

Formula for calculating the $\gamma N \rightarrow MB$ cross sections
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We use the standard definitions of the multipole amplitudes of photoproduction of pseudo-scalar mesons ($M = \pi, \eta, K$) on the nucleon (N). In the center of mass system, the formula for calculating the un-polarized differential cross section of $\gamma(\vec{q}) + N(-\vec{q}) \rightarrow M(\vec{k}) + N(-\vec{k})$ is

$$\frac{d\sigma_0}{d\Omega} = \frac{1}{4} \sum_{m_{s_N}=\pm 1/2} \sum_{m'_{s_N}=\pm 1/2} \sum_{\lambda=\pm 1} \frac{k}{q} | \langle m'_{s_N} | F_{CGLN} | m_{s_N} \rangle |^2, \quad (1)$$

where the Chew-Goldberger-Low-Nambu (CGLN) amplitude is defined as

$$F_{CGLN} = \sum_{i=1,4} O_i F_i(\theta, W). \quad (2)$$

Here we have defined $W = q + E_N(q) = E_M(k) + E_N(k)$, where $E_\alpha(p) = \sqrt{m_\alpha^2 + p^2}$ and m_α is the mass of particle α , and θ is the angle between \vec{k} and \vec{q} . The operators in the above equation are:

$$\begin{aligned} O_1 &= -i\vec{\sigma} \cdot \vec{\epsilon}_\lambda \\ O_2 &= -[\vec{\sigma} \cdot \hat{k}][\vec{\sigma} \cdot (\hat{q} \times \vec{\epsilon}_\lambda)] \\ O_3 &= -i[\vec{\sigma} \cdot \hat{q}][\hat{k} \cdot \vec{\epsilon}_\lambda] \\ O_4 &= -i[\vec{\sigma} \cdot \hat{k}][\hat{k} \cdot \vec{\epsilon}_\lambda] \end{aligned}$$

where $\hat{k} = \vec{k}/|\vec{k}|$ and $\hat{q} = \vec{q}/|\vec{q}| = \hat{z}$, and $\vec{\sigma}$ is the standard Pauli operator. The quantization direction is chosen to be in the \hat{z} -direction, and the photon polarization vectors are $\vec{\epsilon}_\pm = \frac{\mp}{\sqrt{2}}[\hat{x} \pm i\hat{y}]$.

The CGLN amplitudes are related to multipole amplitudes by

$$\begin{aligned} F_1(\theta, W) &= \sum_l [P'_{l+1}(x)E_{l+}(W) + P'_{l-1}(x)E_{l-}(W) + lP'_{l+1}(x)M_{l+}(W) + (l+1)P'_{l-1}(x)M_{l-}(W)] \\ F_2(\theta, W) &= \sum_l [(l+1)P'_l(x)M_{l+}(W) + lP'_l(x)M_{l-}(W)] \\ F_3(\theta, W) &= \sum_l [P''_{l+1}(x)E_{l+}(W) + P''_{l-1}(x)E_{l-}(W) - P''_{l+1}(x)M_{l+}(W) + P''_{l-1}(x)M_{l-}(W)] \\ F_4(\theta, W) &= \sum_l [-P''_l(x)E_{l+}(W) - P''_l(x)E_{l-}(W) + P''_l(x)M_{l+}(W) - P''_l(x)M_{l-}(W)] \end{aligned}$$

where $x = \hat{k} \cdot \hat{q} = \cos\theta$, l is the orbital angular momentum of the final πN system, $P'_l(x) = dP_l(x)/dx$ and $P''_l(x) = d^2P_l(x)/d^2x$ are the derivatives of the Legendre function $P_l(x)$ with the understanding that $P'_{-1} = P''_{-1} = 0$.

The formula for using the multipole amplitudes to calculate the polarization observables can be found in "A. Sandorfi, S. Hoblit, H. Kamano, T.-S. H. Lee, J.Phys. G38, 053001 (2011)

The multipole amplitudes $E_{l\pm}(W)$ and $M_{l\pm}(W)$ for $W = 1080 - 2000$ MeV and $\gamma N \rightarrow \pi N, \eta N, K\Lambda, K\Sigma$ from the ANL-Osaka model are presented on the webpage.