Emergent Phenomena via Dyson-Schwinger Equations

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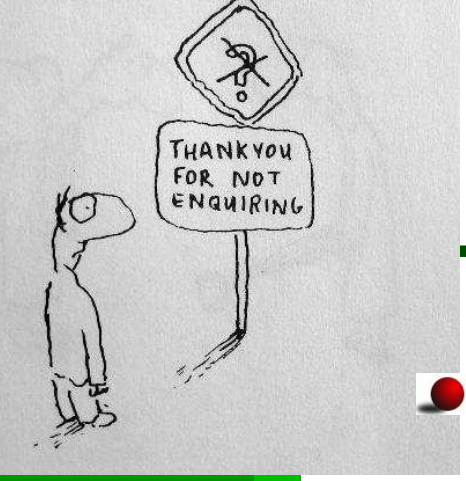
http://www.phy.anl.gov/theory/FritzFest/Fritz.html

QCD's Emergent Phenomena



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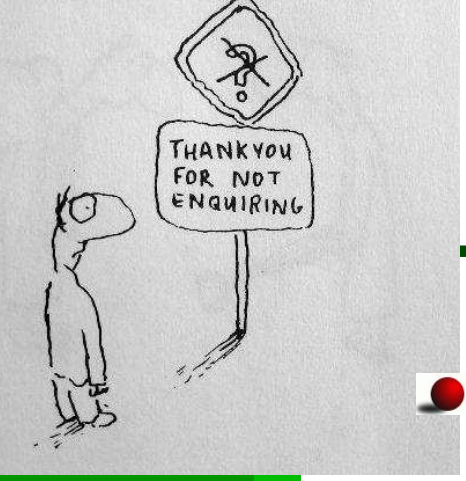

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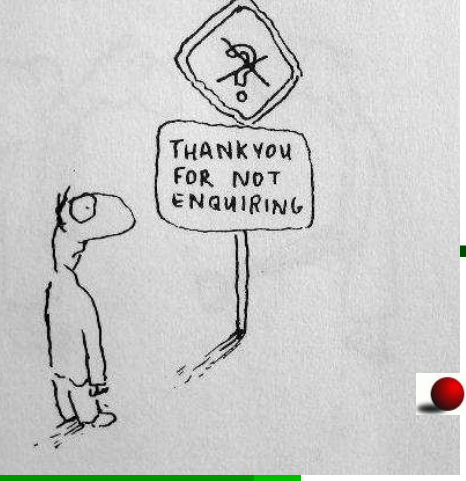

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- NSAC – Understanding these phenomena is one of the greatest intellectual challenges in physics

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Using DSEs, we've provided this.

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  - Allowed ... $J^{PC} = 0^{++}, 0^{-+}, 1^{--}, 1^{+-}, 1^{++}, 2^{++}$
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- Constituent-like gluon & $\bar{Q}$ & Q can also be combined to produce the *Allowed* quantum numbers
  - *Hybrids* or *glueballs*.

Pseudoscalar & Vector Mesons

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Hence, the question: What is the long-range interaction between light-quarks; viz., what is the Hamiltonian?

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NB. Hadron Physics Milestone, 2012: Measure the electromagnetic excitations of low-lying hadrons and their transition form factors.

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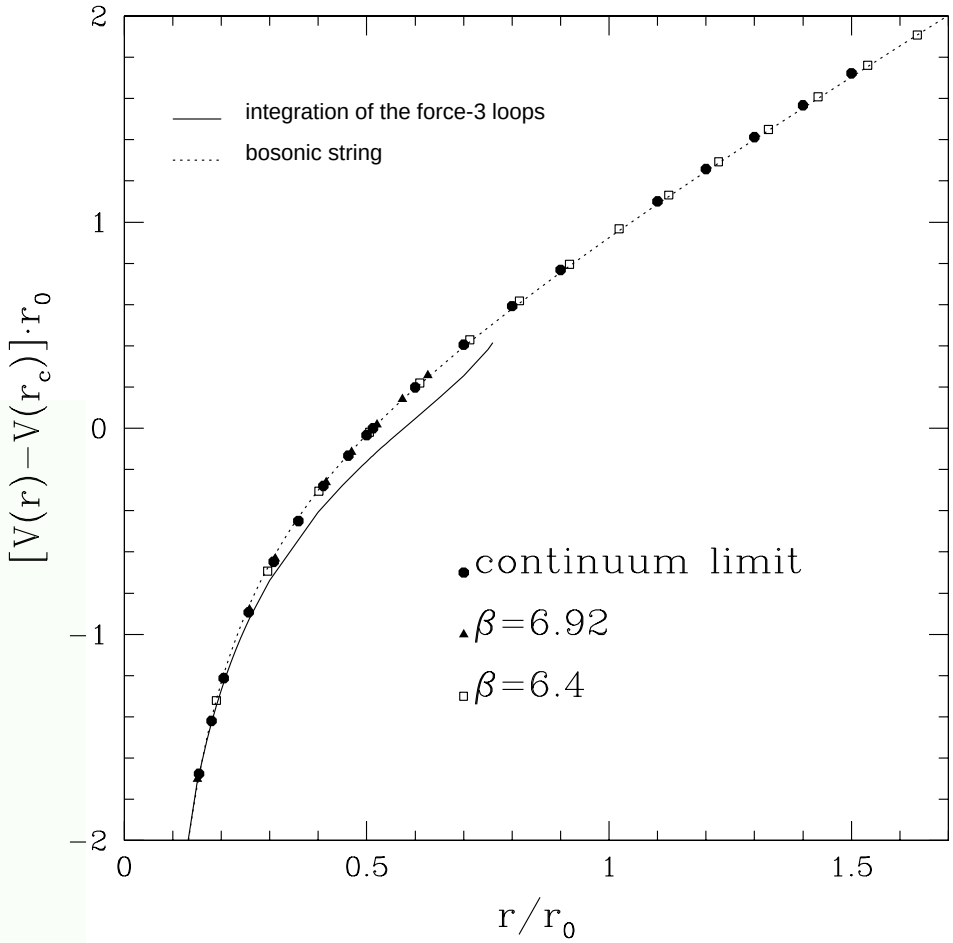
- Infinitely Heavy Quarks . . . Picture in Quantum Mechanics



$$V(r) = \sigma\, r - \frac{\pi}{12}\,\frac{1}{r}$$

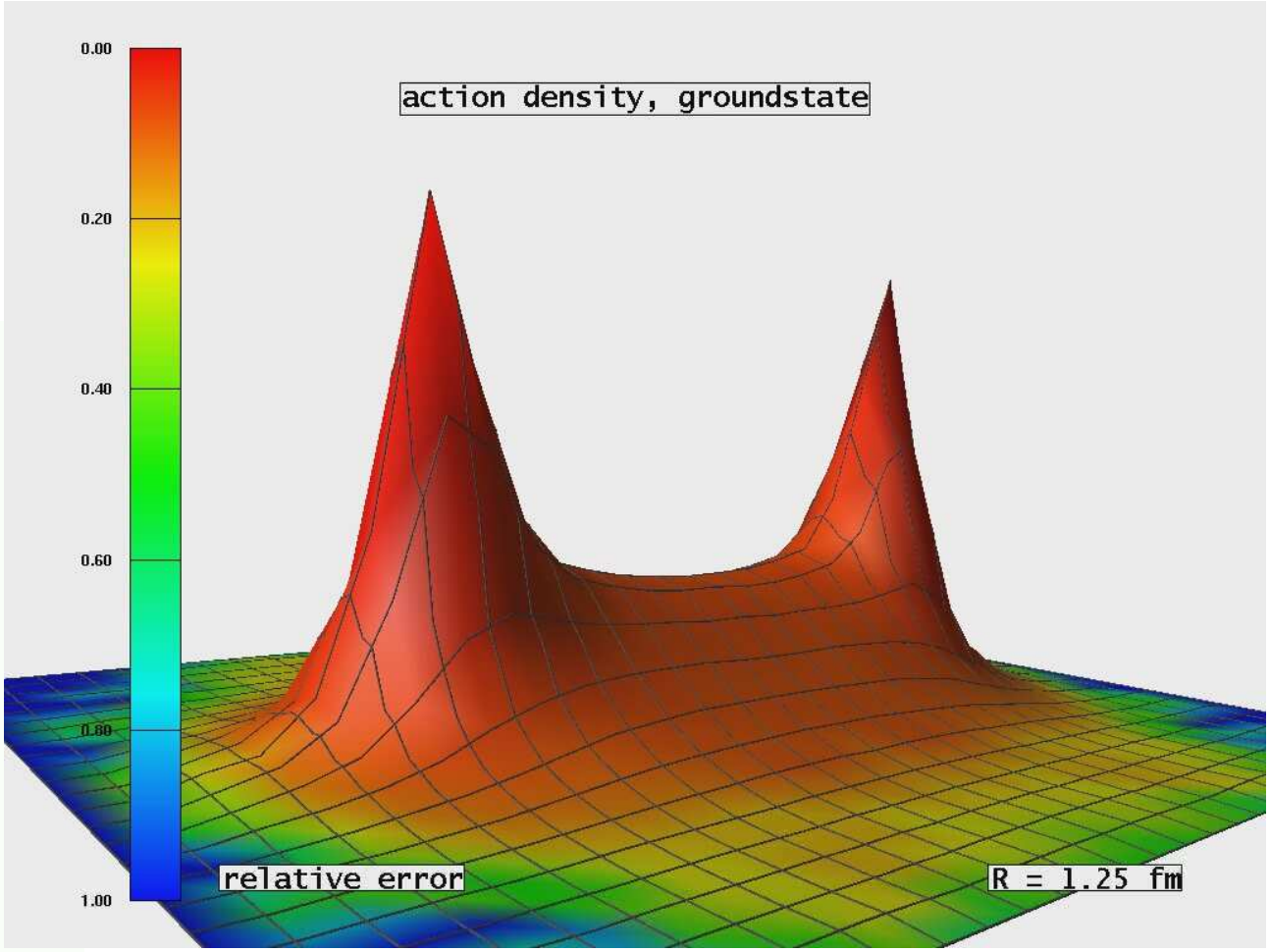
$\sigma \sim 470\,\text{MeV}$

Necco & Sommer

he-la/0108008

Confinement

- Illustrate this in terms of the action density . . . analogous to plotting the Force $= F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12}\frac{1}{r^2}$



Bali, *et al.*
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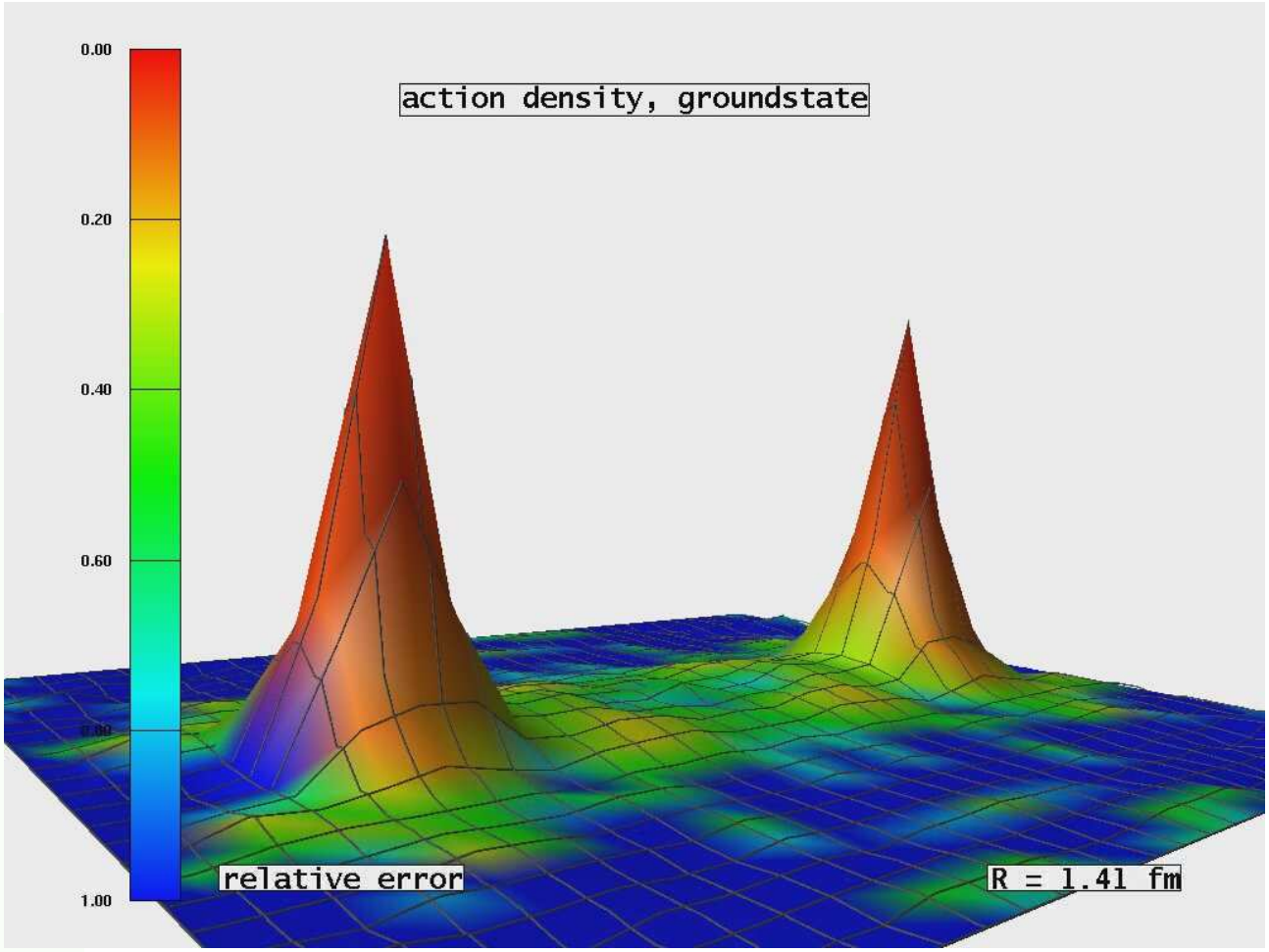

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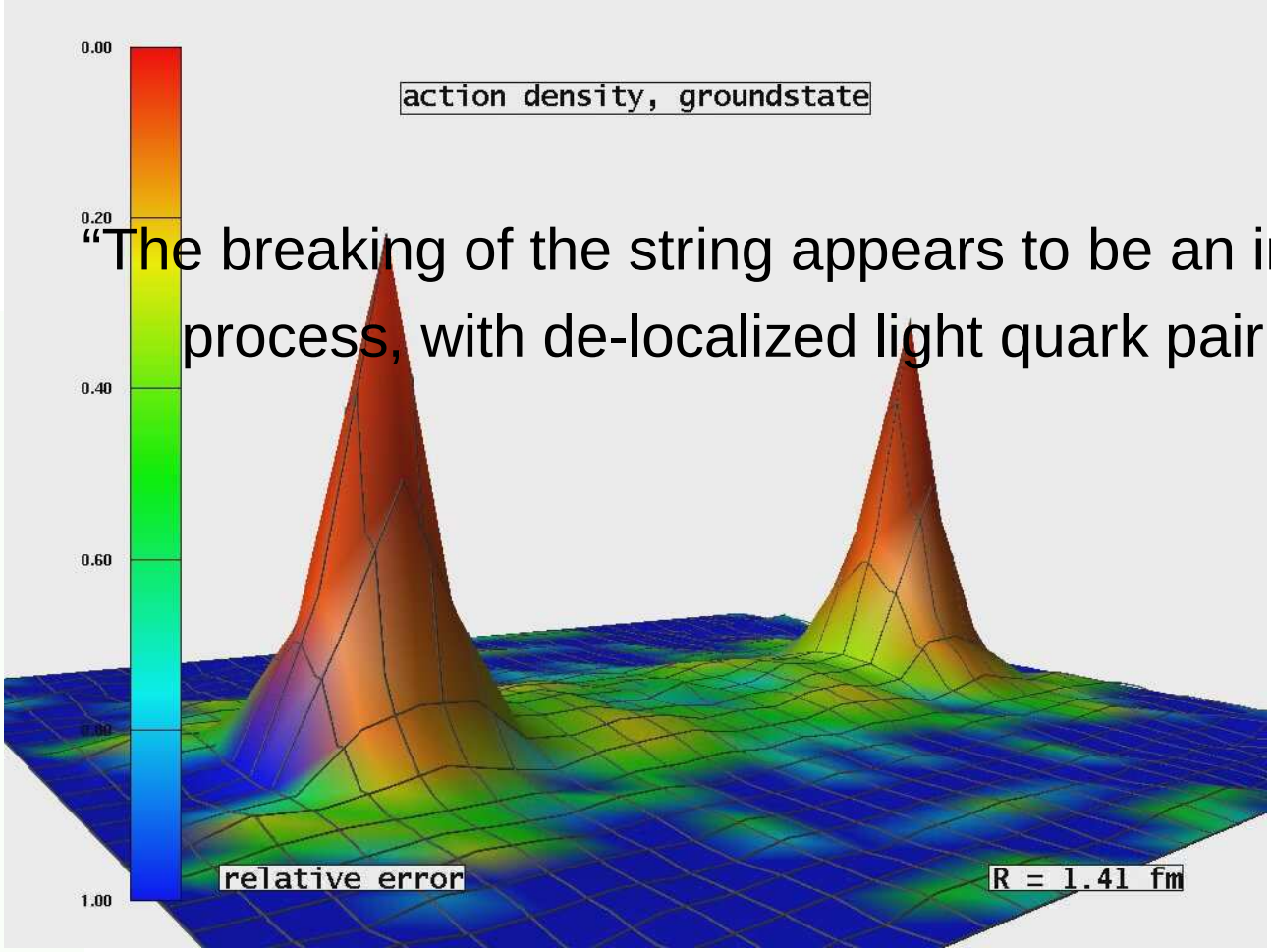

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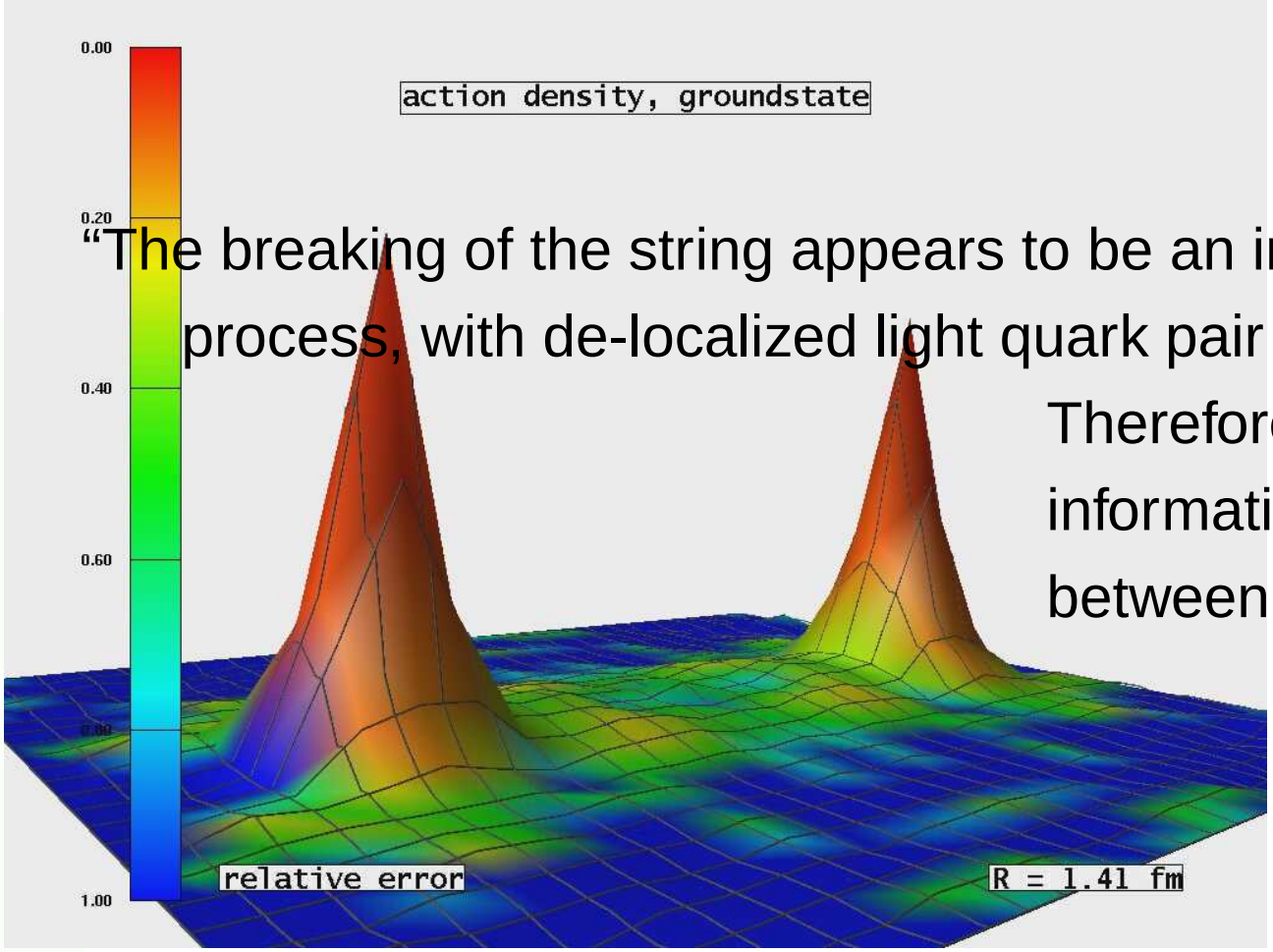

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Therefore . . . No information on *potential* between light-quarks.

a_1 & b_1 Decays

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- First three decays have $J = 1$ meson in both initial and final states ... they therefore proceed in two partial waves: S , D

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- Conclusion: D/S ratio is very sensitive to Hamiltonian's structure and $^3P_0$ model is superior.

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Lattice studies McNeile; he-la/0307027

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- Signal to noise ratio $S/N \sim \exp[-(m_H - m_\pi)]$:
  - Smaller for P-wave mesons than for ground-state S-waves because P-wave mesons are heavier;
  - Especially bad for realistic current-quark masses.

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  - large errors;
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- Result
  - $\boldsymbol{a_1(1230 \pm 20)}$
  - $\boldsymbol{b_1(1300 \pm 20)}$

NB. The *quoted* errors are better than experimental!

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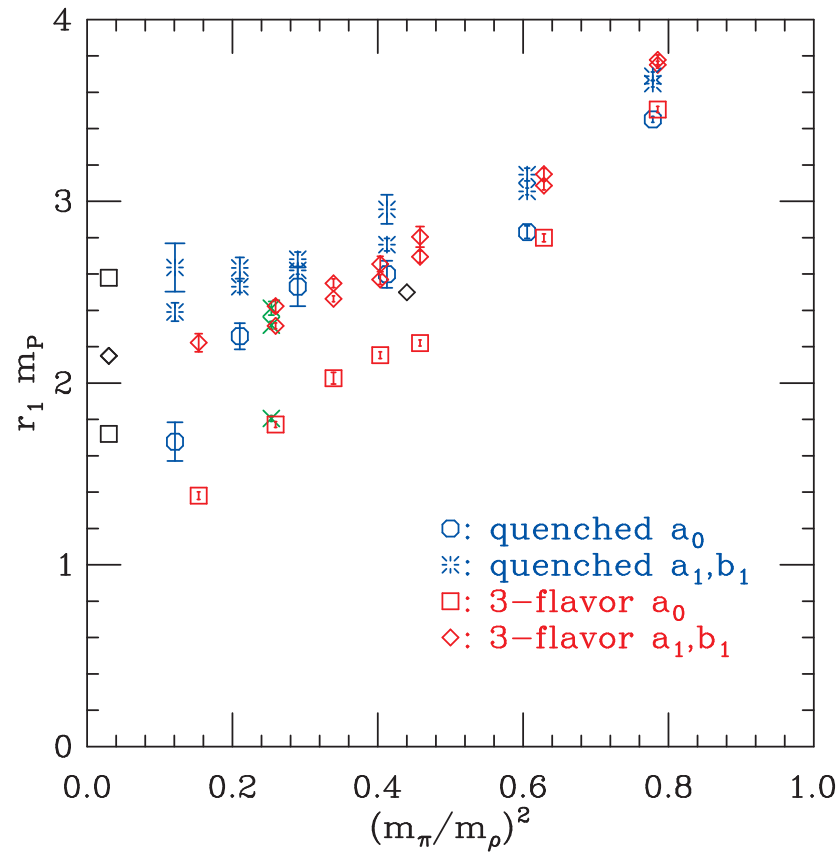
- MILC; he-la/0104002, $20^3 \times 64$, $1/r_1 = 0.563\,\mathrm{GeV}$
  - large errors;
  - small minimum distances to get good fits;
  - plateaux in the effective mass are short.
- Result
  - $\boldsymbol{a_1(1230 \pm 20)}$
  - $\boldsymbol{b_1(1300 \pm 20)}$

NB. The *quoted* errors are better than experimental! However, the figures presented indicate that uncontrolled systematic error much larger than statistical.



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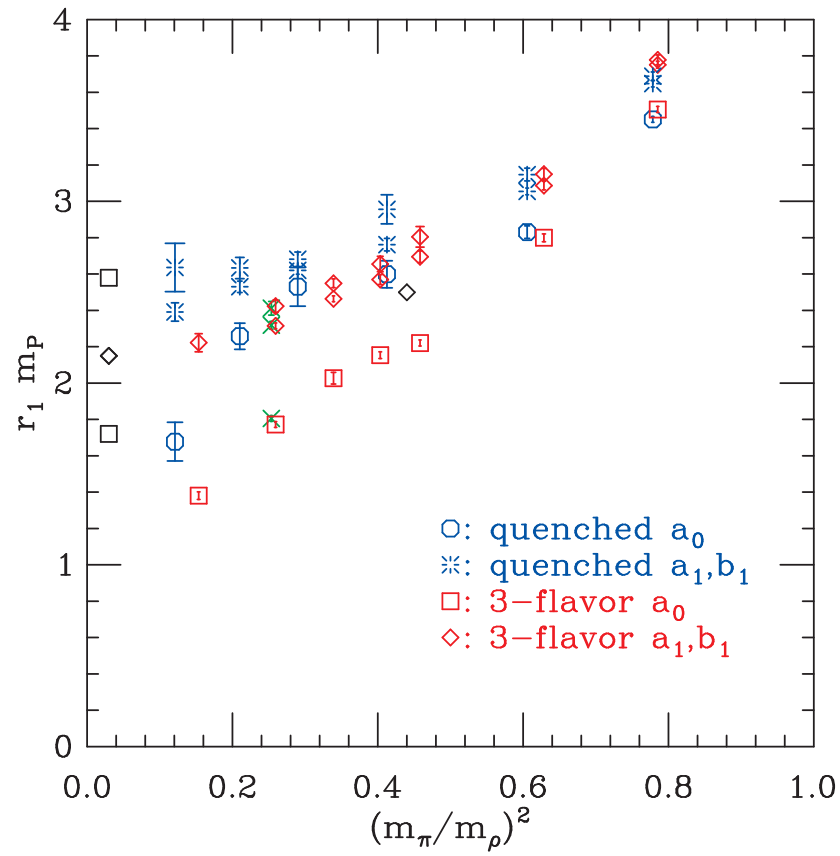

NB. $a_1 \to \rho\pi$ impossible with parameters of this simulation.

Dyson-Schwinger Equations

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- $\Rightarrow$ Understanding InfraRed (long-range)
  ..... behaviour of $\alpha_s(Q^2)$

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- Method yields Schwinger Functions $\equiv$ Propagators

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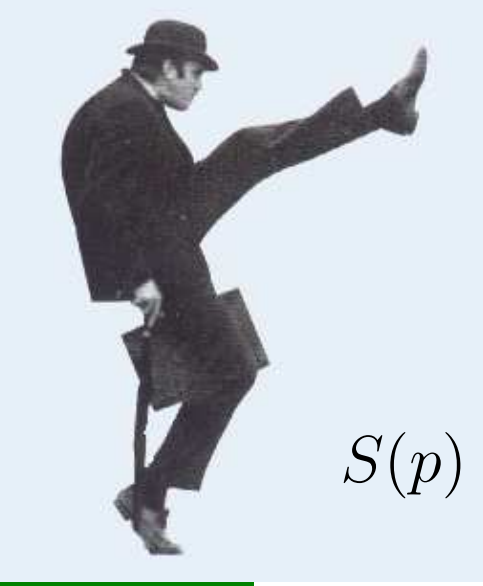
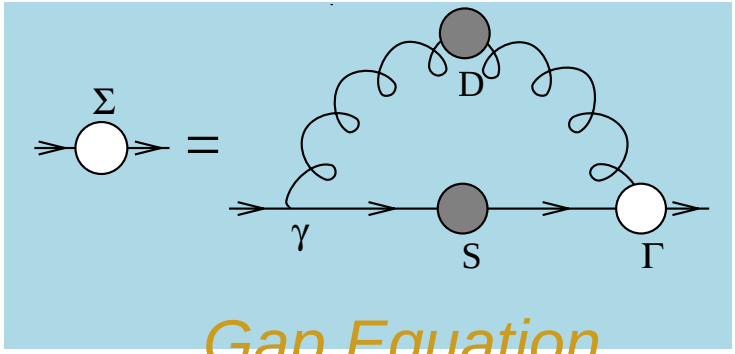
Cross-Sections built from Schwinger Functions

Perturbative Dressed-quark Propagator

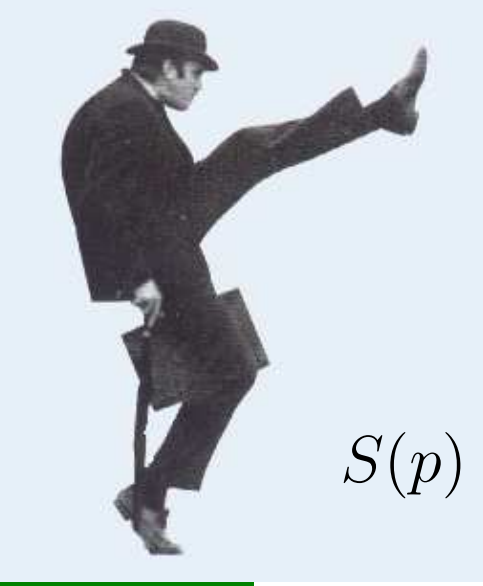


Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation



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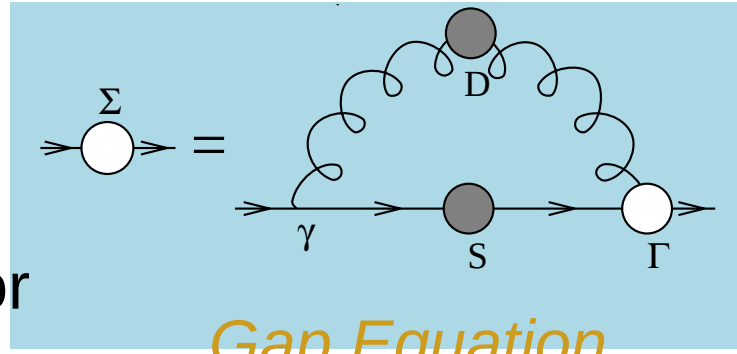

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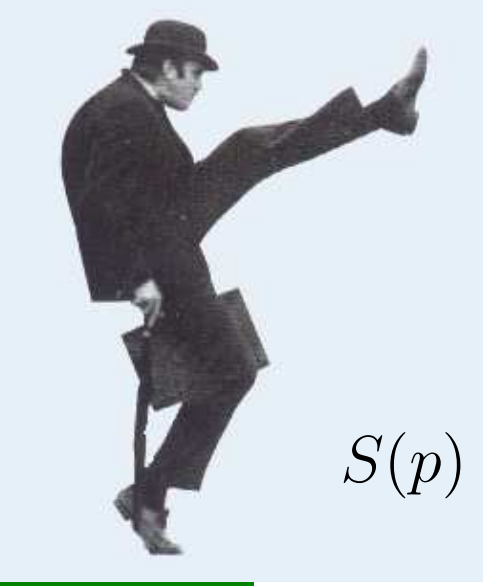

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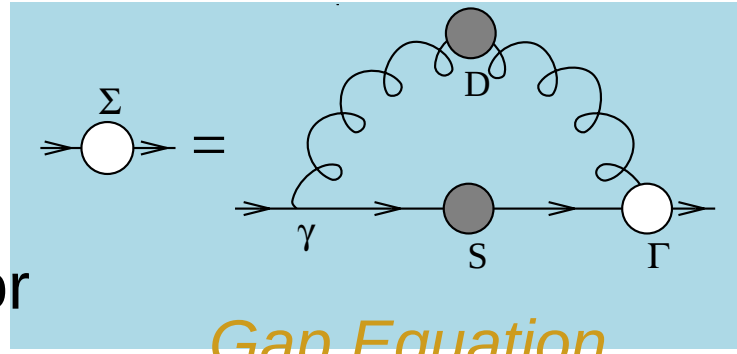

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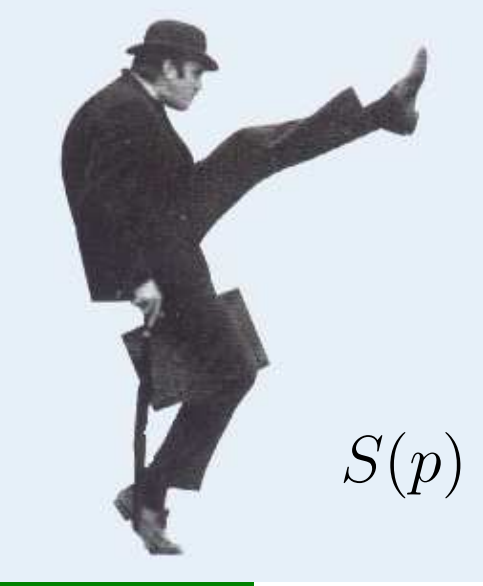

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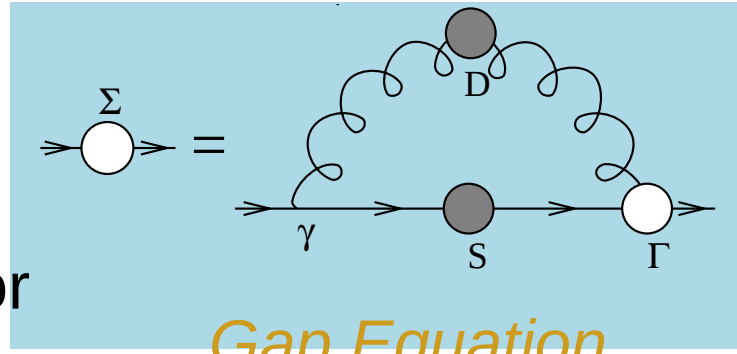

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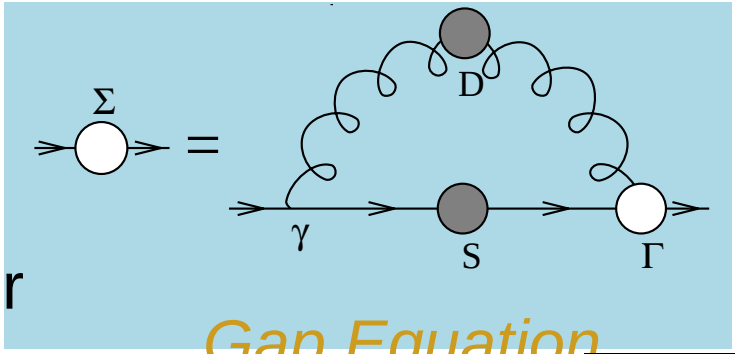

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No DCSB Here!

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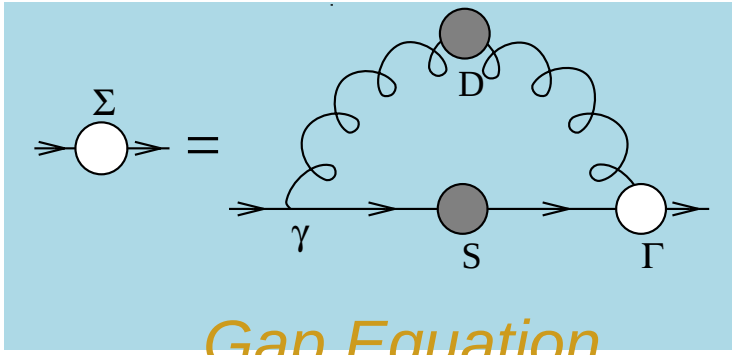

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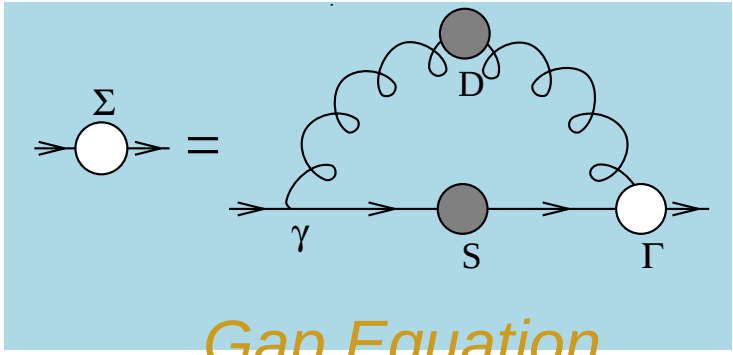

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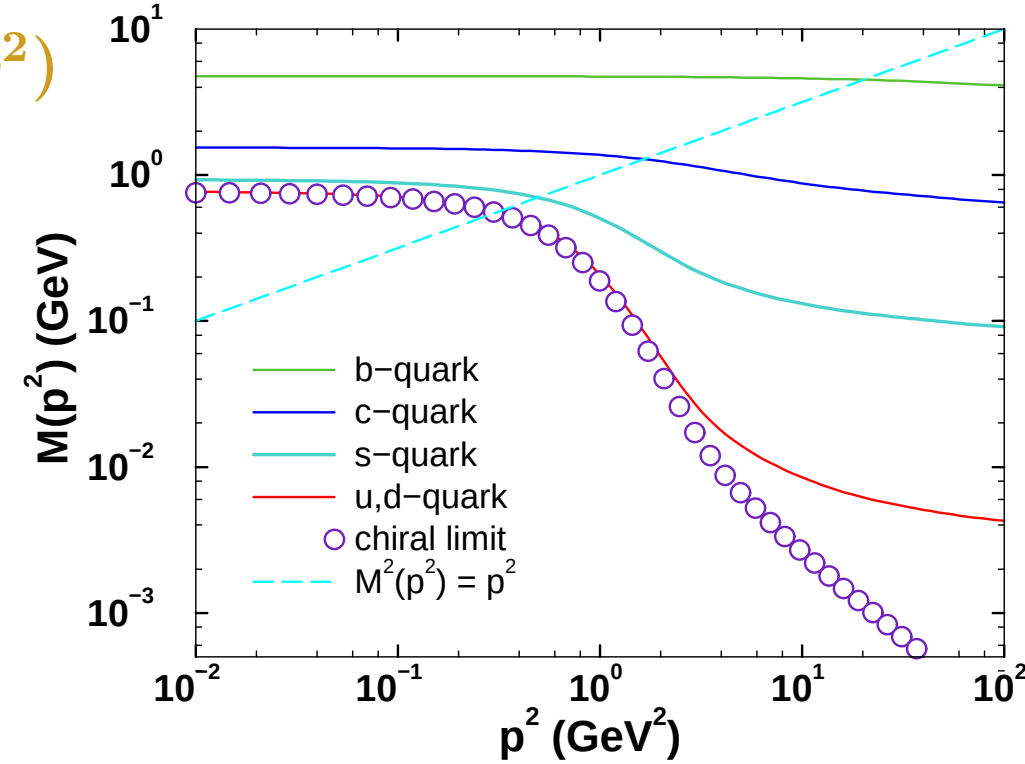
$\Rightarrow$ IR Enhancement of $M(p^2)$



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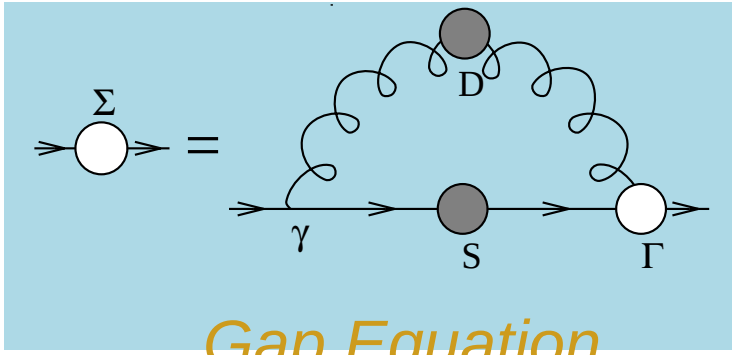

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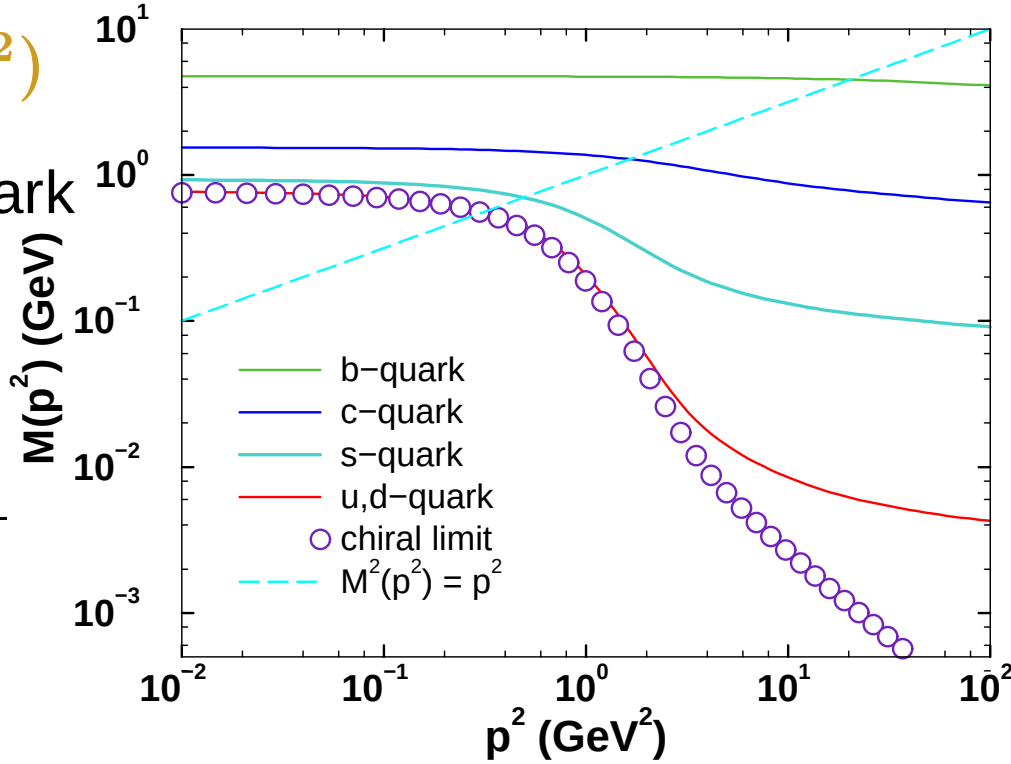
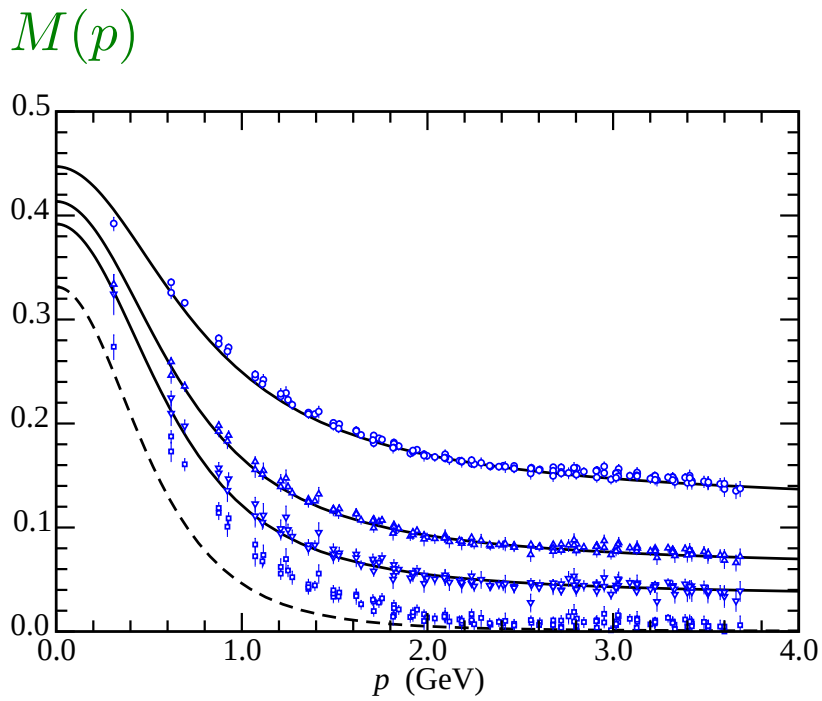
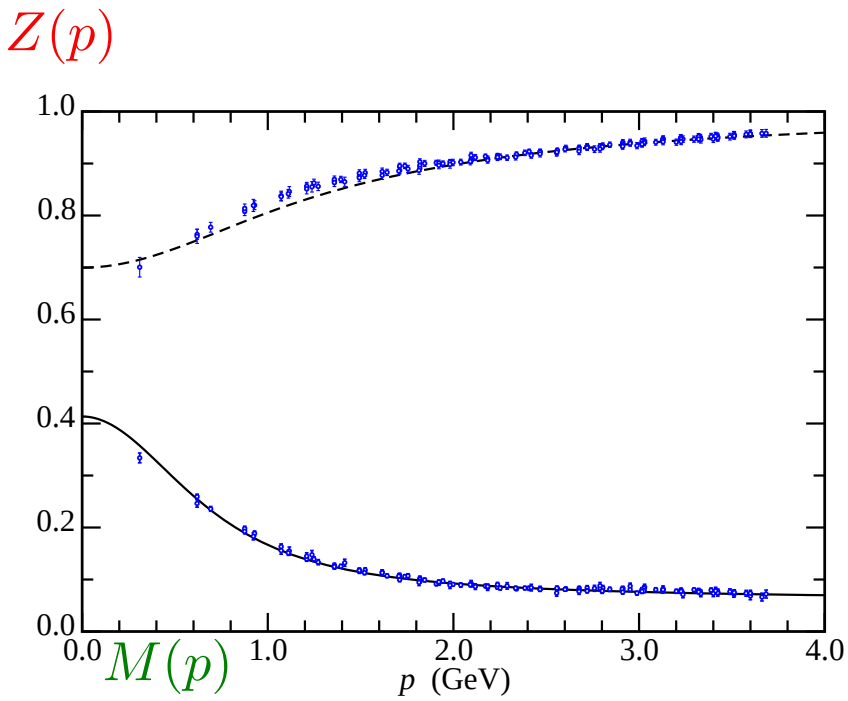
- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



Quenched-QCD Dressed-Quark Propagator





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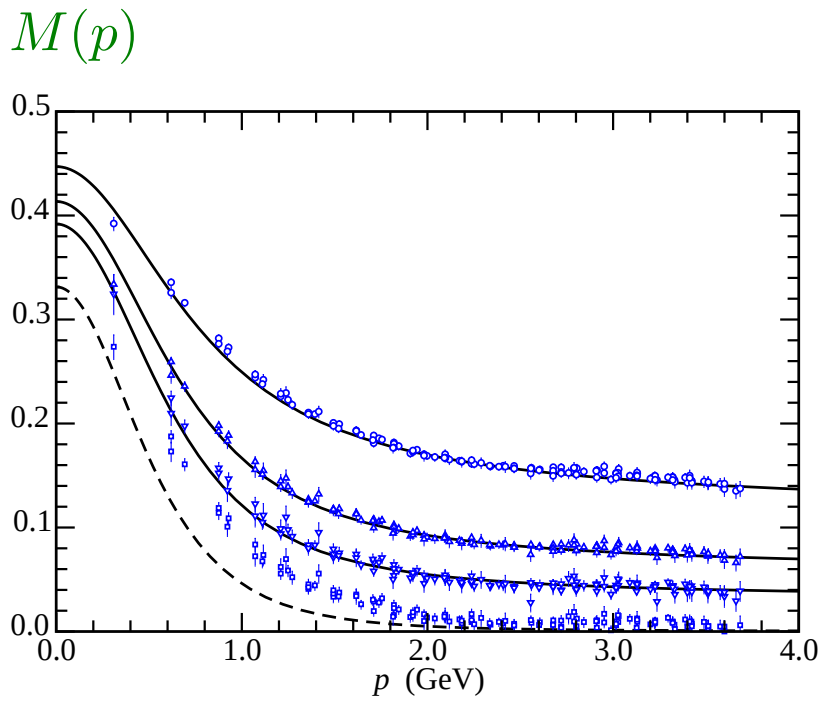
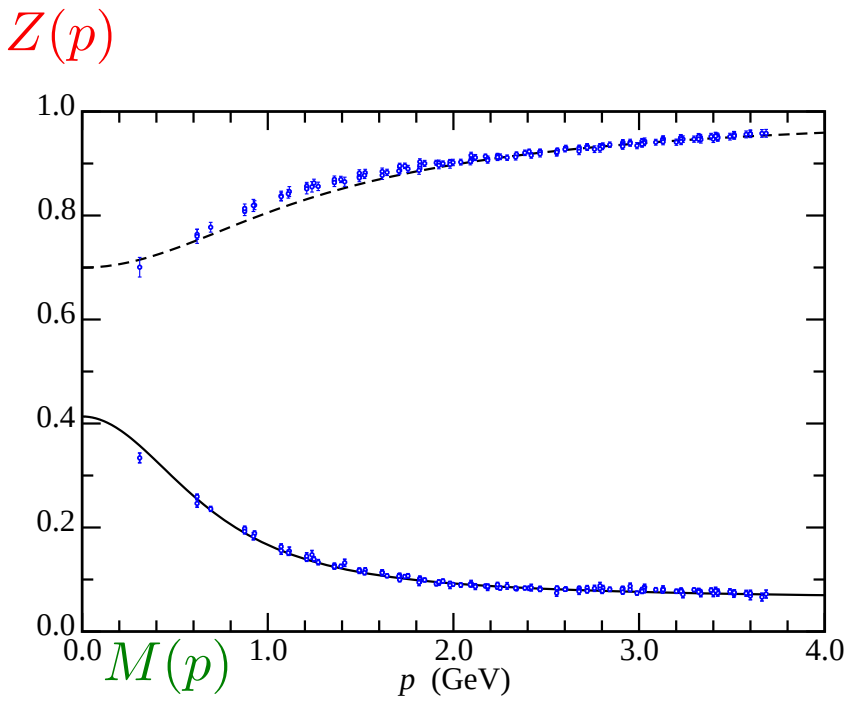
2002





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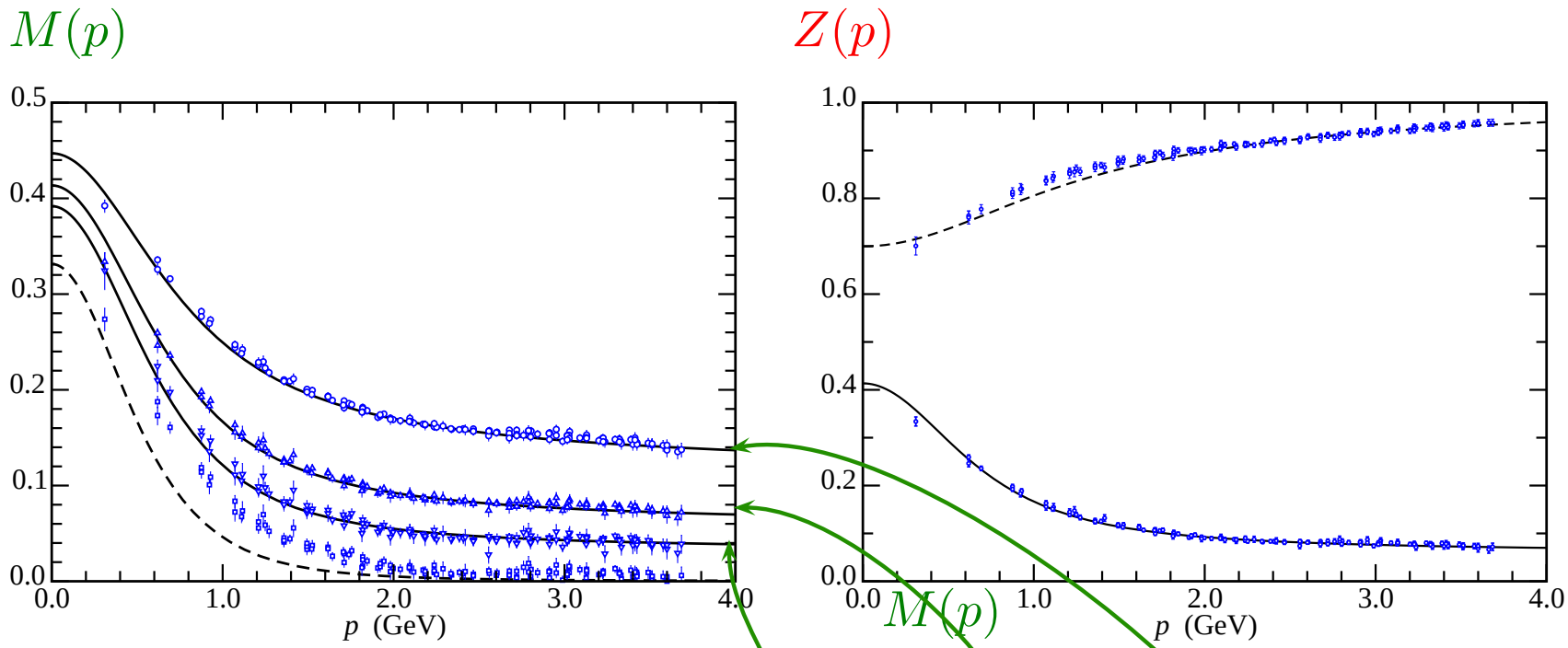

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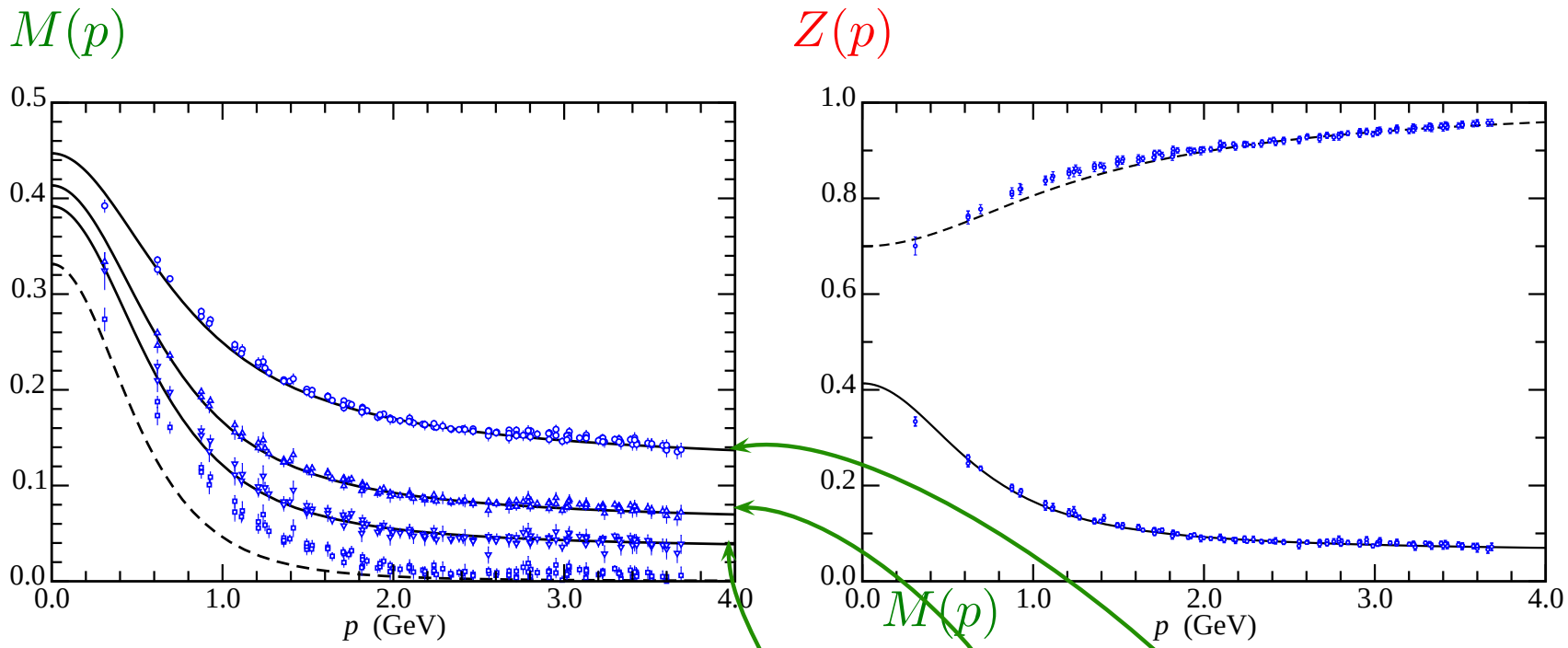

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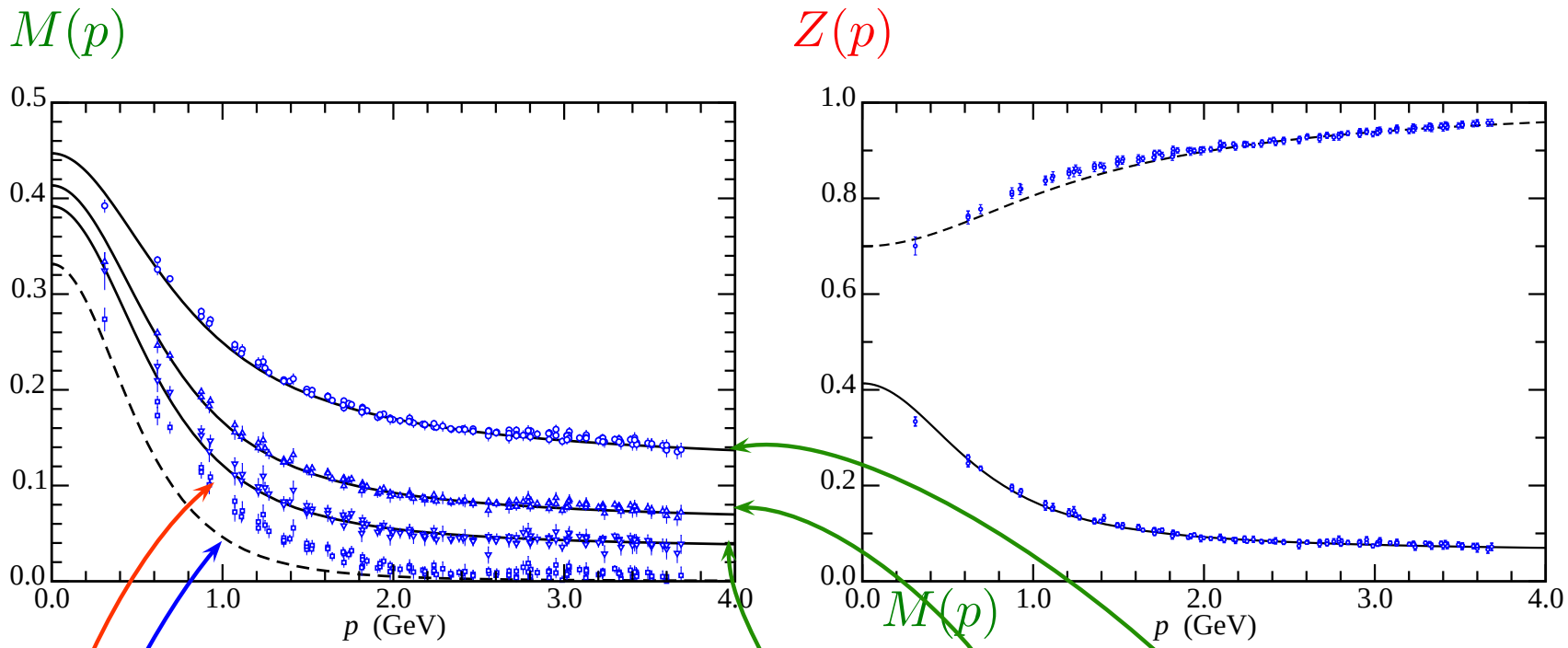

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- Linear extrapolation of lattice data to chiral limit is inaccurate

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- Established understanding
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- What about bound states?

Hadrons

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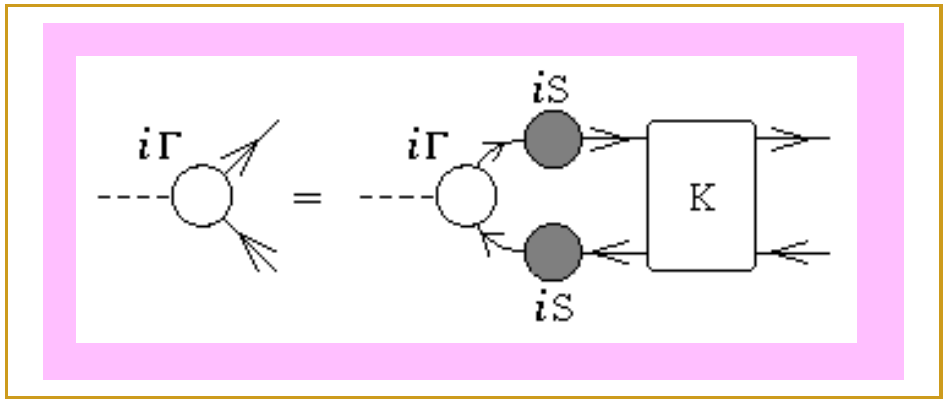

QFT Generalisation of Lippman-Schwinger Equation.

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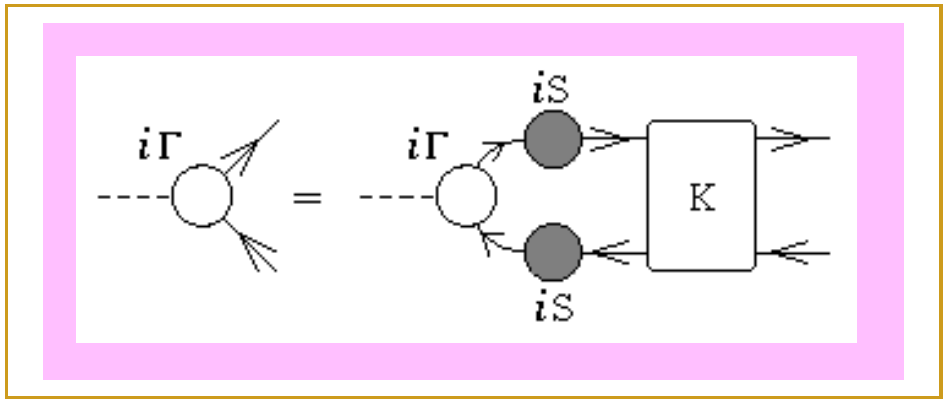

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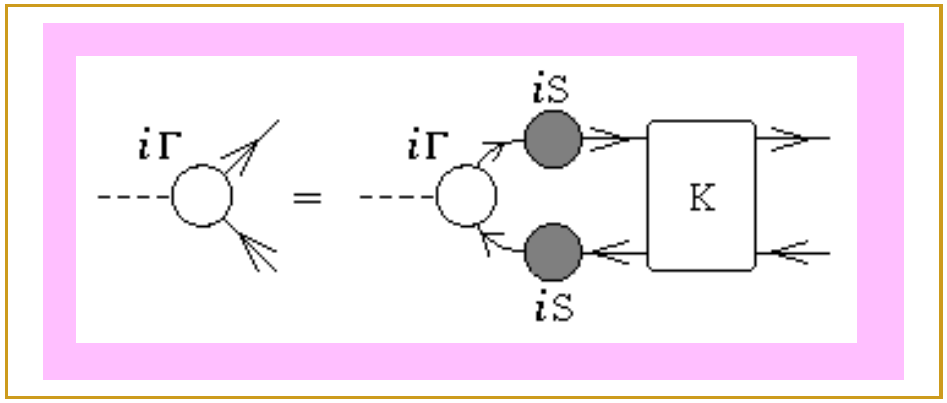

QFT Generalisation of Lippman-Schwinger Equation.

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or What is the long-range potential in QCD?

Bethe-Salpeter Kernel

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- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta\, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

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Satisfies BSE Satisfies DSE

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- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

Radial Excitations & Chiral Symmetry

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(Maris, Roberts, Tandy nu-th/9707003)

$$f_H \quad m_H^2 = - \quad \rho_\zeta^H \quad \mathcal{M}_H$$

Radial Excitations & Chiral Symmetry

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- Mass^2 of pseudoscalar hadron

Radial Excitations & Chiral Symmetry

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$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

$$\mathcal{M}_H := \mathrm{tr}_{\mathrm{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

Radial Excitations & Chiral Symmetry

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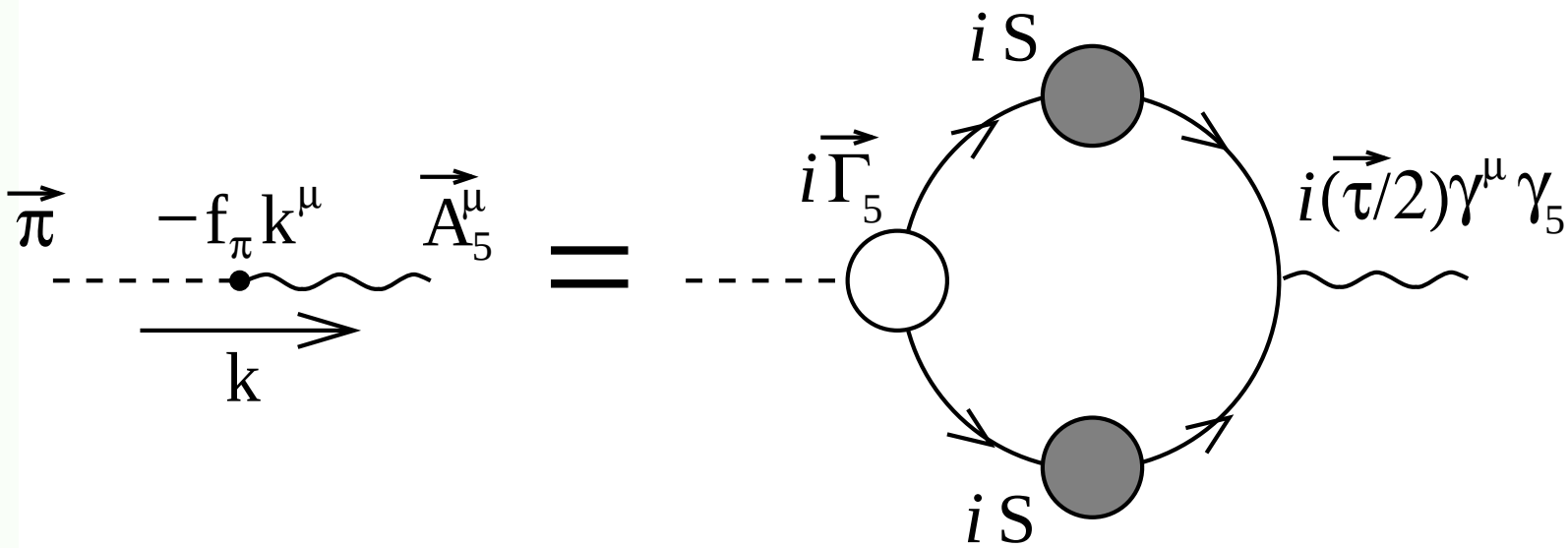
- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



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$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \dfrac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q$ ...GMOR relation, a corollary

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- Heavy-quark + light-quark

$\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $m_H \propto m_q$

... QCD Proof of Potential Model result

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

$$f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H$$

- Valid for ALL Pseudoscalar mesons

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- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

Radial Excitations

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- Spectrum contains 3 pseudoscalars [$I^G(J^P)L = 1^-(0^-)S$]

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

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- But $\pi(1800)$ is narrow ($\Gamma = 207 \pm 13$) & decay pattern might indicate some "flux tube angular momentum" content:
  $S_{\bar{Q}Q} = 1 \oplus L_F = 1 \Rightarrow J = 0$
  & $L_F = 1 \Rightarrow\ ^3S_1 \oplus\ ^3S_1$ ($\bar{Q}Q$) decays suppressed?

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- NSAC Long-Range Plan, 2002: . . . an understanding of confinement "remains one of the greatest intellectual challenges in physics"

Radial Excitations & Chiral Symmetry

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Höll, Krassnigg, Roberts
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- Fundamental properties of QCD

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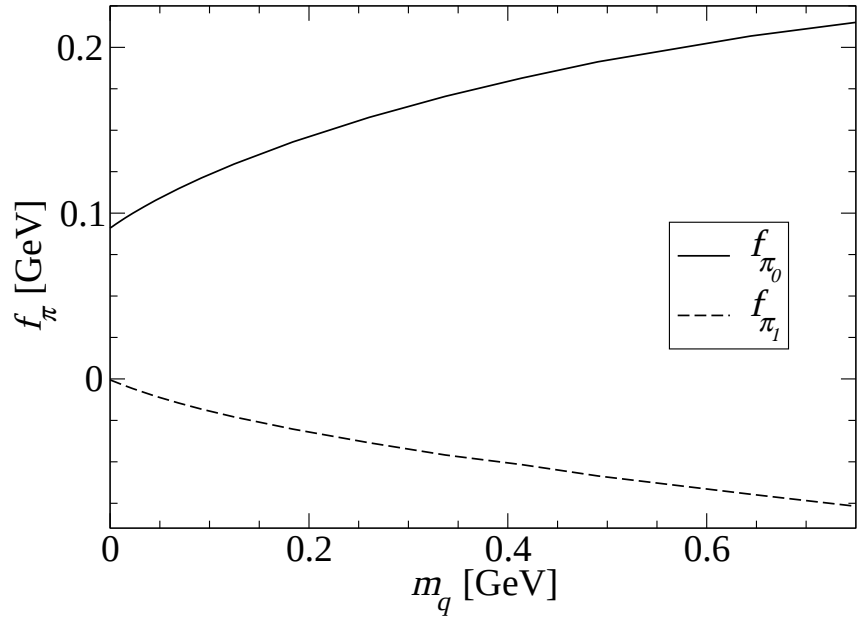

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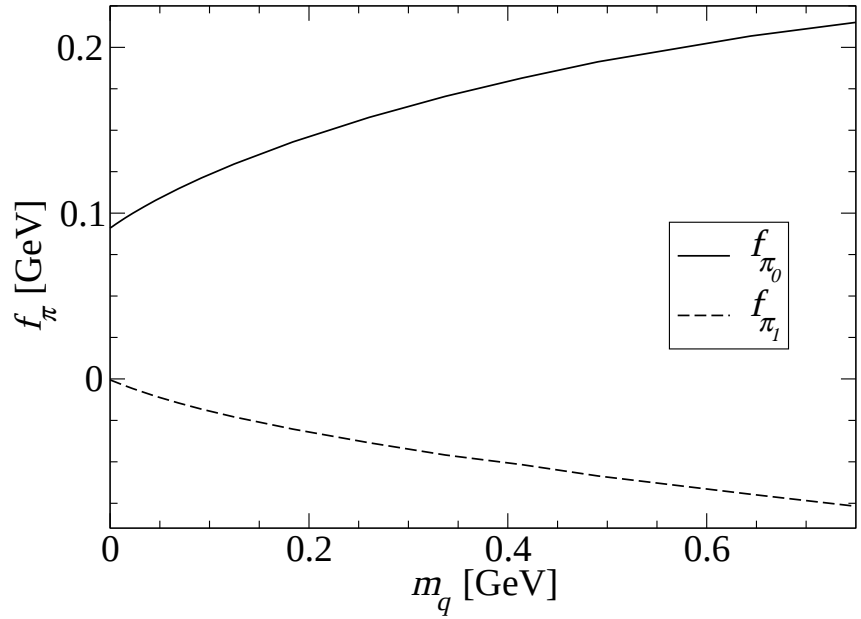

a_1 & b_1 Decays

First DSE study

- Bloch *et al*., nu-th/9906038
  - Rank-2 separable model for the Bethe-Salpeter Kernel
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\Gamma^{\pi}(\bar{k};\hat{P}) &= \vec{\tau}\,\gamma_5\left(ie_1^{\pi} - e_2^{\pi}\gamma\cdot\hat{P}\right)\bar{G}(x),\\
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\bar{F}(x) := \frac{1}{a}\left[\bar{A}(x) - 1\right],\ \bar{G}(x) := \frac{1}{b}\left[\bar{B}(x) - \bar{m}\right],
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from gap equation, $a = 0.129$, $b = 0.0877$

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- Separable model results . . . isospin partners degenerate in rainbow-ladder-like truncations:

	$m_{\rm exp}$(GeV)	m(GeV)	e_1	e_2	e_3
π	0.139	0.139	0.9973	-0.0732	
ω	0.783	0.736	-0.2270	0.9610	-0.1580
ρ	0.77	0.736	-0.2270	0.9610	-0.1580
h_1	1.17 ± 0.020	1.24	0.9708	0.2400	
b_1	1.2295 ± 0.0032	1.24	0.9708	0.2400	
f_1	1.2818 ± 0.0006	1.34	0.1991	0.9800	
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- Notes
  - No free parameters . . . all fixed elsewhere
  - a_1 has $e_2 \gg e_1$ & $e_2^{a_1} \approx e_2^{\rho}$ . . . thus model predicts a_1 is predominantly orbital excitation, analogous to constituent-quark model.
    NB. In point-meson field theory models, $e_2^{a_1} \equiv 0$.

a_1 & b_1 Decays

First DSE study nu-th/9906038

- Results

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- Results

	Obs.	I	II
$D/S\|_{h_1\to\rho\pi}$ =: R_{h_1}		0.81	0.25
$D/S\|_{a_1\to\rho\pi}$ =: R_{a_1}	-0.1 $\pm$ 0.028	-0.092	-0.075
$D/S\|_{b_1\to\omega\pi}$ =: R_{b_1}	0.29 $\pm$ 0.04	0.97	0.31
R_{a_1}/R_{b_1}	-0.34 $\pm$ 0.11	-0.095	-0.25
R_{h_1}/R_{b_1}		0.84	0.83
$g_{\rho\pi\pi}$	6.05 $\pm$ 0.02	9.58	8.18
$\Gamma_{\rho\to\pi\pi}$	0.151 $\pm$ 0.001	0.356	0.259
$\Gamma_{a_1\to\rho\pi}$	0.25 - 0.60	0.402	0.385
$\Gamma_{b_1\to\omega\pi}$	0.142 $\pm$ 0.009	0.308	0.146
$\Gamma_{h_1\to\rho\pi}$	0.360 $\pm$ 0.040	0.573	0.301
f_{a_1}	0.203 $\pm$ 0.018	0.121	0.221
f_ρ	0.216 $\pm$ 0.005	0.189	0.223
f_π	0.1307 $\pm$ 0.0004	0.136	0.148
		30%	

Decay constants are broadly acceptable.

D/S ratios have the correct sign.

Widths are too large.

Results are very sensitive to the details Bethe-Salpeter amplitude, which in turn are are determined by the infrared behaviour of the interaction.

a_1 & b_1 Decays

First DSE study nu-th/9906038

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		30%	17%

Is there a form for the long-range interaction that can provide a better description?

Separable model . . . relax constraints . . . allow one parameter variation in Bethe-Salpeter amplitudes.

b_2^+	b_2^π	b_0^ρ
0.60	0.60	0.13
0.86	0.89	0.69

b_1^ρ	$e_2^{b_1}$
2.90	0.24
3.43	-0.67

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Puzzles

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  - Alkofer, Watson, Weigel; hep-ph/0202053 & Jarecke, Maris, Tandy; nu-th/0208019

Calculated masses only; they are far too small: 800 – 900 MeV.

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- These states are a contemporary challenge for theory.
- Exp.: $\Gamma_{a_1} = 0.25 - 0.60$ & D/S ratios not known better than 10%

Hall D

http://www.jlab.org/Hall-D/

- Key Part of the 12 GeV upgrade of JLab.
  One of the International Efforts to search for Exotics and Hybrids.
  FAIR at GSI will also have this capacity

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  - "Clean" source . . . hopefully least model-dependence
  - Pseudoscalar and scalar hybrids notionally have $S_{\bar{q}q} = 1$ and hence hybrids/exotics should be accessible via photons.
  - Moreover . . . haven't seen too many hybrids with other beams, so seems sensible to try with photons.

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- That is what we're after . . . *essentially* nonperturbative dynamics . . . only Emergent Phenomena in QCD can support the existence of hybrid/exotic mesons

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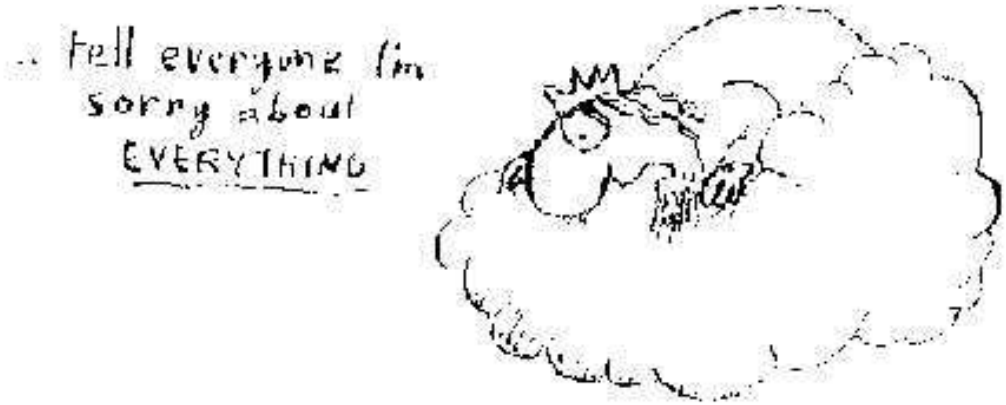

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- On the surface, Pion is peculiar
  - dressed-quark-antiquark bound state
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- Dyson-Schwinger equations provide fundamental understanding of these particularities
  - Dynamical Chiral Symmetry Breaking
  - How this connects current-quark with constituent-quark

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- Physics of hadrons is dominated by qualities that are not apparent in QCD's action; namely, Emergent Phenomena
- Existence of systematic, symmetry preserving DSE truncation
  - foundation for proof of exact results *e.g.*
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- Physics of hadrons is dominated by qualities that are not apparent in QCD's action; namely, Emergent Phenomena
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- Experimental and theoretical programme
  - map out interaction between light-quarks at long range.

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20. a_1 & b_1 DSE
21. a_1 & b_1 DSE-2
22. a_1 & b_1 Results
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