Symmetry methods for exotic nuclei

P. Van Isacker, GANIL, France

Role of symmetries in

- The nuclear shell model
- The interacting boson model

Their relevance for RIBs

RIA Theory meeting, Argonne, April 2006

ECT* doctoral training programme

- Title: “Nuclear structure and reactions” (spring 2007, ±3 months, for PhD students).
- Lecture series on shell model, mean-field approaches, nuclear astrophysics, fundamental interactions, symmetries in nuclei, reaction theory, exotic nuclei,…
- Workshops related to these topics.
- Please:
  – Encourage students to apply;
  – Submit workshop proposals to ECT*.

Nuclear superfluidity

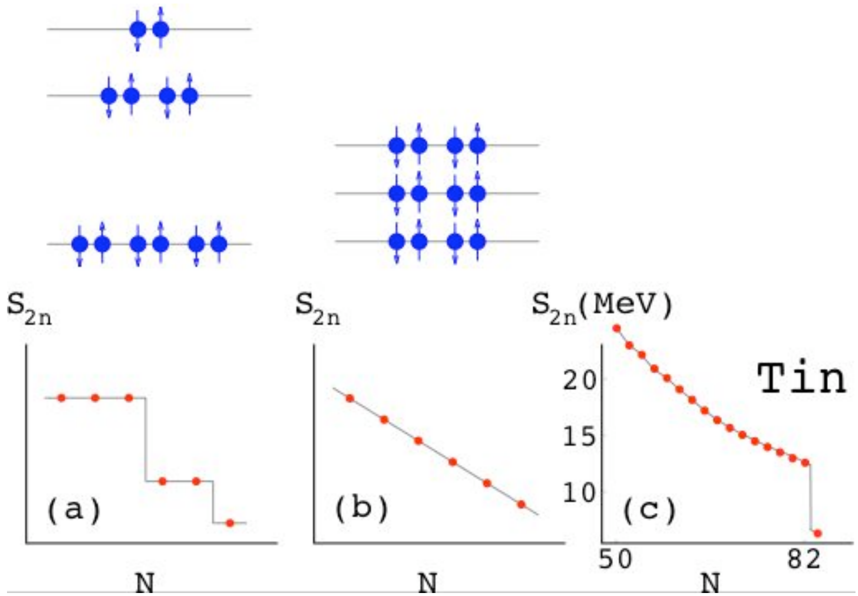
- Ground states of pairing hamiltonian have the following *correlated* character:
  - Even-even nucleus (υ=0): $\left(\hat{S}_+\right)^{n/2}|o\rangle, \quad \hat{S}_+ = \sum_{m>0} \hat{a}_m^+ \hat{a}_{\bar{m}}^+$
  - Odd-mass nucleus (υ=1): $\hat{a}_m^+ \left(\hat{S}_+\right)^{n/2}|o\rangle$
- Nuclear superfluidity leads to
  - Constant energy of first 2^+ in even-even nuclei.
  - Odd-even staggering in masses.
  - Smooth variation of two-nucleon separation energies with nucleon number.
  - Two-particle (2n or 2p) transfer enhancement.

Two-nucleon separation energies

a. Shell splitting dominates over interaction.

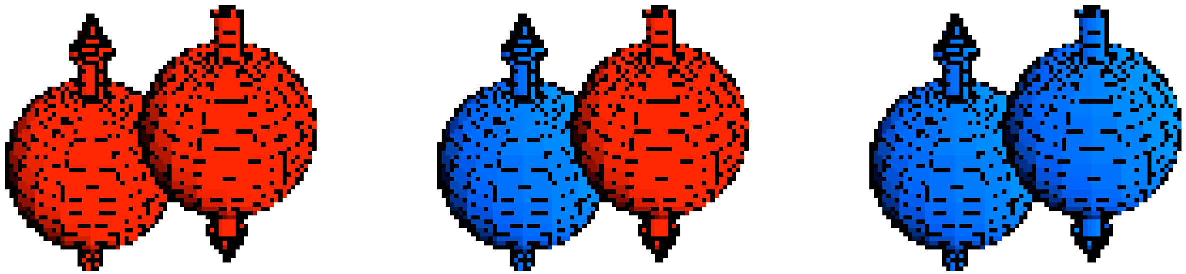
b. Interaction dominates over shell splitting.

c. S_{2n} in tin isotopes.

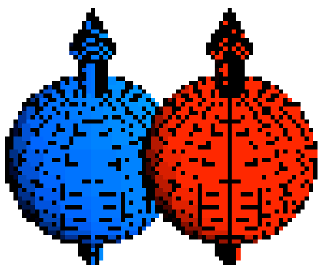


Pairing with neutrons and protons

- For neutrons and protons *two* pairs and hence *two* pairing interactions are possible:
  - $^1S_0$ isovector or spin singlet (S=0,T=1): $\hat{S}_+ = \sum_{m>0} \hat{a}^+_{m\downarrow}\hat{a}^+_{\bar{m}\uparrow}$



  - $^3S_1$ isoscalar or spin triplet (S=1,T=0): $\hat{P}_+ = \sum_{m>0} \hat{a}^+_{m\uparrow}\hat{a}^+_{\bar{m}\uparrow}$



Neutron-proton pairing hamiltonian

- The nuclear hamiltonian has two pairing interactions

$$\hat{V}_{\text{pairing}} = -g_0 \hat{S}_+ \cdot \hat{S}_- - g_1 \hat{P}_+ \cdot \hat{P}_-$$

- SO(8) algebraic structure.
- Integrable and solvable for g_0=0, g_1=0 and g_0=g_1.

B.H. Flowers & S. Szpikowski, Proc. Phys. Soc. **84** (1964) 673

Quartetting in N=Z nuclei

- Pairing ground state of an N=Z nucleus:

$$\left(\cos\theta\,\hat{S}_{+}\cdot\hat{S}_{+}-\sin\theta\,\hat{P}_{+}\cdot\hat{P}_{+}\right)^{n/4}\left|\mathrm{o}\right\rangle$$

- $\Rightarrow$ Condensate of "α-like" objects.
- Observations:
  – Isoscalar component in condensate survives only in N~Z nuclei, if anywhere at all.
  – Spin-orbit term *reduces* isoscalar component.

Generalized pairing models

- Pairing in degenerate orbits between identical particles has SU(2) symmetry.
- Richardson-Gaudin models can be generalized to higher-rank algebras:

$$\hat{R}_i = \hat{H}_i^s + g_0 \sum_{j(\neq i)}^{L} \sum_{\mu,\nu} \frac{\hat{X}_i^\mu g_{\mu\nu} \hat{X}_j^\nu}{2\varepsilon_i - 2\varepsilon_j}$$

$$g_0 \sum_{i=1}^{L} \frac{\Lambda_i^a}{e_{a\alpha} - 2\varepsilon_i} - g_0 \sum_{b=1}^{r} \sum_{\beta=1}^{M_b} \frac{A_{ba}}{e_{a\alpha} - e_{b\beta}} = \delta_{as}$$

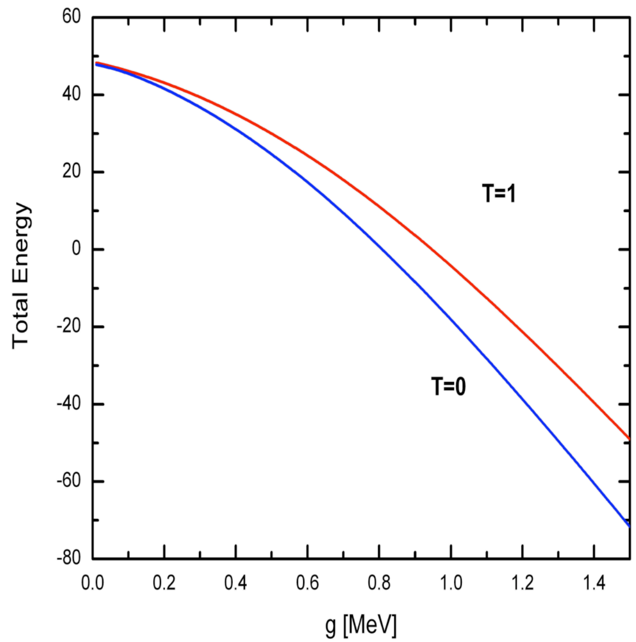
J. Dukelsky *et al.*, to be published

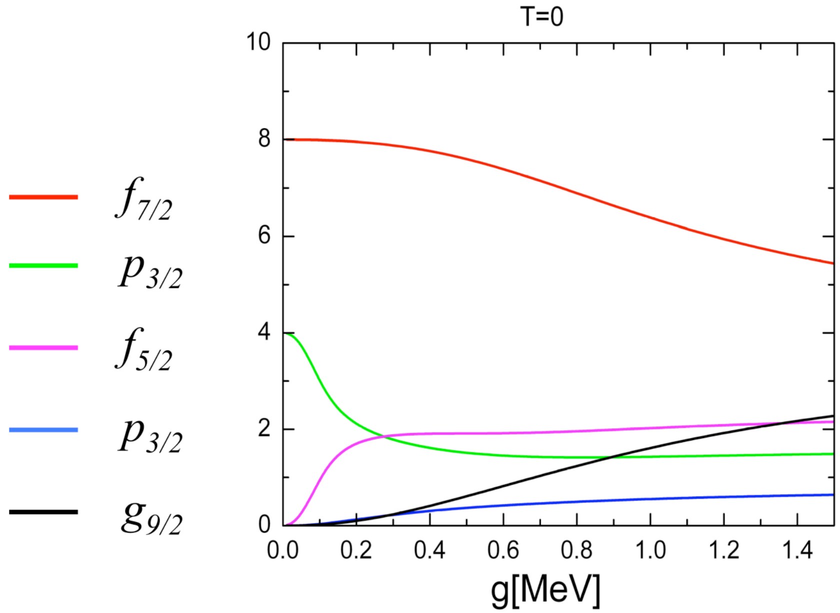
SO(5) pairing

- Hamiltonian:

$$\hat{H} = \sum_j \varepsilon_j \hat{n}_j - g_0 \hat{S}_+ \cdot \hat{S}_-$$

- "Quasi-spin" algebra is SO(5) (rank 2).
- Example: ^{64}Ge in $pfg_{9/2}$ shell (d~$9\cdot10^{14}$).





J. Dukelsky *et al.*, Phys. Rev. Lett. **96** (2006) 072503

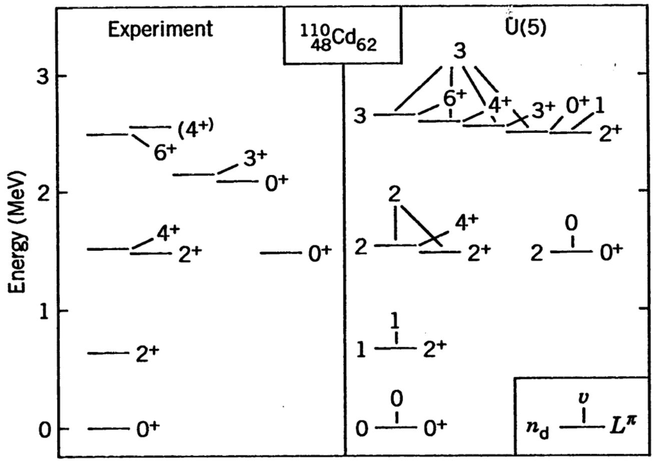
The interacting boson model

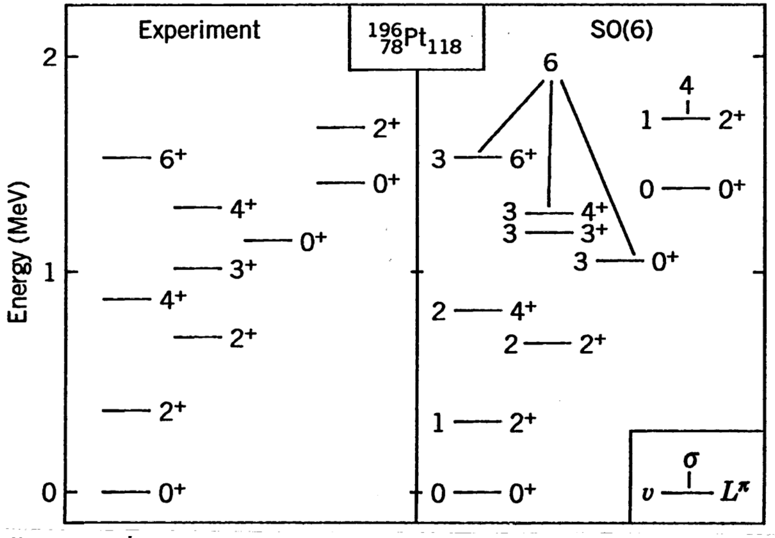
- Spectrum generating algebra for the nucleus is U(6). All physical observables (hamiltonian, transition operators,…) are expressed in terms of *s* and *d* bosons.
- Justification from
  - Shell model: *s* and *d* bosons are associated with *S* and *D* fermion (*Cooper*) pairs.
  - Geometric model: for large boson number the IBM reduces to a liquid-drop hamiltonian.

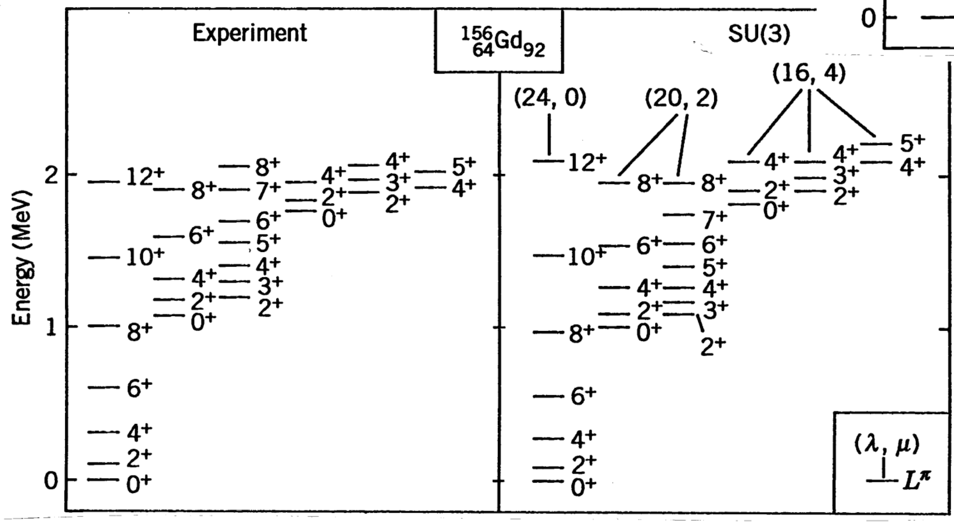
A. Arima & F. Iachello, Ann. Phys. (NY) **99** (1976) 253; **111** (1978) 201; **123** (1979) 468

The IBM symmetries

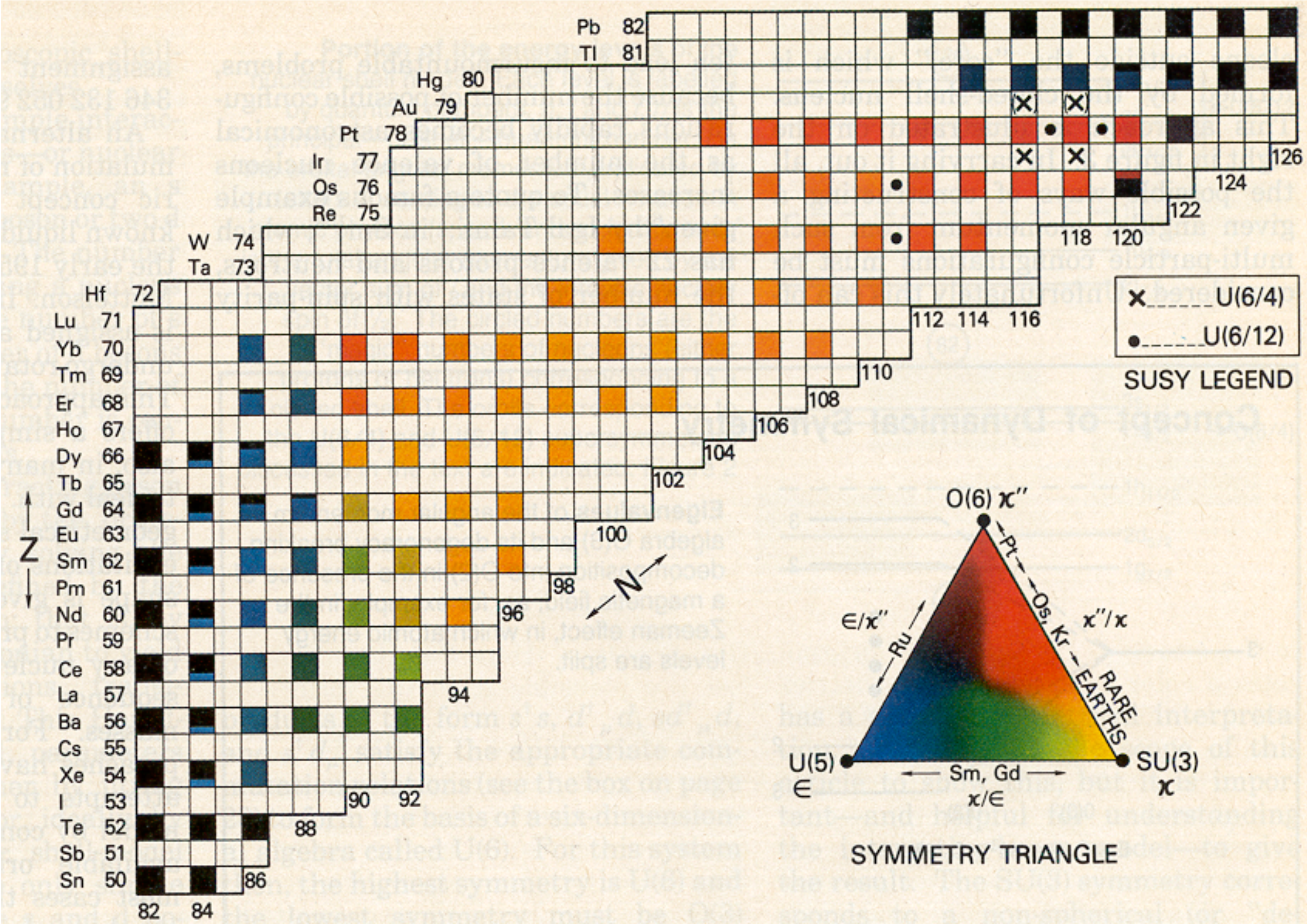
- Three analytic solutions: U(5), SU(3) & SO(6).



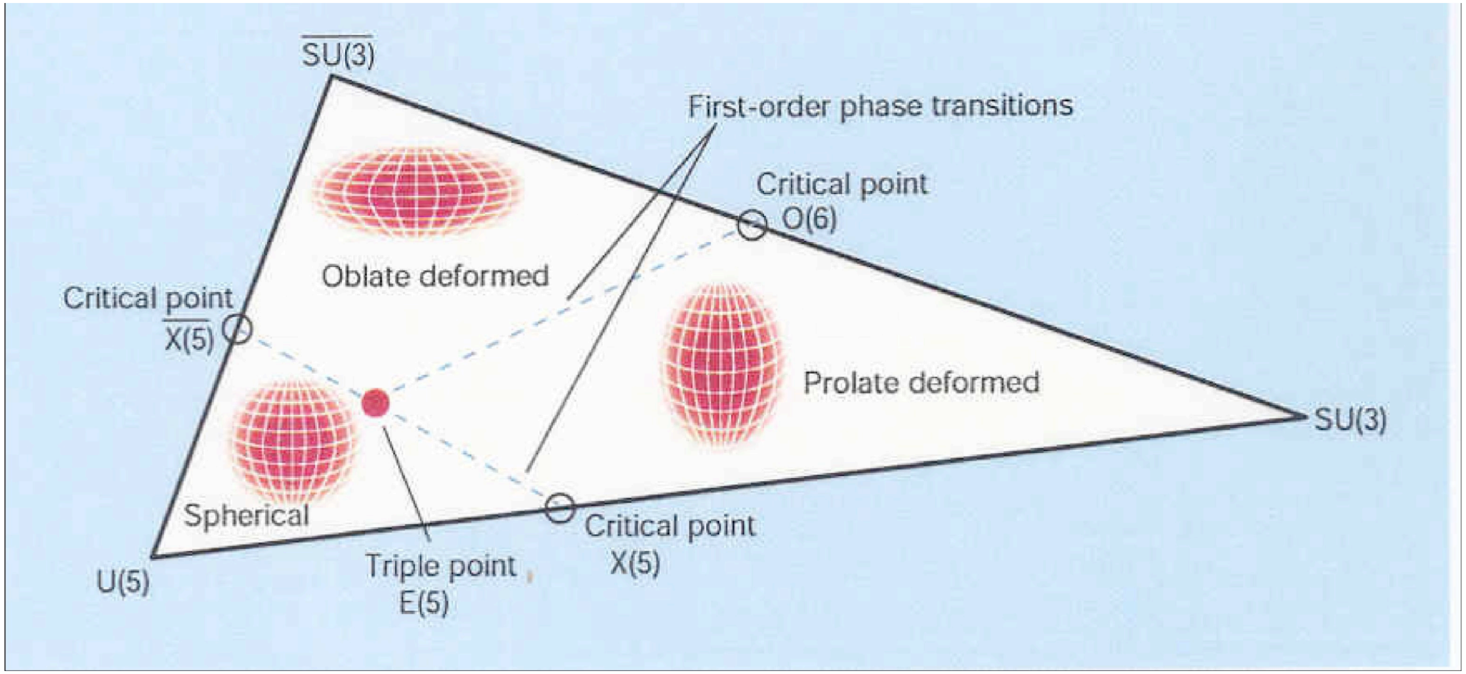




Applications of IBM



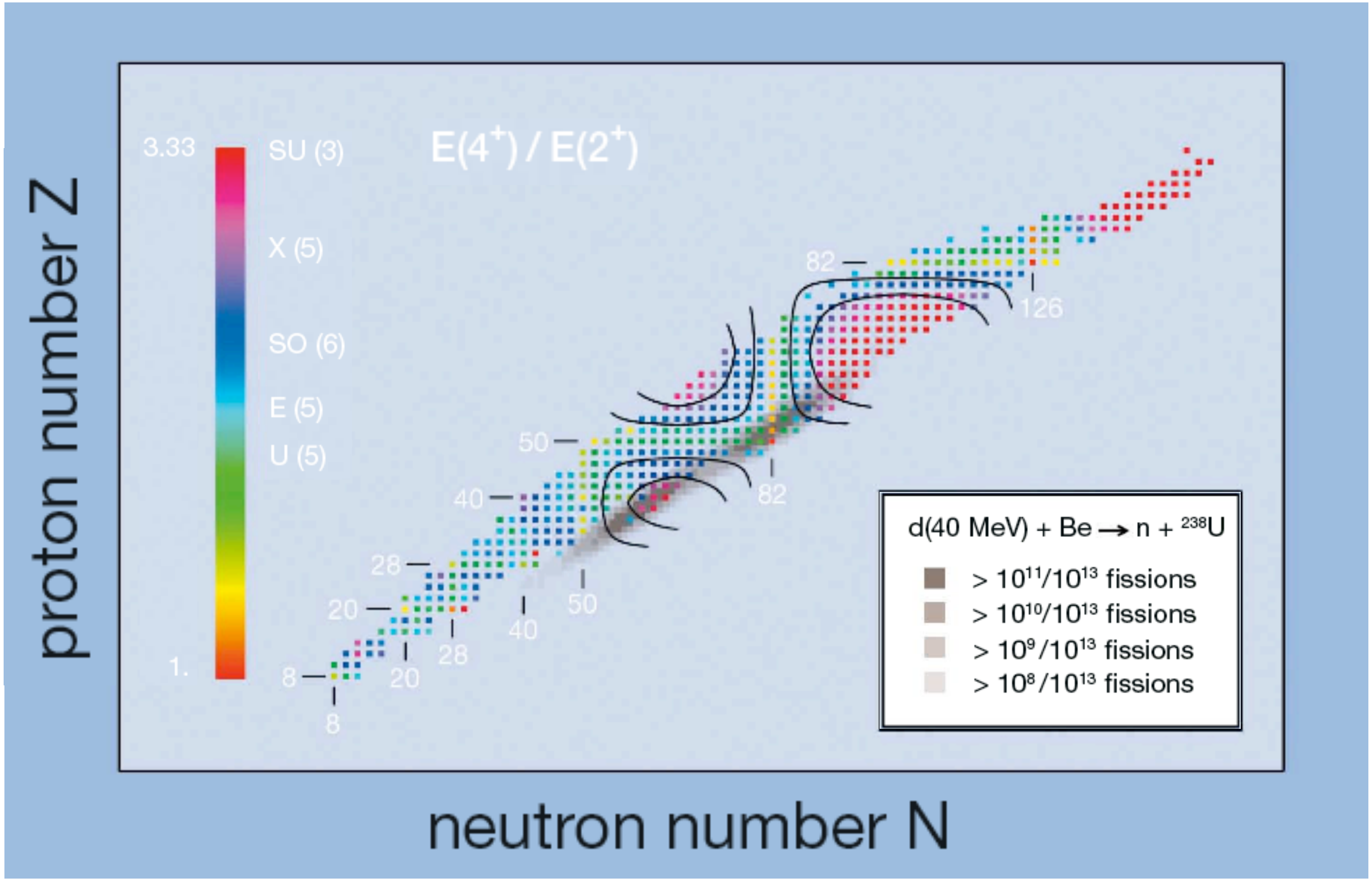
IBM symmetries and phases



- Open problems:
  - Symmetries and phases of two fluids (IBM-2).
  - Coexisting phases?
  - Existence of three-fluid systems?

D.D. Warner, Nature **420** (2002) 614

Symmetry chart (SPIRAL-2)



Model with L=0 vector bosons

- Correspondence: $\hat{S}_+ \to b^+_{T=1} \equiv s^+ \qquad \hat{P}_+ \to b^+_{T=0} \equiv p^+$
- Algebraic structure is U(6).
- Symmetry *lattice* of U(6):

$$\mathrm{U}(6) \supset \left\{ \begin{matrix} \mathrm{U}_S(3) \otimes \mathrm{U}_T(3) \\ \mathrm{SU}(4) \end{matrix} \right\} \supset \mathrm{SO}_S(3) \otimes \mathrm{SO}_T(3)$$

- Boson mapping is *exact* in the symmetry limits [for fully paired states of the SO(8)].

Masses of *N*~*Z* nuclei

- Neutron-proton pairing hamiltonian in *non-degenerate* shells:

$$\hat{H}_{\mathrm{F}} = \sum_j \varepsilon_j \hat{n}_j - g_0 \hat{S}_+ \cdot \hat{S}_- - g_1 \hat{P}_+ \cdot \hat{P}_-$$

- H_{F} maps into the boson hamiltonian:

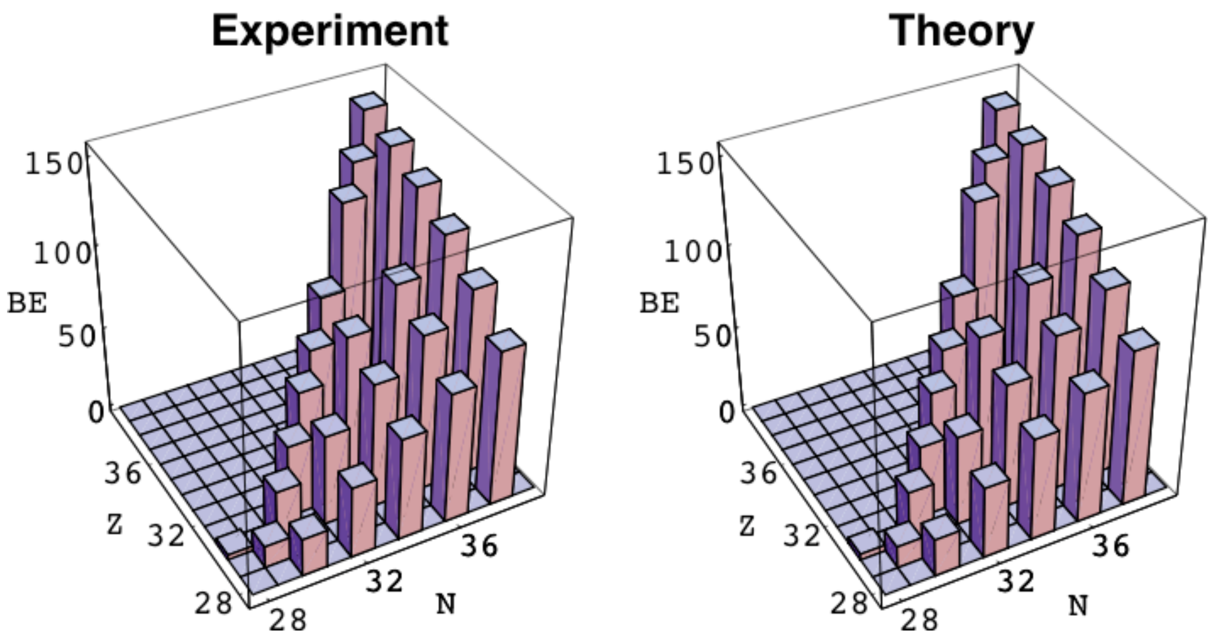
$$\hat{H}_{\mathrm{B}} = a\hat{C}_2[\mathrm{SU}(4)] + b\hat{C}_1[\mathrm{U}_S(3)] + c_1\hat{C}_1[\mathrm{U}(6)] + c_2\hat{C}_2[\mathrm{U}(6)] + d\hat{C}_2[\mathrm{SO}_T(3)]$$

- H_{B} describes masses of *N*~*Z* nuclei.

E. Baldini-Neto *et al.,* Phys. Rev. C **65** (2002) 064303

Masses of *pf*-shell nuclei

- Root-mean-square deviation is 254 keV.
- Parameter ratio: $b/a \approx 5$.

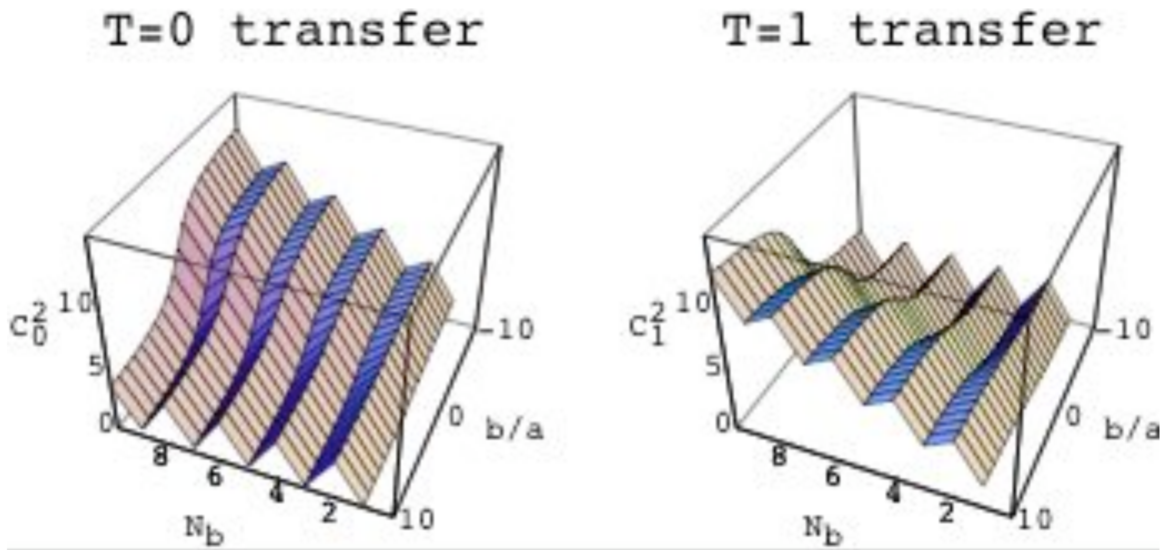


Deuteron transfer in N=Z nuclei

Deuteron Transfer in $N = Z$ Nuclei

P. Van Isacker,[1] D. D. Warner,[2] and A. Frank[3]
[1]*Grand Accélérateur National d'Ions Lourds, B.P. 55027, F-14076 Caen Cedex 5, France*
[2]*CCLRC Daresbury Laboratory, Daresbury, Warrington WA4 4AD, United Kingdom*
[3]*Instituto de Ciencias Nucleares, UNAM, Apdo. Postal 70-543, 04510 México, D.F. Mexico*
(Received 14 September 2004; published 29 April 2005)

Predictions are obtained for $T = 0$ and $T = 1$ deuteron-transfer intensities between self-conjugate $N = Z$ nuclei on the basis of a simplified interacting boson model which considers bosons without orbital angular momentum but with full spin-isospin structure. These transfer predictions can be correlated with nuclear binding energies in specific regions of the mass table.

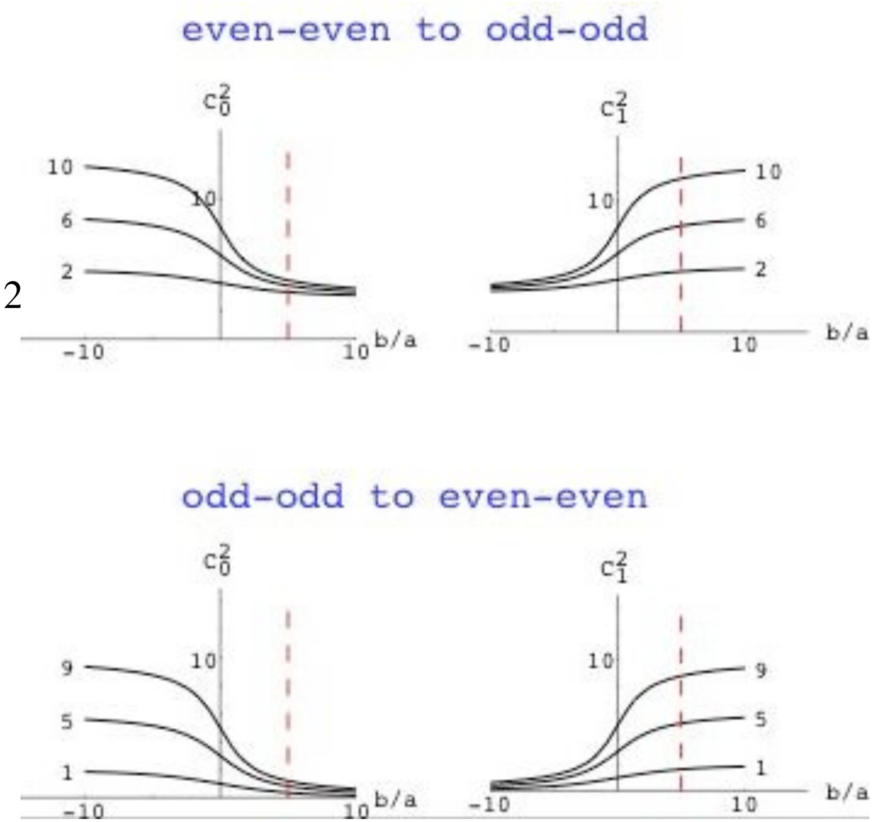


Deuteron transfer in *N*=*Z* nuclei

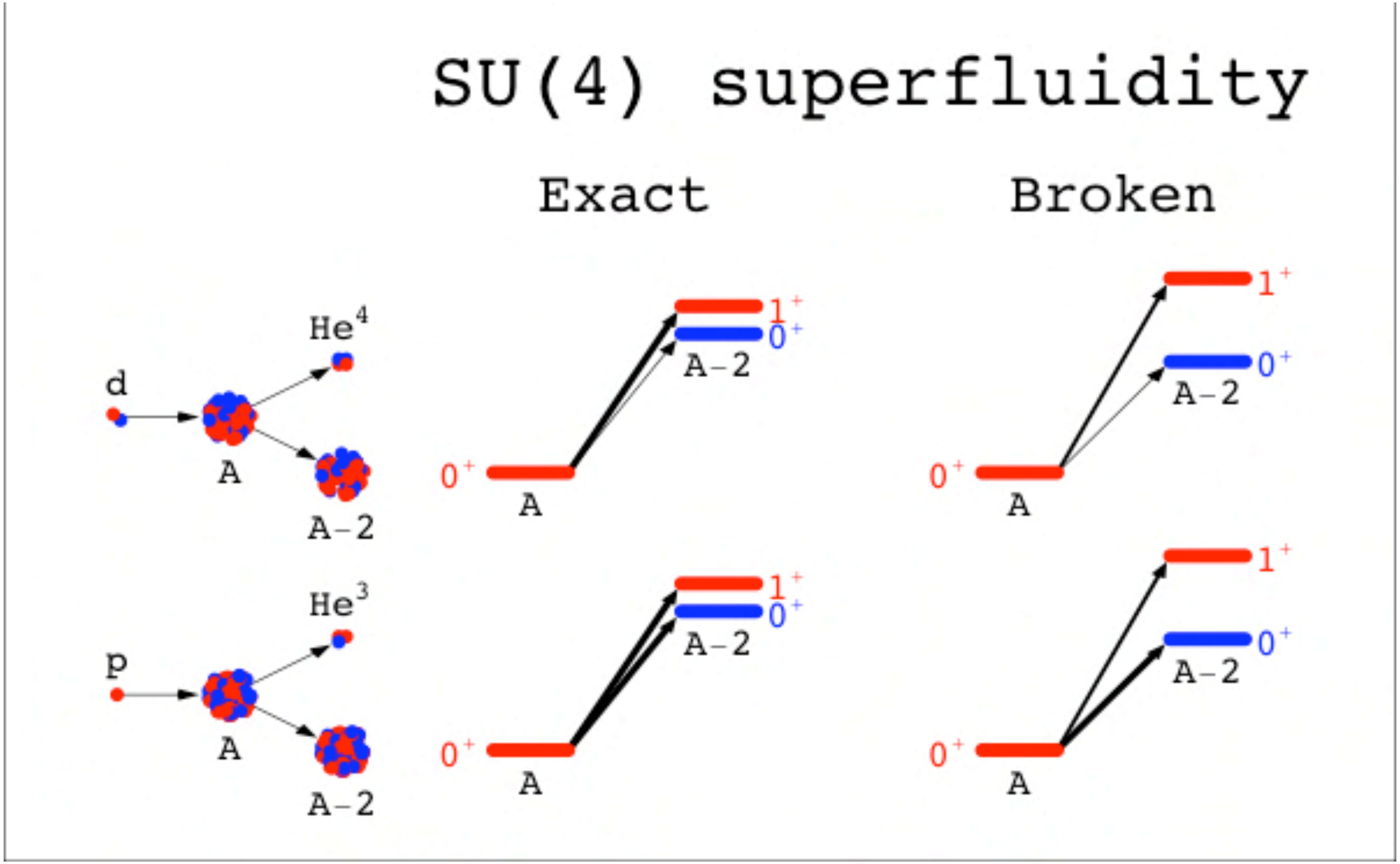
- Deuteron-transfer intesity $c_T{}^2$ calculated in *sp*-IBM based on SO(8).

$$c_T^2 = \left\langle [N_\mathrm{b}+1]\phi_\mathrm{B} \middle\| b_{TS}^{+} \middle\| [N_\mathrm{b}]\phi_\mathrm{A} \right\rangle^2$$

- Ratio *b*/*a* fixed from masses in lower half of 28-50 shell.



(d,α) and (p,^{3}He) transfer



Collective modes in n-rich nuclei

- New collective modes in nuclei with a neutron-skin?
- Algebraic model via

$$\begin{array}{ccccc} U_\nu(6) & \otimes & U_\pi(6) & \otimes & U_{\nu_S}(6) \\ \downarrow & & \downarrow & & \downarrow \\ [N_\nu] & & [N_\pi] & & [N_{\nu_S}] \end{array}$$

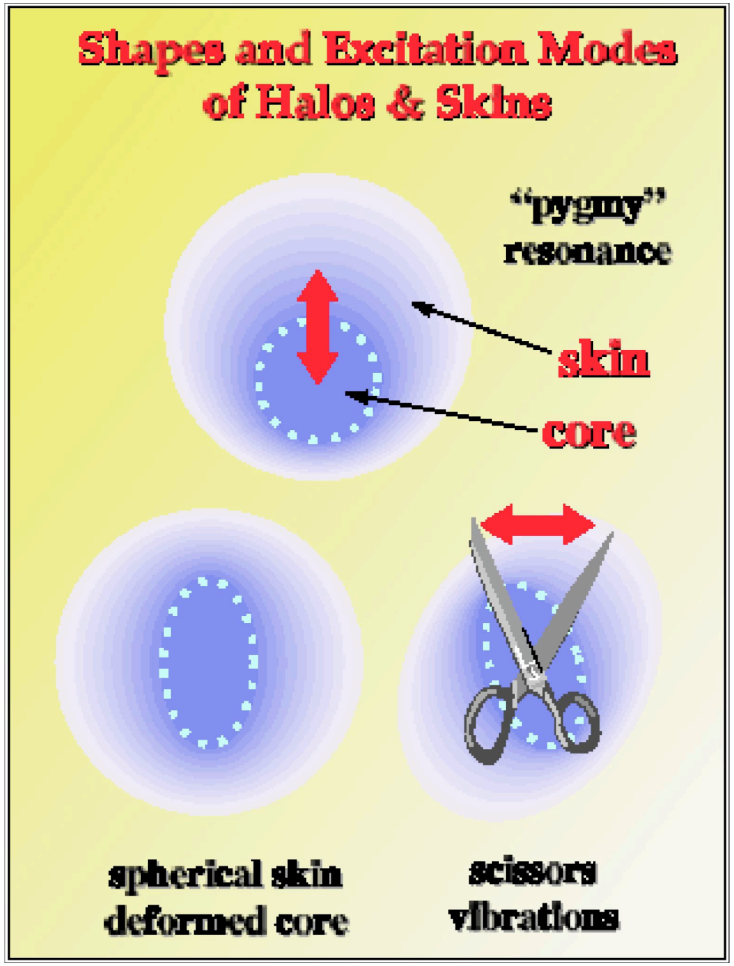
- Expressions for M1 strength:

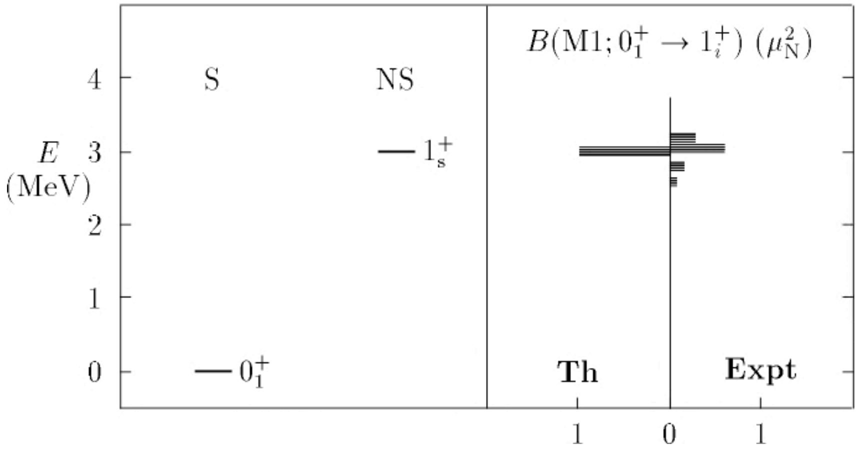
$$B(\mathrm{M1};0_1^+ \to 1_\mathrm{S}^+) = \frac{3}{4\pi}(g_\nu - g_\pi)^2 f(N) N_\nu N_\pi$$

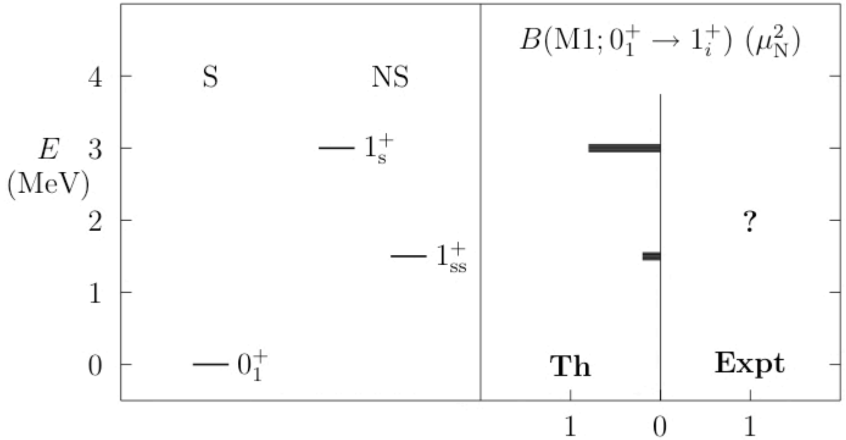
$$B(\mathrm{M1};0_1^+ \to 1_\mathrm{SS}^+) = \frac{3}{4\pi}(g_\nu - g_\pi)^2 f(N) \frac{N_{\nu_S} N_\pi^2}{N_\nu + N_\pi}$$

D.D. Warner & P. Van Isacker, Phys. Lett. B **395** (1997) 145

‘Soft scissors’ excitation







Conclusion

Sir Denys in *Blood, Birds and the Old Road*:

« Accelerators rarely carry out the program on the basis of which their funding was granted: something more exciting always comes along. The lesson is that what matters most is enthusiasm and commitment: the fire in the belly. »

D. Wilkinson, Annu. Rev. Nucl. Part. Sci. 45 (1995) 1