Formula for calculating meson-baryon scattering cross sections
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We follow the convention of Goldberger and Watson to define the meson-baryon scattering amplitudes. The normalization of states are : $<\vec{k}|\vec{k}\,'>=\delta(\vec{k}-\vec{k}\,')$ for plane wave states, and $<\phi_\alpha|\phi_\beta>=\delta_{\alpha,\beta}$ for bound states. The T-matrix elements are related to S-matrix elements by

$$S_{MB,M'B'}=\delta_{MB,M'B'}-2\pi\, i\,\delta(E_{MB}-E_{M'B'})\,T_{MB,M'B'} \tag{1}$$

Note that the " $-$ " sign in the right side of the above equation is opposite to the " $+$ " sign used by the other partial-wave analysis groups such as SAID.

The formula for calculating the meson-baryon scattering cross sections given here are in the center of mass system. For the process $M(\vec{k})+B(-\vec{k})\rightarrow M'(\vec{k}\,')+B'(-\vec{k}\,')$ the differential cross section can be written as

$$\frac{d\sigma_{MB\rightarrow M'B'}}{d\Omega_{k'}}=\frac{(4\pi)^2}{k^2}\rho_{M'B'}(k')\rho_{MB}(k)\frac{1}{(2j_M+1)(2j_B+1)}\sum_{m_{j_M}m_{j_B}}\sum_{m'_{j_M}m'_{j_B}}|<M'B'|t(W)|MB>|^2\ , \tag{2}$$

iIn the above equation, the incoming and outgoing momenta k and k' are defined by the invariant mass W

$$W=E_M(k)+E_B(k)=E_{M'}(k')+E_{B'}(k')\ , \tag{3}$$

where $E_\alpha(k)=\sqrt{m_\alpha^2+\vec{k}^{\,2}}$ with m_α being the mass of particle α, and the phase-space factor is

$$\rho_{MB}(k)=\pi\frac{kE_M(k)E_B(k)}{W}\ . \tag{4}$$

The scattering amplitude $<M'B'|t(W)|MB>$ in Eq. (2) can be calculated from the partial-wave amplitudes $t^{JT}_{L'S'M'B',LSMB}(k',k,W)$ as

$$\begin{aligned}
&<M'B'|t(W)|MB>\\
&=\sum_{JM_J,T}\ \sum_{L'M'_L,S'M'_S}\ \sum_{LM_L,SM_S} t^{JT}_{L'S'M'B',`LSMB}(k',k,W)\\
&\times[<TM_T|i'_M\tau'_Bm'_{i_M}m'_{\tau_B}><JM_J|L'S'm'_Lm'_S><S'm'_S|j'_Mj'_Bm'_{j_M}m'_{j_B}>Y^*_{L'm'_L}(\hat{k}\,')]\\
&\times[<TM_T|i_M\tau_Bm_{i_M}m_{\tau_B}><JM_J|LSm_Lm_S><Sm_S|j_Mj_Bm_{j_M}m_{j_B}>Y_{Lm_L}(\hat{k})]\ ,
\end{aligned} \tag{5}$$

where $<jm_j|j_1j_2m_{j_1}m_{j_2}>$ is the Clebsch-Gordon coefficient for the $\vec{j}_1+\vec{j}_2=\vec{j}$ coupling, $[(j_Mm_{j_M}),(i_Mm_{i_M})]$ and $[(j_Bm_{j_B}),(\tau_Bm_{\tau_B})]$ are the spin-isospin quantum numbers of mesons and baryons, respectively; $(JM_J)((TM_T))$ are the total angular momentum (total isospin), (LM_L) $((SM_S))$ are the relative orbital angular momentum (total spin) of the considered two-body systems.

By choosing the incoming momentum $\vec{k}$ in the quantization z-component, the total $MB\rightarrow M'B'$ cross sections are

$$\sigma^{tot}_{MB\rightarrow M'B'}(W)=\int d\Omega_{k'}\frac{d\sigma_{MB\rightarrow M'B'}}{d\Omega_{k'}}\ . \tag{6}$$

By optical theorem and the above partial-wave expansion, one can get the $\pi N \to X$ total cross sections averaged over the initial spins:

$$\sigma^{tot}_{\pi N\to X}(W) = \frac{-4\pi}{(2s_N+1)k^2}\sum_{J,T,L}(2J+1)\,\rho_{\pi N}(k)\,Im[t^{JT}_{L\frac{1}{2}\pi N,L\frac{1}{2}\pi N}(k,k,W)] \times [< TM_T|1\frac{1}{2}m_{i_\pi}m_{\tau_N} >]^2 \ , \tag{7}$$

where $M_T = m_{i_\pi} + m_{\tau_N}$, and $s_N = 1/2$ is the nucleon spin.

The ANL-Osaka partial-wave amplitudes $t^{JT}_{L'S'M'B',LSMB}(k',k,W)$ can be obtained from the following quantities presented on the webpage:

$$< M'B'|T(W)|MB >= -\rho^{1/2}_{M'B'}(k')t^{JT}_{L'S'M'B',LSMB}(k',k,W)\rho^{1/2}_{MB}(k) \ , \tag{8}$$

for $MB, M'B' = \pi N, \eta N, K\Lambda, K\Sigma$ and $W = W_{th} - 2000$ MeV, where W_{th} is the lower one of the two threshold energies $m_M + m_B$ and $m_{M'} + m_{B'}$. The phase space factors $\rho_{MB}(k)$ and $\rho_{M'B'}(k')$ are defined by Eq.(4).