Probing the gluon self-interaction in light mesons

Christian S. Fischer

TU Darmstadt and GSI

18th June 2009

C.F. and R. Williams, arXiv:0905.2291 [hep-ph]
C.F. and R. Williams, Phys. Rev. D **78** (2008) 074006

C.F., arXiv:0904.2700 [hep-ph].

Content

1. Introduction
2. Properties of Yang-Mills theory
3. Dynamical chiral symmetry breaking: Quarks and mesons
4. QCD phase transition

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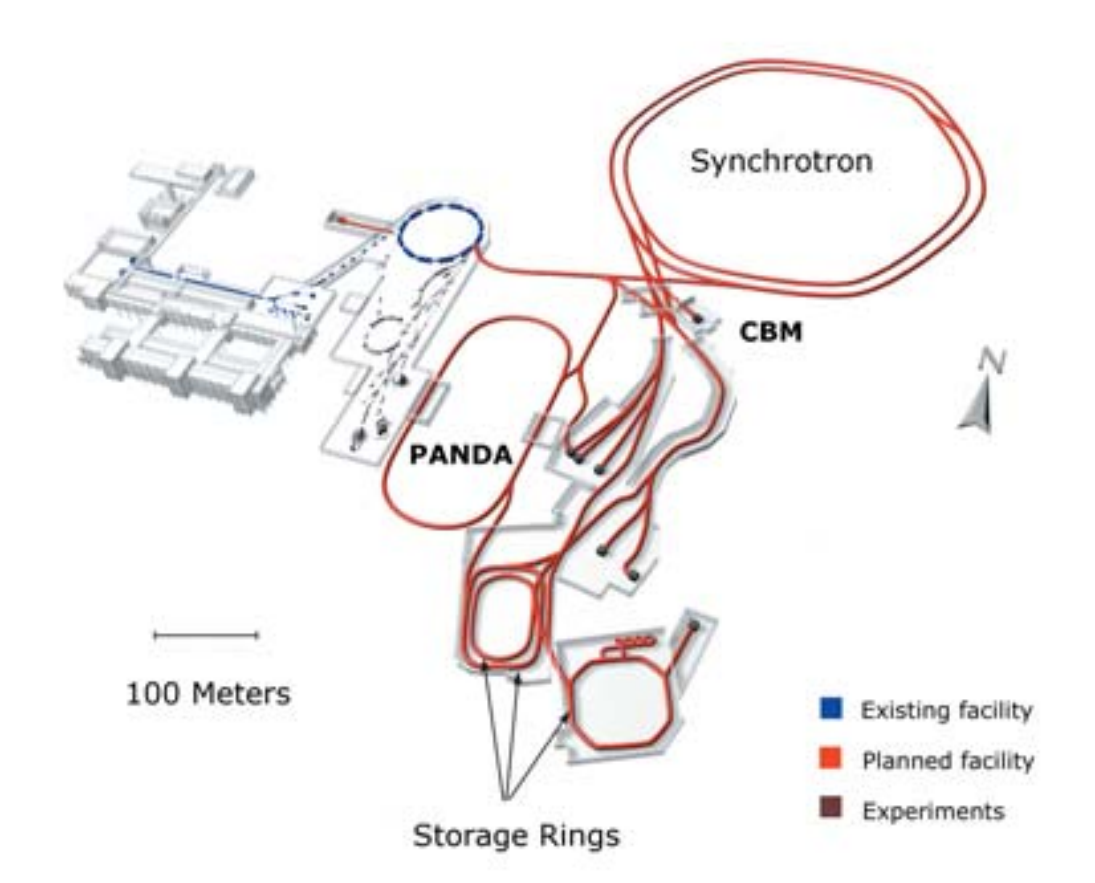
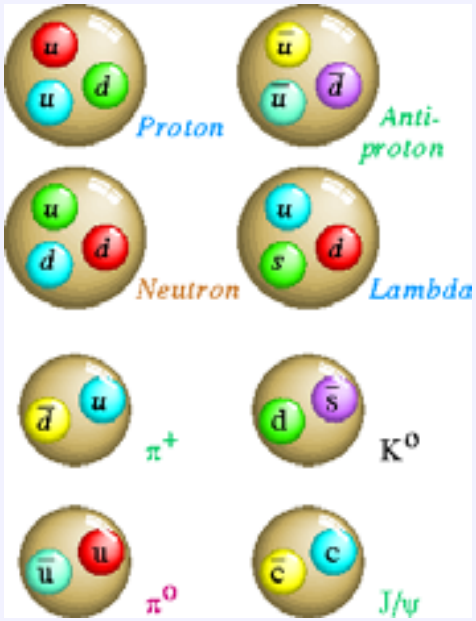
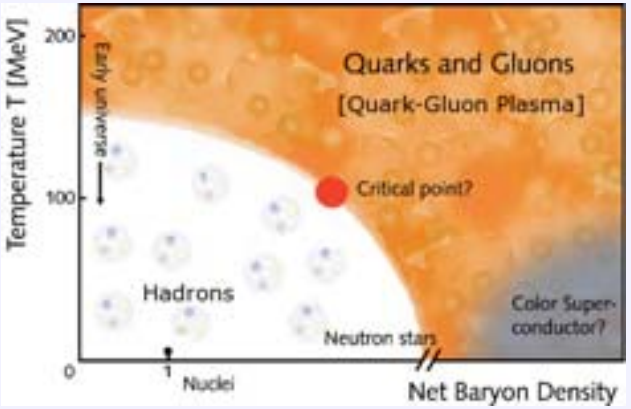
FAIR: CBM and PANDA

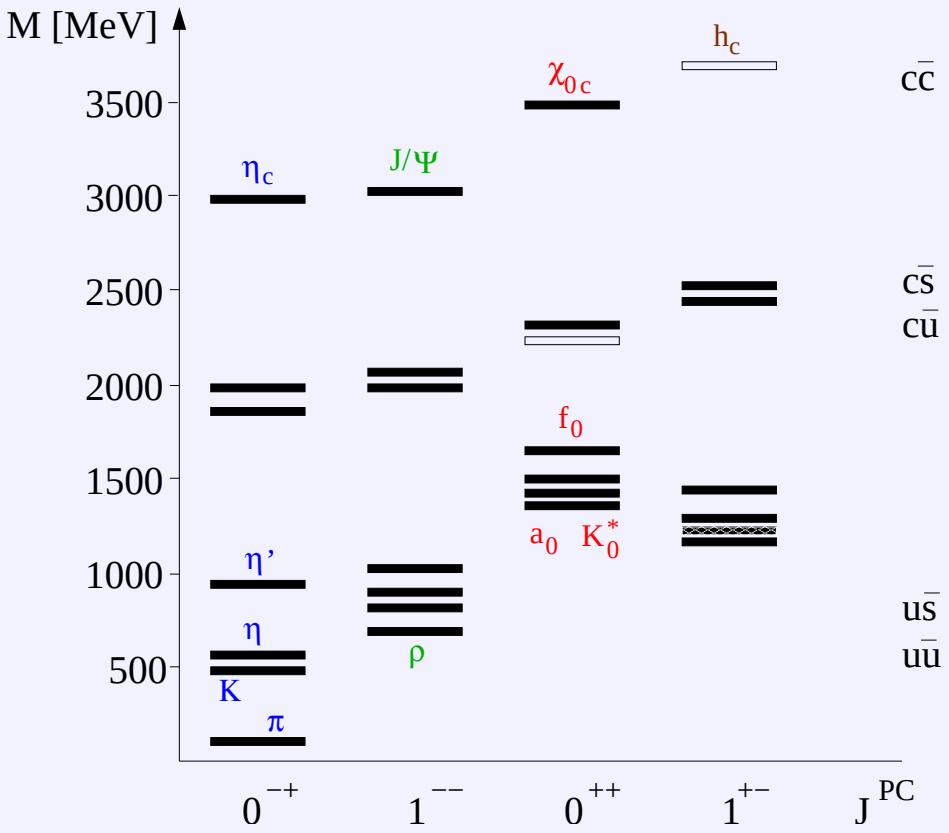






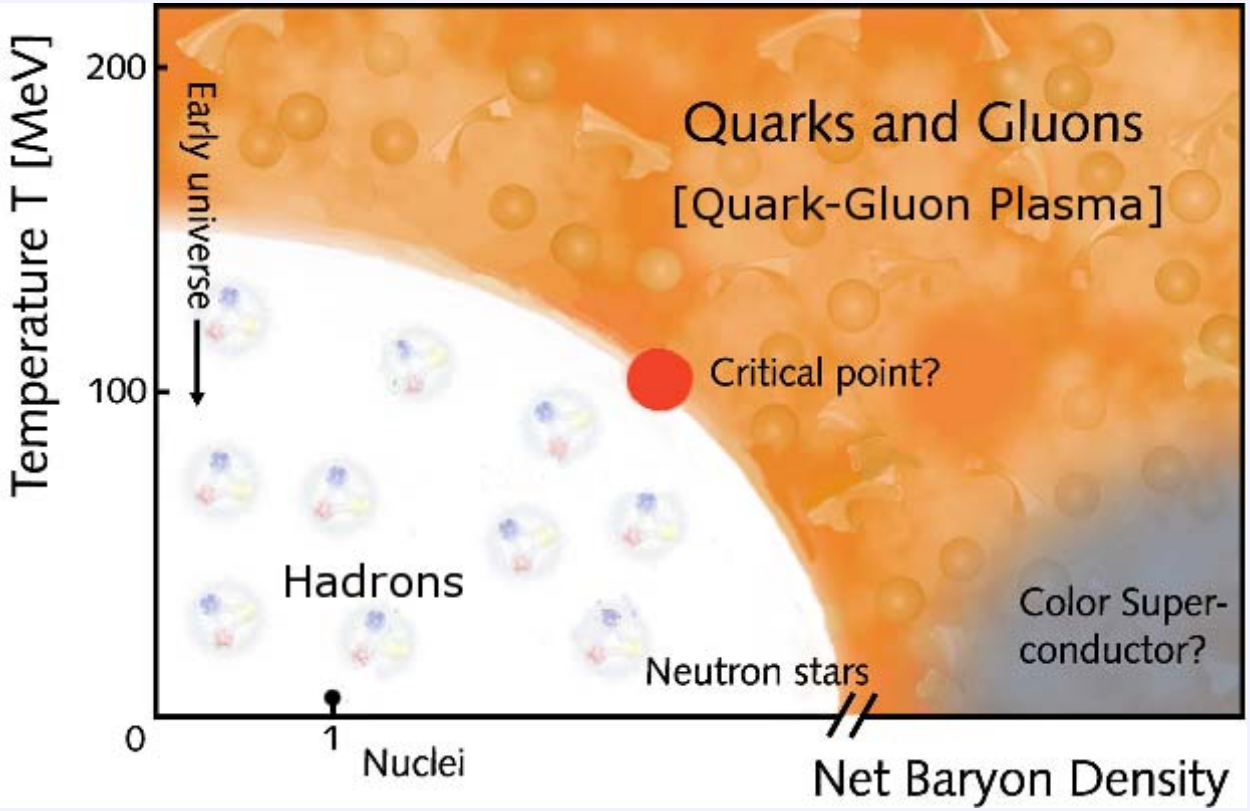
Spectrum of mesons

Experiment:



Goldstone-Bosons $\leftrightarrow$ **Bound pseudoscalar Quark-Antiquark-states:** $\pi^{\pm,0}, K^{\pm,0,\bar{0}}, \eta$

QCD phase transitions



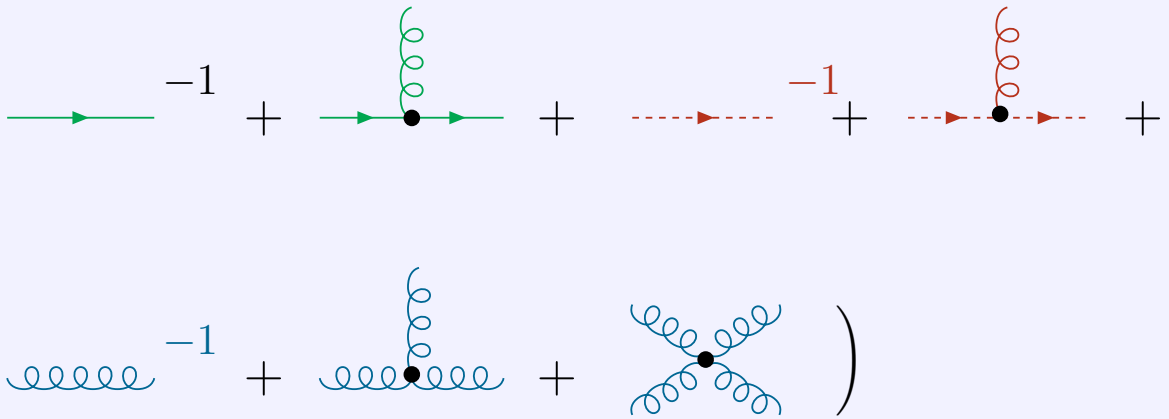
- Chiral limit ($M_{weak} = 0$): order parameter chiral condensate
- Heavy quarks ($M_{weak} = \infty$): order parameter Polyakov-loop

QCD in covariant gauge

quarks, gluons and ghosts:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \; \exp\left\{ - \int d^4x \left(\overline{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

$S_{QCD} = \int d^4x$

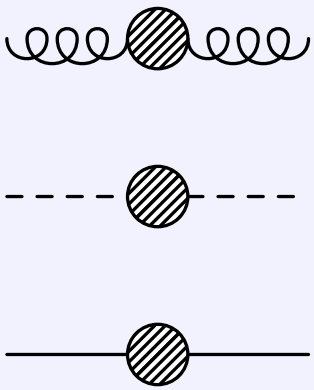


QCD in covariant gauge

quarks, gluons and ghosts:

$$\mathcal{Z}_{QCD} = \int \mathcal{D}[\Psi, A, c] \; \exp\left\{ - \int d^4x \left(\bar{\Psi}\,(i\not{D} - m)\,\Psi - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2 + \frac{(\partial A)^2}{2\xi} + \bar{c}(-\partial D)c \right) \right\}$$

Propagators in momentum space:



$$D^{\text{Gluon}}_{\mu\nu}(p) = \frac{\mathbf{Z}(p^2)}{p^2}\left(\delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)$$

$$D^{\text{Ghost}}(p) = -\frac{\mathbf{G}(p^2)}{p^2}$$

$$S^{\text{Quark}}(p) = \frac{Z_f(p^2)}{-i\not{p} + M(p^2)}$$

Green's functions

QCD Green's functions

- are connected to confinement:
  - Gribov-Zwanziger and Kugo-Ojima scenarios
  - Positivity
  - Running Coupling
  - Polyakov Loop
- encode DχSB
- are ingredients for hadron phenomenology
  - Bound state equations:
    Bethe–Salpeter equation / Faddeev equation

The Goal:

Gauge invariant information from gauge fixed functional approach

The Tool:

Dyson-Schwinger and Bethe-Salpeter-equations (DSE/BSE)

Lattice vs. DSE/FRG/BSE: Complementary!

- Lattice simulations
  - Ab initio
  - Gauge invariant
- Functional approaches:
  Dyson-Schwinger equations (DSE)
  Functional renormalisation group (FRG)
  Bethe-Salpeter-equations (BSE)
  - Analytic solutions at small momenta
  - Chiral symmetry: light quarks and mesons
  - Space-Time-Continuum
  - Chemical potential: no sign problem

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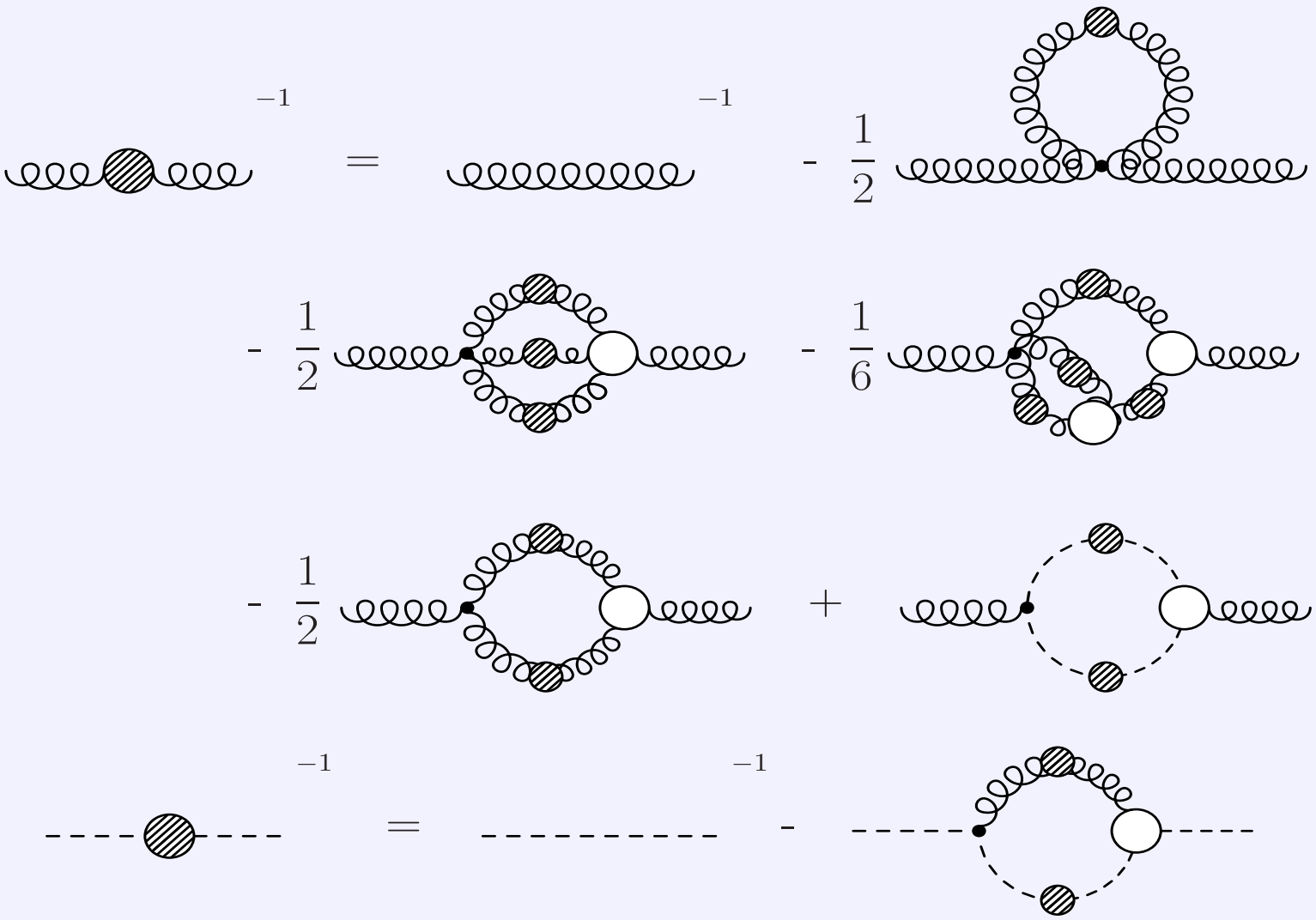
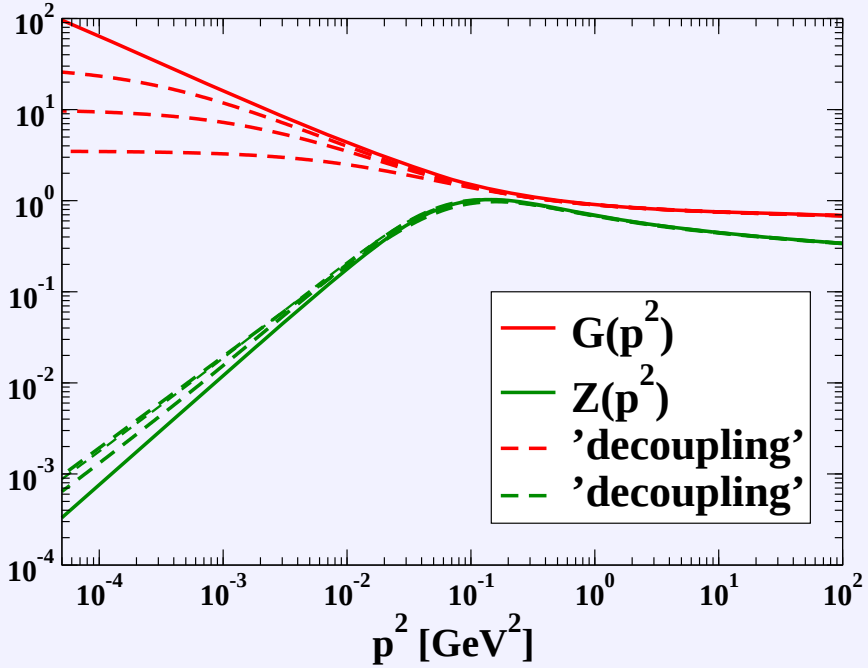
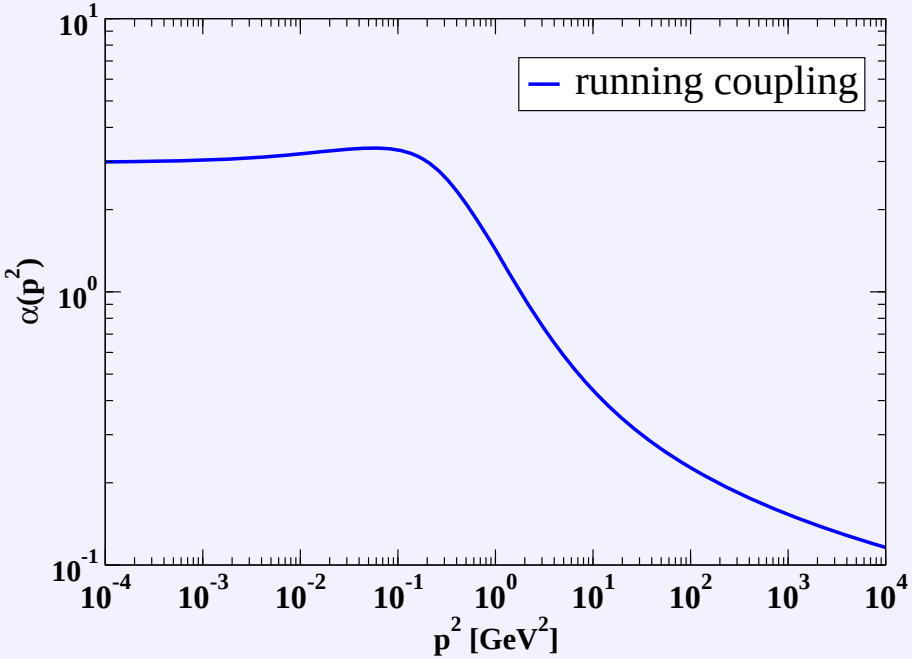
Dyson-Schwinger equations (DSEs)



Ghost, Glue and Coupling





- dynamically generated scale
- fixed point of coupling $\alpha(p^2) = g^2/(4\pi)Z(p^2)G^2(p^2)$
- deep infrared ($< 50MeV$): scaling vs. decoupling

CF and Alkofer, PLB 536 (2002) 177.

C.F., A. Maas and J. M. Pawlowski, arXive 0810.1987 [hep-ph]

Boucaud, Leroy, Yaouanc, Micheli, Pene, Rodriguez-Quintero, JHEP **0806** (2008) 012.

Infrared Structure of YM-theory: $p^2 << \Lambda_{QCD}$

Two type of analytic solutions for complete tower of DSEs:

Scaling:

n ghost, m gluon legs

$$\Gamma^{n,m}(p^2) \sim (p^2)^{(n/2-m)\kappa}$$

- $G(0) = \infty$
- $\kappa > 0$
- Kugo Ojima confinement scenario supported!

Decoupling:

n ghost, m gluon legs

$$\Gamma^{0,2}(p^2) \sim (p^2)$$

others finite

- $G(0) = \textit{finite}$
- gluon 'mass' $(Z(p^2)/p^2)_{p^2=0}$ not determined
- BRST symmetry broken

R. Alkofer, C. F., F. Llanes-Estrada, Phys. Lett. B **611** (2005)

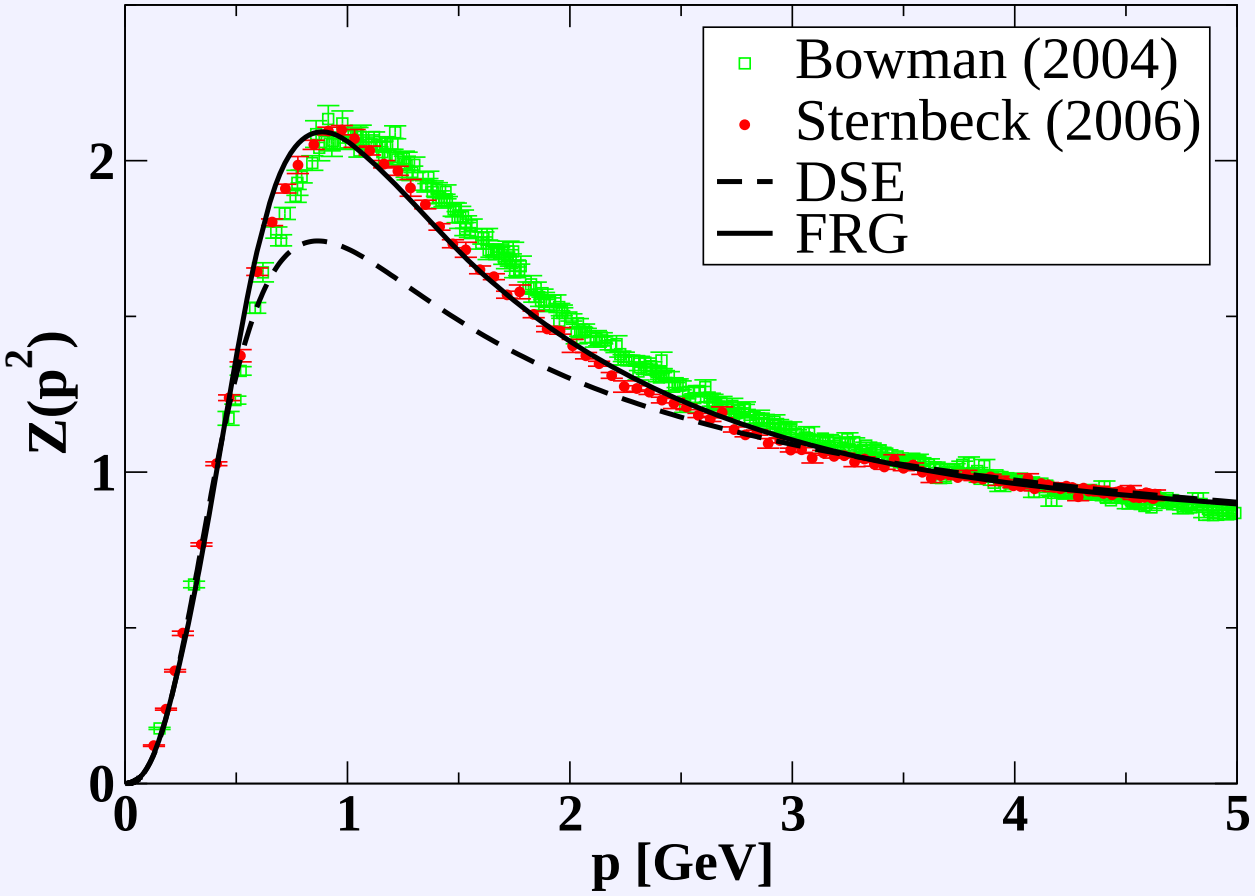
C.F. and J. M. Pawlowski, Phys. Rev. D **75** (2007) 025012; arXiv:0903.2193 [hep-th]

Boucaud, Leroy, Yaouanc, Micheli, Pene, Rodriguez-Quintero, JHEP **0806** (2008) 012.

Aguilar, Binosi, Papavassiliou, PRD **78**, 025010 (2008).

C.F., A. Maas and J. M. Pawlowski, arXive 0810.1987 [hep-ph].

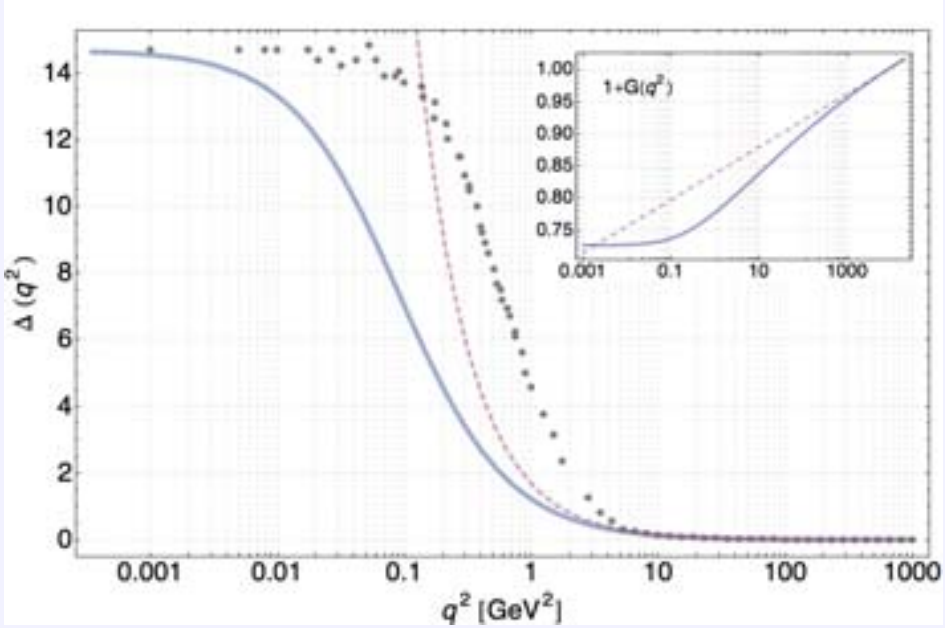
DSEs vs Lattice



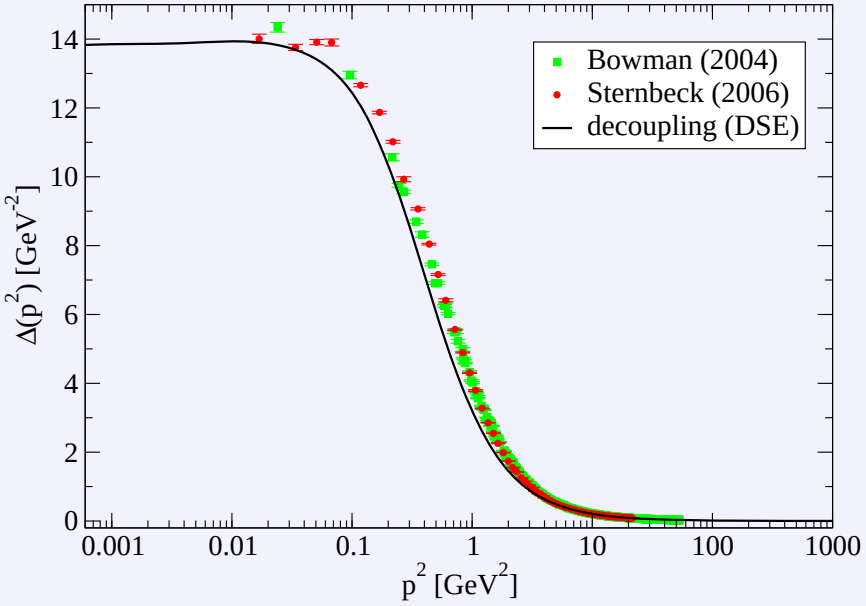
- $p^2 \approx 1GeV$: Systematic improvement possible for **DSEs**
- Deep infrared: Interesting and subtle questions

C.F., A. Maas and J. M. Pawlowski, arXive:0810.1987 [hep-ph]

Gluon propagator



Aguilar, Binosi, Papavassiliou, PRD **78**, 025010 (2008).



C.F., A. Maas and J. M. Pawlowski, arXive 0810.1987[hep-ph].

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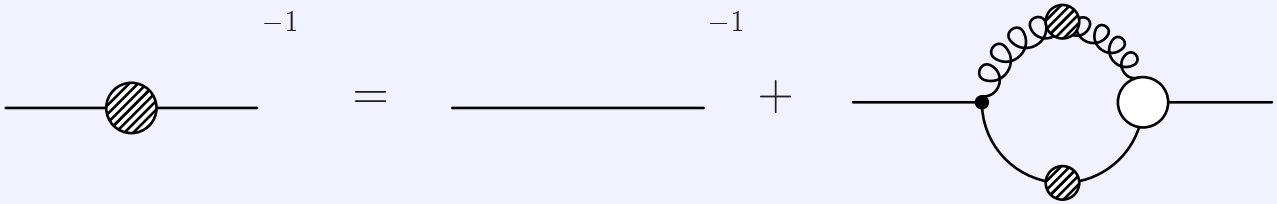
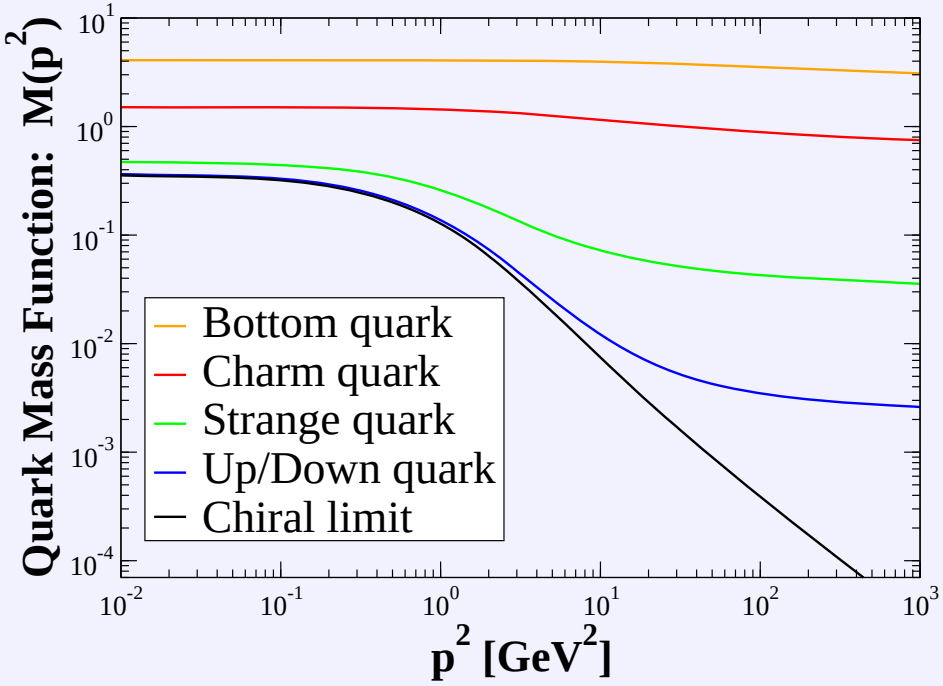
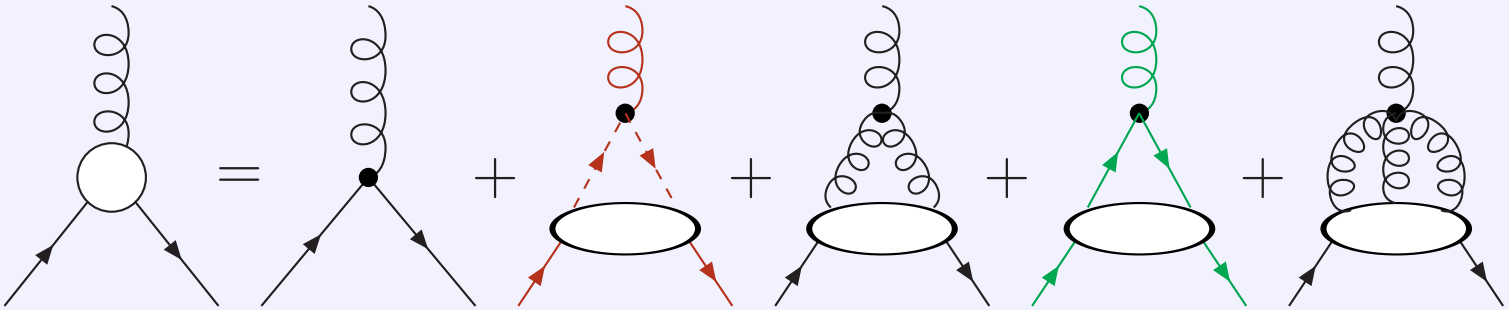
Explicit vs. dynamical chiral symmetry breaking



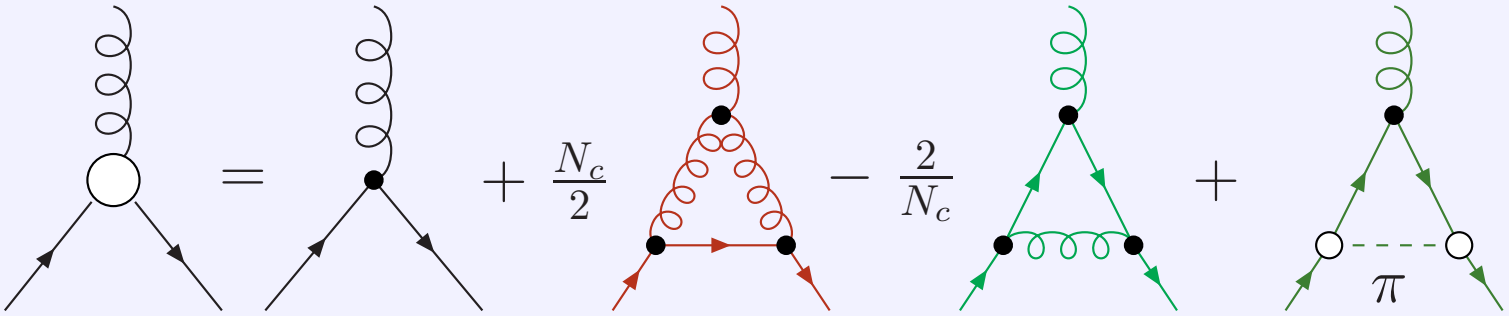


- $M(p^2)$: momentum dependent!
- Dynamical masses $M_{strong}(0) \approx 350 MeV$
- Flavour dependence because of M_{weak}

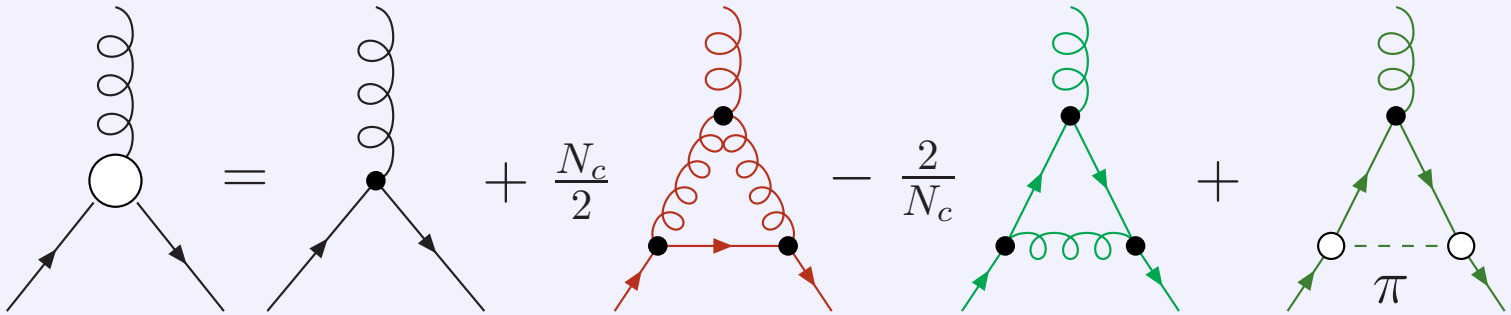
Quark-gluon vertex I



- Truncation via skeleton expansion leads to:



Quark-gluon vertex II



- Abelian diagram extensively explored

  Roberts, Tandy, Thomas, Watson *et al.*

- Gluon self-interaction leading in large N_c
- Gluon self-interaction also leading at $N_c = 3$

  Alkofer, C.F., Llanes-Estrada, Schwenzer, Annals Phys.324:106-172,2009.

  C.F, R. Williams, arXiv:0905.2291 [hep-ph]

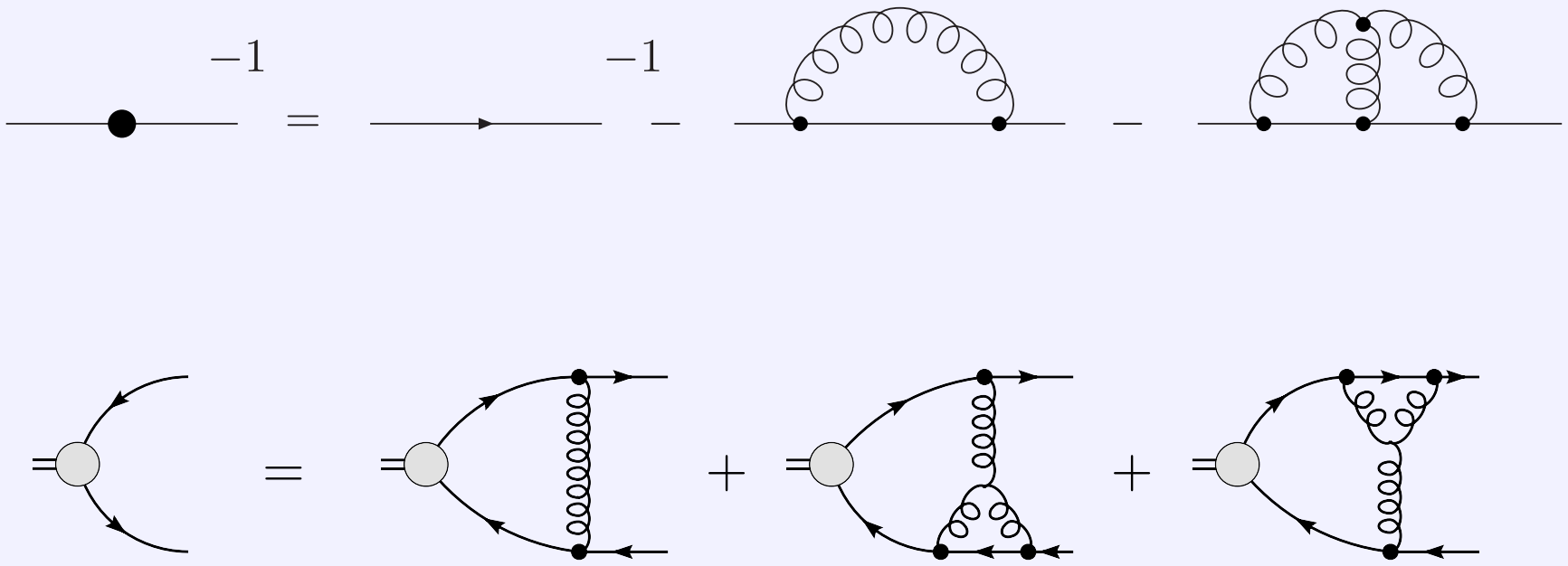
- Pion backreaction $\longrightarrow$ pion cloud effects

  C.F, D. Nickel and J. Wambach, PRD **76** (2007) 094009

  C.F., D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

  C.F. and R. Williams, PRD **78**, 074006 (2008).

Gluon self-interaction and Bethe-Salpeter equation



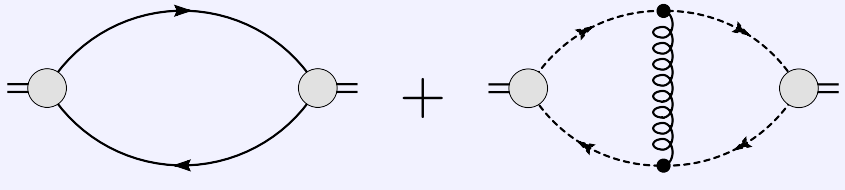
- Axial Ward-Takahashi identity satisfied
- Solve both equations in the complex plane !
- Include full momentum dependence of gluon !

C.F. and R. Williams, arXiv:0905.2291 [hep-ph]

Normalisation

Either use standard method:

$$\mathbb{1} = 2\frac{\partial}{\partial P^2}\,\mathrm{tr}\,\Bigg[$$



$$\Bigg]$$

or alternatively (and much cheaper):

$$\left(\frac{d\ln(\lambda)}{dP^2}\right)^{-1} = \mathrm{tr}\int_k \bar{\Gamma}(k,-P)\;S(k_+)\;\Gamma(k,P)\;S(k_-)\,.$$

N. Nakanishi, Phys. Rev. **138** (1965) B1182.

Gluon self-interaction effects in light mesons

	RL	BTR	shift	Experiment
M_π	138	138	0	138
f_π	94	111	+17	93
M_ρ	758	881	+123	776
f_ρ	154	176	+22	162
M_σ	645	884	+239	$400 - 1200$
M_{a_1}	926	1055	+129	1230
M_{b_1}	912	972	+60	1229

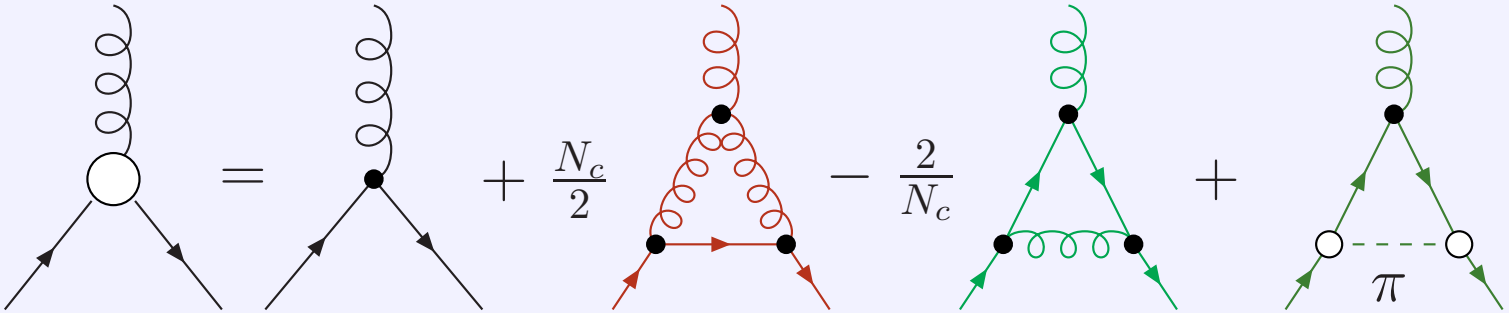
- Use Maris-Tandy-interaction for rainbow-ladder (RL) gluon

  Alkofer, Watson, Weigel, PRD **65** (2002) 094026.

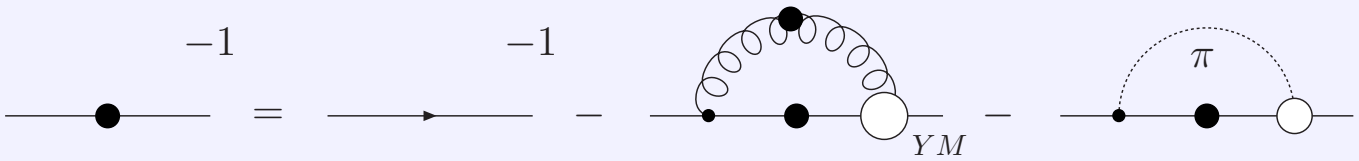
- BTR ρ-meson: Repulsive effects of order of 100-150 MeV

  CF and R. Williams, arXiv:0905.2291 [hep-ph]

Quark-gluon vertex



- Pion cloud effects

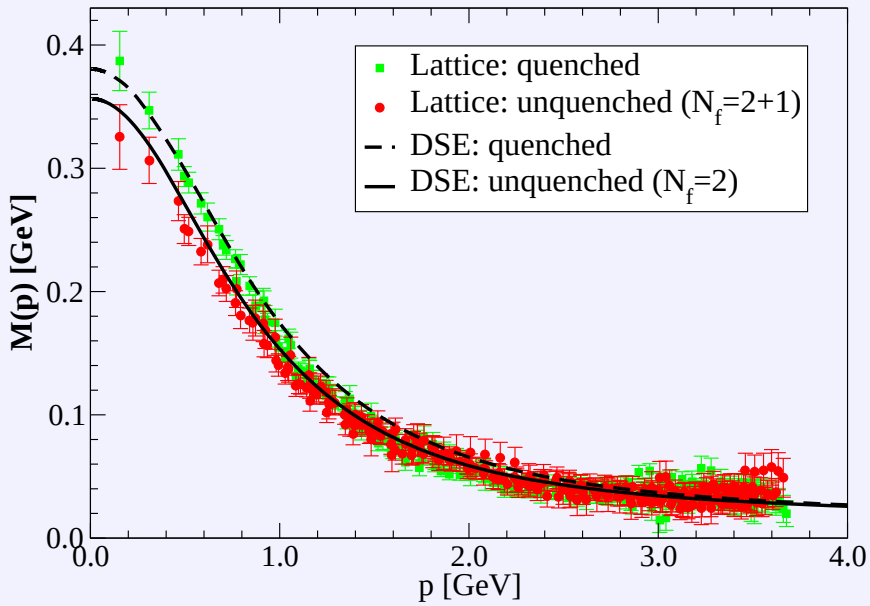


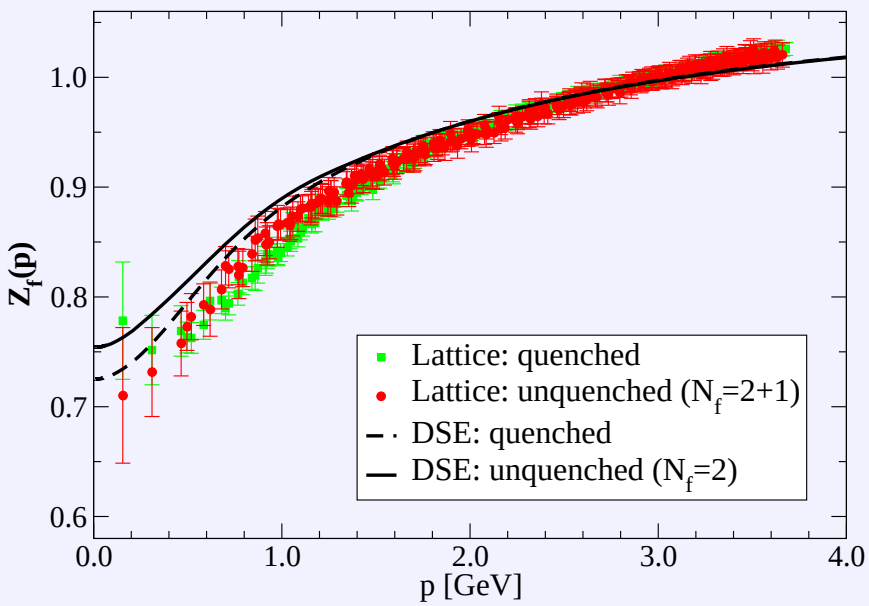
  - Visible pion cloud effects in quark propagator
  - Large pion cloud effects in light meson spectrum

C.F., D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

C.F. and R. Williams, PRD **78**, 074006 (2008).

Pion cloud effects in the quark propagator



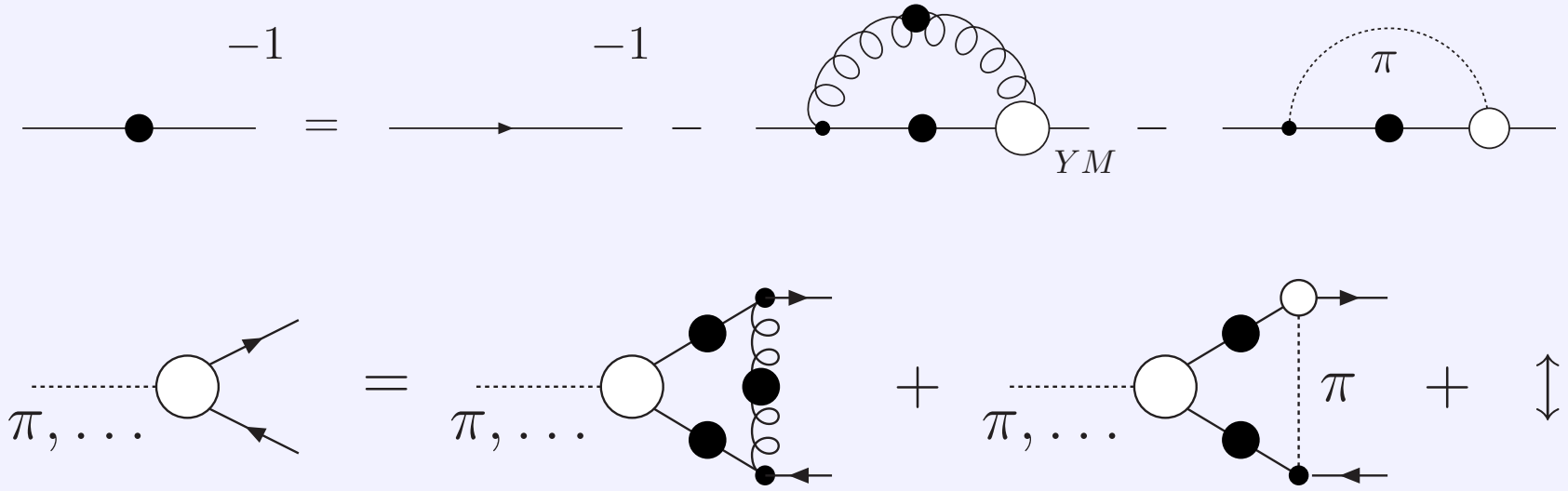


CF, D. Nickel and R. Williams, EPJC **60**, 1434 (2008)

- Unquenching effects of similar size as lattice

  P. O. Bowman, et al. Phys. Rev. D **71** (2005) 054507

Bethe-Salpeter equation



- Axial Ward-Takahashi identity satisfied:
  - identically in chiral limit
  - possible violations below 0.5 % at physical point

Pion cloud effects in light mesons

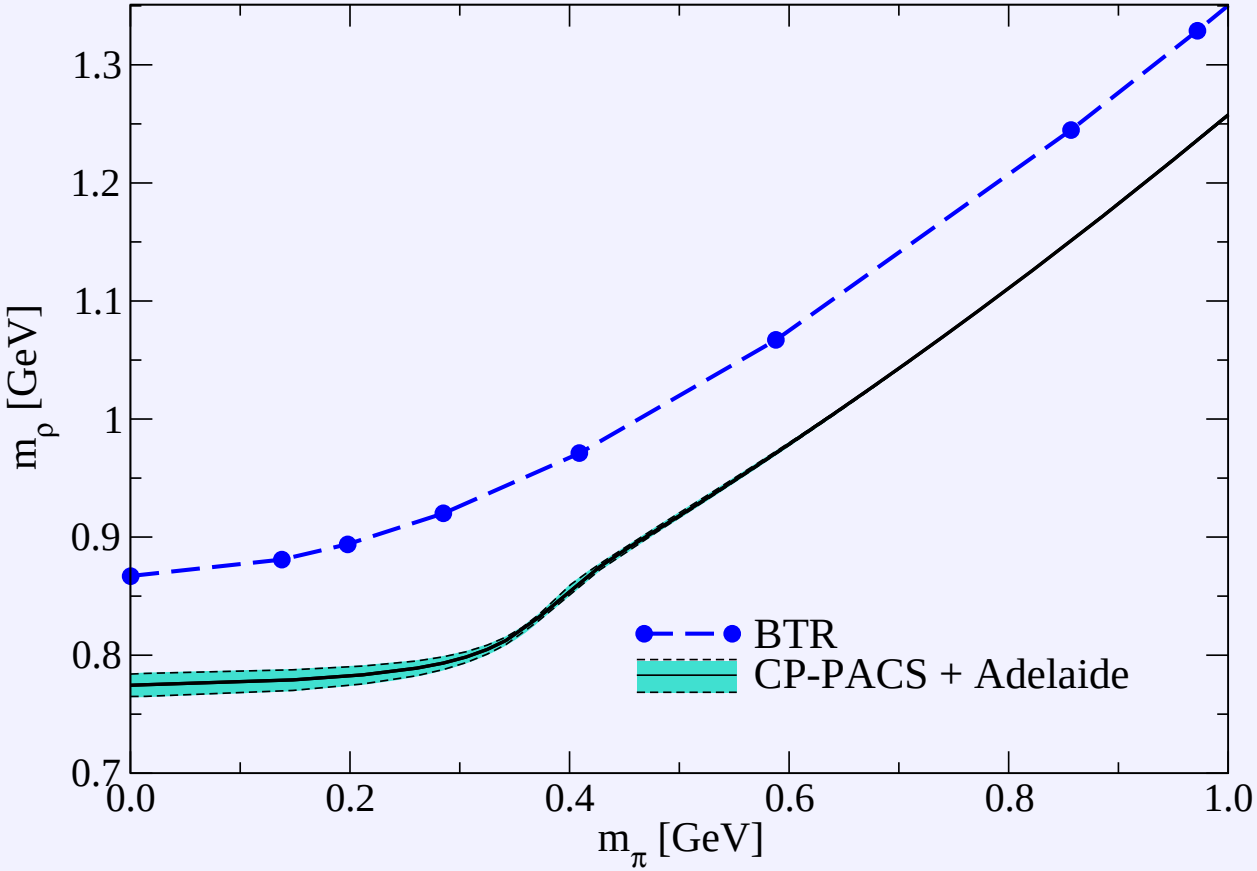
	RL	BTR	BTR+π	shift	Experiment
M_π	138	138	138	0	138
f_π	94	111	105	-6	93
M_ρ	758	881	805	-76	776
f_ρ	154	176	168	-8	162
M_σ	645	884	820	-64	400 – 1200
M_{a_1}	926	1055	1040	-15	1230
M_{b_1}	912	972	940	-31	1229

- Attractive effects of 'pion cloud'
- Corrections from Abelian diagram and decay channels still missing

CF and R. Williams, PRD **78**, 074006 (2008).

CF and R. Williams, in preparation

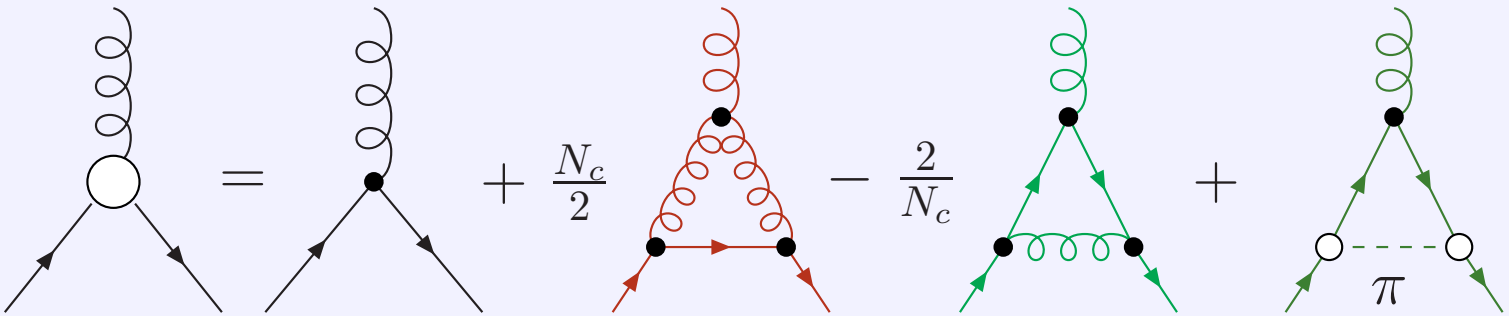
Compare with lattice I



- BTR: Our calculation **without** pion effects
- CP-PACS+Adelaide: Chiral extrapolation based on (corrected) lattice data

Allton,Armour,Leinweber,Thomas and Young, PLB **628** (2005) 125.

Quark-gluon vertex



- Gluon self-interaction: Infrared leading
  - infrared slavery (quark-gluon coupling)
  - generation of topological mass of η' ($U_A(1)$-problem)

R. Alkofer, C.F., F. Llanes-Estrada, Kai Schwenzer, Annals Phys. **324**, 106 (2009).

R. Alkofer, C.F, R. Williams, Eur. Phys. J. A **38**, 53 (2008).

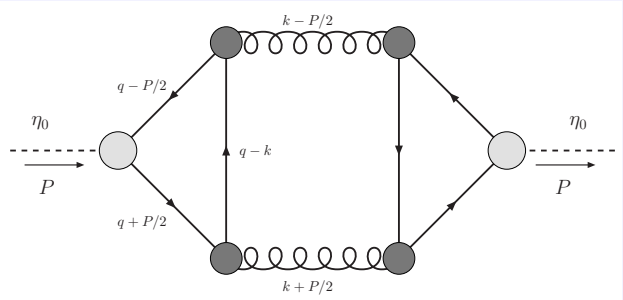
The $U_A(1)$-problem

η' too heavy for a Goldstone boson $\longrightarrow$ $m^{\chi}_{\eta'} \neq 0$ in chiral limit

- Conjecture: Coloured infrared singularities generate $m^{\chi}_{\eta'} \neq 0$

  J. B. Kogut and L. Susskind, Phys. Rev. D **10** (1974) 3468.

- $\Gamma^{quark-gluon}(p^2) \sim (p^2)^{-1/2-\kappa}$ provides correct IR-strength



- our results:

$m^{\chi}_{\eta'}$ [MeV]	$Top.susc.$ [MeV^4]	θ	m_η [MeV]	$m_{\eta'}$ [MeV]
748	169	-23.2	479	906

R. Alkofer, C.F. and R. Williams, Eur. Phys. J. A **38**, 53 (2008)

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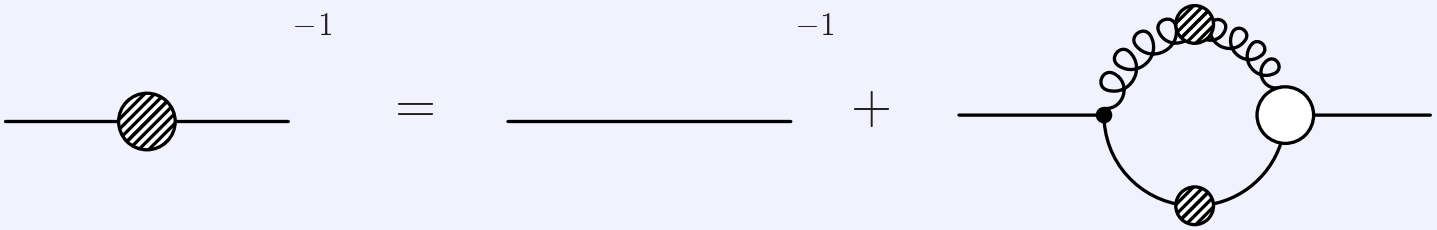
Finite Temperature: framework



- Quark propagator:

$$S(p_{\vec{p},\omega_p}) = [i\,\vec{\gamma}\cdot\vec{p}\,A(\vec{p},\omega_p) + i\,\gamma_4\omega_n\,C(\vec{p},\omega_p) + B(\vec{p},\omega_p)]^{-1}$$

- consider DSE on torus with $V = 1/T \times L^3$
- spatial directions: periodic boundary conditions
- temporal direction: antiperiodic boundary condition
- 'Order parameter' for chiral transition:

$$\langle\bar{\psi}\psi\rangle = Z_2 N_c \frac{T}{L^3} Tr_D \sum_{\vec{p},\omega_p} S(p_{\vec{p},\omega_p})$$

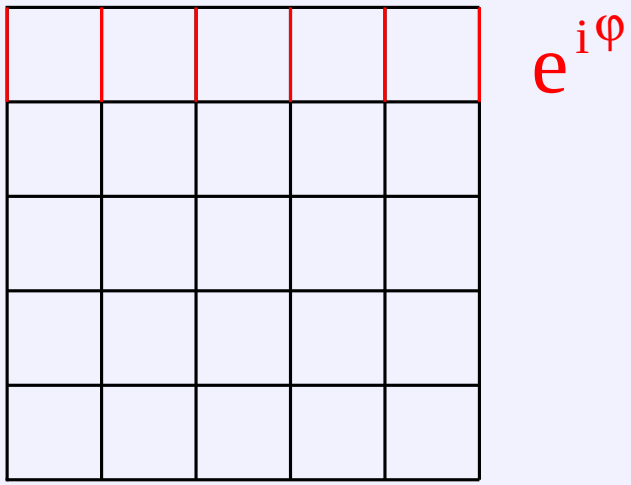
The dual condensate I

Consider general $U(1)$-valued boundary conditions in temporal direction for quark fields ψ:

$$\psi(\vec{x}, 1/T) = e^{i\varphi}\psi(\vec{x}, 0)$$

Matsubara frequencies: $\omega_p(n_t) = (2\pi T)(n_t + \varphi/2\pi)$

Lattice:



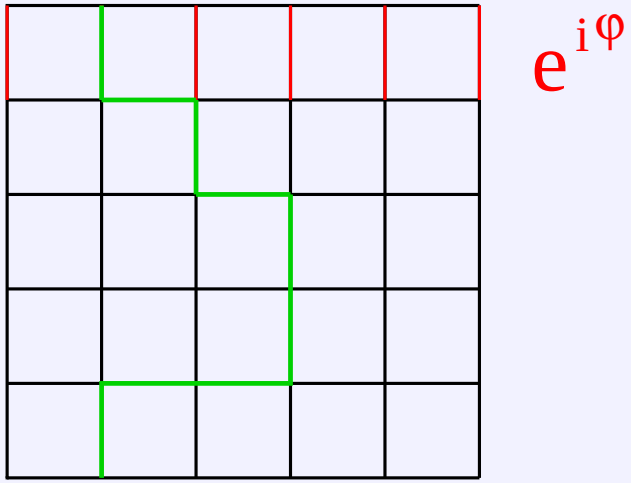
E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007..

The dual condensate II

Relation of condensate to loops of link variables $U_\mu(x)$:

$$\langle\overline{\psi}\psi\rangle_\varphi = Tr\,[m + D_\varphi]^{-1} = \frac{1}{Vm}\sum_{l\in\mathcal{L}}\frac{s(l)e^{i\varphi q(l)}}{(2am)^{|l|}}Tr_c\prod_{(x,\mu)\in l}U_\mu(x)\,.$$

- geometric series of inverse staggered Dirac operator
- winding number $q(l)$ of loop l around temporal direction



E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.

The dual condensate III

Then define dual condensate Σ_n:

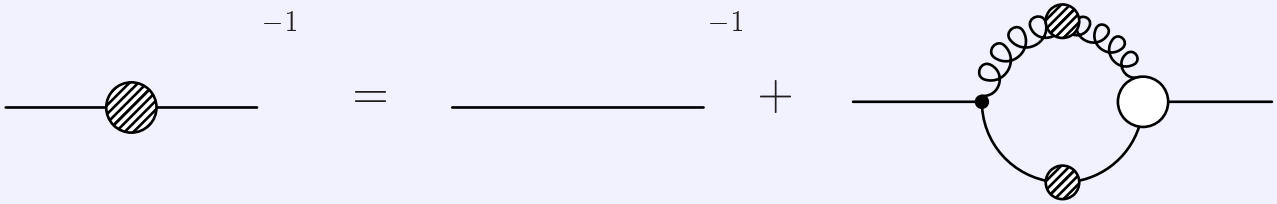
$$\Sigma_n = -\int_0^{2\pi} \frac{d\varphi}{2\pi} e^{-i\varphi n} \langle \overline{\psi}\psi \rangle_\varphi$$

- $n = 1$ projects out loops with $q(l) = 1$: dressed Polyakov loop
- transforms under center transformation exactly like ordinary Polyakov loop
- Σ_1 is order parameter for center symmetry/deconfinement

  E. Bilgici, F. Bruckmann, C. Gattringer and C. Hagen, PRD **77** (2008) 094007.
- Σ_1 is accessible with functional methods

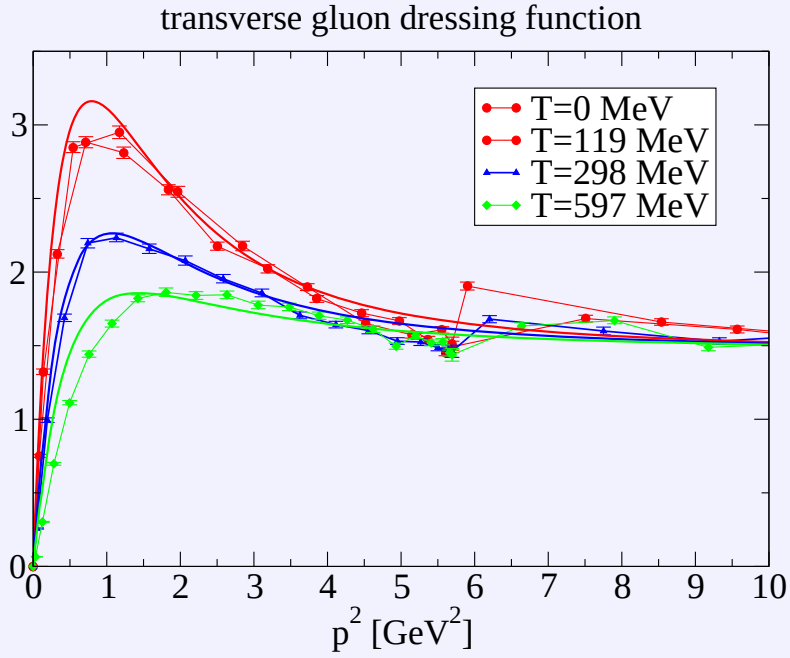
  C.F., arXiv:0904.2700 [hep-ph]

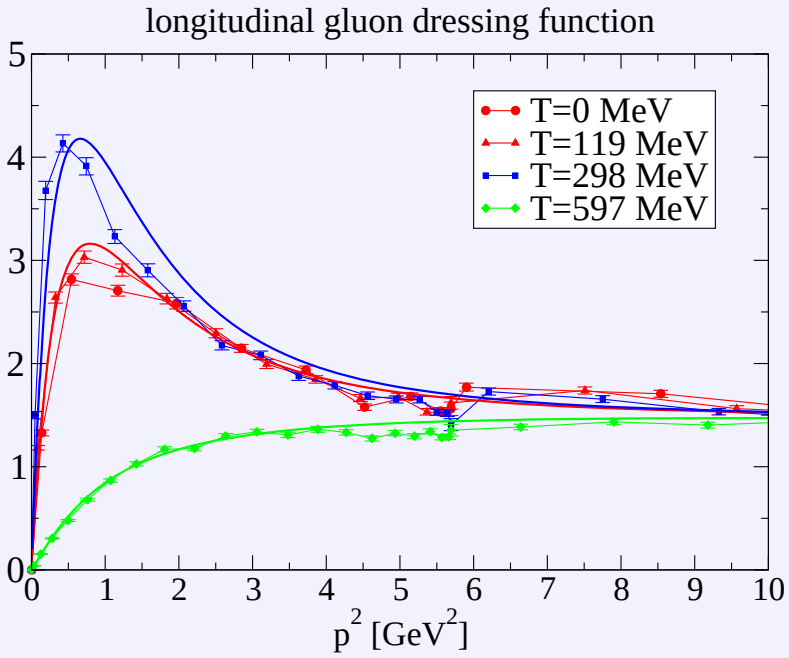
Input into quark-DSE



- T-dependent gluon propagator from lattice data

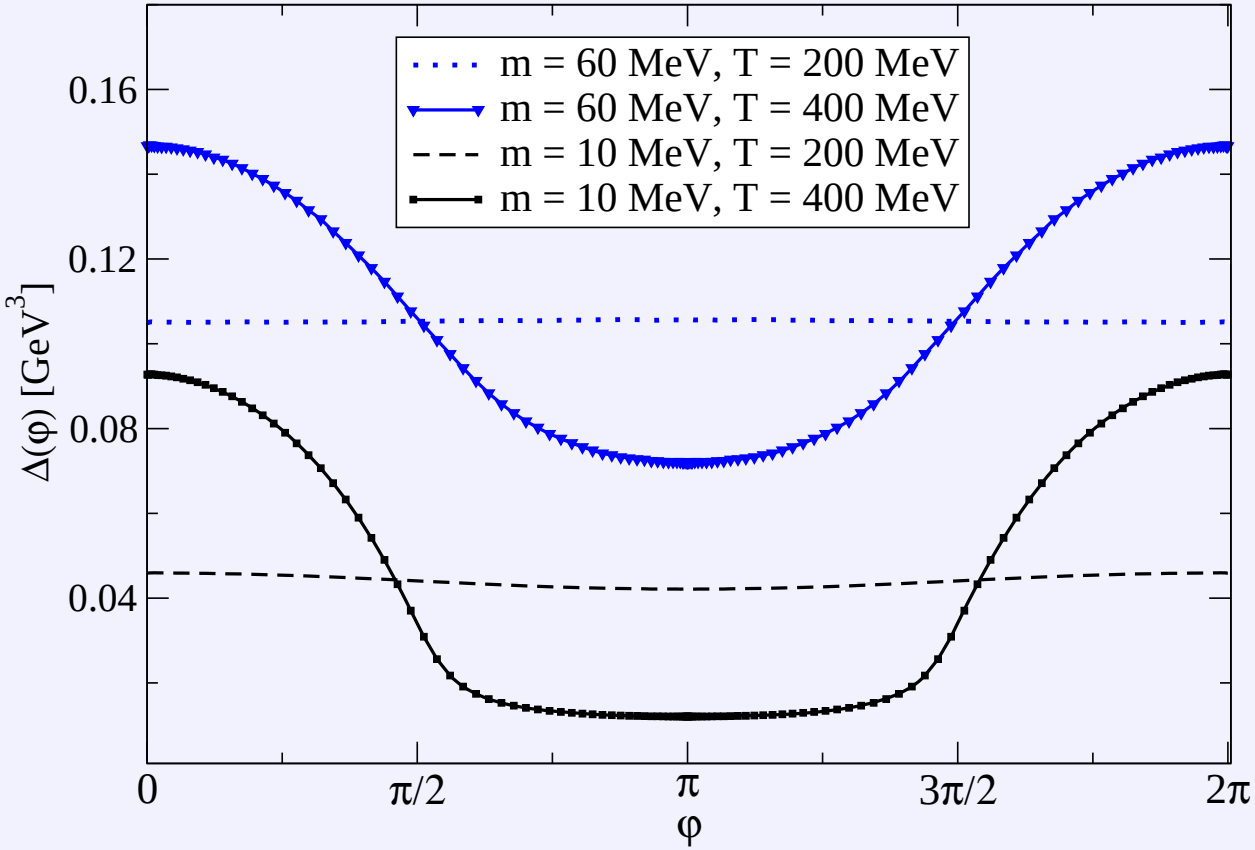
Cucchieri, Maas, Mendes, PRD75 (2007)





- T-dependent ansatz for quark-gluon vertex

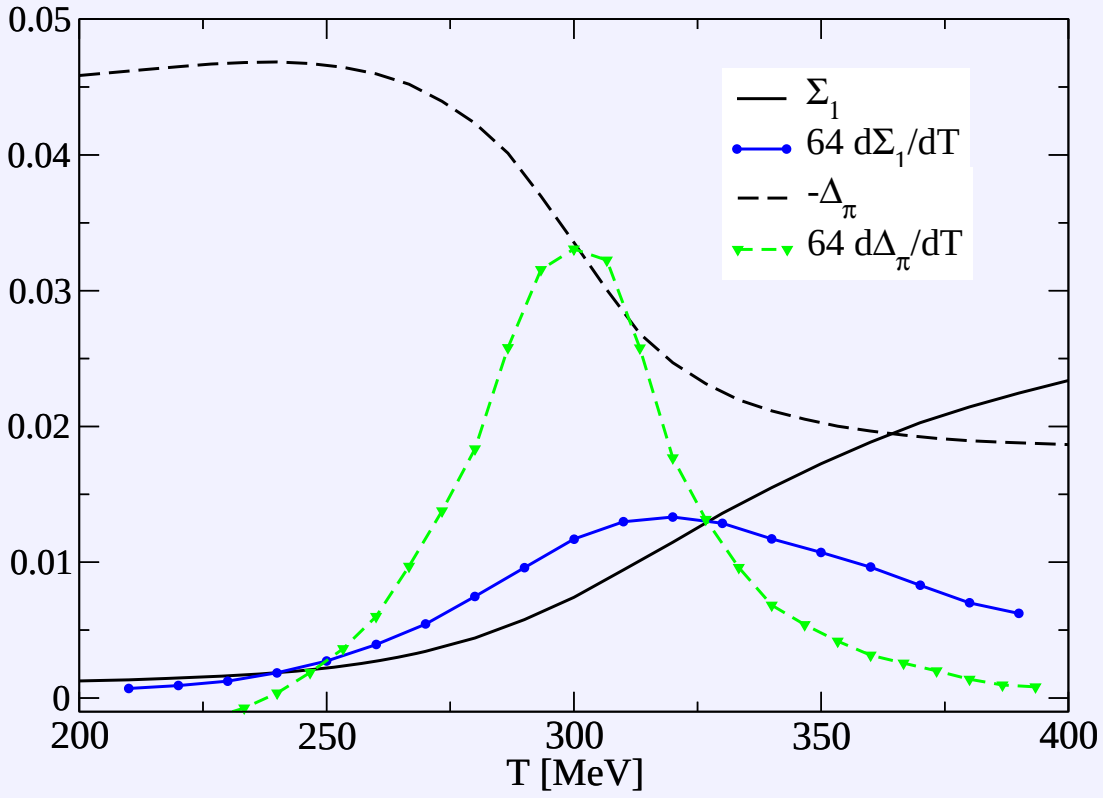
Results: angular dependence of condensate



$$\Delta(\varphi) \equiv \langle\overline{\psi}\psi\rangle_\varphi = Tr\,[m + D_\varphi]^{-1} = \frac{1}{Vm}\sum_{l\in\mathcal{L}} \frac{s(l)e^{i\varphi q(l)}}{(2am)^{|l|}} Tr_c \prod_{(x,\mu)\in l} U_\mu(x)\,.$$

- Smaller mass: more contributions from loop with larger $q(l)$!

Results: dressed Polyakov loop Σ_1



- Deconfinement transition from functional methods
- Insensitive to scaling vs decoupling

see also J. Braun, H. Gies and J. M. Pawlowski, arXiv:0708.2413 [hep-th].

C.F., arXiv:0904.2700 [hep-ph]

Summary

Light mesons:

- Bethe-Salpeter kernel including
  - gluon self-interaction
  - pion cloud effects

  beyond rainbow-ladder
- Quark-gluon vertex and quark-DSE solved in complex plane!
- Large repulsive effects from gluon self-interaction
- Large attractive unquenching ('pion cloud') effects

Finite temperature:

- Dual condensate $\rightarrow$ order parameter for center symmetry
- Calculable with functional methods!

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HELMHOLTZ GEMEINSCHAFT

GSI

HIC for FAIR

LOEWE – Landes-Offensive zur Entwicklung Wissenschaftlich-ökonomischer Exzellenz

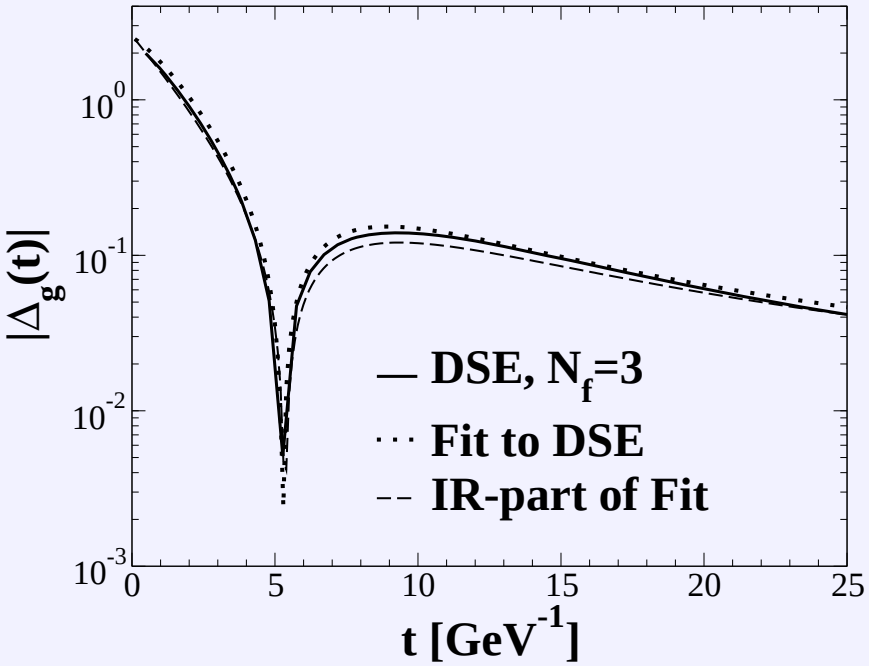
EMMI

Helmholtz-Alliance: Extremes of density and temperature; cosmic matter in the laboratory

Gluon confinement

- $$\int \frac{d^4x}{(2\pi)^4} \Delta_g(x^2) = \left(\frac{Z(p^2)}{p^2}\right)\Big|_{p^2=0} = \left(\frac{(p^2)^{2\kappa}}{p^2}\right)\Big|_{p^2=0} = 0 \quad \text{for } \kappa > 1/2$$

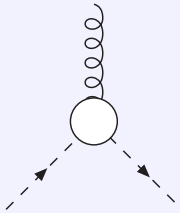
$$\Delta_g(t) := \int d^3x \int \frac{d^4p}{(2\pi)^4} e^{i(tp_4+\vec{p}\vec{x})} \frac{Z(p^2)}{p^2}$$



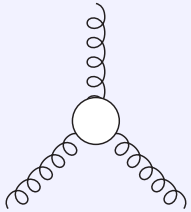
- Violation of positivity $\Rightarrow$ Signal for **confined gluons**

R. Alkofer, W. Detmold, C. F., P. Maris, Phys. Rev. D **70** (2004) 014014

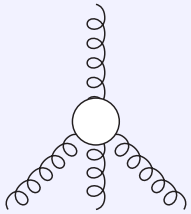
Consequence of scaling: Running coupling for $p^2 \to 0$



$$\alpha^{gg}(p^2) = \frac{g^2}{4\pi} G^2(p^2) Z(p^2) = const_{gg} \approx 2.97$$



$$\alpha^{3g}(p^2) = \frac{g^2}{4\pi} [\Gamma^{3g}(p^2)]^2 Z^3(p^2) = const_{3g}$$



$$\alpha^{4g}(p^2) = \frac{g^2}{4\pi} [\Gamma^{4g}(p^2)] Z^2(p^2) = const_{4g} \approx 0.0029$$

Qualitatively universal fixed point!

C. Lerche and L. v. Smekal, Phys. Rev. D **65** (2002) 125006

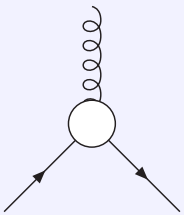
R. Alkofer, C.F., F. Llanes-Estrada, Phys.Lett.B611 (2005) 279-288.

C. Kellermann and C.F., PRD **78**,(2008) 025015.

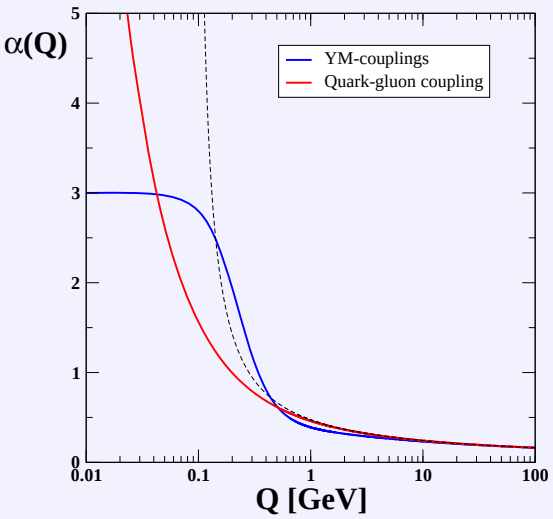
Infrared slavery (quenched QCD)

Quark-gluon vertex: $\Gamma^{quark-gluon}(p^2) \sim \{ \ (p^2)^{-1/2-\kappa} : \chi SB$

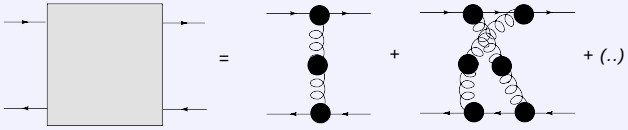
- Quark-gluon coupling:



$$\alpha^{qg}(p^2) \sim \left\{ \ \frac{1}{p^2} : \chi SB \right.$$

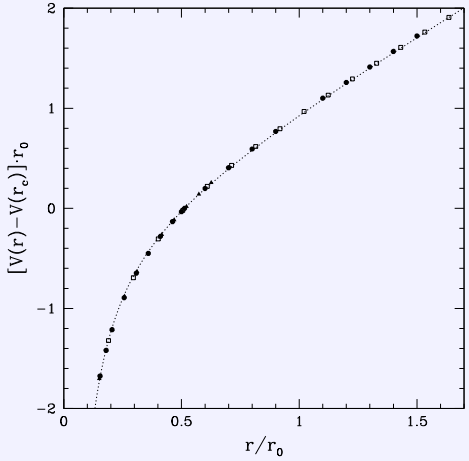


- 'Quark-Antiquark potential'

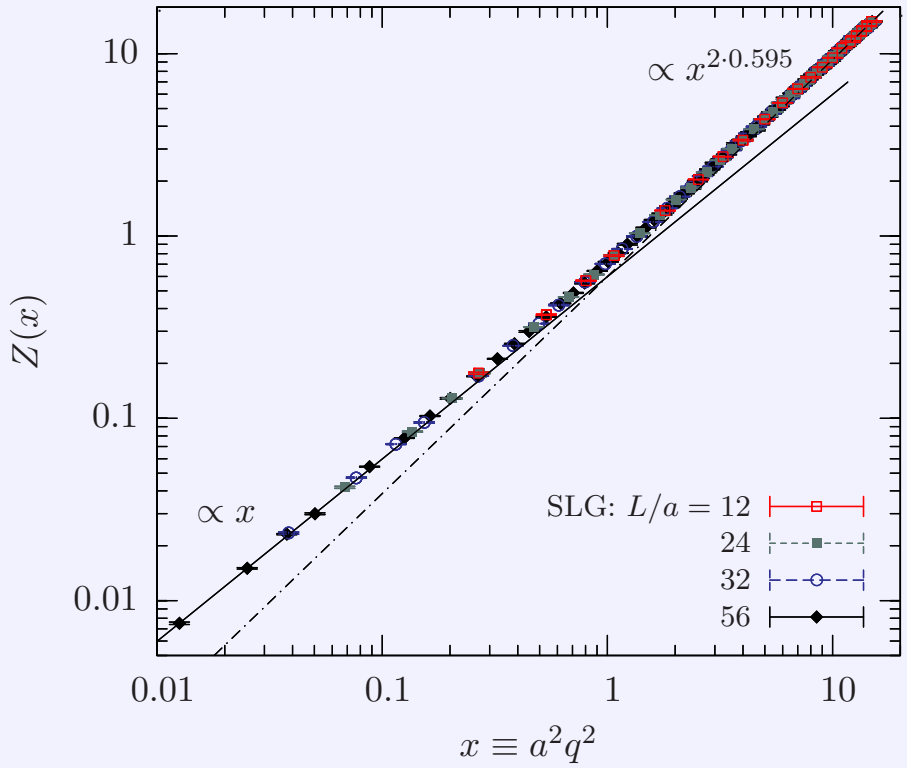


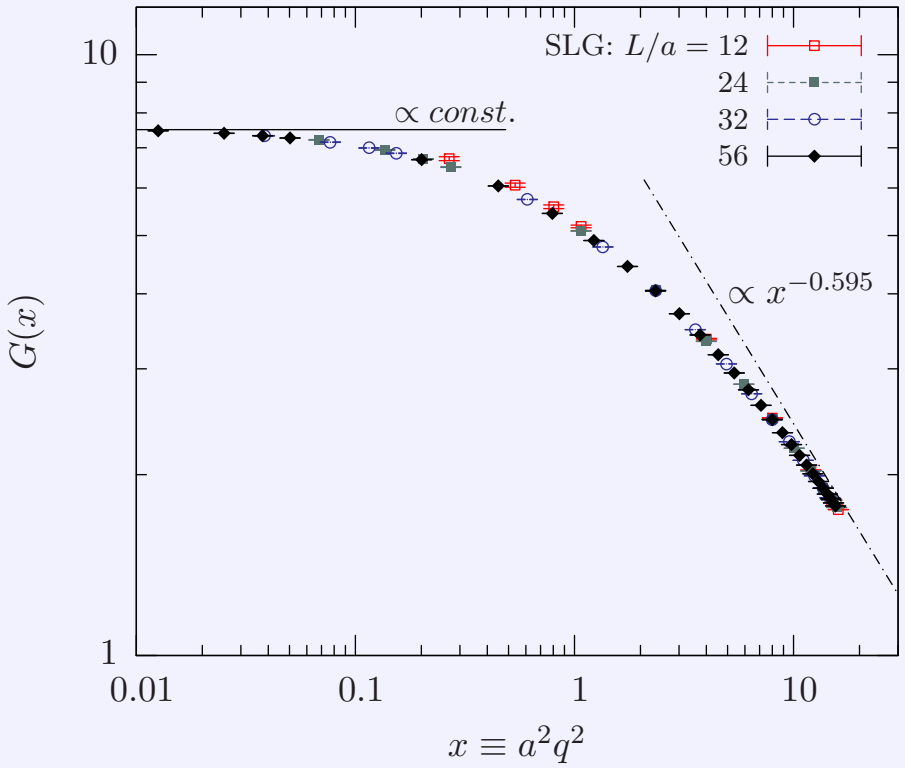
$$\sim \frac{1}{p^4}$$

$$\rightarrow V(\mathbf{r}) \ \sim \{ \ |r| \ : \ D\chi SB$$



Lattice-results $\beta = 0$ I

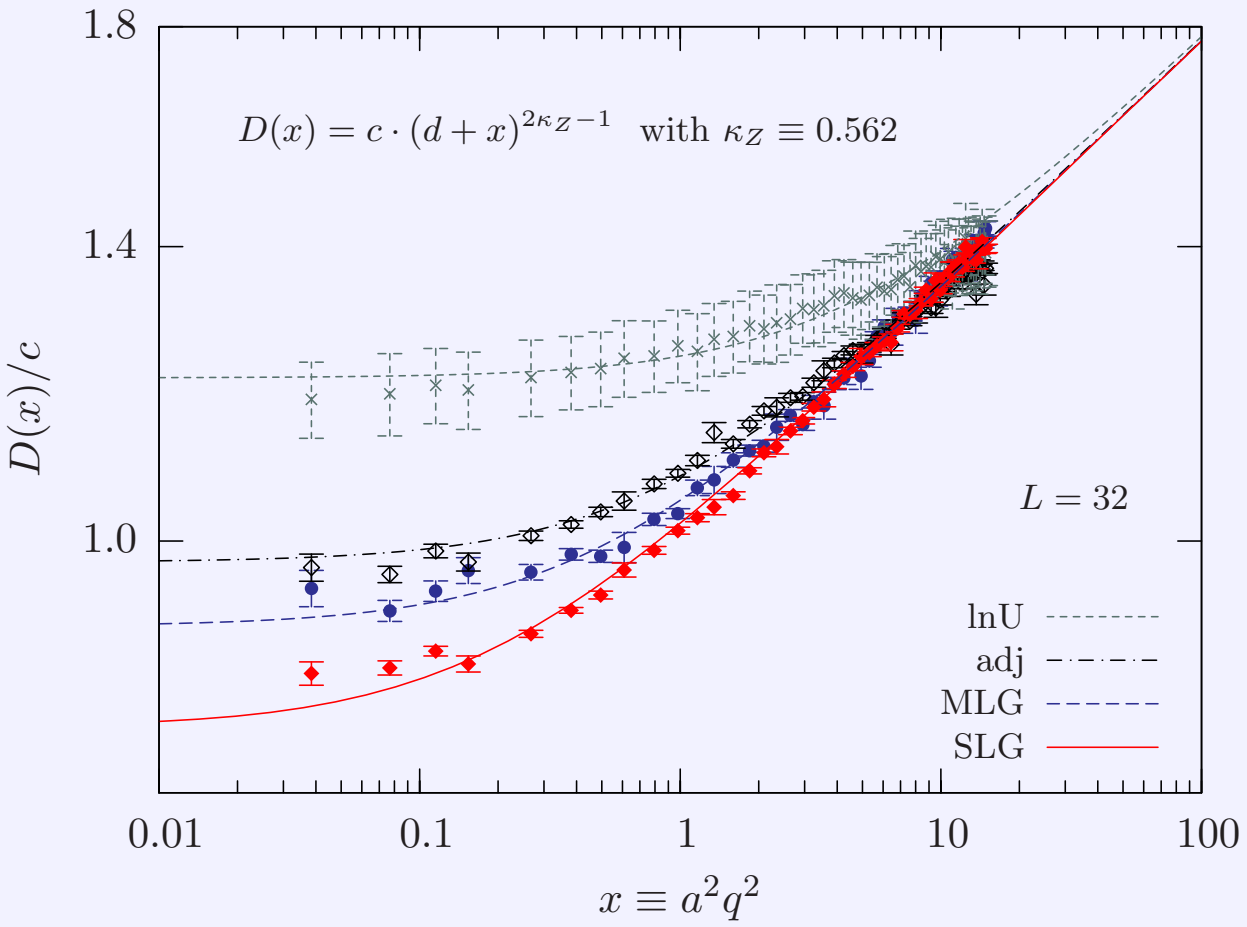




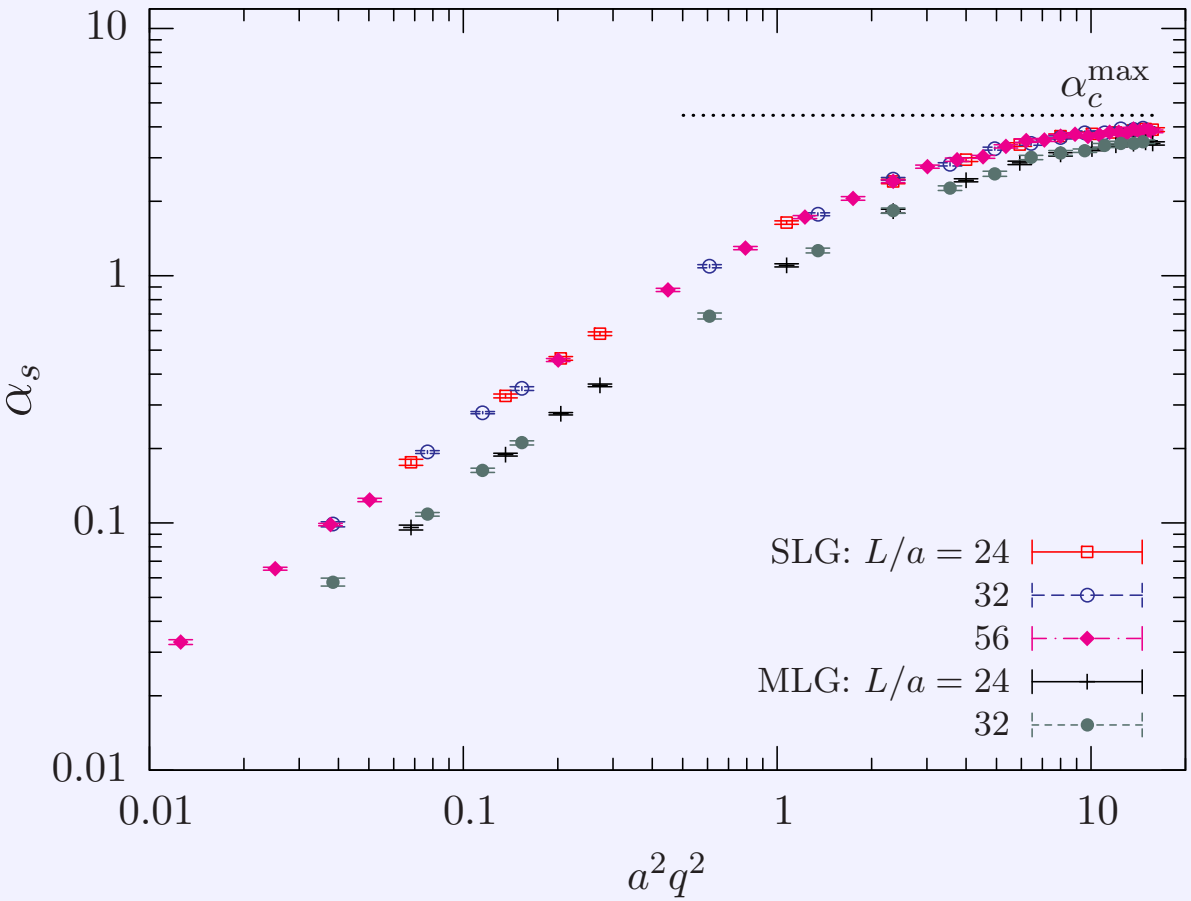
- Scale ??
- Scaling vs decoupling

Sternbeck, von Smekal,arXiv:0811.4300 [hep-lat].

Lattice-results $\beta = 0$ II



- 'Gluon mass' depends on definition of gauge field!

Sternbeck, von Smekal,arXiv:0811.4300 [hep-lat].

Lattice-results $\beta = 0$ III