Symmetry methods
for exotic nuclei

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Role of symmetries in
The nuclear shell model
The interacting boson model
Their relevance for RIBs

RIA Theory meeting, Argonne, April 2006
ECT* doctoral training programme

• Title: “Nuclear structure and reactions” (spring 2007, ±3 months, for PhD students).
• Lecture series on shell model, mean-field approaches, nuclear astrophysics, fundamental interactions, symmetries in nuclei, reaction theory, exotic nuclei,…
• Workshops related to these topics.
• Please:
  – Encourage students to apply;
  – Submit workshop proposals to ECT*. 

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Nuclear superfluidity

• Ground states of pairing hamiltonian have the following correlated character:
  – Even-even nucleus ($\nu=0$): $\left(\hat{S}_+\right)^{n/2}|o\rangle$, $\hat{S}_+ = \sum_{m>0} \hat{a}_m^+ \hat{a}_m^+$
  – Odd-mass nucleus ($\nu=1$): $\hat{a}_m^+ \left(\hat{S}_+\right)^{n/2}|o\rangle$

• Nuclear superfluidity leads to
  – Constant energy of first $2^+$ in even-even nuclei.
  – Odd-even staggering in masses.
  – Smooth variation of two-nucleon separation energies with nucleon number.
  – Two-particle (2n or 2p) transfer enhancement.
Two-nucleon separation energies

a. Shell splitting dominates over interaction.

b. Interaction dominates over shell splitting.

c. $S_{2n}$ in tin isotopes.
Pairing with neutrons and protons

- For neutrons and protons *two* pairs and hence *two* pairing interactions are possible:
  - $^1S_0$ isovector or spin singlet ($S=0,T=1$): $\hat{S}_+ = \sum_{m>0} \hat{a}^+_{m\downarrow} \hat{a}^+_{m\uparrow}$
  - $^3S_1$ isoscalar or spin triplet ($S=1,T=0$): $\hat{P}_+ = \sum_{m>0} \hat{a}^+_{m\uparrow} \hat{a}^+_{m\uparrow}$
Neutron-proton pairing hamiltonian

- The nuclear hamiltonian has two pairing interactions
  \[ \hat{V}_{\text{pairing}} = -g_0 \hat{S}_+ \cdot \hat{S}_- - g_1 \hat{P}_+ \cdot \hat{P}_- \]

- SO(8) algebraic structure.

- Integrable and solvable for \( g_0 = 0, g_1 = 0 \) and \( g_0 = g_1 \).


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Quartetting in $N=Z$ nuclei

• Pairing ground state of an $N=Z$ nucleus:

\[
\left( \cos \theta \hat{S}_+ \cdot \hat{S}_+ - \sin \theta \hat{P}_+ \cdot \hat{P}_+ \right)^{n/4} \left| o \right>
\]

• $\Rightarrow$ Condensate of “$\alpha$-like” objects.

• Observations:
  – Isoscalar component in condensate survives only in $N\sim Z$ nuclei, if anywhere at all.
  – Spin-orbit term reduces isoscalar component.
Generalized pairing models

- Pairing in degenerate orbits between identical particles has SU(2) symmetry.
- Richardson-Gaudin models can be generalized to higher-rank algebras:

$$\hat{R}_i = \hat{H}_i^s + g_0 \sum_{j \neq i}^{L} \sum_{\mu,\nu} \frac{\hat{X}_i^{\mu} g_{\mu\nu} \hat{X}_j^{\nu}}{2\epsilon_i - 2\epsilon_j}$$

$$g_0 \sum_{i=1}^{L} \frac{\Lambda_i^a}{e_{a\alpha} - 2\epsilon_i} - g_0 \sum_{b=1}^{r} \sum_{\beta=1}^{M_b} \frac{A_{ba}}{e_{a\alpha} - e_{b\beta}} = \delta_{as}$$

J. Dukelsky et al., to be published
SO(5) pairing

• Hamiltonian:

\[ \hat{H} = \sum_j \epsilon_j \hat{n}_j - g_0 \hat{S}_+ \cdot \hat{S}_- \]

• “Quasi-spin” algebra is SO(5) (rank 2).

• Example: \(^{64}\text{Ge}\) in \(pf_g9/2\) shell (\(d \sim 9 \cdot 10^{14}\)).

The interacting boson model

• Spectrum generating algebra for the nucleus is $U(6)$. All physical observables (hamiltonian, transition operators,…) are expressed in terms of $s$ and $d$ bosons.

• Justification from
  – Shell model: $s$ and $d$ bosons are associated with $S$ and $D$ fermion ($Cooper$) pairs.
  – Geometric model: for large boson number the IBM reduces to a liquid-drop hamiltonian.

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The IBM symmetries

- Three analytic solutions: U(5), SU(3) & SO(6).
Applications of IBM
IBM symmetries and phases

- Open problems:
  - Symmetries and phases of two fluids (IBM-2).
  - Coexisting phases?
  - Existence of three-fluid systems?


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Symmetry chart (SPIRAL-2)
Model with $L=0$ vector bosons

- **Correspondence:** $\hat{S}_+ \rightarrow b_{T=1}^+ \equiv s^+$, $\hat{P}_+ \rightarrow b_{T=0}^+ \equiv p^+$
- **Algebraic structure is** $U(6)$.
- **Symmetry lattice of** $U(6)$:

$$U(6) \supset \left\{ U_s(3) \otimes U_T(3) \right\} \supset SU(4) \supset SO_s(3) \otimes SO_T(3)$$

- **Boson mapping is exact in the symmetry limits** [for fully paired states of the SO(8)].
Masses of $N\sim Z$ nuclei

- Neutron-proton pairing hamiltonian in \textit{non-degenerate} shells:
  \[
  \hat{H}_F = \sum_j \epsilon_j \hat{n}_j - g_0 \hat{S}_+ \cdot \hat{S}_- - g_1 \hat{P}_+ \cdot \hat{P}_-
  \]

- $H_F$ maps into the boson hamiltonian:
  \[
  \hat{H}_B = a\hat{C}_2[SU(4)] + b\hat{C}_1[U_S(3)]
  \]
  \[
  + c_1\hat{C}_1[U(6)] + c_2\hat{C}_2[U(6)] + d\hat{C}_2[SO_T(3)]
  \]

- $H_B$ describes masses of $N\sim Z$ nuclei.
Masses of \textit{pf}-shell nuclei

- Root-mean-square deviation is 254 keV.
- Parameter ratio: $b/a \approx 5$. 

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Deuteron transfer in $N=Z$ nuclei

Deuteron Transfer in $N = Z$ Nuclei

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Predictions are obtained for $T = 0$ and $T = 1$ deuteron-transfer intensities between self-conjugate $N = Z$ nuclei on the basis of a simplified interacting boson model which considers bosons without orbital angular momentum but with full spin-isospin structure. These transfer predictions can be correlated with nuclear binding energies in specific regions of the mass table.

$T=0$ transfer

$T=1$ transfer

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Deuteron transfer in $N=Z$ nuclei

- Deuteron-transfer intensity $c_T^2$ calculated in $sp$-IBM based on $SO(8)$.

$$c_T^2 = \langle [N_b + 1] \phi_B | b_{TS}^+ | [N_b] \phi_A \rangle^2$$

- Ratio $b/a$ fixed from masses in lower half of 28-50 shell.
(d,α) and (p,³He) transfer
Collective modes in n-rich nuclei

- New collective modes in nuclei with a neutron-skin?

- Algebraic model via

- Expressions for M1 strength:

\[
B\left(\text{M1}; 0^+_1 \rightarrow 1^+_S\right) = \frac{3}{4\pi} \left(g_\nu - g_\pi\right)^2 f(N) N_\nu N_\pi
\]

\[
B\left(\text{M1}; 0^+_1 \rightarrow 1^+_SS\right) = \frac{3}{4\pi} \left(g_\nu - g_\pi\right)^2 f(N) \frac{N_{\nu_S} N_\pi^2}{N_\nu + N_\pi}
\]

‘Soft scissors’ excitation
Conclusion

Sir Denys in *Blood, Birds and the Old Road*:

« Accelerators rarely carry out the program on the basis of which their funding was granted: something more exciting always comes along. The lesson is that what matters most is enthusiasm and commitment: the fire in the belly. »


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