Aspects of Hadron Physics

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Nucleon Properties

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- Maris & Tandy . . . series of five papers . . . excellent description of light pseudoscalar and vector mesons . . . basket of 31 masses/couplings/radii with r.m.s. error of 15% . . . moreover, prediction of $F_\pi(Q^2)$ measured in Hall A.

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Pieter Maris

Peter Tandy

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- One parameter model . . . parameter specifies long-range interaction between light-quarks . . . model-independent results in ultraviolet
- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks

Nucleon Properties

- Another Direction . . . Also want/need information about three-quark systems

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- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.

Proton Form Factors, Reprise

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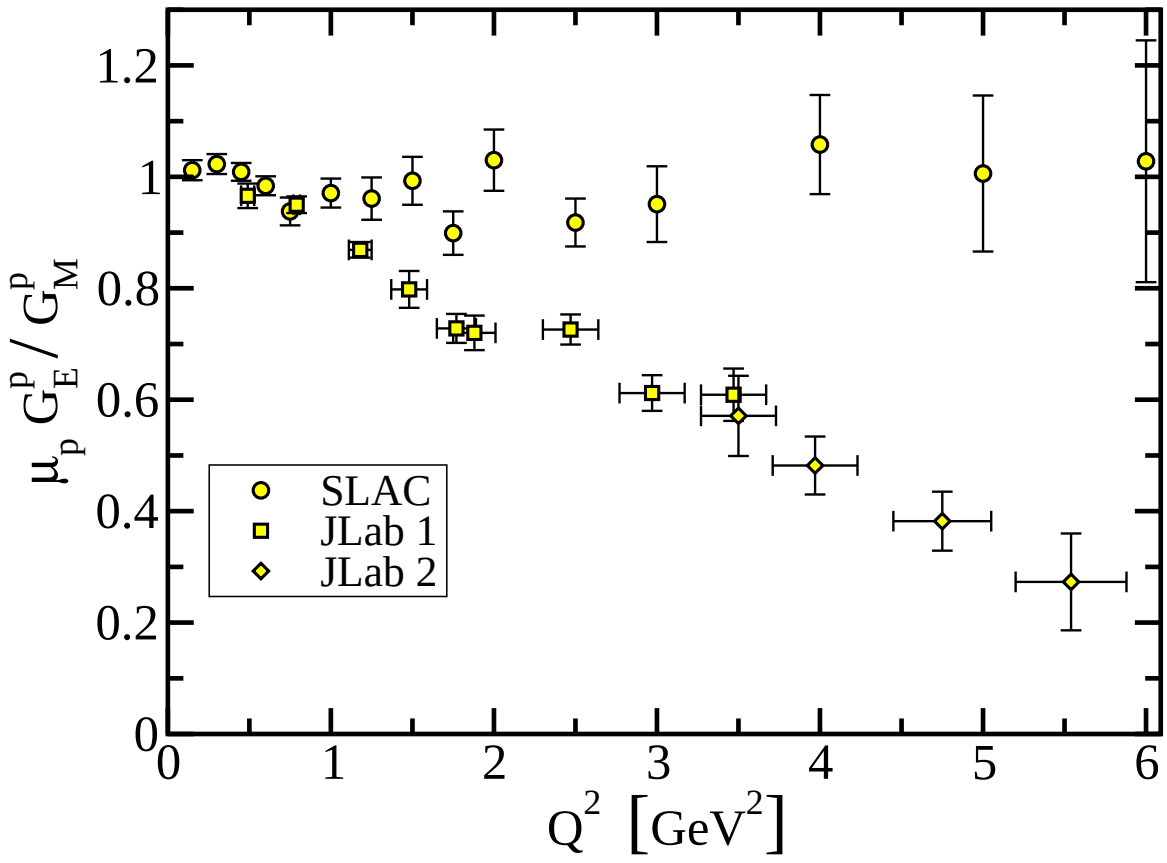
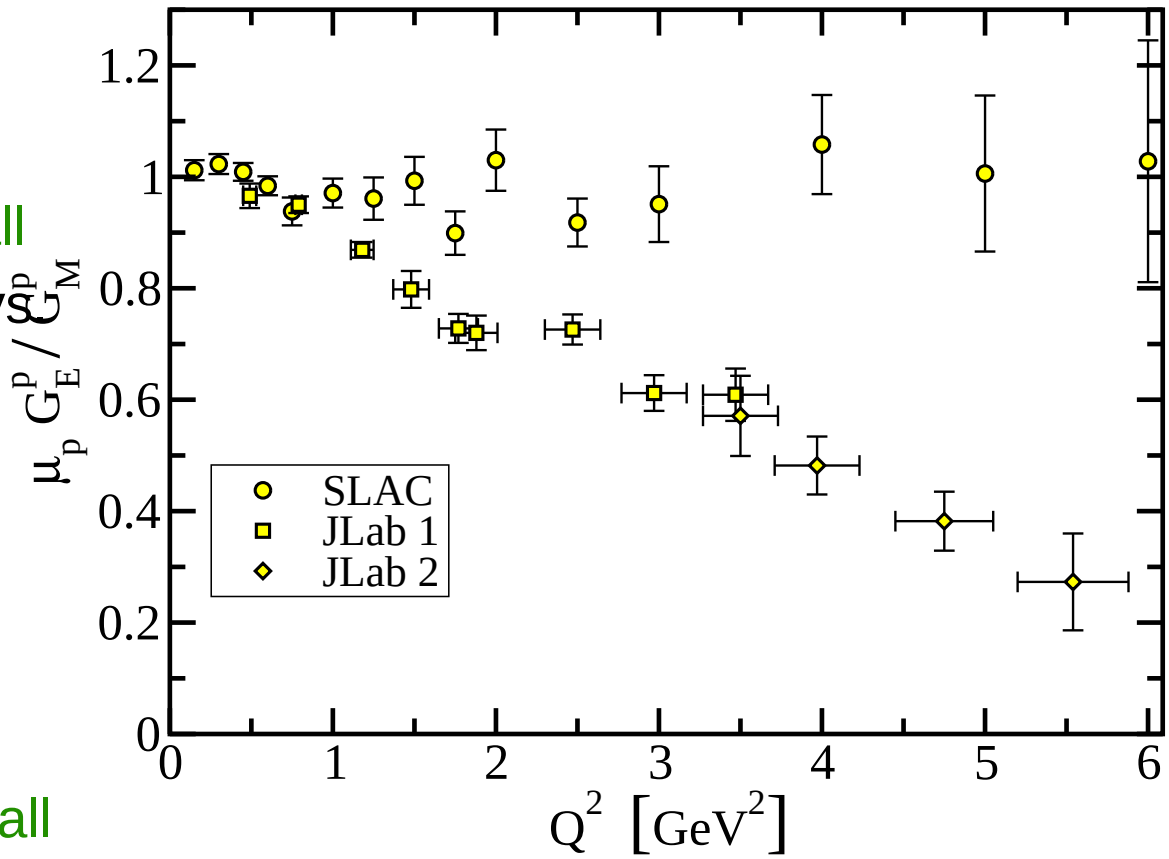
- SLAC and JLab have Measured Ratio of Proton's Electric and Magnetic Form Factors

Proton Form Factors, Reprise



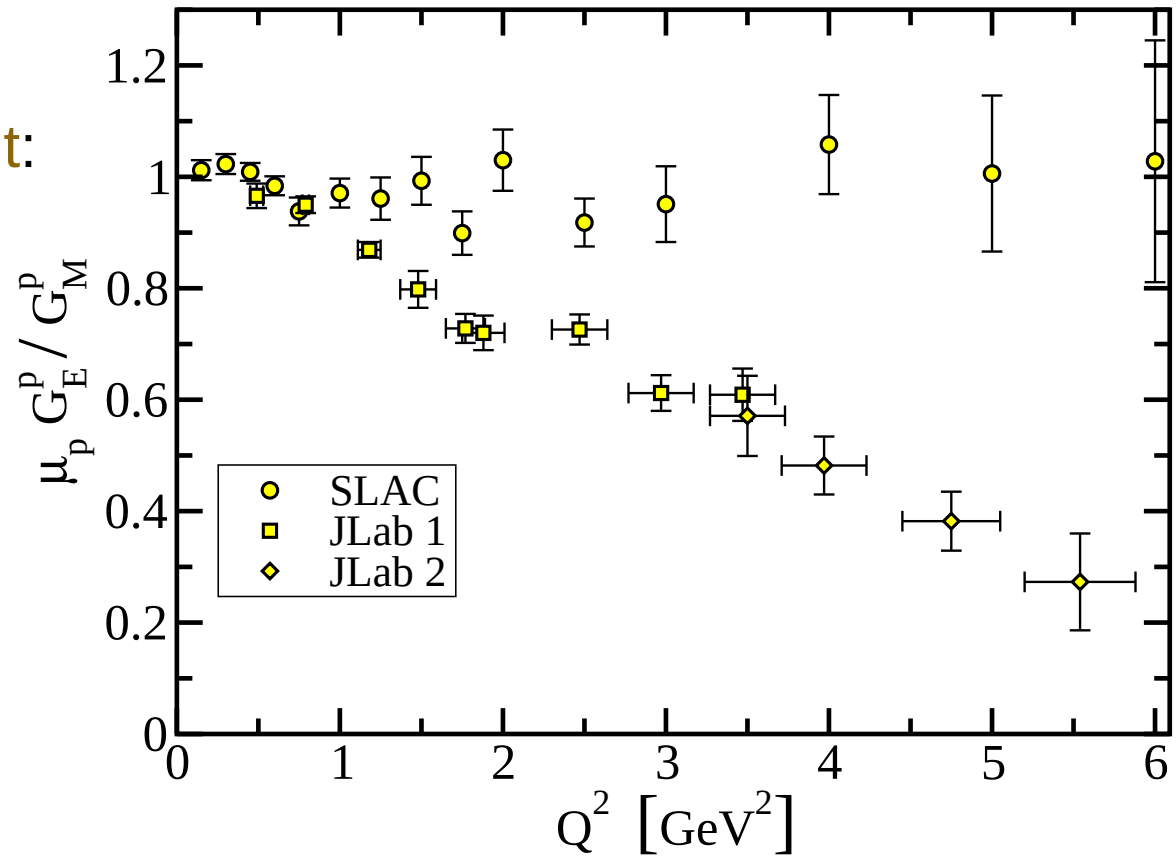
Proton Form Factors, Reprise

- Walker *et al.*, Phys. Rev. **D 49**, 5671 (1994). (SLAC)
- Jones *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **84**, 1398 (2000)
- Gayou, *et al.*, Phys. Rev. **C 64**, 038202 (2001)
- Gayou, *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **88** 092301 (2002)



Proton Form Factors, Reprise

- If JLab Correct, then Completely Unexpected Result: In the Proton
  - On Relativistic Domain
  - Distribution of Quark-Charge Not Equal Distribution of Quark-Current!



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- 2010: Determine all nucleon electromagnetic form factors accurately to a momentum transfer of 3.5 GeV^2

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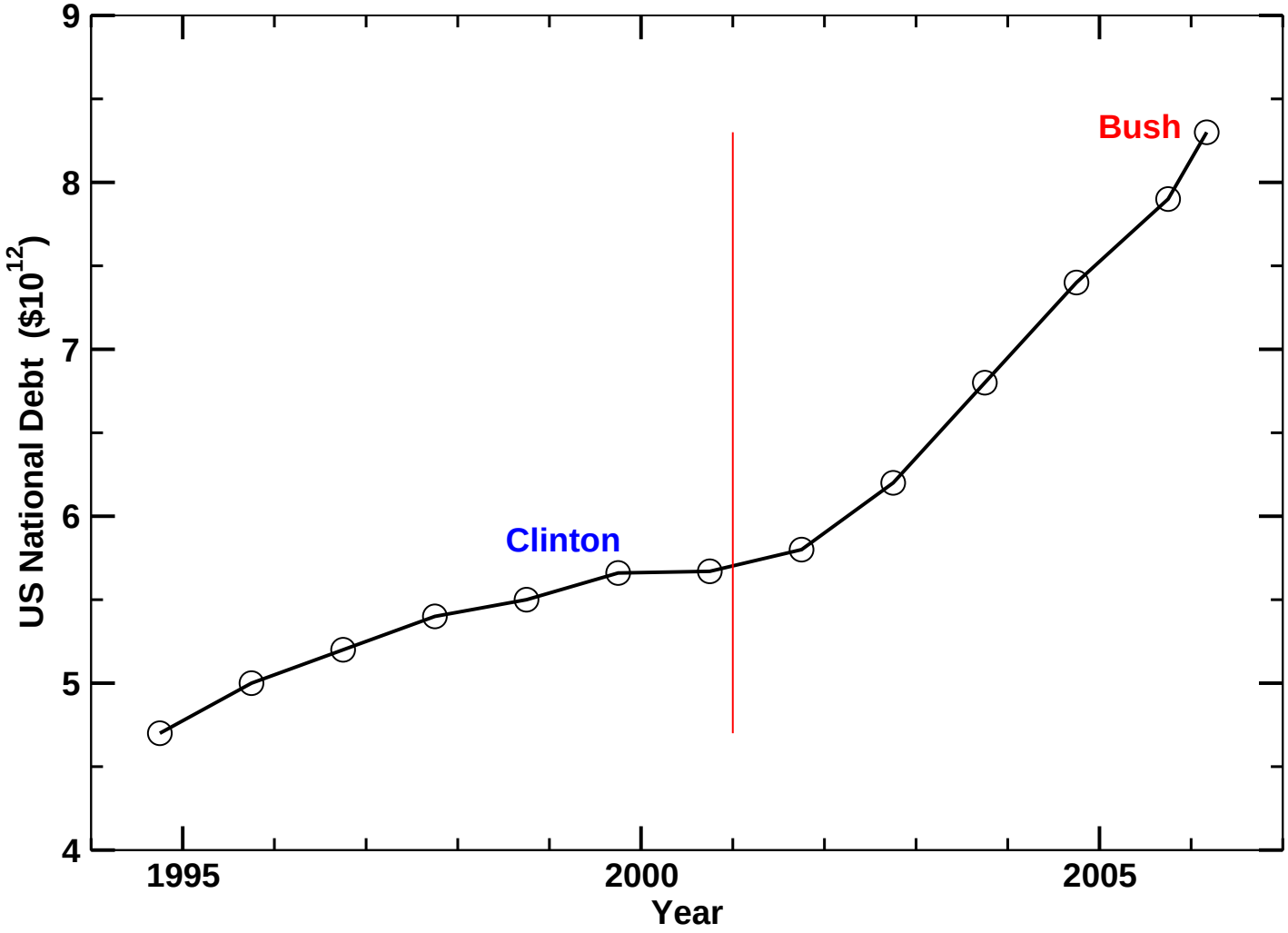
- 2010: Determine all nucleon electromagnetic form factors accurately to a momentum transfer of 3.5 GeV^2
- 2014:
  - Calculate nucleon electromagnetic form factors via Numerical Simulations of lattice-regularised full-QCD
  - Carry out ab initio microscopic studies of the structure and dynamics of light nuclei based on two-nucleon and many-nucleon forces and lattice QCD calculations of hadron interaction mechanisms relevant to the origin of the nucleon-nucleon interaction.

Administration's Milestones



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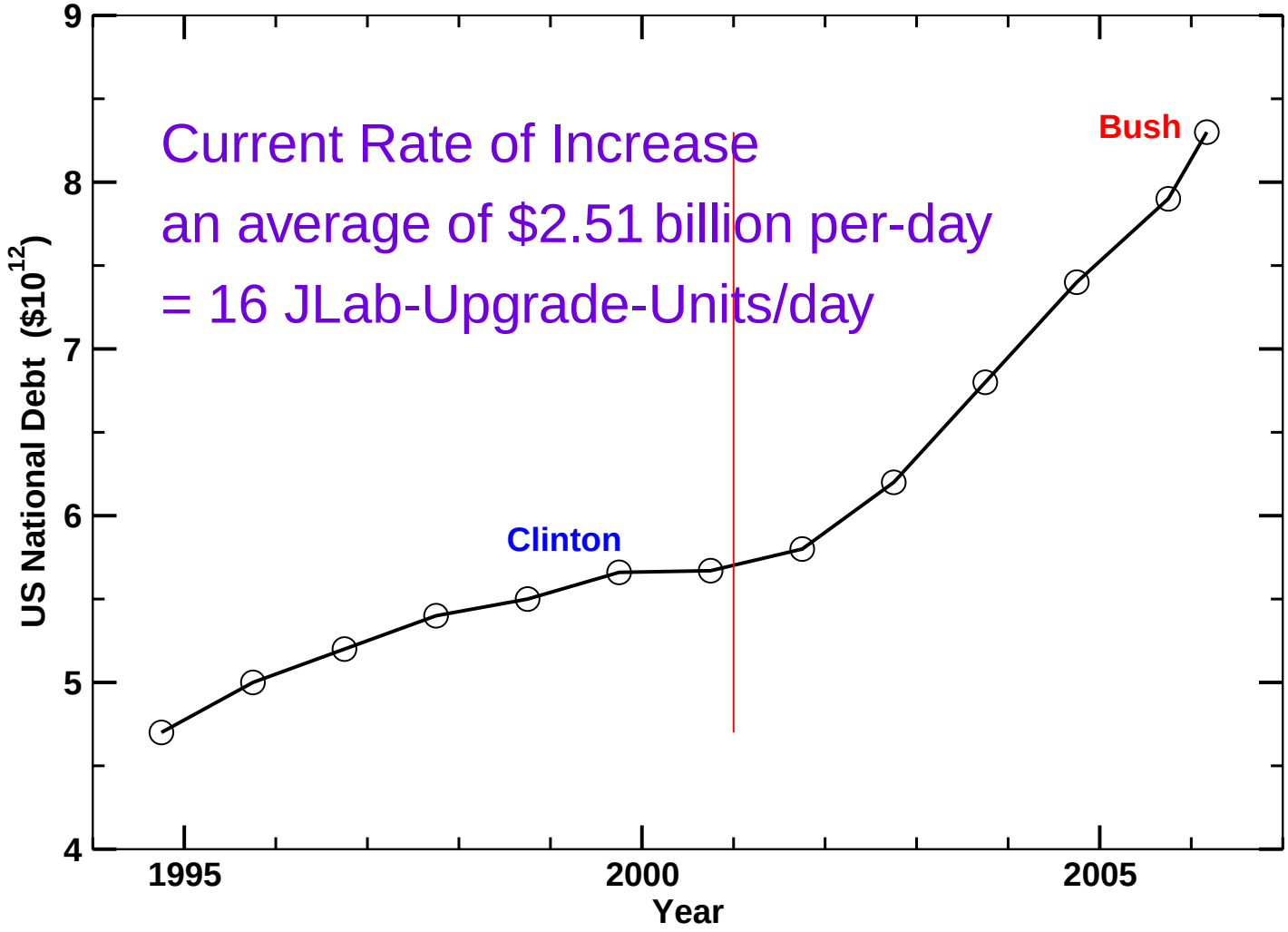

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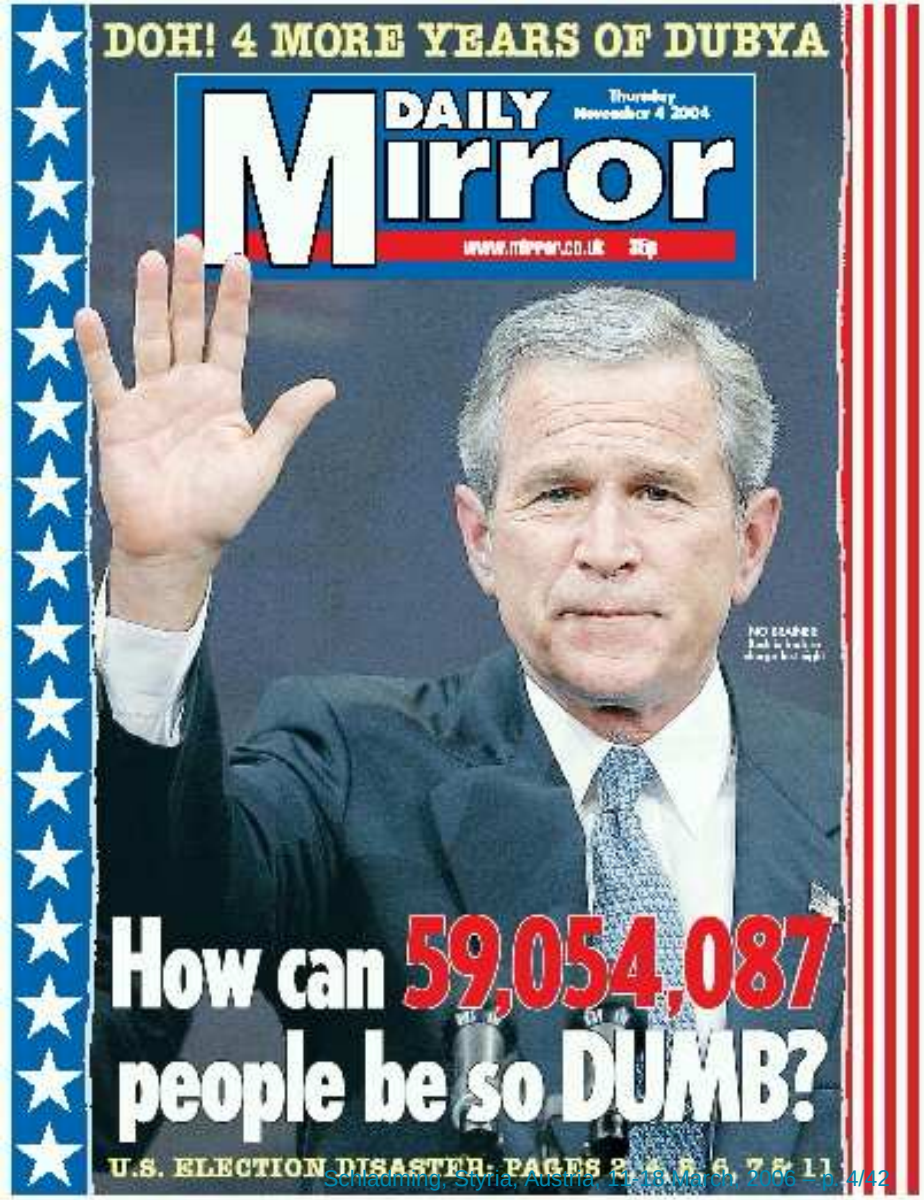
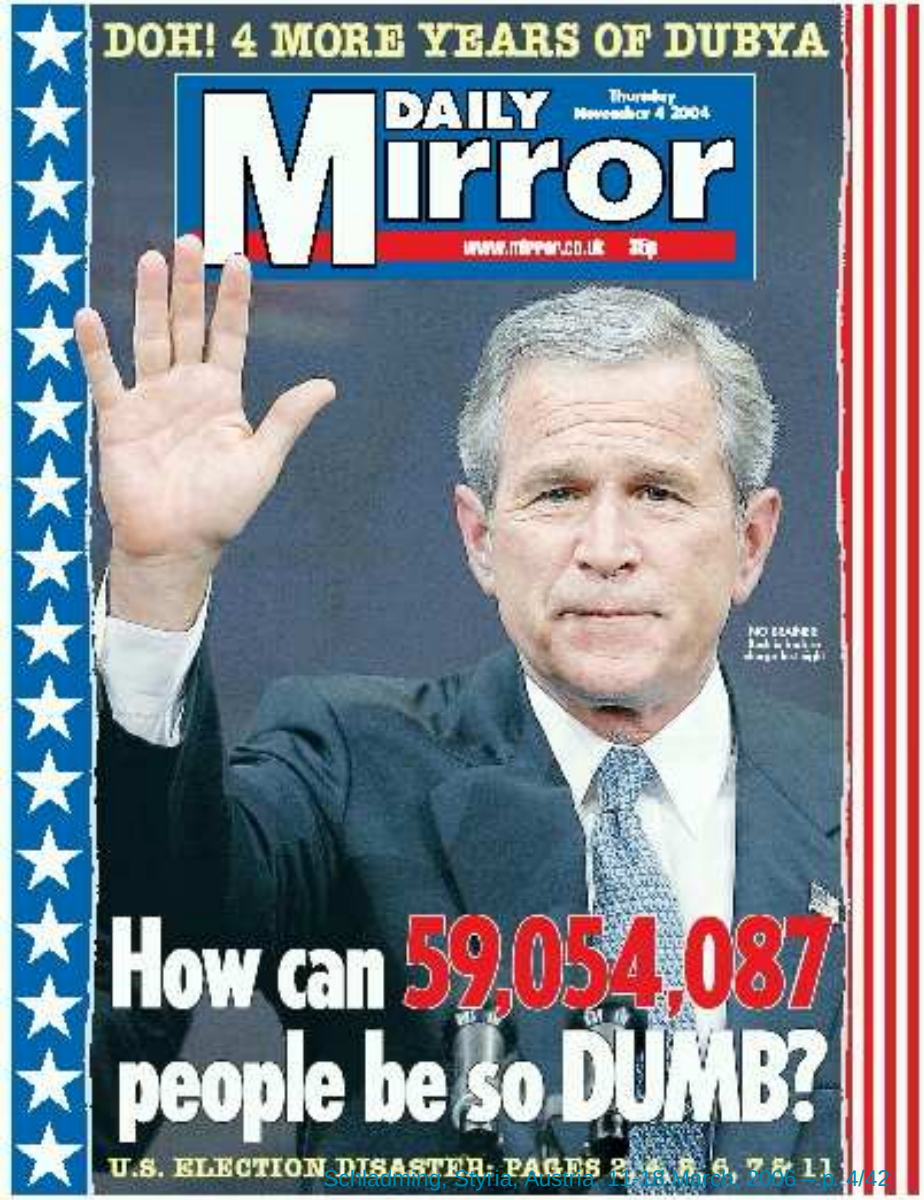
- FY07: Administration's commitment to double federal investment basic research programs over the next ten years
- Is planning possible? . . . Remember FY05: Men on Mars?

Administration's Milestones



Administration's Milestones

- That, too, is a fair question



Nucleon EM Form Factors: A Précis

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- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

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- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

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m_{qq} (GeV)	0.74	0.88

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m_{qq} (GeV)	0.95	1.05	1.13

$(qq)_{J^P}$	$(uu)_{1-}$	$(us)_{1-}$	$(ss)_{1-}$
m_{qq} (GeV)	1.47	1.53	1.64

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- "Energy denominators:" 0^+ and 1^+ are dominant

Faddeev equation

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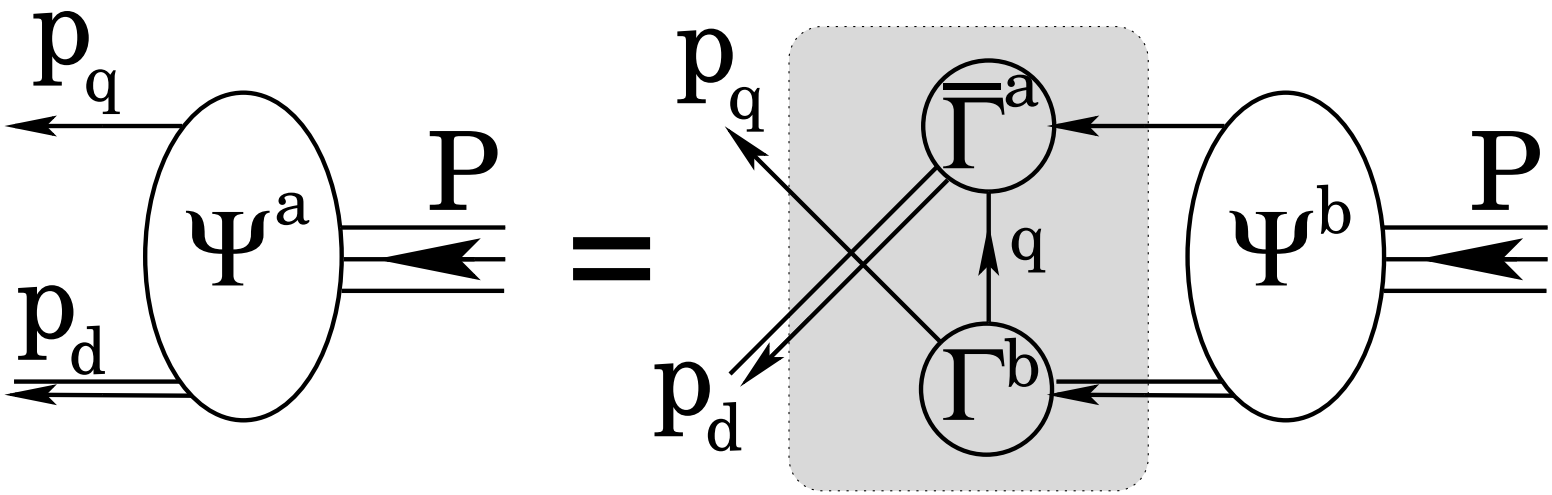

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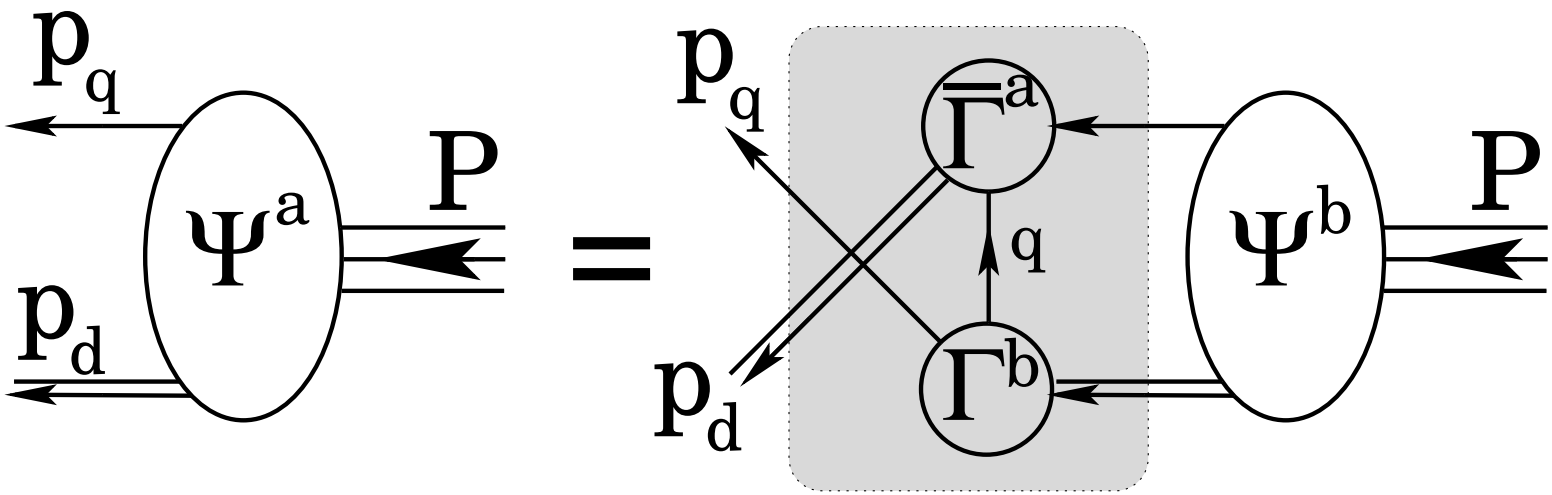
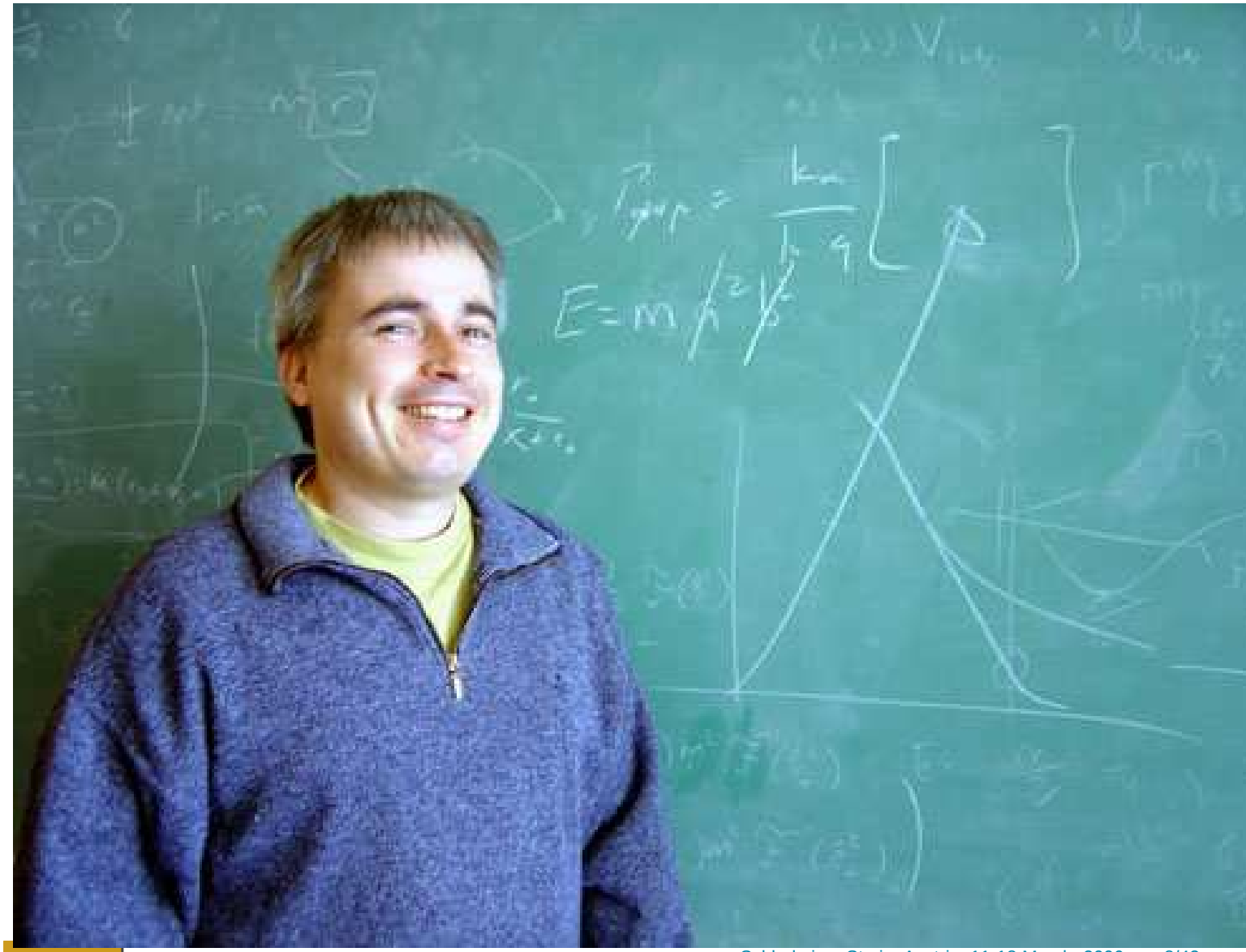


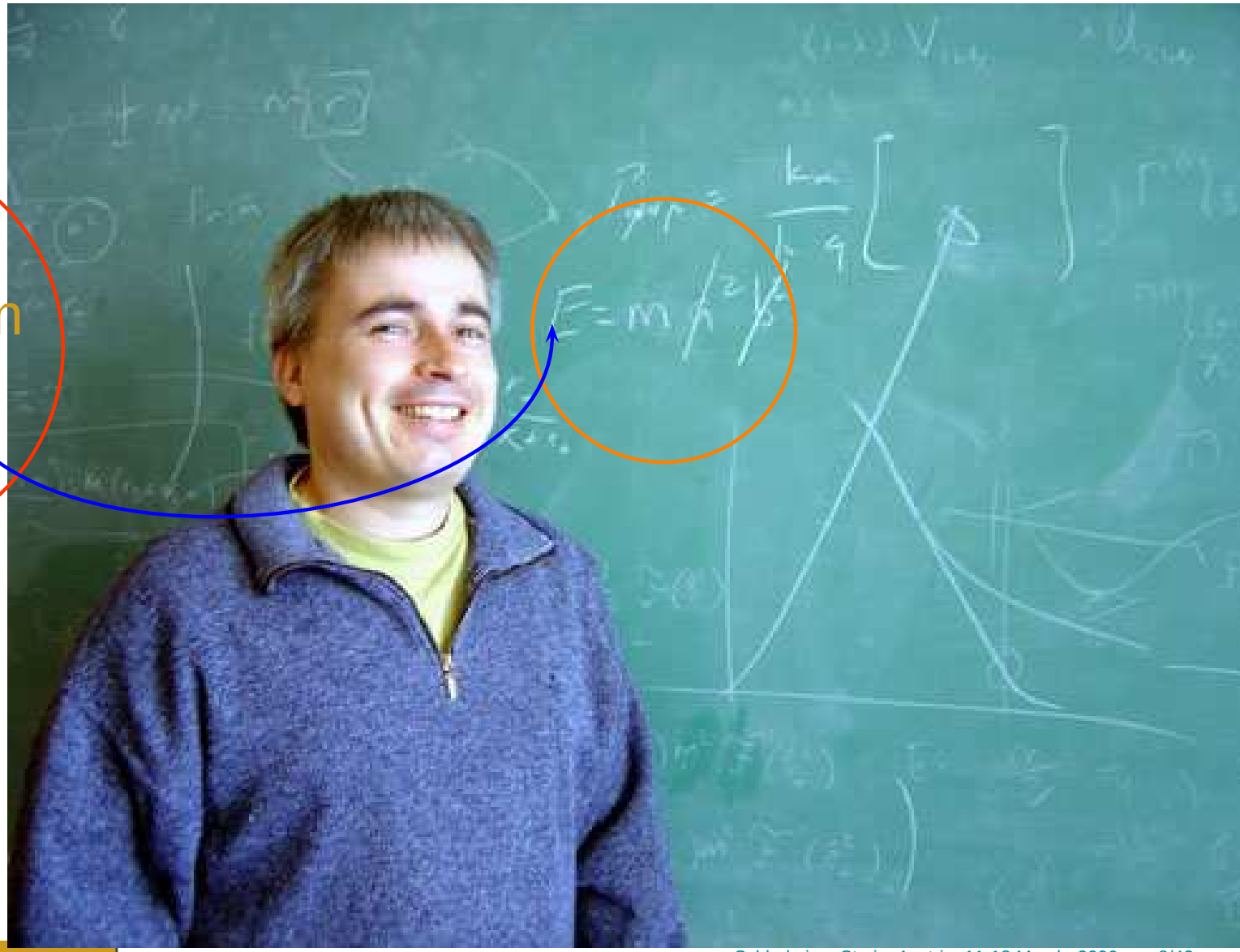
- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Arne Höll



Arne Höll

Closing in on something



Parametrising diquark properties

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- Dressed-quark . . . fixed by DSE and Meson Studies

  . . . Burden, Roberts, Thomson, Phys. Lett. **B 371**, 163 (1996)

Parametrising diquark properties

- Bethe-Salpeter-Like Amplitudes

$$\Gamma^{0^+}(k;K) = \frac{1}{\mathcal{N}^{0^+}} H^a \, C i\gamma_5 \, i\tau_2 \, \mathcal{F}(k^2/\ \omega_{0^+}^2\ ),$$

$$\mathtt{t}^i \Gamma_\mu^{1^+}(k;K) = \frac{1}{\mathcal{N}^{1^+}} H^a \, i\gamma_\mu C \, \mathtt{t}^i \, \mathcal{F}(k^2/\ \omega_{1^+}^2\ )$$

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- Colour matrices:

$$\{H^1 = i\lambda^7, H^2 = -i\lambda^5, H^3 = i\lambda^2\}, \epsilon_{c_1 c_2 c_3} = (H^{c_3})_{c_1 c_2}$$

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- Two parameters: ω_{0^+}, ω_{1^+}
  - $\sim$ Inverse of diquarks' configuration-space size

Parametrising diquark properties

- Pseudoparticle Propagators

$$\Delta^{0^+}(K) = \frac{1}{m_{0^+}^2} \mathcal{F}(K^2/\omega_{0^+}^2)\,,$$

$$\Delta^{1^+}_{\mu\nu}(K) = \left(\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1^+}^2}\right) \frac{1}{m_{1^+}^2} \mathcal{F}(K^2/\omega_{1^+}^2)$$

- $$\mathcal{F}(x) = \frac{1-\exp(-x)}{x}$$
  - Absence of a Spectral Representation
  - Realisation of Confinement

Parametrising diquark properties

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- Two parameters: m_{0^+}, m_{1^+}

  $\sim$ Inverse of diquarks' configuration-space correlation length

Parametrising diquark properties

- Total of four parameters

. . . reduce that via Normalisation Condition

$$\frac{d}{dK^2}\left(\frac{1}{m_{J^P}^2}\,\mathcal{F}(K^2/\omega_{J^P}^2)\right)^{-1}\Bigg|_{K^2=0} = 1 \;\Rightarrow\; \omega_{J^P}^2 = \frac{1}{2}\,m_{J^P}^2\,,$$

Accentuates free-particle-like propagation characteristics of the diquarks within hadron.

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- Solve Faddeev Equation
- Vary m_{0^+} and m_{1^+} to obtain desired masses for N and Δ

Nucleon's self-energy - pion loop

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$$
\begin{aligned}
\Sigma(P) &= 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2(P,k)\,\Delta_\pi((P-k)^2) \\
&\quad \times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5} \\
&= i\gamma\cdot k\,[\mathcal{A}(k^2)-1] + \mathcal{B}(k^2)
\end{aligned}
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- Pseudovector coupling
- Completely equivalent to pseudoscalar coupling

  IF that is treated completely

  - Tadpole contribution can't be neglected

(Hecht, Oettel, Roberts, Schmidt, Tandy, Thomas: nucl-th/0201084)

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- $g_{PV}(P,k)$, πN vertex function
- Calculated using Γ_π and Ψ_N
  - Always soft: Monopole $\lambda \sim 0.6$ GeV

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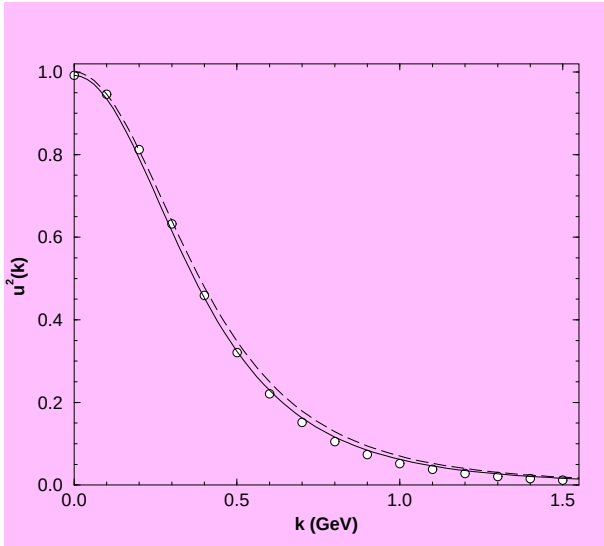
  - Always soft: Monopole $\lambda \sim 0.6$ GeV
    - Corresponds to range $r_\lambda \sim 0.8$ fm
      … pion cloud does not penetrate deeply within nucleon.



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\end{aligned}
$$

$$
\begin{aligned}
G(k) &= 1/[i\gamma\cdot k + M + \Sigma(P)] \\
&= -i\gamma\cdot k\,\sigma_{\mathcal{V}}(k^2) + \sigma_{\mathcal{S}}(k^2)
\end{aligned}
$$

Pole Position Not Known *a priori*

- Mass shift calculated via self-consistent solution

Nucleon's self-energy - pion loop

$$\Sigma(P) = 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2(\text{const.})\, \Delta_\pi((P-k)^2)$$
$$\times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5}$$
$$= i\gamma\cdot k\,[\mathcal{A}(k^2)-1] + \mathcal{B}(k^2)$$

Obtain Integral Equation Kernels

$$\int d\Omega_k\, f((P-k)^2) = \frac{2}{\pi}\int_{-1}^{1} dz\sqrt{1-z^2}\, f(P^2+k^2-2Pkz)$$

E.g.

$$\omega_{\mathcal{B}}(P^2,k^2) = \int d\Omega_k \frac{(P-k)^2}{(P-k)^2+m_\pi^2} = 1 - \frac{2\,m_\pi^2}{a+\sqrt{a^2-b^2}}\,,$$

$$a = P^2+k^2+m_\pi^2,\ \mathrm{b} = 2Pk$$

Nucleon's self-energy - pion loop

$$\begin{aligned}\Sigma(P) &= 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2(P,k)\,\Delta_\pi((P-k)^2) \\ &\quad \times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5} \\ &= i\gamma\cdot k\,[\mathcal{A}(k^2)-1] + \mathcal{B}(k^2)\end{aligned}$$

- But $g_{PV} = g_{PV}(P^2, k^2, P\cdot k)$

  Therefore, In General, Kernel only known Numerically

- Complicates analysis . . .

  locating, incorporating poles in integrand

Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

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- Let's look what happens when $m_\pi \longrightarrow 0$
  - Minkowski Space
  - Pseudovector Coupling

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  - Minkowski Space
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- One-loop nucleon self energy

$$\boxed{\Sigma(P) = 3i\frac{g^2}{4M^2}\int \frac{d^4k}{(2\pi)^4}\,\Delta(k^2, m_\pi^2)\,\not{k}\gamma_5\,G_0(P-k)\,\not{k}\gamma_5\,.}$$

This integral is divergent. Assume a Poincaré covariant regularisation, characterised by a mass-scale λ

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- Decompose nucleon propagator into positive and negative energy components

$$\begin{aligned} G_0(P) &= \frac{1}{\not{P} - M_0} = G_0^+(P) + G_0^-(P) \\ &= \frac{M}{\omega_N(\vec{P})}\left[\Lambda_+(\vec{P})\frac{1}{P_0 - \omega_N(\vec{P}) + i\varepsilon} + \Lambda_-(\vec{P})\frac{1}{P_0 + \omega_N(\vec{P}) - i\varepsilon}\right] \end{aligned} \quad (4)$$

$\omega_N^2(\vec{P}) = \vec{P}^2 + M^2$, and $\Lambda_\pm(\vec{P}) = (\tilde{\not{P}} \pm M)/(2M)$, $\tilde{P} = (\omega(\vec{P}), \vec{P})$

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- Shift in the mass of a positive energy nucleon nucleon:

$$\delta M_+ = \tfrac{1}{2}\mathrm{tr}_D\left[\Lambda_+(\vec{P}=0)\,\Sigma(P_0 = M, \vec{P}=0)\right]$$

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- Focus on positive energy nucleon's contribution to the loop integral; i.e., $\Delta(k)\,G^+(P-k)$, which we denote: $\delta_F\,M_+^+$

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- Focus on positive energy nucleon's contribution to the loop integral; i.e., $\Delta(k)\,G^+(P-k)$, which we denote: $\delta_F M_+^+$
  - To evaluate k_0 integral, close contour in lower half-plane, thereby encircling only the positive-energy pion pole.

$$\delta_F M_+^+ = -3g^2\int \frac{d^3k}{(2\pi)^3}\,\frac{\omega_N(\vec{k}^2) - M_0}{4\,\omega_N(\vec{k}^2)}\,\frac{1}{\omega_\pi(\vec{k}^2)\left[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_0\right]} \tag{9}$$

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- On the domain for which the regularised integral has significant support, assume that M_0 is very much greater than all other mass scales.

$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \tag{15}$$

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- Then

$$\delta_F M_+^+ \approx -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{8\,M^2} \frac{1}{\omega_{\lambda_i}^2(\vec{k}^2)} \tag{20}$$

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- So that

$$\frac{d^2\,\delta_F M_+^+}{(dm_\pi^2)^2} \approx -\frac{3g^2}{4M^2} \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{\omega_\pi^6(\vec{k}^2)} = -\frac{9}{128\pi} \frac{g^2}{M^2} \frac{1}{m_\pi}\,. \tag{25}$$

Nucleon Self Energy: Chiral Limit

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$$\delta_F M_+^+ = -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\omega_N(\vec{k}^2) - M_0}{4\,\omega_N(\vec{k}^2)} \frac{1}{\omega_\pi(\vec{k}^2)\,[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_0]} \tag{26}$$

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$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \tag{27}$$

- Then

$$\delta_F M_+^+ \approx -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{8\,M^2} \frac{1}{\omega^2_{\lambda_i}(\vec{k}^2)} \tag{28}$$

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- Namely $\delta_F M_+^+ = -\frac{3}{32\pi}\frac{g^2}{M^2} m_\pi^3 + f^+_{(1)}(\lambda_1,\lambda_2)\, m_\pi^2 + f^+_{(0)}(\lambda_1,\lambda_2)$ where the last two terms express the necessary contribution from the regulator.

Nucleon Self Energy: Chiral Limit

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  - This is the Leading Nonanalytic Contribution much touted in effective field theory.
  - Its form is completely fixed by chiral symmetry and the pattern of its dynamical breaking.

NB. Contribution from negative energy nucleon is $\propto \dfrac{1}{M^3}$.

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- The remaining terms are regular in the current-quark mass. Their exact nature depends on the explicit form of regularisation procedure.

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- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.
- The Leading Nonanalytic Contribution is a somewhat magical model-independent result.

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- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.
- The Leading Nonanalytic Contribution is a somewhat magical model-independent result.
- Unfortunately, it is not of much relevance in the real world. The actual value of the pion loop contribution to the nucleon's mass is completely determined by the regularisation dependent terms.
  - It is essential for a framework to veraciously express the leading nonanalytic contribution . . . it serves as a check that DCSB is truly described.
  - However, beyond that, one must accept that the world is messy.
    - The pion has a finite size. So does the nucleon.
    - These sizes set the mass-scale which determines the nucleon's mass shift.

Model pion-nucleon coupling

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- $\mathcal{B}$-Kernel

$$\int d\Omega_k\, g_{PV}^2((P-k)^2)\,\left[1 - \frac{2m_\pi^2}{(P-k)^2 + m_\pi^2}\right]$$

Clearly the sum of two independent terms.

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$$\int d\Omega_k\, g_{PV}^2((P-k)^2)\,\left[1-\frac{2m_\pi^2}{(P-k)^2+m_\pi^2}\right]$$

- First term can be evaluated exactly

$$\begin{aligned}\bar{g}_{PV}^2(P^2,k^2) &= \int d\Omega_k\, g_{PV}^2((P-k)^2)\\ &= \frac{g^2}{4M^2}\,\mathrm{e}^{-2(P^2+k^2)/\Lambda^2}\,\frac{\Lambda^2}{2Pk}\,\mathrm{I}_1(4Pk/\Lambda^2)\,,\end{aligned}$$

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$$\int d\Omega_k\, g_{PV}^2((P-k)^2)\,\left[1 - \frac{2m_\pi^2}{(P-k)^2 + m_\pi^2}\right]$$

- Second term can be approximated

$$\begin{aligned}\omega_{g^2}(P^2,k^2) &= 2\,m_\pi^2 \int d\Omega_k\, \frac{g_{PV}^2((P-k)^2)}{(P-k)^2 + m_\pi^2} \\ &\approx g_{PV}^2(|P-k|^2)\frac{2\,m_\pi^2}{a + \sqrt{a^2 - b^2}}\end{aligned}$$

- Reliable when analytic structure of g_{PV} is not key to that of solution

Model pion-nucleon coupling

$$g_{PV}(P,k) = \frac{g}{2M}\,\exp(-(P-k)^2/\Lambda^2)$$

- $\mathcal{B}$-Kernel

$$\int d\Omega_k\, g_{PV}^2((P-k)^2)\,\left[1 - \frac{2m_\pi^2}{(P-k)^2 + m_\pi^2}\right]$$

- Total Kernel:

$$\begin{aligned} &\approx \quad \bar{g}_{PV}^2(P^2,k^2) - g_{PV}^2(|P^2-k^2|)\frac{2\,m_\pi^2}{a+\sqrt{a^2-b^2}}\,, \\ &=: \quad \bar{g}_{PV}^2(P^2,k^2) - \tilde{g}_{PV}^2(P^2,k^2))\frac{2\,m_\pi^2}{a+\sqrt{a^2-b^2}}\,, \end{aligned}$$

- Analytic structure is transparent

Nucleon's self energy and mass shift

Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$
\begin{aligned}
M_D^2 \mathcal{A}^2(-M_D^2) &= [M + \mathcal{B}(-M_D^2)]^2 \\
\delta M &= M_D - M
\end{aligned}
$$

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- Vector self energy

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- Scalar self energy

Nucleon's self energy and mass shift

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$$

	(Λ, Λ_N)	(Λ, Λ_N)	(Λ, Λ_N)
	$(0.9, \infty)$	$(0.9, 1.5)$	$(0.9, 2.0)$
$-\delta M$ (MeV)	222	61	99

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- No suppression for nucleon off-shell in self-energy loop; i.e, $g_{PV}((P - k^2), P^2, k^2)$

Neglected this dependence

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$$g_{PV}(P^2, k^2, P \cdot k) = \frac{g}{2M}\, \mathrm{e}^{-(P-k)^2/\Lambda^2}\, \mathrm{e}^{-(P^2+M^2+k^2+M^2)/\Lambda_N^2}$$

- Correct on-shell limit:

$$g_{PV}(P^2 = -M^2, k^2 = -M^2, (P-k)^2 = 0) = \frac{g}{2M}$$

Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$M_D^2 \mathcal{A}^2(-M_D^2) = [M + \mathcal{B}(-M_D^2)]^2$$
$$\delta M = M_D - M$$

Range from meson exchange model phen.

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- $\Lambda_N \to \infty \Rightarrow$ pointlike nucleon

Pion loop's effect

- Nonpointlike πN-loop

  ... reduces nucleon's mass by ~ 100 MeV

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  ... reduces nucleon's mass by ~ 100 MeV

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  ... reduces nucleon's mass by not more than 100 MeV

- $$\boxed{-\delta M_N \sim 200\,\text{MeV}}$$

- Qualitative effect of this?

Too much of a good thing

Too much of a good thing

- Refit Faddeev model parameters,

  allowing for heavier “quark-core” mass

Too much of a good thing

	ω_{0+}	ω_{1+}	M_N	M_Δ	ω_{f_1}	ω_{f_2}	R
0^+	0.45	-	1.44	-	0.36	0.35	2.32
0^+ & 1^+	0.45	1.36	1.14	1.33	0.44	0.36	0.54
0^+	0.64	-	1.59	-	0.39	0.41	1.28
0^+ & 1^+	0.64	1.19	0.94	1.23	0.49	0.44	0.25

50% reduction in role of axial-vector diquark

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- 50% reduction in role of axial-vector diquark
- 10% increase in role of scalar diquark

Too much of a good thing

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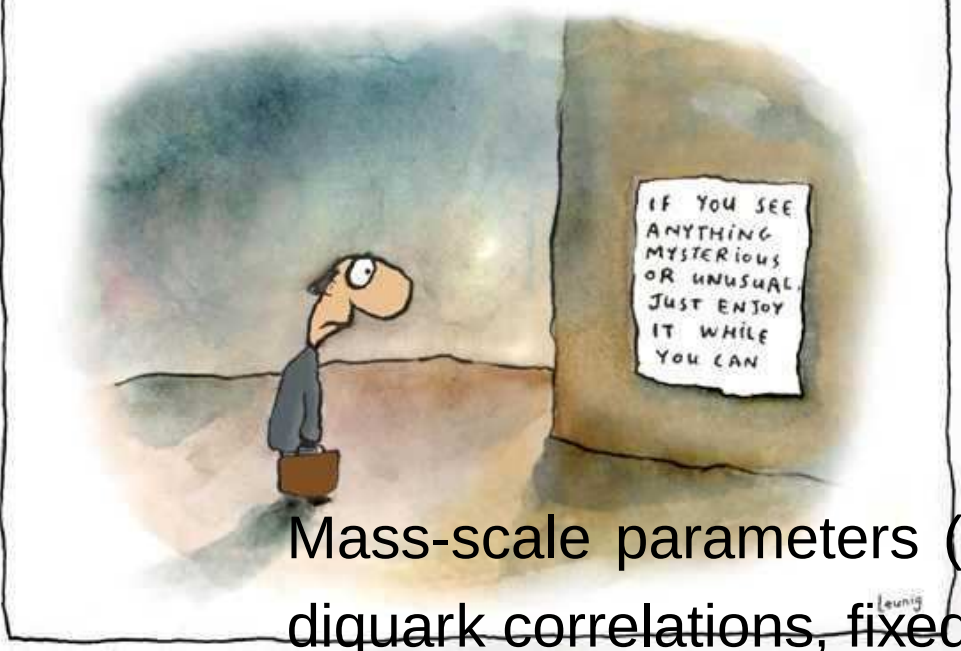
Unsurprisingly:

Requiring Exact Fit to N, Δ masses

with only q, $(qq)_{J^P}$ Degrees of Freedom

$\Rightarrow$ Forces 1^+ to mimic, in part, effect of π

Results: Nucleon and Δ Masses



Results: Nucleon and Δ Masses

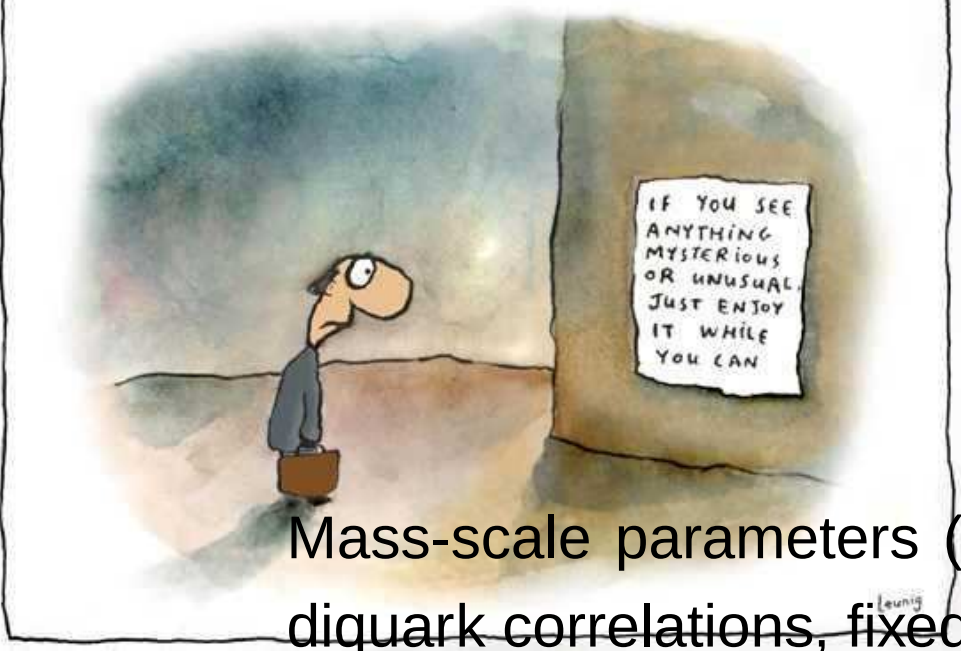
Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV



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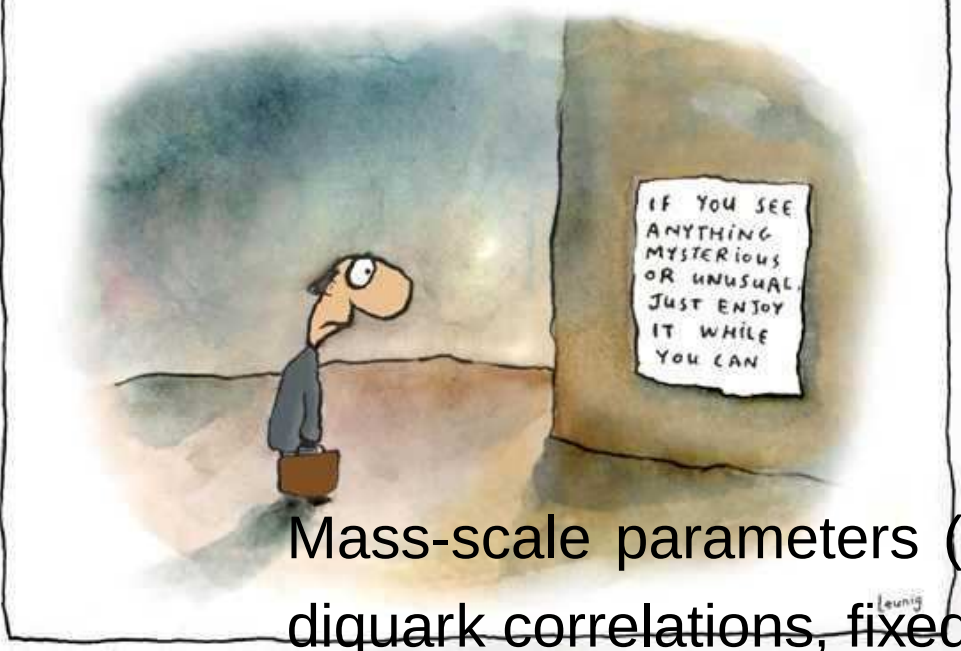
- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV
- Axial-vector diquark provides significant attraction



Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV
- Constructive Interference: 1^{++}-diquark $+ \partial_\mu \pi$

Sigma Term

Höll, *et al*., nu-th/0510075

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- Important for numerous reasons, some of longstanding.

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- Derivative couplings vanish at zero pion 4-momentum (chiral limit)
  - Therefore non-derivative coupling can be isolated by extrapolating (dispersion relations) isospin-even πN-scattering amplitude to the Cheng-Dashen point: $s = u = M_N,\, t = 2m_\pi^2$

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$$\sigma_N = \sigma_N(t=0) = \Sigma_N - 15\,\text{MeV} = 67 \pm 8$$

Return to Pion Sigma Term

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- The pion's σ-term is defined by

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\begin{aligned}
2\, m_\pi\, \sigma_\pi \quad &:= \quad s_\pi(Q^2 = 0) \\
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&\quad \times \Gamma_\pi(\ell_{-\frac{1}{2},0}; -P)\, \mathcal{S}(\ell)\, \Gamma_\pi(\ell_{-\frac{1}{2},0}; P)
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- Canonical normalisation condition for Bethe-Salpeter amplitude

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2P_\mu = \mathrm{tr}_{CDF} \int_q^\Lambda \Gamma_\pi(q; -P) \frac{\partial}{\partial P_\mu} \mathcal{S}(q + Q/2) \Gamma_\pi(q; P) \mathcal{S}(q - Q/2) + \text{sym}
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Feynman-Hellmann Theorem

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- In deriving $\boxed{\sigma_\pi = \bar{m}(\zeta)\frac{\partial m_\pi}{\partial \bar{m}(\zeta)}}$, I have depended heavily upon the fact that the rainbow-ladder expression is the leading term in a systematic, nonperturbative and symmetry preserving truncation.

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  - Present case: theorem states that response of an eigenvalue of the QCD mass^2-operator to a change in a parameter in that operator is given by expectation value of the derivative of the mass^2-operator operator with respect to the parameter.

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$$\boxed{\frac{\partial E_{M^2}}{\partial \lambda} = \left\langle \frac{\partial M^2}{\partial \lambda} \right\rangle}$$

Derived by Feynman, when 21, in his final year as an undergraduate. Has played an important role in theoretical chemistry and condensed matter physics.

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$$\boxed{\frac{\partial E_{M^2}}{\partial \lambda} = \left\langle \frac{\partial M^2}{\partial \lambda} \right\rangle}$$

- The result is valid in this form for all mesons; i.e.,

$$\boxed{2\, m_M \sigma_M := s_M(0) = \bar{m}(\zeta)\frac{\partial m_M^2}{\partial \bar{m}(\zeta)} \;\Rightarrow\; \sigma_M = \bar{m}(\zeta)\frac{\partial m_M}{\partial \bar{m}(\zeta)}}$$

NB. The σ-term is a renormalisation point invariant, in general and also in the explicit calculation, so long as a RGI rainbow-ladder truncation is used.

Constituent-Quark Sigma Term

Wright, *et al*., nu-th/0512048

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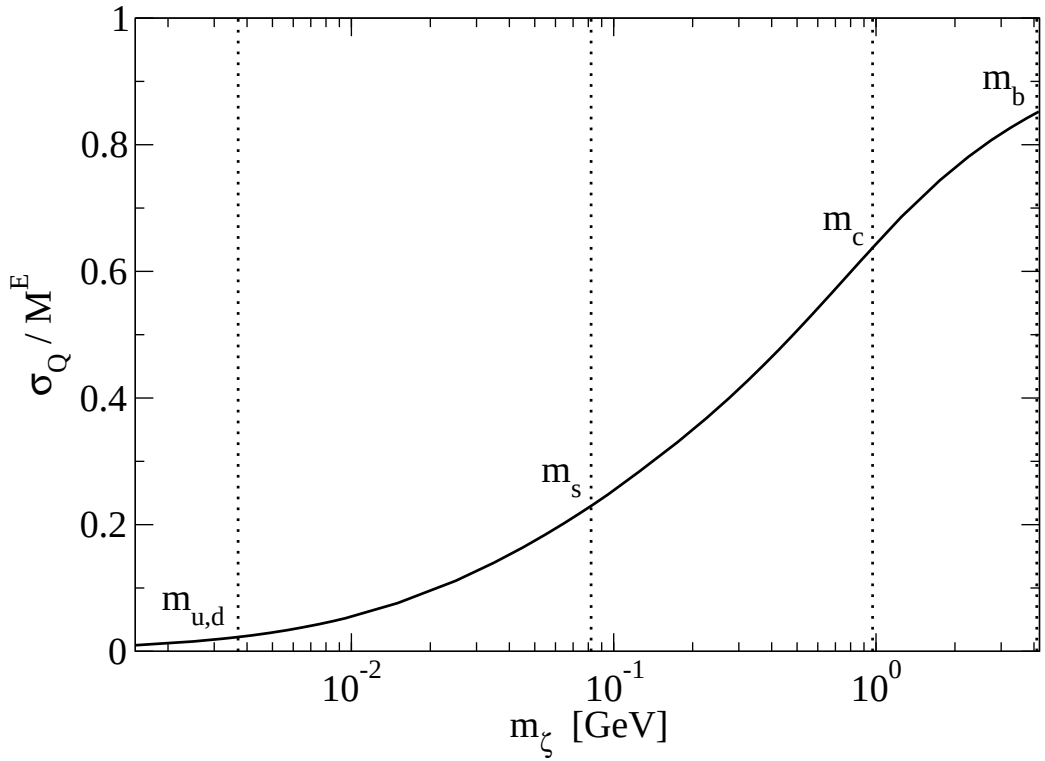

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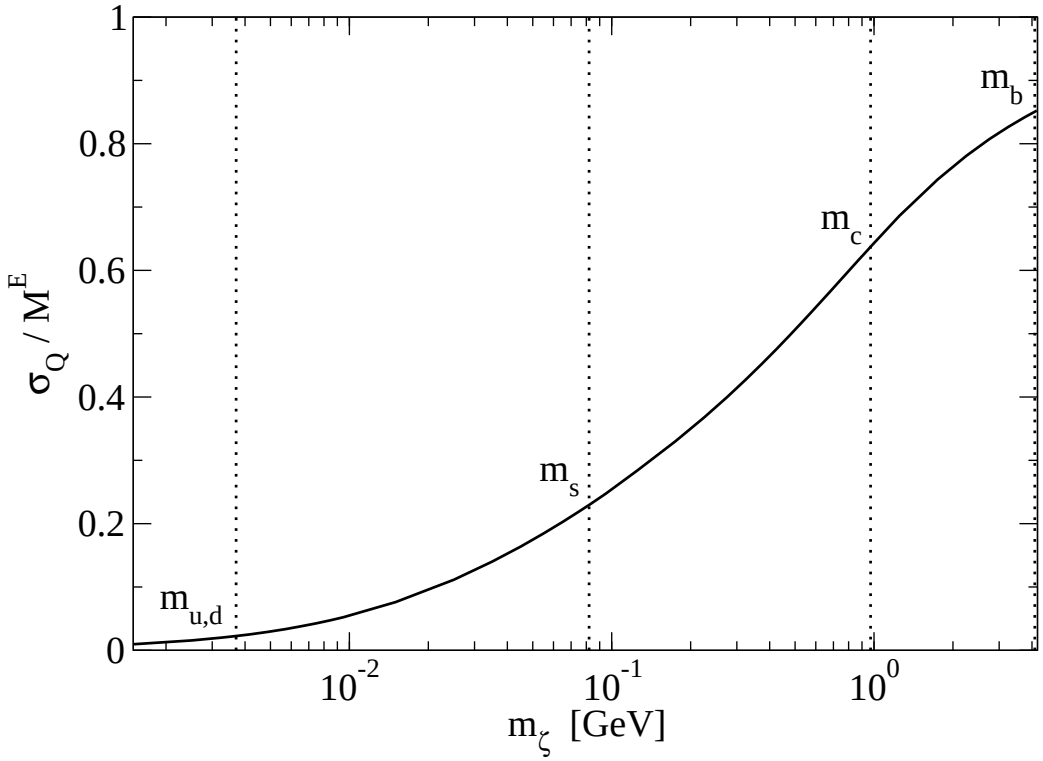

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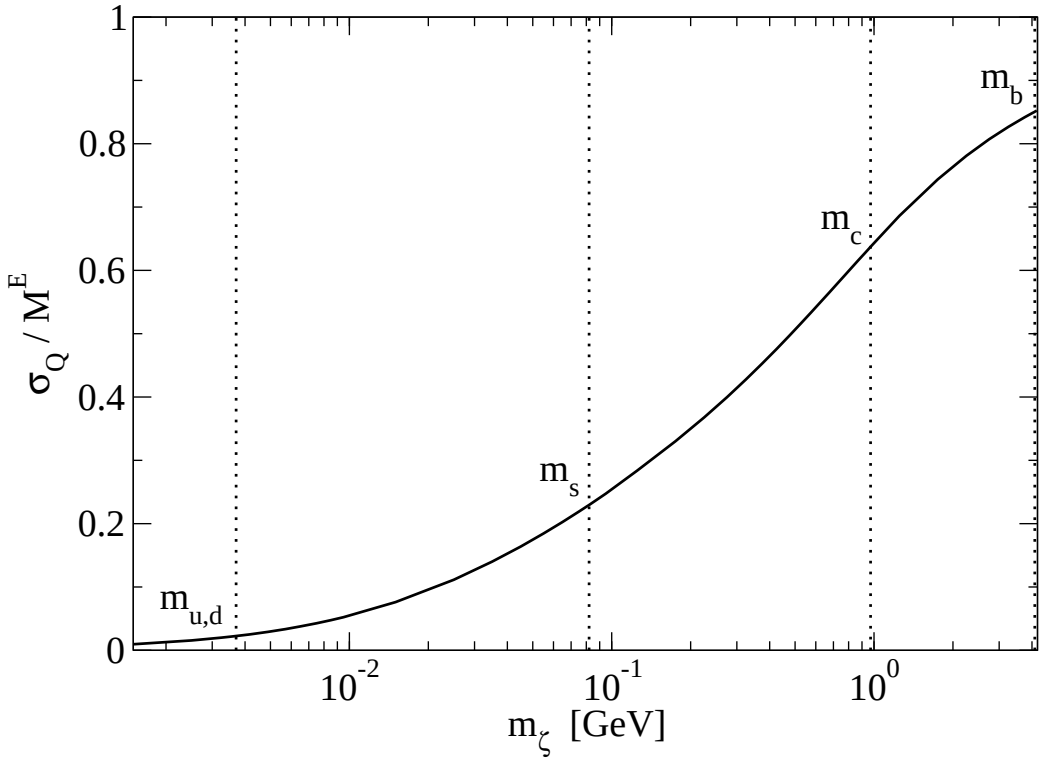

$$\frac{\sigma_{u,d}}{M_{u,d}^E} := \frac{m_{u,d}(\zeta)}{M_{u,d}^E}\,\frac{\partial M_{u,d}^E}{\partial m_{u,d}(\zeta)} = 0.023$$

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- σ-terms of the nucleon and Δ can thus be determined in any framework that provides for a calculation of the current-quark-mass dependence of M_N and M_Δ.

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  - $\sigma_Q := m(\zeta)\,\dfrac{\partial M^E}{\partial m(\zeta)} = 6.2\,\mathrm{MeV}$
  - $m_{qq}(m) = m^0_{qq} + m\,\sigma_{qq}$
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Set	M_N	M_Δ	σ_N	σ_Δ
A	0.94	1.23	0.046	0.042
B	1.18	1.33	0.047	0.042
$B_{\tilde{\sigma}}$	1.18	1.33	0.057	0.051

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- $N \longrightarrow \pi N \longrightarrow N$-loop correction to nucleon's mass

$$\delta M_N = -6\pi \frac{f_{NN\pi}^2}{m_\pi^2} \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2\, u^2(\vec{k}^2)}{\omega_\pi(\vec{k}^2)[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_N]}$$

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- Input . . .
  - $f_{NN\pi}^2 = g_{NN\pi}^2 m_\pi^2/(16\pi M_N^2)$,
    $(f_{NN\pi}/M_N) = (g_A/f_\pi) = 13.6\,\text{GeV}^{-1}$
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  - $\omega_\pi(\vec{k}^2) = \sqrt{\vec{k}^2 + m_\pi^2}$
  - $u(\vec{k}^2) = 1/(1 + \vec{k}^2/\lambda^2)$
- And . . . Calculated results:
  - $m_\pi^2 = -2\,m\,\dfrac{\langle \bar{q}q\rangle^\pi}{f_\pi^2}$
  - $M_N^B = 1.13 + 9.20\,m\,,\ m \in [0.0045, 0.0055]$

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- Shift in M_N owing to $\pi\Delta$ self-energy has same sign and is no larger in magnitude $\Rightarrow \delta\sigma_N^{\pi N+\pi\Delta} \simeq 7\,\mathrm{MeV}$
- Shift in M_Δ owing to πN and $\pi\Delta$ self-energies is approximately one-half of correction to $M_N \Rightarrow \delta\sigma_\Delta^{\pi N+\pi\Delta} \simeq 3\,\mathrm{MeV}$

Sigma Terms:

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Summary ... $\dfrac{\delta m_H}{m_H} = \dfrac{\sigma_H}{m_H}\dfrac{\delta \bar{m}}{\bar{m}}$

	π	π_1	σ	ρ	ω	N	Δ
$\dfrac{\sigma_H}{m_H}$	0.498	0.017	0.013	0.021	0.034	0.064	0.041

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- Extensive Expertise in Nuclear Physics Community

Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

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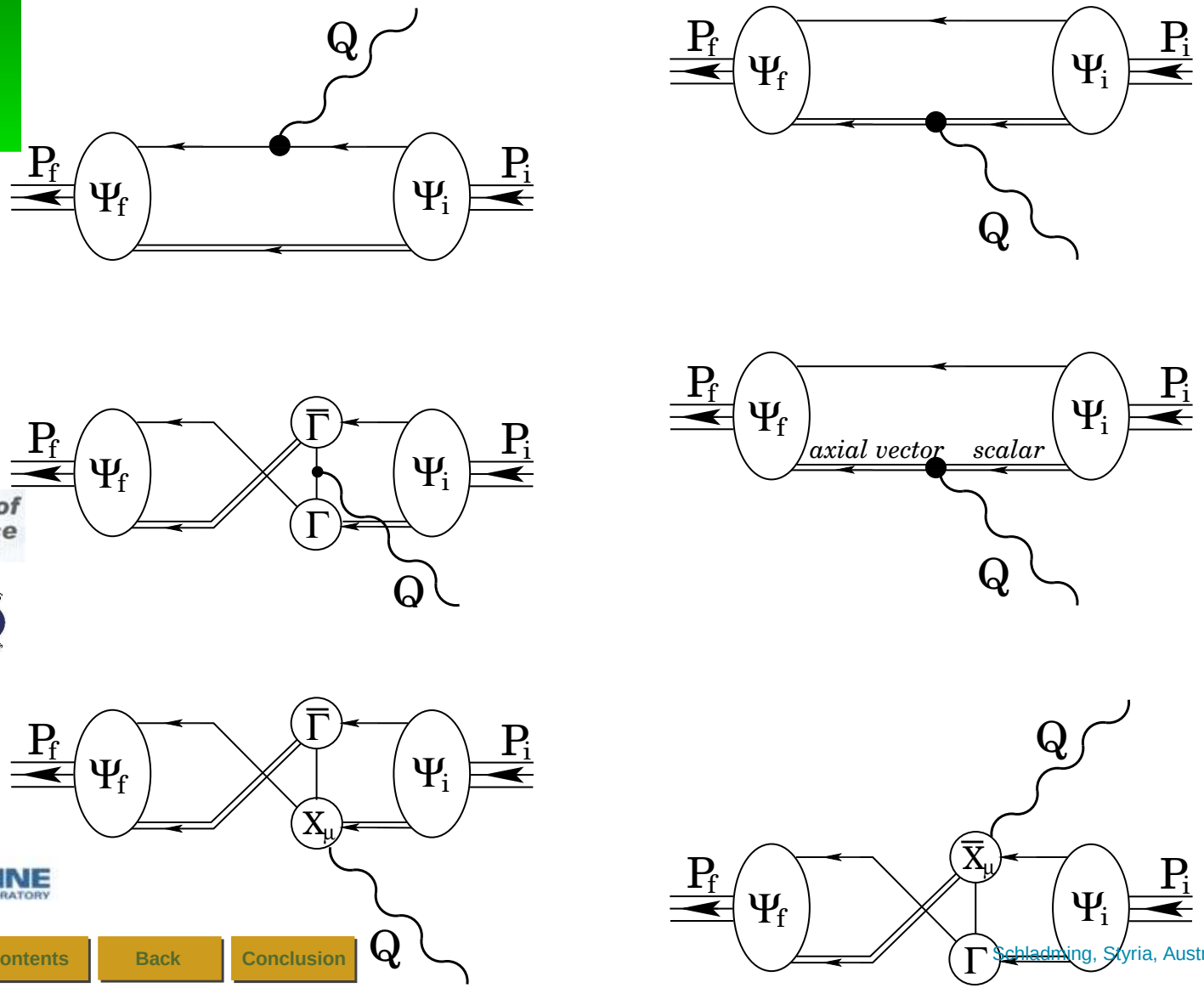

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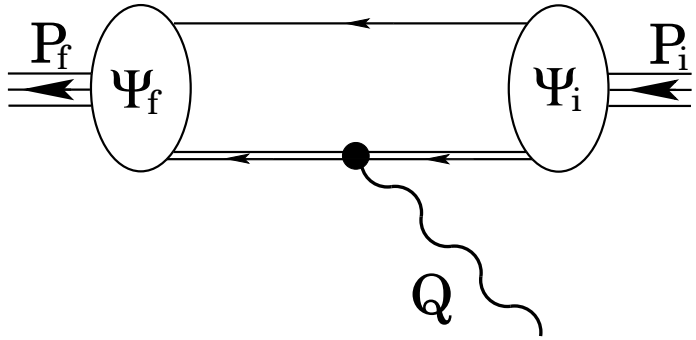

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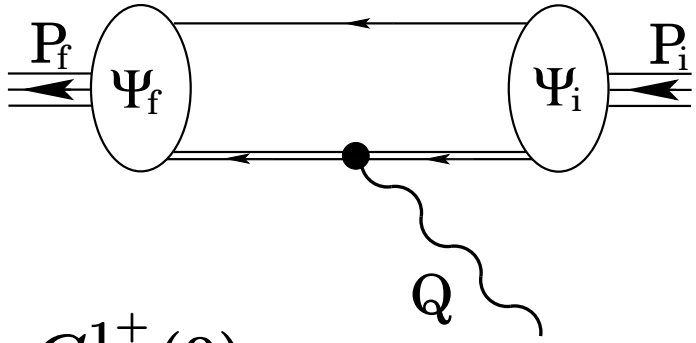

- Three Form Factors

  Two parameters ...

  $$G_{\mathcal{E}}^{1^+}(0) = 1\,,\; G_{\mathcal{M}}^{1^+}(0) = \mu_{1^+}\,,\; G_{\mathcal{Q}}^{1^+}(0) = -\chi_{1^+}\,.$$

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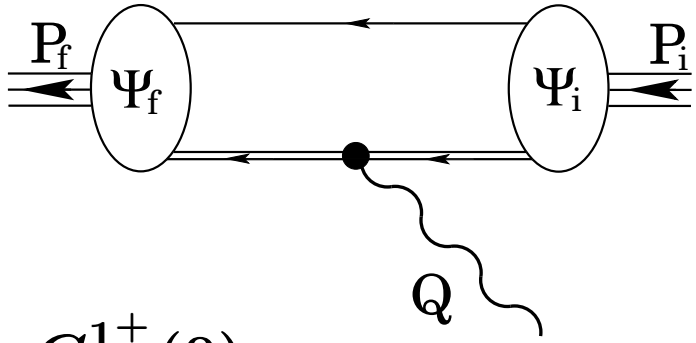

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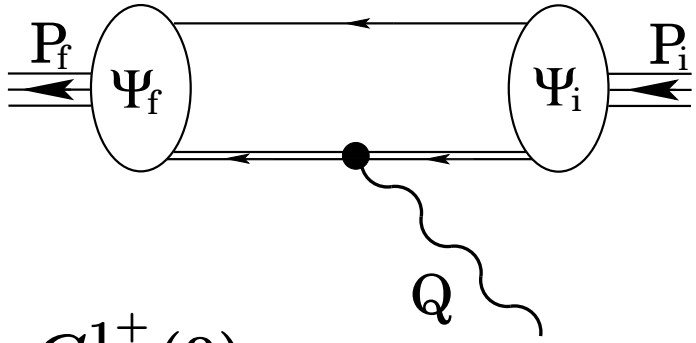

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$$G_{\mathcal{E}}^{1^+}(Q^2) : G_{\mathcal{M}}^{1^+}(Q^2) : G_{\mathcal{Q}}^{1^+}(Q^2) \overset{Q^2\to\infty}{=} (1 - \frac{2}{3}\frac{Q^2}{4m_{1^+}^2}) : 2 : -1$$

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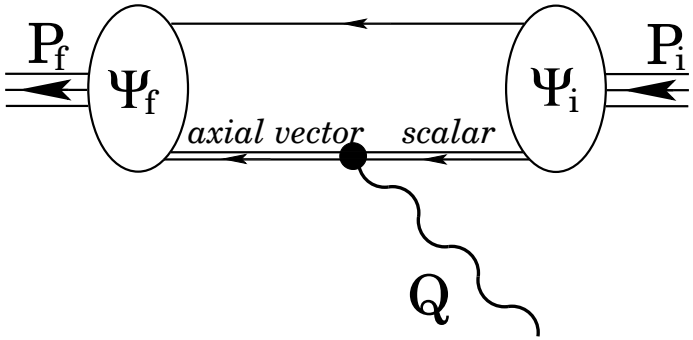

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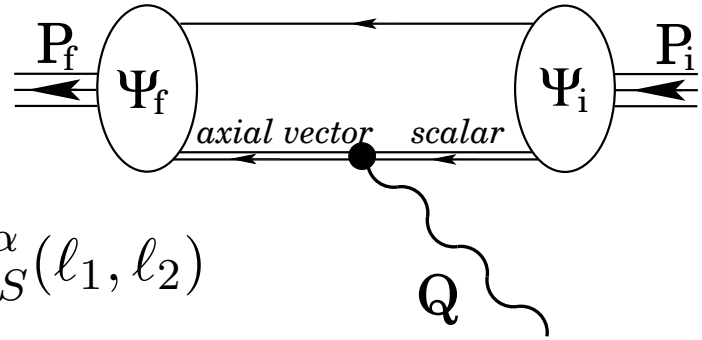

- One parameter . . .

$$\Gamma^{\gamma\alpha}_{SA}(\ell_1,\ell_2) = -\Gamma^{\gamma\alpha}_{AS}(\ell_1,\ell_2) = \frac{i}{M_N}\,\kappa_{\mathcal{T}}\,\varepsilon_{\gamma\alpha\rho\lambda}\ell_{1\rho}\ell_{2\lambda}$$

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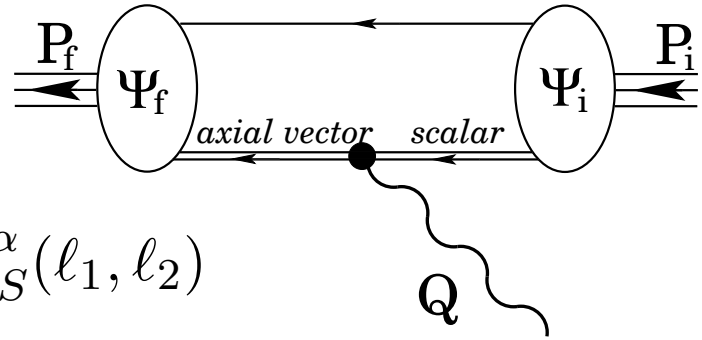

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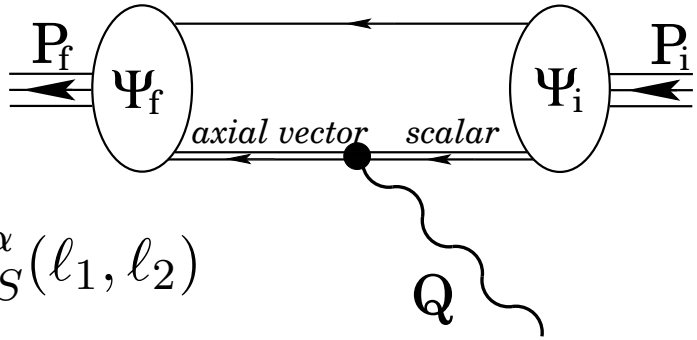

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- DSE-based calculations yield: $\kappa_{\mathcal{T}} = 2$
- All other couplings fixed through experience with DSE studies of mesons
- seagull terms fixed by current-conservation

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Contrast

Magnetic Moments

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- "On Nucleon Electromagnetic Form Factors," Arne Höll, et al., nu-th/0412046, has many tables

Magnetic Moments

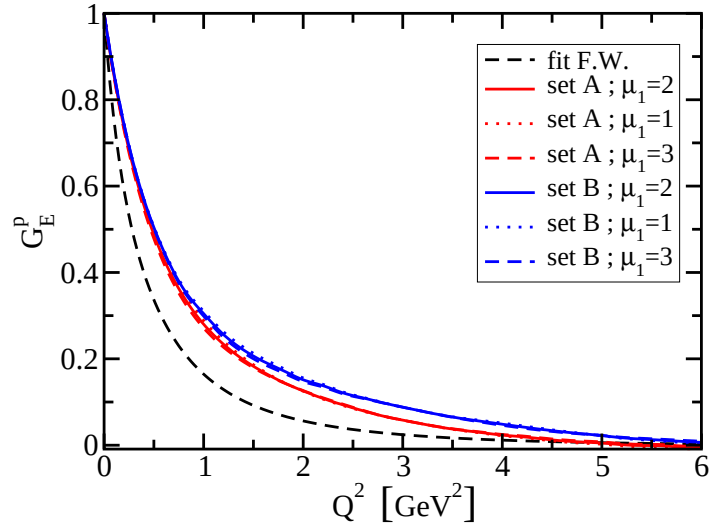
- Illustrate results with magnetic moments

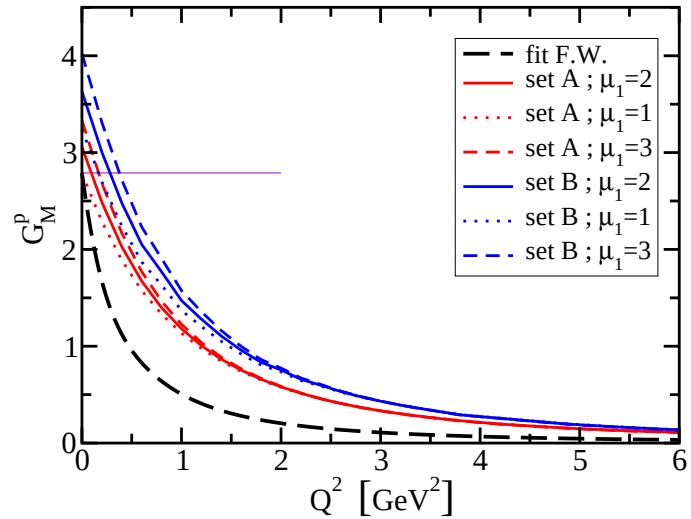
			Set A (no π cloud)				Set B (room for π cloud)			
μ_{1+}	χ_{1+}	$\kappa_{\mathcal{T}}$	κ_p	$\sigma^A_{\kappa_p}$	$\lvert\mu_n\rvert$	$\sigma^A_{\lvert\mu_n\rvert}$	κ_p	$\sigma^B_{\kappa_p}$	$\lvert\mu_n\rvert$	$\sigma^B_{\lvert\mu_n\rvert}$
1	1	2	1.79	-15.3	1.70	-5.1	2.24	-21.9	2.00	-6.2
2	1	2	2.06		1.79		2.63		2.13	
3	1	2	2.33	15.4	1.88	5.1	3.02	21.9	2.26	6.1
2	0	2	2.06	0.0	1.79	0.0	2.63	0.0	2.13	0.0
2	2	2	2.06	0.0	1.79	0.0	2.63	0.0	2.13	0.0
2	1	1	1.91	-8.4	1.64	-8.4	2.45	-10.1	1.95	-8.5
2	1	3	2.21	8.4	1.85	8.3	2.82	10.1	2.31	8.5

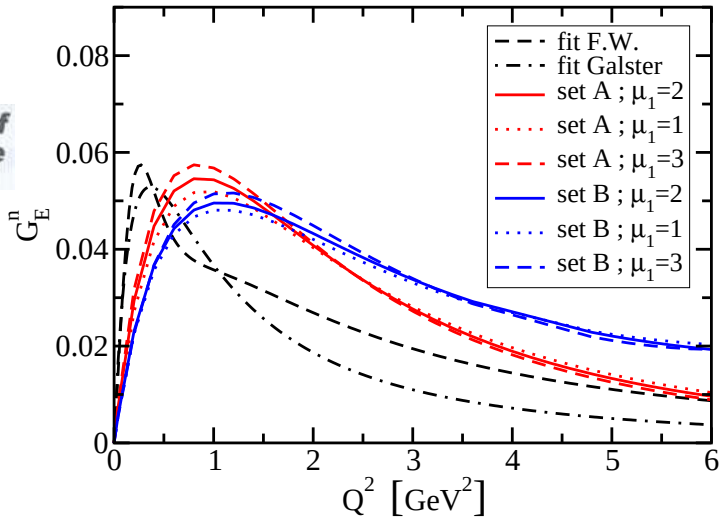
Form Factors

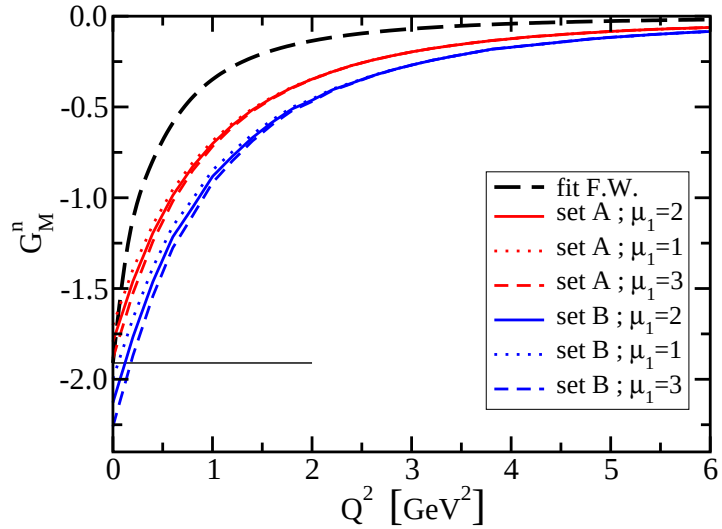
Comparison

Form Factors









Variations in $\mu_{1+} = 1, 2, 3$; $\chi_{1+} = 1$, $\kappa_{\mathcal{T}} = 2$

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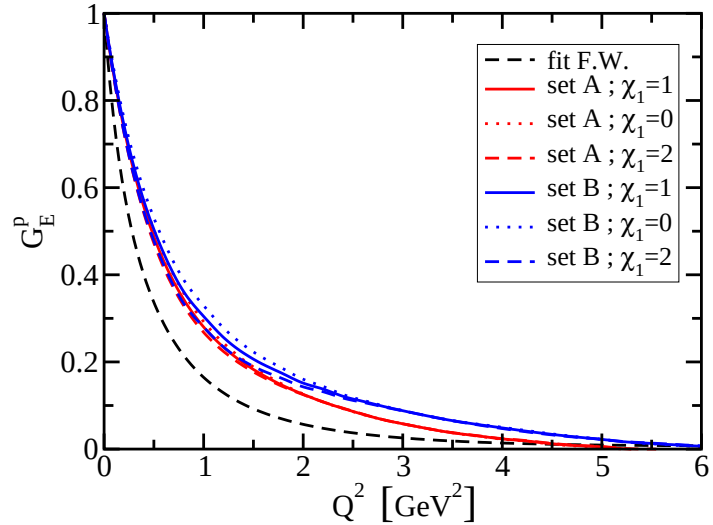

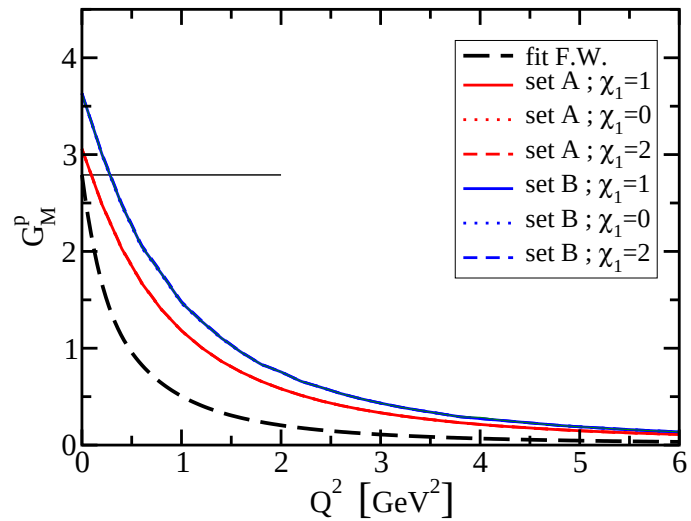

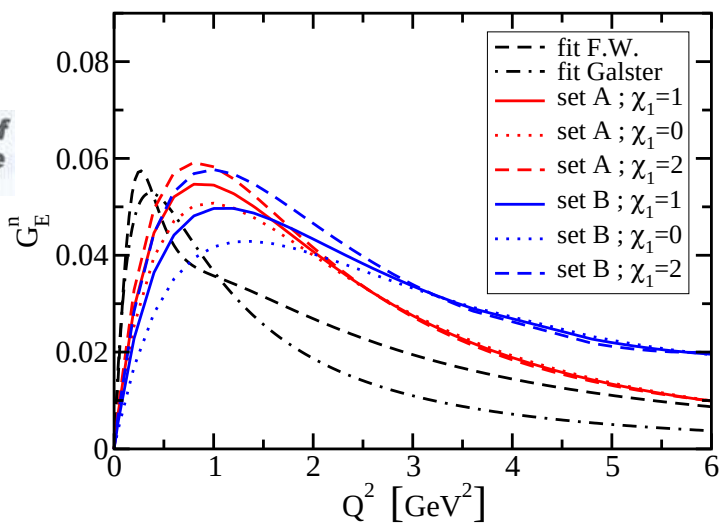

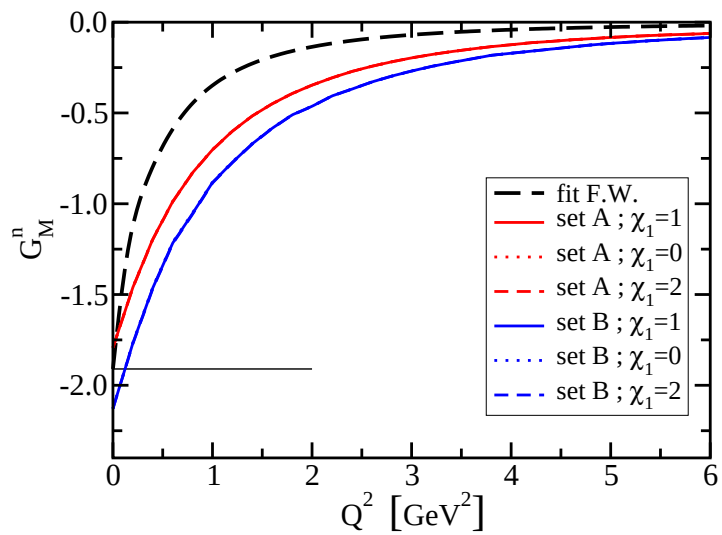

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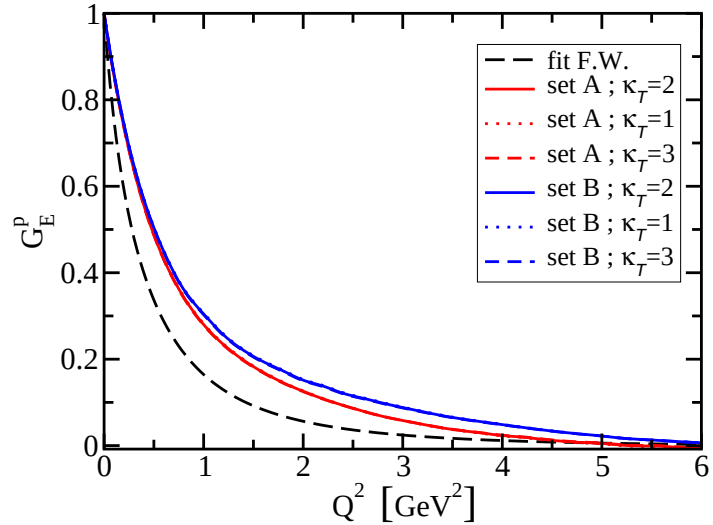

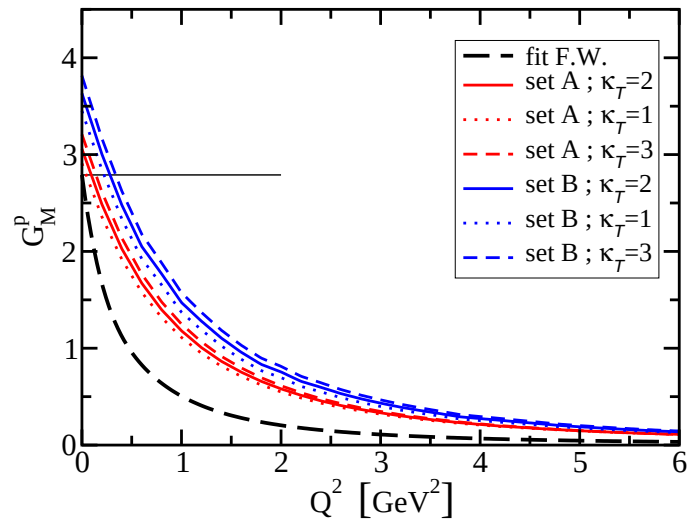

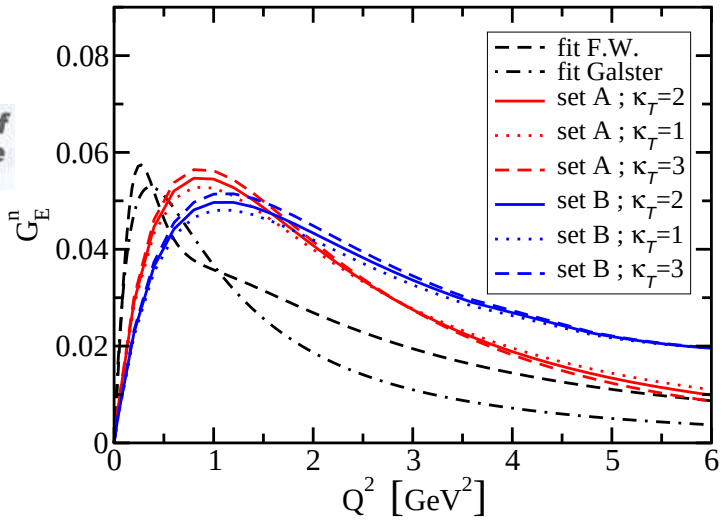

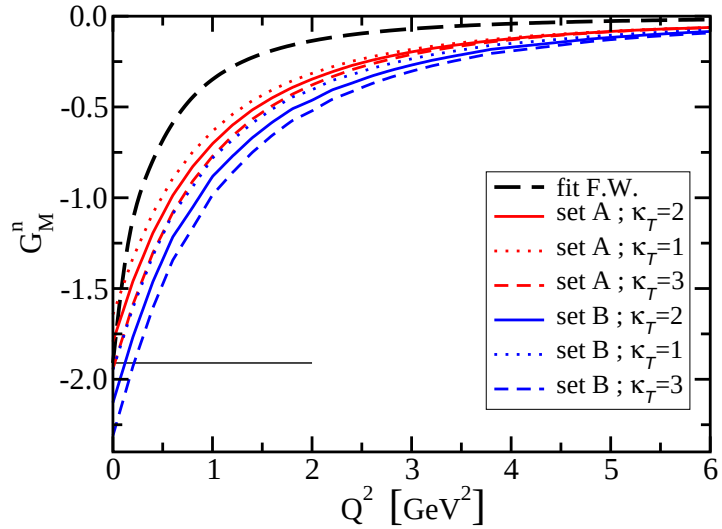

Variations in $\kappa_T = 1, 2, 3$; $\mu_{1+} = 2$, $\chi_{1+} = 1$

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  - No sensitivity to diquark parameters for $Q^2 \gtrsim 4\,\mathrm{GeV}^2$ … construction respects pQCD limit
- Cannot readily tune model to uniformly good account of nucleon electromagnetic properties
  - Something more than dressed-quark and -diquark degrees of freedom is required

Chiral Corrections

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$$\langle r^2_{\substack{p\\n}} \rangle^{1-loop^R}_{NA} = \mp \frac{1+5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2+\lambda^2}\right),$$

$$\langle (r^\mu_N)^2 \rangle^{1-loop^R}_{NA} = -\frac{1+5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2+\lambda^2}\right) + \frac{g_A^2 M_N}{16\pi f_\pi^2 \mu_v} \frac{1}{m_\pi} \frac{2}{\pi} \arctan\left(\frac{\lambda}{m_\pi}\right),$$

$$(\mu_{\substack{p\\n}})^{1-loop^R}_{NA} = \mp \frac{g_A^2 M_N}{4\pi^2 f_\pi^2} m_\pi \frac{2}{\pi} \arctan\left(\frac{\lambda^3}{m_\pi^3}\right),$$

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	r_p	r_n	r_p^μ	r_n^μ	μ_p	$-\mu_n$	ς
q-(qq) core	0.595	0.169	0.449	0.449	3.63	2.13	0.39
$+\pi$-loop correction	0.762	0.506	0.761	0.761	3.05	1.55	0.23
experiment	0.847	0.336	0.836	0.889	2.79	1.91	

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- Complements nucleon mass considerations

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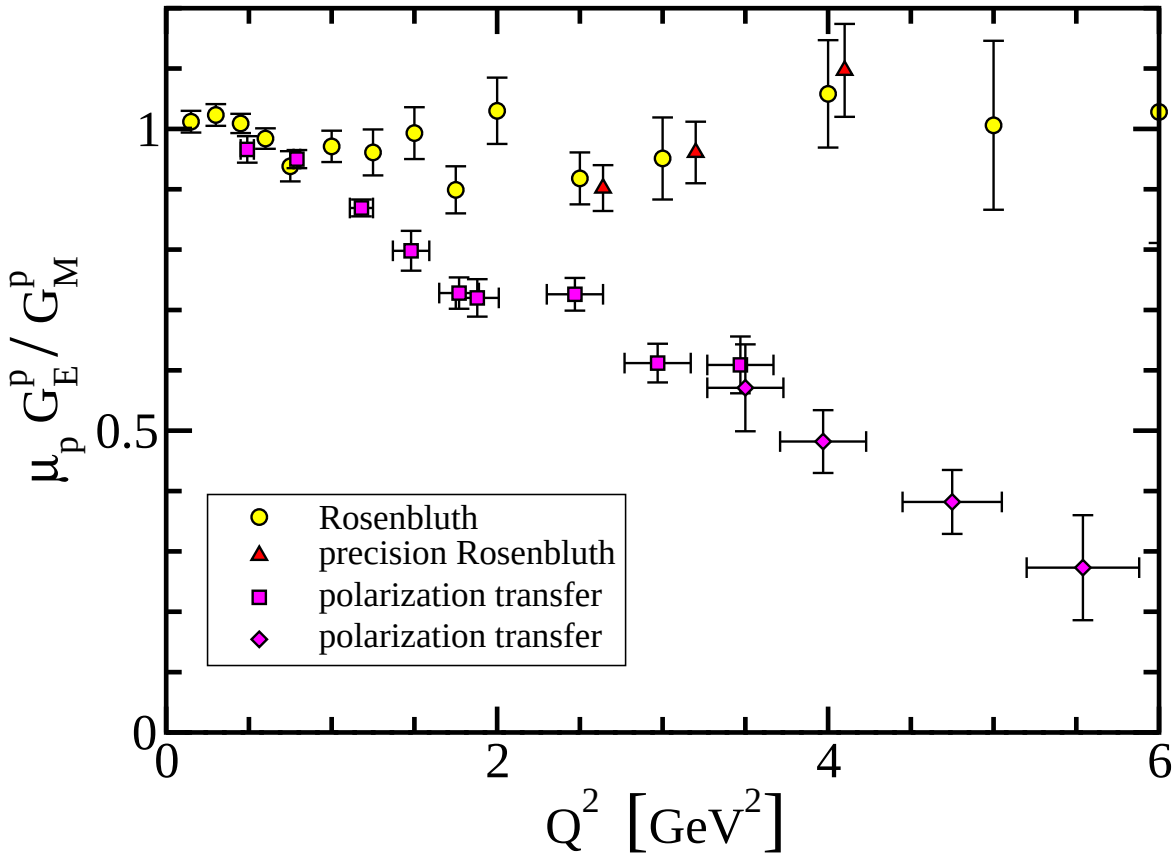
Form Factor Ratio: GE/GM

- Rosenbluth and Polarization-Transfer Extractions of Ratio of Proton's Electric and Magnetic Form Factors



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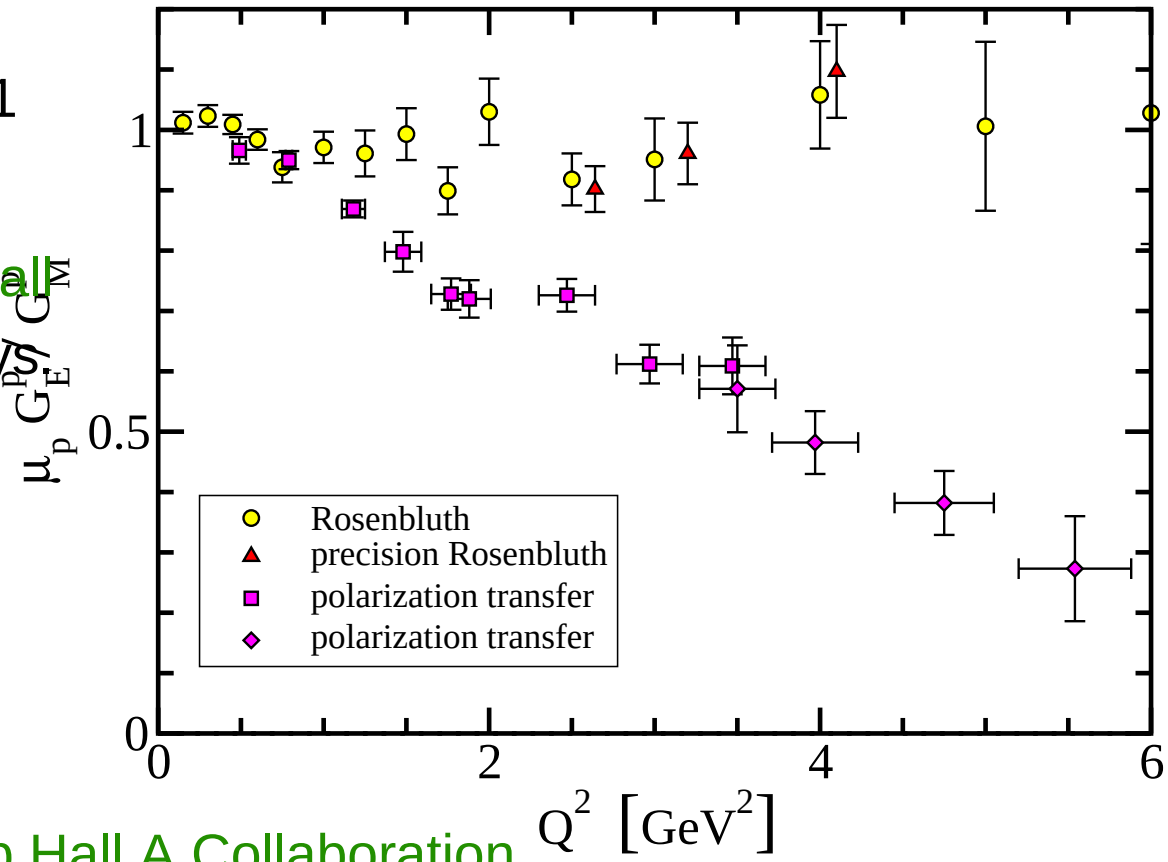
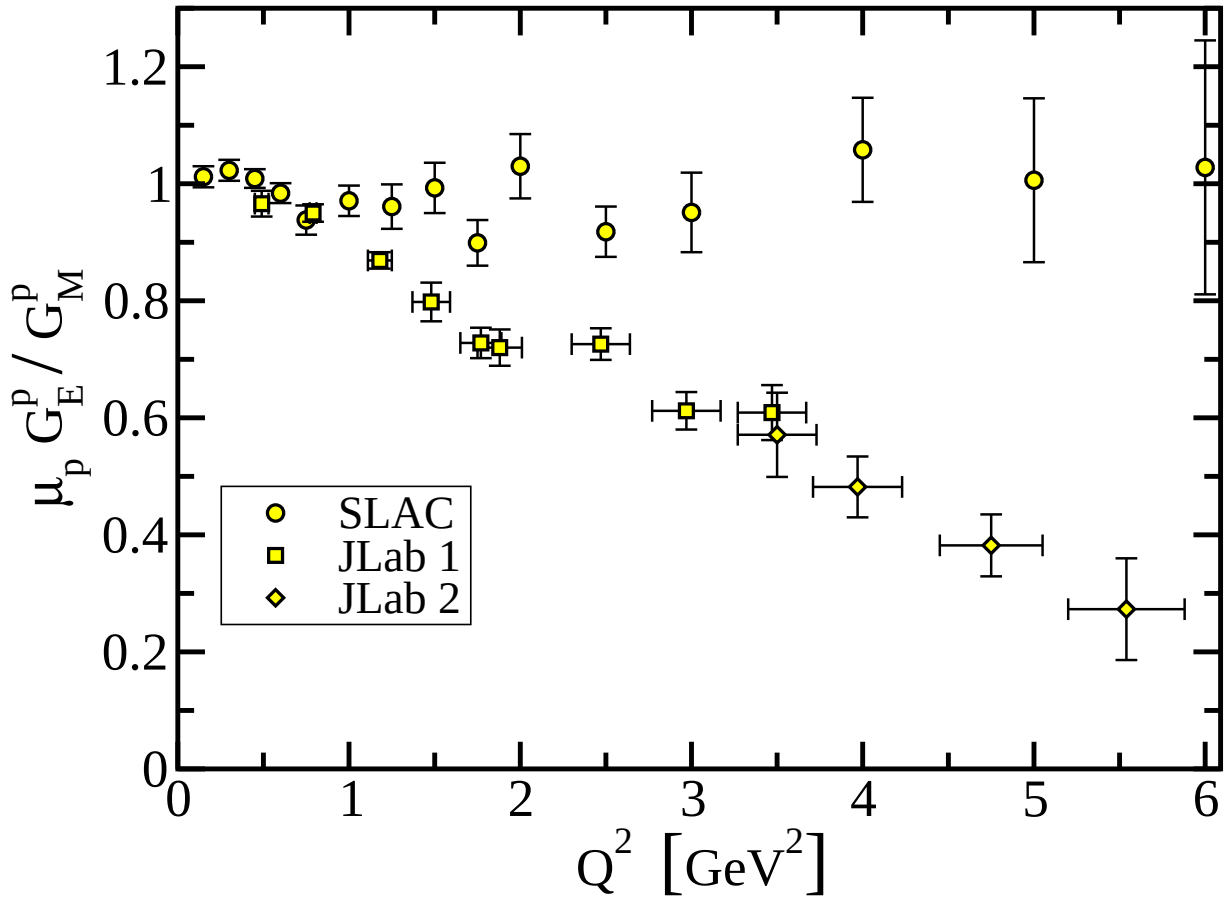
- Walker *et al.*, Phys. Rev. **D 49**, 5671 (1994)
- Qattan *et al.*, Phys. Rev. Lett. **94** 142301 (2005)
- Jones *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **84**, 1398 (2000)
- Gayou, *et al.*, Phys. Rev. **C 64**, 038202 (2001)
- Gayou, *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **88** 092301 (2002)

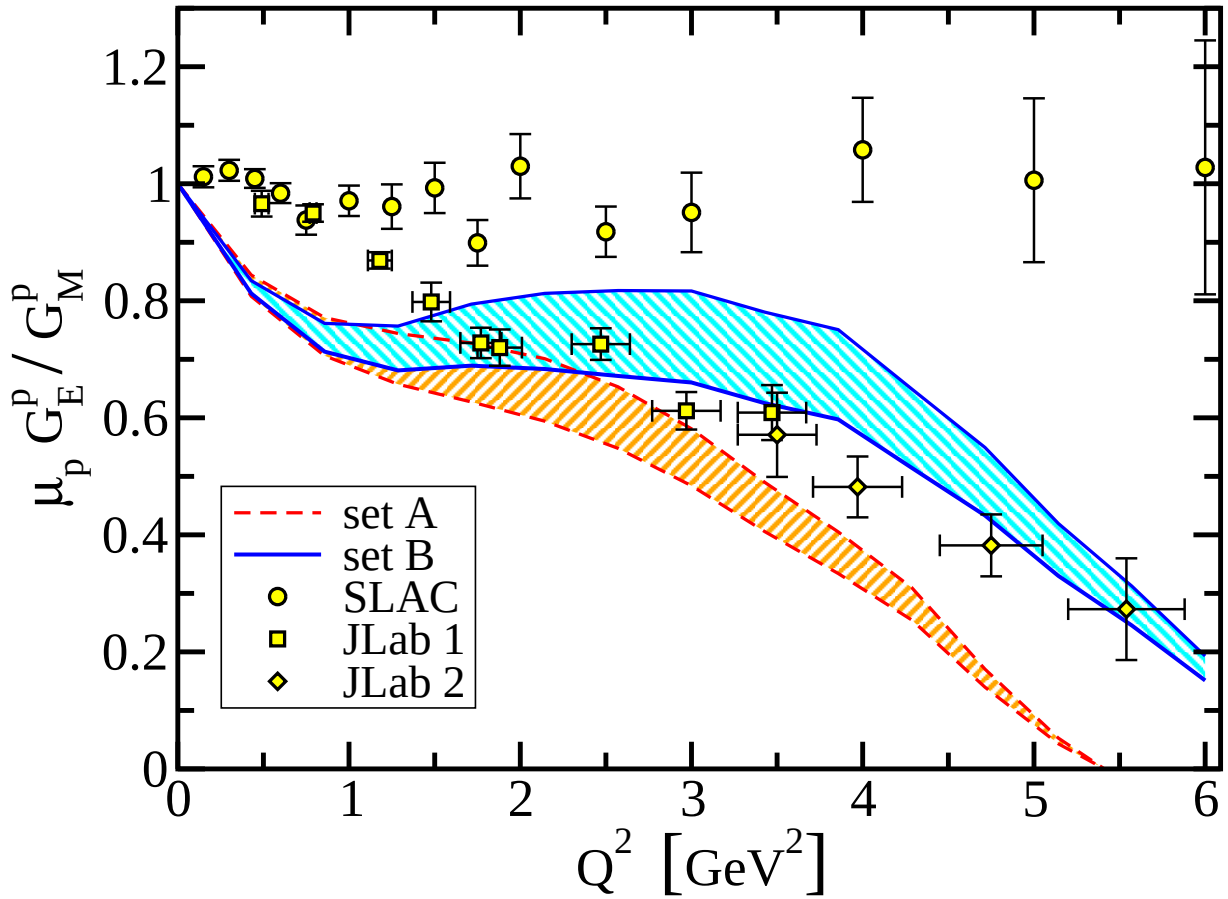


Form Factor Ratio: GE/GM



Form Factor Ratio: GE/GM

- Small Q^2



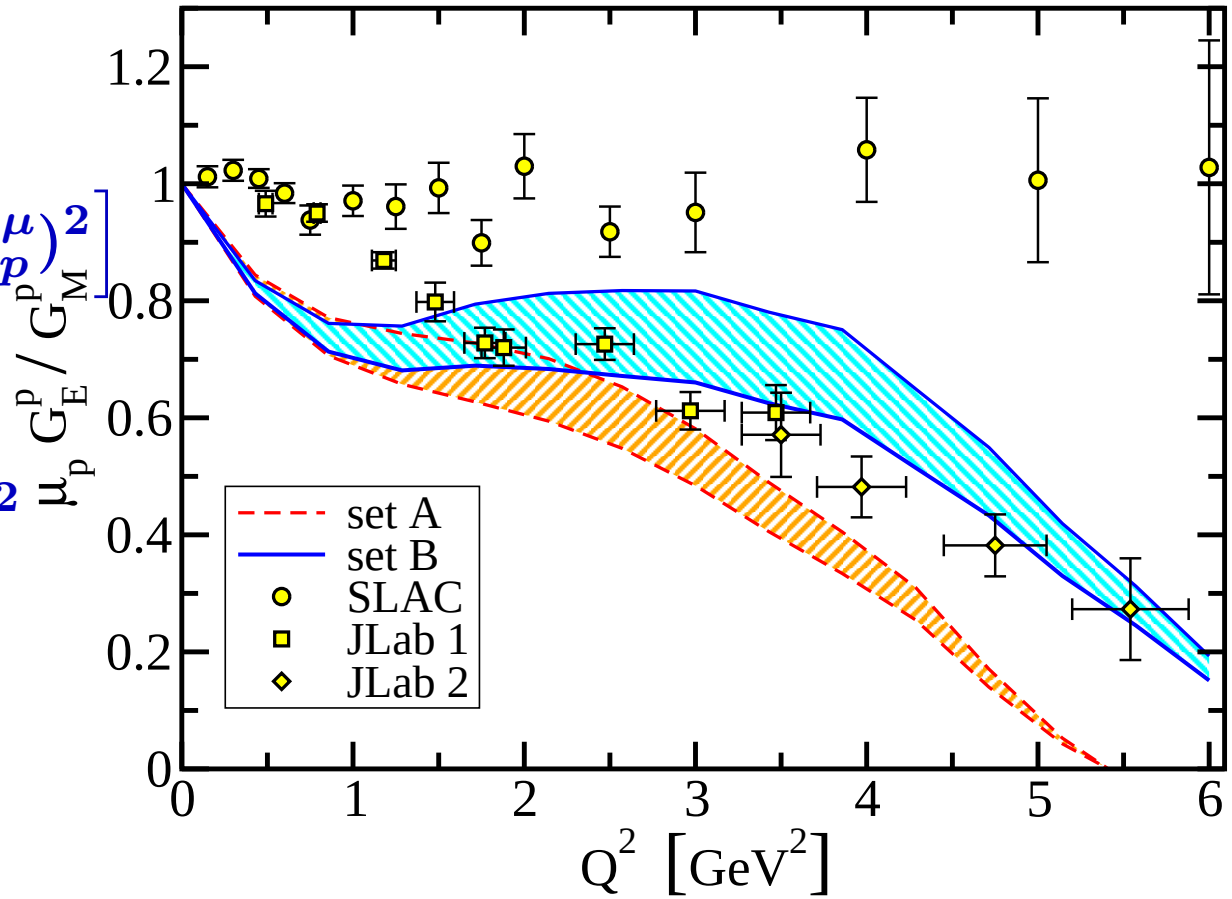
Form Factor Ratio: GE/GM

- Small Q^2

$$\mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)} = 1 - \frac{Q^2}{6}\left[(r_p)^2 - (r_p^\mu)^2\right]$$

$r_p \approx r_p^\mu$

$\Rightarrow$ ratio varies $< 10\%$ on $0 < Q^2 < 0.6$ GeV^2



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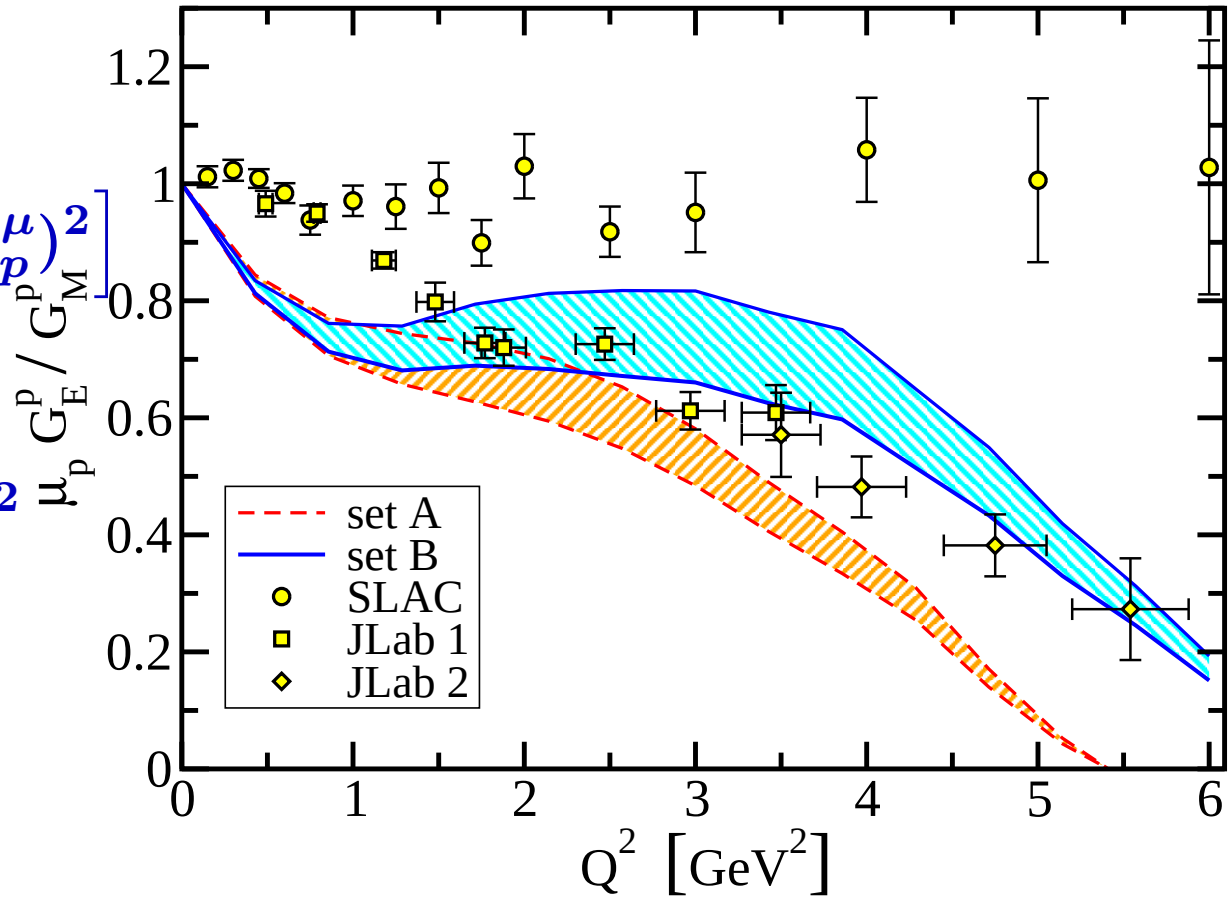
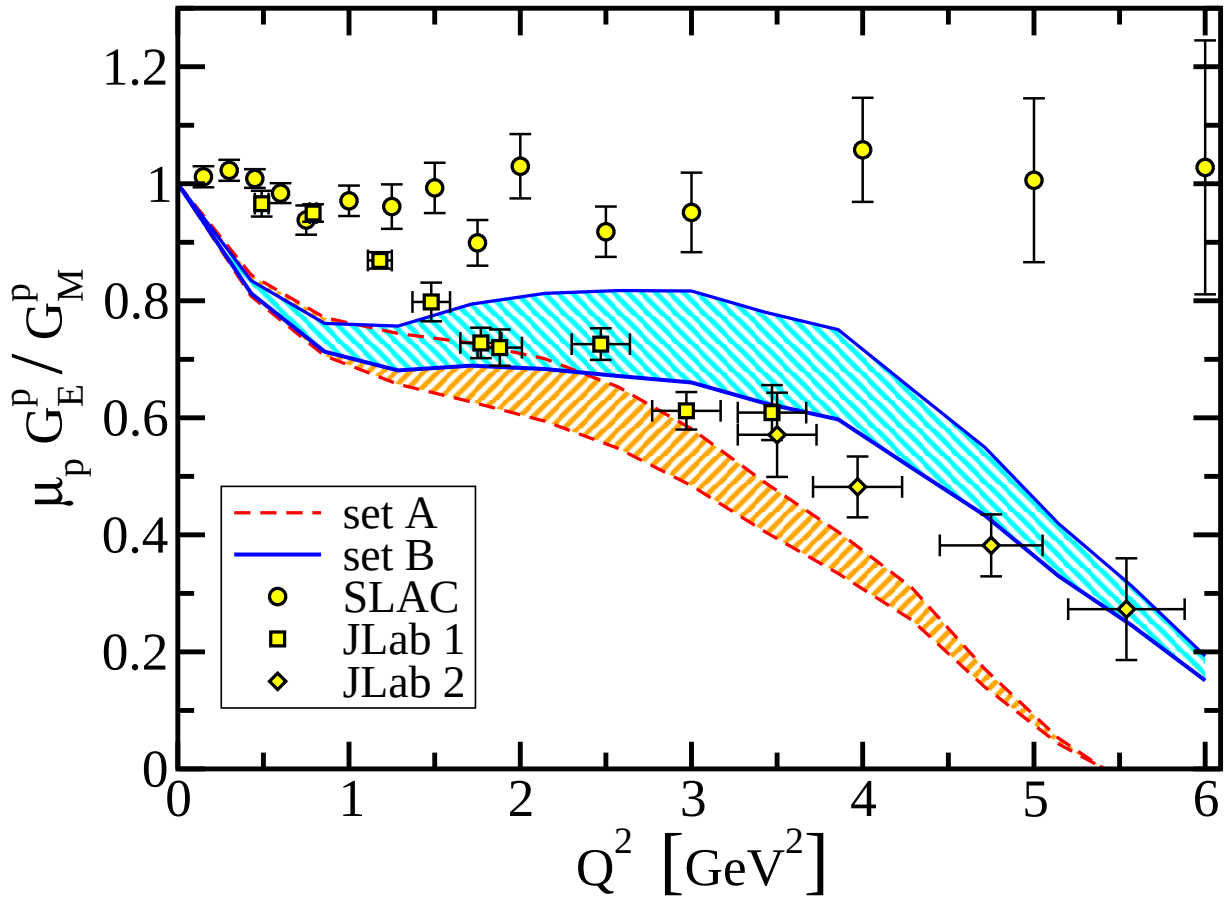


Form Factor Ratio: GE/GM

- $Q^2 \gtrsim 2\,\mathrm{GeV}^2$



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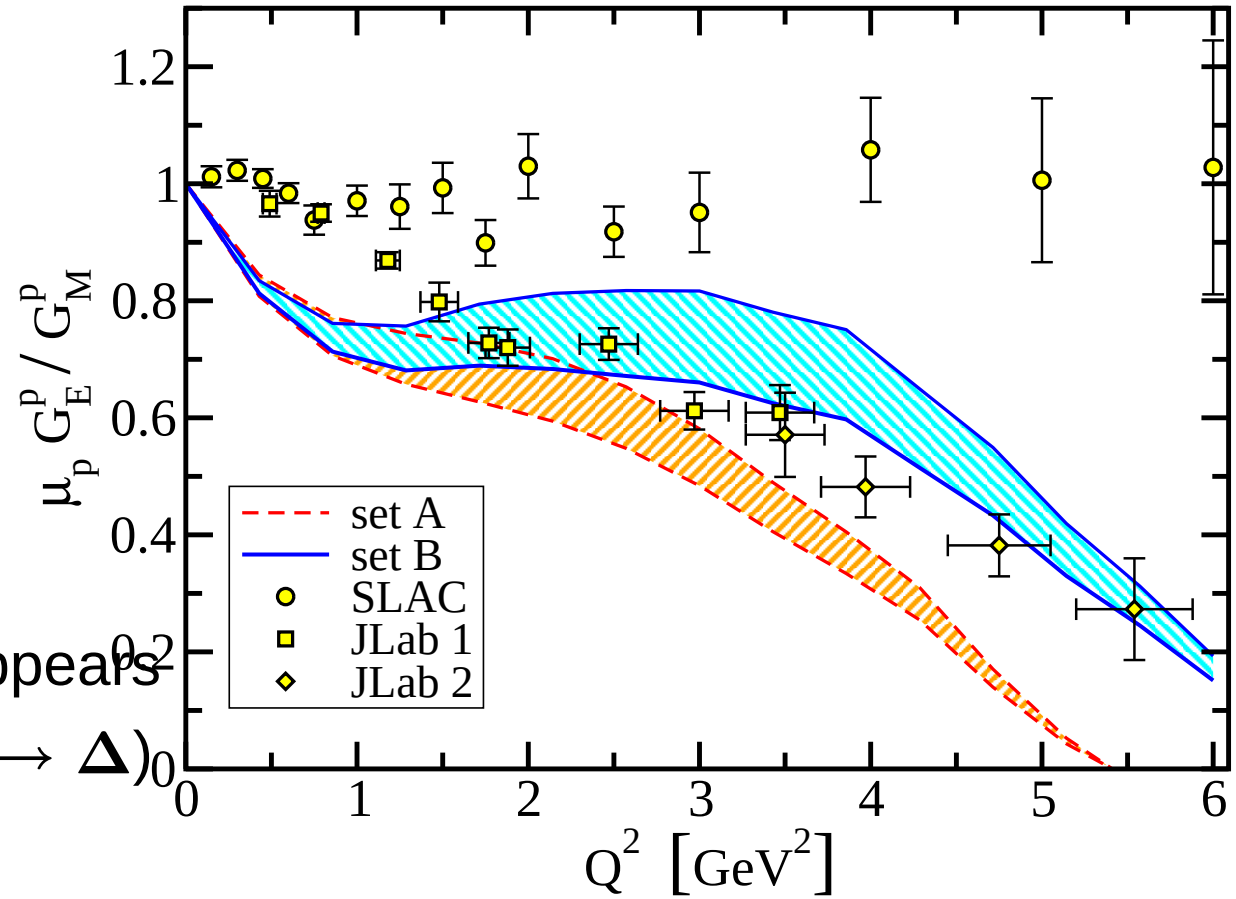
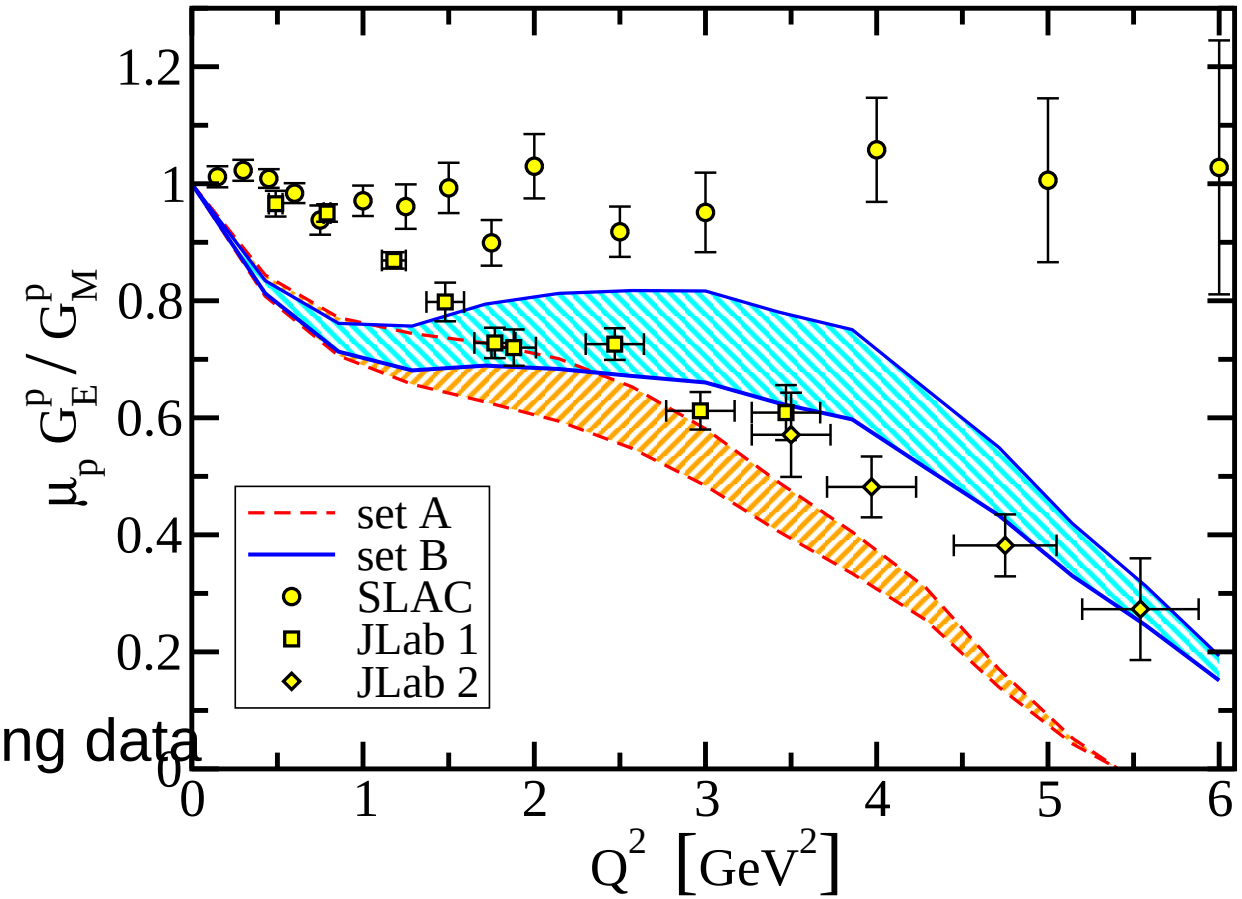


- $Q^2 \gtrsim 2\,\mathrm{GeV}^2$
  - π not pointlike
  - Contribution disappears (Lee & Sato $\pi N \longrightarrow \Delta$)
  - Quark-core

… represented by Faddeev Amplitude … becomes dominant

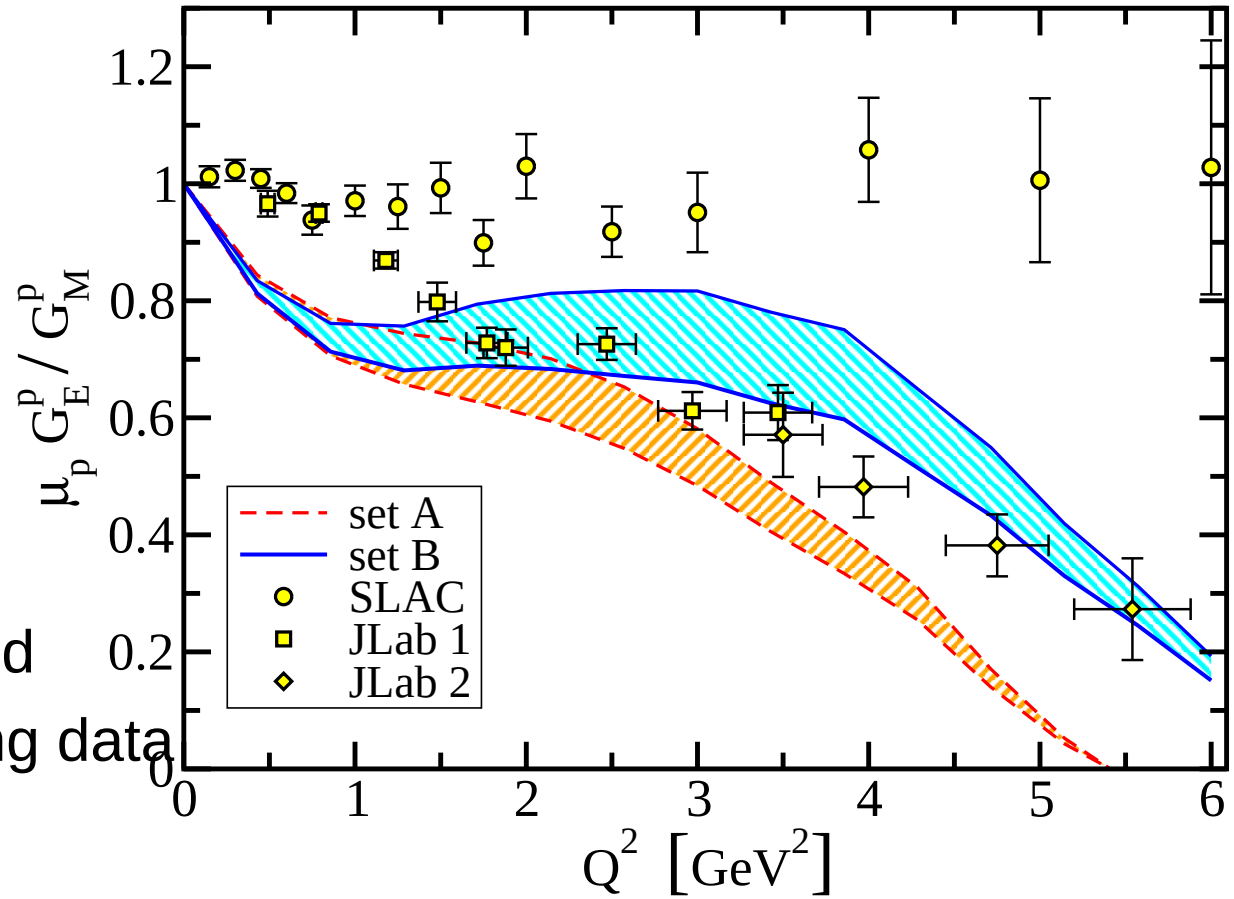
Form Factor Ratio: GE/GM

- $Q^2 \gtrsim 2\,\mathrm{GeV}^2$
- Set A
  - no π-cloud
  - ruled-out by existing data



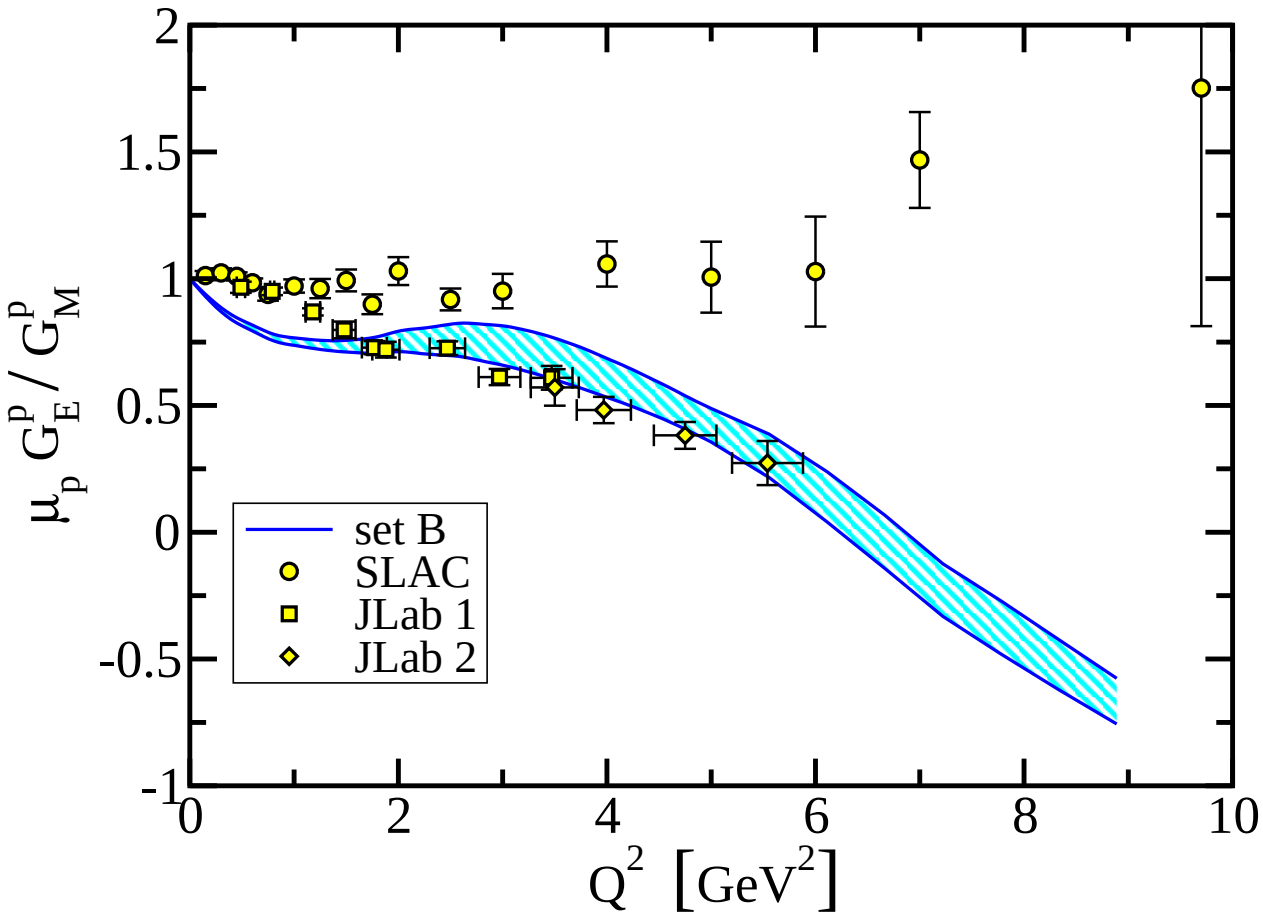
Form Factor Ratio: GE/GM

- $Q^2 \gtrsim 2\,\mathrm{GeV}^2$
- Set B
  anticipates π-cloud
  not fitted to existing data
  but in agreement



Form Factor Ratio: GE/GM

- $Q^2 \gtrsim 2\,\mathrm{GeV}^2$
- Set B Zero at $Q^2 \approx 6.5\mathrm{GeV}^2$



Epilogue

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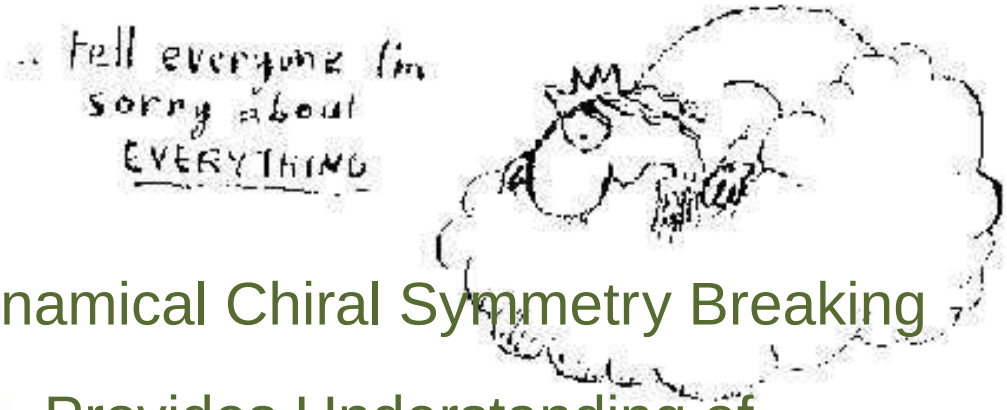


Epilogue

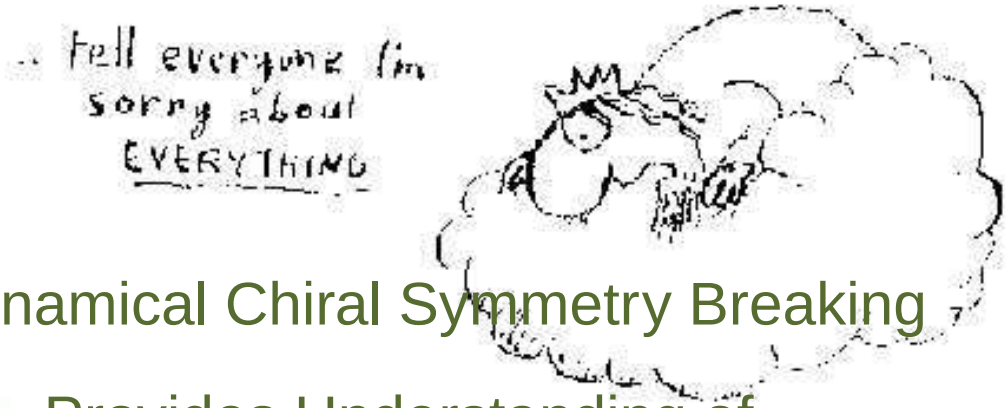
Dynamical Chiral Symmetry Breaking

- Provides Understanding of

  Appearance of Substantial Mass from "Empty Space":

  - proton -

$$3 \times 5 \overset{\text{virtual particles}}{=} 950$$



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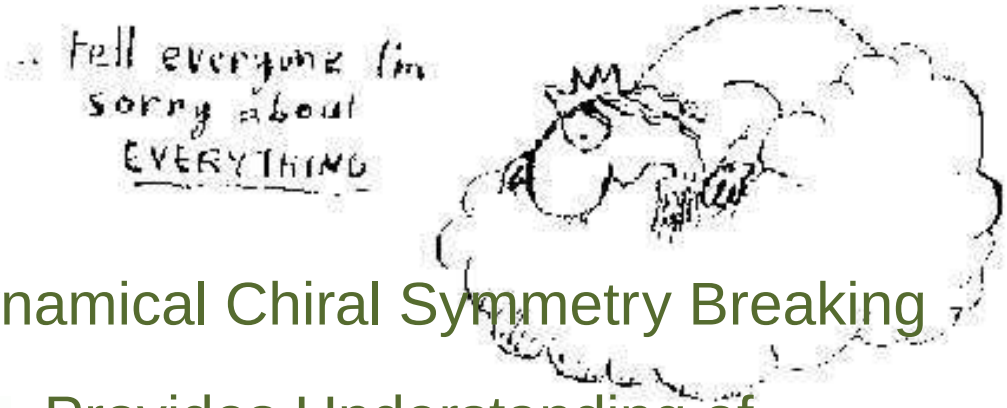
  Conspiracy of Weightlessness

  - pion -

$$2 \times 350 \overset{\text{symmetry cancellation of virtual particles}}{=} 140$$



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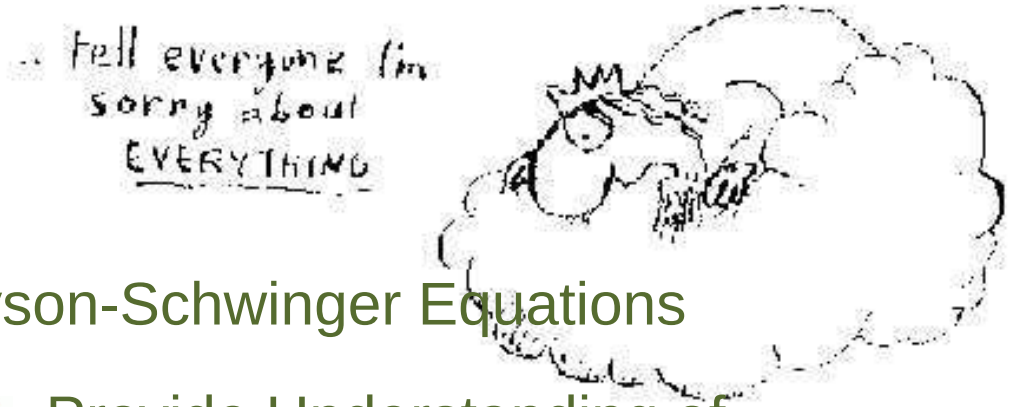
- Both Effects *impossible* in Quantum Mechanics

  – Require Machinery of Relativistic Quantum Field Theory



Epilogue

Dyson-Schwinger Equations

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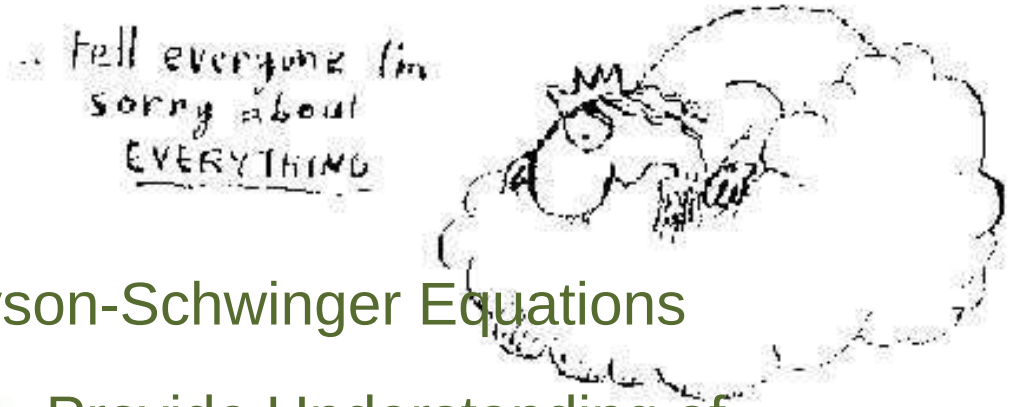
  $\Rightarrow$ π is quark-antiquark Bound State

  AND QCD's Goldstone Mode



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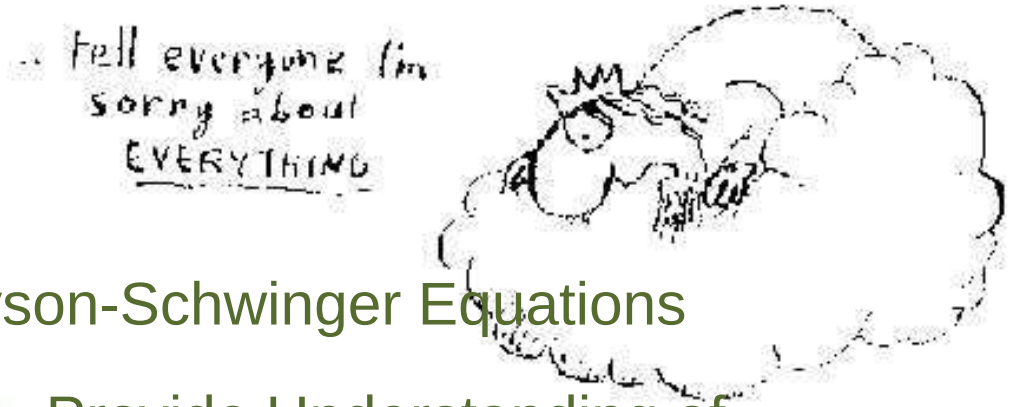
  Exact Results in QCD

  e.g., Quark Goldberger-Treiman

  Properties of Pseudoscalar Mesons



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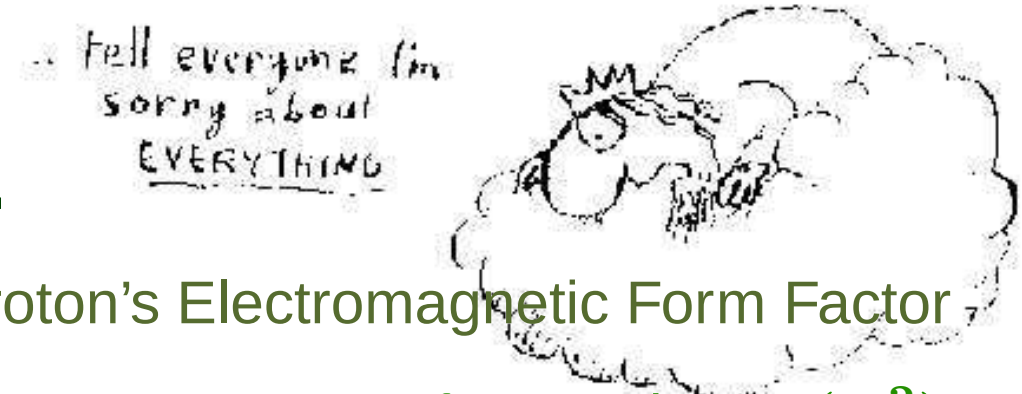
  Properties of Pseudoscalar Mesons

- Renormalisation-Group-Improved Rainbow-Ladder

  $\Rightarrow$ Practical Phenomenological Tool

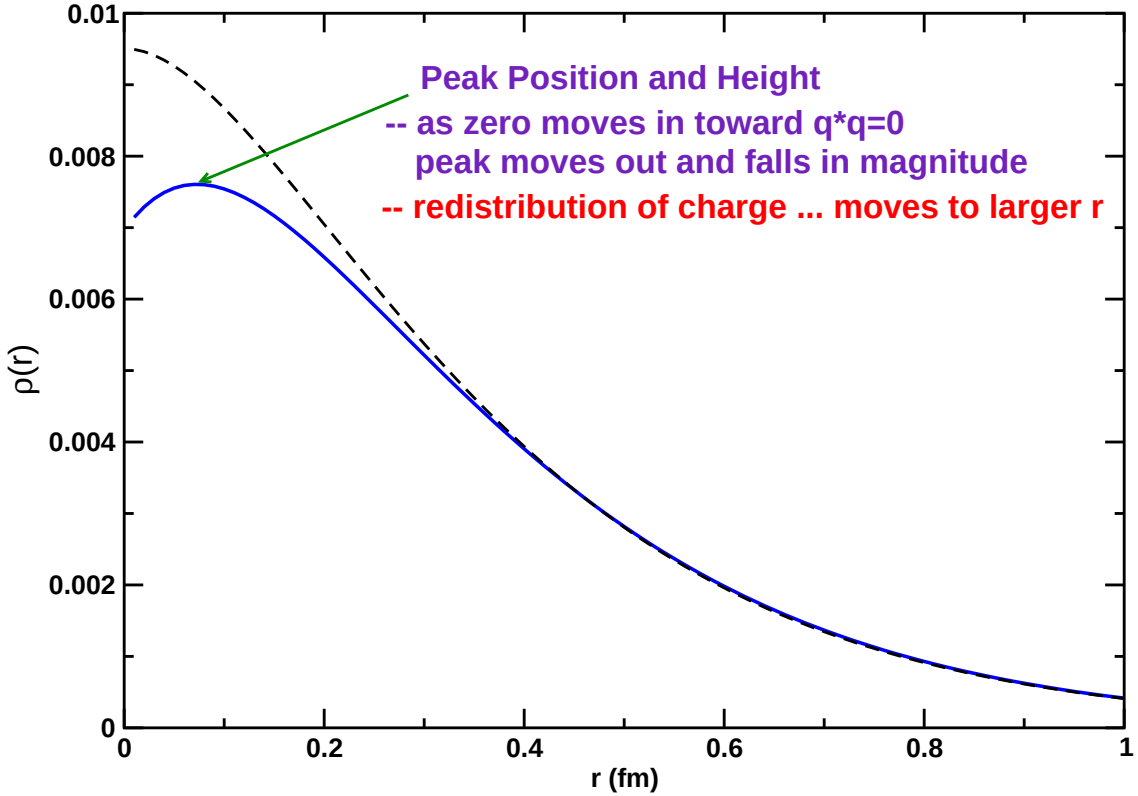
  Corrections Quantifiable

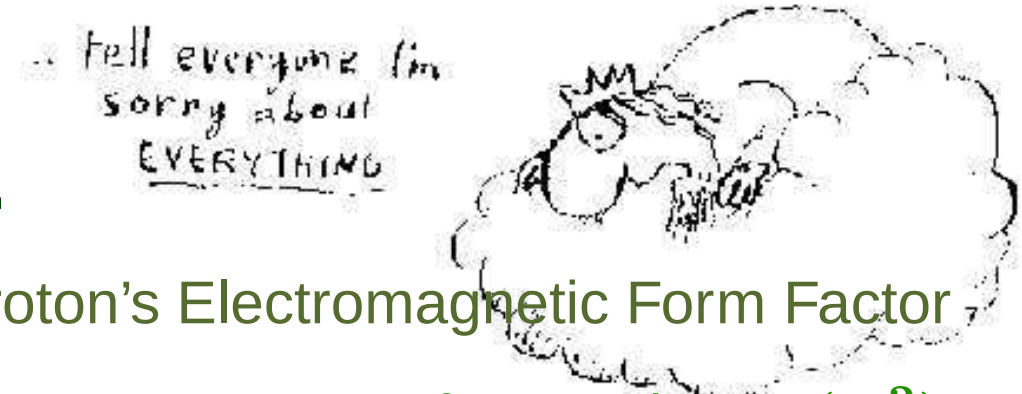


Epilogue

Proton's Electromagnetic Form Factor

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected





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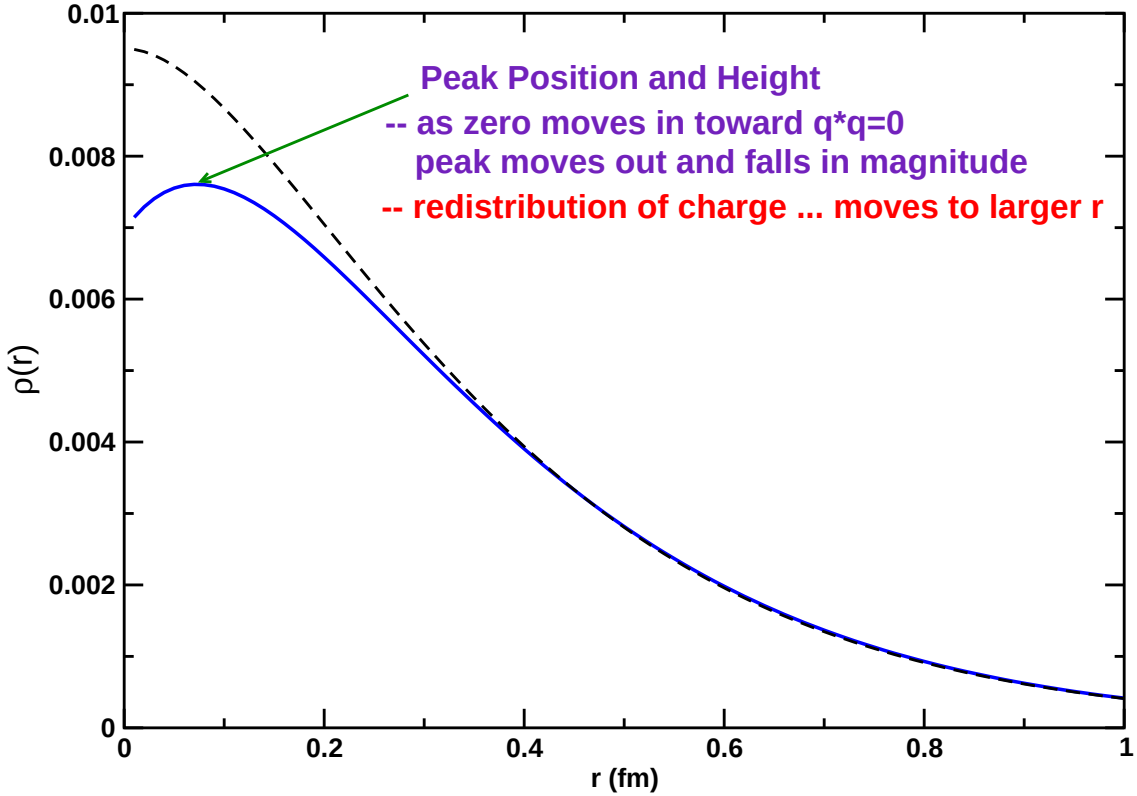
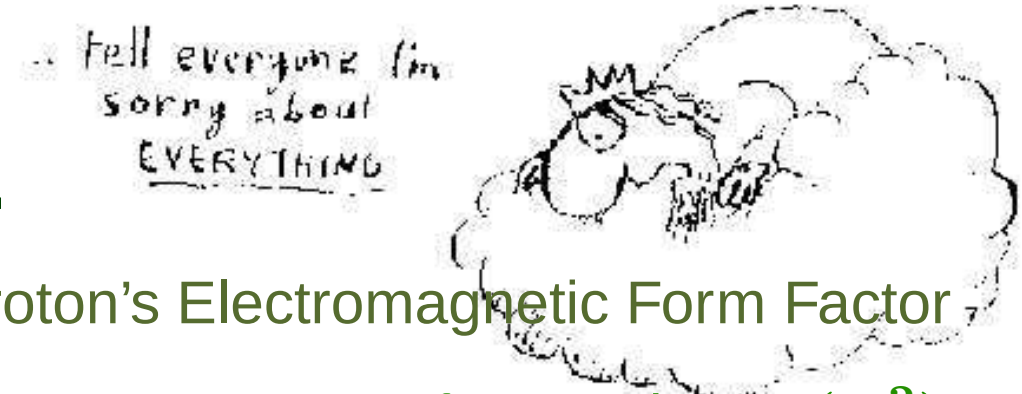


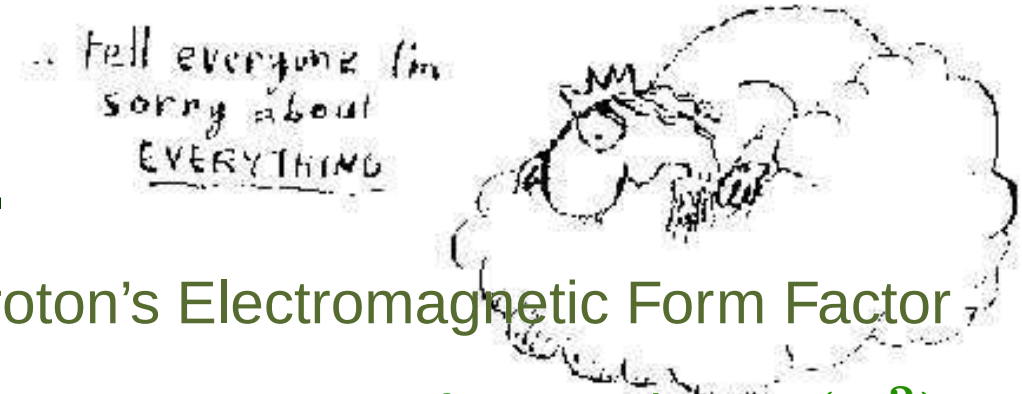
- However, Current Density remains peaked at $r = 0$!



Epilogue

Proton's Electromagnetic Form Factor

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected
- Wave Function is complex and correlated mix of virtual particles and antiparticles: $s-$, $p-$ and $d-$waves



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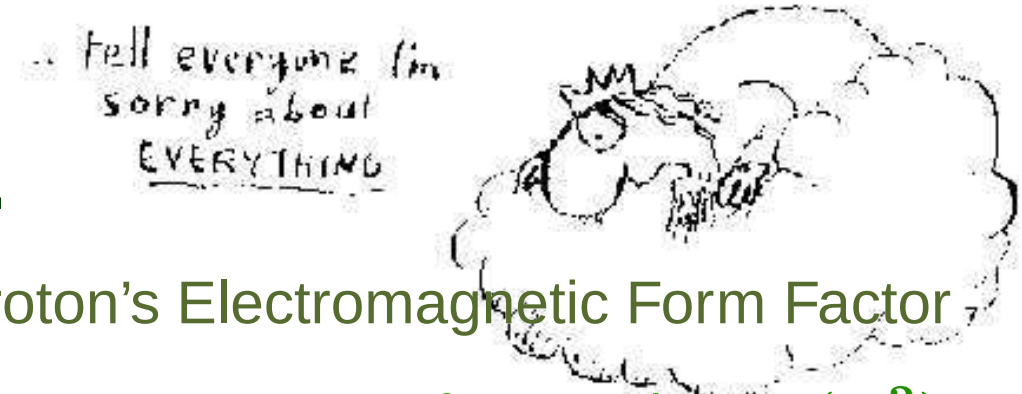

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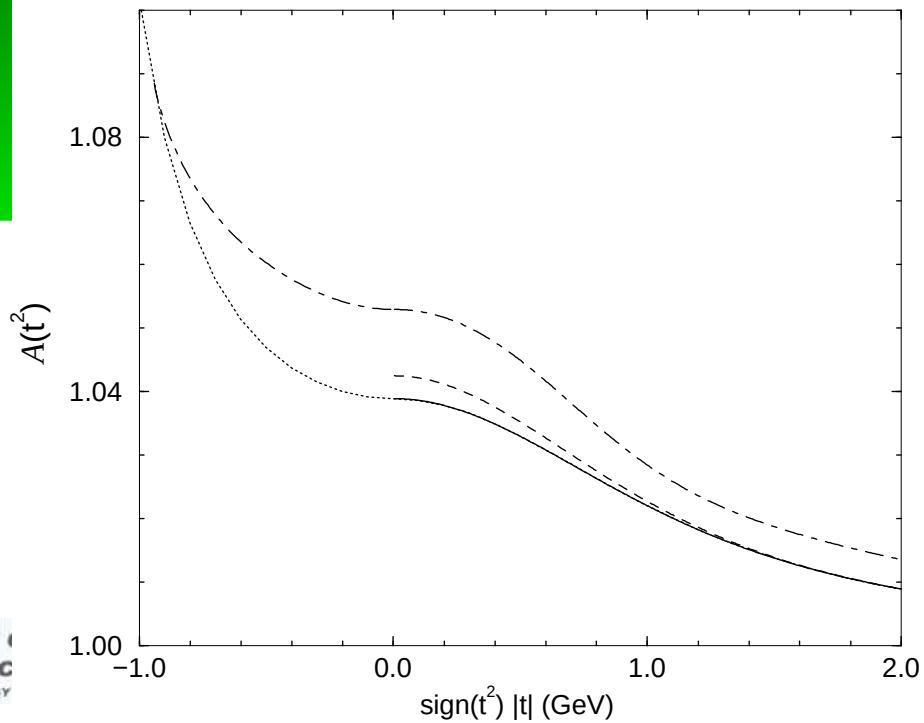
- Can Expect

  Contemporary Nuclear Physics
  to Yield *Many More Surprises*

Vector piece of nucleon's self energy



$M = 0.94$, $m_\pi = 0.14$

$\Lambda = 0.9$ GeV, $g_A = 1$

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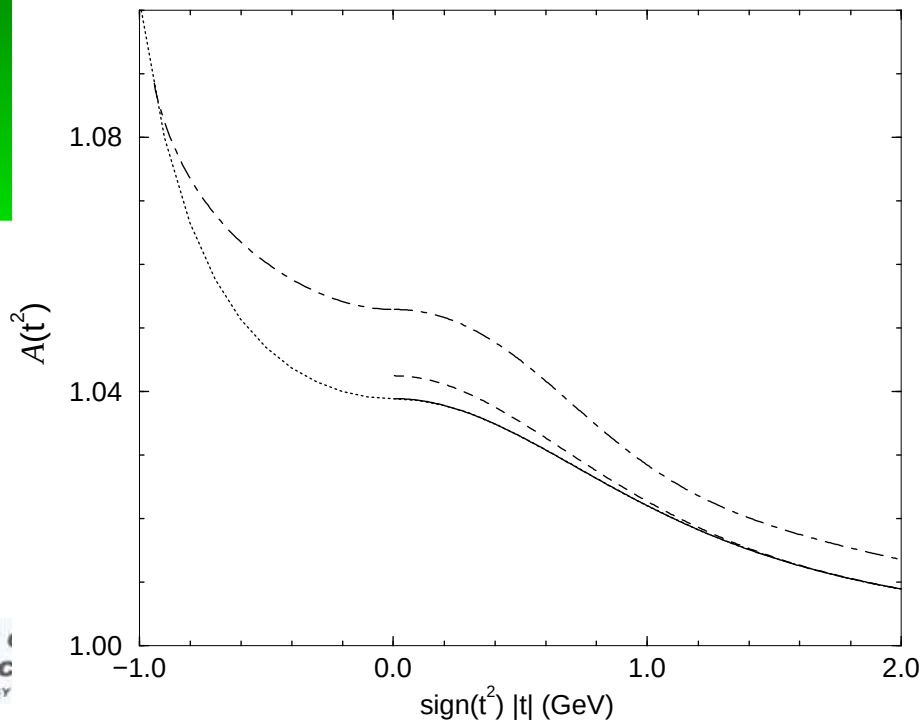

- Solid line: One-loop; numerically evaluated, exact (numerical) kernel

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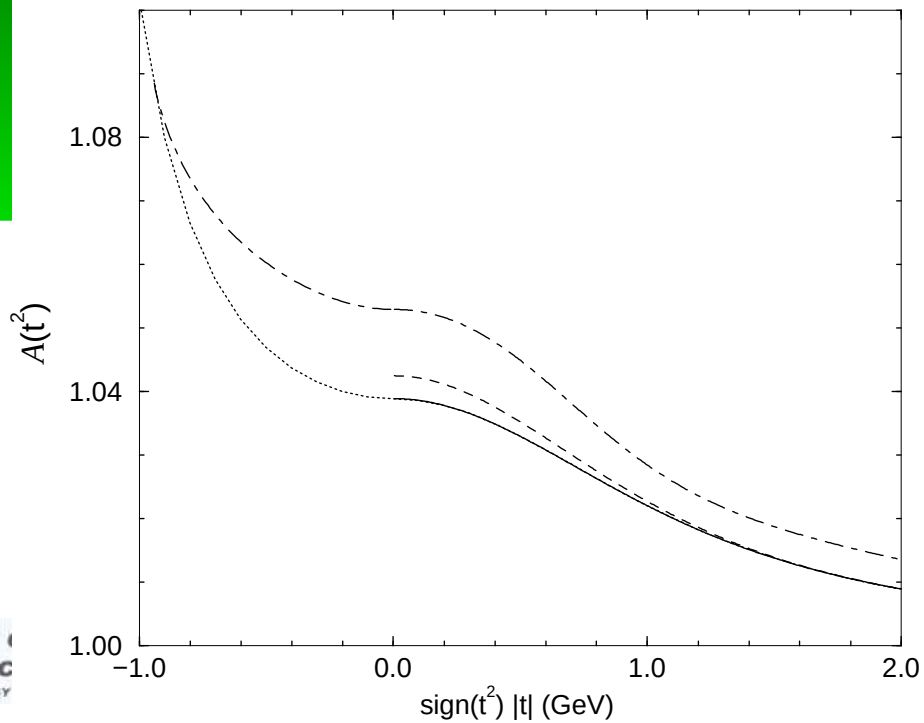

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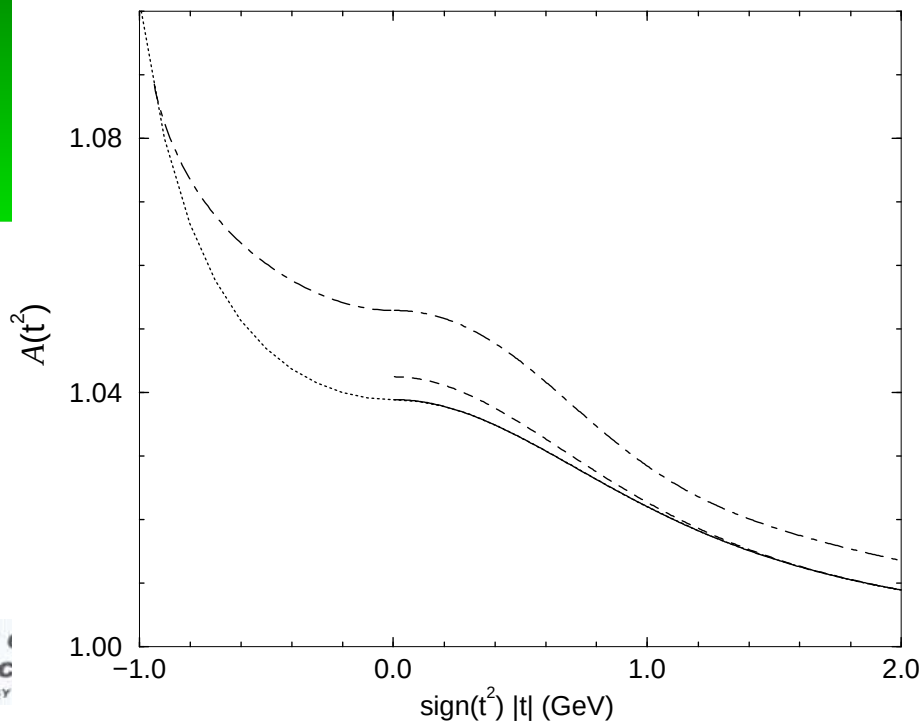

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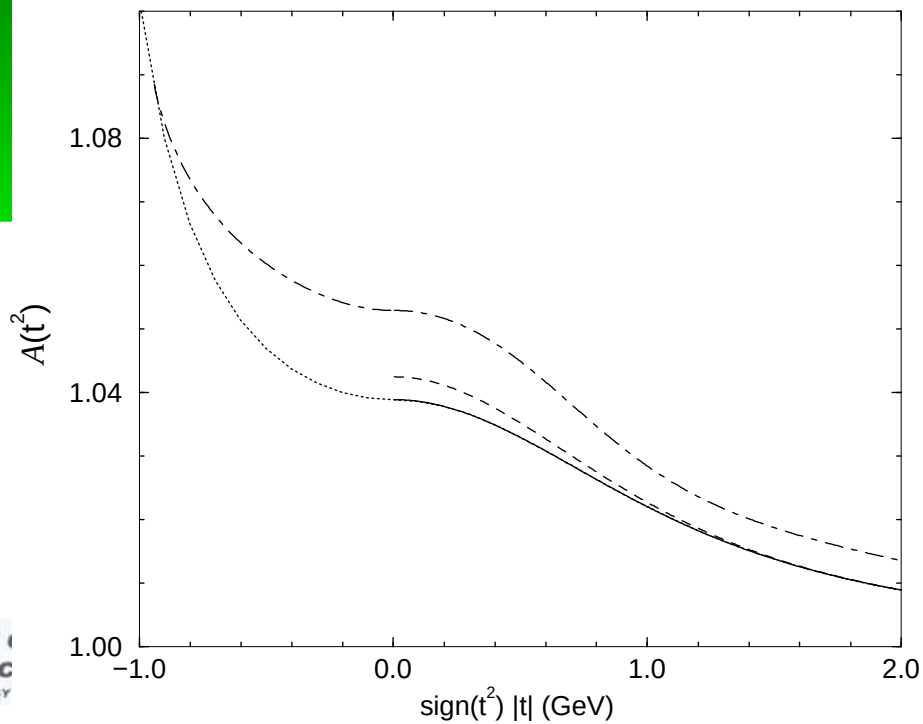

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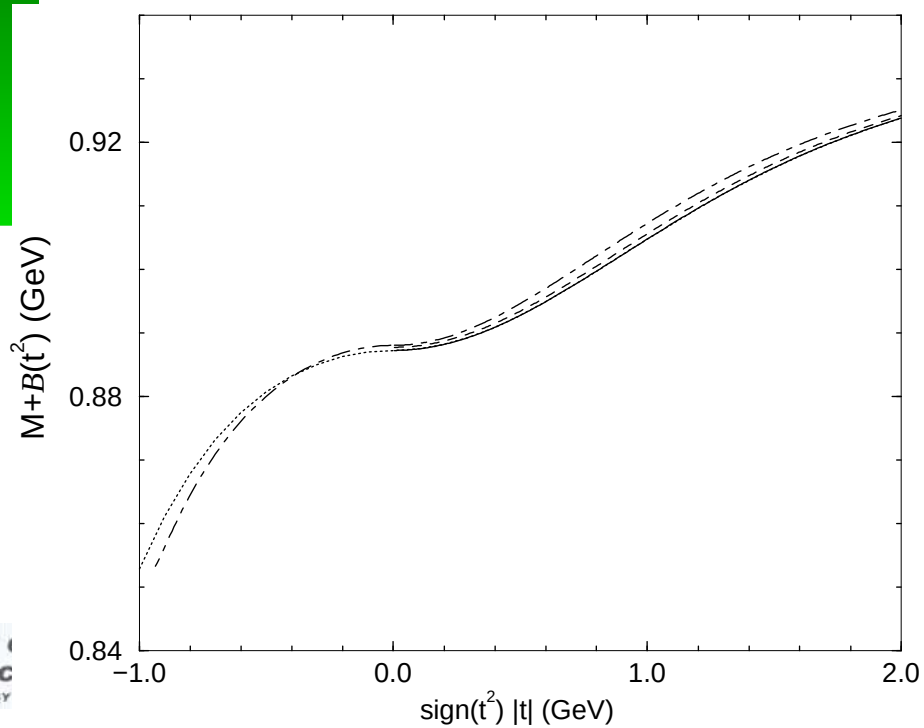
return

Scalar piece of nucleon's self energy



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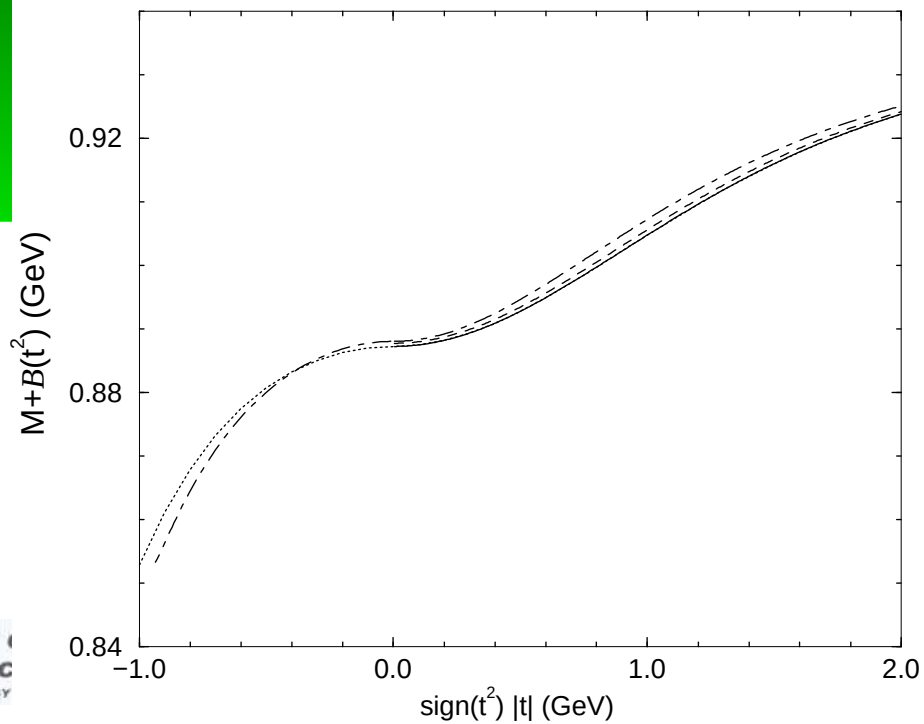

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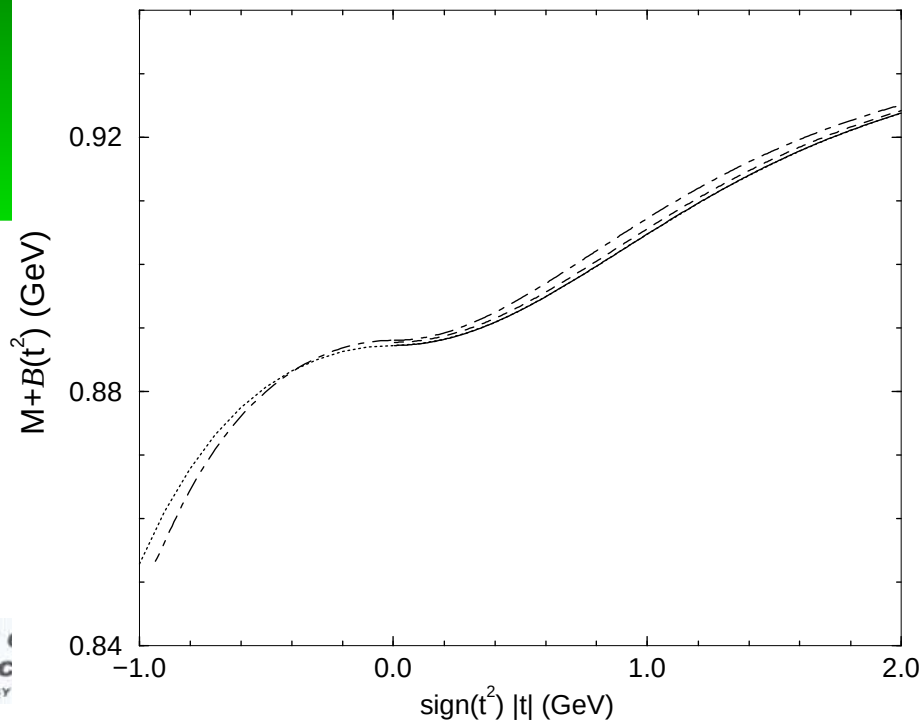

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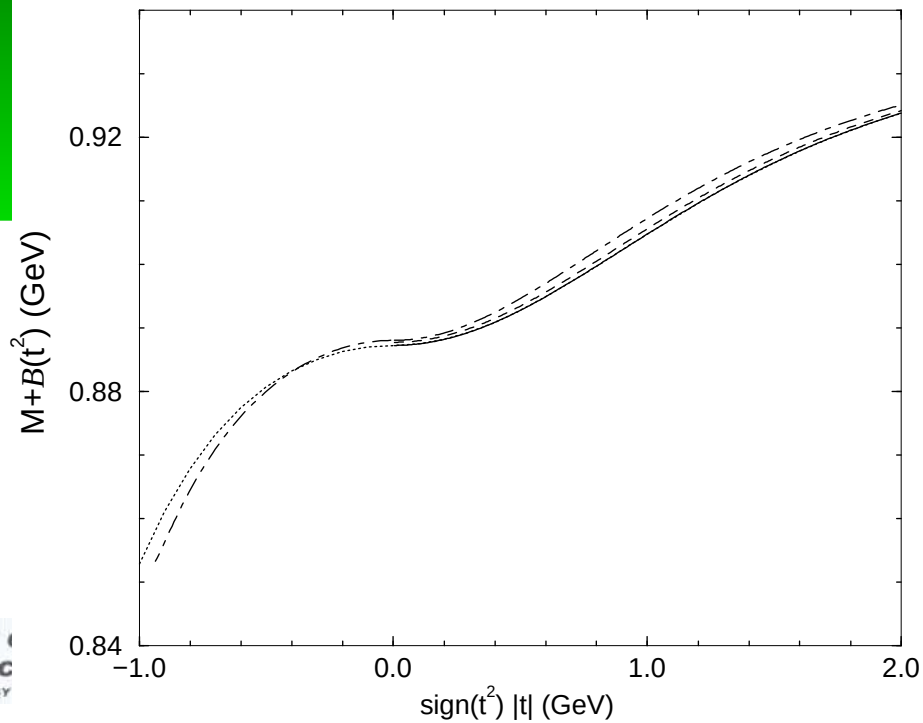

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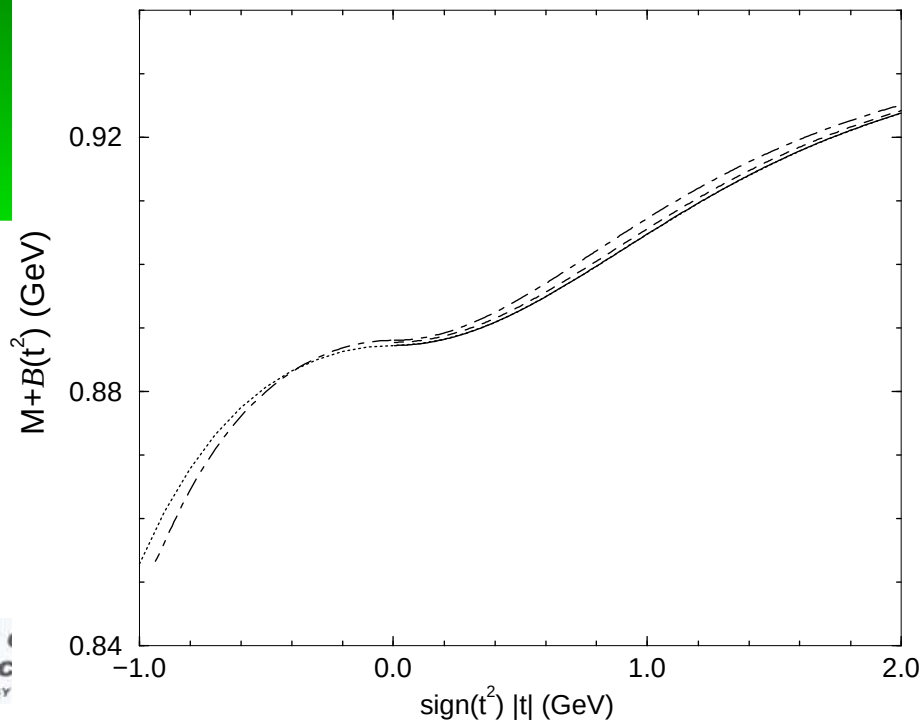

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Form Factor Ratio: $Q * F2/F1$

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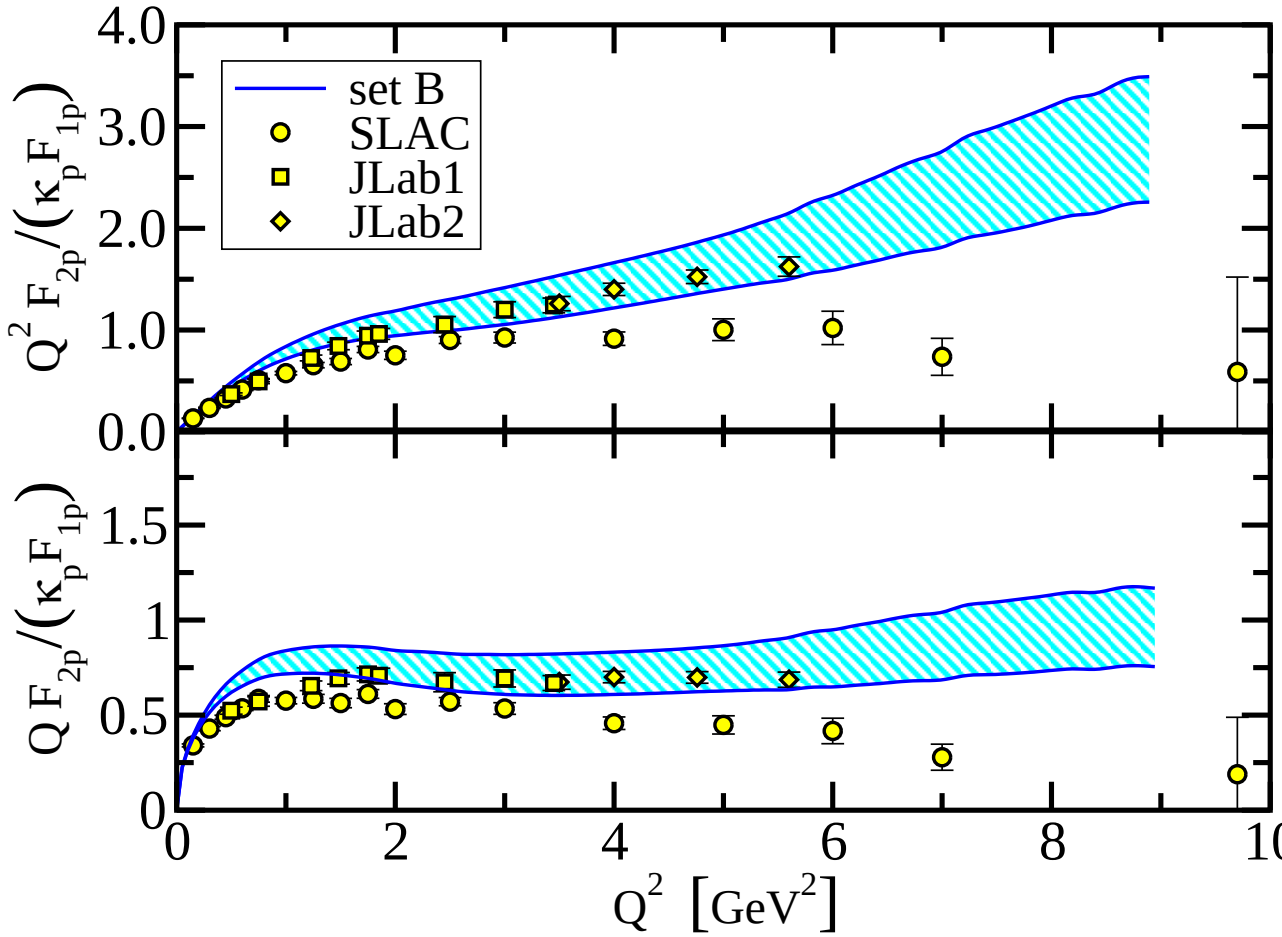
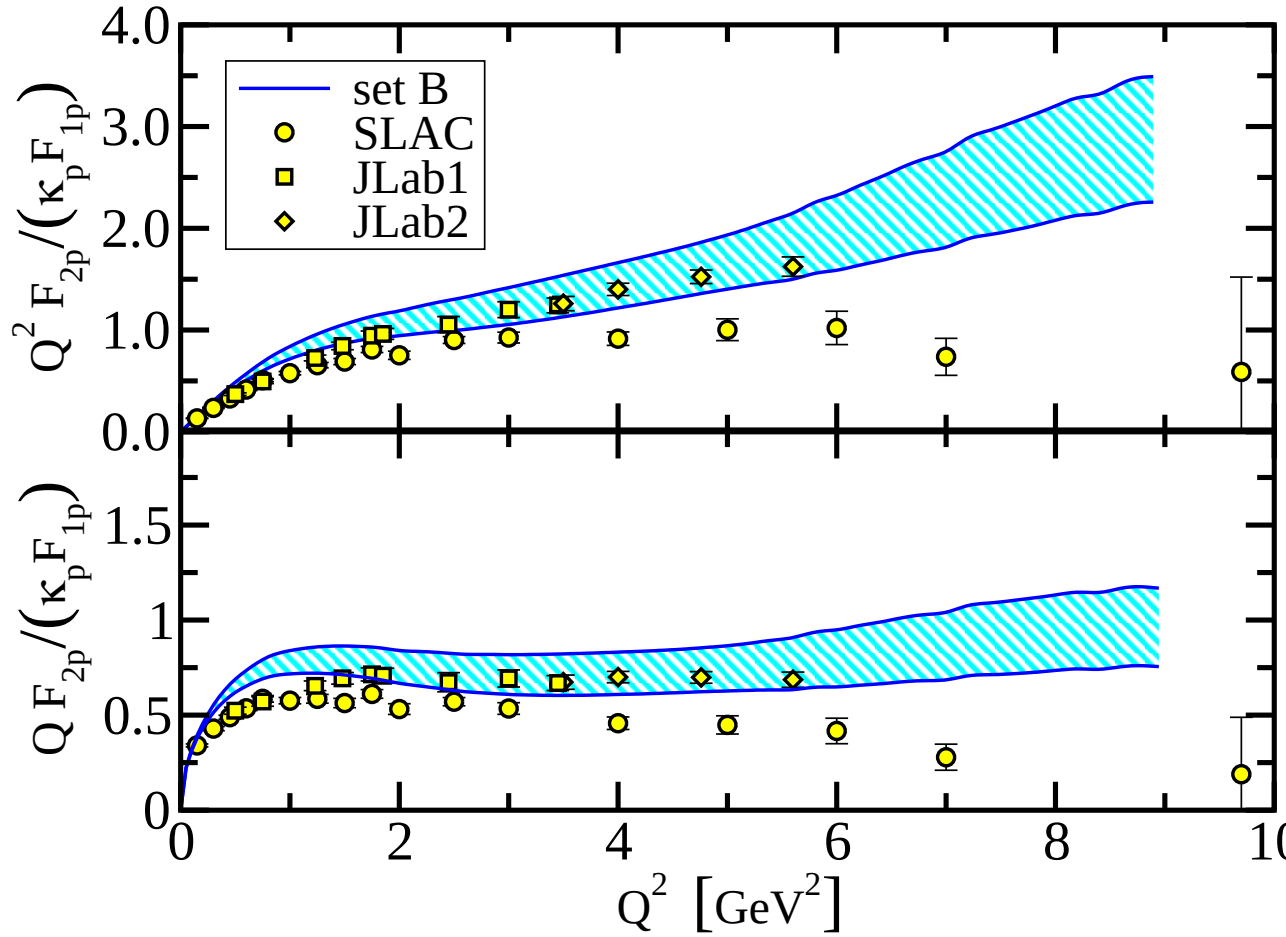


Form Factor Ratio: $Q * F2/F1$

- Perhaps $\approx$ constant for $2 \lesssim Q^2 \lesssim 6\text{GeV}^2$



Form Factor Ratio: alternative F2/F1

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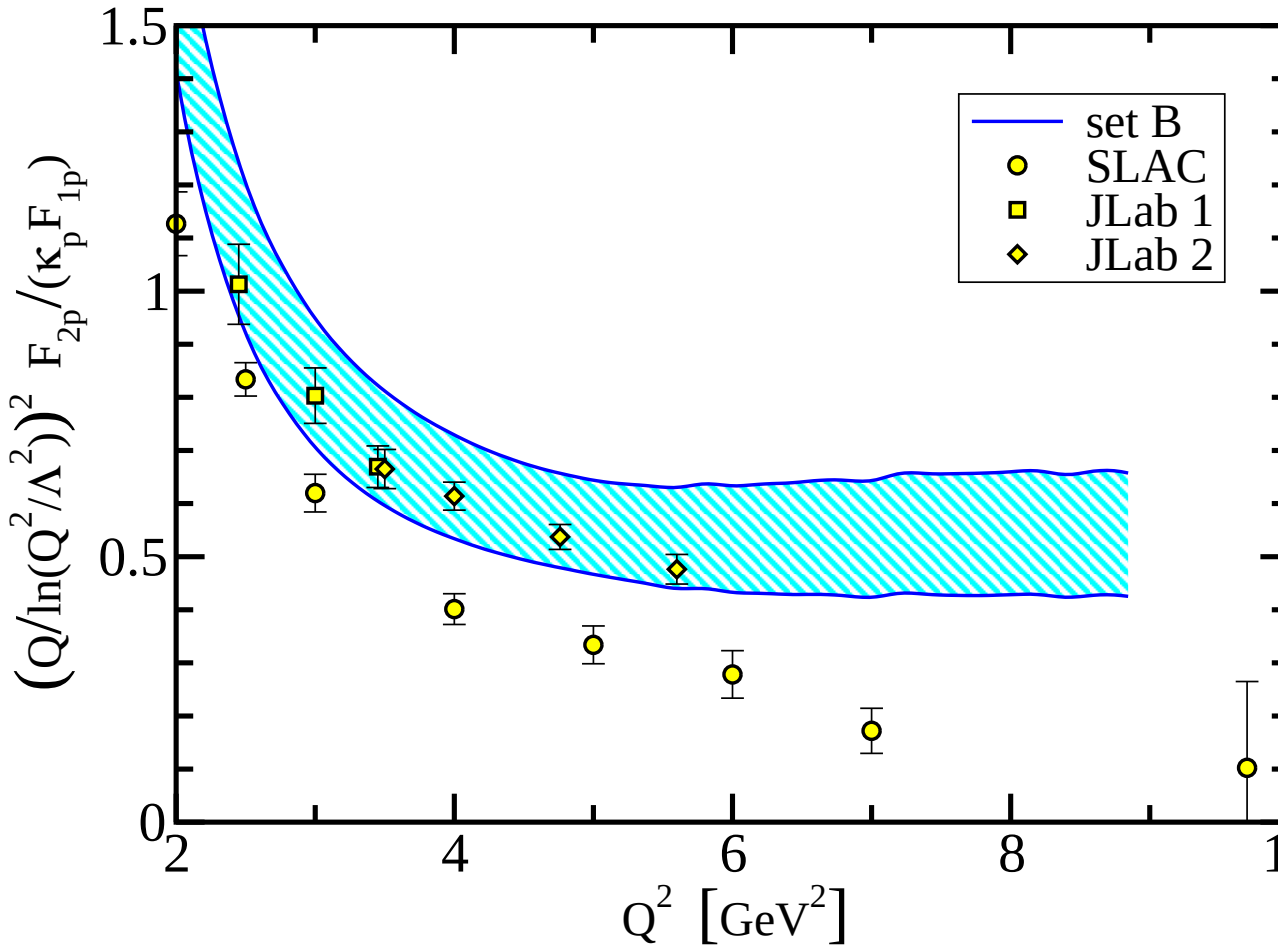
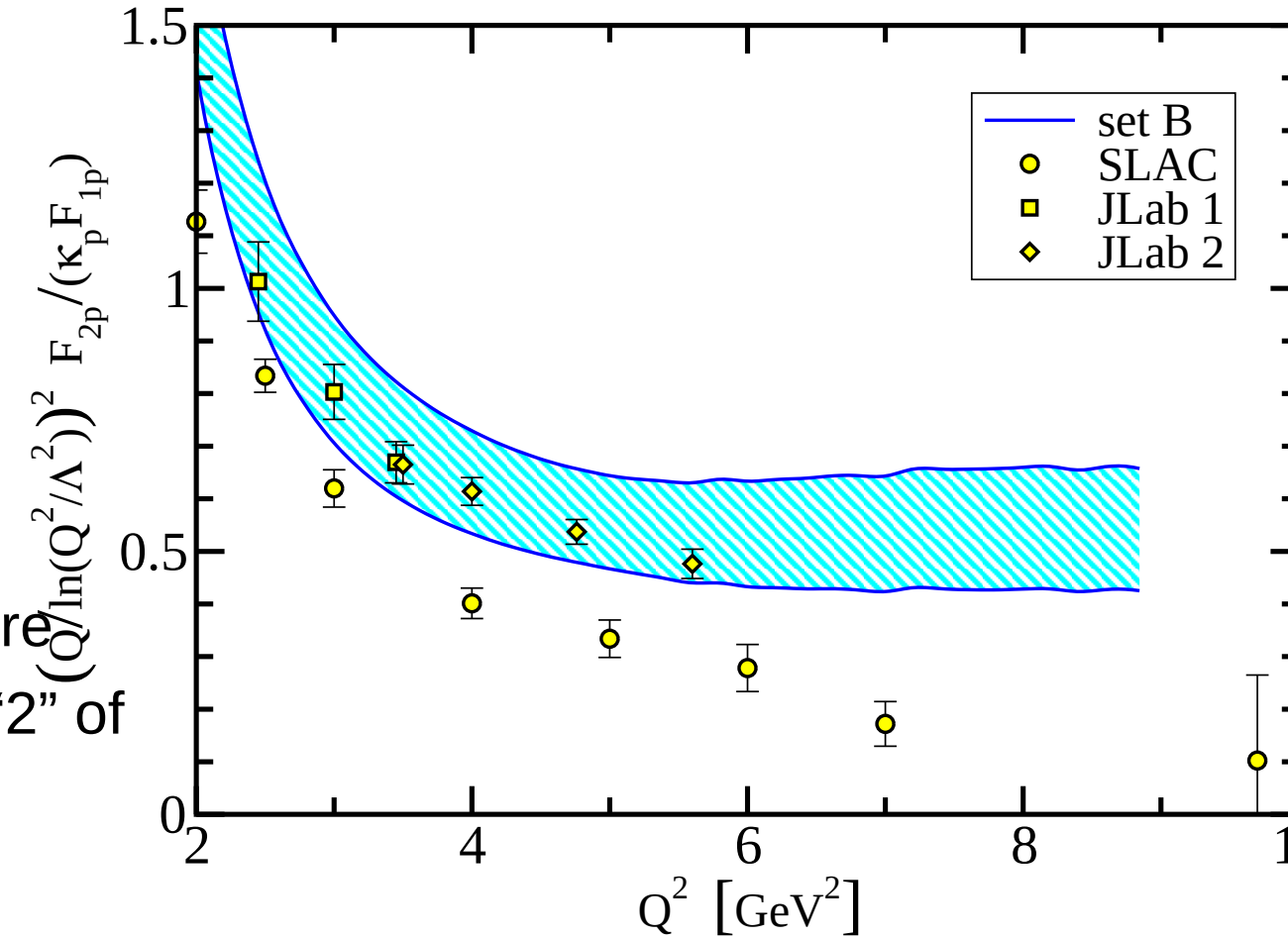


Form Factor Ratio: alternative F2/F1

- $$\frac{Q^2}{[\ln Q^2/\Lambda^2]^2}\,\frac{F_2(Q^2)}{F_1(Q^2)} = \text{constant}, \quad Q^2 \gg \Lambda^2 \approx M_N^2$$

Suggestive

NB. Framework constructed to give quark-counting i.e., "pQCD" *but* with wrong anomalous dimensions *but* they're ignored in *ln*-power "2" of this ratio