Hadron Physics & DSE Perspective

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http://www.phy.anl.gov/theory/staff/cdr.html

QCD’s Challenges

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- Quark and Gluon Confinement
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QCD's Challenges

Understand Emergent Phenomena

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- QCD – Complex behaviour
  arises from apparently simple rules

Dichotomy of Pion
– *Goldstone Mode* and *Bound state*

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  . . . . from two massive constituent-quarks?

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Must exhibit $\boxed{m_\pi^2 \propto m_q}$

Current Algebra . . . 1968

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Highly Nontrivial

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  - detailed understanding of connection between Current-quark and Constituent-quark masses;
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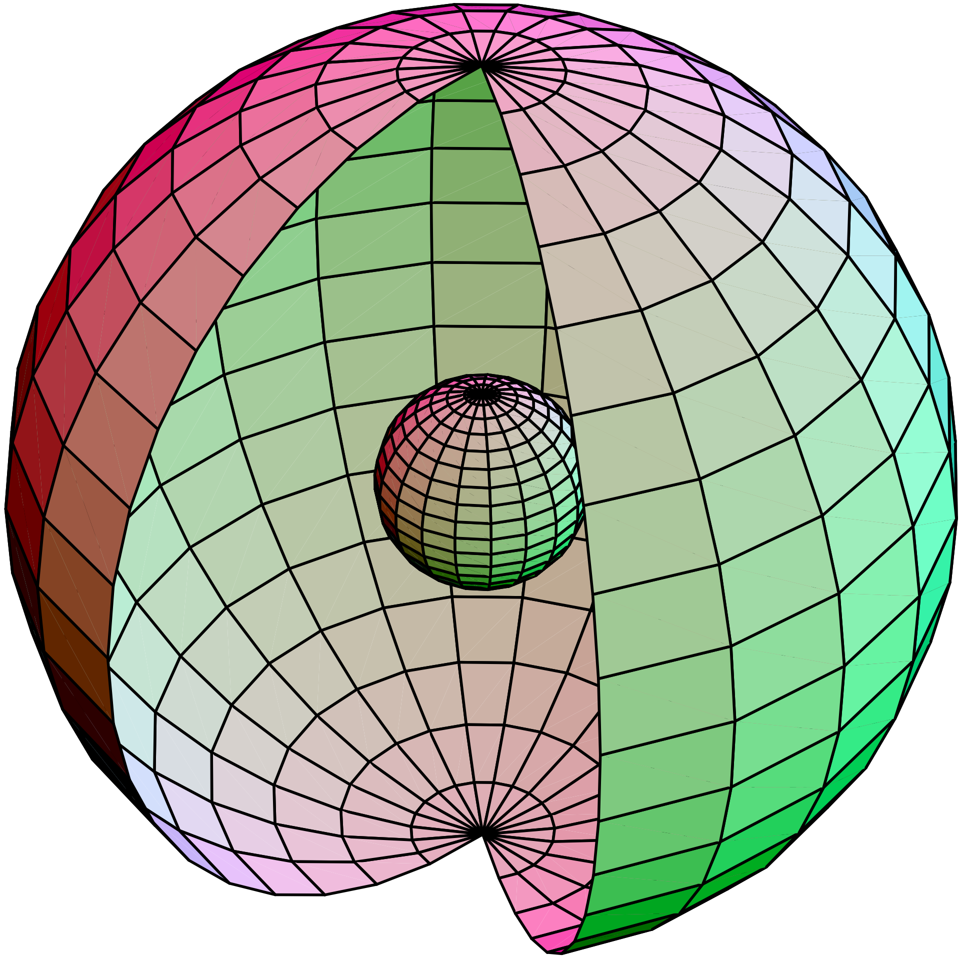
  - Interaction between quarks – the Interquark "Potential" – unknown throughout $>$ 98% of a hadron's volume

Intranucleon Interaction

Intranucleon Interaction



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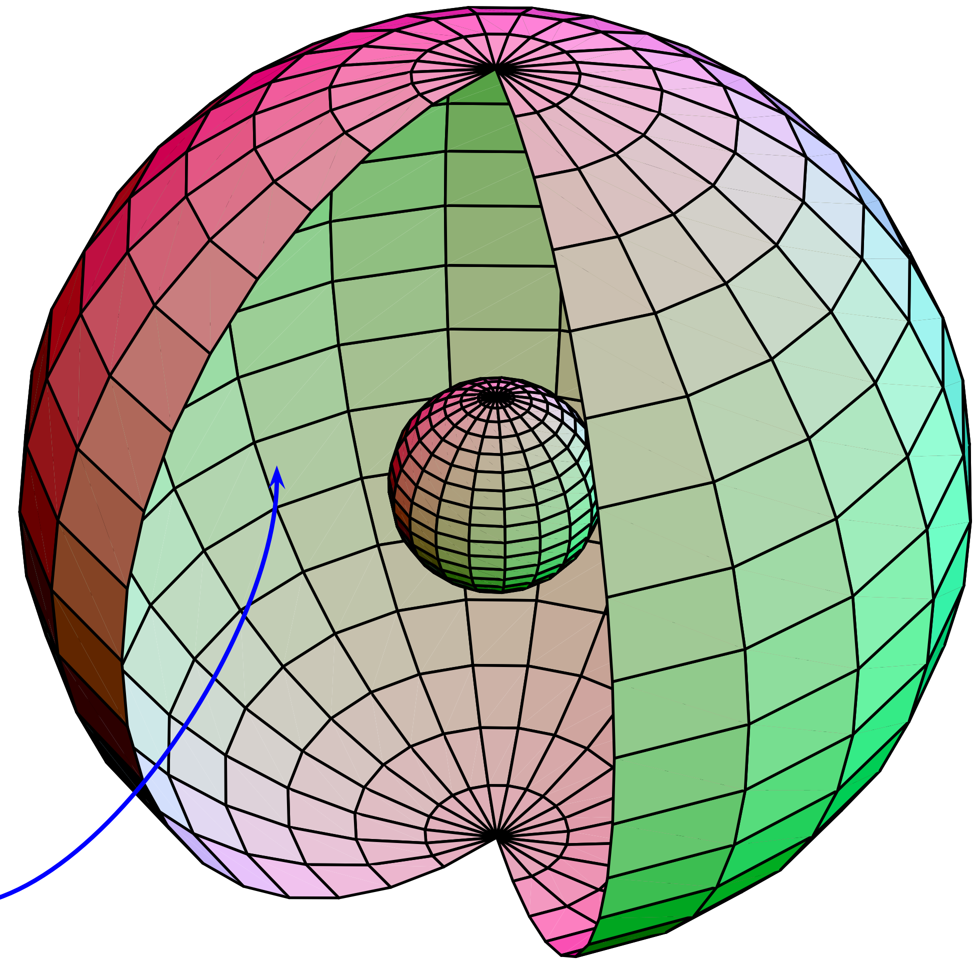
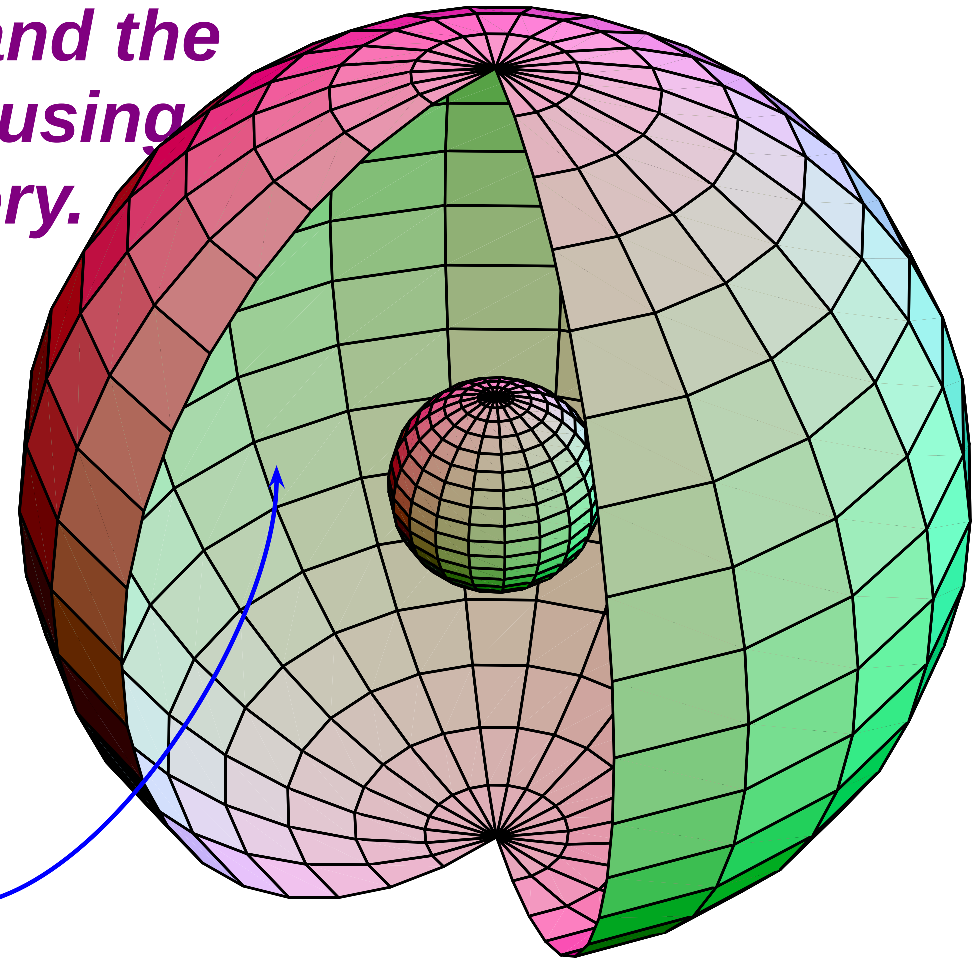


98% of the volume

The question must be rigorously defined, and the answer mapped out using experiment and theory.



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    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking

        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement

        – Coloured objects not detected, not detectable?

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- $\Rightarrow$ Understanding InfraRed (long-range)
  . . . . behaviour of $\alpha_s(Q^2)$

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- Method yields Schwinger Functions $\equiv$ Propagators

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Cross-Sections built from Schwinger Functions

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  - opportunity for comparisons at pre-experimental level . . . cross-fertilisation
- Proving fruitful.

World . . .

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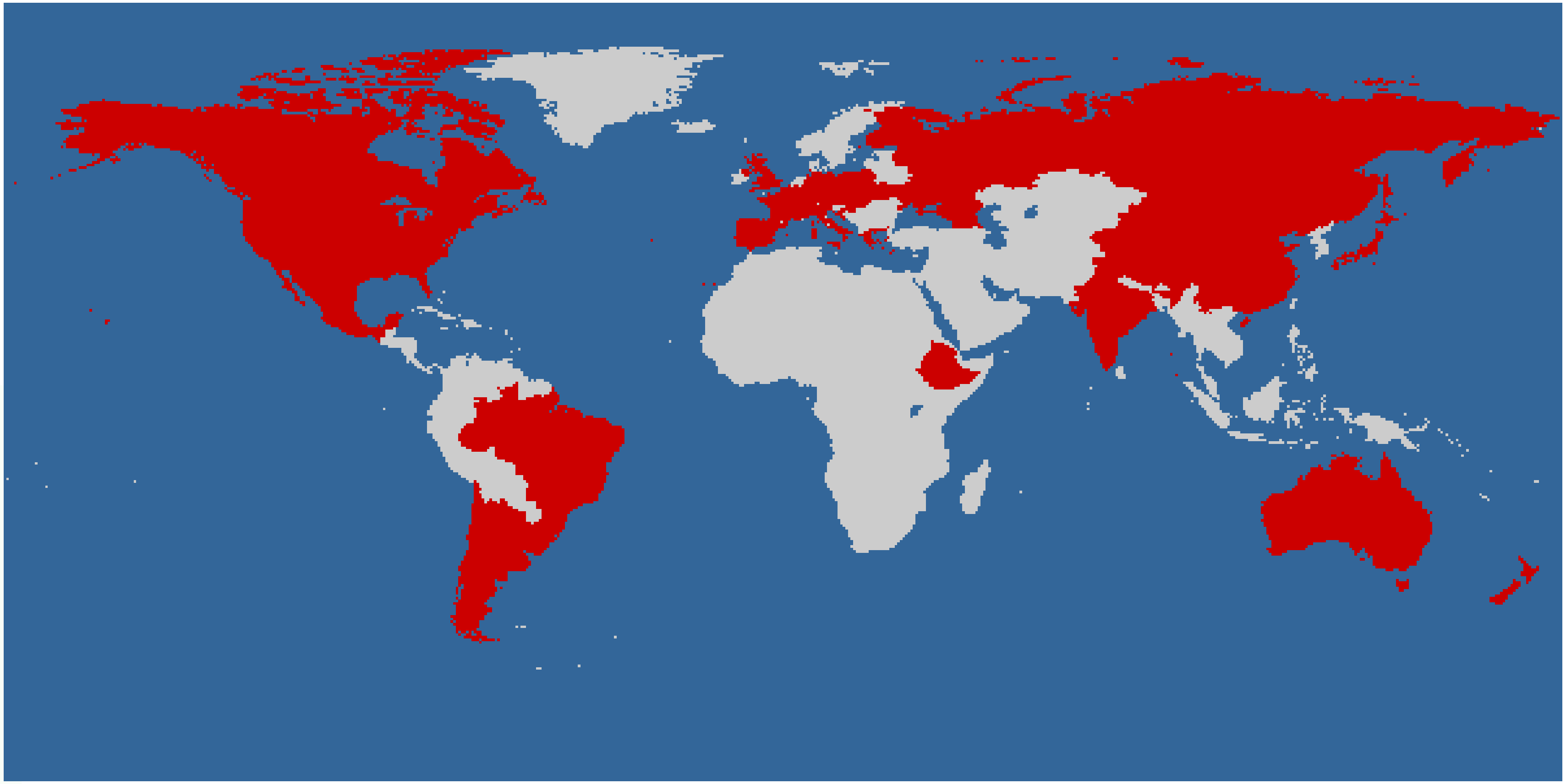
DSE Perspective



Persistent Challenge

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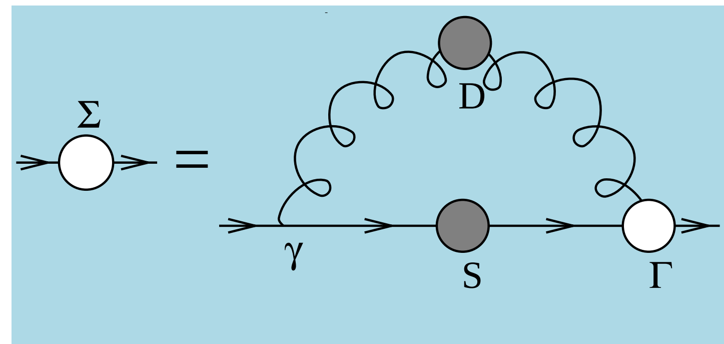
- Infinitely Many Coupled Equations



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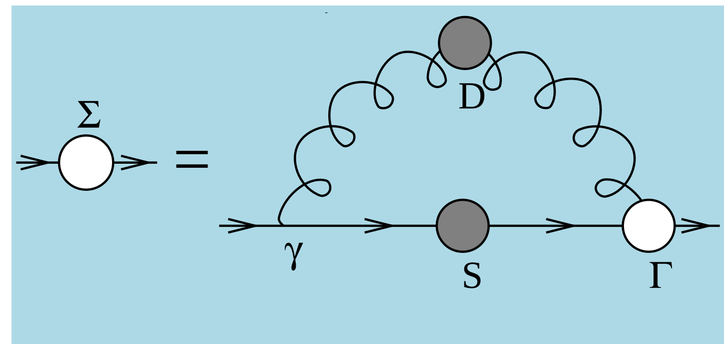

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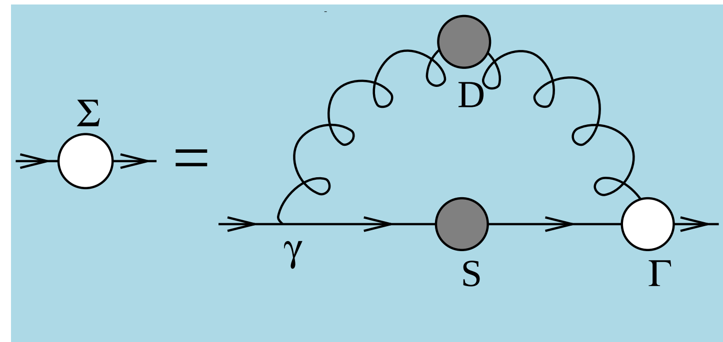

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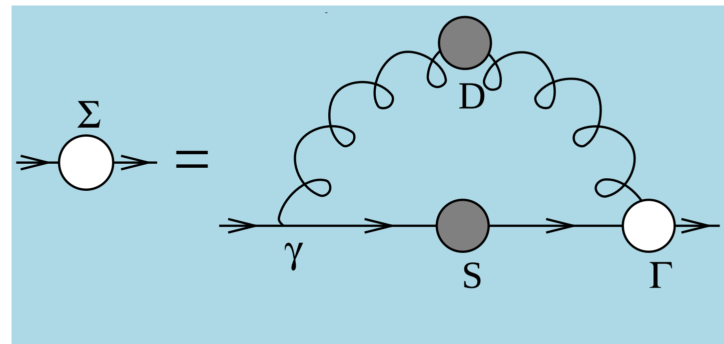

- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
    Not useful for the nonperturbative problems
    in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

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  - Make Predictions with Readily Quantifiable Errors

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Perturbative Dressed-quark Propagator

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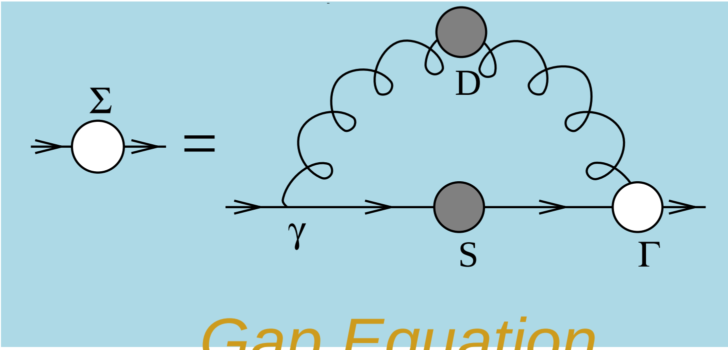
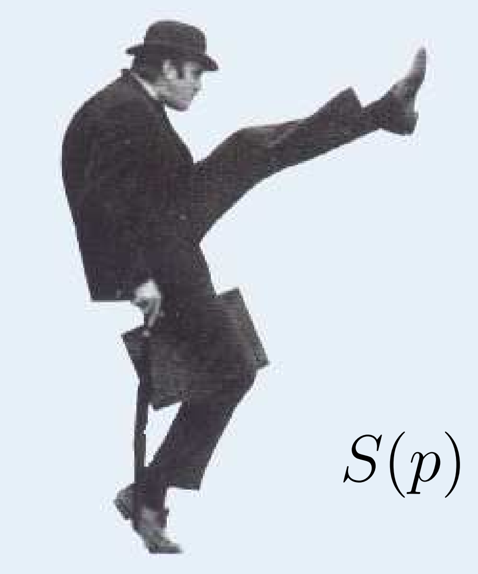
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation



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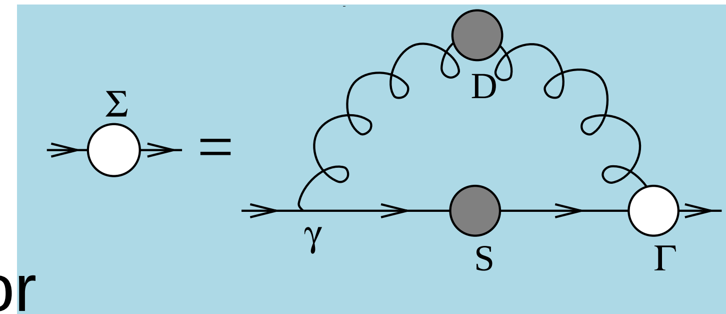

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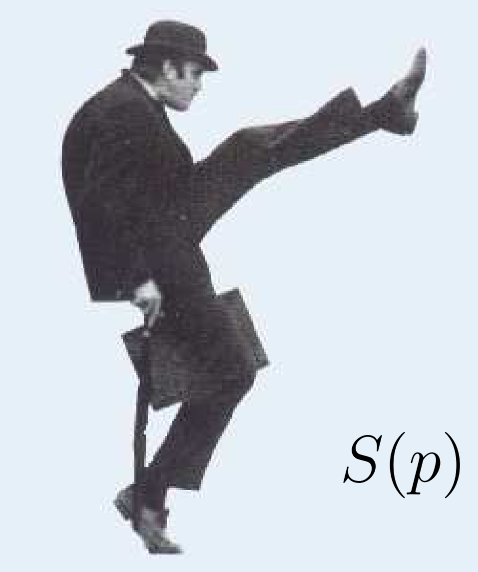

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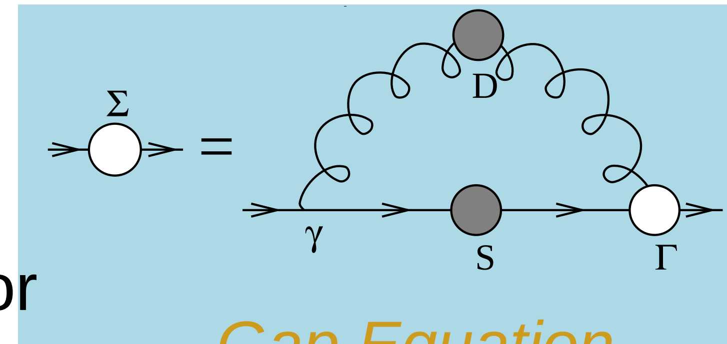

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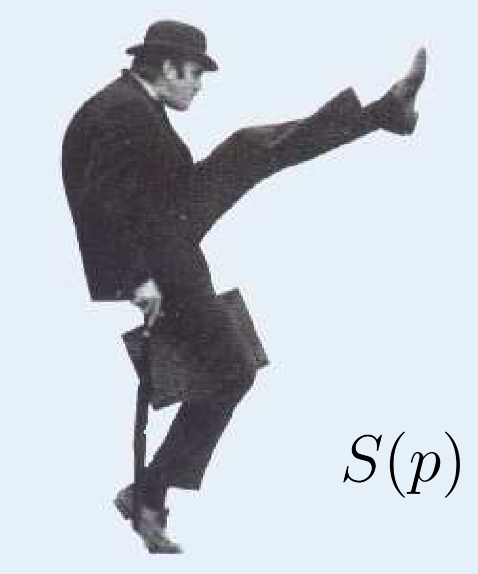

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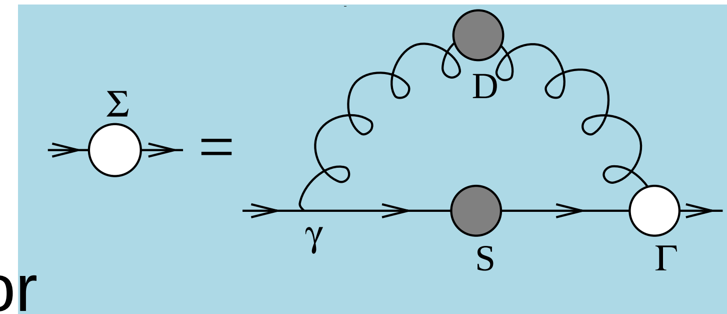

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$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\rightarrow} 0$$

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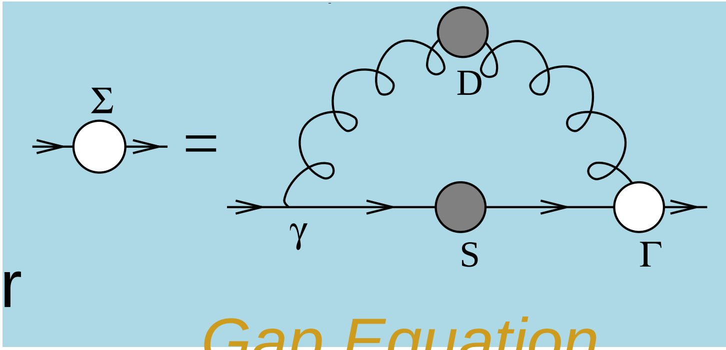

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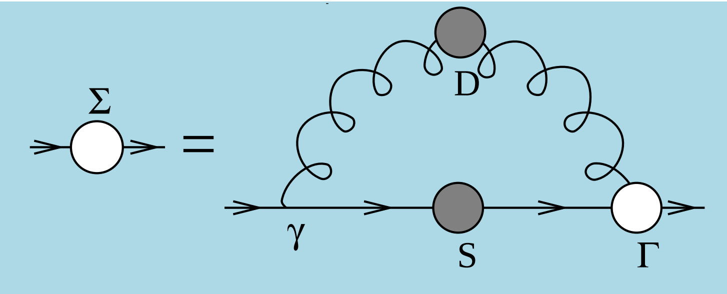

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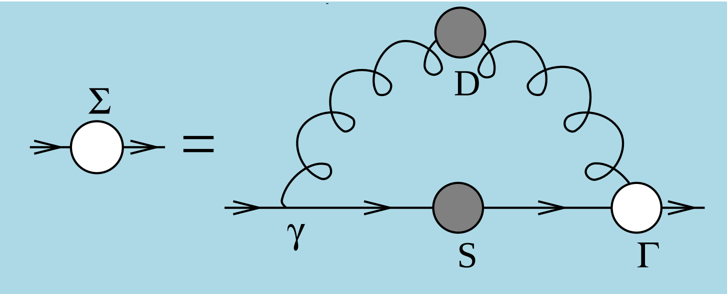
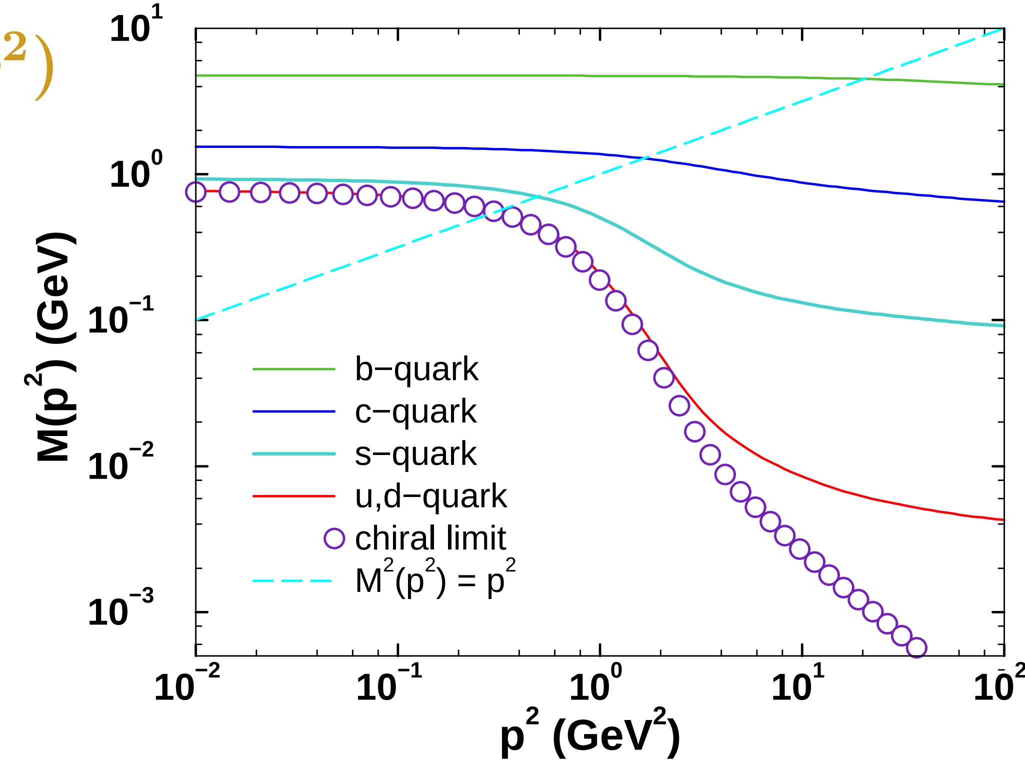


Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



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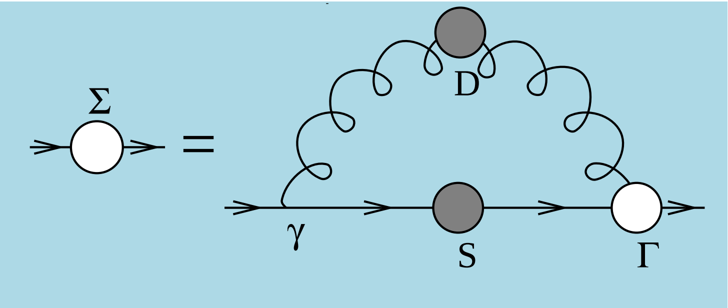

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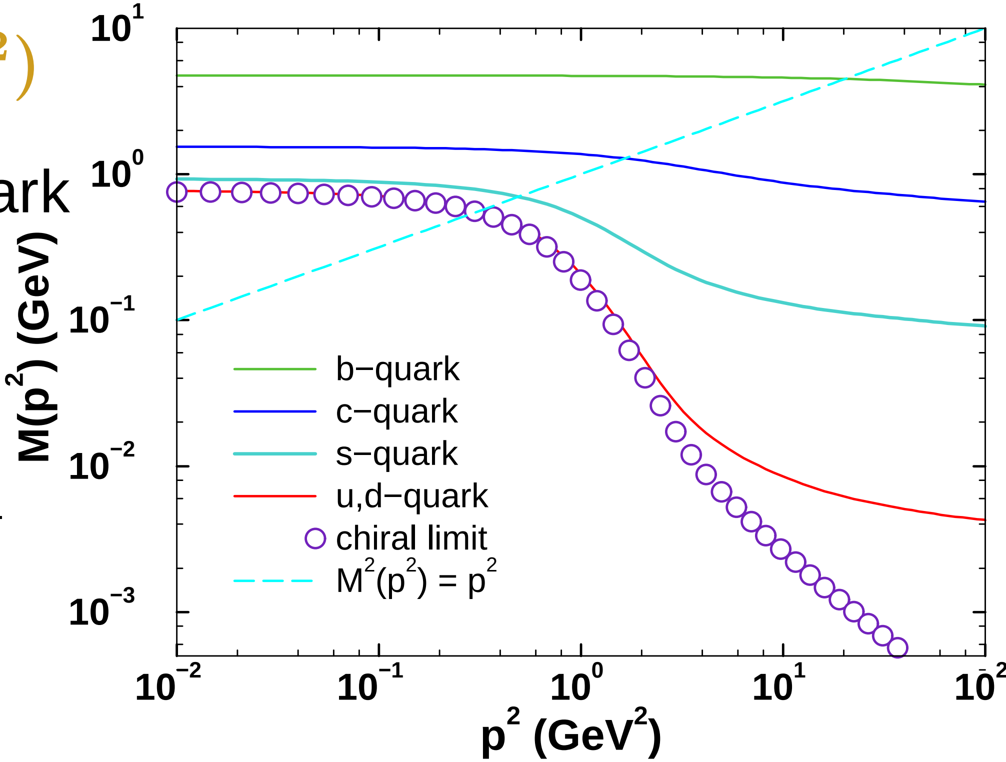
- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



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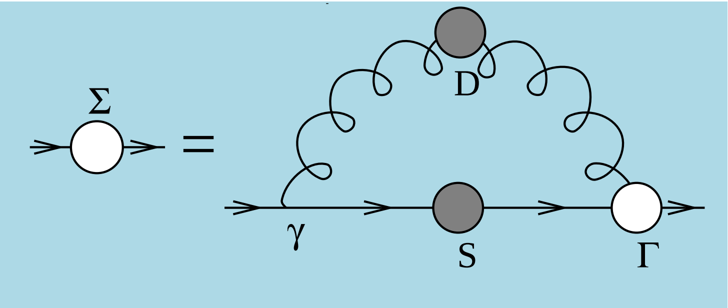

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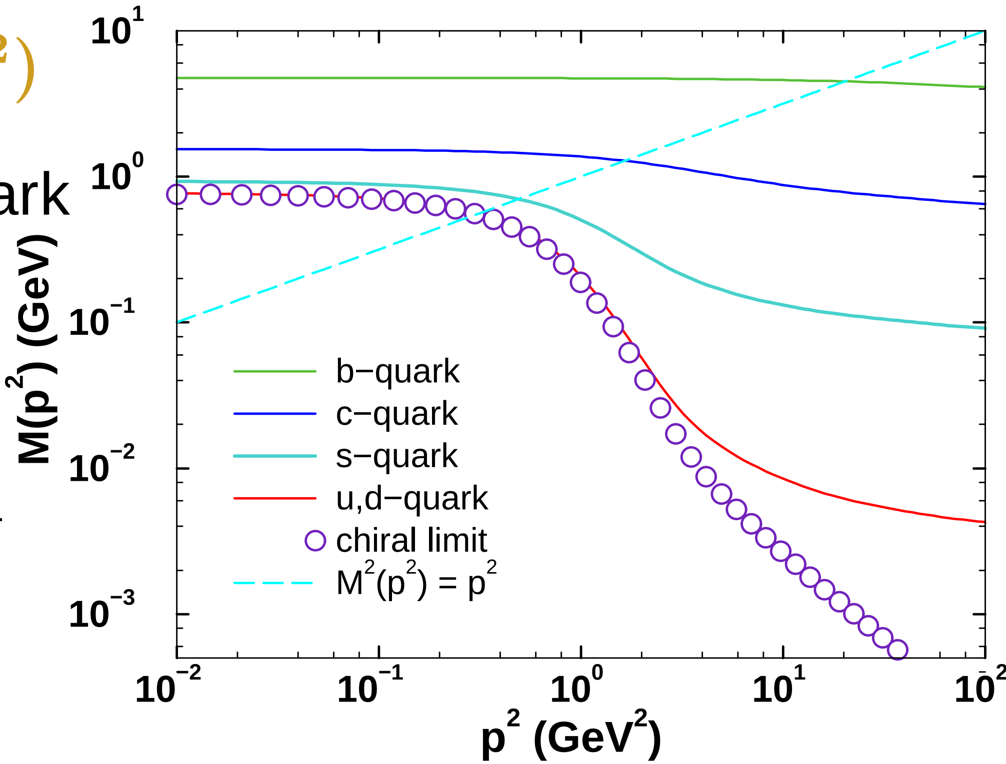

Predictions confirmed in numerical simulations of lattice-QCD

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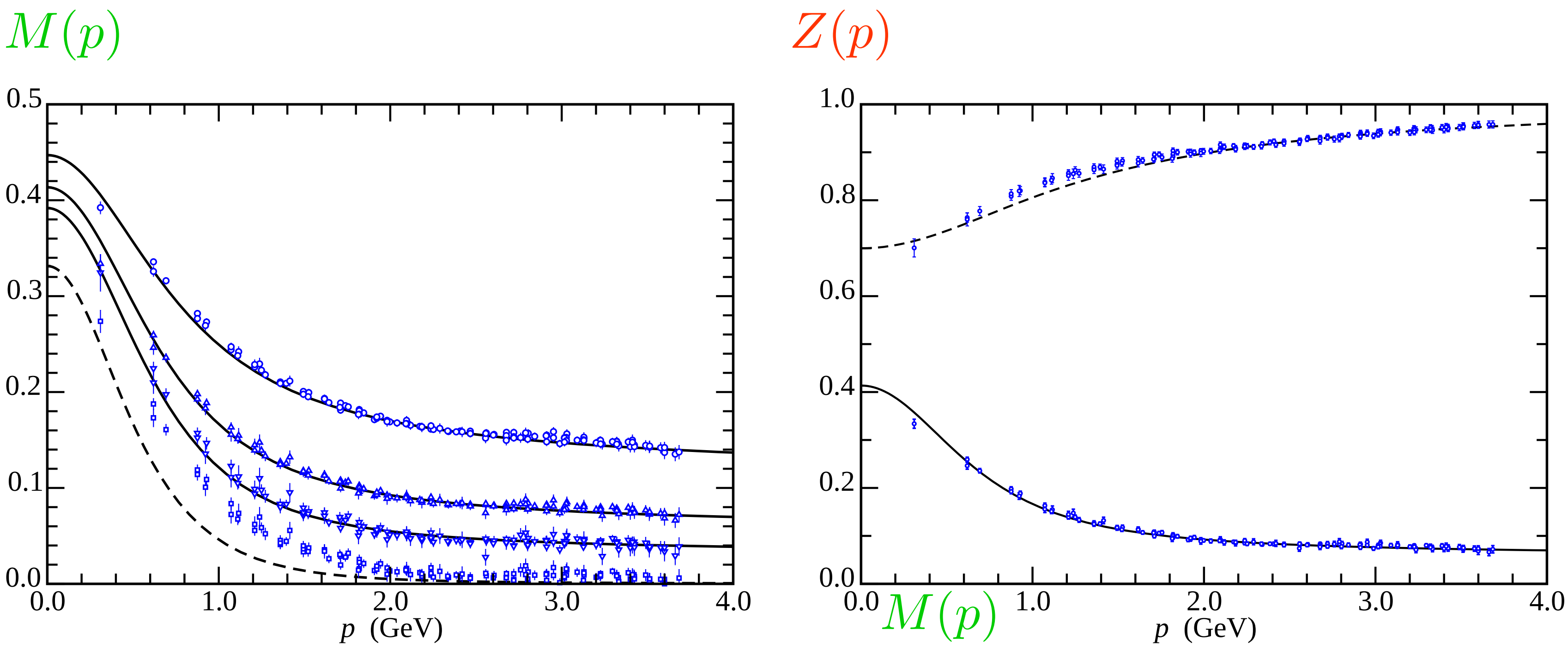
Quenched-QCD Dressed-Quark Propagator



2002 Quenched-QCD Dressed-Quark Propagator

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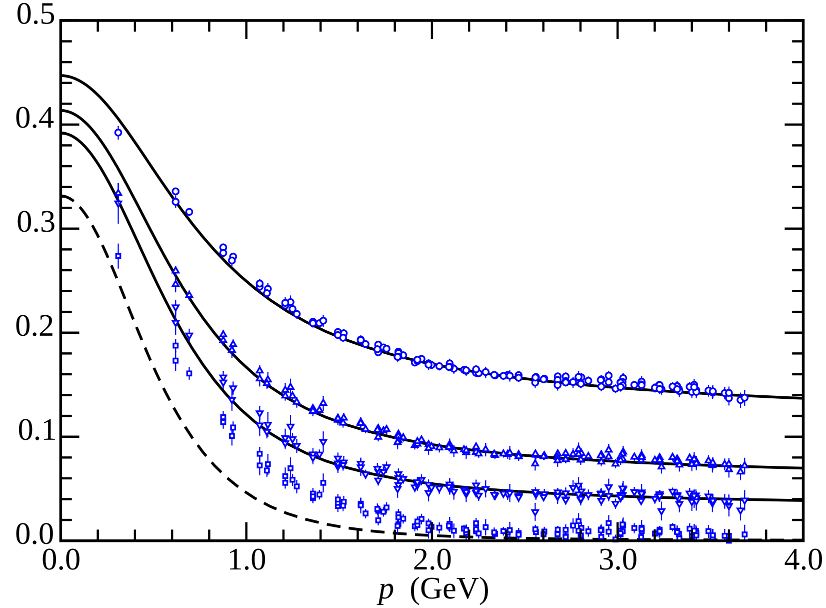

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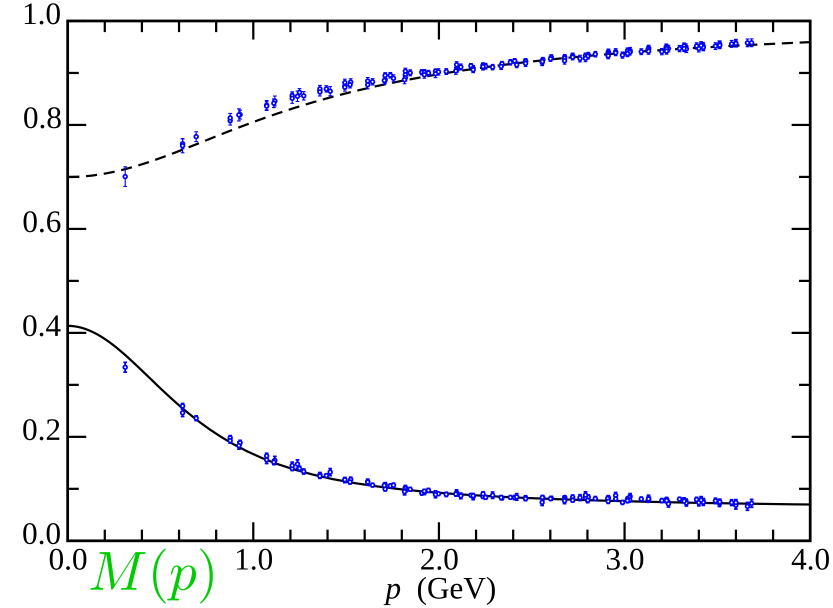

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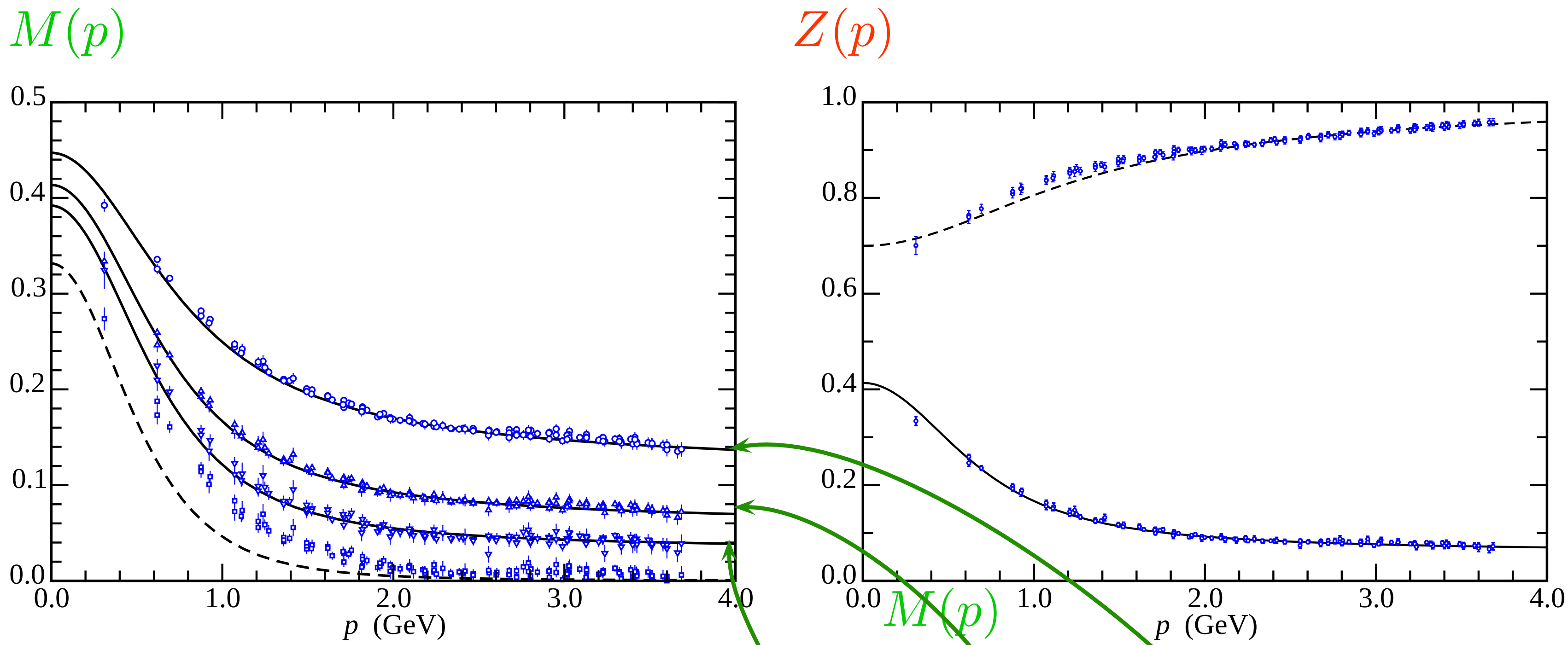
2002



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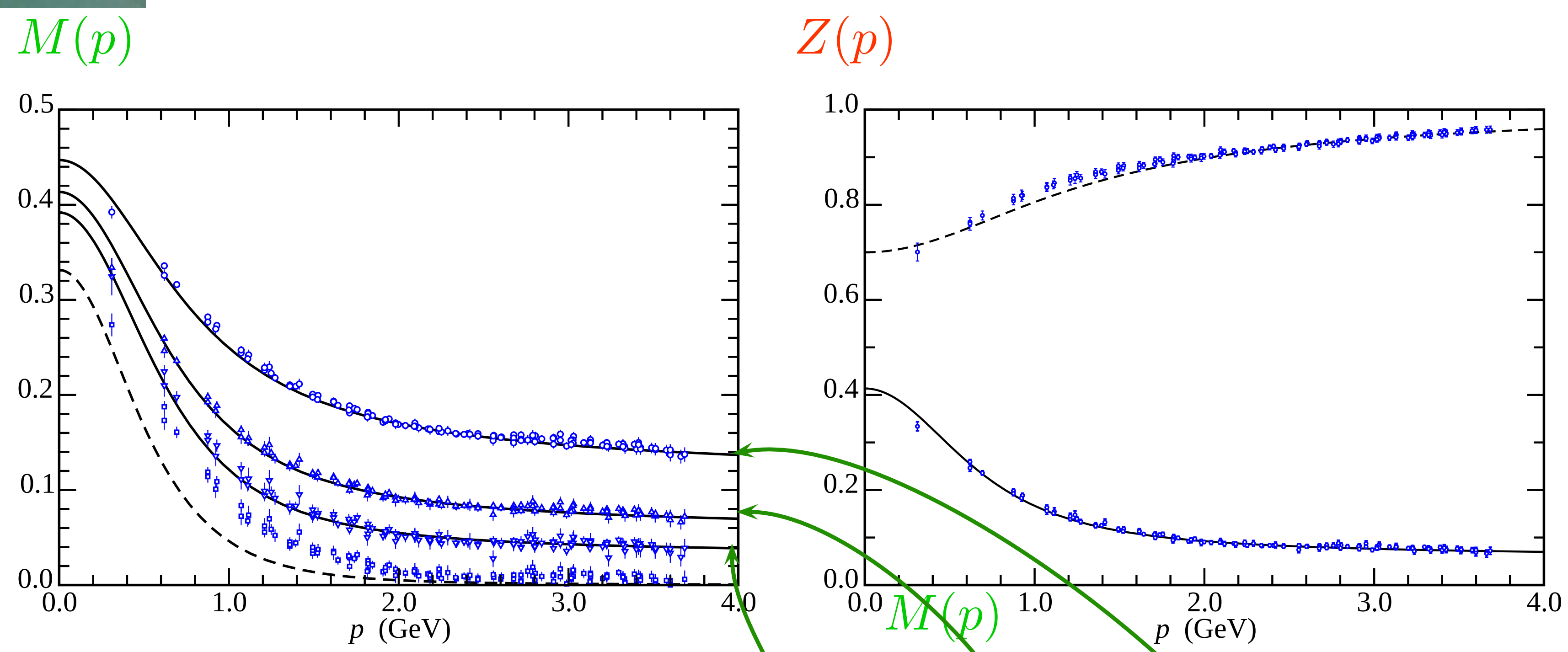

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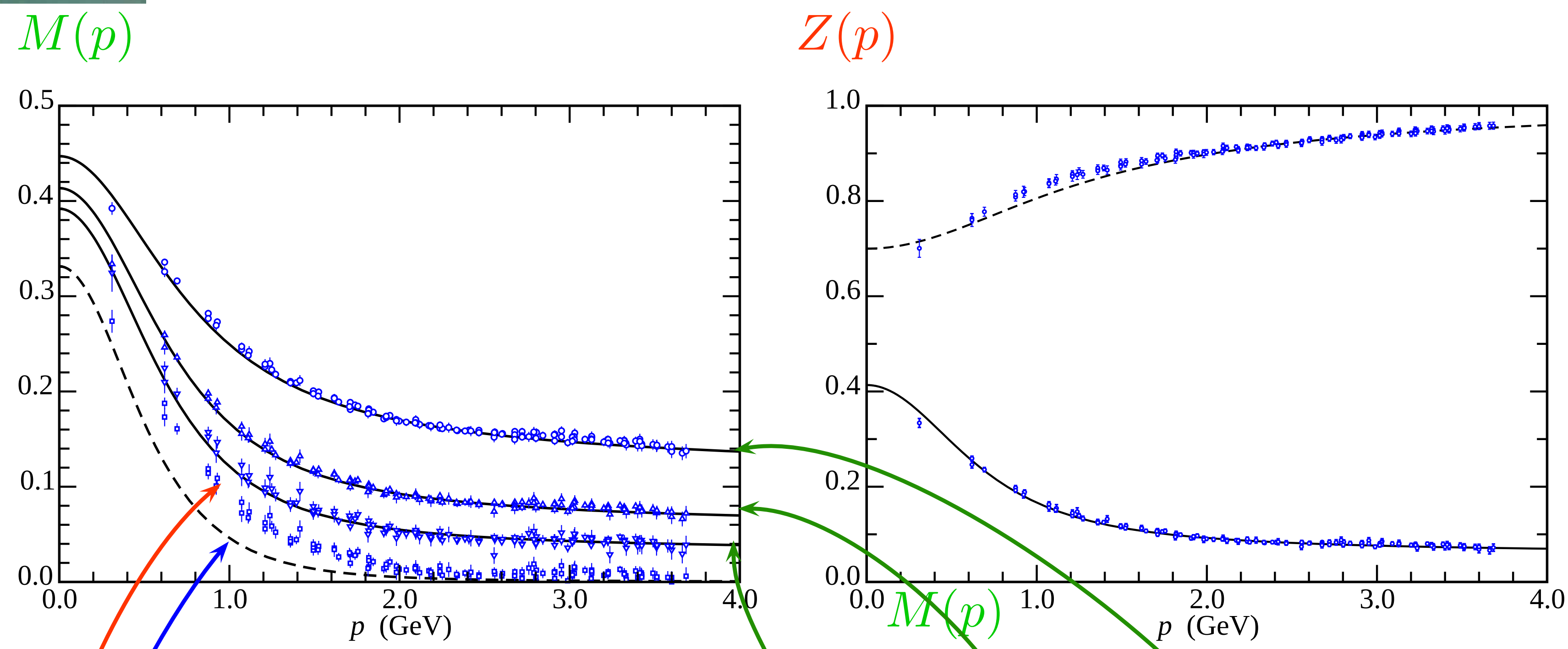

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- Linear extrapolation of lattice data to chiral limit is inaccurate

QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
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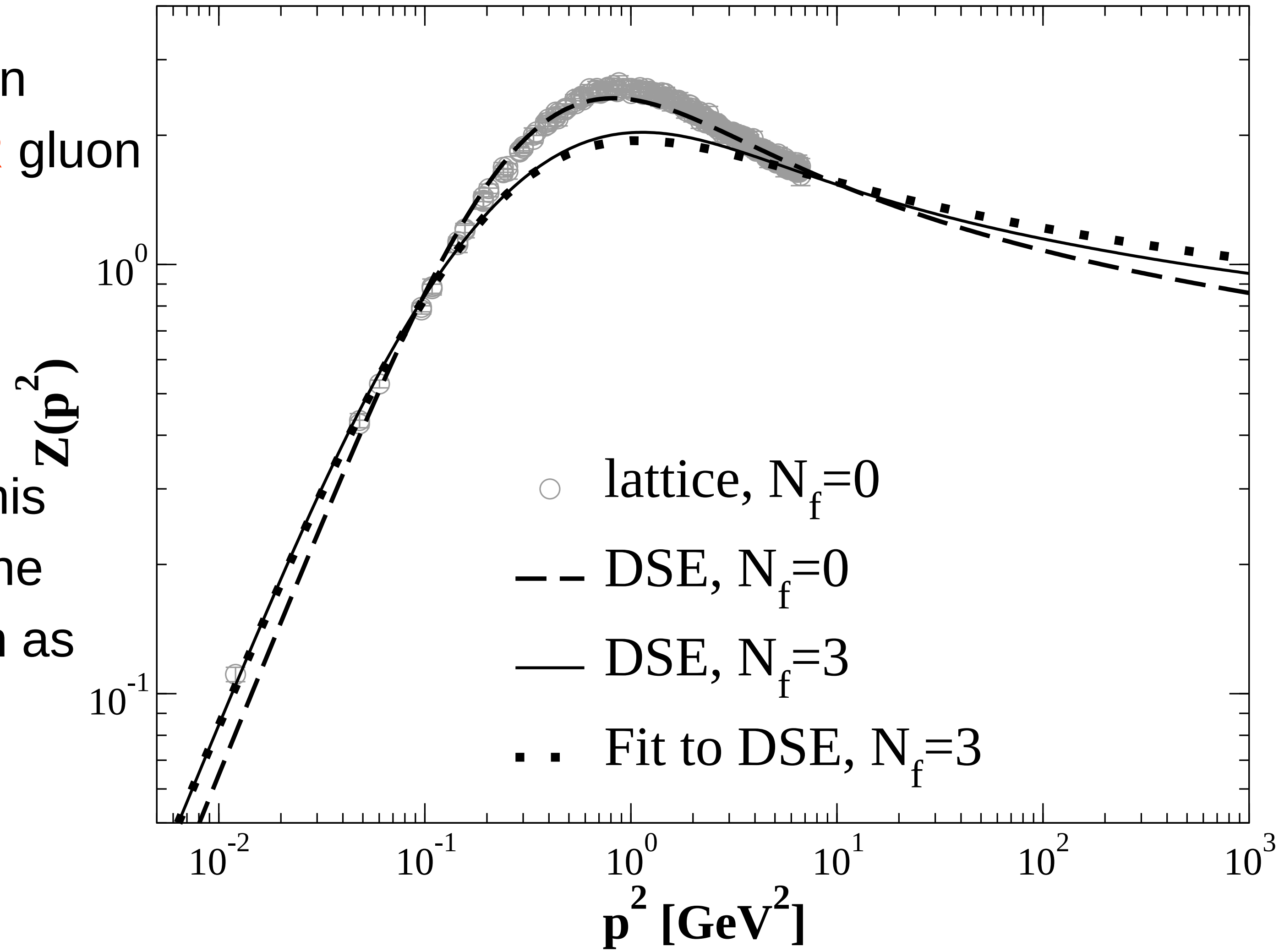
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$$

- Suppression means ∃ IR gluon mass-scale $\approx 1\,\text{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



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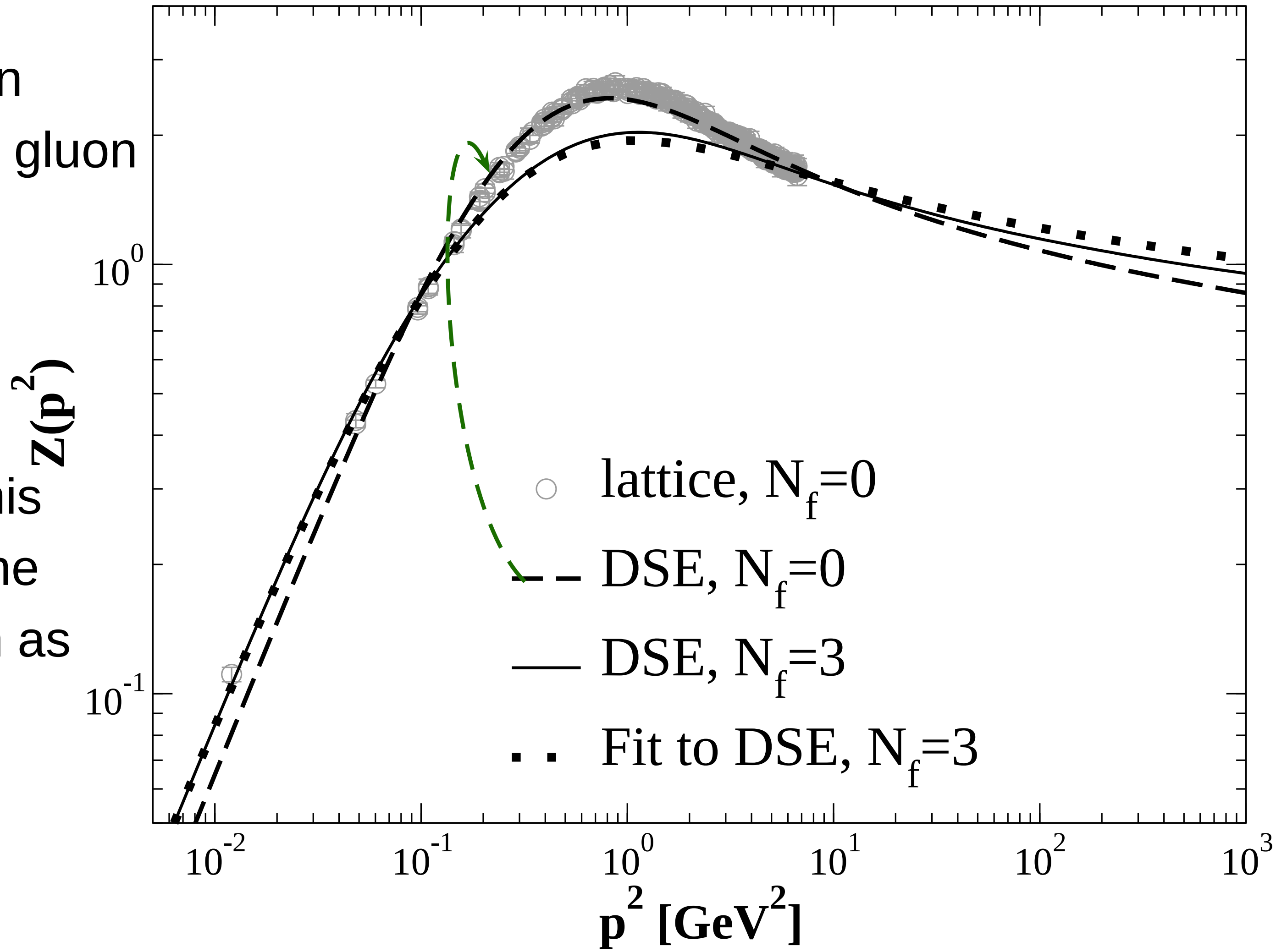

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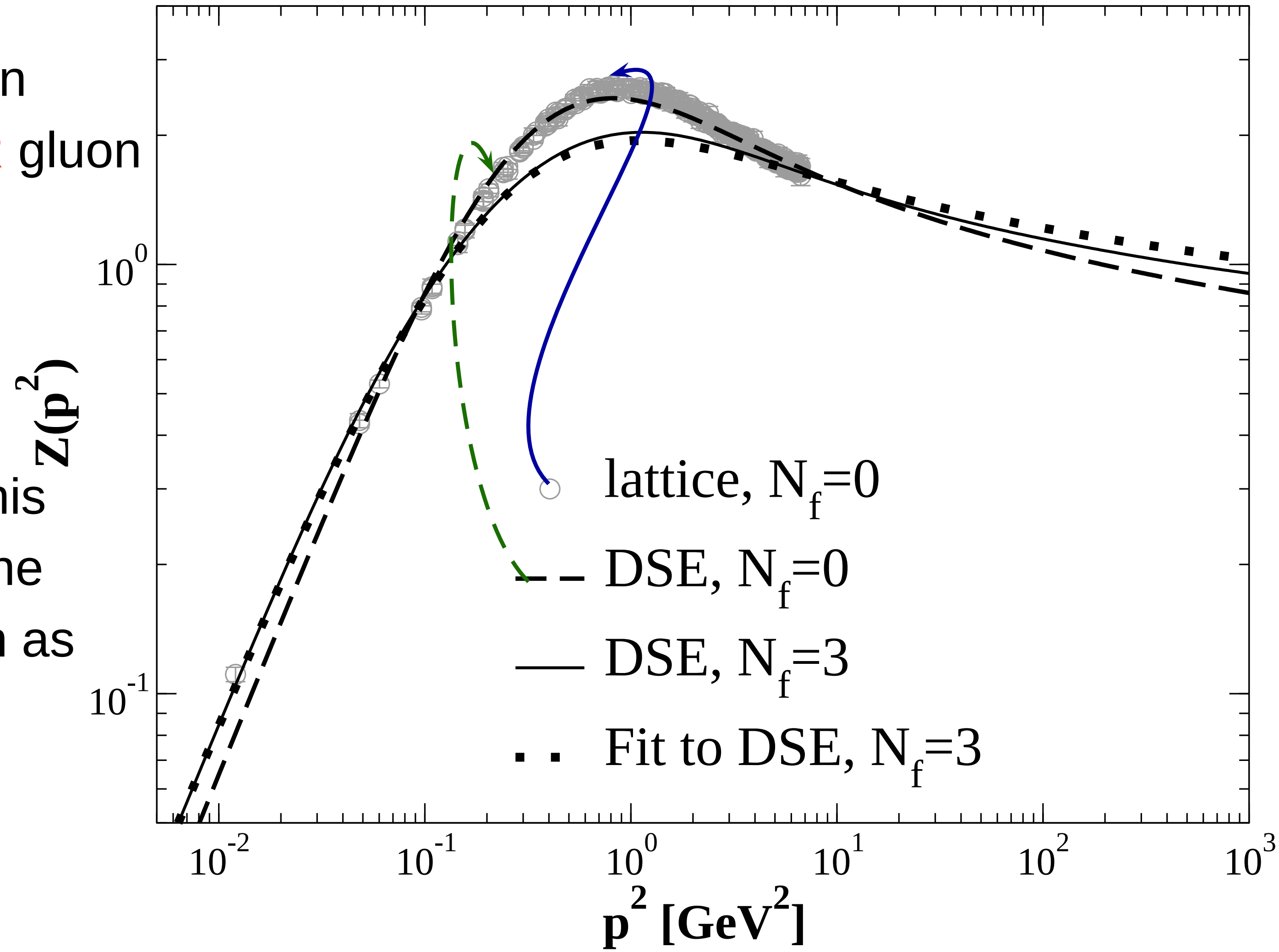

Critical Mass for Chiral Expansion

Lei Chang, *et al*., nucl-th/0605058

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$$\boxed{M(p^2; m = 0) \neq 0}.$$

- Does this mass function have a ***convergent*** expansion in current-quark mass about its nonzero chiral-limit value:

$$M(0;m) = M(0,0) + m \, \frac{\partial}{\partial m} M(0;m)\Big|_{m=0} + \ldots \; ?$$

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- $$M(0;m) = M(0,0) + \sum_{n=1}^{\infty} m^n a_n$$

  Radius of convergence: $m_{\rm rc} = \lim_{n\to\infty} \left(\frac{1}{|a_n|}\right)^{1/n}$

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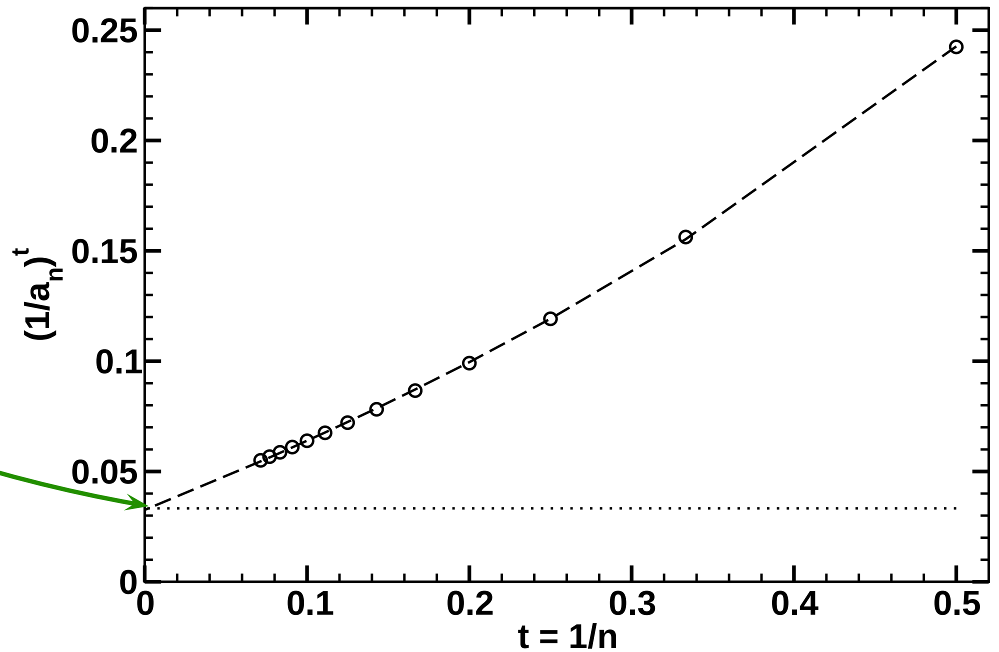
$$m_{\rm rc} = 0.034 \pm 0.001$$



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- Entails, e.g., lattice-QCD simulations *must* have results at $m_\pi^2 < [m_{0^-}^{\rm cr}]^2 \sim 0.2\,\mathrm{GeV}^2$ for reasonable extrapolation via EFT.

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Consituent-quark σ-term

- Impact of Dynamical chiral symmetry breaking … exhibited via constituent-quark σ-term

$$\sigma_f := m_f(\zeta)\,\frac{\partial M_f^E}{\partial m_f(\zeta)}\,,\;(M^E)^2 := s\,|\,s = M(s)^2\,.$$

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- Renormalisation-group-invariant and determined from solutions of the gap equation

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- Unambiguous probe of impact of explicit chiral symmetry breaking on the mass function

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- Ratio $\boxed{\dfrac{\sigma_f}{M_f^E} = \dfrac{\text{EXPLICIT}}{\text{EXPLICIT} + \text{DYNAMICAL}}}$

measures effect of *EXPLICIT* chiral symmetry breaking on dressed-quark mass-function

cf. *SUM* of effects of *EXPLICIT* AND *DYNAMICAL CHIRAL SYMMETRY BREAKING*

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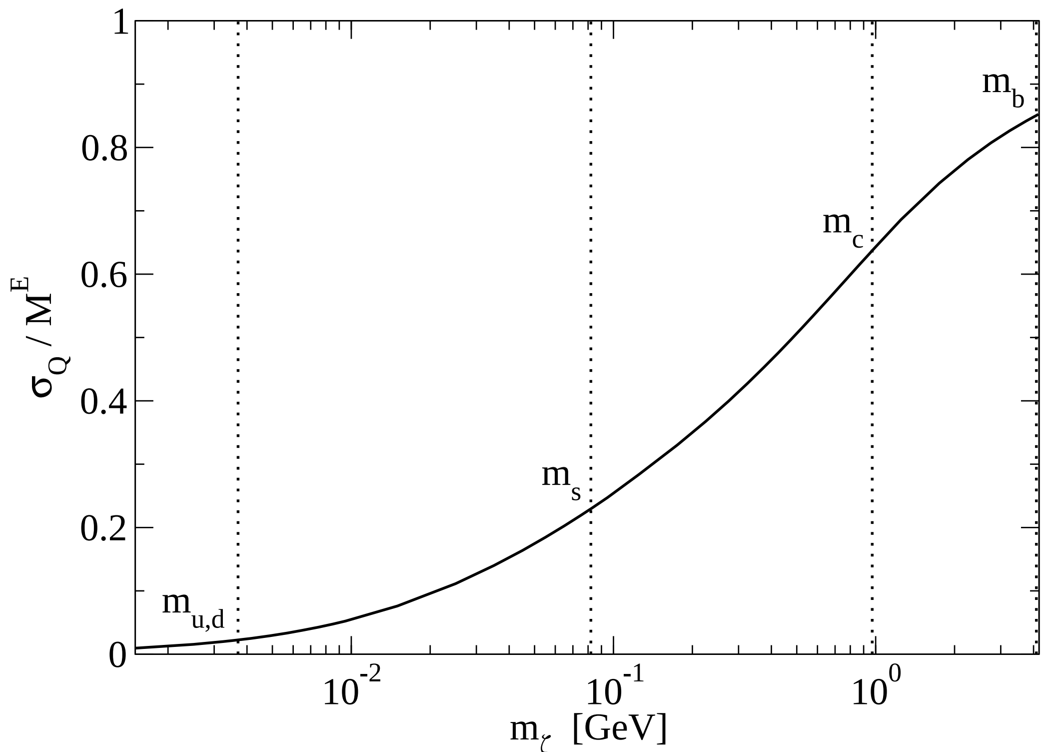

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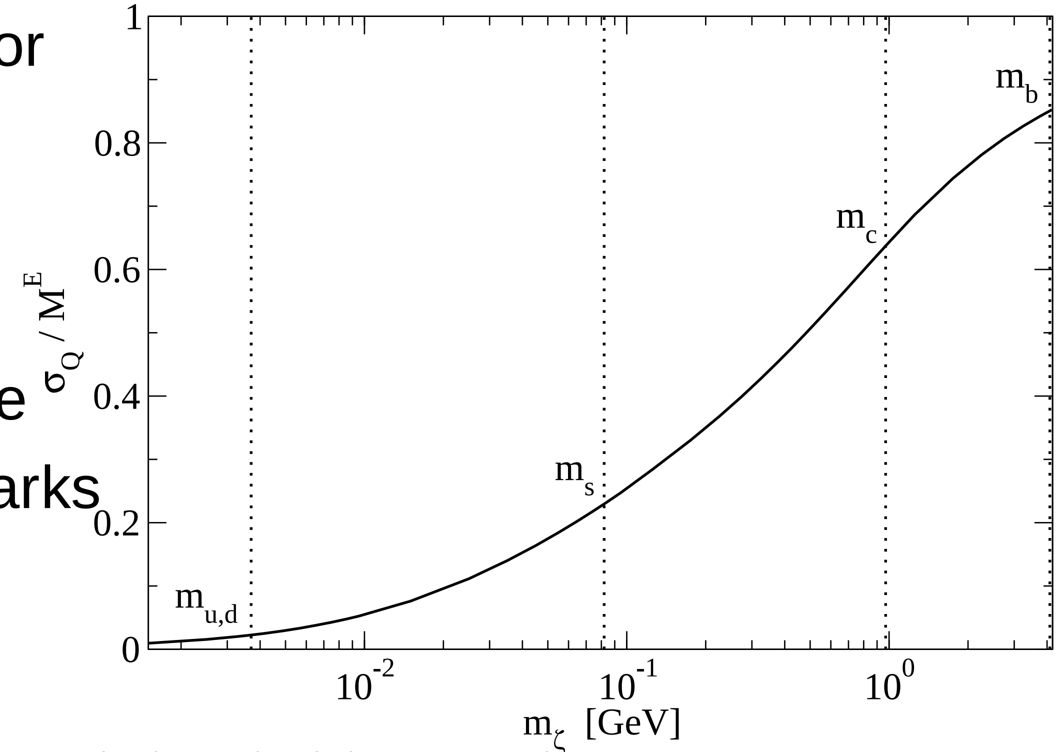
Obvious: ratio vanishes for light-quarks because magnitude of their constituent-mass owes primarily to DCSB. On the other hand, for heavy-quarks it approaches one.



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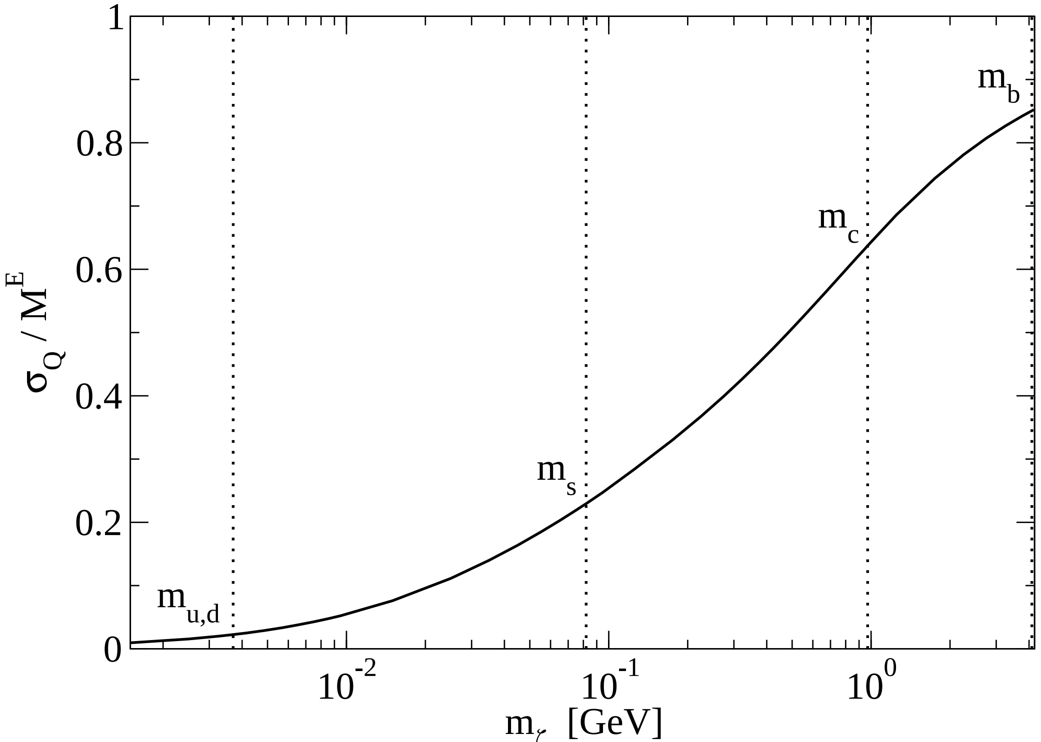
Essentially dynamical component of chiral symmetry breaking, and manifestation in all its order parameters, vanishes with increasing current-quark mass



Hadrons

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- What about bound states?

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- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

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- Without bound states, Comparison with experiment is impossible
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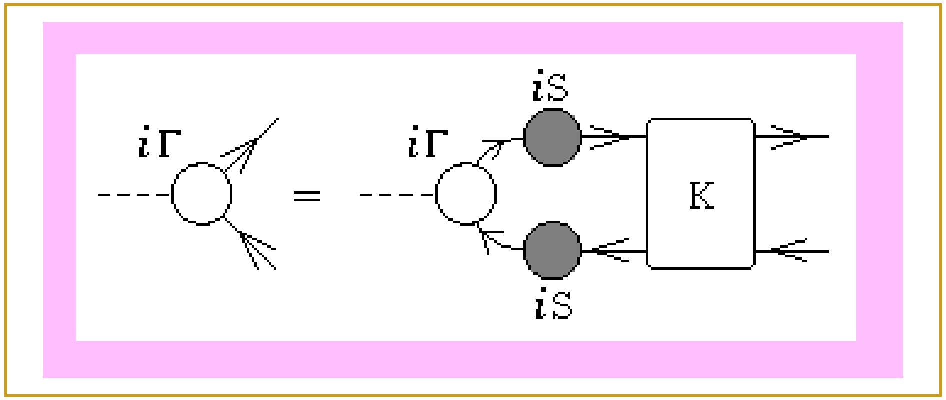

QFT Generalisation of Lippmann-Schwinger Equation.

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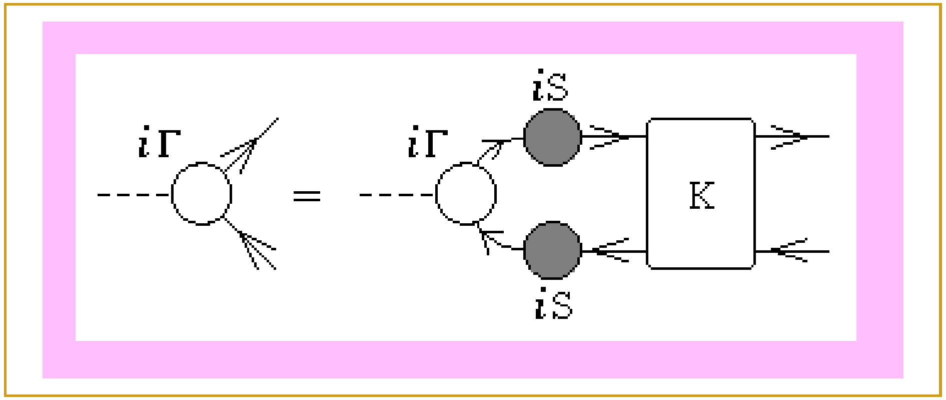

QFT Generalisation of Lippmann-Schwinger Equation.

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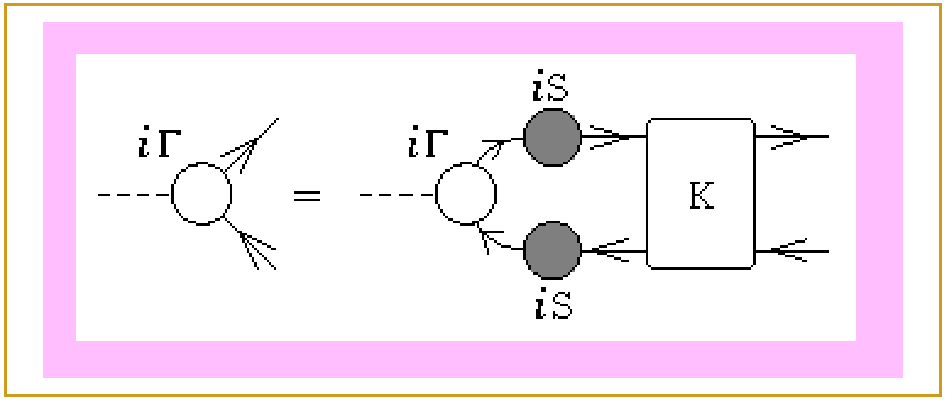

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or

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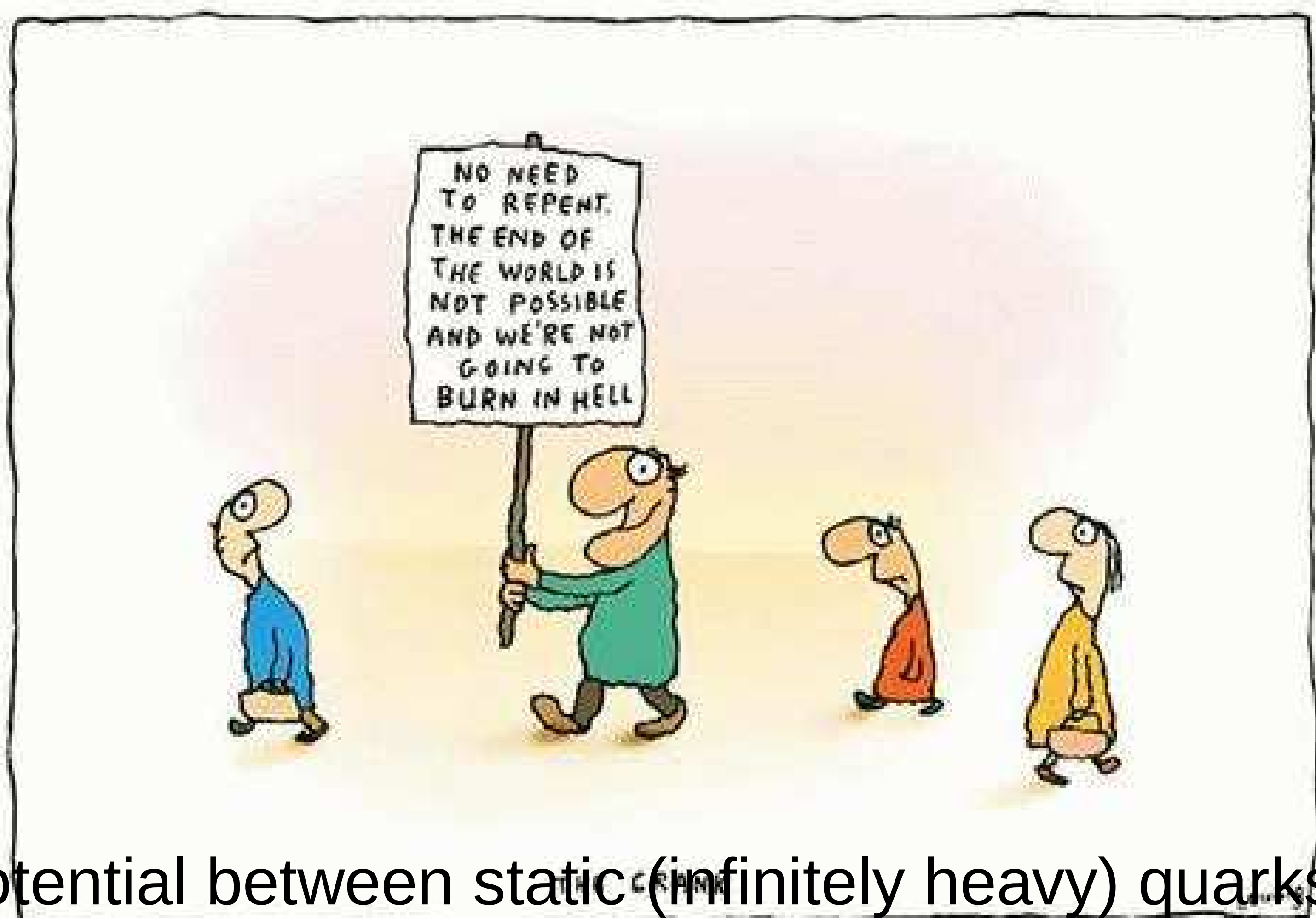

Potential between static (infinitely heavy) quarks measured in numerical simulations of lattice-QCD ***is not related*** in any simple way to the light-quark interaction.

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

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- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Radial Excitations
& Chiral Symmetry

(Maris, Roberts, Tandy
nu-th/9707003)

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

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- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

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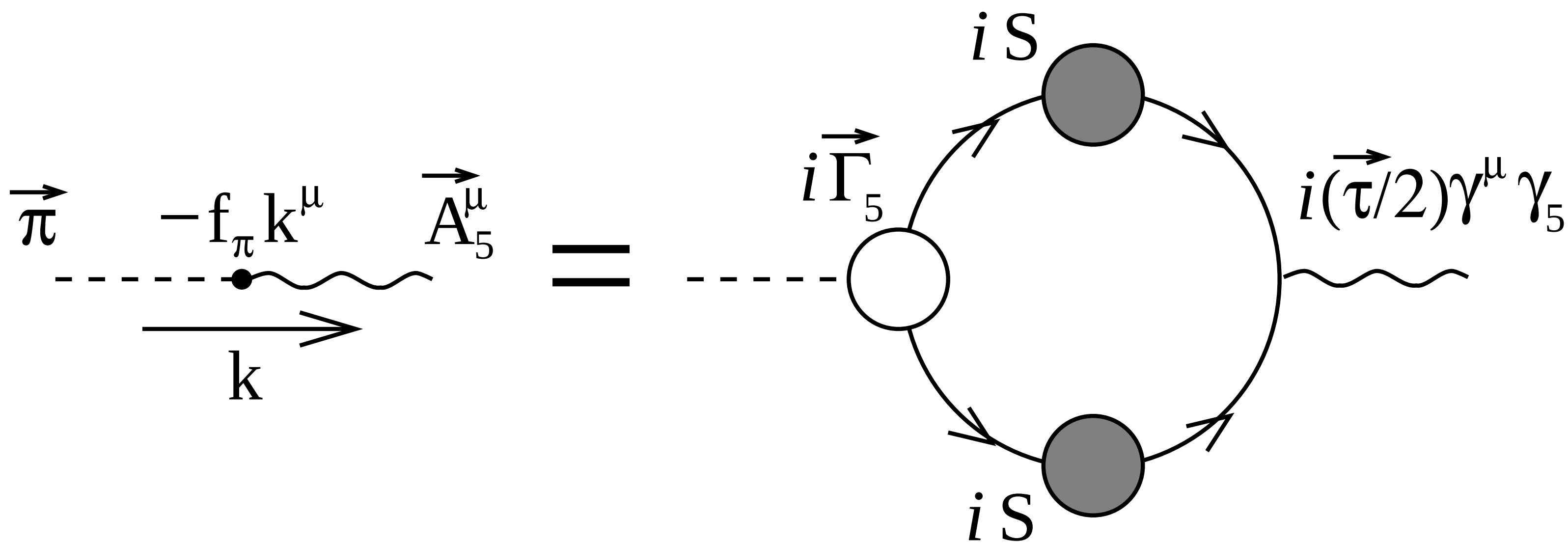
- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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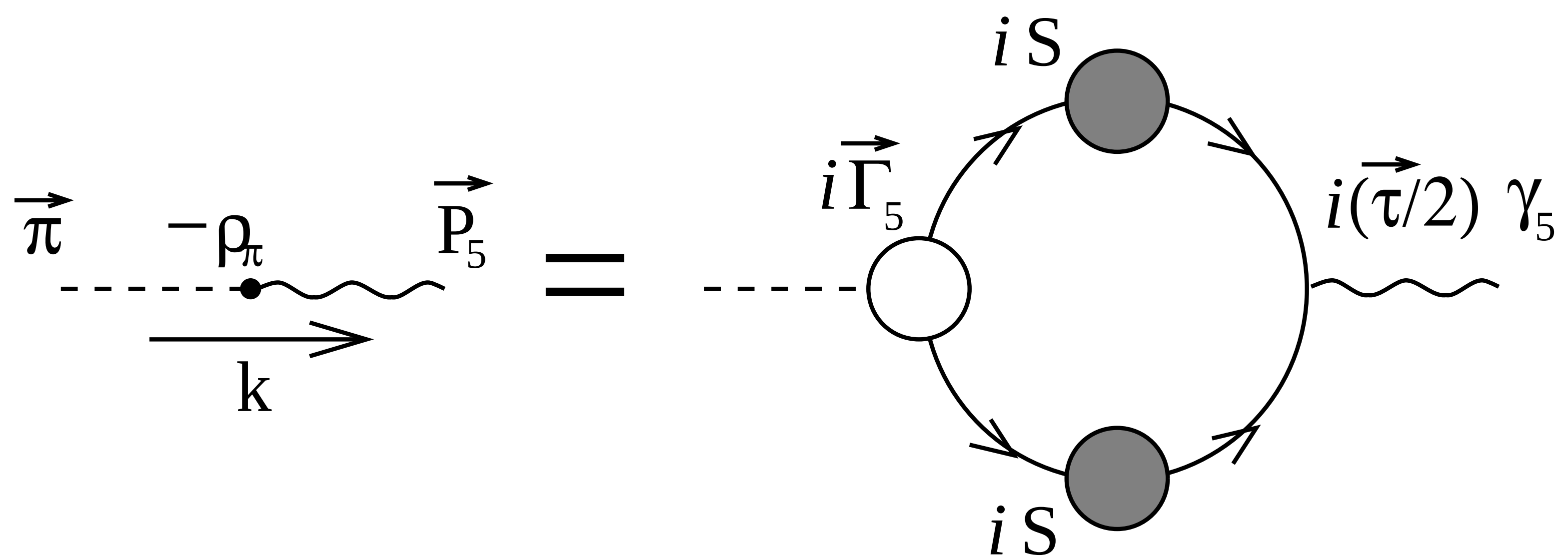

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- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $\boxed{m_H \propto m_q}$

... QCD Proof of Potential Model result

Höll, Krassnigg, Roberts
nu-th/0406030

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- Dynamical Chiral Symmetry Breaking

  – Goldstone's Theorem –

  impacts upon *every* pseudoscalar meson

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

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  $16^3 \times 32$,
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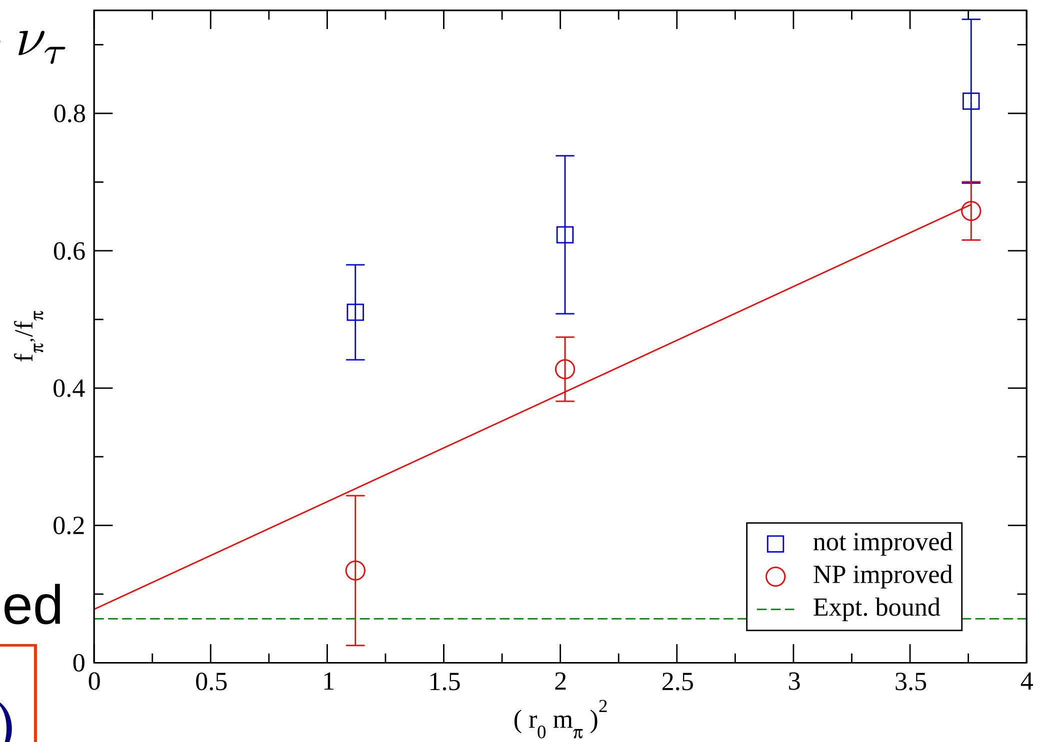
$$\boxed{\Rightarrow \frac{f_{\pi_1}}{f_\pi} = \mathbf{0.078\,(93)}}$$



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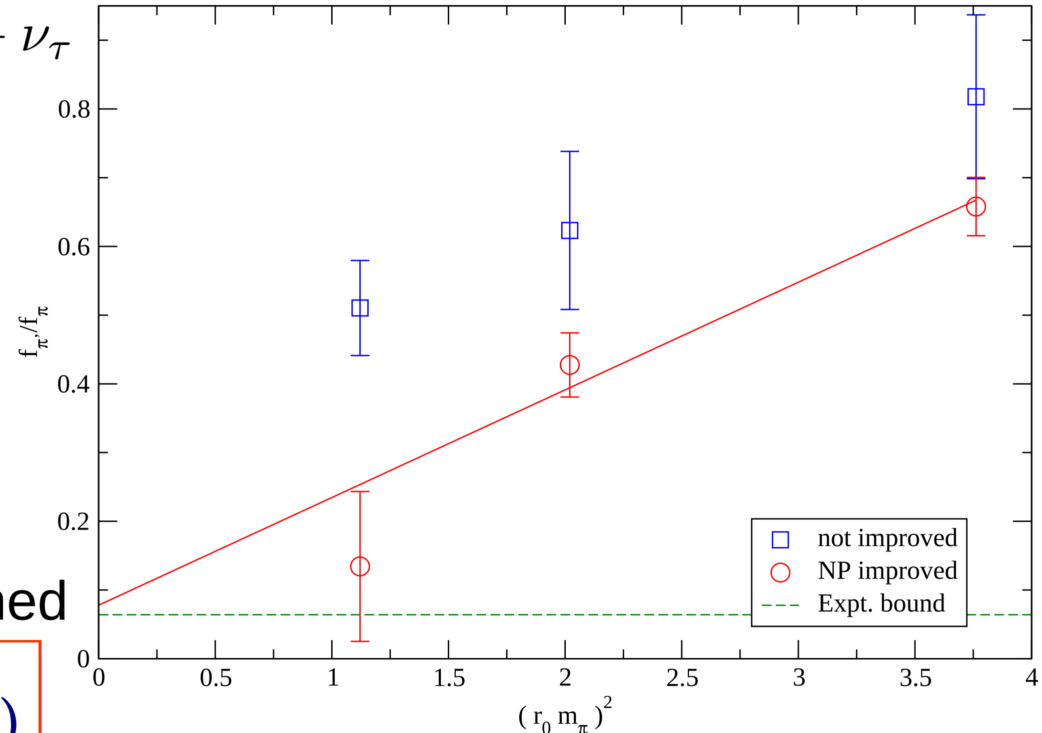

- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

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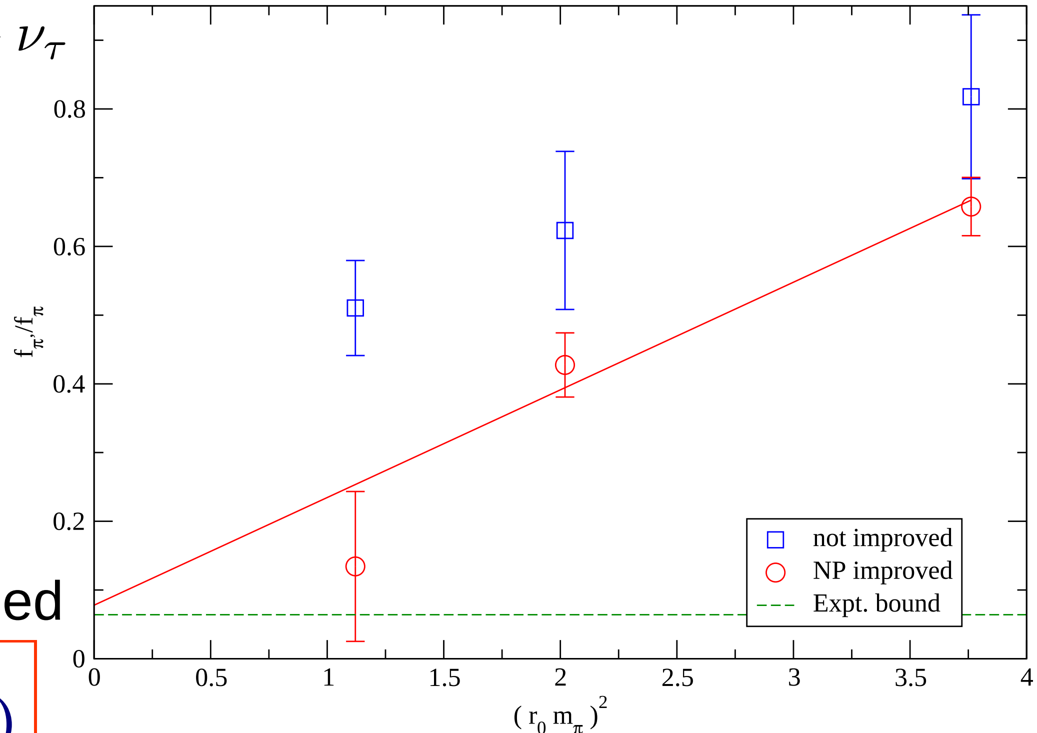

- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Pion . . . $J = 0$ but . . .

- Orbital angular momentum is not a Poincaré invariant. However, if absent in a particular frame, it will appear in another frame related via a Poincaré transformation.

Pion . . . $J = 0$ but . . .

- Nonzero quark orbital angular momentum is thus a necessary outcome of a Poincaré covariant description.

Pion . . . $J = 0$ but . . .

- Pseudoscalar meson Bethe-Salpeter amplitude

$$\begin{aligned}\chi_\pi(k;P) &= \gamma_5\,[i\mathcal{E}_{\pi_n}(k;P) + \gamma\cdot P\mathcal{F}_{\pi_n}(k;P)\\ &\quad \gamma\cdot k\, k\cdot P\,\mathcal{G}_{\pi_n}(k;P) + \sigma_{\mu\nu}\,k_\mu P_\nu\,\mathcal{H}_{\pi_n}(k;P)]\end{aligned}$$

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- $J = 0 \ldots$ *but* while $\mathcal{E}$ and $\mathcal{F}$ are purely $L = 0$ in the rest frame, the $\mathcal{G}$ and $\mathcal{H}$ terms are associated with $L = 1$. Thus a pseudoscalar meson Bethe-Salpeter wave function *always* contains both S- and P-wave components.

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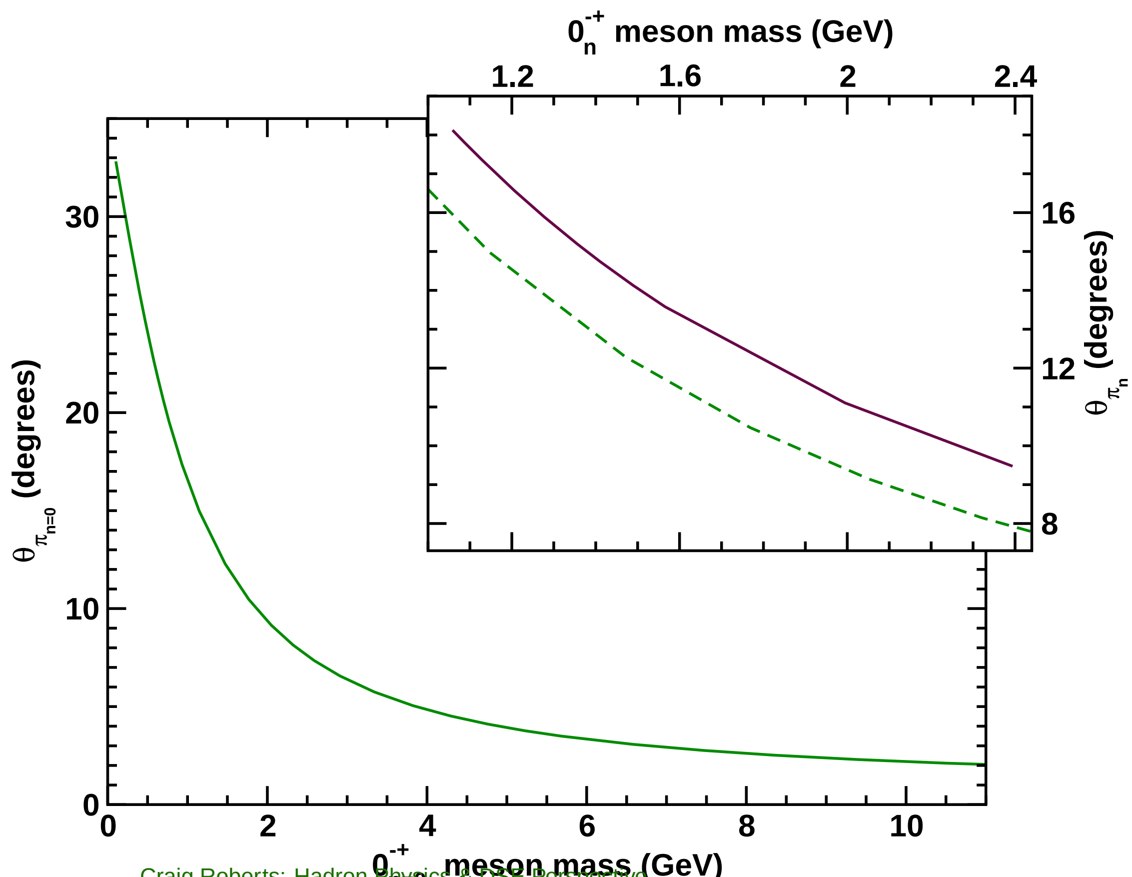

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Introduce mixing angle θ_π such that

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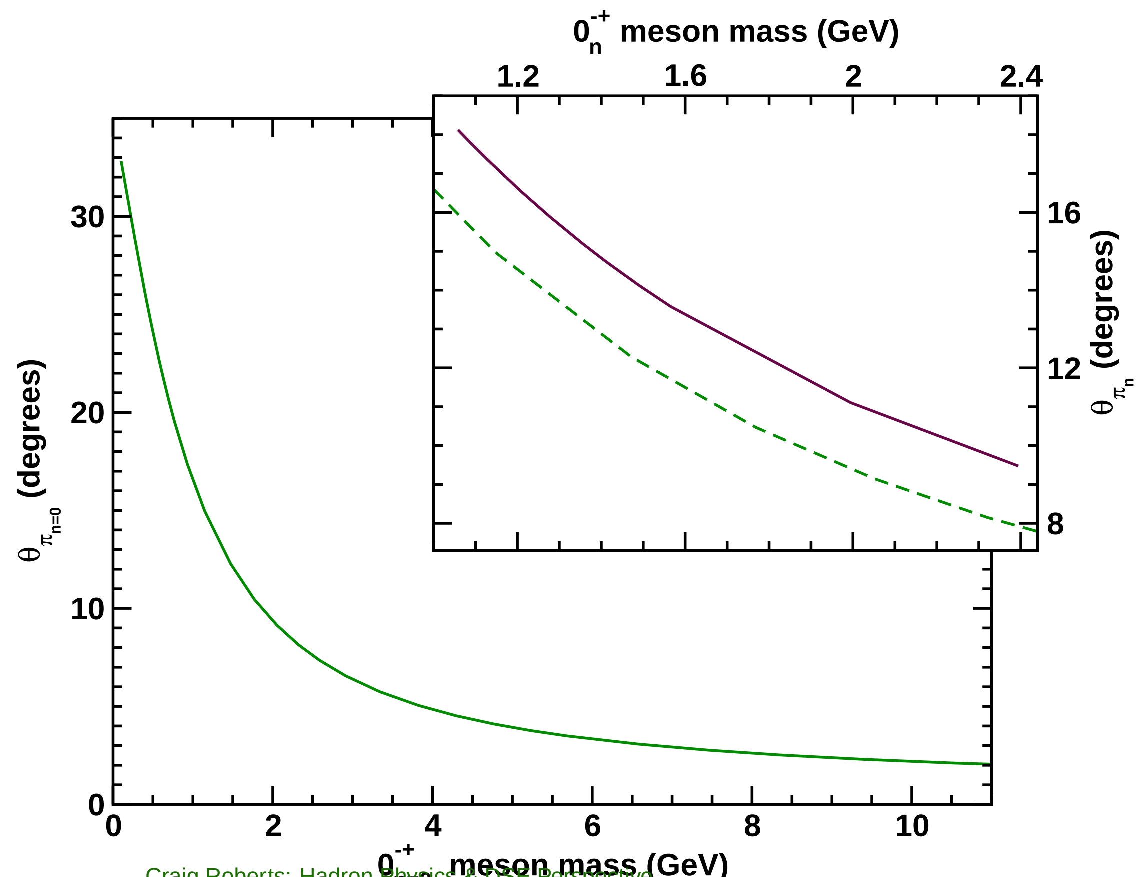
L is significant in the neighbourhood of the chiral limit, and decreases with increasing current-quark mass.



Charge Neutral Pseudoscalar Mesons

$$
\begin{aligned}
P_\mu \Gamma^a_{5\mu}(k;P) &= \mathcal{S}^{-1}(k_+) i\gamma_5 \mathcal{F}^a + i\gamma_5 \mathcal{F}^a \mathcal{S}^{-1}(k_-) \\
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- $\{\mathcal{F}^a \,|\, a = 0,\ldots,N_f^2-1\}$ are the generators of $U(N_f)$
- $\mathcal{S} =$ diag$[S_u, S_d, S_s, S_c, S_b, \ldots]$
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Charge Neutral Pseudoscalar Mesons

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- The final term in the second line expresses the non-Abelian axial anomaly.

Charge Neutral Pseudoscalar Mesons

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… The topological charge density operator.

Charge Neutral Pseudoscalar Mesons

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  … The topological charge density operator.
  (Trace is over colour indices & $F_{\mu\nu} = \frac{1}{2}\lambda^a F^a_{\mu\nu}$.)

Office of Science U.S. DEPARTMENT OF ENERGY

Office of Nuclear Physics

Argonne NATIONAL LABORATORY

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  … The topological charge density operator.
- Important that only $\mathcal{A}^{a=0}$ is nonzero.

Charge Neutral Pseudoscalar Mesons

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  … The topological charge density operator.
- NB. While $\mathcal{Q}(x)$ is gauge invariant, the associated Chern-Simons current, K_μ, is not $\Rightarrow$ in QCD *no physical* boson can couple to K_μ and hence *no physical* states can contribute to resolution of U$_A$(1) problem.

Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting

Bhagwat, Chang, Liu, Roberts, Tandy
nucl-th/arXiv:0708.1118

Charge Neutral Pseudoscalar Mesons

- Only $\mathcal{A}^0 \not\equiv 0$ is interesting ... otherwise all pseudoscalar mesons are Goldstone Modes!

Charge Neutral Pseudoscalar Mesons

- Anomaly term has structure

$$\mathcal{A}^0(k;P) = \mathcal{F}^0\gamma_5\left[i\mathcal{E}_{\mathcal{A}}(k;P) + \gamma\cdot P\mathcal{F}_{\mathcal{A}}(k;P)\right.$$
$$\left.+\gamma\cdot k k\cdot P\mathcal{G}_{\mathcal{A}}(k;P) + \sigma_{\mu\nu}k_\mu P_\nu\mathcal{H}_{\mathcal{A}}(k;P)\right]$$

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Charge Neutral Pseudoscalar Mesons

- AVWTI gives generalised Goldberger-Treiman relations

$$\begin{aligned} 2f^0_{\eta'} E_{BS}(k;0) &= 2B_0(k^2) - \mathcal{E}_{\mathcal{A}}(k;0), \\ F^0_R(k;0) + 2f^0_{\eta'} F_{BS}(k;0) &= A_0(k^2) - \mathcal{F}_{\mathcal{A}}(k;0), \\ G^0_R(k;0) + 2f^0_{\eta'} G_{BS}(k;0) &= 2A'_0(k^2) - \mathcal{G}_{\mathcal{A}}(k;0), \\ H^0_R(k;0) + 2f^0_{\eta'} H_{BS}(k;0) &= -\mathcal{H}_{\mathcal{A}}(k;0), \end{aligned}$$

A_0, B_0 characterise gap equation's chiral limit solution.

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A_0, B_0 characterise gap equation's chiral limit solution.

- Follows that $\boldsymbol{\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)}$ is necessary and sufficient condition for absence of massless $\boldsymbol{\eta'}$ bound-state.

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Charge Neutral Pseudoscalar Mesons

- $\mathcal{E}_{\mathcal{A}}(k;0) = 2B_0(k^2)$
  Discussing the chiral limit
  - $B_0(k^2) \neq 0$ if, and only if, chiral symmetry is dynamically broken.
  - Hence, absence of massless η' bound-state is only assured through existence of intimate connection between DCSB and an expectation value of the topological charge density.

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- Further highlighted ... proved

$$\begin{aligned}\langle \bar{q}q \rangle_\zeta^0 &= -\lim_{\Lambda\to\infty} Z_4(\zeta^2,\Lambda^2)\, \mathrm{tr}_{\mathrm{CD}} \int_q^\Lambda S^0(q,\zeta) \\ &= N_f \int d^4x\, \langle \bar{q}(x) i\gamma_5 q(x) \mathcal{Q}(0)\rangle^0 .\end{aligned}$$

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Charge Neutral Pseudoscalar Mesons

- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons

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- Employed in an analysis of pseudoscalar- and vector-meson bound-states

Charge Neutral Pseudoscalar Mesons

Bhagwat, Chang, Liu, Roberts, Tandy
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- AVWTI $\Rightarrow$ QCD mass formulae for neutral pseudoscalar mesons
- Implications of mass formulae illustrated using elementary dynamical model, which includes *Ansatz* for that part of the Bethe-Salpeter kernel related to the non-Abelian anomaly
- Despite its simplicity, model is elucidative and phenomenologically efficacious; e.g., it predicts
  - η–η' mixing angles of $\sim -15^\circ$ (Expt.: $-13.3^\circ \pm 1.0^\circ$)
  - π^0–η angles of $\sim 1.2^\circ$ (Expt. $p\,d \to {}^3$He$\,\pi^0$: $0.6^\circ \pm 0.3^\circ$)
  - Strong neutron-proton mass difference . . .

    $\lesssim 75\,\%$ current-quark mass-difference

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New Challenges

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- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks.

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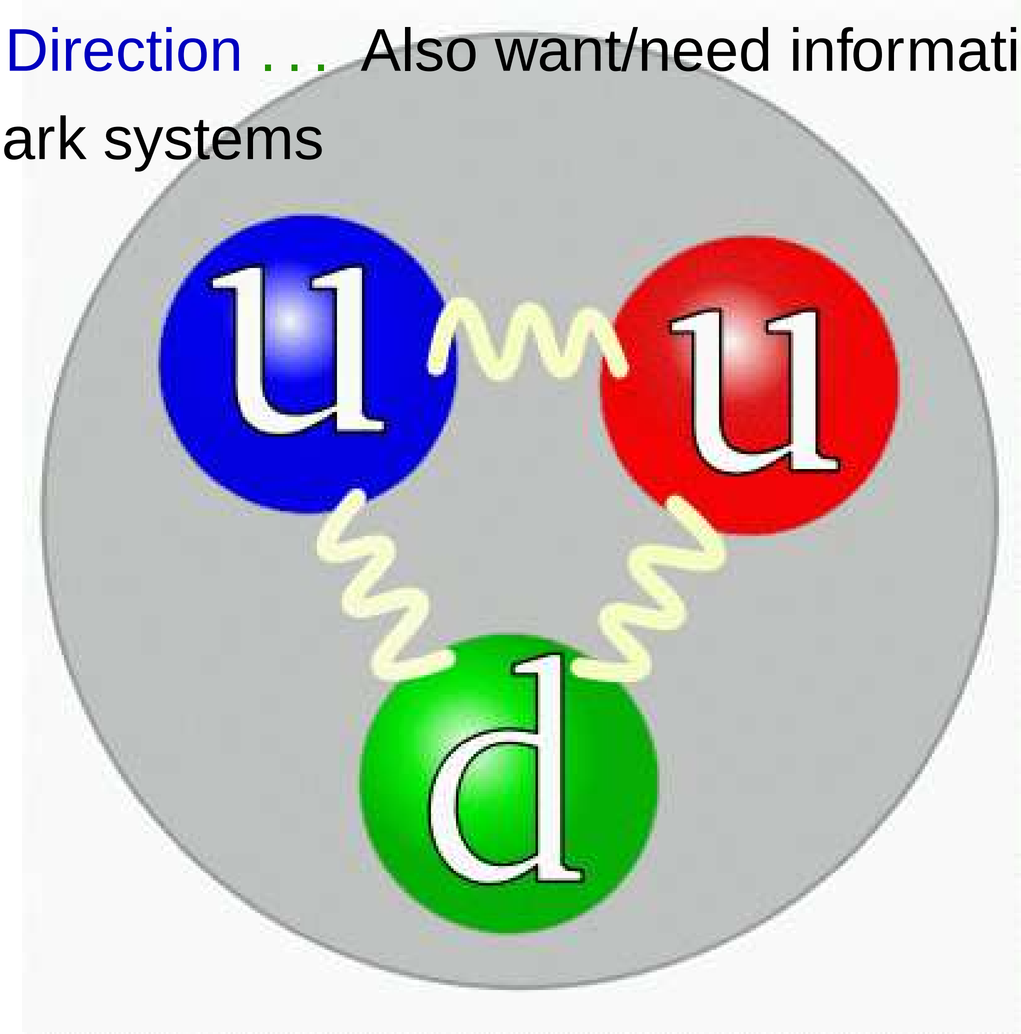
- Move on to the problem of a symmetry preserving treatment of hybrids and exotics.

New Challenges

- Another Direction . . . Also want/need information about three-quark systems



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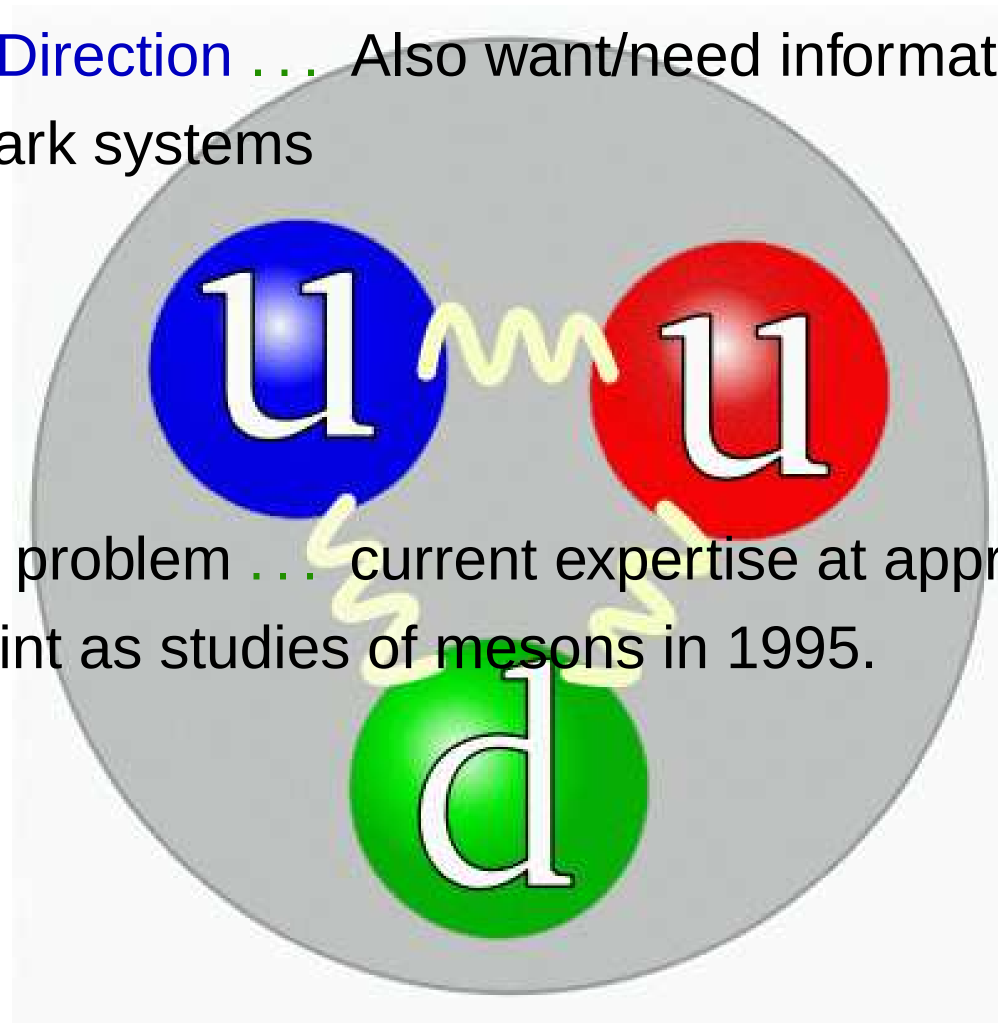

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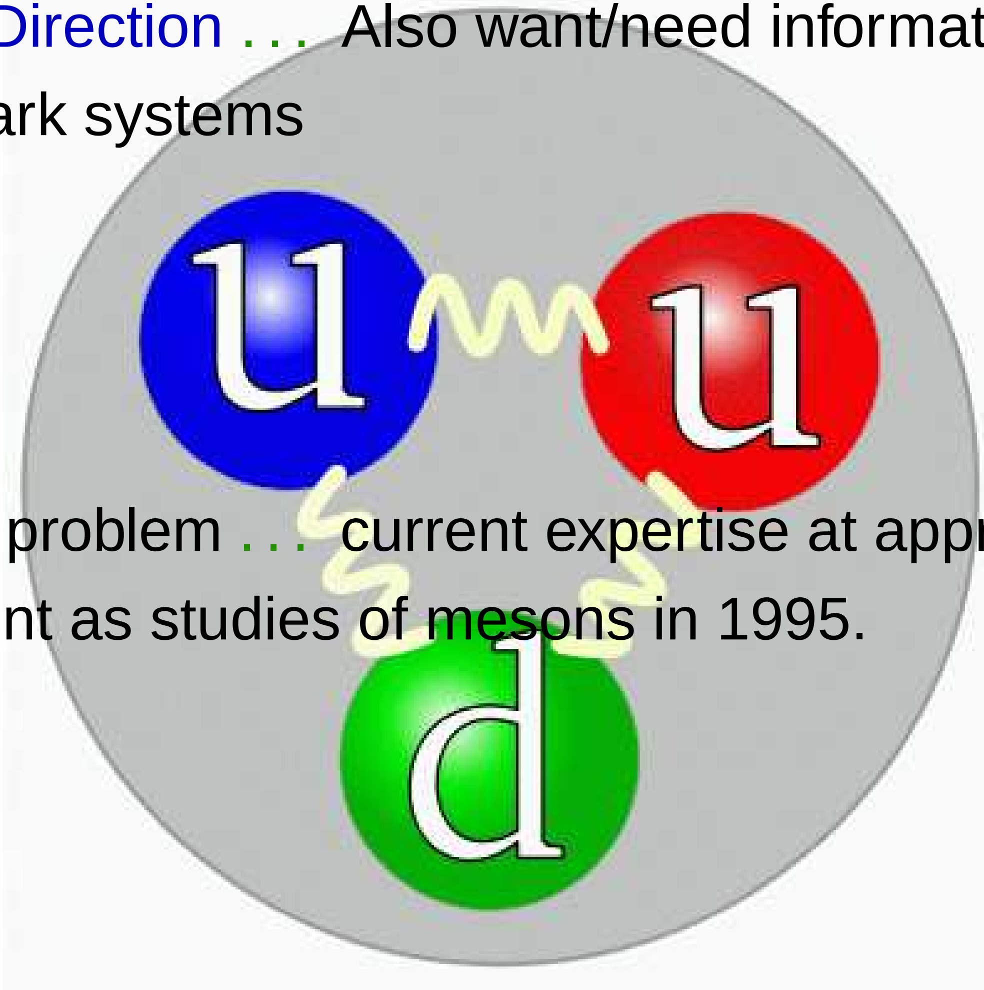

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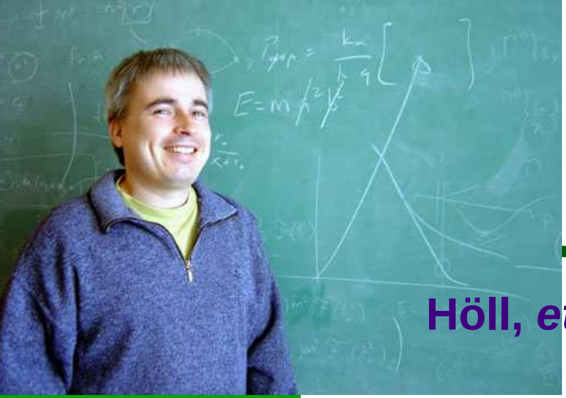
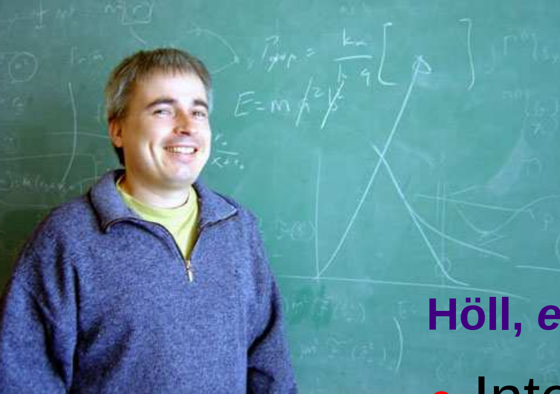
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.



Nucleon EM Form Factors: A Précis

Höll, *et al.*: nu-th/0412046 & nu-th/0501033



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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

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- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

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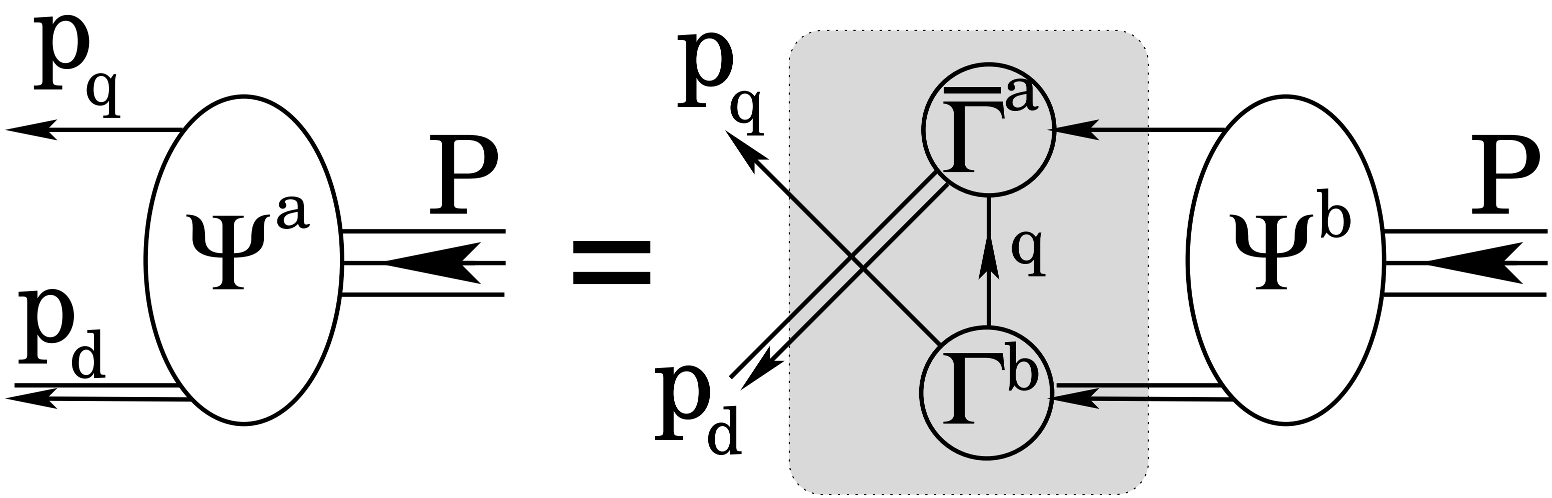
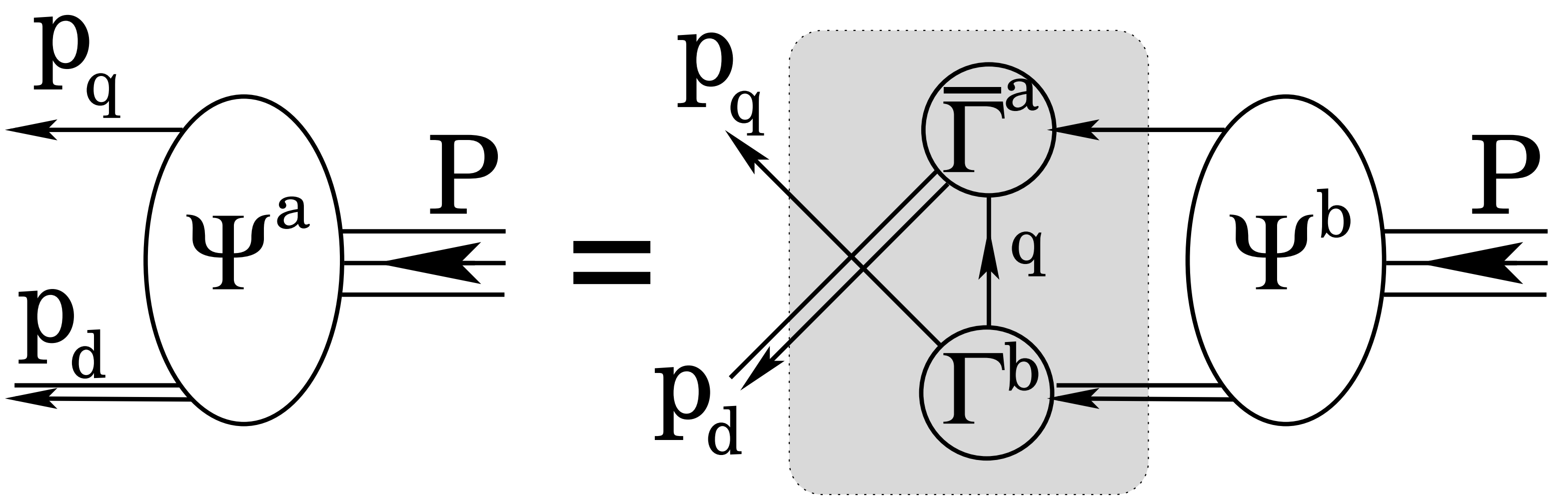
Faddeev equation

Faddeev equation

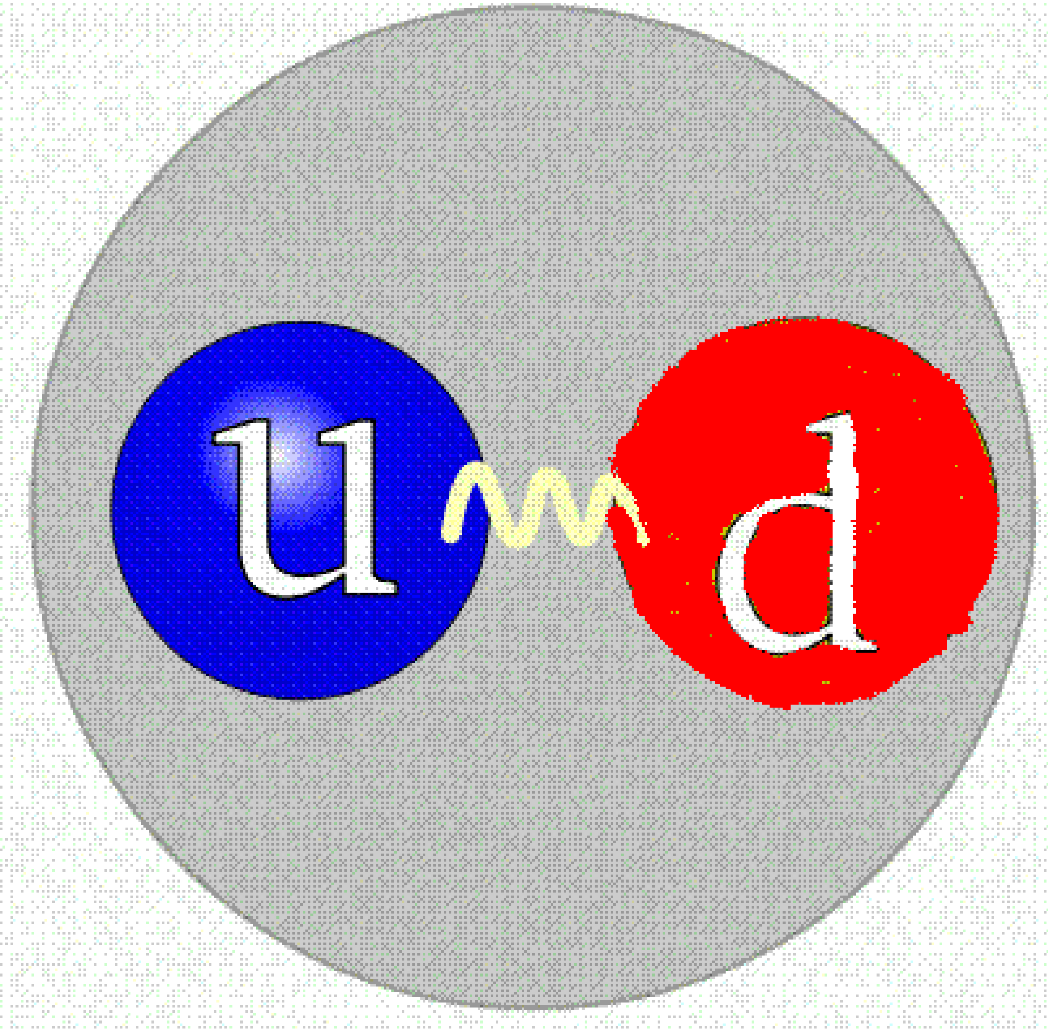


Faddeev equation



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Diquark correlations



QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red

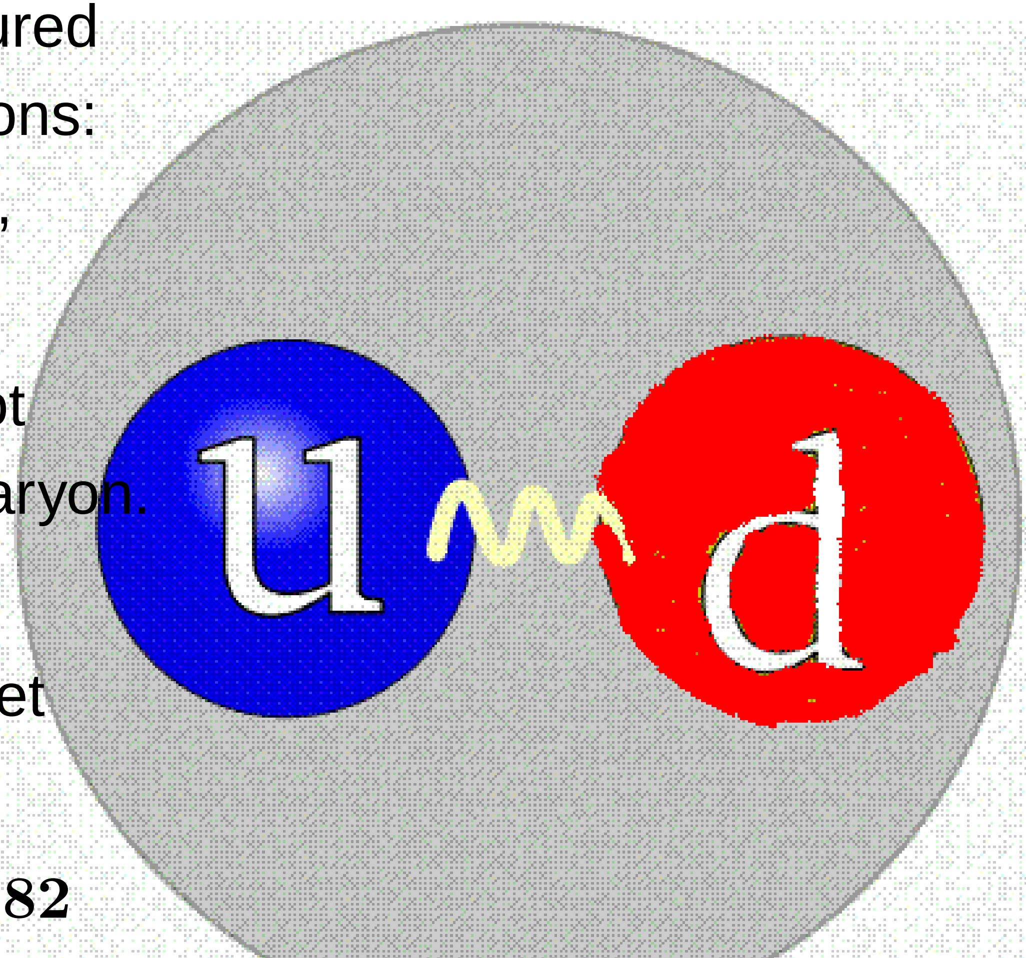
- Confined . . . Does not escape from within baryon.

- Scalar is isosinglet, Axial-vector is isotriplet

- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

Harry Lee

Pions and Form Factors

Pions and Form Factors

- Dynamical coupled-channels model ... Analyzed extensive JLab data ... Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e,e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)

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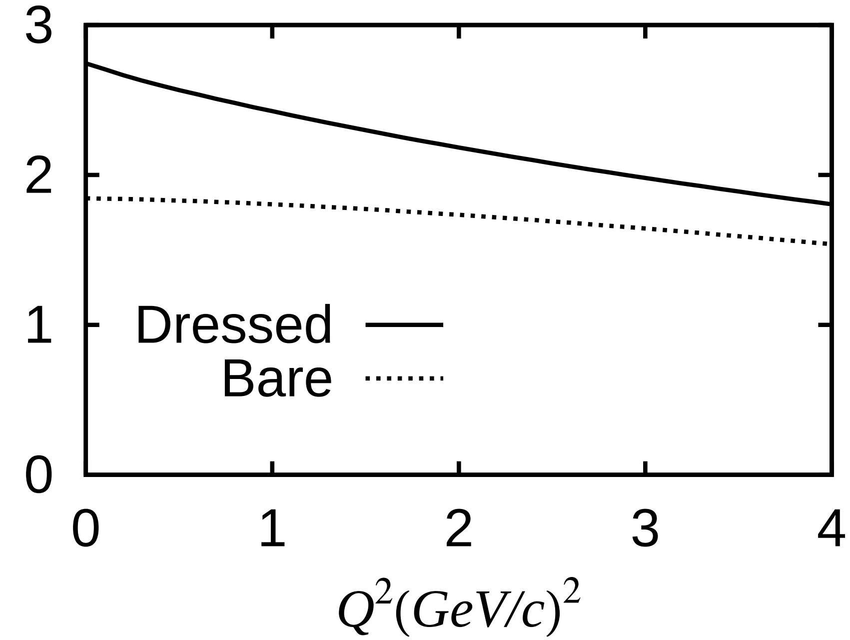
Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



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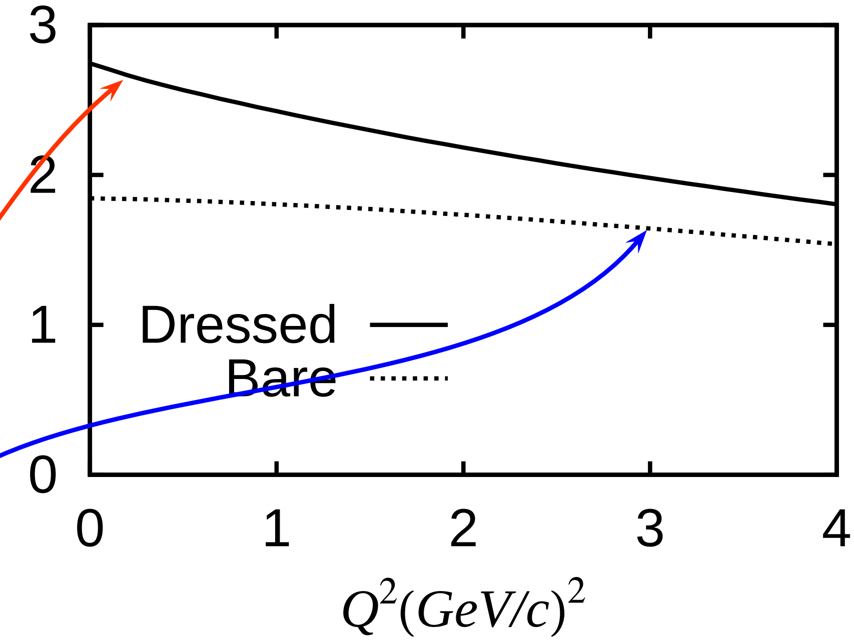

Quark Core

- Responsible for only 2/3 of result at small Q^2
- Dominant for $Q^2 >$ 2 – 3 GeV^2

Results: Nucleon and Δ Masses

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Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for
Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

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- Axial-vector diquark provides significant attraction

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- Constructive Interference: 1^{++}-diquark $+ \partial_\mu \pi$

Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

for on-shell nucleons described by Faddeev Amplitude

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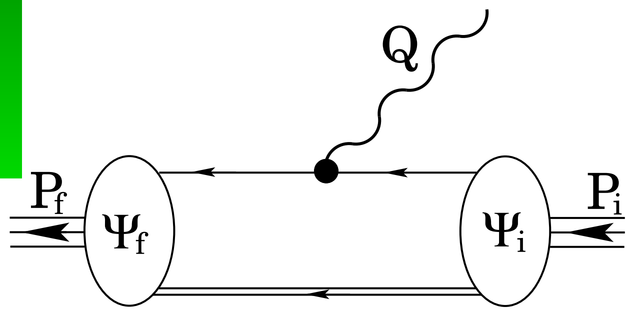
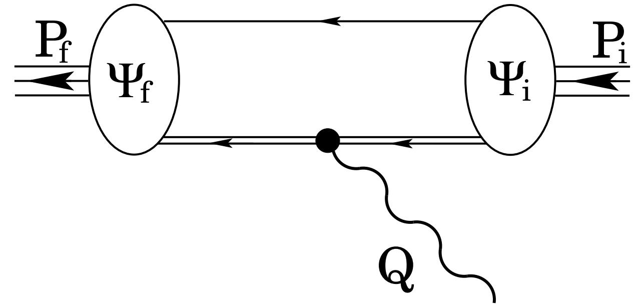
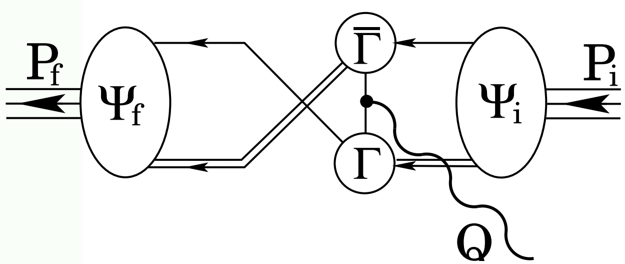
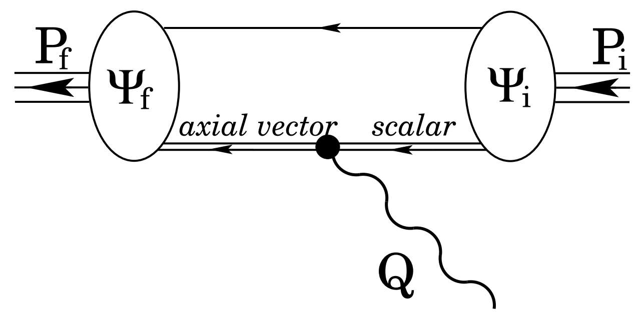
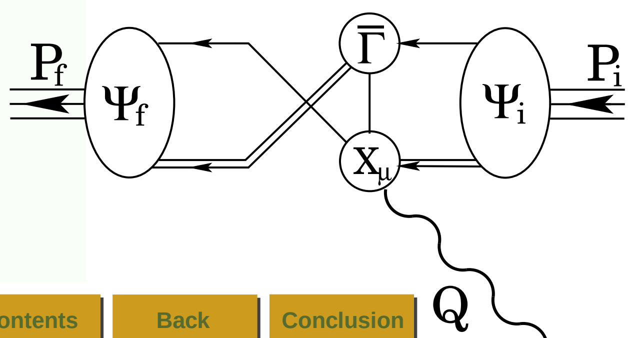
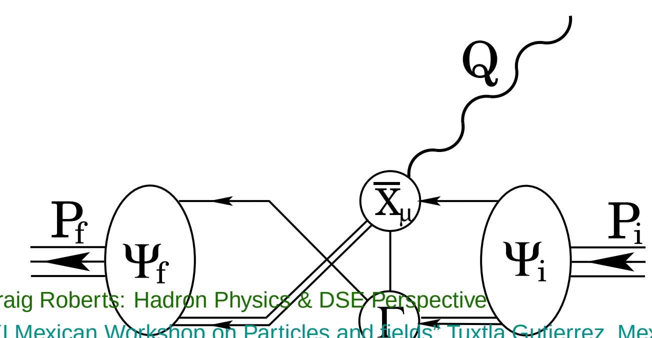




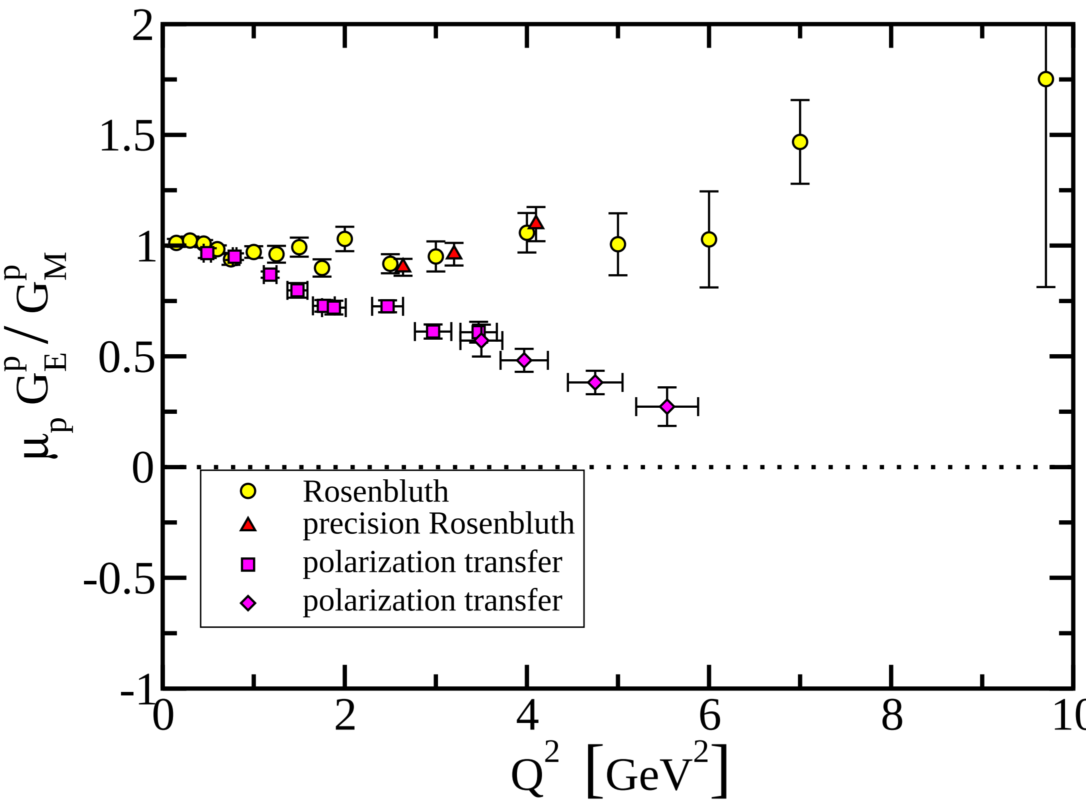








Form Factor Ratio: GE/GM



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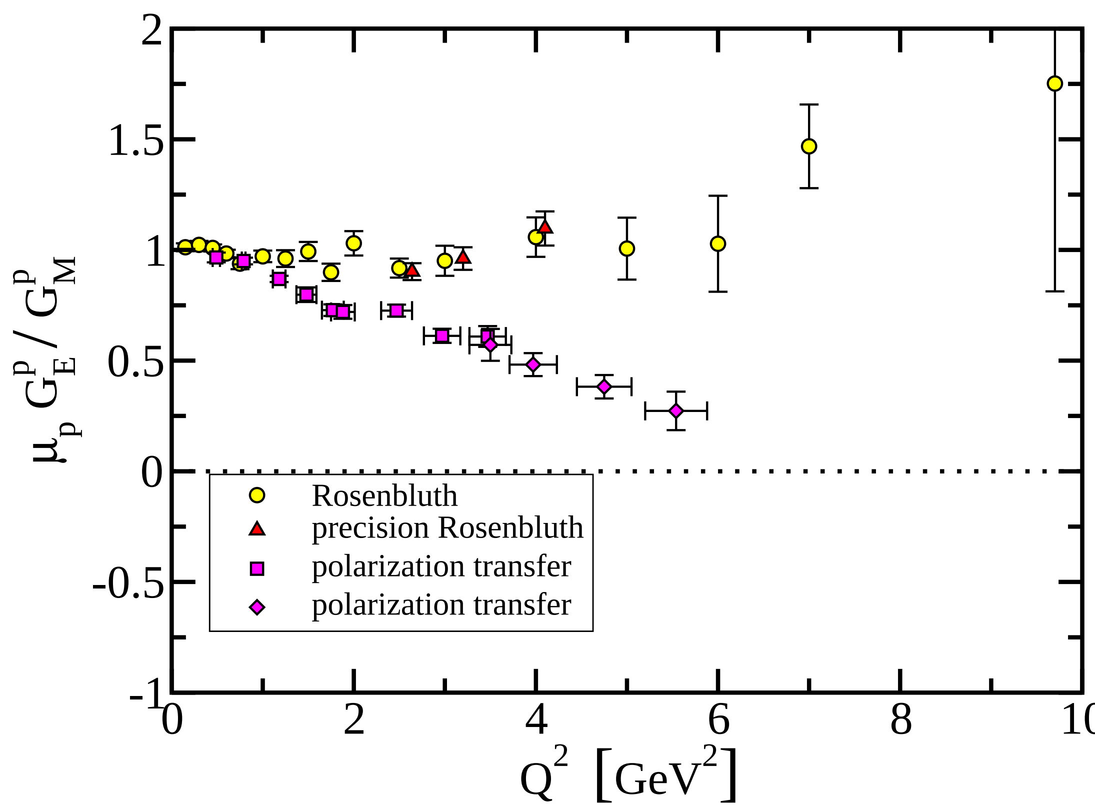

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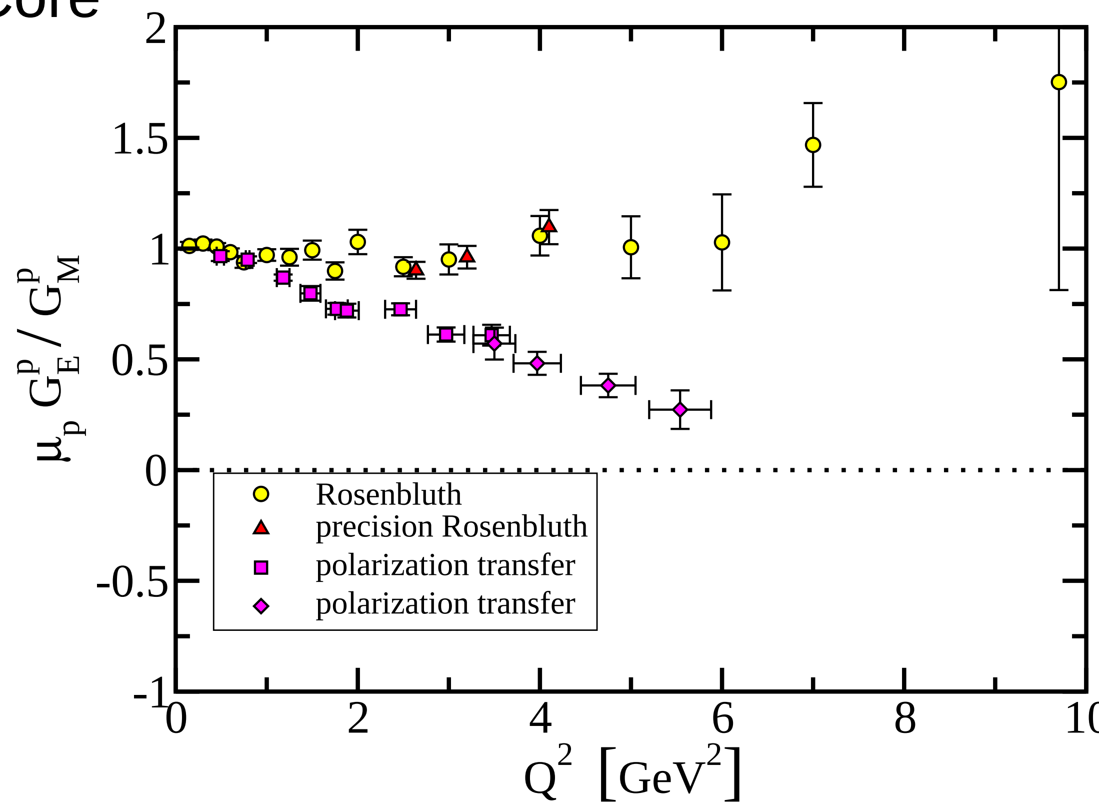

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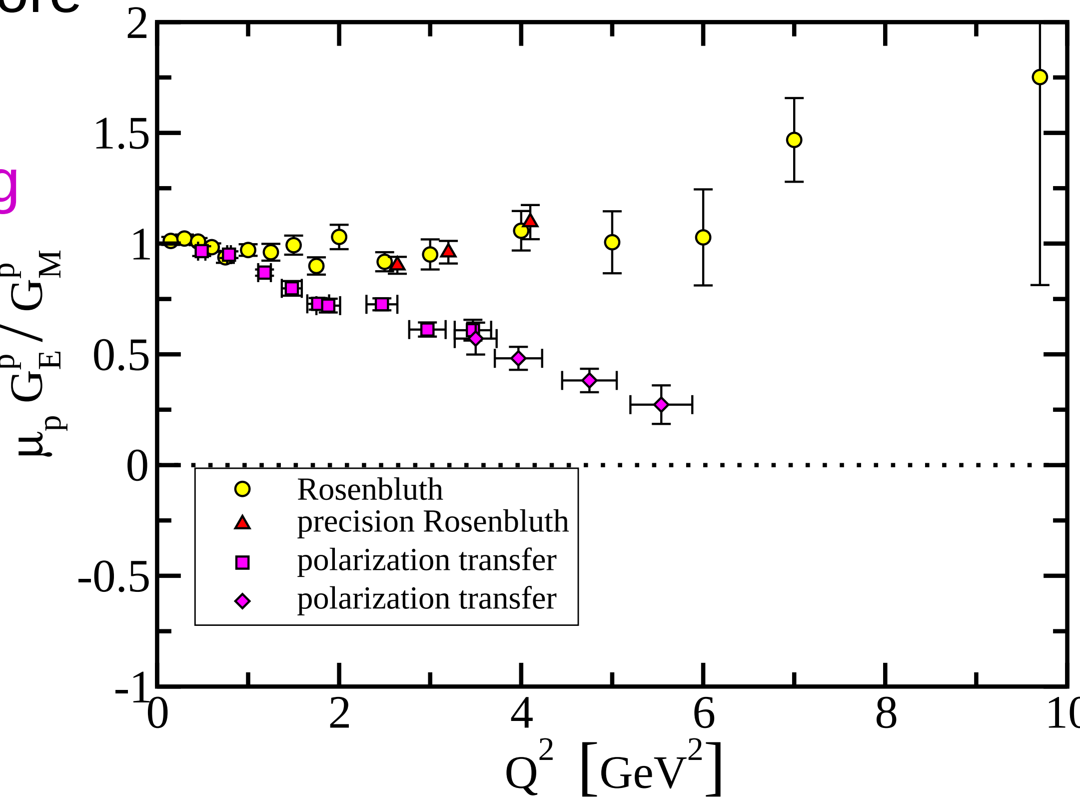

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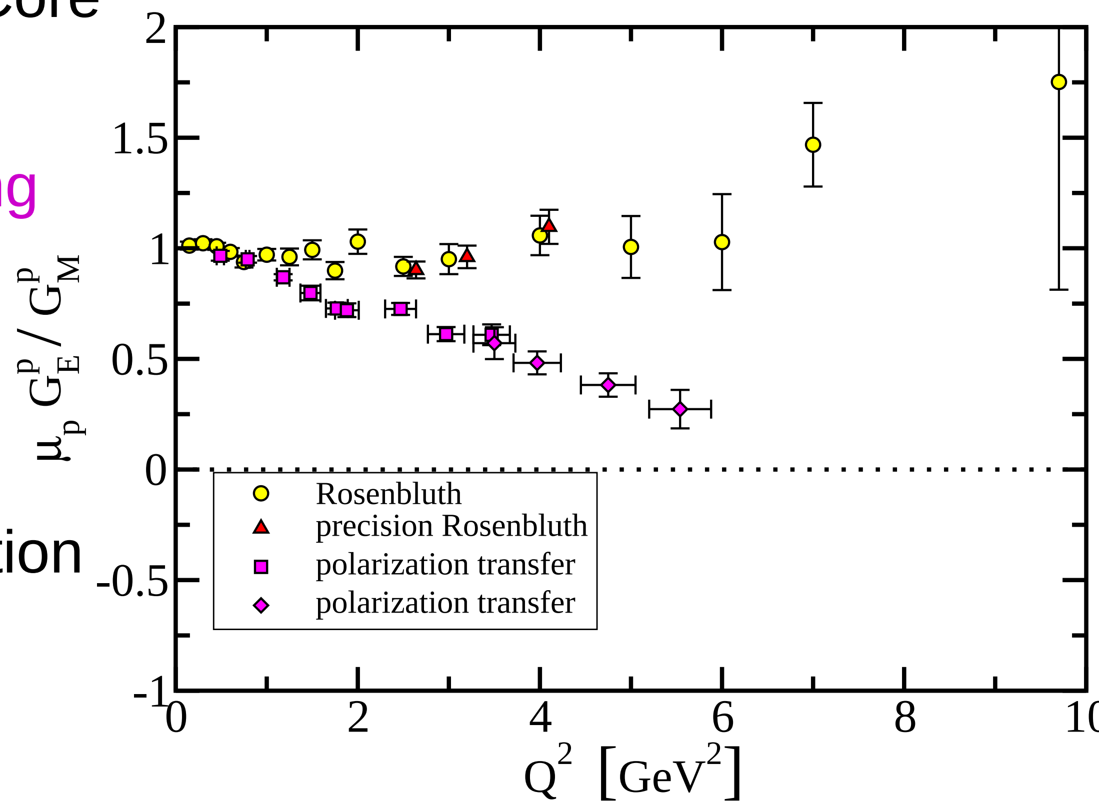

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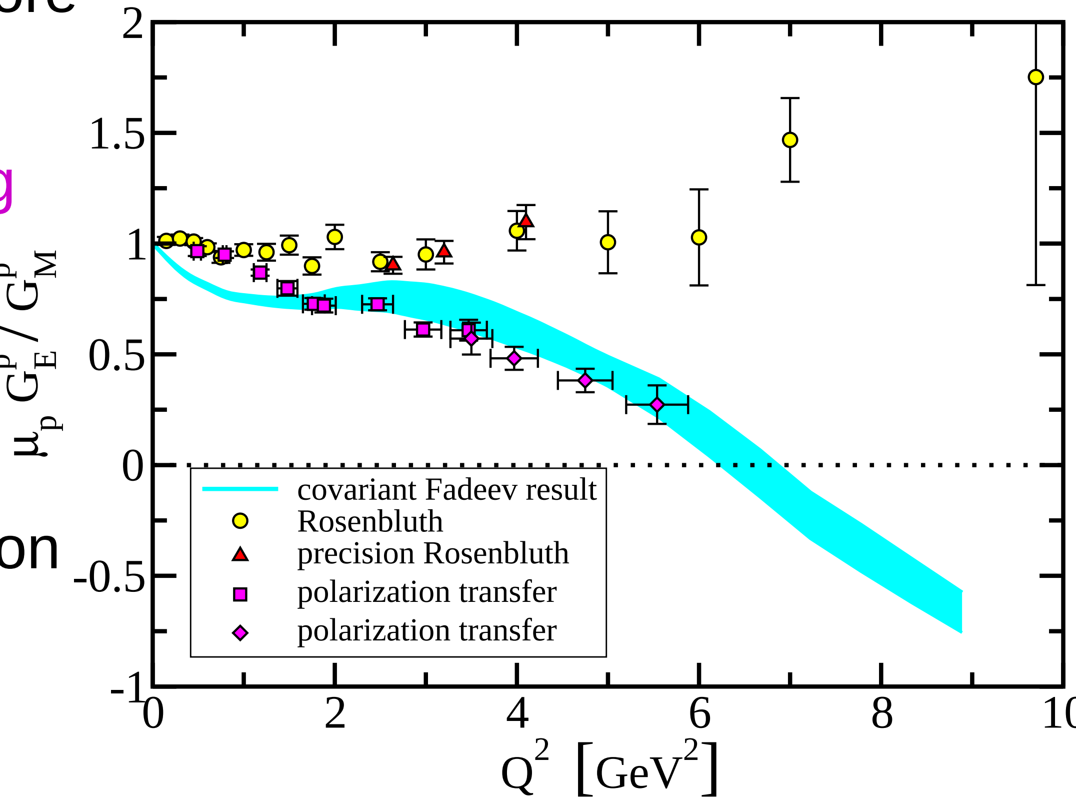

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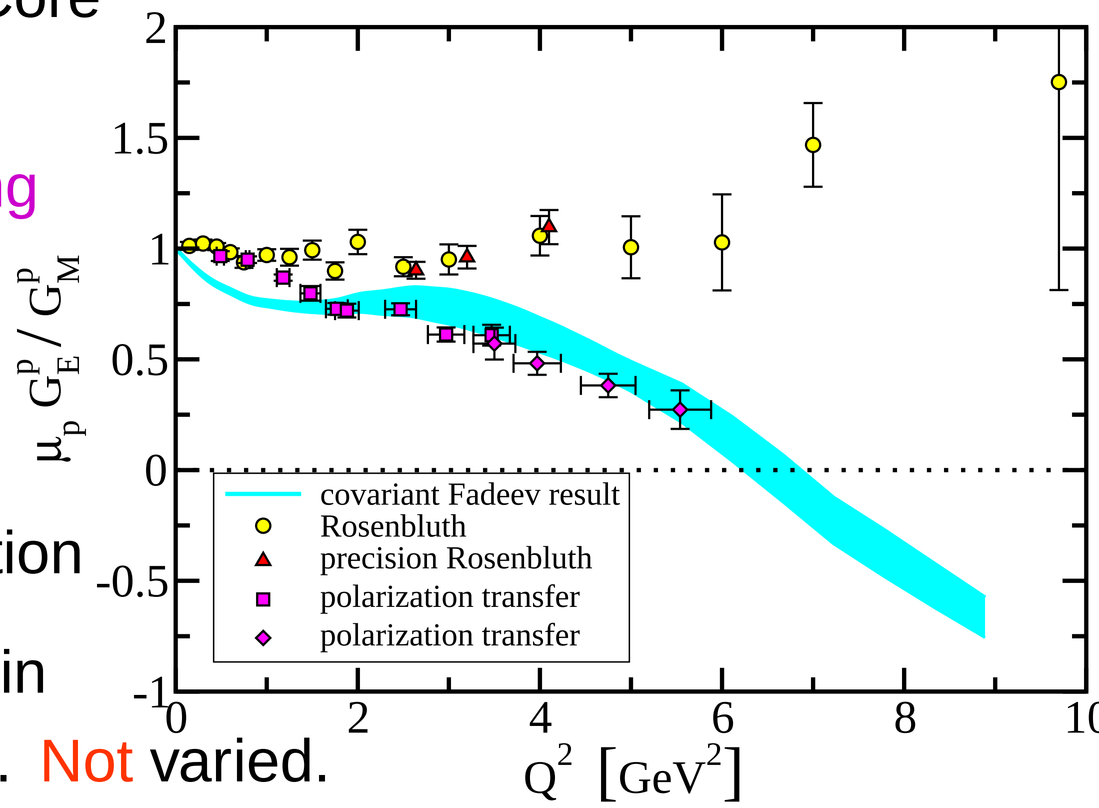

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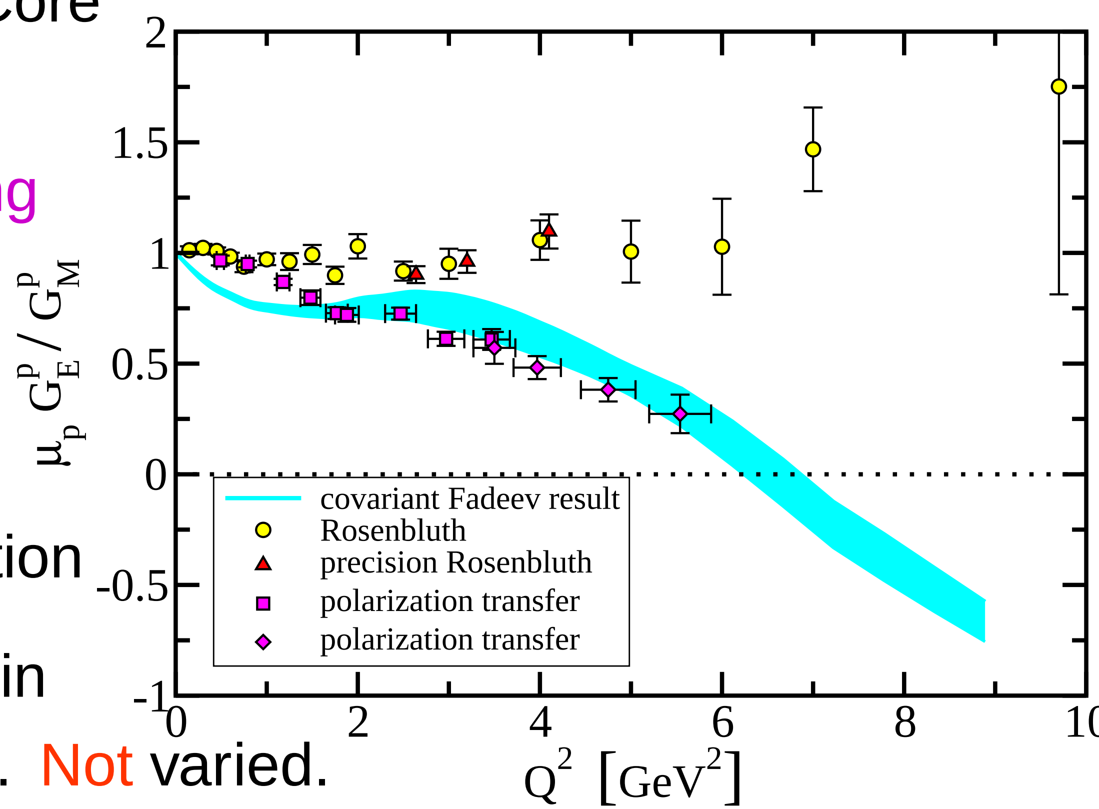

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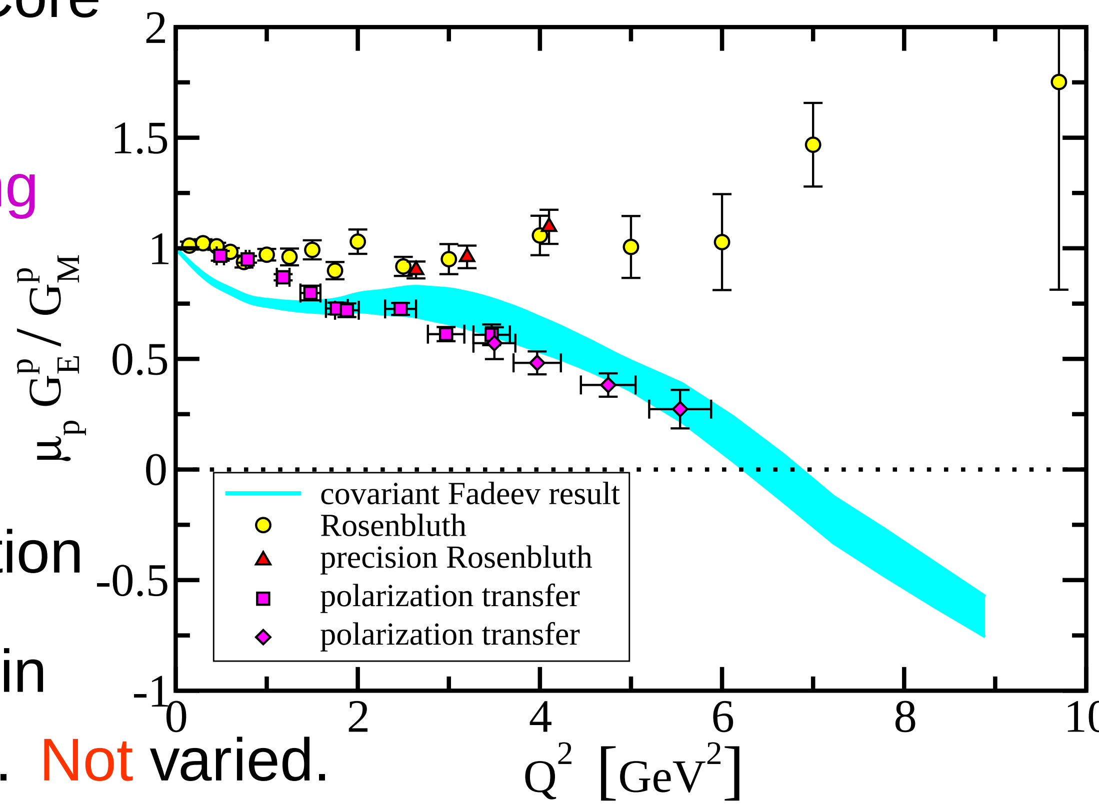

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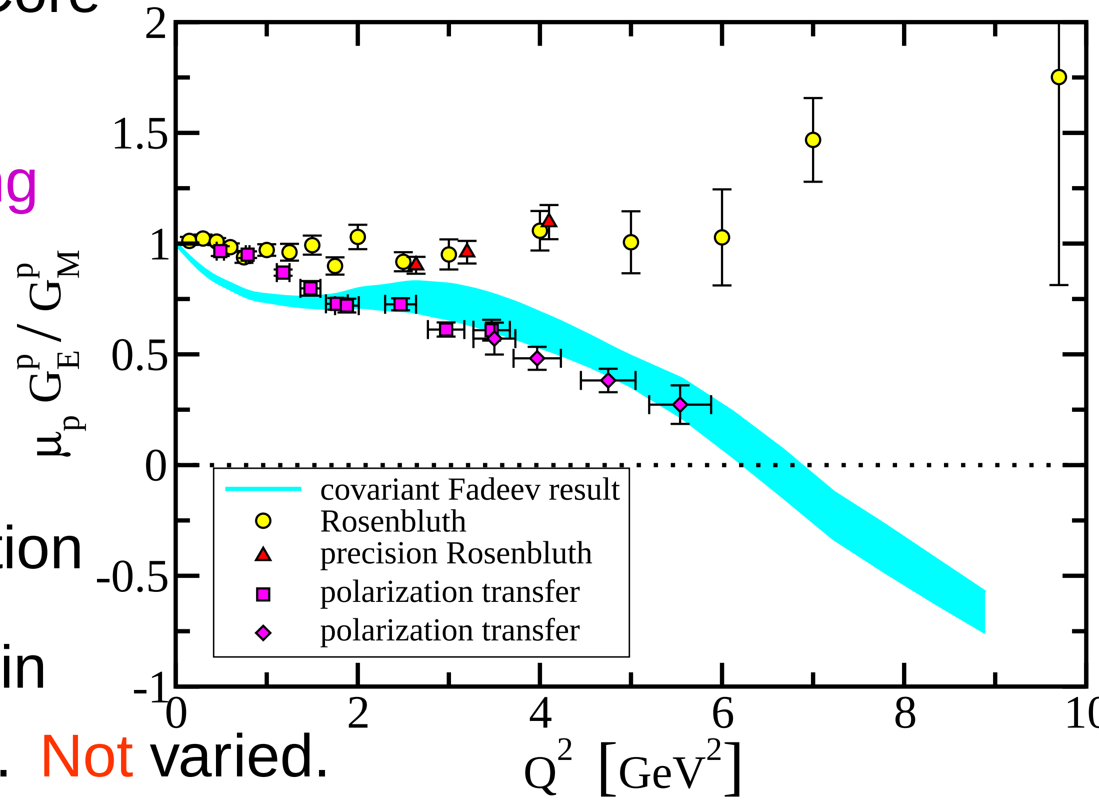

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  - Predict Zero at $Q^2 \approx 6.5$GeV^2

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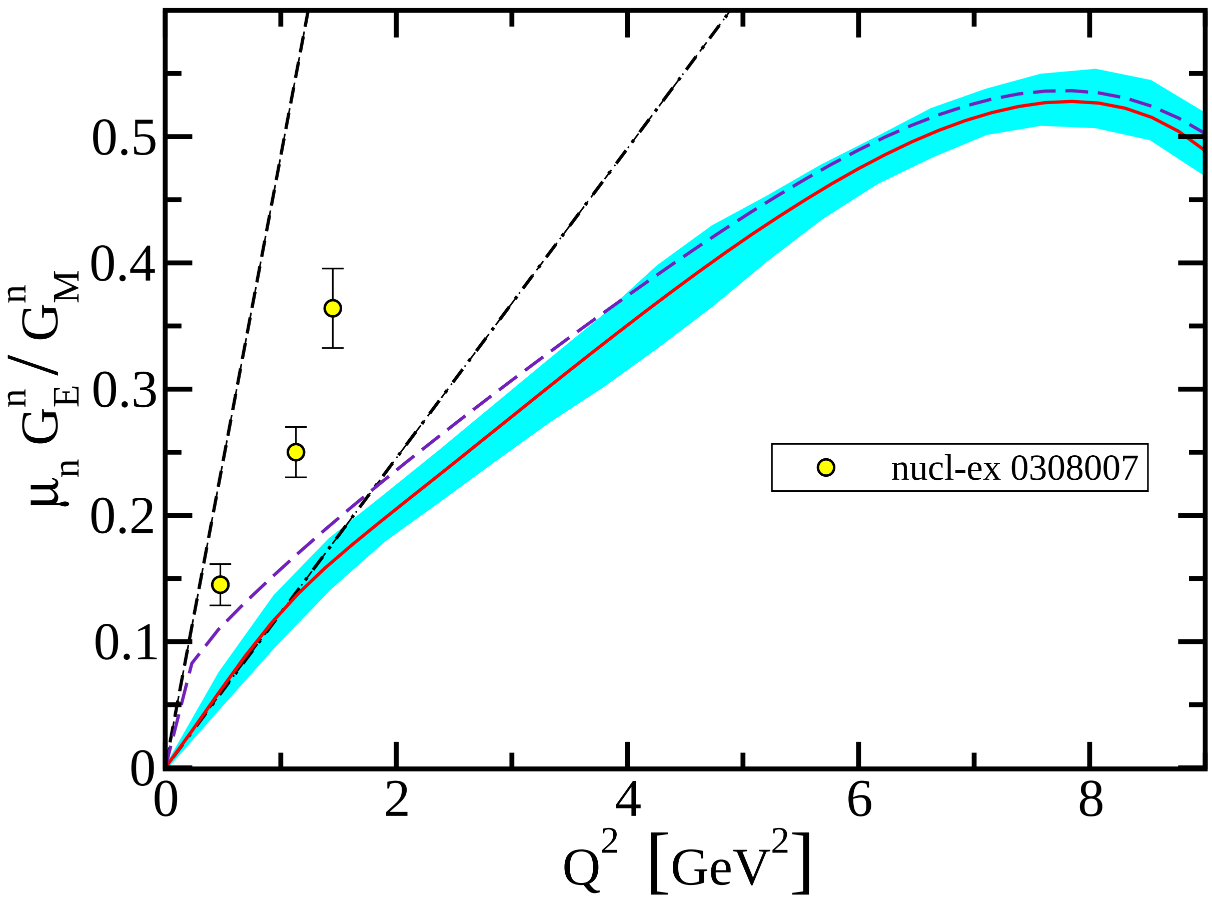
Neutron Form Factors



Neutron Form Factors

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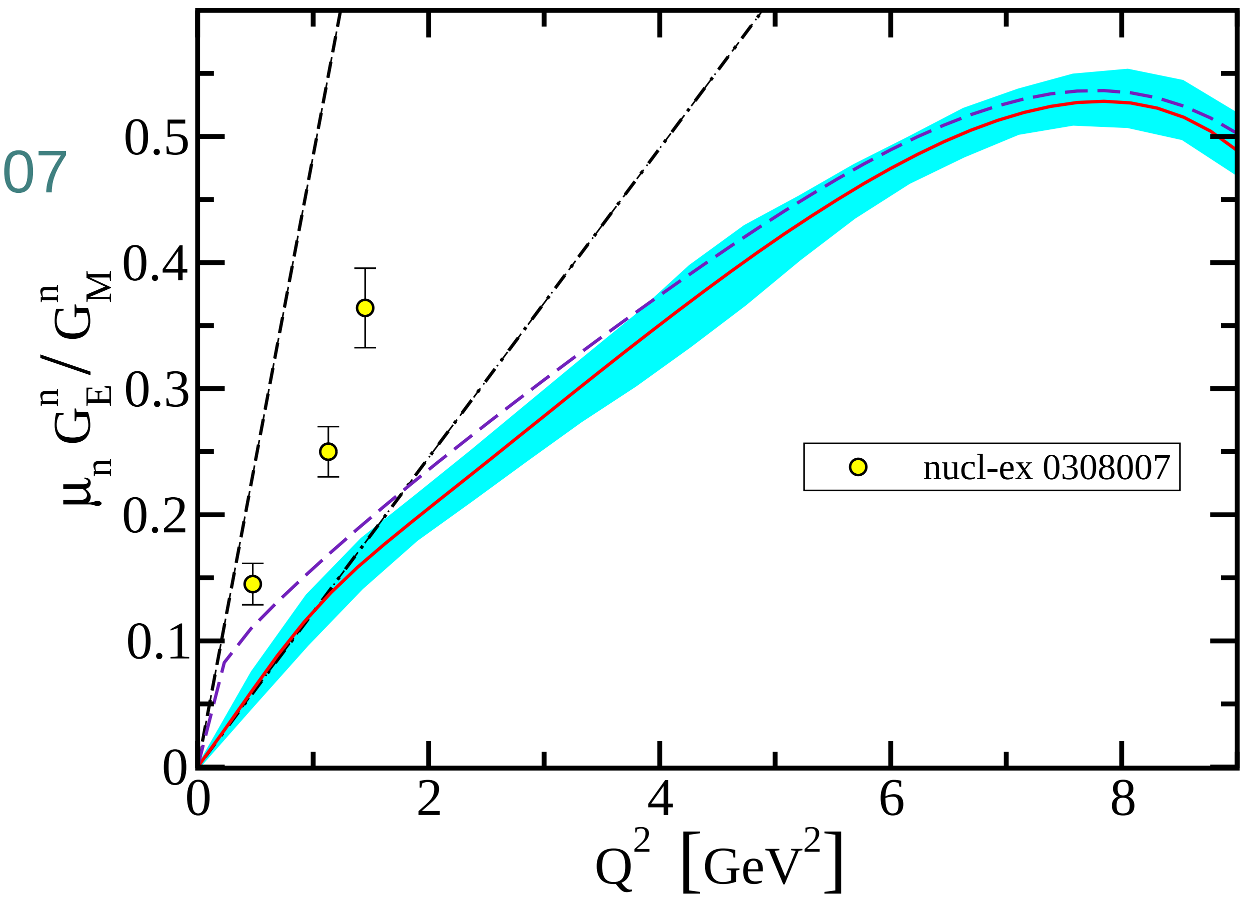

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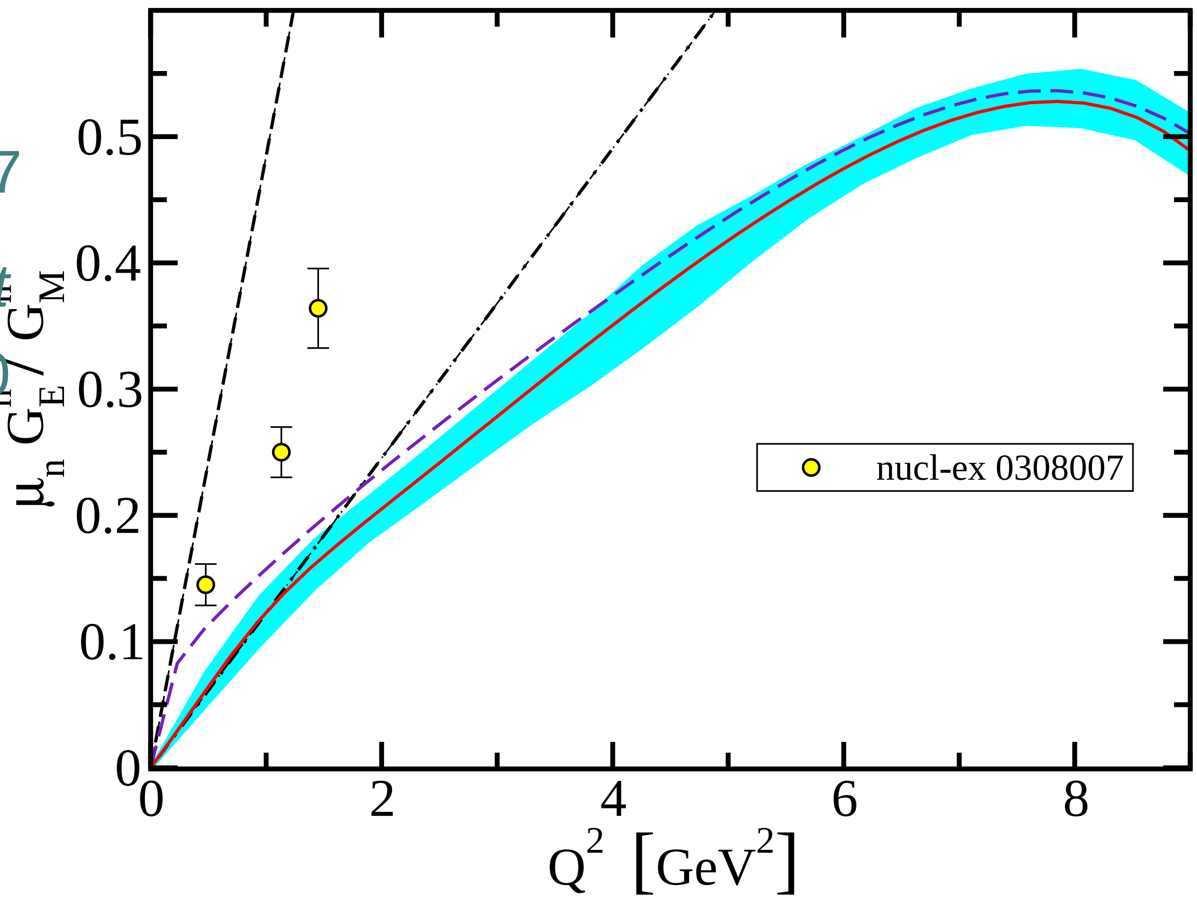
- $\mu_p \frac{G_E^n(Q^2)}{G_M^n(Q^2)} = -\frac{r_n^2}{6} Q^2$

Valid for $r_n^2 Q^2 \lesssim 1$



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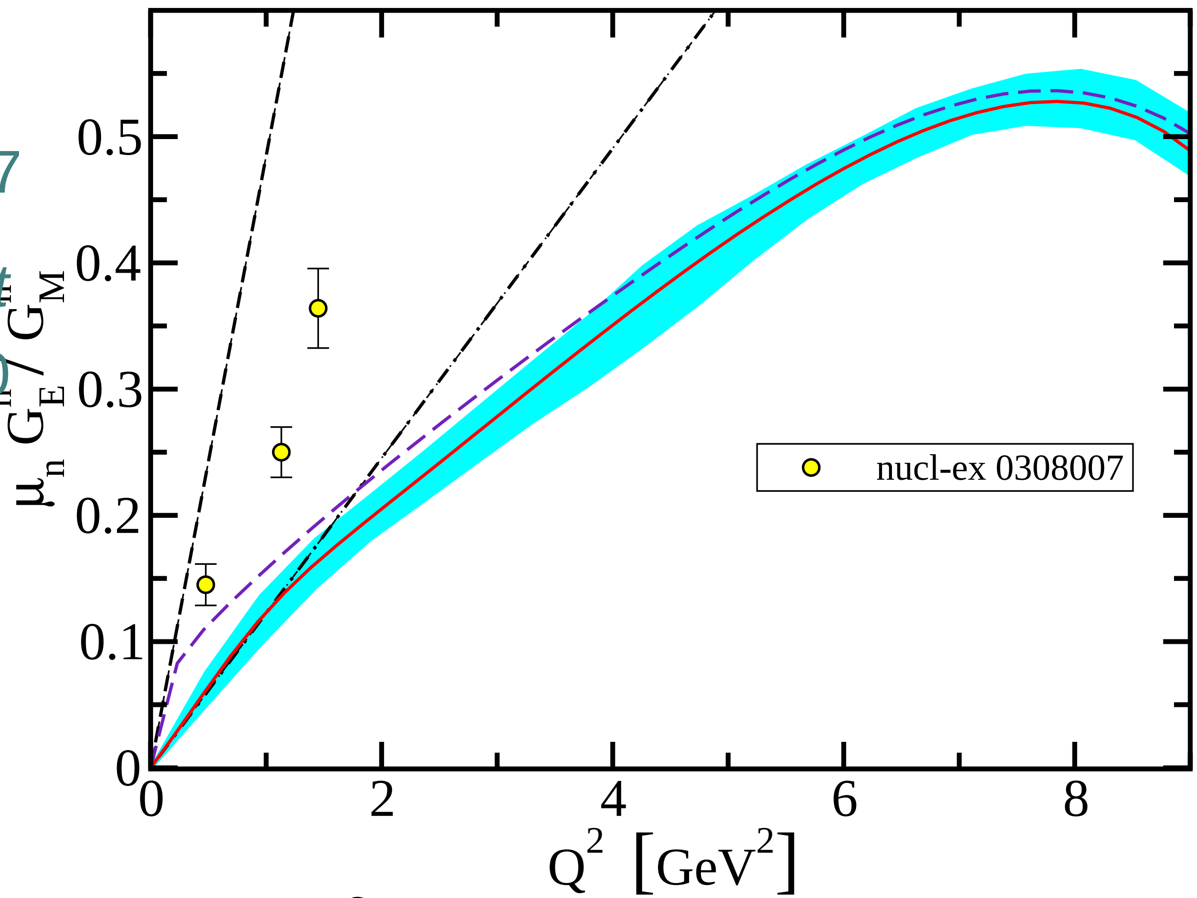

- No sign yet of a zero in $G_E^n(Q^2)$, even though calculation predicts $G_E^p(Q^2 \approx 6.5\,\mathrm{GeV}^2) = 0$
- Data to $Q^2 = 3.4$ GeV^2 is being analysed (JLab E02-013)

Epilogue

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  - It is manifest in dressed propagators and vertices
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  - Expressed and realised in dressed propagators and vertices associated with elementary excitations
  - Observables can be used to explore model realisations
- DSEs . . . contemporary tool that describes and explains these phenomena, and connects them with prediction of observables

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Contemporary Reviews

- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD
  C.D. Roberts and S.M. Schmidt, nu-th/0005064,
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- The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .
  R. Alkofer and L. von Smekal, he-ph/0007355,
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- Dyson-Schwinger equations: A Tool for Hadron Physics
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- Infrared properties of QCD from Dyson-Schwinger equations.
  C. S. Fischer, he-ph/0605173,
  J. Phys. **G 32** (2006) pp. R253-R291
- Nucleon electromagnetic form factors
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  J. Phys. **G 34** (2007) pp. S23-S52.

Colour-singlet Bethe-Salpeter equation

Detmold *et al*., nu-th/0202082

Bhagwat, *et al*., nu-th/0403012

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Colour-singlet Bethe-Salpeter equation

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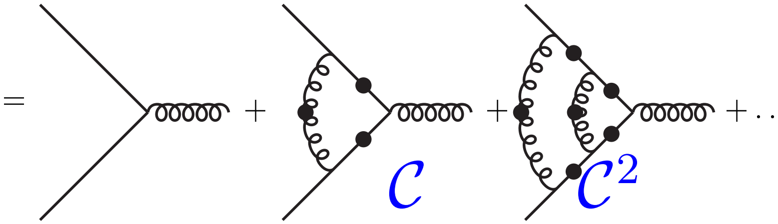
- Coupling-modified dressed-ladder vertex

$$\Gamma^a_\mu(k,p) =$$



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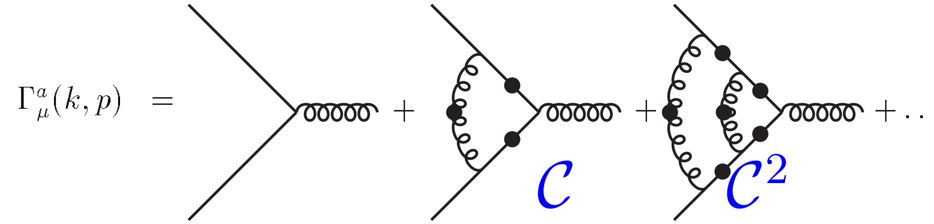

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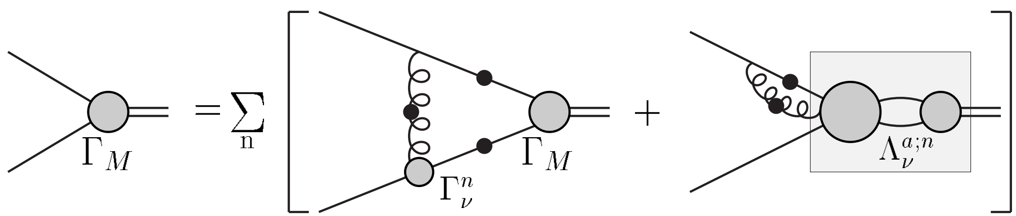

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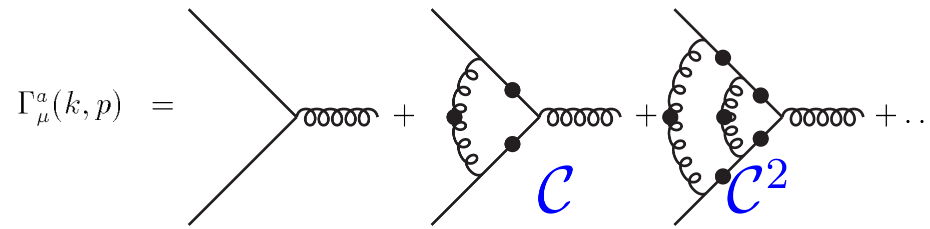

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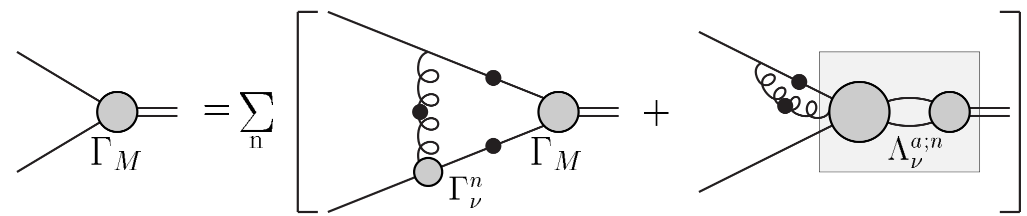
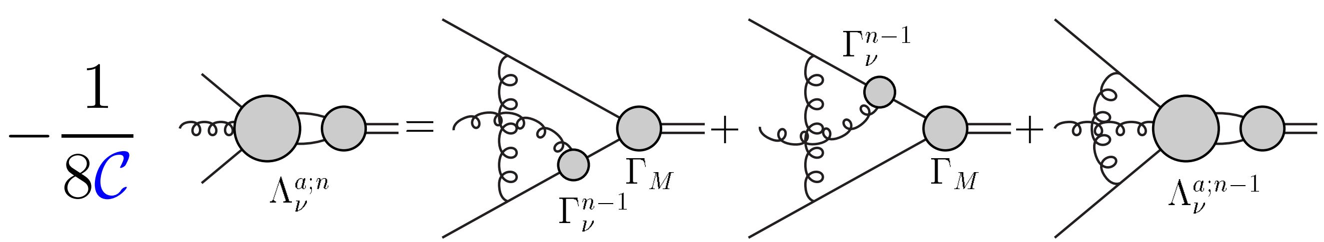


- Bethe-Salpeter kernel . . . recursion relation



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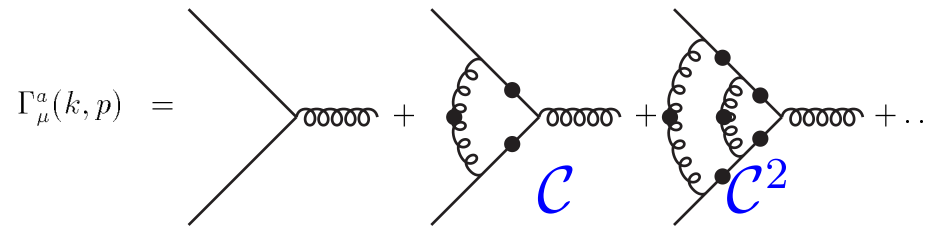

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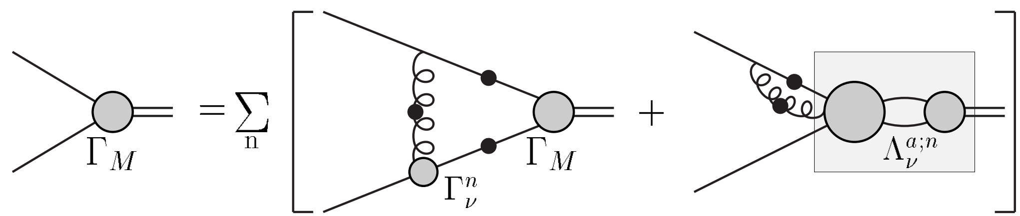

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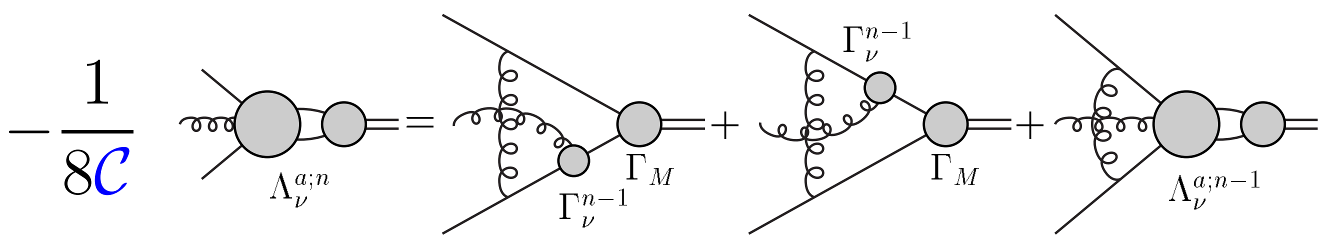

- Kernel necessarily non-planar,

even with planar vertex

π and ρ mesons

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π, $m = 0.011$	0.147	0.135	0.139	0.138
ρ, $m = 0$	0.920	0.648	0.782	0.754
ρ, $m = 0.011$	0.936	0.667	0.798	0.770

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π and ρ mesons

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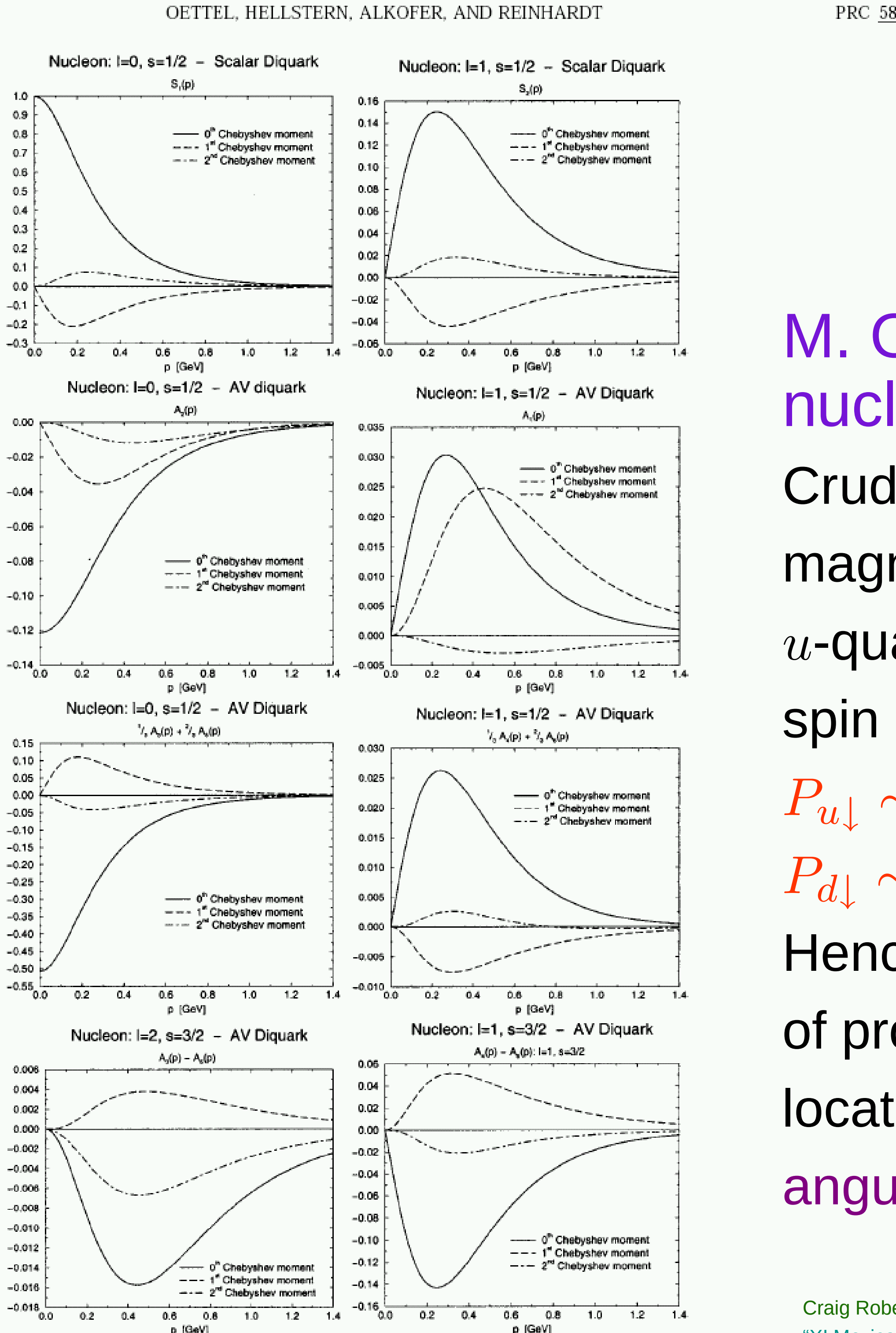


Angular Momentum Rest Frame

M. Oettel, *et al.*
nucl-th/9805054

Crude estimate based on magnitudes ⇒ probability for a u-quark to carry the proton's spin is $P_{u\uparrow} \sim 80\,\%$, with $P_{u\downarrow} \sim 5\,\%$, $P_{d\uparrow} \sim 5\,\%$, $P_{d\downarrow} \sim 10\,\%$.

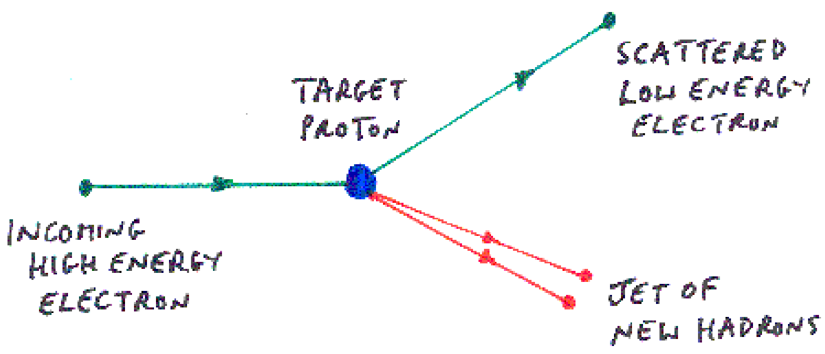
Hence, by this reckoning $\sim 30\%$ of proton's rest-frame spin is located in dressed-quark angular momentum.

Deep-inelastic scattering

Deep-inelastic scattering

- Looking for Quarks

Deep-inelastic scattering



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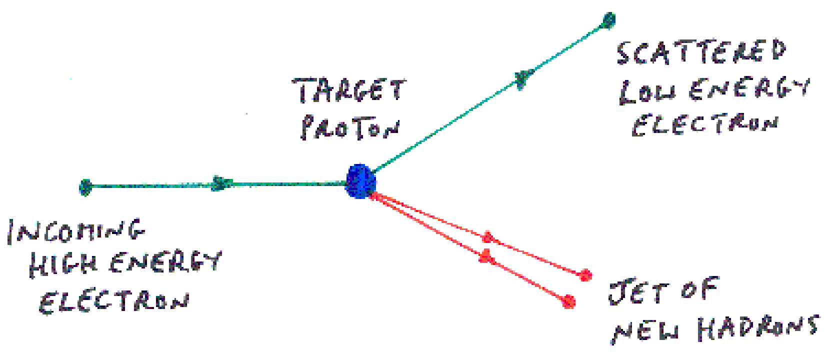

- Looking for Quarks

- Signature Experiment for QCD:
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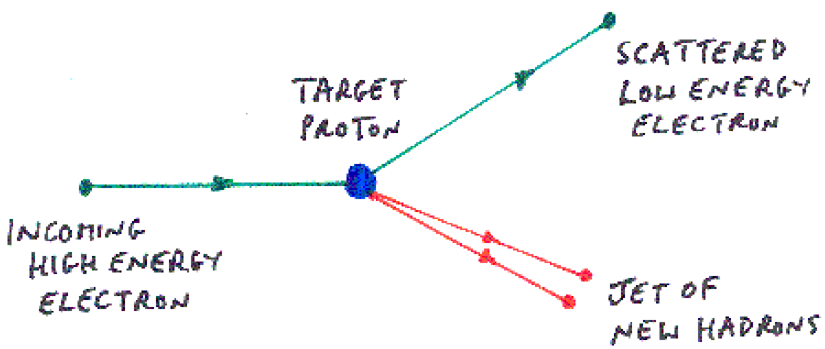
Deep-inelastic scattering



- Looking for Quarks

- Signature Experiment for QCD:
  Discovery of Quarks at SLAC
- Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x)$, $g(x)$

Pion's valence quark distn

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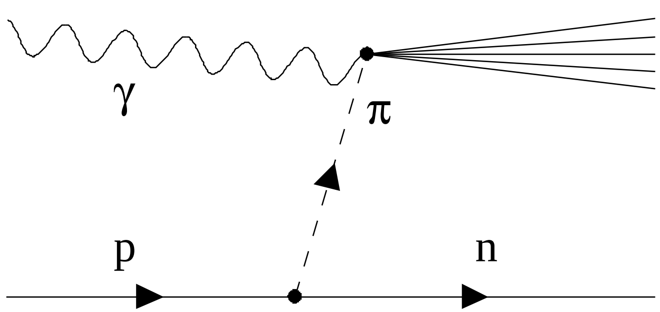
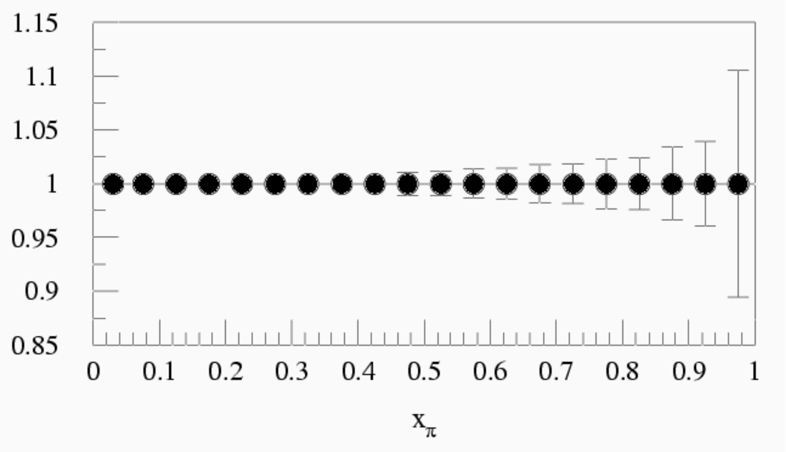
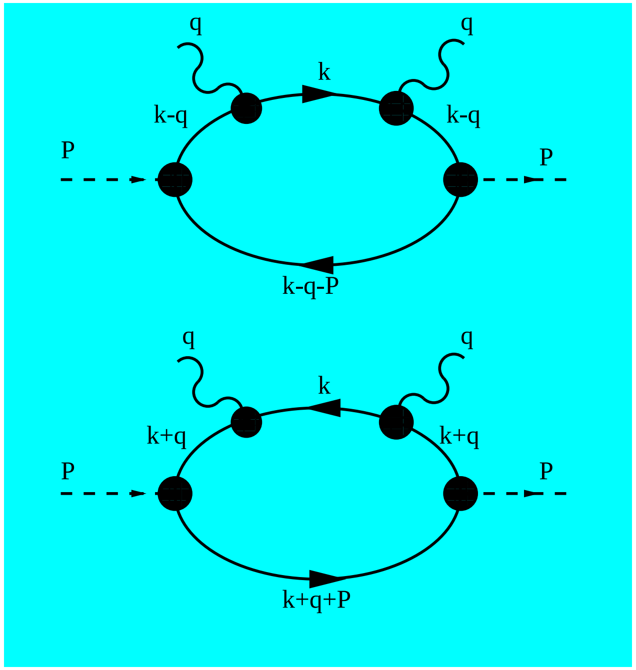
- Proposal `(Holt & Reimer, ANL, nu-ex/0010004)`

$e^-_{5\mathrm{GeV}} - p_{25\,\mathrm{GeV}}$ Collider $\to$ Accurate "Measurement"

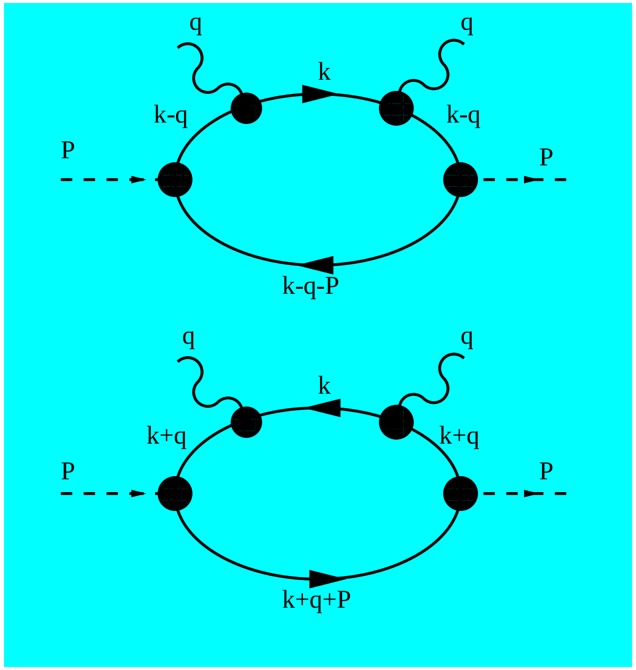




Handbag diagrams



Handbag diagrams



$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

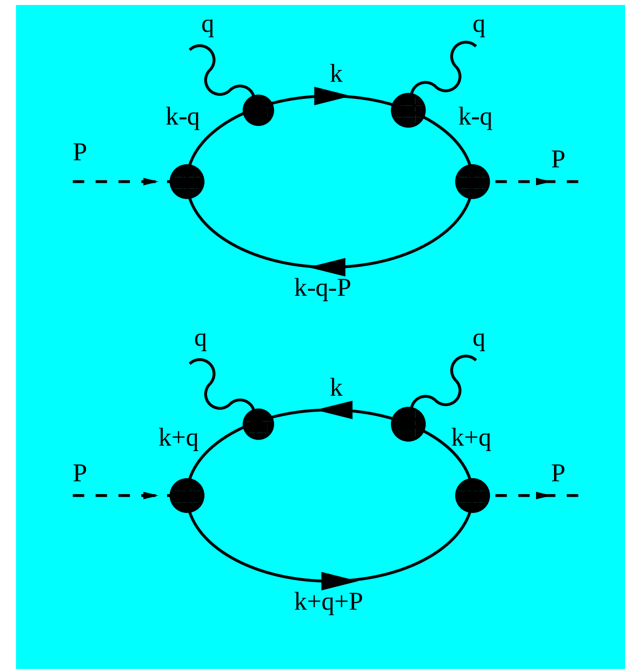
$$T^{+}_{\mu\nu}(q,P) = \mathrm{tr}\int\frac{d^4k}{(2\pi)^4}\,\tau_-\bar{\Gamma}_\pi(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_\nu(k_{-0},k)$$
$$\times\, S(k)\,ieQ\Gamma_\mu(k,k_{-0})\,S(k_{-0})\tau_+\Gamma_\pi(k_{-\frac{1}{2}};P)\,S(k_{--})$$

Handbag diagrams

Bjorken Limit: $q^2 \to \infty\,,\quad P\cdot q \to -\infty$

but $x := -\dfrac{q^2}{2P\cdot q}$ fixed.

Numerous algebraic simplifications



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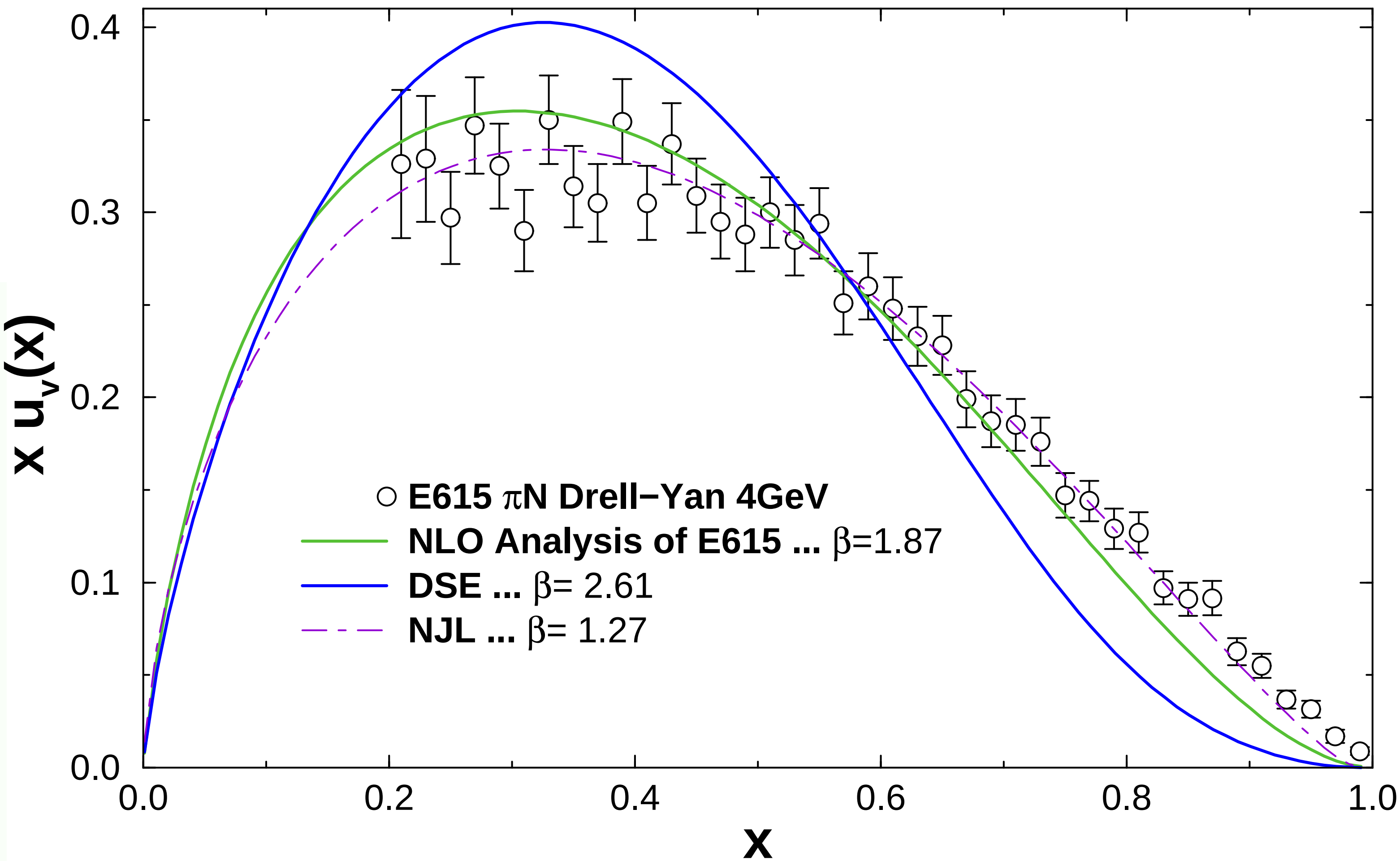
Extant theory vs. experiment

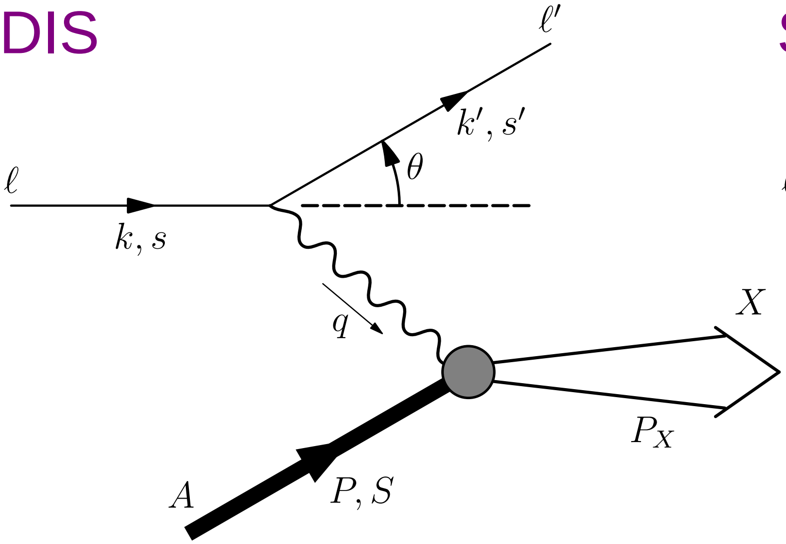
K. Wijersooriya, P. Reimer and R. Holt,
nu-ex/0509012 ... Phys. Rev. C (Rapid)



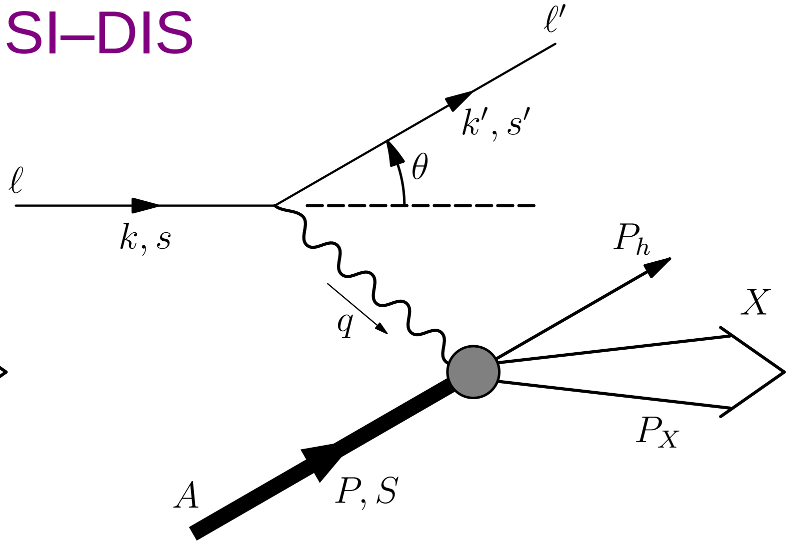
Quark
Distribution Functions

Quark Distribution Functions

DIS



SI–DIS



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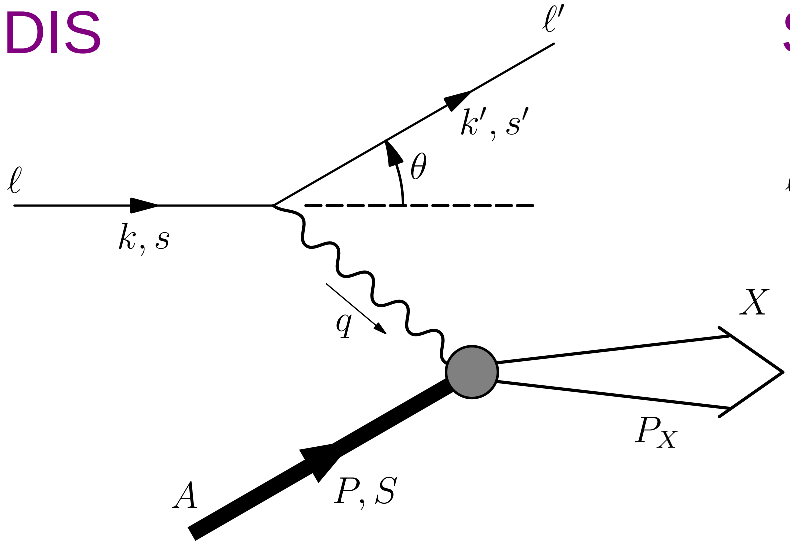
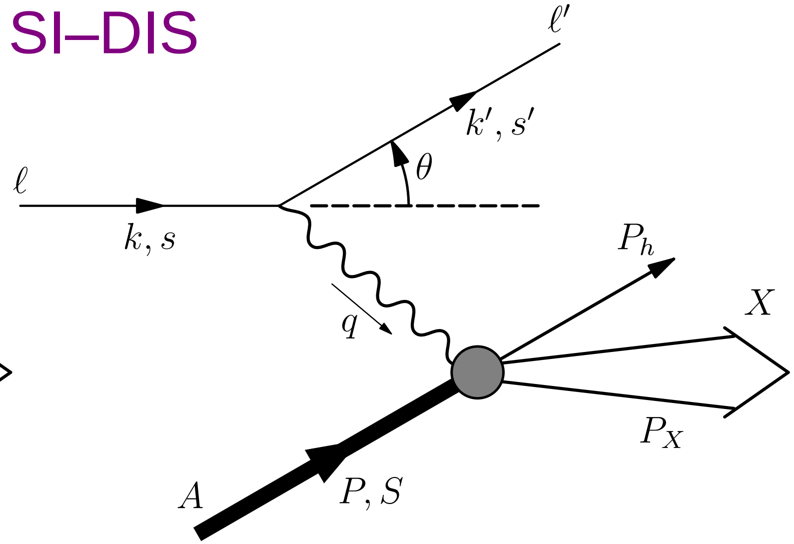
Quark Distribution Functions





- Three twist-2 parton distributions $(k_\perp = 0)$:
  - Spin-Independent: $q(x)$
  - Helicity: $\Delta q(x)$
  - Transversity: $\Delta_T q(x)$
- All distributions have probability interpretation.
- By definition, contain essentially non-perturbative information about a given process.

Definition and Sum Rules

Definition and Sum Rules

- Light-cone Fourier transforms :

$$\Delta_T q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{i\,x\,p^+\xi^-} \langle p, s | \overline{\psi}_q(0) \gamma^+ \gamma^1 \gamma_5 \psi_q(\xi^-) | p, s \rangle_c$$

$$q(x) = \langle \gamma^+ \rangle, \qquad \Delta q(x) = \langle \gamma^+ \gamma_5 \rangle$$

- Related to the nucleon axial & tensor charges via

$$g_A = \int dx [\Delta u(x) - \Delta d(x)], \quad g_T = \int dx [\Delta_T u(x) - \Delta_T d(x)],$$

- Must satisfy: positivity constraints and Soffer bound

$$\Delta q(x), \Delta_T q(x) \leqslant q(x), \quad q(x) + \Delta q(x) \geqslant 2\,|\Delta_T q(x)|$$

Ian Cloët

JLab, now ANL

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JLab, now ANL

Once more on the one that got away.

Cloët, Bentz, Thomas

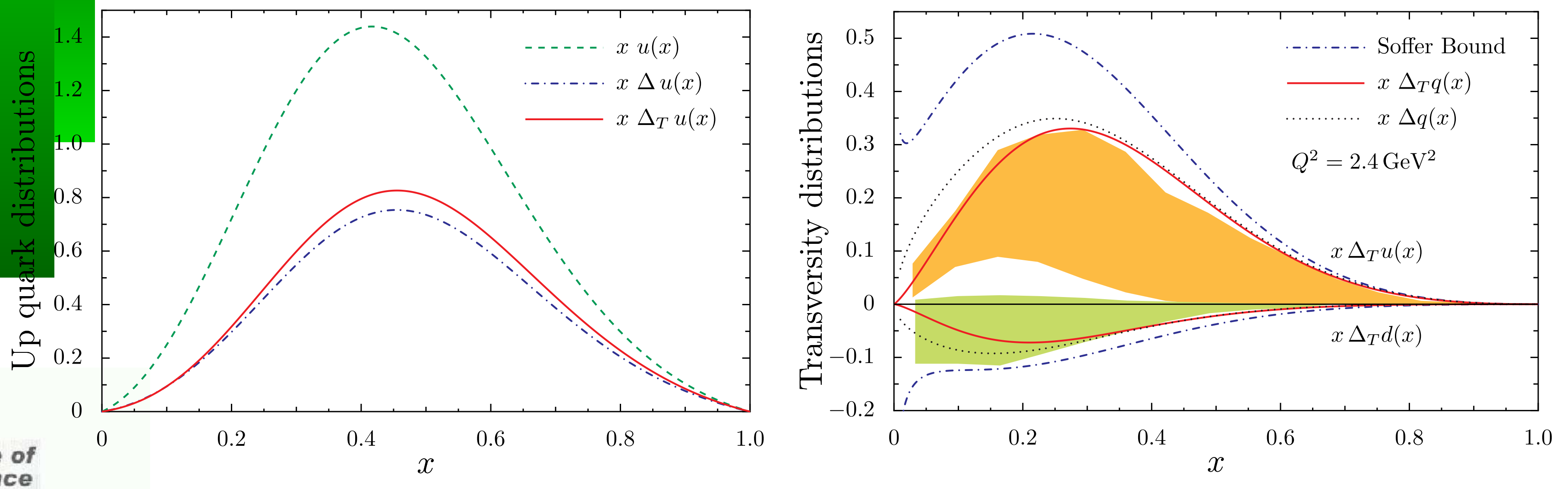
arXiv:0708.3246 [hep-ph]

Model predictions

Cloët, Bentz, Thomas
arXiv:0708.3246 [hep-ph]

Model predictions

- Simplified Faddeev equation



- Satisfy: Soffer bound, baryon & momentum SRs.

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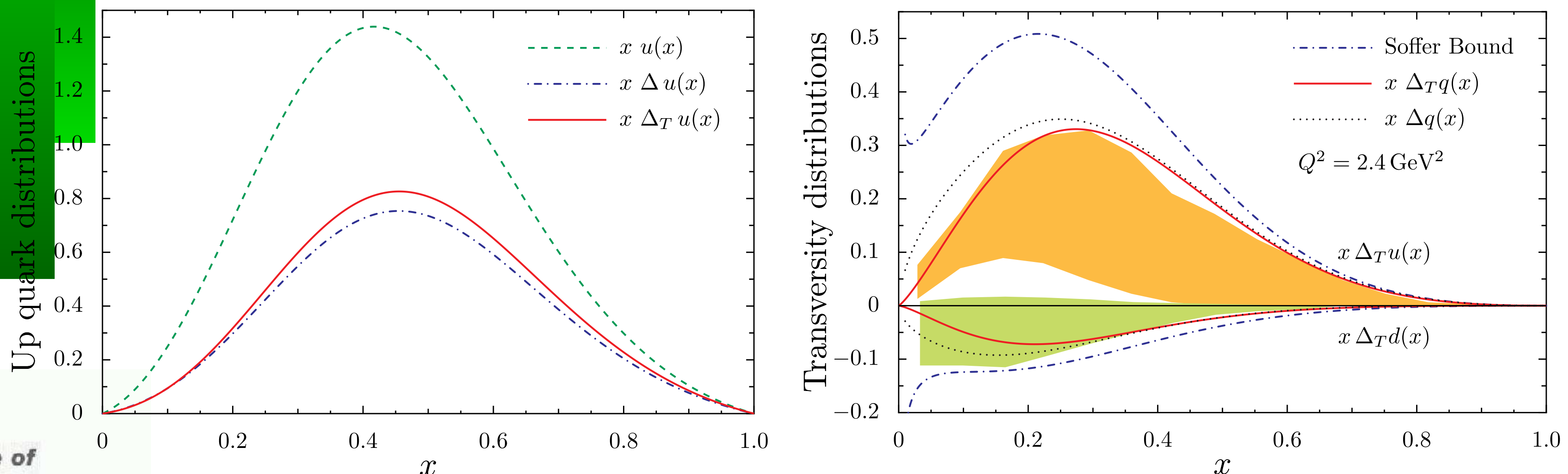

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- Moments at $Q^2 = 0.16$ GeV^2:

$$\Delta u = 0.97, \quad \Delta d = -0.30 \quad \Longrightarrow \quad g_A = 1.267$$

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Model constraint

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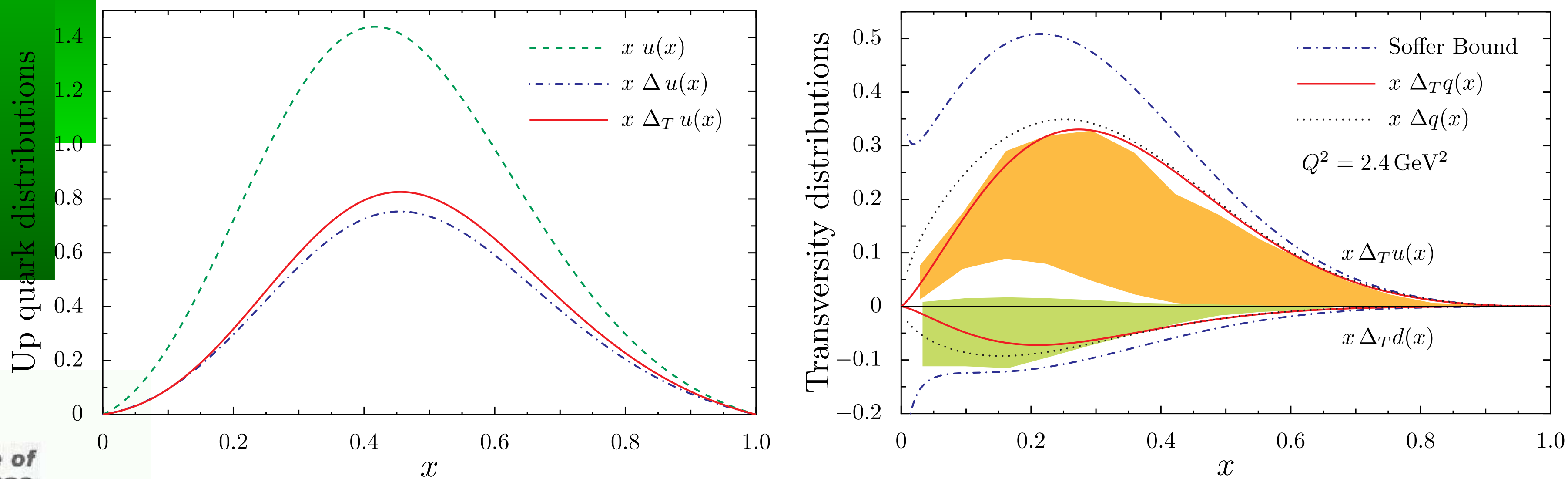

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- $\Delta q(x) \sim \Delta_T q(x)$ in valence region for $Q^2 \lesssim 10\,$GeV^2