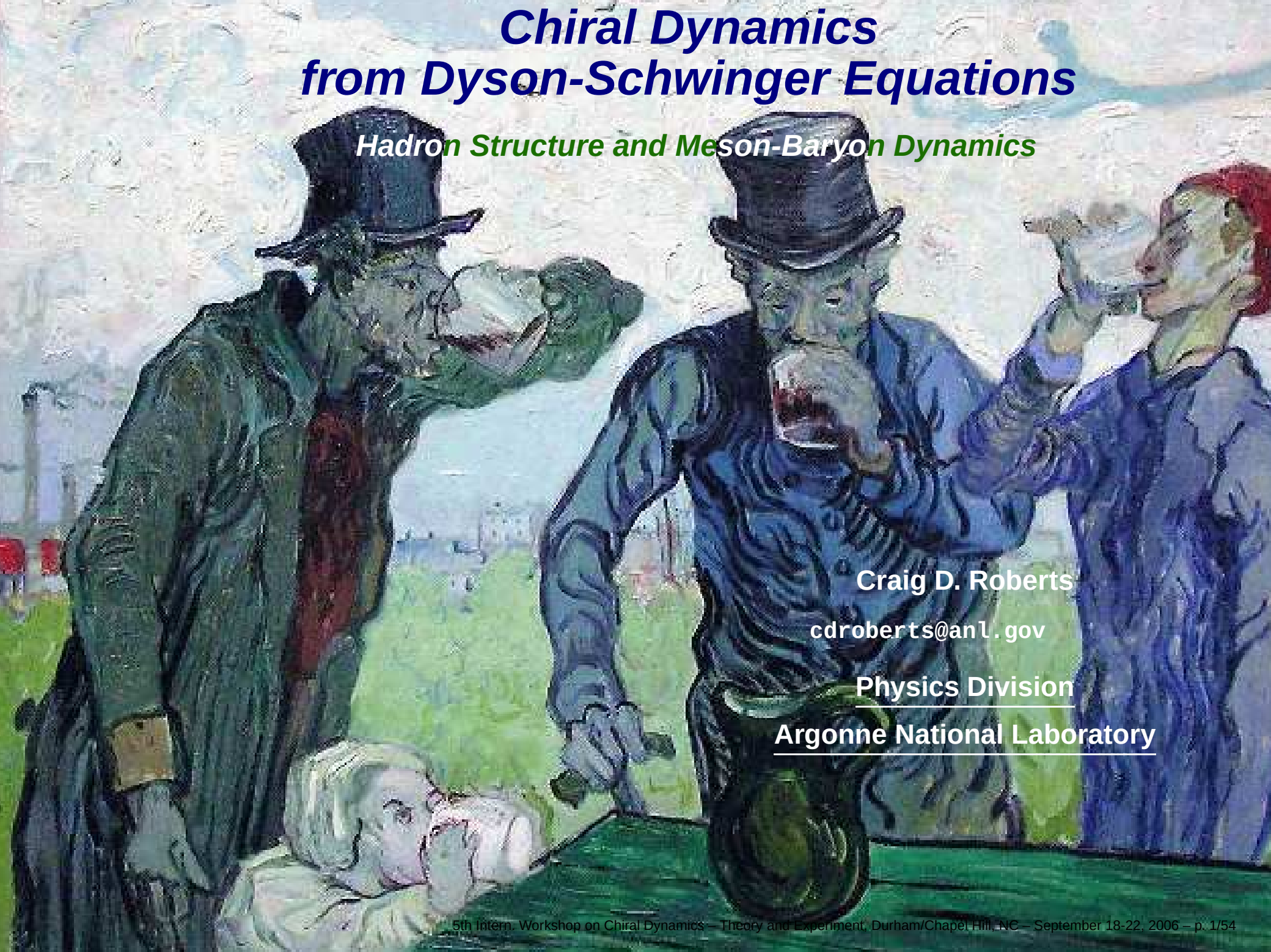
Chiral Dynamics from Dyson-Schwinger Equations

Hadron Structure and Meson-Baryon Dynamics



Craig D. Roberts

cdroberts@anl.gov

Physics Division

Argonne National Laboratory

Dichotomy of the Pion

Dichotomy of the Pion

- How does one make an almost massless particle
  ..... from two massive constituent-quarks?

Dichotomy of the Pion

- How does one make an almost massless particle
  . . . . . from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

  Must exhibit $\boxed{m_\pi^2 \propto m_q}$

  Current Algebra . . . 1968

Dichotomy of the Pion

- How does one make an almost massless particle
  . . . . . from two massive constituent-quarks?

- Not Allowed to do it by fine-tuning a potential

  Must exhibit $\boxed{m_\pi^2 \propto m_q}$

  Current Algebra . . . 1968

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a well-defined and valid chiral limit, and an accurate realisation of dynamical chiral symmetry breaking.

Dichotomy of the Pion

- How does one make an almost massless particle ..... from two massive constituent-quarks?

- Not Allowed to do it by fine-tuning a potential

  Must exhibit $\boxed{m_\pi^2 \propto m_q}$

  Current Algebra . . . 1968

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a well-defined and valid chiral limit, and an accurate realisation of dynamical chiral symmetry breaking.

- Requires detailed understanding of Connection between Current-quark and Constituent-quark masses

Dichotomy of the Pion

- How does one make an almost massless particle
  . . . . . from two massive constituent-quarks?
- Not Allowed to do it by fine-tuning a potential

  Must exhibit $m_\pi^2 \propto m_q$

  Current Algebra . . . 1968

The correct understanding of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a well-defined and valid chiral limit, and an accurate realisation of dynamical chiral symmetry breaking.

- Requires detailed understanding of Connection between Current-quark and Constituent-quark masses

Using DSEs, we've provided this.

Dyson-Schwinger Equations

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?
- $\Rightarrow$ Understanding InfraRed (long-range)
  ..... behaviour of $\alpha_s(Q^2)$

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory
  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking
        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement
        – Coloured objects not detected, not detectable?
- Method yields Schwinger Functions $\equiv$ Propagators

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory

  ..... Materially Reduces Model Dependence
- NonPerturbative, Continuum approach to QCD
  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking

        – Generation of fermion mass from *nothing*
      - Quark & Gluon Confinement

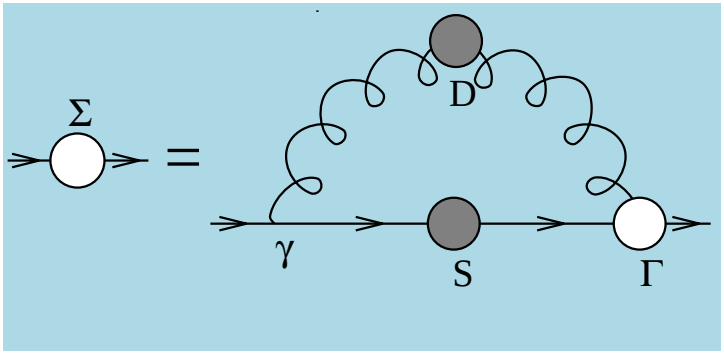
        – Coloured objects not detected, not detectable?

Cross-Sections built from Schwinger Functions

Persistent Challenge

Persistent Challenge

- Infinitely Many Coupled Equations



Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
  - Same VEVs measured in Lattice-QCD simulations

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
  - Same VEVs measured in Lattice-QCD simulations
- Coupling between equations necessitates truncation

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
  - Same VEVs measured in Lattice-QCD simulations
- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
  - Same VEVs measured in Lattice-QCD simulations
- Coupling between equations necessitates truncation
  - Weak coupling expansion $\Rightarrow$ Perturbation Theory Not useful for the nonperturbative problems in which we're interested

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
  - Same VEVs measured in Lattice-QCD simulations
- We introduced a systematic nonperturbative, symmetry-preserving truncation scheme *Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*, A. Bender, C. D. Robert and L. von Smekal, Phys. Lett. B **380** (1996) 7

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
  - Same VEVs measured in Lattice-QCD simulations
- We introduced a systematic nonperturbative, symmetry-preserving truncation scheme *Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*, A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
- Has Enabled Proof of EXACT Results in QCD

Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
  - Same VEVs measured in Lattice-QCD simulations
- We introduced a systematic nonperturbative, symmetry-preserving truncation scheme *Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*, A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
- Has Enabled Proof of EXACT Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Illustrate Exact Results

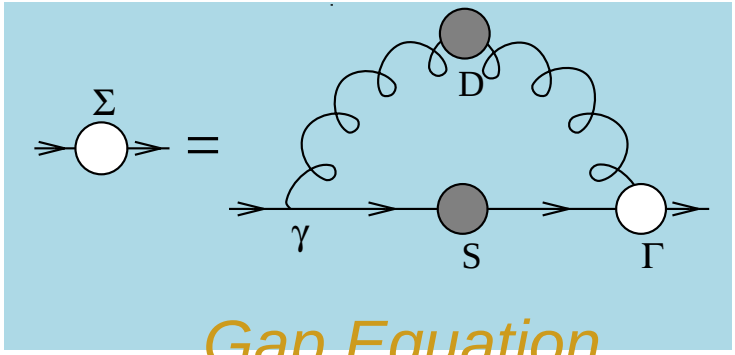
Persistent Challenge

- Infinitely Many Coupled Equations
  - Solutions are Schwinger Functions (Euclidean Green Functions)
  - Same VEVs measured in Lattice-QCD simulations
- We introduced a systematic nonperturbative, symmetry-preserving truncation scheme *Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation*, A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
- Has Enabled Proof of EXACT Results in QCD
- And Formulation of Practical Phenomenological Tool to
  - Make Predictions with Readily Quantifiable Errors

Dressed-Quark Propagator

Dressed-Quark Propagator

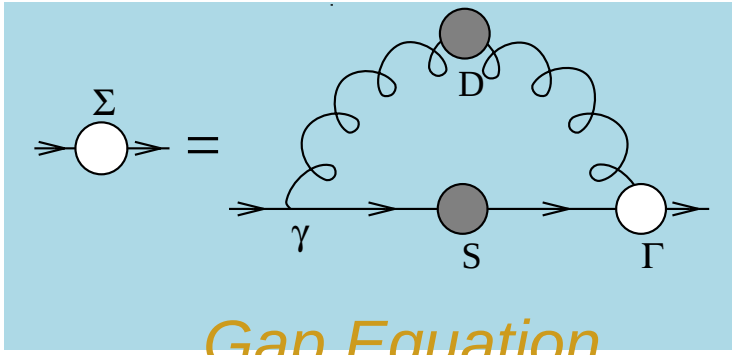
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

Dressed-Quark Propagator

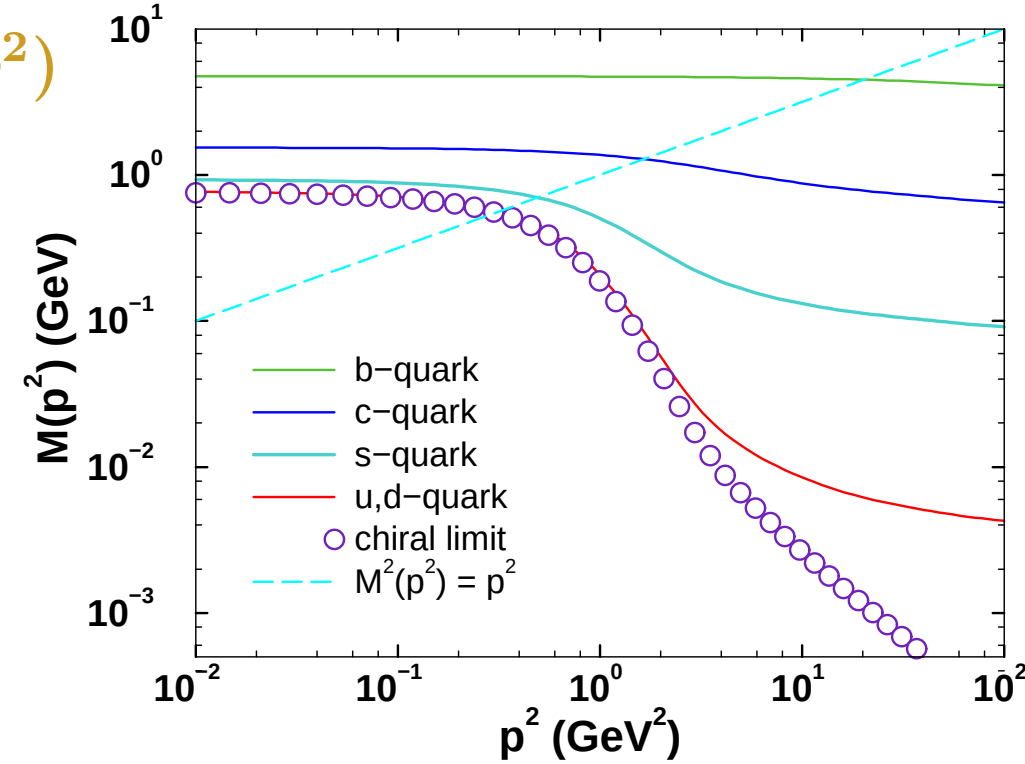
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

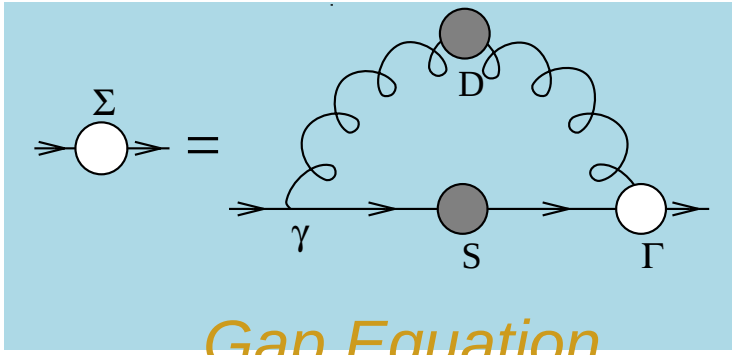
- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$



Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



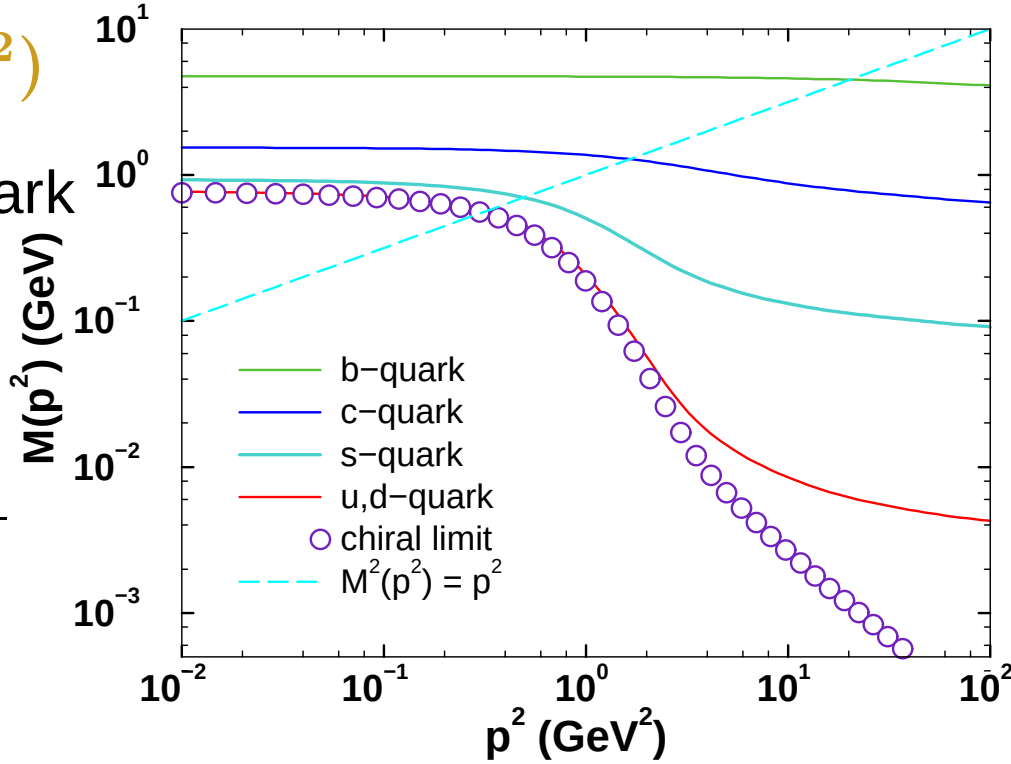
Gap Equation

- Gap Equation's Kernel Enhanced on IR domain

$\Rightarrow$ IR Enhancement of $M(p^2)$

- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



Dressed-Quark Propagator

- Longstanding Prediction of Dyson-Schwinger Equation Studies

Dressed-Quark Propagator

- Longstanding Prediction of Dyson-Schwinger Equation Studies
  - E.g., *Dyson-Schwinger equations and their application to hadronic physics*, C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477

Dressed-Quark Propagator

- Long used as basis for efficacious hadron physics phenomenology

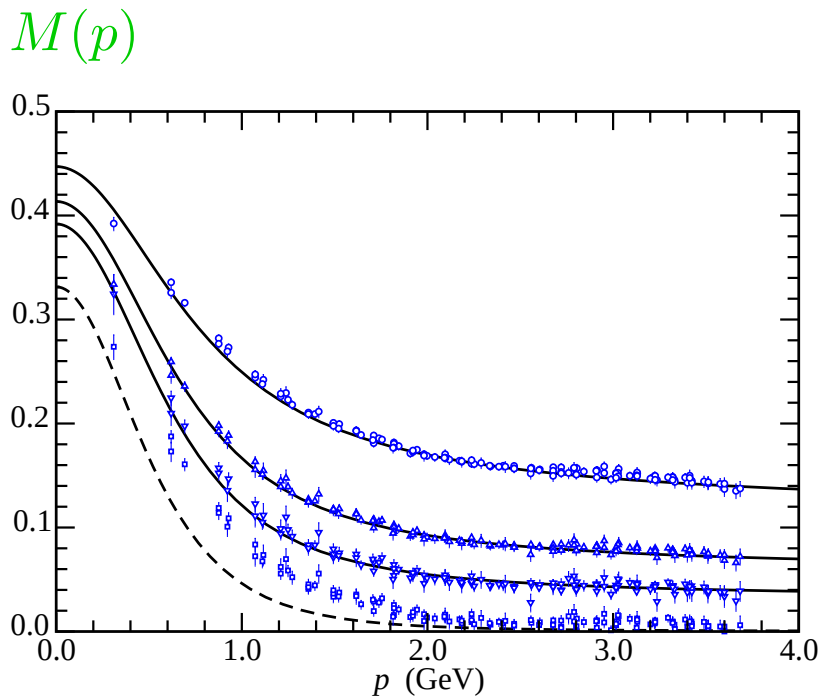
- Longstanding Prediction of Dyson-Schwinger Equation Studies
  - E.g., *Dyson-Schwinger equations and their application to hadronic physics*, C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477

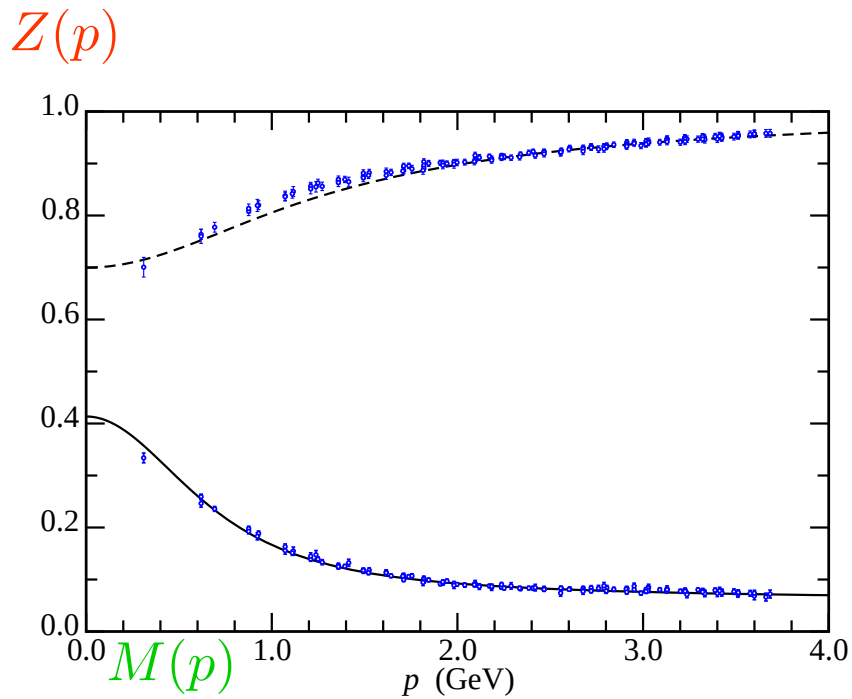
Dressed-Quark Propagator

- Long used as basis for efficacious hadron physics phenomenology
  - *Electromagnetic pion form-factor and neutral pion decay width*, C. D. Roberts, Nucl. Phys. A **605** (1996) 475

- Longstanding Prediction of Dyson-Schwinger Equation Studies
  - E.g., *Dyson-Schwinger equations and their application to hadronic physics*, C. D. Roberts and A. G. Williams, Prog. Part. Nucl. Phys. **33** (1994) 477

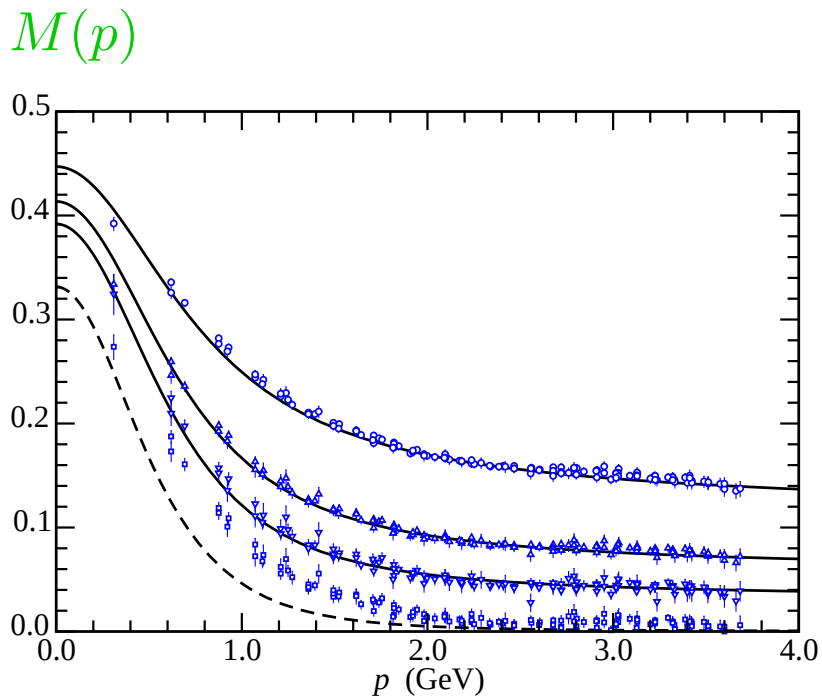
Quenched-QCD
Dressed-Quark Propagator

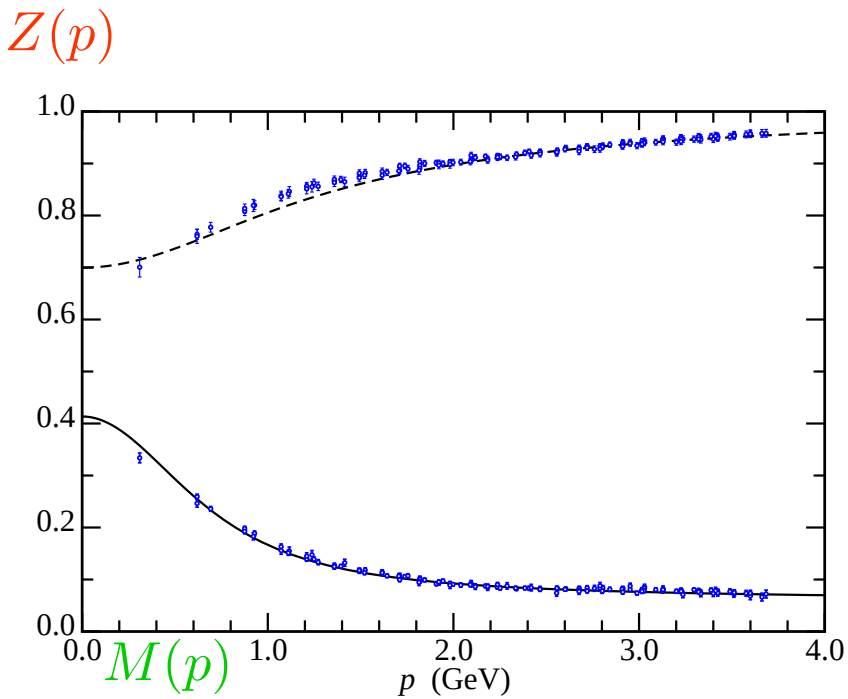




Quenched-QCD Dressed-Quark Propagator

2002

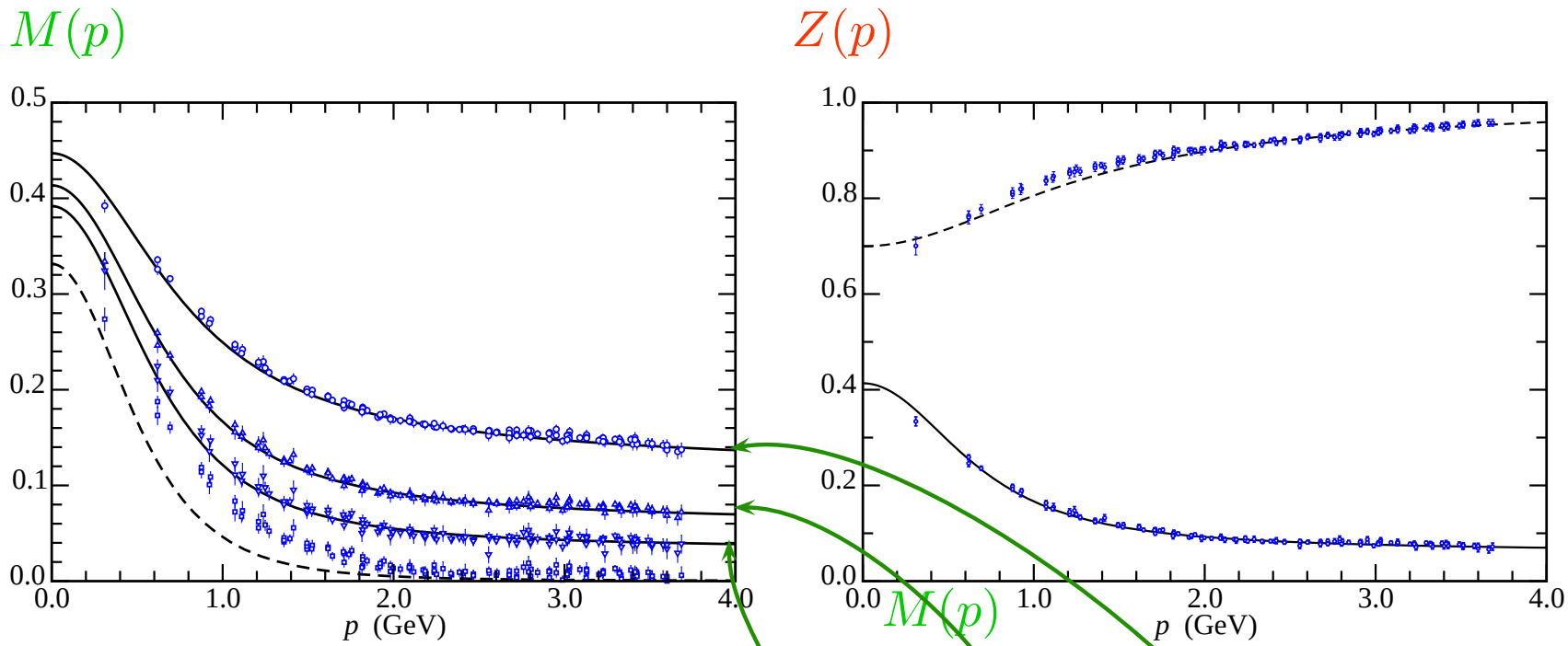




- "*data*:" Quenched Lattice Meas.
  - Bowman, Heller, Leinweber, Williams: he-lat/0209129

Quenched-QCD Dressed-Quark Propagator

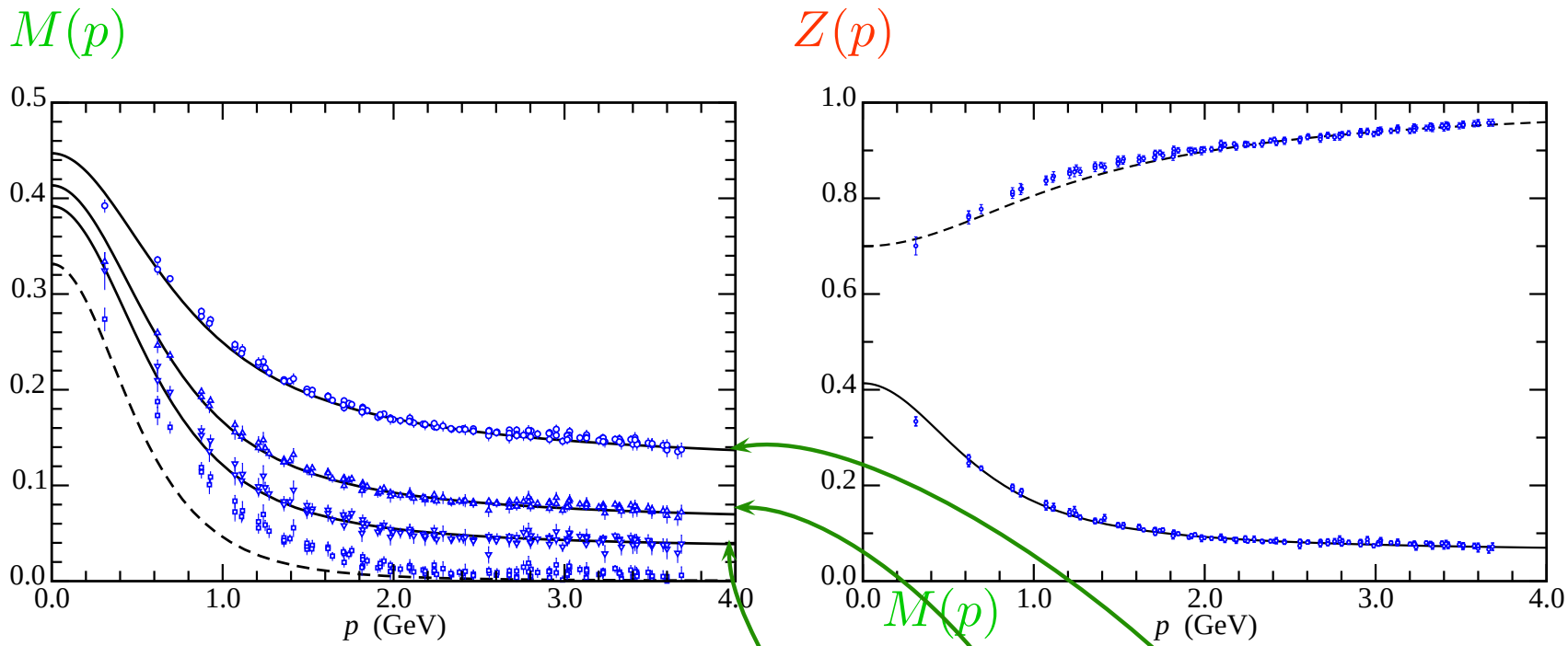
2002



- "*data*:" Quenched Lattice Meas.
  – Bowman, Heller, Leinweber, Williams: he-lat/0209129
  current-quark masses: 30 MeV, 50 MeV, 100 MeV

Quenched-QCD Dressed-Quark Propagator

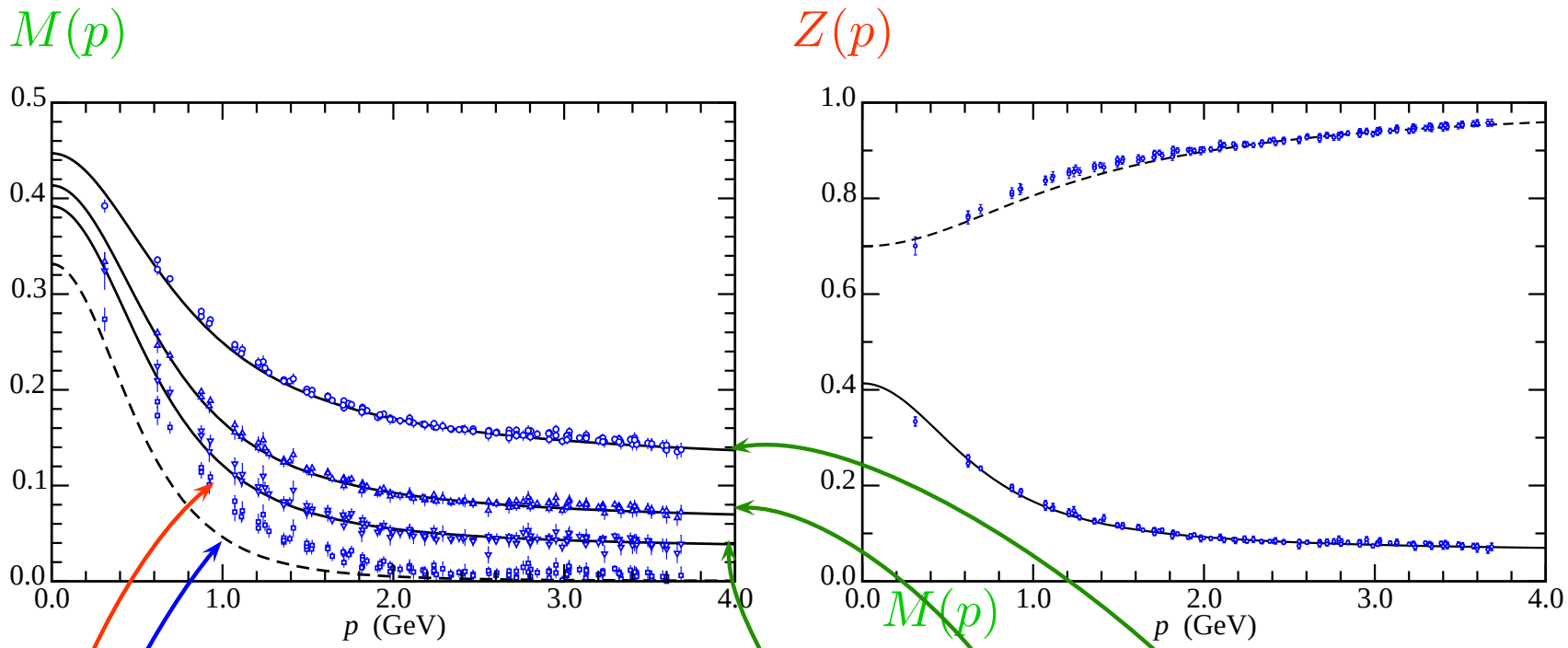
2002



- "*data*:" Quenched Lattice Meas.
  – Bowman, Heller, Leinweber, Williams: he-lat/0209129
  current-quark masses: 30 MeV, 50 MeV, 100 MeV
- *Curves*: Quenched DSE Cal.
  – Bhagwat, Pichowsky, Roberts, Tandy nu-th/0304003

Quenched-QCD Dressed-Quark Propagator

2002



- "*data*:" Quenched Lattice Meas.
  - – Bowman, Heller, Leinweber, Williams: he-lat/0209129
  - current-quark masses: 30 MeV, 50 MeV, 100 MeV
- *Curves*: Quenched DSE Cal.
  - – Bhagwat, Pichowsky, Roberts, Tandy nu-th/0304003
- Linear extrapolation of lattice data to chiral limit is inaccurate

QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
  - R. Alkofer and L. von Smekal, *The infrared behavior of QCD Green's functions* . . . , Phys. Rept. **353**, 281 (2001).

QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
  - R. Alkofer and L. von Smekal, *The infrared behavior of QCD Green's functions* . . . , Phys. Rept. **353**, 281 (2001).
- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
  - D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD Collaboration], *Asymptotic scaling and infrared behavior of the gluon propagator*, Phys. Rev. D **60**, 094507 (1999) [Erratum-ibid. D **61**, 079901 (2000)].
- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

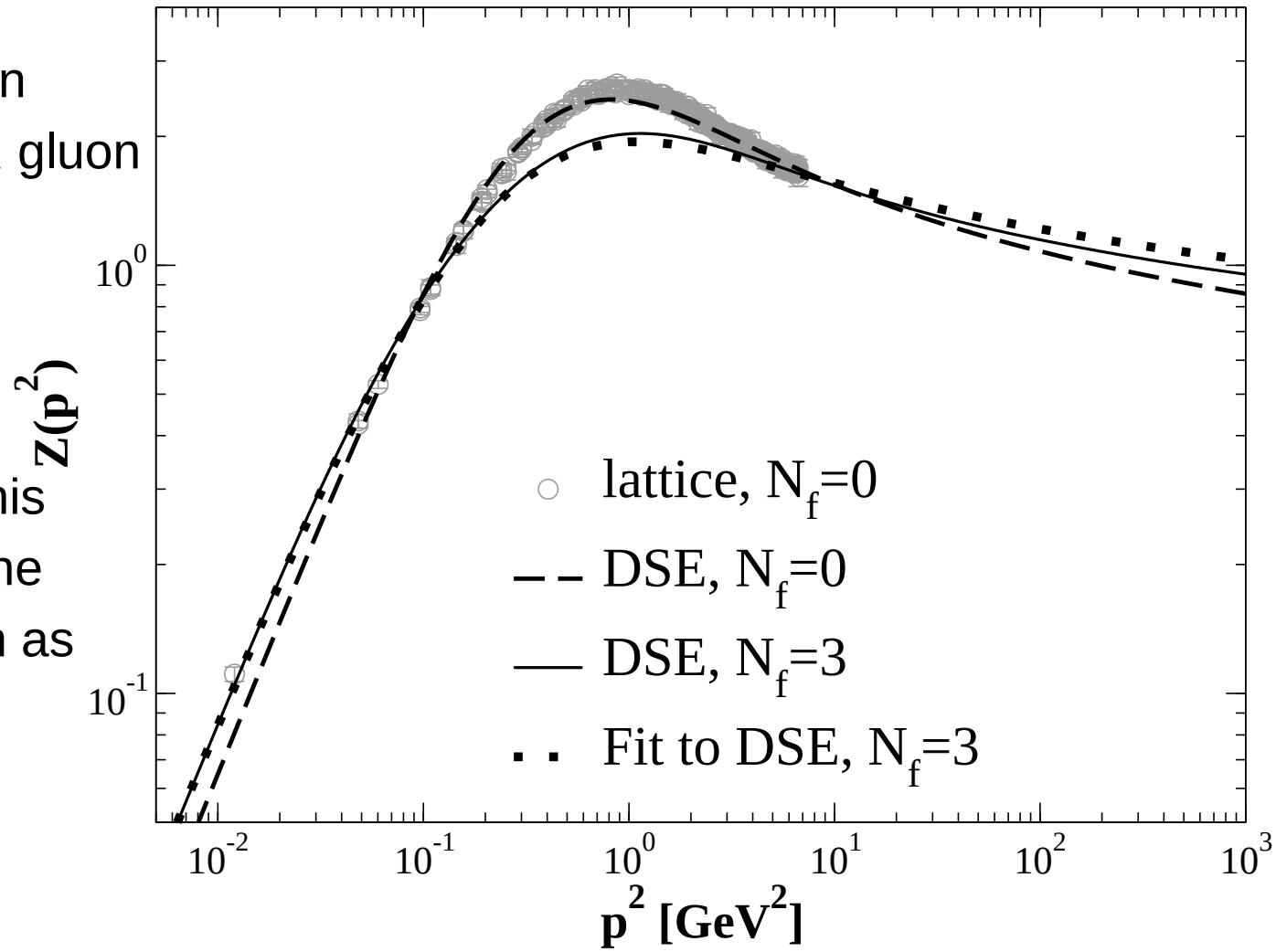
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



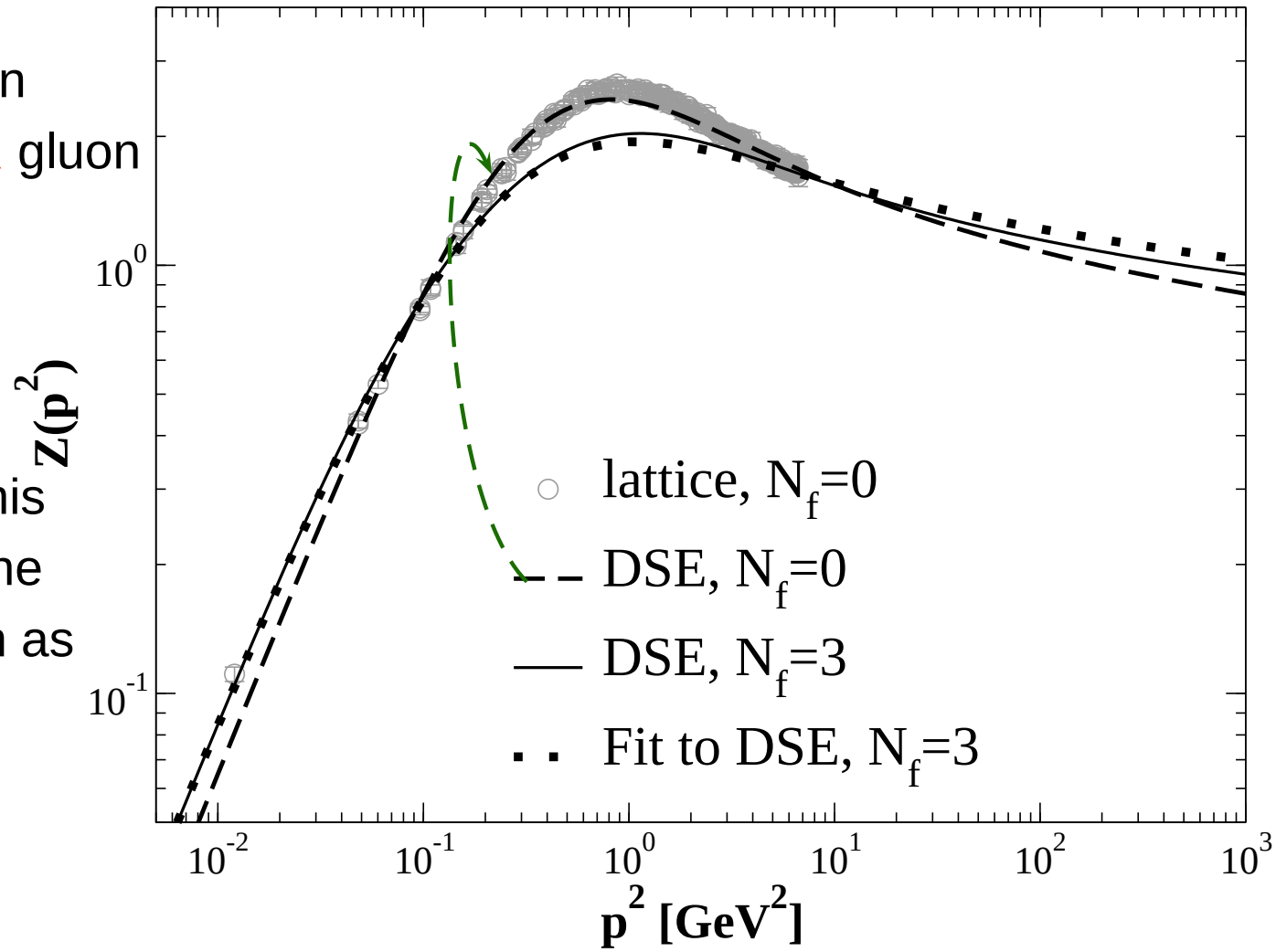
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



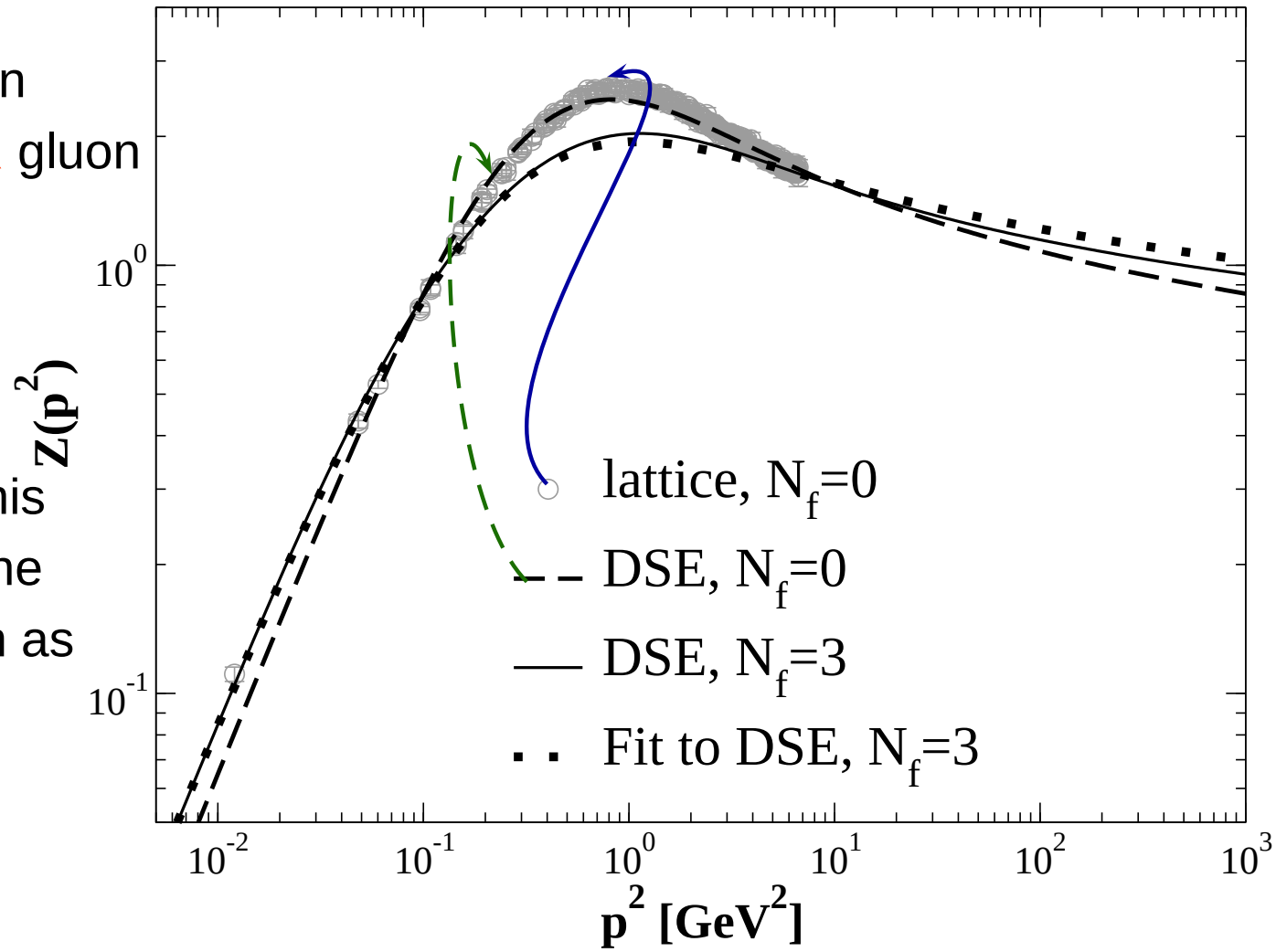
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx 1\,\mathrm{GeV}$

- Naturally, this scale has the same origin as Λ_{QCD}



Hadrons

- Established understanding
  of two- and three-point functions

Hadrons

- Established understanding of two- and three-point functions
- What about bound states?

Hadrons

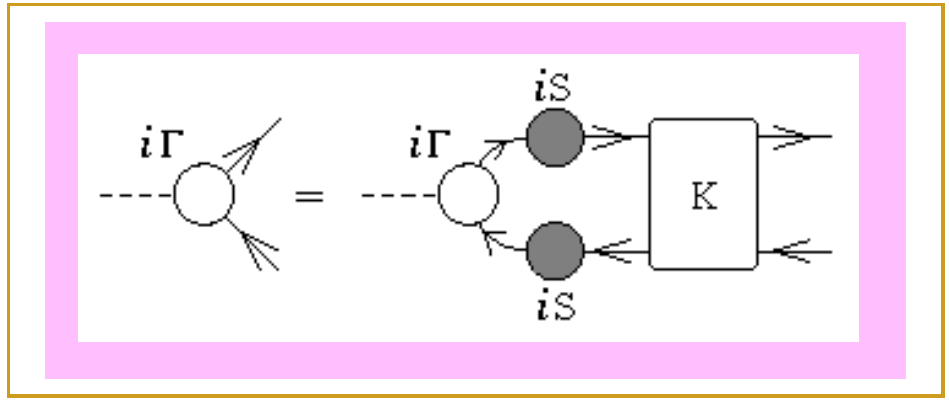
- Without bound states, Comparison with experiment is impossible

Hadrons

- Without bound states, Comparison with experiment is impossible
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

Hadrons

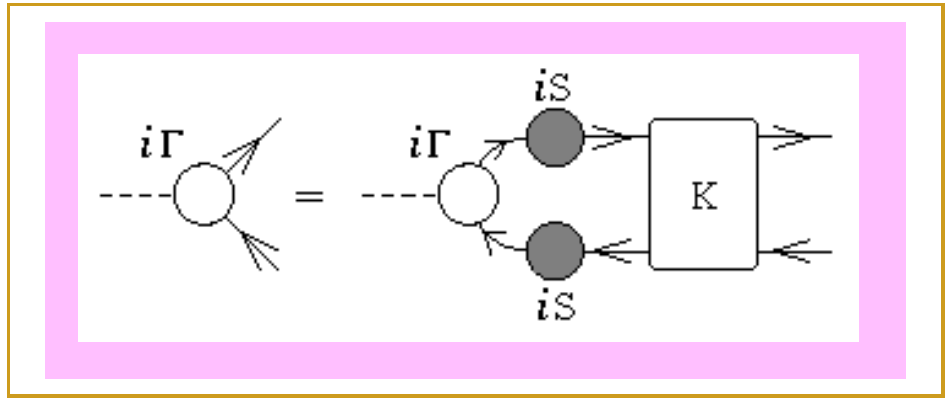
- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippman-Schwinger Equation.

Hadrons

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation

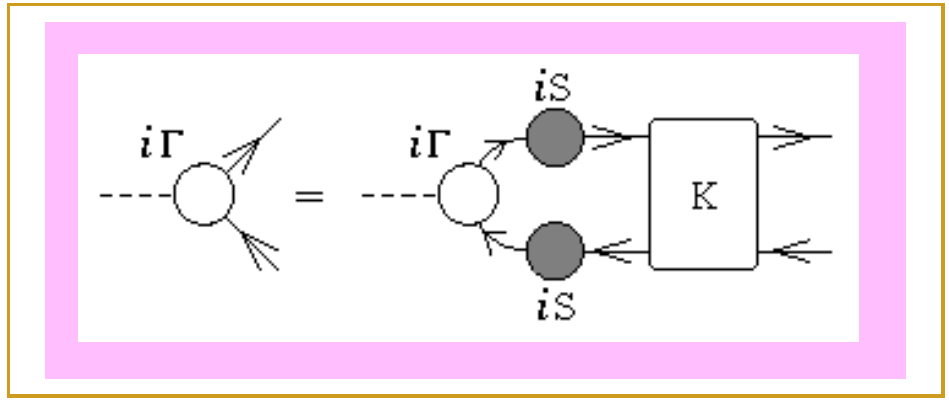


QFT Generalisation of Lippman-Schwinger Equation.

- What is the kernel, K?

Hadrons

- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation



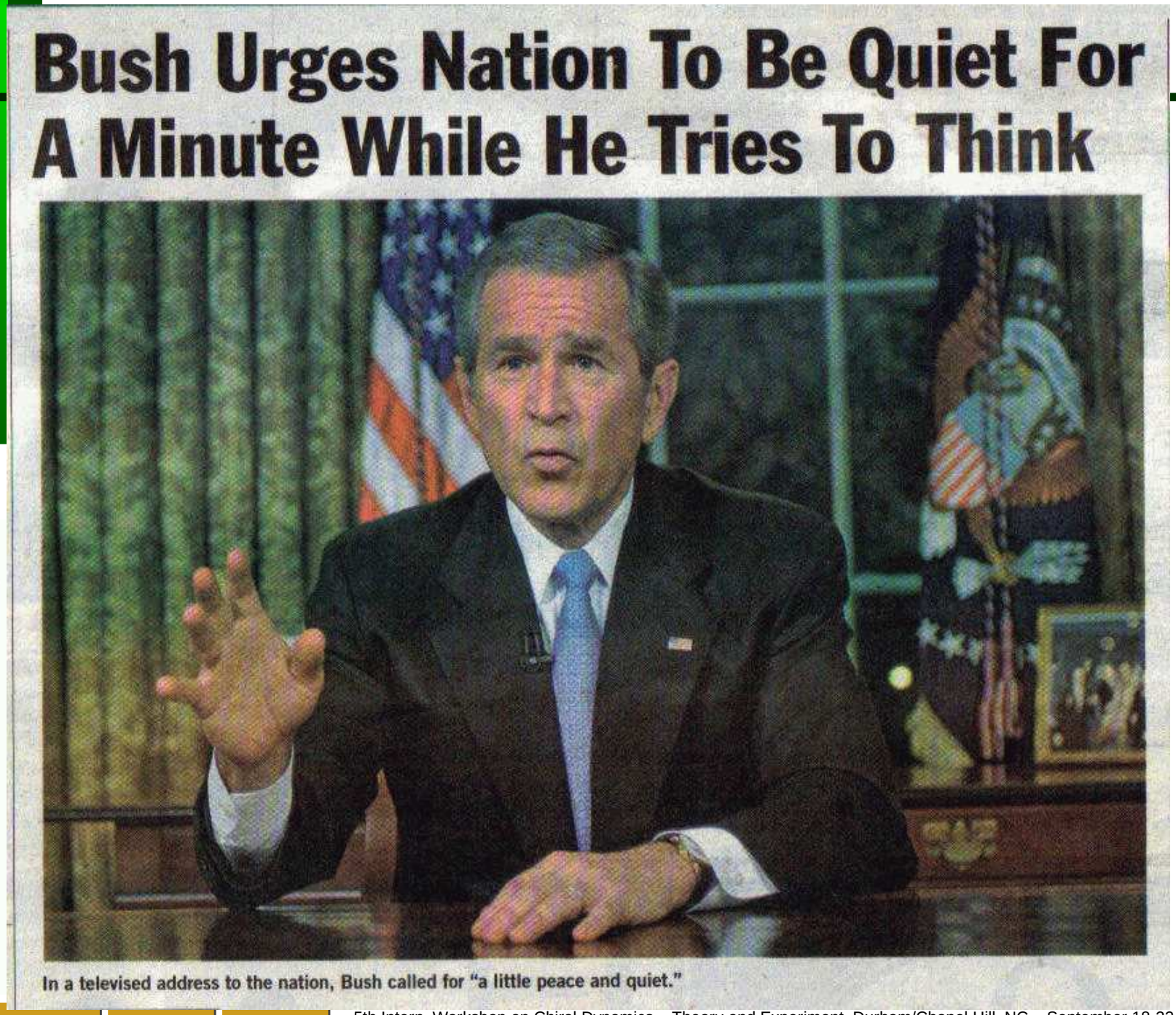
QFT Generalisation of Lippman-Schwinger Equation.

- What is the kernel, K?

or What is the long-range potential in QCD?

Chiral Dynamics?

Chiral Dynamics?



Bush Urges Nation To Be Quiet For A Minute While He Tries To Think

In a televised address to the nation, Bush called for "a little peace and quiet."

Bethe-Salpeter Kernel

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels must be intimately related

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation
- Nontrivial constraint

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma^l_{5\mu}(k;P) = \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5\,\mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

Satisfies BSE Satisfies DSE

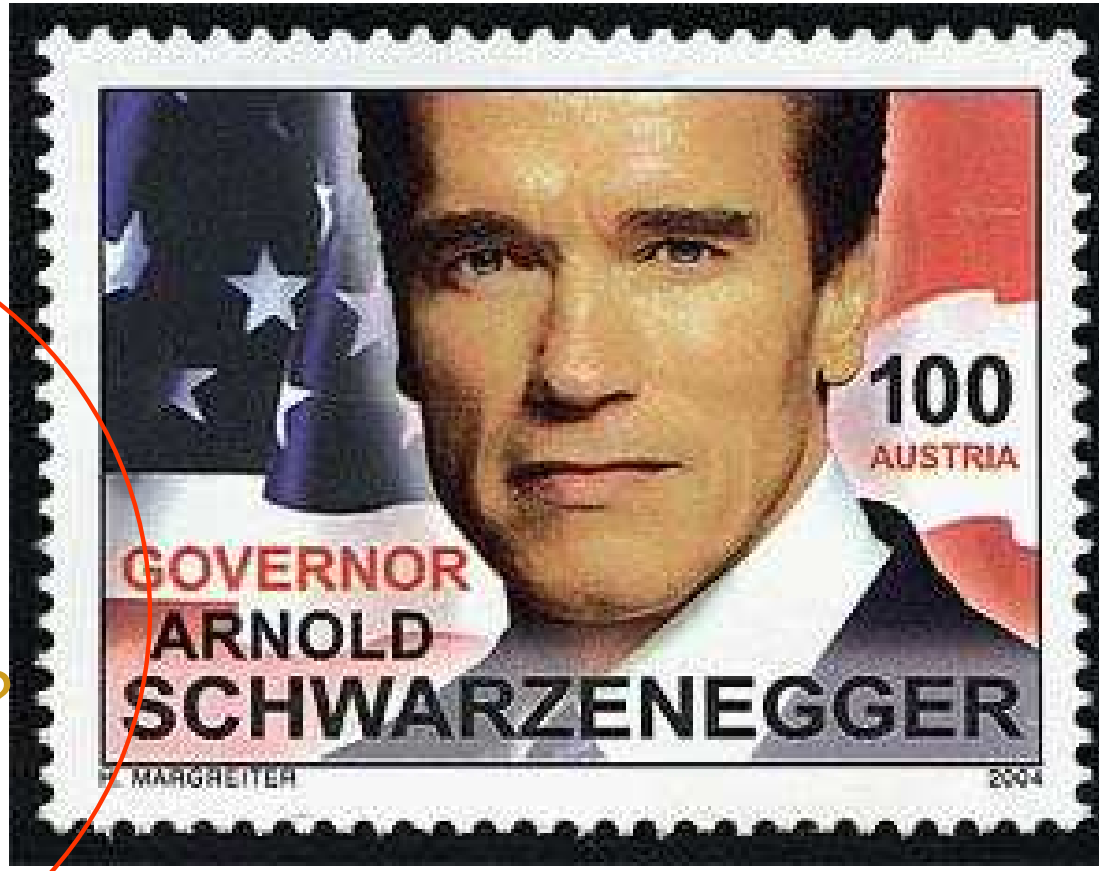
Kernels must be intimately related

- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Andreas Krassnigg

FWF “Erwin Schrödinger Fellow,” ANL 2003-2005

Almost Blood
Relative of Arnold
. . . Future
President?
. . . Executioner?



Radial Excitations & Chiral Symmetry

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

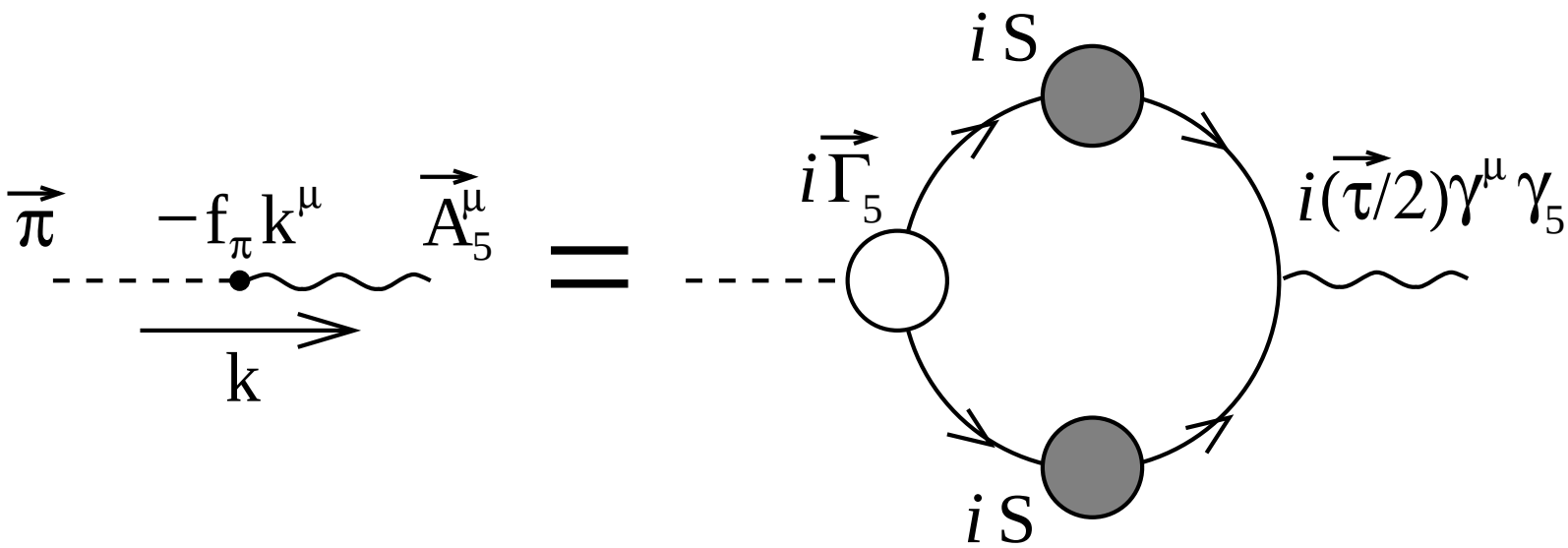
Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H\, m_H^2 = -\ \rho_\zeta^H\ \mathcal{M}_H$$

$$f_H\, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5\gamma_\mu\, \mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-) \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = -\rho_\zeta^H \mathcal{M}_H$$

$$i\rho_\zeta^H = Z_4 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5 \mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-) \right\}$$

- Pseudoscalar projection of BS wave function at $x = 0$

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ ...GMOR relation, a corollary

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $\boxed{m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q}$ ...GMOR relation, a corollary

- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $\boxed{m_H \propto m_q}$

... QCD Proof of Potential Model result

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson,
  $\boldsymbol{m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0}$, in chiral limit

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson,
  $\boldsymbol{m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0}$, in chiral limit
- $\Rightarrow \boldsymbol{f_H = 0}$
  ALL pseudoscalar mesons except $\boldsymbol{\pi(140)}$ in chiral limit

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons
- $\rho_H \Rightarrow$ finite, nonzero value in chiral limit, $\mathcal{M}_H \to 0$
- "radial" excitation of π-meson,
  $m^2_{\pi_{n\neq 0}} > m^2_{\pi_{n=0}} = 0$, in chiral limit
- $\Rightarrow f_H = 0$
  ALL pseudoscalar mesons except $\pi(140)$ in chiral limit
- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

Radial Excitations

Radial Excitations

- Spectrum contains 3 pseudoscalars [$I^G(J^P)L = 1^-(0^-)S$]

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

Radial Excitations

- Spectrum contains 3 pseudoscalars $[I^G(J^P)L = 1^-(0^-)S]$

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

- The Pion

- Consituent-Q Model: 1st three members of $\boldsymbol{n\,{}^1S_0}$ trajectory; i.e., ground state plus radial excitations?

Radial Excitations

- Spectrum contains 3 pseudoscalars $[I^G(J^P)L = 1^-(0^-)S]$

  masses below 2 GeV: $\pi(140)$; $\pi(1300)$; and $\pi(1800)$

- The Pion
- Consituent-Q Model: 1$^{\rm st}$ three members of $n\,^1S_0$ trajectory; i.e., ground state plus radial excitations?
- But $\pi(1800)$ is narrow ($\Gamma = 207 \pm 13$) & decay pattern might indicate some "flux tube angular momentum" content:
  $S_{\bar{Q}Q} = 1 \oplus L_F = 1 \Rightarrow J = 0$
  & $L_F = 1 \Rightarrow\ ^3S_1 \oplus\ ^3S_1$ ($\bar{Q}Q$) decays suppressed?

Radial Excitations

- Spectrum contains 3 pseudoscalars $[I^G(J^P)L = 1^-(0^-)S]$

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

- The Pion
- Consituent-Q Model: 1st three members of $\boldsymbol{n\,^1S_0}$ trajectory; i.e., ground state plus radial excitations?
- But $\boldsymbol{\pi(1800)}$ is narrow $(\boldsymbol{\Gamma = 207 \pm 13})$ & decay pattern might indicate some "flux tube angular momentum" content:
- Radial excitations & Hybrids & Exotics $\Rightarrow$ Long-range radial wave functions $\Rightarrow$ sensitive to confinement

Radial Excitations

- Spectrum contains 3 pseudoscalars $[I^G(J^P)L = 1^-(0^-)S]$

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

- The Pion
- Consituent-Q Model: 1st three members of $\boldsymbol{n\,^1S_0}$ trajectory; i.e., ground state plus radial excitations?
- But $\boldsymbol{\pi(1800)}$ is narrow $(\boldsymbol{\Gamma = 207 \pm 13})$ & decay pattern might indicate some "flux tube angular momentum" content:
- Radial excitations & Hybrids & Exotics $\Rightarrow$ Long-range radial wave functions $\Rightarrow$ sensitive to confinement
- NSAC Long-Range Plan, 2002: . . . an understanding of confinement "remains one of the greatest intellectual challenges in physics"

Radial Excitations & Chiral Symmetry

Radial Excitations & Chiral Symmetry

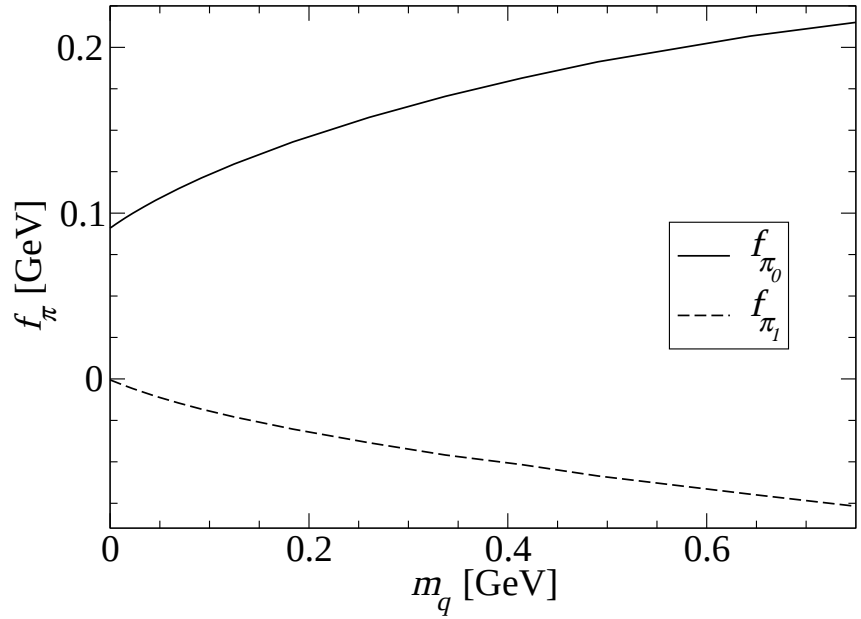
Höll, Krassnigg, Roberts
nu-th/0406030

- Fundamental properties of QCD

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

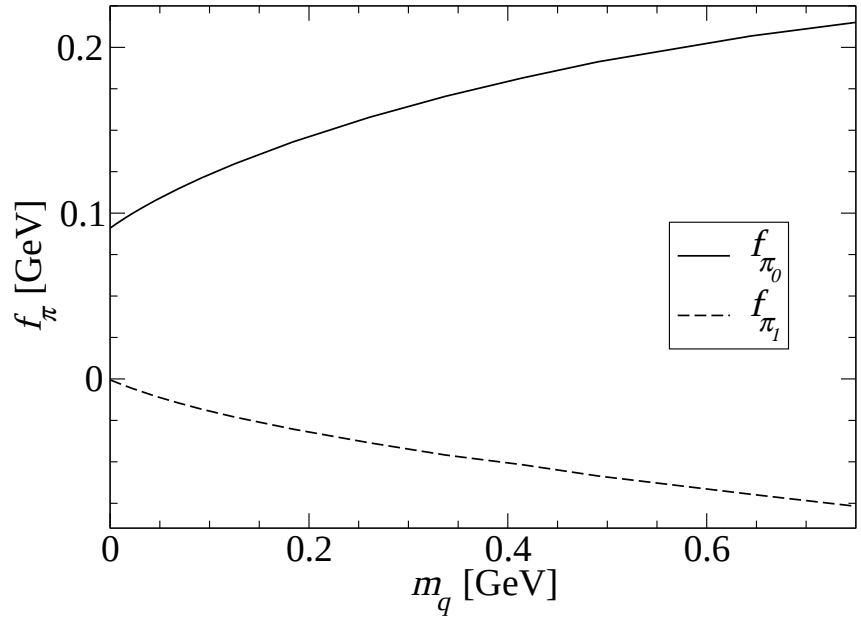
- Fundamental properties of QCD
  - If chiral symmetry is dynamically broken, then in the chiral limit every pseudoscalar meson is blind to the weak interaction *except* $\pi(140)$.



Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

- Fundamental properties of QCD
  - If chiral symmetry is dynamically broken, then in the chiral limit every pseudoscalar meson is blind to the weak interaction *except* $\pi(140)$.
  - If chiral symmetry is not broken, then *NO pseudoscalar meson experiences the weak interaction.*



Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.

Radial Excitations & Lattice-QCD

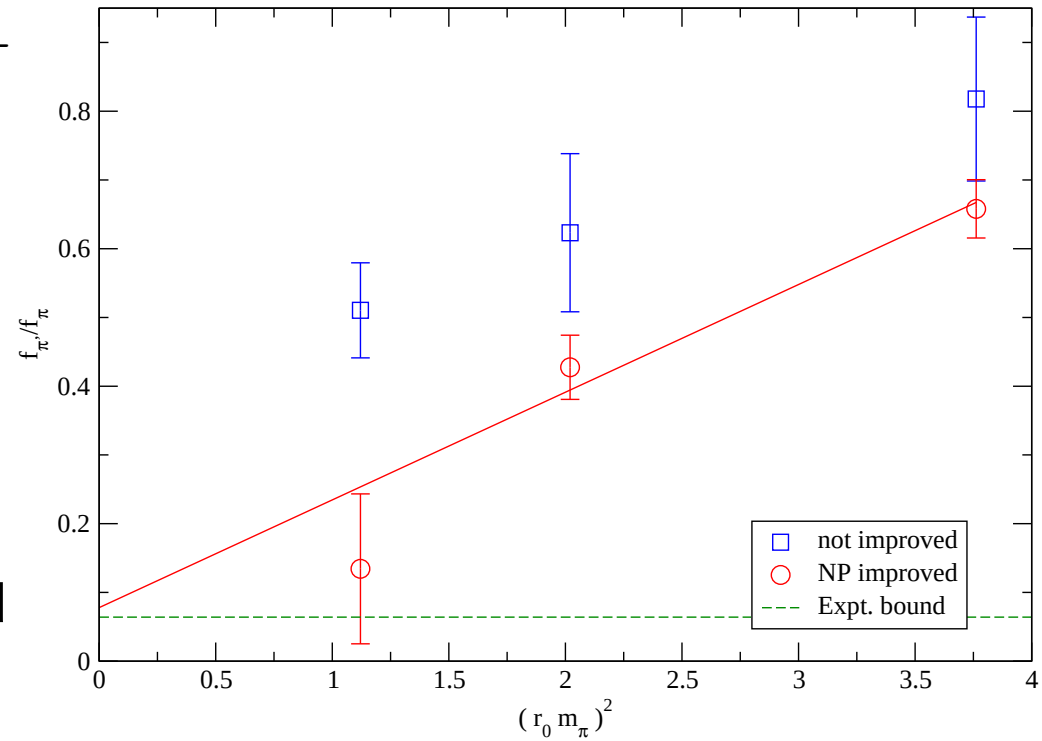
McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

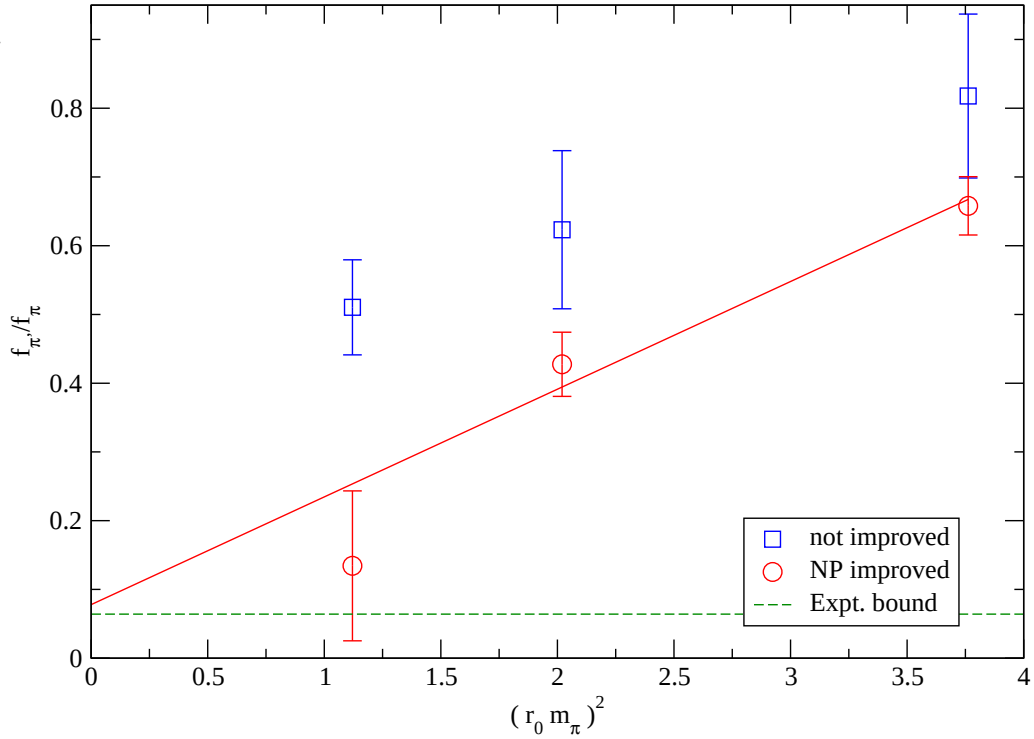
- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched
  $\Rightarrow \dfrac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$



Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable"*.
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,$MeV
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,$fm,
  two-flavour, unquenched
  $\Rightarrow \dfrac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$

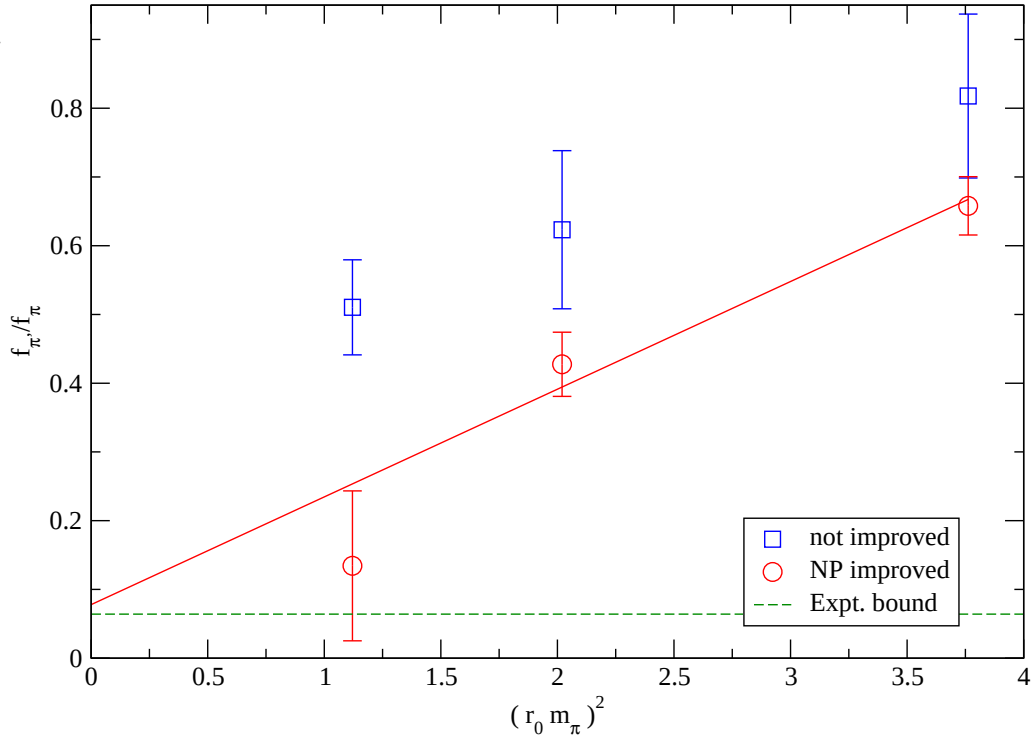


- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of "that is remarkable" and "unbelievable".*
- CLEO: $\tau \to \pi(1300) + \nu_\tau$
  $\Rightarrow f_{\pi_1} < 8.4\,\text{MeV}$
  Diehl & Hiller
  he-ph/0105194
- Lattice-QCD check:
  $16^3 \times 32$,
  $a \sim 0.1\,\text{fm}$,
  two-flavour, unquenched
  $\Rightarrow \dfrac{f_{\pi_1}}{f_\pi} = 0.078\,(93)$



- *The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.*

Are we there yet?

Nucleon Properties

Nucleon Properties

- Maris & Tandy . . . series of five papers . . . excellent description of light pseudoscalar and vector mesons . . . basket of 31 masses/couplings/radii with r.m.s. error of 15% . . . moreover, prediction of $F_\pi(Q^2)$ measured in Hall A.

Pieter Maris

Peter Tandy

Nucleon Properties

- Maris & Tandy . . . series of five papers . . . excellent description of light pseudoscalar and vector mesons . . . basket of 31 masses/couplings/radii with r.m.s. error of 15% . . . moreover, prediction of $F_\pi(Q^2)$ measured in Hall A.
- One parameter model . . . parameter specifies long-range interaction between light-quarks . . . model-independent results in ultraviolet

Nucleon Properties

- Maris & Tandy . . . series of five papers . . . excellent description of light pseudoscalar and vector mesons . . . basket of 31 masses/couplings/radii with r.m.s. error of 15% . . . moreover, prediction of $F_\pi(Q^2)$ measured in Hall A.
- One parameter model . . . parameter specifies long-range interaction between light-quarks . . . model-independent results in ultraviolet
- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks

Nucleon Properties

- Another Direction . . . Also want/need information about three-quark systems

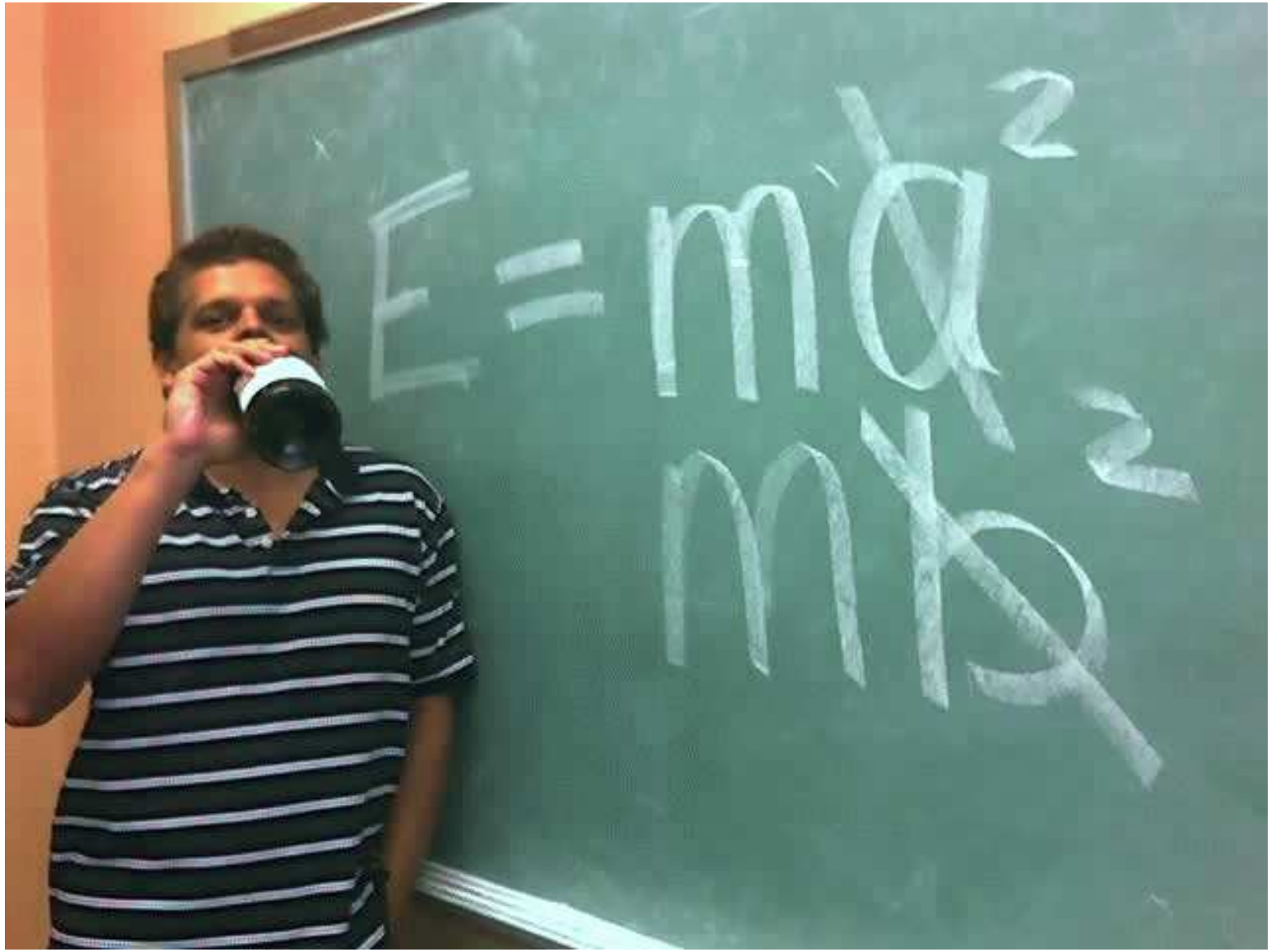
Nucleon Properties

- Another Direction . . . Also want/need information about three-quark systems
- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

Nucleon Properties

- Another Direction . . . Also want/need information about three-quark systems
- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.

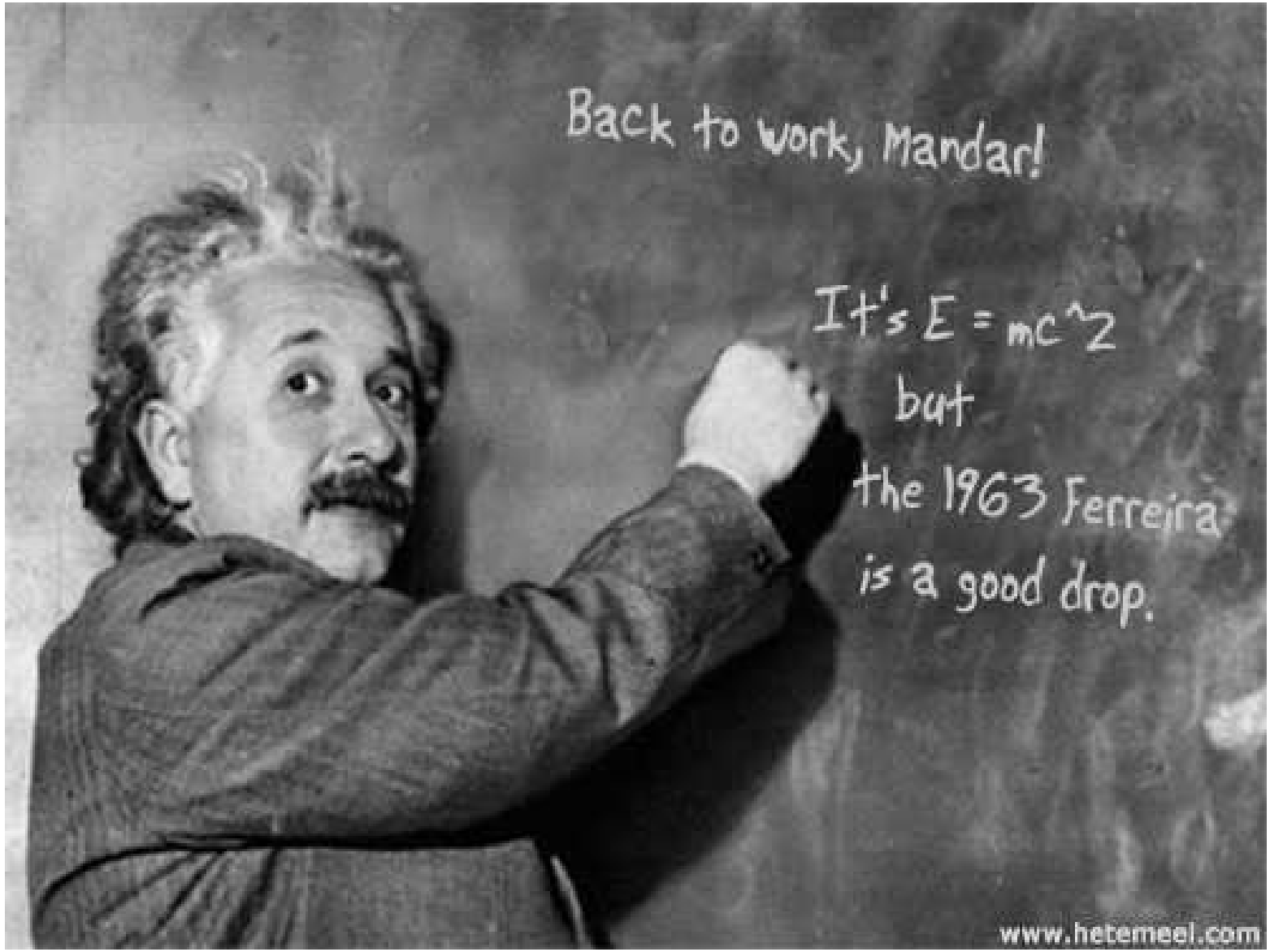
Mandar Bhagwat



Emulating supervisor's approach to physics

Mandar Bhagwat

Mandar Bhagwat



Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

- But is that good?

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

- But is that good?
  - Cloudy Bag: $\delta M_+^{\pi-\rm loop} = -300$ to -400 MeV!

Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons
  $\Rightarrow$ Covariant dressed-quark Faddeev Equation
- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

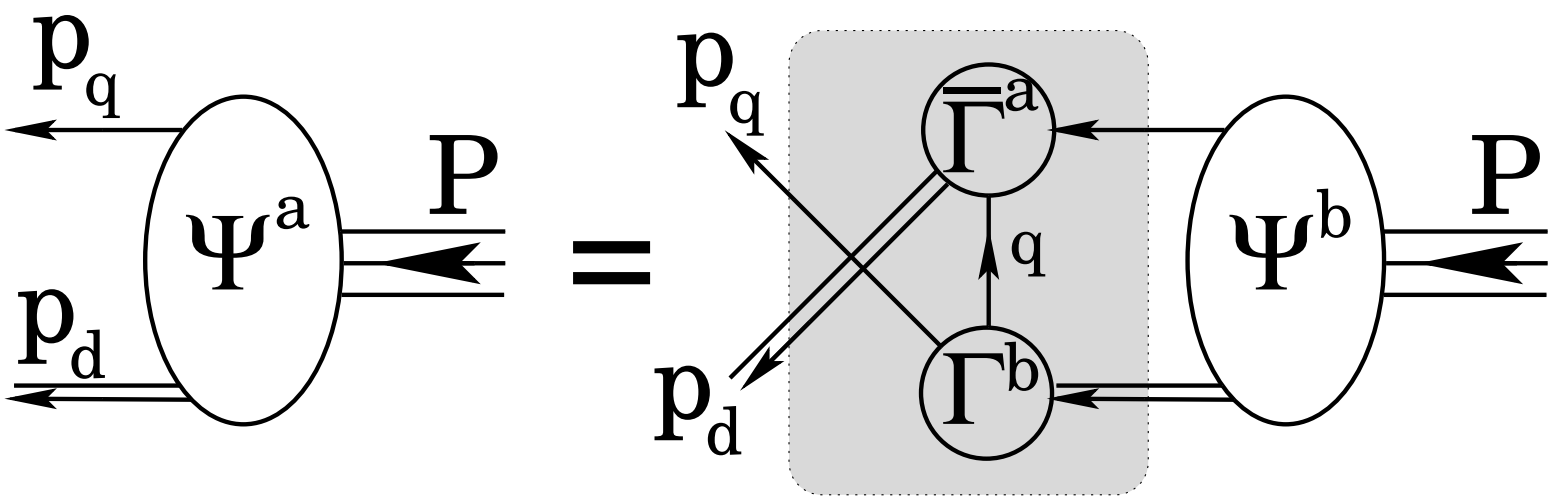
$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

- But is that good?
  - Cloudy Bag: $\delta M_+^{\pi-\rm loop} = -300$ to -400 MeV!
- Critical to anticipate pion cloud effects

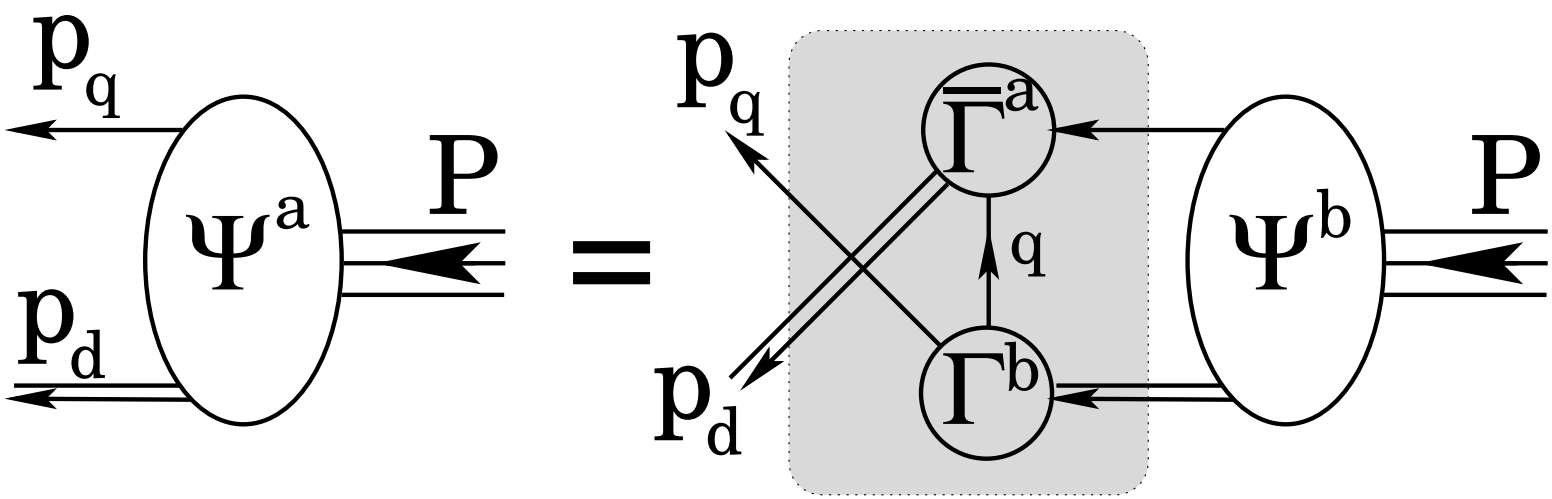
Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

Faddeev equation

Faddeev equation



Faddeev equation



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . . In Nucleon's Rest Frame Amplitude has . . . $s-$, $p-$ & $d-$wave correlations

Parametrising diquark properties

Parametrising diquark properties

- Dressed-quark . . . fixed by DSE and Meson Studies

  . . . Burden, Roberts, Thomson, Phys. Lett. **B 371**, 163 (1996)

Parametrising diquark properties

- Dressed-quark . . . fixed by DSE and Meson Studies

  . . . Burden, Roberts, Thomson, Phys. Lett. **B 371**, 163 (1996)

- Non-pointlike scalar and pseudovector colour-antitriplet diquark correlations – described by
  - Bethe-Salpeter amplitudes . . . width for each – ω_{J^P}
  - Confining propagators . . . mass for each – m_{J^P}

Parametrising diquark properties

- Dressed-quark . . . fixed by DSE and Meson Studies

  . . . Burden, Roberts, Thomson, Phys. Lett. **B 371**, 163 (1996)

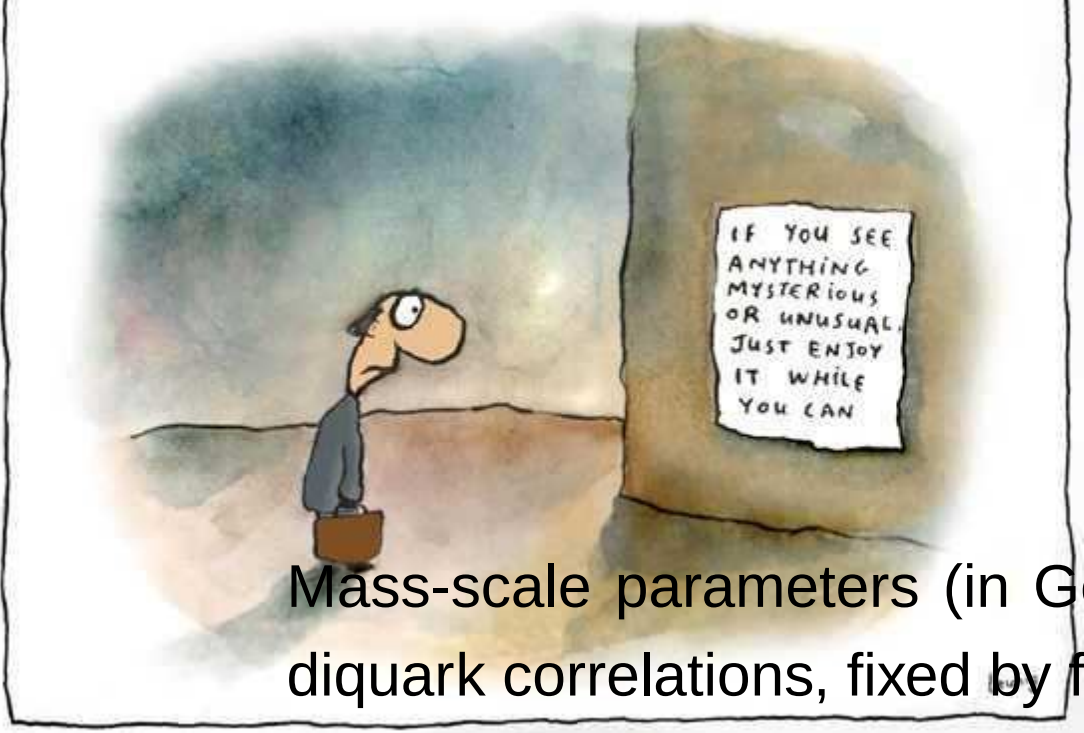
- Non-pointlike scalar and pseudovector colour-antitriplet diquark correlations – described by
  - Bethe-Salpeter amplitudes . . . width for each – ω_{J^P}
  - Confining propagators . . . mass for each – m_{J^P}

  Widths fixed by "asymptotic freedom" condition –

$$\frac{d}{dK^2}\left(\frac{1}{m_{J^P}^2}\,\mathcal{F}(K^2/\omega_{J^P}^2)\right)^{-1}\Bigg|_{K^2=0} = 1 \;\Rightarrow\; \omega_{J^P}^2 = \frac{1}{2}\,m_{J^P}^2\,,$$

  Only two parameters; viz., diquark "masses": m_{J^P}

Results: Nucleon and Δ Masses



Results: Nucleon and Δ Masses

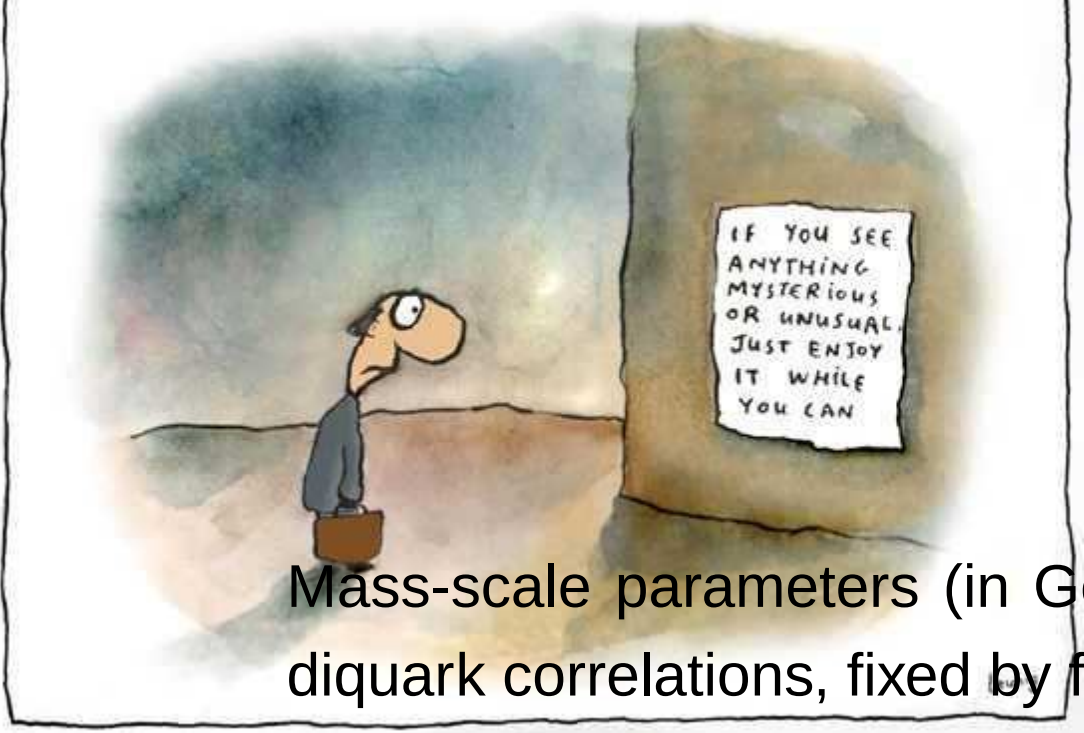
Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV



Results: Nucleon and Δ Masses

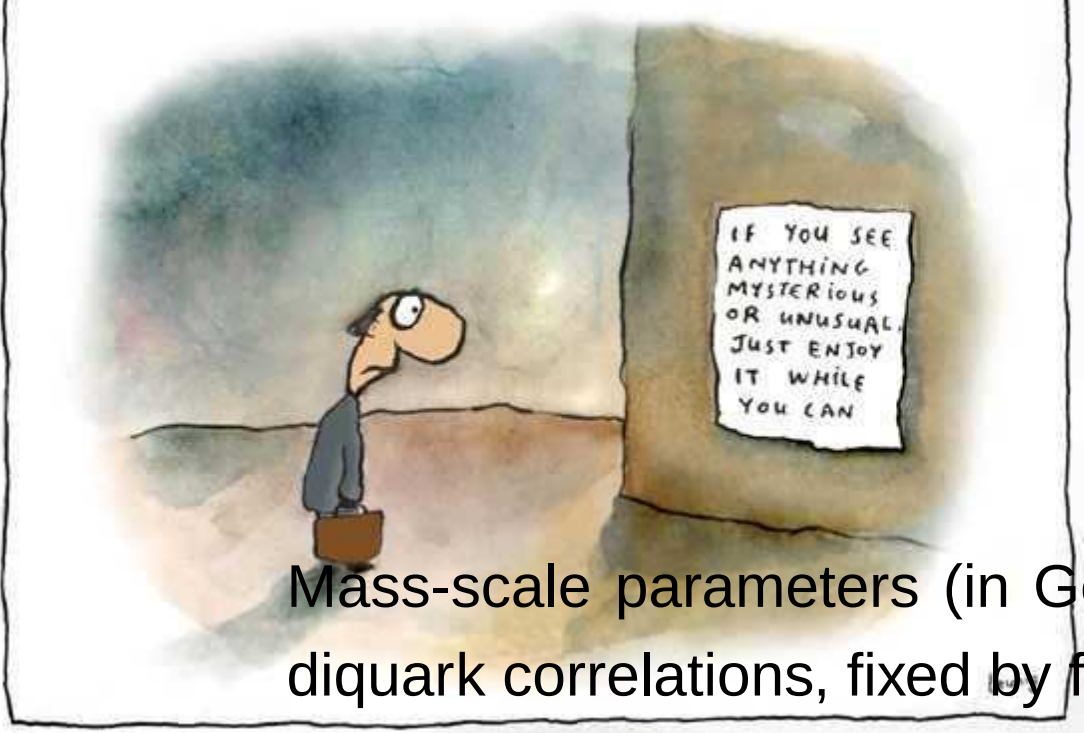
Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV
- Axial-vector diquark provides significant attraction



Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV
- Constructive Interference: 1^{++}-diquark $+ \partial_\mu \pi$

Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

for on-shell nucleons described by Faddeev Amplitude

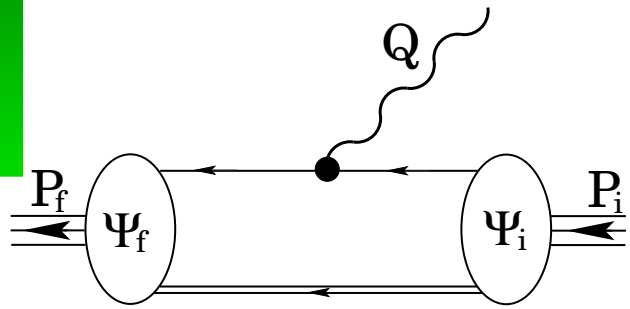
M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

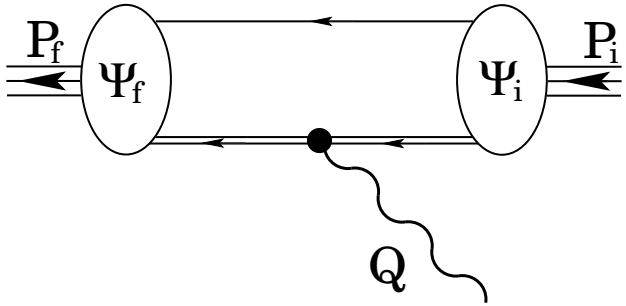
Nucleon-Photon Vertex

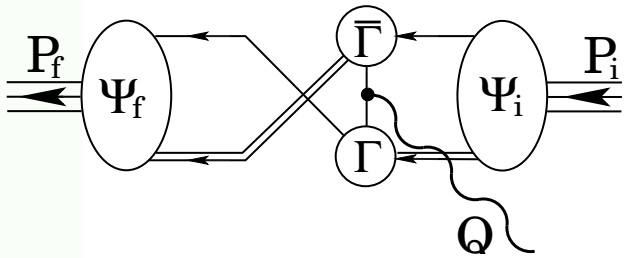
6 terms . . .

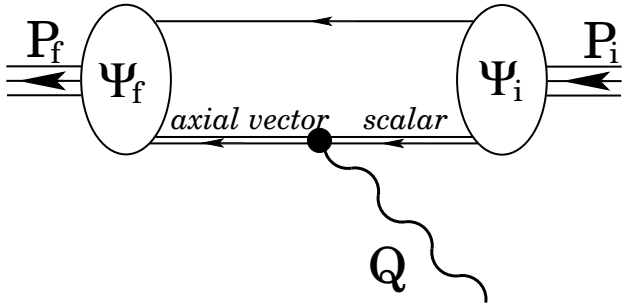
constructed systematically . . . current conserved automatically

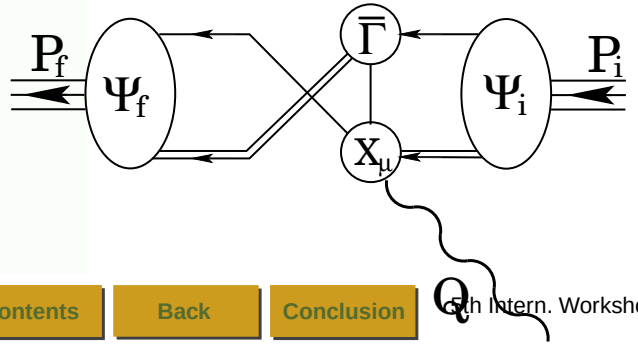
for on-shell nucleons described by Faddeev Amplitude

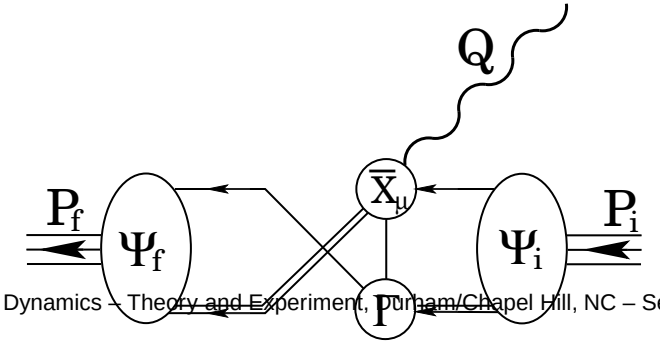




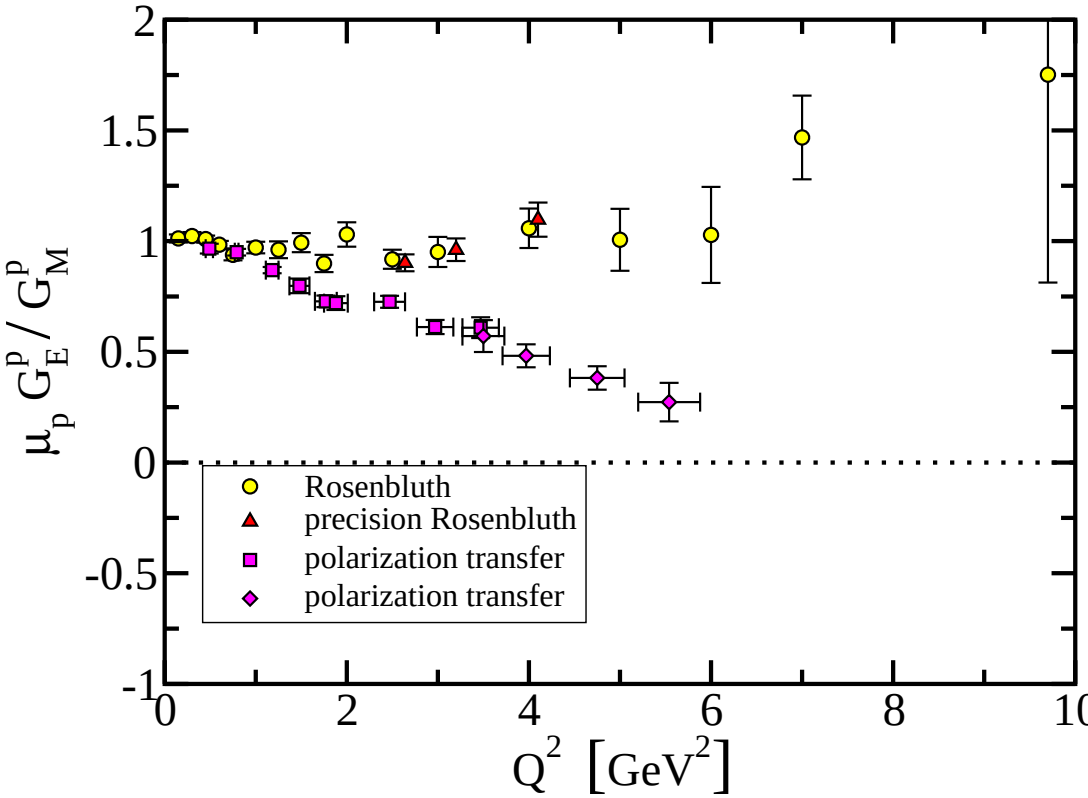






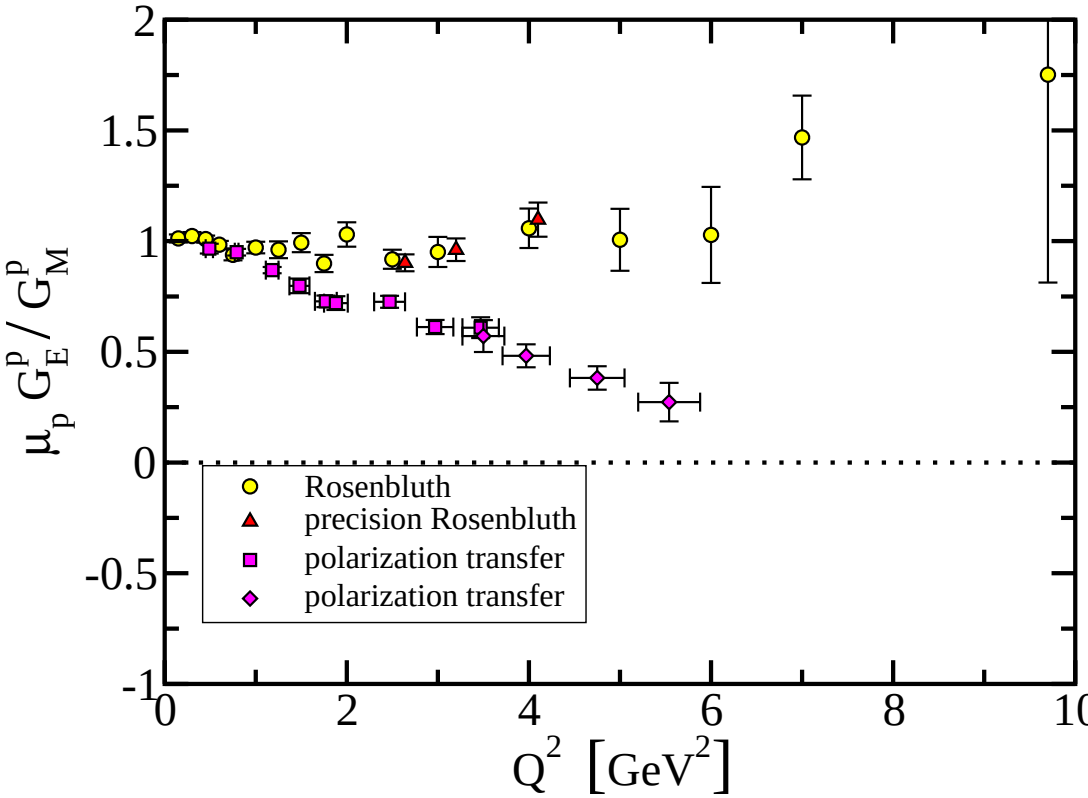


Form Factor Ratio: GE/GM



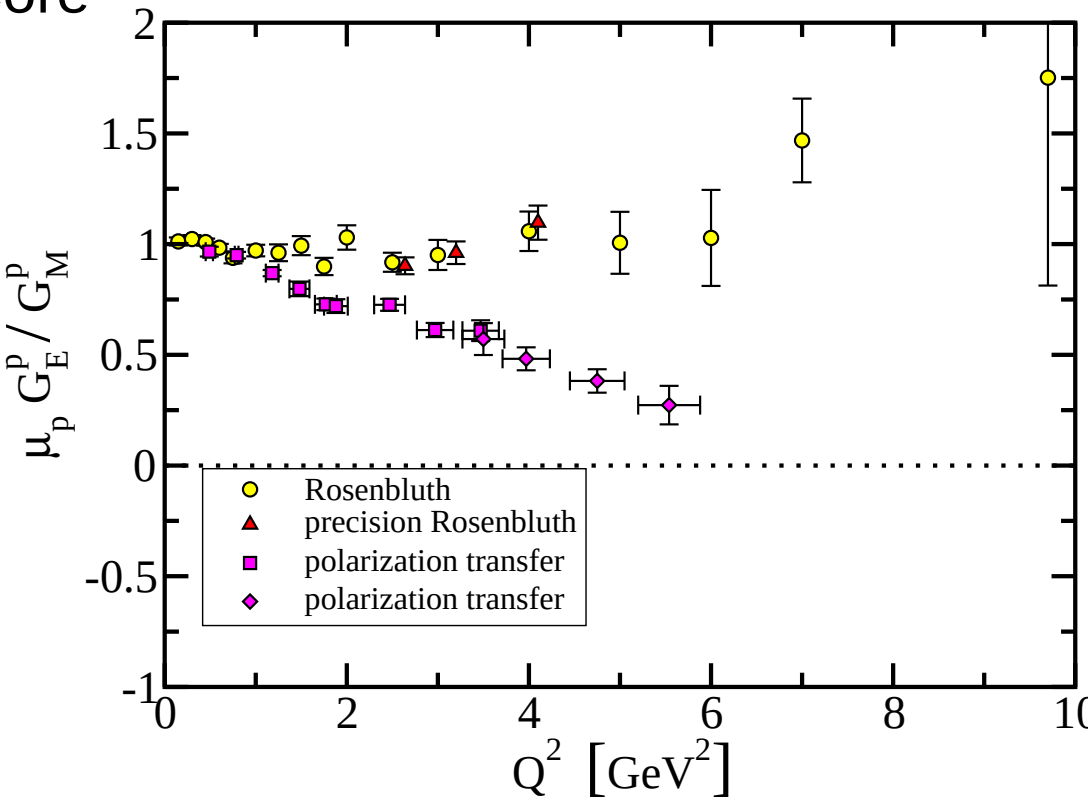
Form Factor Ratio: GE/GM

- Combine these elements . . .



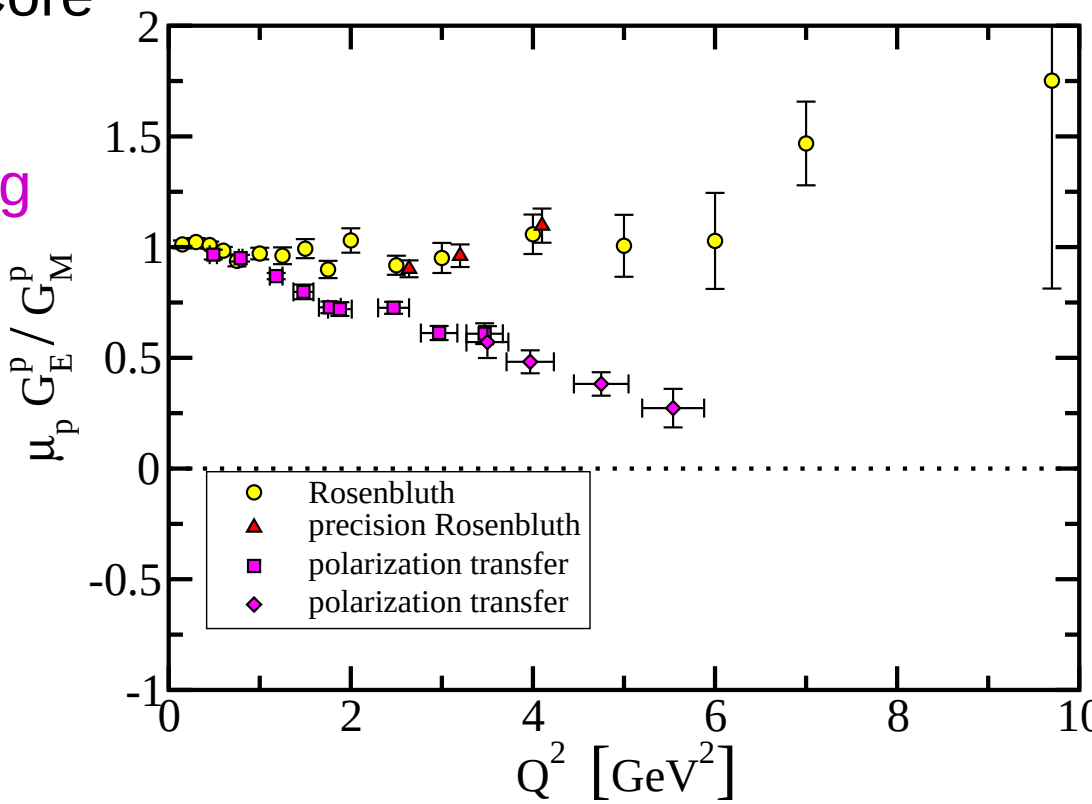
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core



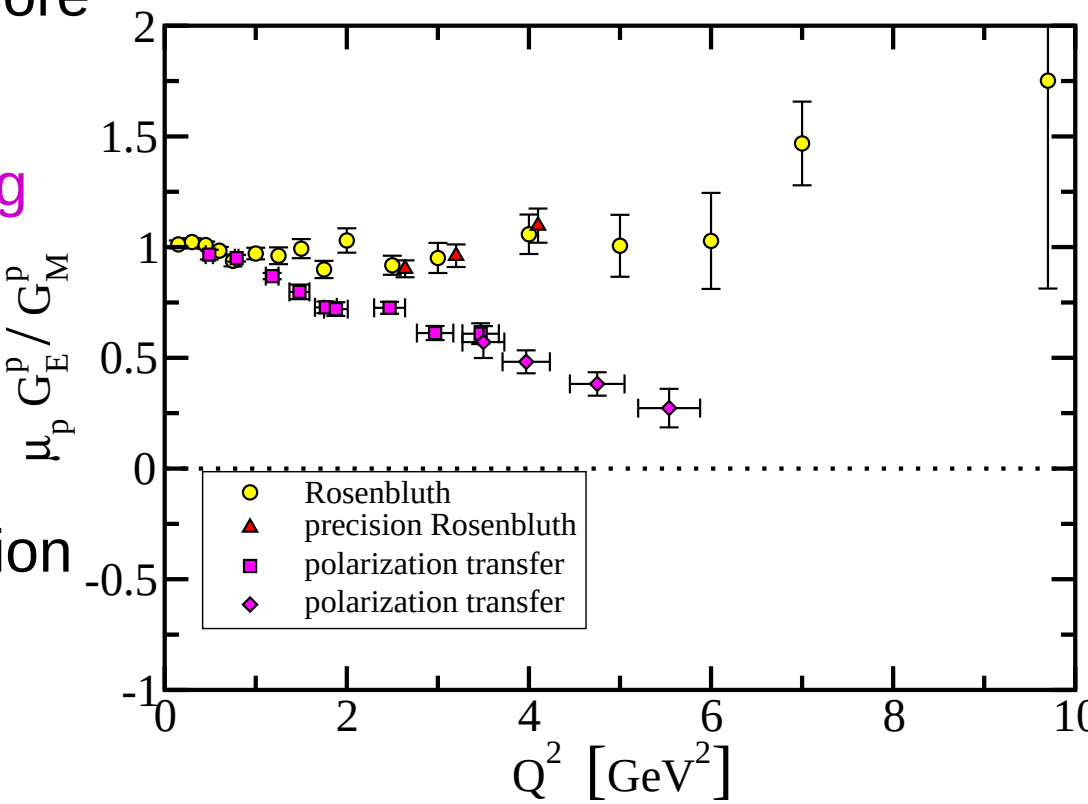
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current



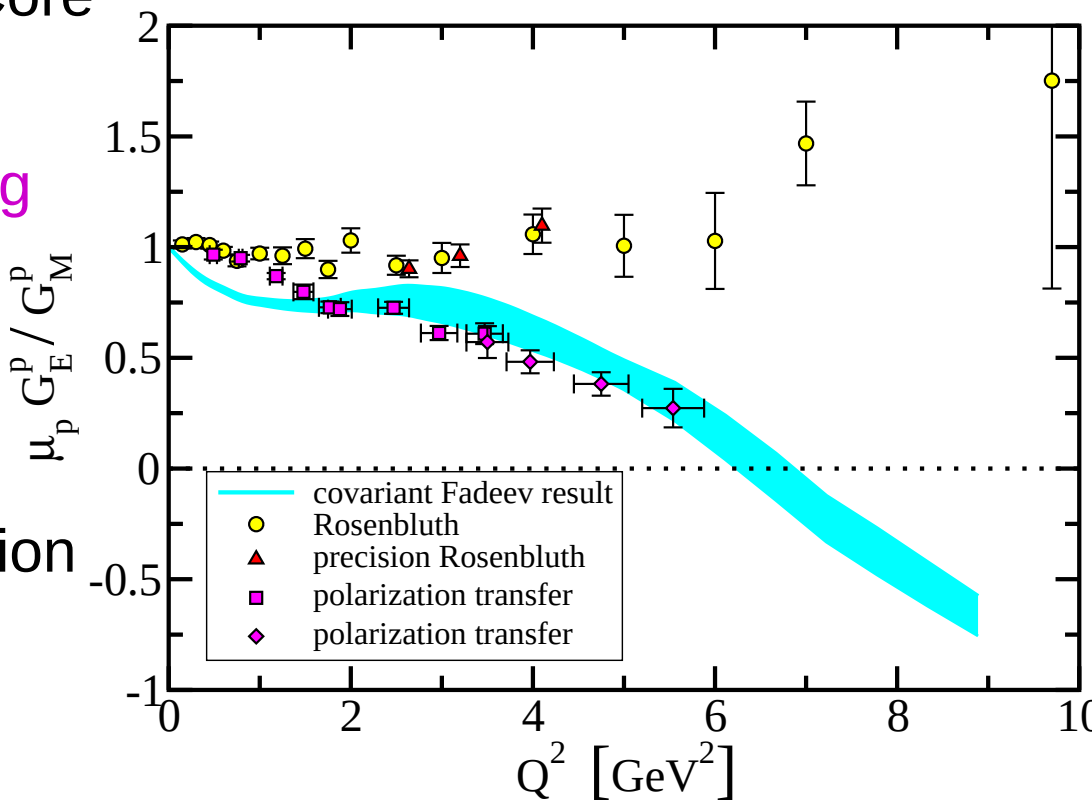
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



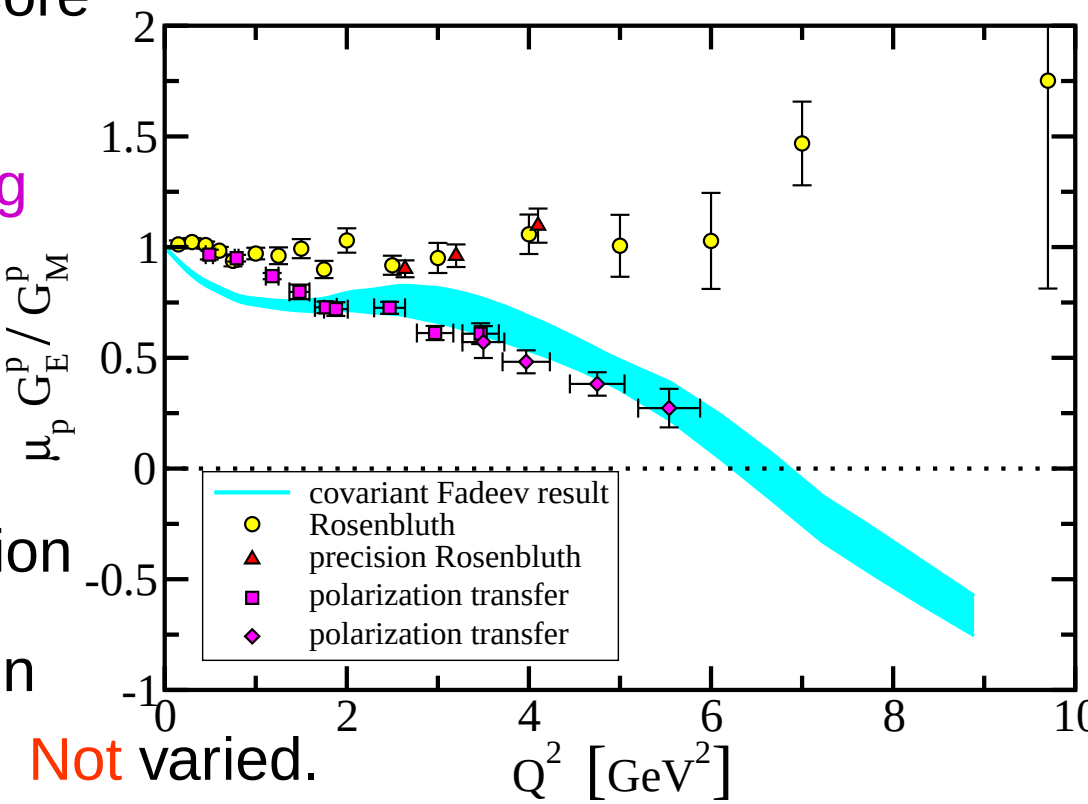
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



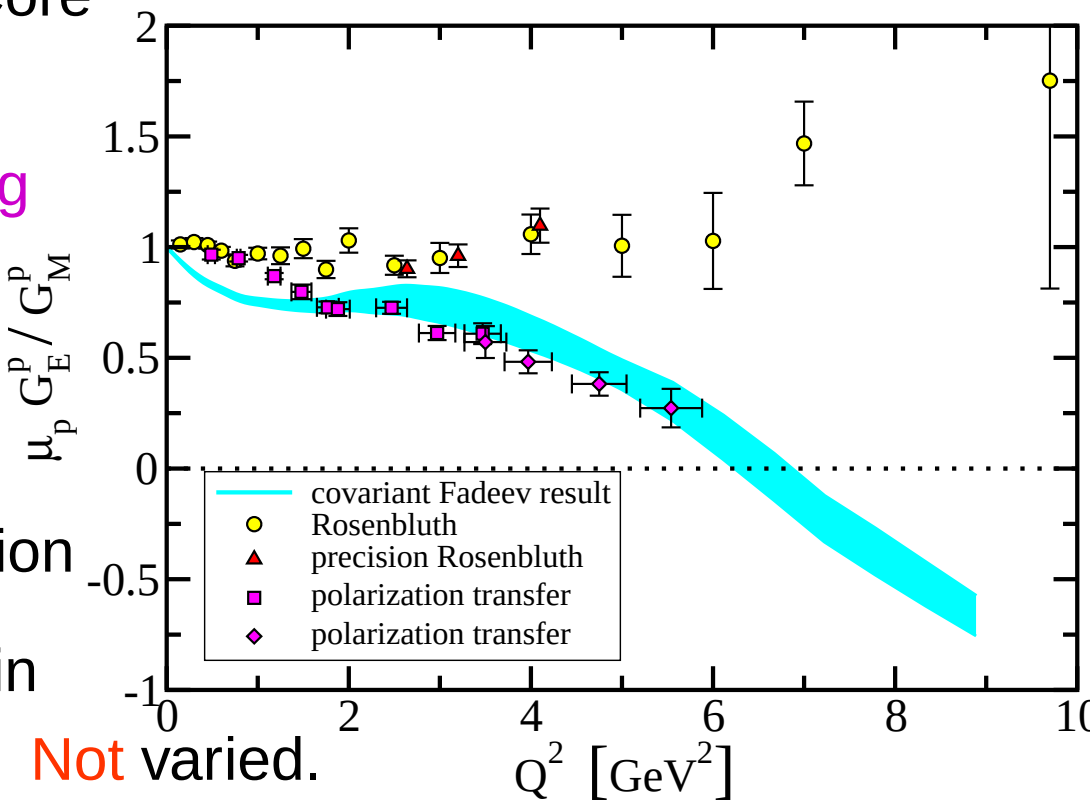
Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution
- All parameters fixed in other applications . . . Not varied.



Form Factor Ratio: GE/GM

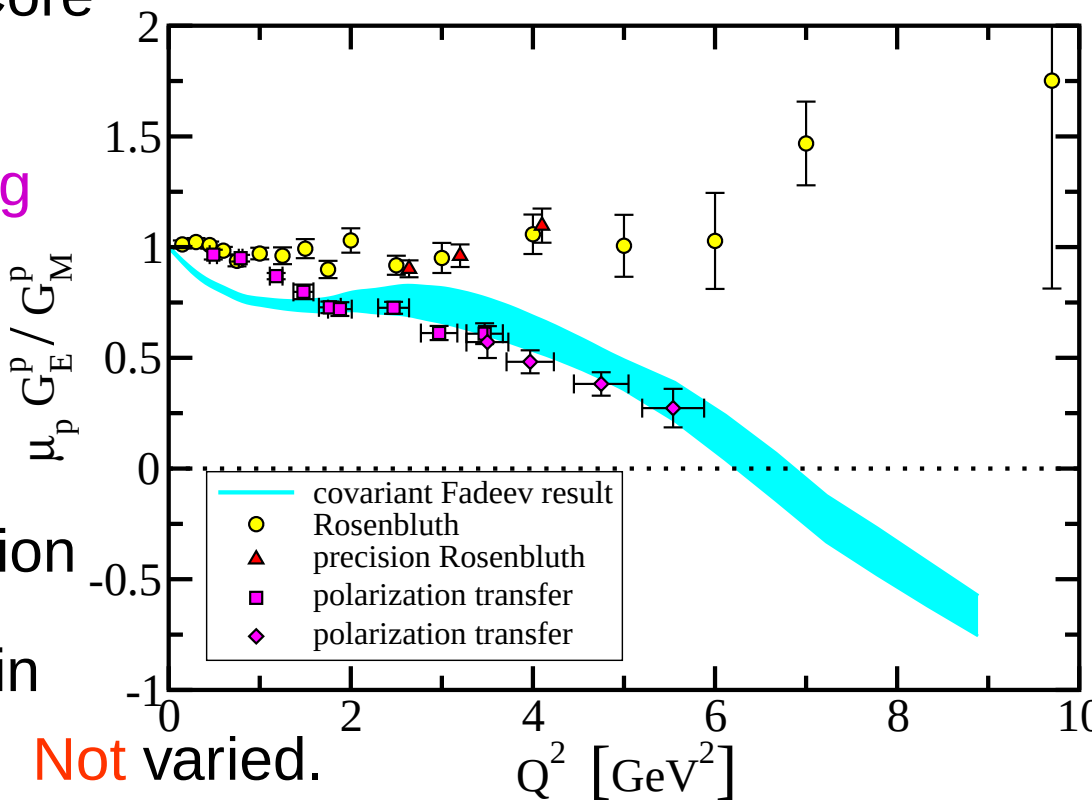
- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



- All parameters fixed in other applications . . . Not varied.
  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\,\mathrm{GeV}^2$

Form Factor Ratio: GE/GM

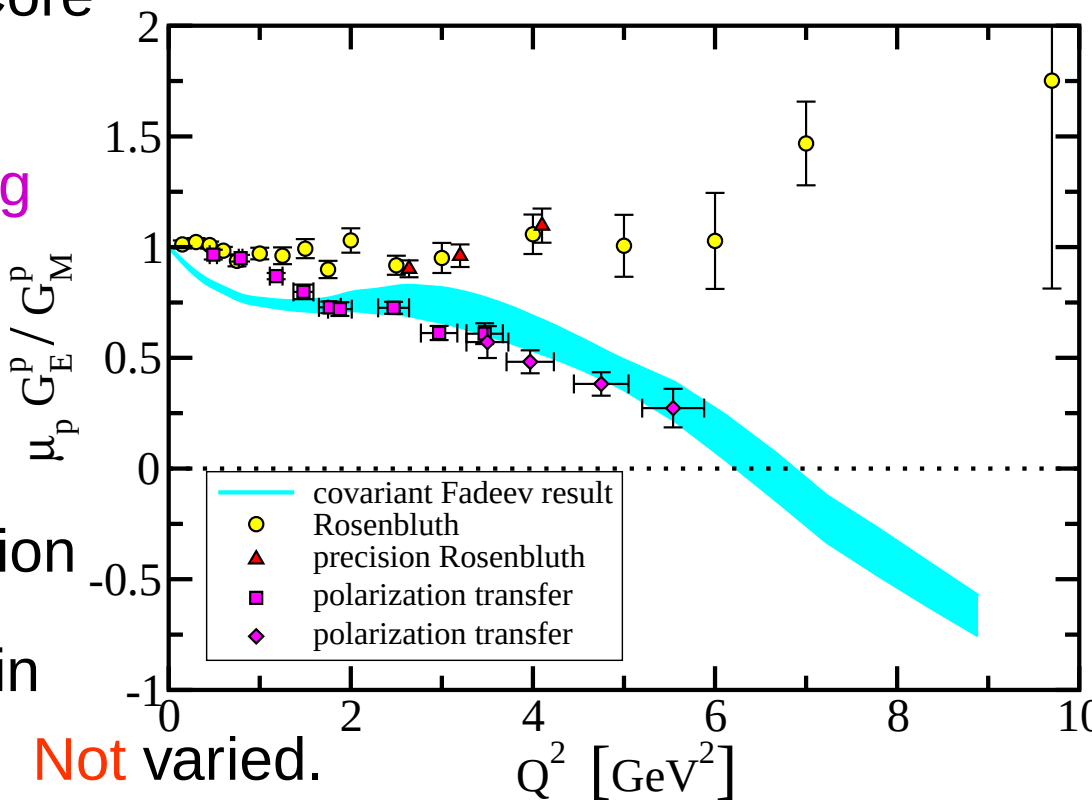
- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



- All parameters fixed in other applications . . . Not varied.
  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\,\mathrm{GeV}^2$
  - Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement

Form Factor Ratio: GE/GM

- Combine these elements . . .
  - Dressed-Quark Core
  - *Ward-Takahashi* Identity preserving current
  - Anticipate and Estimate Pion Cloud's Contribution



- All parameters fixed in other applications . . . Not varied.
  - Agreement with Pol. Trans. data at $Q^2 \gtrsim 2\,\mathrm{GeV}^2$
  - Correlations in Faddeev amplitude – quark orbital angular momentum – essential to that agreement
  - Predict Zero at $Q^2 \approx 6.5\mathrm{GeV}^2$

Chiral Corrections

Chiral Corrections

- Thus far, omitted pion cloud contribution to current

Chiral Corrections

- Thus far, omitted pion cloud contribution to current
- Include loops following method of
  - . . . Ashley, Leinweber, Thomas, Young, he-lat/0308024
  - . . . finite-range regularisation of loop corrections
  - . . . $\lambda =$ regularisation mass-scale

Chiral Corrections

- Thus far, omitted pion cloud contribution to current
- Include loops following method of
  - . . . Ashley, Leinweber, Thomas, Young, he-lat/0308024
  - . . . finite-range regularisation of loop corrections
  - . . . $\lambda =$ regularisation mass-scale

$$\langle r^2_{\substack{p\\n}} \rangle^{1-loop^R}_{NA} = \mp \frac{1+5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2+\lambda^2}\right),$$

$$\langle (r^\mu_N)^2 \rangle^{1-loop^R}_{NA} = -\frac{1+5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2+\lambda^2}\right) + \frac{g_A^2 M_N}{16\pi f_\pi^2 \mu_v} \frac{1}{m_\pi} \frac{2}{\pi} \arctan\left(\frac{\lambda}{m_\pi}\right),$$

$$(\mu_{\substack{p\\n}})^{1-loop^R}_{NA} = \mp \frac{g_A^2 M_N}{4\pi^2 f_\pi^2} m_\pi \frac{2}{\pi} \arctan\left(\frac{\lambda^3}{m_\pi^3}\right),$$

Chiral Corrections

- Thus far, omitted pion cloud contribution to current
- Include loops following method of
  - . . . Ashley, Leinweber, Thomas, Young, he-lat/0308024
  - . . . finite-range regularisation of loop corrections
  - . . . $\frac{1}{\lambda} = \frac{2}{3}$ fm

	r_p	r_n	r_p^μ	r_n^μ	μ_p	$-\mu_n$	ς
q-(qq) core	0.595	0.169	0.449	0.449	3.63	2.13	0.39
$+\pi$-loop correction	0.762	0.506	0.761	0.761	3.05	1.55	0.23
experiment	0.847	0.336	0.836	0.889	2.79	1.91	

Chiral Corrections

- Thus far, omitted pion cloud contribution to current
- Include loops following method of
  - . . . Ashley, Leinweber, Thomas, Young, he-lat/0308024
  - . . . finite-range regularisation of loop corrections
  - . . . $\frac{1}{\lambda} = \frac{2}{3}$ fm

$$\mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)} = 1 - \frac{Q^2}{6}\left[(r_p)^2 - (r_p^\mu)^2\right]$$

$r_p \approx r_p^\mu \Rightarrow$ ratio varies $< 10\%$ on $0 < Q^2 < 0.6$ GeV^2

Chiral Corrections

- Thus far, omitted pion cloud contribution to current
- Include loops following method of
  - . . . Ashley, Leinweber, Thomas, Young, he-lat/0308024
  - . . . finite-range regularisation of loop corrections
  - . . . $\frac{1}{\lambda} = \frac{2}{3}$ fm

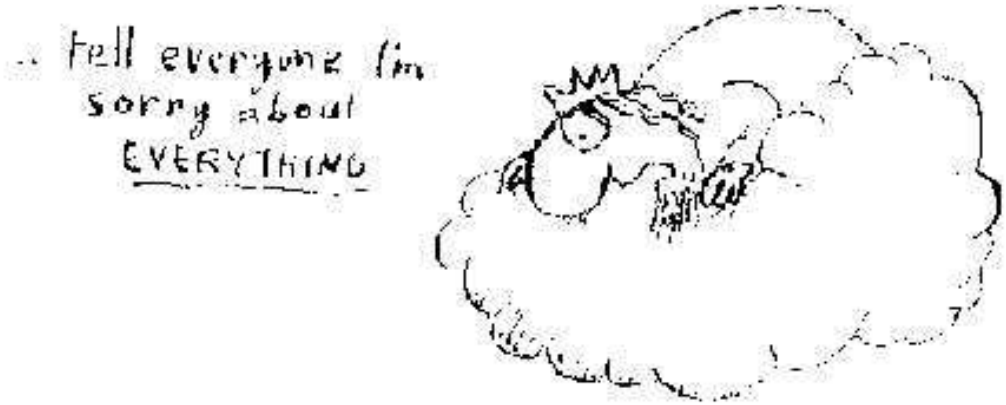
$$\mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)} = 1 - \frac{Q^2}{6}\left[(r_p)^2 - (r_p^\mu)^2\right]$$

$r_p \approx r_p^\mu \Rightarrow$ ratio varies $< 10\%$ on $0 < Q^2 < 0.6$ GeV^2

- Complements nucleon mass considerations
  - . . . veracious understanding of all nucleon properties
  - . . . impossible without intelligent incorporation of chiral corrections

Epilogue

Epilogue



Epilogue

Dyson-Schwinger Equations

- Provide Understanding of

  Dynamical Chiral Symmetry Breaking:

  $\Rightarrow$ π is quark-antiquark Bound State

  AND QCD's Goldstone Mode

Epilogue

Dyson-Schwinger Equations

- Provide Understanding of
  Dynamical Chiral Symmetry Breaking:

  $\Rightarrow \pi$ is quark-antiquark Bound State

  AND QCD's Goldstone Mode

- Foundation for Proof of
  Exact Results in QCD

  e.g., Quark Goldberger-Treiman

  Properties of Pseudoscalar Mesons

Epilogue

Dyson-Schwinger Equations

- Provide Understanding of
  Dynamical Chiral Symmetry Breaking:

  $\Rightarrow$ π is quark-antiquark Bound State

  AND QCD's Goldstone Mode

- Foundation for Proof of
  Exact Results in QCD

  e.g., Quark Goldberger-Treiman

  Properties of Pseudoscalar Mesons

- Renormalisation-Group-Improved Rainbow-Ladder

  $\Rightarrow$ Practical Phenomenological Tool

  Corrections Quantifiable

Epilogue

- Poincaré Covariant Faddeev Equation

Epilogue

- Poincaré Covariant Faddeev Equation
  - Nonpointlike scalar and axial-vector diquark correlations

Epilogue

- Poincaré Covariant Faddeev Equation
  - Nonpointlike scalar and axial-vector diquark correlations
  - $s-$, $p-$, $d-$wave quark angular momentum

Epilogue

- Poincaré Covariant Faddeev Equation
  - Nonpointlike scalar and axial-vector diquark correlations
  - $s-$, $p-$, $d-$wave quark angular momentum
- Quark core, relaxed to allow for pion cloud

Epilogue

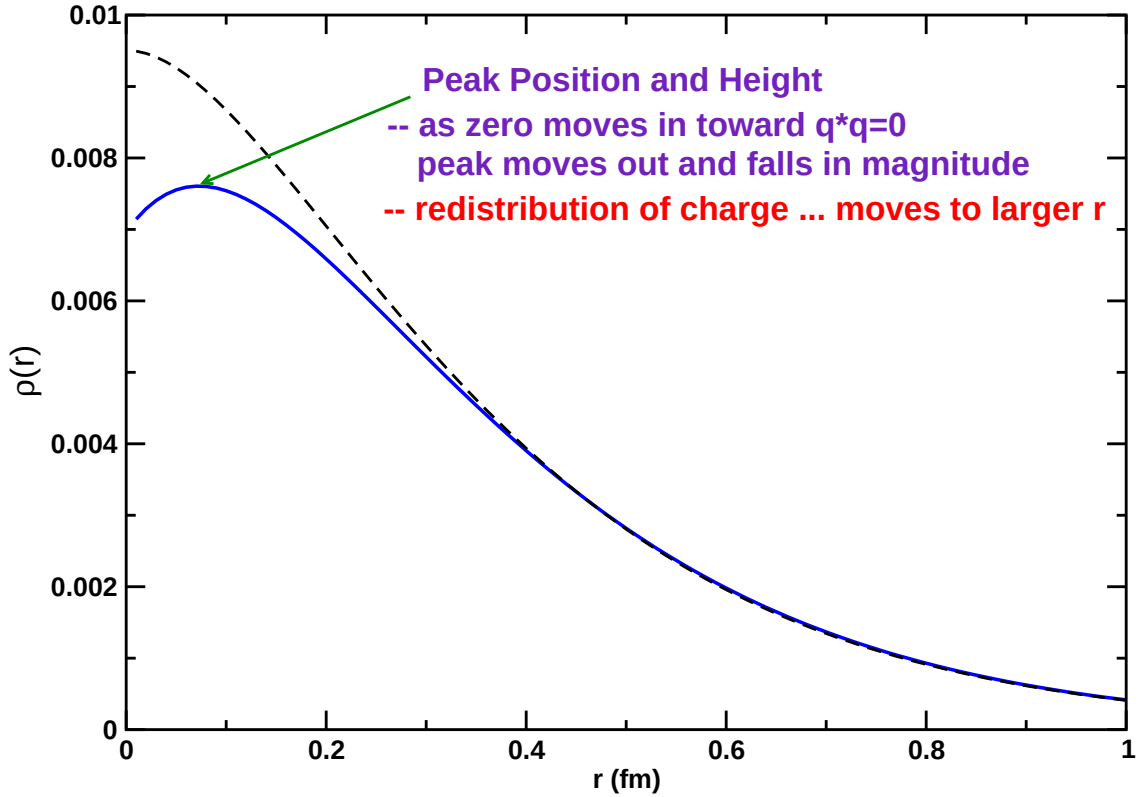
- Poincaré Covariant Faddeev Equation
  - Nonpointlike scalar and axial-vector diquark correlations
  - $s-$, $p-$, $d-$wave quark angular momentum
- Quark core, relaxed to allow for pion cloud
  - Predicts zero in $G_E^P(Q^2)$ at $Q^2 \approx 6.5\,\mathrm{GeV}^2$

Epilogue

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected

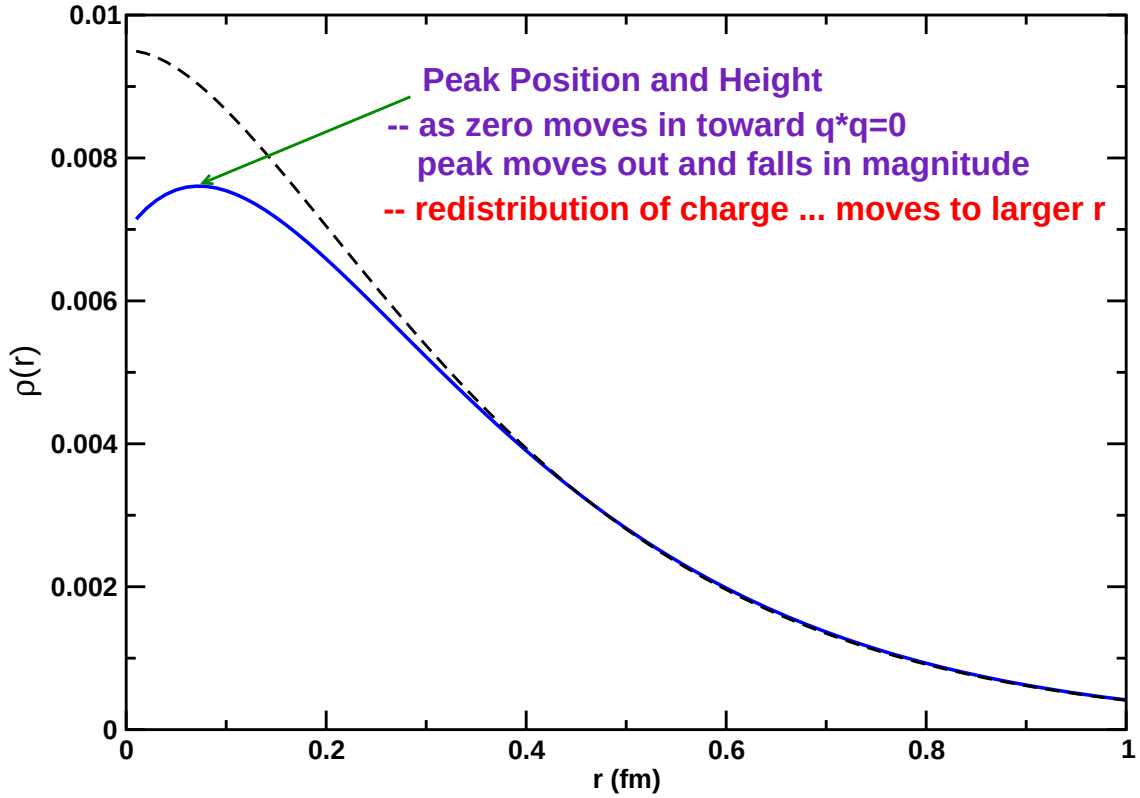
Epilogue

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected



Epilogue

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected



- However, Current Density remains peaked at $r = 0$!

Epilogue

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected
- Wave Function is complex and correlated mix of virtual particles and antiparticles: $s-$, $p-$ and $d-$waves

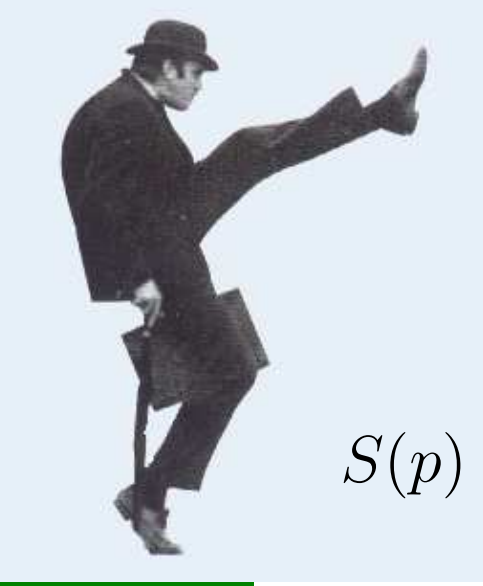
Contents

1. Pion Dichotomy
2. DSEs
3. Persistent Challenge
4. Dressed-Quark Propagator
5. Lattice cf. DSE
6. Light-Quark Interaction
7. Dressed-gluon Propagator
8. Hadrons
9. Bethe-Salpeter Kernel
10. Excitations & Chiral Symmetry
11. Radial Excitations
12. Radial Excitations II
13. Lattice Radial
14. Nucleon EM Form Factors
15. Faddeev equation
16. Parametrising diquark properties
17. Results: Nucleon & Δ Masses
18. Nucleon-Photon Vertex
19. Form Factor Ratio: GE/GM
20. Chiral Corrections
21. Contemporary Reviews
22. Perturbative Quark Propagator
23. Dressed-Vertex
24. Dressed-Vertex II
25. Nonpert. Trunc. – Contributions
26. Goldberger-Treiman for pion
27. Colour-singlet BSE
28. Two-photon Couplings
29. Two-photon: small Q^2
30. E.M. Charge Radii
31. Pions and Form Factors
32. Parametrising diquark properties
33. Form Factor Ratio: Q $*$ F2/F1
34. Form Factor Ratio: alternative F2/F1
35. DIS
36. Valence Distribution
37. Handbag Diagrams
38. General Calc. cf. Data

Contemporary Reviews

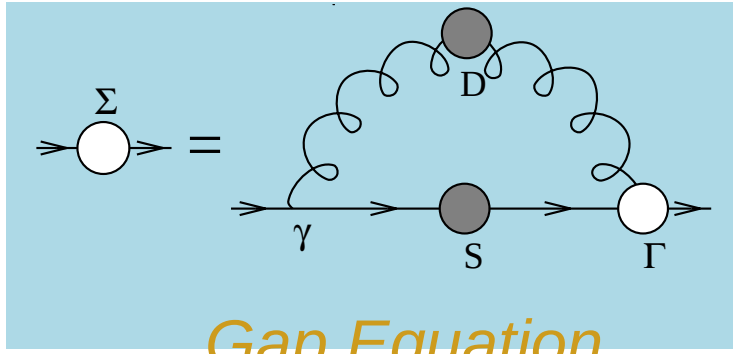
- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD
  C.D. Roberts and S.M. Schmidt, nu-th/0005064,
  Prog. Part. Nucl. Phys. **45** (2000) S1
- The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .
  R. Alkofer and L. von Smekal, he-ph/0007355,
  Phys. Rept. **353** (2001) 281
- Dyson-Schwinger equations: A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049,
  Int. J. Mod. Phys. **E 12** (2003) pp. 297-365
- Infrared properties of QCD from Dyson-Schwinger equations.
  C. S. Fischer, he-ph/0605173,
  J. Phys. **G 32** (2006) pp. R253-R291

Perturbative Dressed-quark Propagator



Perturbative Dressed-quark Propagator

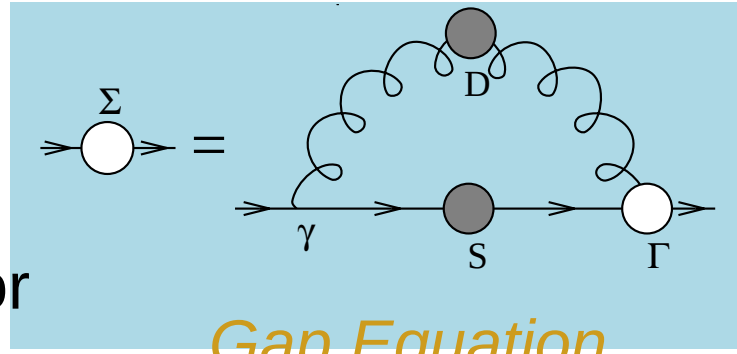
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

Perturbative Dressed-quark Propagator

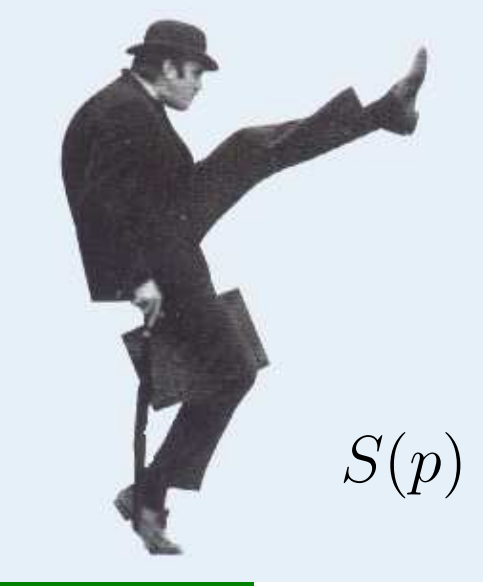
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

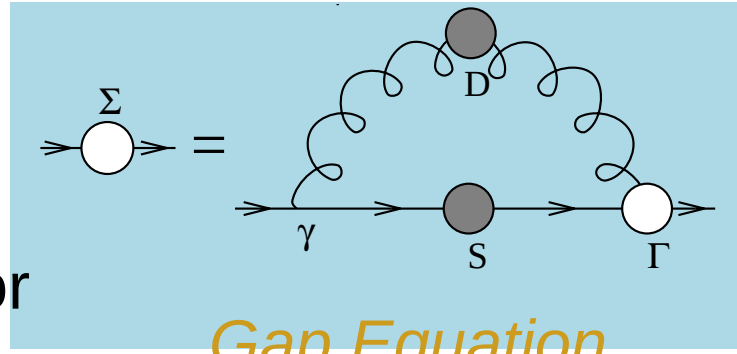
- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$



Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



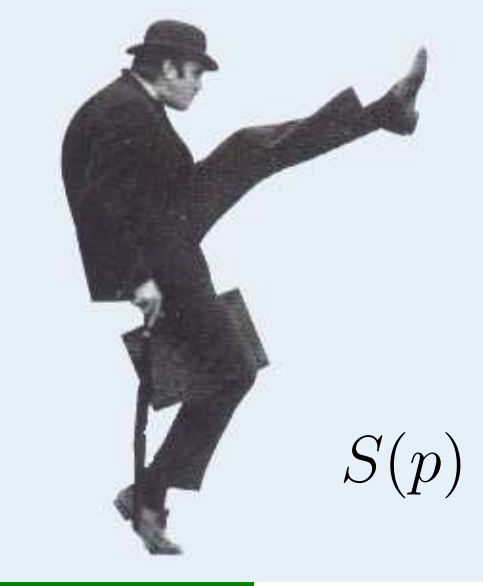
Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

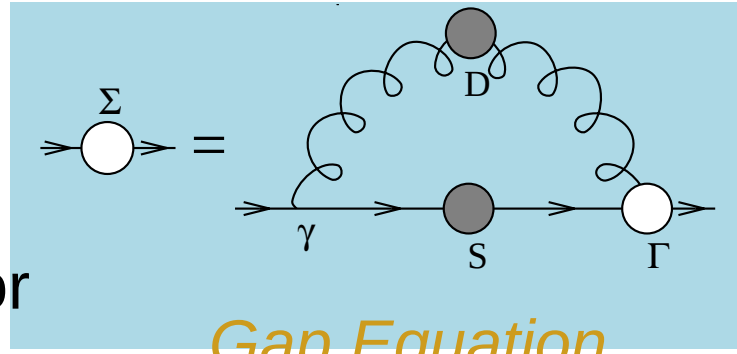
- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory



Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

- Weak Coupling Expansion

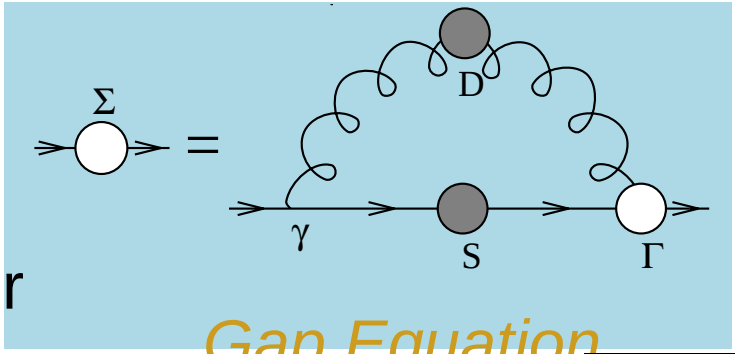
  Reproduces Every Diagram in Perturbation Theory

- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \overset{m\to 0}{\longrightarrow} 0$$

Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

No DCSB Here!

- Weak Coupling Expansion

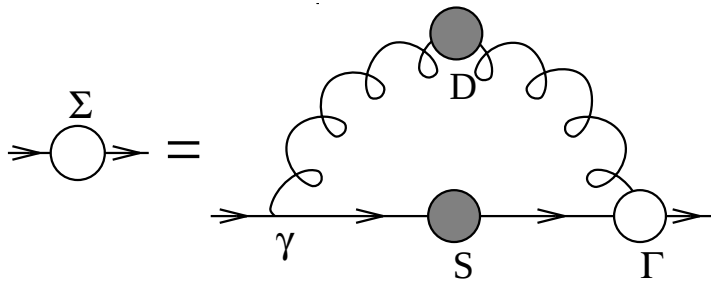
  Reproduces Every Diagram in Perturbation Theory

- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \xrightarrow{m\to 0} 0$$

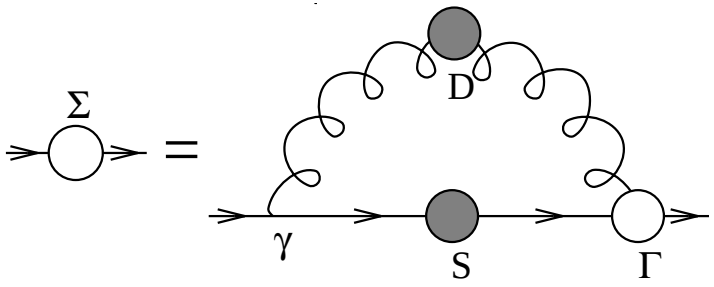
Dressed-quark-gluon Vertex

- How is this agreement achieved via the gap equation if the gluon propagator is IR-suppressed?



Dressed-quark-gluon Vertex

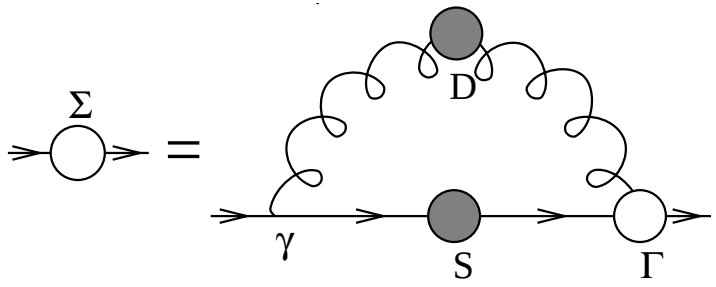
- How is this agreement achieved via the gap equation if the gluon propagator is IR-suppressed?



- F. T. Hawes, P. Maris and C. D. Roberts, *Infrared behaviour of propagators and vertices* Phys. Lett. B **440**, 353 (1998): “The vertex must possess an IR-enhancement.”

Dressed-quark-gluon Vertex

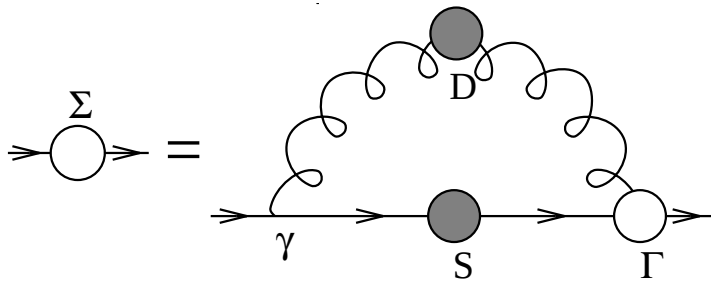
- How is this agreement achieved via the gap equation if the gluon propagator is IR-suppressed?



- F. T. Hawes, P. Maris and C. D. Roberts, *Infrared behaviour of propagators and vertices* Phys. Lett. B **440**, 353 (1998): "The vertex must possess an IR-enhancement."
  - Bhagwat, *et al*., nu-th 0304003, found that such behaviour was indeed required.
    Others (Alkofer, *et al*., he-ph/0309077) since have too.

Dressed-quark-gluon Vertex

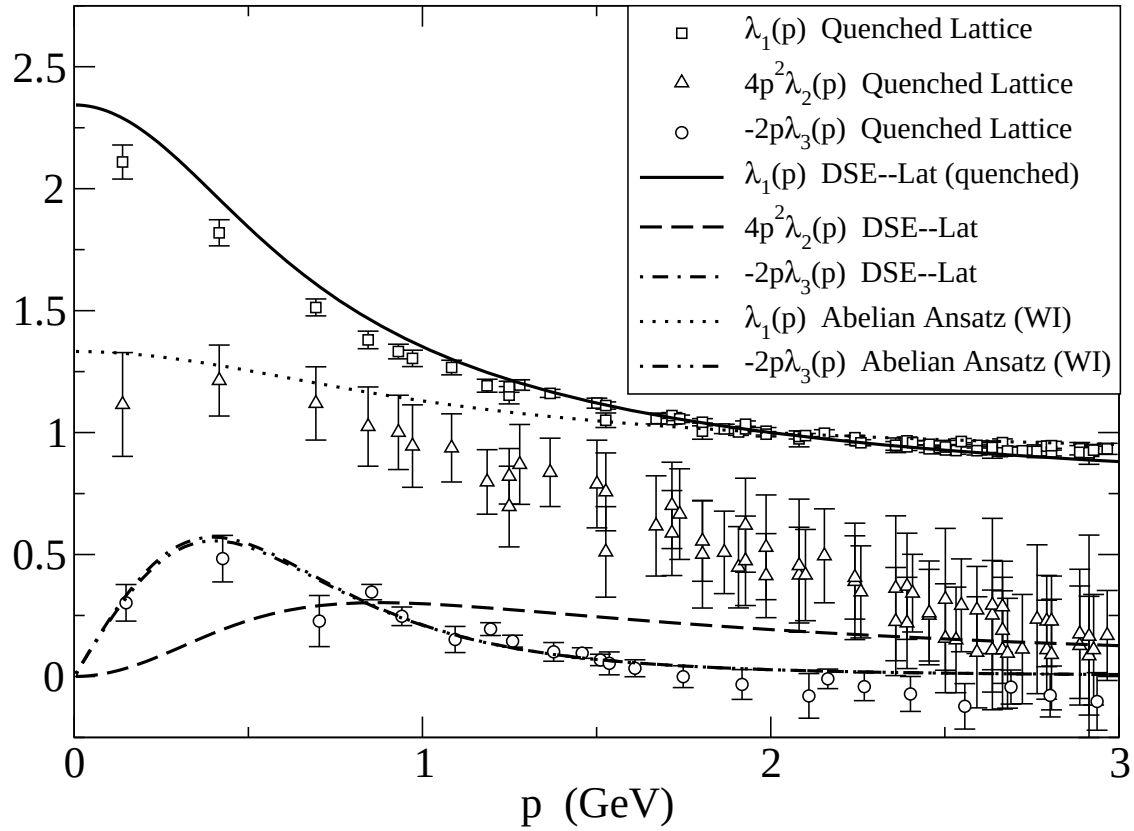
- How is this agreement achieved via the gap equation if the gluon propagator is IR-suppressed?



- F. T. Hawes, P. Maris and C. D. Roberts, *Infrared behaviour of propagators and vertices* Phys. Lett. B **440**, 353 (1998): "The vertex must possess an IR-enhancement."
  - Bhagwat, *et al*., nu-th 0304003, found that such behaviour was indeed required.
    Others (Alkofer, *et al*., he-ph/0309077) since have too.
- Exact nature of the enhancement is subject of ongoing research.

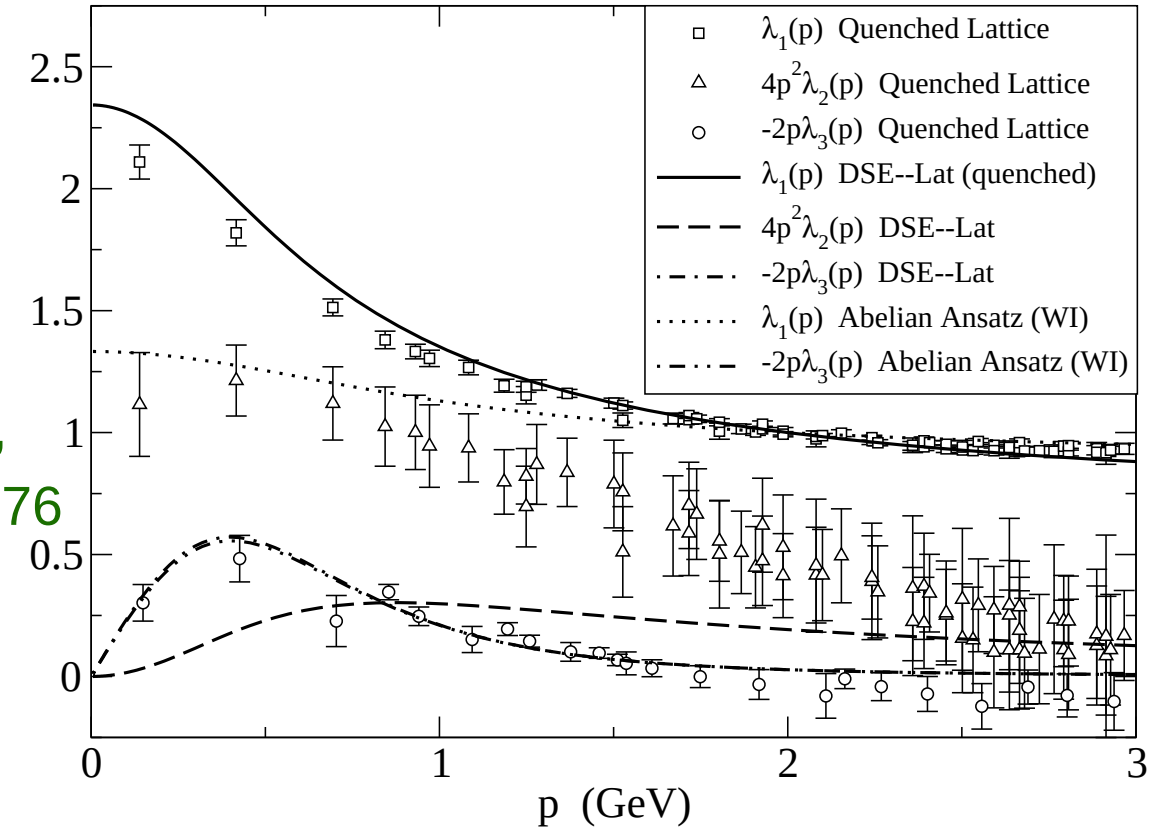
Quenched-QCD
Dressed-quark-gluon Vertex

- Bhagwat, *et al.*:
  - nu-th/0304003
  - nu-th/0403012
  - he-ph/0407163



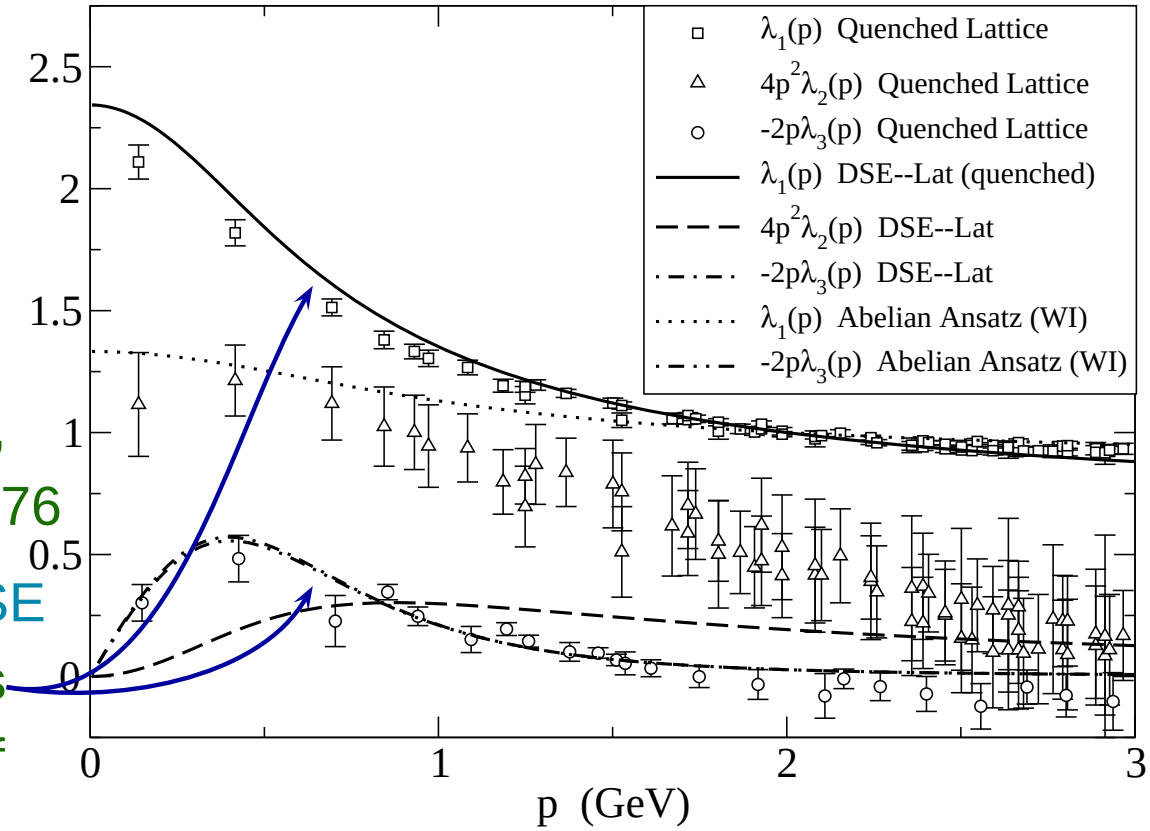
Quenched-QCD
Dressed-quark-gluon Vertex

- Bhagwat, *et al.*:
  - nu-th/0304003
  - nu-th/0403012
  - he-ph/0407163
- Lattice – Skullerud, *et al.*: he-ph/0303176



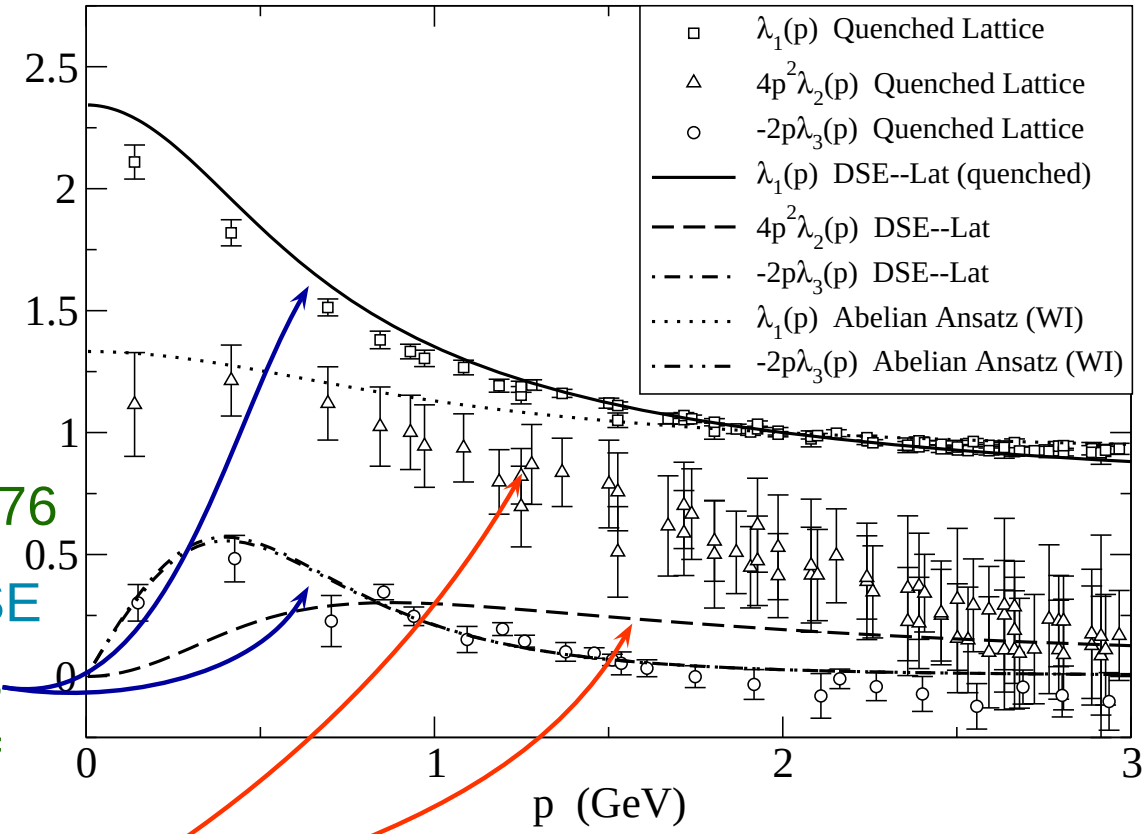
Quenched-QCD
Dressed-quark-gluon Vertex

- Bhagwat, *et al.*:
  - nu-th/0304003
  - nu-th/0403012
  - he-ph/0407163
- Lattice – Skullerud, *et al.*: he-ph/0303176
- Parameter Free DSE Prediction confirms lattice simulation of λ_1, λ_3



Quenched-QCD
Dressed-quark-gluon Vertex

- Bhagwat, *et al.*:
  - nu-th/0304003
  - nu-th/0403012
  - he-ph/0407163
- Lattice – Skullerud, *et al.*: he-ph/0303176
- Parameter Free DSE Prediction confirms lattice simulation of λ_1, λ_3
- Parameter Free DSE Prediction suggests lattice result for λ_2 erroneous – owing to systematic errors



Systematic Nonpert. Trunc. – Key Contributions

Systematic Nonpert. Trunc. – Key Contributions

- Existence . . .
  - Munczek, Phys. Rev. D **52** (1995) 4736;
  - Bender, Roberts, von Smekal, Phys. Lett. **B 380** (1996) 7.

Systematic Nonpert. Trunc. – Key Contributions

- Existence . . .
  - Munczek, Phys. Rev. D **52** (1995) 4736;
  - Bender, Roberts, von Smekal, Phys. Lett. **B 380** (1996) 7.
- Convergence . . .
  - Bender, Detmold, Roberts, Thomas, Phys. Rev. C **65** (2002) 065203.

Systematic Nonpert. Trunc. – Key Contributions

- Existence . . .
  - Munczek, Phys. Rev. D **52** (1995) 4736;
  - Bender, Roberts, von Smekal, Phys. Lett. **B 380** (1996) 7.
- Convergence . . .
  - Bender, Detmold, Roberts, Thomas, Phys. Rev. C **65** (2002) 065203.
- Interactions . . .
  - P. Bicudo, Phys. Rev. C **67** (2003) 035201;
  - Höll, Krassnigg, Maris, Roberts, Wright, Phys. Rev. C **71** (2005) 065204

Goldberger-Treiman for pion

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$
- Axial-vector Ward-Takahashi identity

$\Rightarrow$ $$f_\pi E_\pi(k;P=0) = B(p^2)$$

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$
- Axial-vector Ward-Takahashi identity

$$\Rightarrow \quad \begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

Pseudovector components necessarily nonzero

- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$
- Axial-vector Ward-Takahashi identity

$$\begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

$\Rightarrow$ Exact in Chiral QCD

Colour-singlet Bethe-Salpeter equation

Detmold *et al*., nu-th/0202082

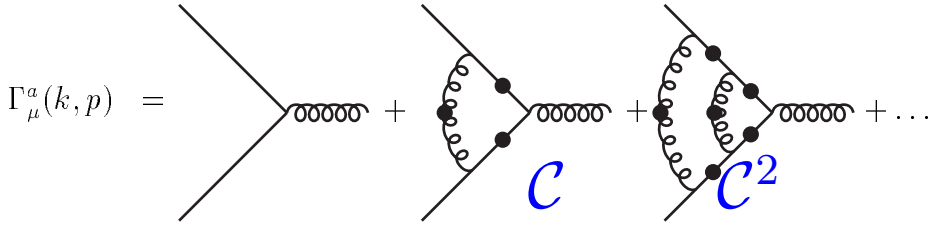
Bhagwat, *et al*., nu-th/0403012

Colour-singlet Bethe-Salpeter equation

Detmold *et al*., nu-th/0202082

Bhagwat, *et al*., nu-th/0403012

- Coupling-modified dressed-ladder vertex

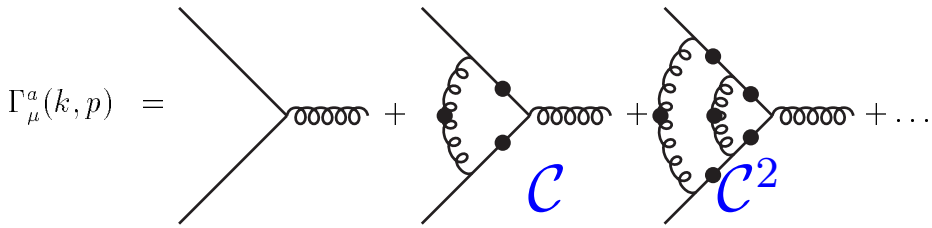


Colour-singlet Bethe-Salpeter equation

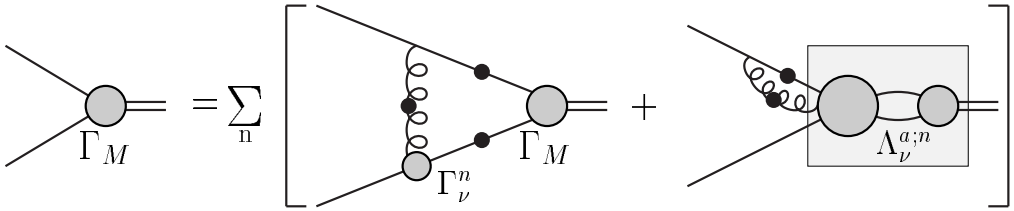
Detmold *et al*., nu-th/0202082

Bhagwat, *et al*., nu-th/0403012

- Coupling-modified dressed-ladder vertex



- BSE consistent with vertex

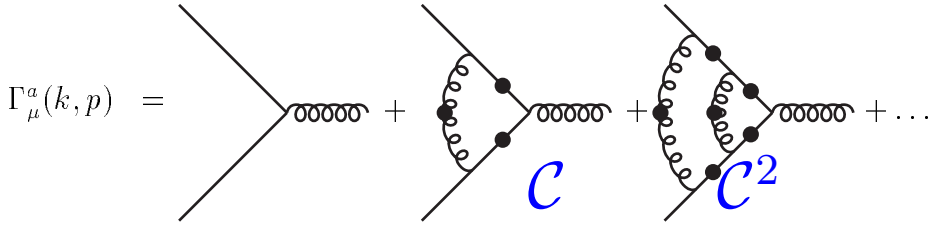


Colour-singlet Bethe-Salpeter equation

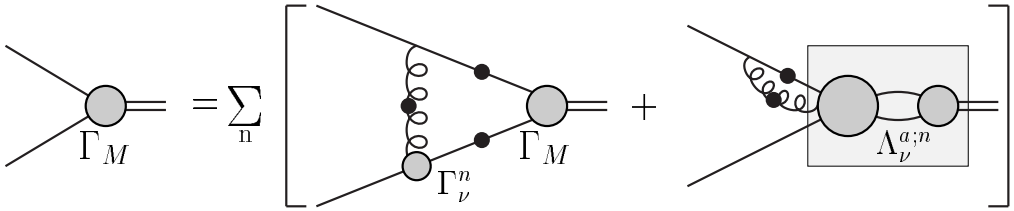
Detmold *et al*., nu-th/0202082

Bhagwat, *et al*., nu-th/0403012

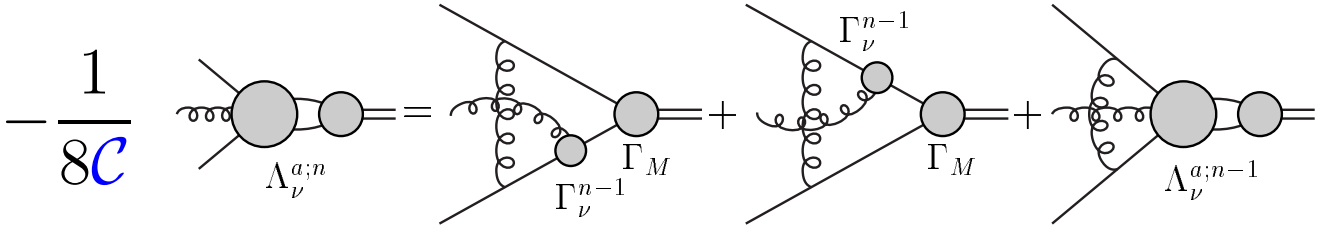
- Coupling-modified dressed-ladder vertex



- BSE consistent with vertex



- Bethe-Salpeter kernel . . . recursion relation

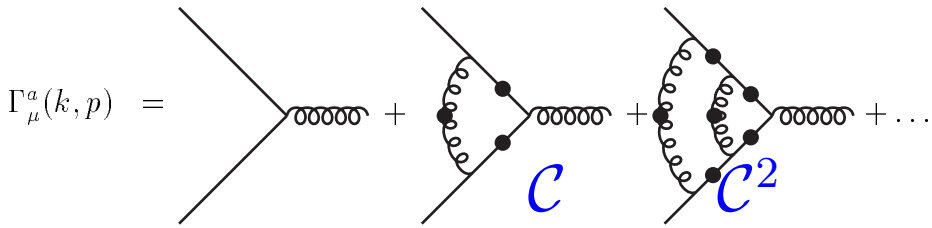


Colour-singlet Bethe-Salpeter equation

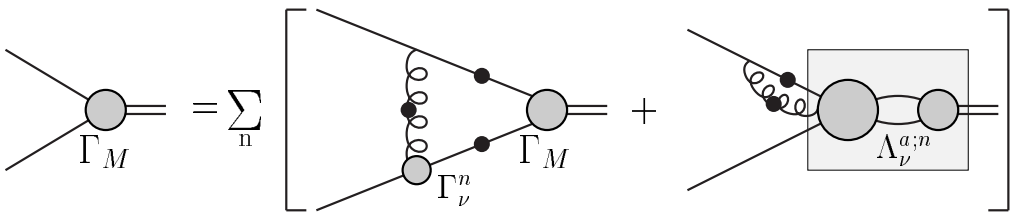
Detmold *et al.*, nu-th/0202082

Bhagwat, *et al.*, nu-th/0403012

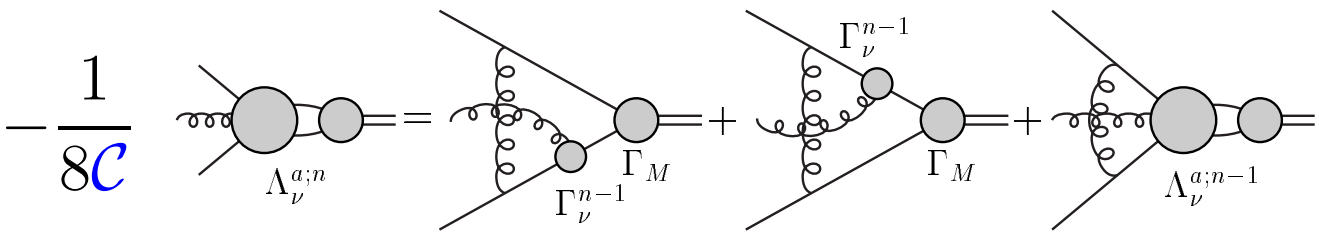
- Coupling-modified dressed-ladder vertex



- BSE consistent with vertex



- Bethe-Salpeter kernel . . . recursion relation



- Kernel necessarily non-planar,

  even with planar vertex

π and ρ mesons

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m = 0$	0	0	0	0
π, $m = 0.011$	0.147	0.135	0.139	0.138
ρ, $m = 0$	0.920	0.648	0.782	0.754
ρ, $m = 0.011$	0.936	0.667	0.798	0.770

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m = 0$	0	0	0	0
π, $m = 0.011$	0.147	0.135	0.139	0.138
ρ, $m = 0$	0.920	0.648	0.782	0.754
ρ, $m = 0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m = 0$	0	0	0	0
π, $m = 0.011$	0.147	0.135	0.139	0.138
ρ, $m = 0$	0.920	0.648	0.782	0.754
ρ, $m = 0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning
- ALL π-ρ mass splitting present in chiral limit

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m = 0$	0	0	0	0
π, $m = 0.011$	0.147	0.135	0.139	0.138
ρ, $m = 0$	0.920	0.648	0.782	0.754
ρ, $m = 0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning
- ALL π-ρ mass splitting present in chiral limit and with the Simplest kernel

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m = 0$	0	0	0	0
π, $m = 0.011$	0.147	0.135	0.139	0.138
ρ, $m = 0$	0.920	0.648	0.782	0.754
ρ, $m = 0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning
- π-ρ mass splitting driven by DχSB mechanism

  Not constituent-quark-model-like hyperfine splitting

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m = 0$	0	0	0	0
π, $m = 0.011$	0.147	0.135	0.139	0.138
ρ, $m = 0$	0.920	0.648	0.782	0.754
ρ, $m = 0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning
- π-ρ mass splitting driven by DχSB mechanism
  Not constituent-quark-model-like hyperfine splitting
- Extending kernel

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning
- π-ρ mass splitting driven by DχSB mechanism
  Not constituent-quark-model-like hyperfine splitting
- Extending kernel: NO effect on m_π

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . **NO** Fine Tuning
- π-ρ mass splitting driven by DχSB mechanism

  Not constituent-quark-model-like hyperfine splitting
- Extending kernel: NO effect on m_π

  For m_ρ – zeroth order, accurate to 20%

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . NO Fine Tuning
- π-ρ mass splitting driven by DχSB mechanism
  Not constituent-quark-model-like hyperfine splitting
- Extending kernel: NO effect on m_π
  For m_ρ – zeroth order, accurate to 20%
  – one loop, accurate to 13%

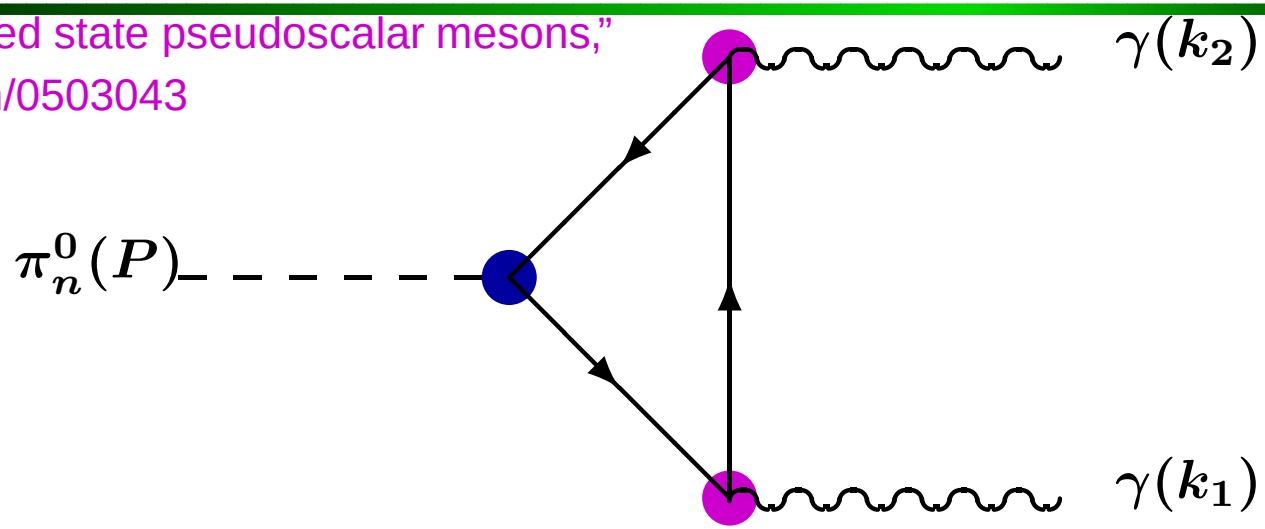
π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

- π massless in chiral limit . . . **NO** Fine Tuning
- π-ρ mass splitting driven by DχSB mechanism
  Not constituent-quark-model-like hyperfine splitting
- Extending kernel: NO effect on m_π
  For m_ρ – zeroth order, accurate to 20%
  – one loop, accurate to 13%
  – two loop, accurate to 4%

Two-photon Couplings of Pseudoscalar Mesons

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



$$T^{\pi_n^0}_{\mu\nu}(k_1,k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}\, G^{\pi_n^0}(k_1,k_2)$$

- Define: $\mathcal{T}_{\pi_n^0}(P^2,Q^2) = G^{\pi_n^0}(k_1,k_2)\Big|_{k_1^2=Q^2=k_2^2}$

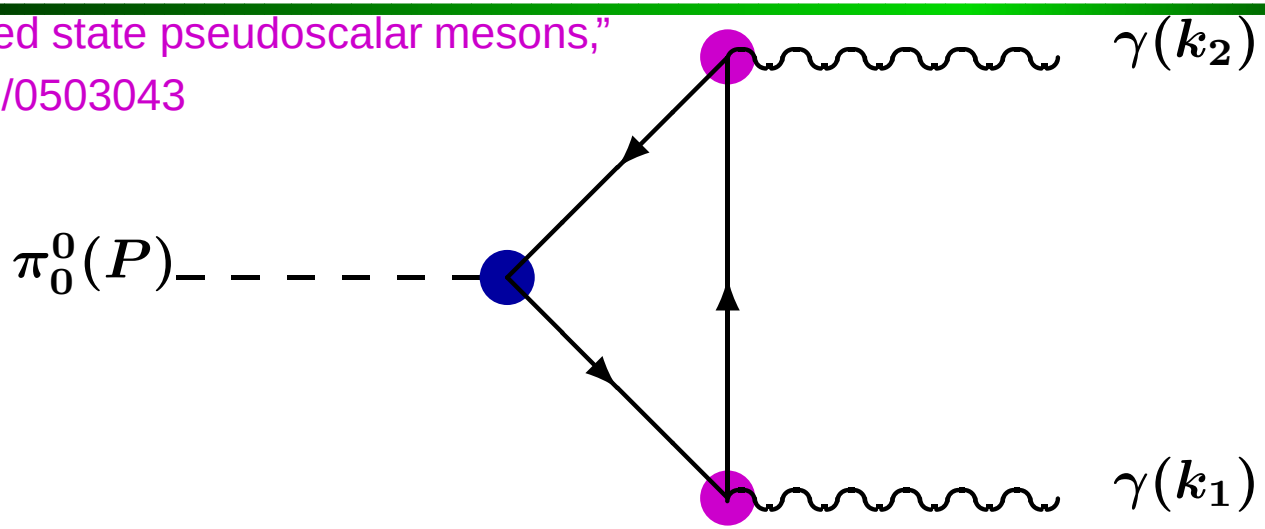
  This is a transition form factor.
- Physical Processes described by couplings:

  $g_{\pi_0^0\gamma\gamma} := \mathcal{T}_{\pi_0^0}(-m^2_{\pi_0^0},0)$

  Width: $\Gamma_{\pi_n^0\gamma\gamma} = \alpha^2_{\rm em}\,\dfrac{m^3_{\pi_n}}{16\pi^3}\, g^2_{\pi_n\gamma\gamma}$

Two-photon Couplings: Goldstone Mode

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



$$T^{\pi_0^0}_{\mu\nu}(k_1,k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}\, G^{\pi_0^0}(k_1,k_2)$$

Chiral limit, model-independent and algebraic result

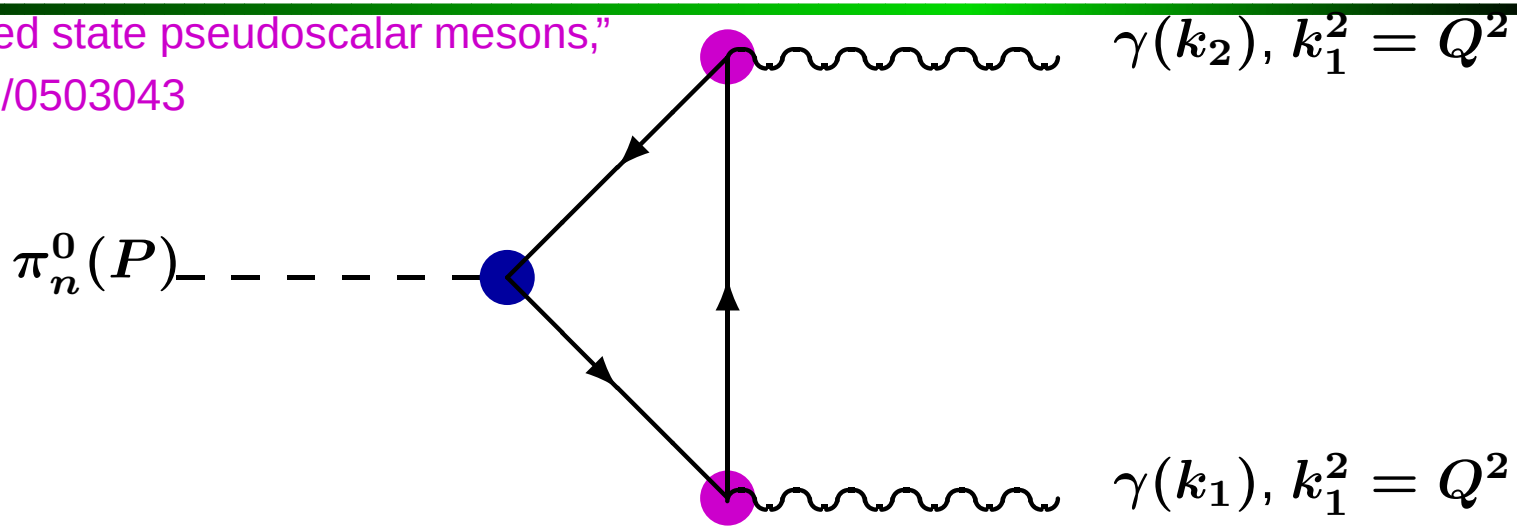
$$g_{\pi_0^0\gamma\gamma} := \mathcal{T}_{\pi_0^0}(-m^2_{\pi_0^0} = 0, 0) = \frac{1}{2}\frac{1}{f_{\pi_0}}$$

So long as truncation preserves chiral symmetry and the pattern of its dynamical breakdown

The most widely known consequence of the Abelian anomaly

Two-photon Couplings: Transition Form Factor

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



$$T_{\mu\nu}^{\pi_n^0}(k_1,k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}\, G^{\pi_n^0}(k_1,k_2)$$

So long as truncation preserves chiral symmetry and the pattern of its dynamical breakdown, and the one-loop renormalisation group properties of QCD: model-independent result – $\forall n$:

$$\mathcal{T}_{\pi_n^0}(P^2,Q^2) = G^{\pi_n^0}(k_1,k_2)\Big|_{k_1^2=Q^2=k_2^2} \overset{Q^2 \gg \Lambda_{\rm QCD}^2}{=} \frac{4\pi^2}{3}\frac{f_{\pi_n}}{Q^2}$$

Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$

Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$
- BUT, $f_{\pi_n} \equiv 0, \forall n$!

Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$
- BUT, $f_{\pi_n} \equiv 0, \forall n$!
- Model-independent result, in chiral limit: $\forall n \geq 1$

$$\lim_{\hat{m}\to 0} \mathcal{T}_{\pi_n^0}(-m_{\pi_n}^2, Q^2) \overset{Q^2 \gg \Lambda_{\rm QCD}^2}{=} \frac{4\pi^2}{3} F_n^{(2)}(-m_{\pi_n}^2) \frac{\ln^\gamma Q^2/\omega_{\pi_n}^2}{Q^4}\Bigg|_{\hat{m}=0}$$

where:

- γ is an anomalous dimension
- ω_{π_n} is a width mass-scale

both determined, in part, by properties of the meson's Bethe-Salpeter wave function.

Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$
- BUT, $f_{\pi_n} \equiv 0, \forall n$!
- Model-independent result, in chiral limit: $\forall n \geq 1$

$$\lim_{\hat{m}\to 0} \mathcal{T}_{\pi_n^0}(-m_{\pi_n}^2, Q^2) \overset{Q^2 \gg \Lambda_{\rm QCD}^2}{=} \frac{4\pi^2}{3} F_n^{(2)}(-m_{\pi_n}^2) \frac{\ln^\gamma Q^2/\omega_{\pi_n}^2}{Q^4}\Bigg|_{\hat{m}=0}$$

where:

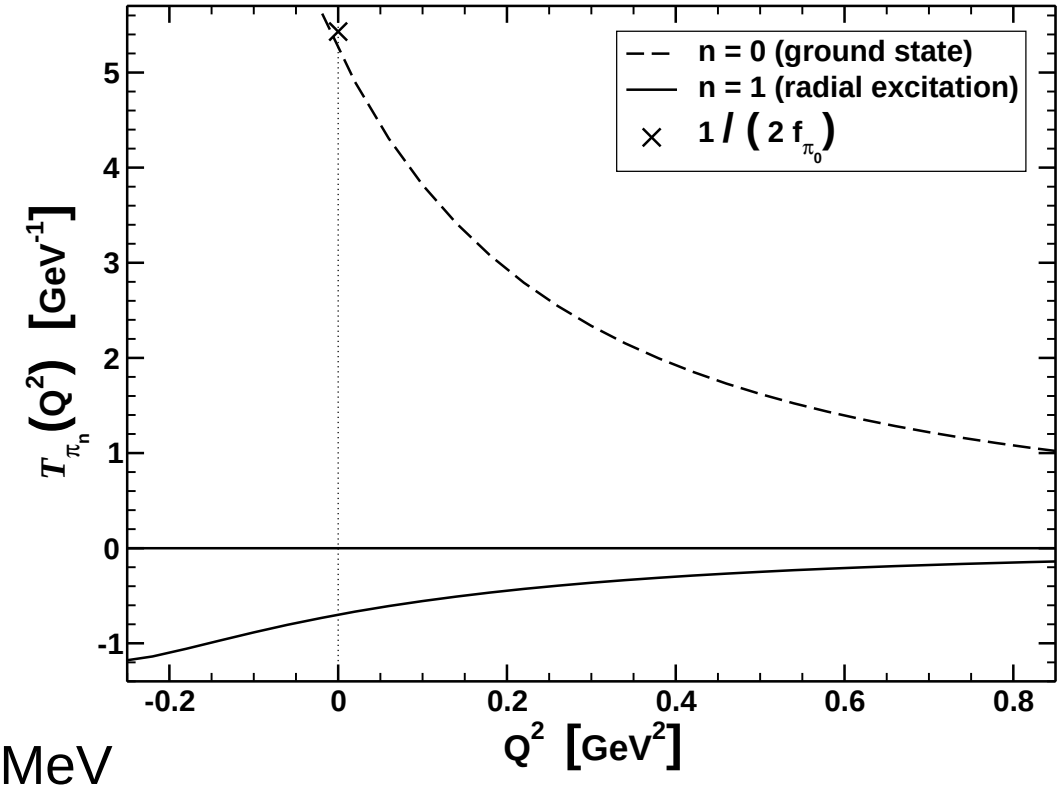
  - γ is an anomalous dimension
  - ω_{π_n} is a width mass-scale

  both determined, in part, by properties of the meson's Bethe-Salpeter wave function.

- Importantly, $F_n^{(2)}(-m_{\pi_n}^2) \not\propto f_{\pi_n}$. Instead, it is determined by DCSB mass-scales for π_n that do not vanish in the chiral limit.

Calculated Transition Form Factor: RGI Rainbow-Ladder

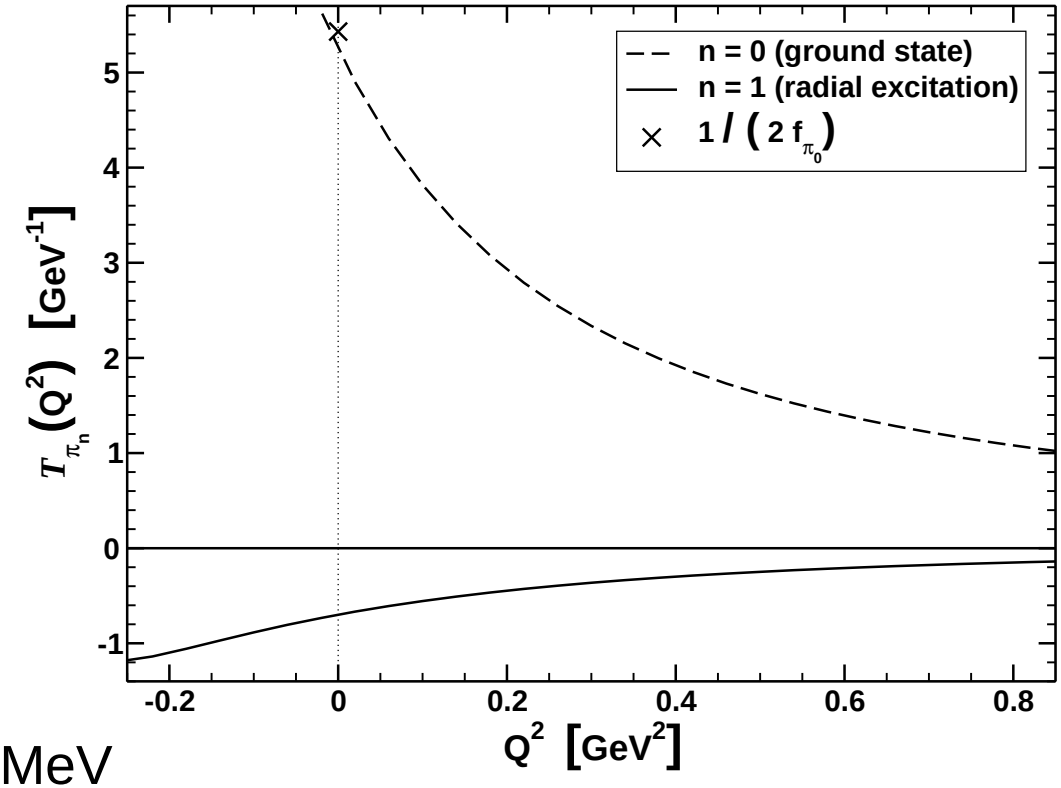
Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5\,\mathrm{MeV}$

Calculated Transition Form Factor: RGI Rainbow-Ladder

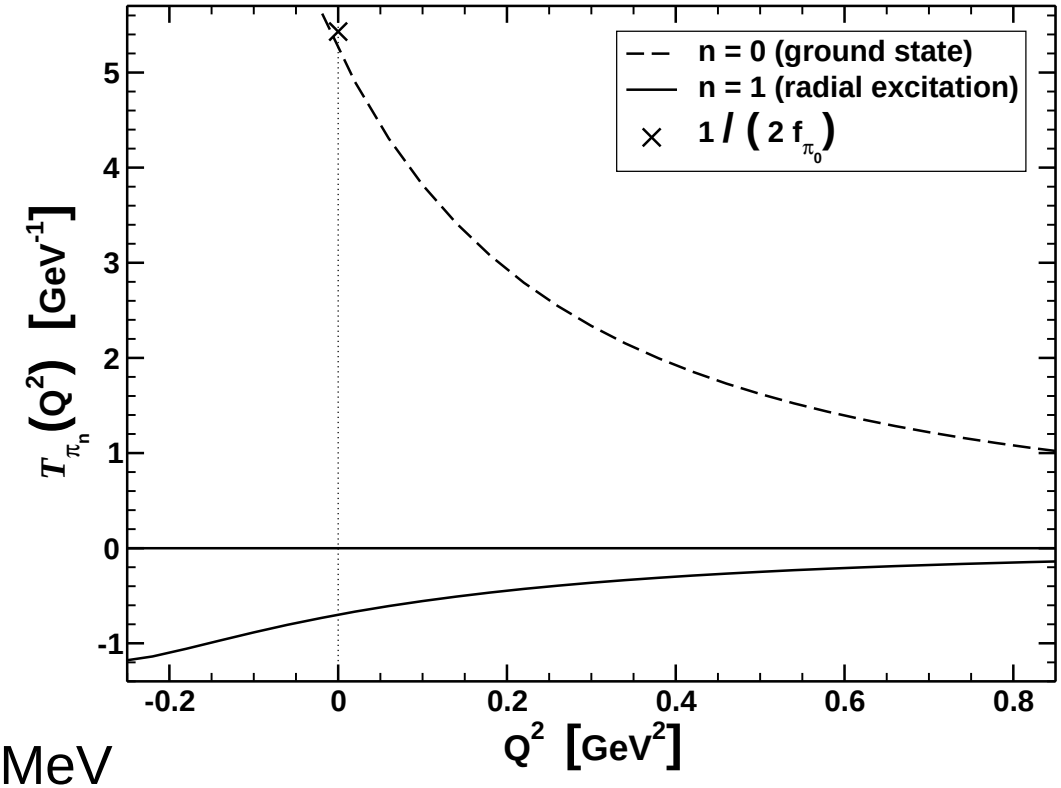
Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5\,\mathrm{MeV}$
- $\mathcal{T}_{\pi_1^0}(-m_{\pi_1}^2, Q^2) < 0\,,\ Q^2 \geq -m_{\pi_1}^2/4$; viz., it is negative on the entire kinematically accessible domain.

Calculated Transition Form Factor: RGI Rainbow-Ladder

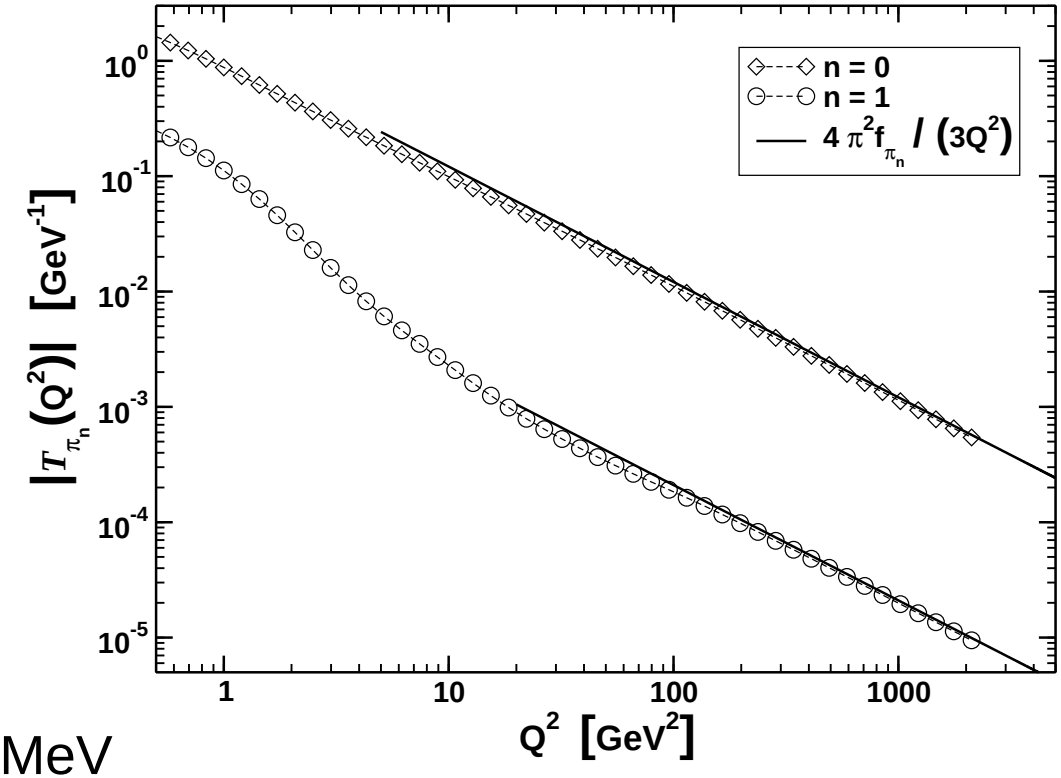
Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5$ MeV
- $\mathcal{T}_{\pi_1^0}(-m_{\pi_1}^2, Q^2) < 0\,,\ Q^2 \geq -m_{\pi_1}^2/4$; viz., it is negative on the entire kinematically accessible domain.
- $\Gamma_{\pi_0^0\gamma\gamma} = 7.9$ eV, $\Gamma_{\pi_1^0\gamma\gamma} = 240$ eV

Calculated Transition Form Factor: RGI Rainbow-Ladder

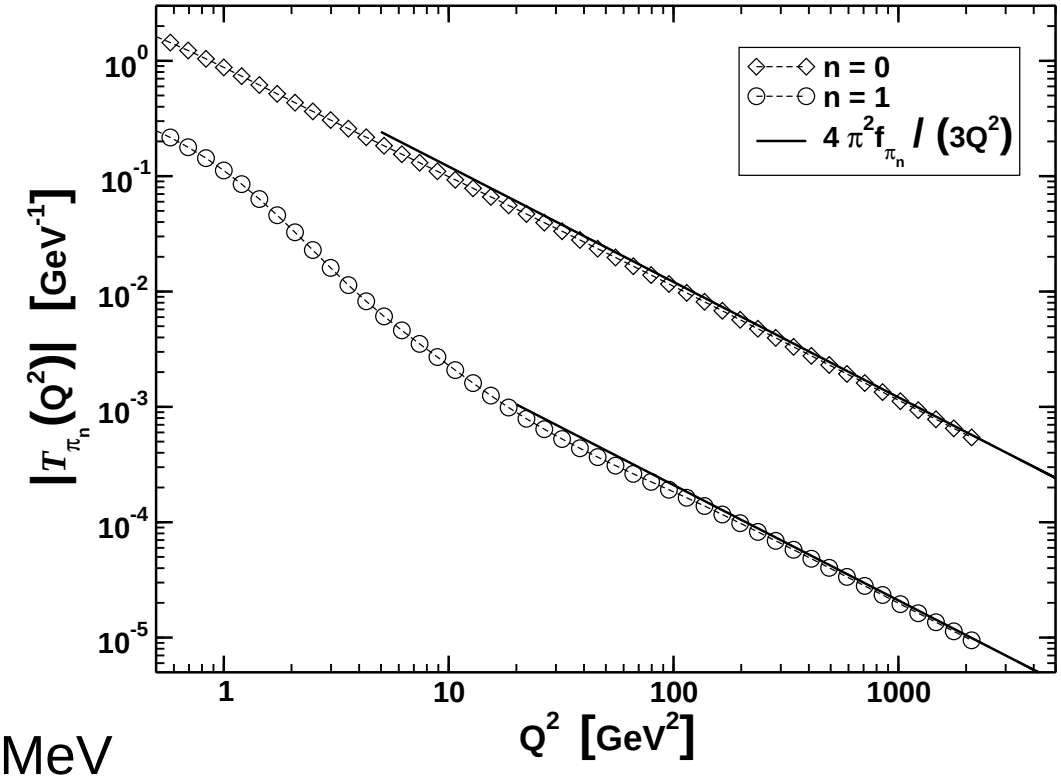
Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5\,\mathrm{MeV}$

Calculated Transition Form Factor: RGI Rainbow-Ladder

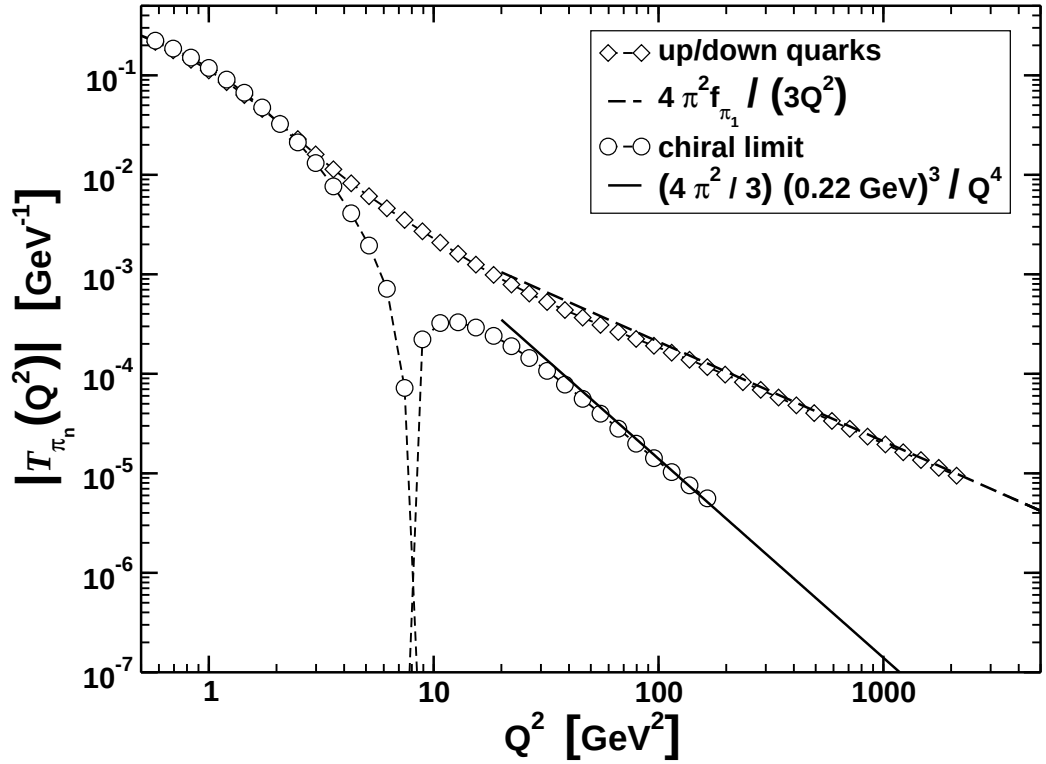
Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $m_u(1\,\mathrm{GeV})$
  $= m_d(1\,\mathrm{GeV}) = 5.5$ MeV
- Predicted UV-behaviour is abundantly clear
  - precise for $Q^2 > 120\,\mathrm{GeV}^2$

Transition Form Factor (Chiral): RGI Rainbow-Ladder

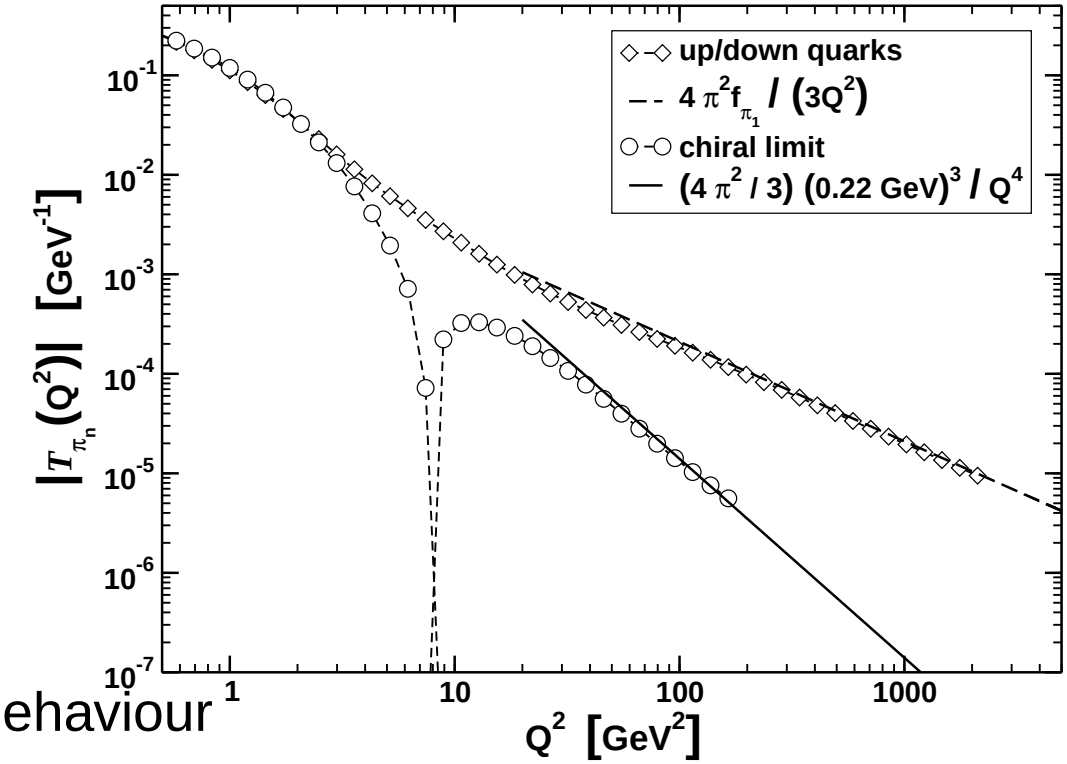
Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 0$

Transition Form Factor (Chiral): RGI Rainbow-Ladder

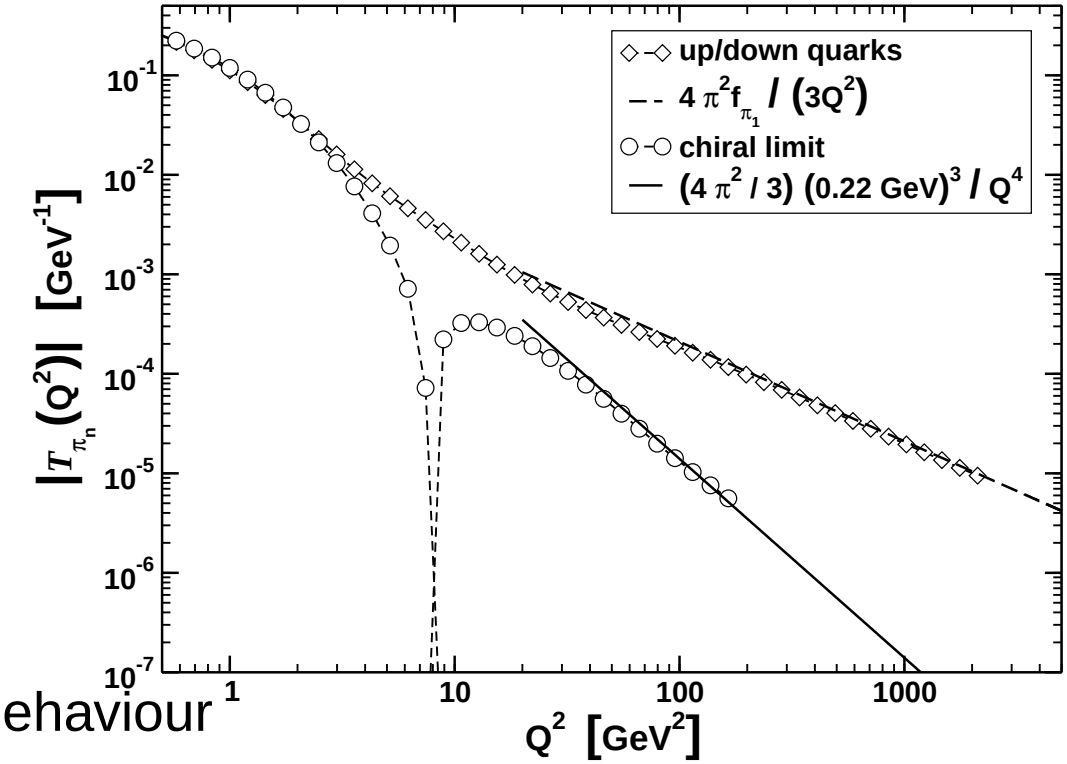
Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 0$
- Again, Predicted UV-behaviour is abundantly clear
  - precise for $Q^2 > 120\,\mathrm{GeV}^2$

Transition Form Factor (Chiral): RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043

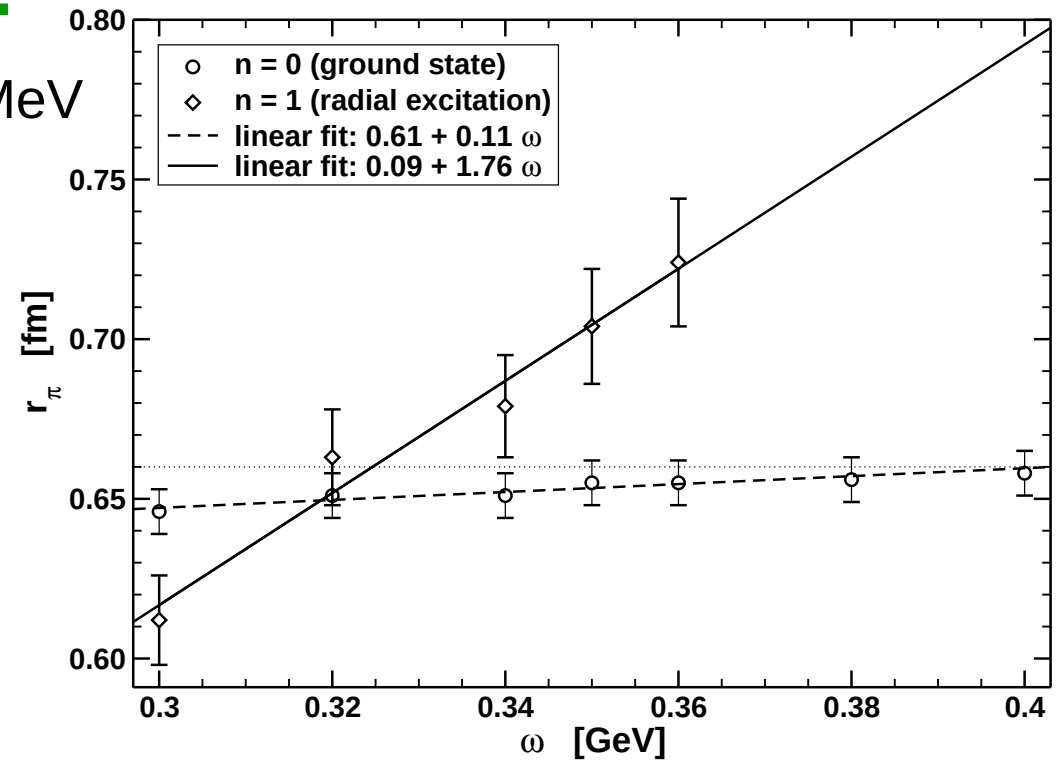


- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 0$
- Again, Predicted UV-behaviour is abundantly clear
  - precise for $Q^2 > 120\,\mathrm{GeV}^2$
- $$F_1^{(2)}(-m_{\pi_1}^2)\,\ln^\gamma Q^2/\omega_{\pi_1}^2\Big|_{\hat{m}=0} \approx (0.22\,\mathrm{GeV})^3 \simeq -\langle\bar{q}q\rangle^0 \quad (3)$$

Electromagnetic Charge Radii – RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al.*, nu-th/0503043

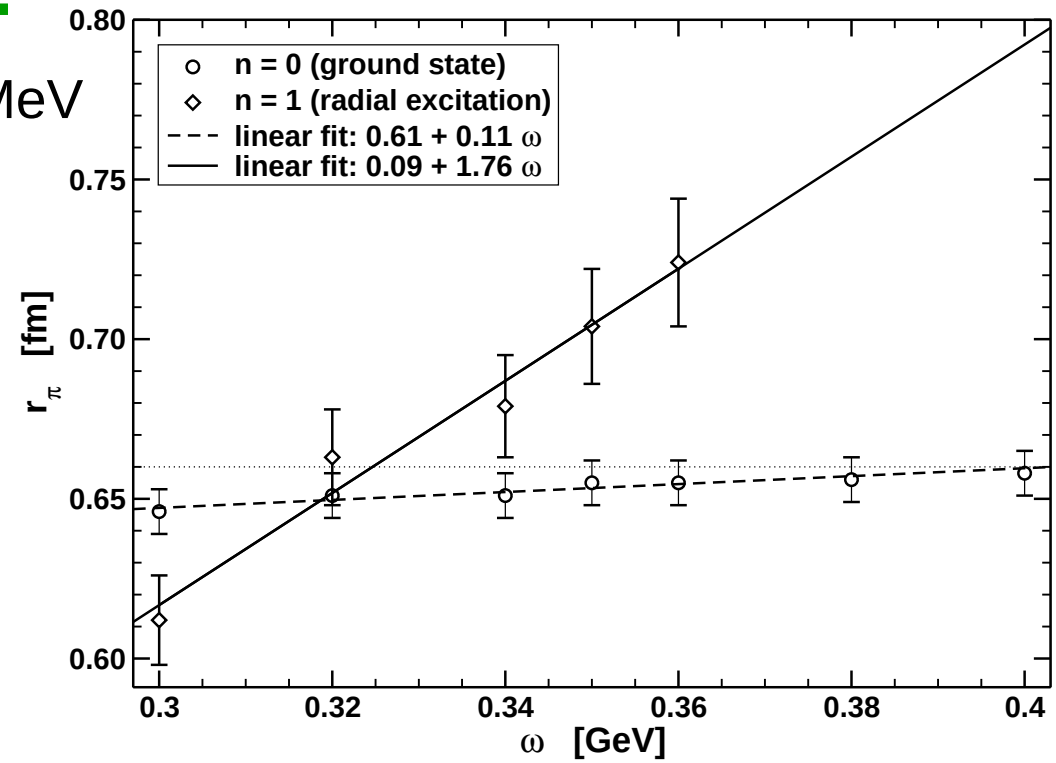
- $m_{u,d}(1\,\mathrm{GeV}) = 5.5\,\mathrm{MeV}$



Electromagnetic Charge Radii – RGI Rainbow-Ladder

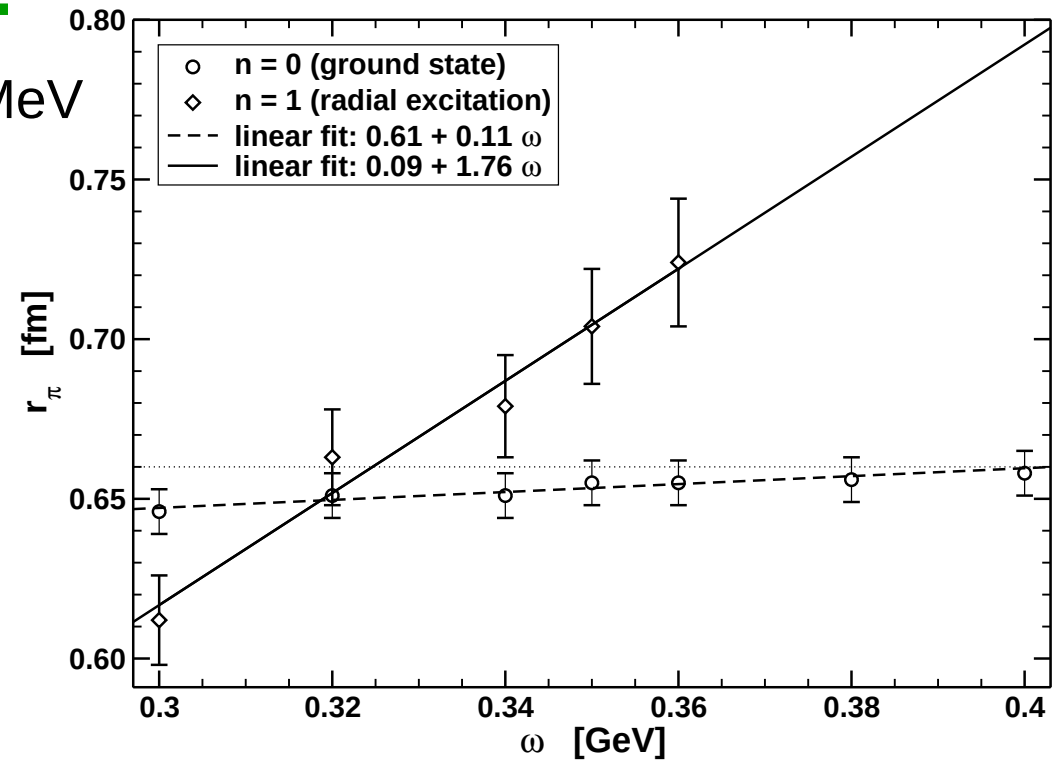
Höll, Krassnigg, Maris, *et al.*, nu-th/0503043

- $m_{u,d}(1\,\mathrm{GeV}) = 5.5$ MeV
- Reminder: MT-model has one IR-mass-scale – ω
  - $r_a := 1/\omega$ gauges the range of strong attraction



Electromagnetic Charge Radii – RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al.*, nu-th/0503043

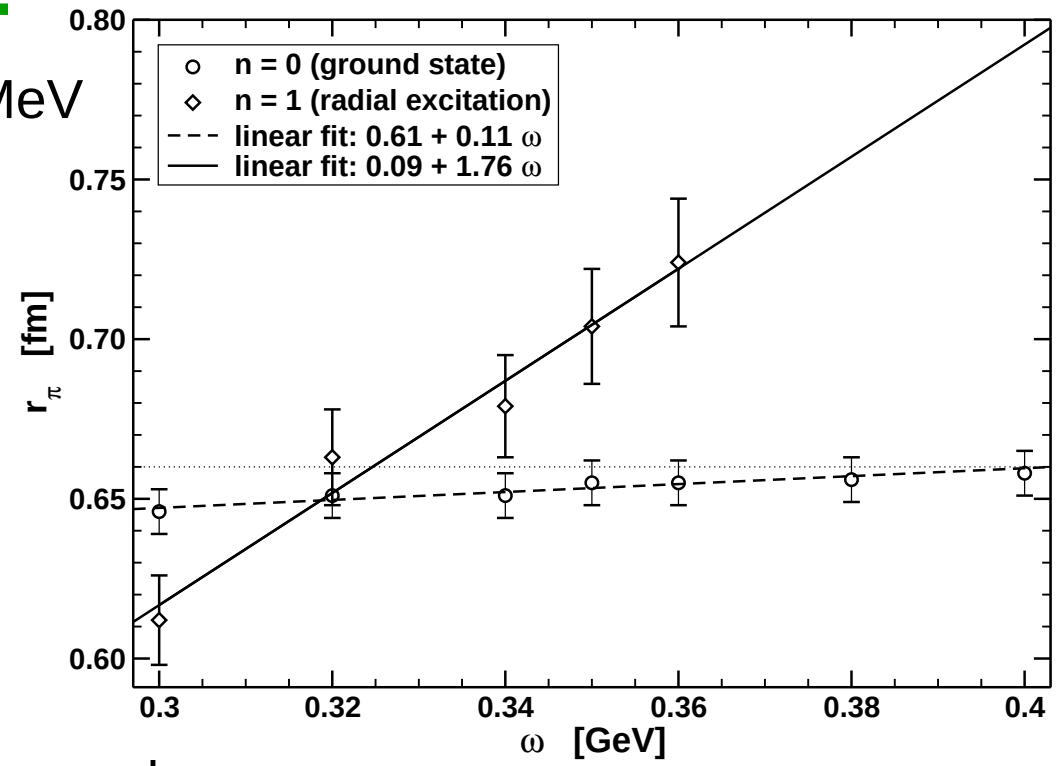


- $m_{u,d}(1\,\mathrm{GeV}) = 5.5$ MeV
- Reminder: MT-model has one IR-mass-scale – ω
  - $r_a := 1/\omega$ gauges the range of strong attraction
- Goldstone Mode's properties are insensitive to r_a
  - Expected cf. $T \neq 0$, Goldstone mode's properties do not change until very near chiral symmetry restoration temperature.

Electromagnetic Charge Radii – RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al.*, nu-th/0503043

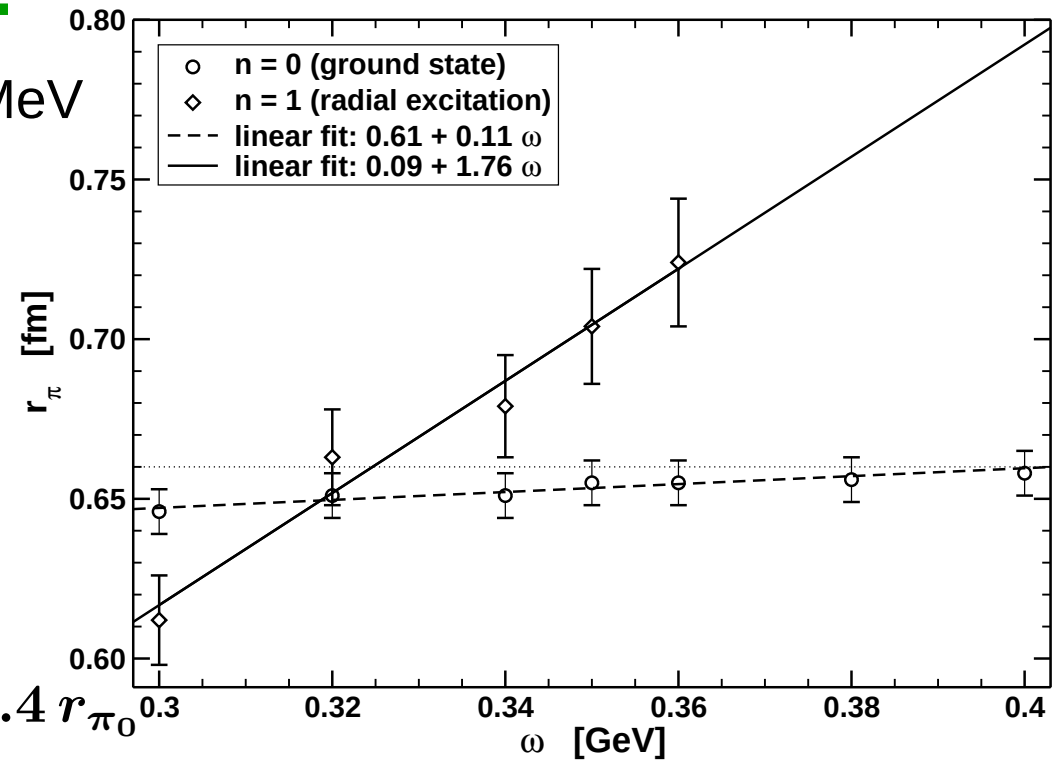
- $m_{u,d}(1\,\mathrm{GeV}) = 5.5$ MeV
- Reminder: MT-model has one IR-mass-scale – ω
  - $r_a := 1/\omega$ gauges the range of strong attraction
- 1st excited state: orthogonal to Goldstone mode
  - Not protected . . . properties very sensitive to r_a



Electromagnetic Charge Radii – RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al.*, nu-th/0503043

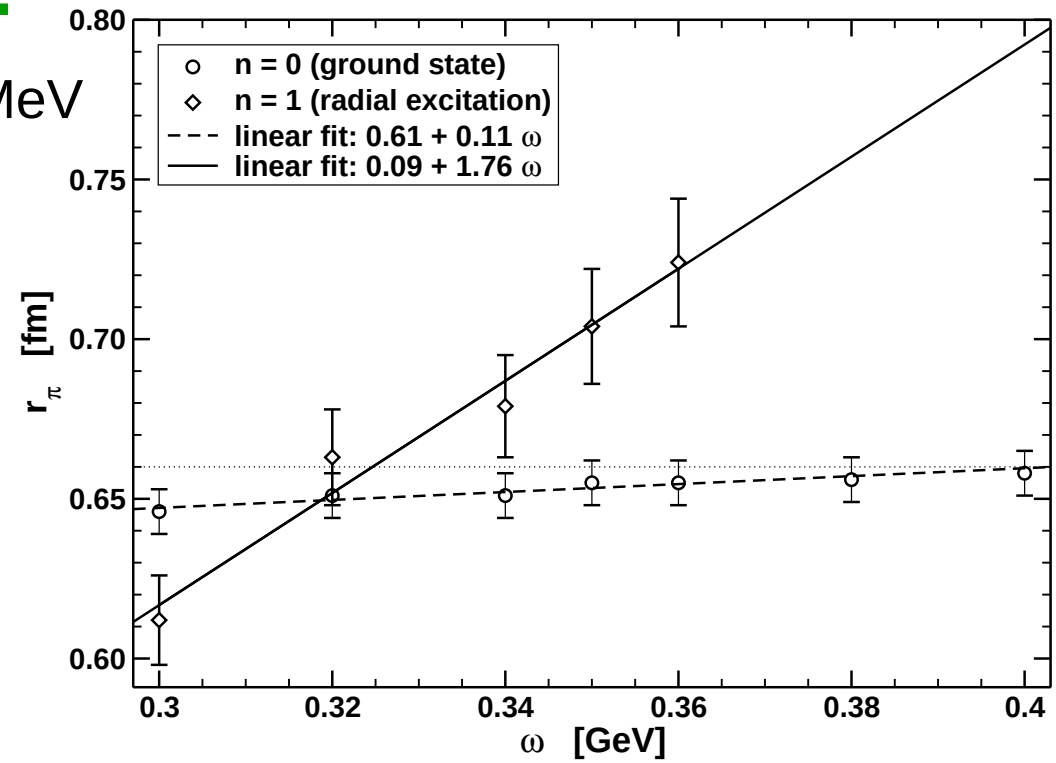
- $m_{u,d}(1\,\mathrm{GeV}) = 5.5$ MeV
- Reminder: MT-model has one IR-mass-scale – ω
  - $r_a := 1/\omega$ gauges the range of strong attraction
- Best estimate $r_{\pi_1} = 1.4\, r_{\pi_0}$
  - But $r_{\pi_1} < r_{\pi_0}$ is possible if confinement force is very strong



Electromagnetic Charge Radii – RGI Rainbow-Ladder

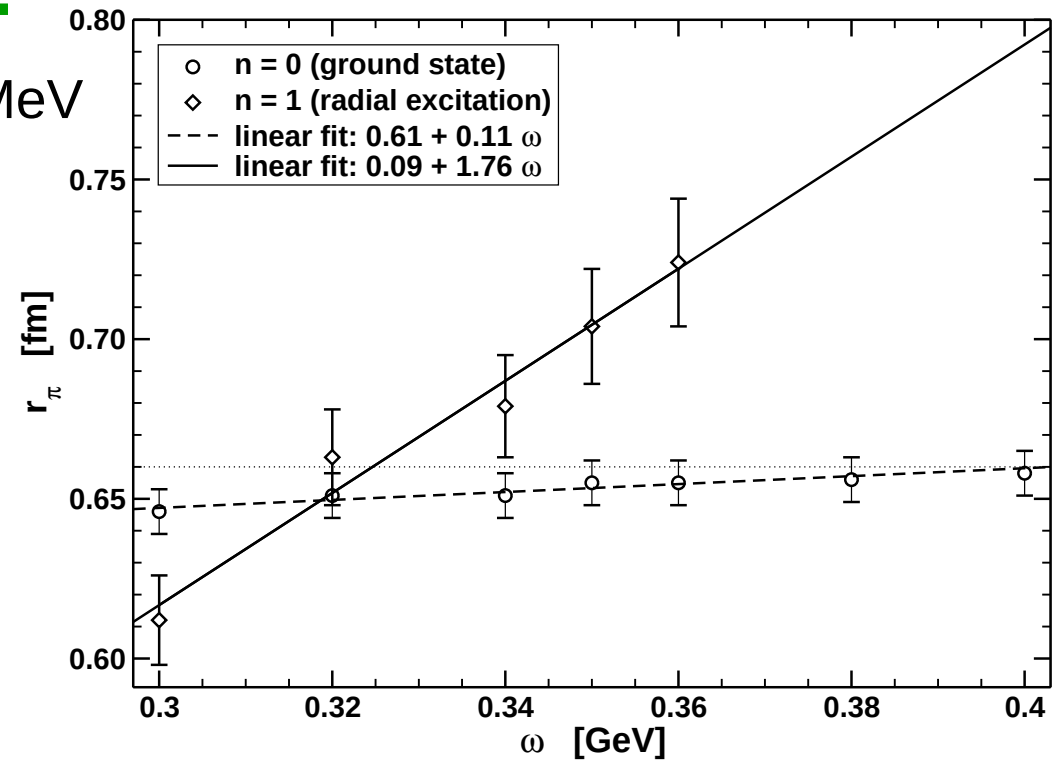
Höll, Krassnigg, Maris, *et al.*, nu-th/0503043

- $m_{u,d}(1\,\mathrm{GeV}) = 5.5$ MeV
- Reminder: MT-model has one IR-mass-scale – ω
  - $r_a := 1/\omega$ gauges the range of strong attraction
- Radial excitations are plainly useful to map out the long-range part of interaction between light-quarks.



Electromagnetic Charge Radii – RGI Rainbow-Ladder

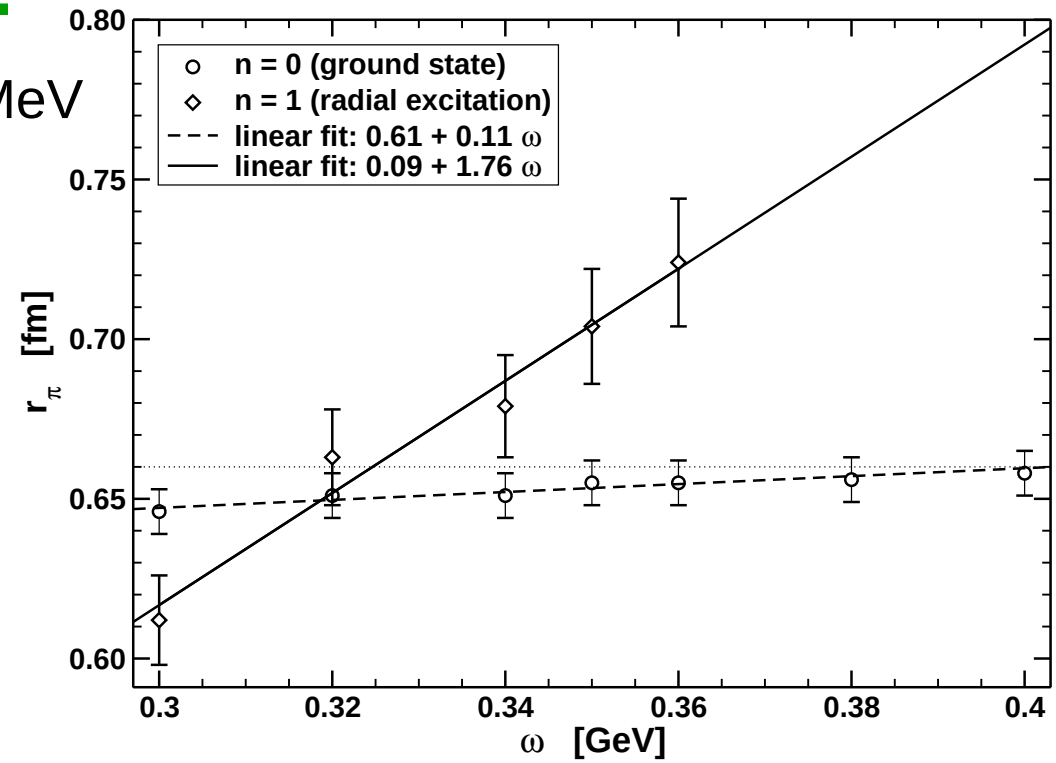
Höll, Krassnigg, Maris, *et al.*, nu-th/0503043



- $m_{u,d}(1\,\mathrm{GeV}) = 5.5$ MeV
- Reminder:
  MT-model has one IR-mass-scale – ω
  - $r_a := 1/\omega$ gauges the range of strong attraction
- Radial excitations are plainly useful to map out the long-range part of interaction between light-quarks.
- Same is true of orbital excitations; e.g., axial-vector mesons.

Electromagnetic Charge Radii – RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al.*, nu-th/0503043



- $m_{u,d}(1\,\mathrm{GeV}) = 5.5$ MeV
- Reminder:
  MT-model has one IR-mass-scale – ω
  - $r_a := 1/\omega$ gauges the range of strong attraction
- Radial excitations are plainly useful to map out the long-range part of interaction between light-quarks.
- Same is true of orbital excitations; e.g., axial-vector mesons.
- Hall-D at JLab

Pions and Form Factors

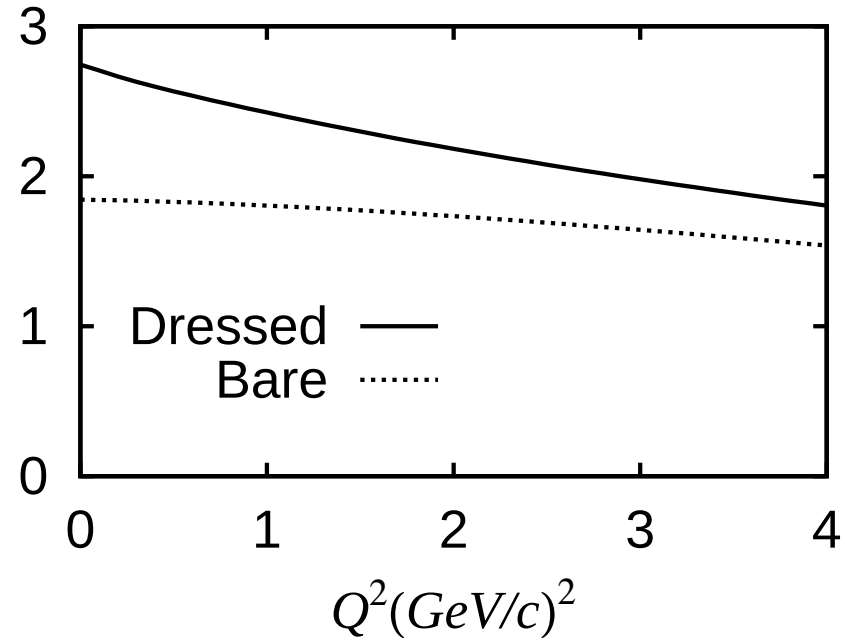
Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e, e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)

Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e, e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)
- Pion cloud effects are large in the low Q^2 region.

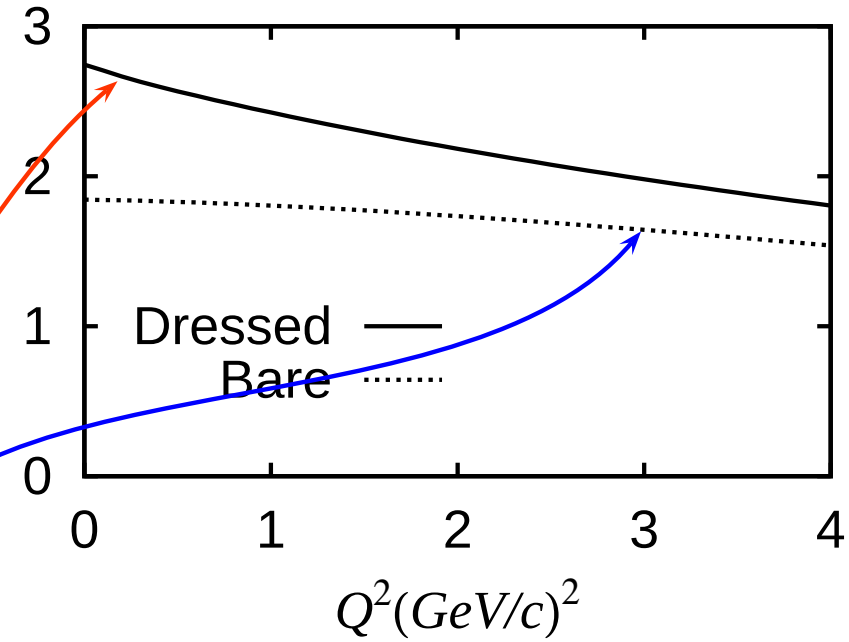
Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



Pions and Form Factors

- Dynamical coupled-channels model . . . Analyzed extensive JLab data . . . Completed a study of the $\Delta(1236)$
  - *Meson Exchange Model for πN Scattering and $\gamma N \to \pi N$ Reaction*, T. Sato and T.-S. H. Lee, Phys. Rev. C **54**, 2660 (1996)
  - *Dynamical Study of the Δ Excitation in $N(e, e'\pi)$ Reactions*, T. Sato and T.-S. H. Lee, Phys. Rev. C 63, 055201/1-13 (2001)
- Pion cloud effects are large in the low Q^2 region.

Ratio of the M1 form factor in $\gamma N \to \Delta$ transition and proton dipole form factor G_D. Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



Quark Core

- Responsible for only 2/3 of result at small Q^2
- Dominant for $Q^2 >$2 – 3 GeV^2

Parametrising diquark properties

Parametrising diquark properties

- Dressed-quark . . . fixed by DSE and Meson Studies

  . . . Burden, Roberts, Thomson, Phys. Lett. **B 371**, 163 (1996)

Parametrising diquark properties

- Bethe-Salpeter-Like Amplitudes

$$\Gamma^{0^+}(k;K) = \frac{1}{\mathcal{N}^{0^+}} H^a \, C i\gamma_5 \, i\tau_2 \, \mathcal{F}(k^2/\ \omega_{0^+}^2\ ),$$

$$\mathtt{t}^i \Gamma_\mu^{1^+}(k;K) = \frac{1}{\mathcal{N}^{1^+}} H^a \, i\gamma_\mu C \, \mathtt{t}^i \, \mathcal{F}(k^2/\ \omega_{1^+}^2\ )$$

Parametrising diquark properties

- Bethe-Salpeter-Like Amplitudes

$$\Gamma^{0^+}(k;K) = \frac{1}{\mathcal{N}^{0^+}} H^a \, C i\gamma_5 \, i\tau_2 \, \mathcal{F}(k^2/\ \omega_{0^+}^2\ ),$$

$$\mathtt{t}^i \Gamma_\mu^{1^+}(k;K) = \frac{1}{\mathcal{N}^{1^+}} H^a \, i\gamma_\mu C \, \mathtt{t}^i \, \mathcal{F}(k^2/\ \omega_{1^+}^2\ )$$

- Colour matrices:

$$\{H^1 = i\lambda^7, H^2 = -i\lambda^5, H^3 = i\lambda^2\}, \epsilon_{c_1 c_2 c_3} = (H^{c_3})_{c_1 c_2}$$

Parametrising diquark properties

- Bethe-Salpeter-Like Amplitudes

$$\Gamma^{0^+}(k;K) = \frac{1}{\mathcal{N}^{0^+}} H^a C i\gamma_5 \, i\tau_2 \, \mathcal{F}(k^2/\omega_{0^+}^2) \,,$$

$$\mathtt{t}^i \Gamma_\mu^{1^+}(k;K) = \frac{1}{\mathcal{N}^{1^+}} H^a \, i\gamma_\mu C \, \mathtt{t}^i \, \mathcal{F}(k^2/\omega_{1^+}^2)$$

- Two parameters: ω_{0^+}, ω_{1^+}

  $\sim$ Inverse of diquarks' configuration-space size

Parametrising diquark properties

- Pseudoparticle Propagators

$$\Delta^{0^+}(K) = \frac{1}{m_{0^+}^2} \mathcal{F}(K^2/\omega_{0^+}^2)\,,$$

$$\Delta^{1^+}_{\mu\nu}(K) = \left(\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1^+}^2}\right) \frac{1}{m_{1^+}^2} \mathcal{F}(K^2/\omega_{1^+}^2)$$

- $\mathcal{F}(x) = \dfrac{1-\exp(-x)}{x}$
  - Absence of a Spectral Representation
  - Realisation of Confinement

Parametrising diquark properties

- Pseudoparticle Propagators

$$\Delta^{0^+}(K) = \frac{1}{m_{0+}^2}\mathcal{F}(K^2/\omega_{0+}^2)\,,$$

$$\Delta^{1^+}_{\mu\nu}(K) = \left(\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1+}^2}\right)\frac{1}{m_{1+}^2}\mathcal{F}(K^2/\omega_{1+}^2)$$

- Two parameters: m_{0+}, m_{1+}

  $\sim$ Inverse of diquarks' configuration-space correlation length

Parametrising diquark properties

- Total of four parameters

. . . reduce that via Normalisation Condition

$$\frac{d}{dK^2}\left(\frac{1}{m_{J^P}^2}\,\mathcal{F}(K^2/\omega_{J^P}^2)\right)^{-1}\Bigg|_{K^2=0} = 1 \;\Rightarrow\; \omega_{J^P}^2 = \frac{1}{2}\,m_{J^P}^2\,,$$

Accentuates free-particle-like propagation characteristics of the diquarks within hadron.

Parametrising diquark properties

- Total of four parameters

. . . reduce that via Normalisation Condition

$$\frac{d}{dK^2}\left(\frac{1}{m_{J^P}^2}\,\mathcal{F}(K^2/\omega_{J^P}^2)\right)^{-1}\Bigg|_{K^2=0} = 1 \;\Rightarrow\; \omega_{J^P}^2 = \frac{1}{2}\,m_{J^P}^2\,,$$

Accentuates free-particle-like propagation characteristics of the diquarks within hadron.

- Two Parameter Faddeev Equation Model of Nucleon

Parametrising diquark properties

- Total of four parameters

. . . reduce that via Normalisation Condition

$$\frac{d}{dK^2}\left(\frac{1}{m_{J^P}^2}\,\mathcal{F}(K^2/\omega_{J^P}^2)\right)^{-1}\Bigg|_{K^2=0} = 1 \;\Rightarrow\; \omega_{J^P}^2 = \frac{1}{2}\,m_{J^P}^2\,,$$

Accentuates free-particle-like propagation characteristics of the diquarks within hadron.

- Two Parameter Faddeev Equation Model of Nucleon
- Solve Faddeev Equation

Parametrising diquark properties

- Total of four parameters

. . . reduce that via Normalisation Condition

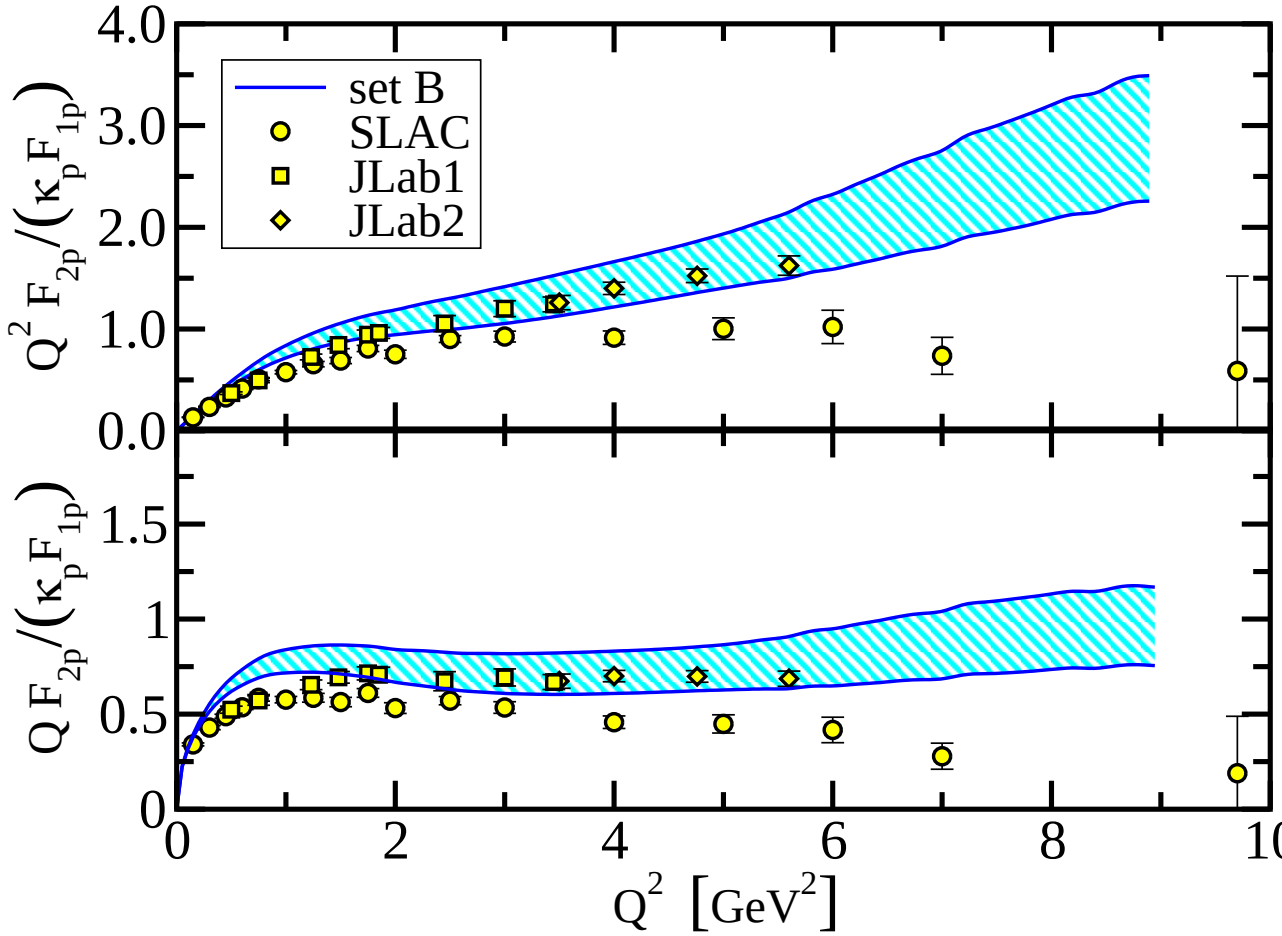
$$\frac{d}{dK^2}\left(\frac{1}{m_{J^P}^2}\,\mathcal{F}(K^2/\omega_{J^P}^2)\right)^{-1}\Bigg|_{K^2=0} = 1 \;\Rightarrow\; \omega_{J^P}^2 = \frac{1}{2}\,m_{J^P}^2\,,$$

Accentuates free-particle-like propagation characteristics of the diquarks within hadron.

- Two Parameter Faddeev Equation Model of Nucleon
- Solve Faddeev Equation
- Vary m_{0^+} and m_{1^+} to obtain desired masses for N and Δ

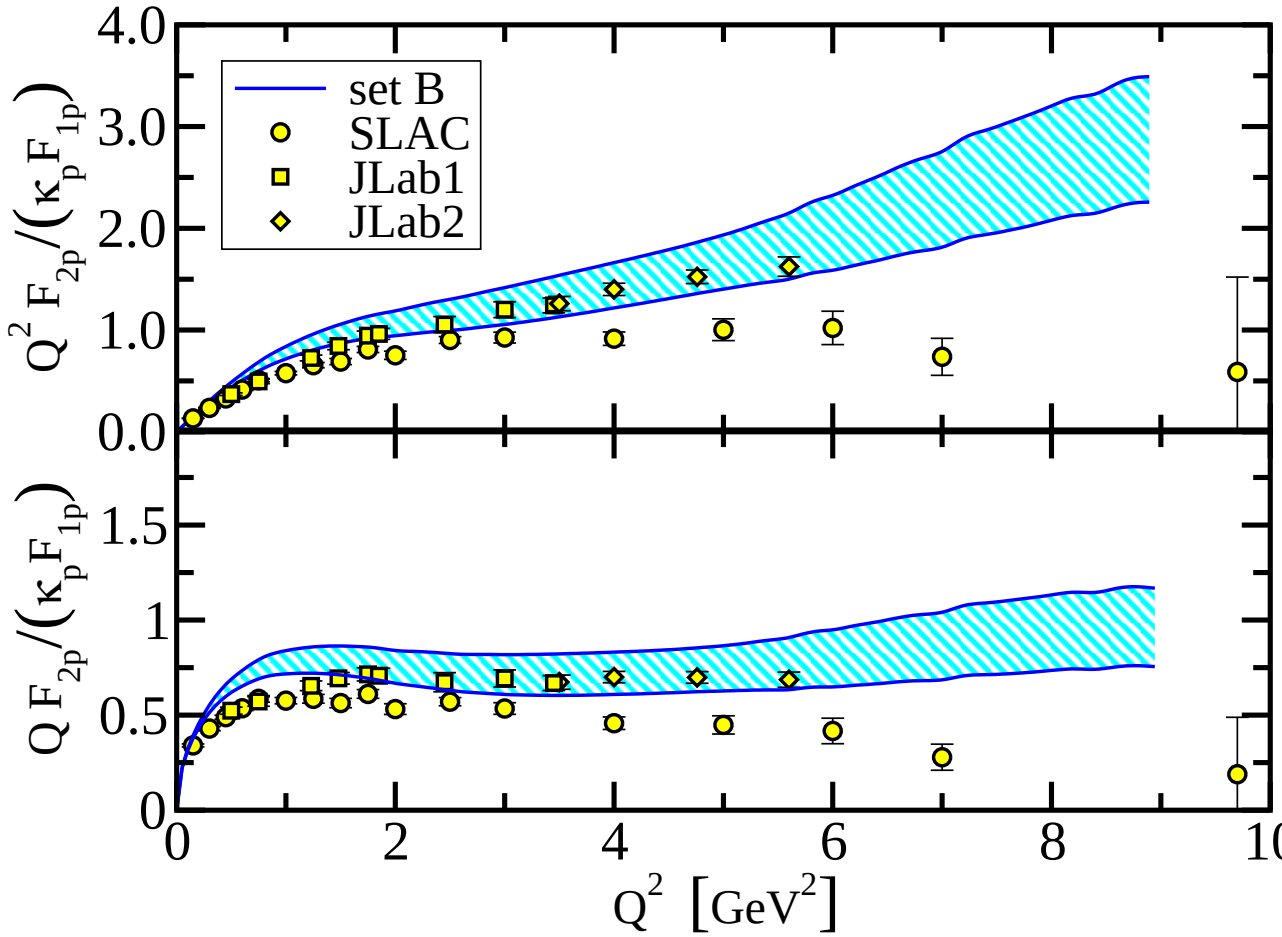
Form Factor Ratio: $Q * F2/F1$

Form Factor Ratio: $Q * F2/F1$



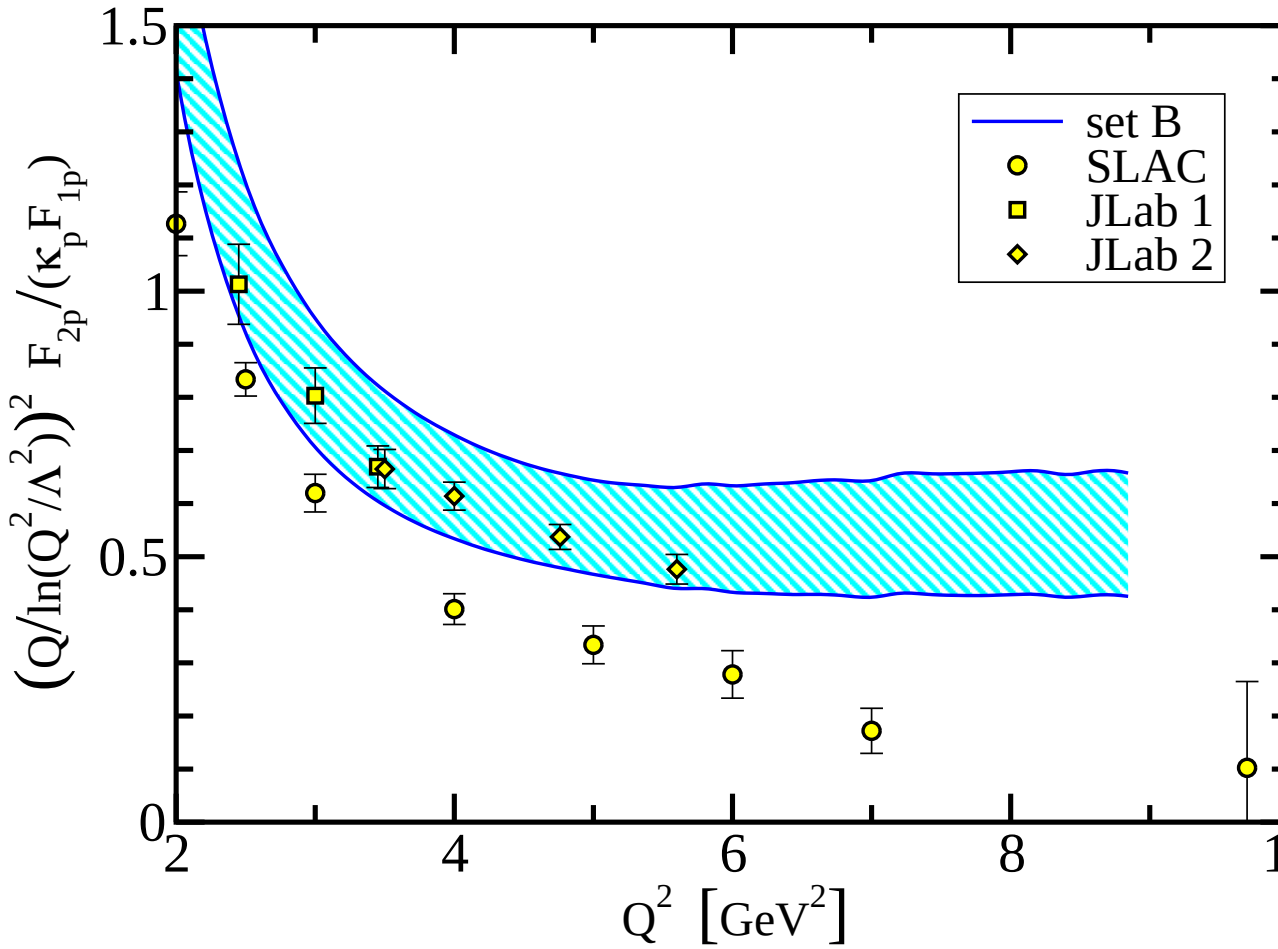
Form Factor Ratio: $Q * F2/F1$

- Perhaps $\approx$ constant for $2 \lesssim Q^2 \lesssim 6\text{GeV}^2$



FormIn Factor Ratio: alternative F2/F1

FormIn Factor Ratio: alternative F2/F1

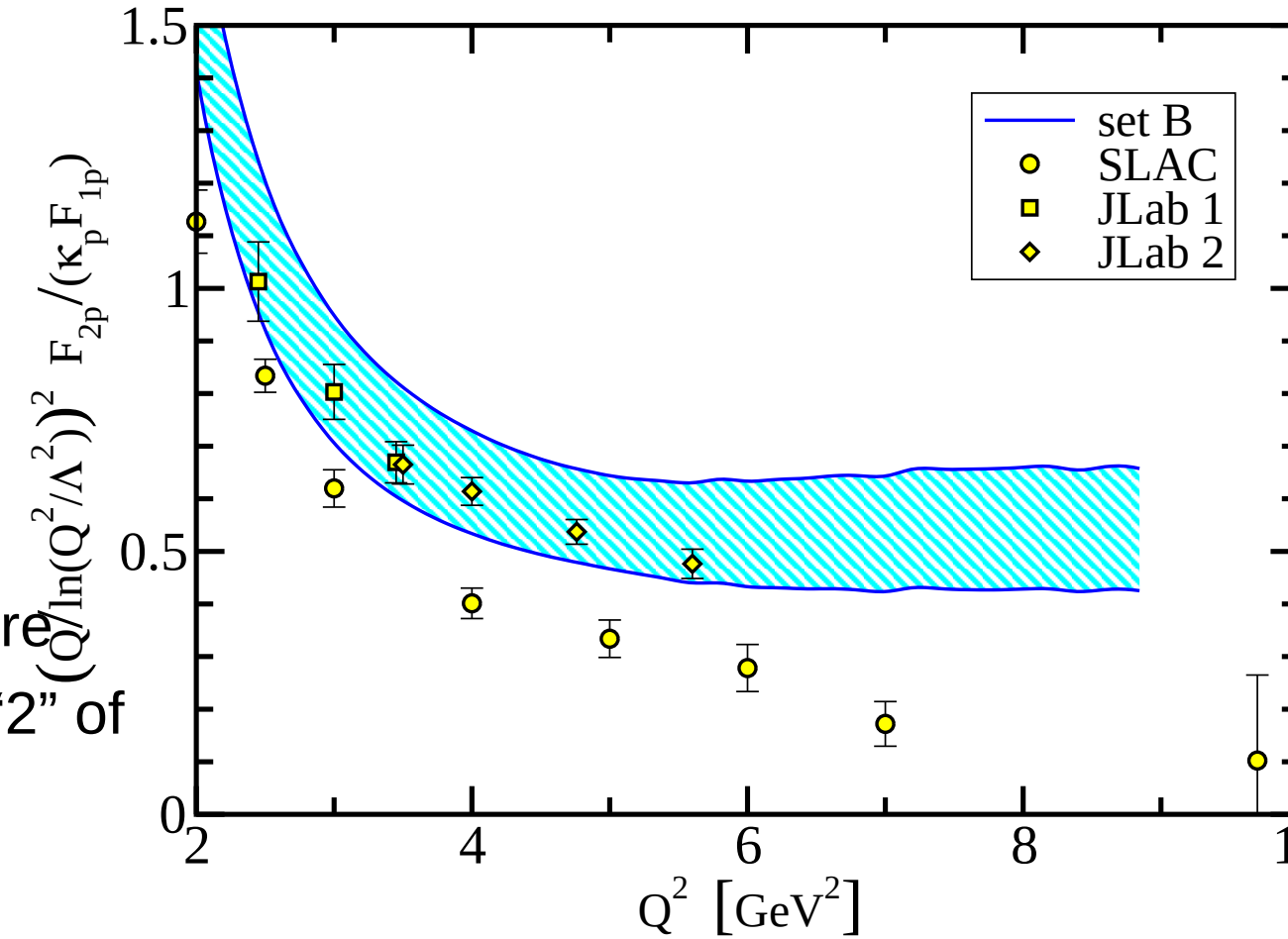


FormIn Factor Ratio: alternative F2/F1

- $$\frac{Q^2}{[\ln Q^2/\Lambda^2]^2}\,\frac{F_2(Q^2)}{F_1(Q^2)} = \text{constant}, \quad Q^2 \gg \Lambda^2 \approx M_N^2$$

Suggestive

NB. Framework constructed to give quark-counting i.e., "pQCD" *but* with wrong anomalous dimensions *but* they're ignored in *ln*-power "2" of this ratio

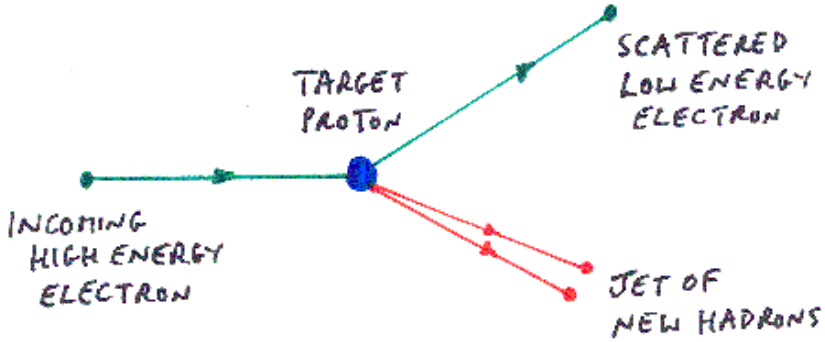


Deep-inelastic scattering

Deep-inelastic scattering

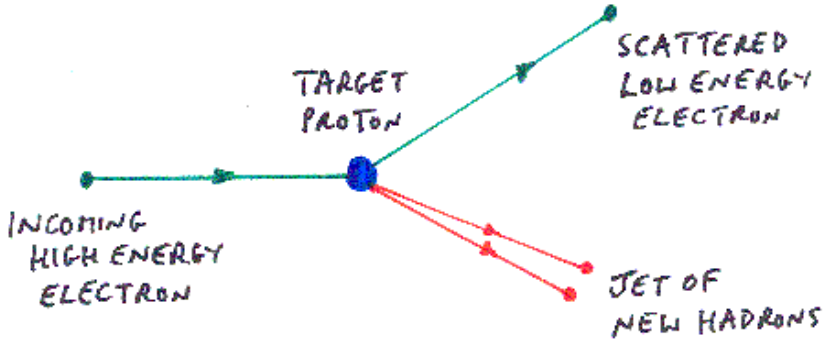
- Looking for Quarks

Deep-inelastic scattering



- Looking for Quarks

Deep-inelastic scattering

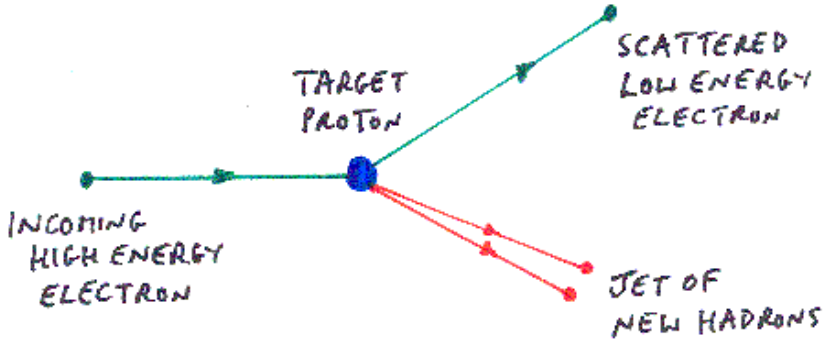


- Looking for Quarks

- Signature Experiment for QCD:

  Discovery of Quarks at SLAC

Deep-inelastic scattering



- Looking for Quarks

- Signature Experiment for QCD:

  Discovery of Quarks at SLAC

- Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x)$, $g(x)$

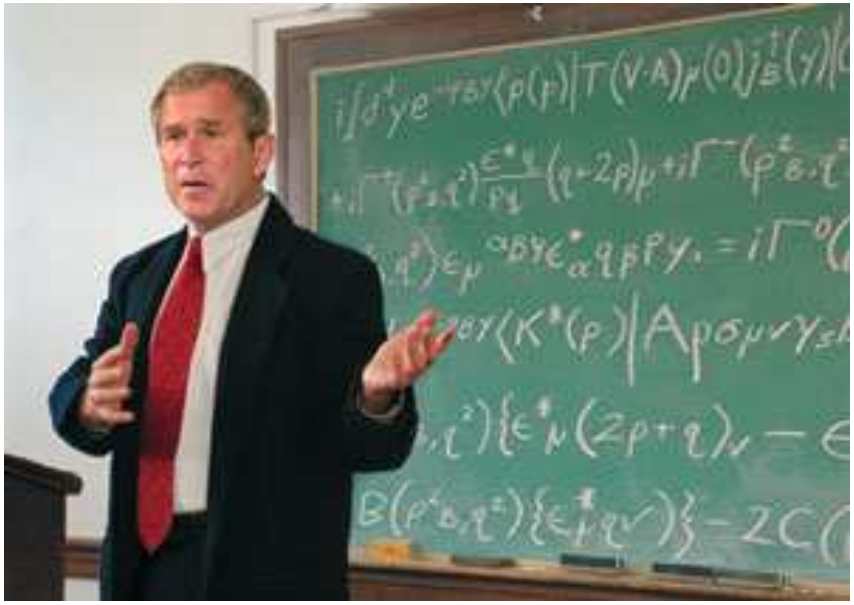
Pion's valence quark distn

Pion's valence quark distn

- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!

Pion's valence quark distn

- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!
- Proved on 22/July/2002, ANL



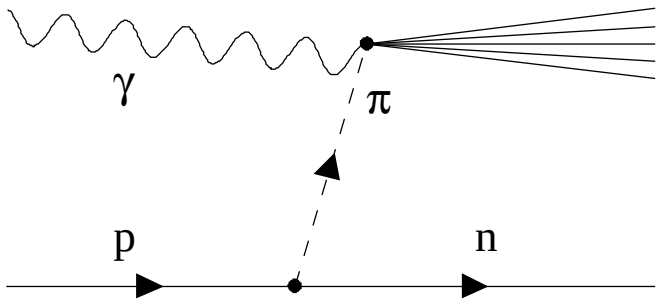
Pion's valence quark distn

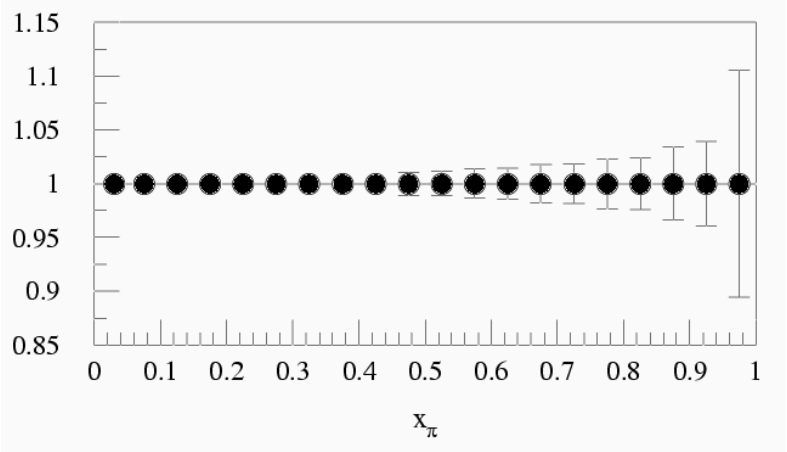
- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!
- Existing Measurement Inferred from Drell-Yan:
  $\pi N \to \mu^+ \mu^- X$

Pion's valence quark distn

- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!
- Existing Measurement Inferred from Drell-Yan:
  $\pi N \to \mu^+\mu^- X$
- Proposal (Holt & Reimer, ANL, nu-ex/0010004)

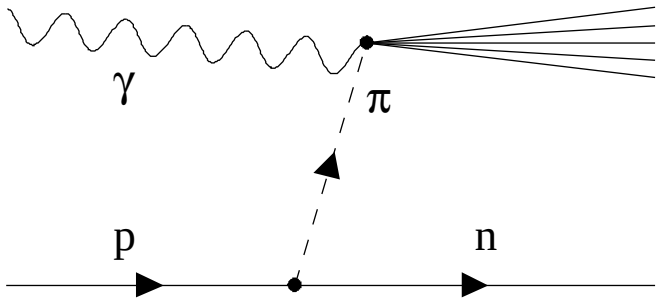
$e^-_{5\mathrm{GeV}} - p_{25\,\mathrm{GeV}}$ Collider $\to$ Accurate "Measurement"

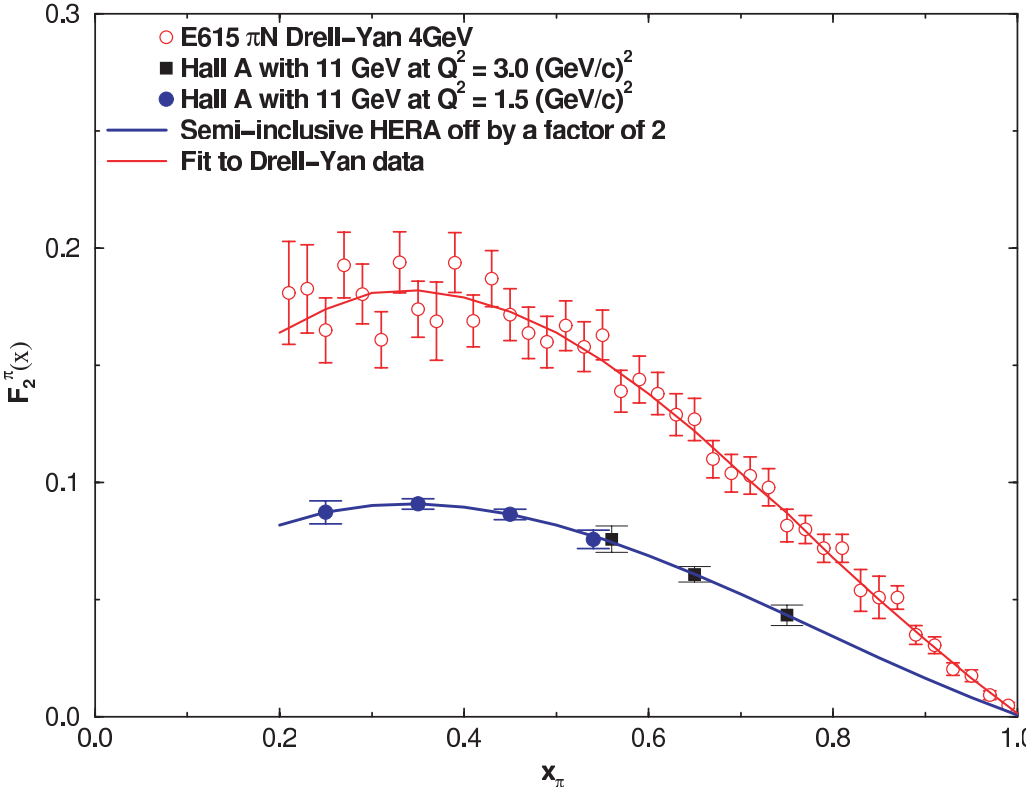




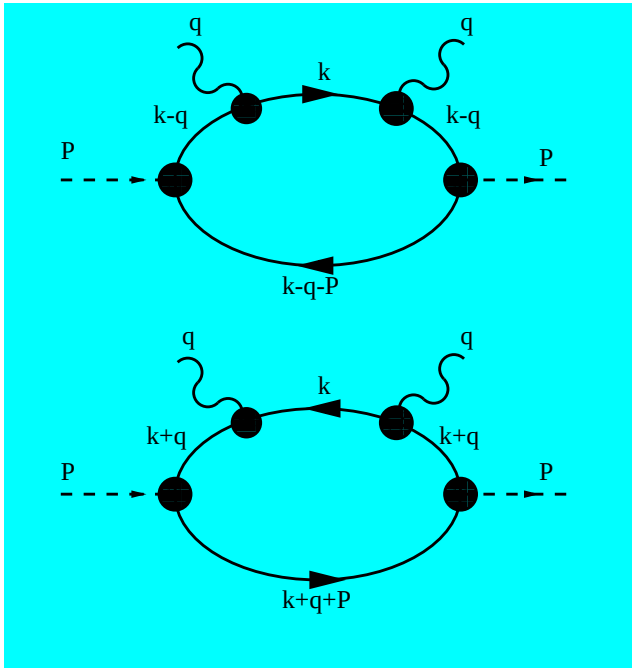
Pion's valence quark distn

- Proposal at JLab
  (Holt, Reimer, Wijesooriya, et al., JLab at 12 GeV)

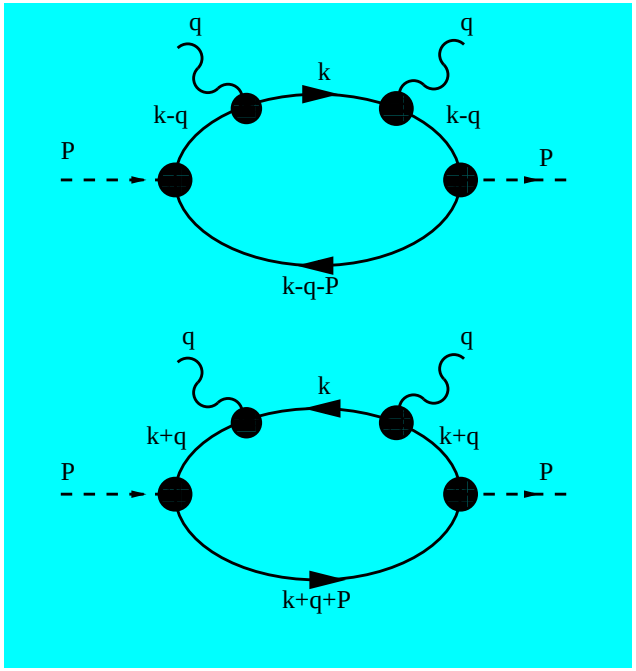




Handbag diagrams



Handbag diagrams



$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

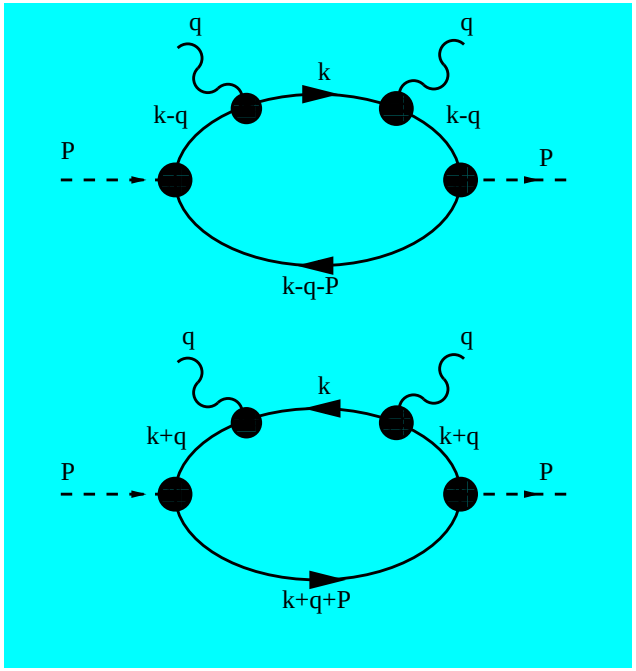
$$T^{+}_{\mu\nu}(q,P) = \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\,\tau_{-}\bar{\Gamma}_{\pi}(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_{\nu}(k_{-0},k)$$
$$\times\, S(k)\,ieQ\Gamma_{\mu}(k,k_{-0})\,S(k_{-0})\tau_{+}\Gamma_{\pi}(k_{-\frac{1}{2}};P)\,S(k_{--})$$

Handbag diagrams

Bjorken Limit: $q^2 \to \infty\,, \quad P \cdot q \to -\infty$

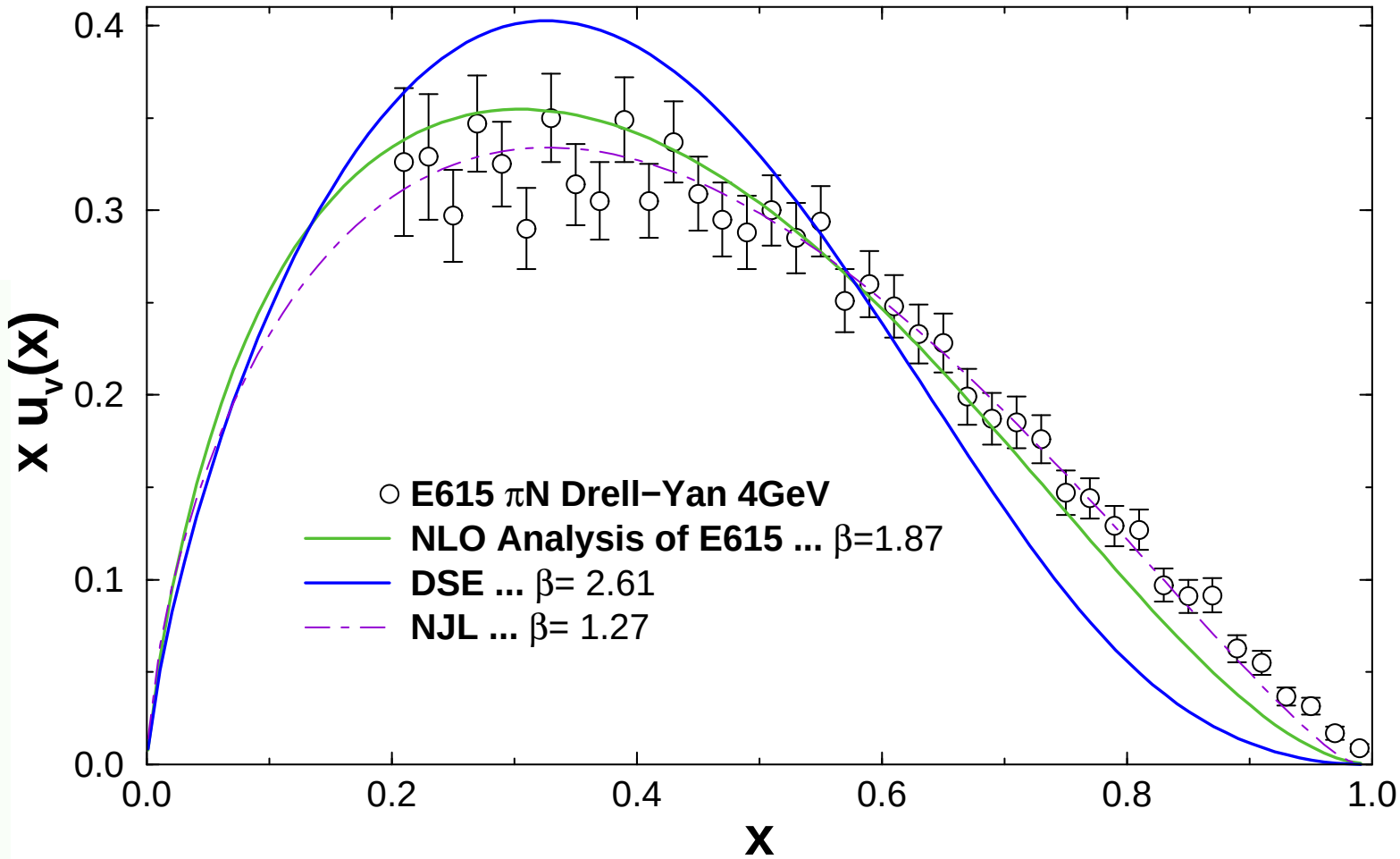
but $x := -\dfrac{q^2}{2P \cdot q}$ fixed.

Numerous algebraic simplifications



$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

$$T^{+}_{\mu\nu}(q,P) = \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\,\tau_{-}\bar{\Gamma}_{\pi}(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_{\nu}(k_{-0},k)$$
$$\times\, S(k)\,ieQ\Gamma_{\mu}(k,k_{-0})\,S(k_{-0})\tau_{+}\Gamma_{\pi}(k_{-\frac{1}{2}};P)\,S(k_{--})$$

Extant theory vs. experiment

K. Wijersooriya, P. Reimer and R. Holt, nu-ex/0509012 ... Phys. Rev. C (Rapid)