Aspects of Hadron Physics

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Dyson-Schwinger Equations

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- A Modern Method for Relativistic Quantum Field Theory

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- NonPerturbative, Continuum approach to QCD

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- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence

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- Method yields Schwinger Functions $\equiv$ Propagators

Cross-Sections built from Schwinger Functions

Perturbative Dressed-quark Propagator

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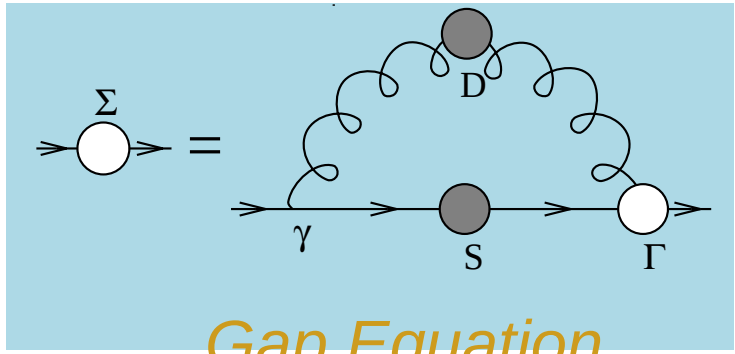
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



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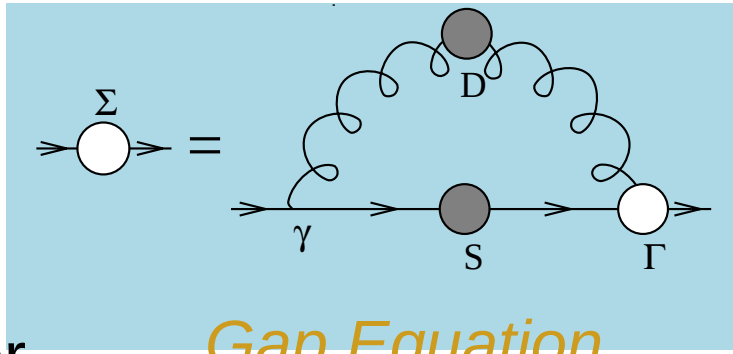

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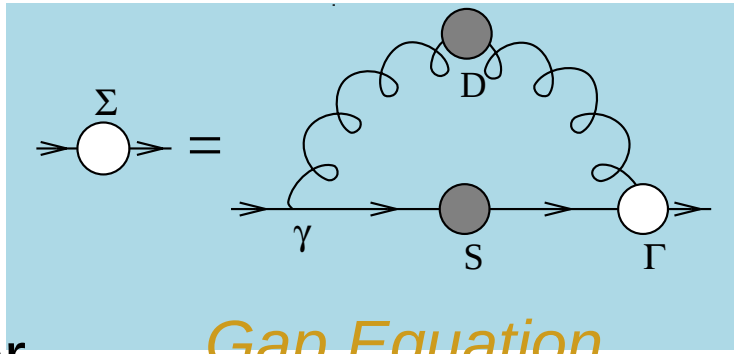

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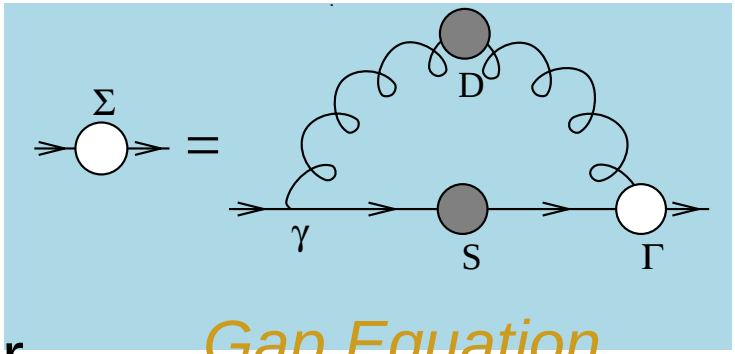

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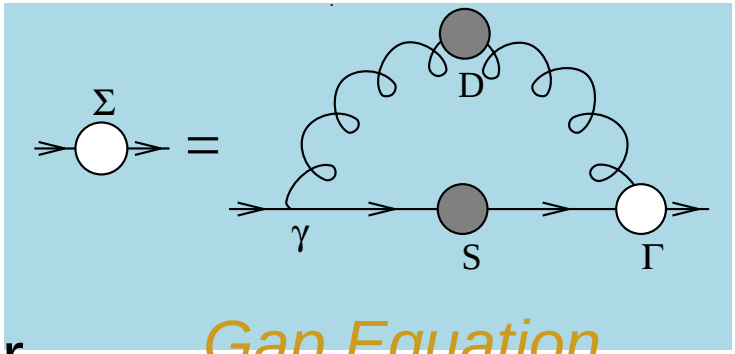

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QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
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- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
  - D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD Collaboration], *Asymptotic scaling and infrared behavior of the gluon propagator*, Phys. Rev. D **60**, 094507 (1999) [Erratum-ibid. D **61**, 079901 (2000)].
- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

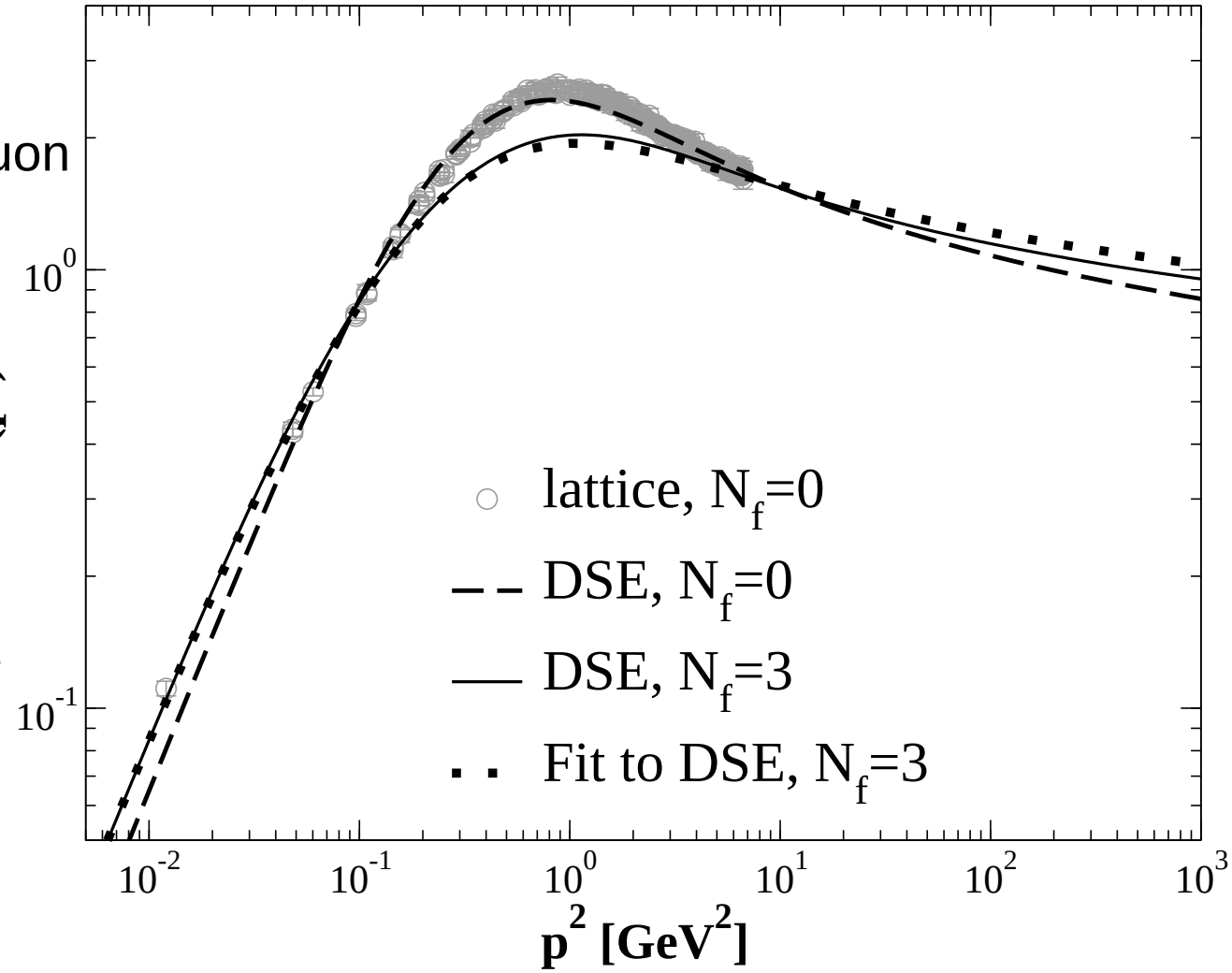
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



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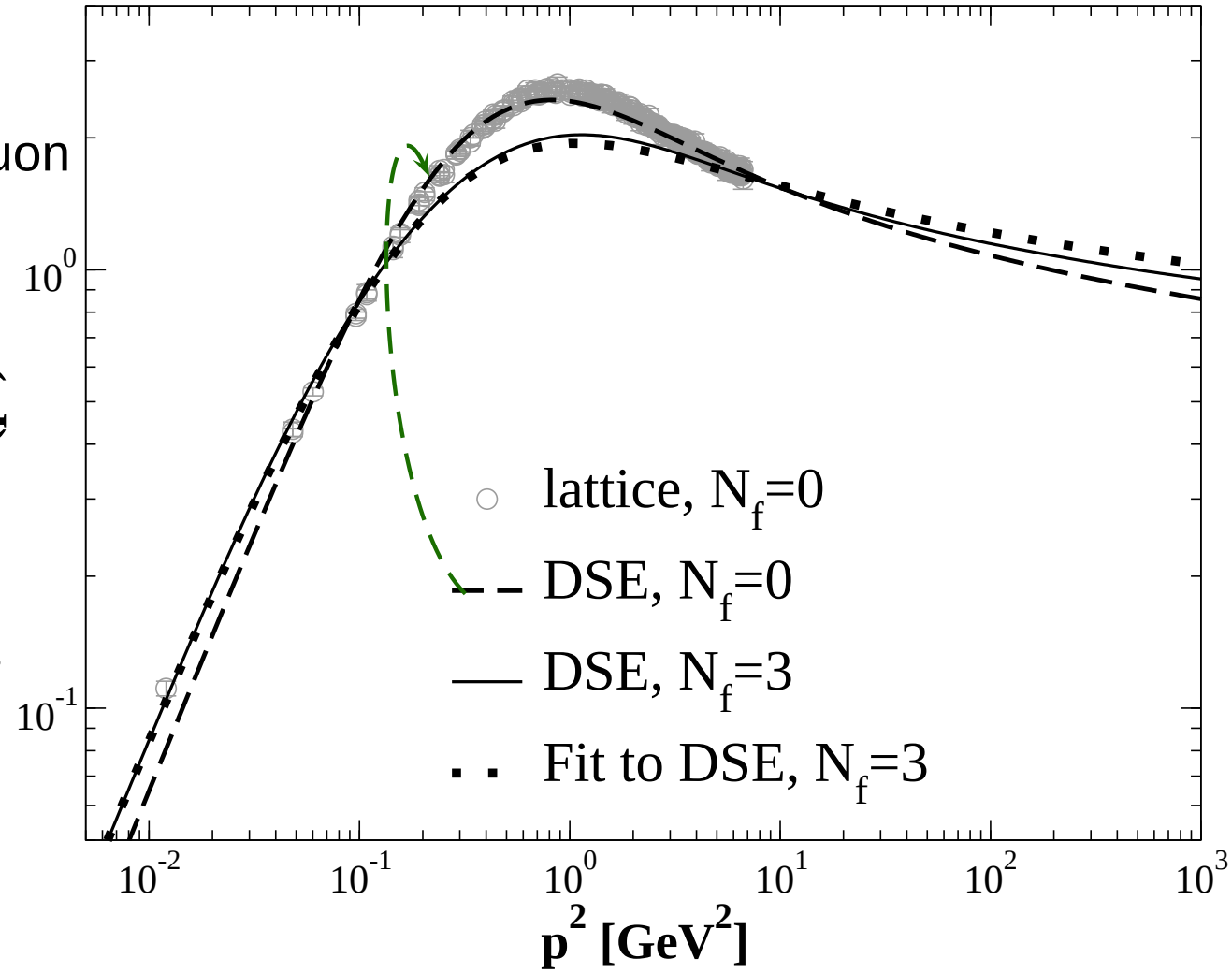

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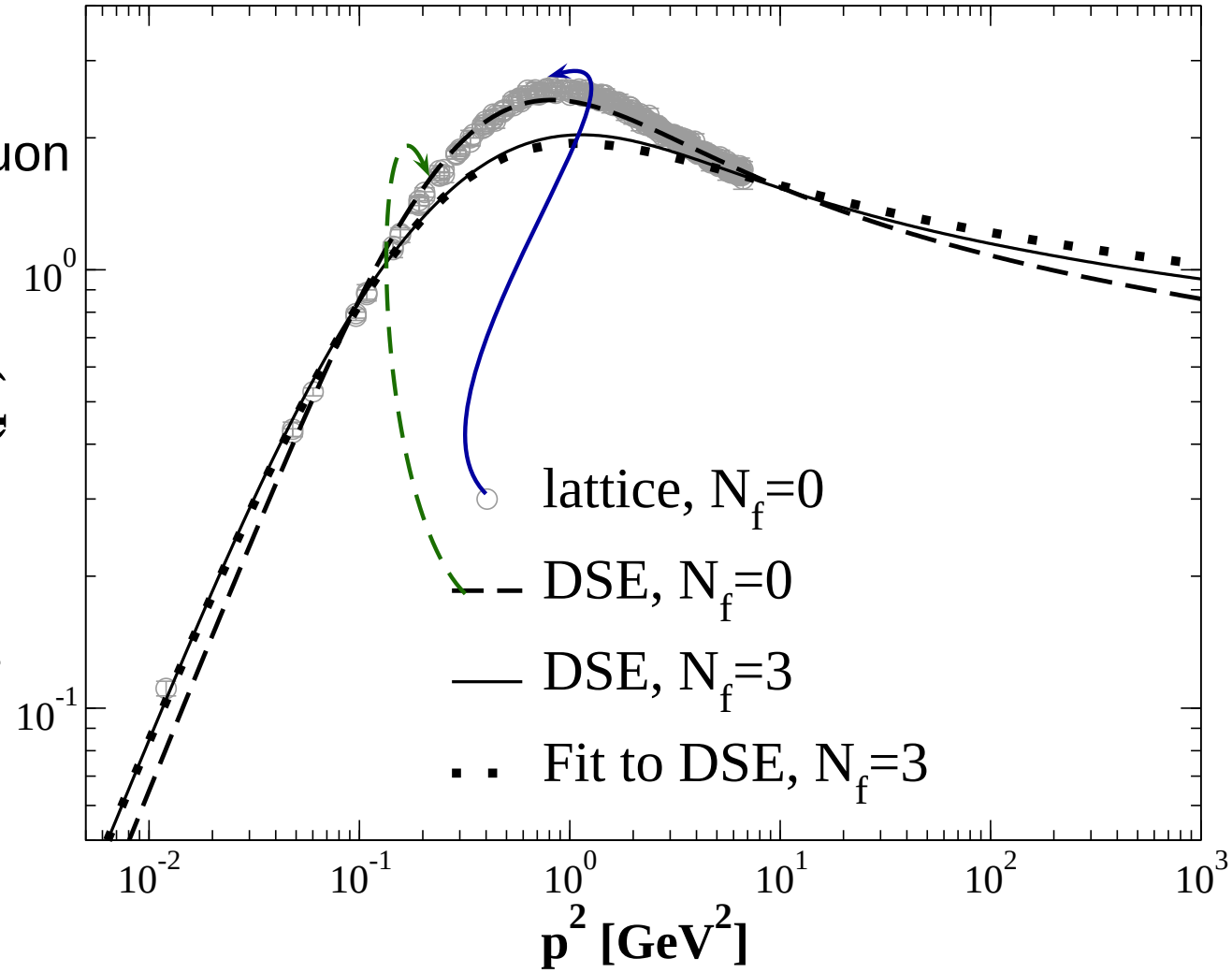
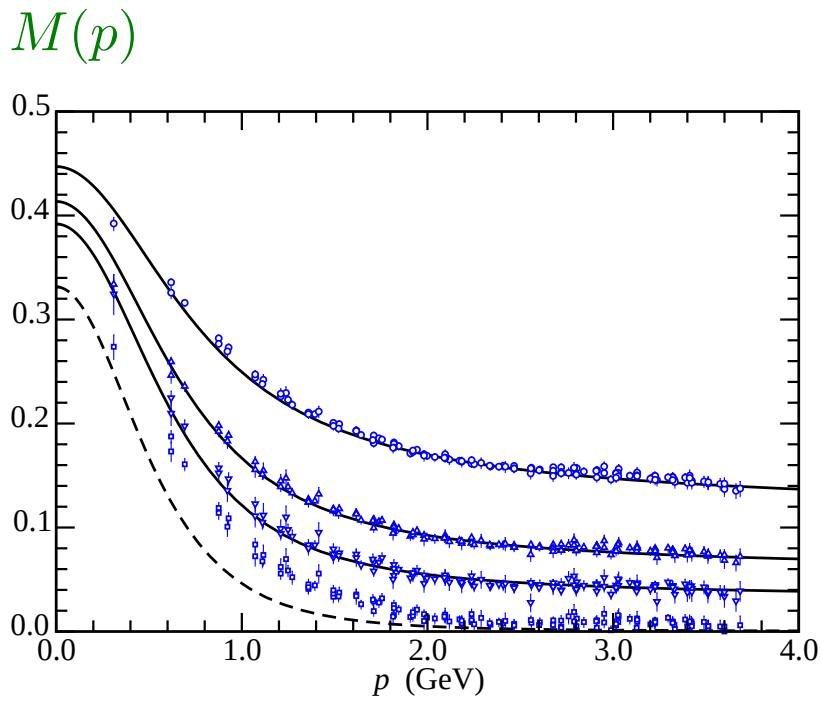
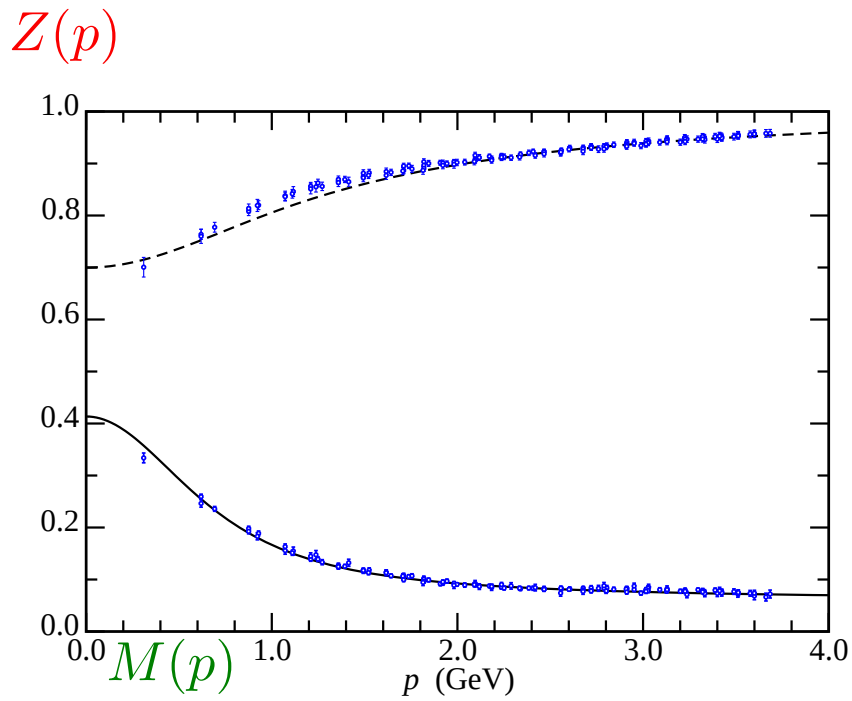


Dressed-quark Propagator

- $S(p) = \dfrac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$
  - Wave function renormalisation: $Z(p^2)$
    - Free particle: $Z(p^2) \equiv 1$
  - Mass function: $M(p^2)$
    - Free particle: $M(p^2) \equiv m_{\rm current}$
- Behaviour of these functions in QCD is a longstanding prediction of DSE studies. For example, it could have been anticipated from; e.g.,
  - K. D. Lane, *Asymptotic Freedom And Goldstone Realization Of Chiral Symmetry*, Phys. Rev. D **10**, 2605 (1974);
  - H. D. Politzer, *Effective Quark Masses In The Chiral Limit*, Nucl. Phys. B **117**, 397 (1976).

Quenched-QCD Dressed-Quark Propagator





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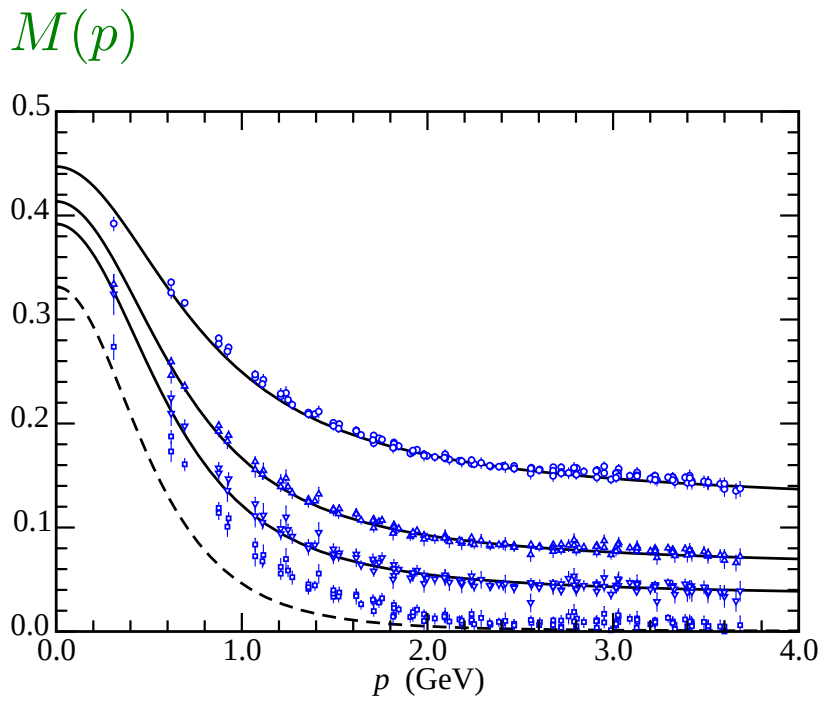

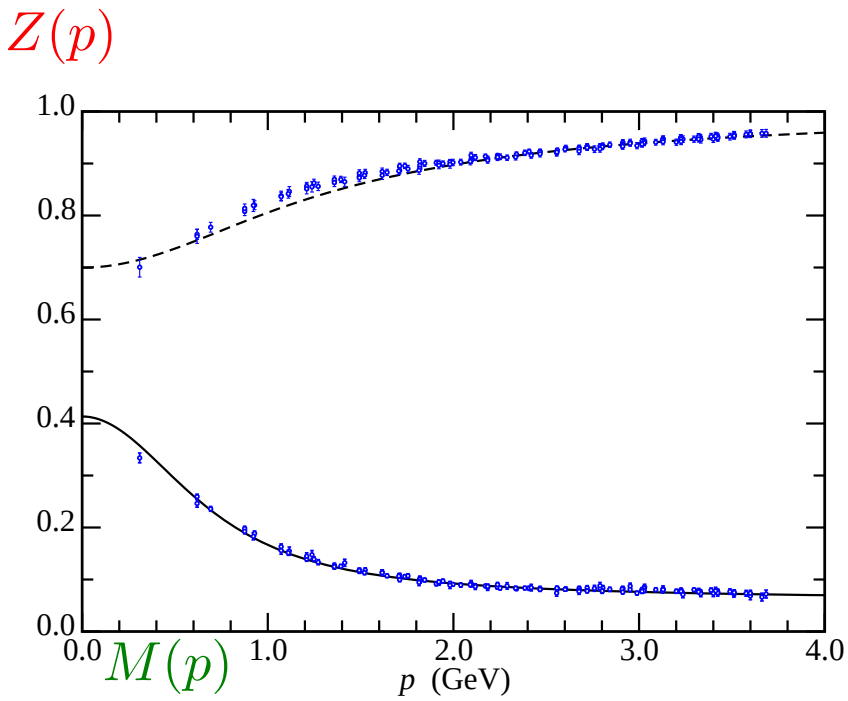

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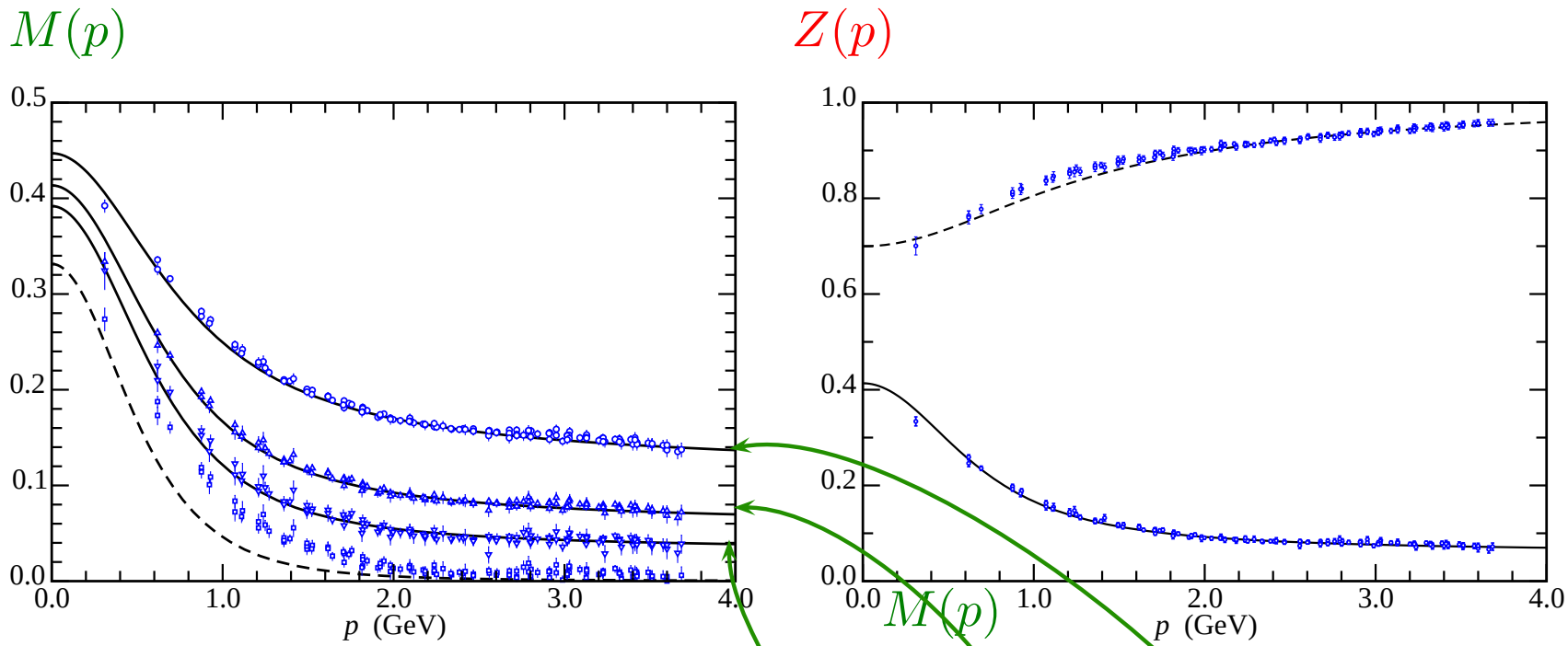

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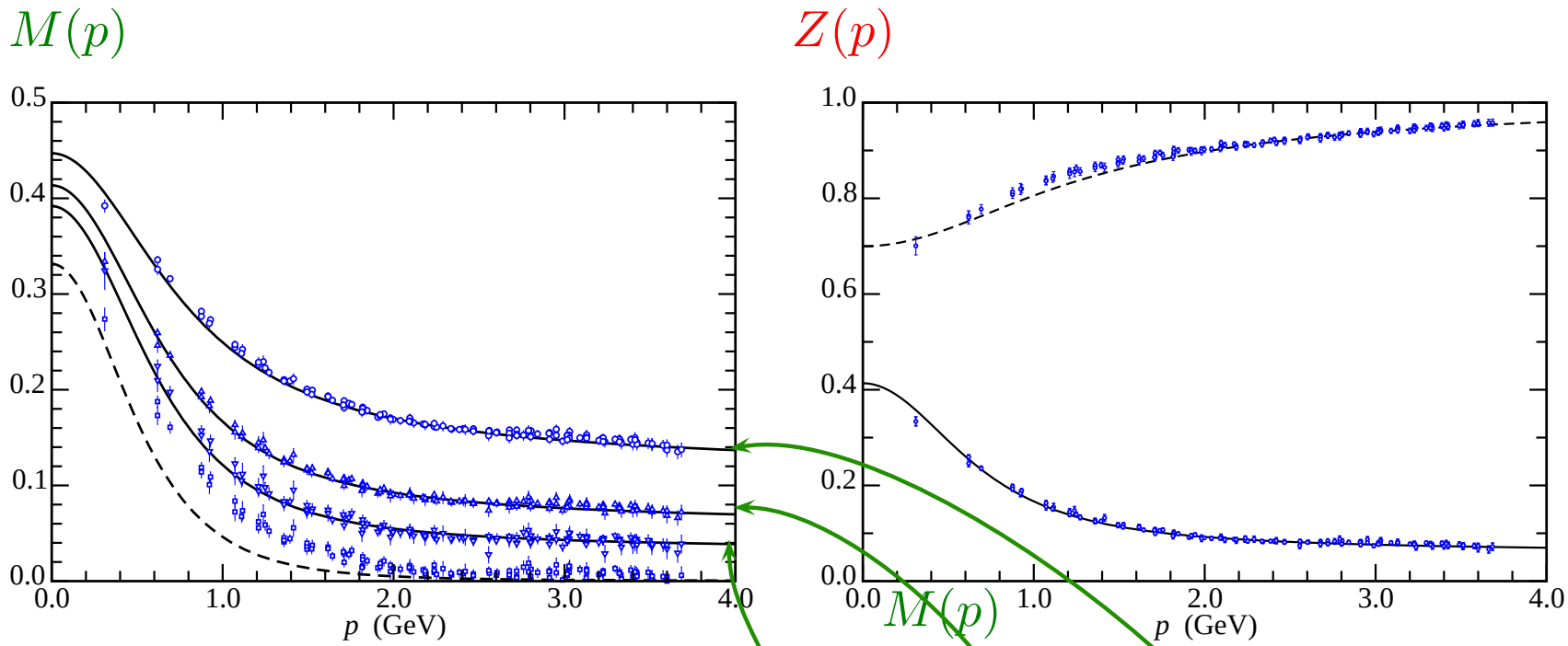

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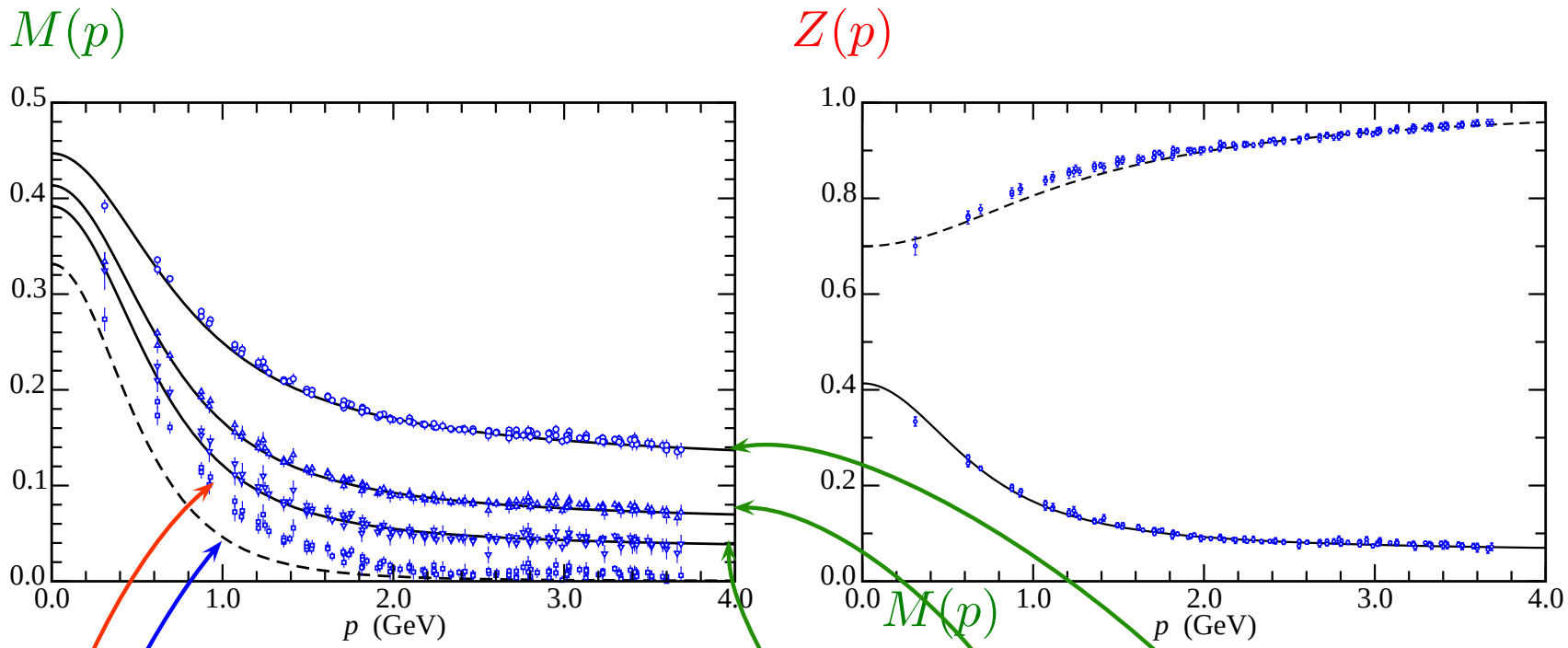

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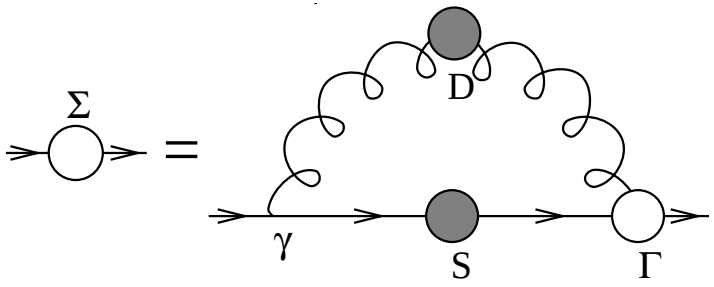
- Linear extrapolation of lattice data to chiral limit is inaccurate

Dressed-quark-gluon Vertex

- How is this agreement achieved via the gap equation if the gluon propagator is IR-suppressed?



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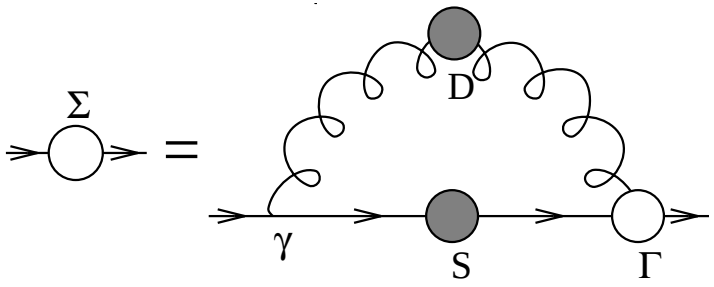

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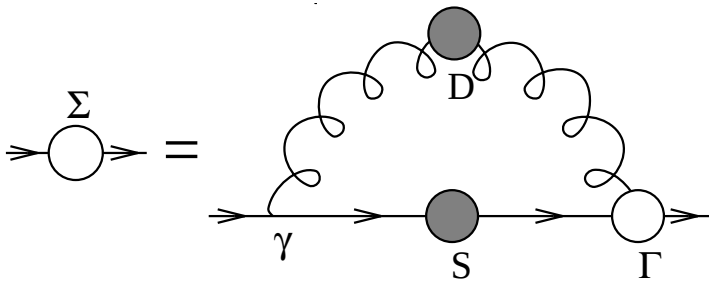

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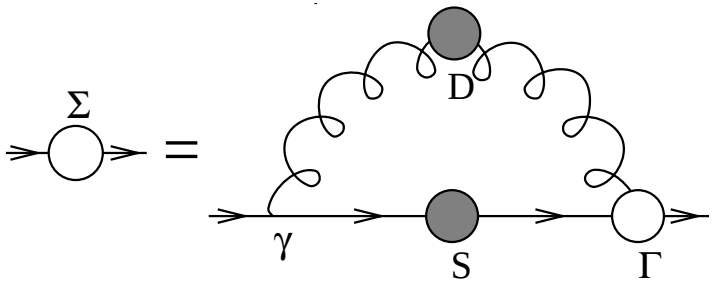

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- Exact nature of the enhancement is subject of ongoing research.

Hadrons

- Suppose one has established an understanding of two- and three-point functions

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- What about bound states?

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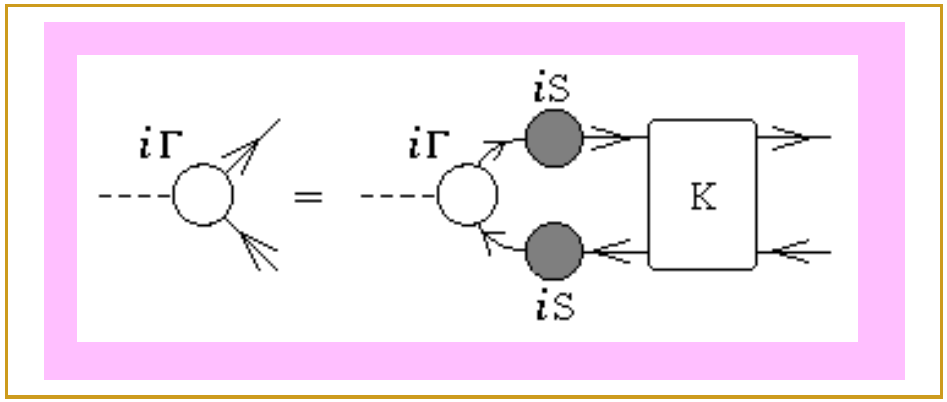

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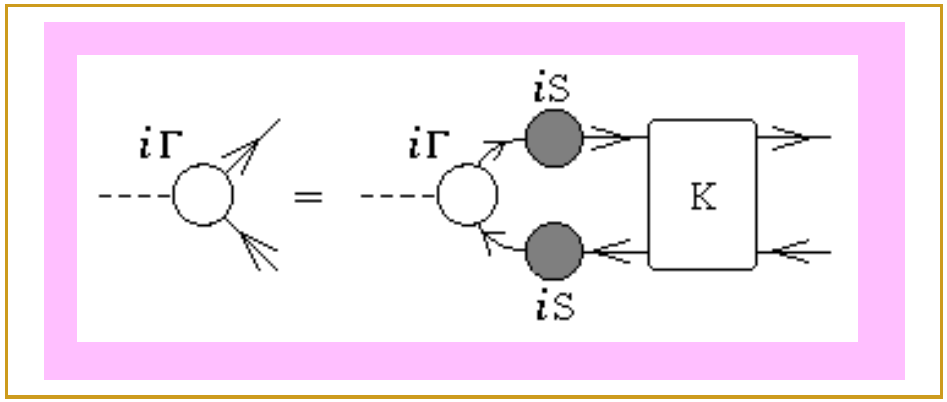

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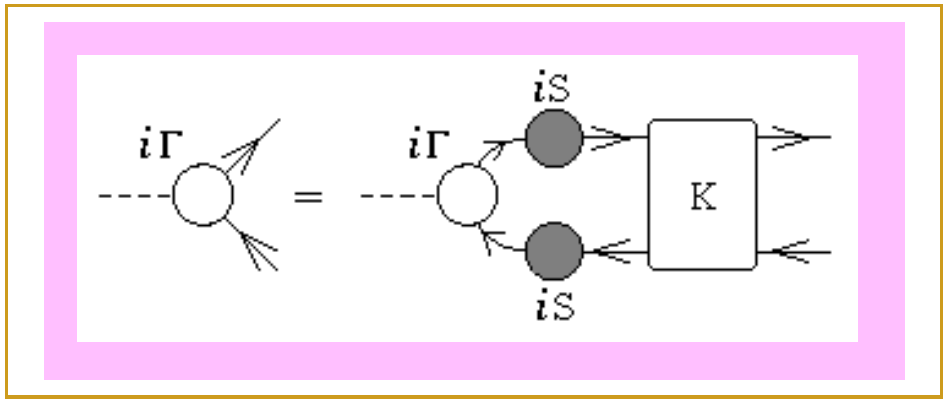

QFT Generalisation of Lippman-Schwinger Equation.

- or What is the long-range potential in QCD?

Bethe-Salpeter Kernel

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- Axial-vector Ward-Takahashi identity

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$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

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- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Goldstone's Theorem

- In the chiral limit the QCD Action possesses chiral symmetry
- The chiral limit is a good approximation in QCD for u- and d-quarks
- If this $SU(N_f = 2)$ chiral symmetry is dynamically broken, then there is a massless composite particle associated with each generator of chiral transformations; i.e., three Goldstone Bosons
- These three Goldstone Bosons have long been identified with the pions: π^+, π^0, π^-

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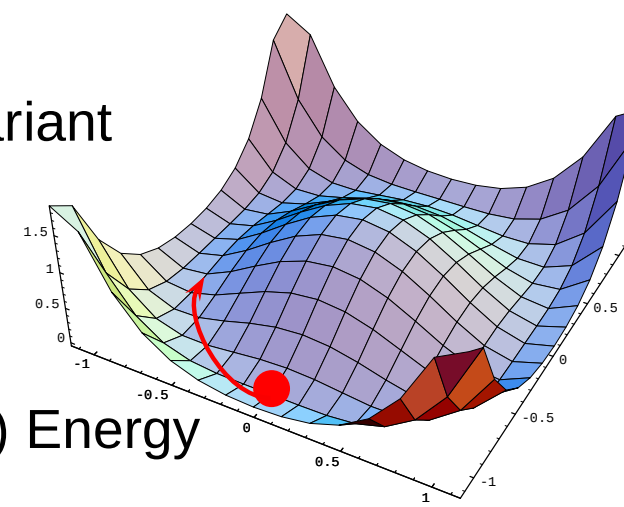
  E.g., $V(x, y) = (\sigma^2 + \pi^2 - 1)^2$

  – Hamiltonian: $T + V$, is Rotationally Invariant

  Ground State
  - Ball at any (σ, π) for which $\sigma^2 + \pi^2 = 1$
    - All Positions have Same (Minimum) Energy
    - But not invariant under rotations



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- These three Goldstone Bosons have long been identified with the pions: $\boldsymbol{\pi^+}$, $\boldsymbol{\pi^0}$, $\boldsymbol{\pi^-}$
- If one assumes the s-quark is also light; namely, assumes that $SU(N_f = 3)$ chiral symmetry is a good approximation, then the kaons are four more Goldstone Bosons

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- Requires detailed understanding of Connection between Current-quark and Constituent-quark masses

Goldberger-Treiman for pion

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- Pseudoscalar Bethe-Salpeter amplitude

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Pseudovector components necessarily nonzero

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$\Rightarrow$ Exact in Chiral QCD

Radial Excitations

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- Spectrum contains 3 pseudoscalars $[I^G(J^P)L = 1^-(0^-)S]$

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

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- But $\pi(1800)$ is narrow ($\Gamma = 207 \pm 13$) & decay pattern might indicate some "flux tube angular momentum" content:
  $S_{\bar{Q}Q} = 1 \oplus L_F = 1 \Rightarrow J = 0$
  & $L_F = 1 \Rightarrow\ ^3S_1 \oplus\ ^3S_1$ ($\bar{Q}Q$) decays suppressed?

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- NSAC Long-Range Plan, 2002: . . . an understanding of confinement "remains one of the greatest intellectual challenges in physics"

Andreas Krassnigg

FWF “Erwin Schrödinger Fellow,” ANL 2003-2005

Andreas Krassnigg

Future President
. . . almost Blood
Relative of Arnold

Radial Excitations & Chiral Symmetry

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(Maris, Roberts, Tandy nu-th/9707003)

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- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

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$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

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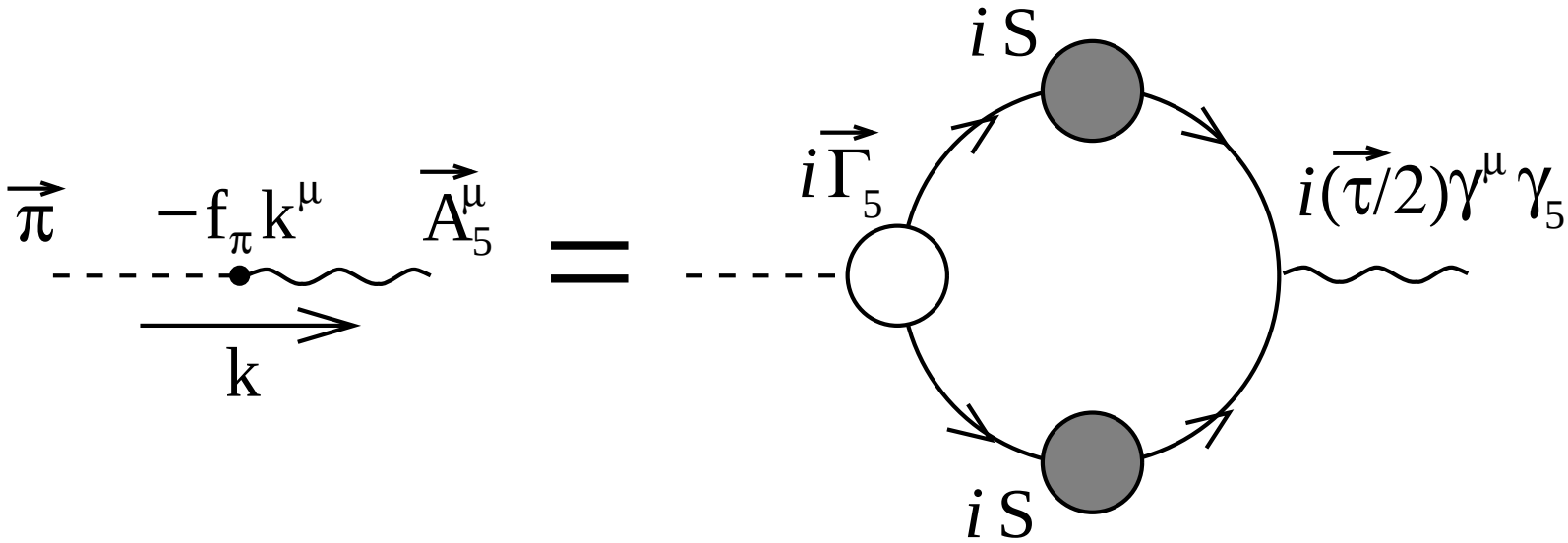
- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ ...GMOR relation, a corollary

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- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $\boxed{m_H \propto m_q}$

... QCD Proof of Potential Model result

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

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- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

Radial Excitations & Chiral Symmetry

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- Fundamental properties of QCD

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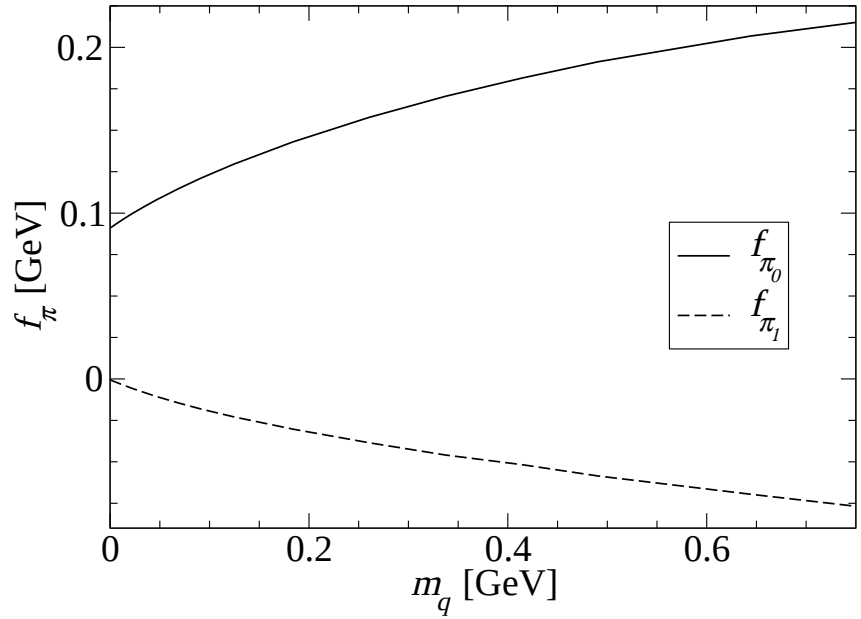
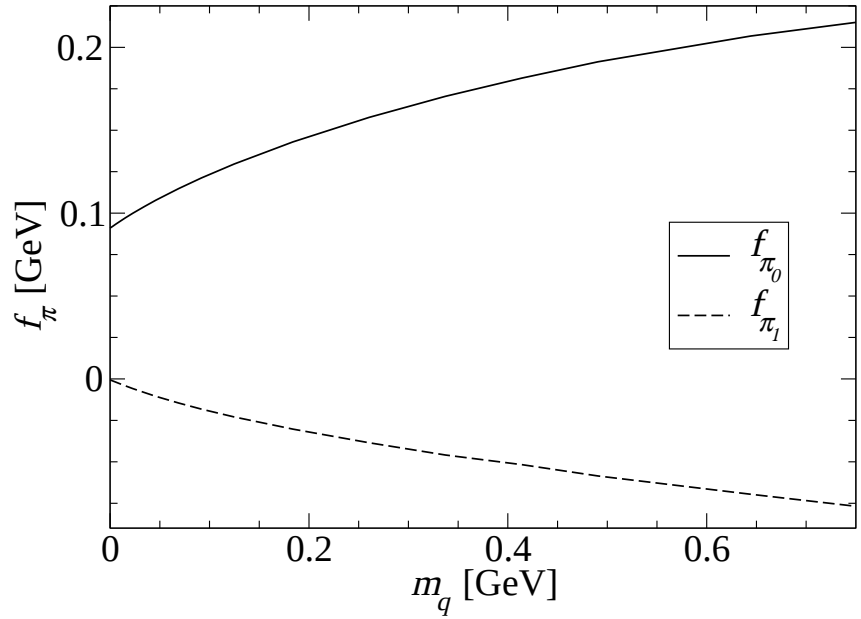


Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

- Fundamental properties of QCD
  - If chiral symmetry is dynamically broken, then in the chiral limit every pseudoscalar meson is blind to the weak interaction *except* $\pi(140)$.
  - If chiral symmetry is not broken, then *NO pseudoscalar meson experiences the weak interaction.*



Colour-singlet Bethe-Salpeter equation

Detmold *et al*., nu-th/0202082

Bhagwat, *et al*., nu-th/0403012

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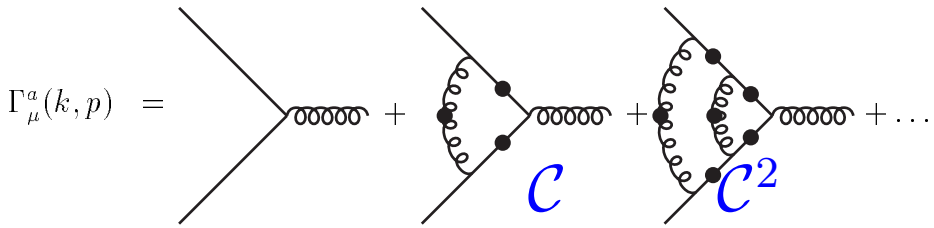
- Coupling-modified dressed-ladder vertex



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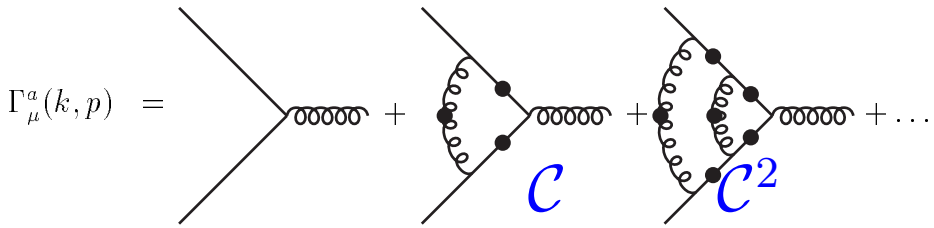

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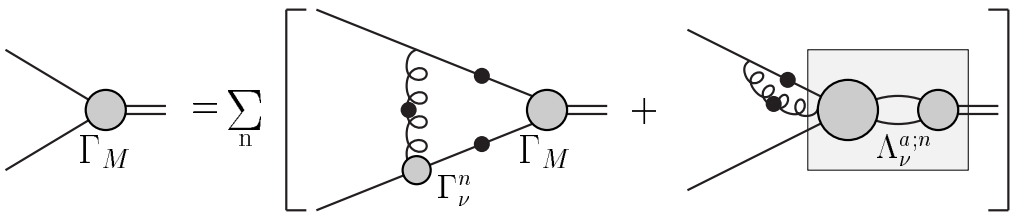

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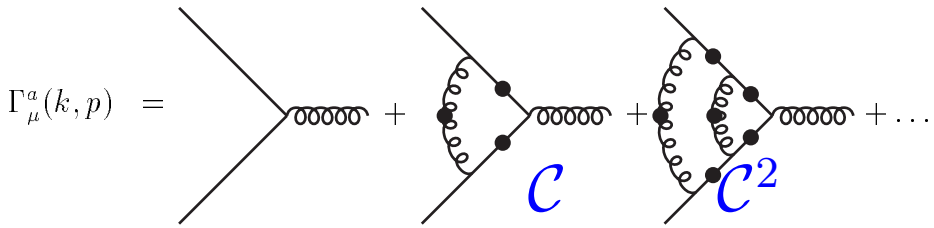

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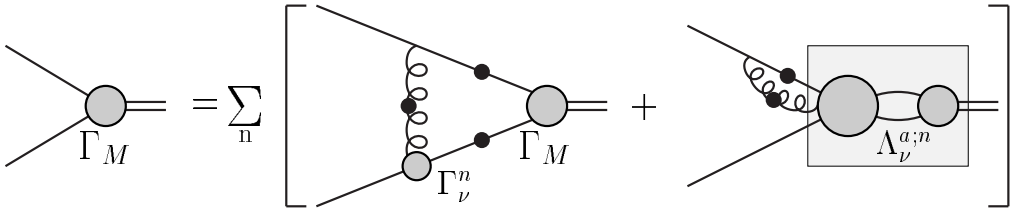
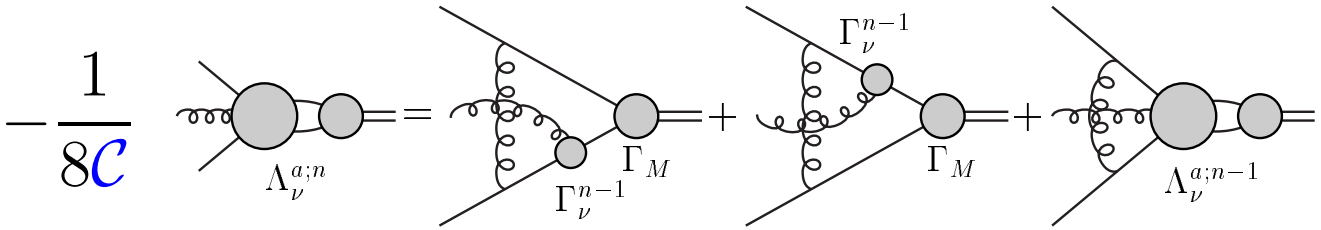


- Bethe-Salpeter kernel . . . recursion relation



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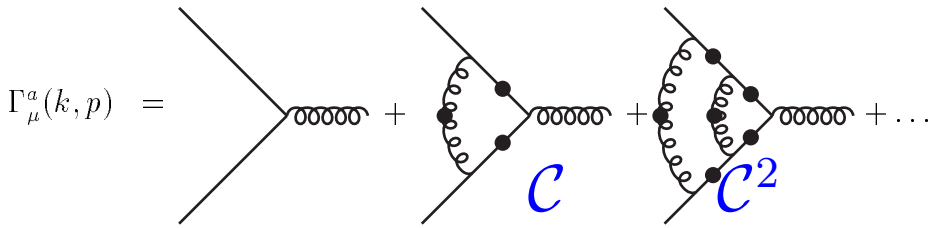

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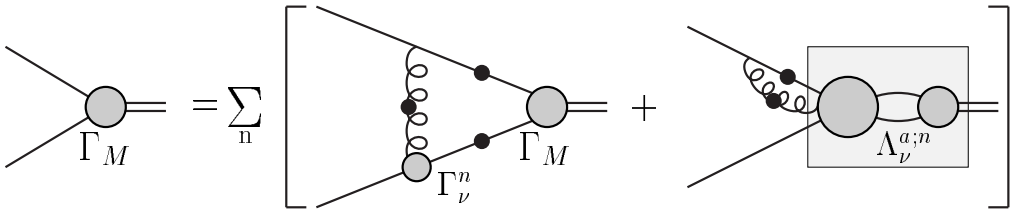

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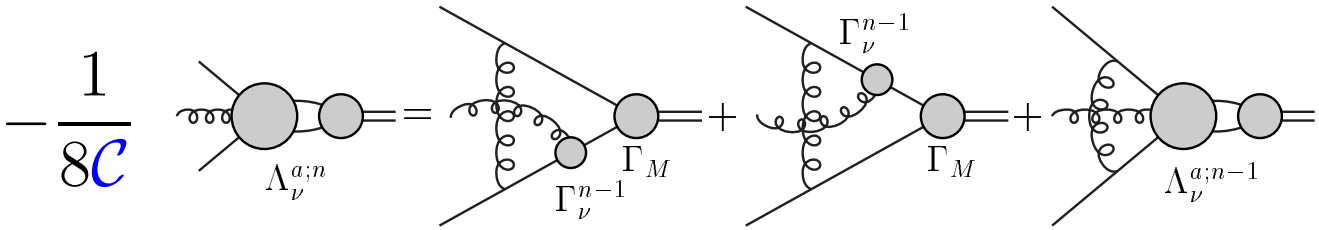

- Kernel necessarily non-planar,

  even with planar vertex

π and ρ mesons

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m = 0$	0	0	0	0
π, $m = 0.011$	0.147	0.135	0.139	0.138
ρ, $m = 0$	0.920	0.648	0.782	0.754
ρ, $m = 0.011$	0.936	0.667	0.798	0.770

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Ab-Initio **Calculations**

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Pieter Maris

Peter Tandy

Ab-Initio **Calculations**

Maris & Tandy, Series of Five Articles: 1999 – Present

Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

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- Rainbow-Ladder $=$ First Order in Truncation Described Above

- Anticipate Accurate for 0^- & 1^- Mesons

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  Characterises DCSB and light-quark Confinement

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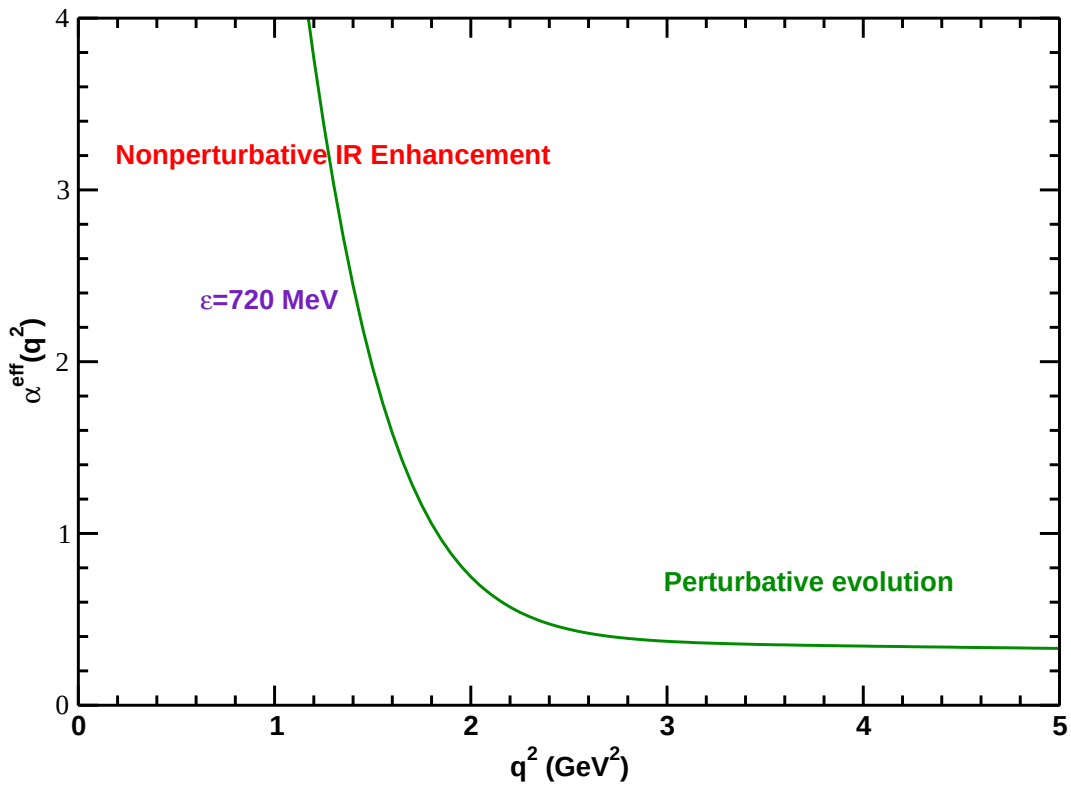
- *Dyson-Schwinger equations:*
  A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049

Interaction

Kernel of Bethe-Salpeter Equation

$$K(p,k;P) \approx \frac{\alpha^{\rm eff}((p-k)^2)}{(p-k)^2}$$



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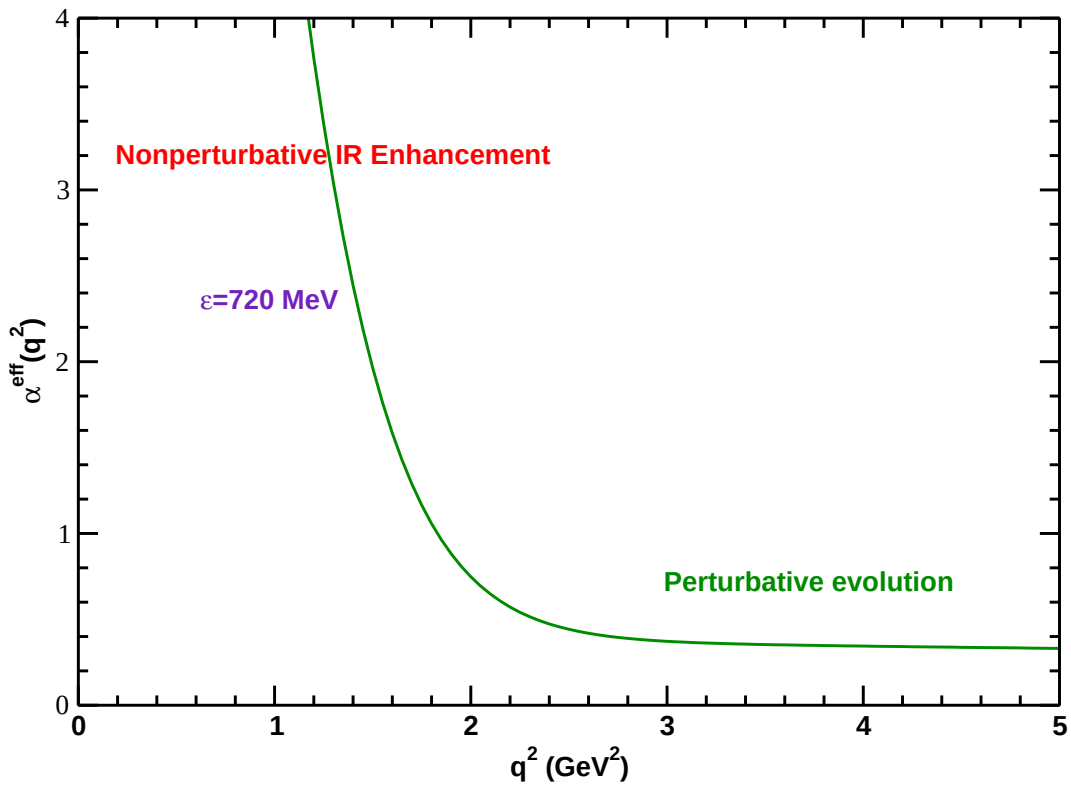
Prescribes Gap Equation's Kernel



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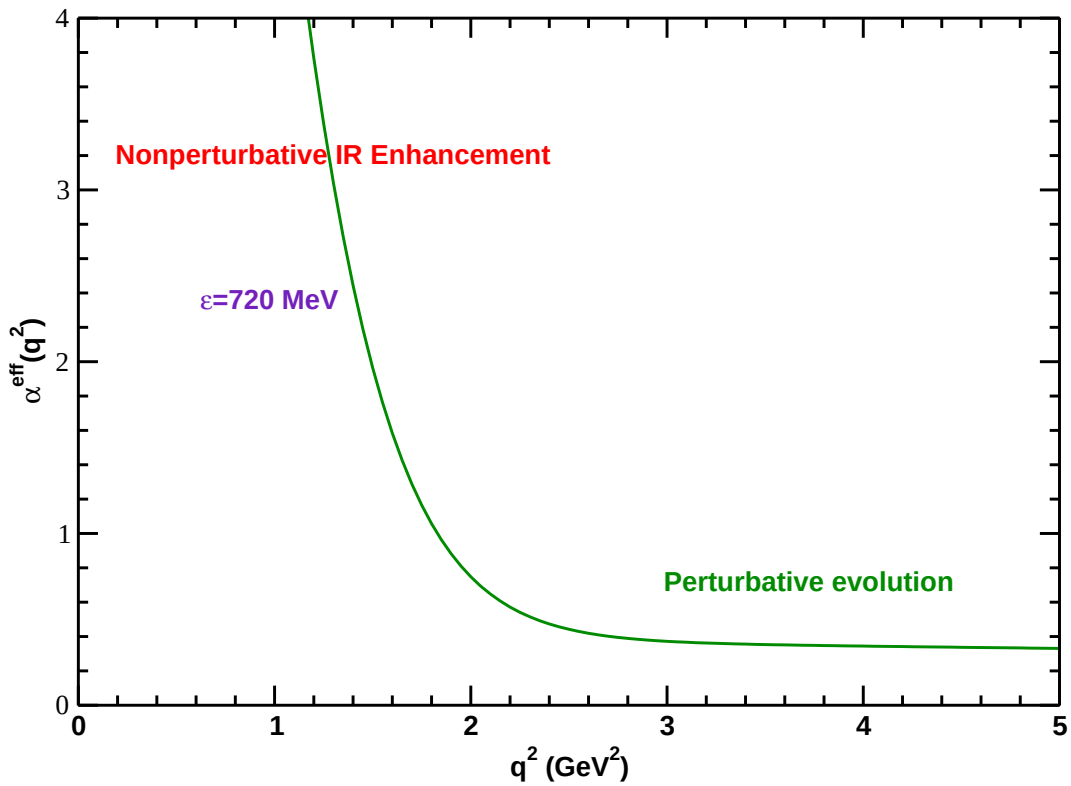

Prescribes Gap Equation's Kernel

Connects *Ansatz* for long-range part of QCD's interaction with Observables.

Interaction

Kernel of Bethe-Salpeter Equation

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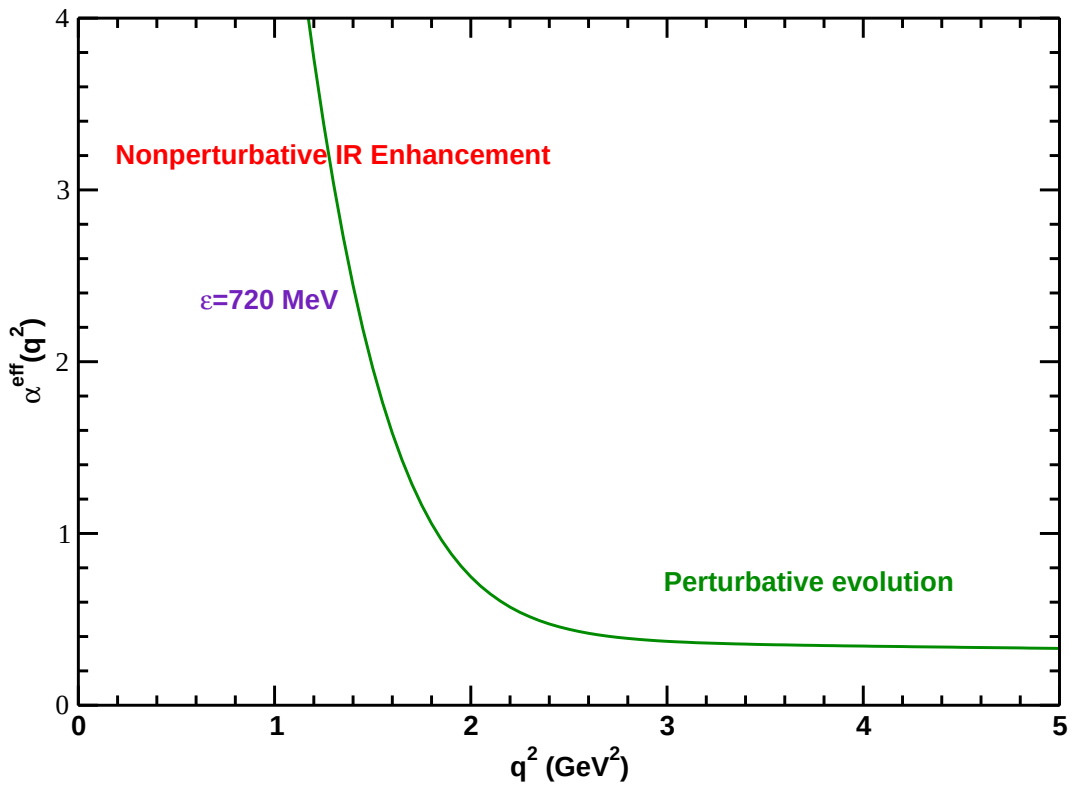


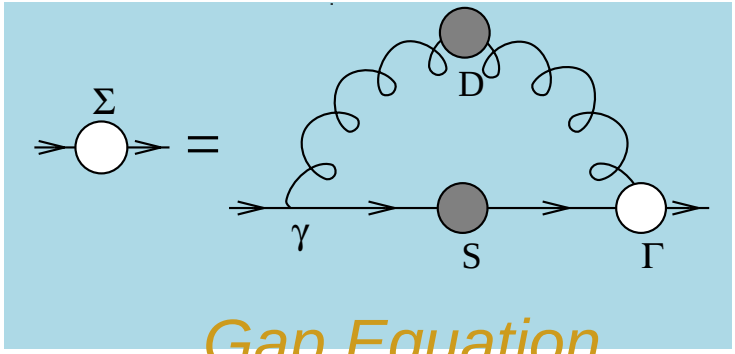
Prescribes Gap Equation's Kernel

IR-*Enhancement* at long-range agrees semi-quantitatively with Bhagwat, *et al*.

Dressed-quark Propagator

Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



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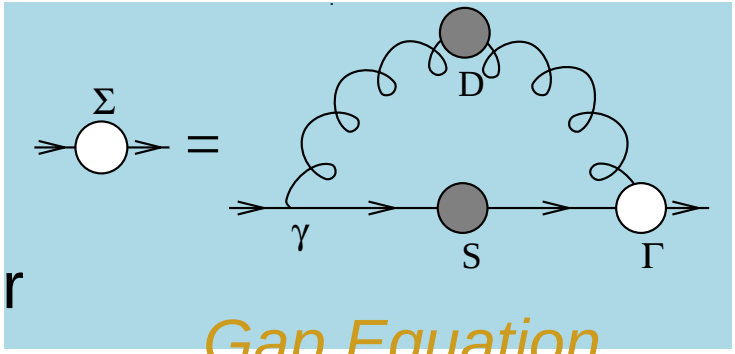
- dressed-quark propagator



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$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

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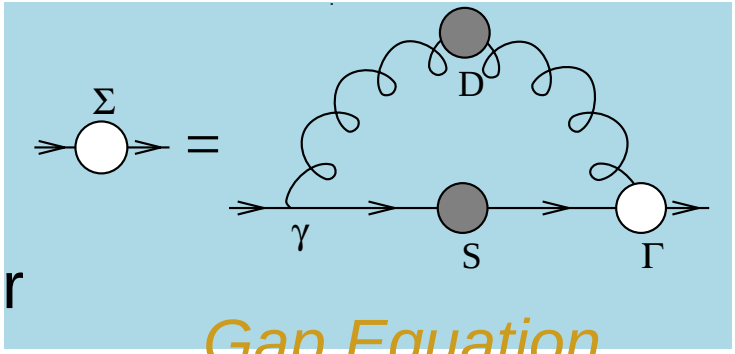

- dressed-quark propagator

Gap Equation

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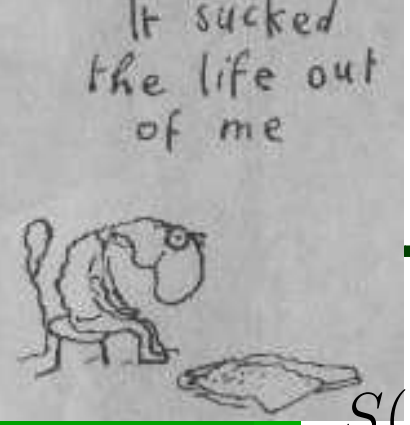
- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory



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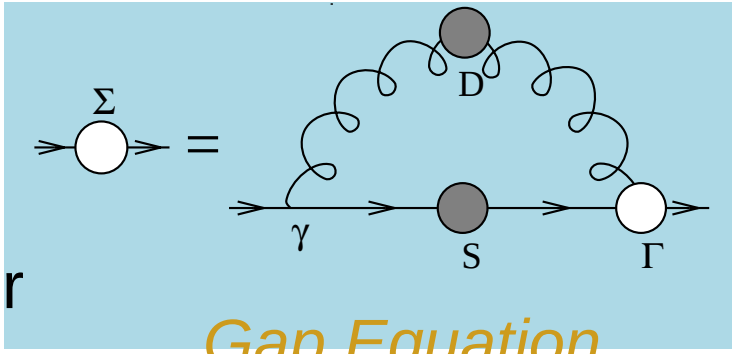

Gap Equation

- dressed-quark propagator

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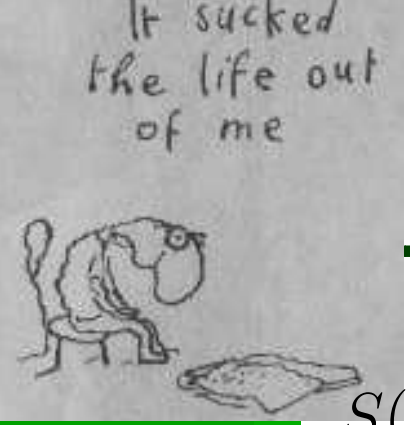
- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \overset{m\to 0}{\to} 0$$



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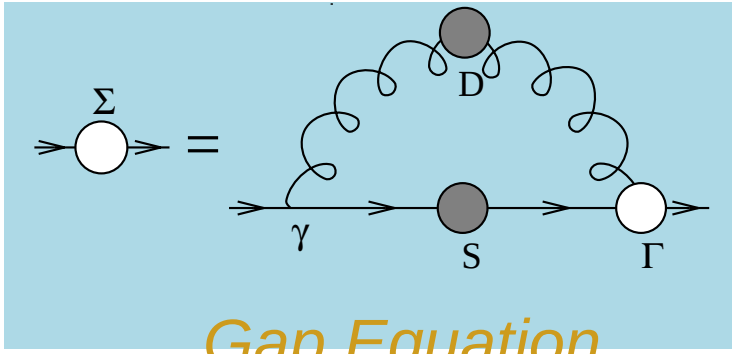
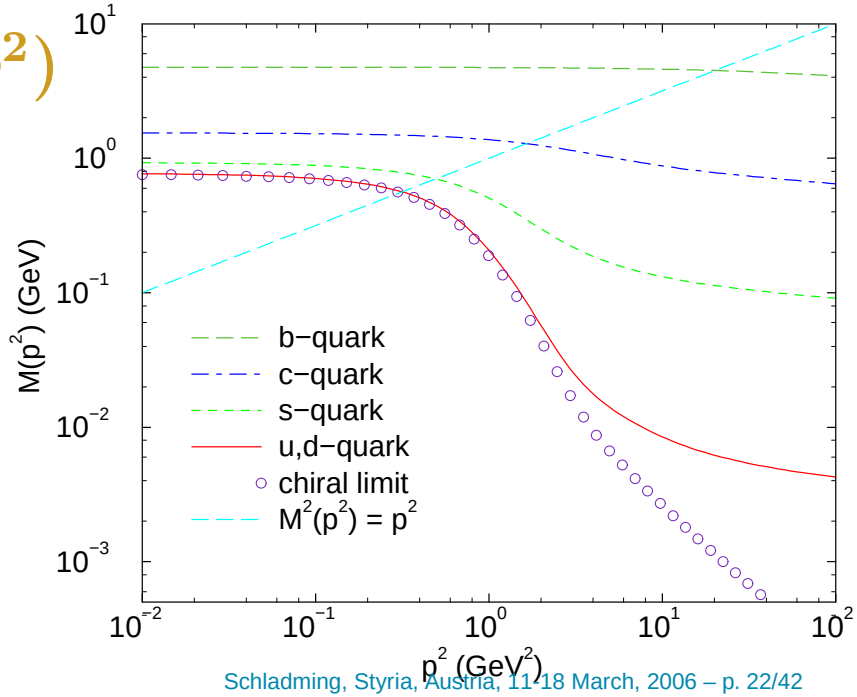
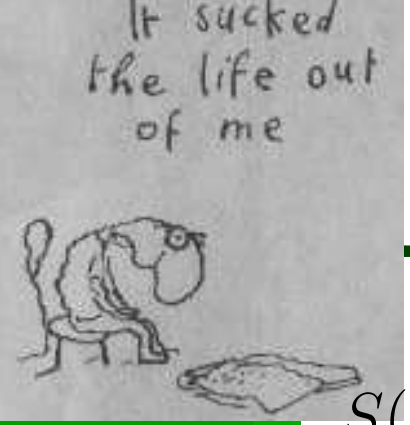


Gap Equation

- Enhancement of Gap Equation's Kernel on IR domain

  $\Rightarrow$ IR Enhancement of $M(p^2)$





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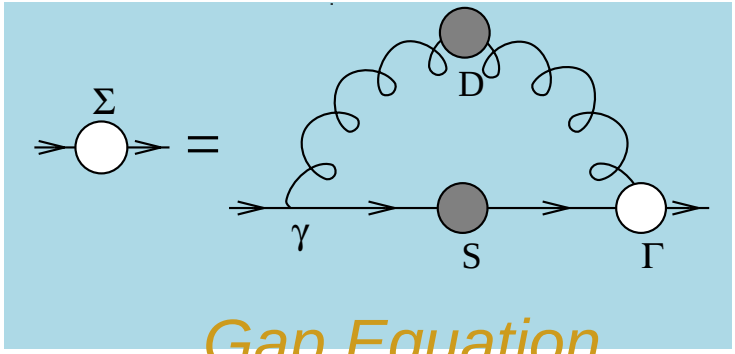

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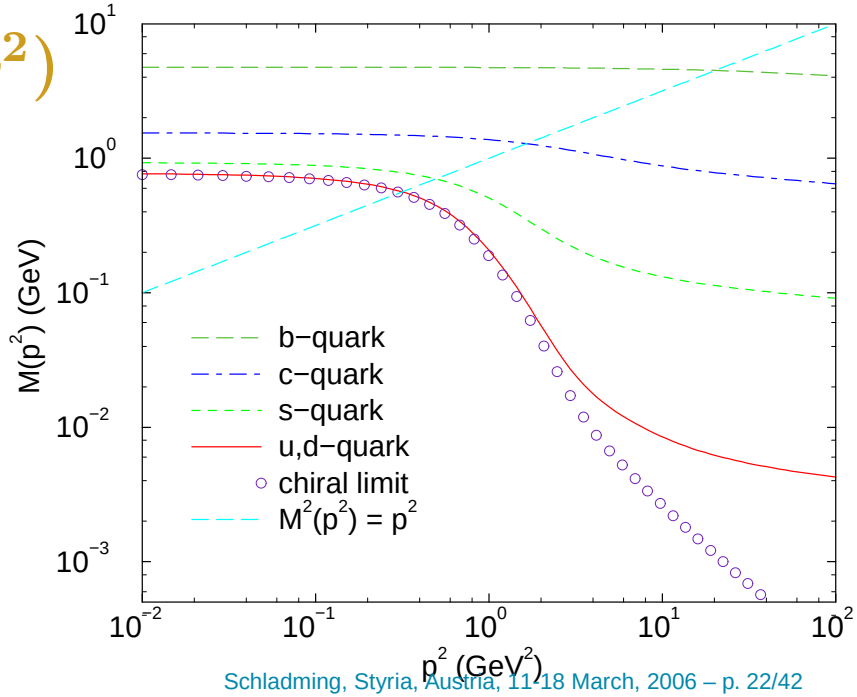

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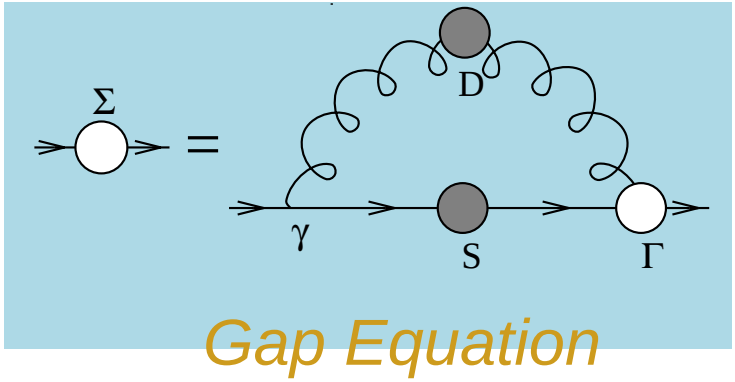

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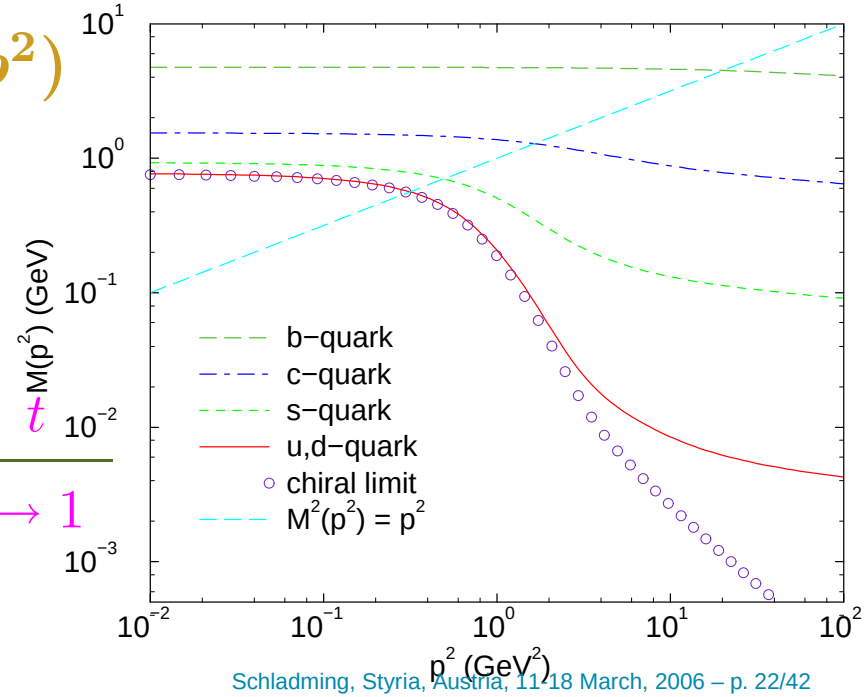
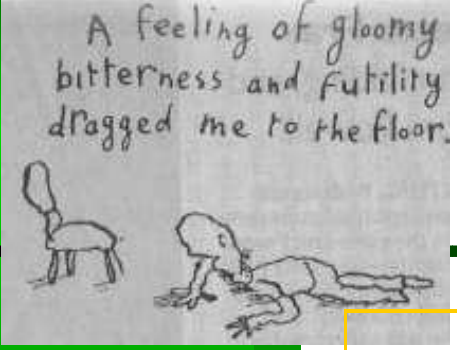
flavour	u/d	s	c	b	t
$\frac{M^E}{m_{\mu\sim 20\,\mathrm{GeV}}}$	$\sim 10^2$	~ 10	1.4	1.1	$\to 1$
			-2.2	-1.2	



Vacuum quark condensate

Vacuum quark condensate

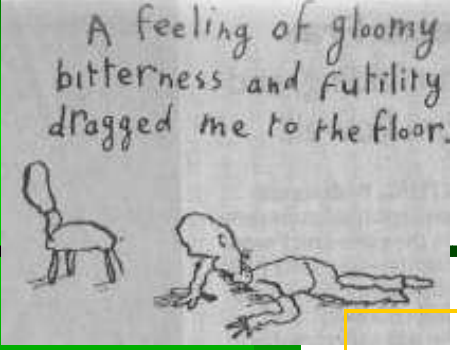
Trace of the Chiral-limit dressed-quark propagator



Vacuum quark condensate

$$-\langle \bar{q}q \rangle^0_\zeta \Big|_{\zeta=19\,\mathsf{GeV}} =$$

$$Z_4(\zeta,\Lambda)\;\; N_c \int_q^\Lambda \mathrm{tr}_{\mathrm{Dirac}}\left[S_{\hat{m}=0}(q)\right] = \;\; (0.275\,\mathrm{GeV})^3$$



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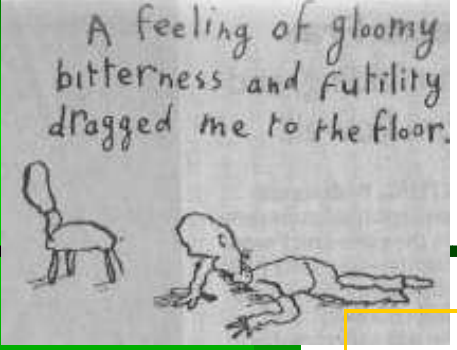
- Mass renormalisation constant

Ensures finiteness and correct renormalisation group flow

$$m(\zeta)\,\langle\bar{q}q\rangle^0_\zeta = \text{const.}$$



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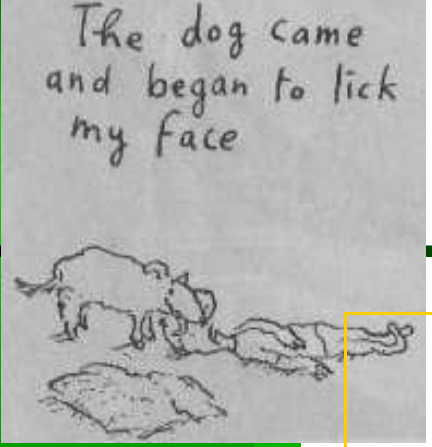
AND ensures Gauge Invariance



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Numerical Value

π and K observables

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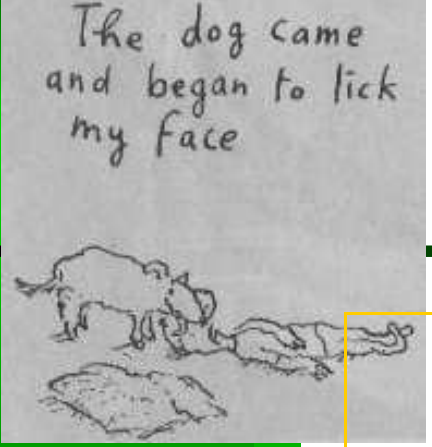

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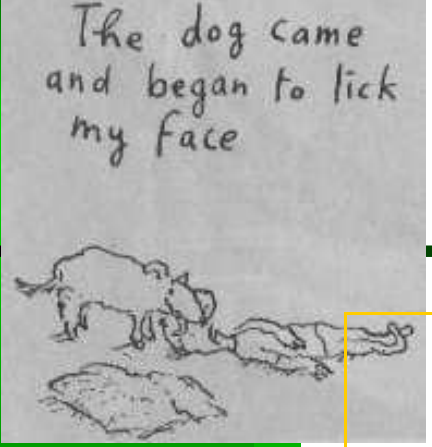

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Close packed spheres: $1.8\,\mathrm{fm}^{-3}$

$$\Rightarrow V_{\langle\bar{q}q\rangle} = 0.55\,\mathrm{fm}^3 \Rightarrow r_{\langle\bar{q}q\rangle} = 0.51\,\mathrm{fm} = 0.77\,r_\pi = 0.58\,r_p$$

QCD's Mass Gap

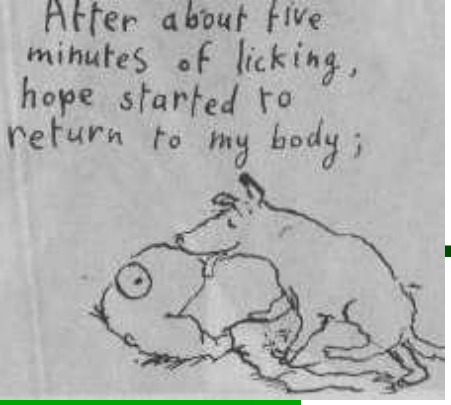
Sea

of Virtual Particle-Antiparticle Pairs

cf. Quenched lattice simulations

Contemporary Supercomputer Resources

– 1 Teraflop Sustained



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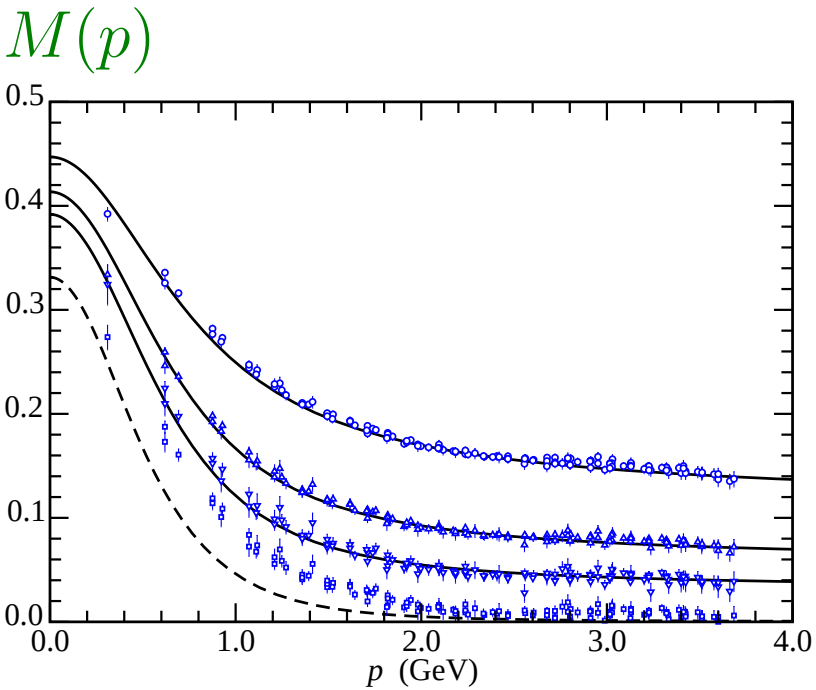
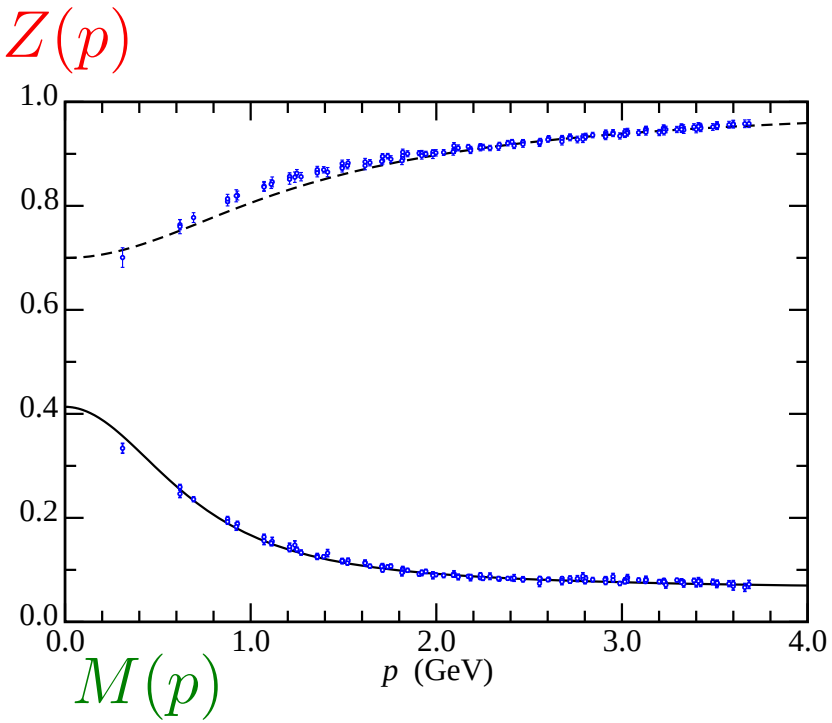
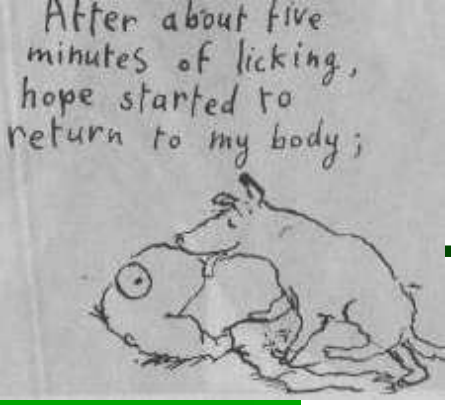




- Lattice Meas.
  - Bowman, Heller, Leinweber, Williams: he-lat/0209129



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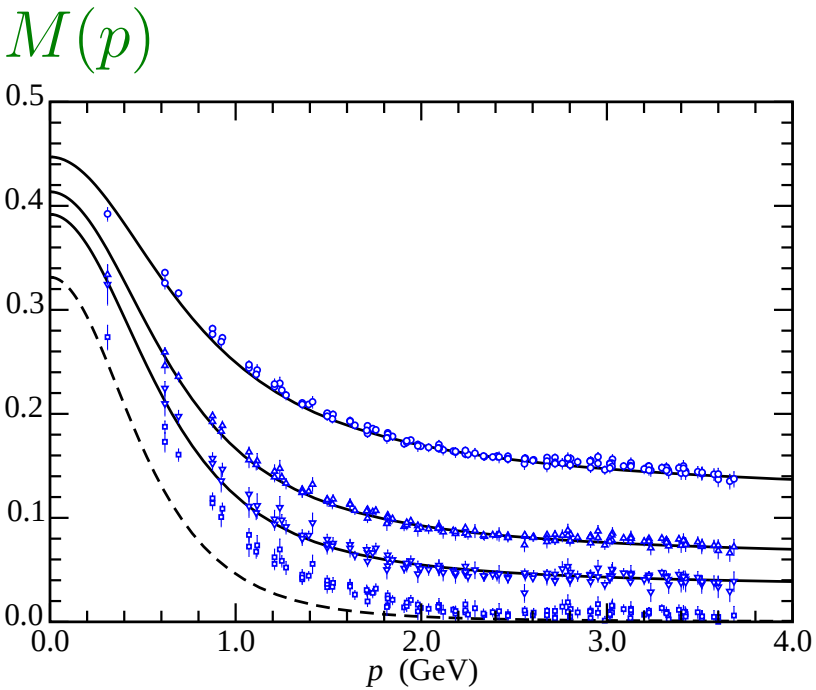

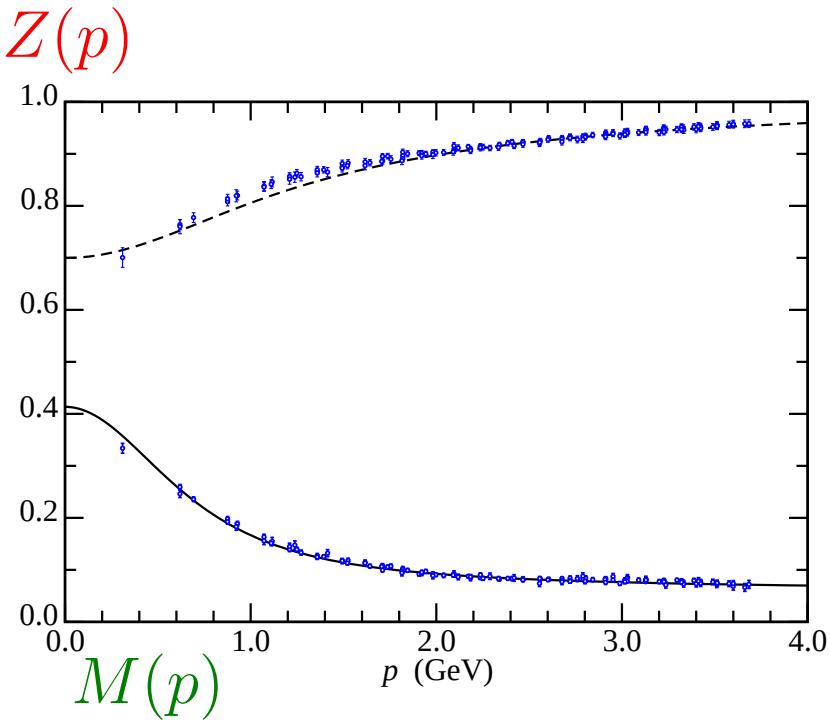

- DSE Cal.– Bhagwat, Pichowsky, CDR, Tandy nu-th/0304003

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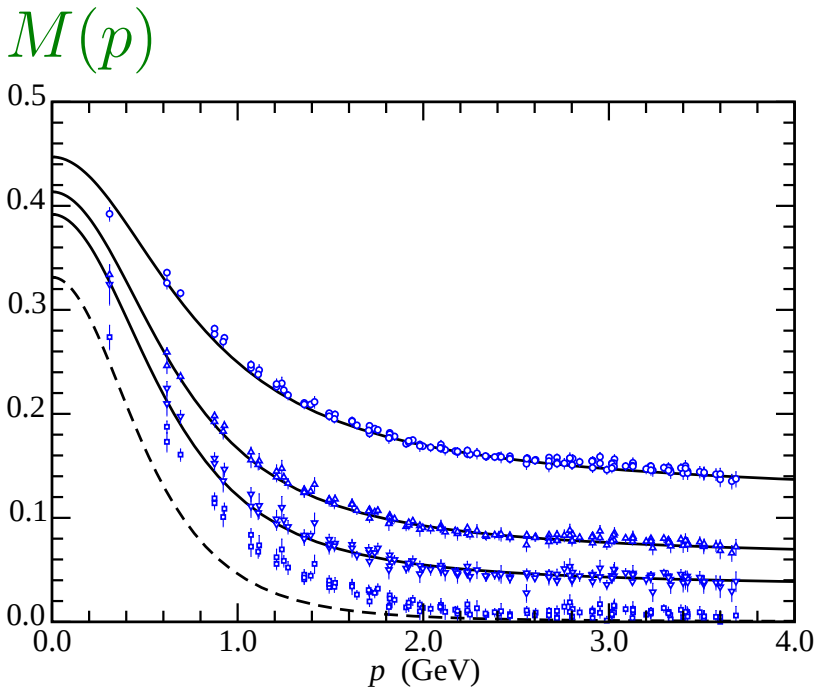
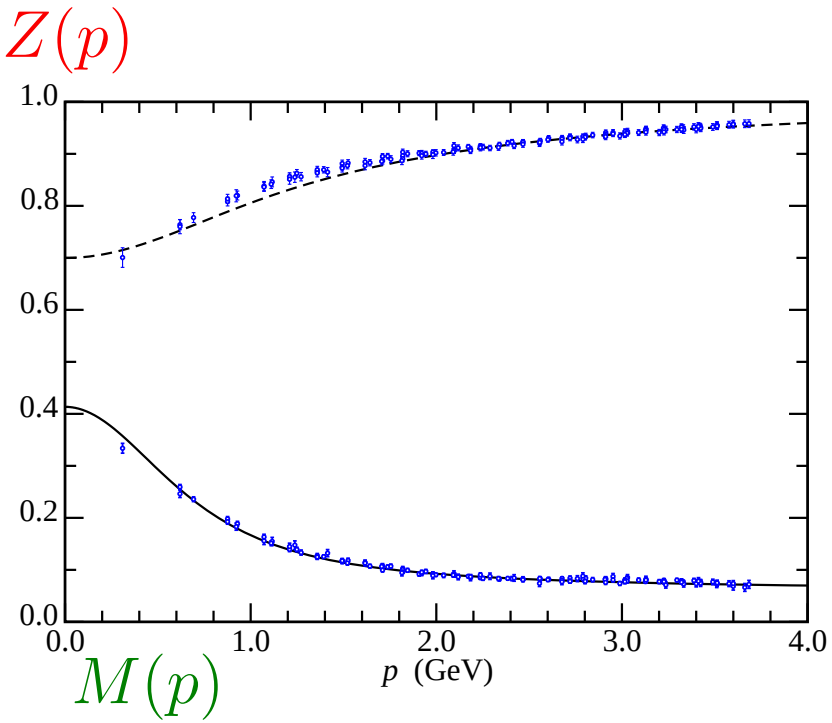
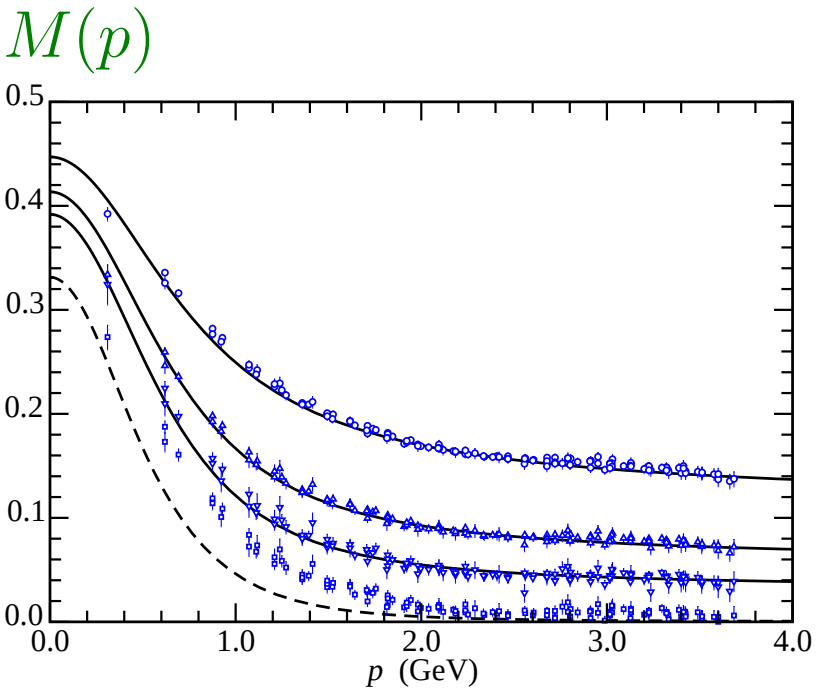


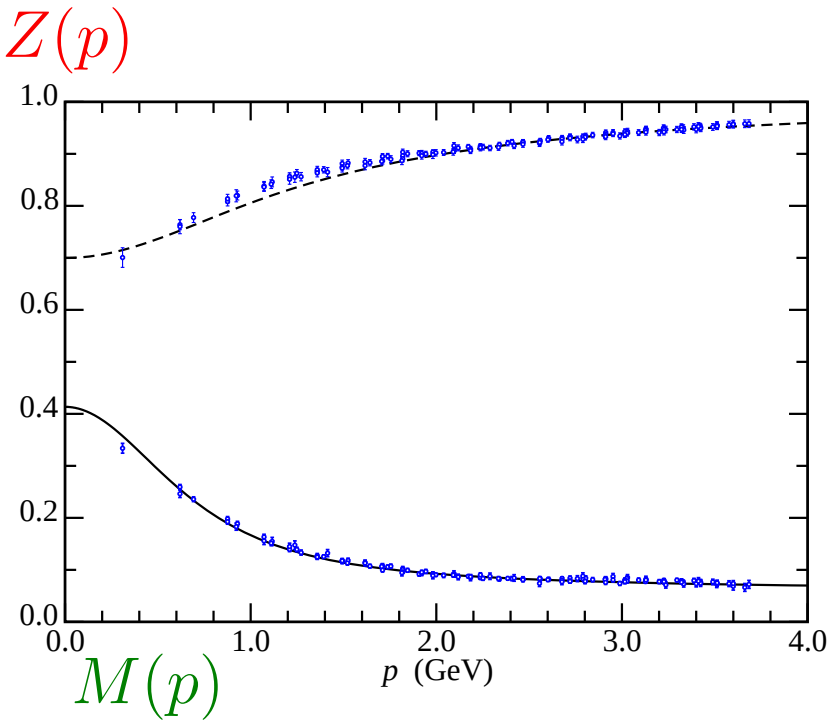


- DSE Cal.– Bhagwat, Pichowsky, CDR, Tandy nu-th/0304003

$$f_\pi^0 = 0.068\,\mathrm{GeV} \quad \langle \bar{q}q \rangle^0_{1\,\mathrm{GeV}} = (-0.19\,\mathrm{GeV})^3 \,.$$

cf. Quenched lattice simulations

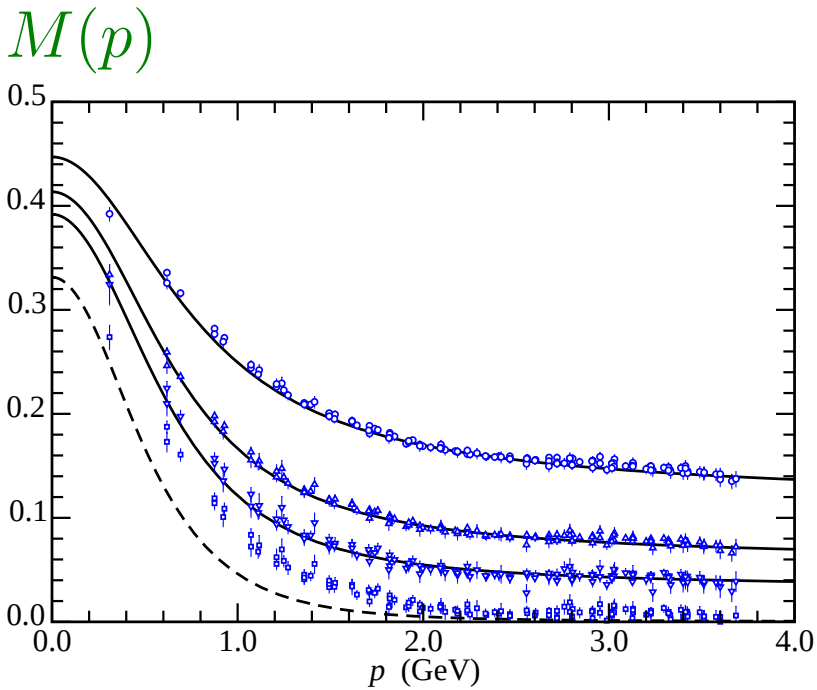


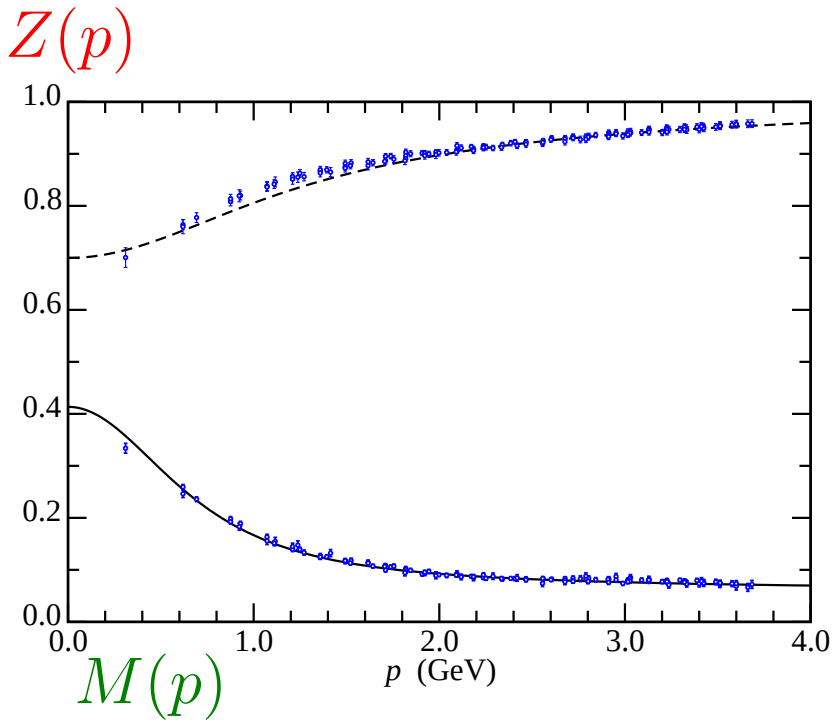


- *ab initio* support for DSE conjecture

  Mass from Nothing is a Reality!

cf. Quenched lattice simulations





- *ab initio* support for DSE conjecture

  Mass from Nothing is a Reality!

- More than 98% of Spectrum/Constituent-Quark's Mass = cloud of strongly-interacting virtual quanta

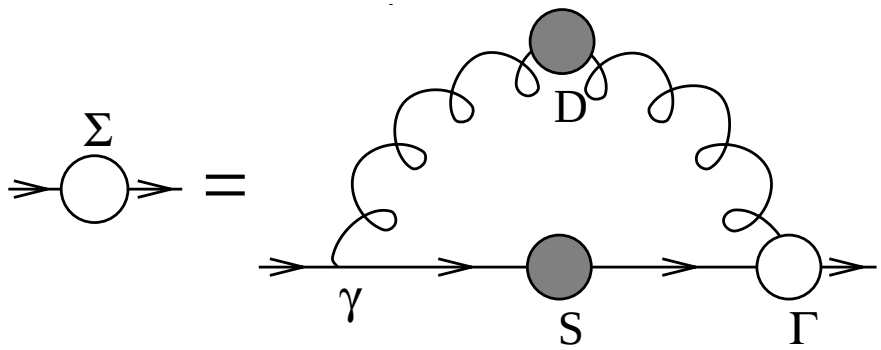
Pion Form Factor

Procedure Now Straightforward

Pion Form Factor

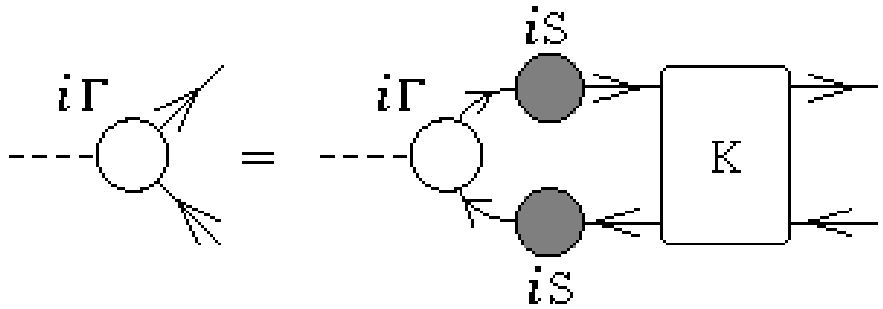
- Solve Gap Equation

  $\Rightarrow$ Dressed-Quark Propagator, $S(p)$



Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, K
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π



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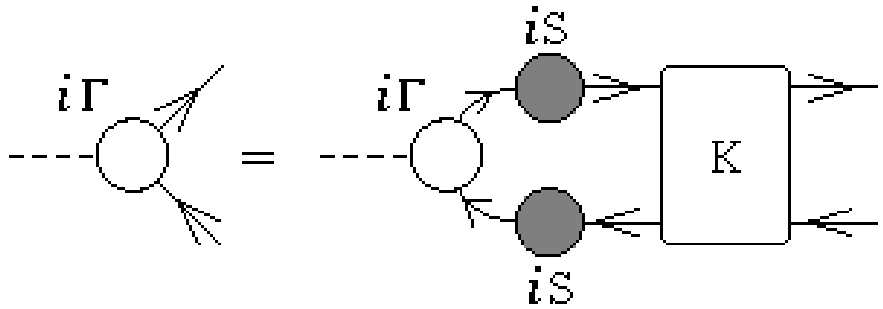
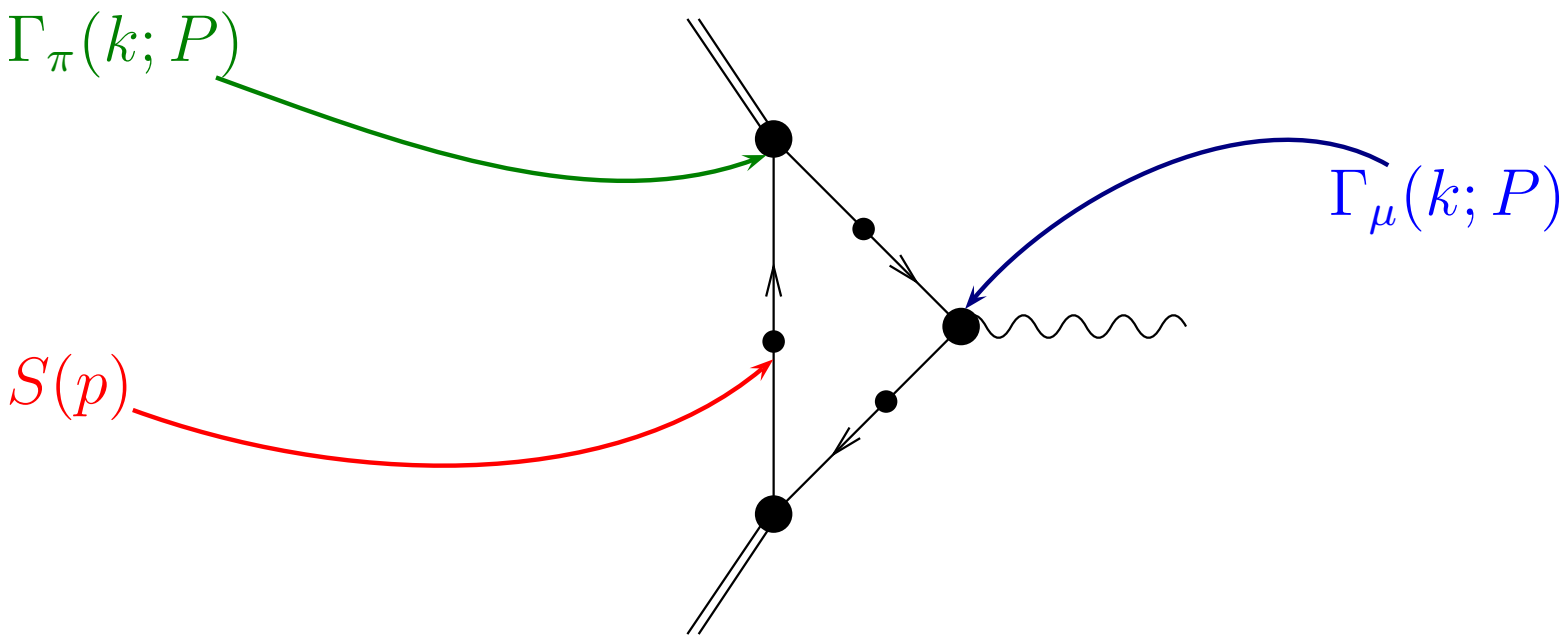


- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Gluon Vertex, Γ_μ

Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



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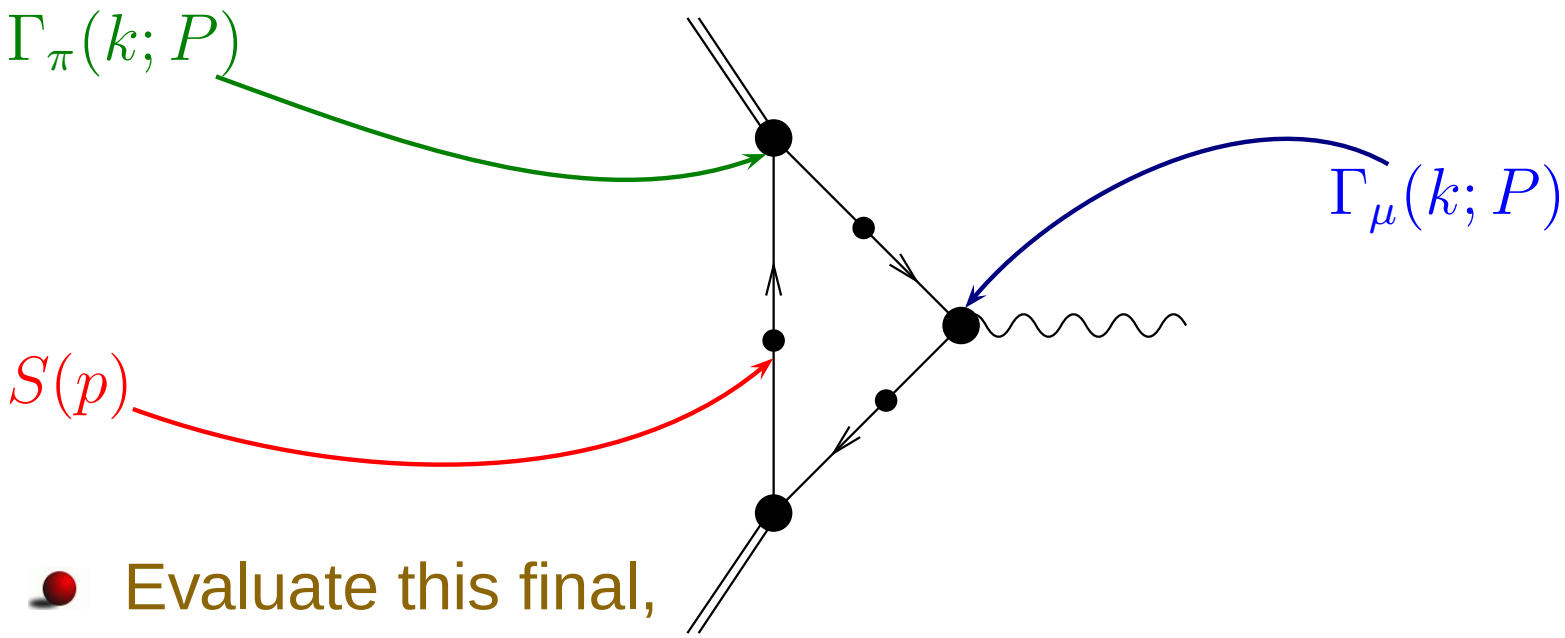
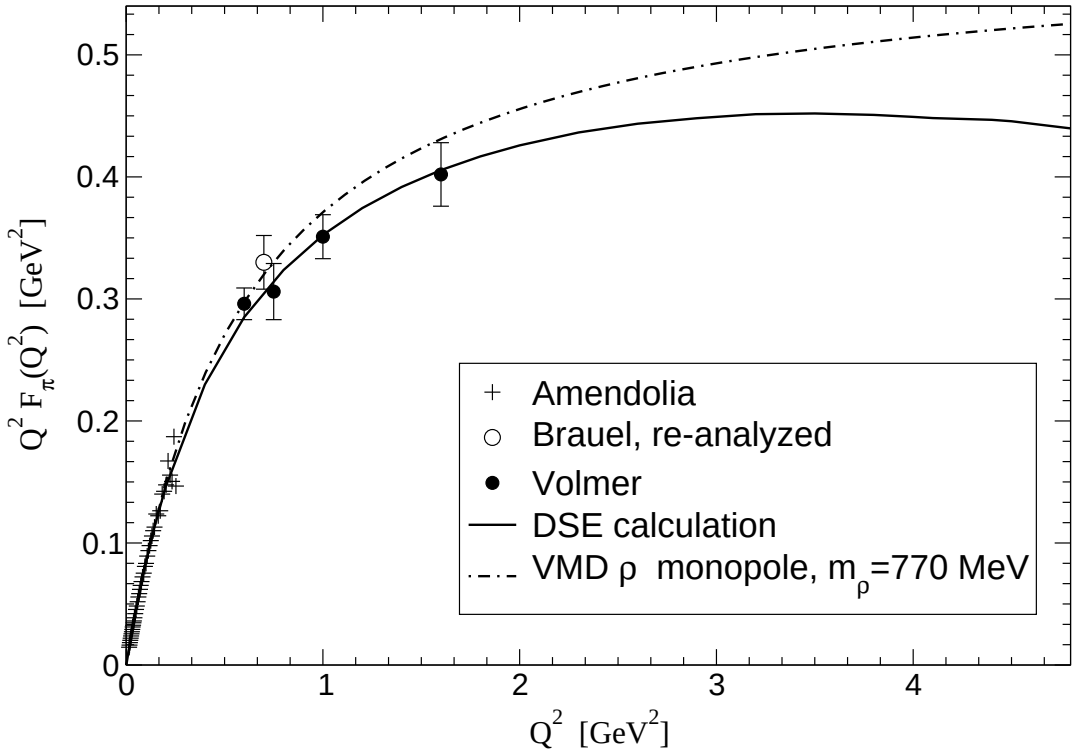


- Evaluate this final, three-dimensional integral

Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied



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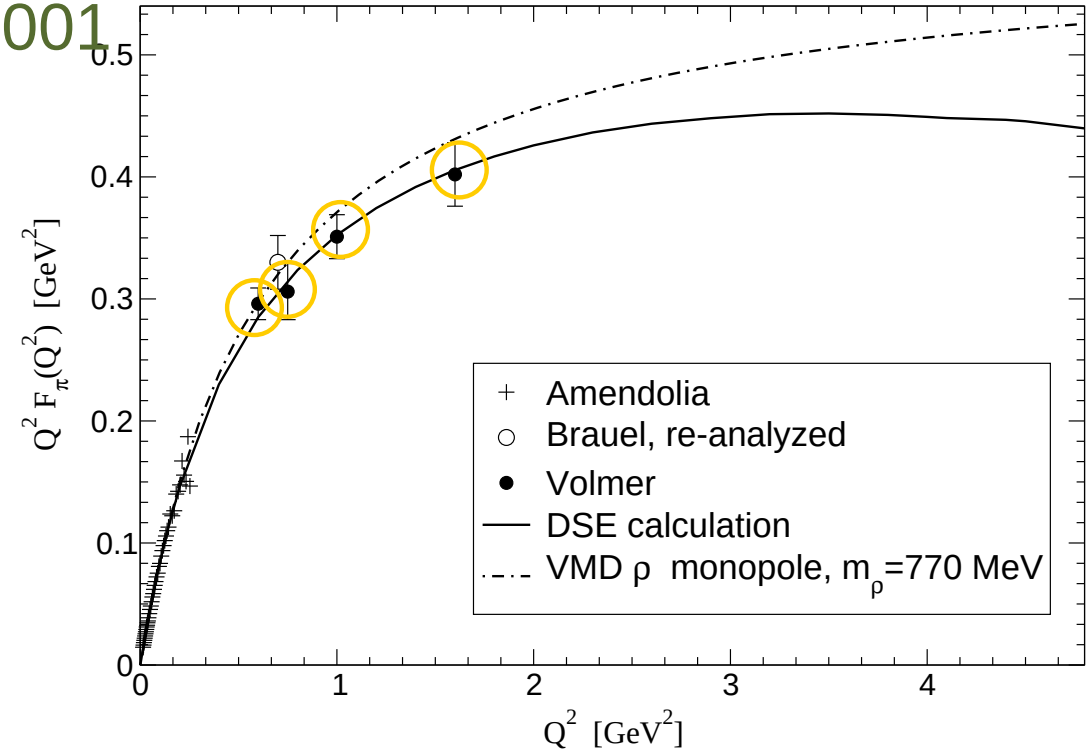
Calculation published in 1999; No Parameters Varied

Data published in 2001



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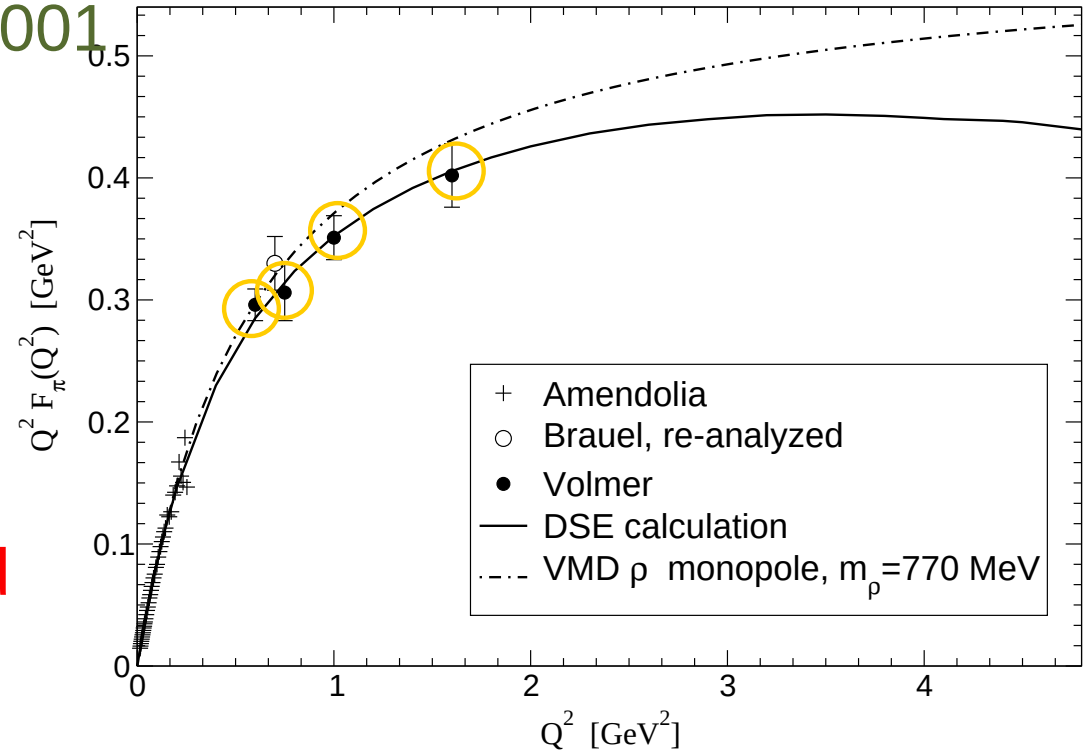
Calculation published in 1999; No Parameters Varied

Data published in 2001



Many subsequent successful applications

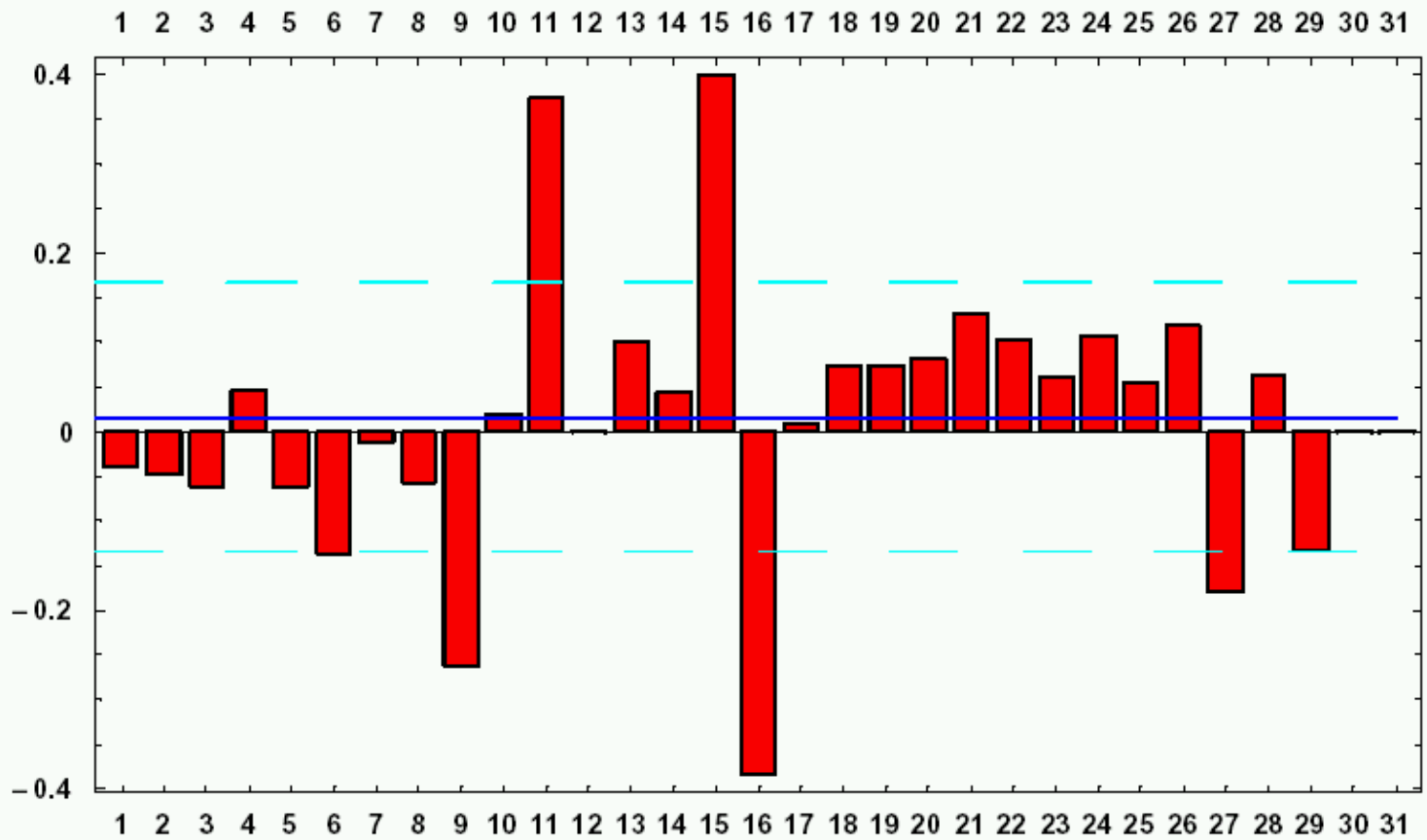
Again, parameters Fixed

Notably $\pi\ \pi$ Scattering

Maris, et al., Phys. Rev. D 65, 076008

Bicudo, Phys. Rev. C 67, 035201

Maris & Tandy
Relative Error



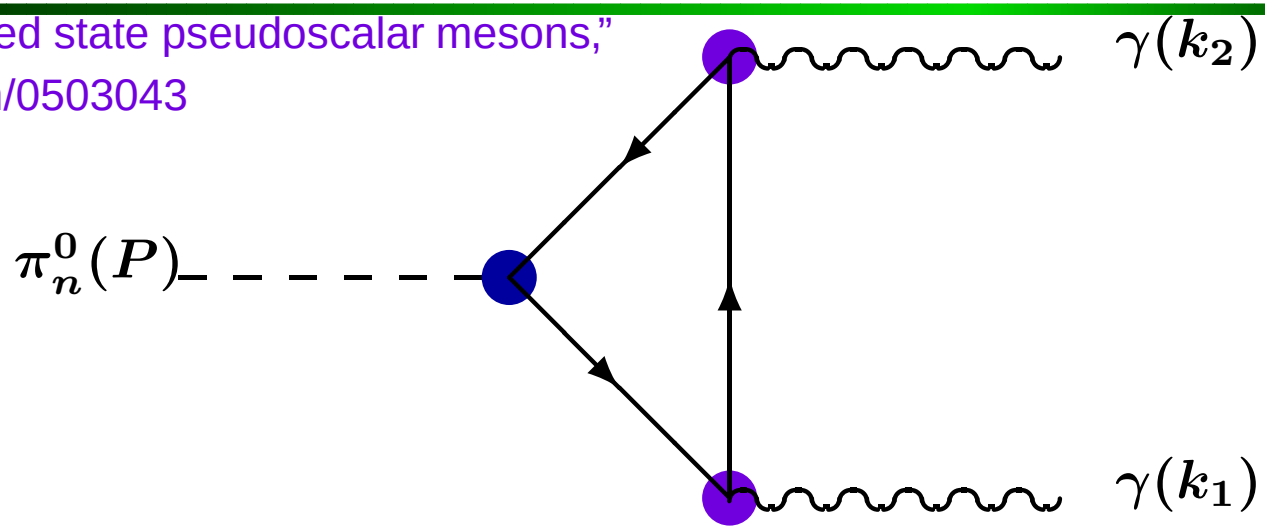
Relative Error, Predictions of Maris and Tandy Model

All tabulated quantities in nu-th/0301049

<error> = 1.6%, Sqrt[<error^2>] = 15%

Two-photon Couplings of Pseudoscalar Mesons

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $$T^{\pi_n^0}_{\mu\nu}(k_1,k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}\, G^{\pi_n^0}(k_1,k_2)$$
  - Define: $\mathcal{T}_{\pi_n^0}(P^2,Q^2) = G^{\pi_n^0}(k_1,k_2)\Big|_{k_1^2=Q^2=k_2^2}$

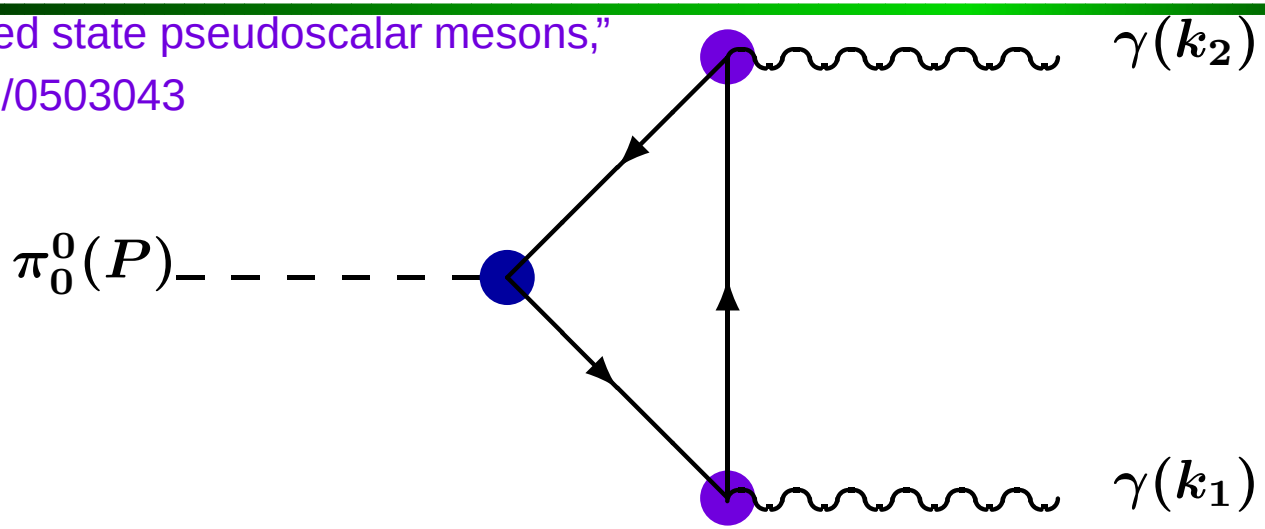
    This is a transition form factor.
  - Physical Processes described by couplings:

    $g_{\pi_0^0\gamma\gamma} := \mathcal{T}_{\pi_0^0}(-m^2_{\pi_0^0},0)$

    Width: $\Gamma_{\pi_n^0\gamma\gamma} = \alpha^2_{\rm em}\,\dfrac{m^3_{\pi_n}}{16\pi^3}\, g^2_{\pi_n\gamma\gamma}$

Two-photon Couplings: Goldstone Mode

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $$T^{\pi_0^0}_{\mu\nu}(k_1,k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}\, G^{\pi_0^0}(k_1,k_2)$$

- Chiral limit, model-independent and algebraic result

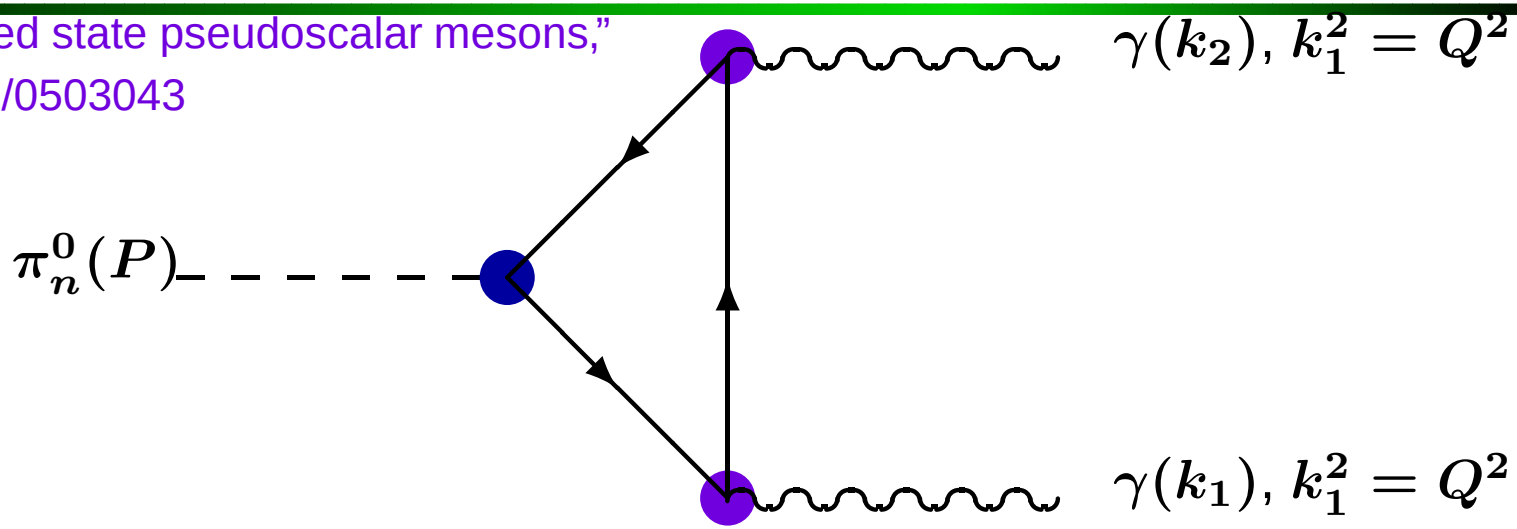
  $$g_{\pi_0^0\gamma\gamma} := \mathcal{T}_{\pi_0^0}(-m^2_{\pi_0^0}=0,0) = \frac{1}{2}\frac{1}{f_{\pi_0}}$$

  So long as truncation preserves chiral symmetry and the pattern of its dynamical breakdown

- The most widely known consequence of the Abelian anomaly

Two-photon Couplings: Transition Form Factor

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $$T_{\mu\nu}^{\pi_n^0}(k_1,k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}\, G^{\pi_n^0}(k_1,k_2)$$

- So long as truncation preserves chiral symmetry and the pattern of its dynamical breakdown, and the one-loop renormalisation group properties of QCD: model-independent result – $\forall n$:

$$\mathcal{T}_{\pi_n^0}(P^2,Q^2) = G^{\pi_n^0}(k_1,k_2)\Big|_{k_1^2=Q^2=k_2^2} \overset{Q^2 \gg \Lambda_{\rm QCD}^2}{=} \frac{4\pi^2}{3}\frac{f_{\pi_n}}{Q^2}$$

Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$

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- BUT, $f_{\pi_n} \equiv 0, \forall n$!
- Model-independent result, in chiral limit: $\forall n \geq 1$

$$\lim_{\hat{m}\to 0} \mathcal{T}_{\pi_n^0}(-m_{\pi_n}^2, Q^2) \overset{Q^2 \gg \Lambda_{\rm QCD}^2}{=} \frac{4\pi^2}{3} F_n^{(2)}(-m_{\pi_n}^2) \frac{\ln^\gamma Q^2/\omega_{\pi_n}^2}{Q^4}\Bigg|_{\hat{m}=0}$$

where:

- γ is an anomalous dimension
- ω_{π_n} is a width mass-scale

both determined, in part, by properties of the meson's Bethe-Salpeter wave function.

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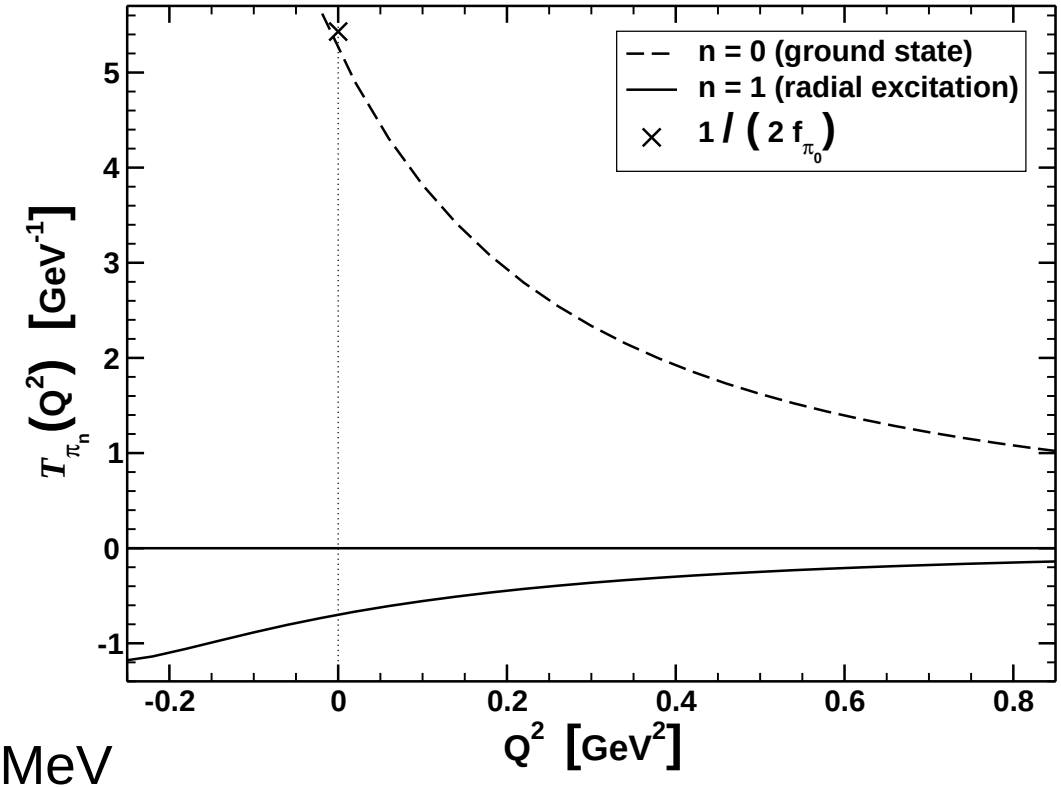
- Importantly, $F_n^{(2)}(-m_{\pi_n}^2) \not\propto f_{\pi_n}$. Instead, it is determined by DCSB mass-scales for π_n that do not vanish in the chiral limit.

Calculated Transition Form Factor: RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5\,\mathrm{MeV}$

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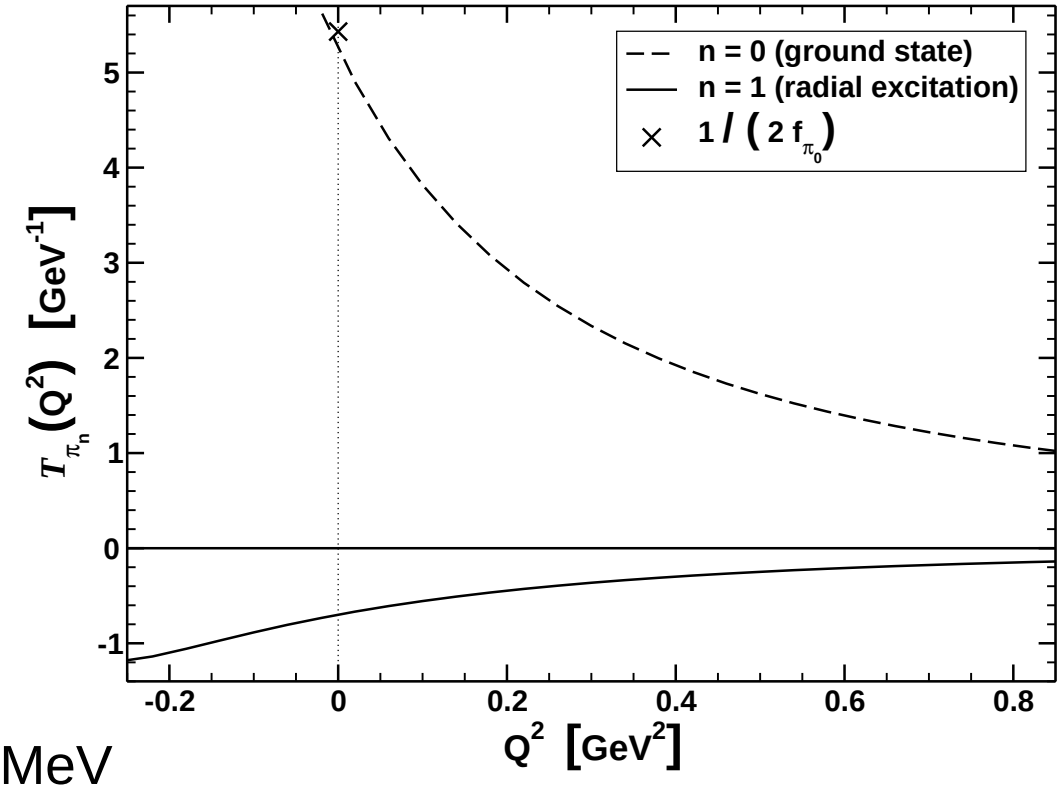

- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5$ MeV
- $\mathcal{T}_{\pi_1^0}(-m_{\pi_1}^2, Q^2) < 0\,,\ Q^2 \geq -m_{\pi_1}^2/4$; viz., it is negative on the entire kinematically accessible domain.

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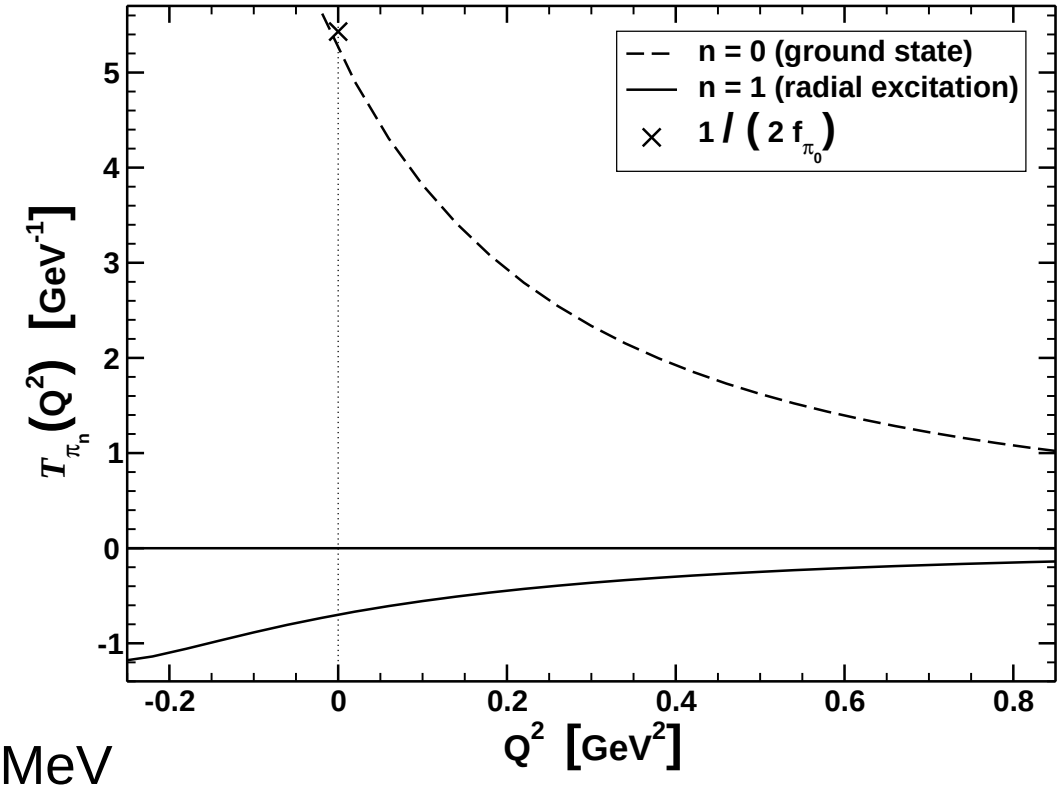

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- $\Gamma_{\pi_0^0\gamma\gamma} = 7.9$ eV, $\Gamma_{\pi_1^0\gamma\gamma} = 240$ eV

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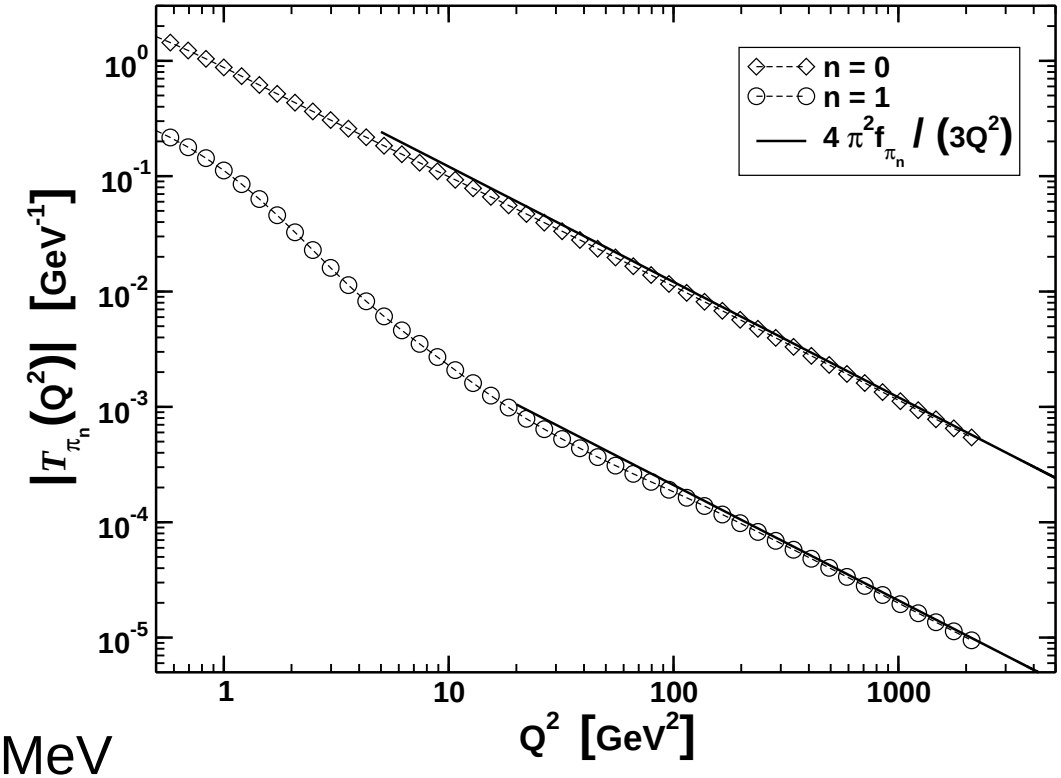

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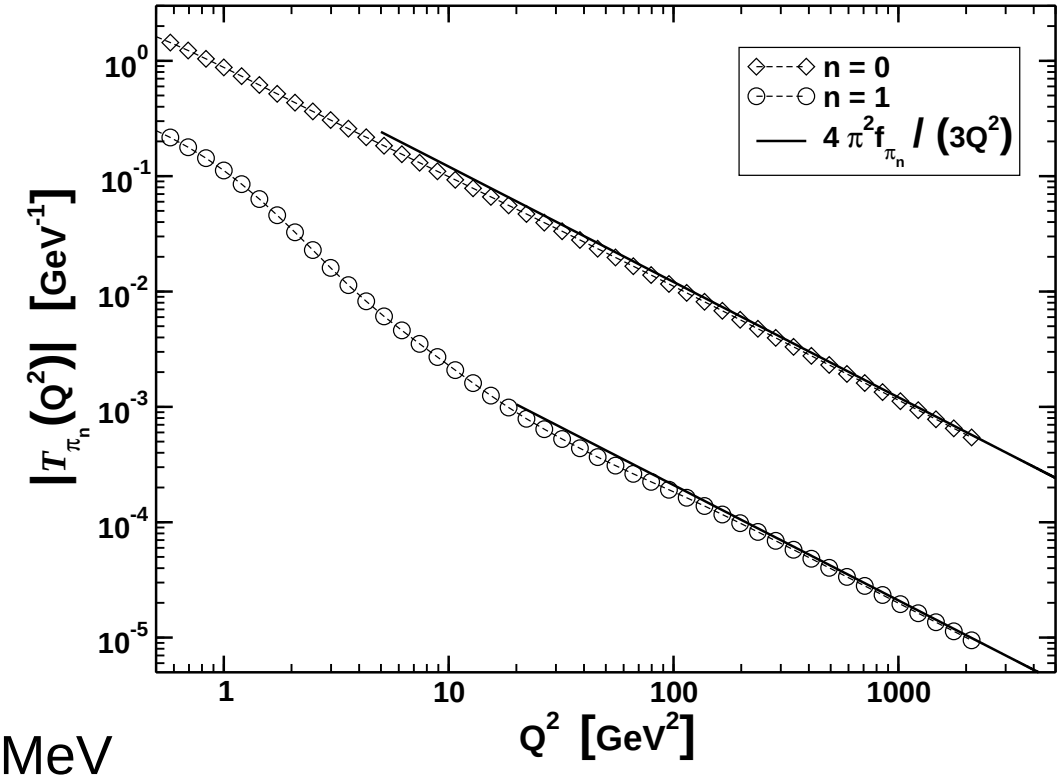
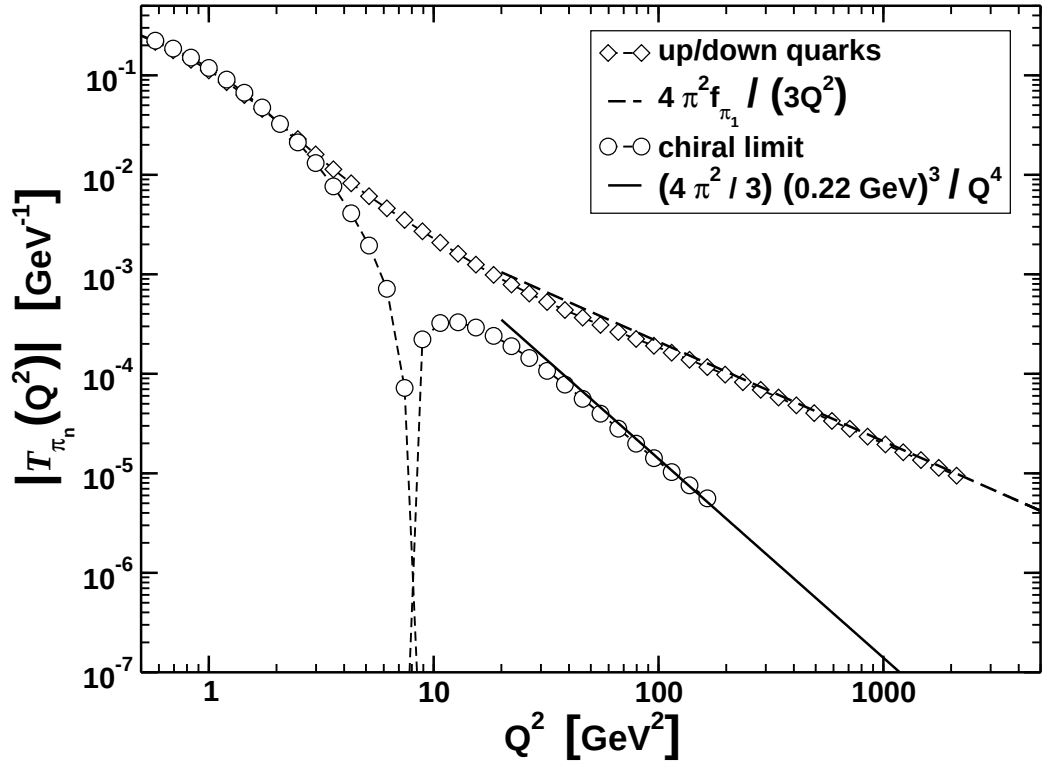


- $m_u(1\,\mathrm{GeV})$
  $= m_d(1\,\mathrm{GeV}) = 5.5$ MeV
- Predicted UV-behaviour is abundantly clear
  - precise for $Q^2 > 120\,\mathrm{GeV}^2$

Transition Form Factor (Chiral): RGI Rainbow-Ladder

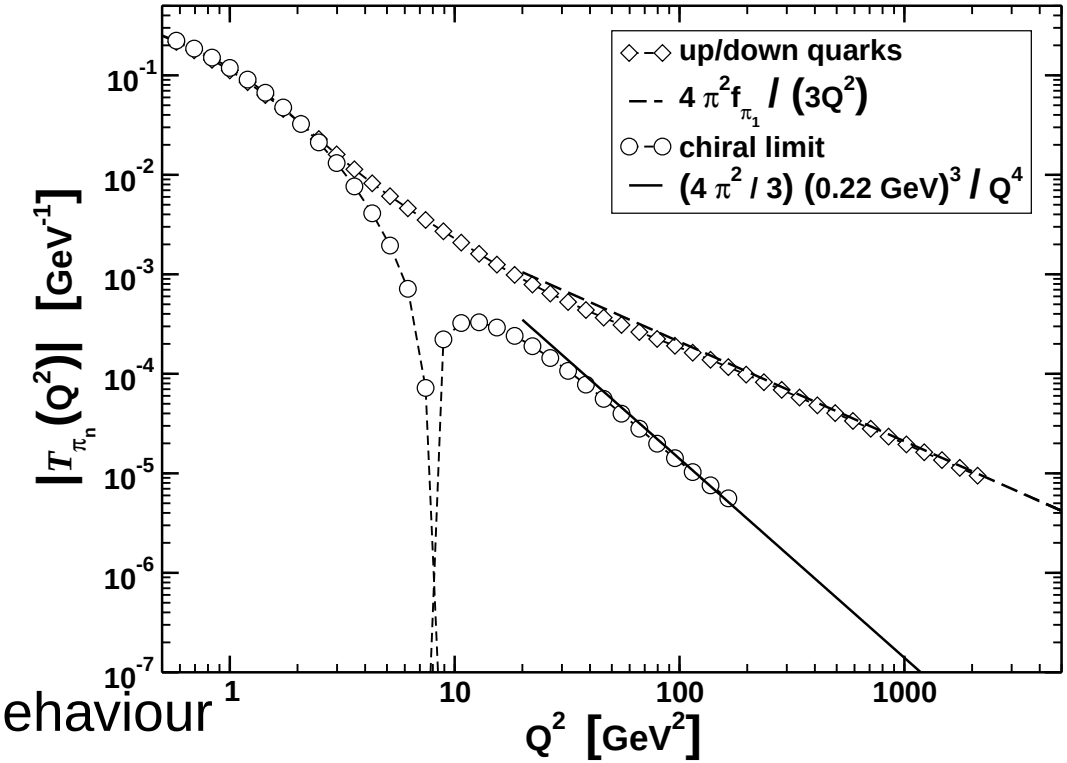
Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 0$

Transition Form Factor (Chiral): RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



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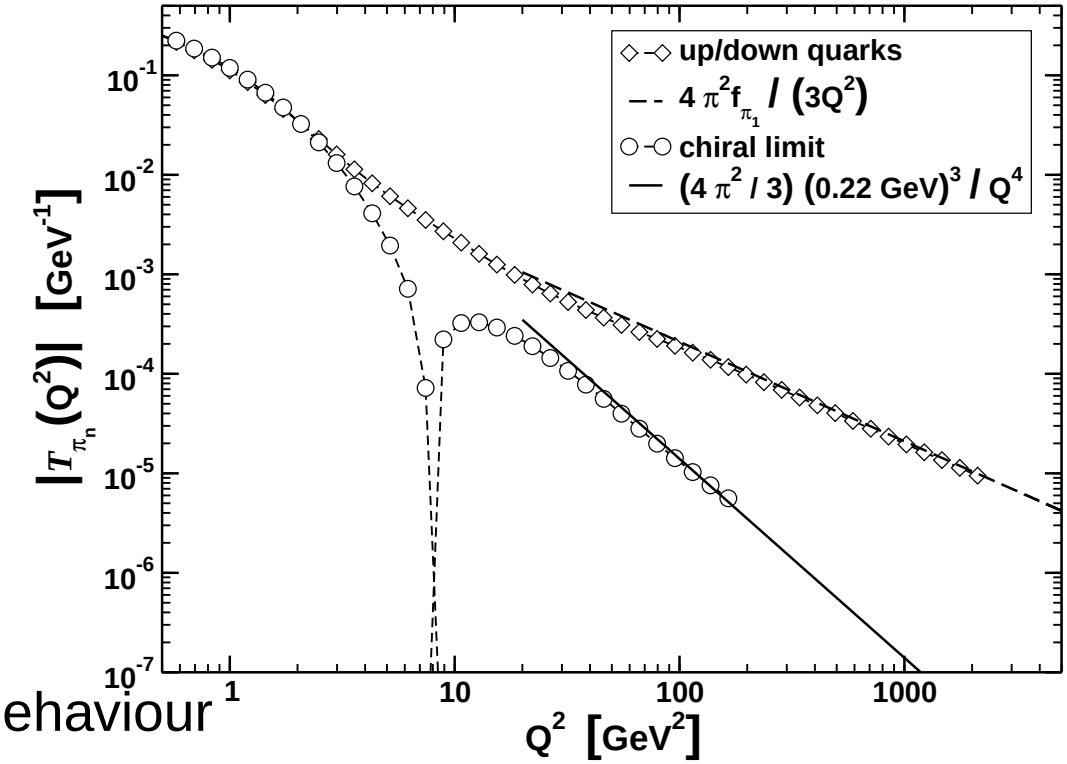
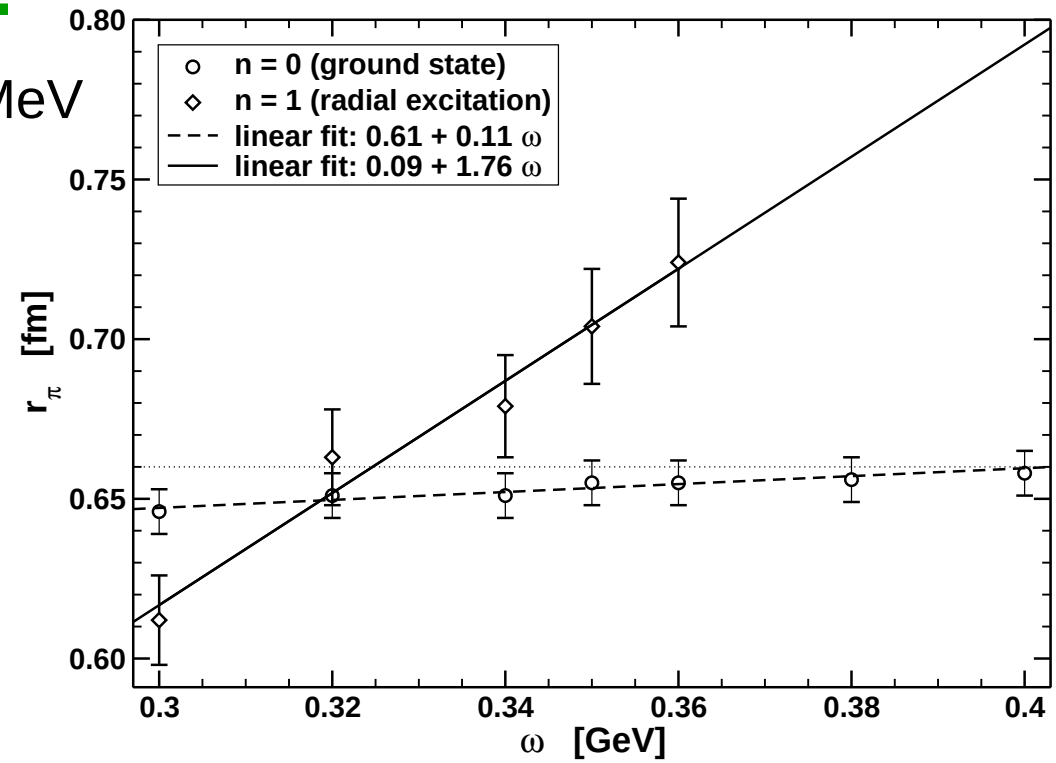


- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 0$
- Again, Predicted UV-behaviour is abundantly clear
  - precise for $Q^2 > 120\,\mathrm{GeV}^2$
- $$F_1^{(2)}(-m_{\pi_1}^2)\,\ln^\gamma Q^2/\omega_{\pi_1}^2\Big|_{\hat{m}=0} \approx (0.22\,\mathrm{GeV})^3 \simeq -\langle\bar{q}q\rangle^0 \quad (3)$$

Electromagnetic Charge Radii – RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al.*, nu-th/0503043

- $m_{u,d}(1\,\mathrm{GeV}) = 5.5\,\mathrm{MeV}$



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- $m_{u,d}(1\,\mathrm{GeV}) = 5.5$ MeV
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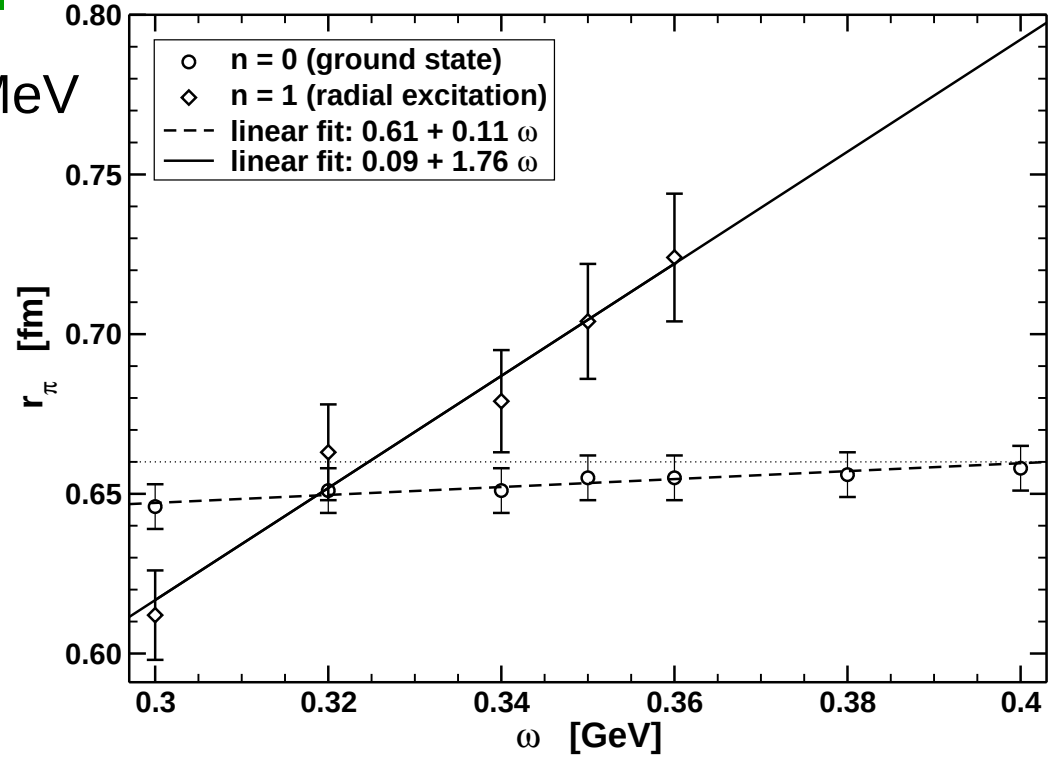
  - $r_a := 1/\omega$ gauges the range of strong attraction



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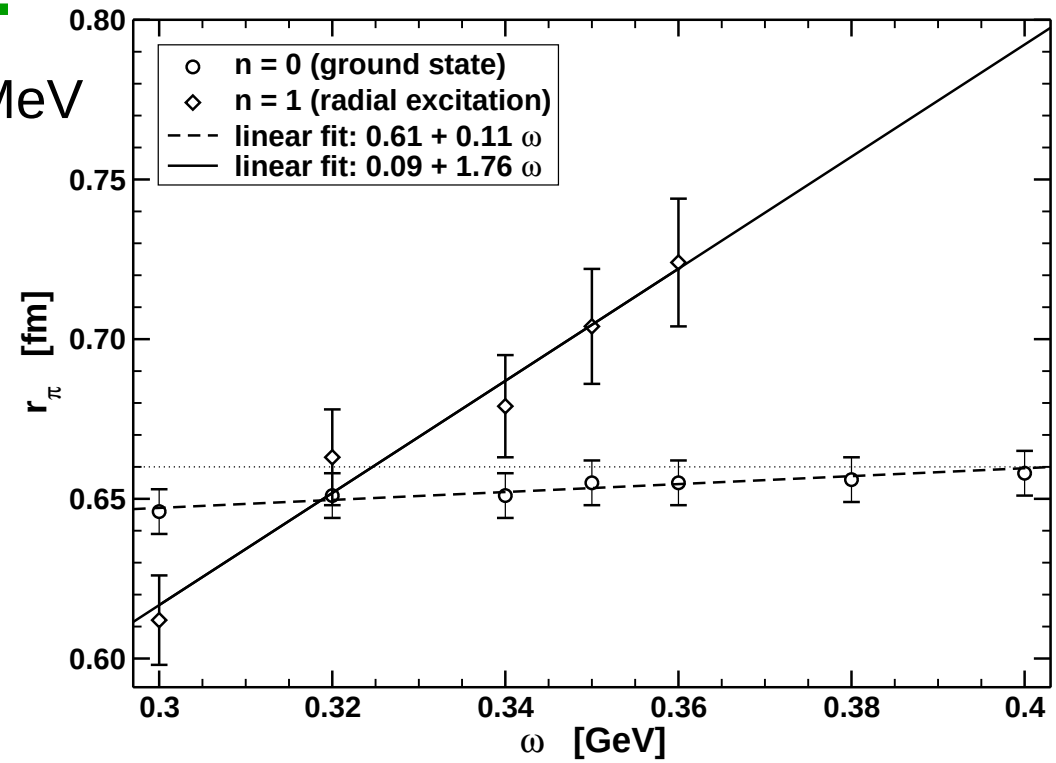
- Goldstone Mode's properties are insensitive to r_a
  - Expected cf. $T \neq 0$, Goldstone mode's properties do not change until very near chiral symmetry restoration temperature.



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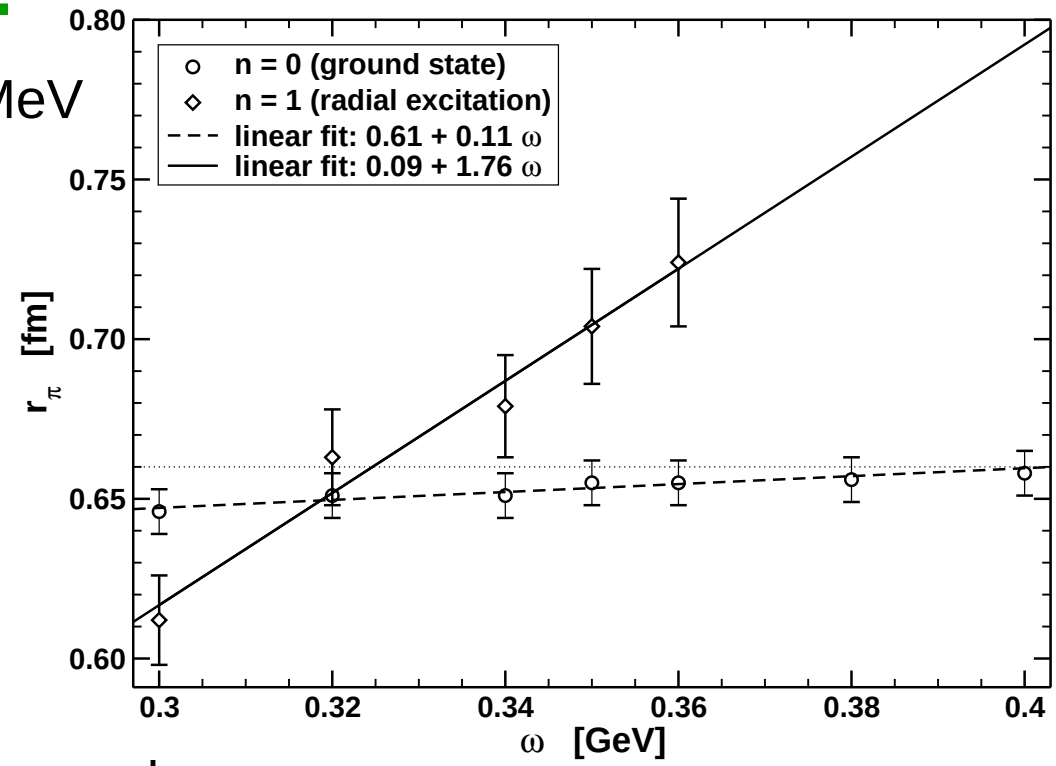
- 1st excited state: orthogonal to Goldstone mode
  - Not protected . . . properties very sensitive to r_a



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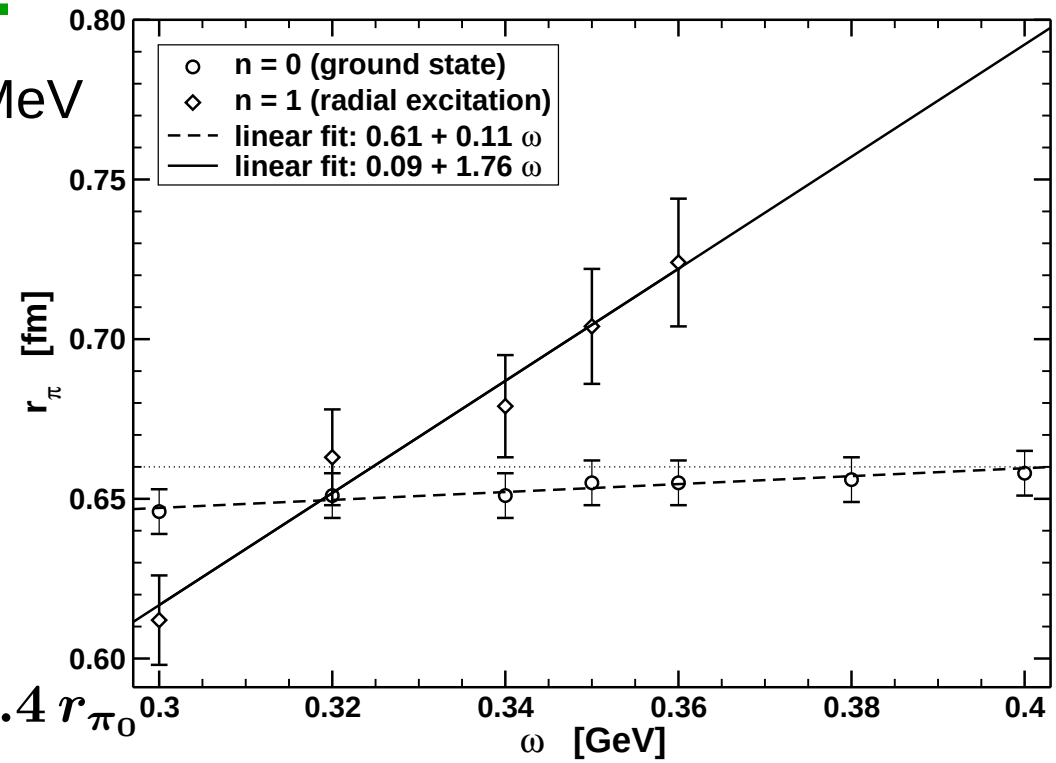
- Best estimate $r_{\pi_1} = 1.4\, r_{\pi_0}$
  - But $r_{\pi_1} < r_{\pi_0}$ is possible if confinement force is very strong



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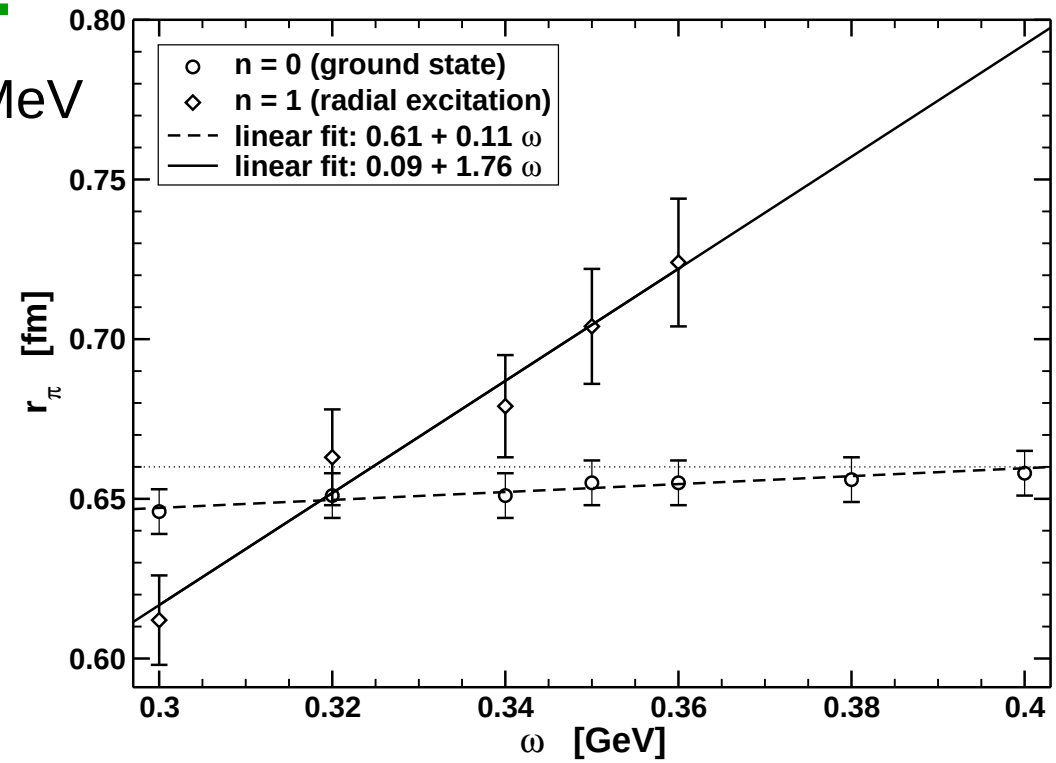

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- Radial excitations are plainly useful to map out the long-range part of interaction between light-quarks.

Electromagnetic Charge Radii – RGI Rainbow-Ladder

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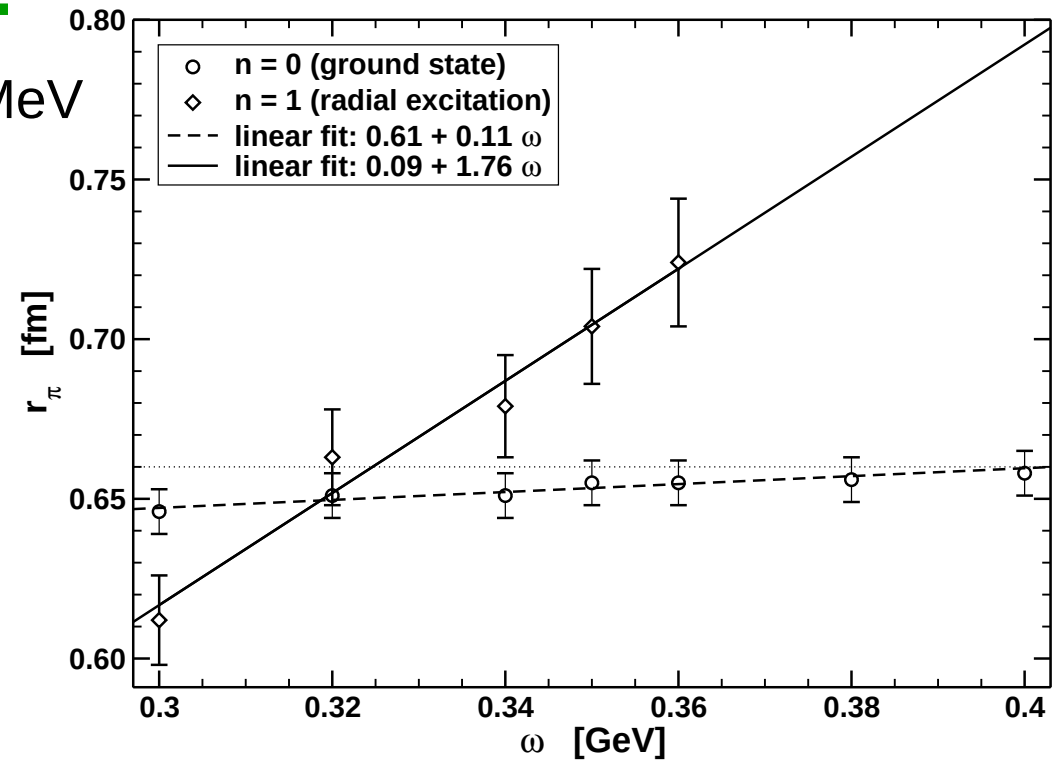

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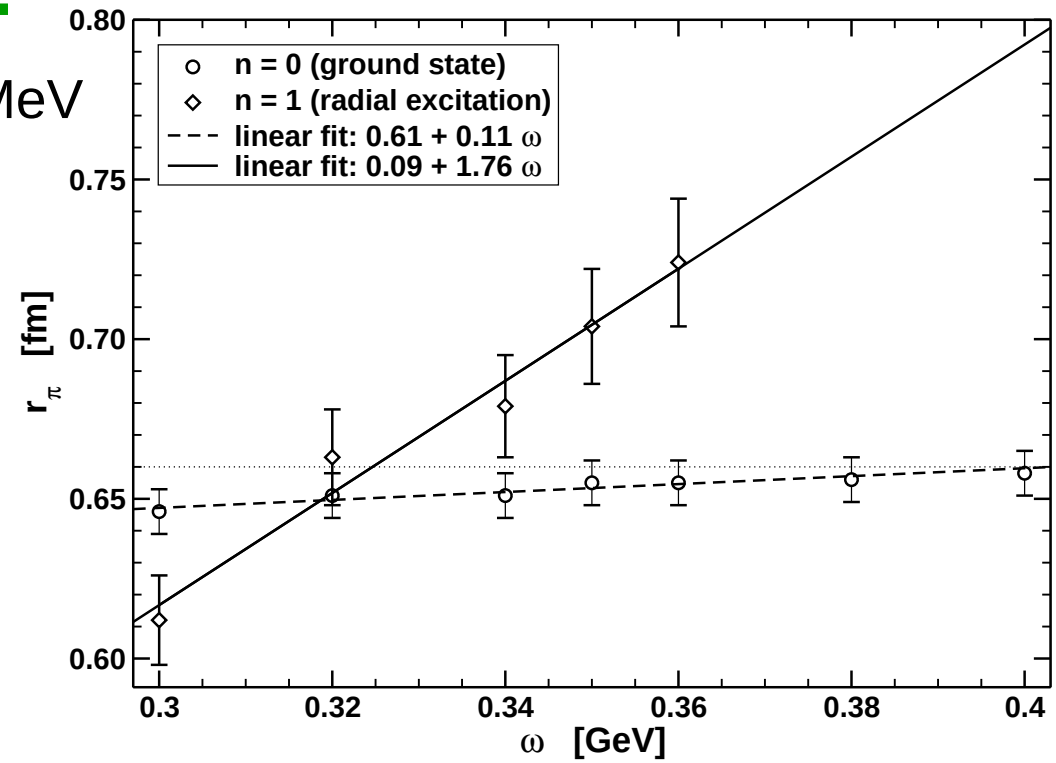

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- Hall-D at JLab

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- What about Baryons?

Current-quark mass-dependence of meson masses

Bhagwat, Höll, Krassnigg, *et al*., “Aspects and consequences of a dressed-quark-gluon vertex,” nu-th/0403012

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$m_{u,d} = 0.01$	$m_s = 0.166$	$m_c = 1.33$	$m_b = 4.62$
$m_\rho = 0.77$	$m_\phi = 1.02$	$m_{J/\psi} = 3.10$	$m_{\Upsilon(1S)} = 9.46$
$m_\pi = 0.14$	$m_{0^-_{s\bar{s}}} = 0.63$	$m_{\eta_c} = 2.97$	$m_{\eta_b} = 9.42$

- Fit current-quark masses to vector meson spectrum

Current-quark mass-dependence of meson masses

Bhagwat, Höll, Krassnigg, *et al.*, "Aspects and consequences of a dressed-quark-gluon vertex," nu-th/0403012

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- Fit current-quark masses to vector meson spectrum
- Predictions $m_{\eta_c} = 2.97$ and $m_{\eta_b} = 9.42$

  cf. $m_{\eta_c}^{\rm expt.} = 2.98$ and $m_{\eta_b}^{\rm expt.} = 9.30 \pm 0.03$

  NB. $m_{\Upsilon(1S)} - m_{\eta_b}^{\rm expt.}$
  - Calc.: 40 MeV
  - Expt.: 160 MeV, which is $>$ Expt. $m_{J/\psi} - m_{\eta_c}^{\rm expt.}$
  - Unlikely

Current-quark mass-dependence of meson masses

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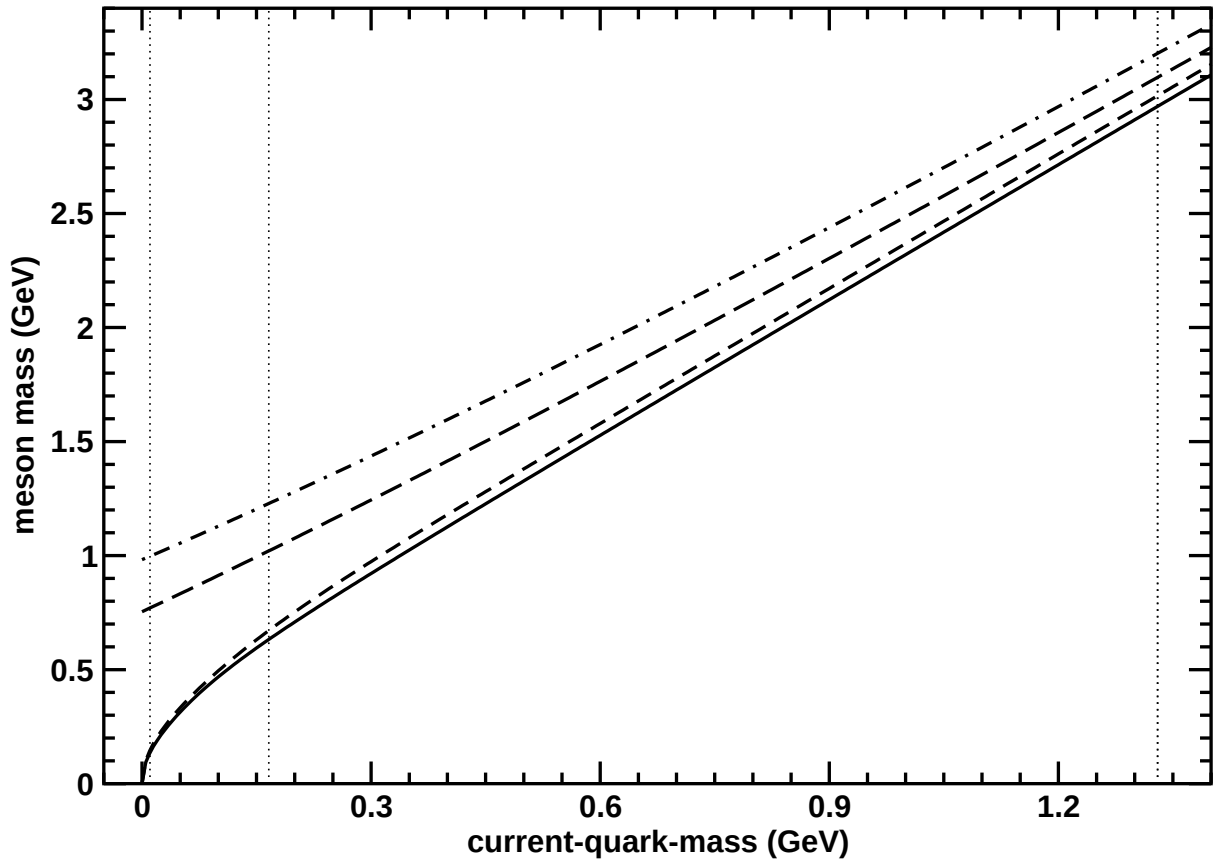
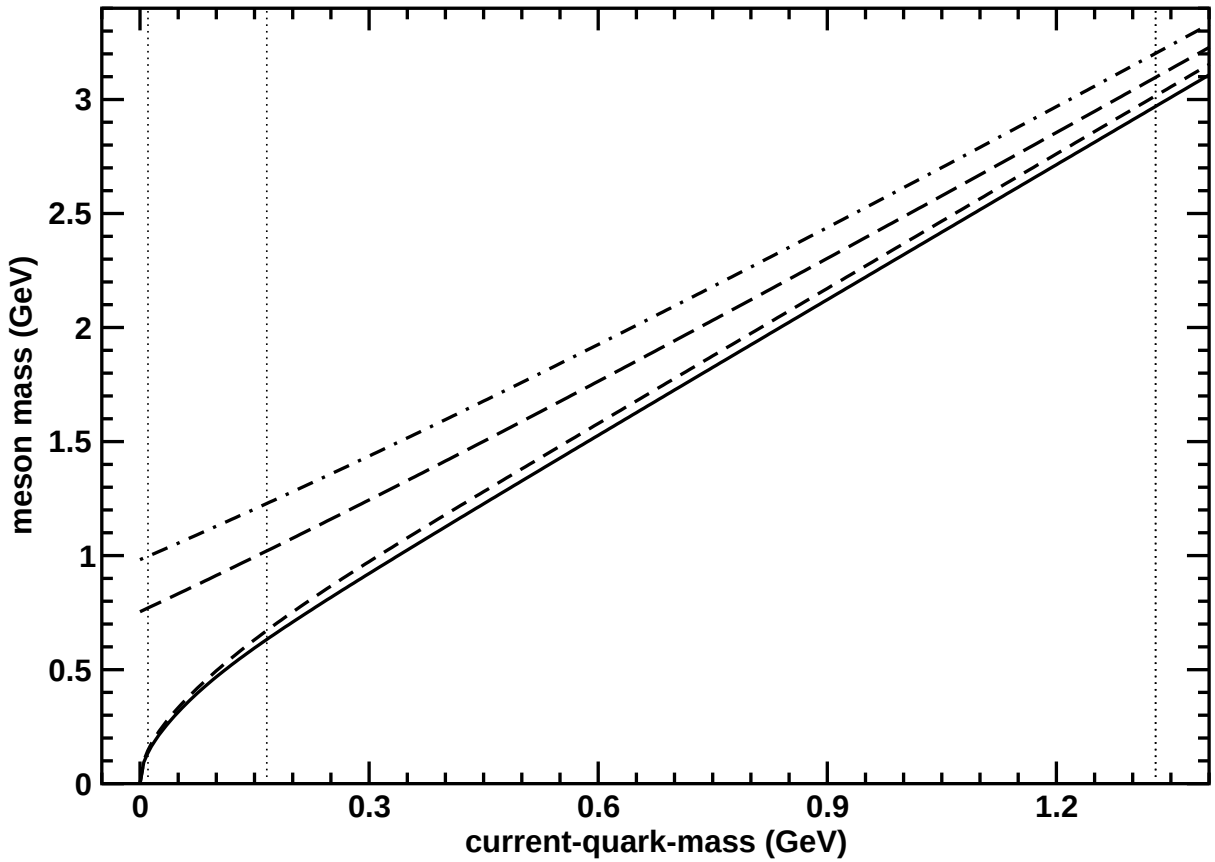


Current-quark mass-dependence of meson masses

Bhagwat, Höll, Krassnigg, *et al*., "Aspects and consequences of a dressed-quark-gluon vertex," nu-th/0403012

- For $m \ll \mathcal{G}$

  $m_{0^-}^2 = 1.33\, m$



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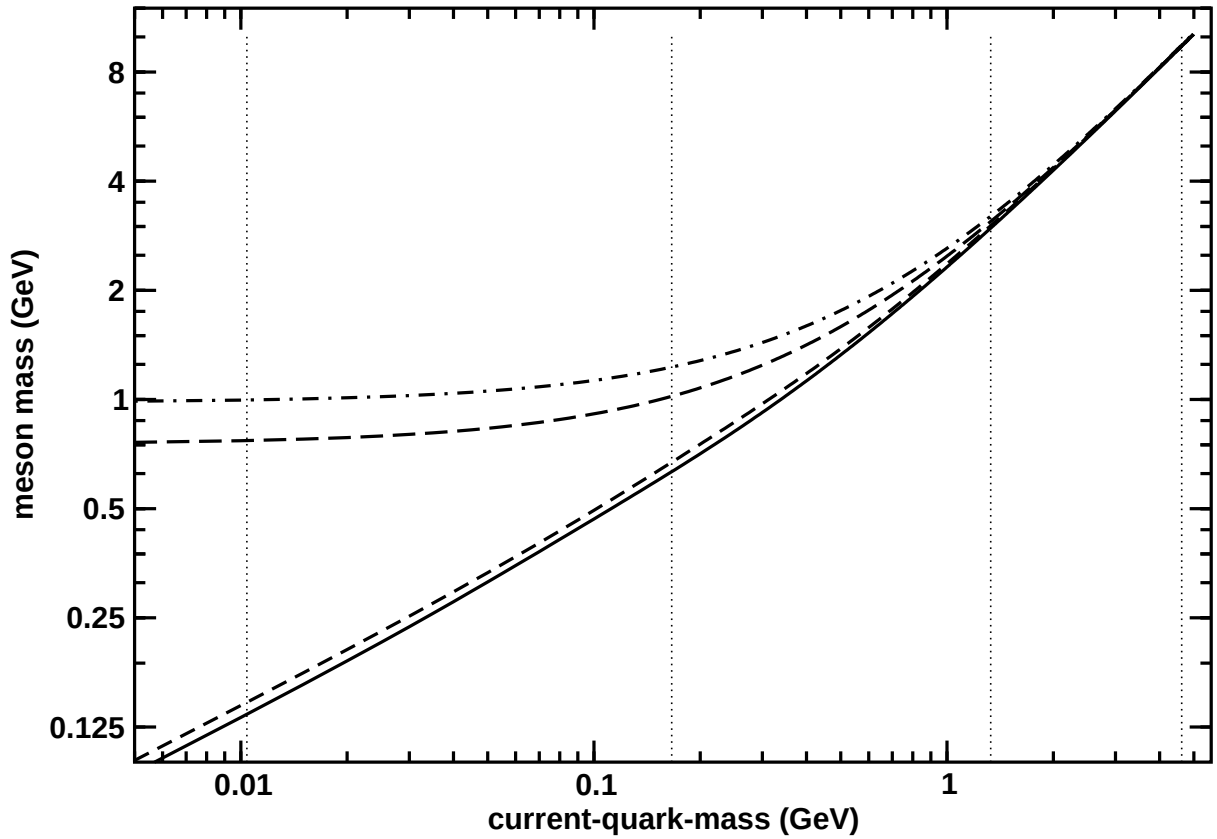

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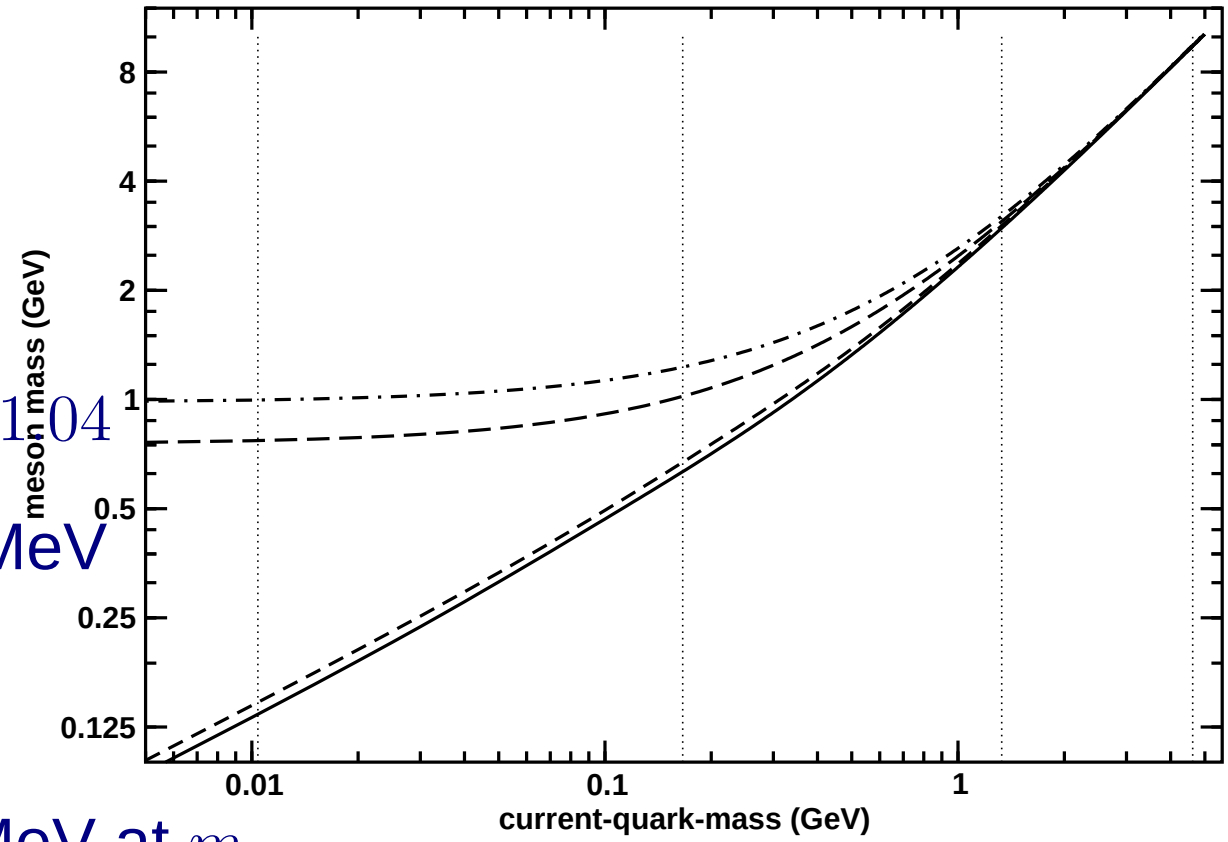
- For $m \gg \mathcal{G}$

  $m_{0^-} = m_{1^+}$

- $\left.\dfrac{m_{1^-}}{m_{0^-}}\right|_{m=m_c} = 1.04$

  splitting 130 MeV



- Drops to 40 MeV at m_b

  just 5% of its chiral limit value

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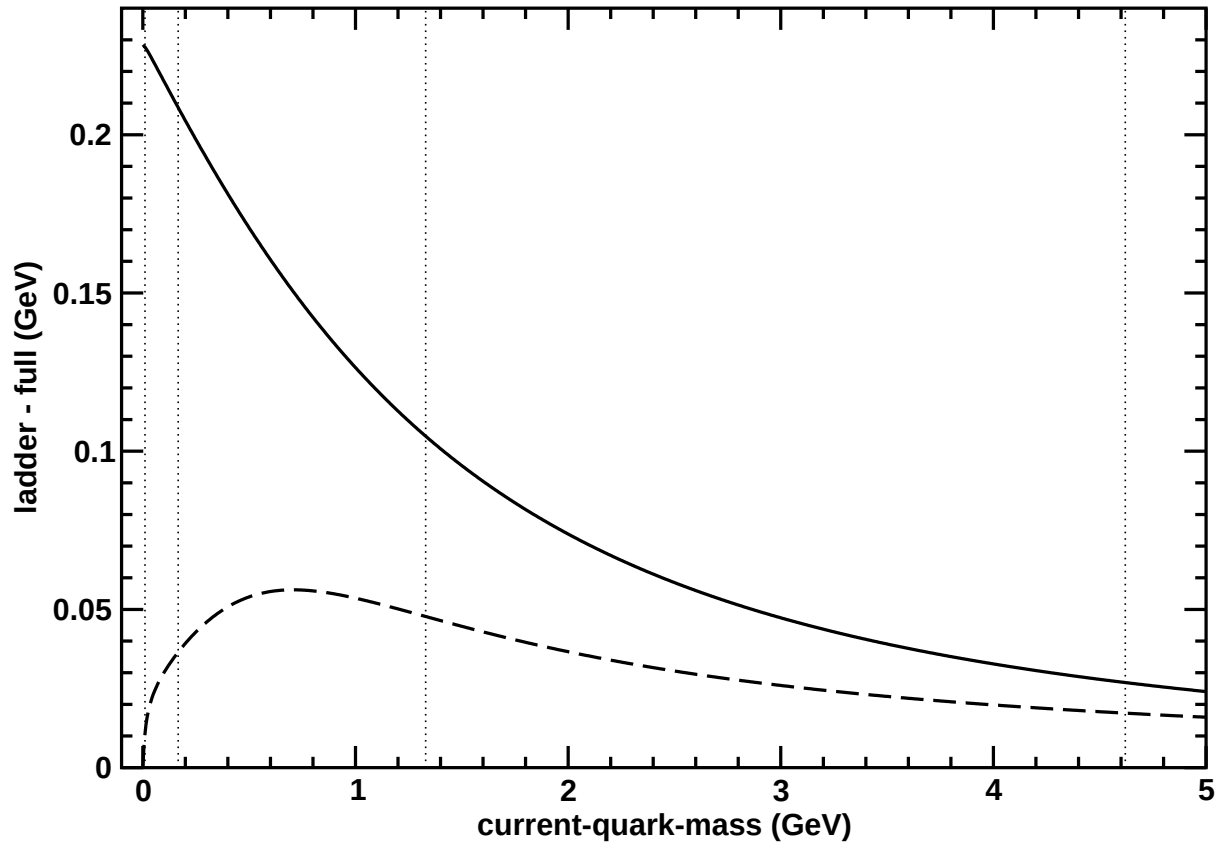

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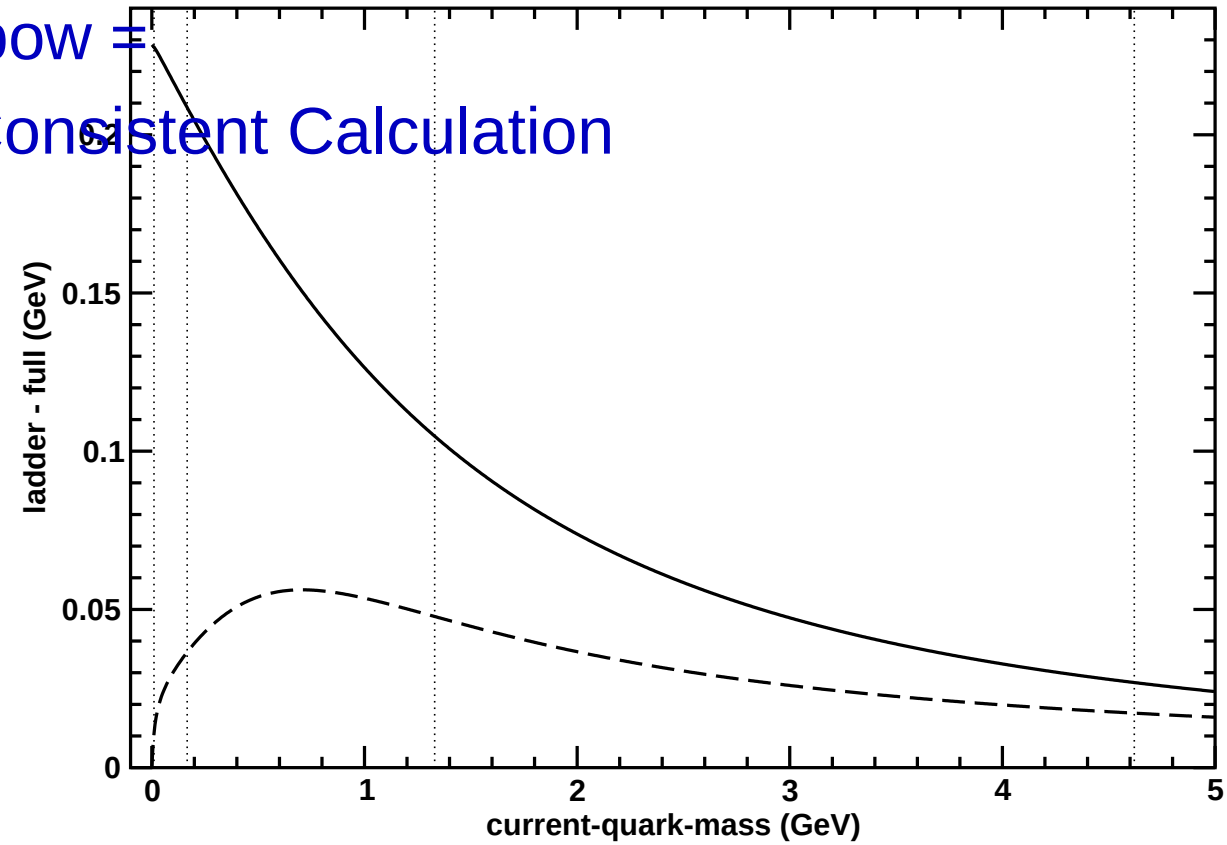

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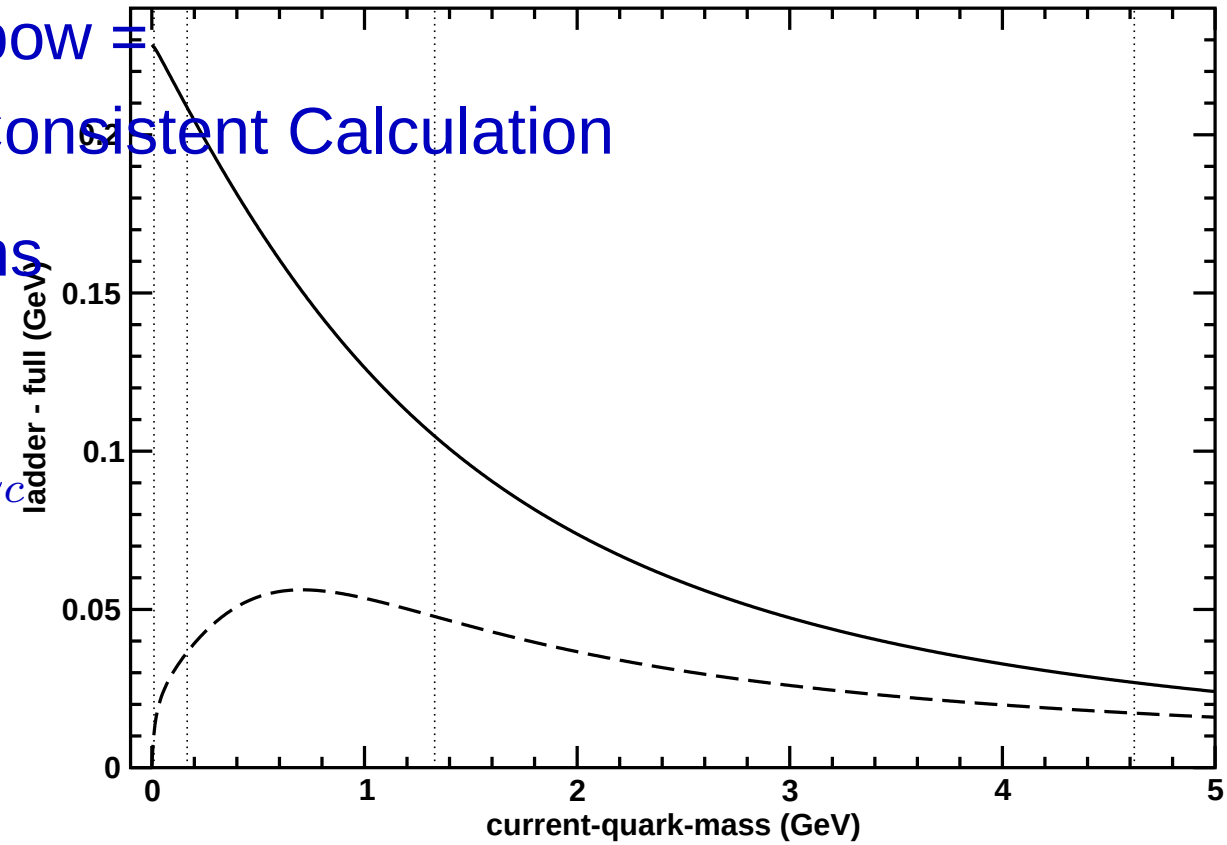

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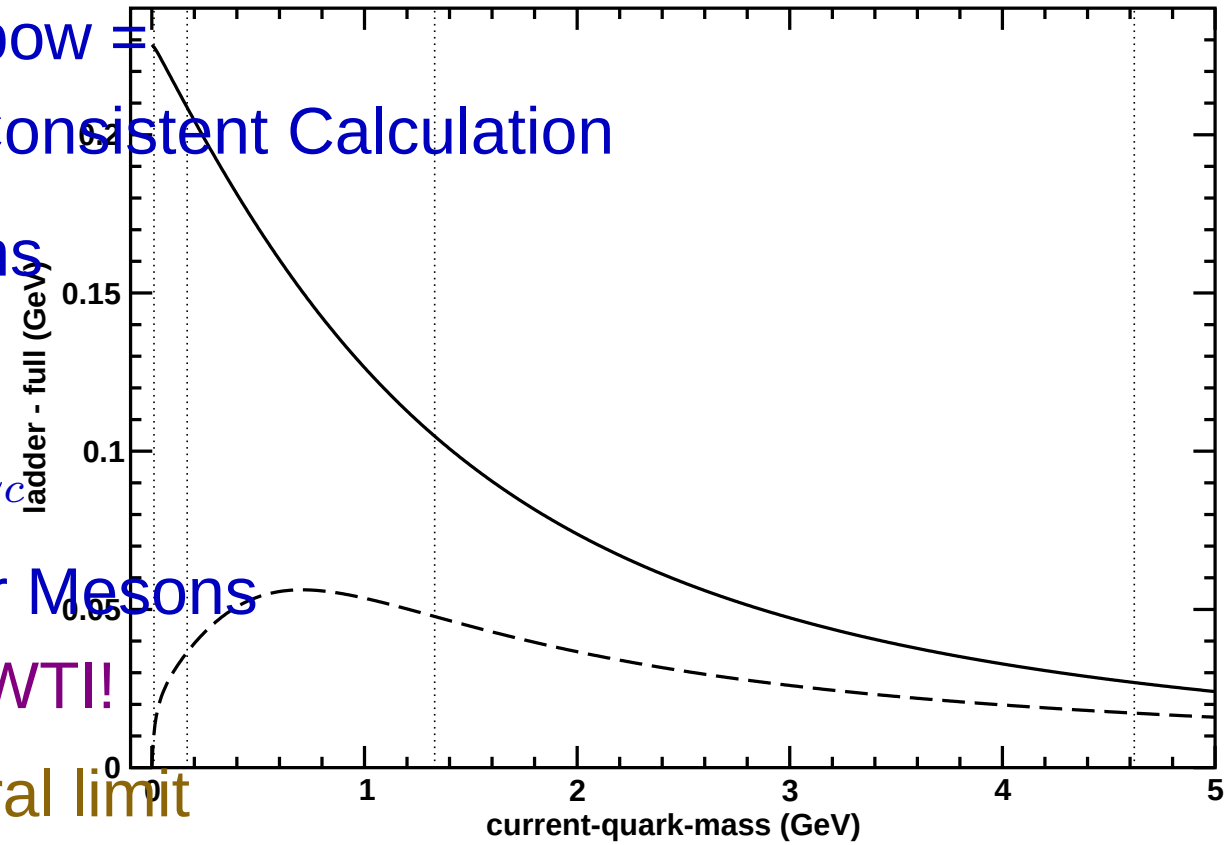

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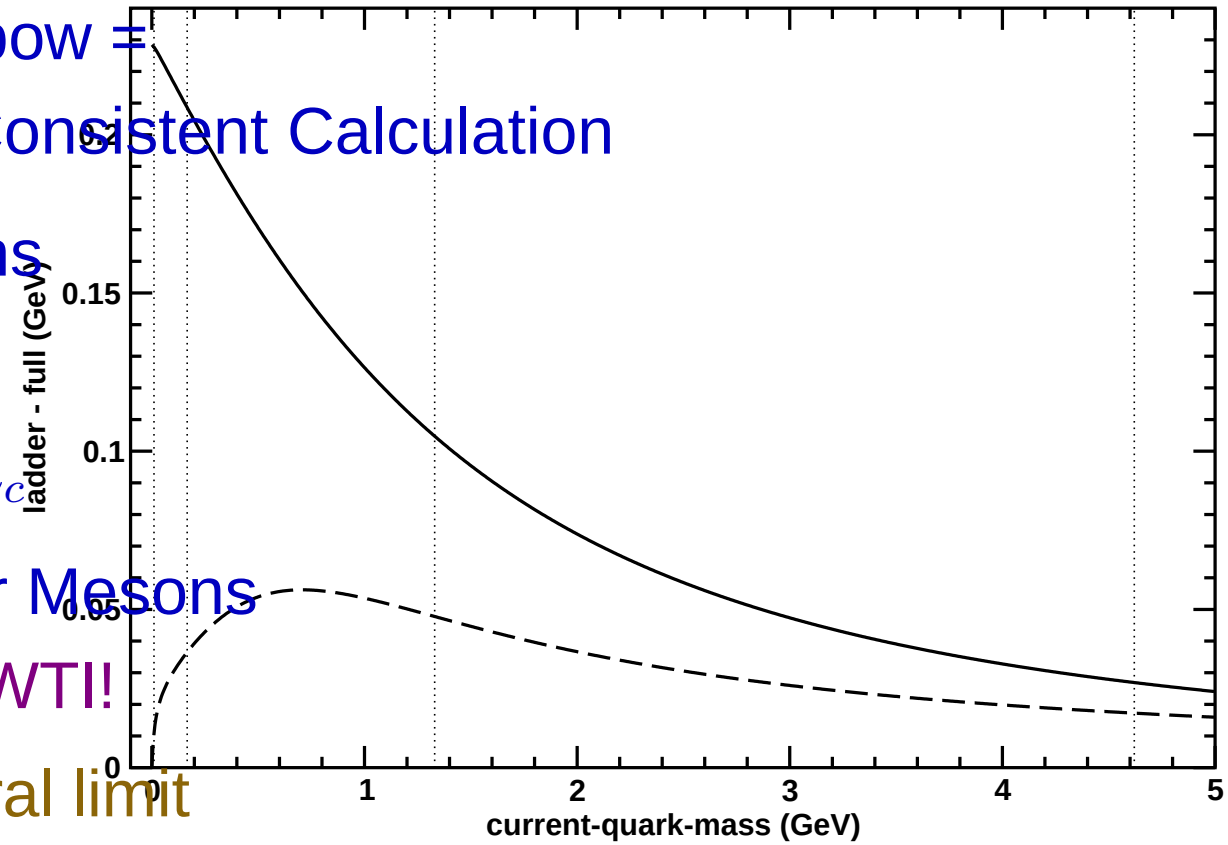

Explains Pattern of Maris-Tandy Success

Deep-inelastic scattering

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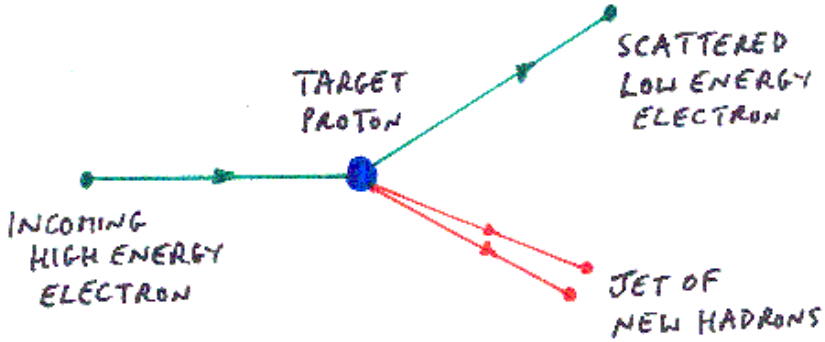
- Looking for Quarks

Deep-inelastic scattering



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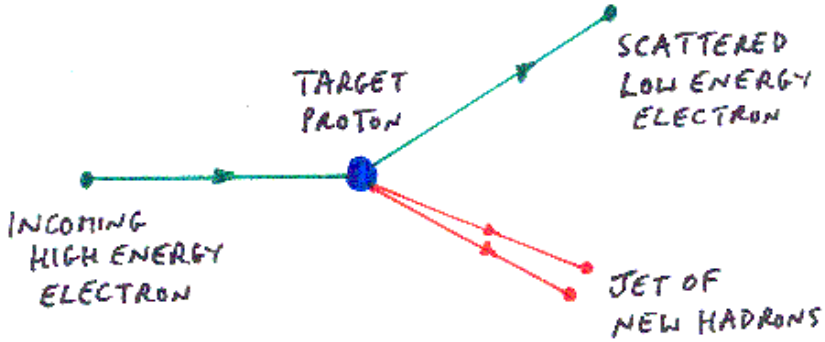

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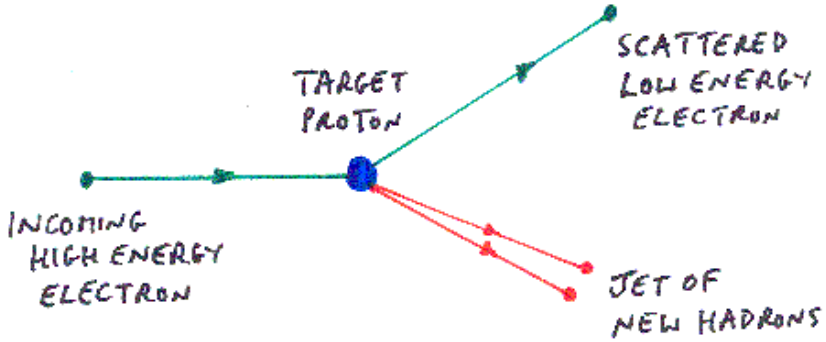

- Looking for Quarks

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- Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x)$, $g(x)$

Pion's valence quark distn

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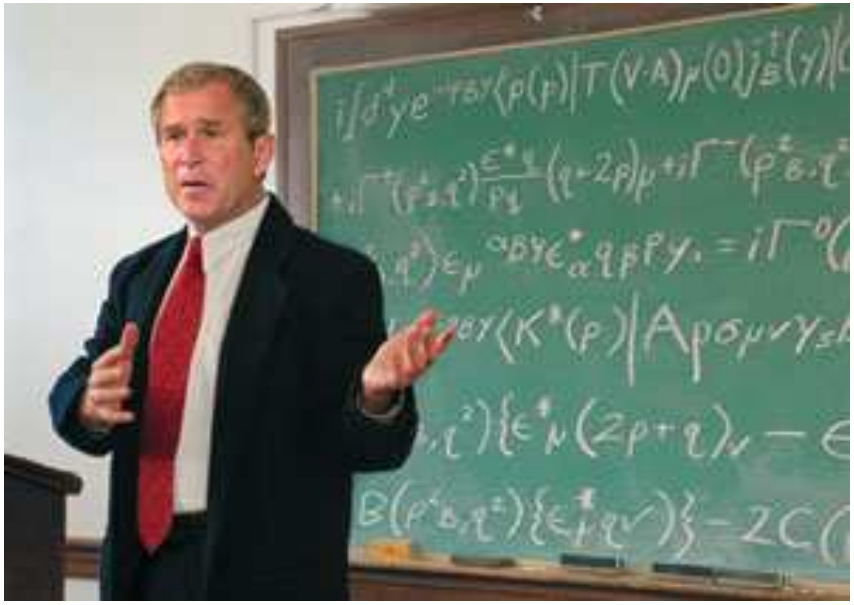

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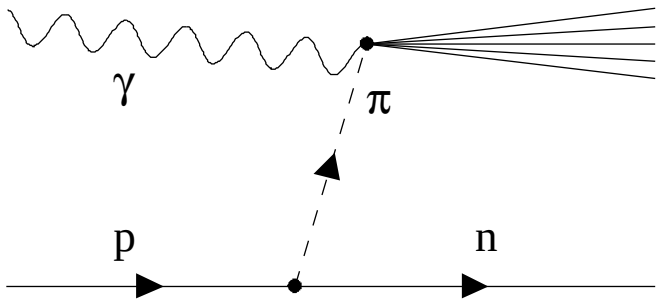
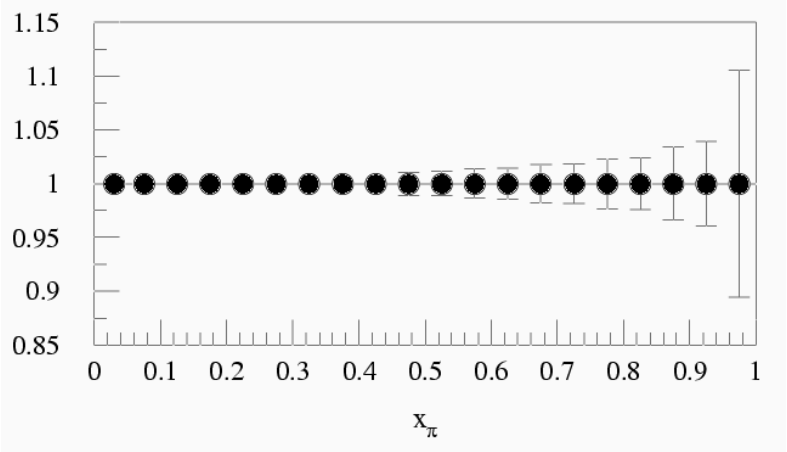
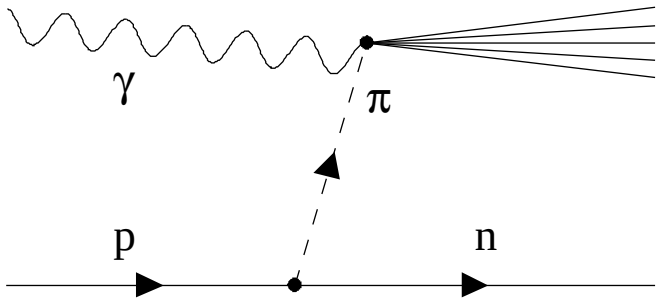
$e^-_{5\mathrm{GeV}} - p_{25\,\mathrm{GeV}}$ Collider $\to$ Accurate "Measurement"

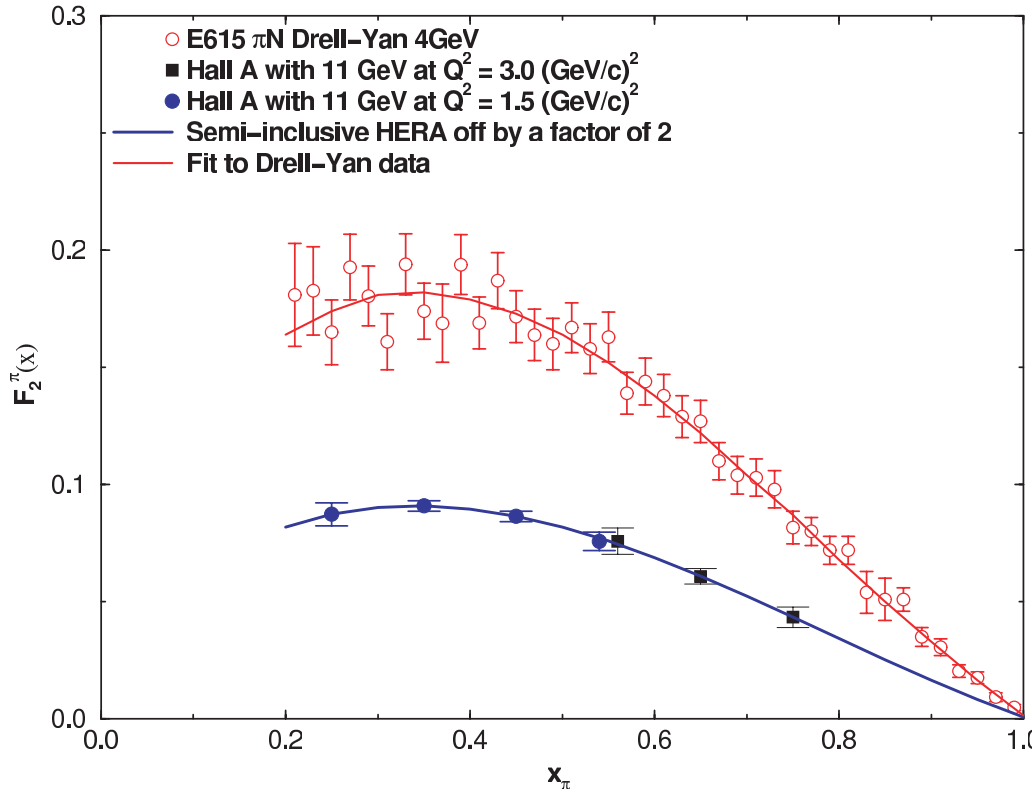




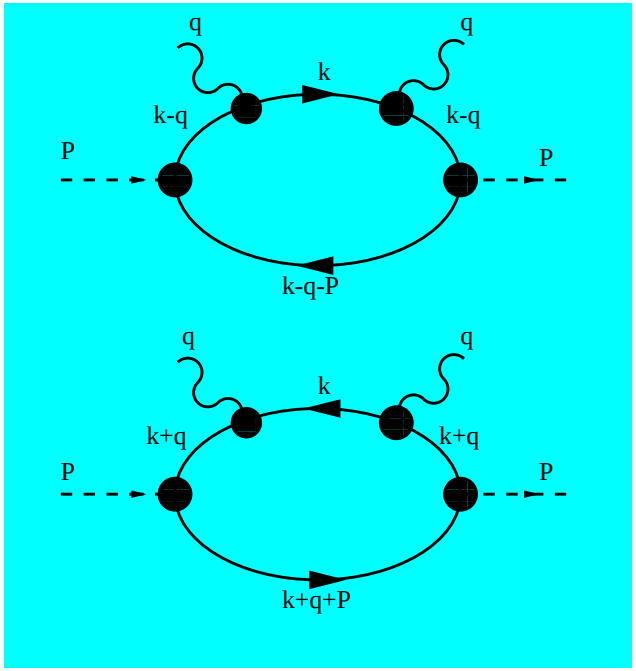
Pion's valence quark distn

- Proposal at JLab
  (Holt, Reimer, Wijesooriya, et al., JLab at 12 GeV)





Handbag diagrams



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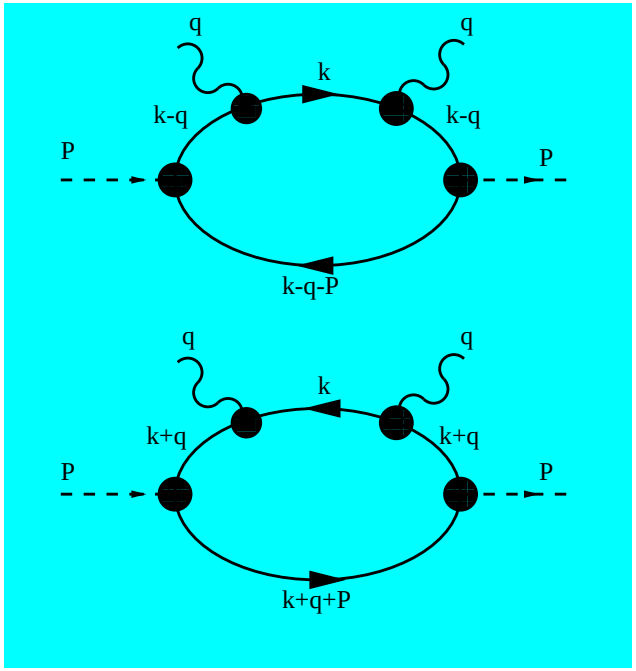

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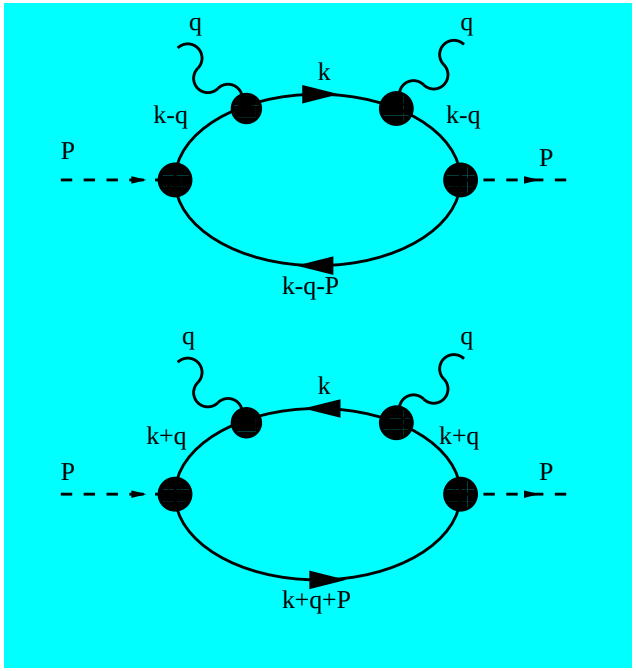
Handbag diagrams

Bjorken Limit: $q^2 \to \infty\,, \quad P \cdot q \to -\infty$

but $x := -\dfrac{q^2}{2P \cdot q}$ fixed.

Numerous algebraic simplifications



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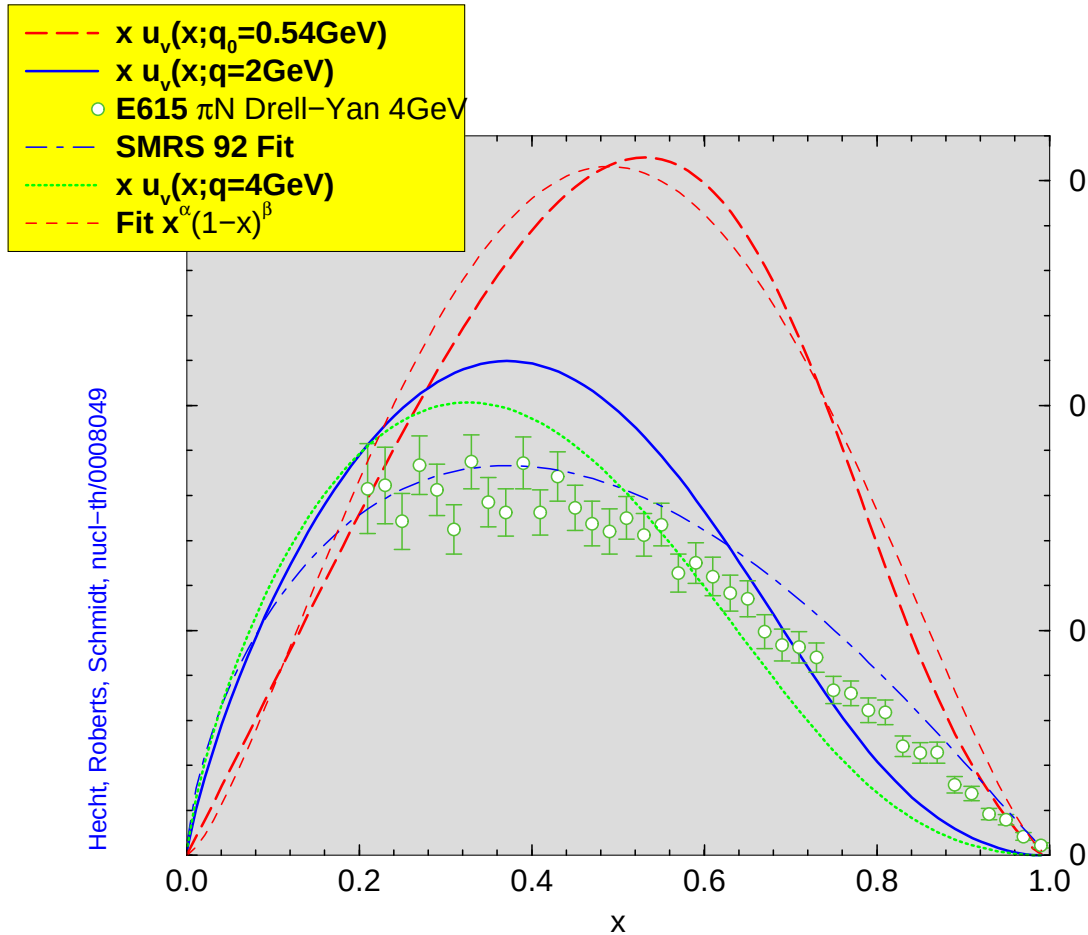

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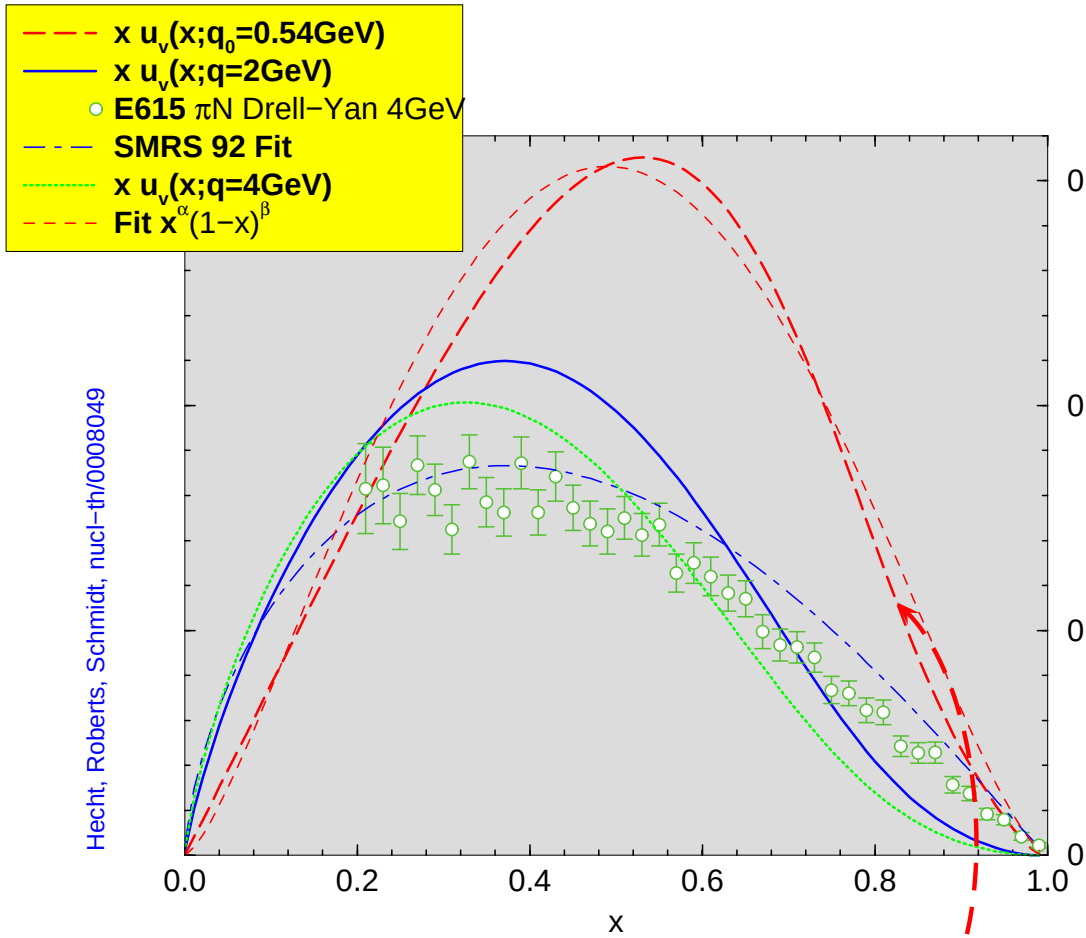
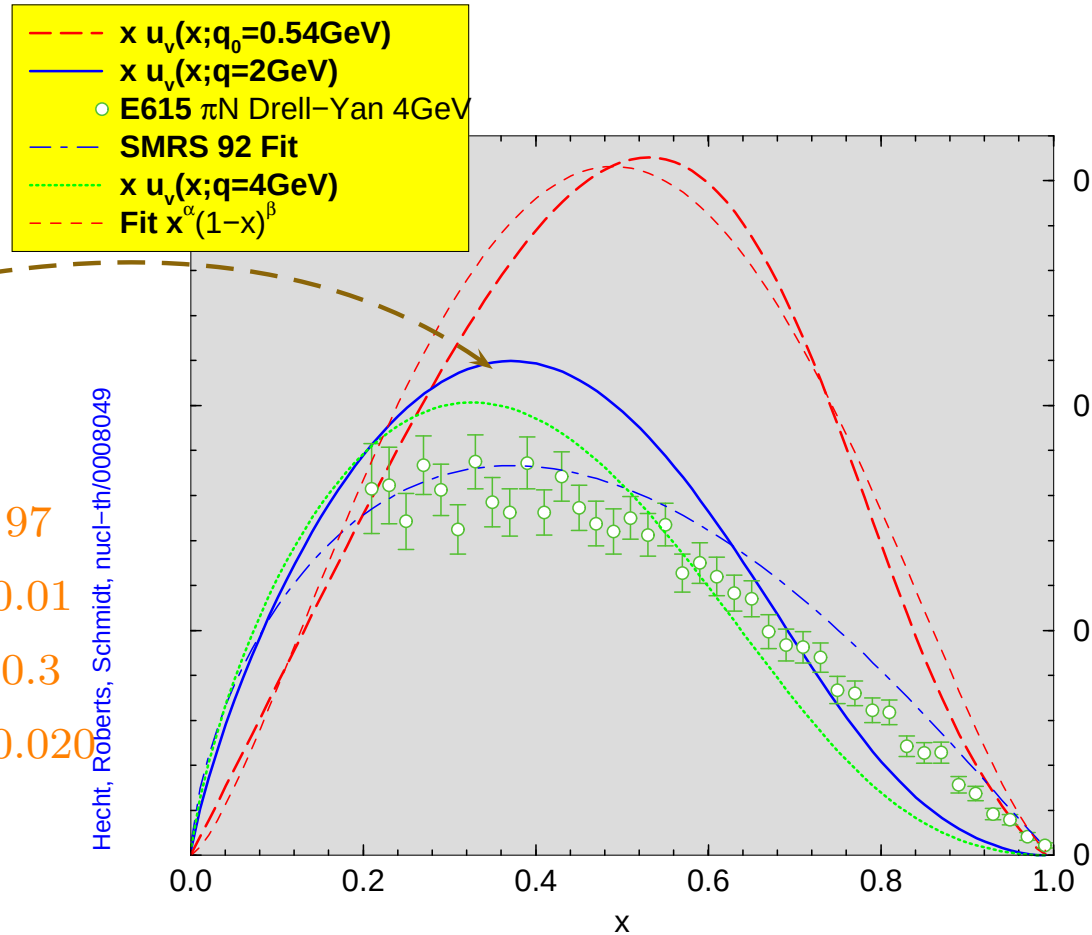
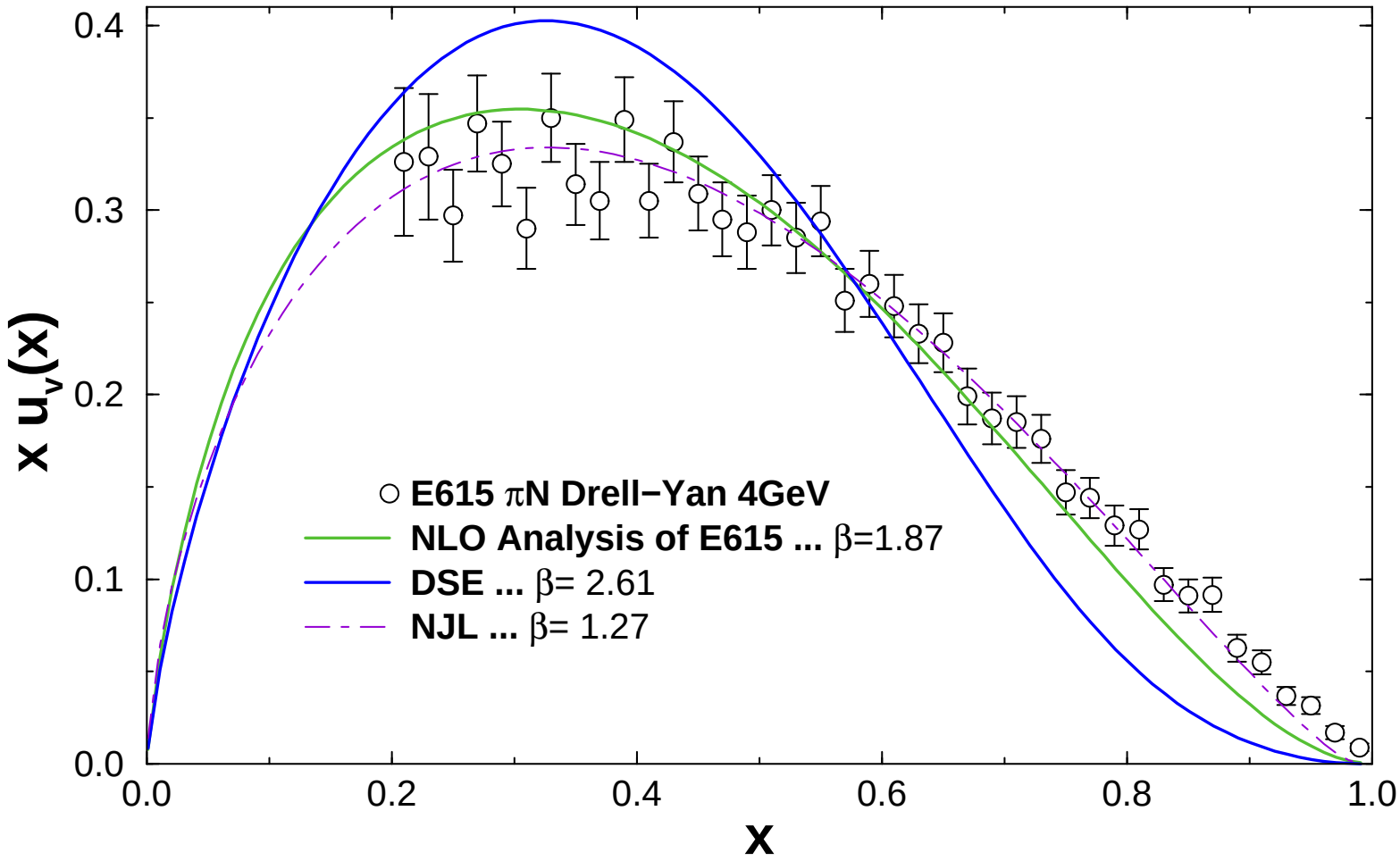


Resolving Scale: $q_0 = 0.54\,\mathrm{GeV} = 1/(0.37\,\mathrm{fm})$

Calc. $u_V(x)$ cf. Drell-Yan data

$q =$ 2 GeV	Calc.	Fit, 92	Latt., 97
$\langle x \rangle_q$	0.24	0.24 ± 0.01	0.27 ± 0.01
$\langle x^2 \rangle_q$	0.10	0.10 ± 0.01	0.11 ± 0.3
$\langle x^3 \rangle_q$	0.050	0.058 ± 0.004	0.048 ± 0.020



Extant theory vs. experiment

K. Wijersooriya, P. Reimer and R. Holt, nu-ex/0509012 ... Phys. Rev. C (Rapid)