Hadron Physics and Dyson-Schwinger Equations

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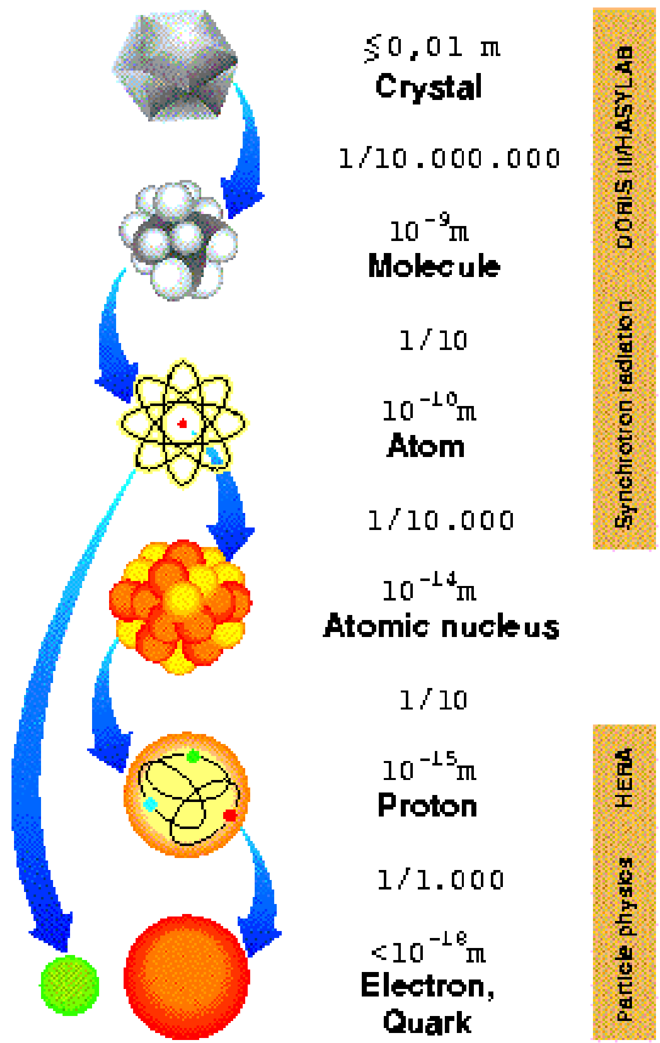
Argonne National Laboratory

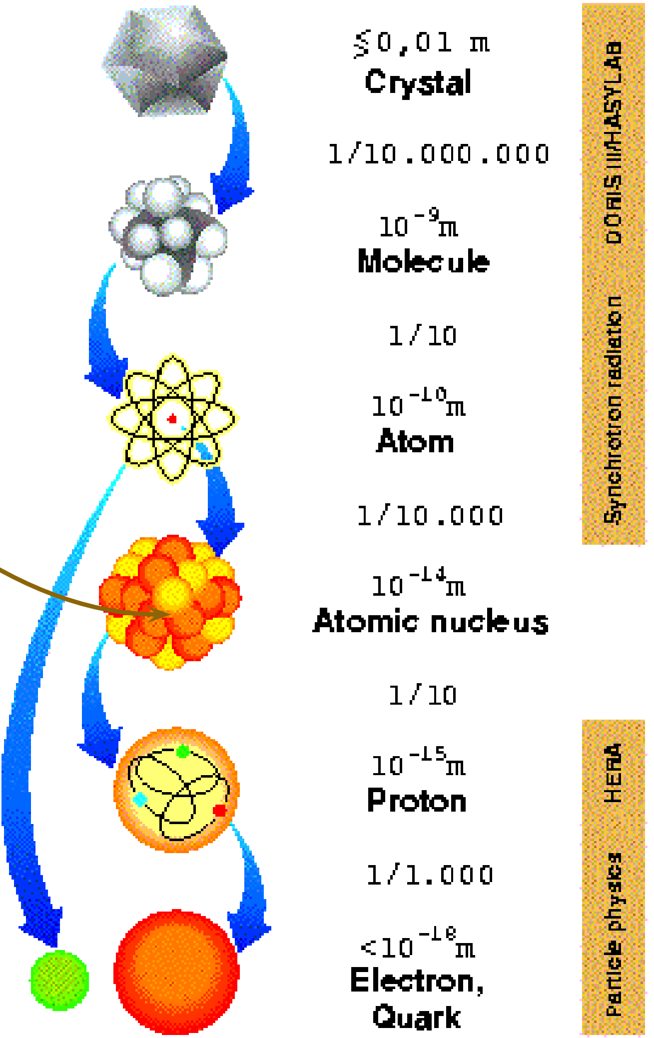
http://www.phy.anl.gov/theory/staff/cdr.html

Scales in Modern Physics



Scales in Modern Physics

Nuclear Physics



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Scales in Modern Physics

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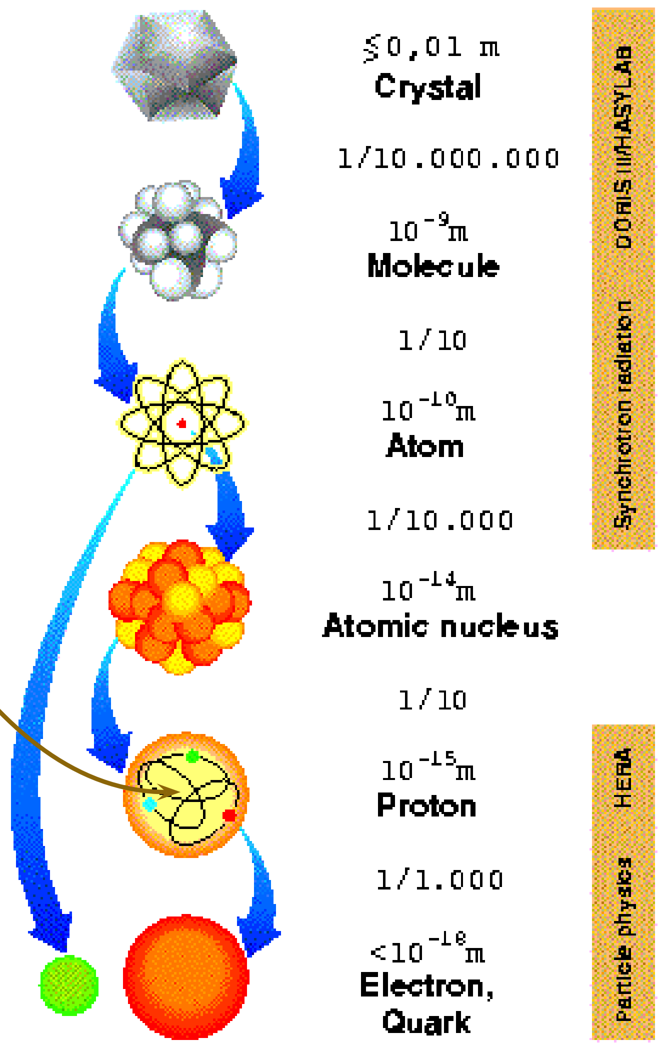

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Nucleon = Proton and Neutron

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- Stern (1933) – $\mu_p = (1 + 1.79)\frac{e\hbar}{2M}$
  - Big Hint that Proton is not a point particle
  - Proton has constituents
  - These are Quarks and Gluons
    – the elementary quanta of Quantum Chromo-dynamics

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Nucleon Form Factors

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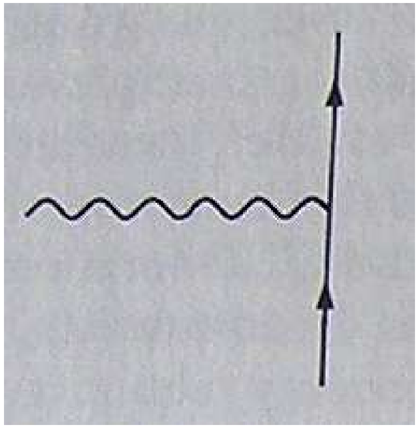
- Electron's relativistic electromagnetic current:

$$\begin{aligned} j_\mu(P',P) &= ie\,\bar{u}_e(P')\,\Lambda_\mu(Q,P)\,u_e(P)\,,\; Q = P' - P \\ &= ie\,\bar{u}_e(P')\,\gamma_\mu(-1)\,u_e(P) \end{aligned}$$



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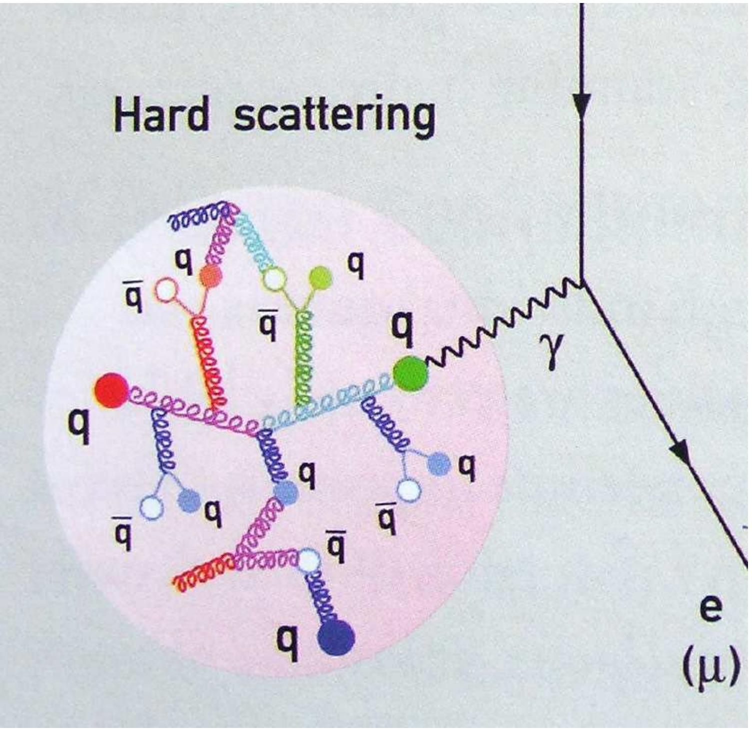

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$$G_E(Q^2)=F_1(Q^2)-\frac{Q^2}{4M^2}F_2(Q^2)\,,\;G_M(Q^2)=F_1(Q^2)+F_2(Q^2)\,.$$

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$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\ G_M(Q^2) = F_1(Q^2) + F_2(Q^2)\,.$$

Point-particle: $\boldsymbol{F_2 \equiv 0 \Rightarrow G_E \equiv G_M}$

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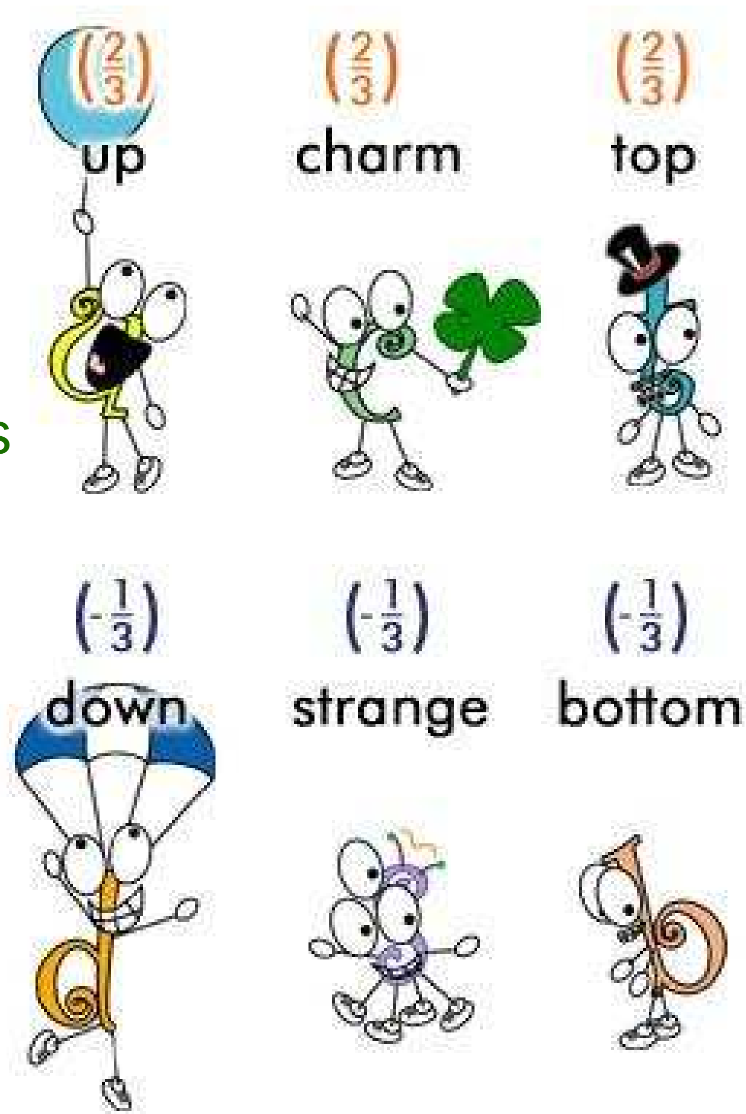
Quarks and Nuclear Physics

Quarks and Nuclear Physics

Standard Model

of Particle Physics

Six Flavours



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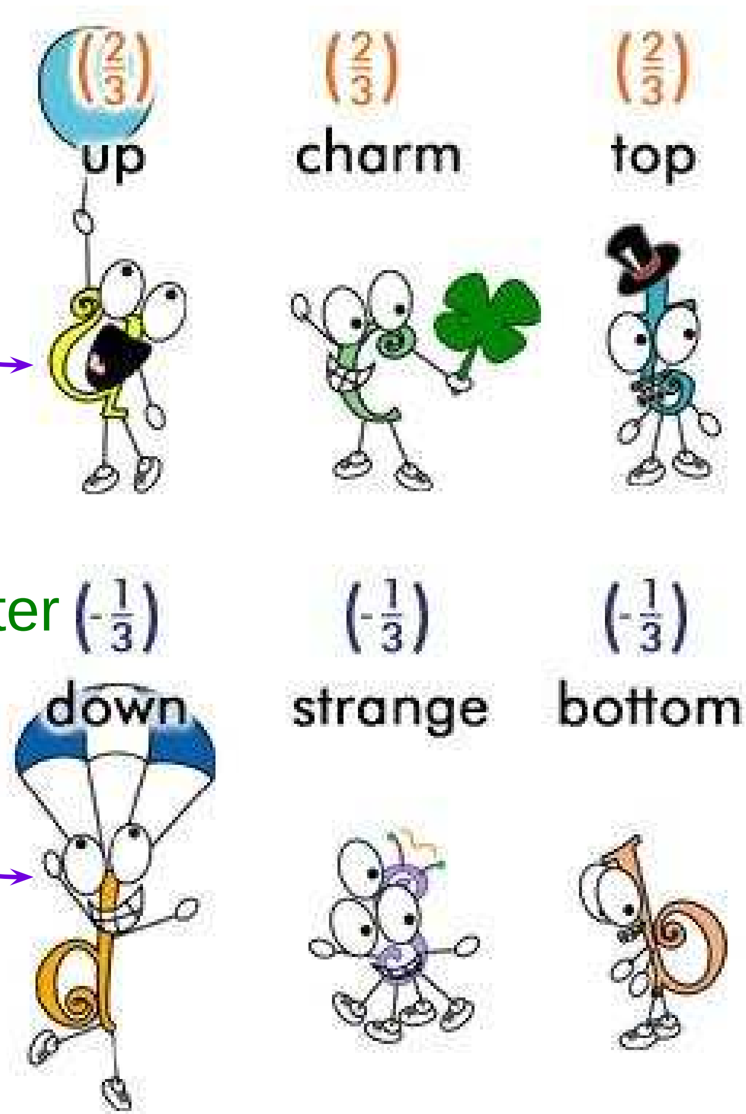
Real World

Often . . . Only

Two Flavours Matter



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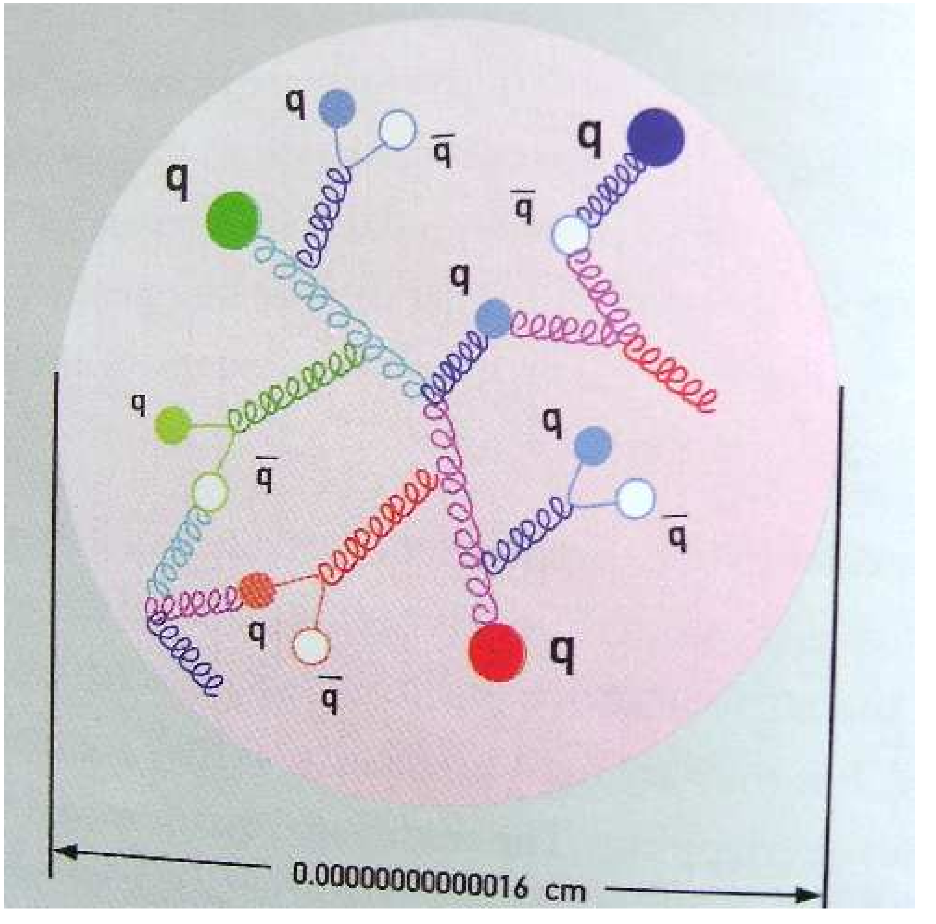
NSAC Long Range Plan

A central goal of nuclear physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD



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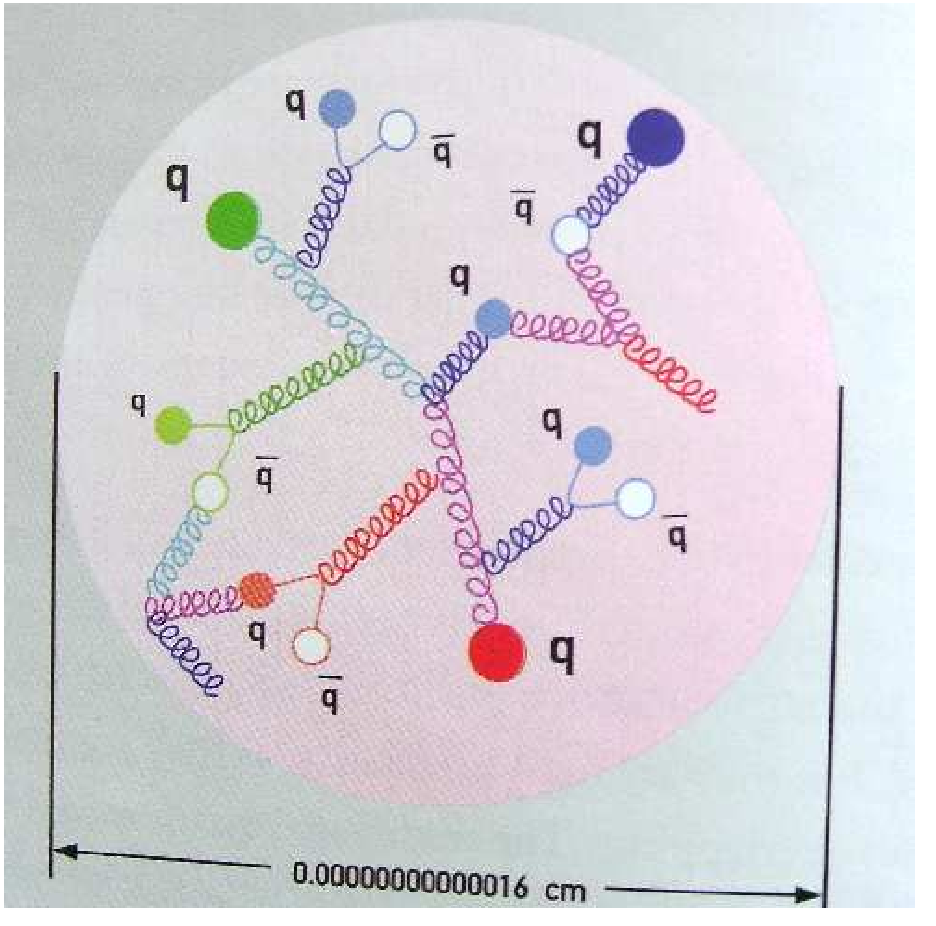

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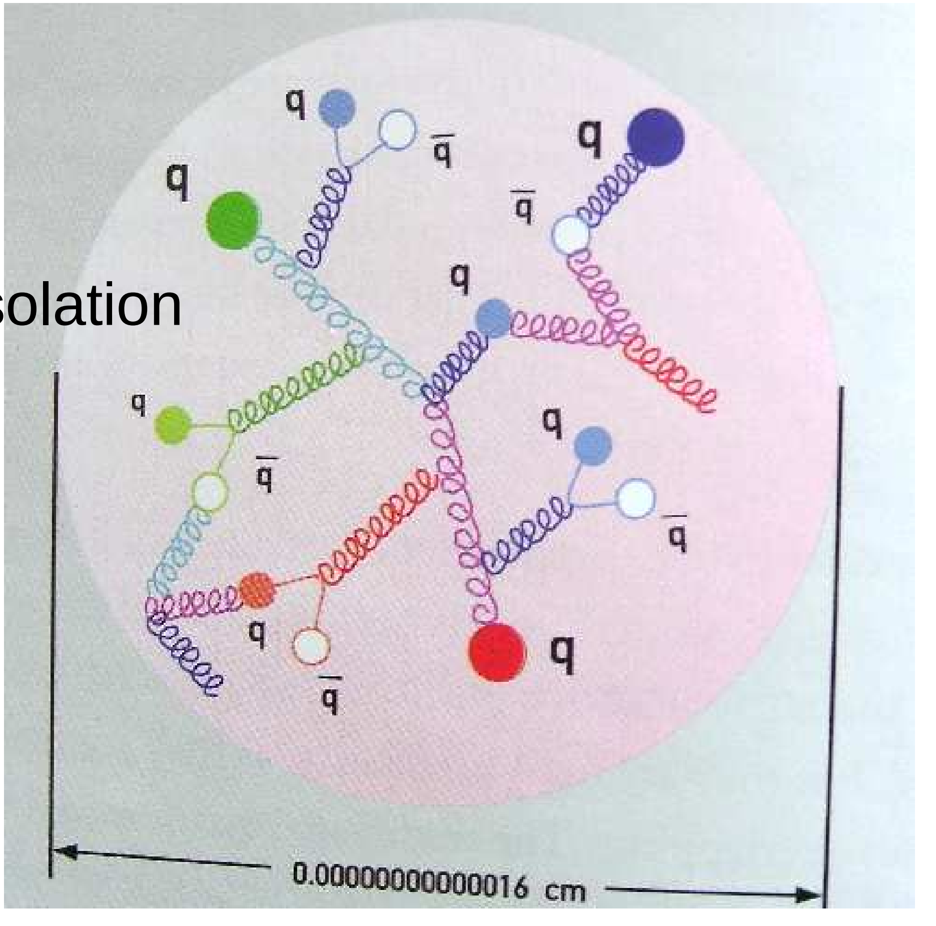
- Confinement
  – No quark ever seen in isolation



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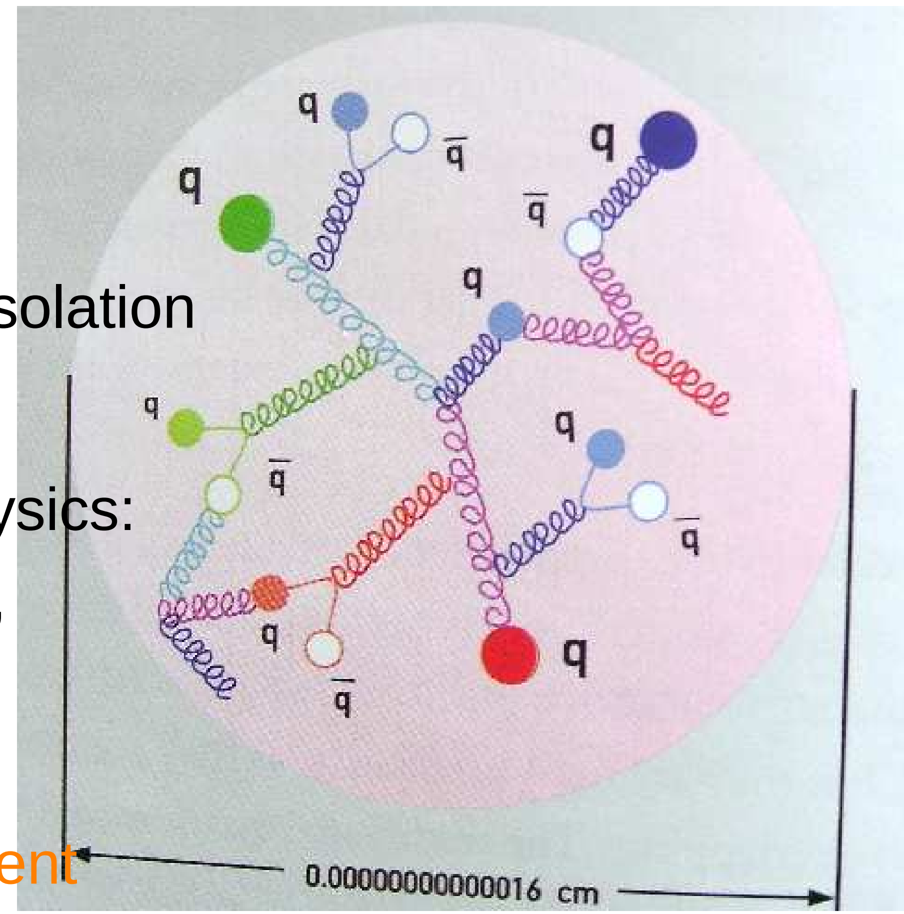

- Confinement
  – No quark ever seen in isolation
- Weightlessness
  – 2004 Nobel Prize in Physics:
  Mass of $u-$ & $d-$quarks,
  each just 5 MeV;
  Proton Mass is 940 MeV
  $\Rightarrow$ No Explanation Apparent
  for 98.4 % of Mass

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Meson Spectrum

LIGHT UNFLAVORED ($S = C = B = 0$)	$I^G(J^{PC})$		$I^G(J^{PC})$	STRANGE ($S = \pm1, C = B = 0$)	$I(J^P)$
• $\pi^\pm$ 140 MeV	$1^-(0^-)$	• $\phi(1680)$	$0^-(1^{--})$	• $K^\pm$	$1/2(0^-)$
• π^0	$1^-(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$	• K^0	$1/2(0^-)$
• η	$0^+(0^{-+})$	• $\rho(1700)$	$1^+(1^{--})$	• K^0_S	$1/2(0^-)$
• $f_0(600)$	$0^+(0^{++})$	$a_2(1700)$	$1^-(2^{++})$	• K^0_L	$1/2(0^-)$
• $\rho(770)$ 770	$1^+(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$	• $K^*(892)$	$1/2(1^-)$
• $\omega(782)$	$0^-(1^{--})$	$\eta(1760)$	$0^+(0^{-+})$	• $K_1(1270)$	$1/2(1^+)$
• $\eta'(958)$	$0^+(0^{-+})$	• $\pi(1800)$	$1^-(0^{-+})$	• $K_1(1400)$	$1/2(1^+)$
• $f_0(980)$	$0^+(0^{++})$	$f_2(1810)$	$0^+(2^{++})$	• $K^*(1410)$	$1/2(1^-)$
• $a_0(980)$	$1^-(0^{++})$	• $\phi_3(1850)$	$0^-(3^{--})$	• $K^*_0(1430)$	$1/2(0^+)$
• $\phi(1020)$	$0^-(1^{--})$	$\eta_2(1870)$	$0^+(2^{-+})$	• $K^*_2(1430)$	$1/2(2^+)$
• $h_1(1170)$	$0^-(1^{+-})$	$\rho(1900)$	$1^+(1^{--})$	$K(1460)$	$1/2(0^-)$
• $b_1(1235)$	$1^+(1^{+-})$	$f_2(1910)$	$0^+(2^{++})$	$K_2(1580)$	$1/2(2^-)$
• $a_1(1260)$	$1^-(1^{++})$	$f_2(1950)$	$0^+(2^{++})$	$K(1630)$	$1/2(?^?)$
• $f_2(1270)$	$0^+(2^{++})$	$\rho_3(1990)$	$1^+(3^{--})$	$K_1(1650)$	$1/2(1^+)$
• $f_1(1285)$	$0^+(1^{++})$	$X(2000)$	$1^-(?^{?+})$	• $K^*(1680)$	$1/2(1^-)$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(2010)$	$0^+(2^{++})$	• $K_2(1770)$	$1/2(2^-)$
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Modern Miracles in Hadron Physics

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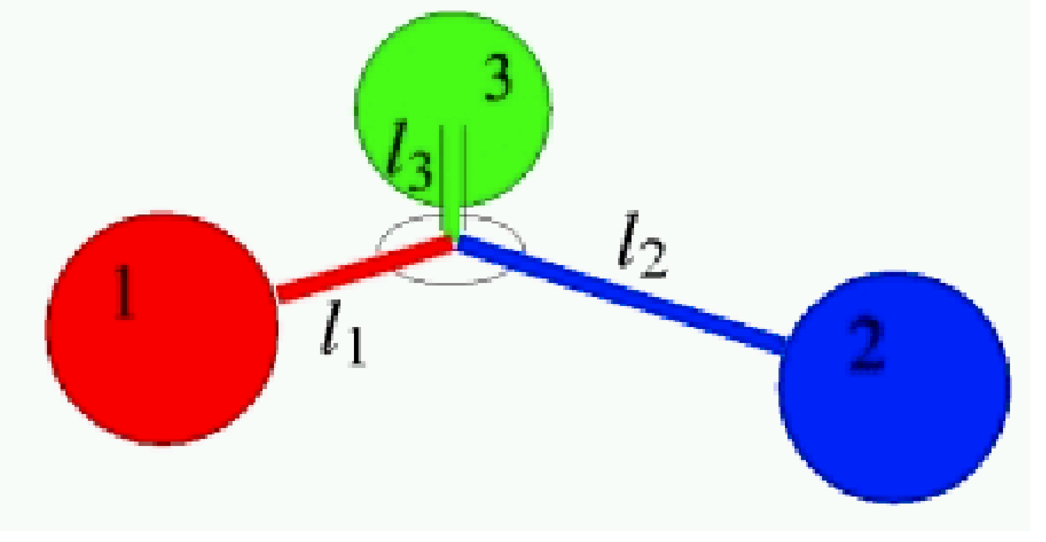
- proton = three constituent quarks



Modern Miracles in Hadron Physics

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- proton $=$ three constituent quarks
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- guess $M_{\mathbf{constituent-quark}} \approx \dfrac{\mathbf{1\,GeV}}{\mathbf{3}} \approx \mathbf{350\,MeV}$

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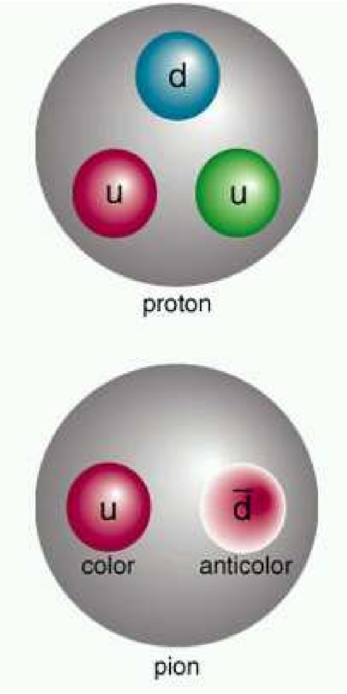

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- WRONG . . . . $\boxed{M_{\text{pion}} = 140\,\text{MeV}}$
- Another meson:
  . . . . $M_\rho = 770\,\text{MeV}$ . . . . No Surprises Here

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Modern Miracles in Hadron Physics

- proton $=$ three constituent quarks
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- WRONG .... $\boxed{M_{\text{pion}} = 140\,\text{MeV}}$
- What is "wrong" with the pion?

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Is this . . . Mass Destruction?

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Thomas Jefferson National Accelerator Facility

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- World’s Premier Hadron Physics Facility

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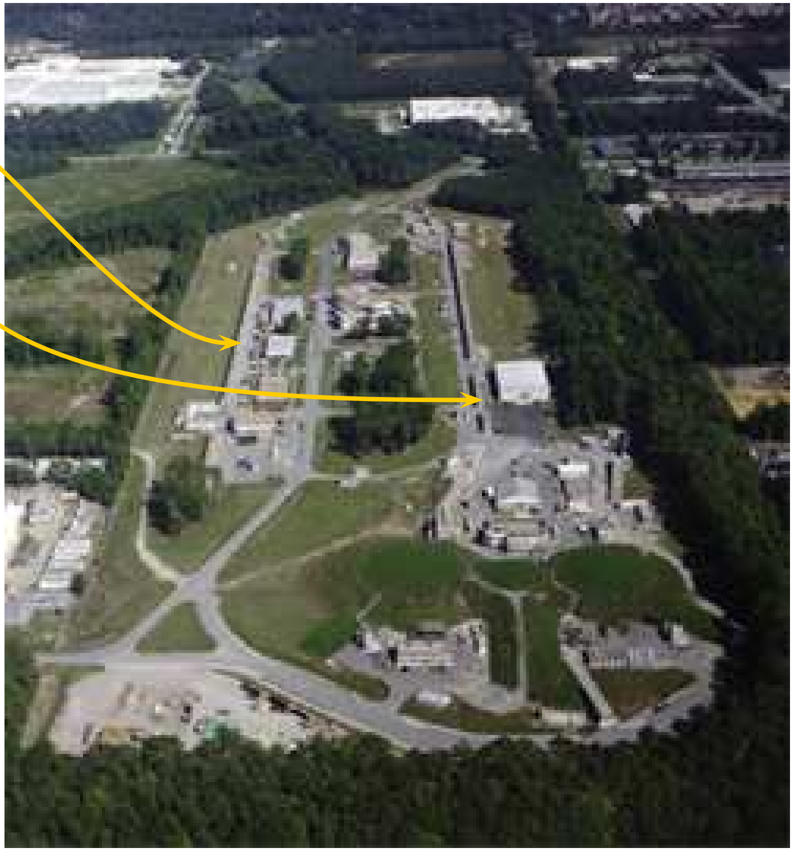

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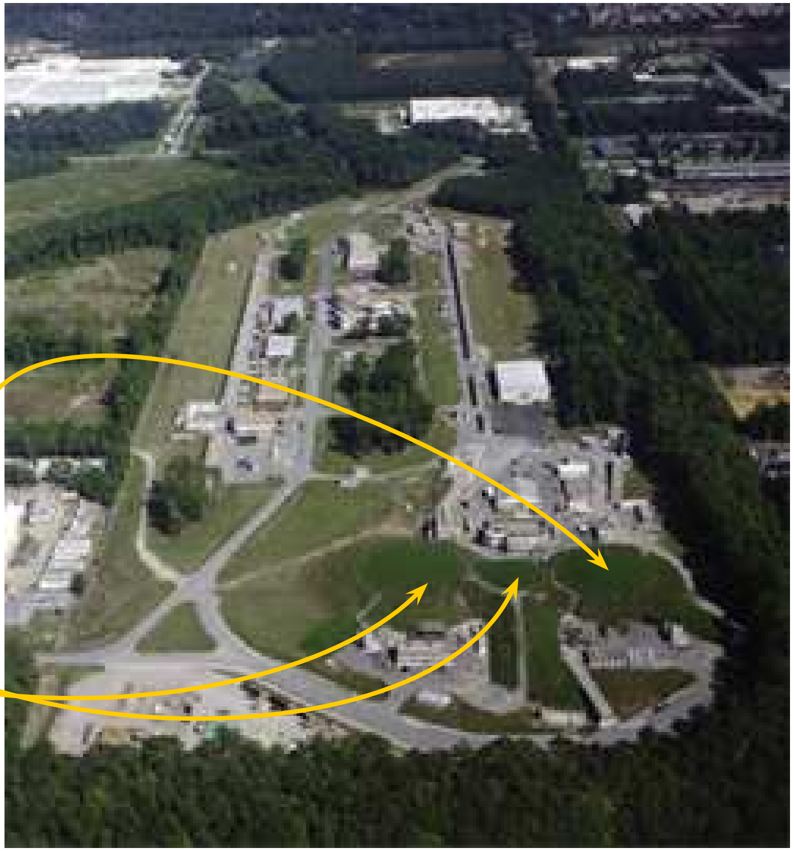

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Thomas Jefferson National Accelerator Facility

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- Design goal (4 GeV) experiments began in 1995
- Electrons accelerated by repeated journeys along *linacs*
- Once desired energy is reached, Beam is directed into Experimental Halls A, B and C
- Current Peak Electron Beam Energy Nearly 6 GeV

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JLab Hall-A

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- Measured Ratio of
  Proton's Electric and Magnetic Form Factors

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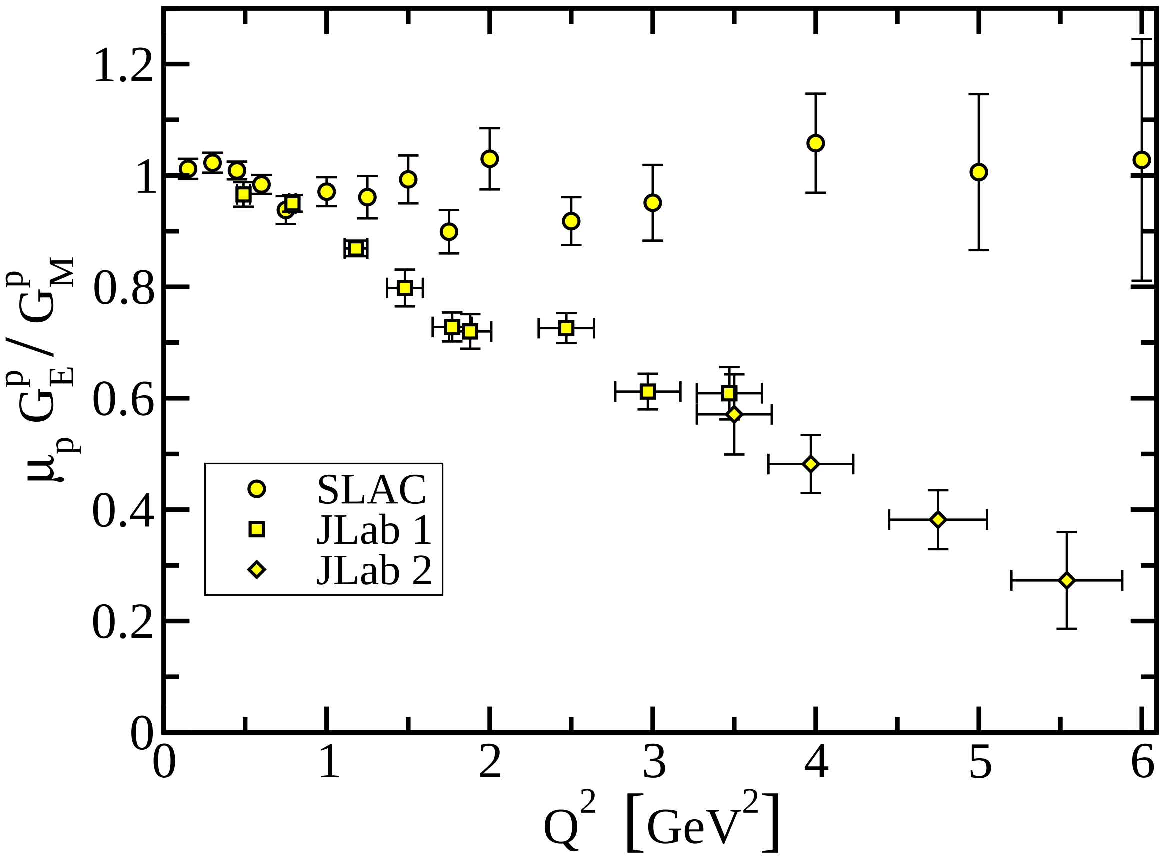
JLab Hall-A



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JLab Hall-A

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- Jones *et al.*, JLab Hall A Collaboration, Phys. Rev. Lett. **84**, 1398 (2000)
- Gayou, *et al.*, Phys. Rev. **C 64**, 038202 (2001)
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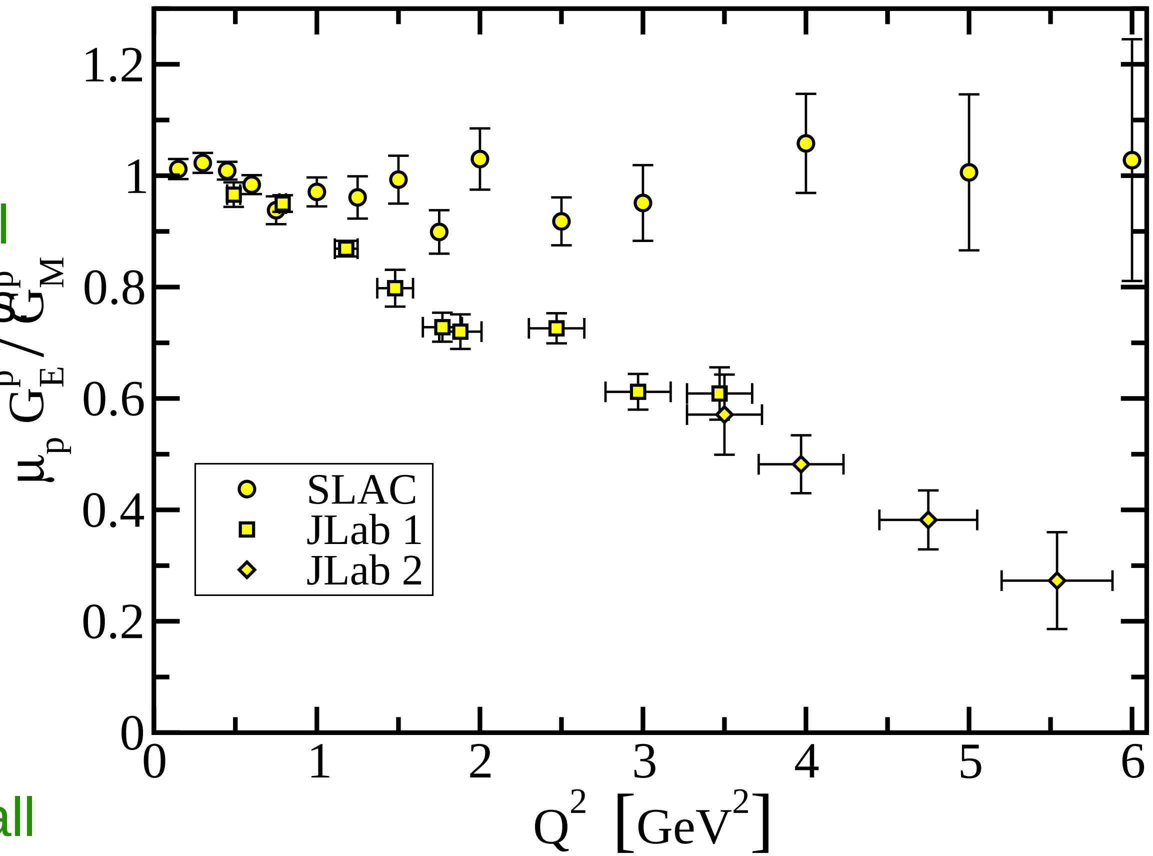
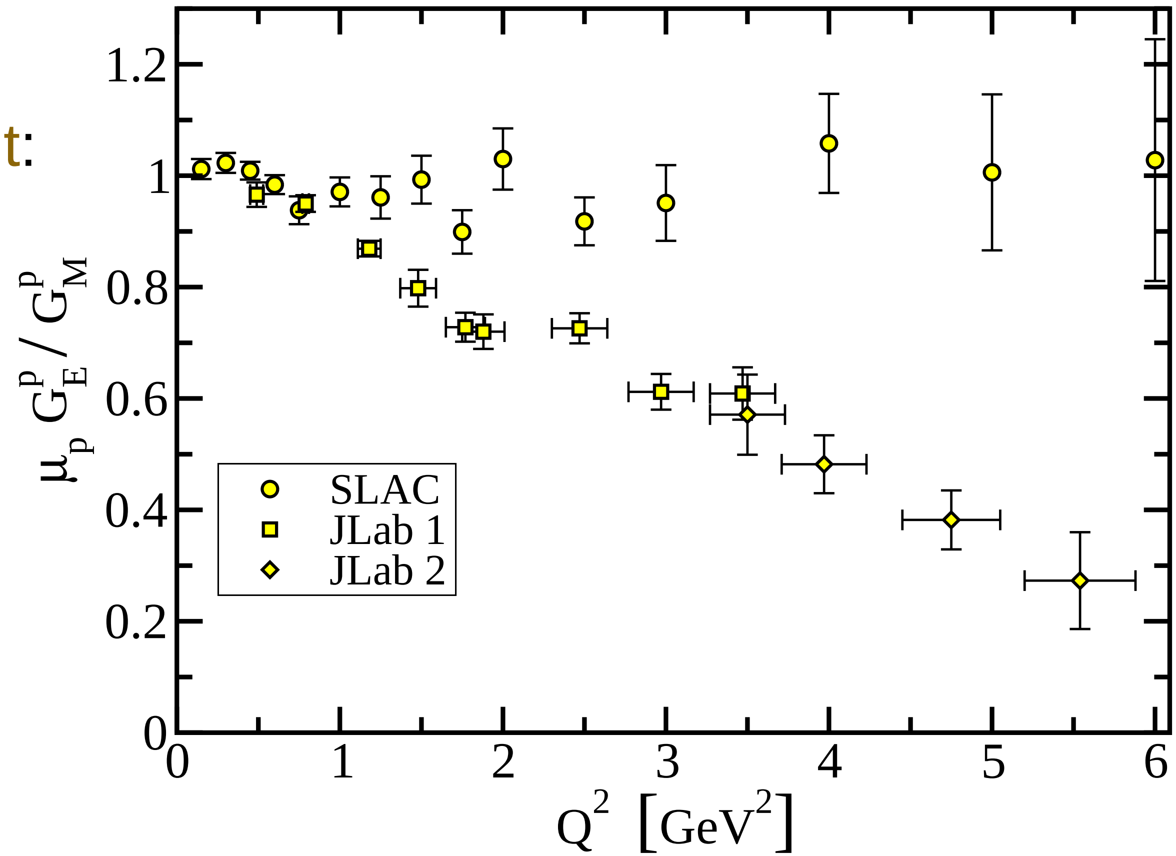


JLab Hall-A

- If JLab Correct, then Completely Unexpected Result: In the Proton
  - – On Relativistic Domain
  - – Distribution of Quark-Charge Not Equal Distribution of Quark-Current!



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What's the Problem?

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- Must calculate the proton's *wave function*
  – Can't be done using perturbation theory

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  - Here relativistic effects are crucial

    – *virtual particles*

    Quintessence of Relativistic Quantum Field Theory

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  - Interaction between quarks – the Interquark Potential –
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- Modern Physics & Mathematics
  – Still quite some way from being able to do that

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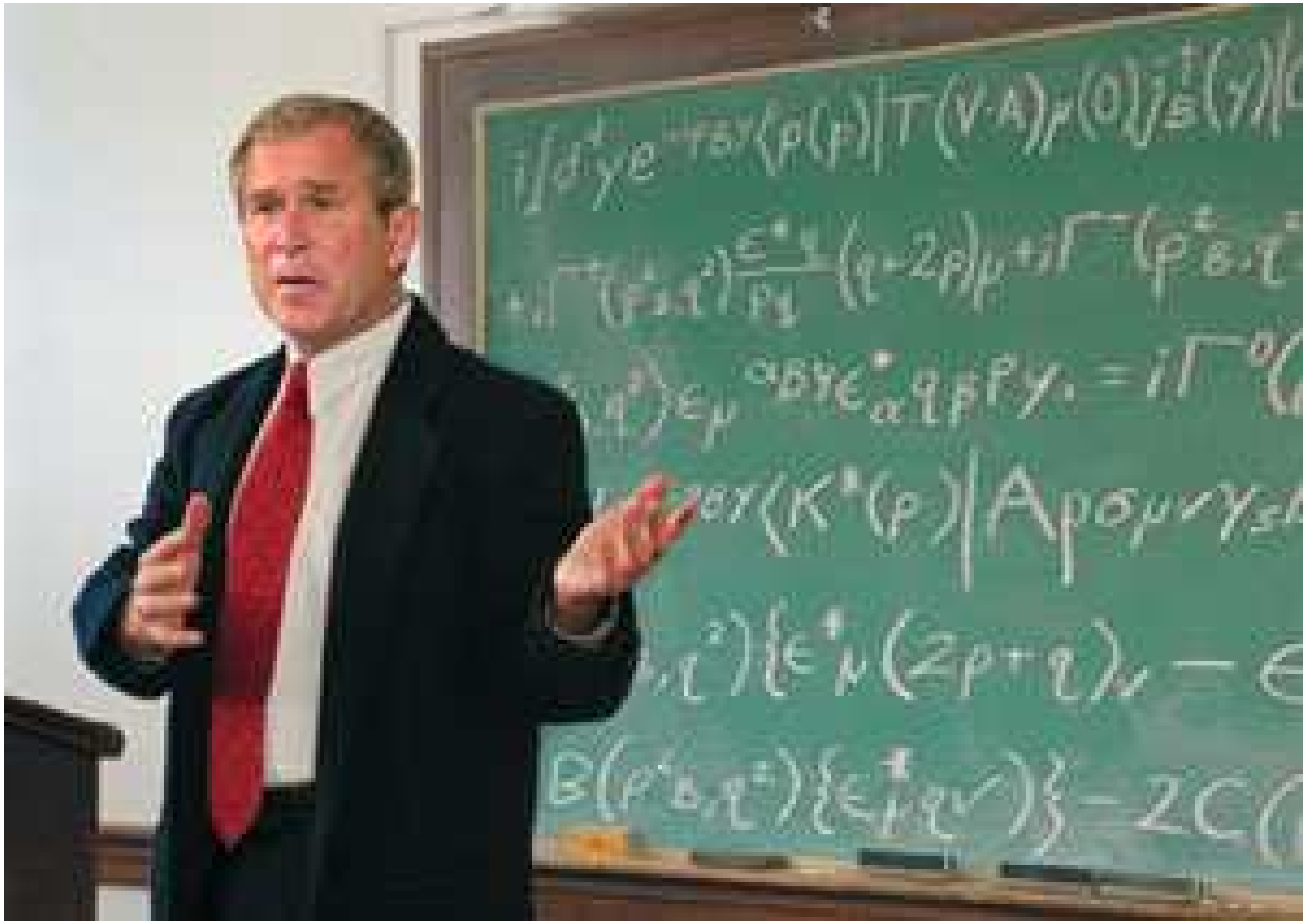
Explanation?



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QCD

- Action, in terms of local Lagrangian density:

$$S[A^a_\mu, \bar{q}, q] = \int d^4x \left\{ \frac{1}{4} F^a_{\mu\nu}(x) F^a_{\mu\nu}(x) + \frac{1}{2\xi} \partial_\mu A^a_\mu(x)\, \partial_\nu A^a_\nu(x) + \bar{q}(x) \left[\gamma_\mu D_\mu + M \right] q(x) \right\} \tag{1}$$

  - Chromomagnetic Field Strength Tensor –
    $\partial_\mu A^a_\nu(x) - \partial_\nu A^a_\mu(x) + g f^{abc} A^b_\mu(x) A^c_\nu(x)$
  - Covariant Derivative – $D_\mu = \partial_\mu - ig \dfrac{\lambda^a}{2} A^a_\mu(x)$
  - Current-quark Mass matrix: $\begin{pmatrix} m_u & 0 & 0 & \ldots \\ 0 & m_d & 0 & \ldots \\ 0 & 0 & m_s & \ldots \\ \vdots & \vdots & \vdots & \end{pmatrix}$

- Understanding JLab Observables means knowing all that this Action predicts.
- Perturbation Theory (asymptotic freedom) is not enough!
  - Bound states are not perturbative
  - Confinement is not perturbative
  - DCSB is not perturbative

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Euclidean Metric

- Almost all nonperturbative studies in relativistic quantum field theory employ a Euclidean Metric. (NB. Remember the Wick Rotation?)
- It is possible to view the Euclidean formulation of a quantum field theory as definitive; e.g.,
  - Symanzik, K. (1963) in *Local Quantum Theory* (Academic, New York) edited by R. Jost.
  - Streater, R.F. and Wightman, A.S. (1980), *PCT, Spin and Statistics, and All That* (Addison-Wesley, Reading, Mass, 3rd edition).
  - Glimm, J. and Jaffee, A. (1981), *Quantum Physics. A Functional Point of View* (Springer-Verlag, New York).
  - Seiler, E. (1982), *Gauge Theories as a Problem of Constructive Quantum Theory and Statistical Mechanics* (Springer-Verlag, New York).
- That decision is crucial when a consideration of nonperturbative effects becomes important. In addition, the discrete lattice formulation in Euclidean space has allowed some progress to be made in attempting to answer existence questions for interacting gauge field theories.
  - A lattice formulation is impossible in Minkowski space – the integrand is not non-negative and hence does not provid a probability measure.

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Euclidean Metric: Transcription Formulae

- To make clear our conventions: for 4-vectors a, b: $a \cdot b := a_\mu \, b_\nu \, \delta_{\mu\nu} := \sum_{i=1}^{4} a_i \, b_i$,

  Hence, a spacelike vector, Q_μ, has $Q^2 > 0$.
- Dirac matrices:
  - Hermitian and defined by the algebra $\{\gamma_\mu, \gamma_\nu\} = 2\,\delta_{\mu\nu}$;
  - we use $\gamma_5 := -\,\gamma_1\gamma_2\gamma_3\gamma_4$, so that $\mathrm{tr}\,[\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma] = -4\,\varepsilon_{\mu\nu\rho\sigma}$, $\varepsilon_{1234} = 1$.
  - The Dirac-like representation of these matrices is:

$$\vec{\gamma} = \begin{pmatrix} 0 & -i\vec{\tau} \\ i\vec{\tau} & 0 \end{pmatrix}, \; \gamma_4 = \begin{pmatrix} \tau^0 & 0 \\ 0 & -\tau^0 \end{pmatrix}, \tag{2}$$

where the 2×2 Pauli matrices are:

$$\tau^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; \tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

Euclidean Metric: Transcription Formulae

- It is possible to derive every equation introduced above assuming certain analytic properties of the integrands. However, the derivations can be sidestepped using the following *transcription rules*:

Configuration Space	Momentum Space
1. $\int^M d^4x^M \to -i \int^E d^4x^E$	1. $\int^M d^4k^M \to i \int^E d^4k^E$
2. $\not\partial \to i\gamma^E \cdot \partial^E$	2. $\not k \to -i\gamma^E \cdot k^E$
3. $\not A \to -i\gamma^E \cdot A^E$	3. $\not A \to -i\gamma^E \cdot A^E$
4. $A_\mu B^\mu \to -A^E \cdot B^E$	4. $k_\mu q^\mu \to -k^E \cdot q^E$
5. $x^\mu \partial_\mu \to x^E \cdot \partial^E$	5. $k_\mu x^\mu \to -k^E \cdot x^E$

- These rules are valid in perturbation theory; i.e., the correct Minkowski space integral for a given diagram will be obtained by applying these rules to the Euclidean integral: they take account of the change of variables and rotation of the contour. However, for diagrams that represent DSEs which involve dressed n-point functions, whose analytic structure is not known *a priori*, the Minkowski space equation obtained using this prescription will have the right appearance but it's solutions may bear no relation to the analytic continuation of the solution of the Euclidean equation. Any such differences will be nonperturbative in origin.

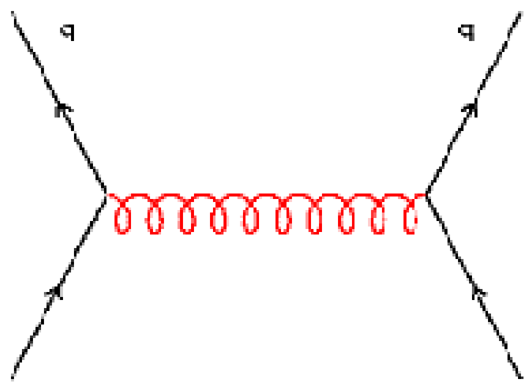
What is QCD?

What is QCD?

- Gauge Theory:

  Interactions Mediated by massless vector bosons

Feynman Diagram of Quark–Quark Scattering



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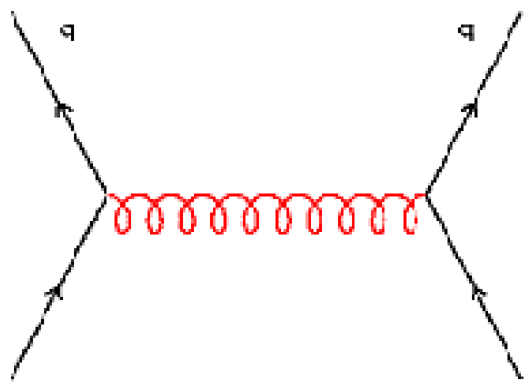

- Similar interaction in QED

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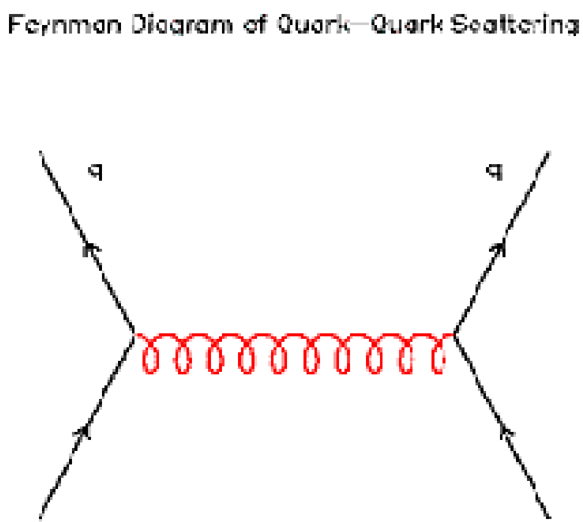

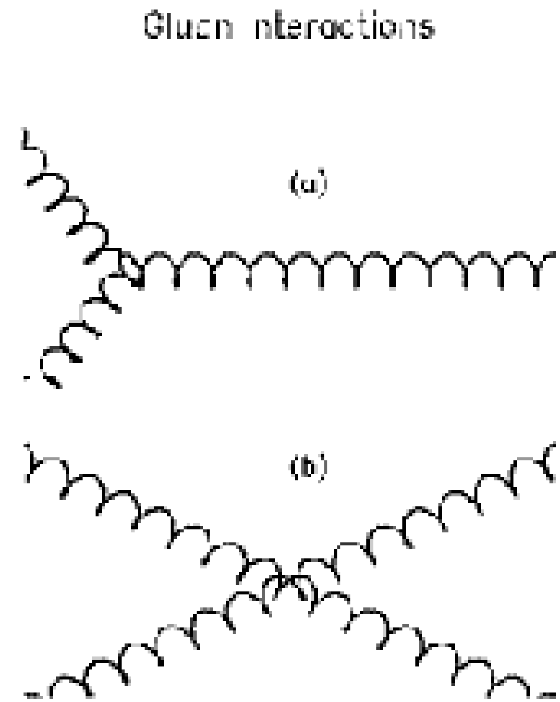

- Similar interaction in QED

- Special Feature of QCD – gluon self-interactions

  Completely Change the Character of the Theory

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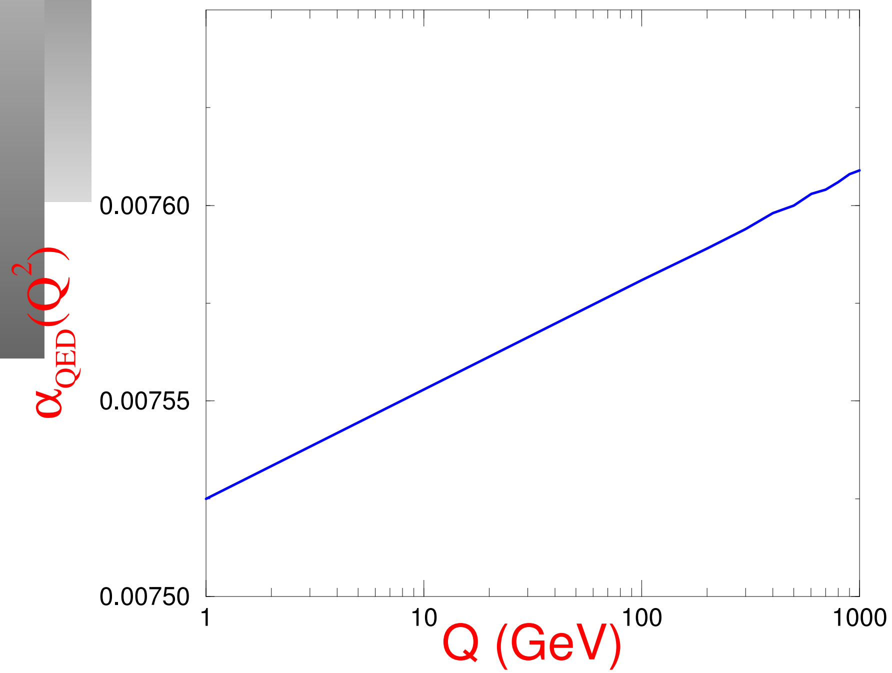
QED cf. QCD

QED cf. QCD



$$\alpha_{\mathrm{QED}} = \frac{\alpha}{1 - \alpha/3\pi \ln\left(Q^2/m_e^2\right)}$$

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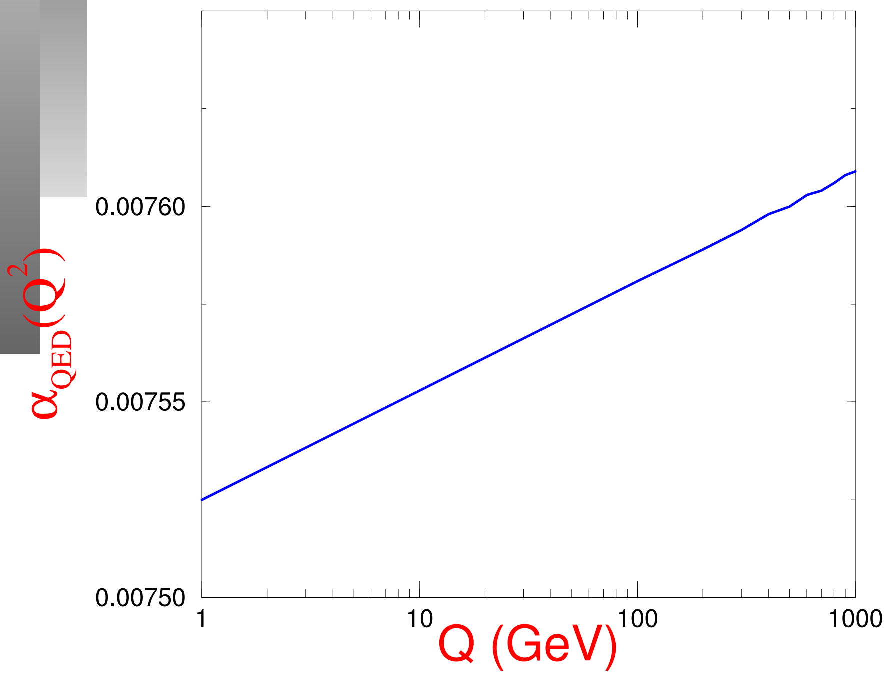
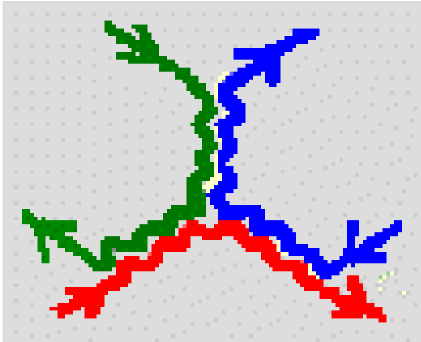
QED cf. QCD



Add three-gluon interaction



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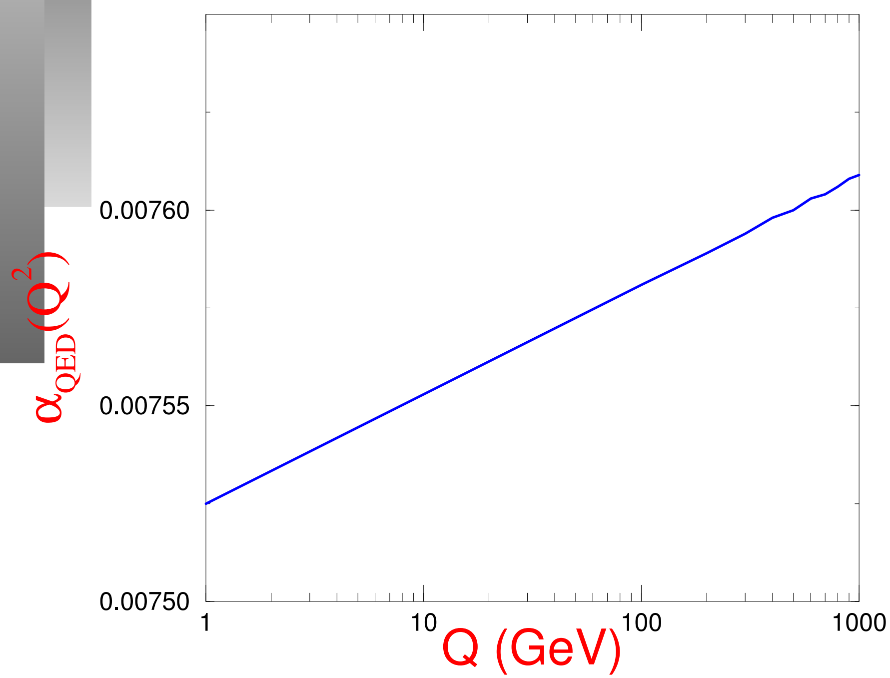

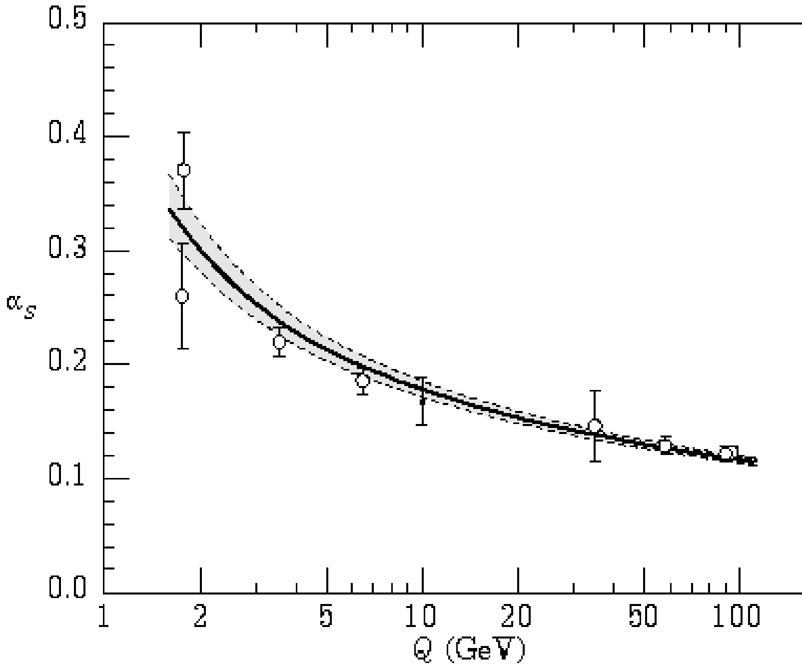

Figure 9.2: Summary of the values of $\alpha_s(Q)$ at the value measured. The lines show the central values and the $\pm 1\sigma$ lin figure clearly shows the decrease in $\alpha_s(Q)$ with increasing Q

$$\alpha_{\mathrm{QED}} = \frac{\alpha}{1 - \alpha/3\pi \ln\left(Q^2/m_e^2\right)}$$

$$\alpha_{\mathrm{QCD}} = \frac{12\pi}{(33 - 2N_f)\ln\left(Q^2/\Lambda^2\right)}$$

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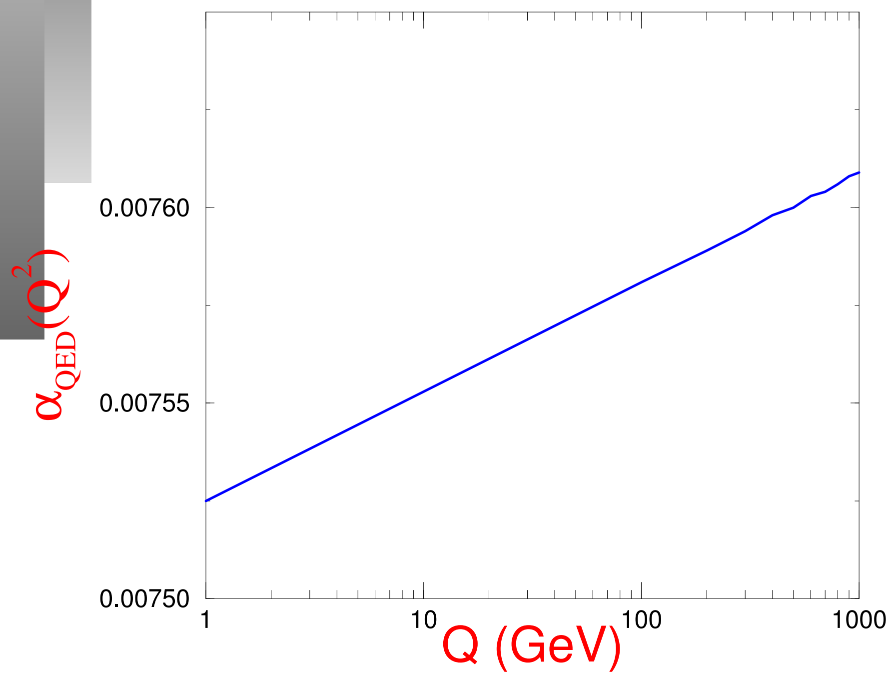
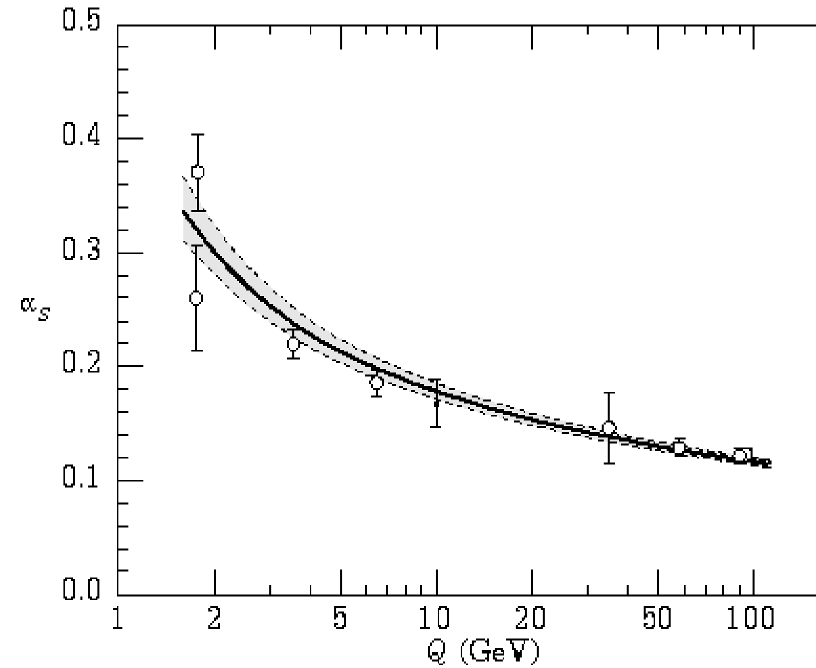
2004 Nobel Prize in Physics: Gross, Politzer and Wilczek





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Closer look at Spectrum

- Features of the Spectrum:
  - $\dfrac{m_\rho^2}{m_\pi^2} = 30$
  - $\dfrac{m_{a_1}^2}{m_{a_0}^2} = 1.7$
  - ? Hyperfine Splitting

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• $\dfrac{m_N}{m_\pi} \approx 7$	• $\dfrac{m_N}{m_\rho} = \dfrac{5}{4} \approx \dfrac{3}{2}$	? Quark Counting

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Something Very Odd About the Pion

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Dichotomy of the Pion

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Dichotomy of the Pion

- Pion responsible for long-range part of nucleon-nucleon potential

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Dichotomy of the Pion

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..................Pion better be light for long-range potential

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The correct understanding of hadron observables must explain why the pion is light but the proton is heavy.

- Requires explanation of Connection between pQCD-quark and Spectrum/Constituent-quark

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QCD's Emergent Phenomena

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- Quark and Gluon Confinement
  - No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon

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  - Very unnatural pattern of bound state masses

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- Neither of these phenomena is apparent in QCD's Lagrangian yet they are the dominant determining characteristics of real-world QCD.
- NSAC – Understanding these phenomena is one of the greatest intellectual challenges in physics

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Chiral Symmetry

Gauge Theories with Massless Fermions have

$\boxed{\text{CHIRAL SYMMETRY}}$

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Chiral Symmetry

- Helicity $\lambda \propto J \cdot p$
  - Projection of Spin onto Direction of Motion
  - Poincaré Invariant Spin-Observable
  - $\lambda = \pm$ ($\|$ or anti-$\|$ to p_μ)

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Chiral Symmetry

- Helicity $\lambda \propto J \cdot p$
  - Projection of Spin onto Direction of Motion
  - Poincaré Invariant Spin-Observable
  - $\lambda = \pm$ ($\|$ or anti-$\|$ to p_μ)

- Chirality Operator: γ_5
  - Chiral Transformation $q(x) \to \mathrm{e}^{i\gamma_5\theta}\, q(x)$
  - Chiral Rotation $\theta = \dfrac{\pi}{2}$
    - $q_{\lambda=+} \to q_{\lambda=+}$, $q_{\lambda=-} \to -q_{\lambda=-}$
    - Composite Particles: $J^{P=+} \leftrightarrow J^{P=-}$
  - Equivalent to "Parity Conjugation" Operation

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Chiral Symmetry

- A Prediction of Chiral Symmetry
  - Degeneracy between Parity Partners

    $N(\frac{1}{2}^+, 938) = N(\frac{1}{2}^-, 1535)$, $\pi(0^-, 140) = a_0(0^+, 980)$,

    $\rho(1^-, 770) = a_1(1^+, 1260)$
- Doesn't Look too good

  Predictions *not* Valid – Violations *too* Large.
- Appears to suggest quarks are Very Heavy

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How can pion mass be so small

If quarks are so heavy?!

Propagators

- Extraordinary Effects in QCD Tied to Properties of *Dressed*-Quark and -Gluon Propagators

Quark	Gluon
$S_f(x-y) \equiv \langle q_f(x)\bar{q}_f(y)\rangle$	$D_{\mu\nu}(x-y) \equiv \langle A_\mu(x)A_\nu(y)\rangle$

- Describe in-Medium *Propagation Characteristics* of Elementary Particles

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Propagators

- Example: Solid-State Physics
  - γ propagating in a Dense e$^-$ Gas
  - Acquires a Debye Mass
    $$m_{\mathrm{D}}^2 \propto k_F^2: \; \frac{1}{Q^2} \to \frac{1}{Q^2 + m_{\mathrm{D}}^2}$$
  - γ develops an Effective-mass

Propagators

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    $$m_{\rm D}^2 \propto k_F^2 : \ \frac{1}{Q^2} \to \frac{1}{Q^2 + m_{\rm D}^2}$$

  - γ develops an Effective-mass
  - Leads to Screening of the Interaction: $r \propto \dfrac{1}{m_D}$
- Quark and Gluon Propagators:

  Modified in a similar way -

  Momentum Dependent Effective Masses
- The Effect of this is Observable in QCD

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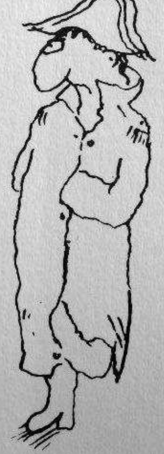
Explicit Chiral Symmetry Breaking

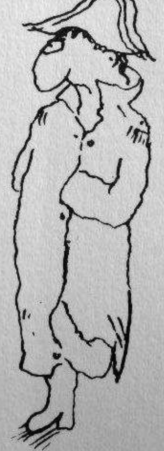


Explicit Chiral Symmetry Breaking

- Chiral Symmetry

  Can be discussed in terms of Quark Propagator

  - Free Quark Propagator $S_0(p) = \dfrac{-i\gamma \cdot p + m}{p^2 + m^2}$



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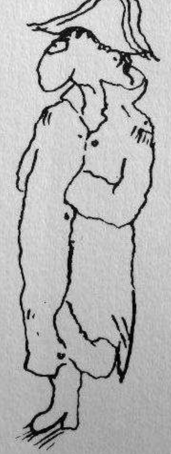
  - Free Quark Propagator $S_0(p) = \dfrac{-i\gamma \cdot p + m}{p^2 + m^2}$

- Chiral Transformation

$$\begin{aligned} \mathbf{S}_0(p) &\rightarrow e^{i\gamma_5\theta} S_0(p) e^{i\gamma_5\theta} \\ &= \frac{-i\gamma \cdot p}{p^2+m^2} + e^{2i\gamma_5\theta} \frac{m}{p^2+m^2} \end{aligned}$$

  - Symmetry Violation $\propto m$
  - $\mathbf{m} = 0$: $S_0(p) \rightarrow S_0(p)$



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  $$\langle \bar{q} q \rangle_\mu \equiv \int_\mu^\Lambda \frac{d^4 p}{(2\pi)^4} \, \mathrm{tr}\,[S(p)] \propto \int_\mu^\Lambda \frac{d^4 p}{(2\pi)^4} \frac{m}{p^2 + m^2}$$

  - A Measure of the Chiral Symmetry Violating Term

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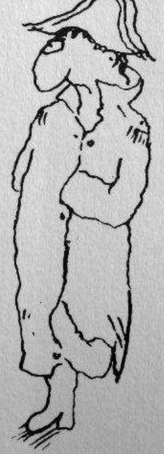

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  - A Measure of the Chiral Symmetry Violating Term
  - Perturbative QCD: Vanishes if $m = 0$

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Dynamical Symmetry Breaking

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Dynamical Symmetry Breaking

$$V(x,y) = (\sigma^2 + \pi^2 - 1)^2$$

Hamiltonian: $T + V$, is Rotationally Invariant

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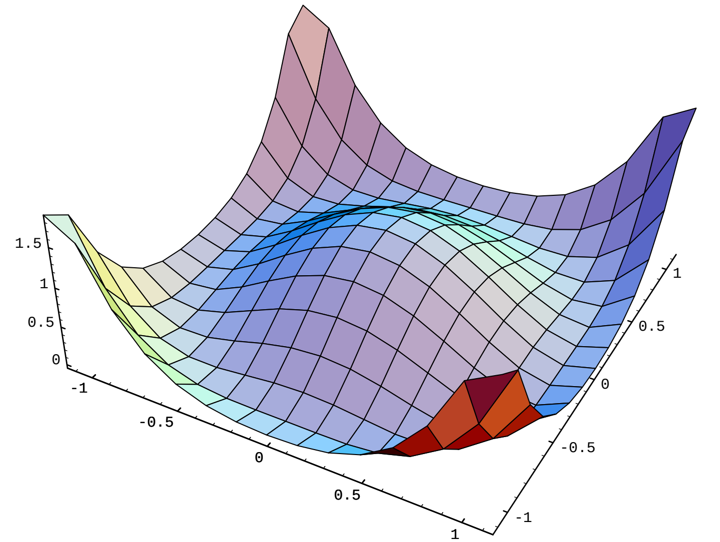

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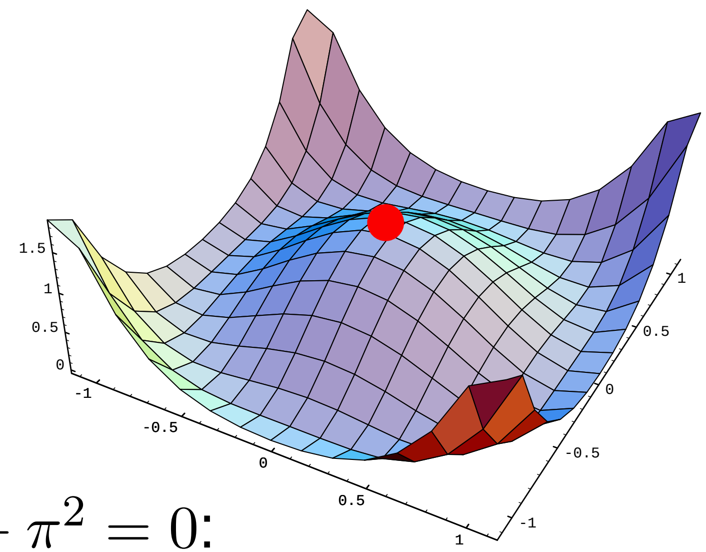

Ground State?

- **Ball** at (σ, π) for which $\sigma^2 + \pi^2 = 0$:
  - Rotationally Invariant

UNSTABLE

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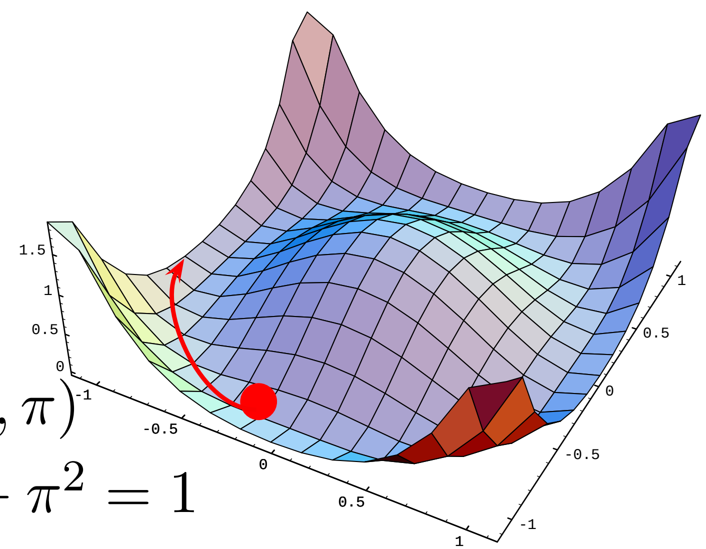

Ground State

- Ball at any (σ, π) for which $\sigma^2 + \pi^2 = 1$
  - All Positions have Same (Minimum) Energy
  - But not invariant under rotations

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Dynamical Symmetry Breaking

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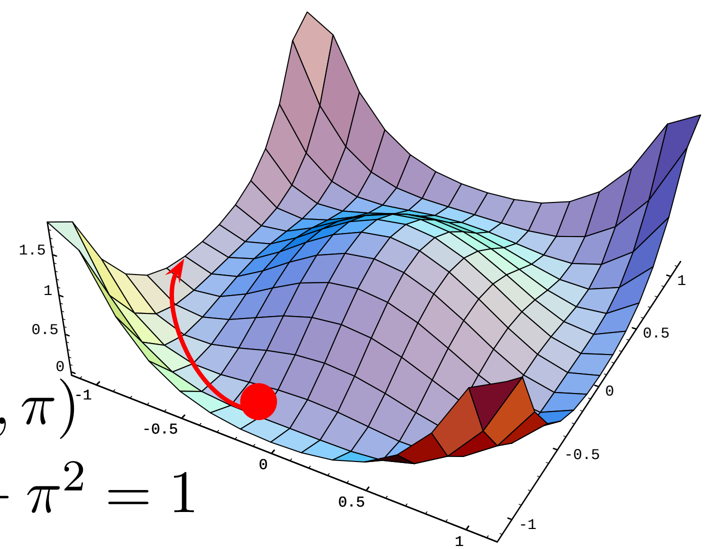
Symmetry of Ground State $\neq$ Symmetry of Hamiltonian

Ground State



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Dynamics and Symmetries

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  Quarks are either left- or right-handed

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  Dynamical Symmetry Breaking and Confinement
  Start with Massless Quarks and
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Dynamics and Symmetries

Very Nonperturbative Problem

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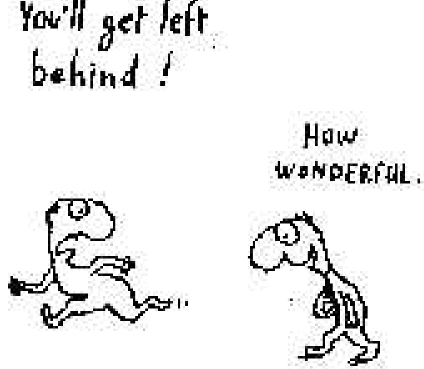
Dyson-Schwinger Equations

Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory



Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory
- NonPerturbative, Continuum approach to QCD

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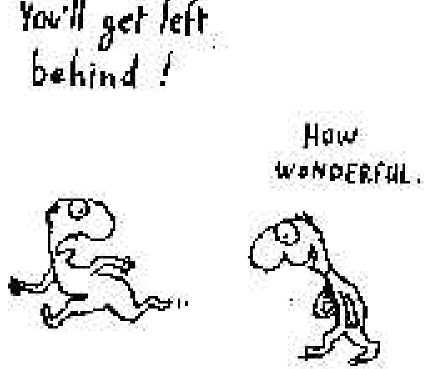

Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory
- NonPerturbative, Continuum approach to QCD
- Simplest level: Generating Tool for Perturbation Theory

  .... Materially Reduces Model Dependence

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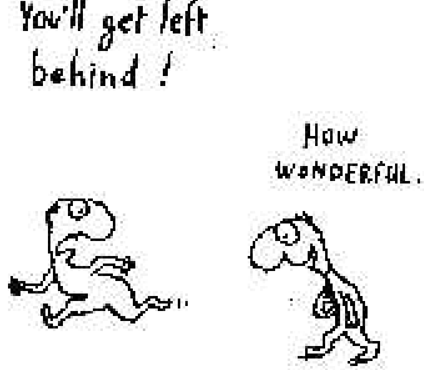

Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory
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  - Hadrons as Composites of Quarks and Gluons

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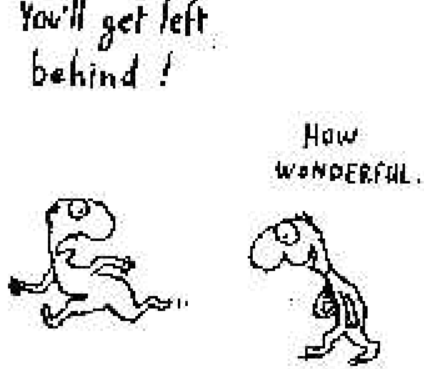

Dyson-Schwinger Equations

- A Modern Method for Relativistic Quantum Field Theory
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  - Hadrons as Composites of Quarks and Gluons
    - Qualitative and Quantitative Importance of:
      - Dynamical Chiral Symmetry Breaking

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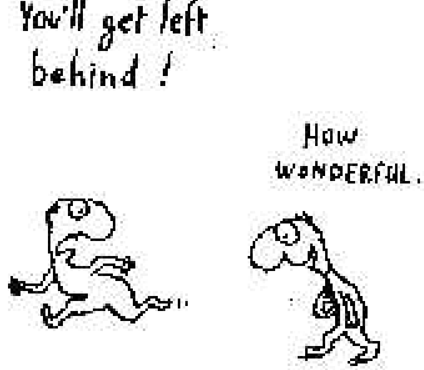

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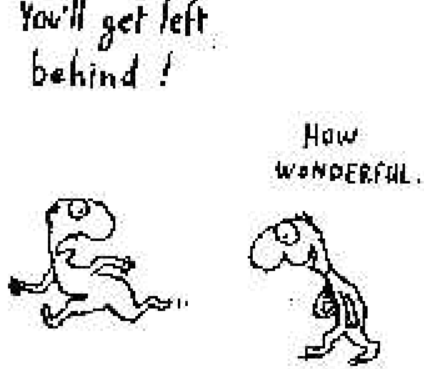

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    - $\Rightarrow$ Understanding InfraRed (long-range)

  .... behaviour of $\alpha_s(Q^2)$

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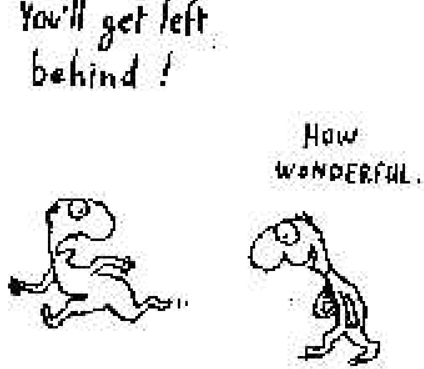

Dyson-Schwinger Equations

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    - $\Rightarrow$ Understanding InfraRed (long-range) .... behaviour of $\alpha_s(Q^2)$
- Method yields Schwinger Functions $\equiv$ Propagators

Cross-Sections built from Schwinger Functions

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Contemporary Reviews

- Dyson-Schwinger Equations:
  Density, Temperature and Continuum Strong QCD
  C.D. Roberts and S.M. Schmidt, nu-th/0005064,
  Prog. Part. Nucl. Phys. **45** (2000) S1
- The IR behavior of QCD Green's functions:
  Confinement, DCSB, and hadrons . . .
  R. Alkofer and L. von Smekal, he-ph/0007355,
  Phys. Rept. **353** (2001) 281
- Dyson-Schwinger equations:
  A Tool for Hadron Physics
  P. Maris and C.D. Roberts, nu-th/0301049,
  Int. J. Mod. Phys. **E 12** (2003) pp. 297-365

Dressed-quark Propagator

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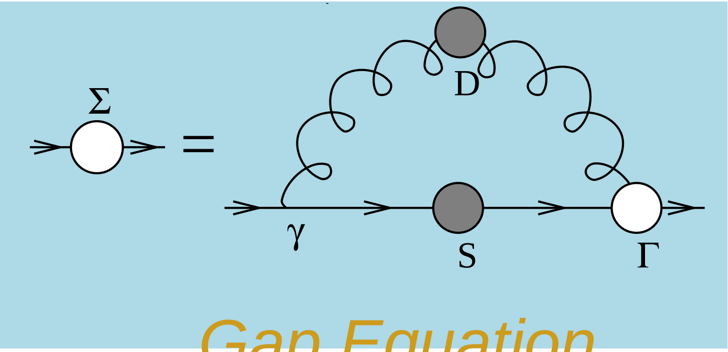
Dressed-quark Propagator

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Gap Equation

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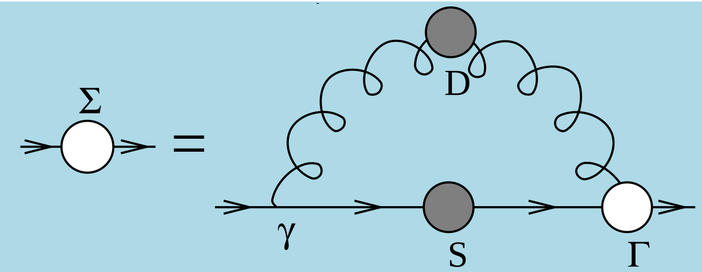

Gap Equation

- dressed-quark propagator

$$S(p) \quad = \quad \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

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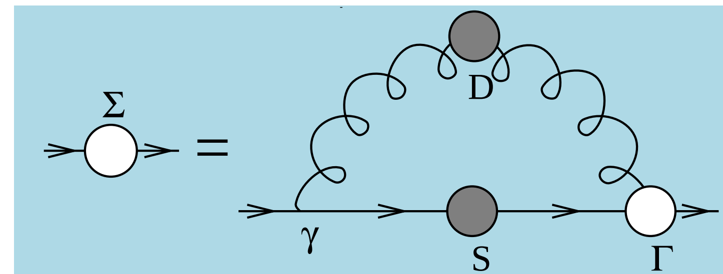

Gap Equation

- dressed-quark propagator

$$S(p) \quad = \quad \frac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$$

- Weak Coupling Expansion

  Reproduces Every Diagram in Perturbation Theory

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Dressed-quark Propagator

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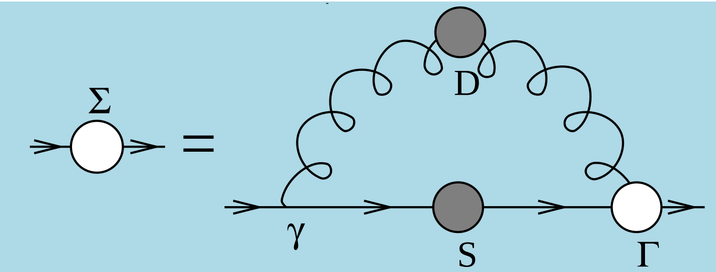

Gap Equation

- dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

- But in Perturbation Theory

$$B(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{m^2}\right] + \ldots\right) \stackrel{m\to 0}{\to} 0$$

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Nambu–Jona-Lasinio Model

- Recall the Gap Equation:

$$S^{-1}(p) = i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m$$
$$+ \int^{\Lambda} \frac{d^4\ell}{(2\pi)^4}\, g^2\, D_{\mu\nu}(p-\ell)\, \gamma_\mu \frac{\lambda^a}{2} \frac{1}{i\gamma\cdot\ell A(\ell^2)+B(\ell^2)} \Gamma^a_\nu(\ell,p) \tag{4}$$

- NJL: $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu \dfrac{\lambda^a}{2}$;

$$g^2 D_{\mu\nu}(p-\ell) \to \delta_{\mu\nu}\, \frac{1}{m_G^2}\, \theta(\Lambda^2-\ell^2) \tag{5}$$

  - Model is not renormalisable
    $\Rightarrow$ regularisation parameter (Λ) plays a dynamical role.

- NJL Gap Equation

$$i\gamma\cdot p\, A(p^2) + B(p^2)$$
$$= \quad i\gamma\cdot p + m + \frac{4}{3}\frac{1}{m_G^2} \int \frac{d^4\ell}{(2\pi)^4}\, \theta(\Lambda^2-\ell^2)\, \gamma_\mu \frac{-i\gamma\cdot\ell A(\ell^2)+B(\ell^2)}{\ell^2 A^2(\ell^2)+B^2(\ell^2)}\, \gamma_\mu \tag{6}$$

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Solving NJL Gap Equation

- Multiply Eq. (6) by $(-i\gamma \cdot p)$; trace over Dirac indices:

$$p^2\, A(p^2) = p^2 + \frac{8}{3}\,\frac{1}{m_G^2}\int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2-\ell^2)\, p\cdot\ell\, \frac{A(\ell^2)}{\ell^2 A^2(\ell^2)+B^2(\ell^2)} \tag{7}$$

  - Angular integral vanishes, therefore

$$A(p^2) \equiv 1\,. \tag{8}$$

This owes to the the fact that NJL model is defined by four-fermion contact interaction in configuration space, entails momentum-independence of interaction in momentum space.

- Tracing over Dirac indices; use Eq. (8):

$$B(p^2) = m + \frac{16}{3}\,\frac{1}{m_G^2}\int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2-\ell^2)\,\frac{B(\ell^2)}{\ell^2+B^2(\ell^2)}\,, \tag{9}$$

  - Integral is p^2-independent.
  - Therefore $B(p^2) = \mathrm{constant} = M$ is the only solution.

NJL Mass Gap

- Evaluate integrals; Eq. (9) becomes

$$M = m + M \, \frac{1}{3\pi^2} \, \frac{1}{m_G^2} \, \mathcal{C}(M^2, \Lambda^2) \,, \tag{10}$$

$$\mathcal{C}(M^2, \Lambda^2) = \Lambda^2 - M^2 \ln \left[1 + \Lambda^2/M^2\right] \,. \tag{11}$$

- Λ defines model's mass-scale. Henceforth set $\boxed{\Lambda = 1}$. Then all other dimensioned quantities are given in units of this scale, in which case the gap equation can be written

$$M = m + M \, \frac{1}{3\pi^2} \, \frac{1}{m_G^2} \, \mathcal{C}(M^2, 1) \,. \tag{12}$$

- Chiral limit: $m = 0$, $\boxed{M = M \, \frac{1}{3\pi^2} \, \frac{1}{m_G^2} \, \mathcal{C}(M^2, 1)}$
  - Solved if $M \equiv 0$

    … This is the $\boxed{\text{perturbative result}}$: start with no mass, end up with no mass.
  - Suppose $M \neq 0$
    - Solved *iff* $\boxed{1 = \frac{1}{3\pi^2} \, \frac{1}{m_G^2} \, \mathcal{C}(M^2, 1)}$.

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NJL Dynamical Mass

- Can one satisfy $\boxed{1 = \frac{1}{3\pi^2}\,\frac{1}{m_G^2}\,\mathcal{C}(M^2,1)}$?
- $\mathcal{C}(M^2,1) = 1 - M^2 \ln\left[1 + 1/M^2\right]$
  - Monotonically decreasing function of M
  - Maximum value at $M = 0$: $\mathcal{C}(0,1) = 1$.
- Consequently $\exists\ M \neq 0$ solution *iff* $\boxed{\frac{1}{3\pi^2}\,\frac{1}{m_G^2} > 1}$
  - Typical scale for hadron physics $\Lambda \sim 1$ GeV.
    - $M \neq 0$ solution *iff* $\boxed{m_G^2 < \frac{\Lambda^2}{3\pi^2} \simeq (0.2\,GeV)^2}$
- Interaction Strength is proportional to $\frac{1}{m_G^2}$
  - When interaction is strong enough,
    one can start with no mass but end up with a massive quark.

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NJL Dynamical Mass

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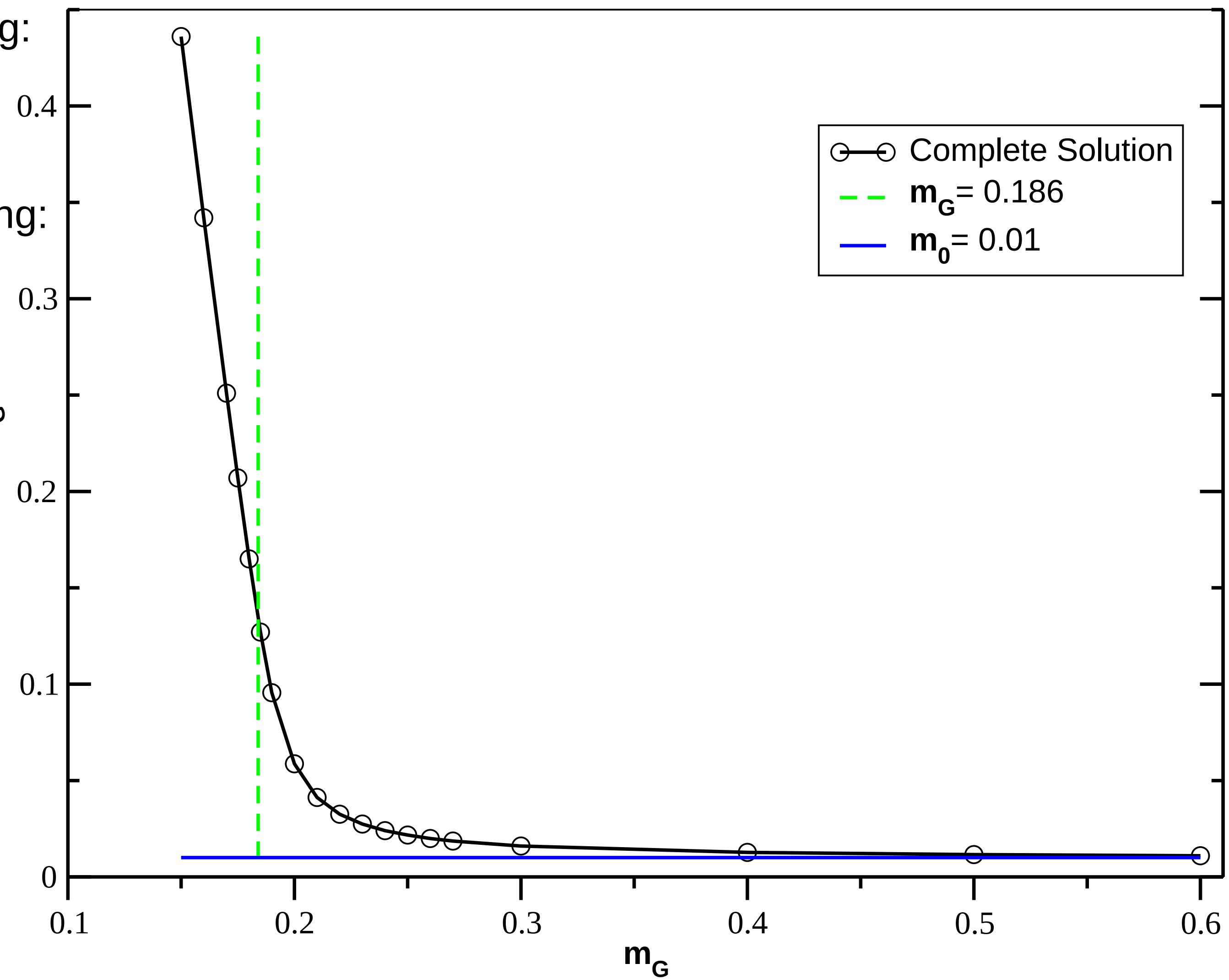
Dynamical Chiral Symmetry Breaking

NJL Dynamical Mass

Solve $$\boxed{M = m_0 + M\,\frac{1}{3\pi^2}\,\frac{1}{m_G^2}\,\mathcal{C}(M^2,1)}$$

- Weak coupling:
  $\Leftrightarrow m_G$ large
  $M \sim m_0$
- Strong coupling:
  $\Leftrightarrow m_G$ small
  $M \gg m_0$
  This is the essential characteristic of DCSB

NJL Mass Gap



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NJL Model and Confinement?

- **Confinement** – no free-particle-like quarks

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NJL Model and Confinement?

- **Confinement** – no free-particle-like quarks
- Fully-dressed NJL propagator

$$S(p)^{\mathrm{NJL}} = \frac{1}{i\gamma\cdot p[A(p^2)=1]+[B(p^2)=M]} = \frac{-i\gamma\cdot p+M}{p^2+M^2} \qquad (15)$$

NJL Model and Confinement?

- **Confinement** – no free-particle-like quarks
- Fully-dressed NJL propagator

$$S(p)^{\rm NJL} = \frac{1}{i\gamma\cdot p[A(p^2)=1] + [B(p^2)=M]} = \frac{-i\gamma\cdot p + M}{p^2+M^2} \tag{17}$$

- This is merely a free-particle-like propagator with a shifted mass:

$$p^2 + M^2 = 0 \Rightarrow \text{ Minkowski-space mass} = M. \tag{18}$$

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NJL Model and Confinement?

- **Confinement** – no free-particle-like quarks
- Fully-dressed NJL propagator

$$S(p)^{\rm NJL} = \frac{1}{i\gamma\cdot p[A(p^2)=1]+[B(p^2)=M]} = \frac{-i\gamma\cdot p + M}{p^2+M^2} \quad (19)$$

- This is merely a free-particle-like propagator with a shifted mass:

$$p^2 + M^2 = 0 \Rightarrow \text{ Minkowski-space mass} = M. \quad (20)$$

- Hence, while **NJL Model** certainly contains DCSB, it **does not exhibit confinement**.

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Munczek-Nemirovsky Model

- Munczek, H.J. and Nemirovsky, A.M. (1983), "The Ground State $q\bar{q}$ Mass Spectrum In QCD," *Phys. Rev*. **D 28**, 181.
- $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu \, \frac{\lambda^a}{2};$

$$g^2 D_{\mu\nu}(k) \to (2\pi)^4 \, G \, \delta^4(k) \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right] \quad (21)$$

Here G defines the model's mass-scale.

  - δ-function in momentum space
    cf. NJL, which has δ-function in configuration space.

- Gap equation

$$i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m + G\,\gamma_\mu \, \frac{-i\gamma\cdot p\, A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)} \, \gamma_\mu \quad (22)$$

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MN Model's Gap Equation

- The gap equation yields the following two coupled equations (set the mass-scale $G=1$):

$$A(p^2) = 1 + 2\,\frac{A(p^2)}{p^2 A^2(p^2) + B^2(p^2)} \tag{23}$$

$$B(p^2) = m + 4\,\frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,, \tag{24}$$

- Consider the chiral limit equation for $B(p^2)$:

$$B(p^2) = 4\,\frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,. \tag{25}$$

  - Obviously, $B \equiv 0$ is a solution.
  - Is there another?

DCSB in MN Model

- The existence of a $B \not\equiv 0$ solution; i.e., a solution that dynamically breaks chiral symmetry, requires (in units of G)

$$p^2 A^2(p^2) + B^2(p^2) = 4\,. \tag{26}$$

- Substituting this identity into equation Eq. (23), one finds

$$A(p^2) - 1 = \frac{1}{2}\,A(p^2) \;\Rightarrow\; A(p^2) \equiv 2\,, \tag{27}$$

which in turn entails

$$B(p^2) = 2\,\sqrt{1-p^2}\,. \tag{28}$$

- Physical requirement: quark self energy is real on the spacelike domain $\Rightarrow$ complete chiral-limit solution –

$$A(p^2) = \left\{ \begin{array}{ll} 2\,; & p^2 \leq 1 \\ \frac{1}{2}\left(1+\sqrt{1+8/p^2}\right)\,; & p^2 > 1 \end{array} \right. \tag{29}$$

$$B(p^2) = \left\{ \begin{array}{ll} \sqrt{1-p^2}\,; & p^2 \leq 1 \\ 0\,; & p^2 > 1\,. \end{array} \right. \tag{30}$$

- NB. Dressed-quark self-energy is momentum dependent, as is the case in QCD.

Confinement in MN Model

- Solution is continuous and defined for all p^2, even $p^2 < 0$; namely, timelike momenta.
- Examine the propagator's denominator:

$$p^2 A^2(p^2) + B^2(p^2) > 0\,,\ \forall\, p^2\,. \tag{31}$$

This is positive definite ... there are *no zeros*

- This is nothing like a free-particle propagator. It can be interpreted as describing a **confined** degree-of-freedom
- Note that, in addition there is no critical coupling: the nontrivial solution exists so long as **G** > 0.
- Conjecture: **All confining theories exhibit DCSB**.
  - NJL model demonstrates that converse is not true.

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Massive Solution in MN Model

- In the chirally asymmetric case the gap equation yields

$$A(p^2) = \frac{2\,B(p^2)}{m + B(p^2)}\,, \tag{32}$$

$$B(p^2) = m + \frac{4\,[m + B(p^2)]^2}{B(p^2)([m + B(p^2)]^2 + 4p^2)}\,. \tag{33}$$

  - Second is a quartic equation for $B(p^2)$.
  - Can be solved algebraically with four solutions, available in a closed form.
  - Only one has the correct $p^2 \to \infty$ limit: $B(p^2) \to m$.
  - NB. The equations and their solutions always have a smooth $m \to 0$ limit, a result owing to the persistence of the DCSB solution.

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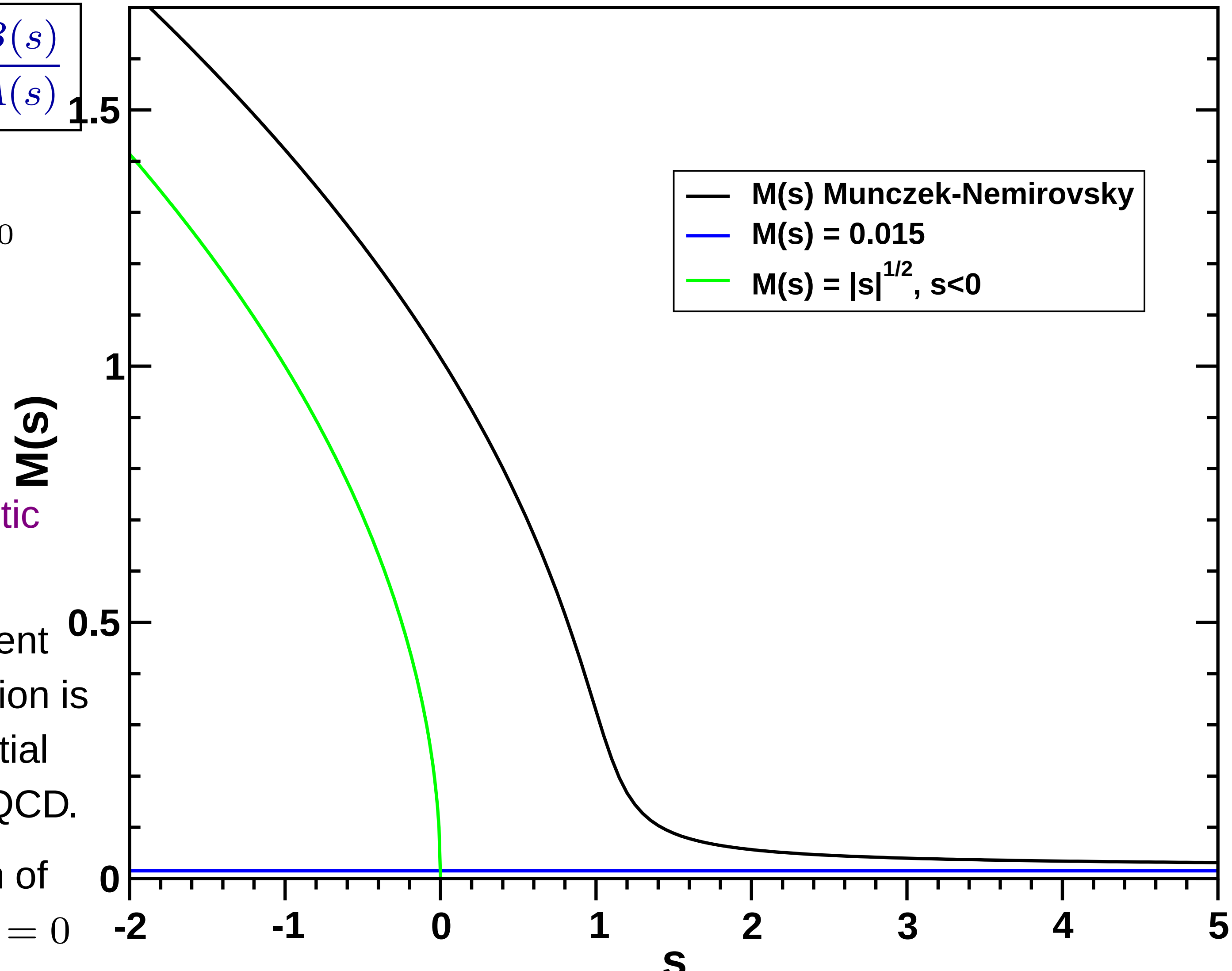
MN Dynamical Mass

$$M(s = p^2) = \frac{B(s)}{A(s)}$$

- Large s:
  $M(s) \sim m_0$
- Small s
  $M \gg m_0$
  This is the essential characteristic of DCSB
- p^2-dependent mass function is quintessential feature of QCD.
- No solution of $s + M(s)^2 = 0$ ⇒ **confinement**.



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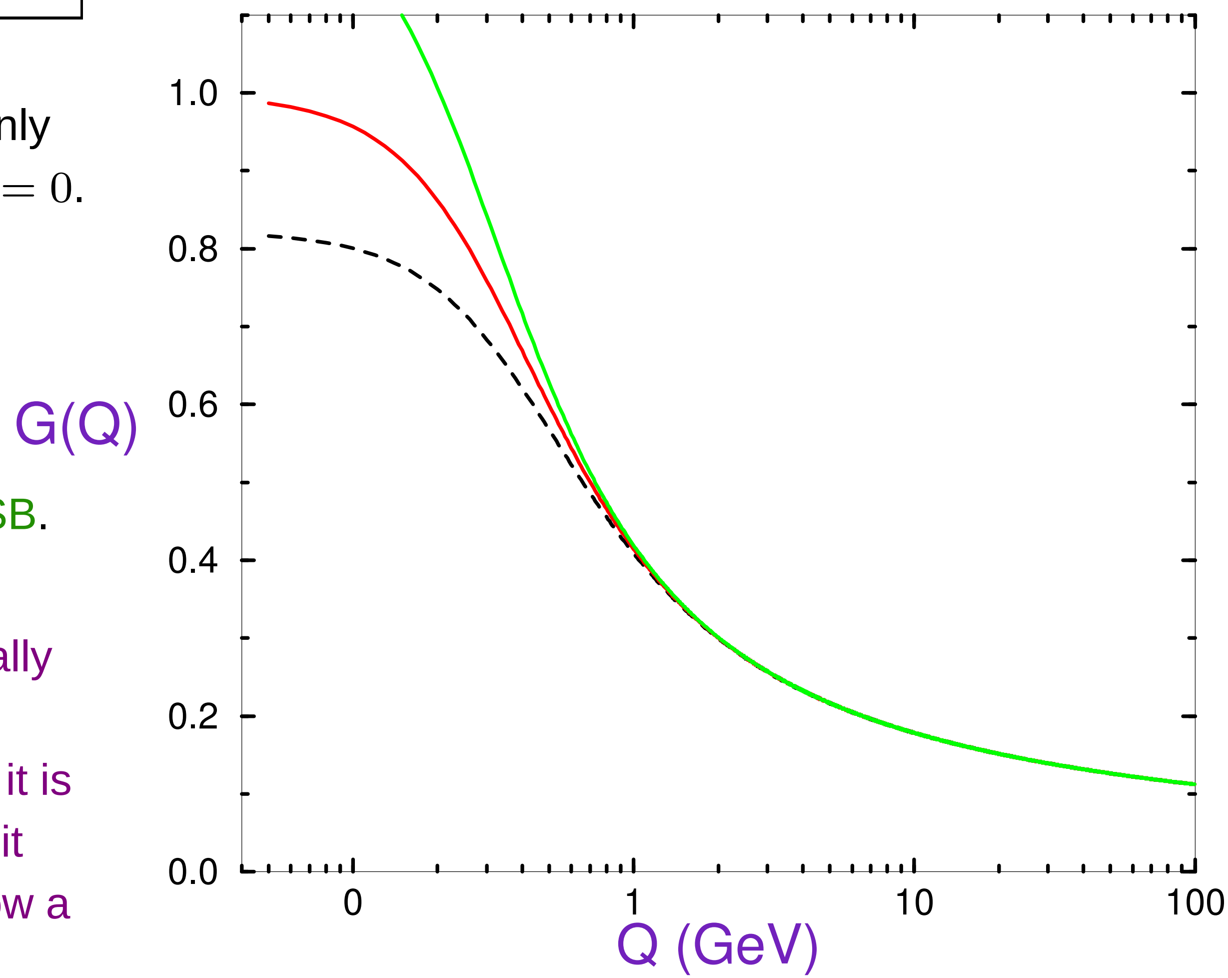
Real World Alternatives

$$\boxed{g^2 D(Q^2) = 4\pi \frac{G(Q^2)}{Q^2}}$$

- $G(0) < 1$: $M(s) \equiv 0$ is only solution for $m = 0$.
- $G(0) \geq 1$ $M(s) \neq 0$ is possible and energetically favoured: DCSB.
- $M(0) \neq 0$ is a new, dynamically generated mass-scale. If it is large enough, it can explain how a theory that is apparently massless (in the Lagrangian) possesses the spectrum of a massive theory.



Overview

- Confinement and Dynamical Chiral Symmetry Breaking are Key Emergent Phenomena in QCD

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Overview

- Confinement and Dynamical Chiral Symmetry Breaking are Key Emergent Phenomena in QCD
- Understanding requires Nonperturbative Solution of Fully-Fledged Relativistic Quantum Field Theory
  - Mathematics and Physics still far from being able to accomplish that

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Overview

- Confinement and Dynamical Chiral Symmetry Breaking are Key Emergent Phenomena in QCD
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- Confinement and DCSB are expressed in QCD's propagators and vertices

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  - Nonperturbative modifications should have observable consequences

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  - DCSB $\not\Rightarrow$ Confinement

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- Simple models (MN) can exhibit Confinement
  - Confinement $\Rightarrow$ DCSB
- What's the story in QCD?

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