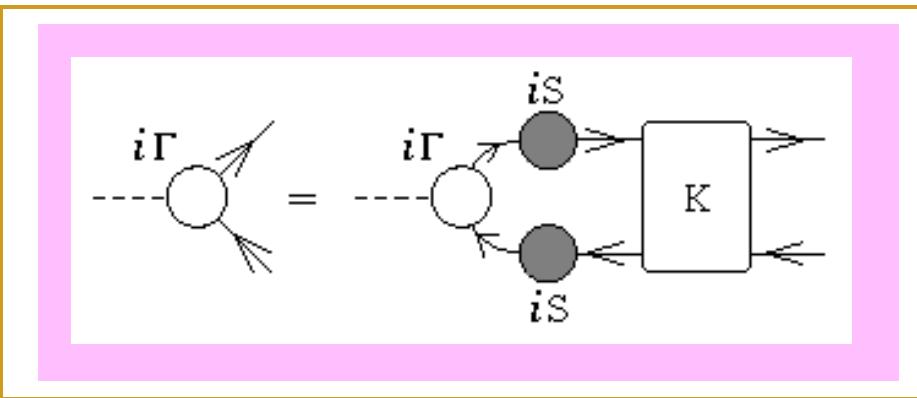
- Without bound states,  
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QFT Generalisation of Lippmann-Schwinger Equation.



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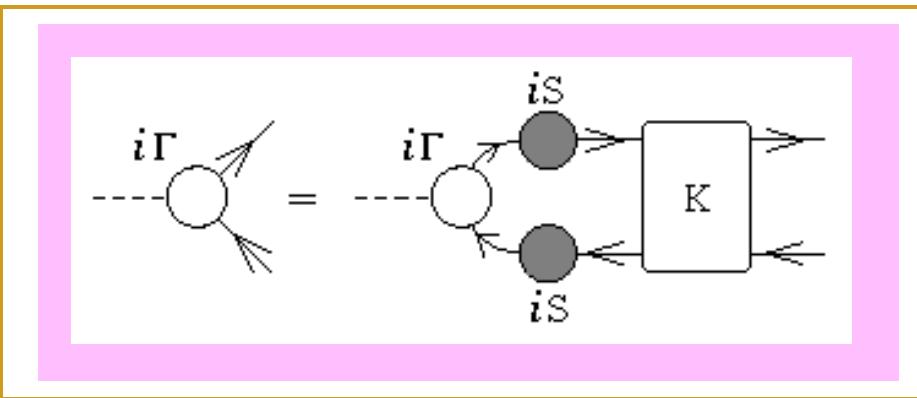


QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel,  $K$ ?



- Without bound states,  
Comparison with experiment is  
impossible
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QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel,  $K$ ?

or



# *What is the Long-Range Potential?*



First

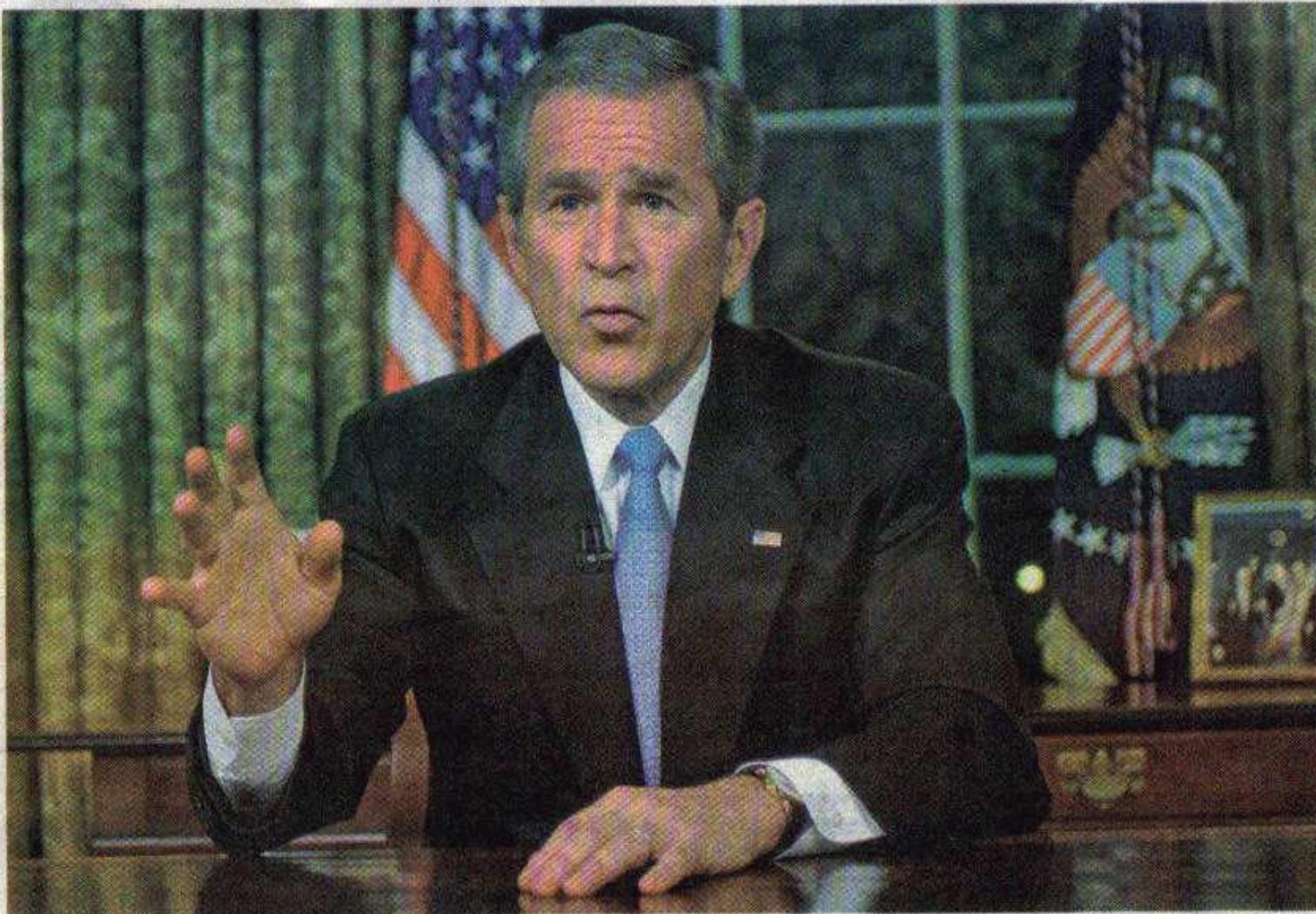
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# *What is the Long-Range Potential?*

## **Bush Urges Nation To Be Quiet For A Minute While He Tries To Think**



In a televised address to the nation, Bush called for "a little peace and quiet."



# Bethe-Salpeter Kernel



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# Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^l(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f^l i\gamma_5 + \frac{1}{2} \lambda_f^l i\gamma_5 \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta i\Gamma_5^l(k; P) - i\Gamma_5^l(k; P) M_\zeta$$

## QFT Statement of Chiral Symmetry



# Bethe-Salpeter Kernel

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Satisfies BSE

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- **Nontrivial** constraint





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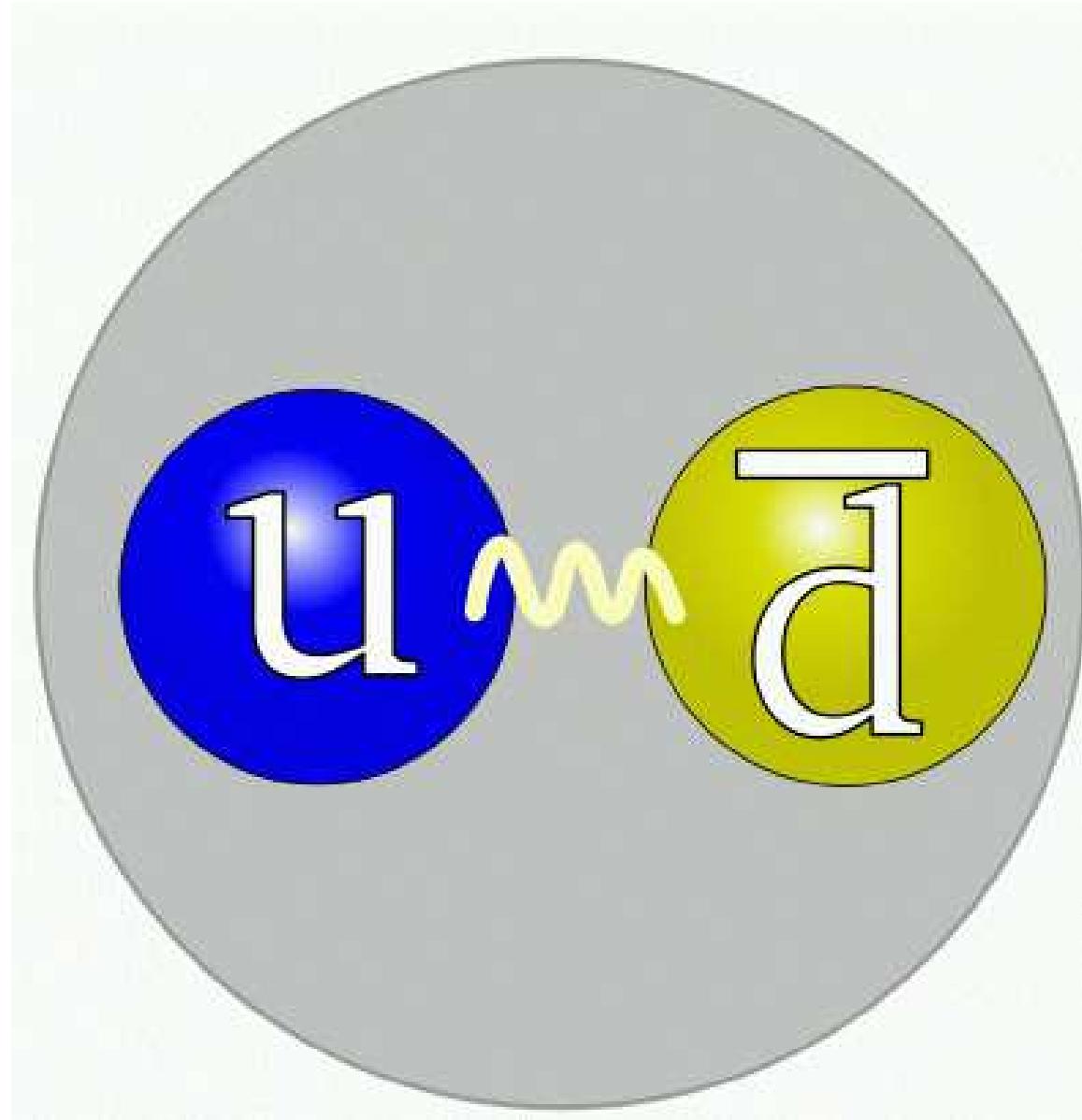
Satisfies DSE

Kernels must be **intimately** related

- Relation **must** be preserved by truncation
- **Failure**  $\Rightarrow$  Explicit Violation of QCD's Chiral Symmetry

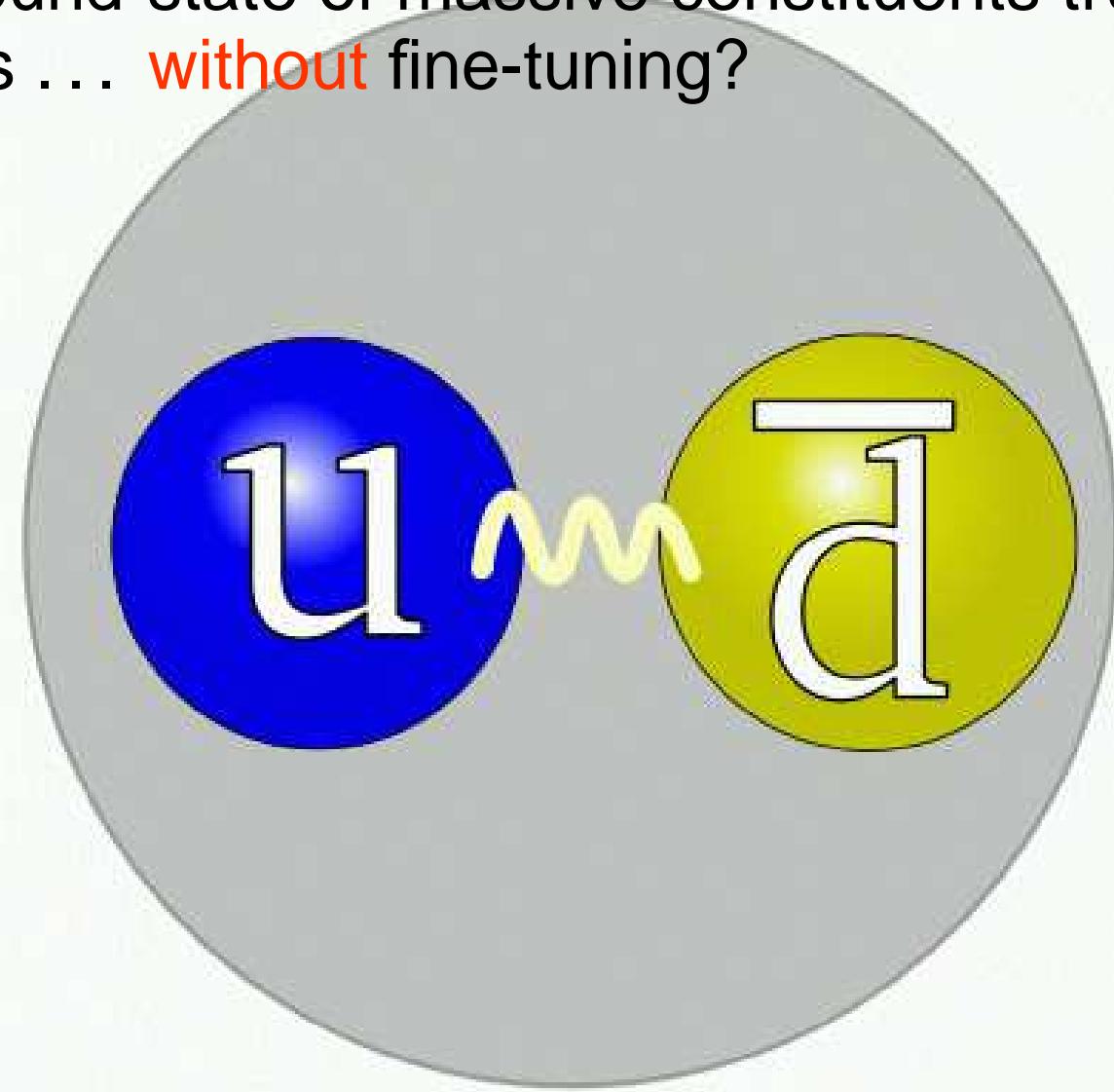


## Pseudoscalar Mesons?

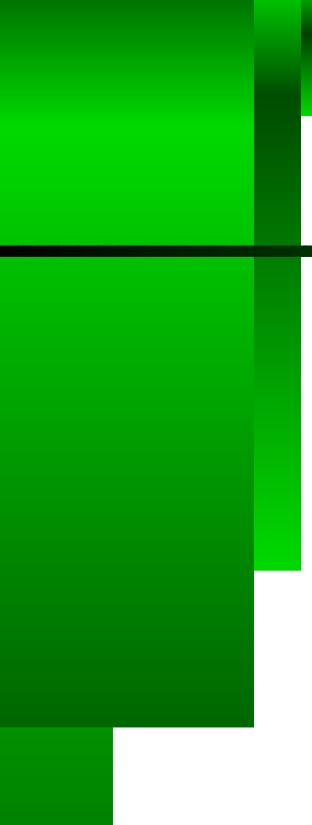


## Pseudoscalar Mesons?

Can a bound-state of massive constituents truly be massless ... **without** fine-tuning?



# *Radial Excitations & Chiral Symmetry*



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(Maris, Roberts, Tandy  
nu-th/9707003 )

$$f_H \ m_H^2 = - \rho_{\zeta}^H \ \mathcal{M}_H$$



# Radial Excitations & Chiral Symmetry

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$$f_H \quad m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

- Mass<sup>2</sup> of pseudoscalar hadron



# Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy  
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$$f_H \quad m_H^2 = - \quad \rho_{\zeta}^H \quad \mathcal{M}_H$$

$$\mathcal{M}_H := \text{tr}_{\text{flavour}} \left[ M_{(\mu)} \left\{ T^H, (T^H)^t \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g.,  $T^{K^+} = \frac{1}{2} (\lambda^4 + i\lambda^5)$



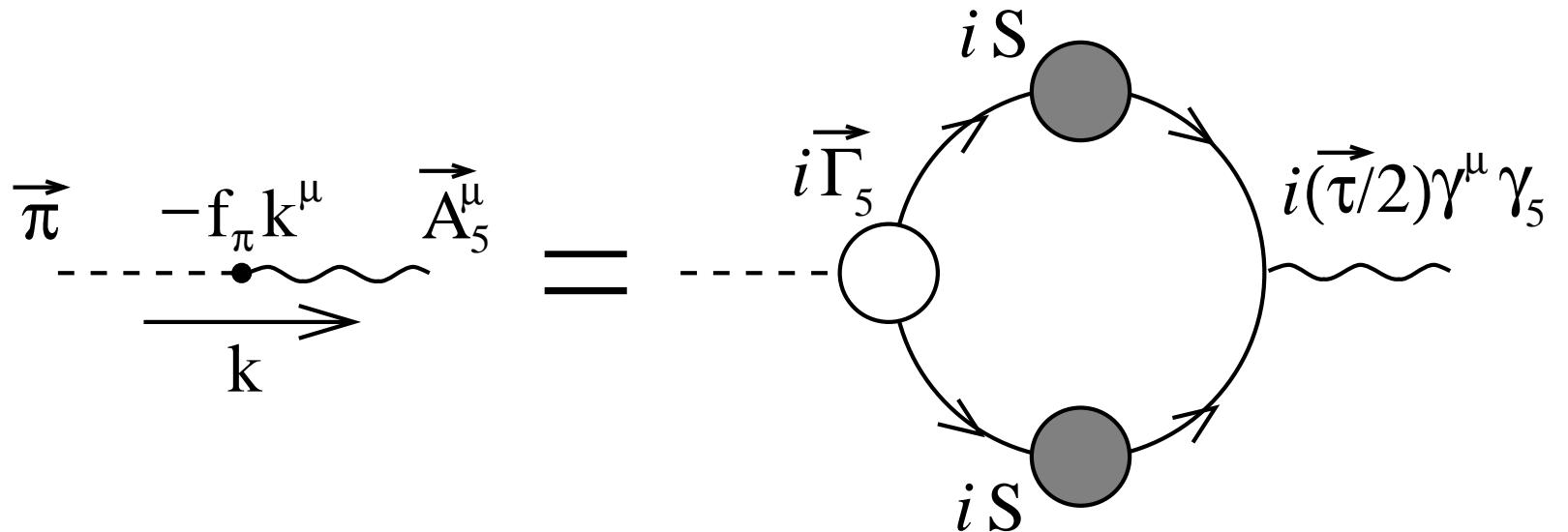
# Radial Excitations & Chiral Symmetry

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$$f_H p_\mu = Z_2 \int_q^\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 \gamma_\mu \boxed{\mathcal{S}(q_+) \Gamma_H(q; P) \mathcal{S}(q_-)} \right\}$$

$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$

- Pseudovector projection of BS wave function at  $x = 0$
- Pseudoscalar meson's leptonic decay constant



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- Light-quarks; i.e.,  $m_q \sim 0$

- $f_H \rightarrow f_H^0$  &  $\rho_\zeta^H \rightarrow \frac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$ , Independent of  $m_q$

Hence  $m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$  ... GMOR relation, a corollary



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- Heavy-quark + light-quark

$$\Rightarrow f_H \propto \frac{1}{\sqrt{m_H}} \text{ and } \rho_\zeta^H \propto \sqrt{m_H}$$

Hence,  $m_H \propto m_q$

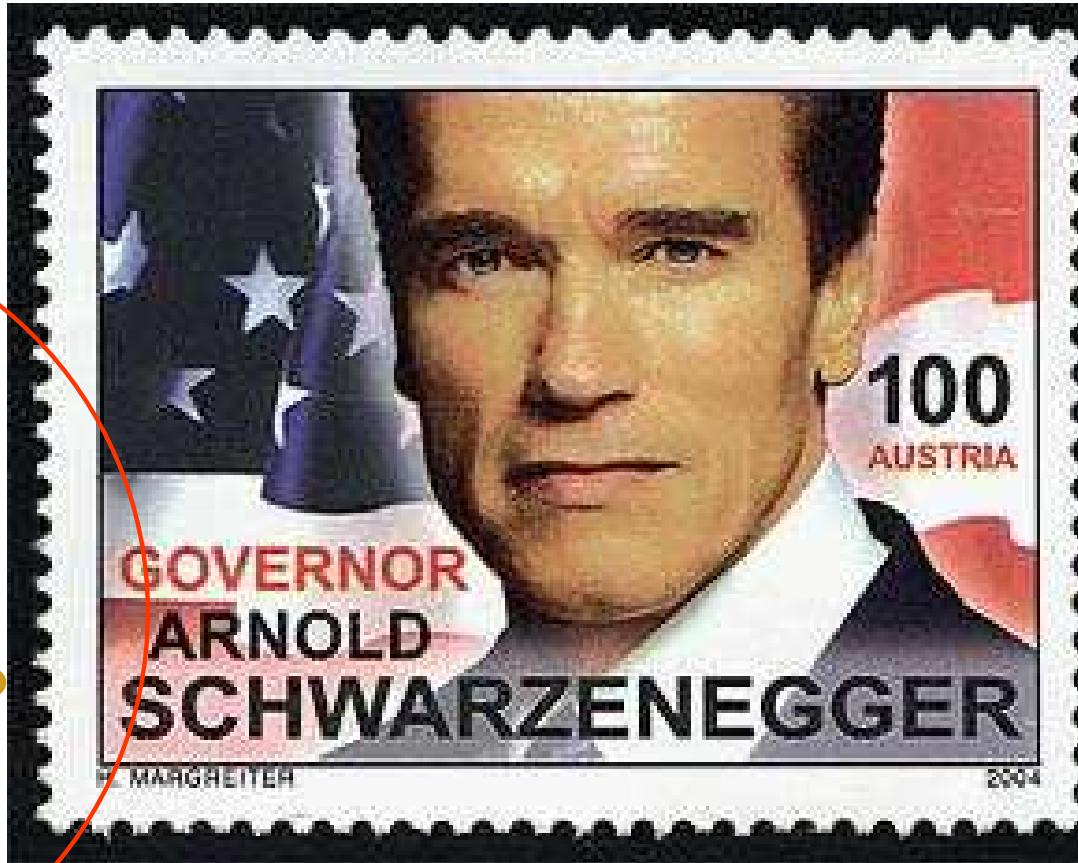
... QCD Proof of Potential Model result

# *Andreas Krassnigg*

FWF “Erwin  
Schrödinger Fellow,”  
ANL 2003-2005



Almost Blood  
Relative of Arnold  
... Future  
President?  
... Executioner?



# Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts  
nu-th/0406030

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- Valid for ALL Pseudoscalar mesons



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 $m_{\pi_n \neq 0}^2 > m_{\pi_n = 0}^2 = 0$ , in chiral limit



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ALL pseudoscalar mesons except  $\pi(140)$  in chiral limit
- Dynamical Chiral Symmetry Breaking
  - Goldstone’s Theorem –impacts upon *every* pseudoscalar meson



# *Radial Excitations*



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# *Radial Excitations*

- Spectrum contains 3 pseudoscalars [ $I^G(J^P)L = 1^-(0^-)S$ ]

masses below 2 GeV:  $\pi(140)$  ;  $\pi(1300)$ ; and  $\pi(1800)$



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- But  $\pi(1800)$  is narrow ( $\Gamma = 207 \pm 13$ ) & decay pattern might indicate some “flux tube angular momentum” content:  
 $S_{\bar{Q}Q} = 1 \oplus L_F = 1 \Rightarrow J = 0$   
&  $L_F = 1 \Rightarrow \ ^3S_1 \oplus \ ^3S_1 (\bar{Q}Q)$  decays suppressed?



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- NSAC Long-Range Plan, 2002: . . . an understanding of confinement “remains one of the greatest intellectual challenges in physics”



# *Radial Excitations*

## *& Lattice-QCD*

McNeile and Michael  
he-la/0607032



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# Radial Excitations & Lattice-QCD

McNeile and Michael  
he-la/0607032

- When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.



# Radial Excitations & Lattice-QCD

McNeile and Michael  
he-la/0607032

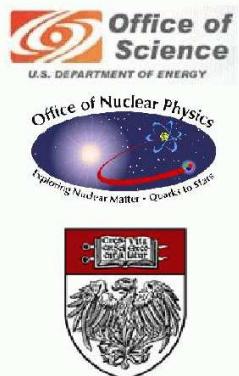
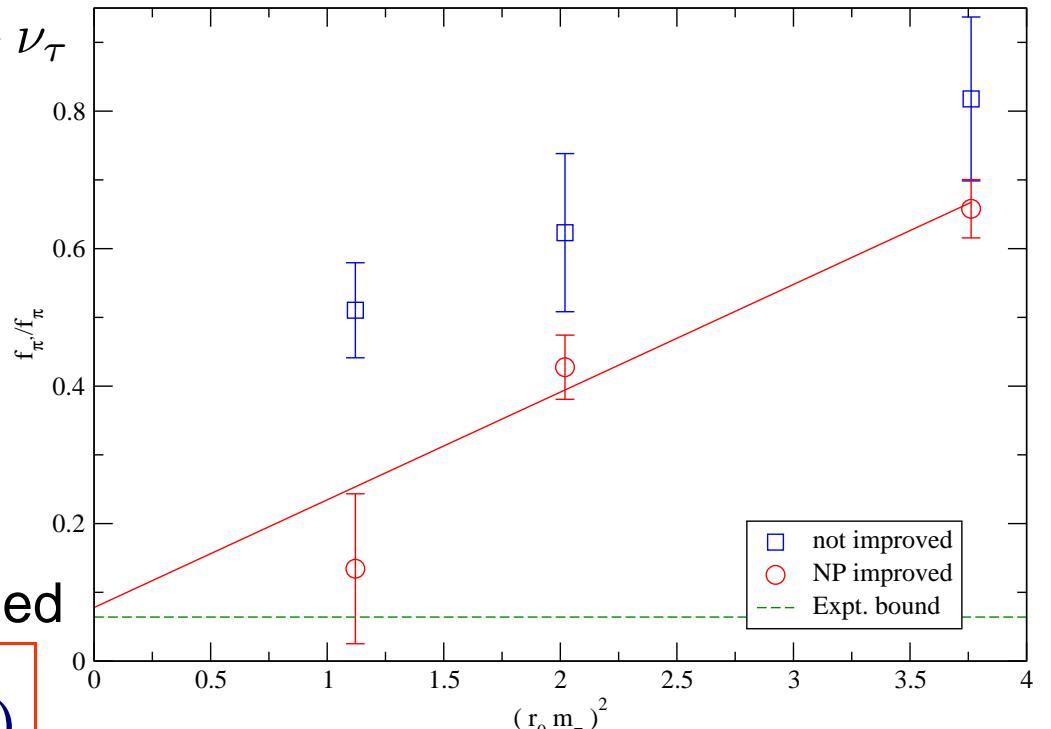
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*Diehl & Hiller*  
*he-ph/0105194*



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- Lattice-QCD check:  
 $16^3 \times 32$ ,  
 $a \sim 0.1 \text{ fm}$ ,  
two-flavour, unquenched  
 $\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078(93)$



# Radial Excitations & Lattice-QCD

McNeile and Michael  
he-la/0607032

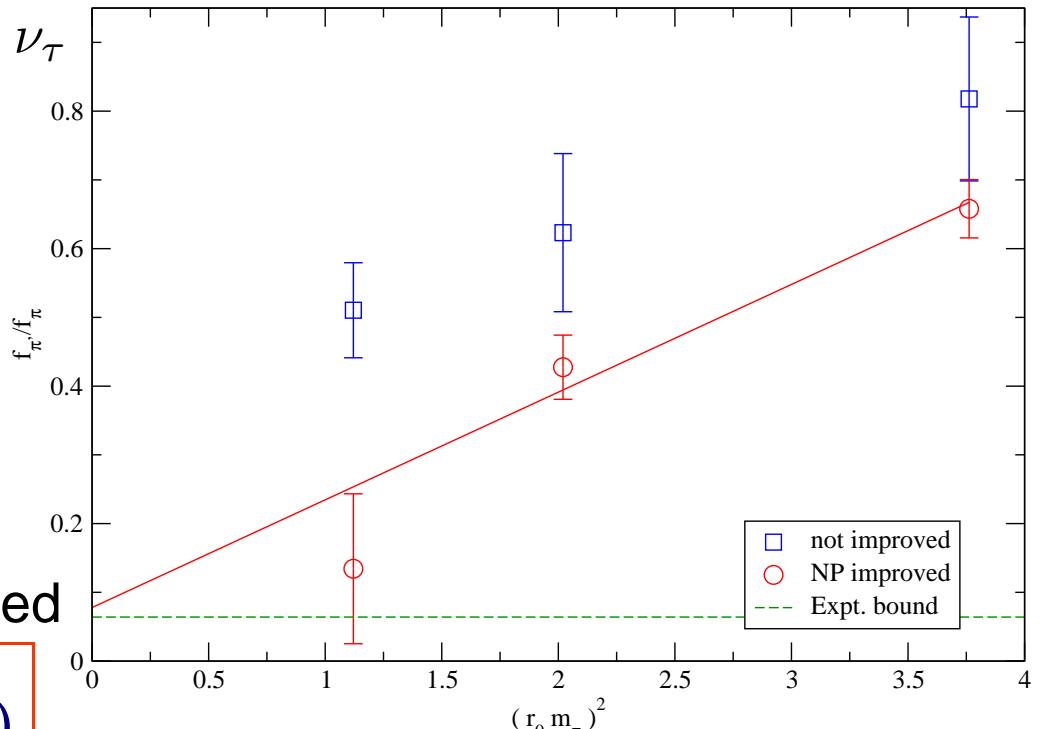
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- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

# Radial Excitations & Lattice-QCD

McNeile and Michael  
he-la/0607032

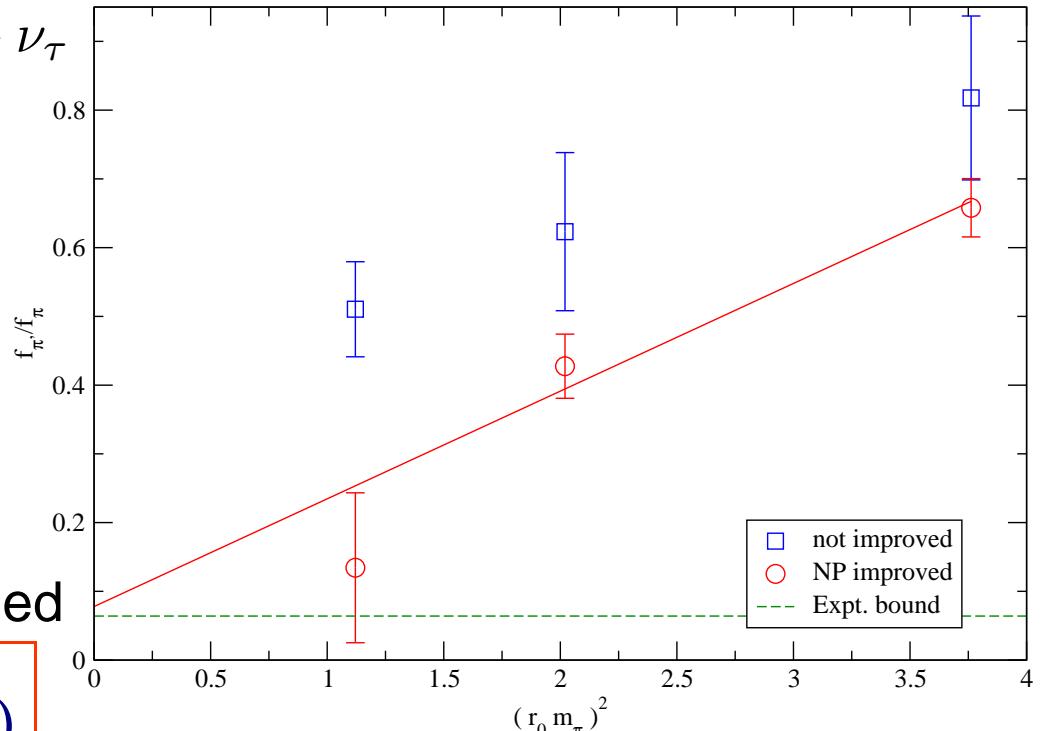
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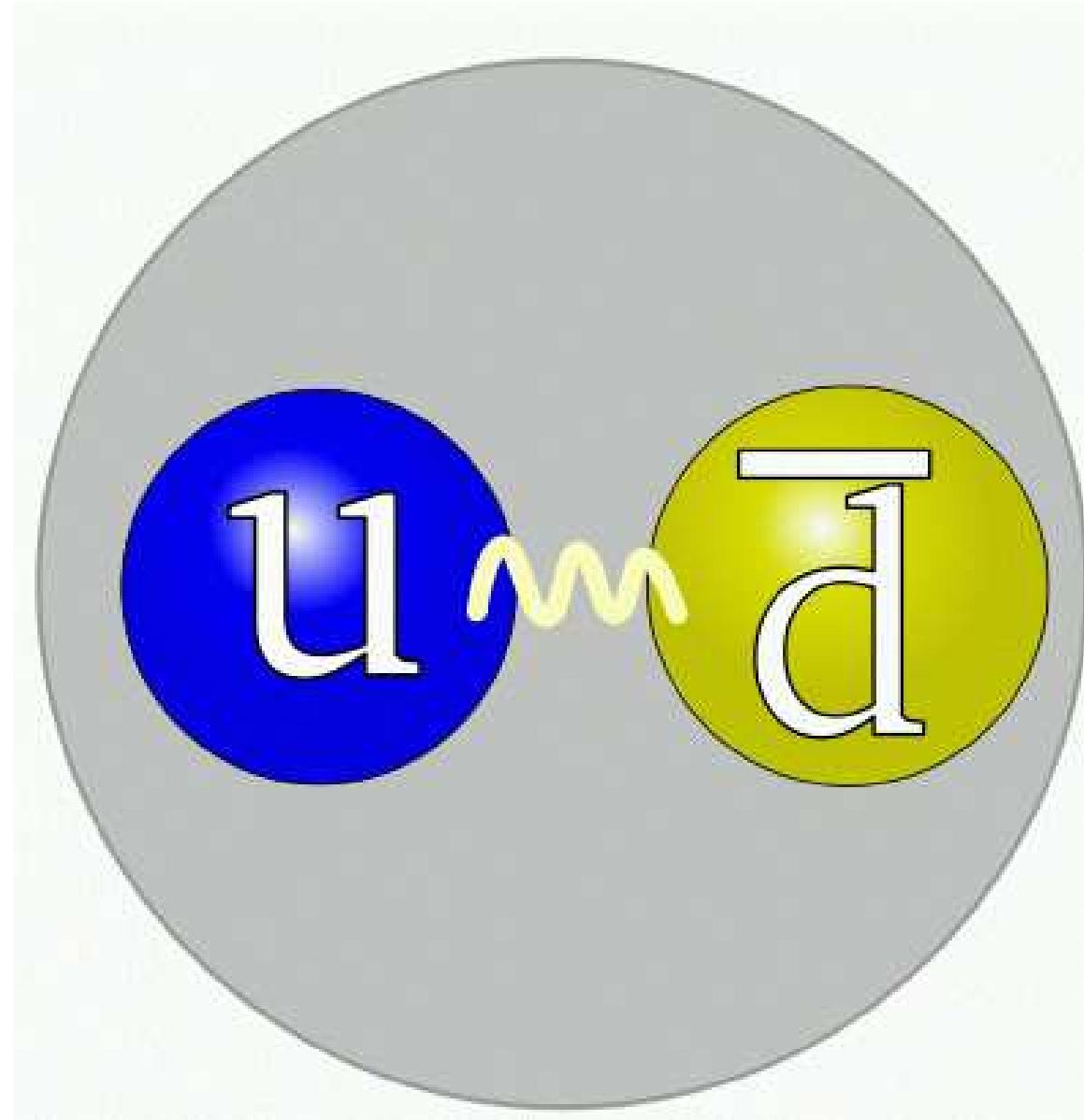
$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078 (93)$$



- The suppression of  $f_{\pi_1}$  is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.

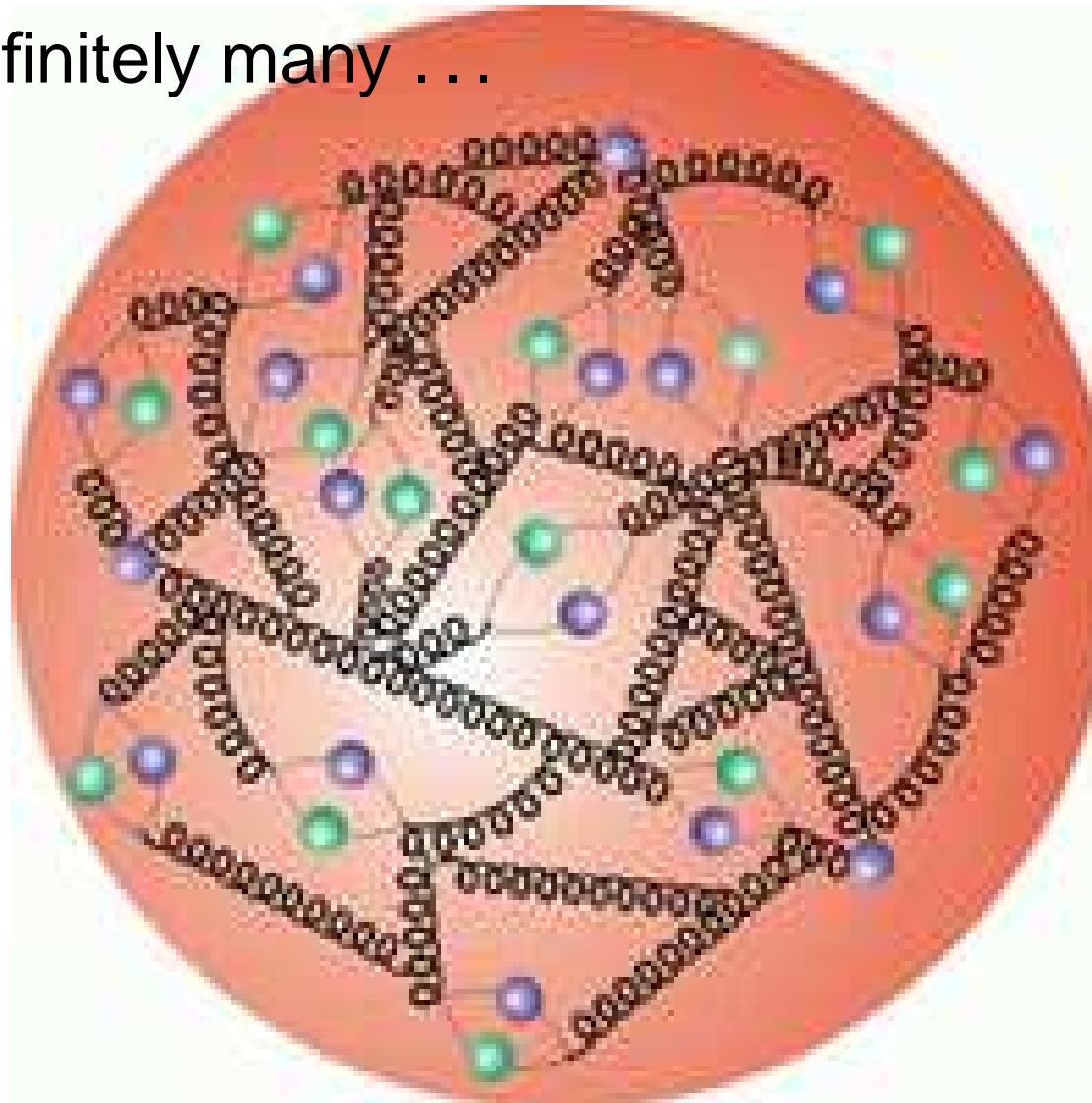


# *Answer for the pion*



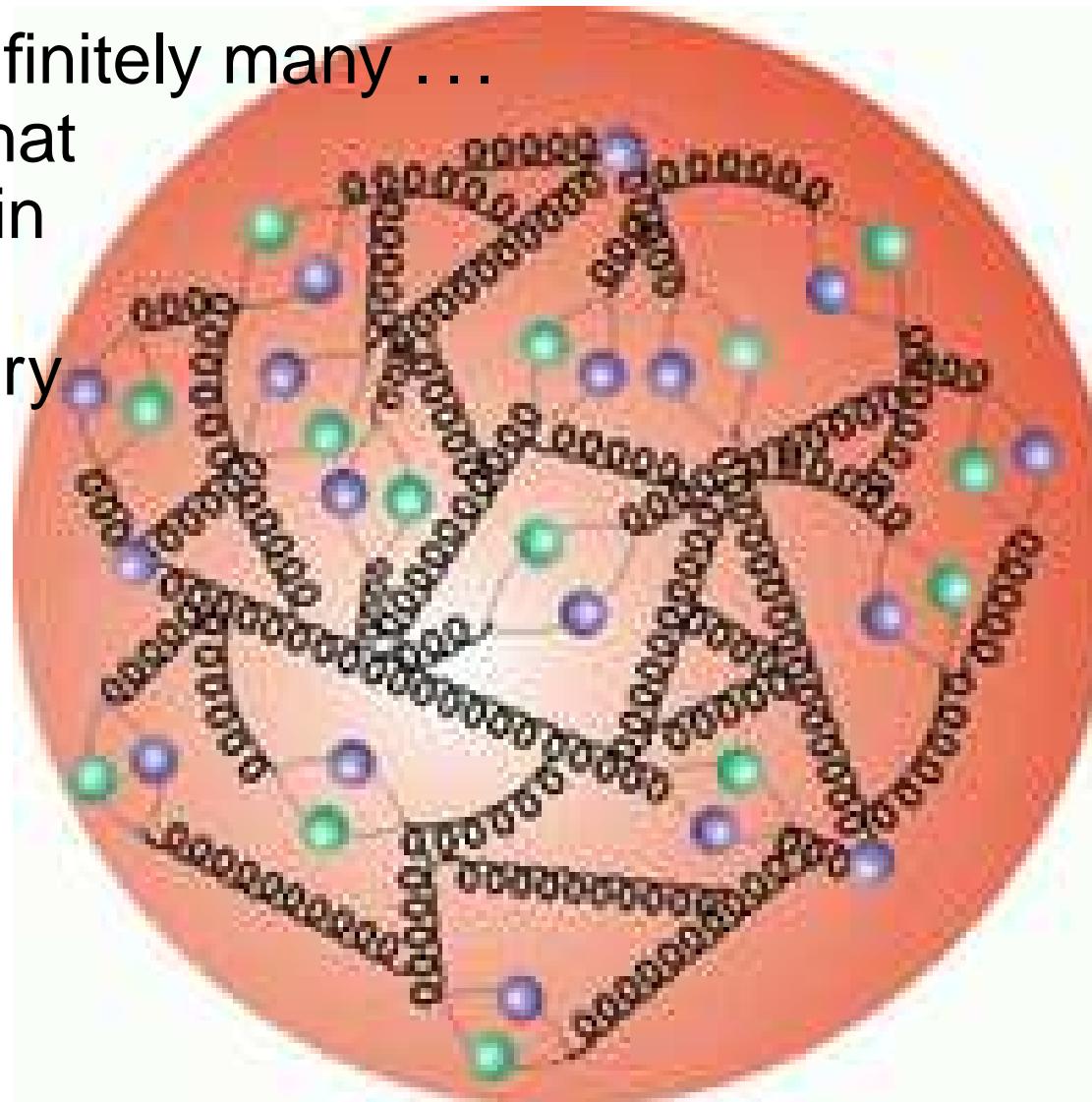
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# *Answer for the pion*

Two → Infinitely many . . .  
Handle that  
properly in  
quantum  
field theory

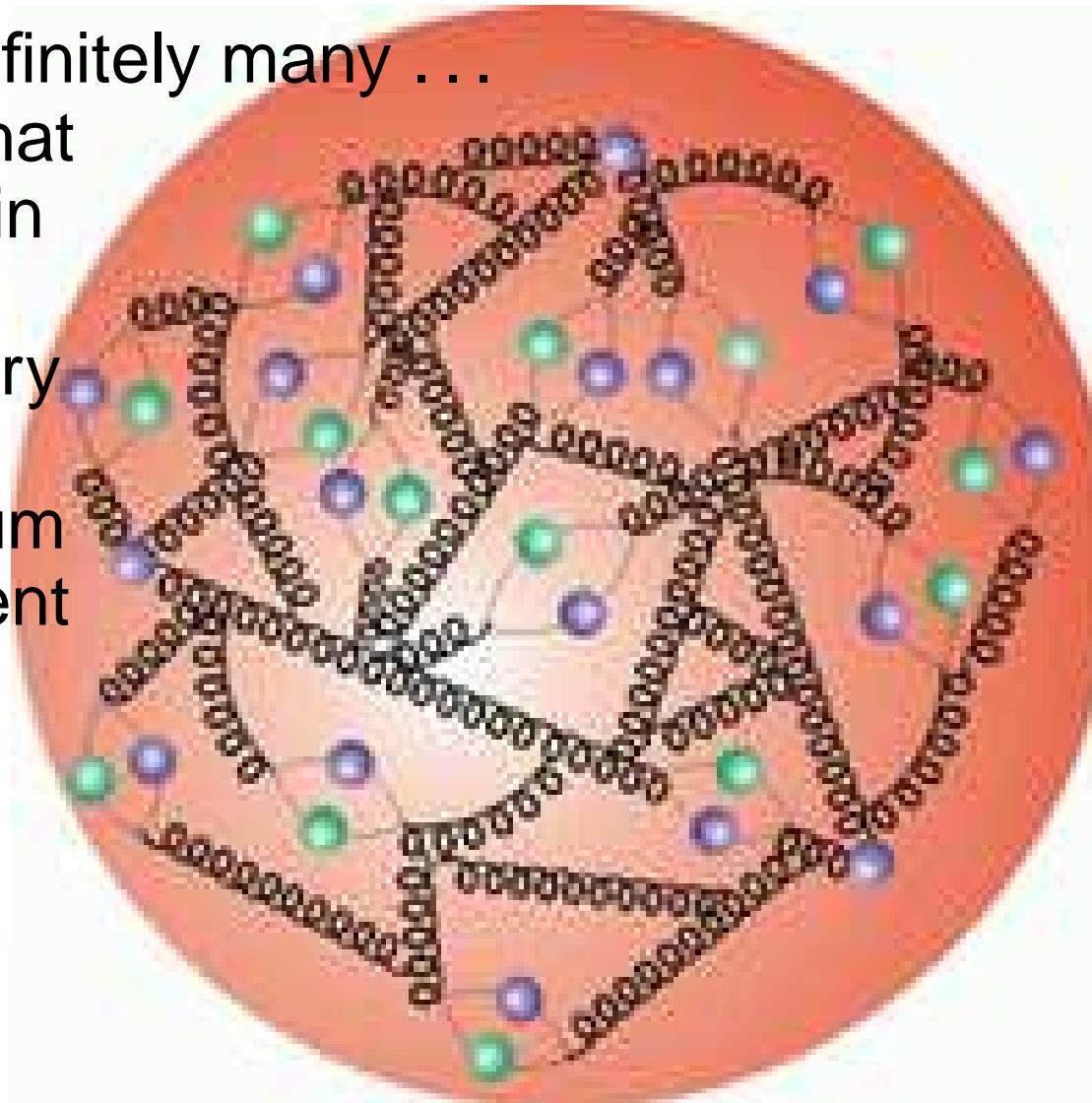


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field theory

...  
momentum  
-dependent  
dressing



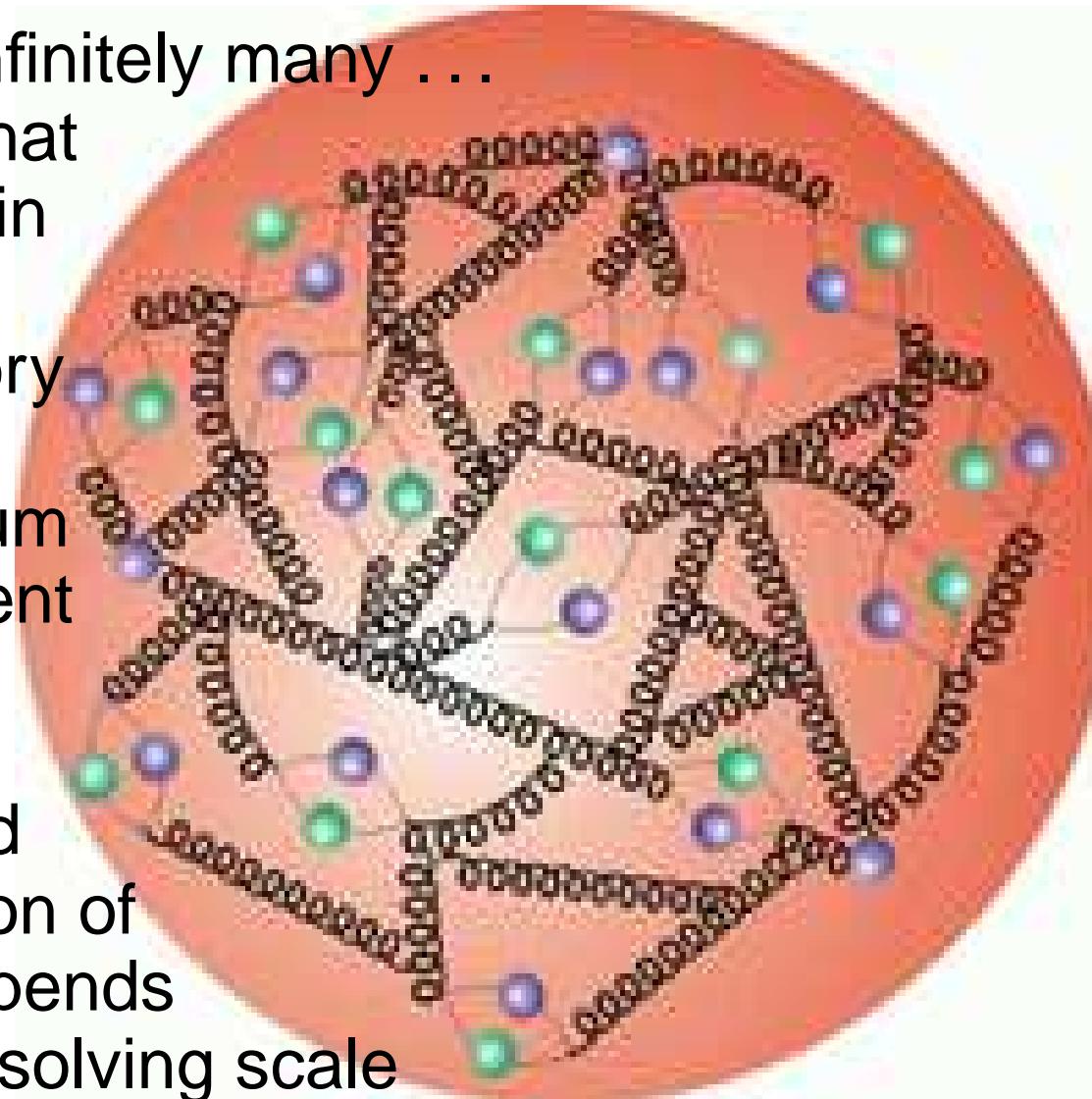
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...  
momentum  
-dependent  
dressing

...  
perceived  
distribution of  
mass depends  
on the resolving scale



***but ...***

- Orbital angular momentum is not a Poincaré invariant.  
However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.



***but ...***

- Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.



**but ...**

- Pseudoscalar meson Bethe-Salpeter amplitude

$$\begin{aligned}\chi_{\pi}(k; P) &= \gamma_5 [i\mathcal{E}_{\pi_n}(k; P) + \gamma \cdot P \mathcal{F}_{\pi_n}(k; P) \\ &\quad \gamma \cdot k \, k \cdot P \mathcal{G}_{\pi_n}(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} \mathcal{H}_{\pi_n}(k; P)]\end{aligned}$$



***but ...***

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- $J = 0 \dots$  *but* while  $\mathcal{E}$  and  $\mathcal{F}$  are purely  $L = 0$  in the rest frame, the  $\mathcal{G}$  and  $\mathcal{H}$  terms are associated with  $L = 1$ . Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both  $S$ - and  $P$ -wave components.



**but ...**

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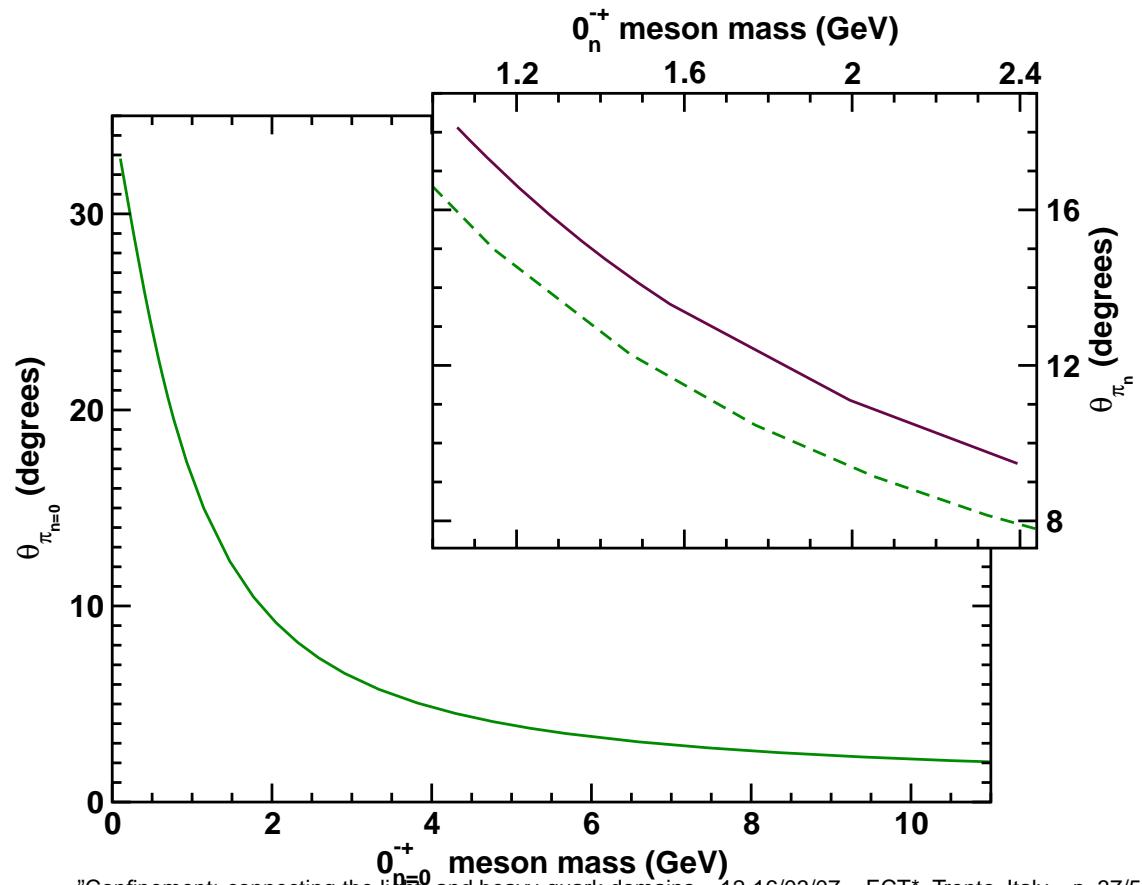


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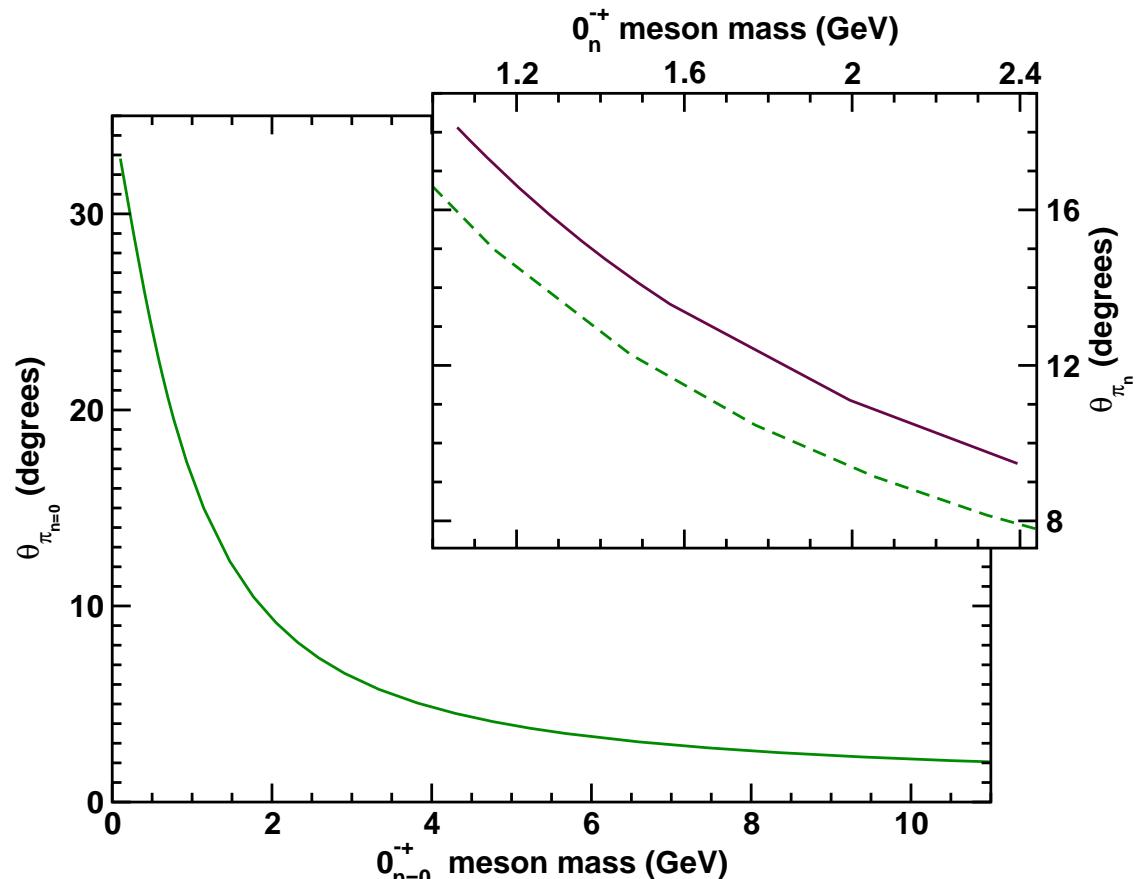
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$L$  is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.



# Are we *there* yet?



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# New Challenges



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# New Challenges

- Maris & Tandy ... series of five papers ... excellent description of light pseudoscalar and vector mesons ... basket of 31 masses/couplings/radii with r.m.s. error of 15% ... moreover, prediction of  $F_\pi(Q^2)$  measured in Hall C.



Pieter Maris

Peter Tandy



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- One parameter model ... parameter specifies long-range interaction between light-quarks ... model-independent results in ultraviolet



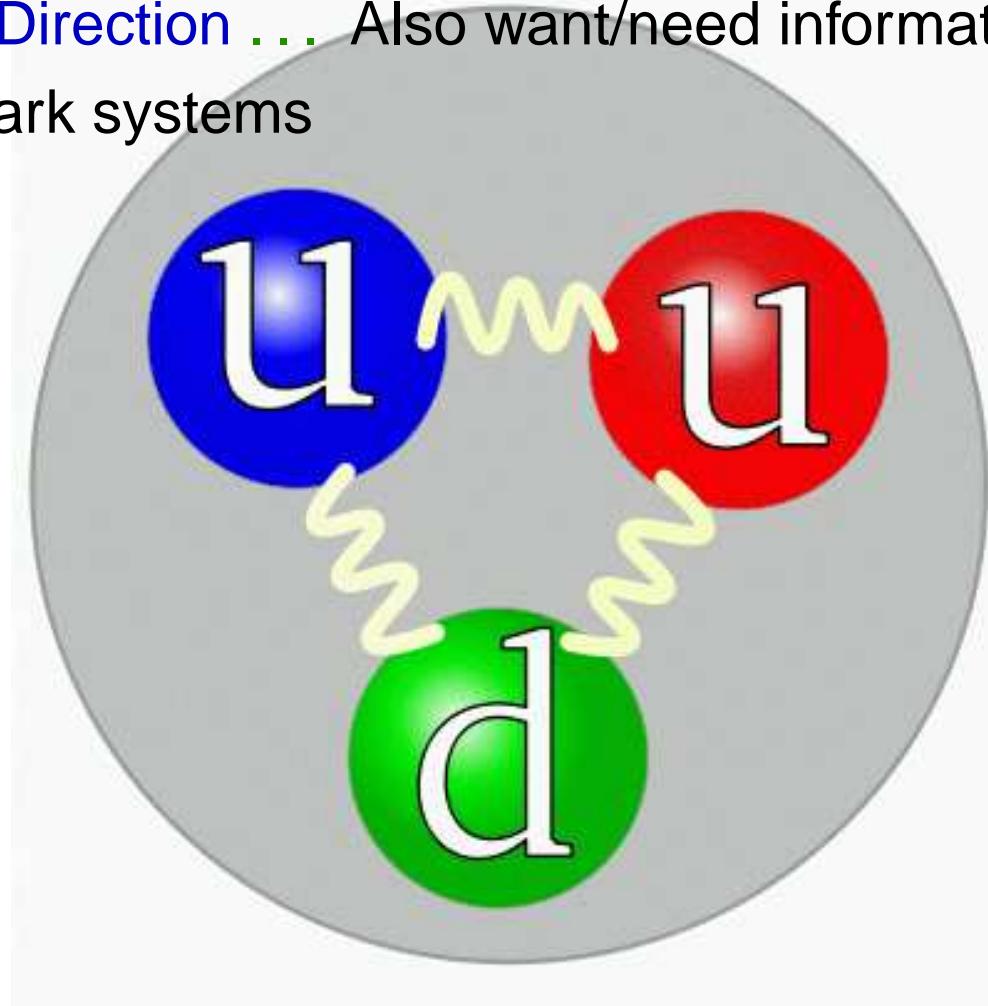
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- One parameter model ... parameter specifies long-range interaction between light-quarks ... model-independent results in ultraviolet
- Next Steps ... Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks



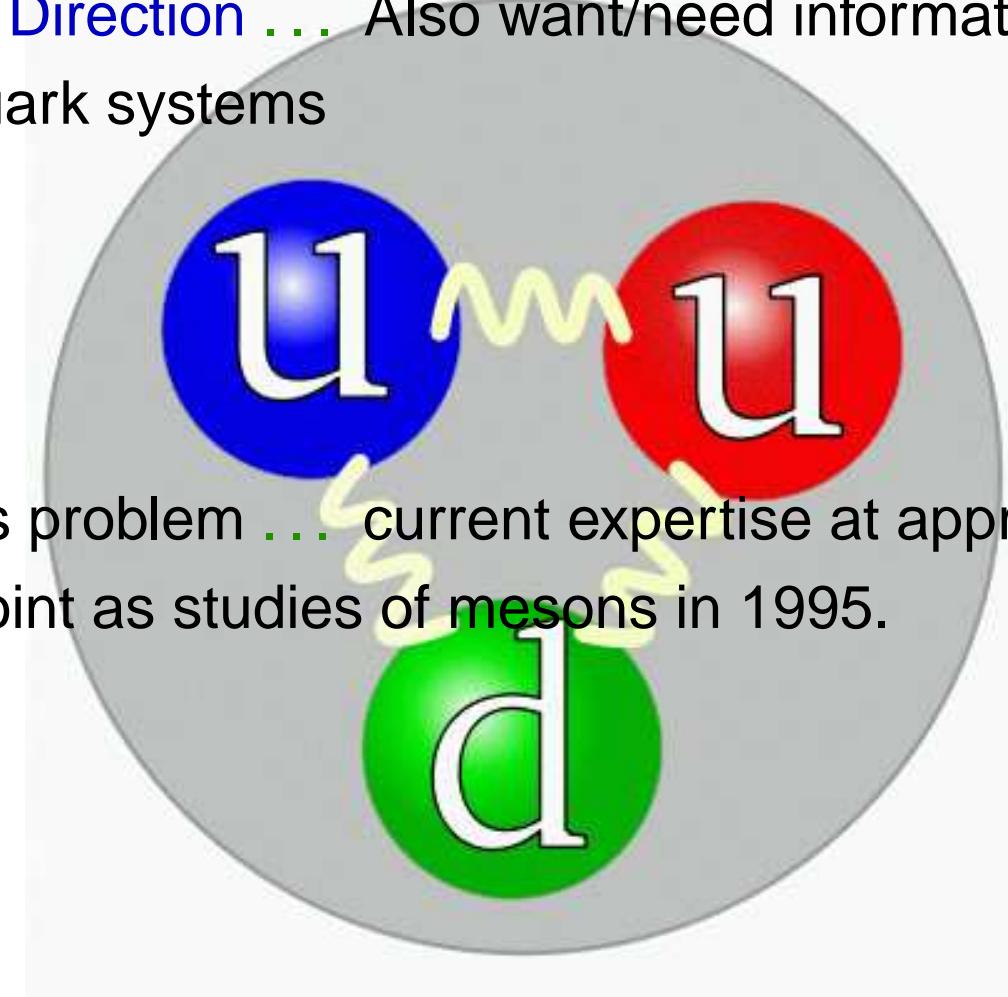
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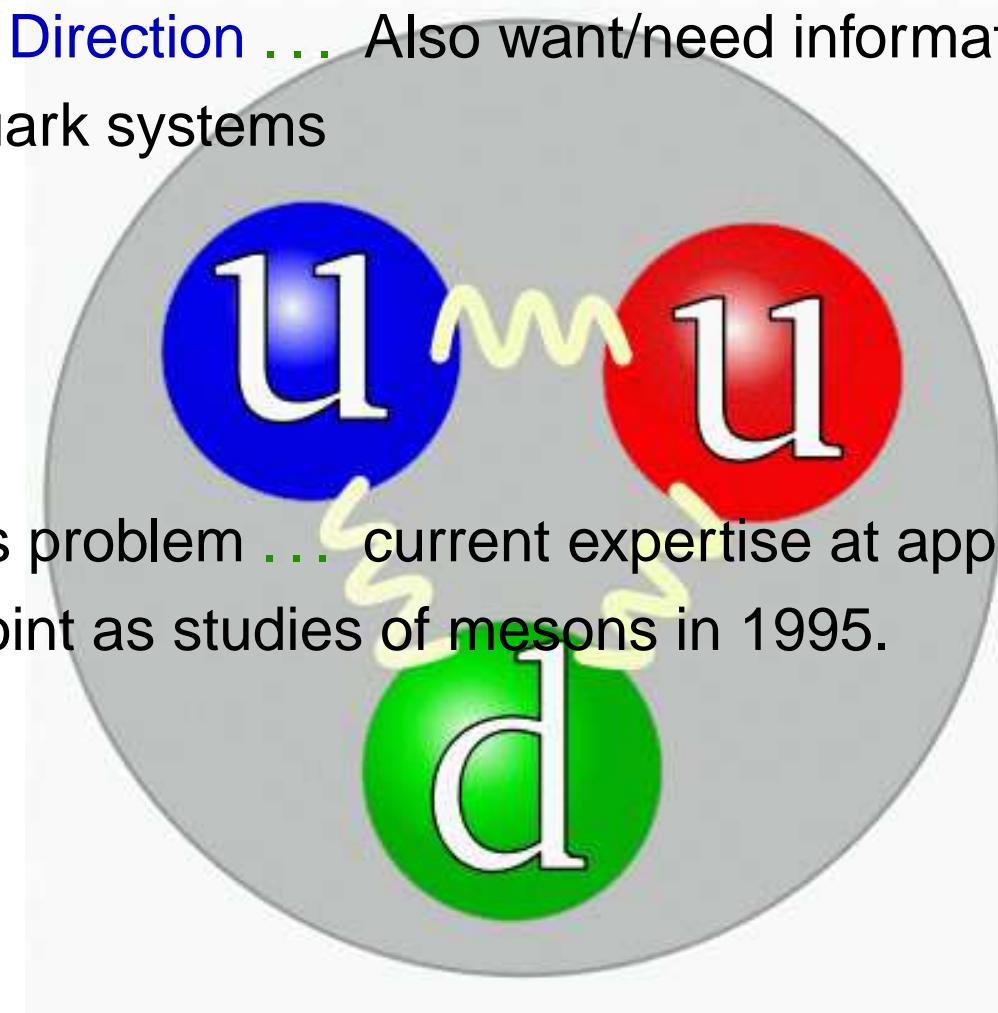


- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.



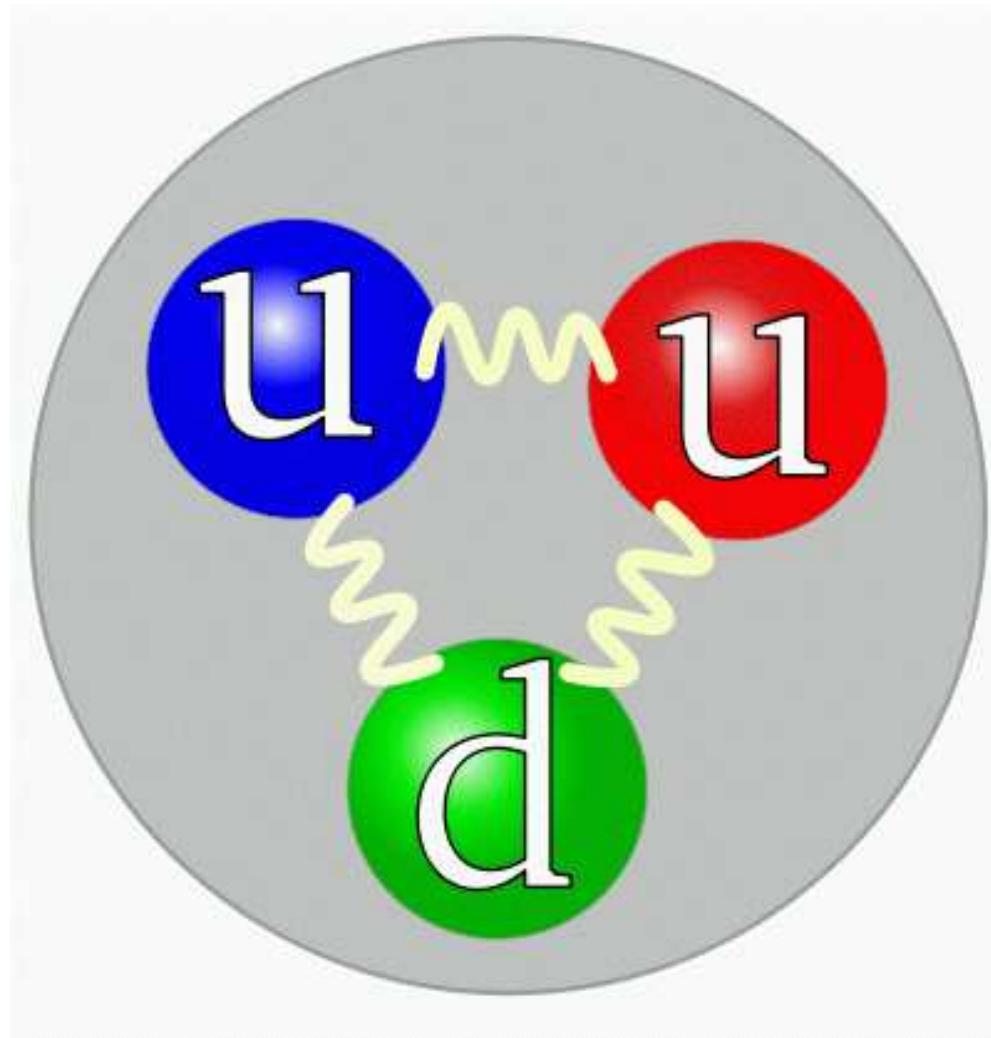
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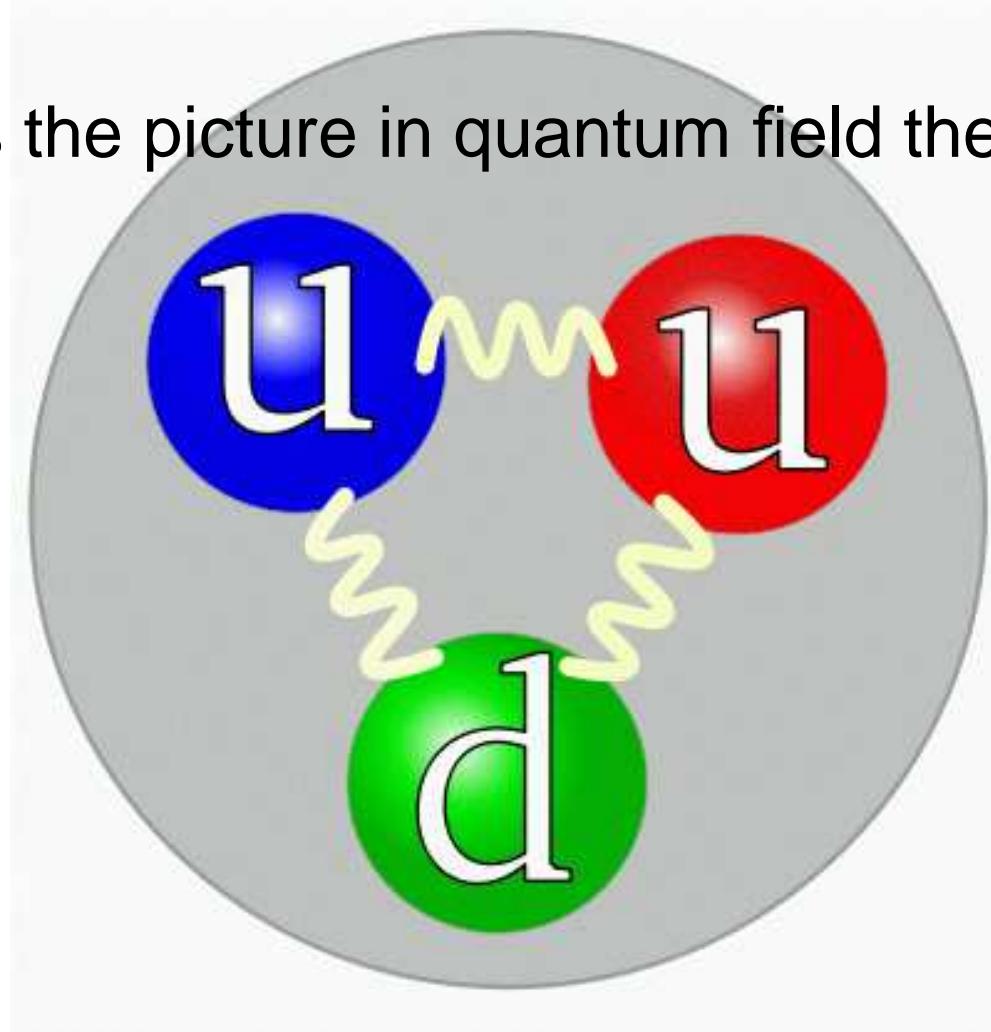
- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.

## Three-body Problem?



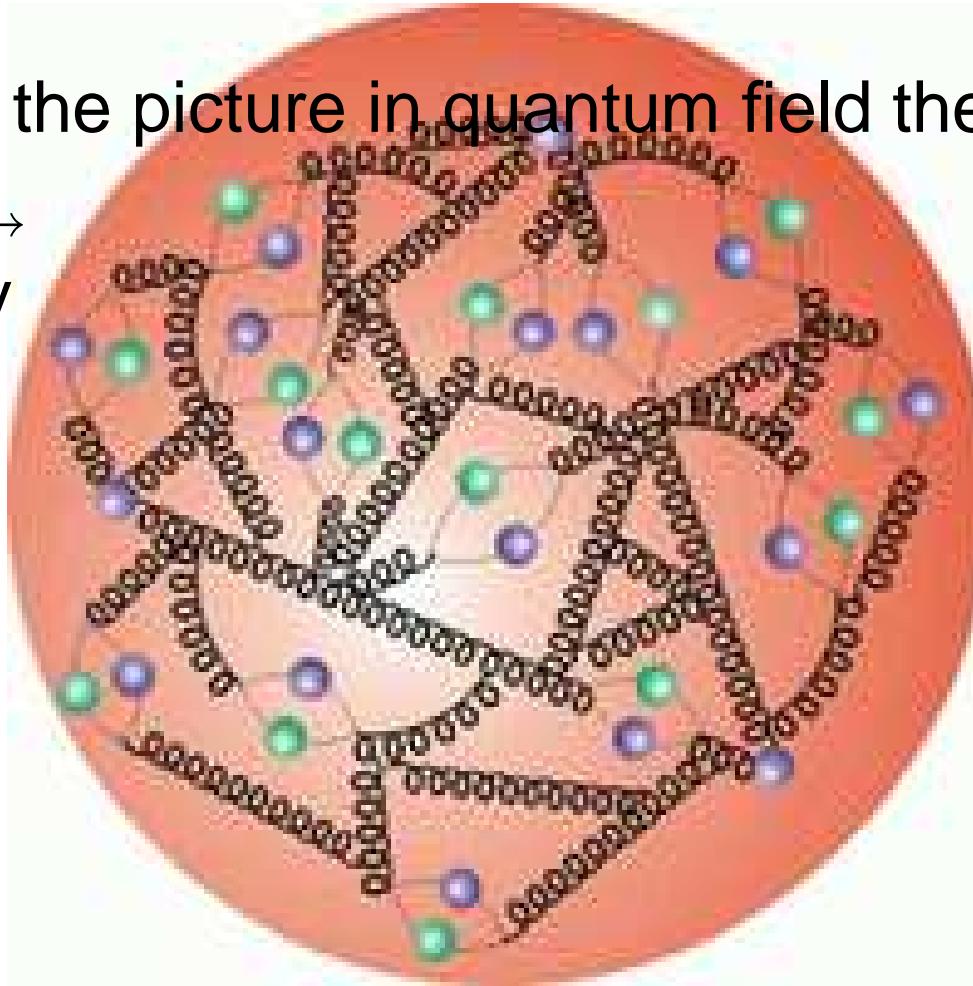
## Three-body Problem?

- What is the picture in quantum field theory?



## Three-body Problem?

- What is the picture in quantum field theory?
- Three → infinitely many!



# *Faddeev equation*



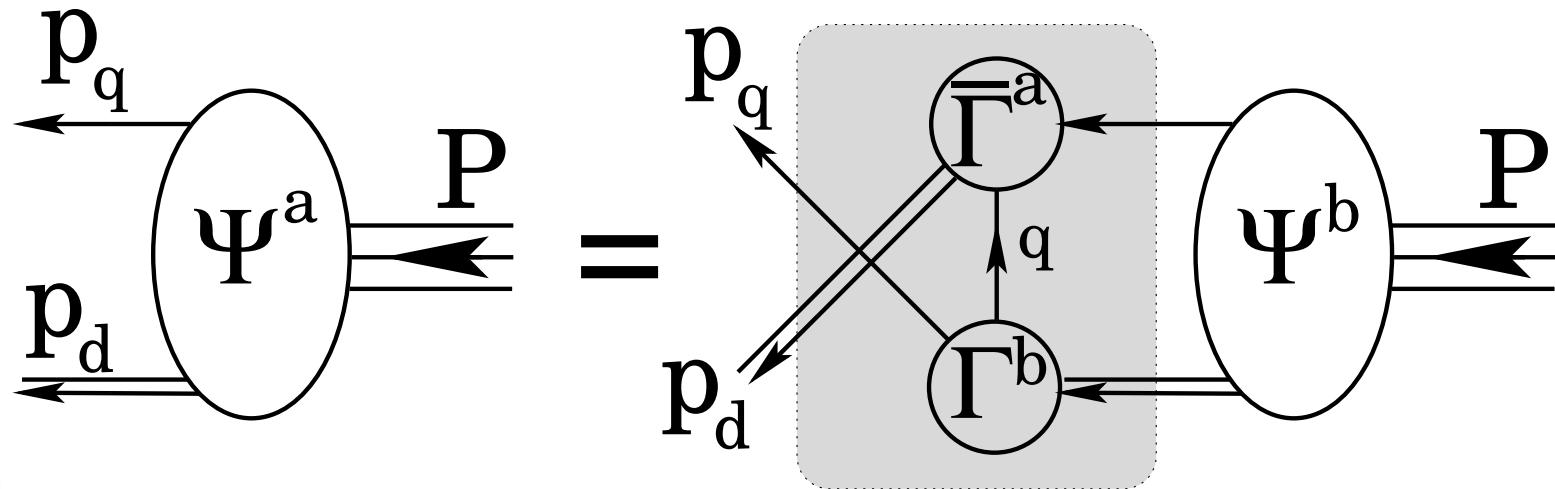
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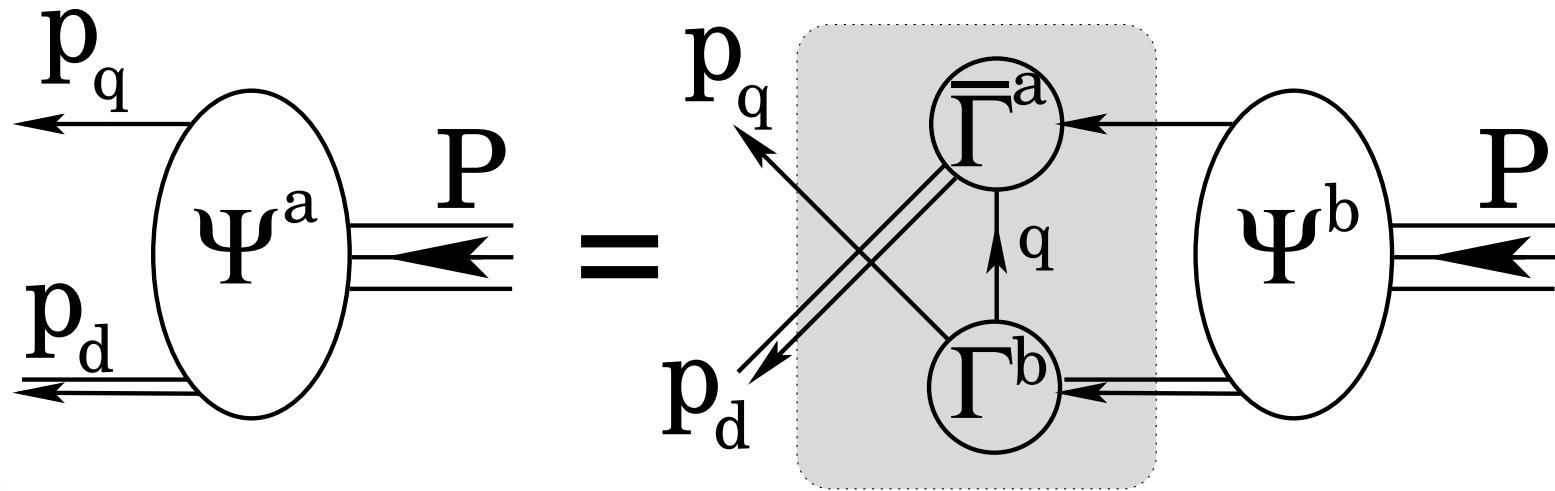
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# Faddeev equation



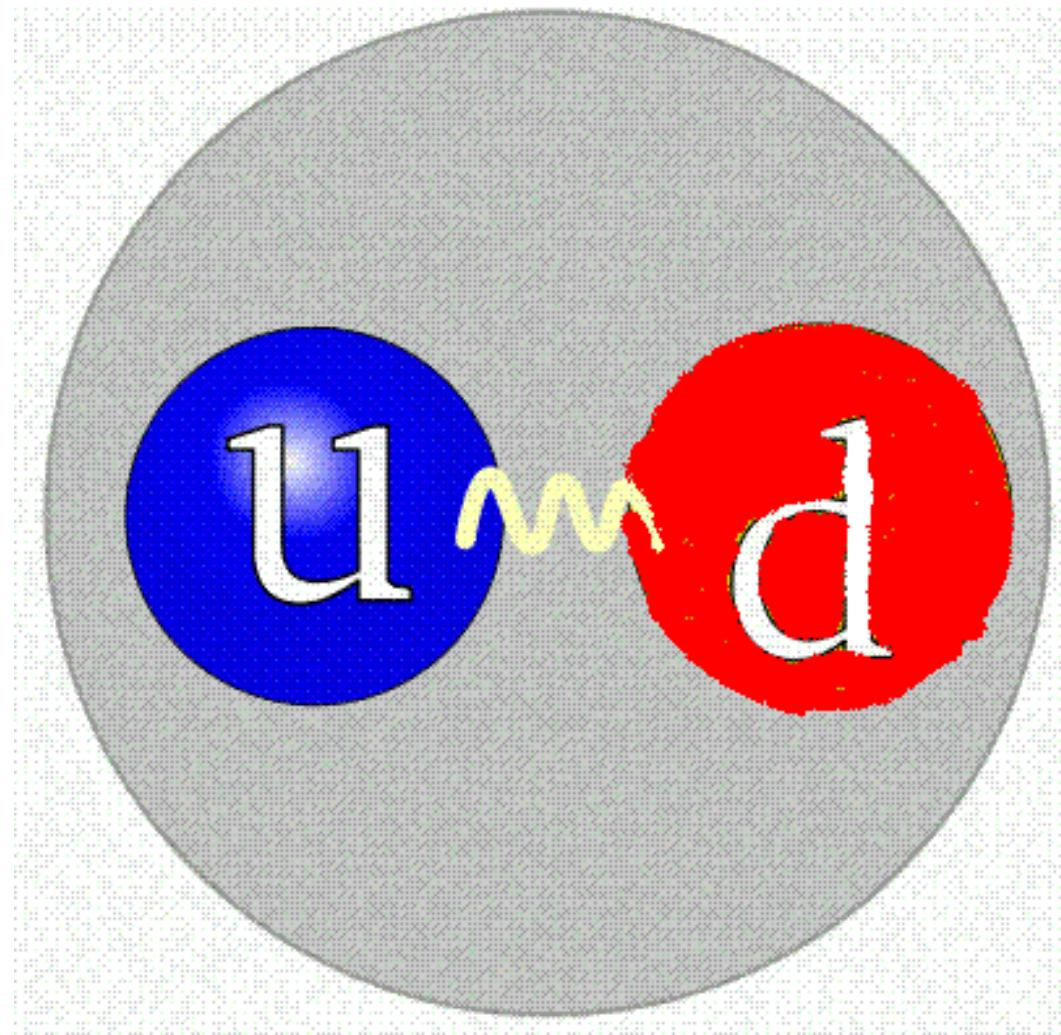
# Faddeev equation



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (**Poincaré Covariant Faddeev Amplitude**) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame **Amplitude** has ... *s*–, *p*– & *d*–wave correlations



# *Diquark correlations*



QUARK-QUARK



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# Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations:  
**blue-red, blue-green,  
green-red**

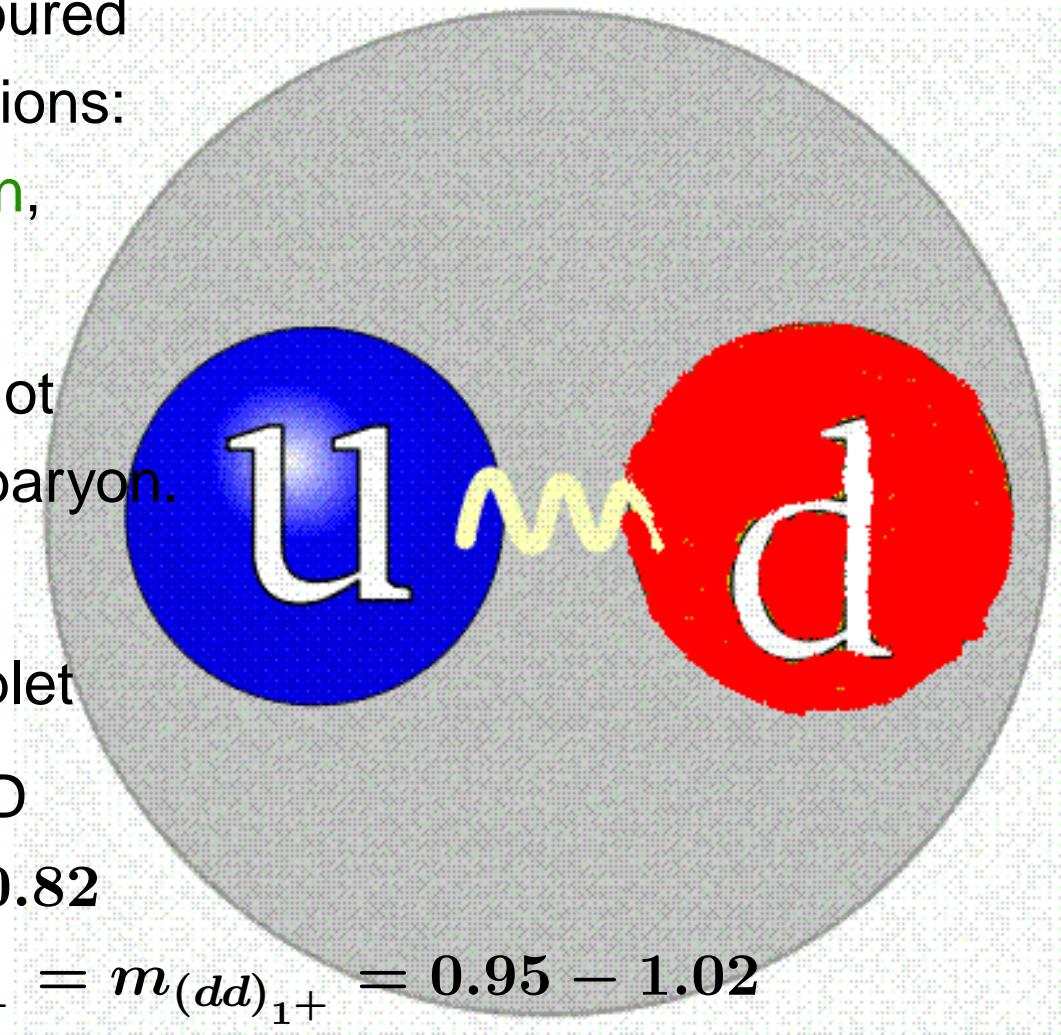
- Confined ... Does not escape from within baryon.

- Scalar is isosinglet,  
Axial-vector is isotriplet

- DSE and lattice-QCD

$$m_{[ud]_0^+} = 0.74 - 0.82$$

$$m_{(uu)_1^+} = m_{(ud)_1^+} = m_{(dd)_1^+} = 0.95 - 1.02$$



QUARK-QUARK

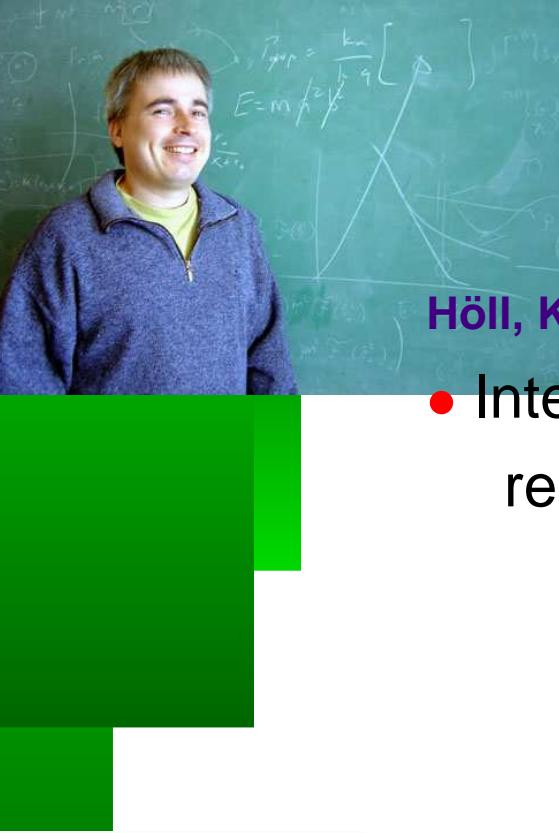




# Nucleon EM Form Factors: A Précis

Höll, Kloker, et al.: nu-th/0412046 & nu-th/0501033





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**Easily** obtained:

$$\left( \frac{1}{N_H} \sum_H \frac{[M_H^{\text{exp}} - M_H^{\text{calc}}]^2}{[M_H^{\text{exp}}]^2} \right)^{1/2} = 2\%$$



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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)



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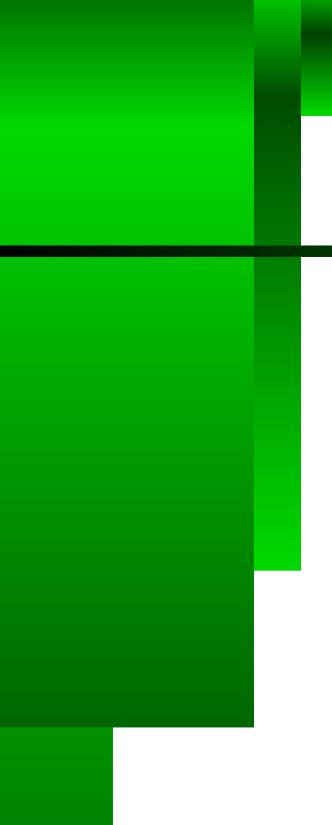
- **But** is that good?
    - Cloudy Bag:  $\delta M_+^{\pi\text{-loop}} = -300$  to  $-400$  MeV!
  - **Critical** to anticipate pion cloud effects
- Roberts, Tandy, Thomas, et al., nu-th/02010084



*Harry Lee*

# *Pions and Form Factors*

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## Pions and Form Factors

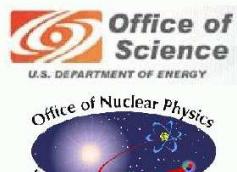
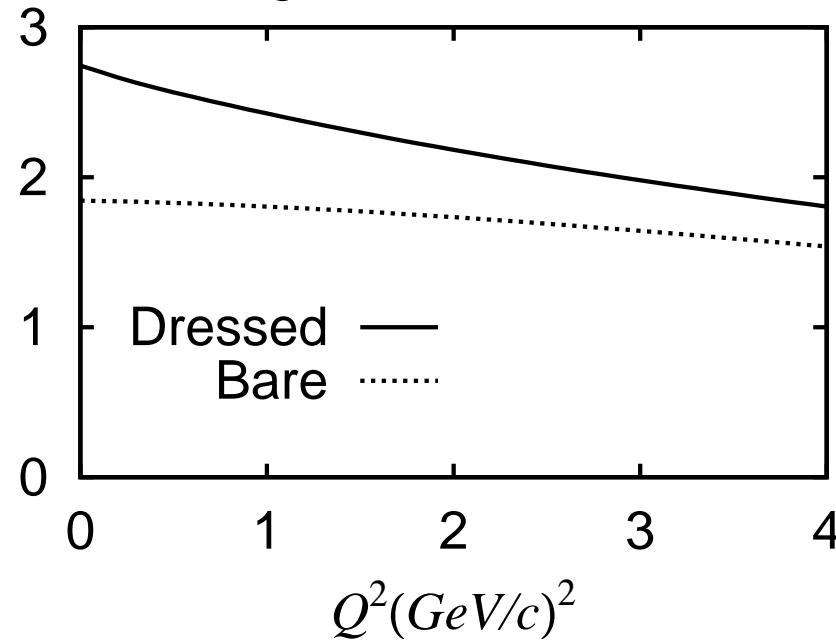
- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the  $\Delta(1236)$ 
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*Ratio of the M1 form factor in  $\gamma N \rightarrow \Delta$  transition and proton dipole form factor  $G_D$ . Solid curve is  $G_M^*(Q^2)/G_D(Q^2)$  including pions; Dotted curve is  $G_M(Q^2)/G_D(Q^2)$  without pions.*



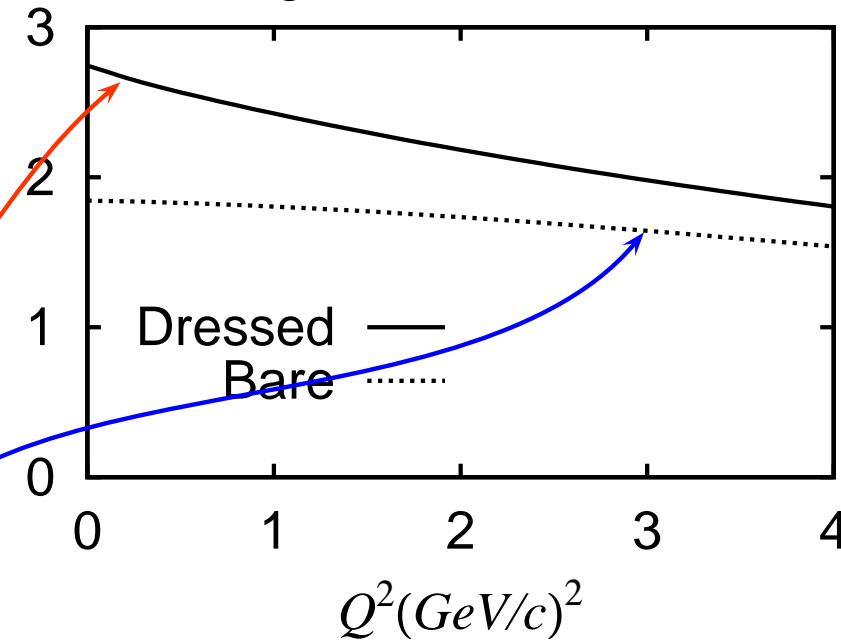
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## Quark Core

- Responsible for only 2/3 of result at small  $Q^2$
- Dominant for  $Q^2 > 2 - 3 \text{ GeV}^2$



# Results: Nucleon and $\Delta$ Masses

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Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and  $\Delta$  masses



**Set A** – fit to the actual masses was required; whereas for  
**Set B** – fitted mass was offset to allow for “ $\pi$ -cloud” contributions

set	$M_N$	$M_\Delta$	$m_{0+}$	$m_{1+}$	$\omega_{0+}$	$\omega_{1+}$
A	0.94	1.23	0.63	0.84	$0.44=1/(0.45 \text{ fm})$	$0.59=1/(0.33 \text{ fm})$
B	1.18	1.33	0.79	0.89	$0.56=1/(0.35 \text{ fm})$	$0.63=1/(0.31 \text{ fm})$

- $m_{1+} \rightarrow \infty$ :  $M_N^A = 1.15 \text{ GeV}$ ;  $M_N^B = 1.46 \text{ GeV}$



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- Axial-vector diquark provides significant attraction



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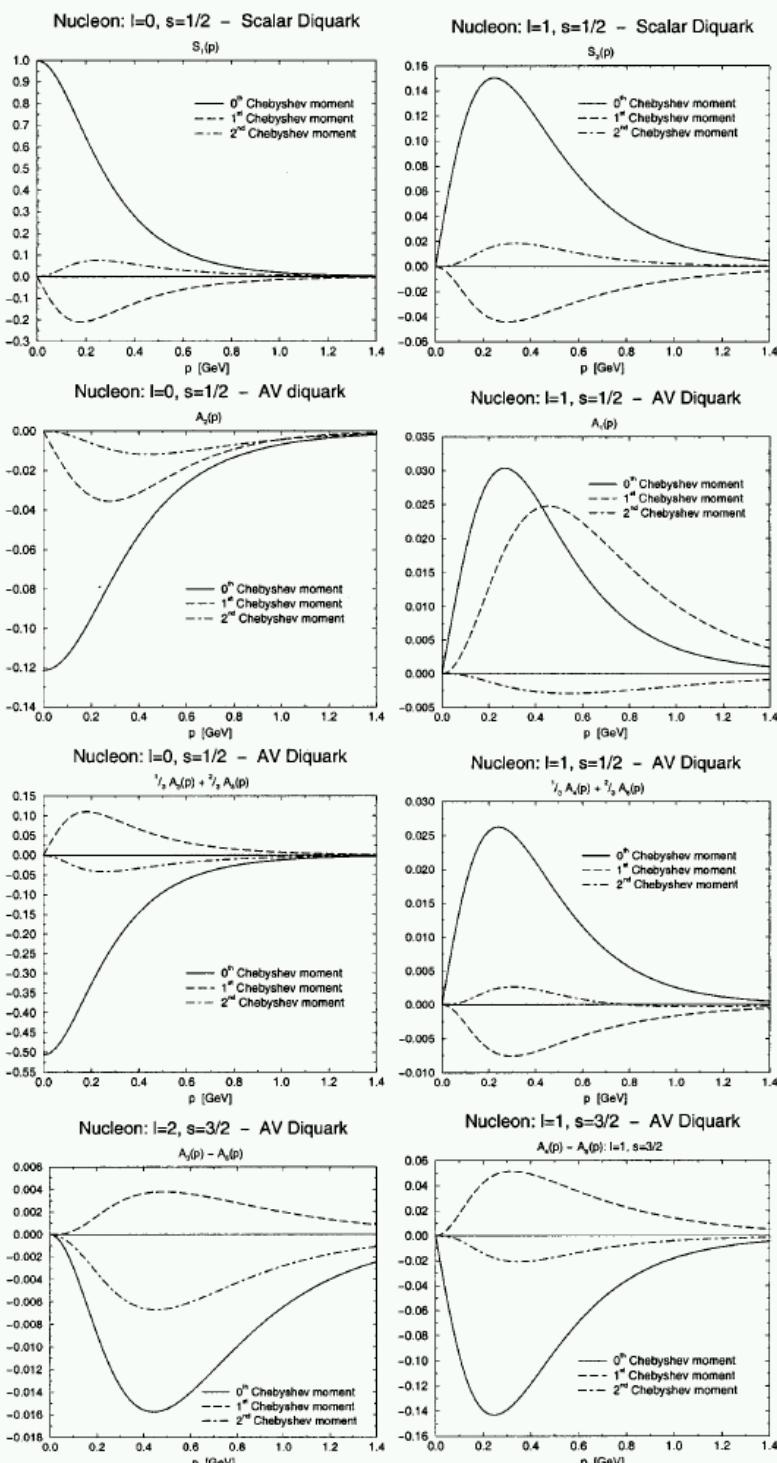
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- **Constructive Interference**:  $1^{++}$ -diquark +  $\partial_\mu \pi$



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# Angular Momentum Rest Frame

---

M. Oettel, et al.  
nucl-th/9805054

Crude estimate based on magnitudes  $\Rightarrow$  probability for a  $u$ -quark to carry the proton's spin is  $P_{u\uparrow} \sim 80\%$ , with

$P_{u\downarrow} \sim 5\%$ ,  $P_{d\uparrow} \sim 5\%$ ,  
 $P_{d\downarrow} \sim 10\%$ .

Hence, by this reckoning  $\sim 30\%$  of proton's rest-frame spin is located in dressed-quark angular momentum.

# Nucleon-Photon Vertex



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M. Oettel, M. Pichowsky  
and L. von Smekal, nu-th/9909082

6 terms ...

## Nucleon-Photon Vertex

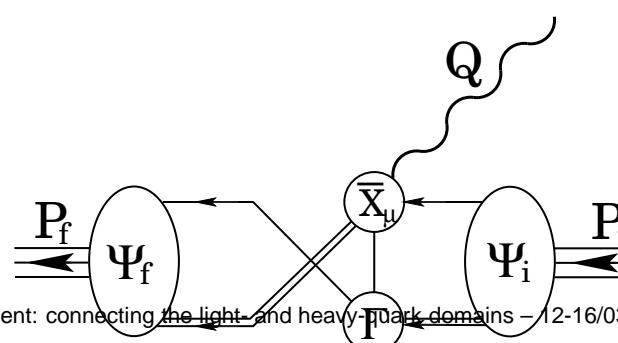
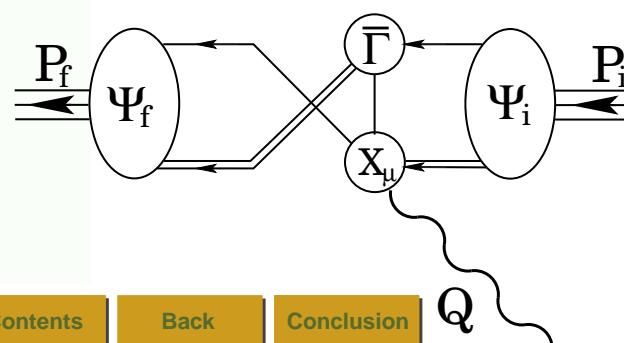
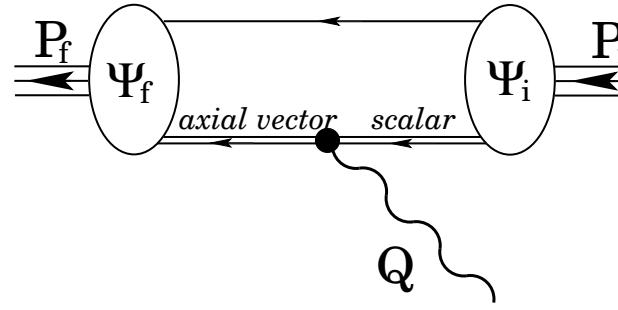
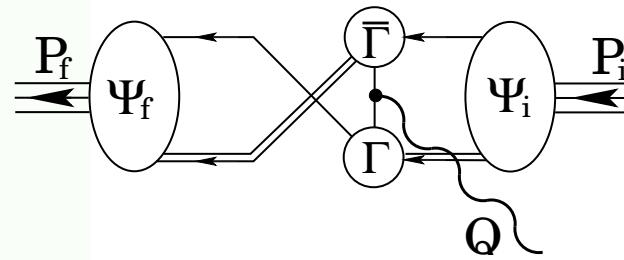
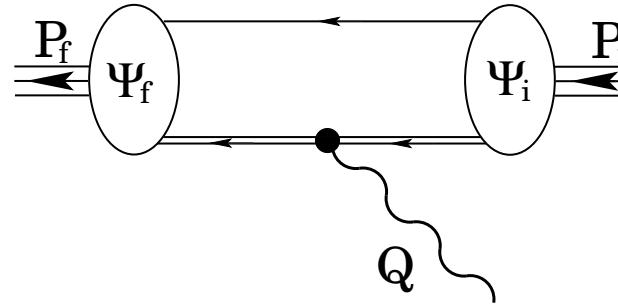
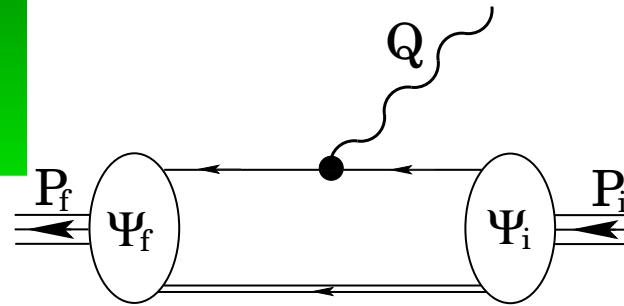
constructed systematically ... current conserved automatically  
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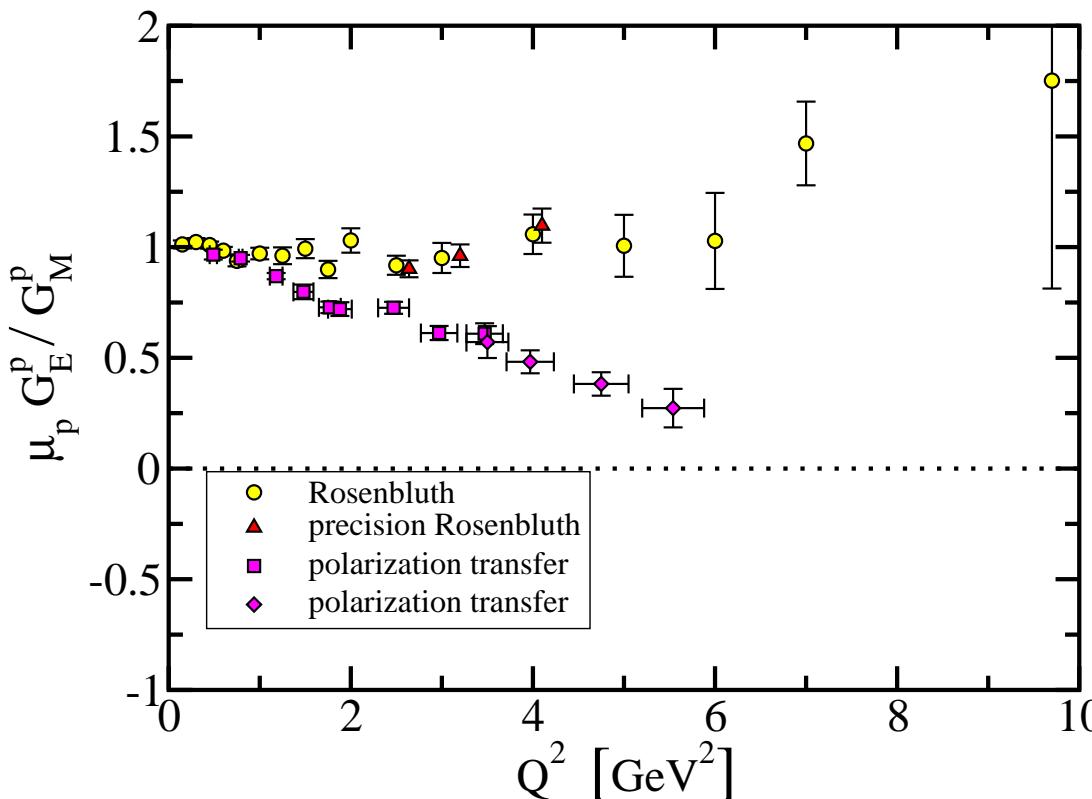
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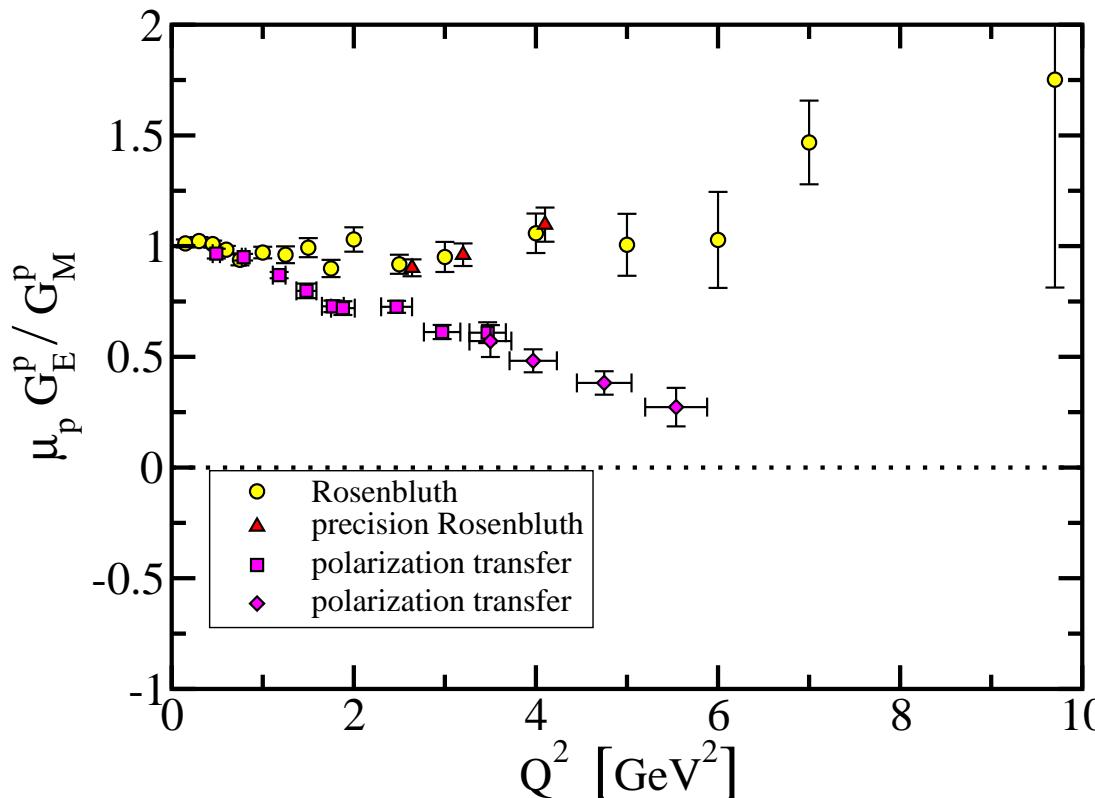


# Form Factor Ratio: $GE/GM$



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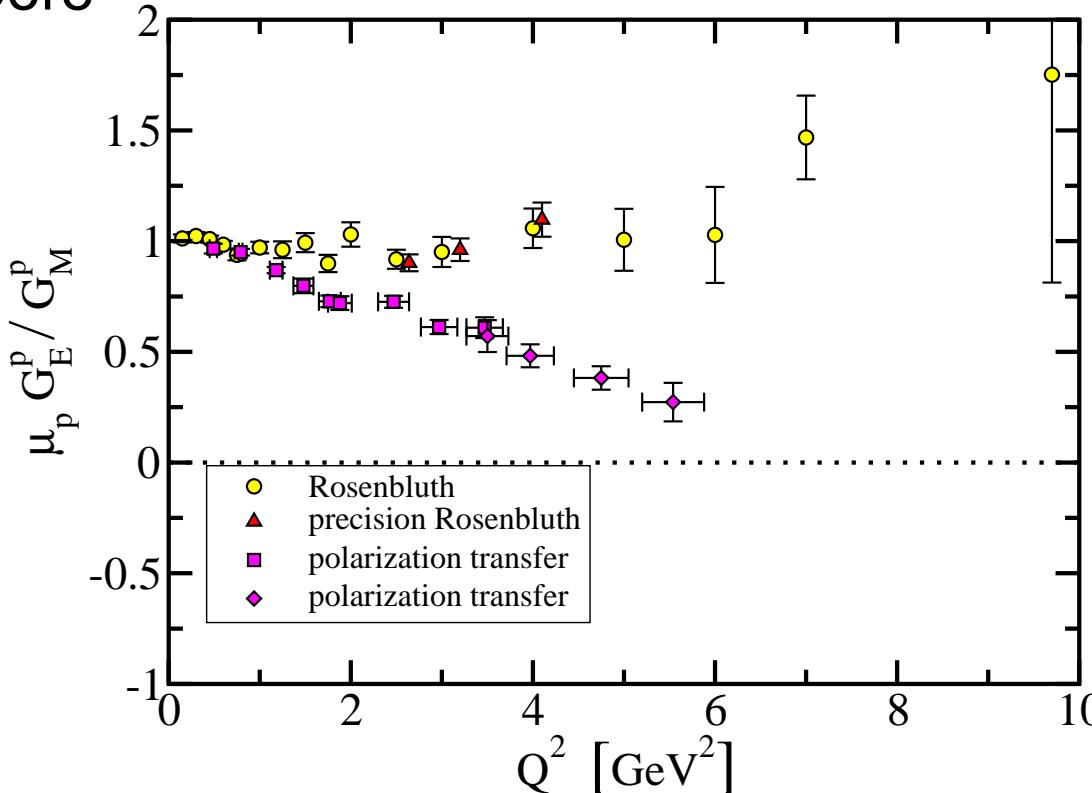
- Combine these elements ...



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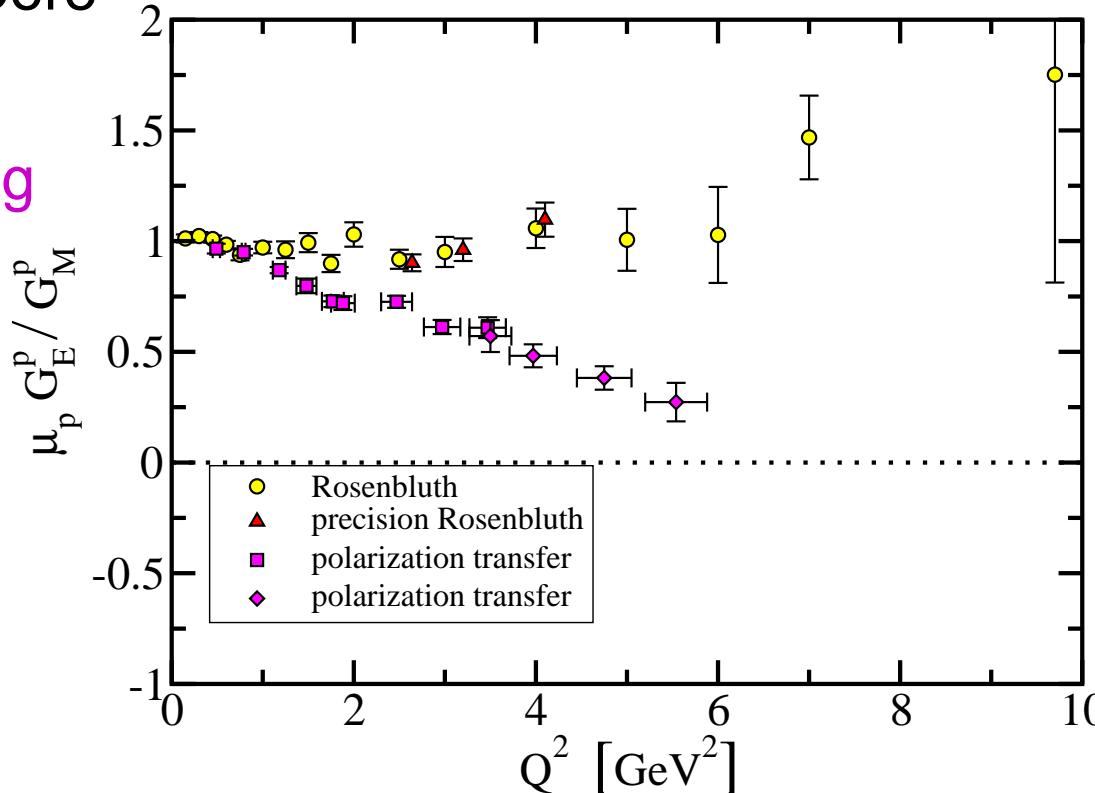
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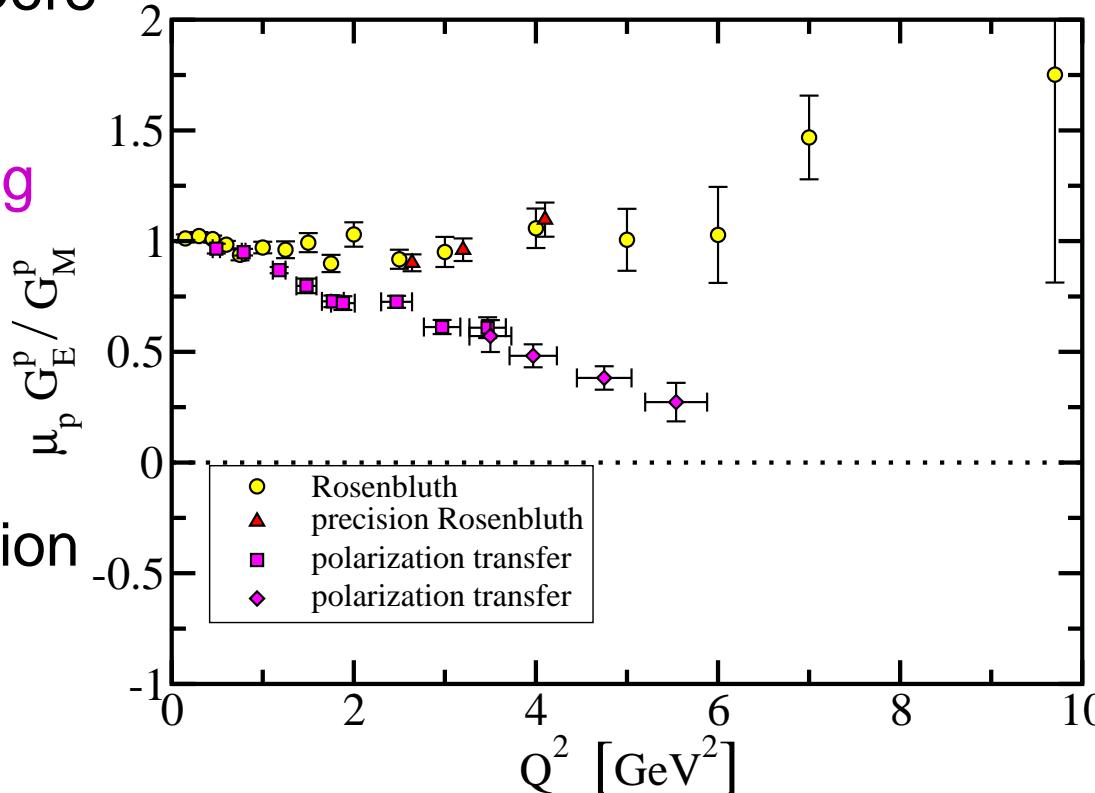
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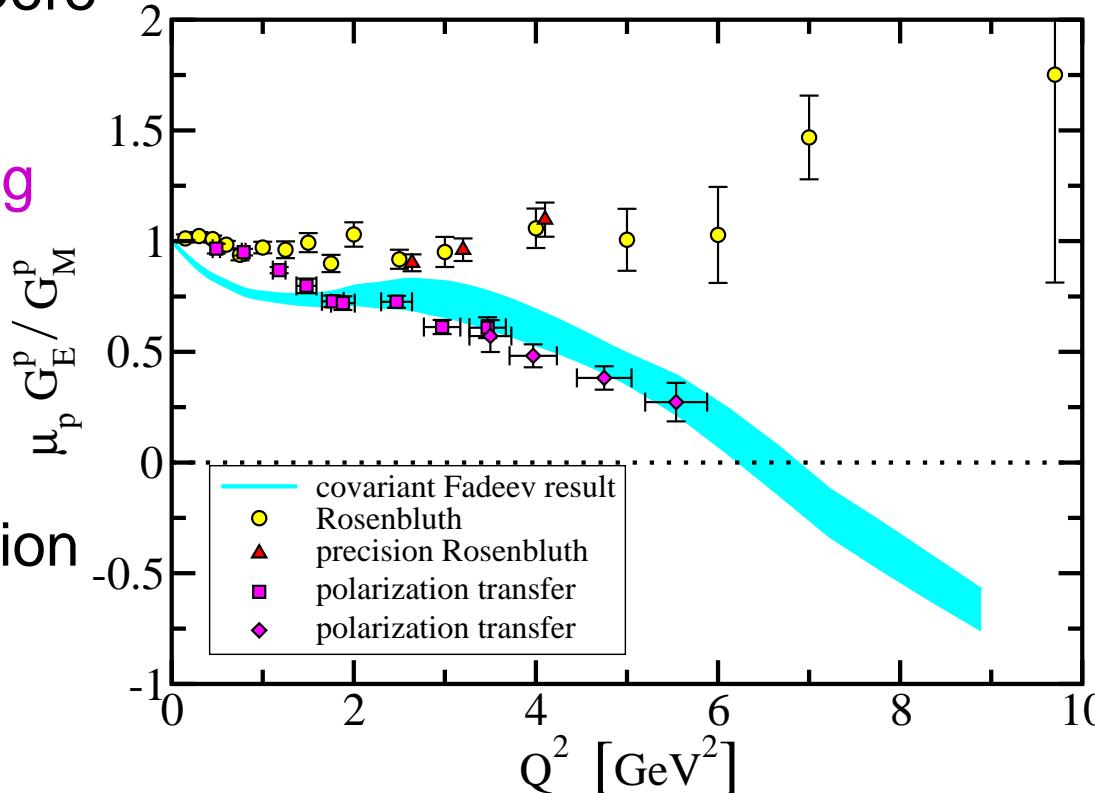
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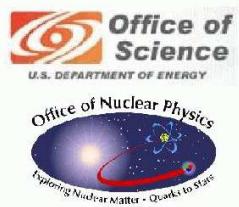
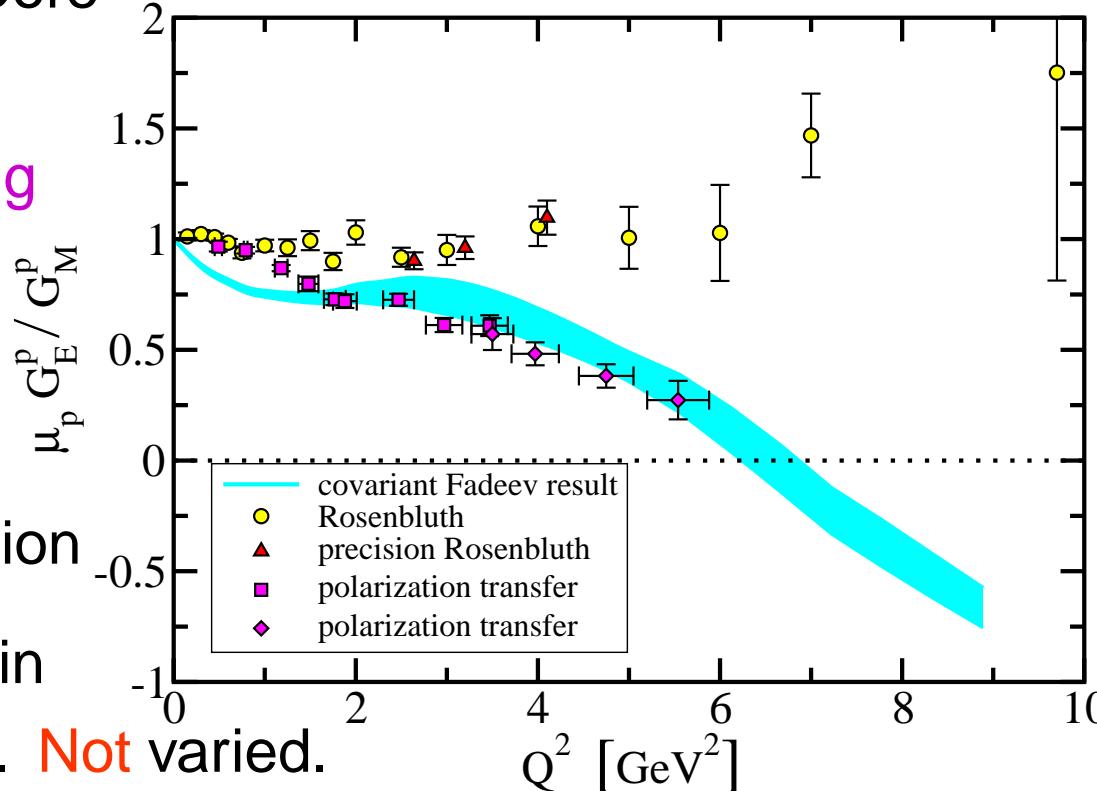


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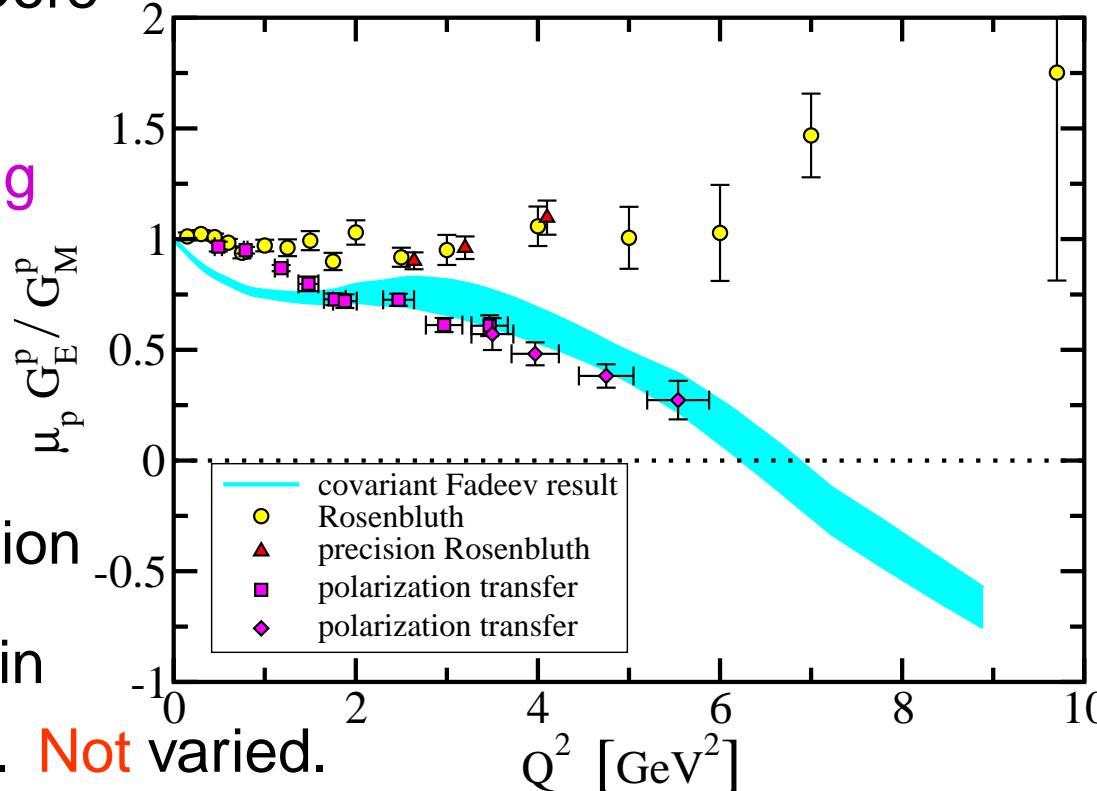
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- Agreement with Pol. Trans. data at  $Q^2 \gtrsim 2 \text{ GeV}^2$

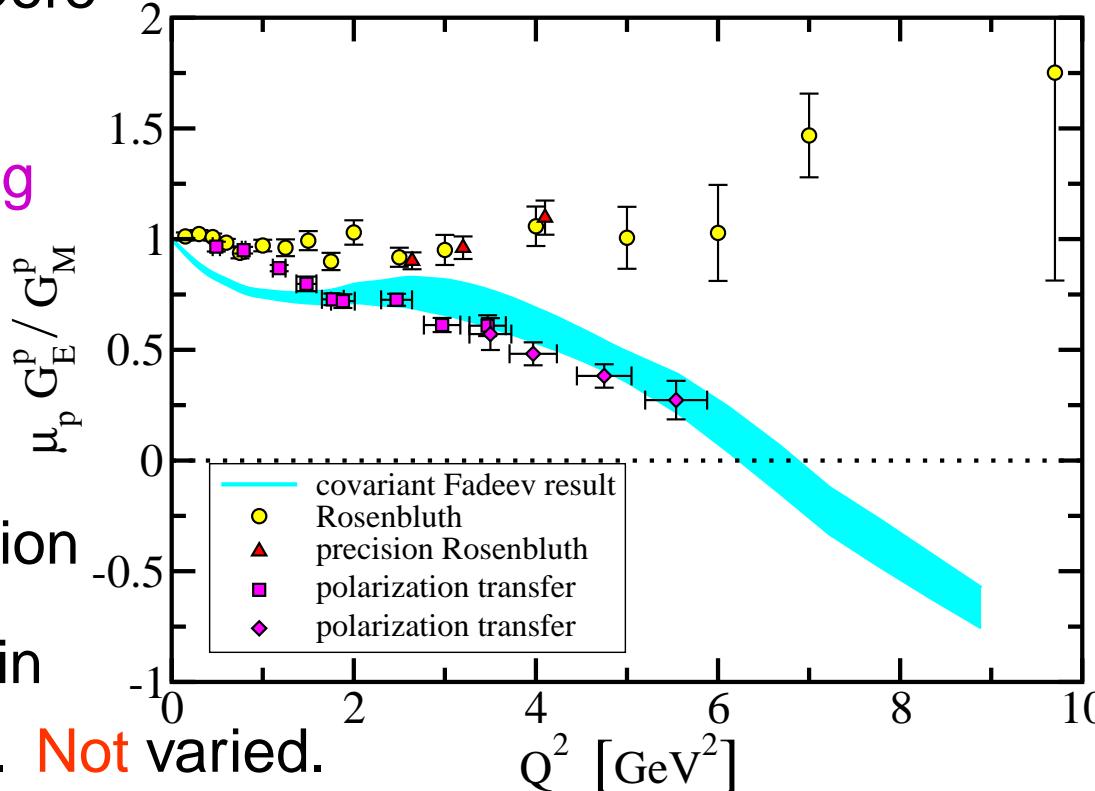


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- Ward-Takahashi*  
Identity preserving  
current
- Anticipate and  
Estimate Pion  
Cloud's Contribution

- All parameters fixed in  
other applications ... **Not varied.**
  - Agreement with Pol. Trans. data at  $Q^2 \gtrsim 2 \text{ GeV}^2$
  - Correlations in Faddeev amplitude – quark orbital  
angular momentum – essential to that agreement

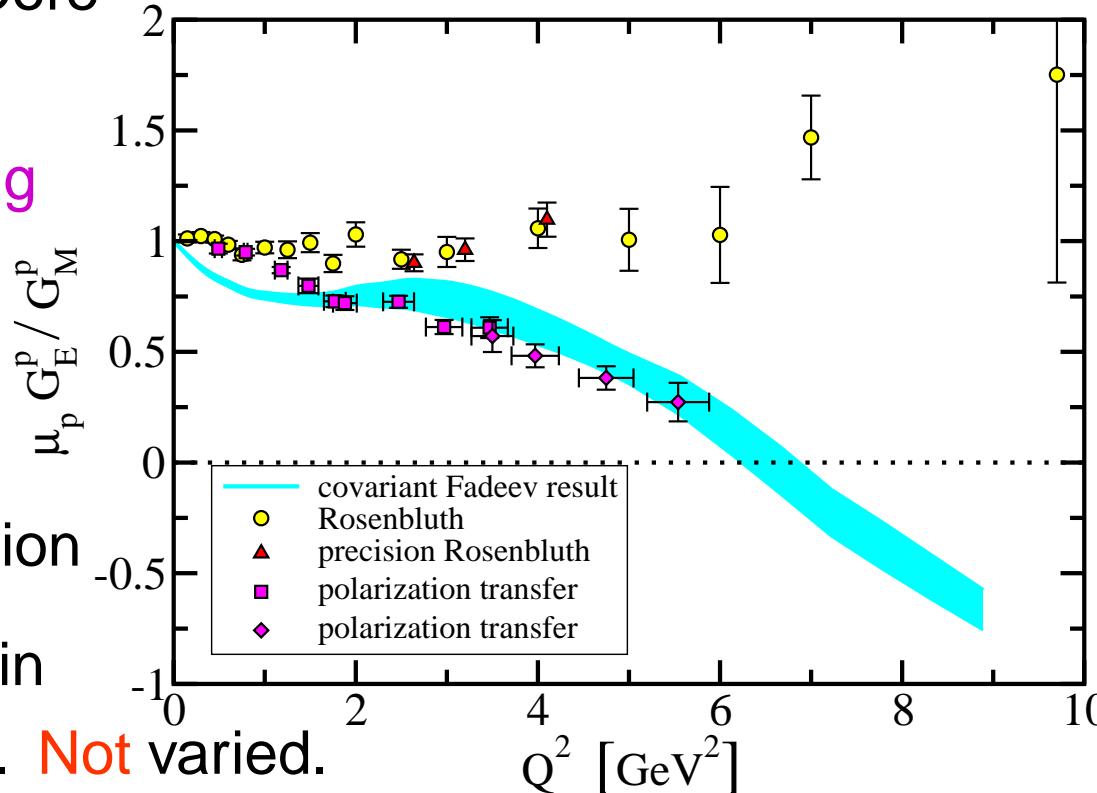


# Form Factor Ratio: GE/GM

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  - Correlations in Faddeev amplitude – quark orbital  
angular momentum – essential to that agreement
  - Predict Zero at  $Q^2 \approx 6.5 \text{ GeV}^2$



# *Chiral Corrections*



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# *Chiral Corrections*

- Thus far, omitted pion cloud contribution to current



# *Chiral Corrections*

- Thus far, omitted pion cloud contribution to current
- Include loops following method of
  - ... **Ashley**, Leinweber, Thomas, Young, [he-lat/0308024](#)
  - ... finite-range regularisation of loop corrections
  - ...  $\lambda$  = regularisation mass-scale



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$$\langle r_p^2 \rangle_{NA}^{1-loop^R} = \mp \frac{1 + 5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \lambda^2}\right),$$

$$\begin{aligned} \langle (r_N^\mu)^2 \rangle_{NA}^{1-loop^R} &= - \frac{1 + 5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \lambda^2}\right) \\ &\quad + \frac{g_A^2 M_N}{16\pi f_\pi^2 \mu_v} \frac{1}{m_\pi} \frac{2}{\pi} \arctan\left(\frac{\lambda}{m_\pi}\right), \end{aligned}$$

$$(\mu_n^p)_{NA}^{1-loop^R} = \mp \frac{g_A^2 M_N}{4\pi^2 f_\pi^2} m_\pi \frac{2}{\pi} \arctan\left(\frac{\lambda^3}{m_\pi^3}\right),$$



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  - ...  $\frac{1}{\lambda} = \frac{2}{3} \text{ fm}$

	$r_p$	$r_n$	$r_p^\mu$	$r_n^\mu$	$\mu_p$	$-\mu_n$	$\zeta$
$q-(qq)$ core	0.595	0.169	0.449	0.449	3.63	2.13	0.39
+ $\pi$ -loop correction	0.762	0.506	0.761	0.761	3.05	1.55	0.23
experiment	0.847	0.336	0.836	0.889	2.79	1.91	



# Chiral Corrections

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- Include loops following method of
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$$\dots \frac{1}{\lambda} = \frac{2}{3} \text{ fm}$$

$$\mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)} = 1 - \frac{Q^2}{6} \left[ (r_p)^2 - (r_p^\mu)^2 \right]$$

$$r_p \approx r_p^\mu \Rightarrow \text{ratio varies} < 10\% \text{ on } 0 < Q^2 < 0.6 \text{ GeV}^2$$



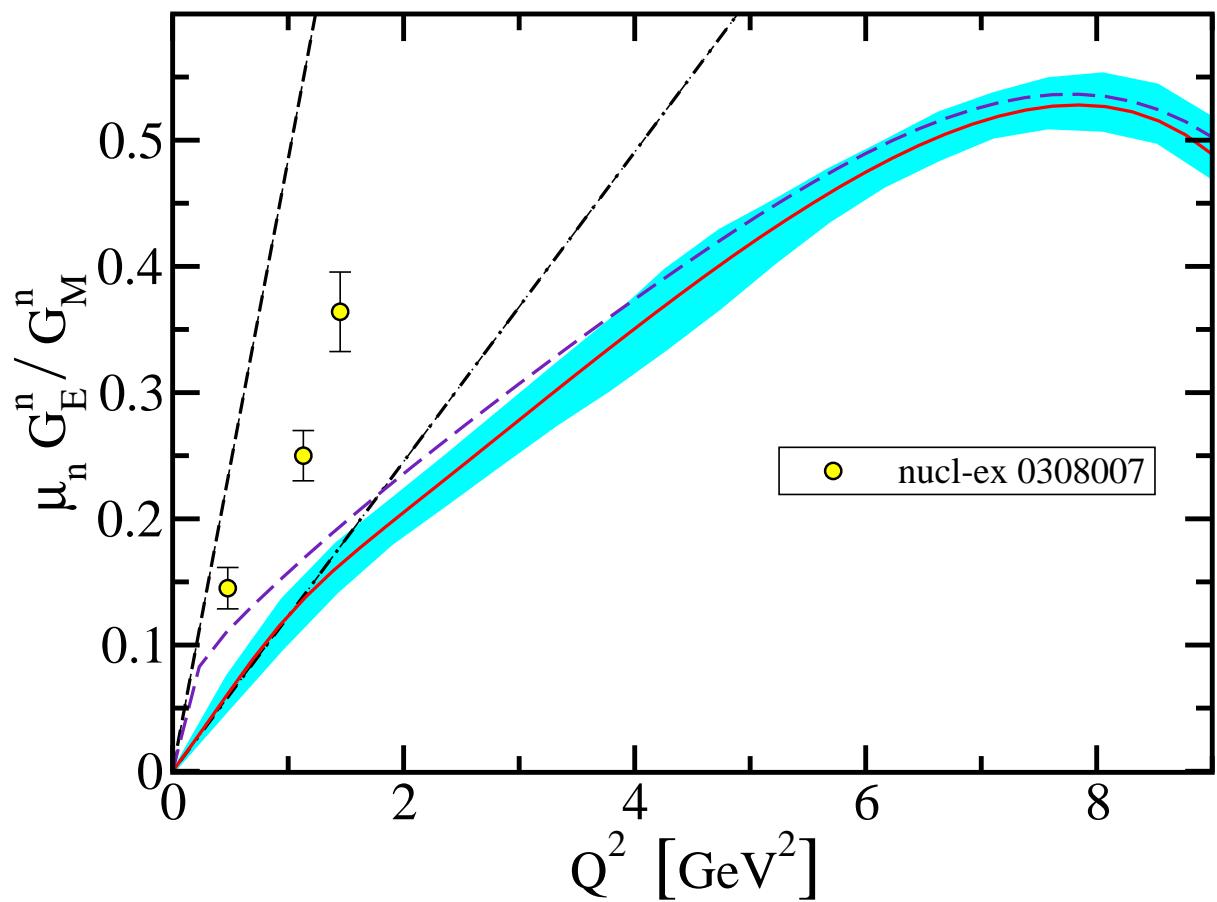
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- Thus far, omitted pion cloud contribution to current
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    - $\frac{1}{\lambda} = \frac{2}{3} \text{ fm}$
    - $\mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)} = 1 - \frac{Q^2}{6} \left[ (r_p)^2 - (r_p^\mu)^2 \right]$
    - $r_p \approx r_p^\mu \Rightarrow \text{ratio varies} < 10\% \text{ on } 0 < Q^2 < 0.6 \text{ GeV}^2$
- Complements nucleon mass considerations
  - veracious understanding of all nucleon properties
  - impossible without intelligent incorporation

of chiral corrections



# Neutron Form Factors



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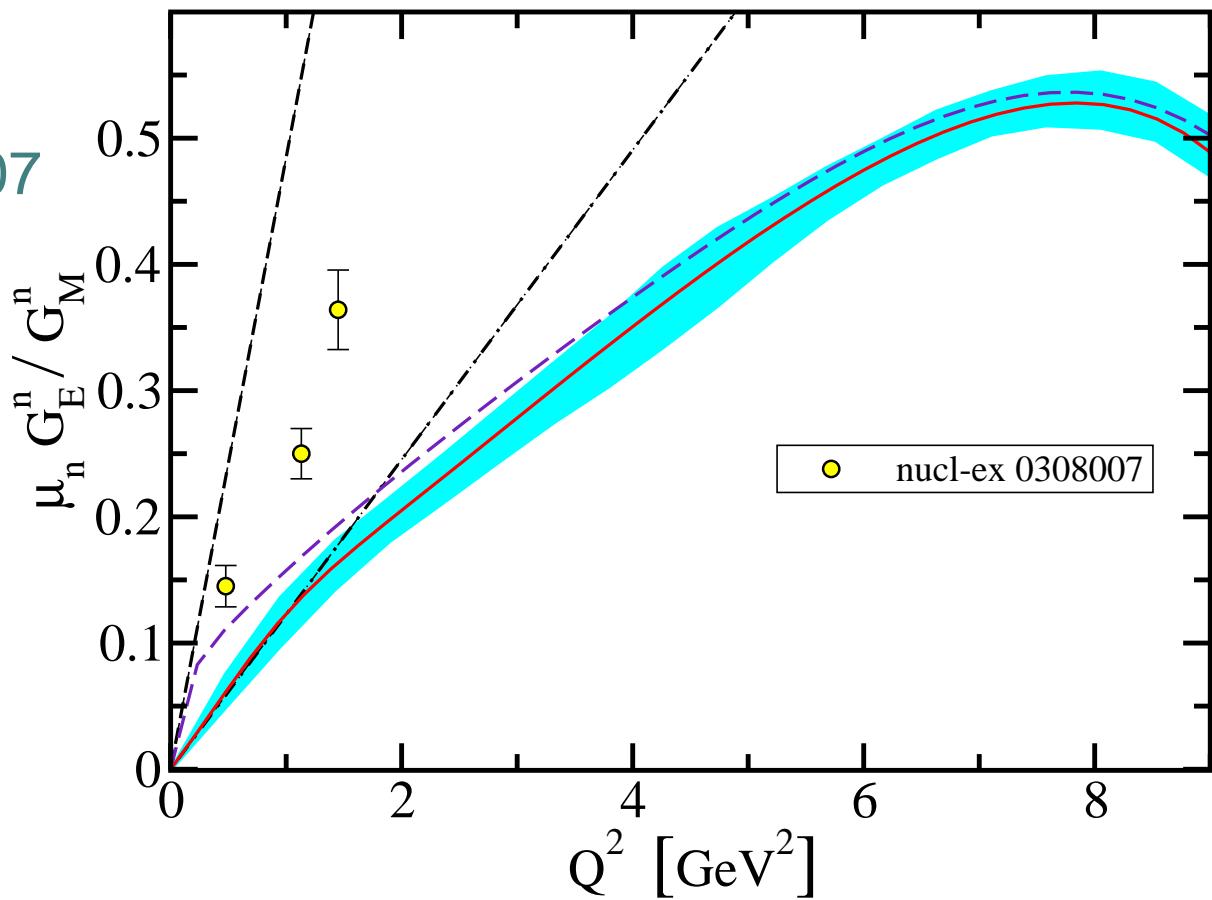
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# Neutron Form Factors

- Expt. Madey, et al. nu-ex/0308007

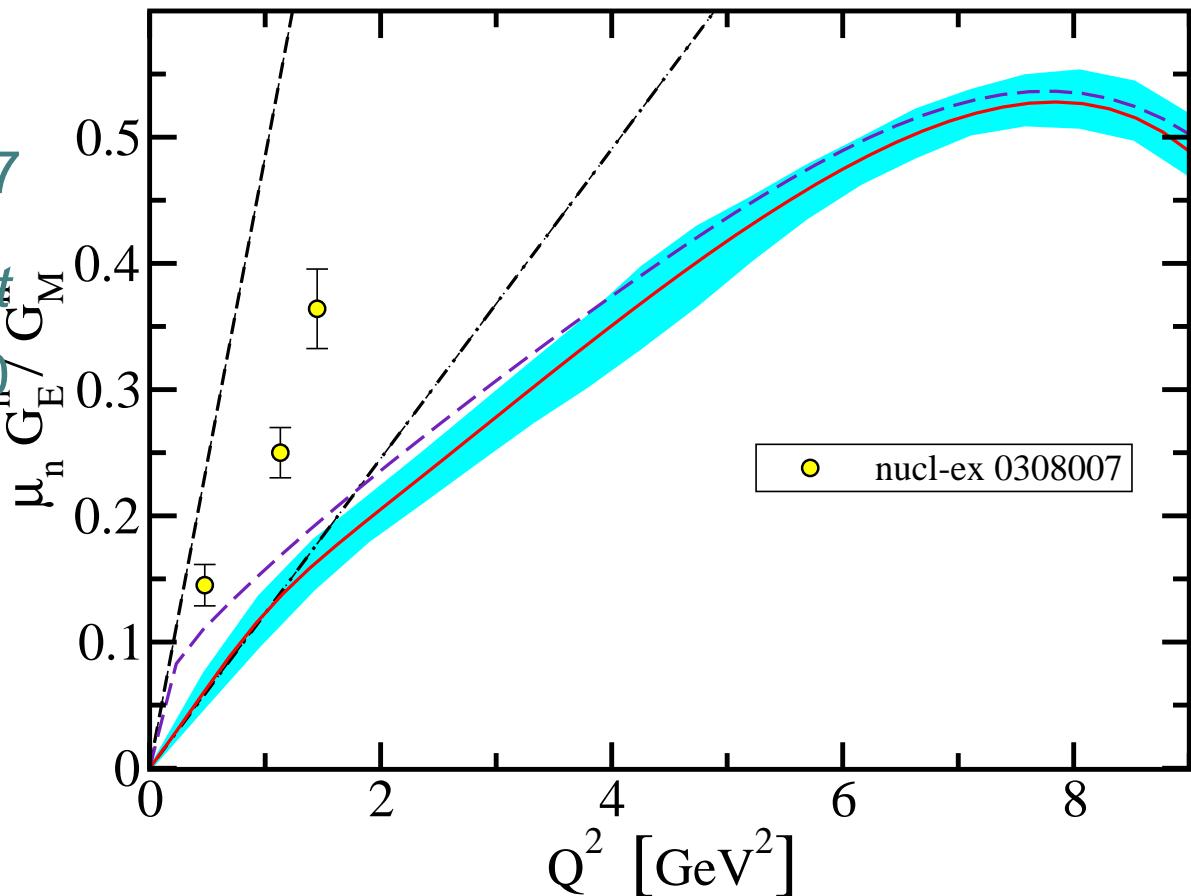


# Neutron Form Factors

- Expt. Madey, et al. nu-ex/0308007
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$$\mu_p \frac{G_E^n(Q^2)}{G_M^n(Q^2)} = -\frac{r_n^2}{6} Q^2$$

Valid for  $r_n^2 Q^2 \lesssim 1$

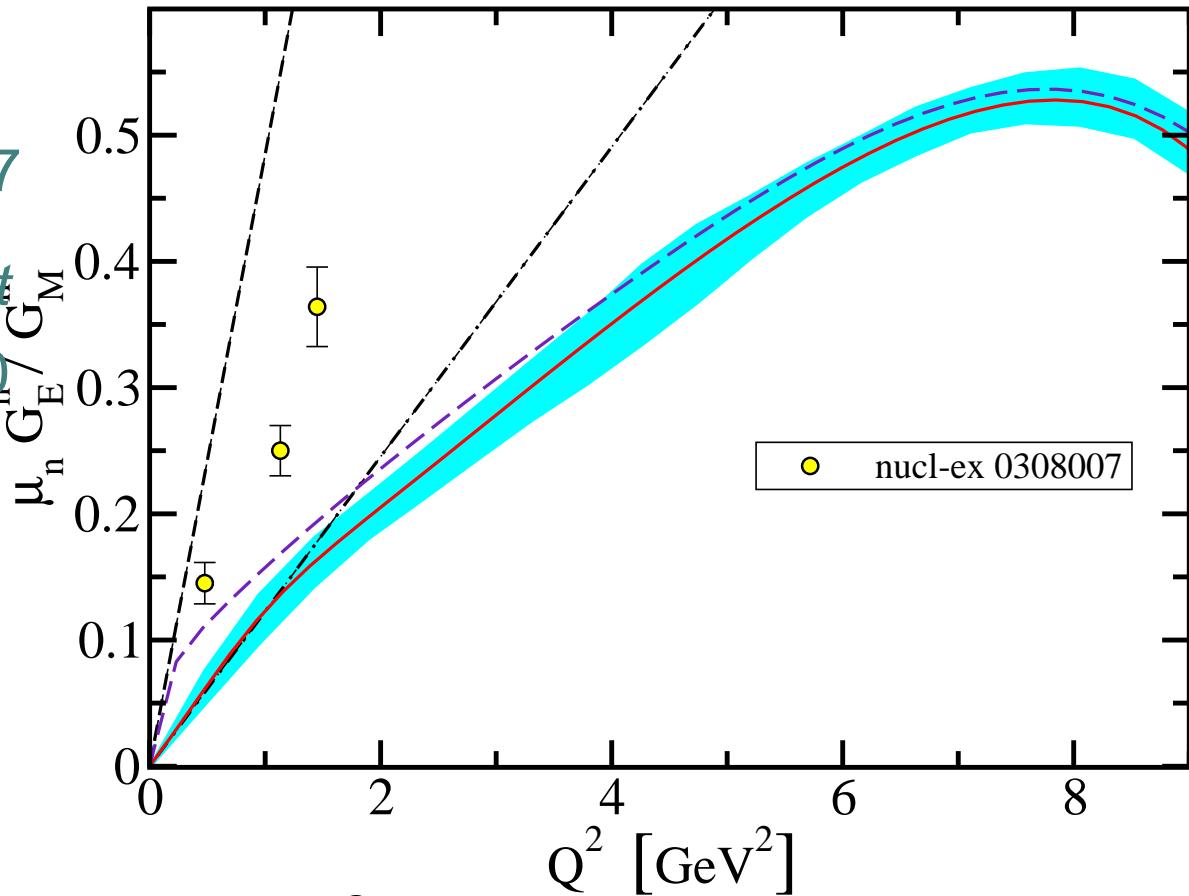


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Valid for  $r_n^2 Q^2 \lesssim 1$



- No sign yet of a zero in  $G_E^n(Q^2)$ , even though calculation predicts  $G_E^p(Q^2 \approx 6.5 \text{ GeV}^2) = 0$
- Data to  $Q^2 = 3.4 \text{ GeV}^2$  is being analysed (JLab E02-013)



# *Epilogue*



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... tell everyone I'm  
sorry about  
EVERYTHING



# Epilogue



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## Epilogue

- DCSB exists in QCD.



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## Epilogue

- DCSB exists in QCD.
  - It is manifest in the dressed light-quark propagator.
  - It impacts dramatically upon observables.



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  - Can be realised in dressed propagators of elementary excitations
  - Observables can be used to explore model realisations



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- An excellent way to test conjectures and constrain the possibilities
- Physics is an Experimental science



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- Dressed-quark . . . fixed by DSE and Meson Studies  
    . . . Burden, Roberts, Thomson, Phys. Lett. **B 371**, 163 (1996)



# Parametrising diquark properties

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Widths fixed by “asymptotic freedom” condition –

$$\left. \frac{d}{dK^2} \left( \frac{1}{m_{JP}^2} \mathcal{F}(K^2/\omega_{JP}^2) \right)^{-1} \right|_{K^2=0} = 1 \Rightarrow \omega_{JP}^2 = \frac{1}{2} m_{JP}^2 ,$$

Only two parameters; viz., diquark “masses”:  $m_{JP}$



# *Contemporary Reviews*

- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD  
C.D. Roberts and S.M. Schmidt, nu-th/0005064,  
Prog. Part. Nucl. Phys. **45** (2000) S1
- The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .  
R. Alkofer and L. von Smekal, he-ph/0007355,  
Phys. Rept. **353** (2001) 281
- Dyson-Schwinger equations: A Tool for Hadron Physics  
P. Maris and C.D. Roberts, nu-th/0301049,  
Int. J. Mod. Phys. **E 12** (2003) pp. 297-365
- Infrared properties of QCD from Dyson-Schwinger equations.  
C. S. Fischer, he-ph/0605173,  
J. Phys. **G 32** (2006) pp. R253-R291

