A. cross section formula

For the pion electroproduction ($e(p_e)+N(p_N)\rightarrow e'(p'_e)+\pi(k)+N(p'_N)$), the differential cross section is given as

$$\frac{d\sigma}{dE'_e d\Omega_{e'} d\Omega_\pi} = \Gamma\frac{d\sigma^v}{d\Omega_\pi} \quad (1)$$

where

$$\Gamma = \frac{\alpha q_L^\gamma}{2\pi^2 Q^2}\frac{E'_e}{E_e}\frac{1}{1-\epsilon}. \quad (2)$$

$$\begin{aligned} q^\mu &= (p_e - p'_e)^\mu && (3)\\ W &= \sqrt{(p_N+q)^2} && (4)\\ Q^2 &= -q^2 && (5)\\ q_L^\gamma &= \frac{W^2-M_N^2}{2M_N} && (6)\\ \epsilon &= [1+\frac{2_L^2}{Q^2}\tan^2\frac{\theta_e}{2}]^{-1} && (7)\end{aligned}$$

The pion virtual photon production cross section is given as

$$\begin{aligned}\frac{d\sigma^v}{d\Omega_\pi} &= \frac{d\sigma_T}{d\Omega_\pi}+\epsilon\frac{d\sigma_L}{d\Omega_\pi}+\epsilon\frac{d\sigma_{TT}}{d\Omega_\pi}\cos 2\phi_\pi+\sqrt{\epsilon(\epsilon+1)}\frac{d\sigma_{LT}}{d\Omega_\pi}\cos\phi_\pi && (8)\\ &+h_e\sqrt{2\epsilon(1-\epsilon)}\frac{d\sigma_{LT'}}{d\Omega_\pi}\sin\phi_\pi && (9)\end{aligned}$$

Here ϕ_π is pion angle from scattering plane of electron.

$$\begin{aligned}\frac{d\sigma_T}{d\Omega_\pi} &= \frac{|k|}{|q|}\sum_{spin}\frac{F^{xx}+F^{yy}}{2} && (10)\\ \frac{d\sigma_L}{d\Omega_\pi} &= \frac{|k|}{|q|}\sum_{spin}\frac{Q^2}{q^2}F^{00} && (11)\\ \frac{d\sigma_P}{d\Omega_\pi} &= \frac{|k|}{|q|}\sum_{spin}\frac{F^{xx}-F^{yy}}{2} && (12)\\ \frac{d\sigma_I}{d\Omega_\pi} &= \frac{|k|}{|q|}\sum_{spin}(-1)\sqrt{\frac{Q^2}{q^2}}Re(F^{x0}) && (13)\\ \frac{d\sigma_e}{d\Omega_\pi} &= \frac{|k|}{|q|}\sum_{spin}\sqrt{\frac{Q^2}{q^2}}Im(F^{x0}). && (14)\end{aligned}$$

$$\sum_{spin}F^{ij} = \frac{1}{2}\sum_{s_i,s_f} < s_f|F^i|s_i >< s_f|F^j|s_i >^* \quad (15)$$

Here q and k are momentum transfer to nucleon and pion momentum in the center of mass system.

$$|q| = \frac{W^2 - M_N^2}{2W} \tag{16}$$

$$|k| = \sqrt{(\frac{W^2 + m_\pi^2 - M_N^2}{2W})^2 - m_\pi^2} \tag{17}$$

$$\hat{q} = q/|q| = \hat{z} \tag{18}$$

$$\hat{k} = k/|k| = \cos\theta\hat{z} + \sin\theta\hat{x} \tag{19}$$

$$\tag{20}$$

The CGLN amplitude F is given as

$$\begin{aligned} F = & i\sigma\cdot\epsilon_\perp F_1 + \sigma\cdot\hat{k}\sigma\cdot\hat{q}\times\epsilon_\perp F_2 + i\sigma\cdot\hat{q}\hat{k}\cdot\epsilon_\perp F_3 + i\sigma\cdot\hat{k}\hat{k}\cdot\epsilon_\perp F_4 \\ & + i\sigma\cdot\hat{q}\hat{q}\cdot\epsilon F_5 + i\sigma\cdot\hat{k}\hat{q}\cdot\epsilon F_6 - i\sigma\cdot\hat{k}\epsilon_0 F_7 - i\sigma\cdot\hat{q}\epsilon_0 F_8\,, \end{aligned} \tag{21}$$

where $\epsilon_\perp = \hat{q}\times(\epsilon\times\hat{q})$.

$$F^x = i\sigma_x(F_1 - \cos\theta F_2 + \sin^2\theta F_4) + i\sigma_z\sin\theta(F_2 + F_3 + \cos\theta F_4) \tag{22}$$

$$F^y = i\sigma_y(F_1 - \cos\theta F_2) + \sin\theta F_2 \tag{23}$$

$$F^0 = i\sigma^z F_8 + i\sigma\cdot\hat{k}F_7 \tag{24}$$

Finally the amplitudes F_i are expressed in terms of multipole amplitudes $E_{l\pm}, M_{l\pm}, S_{l\pm}$ and $L_{l\pm}$.

$$F_1 = \sum_l [P'_{l+1}E_{l+} + P'_{l-1}E_{l-} + lP'_{l+1}M_{l+} + (l+1)P'_{l-1}M_{l-}]\,, \tag{25}$$

$$F_2 = \sum_l [(l+1)P'_l M_{l+} + lP'_l M_{l-}]\,, \tag{26}$$

$$F_3 = \sum_l [P''_{l+1}E_{l+} + P''_{l-1}E_{l-} - P''_{l+1}M_{l+} + P''_{l-1}M_{l-}]\,, \tag{27}$$

$$F_4 = \sum_l [-P''_l E_{l+} - P''_l E_{l-} + P''_l M_{l+} - P''_l M_{l-}]\,, \tag{28}$$

$$F_5 = \sum_l [(l+1)P'_{l+1}L_{l+} - lP'_{l-1}L_{l-}]\,, \tag{29}$$

$$F_6 = \sum_l [-(l+1)P'_l L_{l+} + lP'_l L_{l-}]\,, \tag{30}$$

$$F_7 = \sum_l [-(l+1)P'_l S_{l+} + lP'_l S_{l-}]\,, \tag{31}$$

$$F_8 = \sum_l [(l+1)P'_{l+1}S_{l+} - lP'_{l-1}S_{l-}]\,, \tag{32}$$

$P_L(x)$ is Legendre function and $x = \hat{k}\cdot\hat{q}$.