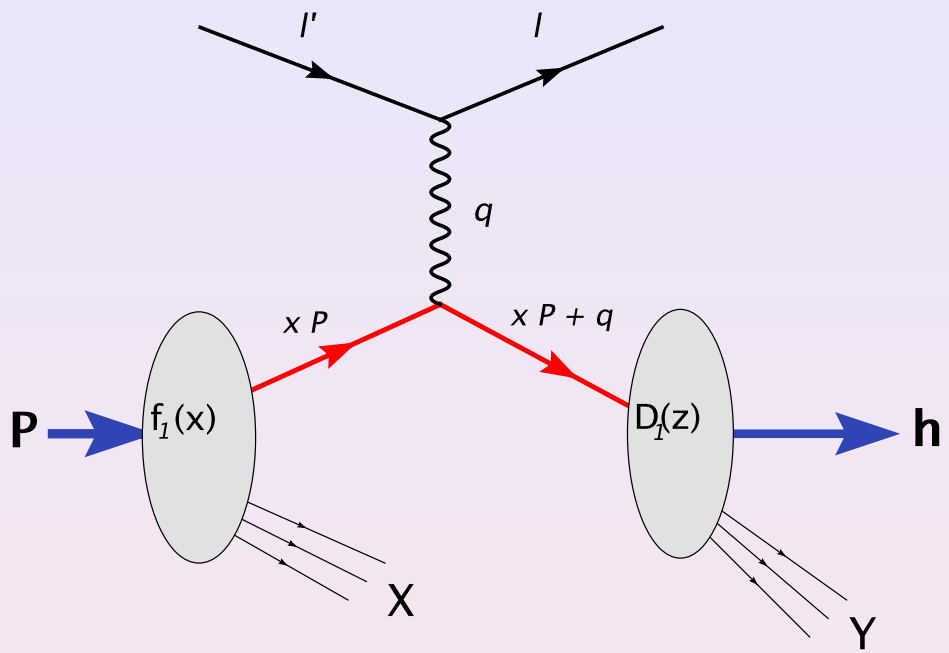
# Parton intrinsic motion and the spin structure of the proton

Alexei Prokudin

University of Turin and INFN Sezione di Torino, Italy

**QCD Bound States Workshop**

# Semi Inclusive Deep Inelastic Scattering




$l + P \to l' + h + X$

Kinematical variables:

$Q^2 = -q^2$, $x = \frac{Q^2}{2P\cdot q}$,

$y = \frac{Q^2}{sx}$, $z = \frac{P\cdot h}{P\cdot q}$

$f_1(x)$ – parton density function (PDF)

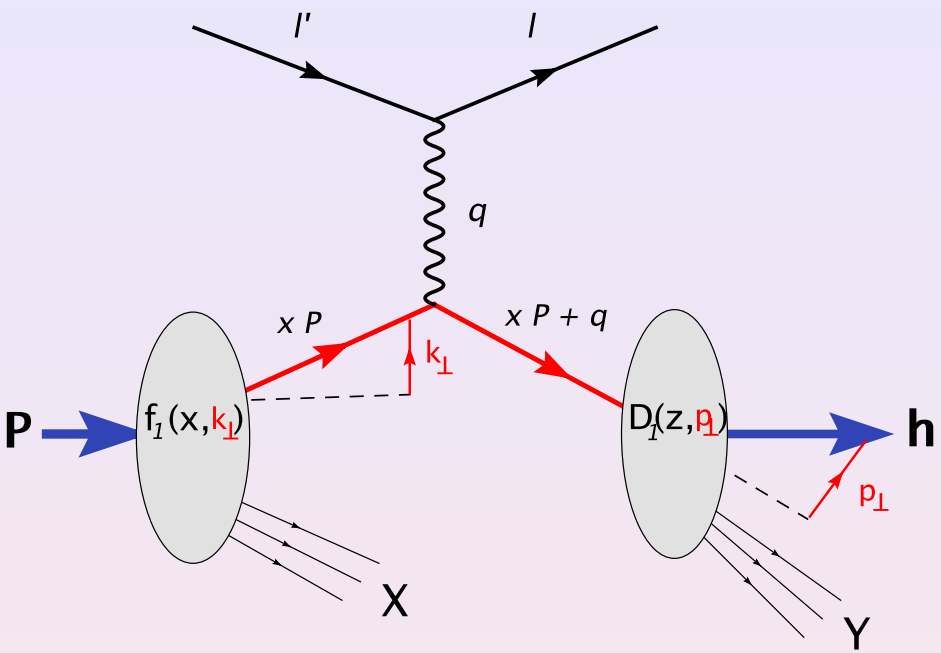
$D_1(z)$ – fragmentation function (FF)

$$\sigma = \sum_q f_{q/p}(x, Q^2) \otimes \sigma^{lq\to lq'} \otimes D_q^h(z, Q^2)$$

Partons are considered as collinear with respect to the incident proton.

$$P = \frac{\sqrt{s}}{2}(1, 0, 0, -1) \quad h_q = x\frac{\sqrt{s}}{2}(1, 0, 0, -1)$$

# Semi Inclusive Deep Inelastic Scattering




$l + P \to l' + h + X$

$f_1(x, \mathbf{k}_\perp)$ – transverse momentum dependent parton density function (TMD PDF)

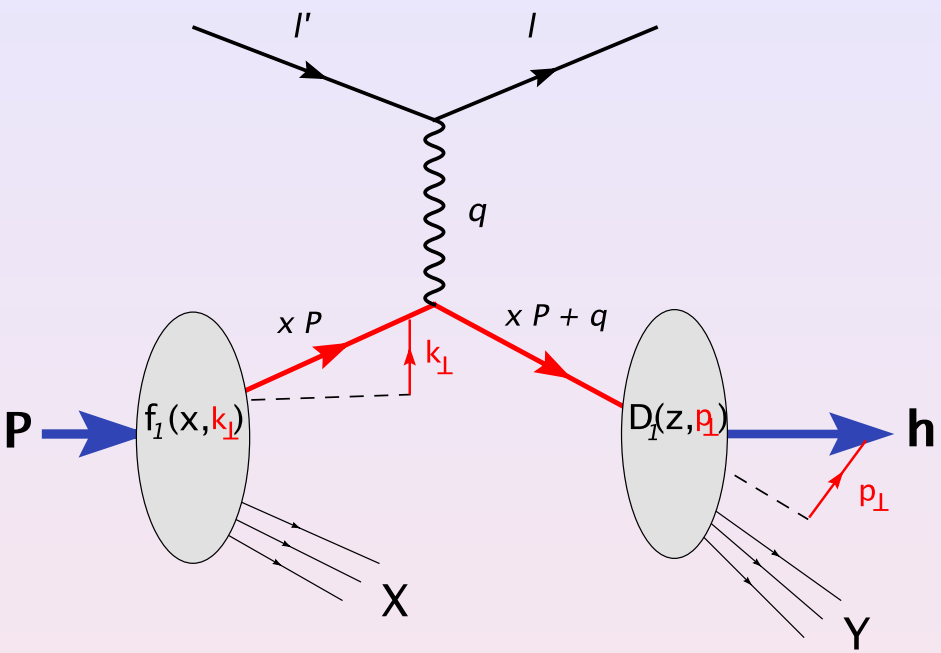
$D_1(z, \mathbf{p}_\perp)$ – transverse momentum dependent fragmentation function (TMD FF)

$$\sigma = \sum_q f_{q/p}(x, \mathbf{k}_\perp, Q^2) \otimes \sigma^{lq \to lq'} \otimes D_q^h(z, \mathbf{p}_\perp, Q^2)$$

Non collinear partons R.P. Feynman, R.D. Field, G.C. Fox, Nucl.Phys.B128:1-65,1977

$$P = \frac{\sqrt{s}}{2}(1, 0, 0, -1) \quad h_q = x\frac{\sqrt{s}}{2}(1, k_\perp^x, k_\perp^y, -1)$$

# Semi Inclusive Deep Inelastic Scattering




$l + P \to l' + h + X$

$f_1(x, \mathbf{k}_\perp)$ – transverse momentum dependent parton density function (TMD PDF)

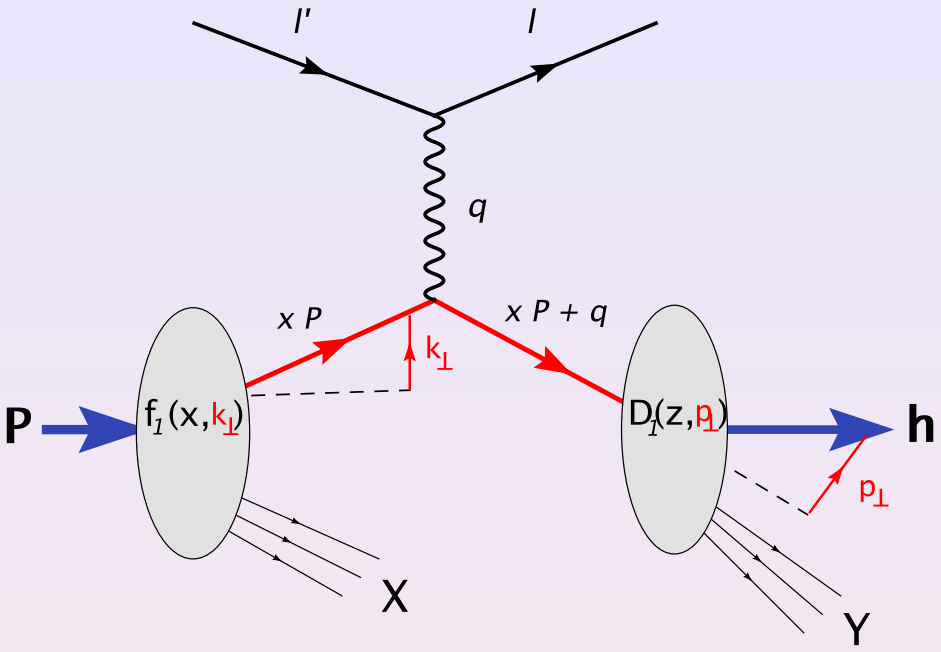
$D_1(z, \mathbf{p}_\perp)$ – transverse momentum dependent fragmentation function (TMD FF)

$$\sigma = \sum_q f_{q/p}(x, \mathbf{k}_\perp, Q^2) \otimes \sigma^{lq \to lq'} \otimes D_q^h(z, \mathbf{p}_\perp, Q^2)$$

Due to the finite size of the target and the produced hadron all the partons are non collinear: $\langle k_\perp^2 \rangle \sim 1/R^2$

$\sigma^{lq \to lq'}$ now depends on $k_\perp$ and aquire azimuthal modulations. These effects are non perturbative and important in the region of small $P_T \lesssim 1$ GeV.

# Semi Inclusive Deep Inelastic Scattering




$l + P \rightarrow l' + h + X$

QCD corrections must be taken into account for high $P_T$ hadrons.

Higher $\alpha_s$ QCD corrections are important and dominating in the region of high $P_T >> 1$ GeV.

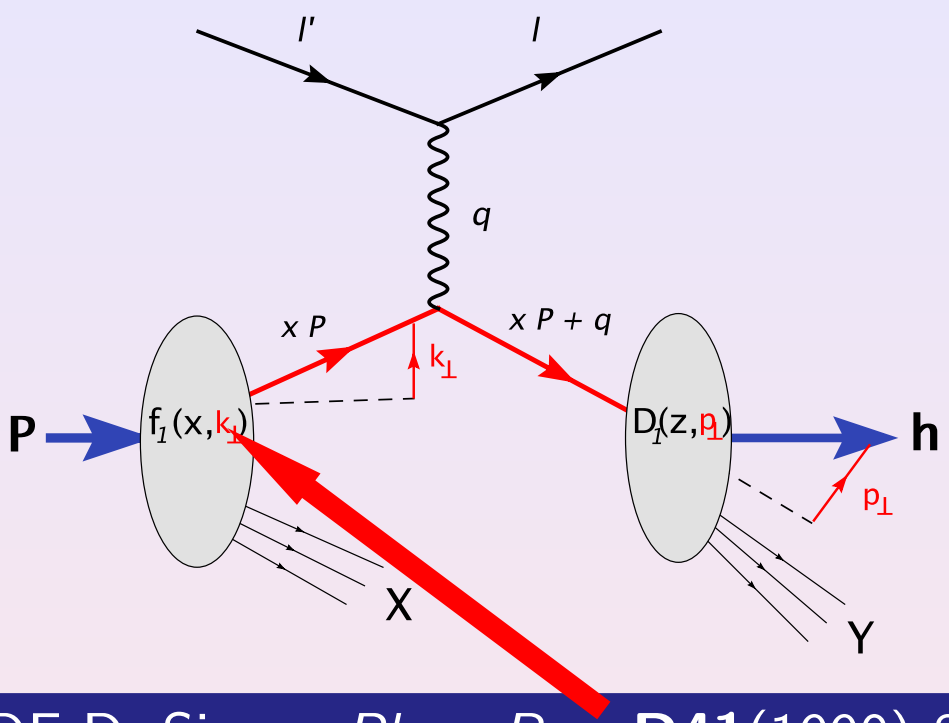

H. Georgi and H. D. Politzer, *Phys. Rev.* **D40** 3 (1978) LO

J. Chay, S. D. Ellis and W. J. Stirling, *Phys. Rev.* **D45** 45 (1992) LO

A. Daleo, D. de Florian and R. Sassot, *Phys. Rev.* **D71** (2005) 034013 NLO

B.A. Kniehl, G. Kramer, M. Maniatis, *Nucl. Phys.* **B711** (2005) 345. NLO

# Semi Inclusive Deep Inelastic Scattering



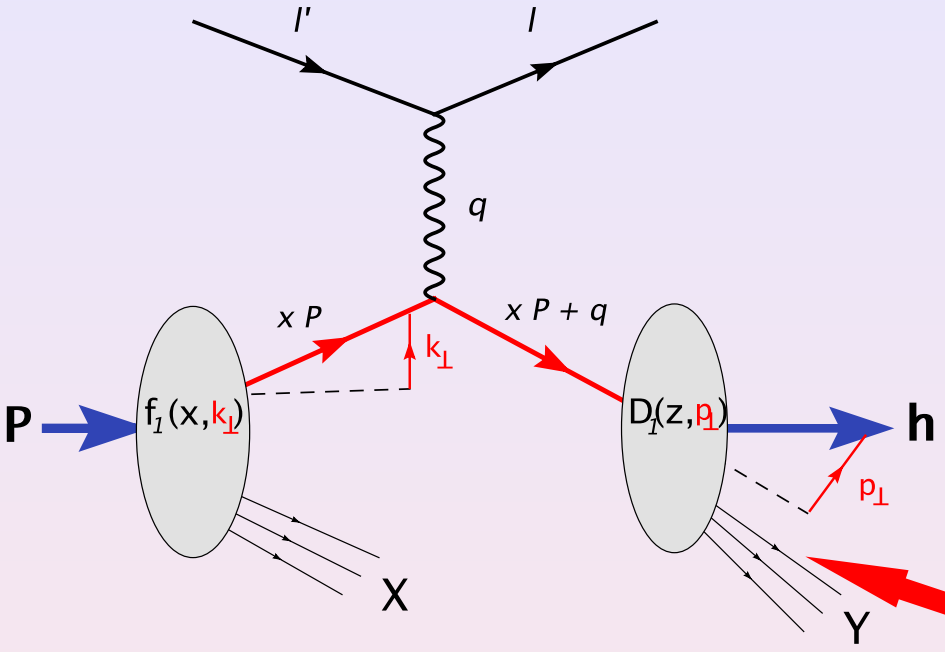

$l + P \to l' + h + X$

In polarized case new spin dependent functions arise. Correlation between the spin of the proton and the parton angular momentum: SIVERS function $\Delta^N f_{q/p^\uparrow}(x, k_\perp)$

**PDF D. Sivers, *Phys. Rev.* D41(1990) 83**

$$f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = f_{q/p}(x, k_\perp) + \frac{1}{2}\Delta^N f_{q/p^\uparrow}(x, k_\perp)\mathbf{S}\cdot(\hat{\mathbf{P}} \times \hat{\mathbf{k}_\perp})$$

# Semi Inclusive Deep Inelastic Scattering




$l + P \to l' + h + X$

In polarized case new spin dependent functions arise, correlations between fragmenting quark and jet remnants:

COLLINS fragmentation function $\Delta^N D_{h/q^\uparrow}(z, \mathbf{p}_\perp)$

**FF** J. C. Collins, *Nucl. Phys.* **B396** (1993) 161

$$D_{h/q^\uparrow}(z, \mathbf{p}_\perp) = D_{h/q}(z, |p_\perp|) + \frac{1}{2}\Delta^N D_{h/q^\uparrow}(z, |p_\perp|) S_q \cdot (\hat{p}_q x \hat{\mathbf{p}}_\perp)$$

# Unpolarized **SIDIS**

## Cross section of **SIDIS**

$$\mathrm{d}\sigma^{lp\to lhX} = \sum_q f_q(x, Q^2) \otimes \mathrm{d}\sigma^{lq\to lq} \otimes D_q^h(z, Q^2)\ ,$$

where $f_q$ is the parton $q$ distribution function, $D_q^h$ is the fragmentation function of parton $q$ into a hadron $h$.

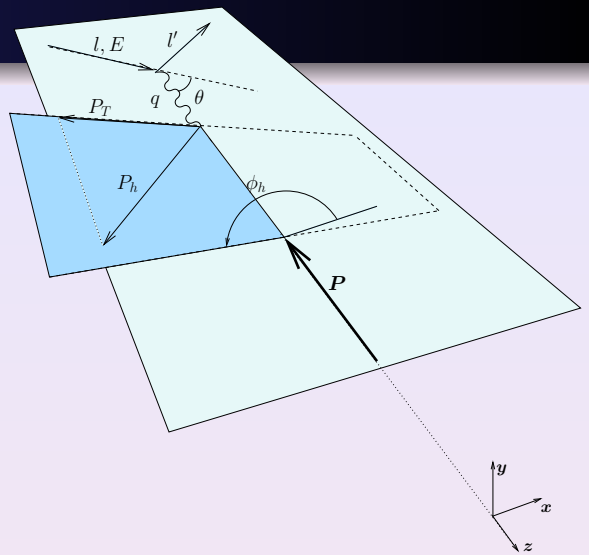
In collinear parton model we have

$$\sigma^{lq\to lq} \propto \hat{s}^2 + \hat{u}^2 \propto 1 + (1-y)^2$$

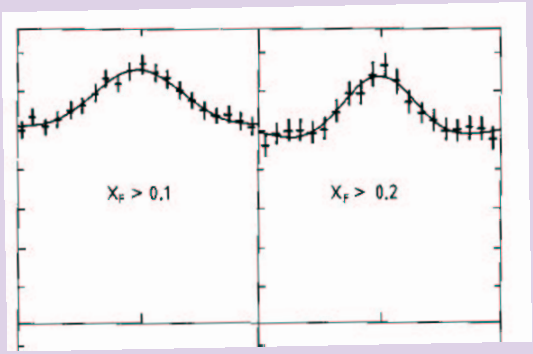
thus no dependence on azimuthal angle $\phi_h$ at first order of PT.

The experimental data reveal that

$$\mathrm{d}\sigma^{lp\to lh^{\pm}X}/\mathrm{d}\phi_h \propto \mathrm{A} + \mathrm{B}\cdot\cos(\phi_\mathrm{h}) + \mathrm{D}\cdot\cos(2\phi_\mathrm{h})$$



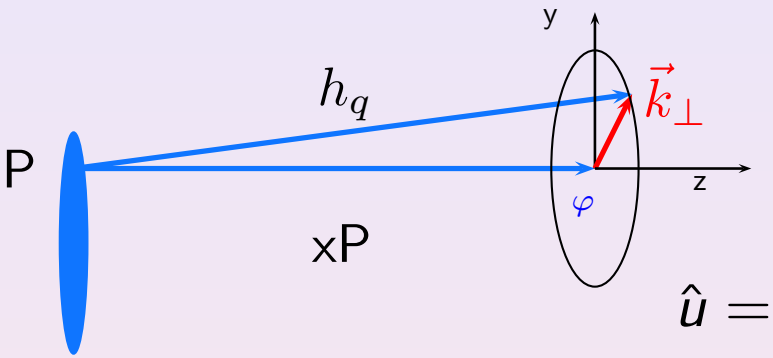

## EMC data




M. Arneodo et al (EMC): Measurement of hadron azimuthal distributions, Z. Phys. C 34 (1987) 277

# Intrinsic $k_\perp$

Robert Cahn[1] introduced parton intrinsic transverse momentum $k_\perp$, parton momentum $h_q = xP + k_\perp$, where $k_\perp = (0, k_\perp cos(\varphi), k_\perp sin(\varphi), 0)$




$$\hat{s} = sx\left[1 - \frac{2k_\perp}{Q}\sqrt{1-y}\cdot\cos(\varphi)\right] + \mathcal{O}\left(\frac{k_\perp^2}{Q}\right)$$

$$\hat{u} = sx(1-y)\left[1 - \frac{2k_\perp}{Q\sqrt{1-y}}\cdot\cos(\varphi)\right] + \mathcal{O}\left(\frac{k_\perp^2}{Q}\right)$$

Hence (assuming collinear fragmentation, $\phi_h = \varphi$)

$$\frac{d\sigma_{ep\to ehX}}{d\phi_h} \propto \hat{s}^2 + \hat{u}^2 \propto \mathrm{A} + \mathrm{B}\cdot\cos(\phi_\mathrm{h}) + \mathrm{D}\cdot\cos(2\phi_\mathrm{h})$$

and these modulations of the cross section with azimuthal angle are called **Cahn effect**.

[1] R. Cahn, Phys. Lett. B 78 (1978) 269; Phys. Rev. D 40 (1989) 3107

Let us assume that $k_\perp$ and $p_\perp$ distributions have the following form

$$f_q(x, k_\perp^2) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-\frac{k_\perp^2}{\langle k_\perp^2 \rangle}} ,$$

$$D_h^q(z, p_\perp^2) = D_h^q(z) \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-\frac{p_\perp^2}{\langle p_\perp^2 \rangle}} ,$$

then we can integrate the previous formula and obtain

$$\frac{\mathrm{d}^5\sigma^{ep\to ehX}}{\mathrm{d}x\mathrm{d}y\mathrm{d}zP_T\mathrm{d}P_T\mathrm{d}\phi_h} \propto \{[1+(1-y)^2] - 4\frac{\sqrt{1-y}(2-y)\langle k_\perp^2\rangle zP_T}{(\langle p_\perp^2\rangle + z^2\langle k_\perp^2\rangle)Q}\cos(\phi_\mathrm{h})\} \cdot$$

$$\cdot f_q(x) D_h^q(z) \frac{1}{\pi\langle P_T^2\rangle} e^{-\frac{P_T^2}{\langle P_T^2\rangle}} ,$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z^2\langle k_\perp^2\rangle$$

$\langle p_\perp^2\rangle$ &$\langle k_\perp^2\rangle$ are essential ingredients for description of asymmetries in SIDIS.

One must describe the data on unpolarized SIDIS before describing SSA.

Let us assume that $k_\perp$ and $p_\perp$ distributions have the following form

$$f_q(x, k_\perp^2) = f_q(x) \frac{1}{\pi \langle k_\perp^2 \rangle} e^{-\frac{k_\perp^2}{\langle k_\perp^2 \rangle}} ,$$

$$D_h^q(z, p_\perp^2) = D_h^q(z) \frac{1}{\pi \langle p_\perp^2 \rangle} e^{-\frac{p_\perp^2}{\langle p_\perp^2 \rangle}} ,$$

then we can integrate the previous formula and obtain

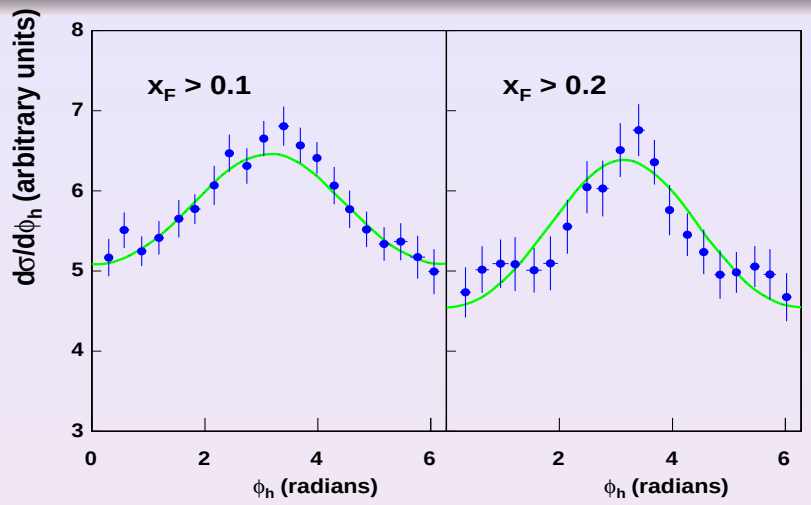
$$\frac{\mathrm{d}^5\sigma^{ep\to ehX}}{\mathrm{d}x\mathrm{d}y\mathrm{d}zP_T\mathrm{d}P_T\mathrm{d}\phi_h} \propto \{[1+(1-y)^2] - 4\frac{\sqrt{1-y}(2-y)\langle k_\perp^2\rangle zP_T}{(\langle p_\perp^2\rangle + z^2\langle k_\perp^2\rangle)Q}\cos(\phi_\mathrm{h})\} \cdot$$

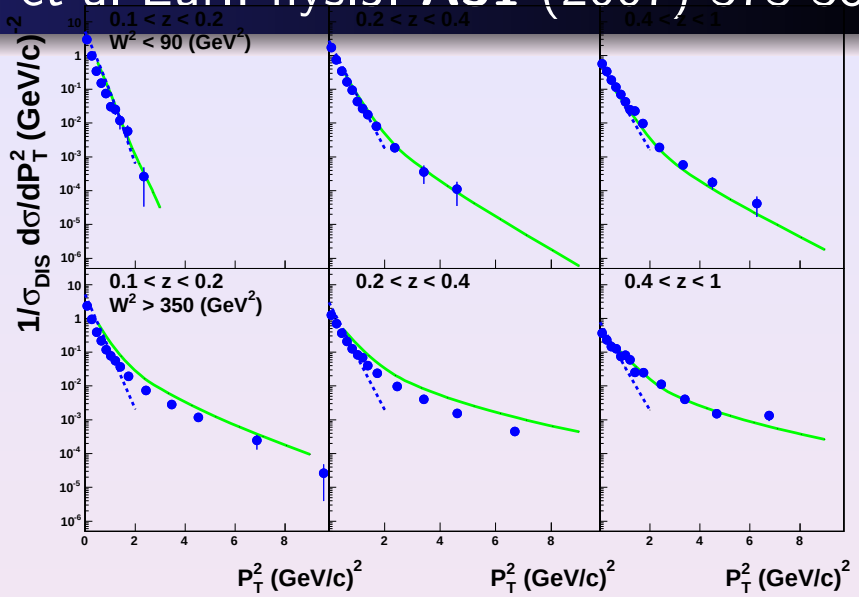
$$\cdot f_q(x) D_h^q(z) \frac{1}{\pi\langle P_T^2\rangle} e^{-\frac{P_T^2}{\langle P_T^2\rangle}} ,$$

$$\langle P_T^2\rangle = \langle p_\perp^2\rangle + z^2\langle k_\perp^2\rangle$$

$\langle p_\perp^2\rangle$ &$\langle k_\perp^2\rangle$ are essential ingredients for description of asymmetries in SIDIS. One must describe the data on unpolarized SIDIS before describing SSA.

# From low to high $P_T$ Anselmino et al Eur.Phys.J. **A31** (2007) 373-381.




QCD cross section is included:

$$d\sigma = \alpha_s^0\, d\sigma_0 + \alpha_s^1\, d\sigma_1 + \alpha_s^2\, d\sigma_2 \simeq$$
$$\alpha_s^0\, d\sigma_0 \quad + \quad K\alpha_s^1\, d\sigma_1$$

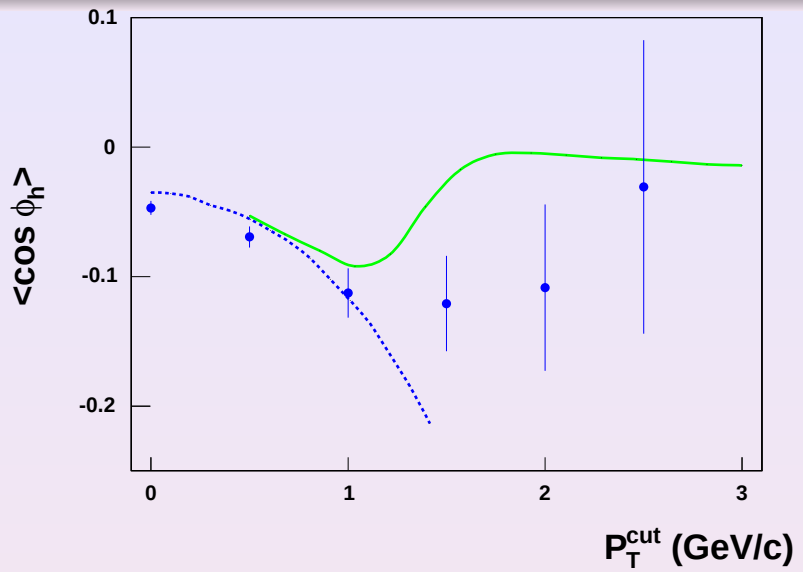
$K \simeq 5$, The data are from **EMC** at CERN.

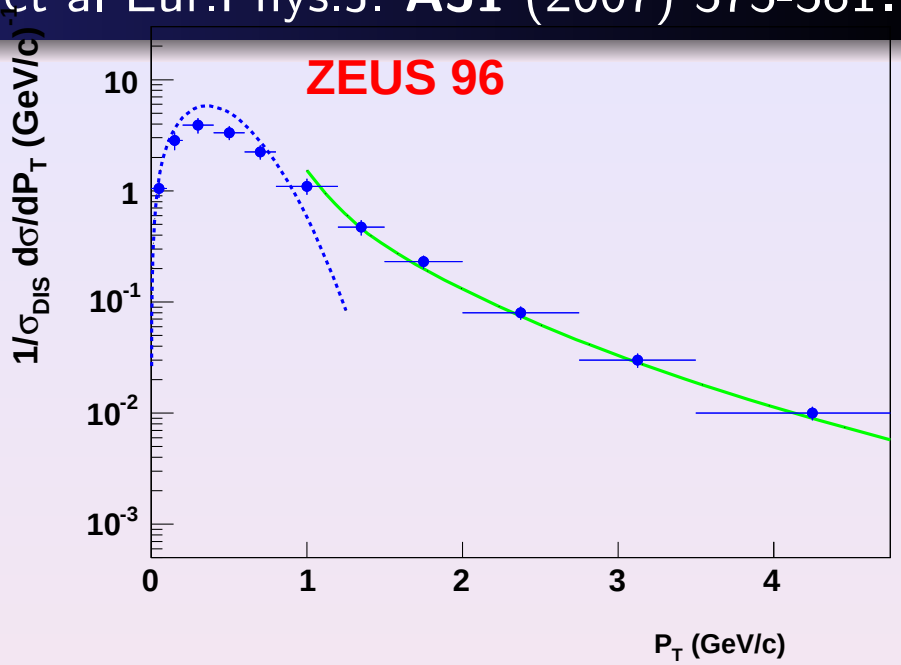

EMC Collaboration, *Z. Phys.* **C34** (1987) 277.

A. Daleo, D. de Florian and R. Sassot, *Phys. Rev.* **D71** (2005) 034013

B.A. Kniehl, G. Kramer, M. Maniatis, *Nucl. Phys.* **B711** (2005) 345.

# From low to high $P_T$ Anselmino et al Eur.Phys.J. **A31** (2007) 373-381.




QCD cross section is included:

$$d\sigma = \alpha_s^0\, d\sigma_0 + \alpha_s^1\, d\sigma_1 + \alpha_s^2\, d\sigma_2 \simeq$$
$$\alpha_s^0\, d\sigma_0 \quad + \quad K\alpha_s^1\, d\sigma_1$$

$K \simeq 1.5$, The data are from **ZEUS** and **E665**.

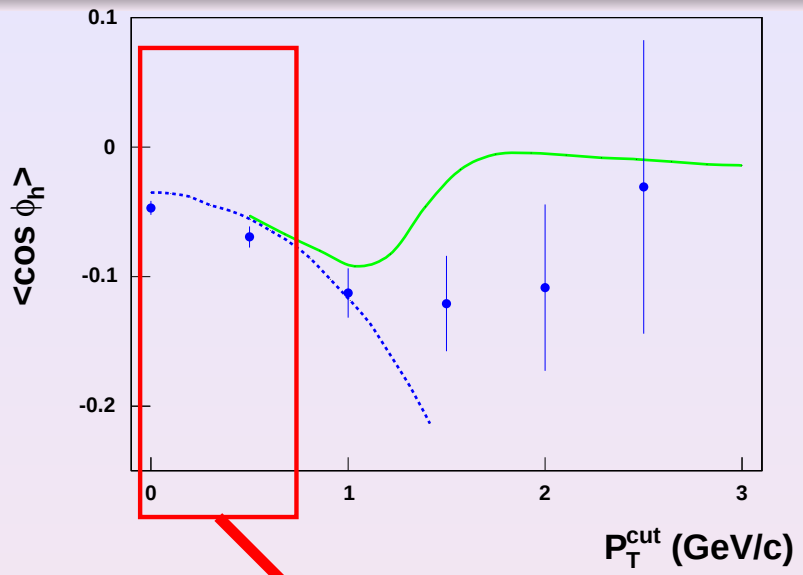

Fermilab E665 Collaboration, M.R. Adams *et al.*, *Phys. Rev.* **D48** (1993) 5057.

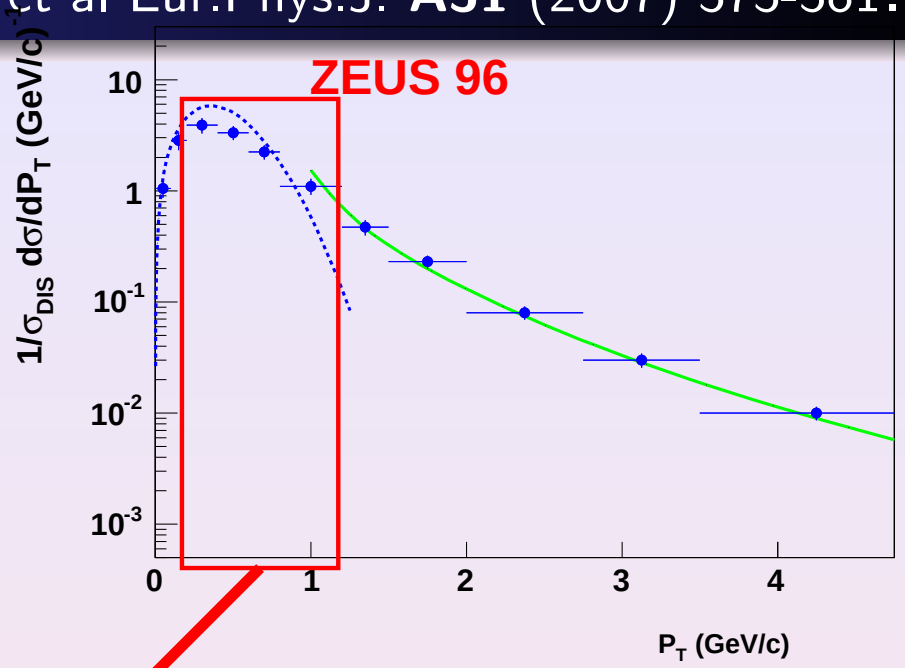
M. Derrick *et al*, ZEUS Collaboration, *Z. Phys.* **C70** (1996) 1.

A. Daleo, D. de Florian and R. Sassot, *Phys. Rev.* **D71** (2005) 034013

B.A. Kniehl, G. Kramer, M. Maniatis, *Nucl. Phys.* **B711** (2005) 345.

# From low to high $P_T$ Anselmino et al Eur.Phys.J. **A31** (2007) 373-381.



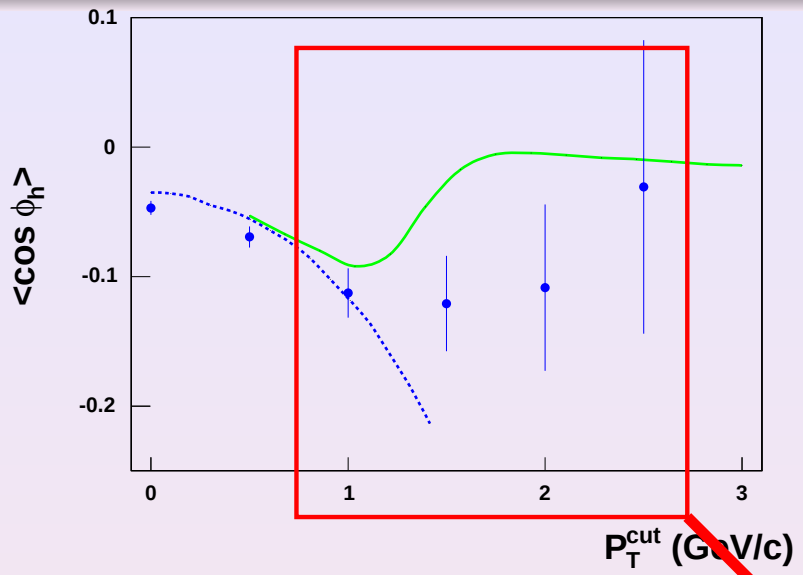

QCD cross section is included:

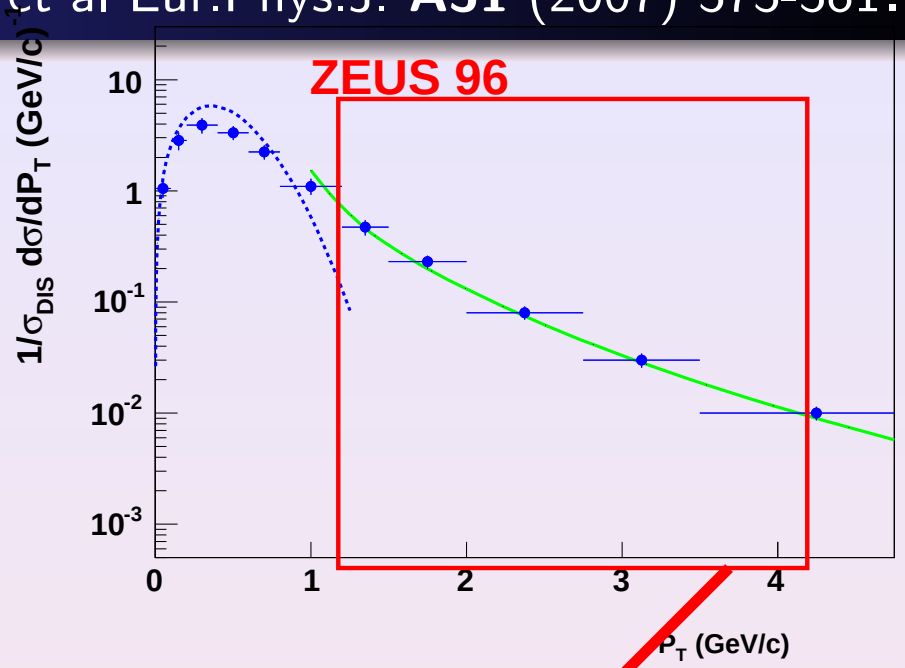
$$d\sigma = \alpha_s^0\, d\sigma_0 + \alpha_s^1\, d\sigma_1 + \alpha_s^2\, d\sigma_2 \simeq$$
$$\alpha_s^0\, d\sigma_0 \quad + \quad K\alpha_s^1\, d\sigma_1$$

$\langle k_\perp^2 \rangle = 0.25$ GeV$^2$ $\langle p_\perp^2 \rangle = 0.2$ GeV$^2$

in accordance with lattice calculation

# From low to high $P_T$ Anselmino et al Eur.Phys.J. **A31** (2007) 373-381.




QCD cross section is included:

$$d\sigma = \alpha_s^0\, d\sigma_0 + \alpha_s^1\, d\sigma_1 + \alpha_s^2\, d\sigma_2 \simeq$$
$$\alpha_s^0\, d\sigma_0 + K\alpha_s^1\, d\sigma_1$$

$\langle k_\perp^2 \rangle = 0.25$ GeV$^2$ $\langle p_\perp^2 \rangle = 0.2$ GeV$^2$

in accordance with lattice calculation

# The fundamental distributions of partons inside a nucleon

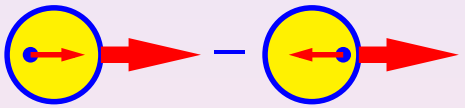
**Unpolarised Distribution**

$f_1(x)$ or $q(x)$

Distribution of unpolarised partons in an unpolarised nucleon.
Well known

**Helicity Distribution**

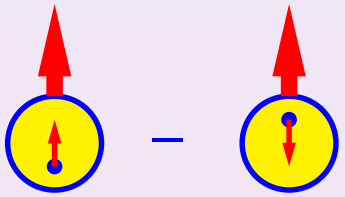
$g_1(x)$ or $\Delta q(x)$



Distribution of longitudinally polarised partons in a longitudinally polarised nucleon.
Known

**Transversity Distribution**

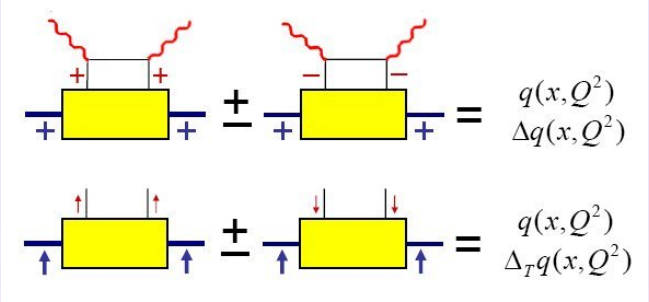
$h_1(x)$ or $\Delta_T q(x)$



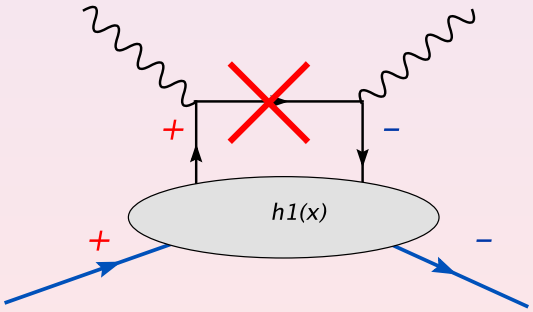
Distribution of transversely polarised quarks in a transversely polarised nucleon.
Little known!
HERMES and COMPASS experimental measurements

# TRANSVERSITY

Transversity and helicity distributions



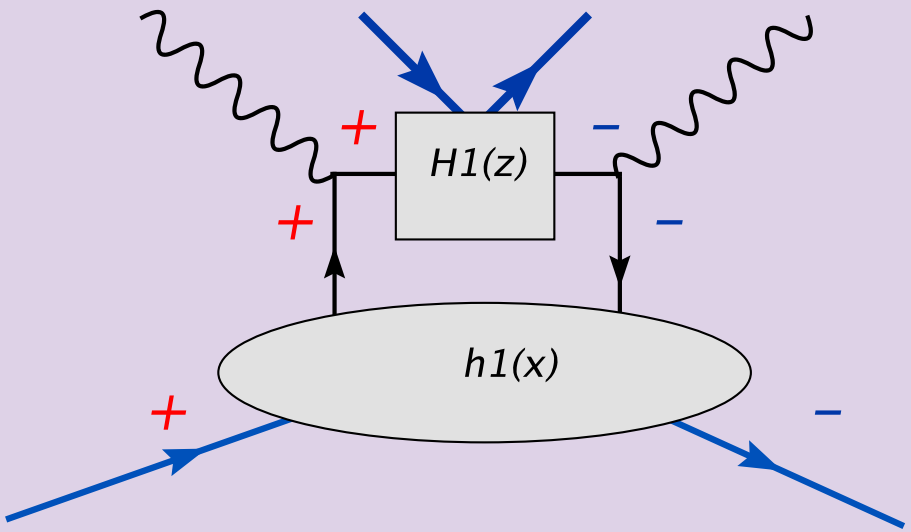

In helicity basis $|\uparrow(\downarrow)\rangle = \frac{1}{\sqrt{2}}(|+\rangle \pm i|-\rangle)$
Transversity cannot be studied in DIS as QED and QCD interactions conserve helicity up to corrections $\mathcal{O}(m_q/E)$.




Another chiral-odd function is needed to study transversity.

# Transversity in SIDIS

## Transversity in Semi inclusive Deep Inelastic Scattering $lN \to l'\pi X$




Transversely polarised quark fragments into an unpolarised hadron.

Asymmetry $= \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}} \sim sin(\phi_H + \phi_S) \cdot \Delta_T q(x, Q^2) \otimes \Delta^N D_{h/q^{\uparrow}}(z, Q^2)$

Transversity is convoluted with Collis FF.

Collins FF can be extracted from $e^+e^- \to \pi\pi X$.

Cleanest test would be DY in $\bar{p}p \to l^+l^-X$ at GSI $>$ 2016.

# Parton distribution functions

- Collinear Parton Distribution Functions:
  $f_{a/A}(x_a)$, $\Delta q(x_a)$, $\Delta_T q(x_a)$
- Transverse Momentum Dependent (TMD) PDFs
  1. Hadron $A$ unpolarized: $\hat{f}_{a/A}(x_a, k_{\perp a})$, $\underbrace{\Delta\hat{f}_{s_y/A}(x_a, \mathbf{k}_{\perp a})}_{\text{Boer-Mulders} \quad h_1^\perp}$
  2. Transversely polarised hadron $A^\uparrow$:
     $\underbrace{\Delta\hat{f}_{a/A^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{\text{Sivers} \quad f_{1T}^\perp}$, $\underbrace{\Delta\hat{f}^q_{s_x/A^\uparrow}(x_a, \mathbf{k}_{\perp a}), \Delta^-\hat{f}_{s_y/A^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{\text{Transversity} \quad h_1, h_{1T}^\perp}$,
     $\underbrace{\Delta\hat{f}_{s_z/A^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{g_{1T}^\perp}$
  3. Longitudinally polarised hadron $A^\rightarrow$:
     $\underbrace{\Delta\hat{f}_{s_z/+}(x_a, \mathbf{k}_{\perp a})}_{\Delta q}$, $\underbrace{\Delta\hat{f}_{s_x/+}(x_a, \mathbf{k}_{\perp a})}_{h_{1L}^\perp}$


M. Anselmino et al, Phys.Rev.D73:014020,2006

Mulders, Tangerman, Nucl.Phys.B461:197-237,1996

# Parton distribution functions

- Collinear Parton Distribution Functions:
  $f_{a/A}(x_a)$, $\Delta q(x_a)$, $\Delta_T q(x_a)$
- Transverse Momentum Dependent (TMD) PDFs
  1. Hadron $A$ unpolarized: $\hat{f}_{a/A}(x_a, k_{\perp a})$, $\underbrace{\Delta\hat{f}_{s_y/A}(x_a, \mathbf{k}_{\perp a})}_{\text{Boer-Mulders} \quad h_1^\perp}$
  2. Transversely polarised hadron $A^\uparrow$:
     $\underbrace{\Delta\hat{f}_{a/A^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{\text{Sivers} \quad f_{1T}^\perp}$, $\underbrace{\Delta\hat{f}^q_{s_x/A^\uparrow}(x_a, \mathbf{k}_{\perp a}), \Delta^-\hat{f}_{s_y/A^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{\text{Transversity} \quad h_1, h_{1T}^\perp}$,
     $\underbrace{\Delta\hat{f}_{s_z/A^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{g_{1T}^\perp}$
  3. Longitudinally polarised hadron $A^\rightarrow$:
     $\underbrace{\Delta\hat{f}_{s_z/+}(x_a, \mathbf{k}_{\perp a})}_{\Delta q}$, $\underbrace{\Delta\hat{f}_{s_x/+}(x_a, \mathbf{k}_{\perp a})}_{h_{1L}^\perp}$


M. Anselmino et al, Phys.Rev.D73:014020,2006

Mulders, Tangerman, Nucl.Phys.B461:197-237,1996

# Parton distribution functions

- Collinear Parton Distribution Functions:
  $f_{a/A}(x_a)$, $\Delta q(x_a)$, $\Delta_T q(x_a)$
- Transverse Momentum Dependent (TMD) PDFs
  1. Hadron $A$ unpolarized: $\hat{f}_{a/A}(x_a, k_{\perp a})$, $\underbrace{\Delta\hat{f}_{s_y/A}(x_a, \mathbf{k}_{\perp a})}_{\text{Boer-Mulders} \quad h_1^\perp}$
  2. Transversely polarised hadron $A^\uparrow$:
     $\underbrace{\Delta\hat{f}_{a/A^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{\text{Sivers} \quad f_{1T}^\perp}$, $\underbrace{\Delta\hat{f}^q_{s_x/A^\uparrow}(x_a, \mathbf{k}_{\perp a}),\ \Delta^-\hat{f}_{s_y/A^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{\text{Transversity} \quad h_1,\, h_{1T}^\perp}$,
     $\underbrace{\Delta\hat{f}_{s_z/A^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{g_{1T}^\perp}$
  3. Longitudinally polarised hadron $A^\rightarrow$:
     $\underbrace{\Delta\hat{f}_{s_z/+}(x_a, \mathbf{k}_{\perp a})}_{\Delta q}$, $\underbrace{\Delta\hat{f}_{s_x/+}(x_a, \mathbf{k}_{\perp a})}_{h_{1L}^\perp}$


M. Anselmino et al, Phys.Rev.D73:014020,2006

Mulders, Tangerman, Nucl.Phys.B461:197-237,1996

# Parton distribution functions

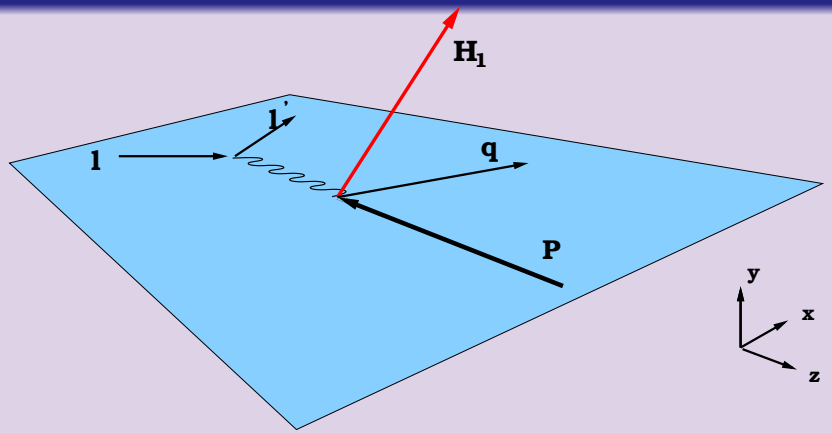
- Collinear Parton Distribution Functions:
  $f_{a/A}(x_a)$, $\Delta q(x_a)$, $\Delta_T q(x_a)$
- Transverse Momentum Dependent (TMD) PDFs
  1. Hadron $A$ unpolarized: $\hat{f}_{a/A}(x_a, k_{\perp a})$, $\underbrace{\Delta\hat{f}_{s_y/A}(x_a, \mathbf{k}_{\perp a})}_{\text{Boer-Mulders} \quad h_1^\perp}$
  2. Transversely polarised hadron $A^\uparrow$:
     $\underbrace{\Delta\hat{f}_{a/A^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{\text{Sivers} \quad f_{1T}^\perp}$, $\underbrace{\Delta\hat{f}^q_{s_x/A^\uparrow}(x_a, \mathbf{k}_{\perp a}),\ \Delta^-\hat{f}_{s_y/A^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{\text{Transversity} \quad h_1,\, h_{1T}^\perp}$,
     $\underbrace{\Delta\hat{f}_{s_z/A^\uparrow}(x_a, \mathbf{k}_{\perp a})}_{g_{1T}^\perp}$
  3. Longitudinally polarised hadron $A^\rightarrow$:
     $\underbrace{\Delta\hat{f}_{s_z/+}(x_a, \mathbf{k}_{\perp a})}_{\Delta q}$, $\underbrace{\Delta\hat{f}_{s_x/+}(x_a, \mathbf{k}_{\perp a})}_{h_{1L}^\perp}$


M. Anselmino et al, Phys.Rev.D73:014020,2006

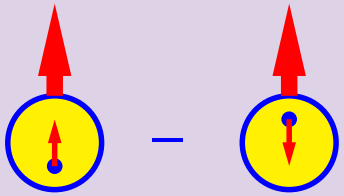
Mulders, Tangerman, Nucl.Phys.B461:197-237,1996

# SIDIS and $e^+e^-$ annihilation

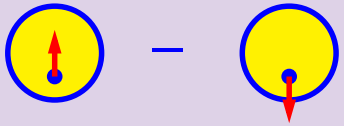
## SIDIS $lN \to l' H_1 X$




Collins effect gives rise to azimuthal Single Spin Asymmetry



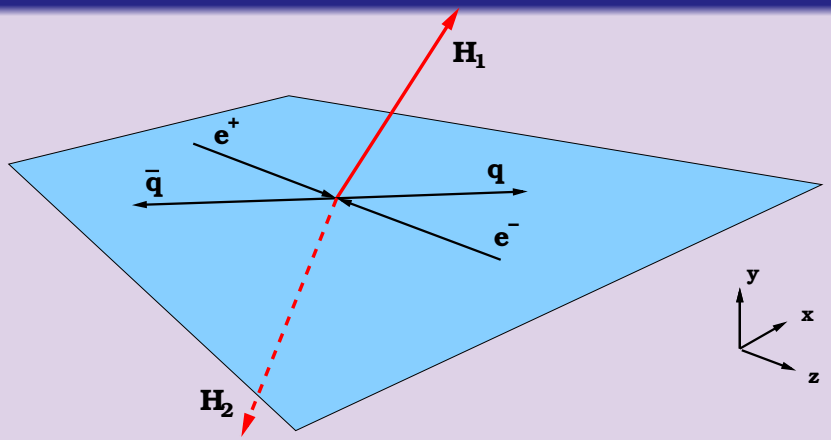
$$= \Delta_T q(x, Q^2)$$



$$= \Delta^N D_{h/q^\uparrow}(z, Q^2)$$

J. C. Collins, *Nucl. Phys.* **B396** (1993) 161

## $e^+e^- \to H_1 H_2 X$




Collins effect gives rise to azimuthal asymmetry, $q$ and $\bar{q}$ Collins functions are present in the process:

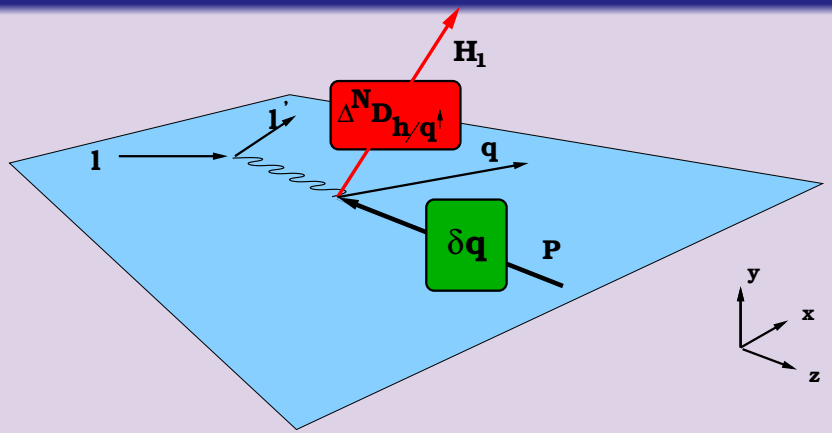
$\Delta^N D_{h/q^\uparrow}(z_1, Q^2)$

$\Delta^N D_{h/\bar{q}^\uparrow}(z_2, Q^2)$

D. Boer, R.Jacob and P. J. Mulders *Nucl. Phys.* **B504** (1997) 345

# SIDIS and $e^+e^-$ annihilation

## SIDIS $lN \to l' H_1 X$

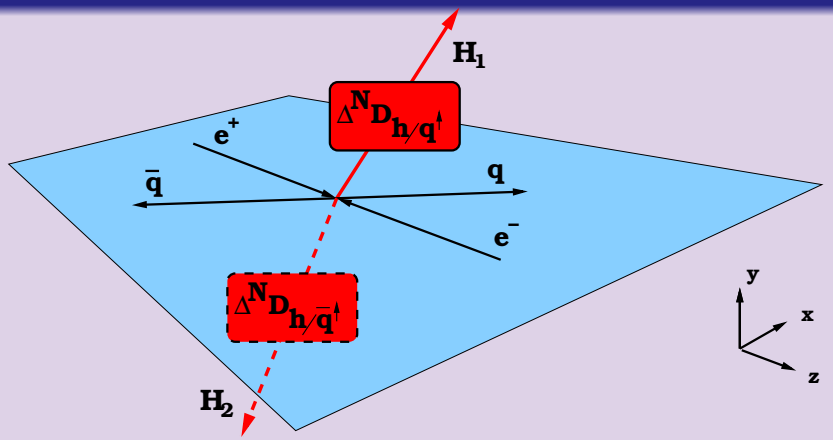



Cross Section $\sim sin(\phi_H + \phi_S) \cdot$
$\Delta_T q(x, Q^2) \otimes \Delta^N D_{h/q^\uparrow}(z, Q^2)$

## We extract (PRD75:054032,2007)

$\Delta_T q(x, Q^2)$ , $\Delta^N D_{h/q^\uparrow}(z, Q^2)$

## $e^+e^- \to H_1 H_2 X$




Cross Section $\sim cos(\phi_{H_1} + \phi_{H_2}) \cdot$
$\Delta^N D_{h/q^\uparrow}(z_1) \otimes \Delta^N D_{h/\bar{q}^\uparrow}(z_2)$

## We extract (PRD75:054032,2007)

$\Delta^N D_{h/q^\uparrow}(z_1, Q^2)$ ,
$\Delta^N D_{h/\bar{q}^\uparrow}(z_2, Q^2)$

# Collins function and transversity distribution

## Model for Collins FF

For $\Delta^N D_{h/q^\uparrow}(z, |p_\perp|) = \frac{2|p_\perp|}{zM_\pi} H_1^{\perp q}(z, |p_\perp|)$ we use factorized $z$ and $p_\perp$ and Gaussian dependence on $p_\perp$, $\Delta^N D_{h/q^\uparrow} \propto z^\gamma (1-z)^\delta$, positivity constraint $|\Delta^N D_{h/q^\uparrow}(z, \mathbf{p}_\perp)| \leq 2D_{h/q}(z, \mathbf{p}_\perp)$ is fulfilled.

## Model for Transversity distribution

$$\Delta_T q(x, k_\perp) = \frac{1}{2}\left[f_{q/p}(x) + \Delta q(x)\right] \mathcal{N}_q^T(x) \frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_T}}{\pi\langle k_\perp^2\rangle_T},$$

$\Delta_T q(x) \propto x^\alpha (1-x)^\beta$, Soffer bound

$$|\Delta_T q(x)| \leq \frac{1}{2}\left[f_{q/p}(x) + \Delta q(x)\right]$$

is fulfilled.

# Description of $A_{UT}^{sin(\phi_h+\phi_S)}$

We use HERMES and COMPASS data sets on $A_{UT}^{sin(\phi_h+\phi_S)}$ and BELLE data in the fitting procedure.
Favored and unfavored fragmentation functions are defined as follows:

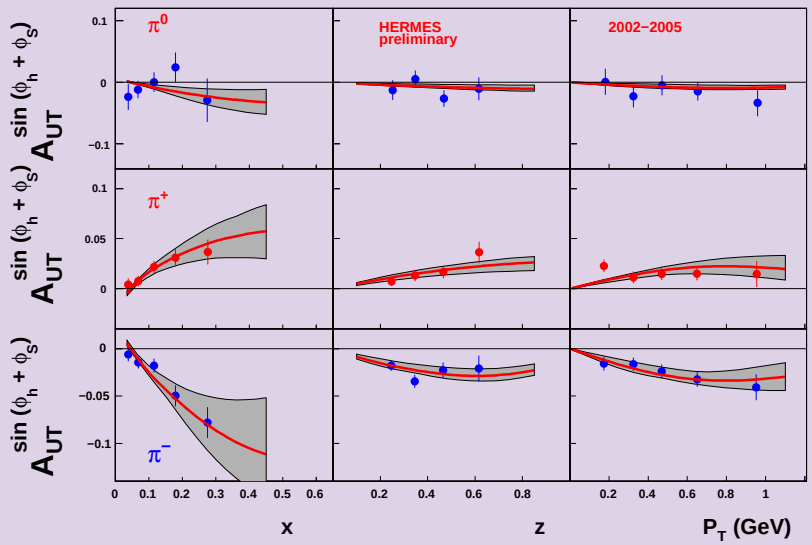
$$D^{fav}(z) \equiv D^{u\to\pi^+}(z) = D^{d\to\pi^-}(z) = D^{\bar{u}\to\pi^-}(z) = D^{\bar{d}\to\pi^+}(z)$$
$$D^{unfav}(z) \equiv D^{u\to\pi^-}(z) = D^{d\to\pi^+}(z) = D^{\bar{u}\to\pi^+}(z) = D^{\bar{d}\to\pi^-}(z)$$

For simplicity we assume that Collins FFs have universal $z$ behaviour and transversity for $u$ and $d$ quarks have universal $x$ behaviour:

$$\alpha_u = \alpha_d \equiv \alpha, \beta_u = \beta_d \equiv \beta$$
$$\gamma_{fav} = \gamma_{unfav} \equiv \gamma, \delta_{fav} = \delta_{unfav} \equiv \delta$$

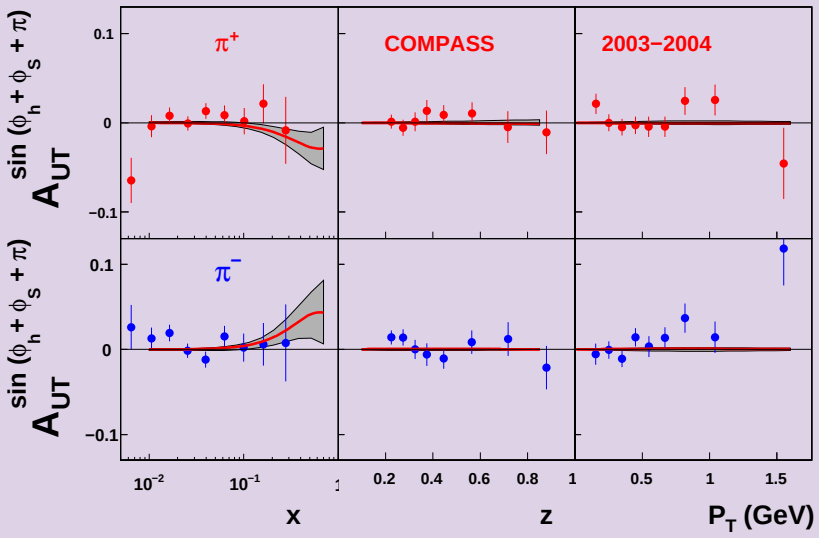
# Description of the data

## HERMES $A_{UT}^{sin(\phi_h+\phi_S)}$




$ep \to e\pi X$, $p_{lab} = 27.57$ GeV.

## COMPASS $A_{UT}^{sin(\phi_h+\phi_S+\pi)}$




$\mu D \to \mu\pi X$, $p_{lab} = 160$ GeV

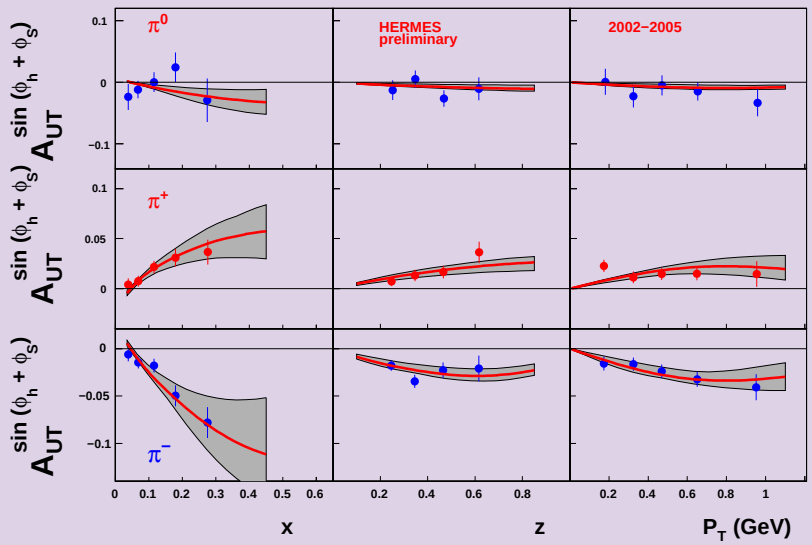

M.Anselmino et al, Nucl.Phys.Proc.Suppl.191:98-107,2009

HERMES, M. Diefenthaler, (2007),arXiv:0706.2242

COMPASS, M. Alekseev et al., (2008), Phys.Lett.B673:127-135,2009

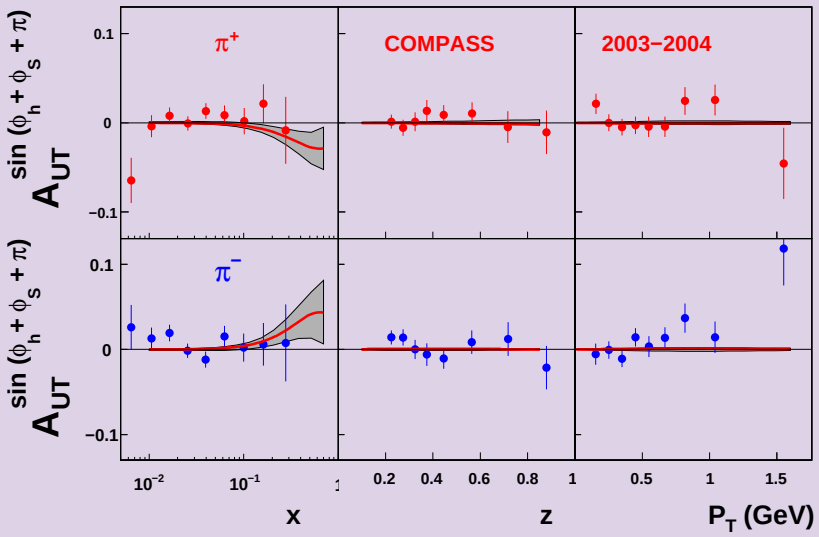
# Description of the data

## HERMES $A_{UT}^{sin(\phi_h+\phi_S)}$




$ep \to e\pi X$, $p_{lab} = 27.57$ GeV.

## COMPASS $A_{UT}^{sin(\phi_h+\phi_S+\pi)}$



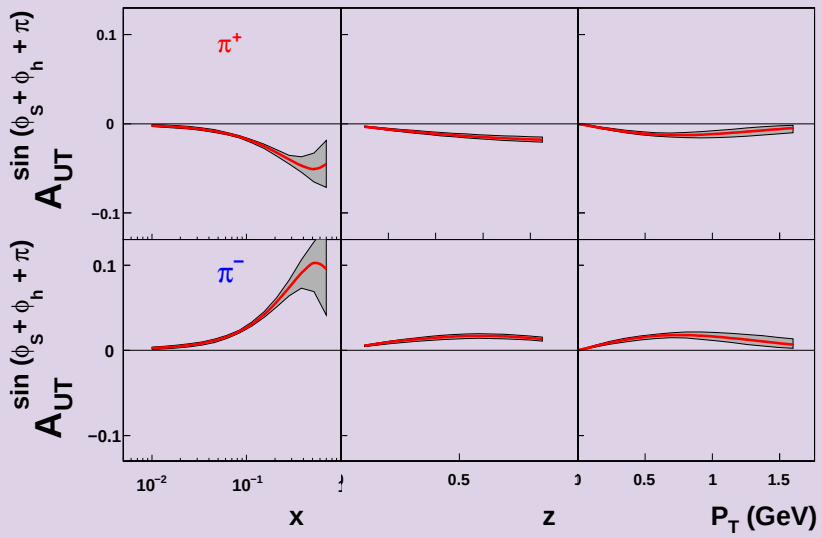

$\mu D \to \mu\pi X$, $p_{lab} = 160$ GeV

$$lp^\uparrow \to l\pi^+X \simeq \Delta_T u \otimes \Delta^N D_{u/\pi^+} > 0$$
$$lp^\uparrow \to l\pi^-X \simeq \Delta_T u \otimes \Delta^N D_{u/\pi^-} < 0$$
$$lD^\uparrow \to l\pi^+X \simeq (\Delta_T u + \Delta_T d) \otimes \Delta^N D_{u/\pi^+} \simeq 0$$

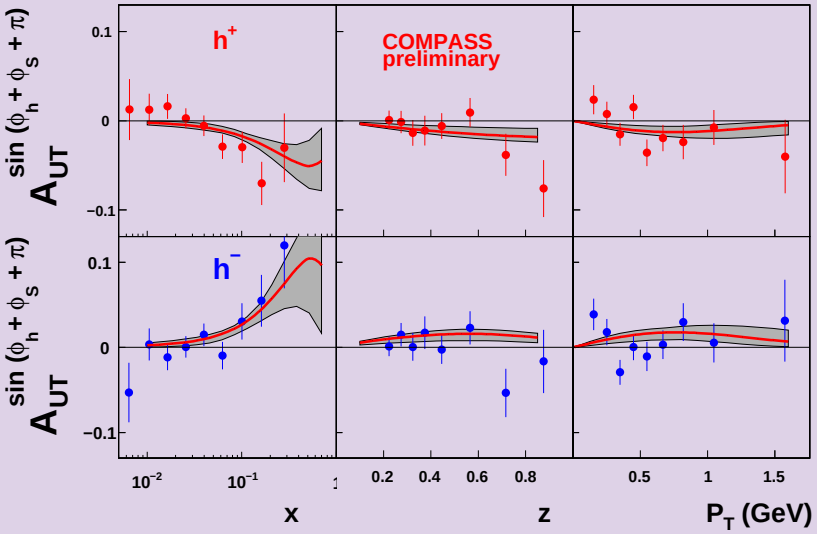
# Description of the data

Predictions for COMPASS operating on PROTON target

## COMPASS $A_{UT}^{sin(\phi_h+\phi_S+\pi)}$




## COMPASS $A_{UT}^{sin(\phi_h+\phi_S+\pi)}$

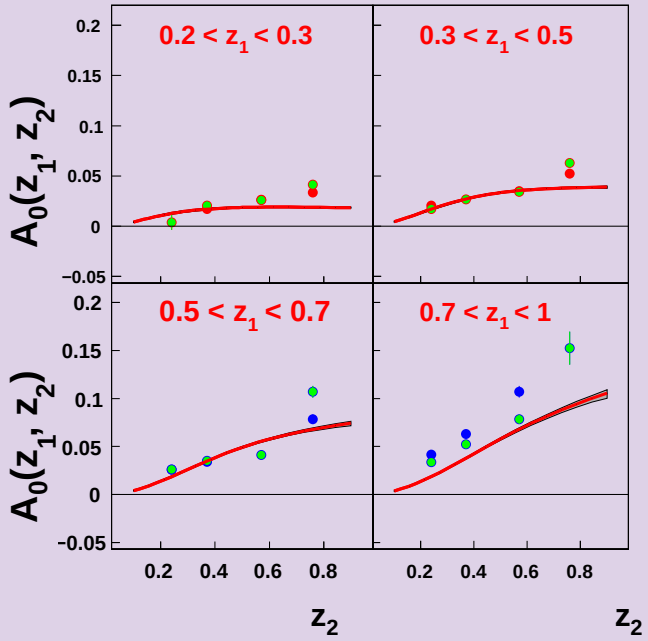



Comparison with preliminary COMPASS data arXiv:0808.0086

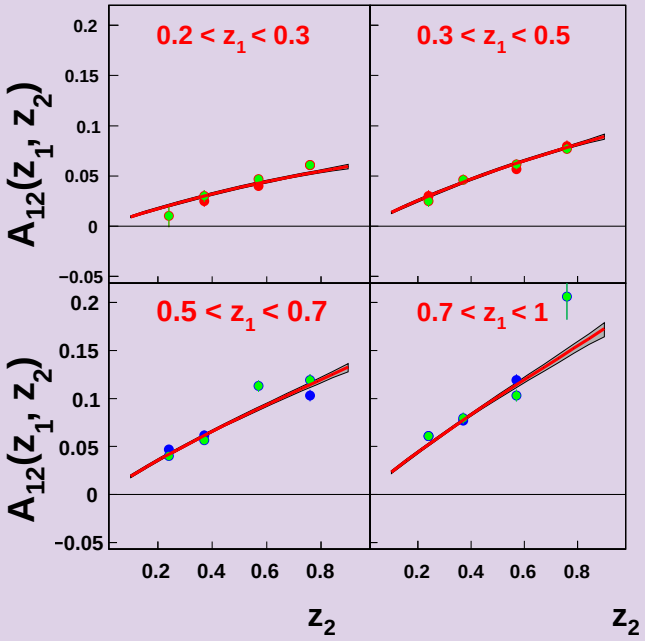
M.Anselmino et al, Nucl.Phys.Proc.Suppl.191:98-107,2009

# Description of the data $e^+e^-$

## BELLE $\cos(2\varphi_0)$



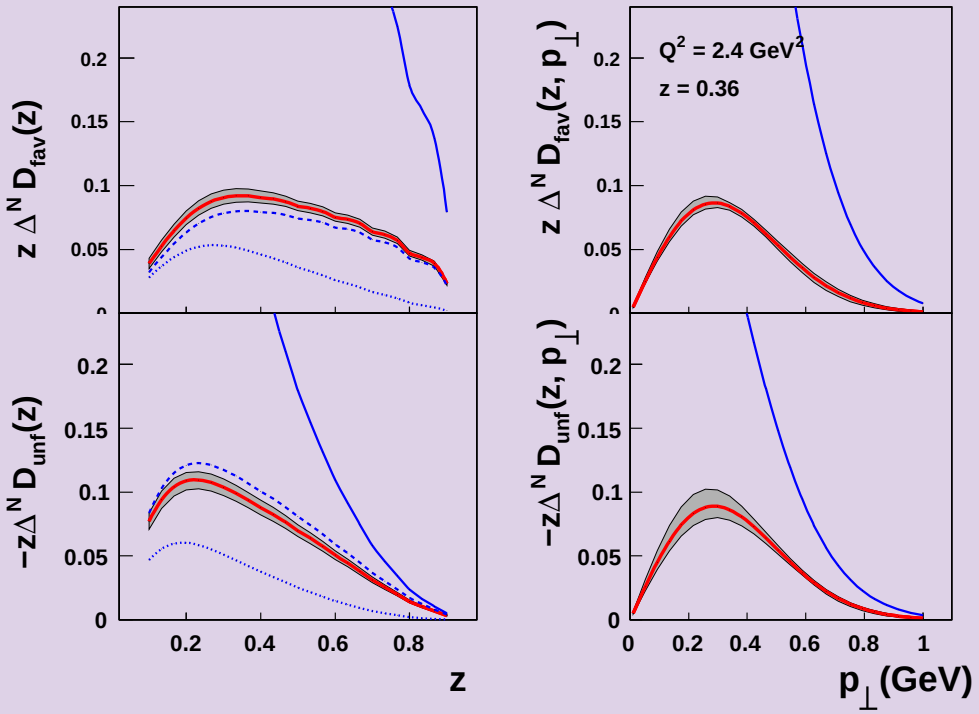

## BELLE $\cos(\varphi_1 + \varphi_2)$




$e^+e^- \to \pi\pi X$, $\sqrt{s} = 10.58$ GeV

Belle, K. Abe et al., Phys. Rev. Lett. 96 (2006) 232002

# Collins fragmentation function

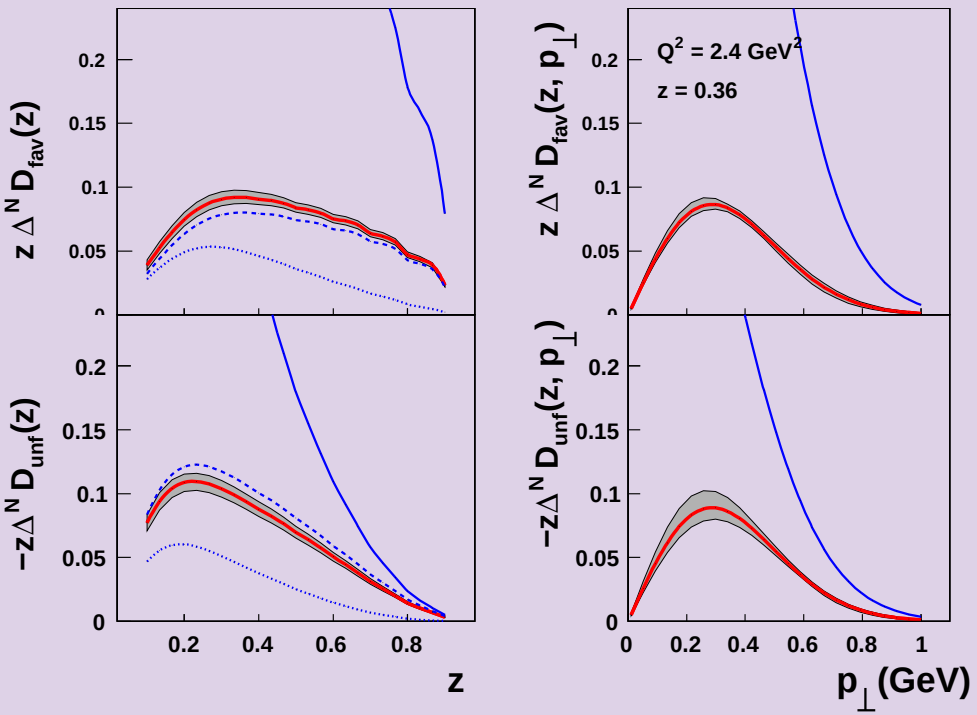



compared to Ref. [1] (dashed line), Ref. [2] (dotted line)


[1] A. V. Efremov, K. Goeke, and P. Schweitzer, Phys. Rev. **D73**, 094025 (2006).

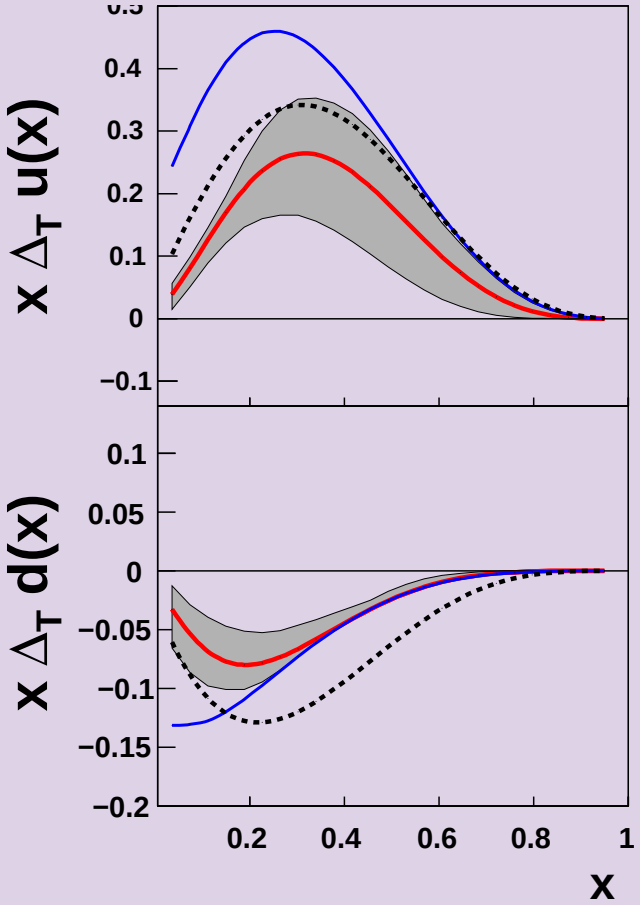
[2] W. Vogelsang and F. Yuan, Phys. Rev. **D72**, 054028 (2005).

# Collins fragmentation function




$$\Delta^N D_{fav} = \Delta^N D_{u/\pi^+} > 0$$
$$\Delta^N D_{unfav} = \Delta^N D_{u/\pi^-} < 0$$
$$|\Delta^N D_{fav}| \simeq |\Delta^N D_{unfav}|$$

# Transversity vs. helicity




1. Solid red line – transversity distribution

$$\Delta_T q(x)$$

this analysis at $Q^2 = 2.4$ GeV$^2$.

2. Solid blue line – Soffer bound

$$\frac{q(x) + \Delta q(x)}{2}$$

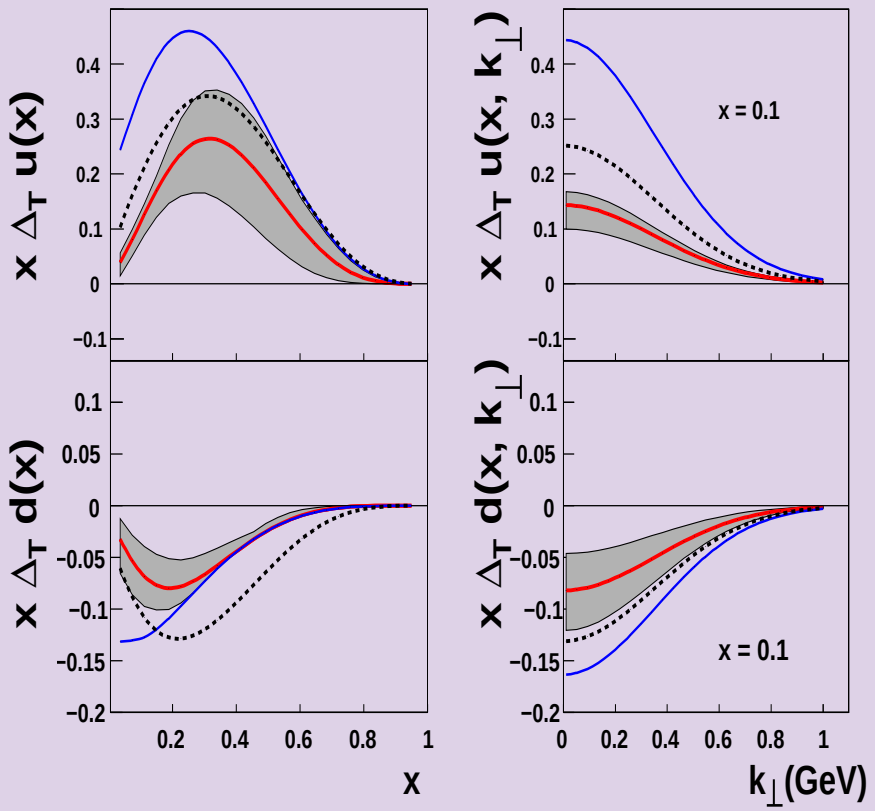
GRV98LO + GRSV98LO

3. Dashed line – helicity distribution

$$\Delta q(x)$$

GRSV98LO

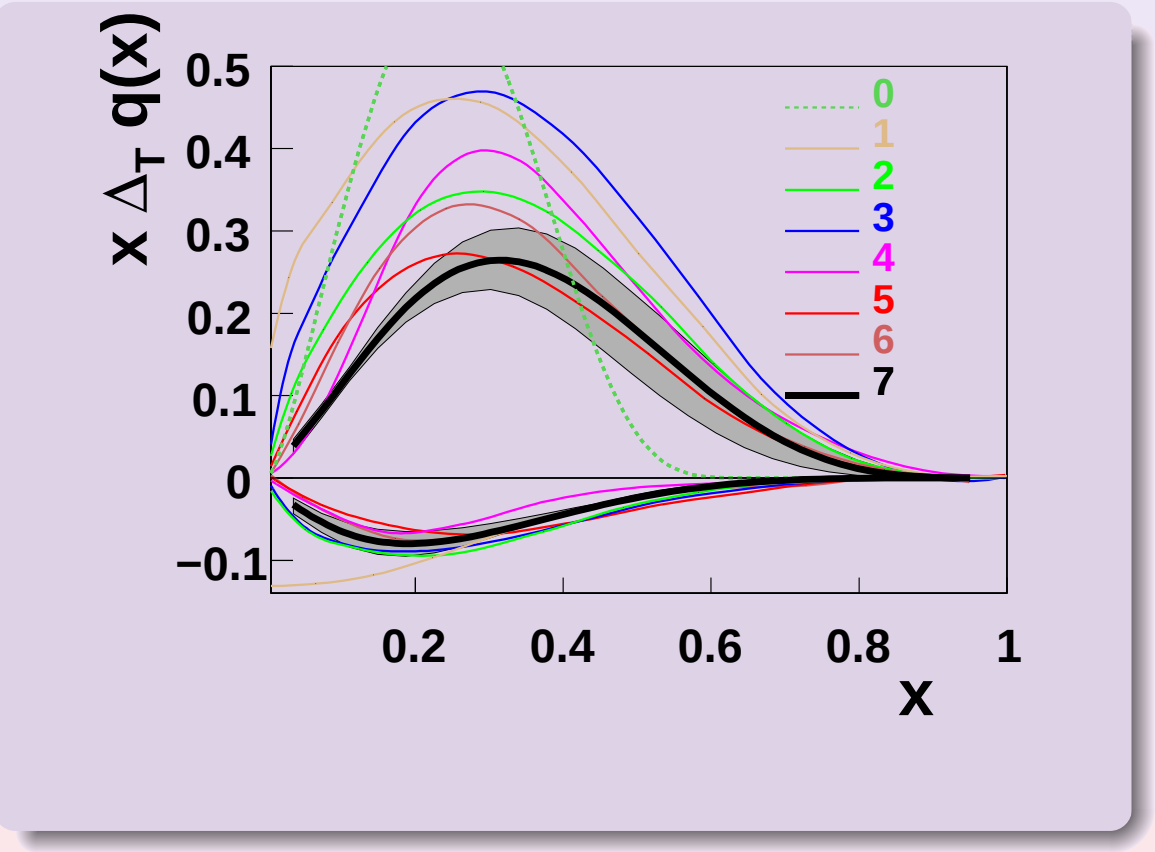
# Transversity




- This is the extraction of transversity from new experimental data. PRD75:054032,2007, arXiv:0812.4366
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- $|\Delta_T q(x)| < |\Delta q(x)|$.

# Transversity, comparison with models

New extraction is close to most models.

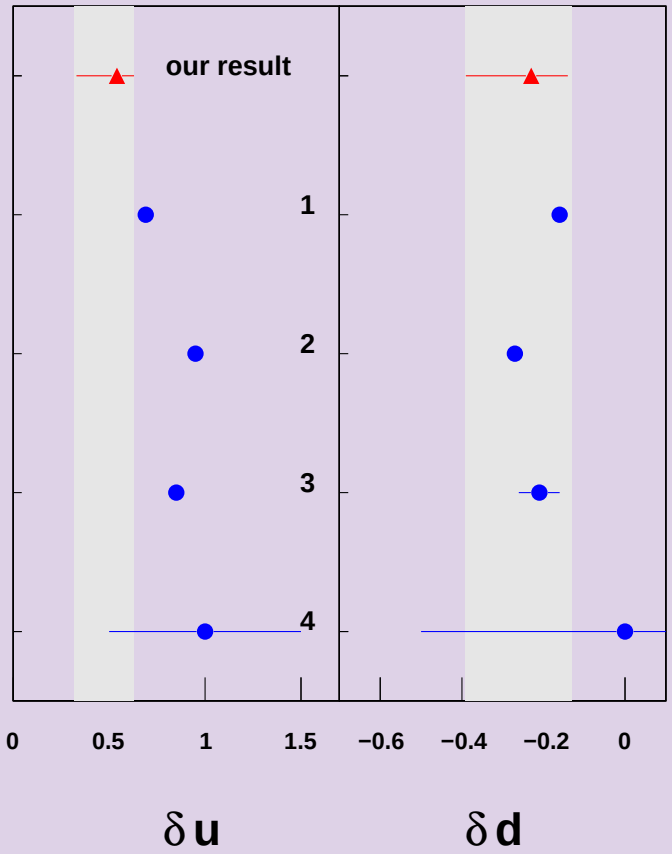



0. Barone, Calarco, Drago PLB 390 287 (97)
1. Soffer et al. PRD 65 (02)
2. Korotkov et al. EPJC 18 (01)
3. Schweitzer et al. PRD 64 (01)
4. Wakamatsu, PLB B653 (07)
5. Pasquini et al., PRD 72 (05)
6. Cloet, Bentz and Thomas PLB 659 (08)
7. This analysis.

# Tensor charges

$$\delta q = \int_0^1 dx\,(\Delta_T q - \Delta_T \bar{q}) = \int_0^1 dx\,\Delta_T q$$

$$\Delta_T u = 0.54^{+0.09}_{-0.22},\ \Delta_T d = -0.23^{+0.09}_{-0.16} \text{ at } Q^2 = 0.8 \text{ GeV}^2$$




1. Quark-diquark model:
   Cloet, Bentz and Thomas
   PLB **659**, 214 (2008), $Q^2 = 0.4$ GeV$^2$
2. CQSM:
   M. Wakamatsu, PLB B **653** (2007) 398
   $Q^2 = 0.3$ GeV$^2$
3. Lattice QCD:
   M. Gockeler et al.,
   Phys.Lett.B627:113-123,2005 , $Q^2$ =
   GeV$^2$
4. QCD sum rules:
   Han-xin He, Xiang-Dong Ji,
   PRD 52:2960-2963,1995, $Q^2 \sim 1$ GeV$^2$

# Sivers effect

The azimuthal asymmetry $A_{UT}^{sin(\phi_h-\phi_S)}$ arises due to Sivers function

$$f_{q/p^\uparrow}(x, \mathbf{k}_\perp) \quad = \quad f_{q/p}(x, k_\perp) + \frac{1}{2}\,\Delta^N f_{q/p^\uparrow}(x, k_\perp)\,\mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \hat{\mathbf{k}}_\perp)$$

Asymmetry $\sim sin(\phi_H - \phi_S) \cdot \Delta^N f_{q/p^\uparrow}(x, Q^2) \otimes D_{h/q}(z, Q^2)$

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + < L_z^{q,\bar{q}} > + < L_z^G >$$

EMC result on $\Delta\Sigma = \sum_{q,\bar{q}} \Delta q \simeq 0.3$ triggered so called "Spin crisis"

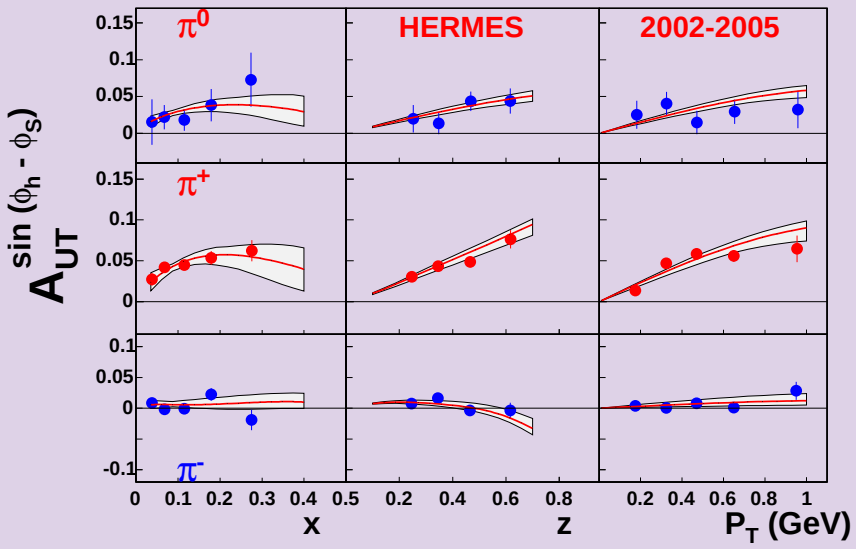
Leader, Anselmino ''A Crisis In The Parton Model: Where, Oh Where Is The Proton's Spin?'' Z.Phys.C41:239,1988

Correlation between spin and angular momentum implies non zero contribution $< L_z^{q,\bar{q}} > \neq 0$

# HERMES and COMPASS DATA

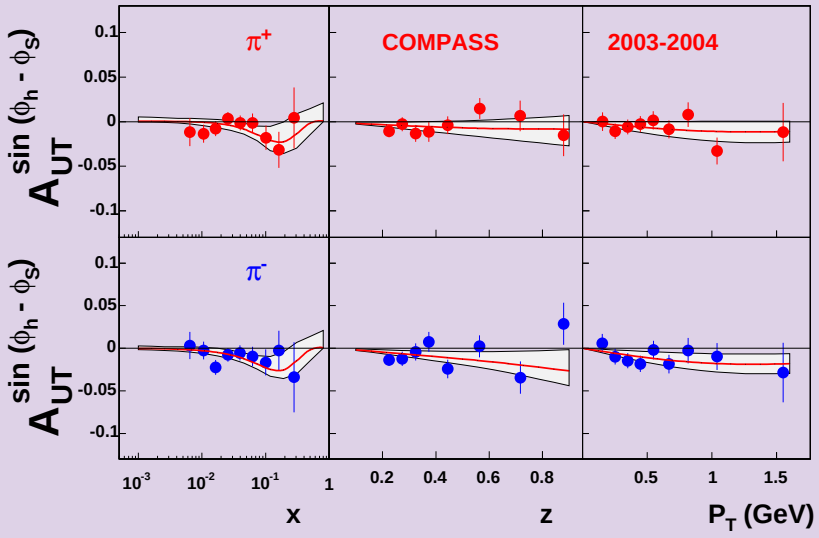
## HERMES

$ep \to e\pi X$, $p_{lab} = 27.57$ GeV.




## COMPASS

$\mu D \to \mu\pi X$, $p_{lab} = 160$ GeV.

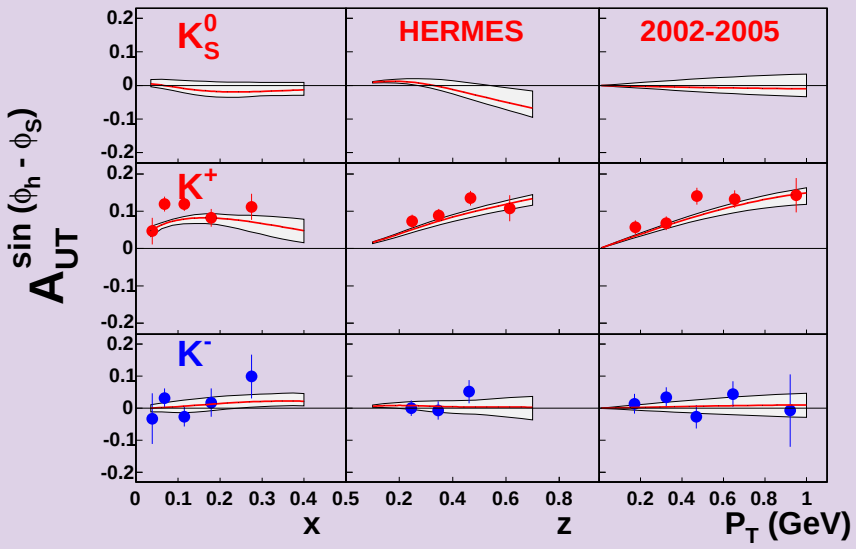



$$lp^{\uparrow} \to l\pi^{+}X \simeq \Delta^{N}u \otimes D_{u/\pi^{+}} > 0$$
$$lp^{\uparrow} \to l\pi^{-}X \simeq 4\Delta^{N}u \otimes D_{u/\pi^{-}} + \Delta^{N}d \otimes D_{d/\pi^{-}} \simeq 0$$
$$lD^{\uparrow} \to l\pi^{+}X \simeq (\Delta^{N}u + \Delta^{N}d) \otimes D_{u/\pi^{+}} \simeq 0$$

# HERMES and COMPASS DATA

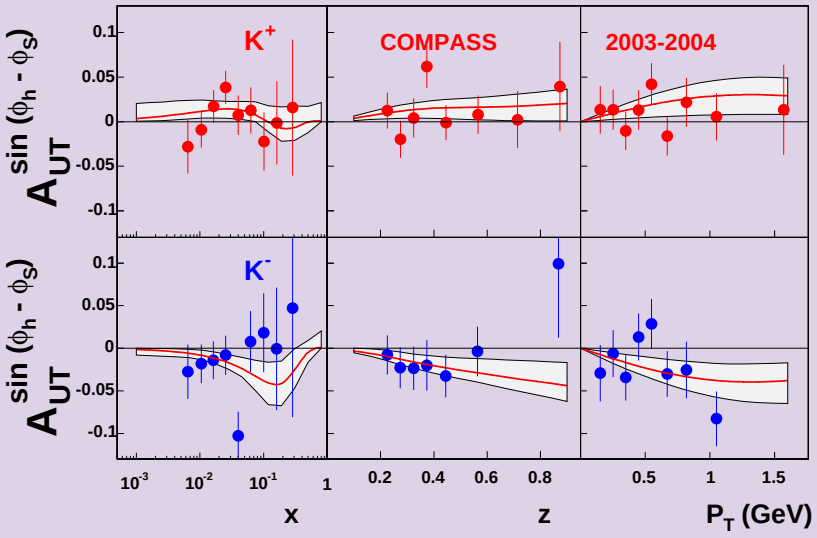
## HERMES

$ep \to eKX$, $p_{lab} = 27.57$ GeV.




## COMPASS

$\mu D \to \mu KX$, $p_{lab} = 160$ GeV.




Kaon data allow for light antiquark Sivers function extraction.

M. Anselmino et al Phys.Rev.D79:054010,2009

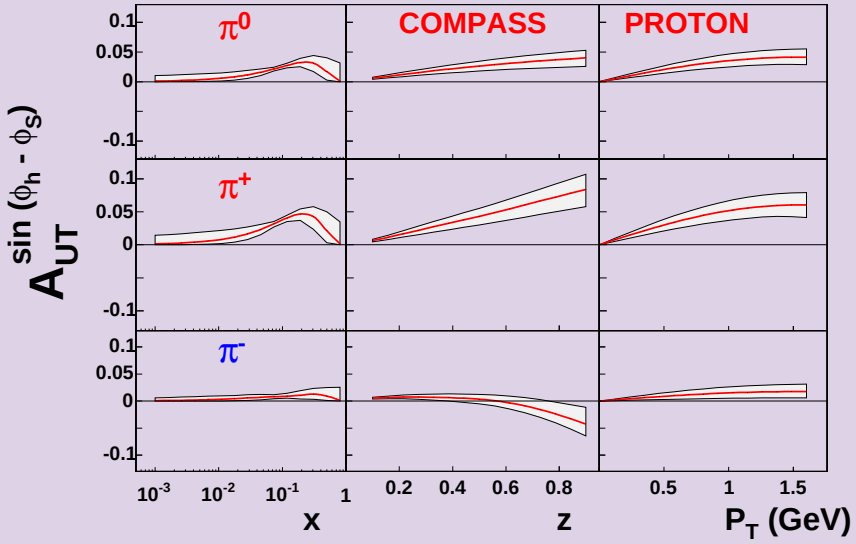
Kaon FF

De Florian D., Sassot R., and Stratmann M. Phys. Rev. **D75** 114010 (2007)

# PREDICTIONS

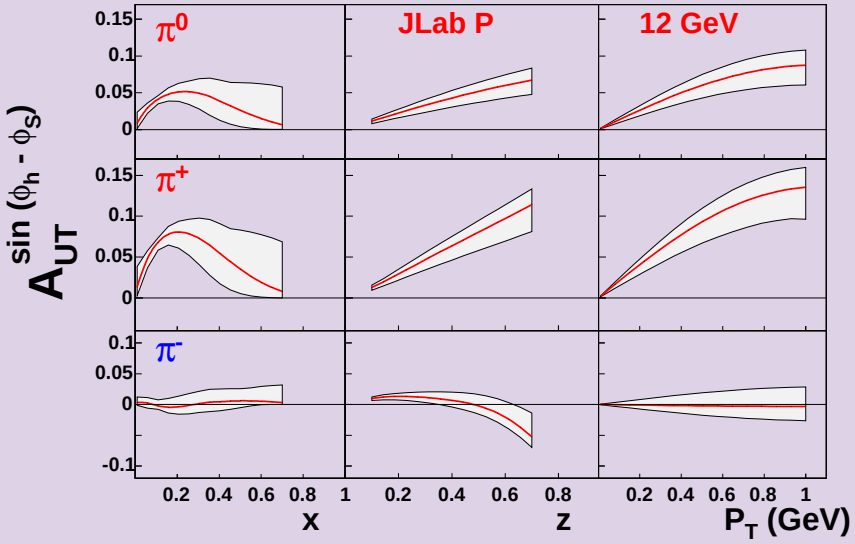
## COMPASS on PROTON

$\mu p \to \mu \pi X$, $p_{lab} = 160$ GeV.




## JLAB12

$ep \to e\pi X$, $p_{lab} = 12$ GeV.

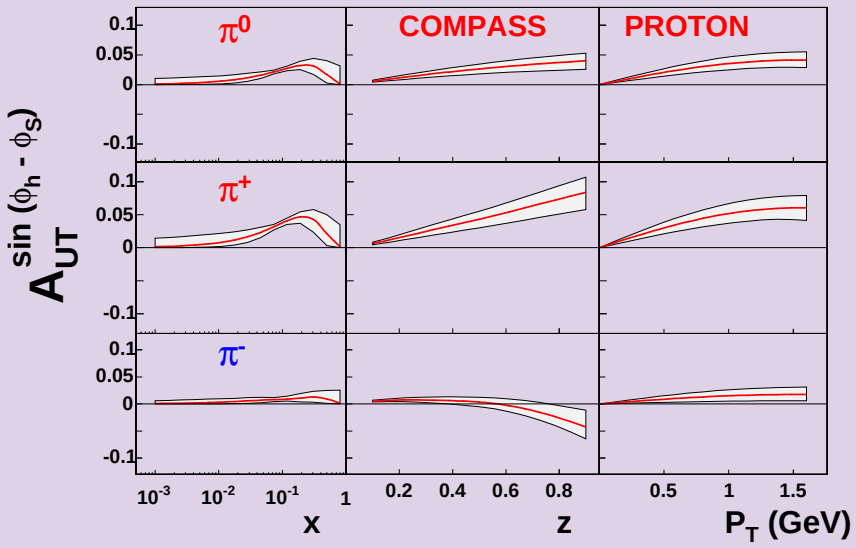



JLab can improve our knowledge of Sivers function in high $x$ region. COMPASS operating on proton target is expected to measure 5% asymmetry for $h^+$.

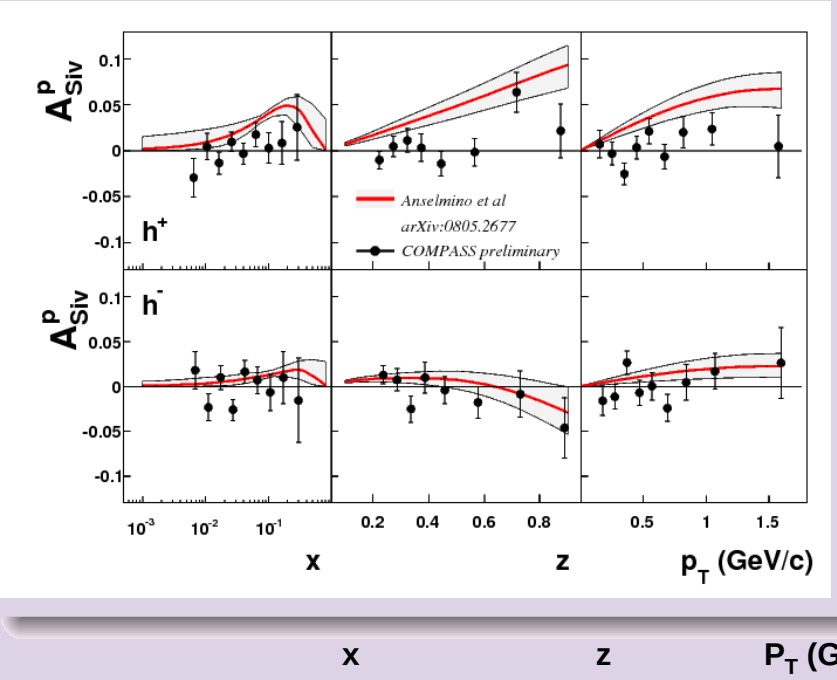
# PREDICTIONS

## COMPASS on PROTON

$\mu p \to \mu \pi X$, $p_{lab} = 160$ GeV.




## Comparison with preliminary COMPASS data arXiv:0808.0086

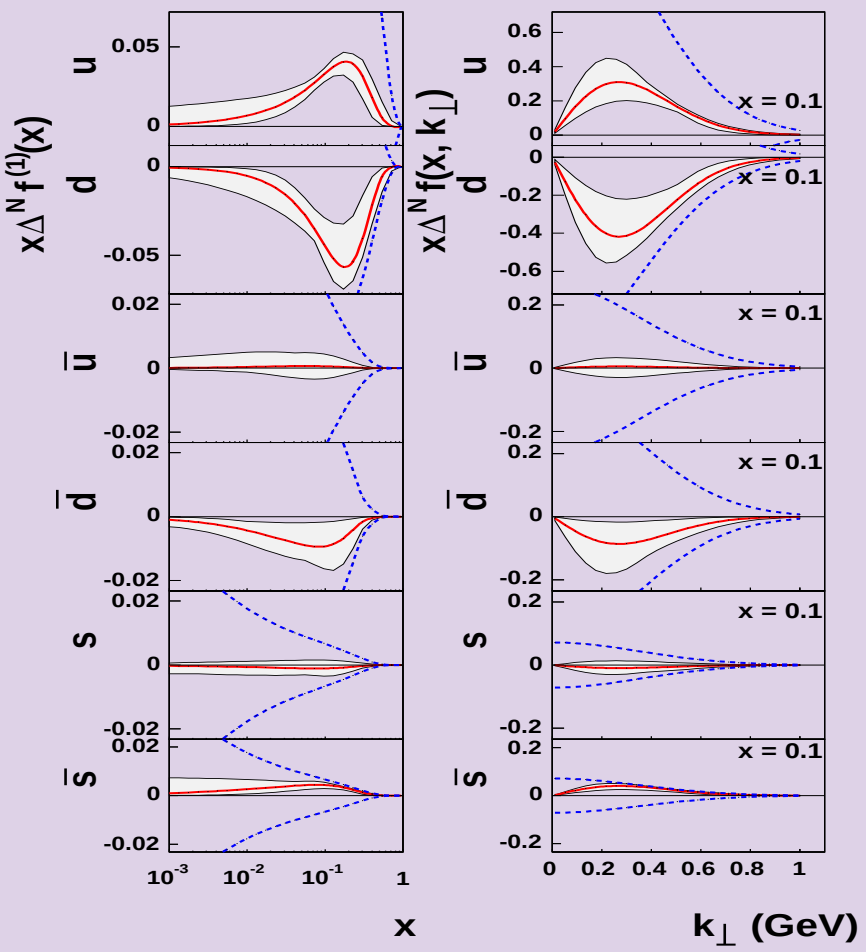



x z $P_T$ (GeV)

COMPASS data on proton target need to be explained as predictions are not supported by the preliminary data.

# Sivers functions

$$\Delta^N f_q^{(1)}(x) \equiv \int d^2\,\mathbf{k}_\perp \, \frac{k_\perp}{4m_p} \, \Delta^N f_{q/p^\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x)\,.$$




Sivers functions for *u*, *d* and *sea* quarks are extracted from HERMES and COMPASS data.

$\Delta^N f_u > 0$, $\Delta^N f_d < 0$, first hints on nonzero sea quark Sivers functions.

# Timeline

- SIDIS experiments $lH^{\uparrow} \to lhX$:
  HERMES (terminated) COMPASS (ongoing) JLAB (ongoing/planned) ERICH/EIC etc (planned)
  Sivers, Collins FF, transversity (studied) Boer-Mulders, unpolarised cross section (ongoing), $h_{1T}^{\perp}$, $h_{1L}^{\perp}$, $g_{1T}^{\perp}$ (possible)
- DY experiments $H_1^{(\uparrow)} H_2^{\uparrow} \to l^+ l^- X$
  COMPASS $\pi D^{\uparrow} > 2010$, PAX, GSI $\bar{P}^{\uparrow} P^{\uparrow} > 2016$, RHIC collider $P^{\uparrow} P^{\uparrow} > 2013$
  Distribution function study, transversity at $\bar{P}P$ is the main goal.
- Hadron experiments $H_1^{(\uparrow)} H_2^{\uparrow} \to hX$
  RHIC collider $P^{\uparrow} P^{\uparrow}$ is a rich source of new data $\sqrt{s} = 200$ GeV.
  PROTVINO $PP^{\uparrow}$ at low energy $\sqrt{s} \simeq 10$ GeV.
- $e^+ e^- \to h_1 h_2 X$
  BELLE ongoing data analysis, BABAR planned data analysis.

# CONCLUSIONS

- Spin structure of the proton contains eight twist two structure functions which can be studied experimentally.
- Extraction of transversity for $u$ and $d$ quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$.
- The Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained.
  $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for $u$, $d$ and *sea* quarks are extracted from HERMES and COMPASS data.
- JLAB & 11 GeV and other planned experiments will improve our knowledge of TMDs

# CONCLUSIONS

- Spin structure of the proton contains eight twist two structure functions which can be studied experimentally.
- Extraction of transversity for $u$ and $d$ quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$.
- The Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained.
  $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for $u$, $d$ and *sea* quarks are extracted from HERMES and COMPASS data.
- JLAB & 11 GeV and other planned experiments will improve our knowledge of TMDs

# CONCLUSIONS

- Spin structure of the proton contains eight twist two structure functions which can be studied experimentally.
- Extraction of transversity for $u$ and $d$ quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$.
- The Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained.
  $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for $u$, $d$ and *sea* quarks are extracted from HERMES and COMPASS data.
- JLAB & 11 GeV and other planned experiments will improve our knowledge of TMDs

# CONCLUSIONS

- Spin structure of the proton contains eight twist two structure functions which can be studied experimentally.
- Extraction of transversity for $u$ and $d$ quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$.
- The Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained.
  $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for $u$, $d$ and *sea* quarks are extracted from HERMES and COMPASS data.
- JLAB & 11 GeV and other planned experiments will improve our knowledge of TMDs

# CONCLUSIONS

- Spin structure of the proton contains eight twist two structure functions which can be studied experimentally.
- Extraction of transversity for $u$ and $d$ quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$.
- The Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained.
  $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for $u$, $d$ and *sea* quarks are extracted from HERMES and COMPASS data.
- JLAB & 11 GeV and other planned experiments will improve our knowledge of TMDs

# CONCLUSIONS

- Spin structure of the proton contains eight twist two structure functions which can be studied experimentally.
- Extraction of transversity for $u$ and $d$ quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$.
- The Collins fragmentation functions for favoured and unfavoured fragmentation have been obtained.
  $\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$
- Sivers functions for $u$, $d$ and *sea* quarks are extracted from HERMES and COMPASS data.
- JLAB & 11 GeV and other planned experiments will improve our knowledge of TMDs

# CONCLUSIONS

- Spin structure of the proton contains eight twist two structure functions which can be studied experimentally.
- Extraction of transversity for $u$ and $d$ quarks, $\Delta_T u(x)$ and $\Delta_T d(x)$, from HERMES, COMPASS and BELLE data is presented.

**THANK YOU!**

fragmentation have been obtained.
$\Delta^N D_h^{fav}(z, |p_\perp|) > 0$ and $\Delta^N D_h^{unf}(z, |p_\perp|) < 0$

- Sivers functions for $u$, $d$ and *sea* quarks are extracted from HERMES and COMPASS data.
- JLAB & 11 GeV and other planned experiments will improve our knowledge of TMDs