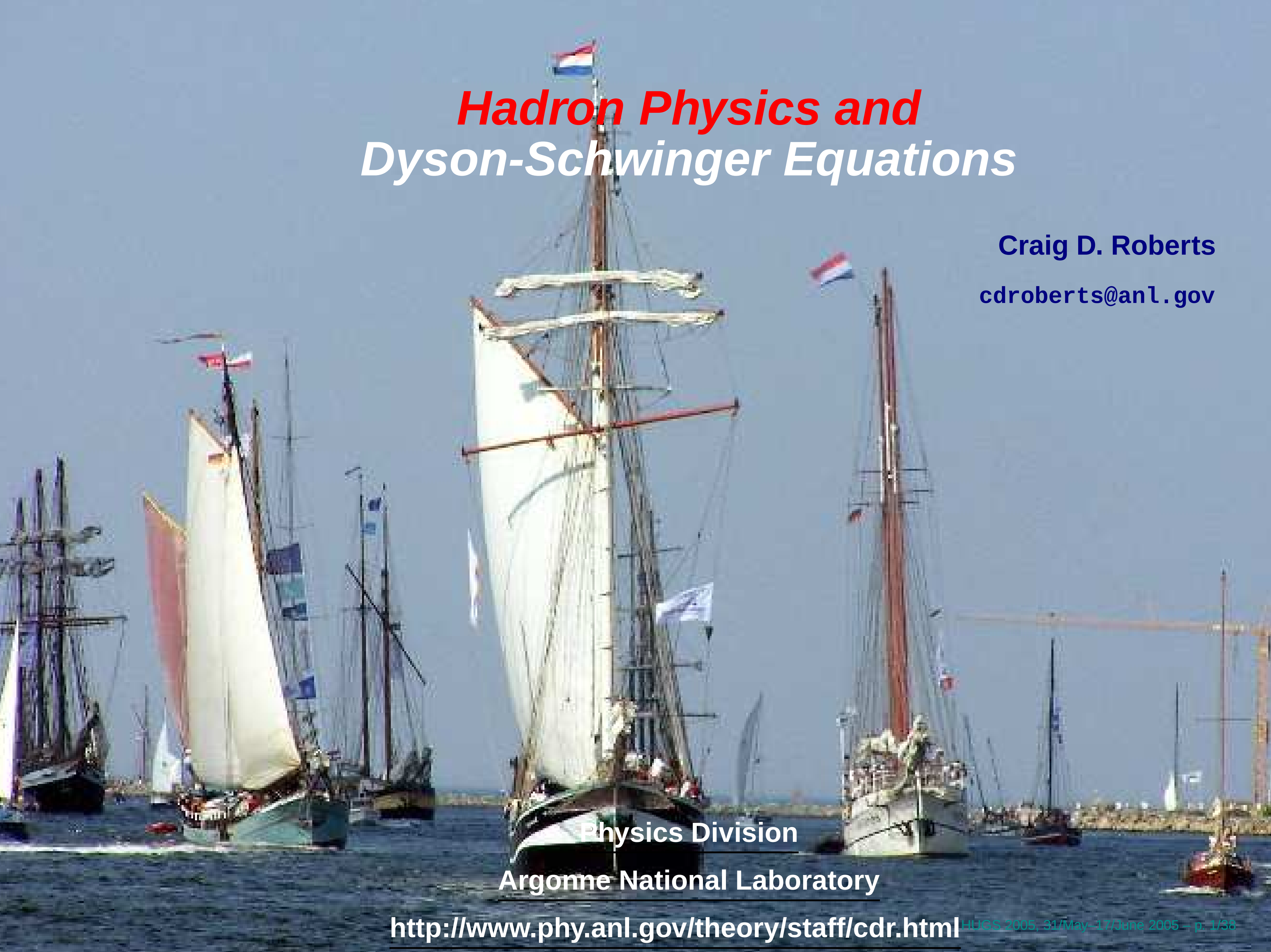


Hadron Physics and Dyson-Schwinger Equations

Craig D. Roberts

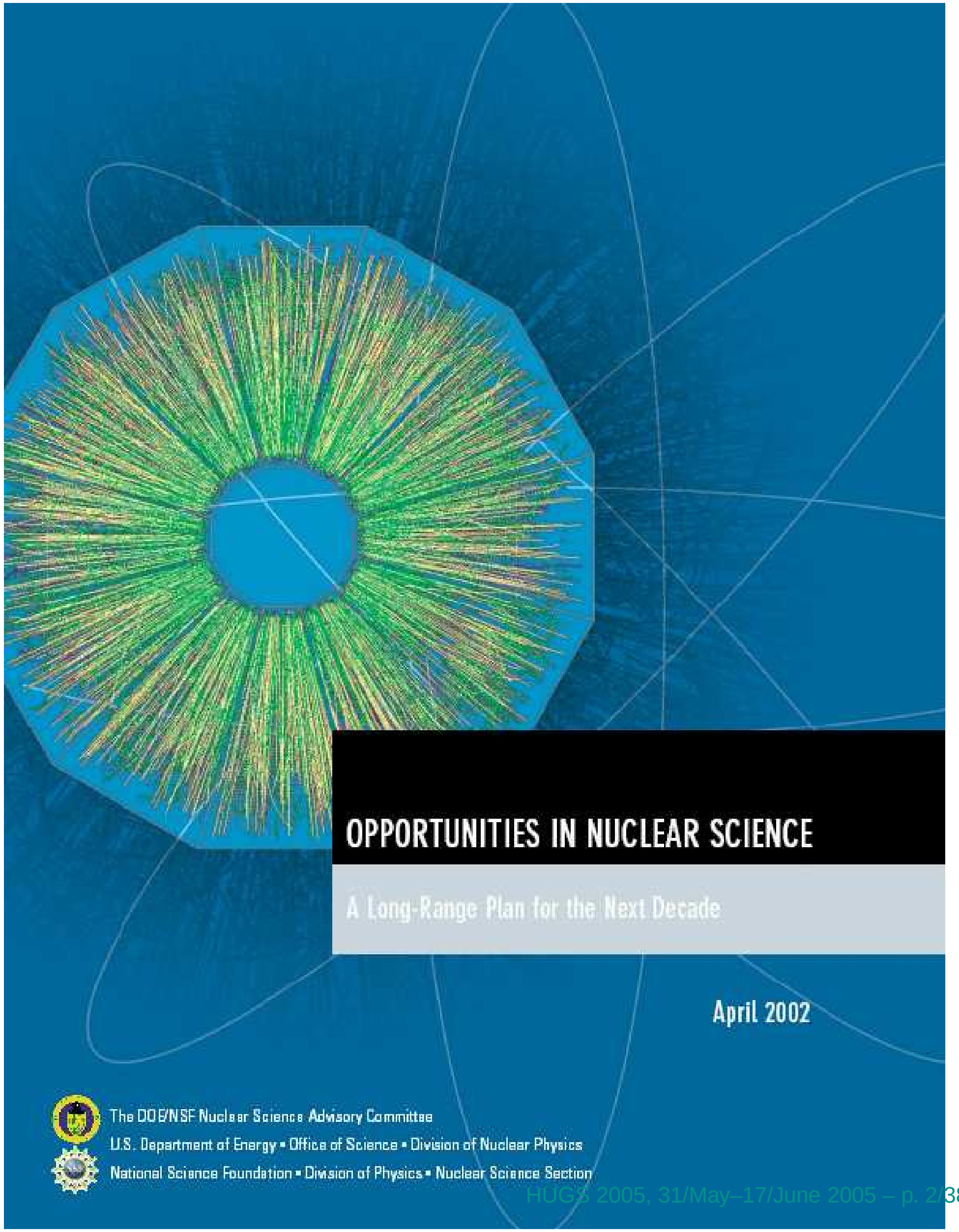
`cdroberts@anl.gov`

Physics Division

Argonne National Laboratory

http://www.phy.anl.gov/theory/staff/cdr.html

Long Range Plan



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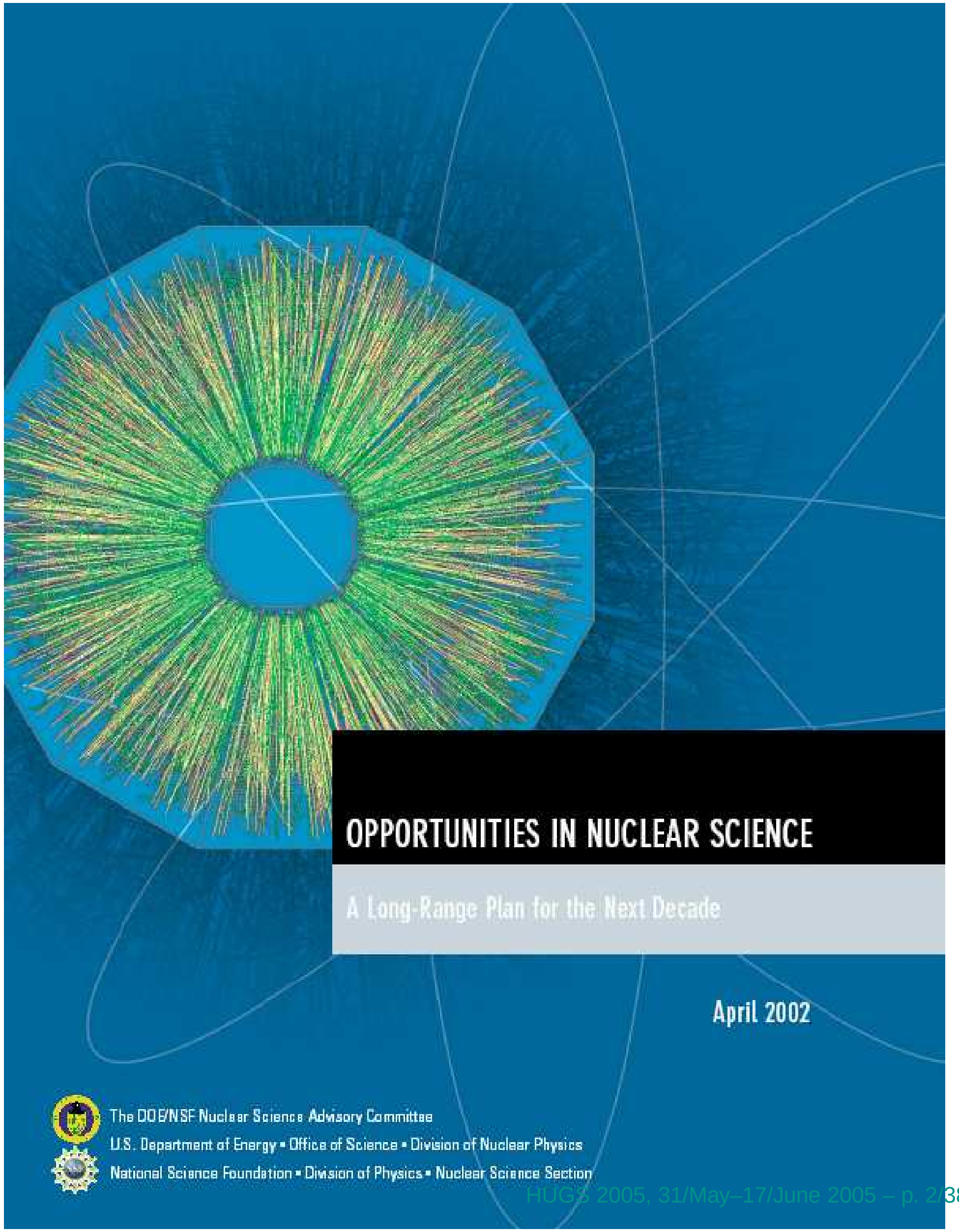
Long Range Plan

- US Plans
  in Nuclear Science
  for the Next Decade



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Long Range Plan

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  DOE-NSF
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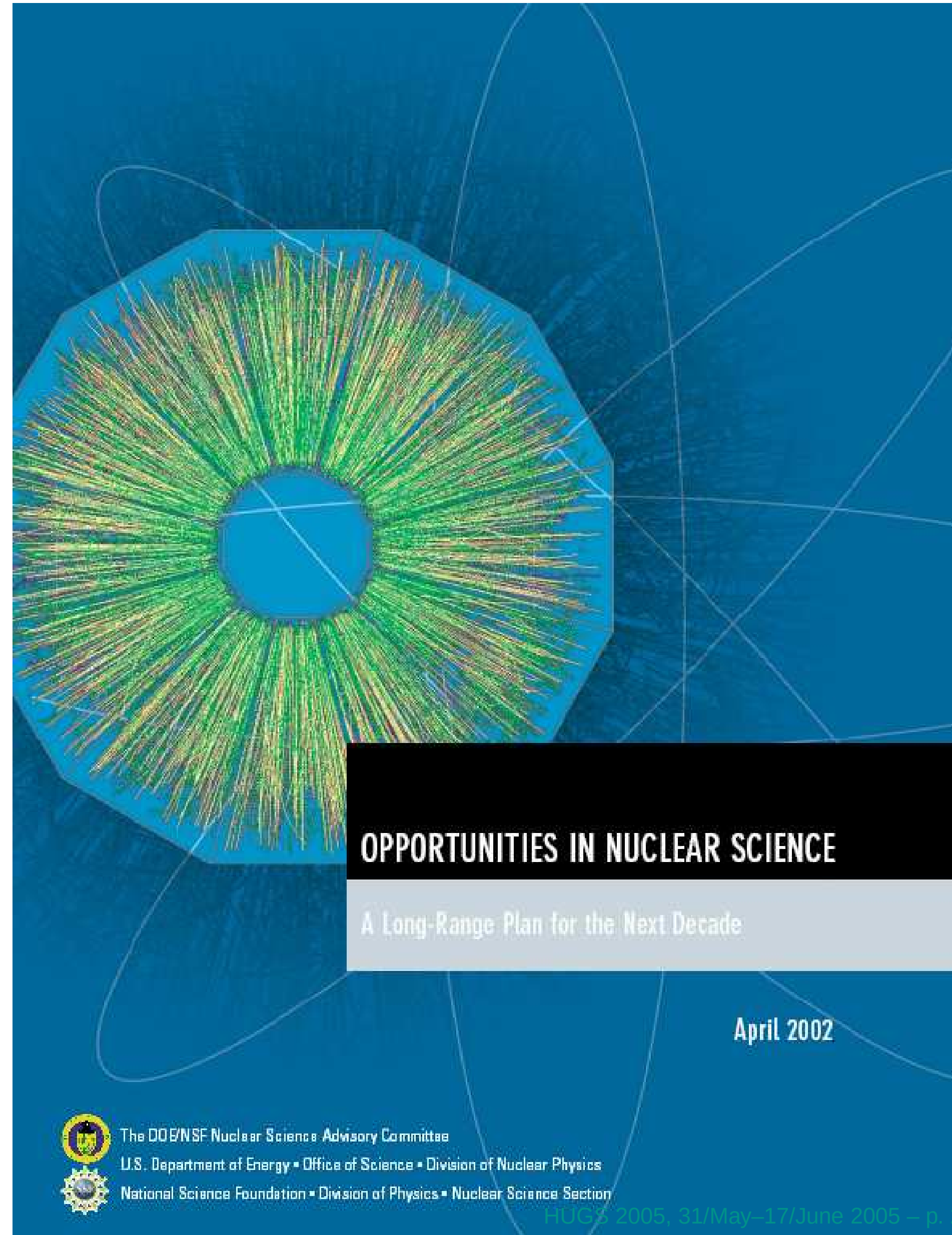

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Long Range Plan

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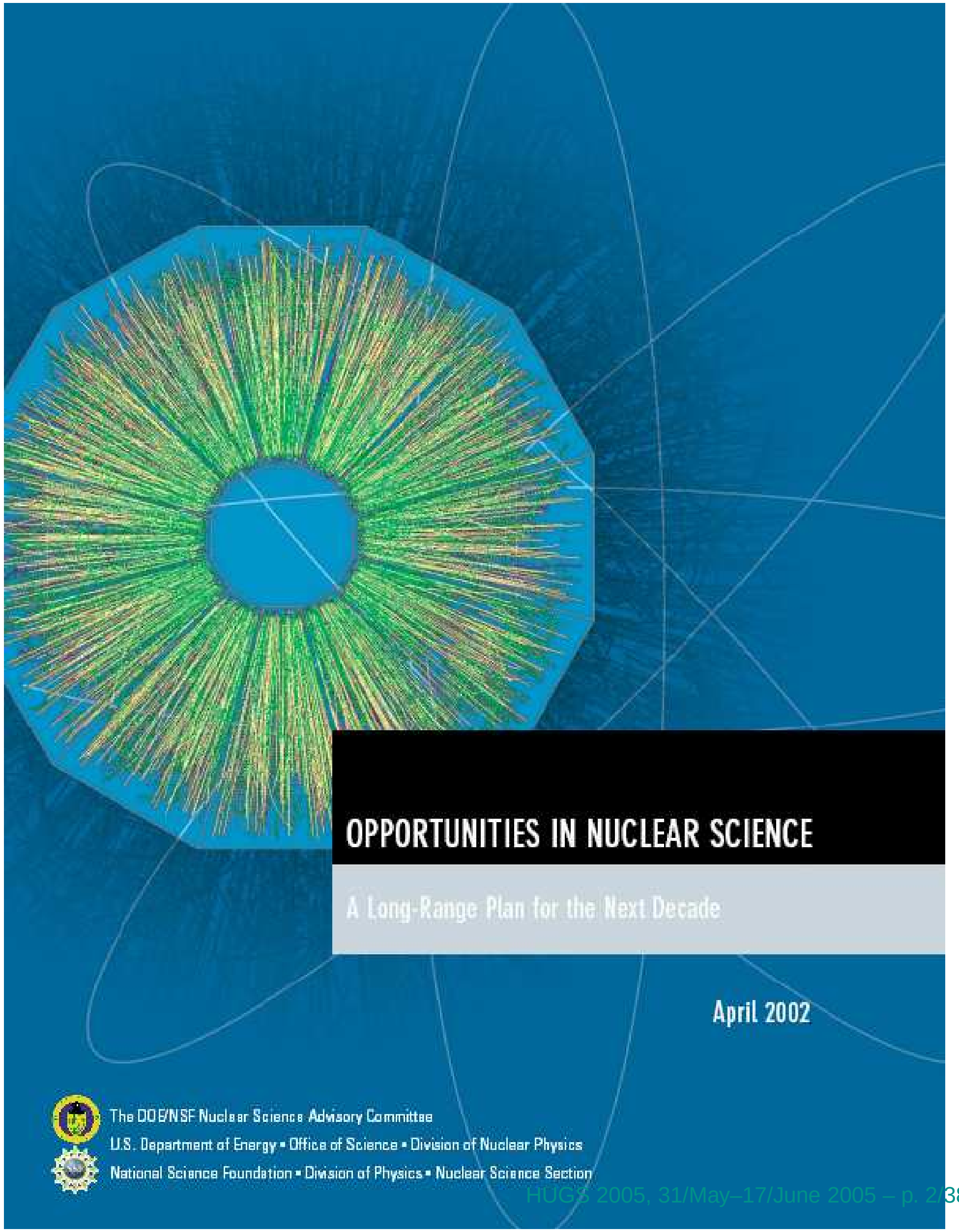

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Long Range Plan

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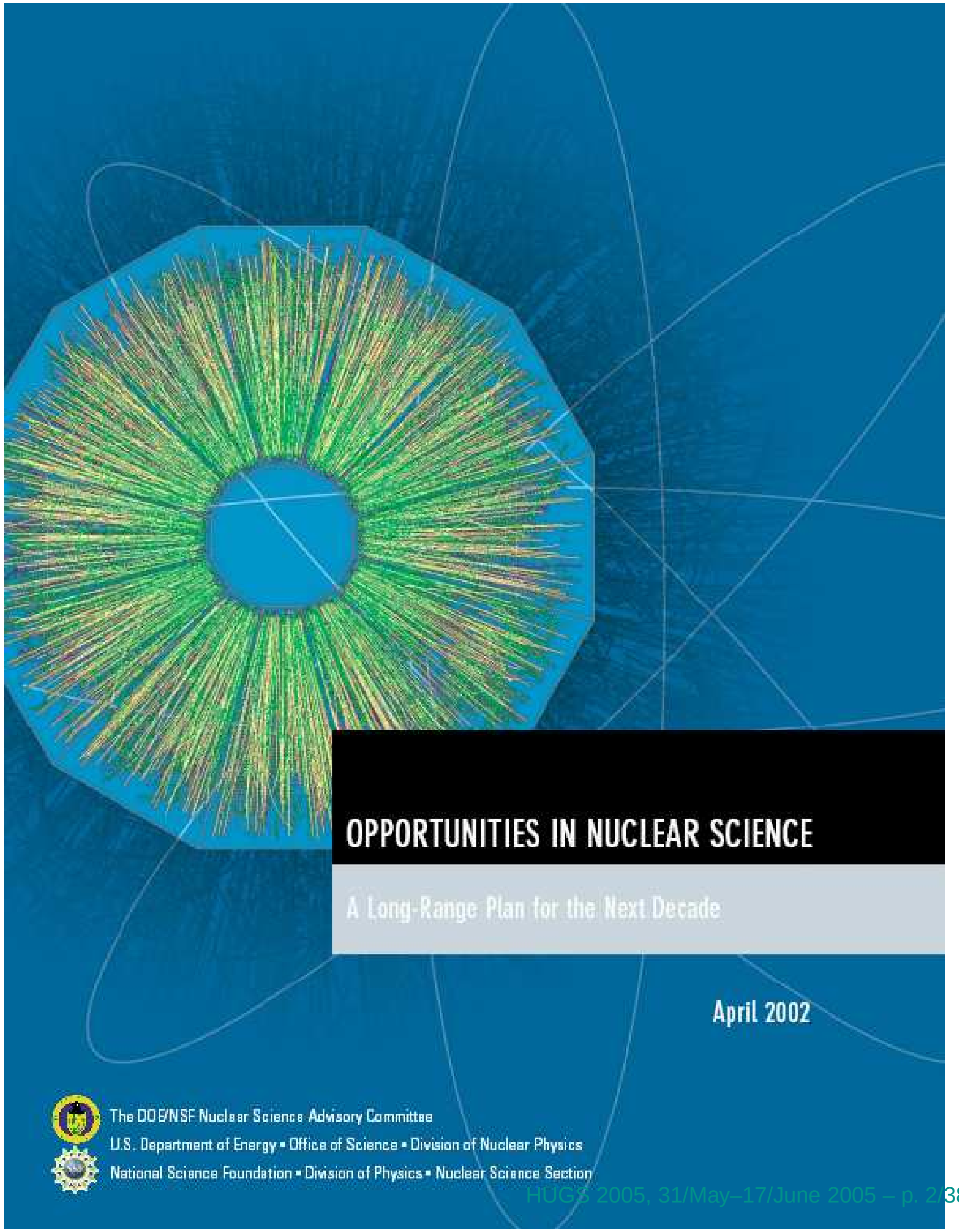
- Prepared by
  DOE-NSF
  Nuclear Science
  Advisory Committee
- Based on Community
  Input: Town Meetings
  2000 & 2001



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Long Range Plan

- US Plans
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  Nuclear Science
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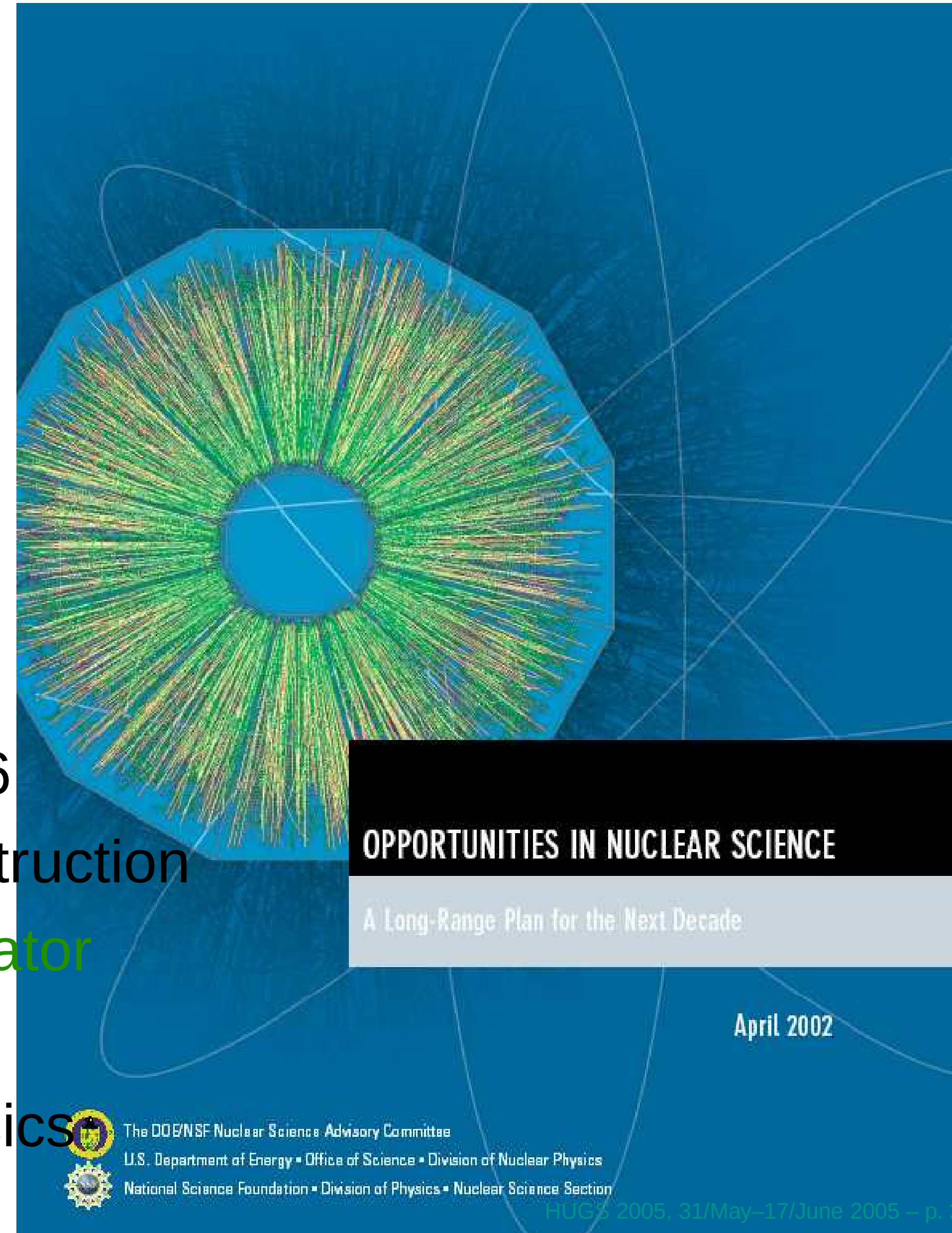
- Previous Plan in 1996
  Recommended Construction
  Rare Isotope Accelerator
  Quantitative Tool
  for Nuclear Astrophysics



Long Range Plan

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Long Range Plan

- Recommendation One: “Significantly increase funding for nuclear theory, which is essential for developing the full potential of the scientific program”

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Long Range Plan

- Recommendation One: “Significantly increase funding for nuclear theory, which is essential for developing the full potential of the scientific program”
- Recommendation Two: “RIA is our highest priority for major new construction. RIA will be the world-leading facility in nuclear structure and nuclear astrophysics”

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Long Range Plan

- Recommendation One: “Significantly increase funding for nuclear theory, which is essential for developing the full potential of the scientific program”
- Recommendation Two: “RIA is our highest priority for major new construction. RIA will be the world-leading facility in nuclear structure and nuclear astrophysics”
- Recommendation Three: “We strongly recommend immediate construction of the world’s deepest underground science laboratory. This laboratory will provide a compelling opportunity for nuclear scientists to explore fundamental questions in neutrino physics and astrophysics.”

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Long Range Plan

- Recommendation One: "Significantly increase funding for nuclear theory, which is essential for developing the full potential of the scientific program"
- Recommendation Two: "RIA is our highest priority for major new construction. RIA will be the world-leading facility in nuclear structure and nuclear astrophysics"
- Recommendation Four: "We strongly recommend the upgrade of CEBAF at Jefferson Laboratory to 12 GeV as soon as possible. [This upgrade] is critical for our continued leadership in the experimental study of hadronic matter."

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Long Range Plan

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Meantime . . Other Long Range Plans

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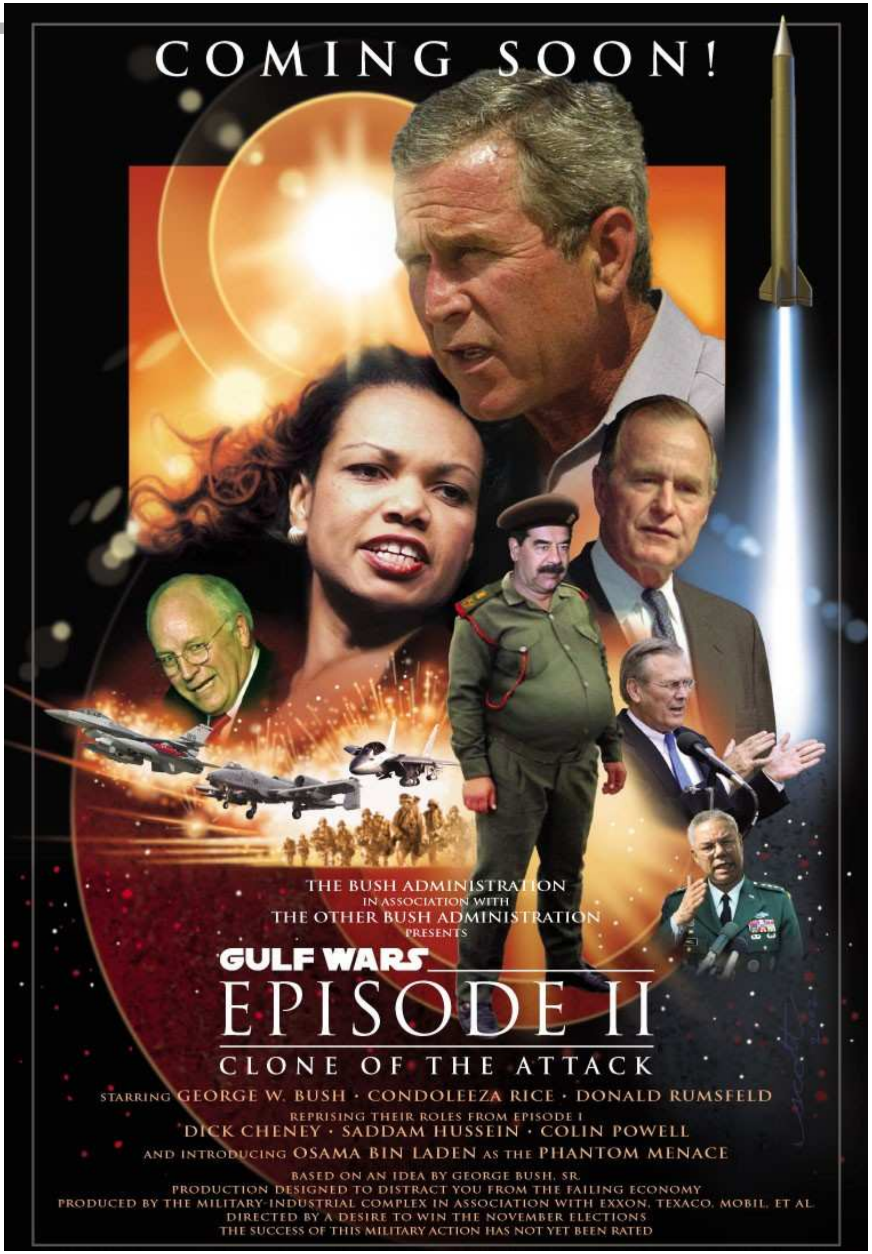
Meantime . . Other Long Range Plans



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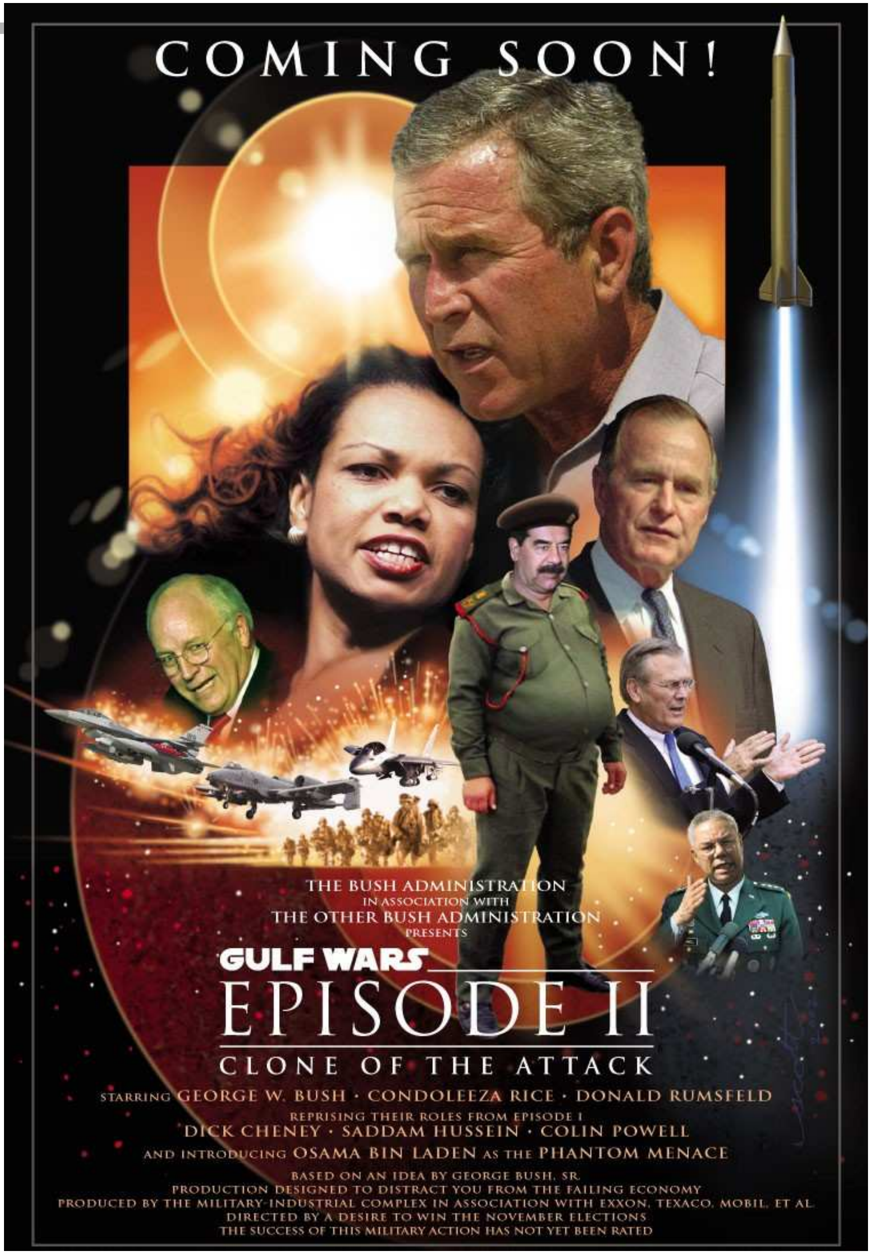
Meantime . . Other Long Range Plans

Mere Financial Cost:

- One CEBAF upgrade

  . . . per day



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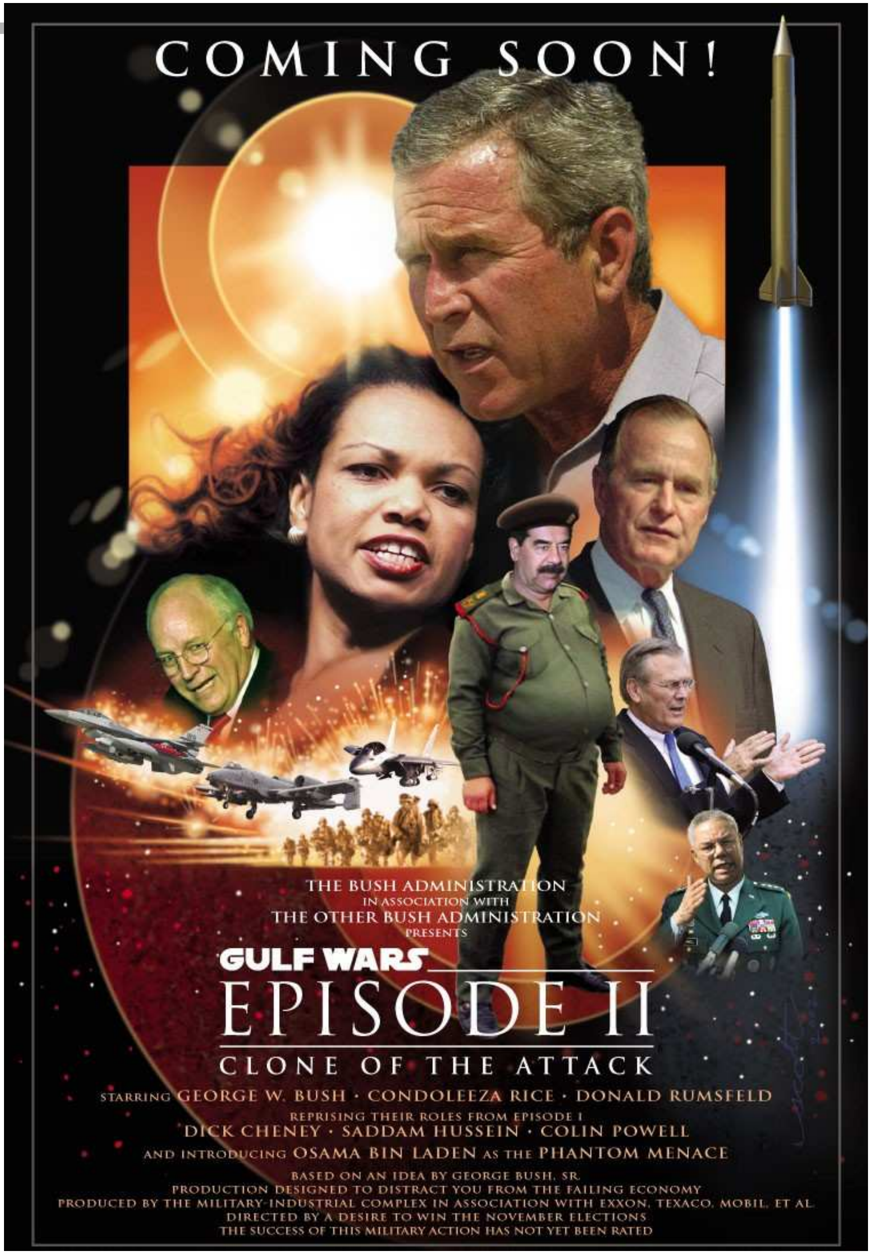
Meantime . . Other Long Range Plans

Mere Financial Cost:

- One CEBAF upgrade
  . . . per day
- One RIA
  . . . per week



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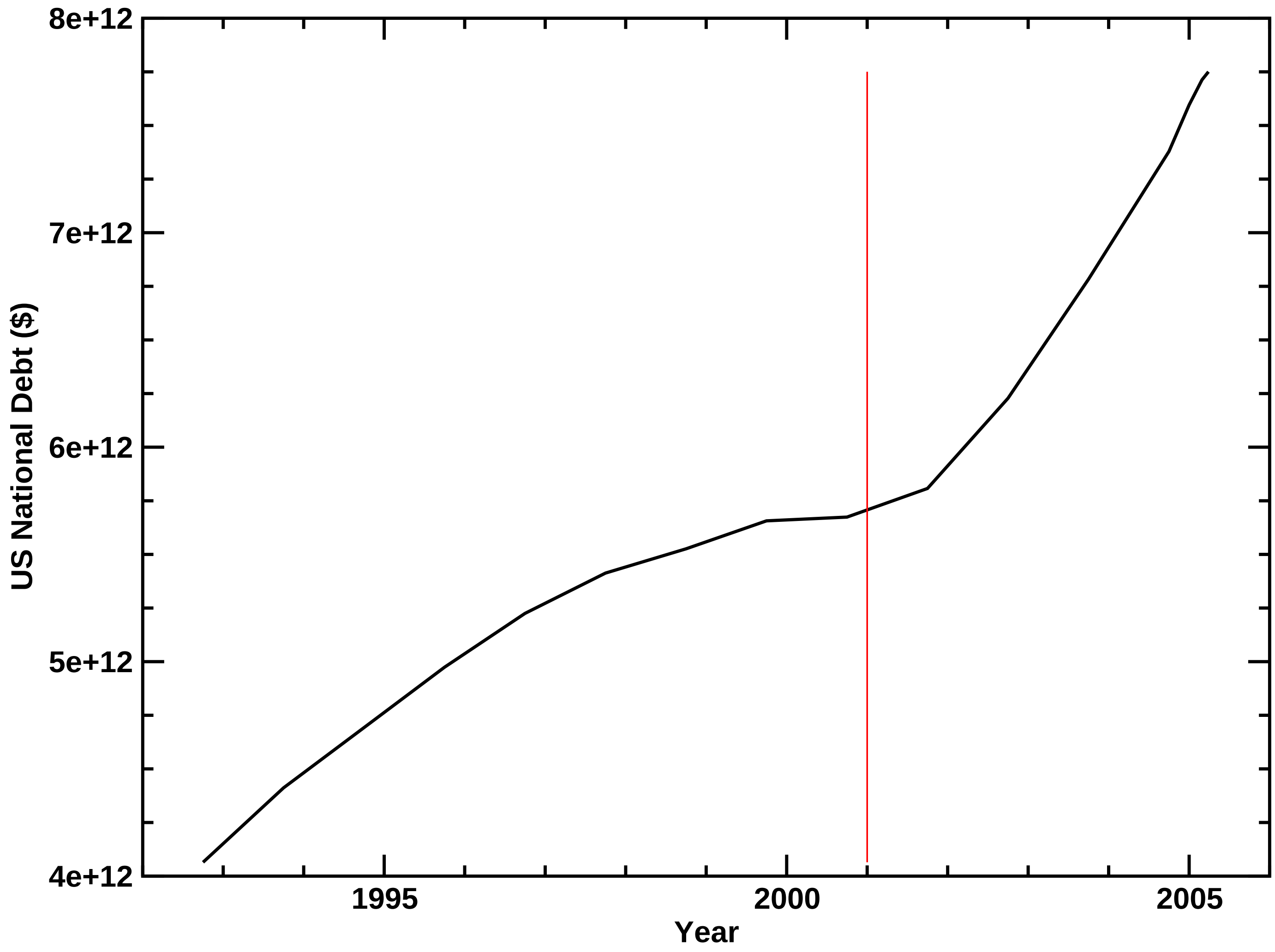
Another Long Range Plan

Another Long Range Plan



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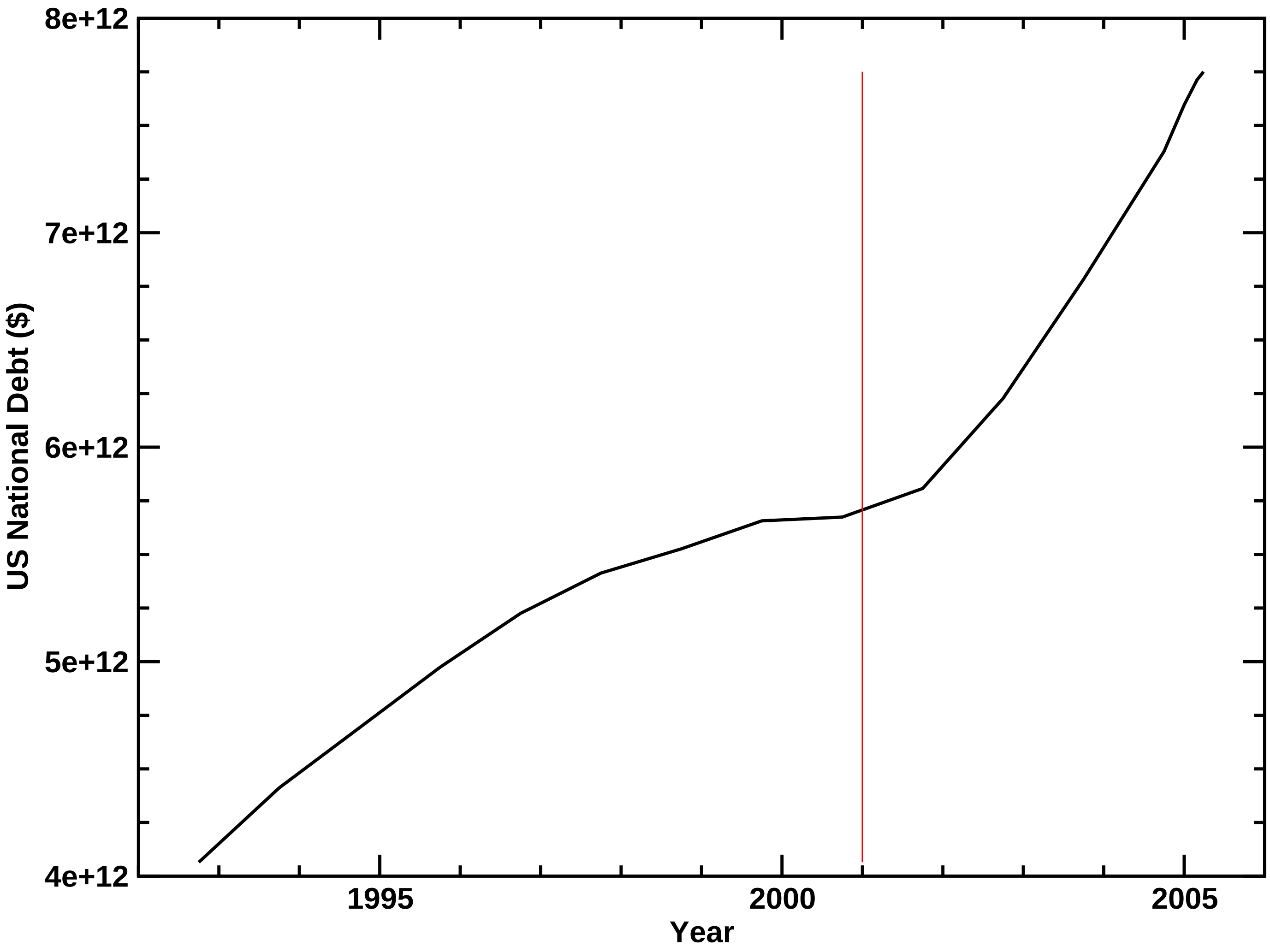
Another Long Range Plan

- US National Debt



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Another Long Range Plan

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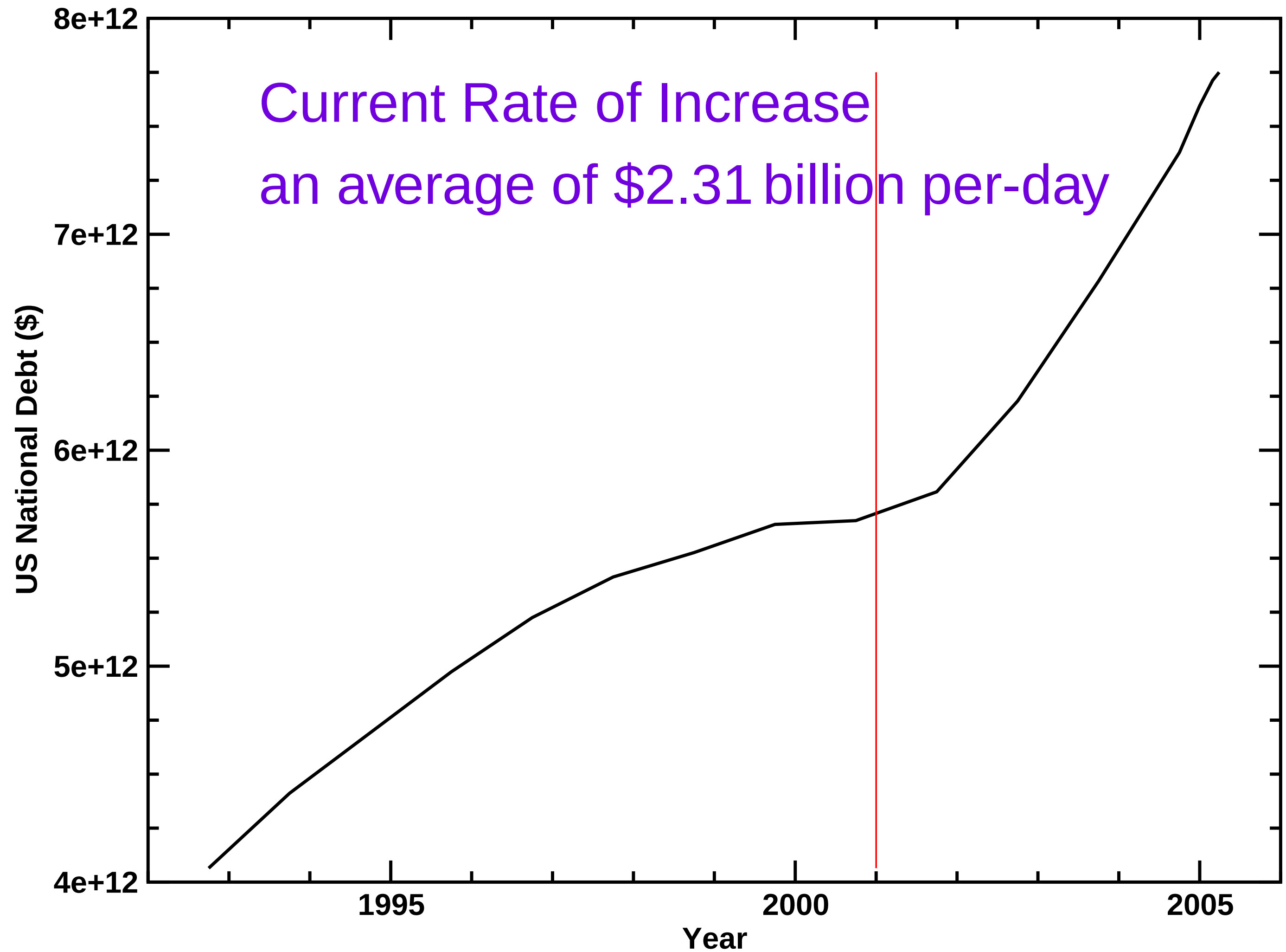

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Current Long Range Plan

Current Long Range Plan

- President's Budget was sent to Congress on Monday, 7/Feb.

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Current Long Range Plan

- President's Budget was sent to Congress on Monday, 7/Feb.
- Proposes to reduce record federal deficits without increasing taxes.

Current Long Range Plan

- President's Budget was sent to Congress on Monday, 7/Feb.
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  e.g., *Nuclear Theory*

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Current Long Range Plan

- President's Budget was sent to Congress on Monday, 7/Feb.
- Proposes to reduce record federal deficits without increasing taxes.
- Sweeping funding cuts across the board; e.g., *Nuclear Theory*
  - Universities . . . Graduate student and postdoctoral support is a major element of this program. Funding is decreased by 13.6% compared with FY 2005 resulting in ∼14% reduction in the number of Ph.D. researchers and graduate students supported in FY 2006.

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Current Long Range Plan

- President's Budget was sent to Congress on Monday, 7/Feb.
- Proposes to reduce record federal deficits without increasing taxes.
- Sweeping funding cuts across the board; e.g., *Nuclear Theory*
  - National Laboratory Research . . . Research programs supported at 7 national laboratories (ANL, BNL, LANL, LBNL, LLNL, ORNL and TJNAF). The larger size and diversity of the national laboratory groups make them particularly good sites for the training of nuclear theory postdoctoral associates. Funding for scientific/technical staff is decreased by 8.3% compared with FY 2005.

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Current Long Range Plan

- President's Budget was sent to Congress on Monday, 7/Feb.
- Proposes to reduce record federal deficits without increasing taxes.
- NSAC was reconvened to reconsider the Long Range Plan

"The FY06 President's Budget Request for Nuclear Physics is an 8.4% reduction from the FY05 Appropriations and this funding level projected into the future is not sufficient to maintain the scope of the present Nuclear Physics program. In light of these budget stringencies and their implications for the U.S. Nuclear Physics program, NSAC is being asked how to implement the highest priority science in the context of available funding and world wide capabilities. NSAC is asked to provide recommendations on priorities for an optimized DOE nuclear science program over the next five years (FY07-FY11) . . ."

- Subpanel met in Chicago on 4-6/May.
- 9/May, DNP membership informed that Panel Members may not disclose nature of discussions nor preliminary conclusions.
- Full report due on 30/June

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Nucleon Properties

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Nucleon Properties

- Maris & Tandy . . . series of five papers . . . excellent description of light pseudoscalar and vector mesons . . . basket of 31 masses/couplings/radii with r.m.s. error of 15% . . . moreover, prediction of $F_\pi(Q^2)$ measured in Hall A.

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Pieter Maris

Peter Tandy

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- One parameter model . . . parameter specifies long-range interaction between light-quarks . . . model-independent results in ultraviolet

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- One parameter model . . . parameter specifies long-range interaction between light-quarks . . . model-independent results in ultraviolet
- Next Steps . . . Applications to excited states and axial-vector mesons, e.g., will improve understanding of confinement interaction between light-quarks

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Nucleon Properties

- Another Direction . . . Also want/need information about three-quark systems

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Nucleon Properties

- Another Direction . . . Also want/need information about three-quark systems
- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.

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Nucleon Properties

- Another Direction . . . Also want/need information about three-quark systems
- With this problem . . . current expertise at approximately same point as studies of mesons in 1995.
- Namely . . . Model-building and Phenomenology, constrained by the DSE results outlined already.

Proton Form Factors, Reprise

Proton Form Factors, Reprise

- SLAC and JLab have Measured Ratio of Proton's Electric and Magnetic Form Factors

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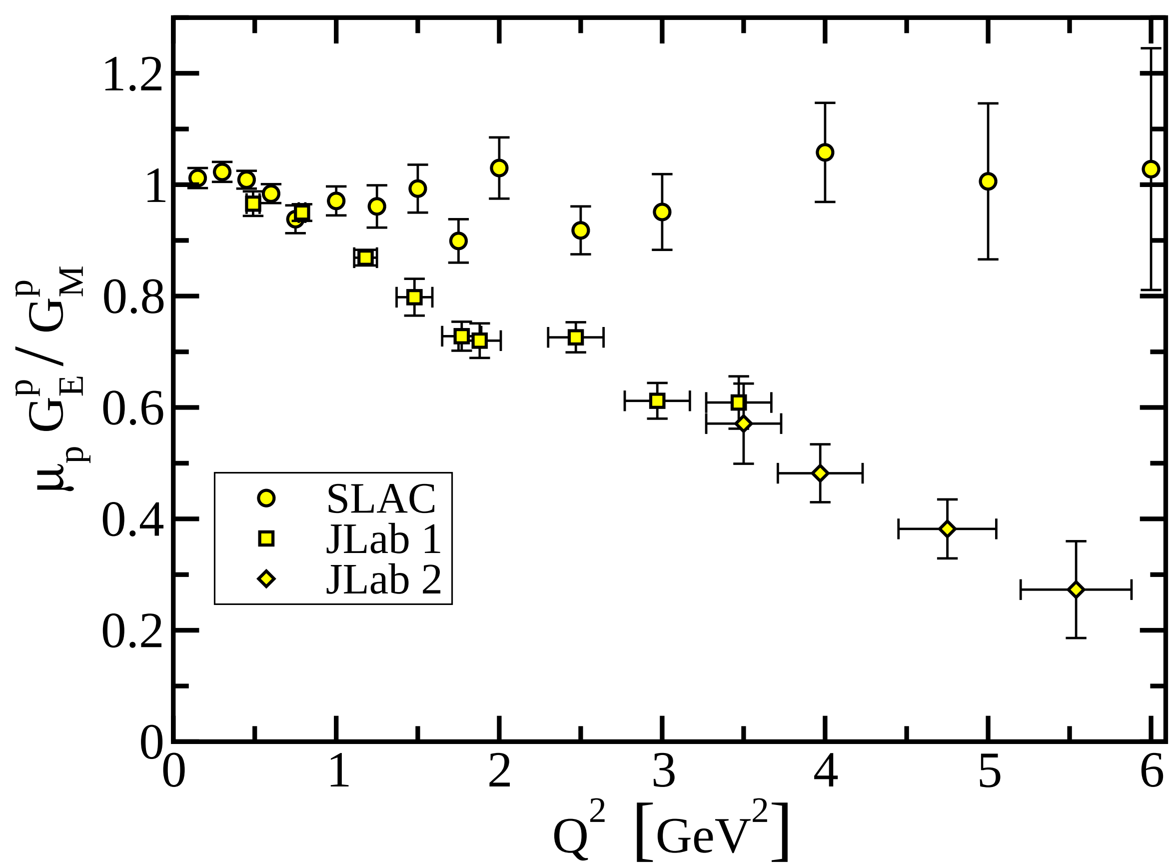
Proton Form Factors, Reprise



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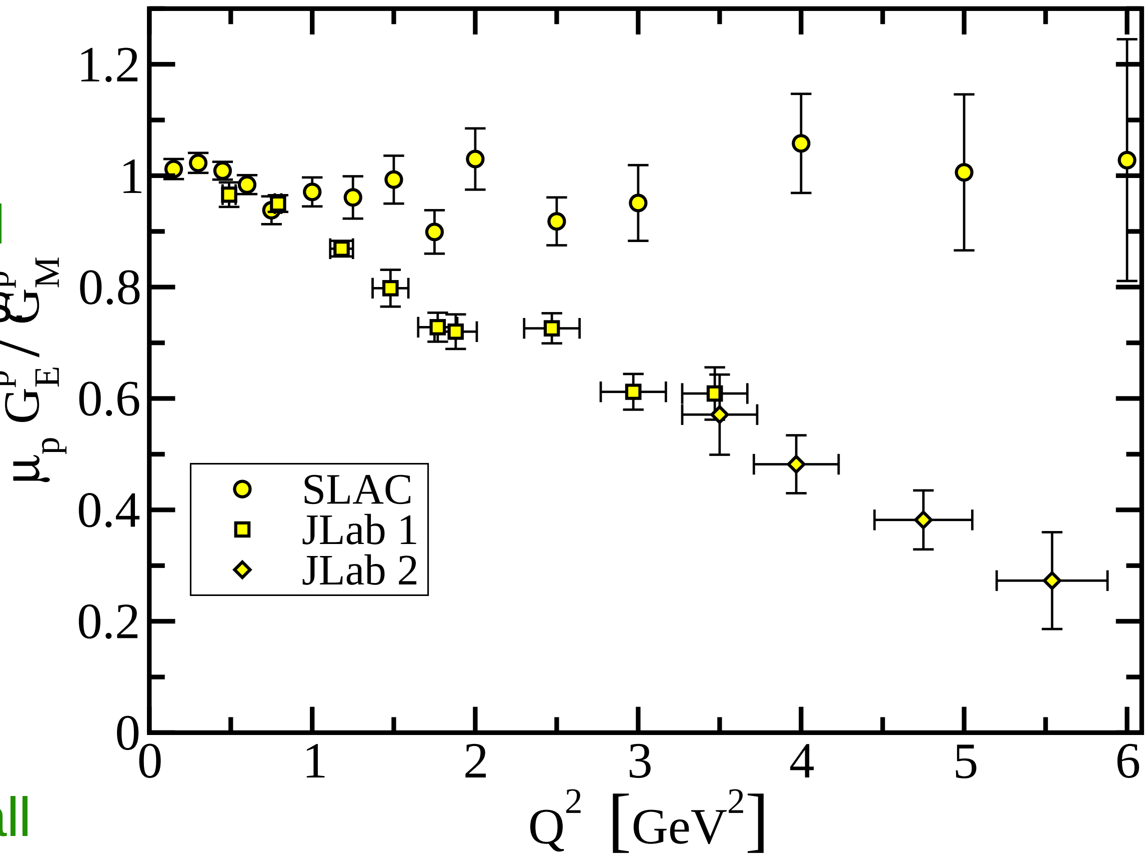
Proton Form Factors, Reprise

- Walker *et al.*, Phys. Rev. **D 49**, 5671 (1994). (SLAC)
- Jones *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **84**, 1398 (2000)
- Gayou, *et al.*, Phys. Rev. **C 64**, 038202 (2001)
- Gayou, *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **88** 092301 (2002)



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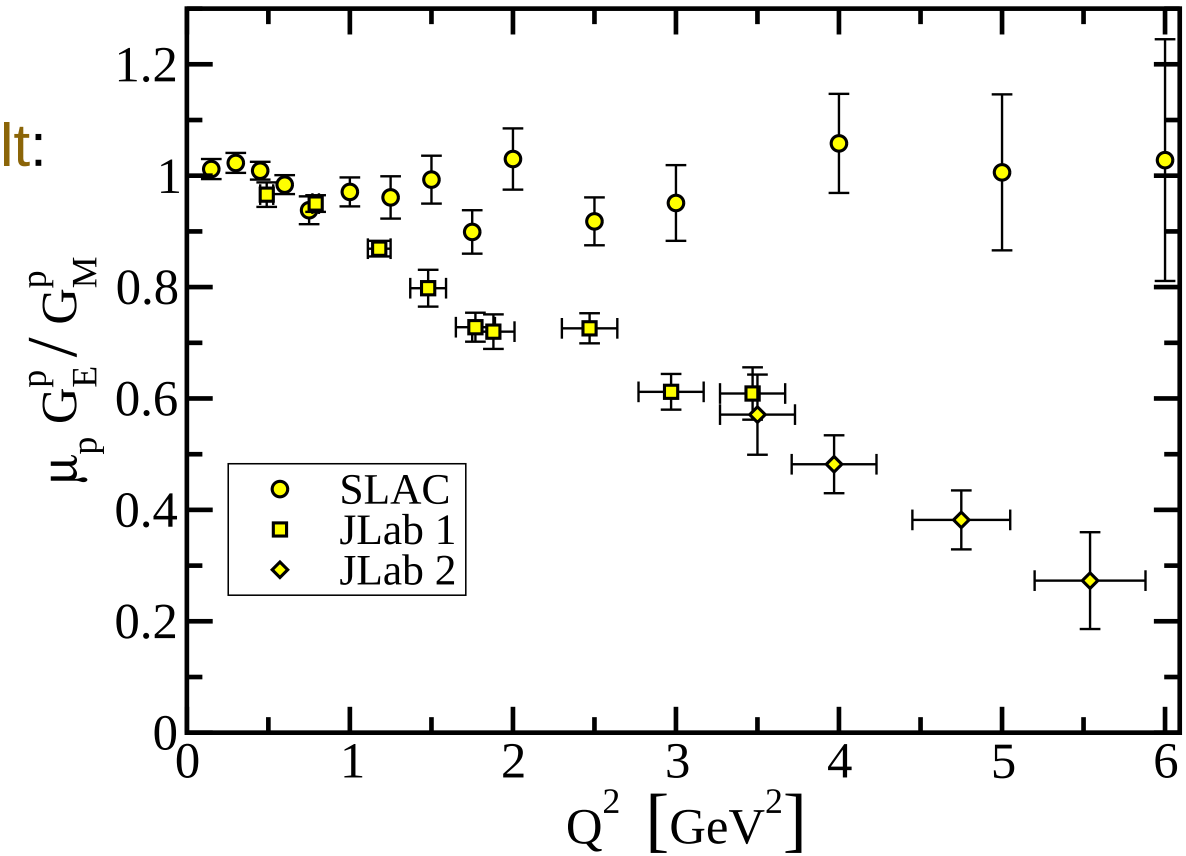
Proton Form Factors, Reprise

- If JLab Correct, then Completely Unexpected Result: In the Proton
  – On Relativistic Domain
  – Distribution of Quark-Charge Not Equal Distribution of Quark-Current!



Administration's Milestones

Basic research, too, must be results-oriented

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Administration's Milestones

Basic research, too, must be results-oriented

- 2010: Determine all nucleon electromagnetic form factors accurately to a momentum transfer of 3.5 GeV^2

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Administration's Milestones

Basic research, too, must be results-oriented

- 2010: Determine all nucleon electromagnetic form factors accurately to a momentum transfer of 3.5 GeV^2
- 2014:
  - Calculate nucleon electromagnetic form factors via Numerical Simulations of lattice-regularised full-QCD
  - Carry out ab initio microscopic studies of the structure and dynamics of light nuclei based on two-nucleon and many-nucleon forces and lattice QCD calculations of hadron interaction mechanisms relevant to the origin of the nucleon-nucleon interaction.

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Administration's Milestones

Can these goals be met subject to the Administration's other stated goal of 10% Reduction in Force (*euphemism for sackings*) and Operations?

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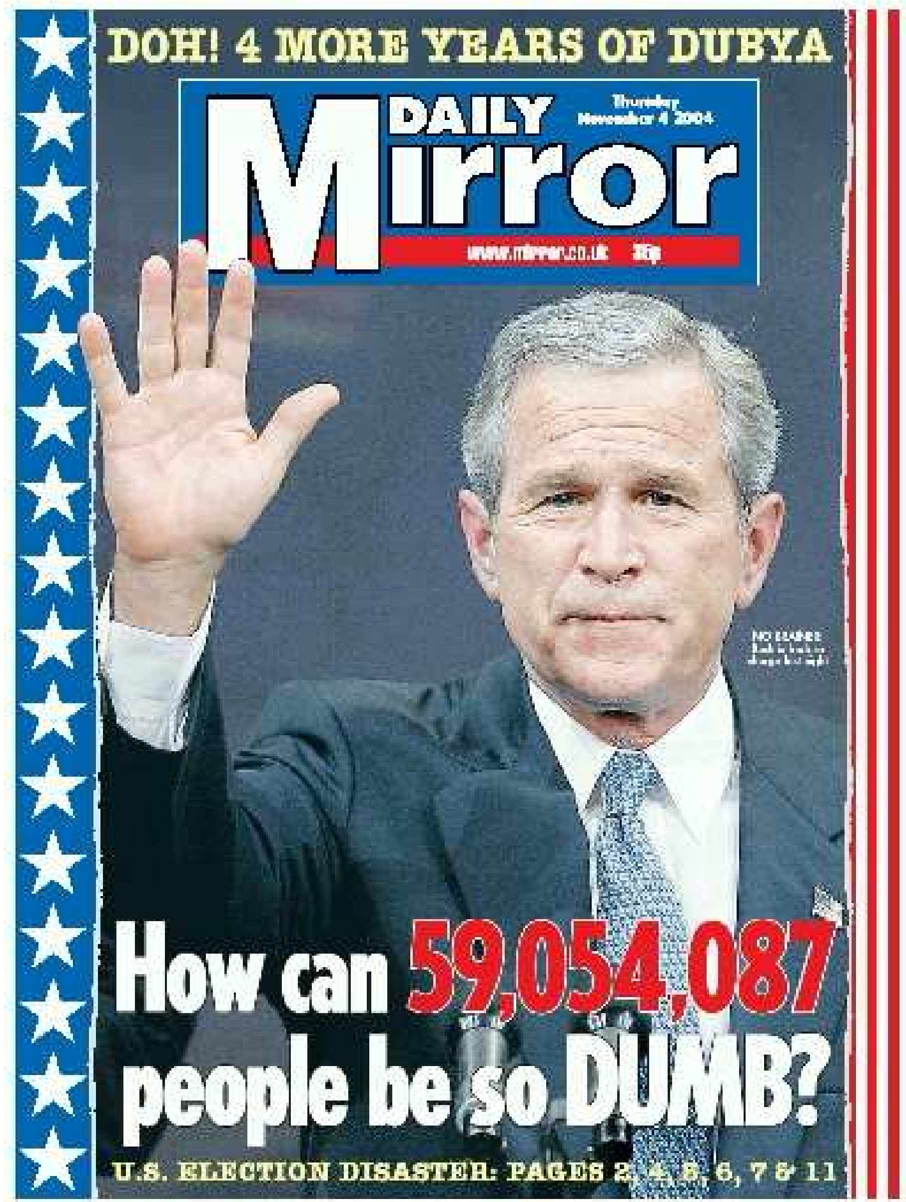
Administration's Milestones


DOH! 4 MORE YEARS OF DUBYA

DAILY Mirror

Thursday November 4 2004

www.mirror.co.uk

How can 59,054,087 people be so DUMB?

U.S. ELECTION DISASTER: PAGES 2, 4, 5, 6, 7 & 11

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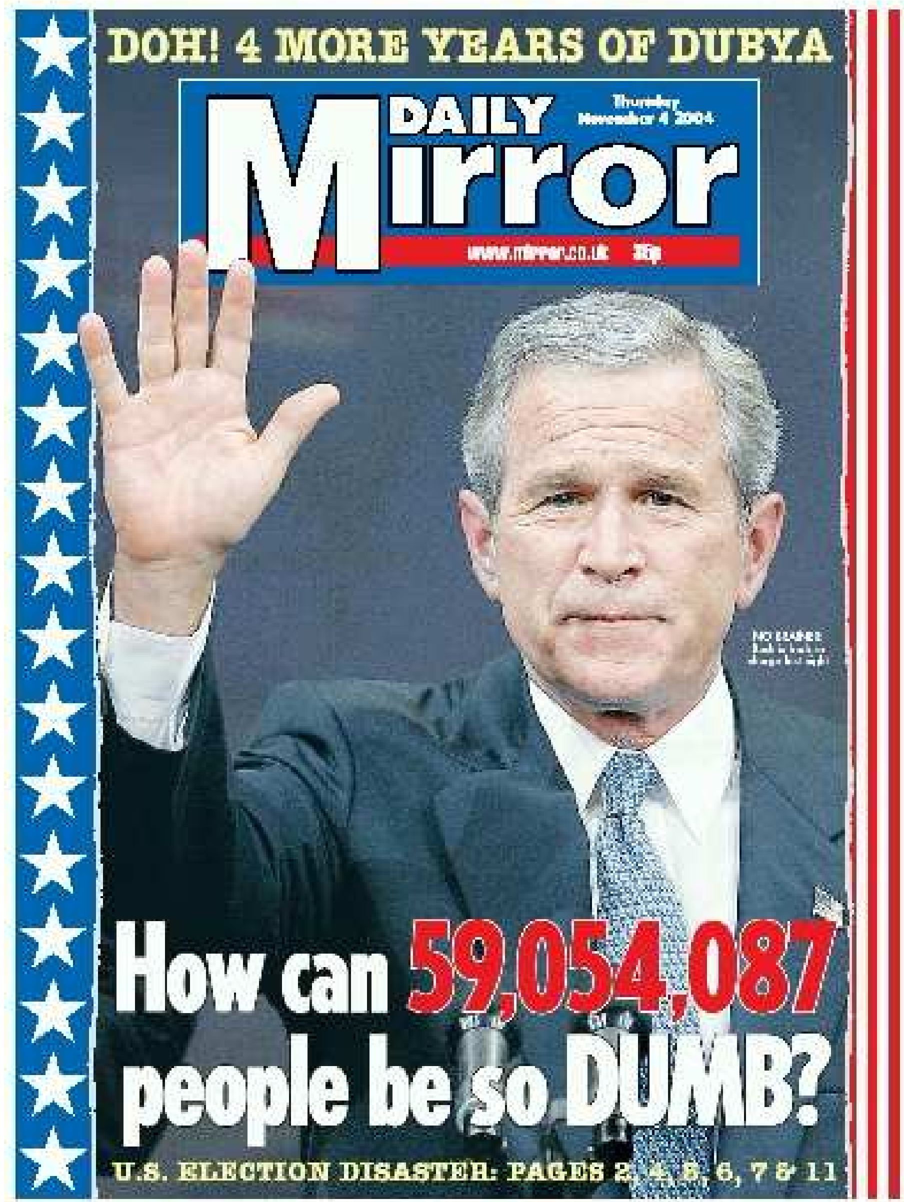
Administration's Milestones

- That, too, is a fair question



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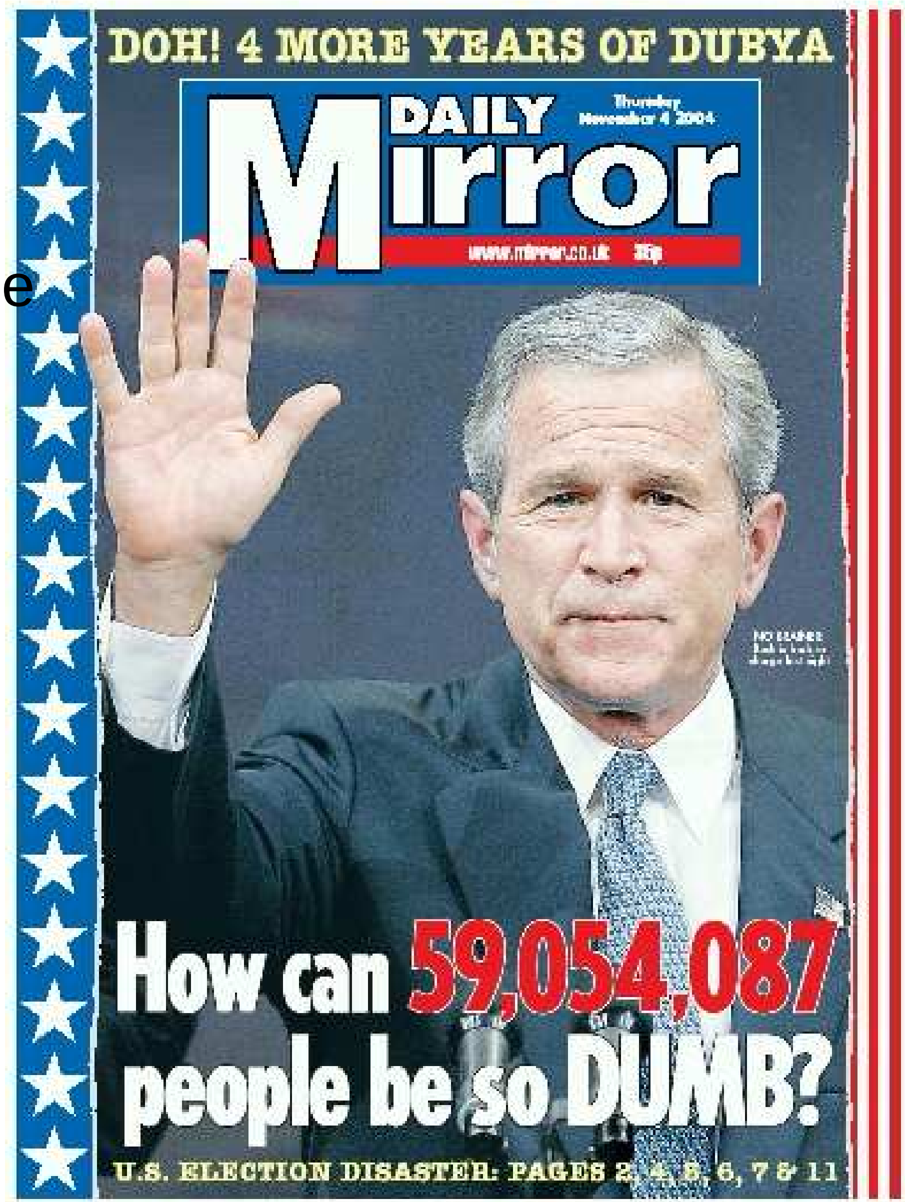
Administration's Milestones

- That, too, is a fair question
- The Community awaits the NSAC Subcommittee's report



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Nucleon EM Form Factors: A Précis

Höll, *et al.* ...nucl-th/0412046 & nucl-th/0501033

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Nucleon EM Form Factors: A Précis

Höll, *et al.* ...nucl-th/0412046 & nucl-th/0501033

- Interpreting expts. with GeV electromagnetic probes requires Poincaré covariant treatment of baryons

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Nucleon EM Form Factors: A Précis

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- Excellent mass spectrum (octet and decuplet)
  Easily obtained:

$$\left(\frac{1}{N_H} \sum_H \frac{[M_H^{\rm exp} - M_H^{\rm calc}]^2}{[M_H^{\rm exp}]^2} \right)^{1/2} = 2\%$$

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(Oettel, Hellstern, Alkofer, Reinhardt: nucl-th/9805054)

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  - Cloudy Bag: $\delta M_+^{\pi-\mathrm{loop}} = -300$ to -400 MeV!

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- But is that good?
  - Cloudy Bag: $\delta M_+^{\pi-\rm loop} = -300$ to -400 MeV!
- Critical to anticipate pion cloud effects
  Roberts, Tandy, Thomas, *et al.*, nu-th/02010084

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Covariant Faddeev equation

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Covariant Faddeev equation

- Is there a viable two-body correlation in three dressed-quark system?

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Covariant Faddeev equation

- Is there a viable two-body correlation in three dressed-quark system?
- Look at correlation lengths in diquark channels: $\ell_{qq} := \dfrac{1}{m_{qq}}$

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Covariant Faddeev equation

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$(qq)_{J^P}$	$(ud)_{0^+}$	$(us)_{0^+}$	
m_{qq} (GeV)	0.74	0.88	
$(qq)_{J^P}$	$(uu)_{1^+}$	$(us)_{1^+}$	$(ss)_{1^+}$
m_{qq} (GeV)	0.95	1.05	1.13
$(qq)_{J^P}$	$(uu)_{1^-}$	$(us)_{1^-}$	$(ss)_{1^-}$
m_{qq} (GeV)	1.47	1.53	1.64
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- octet: 1.0-1.3 GeV, decuplet: 1.2-1.7 GeV

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- octet: 1.0-1.3 GeV,
  decuplet: 1.2-1.7 GeV
- "Energy denominators:"
  0^+ and 1^+
  are dominant

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Faddeev equation

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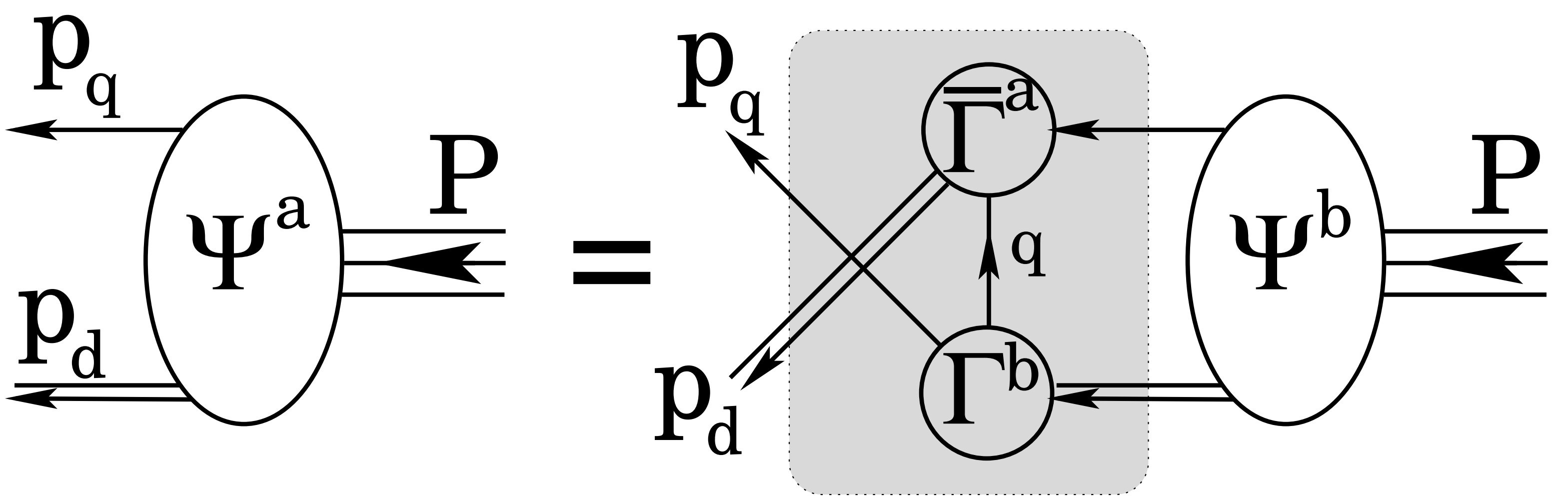
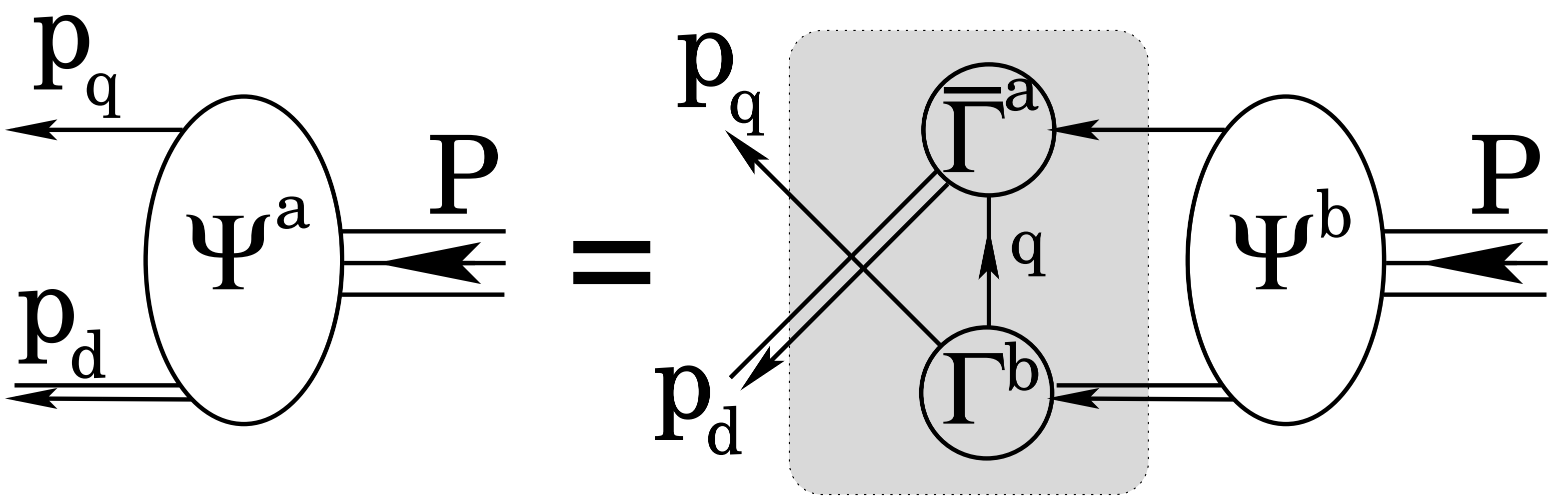
Faddeev equation



Faddeev equation



- Linear, Homogeneous Matrix equation
  - Yields *wave function* (Poincaré Covariant Faddeev Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks ... In Nucleon's Rest Frame Amplitude has ... $s-$, $p-$ & $d-$wave correlations

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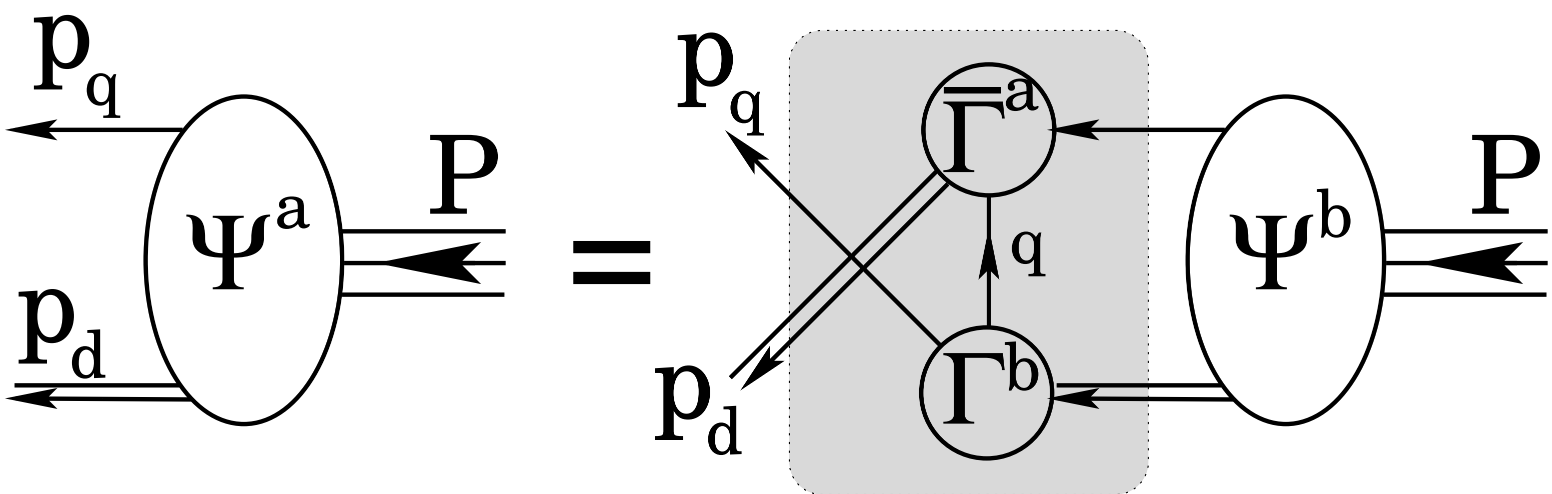
Faddeev equation



$$\begin{bmatrix} \mathcal{S}(k;P)\,u(P) \\ \mathcal{A}^i_\mu(k;P)\,u(P) \end{bmatrix} = -4\int \frac{d^4\ell}{(2\pi)^4}\,\mathcal{M}(k,\ell;P) \begin{bmatrix} \mathcal{S}(\ell;P)\,u(P) \\ \mathcal{A}^j_\nu(\ell;P)\,u(P) \end{bmatrix}$$

- $\mathcal{M}(k,\ell;P) = \begin{bmatrix} \mathcal{M}_{00} & (\mathcal{M}_{01})^j_\nu \\ (\mathcal{M}_{10})^i_\mu & (\mathcal{M}_{11})^{ij}_{\mu\nu} \end{bmatrix}$

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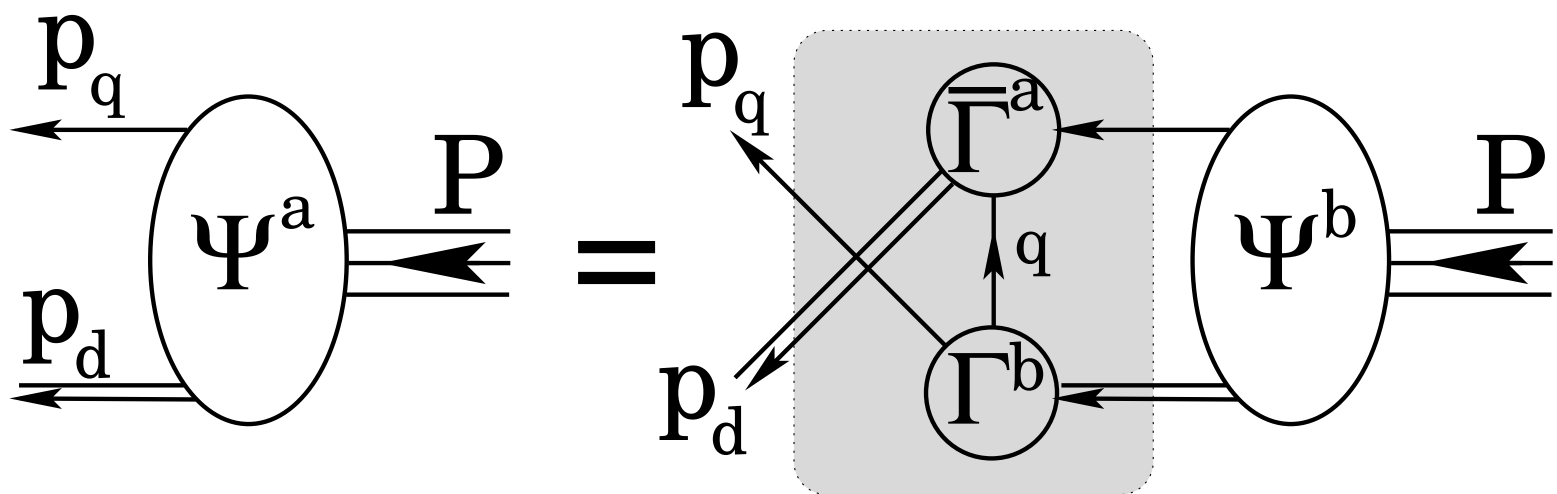
Faddeev equation



$$\mathcal{M}_{00} = \Gamma^{0^+}(k_q - \ell_{qq}/2; \ell_{qq})\, S^{\rm T}(\ell_{qq} - k_q)$$
$$\times \bar\Gamma^{0^+}(\ell_q - k_{qq}/2; -k_{qq})\, S(\ell_q)\, \Delta^{0^+}(\ell_{qq})$$

$$\ell_q = \ell + P/3,\; k_q = k + P/3,$$
$$\ell_{qq} = -\ell + 2P/3,\; k_{qq} = -k + 2P/3$$

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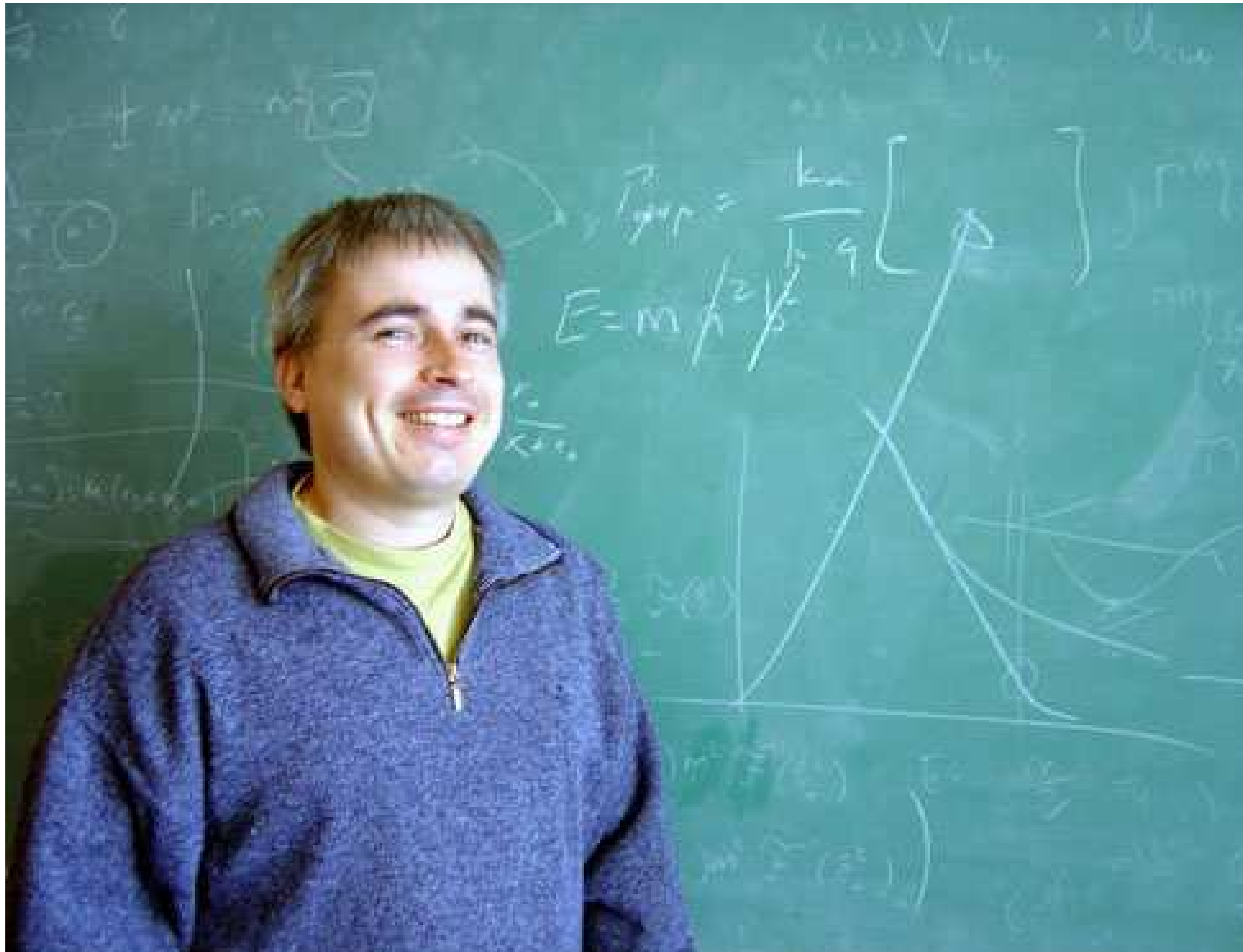

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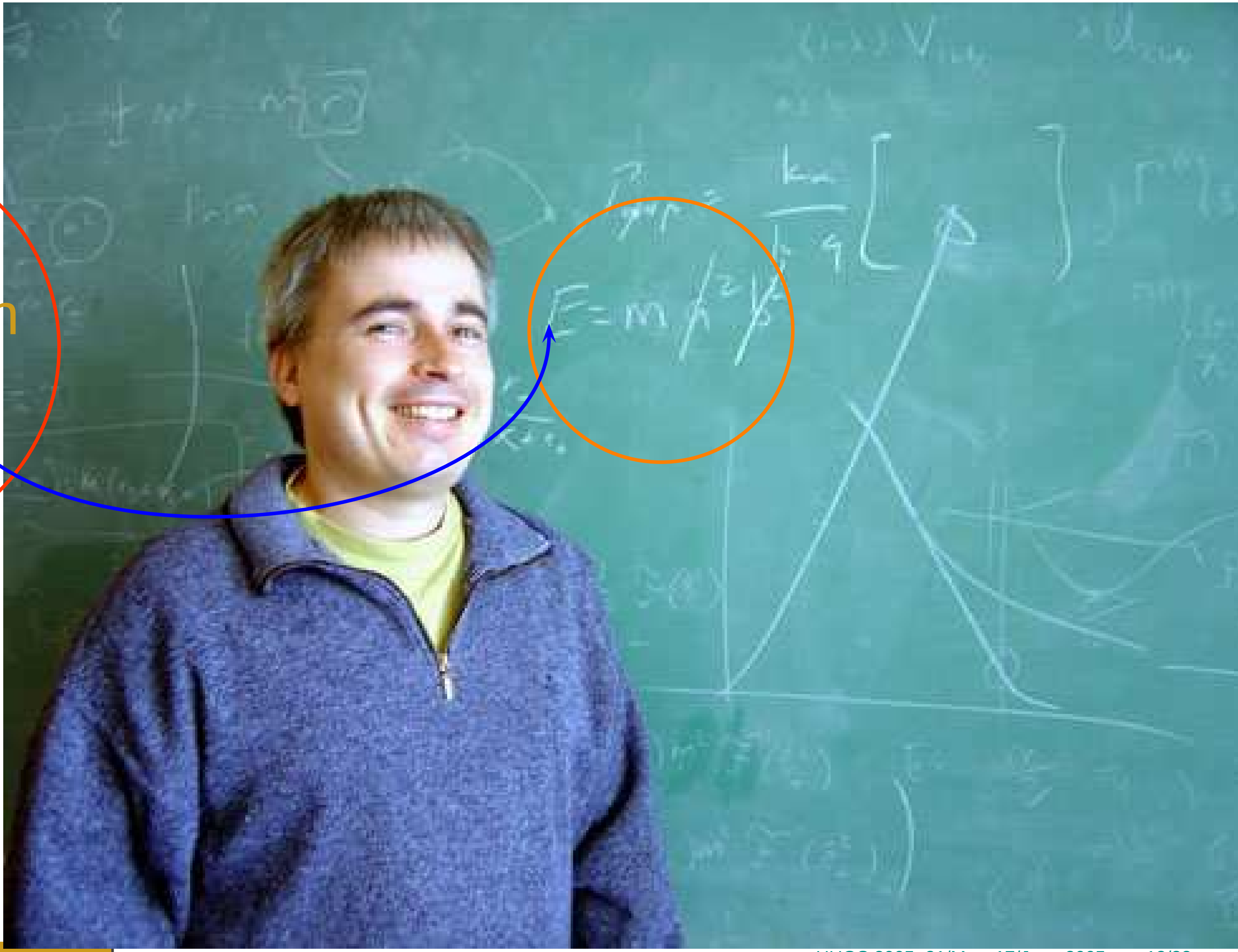

Closing in on something

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Parametrising diquark properties

Parametrising diquark properties

- Dressed-quark . . . fixed by DSE and Meson Studies

  . . . Burden, Roberts, Thomson, Phys. Lett. **B 371**, 163 (1996)

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First Contents Back Conclusion

Parametrising diquark properties

- Bethe-Salpeter-Like Amplitudes

$$\Gamma^{0^+}(k;K) = \frac{1}{\mathcal{N}^{0^+}} H^a \, C i\gamma_5 \, i\tau_2 \, \mathcal{F}(k^2/\ \omega_{0^+}^2\ ),$$

$$\mathtt{t}^i \Gamma_\mu^{1^+}(k;K) = \frac{1}{\mathcal{N}^{1^+}} H^a \, i\gamma_\mu C \, \mathtt{t}^i \, \mathcal{F}(k^2/\ \omega_{1^+}^2\ )$$

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Parametrising diquark properties

- Bethe-Salpeter-Like Amplitudes

$$\Gamma^{0^+}(k;K) = \frac{1}{\mathcal{N}^{0^+}} H^a \, C i\gamma_5 \, i\tau_2 \, \mathcal{F}(k^2/\ \omega_{0^+}^2\ ) \,,$$

$$\mathtt{t}^i \Gamma_\mu^{1^+}(k;K) = \frac{1}{\mathcal{N}^{1^+}} H^a \, i\gamma_\mu C \, \mathtt{t}^i \, \mathcal{F}(k^2/\ \omega_{1^+}^2\ )$$

- Colour matrices:

$$\{H^1 = i\lambda^7, H^2 = -i\lambda^5, H^3 = i\lambda^2\} \,,\ \epsilon_{c_1 c_2 c_3} = (H^{c_3})_{c_1 c_2}$$

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Parametrising diquark properties

- Bethe-Salpeter-Like Amplitudes

$$\Gamma^{0^+}(k;K) = \frac{1}{\mathcal{N}^{0^+}} H^a C i\gamma_5 i\tau_2 \mathcal{F}(k^2/\omega_{0^+}^2),$$

$$\mathtt{t}^i\Gamma_\mu^{1^+}(k;K) = \frac{1}{\mathcal{N}^{1^+}} H^a i\gamma_\mu C \mathtt{t}^i \mathcal{F}(k^2/\omega_{1^+}^2)$$

- Two parameters: ω_{0^+}, ω_{1^+}

  $\sim$ Inverse of diquarks' configuration-space size

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Parametrising diquark properties

- Pseudoparticle Propagators

$$\Delta^{0^+}(K) = \frac{1}{m_{0^+}^2}\,\mathcal{F}(K^2/\omega_{0^+}^2)\,,$$

$$\Delta^{1^+}_{\mu\nu}(K) = \left(\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1^+}^2}\right)\frac{1}{m_{1^+}^2}\,\mathcal{F}(K^2/\omega_{1^+}^2)$$

- $\mathcal{F}(x) = \frac{1-\exp(-x)}{x}$
  - Absence of a Spectral Representation
  - Realisation of Confinement

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Parametrising diquark properties

- Pseudoparticle Propagators

$$\Delta^{0^+}(K) = \frac{1}{m_{0^+}^2}\mathcal{F}(K^2/\omega_{0^+}^2)\,,$$

$$\Delta^{1^+}_{\mu\nu}(K) = \left(\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1^+}^2}\right)\frac{1}{m_{1^+}^2}\mathcal{F}(K^2/\omega_{1^+}^2)$$

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- Two parameters: m_{0^+}, m_{1^+}

  $\sim$ Inverse of diquarks' configuration-space correlation length

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Parametrising diquark properties

- Total of four parameters

... reduce that via Normalisation Condition

$$\frac{d}{dK^2}\left(\frac{1}{m_{JP}^2}\,\mathcal{F}(K^2/\omega_{JP}^2)\right)^{-1}\Bigg|_{K^2=0}=1\;\Rightarrow\;\omega_{JP}^2=\frac{1}{2}\,m_{JP}^2\,,$$

Accentuates free-particle-like propagation characteristics of the diquarks within hadron.

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Parametrising diquark properties

- Total of four parameters

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$$\frac{d}{dK^2}\left(\frac{1}{m_{JP}^2}\mathcal{F}(K^2/\omega_{JP}^2)\right)^{-1}\Bigg|_{K^2=0} = 1 \;\Rightarrow\; \omega_{JP}^2 = \frac{1}{2}\,m_{JP}^2\,,$$

Accentuates free-particle-like propagation characteristics of the diquarks within hadron.

- Two Parameter Faddeev Equation Model of Nucleon

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Parametrising diquark properties

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Accentuates free-particle-like propagation characteristics of the diquarks within hadron.

- Two Parameter Faddeev Equation Model of Nucleon
- Solve Faddeev Equation

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Parametrising diquark properties

- Total of four parameters

. . . reduce that via Normalisation Condition

$$\frac{d}{dK^2}\left(\frac{1}{m_{JP}^2}\mathcal{F}(K^2/\omega_{JP}^2)\right)^{-1}\Bigg|_{K^2=0} = 1 \;\Rightarrow\; \omega_{JP}^2 = \frac{1}{2}m_{JP}^2\,,$$

Accentuates free-particle-like propagation characteristics of the diquarks within hadron.

- Two Parameter Faddeev Equation Model of Nucleon
- Solve Faddeev Equation
- Vary m_{0^+} and m_{1^+} to obtain desired masses for N and Δ

Nucleon's self-energy - pion loop

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Nucleon's self-energy - pion loop

$$\begin{aligned}\Sigma(P) &= 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2(P,k)\,\Delta_\pi((P-k)^2) \\ &\quad \times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5} \\ &= i\gamma\cdot k\,[\mathcal{A}(k^2)-1] + \mathcal{B}(k^2)\end{aligned}$$

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Nucleon's self-energy - pion loop

$$\Sigma(P) = 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2(P,k)\,\Delta_\pi((P-k)^2) \times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5}$$

$$= i\gamma\cdot k\,[\mathcal{A}(k^2)-1]+\mathcal{B}(k^2)$$

- Pseudovector coupling

Nucleon's self-energy - pion loop

$$\begin{aligned}\Sigma(P) &= 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2(P,k)\,\Delta_\pi((P-k)^2) \\ &\quad \times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5} \\ &= i\gamma\cdot k\,[\mathcal{A}(k^2)-1]+\mathcal{B}(k^2)\end{aligned}$$

- Pseudovector coupling
- Completely equivalent to pseudoscalar coupling
  IF that is treated completely
  - Tadpole contribution can't be neglected

(Hecht, Oettel, Roberts, Schmidt, Tandy, Thomas: nucl-th/0201084)

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Nucleon's self-energy - pion loop

$$\begin{aligned}\Sigma(P) &= 3\int \frac{d^4k}{(2\pi)^4}\, g^2_{PV}(P,k)\, \Delta_\pi((P-k)^2) \\ &\quad \times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5} \\ &= i\gamma\cdot k\,[\mathcal{A}(k^2)-1]+\mathcal{B}(k^2)\end{aligned}$$

- $g_{PV}(P,k)$, πN vertex function
- Calculated using Γ_π and Ψ_N
  - Always soft: Monopole $\lambda \sim 0.6$ GeV

Nucleon's self-energy - pion loop

$$\Sigma(P) = 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2(P,k)\,\Delta_\pi((P-k)^2)$$
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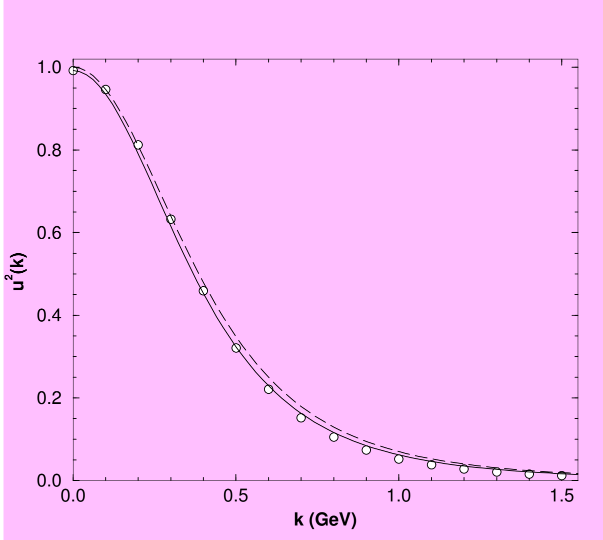
- $g_{PV}(P,k)$, πN vertex function
- Calculated using Γ_π and Ψ_N
  - Always soft: Monopole $\lambda \sim 0.6$ GeV
    - Corresponds to range $r_\lambda \sim 0.8$ fm … pion cloud does not penetrate deeply within nucleon.



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Nucleon's self-energy - pion loop

$$\begin{aligned}\Sigma(P) &= 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2(P,k)\,\Delta_\pi((P-k)^2) \\ &\quad \times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5} \\ &= i\gamma\cdot k\,[\mathcal{A}(k^2)-1]+\mathcal{B}(k^2)\end{aligned}$$

- $$\begin{aligned}G(k) &= 1/[i\gamma\cdot k + M + \Sigma(P)] \\ &= -i\gamma\cdot k\,\sigma_{\mathcal{V}}(k^2)+\sigma_{\mathcal{S}}(k^2)\end{aligned}$$
  Pole Position Not Known *a priori*
- Mass shift calculated via self-consistent solution

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Nucleon's self-energy - pion loop

$$\Sigma(P) = 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2\, (\text{const.})\, \Delta_\pi((P-k)^2)$$
$$\times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5}$$
$$= i\gamma\cdot k\,[\mathcal{A}(k^2)-1] + \mathcal{B}(k^2)$$

- Obtain Integral Equation Kernels

$$\int d\Omega_k\, f((P-k)^2) = \frac{2}{\pi}\int_{-1}^{1} dz\sqrt{1-z^2}\, f(P^2+k^2-2Pkz)$$

E.g.

$$\omega_{\mathcal{B}}(P^2,k^2) = \int d\Omega_k\, \frac{(P-k)^2}{(P-k)^2+m_\pi^2} = 1 - \frac{2\,m_\pi^2}{a+\sqrt{a^2-b^2}}\,,$$

$a = P^2 + k^2 + m_\pi^2$, b $= 2Pk$

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Nucleon's self-energy - pion loop

$$\Sigma(P) = 3\int \frac{d^4k}{(2\pi)^4}\, g_{PV}^2(P,k)\, \Delta_\pi((P-k)^2) \times \boxed{\gamma\cdot(P-k)\gamma_5}\, G(k)\, \boxed{\gamma\cdot(P-k)\gamma_5}$$

$$= i\gamma\cdot k\,[\mathcal{A}(k^2)-1] + \mathcal{B}(k^2)$$

- But $g_{PV} = g_{PV}(P^2, k^2, P\cdot k)$

  Therefore, In General, Kernel only known Numerically

- Complicates analysis . . .

  locating, incorporating poles in integrand

Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

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Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

- Let's look what happens when $m_\pi \longrightarrow 0$
  - Minkowski Space
  - Pseudovector Coupling

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Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

- Let's look what happens when $m_\pi \longrightarrow 0$
  - Minkowski Space
  - Pseudovector Coupling
- One-loop nucleon self energy

$$\Sigma(P) = 3i\frac{g^2}{4M^2}\int \frac{d^4k}{(2\pi)^4}\,\Delta(k^2, m_\pi^2)\,\not{k}\gamma_5\,G_0(P-k)\,\not{k}\gamma_5\,.$$

This integral is divergent. Assume a Poincaré covariant regularisation, characterised by a mass-scale λ

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Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

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This integral is divergent. Assume a Poincaré covariant regularisation, characterised by a mass-scale λ

- Decompose nucleon propagator into positive and negative energy components

$$\begin{aligned} G_0(P) &= \frac{1}{\not P - M_0} = G_0^+(P) + G_0^-(P) \\ &= \frac{M}{\omega_N(\vec P)}\left[\Lambda_+(\vec P)\frac{1}{P_0-\omega_N(\vec P)+i\varepsilon} + \Lambda_-(\vec P)\frac{1}{P_0+\omega_N(\vec P)-i\varepsilon}\right] \end{aligned} \quad (4)$$

$\omega_N^2(\vec P) = \vec P^2 + M^2$, and $\Lambda_\pm(\vec P) = (\tilde{\not P} \pm M)/(2M)$, $\tilde P = (\omega(\vec P), \vec P)$

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Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

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Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

- One-loop nucleon self energy

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Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

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- Shift in the mass of a positive energy nucleon nucleon:

$$\delta M_+ = \tfrac{1}{2}\mathrm{tr}_D\left[\Lambda_+(\vec{P}=0)\,\Sigma(P_0=M,\vec{P}=0)\right]$$

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Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

- One-loop nucleon self energy

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- Focus on positive energy nucleon's contribution to the loop integral; i.e., $\Delta(k)\,G^+(P-k)$, which we denote: $\delta_F M_+^+$

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Nucleon Self Energy: Chiral Limit

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- One-loop nucleon self energy

$$\Sigma(P) = 3i\frac{g^2}{4M^2}\int\frac{d^4k}{(2\pi)^4}\,\Delta(k^2,m_\pi^2)\,\not{k}\,\gamma_5\,G_0(P-k)\,\not{k}\,\gamma_5\,.$$

- Shift in the mass of a positive energy nucleon nucleon:
$$\delta M_+ = \tfrac{1}{2}\mathrm{tr}_D\left[\Lambda_+(\vec{P}=0)\,\Sigma(P_0=M,\vec{P}=0)\right]$$

- Focus on positive energy nucleon's contribution to the loop integral; i.e., $\Delta(k)\,G^+(P-k)$, which we denote: $\delta_F M_+^+$
  - To evaluate k_0 integral, close contour in lower half-plane, thereby encircling only the positive-energy pion pole.

$$\delta_F M_+^+ = -3g^2\int\frac{d^3k}{(2\pi)^3}\,\frac{\omega_N(\vec{k}^2)-M_0}{4\,\omega_N(\vec{k}^2)}\,\frac{1}{\omega_\pi(\vec{k}^2)\,[\omega_\pi(\vec{k}^2)+\omega_N(\vec{k}^2)-M_0]} \qquad (9)$$

Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

$$\delta_F M_+^+ = -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\omega_N(\vec{k}^2) - M_0}{4\,\omega_N(\vec{k}^2)} \frac{1}{\omega_\pi(\vec{k}^2)\,[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_0]} \tag{10}$$

Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

$$\delta_F M_+^+ = -3g^2 \int \frac{d^3k}{(2\pi)^3}\, \frac{\omega_N(\vec{k}^2) - M_0}{4\,\omega_N(\vec{k}^2)}\, \frac{1}{\omega_\pi(\vec{k}^2)\,[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_0]} \tag{14}$$

- On the domain for which the regularised integral has significant support, assume that M_0 is very much greater than all other mass scales.

$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \tag{15}$$

Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

$$\delta_F M_+^+ = -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\omega_N(\vec{k}^2) - M_0}{4\,\omega_N(\vec{k}^2)} \frac{1}{\omega_\pi(\vec{k}^2)\,[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_0]} \tag{18}$$

- On the domain for which the regularised integral has significant support, assume that M_0 is very much greater than all other mass scales.

$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \tag{19}$$

- Then

$$\delta_F M_+^+ \approx -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{8\,M^2} \frac{1}{\omega_{\lambda_i}^2(\vec{k}^2)} \tag{20}$$

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Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

$$\delta_F M_+^+ = -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\omega_N(\vec{k}^2) - M_0}{4\,\omega_N(\vec{k}^2)} \frac{1}{\omega_\pi(\vec{k}^2)\,[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_0]} \qquad (22)$$

- On the domain for which the regularised integral has significant support, assume that M_0 is very much greater than all other mass scales.

$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \qquad (23)$$

- Then
$$\delta_F M_+^+ \approx -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{8\,M^2} \frac{1}{\omega_{\lambda_i}^2(\vec{k}^2)} \qquad (24)$$

- So that
$$\frac{d^2\,\delta_F M_+^+}{(dm_\pi^2)^2} \approx -\frac{3g^2}{4M^2} \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{\omega_\pi^6(\vec{k}^2)} = -\frac{9}{128\pi} \frac{g^2}{M^2} \frac{1}{m_\pi}\,. \qquad (25)$$

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Nucleon Self Energy: Chiral Limit

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$$\delta_F M_+^+ = -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\omega_N(\vec{k}^2) - M_0}{4\,\omega_N(\vec{k}^2)} \frac{1}{\omega_\pi(\vec{k}^2)\,[\omega_\pi(\vec{k}^2) + \omega_N(\vec{k}^2) - M_0]} \tag{26}$$

- On the domain for which the regularised integral has significant support, assume that M_0 is very much greater than all other mass scales.

$$\omega_N(\vec{k}^2) - M \approx \frac{\vec{k}^2}{2\,M} \tag{27}$$

- Then $$\delta_F M_+^+ \approx -3g^2 \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{8\,M^2} \frac{1}{\omega_{\lambda_i}^2(\vec{k}^2)} \tag{28}$$

- So that $$\frac{d^2\,\delta_F M_+^+}{(dm_\pi^2)^2} \approx -\frac{3g^2}{4M^2} \int \frac{d^3k}{(2\pi)^3} \frac{\vec{k}^2}{\omega_\pi^6(\vec{k}^2)} = -\frac{9}{128\pi} \frac{g^2}{M^2} \frac{1}{m_\pi}\,. \tag{29}$$

- Namely $\delta_F M_+^+ = -\frac{3}{32\pi}\frac{g^2}{M^2} m_\pi^3 + f_{(1)}^+(\lambda_1, \lambda_2)\, m_\pi^2 + f_{(0)}^+(\lambda_1, \lambda_2)$ where the last two terms express the necessary contribution from the regulator.

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Nucleon Self Energy: Chiral Limit

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Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

- Nucleon's self energy

$$\delta_F M_+^+ = -\frac{3}{32\pi}\frac{g^2}{M^2} m_\pi^3 + f_{(1)}^+(\lambda_1,\lambda_2)\, m_\pi^2 + f_{(0)}^+(\lambda_1,\lambda_2)$$

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Nucleon Self Energy: Chiral Limit

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- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.

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$$\delta_F M_+^+ = -\frac{3}{32\pi}\frac{g^2}{M^2} m_\pi^3 + f_{(1)}^+(\lambda_1,\lambda_2)\, m_\pi^2 + f_{(0)}^+(\lambda_1,\lambda_2)$$

- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.
  - This is the Leading Nonanalytic Contribution much touted in effective field theory.
  - Its form is completely fixed by chiral symmetry and the pattern of its dynamical breaking.

NB. Contribution from negative energy nucleon is $\propto \frac{1}{M^3}$.

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Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

- Nucleon's self energy

$$\delta_F M_+^+ = -\frac{3}{32\pi}\frac{g^2}{M^2} m_\pi^3 + f_{(1)}^+(\lambda_1,\lambda_2)\, m_\pi^2 + f_{(0)}^+(\lambda_1,\lambda_2)$$

- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.
  - This is the Leading Nonanalytic Contribution much touted in effective field theory.
  - Its form is completely fixed by chiral symmetry and the pattern of its dynamical breaking.

  NB. Contribution from negative energy nucleon is $\propto \dfrac{1}{M^3}$.

- The remaining terms are regular in the current-quark mass. Their exact nature depends on the explicit form of regularisation procedure.

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Nucleon Self Energy: Chiral Limit

Hecht, *et al.*, nu-th/0201084

- Nucleon's self energy
  $$\delta_F M_+^+ = -\frac{3}{32\pi}\frac{g^2}{M^2} m_\pi^3 + f_{(1)}^+(\lambda_1,\lambda_2)\, m_\pi^2 + f_{(0)}^+(\lambda_1,\lambda_2)$$
- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.
- The Leading Nonanalytic Contribution is a somewhat magical model-independent result.

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Nucleon Self Energy: Chiral Limit

Hecht, *et al*., nu-th/0201084

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- Given that $m_\pi^2 \propto \hat{m}$ in the neighbourhood of the chiral limit, the m_π^3 is nonanalytic in the current-quark mass on this domain.
- The Leading Nonanalytic Contribution is a somewhat magical model-independent result.
- Unfortunately, it is not of much relevance in the real world. The actual value of the pion loop contribution to the nucleon's mass is completely determined by the regularisation dependent terms.
  - It is essential for a framework to veraciously express the leading nonanalytic contribution . . . it serves as a check that DCSB is truly described.
  - However, beyond that, one must accept that the world is messy.
    - The pion has a finite size. So does the nucleon.
    - These sizes set the mass-scale which determines the nucleon's mass shift.

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Model pion-nucleon coupling

Model pion-nucleon coupling

$$g_{PV}(P,k) = \frac{g}{2M}\, \exp(-(P-k)^2/\Lambda^2)$$

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Model pion-nucleon coupling

$$g_{PV}(P,k) = \frac{g}{2M}\ \exp(-(P-k)^2/\Lambda^2)$$

- $\mathcal{B}$-Kernel

$$\int d\Omega_k\, g_{PV}^2((P-k)^2)\ \left[1 - \frac{2m_\pi^2}{(P-k)^2 + m_\pi^2}\right]$$

Clearly the sum of two independent terms.

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- $\mathcal{B}$-Kernel

$$\int d\Omega_k\, g_{PV}^2((P-k)^2)\,\left[1 - \frac{2m_\pi^2}{(P-k)^2+m_\pi^2}\right]$$

- First term can be evaluated exactly

$$\begin{aligned}\bar{g}_{PV}^2(P^2,k^2) &= \int d\Omega_k\, g_{PV}^2((P-k)^2)\\ &= \frac{g^2}{4M^2}\,\mathrm{e}^{-2(P^2+k^2)/\Lambda^2}\,\frac{\Lambda^2}{2Pk}\,\mathrm{I}_1(4Pk/\Lambda^2)\,,\end{aligned}$$

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$$\int d\Omega_k\, g_{PV}^2((P-k)^2)\,\left[1 - \frac{2m_\pi^2}{(P-k)^2 + m_\pi^2}\right]$$

- Second term can be approximated

$$\begin{aligned}\omega_{g^2}(P^2,k^2) &= 2\,m_\pi^2 \int d\Omega_k\, \frac{g_{PV}^2((P-k)^2)}{(P-k)^2+m_\pi^2} \\ &\approx g_{PV}^2(|P-k|^2)\frac{2\,m_\pi^2}{a+\sqrt{a^2-b^2}}\end{aligned}$$

- Reliable when analytic

structure of g_{PV} is not key to that of solution

Model pion-nucleon coupling

$$g_{PV}(P,k) = \frac{g}{2M}\, \exp(-(P-k)^2/\Lambda^2)$$

- $\mathcal{B}$-Kernel

$$\int d\Omega_k \, g_{PV}^2((P-k)^2)\, \left[1 - \frac{2m_\pi^2}{(P-k)^2 + m_\pi^2}\right]$$

- Total Kernel:

$$\begin{aligned} &\approx \quad \bar{g}_{PV}^2(P^2,k^2) - g_{PV}^2(|P^2-k^2|)\frac{2\,m_\pi^2}{a+\sqrt{a^2-b^2}}\,, \\ &=: \quad \bar{g}_{PV}^2(P^2,k^2) - \tilde{g}_{PV}^2(P^2,k^2))\frac{2\,m_\pi^2}{a+\sqrt{a^2-b^2}}\,, \end{aligned}$$

- Analytic structure is transparent

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Nucleon's self energy and mass shift

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Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$M_D^2 \mathcal{A}^2(-M_D^2) = [M + \mathcal{B}(-M_D^2)]^2$$

$$\delta M = M_D - M$$

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Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$M_D^2 \mathcal{A}^2(-M_D^2) = [M + \mathcal{B}(-M_D^2)]^2$$
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- Vector self energy

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Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$
\begin{aligned}
M_D^2 \mathcal{A}^2(-M_D^2) &= [M + \mathcal{B}(-M_D^2)]^2 \\
\delta M &= M_D - M
\end{aligned}
$$

- Scalar self energy

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Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$M_D^2 \mathcal{A}^2(-M_D^2) = [M + \mathcal{B}(-M_D^2)]^2$$

$$\delta M = M_D - M$$

	(Λ, Λ_N)	(Λ, Λ_N)	(Λ, Λ_N)
	$(0.9, \infty)$	$(0.9, 1.5)$	$(0.9, 2.0)$
$-\,\delta M$ (MeV)	222	61	99

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- No suppression for nucleon off-shell in self-energy loop; i.e, $g_{PV}((P-k^2), P^2, k^2)$

Neglected this dependence

Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

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$$g_{PV}(P^2, k^2, P\cdot k) = \frac{g}{2M}\, e^{-(P-k)^2/\Lambda^2}\, e^{-(P^2+M^2+k^2+M^2)/\Lambda_N^2}$$

- Correct on-shell limit:

$$g_{PV}(P^2 = -M^2, k^2 = -M^2, (P-k)^2 = 0) = \frac{g}{2M}$$

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Nucleon's self energy and mass shift

- Solve DSE Nonperturbatively

$$M_D^2\mathcal{A}^2(-M_D^2) = [M+\mathcal{B}(-M_D^2)]^2$$
$$\delta M = M_D - M$$

Range from meson exchange model phen.

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- $\Lambda_N \to \infty \Rightarrow$ pointlike nucleon

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Pion loop's effect

- Nonpointlike πN-loop

  ... reduces nucleon's mass by ~ 100 MeV

Pion loop's effect

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  ... reduces nucleon's mass by ~ 100 MeV
- There's also a $\pi\Delta$-loop

  ... reduces nucleon's mass by not more than 100 MeV
- $$-\delta M_N \sim 200\,\text{MeV}$$
- Qualitative effect of this?

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Too much of a good thing

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Too much of a good thing

- Refit Faddeev model parameters,

  allowing for heavier "quark-core" mass

Too much of a good thing

	ω_{0^+}	ω_{1^+}	M_N	M_Δ	ω_{f_1}	ω_{f_2}	R
0^+	0.45	-	1.44	-	0.36	0.35	2.32
0^+ & 1^+	0.45	1.36	1.14	1.33	0.44	0.36	0.54
0^+	0.64	-	1.59	-	0.39	0.41	1.28
0^+ & 1^+	0.64	1.19	0.94	1.23	0.49	0.44	0.25

- 50% reduction in role of axial-vector diquark

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- 50% reduction in role of axial-vector diquark
- 10% increase in role of scalar diquark

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Unsurprisingly:

Requiring Exact Fit to N, Δ masses

with only q, $(qq)_{J^P}$ Degrees of Freedom

$\Rightarrow$ Forces 1^+ to mimic, in part, effect of π

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Results: Nucleon and Δ Masses

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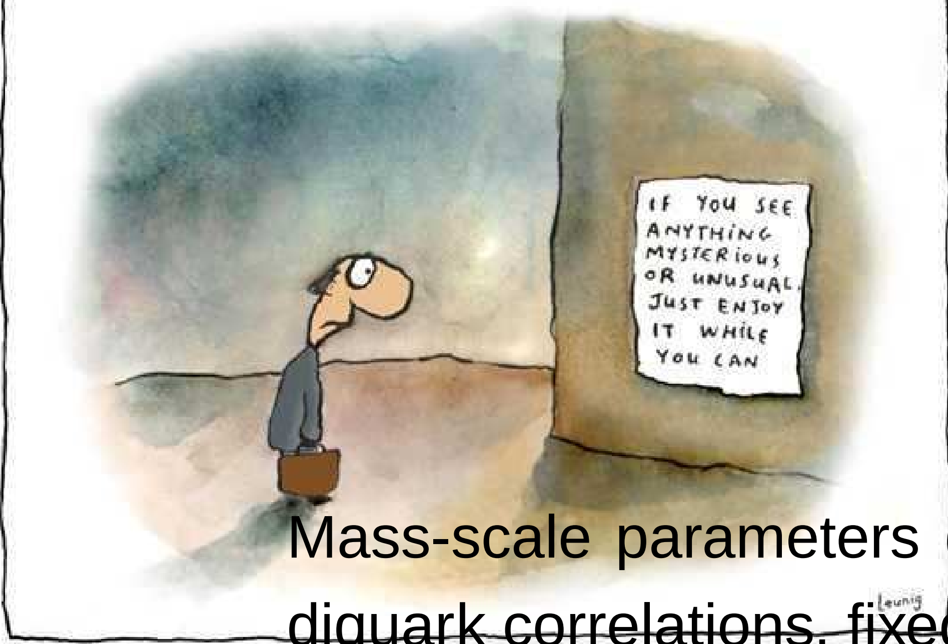
Results: Nucleon and Δ Masses



Mass-scale parameters (in GeV) for the scalar and axial-vector diquark correlations, fixed by fitting nucleon and Δ masses

Set A – fit to the actual masses was required; whereas for

Set B – fitted mass was offset to allow for "π-cloud" contributions

set	M_N	M_Δ	m_{0+}	m_{1+}	ω_{0+}	ω_{1+}
A	0.94	1.23	0.63	0.84	0.44=1/(0.45 fm)	0.59=1/(0.33 fm)
B	1.18	1.33	0.79	0.89	0.56=1/(0.35 fm)	0.63=1/(0.31 fm)

- $m_{1+} \to \infty$: $M_N^A = 1.15$ GeV; $M_N^B = 1.46$ GeV

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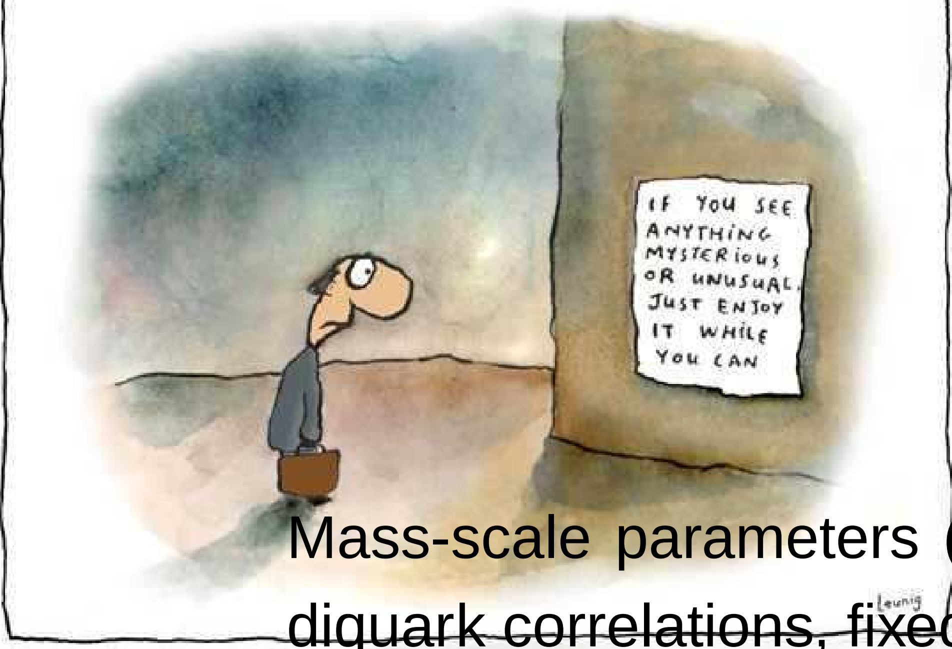

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- Axial-vector diquark provides significant attraction

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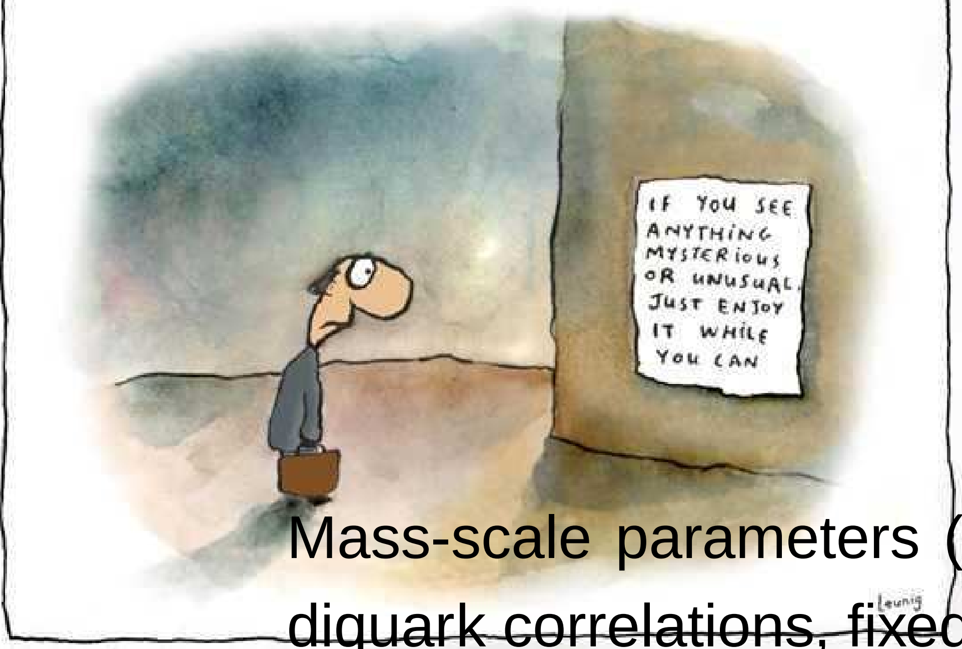

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- Constructive Interference: $\mathbf{1}^{++}$-diquark $+ \partial_\mu \pi$

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Nucleon-Photon Current

Nucleon-Photon Current

- Now have Poincaré Covariant *Wave Function* for Nucleon

Nucleon-Photon Current

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$$J_\mu(P',P) = ie\,\bar{u}(P')\,\Lambda_\mu(Q,P)\,u(P)\,,\; Q = P' - P$$

$$= \; ie\,\bar{u}(P')\left(\gamma_\mu F_1(Q^2) + \frac{1}{2M}\,\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right)u(P)$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\; G_M(Q^2) = F_1(Q^2) + F_2(Q^2)\,.$$

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- Question: . . . What is $\boldsymbol{\Lambda_\mu(Q,P)}$?

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- Question: . . . What is $\boldsymbol{\Lambda_\mu(Q,P)}$?
- Extensive Expertise in Nuclear Physics Community

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Nucleon-Photon Vertex

M. Oettel, M. Pichowsky
and L. von Smekal, nu-th/9909082

Nucleon-Photon Vertex

6 terms . . .

constructed systematically . . . current conserved automatically

for on-shell nucleons described by Faddeev Amplitude

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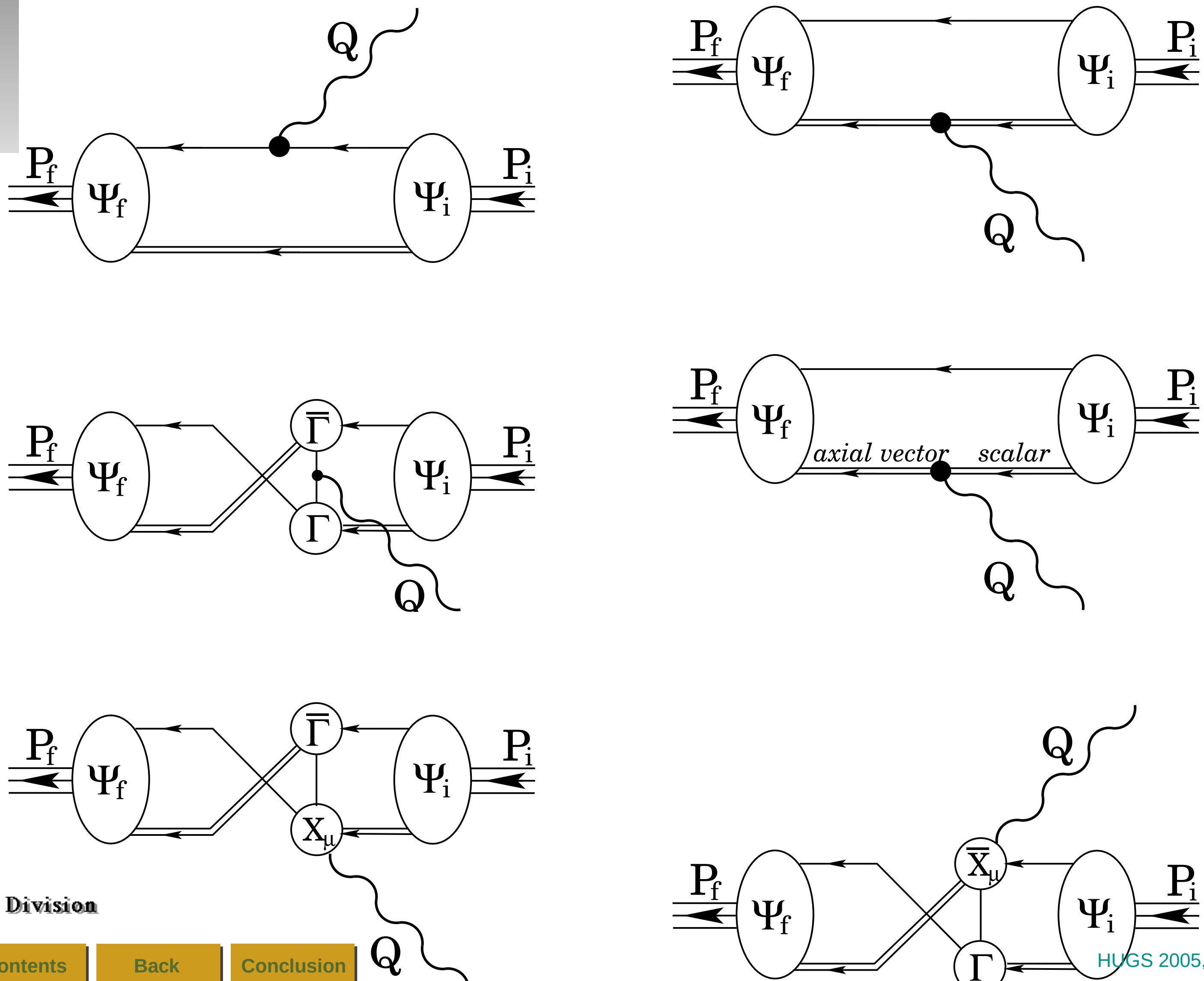

Parameters: Nucleon-Photon Vertex

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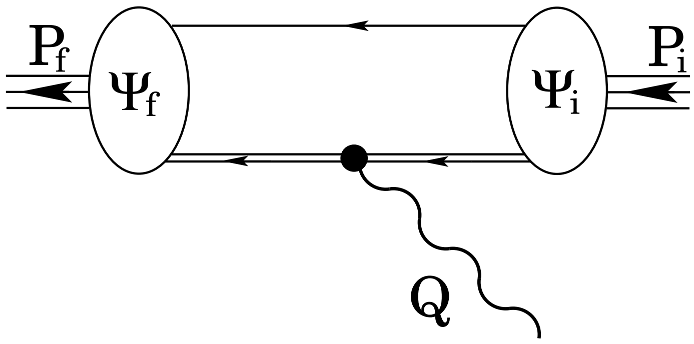
Parameters: Nucleon-Photon Vertex

- Photon–Axial-vector-diquark coupling



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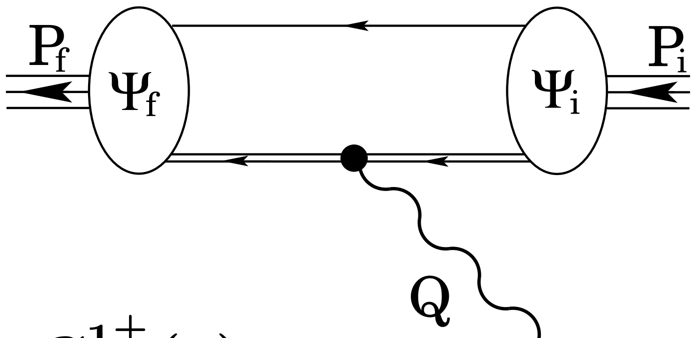

- Three Form Factors

  Two parameters . . .

  $$G^{1^+}_{\mathcal{E}}(0) = 1\,,\; G^{1^+}_{\mathcal{M}}(0) = \mu_{1^+}\,,\; G^{1^+}_{\mathcal{Q}}(0) = -\chi_{1^+}\,.$$

  Charge correctly normalised

  Magnetic (μ_{1^+}) and Quadrupole (χ_{1^+}) Moments

Parameters: Nucleon-Photon Vertex

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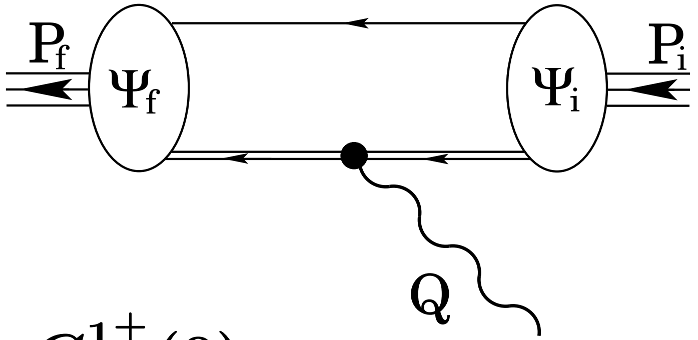

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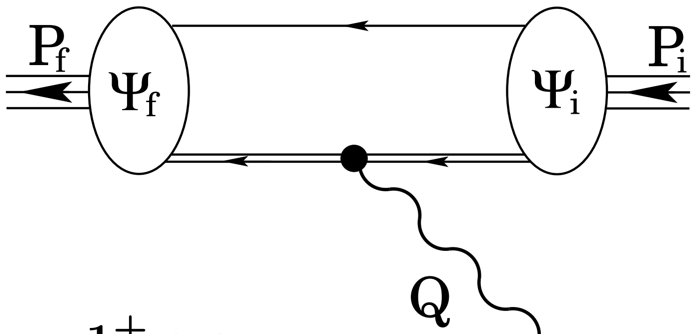

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  Charge correctly normalised

  Magnetic (μ_{1^+}) and Quadrupole (χ_{1^+}) Moments

- pointlike axial-vector: $\mu_{1^+} = 2$; and $\chi_{1^+} = 1$
- pQCD ultraviolet constraint:

$$G_{\mathcal{E}}^{1^+}(Q^2) : G_{\mathcal{M}}^{1^+}(Q^2) : G_{\mathcal{Q}}^{1^+}(Q^2) \stackrel{Q^2\to\infty}{=} (1 - \frac{2}{3}\frac{Q^2}{4m_{1^+}^2}) : 2 : -1$$

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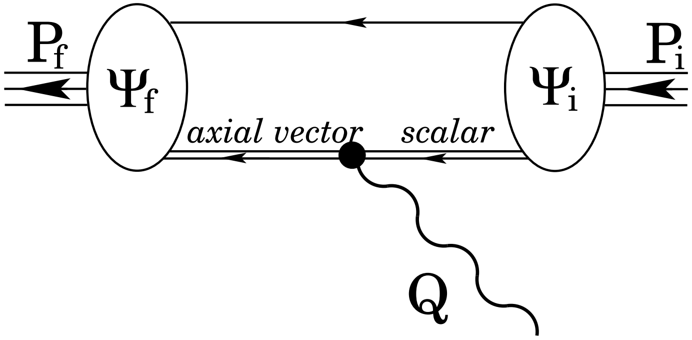
Parameters: Nucleon-Photon Vertex

- Axial-vector-diquark $\leftrightarrow$ Scalar-diquark Transition coupling



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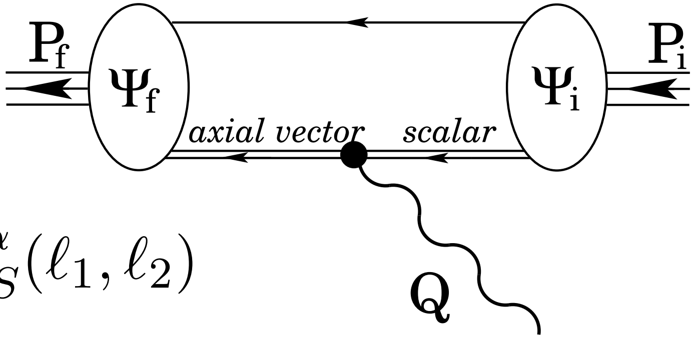

- One parameter ...

$$\Gamma^{\gamma\alpha}_{SA}(\ell_1,\ell_2) = -\Gamma^{\gamma\alpha}_{AS}(\ell_1,\ell_2) = \frac{i}{M_N}\,\kappa_{\mathcal{T}}\,\varepsilon_{\gamma\alpha\rho\lambda}\ell_{1\rho}\ell_{2\lambda}$$

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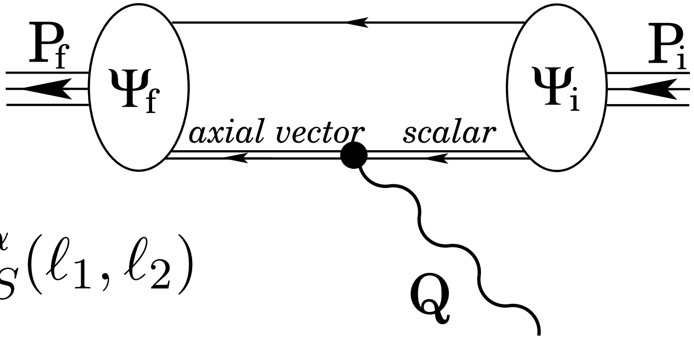

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- DSE-based calculations yield: $\kappa_{\mathcal{T}} = 2$

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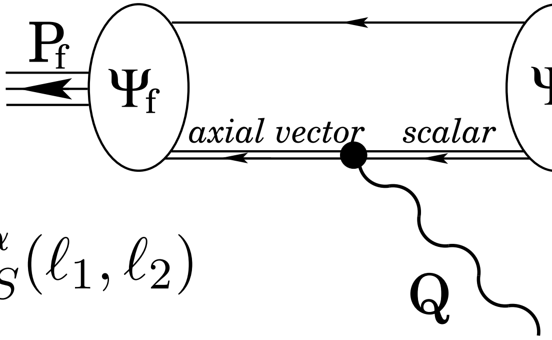

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$$\begin{aligned}\Gamma^{\gamma\alpha}_{SA}(\ell_1,\ell_2) &= -\Gamma^{\gamma\alpha}_{AS}(\ell_1,\ell_2)\\ &= \frac{i}{M_N}\,\kappa_{\mathcal{T}}\,\varepsilon_{\gamma\alpha\rho\lambda}\ell_{1\rho}\ell_{2\lambda}\end{aligned}$$

- DSE-based calculations yield: $\kappa_{\mathcal{T}} = 2$
- All other couplings fixed through experience with DSE studies of mesons
- seagull terms fixed by current-conservation

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Exegesis

Exegesis

- Dressed-quark propagators
  ... fixed in calculation of meson observables
  ... no free parameters

Exegesis

- Dressed-quark propagators

  . . . fixed in calculation of meson observables

  . . . no free parameters

- Diquark propagators and Bethe-Salpeter-like amplitudes

  . . . two parameters, fixed in fitting desired N and Δ masses

  . . . no free parameters

  . . . compare Set A cf. Set B $\left\{ \textbf{internal} \atop \textbf{structure} \right\}$

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Exegesis

- Dressed-quark propagators
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- Nucleon's electromagnetic vertex
  - . . . three free parameters
  - . . . all tied to e.m. properties of axial-vector diquark
  - . . . explore sensitivity to $\mathbf{1}^{++}$ parameters

Exegesis

- Dressed-quark propagators
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  ... compare Set A cf. Set B $\left\{ \begin{array}{c} \textbf{internal} \\ \textbf{structure} \end{array} \right\}$
- Nucleon's electromagnetic vertex — Contrast
  ... three free parameters
  ... all tied to e.m. properties of axial-vector diquark
  ... explore sensitivity to 1^{++} parameters

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Magnetic Moments

Magnetic Moments

- "On Nucleon Electromagnetic Form Factors,"
  Arne Höll, et al., nu-th/0412046, has many tables

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Magnetic Moments

- Illustrate results with magnetic moments

			Set A (no π cloud)				Set B (room for π cloud)			
μ_{1^+}	χ_{1^+}	$\kappa_{\mathcal{T}}$	κ_p	$\sigma^A_{\kappa_p}$	$\|\mu_n\|$	$\sigma^A_{\|\mu_n\|}$	κ_p	$\sigma^B_{\kappa_p}$	$\|\mu_n\|$	$\sigma^B_{\|\mu_n\|}$
1	1	2	1.79	-15.3	1.70	-5.1	2.24	-21.9	2.00	-6.2
2	1	2	2.06		1.79		2.63		2.13	
3	1	2	2.33	15.4	1.88	5.1	3.02	21.9	2.26	6.1
2	0	2	2.06	0.0	1.79	0.0	2.63	0.0	2.13	0.0
2	2	2	2.06	0.0	1.79	0.0	2.63	0.0	2.13	0.0
2	1	1	1.91	-8.4	1.64	-8.4	2.45	-10.1	1.95	-8.5
2	1	3	2.21	8.4	1.85	8.3	2.82	10.1	2.31	8.5

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Form Factors

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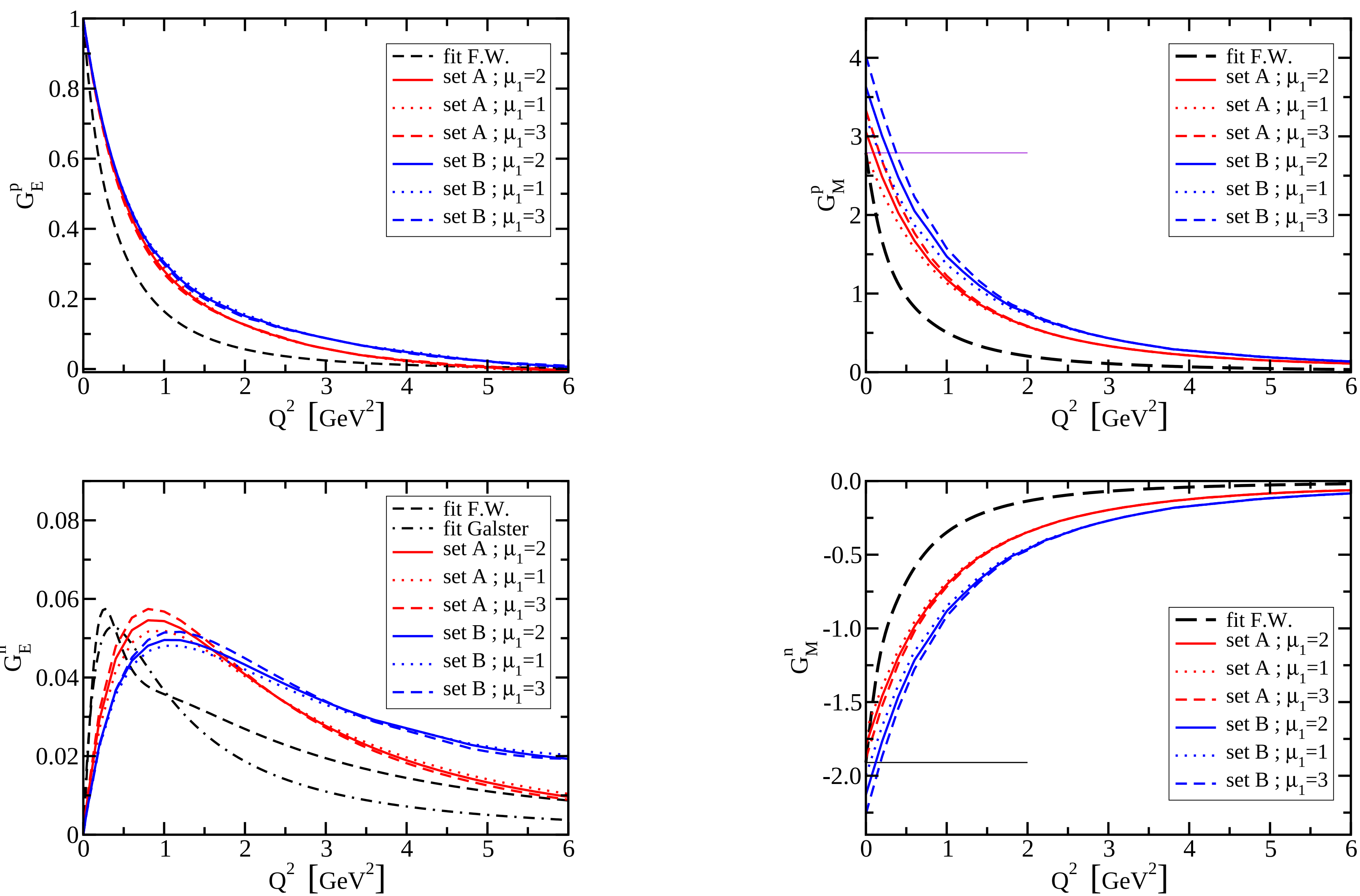
Comparison

Form Factors



Variations in $\mu_{1+} = 1, 2, 3$; $\chi_{1+} = 1$, $\kappa_{\mathcal{T}} = 2$

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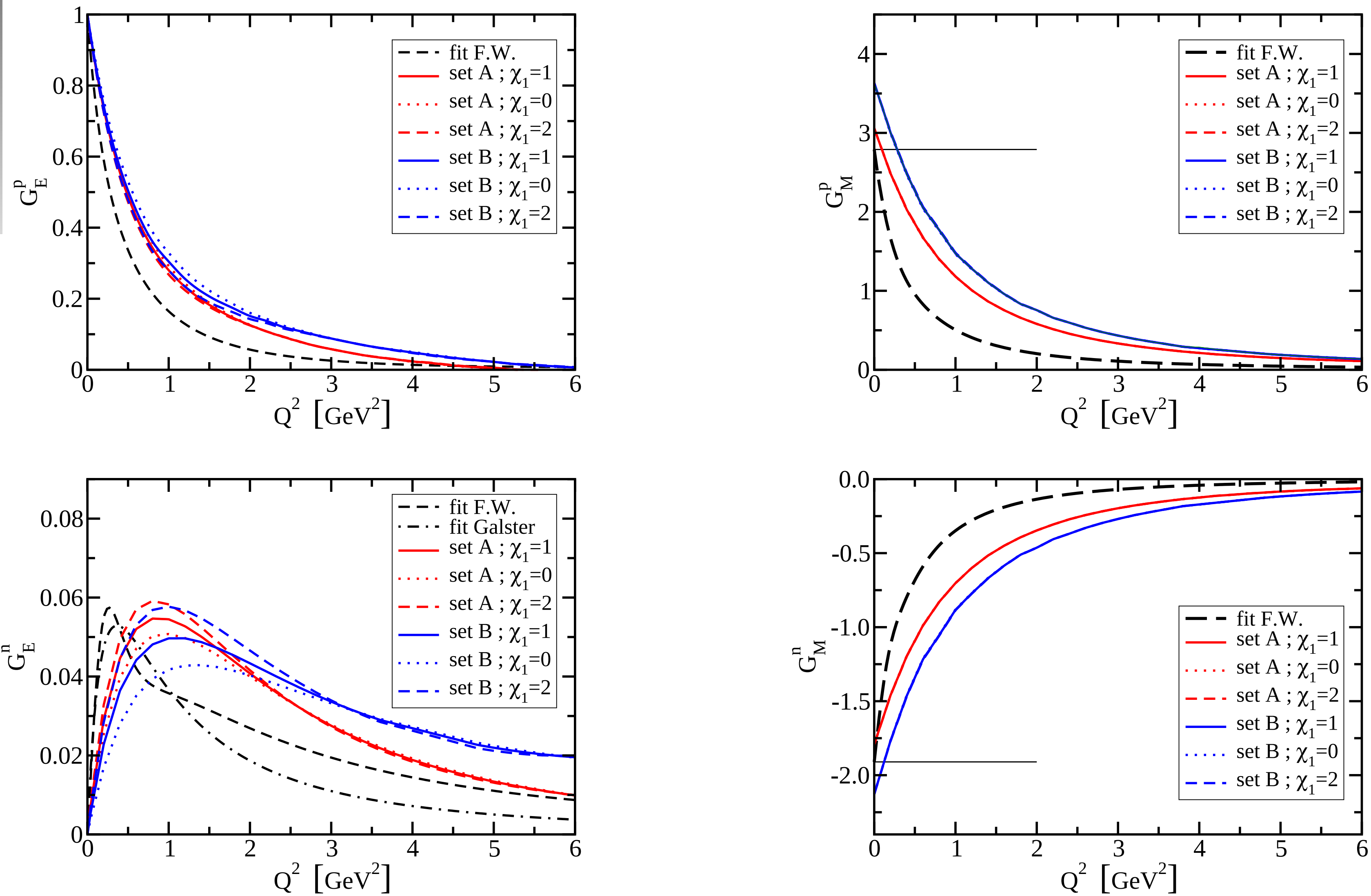
Comparison

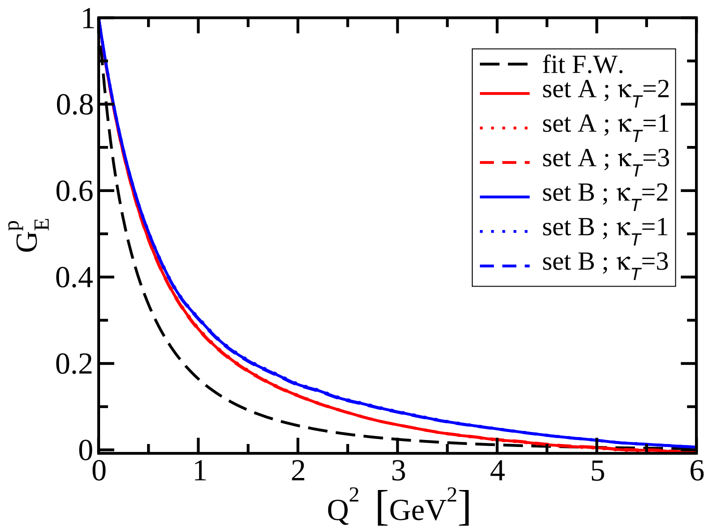
Form Factors

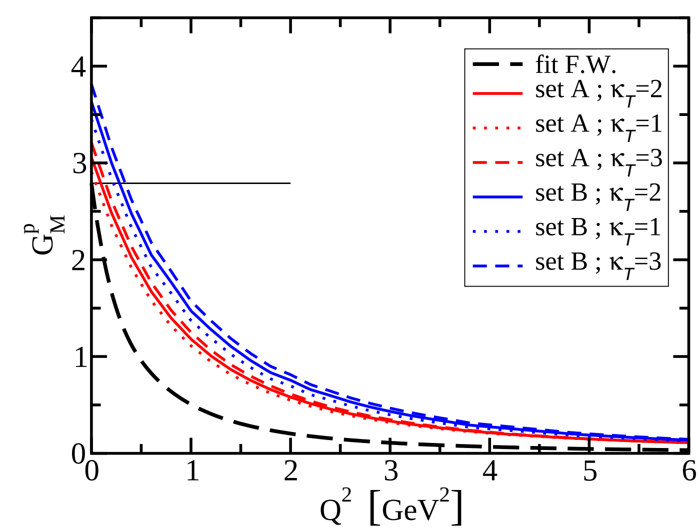


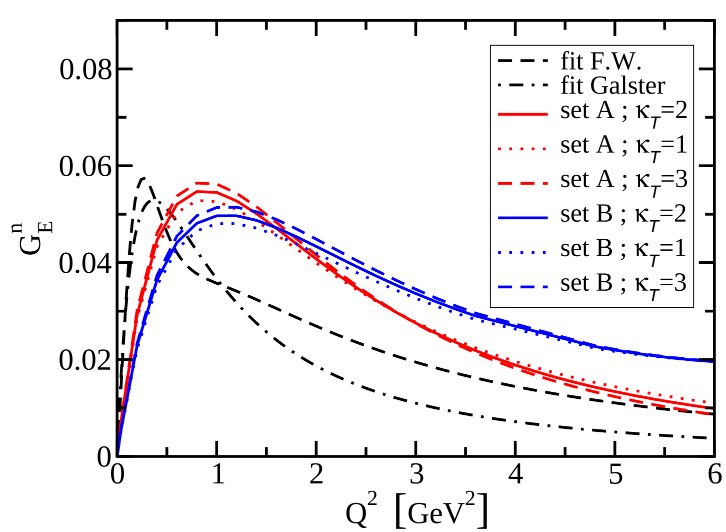
Variations in $\chi_{1+} = 0, 1, 2$; $\mu_{1+} = 2$, $\kappa_{\mathcal{T}} = 2$

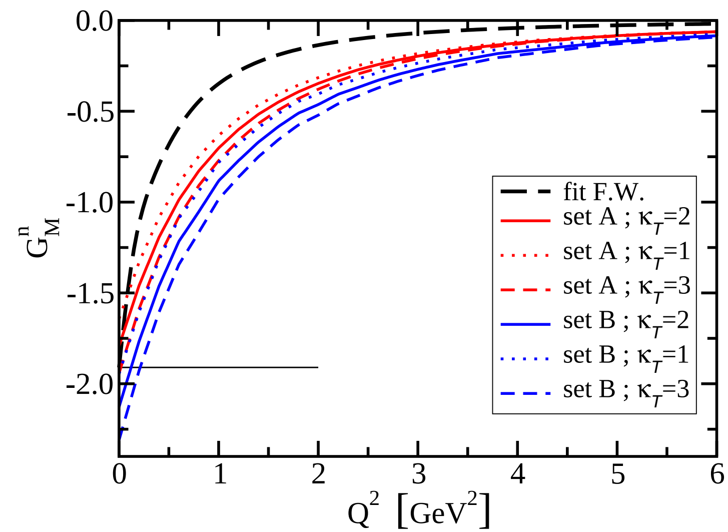
Comparison

Form Factors









Variations in $\kappa_{\mathcal{T}} = 1, 2, 3$; $\mu_{1+} = 2$, $\chi_{1+} = 1$

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Form Factors

- Behaviour of Form Factors is primarily determined by information expressed in Poincaré Covariant Faddeev Amplitudes
  - Differences between Set A and Set B outweigh dependence on electromagnetic parameters of axial-vector diquark
  - No sensitivity to diquark parameters for $Q^2 \gtrsim 4$ GeV^2 ... construction respects pQCD limit

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Form Factors

- Behaviour of Form Factors is primarily determined by information expressed in Poincaré Covariant Faddeev Amplitudes
  - Differences between Set A and Set B outweigh dependence on electromagnetic parameters of axial-vector diquark
  - No sensitivity to diquark parameters for $Q^2 \gtrsim 4\,\text{GeV}^2$ . . . construction respects pQCD limit
- Cannot readily tune model to uniformly good account of nucleon electromagnetic properties
  - Something more than dressed-quark and -diquark degrees of freedom is required

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Chiral Corrections

Chiral Corrections

- Thus far, omitted pion cloud contribution to current

Chiral Corrections

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- Include loops following method of
  ... Ashley, Leinweber, Thomas, Young, he-lat/0308024
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$$\langle r^2_{\substack{p\\n}} \rangle^{1-loop^R}_{NA} = \mp \frac{1+5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2+\lambda^2}\right),$$

$$\langle (r^\mu_N)^2 \rangle^{1-loop^R}_{NA} = -\frac{1+5g_A^2}{32\pi^2 f_\pi^2} \ln\left(\frac{m_\pi^2}{m_\pi^2+\lambda^2}\right) + \frac{g_A^2 M_N}{16\pi f_\pi^2 \mu_v} \frac{1}{m_\pi} \frac{2}{\pi} \arctan\left(\frac{\lambda}{m_\pi}\right),$$

$$(\mu_{\substack{p\\n}})^{1-loop^R}_{NA} = \mp \frac{g_A^2 M_N}{4\pi^2 f_\pi^2} m_\pi \frac{2}{\pi} \arctan\left(\frac{\lambda^3}{m_\pi^3}\right),$$

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  ... $\frac{1}{\lambda} = \frac{2}{3}$ fm

	r_p	r_n	r_p^μ	r_n^μ	μ_p	$-\mu_n$	ς
q-(qq) core	0.595	0.169	0.449	0.449	3.63	2.13	0.39
$+\pi$-loop correction	0.762	0.506	0.761	0.761	3.05	1.55	0.23
experiment	0.847	0.336	0.836	0.889	2.79	1.91	

Chiral Corrections

- Thus far, omitted pion cloud contribution to current
- Include loops following method of
  ... Ashley, Leinweber, Thomas, Young, he-lat/0308024
  ... finite-range regularisation of loop corrections
  ... $\frac{1}{\lambda} = \frac{2}{3}$ fm
- Complements nucleon mass considerations
  ... veracious understanding of all nucleon properties
  ... impossible without intelligent incorporation
  of chiral corrections

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Form Factor Ratio: GE/GM

- Walker *et al.* Phys. Rev. **D 49**, 5671 (1994).
- Jones *et al.*
  JLab Hall A Collaboration
  Phys. Rev. Lett. **84**, 1398 (2000)
- Gayou, *et al.*
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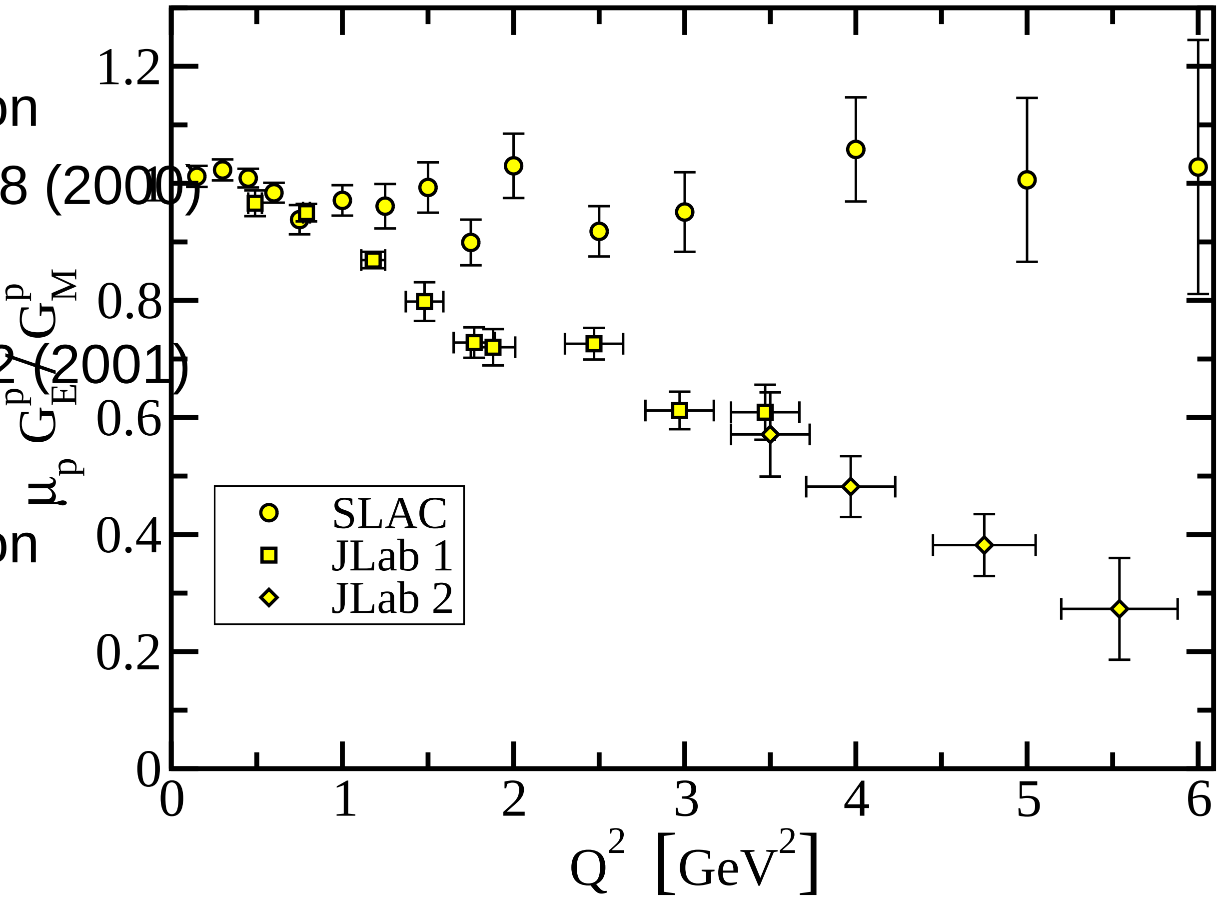

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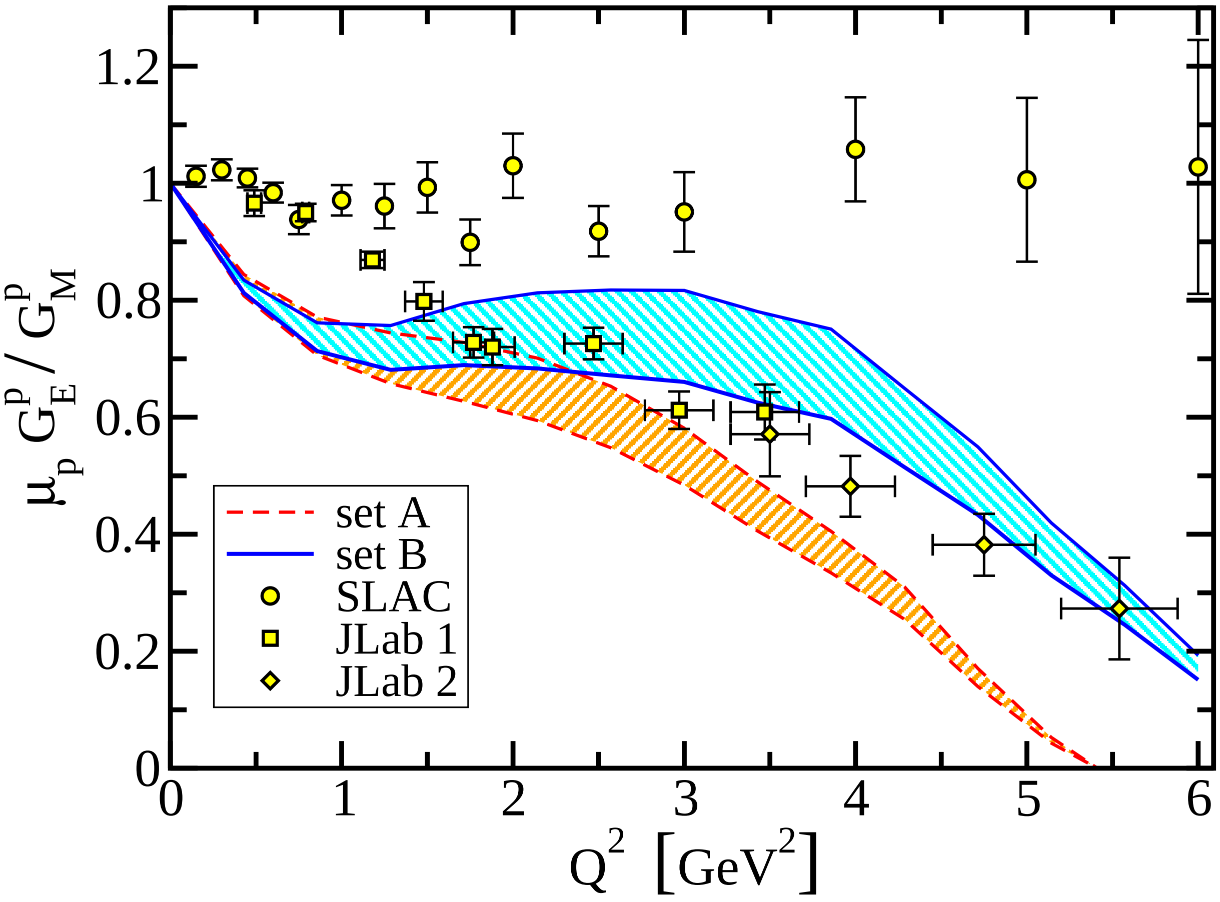
Form Factor Ratio: GE/GM

- Small Q^2



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Form Factor Ratio: GE/GM

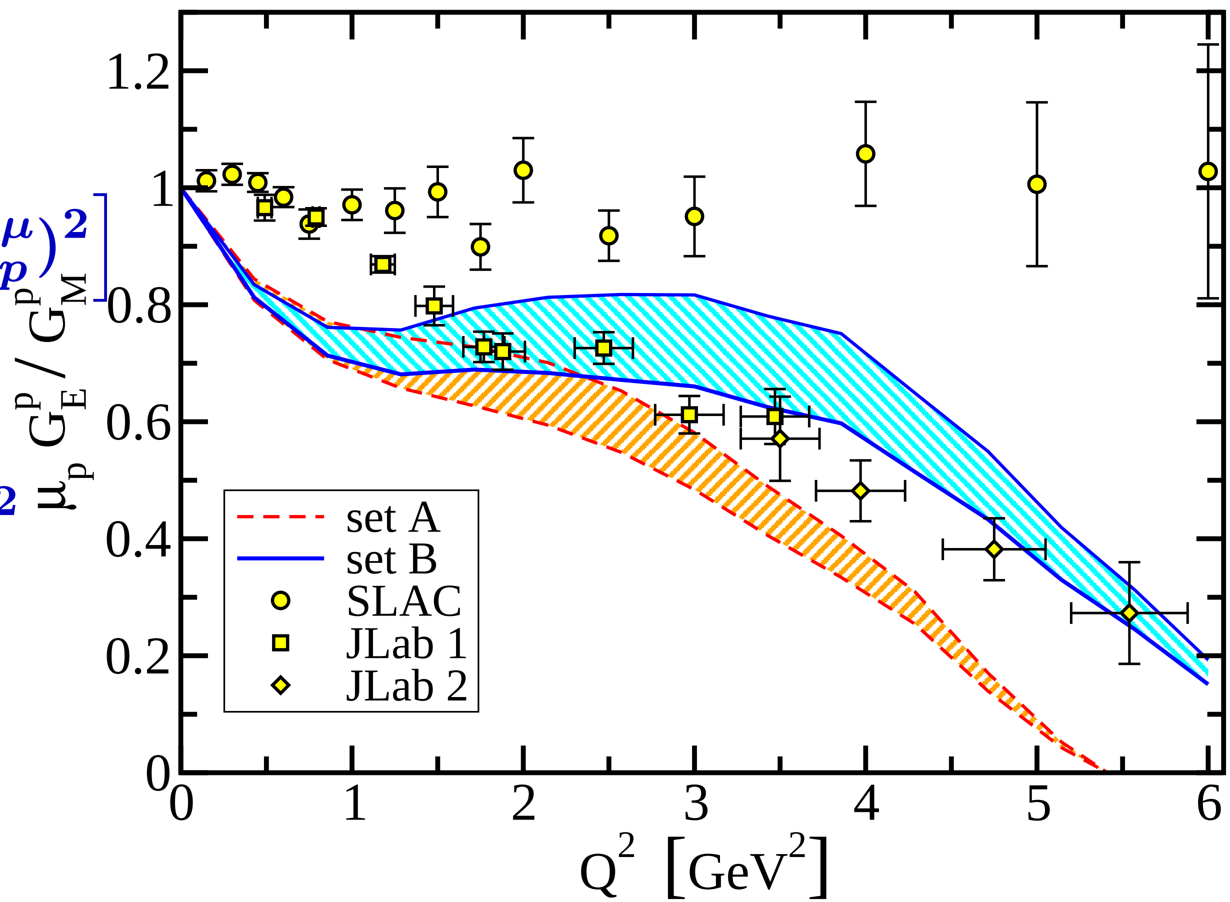
- Small Q^2

$$\mu_p \frac{G_E^p(Q^2)}{G_M^p(Q^2)} = 1 - \frac{Q^2}{6}\left[(r_p)^2 - (r_p^\mu)^2\right]$$

$r_p \approx r_p^\mu$

$\Rightarrow$ ratio varies $< 10\%$

on $0 < Q^2 < 0.6$ GeV^2



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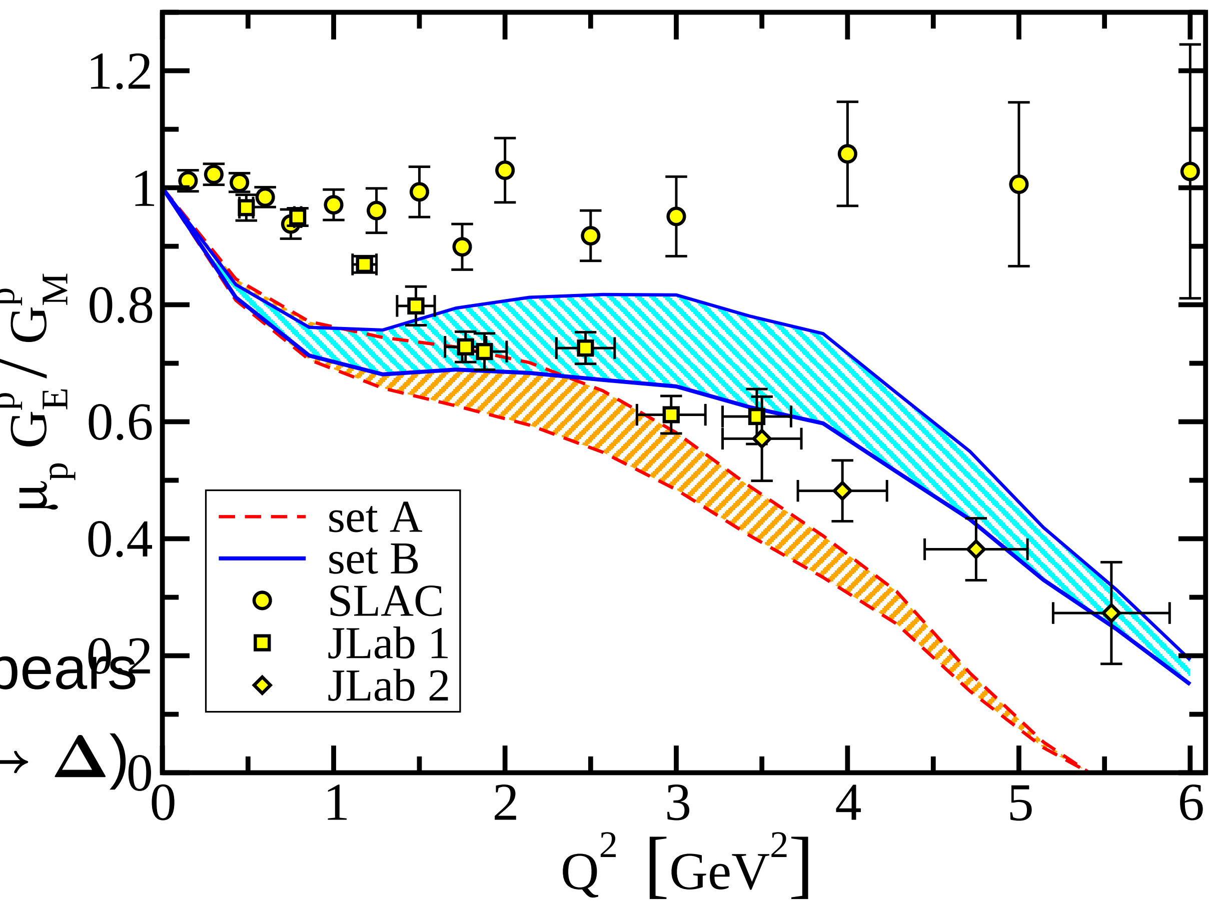

- $Q^2 \gtrsim 2\,\text{GeV}^2$
  - π not pointlike
  - Contribution disappears (Lee & Sato $\pi N \to \Delta$)
  - Quark-core

. . . represented by Faddeev Amplitude . . . becomes dominant

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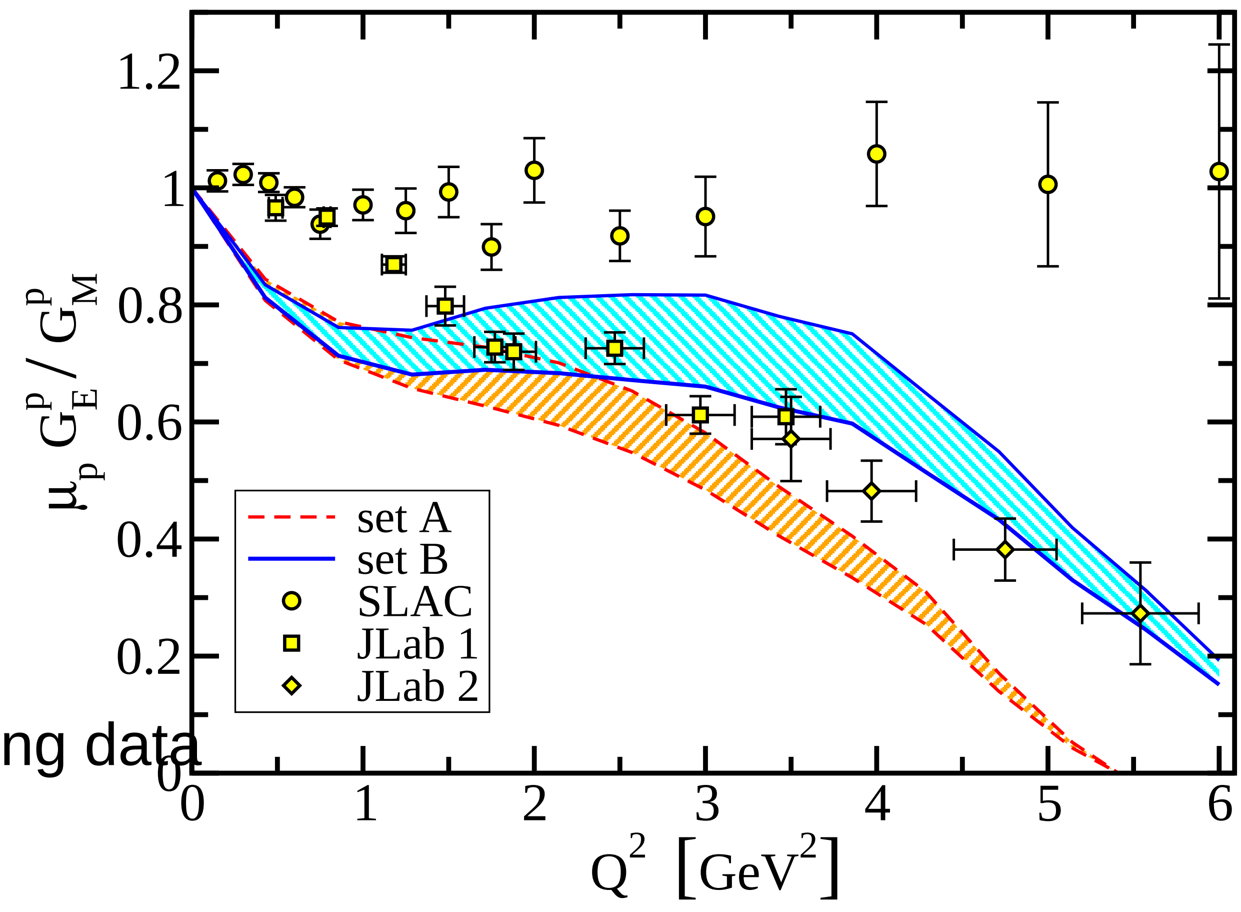
Form Factor Ratio: GE/GM

- $Q^2 \gtrsim 2\,\text{GeV}^2$
  - Set A

    no π-cloud

    ruled-out by existing data



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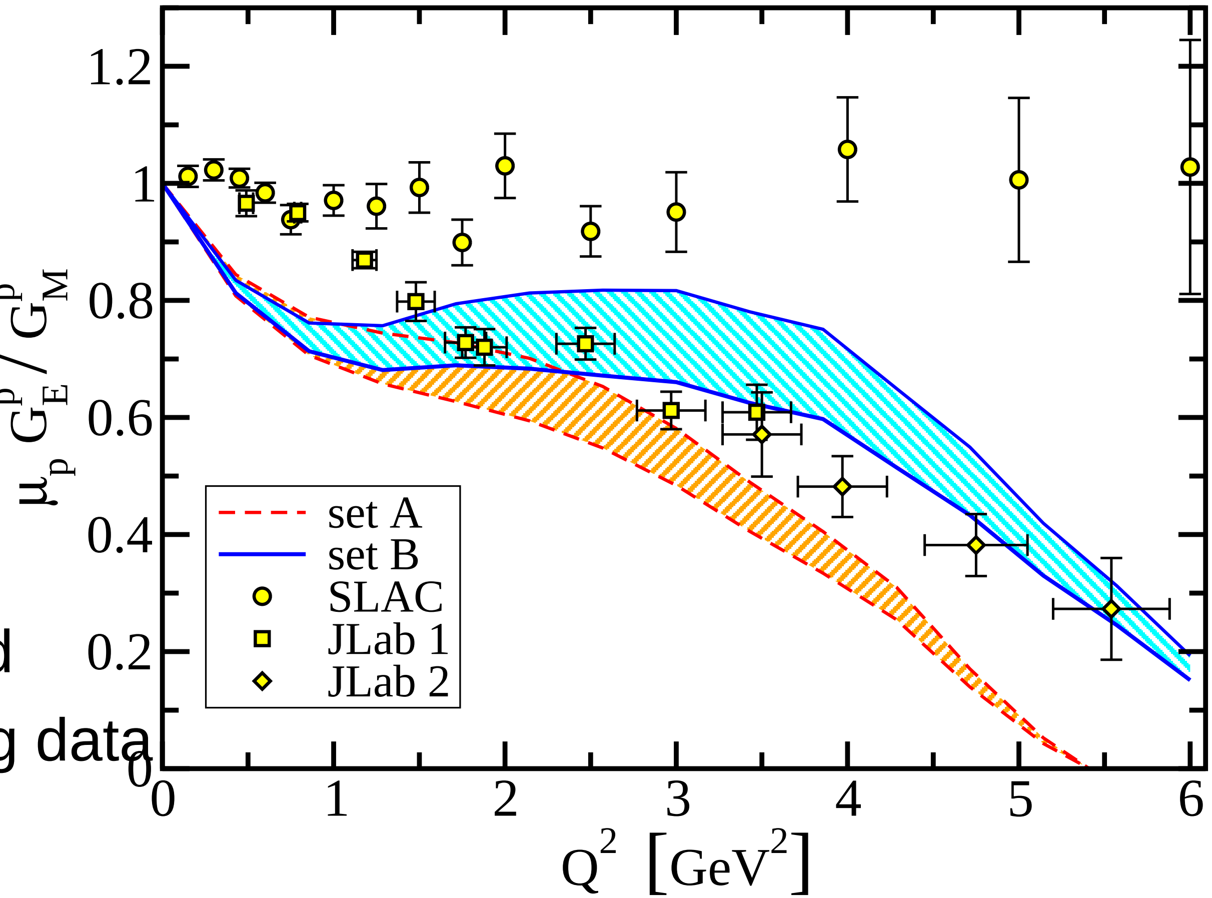
Form Factor Ratio: GE/GM

- $Q^2 \gtrsim 2\,\mathrm{GeV}^2$
  - Set B

    anticipates π-cloud

    not fitted to existing data

    but in agreement



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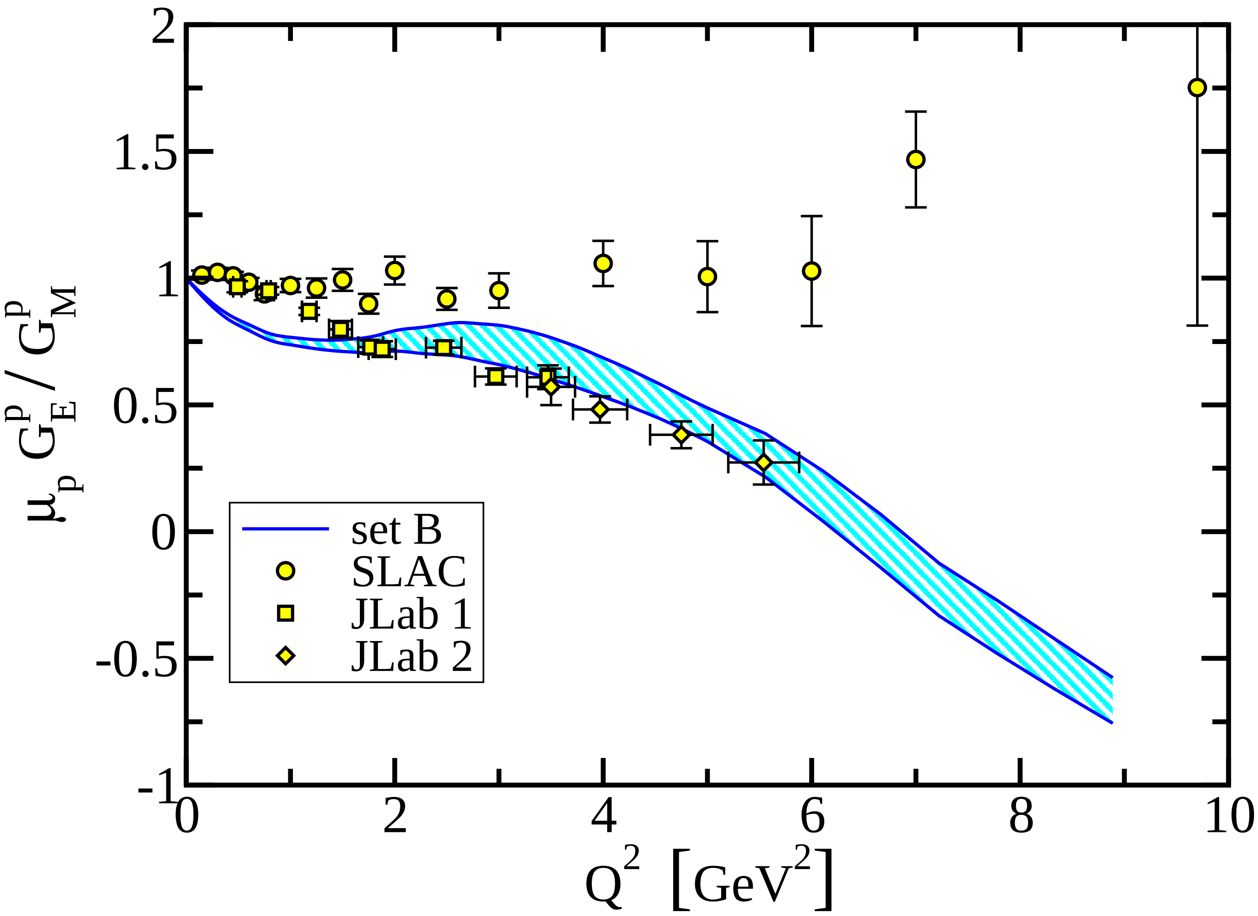
Form Factor Ratio: GE/GM

- $Q^2 \gtrsim 2\,\mathrm{GeV}^2$
  - Set B Zero at $Q^2 \approx 6.5\mathrm{GeV}^2$



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Form Factor Ratio: $Q * F2/F1$

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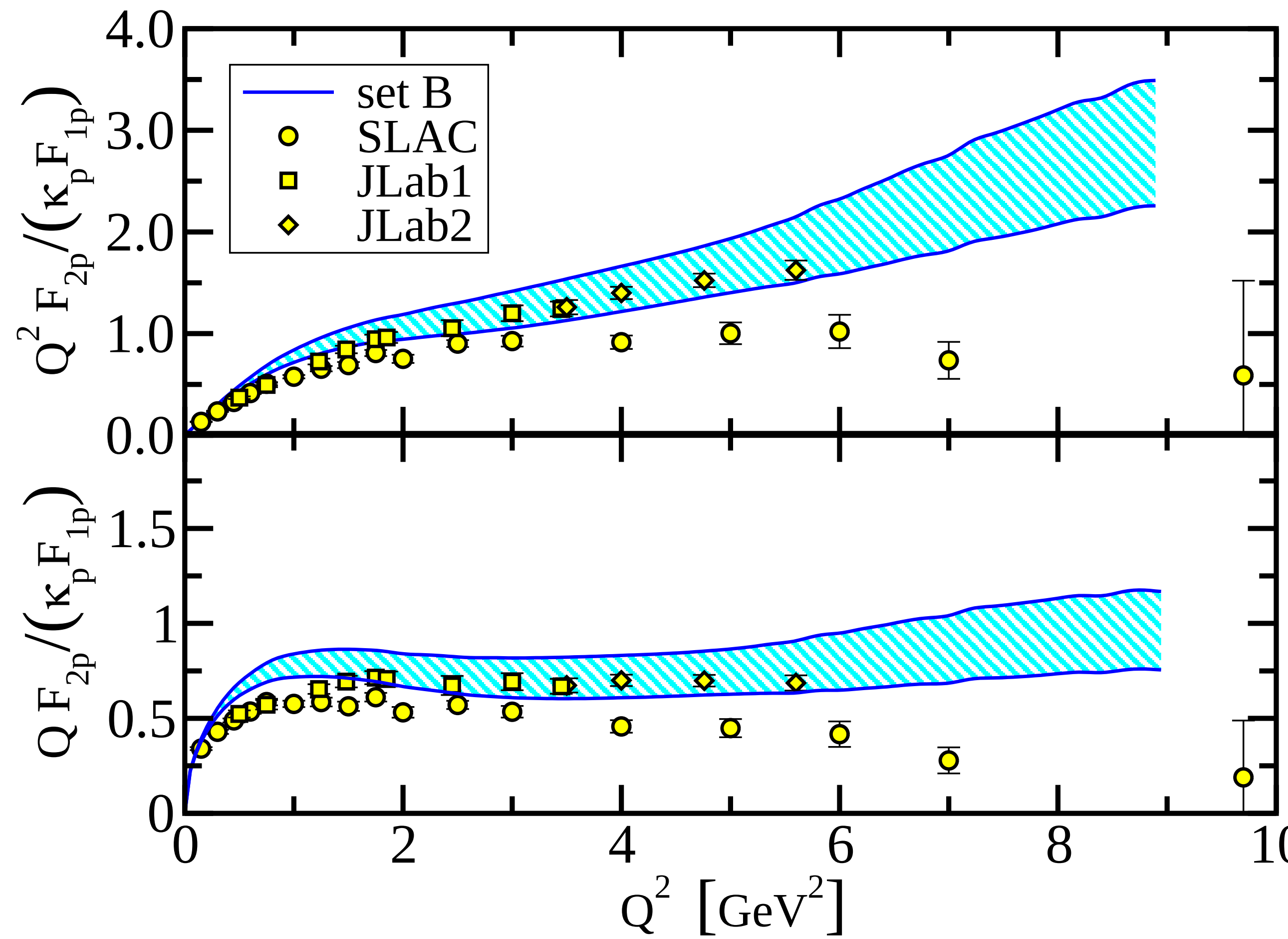

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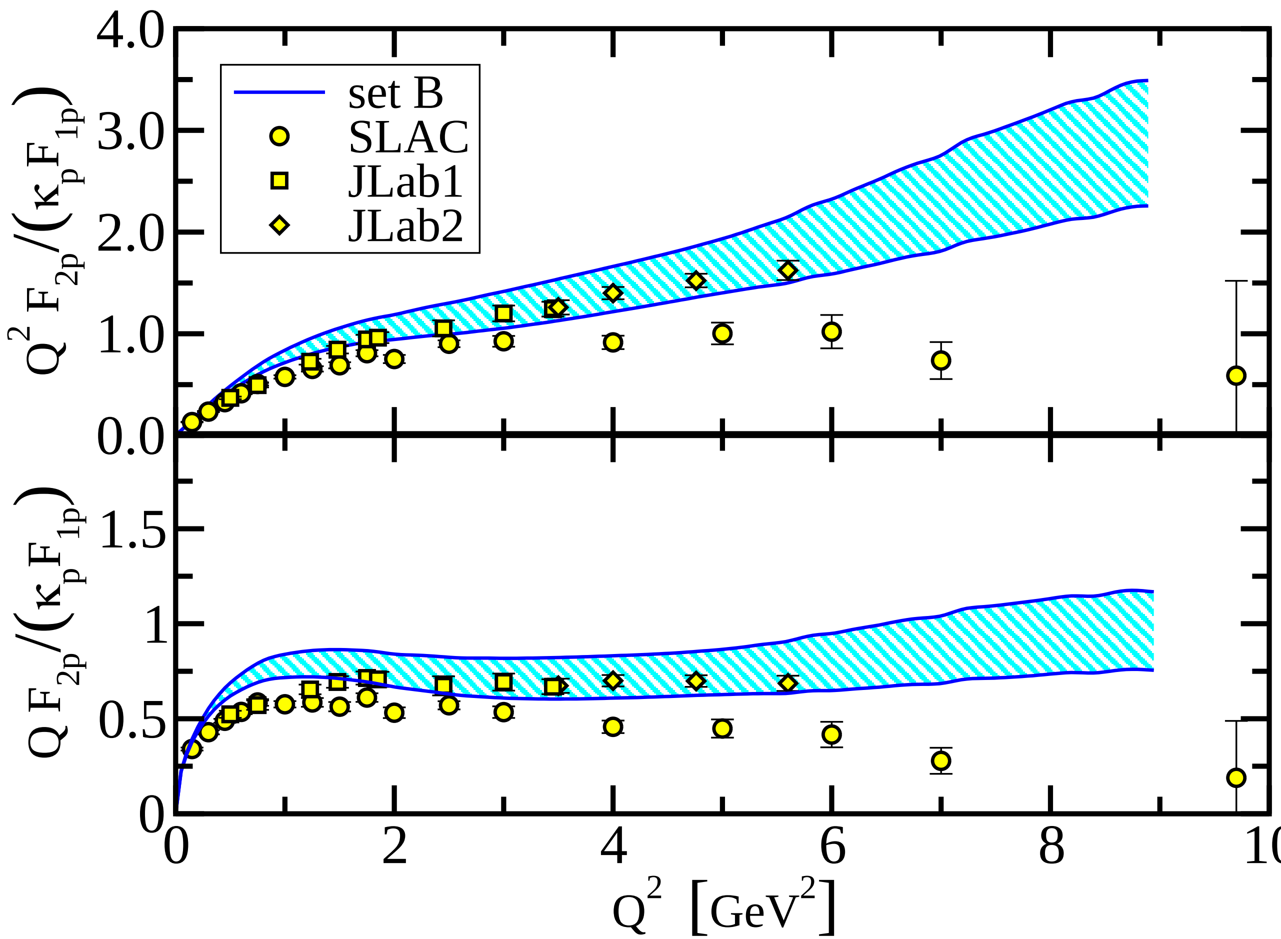
Form Factor Ratio: $Q * F2/F1$

- Perhaps $\approx$ constant for $2 \lesssim Q^2 \lesssim 6\text{GeV}^2$



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Form Factor Ratio: alternative F2/F1

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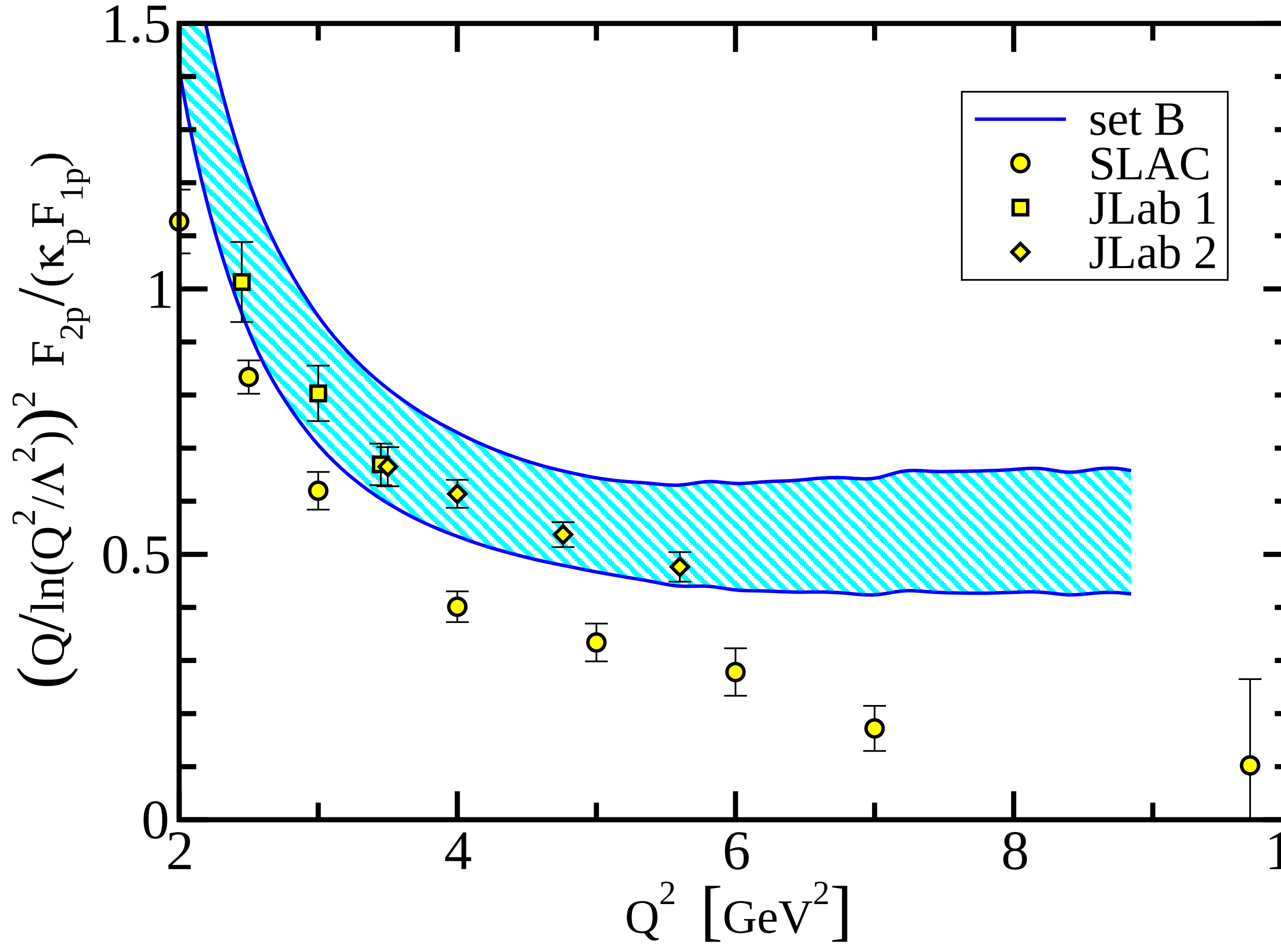
Form Factor Ratio: alternative F2/F1

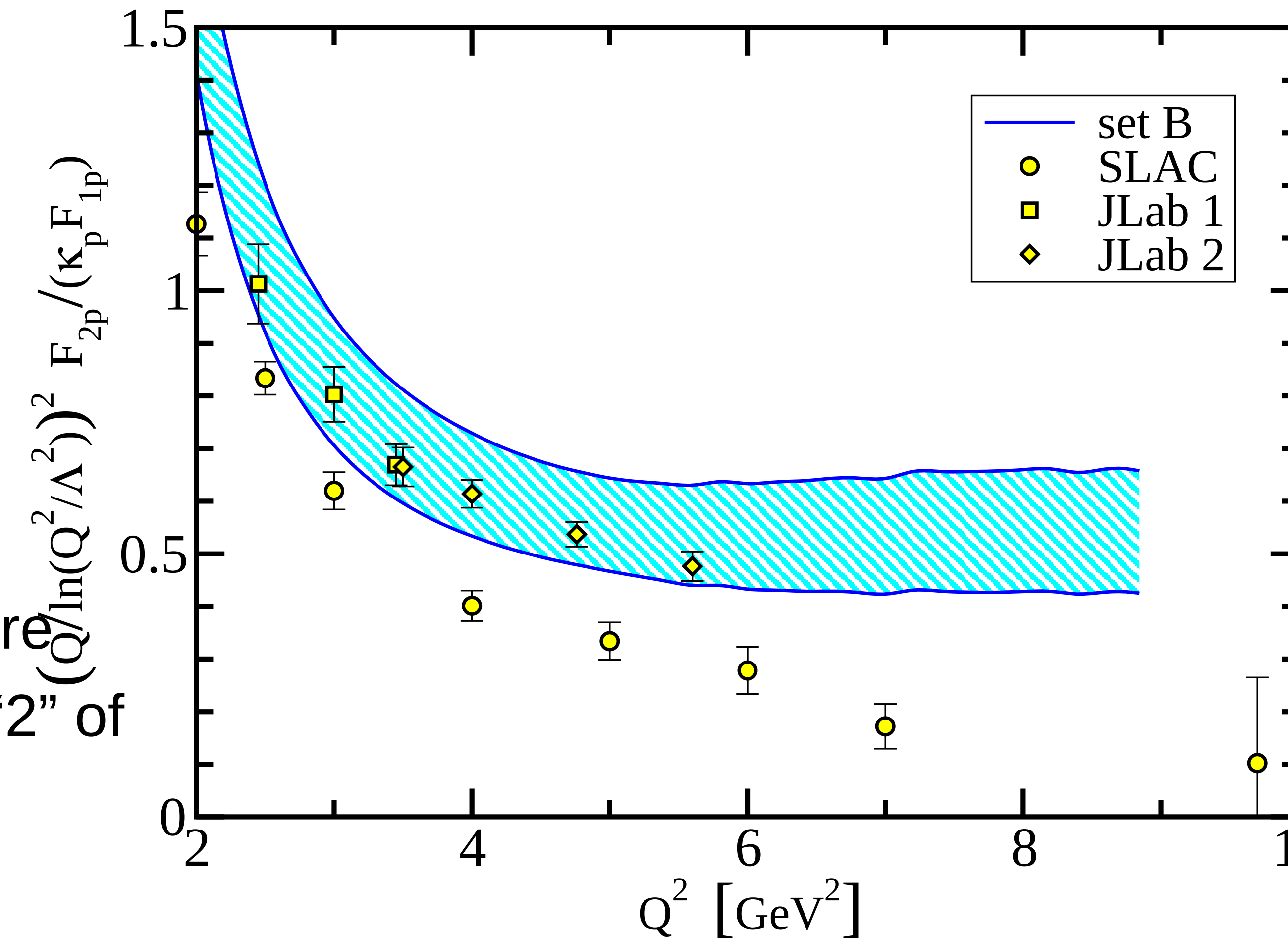


Form Factor Ratio: alternative F2/F1

- $$\frac{Q^2}{[\ln Q^2/\Lambda^2]^2}\frac{F_2(Q^2)}{F_1(Q^2)} = \text{constant}, \;\; Q^2 \gg \Lambda^2 \approx M_N^2$$

  Suggestive

NB. Framework constructed to give quark-counting i.e., "pQCD" *but* with wrong anomalous dimensions *but* they're ignored in *ln*-power "2" of this ratio



Epilogue

Epilogue

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Epilogue

Dynamical Chiral Symmetry Breaking

- Provides Understanding of
  Appearance of Substantial Mass from "Empty Space":
  - proton -

$$3 \times 5 \stackrel{\text{virtual particles}}{=} 950$$

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Epilogue

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$$2 \times 350 \overset{\text{symmetry cancellation of virtual particles}}{=} 140$$

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- Both Effects *impossible* in Quantum Mechanics

  – Require Machinery of Relativistic Quantum Field Theory

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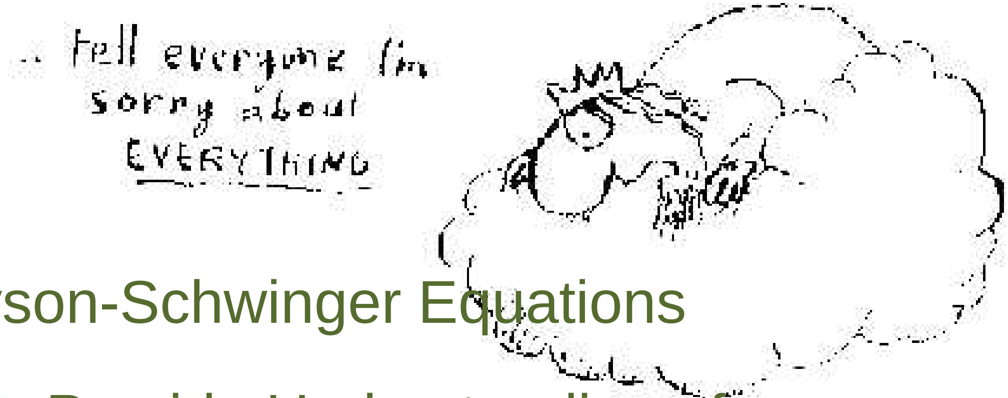

Epilogue

Dyson-Schwinger Equations

- Provide Understanding of

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  $\Rightarrow \pi$ is quark-antiquark Bound State

  AND QCD's Goldstone Mode

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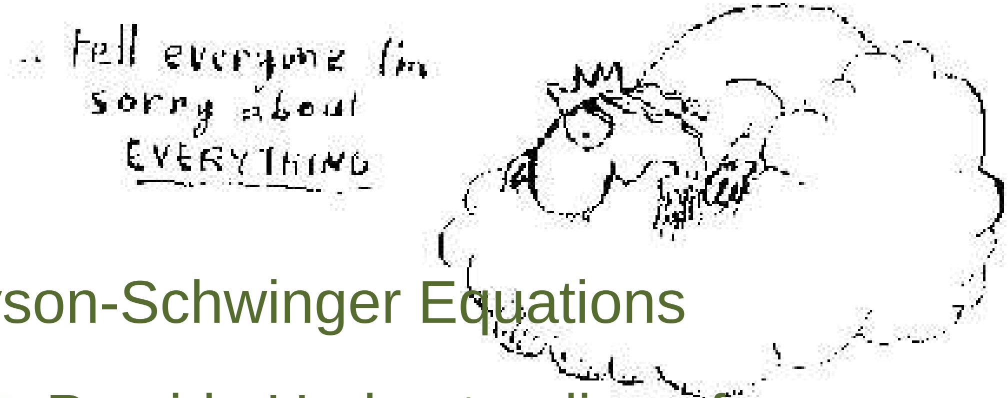

Epilogue

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- Foundation for Proof of
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  e.g., Quark Goldberger-Treiman

  Properties of Pseudoscalar Mesons

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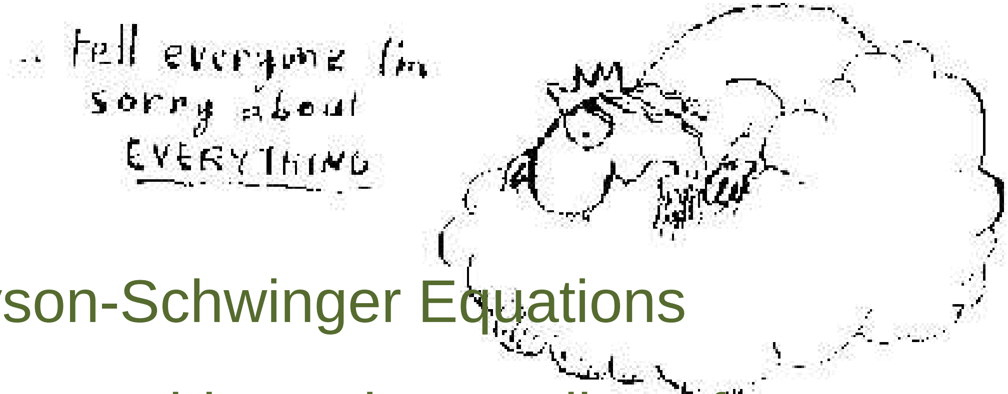

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  e.g., Quark Goldberger-Treiman
  Properties of Pseudoscalar Mesons

- Renormalisation-Group-Improved Rainbow-Ladder

  $\Rightarrow$ Practical Phenomenological Tool
  Corrections Quantifiable

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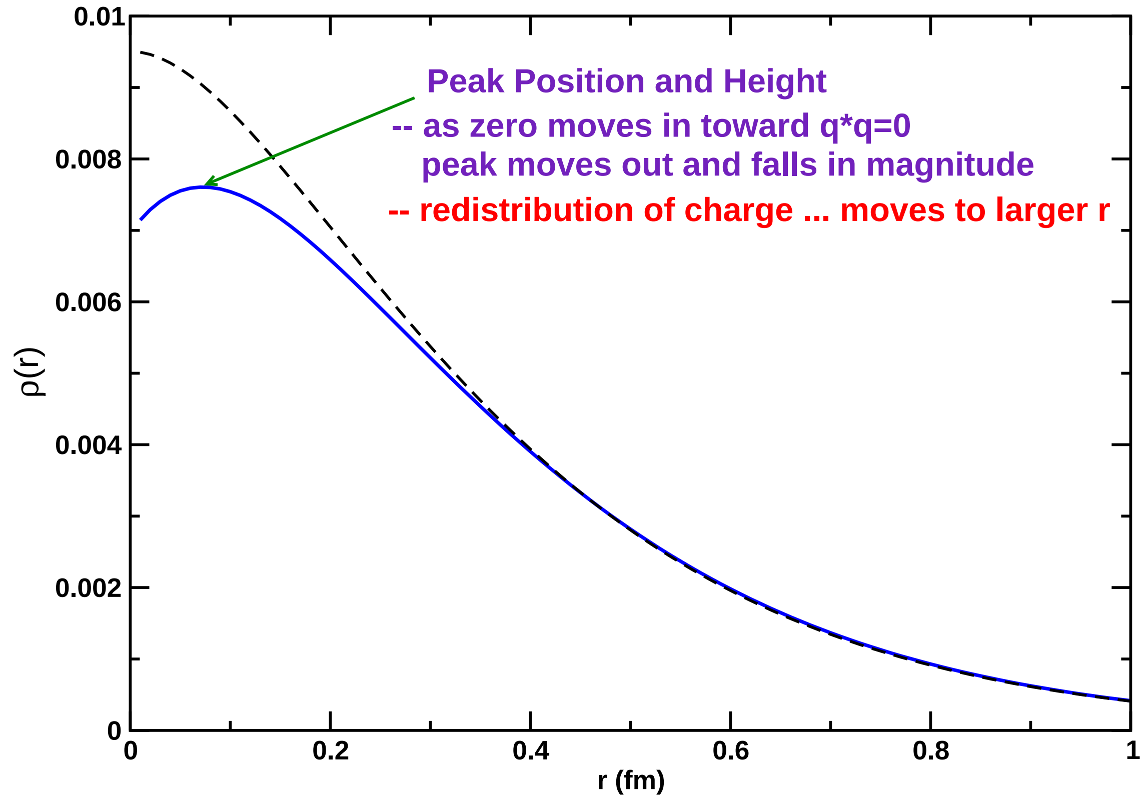
Epilogue

Proton’s Electromagnetic Form Factor

- Appearance of a zero in $G_E(Q^2)$ – Completely Unexpected



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Epilogue

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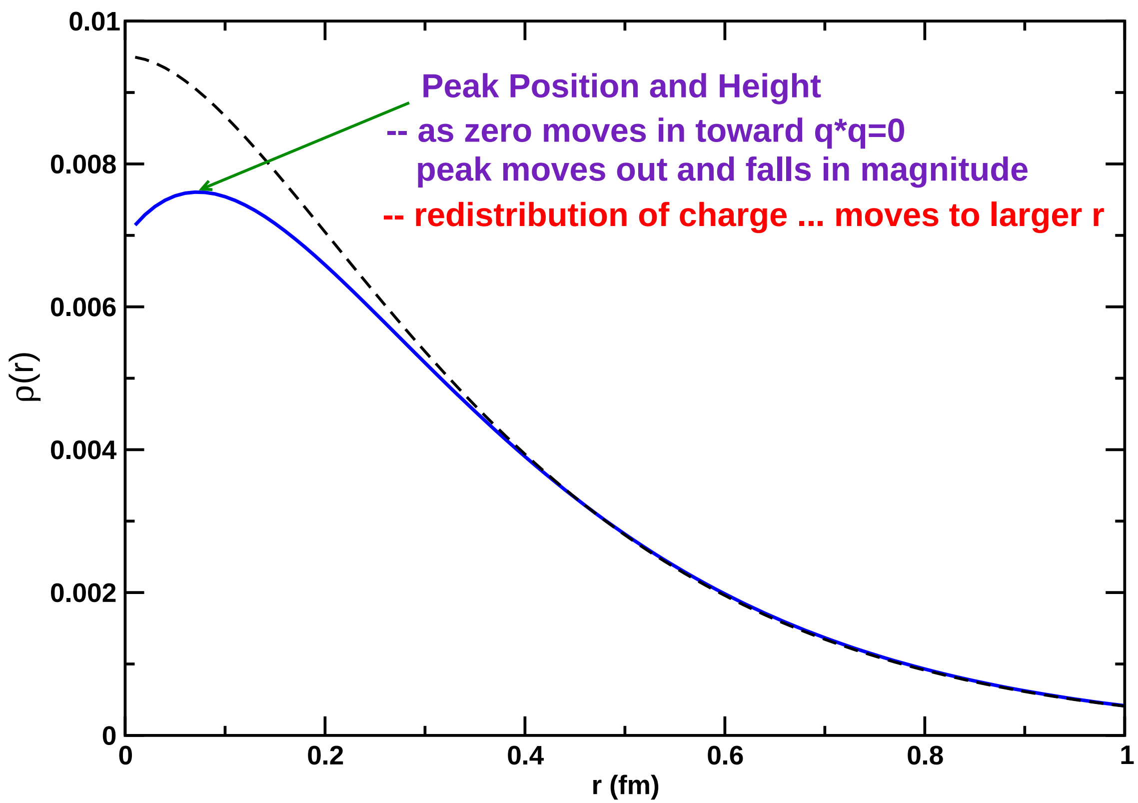

- However, Current Density remains peaked at $r = 0$!

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Epilogue

Proton's Electromagnetic Form Factor

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- Wave Function is complex and correlated mix of virtual particles and antiparticles: $s-$, $p-$ and $d-$waves

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- Can Expect

Contemporary Nuclear Physics
to Yield *Many More Surprises*

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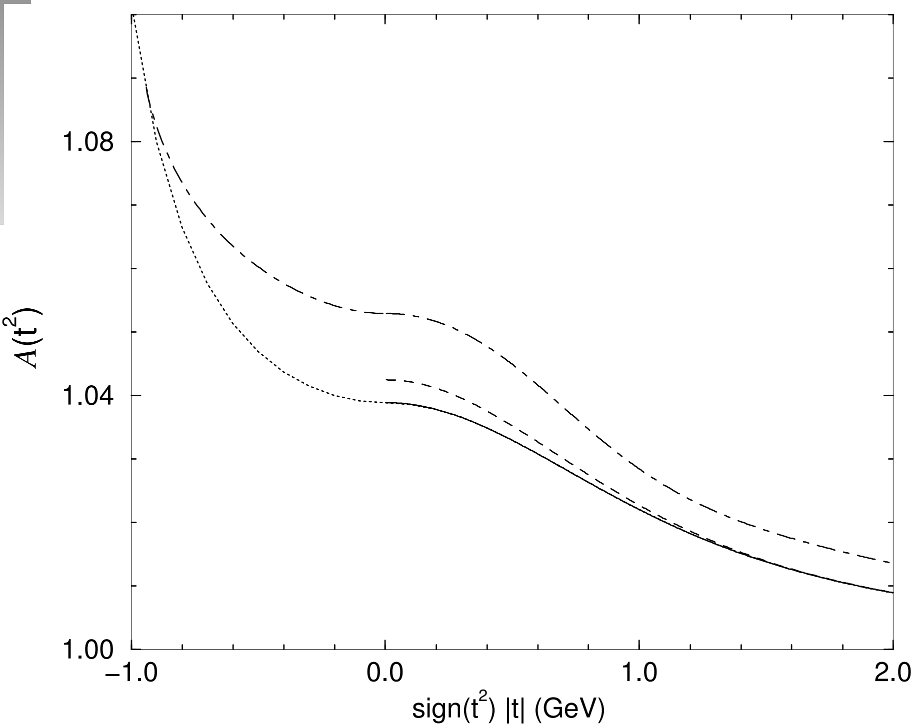
Vector piece of nucleon's self energy



$M = 0.94$, $m_\pi = 0.14$

$\Lambda = 0.9\,$GeV, $g_A = 1$

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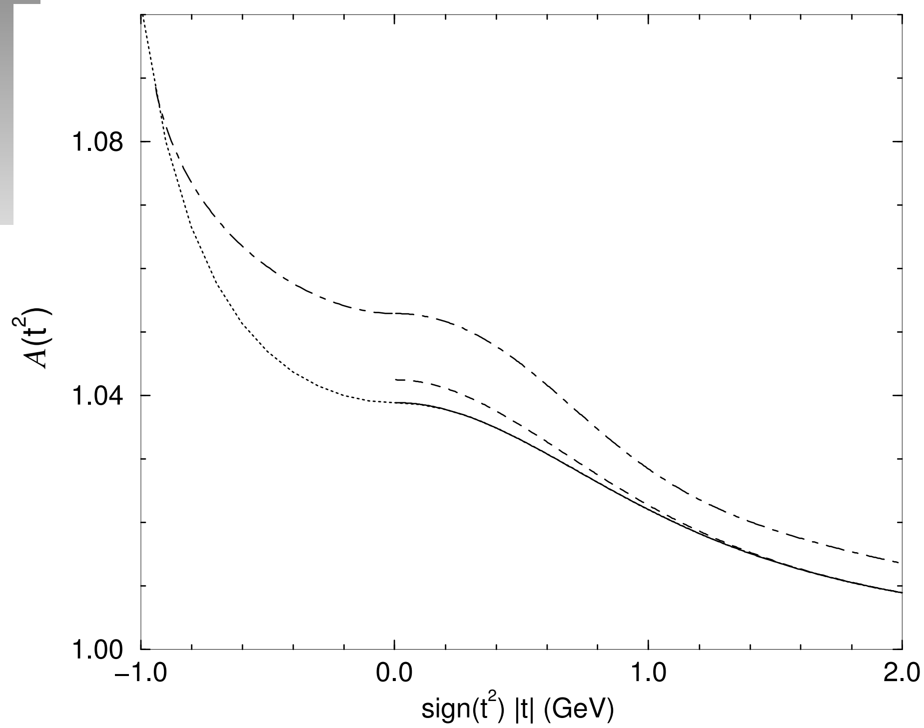
Vector piece of nucleon's self energy



- Solid line: One-loop; numerically evaluated, exact (numerical) kernel

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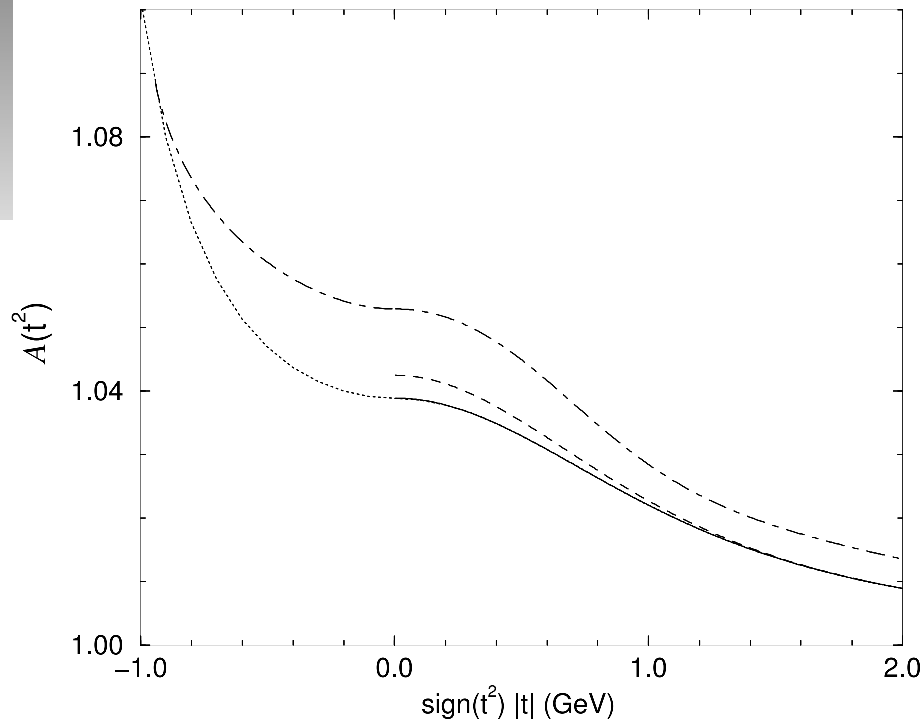

- Solid line: One-loop; numerically evaluated, exact (numerical) kernel
- Dotted line: One-loop; algebraic; code test

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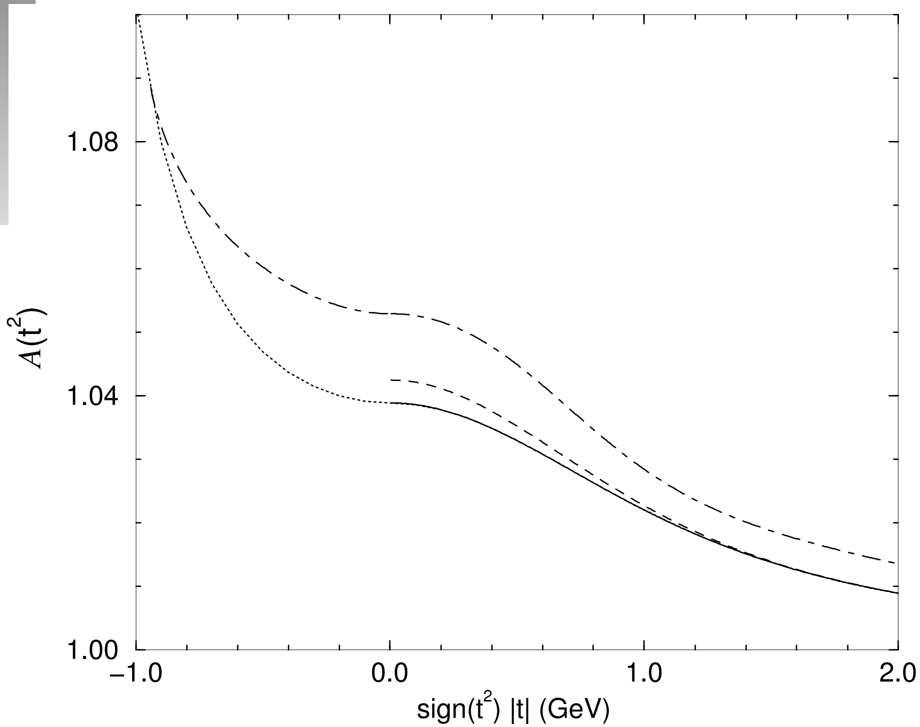

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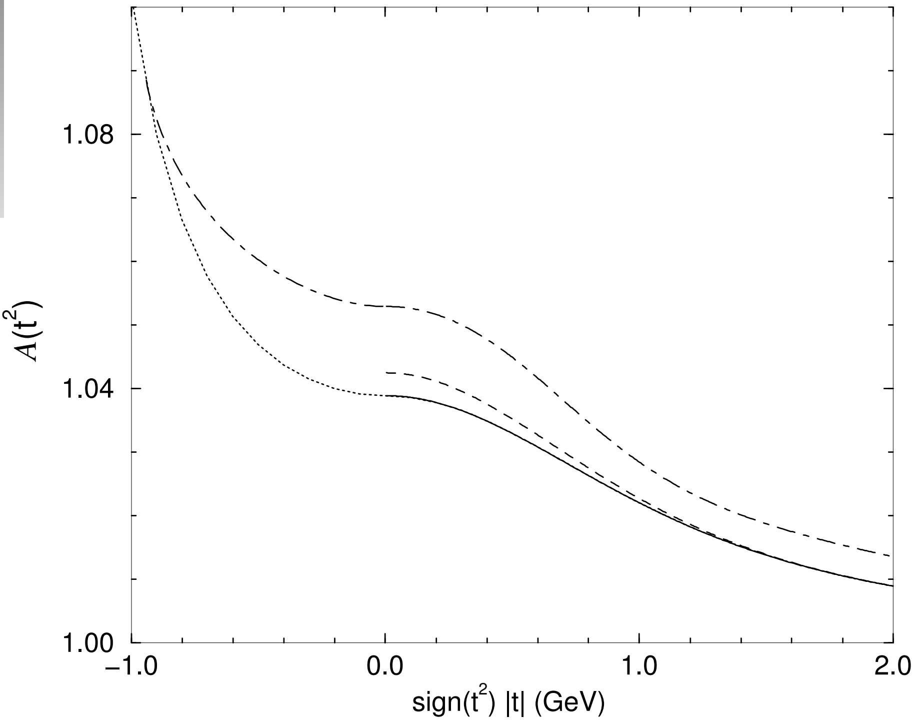
- Dashed line: Fully self-consistent numerical solution; extra pion loops do little

Vector piece of nucleon's self energy



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- Dotted line: One-loop; algebraic; code test
- Dot-dashed line: Fully self-consistent numerical solution; continuation to timelike region

return

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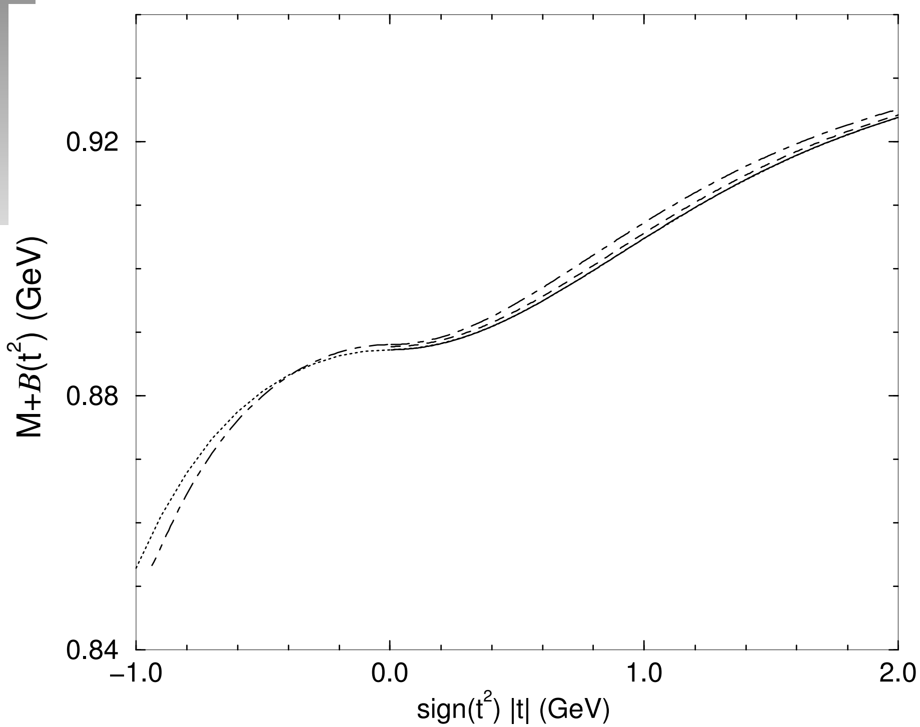
Scalar piece of nucleon's self energy



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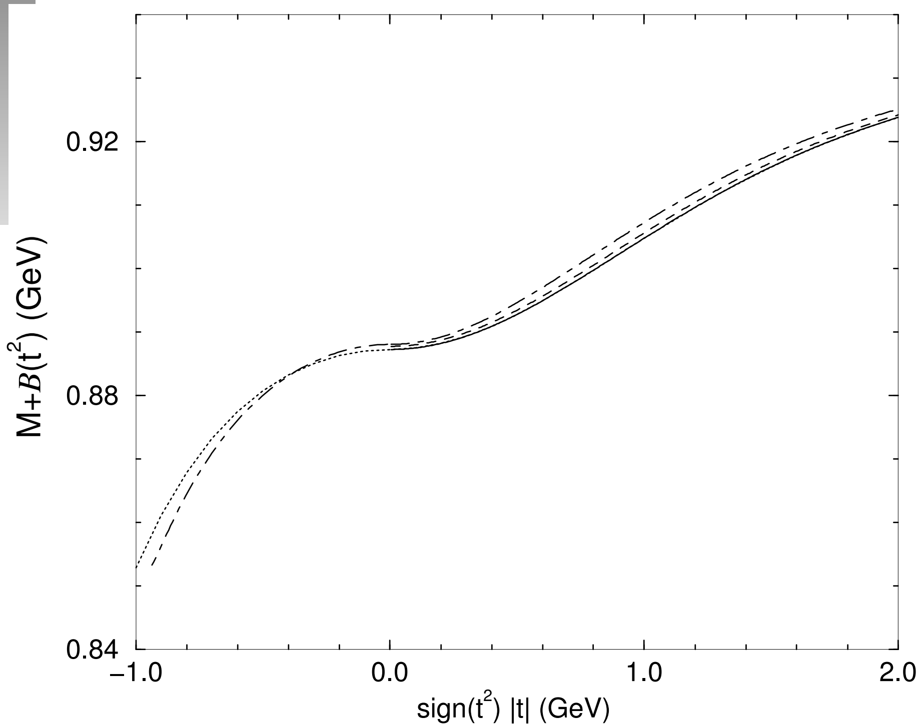
Scalar piece of nucleon's self energy



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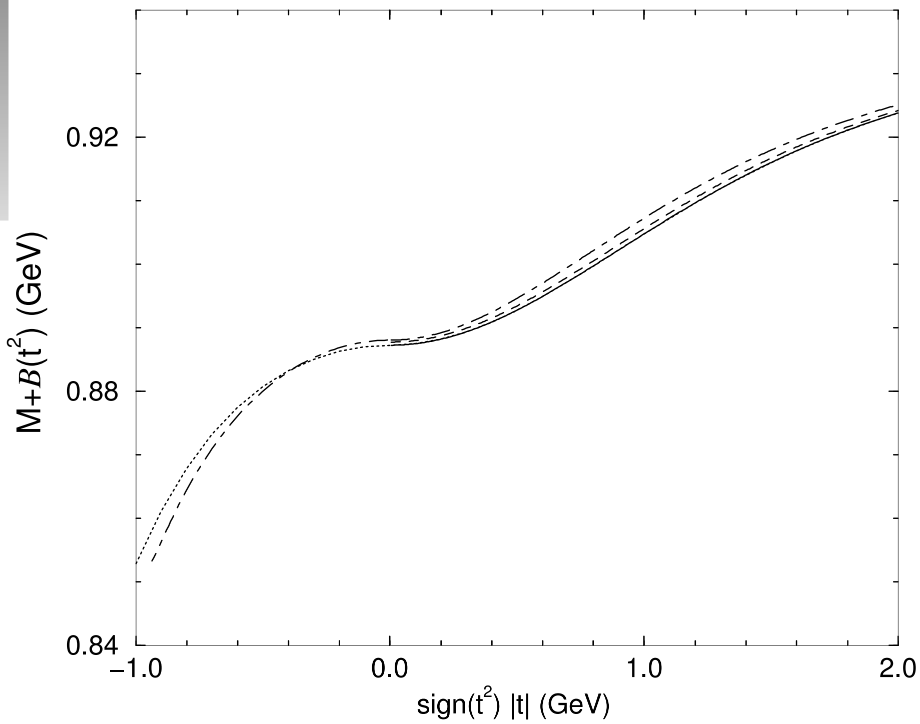

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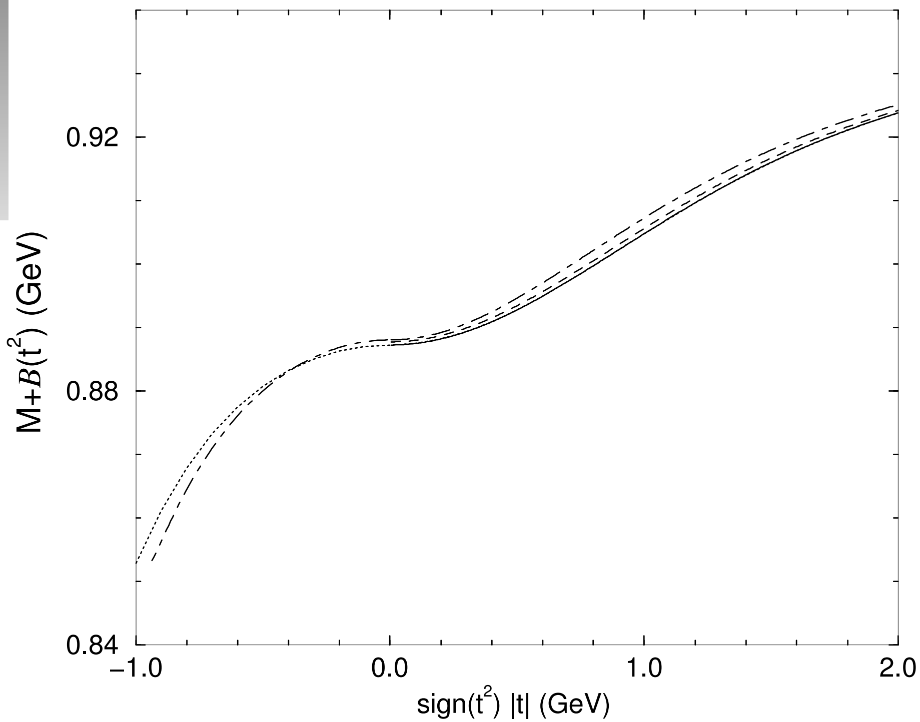
Scalar piece of nucleon's self energy



$M = 0.94$, $m_\pi = 0.14$
$\Lambda = 0.9$ GeV, $g_A = 1$

- Solid line: One-loop; numerically evaluated, exact (numerical) kernel
- Dotted line: One-loop; algebraic; code test
- Dashed line: Fully self-consistent numerical solution; extra pion loops do little

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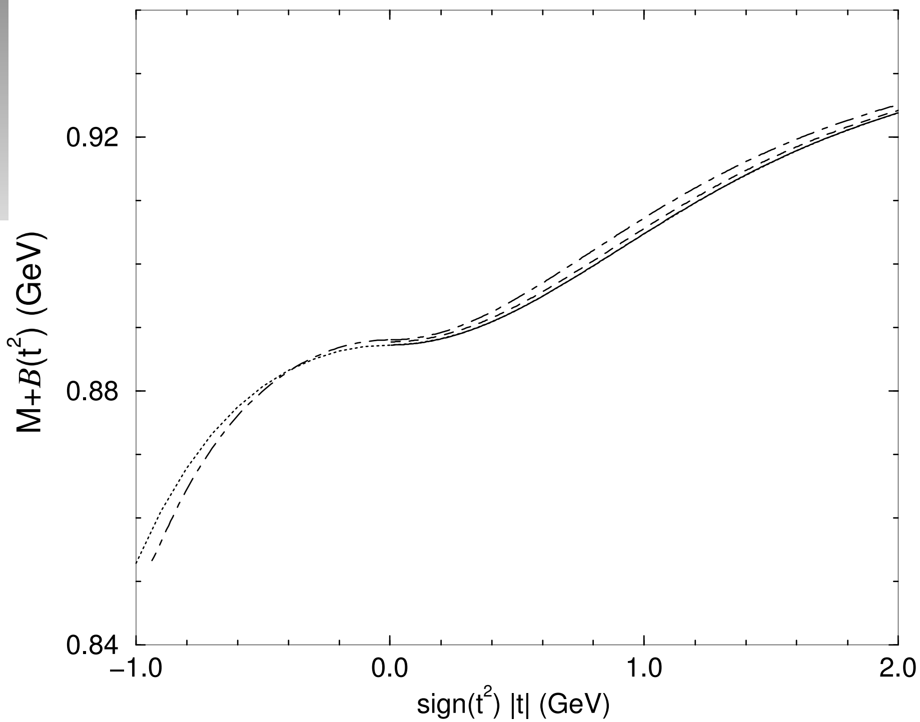
Scalar piece of nucleon's self energy



$M = 0.94$, $m_\pi = 0.14$

$\Lambda = 0.9$ GeV, $g_A = 1$

- Solid line: One-loop; numerically evaluated, exact (numerical) kernel
- Dotted line: One-loop; algebraic; code test
- Dot-dashed line: Fully self-consistent numerical solution; continuation to timelike region

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