QCD Bound-State Problem:
Off-shell Persistence of Composite Pions and Kaons

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Argonne National Laboratory
1 Background: What are bound-states?

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1 Background: Why do we study bound-states?

“Easy” objects involving interactions + etc.
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Experiment

“Easy” objects involving interactions

Theory

“Simple” objects involving dynamics

- Newtonian Mechanics
- Quantum Mechanics
- Quantum Field Theory
+ etc.
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**Experiment**

"Easy" objects involving interactions

**Theory**

What matter is possible & how is it constituted?

"Simple" objects involving dynamics

- Newtonian Mechanics
- Quantum Mechanics
- Quantum Field Theory
  + etc.
1 Background: How do we study bound-states?

e^+ e^- hadronic annihilation
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**Quantum Field Theory**

- **Green functions**
- **Bethe-Salpeter equation**

\[
\begin{align*}
\Psi &= K \Psi \\
\end{align*}
\]

**e^+ e^- hadronic annihilation**

**Standard Hadrons**
- **Meson**
- **Baryon**

**Exotic Hadrons**

**Fig. 2.18**

\[
\begin{align*}
\sigma &\propto \left( \frac{M}{\sqrt{s}} \right)^n \\
R &\propto \left( \frac{M}{\sqrt{s}} \right)^n \\
\sqrt{s} \text{ [GeV]} &\propto \left( \frac{M}{\sqrt{s}} \right)^n \\
\end{align*}
\]
1 Background: Why is QCD bound-state problem difficult?

- **Relativistic bound states**

  “These problems are those involving bound states [...] such problems necessarily involve a breakdown of ordinary perturbation theory. [...] The pole therefore can only arise from a divergence of the sum of all diagrams [...]”

  The QFT book vol1 p564 Weinberg

- **Strongly coupled systems**

  - Asymptotic freedom: Bonds between particles become asymptotically weaker as energy increases and distance decreases (Nobel Prize).
  
  - Quark and Gluon Confinement: No matter how hard one strikes the proton, one cannot liberate an individual quark or gluon.
  
  - Dynamical Chiral Symmetry Breaking: Mystery of bound state masses, e.g., current quark mass (Higgs) is small, and no degeneracy between parity partners.
1 Background: Non-perturbative approaches of QCD
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- **Full QCD**

- **Observables**

  - **Physics**
  - **Simplicity**
  - **Power**
1 Background: Non-perturbative approaches of QCD

Lattice QCD, Dyson-Schwinger equations, chiral perturbation, AdS/QCD, NJL model, ...
2 DSE: EoM of QCD's Green functions

- Classical Mechanics
- Quantum Field Theory

- Degrees of freedom

- Generalized coord.
- Fields on spacetime

- Principle of Least Action

- Equations of Motion (EoM)

- Euler-Lagrange Equation
- Dyson-Schwinger Equations
2 DSE: EoM of QCD’s Green functions
Most equations are very complicated.

Green functions of different orders couple together.
Most equations are very complicated.

- **Modeling**

- Green functions of different orders couple together.

- **Truncation**
2 DSE: Most frequently used equations

- **One-body gap equation**

\[
\begin{align*}
\bar{\psi} \Gamma^\mu \psi & = 2 \bar{\psi} \gamma^\mu \psi
\end{align*}
\]

- **Two-body Bethe-Salpeter equation**

\[
T = K + KT
\]

- **Three-body form factor equation**

\[
2P_\mu F(Q^2) = G^{(6)}
\]
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- **One-body gap equation**

\[ T^{-1} = G^{-1} + K T \]

- **Two-body Bethe-Salpeter equation**

\[ T = K + K T \]

- **Three-body form factor equation**

\[ 2P_\mu F(Q^2) = G^{(6)} \]
2 DSE: Most frequently used equations

• **One-body gap equation**

\[
-1 = -1 + \Gamma
\]

• **Two-body Bethe-Salpeter equation**

\[
T = K + K T
\]

\[
\Xi \rightarrow M^2 \rightarrow \Xi
\]

\[
\Xi = K \Xi
\]

• **Three-body form factor equation**

\[
2P_\mu F(Q^2) = \Gamma
\]

\[
\left(\begin{array}{c} \psi \psi \\ \bar{\psi} \bar{\psi} \end{array}\right)
\]

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\[
G^{(6)}
\]
2 DSE: Simplest approximation of QCD DSEs

I. Gluon propagator

II. Quark-gluon vertex

III. Scattering kernel

IV. 6-point Green function
2 DSE: Simplest approximation of QCD DSEs

I. Gluon propagator

massive gluon model

\[ g^2 D_{\mu\nu}^{ab}(k) = \delta_{ab} D_{\mu\nu}^{\text{free}}(k) G(k^2) \]

II. Quark-gluon vertex

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triangle diagram

\[ \Lambda_\mu(P, Q) = 2P_\mu F(Q^2) \]
In the chiral limit, the color-singlet av-WGTI (chiral symmetry) is written as

\[ P_\mu \Gamma_{5\mu}(k, P) = S^{-1} \left( k + \frac{P}{2} \right) i \gamma_5 + i \gamma_5 S^{-1} \left( k - \frac{P}{2} \right) \]
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- Assuming DCSB, i.e., the mass function is nonzero, we have the following identity

\[ \lim_{P \to 0} P_\mu \Gamma_5(\mu, \nu) = 2i\gamma_5 B(k^2) \neq 0 \]
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✧ The axial-vector vertex must involve a pseudo scalar pole (Goldstone theorem)

\[ \Gamma_{5\mu}(k, 0) \sim \frac{2i\gamma_5 f_\pi E_\pi(k^2) P_\mu}{P^2} \propto \frac{P_\mu}{P^2} \quad f_\pi E_\pi(k^2) = B(k^2) \]
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Assuming there is a radially excited pion, its decay constant vanishes

\[ \lim_{P^2 \to M_{\pi n}^2} \Gamma_{5\mu}(k, P) \sim \frac{2i\gamma_5 f_{\pi n} E_{\pi n}(k, P) P_\mu}{P^2 + M_{\pi n}^2} < \infty \quad f_{\pi n} = 0 \]
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- Assuming there is a radially excited pion, its decay constant vanishes

\[ \lim_{P^2 \to M_{\pi_n}^2} \Gamma_{5\mu}(k, P) \sim \frac{2i \gamma_5 f_\pi E_{\pi_n}(k, P) P_\mu}{P^2 + M_{\pi_n}^2} < \infty \quad f_{\pi_n} = 0 \]

DCSB means much more than massless pseudo-scalar meson.
2 DSE: Summary

- **Gluon propagator**: Solve the gluon DSE or extract information from lattice QCD. The dressing function of gluon has a mass scale as that of quark.

- **Quark-gluon vertex + Scattering kernel**: Analyze continuous (WGTIs or STIs) & discrete symmetries. The kernel (RL) preserves the chiral symmetry which makes pion to play a twofold role: Bound-state and Goldstone boson.

- **Form factor**: Generalize the wave function normalization condition. The form factor (the triangle diagram) preserves the current conservation.
3 Application: Realization of DCSB & Confinement

**DCSB:**
1. The quark's **effective mass** runs with its momentum.
2. The most **constituent mass** of a light quark comes from a cloud of gluons.

**Confinement:**
Although we exactly know few knowledge about confinement, the **positivity violation** of quark spectral density supports a fact that a asymptotically free quark is unphysical. In this sense, we say that quarks are **confined**.

\[
S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)} = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}
\]
3 Application: Off-Shell pions and kaons

Experiments use a nucleon’s virtual pion cloud as a pion target, e.g., the processes are usually involved:

\[ \pi^* + \gamma \rightarrow \pi \]
\[ \pi^* + \gamma \rightarrow X \]

Sullivan processes, in which a nucleon’s pion cloud is used to provide access to the pion’s (a) elastic form factor and (b) parton distribution functions.
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✦ How does the pion’s virtuality affect its properties and further affect the related processes?

✦ Is there a critical virtuality above which a Sullivan-like process cannot provide reliable access to a meson target?
3 Application: Off-Shell pions and kaons

- In QFT, bound-states are encoded in Green functions.

\[ G^{(4)}(4) = G_0^{(4)} + G_0^{(4)} \cdot K^{(2)} \cdot G^{(4)} \]
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\[ G^{(4)} = G_0^{(4)} + G_0^{(4)} \cdot K^{(2)} \cdot G^{(4)} \]

The kernel can be decomposed by its orthogonal eigenbasis, which are classified by \( J^P \) quantum number and radial quantum number \( n_r \).

\[ K^{(2)} = \sum_i \lambda_i^{-1} |\Gamma_i\rangle \langle \Gamma_i| \quad |\Gamma_i\rangle = \lambda_i \cdot K^{(2)} \cdot G_0^{(4)} \cdot |\Gamma_i\rangle \quad \langle \Gamma_i | G_0^{(4)} | \Gamma_j \rangle = \delta_{ij} \]
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- In QFT, bound-states are encoded in Green functions.

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- Accordingly, the four-point Green function can be decomposed:

\[ G^{(4)} = G^{(4)}_0 + \sum_i |\chi_i\rangle \frac{1}{\lambda_i(P^2) - 1} \langle \chi_i| \]
3 Application: Off-Shell pions and kaons

- The wave function of the bound state has to satisfy the following condition

\[
\lim_{\text{on-shell}} \frac{1}{P^2 + M^2} \left\{ \begin{array}{c}
\begin{array}{c}
\leftrightarrow
\end{array}
\end{array} \right. \left[ \begin{array}{c}
\begin{array}{c}
\leftrightarrow
\end{array}
\end{array} \right]^{-1} - \begin{array}{c}
\begin{array}{c}
\leftrightarrow
\end{array}
\end{array} K^{(2)} \right\} = 1
\]
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- The wave function of the bound state has to satisfy the following condition

\[
\lim_{\text{on-shell}} \frac{1}{P^2 + M^2} \left\{ \left[ \left( \begin{array}{c}
\end{array} \right) \right]^{-1} - \left( \begin{array}{c}
\end{array} \right) \right\} = 0 \Rightarrow 1
\]
### 3 Application: Off-Shell pions and kaons

- The **wave function** of the bound state has to satisfy the following condition

\[
\lim_{\text{on-shell}} \frac{1}{P^2 + M^2} \left\{ \left( \begin{array}{c}
\text{Diagram 1} \n\end{array} \right) - K^{(2)} \right\} = 0
\]

\[
\lim_{\text{on-shell}} \frac{1}{P^2 + M^2} \left\{ \left( \begin{array}{c}
\text{Diagram 2} \n\end{array} \right) - K^{(2)} \right\} = 1
\]

- The generalized homogeneous **Bethe-Salpeter** equation can be obtained as

\[
\left( \begin{array}{c}
\text{Diagram 3} \n\end{array} \right) = K^{(2)} \times \lambda(P^2)
\]
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\[
\lim_{\text{on-shell}} \frac{1}{P^2 + M^2} \left\{ \begin{array}{c}
\left[ \left( \begin{array}{cc}
\overline{F} \\
F
\end{array} \right) \right]^{-1} - K^{(2)} \\
\end{array} \right. = 0
\]

\[
\right. \left. \begin{array}{c}
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F
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F
\end{array} \right)^{-1} - K^{(2)} \\
\end{array} \right. = \lambda(P^2)
\]

- The solved wave function must be normalized as following:

\[
\left\{ \frac{\partial}{\partial P_\mu} \left[ \left( \begin{array}{cc}
\overline{F} \\
F
\end{array} \right) \left( \begin{array}{cc}
\overline{F} \\
F
\end{array} \right)^{-1} - K^{(2)} \right] \right\} = 2P_\mu.
\]
3 Application: Off-Shell pions and kaons

- The eigenvalue is linear to the virtuality less than 45 ($P^2 = (\nu - 1)m^2_\pi$)

$$\lambda(\nu) = 1 + 0.016 \nu$$
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- Recalling the Green function's structure

\[
\frac{1}{\lambda(P^2) - 1} \sim \frac{1}{P^2 + m_{\pi}^2}
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the change in $\lambda$ is purely kinematic and, hence, the pion pole dominates the quark-antiquark scattering matrix.
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the change in $\lambda$ is purely kinematic and, hence, the pion pole dominates the quark-antiquark scattering matrix.

- The UV shifts of the BS amplitudes grow with the virtuality less than $31$ and that growths are almost linear. This leads to a linear growth of the in-pion condensate:

$$\kappa_\pi^\xi(\nu) \approx \kappa_\pi^\xi(0)[1 + 0.032\nu]$$
3 Application: Off-Shell pions and kaons

✧ With the virtuality increasing, the pion has a smaller radius and becomes more point-like.
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✧ With the virtuality increasing, the pion has a smaller radius and becomes more point-like.

✧ The computed form factor can be interpolated by a monopole multiplied by a simple factor that restores the correct QCD anomalous dimension.

\[
F_\pi^*(Q^2, \nu) = \frac{1}{1 + Q^2/m_0^2} \mathcal{A}(Q^2, \nu)
\]

\[
\mathcal{A}(Q^2, \nu) = \frac{1 + Q^2 a_0^2(\nu)}{1 + Q^2 [a_0^2(\nu)/b_0^2(\nu)] \ln(1 + Q^2/\Lambda_{\text{QCD}}^2)}
\]
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\]

- For \( \nu \lesssim \nu_S \), the \( Q^2 \gtrsim 10 \text{GeV}^2 \) form factor responds linearly to changes in the BS amplitudes and such modifications should become evident on this domain.
3 Application: Off-Shell pions and kaons

The pion’s twist-two valence-quark PDA is connected with the large-$Q^2$ form factor:

\[ Q^2 F_\pi(Q^2) \approx \Lambda_{QCD}^2 \frac{16\pi\alpha_s(Q^2)}{f_\pi^2} w_\varphi^2, \]

\[ w_\varphi = \frac{1}{3} \int_0^1 dx \frac{1}{x} \varphi(x), \]

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$$Q^2 F_\pi(Q^2) \approx \frac{1}{Q^2} \frac{16\pi\alpha_s(Q^2) f_\pi^2 w_\varphi^2}{\lambda_{QCD}^3},$$

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- The pion’s twist-two valence-quark PDA is connected with the large-$Q^2$ form factor:

\[ Q^2 F_\pi(Q^2) \approx 16 \pi \alpha_s(Q^2)f_\pi^2 \omega_\varphi^2, \]

where \( \omega_\varphi = \frac{1}{3} \int_0^1 dx \frac{1}{x} \varphi_{\pi}(x) \).

- we can use GPDs to translate the behavior of \( F_\pi^*(Q^2, \nu) \) into insights regarding the impact of virtuality on extractions of the pion’s valence-quark PDF:

\[ F_\pi(Q^2) = \int_{-1}^{1} dx H_{\pi+}^u(x, 0, Q^2), \]

\[ u_{\pi}(x) = H_{\pi+}^u(x > 0, 0, 0) , \]
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- The pion’s twist-two valence-quark PDA is connected with the large-$Q^2$ form factor:

\[
Q^2 F_\pi(Q^2) \overset{Q^2 \gg \Lambda_{QCD}^2}{\approx} 16\pi\alpha_s(Q^2) f_\pi^2 \omega_\varphi^2,
\]

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\]

\[
u(x) = H^u_{\pi+}(x > 0,0,0),
\]

- The critical virtuality, below which the virtual particles serve as a valid target, is

\[
-u\bar{u} \quad u\bar{s} \quad s\bar{s}
\]

\[-t \lesssim 0.6 \text{ GeV}^2 \quad -t \lesssim 0.9 \text{ GeV}^2 \quad -t \lesssim 1.3 \text{ GeV}^2\]
Summary

- **Bound-states** are **ideal** objects connecting experiments and theories. **QCD bound-state** problems are difficult because of its relativistic and strongly-couple properties.

- Based on LQCD and QCD’s symmetries, the **simplest method** to construct the **gluon propagator, quark-gluon vertex, scattering kernel, and form factor**, is demonstrated.

- A **model-independent** scheme to study the **off-shell** bound state is proposed. Off-shell pions and kaons are studied to suggest critical **virtualities** for experiments.
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Outlook

- With the **sophisticated method** to solve the DSEs, we can push the approach to a wide range of applications in **QCD bound-state** problems.

- Hopefully, after more and more **successful applications** are presented, the DSEs may provide a **faithful path** to understand **QCD** and a powerful tool for general physics.