Hadron Physics and Dyson-Schwinger Equations

Craig D. Roberts

cdroberts@anl.gov

HUGS Lectures . . . http://www.phy.anl.gov/theory/z05HUGS

Physics Division

Argonne National Laboratory

http://www.phy.anl.gov/theory/staff/cdr.html

Dyson-Schwinger Equations

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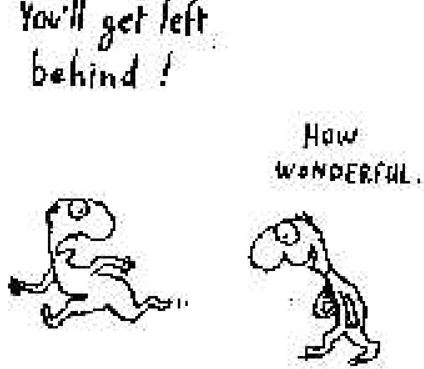
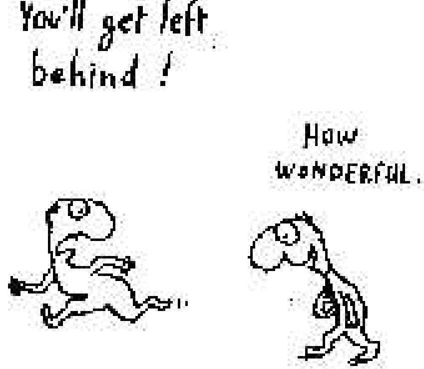


Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory



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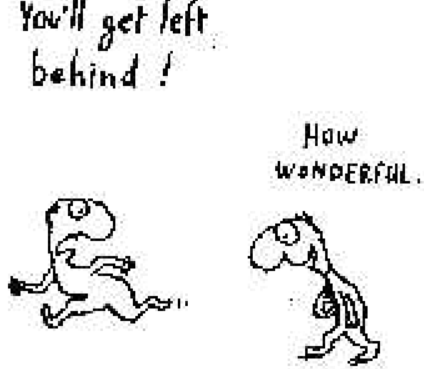

Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory
- NonPerturbative, Continuum approach to QCD
- Simplest level: Generating Tool for Perturbation Theory

  .... Materially Reduces Model Dependence

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Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory
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  - Hadrons as Composites of Quarks and Gluons

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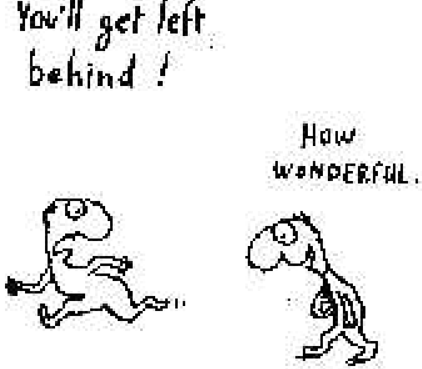

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    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking

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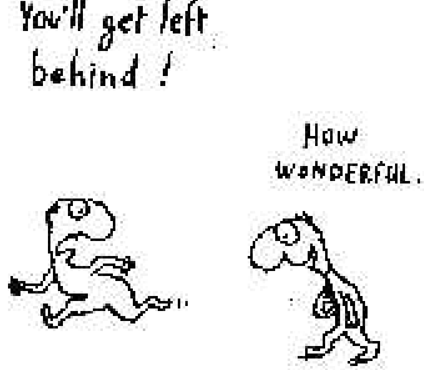

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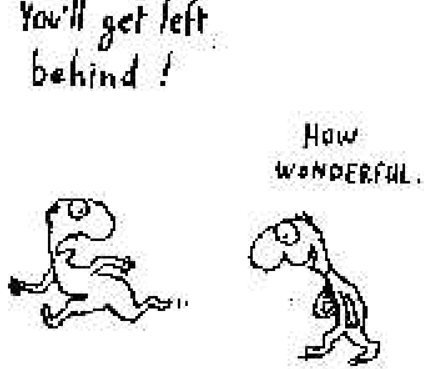

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    - $\Rightarrow$ Understanding InfraRed (long-range)

      .... behaviour of $\alpha_s(Q^2)$

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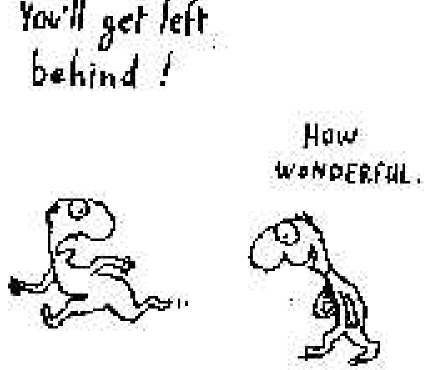

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      .... behaviour of $\alpha_s(Q^2)$
- Method yields Schwinger Functions $\equiv$ Propagators

Cross-Sections built from Schwinger Functions

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Dressed-quark Propagator

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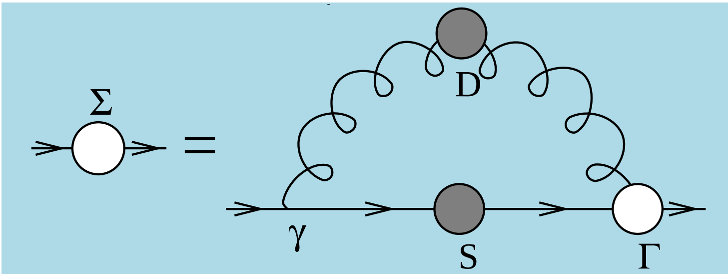
Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

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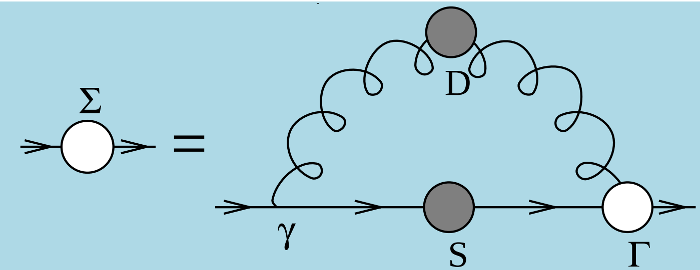

Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

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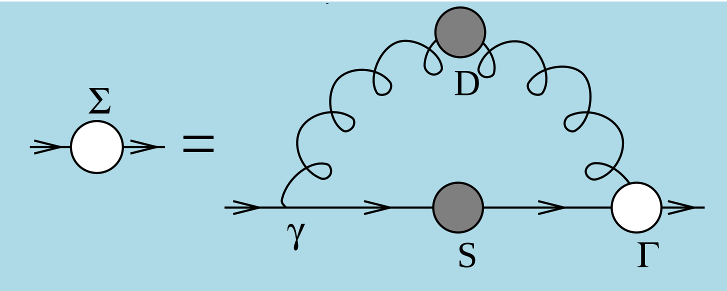

Gap Equation

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$$S(p) = \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory

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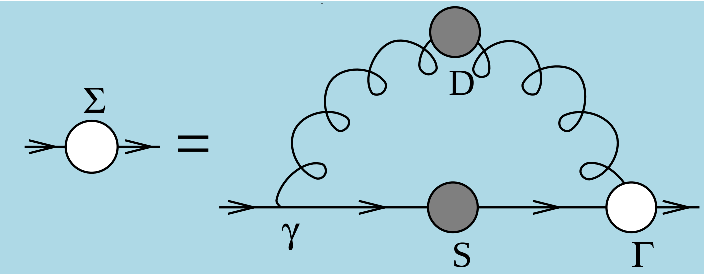

Gap Equation

- dressed-quark propagator

$$S(p) \;=\; \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\to} 0$$

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QCD & Interaction Between Light-Quarks

- Kernel of Gap Equation: $D_{\mu\nu}(p-q)\,\Gamma_\nu(q)$
  Dressed-gluon propagator and dressed-quark-gluon vertex
- Reliable DSE studies of Dressed-gluon propagator:
  - R. Alkofer and L. von Smekal, *The infrared behavior of QCD Green's functions* . . . , Phys. Rept. **353**, 281 (2001).
- Dressed-gluon propagator – lattice-QCD simulations confirm that behaviour:
  - D. B. Leinweber, J. I. Skullerud, A. G. Williams and C. Parrinello [UKQCD Collaboration], *Asymptotic scaling and infrared behavior of the gluon propagator*, Phys. Rev. D **60**, 094507 (1999) [Erratum-ibid. D **61**, 079901 (2000)].
- Exploratory DSE and lattice-QCD studies
  of dressed-quark-gluon vertex

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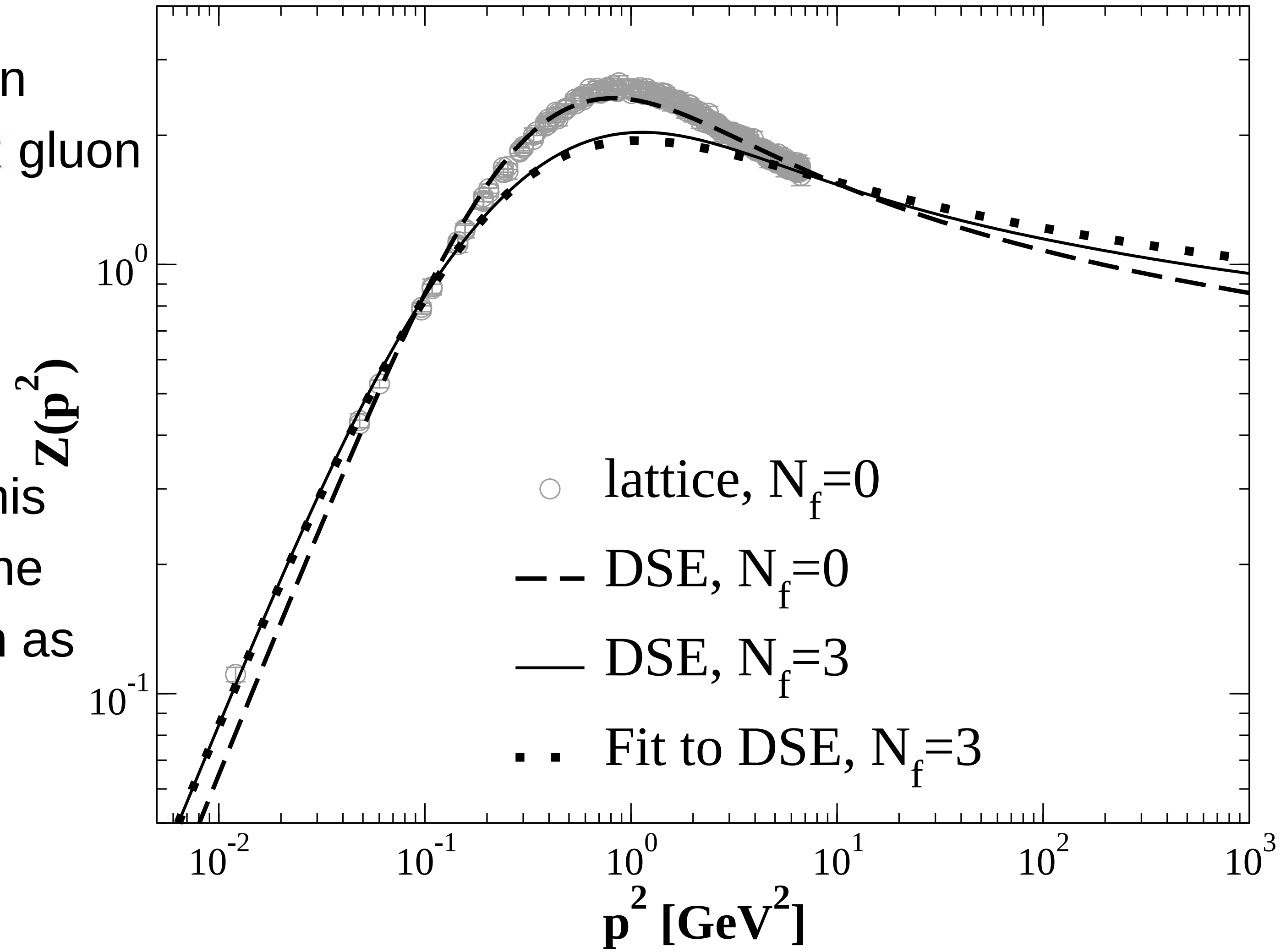
Alkofer, Detmold, Fischer, Maris: he-ph/0309078

Dressed-gluon Propagator

- $$D_{\mu\nu}(k) = \left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\frac{Z(k^2)}{k^2}$$

- Suppression means $\exists$ IR gluon mass-scale $\approx$ 1 GeV

- Naturally, this scale has the same origin as $\Lambda_{\rm QCD}$



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Dressed-quark Propagator

- $S(p) = \dfrac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$
  - Wave function renormalisation: $Z(p^2)$
    - Free particle: $Z(p^2) \equiv 1$
  - Mass function: $M(p^2)$
    - Free particle: $M(p^2) \equiv m_{\rm current}$
- Behaviour of these functions in QCD is a longstanding prediction of DSE studies. For example, it could have been anticipated from; e.g.,
  - K. D. Lane, *Asymptotic Freedom And Goldstone Realization Of Chiral Symmetry*, Phys. Rev. D **10**, 2605 (1974);
  - H. D. Politzer, *Effective Quark Masses In The Chiral Limit*, Nucl. Phys. B **117**, 397 (1976).

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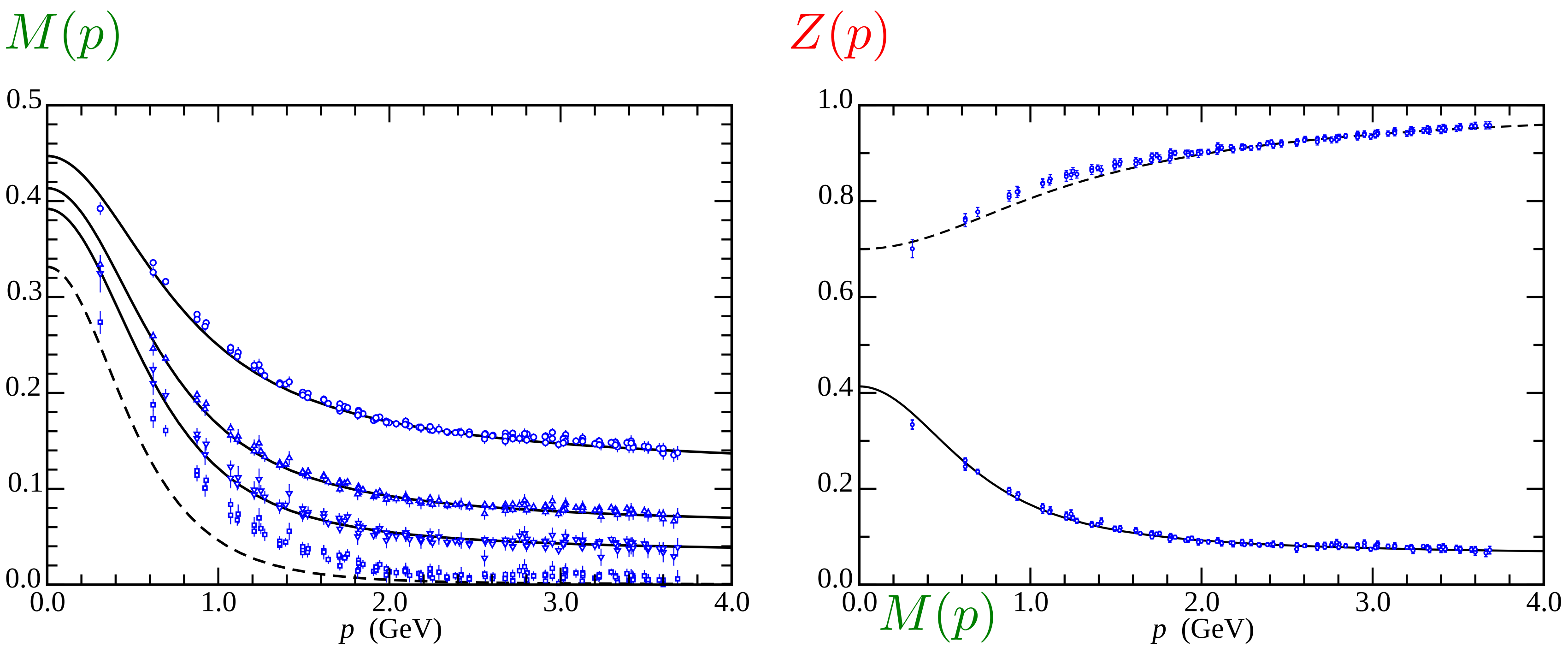
Dressed-quark Propagator



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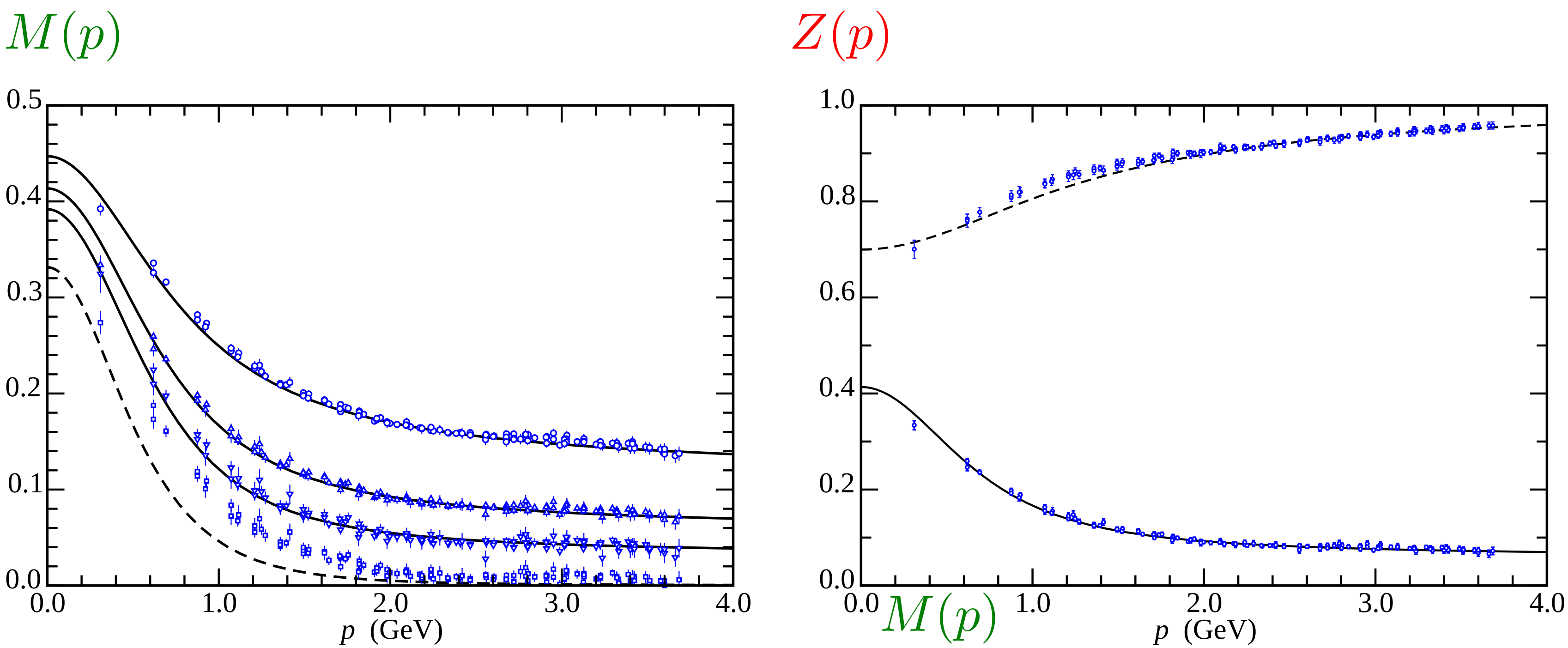

- Lattice Meas.
  – Bowman, Heller, Leinweber, Williams: he-lat/0209129

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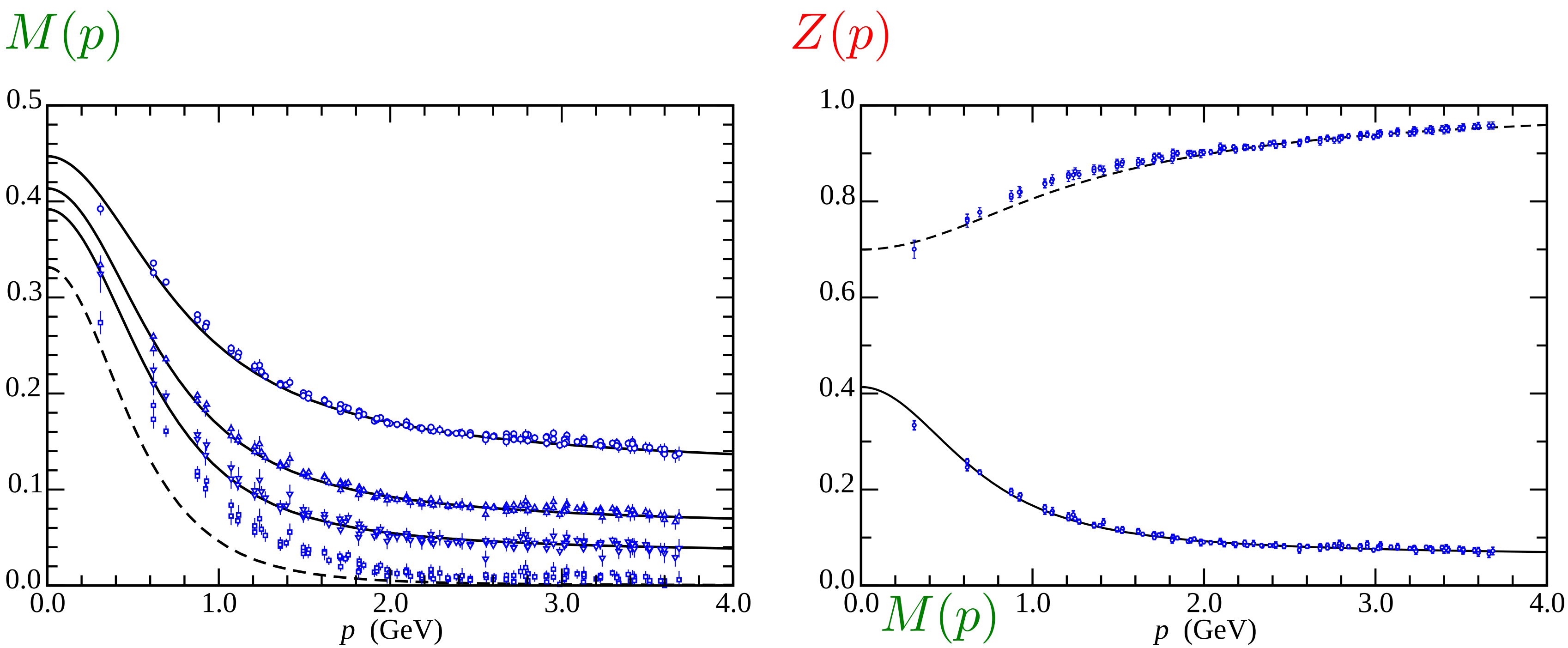

- Lattice Meas.
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- DSE Cal.– Bhagwat, Pichowsky, CDR, Tandy nu-th/0304003

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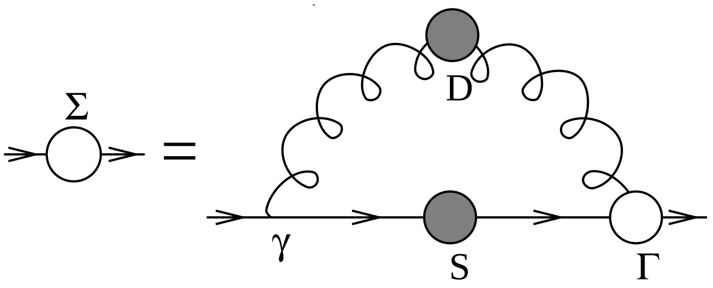
Dressed-quark Propagator

- How is this agreement achieved via the gap equation if the gluon propagator is IR-suppressed?



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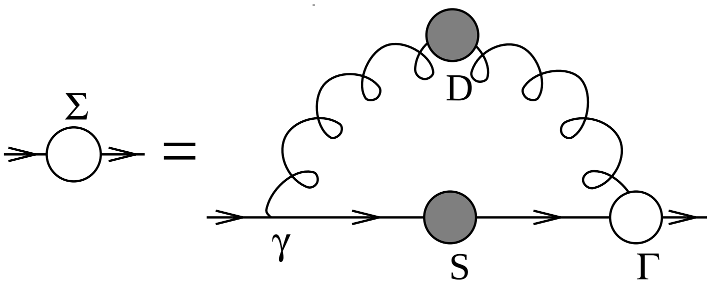

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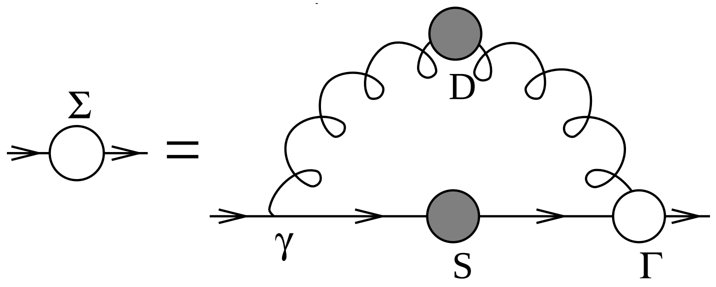

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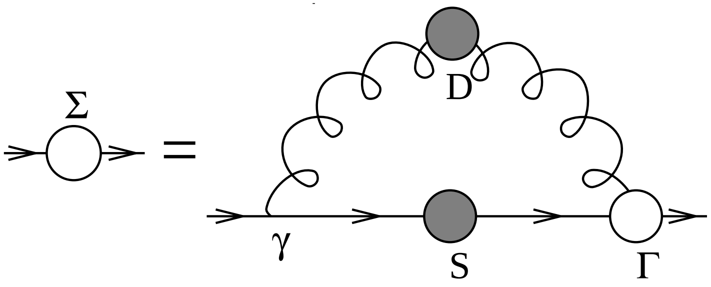

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- Exact nature of the enhancement is subject of ongoing research.

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Hadrons

- Suppose one has established an understanding of two- and three-point functions

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- What about bound states?

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- Without bound states, Comparison with experiment is impossible

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- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions

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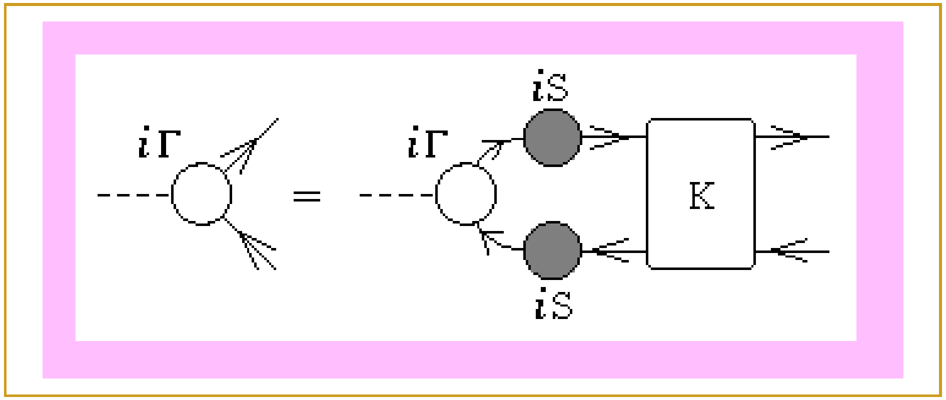
- Bethe-Salpeter Equation



QFT Generalisation of Lippman-Schwinger Equation.

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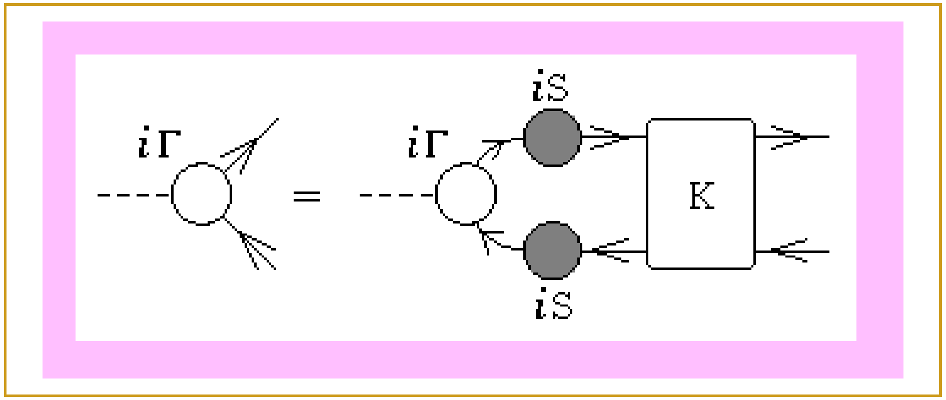

QFT Generalisation of Lippman-Schwinger Equation.

- What is the kernel, K?

- Without bound states, Comparison with experiment is impossible
- They appear as pole contributions to $n \geq 3$-point colour-singlet Schwinger functions
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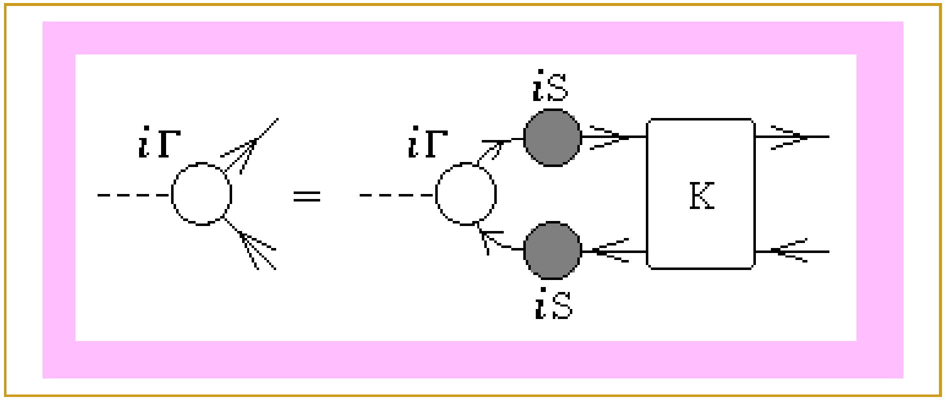

QFT Generalisation of Lippman-Schwinger Equation.

- or What is the long-range potential in QCD?

Bethe-Salpeter Kernel

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Bethe-Salpeter Kernel

- Axial-vector Ward-Takahashi identity

$$P_\mu \; \Gamma^l_{5\mu}(k;P) \quad = \quad \mathcal{S}^{-1}(k_+)\,\frac{1}{2}\lambda^l_f i\gamma_5 + \frac{1}{2}\lambda^l_f i\gamma_5 \;\; \mathcal{S}^{-1}(k_-)$$

$$-M_\zeta \, i\Gamma^l_5(k;P) - i\Gamma^l_5(k;P)\, M_\zeta$$

QFT Statement of Chiral Symmetry

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Satisfies BSE

Satisfies DSE

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Satisfies BSE Satisfies DSE

Kernels must be intimately related

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- Relation must be preserved by truncation

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Satisfies BSE — Satisfies DSE

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- Nontrivial constraint

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Satisfies BSE — Satisfies DSE

Kernels must be intimately related

- Relation must be preserved by truncation
- Failure $\Rightarrow$ Explicit Violation of QCD's Chiral Symmetry

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Goldstone's Theorem

- In the chiral limit the QCD Action possesses chiral symmetry
- The chiral limit is a good approximation in QCD for u- and d-quarks
- If this $SU(N_f = 2)$ chiral symmetry is dynamically broken, then there is a massless composite particle associated with each generator of chiral transformations; i.e., three Goldstone Bosons
- These three Goldstone Bosons have long been identified with the pions: $\boldsymbol{\pi}^+, \boldsymbol{\pi}^0, \boldsymbol{\pi}^-$

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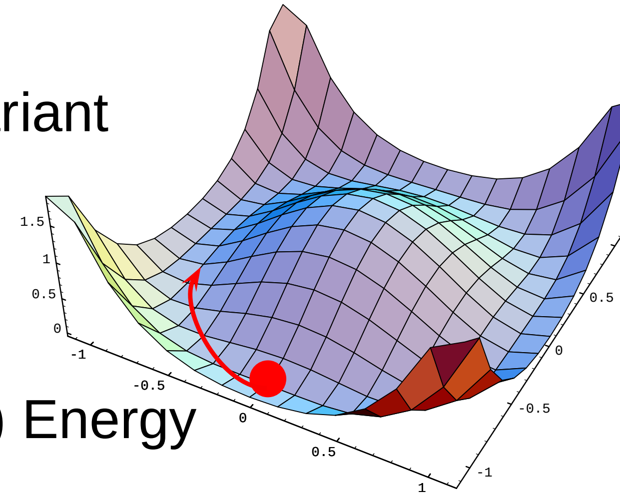
  E.g., $V(x,y) = (\sigma^2 + \pi^2 - 1)^2$

  – Hamiltonian: $T + V$, is Rotationally Invariant Ground State

  - Ball at any (σ, π) for which $\sigma^2 + \pi^2 = 1$
    - All Positions have Same (Minimum) Energy
    - But not invariant under rotations

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- These three Goldstone Bosons have long been identified with the pions: $\boldsymbol{\pi}^{+}, \boldsymbol{\pi}^{0}, \boldsymbol{\pi}^{-}$
- If one assumes the s-quark is also light; namely, assumes that $SU(N_f = 3)$ chiral symmetry is a good approximation, then the kaons are four more Goldstone Bosons

Dichotomy of the Pion

Dichotomy of the Pion

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Dichotomy of the Pion

- How does one make an almost massless particle
 . . . . from two heavy constituents?

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Dichotomy of the Pion

- How does one make an almost massless particle
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- Not Allowed to do it by fine-tuning

Must exhibit $\boxed{m_\pi^2 \propto m_q}$

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The correct understanding of pion observables;
e.g. mass, decay constant and form factors,
requires an approach to contain
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  The correct understanding of pion observables;
  e.g. mass, decay constant and form factors,
  requires an approach to contain
  a well-defined and valid chiral limit.
- Requires detailed understanding of Connection between Current-quark and Constituent-quark masses

Goldberger-Treiman for pion

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Goldberger-Treiman for pion

- Pseudoscalar Bethe-Salpeter amplitude

$$\begin{aligned}\Gamma_{\pi^j}(k;P) &= \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P)+\gamma\cdot P F_\pi(k;P)\\ &\quad +\gamma\cdot k\,k\cdot P\,G_\pi(k;P)+\sigma_{\mu\nu}\,k_\mu P_\nu\,H_\pi(k;P)\Big]\end{aligned}$$

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Goldberger-Treiman for pion

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- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

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- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$
- Axial-vector Ward-Takahashi identity

$$\Rightarrow \quad f_\pi E_\pi(k;P=0) = B(p^2)$$

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- Dressed-quark Propagator: $S(p)=\dfrac{1}{i\gamma\cdot p\,A(p^2)+B(p^2)}$
- Axial-vector Ward-Takahashi identity

$$\Rightarrow \quad \begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2)\\ F_R(k;0)+2\,f_\pi F_\pi(k;0) &= A(k^2)\\ G_R(k;0)+2\,f_\pi G_\pi(k;0) &= 2A'(k^2)\\ H_R(k;0)+2\,f_\pi H_\pi(k;0) &= 0\end{aligned}$$

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Goldberger-Treiman for pion

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$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

Pseudovector components necessarily nonzero

- Dressed-quark Propagator: $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$
- Axial-vector Ward-Takahashi identity

$\Rightarrow$

Exact in Chiral QCD

$$\begin{aligned} f_\pi E_\pi(k;P=0) &= B(p^2) \\ F_R(k;0) + 2\, f_\pi F_\pi(k;0) &= A(k^2) \\ G_R(k;0) + 2\, f_\pi G_\pi(k;0) &= 2A'(k^2) \\ H_R(k;0) + 2\, f_\pi H_\pi(k;0) &= 0 \end{aligned}$$

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Radial Excitations

Radial Excitations

- Spectrum contains 3 pseudoscalars [$I^G(J^P)L = 1^-(0^-)S$]

  masses below 2 GeV: $\boldsymbol{\pi(140)}$; $\boldsymbol{\pi(1300)}$; and $\boldsymbol{\pi(1800)}$

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- Consituent-Q Model: $1^{\rm st}$ three members of $n\,{}^1S_0$ trajectory; i.e., ground state plus radial excitations
- Radial excitations $\Rightarrow$ Long-range radial wave functions $\Rightarrow$ sensitive to confinement
- NSAC Long-Range Plan, 2002:
  ... an understanding of confinement "remains one of the greatest intellectual challenges in physics"

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Andreas Krassnigg

FWF “Erwin Schrödinger Fellow,” ANL 2003-2005

ANL Physics Division

Andreas Krassnigg

Future President
. . . almost Blood
Relative of Arnold

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Radial Excitations & Chiral Symmetry

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Radial Excitations & Chiral Symmetry

$$\boxed{f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H}$$

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(Maris, Roberts, Tandy
nu-th/9707003)

$$\boxed{f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H}$$

- Mass^2 of pseudoscalar hadron

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Radial Excitations & Chiral Symmetry

$$\boxed{f_H \; m_H^2 = - \; \rho_\zeta^H \; \mathcal{M}_H}$$

$$\mathcal{M}_H := \mathrm{tr}_{\text{flavour}} \left[M_{(\mu)} \left\{ T^H, \left(T^H\right)^{\mathrm{t}} \right\} \right] = m_{q_1} + m_{q_2}$$

- Sum of constituents' current-quark masses
- e.g., $T^{K^+} = \frac{1}{2}\left(\lambda^4 + i\lambda^5\right)$

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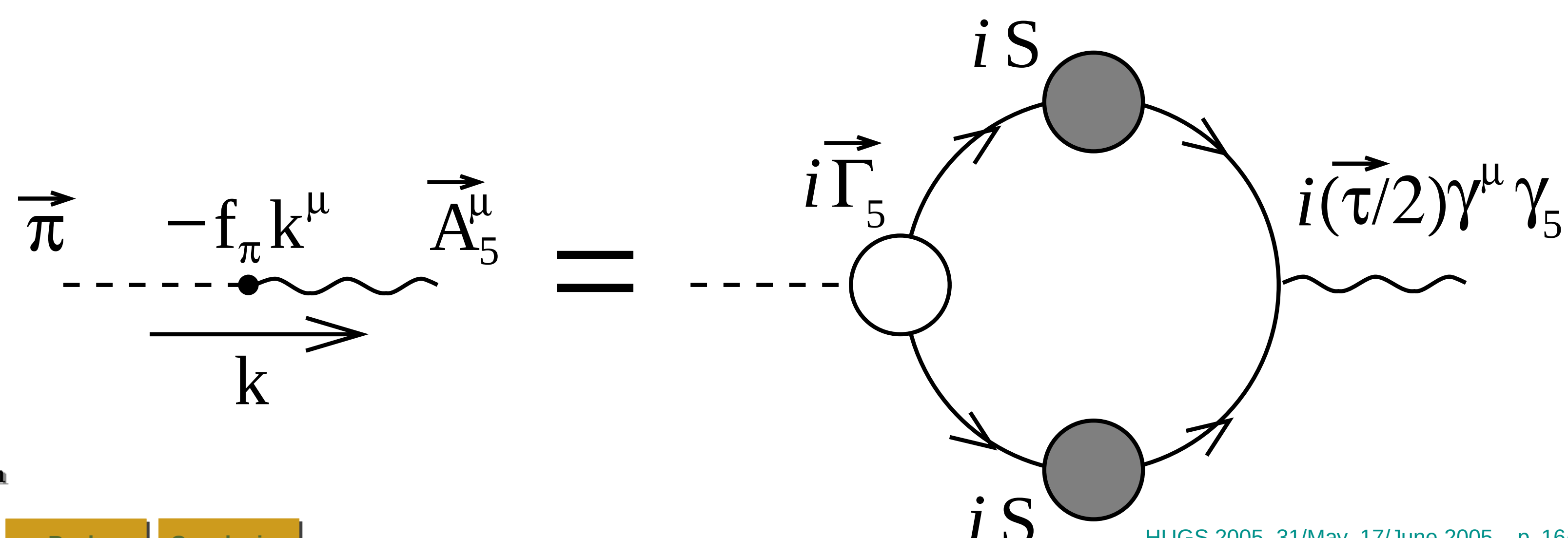
(Maris, Roberts, Tandy nu-th/9707003)

$$\boxed{f_H\, m_H^2 = -\;\rho_\zeta^H\;\mathcal{M}_H}$$

$$f_H\, p_\mu = Z_2 \int_q^\Lambda \tfrac{1}{2}\mathrm{tr}\left\{ \left(T^H\right)^{\mathrm{t}} \gamma_5\gamma_\mu \boxed{\mathcal{S}(q_+)\Gamma_H(q;P)\mathcal{S}(q_-)} \right\}$$

- Pseudovector projection of BS wave function at $x=0$
- Pseudoscalar meson's leptonic decay constant



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Radial Excitations & Chiral Symmetry

(Maris, Roberts, Tandy nu-th/9707003)

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Radial Excitations & Chiral Symmetry

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

- Light-quarks; i.e., $m_q \sim 0$
  - $f_H \to f_H^0$ & $\rho_\zeta^H \to \dfrac{-\langle \bar{q}q\rangle_\zeta^0}{f_H^0}$, Independent of m_q

Hence $\boxed{m_H^2 = \dfrac{-\langle \bar{q}q\rangle_\zeta^0}{(f_H^0)^2}\, m_q}$

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- Heavy-quark + light-quark

  $\Rightarrow f_H \propto \dfrac{1}{\sqrt{m_H}}$ and $\rho_\zeta^H \propto \sqrt{m_H}$

  Hence, $\boxed{m_H \propto m_q}$

Höll, Krassnigg, Roberts
nu-th/0406030

$$\boxed{f_H \;\; m_H^2 = - \;\; \rho_\zeta^H \;\; \mathcal{M}_H}$$

- Valid for ALL Pseudoscalar mesons

Radial Excitations & Chiral Symmetry

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- $\Rightarrow \boldsymbol{f_H = 0}$
  ALL pseudoscalar mesons except $\boldsymbol{\pi(140)}$ in chiral limit
- Dynamical Chiral Symmetry Breaking
  – Goldstone's Theorem –
  impacts upon *every* pseudoscalar meson

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Radial Excitations
& Chiral Symmetry

Radial Excitations & Chiral Symmetry

- Fundamental properties of QCD

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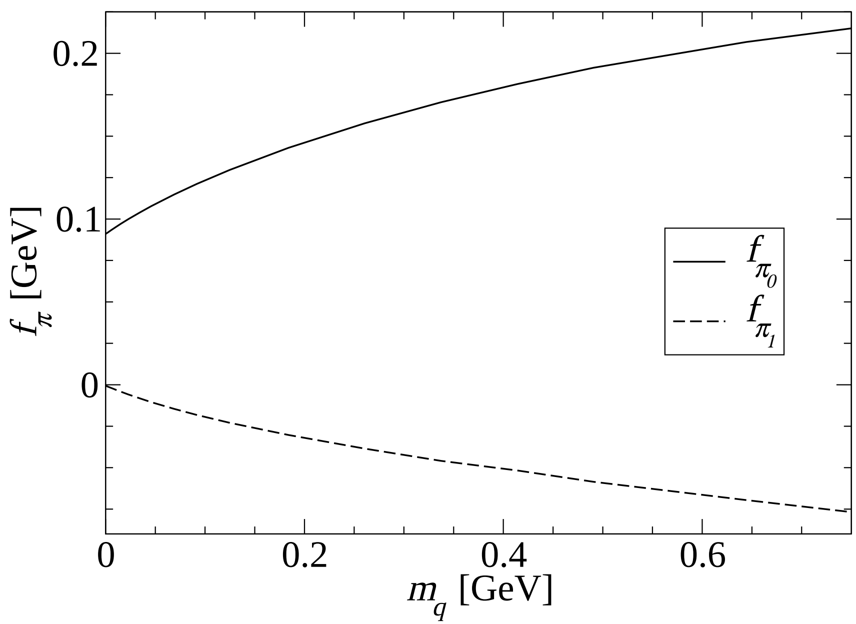
Radial Excitations & Chiral Symmetry

- Fundamental properties of QCD
  - If chiral symmetry is dynamically broken, then in the chiral limit every pseudoscalar meson is blind to the weak interaction *except* $\pi(140)$.



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Radial Excitations & Chiral Symmetry

- Fundamental properties of QCD
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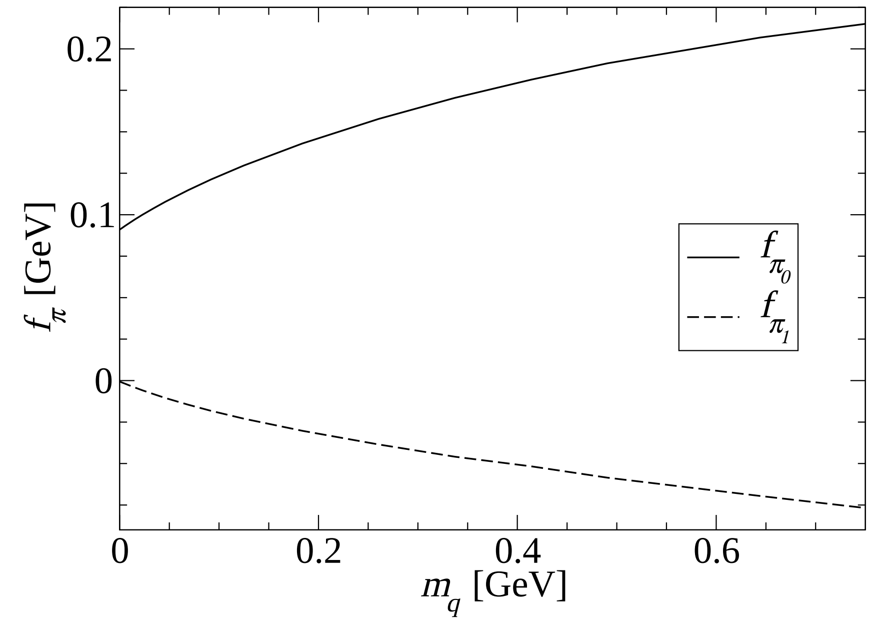
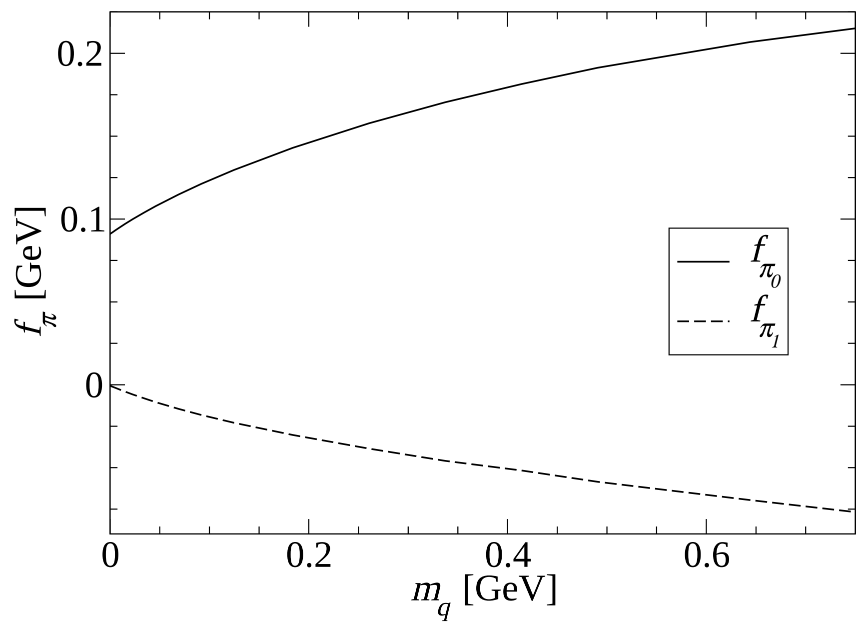
  - If chiral symmetry is not broken, then *not one single pseudoscalar meson experiences the weak interaction.*



Radial Excitations & Chiral Symmetry

- Fundamental properties of QCD
  - If chiral symmetry is dynamically broken, then in the chiral limit every pseudoscalar meson is blind to the weak interaction *except* $\pi(140)$.
  - If chiral symmetry is not broken, then *not one single pseudoscalar meson experiences the weak interaction.*
- All theories with confinement exhibit Dynamical Chiral Symmetry Breaking



Colour-singlet Bethe-Salpeter equation

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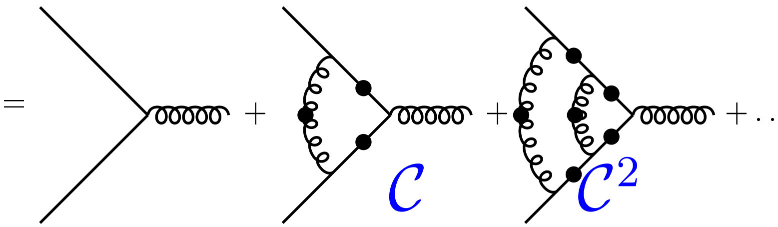
Colour-singlet Bethe-Salpeter equation

- Coupling-modified dressed-ladder vertex

$$\Gamma^a_\mu(k,p) \;=$$



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Colour-singlet Bethe-Salpeter equation

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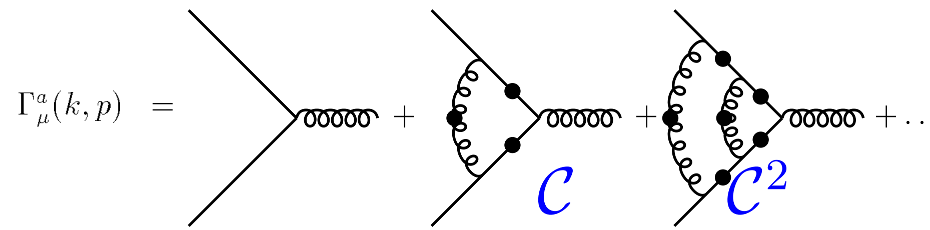
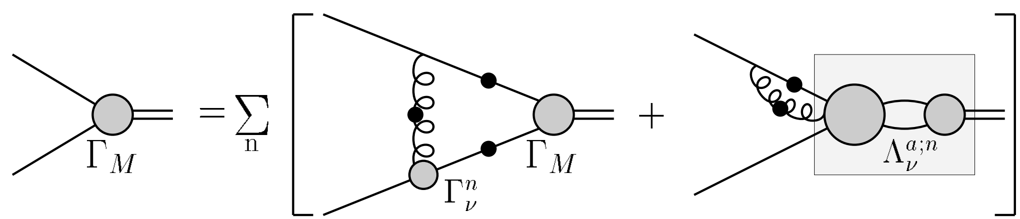


- BSE consistent with vertex



Colour-singlet Bethe-Salpeter equation

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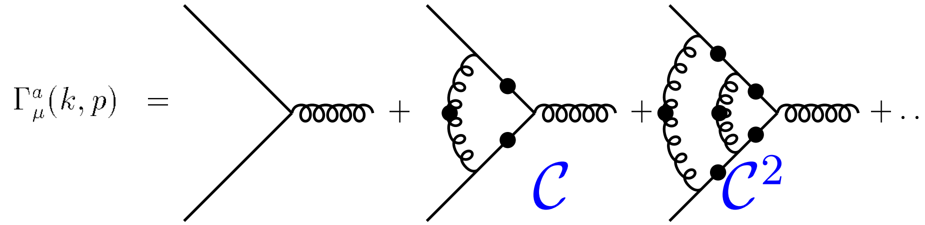

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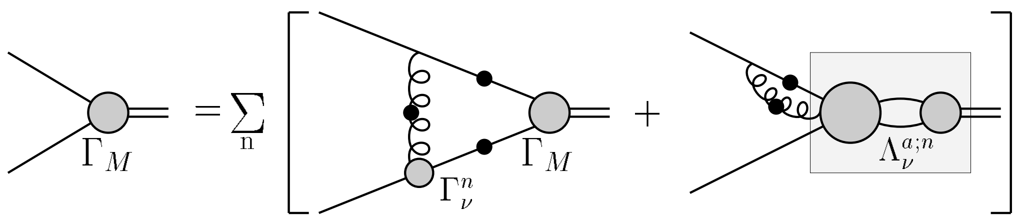
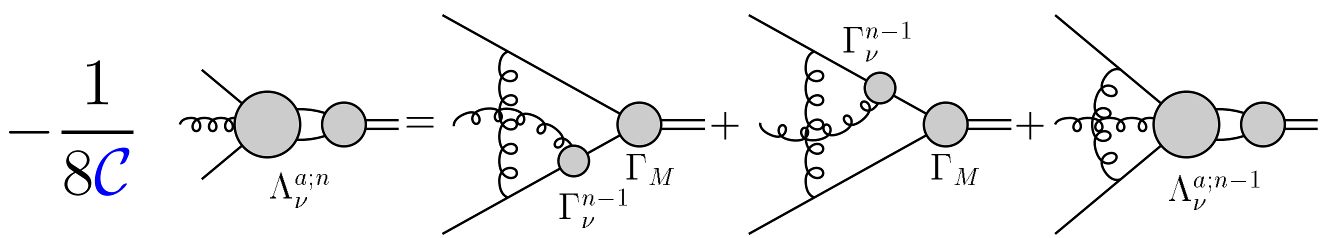


- Bethe-Salpeter kernel . . . recursion relation



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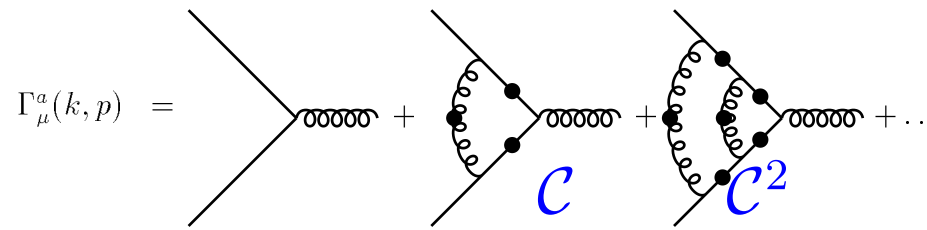

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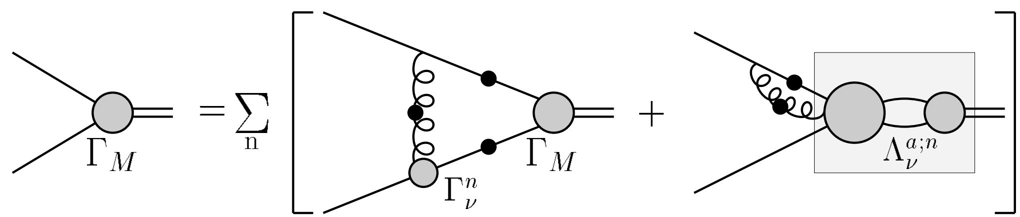

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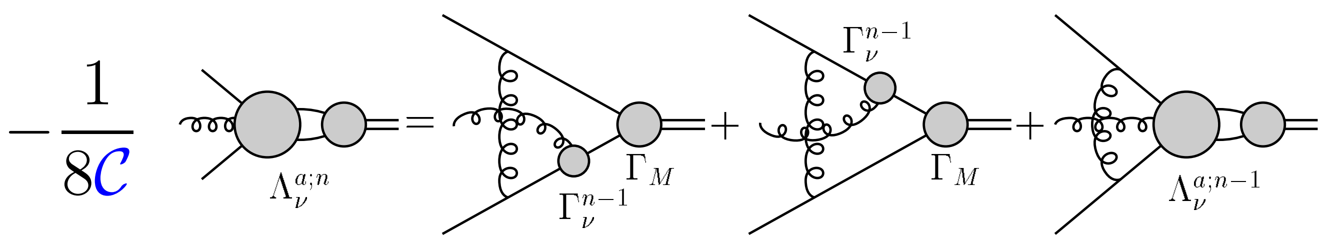

- Kernel necessarily non-planar,

  even with planar vertex

π and ρ mesons

π and ρ mesons

	$M_H^{n=0}$	$M_H^{n=1}$	$M_H^{n=2}$	$M_H^{n=\infty}$
π, $m=0$	0	0	0	0
π, $m=0.011$	0.147	0.135	0.139	0.138
ρ, $m=0$	0.920	0.648	0.782	0.754
ρ, $m=0.011$	0.936	0.667	0.798	0.770

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  Not constituent-quark-model-like hyperfine splitting

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- Extending kernel

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  For $\boldsymbol{m_\rho}$ – zeroth order, accurate to 20%

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  – one loop, accurate to 13%
  – two loop, accurate to 4%

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Ab-Initio **Calculations**

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Ab-Initio **Calculations**

Pieter Maris

Peter Tandy

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Ab-Initio **Calculations**

Maris & Tandy, Series of Five Articles: 1999 – Present

Perfected a Renormalisation-Group Improved Rainbow-Ladder Model of Quark-Quark Interaction

Ab-Initio **Calculations**

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- Rainbow-Ladder $=$ First Order in Truncation Described Above
- Anticipate Accurate for 0^- & 1^- Mesons

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  $$\boxed{\mathcal{E} \approx 700\,\text{MeV}}$$

- Dressed-Glue Mass scale:

  Characterises DCSB and light-quark Confinement

- Both Phenomena Disappear for $\mathcal{E} \lesssim 200\,\text{MeV}$

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- *Dyson-Schwinger equations: A Tool for Hadron Physics*

  P. Maris and C.D. Roberts, nu-th/0301049

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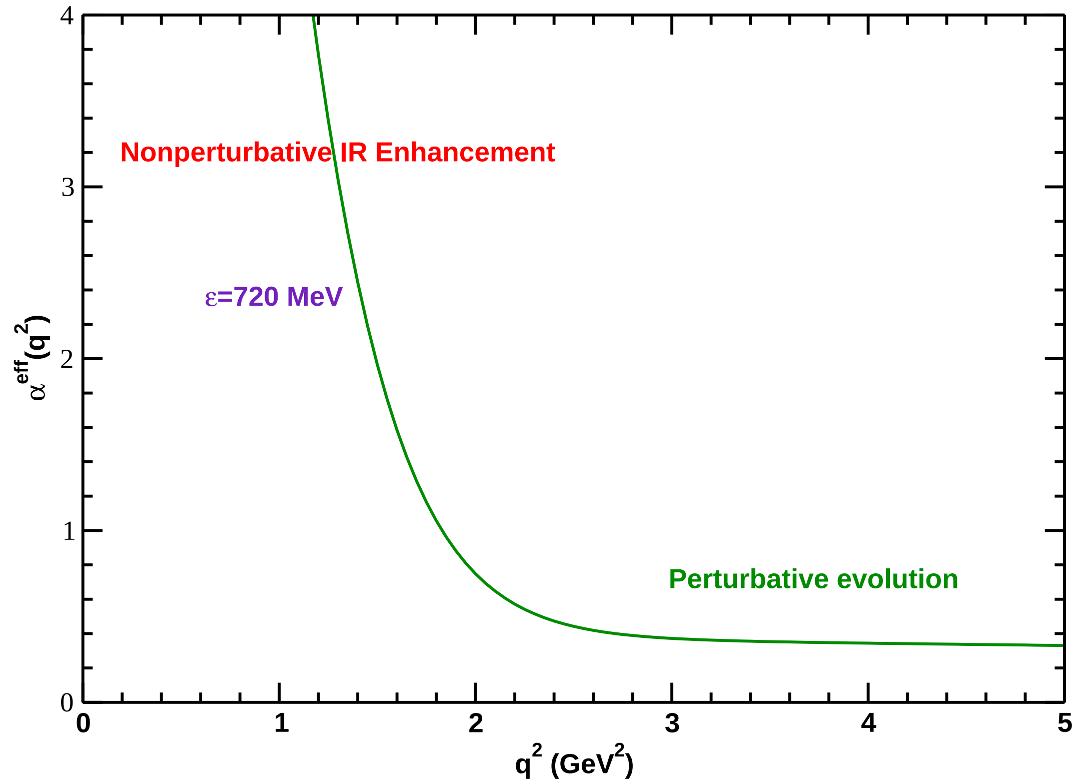
Interaction

Kernel of Bethe-Salpeter Equation

$$K(p,k;P) \approx \frac{\alpha^{\rm eff}((p-k)^2)}{(p-k)^2}$$



Interaction

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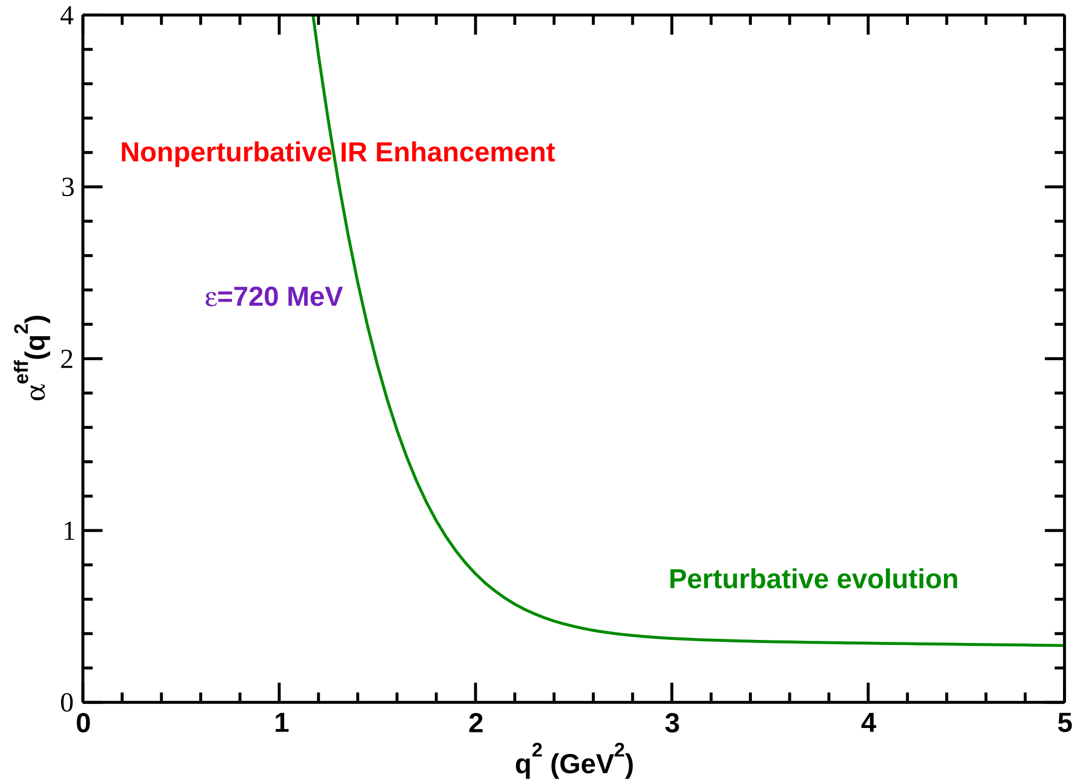

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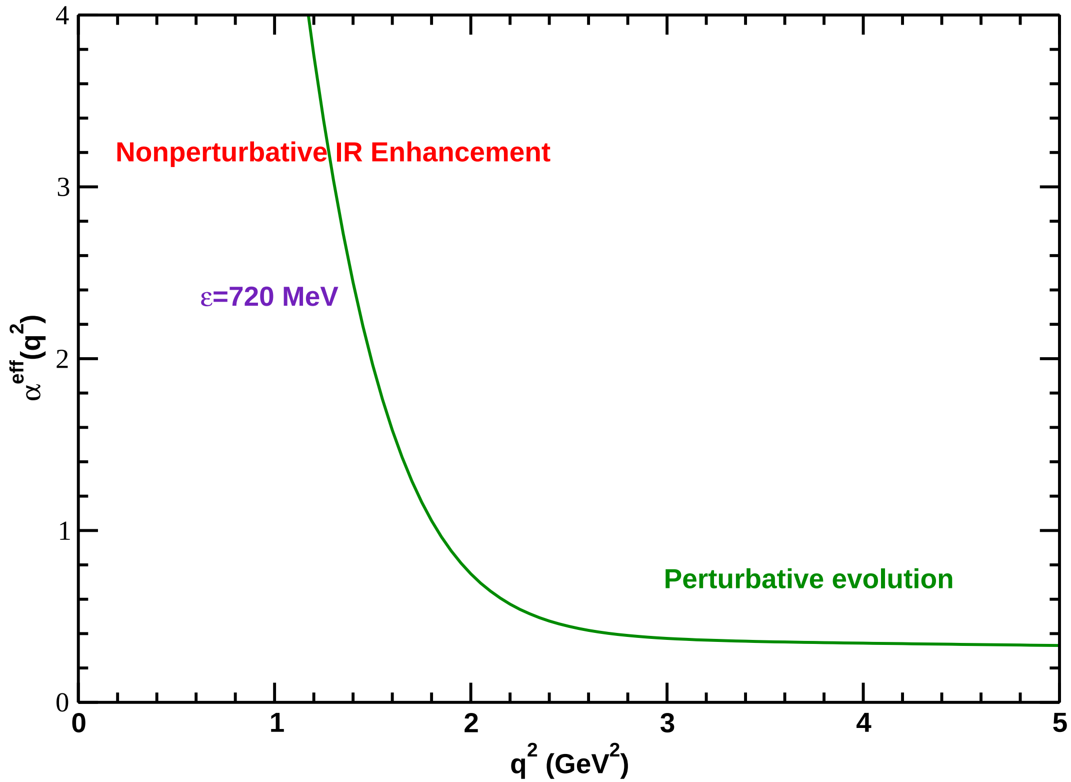


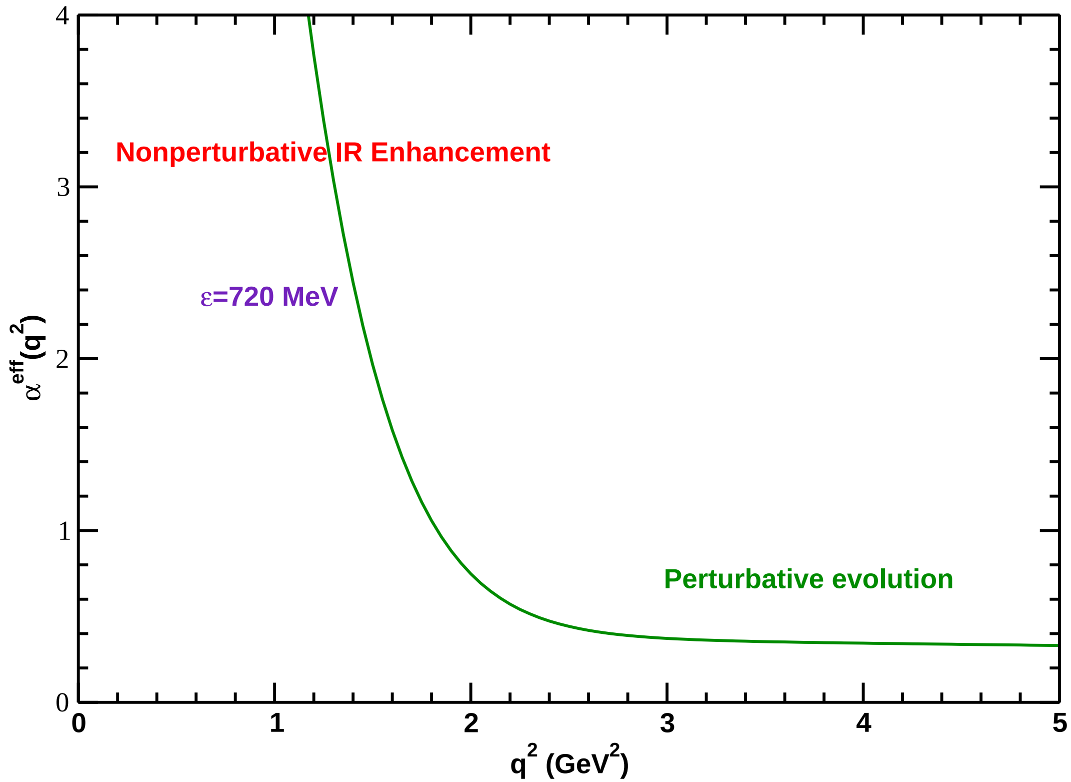
Prescribes Gap Equation's Kernel

Connects *Ansatz* for long-range part of QCD's interaction with Observables.

Kernel of Bethe-Salpeter Equation

$$K(p,k;P) \approx \frac{\alpha^{\text{eff}}((p-k)^2)}{(p-k)^2}$$

Prescribes Gap Equation's Kernel



IR-*Enhancement* at long-range agrees semi-quantitatively with Bhagwat, *et al*.

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Dressed-quark Propagator

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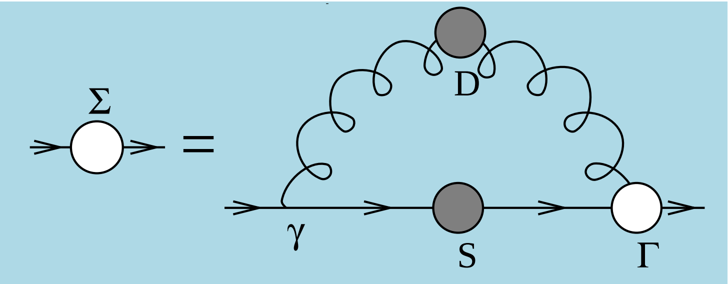
Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

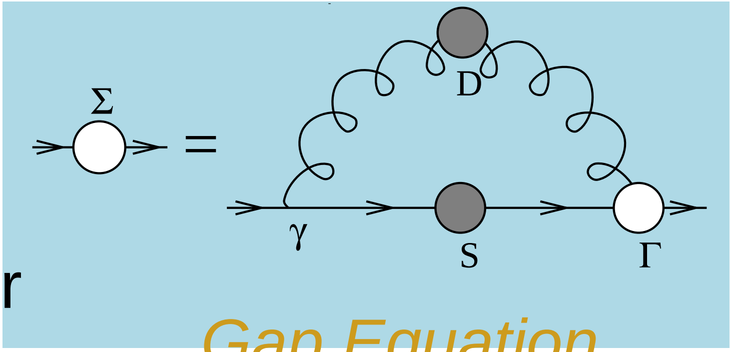


Gap Equation

Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

- dressed-quark propagator

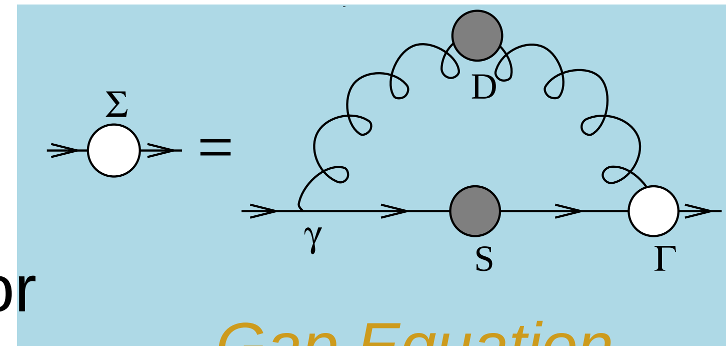


Gap Equation

$$S(p) \;=\; \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$



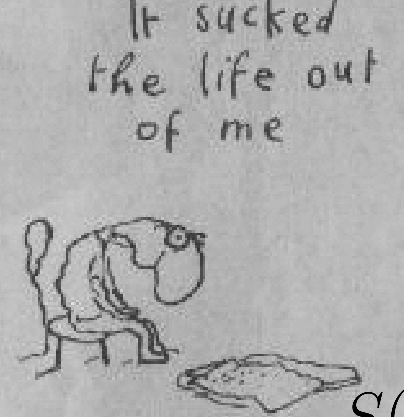
Gap Equation

- dressed-quark propagator

$$S(p) \quad = \quad \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory



Dressed-quark Propagator

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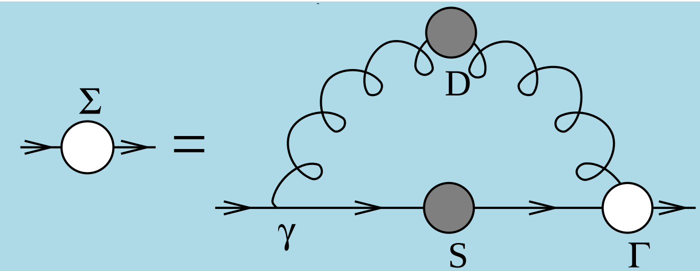

Gap Equation

- dressed-quark propagator

$$S(p) \;=\; \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

- But in Perturbation Theory

$$B(p^2) = m \left(1 - \frac{\alpha}{\pi} \ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\to} 0$$

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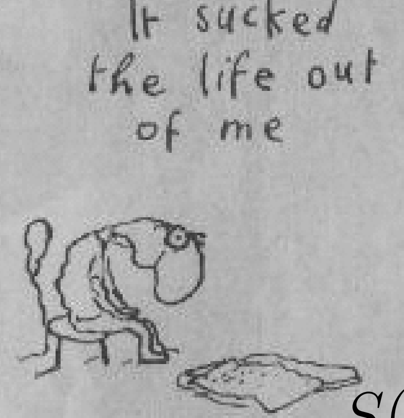

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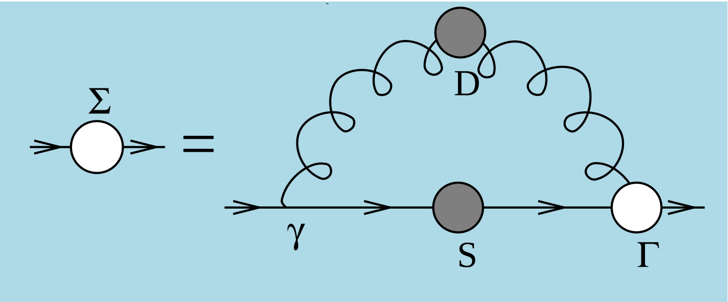
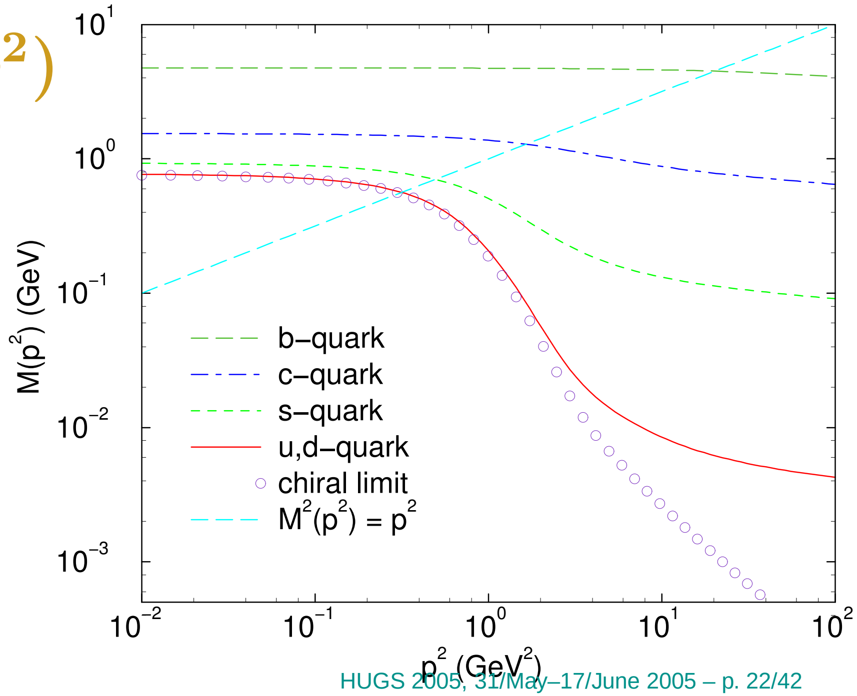


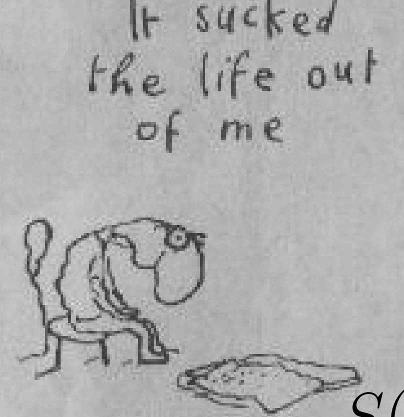
Gap Equation

- Enhancement of Gap Equation's Kernel on IR domain

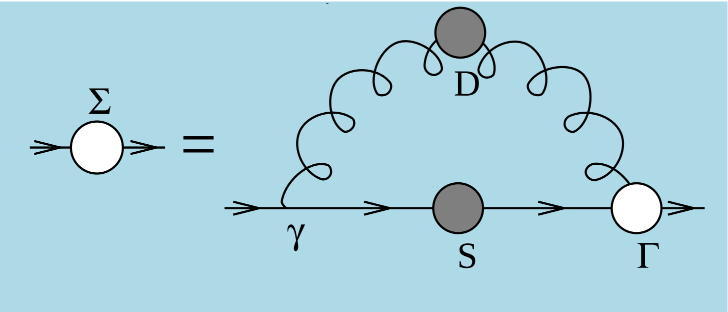
  $\Rightarrow$ IR Enhancement of $M(p^2)$



Dressed-quark Propagator



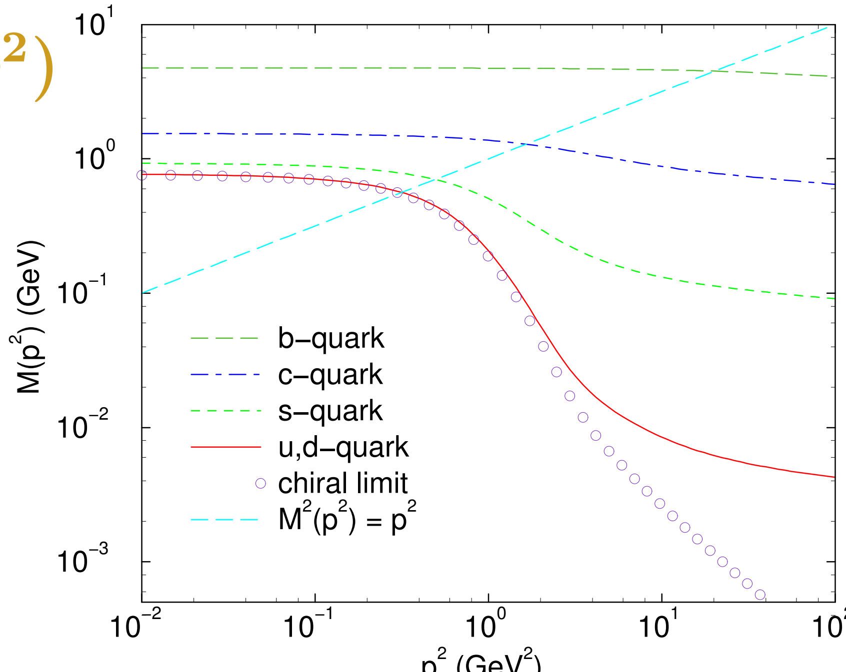
$$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$



Gap Equation

- Enhancement of Gap Equation's Kernel on IR domain
  $\Rightarrow$ IR Enhancement of $M(p^2)$

- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$



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Dressed-quark Propagator

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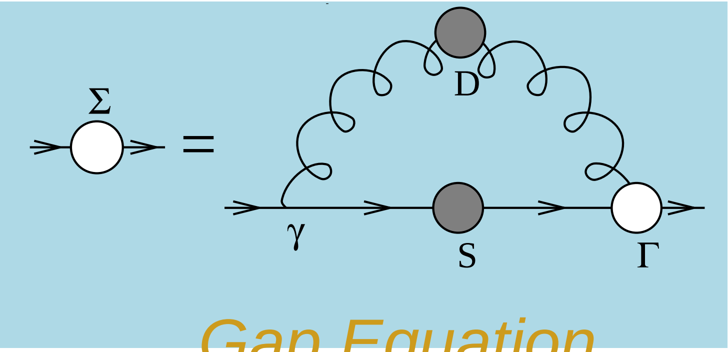
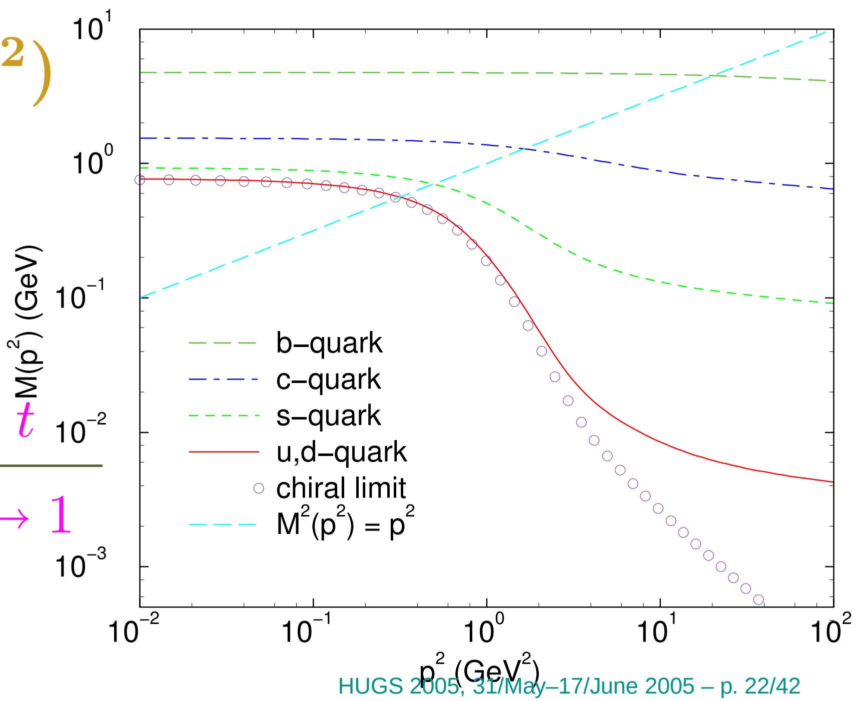


Gap Equation

- Enhancement of Gap Equation's Kernel on IR domain
  $\Rightarrow$ IR Enhancement of $M(p^2)$

- Euclidean Constituent–Quark Mass: M_f^E: $p^2 = M(p^2)^2$

flavour	u/d	s	c	b	t
$\frac{M^E}{m_{\mu\sim 20\ \mathrm{GeV}}}$	$\sim 10^2$	~ 10	1.4	1.1	$\to 1$
			-2.2	-1.2	



Vacuum quark condensate

Vacuum quark condensate

We've seen this before

. . . Trace of the Chiral-limit dressed-quark propagator

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Vacuum quark condensate

$$-\left.\langle \bar{q}q\rangle_\zeta^0\right|_{\zeta=19\,\mathsf{GeV}} =$$

$$Z_4(\zeta,\Lambda)\;\; N_c \int_q^\Lambda \mathrm{tr}_{\mathrm{Dirac}}\left[S_{\hat{m}=0}(q)\right] = \;\; (0.275\,\mathrm{GeV})^3$$

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Vacuum quark condensate

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$$Z_4(\zeta,\Lambda)\, N_c \int_q^\Lambda \mathrm{tr}_{\mathrm{Dirac}} \left[S_{\hat{m}=0}(q) \right] = \; (0.275\,\mathrm{GeV})^3$$

- Mass renormalisation constant

  Ensures finiteness and correct renormalisation group flow

  $m(\zeta)\,\langle \bar{q} q \rangle_\zeta^0 =$ const.

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Vacuum quark condensate

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- Mass renormalisation constant

  Ensures finiteness and correct renormalisation group flow

  $m(\zeta)\,\langle \bar{q}q \rangle_\zeta^0 =$ const.

  AND ensures Gauge Invariance

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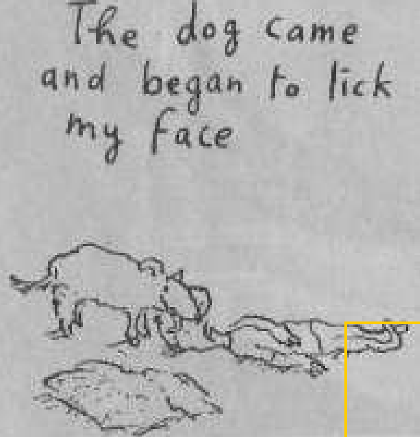


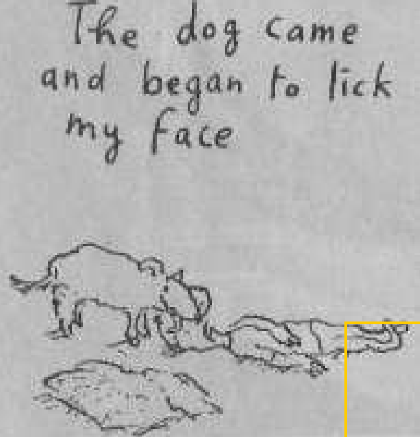
Vacuum quark condensate

$$-\left.\langle \bar{q} q \rangle_\zeta^0\right|_{\zeta=19\,\mathsf{GeV}} =$$

Numerical Value

π and K observables

$$Z_4(\zeta,\Lambda)\ \ N_c \int_q^\Lambda \mathrm{tr}_{\mathrm{Dirac}} \left[S_{\hat{m}=0}(q)\right] = (0.275\,GeV)^3$$



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Corresponds to

$$-\left.\langle\bar{q}q\rangle_\zeta^0\right|_{\zeta=1\,\mathrm{GeV}} := \left(\ln\left[1/\Lambda_{\rm QCD}\right]\right)^{\gamma_m}\;\langle\bar{q}q\rangle^0 = (0.241\,\mathrm{GeV})^3$$

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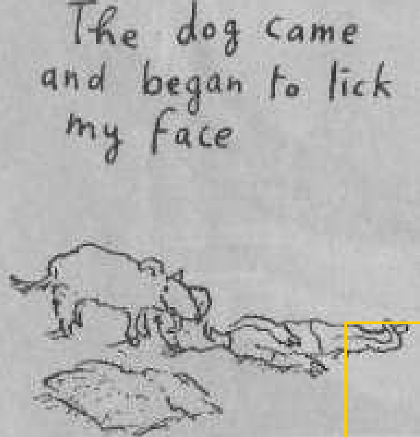

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Close packed spheres: $1.8\,\mathrm{fm}^{-3}$

$$\Rightarrow V_{\langle\bar{q}q\rangle} = 0.55\,\mathrm{fm}^3 \Rightarrow r_{\langle\bar{q}q\rangle} = 0.51\,\mathrm{fm} = 0.77\,r_\pi = 0.58\,r_p$$

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QCD's Mass Gap

Sea

of Virtual Particle-Antiparticle Pairs

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cf. Quenched lattice simulations

Contemporary Supercomputer Resources

– 1 Teraflop Sustained

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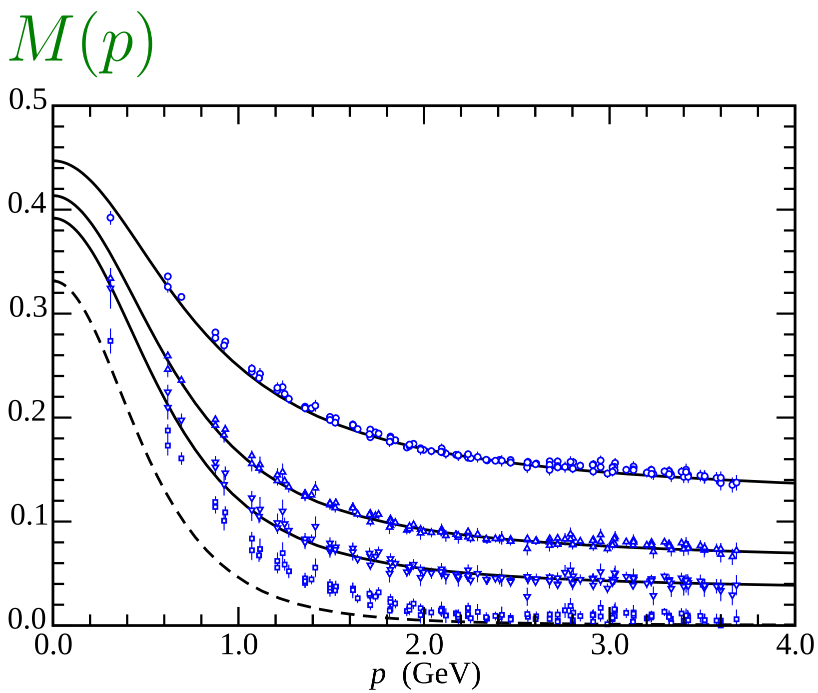
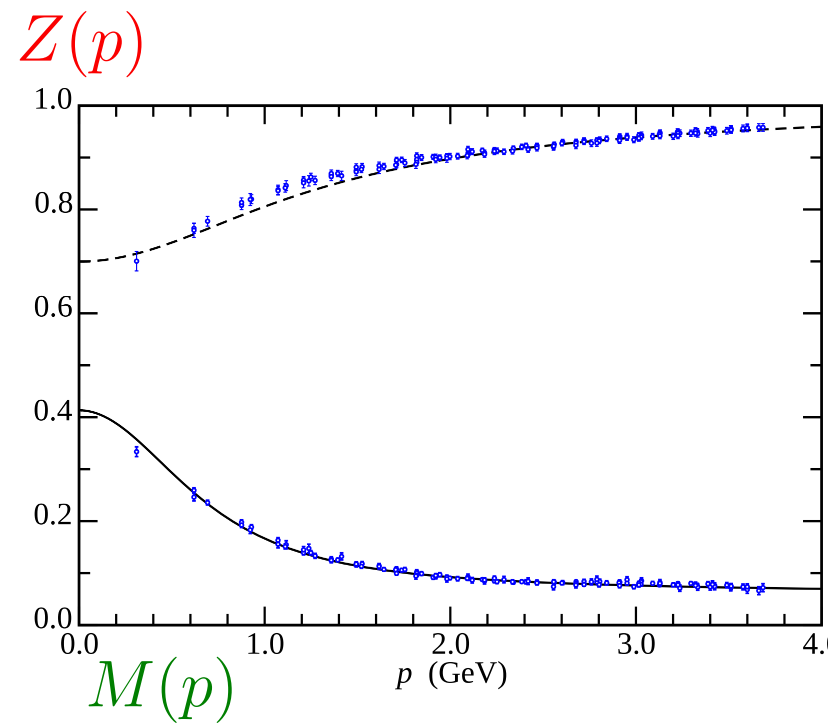
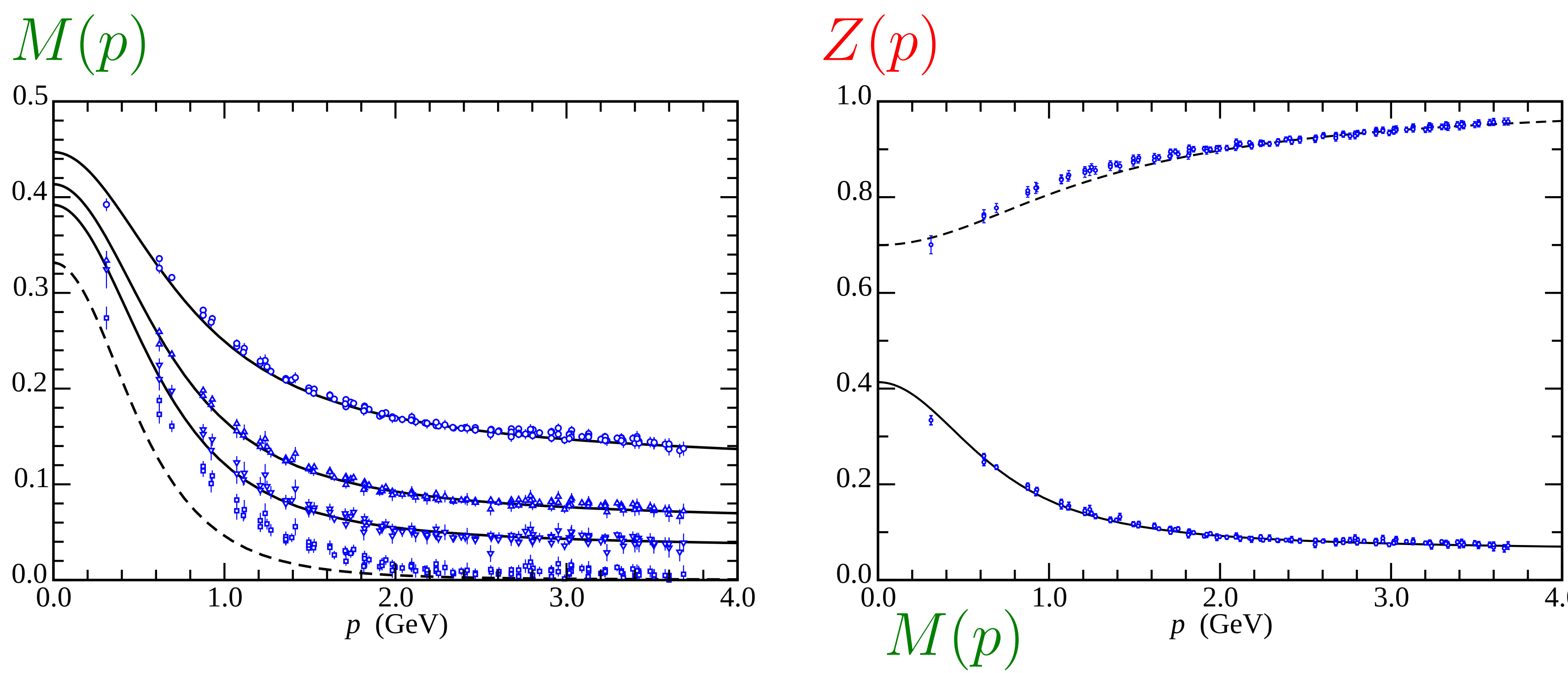
cf. Quenched lattice simulations





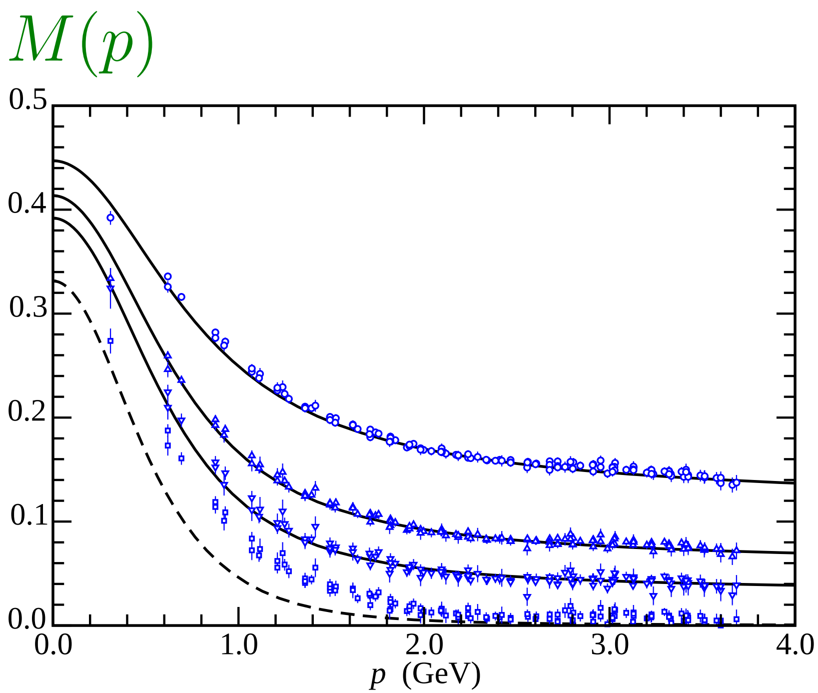
- Lattice Meas.
  - – Bowman, Heller, Leinweber, Williams: he-lat/0209129

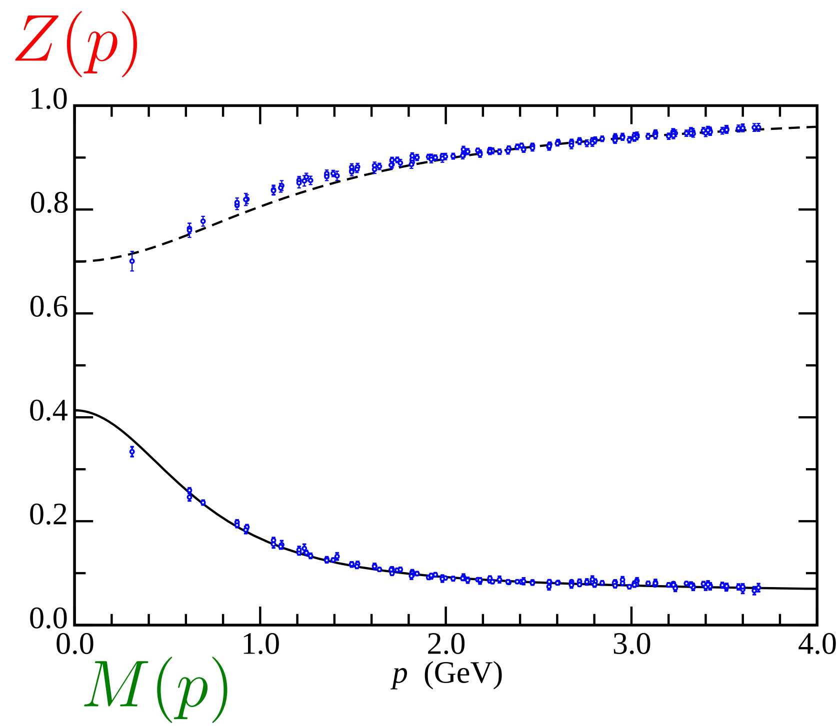
cf. Quenched lattice simulations



- DSE Cal.– Bhagwat, Pichowsky, CDR, Tandy nu-th/0304003

cf. Quenched lattice simulations





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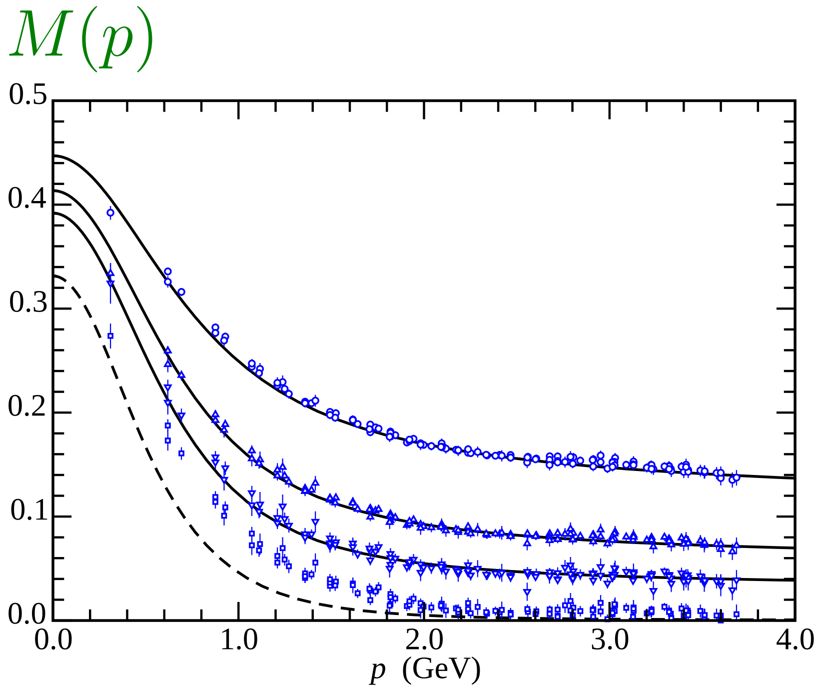
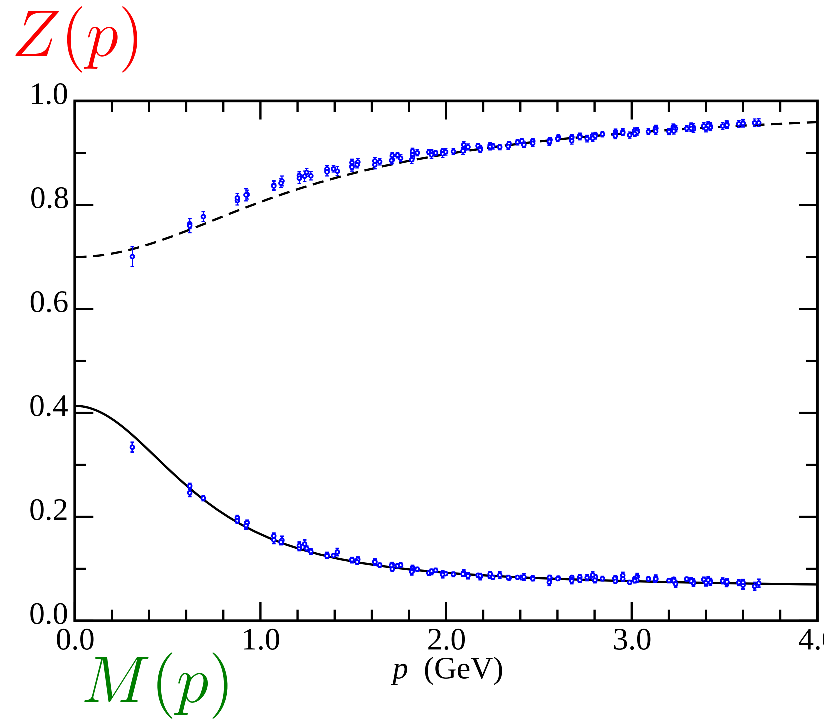
- DSE Cal.– Bhagwat, Pichowsky, CDR, Tandy nu-th/0304003

$$f_\pi^0 = 0.068\,\mathrm{GeV} \quad \langle \bar{q}q \rangle^0_{1\,\mathrm{GeV}} = (-0.19\,\mathrm{GeV})^3\,.$$

cf. Quenched lattice simulations





- *ab initio* support for DSE conjecture

  Mass from Nothing is a Reality!

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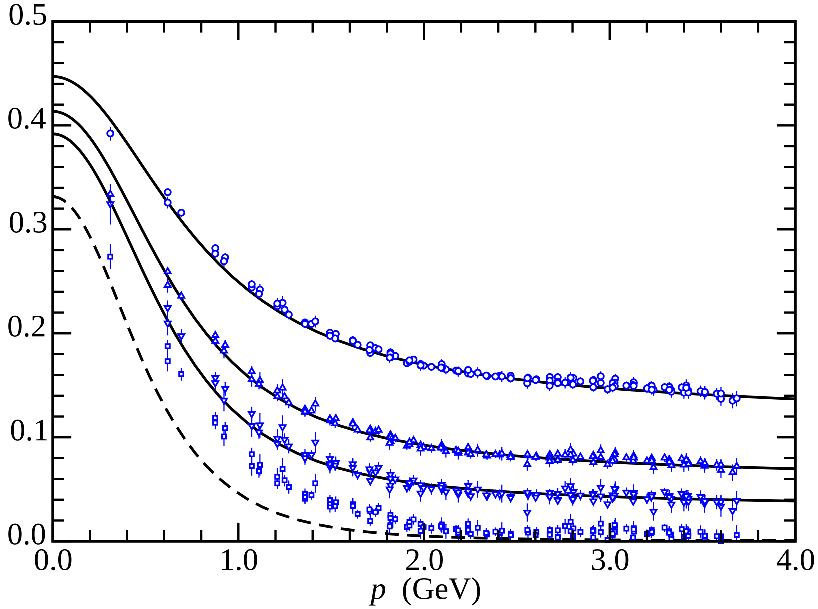
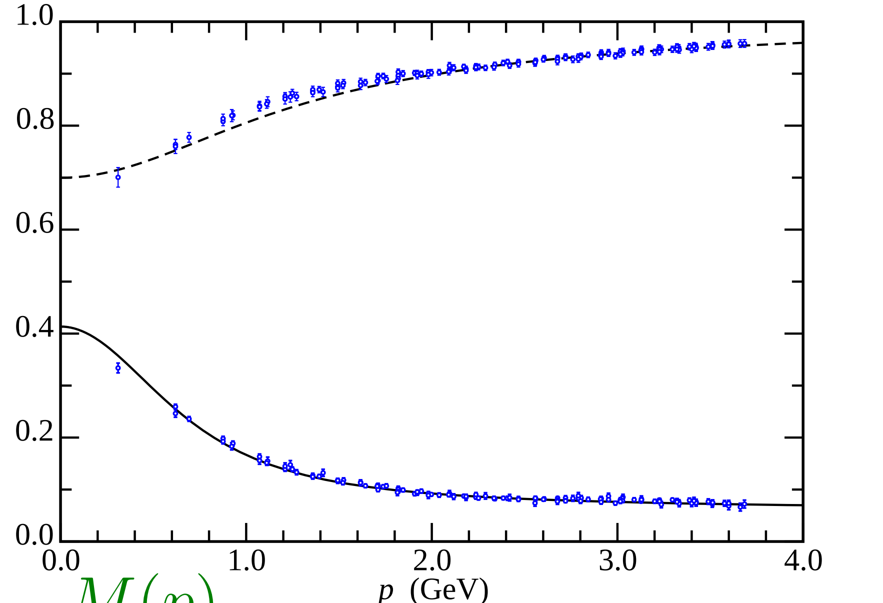
cf. Quenched lattice simulations

$M(p)$



$Z(p)$



- *ab initio* support for DSE conjecture
  Mass from Nothing is a Reality!
- More than 98% of Spectrum/Constituent-Quark's Mass
  = cloud of strongly-interacting virtual quanta

Pion Form Factor

Procedure Now Straightforward

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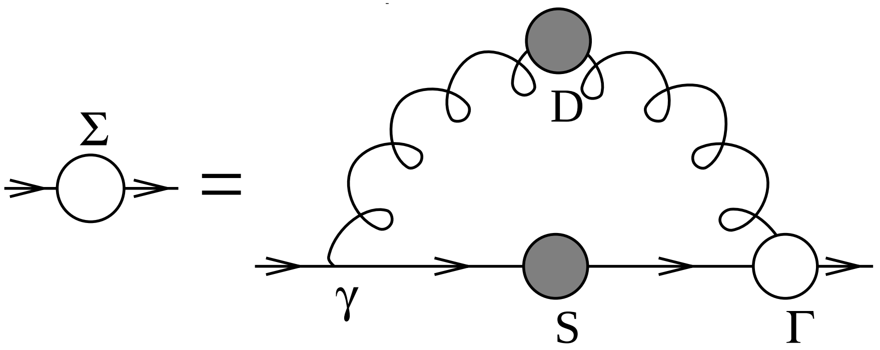
Pion Form Factor

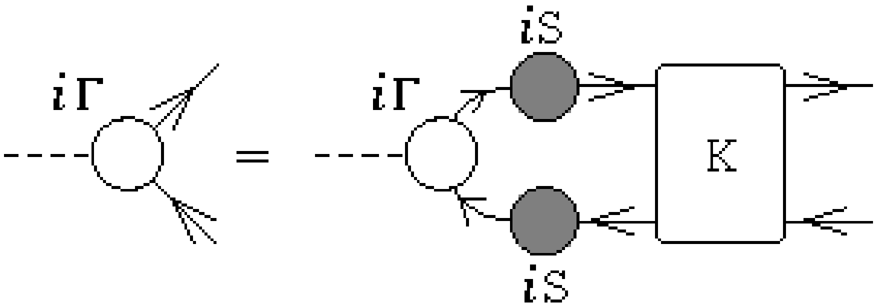
- Solve Gap Equation

  $\Rightarrow$ Dressed-Quark Propagator, $S(p)$



Pion Form Factor

- Use that to Complete Bethe Salpeter Kernel, K
- Solve Homogeneous Bethe-Salpeter Equation for Pion Bethe-Salpeter Amplitude, Γ_π



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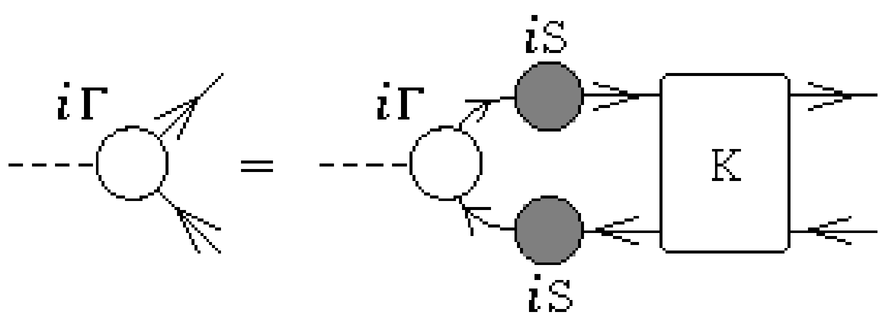
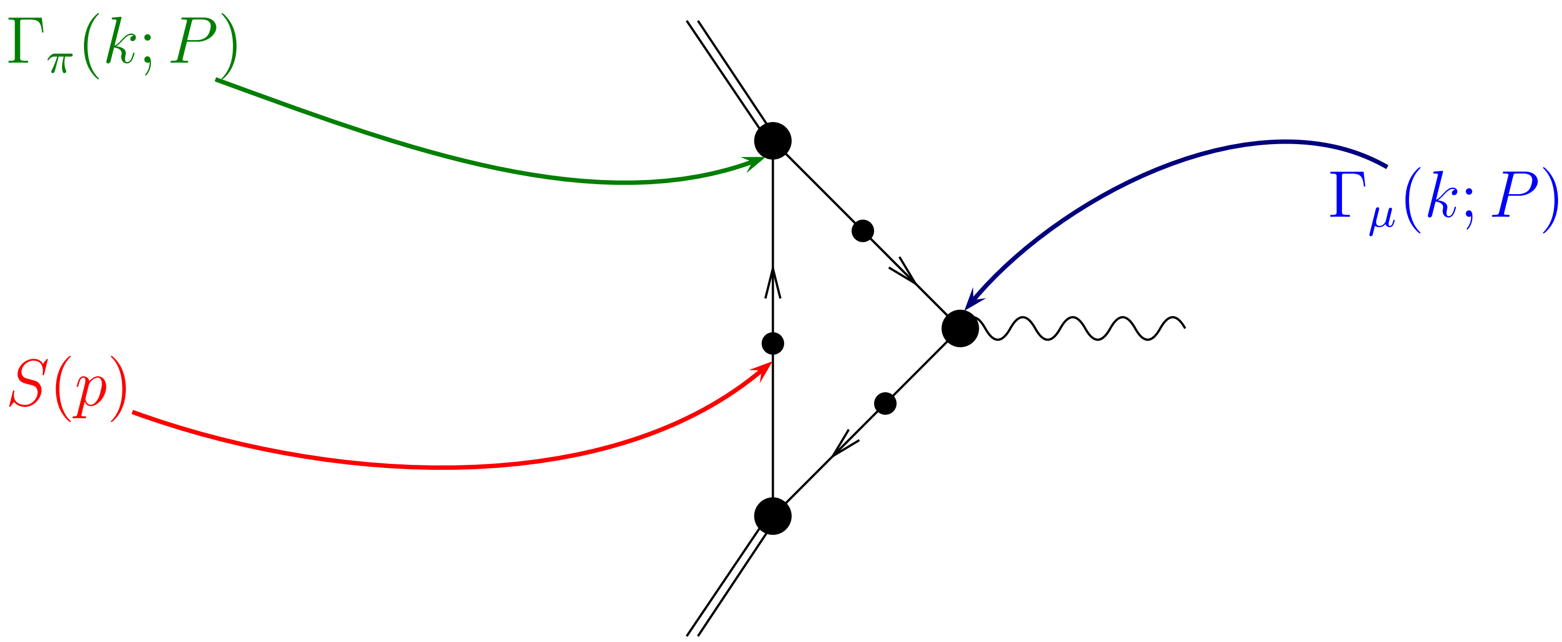


- Solve Inhomogeneous Bethe-Salpeter Equation for Dressed-Quark-Gluon Vertex, Γ_μ

Pion Form Factor

- Now have all elements for Impulse Approximation to Electromagnetic Pion Form factor



Pion Form Factor

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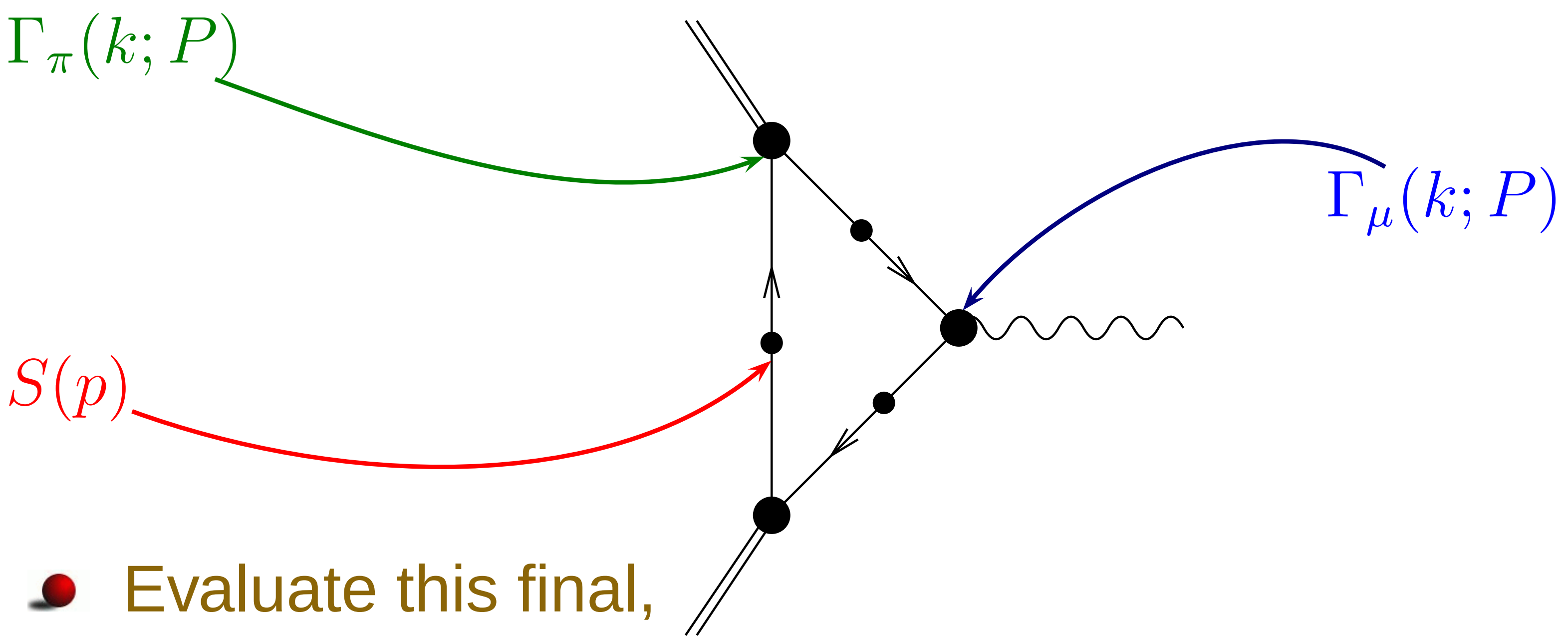
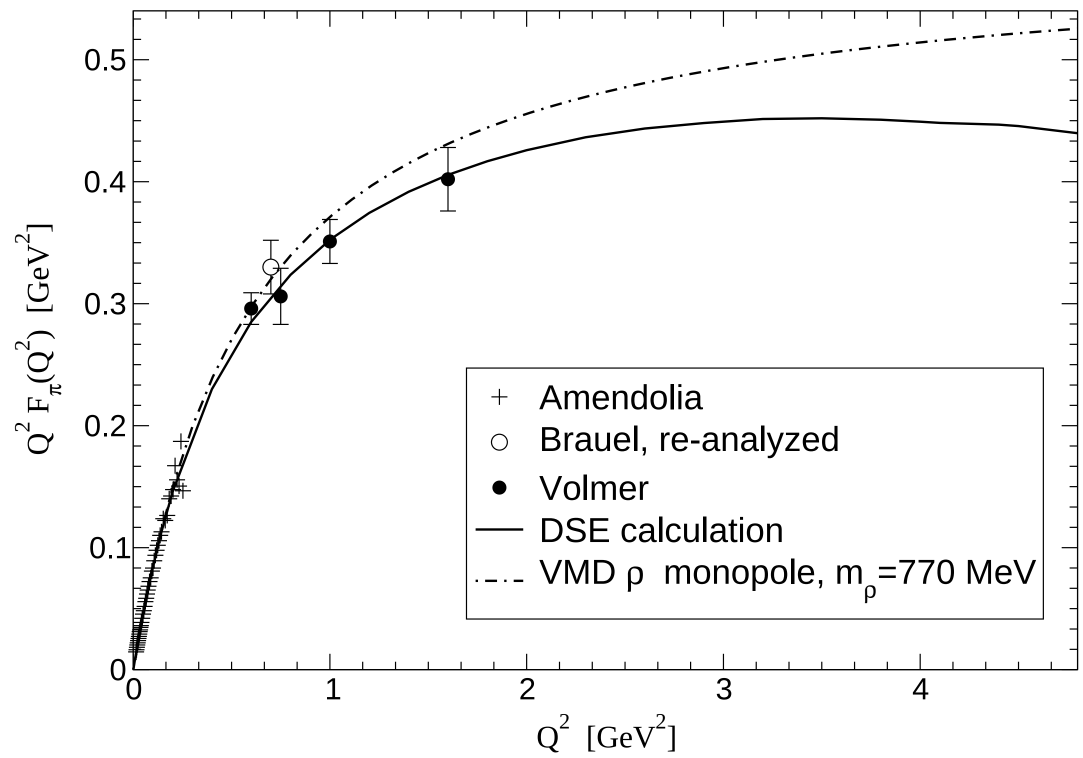


- Evaluate this final, three-dimensional integral

Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied



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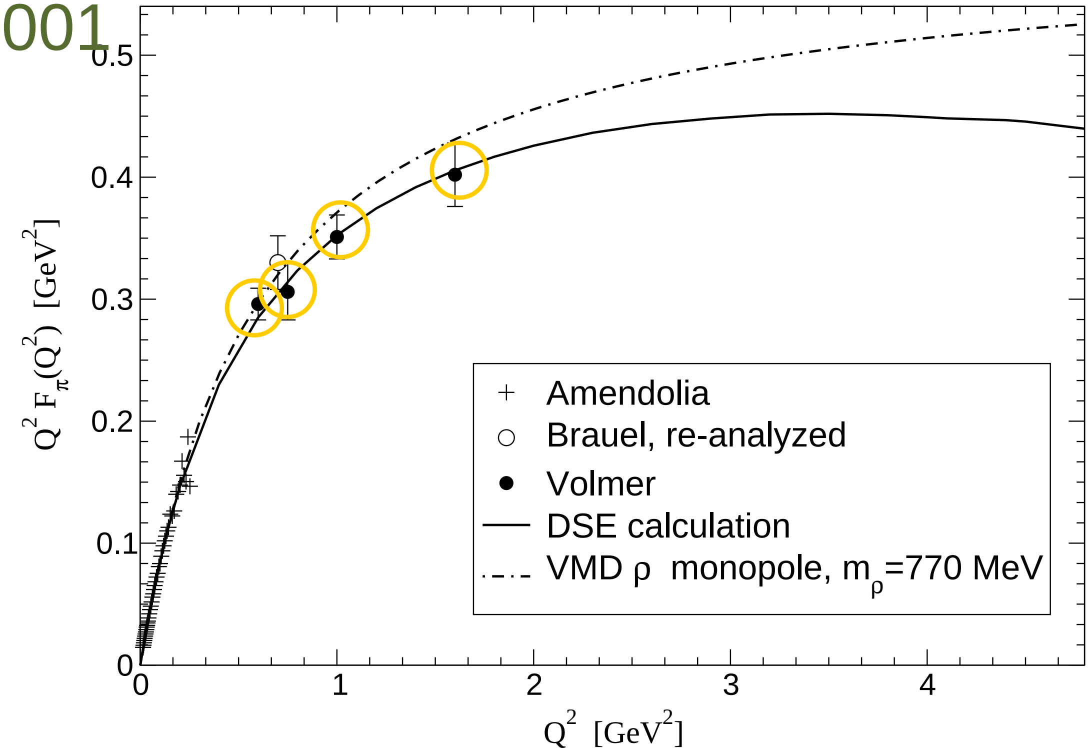
Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

Data published in 2001



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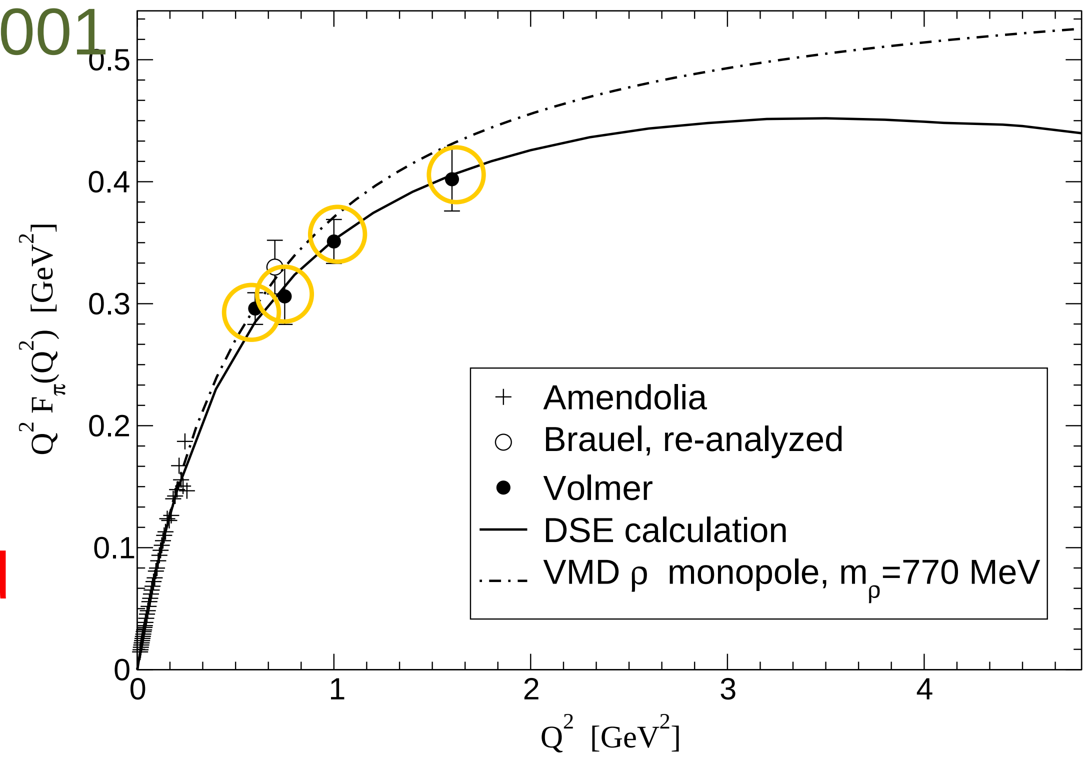
Calculated Pion Form Factor

Calculation published in 1999; No Parameters Varied

Data published in 2001



Many subsequent successful applications

. . Again, parameters Fixed

Notably $\pi\ \pi$ Scattering

Maris, et al., Phys. Rev. D 65, 076008

Bicudo, Phys. Rev. C 67, 035201

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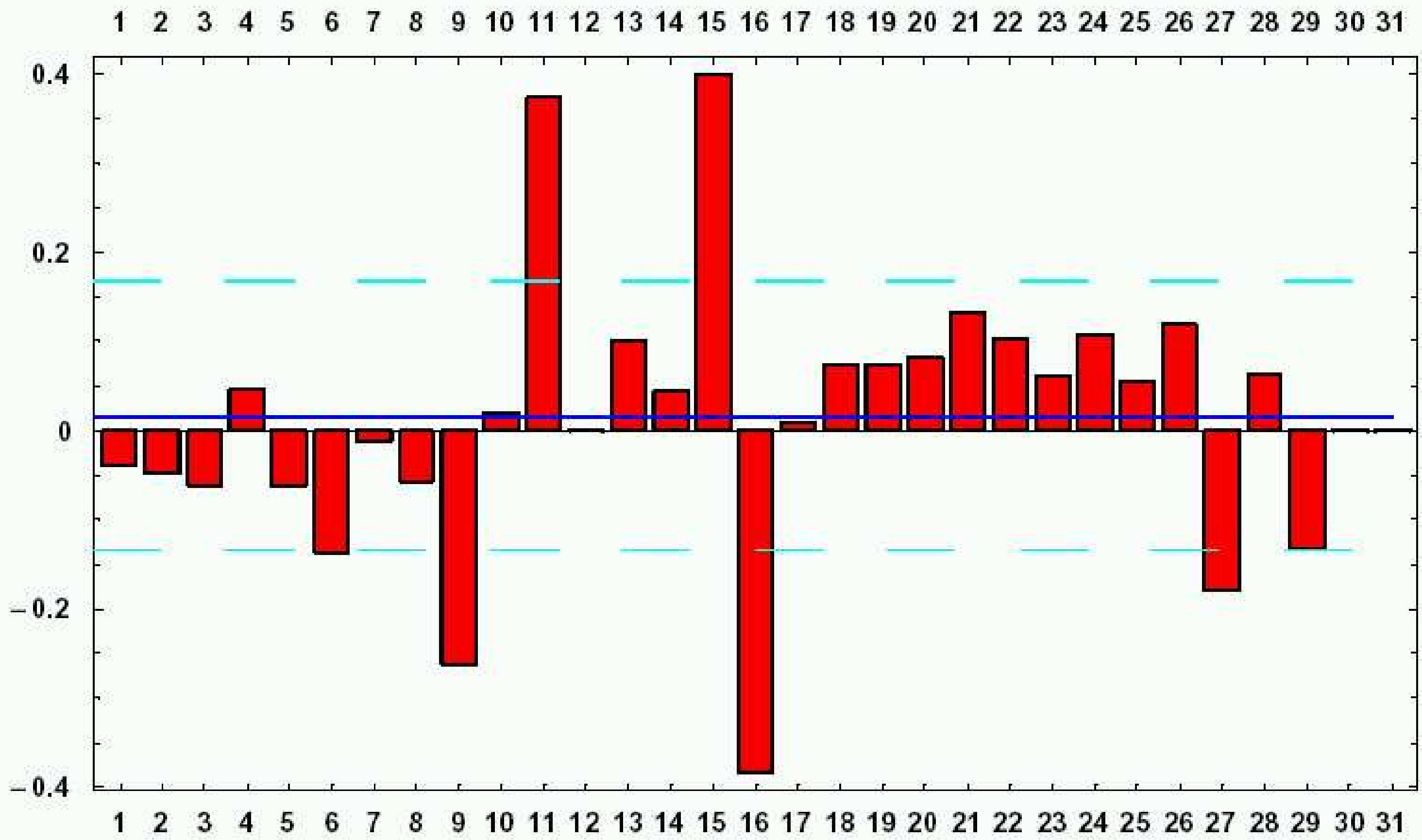
Maris & Tandy
Relative Error



Relative Error, Predictions of Maris and Tandy Model
All tabulated quantities in nu-th/0301049
<error> = 1.6%, Sqrt[<error^2>] = 15%

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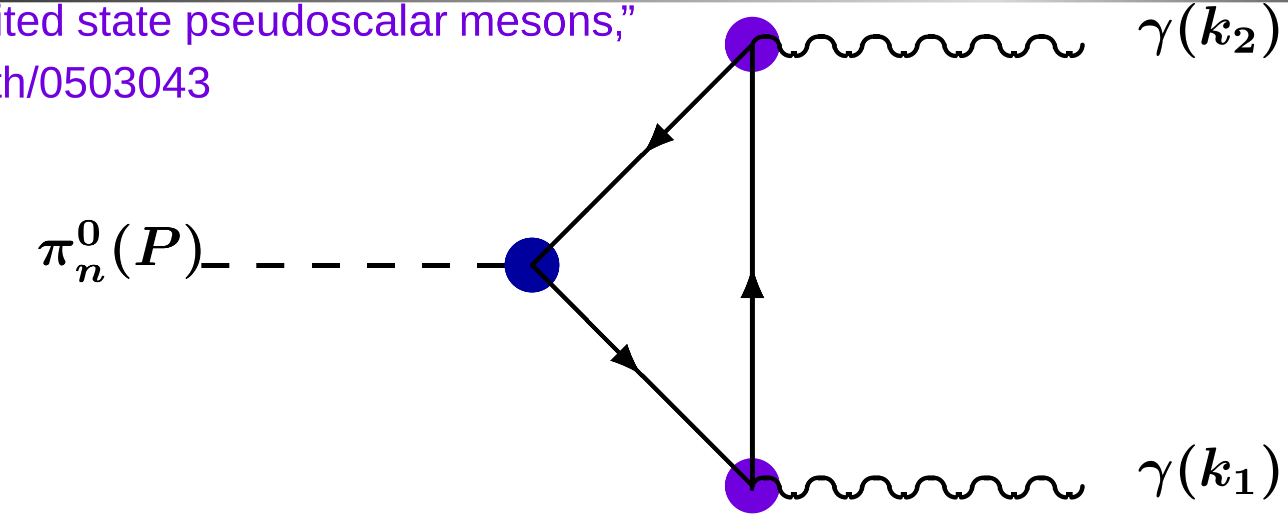
Two-photon Couplings of Pseudoscalar Mesons

Höll, Krassnigg, Maris, *et al.*, "Electromagnetic properties of ground and excited state pseudoscalar mesons," nu-th/0503043



- $$T^{\pi^0_n}_{\mu\nu}(k_1,k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}\, G^{\pi^0_n}(k_1,k_2)$$
  - Define: $\mathcal{T}_{\pi^0_n}(P^2,Q^2) = G^{\pi^0_n}(k_1,k_2)\Big|_{k_1^2=Q^2=k_2^2}$

    This is a transition form factor.
  - Physical Processes described by couplings:

    $g_{\pi^0_0\gamma\gamma} := \mathcal{T}_{\pi^0_0}(-m^2_{\pi^0_0}, 0)$

    Width: $\Gamma_{\pi^0_n\gamma\gamma} = \alpha^2_{\rm em}\,\frac{m^3_{\pi_n}}{16\pi^3}\, g^2_{\pi_n\gamma\gamma}$

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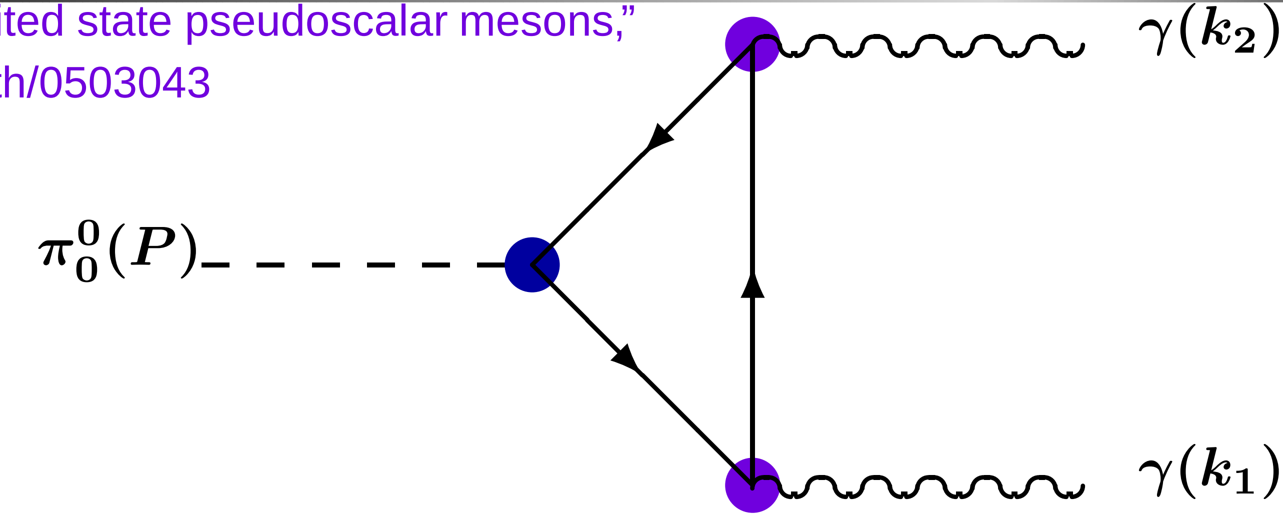
Two-photon Couplings: Goldstone Mode

Höll, Krassnigg, Maris, *et al.*,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043



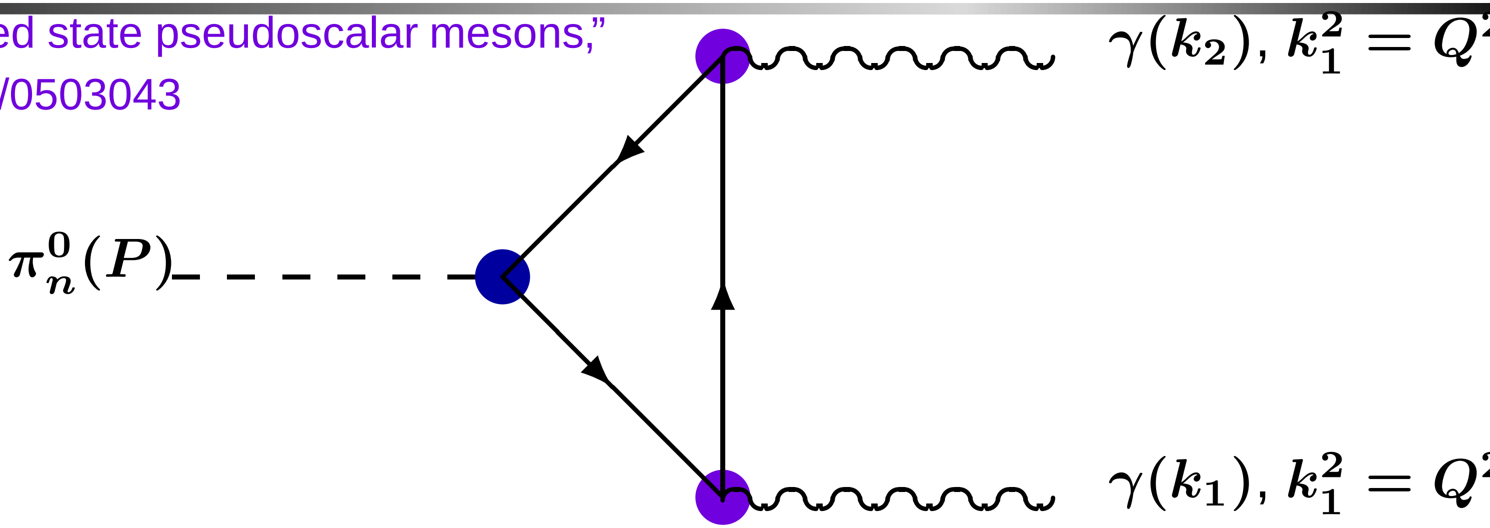
- $$T_{\mu\nu}^{\pi_0^0}(k_1,k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}\, G^{\pi_0^0}(k_1,k_2)$$
  - Chiral limit, model-independent and algebraic result

    $$g_{\pi_0^0\gamma\gamma} := \mathcal{T}_{\pi_0^0}(-m_{\pi_0^0}^2 = 0, 0) = \frac{1}{2}\frac{1}{f_{\pi_0}}$$

    So long as truncation preserves chiral symmetry and the pattern of its dynamical breakdown
- The most widely known consequence of the Abelian anomaly

Two-photon Couplings: Transition Form Factor

Höll, Krassnigg, Maris, *et al*.,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043



- $$T_{\mu\nu}^{\pi_n^0}(k_1,k_2) = \frac{\alpha}{\pi} i\varepsilon_{\mu\nu\rho\sigma} k_{1\rho} k_{2\sigma}\, G^{\pi_n^0}(k_1,k_2)$$
  - So long as truncation preserves chiral symmetry and the pattern of its dynamical breakdown, and the one-loop renormalisation group properties of QCD: model-independent result – $\forall n$:
  $$\mathcal{T}_{\pi_n^0}(P^2,Q^2) = G^{\pi_n^0}(k_1,k_2)\Big|_{k_1^2=Q^2=k_2^2} \overset{Q^2\gg\Lambda_{\rm QCD}^2}{=} \frac{4\pi^2}{3}\frac{f_{\pi_n}}{Q^2}$$

Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al*.,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$

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Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al.*,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$
- BUT, $f_{\pi_n} \equiv 0$, $\forall n$!

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Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al*.,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$
- BUT, $f_{\pi_n} \equiv 0$, $\forall n$!
- Model-independent result, in chiral limit: $\forall n \geq 1$

$$\lim_{\hat{m}\to 0} \mathcal{T}_{\pi_n^0}(-m_{\pi_n}^2, Q^2) \overset{Q^2 \gg \Lambda_{\rm QCD}^2}{=} \frac{4\pi^2}{3} F_n^{(2)}(-m_{\pi_n}^2) \left.\frac{\ln^\gamma Q^2/\omega_{\pi_n}^2}{Q^4}\right|_{\hat{m}=0}$$

where:

- γ is an anomalous dimension
- ω_{π_n} is a width mass-scale

both determined, in part, by properties of the meson's Bethe-Salpeter wave function.

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Transition Form Factor: Chiral limit

Höll, Krassnigg, Maris, *et al*.,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043

- Chiral limit with DCSB: $f_{\pi_0} \neq 0$
- BUT, $f_{\pi_n} \equiv 0$, $\forall n$!
- Model-independent result, in chiral limit: $\forall n \geq 1$

$$\lim_{\hat{m}\to 0} \mathcal{T}_{\pi_n^0}(-m_{\pi_n}^2, Q^2)$$

$$\stackrel{Q^2 \gg \Lambda_{\rm QCD}^2}{=} \frac{4\pi^2}{3} F_n^{(2)}(-m_{\pi_n}^2) \frac{\ln^\gamma Q^2/\omega_{\pi_n}^2}{Q^4}\bigg|_{\hat{m}=0}$$

where:
  - γ is an anomalous dimension
  - ω_{π_n} is a width mass-scale

both determined, in part, by properties of the meson's Bethe-Salpeter wave function.

- Importantly, $F_n^{(2)}(-m_{\pi_n}^2) \not\propto f_{\pi_n}$. Instead, it is determined by DCSB mass-scales for π_n that do not vanish in the chiral limit.

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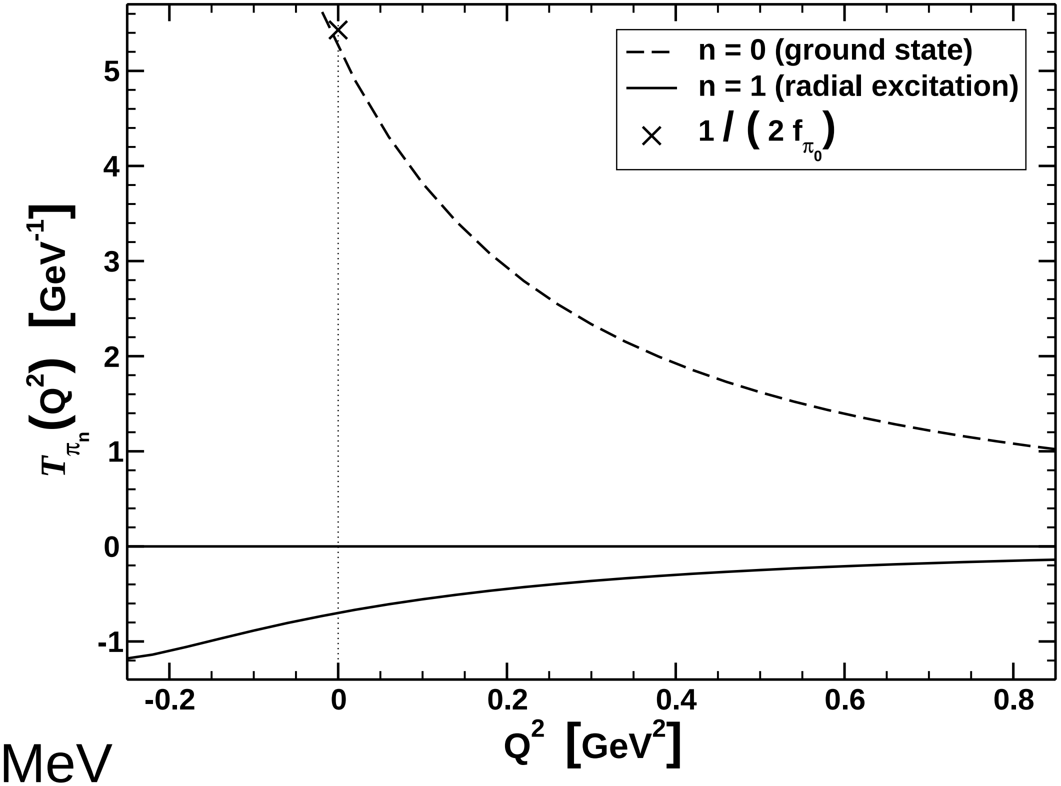
Calculated Transition Form Factor: RGI Rainbow-Ladder

Höll, Krassnigg, Maris, *et al*.,
"Electromagnetic properties of ground and excited state pseudoscalar mesons,"
nu-th/0503043



- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5$ MeV

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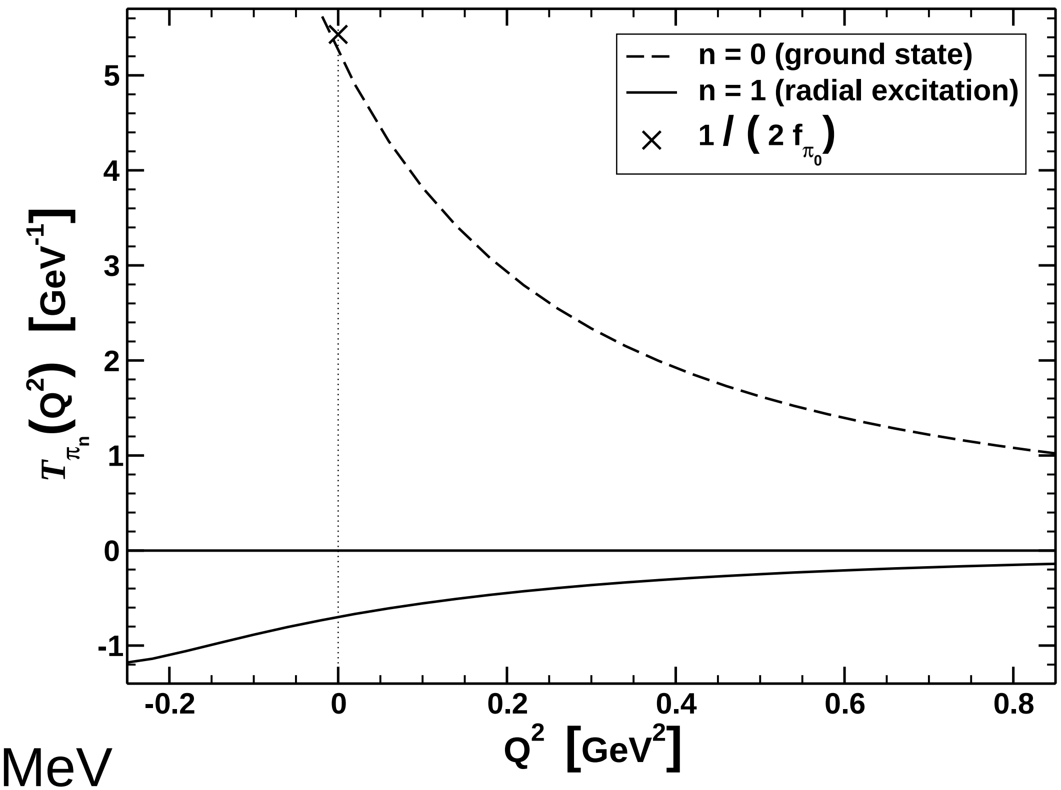

- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5$ MeV
- $\mathcal{T}_{\pi_1^0}(-m_{\pi_1}^2, Q^2) < 0\,,\ Q^2 \geq -m_{\pi_1}^2/4$;
  viz., it is negative on the entire kinematically accessible domain.

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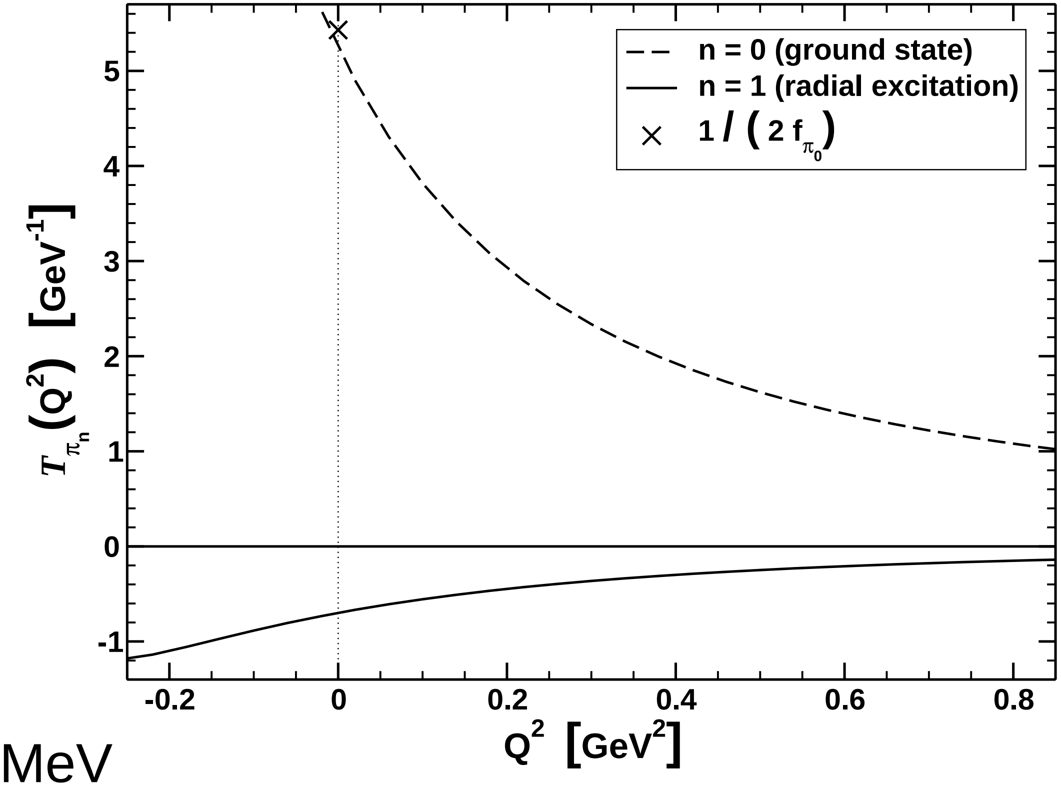

- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5$ MeV
- $\mathcal{T}_{\pi_1^0}(-m_{\pi_1}^2, Q^2) < 0\,,\ Q^2 \geq -m_{\pi_1}^2/4;$ viz., it is negative on the entire kinematically accessible domain.
- $\Gamma_{\pi_0^0\gamma\gamma} = 7.9$ eV, $\Gamma_{\pi_1^0\gamma\gamma} = 240$ eV

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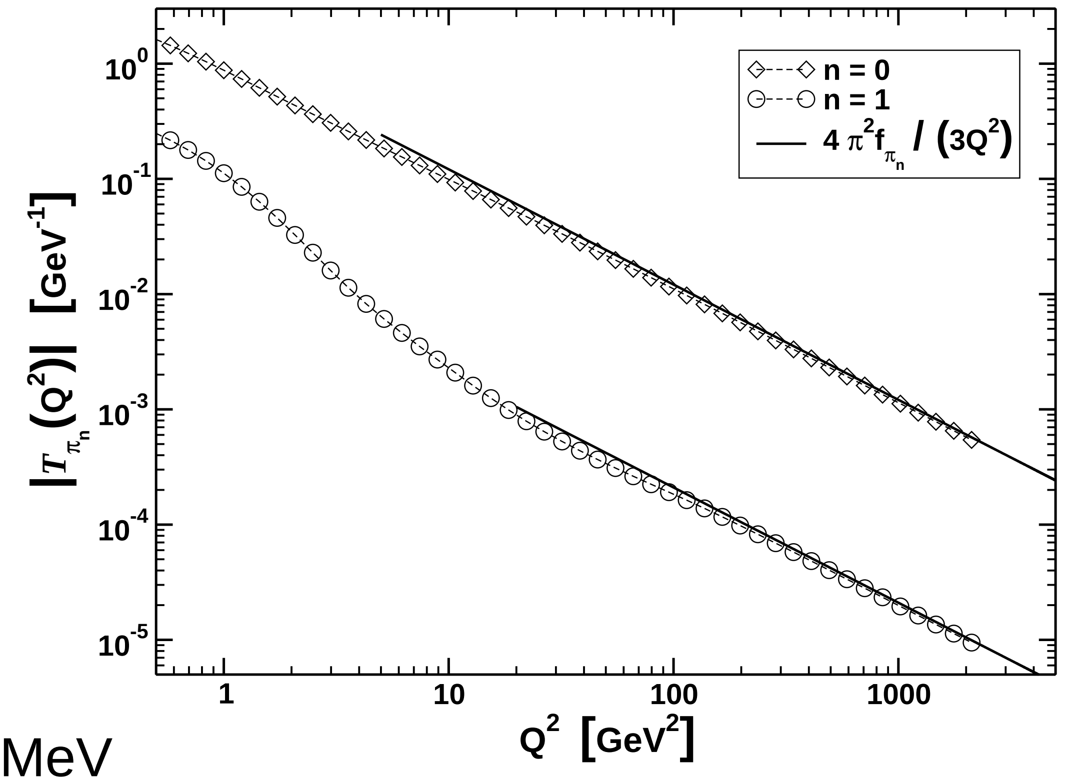

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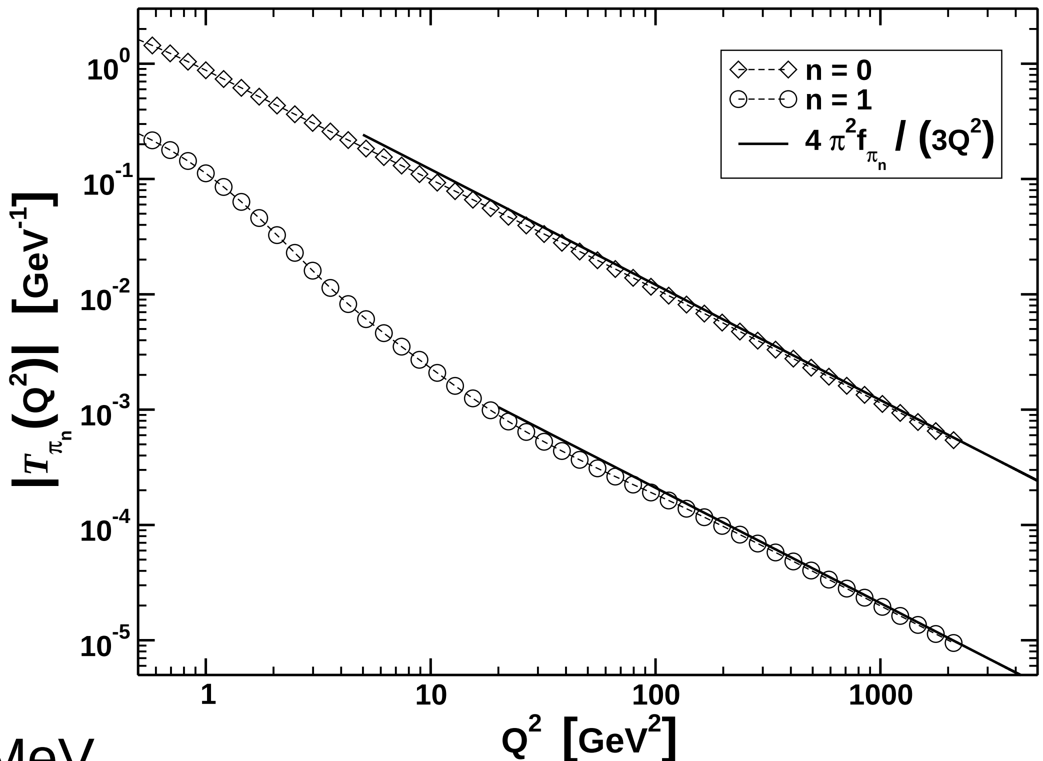

- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5$ MeV
- Predicted UV-behaviour is abundantly clear
  - precise for $Q^2 > 120\,$GeV^2

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Transition Form Factor (Chiral): RGI Rainbow-Ladder

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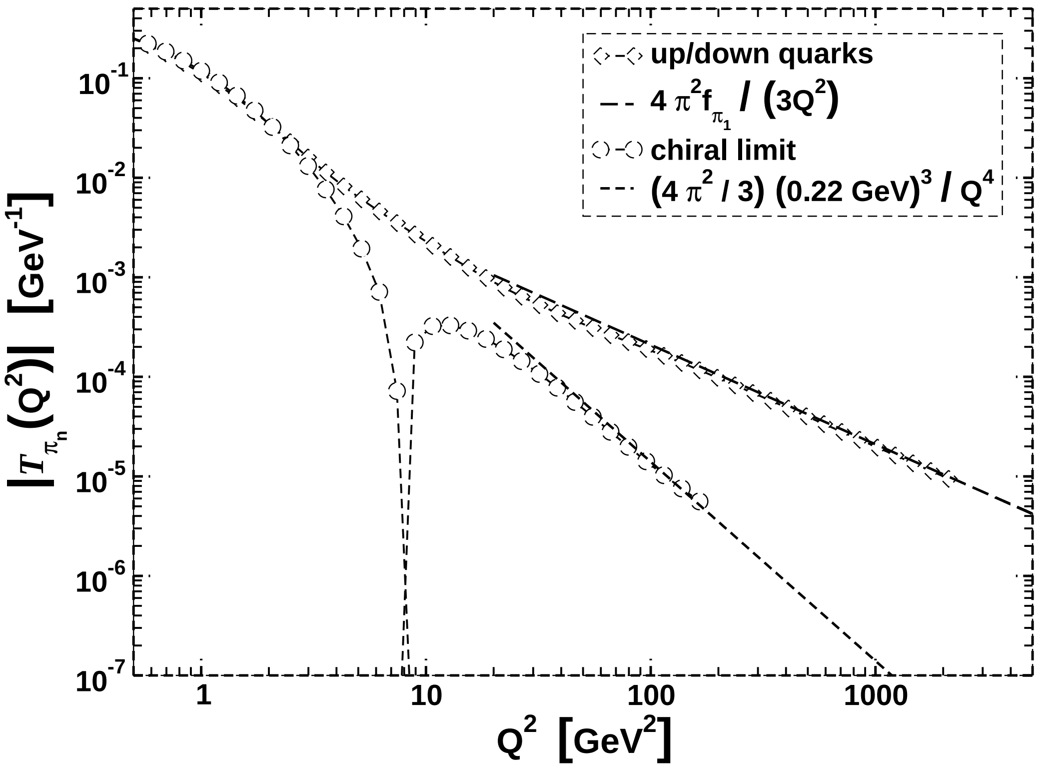

- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 0$

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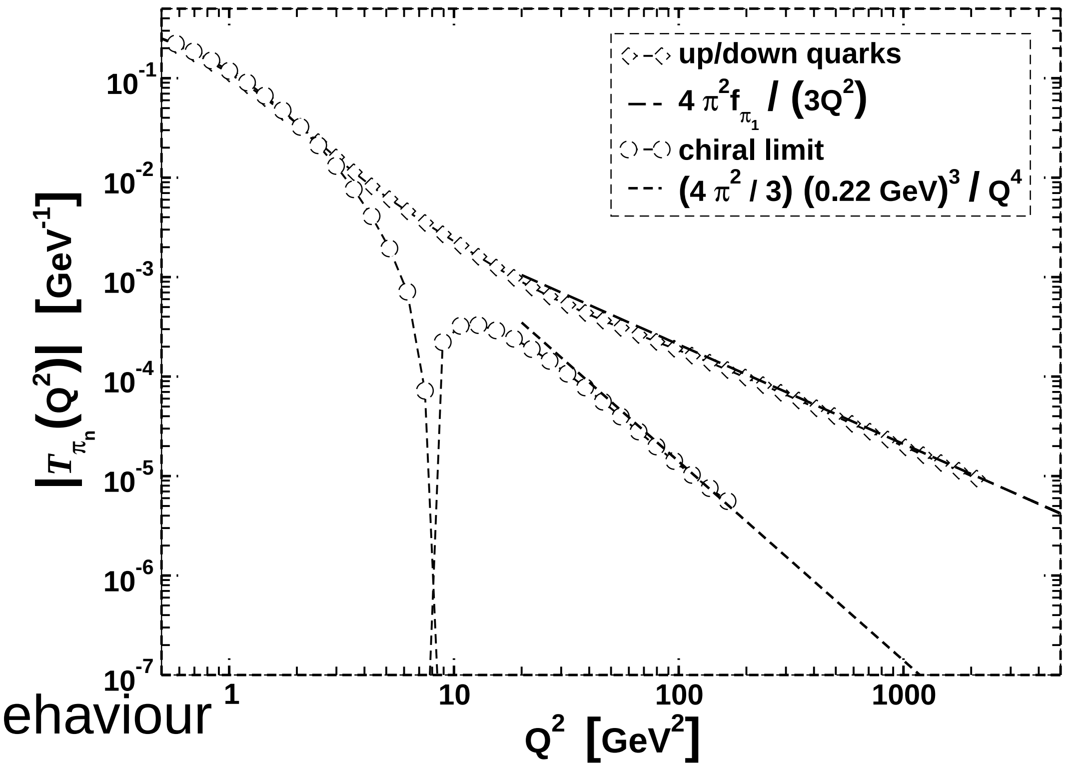

- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 0$
- Again, Predicted UV-behaviour is abundantly clear
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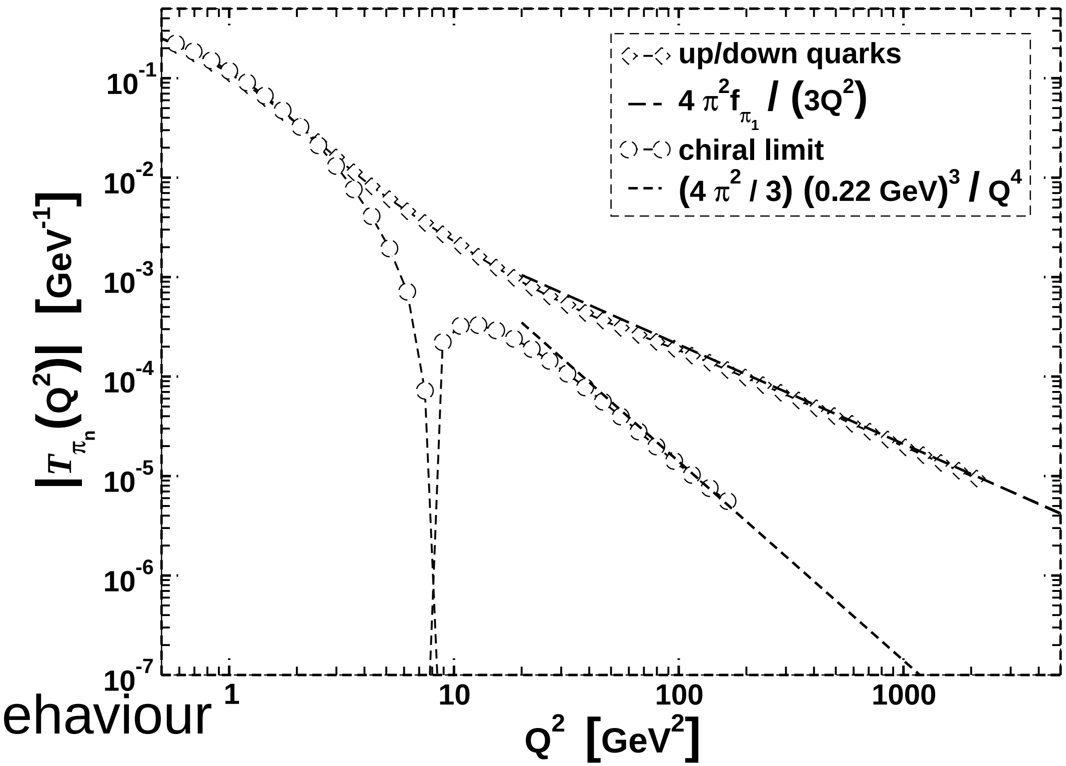

- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 0$
- Again, Predicted UV-behaviour is abundantly clear
  - precise for $Q^2 > 120\,\mathrm{GeV}^2$
- $$F_1^{(2)}(-m_{\pi_1}^2)\;\ln^\gamma Q^2/\omega_{\pi_1}^2\Big|_{\hat{m}=0} \approx (0.22\,\mathrm{GeV})^3 \simeq -\langle\bar{q}q\rangle^0 \quad (3)$$

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Electromagnetic Charge Radii: RGI Rainbow-Ladder

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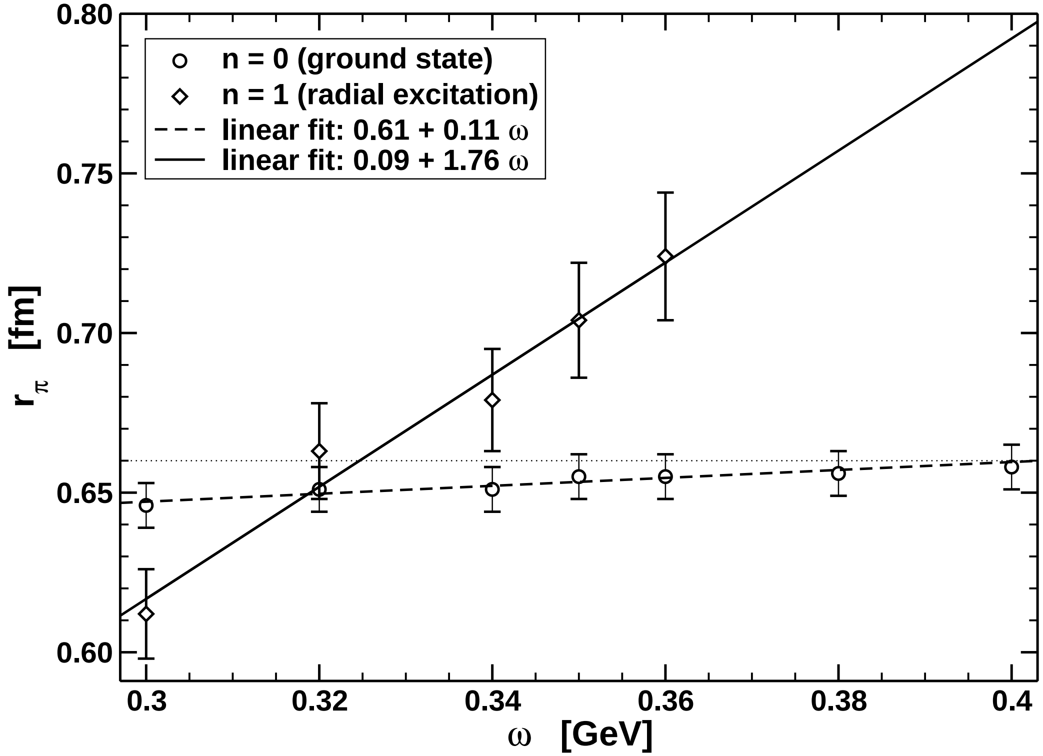

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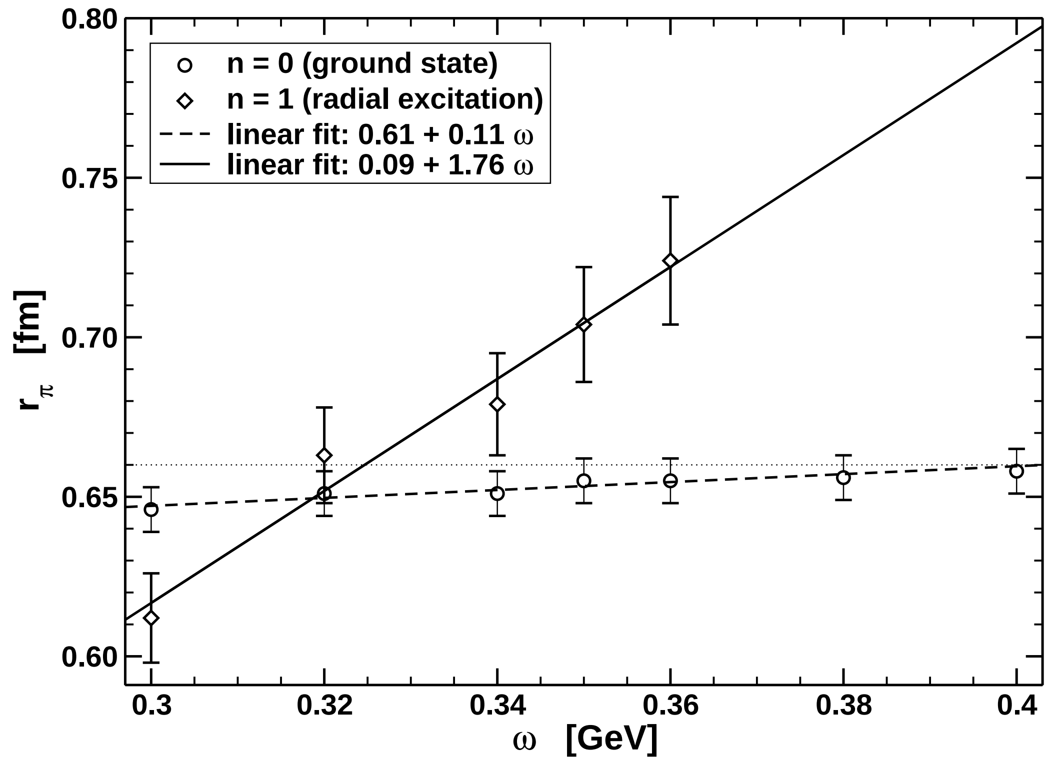
- $m_u(1\,\mathrm{GeV}) = m_d(1\,\mathrm{GeV}) = 5.5$ MeV
- Reminder:
  MT-model has one IR-mass-scale – ω
  - $r_a := 1/\omega$ gauges the range of strong attraction



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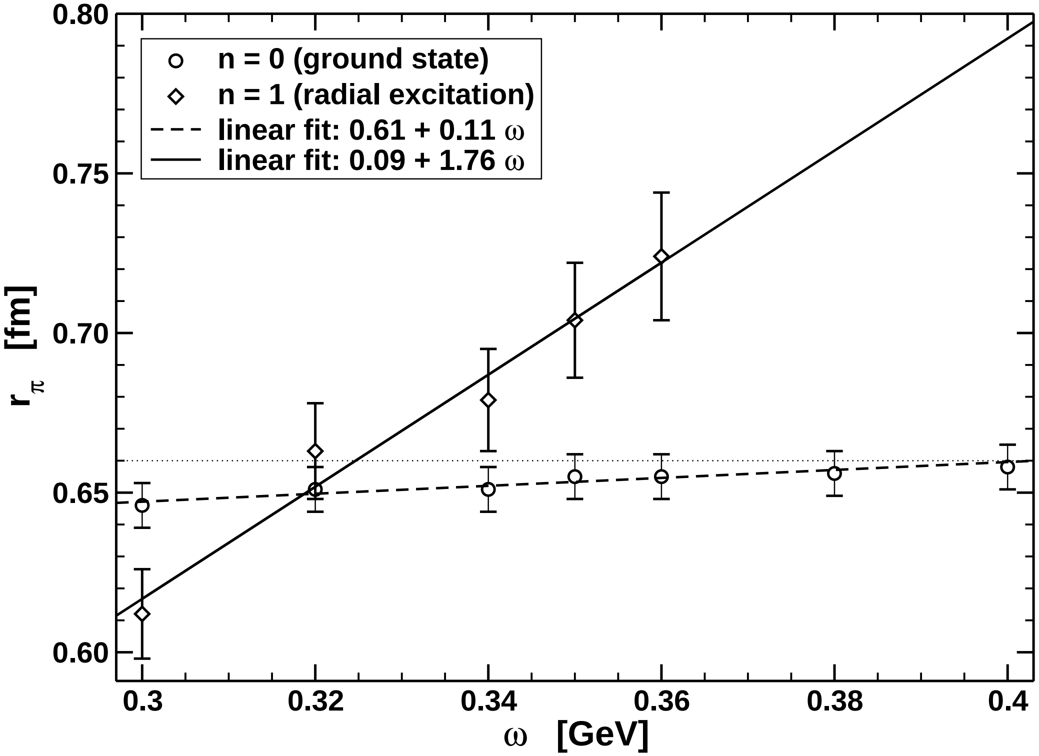

- Goldstone Mode's properties are insensitive to r_a
  - Expected cf. $T \neq 0$, Goldstone mode's properties do not change until very near chiral symmetry restoration temperature.

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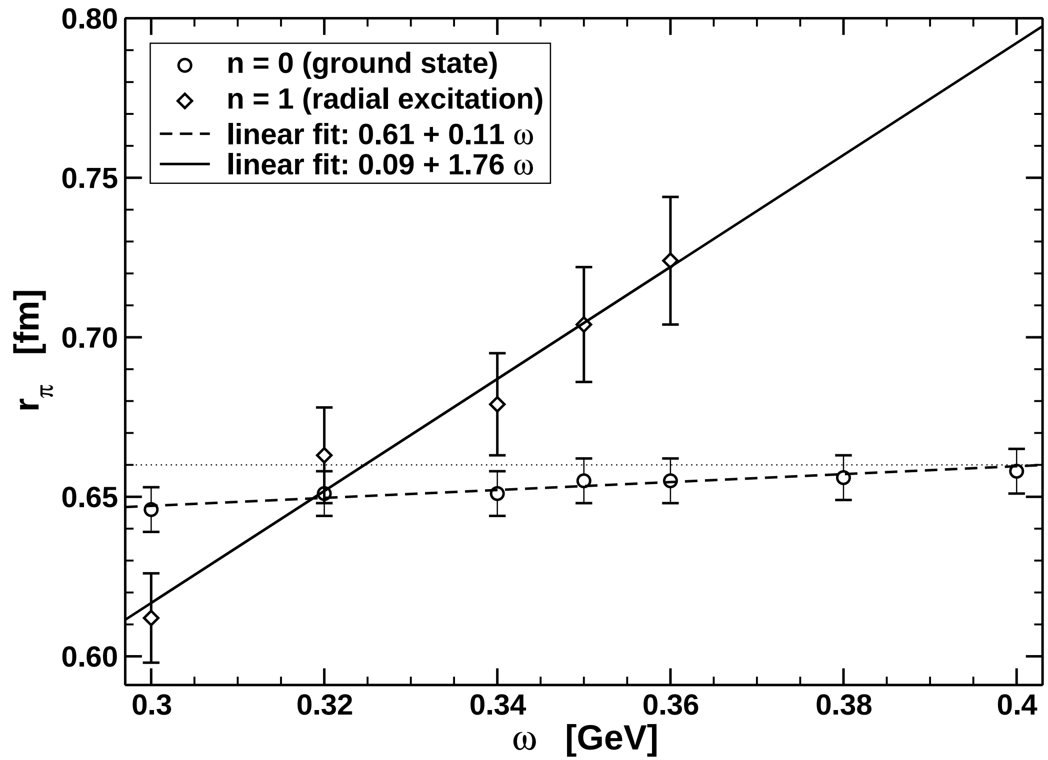

- 1st excited state: orthogonal to Goldstone mode
  - Not protected . . . properties very sensitive to r_a

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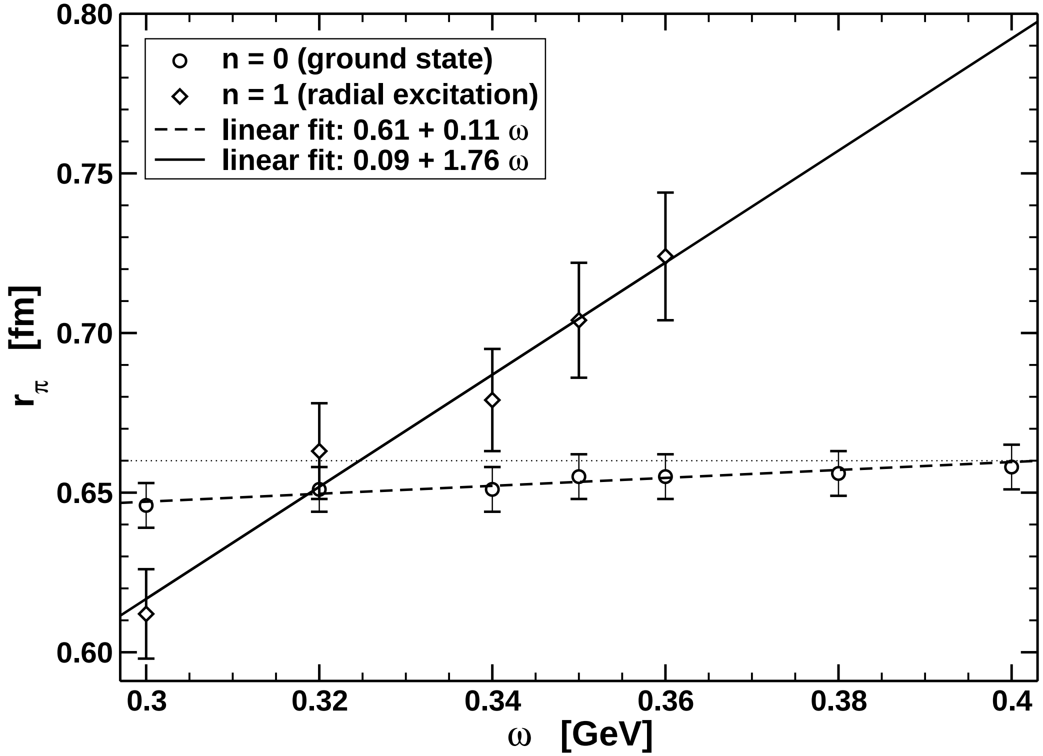

- Best estimate $r_{\pi_1} = 1.4\, r_{\pi_0}$
  - But $r_{\pi_1} < r_{\pi_0}$ is possible if confinement force is very strong

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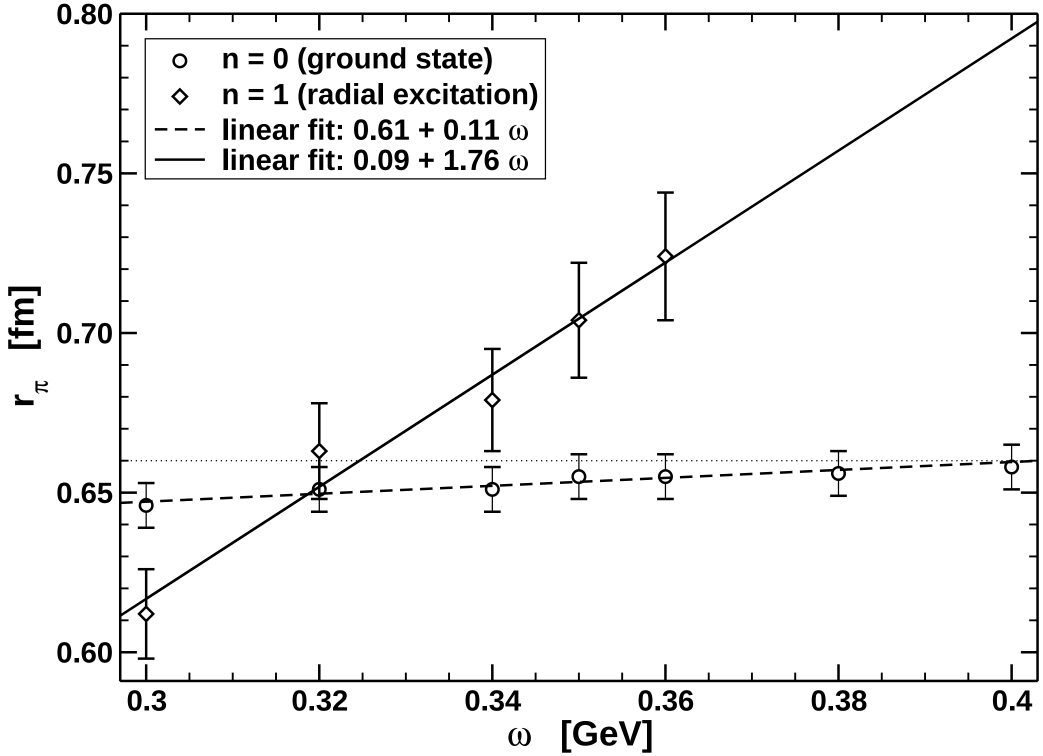

- Radial excitations are plainly useful to map out the long-range part of interaction between light-quarks.

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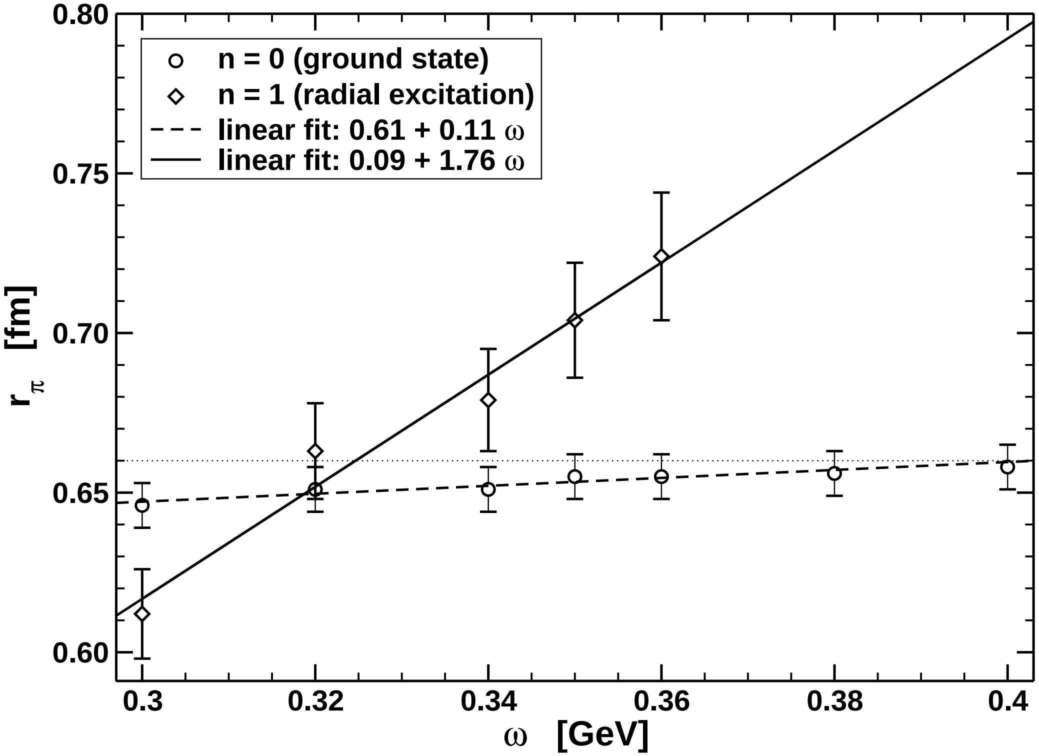

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- Same is true of orbital excitations; e.g., axial-vector mesons.

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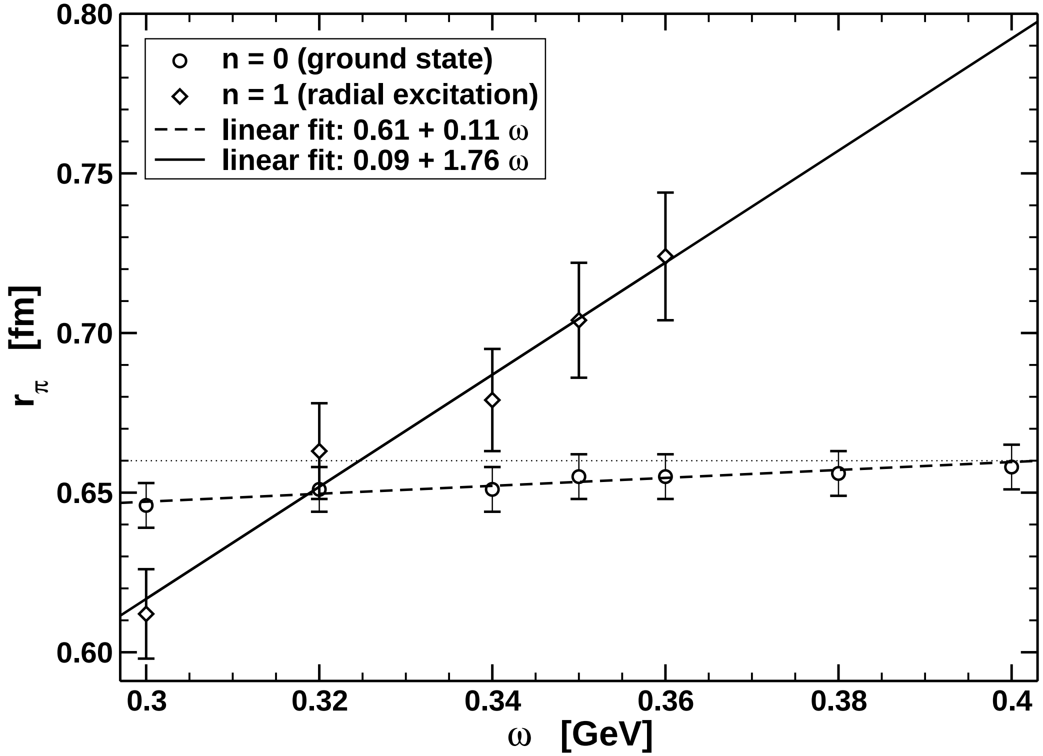

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- Hall-D at JLab

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Current-quark mass-dependence of meson masses

Bhagwat, Höll, Krassnigg, *et al*., "Aspects and consequences of a dressed-quark-gluon vertex," nu-th/0403012

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$$m_{u,d} = 0.01 \quad m_s = 0.166 \quad m_c = 1.33 \quad m_b = 4.62$$

$$m_\rho = 0.77 \quad m_\phi = 1.02 \quad m_{J/\psi} = 3.10 \quad m_{\Upsilon(1S)} = 9.46$$

$$m_\pi = 0.14 \quad m_{0^-_{s\bar{s}}} = 0.63 \quad m_{\eta_c} = 2.97 \quad m_{\eta_b} = 9.42$$

- Fit current-quark masses to vector meson spectrum

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- Fit current-quark masses to vector meson spectrum
- Predictions $m_{\eta_c} = 2.97$ and $m_{\eta_b} = 9.42$

  cf. $m_{\eta_c}^{\rm expt.} = 2.98$ and $m_{\eta_b}^{\rm expt.} = 9.30 \pm 0.03$

NB. $m_{\Upsilon(1S)} - m_{\eta_b}^{\rm expt.}$

- Calc.: 40 MeV
- Expt.: 160 MeV, which is $>$ Expt. $m_{J/\psi} - m_{\eta_c}^{\rm expt.}$
- Unlikely

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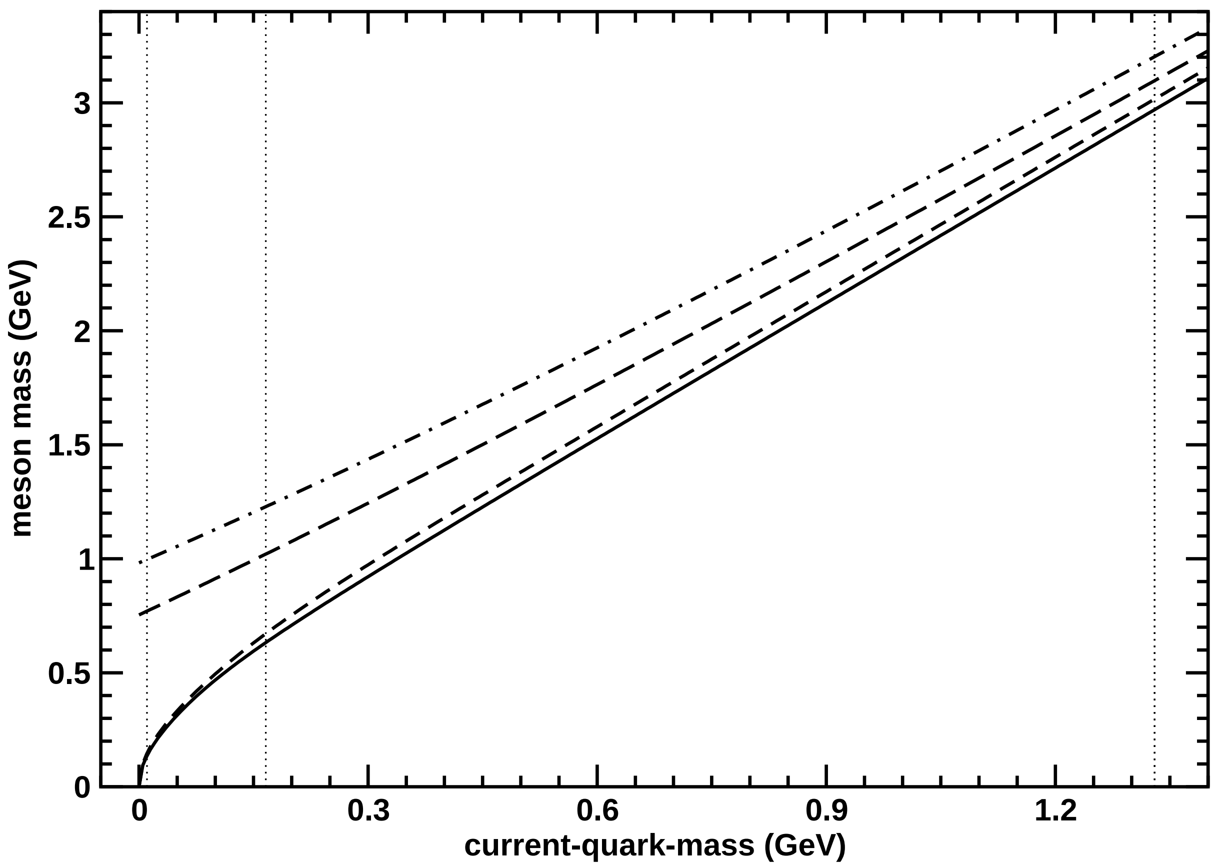

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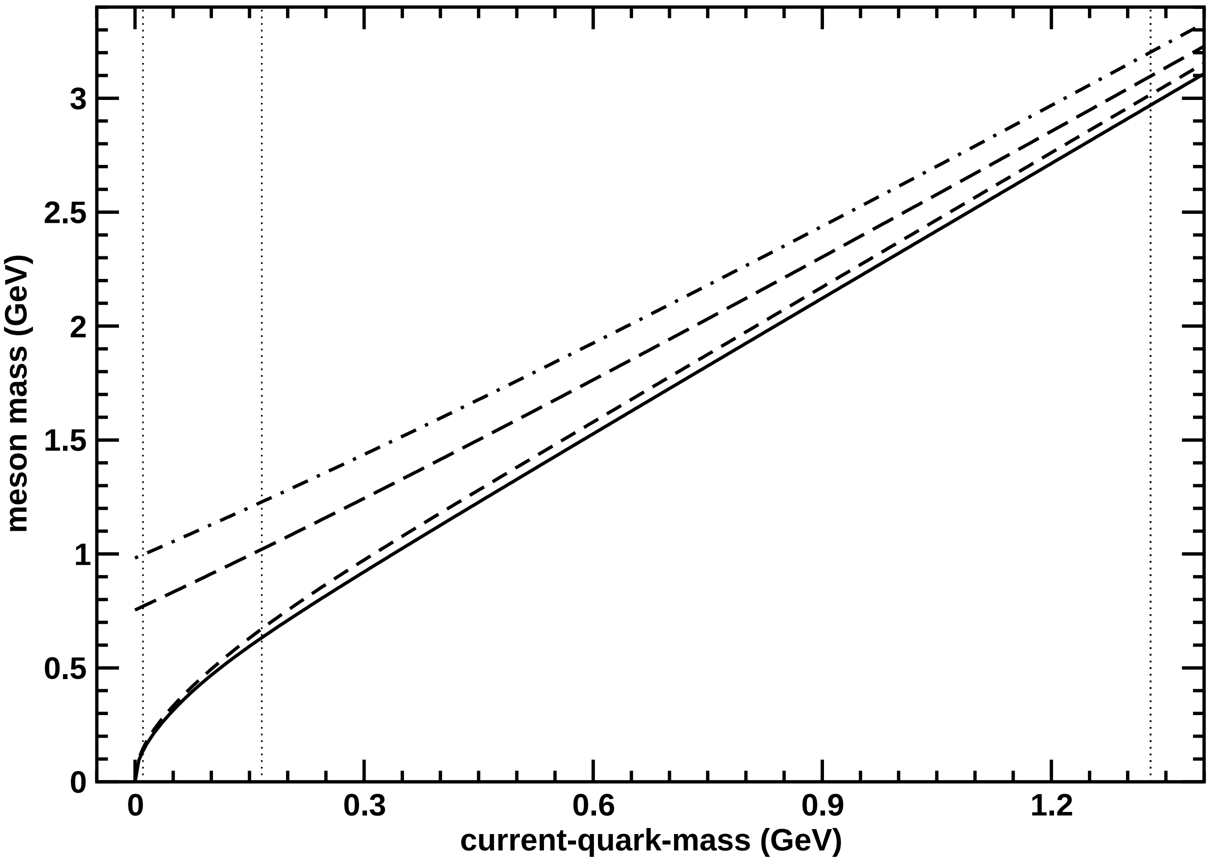
- For $m \ll \mathcal{G}$
  $m_{0^-}^2 = 1.33\, m$



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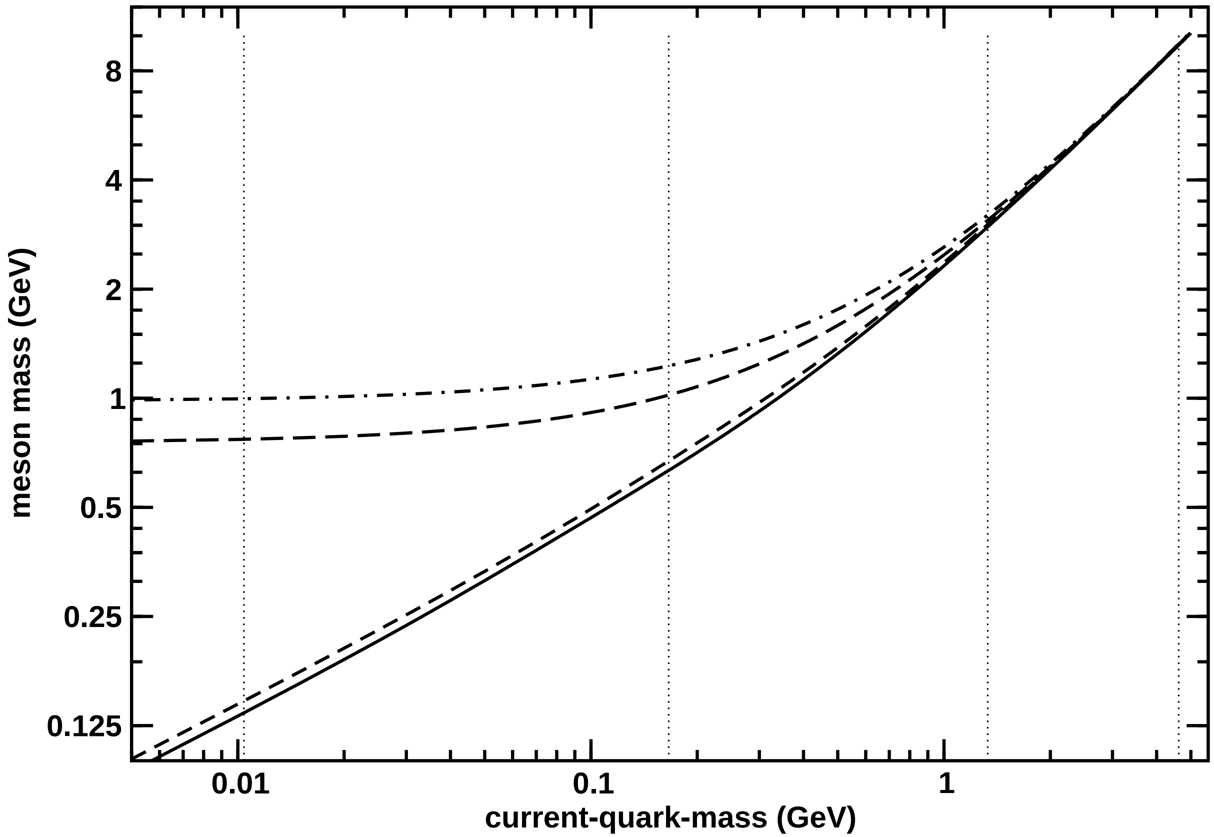

Current-quark mass-dependence of meson masses

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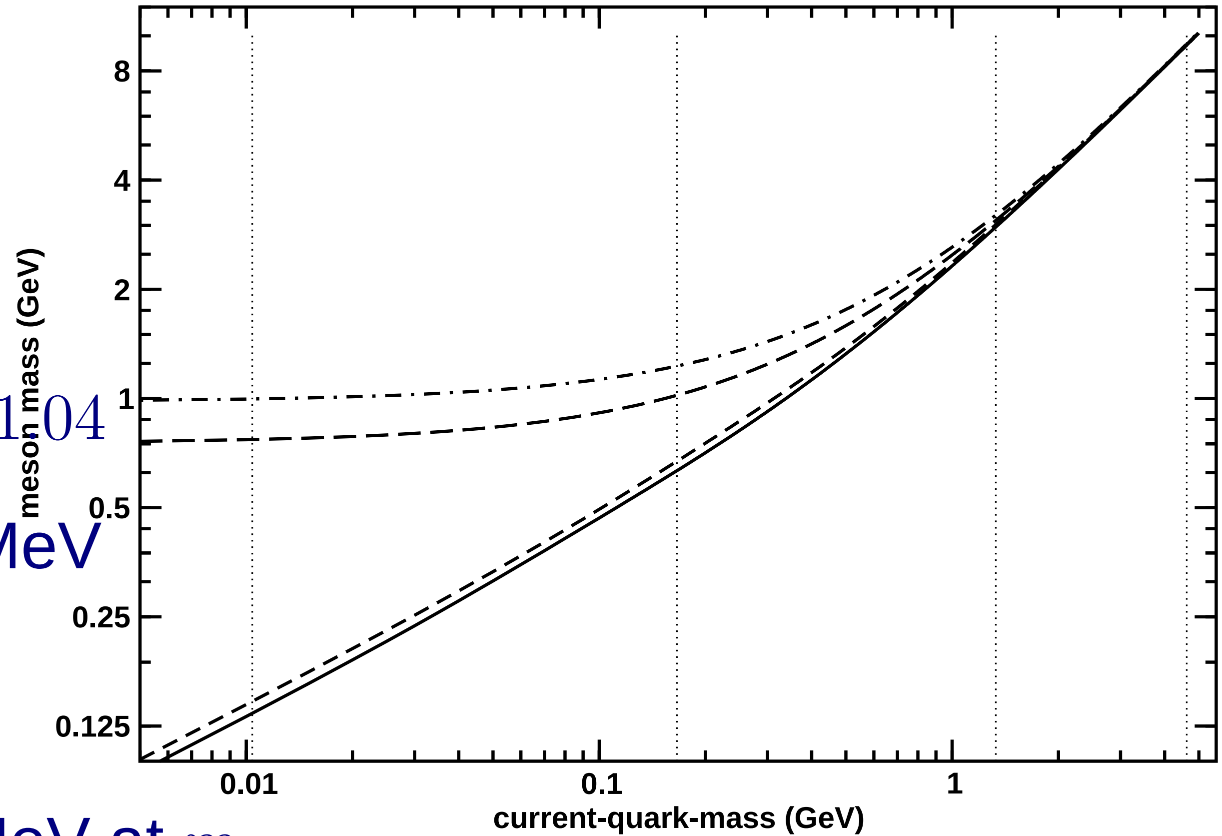
- For $m \gg \mathcal{G}$

  $m_{0^-} = m_{1^+}$

- $\left.\dfrac{m_{1^-}}{m_{0^-}}\right|_{m=m_c} = 1.04$

  splitting 130 MeV



- Drops to 40 MeV at m_b

  just 5% of its chiral limit value

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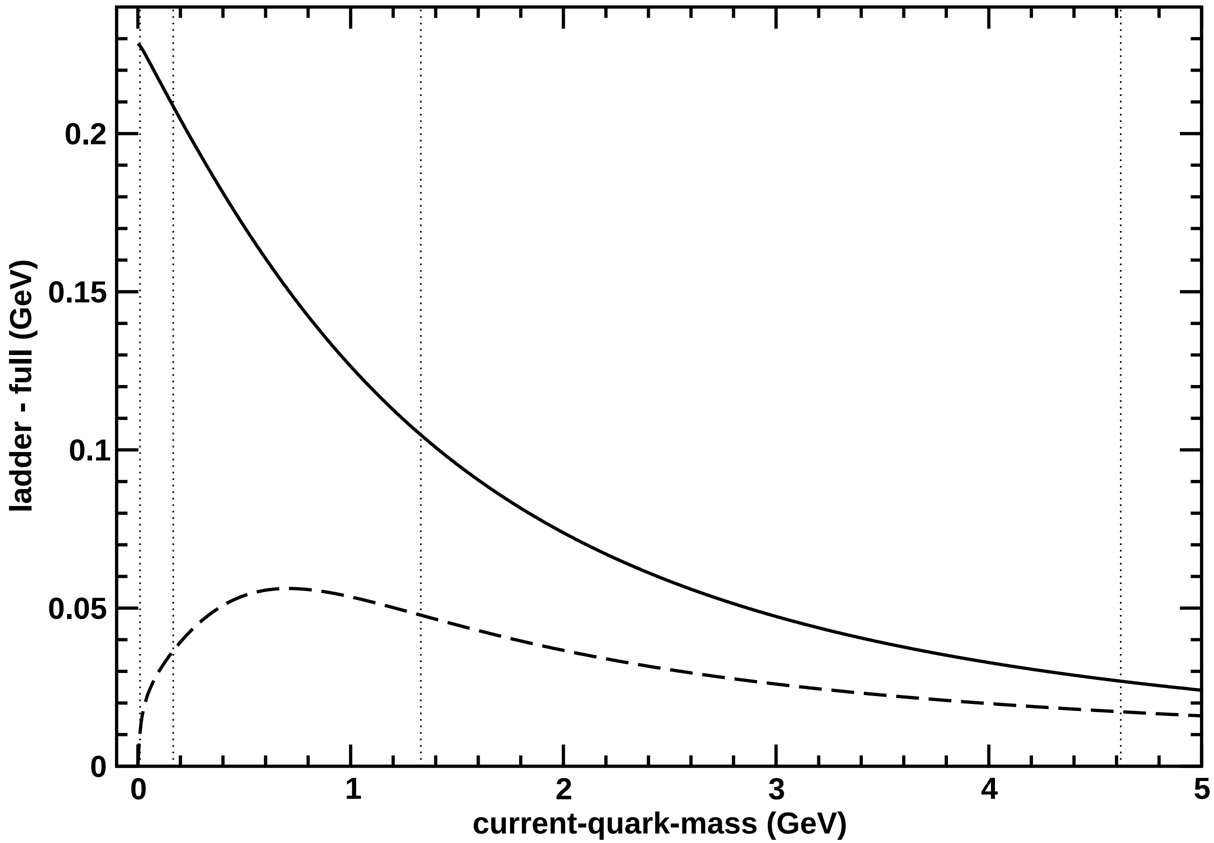

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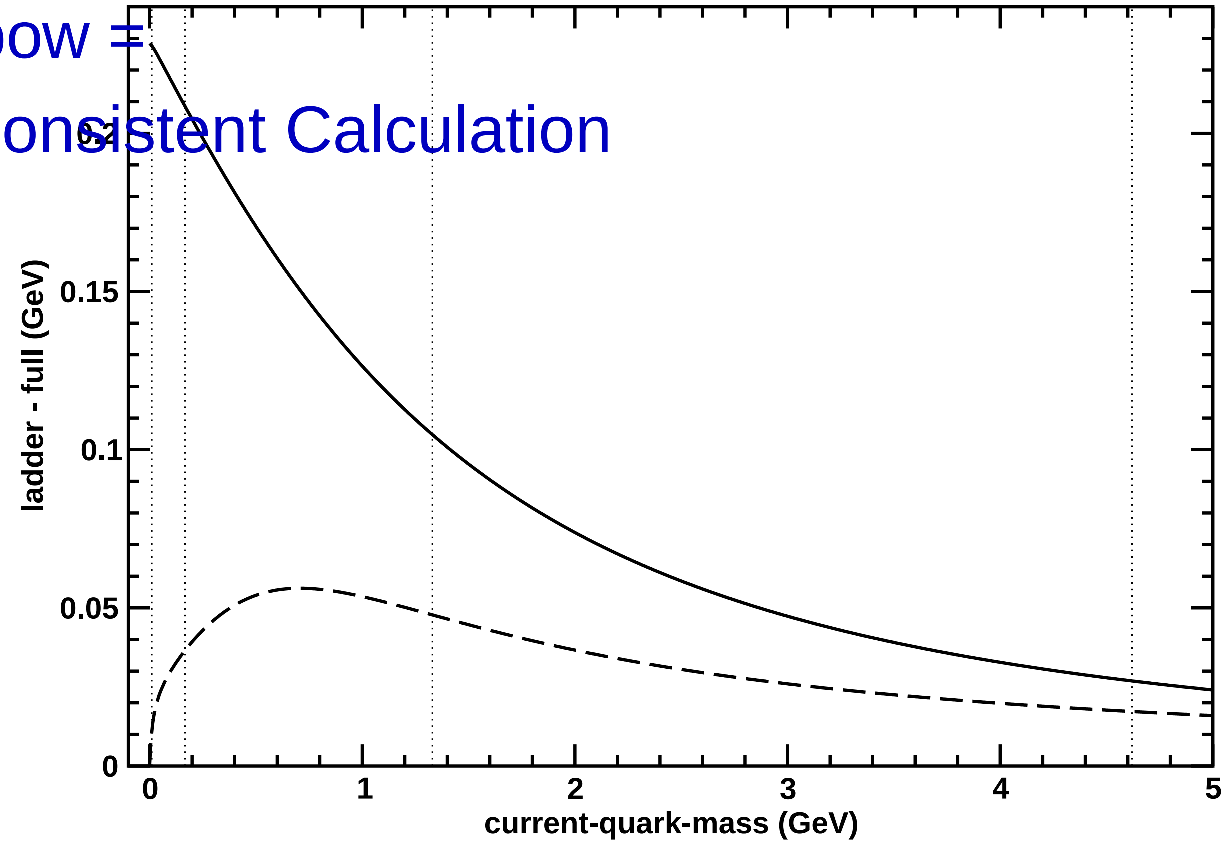

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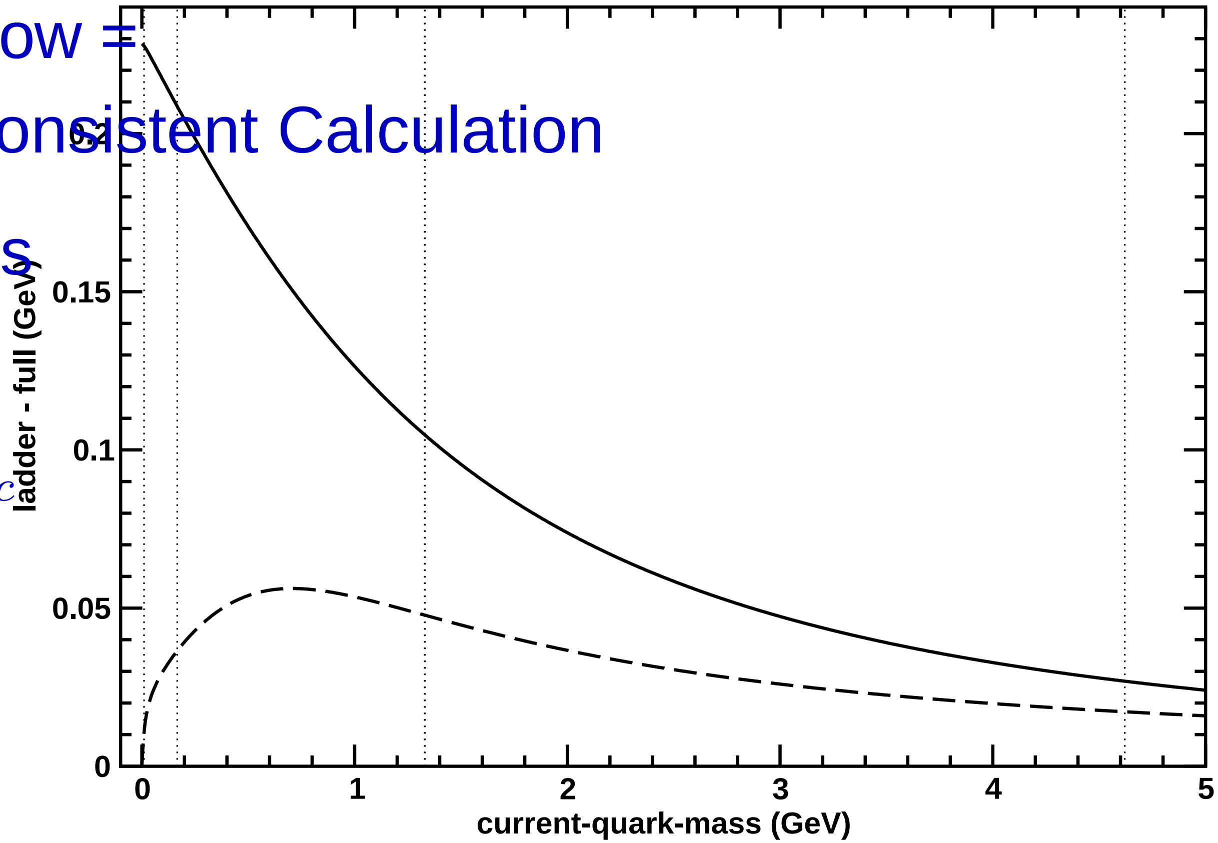

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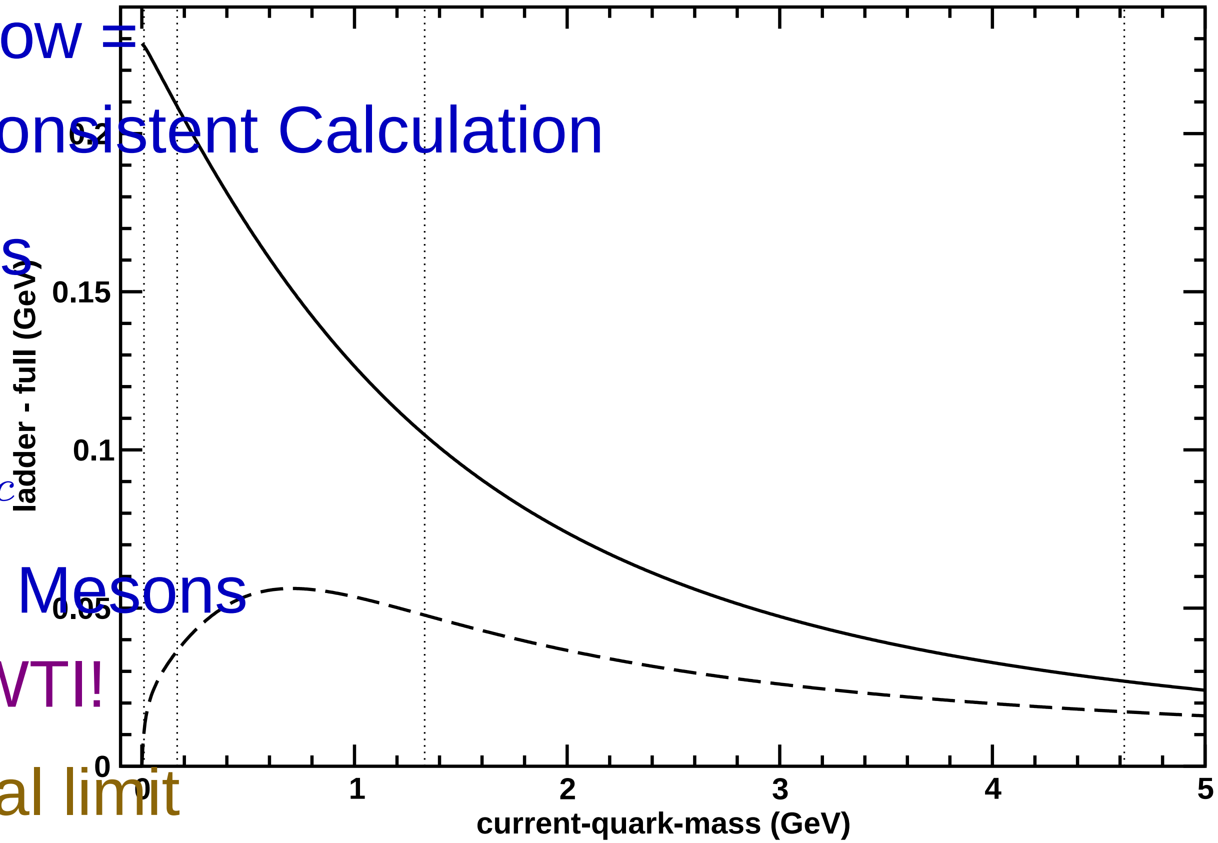
  - 0% in Chiral limit
  - Max err. 3% at 4 m_s



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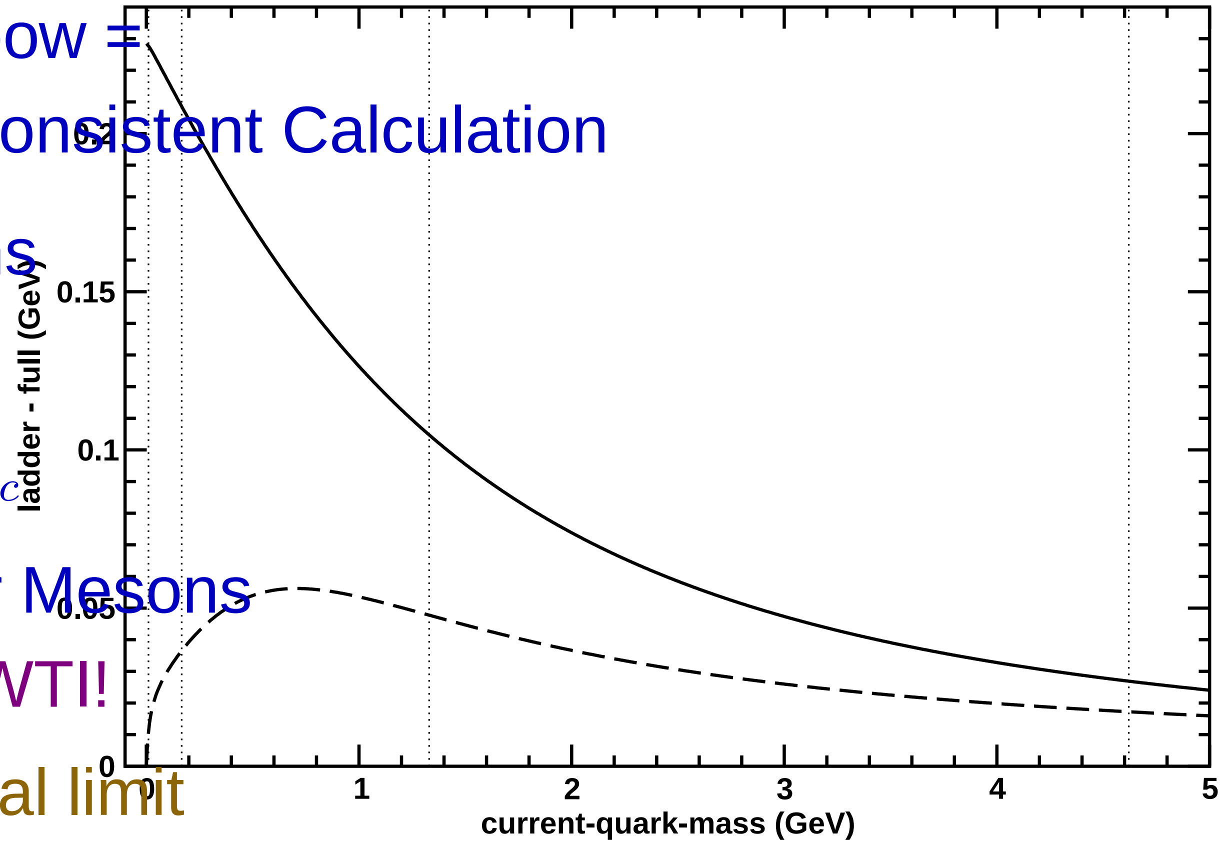

Explains Pattern of Maris-Tandy Success

Deep-inelastic scattering

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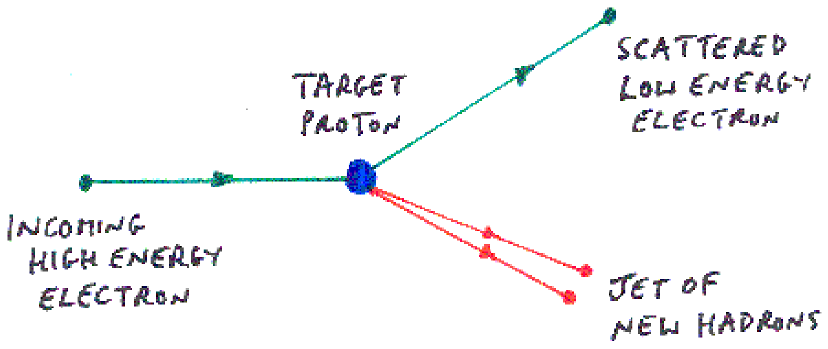
Deep-inelastic scattering

- Looking for Quarks

Deep-inelastic scattering



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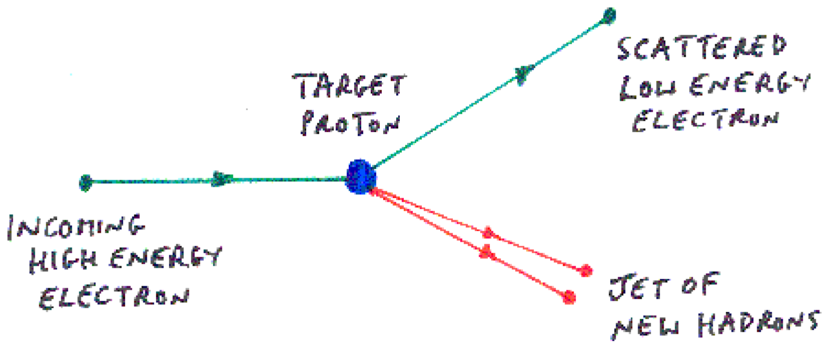

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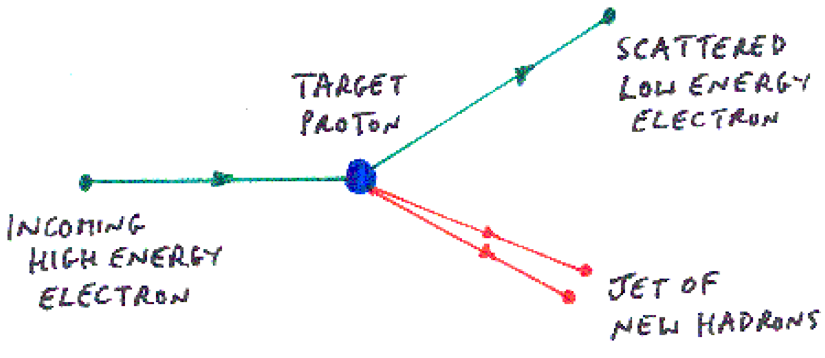

- Looking for Quarks

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- Cross-section: Interpreted as Measurement of Momentum-Fraction Prob. Distribution: $q(x)$, $g(x)$

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Pion's valence quark distn

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Pion's valence quark distn

- π is Two-Body System: "Easiest" Bound State in QCD
- However, NO π Targets!

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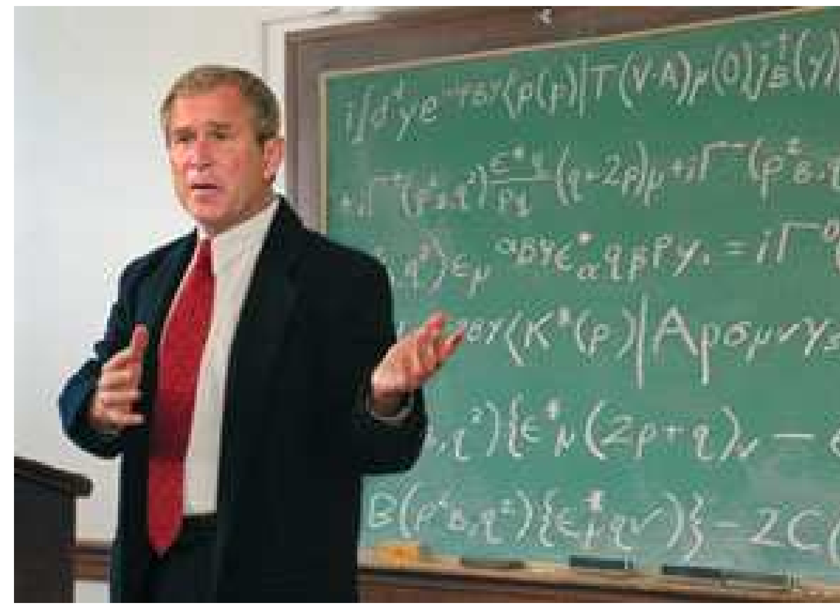
- Proved on
  22/July/2002, ANL



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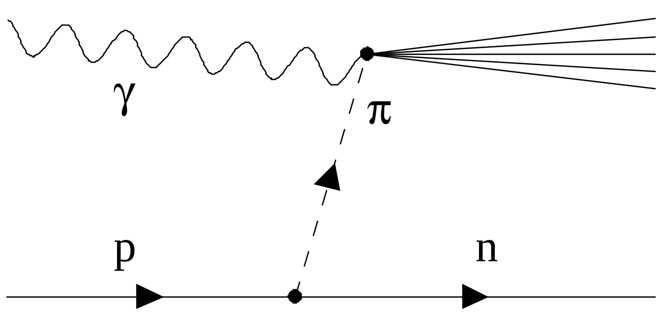
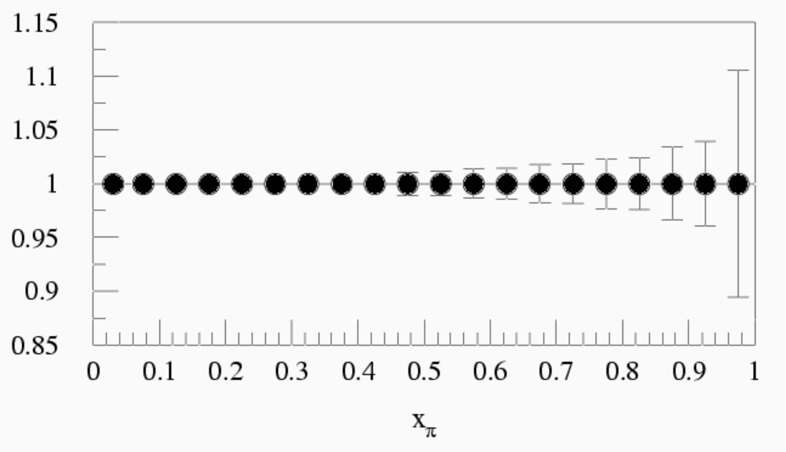
- Proposal `(Holt & Reimer, ANL, nu-ex/0010004)`

  $e^-_{5\,\text{GeV}} - p_{25\,\text{GeV}}$ Collider $\to$ Accurate "Measurement"





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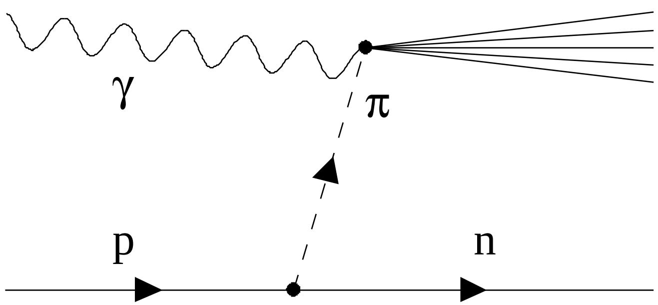
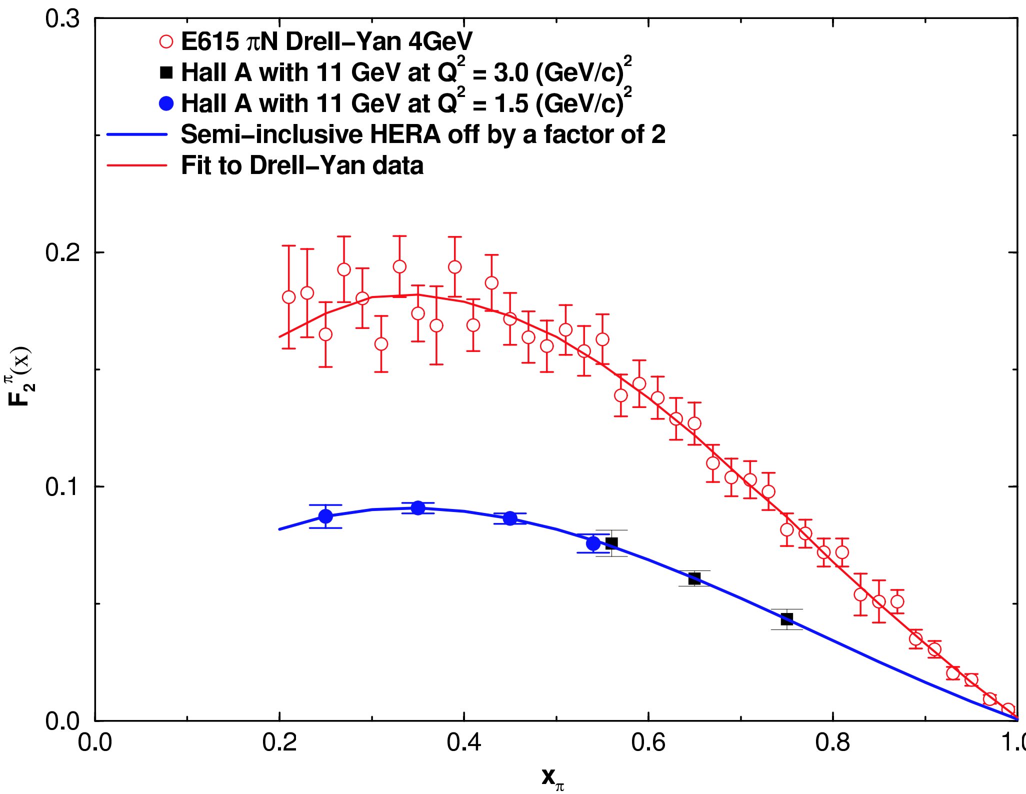
Pion’s valence quark distn

- Proposal at JLab
  (Holt, Reimer, Wijesooriya, et al., JLab at 12 GeV)





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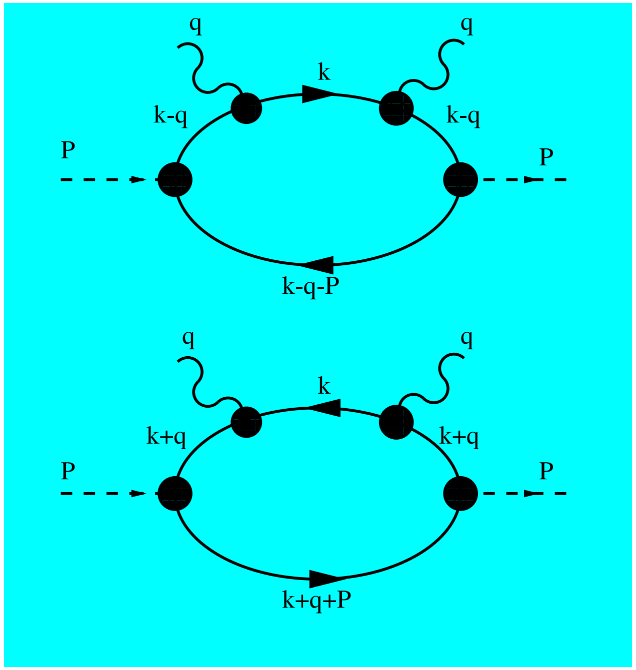
Handbag diagrams



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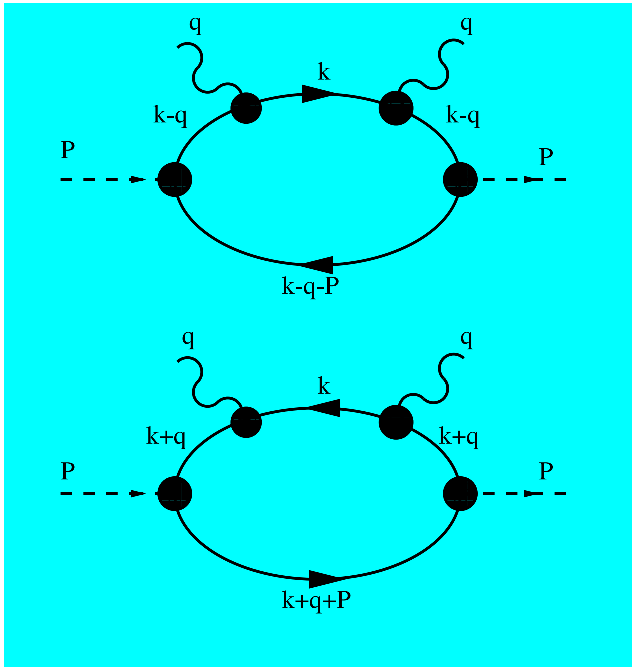
Handbag diagrams



$$W_{\mu\nu}(q;P) \;=\; \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P)+T^{-}_{\mu\nu}(q;P)\right]$$

$$T^{+}_{\mu\nu}(q,P) \;=\; \mathrm{tr}\int\frac{d^4k}{(2\pi)^4}\,\tau_-\bar{\Gamma}_\pi(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_\nu(k_{-0},k)$$
$$\times\, S(k)\,ieQ\Gamma_\mu(k,k_{-0})\,S(k_{-0})\tau_+\Gamma_\pi(k_{-\frac{1}{2}};P)\,S(k_{--})$$

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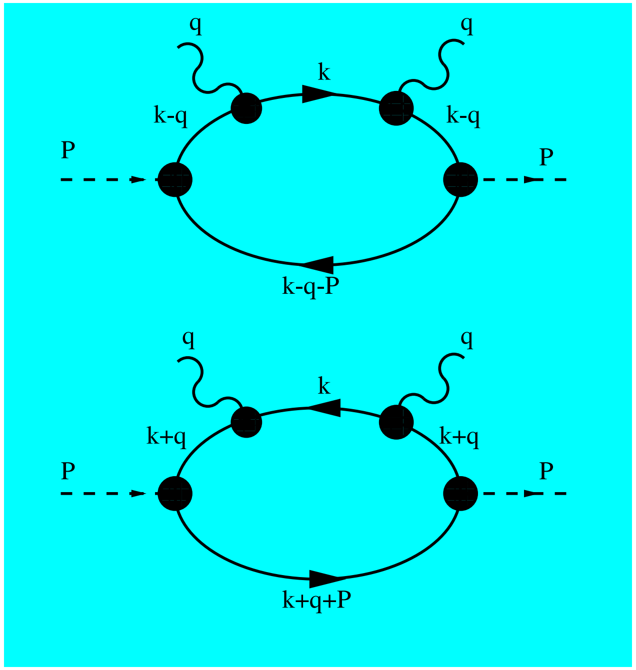
Handbag diagrams

Bjorken Limit: $q^2 \to \infty\,, \quad P\cdot q \to -\infty$

but $x := -\dfrac{q^2}{2P\cdot q}$ fixed.

Numerous algebraic simplifications



$$W_{\mu\nu}(q;P) = \frac{1}{2\pi}\mathrm{Im}\left[T^{+}_{\mu\nu}(q;P) + T^{-}_{\mu\nu}(q;P)\right]$$

$$T^{+}_{\mu\nu}(q,P) = \mathrm{tr}\int \frac{d^4k}{(2\pi)^4}\,\tau_-\bar{\Gamma}_\pi(k_{-\frac{1}{2}};-P)\,S(k_{-0})\,ieQ\Gamma_\nu(k_{-0},k)$$
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Calc. $u_V(x)$ cf. Drell-Yan data

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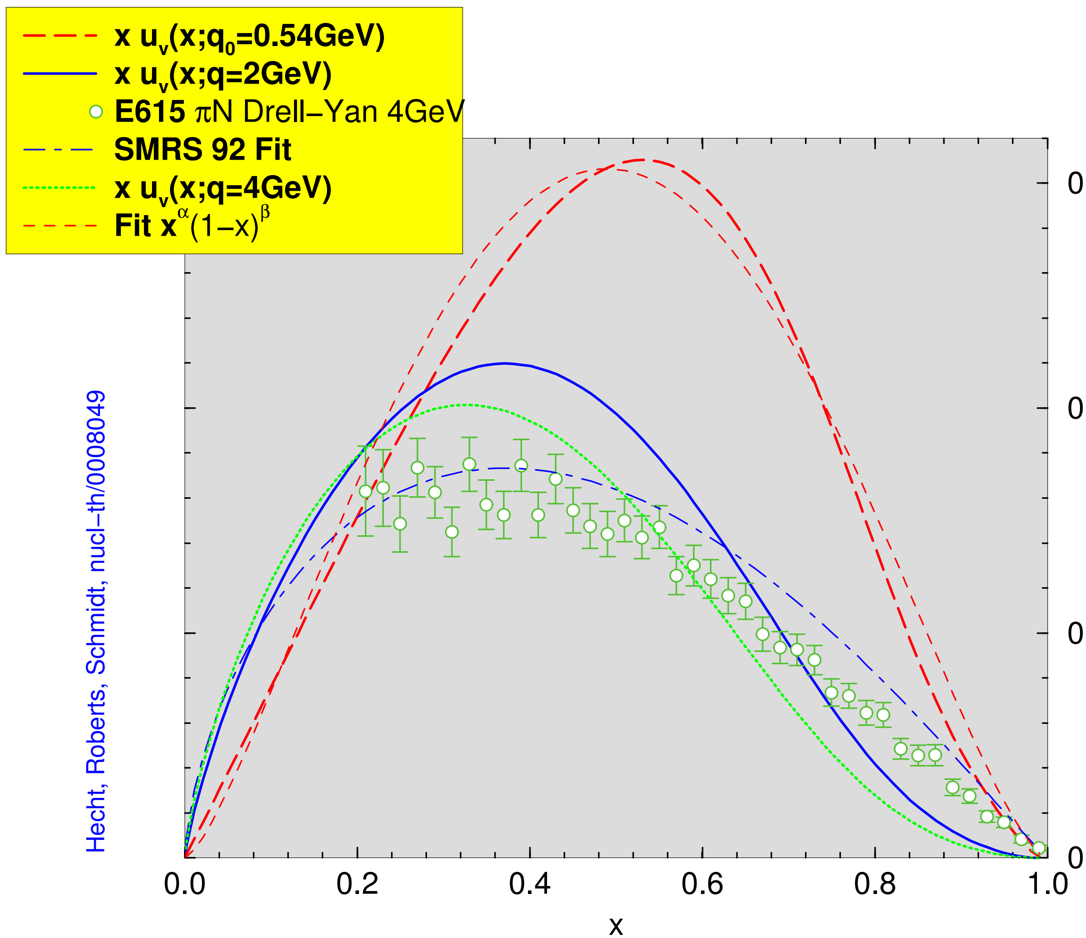
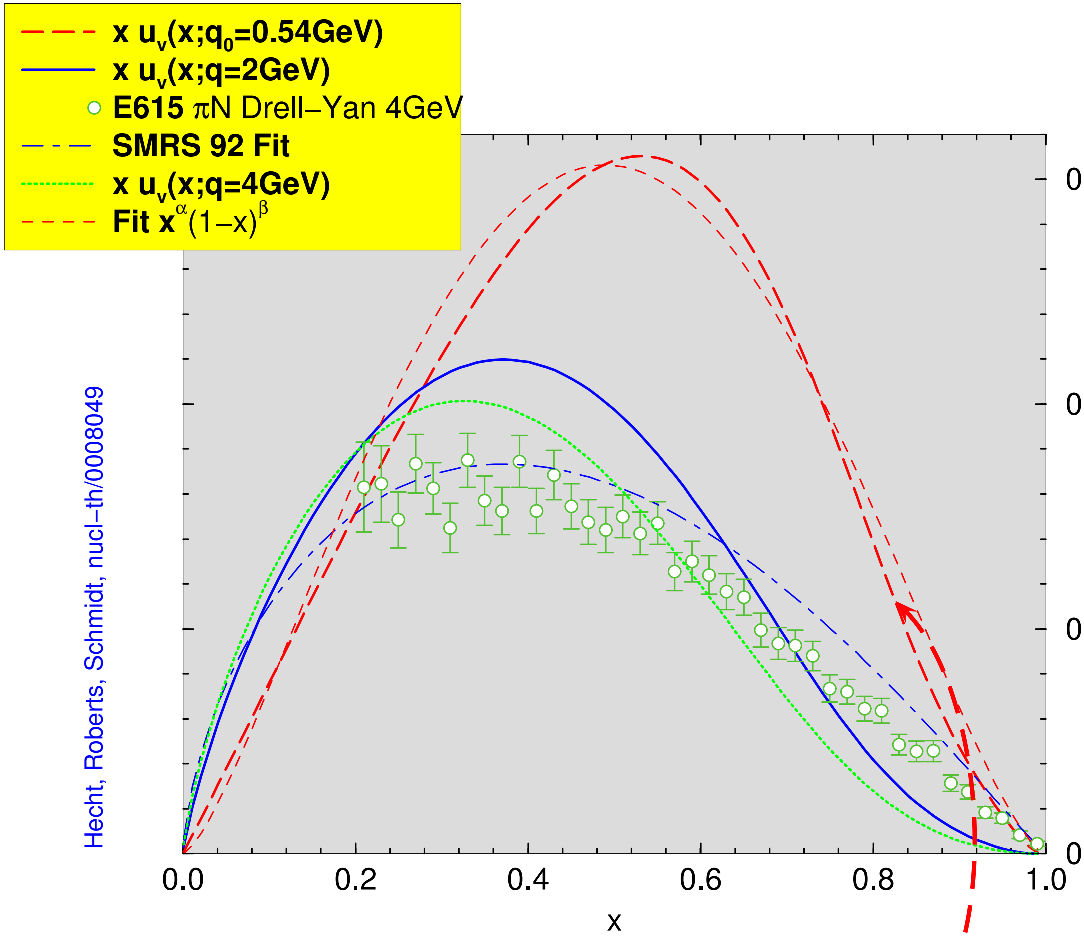


Calc. $u_V(x)$ cf. Drell-Yan data



Resolving Scale: $q_0 = 0.54\,\mathrm{GeV} = 1/(0.37\,\mathrm{fm})$

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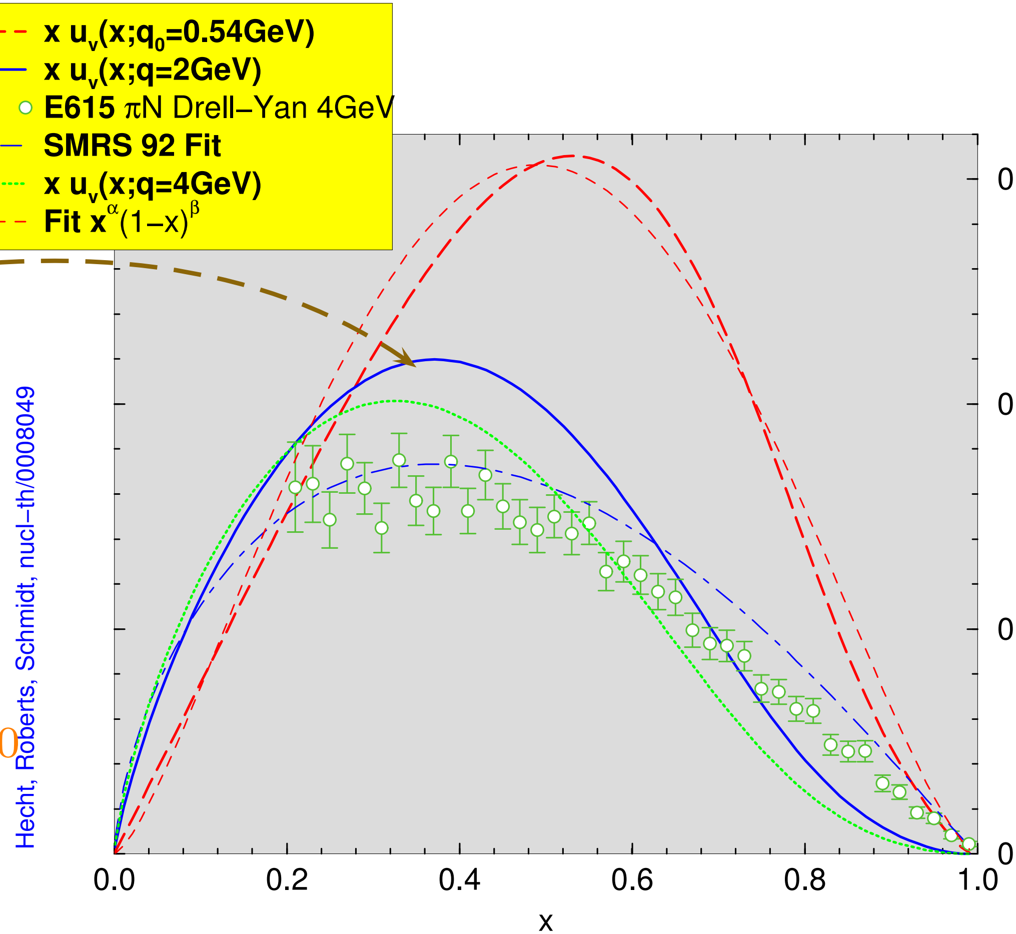
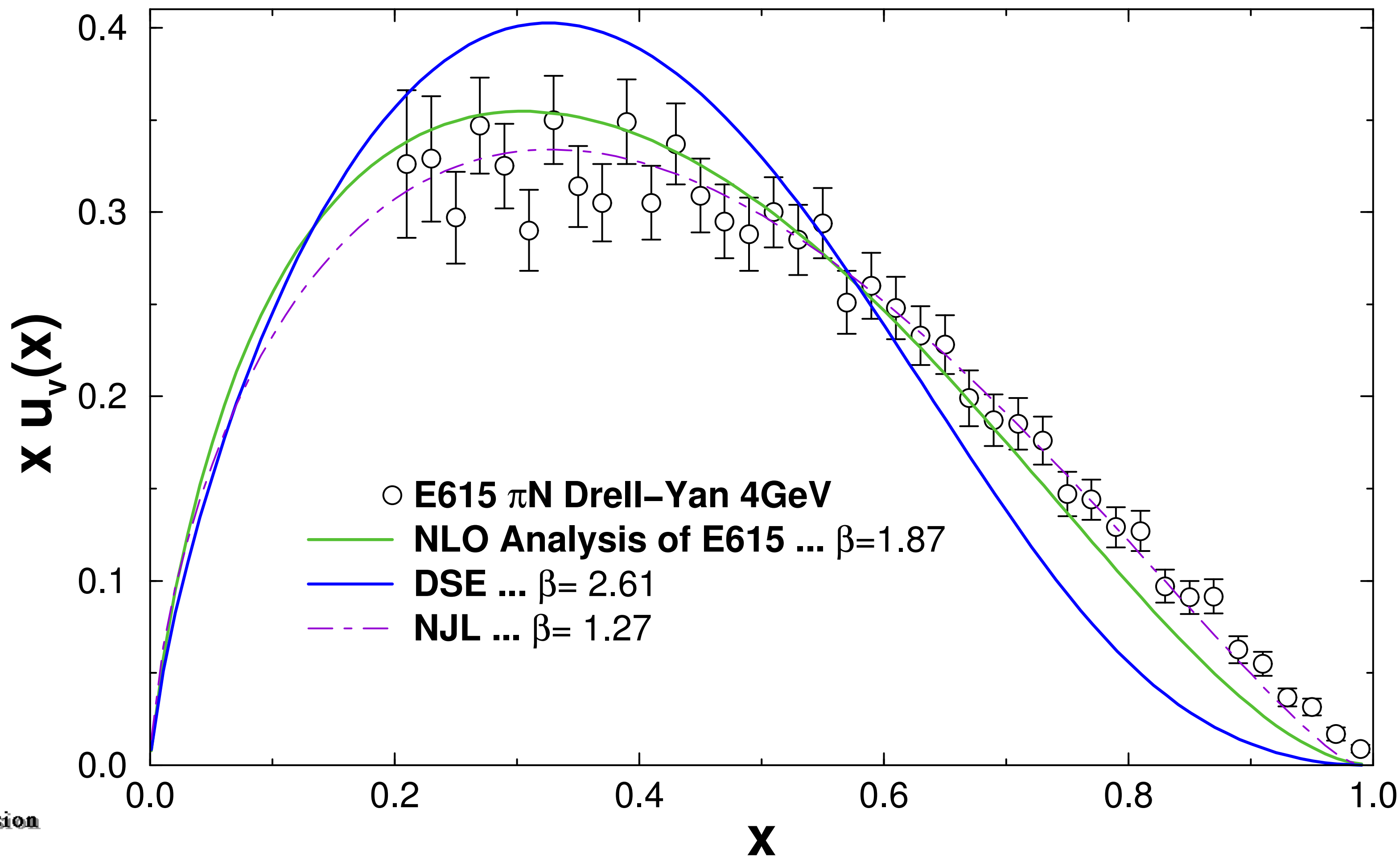
Calc. $u_V(x)$ cf. Drell-Yan data

$q =$ 2 GeV	Calc.	Fit, 92	Latt., 97
$\langle x \rangle_q$	0.24	0.24 ± 0.01	0.27 ± 0.01
$\langle x^2 \rangle_q$	0.10	0.10 ± 0.01	0.11 ± 0.3
$\langle x^3 \rangle_q$	0.050	0.058 ± 0.004	0.048 ± 0.020



Extant theory vs. experiment

Krishni Wijersooriya, Paul Reimer
and Roy Holt, soon to be submitted



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Overview

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- What about Baryons?

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