Perceptions of hadron theory and phenomenology

Craig D. Roberts

cdroberts@anl.gov

Physics Division

Argonne National Laboratory

Nucleon . . . 2 Key Hadrons

= Proton and Neutron

Nucleon . . . 2 Key Hadrons $=$ Proton and Neutron

- Fermions – two static properties:
  proton electric charge $= +1$; and magnetic moment, μ_p

Nucleon . . . 2 Key Hadrons $=$ Proton and Neutron

- Fermions – two static properties:
  proton electric charge $= +1$; and magnetic moment, μ_p
- Magnetic Moment discovered by Otto Stern and collaborators in 1933; Awarded Nobel Prize in 1943
  - Dirac (1928) – pointlike fermion: $\mu_p = \dfrac{e\hbar}{2M}$

Nucleon . . . 2 Key Hadrons $=$ Proton and Neutron

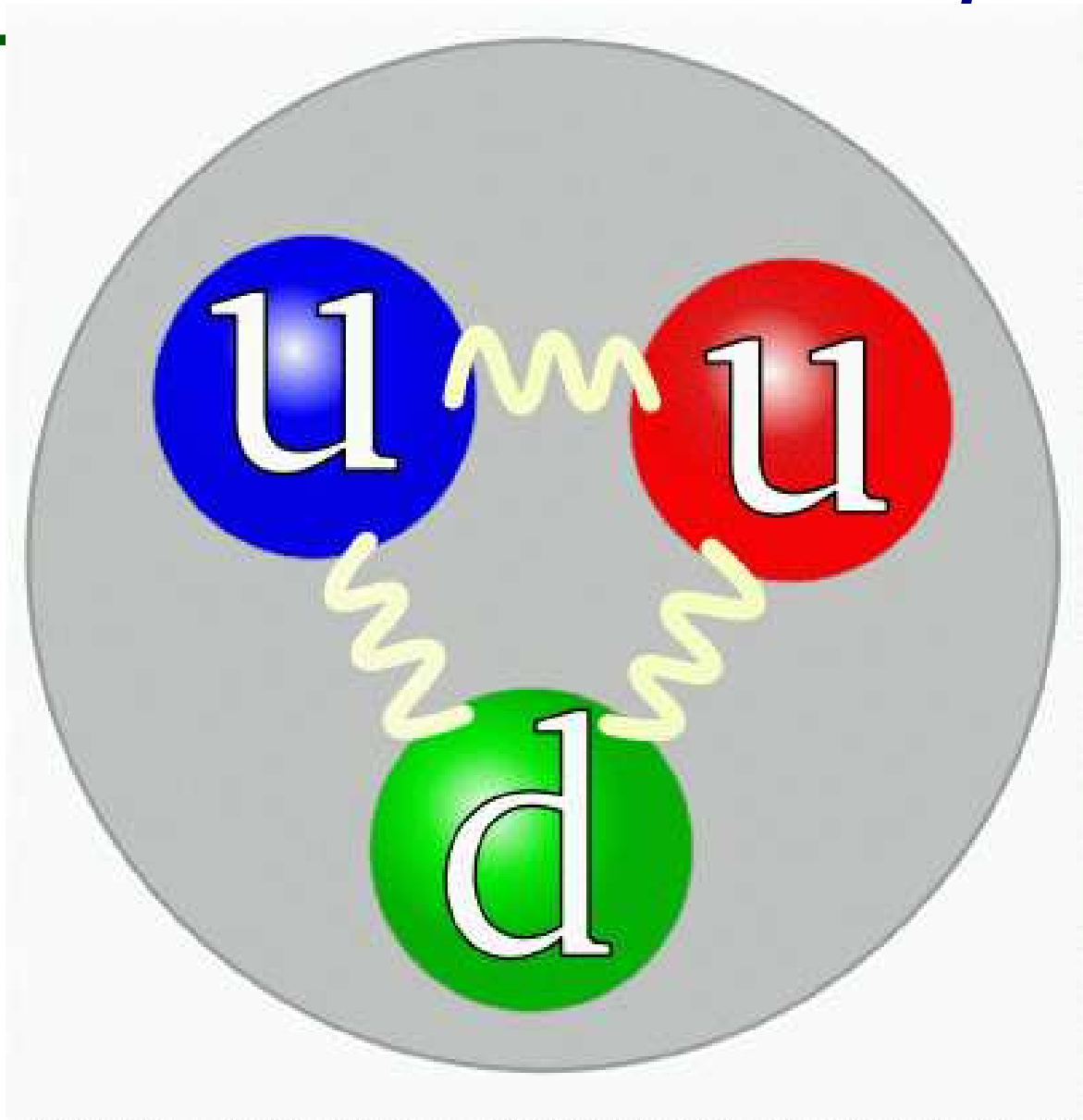
- Fermions – two static properties:
  proton electric charge $= +1$; and magnetic moment, μ_p
- Magnetic Moment discovered by Otto Stern and collaborators in 1933; Awarded Nobel Prize in 1943
  - Dirac (1928) – pointlike fermion: $\mu_p = \dfrac{e\hbar}{2M}$
- Stern (1933) – $\mu_p = (1 + 1.79)\dfrac{e\hbar}{2M}$

Nucleon . . . 2 Key Hadrons = Proton and Neutron

- Fermions – two static properties:
  proton electric charge $= +1$; and magnetic moment, μ_p
- Magnetic Moment discovered by Otto Stern and collaborators in 1933; Awarded Nobel Prize in 1943
  - Dirac (1928) – pointlike fermion: $\mu_p = \dfrac{e\hbar}{2M}$
- Stern (1933) – $\mu_p = (1 + 1.79)\dfrac{e\hbar}{2M}$
  - Big Hint that Proton is not a point particle
  - Proton has constituents
  - These are Quarks and Gluons
    Quark discovery via $e^- p$-scattering at SLAC in 1968
    – the elementary quanta of Quantum Chromo-dynamics

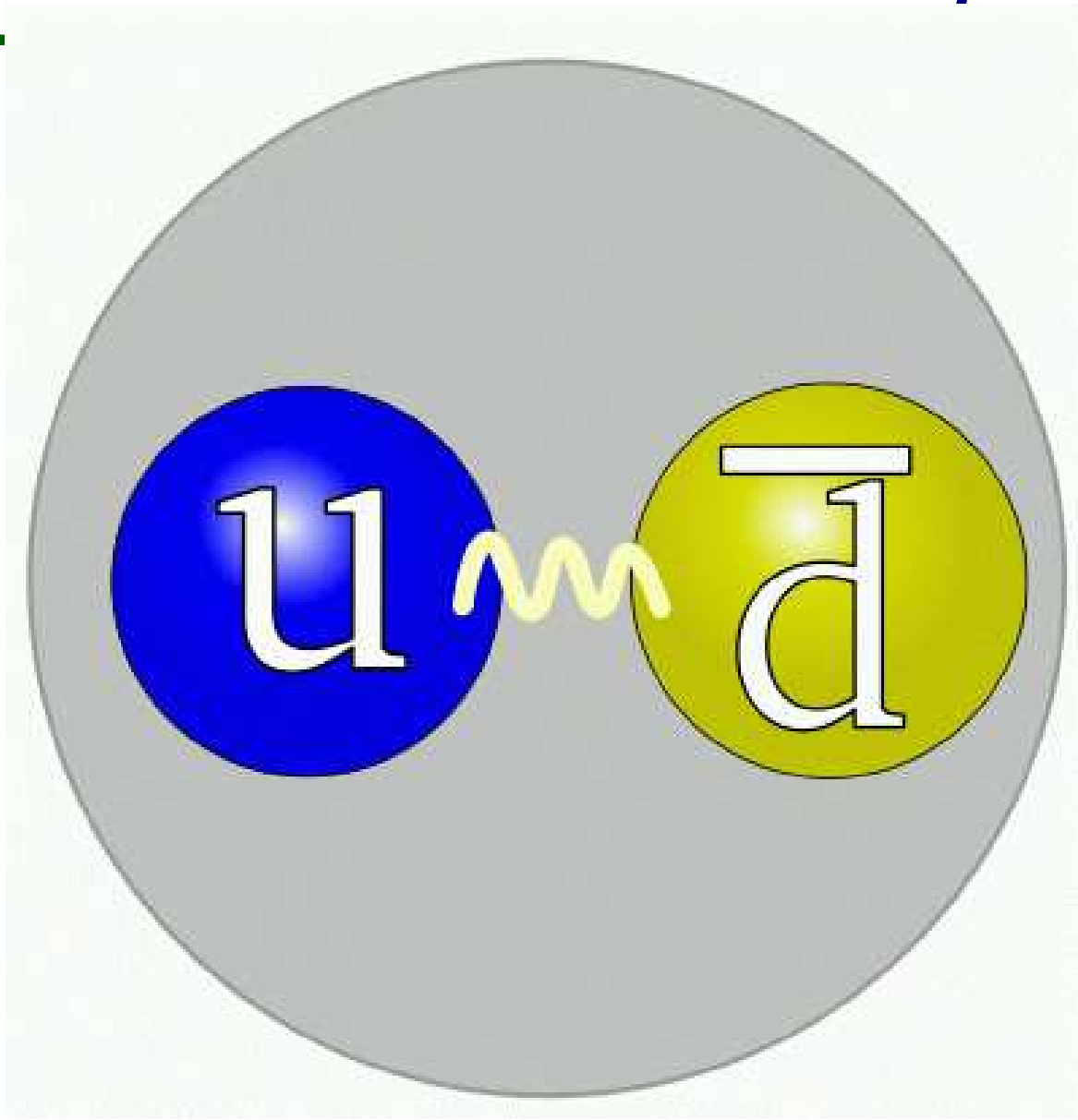
Simple Picture

Simple Picture



PROTON

Simple Picture



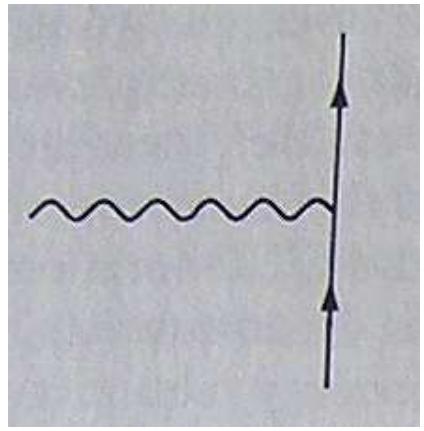
PION

Study Structure via Nucleon Form Factors

Study Structure via Nucleon Form Factors

- Electron's relativistic electromagnetic current:

$$
\begin{aligned}
j_\mu(P', P) &= ie\,\bar{u}_e(P')\,\Lambda_\mu(Q, P)\,u_e(P)\,,\ Q = P' - P \\
&= ie\,\bar{u}_e(P')\,\gamma_\mu(-1)\,u_e(P)
\end{aligned}
$$

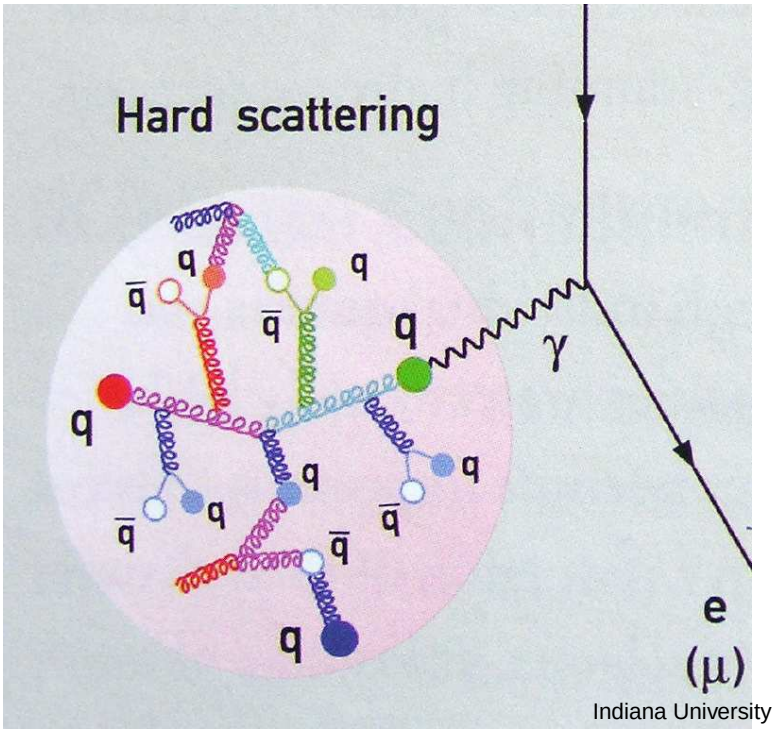


Study Structure via Nucleon Form Factors

- Electron's relativistic electromagnetic current:

$$
\begin{aligned}
j_\mu(P', P) &= ie\,\bar{u}_e(P')\,\Lambda_\mu(Q, P)\,u_e(P)\,,\ Q = P' - P \\
&= ie\,\bar{u}_e(P')\,\gamma_\mu(-1)\,u_e(P)
\end{aligned}
$$

- Nucleon's relativistic electromagnetic current:



Study Structure via Nucleon Form Factors

- Electron's relativistic electromagnetic current:

$$\begin{aligned} j_\mu(P',P) &= ie\,\bar{u}_e(P')\,\Lambda_\mu(Q,P)\,u_e(P)\,,\ Q = P' - P \\ &= ie\,\bar{u}_e(P')\,\gamma_\mu(-1)\,u_e(P) \end{aligned}$$

- Nucleon's relativistic electromagnetic current:

$$\begin{aligned} J_\mu(P',P) &= ie\,\bar{u}_p(P')\,\Lambda_\mu(Q,P)\,u_p(P)\,,\ Q = P' - P \\ &= ie\,\bar{u}_p(P')\left(\gamma_\mu F_1(Q^2) + \frac{1}{2M}\,\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right)u_p(P) \end{aligned}$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\ G_M(Q^2) = F_1(Q^2) + F_2(Q^2)\,.$$

Study Structure via Nucleon Form Factors

- Electron's relativistic electromagnetic current:

$$\begin{aligned} j_\mu(P',P) &= ie\,\bar{u}_e(P')\,\Lambda_\mu(Q,P)\,u_e(P)\,,\ Q=P'-P \\ &= ie\,\bar{u}_e(P')\,\gamma_\mu(-1)\,u_e(P) \end{aligned}$$

- Nucleon's relativistic electromagnetic current:

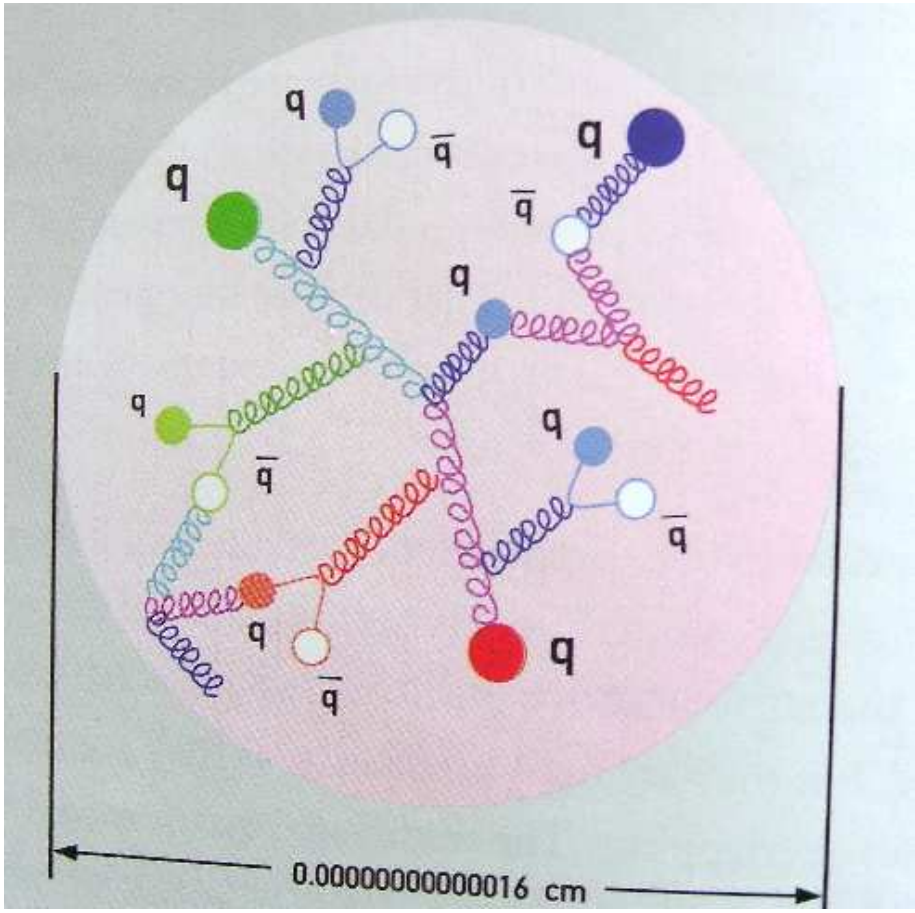
$$\begin{aligned} J_\mu(P',P) &= ie\,\bar{u}_p(P')\,\Lambda_\mu(Q,P)\,u_p(P)\,,\ Q=P'-P \\ &= ie\,\bar{u}_p(P')\left(\gamma_\mu F_1(Q^2)+\frac{1}{2M}\,\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right)u_p(P) \end{aligned}$$

$$G_E(Q^2)=F_1(Q^2)-\frac{Q^2}{4M^2}F_2(Q^2)\,,\ G_M(Q^2)=F_1(Q^2)+F_2(Q^2)\,.$$

Point-particle: $\boldsymbol{F_2 \equiv 0 \Rightarrow G_E \equiv G_M}$

NSAC Long Range Plan

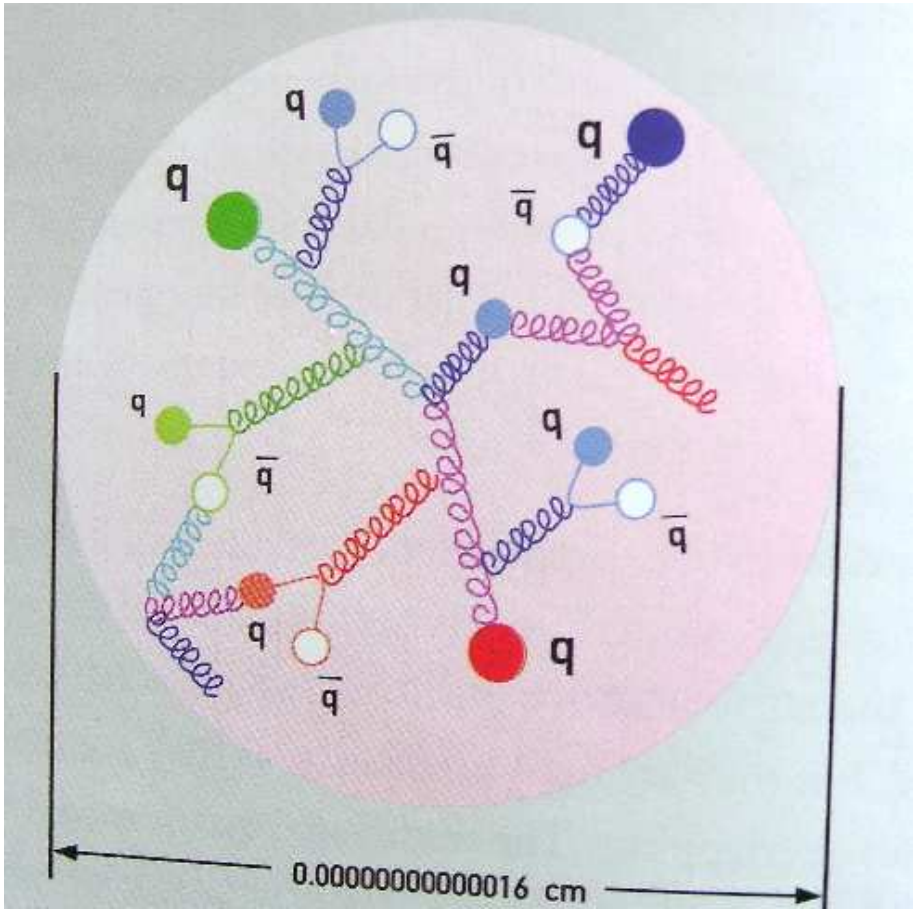
A central goal of nuclear physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD



NSAC Long Range Plan

A central goal of nuclear physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD

So, what's the problem?

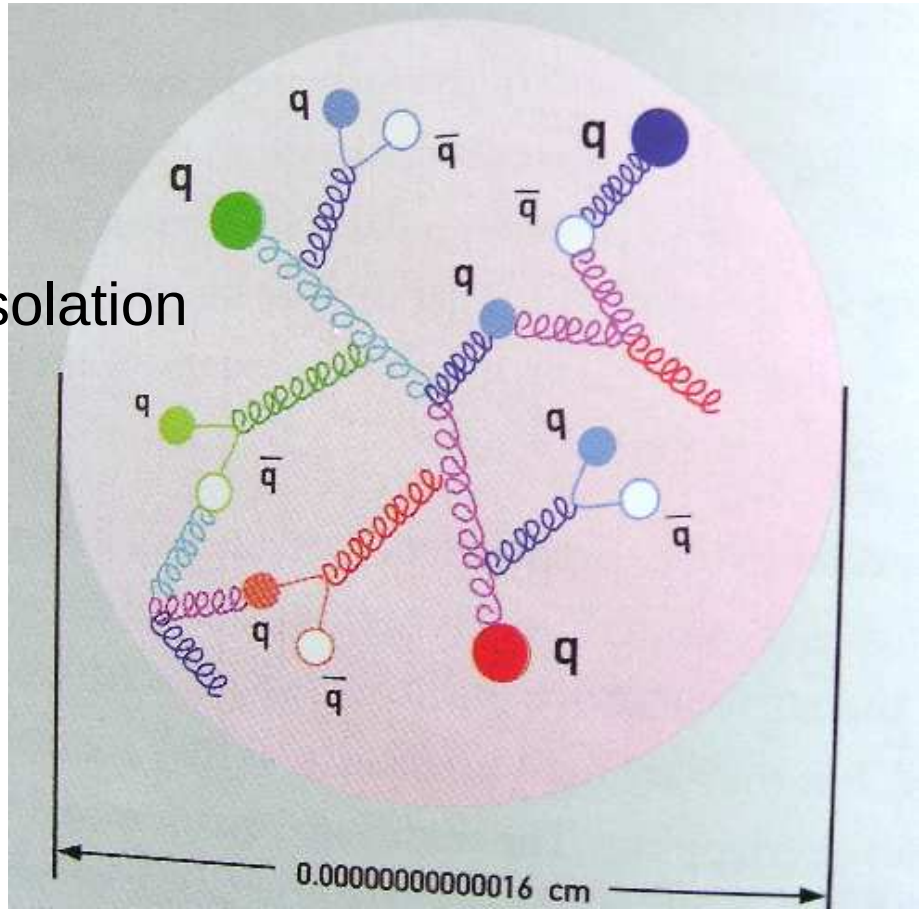


NSAC Long Range Plan

A central goal of nuclear physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD

So, what's the problem?

- Confinement
  - No quark ever seen in isolation

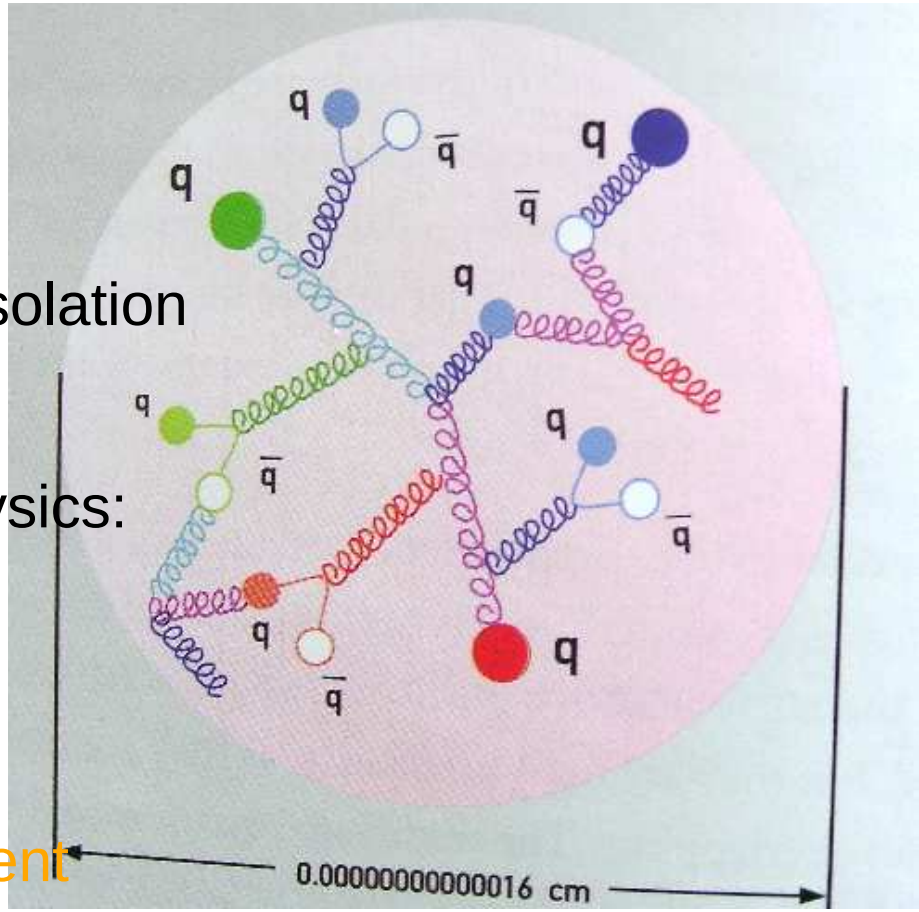


NSAC Long Range Plan

A central goal of nuclear physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD

So, what's the problem?

- Confinement
  – No quark ever seen in isolation
- Weightlessness
  – 2004 Nobel Prize in Physics:
  Mass of $u-$ & $d-$quarks,
  each just 5 MeV;
  Proton Mass is 940 MeV
  $\Rightarrow$ No Explanation Apparent
  for 98.4 % of Mass

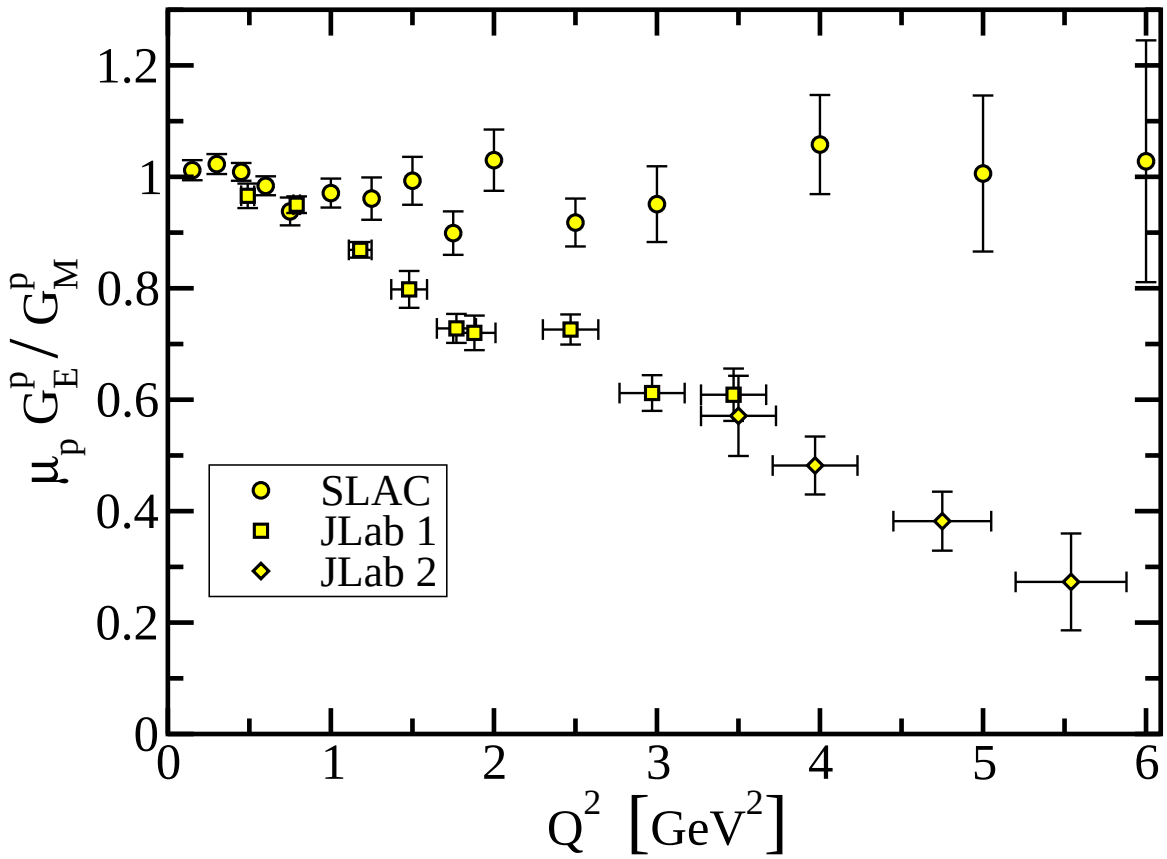


Proton Form Factors, Reprise

Proton Form Factors, Reprise

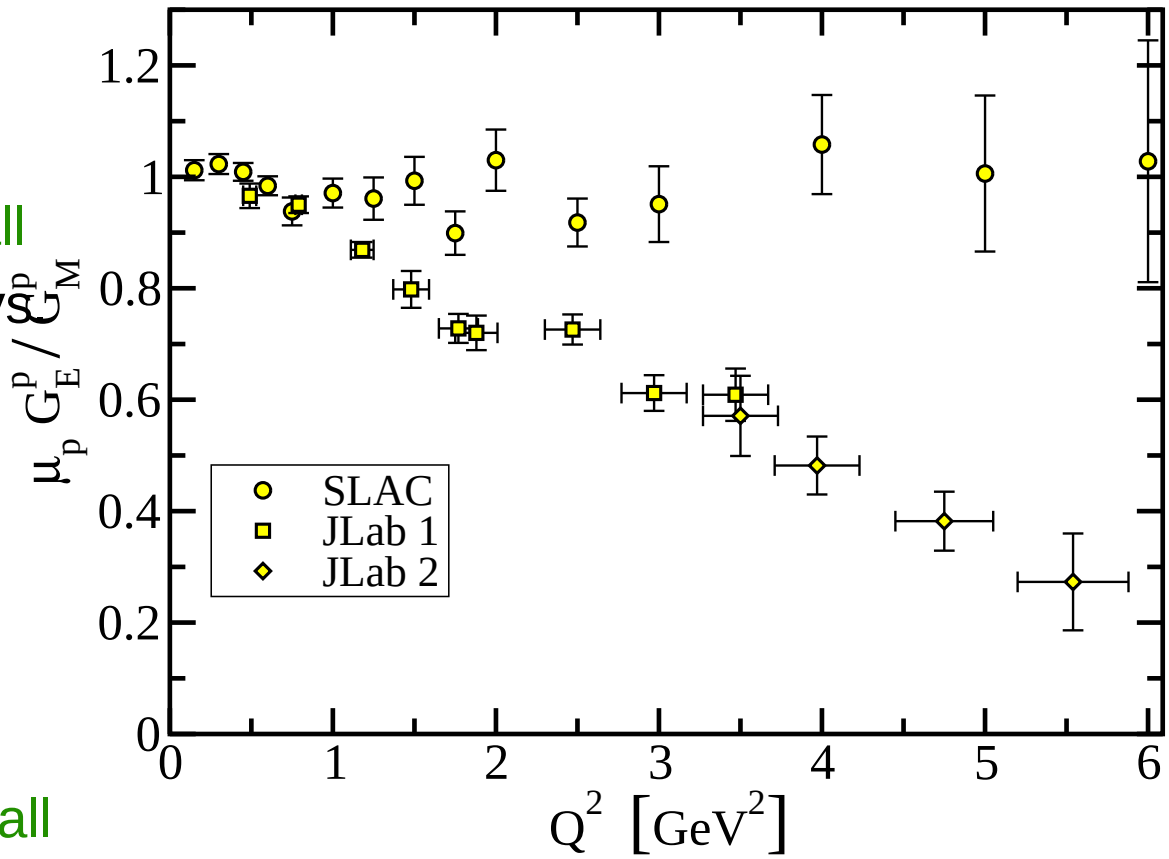
- SLAC and JLab have Measured Ratio of Proton's Electric and Magnetic Form Factors

Proton Form Factors, Reprise



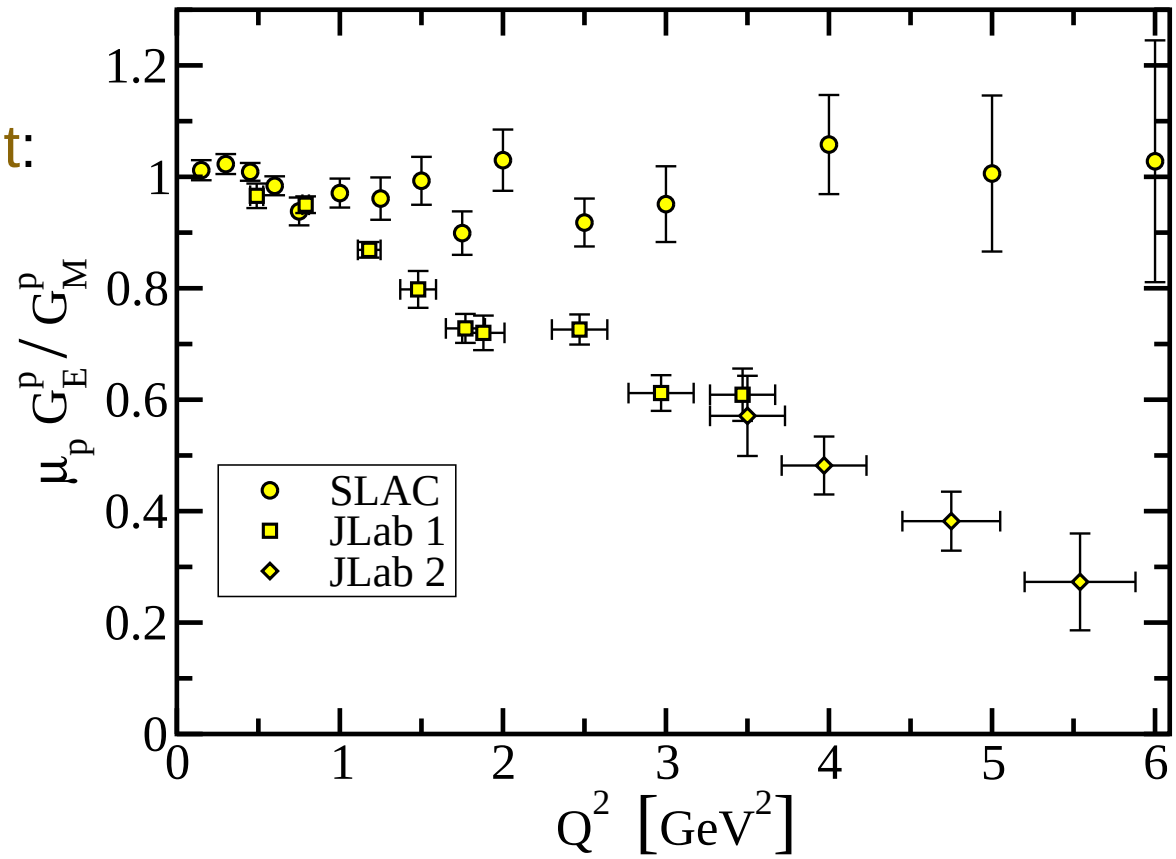
Proton Form Factors, Reprise

- Walker *et al.*, Phys. Rev. **D 49**, 5671 (1994). (SLAC)
- Jones *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **84**, 1398 (2000)
- Gayou, *et al.*, Phys. Rev. **C 64**, 038202 (2001)
- Gayou, *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **88** 092301 (2002)



Proton Form Factors, Reprise

- If JLab Correct, then Completely Unexpected Result: In the Proton
  - On Relativistic Domain
  - Distribution of Quark-Charge Not Equal Distribution of Quark-Current!

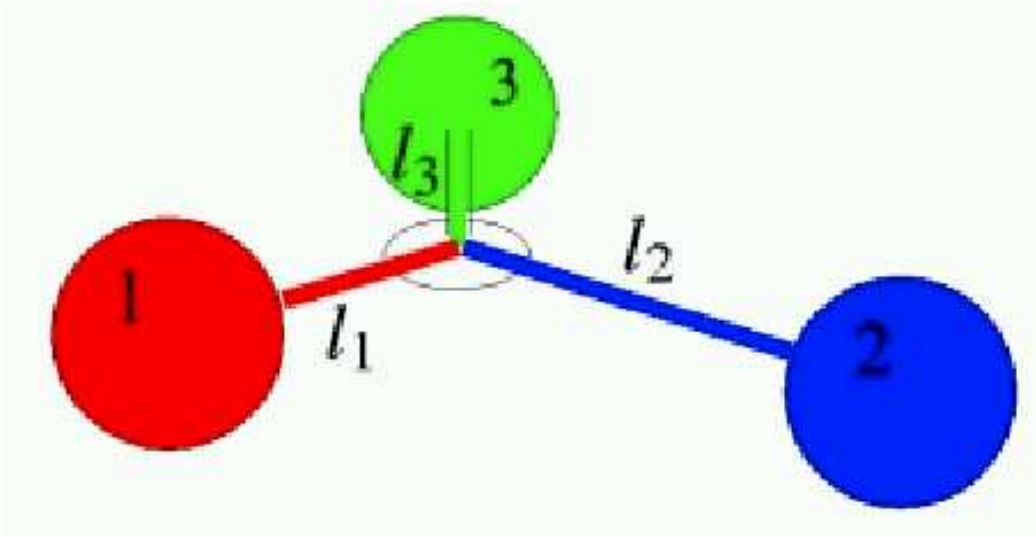


Meson Spectrum

LIGHT UNFLAVORED ($S = C = B = 0$)	$I^G(J^{PC})$		$I^G(J^{PC})$	STRANGE ($S = \pm 1,\ C = B = 0$)	$I(J^P)$
• $\pi^\pm$	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^{-+})$	• $K^\pm$	$1/2(0^-)$
• π^0 140 MeV	$1^-(0^{-+})$	• $\phi(1680)$	$0^-(1^{--})$	• K^0	$1/2(0^-)$
• η	$0^+(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$	• K^0_S	$1/2(0^-)$
• $f_0(600)$	$0^+(0^{++})$	• $\rho(1700)$	$1^+(1^{--})$	• K^0_L	$1/2(0^-)$
• $\rho(770)$ 770	$1^+(1^{--})$	$a_2(1700)$	$1^-(2^{++})$	$K^*_0(800)$	$1/2(0^+)$
• $\omega(782)$	$0^-(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$	• $K^*(892)$	$1/2(1^-)$
• $\eta'(958)$	$0^+(0^{-+})$	$\eta(1760)$	$0^+(0^{-+})$	• $K_1(1270)$	$1/2(1^+)$
• $f_0(980)$	$0^+(0^{++})$	• $\pi(1800)$	$1^-(0^{-+})$	• $K_1(1400)$	$1/2(1^+)$
• $a_0(980)$	$1^-(0^{++})$	$f_2(1810)$	$0^+(2^{++})$	• $K^*(1410)$	$1/2(1^-)$
• $\phi(1020)$	$0^-(1^{--})$	$X(1835)$	$?^?(?^{-+})$	• $K^*_0(1430)$	$1/2(0^+)$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$	• $K^*_2(1430)$	$1/2(2^+)$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$	$K(1460)$	$1/2(0^-)$
• $a_1(1260)$	$1^-(1^{++})$	$\rho(1900)$	$1^+(1^{--})$	$K_2(1580)$	$1/2(2^-)$
• $f_2(1270)$	$0^+(2^{++})$	$f_2(1910)$	$0^+(2^{++})$	$K(1630)$	$1/2(?^?)$
• $f_1(1285)$	$0^+(1^{++})$	• $f_2(1950)$	$0^+(2^{++})$	$K_1(1650)$	$1/2(1^+)$
• $\eta(1295)$	$0^+(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$	• $K^*(1680)$	$1/2(1^-)$
• $\pi(1300)$	$1^-(0^{-+})$	• $f_2(2010)$	$0^+(2^{++})$	• $K_2(1770)$	$1/2(2^-)$

Modern Miracles in Hadron Physics

- proton = three constituent quarks



Modern Miracles in Hadron Physics

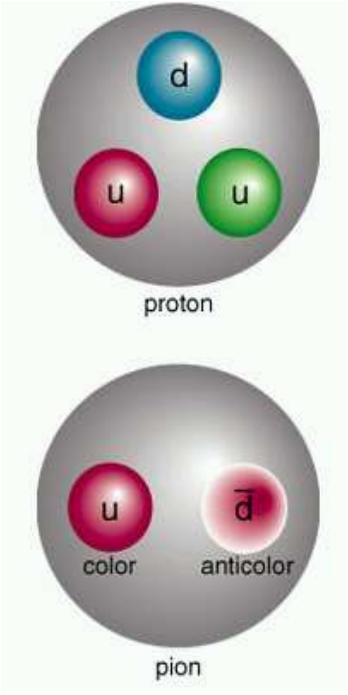
- proton = three constituent quarks
- $M_{\mathbf{proton}} \approx \mathbf{1\,GeV}$

Modern Miracles in Hadron Physics

- proton = three constituent quarks
- $M_{\mathbf{proton}} \approx \mathbf{1\,GeV}$
- guess $M_{\mathbf{constituent-quark}} \approx \dfrac{\mathbf{1\,GeV}}{\mathbf{3}} \approx \mathbf{350\,MeV}$

Modern Miracles in Hadron Physics

- proton = three constituent quarks
- $M_{\mathbf{proton}} \approx \mathbf{1\,GeV}$
- guess $M_{\mathbf{constituent-quark}} \approx \dfrac{\mathbf{1\,GeV}}{\mathbf{3}} \approx \mathbf{350\,MeV}$
- pion =
  constituent quark + constituent antiquark



Modern Miracles in Hadron Physics

- proton = three constituent quarks
- $M_{\mathbf{proton}} \approx \mathbf{1\,GeV}$
- guess $M_{\mathbf{constituent-quark}} \approx \dfrac{\mathbf{1\,GeV}}{\mathbf{3}} \approx \mathbf{350\,MeV}$
- pion =
  constituent quark + constituent antiquark
- guess $M_{\mathbf{pion}} \approx \mathbf{2} \times \dfrac{M_{\mathbf{proton}}}{\mathbf{3}} \approx \mathbf{700\,MeV}$

Modern Miracles in Hadron Physics

- proton = three constituent quarks
- $M_{\mathbf{proton}} \approx \mathbf{1\,GeV}$
- guess $M_{\mathbf{constituent-quark}} \approx \dfrac{\mathbf{1\,GeV}}{\mathbf{3}} \approx \mathbf{350\,MeV}$
- pion =
  constituent quark + constituent antiquark
- guess $M_{\mathbf{pion}} \approx \mathbf{2} \times \dfrac{M_{\mathbf{proton}}}{\mathbf{3}} \approx \mathbf{700\,MeV}$
- WRONG ..... $\boxed{M_{\mathbf{pion}} = \mathbf{140\,MeV}}$

Modern Miracles in Hadron Physics

- proton = three constituent quarks
- $M_{\mathbf{proton}} \approx \mathbf{1\,GeV}$
- guess $M_{\mathbf{constituent-quark}} \approx \dfrac{\mathbf{1\,GeV}}{\mathbf{3}} \approx \mathbf{350\,MeV}$
- pion =
  constituent quark + constituent antiquark
- guess $M_{\mathbf{pion}} \approx \mathbf{2} \times \dfrac{M_{\mathbf{proton}}}{\mathbf{3}} \approx \mathbf{700\,MeV}$
- WRONG ..... $\boxed{M_{\mathbf{pion}} = \mathbf{140\,MeV}}$
- Another meson:
  ..... $M_\rho = \mathbf{770}$ MeV ..... No Surprises Here

Modern Miracles in Hadron Physics

- proton = three constituent quarks
- $M_{\mathbf{proton}} \approx \mathbf{1\,GeV}$
- guess $M_{\mathbf{constituent-quark}} \approx \dfrac{\mathbf{1\,GeV}}{\mathbf{3}} \approx \mathbf{350\,MeV}$
- pion =
  constituent quark + constituent antiquark
- guess $M_{\mathbf{pion}} \approx \mathbf{2} \times \dfrac{M_{\mathbf{proton}}}{\mathbf{3}} \approx \mathbf{700\,MeV}$
- WRONG ..... $\boxed{M_{\mathbf{pion}} = \mathbf{140\,MeV}}$
- What is "wrong" with the pion?

Is this . . . Mass Destruction?

Dichotomy of the Pion

Dichotomy of the Pion

- How does one make an almost massless particle
  . . . . . from two massive constituent-quarks?

Dichotomy of the Pion

- How does one make an almost massless particle
  . . . . . from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

  Must exhibit $\boxed{m_\pi^2 \propto m_q}$

Current Algebra . . . 1968

Dichotomy of the Pion

- How does one make an almost massless particle
  . . . . . from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

  Must exhibit $\boxed{m_\pi^2 \propto m_q}$

  Current Algebra . . . 1968

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a well-defined and valid chiral limit, and an accurate realisation of dynamical chiral symmetry breaking.

Dichotomy of the Pion

- How does one make an almost massless particle ..... from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

  Must exhibit $\boxed{m_\pi^2 \propto m_q}$

  Current Algebra . . . 1968

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a well-defined and valid chiral limit, and an accurate realisation of dynamical chiral symmetry breaking.

- Requires detailed understanding of Connection between Current-quark and Constituent-quark masses

Dichotomy of the Pion

- How does one make an almost massless particle
  . . . . . from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

  Must exhibit $\boxed{m_\pi^2 \propto m_q}$

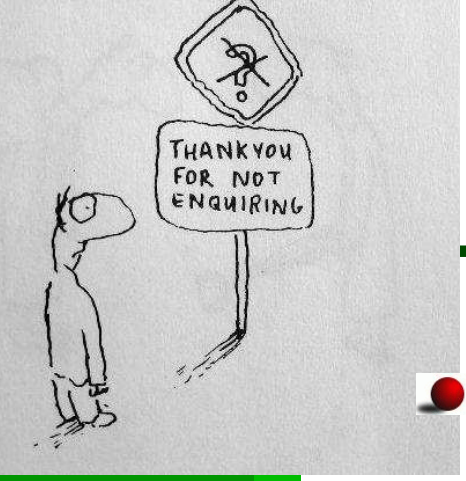
  Current Algebra . . . 1968

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a well-defined and valid chiral limit, and an accurate realisation of dynamical chiral symmetry breaking.

- Requires detailed understanding of Connection between Current-quark and Constituent-quark masses

Using DSEs, we've provided this.

QCD's Emergent Phenomena



QCD's Emergent Phenomena

- Complex behaviour arises from apparently simple rules

QCD's Emergent Phenomena

- Complex behaviour arises from apparently simple rules
- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

QCD's Emergent Phenomena

- Complex behaviour arises from apparently simple rules
- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses

QCD's Emergent Phenomena

- Complex behaviour arises from apparently simple rules
- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
- Neither of these phenomena is apparent in QCD's Lagrangian yet they are the dominant determining characteristics of real-world QCD.

QCD's Emergent Phenomena

- Complex behaviour arises from apparently simple rules
- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon
- Dynamical Chiral Symmetry Breaking
  - Very unnatural pattern of bound state masses
- Neither of these phenomena is apparent in QCD's Lagrangian yet they are the dominant determining characteristics of real-world QCD.
- NSAC – Understanding these phenomena is one of the greatest intellectual challenges in physics

Why should You care?

Why should You care?

$m_\pi = m_\rho \Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!

Why should You care?

$m_\pi = m_\rho \Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!

Under these circumstances,

Why should You care?

$m_\pi = m_\rho \;\Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!

Under these circumstances,

- What is the range: $\dfrac{1}{2\,m_q} \sim 20\,\text{fm}$ or $\dfrac{1}{2\,M_Q} \sim \dfrac{1}{3}\,\text{fm}$?

Why should You care?

$m_\pi = m_\rho \;\Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!

Under these circumstances,

- What is the range: $\dfrac{1}{2\,m_q} \sim 20\,\text{fm}$ or $\dfrac{1}{2\,M_Q} \sim \dfrac{1}{3}\,\text{fm}$?
- Is ^{12}C stable?

Why should You care?

$m_\pi = m_\rho \;\Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!

Under these circumstances,

- What is the range: $\dfrac{1}{2\,m_q} \sim 20\,\text{fm}$ or $\dfrac{1}{2\,M_Q} \sim \dfrac{1}{3}\,\text{fm}$?
- Is ^{12}C stable?
  - Probably not, if range range $\sim \dfrac{1}{2\,M_Q}$

Why should You care?

$m_\pi = m_\rho \Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!
Under these circumstances,

- How does the binding energy of deuterium change?

Why should You care?

$m_\pi = m_\rho \Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!
Under these circumstances,

- How does the binding energy of deuterium change?
- How does the neutron lifetime change?

Why should You care?

$m_\pi = m_\rho \Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!
Under these circumstances,

- How does the binding energy of deuterium change?
- How does the neutron lifetime change?
  - How does $m_u - m_d$ relate to $M_U - M_D$?

Why should You care?

$m_\pi = m_\rho \Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!
Under these circumstances,

- How does the binding energy of deuterium change?
- How does the neutron lifetime change?
  - How does $m_u - m_d$ relate to $M_U - M_D$?
  - Can one guarantee $M_n > M_p$?

Why should You care?

$m_\pi = m_\rho \Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!
Under these circumstances,

- How does the binding energy of deuterium change?
- How does the neutron lifetime change?
  - How does $m_u - m_d$ relate to $M_U - M_D$?
  - Can one guarantee $M_n > M_p$?
- How do such changes affect Big Bang Nucleosynthesis?

Why should You care?

$m_\pi = m_\rho \Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have SAME range and there is No intermediate range attraction!
Under these circumstances,

- How does the binding energy of deuterium change?
- How does the neutron lifetime change?
  - How does $m_u - m_d$ relate to $M_U - M_D$?
  - Can one guarantee $M_n > M_p$?

Is a unique long-range interaction between light-quarks responsible for all this or are there an uncountable infinity of qualitatively equivalent interactions?

What's the Problem?

What's the Problem?

- Must calculate the hadron's *wave function*
  – Can't be done using perturbation theory

What's the Problem?

- Must calculate the hadron's *wave function*
  – Can't be done using perturbation theory
  - So what? Same is true of hydrogen atom

What's the Problem?

- Must calculate the hadron's *wave function*
  – Can't be done using perturbation theory
  - So what? Same is true of hydrogen atom
- Differences

What's the Problem?

- Must calculate the hadron's *wave function*
  – Can't be done using perturbation theory
  - So what? Same is true of hydrogen atom
- Differences
  - Here relativistic effects are crucial
    – *virtual particles*
    Quintessence of Relativistic Quantum Field Theory

What's the Problem?

- Must calculate the hadron's *wave function*
  – Can't be done using perturbation theory
  - So what? Same is true of hydrogen atom
- Differences
  - Here relativistic effects are crucial
    – *virtual particles*
    Quintessence of Relativistic Quantum Field Theory
  - Interaction between quarks – the Interquark Potential – Unknown
    throughout $>$ 98% of the pion's/proton's volume

What's the Problem?

- Must calculate the hadron's *wave function*
  – Can't be done using perturbation theory
  - So what? Same is true of hydrogen atom
- Differences
  - Here relativistic effects are crucial
    – *virtual particles*
    Quintessence of Relativistic Quantum Field Theory
  - Interaction between quarks – the Interquark Potential – Unknown
    throughout $>$ 98% of the pion's/proton's volume
- Determination of proton's wave function requires
  ab initio nonperturbative solution
  of fully-fledged relativistic quantum field theory

What's the Problem?

- Must calculate the proton's *wave function*
  – Can't be done using perturbation theory
  - So what? Same is true of hydrogen atom
- Determination of proton's wave function requires
  ab initio nonperturbative solution
  of fully-fledged relativistic quantum field theory
- Modern Physics & Mathematics
  – Still quite some way from being able to do that

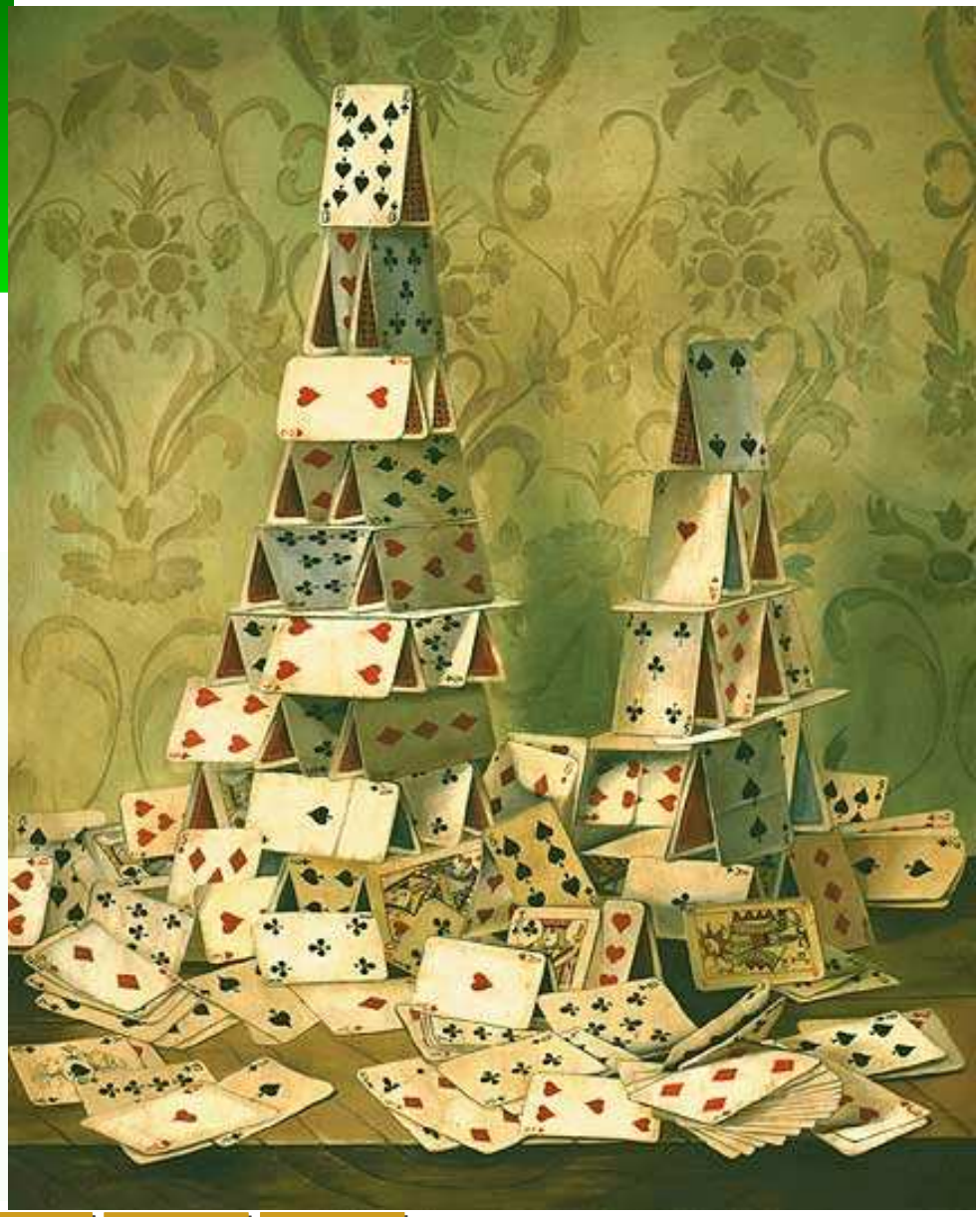
Model QCD

Traditional approach to strong force problem

Model QCD

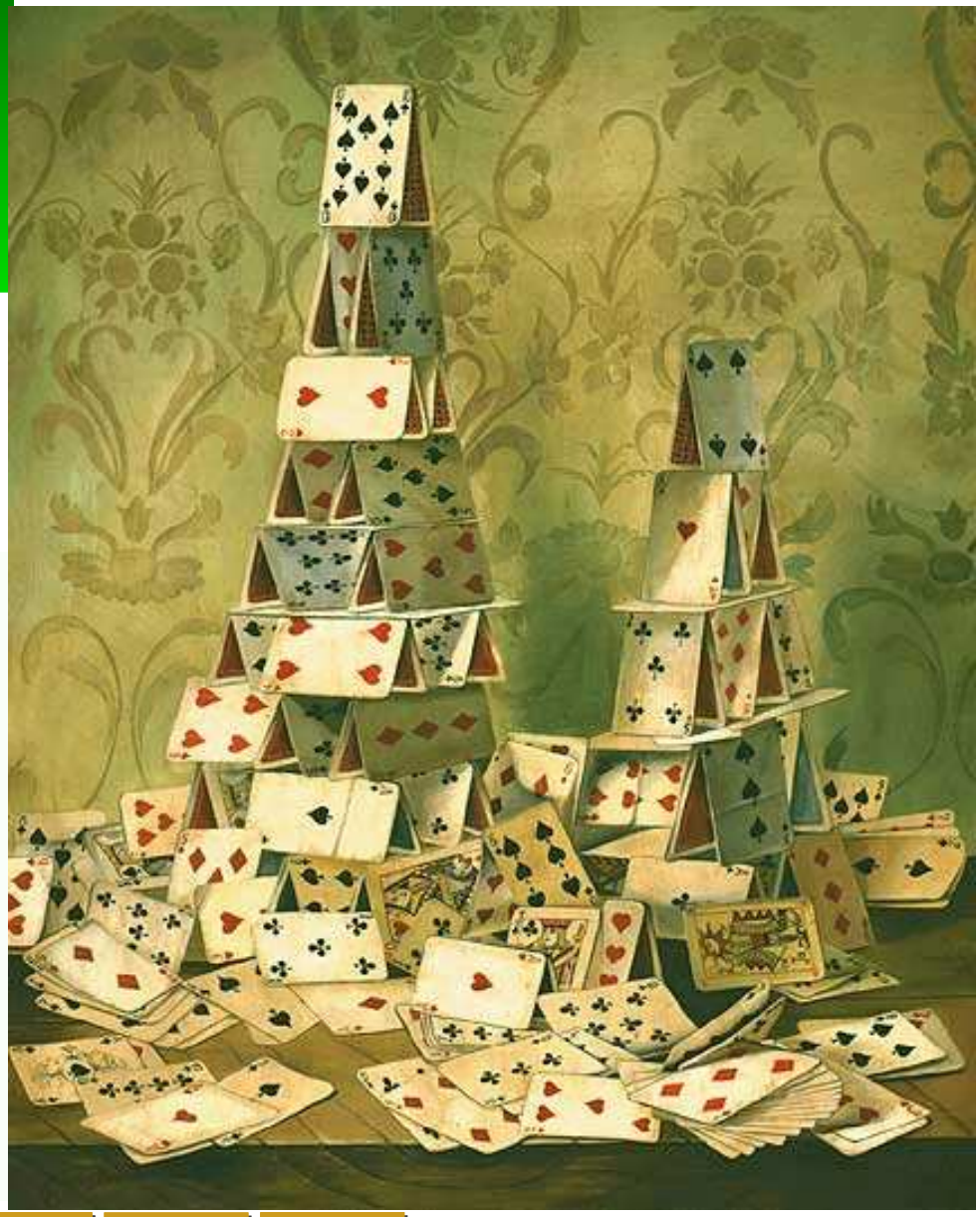
Traditional approach to strong force problem

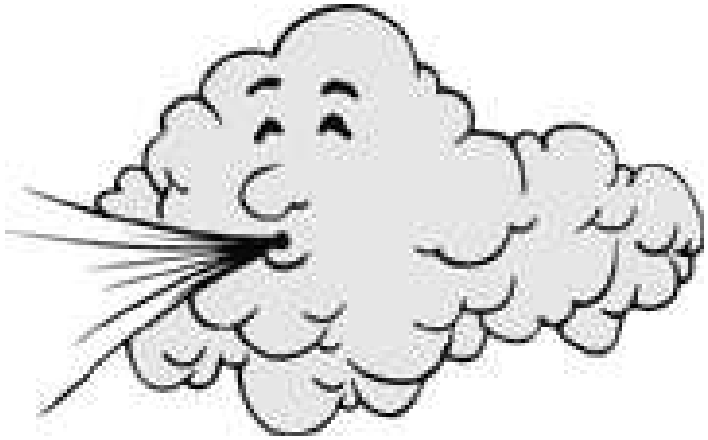
Model QCD



Traditional approach to strong force problem

Model QCD





Lattice QCD

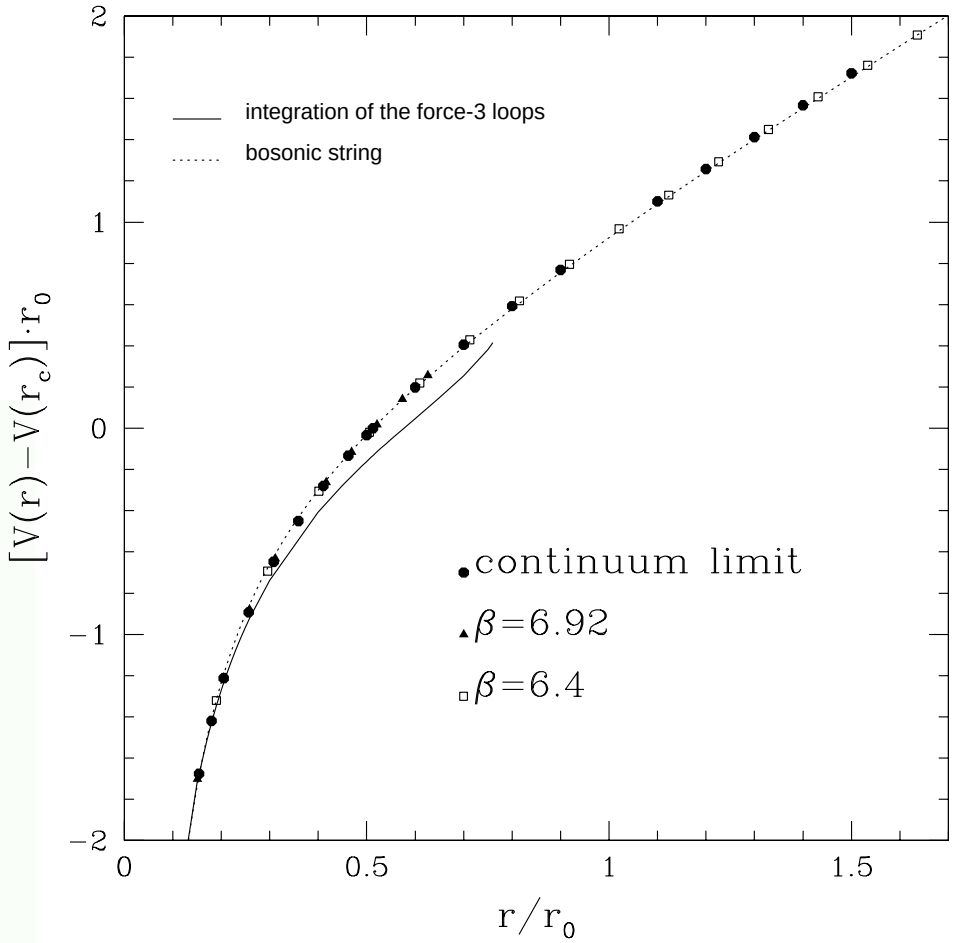
One modern nonperturbative approach ***Lattice QCD***

One modern nonperturbative approach ***Lattice QCD***

Confinement

Confinement

- Infinitely Heavy Quarks . . . Picture in Quantum Mechanics



$$V(r) = \sigma\, r - \frac{\pi}{12}\,\frac{1}{r}$$

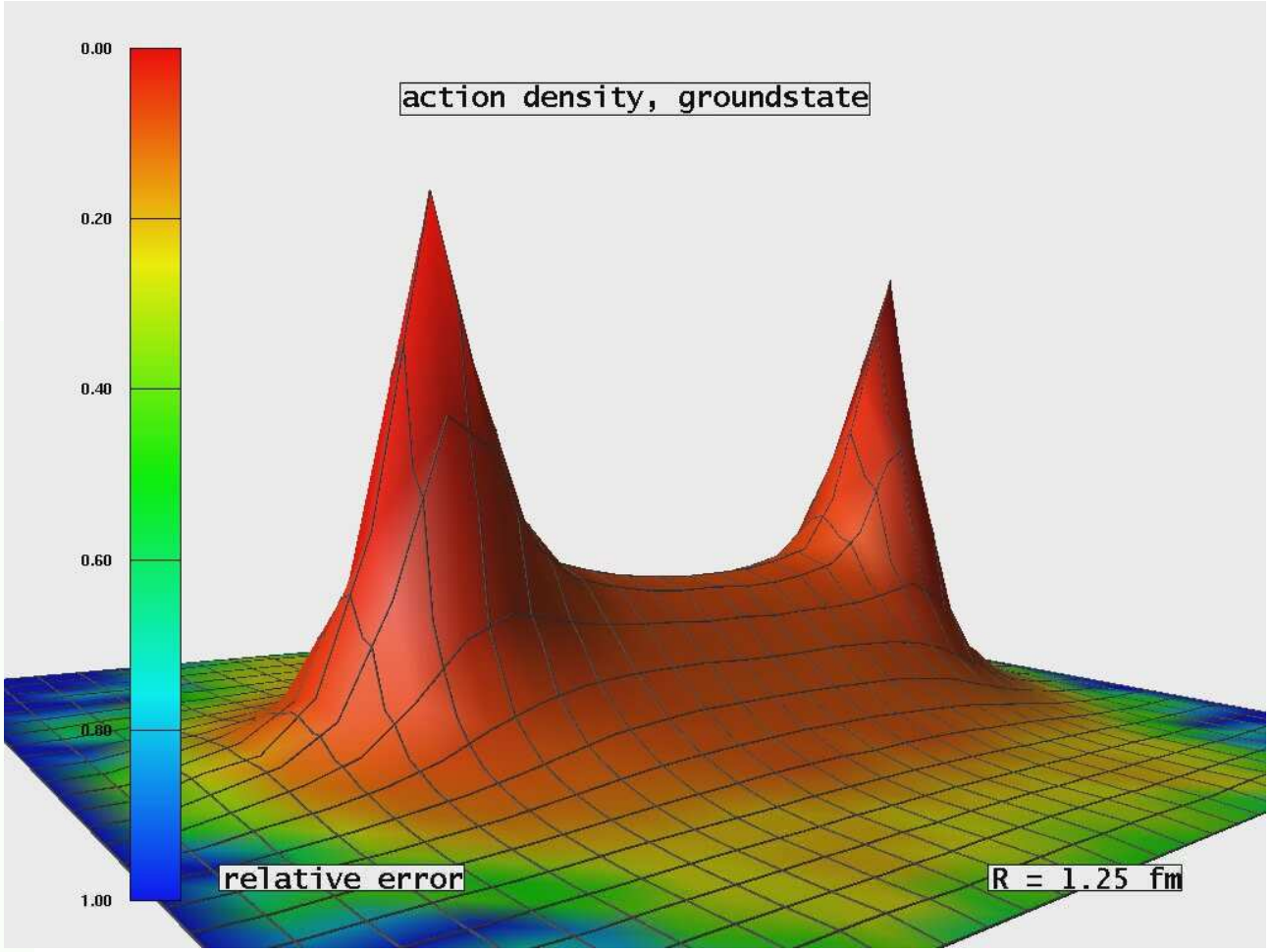
$\sigma \sim 470\,\text{MeV}$

Necco & Sommer

he-la/0108008

Confinement

- Illustrate this in terms of the action density . . . analogous to plotting the Force $= F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12}\frac{1}{r^2}$



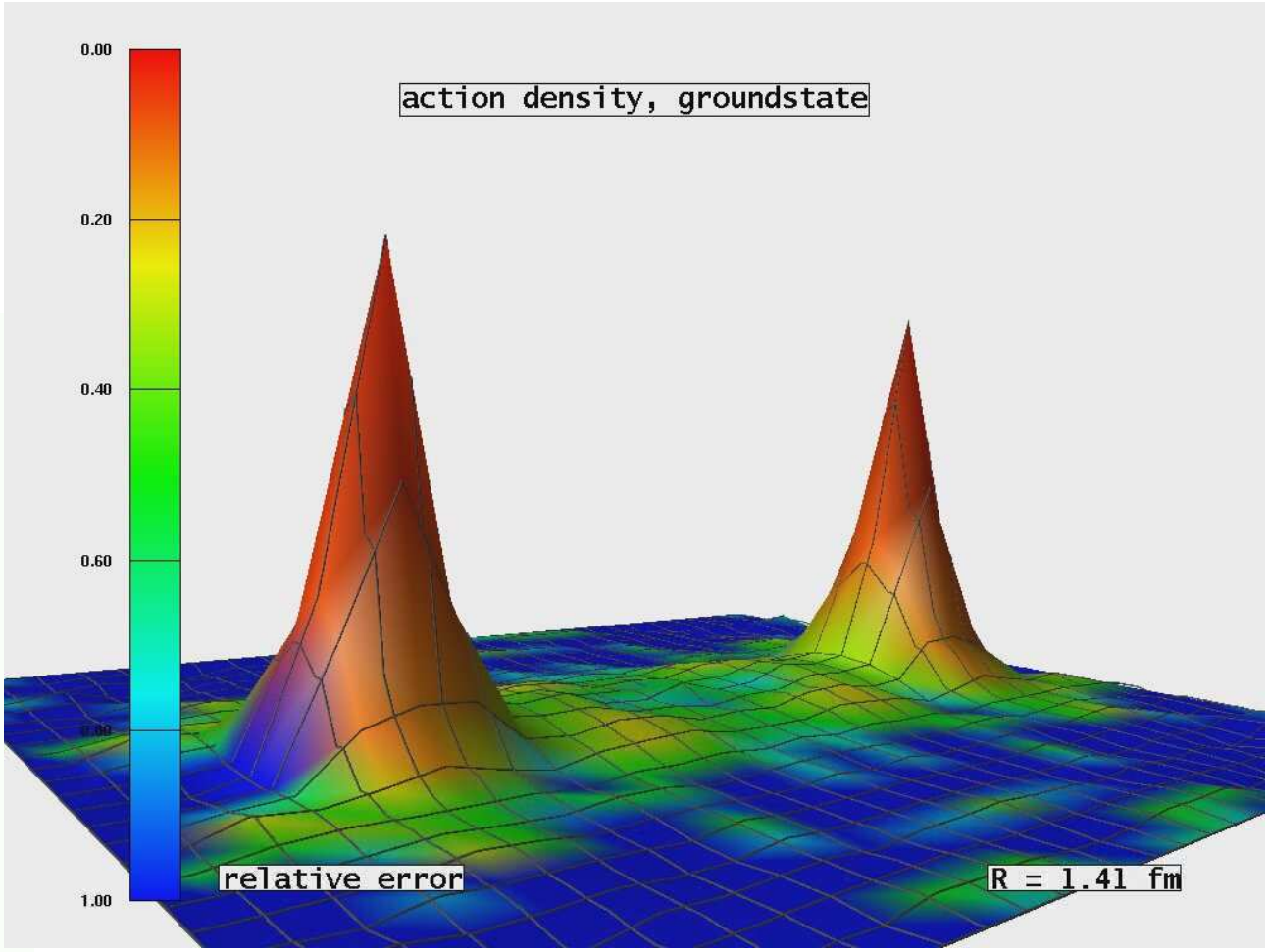
Bali, *et al.*
he-la/0512018

Confinement

- What happens in the real world; namely, in the presence of light-quarks?

Confinement

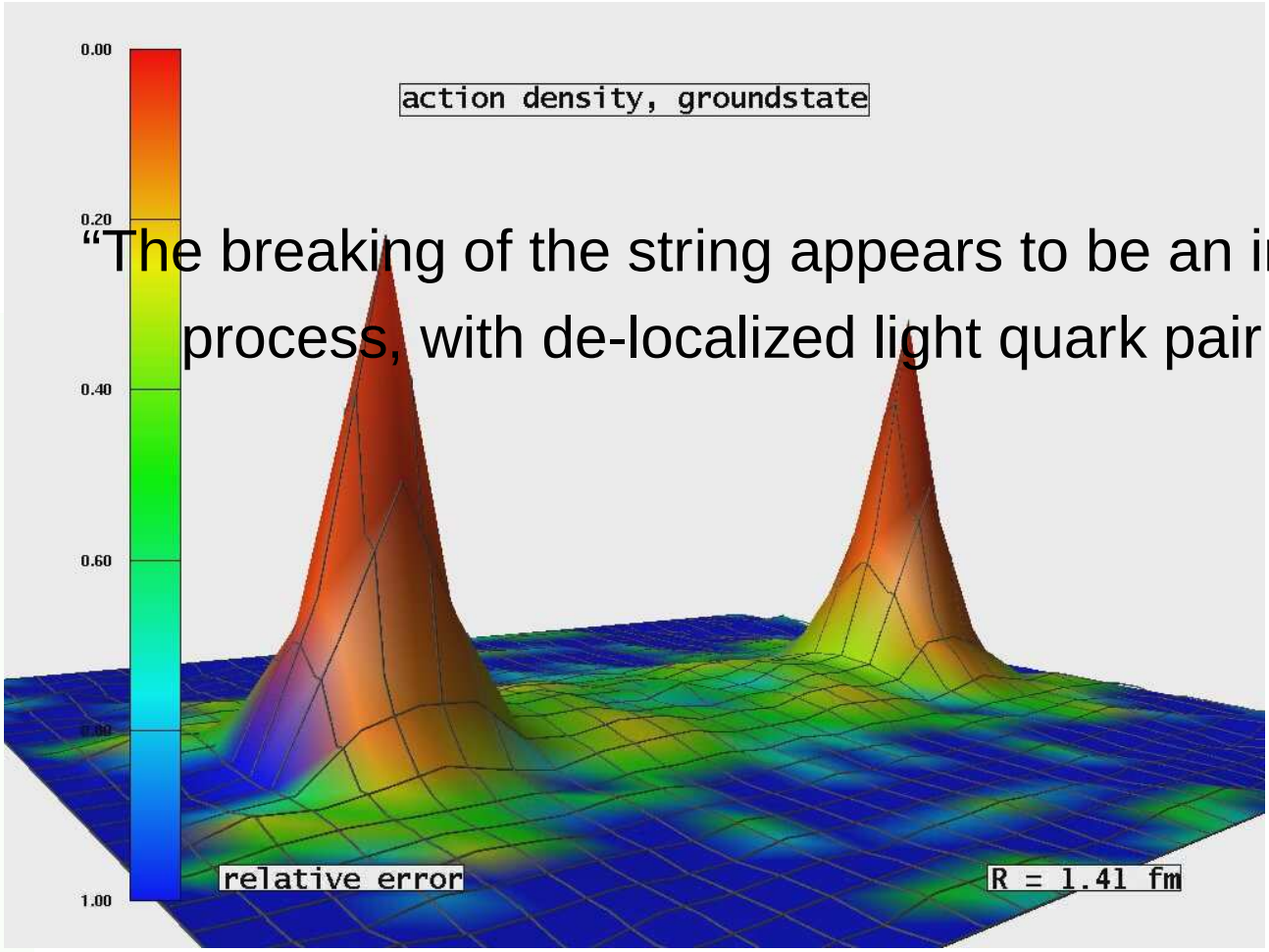
- What happens in the real world; namely, in the presence of light-quarks? No one knows . . . but $\bar{Q}Q + 2 \times \bar{q}q$



Bali, *et al.*
he-la/0512018

Confinement

- What happens in the real world; namely, in the presence of light-quarks? No one knows . . . but $\bar{Q}Q + 2 \times \bar{q}q$

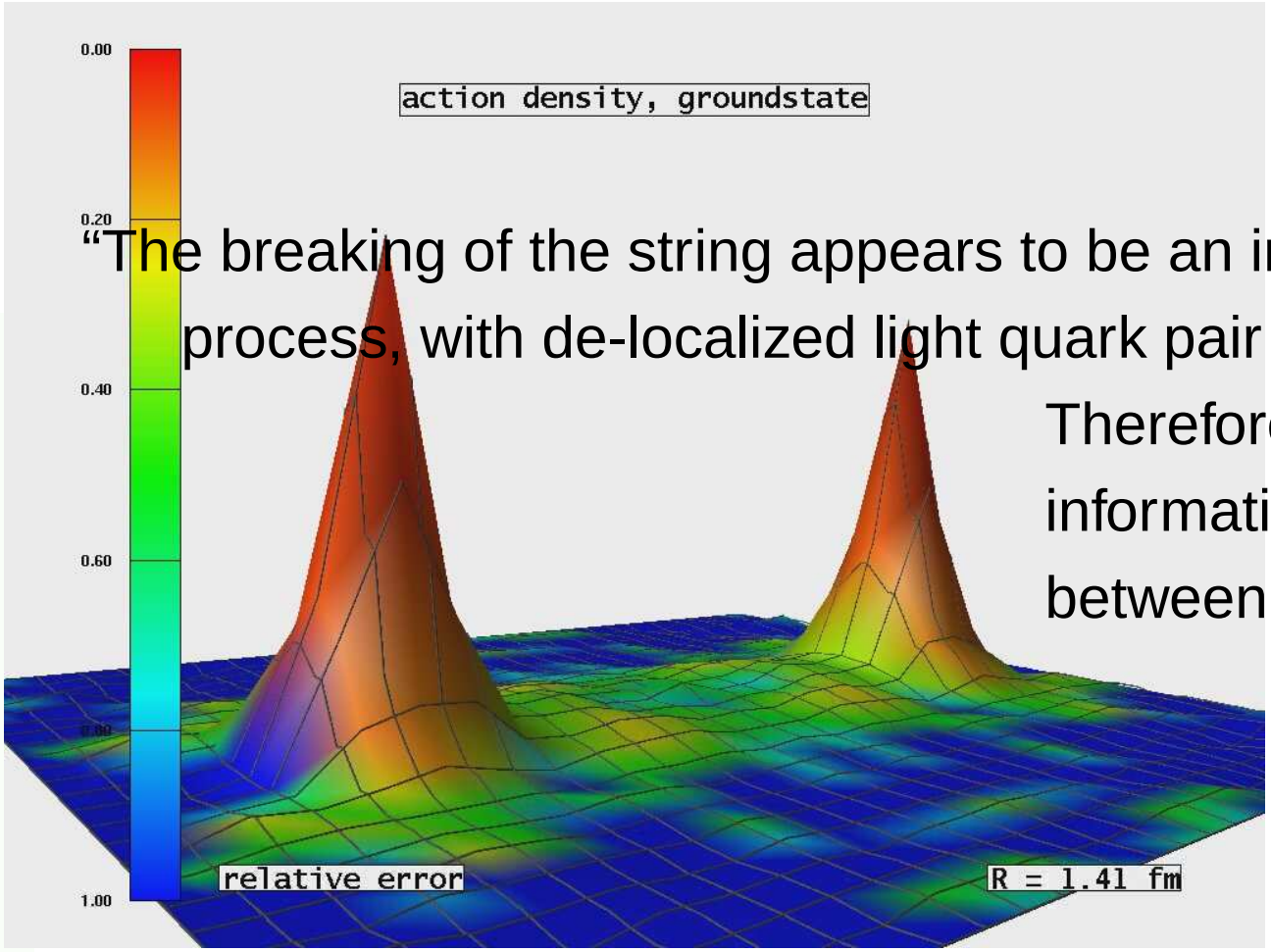


Bali, *et al.*
he-la/0512018

"The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation."

Confinement

- What happens in the real world; namely, in the presence of light-quarks? No one knows . . . but $\bar{Q}Q + 2 \times \bar{q}q$



Bali, *et al.*
he-la/0512018

"The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation."

Therefore . . . No information on *potential* between light-quarks.

A Compromise?

Dyson-Schwinger Equations

A Compromise?
Dyson-Schwinger Equations

- 1994 . . . "*As computer technology continues to improve, lattice gauge theory [LGT] will become an increasingly useful means of studying hadronic physics through investigations of discretised quantum chromodynamics [QCD]. . . . .*"

A Compromise?
Dyson-Schwinger Equations

- 1994 . . . "*However, it is equally important to develop other complementary nonperturbative methods based on continuum descriptions. In particular, with the advent of new accelerators such as CEBAF and RHIC, there is a need for the development of approximation techniques and models which bridge the gap between short-distance, perturbative QCD and the extensive amount of low- and intermediate-energy phenomenology in a single covariant framework. . . .*"

A Compromise?
Dyson-Schwinger Equations

- 1994 . . . "*Cross-fertilisation between LGT studies and continuum techniques provides a particularly useful means of developing a detailed understanding of nonperturbative QCD.*"

A Compromise?
Dyson-Schwinger Equations

- 1994 . . . "*Cross-fertilisation between LGT studies and continuum techniques provides a particularly useful means of developing a detailed understanding of nonperturbative QCD.*"

C. D. Roberts and A. G. Williams, "Dyson-Schwinger equations and their application to hadronic physics," Prog. Part. Nucl. Phys. **33**, 477 (1994) [arXiv:hep-ph/9403224].

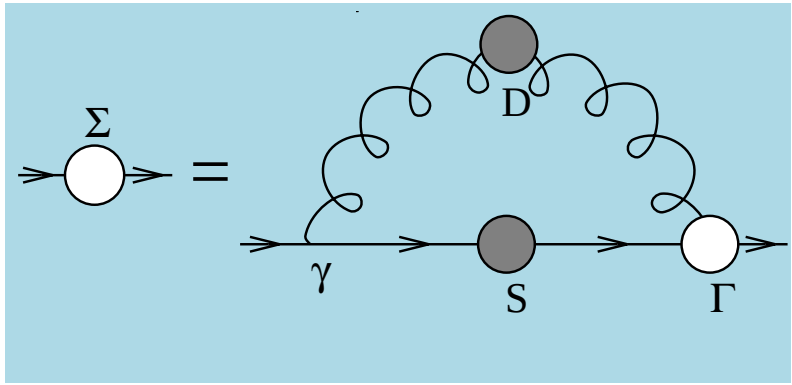
A Compromise?
Dyson-Schwinger Equations

- 1994 . . . "*Cross-fertilisation between LGT studies and continuum techniques provides a particularly useful means of developing a detailed understanding of nonperturbative QCD.*"

C. D. Roberts and ~~A. G. Williams~~, "Dyson-Schwinger equations and their application to hadronic physics," Prog. Part. Nucl. Phys. **33**, 477 (1994) [arXiv:hep-ph/9403224].

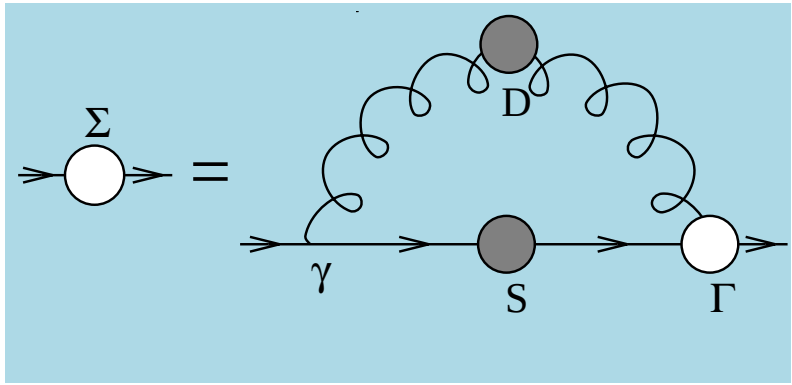
A Compromise?
Dyson-Schwinger Equations

- Dyson (1949) & Schwinger (1951) . . . One can derive a system of coupled integral equations relating the Green functions for the theory to each other.



A Compromise?
Dyson-Schwinger Equations

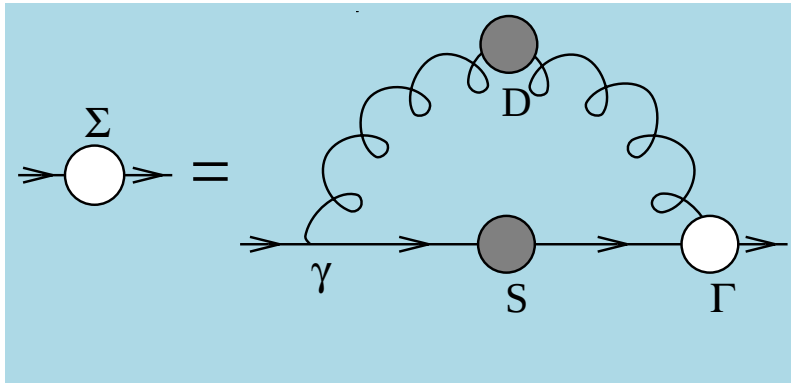
- Dyson (1949) & Schwinger (1951) . . . One can derive a system of coupled integral equations relating the Green functions for the theory to each other.



- These are nonperturbative equivalents in quantum field theory to the Lagrange equations of motion.

A Compromise?
Dyson-Schwinger Equations

- Dyson (1949) & Schwinger (1951) . . . One can derive a system of coupled integral equations relating the Green functions for the theory to each other.



- These are nonperturbative equivalents in quantum field theory to the Lagrange equations of motion.
- Essential in simplifying the general proof of renormalisability of gauge field theories.

Dyson-Schwinger Equations

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?
- $\Rightarrow$ Understanding InfraRed (long-range)
  ..... behaviour of $\alpha_s(Q^2)$

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?
- Method yields Schwinger Functions $\equiv$ Propagators

Dyson-Schwinger Equations

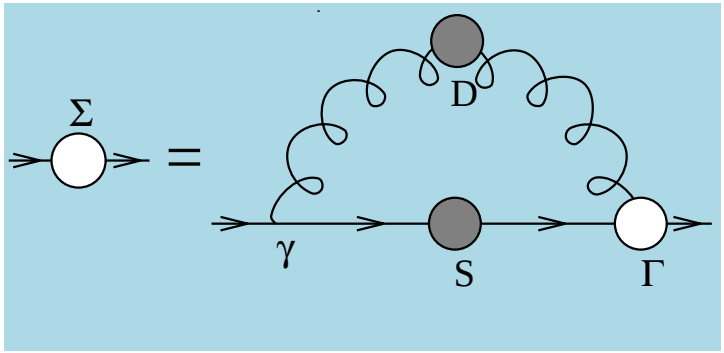
- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?

Cross-Sections built from Schwinger Functions

Persistent Challenge

Persistent Challenge

- Infinitely Many Coupled Equations



Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
  - Not all are Schwinger functions are experimentally observable but all are same VEVs measured in Lattice-QCD simulations . . . opportunity for comparisons at pre-experimental level . . . cross-fertilisation

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory
    Not useful for the nonperturbative problems in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
  H.J. Munczek Phys. Rev. D **52** (1995) 4736
  Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations
  A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
  Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of EXACT Results in QCD

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of EXACT Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results

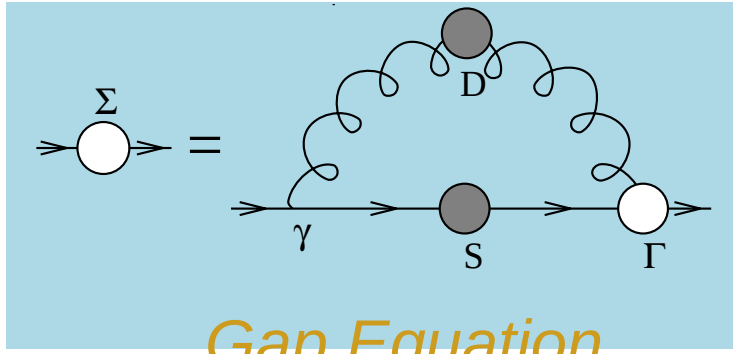
Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
- There is at least one systematic nonperturbative, symmetry-preserving truncation scheme
- Has Enabled Proof of EXACT Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Make Predictions with Readily Quantifiable Errors

Perturbative Dressed-quark Propagator

Perturbative Dressed-quark Propagator

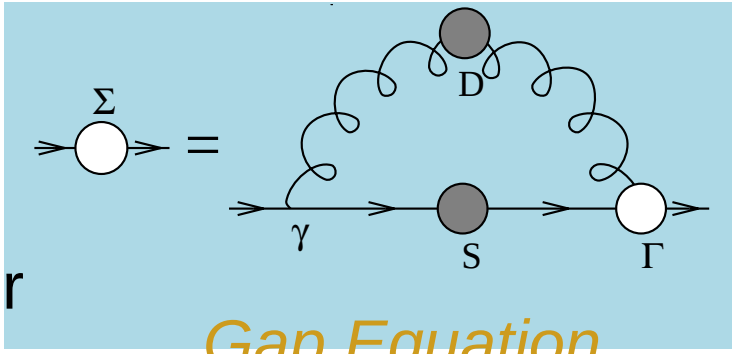
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



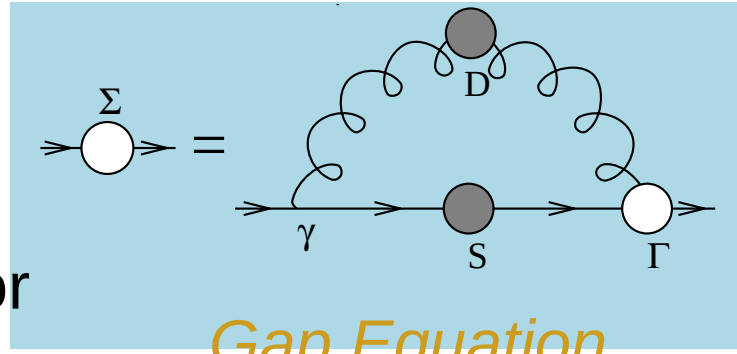
Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

- dressed-quark propagator

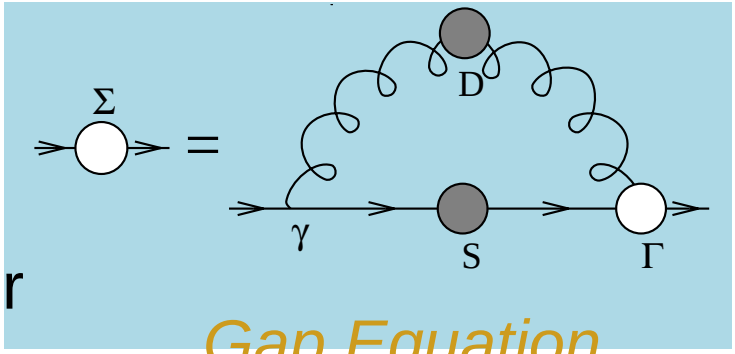
$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory

Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



- dressed-quark propagator

Gap Equation

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

- Weak Coupling Expansion

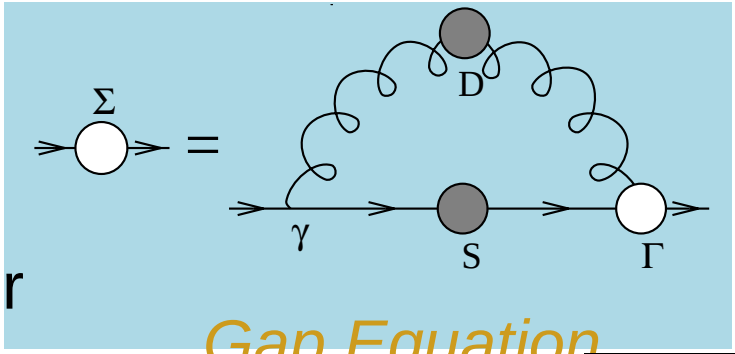
  Reproduces Every Diagram in Perturbation Theory

- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\longrightarrow} 0$$

Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

No DCSB Here!

- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory

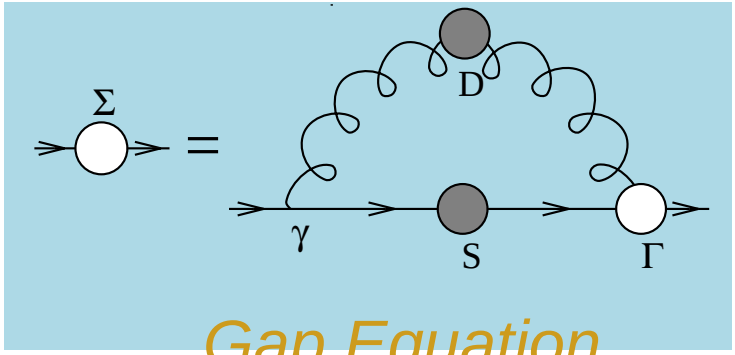
- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \xrightarrow{m\to 0} 0$$

Dressed-Quark Propagator

Dressed-Quark Propagator

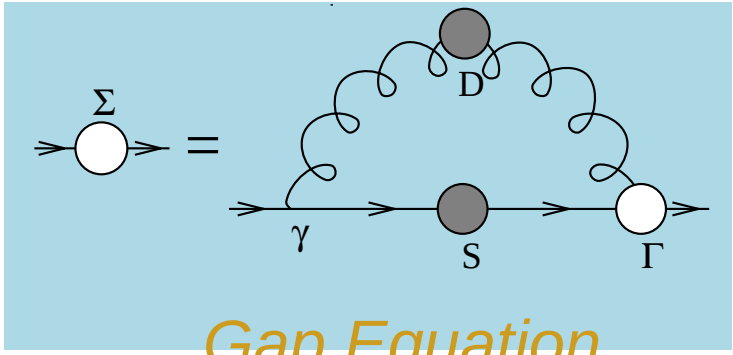
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

Dressed-Quark Propagator

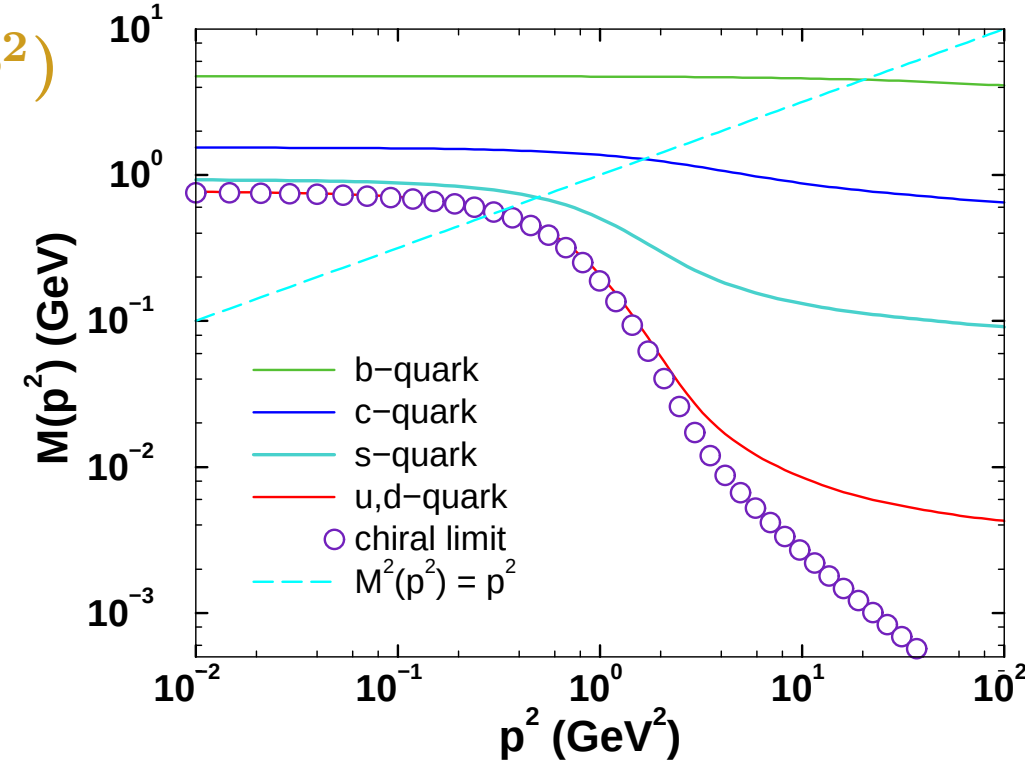
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

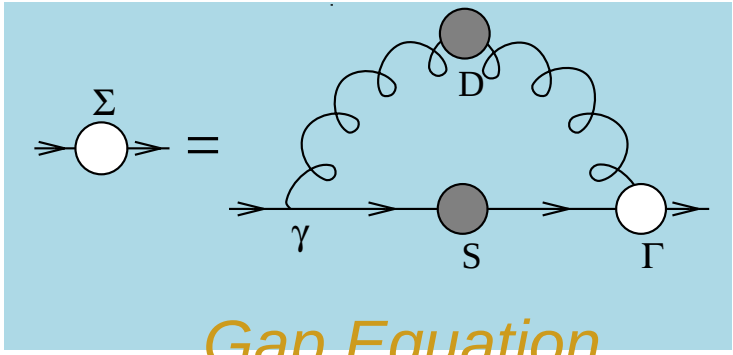
- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



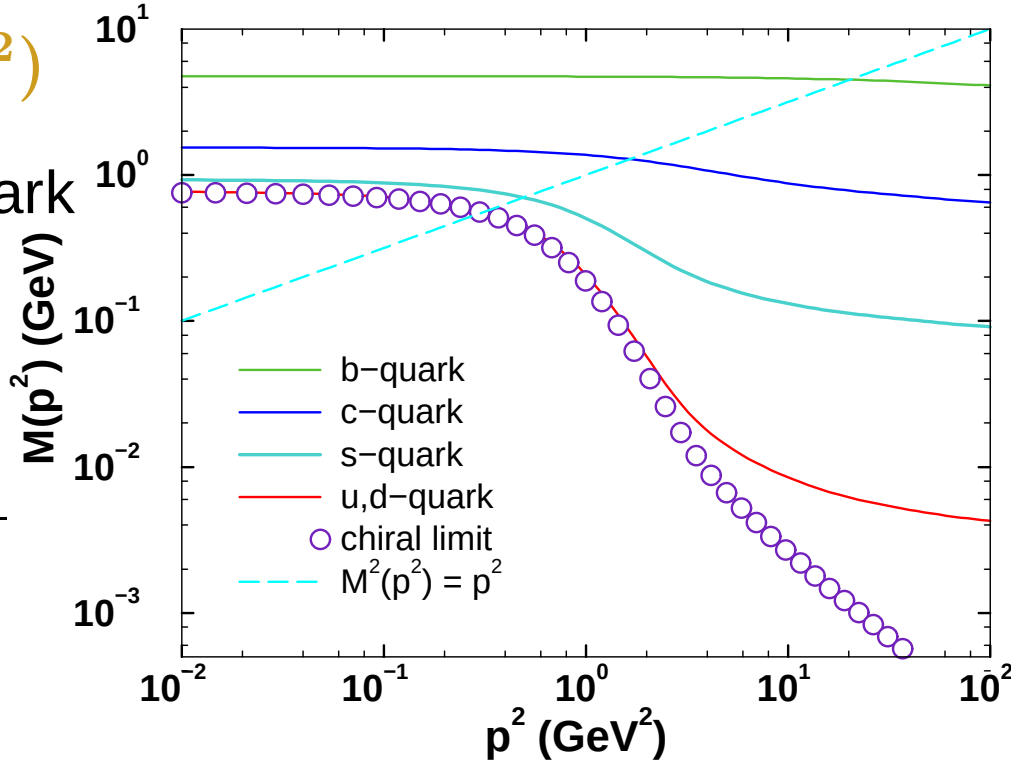
Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$

- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



Dressed-Quark Propagator

- Longstanding Prediction of Dyson-Schwinger Equation Studies

Dressed-Quark Propagator

- Longstanding Prediction of Dyson-Schwinger Equation Studies
  - E.g., *Dyson-Schwinger equations and their application to hadronic physics*, C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477

Dressed-Quark Propagator

- Long used as basis for efficacious hadron physics phenomenology

- Longstanding Prediction of Dyson-Schwinger Equation Studies
  - E.g., *Dyson-Schwinger equations and their application to hadronic physics*, C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477

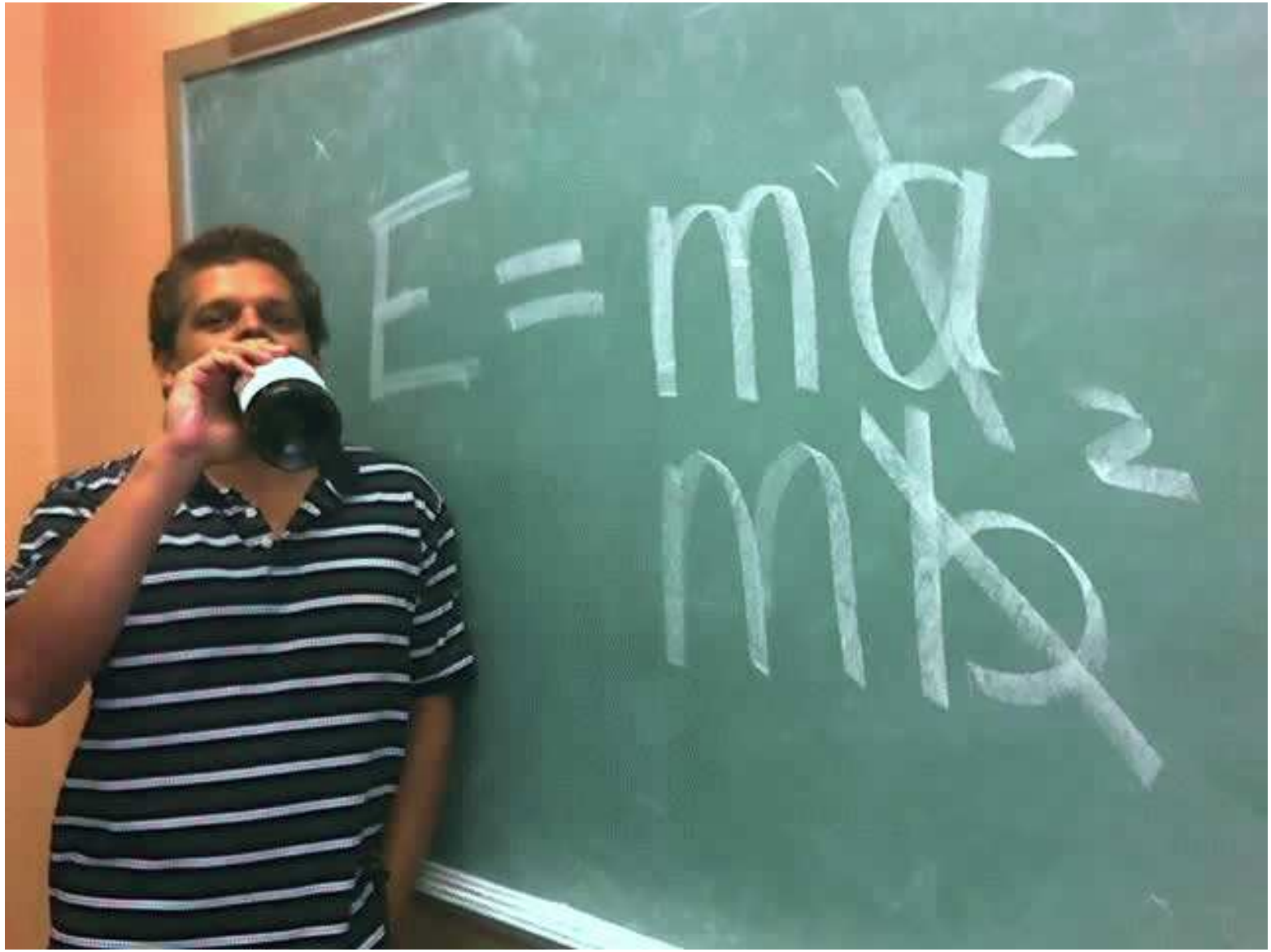
Dressed-Quark Propagator

- Long used as basis for efficacious hadron physics phenomenology
  - *Electromagnetic pion form-factor and neutral pion decay width*, C. D. Roberts, Nucl. Phys. A **605** (1996) 475

- Longstanding Prediction of Dyson-Schwinger Equation Studies
  - E.g., *Dyson-Schwinger equations and their application to hadronic physics*, C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477

Mandar Bhagwat

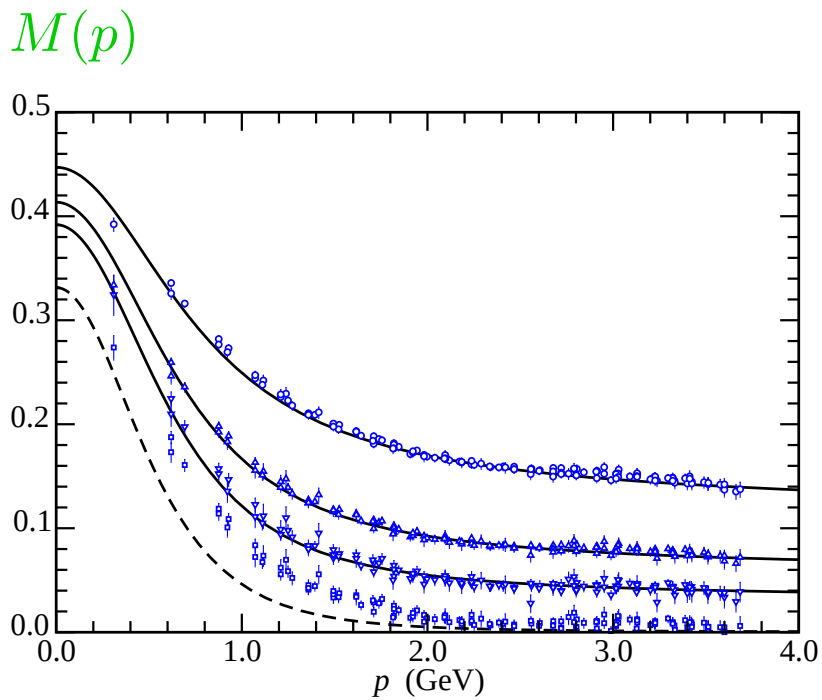
Mandar Bhagwat

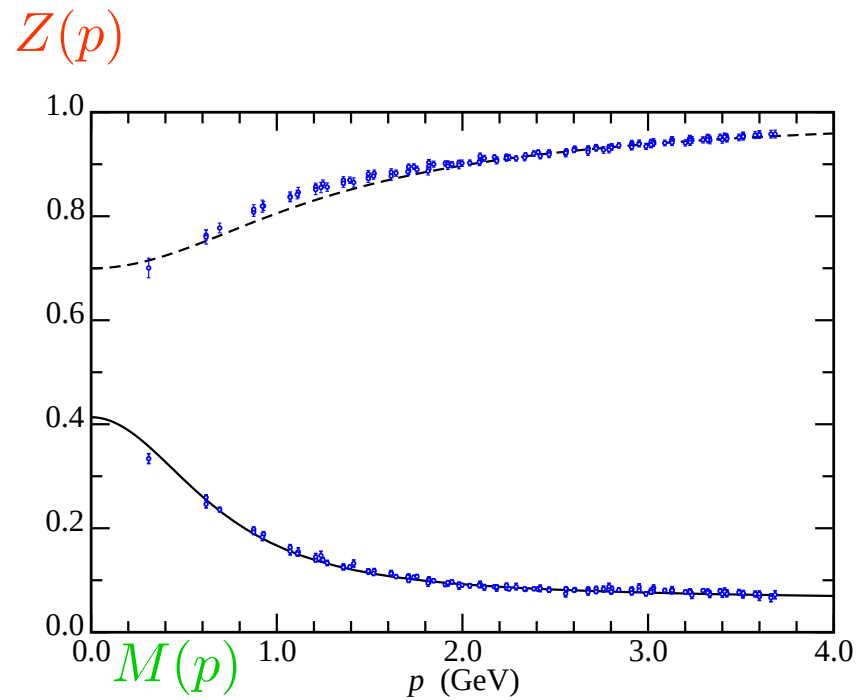


Emulating supervisor's approach to physics

Mandar Bhagwat

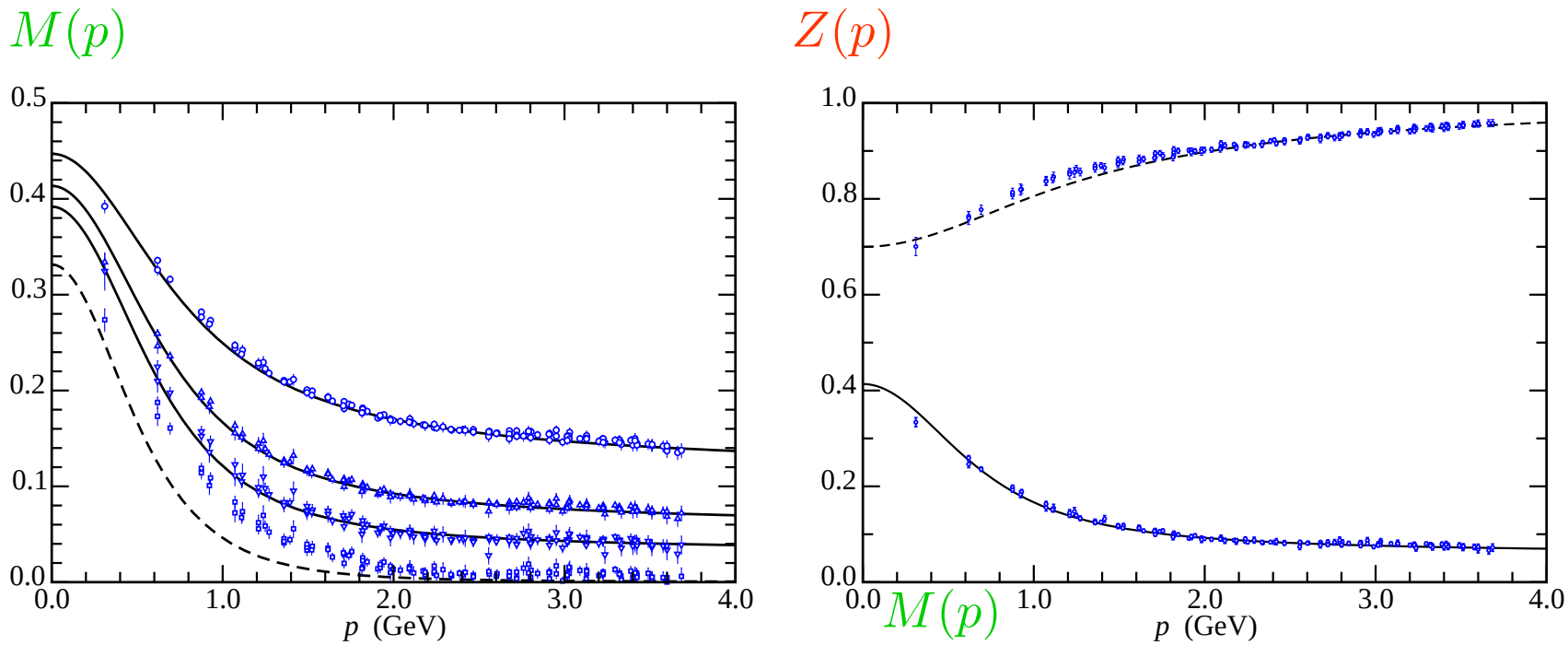
Quenched-QCD
Dressed-Quark Propagator





Quenched-QCD Dressed-Quark Propagator

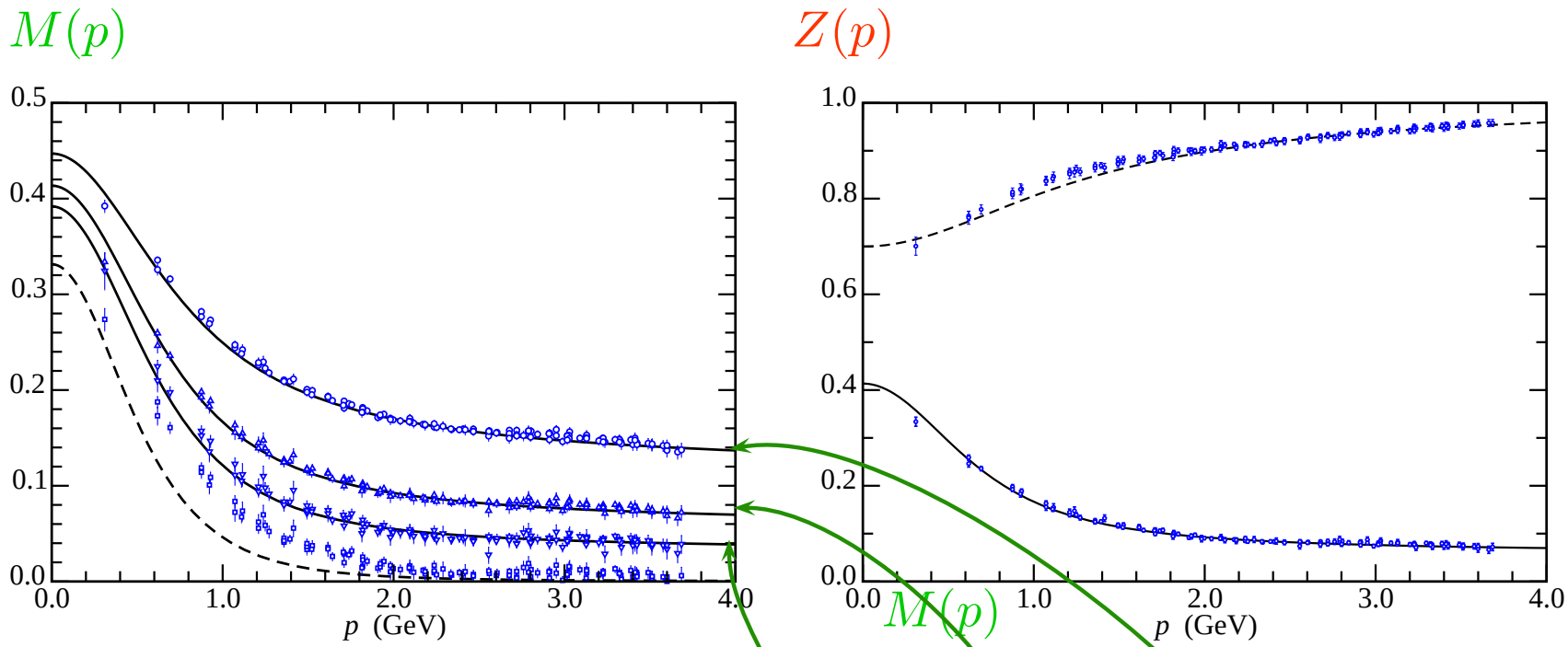
2002



- "*data*:" Quenched Lattice Meas.
  – Bowman, Heller, Leinweber, Williams: he-lat/0209129

Quenched-QCD Dressed-Quark Propagator

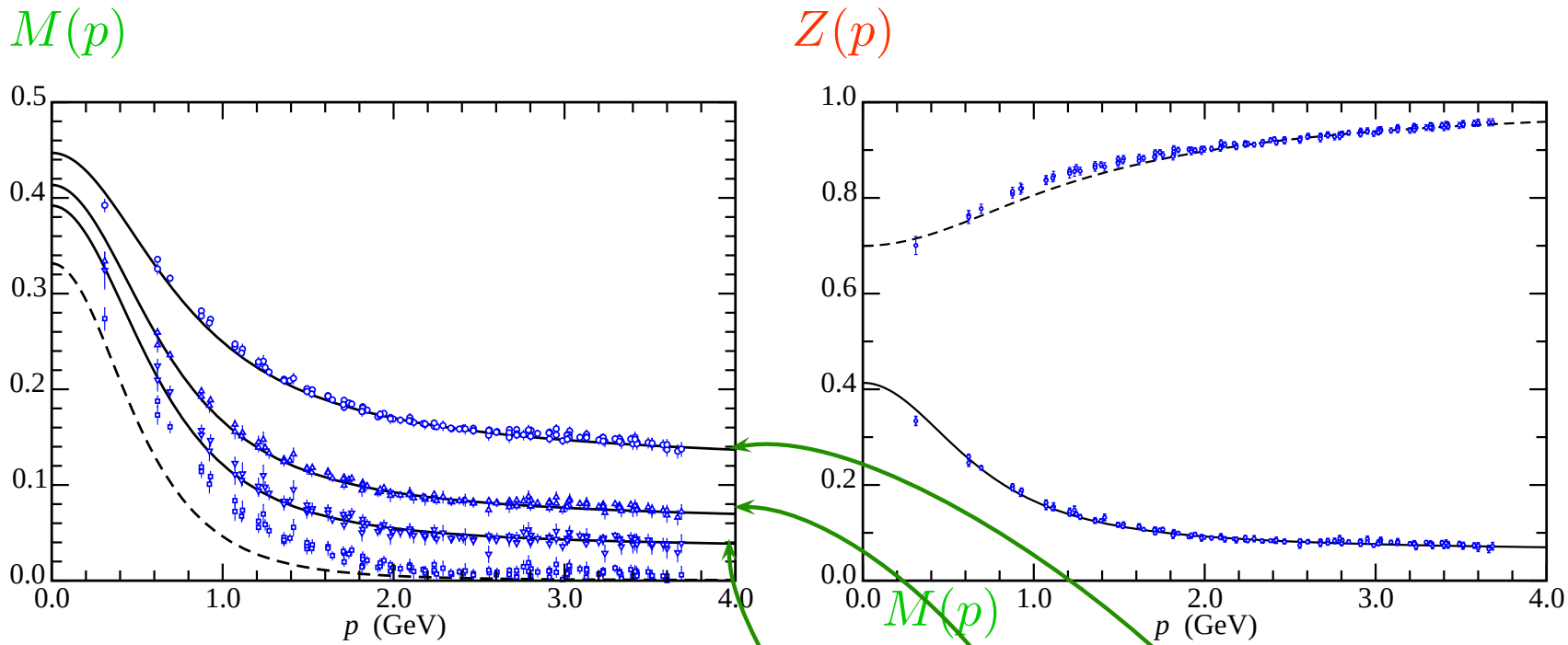
2002



- "*data*:" Quenched Lattice Meas.
  – Bowman, Heller, Leinweber, Williams: he-lat/0209129
  current-quark masses: 30 MeV, 50 MeV, 100 MeV

2002

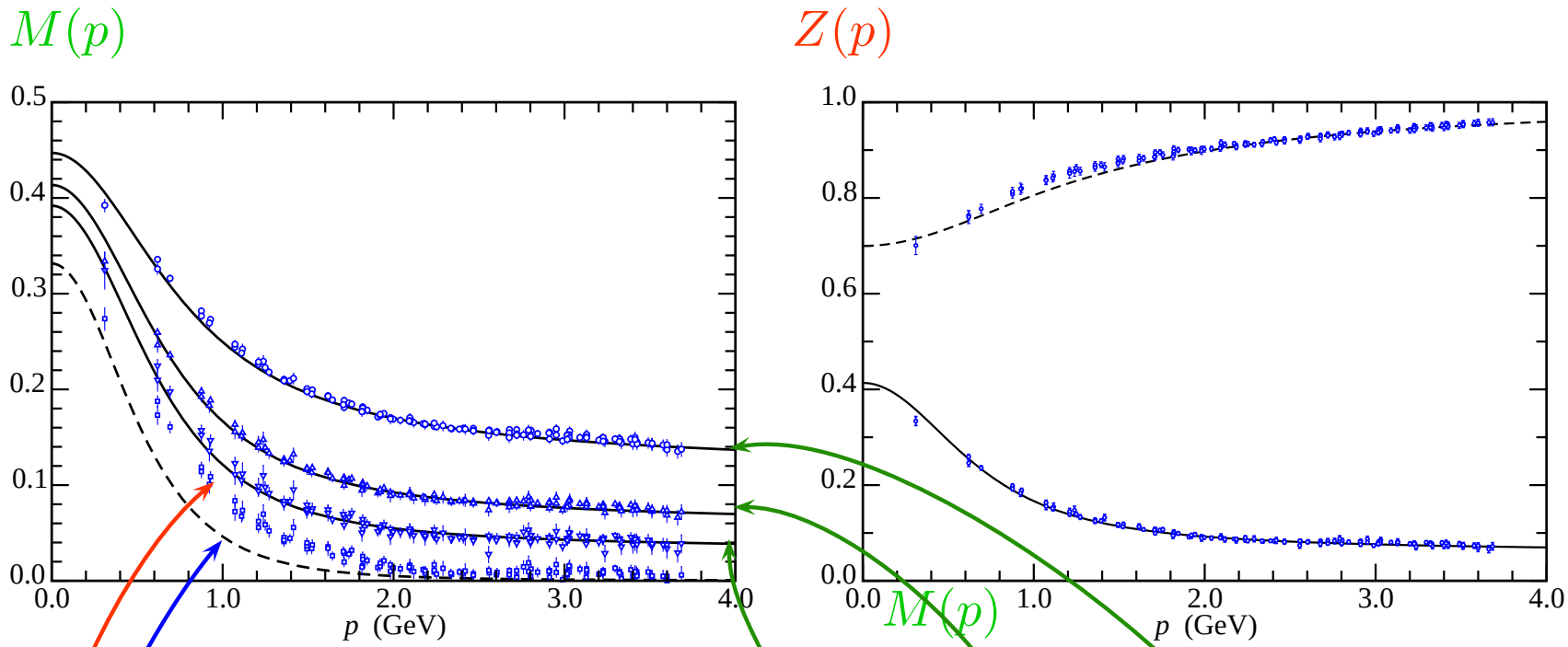
Quenched-QCD Dressed-Quark Propagator



- "*data*:" Quenched Lattice Meas.
  – Bowman, Heller, Leinweber, Williams: he-lat/0209129
  current-quark masses: 30 MeV, 50 MeV, 100 MeV
- *Curves*: Quenched DSE Cal.
  – Bhagwat, Pichowsky, Roberts, Tandy nu-th/0304003

2002

Quenched-QCD Dressed-Quark Propagator



- "*data*:" Quenched Lattice Meas.
  - Bowman, Heller, Leinweber, Williams: he-lat/0209129
  current-quark masses: 30 MeV, 50 MeV, 100 MeV
- *Curves*: Quenched DSE Cal.
  - Bhagwat, Pichowsky, Roberts, Tandy nu-th/0304003
- Linear extrapolation of lattice data to chiral limit is inaccurate

QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
  - R. Alkofer and L. von Smekal, *The infrared behavior of QCD Green's functions* . . . , Phys. Rept. **353**, 281 (2001).

QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
  - R. Alkofer and L. von Smekal, *The infrared behavior of QCD Green's functions* . . . , Phys. Rept. **353**, 281 (2001).
- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
  - D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD Collaboration], *Asymptotic scaling and infrared behavior of the gluon propagator*, Phys. Rev. D **60**, 094507 (1999) [Erratum-ibid. D **61**, 079901 (2000)].
- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

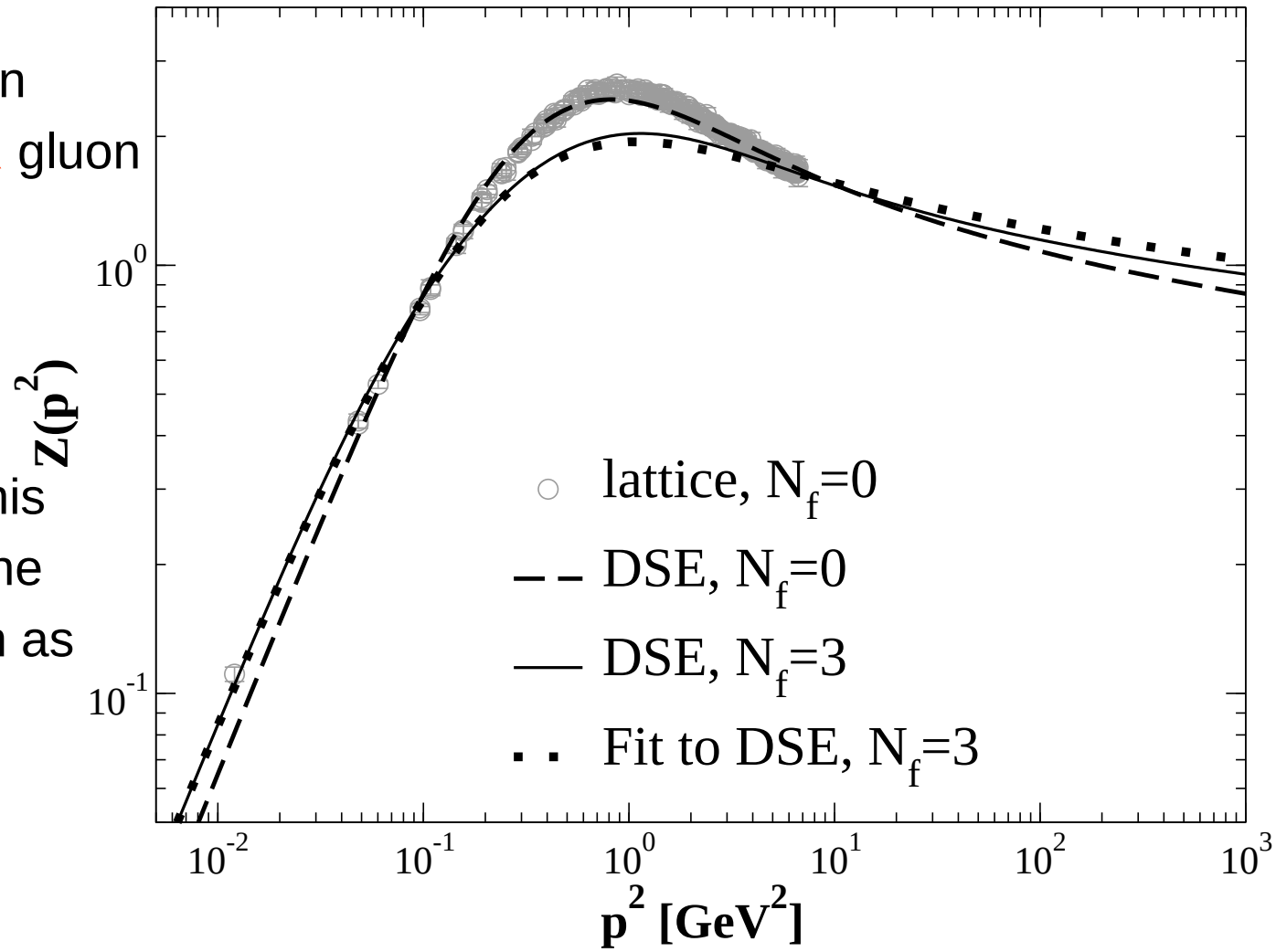
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



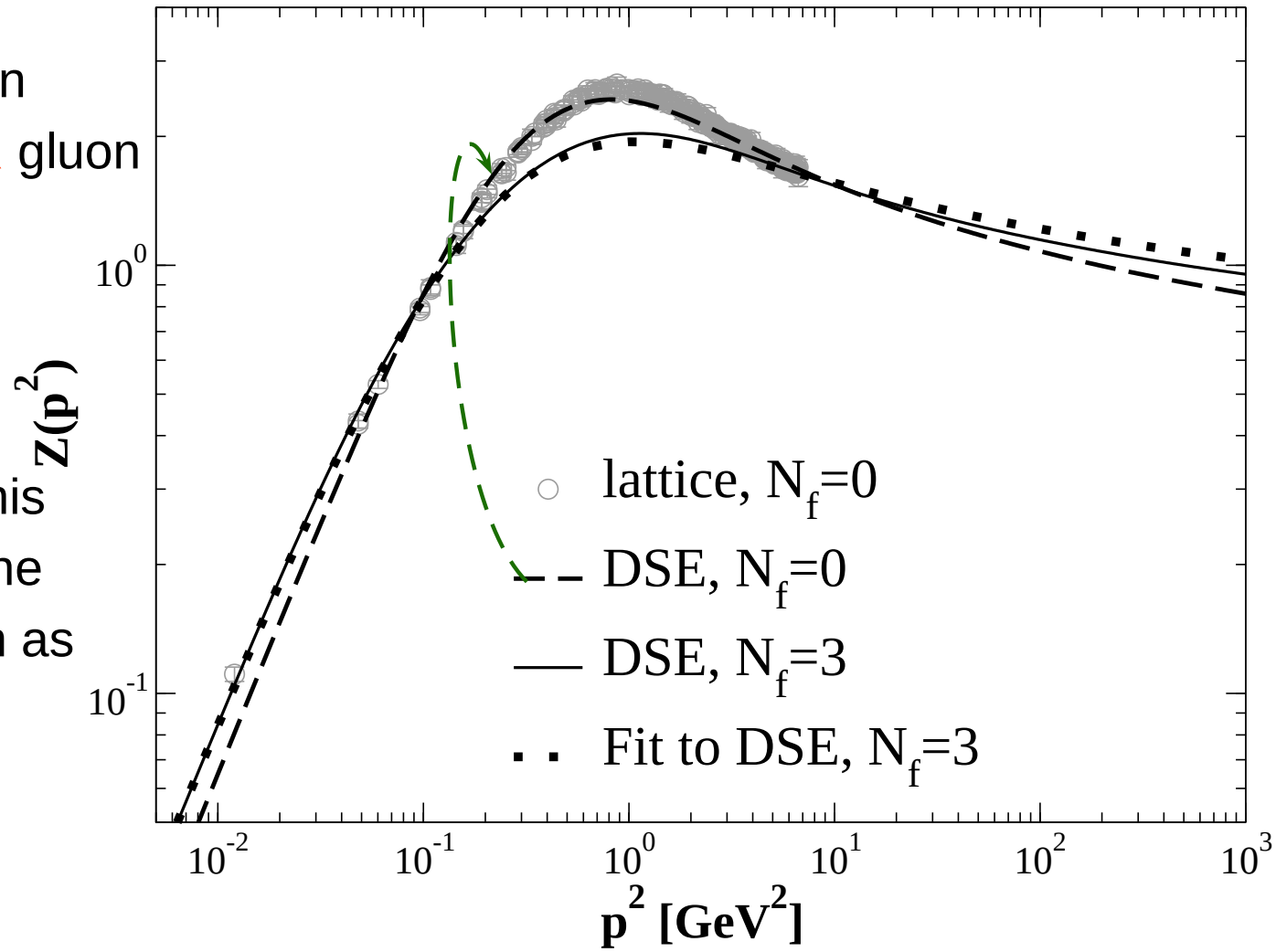
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



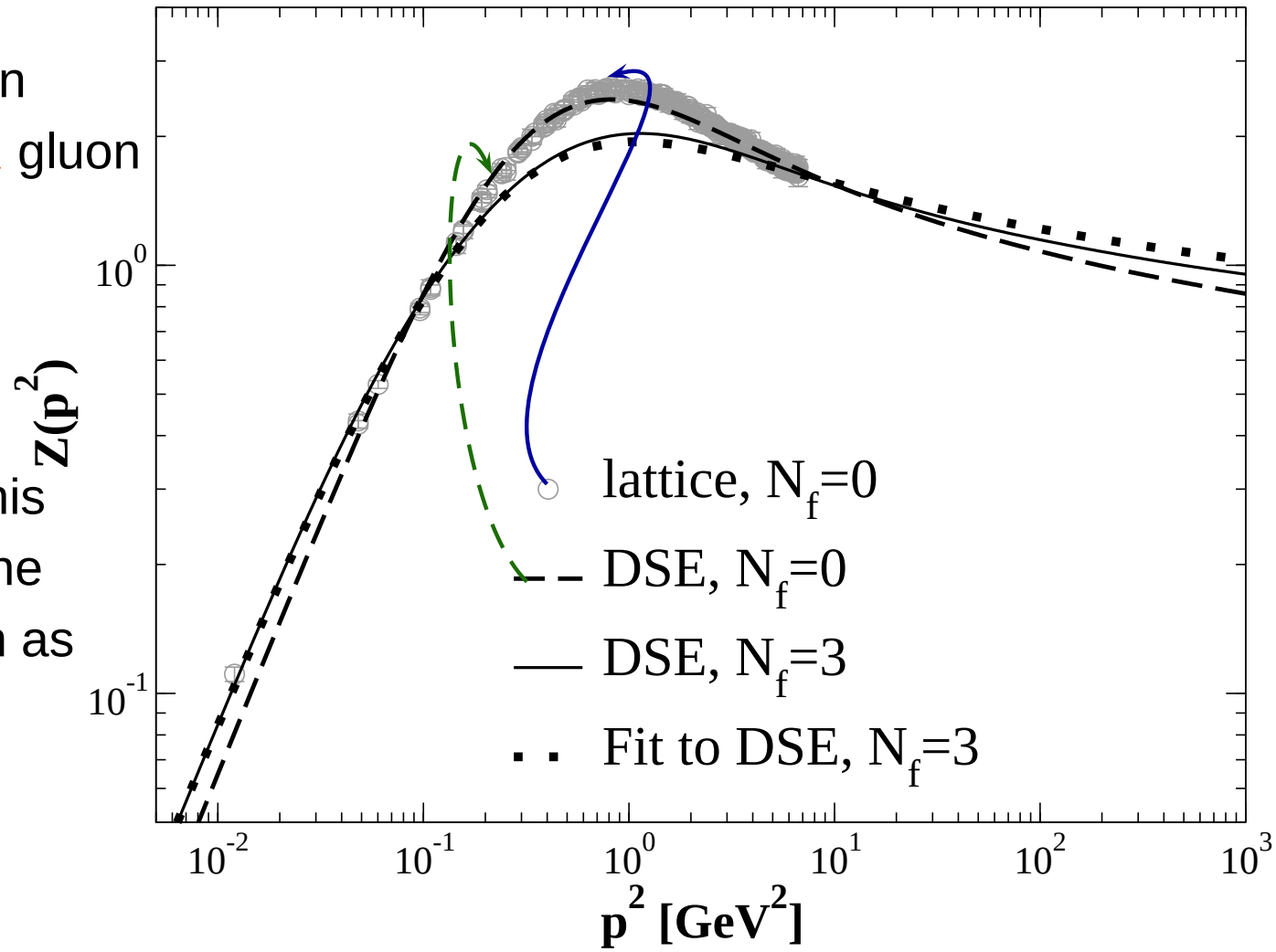
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



Hadrons

- Established understanding of two- and three-point functions

Hadrons

- Established understanding of two- and three-point functions
- What about bound states?

Hadrons

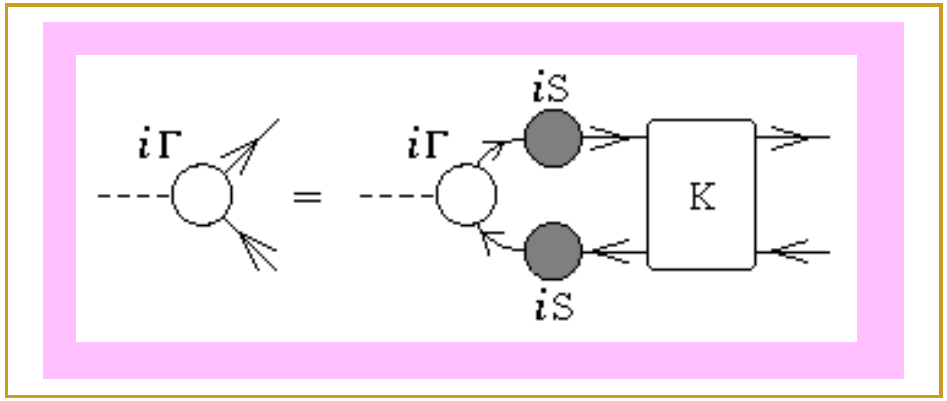
- Without bound states, Comparison with experiment is impossible

Hadrons

- Without bound states, Comparison with experiment is impossible
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

Hadrons

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation

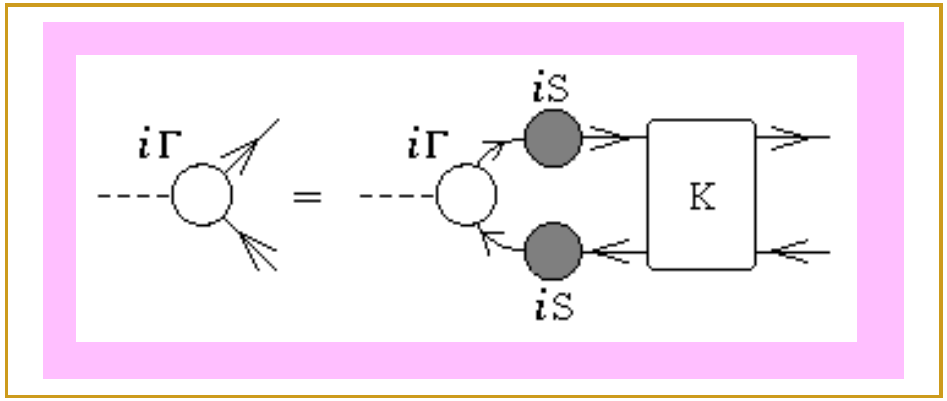


QFT Generalisation of Lippmann-Schwinger Equation.

Hadrons

- Without bound states, Comparison with experiment is impossible

- Bethe-Salpeter Equation



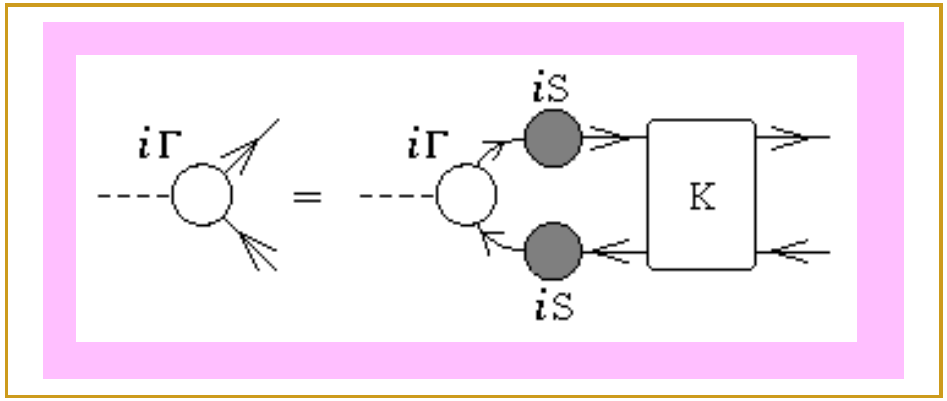
QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

Hadrons

- Without bound states, Comparison with experiment is impossible

- Bethe-Salpeter Equation



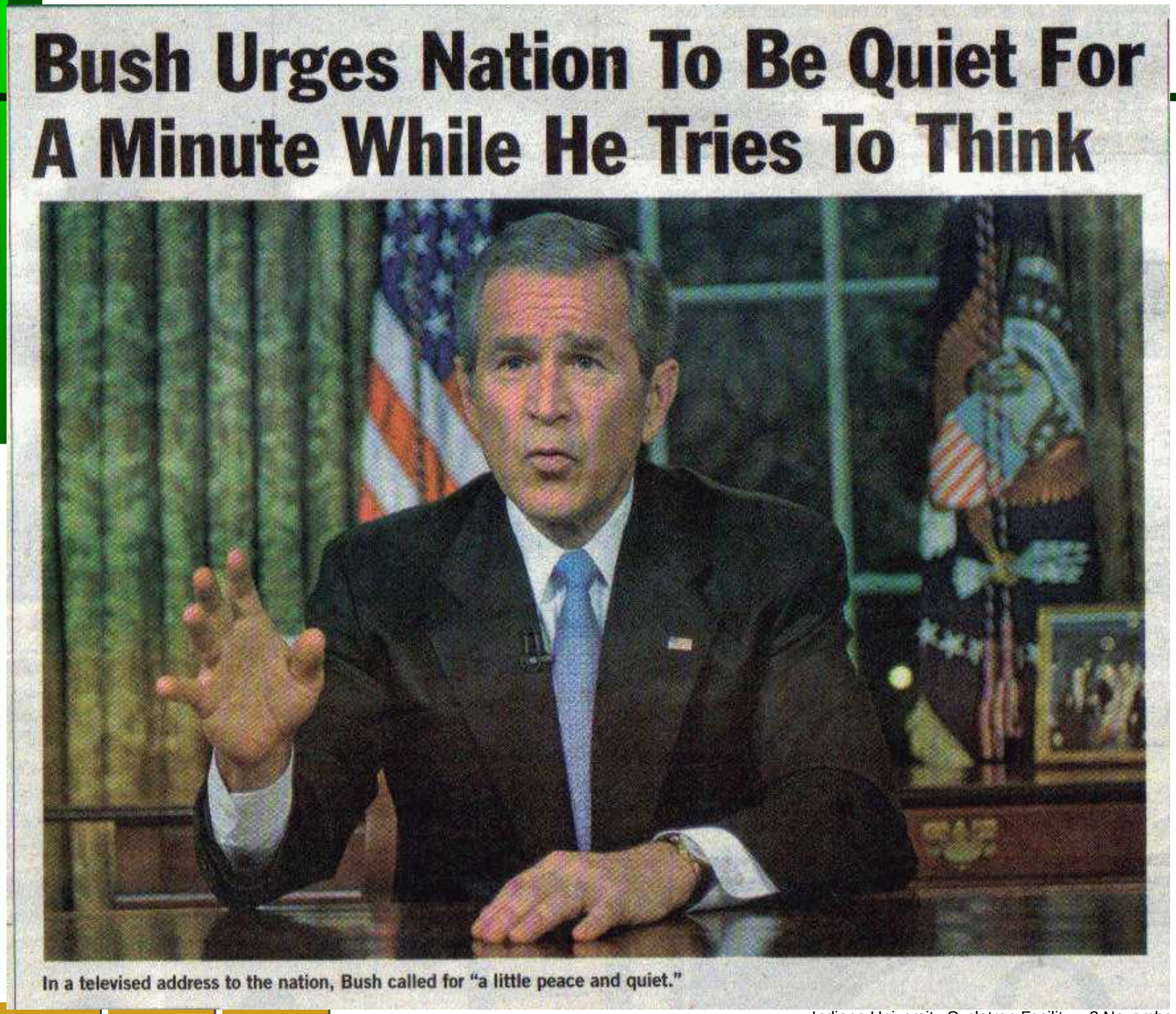
QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K?

or

What is the Long-Range Potential?

What is the Long-Range Potential?



Bush Urges Nation To Be Quiet For A Minute While He Tries To Think

In a televised address to the nation, Bush called for "a little peace and quiet."

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels must be intimately related

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation
- Nontrivial constraint

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

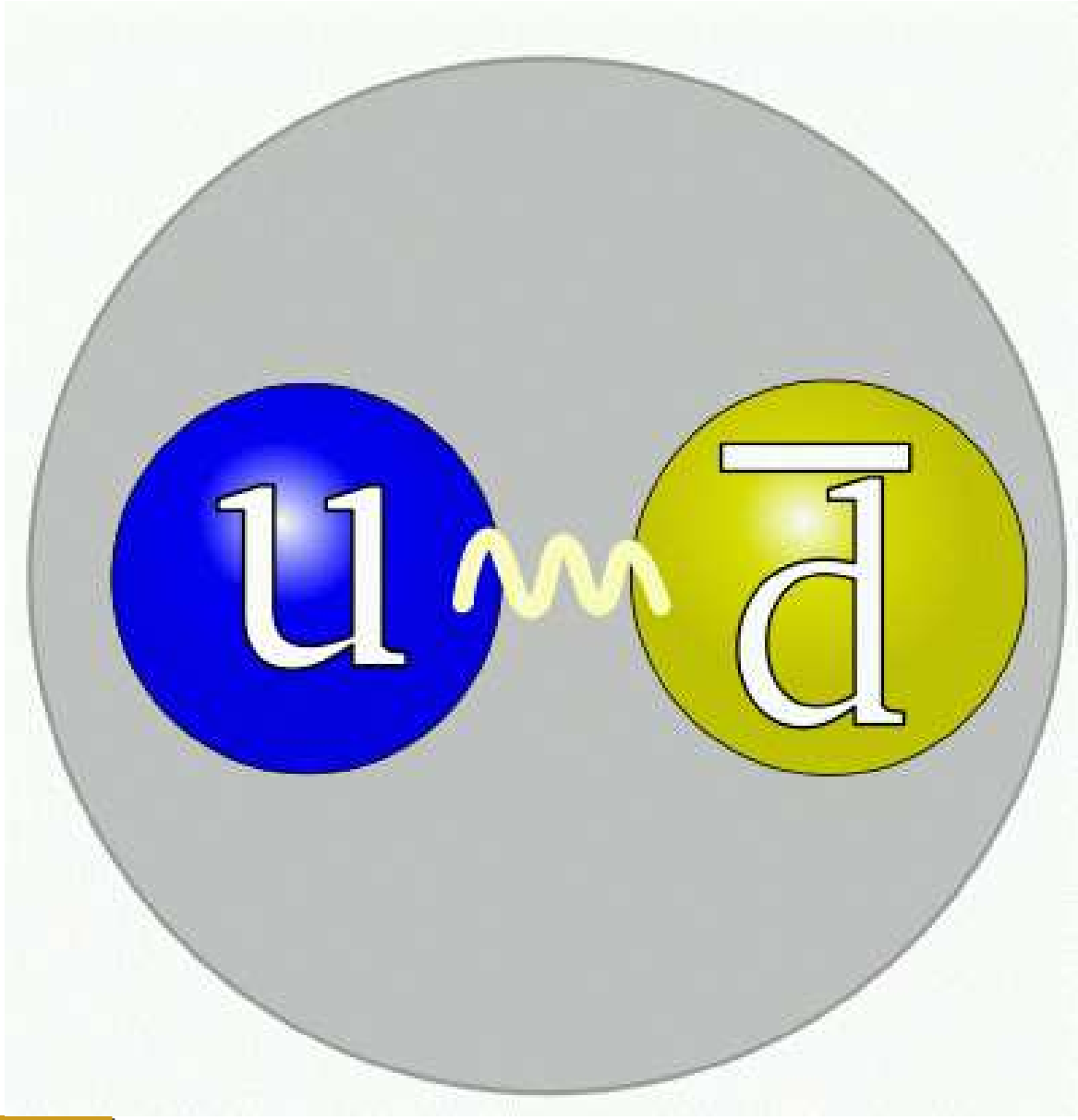
Satisfies BSE Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

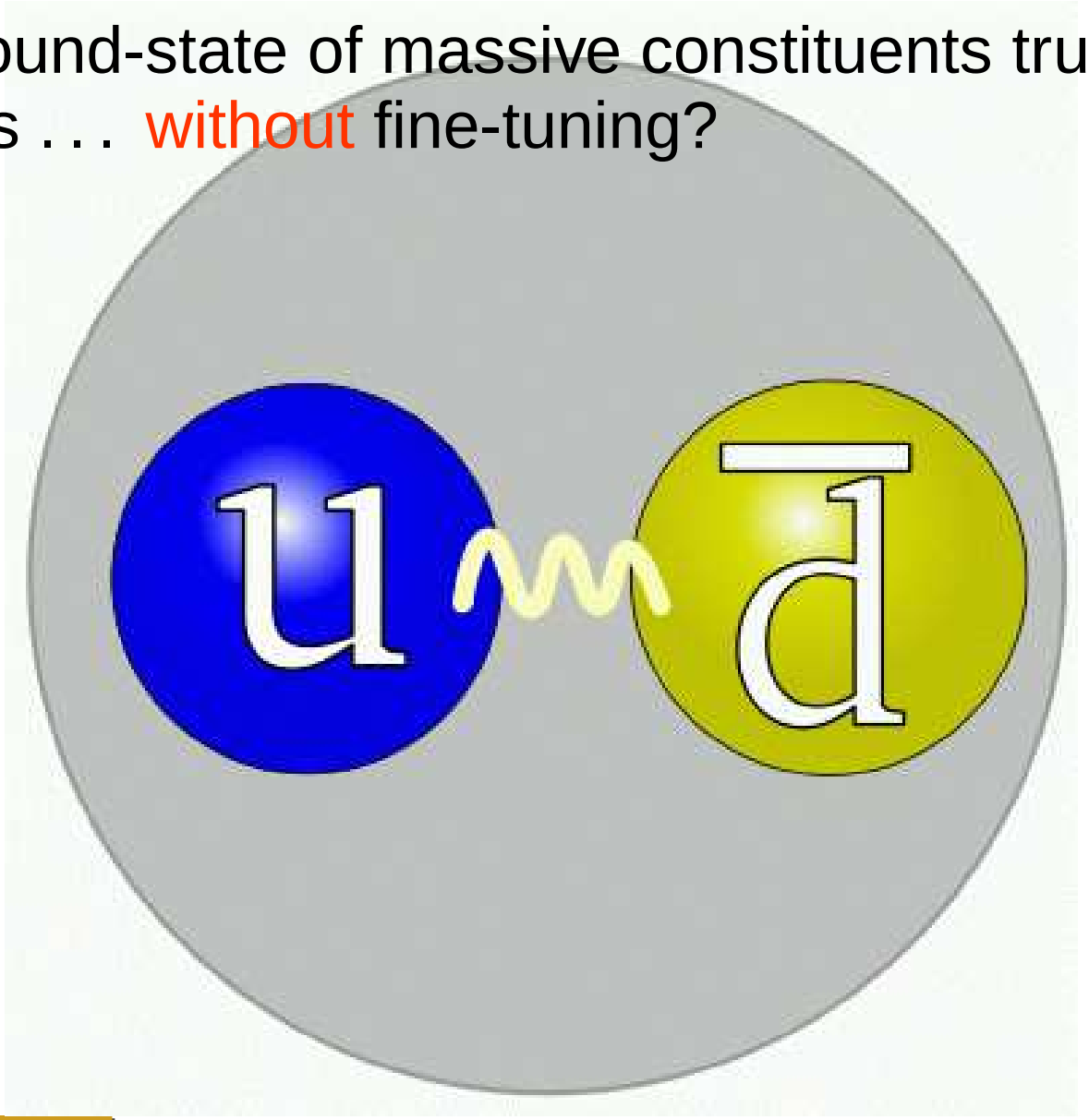
Pion and . . .

Pseudoscalar Mesons?



Pion and . . .
Pseudoscalar Mesons?

Can a bound-state of massive constituents truly be massless . . . without fine-tuning?



Radial Excitations & Chiral Symmetry

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

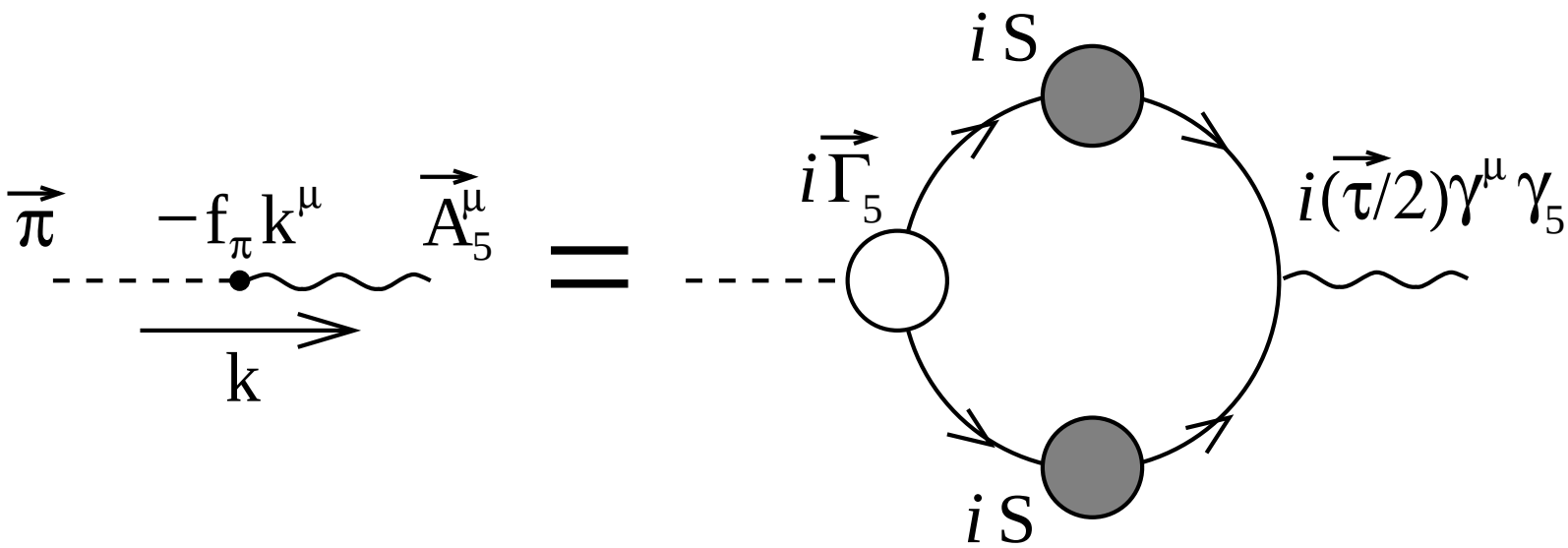
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H\, m_H^2 = -\ \rho_\zeta^H\ \mathcal{M}_H$$

$$f_H\, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5\gamma_\mu\, \mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-) \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = -\rho_\zeta^H \mathcal{M}_H$$

$$i\rho_\zeta^H = Z_4 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5 \mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-) \right\}$$

- Pseudoscalar projection of BS wave function at $x = 0$

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ ...GMOR relation, a corollary

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $\boxed{m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q}$ ...GMOR relation, a corollary

- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

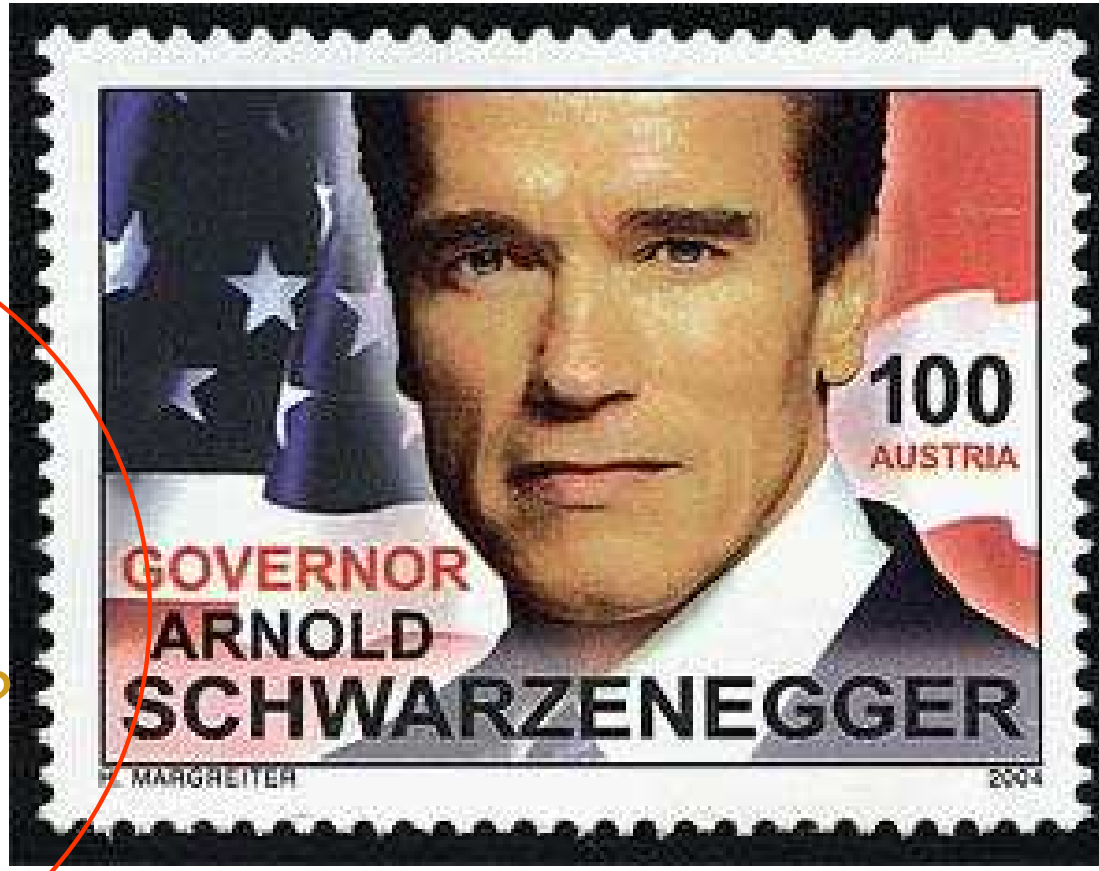
Hence, $\boxed{m_H \propto m_q}$

... QCD Proof of Potential Model result

Andreas Krassnigg

FWF “Erwin Schrödinger Fellow,” ANL 2003-2005

Almost Blood
Relative of Arnold
. . . Future
President?
. . . Executioner?



Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson,
  $\boldsymbol{m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0}$, in chiral limit

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson,
  $\boldsymbol{m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0}$, in chiral limit
- $\Rightarrow \boldsymbol{f_H = 0}$
  ALL pseudoscalar mesons except $\boldsymbol{\pi(140)}$ in chiral limit

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson,
  $m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0$, in chiral limit
- $\Rightarrow f_H = 0$
  ALL pseudoscalar mesons except $\pi(140)$ in chiral limit
- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

Radial Excitations

Radial Excitations

- Spectrum contains 3 pseudoscalars [$I^G(J^P)L = 1^-(0^-)S$]

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

Radial Excitations

- Spectrum contains 3 pseudoscalars $[I^G(J^P)L = 1^-(0^-)S]$

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

- The Pion

- Consituent-Q Model: 1st three members of $\boldsymbol{n\,{}^1S_0}$ trajectory; i.e., ground state plus radial excitations?

Radial Excitations

- Spectrum contains 3 pseudoscalars [$I^G(J^P)L = 1^-(0^-)S$]

  masses below 2 GeV: $\pi(140)$; $\pi(1300)$; and $\pi(1800)$

- The Pion

- Consituent-Q Model: 1$^{\rm st}$ three members of $n\,^1S_0$ trajectory; i.e., ground state plus radial excitations?

- But $\pi(1800)$ is narrow ($\Gamma = 207 \pm 13$) & decay pattern might indicate some "flux tube angular momentum" content:
  $S_{\bar{Q}Q} = 1 \oplus L_F = 1 \Rightarrow J = 0$
  & $L_F = 1 \Rightarrow\ ^3S_1 \oplus\ ^3S_1$ ($\bar{Q}Q$) decays suppressed?

Radial Excitations

- Spectrum contains 3 pseudoscalars $[I^G(J^P)L = 1^-(0^-)S]$

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

- The Pion
- Consituent-Q Model: 1st three members of $\boldsymbol{n\,^1S_0}$ trajectory; i.e., ground state plus radial excitations?
- But $\boldsymbol{\pi(1800)}$ is narrow $(\boldsymbol{\Gamma = 207 \pm 13})$ & decay pattern might indicate some "flux tube angular momentum" content:
- Radial excitations & Hybrids & Exotics $\Rightarrow$ Long-range radial wave functions $\Rightarrow$ sensitive to confinement

Radial Excitations

- Spectrum contains 3 pseudoscalars $[I^G(J^P)L = 1^-(0^-)S]$

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

- The Pion
- Consituent-Q Model: 1st three members of $\boldsymbol{n\,^1S_0}$ trajectory; i.e., ground state plus radial excitations?
- But $\boldsymbol{\pi(1800)}$ is narrow $(\boldsymbol{\Gamma = 207 \pm 13})$ & decay pattern might indicate some "flux tube angular momentum" content:
- Radial excitations & Hybrids & Exotics $\Rightarrow$ Long-range radial wave functions $\Rightarrow$ sensitive to confinement
- NSAC Long-Range Plan, 2002: . . . an understanding of confinement "remains one of the greatest intellectual challenges in physics"

Radial Excitations & Chiral Symmetry

Radial Excitations & Chiral Symmetry

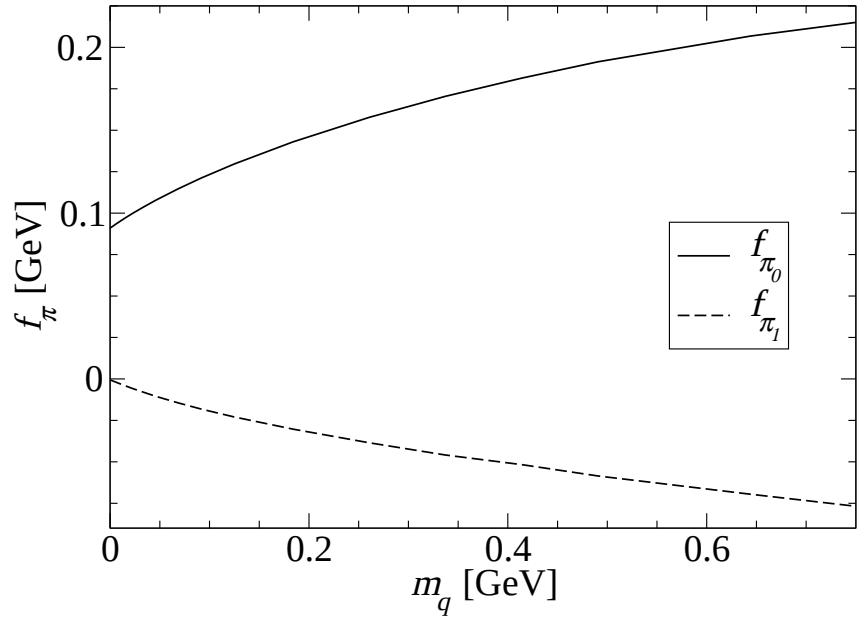
Höll, Krassnigg, Roberts
nu-th/0406030

- Fundamental properties of QCD

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

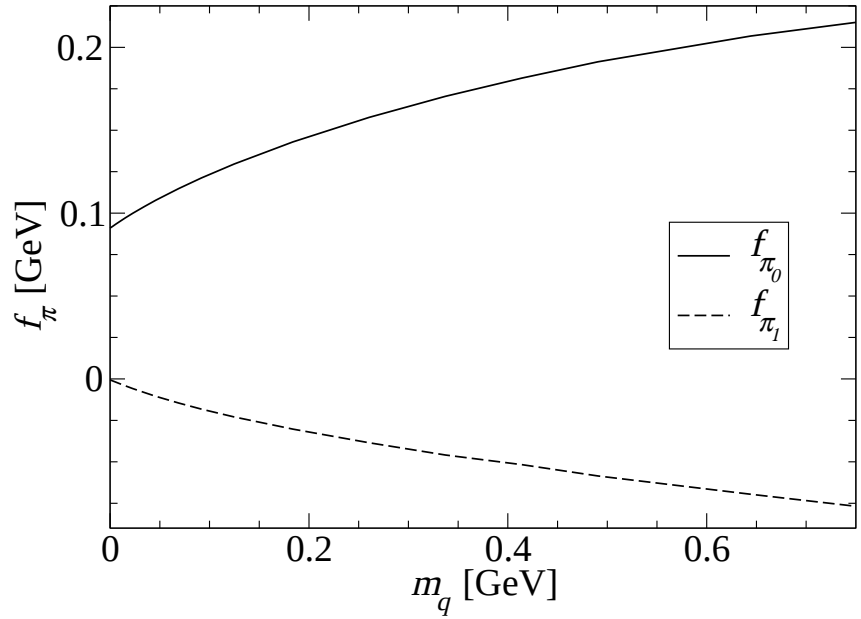
- Fundamental properties of QCD
  - If chiral symmetry is dynamically broken, then in the chiral limit every pseudoscalar meson is blind to the weak interaction *except* $\pi(140)$.



Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

- Fundamental properties of QCD
  - If chiral symmetry is dynamically broken, then in the chiral limit every pseudoscalar meson is blind to the weak interaction *except* $\pi(140)$.
  - If chiral symmetry is not broken, then *NO pseudoscalar meson experiences the weak interaction.*



Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

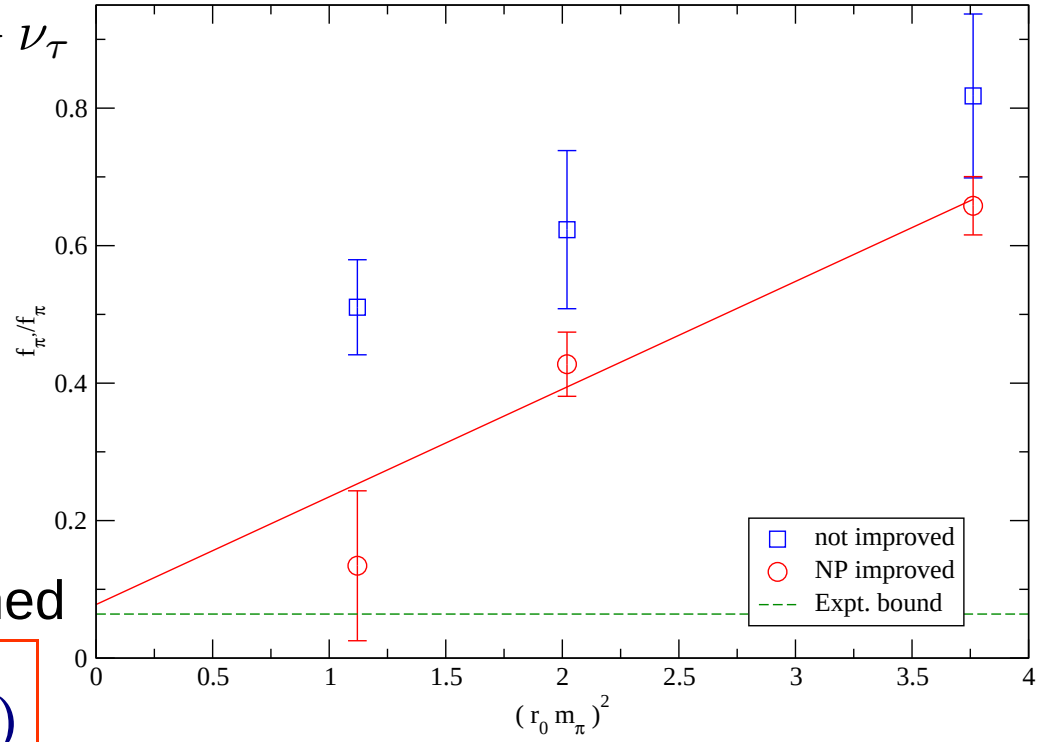
- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$

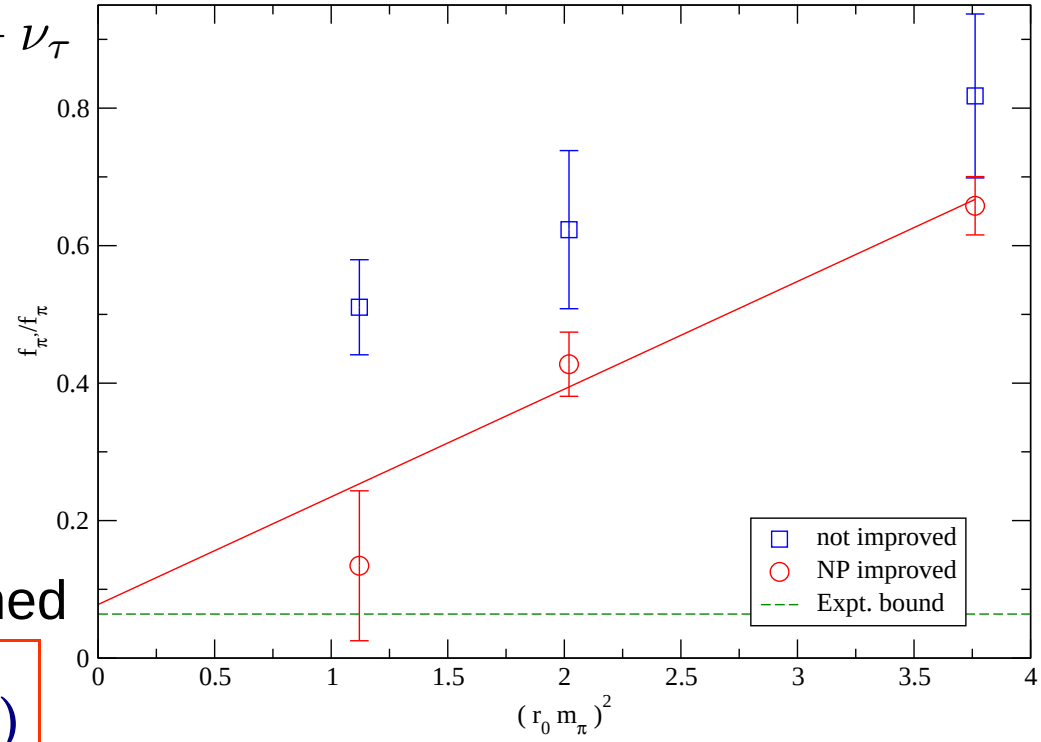


Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$



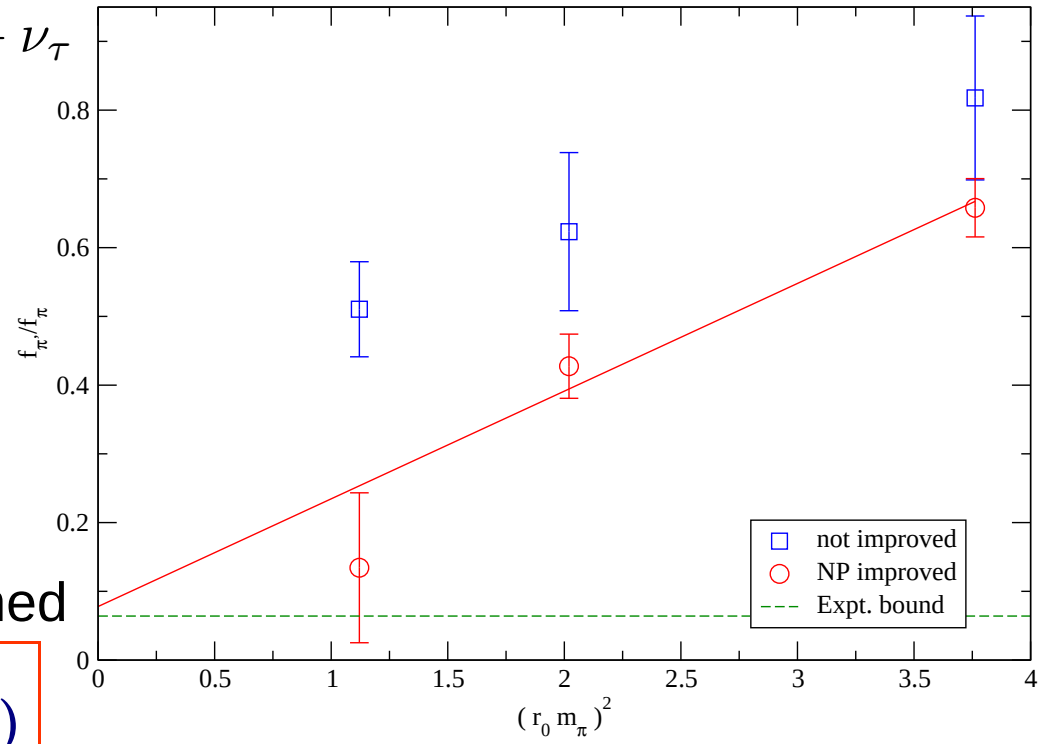
- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

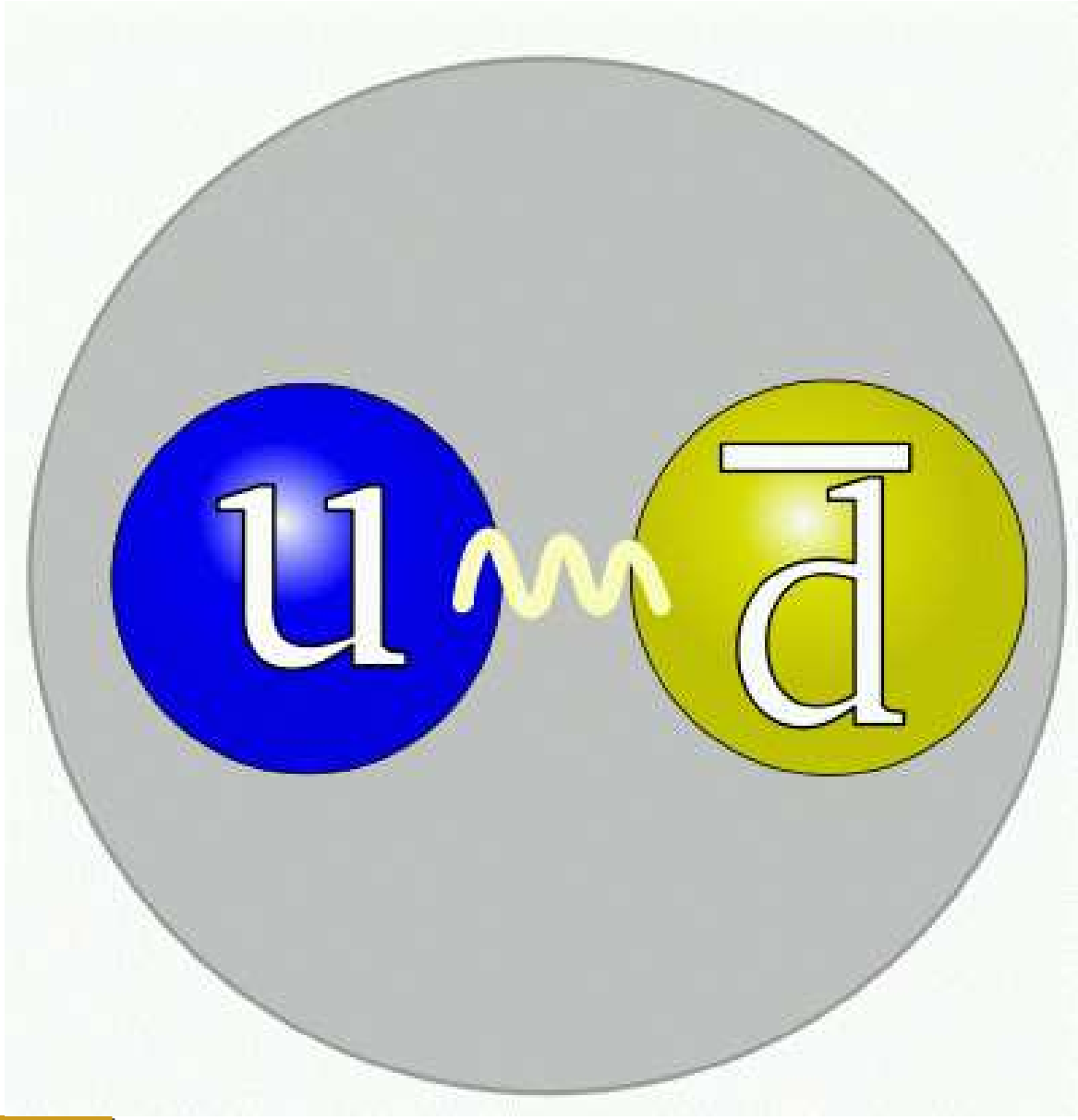
- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$$



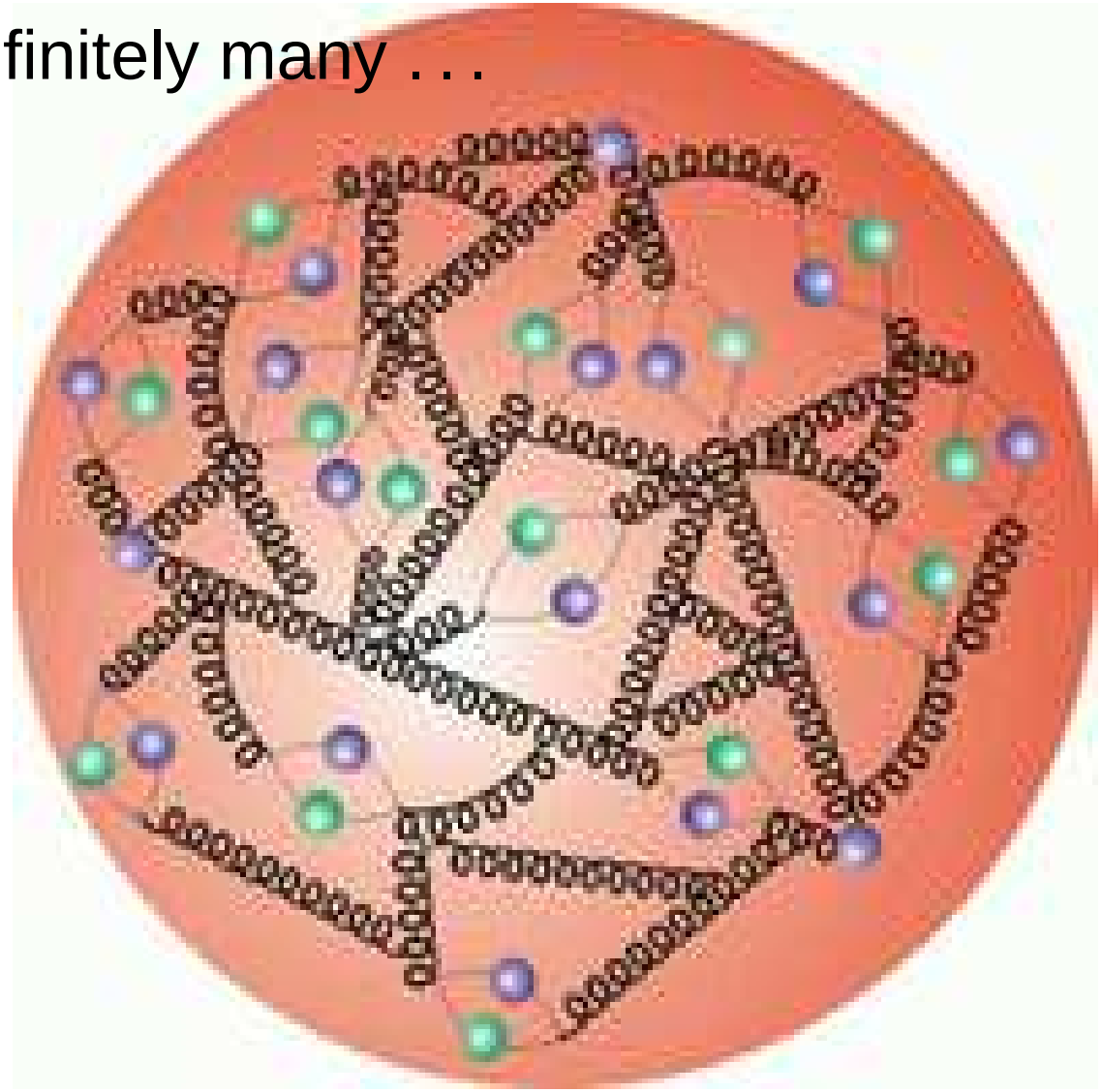
- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Answer for the pion



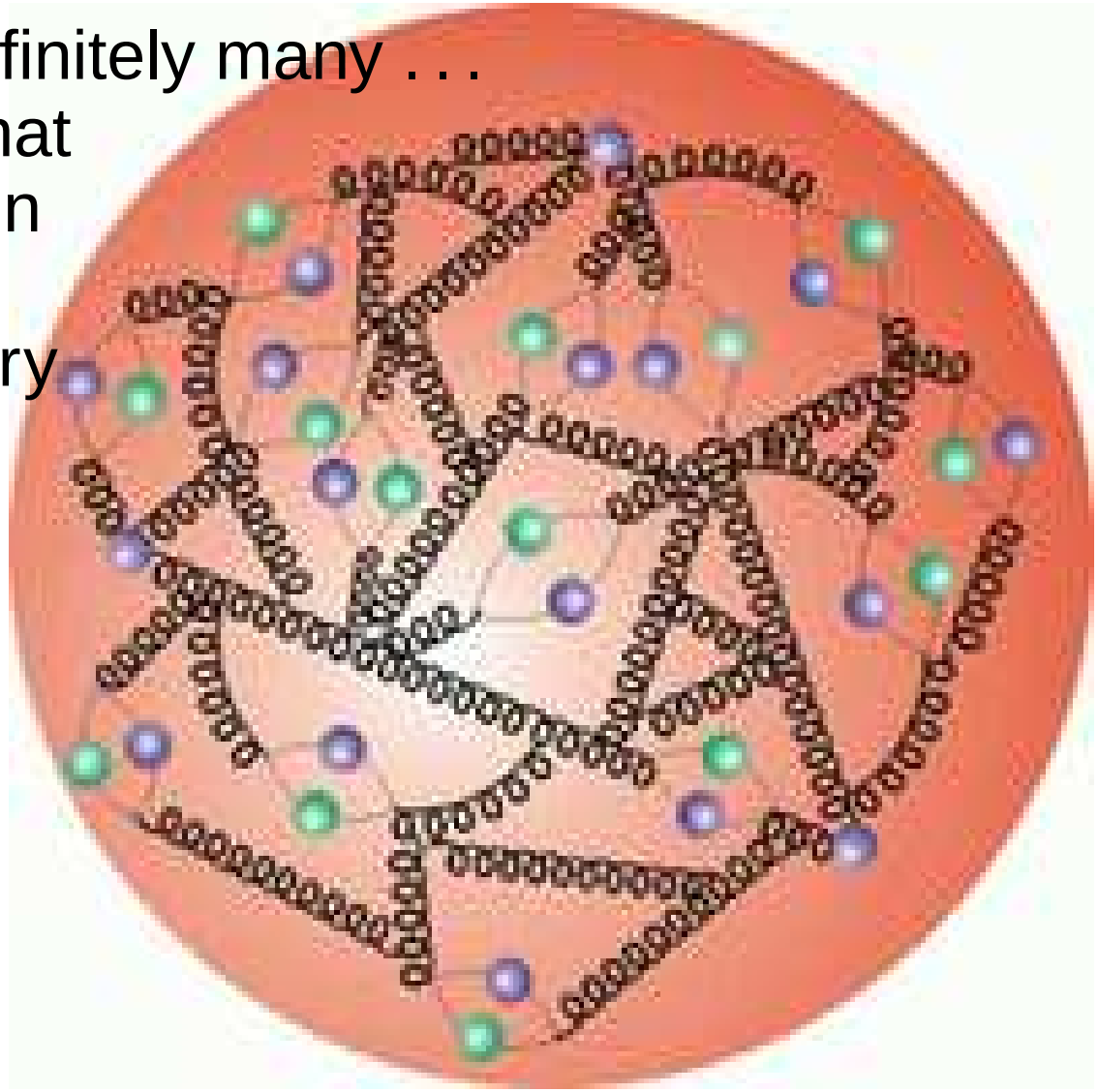
Answer for the pion

Two → Infinitely many . . .



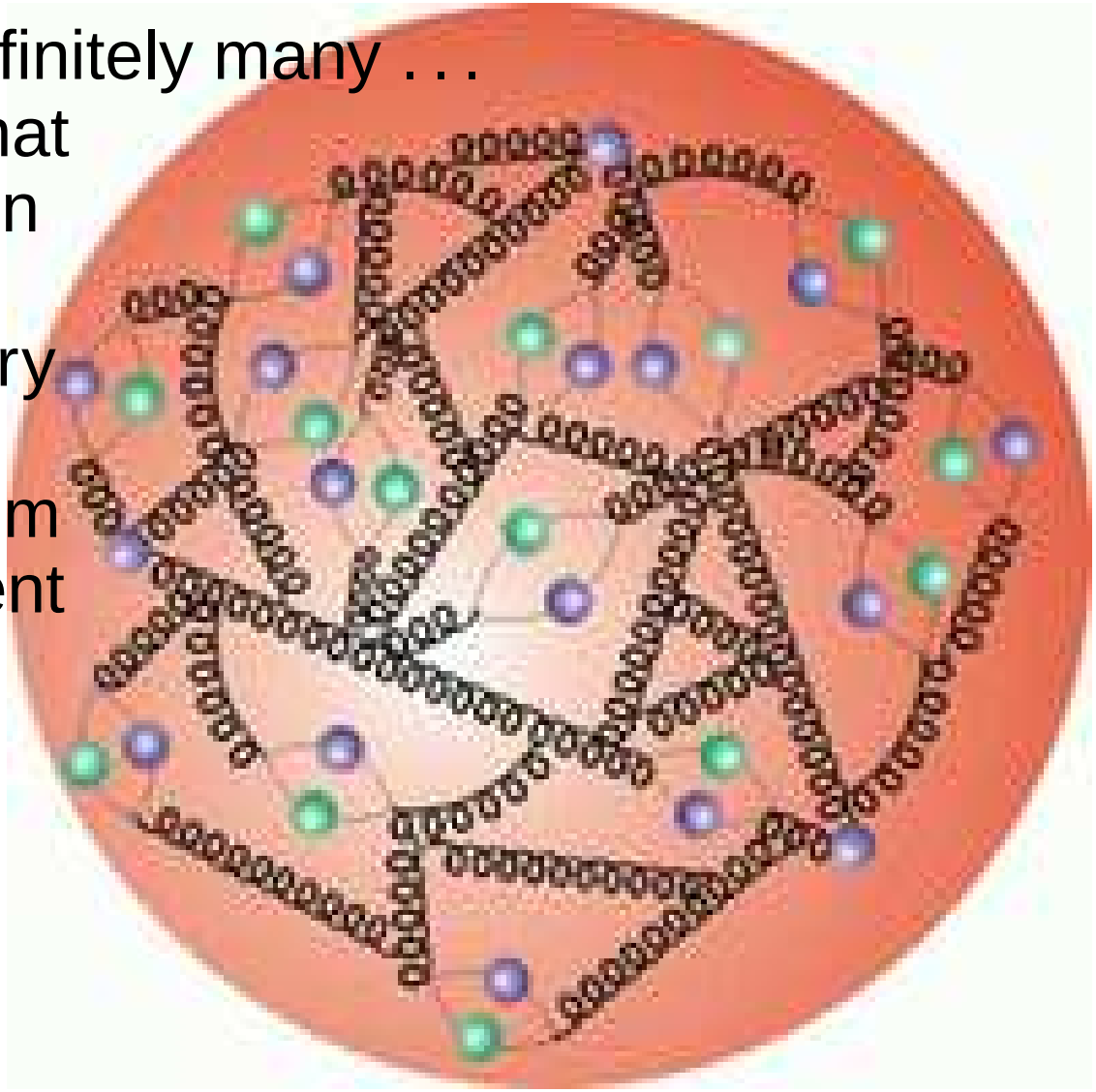
Answer for the pion

Two → Infinitely many . . .
Handle that properly in quantum field theory



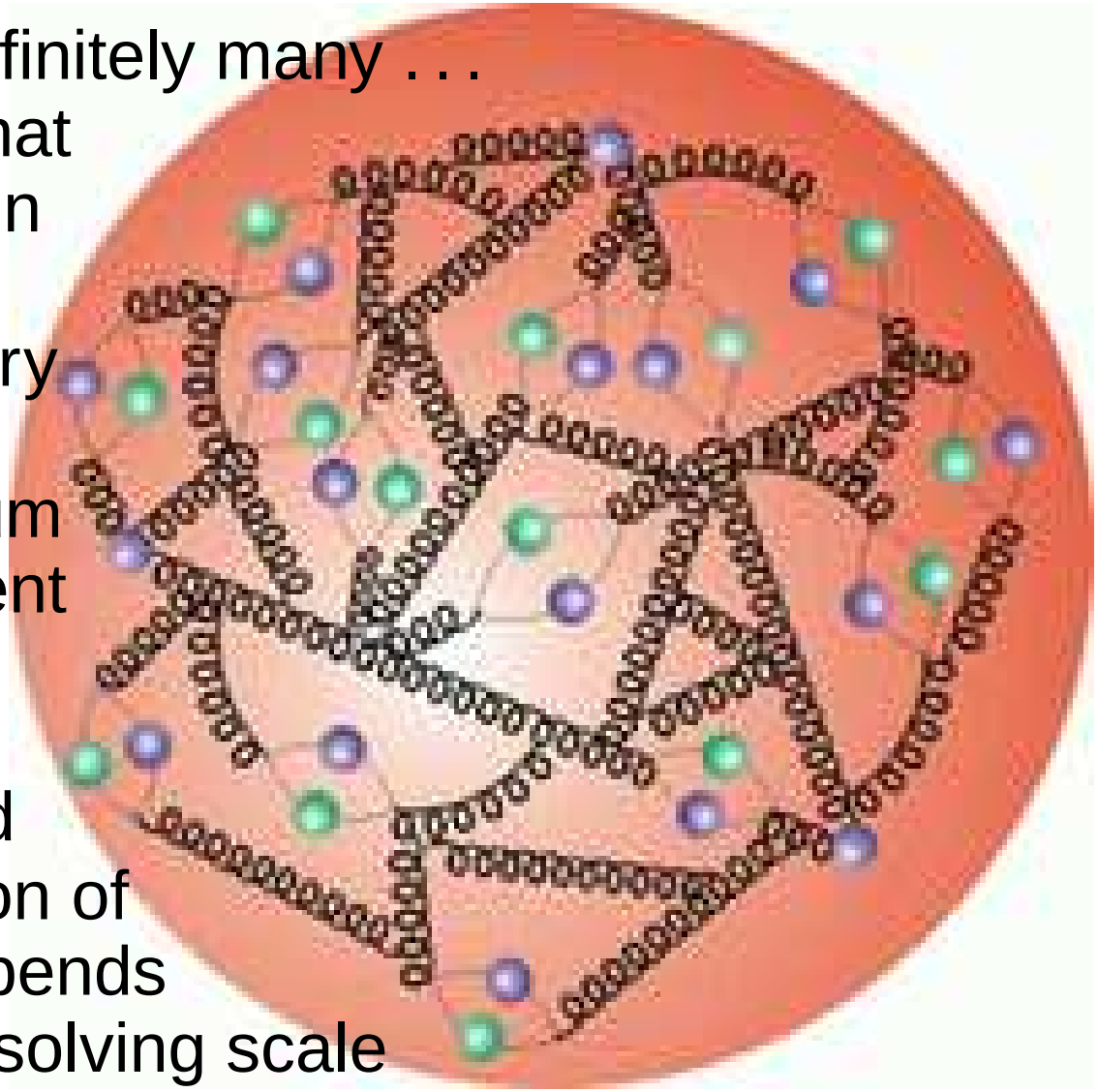
Answer for the pion

Two → Infinitely many . . .
Handle that properly in quantum field theory
. . .
momentum -dependent dressing



Answer for the pion

Two → Infinitely many . . .
Handle that properly in quantum field theory
. . .
momentum -dependent dressing
. . .
perceived distribution of mass depends on the resolving scale



Are we there yet?

On the croc's back: Tony and Trent DeWith negotiate a flooded street in Katherine's town centre in a snappy canoe yesterday

Picture: Patrina Malo

New Challenges

New Challenges

- Maris & Tandy . . . series of five papers . . . excellent description of light pseudoscalar and vector mesons . . . basket of 31 masses/couplings/radii with r.m.s. error of 15% . . . moreover, prediction of $F_\pi(Q^2)$ measured in Hall C.

Pieter Maris

Peter Tandy

New Challenges

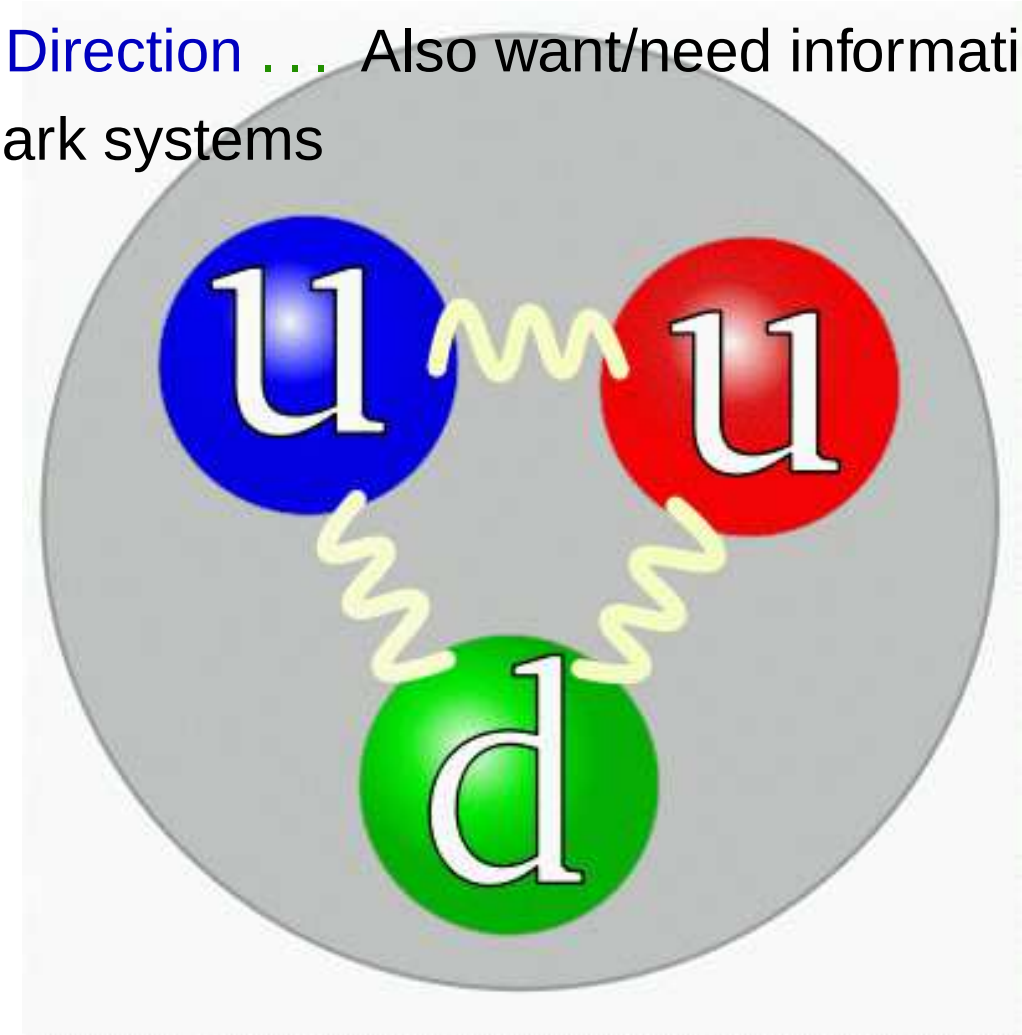
- Maris & Tandy . . . series of five papers . . . excellent description of light pseudoscalar and vector mesons . . . basket of 31 masses/couplings/radii with r.m.s. error of 15% . . . moreover, prediction of $F_\pi(Q^2)$ measured in Hall C.
- One parameter model . . . parameter specifies long-range interaction between light-quarks . . . model-independent results in ultraviolet

New Challenges

- Maris & Tandy . . . series of five papers . . . excellent description of light pseudoscalar and vector mesons . . . basket of 31 masses/couplings/radii with r.m.s. error of 15% . . . moreover, prediction of $F_\pi(Q^2)$ measured in Hall C.
- One parameter model . . . parameter specifies long-range interaction between light-quarks . . . model-independent results in ultraviolet
- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks

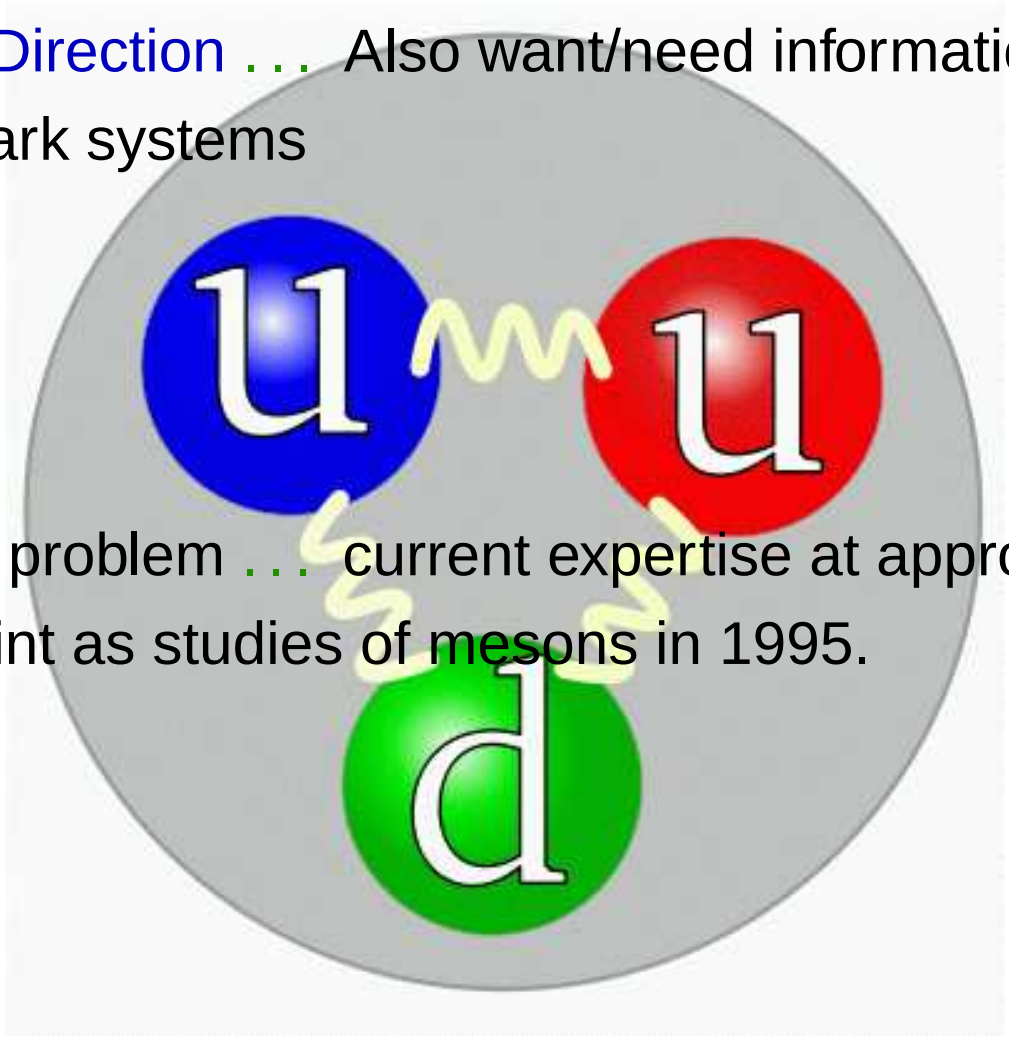
New Challenges

- Another Direction . . . Also want/need information about three-quark systems



New Challenges

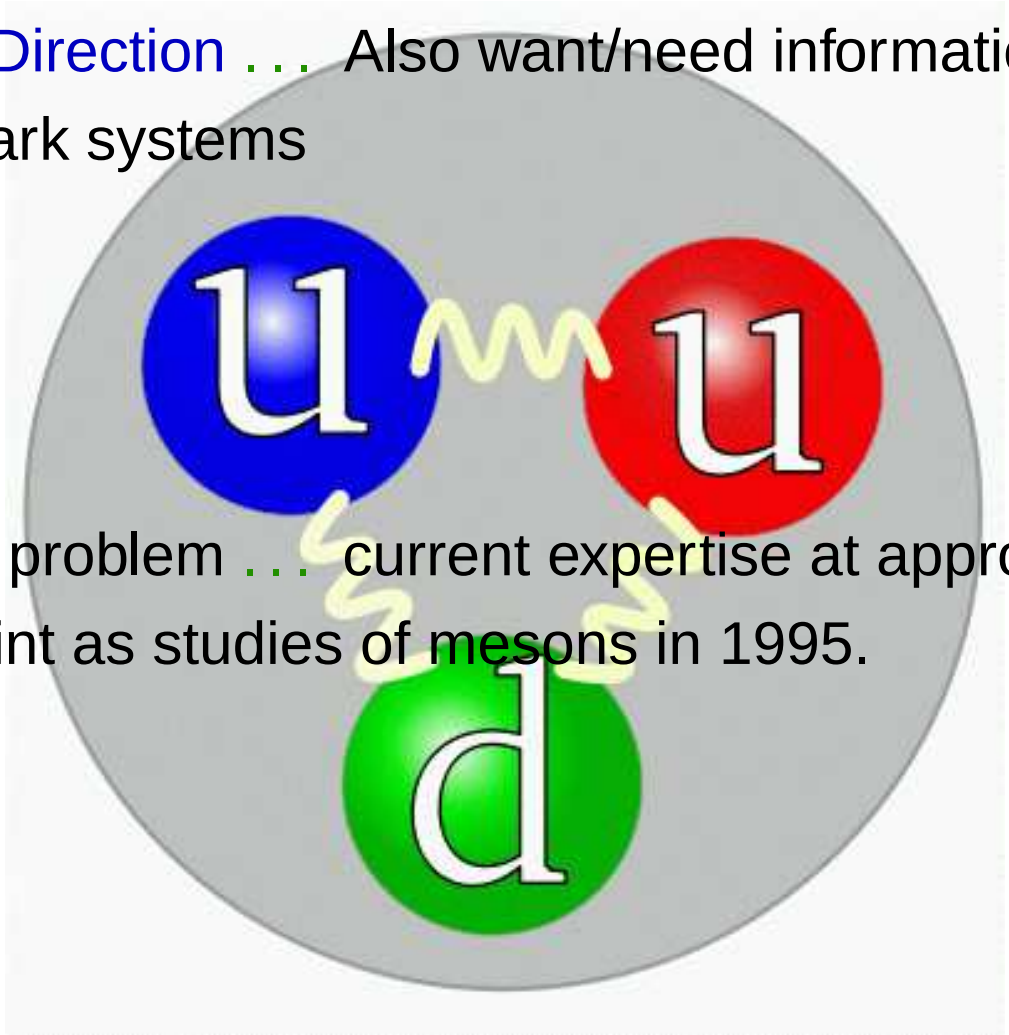
- Another Direction . . . Also want/need information about three-quark systems



- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

New Challenges

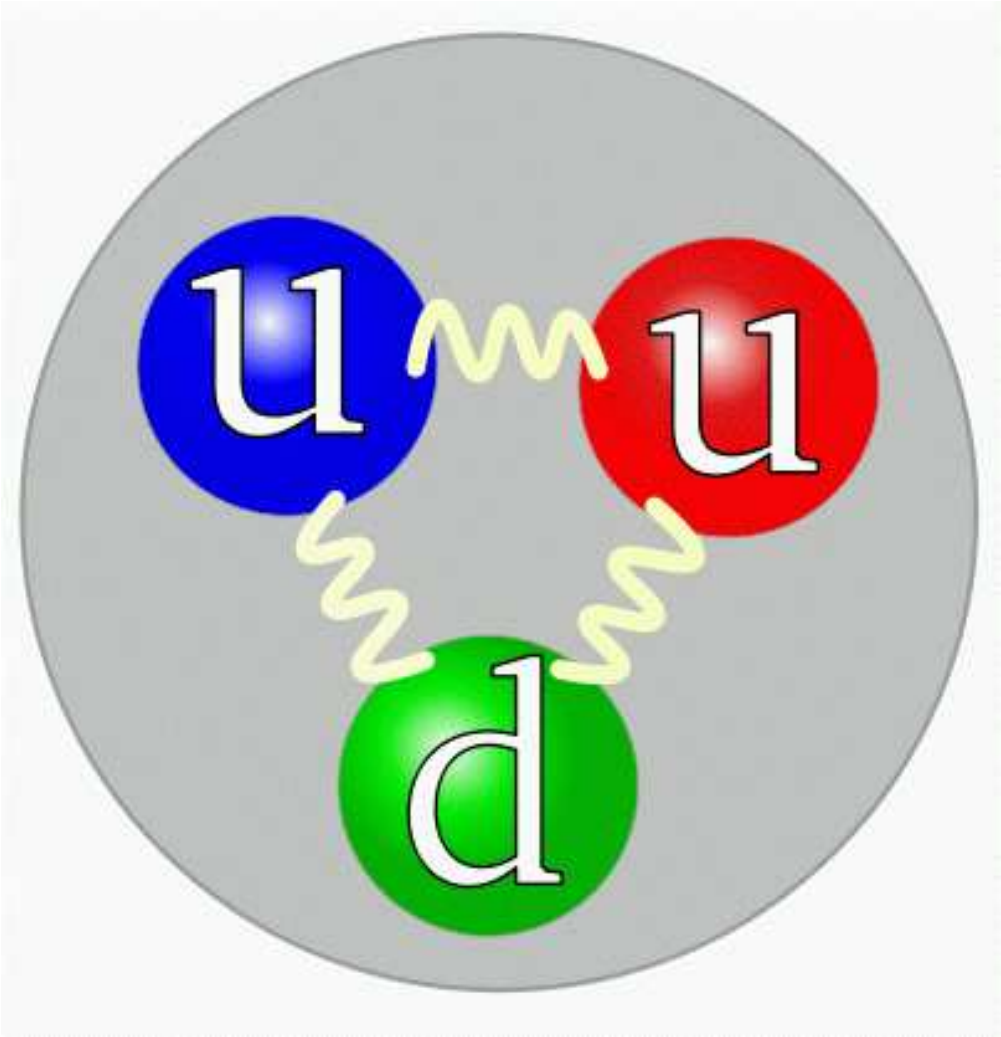
- Another Direction . . . Also want/need information about three-quark systems



- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

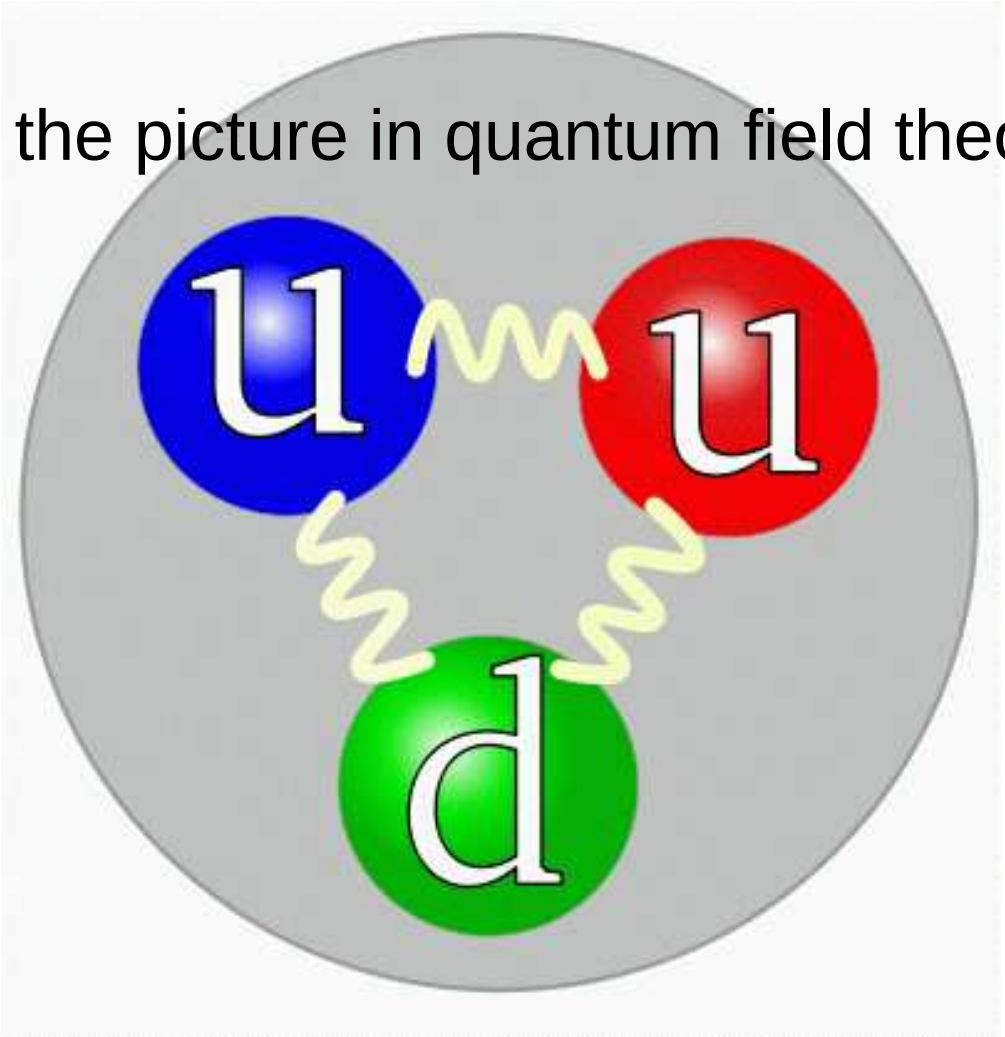
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.

Nucleon . . .
Three-body Problem?



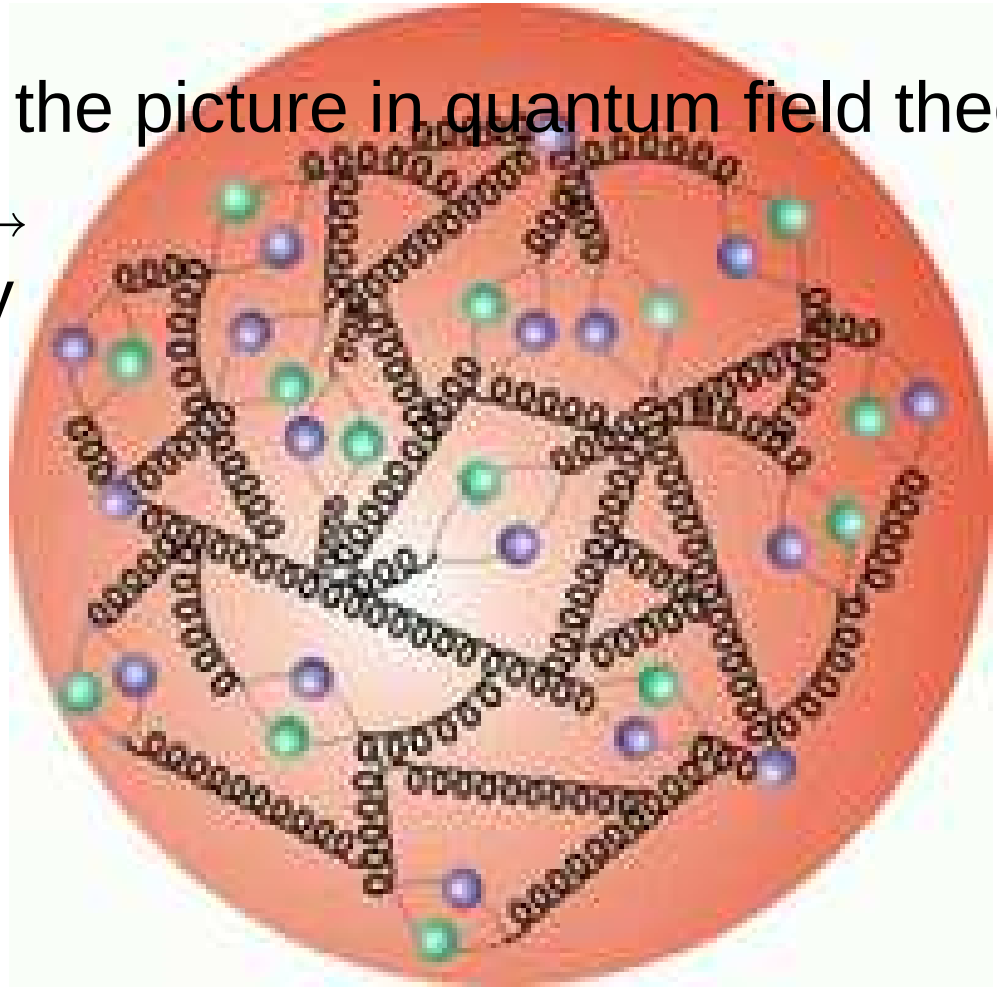
Nucleon . . .
Three-body Problem?

- What is the picture in quantum field theory?



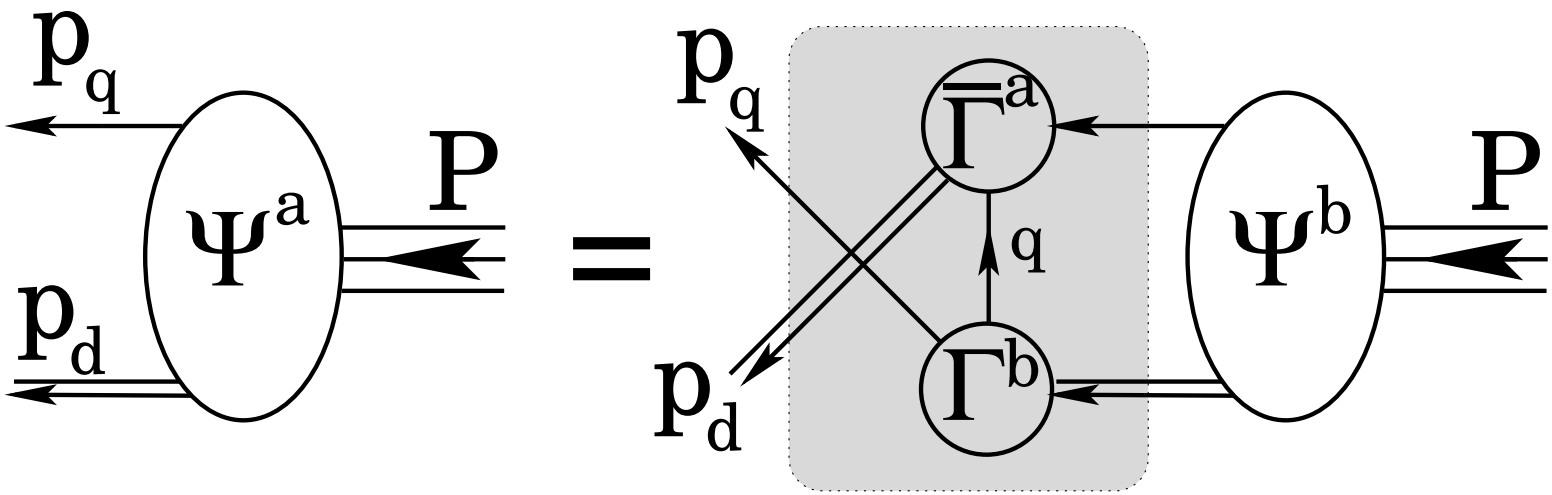
Nucleon . . .
Three-body Problem?

- What is the picture in quantum field theory?
- Three → infinitely many!

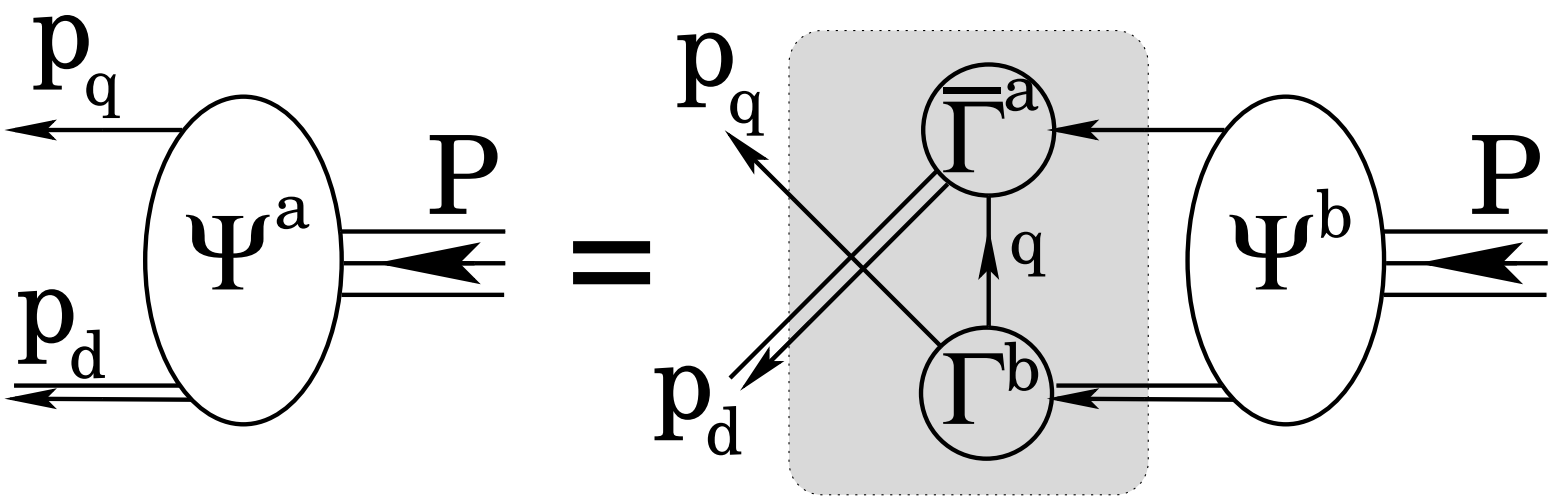


Faddeev equation

Faddeev equation

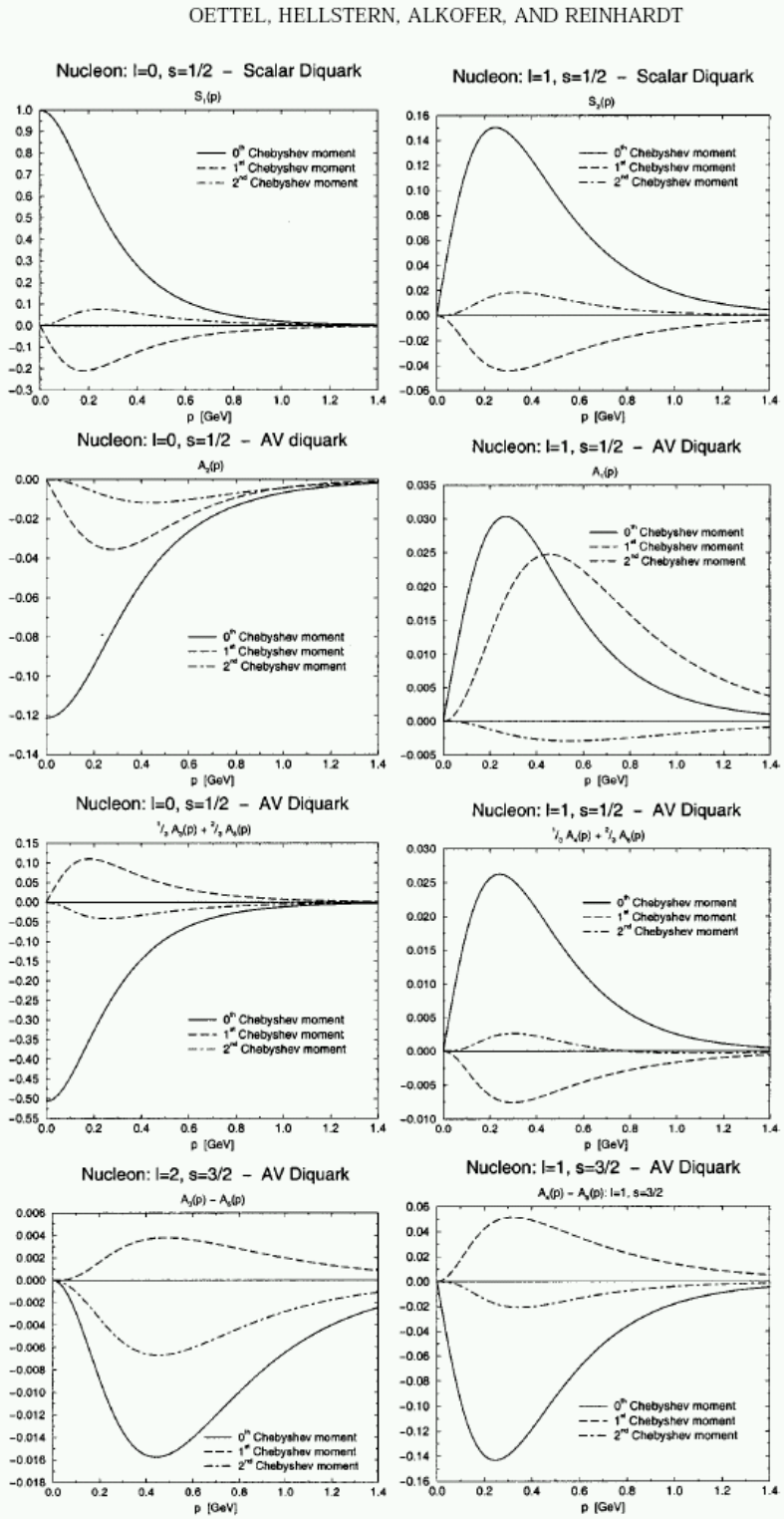


Faddeev equation



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Angular Momentum Rest Frame

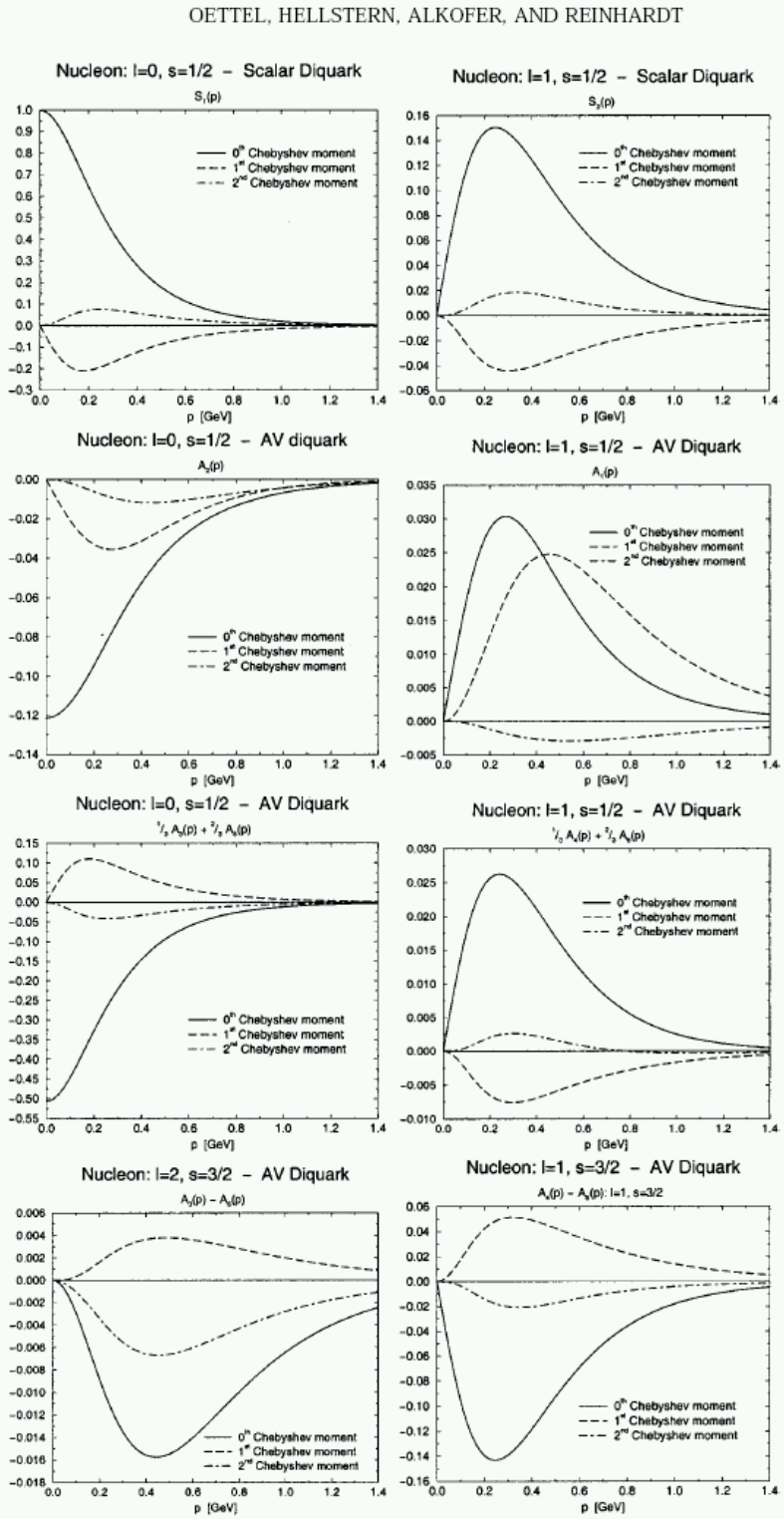


M. Oettel, *et al.*
nucl-th/9805054

Crude estimate based on magnitudes $\Rightarrow$ probability for a u-quark to carry the proton's spin is $P_{u\uparrow} \sim 80\,\%$, with $P_{u\downarrow} \sim 5\,\%$, $P_{d\uparrow} \sim 5\,\%$, $P_{d\downarrow} \sim 10\,\%$.

Hence, by this reckoning $\sim 30\%$ of proton's rest-frame spin is located in dressed-quark angular momentum.

Angular Momentum Rest Frame

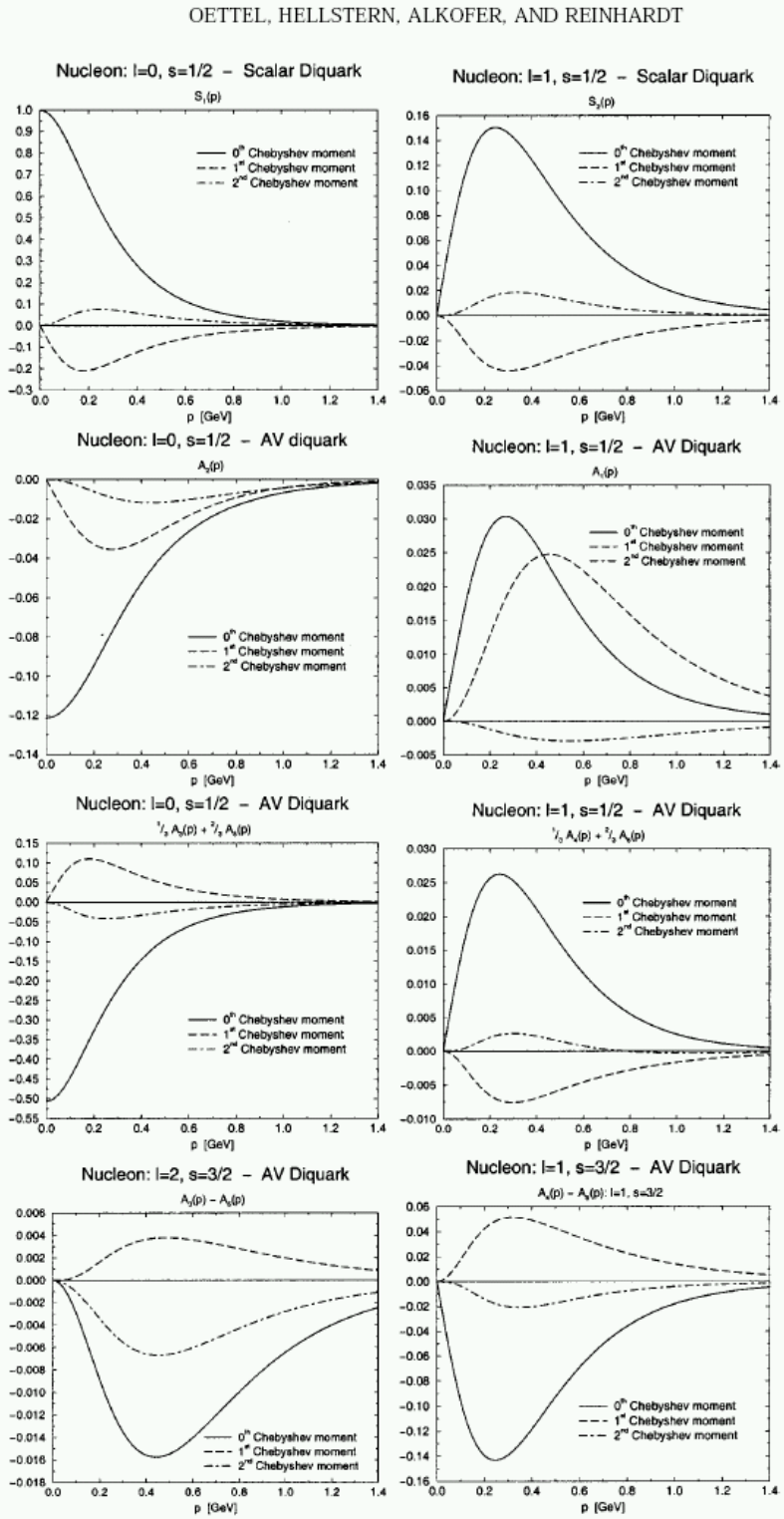


M. Oettel, *et al.*
nucl-th/9805054

Crude estimate based on magnitudes $\Rightarrow$ probability for a u-quark to carry the proton's spin is $P_{u\uparrow} \sim 80\,\%$, with $P_{u\downarrow} \sim 5\,\%$, $P_{d\uparrow} \sim 5\,\%$, $P_{d\downarrow} \sim 10\,\%$.

Hence, by this reckoning $\sim 30\%$ of proton's rest-frame spin is located in dressed-quark angular momentum.

Angular Momentum Rest Frame

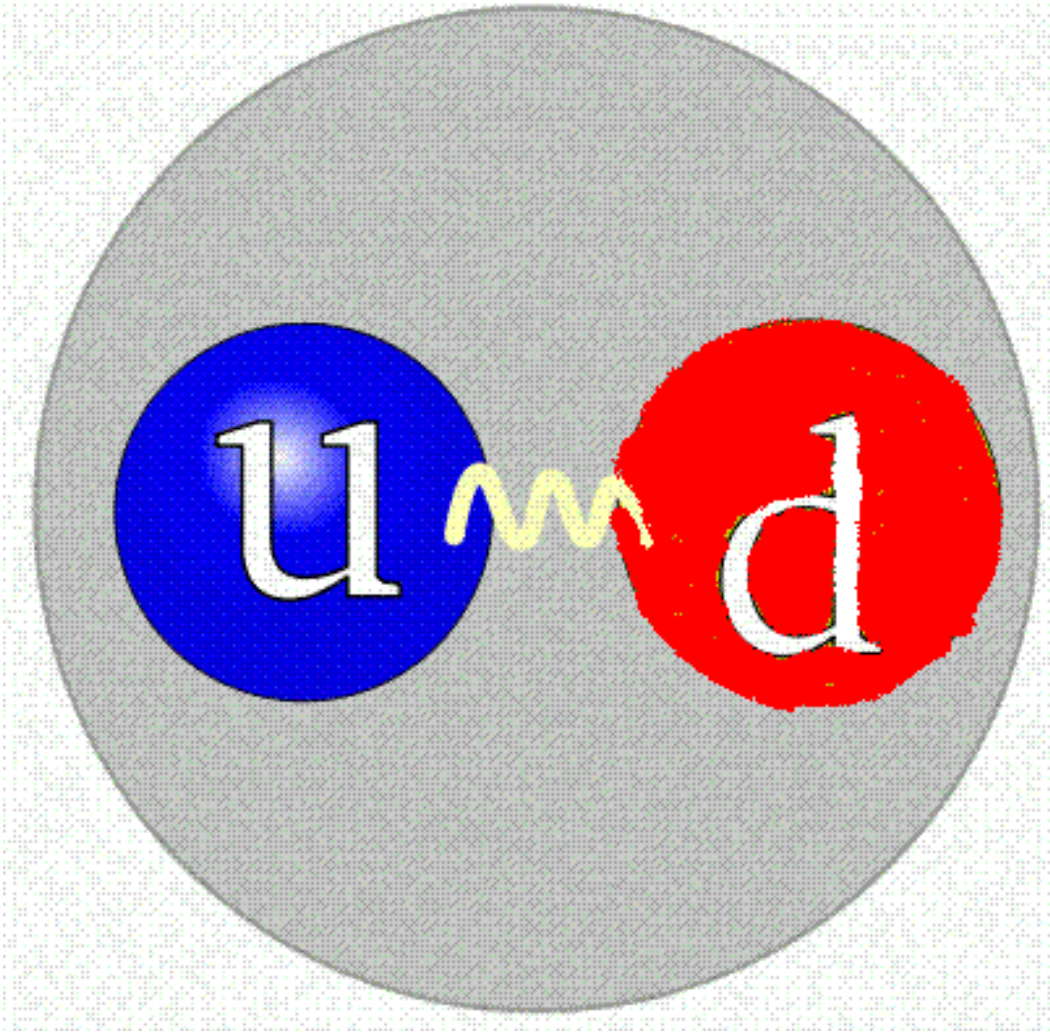


M. Oettel, *et al.*
nucl-th/9805054

Crude estimate based on magnitudes $\Rightarrow$ probability for a u-quark to carry the proton's spin is $P_{u\uparrow} \sim 80\,\%$, with $P_{u\downarrow} \sim 5\,\%$, $P_{d\uparrow} \sim 5\,\%$, $P_{d\downarrow} \sim 10\,\%$.

Hence, by this reckoning $\sim 30\%$ of proton's rest-frame spin is located in dressed-quark angular momentum.

Diquark correlations



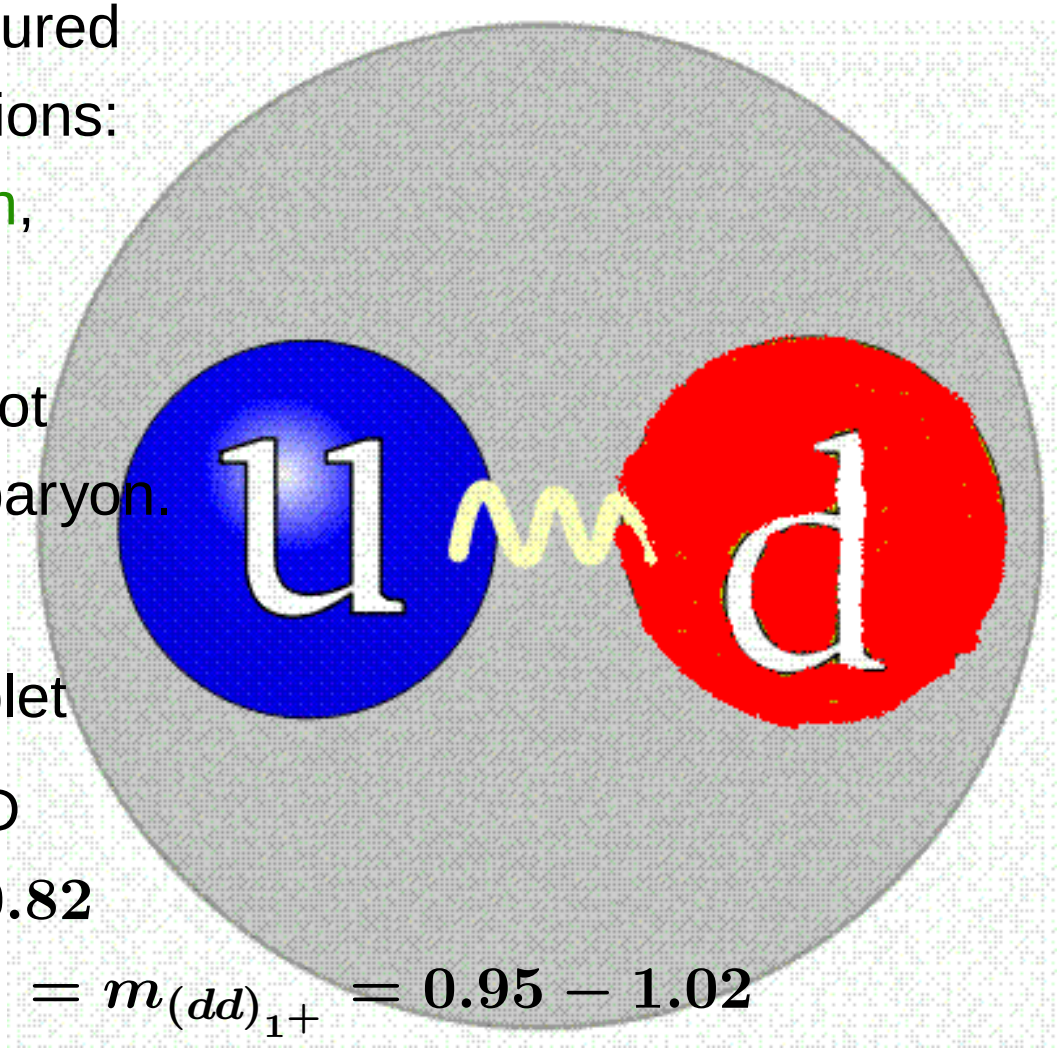
QUARK-QUARK

Diquark correlations

- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue–red, blue–green, green–red
- Confined . . . Does not escape from within baryon.
- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0^+}} = 0.74 - 0.82$$

$$m_{(uu)_{1^+}} = m_{(ud)_{1^+}} = m_{(dd)_{1^+}} = 0.95 - 1.02$$



QUARK-QUARK

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons

  $\Rightarrow$ Covariant dressed-quark Faddeev Equation

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

- But is that good?

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

- But is that good?
  - Cloudy Bag: $\delta M_+^{\pi-\rm loop} = -300$ to -400 MeV!

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

- But is that good?
  - Cloudy Bag: $\delta M_+^{\pi-\rm loop} = -300$ to -400 MeV!
- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

Pions and Form Factors

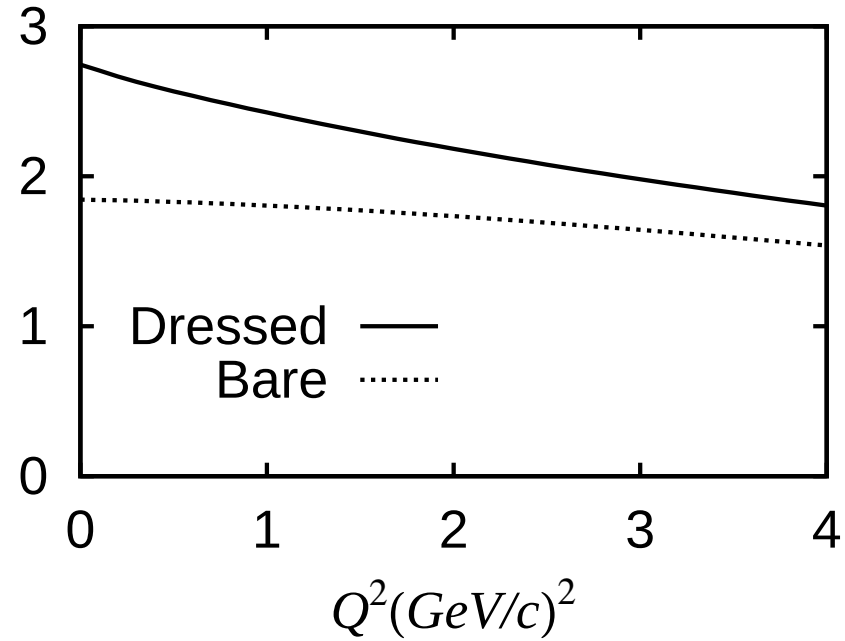
Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e, e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)

Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e, e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)
- Pion cloud effects are large in the low Q^2 region.

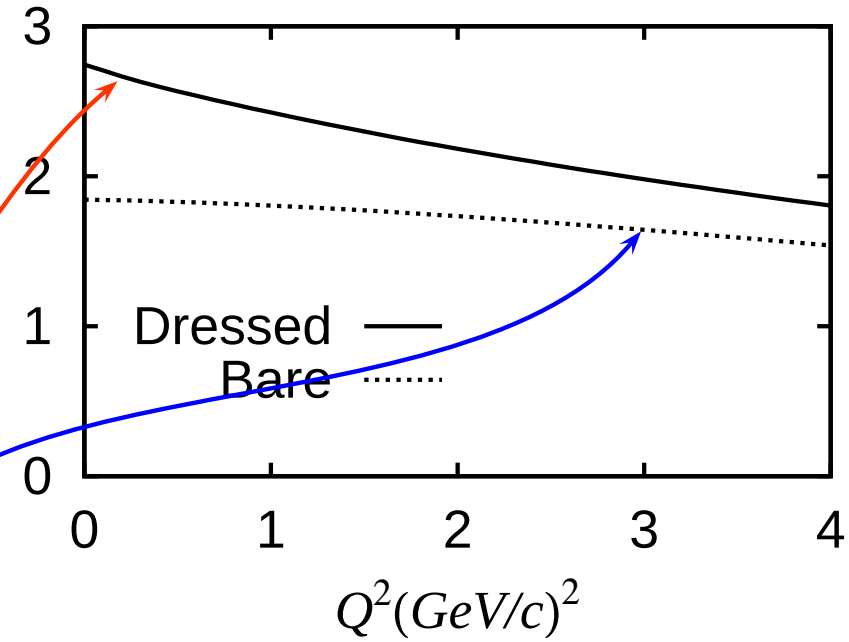
Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e, e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)
- Pion cloud effects are large in the low Q^2 region.

Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



Quark Core

- Responsible for only 2/3 of result at small Q^2
- Dominant for $Q^2 >$2 – 3 GeV^2

Results: Nucleon and Δ Masses

Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV
- Axial-vector diquark provides significant attraction

Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV
- Constructive Interference: 1^{++}-diquark $+ \partial_\mu \pi$

Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

for on-shell nucleons described by Faddeev Amplitude

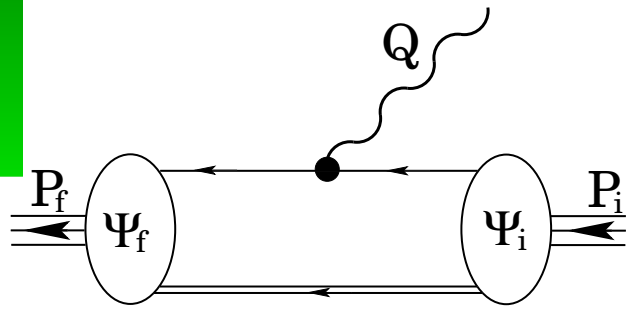
M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

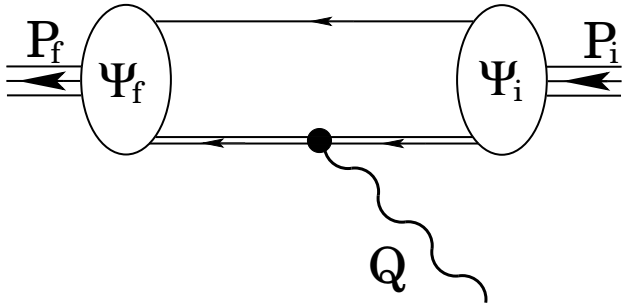
Nucleon-Photon Vertex

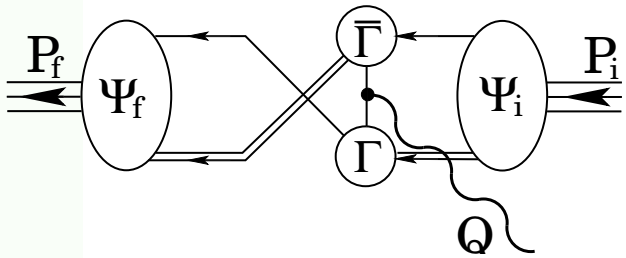
6 terms . . .

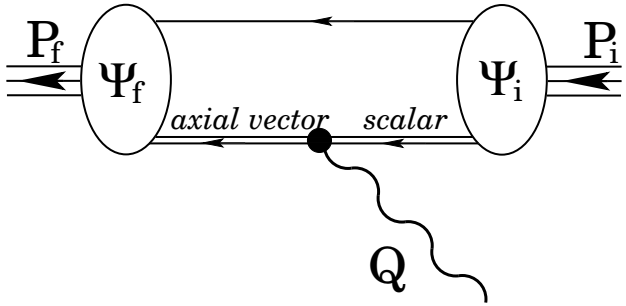
constructed systematically . . . current conserved automatically

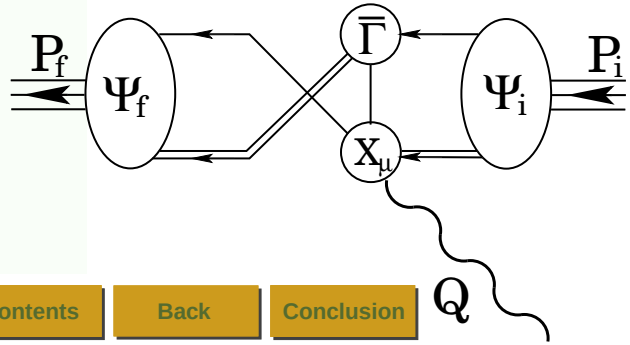
for on-shell nucleons described by Faddeev Amplitude

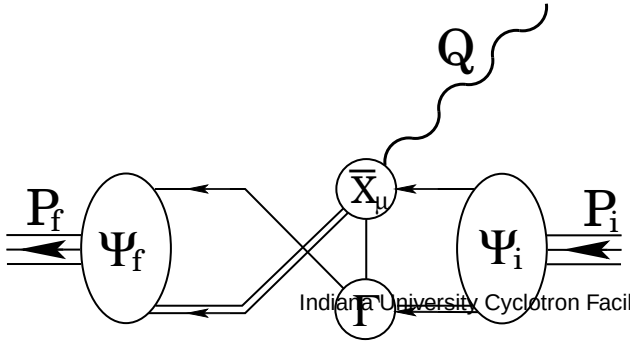




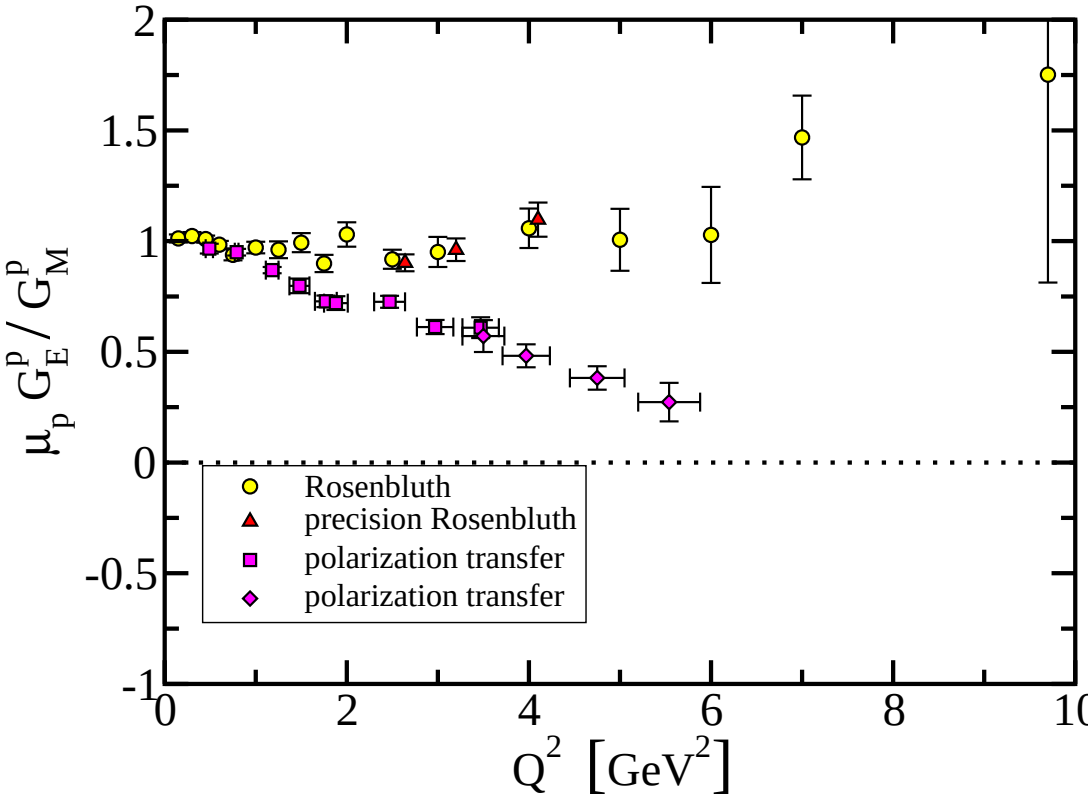






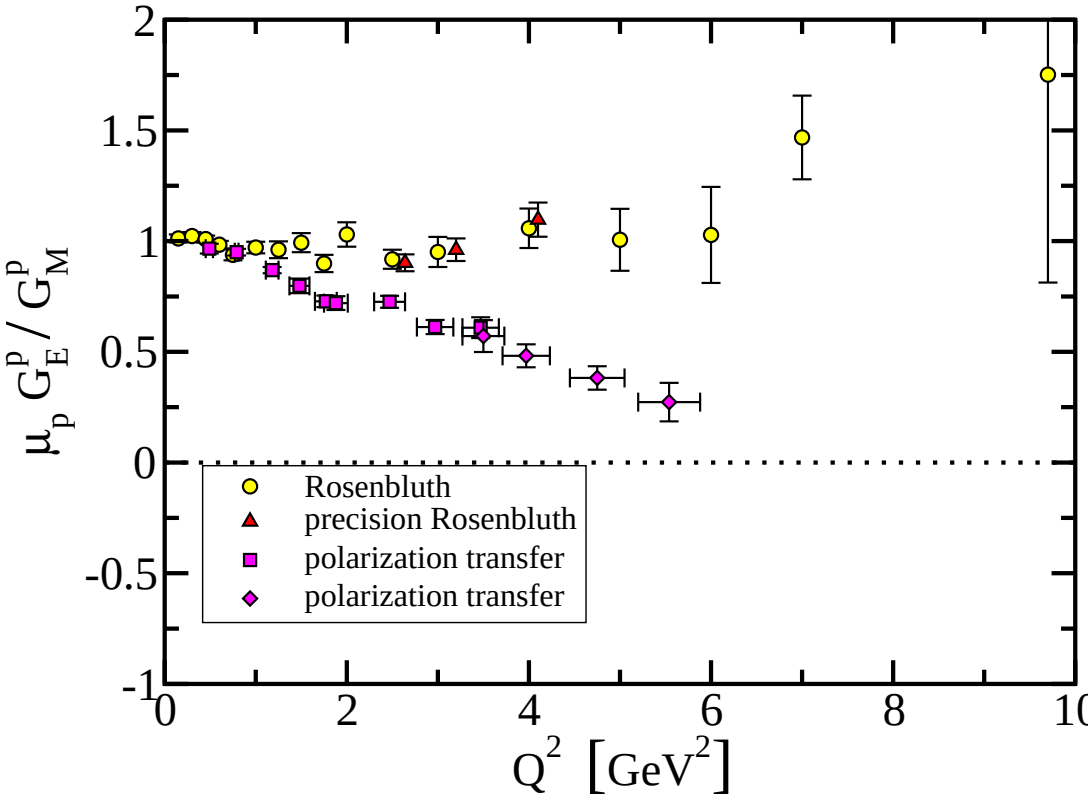


Form Factor Ratio: GE/GM



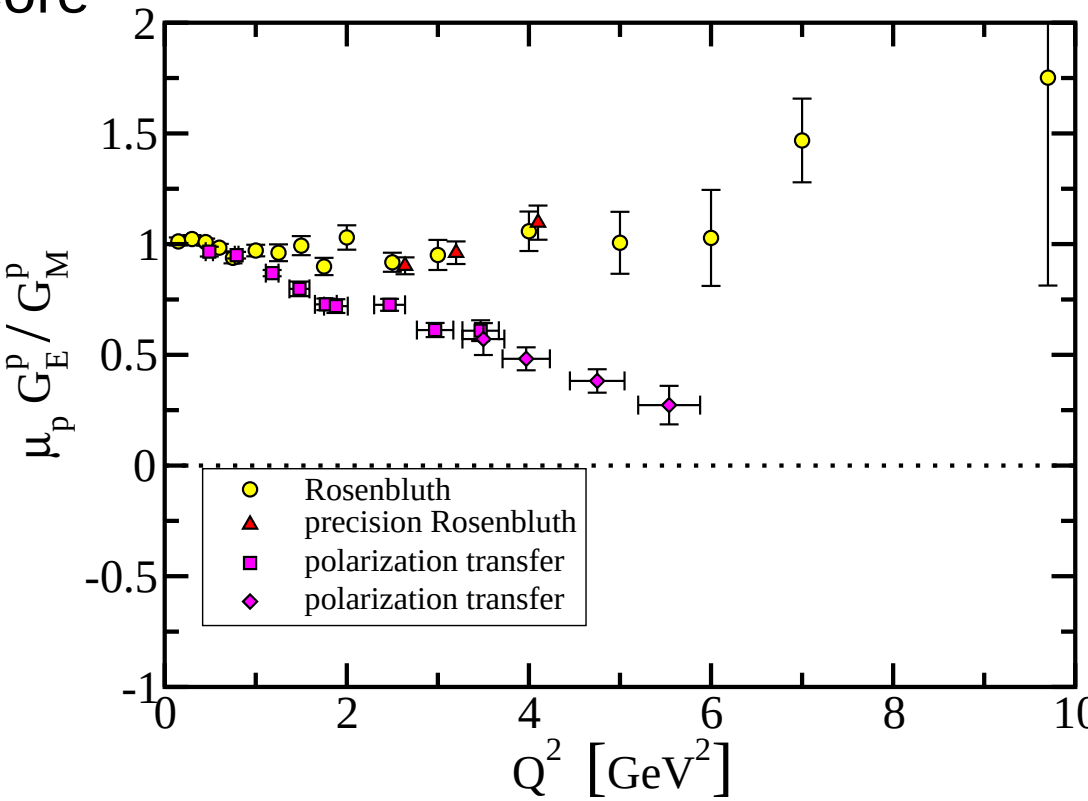
Form Factor Ratio: GE/GM

- Combine these elements . . .



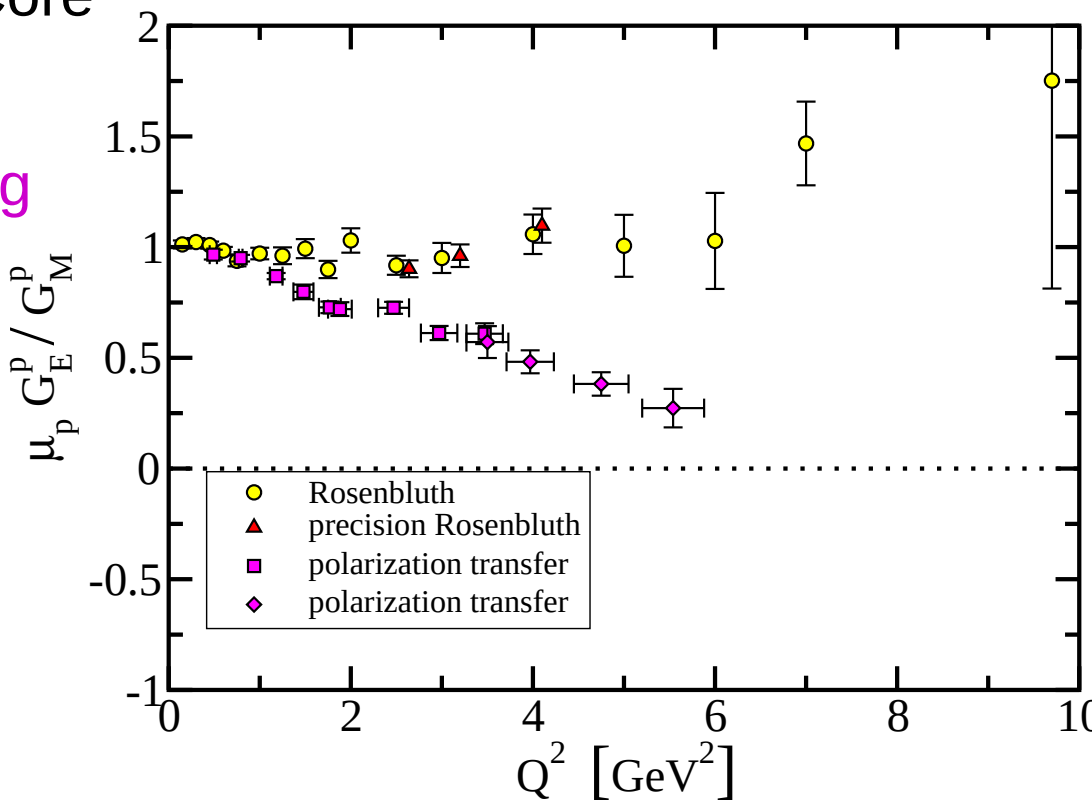
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core



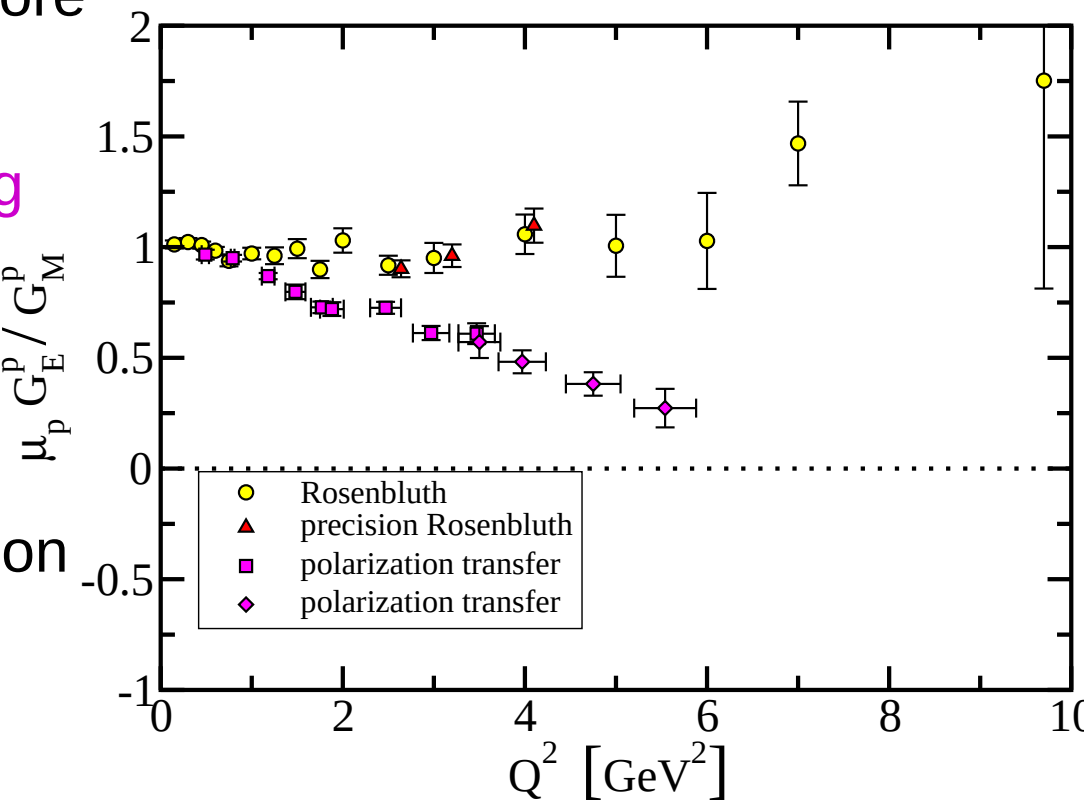
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current



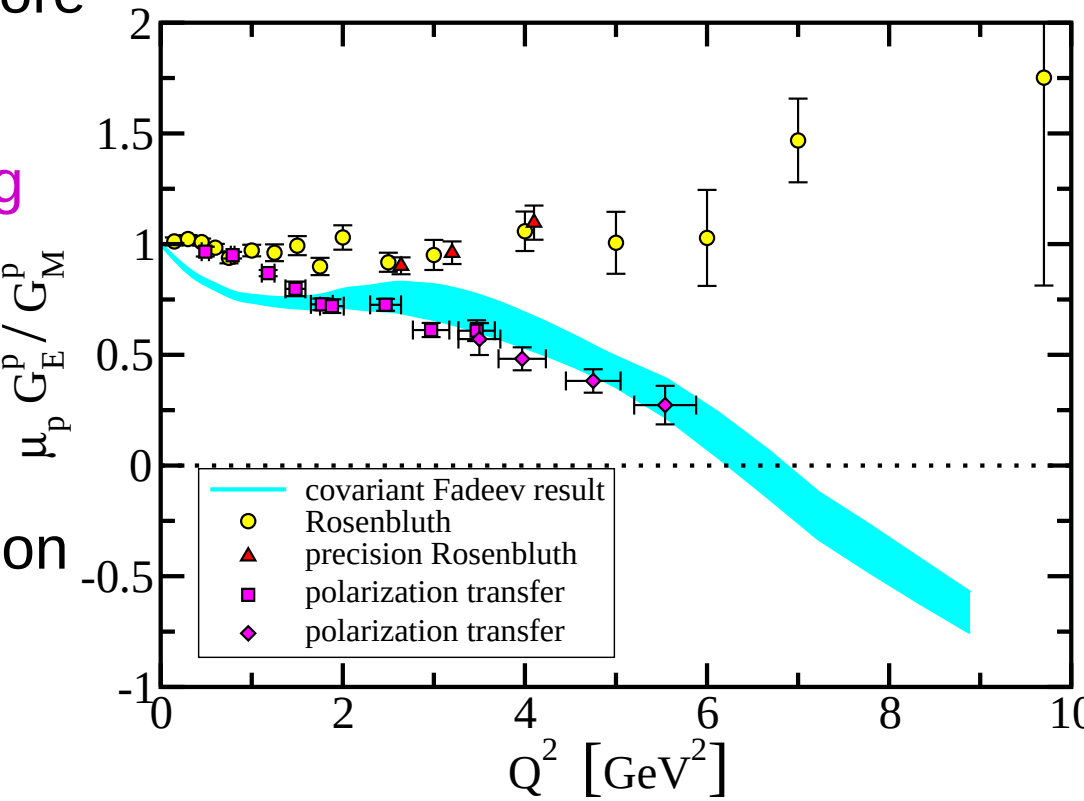
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



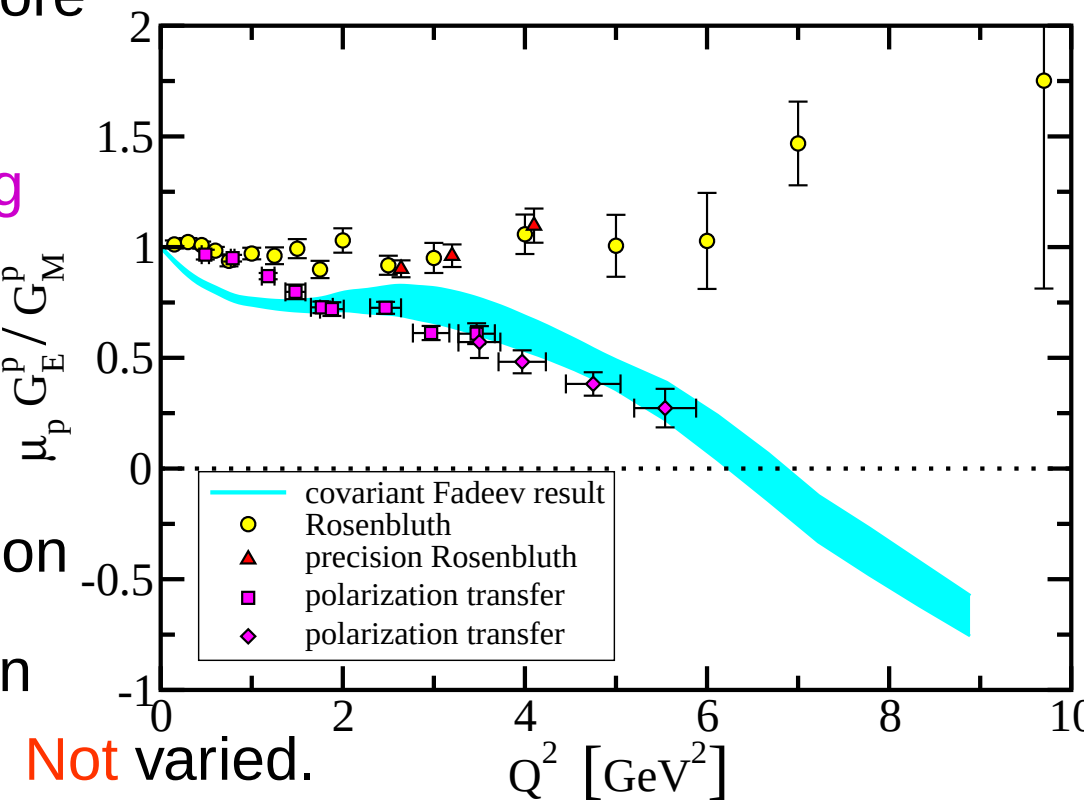
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



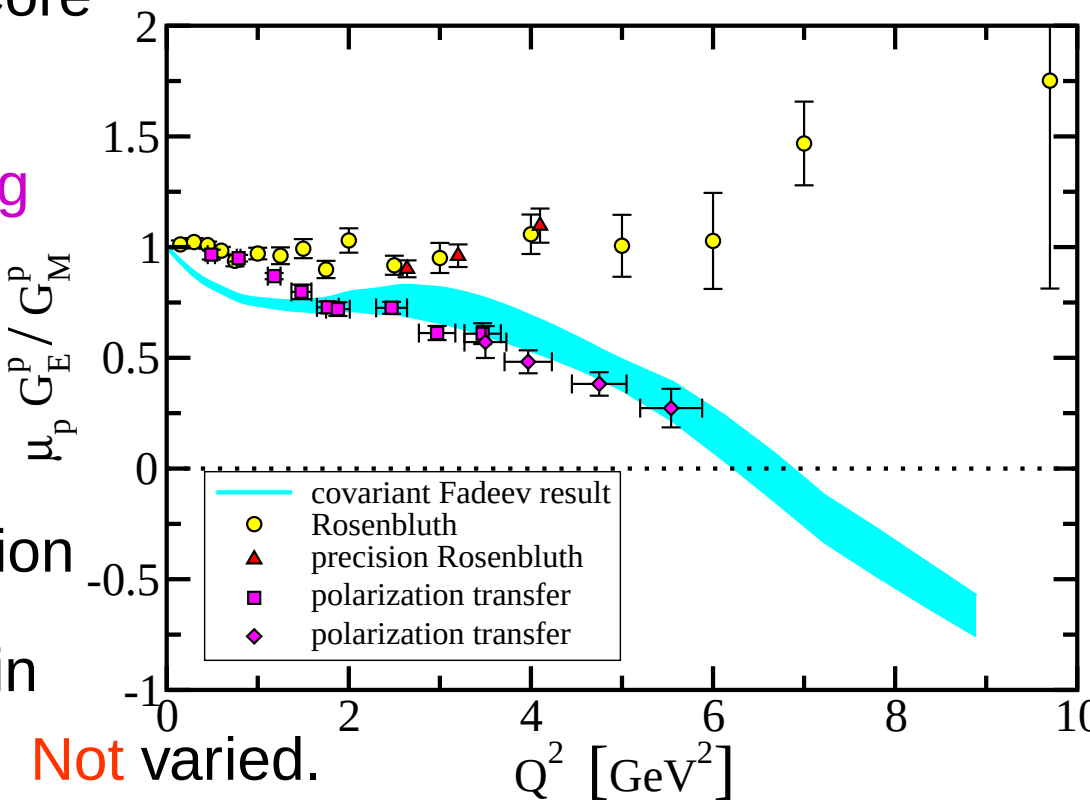
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution
- All parameters fixed in other applications . . . Not varied.



Form Factor Ratio: GE/GM

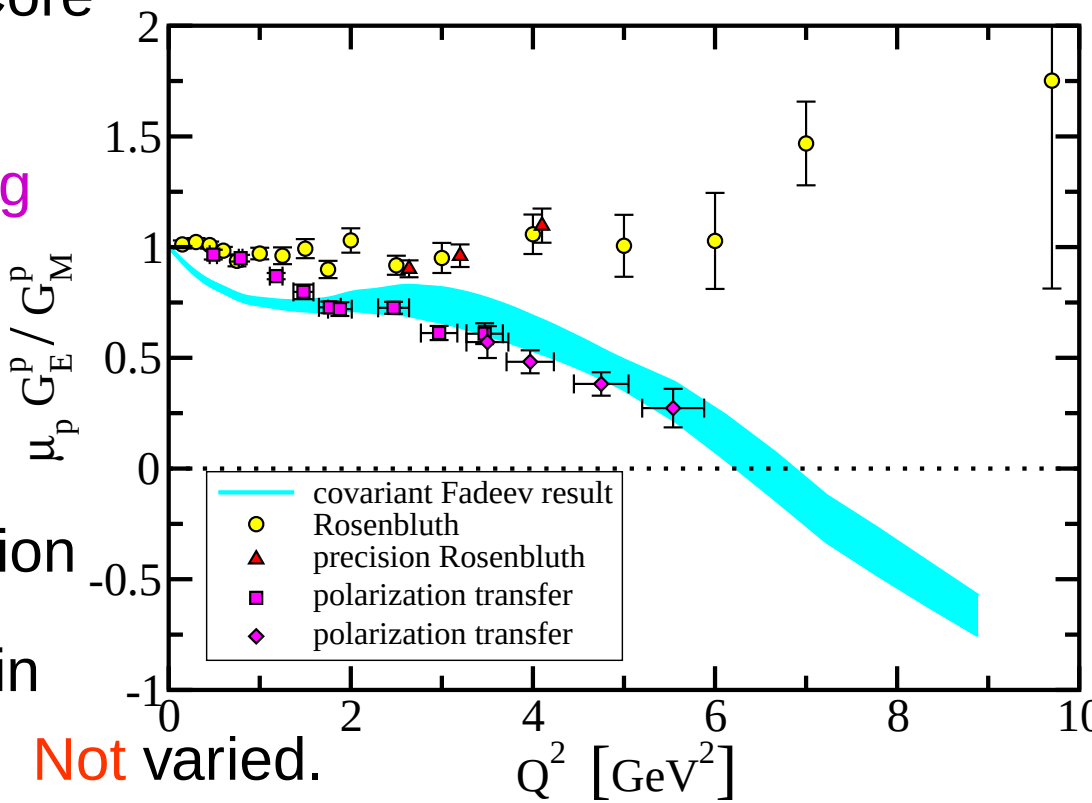
- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



- All parameters fixed in other applications . . . Not varied.
  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\,\mathrm{GeV}^2$

Form Factor Ratio: GE/GM

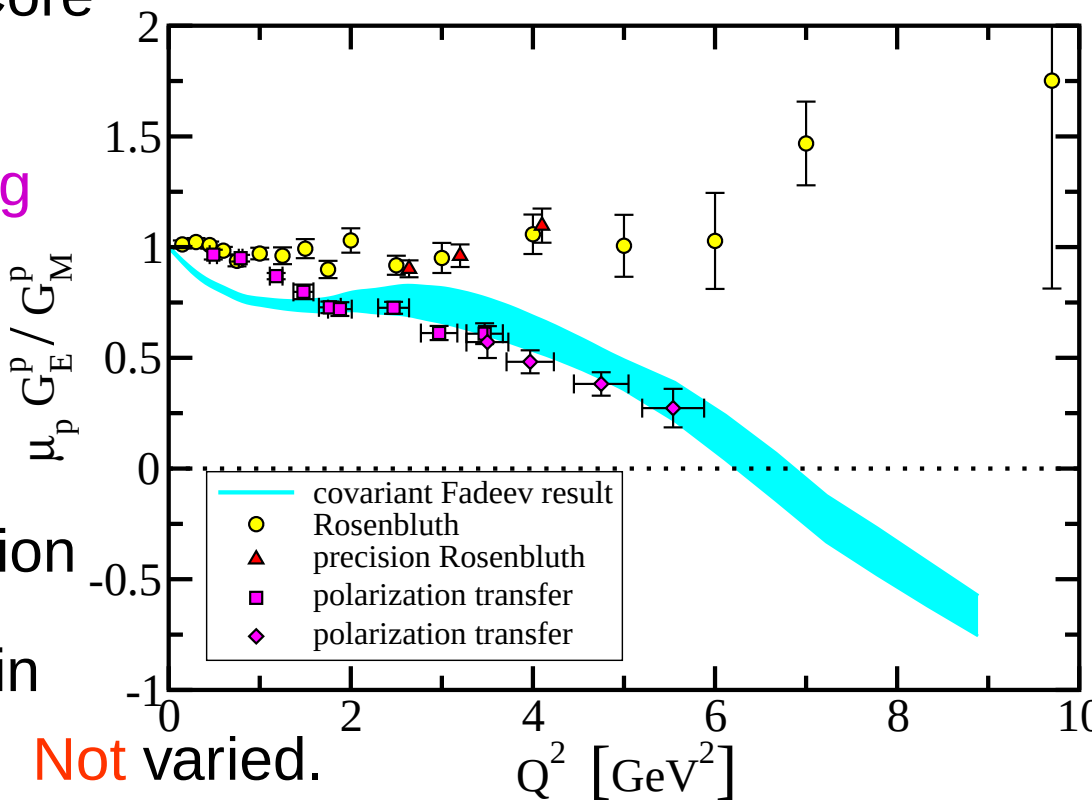
- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



- All parameters fixed in other applications . . . Not varied.
  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\,\mathrm{GeV}^2$
  - Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement

Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



- All parameters fixed in other applications . . . Not varied.
  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\,\mathrm{GeV}^2$
  - Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
  - Predict Zero at $Q^2 \approx 6.5\mathrm{GeV}^2$

Chiral Corrections

Chiral Corrections

- Thus far, omitted pion cloud contribution to current

Chiral Corrections

- Thus far, omitted pion cloud contribution to current
- Include loops following method of
  - . . . Ashley, Leinweber, Thomas, Young, he-lat/0308024
  - . . . finite-range regularisation of loop corrections
  - . . . $\lambda =$ regularisation mass-scale

Chiral Corrections

- Thus far, omitted pion cloud contribution to current
- Include loops following method of
  - . . . Ashley, Leinweber, Thomas, Young, he-lat/0308024
  - . . . finite-range regularisation of loop corrections
  - . . . $\lambda =$ regularisation mass-scale

$$\langle r^2_{\substack{p\\n}} \rangle^{1-loop^R}_{NA} = \mp \frac{1+5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2+\lambda^2}\right),$$

$$\langle (r^\mu_N)^2 \rangle^{1-loop^R}_{NA} = -\frac{1+5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2+\lambda^2}\right) + \frac{g_A^2 M_N}{16\pi f_\pi^2 \mu_v} \frac{1}{m_\pi} \frac{2}{\pi} \arctan\left(\frac{\lambda}{m_\pi}\right),$$

$$(\mu_{\substack{p\\n}})^{1-loop^R}_{NA} = \mp \frac{g_A^2 M_N}{4\pi^2 f_\pi^2} m_\pi \frac{2}{\pi} \arctan\left(\frac{\lambda^3}{m_\pi^3}\right),$$

Chiral Corrections

- Thus far, omitted pion cloud contribution to current
- Include loops following method of
  - . . . Ashley, Leinweber, Thomas, Young, he-lat/0308024
  - . . . finite-range regularisation of loop corrections
  - . . . $\frac{1}{\lambda} = \frac{2}{3}$ fm

	r_p	r_n	r_p^μ	r_n^μ	μ_p	$-\mu_n$	ς
q-(qq) core	0.595	0.169	0.449	0.449	3.63	2.13	0.39
$+\pi$-loop correction	0.762	0.506	0.761	0.761	3.05	1.55	0.23
experiment	0.847	0.336	0.836	0.889	2.79	1.91	

Chiral Corrections

- Thus far, omitted pion cloud contribution to current
- Include loops following method of
  - . . . Ashley, Leinweber, Thomas, Young, he-lat/0308024
  - . . . finite-range regularisation of loop corrections
  - . . . $\frac{1}{\lambda} = \frac{2}{3}$ fm

$$\mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)} = 1 - \frac{Q^2}{6}\left[(r_p)^2 - (r_p^\mu)^2\right]$$

$r_p \approx r_p^\mu \Rightarrow$ ratio varies $< 10\%$ on $0 < Q^2 < 0.6$ GeV^2

Chiral Corrections

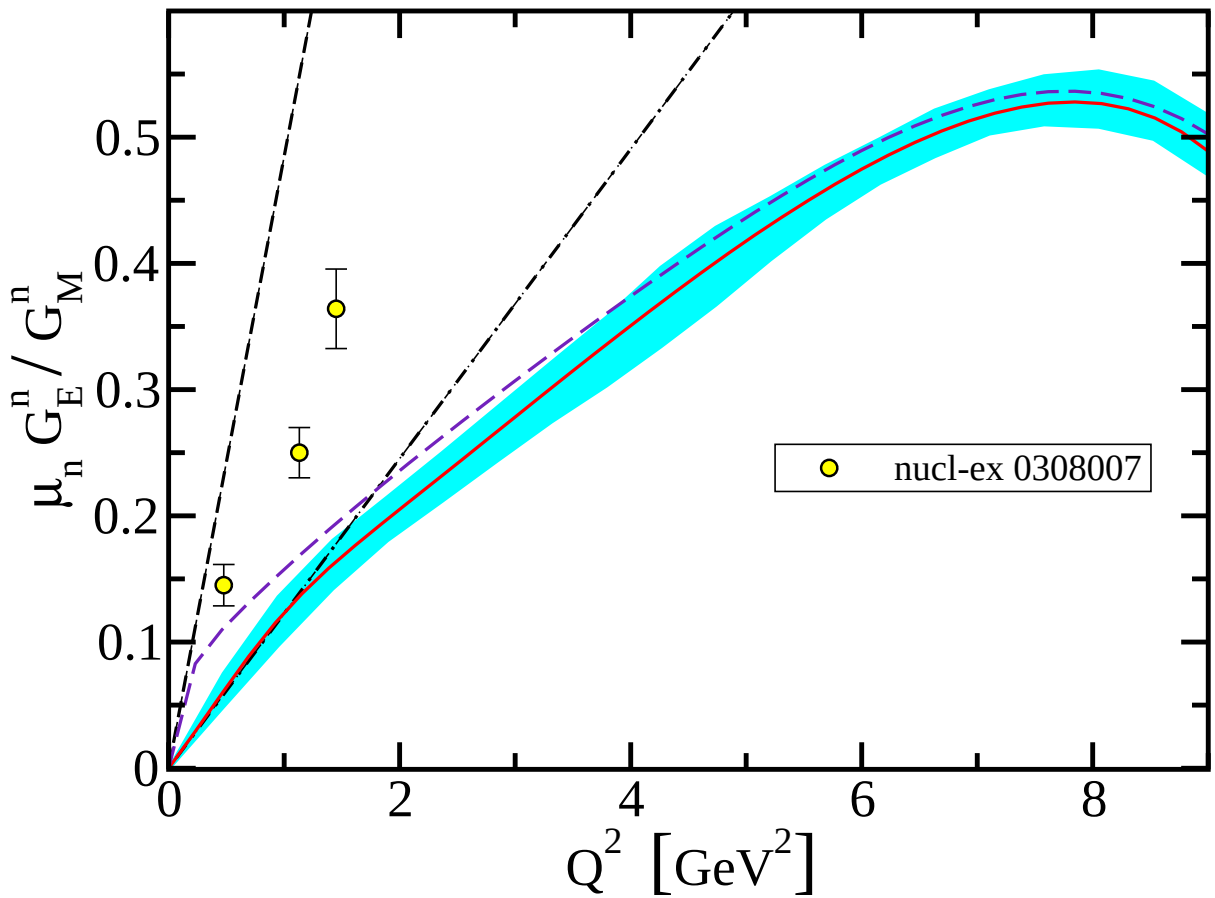
- Thus far, omitted pion cloud contribution to current
- Include loops following method of
  - . . . Ashley, Leinweber, Thomas, Young, he-lat/0308024
  - . . . finite-range regularisation of loop corrections
  - . . . $\frac{1}{\lambda} = \frac{2}{3}$ fm

$$\mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)} = 1 - \frac{Q^2}{6}\left[(r_p)^2 - (r_p^\mu)^2\right]$$

$r_p \approx r_p^\mu \Rightarrow$ ratio varies $< 10\%$ on $0 < Q^2 < 0.6$ GeV^2

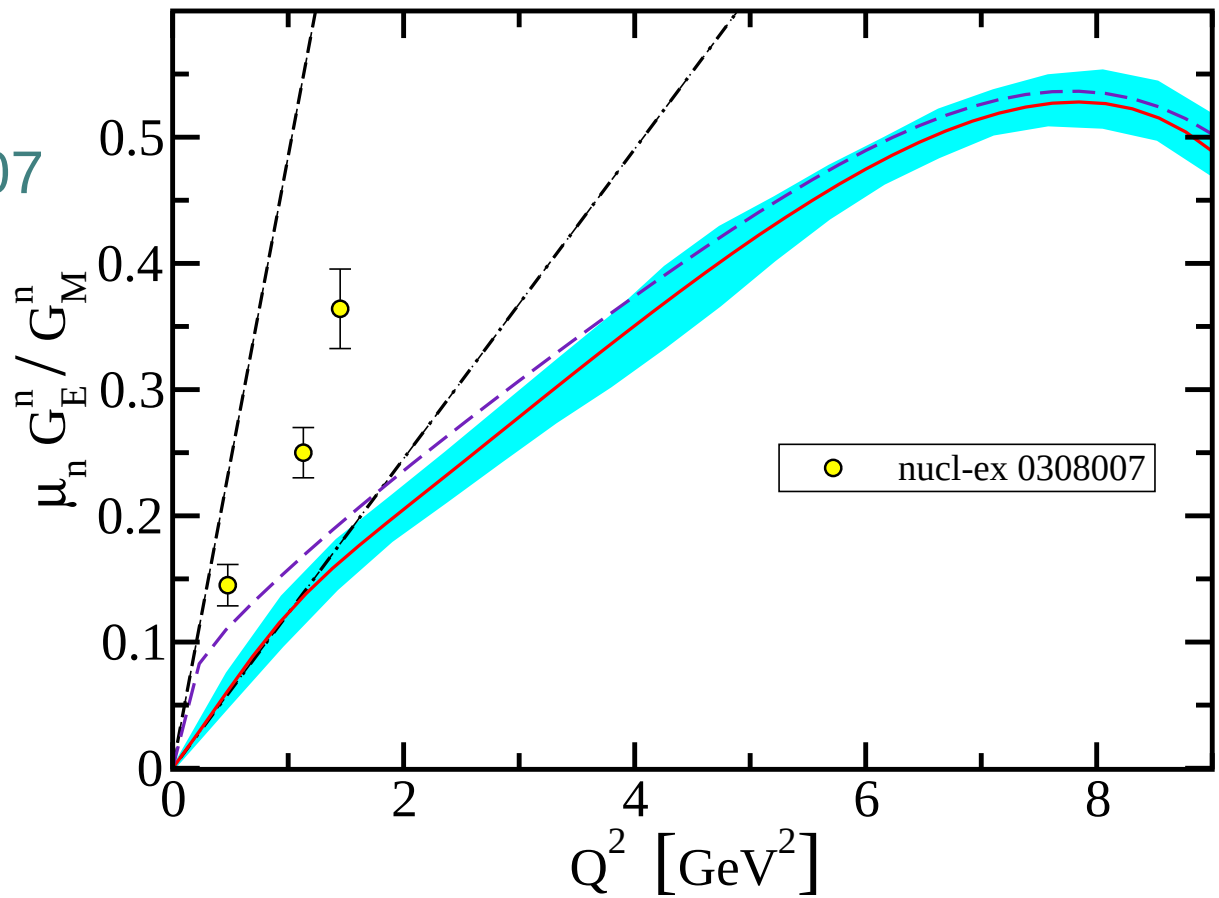
- Complements nucleon mass considerations
  - . . . veracious understanding of all nucleon properties
  - . . . impossible without intelligent incorporation of chiral corrections

Neutron Form Factors



Neutron Form Factors

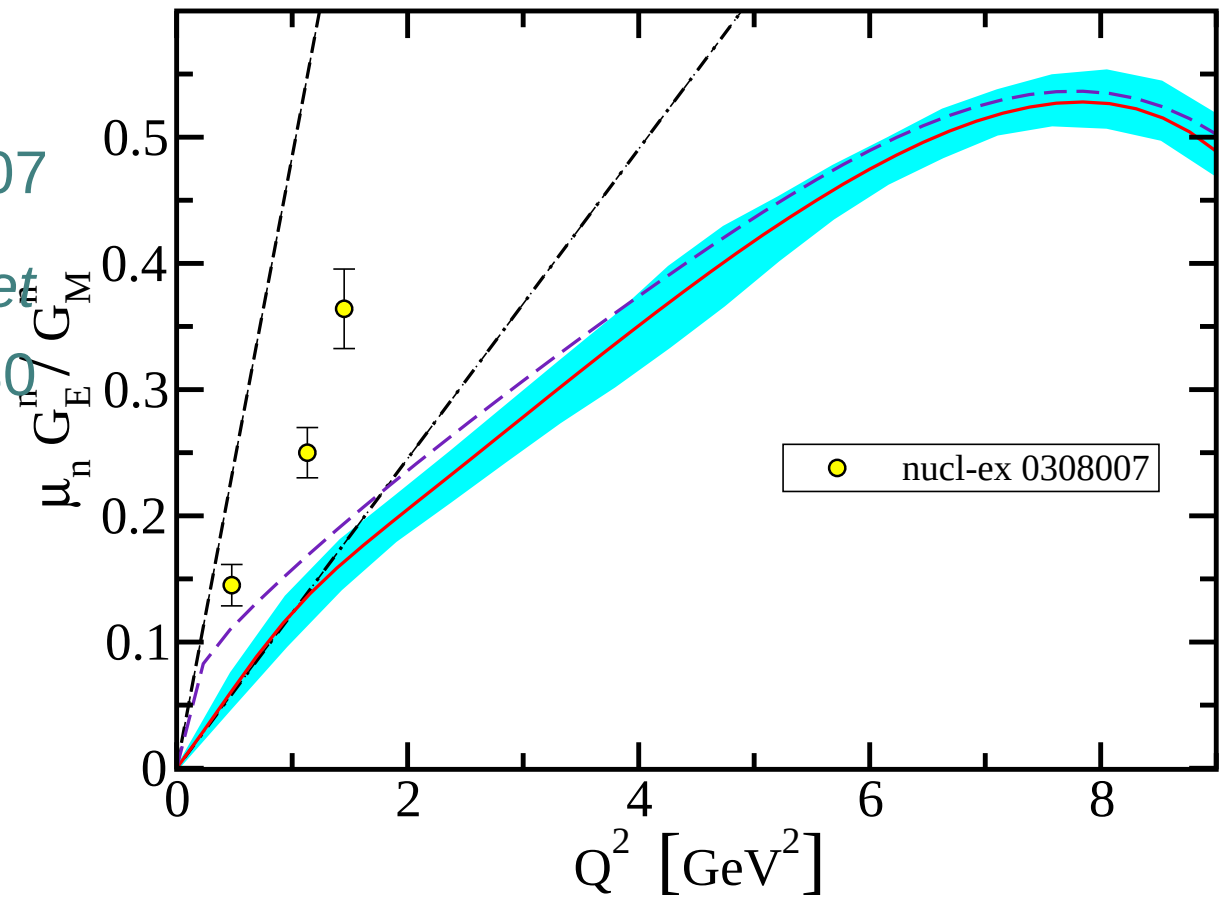
- Expt. Madey, *et al.* nu-ex/0308007



Neutron Form Factors

- Expt. Madey, *et al.* nu-ex/0308007
- Calc. Bhagwat, *et al.* nu-th/0610080
- $\mu_p \dfrac{G_E^n(Q^2)}{G_M^n(Q^2)} = -\dfrac{r_n^2}{6} Q^2$

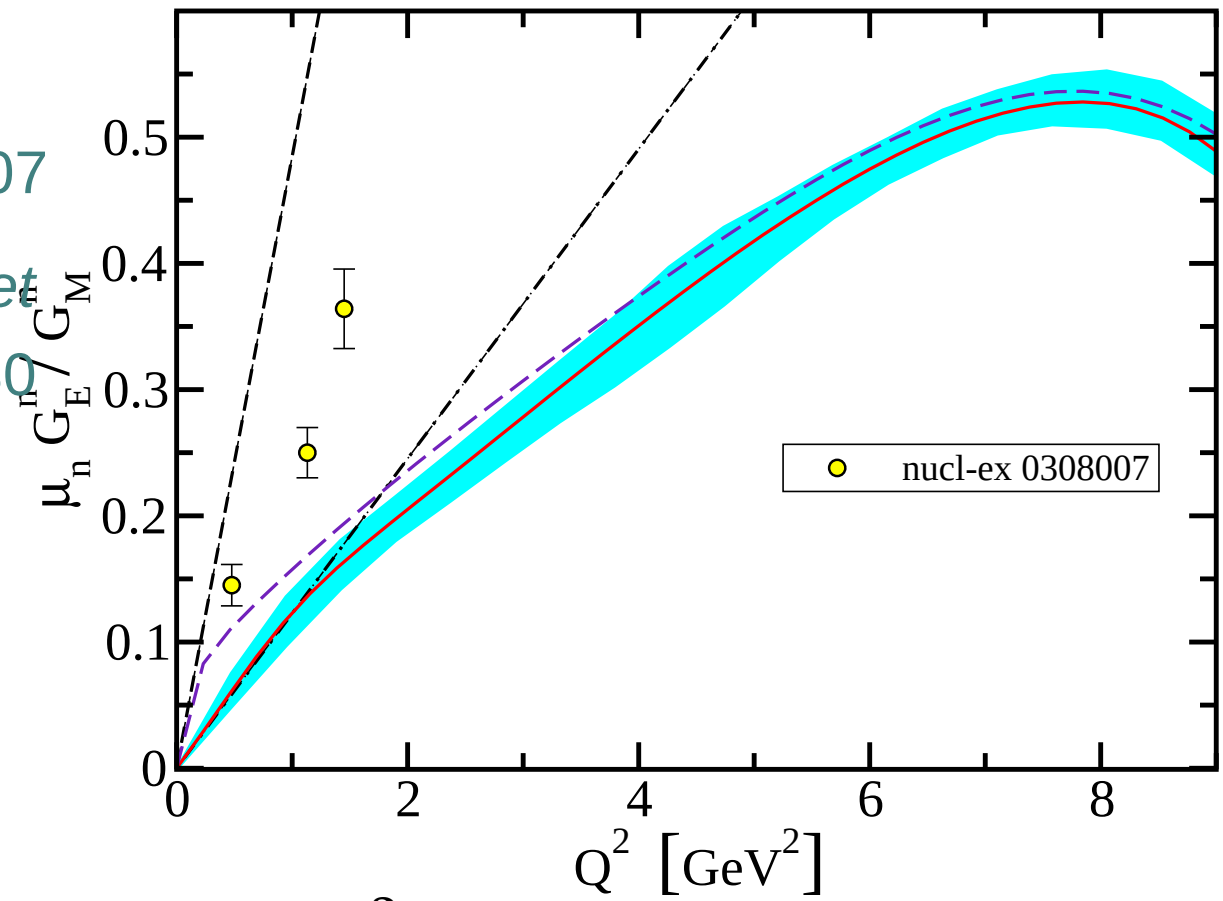
Valid for $r_n^2 Q^2 \lesssim 1$



Neutron Form Factors

- Expt. Madey, *et al.* nu-ex/0308007
- Calc. Bhagwat, *et al.* nu-th/0610080
- $\mu_p \dfrac{G_E^n(Q^2)}{G_M^n(Q^2)} = -\dfrac{r_n^2}{6} Q^2$

Valid for $r_n^2 Q^2 \lesssim 1$



- No sign yet of a zero in $G_E^n(Q^2)$, even though calculation predicts $G_E^p(Q^2 \approx 6.5\,\mathrm{GeV}^2) = 0$
- Data to $Q^2 = 3.4\,\mathrm{GeV}^2$ is being analysed (JLab E02-013)

Epilogue

Epilogue

Epilogue

Dyson-Schwinger Equations

- Provide Understanding of

  Dynamical Chiral Symmetry Breaking:

  $\Rightarrow$ π is quark-antiquark Bound State

  AND QCD's Goldstone Mode

Epilogue

Dyson-Schwinger Equations

- Provide Understanding of
  Dynamical Chiral Symmetry Breaking:

  $\Rightarrow \pi$ is quark-antiquark Bound State

  AND QCD's Goldstone Mode

- Foundation for Proof of
  Exact Results in QCD

  e.g., Quark Goldberger-Treiman

  Properties of Pseudoscalar Mesons

Epilogue

- Poincaré Covariant Faddeev Equation

Epilogue

- Poincaré Covariant Faddeev Equation
  - Nonpointlike scalar and axial-vector diquark correlations

Epilogue

- Poincaré Covariant Faddeev Equation
  - Nonpointlike scalar and axial-vector diquark correlations
  - $s-$, $p-$, $d-$wave quark angular momentum

Epilogue

- Poincaré Covariant Faddeev Equation
  - Nonpointlike scalar and axial-vector diquark correlations
  - $s-$, $p-$, $d-$wave quark angular momentum
- Quark core, relaxed to allow for pion cloud

Epilogue

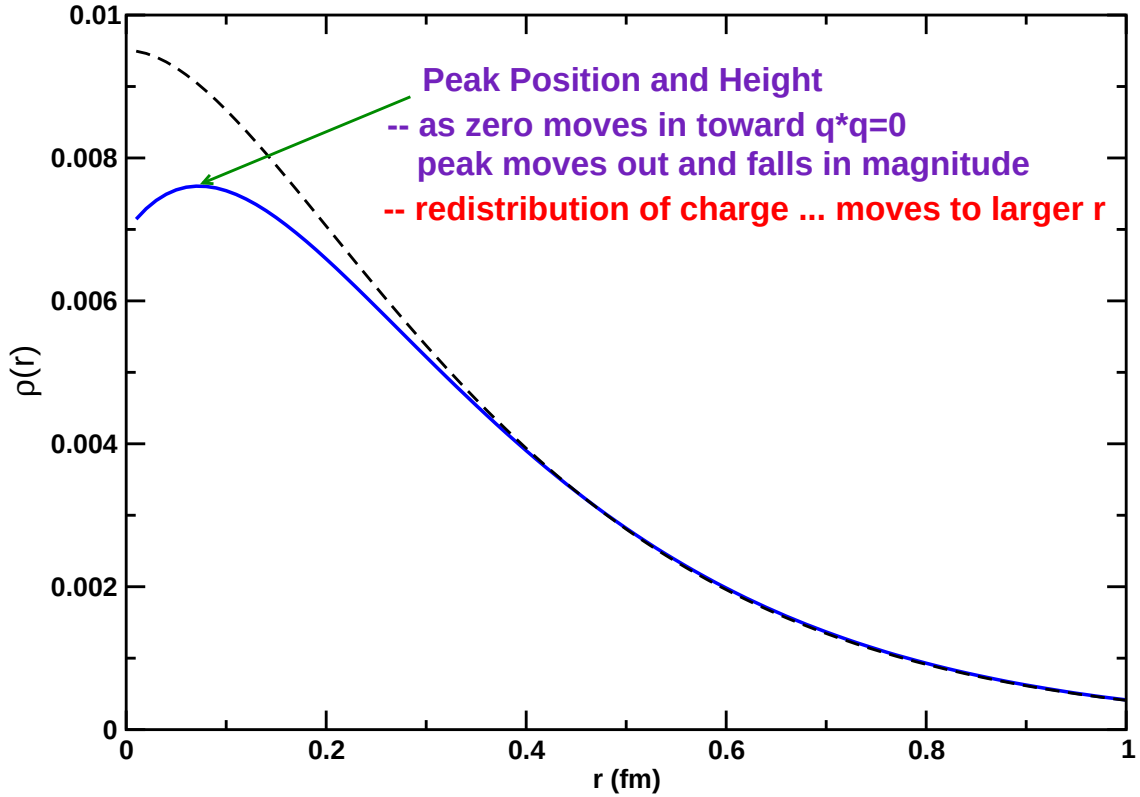
- Poincaré Covariant Faddeev Equation
  - Nonpointlike scalar and axial-vector diquark correlations
  - $s-$, $p-$, $d-$wave quark angular momentum
- Quark core, relaxed to allow for pion cloud
  - Predicts zero in $G_E^P(Q^2)$ at $Q^2 \approx 6.5\,\mathrm{GeV}^2$

Epilogue

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected

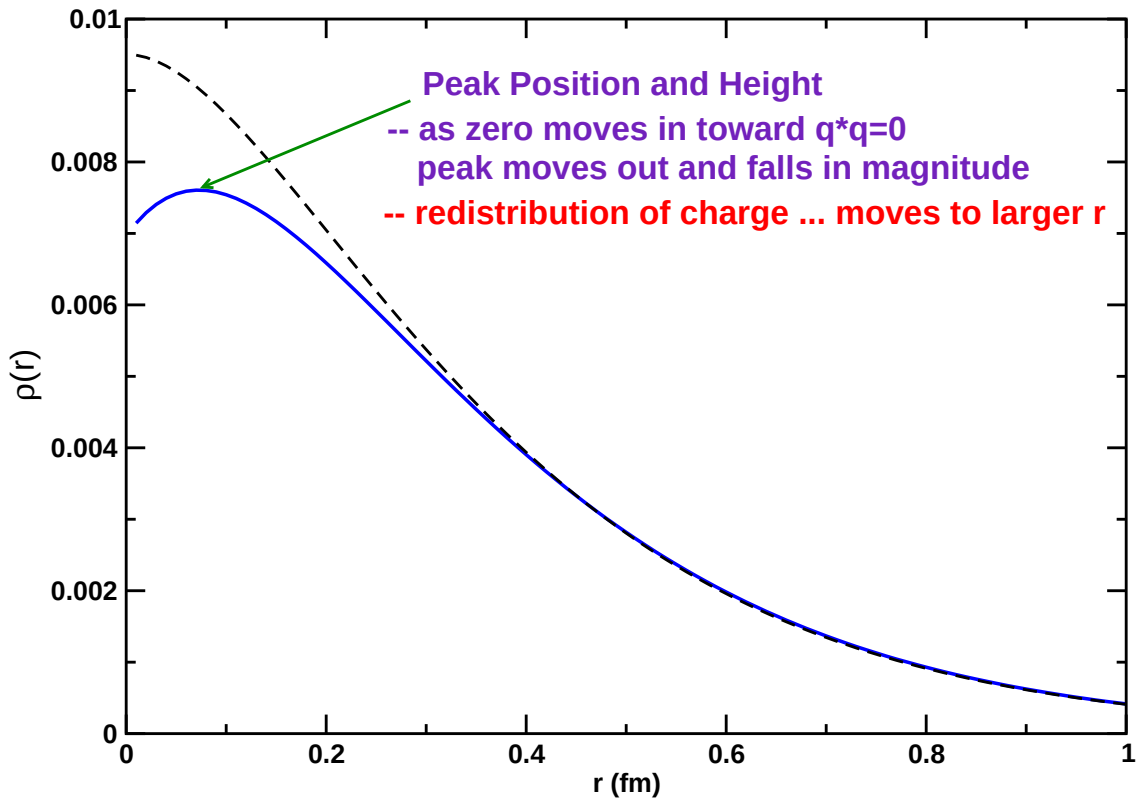
Epilogue

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected



Epilogue

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected



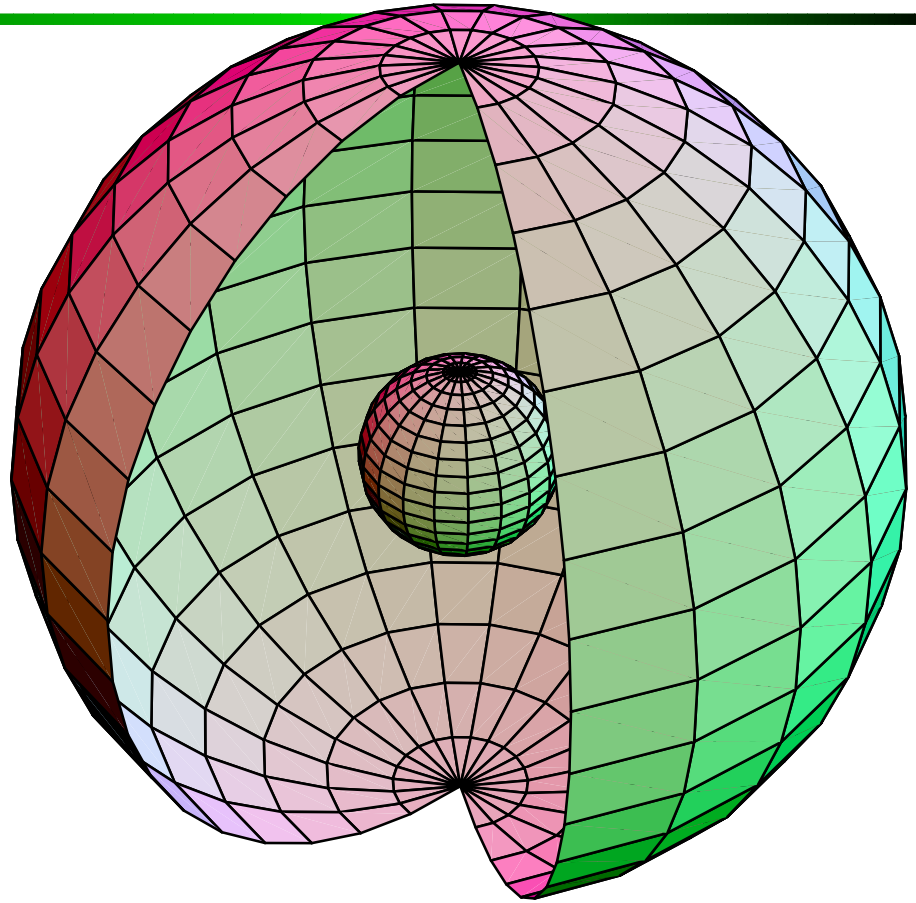
- However, Current Density remains peaked at $r = 0$!

Epilogue

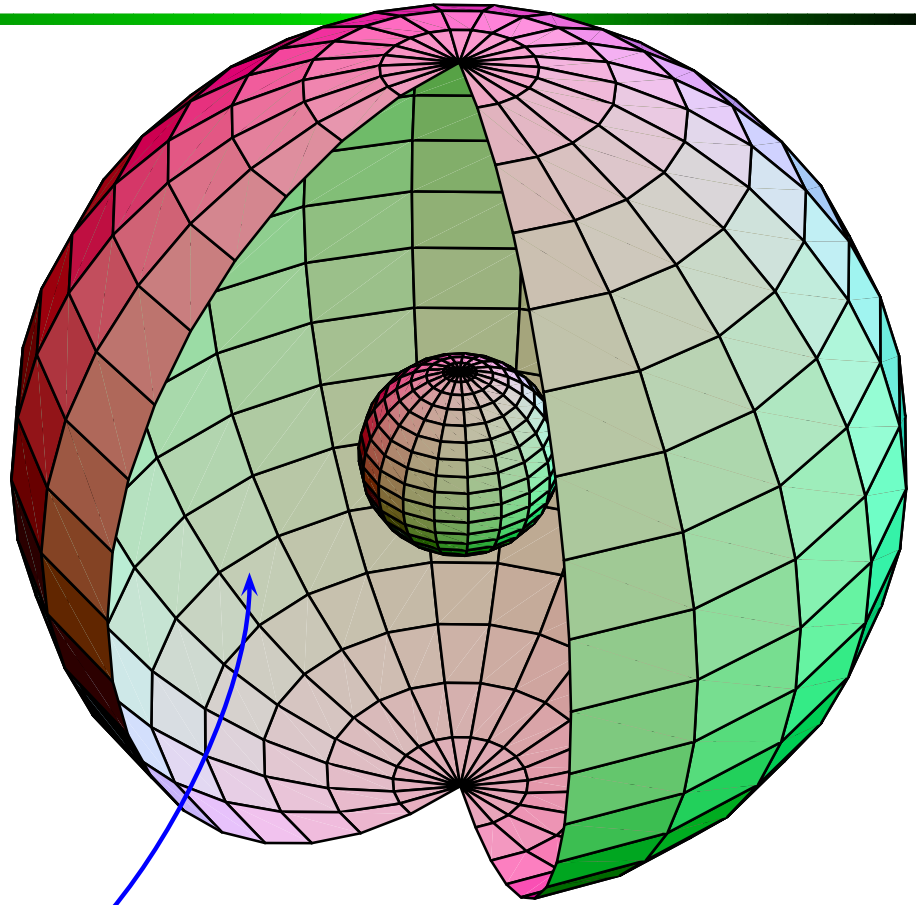
- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected
- Wave Function is complex and correlated mix of virtual particles and antiparticles: $s-$, $p-$ and $d-$waves

Future?

Future?



Future?



98% of the volume

Contents

1. Nucleon . . . 2 Key Hadrons
2. Nucleon Form Factors
3. NSAC Long Range Plan
4. Proton FF, Reprise
5. Meson Spectrum
6. Modern Miracles
7. Pion Dichotomy
8. QCD's Emergent Phenomena
9. Why care?
10. What's the Problem?
11. Confinement
12. A Compromise?
13. DSEs
14. Persistent Challenge
15. Perturbative $S(p)$
16. Dressed-Quark Propagator
17. Lattice cf. DSE
18. Light-Quark Interaction
19. Dressed-gluon Propagator
20. Hadrons
21. Bethe-Salpeter Kernel
22. Excitations & Chiral Symmetry
23. Radial Excitat$^{\rm ns}$ & Chiral Symmetry
24. Radial Excitations
25. Radial Excitations II
26. Lattice Radial
27. Faddeev equation
28. Angular Momentum
29. Nucleon EM Form Factors
30. Pions and Form Factors
31. Results: Nucleon & Δ Masses
32. Nucleon-Photon Vertex
33. Form Factor Ratio: GE/GM
34. Chiral Corrections

---

35. Parametrising diquark properties
36. Contemporary Reviews

Parametrising diquark properties

Parametrising diquark properties

- Dressed-quark . . . fixed by DSE and Meson Studies

  . . . Burden, Roberts, Thomson, Phys. Lett. **B 371**, 163 (1996)

Parametrising diquark properties

- Dressed-quark . . . fixed by DSE and Meson Studies

  . . . Burden, Roberts, Thomson, Phys. Lett. **B 371**, 163 (1996)

- Non-pointlike scalar and pseudovector colour-antitriplet diquark correlations – described by
  - Bethe-Salpeter amplitudes . . . width for each – ω_{J^P}
  - Confining propagators . . . mass for each – m_{J^P}

Parametrising diquark properties

- Dressed-quark . . . fixed by DSE and Meson Studies
  . . . Burden, Roberts, Thomson, Phys. Lett. **B 371**, 163 (1996)
- Non-pointlike scalar and pseudovector colour-antitriplet diquark correlations – described by
  - Bethe-Salpeter amplitudes . . . width for each – ω_{J^P}
  - Confining propagators . . . mass for each – m_{J^P}

Widths fixed by "asymptotic freedom" condition –

$$\frac{d}{dK^2}\left(\frac{1}{m_{J^P}^2}\,\mathcal{F}(K^2/\omega_{J^P}^2)\right)^{-1}\Bigg|_{K^2=0} = 1 \;\Rightarrow\; \omega_{J^P}^2 = \frac{1}{2}\,m_{J^P}^2\,,$$

Only two parameters; viz., diquark "masses": m_{J^P}

Contemporary Reviews

- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD
  C.D. Roberts and S.M. Schmidt, nu-th/0005064,
  Prog. Part. Nucl. Phys. **45** (2000) S1
- The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .
  R. Alkofer and L. von Smekal, he-ph/0007355,
  Phys. Rept. **353** (2001) 281
- Dyson-Schwinger equations: A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049,
  Int. J. Mod. Phys. **E 12** (2003) pp. 297-365
- Infrared properties of QCD from Dyson-Schwinger equations.
  C. S. Fischer, he-ph/0605173,
  J. Phys. **G 32** (2006) pp. R253-R291