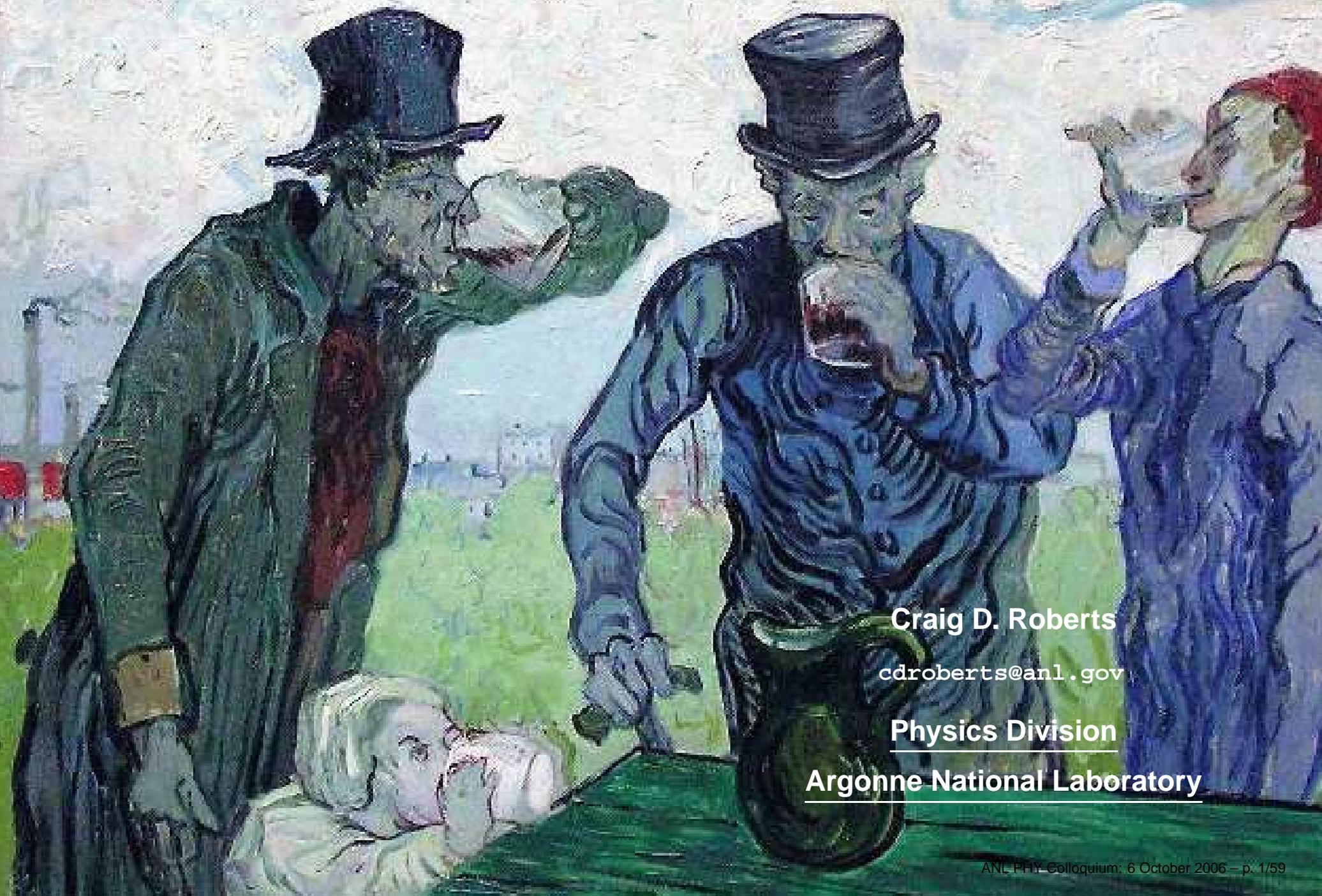


Much Ado About Hadrons



Craig D. Roberts

cdroberts@anl.gov

Physics Division

Argonne National Laboratory

*It that hath a cloud is more than a parton,
And it that hath no cloud is less than an hadron.*

Anon.



[First](#)

[Contents](#)

[Back](#)

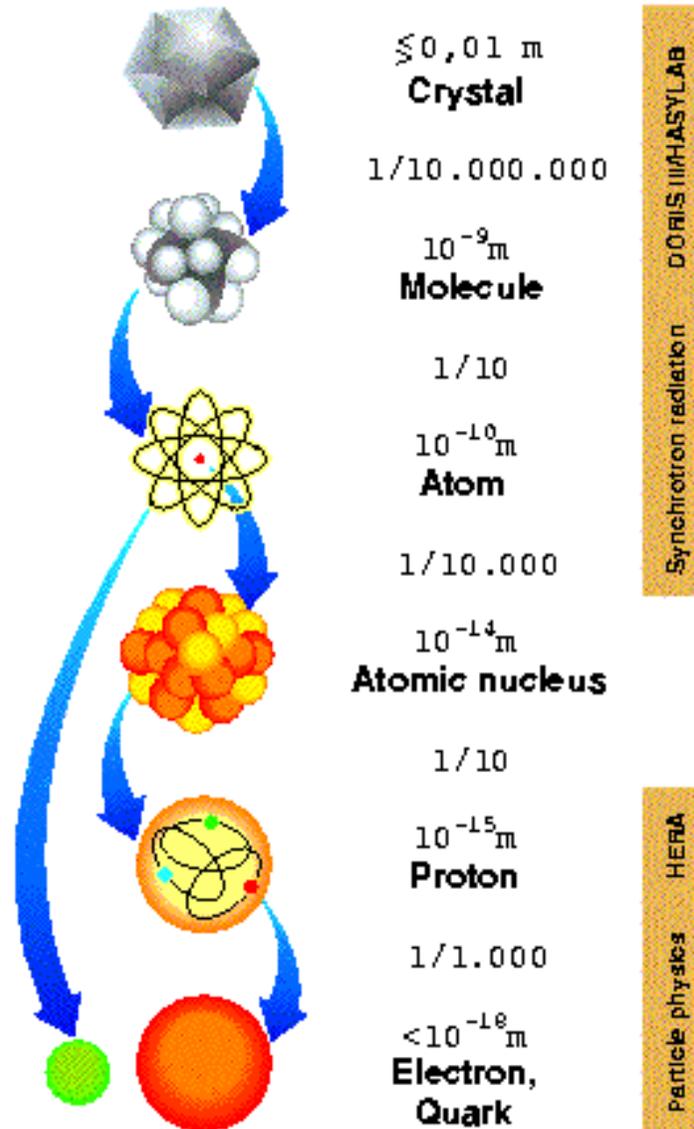
[Conclusion](#)

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Anon.



Hadron Physics



[First](#)

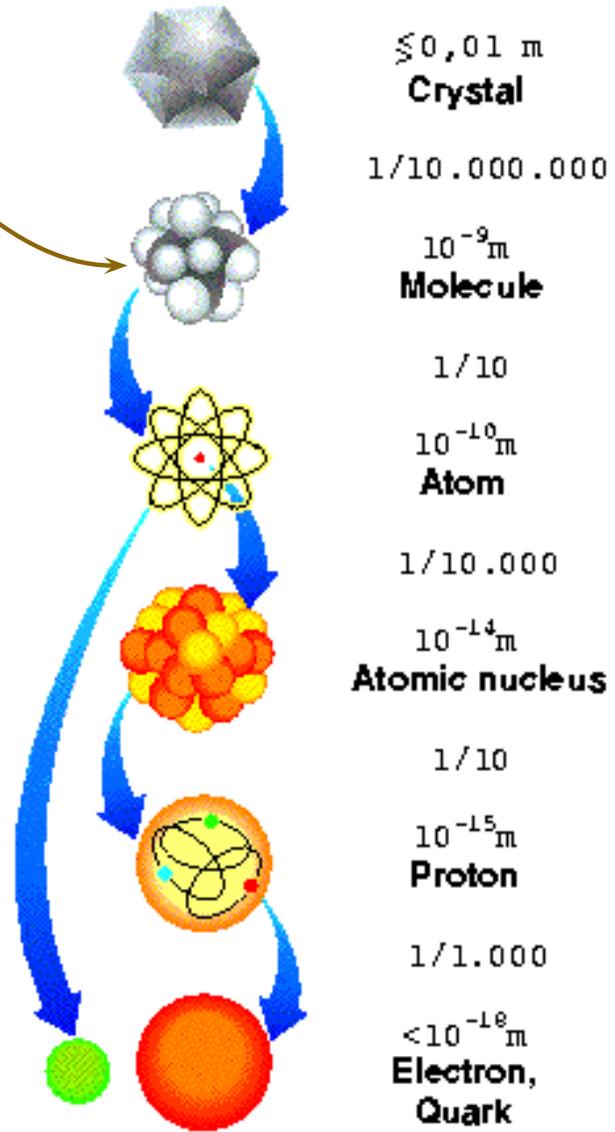
[Contents](#)

[Back](#)

[Conclusion](#)

Hadron Physics

Molecular Physics
Scale = nm

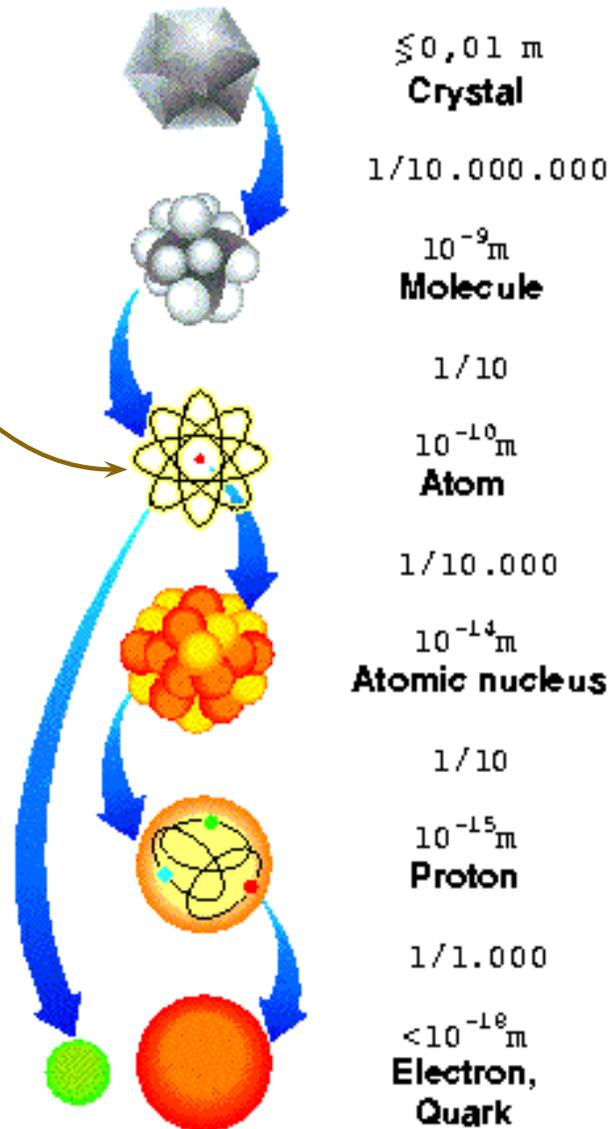


Synchrotron radiation: DORIS III/HASYLAB

Particle physics: HERA

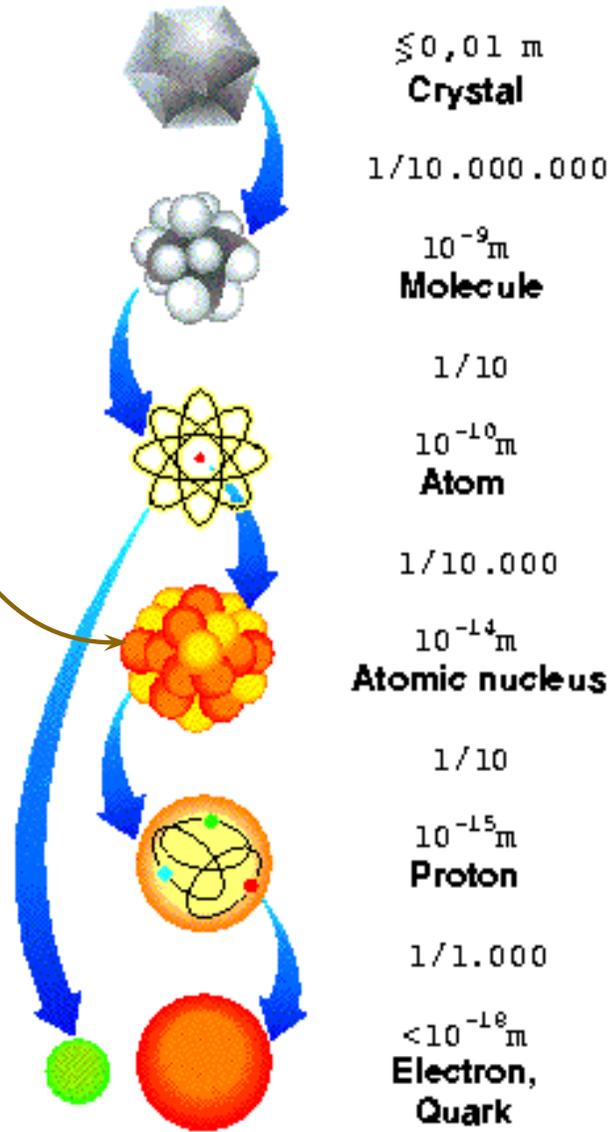
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Atomic Physics
Scale = Å



Hadron Physics

Nuclear Physics
Scale = 10 fm



Synchrotron radiation: DORIS, IHHASYLAB

Particle physics: HERA

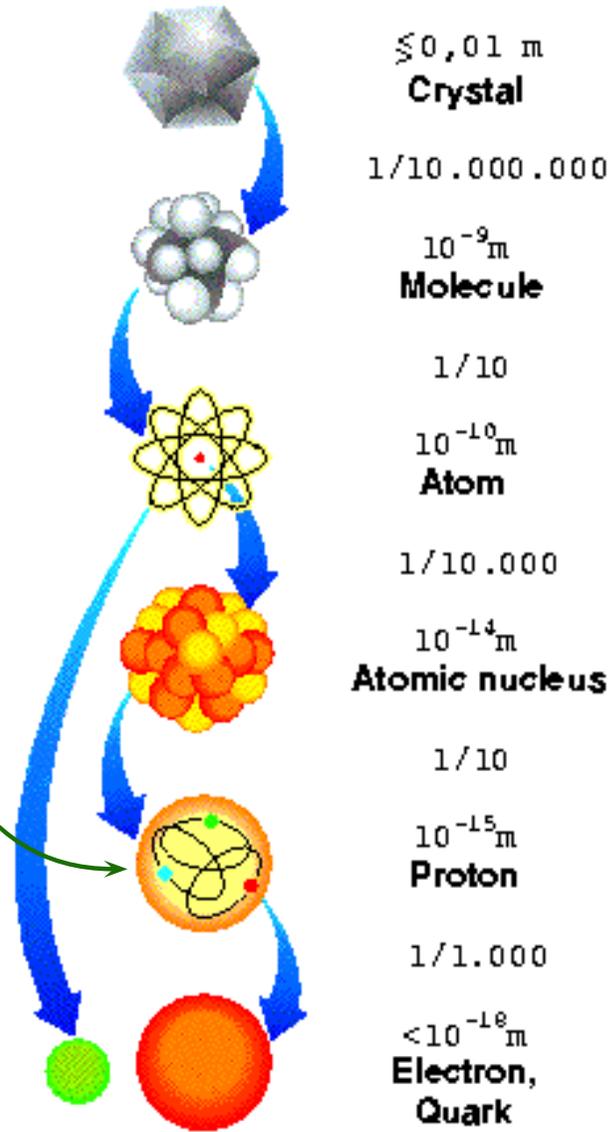
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Hadron Physics

Hadron Physics
Scale = 1 fm



$\lesssim 0,01$ m
Crystal

1/10.000.000

10^{-9} m
Molecule

1/10

10^{-10} m
Atom

1/10.000

10^{-14} m
Atomic nucleus

1/10

10^{-15} m
Proton

1/1.000

$< 10^{-16}$ m
**Electron,
Quark**

Synchrotron radiation: DORIS, IHHASYLAB

Particle physics: HERA

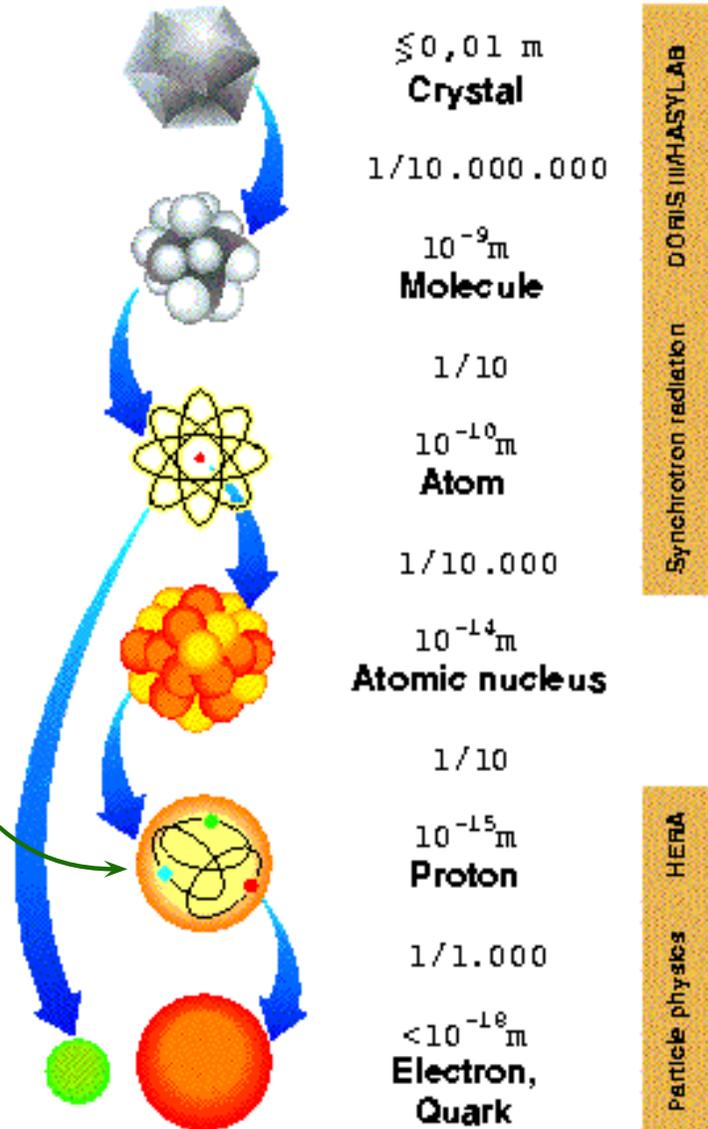
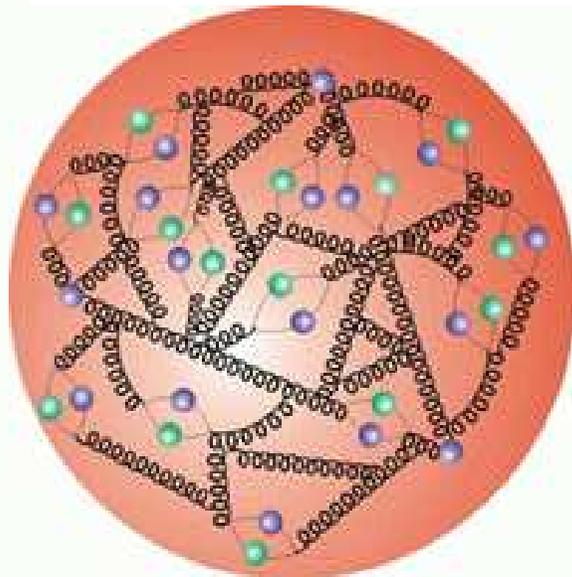
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Hadron Physics

Hadron Physics
Scale = 1 fm



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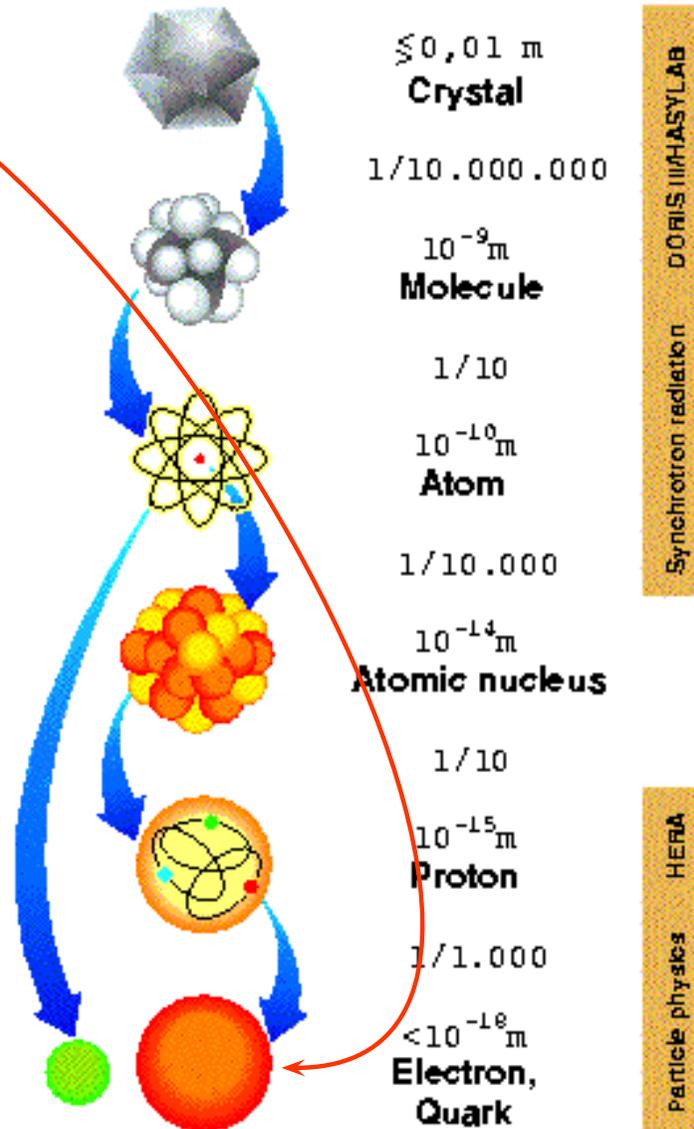





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Hadron Physics

Meta-Physics
 Scale = Limited only
 by Theorists
 Imagination



Nucleon ... 2 Key Hadrons

= Proton and Neutron



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Nucleon ... 2 Key Hadrons

= Proton and Neutron

- Fermions – two static properties:
proton electric charge = +1; and magnetic moment, μ_p



Nucleon ... 2 Key Hadrons = Proton and Neutron

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 - Stern (1933) – $\mu_p = (1 + 1.79) \frac{e\hbar}{2M}$



Nucleon ... 2 Key Hadrons = Proton and Neutron

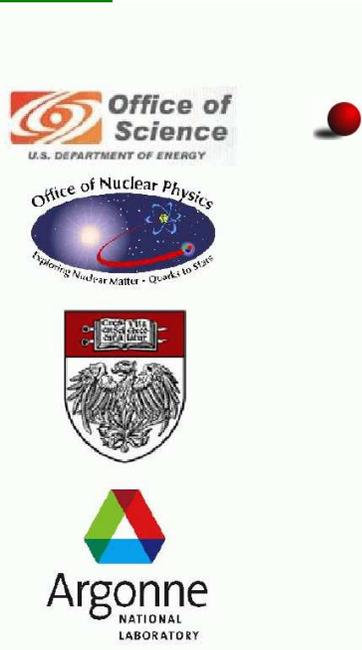
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- Stern (1933) – $\mu_p = (1 + 1.79) \frac{e\hbar}{2M}$

- Big Hint that Proton is not a point particle
- Proton has constituents
- These are Quarks and Gluons

Quark discovery via $e^- p$ -scattering at SLAC in 1968
– the elementary quanta of Quantum Chromo-dynamics



What is QCD?



[First](#)

[Contents](#)

[Back](#)

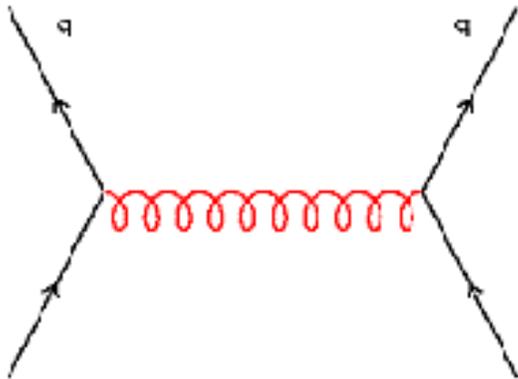
[Conclusion](#)

What is QCD?

- Gauge Theory:

Interactions Mediated by **massless** vector bosons

Feynman Diagram of Quark-Quark Scattering

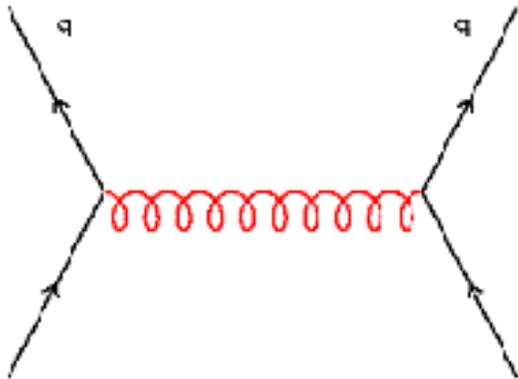


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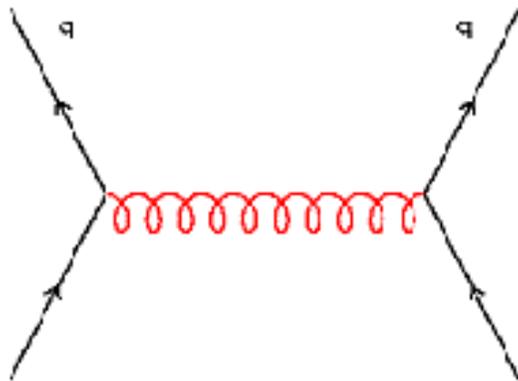
- Similar interaction in QED



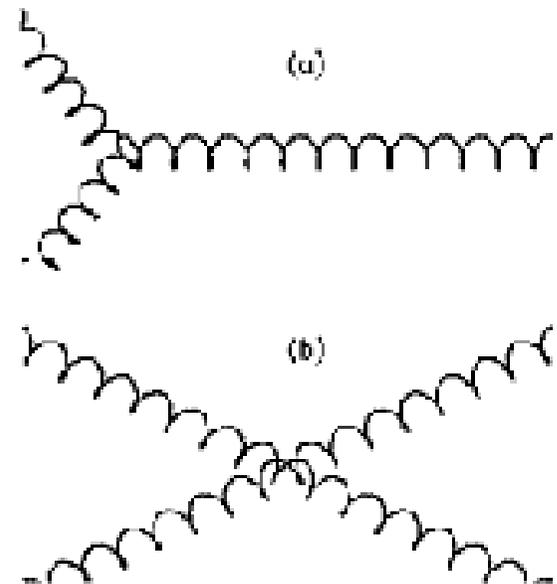
What is QCD?

- Gauge Theory:
Interactions Mediated by **massless** vector bosons

Feynman Diagram of Quark-Quark Scattering



Gluon Interactions



- Similar interaction in QED
- Special Feature of QCD – **gluon self-interactions**

Completely Change the Character of the Theory

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QED cf. QCD



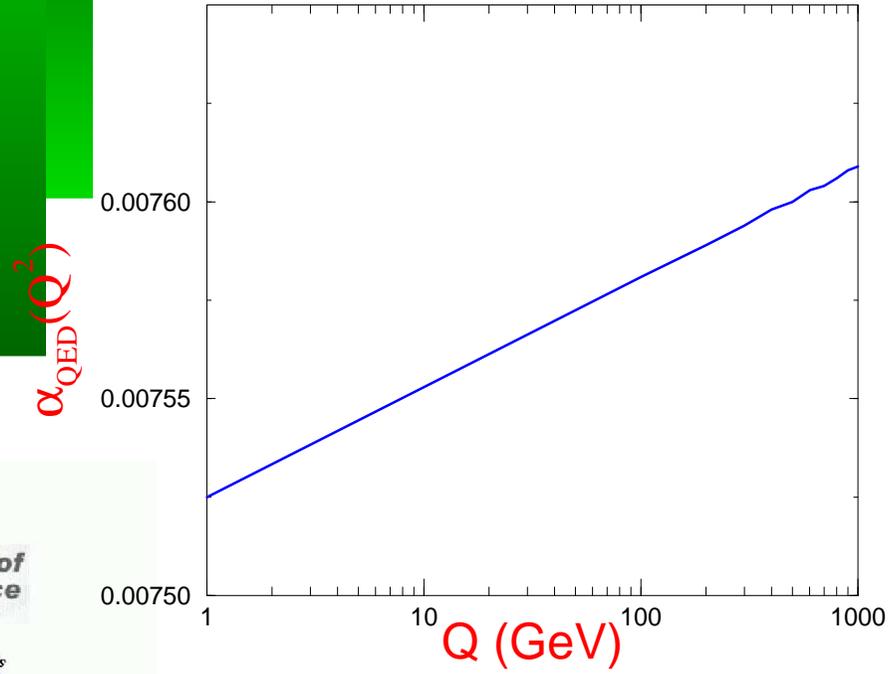
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

QED cf. QCD

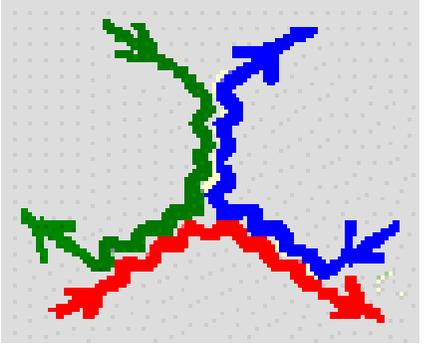
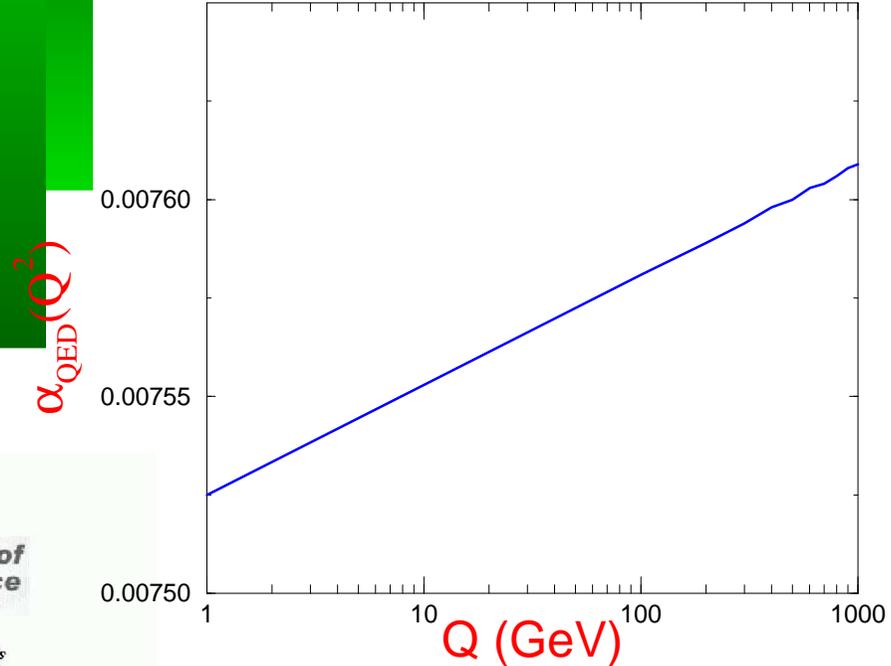


$$\alpha_{\text{QED}} = \frac{\alpha}{1 - \alpha/3\pi \ln(Q^2/m_e^2)}$$



QED cf. QCD

Add three-gluon interaction



$$\alpha_{\text{QED}} = \frac{\alpha}{1 - \alpha/3\pi \ln(Q^2/m_e^2)}$$

QED cf. QCD

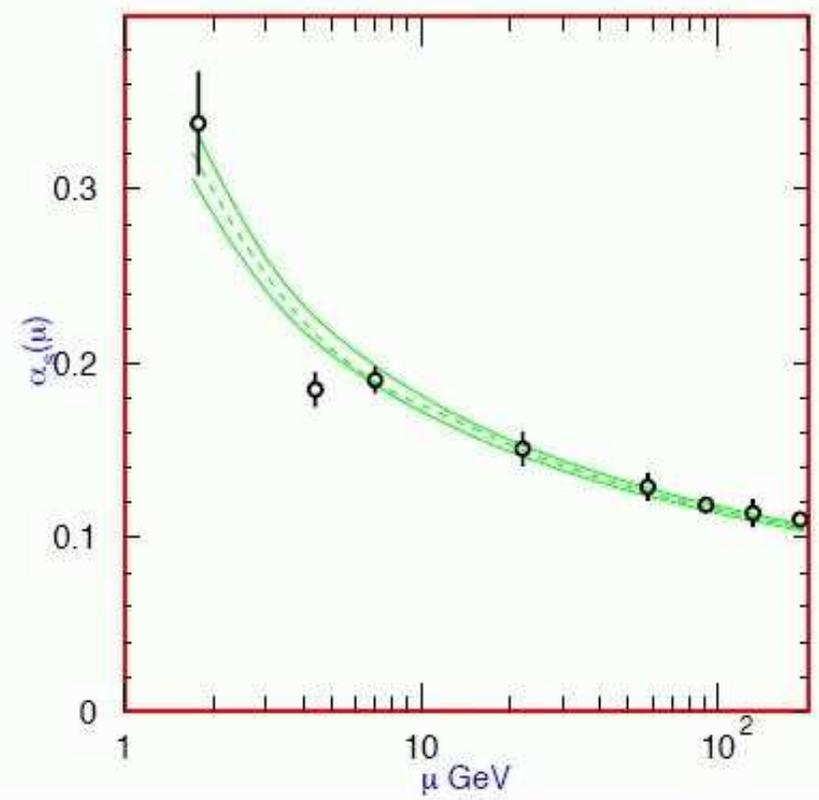
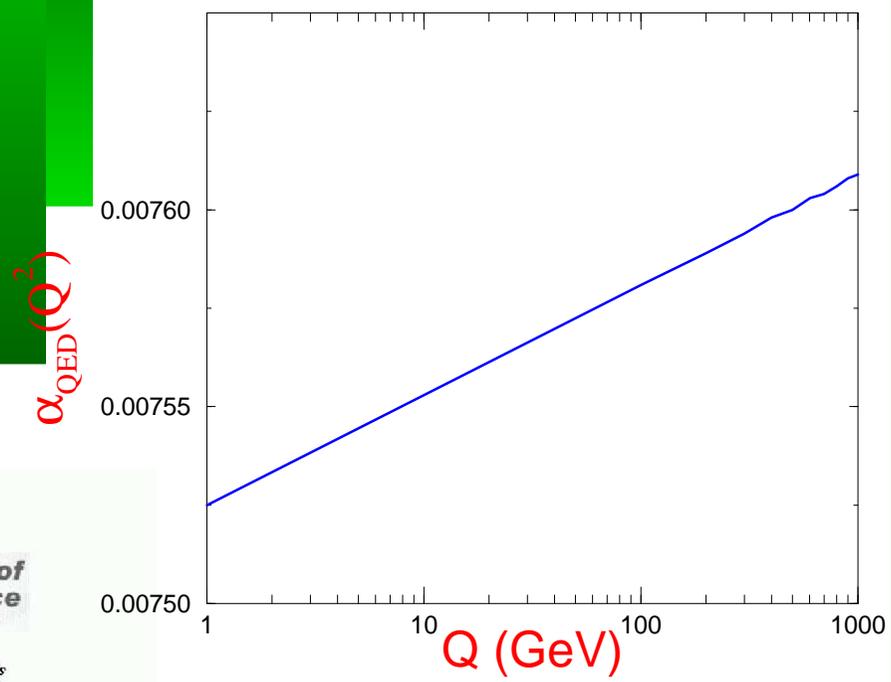
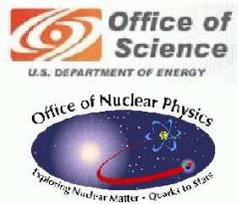


Figure 9.2: Summary of the values of $\alpha_s(\mu)$ at the values of μ where they are measured. The lines show the central values and the $\pm 1\sigma$ limits of our average. The figure clearly shows the decrease in $\alpha_s(\mu)$ with increasing μ . The data are, in increasing order of μ , τ width, Υ decays, deep inelastic scattering, e^+e^- event shapes at 22 GeV from the JADE data, shapes at TRISTAN at 58 GeV, Z width, and e^+e^- event shapes at 135 and 189 GeV.

$$\alpha_{\text{QED}} = \frac{\alpha}{1 - \alpha/3\pi \ln(Q^2/m_e^2)}$$

$$\alpha_{\text{QCD}} = \frac{12\pi}{(33 - 2N_f) \ln(Q^2/\Lambda^2)}$$



2004 Nobel Prize in Physics: Gross, Politzer and Wilczek

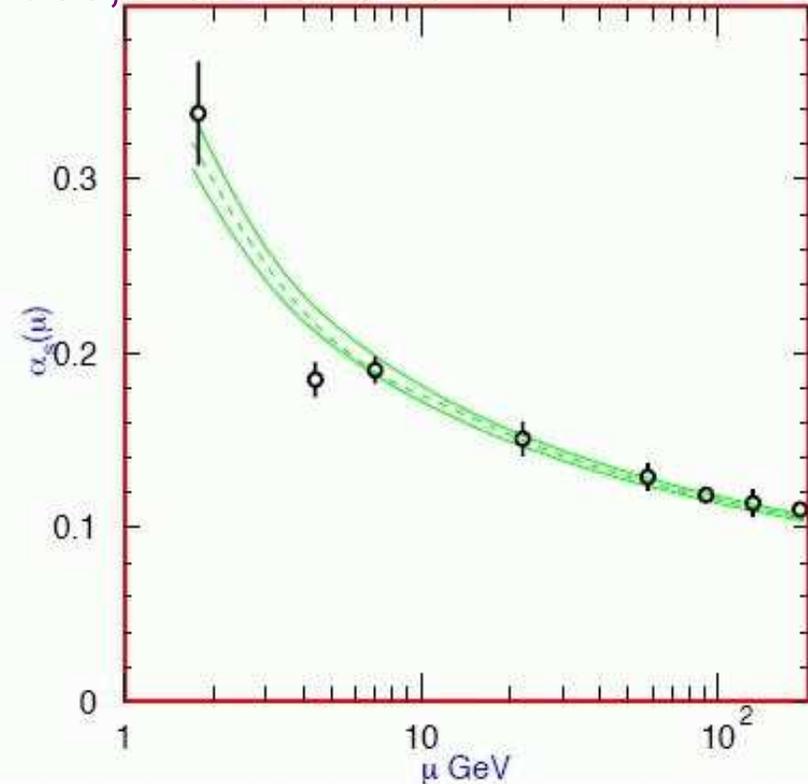
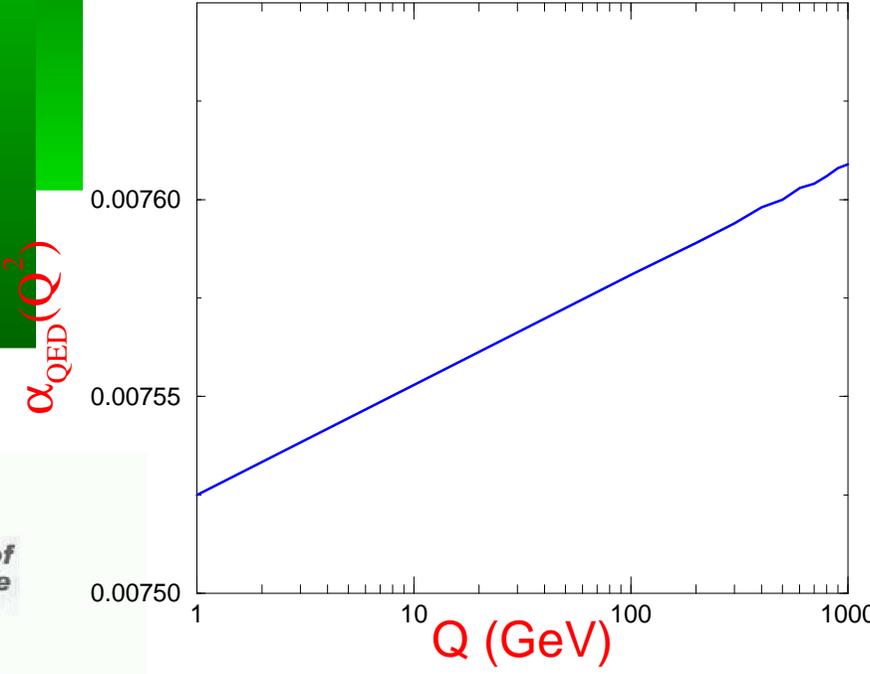


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Quarks and Nuclear Physics



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Quarks and Nuclear Physics

Standard Model of Particle Physics Six Flavours

$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
up



$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
charm

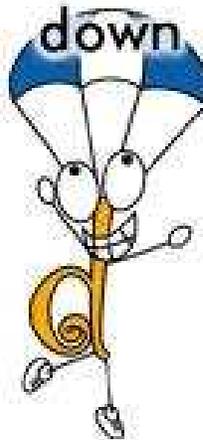


$\begin{pmatrix} 2 \\ 3 \end{pmatrix}$
top



$\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

down



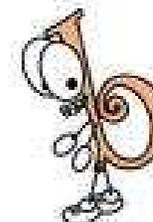
$\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

strange



$\begin{pmatrix} -1 \\ 3 \end{pmatrix}$

bottom



Quarks and Nuclear Physics

Real World
Normal Matter ...
Only Two Light
Flavours Active

$\left(\begin{smallmatrix} 2 \\ 3 \end{smallmatrix}\right)$
up



$\left(\begin{smallmatrix} 2 \\ 3 \end{smallmatrix}\right)$
charm

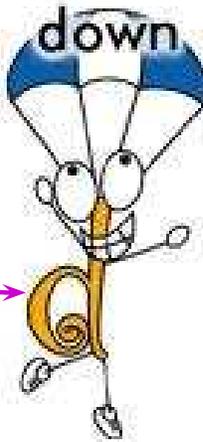


$\left(\begin{smallmatrix} 2 \\ 3 \end{smallmatrix}\right)$
top



$\left(-\frac{1}{3}\right)$

down



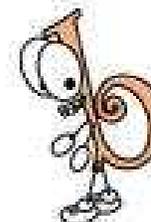
$\left(-\frac{1}{3}\right)$

strange



$\left(-\frac{1}{3}\right)$

bottom



Quarks and Nuclear Physics

Real World
Normal Matter ...
Only Two Light
Flavours Active

or, perhaps, three

$\left(\frac{2}{3}\right)$
up



$\left(\frac{2}{3}\right)$
charm

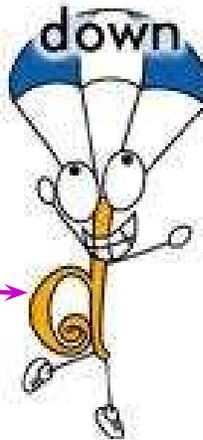


$\left(\frac{2}{3}\right)$
top



$\left(-\frac{1}{3}\right)$

down



$\left(-\frac{1}{3}\right)$

strange



$\left(-\frac{1}{3}\right)$

bottom



Quarks and Nuclear Physics

Real World
Normal Matter ...
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$\left(\frac{2}{3}\right)$
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$\left(\frac{2}{3}\right)$
charm

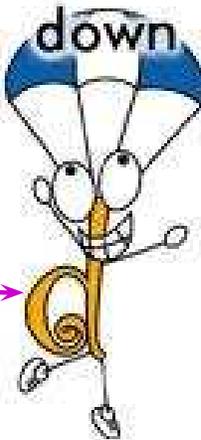


$\left(\frac{2}{3}\right)$
top



$\left(-\frac{1}{3}\right)$

down



$\left(-\frac{1}{3}\right)$

strange



$\left(-\frac{1}{3}\right)$

bottom



For numerous good reasons, much research also focuses on accessible heavy-quarks



Nevertheless, I will focus

Quarks and Nuclear Physics

primarily on the light-quarks.

Real World
Normal Matter ...
Only Two Light
Flavours Active

or, perhaps, three

$\left(\frac{2}{3}\right)$

up



$\left(\frac{2}{3}\right)$

charm



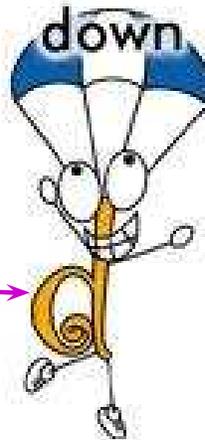
$\left(\frac{2}{3}\right)$

top



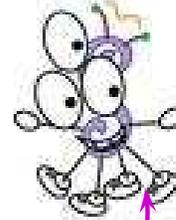
$\left(-\frac{1}{3}\right)$

down



$\left(-\frac{1}{3}\right)$

strange



$\left(-\frac{1}{3}\right)$

bottom



For numerous good reasons, much research also focuses on accessible heavy-quarks



Simple Picture



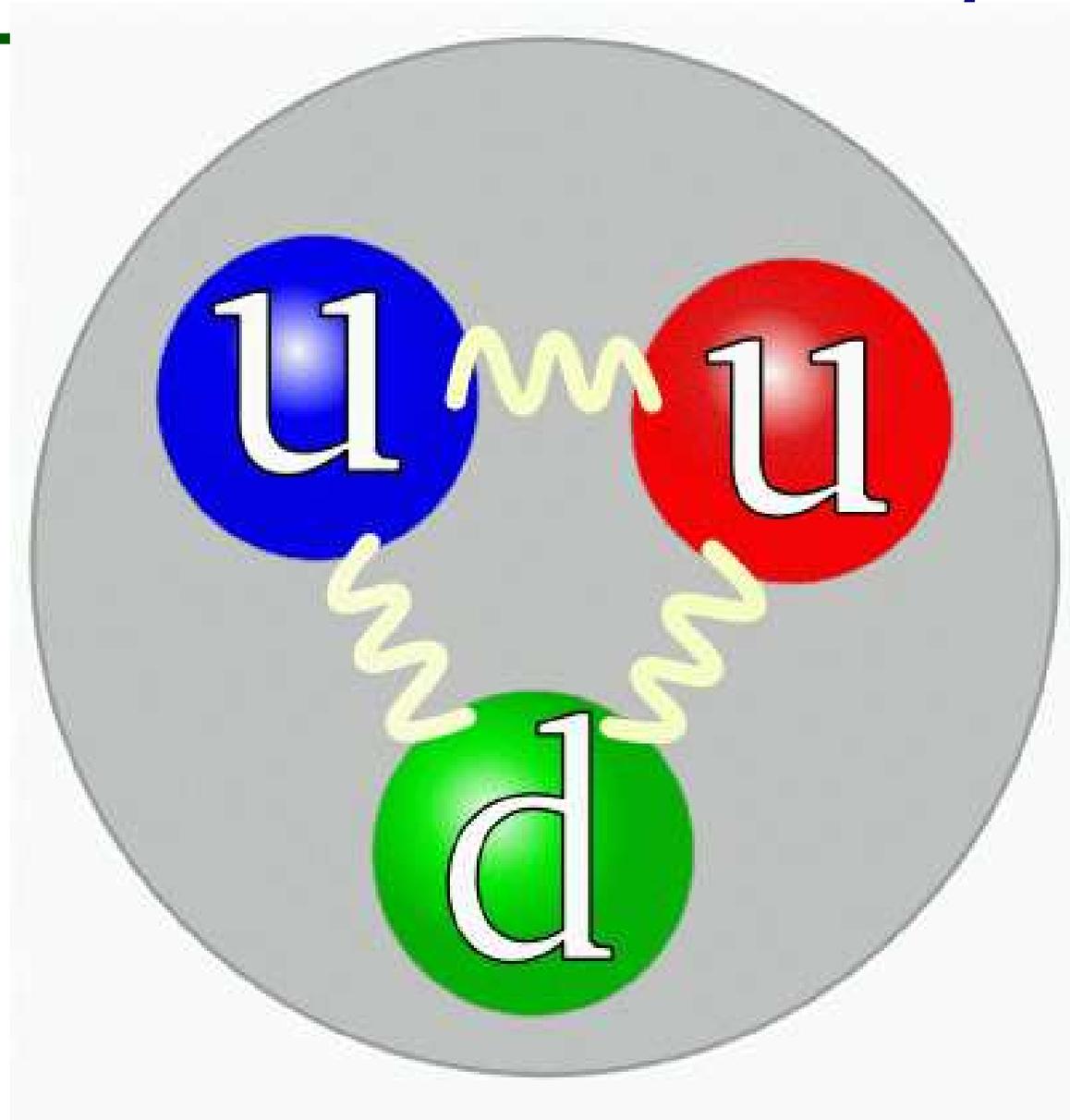
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Simple Picture



PROTON



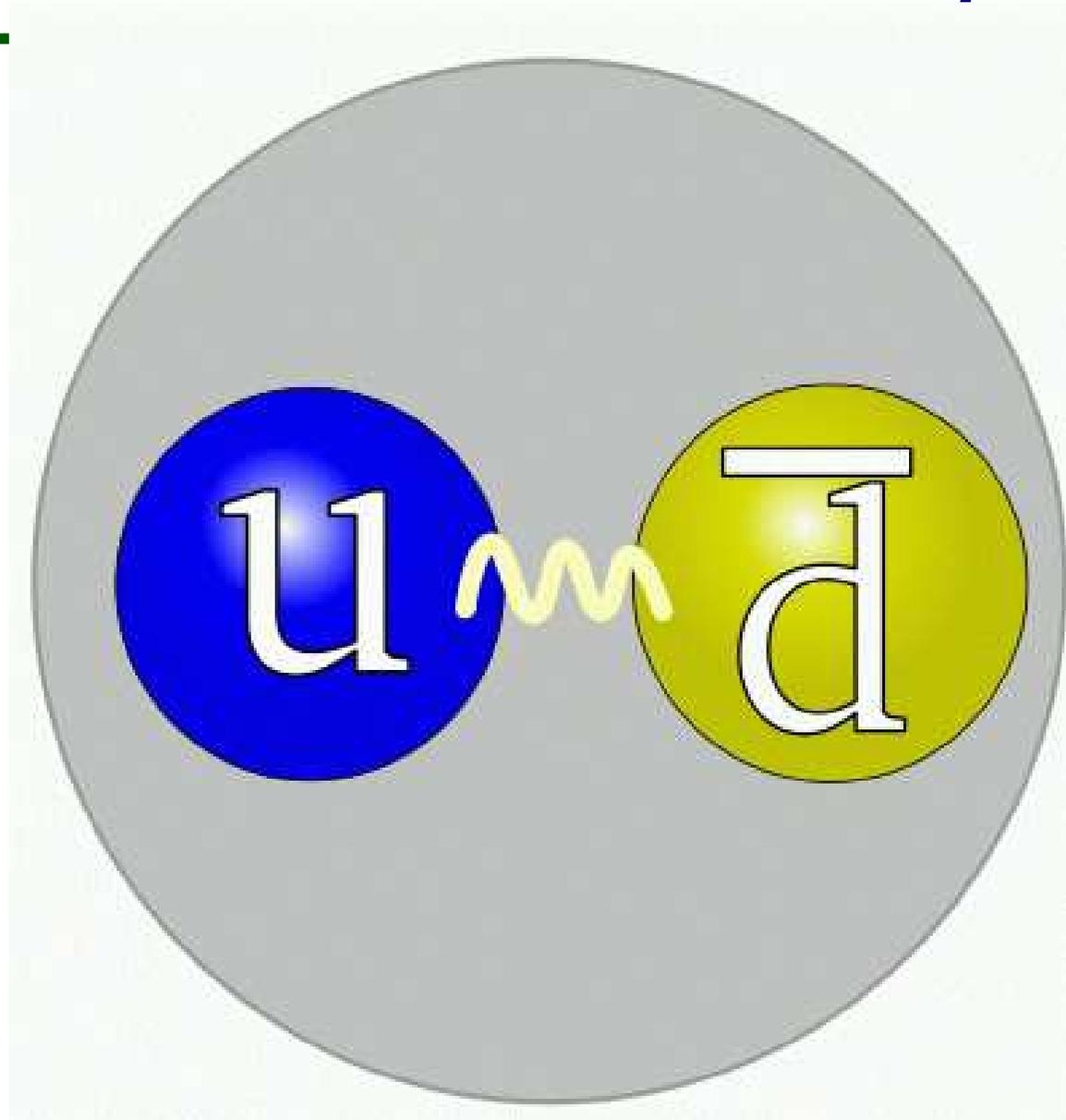
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Simple Picture



PION



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Study Structure via Nucleon Form Factors



[First](#)

[Contents](#)

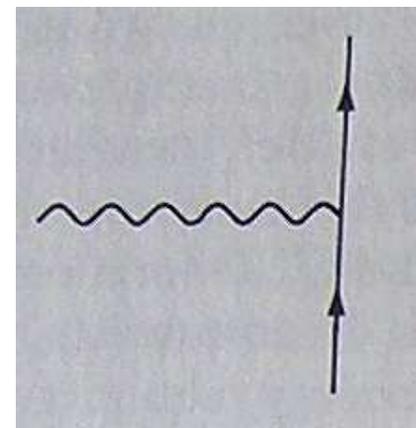
[Back](#)

[Conclusion](#)

Study Structure via Nucleon Form Factors

- Electron's relativistic electromagnetic current:

$$\begin{aligned}j_{\mu}(P', P) &= ie \bar{u}_e(P') \Lambda_{\mu}(Q, P) u_e(P), \quad Q = P' - P \\ &= ie \bar{u}_e(P') \gamma_{\mu}(-1) u_e(P)\end{aligned}$$

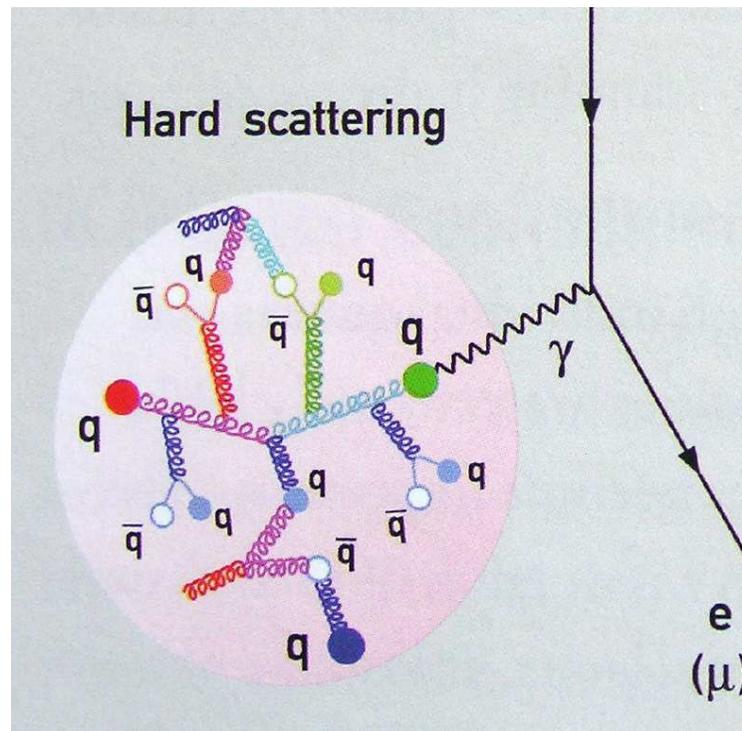


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- Nucleon's relativistic electromagnetic current:



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Study Structure via Nucleon Form Factors

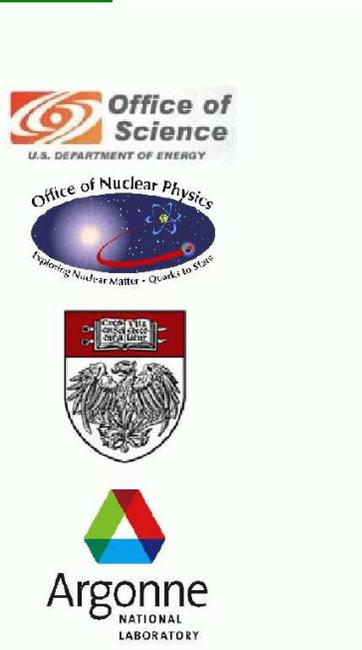
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$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$



Study Structure via Nucleon Form Factors

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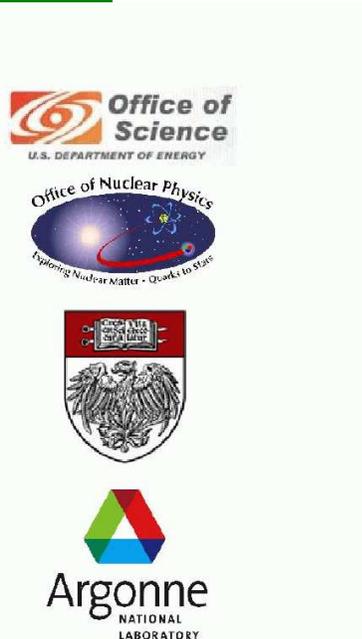
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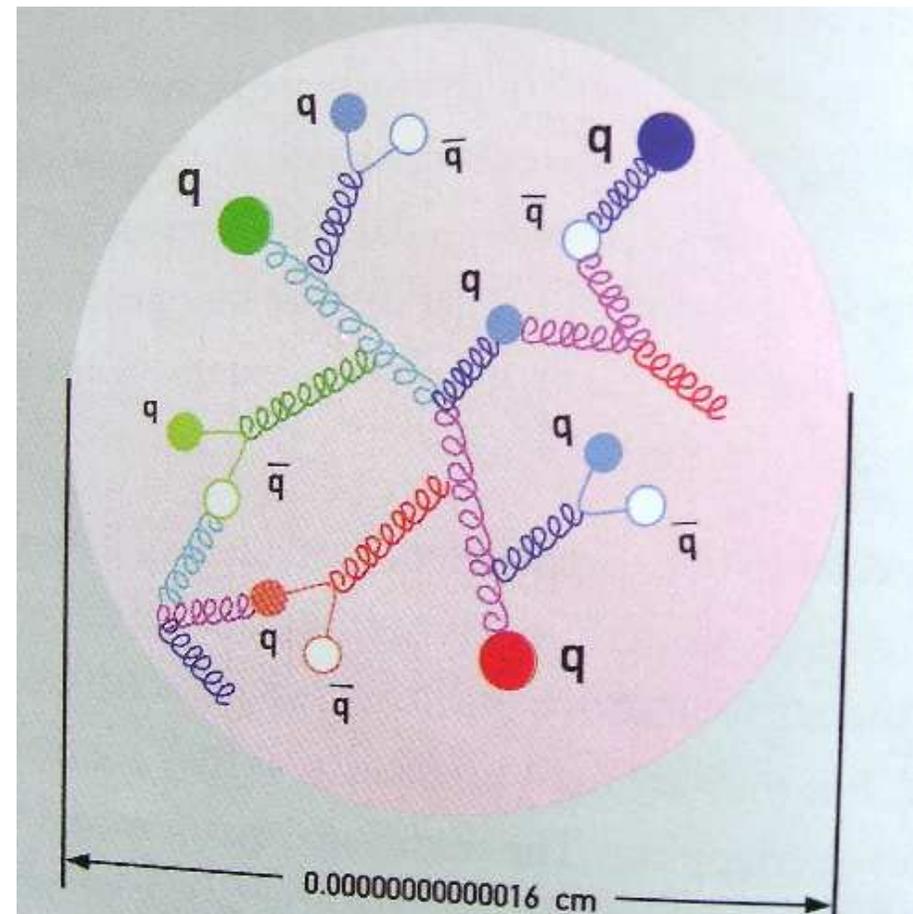
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Point-particle: $F_2 \equiv 0 \Rightarrow G_E \equiv G_M$



NSAC Long Range Plan

A central goal of nuclear physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD



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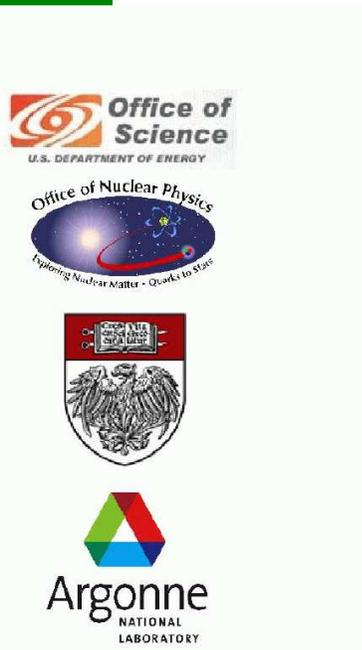
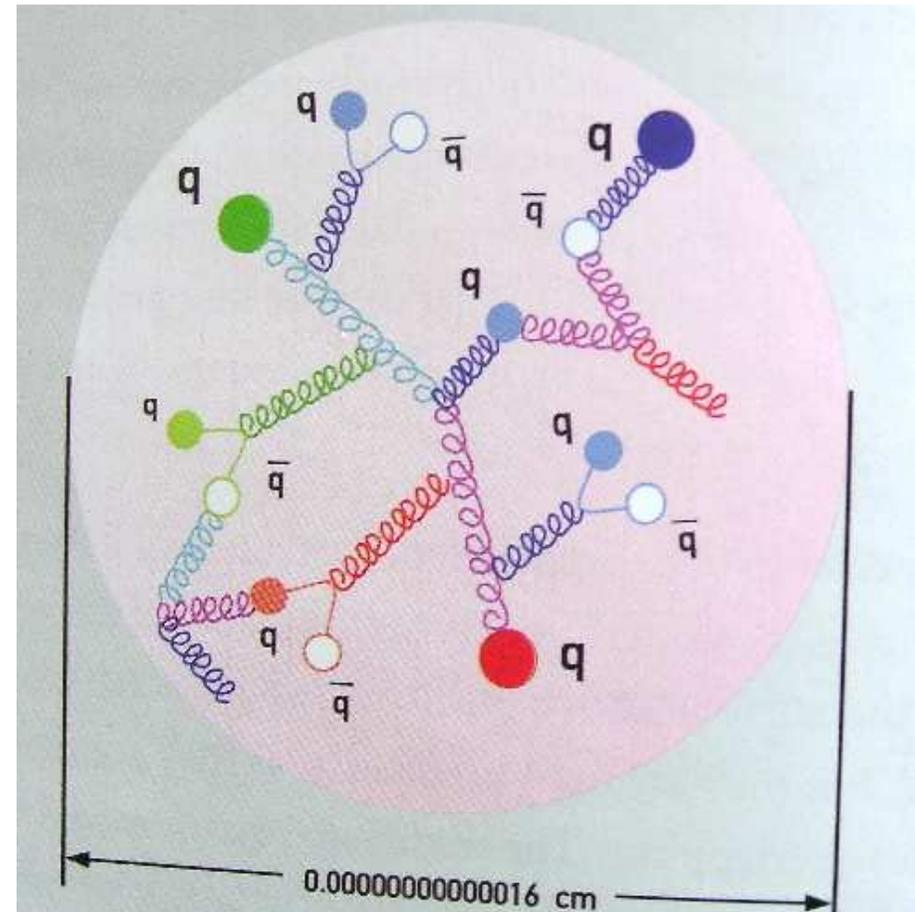
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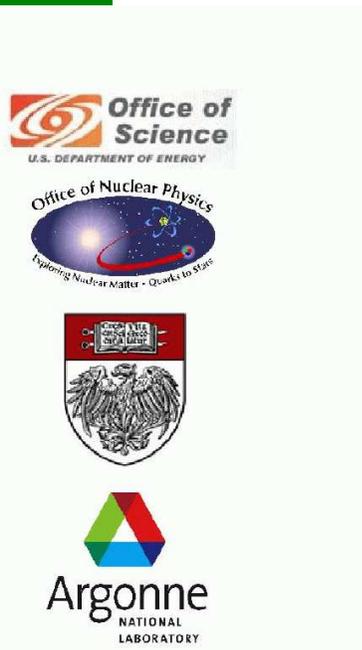
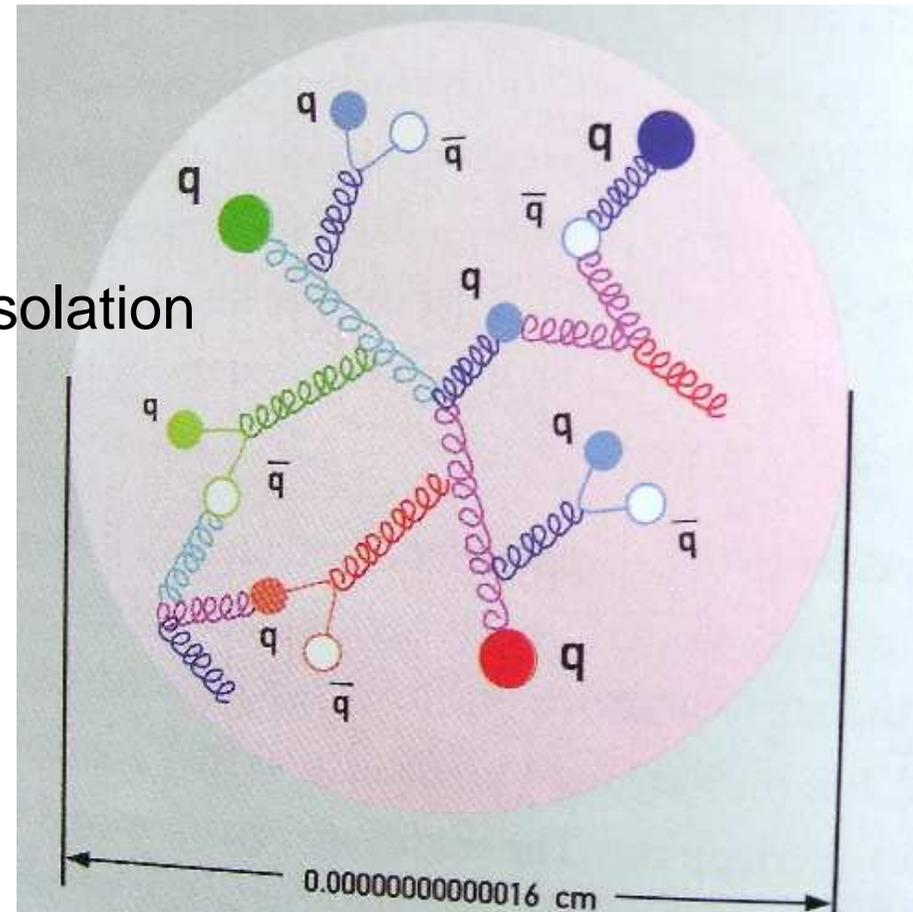


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- **Confinement**
 - No quark ever seen in isolation

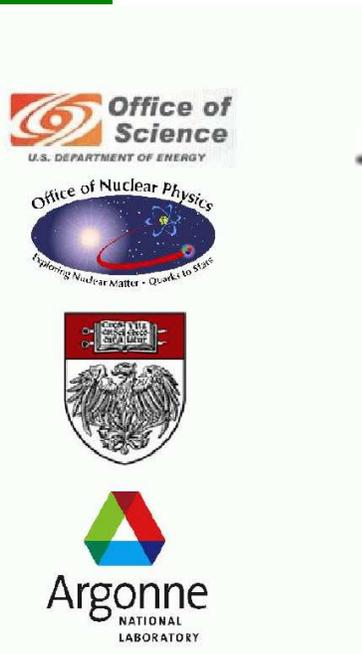
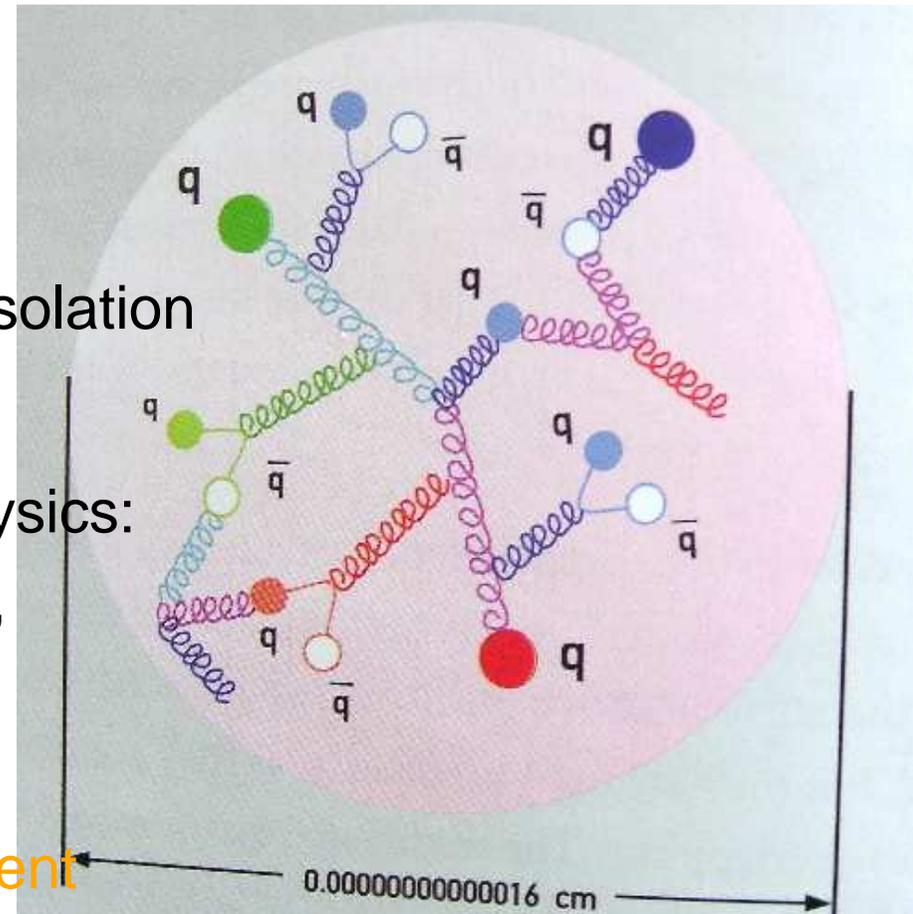


NSAC Long Range Plan

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So, what's the problem?

- **Confinement**
 - No quark ever seen in isolation
- **Weightlessness**
 - 2004 Nobel Prize in Physics:
Mass of u - & d -quarks,
each just 5 MeV;
Proton Mass is 940 MeV
⇒ No Explanation Apparent



for 98.4 % of Mass

Meson Spectrum

LIGHT UNFLAVORED ($S = C = B = 0$)		STRANGE ($S = \pm 1, C = B = 0$)	
	$J^G(J^{PC})$		$J^G(J^{PC})$
• π^\pm	$1^-(0^-)$	• $\pi_2(1670)$	$1^-(2^-+)$
• π^0	$1^-(0^-+)$	• $\phi(1680)$	$0^-(1^-)$
• η	$0^+(0^-+)$	• $\rho_3(1690)$	$1^+(3^-)$
• $f_0(600)$	$0^+(0^++)$	• $\rho(1700)$	$1^+(1^-)$
• $\rho(770)$	$1^+(1^-)$	• $a_2(1700)$	$1^-(2^++)$
• $\omega(782)$	$0^-(1^-)$	• $f_0(1710)$	$0^+(0^++)$
• $\eta'(958)$	$0^+(0^-+)$	• $\eta(1760)$	$0^+(0^-+)$
• $f_0(980)$	$0^+(0^++)$	• $\pi(1800)$	$1^-(0^-+)$
• $a_0(980)$	$1^-(0^++)$	• $f_2(1810)$	$0^+(2^++)$
• $\phi(1020)$	$0^-(1^-)$	• $X(1835)$	$?^?(?^-+)$
• $h_1(1170)$	$0^-(1^+-)$	• $\phi_3(1850)$	$0^-(3^-)$
• $b_1(1235)$	$1^+(1^+-)$	• $\eta_2(1870)$	$0^+(2^-+)$
• $a_1(1260)$	$1^-(1^++)$	• $\rho(1900)$	$1^+(1^-)$
• $f_2(1270)$	$0^+(2^++)$	• $f_2(1910)$	$0^+(2^++)$
• $f_1(1285)$	$0^+(1^++)$	• $f_2(1950)$	$0^+(2^++)$
• $\eta(1295)$	$0^+(0^-+)$	• $\rho_3(1990)$	$1^+(3^-)$
• $\pi(1300)$	$1^-(0^-+)$	• $f_2(2010)$	$0^+(2^++)$
		• K^\pm	$1/2(0^-)$
		• K^0	$1/2(0^-)$
		• K_S^0	$1/2(0^-)$
		• K_L^0	$1/2(0^-)$
		• $K_0^*(800)$	$1/2(0^+)$
		• $K^*(892)$	$1/2(1^-)$
		• $K_1(1270)$	$1/2(1^+)$
		• $K_1(1400)$	$1/2(1^+)$
		• $K^*(1410)$	$1/2(1^-)$
		• $K_0^*(1430)$	$1/2(0^+)$
		• $K_2^*(1430)$	$1/2(2^+)$
		• $K(1460)$	$1/2(0^-)$
		• $K_2(1580)$	$1/2(2^-)$
		• $K(1630)$	$1/2(?^?)$
		• $K_1(1650)$	$1/2(1^+)$
		• $K^*(1680)$	$1/2(1^-)$
		• $K_3(1770)$	$1/2(2^-)$

140 MeV

770



Modern Miracles in Hadron Physics



[First](#)

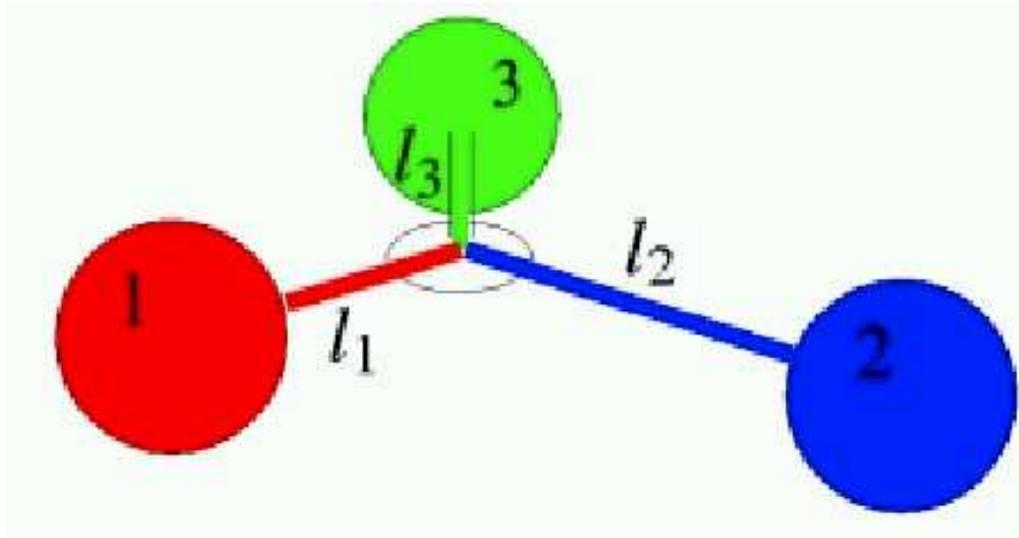
[Contents](#)

[Back](#)

[Conclusion](#)

Modern Miracles in Hadron Physics

- proton = three constituent quarks



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Modern Miracles in Hadron Physics

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- $M_{\text{proton}} \approx 1 \text{ GeV}$



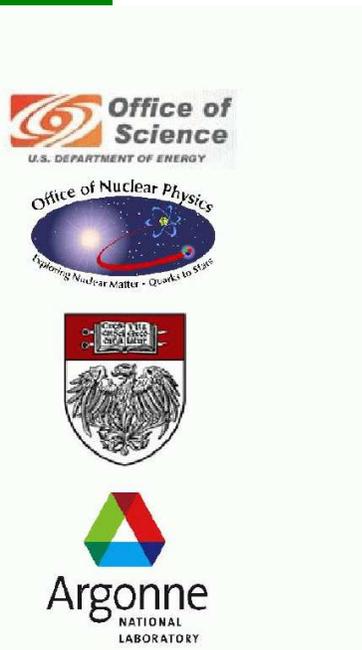
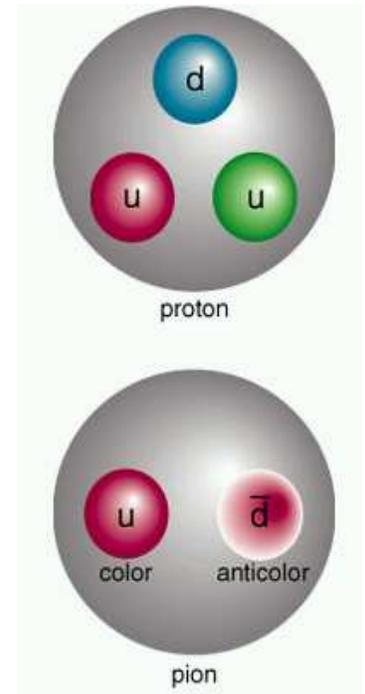
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- Another meson:
..... $M_{\rho} = 770 \text{ MeV}$ No Surprises Here



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- What is “wrong” with the pion?



Mass Destruction? Is this...



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[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Dichotomy of the Pion



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Dichotomy of the Pion

- How does one make an **almost massless** particle from two **massive** constituent-quarks?





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Must exhibit $m_{\pi}^2 \propto m_q$

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The **correct understanding** of pion observables; e.g. **mass**, **decay constant** and **form factors**, **requires** an approach to contain a **well-defined** and **valid chiral limit**, and an **accurate realisation** of dynamical chiral symmetry breaking.





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Using DSEs, we've provided this.



QCD's Emergent Phenomena



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

QCD's Emergent Phenomena

- Complex behaviour arises from apparently simple rules



QCD's Emergent Phenomena

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- Quark and Gluon Confinement
 - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon



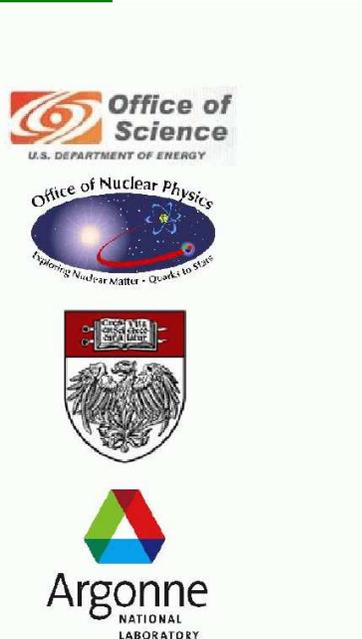
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- NSAC – Understanding these phenomena is one of the greatest intellectual challenges in physics



Why should You care?



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Why should You care?

$m_\pi = m_\rho \Rightarrow$ repulsive and attractive forces in nucleon-nucleon interaction both have **SAME** range and there is **No** intermediate range attraction!



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• Is ^{12}C stable?

• Probably not, if range **range** $\sim \frac{1}{2 M_Q}$



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- How do such changes affect Big Bang Nucleosynthesis?



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Is a unique long-range interaction between light-quarks responsible for all this or are there an uncountable infinity of qualitatively equivalent interactions?



What's the Problem?



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

What's the Problem?

- Must calculate the hadron's *wave function*
 - Can't be done using perturbation theory



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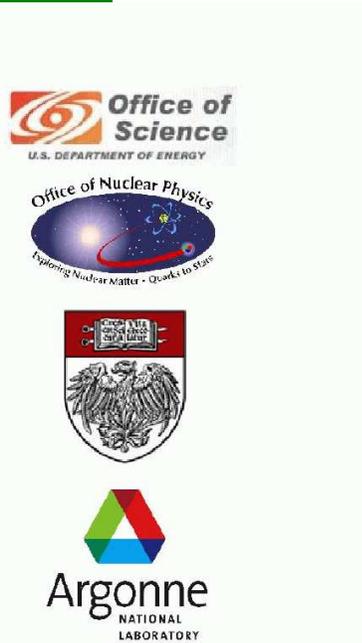
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Quintessence of Relativistic Quantum Field Theory



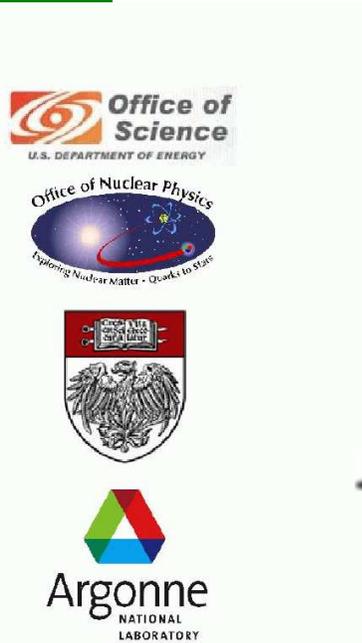
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What's the Problem?

- Must calculate the proton's *wave function*
 - Can't be done using perturbation theory
 - So what? Same is true of hydrogen atom
- Determination of proton's wave function requires *ab initio* nonperturbative solution of fully-fledged relativistic quantum field theory
- Modern Physics & Mathematics
 - Still quite some way from being able to do that



Model QCD



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Traditional approach to strong force problem

Model QCD



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[First](#)

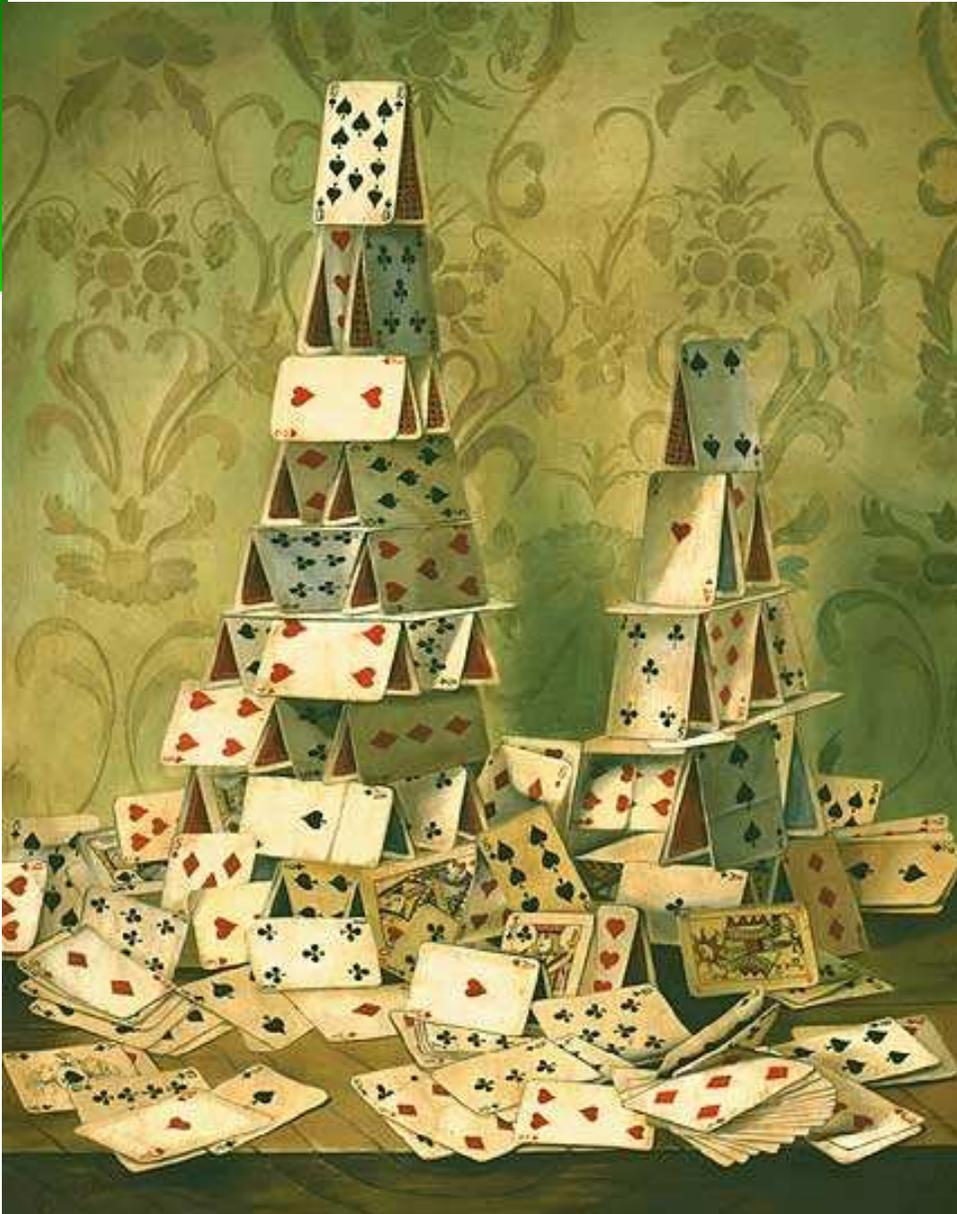
[Contents](#)

[Back](#)

[Conclusion](#)

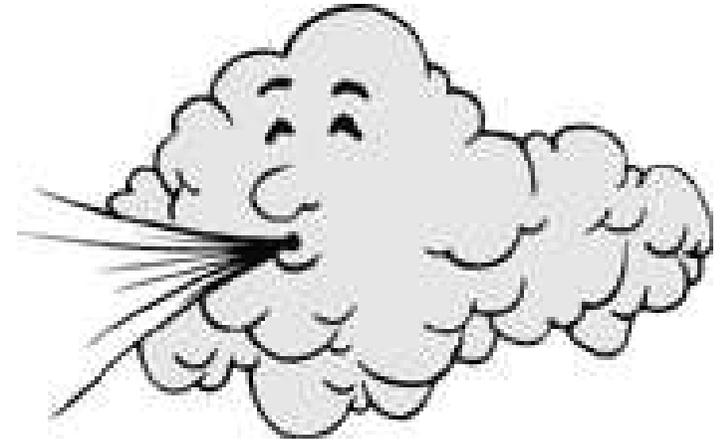
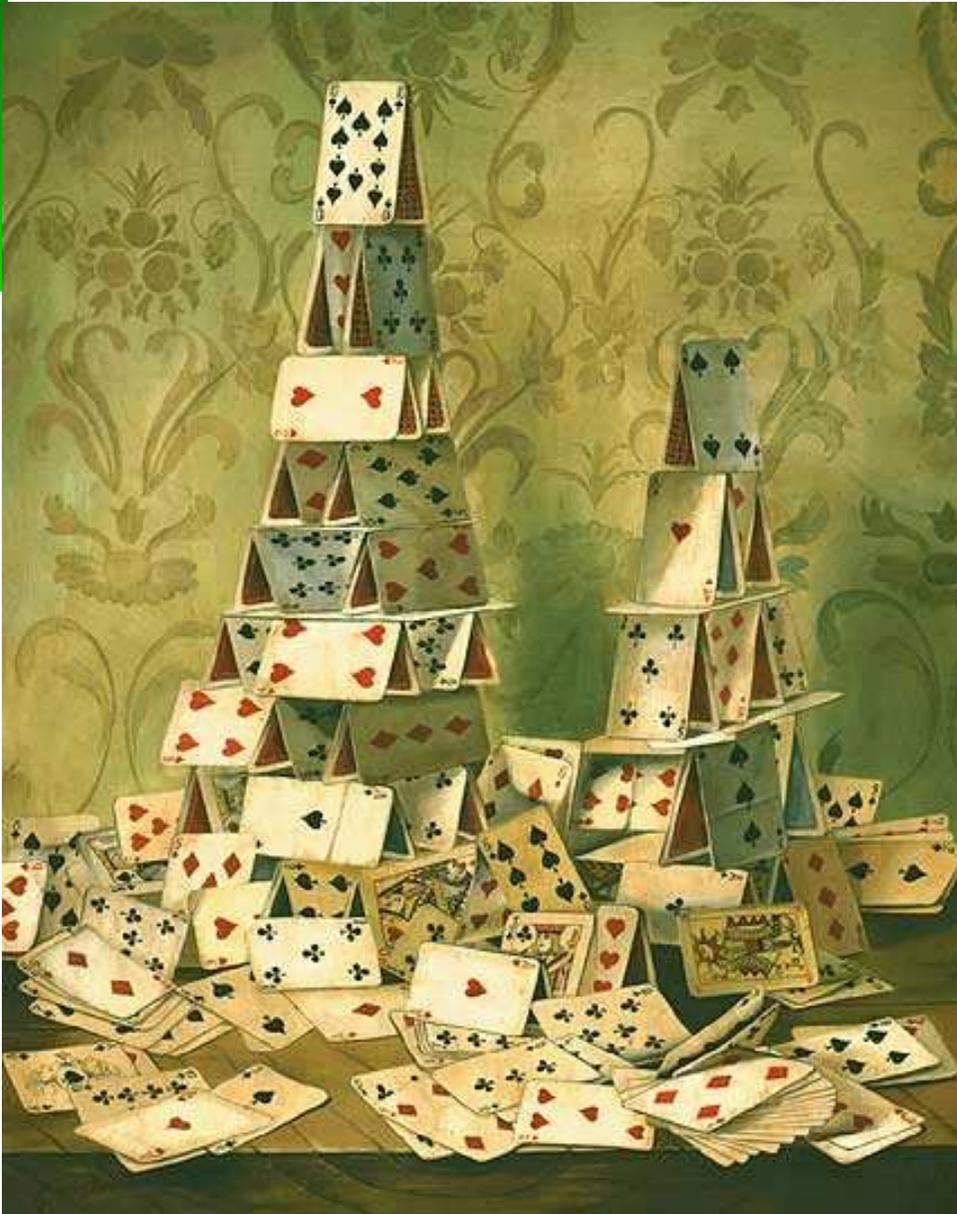
Traditional approach to strong force problem

Model QCD



Traditional approach to strong force problem

Model QCD



Lattice QCD



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

One modern nonperturbative approach *Lattice QCD*



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

One modern nonperturbative approach *Lattice QCD*



Confinement



[First](#)

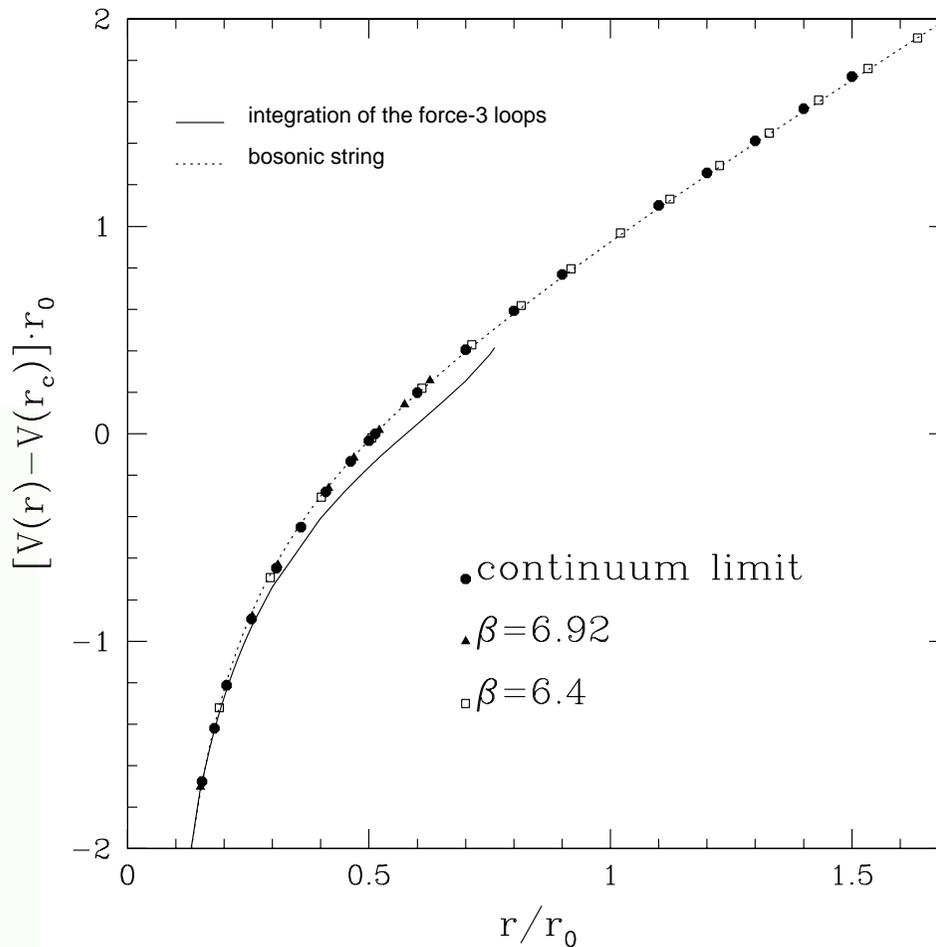
[Contents](#)

[Back](#)

[Conclusion](#)

Confinement

● Infinitely Heavy Quarks ... Picture in Quantum Mechanics



$$V(r) = \sigma r - \frac{\pi}{12} \frac{1}{r}$$

$$\sigma \sim 470 \text{ MeV}$$

Necco & Sommer
he-lq/0108008



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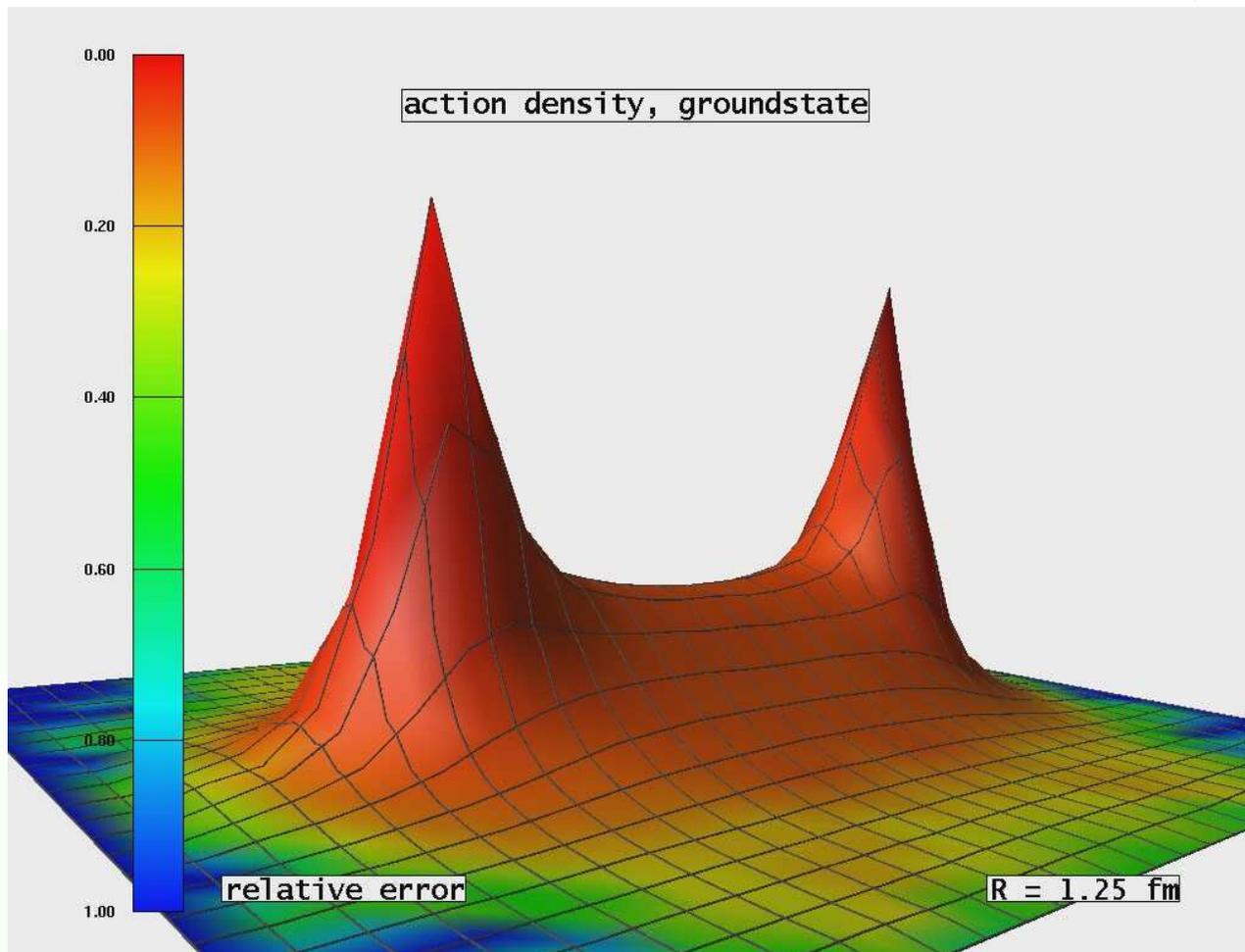
Contents

Back

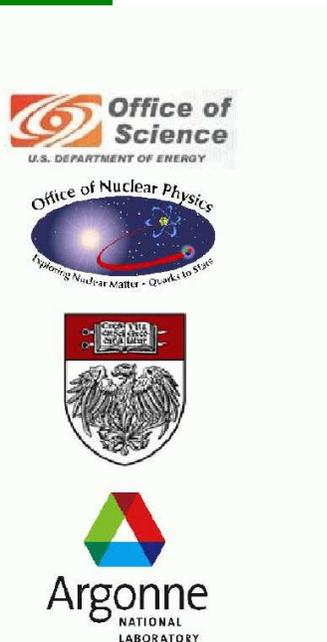
Conclusion

Confinement

- Illustrate this in terms of the action density ... analogous to plotting the Force = $F_{\bar{Q}Q}(r) = \sigma + \frac{\pi}{12} \frac{1}{r^2}$



Bali, *et al.*
he-lq/0512018



Confinement

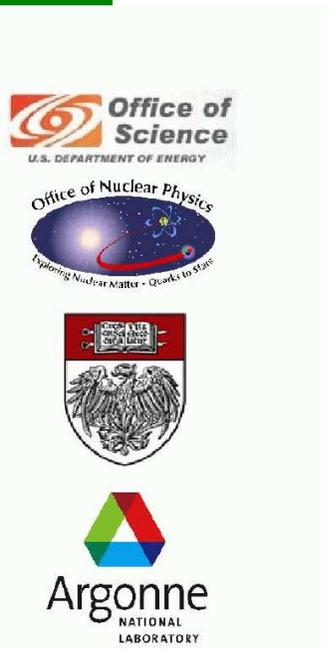
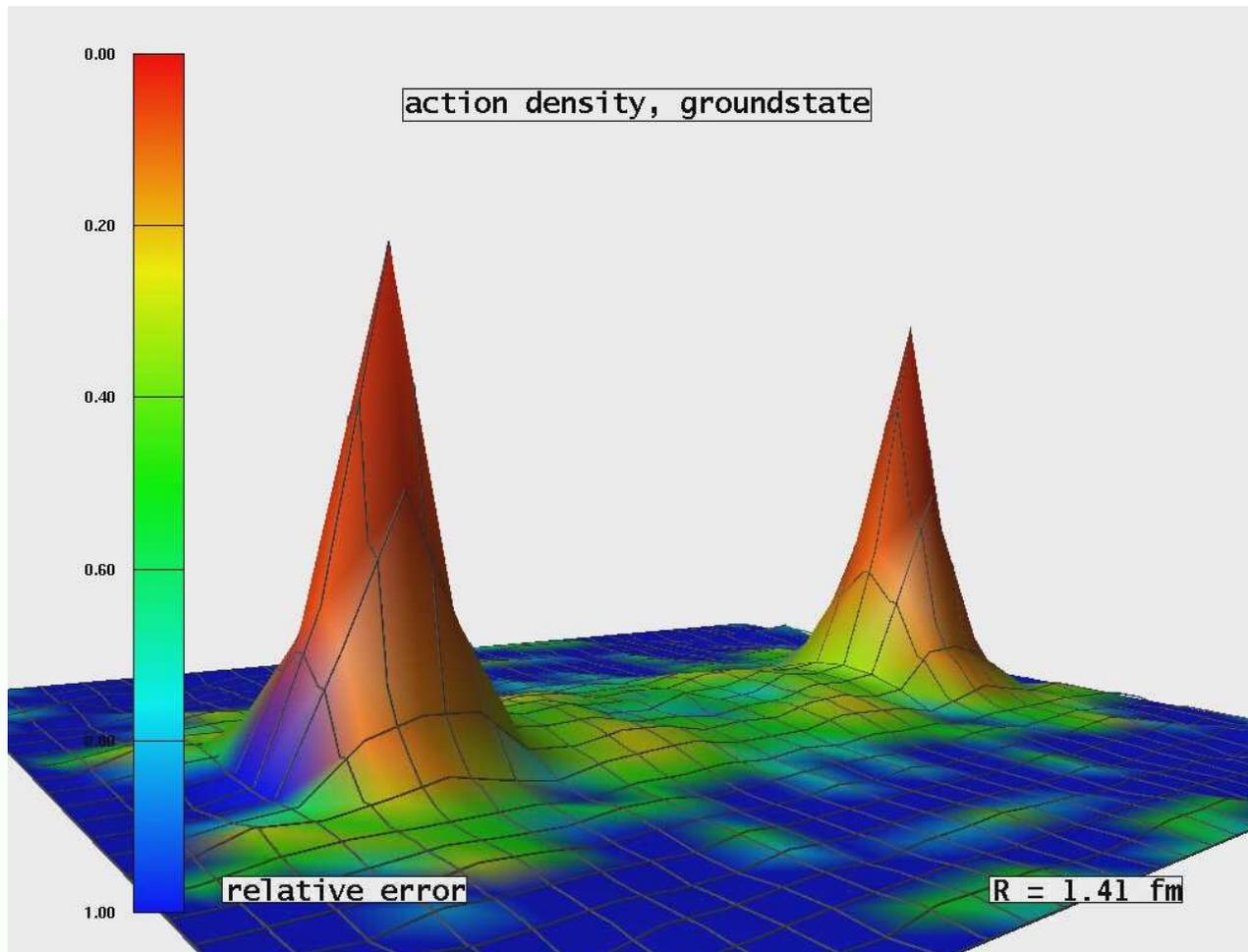
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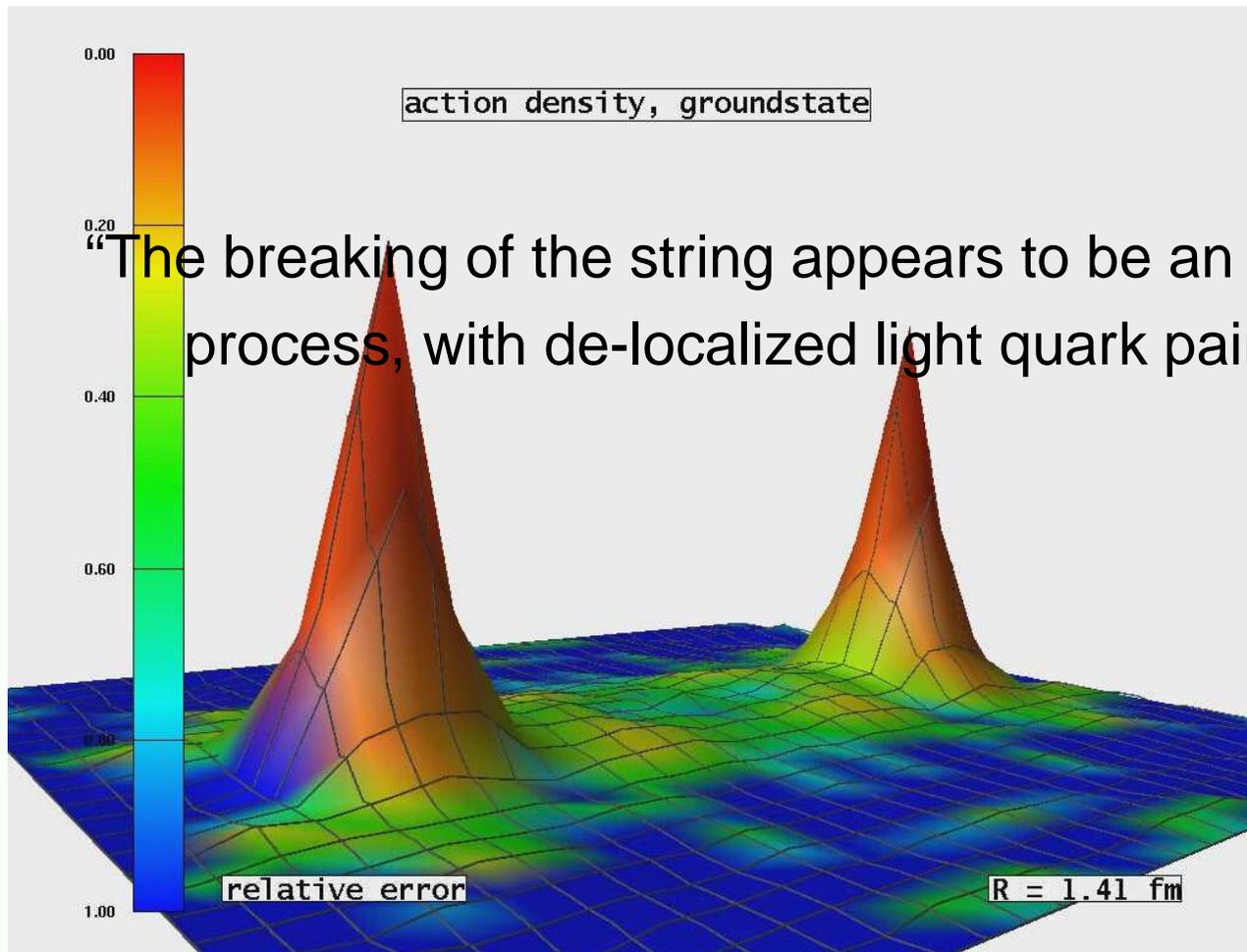
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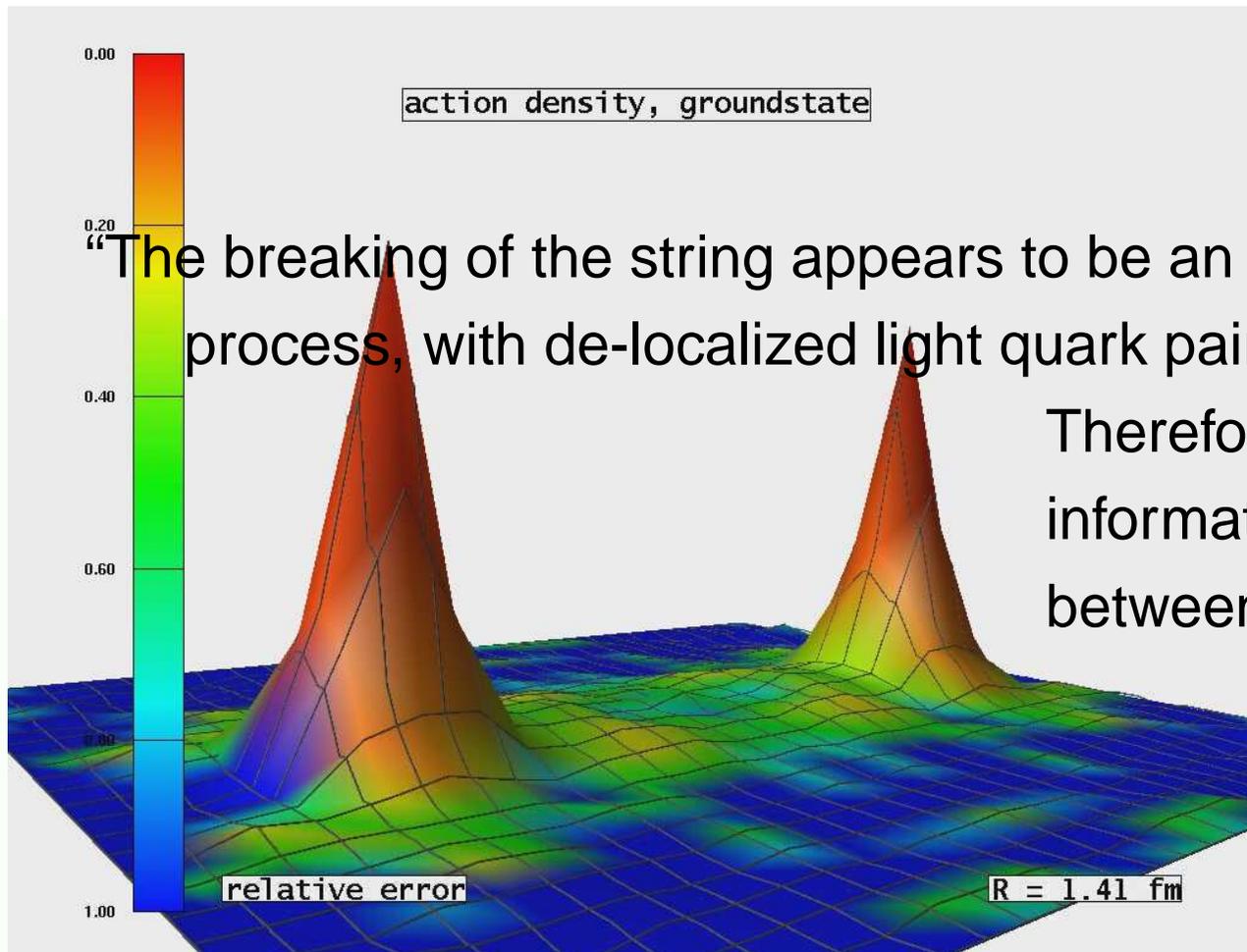
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he-lq/0512018



“The breaking of the string appears to be an instantaneous process, with de-localized light quark pair creation.”

Therefore ... **No** information on *potential* between light-quarks.



A Compromise?

Dyson-Schwinger Equations



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

A Compromise?

Dyson-Schwinger Equations

- 1994 ... *“As computer technology continues to improve, lattice gauge theory [LGT] will become an increasingly useful means of studying hadronic physics through investigations of discretised quantum chromodynamics [QCD]. . . .”*



A Compromise?

Dyson-Schwinger Equations

- 1994 ... *“However, it is equally important to develop other complementary nonperturbative methods based on continuum descriptions. In particular, with the advent of new accelerators such as CEBAF and RHIC, there is a need for the development of approximation techniques and models which bridge the gap between short-distance, perturbative QCD and the extensive amount of low- and intermediate-energy phenomenology in a single covariant framework. . . .”*



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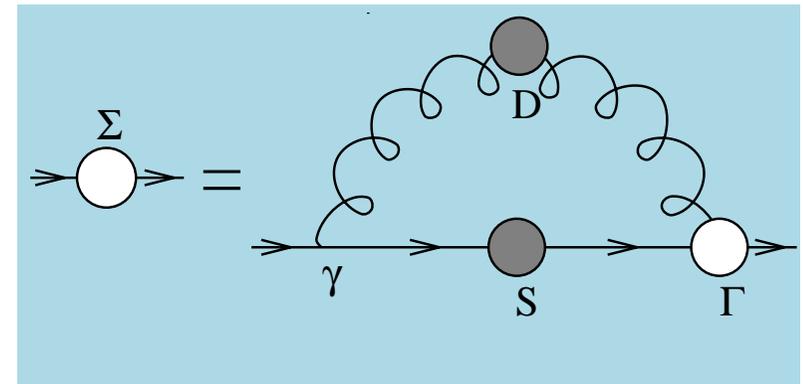
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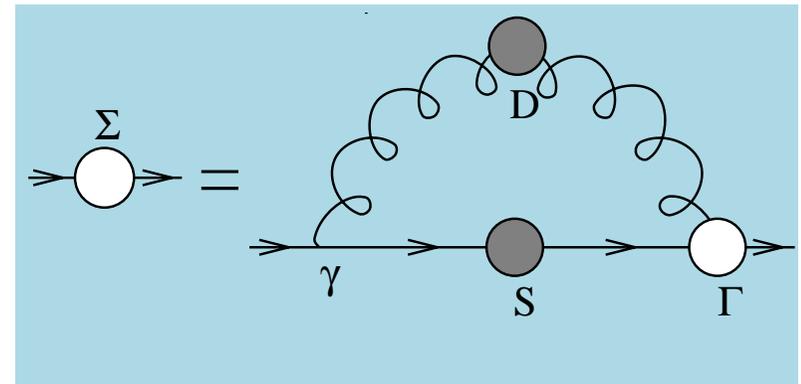
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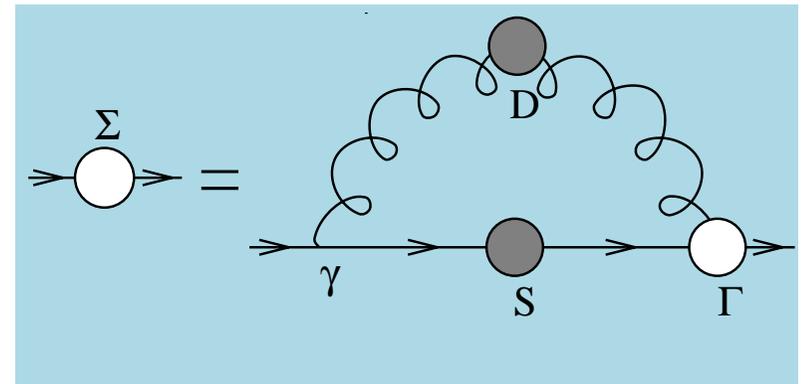
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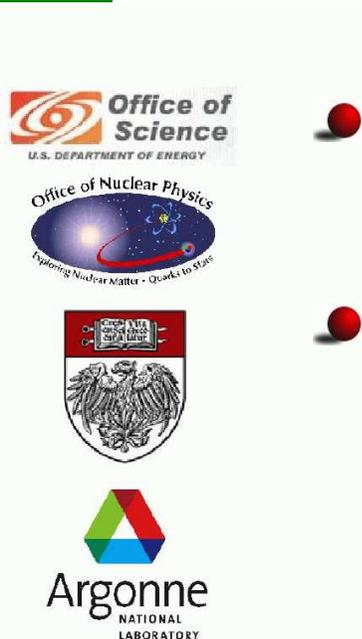
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- These are nonperturbative equivalents in quantum field theory to the Lagrange equations of motion.
- Essential in simplifying the general proof of renormalisability of gauge field theories.



Dyson-Schwinger Equations



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Dyson-Schwinger Equations

- Well suited to Relativistic Quantum Field Theory



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- Simplest level: **Generating Tool for Perturbation Theory**
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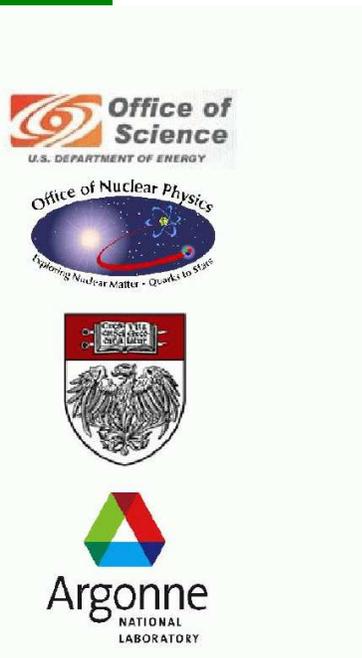
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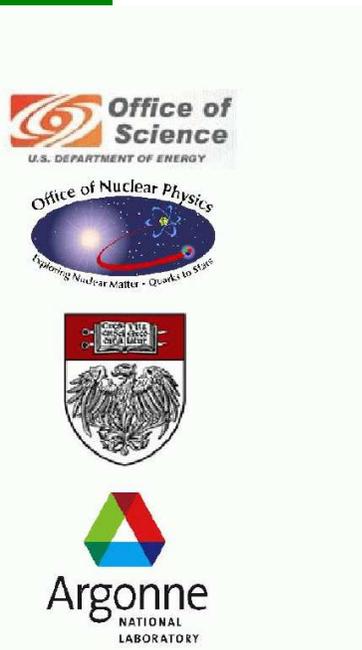
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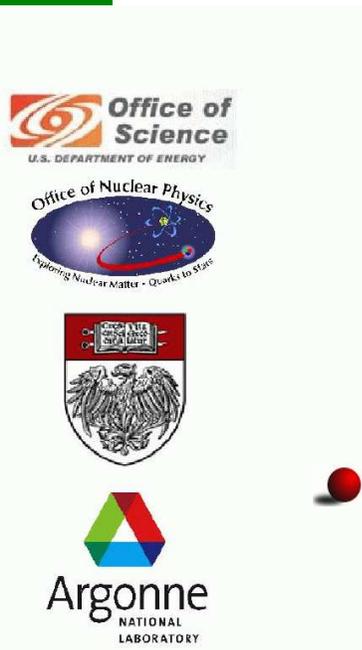
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 - Generation of fermion mass from *nothing*
 - **Quark & Gluon Confinement**
 - Coloured objects not detected, not detectable?



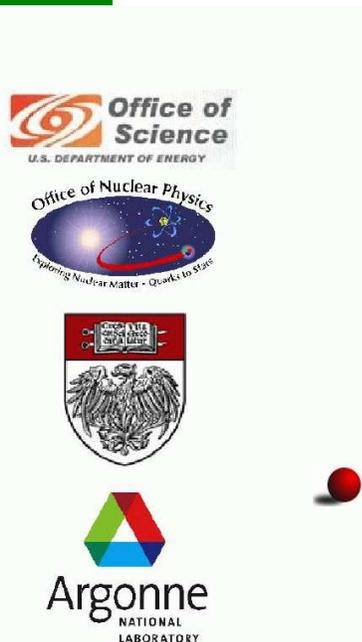
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..... behaviour of $\alpha_s(Q^2)$



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 - Coloured objects not detected, not detectable?

Cross-Sections built from Schwinger Functions



Persistent Challenge



[First](#)

[Contents](#)

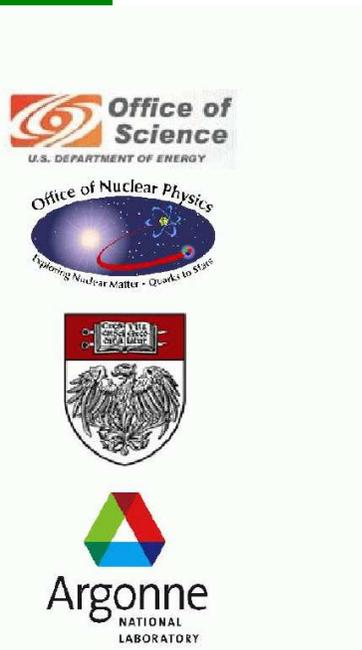
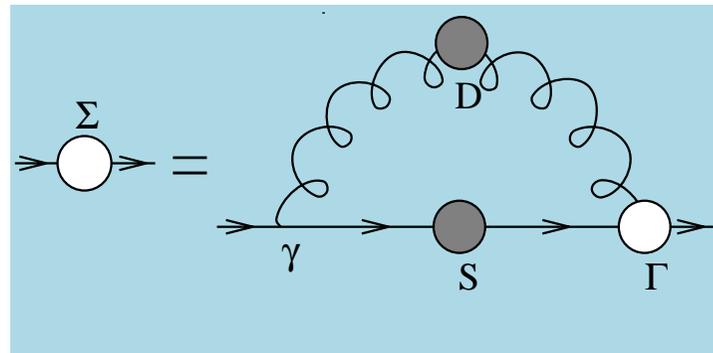
[Back](#)

[Conclusion](#)



Persistent Challenge

- Infinitely Many Coupled Equations



Persistent Challenge



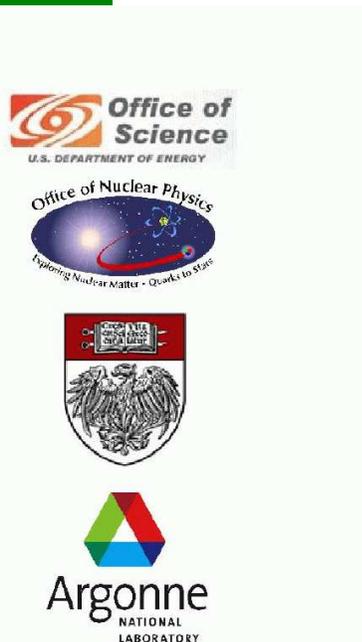
- Infinitely Many Coupled Equations
 - Solutions are Schwinger Functions (Euclidean Green Functions)



Persistent Challenge



- Infinitely Many Coupled Equations
 - Solutions are Schwinger Functions (Euclidean **Green** Functions)
 - Not all are Schwinger functions are experimentally observable but **all are** same VEVs measured in Lattice-QCD simulations . . . opportunity for comparisons at pre-experimental level . . . cross-fertilisation





Persistent Challenge

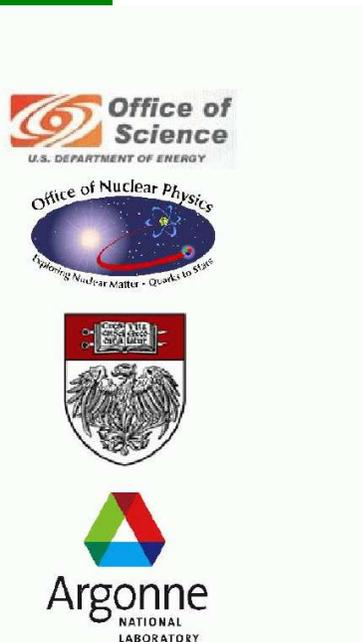
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 - Solutions are Schwinger Functions (Euclidean **Green** Functions)
- Coupling between equations **necessitates** truncation
 - Weak coupling expansion \Rightarrow Perturbation Theory





Persistent Challenge

- Infinitely Many Coupled Equations
 - Solutions are Schwinger Functions (Euclidean **Green** Functions)
- Coupling between equations **necessitates** truncation
 - Weak coupling expansion \Rightarrow Perturbation Theory **Not useful** for the nonperturbative problems in which we're interested



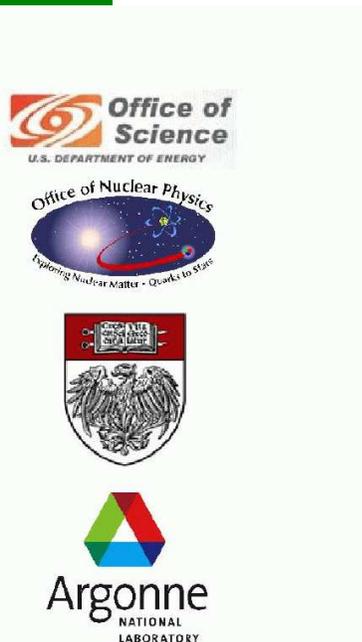


Persistent Challenge

- Infinitely Many Coupled Equations
 - Solutions are Schwinger Functions (Euclidean **Green** Functions)
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme

H.J. Munczek Phys. Rev. D **52** (1995) 4736
Dynamical chiral symmetry breaking, Goldstone's theorem and the consistency of the Schwinger-Dyson and Bethe-Salpeter Equations

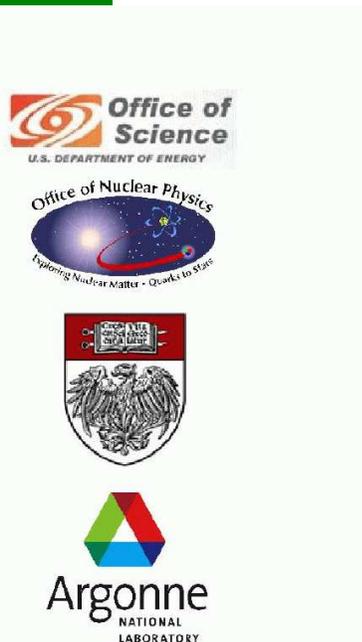
A. Bender, C. D. Roberts and L. von Smekal, Phys. Lett. B **380** (1996) 7
Goldstone Theorem and Diquark Confinement Beyond Rainbow Ladder Approximation





Persistent Challenge

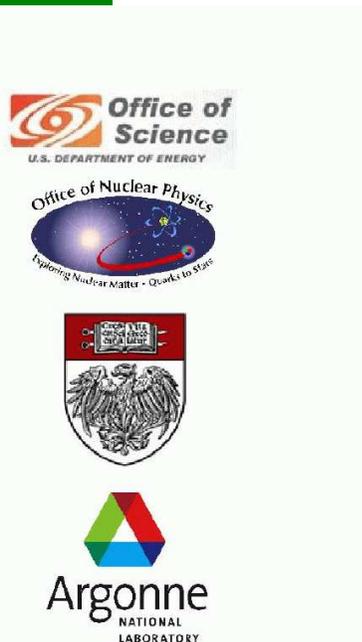
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- Has Enabled Proof of **EXACT** Results in QCD





Persistent Challenge

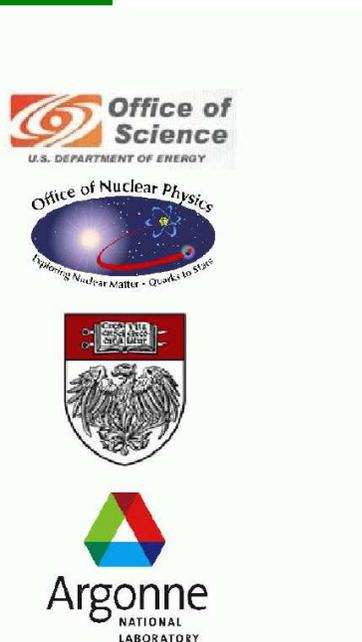
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- And Formulation of Practical Phenomenological Tool to
 - Illustrate Exact Results



Persistent Challenge



- Infinitely Many Coupled Equations
 - Solutions are Schwinger Functions (Euclidean **Green** Functions)
- There is at least one **systematic nonperturbative, symmetry-preserving** truncation scheme
- Has Enabled Proof of **EXACT** Results in QCD
- And Formulation of Practical Phenomenological Tool to
 - Make Predictions with Readily Quantifiable Errors



Perturbative Dressed-quark Propagator



[First](#)

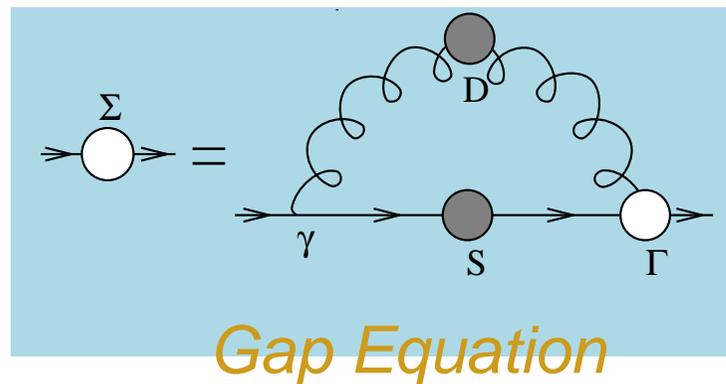
[Contents](#)

[Back](#)

[Conclusion](#)



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



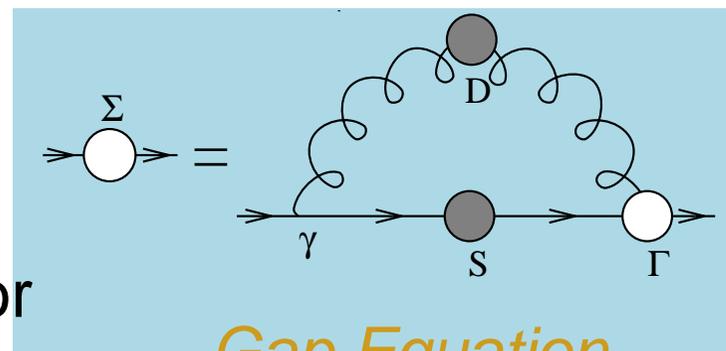
Gap Equation



Argonne
NATIONAL
LABORATORY



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

- dressed-quark propagator

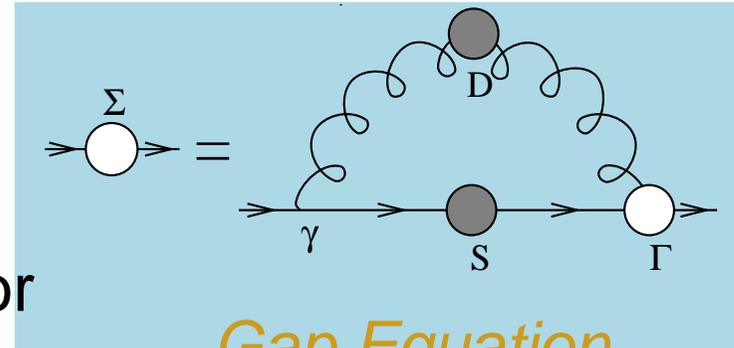
$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$



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$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

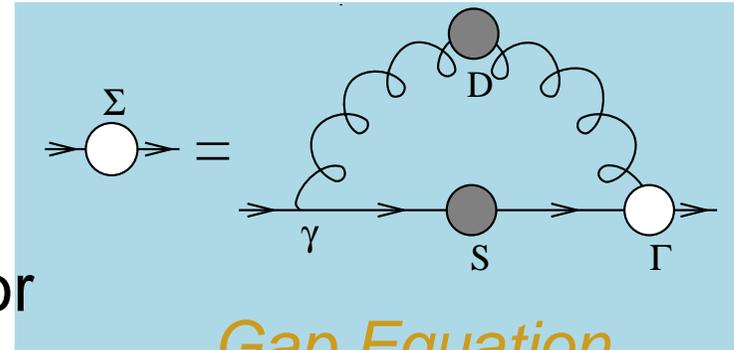
- Weak Coupling Expansion

Reproduces **Every** Diagram in **Perturbation Theory**





$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

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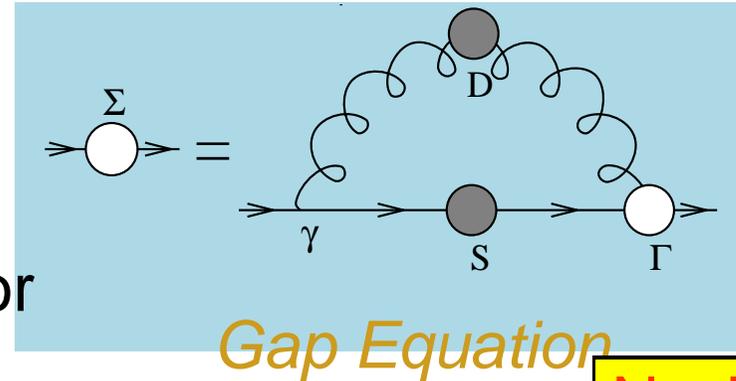
- Weak Coupling Expansion
Reproduces **Every** Diagram in **Perturbation Theory**
- **But** in **Perturbation Theory**

$$B(p^2) = m \left(1 - \frac{\alpha}{\pi} \ln \left[\frac{p^2}{m^2} \right] + \dots \right) \xrightarrow{m \rightarrow 0} 0$$



Perturbative Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

No DCSB
Here!

- Weak Coupling Expansion
Reproduces **Every** Diagram in Perturbation Theory
- But in Perturbation Theory

$$B(p^2) = m \left(1 - \frac{\alpha}{\pi} \ln \left[\frac{p^2}{m^2} \right] + \dots \right) \begin{matrix} m \rightarrow 0 \\ \rightarrow 0 \end{matrix}$$



Dressed-Quark Propagator



[First](#)

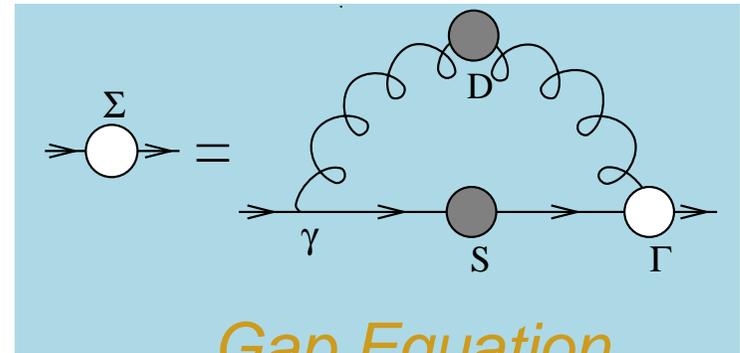
[Contents](#)

[Back](#)

[Conclusion](#)

Dressed-Quark Propagator

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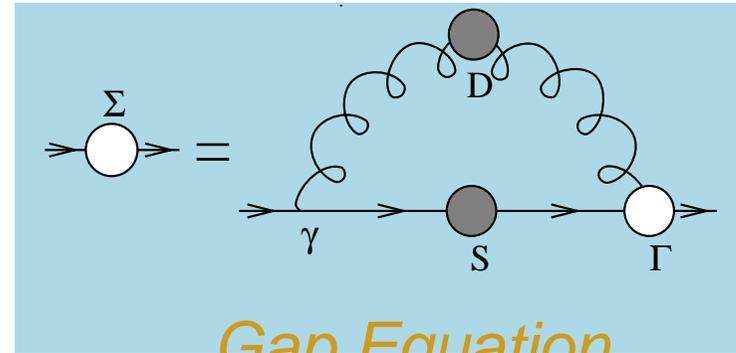


Gap Equation



Dressed-Quark Propagator

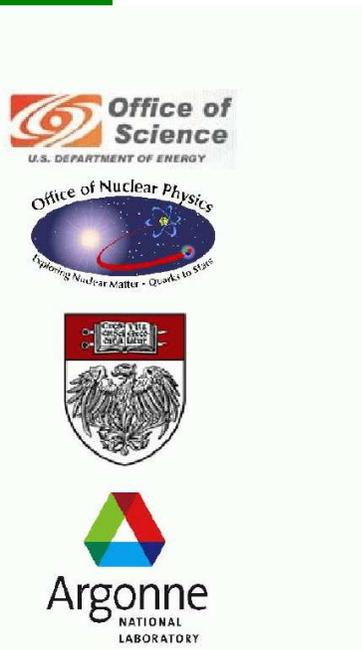
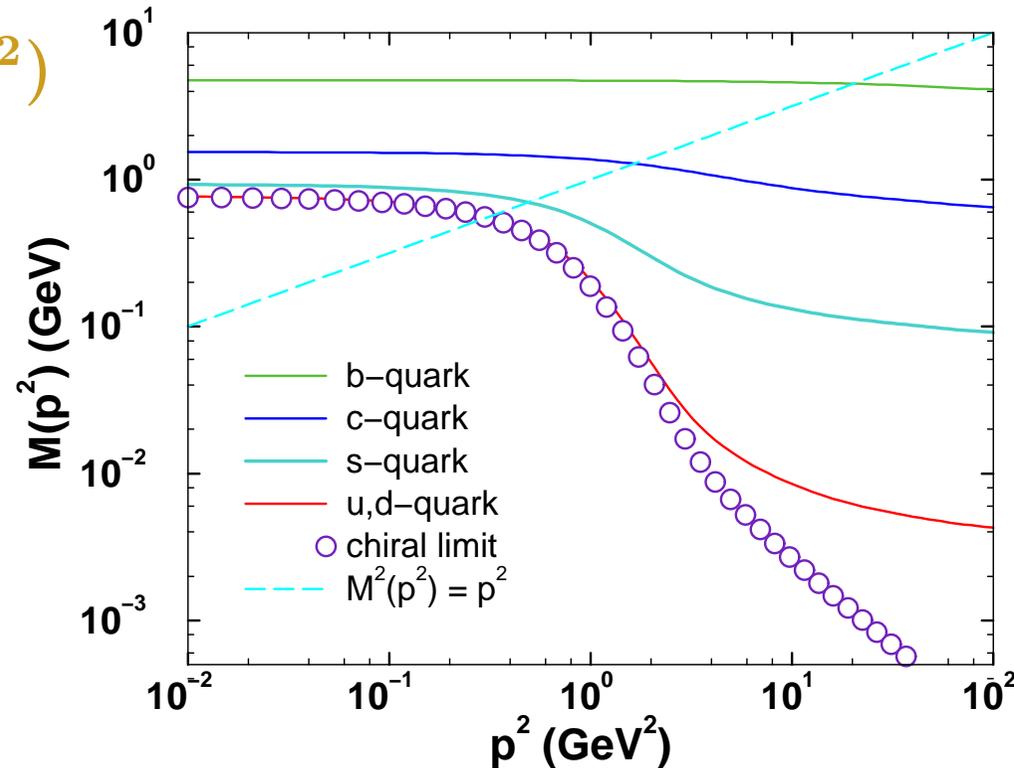
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

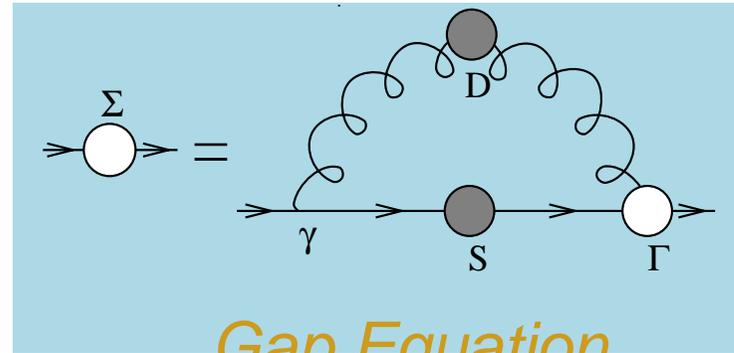
- Gap Equation's Kernel Enhanced on IR domain

⇒ IR Enhancement of $M(p^2)$



Dressed-Quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

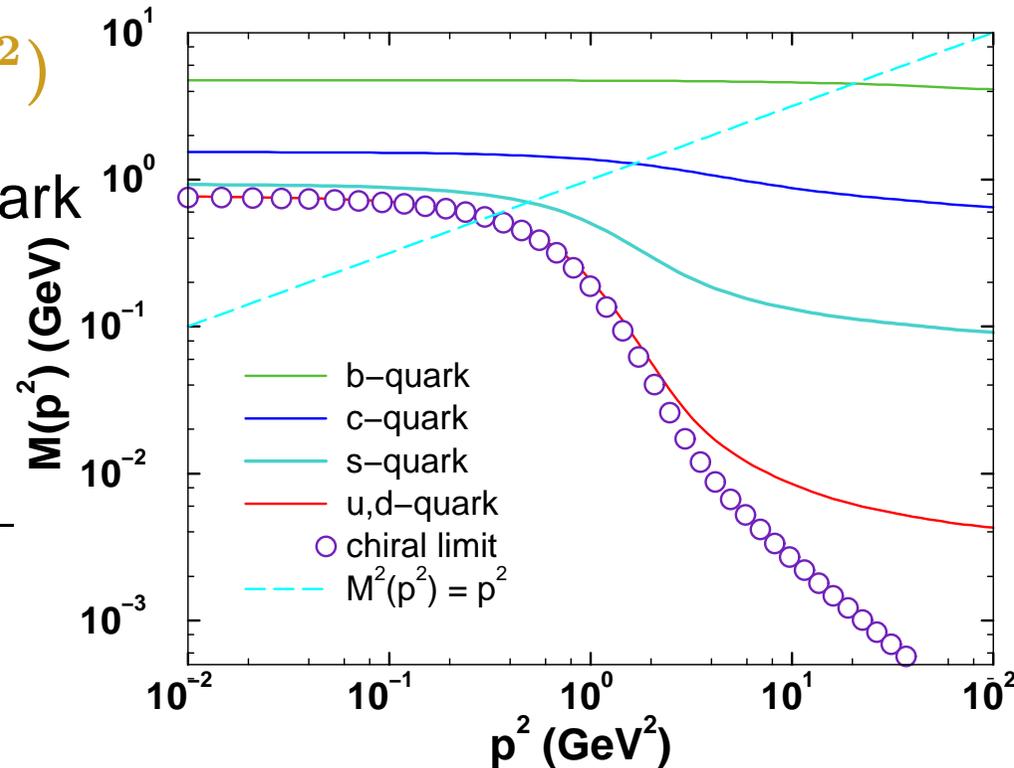
- Gap Equation's Kernel Enhanced on **IR domain**

⇒ **IR Enhancement** of $M(p^2)$

- Euclidean Constituent-Quark

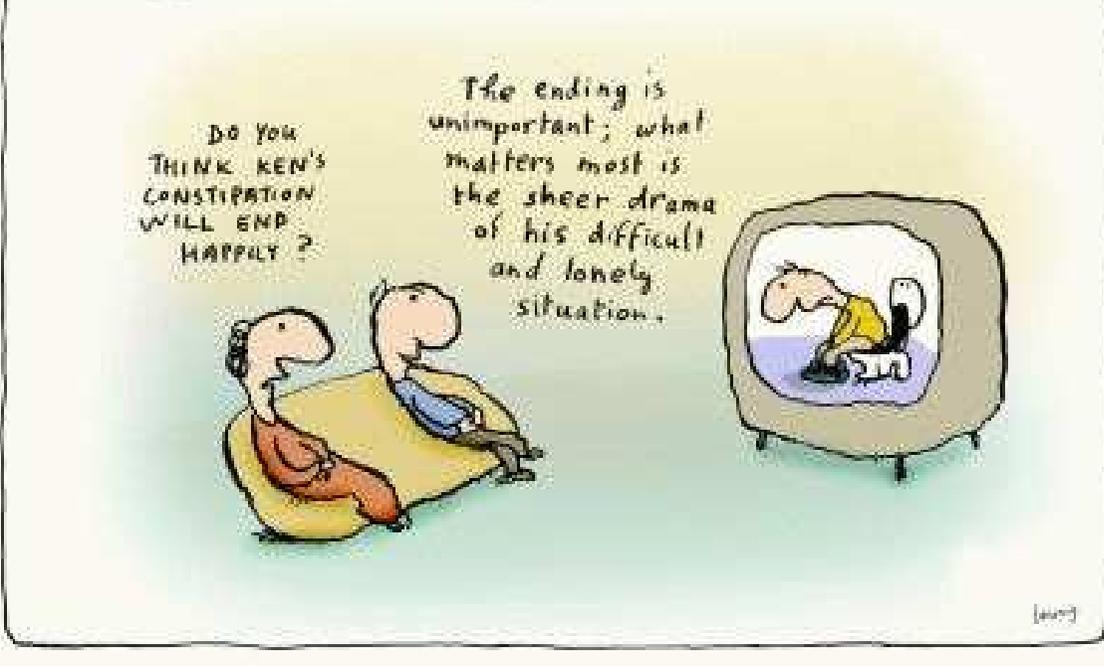
Mass: $M_f^E: p^2 = M(p^2)^2$

flavour	u/d	s	c	b
$\frac{M^E}{m_\zeta}$	$\sim 10^2$	~ 10	~ 1.5	~ 1.1



Dressed-Quark Propagator

- Longstanding Prediction of Dyson-Schwinger Equation Studies



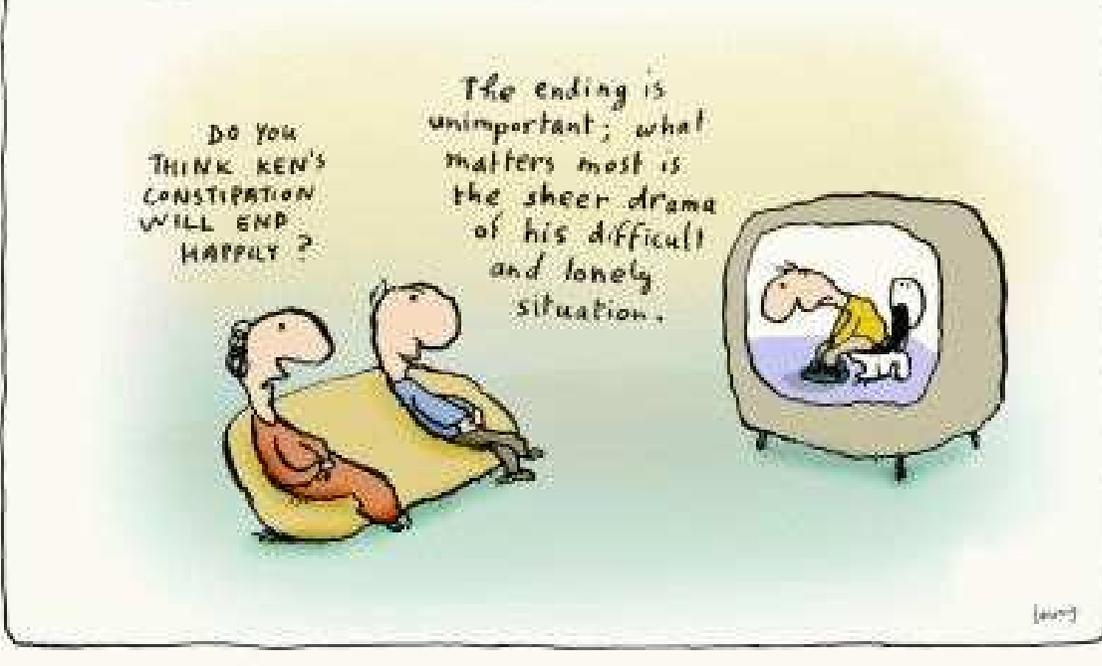
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Dressed-Quark Propagator



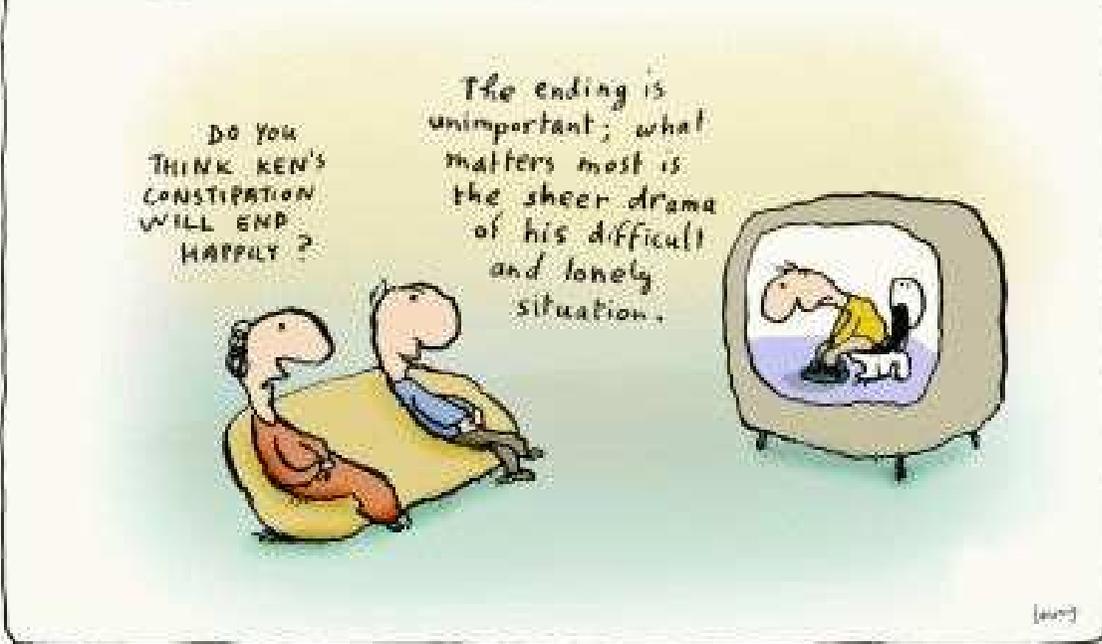
- Longstanding Prediction of Dyson-Schwinger Equation Studies

- E.g., *Dyson-Schwinger equations and their application to hadronic physics,*

C. D. Roberts and
A. G. Williams,
Prog. Part. Nucl. Phys.
33 (1994) 477



Dressed-Quark Propagator



- Long used as basis for efficacious hadron physics phenomenology

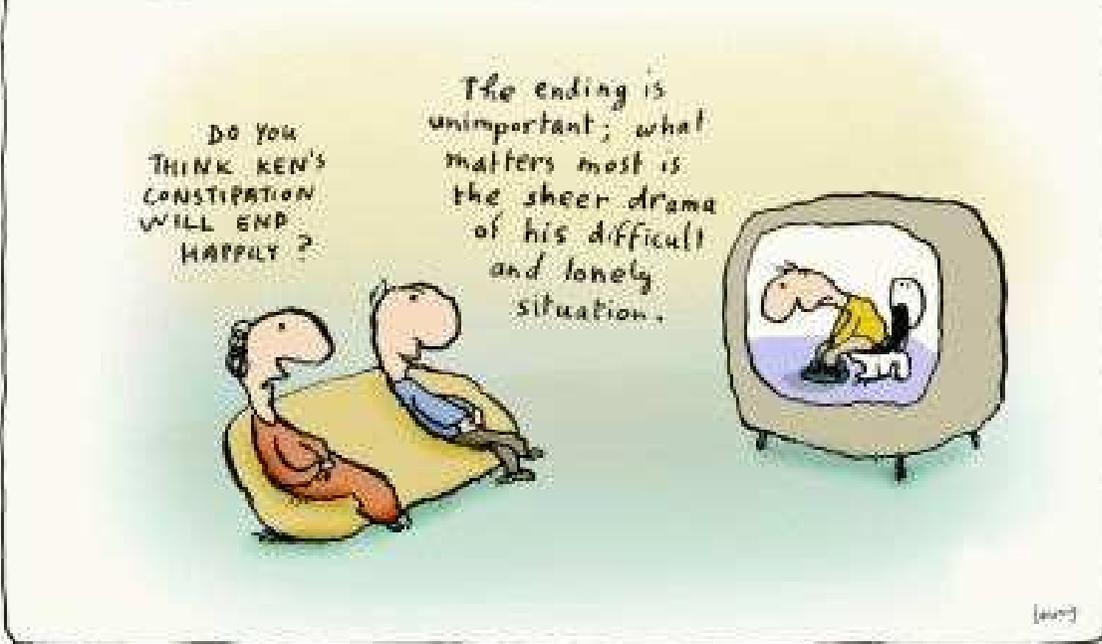
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- *Electromagnetic pion form-factor and neutral pion decay width*,
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(1996) 475

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Mandar Bhagwat



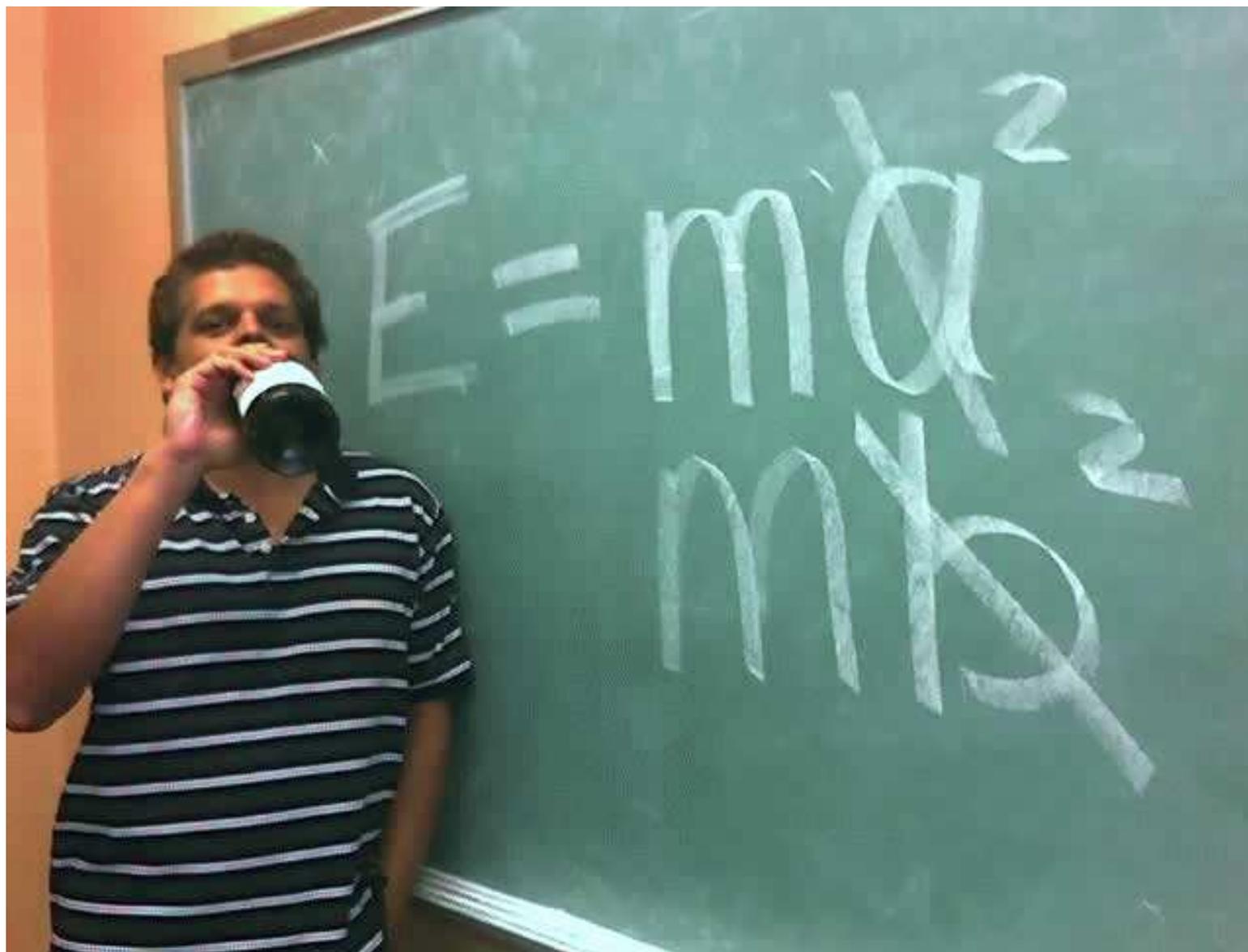
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Mandar Bhagwat



[First](#)

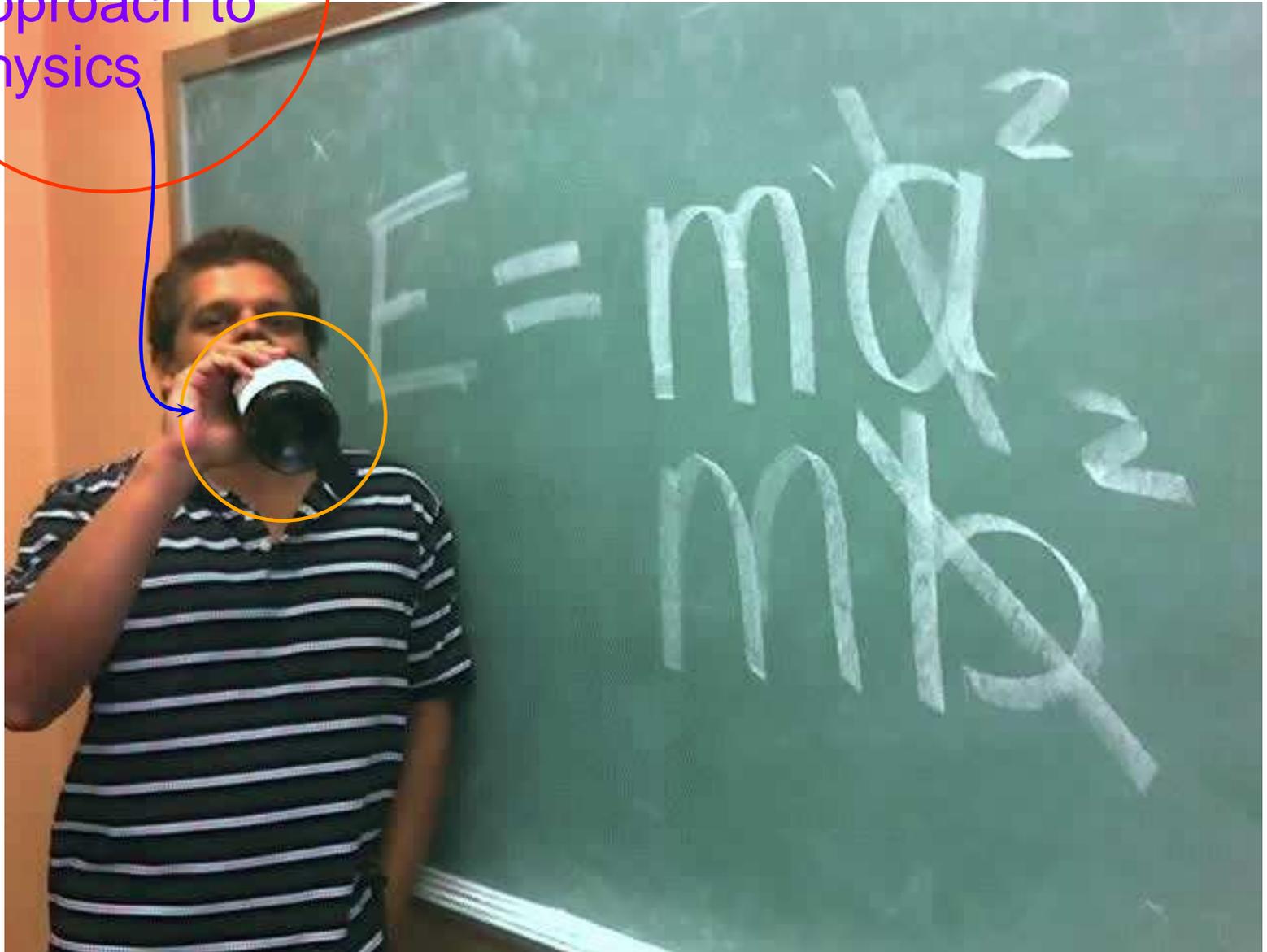
[Contents](#)

[Back](#)

[Conclusion](#)

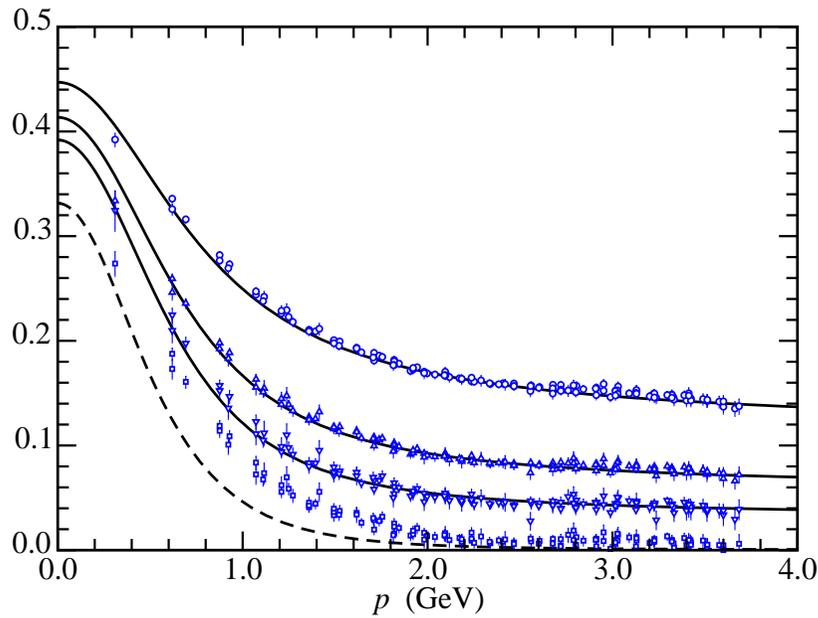
Mandar Bhagwat

Emulating
supervisor's
approach to
physics

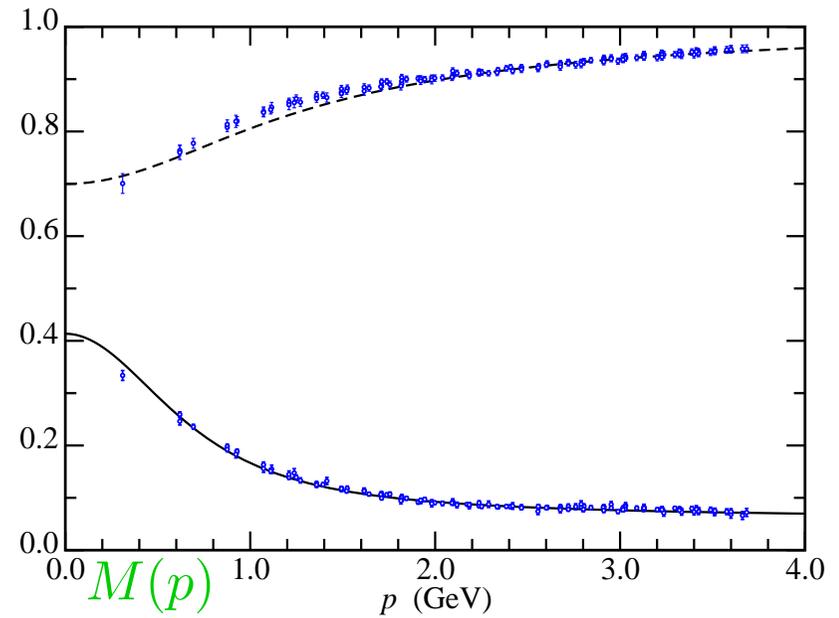


Dressed-Quark Propagator

$M(p)$



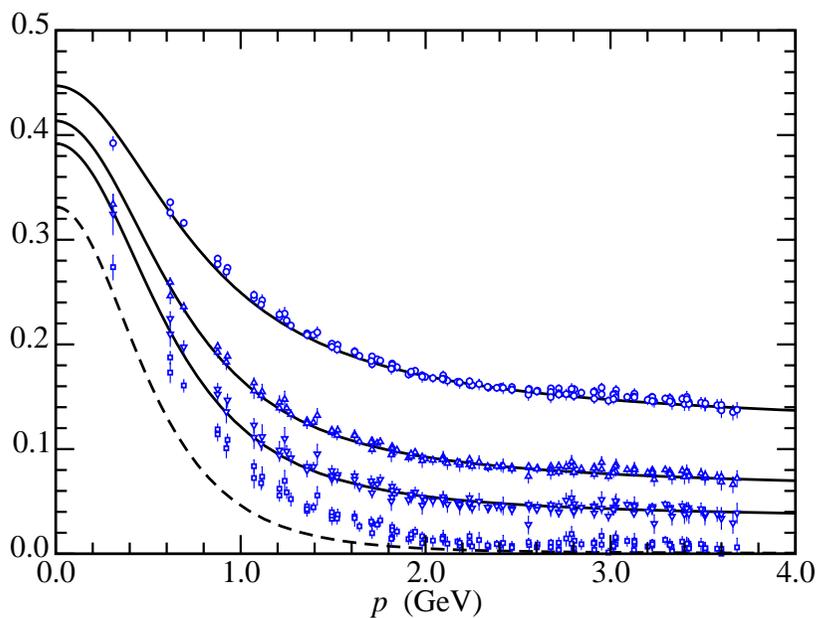
$Z(p)$



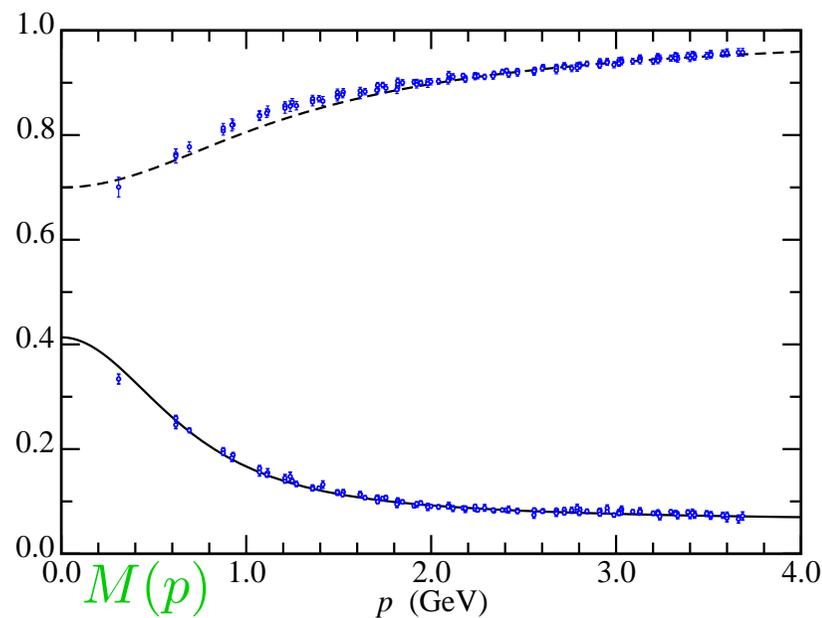
Dressed-Quark Propagator

2002

$M(p)$



$Z(p)$



● “*data*” Quenched Lattice Meas.

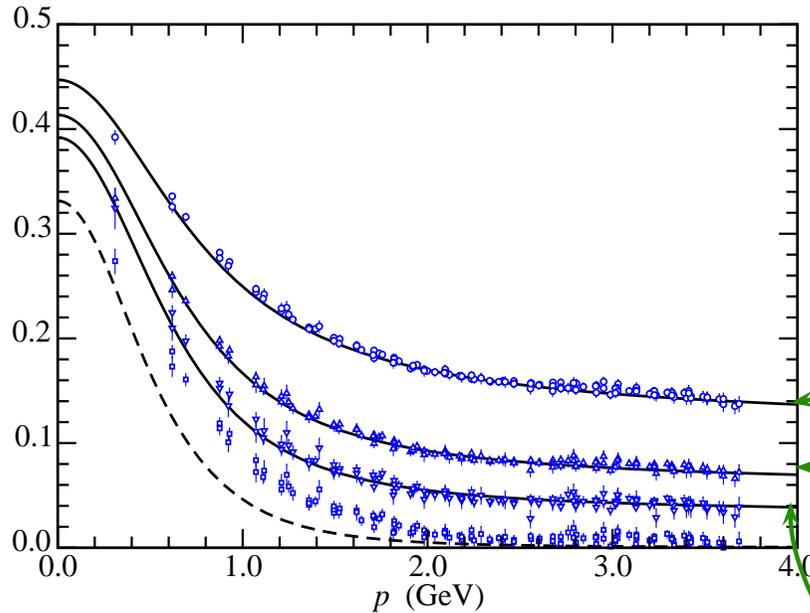
– Bowman, Heller, Leinweber, Williams: he-lat/0209129



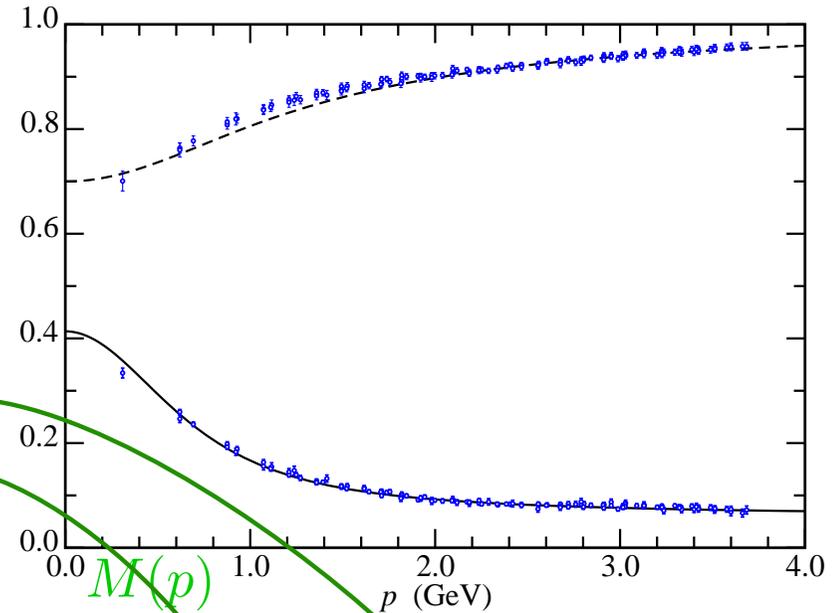
Dressed-Quark Propagator

2002

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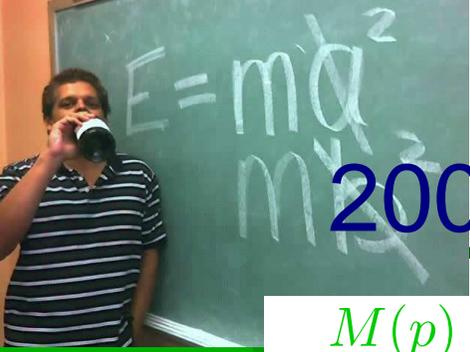


“*data*” Quenched Lattice Meas.

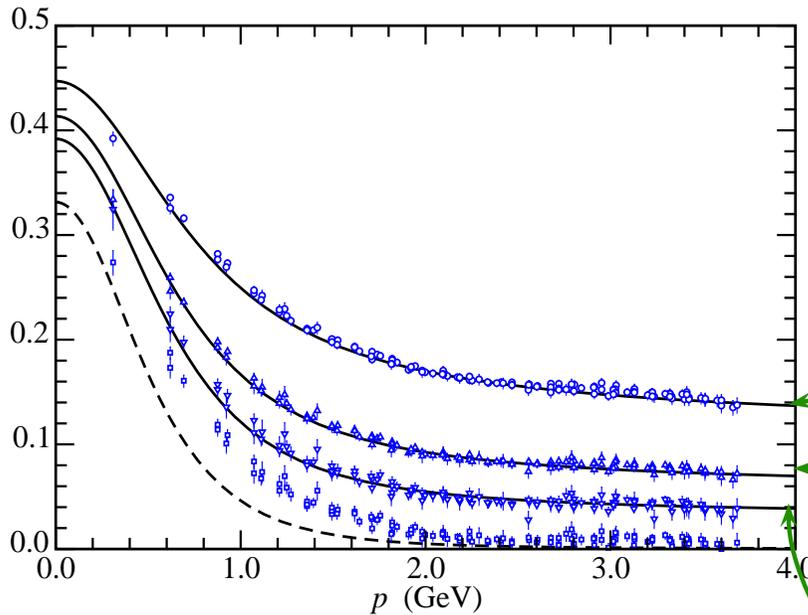
– Bowman, Heller, Leinweber, Williams: [he-lat/0209129](https://arxiv.org/abs/he-lat/0209129)
 current-quark masses: 30 MeV, 50 MeV, 100 MeV



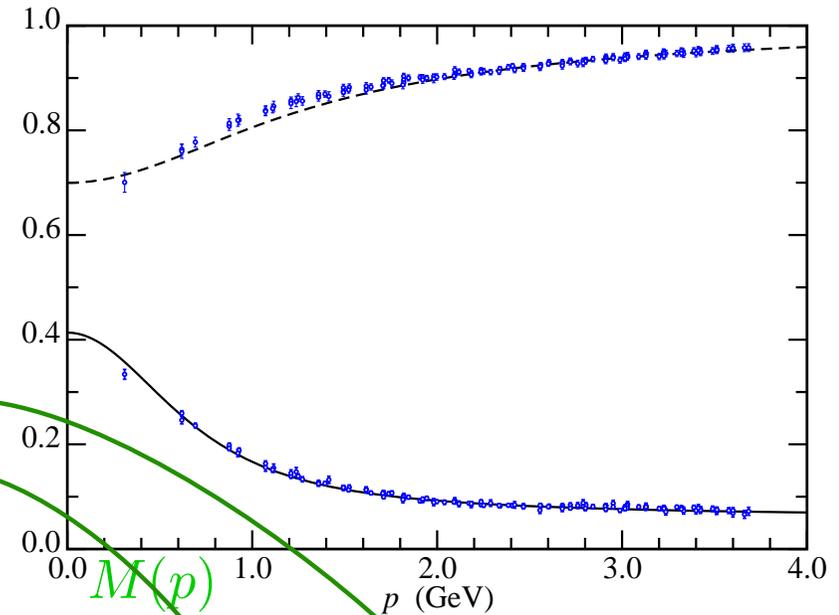
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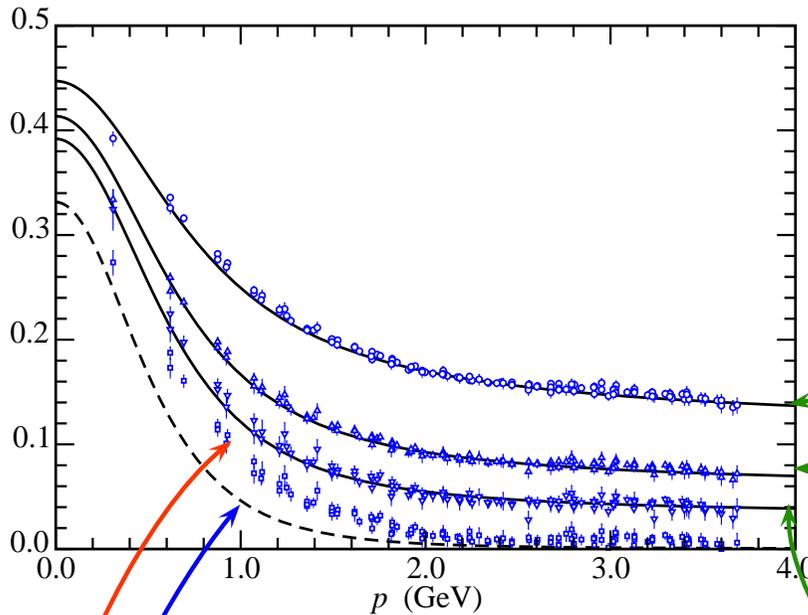
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- **Curves:** Quenched DSE Cal.
 - Bhagwat, Pichowsky, Roberts, Tandy nu-th/0304003



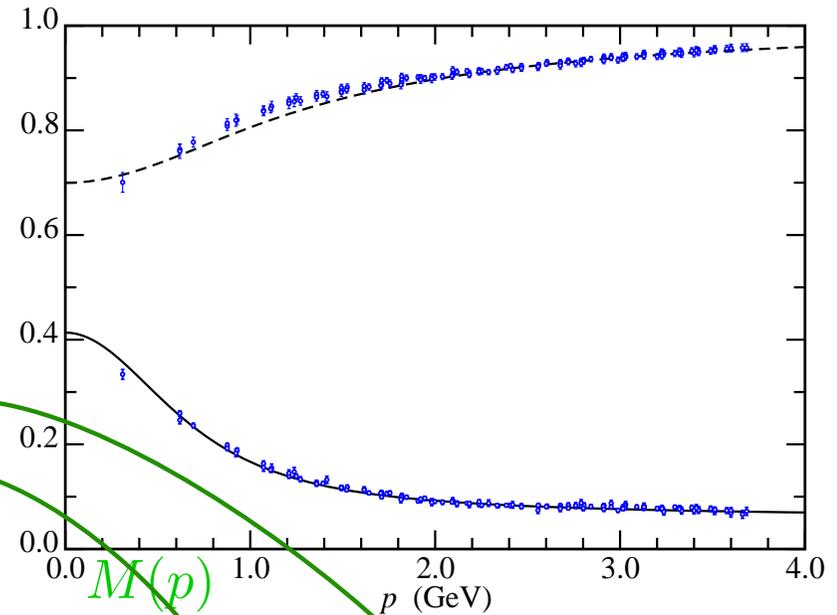
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Linear extrapolation of lattice data to chiral limit is inaccurate



QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p - q) \Gamma_\nu(q)$
Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
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- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
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- Exploratory DSE and lattice-QCD studies of dressed-quark-gluon vertex

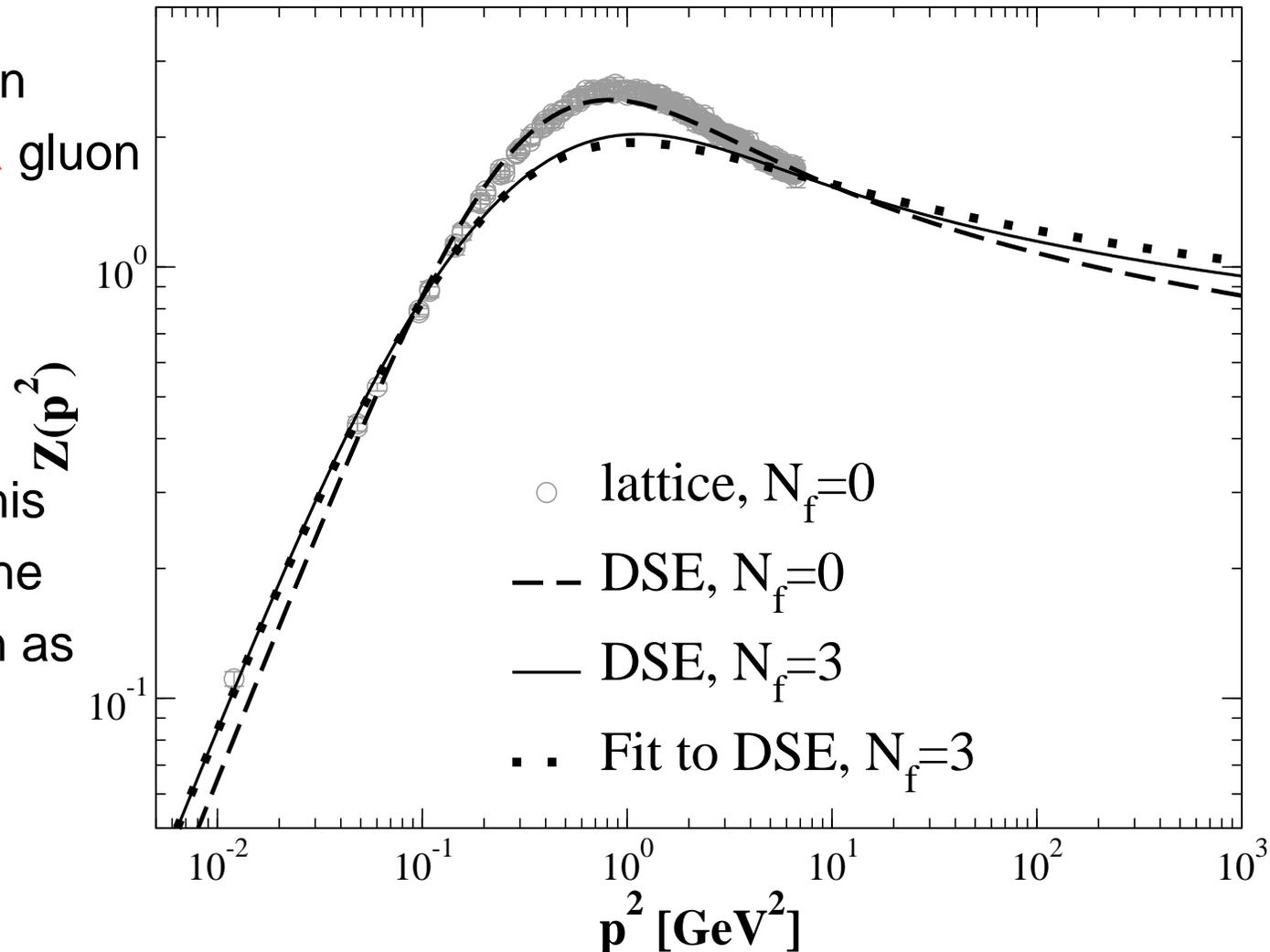


Dressed-gluon Propagator

$$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right) \frac{Z(k^2)}{k^2}$$

- Suppression means \exists IR gluon mass-scale ≈ 1 GeV

- Naturally, this scale has the same origin as Λ_{QCD}



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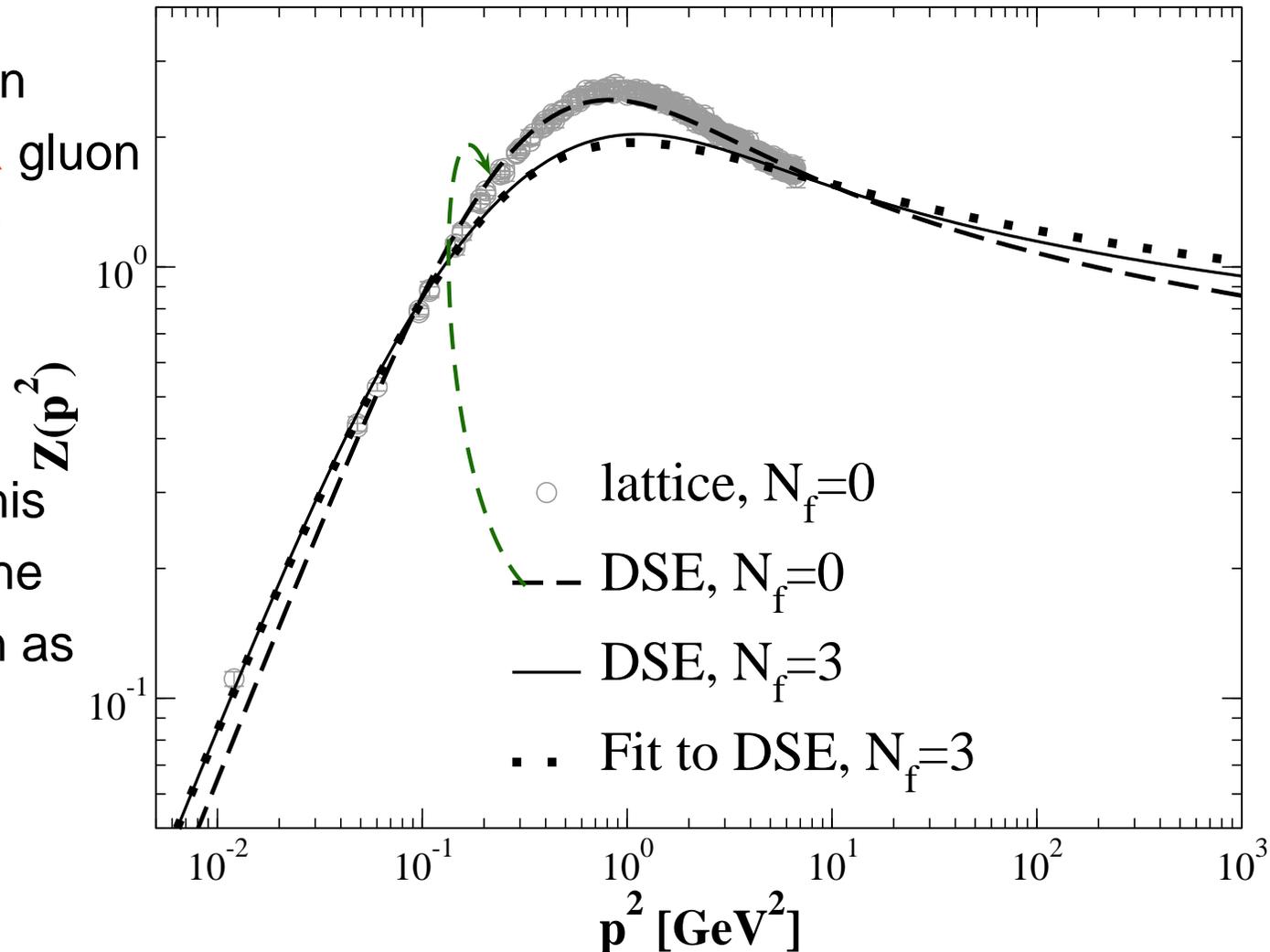
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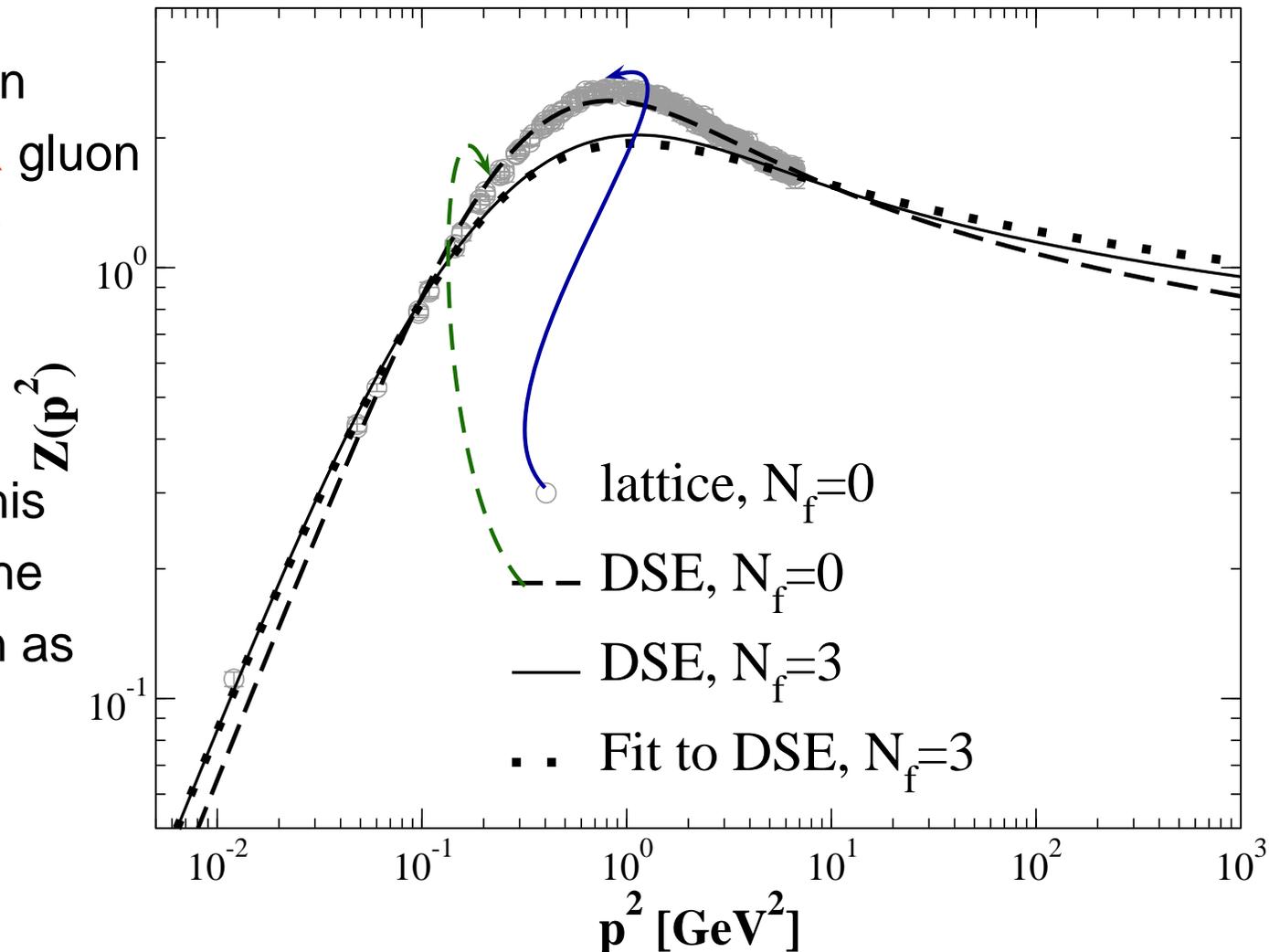
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Hadrons



- Established understanding of two- and three-point functions



Hadrons



- Established understanding of two- and three-point functions
- What about bound states?



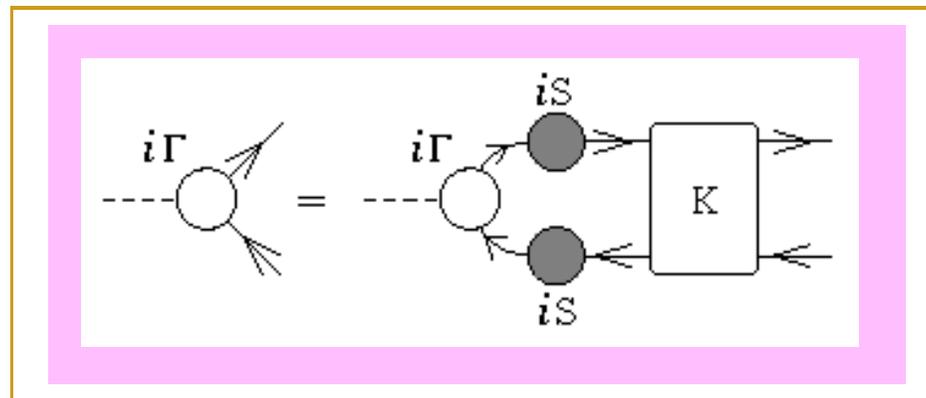
- Without bound states,
Comparison with experiment is
impossible



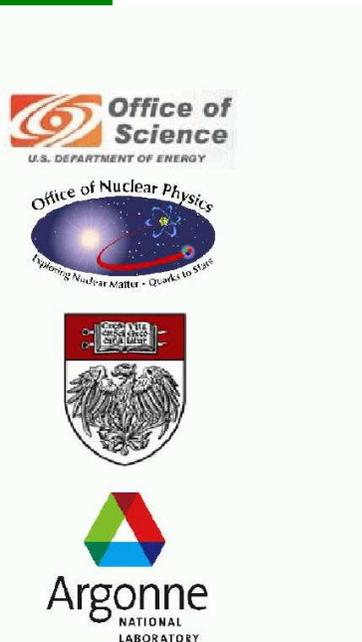
- Without bound states,
Comparison with experiment is
impossible
- They appear as pole contributions
to $n \geq 3$ -point colour-singlet
Schwinger functions



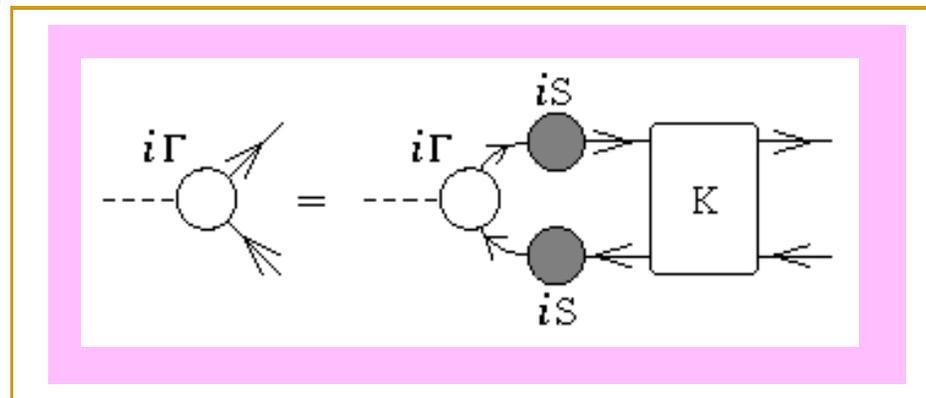
- Without bound states,
Comparison with experiment is
impossible
- Bethe-Salpeter Equation



QFT Generalisation of Lippmann-Schwinger Equation.

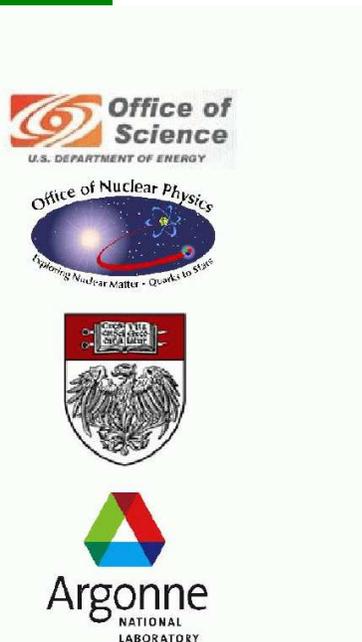


- Without bound states, Comparison with experiment is impossible
- Bethe-Salpeter Equation

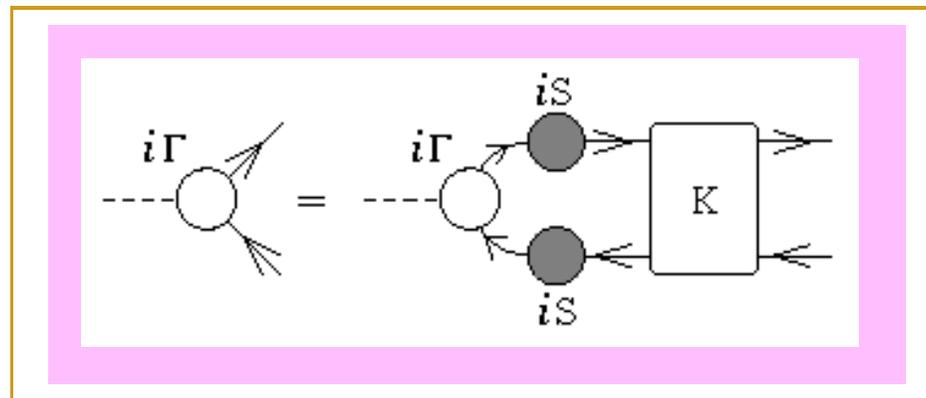


QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K ?



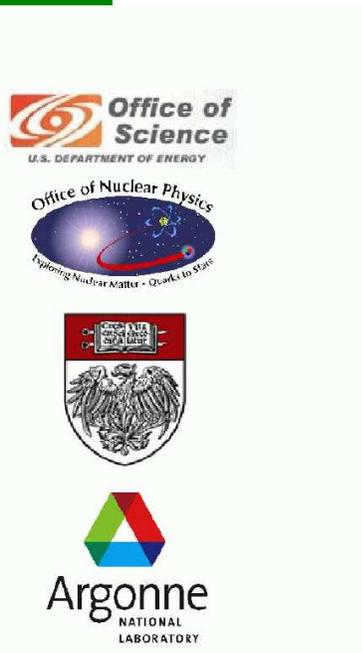
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QFT Generalisation of Lippmann-Schwinger Equation.

- What is the kernel, K ?

or



What is the Long-Range Potential?



[First](#)

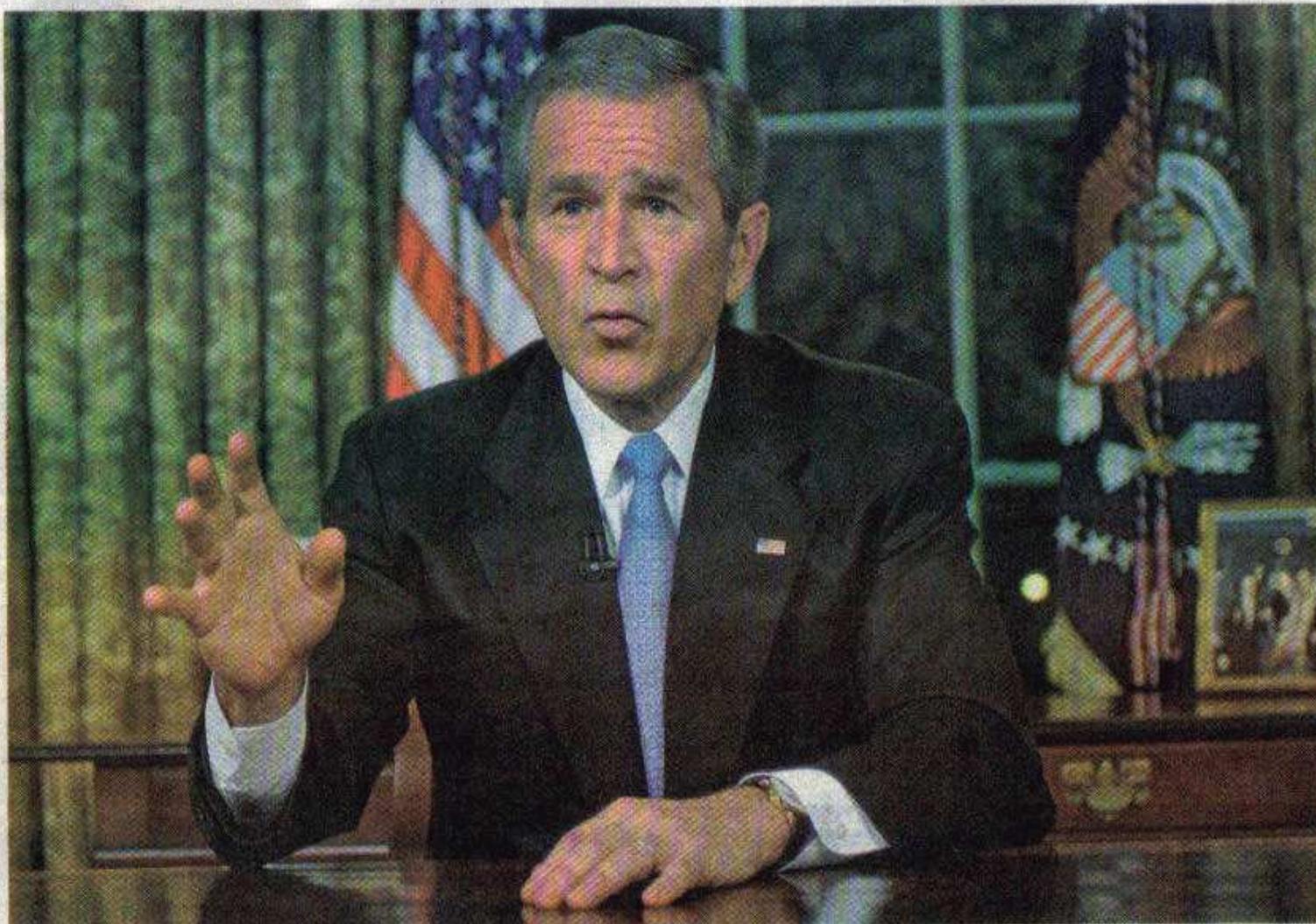
[Contents](#)

[Back](#)

[Conclusion](#)

What is the Long-Range Potential?

Bush Urges Nation To Be Quiet For A Minute While He Tries To Think



In a televised address to the nation, Bush called for "a little peace and quiet."



Bethe-Salpeter Kernel



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^l(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f^l i\gamma_5 + \frac{1}{2} \lambda_f^l i\gamma_5 \mathcal{S}^{-1}(k_-) \\ - M_\zeta i\Gamma_5^l(k; P) - i\Gamma_5^l(k; P) M_\zeta$$

QFT Statement of Chiral Symmetry



Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \Gamma_{5\mu}^l(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f^l i\gamma_5 + \frac{1}{2} \lambda_f^l i\gamma_5 \mathcal{S}^{-1}(k_-) - M_\zeta i\Gamma_5^l(k; P) - i\Gamma_5^l(k; P) M_\zeta$$

Satisfies BSE

Satisfies DSE



Bethe-Salpeter Kernel

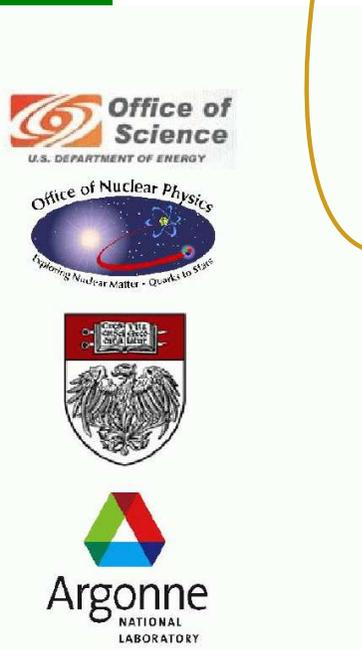
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Satisfies BSE

Satisfies DSE

Kernels must be *intimately* related



Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

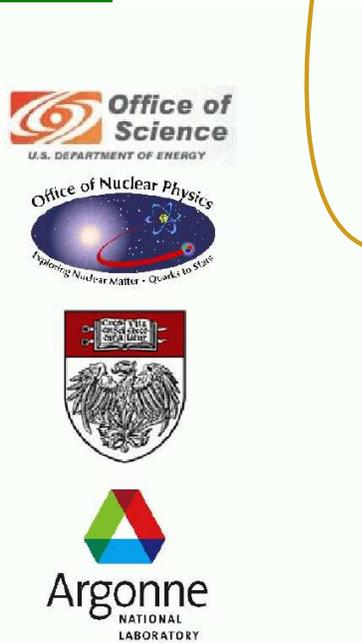
$$P_\mu \Gamma_{5\mu}^l(k; P) = \mathcal{S}^{-1}(k_+) \frac{1}{2} \lambda_f^l i \gamma_5 + \frac{1}{2} \lambda_f^l i \gamma_5 \mathcal{S}^{-1}(k_-) - M_\zeta i \Gamma_5^l(k; P) - i \Gamma_5^l(k; P) M_\zeta$$

Satisfies BSE

Satisfies DSE

Kernels must be *intimately* related

- Relation **must** be preserved by truncation



Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

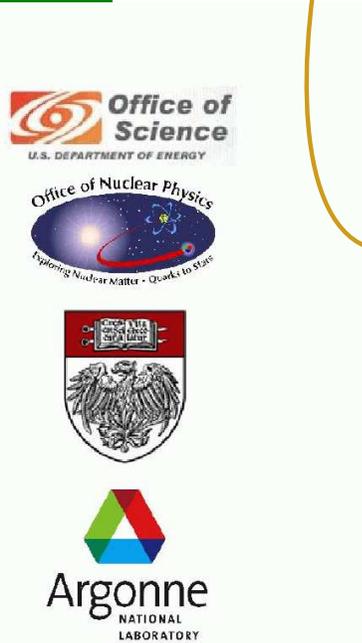
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- **Nontrivial** constraint





Bethe-Salpeter Kernel

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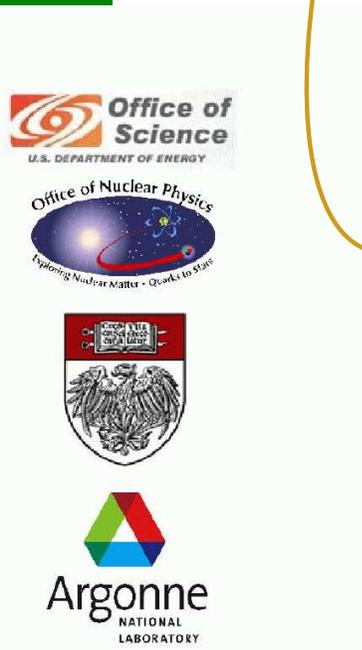
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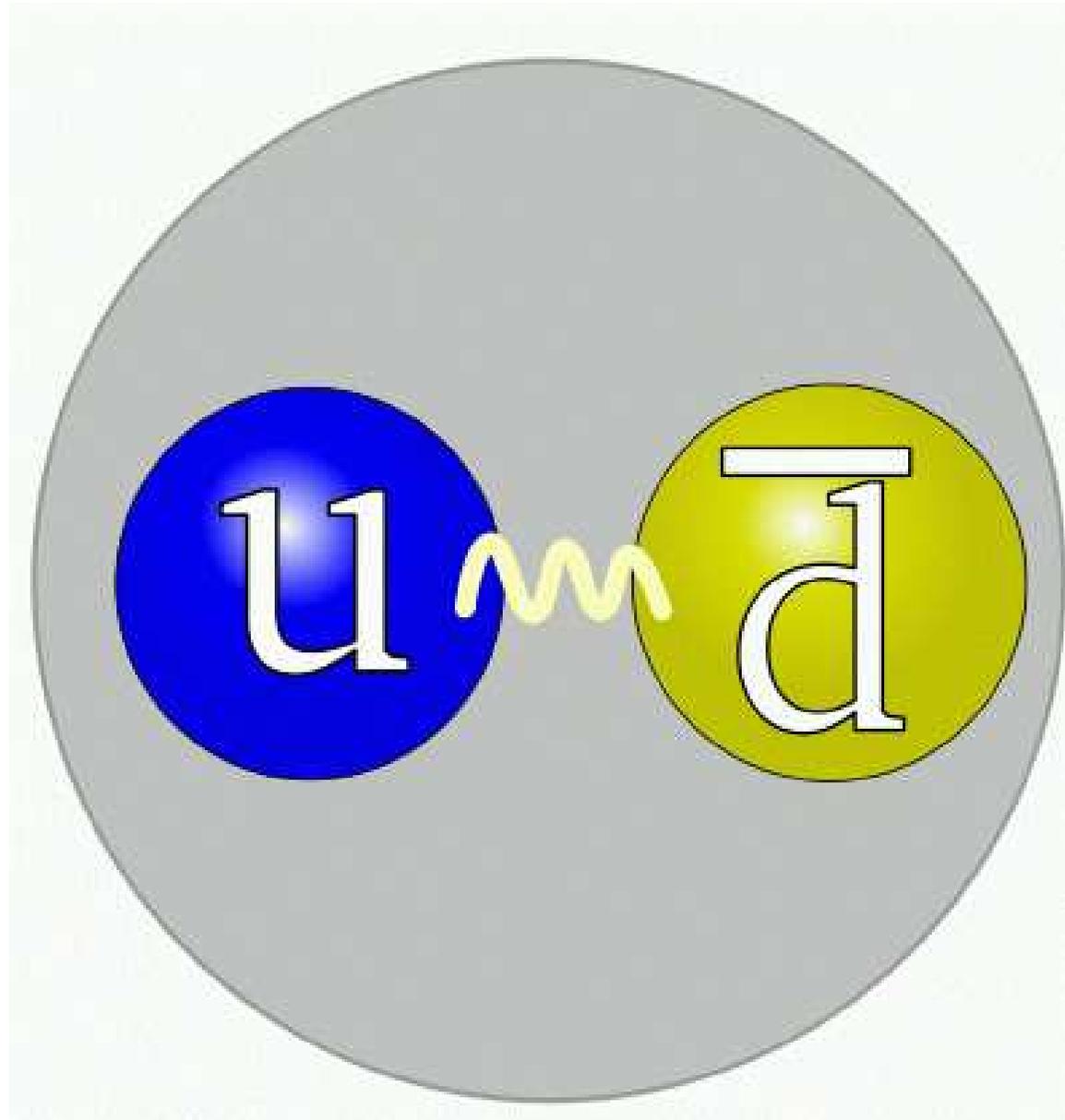
Satisfies DSE

Kernels must be *intimately* related

- Relation **must** be preserved by truncation
- **Failure** \Rightarrow Explicit Violation of QCD's Chiral Symmetry

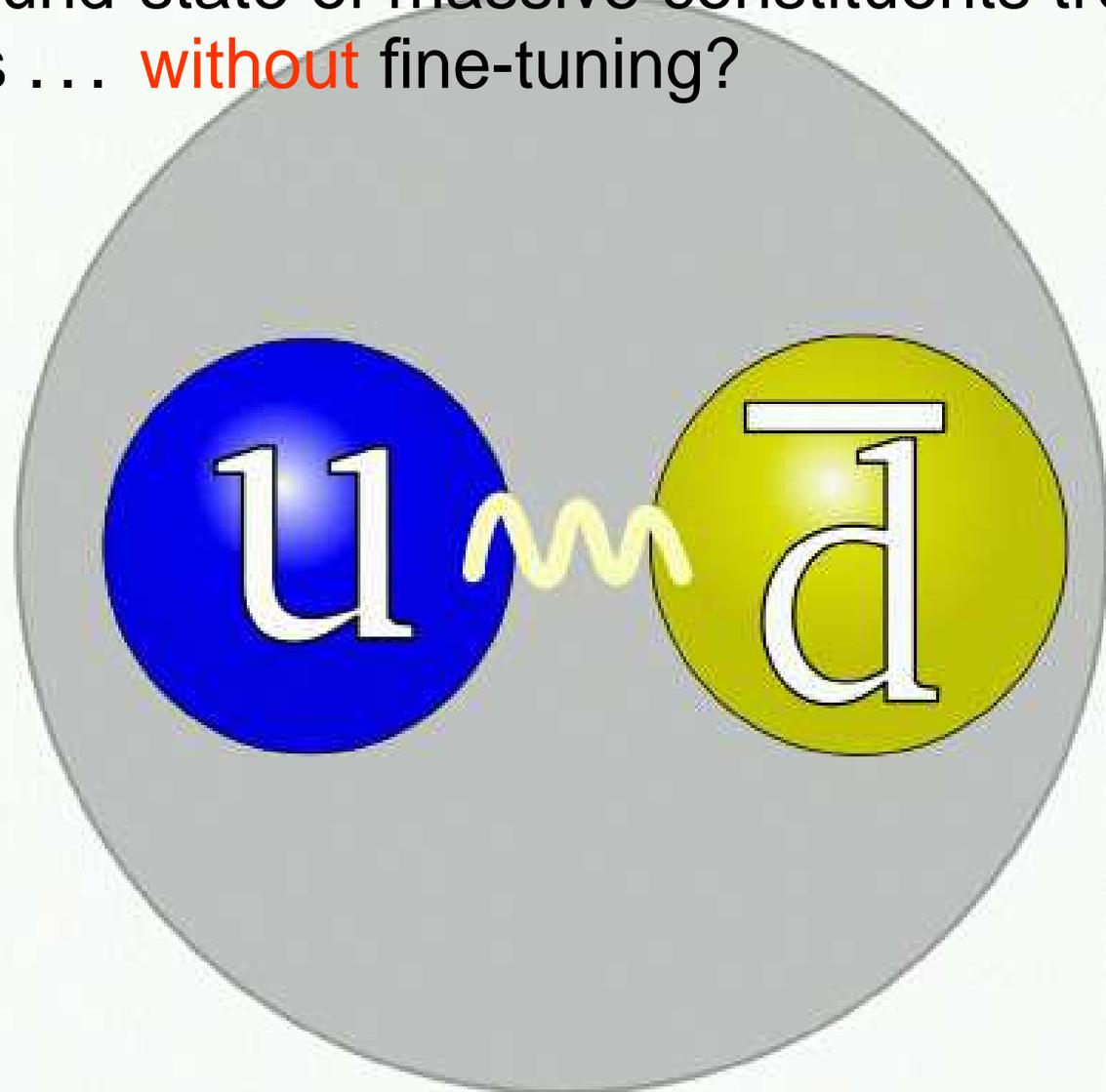


Pion and ... Pseudoscalar Mesons?



Pion and ... Pseudoscalar Mesons?

Can a bound-state of massive constituents truly be massless ... **without** fine-tuning?



A vertical stack of logos on the left side of the slide. From top to bottom: the Office of Science logo (U.S. Department of Energy), the Office of Nuclear Physics logo (Exploring Nuclear Matter - Quarks in Stars), the Argonne National Laboratory logo (featuring an eagle and the text 'Argonne NATIONAL LABORATORY'), and the Argonne National Laboratory logo (featuring a stylized 'A' made of three triangles).

Radial Excitations & Chiral Symmetry



U.S. DEPARTMENT OF ENERGY

Office of Nuclear Physics



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
nu-th/9707003)

$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$



Radial Excitations & Chiral Symmetry

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$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

- Mass² of pseudoscalar hadron



Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
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$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

$$\mathcal{M}_H := \text{tr}_{\text{flavour}} \left[M_{(\mu)} \left\{ T^H, (T^H)^t \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2} (\lambda^4 + i\lambda^5)$



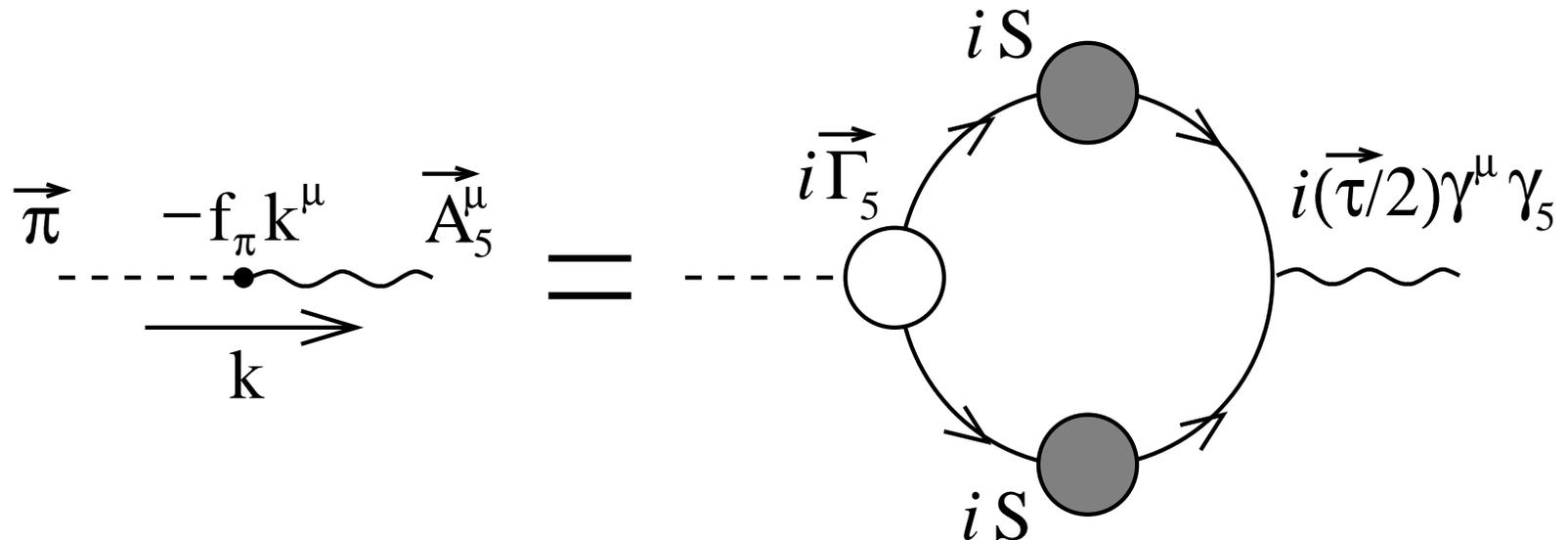
Radial Excitations & Chiral Symmetry

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$$f_H m_H^2 = - \rho_\zeta^H \mathcal{M}_H$$

$$f_H p_\mu = Z_2 \int_q^\Lambda \frac{1}{2} \text{tr} \left\{ (T^H)^t \gamma_5 \gamma_\mu \mathcal{S}(q_+) \Gamma_H(q; P) \mathcal{S}(q_-) \right\}$$

- Pseudovector projection of BS wave function at $x = 0$
- Pseudoscalar meson's leptonic decay constant



Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy
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Radial Excitations & Chiral Symmetry

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- Light-quarks; i.e., $m_q \sim 0$

- $f_H \rightarrow f_H^0$ & $\rho_\zeta^H \rightarrow \frac{-\langle \bar{q}q \rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q \dots$ GMOR relation, a corollary



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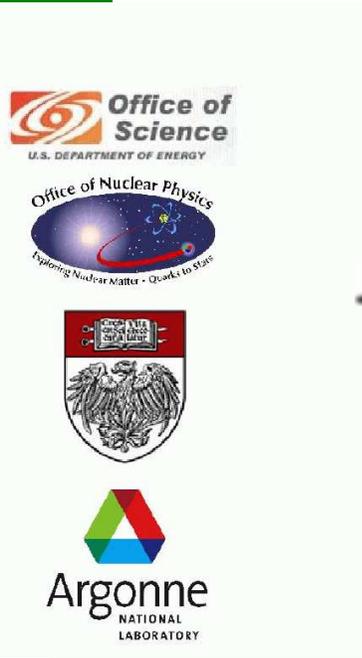
Hence $m_H^2 = \frac{-\langle \bar{q}q \rangle_\zeta^0}{(f_H^0)^2} m_q \dots$ GMOR relation, a corollary

- Heavy-quark + light-quark

$\Rightarrow f_H \propto \frac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

Hence, $m_H \propto m_q$

... QCD Proof of Potential Model result



Andreas Krassnigg

FWF “Erwin
Schrödinger Fellow,”
ANL 2003-2005



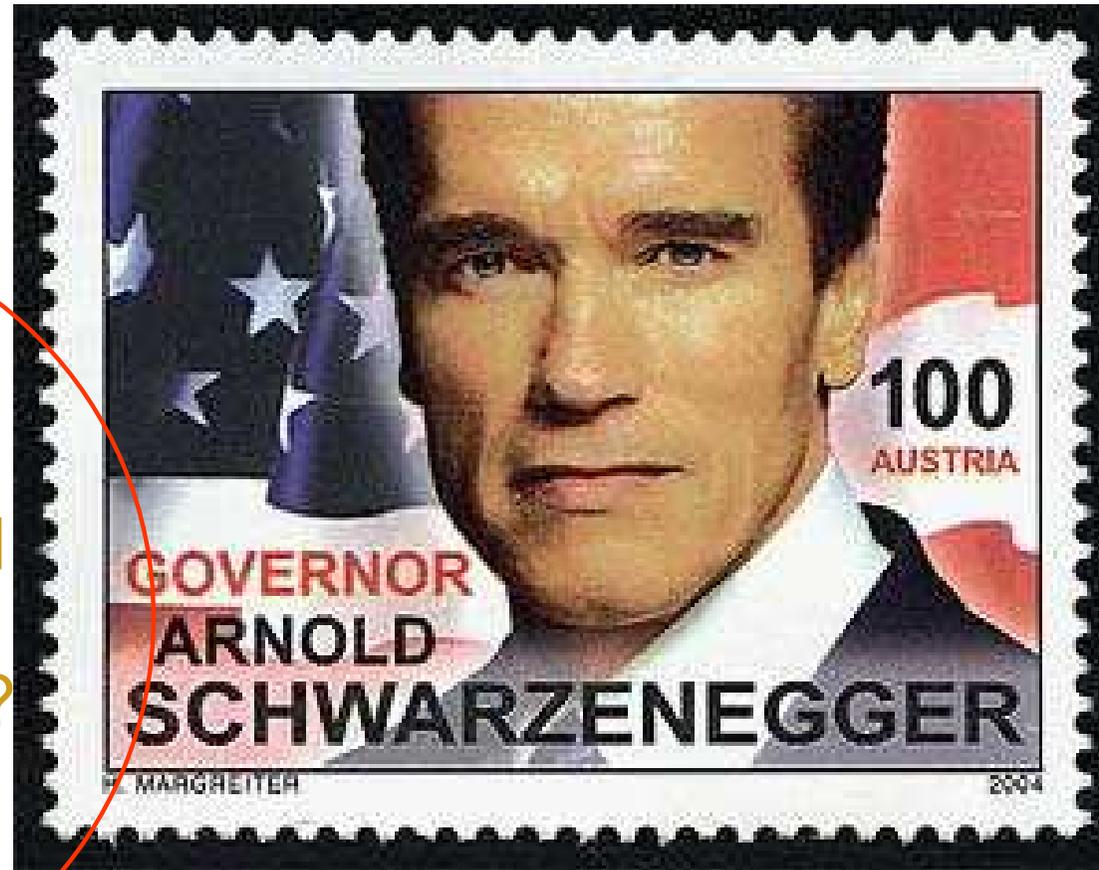
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Almost Blood
Relative of Arnold
... Future
President?
... Executioner?



Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

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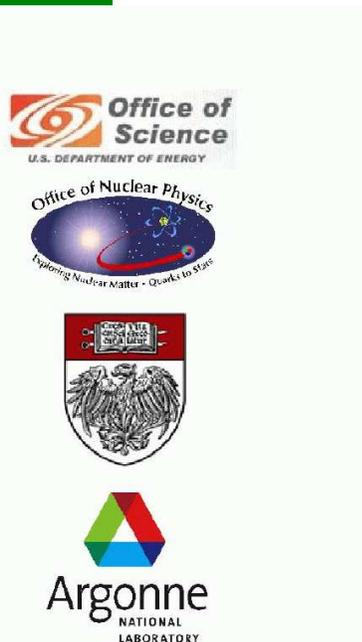


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 $m_{\pi_{n \neq 0}}^2 > m_{\pi_{n=0}}^2 = 0$, in **chiral limit**



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ALL pseudoscalar mesons **except $\pi(140)$** in **chiral limit**



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ALL pseudoscalar mesons **except $\pi(140)$** in **chiral limit**
- **Dynamical Chiral Symmetry Breaking**
– Goldstone’s Theorem –
impacts upon **every pseudoscalar meson**



Radial Excitations



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Radial Excitations

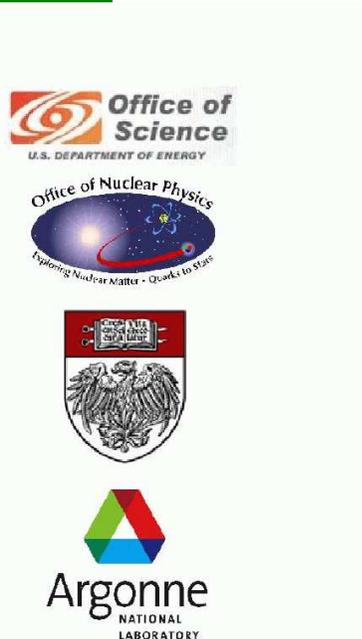
- Spectrum contains 3 pseudoscalars [$I^G(J^P)L = 1^-(0^-)S$]

masses below 2 GeV: $\pi(140)$; $\pi(1300)$; and $\pi(1800)$



Radial Excitations

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- But $\pi(1800)$ is narrow ($\Gamma = 207 \pm 13$) & decay pattern might indicate some “flux tube angular momentum” content:
 $S_{\bar{Q}Q} = 1 \oplus L_F = 1 \Rightarrow J = 0$
& $L_F = 1 \Rightarrow {}^3S_1 \oplus {}^3S_1 (\bar{Q}Q)$ decays suppressed?



Radial Excitations

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- **Radial excitations & Hybrids & Exotics** \Rightarrow Long-range radial wave functions \Rightarrow sensitive to confinement



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- Radial excitations & Hybrids & Exotics \Rightarrow Long-range radial wave functions \Rightarrow sensitive to confinement
- NSAC Long-Range Plan, 2002: ... an understanding of confinement “remains one of the greatest intellectual challenges in physics”



Radial Excitations & Chiral Symmetry



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

- Fundamental properties of QCD



[First](#)

[Contents](#)

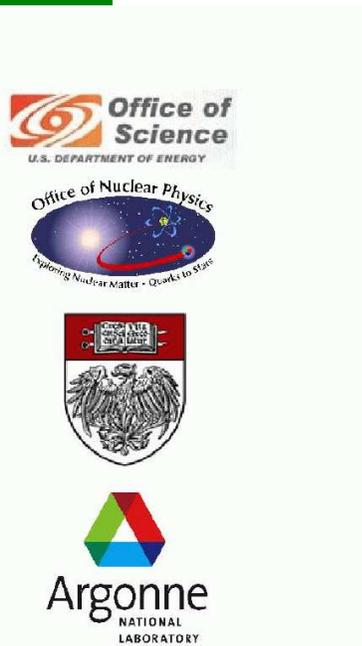
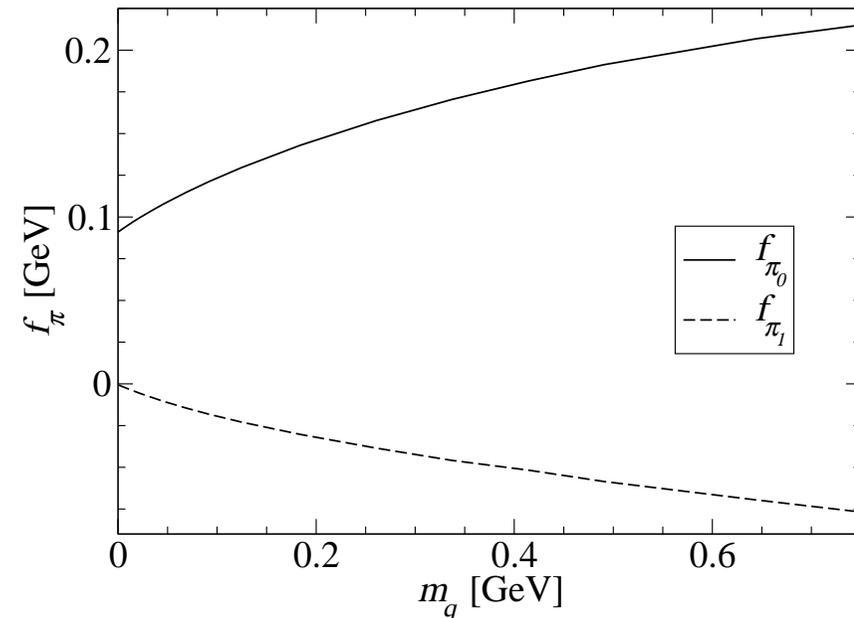
[Back](#)

[Conclusion](#)

Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

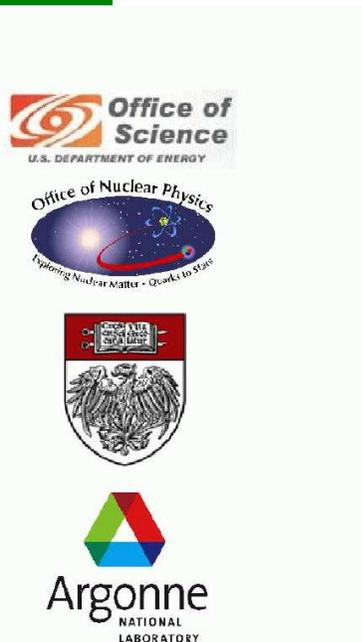
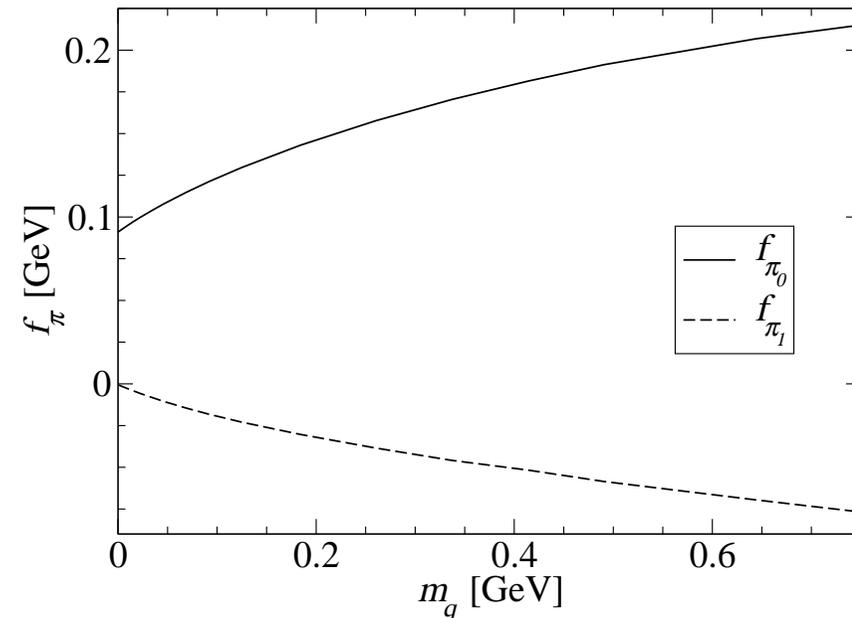
- Fundamental properties of QCD
 - If chiral symmetry is dynamically broken, then in the chiral limit every pseudoscalar meson is blind to the weak interaction *except* $\pi(140)$.



Radial Excitations & Chiral Symmetry

Höll, Krassnigg, Roberts
nu-th/0406030

- Fundamental properties of QCD
 - If chiral symmetry is dynamically broken, then in the chiral limit every pseudoscalar meson is blind to the weak interaction *except* $\pi(140)$.
 - If chiral symmetry is not broken, then *NO pseudoscalar meson experiences the weak interaction.*



Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Radial Excitations & Lattice-QCD

McNeile and Michael
he-la/0607032

- *When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.*



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Radial Excitations & Lattice-QCD

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he-la/0607032

- When we first heard about [this result] our first reaction was a combination of “that is remarkable” and “unbelievable”.
- CLEO: $\tau \rightarrow \pi(1300) + \nu_\tau$
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Diehl & Hiller
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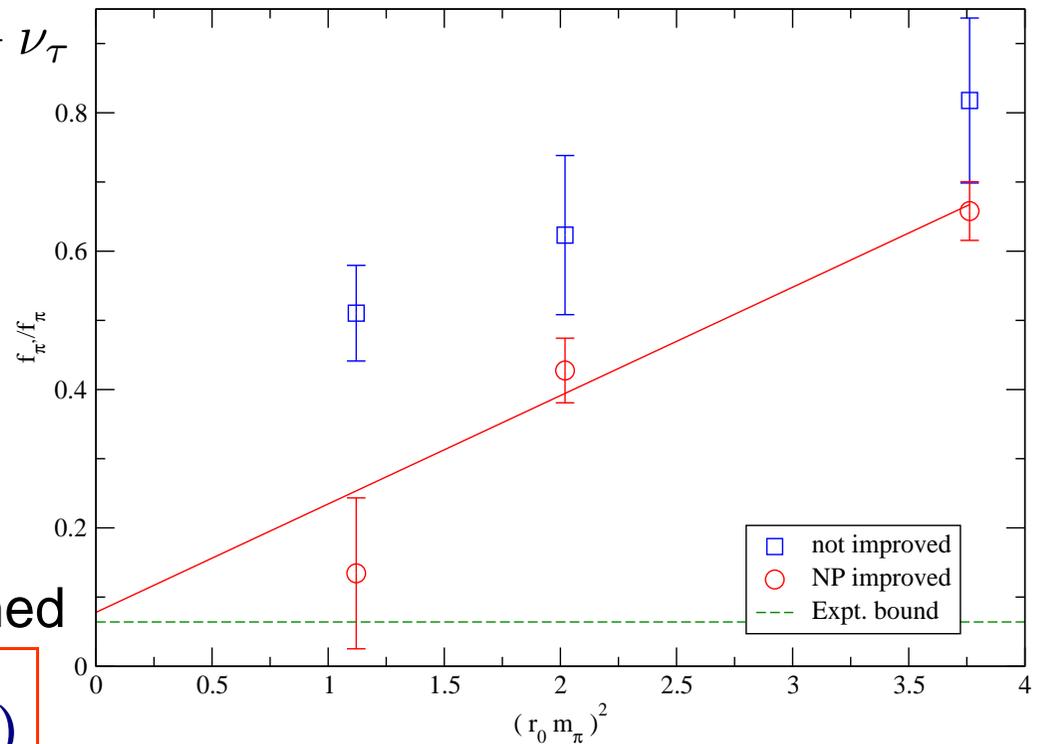
- Lattice-QCD check:

$$16^3 \times 32,$$

$$a \sim 0.1 \text{ fm},$$

two-flavour, unquenched

$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078 (93)$$



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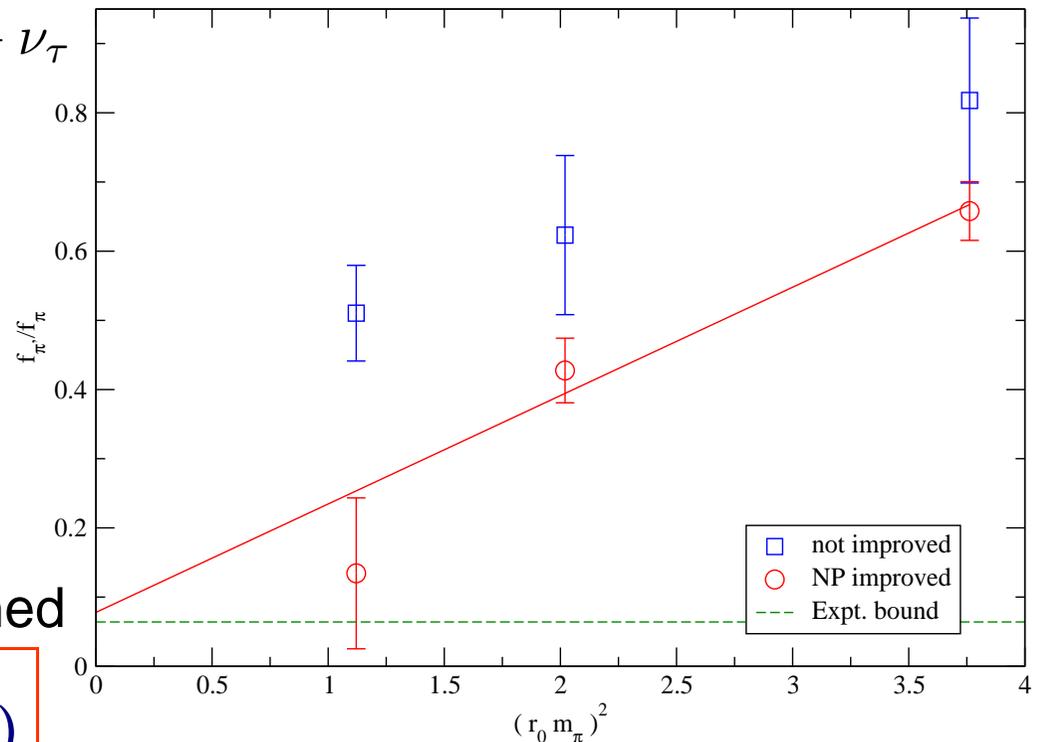
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- Full ALPHA formulation is required to see suppression, because PCAC relation is at the heart of the conditions imposed for improvement (determining coefficients of irrelevant operators)



Radial Excitations & Lattice-QCD

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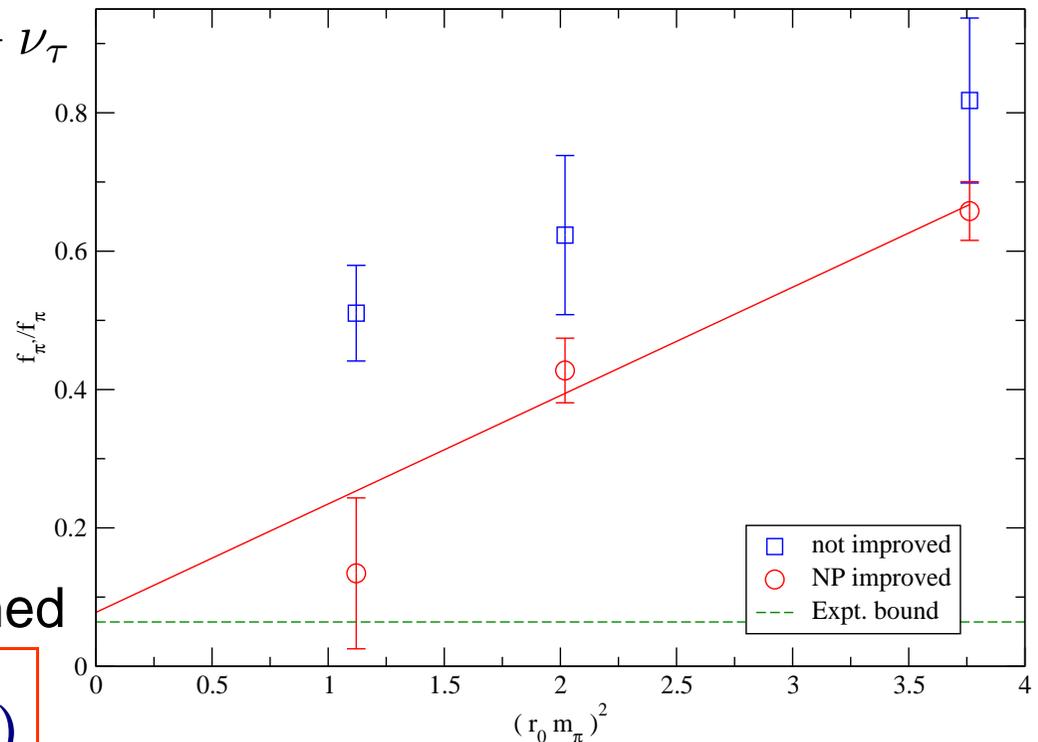
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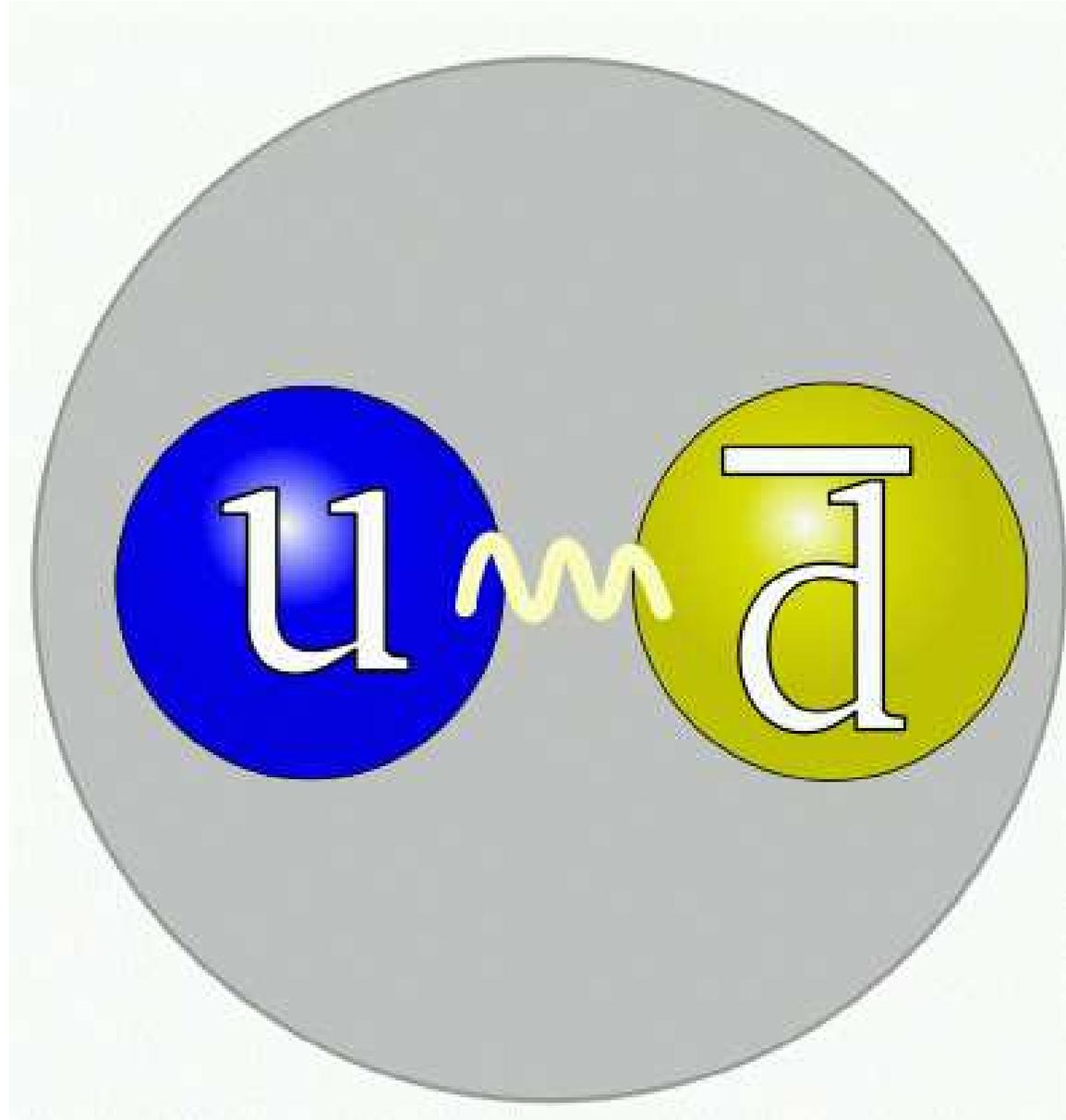
$$\Rightarrow \frac{f_{\pi_1}}{f_\pi} = 0.078 (93)$$



- The suppression of f_{π_1} is a useful benchmark that can be used to tune and validate lattice QCD techniques that try to determine the properties of excited states mesons.



Answer for the pion



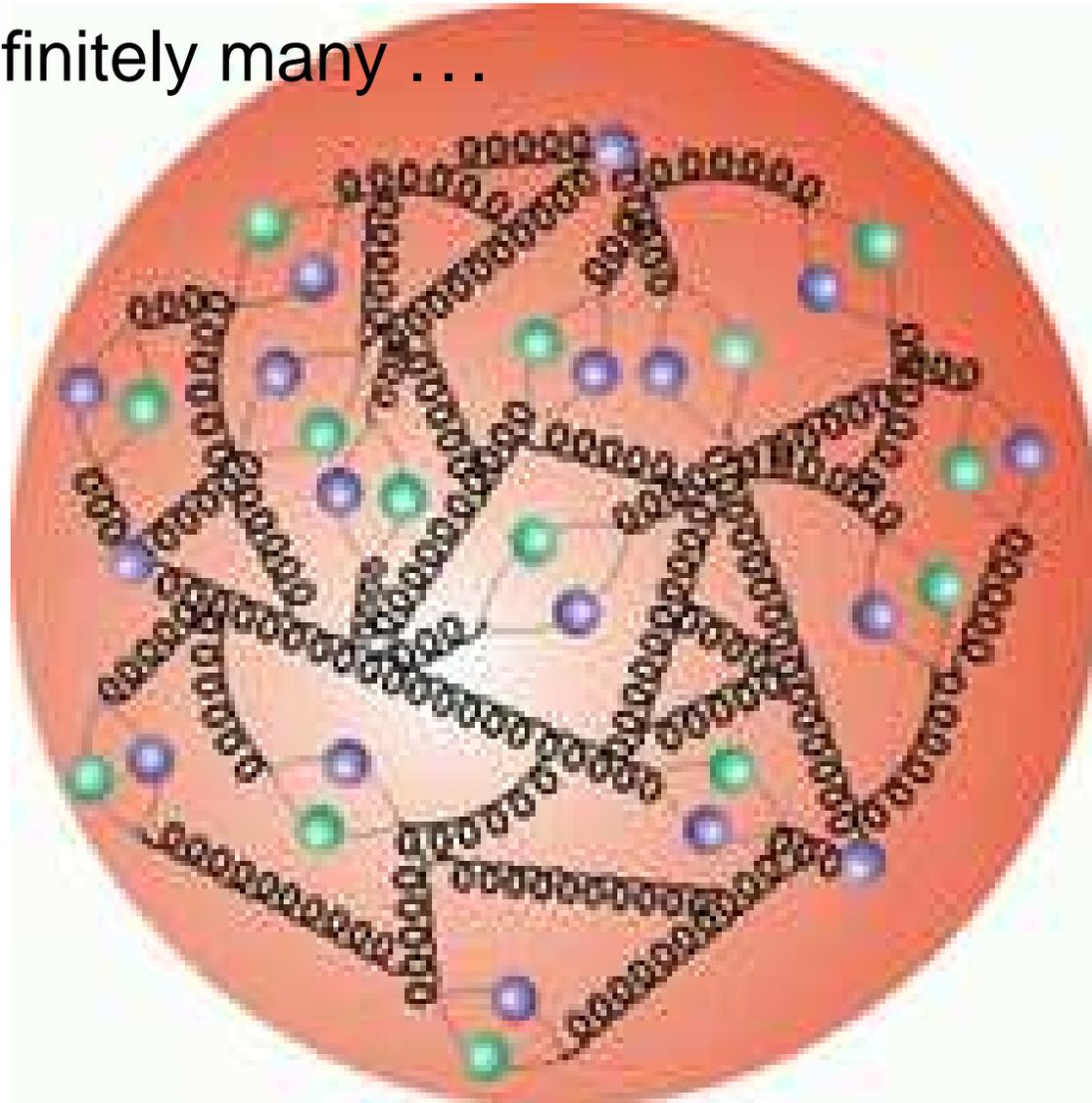
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Answer for the pion

Two \rightarrow Infinitely many ...



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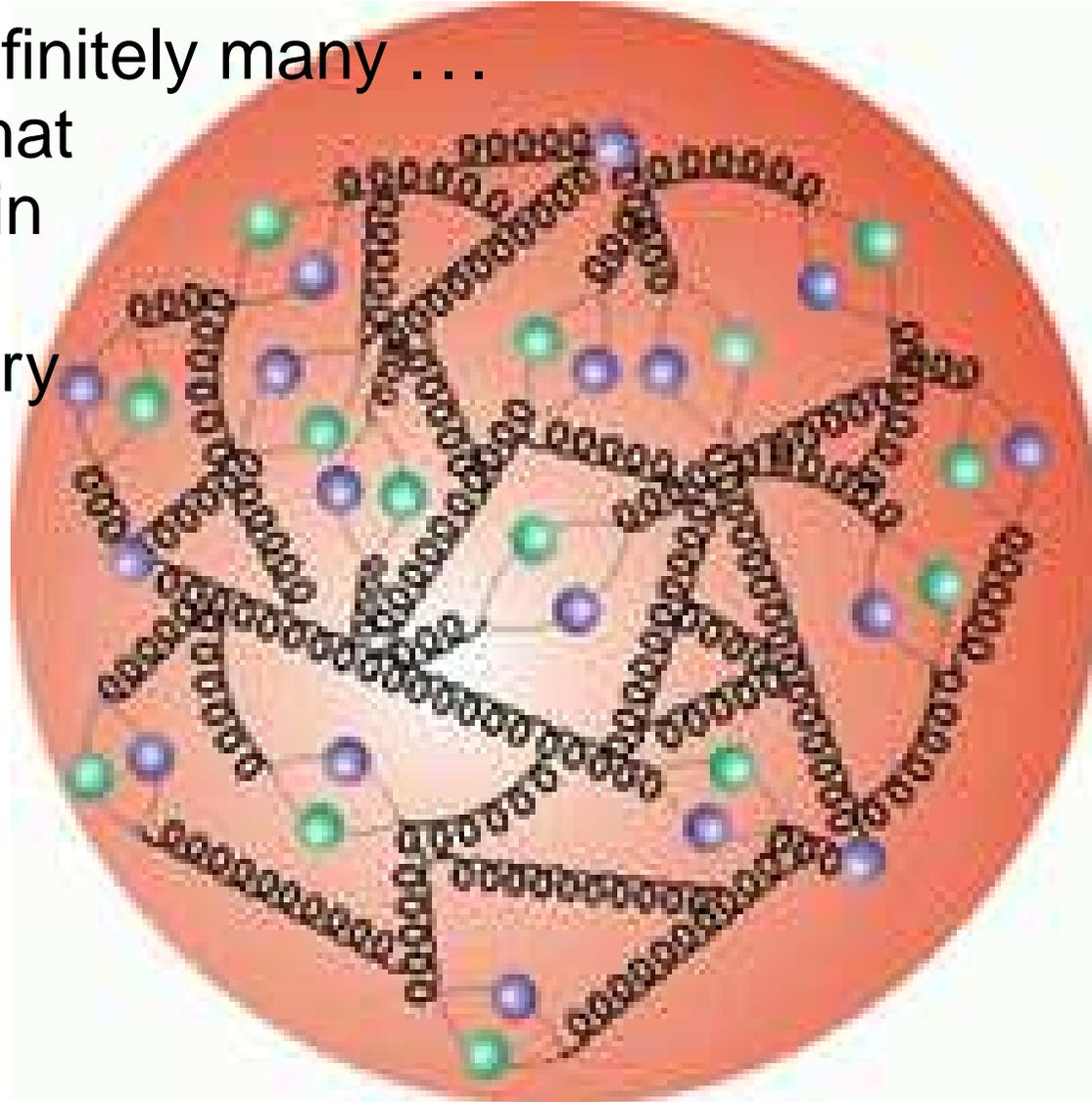
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Handle that
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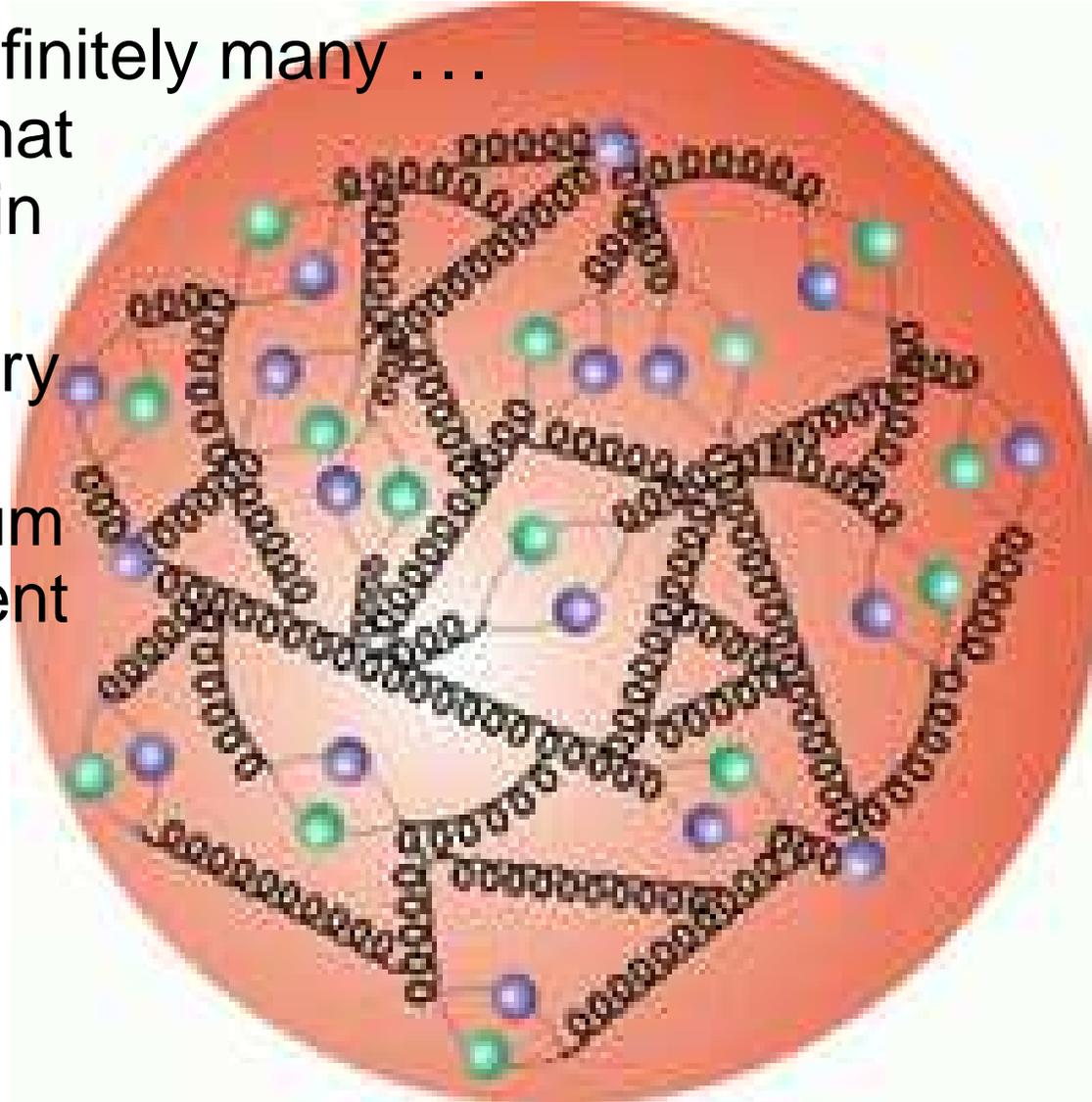
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Answer for the pion

Two \rightarrow Infinitely many ...

Handle that properly in quantum field theory

...
momentum-dependent dressing



Answer for the pion

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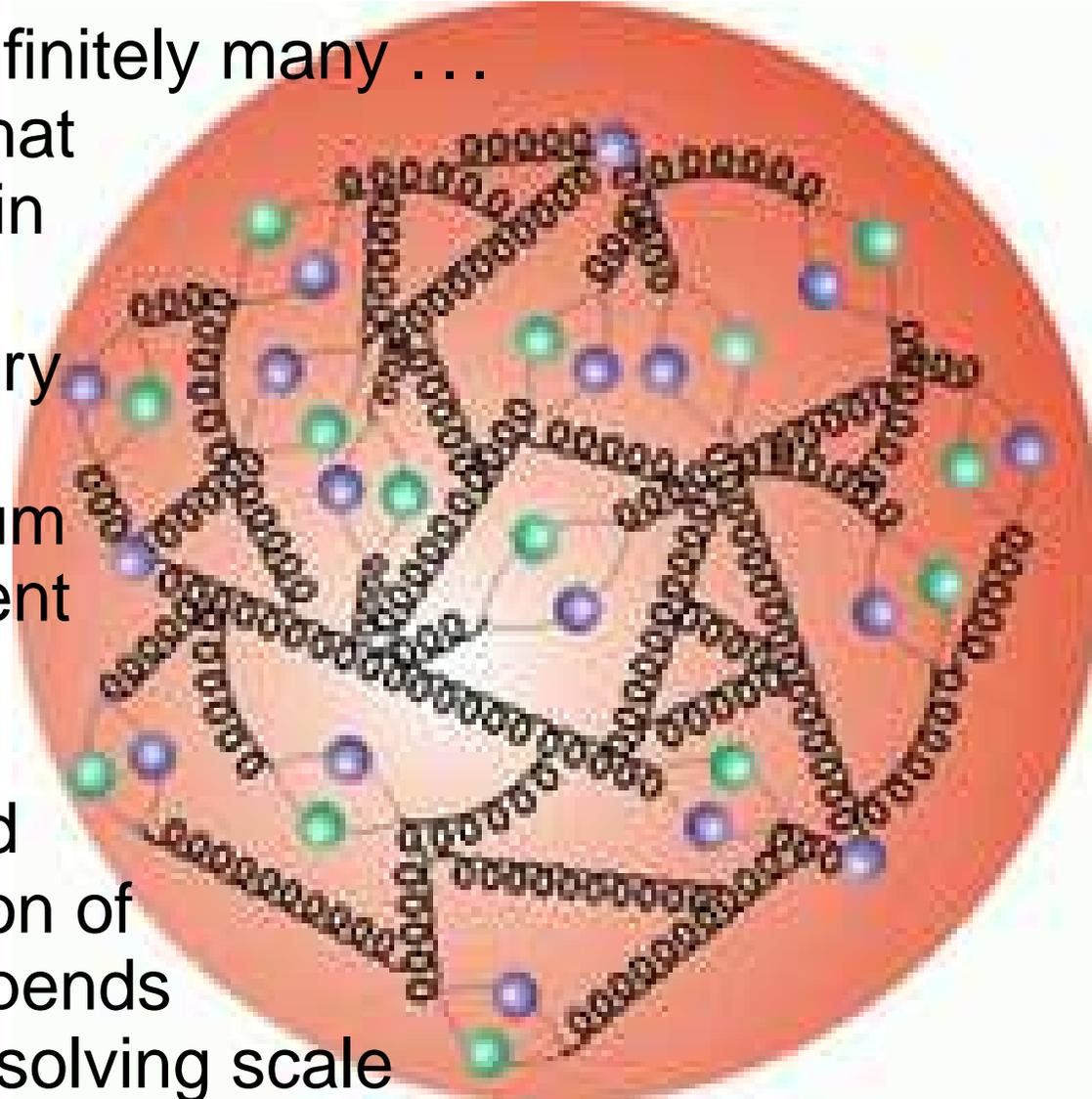
Handle that properly in quantum field theory

...

momentum-dependent dressing

...

perceived distribution of mass depends on the resolving scale



Crocs emerge in town twice bitten



On the croc's back: Tony and Trent DeWith negotiate a flooded street in Katherine's town centre in a snappy canoe yesterday

New Challenges



[First](#)

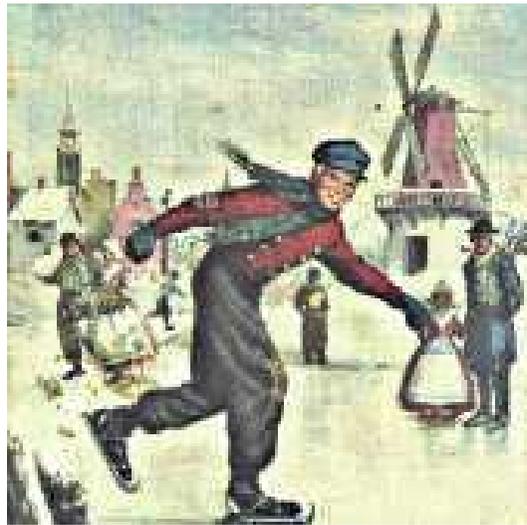
[Contents](#)

[Back](#)

[Conclusion](#)

New Challenges

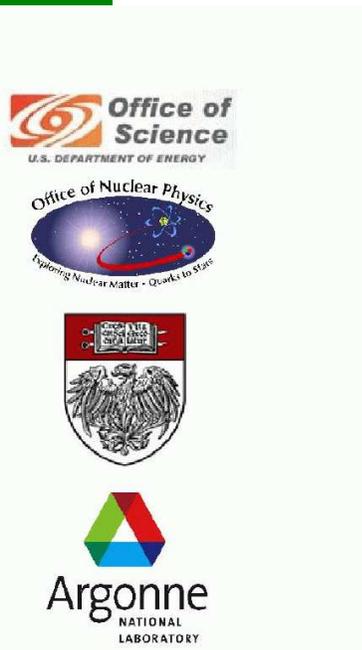
- **Maris & Tandy** ... series of five papers ... excellent description of light pseudoscalar and vector mesons ... basket of 31 masses/couplings/radii with r.m.s. error of 15% ... moreover, prediction of $F_\pi(Q^2)$ measured in Hall C.



Pieter Maris



Peter Tandy



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New Challenges

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- One parameter model . . . parameter specifies long-range interaction between light-quarks . . . model-independent results in ultraviolet



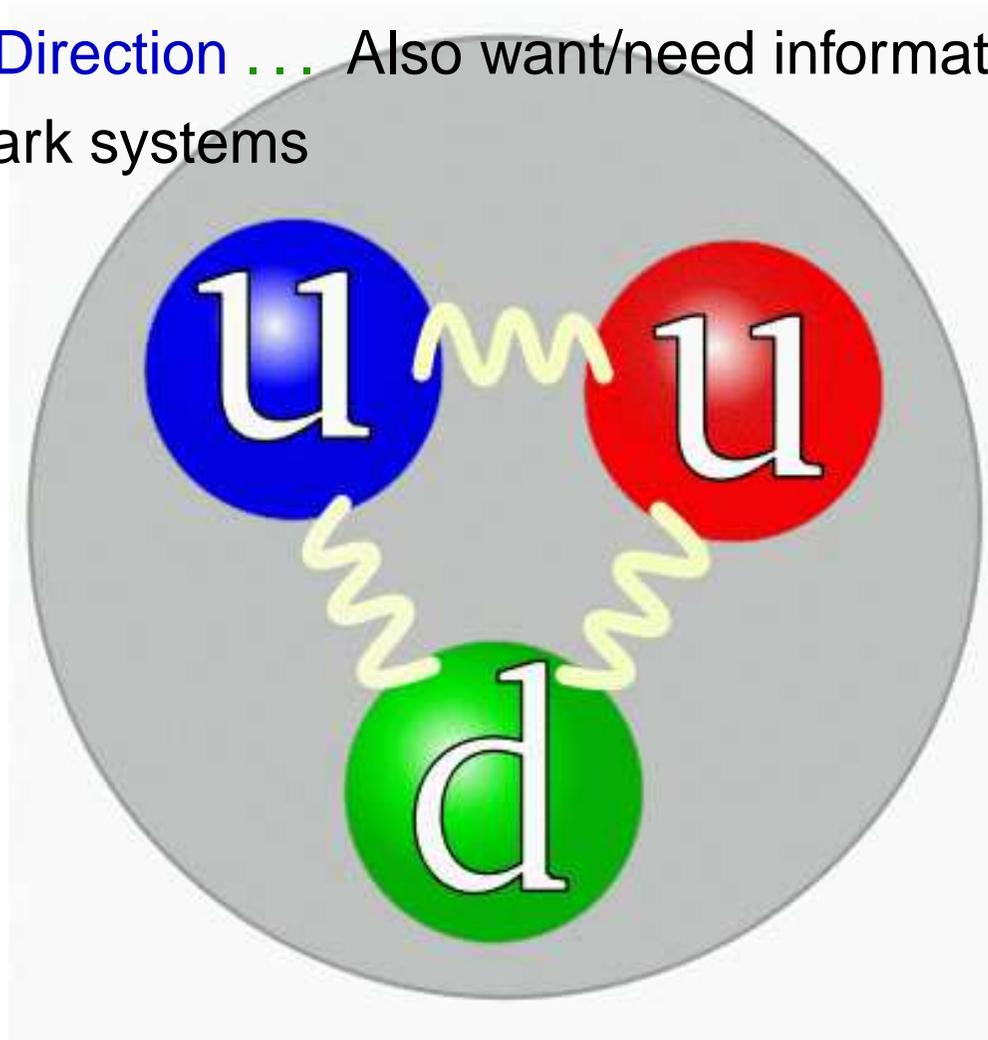
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- One parameter model . . . parameter specifies long-range interaction between light-quarks . . . model-independent results in ultraviolet
- **Next Steps** . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks



New Challenges

- **Another Direction . . .** Also want/need information about three-quark systems



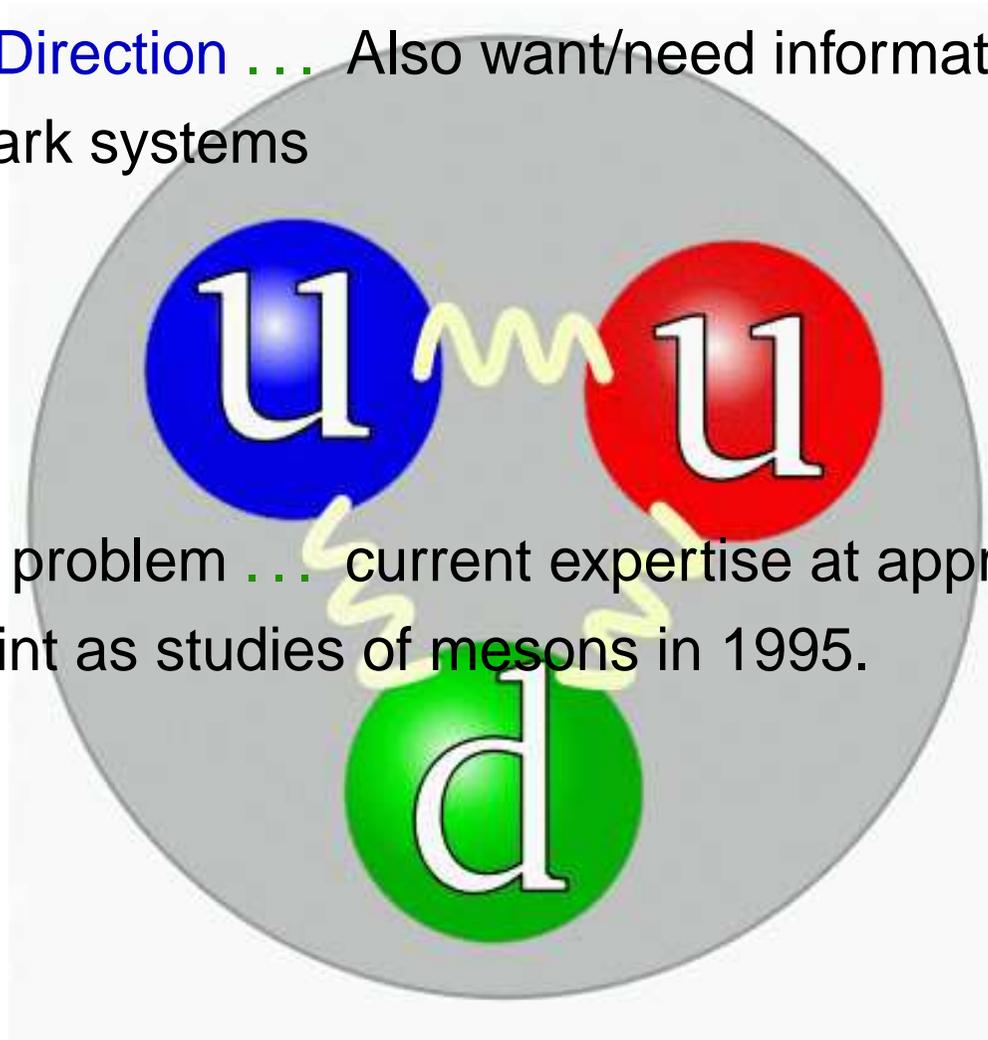
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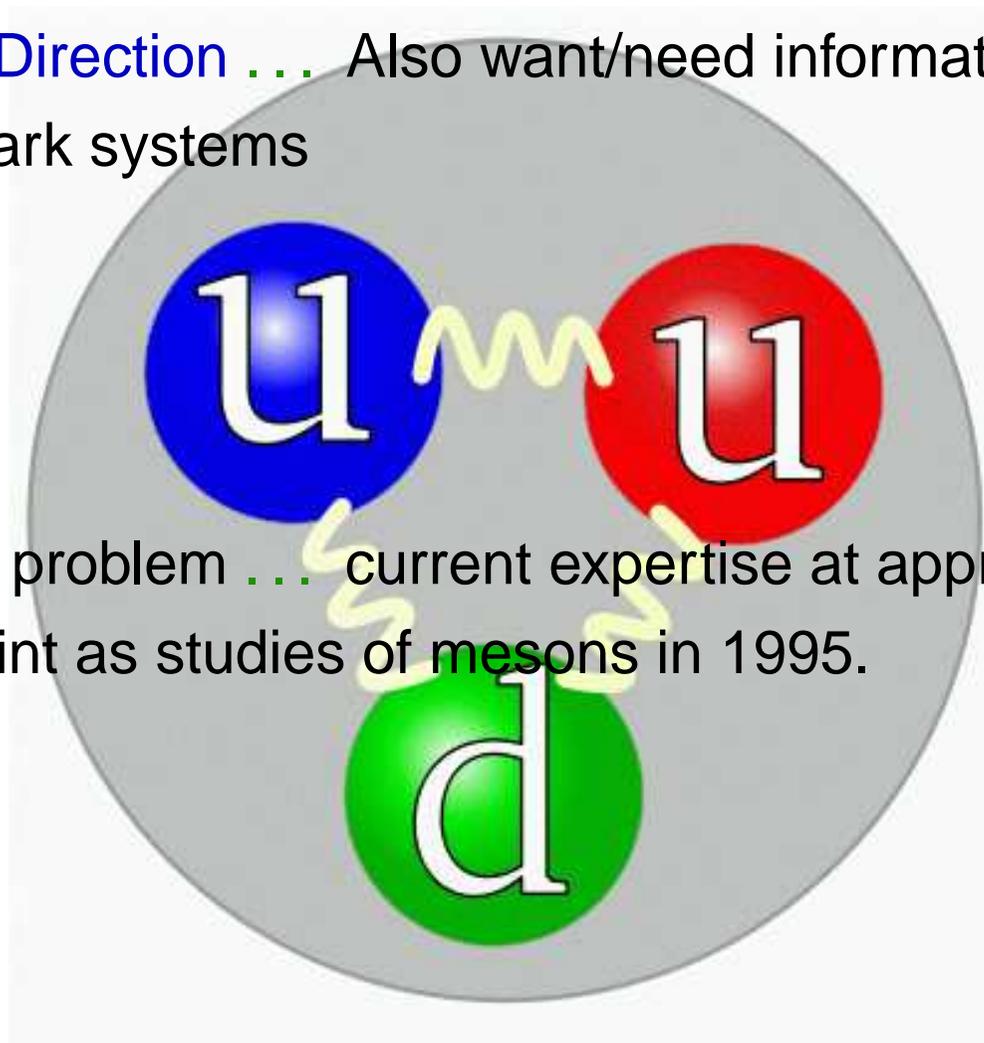


- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.



New Challenges

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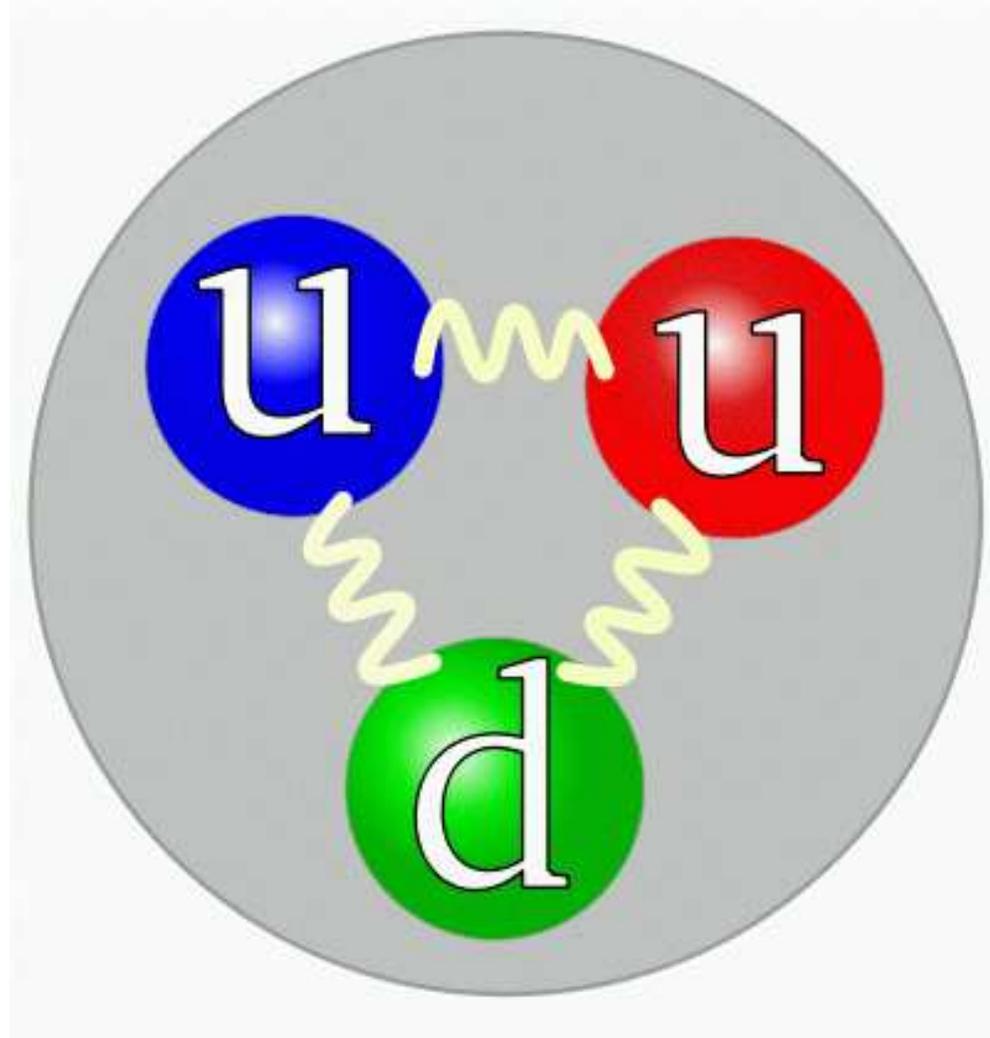


- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.



Nucleon ... Three-body Problem?



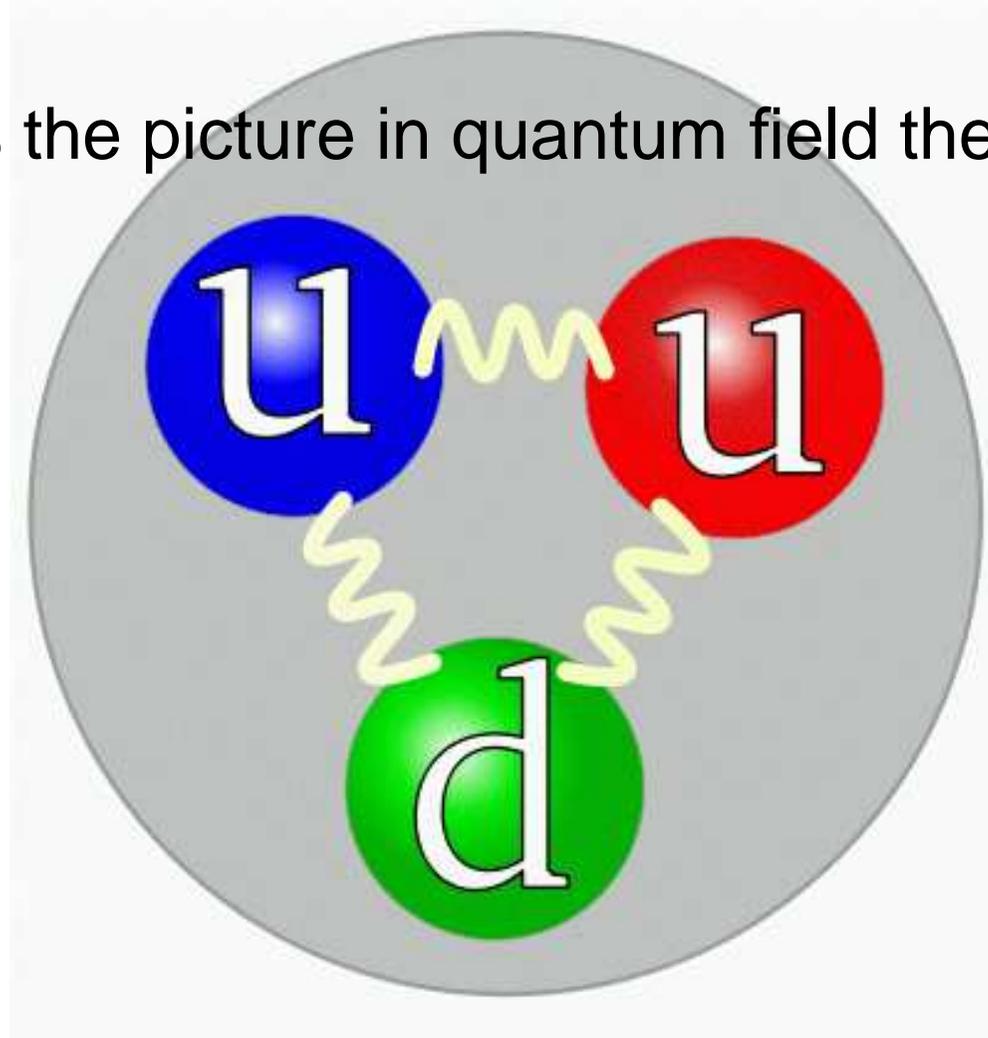
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Nucleon ... Three-body Problem?

- What is the picture in quantum field theory?



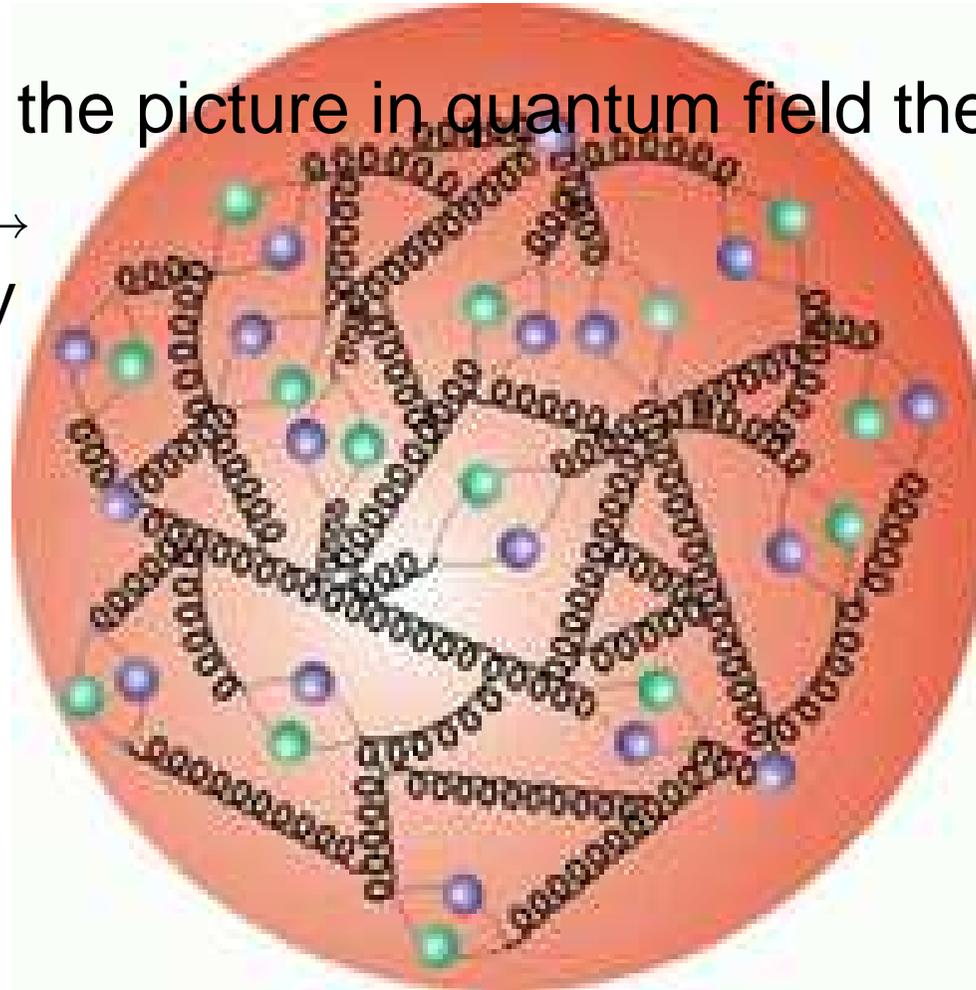
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- Three → infinitely many!



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Faddeev equation



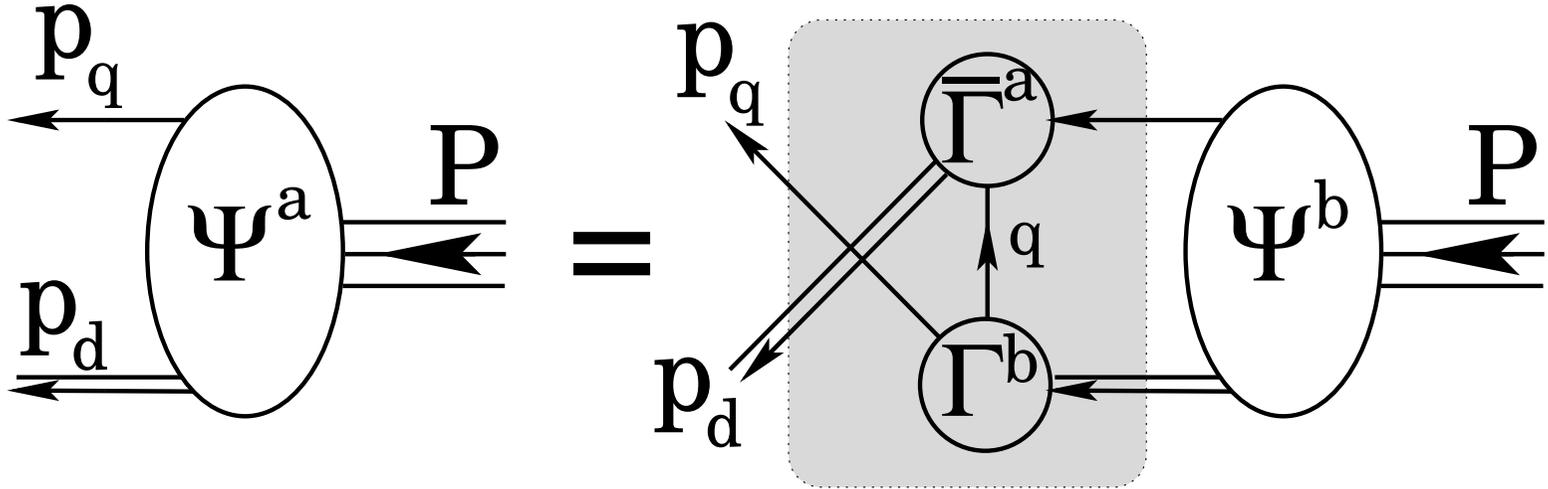
[First](#)

[Contents](#)

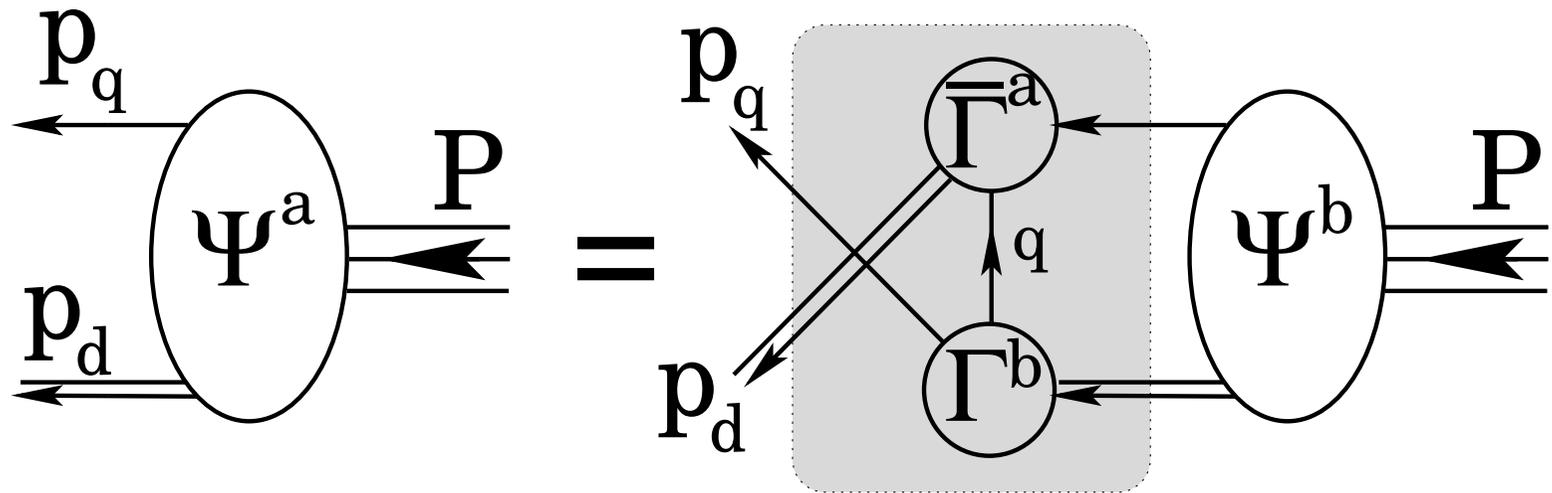
[Back](#)

[Conclusion](#)

Faddeev equation



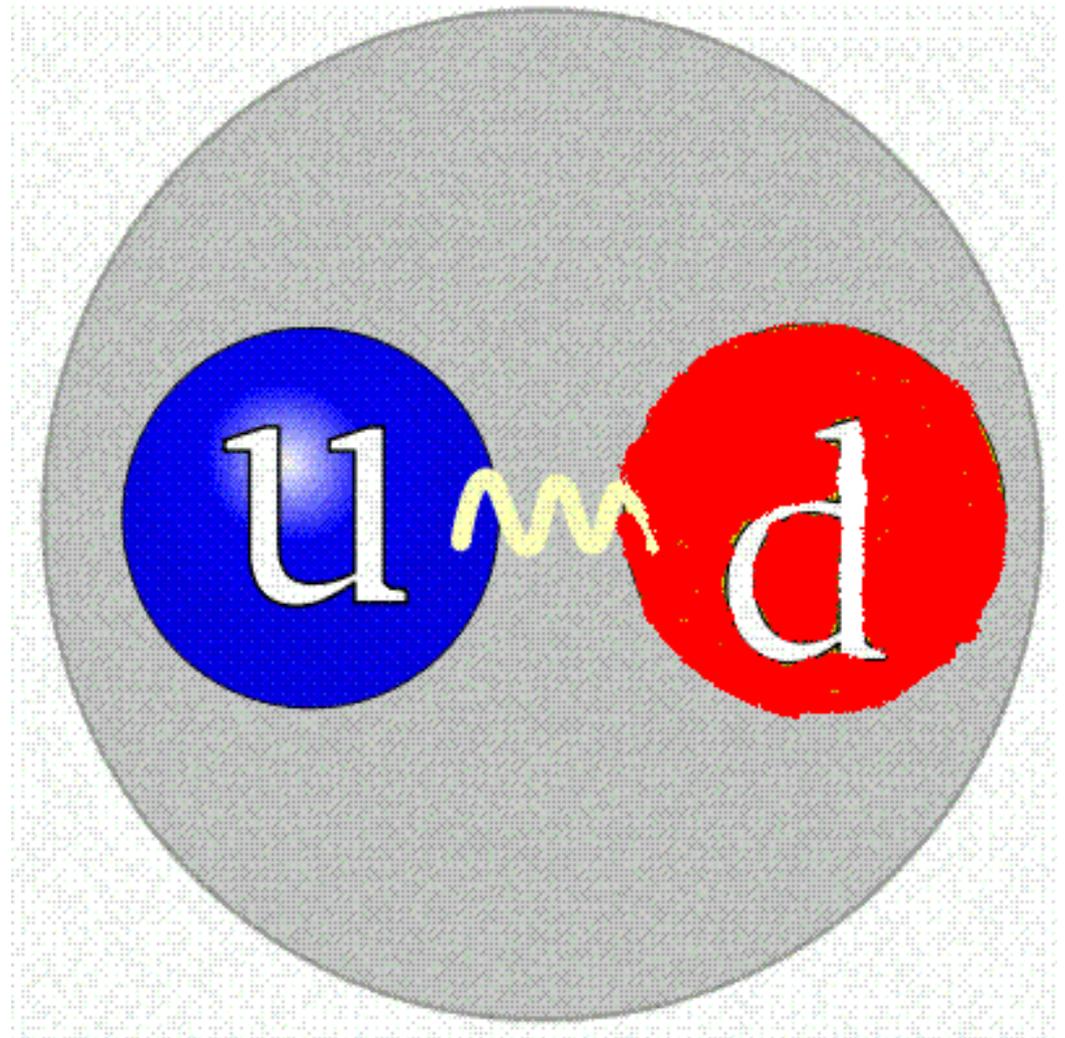
Faddeev equation



- Linear, Homogeneous Matrix equation
 - Yields *wave function* (**Poincaré Covariant Faddeev Amplitude**) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame **Amplitude** has ... *s*-, *p*- & *d*-wave correlations



Diquark correlations



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[First](#)

[Contents](#)

[Back](#)

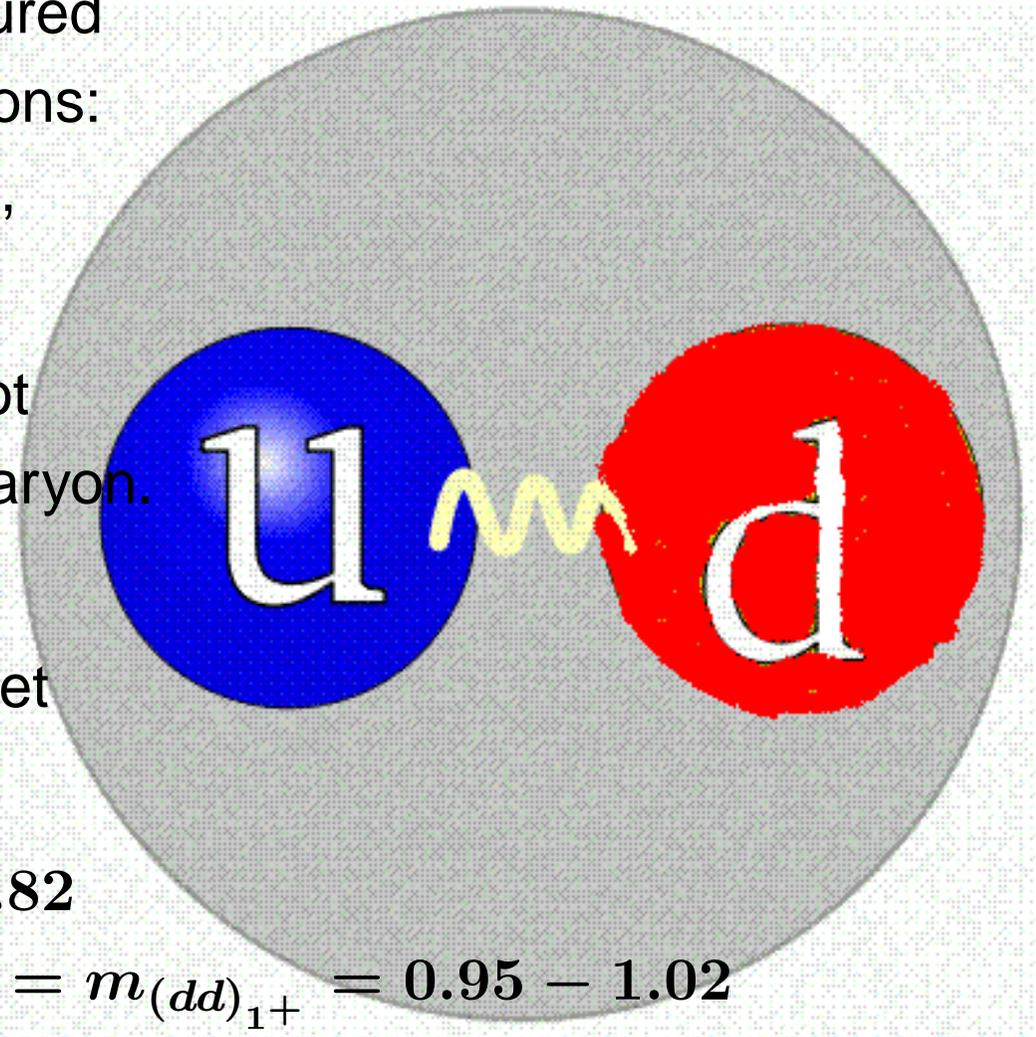
[Conclusion](#)

QUARK-QUARK

ANL PHY Colloquium: 6 October 2006 – p. 47/59

Diquark correlations

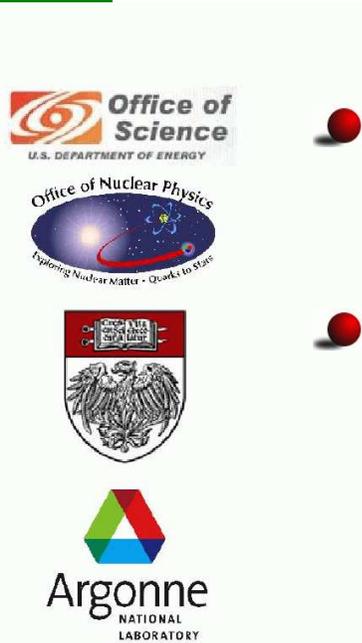
- Same interaction that describes mesons also generates three coloured quark-quark correlations: blue-red, blue-green, green-red
- Confined ... Does not escape from within baryon.



- Scalar is isosinglet, Axial-vector is isotriplet
- DSE and lattice-QCD

$$m_{[ud]_{0+}} = 0.74 - 0.82$$

$$m_{(uu)_{1+}} = m_{(ud)_{1+}} = m_{(dd)_{1+}} = 0.95 - 1.02$$





Nucleon EM Form Factors: A Précis

Höll, Kloker, et al.: nu-th/0412046 & nu-th/0501033



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)



Nucleon EM Form Factors: A Précis

Höll, Kloker, *et al.*: nu-th/0412046 & nu-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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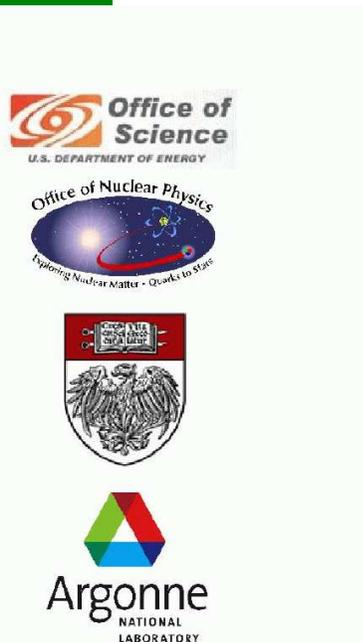
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- Excellent mass spectrum (octet and decuplet)

Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\text{exp}} - M_H^{\text{calc}}]^2}{[M_H^{\text{exp}}]^2} \right)^{1/2} = 2\%$$



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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)



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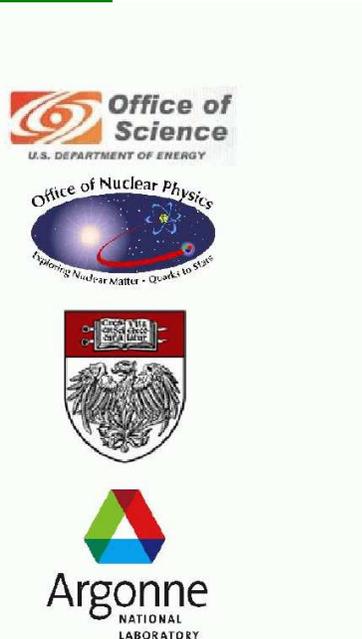
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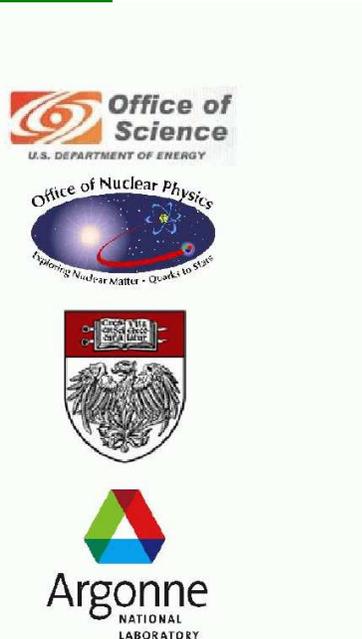
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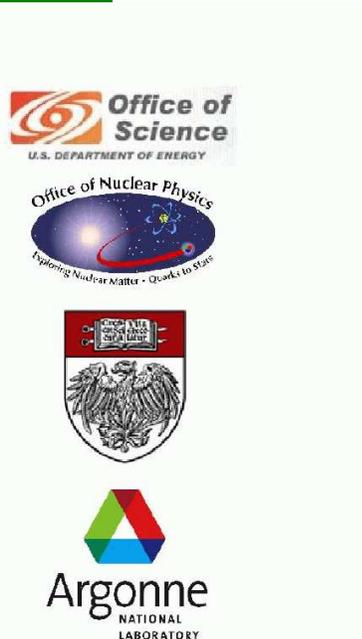
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- **But** is that good?
 - Cloudy Bag: $\delta M_+^{\pi\text{-loop}} = -300$ to -400 MeV!
- **Critical** to anticipate pion cloud effects

Roberts, Tandy, Thomas, *et al.*, nu-th/02010084



Harry Lee

Pions and Form Factors



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Pions and Form Factors

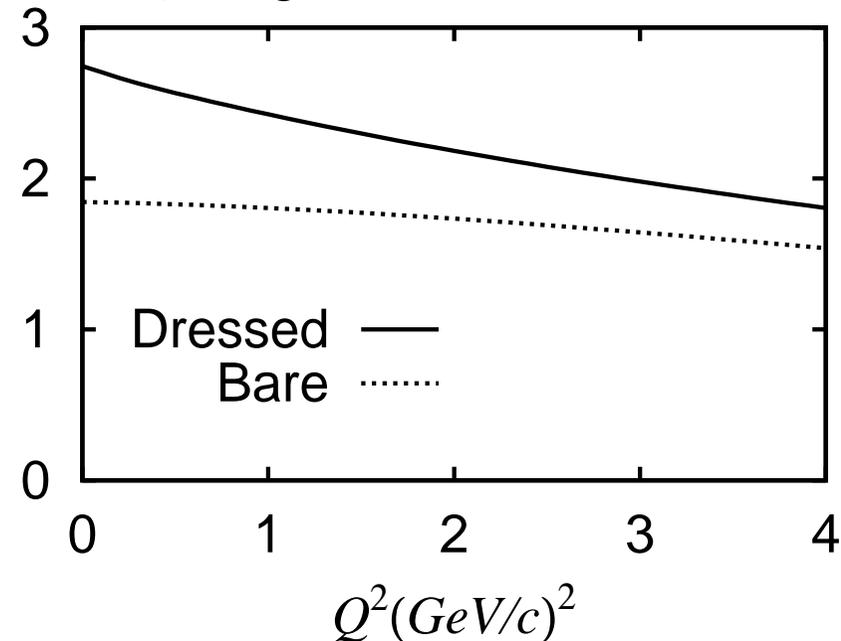
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Ratio of the M1 form factor in $\gamma N \rightarrow \Delta$ transition and proton dipole form factor G_D . Solid curve is $G_M^*(Q^2)/G_D(Q^2)$ including pions; Dotted curve is $G_M(Q^2)/G_D(Q^2)$ without pions.



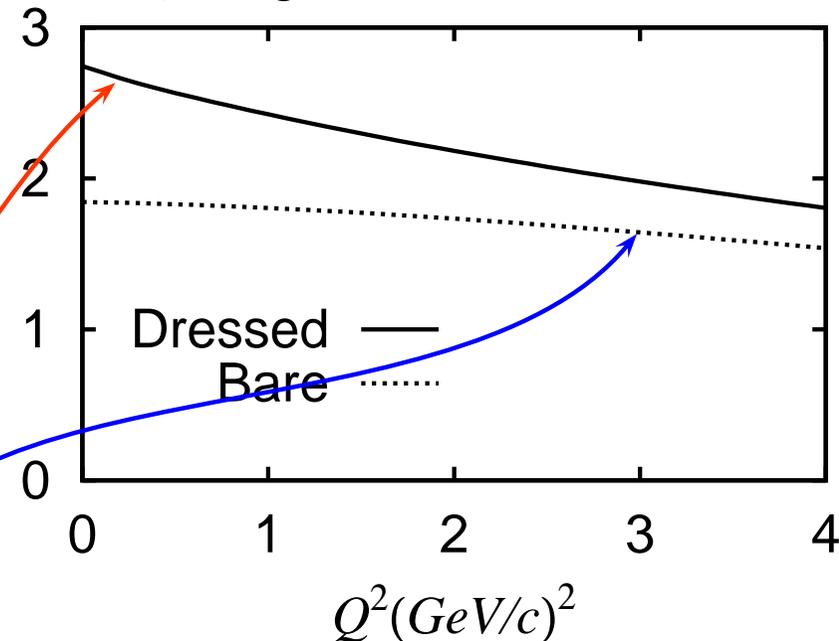
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Quark Core

- Responsible for only 2/3 of result at small Q^2
- Dominant for $Q^2 > 2 - 3 \text{ GeV}^2$



Results: Nucleon and Δ Masses



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Results: Nucleon and Δ Masses

Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses



Set A – fit to the actual masses was required; whereas for **Set B** – fitted mass was offset to allow for “ π -cloud” contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	$0.44=1/(0.45 \text{ fm})$	$0.59=1/(0.33 \text{ fm})$
B	1.18	1.33	0.79	0.89	$0.56=1/(0.35 \text{ fm})$	$0.63=1/(0.31 \text{ fm})$

● $m_{1+} \rightarrow \infty$: $M_N^A = 1.15 \text{ GeV}$; $M_N^B = 1.46 \text{ GeV}$



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• Axial-vector diquark provides significant attraction



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• **Constructive Interference:** 1^{++} -diquark + $\partial_\mu \pi$



Nucleon-Photon Vertex



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

6 terms . . .

Nucleon-Photon Vertex

constructed systematically . . . current conserved automatically
for on-shell nucleons described by Faddeev Amplitude



[First](#)

[Contents](#)

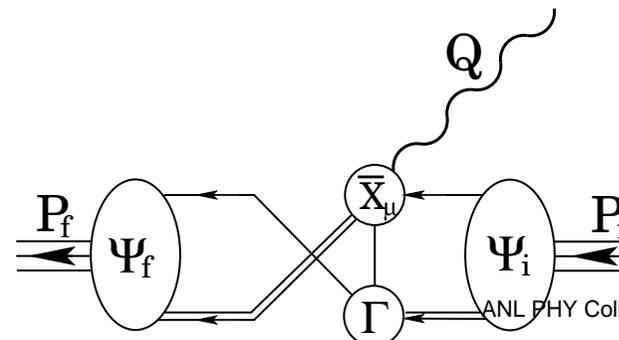
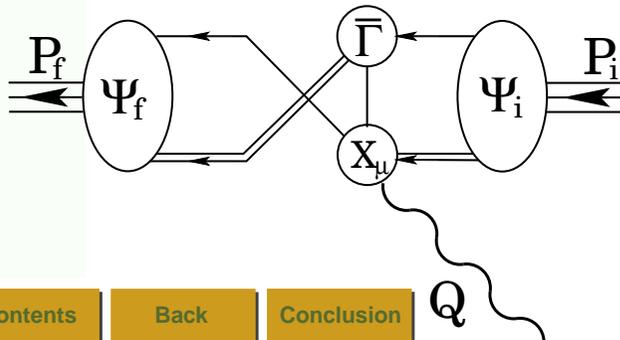
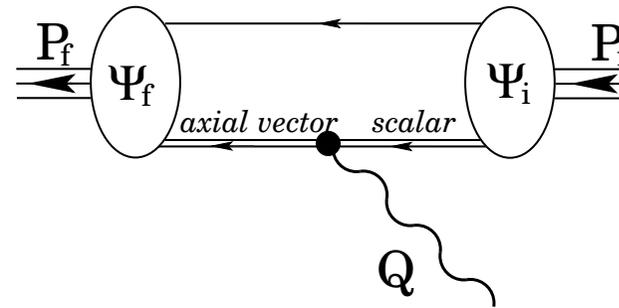
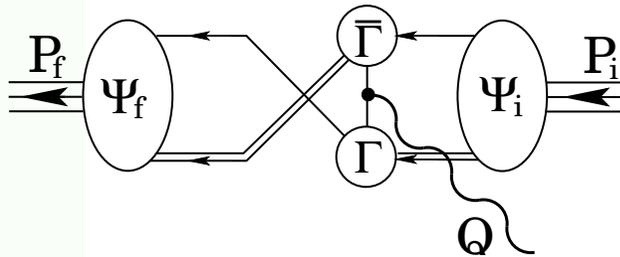
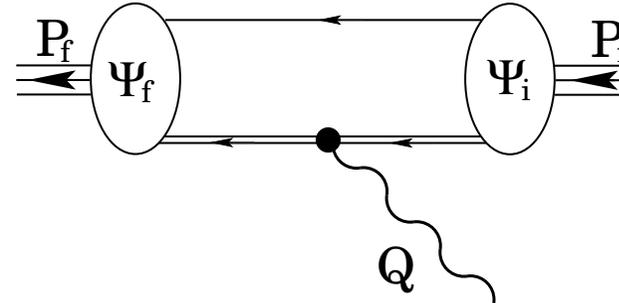
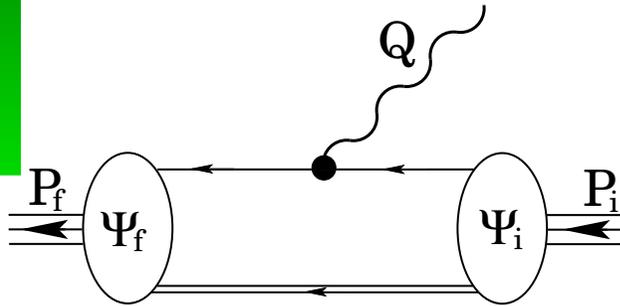
[Back](#)

[Conclusion](#)

6 terms ...

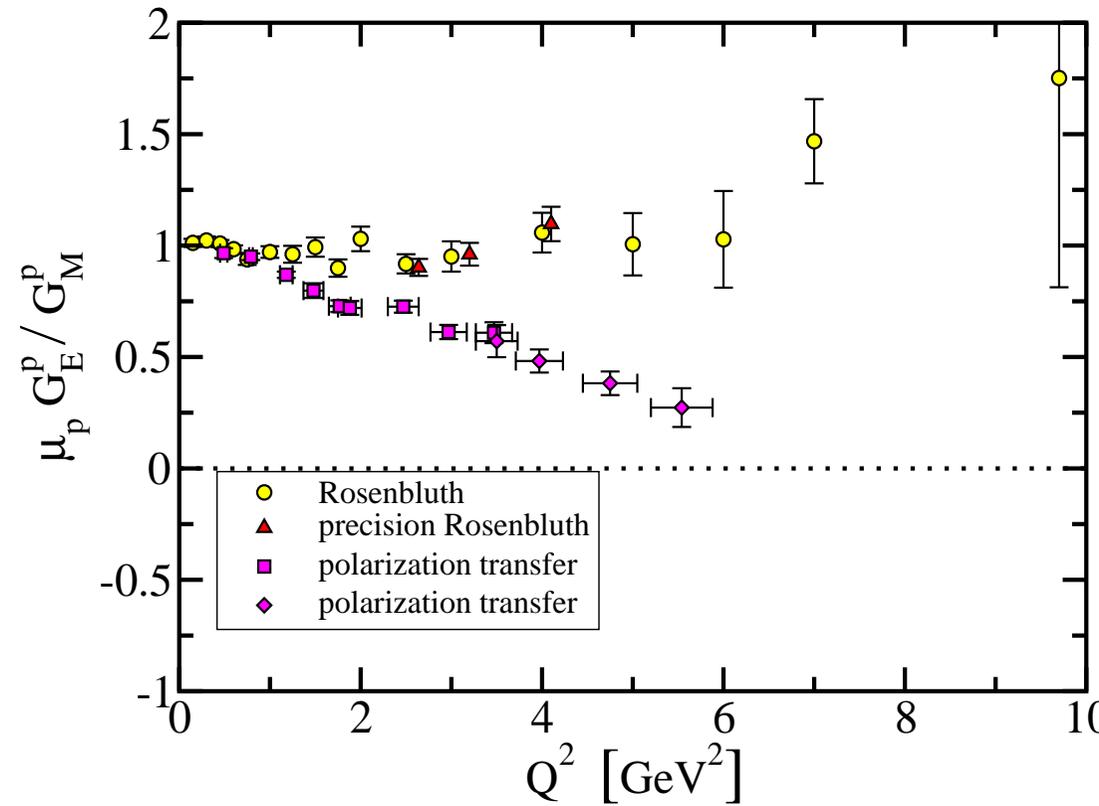
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Form Factor Ratio:

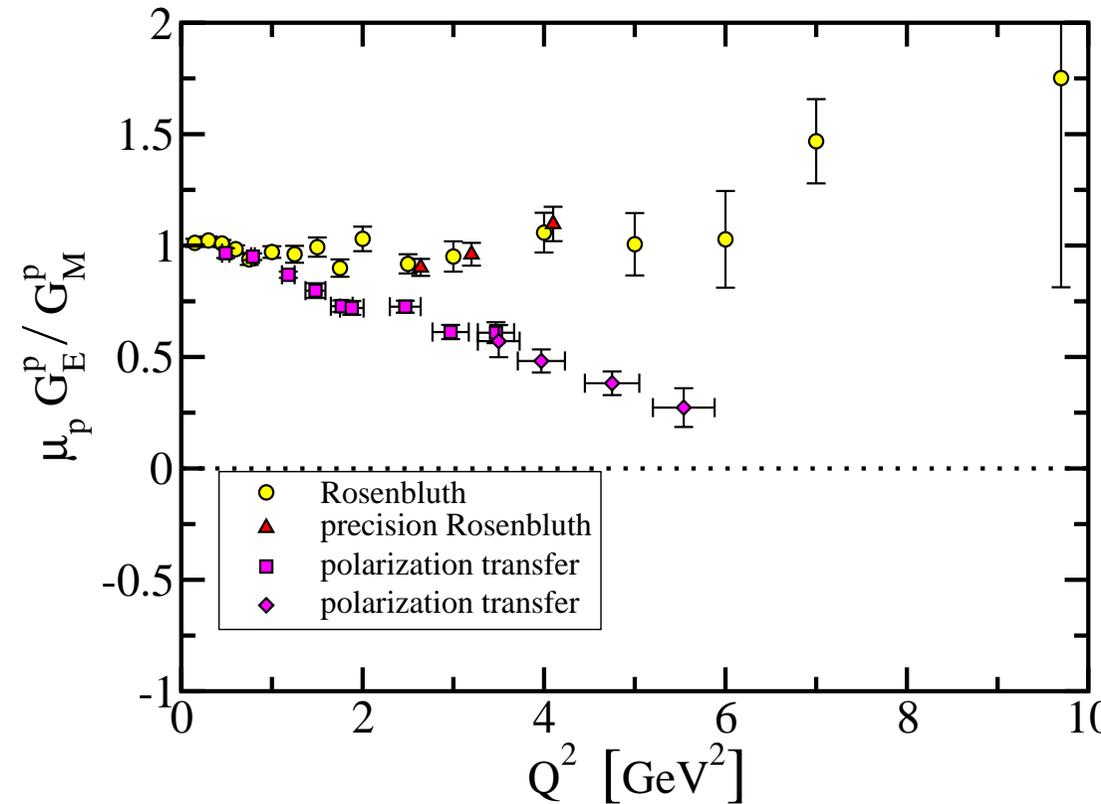
GE/GM



Form Factor Ratio:

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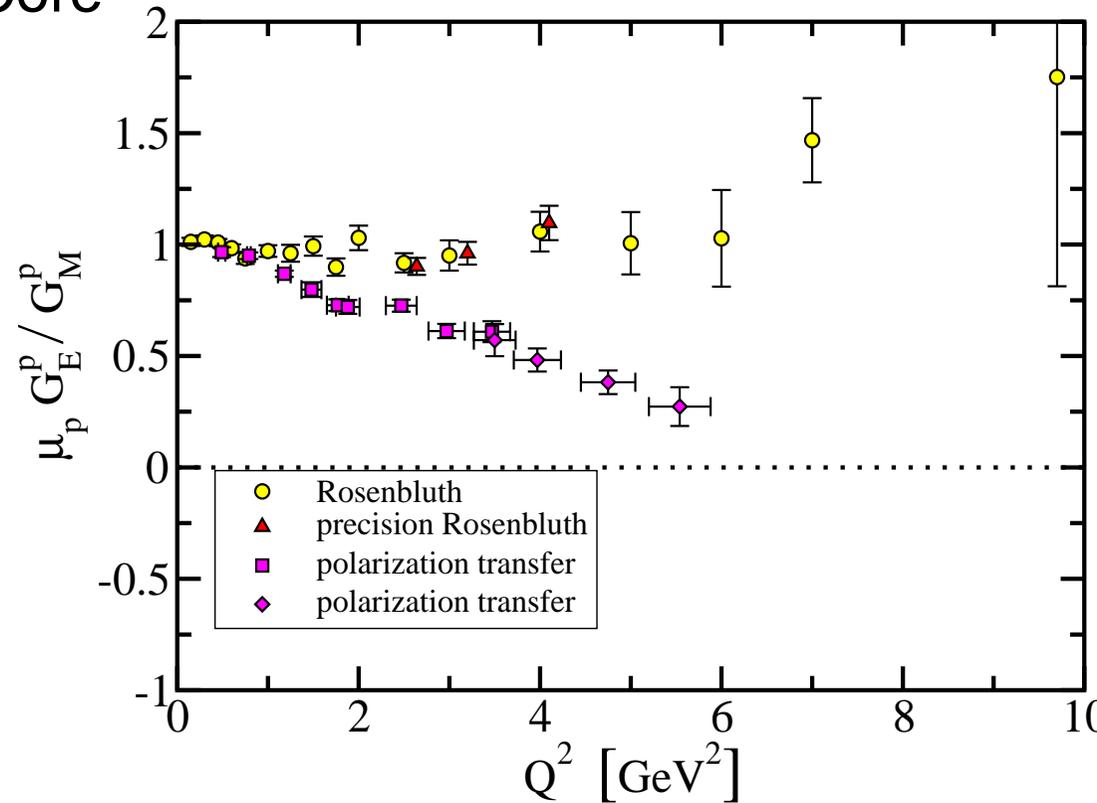


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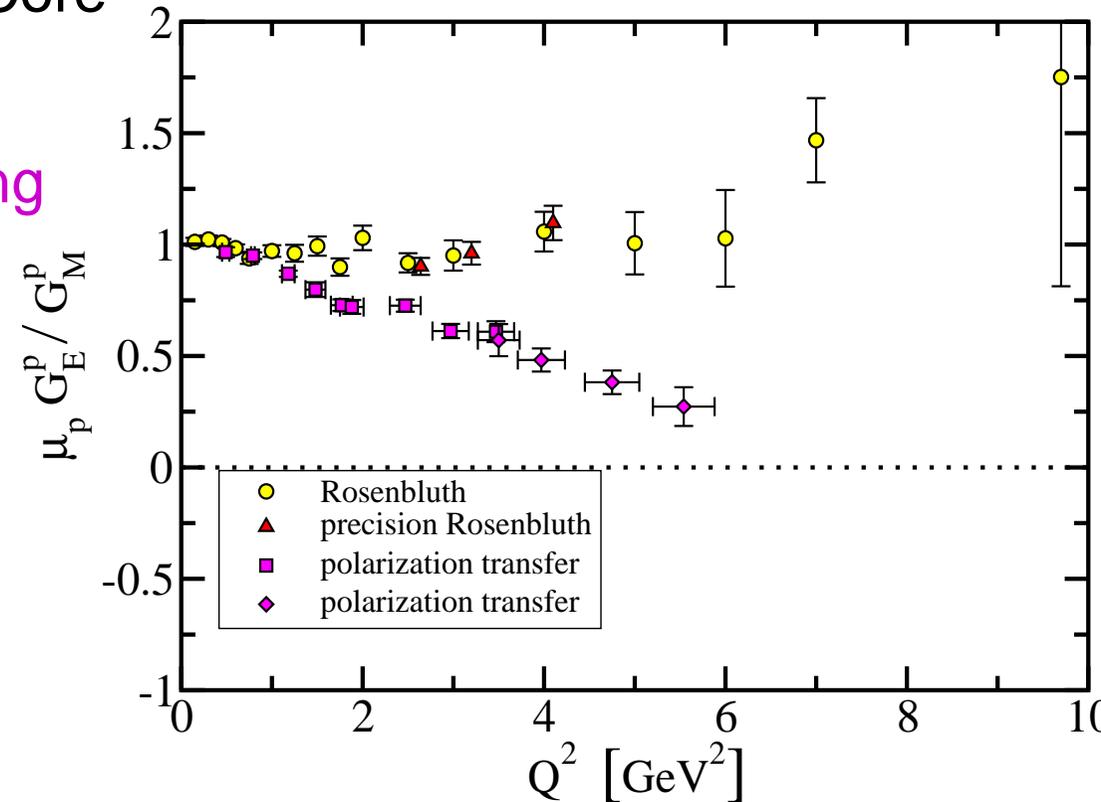


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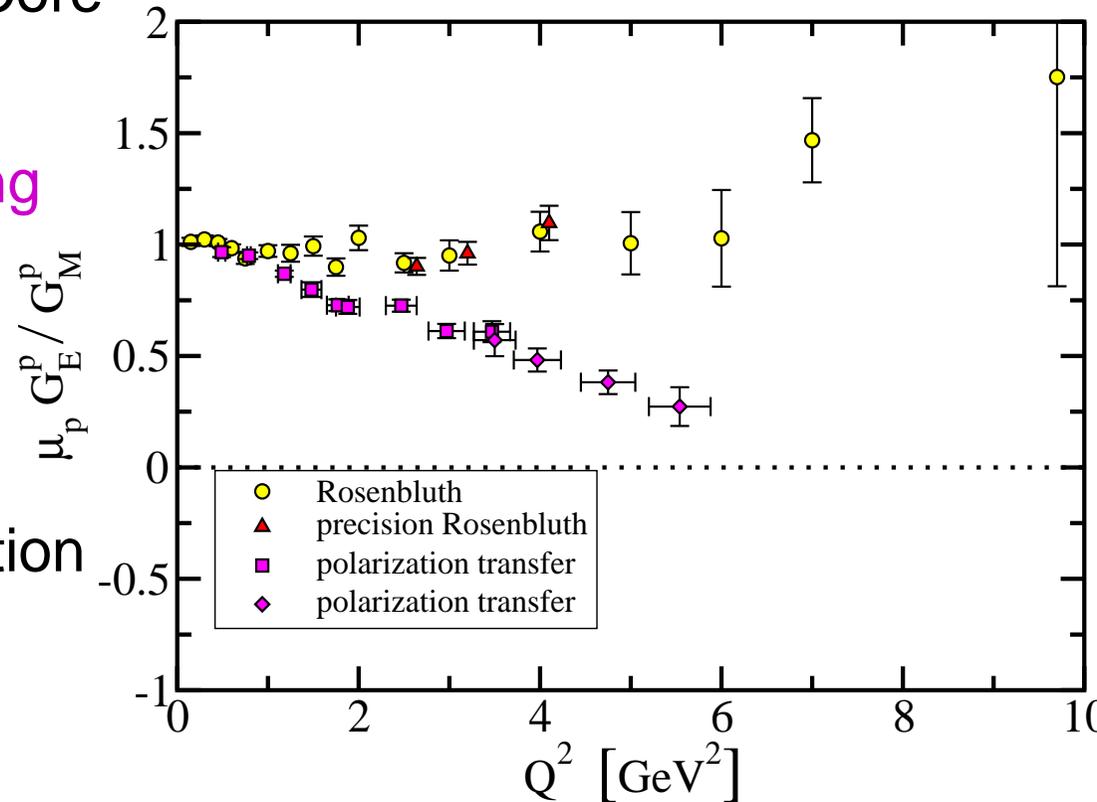


Form Factor Ratio:

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- Anticipate and Estimate Pion Cloud's Contribution



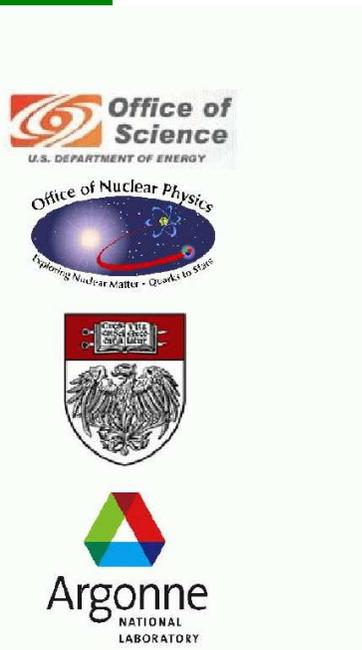
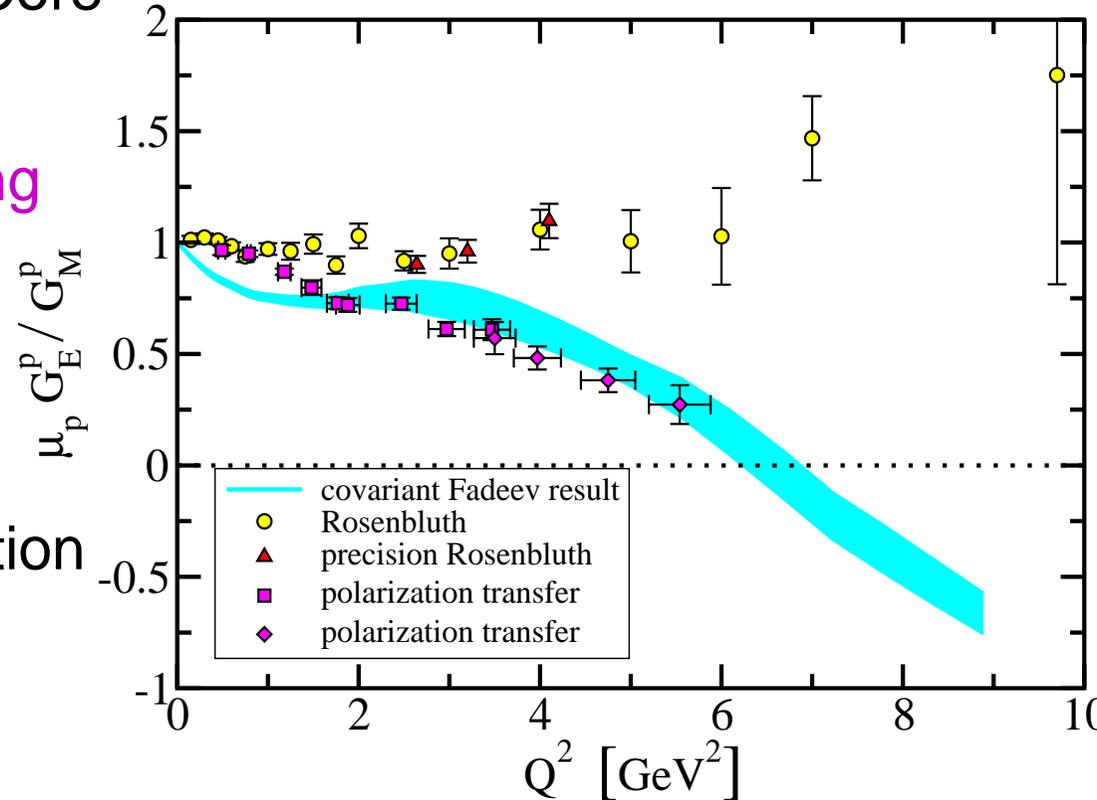
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Office of Nuclear Physics
Exploring Nuclear Matter - Quarks in Stars

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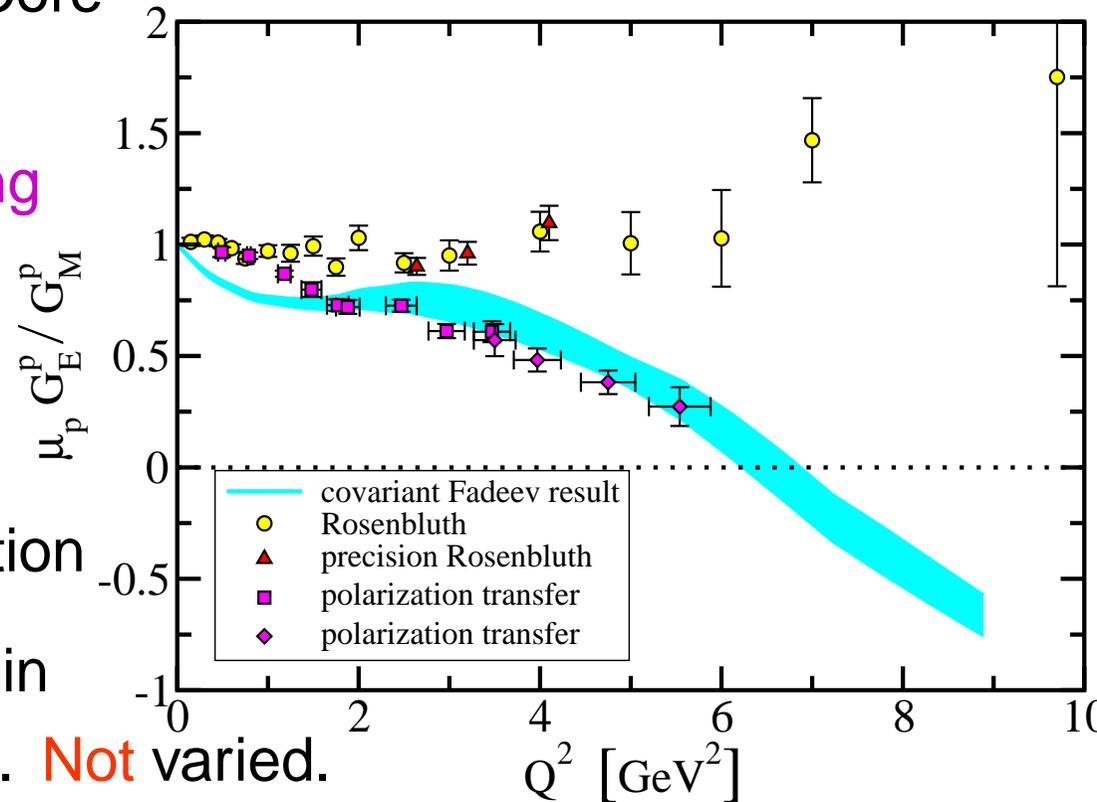
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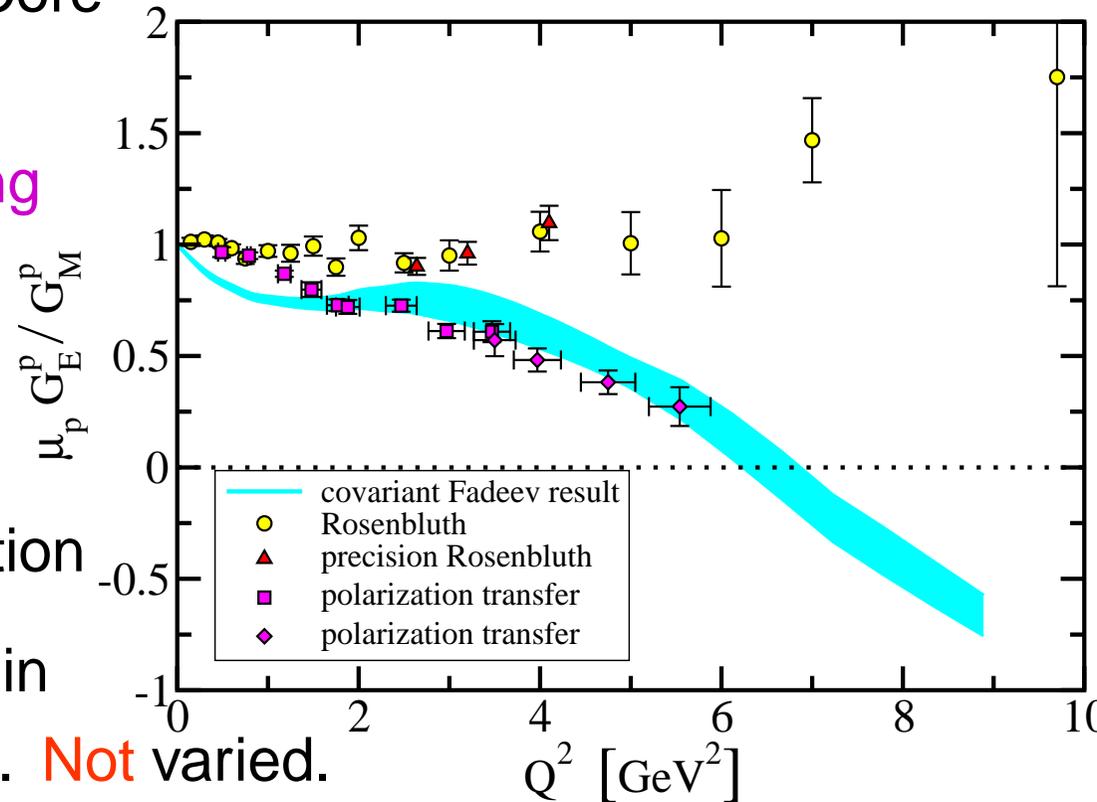


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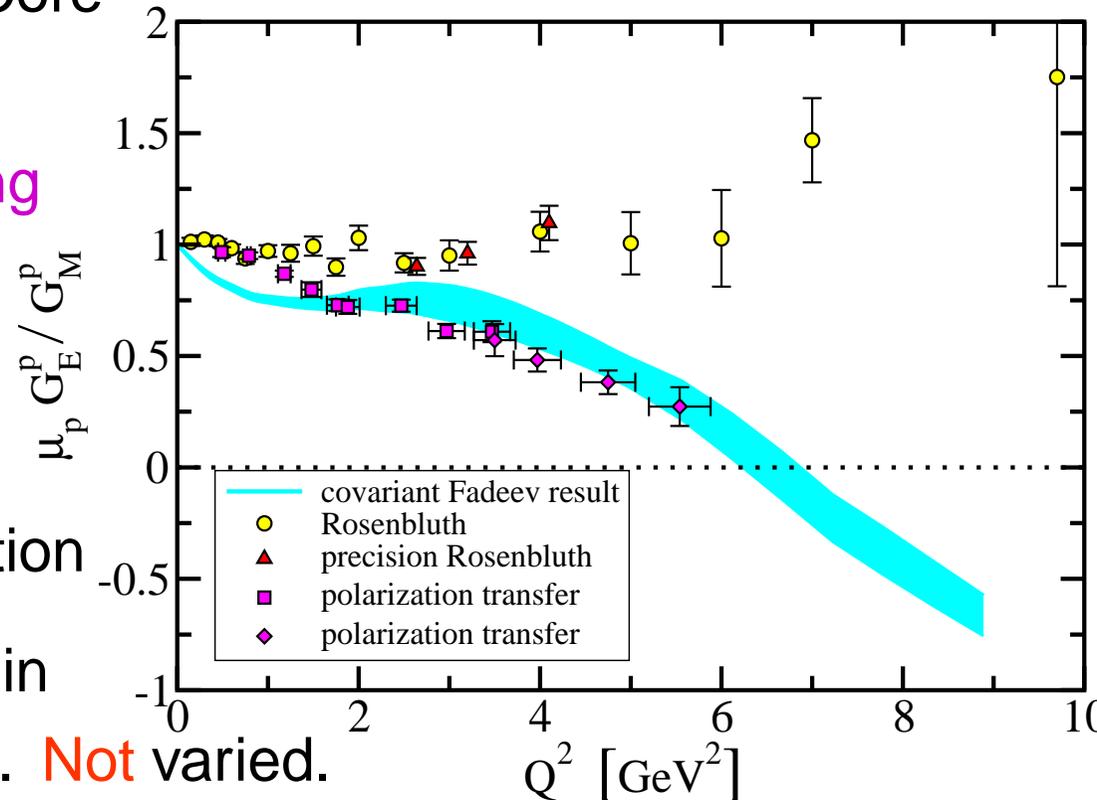


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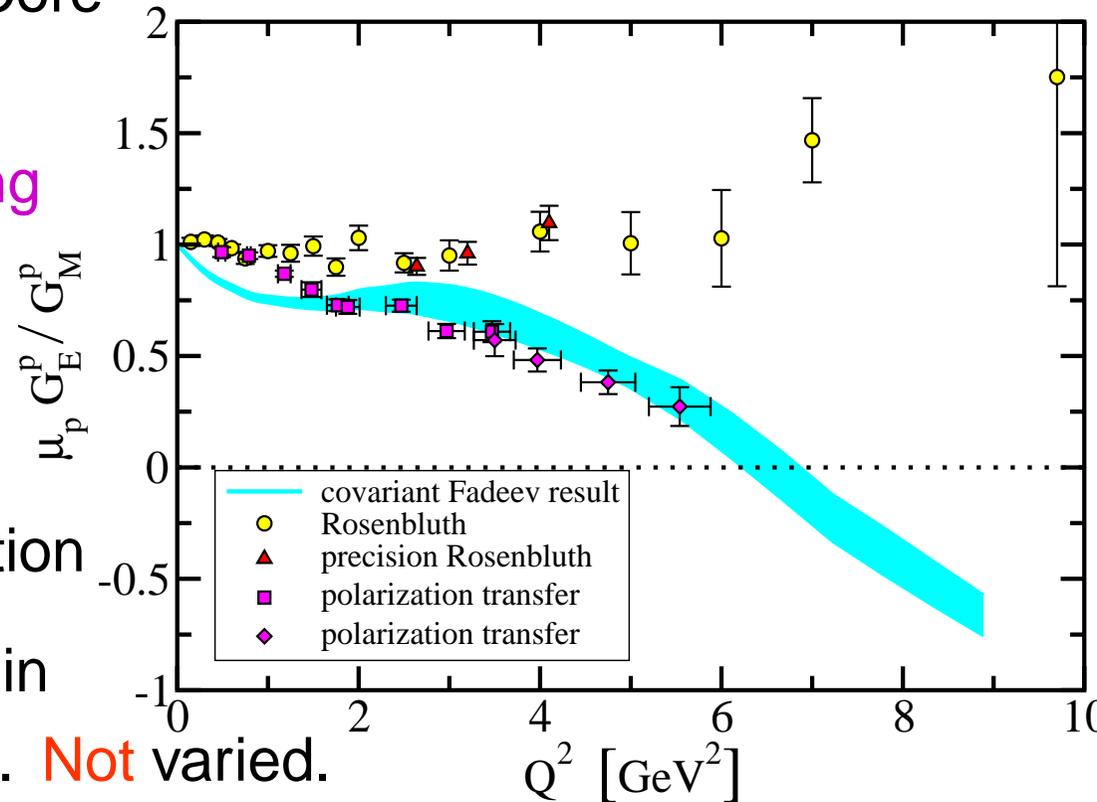
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- Predict Zero at $Q^2 \approx 6.5 \text{ GeV}^2$



Chiral Corrections



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Chiral Corrections

- Thus far, omitted pion cloud contribution to current



Chiral Corrections

- Thus far, omitted pion cloud contribution to current
- Include loops following method of
 - ... Ashley, Leinweber, Thomas, Young, [he-lat/0308024](#)
 - ... finite-range regularisation of loop corrections
 - ... λ = regularisation mass-scale



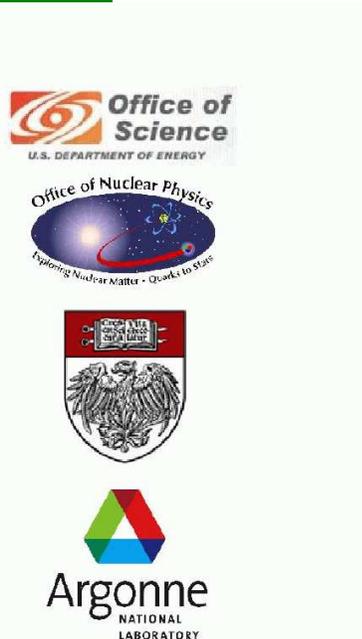
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$$\langle r_p^2 \rangle_{NA}^{1-loop^R} = \mp \frac{1 + 5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \lambda^2}\right),$$

$$\begin{aligned} \langle (r_N^\mu)^2 \rangle_{NA}^{1-loop^R} &= - \frac{1 + 5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2 + \lambda^2}\right) \\ &+ \frac{g_A^2 M_N}{16\pi f_\pi^2 \mu_\nu} \frac{1}{m_\pi} \frac{2}{\pi} \arctan\left(\frac{\lambda}{m_\pi}\right), \end{aligned}$$

$$(\mu_p^n)_{NA}^{1-loop^R} = \mp \frac{g_A^2 M_N}{4\pi^2 f_\pi^2} m_\pi \frac{2}{\pi} \arctan\left(\frac{\lambda^3}{m_\pi^3}\right),$$



Chiral Corrections

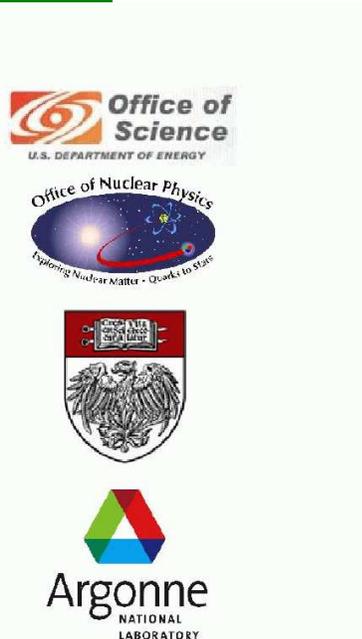
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	r_p	r_n	r_p^μ	r_n^μ	μ_p	$-\mu_n$	ζ
$q-(qq)$ core	0.595	0.169	0.449	0.449	3.63	2.13	0.39
+ π -loop correction	0.762	0.506	0.761	0.761	3.05	1.55	0.23
experiment	0.847	0.336	0.836	0.889	2.79	1.91	



Chiral Corrections

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$$\mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)} = 1 - \frac{Q^2}{6} \left[(r_p)^2 - (r_p^\mu)^2 \right]$$
 - ... $r_p \approx r_p^\mu \Rightarrow \text{ratio varies} < 10\% \text{ on } 0 < Q^2 < 0.6 \text{ GeV}^2$



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- Complements nucleon mass considerations
 - ... veracious understanding of all nucleon properties
 - ... impossible without intelligent incorporation of chiral corrections



Epilogue



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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Epilogue



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

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Epilogue

Dyson-Schwinger Equations

- Provide Understanding of
Dynamical Chiral Symmetry Breaking:

⇒ π is quark-antiquark Bound State

AND QCD's Goldstone Mode



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Dyson-Schwinger Equations

- Provide Understanding of
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AND QCD's Goldstone Mode

- Foundation for Proof of
Exact Results in QCD

e.g., Quark Goldberger-Treiman
Properties of Pseudoscalar Mesons



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- Poincaré Covariant
Faddeev Equation



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Epilogue

- Poincaré Covariant
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 - Nonpointlike scalar and axial-vector diquark correlations



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Epilogue

- Poincaré Covariant
Faddeev Equation
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 - $s-$, $p-$, $d-$ wave quark angular momentum



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Epilogue

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Faddeev Equation
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- Quark core, relaxed to allow for pion cloud



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- Poincaré Covariant
Faddeev Equation
 - Nonpointlike scalar and axial-vector diquark correlations
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- Quark core, relaxed to allow for pion cloud
 - Predicts zero in $G_E^P(Q^2)$ at $Q^2 \approx 6.5 \text{ GeV}^2$



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Epilogue

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected

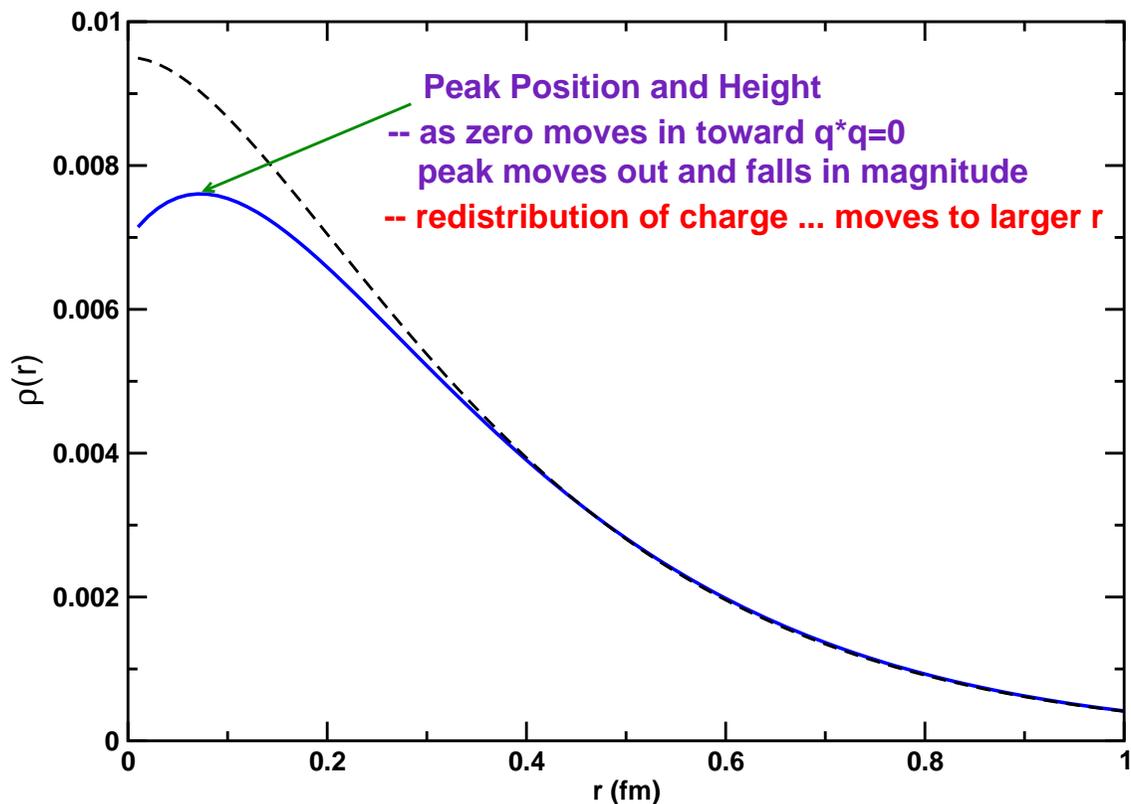


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Office of Nuclear Physics
Exploring Nuclear Matter - Quarks in Stars



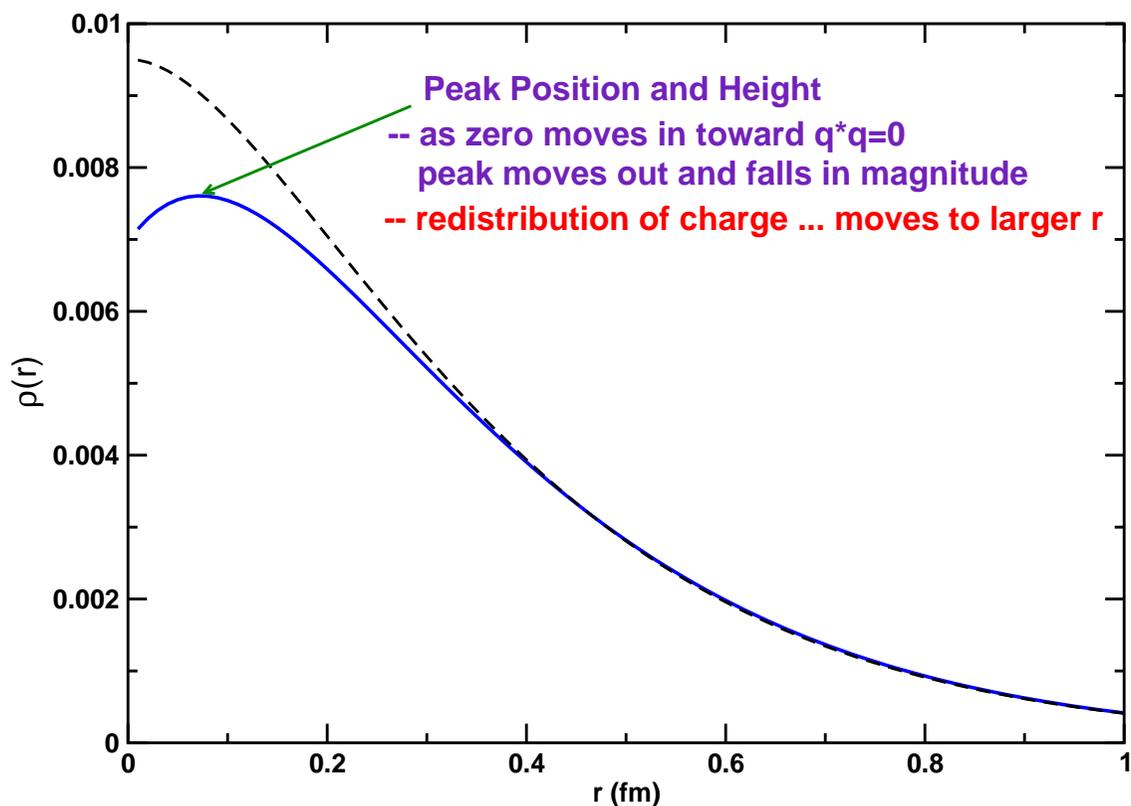
Argonne
NATIONAL
LABORATORY

... tell everyone I'm
sorry about
EVERYTHING



Epilogue

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected



- However, Current Density remains peaked at $r = 0$!



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Epilogue

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected
- Wave Function is complex and correlated mix of virtual particles and antiparticles: s –, p – and d –waves



Future?



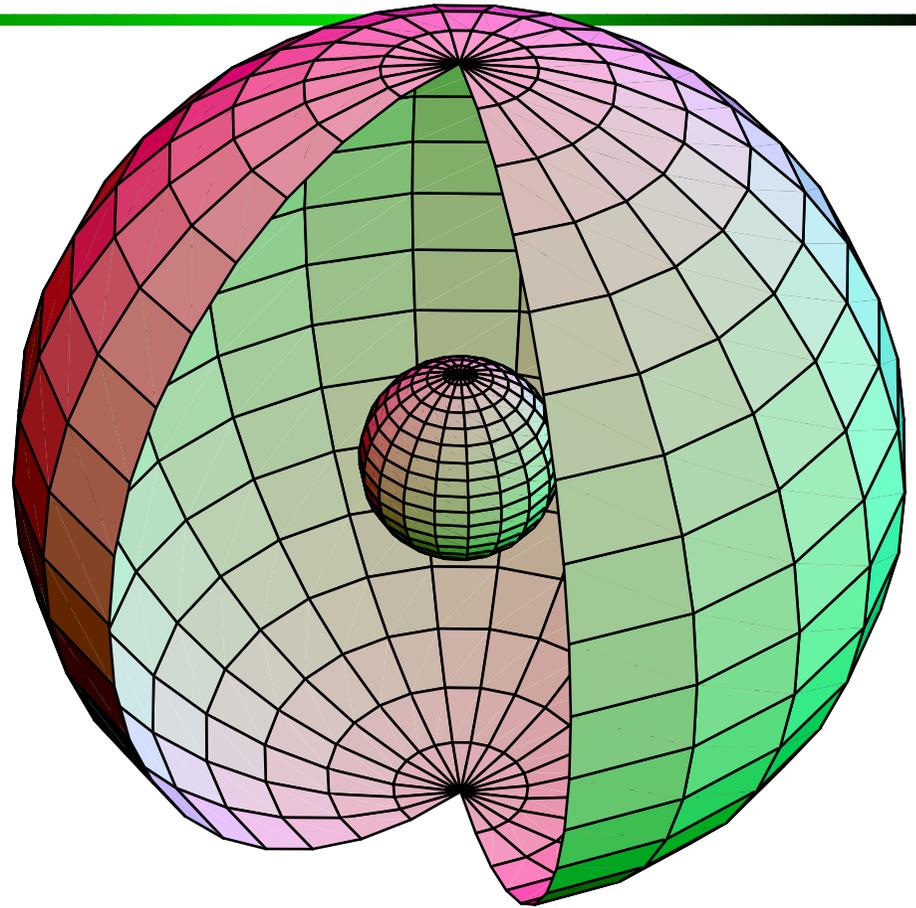
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Future?



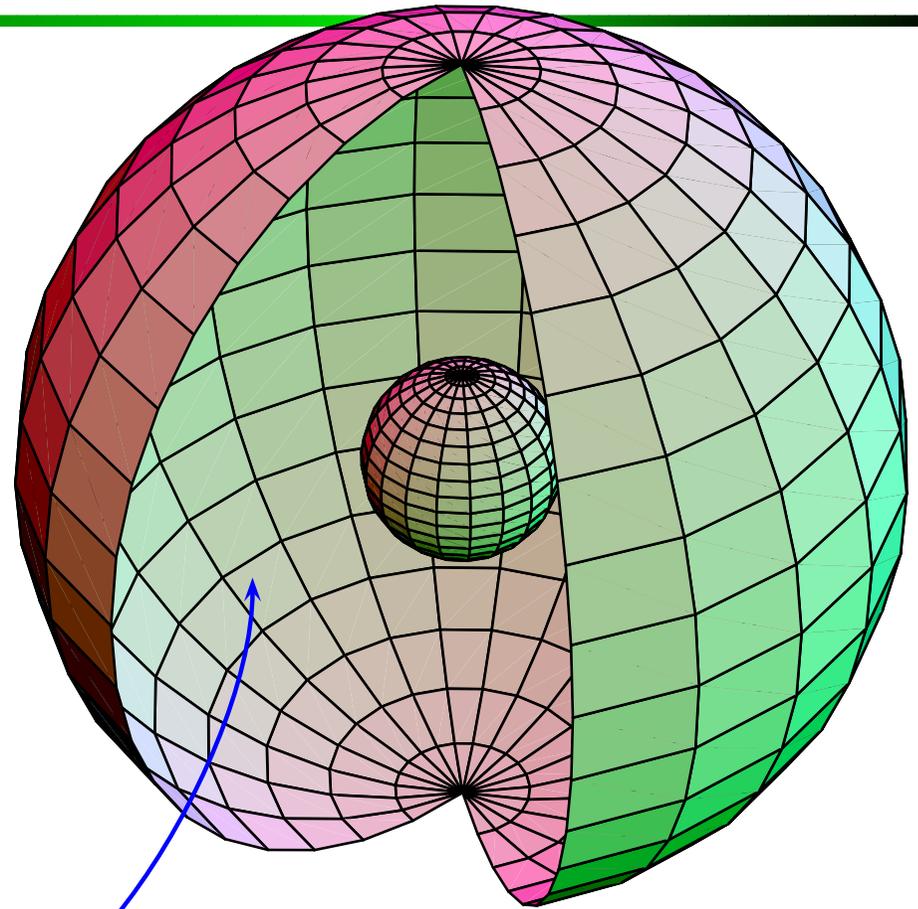
[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

Future?



98% of the volume



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

*It that is more than a parton is not for many,
And it that is less than an hadron,
many are not for it.*

Anon.



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

1. Hadron Physics
2. Nucleon ... 2 Key Hadrons
3. QCD?
4. QED cf. QCD
5. Nucleon Form Factors
6. NSAC Long Range Plan
7. Modern Miracles
8. Pion Dichotomy
9. QCD's Emergent Phenomena
10. Why care?
11. What's the Problem?
12. Confinement
13. A Compromise?
14. DSEs
15. Persistent Challenge
16. Perturbative $S(p)$
17. Dressed-Quark Propagator
18. Lattice cf. DSE

19. Light-Quark Interaction
20. Dressed-gluon Propagator
21. Hadrons
22. Bethe-Salpeter Kernel
23. Excitations & Chiral Symmetry
24. Radial Excitations & Chiral Symmetry
25. Radial Excitations
26. Radial Excitations II
27. Lattice Radial
28. Faddeev equation
29. Nucleon EM Form Factors
30. Pions and Form Factors
31. Results: Nucleon & Δ Masses
32. Nucleon-Photon Vertex
33. Form Factor Ratio: GE/GM
34. Chiral Corrections
35. Parametrising diquark properties
36. Contemporary Reviews

Contents



Parametrising diquark properties



[First](#)

[Contents](#)

[Back](#)

[Conclusion](#)

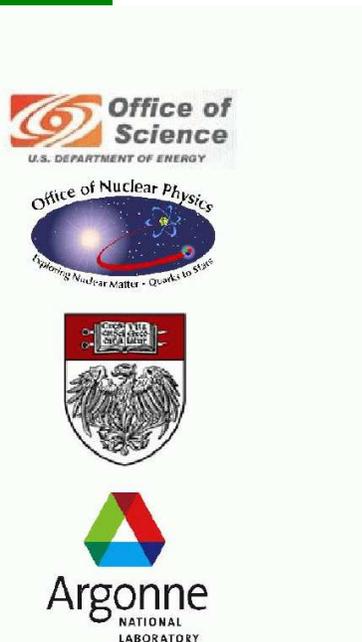
Parametrising diquark properties

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. . . Burden, Roberts, Thomson, Phys. Lett. **B 371**, 163 (1996)



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 - Bethe-Salpeter amplitudes . . . width for each – ω_{JP}
 - Confining propagators . . . mass for each – m_{JP}



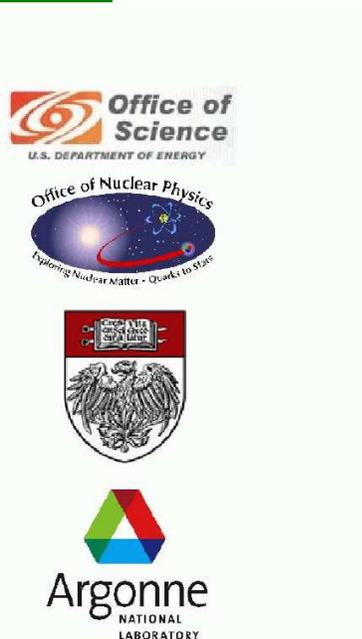
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Widths fixed by “asymptotic freedom” condition –

$$\left. \frac{d}{dK^2} \left(\frac{1}{m_{JP}^2} \mathcal{F}(K^2/\omega_{JP}^2) \right)^{-1} \right|_{K^2=0} = 1 \Rightarrow \omega_{JP}^2 = \frac{1}{2} m_{JP}^2 ,$$

Only two parameters; viz., diquark “masses”: m_{JP}



Contemporary Reviews

- Dyson-Schwinger Equations: Density, Temperature and Continuum Strong QCD
C.D. Roberts and S.M. Schmidt, nu-th/0005064,
Prog. Part. Nucl. Phys. **45** (2000) S1
- The IR behavior of QCD Green's functions: Confinement, DCSB, and hadrons . . .
R. Alkofer and L. von Smekal, he-ph/0007355,
Phys. Rept. **353** (2001) 281
- Dyson-Schwinger equations: A Tool for Hadron Physics
P. Maris and C.D. Roberts, nu-th/0301049,
Int. J. Mod. Phys. **E 12** (2003) pp. 297-365
- Infrared properties of QCD from Dyson-Schwinger equations.
C. S. Fischer, he-ph/0605173,
J. Phys. **G 32** (2006) pp. R253-R291

