# *Aspects of Hadron Physics*


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**http://www.phy.anl.gov/theory/staff/cdr.html**

# Quarks and Nuclear Physics

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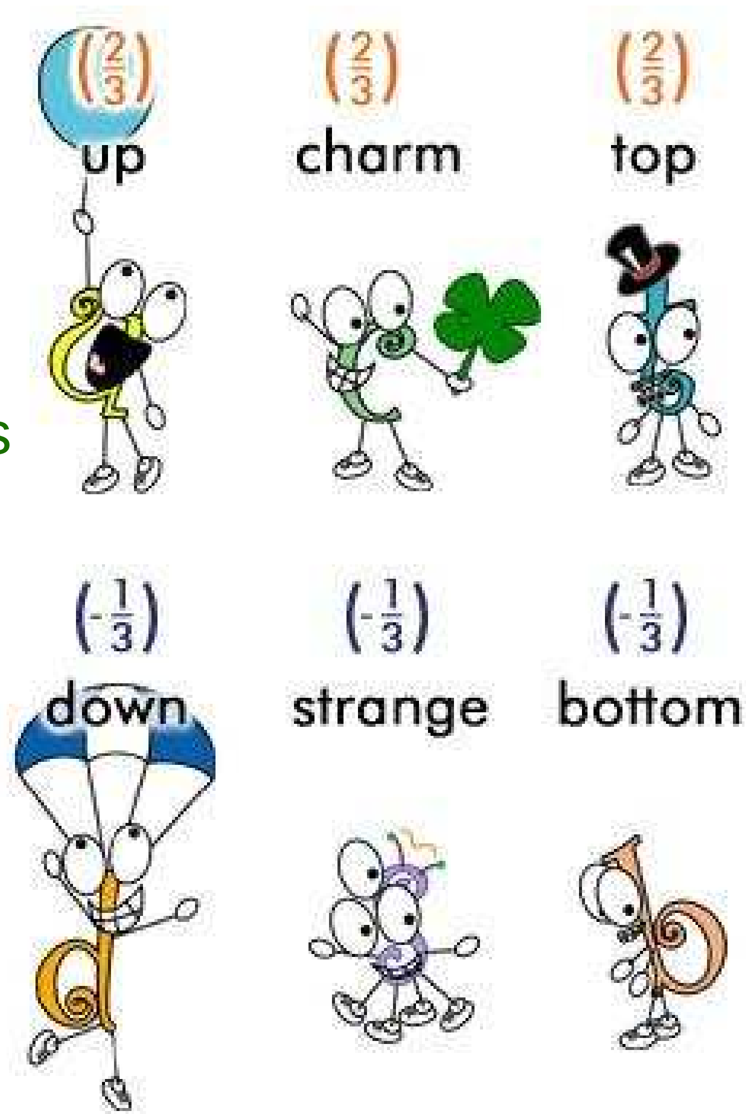
Standard Model
of Particle Physics
Six Flavours



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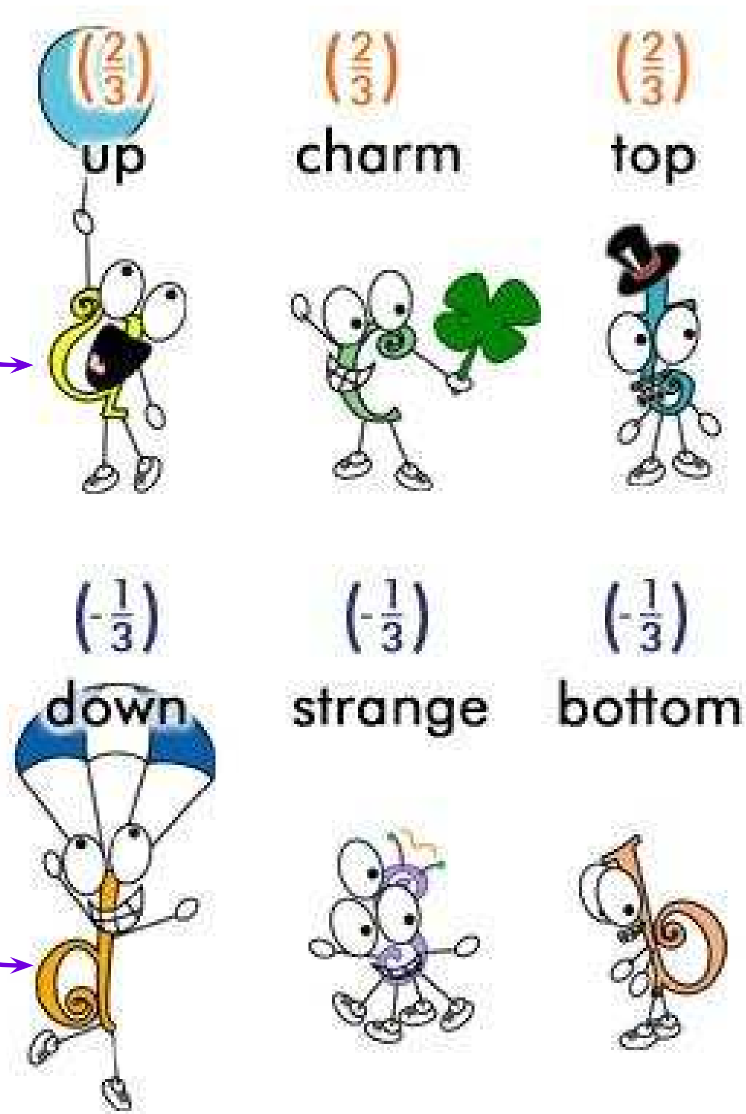
Real World

Normal Matter . . .

Only Two Light

Flavours Active

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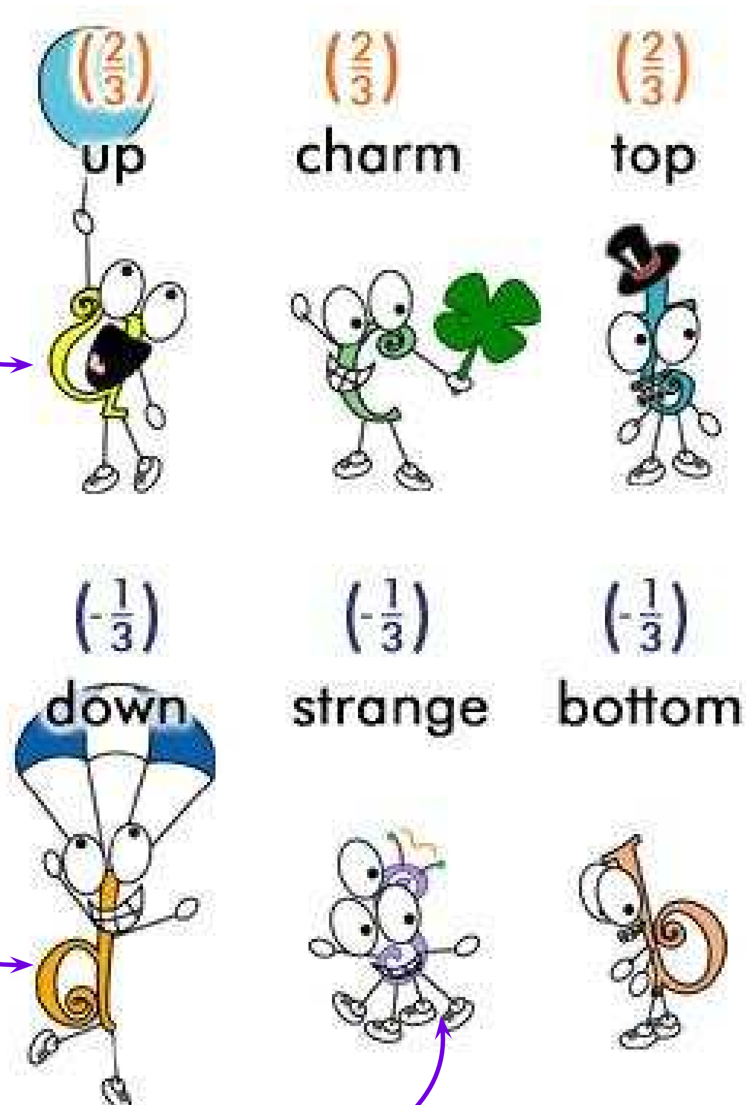

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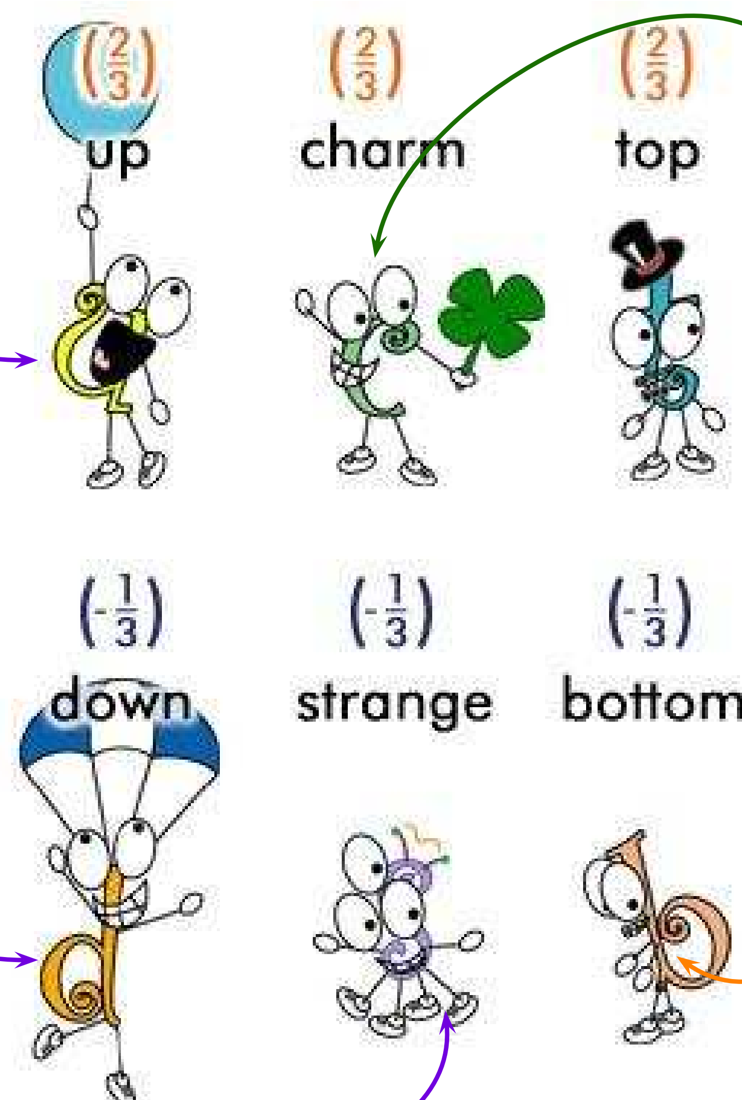
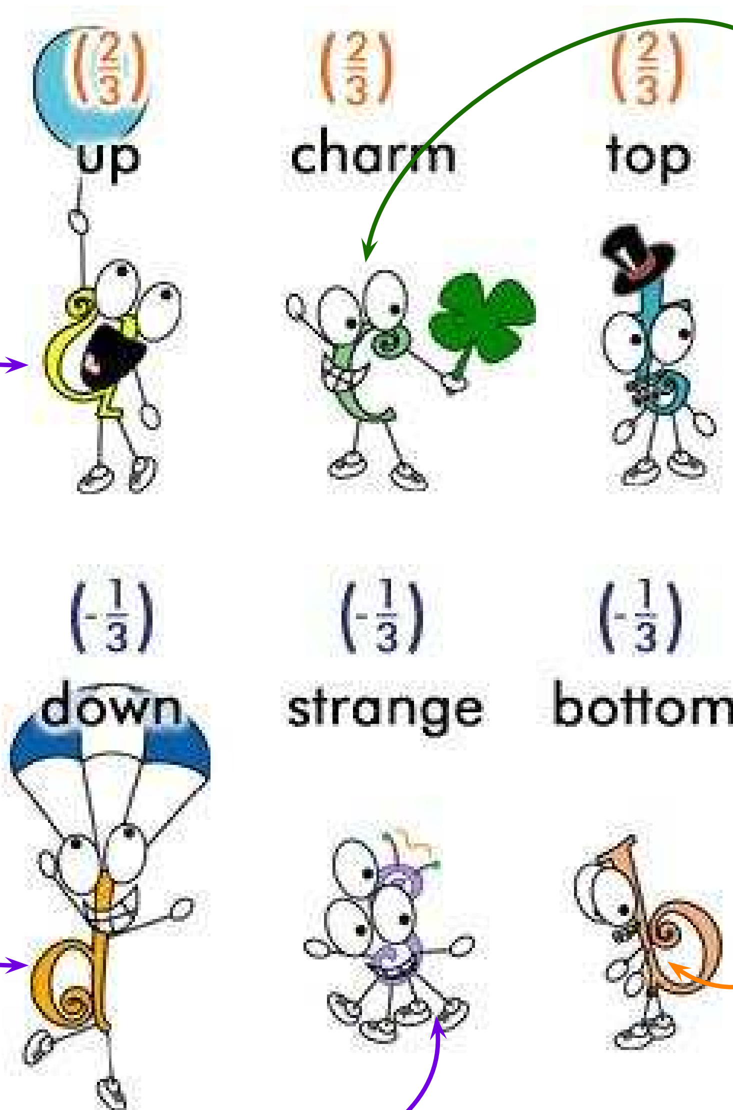



For numerous good reasons, much research also focuses on accessible heavy-quarks

# *Quarks and Nuclear Physics*

Nevertheless, I will focus primarily on the light-quarks.

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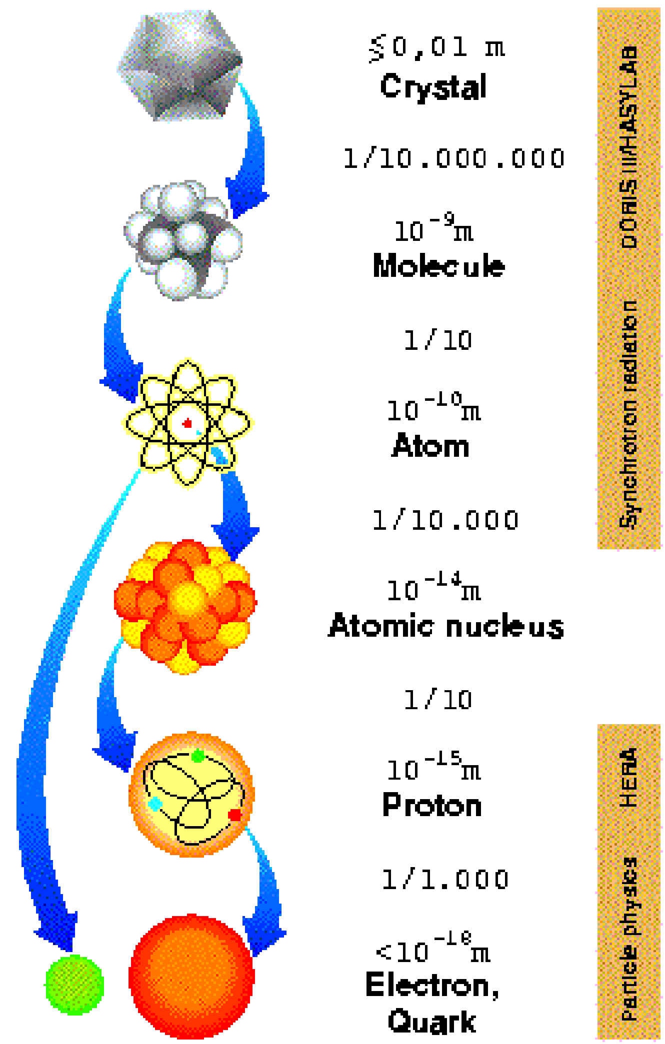
# *Scales in Modern Physics*

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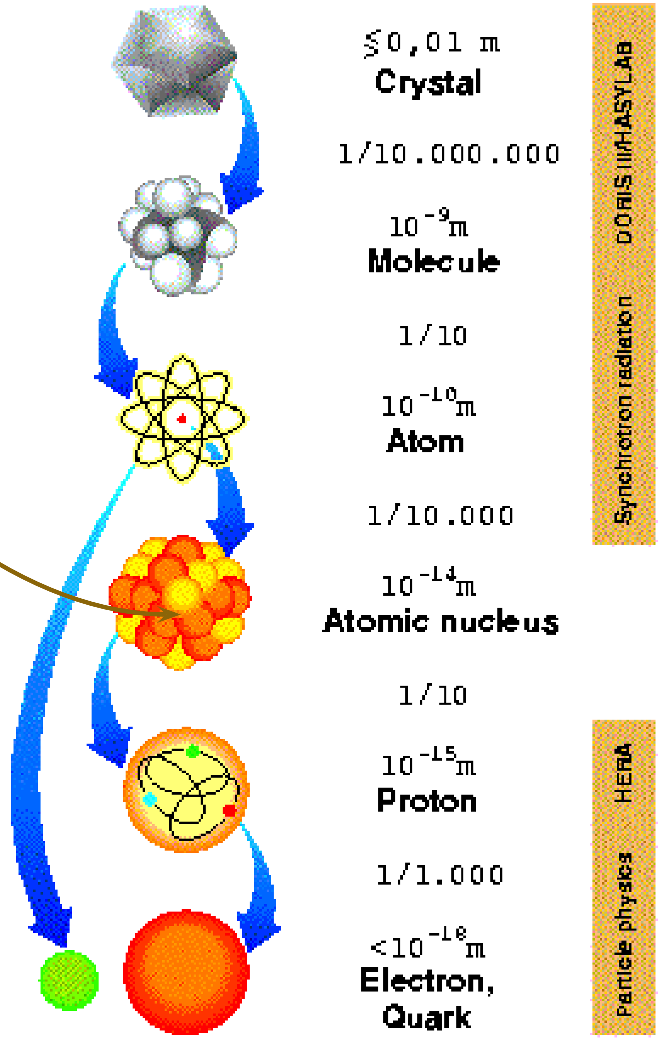


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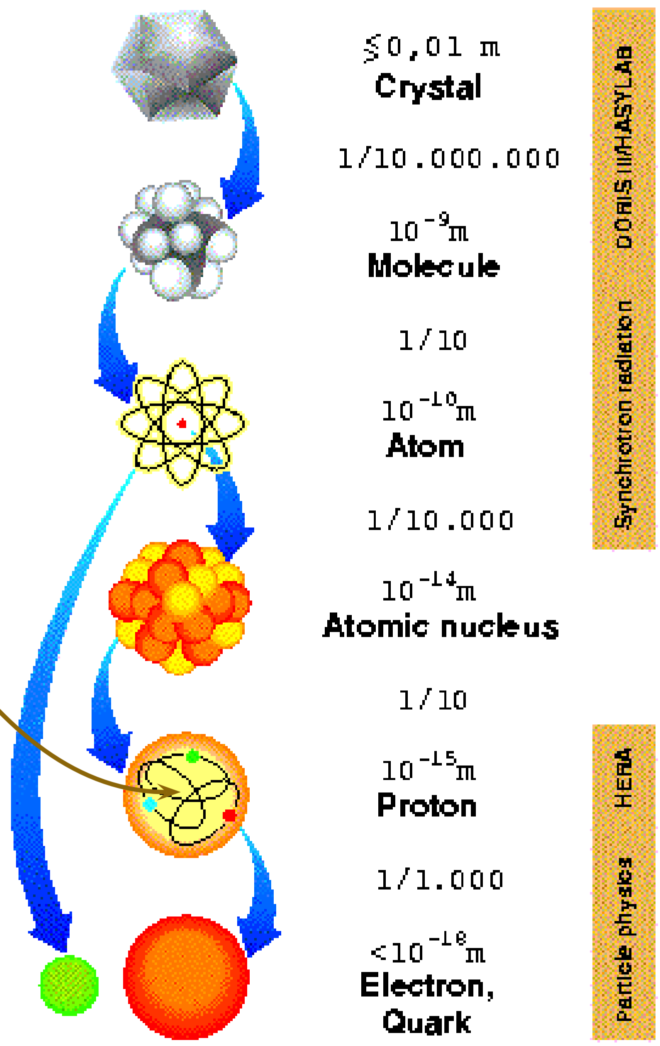


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# Nucleon = Proton and Neutron

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- Fermions – two static properties:
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- Stern (1933) – $\mu_p = (1 + 1.79)\frac{e\hbar}{2M}$
  - Big Hint that Proton is not a point particle
  - Proton has constituents
  - These are Quarks and Gluons
    – the elementary quanta of Quantum Chromo-dynamics

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# Nucleon Form Factors

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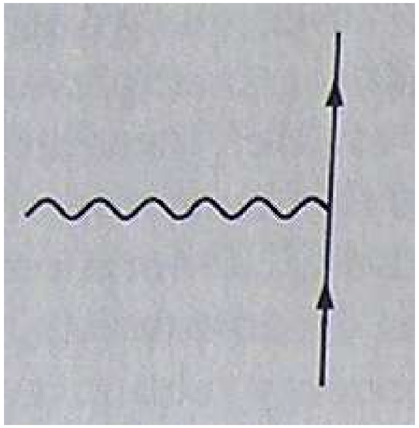
- Electron's relativistic electromagnetic current:

$$\begin{aligned} j_\mu(P',P) &= ie\,\bar{u}_e(P')\,\Lambda_\mu(Q,P)\,u_e(P)\,,\;Q = P' - P \\ &= ie\,\bar{u}_e(P')\,\gamma_\mu(-1)\,u_e(P) \end{aligned}$$



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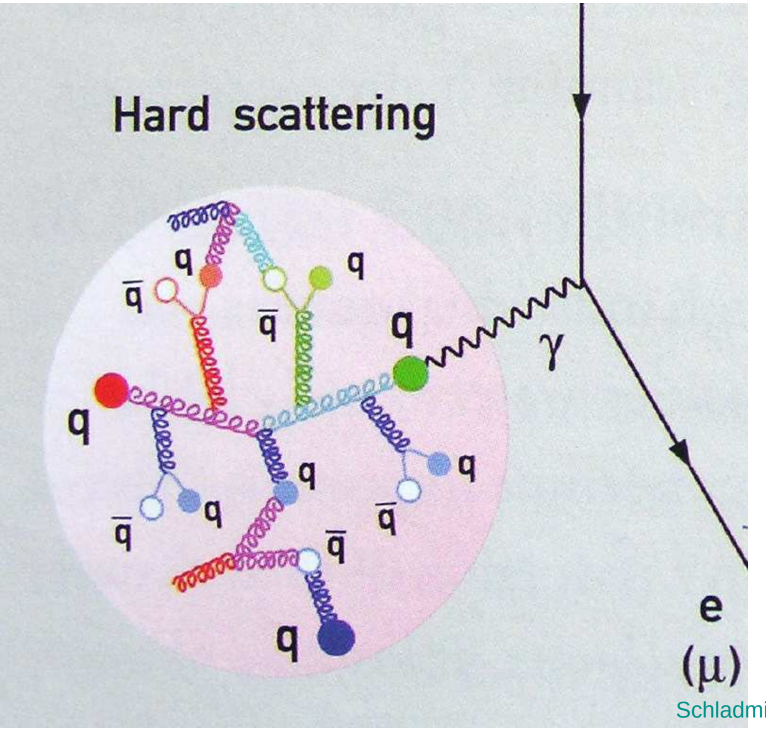

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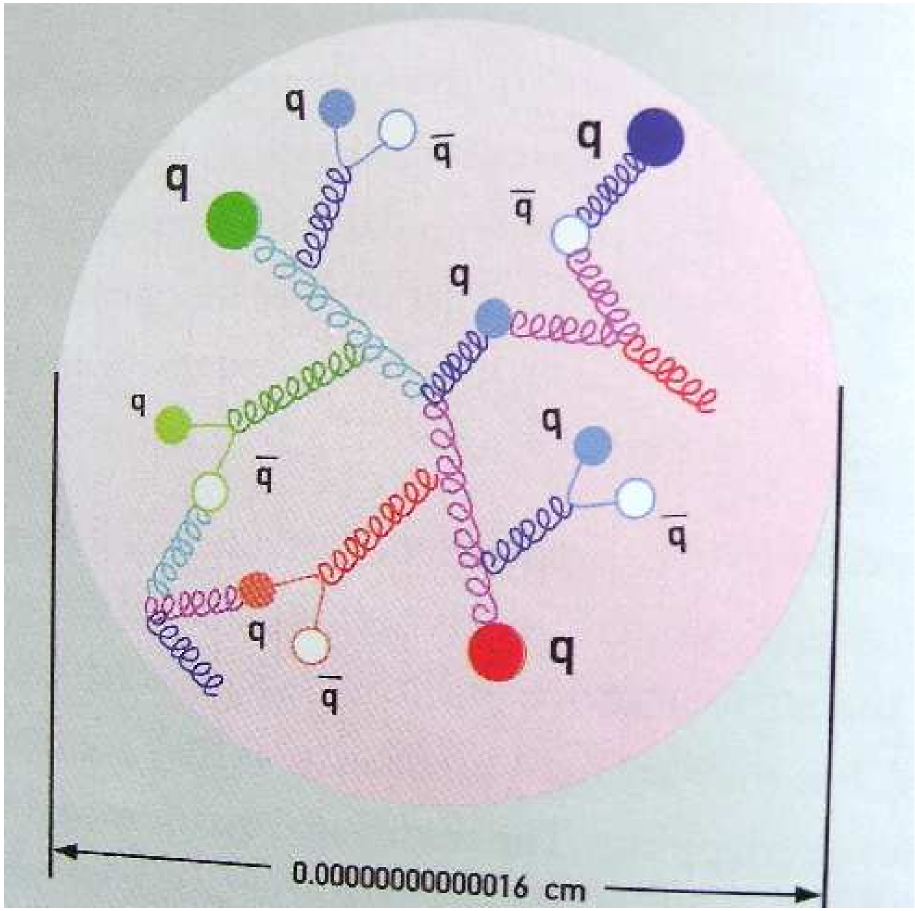
Point-particle: $\boldsymbol{F_2 \equiv 0} \Rightarrow \boldsymbol{G_E \equiv G_M}$

# NSAC Long Range Plan

*A central goal of nuclear physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD*

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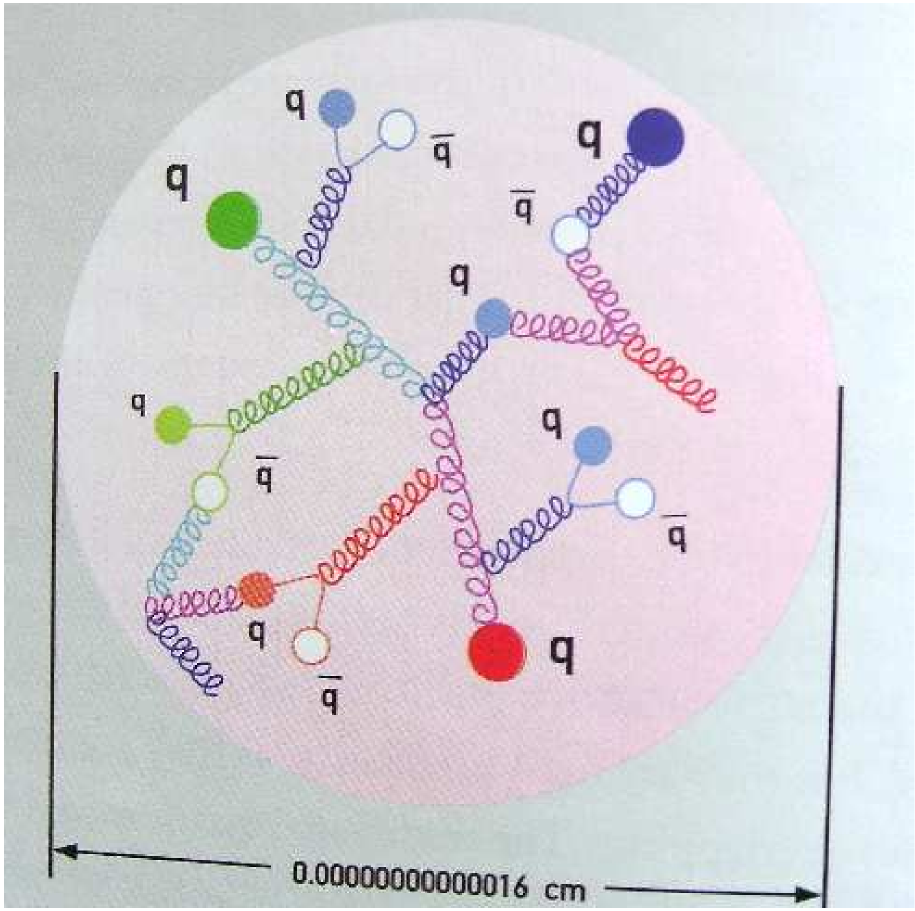

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- Confinement
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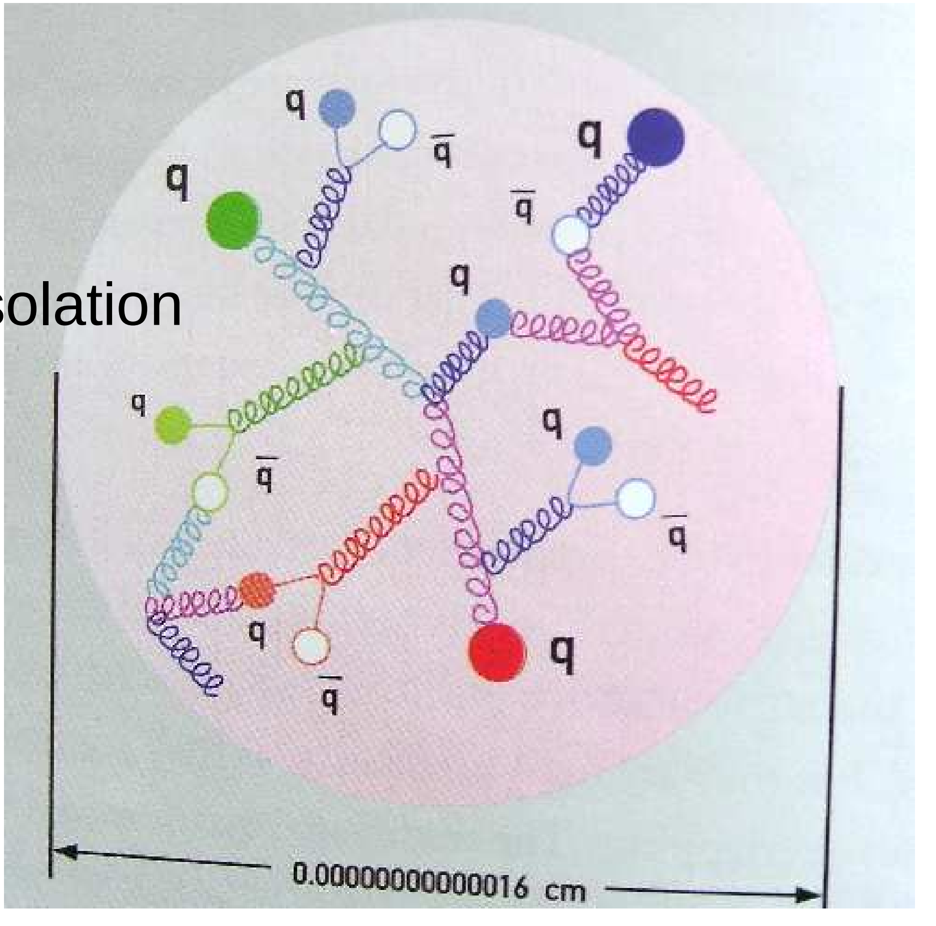

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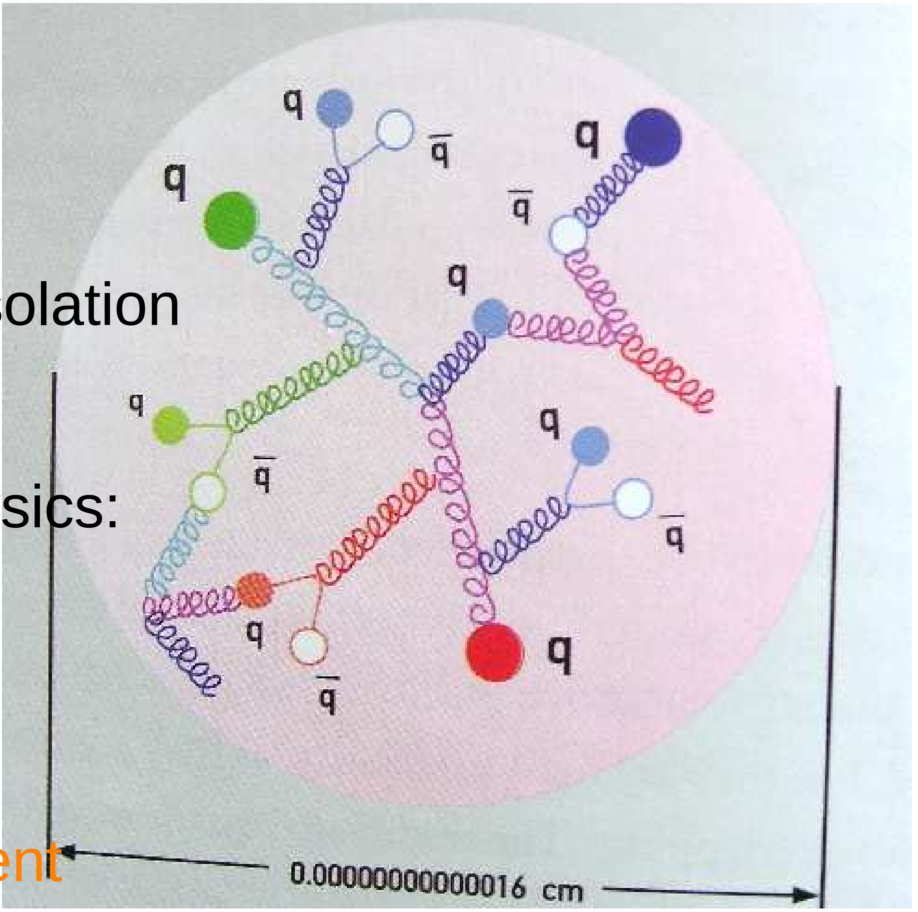


- Confinement
  – No quark ever seen in isolation
- Weightlessness
  – 2004 Nobel Prize in Physics:
  Mass of $u-$ & $d-$quarks,
  each just 5 MeV;
  Proton Mass is 940 MeV
  $\Rightarrow$ No Explanation Apparent
  for 98.4 % of Mass

# Meson Spectrum

| LIGHT UNFLAVORED ($S = C = B = 0$) | | | | STRANGE ($S = \pm 1$, $C = B = 0$) | |
|---|---|---|---|---|---|
| | $I^G(J^{PC})$ | | $I^G(J^{PC})$ | | $I(J^P)$ |
| • $\pi^\pm$ 140 MeV | $1^-(0^-)$ | • $\phi(1680)$ | $0^-(1^{--})$ | • $K^\pm$ | $1/2(0^-)$ |
| • $\pi^0$ | $1^-(0^{-+})$ | • $\rho_3(1690)$ | $1^+(3^{--})$ | • $K^0$ | $1/2(0^-)$ |
| • $\eta$ | $0^+(0^{-+})$ | • $\rho(1700)$ | $1^+(1^{--})$ | • $K^0_S$ | $1/2(0^-)$ |
| • $f_0(600)$ | $0^+(0^{++})$ | $a_2(1700)$ | $1^-(2^{++})$ | • $K^0_L$ | $1/2(0^-)$ |
| • $\rho(770)$ 770 | $1^+(1^{--})$ | • $f_0(1710)$ | $0^+(0^{++})$ | • $K^*(892)$ | $1/2(1^-)$ |
| • $\omega(782)$ | $0^-(1^{--})$ | $\eta(1760)$ | $0^+(0^{-+})$ | • $K_1(1270)$ | $1/2(1^+)$ |
| • $\eta'(958)$ | $0^+(0^{-+})$ | • $\pi(1800)$ | $1^-(0^{-+})$ | • $K_1(1400)$ | $1/2(1^+)$ |
| • $f_0(980)$ | $0^+(0^{++})$ | $f_2(1810)$ | $0^+(2^{++})$ | • $K^*(1410)$ | $1/2(1^-)$ |
| • $a_0(980)$ | $1^-(0^{++})$ | • $\phi_3(1850)$ | $0^-(3^{--})$ | • $K^*_0(1430)$ | $1/2(0^+)$ |
| • $\phi(1020)$ | $0^-(1^{--})$ | $\eta_2(1870)$ | $0^+(2^{-+})$ | • $K^*_2(1430)$ | $1/2(2^+)$ |
| • $h_1(1170)$ | $0^-(1^{+-})$ | $\rho(1900)$ | $1^+(1^{--})$ | $K(1460)$ | $1/2(0^-)$ |
| • $b_1(1235)$ | $1^+(1^{+-})$ | $f_2(1910)$ | $0^+(2^{++})$ | $K_2(1580)$ | $1/2(2^-)$ |
| • $a_1(1260)$ | $1^-(1^{++})$ | $f_2(1950)$ | $0^+(2^{++})$ | $K(1630)$ | $1/2(?^?)$ |
| • $f_2(1270)$ | $0^+(2^{++})$ | $\rho_3(1990)$ | $1^+(3^{--})$ | $K_1(1650)$ | $1/2(1^+)$ |
| • $f_1(1285)$ | $0^+(1^{++})$ | $X(2000)$ | $1^-(?^{?+})$ | • $K^*(1680)$ | $1/2(1^-)$ |
| • $\eta(1295)$ | $0^+(0^{-+})$ | • $f_2(2010)$ | $0^+(2^{++})$ | • $K_2(1770)$ | $1/2(2^-)$ |
| • $\pi(1300)$ | $1^-(0^{-+})$ | $f_0(2020)$ | $0^+(0^{++})$ | • $K^*_3(1780)$ | $1/2(3^-)$ |
| • $a_2(1320)$ | $1^-(2^{++})$ | • $a_4(2040)$ | $1^-(4^{++})$ | • $K_2(1820)$ | $1/2(2^-)$ |
| • $f_0(1370)$ | $0^+(0^{++})$ | • $f_4(2050)$ | $0^+(4^{++})$ | $K(1830)$ | $1/2(0^-)$ |

# Modern Miracles in Hadron Physics

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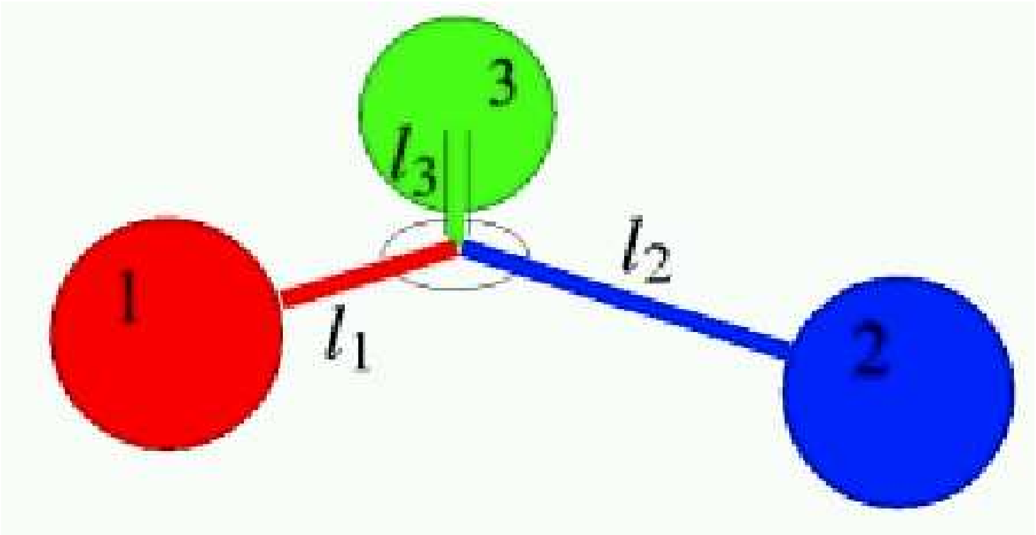

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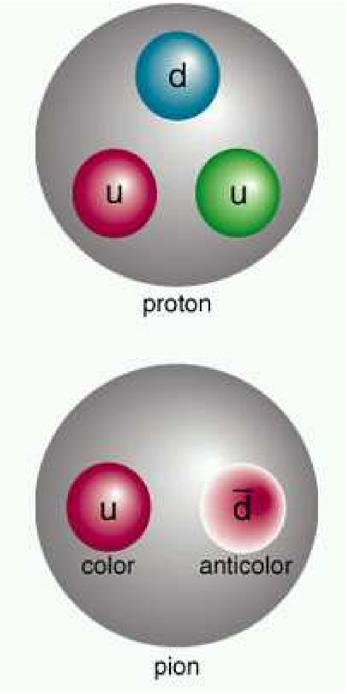

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Office of Nuclear Physics Exploring Nuclear Matter - Quarks to Stars

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- WRONG . . . . . . . . . . . . . . . . . . . . . . . $\boxed{M_{\rm pion} = 140\,{\rm MeV}}$
- Another meson:
  . . . . . . . . . . . $M_\rho = 770\,$MeV . . . . . . . . . . . No Surprises Here

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- WRONG . . . . . . . . . . . . . . . . . . . . . . . . $\boxed{M_{\mathbf{pion}} = \mathbf{140\,MeV}}$
- What is "wrong" with the pion?

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# Is this . . . Mass Destruction?

Thomas Jefferson National Accelerator Facility

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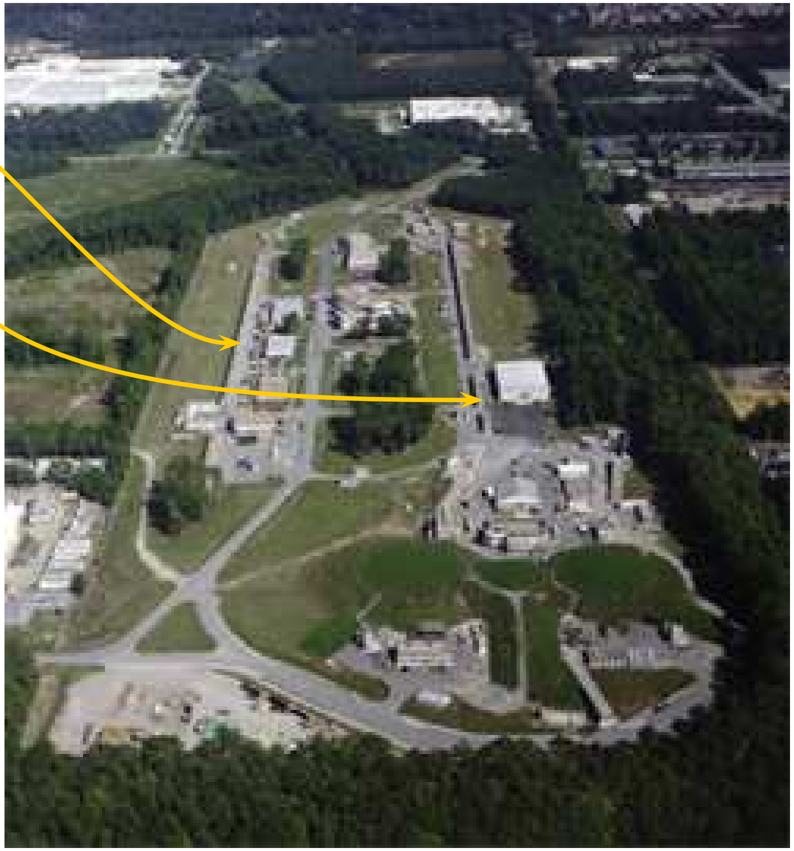

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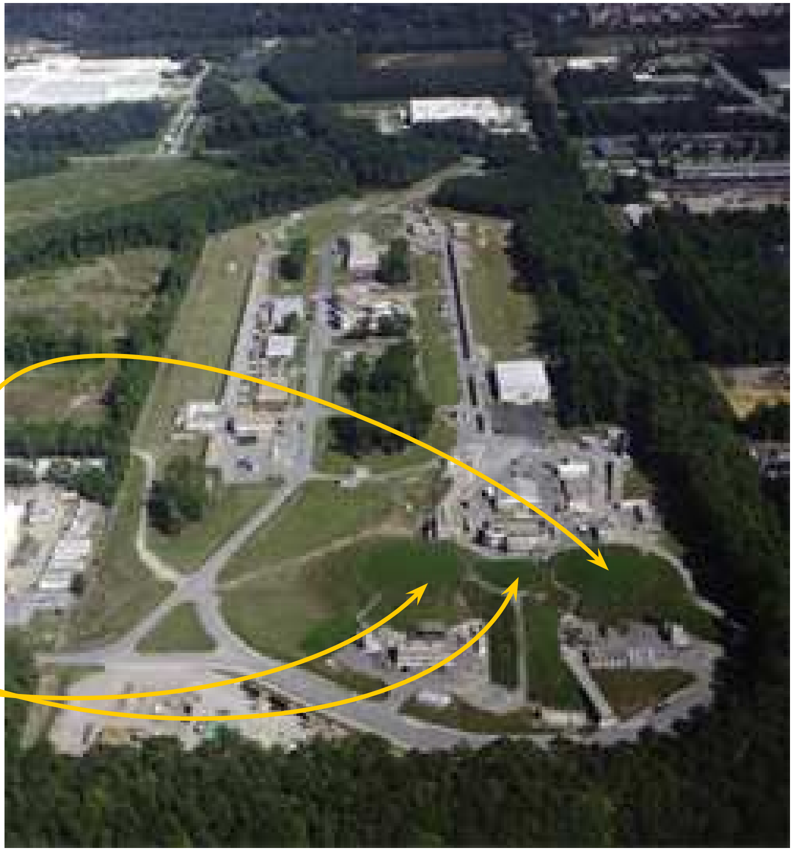

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- Current Peak Electron Beam Energy Nearly 6 GeV

# JLab Hall-A

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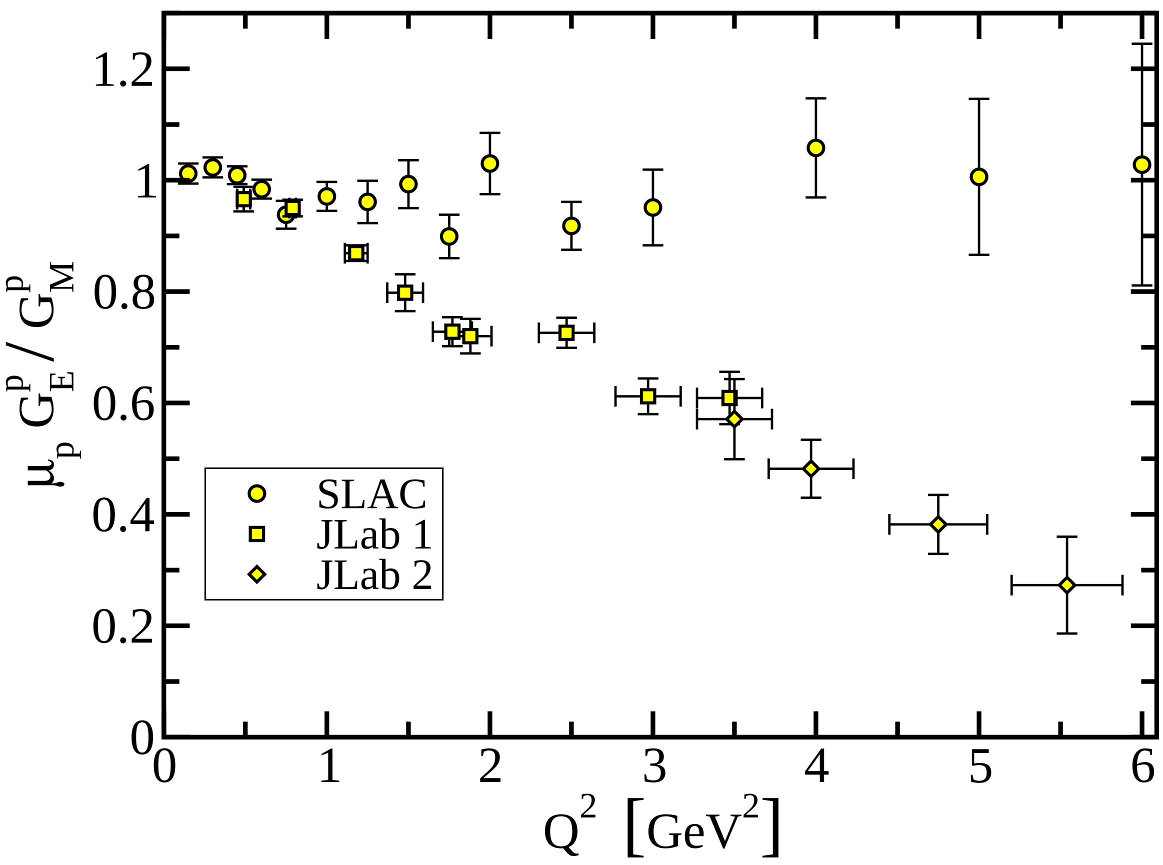
- Measured Ratio of
  Proton’s Electric and Magnetic Form Factors

# JLab Hall-A

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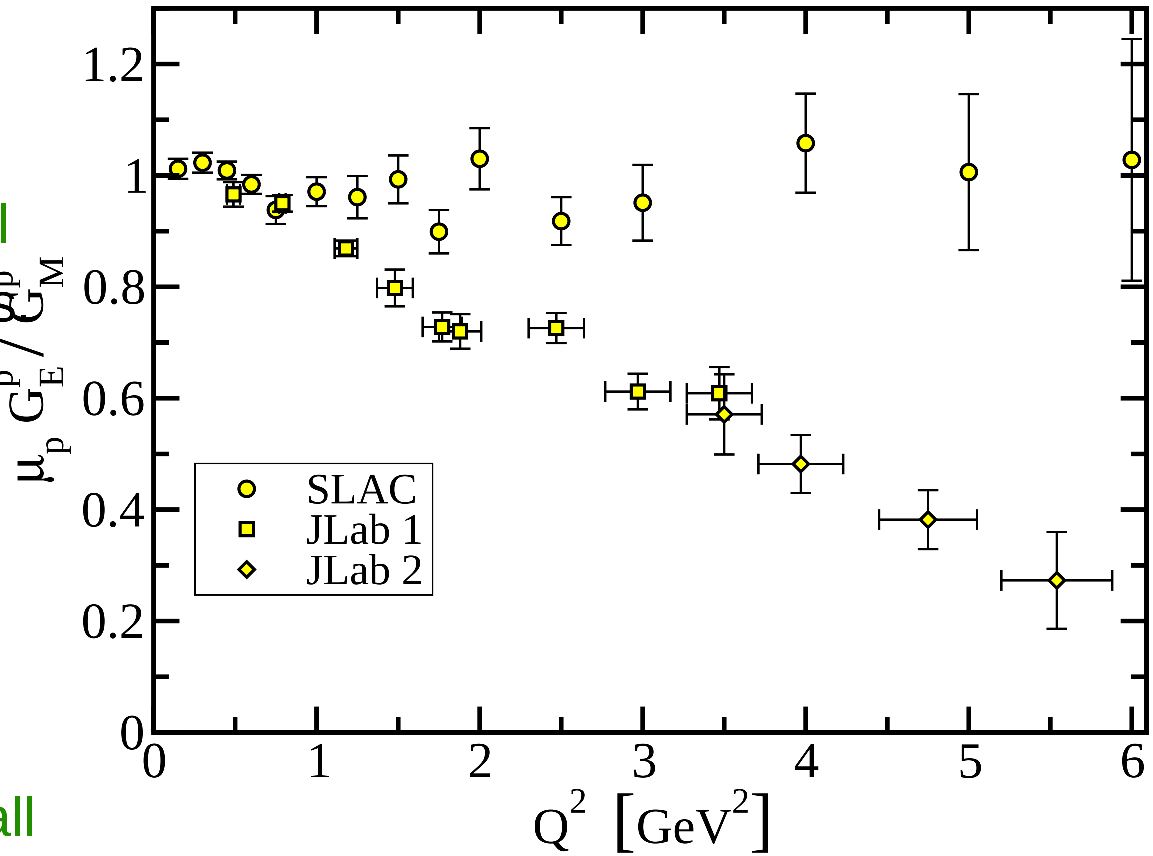
# *JLab Hall-A*

- Walker *et al.*, Phys. Rev. **D 49**, 5671 (1994). (SLAC)
- Jones *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **84**, 1398 (2000)
- Gayou, *et al.*, Phys. Rev. **C 64**, 038202 (2001)
- Gayou, *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **88** 092301 (2002)

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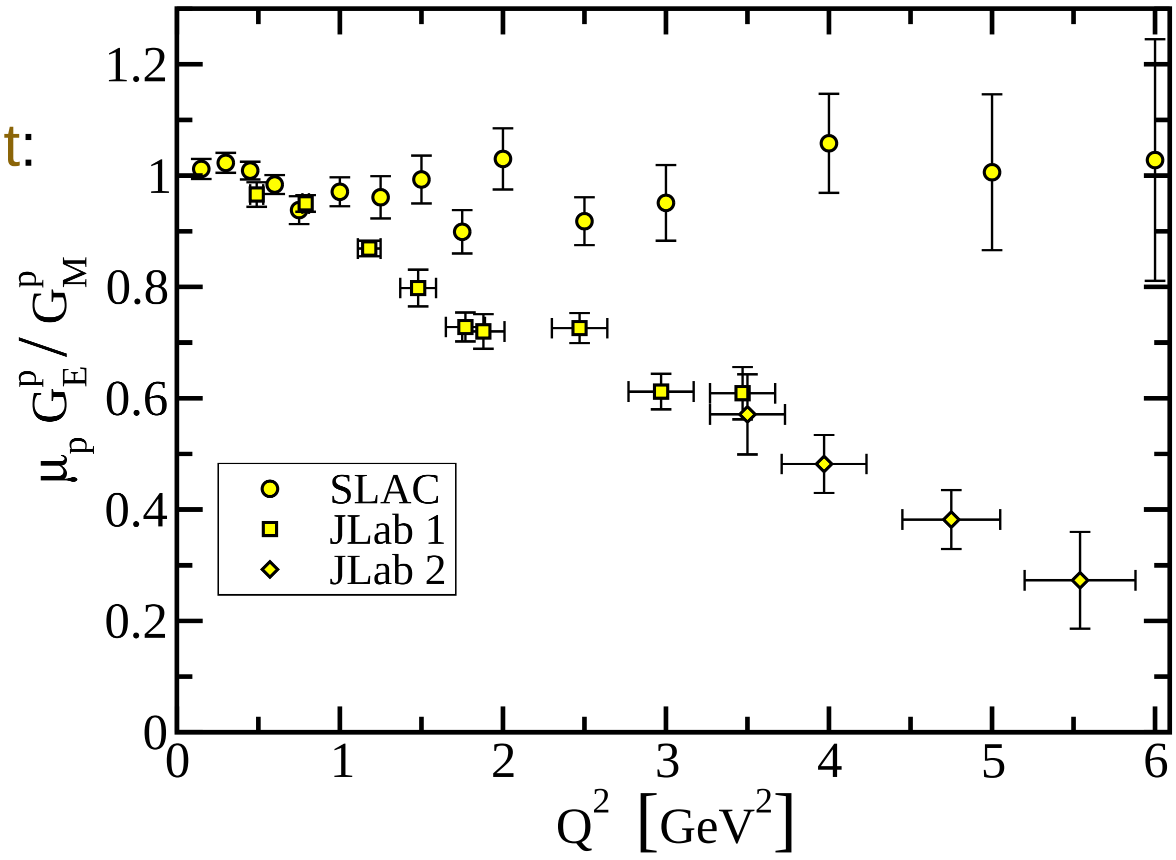
- If JLab Correct, then Completely Unexpected Result: In the Proton
  - On Relativistic Domain
  - Distribution of Quark-Charge Not Equal Distribution of Quark-Current!



# *What's the Problem?*

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- Determination of proton's wave function requires
  *ab initio* nonperturbative solution
  of fully-fledged relativistic quantum field theory
- Modern Physics & Mathematics
  – Still quite some way from being able to do that

# *Explanation?*

# QCD

- Action, in terms of local Lagrangian density:

$$S[A^a_\mu, \bar{q}, q] = \int d^4x \left\{ \frac{1}{4} F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) + \frac{1}{2\xi} \partial_\mu A^a_\mu(x)\, \partial_\nu A^a_\nu(x) + \bar{q}(x) \left[\gamma_\mu D_\mu + M\right] q(x) \right\} \tag{1}$$

  - Chromomagnetic Field Strength Tensor –
    $\partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g f^{abc} A^b_\mu(x) A^c_\nu(x)$
  - Covariant Derivative – $D_\mu = \partial_\mu - ig \dfrac{\lambda^a}{2} A^a_\mu(x)$
  - Current-quark Mass matrix: $\begin{pmatrix} m_u & 0 & 0 & \ldots \\ 0 & m_d & 0 & \ldots \\ 0 & 0 & m_s & \ldots \\ \vdots & \vdots & \vdots & \end{pmatrix}$

- Understanding JLab Observables means knowing all that this Action predicts.
- Perturbation Theory (asymptotic freedom) is not enough!
  - Bound states are not perturbative
  - Confinement is not perturbative
  - DCSB is not perturbative

# Euclidean Metric

- Almost all nonperturbative studies in relativistic quantum field theory employ a Euclidean Metric. (NB. Remember the Wick Rotation?)
- It is possible to view the Euclidean formulation of a quantum field theory as definitive; e.g.,
  - Symanzik, K. (1963) in *Local Quantum Theory* (Academic, New York) edited by R. Jost.
  - Streater, R.F. and Wightman, A.S. (1980), *PCT, Spin and Statistics, and All That* (Addison-Wesley, Reading, Mass, 3rd edition).
  - Glimm, J. and Jaffee, A. (1981), *Quantum Physics. A Functional Point of View* (Springer-Verlag, New York).
  - Seiler, E. (1982), *Gauge Theories as a Problem of Constructive Quantum Theory and Statistical Mechanics* (Springer-Verlag, New York).
- That decision is crucial when a consideration of nonperturbative effects becomes important. In addition, the discrete lattice formulation in Euclidean space has allowed some progress to be made in attempting to answer existence questions for interacting gauge field theories.
  - A lattice formulation is impossible in Minkowski space – the integrand is not non-negative and hence does not provide a probability measure.

# Euclidean Metric: Transcription Formulae

- To make clear our conventions: for 4-vectors $a$, $b$: $a \cdot b := a_\mu\, b_\nu\, \delta_{\mu\nu} := \sum_{i=1}^{4} a_i\, b_i$ ,

  Hence, a spacelike vector, $Q_\mu$, has $Q^2 > 0$.
- Dirac matrices:
  - Hermitian and defined by the algebra $\{\gamma_\mu, \gamma_\nu\} = 2\,\delta_{\mu\nu}$;
  - we use $\gamma_5 := -\,\gamma_1\gamma_2\gamma_3\gamma_4$, so that $\mathrm{tr}\,[\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma] = -4\,\varepsilon_{\mu\nu\rho\sigma}\ ,\ \varepsilon_{1234} = 1$.
  - The Dirac-like representation of these matrices is:

$$\vec{\gamma} = \left( \begin{array}{cc} 0 & -i\vec{\tau} \\ i\vec{\tau} & 0 \end{array} \right),\ \gamma_4 = \left( \begin{array}{cc} \tau^0 & 0 \\ 0 & -\tau^0 \end{array} \right), \tag{2}$$

where the $2 \times 2$ Pauli matrices are:

$$\tau^0 = \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right),\ \tau^1 = \left( \begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array} \right),\ \tau^2 = \left( \begin{array}{cc} 0 & -i \\ i & 0 \end{array} \right),\ \tau^3 = \left( \begin{array}{cc} 1 & 0 \\ 0 & -1 \end{array} \right). \tag{3}$$

First Contents Back Conclusion

# Euclidean Metric: Transcription Formulae

- It is possible to derive every equation introduced above assuming certain analytic properties of the integrands. However, the derivations can be sidestepped using the following *transcription rules*:

| Configuration Space | Momentum Space |
|---|---|
| 1. $\int^M d^4x^M \to -i\int^E d^4x^E$ | 1. $\int^M d^4k^M \to i\int^E d^4k^E$ |
| 2. $\not\partial \to i\gamma^E \cdot \partial^E$ | 2. $\not k \to -i\gamma^E \cdot k^E$ |
| 3. $\not A \to -i\gamma^E \cdot A^E$ | 3. $\not A \to -i\gamma^E \cdot A^E$ |
| 4. $A_\mu B^\mu \to -A^E \cdot B^E$ | 4. $k_\mu q^\mu \to -k^E \cdot q^E$ |
| 5. $x^\mu \partial_\mu \to x^E \cdot \partial^E$ | 5. $k_\mu x^\mu \to -k^E \cdot x^E$ |

- These rules are valid in perturbation theory; i.e., the correct Minkowski space integral for a given diagram will be obtained by applying these rules to the Euclidean integral: they take account of the change of variables and rotation of the contour. However, for diagrams that represent DSEs which involve dressed $n$-point functions, whose analytic structure is not known *a priori*, the Minkowski space equation obtained using this prescription will have the right appearance but it's solutions may bear no relation to the analytic continuation of the solution of the Euclidean equation. Any such differences will be nonperturbative in origin.

# What is QCD?

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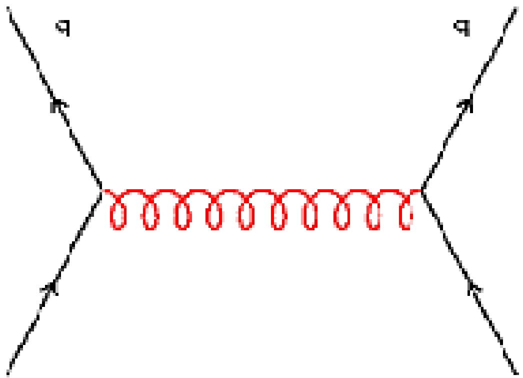
## What is QCD?

- Gauge Theory:

  Interactions Mediated by massless vector bosons

Feynman Diagram of Quark—Quark Scattering



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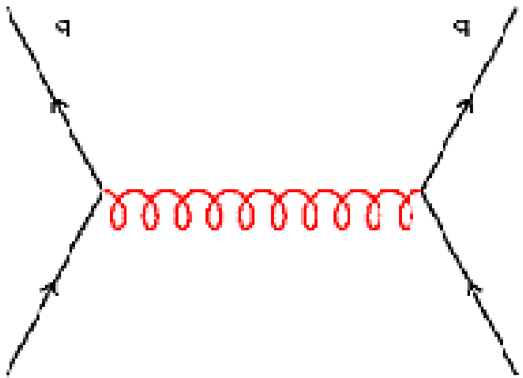


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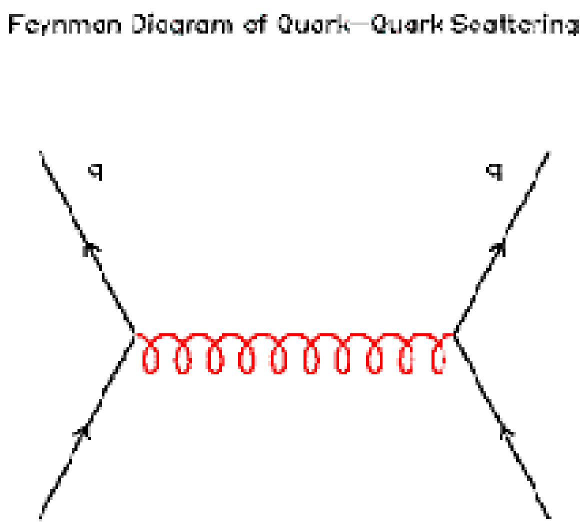


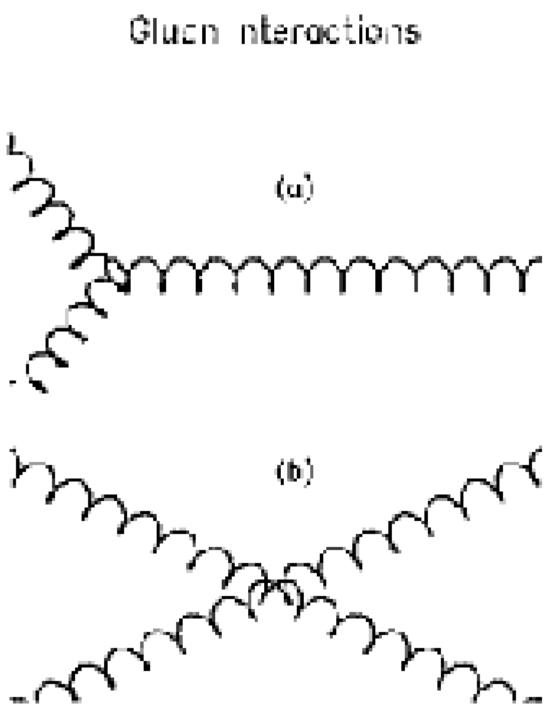


- Similar interaction in QED

- Special Feature of QCD – gluon self-interactions

  Completely Change the Character of the Theory

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# QED cf. QCD

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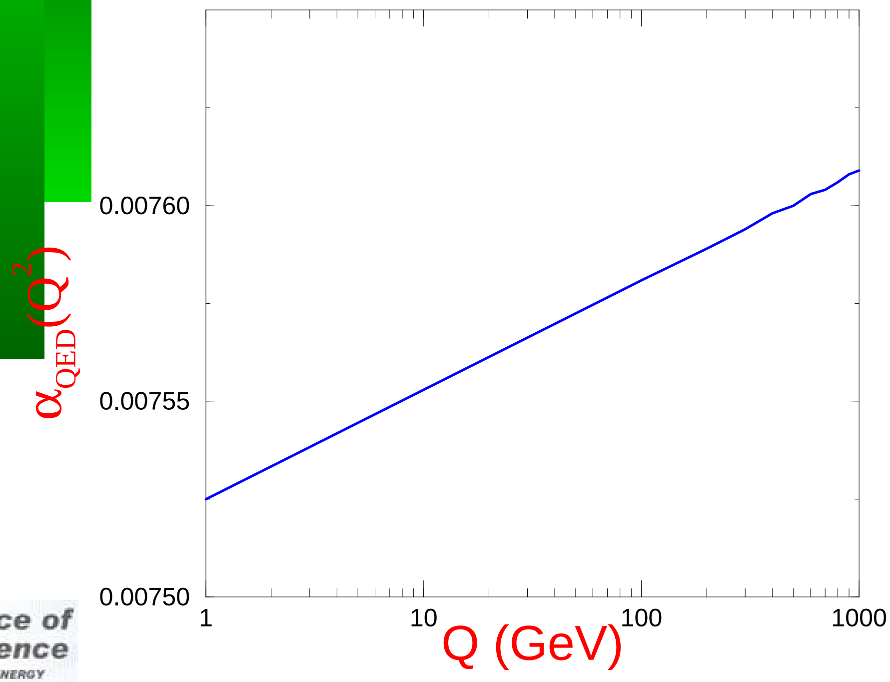
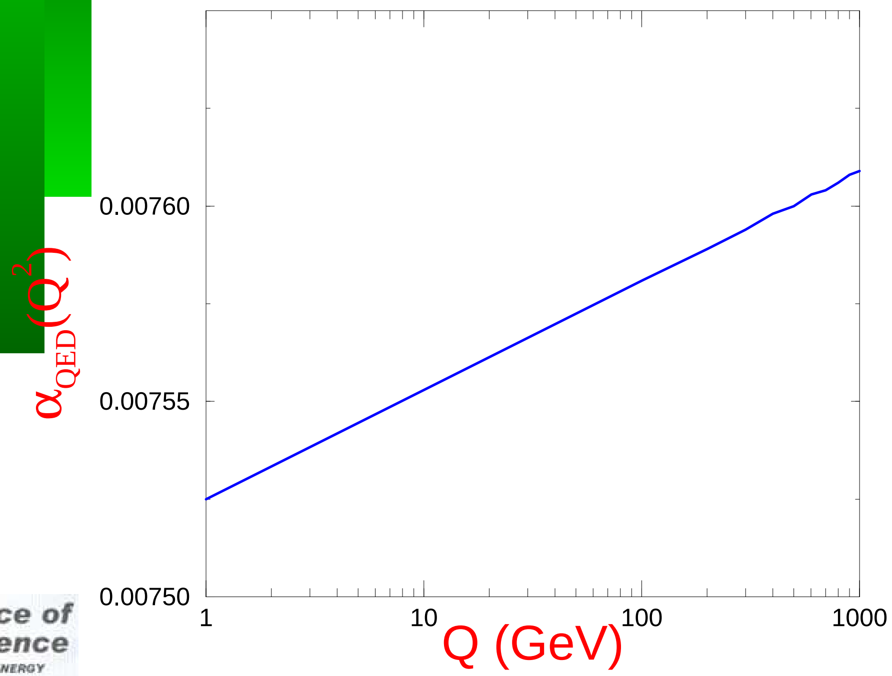
## QED cf. QCD

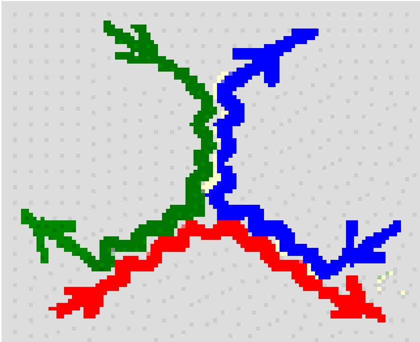



$$\alpha_{\rm QED} = \frac{\alpha}{1 - \alpha/3\pi \ln\left(Q^2/m_e^2\right)}$$

# QED cf. QCD

Add three-gluon interaction



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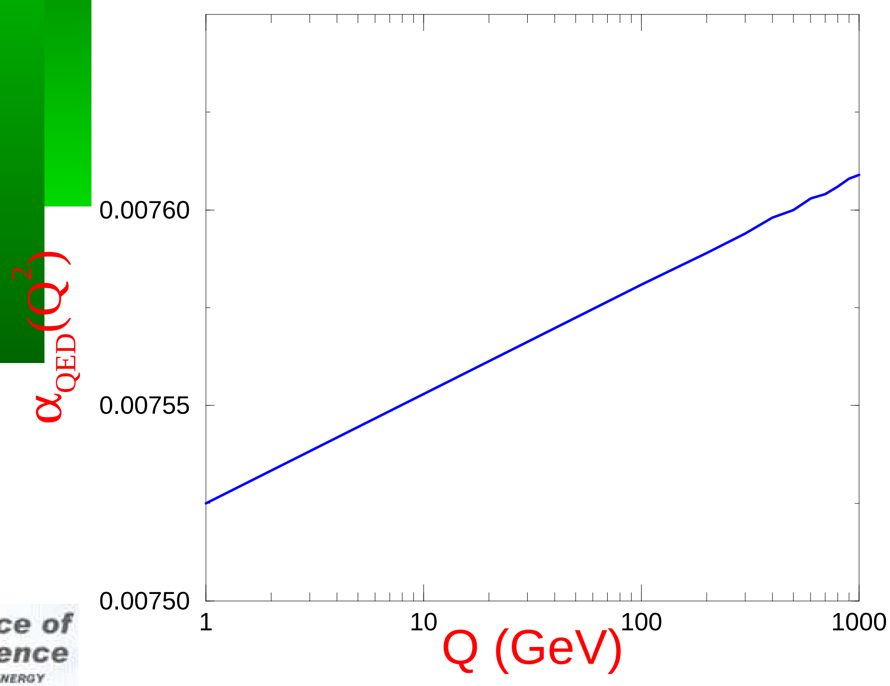


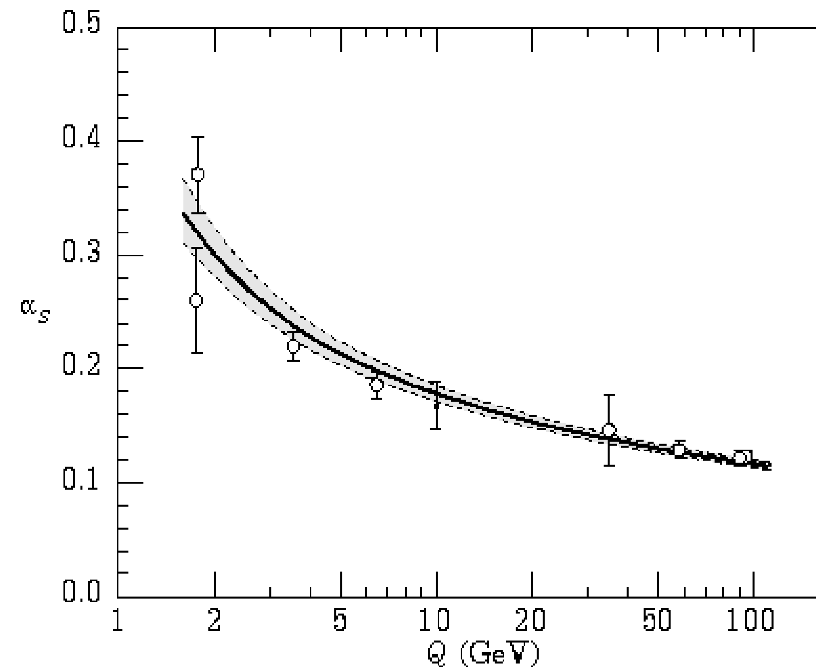


**Figure 9.2:** Summary of the values of $\alpha_s(Q)$ at the value measured. The lines show the central values and the $\pm 1\sigma$ lin figure clearly shows the decrease in $\alpha_s(Q)$ with increasing Q

$$\alpha_{\rm QED} = \frac{\alpha}{1 - \alpha/3\pi \ln\left(Q^2/m_e^2\right)}$$

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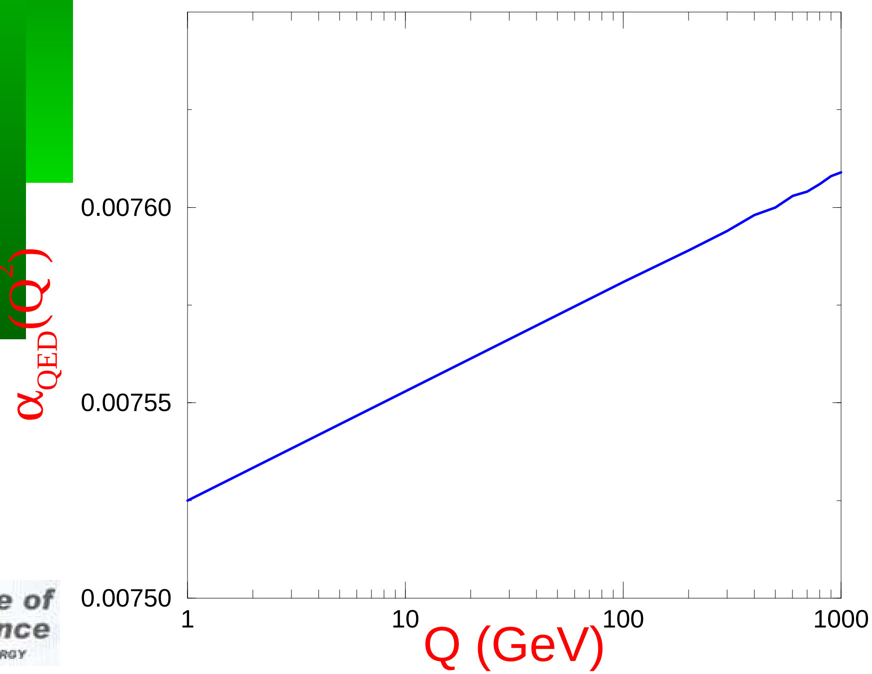
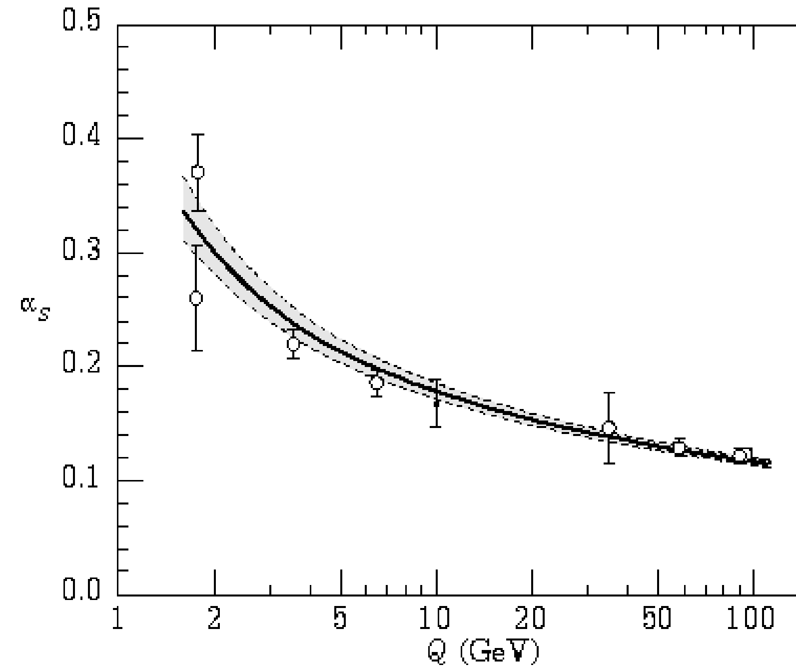
## 2004 Nobel Prize in Physics: Gross, Politzer and Wilczek

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# Closer look at Spectrum

- Features of the Spectrum:
  - $\dfrac{m_\rho^2}{m_\pi^2} = 30$
  - $\dfrac{m_{a_1}^2}{m_{a_0}^2} = 1.7$

? Hyperfine Splitting

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Something Very Odd About the Pion

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# Dichotomy of the Pion

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- Requires explanation of Connection between pQCD-quark and Spectrum/Constituent-quark

# QCD’s Emergent Phenomena

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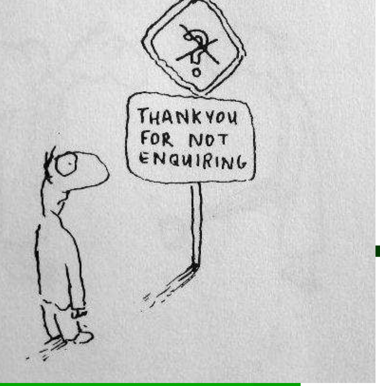


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- Neither of these phenomena is apparent in QCD's Lagrangian yet they are the dominant determining characteristics of real-world QCD.
- NSAC – Understanding these phenomena is one of the greatest intellectual challenges in physics

Gauge Theories with Massless Fermions have

CHIRAL SYMMETRY

# Chiral Symmetry

- Helicity $\lambda \propto J \cdot p$
  - Projection of Spin onto Direction of Motion
  - For massless particles, helicity is a Lorentz invariant *Spin Observable*.
  - $\lambda = \pm$ ($\parallel$ or anti-$\parallel$ to $p_\mu$)

# *Chiral Symmetry*

- Chirality Operator: $\gamma_5$
  - Chiral Transformation $q(x) \to \mathrm{e}^{i\gamma_5\theta}\, q(x)$

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  - Chiral Transformation $q(x) \to \mathrm{e}^{i\gamma_5\theta}\, q(x)$
  - Chiral Rotation $\theta = \dfrac{\pi}{2}$
    - $q_{\lambda=+} \to q_{\lambda=+},\ q_{\lambda=-} \to -q_{\lambda=-}$
    - Hence, a theory invariant under chiral transformations can only contain interactions that are insensitive to a particle's helicity.

# *Chiral Symmetry*

- Chirality Operator: $\gamma_5$
  - Chiral Transformation $q(x) \to \mathrm{e}^{i\gamma_5\theta}\, q(x)$
  - Chiral Rotation $\theta = \dfrac{\pi}{4}$
    - Composite Particles: $J^{P=+} \leftrightarrow J^{P=-}$
    - Equivalent to "Parity Conjugation" Operation

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# Chiral Symmetry

- A Prediction of Chiral Symmetry
  - Degeneracy between Parity Partners

    $N(\frac{1}{2}^+, 938) = N(\frac{1}{2}^-, 1535)$, $\pi(0^-, 140) = a_0(0^+, 980)$,

    $\rho(1^-, 770) = a_1(1^+, 1260)$
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How can pion mass be so small

If quarks are so heavy?!

# Propagators

- Extraordinary Effects in QCD Tied to Properties of *Dressed*-Quark and -Gluon Propagators

| Quark | Gluon |
| --- | --- |
| $S_f(x-y) \equiv \langle q_f(x)\bar{q}_f(y)\rangle$ | $D_{\mu\nu}(x-y) \equiv \langle A_\mu(x)A_\nu(y)\rangle$ |

- Describe in-Medium *Propagation Characteristics* of Elementary Particles

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# Propagators

- Example: Solid-State Physics
  - $\gamma$ propagating in a Dense e$^-$ Gas
  - Acquires a Debye Mass
    $$m_{\mathrm{D}}^2 \propto k_F^2 : \; \frac{1}{Q^2} \to \frac{1}{Q^2 + m_{\mathrm{D}}^2}$$
  - $\gamma$ develops an Effective-mass

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  - $\gamma$ develops an Effective-mass
  - Leads to Screening of the Interaction: $r \propto \dfrac{1}{m_D}$
- Quark and Gluon Propagators:
  Modified in a similar way -
  Momentum Dependent Effective Masses
- The Effect of this is Observable in QCD

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# *Explicit* Chiral Symmetry Breaking

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  Can be discussed in terms of Quark Propagator

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- Chiral Transformation

$$\begin{aligned} \mathbf{S}_0(p) &\rightarrow \mathrm{e}^{i\gamma_5\theta} S_0(p) \mathrm{e}^{i\gamma_5\theta} \\ &= \frac{-i\gamma \cdot p}{p^2 + m^2} + \mathrm{e}^{2i\gamma_5\theta} \frac{m}{p^2 + m^2} \end{aligned}$$

  - Symmetry Violation $\propto m$
  - $\mathbf{m} = 0$: $S_0(p) \rightarrow S_0(p)$

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# *Dynamical Symmetry Breaking*

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$$V(x,y) = (\sigma^2 + \pi^2 - 1)^2$$

Hamiltonian: $T + V$, is Rotationally Invariant

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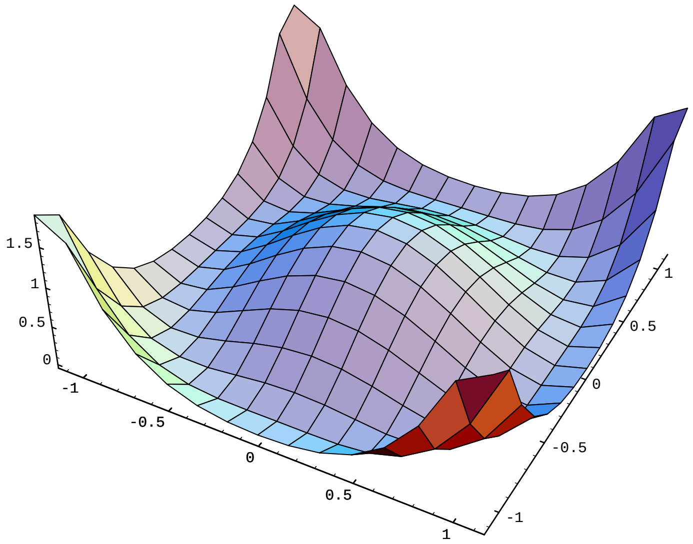


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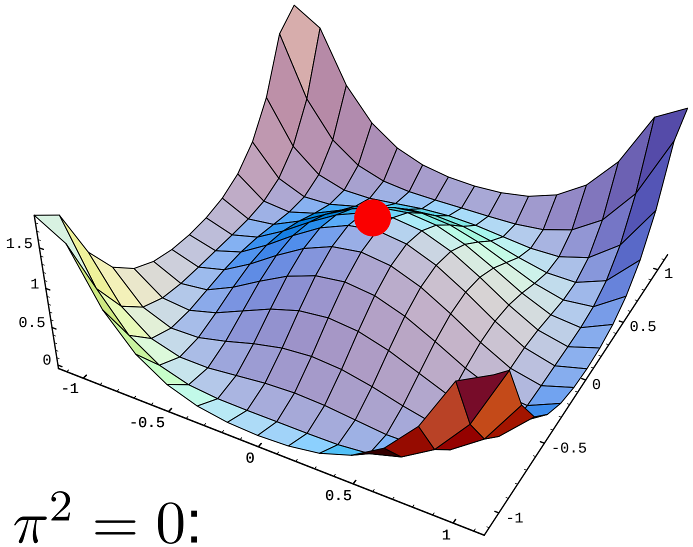


Ground State?

- Ball at $(\sigma, \pi)$
  for which $\sigma^2 + \pi^2 = 0$:
  - Rotationally Invariant

UNSTABLE

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# *Dynamical* **Symmetry Breaking**

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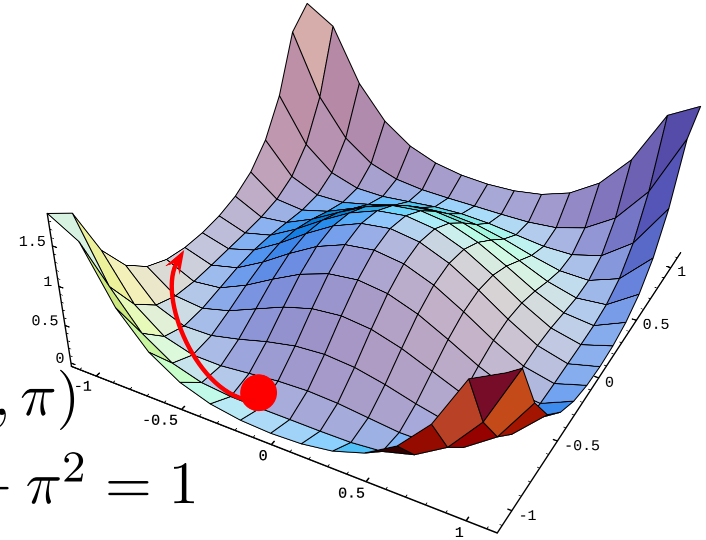


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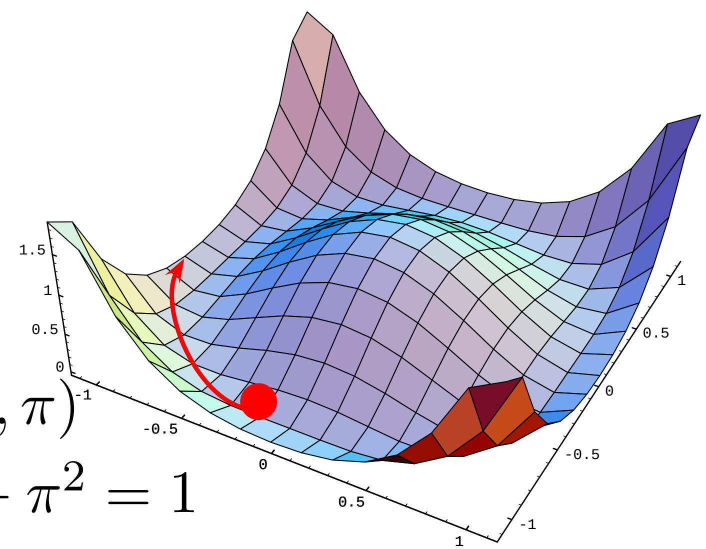
Symmetry of Ground State $\neq$ Symmetry of Hamiltonian

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# Dynamics and Symmetries

Very Nonperturbative Problem

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# Dyson-Schwinger Equations

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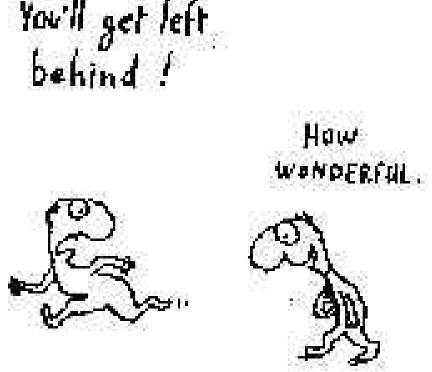


# Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory


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# Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory
- NonPerturbative, Continuum approach to QCD
- Simplest level: Generating Tool for Perturbation Theory

  ...................... Materially Reduces Model Dependence


# Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons

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# Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
      - Quark & Gluon Confinement
    - $\Rightarrow$ Understanding InfraRed (long-range)

      ................................. behaviour of $\alpha_s(Q^2)$


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- Method yields Schwinger Functions $\equiv$ Propagators

Cross-Sections built from Schwinger Functions

# Perturbative Dressed-quark Propagator

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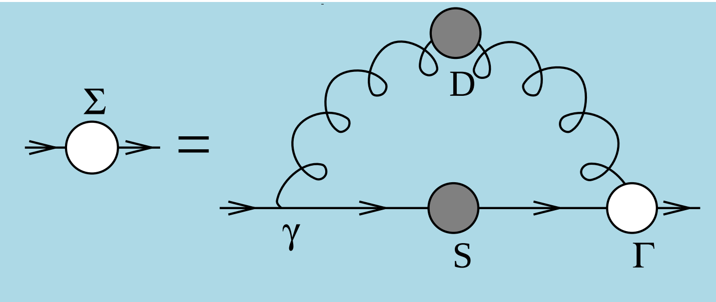
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$




*Gap Equation*

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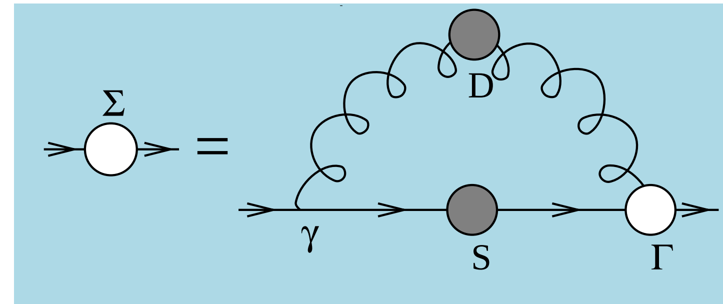


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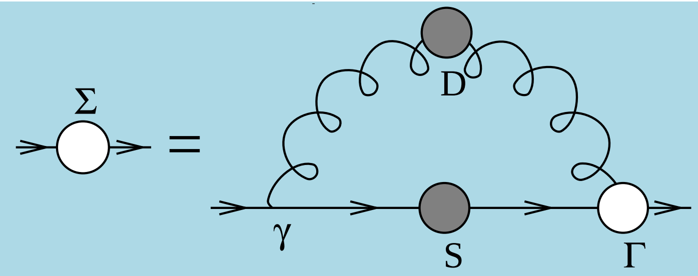


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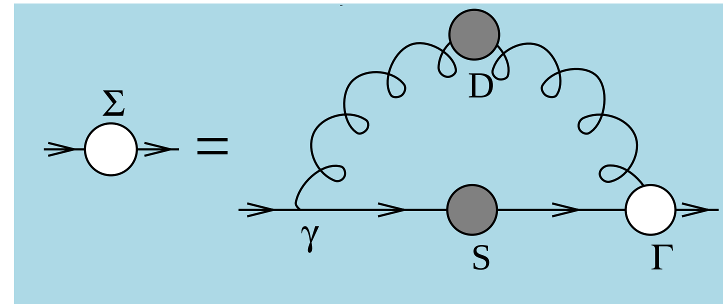


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$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \overset{m\to 0}{\longrightarrow} 0$$

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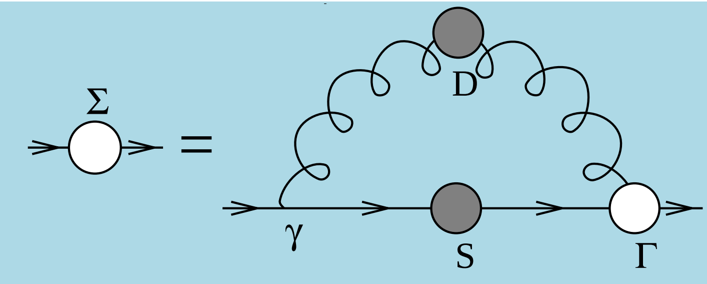


Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p \, A(p^2) + B(p^2)}$$

No DCSB Here!

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# Nambu–Jona-Lasinio Model

- Recall the Gap Equation:

$$S^{-1}(p) = i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m + \int^{\Lambda} \frac{d^4\ell}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-\ell)\,\gamma_\mu \frac{\lambda^a}{2} \frac{1}{i\gamma\cdot\ell A(\ell^2) + B(\ell^2)} \Gamma^a_\nu(\ell,p) \tag{4}$$

- NJL: $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu\, \dfrac{\lambda^a}{2}$;

$$g^2 D_{\mu\nu}(p-\ell) \to \delta_{\mu\nu}\, \frac{1}{m_G^2}\, \theta(\Lambda^2-\ell^2) \tag{5}$$

  - Model is not renormalisable
    $\Rightarrow$ regularisation parameter ($\Lambda$) plays a dynamical role.

- NJL Gap Equation

$$i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m + \frac{4}{3}\frac{1}{m_G^2} \int \frac{d^4\ell}{(2\pi)^4}\, \theta(\Lambda^2-\ell^2)\, \gamma_\mu\, \frac{-i\gamma\cdot\ell A(\ell^2) + B(\ell^2)}{\ell^2 A^2(\ell^2) + B^2(\ell^2)}\, \gamma_\mu \tag{6}$$

# Solving NJL Gap Equation

- Multiply Eq. (6) by $(-i\gamma \cdot p)$; trace over Dirac indices:

$$p^2 A(p^2) = p^2 + \frac{8}{3} \frac{1}{m_G^2} \int \frac{d^4\ell}{(2\pi)^4} \theta(\Lambda^2 - \ell^2)\, p \cdot \ell \, \frac{A(\ell^2)}{\ell^2 A^2(\ell^2) + B^2(\ell^2)} \quad (7)$$

  - Angular integral vanishes, therefore

$$A(p^2) \equiv 1\,. \quad (8)$$

This owes to the the fact that NJL model is defined by four-fermion contact interaction in configuration space, entails momentum-independence of interaction in momentum space.

- Tracing over Dirac indices; use Eq. (8):

$$B(p^2) = m + \frac{16}{3} \frac{1}{m_G^2} \int \frac{d^4\ell}{(2\pi)^4} \theta(\Lambda^2 - \ell^2) \frac{B(\ell^2)}{\ell^2 + B^2(\ell^2)}\,, \quad (9)$$

  - Integral is $p^2$-independent.
  - Therefore $B(p^2) = \text{constant} = M$ is the only solution.

# NJL Mass Gap

- Evaluate integrals; Eq. (9) becomes

$$M = m + M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, \Lambda^2)\,, \tag{10}$$

$$\mathcal{C}(M^2, \Lambda^2) = \Lambda^2 - M^2 \ln\left[1 + \Lambda^2/M^2\right]\,. \tag{11}$$

- $\Lambda$ defines model's mass-scale. Henceforth set $\boxed{\Lambda = 1}$. Then all other dimensioned quantities are given in units of this scale, in which case the gap equation can be written

$$M = m + M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)\,. \tag{12}$$

- Chiral limit: $m = 0$, $\boxed{M = M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)}$
  - Solved if $M \equiv 0$
    ... This is the perturbative result: start with no mass, end up with no mass.
  - Suppose $M \neq 0$
    - Solved *iff* $\boxed{1 = \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)}$.

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# NJL Dynamical Mass

- Can one satisfy $\boxed{1 = \frac{1}{3\pi^2}\,\frac{1}{m_G^2}\,\mathcal{C}(M^2,1)}$?
- $\mathcal{C}(M^2,1) = 1 - M^2 \ln\left[1 + 1/M^2\right]$
  - Monotonically decreasing function of $M$
  - Maximum value at $M=0$: $\mathcal{C}(0,1)=1$.
- Consequently $\exists\ M \neq 0$ solution *iff* $\boxed{\frac{1}{3\pi^2}\,\frac{1}{m_G^2} > 1}$
  - Typical scale for hadron physics $\Lambda \sim 1$ GeV.
    - $M \neq 0$ solution *iff* $\boxed{m_G^2 < \frac{\Lambda^2}{3\pi^2} \simeq (0.2\,GeV)^2}$
- Interaction Strength is proportional to $\frac{1}{m_G^2}$
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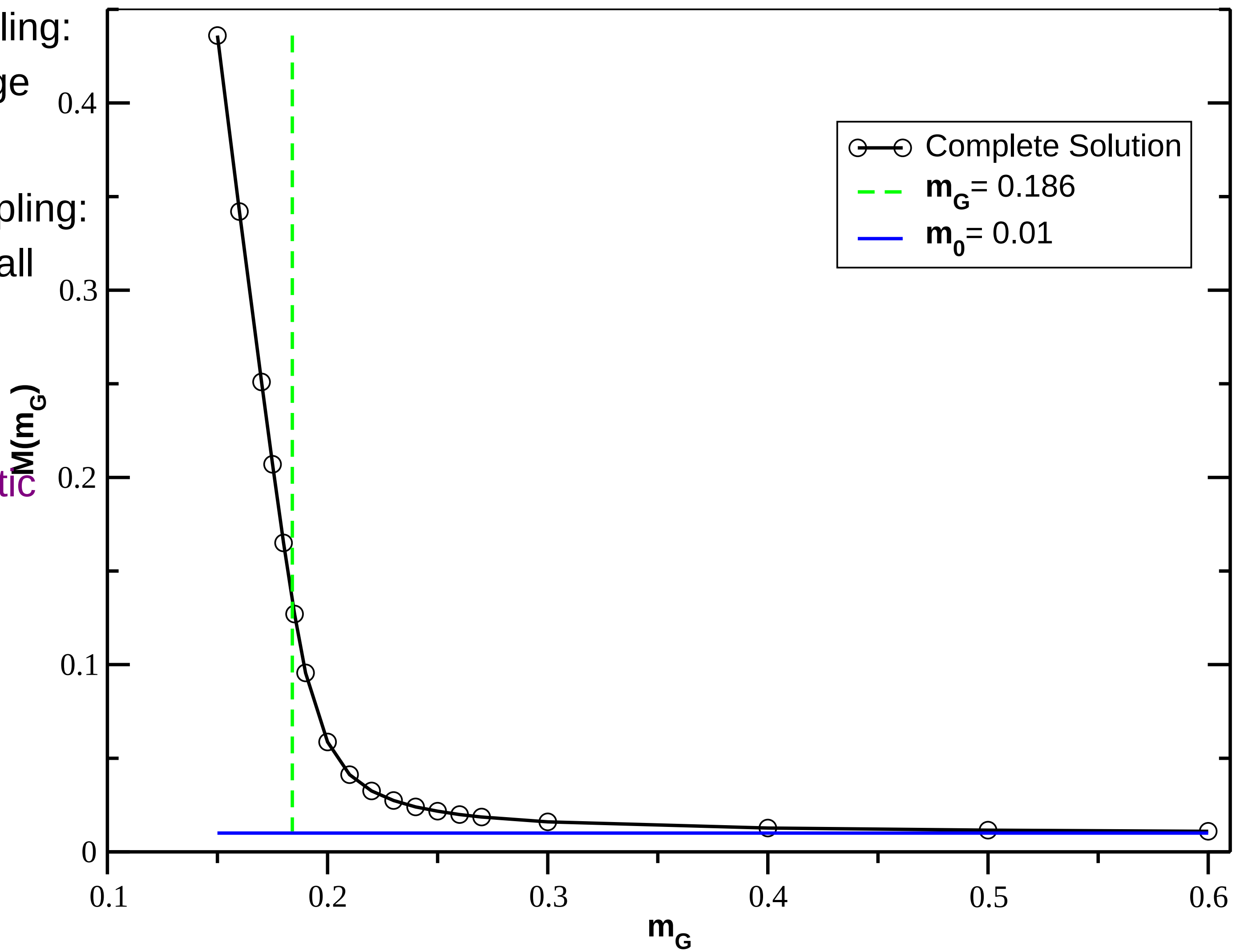
*Dynamical Chiral Symmetry Breaking*

# NJL Dynamical Mass

Solve $\boxed{M = m_0 + M \, \frac{1}{3\pi^2} \, \frac{1}{m_G^2} \, \mathcal{C}(M^2, 1)}$

- Weak coupling:
  $\Leftrightarrow m_G$ large
  $M \sim m_0$
- Strong coupling:
  $\Leftrightarrow m_G$ small
  $M \gg m_0$
  This is the essential characteristic of DCSB

**NJL Mass Gap**



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$$S(p)^{\rm NJL} = \frac{1}{i\gamma\cdot p[A(p^2)=1] + [B(p^2)=M]} = \frac{-i\gamma\cdot p + M}{p^2+M^2} \qquad (15)$$

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- This is merely a free-particle-like propagator with a shifted mass:

$$p^2+M^2=0 \Rightarrow \text{ Minkowski-space mass} = M. \qquad (18)$$

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- This is merely a free-particle-like propagator with a shifted mass:

$$p^2 + M^2 = 0 \Rightarrow \text{ Minkowski-space mass} = M. \qquad (20)$$

- Hence, while **NJL Model** certainly contains DCSB, it **does not exhibit confinement**.

# Munczek-Nemirovsky Model

- Munczek, H.J. and Nemirovsky, A.M. (1983), "The Ground State $q\bar{q}$ Mass Spectrum In QCD," *Phys. Rev.* **D 28**, 181.
- $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu \, \frac{\lambda^a}{2}$;

$$g^2 D_{\mu\nu}(k) \to (2\pi)^4 \, G \, \delta^4(k) \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right] \tag{21}$$

  Here $G$ defines the model's mass-scale.
  - $\delta$-function in momentum space
    cf. NJL, which has $\delta$-function in configuration space.
- Gap equation

$$i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m + G\,\gamma_\mu \, \frac{-i\gamma\cdot p\, A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,\gamma_\mu \tag{22}$$

# MN Model's Gap Equation

- The gap equation yields the following two coupled equations (set the mass-scale $G = 1$):

$$A(p^2) = 1 + 2\,\frac{A(p^2)}{p^2 A^2(p^2) + B^2(p^2)} \tag{23}$$

$$B(p^2) = m + 4\,\frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,, \tag{24}$$

- Consider the chiral limit equation for $B(p^2)$:

$$B(p^2) = 4\,\frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,. \tag{25}$$

  - Obviously, $B \equiv 0$ is a solution.
  - Is there another?

# DCSB in MN Model

- The existence of a $B \not\equiv 0$ solution; i.e., a solution that dynamically breaks chiral symmetry, requires (in units of $G$)

$$p^2 A^2(p^2) + B^2(p^2) = 4\,. \tag{26}$$

- Substituting this identity into equation Eq. (23), one finds

$$A(p^2) - 1 = \frac{1}{2}\, A(p^2) \;\Rightarrow\; A(p^2) \equiv 2\,, \tag{27}$$

which in turn entails

$$B(p^2) = 2\,\sqrt{1-p^2}\,. \tag{28}$$

- Physical requirement: quark self energy is real on the spacelike domain $\Rightarrow$ complete chiral-limit solution –

$$A(p^2) = \left\{ \begin{array}{ll} 2\,; & p^2 \leq 1 \\ \frac{1}{2}\left(1+\sqrt{1+8/p^2}\right)\;; & p^2 > 1 \end{array} \right. \tag{29}$$

$$B(p^2) = \left\{ \begin{array}{ll} \sqrt{1-p^2}\,; & p^2 \leq 1 \\ 0\,; & p^2 > 1\,. \end{array} \right. \tag{30}$$

- NB. Dressed-quark self-energy is momentum dependent, as is the case in QCD.

# Confinement in MN Model

- Solution is continuous and defined for all $p^2$, even $p^2 < 0$; namely, timelike momenta.
- Examine the propagator's denominator:

$$p^2 A^2(p^2) + B^2(p^2) > 0\,,\ \forall\, p^2\,. \tag{31}$$

This is positive definite . . . there are *no zeros*

- This is nothing like a free-particle propagator. It can be interpreted as describing a **confined** degree-of-freedom
- Note that, in addition there is no critical coupling: the nontrivial solution exists so long as **G** $> 0$.
- Conjecture: **All confining theories exhibit DCSB**.
  - NJL model demonstrates that converse is not true.

# Massive Solution in MN Model

- In the chirally asymmetric case the gap equation yields

$$A(p^2) = \frac{2\, B(p^2)}{m + B(p^2)}\,, \tag{32}$$

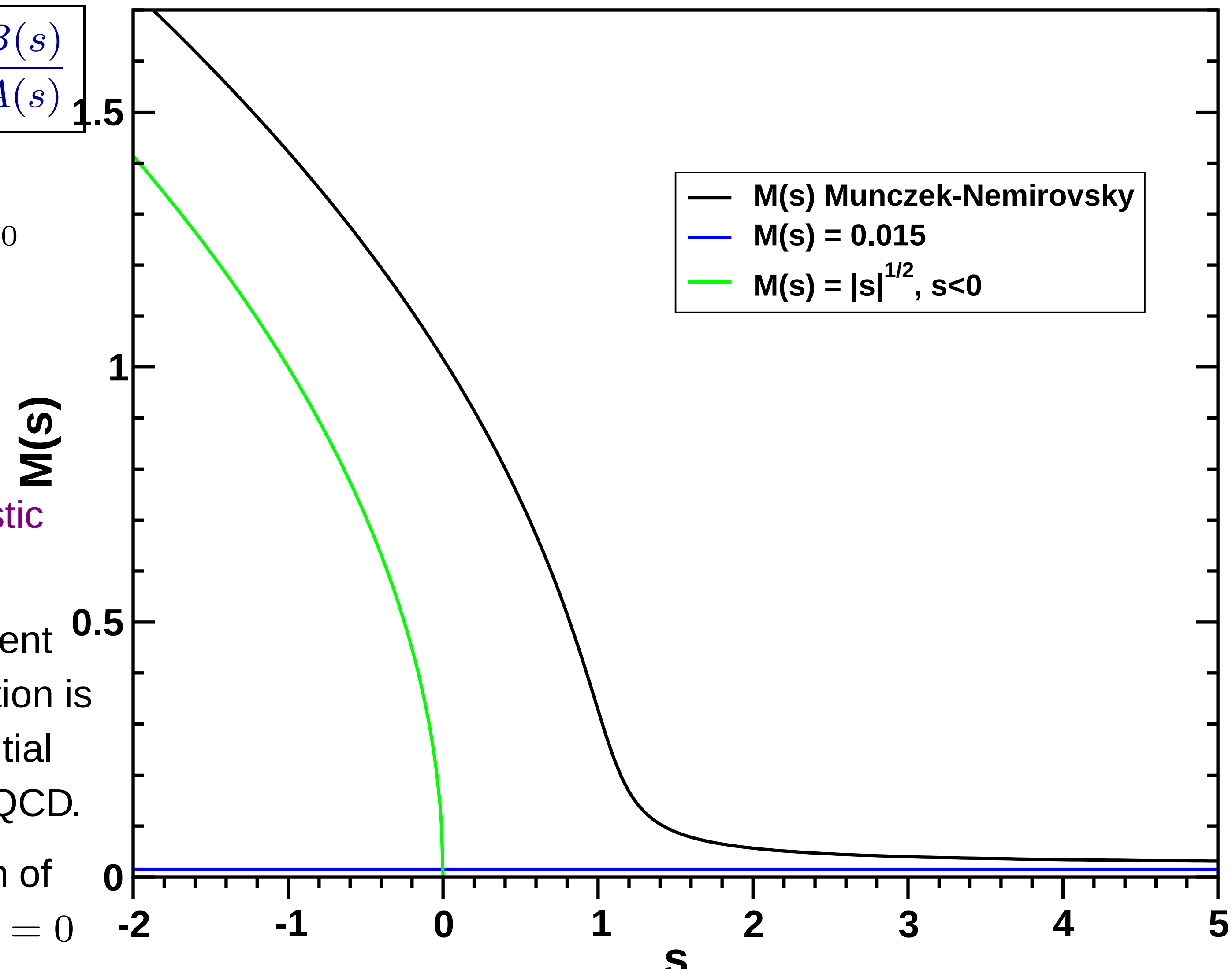
$$B(p^2) = m + \frac{4\,[m + B(p^2)]^2}{B(p^2)([m+B(p^2)]^2 + 4p^2)}\,. \tag{33}$$

  - Second is a quartic equation for $B(p^2)$.
  - Can be solved algebraically with four solutions, available in a closed form.
  - Only one has the correct $p^2 \to \infty$ limit: $B(p^2) \to m$.
  - NB. The equations and their solutions always have a smooth $m \to 0$ limit, a result owing to the persistence of the DCSB solution.

# MN Dynamical Mass

$$M(s=p^2) = \frac{B(s)}{A(s)}$$

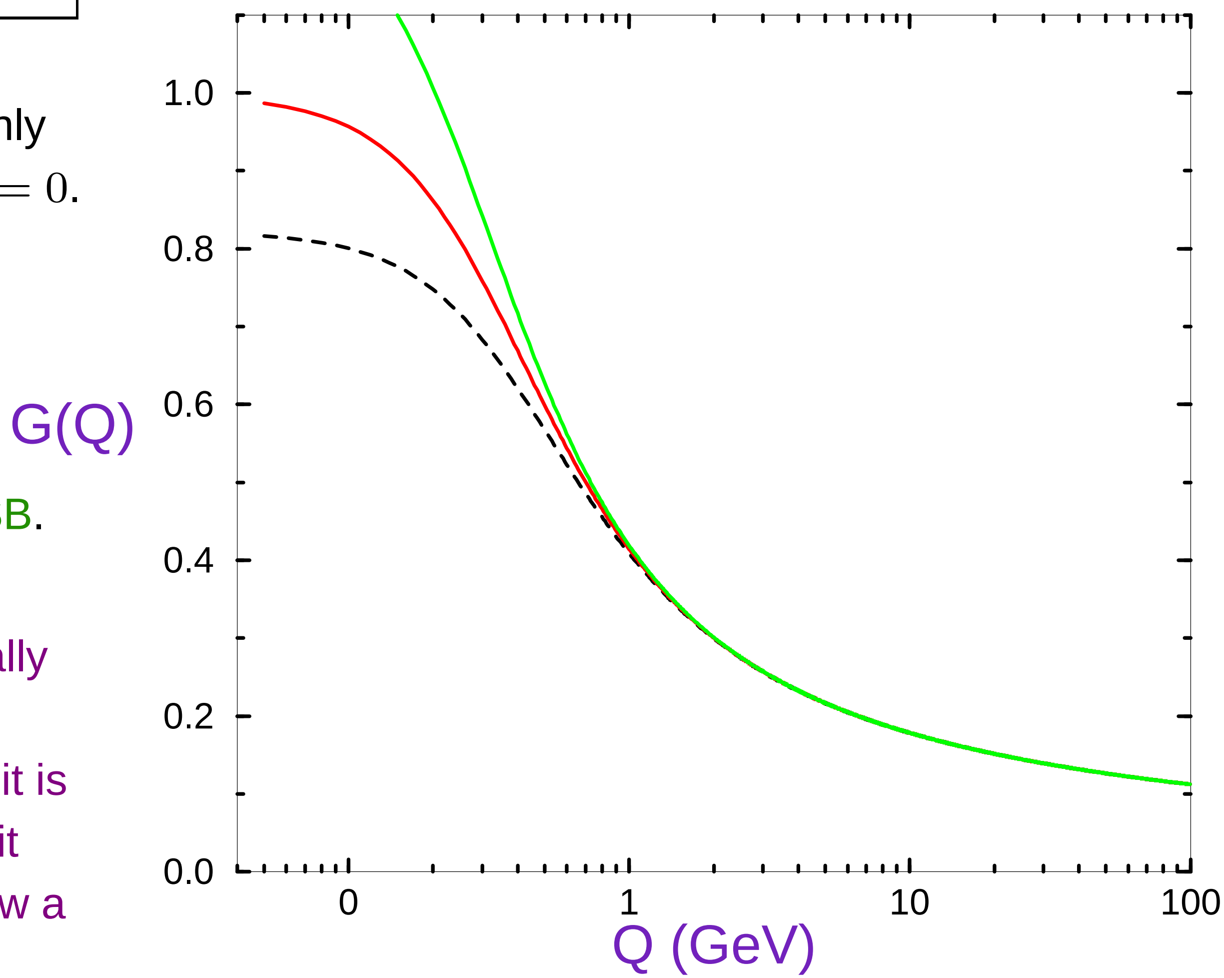
- Large $s$: $M(s) \sim m_0$
- Small $s$ $M \gg m_0$ This is the essential characteristic of DCSB
- $p^2$-dependent mass function is quintessential feature of QCD.
- No solution of $s + M(s)^2 = 0$ **confinement**.



# Real World Alternatives

$$g^2 D(Q^2) = 4\pi \frac{G(Q^2)}{Q^2}$$

- $G(0) < 1$: $M(s) \equiv 0$ is only solution for $m = 0$.
- $G(0) \geq 1$ $M(s) \neq 0$ is possible and energetically favoured: DCSB.
- $M(0) \neq 0$ is a new, dynamically generated mass-scale. If it is large enough, it can explain how a theory that is apparently massless (in the Lagrangian) possesses the spectrum of a massive theory.



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- Confinement and Dynamical Chiral Symmetry Breaking are Key Emergent Phenomena in QCD
- Understanding requires Nonperturbative Solution of Fully-Fledged Relativistic Quantum Field Theory
  - Mathematics and Physics still far from being able to accomplish that

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