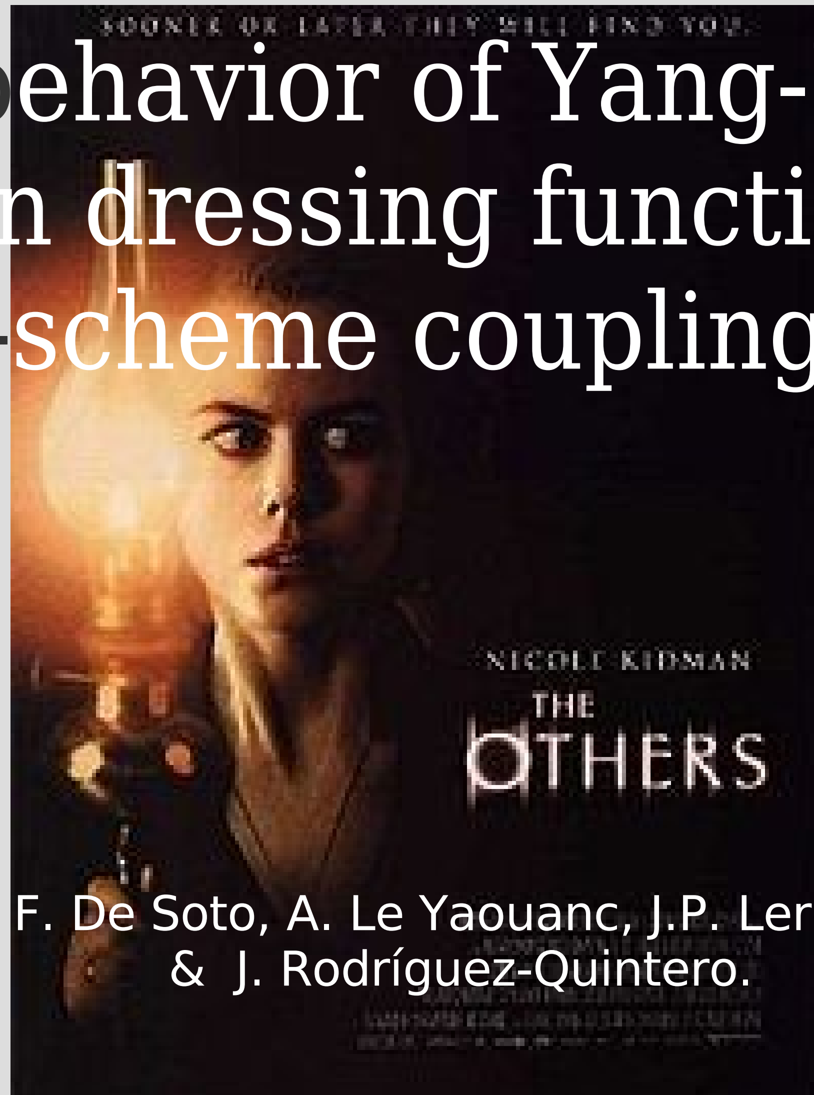
The IR behavior of Yang-Mills ghost and gluon dressing functions and the Taylor-scheme coupling constant

by Ph. Boucaud, F. De Soto, A. Le Yaouanc, J.P. Leroy, J. Micheli, O. Pène
& J. Rodríguez-Quintero.

QCD Bound States, June 15-19, Argonne National Lab

A ghost story

The IR behavior of Yang-Mills ghost and gluon dressing functions and the Taylor-scheme coupling constant



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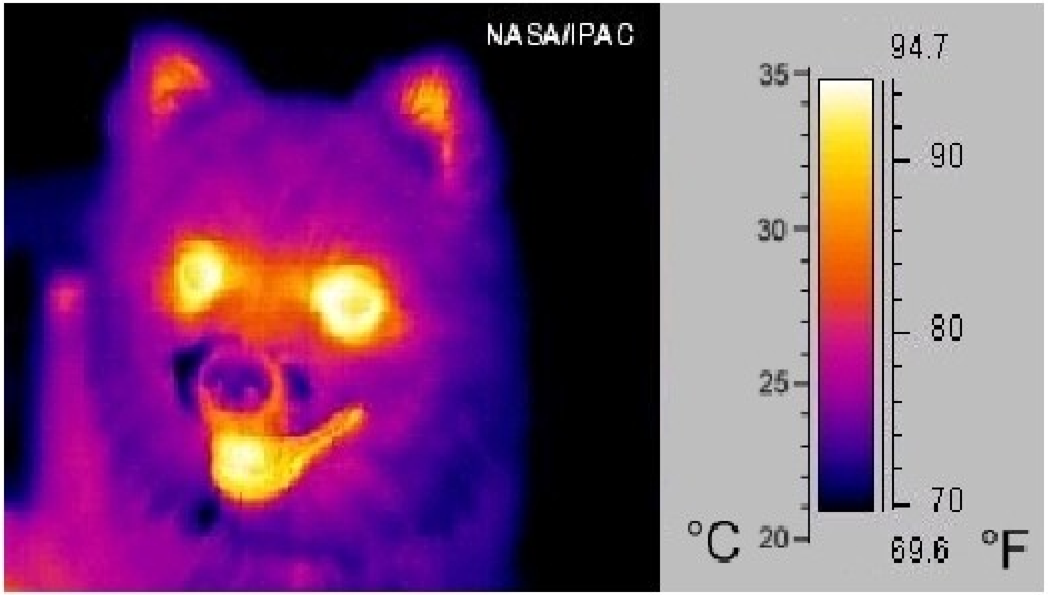
Infra-Red



- Infra-red dog (c'est “chiant” l'infra-rouge!!!)

QCD in the IR(I)

- QCD is « free » in the UV and confining in the IR. Hence the interest in IR behaviour. There exists different models for confinement which usually imply some consequences about the IR behaviour of Green functions.
- Zwanziger's conjecture that confinement has to do with Gribov horizon has such implications.

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Existing tools ?

- There are two sets of very usefus analytic relations to learn about QCD in the IR: Ward-Slavnov-Taylor (WST) identities and the infinite tower of Dyson-Schwinger (DS) integral equations. Lattice QCD give also essential numerical indications.

The best would be to have an analytic solution, however this is not possible:

- WST relates Green-Functions, not enough constraints.
- DS are too complicated, highly non linear, it is not known how many solutions exist, but there is presumably a large number.

Common way out ?

- Use truncated DS with some hypotheses about vertex functions and compare output to LQCD

QCD in the IR(II)

WE PREFER

1- Combine informations from LQCD and analytic methods: not only using LQCD as an a posteriori check, but use it as an input for DSE. We believe that this allows a better control on systematic uncertainties of all methods.

2- Use WST identities (usually overlooked). This however leads today to a unsolved problem.

3- 1 and 2 are complemented with **mild** regularity assumptions about vertex functions

4- Take due care of the UV behaviour (known since QCD is asymptotically free) and use a well defined renormalisation procedure (no renormalisation at μ=0 because of possible IR singularities).

QCD in the IR(III)

Notations

- $G(p^2)$ is the gluon dressing function, $= p^2 G^{(2)}(p^2)$, $G^{(2)}(p^2)$ being the gluon propagator, G LIKE GLUON; $Z_3(\mu^2) = G(\mu^2)$ [MOM renormalisation constant of the gluon propagator]

(frequent notation (fn): $D(p^2)$instead of $G^{(2)}(p^2)$),

- $F(p^2)$ is the ghost dressing function, $= p^2 F^{(2)}(p^2)$, $F^{(2)}(p^2)$ being the ghost propagator, F LIKE FANTÔME; $\widetilde{Z}_3(\mu^2) = F(\mu^2)$ [MOM renormalisation constant of the ghost propagator]

(fn: $G(p^2)$instead of $F^{(2)}(p^2)$)

- In the deep IR it is assumed $G(p^2) \propto (p^2)^{\alpha_G}$

(fn: $p^2 D(p^2) \propto (p^2)^{\alpha_D}$ or $(p^2)^{\delta gl}$; $\alpha_G = 2\kappa$)

- In the deep IR it is assumed $F(p^2) \propto (p^2)^{\alpha_F}$

(fn: $p^2 G(p^2) \propto 1/(p^2)^{\alpha_G}$ or $(p^2)^{\delta gh}$; $\alpha_F = -\kappa$)

QCD in the IR(IV)

NON-PERTURBATIVE DEFINITIONS OF THE STRONG COUPLING CONSTANT

- Compute a three-gluon or ghost-ghost-gluon Green function, in a well defined kinematics depending on a scale μ, and the gluon and ghost propagators.
- **From there compute the corresponding bare vertex function Γ_B**
- Then: $g_R(\mu^2) = g_0 G(\mu^2)^{3/2}\,\Gamma_B$ or $g_R(\mu^2) = g_0\, F(\mu^2)\, G(\mu^2)^{1/2}\,\Gamma_B$
- Special and **preferred** case (Von Smekal) : one vanishing ghost momentum. Taylor: $\Gamma_B=1$

  $g_T(\mu^2) = g_0\, F(\mu^2)\, G(\mu^2)^{1/2}$

  $\alpha_T(\mu^2) = g_0^2/(4\pi)\, F(\mu^2)^2\, G(\mu^2)$

One starting remark:

$F(p^2)^2 G(p^2)$ is thus proportional to $\alpha_T(\mu^2)$

Lattice indicates $\alpha_G \sim 1$, $\alpha_F \sim 0_-$, $F(\mu^2)^2 G(\mu^2) \to 0$, $g^f(\mu) \to 0$

A frequent analysis of the ghost propagator DS equation

Leads to $2\alpha_F + \alpha_G = 0$ (fn: $\alpha_D = 2\,\alpha_G$ or $\delta_{gl} = -2\,\delta_{gl} = 2\kappa$) i.e. $F(p^2)^2 G(p^2) \to ct$

and $F(p^2) \to \infty$

In contradiction with lattice

This is a strong, non truncated DS equation

So what ?

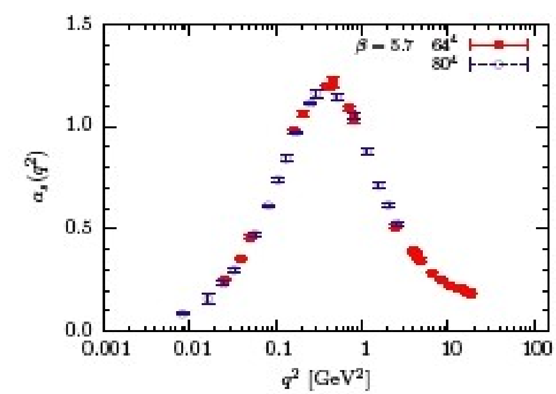
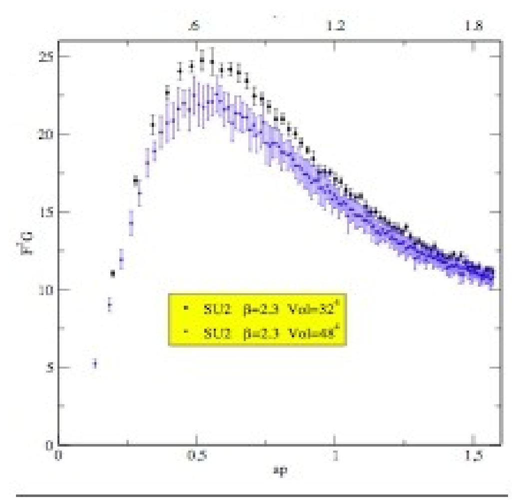
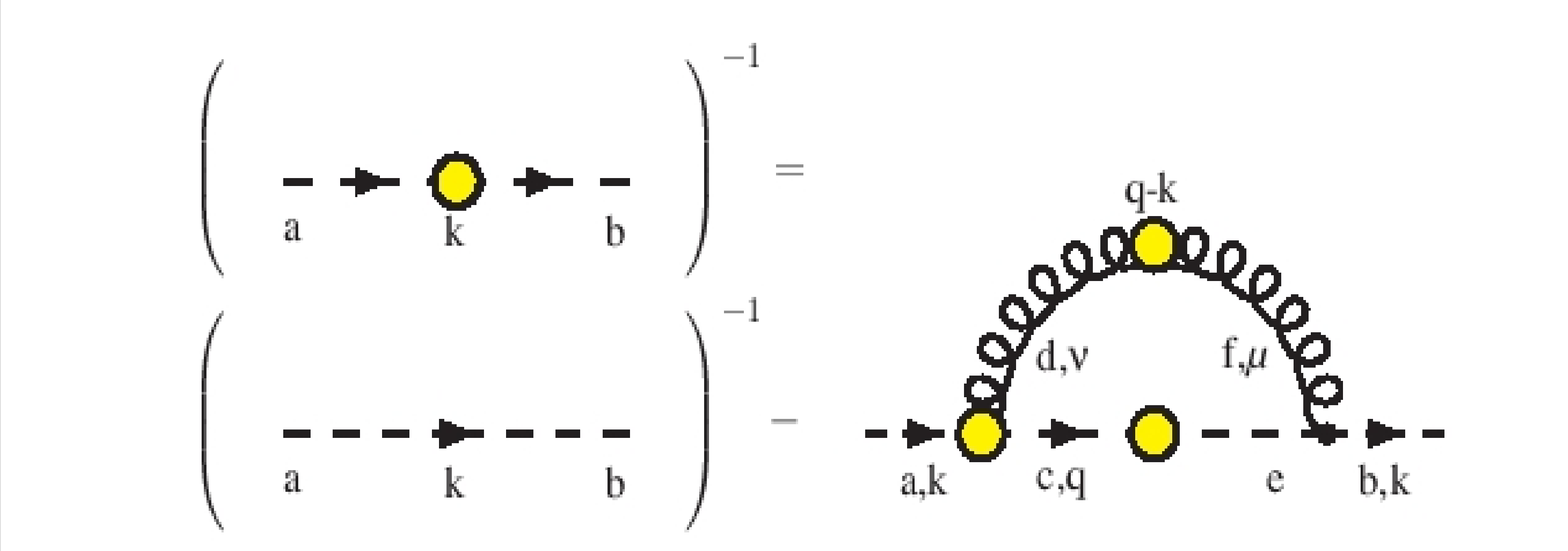


Figure 5: Running coupling $\alpha_s(q^2)$ versus q^2 for lattice sizes 64^4 and 80^4 at $\beta = 5.70$.

Lattice gluodynamics computation of Landau gauge Green's functions in the deep infrared.
I.L. Bogolubsky , E.M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck arXiv:0901.0736



The Ghost-propagator DSE (I)



The Ghost-propagator DSE (I)

$$\frac{1}{F(k^2)} = 1 + g_0^2 N_c \int \frac{d^4q}{(2\pi)^4} \left(\frac{F(q^2)G((q-k)^2)}{q^2(q-k)^4} \left[\frac{(k\cdot q)^2}{k^2} - q^2 \right] H_1(q,k) \right)$$

where:

$$= i g_0 f^{abc} \left(q_\nu H_1(q,k) + (q-k)_\nu H_2(q,k) \right)$$

All in Landau gauge!!

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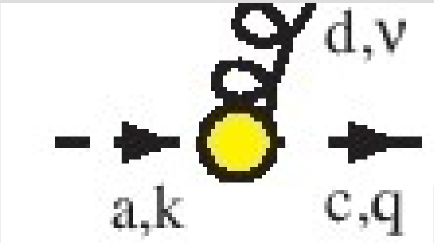

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All in Landau gauge!!

The Ghost-propagator DSE (II)

After MOM renormalization:

$$
\begin{aligned}
g_R^2(\mu^2) &= Z_g^{-2}(\mu^2,\Lambda) g_0^2(\Lambda) \\
G_R(k^2,\mu^2) &= Z_3^{-1}(\mu^2,\Lambda) G(k^2,\Lambda) \\
F_R(k^2,\mu^2) &= \widetilde{Z}_3^{-1}(\mu^2,\Lambda) F(k^2,\Lambda) \\
\widetilde{Z}_1 &= Z_g Z_3^{1/2} \widetilde{Z}_3 \,,
\end{aligned}
$$

One obtains:

$$
\frac{1}{F_R(k^2,\mu^2)} = \widetilde{Z}_3(\mu^2,\Lambda) + N_C \widetilde{Z}_1 \, g_R^2(\mu^2) \, \Sigma_R(k^2,\mu^2;\Lambda)
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$$\Sigma_R(k^2,\mu^2;\Lambda) = \int^{q^2<\Lambda^2} \frac{d^4q}{(2\pi)^4} \times \left(\frac{F_R(q^2,\mu^2) G_R((q-k)^2,\mu^2)}{q^2(q-k)^4} \left[\frac{(k\cdot q)^2}{k^2} - q^2 \right] H_{1,R}(q,k;\mu^2) \right)$$

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$$\int dq^2/q^2 (1 + 11\alpha_S/(2\pi)\log(q/\mu))^{-35/44})$$

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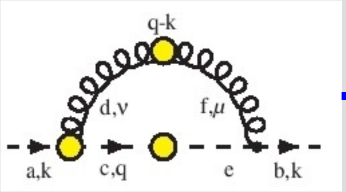

The integral is UV divergent!!

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UV-cutoff dependences cancel to each other!!

(in virtue of the interplay of ghost and gluon anomalous dimensions)

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For not to have to deal with UV, on can subtract:

$$\frac{1}{F_R(\lambda^2 k^2,\mu^2)} - \frac{1}{F_R(\lambda^2\kappa^2 k^2,\mu^2)} = N_C\, g_R^2(\mu^2)\, \widetilde{Z}_1 \left(\Sigma_R(\lambda^2 k^2,\mu^2;\infty) - \Sigma_R(\lambda^2\kappa^2 k^2,\mu^2;\infty)\right)$$

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λ will ultimately go to 0

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Then:

$$\Sigma_R(\lambda^2 k^2,\mu^2;\infty) - \Sigma_R(\lambda^2\kappa^2 k^2,\mu^2;\infty) = I_{\rm IR}(\lambda) + I_{\rm UV}(\lambda)$$

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$$I_{\mathrm{UV}}(\lambda) \sim \lambda^2$$

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$$\Sigma_R(\lambda^2 k^2,\mu^2;\infty) - \Sigma_R(\lambda^2\kappa^2 k^2,\mu^2;\infty) = I_{\rm IR}(\lambda) + I_{\rm UV}(\lambda)$$

$$F_{\rm IR}(q^2,\mu^2) = A(\mu^2)\,(q^2)^{\alpha_F} \quad , \quad G_{\rm IR}(q^2,\mu^2) = B(\mu^2)\,(q^2)^{\alpha_G}$$

$$H_{1,R}(k,q;\mu^2) = 1$$

$$I_{\rm UV}(\lambda) \sim \lambda^2$$

$$I_{\rm IR}(\lambda) \simeq (\lambda^2)^{(\alpha_F+\alpha_G)} A(\mu^2) B(\mu^2) \int^{q^2<\frac{q_0^2}{\lambda^2}} \frac{d^4q}{(2\pi)^4}\, (q^2)^{\alpha_F-1} \left(\frac{(k\cdot q)^2}{k^2} - q^2 \right) \times \left[\left((q-k)^2\right)^{\alpha_G-2} - \left((q-\kappa k)^2\right)^{\alpha_G-2} \right] .$$

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$$\frac{1}{F_R(\lambda^2 k^2,\mu^2)} - \frac{1}{F_R(\lambda^2\kappa^2 k^2,\mu^2)} = N_C\, g_R^2(\mu^2)\,\widetilde{Z}_1 \left(\Sigma_R(\lambda^2 k^2,\mu^2;\infty) - \Sigma_R(\lambda^2\kappa^2 k^2,\mu^2;\infty)\right)$$

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$$\Sigma_R(\lambda^2 k^2,\mu^2;\infty) - \Sigma_R(\lambda^2\kappa^2 k^2,\mu^2;\infty) = I_{\mathrm{IR}}(\lambda) + I_{\mathrm{UV}}(\lambda)$$

$$I_{\mathrm{UV}}(\lambda) \sim \lambda^2$$

$$I_{\mathrm{IR}}(\lambda) \sim \lambda^{2(\alpha_G+\alpha_F)} \int_{\varepsilon}^{q_0/\lambda} dq\, q^{2(\alpha_F+\alpha_G)-3} \sim \begin{cases} \lambda^{2(\alpha_G+\alpha_F)} & \text{if } \alpha_G+\alpha_F<1 \\ \lambda^2 \ln\lambda & \text{if } \alpha_G+\alpha_F=1 \\ \lambda^2 & \text{if } \alpha_G+\alpha_F>1 \end{cases}$$

The Ghost-propagator DSE solutions (I)

$$\frac{1}{F_R(\lambda^2 k^2, \mu^2)} - \frac{1}{F_R(\lambda^2 \kappa^2 k^2, \mu^2)} = N_C\, g_R^2(\mu^2)\, \widetilde{Z}_1 \left(\Sigma_R(\lambda^2 k^2, \mu^2; \infty) - \Sigma_R(\lambda^2 \kappa^2 k^2, \mu^2; \infty) \right)$$

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$$\Sigma_R(\lambda^2 k^2, \mu^2; \infty) - \Sigma_R(\lambda^2 \kappa^2 k^2, \mu^2; \infty) \sim \begin{cases} \lambda^{2(\alpha_G + \alpha_F)} & \text{if } \alpha_G + \alpha_F < 1 \\ \lambda^2 \ln \lambda & \text{if } \alpha_G + \alpha_F = 1 \\ \lambda^2 & \text{if } \alpha_G + \alpha_F > 1 \end{cases}$$

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while ($\alpha_F \neq 0$)

$$\frac{1}{F_R(\lambda^2 k^2,\mu^2)} - \frac{1}{F_R(\lambda^2\kappa^2 k^2,\mu^2)} \simeq \left(1-\kappa^{-2\alpha_F}\right) \frac{\left(\lambda^2 k^2\right)^{-\alpha_F}}{A(\mu^2)}$$

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Then:

$$\Sigma \ldots \quad \begin{cases} \lambda^{2(\alpha_G+\alpha_F)} & \text{if } \alpha_G+\alpha_F<1 \\ \ldots & \ldots = 1 \\ \ldots & \ldots > 1 \end{cases}$$

- If $\alpha_G+\alpha_F>1 \;\Rightarrow\; \alpha_F=-1$ (Excluded by LQCD !!!)
- If $\alpha_G+\alpha_F=1 \;\Rightarrow\;$ No solution !!!
- If $\alpha_G+\alpha_F<1 \;\Rightarrow\; \boxed{2\alpha_F+\alpha_G=0}$

$$\frac{\ldots}{F_R(\lambda^2 k^2,\mu^2)} \quad \frac{\ldots}{F_R(\lambda^2\kappa^2 k^2,\mu^2)} \quad \ldots \quad \frac{\ldots}{A(\mu^2)}$$

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Then:

$\alpha_F \neq 0 \longrightarrow \boxed{2\alpha_F + \alpha_G = 0}$

➔ $F^2(q^2) G(q^2) \to \text{cte., as } q \to 0,$

➔ $N_C\, g_R^2(\mu^2)\, \widetilde{Z}_1 (A(\mu^2))^2 B(\mu^2)\, \phi\left(-\frac{\alpha_G}{2}, \alpha_G\right) \simeq 16\pi^2$

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$$\begin{aligned}
\phi(\alpha_F,\alpha_G) &= -\frac{1}{2}\left(f(\alpha_F,\alpha_G-2) + f(\alpha_F,\alpha_G-1) + f(\alpha_F-1,\alpha_G-1)\right) \\
&+ \frac{1}{4}\left(f(\alpha_F-1,\alpha_G-2) + f(\alpha_F-1,\alpha_G) + f(\alpha_F+1,\alpha_G-2)\right) \\
f(a,b) &= \frac{\Gamma(2+a)\Gamma(2+b)\Gamma(-a-b-2)}{\Gamma(-a)\Gamma(-b)\Gamma(4+a+b)}
\end{aligned}$$

The Ghost-propagator DSE solutions (II)

$$\frac{1}{F_R(\lambda^2 k^2,\mu^2)} - \frac{1}{F_R(\lambda^2\kappa^2 k^2,\mu^2)} = N_C\, g_R^2(\mu^2)\,\widetilde{Z}_1 \left(\Sigma_R(\lambda^2 k^2,\mu^2;\infty) - \Sigma_R(\lambda^2\kappa^2 k^2,\mu^2;\infty)\right)$$

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Then:

$$\Sigma_R(\lambda^2 k^2,\mu^2;\infty) - \Sigma_R(\lambda^2\kappa^2 k^2,\mu^2;\infty) \sim \begin{cases} \lambda^{2(\alpha_G+\alpha_F)} & \text{if } \alpha_G+\alpha_F<1 \\ \lambda^2 \ln\lambda & \text{if } \alpha_G+\alpha_F=1 \\ \lambda^2 & \text{if } \alpha_G+\alpha_F>1 \end{cases}$$

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Then:

$$\Sigma_R(\lambda^2 k^2, \mu^2; \infty) - \Sigma_R(\lambda^2 \kappa^2 k^2, \mu^2; \infty) \sim \begin{cases} \lambda^{2(\alpha_G+\alpha_F)} & \text{if } \alpha_G+\alpha_F < 1 \\ \lambda^2 \ln\lambda & \text{if } \alpha_G+\alpha_F = 1 \\ \lambda^2 & \text{if } \alpha_G+\alpha_F > 1 \end{cases}$$

while ($\alpha_F = 0$)

$$\frac{1}{F_R(\lambda^2 k^2, \mu^2)} - \frac{1}{F_R(\lambda^2 \kappa^2 k^2, \mu^2)} \simeq -\frac{A_2(\mu^2)}{(A(\mu^2))^2} \begin{cases} (\lambda^2 k^2)^{\alpha_F} (1-\kappa^{2\alpha_F}) & \text{if } \alpha_G < 1 \\ k^2(1-\kappa^2)\lambda^2 \ln\lambda^2 & \text{if } \alpha_G = 1 \\ (\lambda^2 k^2)(1-\kappa^2) & \text{if } \alpha_G > 1 \end{cases}$$

The Ghost-propagator DSE solutions (II)

$$\frac{1}{F_R(\lambda^2k^2,\mu^2)} - \frac{1}{F_R(\lambda^2\kappa^2k^2,\mu^2)} = N_C\, g_R^2(\mu^2)\, \widetilde{Z}_1 \left(\Sigma_R(\lambda^2k^2,\mu^2;\infty) - \Sigma_R(\lambda^2\kappa^2k^2,\mu^2;\infty)\right)$$

λ will ultimately go to 0

κ is positive and non-zero

Then:

$$\lambda^{2(\alpha_G+\alpha_F)} \quad \text{if } \alpha_G+\alpha_F<1$$

$$(\lambda^2k^2)(1-\kappa^2) \quad \text{if } \alpha_G>1$$

$$F_{\rm IR}(q^2,\mu^2) = \begin{cases} A(\mu^2)\left(1-\phi(0,\alpha_G)\dfrac{\widetilde{g}^2(\mu^2)}{16\pi^2}A^2(\mu^2)B(\mu^2)(q^2)^{\alpha_G}\right) & \alpha_G<1 \\ A(\mu^2)\left(1+\dfrac{\widetilde{g}^2(\mu^2)}{64\pi^2}A^2(\mu^2)B(\mu^2)\, q^2\ln q^2\right) & \alpha_G=1 \\ A(\mu^2)+A_2(\mu^2)q^2 & \alpha_G>1 \end{cases}$$

with $\widetilde{g}^2(\mu^2) = N_C g_R(\mu^2)\widetilde{Z}_1$

The Ghost-propagator DSE solutions (III)

$$\frac{1}{F_R(\lambda^2 k^2,\mu^2)} - \frac{1}{F_R(\lambda^2\kappa^2 k^2,\mu^2)} = N_C\, g_R^2(\mu^2)\, \widetilde{Z}_1 \left(\Sigma_R(\lambda^2 k^2,\mu^2;\infty) - \Sigma_R(\lambda^2\kappa^2 k^2,\mu^2;\infty)\right)$$

λ will ultimately go to 0

κ is positive and non-zero

Then:

$\alpha_F \neq 0 \rightarrow \boxed{2\alpha_F + \alpha_G = 0}$

$\alpha_F = 0 \rightarrow$

$$F_{\rm IR}(q^2,\mu^2) = \begin{cases} A(\mu^2)\left(1 - \phi(0,\alpha_G)\dfrac{\widetilde{g}^2(\mu^2)}{16\pi^2}A^2(\mu^2)B(\mu^2)(q^2)^{\alpha_G}\right) & \alpha_G < 1 \\ A(\mu^2)\left(1 + \dfrac{\widetilde{g}^2(\mu^2)}{64\pi^2}A^2(\mu^2)B(\mu^2)\; q^2 \ln q^2\right) & \alpha_G = 1 \\ A(\mu^2) + A_2(\mu^2) q^2 & \alpha_G > 1 \end{cases}$$

The Ghost-propagator DSE solutions (III)

$$\frac{1}{F_R(\lambda^2 k^2,\mu^2)} - \frac{1}{F_R(\lambda^2\kappa^2 k^2,\mu^2)} = N_C\, g_R^2(\mu^2)\,\widetilde{Z}_1 \left(\Sigma_R(\lambda^2 k^2,\mu^2;\infty) - \Sigma_R(\lambda^2\kappa^2 k^2,\mu^2;\infty)\right)$$

λ: will ultimately go to 0

κ: is positive and non-zero

Then:

$\alpha_F \neq 0 \rightarrow \boxed{2\alpha_F + \alpha_G = 0}$

LQCD favours

$\alpha_F = 0 \rightarrow$

$$F_{\text{IR}}(q^2,\mu^2) = \begin{cases} A(\mu^2)\left(1 - \phi(0,\alpha_G)\dfrac{\widetilde{g}^2(\mu^2)}{16\pi^2}A^2(\mu^2)B(\mu^2)(q^2)^{\alpha_G}\right) & \alpha_G < 1 \\ A(\mu^2)\left(1 + \dfrac{\widetilde{g}^2(\mu^2)}{64\pi^2}A^2(\mu^2)B(\mu^2)\; q^2\ln q^2\right) & \alpha_G = 1 \\ A(\mu^2) + A_2(\mu^2)q^2 & \alpha_G > 1 \end{cases}$$

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λ will ultimately go to 0

κ is positive and non-zero

Then:

To summarise

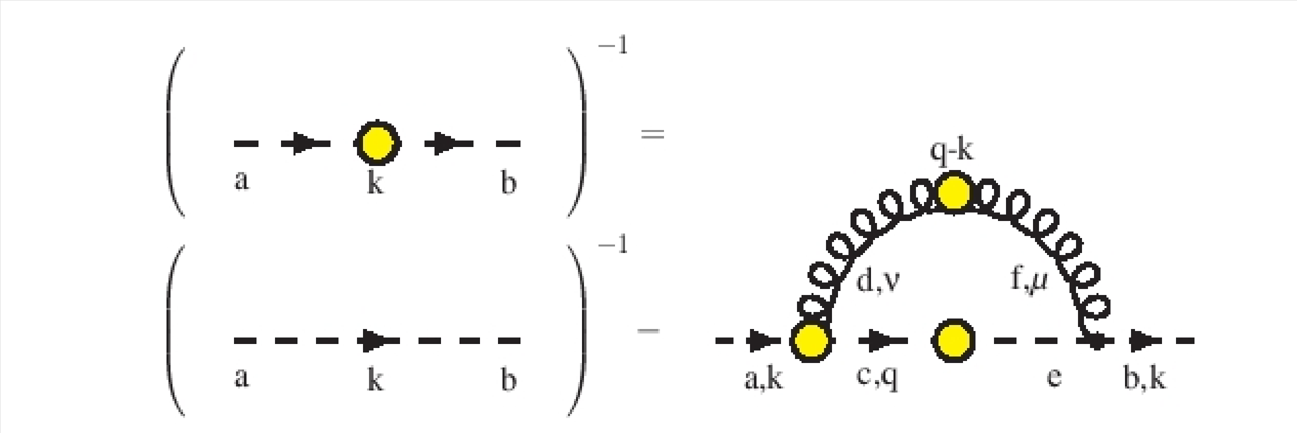
I. If $\alpha_F < 0$, $2\alpha_F + \alpha_G = 0$, $F(p^2)^2 G(p^2) \to ct \neq 0$ and fixed coupling constant at a finite scale; $\alpha_G = -2\,\alpha_F = 2\kappa$

From arXiv:0801.2762, Alkofer et al, $-0.75 \leq \alpha_F \leq -0.5$, $1 \leq \alpha_G \leq 1.5$

II. if $\alpha_F = 0$, $F(p^2) \to ct \neq 0$ and no fixed coupling constant

Notice: solution II agrees rather well with lattice !!

Numerical GPDSE solutions



Numerical GPDSE solutions

$$\frac{1}{F_R(k)} - \frac{1}{F_R(q_0)} = N_c g_R^2 \widetilde{Z_1} \int \frac{d^4q}{(2\pi)^4} \frac{F_R(q^2)}{q^2} \left(\frac{(k.q)^2}{q^2} - q^2 \right)$$
$$\left[\frac{G_R((q-k)^2) H_{1R}(q,k)}{((q-k)^2)^2} - \frac{G_R((q-q_0)^2) H_{1R}(q,q_0)}{((q-q_0)^2)^2} \right]$$

- To solve this equation one needs an input for the gluon propagator G_R (we take it from LQCD, extended to the UV via perturbative QCD) and for the ghost-ghost-gluon vertex H_{1R}: regularity is usually assumed from Taylor identity and confirmed by LQCD.
- To be more specific, we take H_{1R} to be constant, and G_R from lattice data interpolated with the α_G=1 IR power. For simplicity we subtract at k'=0. We take μ=1.5 GeV.

Numerical GPDSE solutions

with $\widetilde{g}^2(\mu^2) = N_C g_R(\mu^2)\widetilde{Z}_1$

0

$$\frac{1}{F_R(k)} - \frac{1}{F_R(q_0)} = N_c g_R^2 \widetilde{Z}_1 \int \frac{d^4 q}{(2\pi)^4} \frac{F_R(q^2)}{q^2} \left(\frac{(k.q)^2}{q^2} - q^2 \right) \left[\frac{G_R((q-k)^2)\cancel{H_{1R}(q,k)}}{((q-k)^2)^2} - \frac{G_R((q-\cancel{q_0})^2)\cancel{H_{1R}(q,q_0)}}{((q-\cancel{q_0})^2)^2} \right]$$

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Numerical GPDSE solutions

with $\widetilde{g}^2(\mu^2) = N_C g_R(\mu^2)\widetilde{Z}_1$

$F_R(k^2) = \widetilde{F}(k^2)/\widetilde{g}(\mu)$

$$\frac{1}{\widetilde{F}(k^2)} = \frac{1}{\widetilde{F}(0)} - \int \frac{d^4q}{(2\pi)^4}\left(1 - \frac{(k.q)^2}{k^2q^2}\right) \times$$

$$\times \left[\frac{G_R((q-k)^2)}{((q-k)^2)^2} - \frac{G_R((q)^2)}{((q)^2)^2}\right]\widetilde{F}(q^2)$$

- To solve this equation one needs an input for the gluon propagator G_R (we take it from LQCD, extended to the UV via perturbative QCD) and for the ghost-ghost-gluon vertex H_{1R}: regularity is usually assumed from Taylor identity and confirmed by LQCD.
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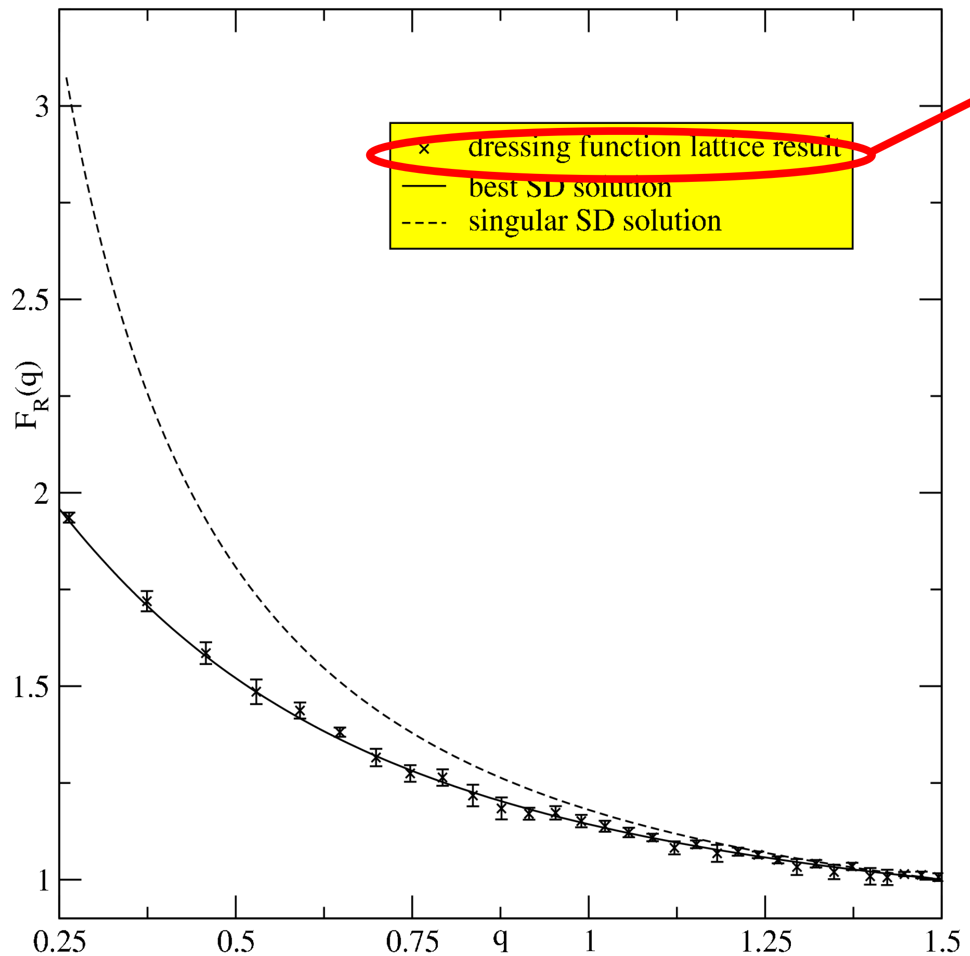
Numerical GPDSE solutions

Dressing function:



$\beta = 5.8, V = 32^4$

Ph. Boucaud, J-P. Leroy, A.Le Yaouanc,
J. Micheli, O. Pene, J. Rodriguez--Quintero
e-Print: arXiv:0801.2721 [hep-ph]

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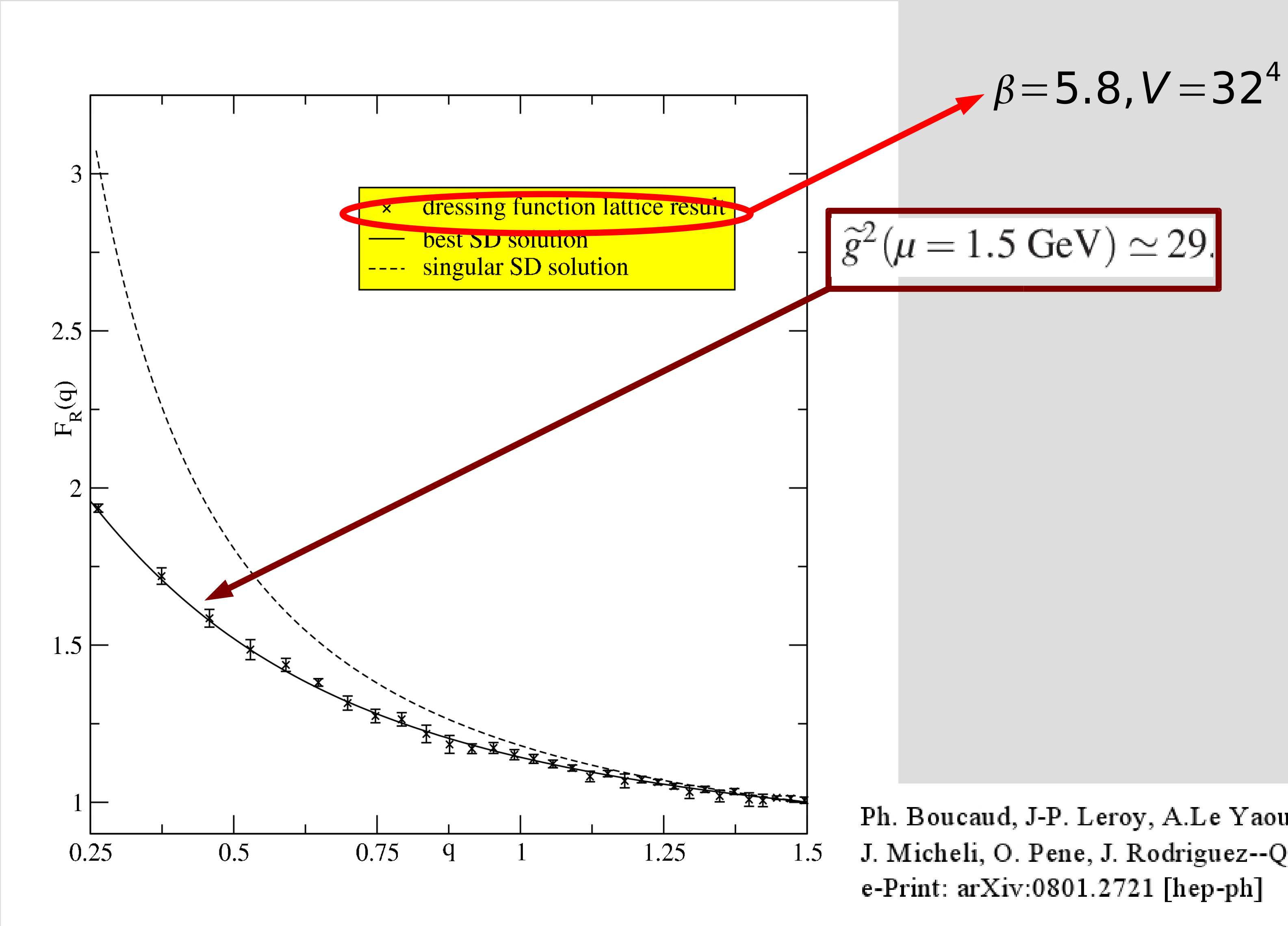

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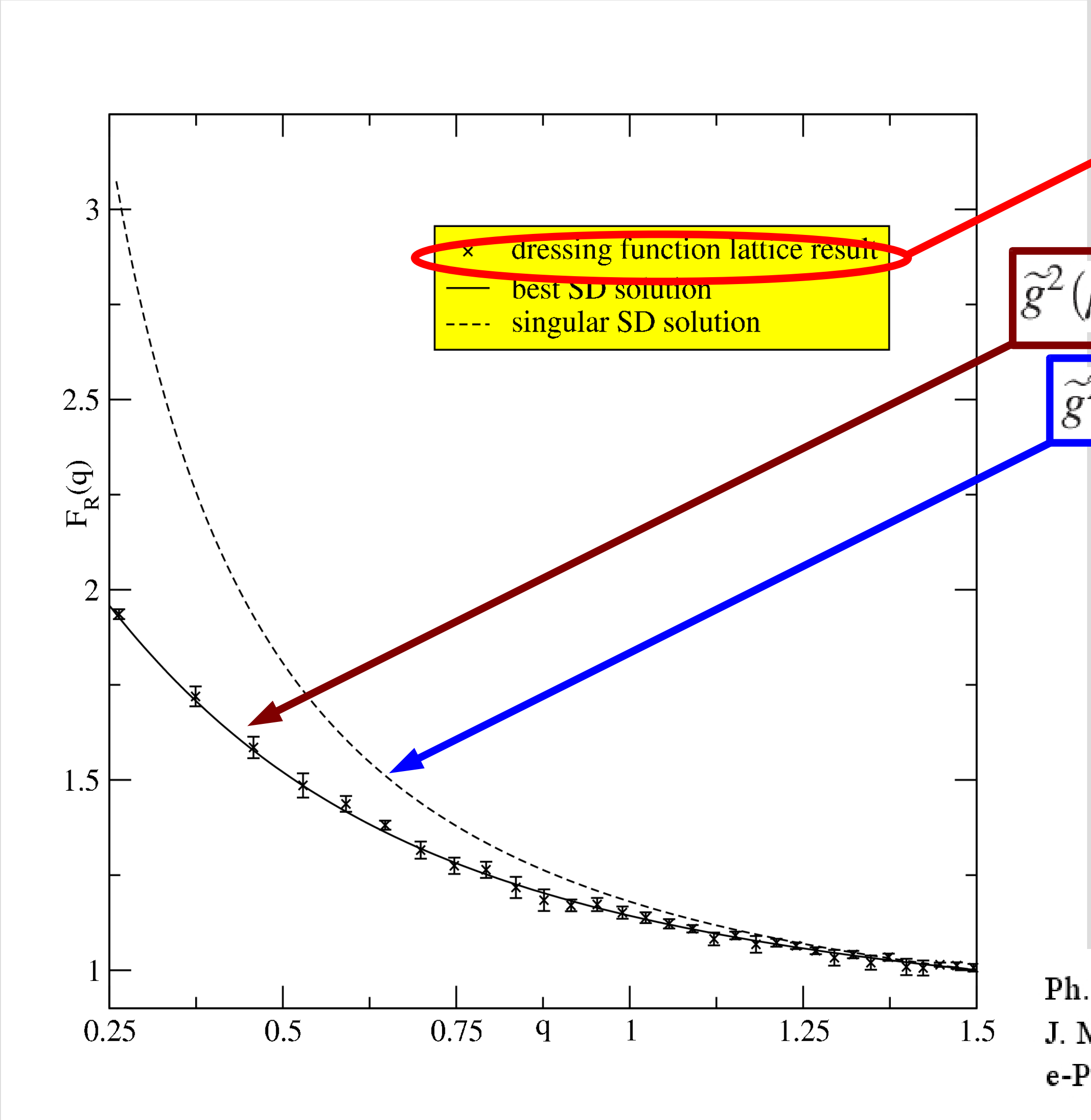

$\beta = 5.8, V = 32^4$

$\widetilde{g}^2(\mu = 1.5\ \mathrm{GeV}) \simeq 29.$

$\widetilde{g}^2(\mu = 1.5\ \mathrm{GeV}) \simeq 33.197$

Ph. Boucaud, J-P. Leroy, A.Le Yaouanc,
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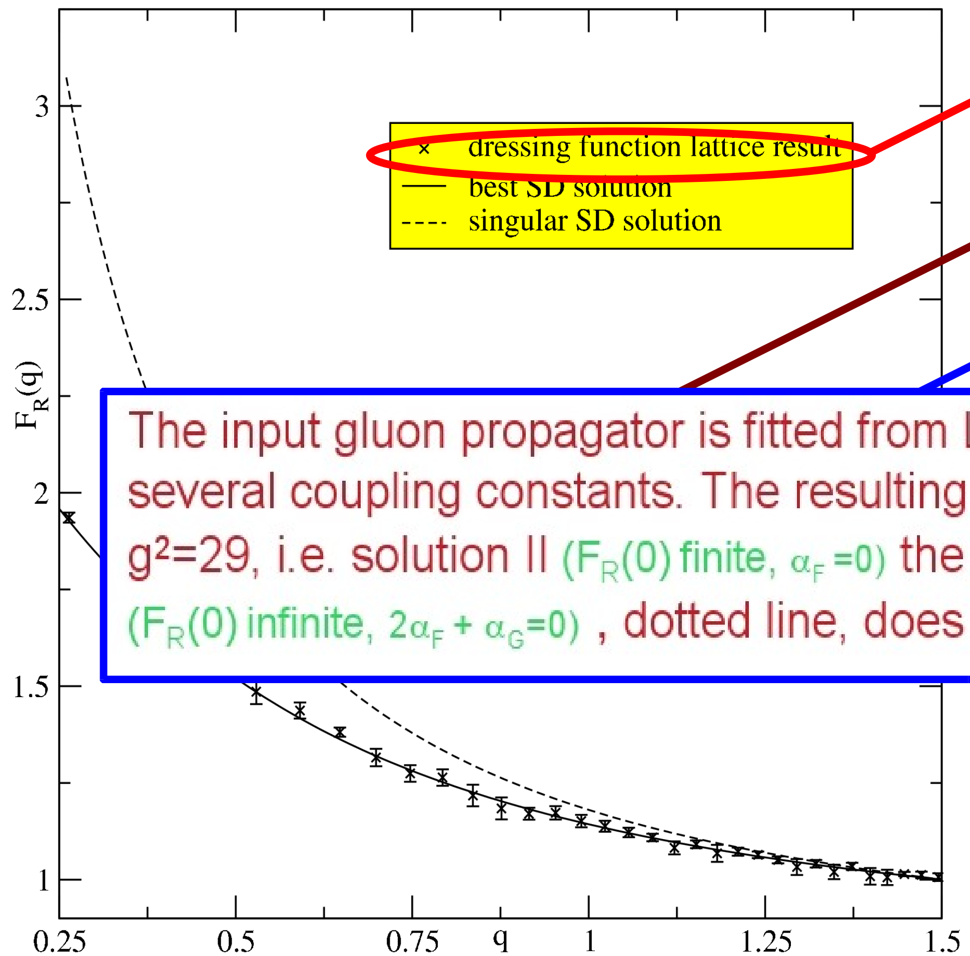


$\beta = 5.8, V = 32^4$

$\tilde{g}^2(\mu = 1.5\ \mathrm{GeV}) \simeq 29$

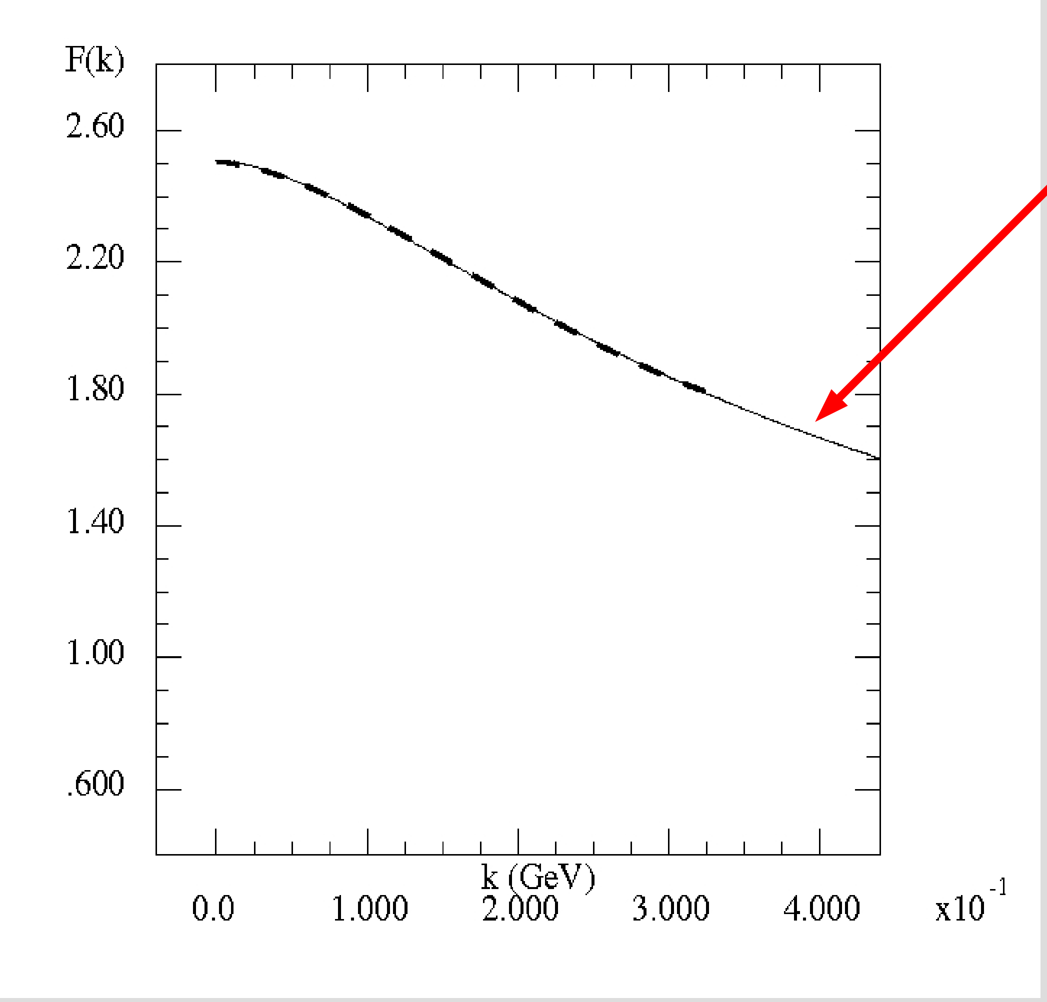
$\tilde{g}^2(\mu = 1.5\ \mathrm{GeV}) \simeq 33.197$

The input gluon propagator is fitted from LQCD. The DSE is solved numerically for several coupling constants. The resulting F_R is compared to lattice results. For $g^2=29$, i.e. solution II ($F_R(0)$ finite, $\alpha_F=0$) the agreement is striking. The solution I ($F_R(0)$ infinite, $2\alpha_F + \alpha_G=0$) , dotted line, does not fit at all.

Ph. Boucaud, J-P. Leroy, A.Le Yaouanc, J. Micheli, O. Pene, J. Rodriguez--Quintero
e-Print: arXiv:0801.2721 [hep-ph]

Numerical GPDSE solutions

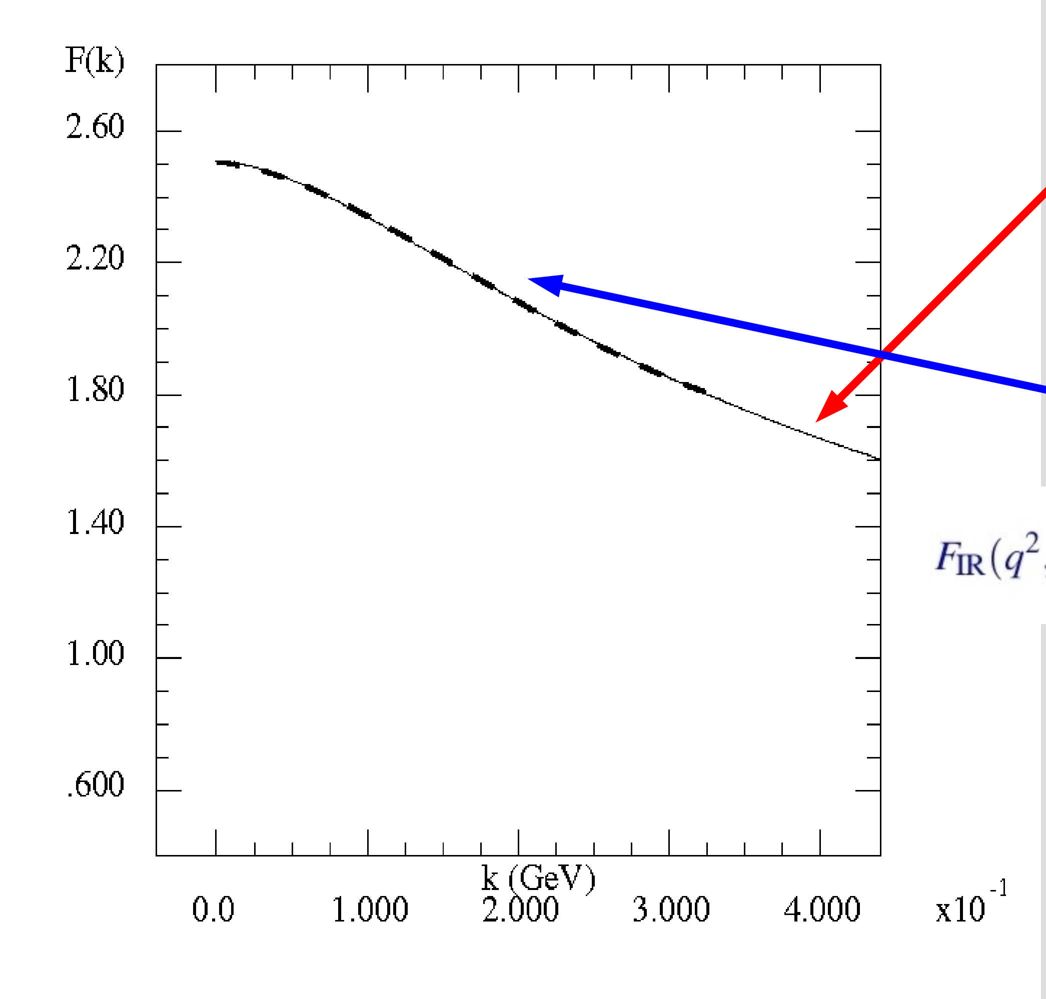
Asymptotic behavior



Solid line is the numerical dressing function previously shown

Numerical GPDSE solutions

Asymptotic behavior



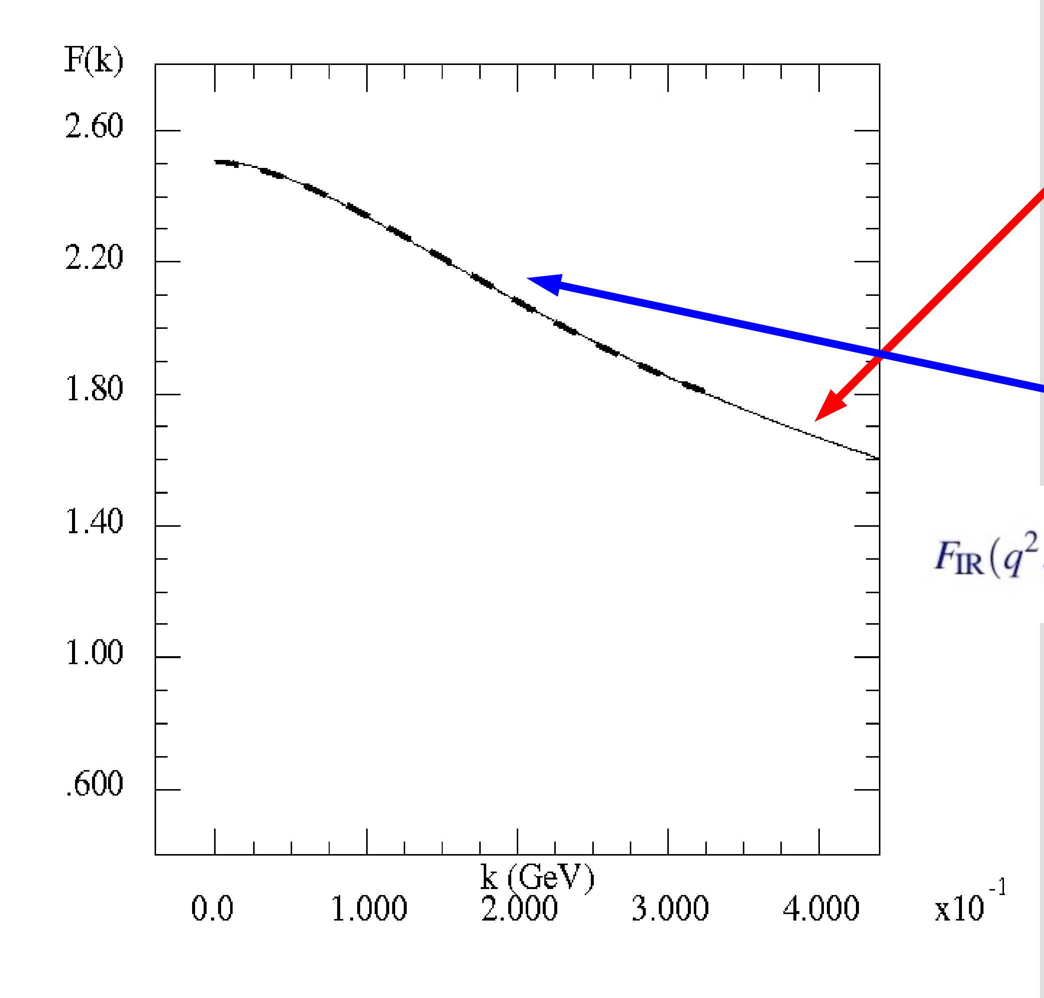
Solid line is the numerical dressing function previously shown

Dotted line is:

$$F_{\mathrm{IR}}(q^2,\mu^2) = A(\mu^2)\left(1 + \frac{\tilde{g}^2(\mu^2)}{64\pi^2} A^2(\mu^2) B(\mu^2)\, q^2 \ln q^2\right)$$

Numerical GPDSE solutions

Asymptotic behavior



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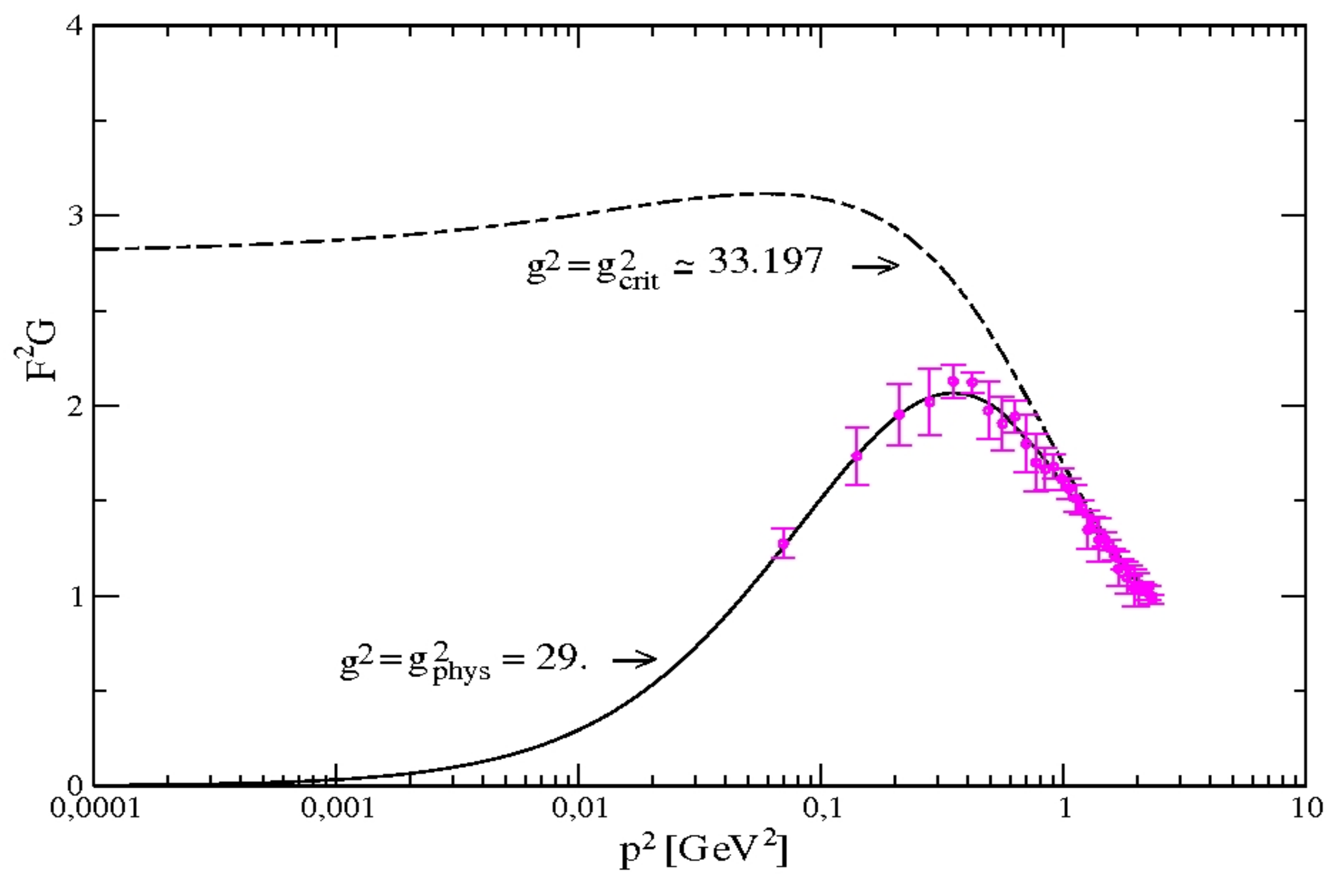
1.64 GeV^{-2}

obtained from the gluon and ghost dressing functions at zero momentum and the value of the coupling applied to solve GPDSE!!!

Numerical GPDSE solutions

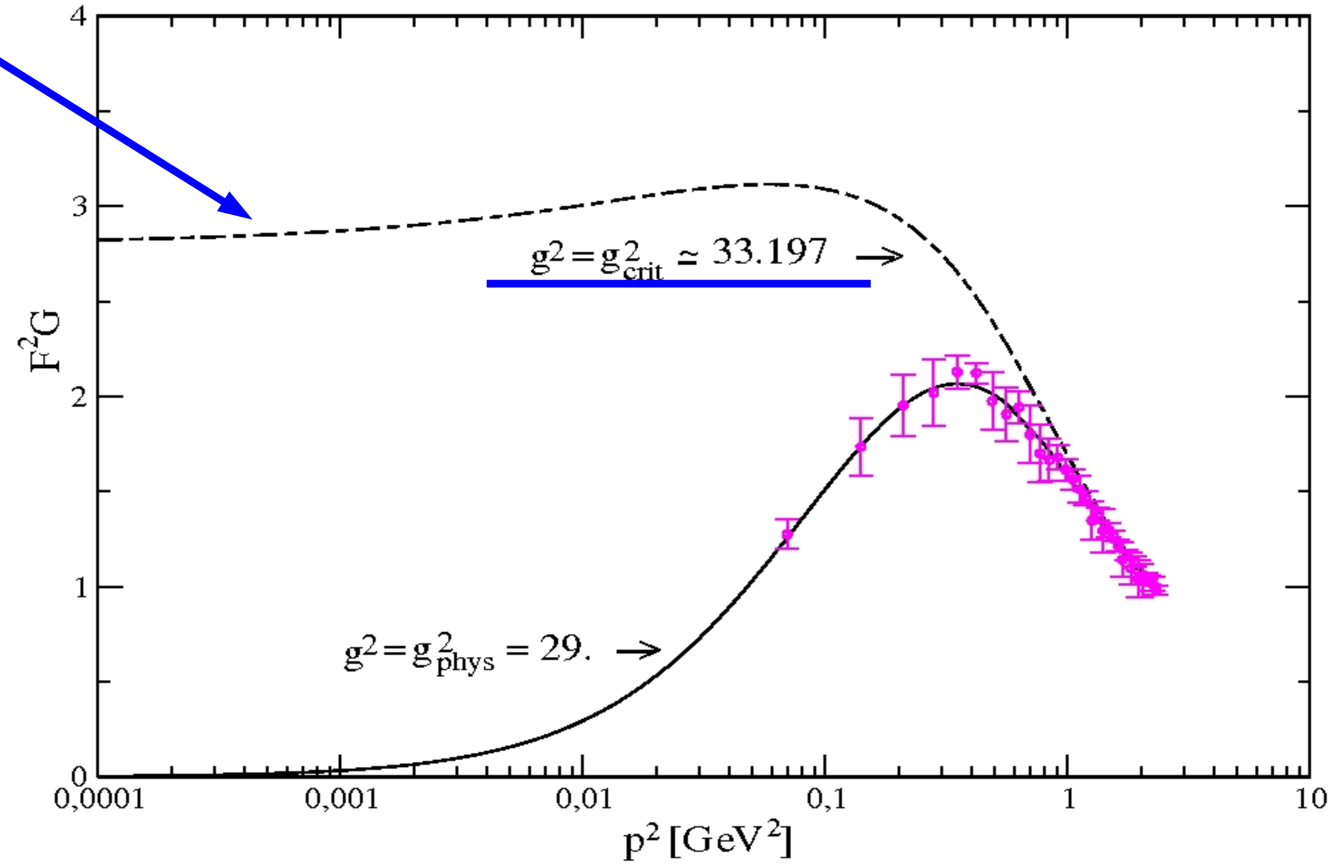
ghost-gluon coupling:



Numerical GPDSE solutions

ghost-gluon coupling:

Solution I (scaling)

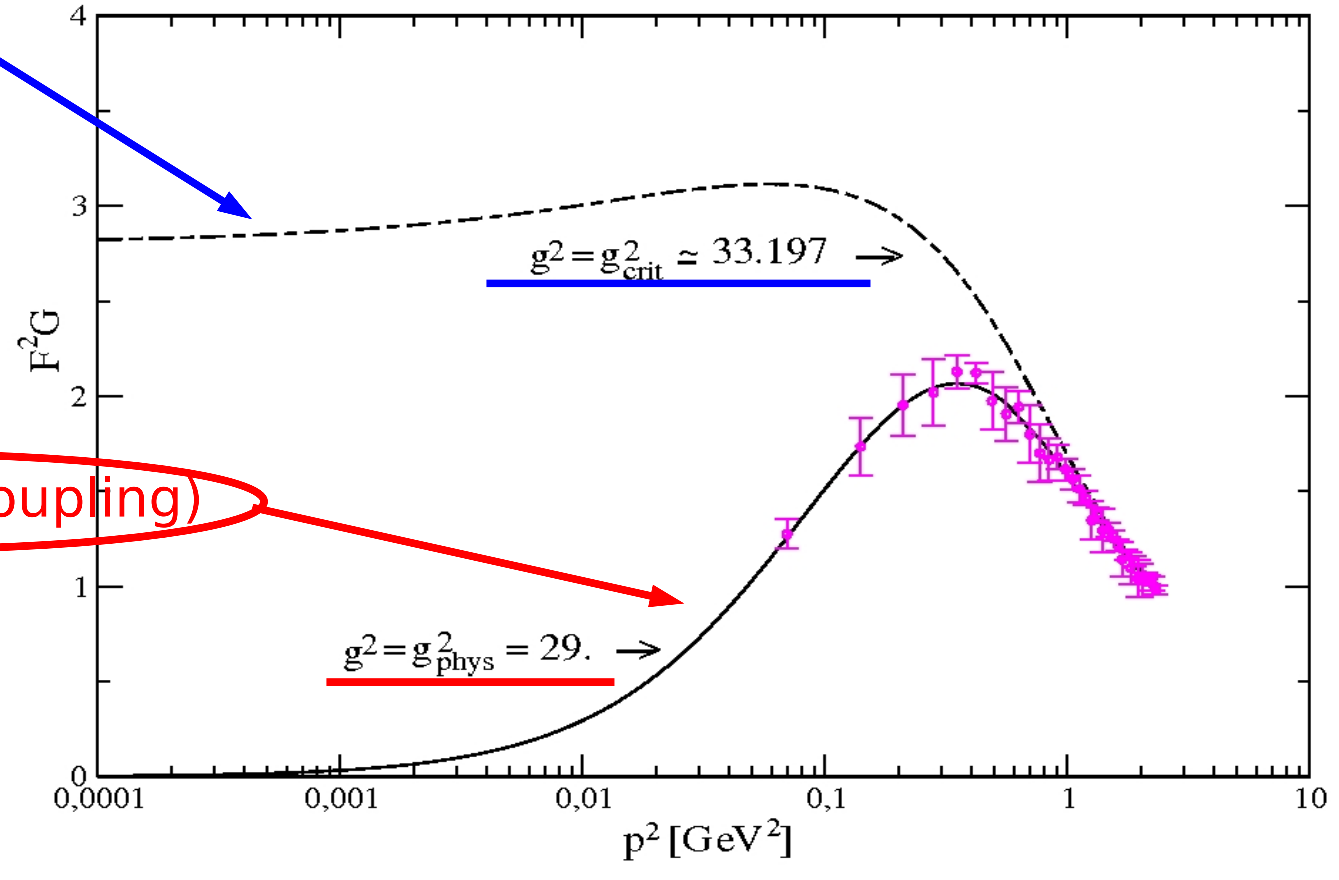


Numerical GPDSE solutions

ghost-gluon coupling:

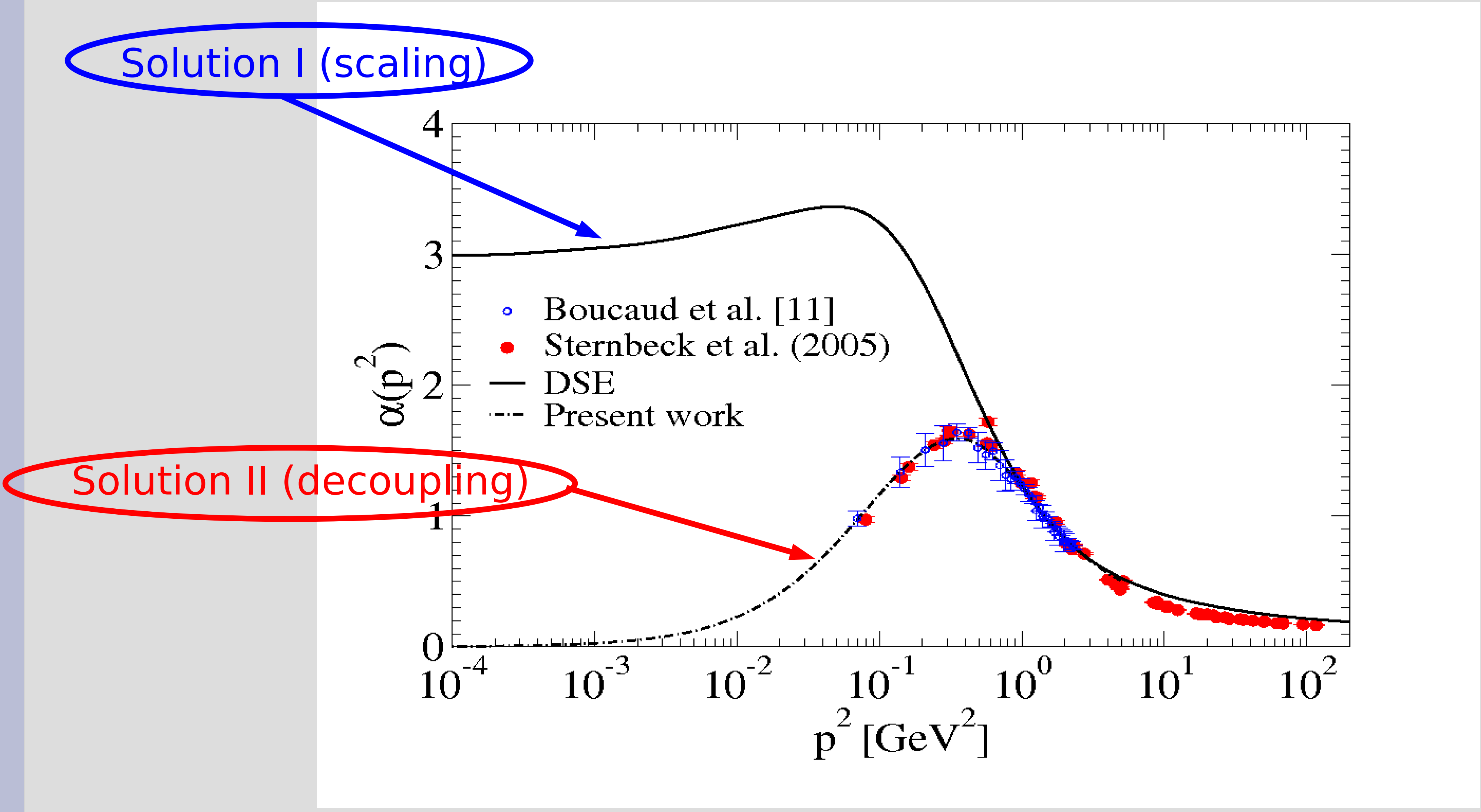
Solution I (scaling)

Solution II (decoupling)



Numerical GPDSE solutions

ghost-gluon coupling:



Numerical GPDSE solutions

To summarize:

- We find one and only one solution for any positive value of $\tilde{F}(0)$. $F(0)=\infty$ corresponds to a critical value:

$$g_c^2 = 10\pi^2/(F_R^2(0)\ G_R^{(2)}(0)) \quad (\text{fn: } 10\pi^2/(D_R(0) \lim p^2 G_R(p^2))$$

J. C. R. Bloch, Few Body Syst. **33** (2003) 111 [arXiv:hep-ph/0303125]

- This critical solution corresponds to $F_R(0)=\infty$, It is the solution I, with $2\alpha_F + \alpha_G = 0$, $F(p^2)^2 G(p^2) \to ct \neq 0$, a diverging ghost dressing function and a fixed coupling constant.

- The non-critical solutions, have $F_R(0)$ finite, i.e. $\alpha_F = 0$, the behaviour $F_R(k^2)=F_R(0) + c\, k^2 \log(k^2)$ has been checked.

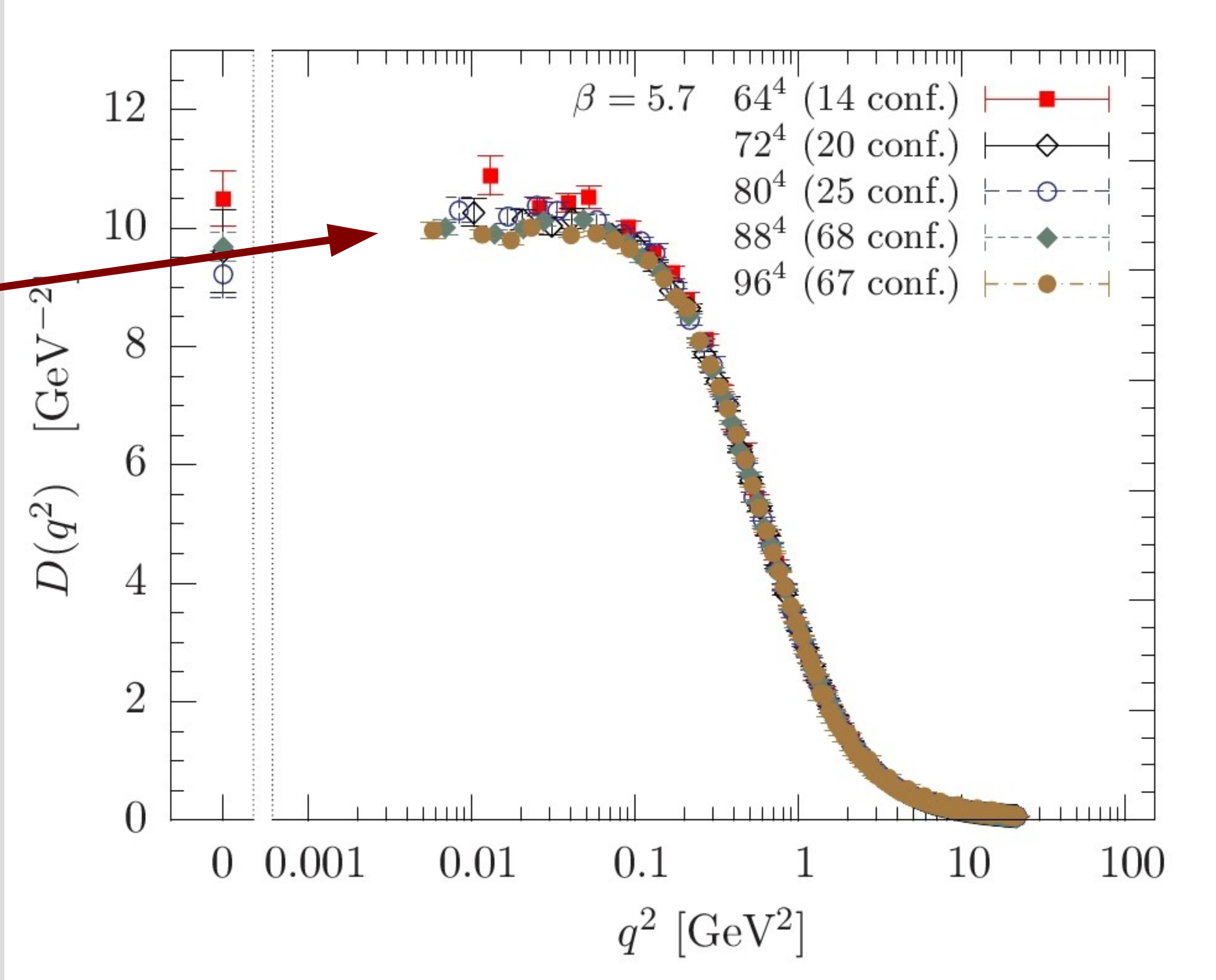
- Not much is changed if we assume a logarithmic divergence of the gluon propagator for $k \to 0$: $F_R(k^2)=F_R(0) - c'\, k^2 \log^2(k^2)$

Lattice (recent) results (I)

Gluon propagator:

$\alpha_G = 1$

(massive solution)



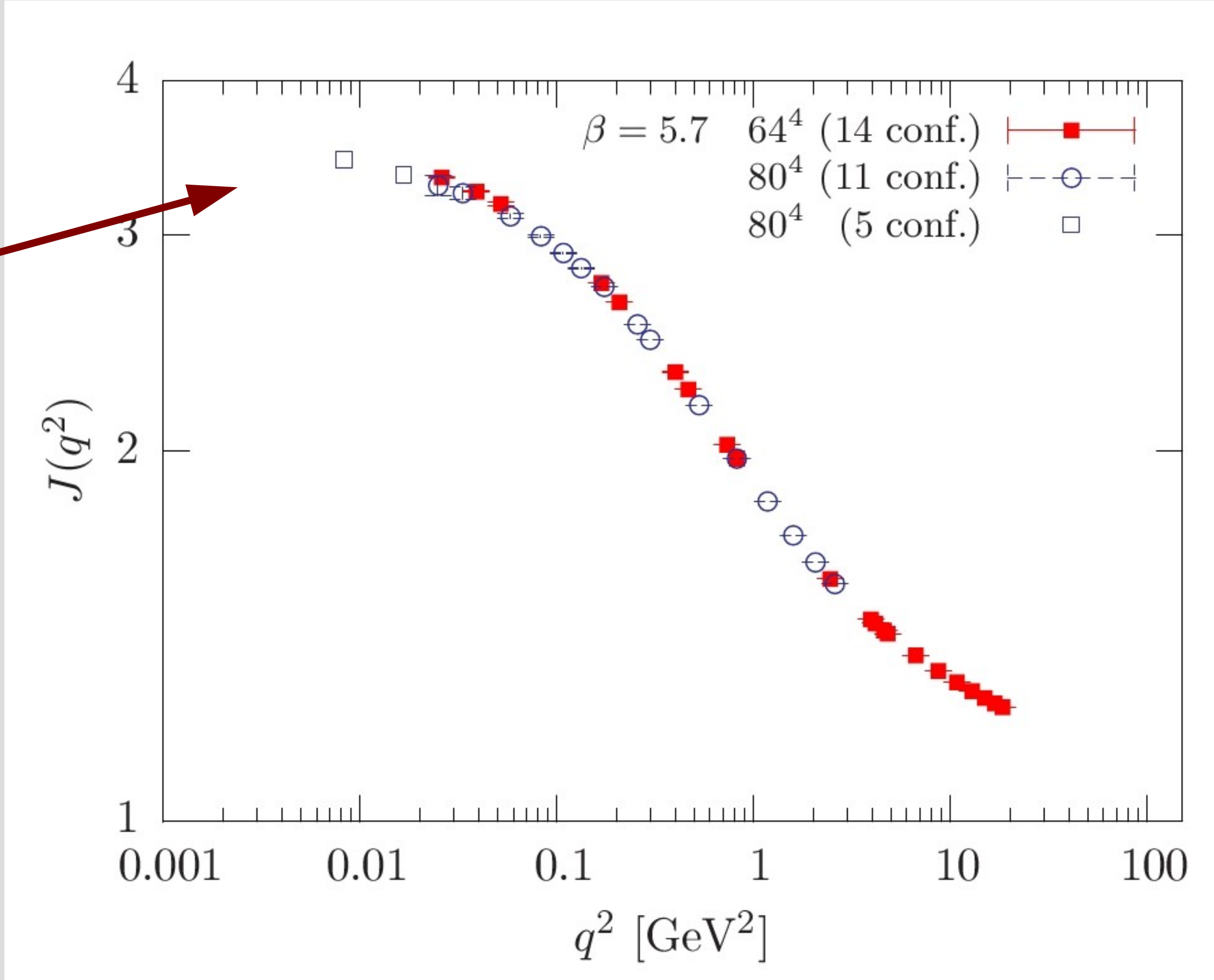
I.L. Bogolubsky, E.M. Ilgenfritz,M. Mulluer-Preussker, A. Sternbeck; arXiv:0901.0736[hep-lat]

Lattice (recent) results (II)

Ghost dressing function:

$\alpha_F = 0$

(solution type II, called also decoupling)



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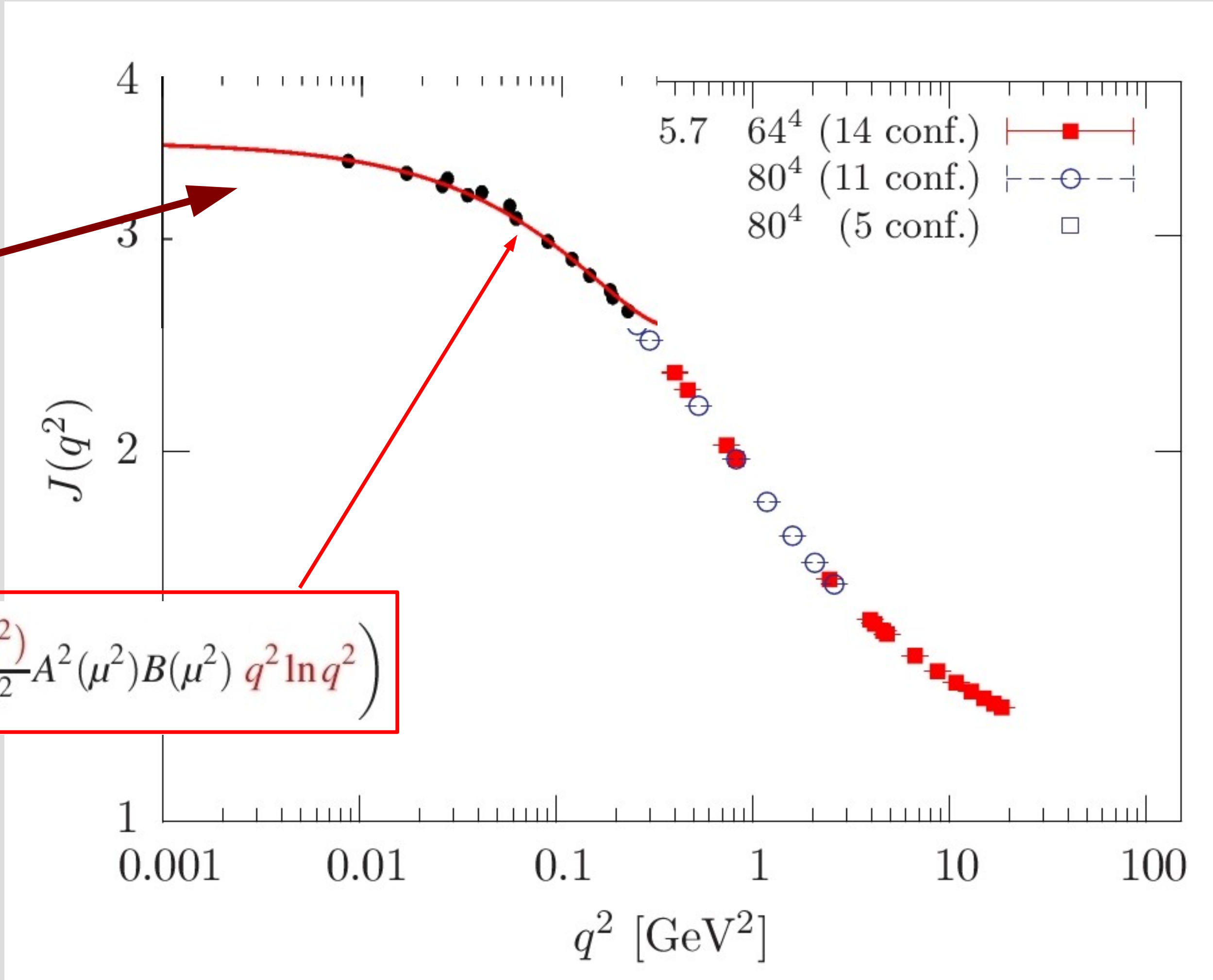


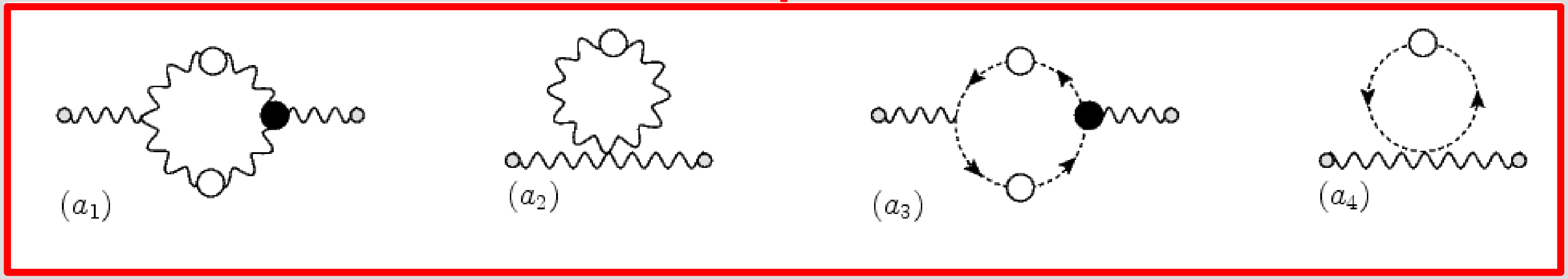
I.L. Bogolubsky, E.M. Ilgenfritz,M. Mulluer-Preussker, A. Sternbeck; arXiv:0901.0736[hep-lat]

Ghost and gluon coupled DSE's system

- In the BFM-PT scheme:

A.C.Aguilar, D.Binosi, J.Papavassiliou; PRD78(2008)025010

$$[1+G(q^2)]^2\Delta^{-1}(q^2)P_{\mu\nu}(q) = q^2P_{\mu\nu}(q) + i\sum_{i=1}^{4}(a_i)_{\mu\nu},$$



Ghost and gluon coupled DSE's system

- In the BFM-PT scheme:

A.C.Aguilar, D.Binosi, J.Papavassiliou; PRD78(2008)025010

- A massive gluon propagator is obtained ($\alpha_G = 1$)
- A finite ghost dressing function at zero momentum is also obtained ($\alpha_F = 0$)

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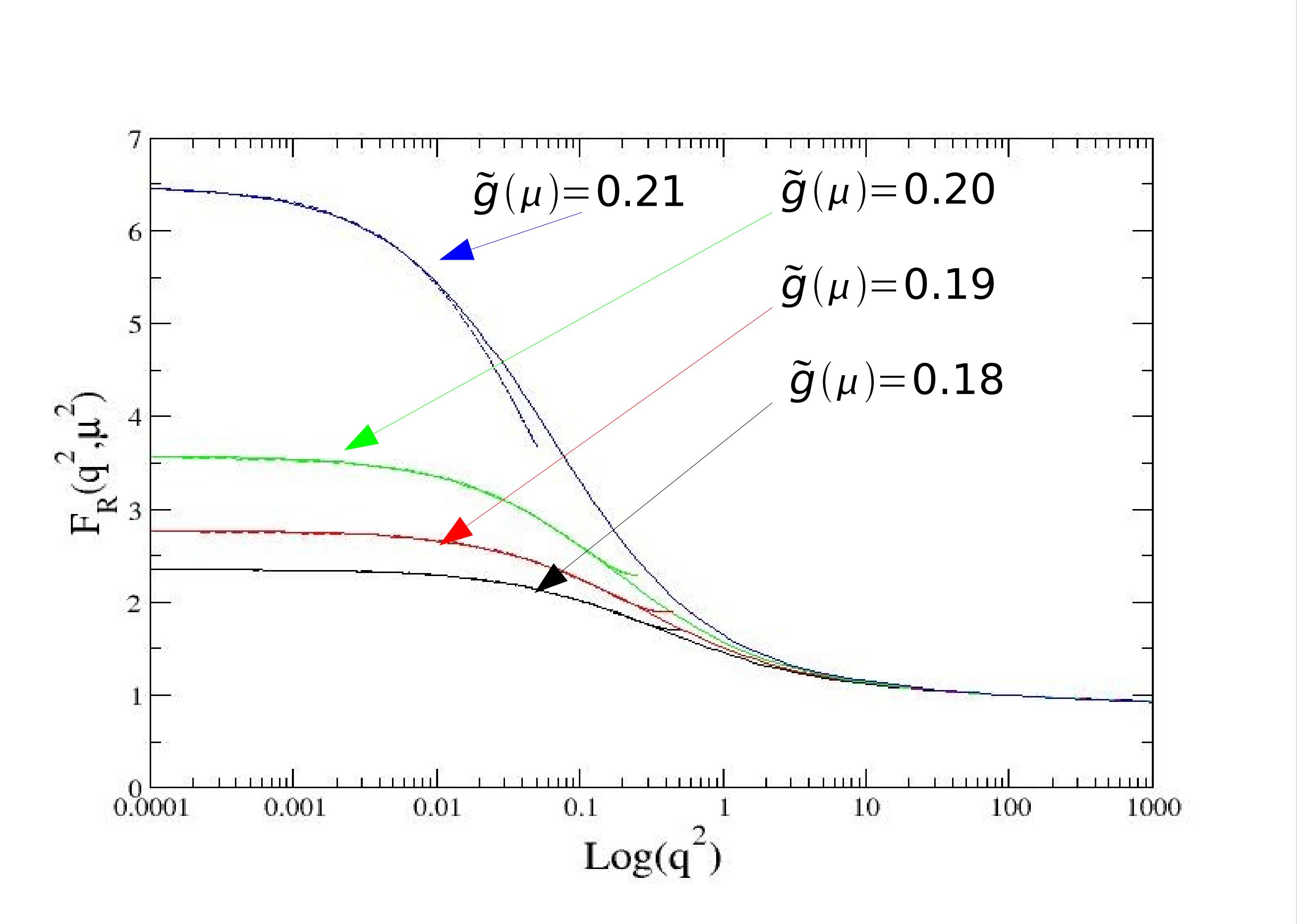
A.C.Aguilar, D.Binosi, J.Papavassiliou; PRD78(2008)025010



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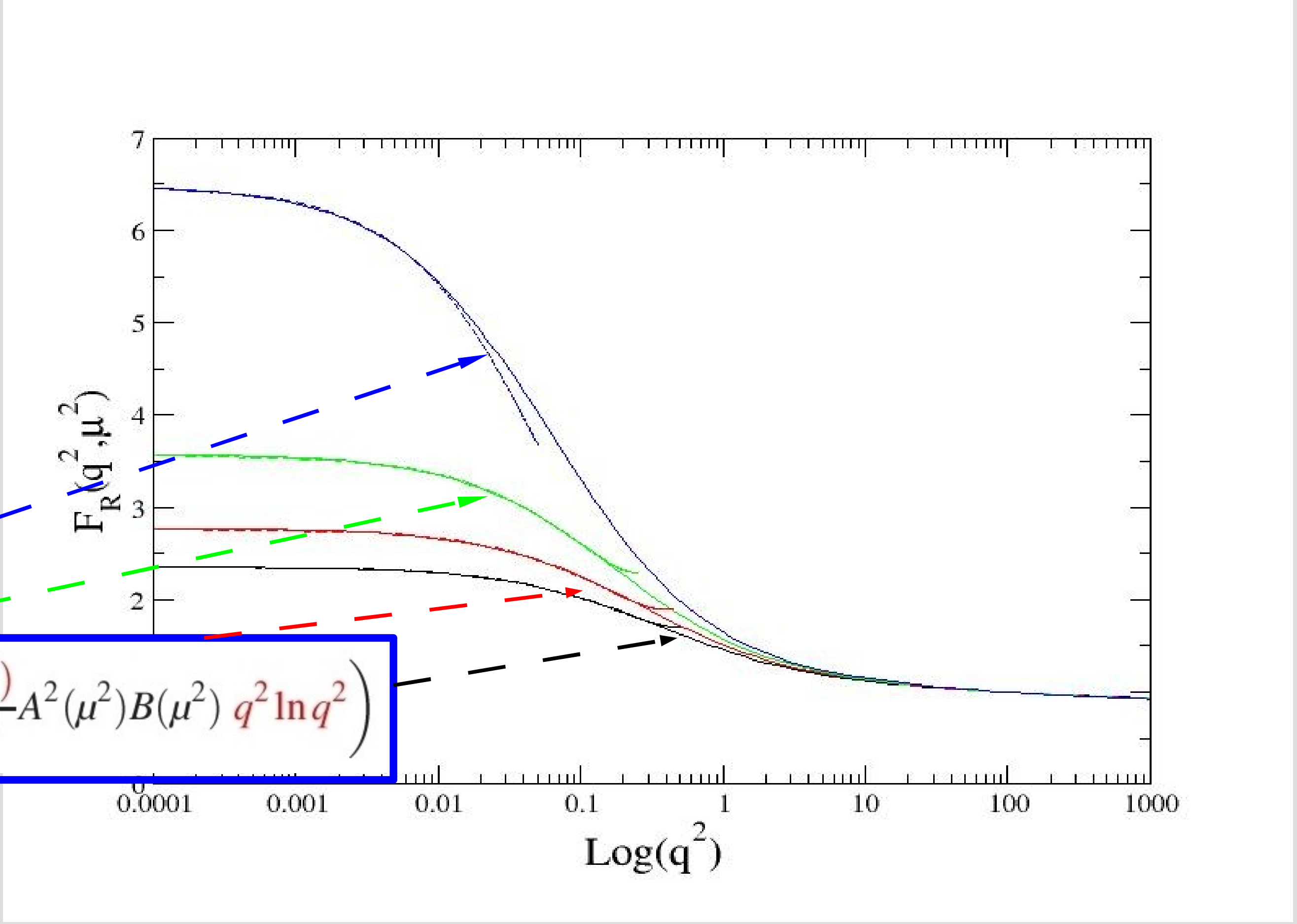

$$\boxed{F_{\mathrm{IR}}(q^2,\mu^2) = A(\mu^2)\left(1+\frac{\widetilde{g}^2(\mu^2)}{64\pi^2}A^2(\mu^2)B(\mu^2)\, q^2\ln q^2\right)}$$

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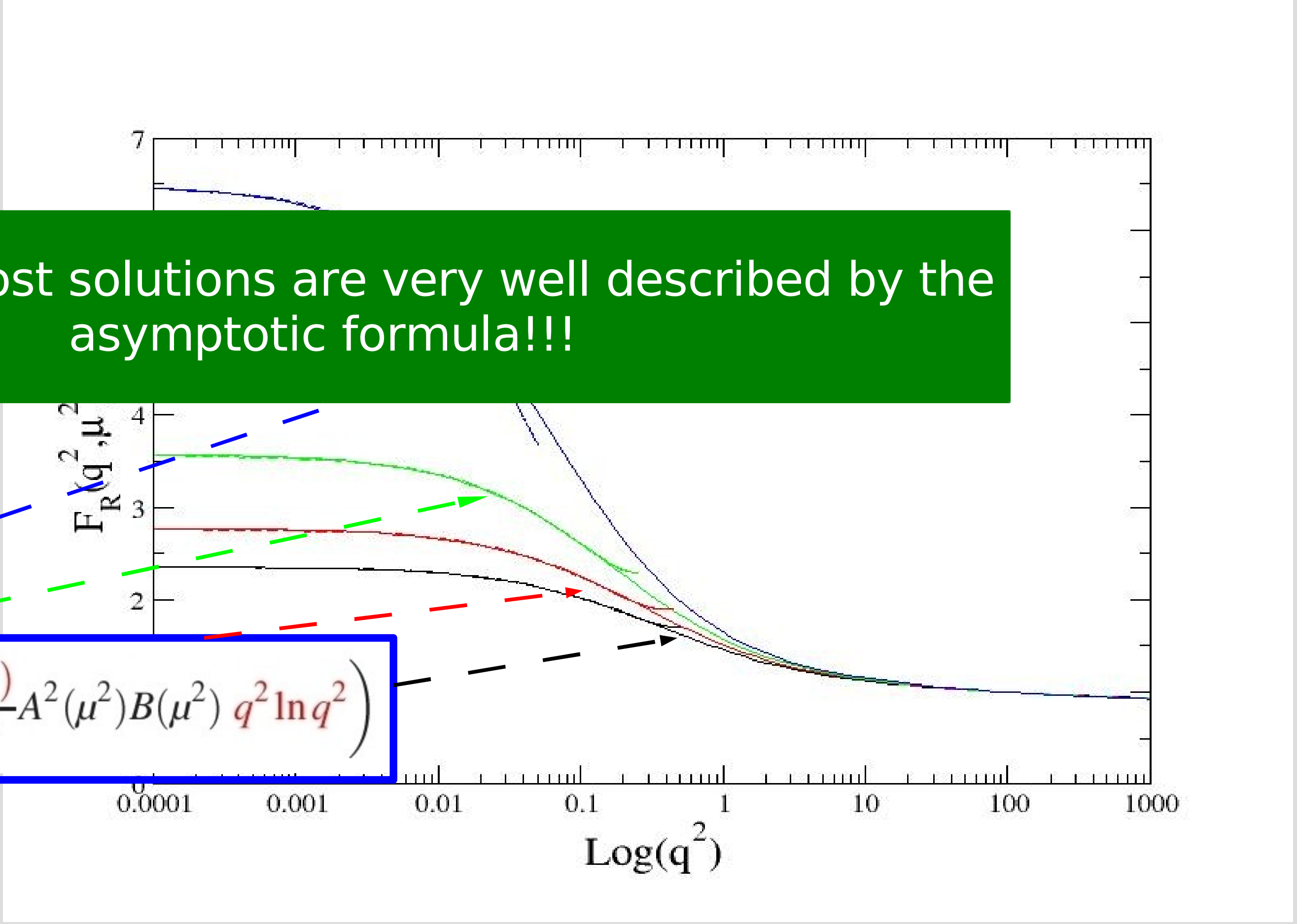

The BFM-PT ghost solutions are very well described by the asymptotic formula!!!

$$F_{\rm IR}(q^2,\mu^2) = A(\mu^2)\left(1+\frac{\widetilde{g}^2(\mu^2)}{64\pi^2}A^2(\mu^2)B(\mu^2)\, q^2\ln q^2\right)$$

Ghost and gluon coupled DSE's system

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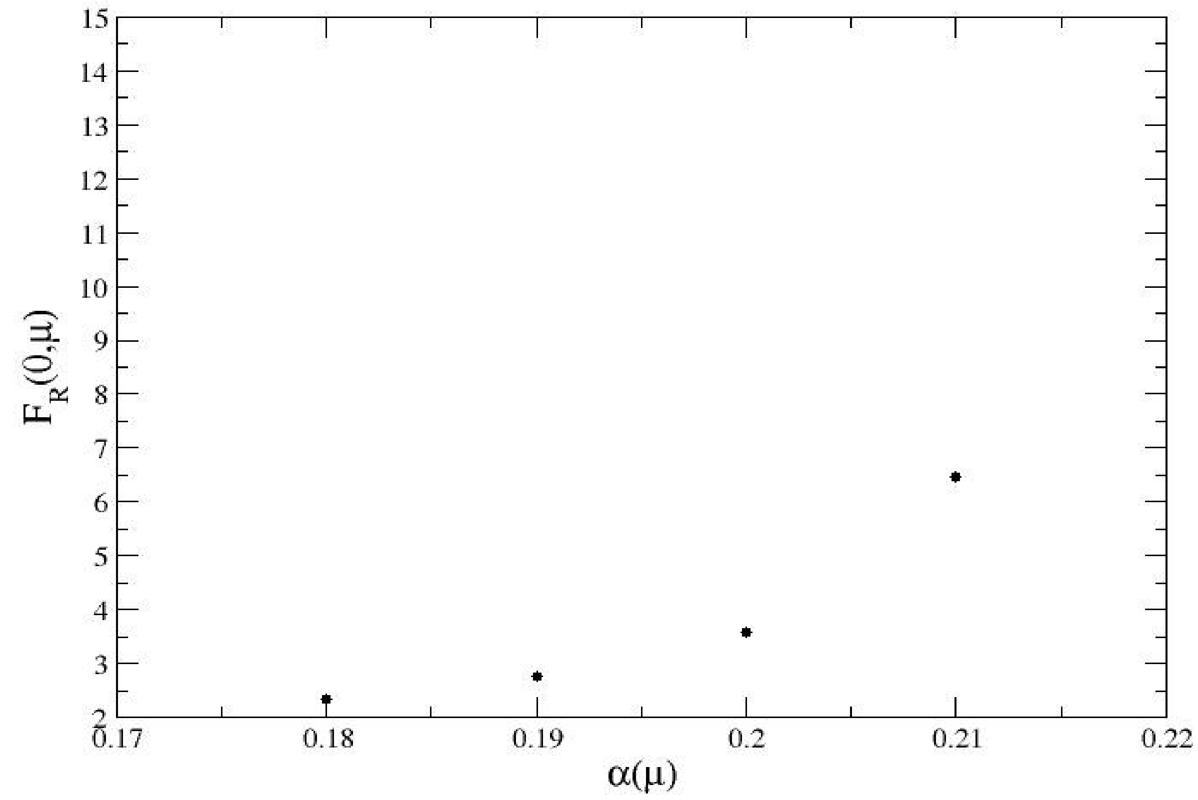

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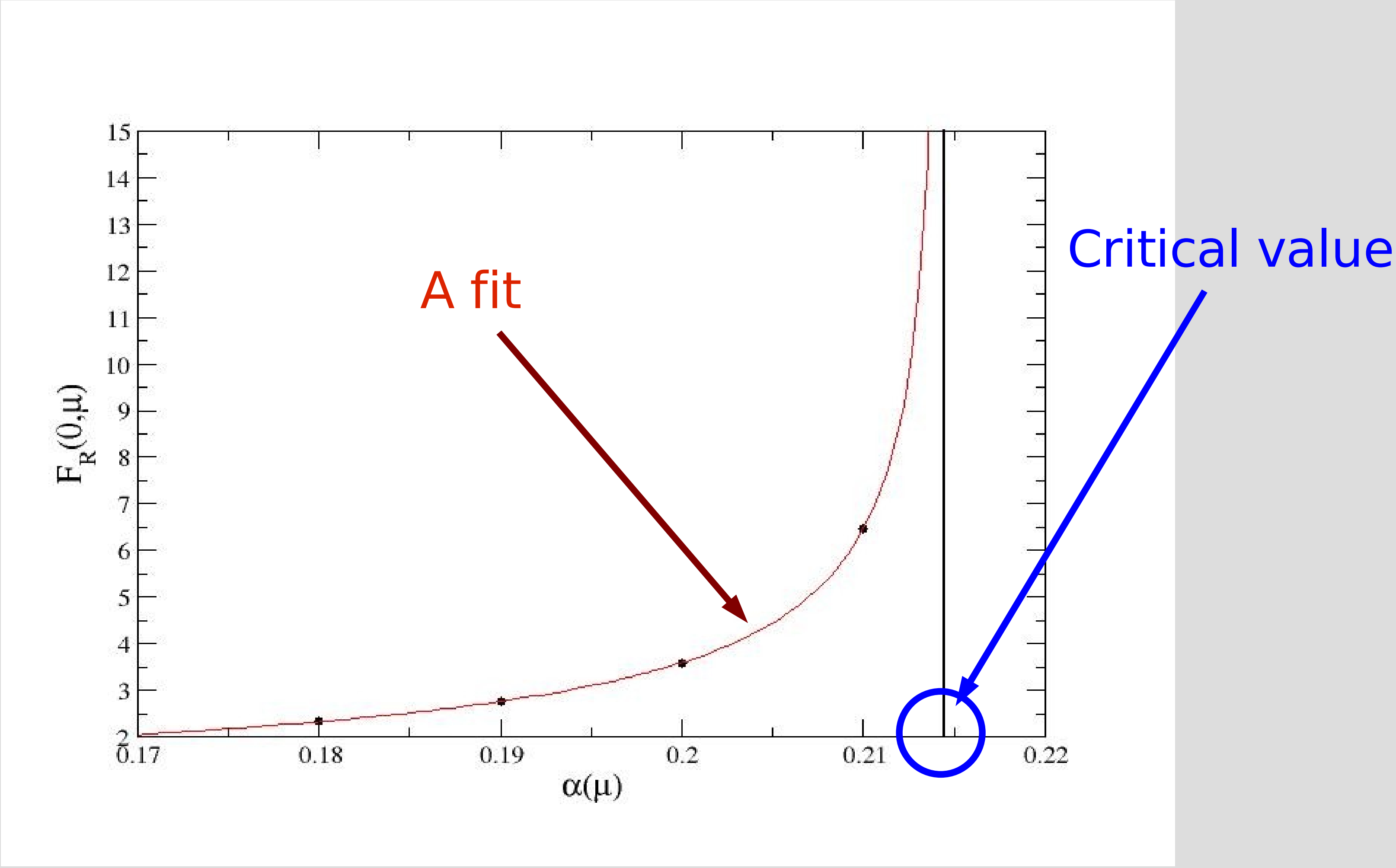

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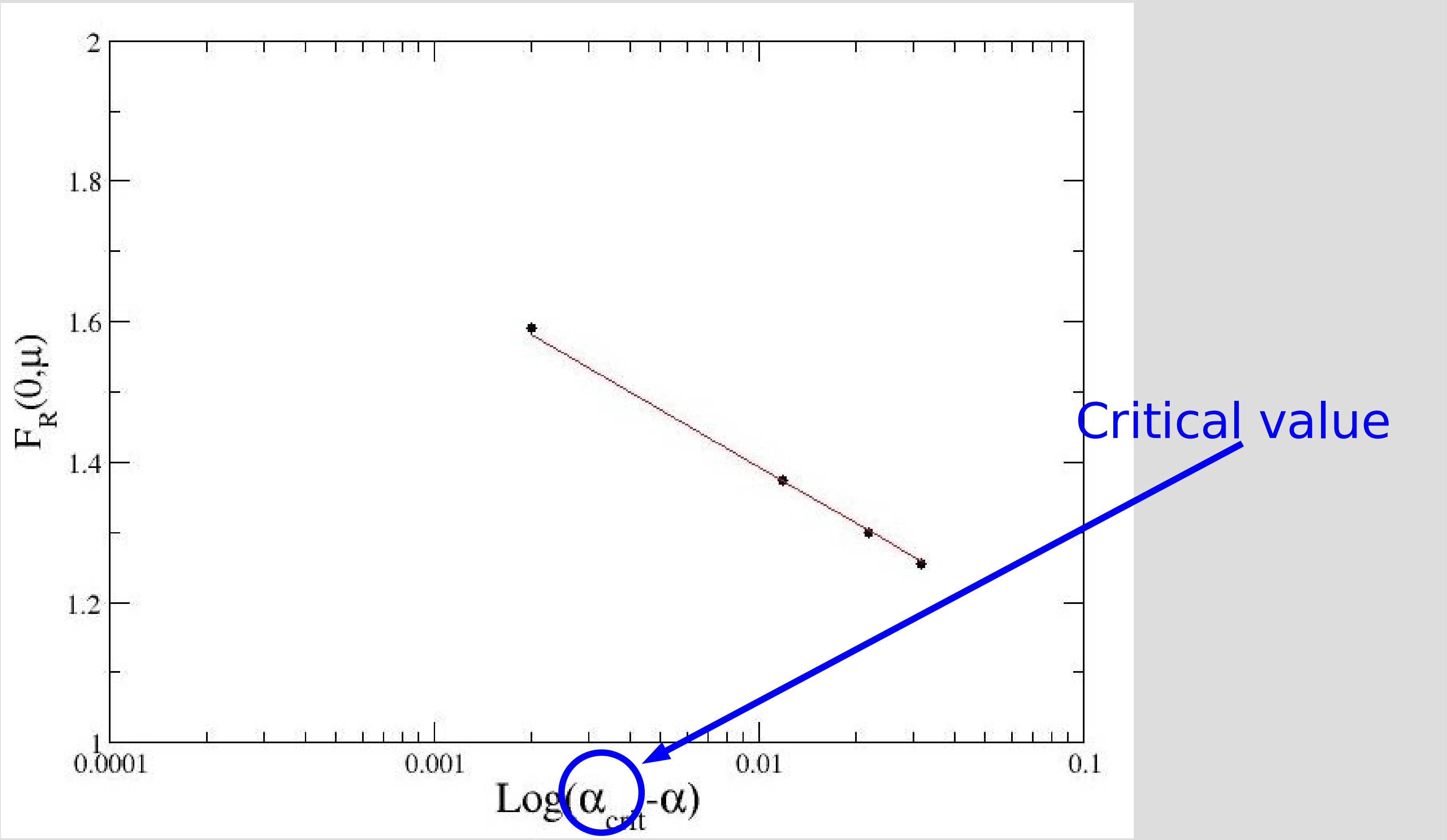

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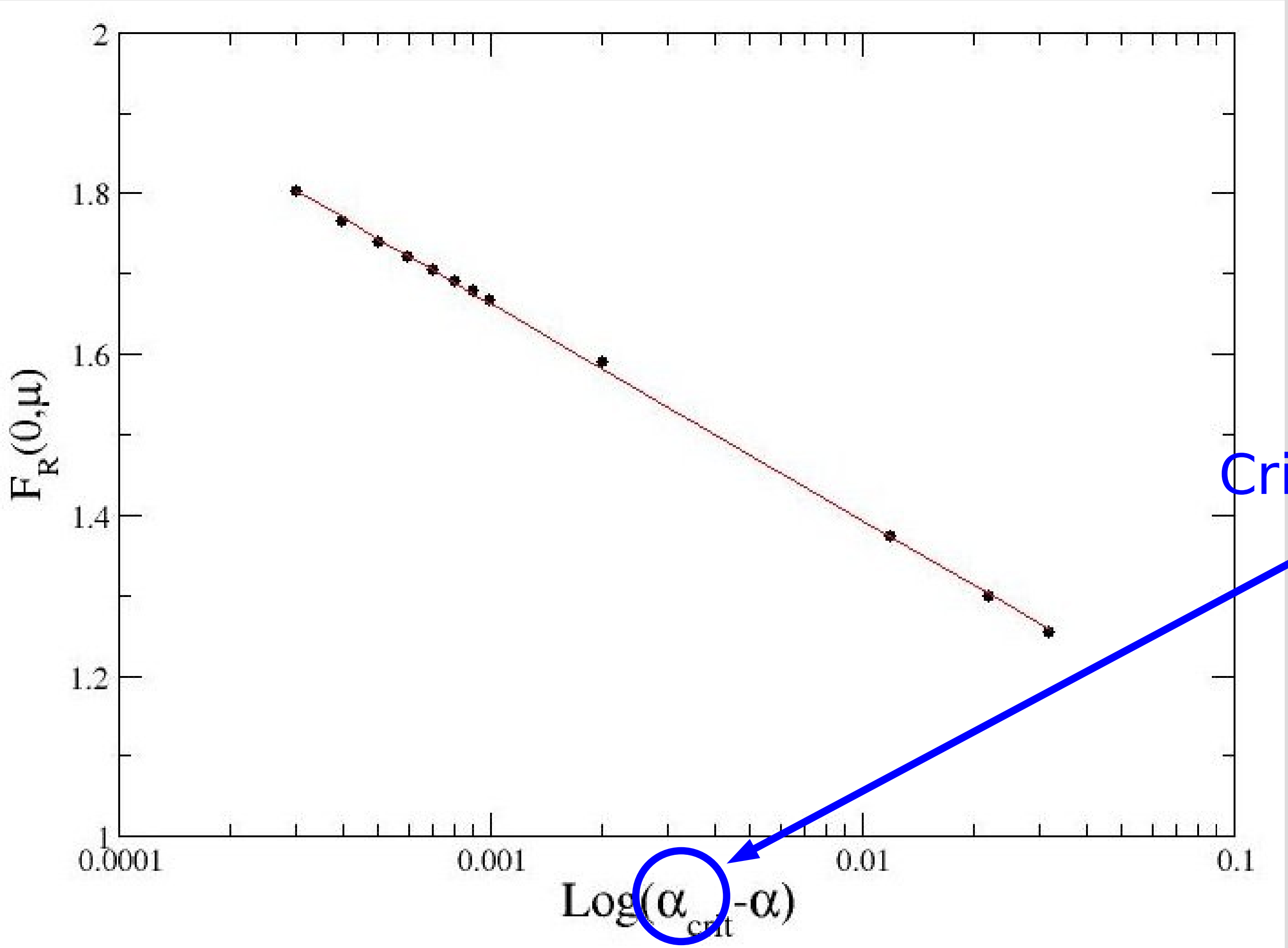
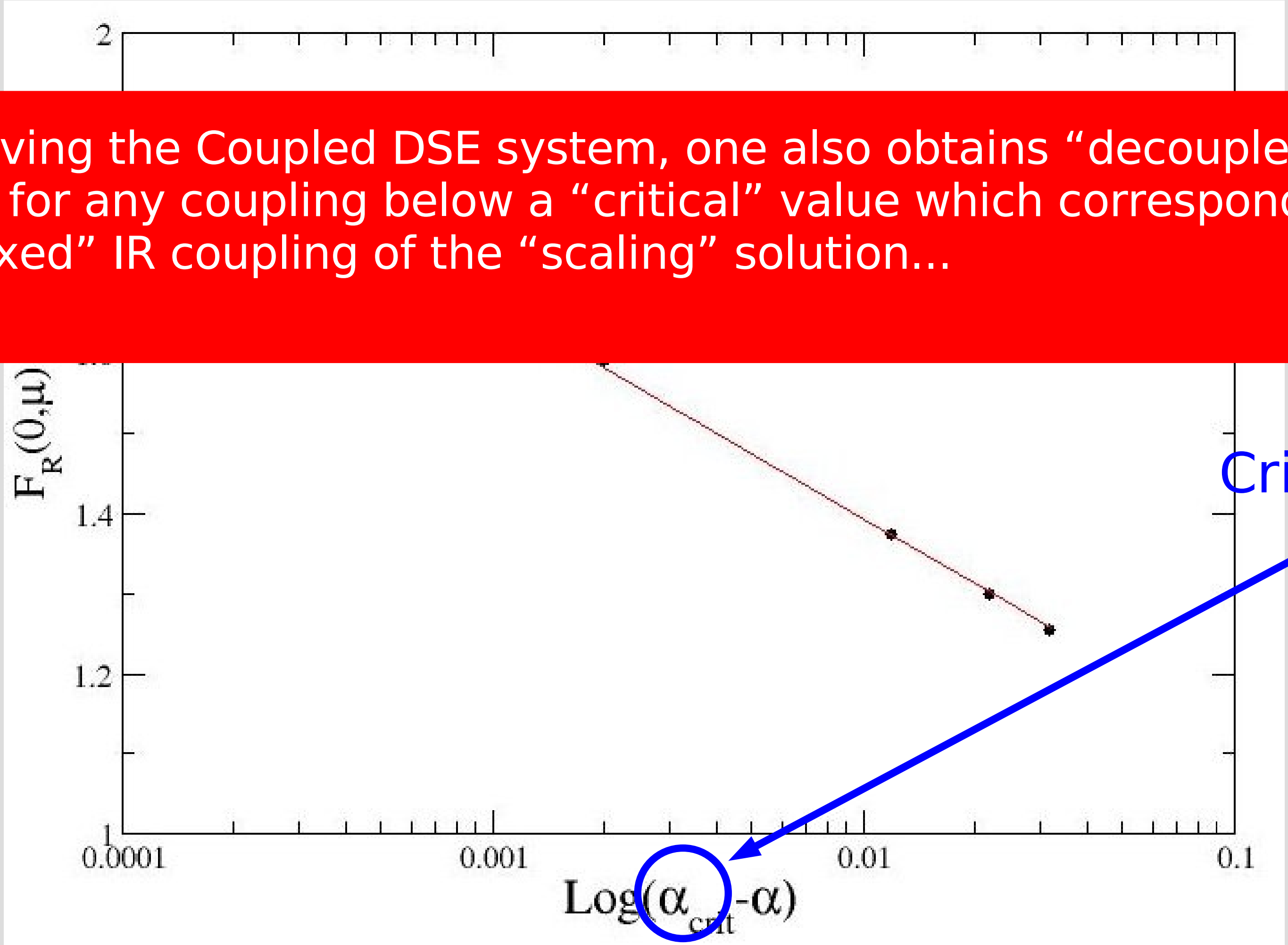


Critical value

Ghost and gluon coupled DSE's system

- In the BFM-PT scheme:

A.C.Aguilar, D.Binosi, J.Papavassiliou; PRD78(2008)025010



When solving the Coupled DSE system, one also obtains “decoupled” solutions for any coupling below a “critical” value which corresponds to the “fixed” IR coupling of the “scaling” solution...

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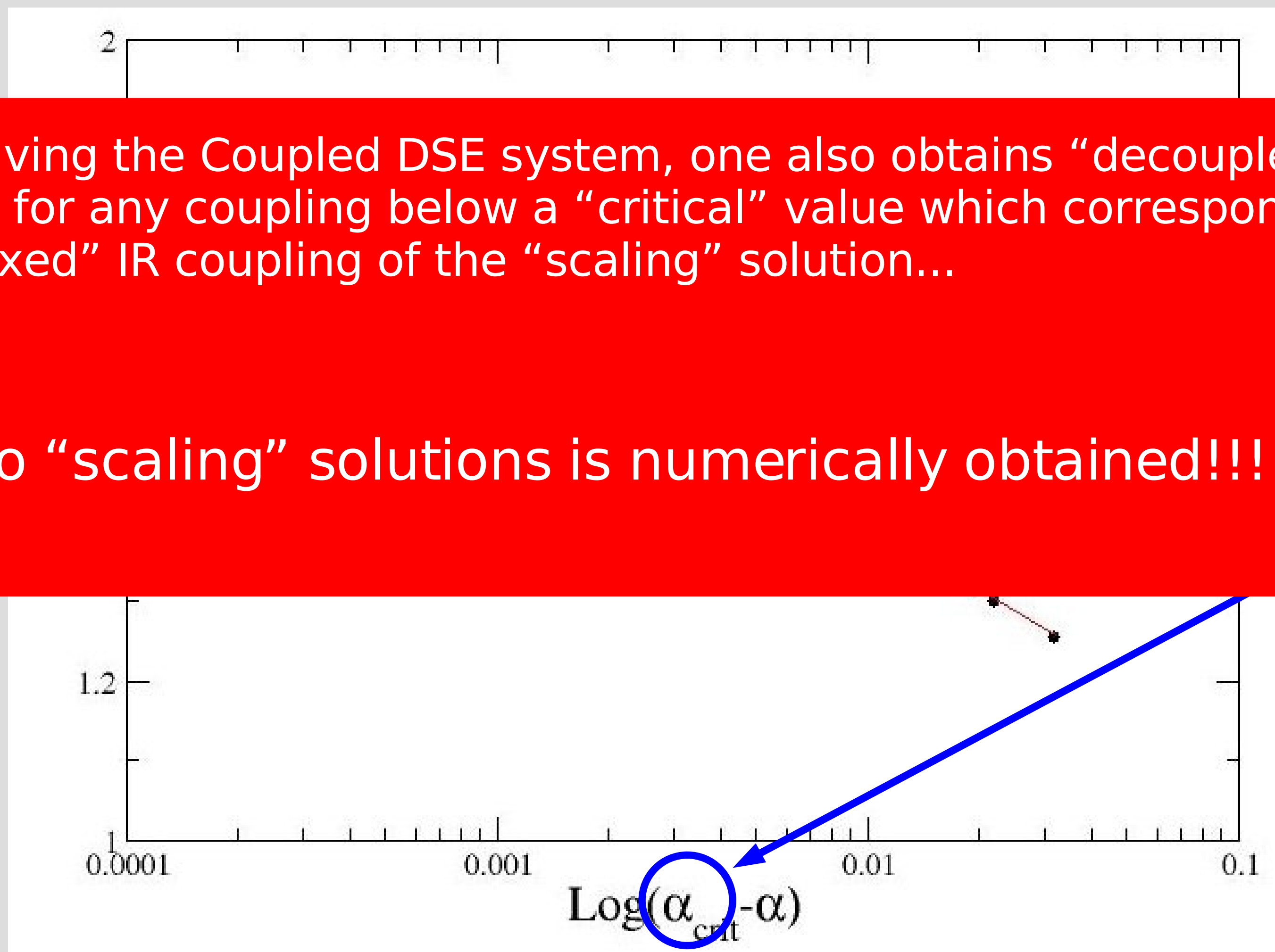
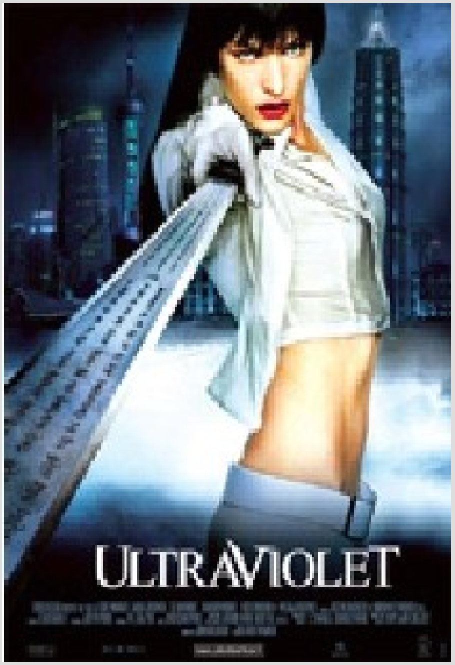


When solving the Coupled DSE system, one also obtains “decoupled” solutions for any coupling below a “critical” value which corresponds to the “fixed” IR coupling of the “scaling” solution...

...but no “scaling” solutions is numerically obtained!!!

Ultra-violet



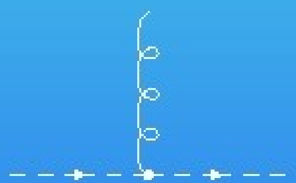
Theory stands here on a much stronger ground

The issue is to compute Λ_{QCD} to be compared to Experiment. There are several ways of computing Λ_{QCD}.

IS THIS UNDER CONTROL ?

The ghost-gluon coupling

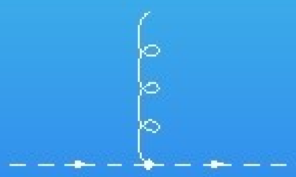
- Another MOM coupling definition (GPDSE)
- In Landau gauge



$$\alpha(\mu^2) = \lim_{\Lambda\to\infty} Z_g^{-2}\left(\mu^2,\Lambda\right)\ \frac{g_0^2(\Lambda)}{4\pi}$$

The ghost-gluon coupling

- Another MOM coupling definition (GPDSE)
- In Landau gauge



$$\alpha(\mu^2) = \lim_{\Lambda\to\infty} \left(\frac{\widetilde{Z}_1(\mu^2)}{\widetilde{Z}_3(\mu^2,\Lambda) Z_3^{1/2}(\mu^2,\Lambda)} \right)^{-2} \frac{g_0^2(\Lambda)}{4\pi}$$

The ghost-gluon coupling

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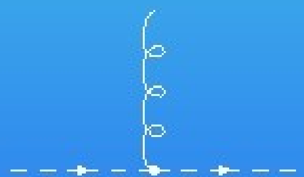

$$\alpha(\mu^2) = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda)}{4\pi} \; \frac{F^2(\mu^2,\Lambda)G(\mu^2,\Lambda)}{\widetilde{Z}_1^2(\mu^2)}$$

The ghost-gluon coupling

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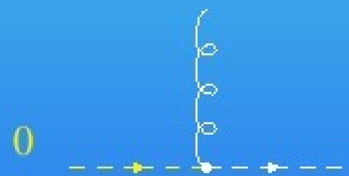

$$\boxed{\alpha(\mu^2) = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda)}{4\pi} F^2(\mu^2,\Lambda)G(\mu^2,\Lambda)}$$

The ghost-gluon coupling

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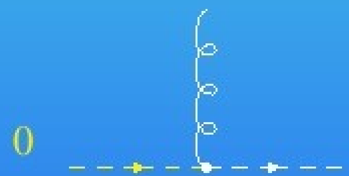


$$\boxed{\alpha(\mu^2) = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda)}{4\pi} F^2(\mu^2,\Lambda) G(\mu^2,\Lambda)}$$

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}[U]\ \mathcal{D}[\Phi] O e^{-S[U,\Phi]}$$

$$A_\mu(x+\frac{\hat{\mu}}{2}) = \frac{1}{2iag_0}\left(U_\mu(x) - U_\mu^\dagger(x)\right)$$

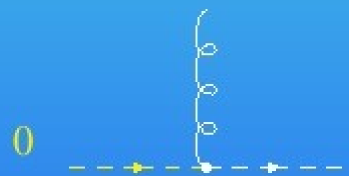
- In Landau gauge: $\left[\text{minimizing} \qquad \mathrm{F_U[g] = Re}\sum_{\mathrm{x}}\sum_{\mu}\left(1 - \frac{1}{\mathrm{N}}\mathrm{g(x)U_\mu(x)g^\dagger(x+\hat{\mu})}\right)\right]$

$$D_{\mu\nu}^{ab}(k) = \int d^4x\, d^4y e^{ik\cdot(x-y)} \langle A_\mu^a(x) A_\nu^b(y)\rangle_U = \frac{G(k^2)}{k^2}\,\delta^{ab}\,\delta_{\mu\nu}^T(k)$$

- In MOM renormalization scheme:

The ghost-gluon coupling

- Another MOM coupling definition (GPDSE)
- In Landau gauge



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$$G^{ab}(k) = \int d^4x\, d^4y e^{ik\cdot(x-y)} \left(M^{-1}\right)^{ab}_{xy} = -\frac{F(k^2)}{k^2}\,\delta^{ab}$$

- In MOM renormalization scheme:

The ghost-gluon coupling

- Another MOM coupling definition (GPDSE)
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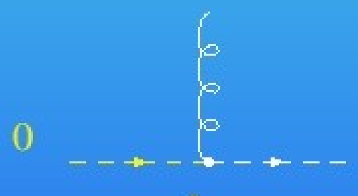

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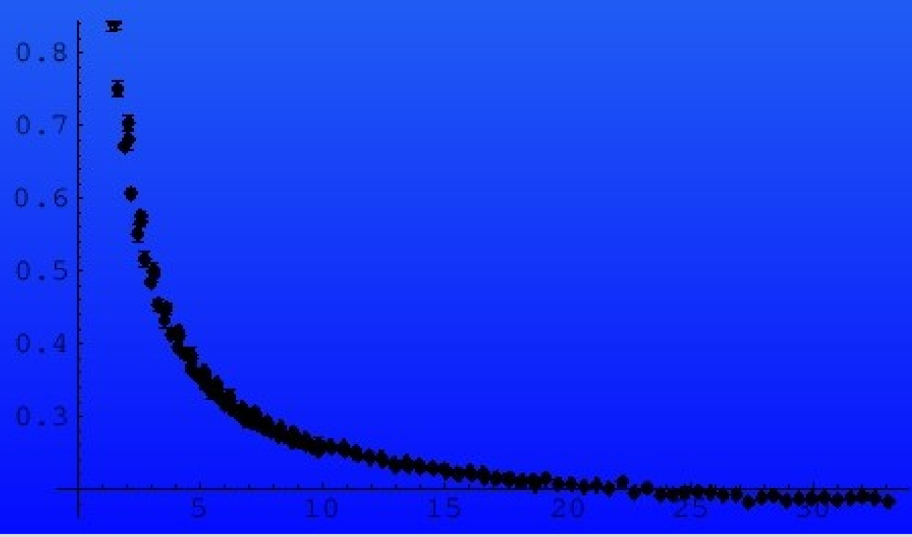

- Lattice data
  $(N_f = 0)$:
  - $\beta = 6.0(16^4)$
  - $\beta = 6.0(24^4)$
  - $\beta = 6.2(24^4)$
  - $\beta = 6.4(32^4)$

The ghost-gluon coupling (II)

$$\boxed{\alpha(\mu^2) = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda)}{4\pi} F^2(\mu^2,\Lambda) G(\mu^2,\Lambda)}$$

$$\alpha_h(\mu) = \overline{\alpha}(\mu) \left(1 + \sum_{i=1} c_i \left(\frac{\overline{\alpha}(\mu)}{4\pi} \right)^i \right)$$

$$\frac{1}{\alpha_h(\mu)} \frac{d\alpha_h(\mu)}{d\overline{\alpha}} = \frac{2\widetilde{\gamma}(\overline{\alpha}) + \gamma(\overline{\alpha})}{\beta_{\overline{MS}}(\overline{\alpha})}$$

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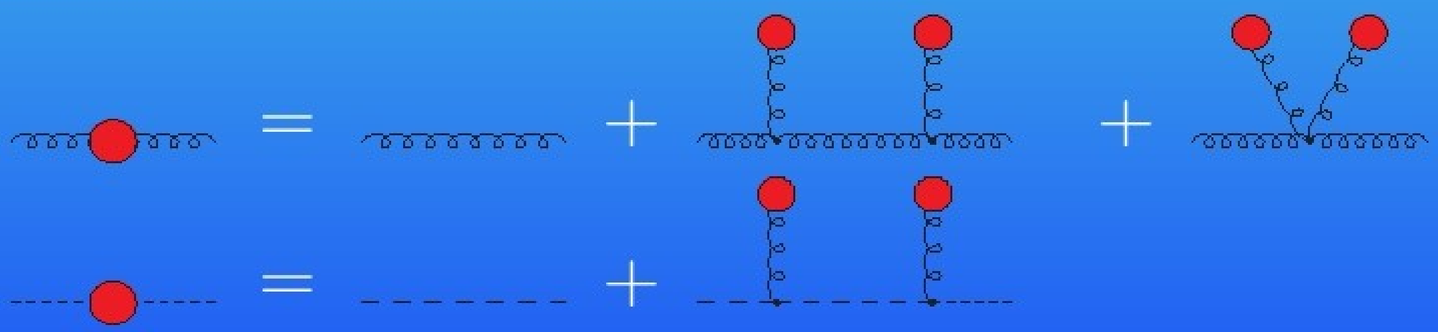
$$\left.\begin{array}{l} \beta_{\overline{\mathrm{MS}}}(\overline{\alpha}) = -4\pi \displaystyle\sum_{i=0} \overline{\beta}_i \left(\frac{\overline{\alpha}}{4\pi} \right)^{i+2} \\ \widetilde{\gamma}_{\mathrm{MOM}}(\overline{\alpha}) = -\displaystyle\sum_{i=0} \widetilde{\gamma}_i \left(\frac{\overline{\alpha}}{4\pi} \right)^{i+1} \\ \gamma_{\mathrm{MOM}}(\overline{\alpha}) = -\displaystyle\sum_{i=0} \gamma_i \left(\frac{\overline{\alpha}}{4\pi} \right)^{i+1} \end{array}\right\} \longrightarrow \left\{\begin{array}{l} c_1 = \dfrac{507 - 40N_f}{36} \\ c_2 = \dfrac{76063}{144} - \dfrac{351}{8}\zeta(3) \\ \quad - \left(\dfrac{1913}{27} + \dfrac{4}{3}\zeta(3) \right) N_f + \dfrac{100}{91} N_f^2 \\ \ldots \end{array}\right.$$

The ghost-gluon coupling: OPE contributions

$$\boxed{\alpha(\mu^2) = \lim_{\Lambda \to \infty} \frac{g_0^2(\Lambda)}{4\pi} F^2(\mu^2, \Lambda) G(\mu^2, \Lambda)}$$



$\alpha^{\mathrm{NP}}(q, \Lambda_{\overline{MS}})$

The ghost-gluon coupling: OPE contributions

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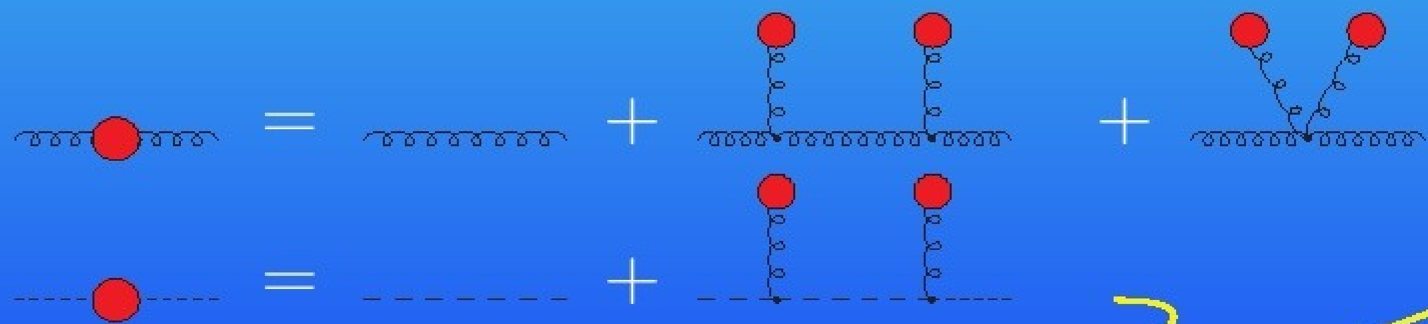


$$\alpha^{NP}(q,\Lambda_{\overline{MS}}) =$$

$$= \boxed{\alpha\left(\ln\frac{q}{\Lambda_{\overline{MS}}}\right)\left(1+\frac{9g_R^2(\mu)\langle A^2\rangle_\mu}{4(N_C^2-1)}\,\frac{1}{q^2}\right)}$$

The ghost-gluon coupling: OPE contributions

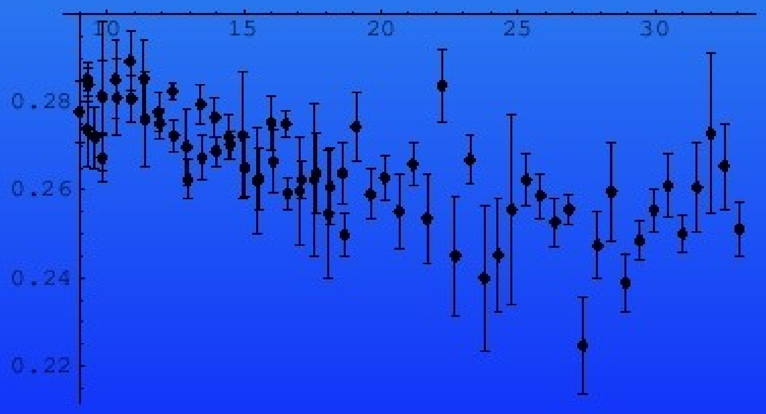
$$\boxed{\alpha(\mu^2) = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda)}{4\pi} F^2(\mu^2,\Lambda)G(\mu^2,\Lambda)}$$

$$\boxed{\alpha^{\mathrm{N}P}(q,\Lambda_{\overline{\mathrm{MS}}}) = \alpha\left(\ln\frac{q}{\Lambda_{\overline{\mathrm{MS}}}}\right)\left(1+\frac{9g_R^2(\mu)\langle A^2\rangle_\mu}{4(N_C^2-1)}\,\frac{1}{q^2}\right)}$$

The ghost-gluon coupling: OPE contributions

$$\alpha(\mu^2) = \lim_{\Lambda\to\infty} \frac{g_0^2(\Lambda)}{4\pi} F^2(\mu^2,\Lambda)G(\mu^2,\Lambda)$$

$$\alpha^{\mathrm{NP}}(q,\Lambda_{\overline{MS}}) = \alpha\left(\ln\frac{q}{\Lambda_{\overline{MS}}}\right)\left(1+\frac{9g_R^2(\mu)\langle A^2\rangle_\mu}{4(N_C^2-1)}\,\frac{1}{q^2}\right)$$



- Lattice data $(N_f = 0)$:
  - $\beta = 6.0(16^4)$
  - $\beta = 6.0(24^4)$
  - $\beta = 6.2(24^4)$
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The ghost-gluon coupling: OPE contributions

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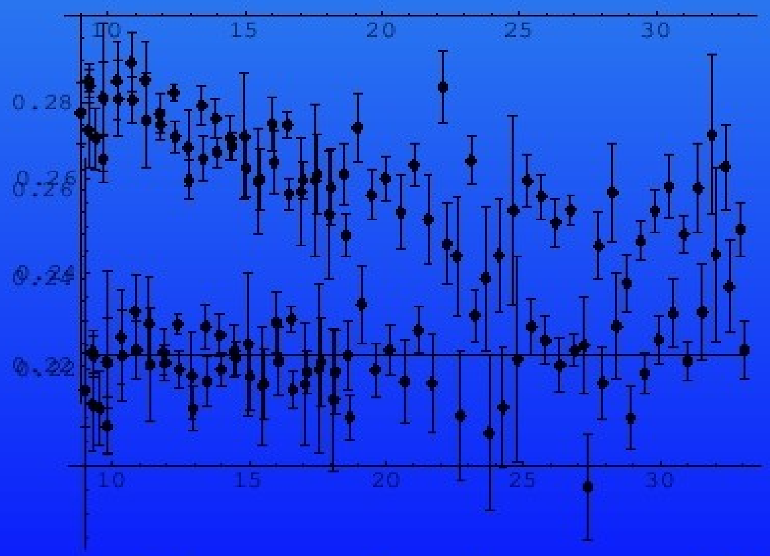

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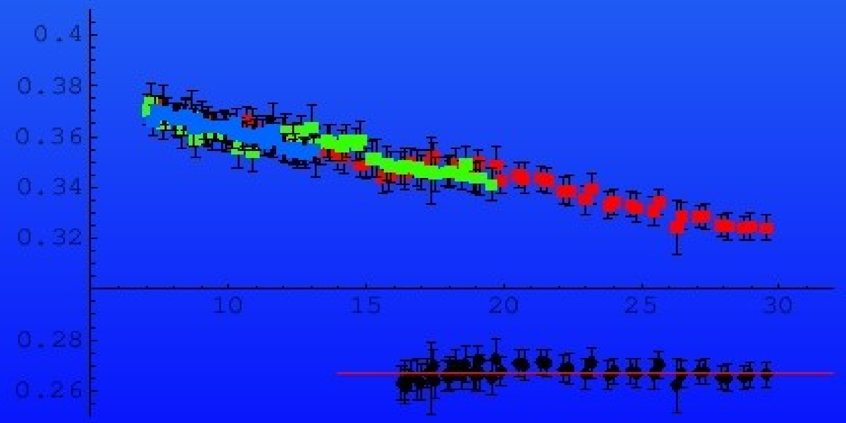
  - $\beta = 6.4(32^4)$

$\Lambda_{\overline{MS}} = 224^{+8}_{-5}$ MeV, $g_R^2 \langle A^2 \rangle_R = 5.1^{+0.8}_{-1.1}$ GeV^2

The ghost-gluon coupling:
Preliminary unquenched results

$$\alpha(\mu^2) = \lim_{\Lambda \to \infty} \frac{g_0^2(\Lambda)}{4\pi} F^2(\mu^2,\Lambda) G(\mu^2,\Lambda)$$

$$\alpha^{NP}(q,\Lambda_{\overline{MS}}) = \alpha\left(\ln\frac{q}{\Lambda_{\overline{MS}}}\right)\left(1 + \frac{9 g_R^2(\mu)\langle A^2\rangle_\mu}{4(N_C^2-1)}\,\frac{1}{q^2}\right)$$



- Lattice data ($N_f = 2$):
  - $\beta = 3.9(24^3 \times 48)$
    $\mu = 0.085$
  - $\beta = 4.05(24^3 \times 48)$
    $\mu = 0.060$
  - $\beta = 4.2(24^3 \times 48)$
    $\mu = 0.002$

$\Lambda_{\overline{MS}} = 267(11)$ MeV, $g_R^2\langle A^2\rangle_R = 9.6^{+1.4}_{-1.1}$ GeV^2

The ghost-gluon coupling:

Preliminary unquenched results

$$\alpha(\mu^2) = \lim_{\Lambda \to \infty} \frac{g_0^2(\Lambda)}{4\pi} F^2(\mu^2,\Lambda) G(\mu^2,\Lambda)$$

$$\alpha^{NP}(q,\Lambda_{\overline{MS}}) = \alpha\left(\ln \frac{q}{\Lambda_{\overline{MS}}}\right)\left(1 + \frac{9 g_R^2(\mu)\langle A^2\rangle_\mu}{4(N_C^2-1)} \frac{1}{q^2}\right)$$

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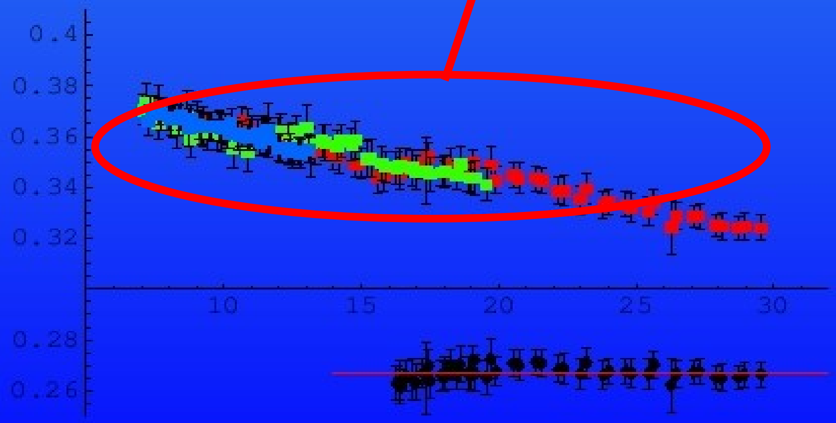
Impressive scaling!!



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$$\Lambda_{\overline{MS}} = 267(11)\ \text{MeV}, \quad g_R^2\langle A^2\rangle_R = 9.6^{+1.4}_{-1.1}\ \text{GeV}^2$$

The ghost-gluon coupling:
Preliminary unquenched results

$$\alpha(\mu^2) = \lim_{\Lambda \to \infty} \frac{g_0^2(\Lambda)}{4\pi} F^2(\mu^2, \Lambda) G(\mu^2, \Lambda)$$

$$\alpha^{NP}(q, \Lambda_{\overline{MS}}) = \alpha\left(\ln \frac{q}{\Lambda_{\overline{MS}}}\right)\left(1 + \frac{9 g_R^2(\mu) \langle A^2 \rangle_\mu}{4(N_C^2 - 1)} \frac{1}{q^2}\right)$$

N_f	$\Lambda^{(3)}_{\mathrm{MS},3g}$ (MeV)	$\Lambda^{(3)}_{\mathrm{MS},F^2G}$ (MeV)	$g_R^2 \langle A^2 \rangle_{R,3g}$	$g_R^2 \langle A^2 \rangle_{R,F^2G}$
0	233(28)	224^{+8}_{-5}	6.7(1.2)	$5.1^{+0.8}_{-1.1}$
2	—	267(11)(?)	—	$9.6^{+1.4}_{-1.1}$(?)

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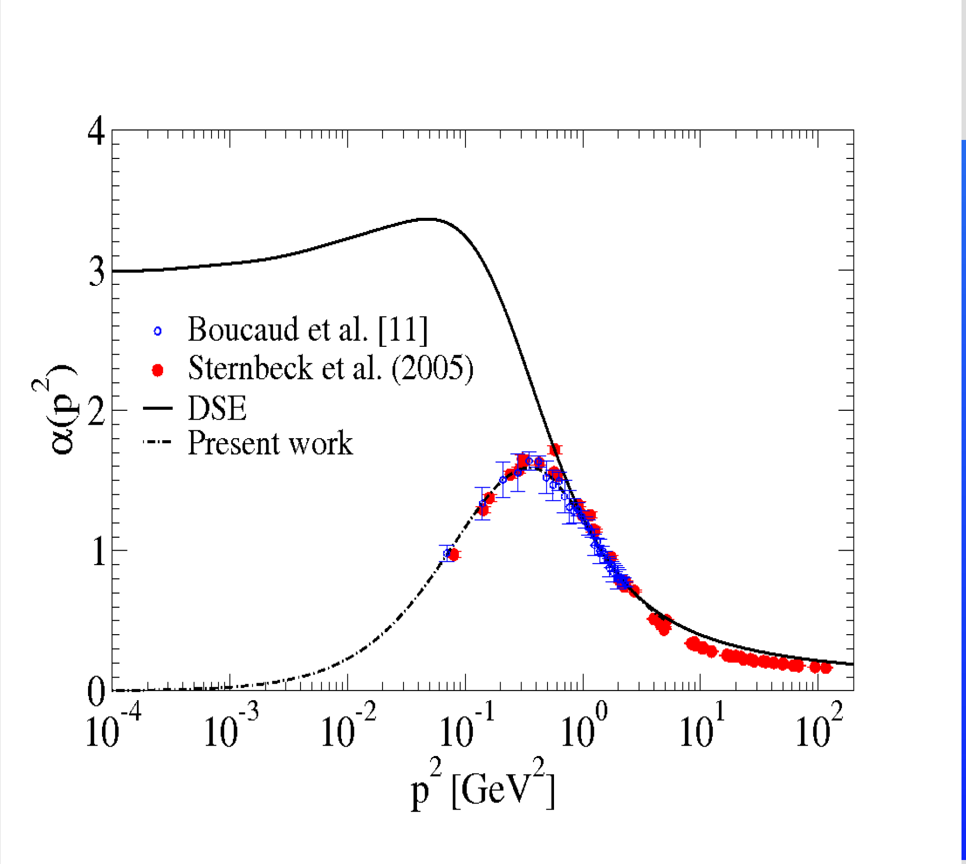
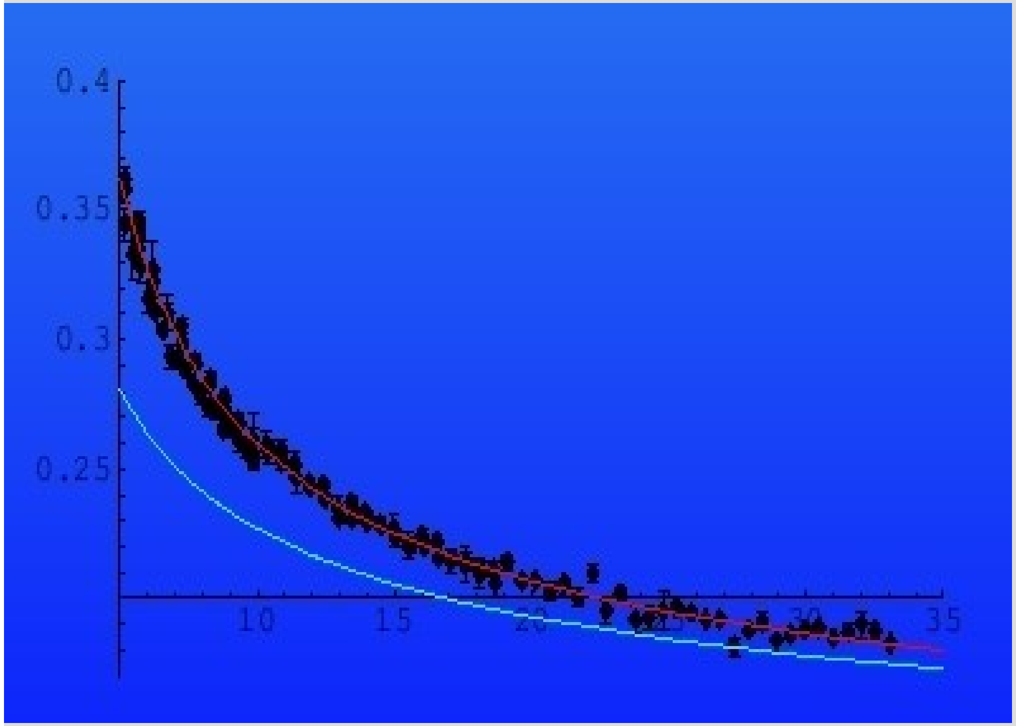
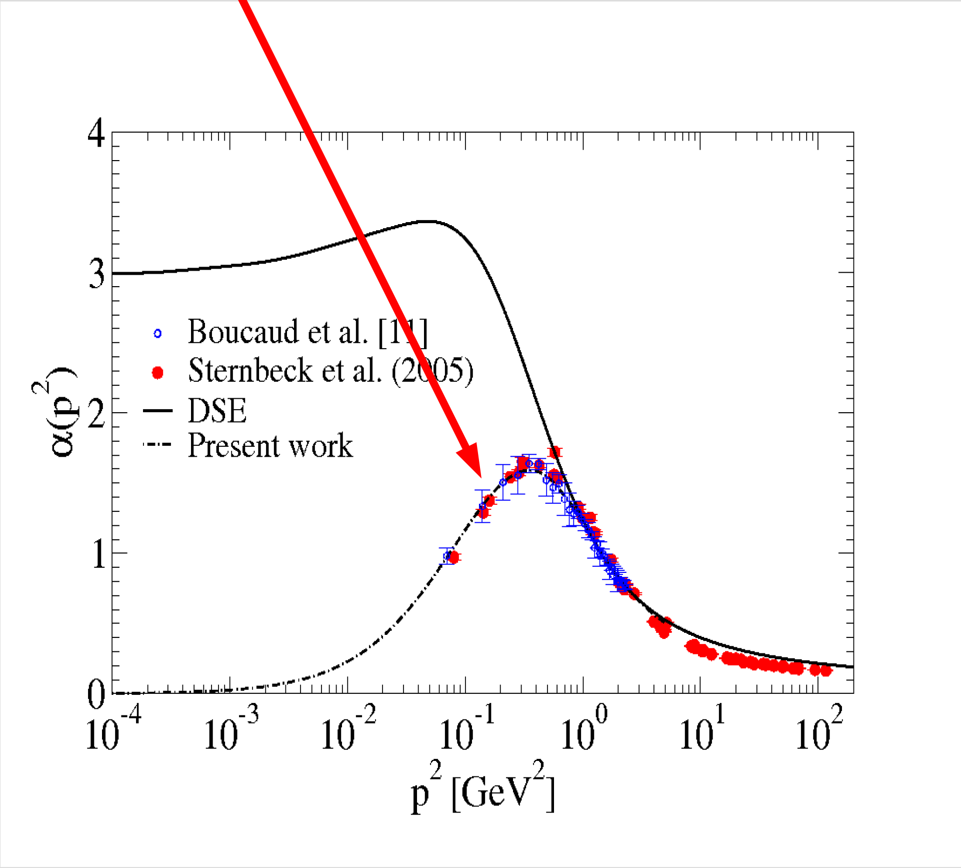
The ghost-gluon coupling: summary

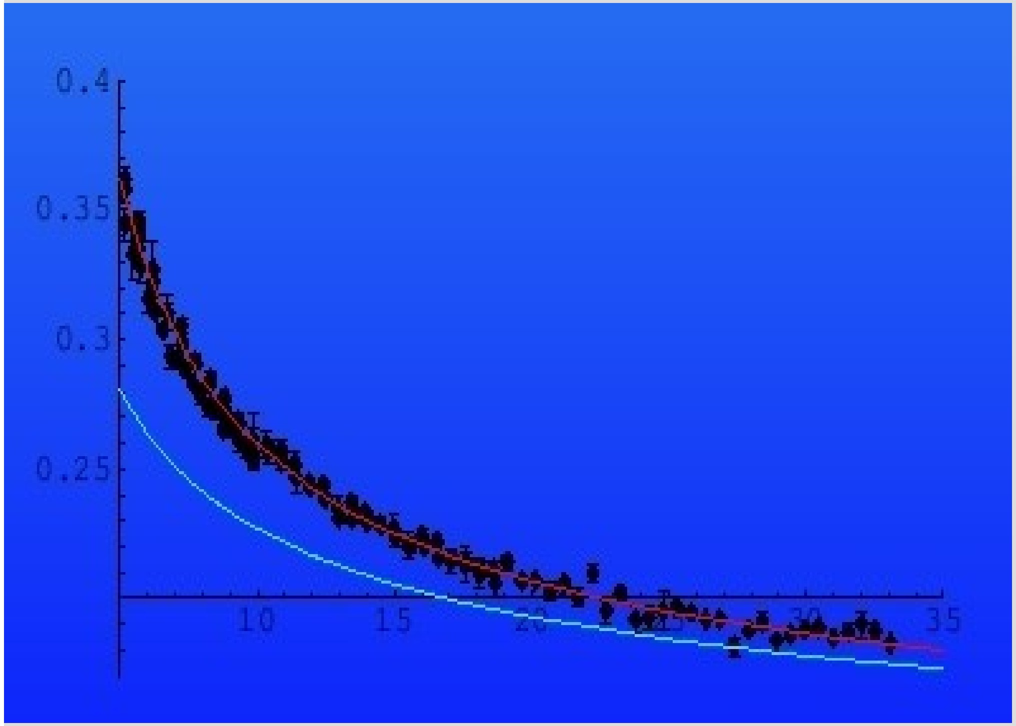




The ghost-gluon coupling: summary

GPDSE solution II





The ghost-gluon coupling: summary

GPDSE solution II

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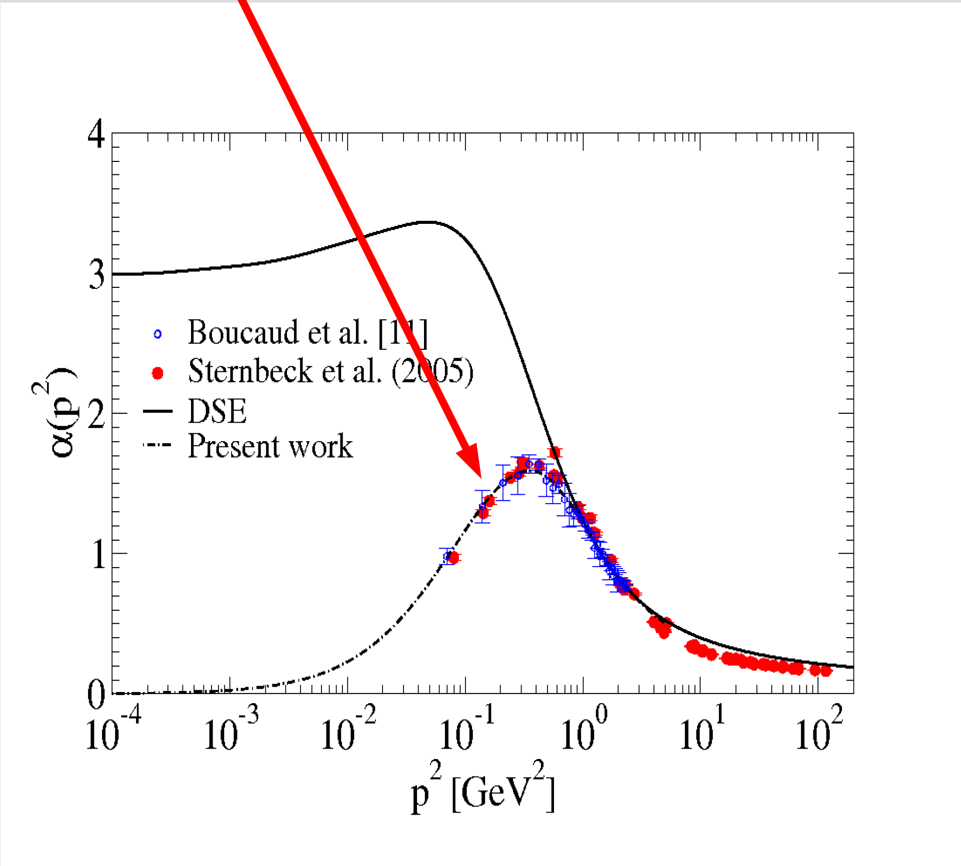

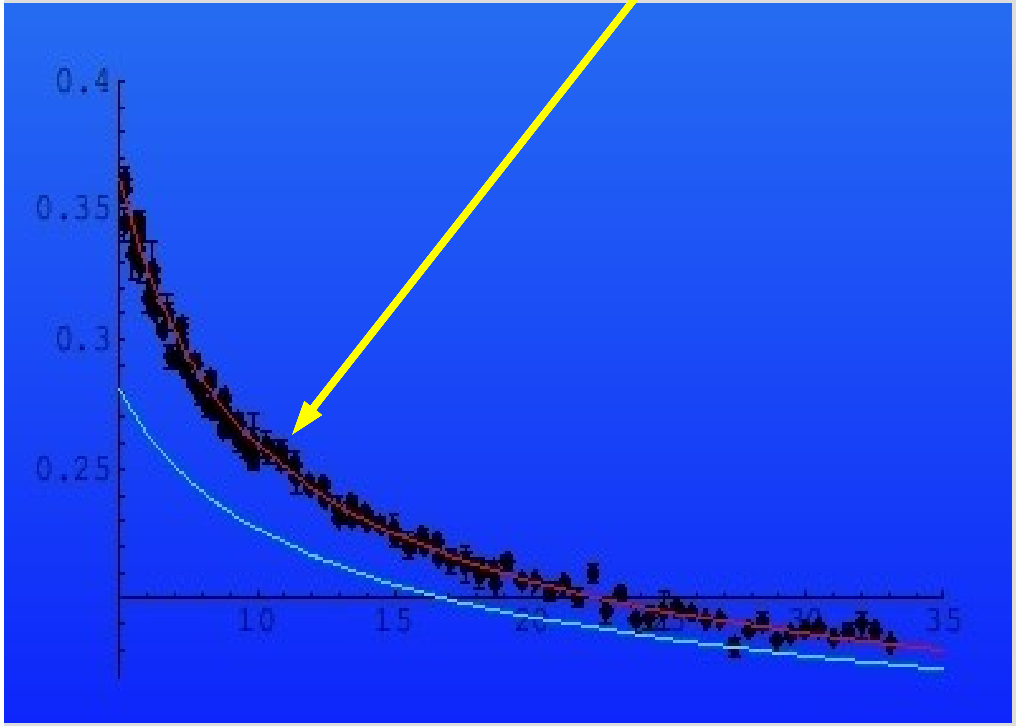

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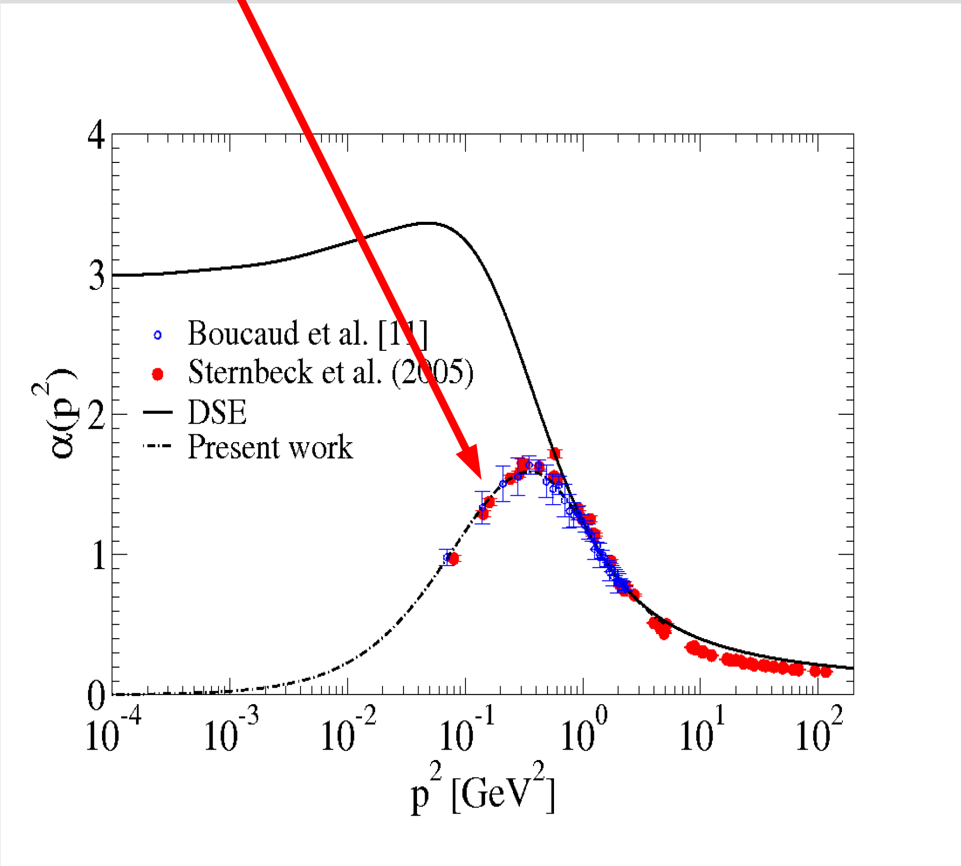
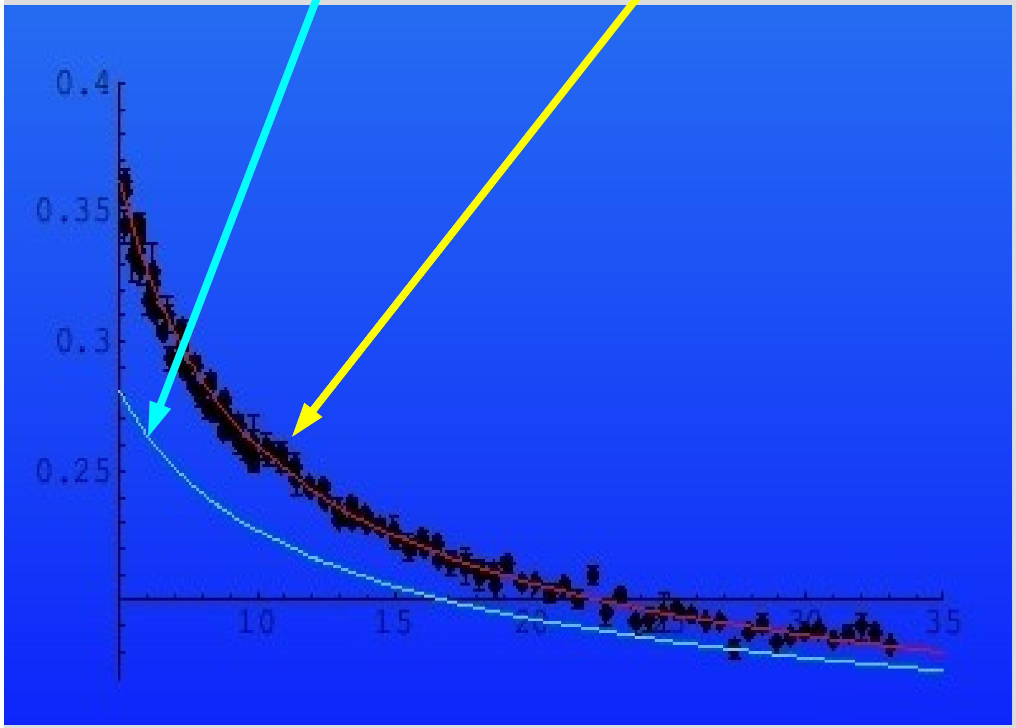




Conclusions:

Conclusions:

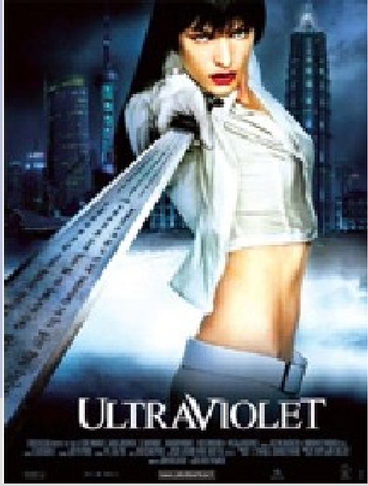
- The ghost propagator DSE allows for solutions observing two types of IR behaviors:
  - I: Divergent ghost dressing function and a finite ghost-gluon coupling at zero momentum ("scaling" or "conformal")
  - II: Finite ghost dressing function and massive gluon propagator, with a vanishing ghost-gluon coupling at zero-momentum.

- (Yang-Mills) Lattice QCD clearly favors the type-II solution (decoupling).

- In the BFM-PT scheme, the DSE coupled system for ghost and gluon propagators, massive gluon and not-enhanced ghost (type-II) solutions emerge.

- A low-momentum asymptotic formula for the ghost dressing function has be obtained for the type-II solutions and checked with numerical evaluations.

Conclusions:



•The MOM ghost-gluon coupling constant in Taylor scheme is computed only from gluon and ghost propagator (involving no three-point function) on the lattice.

•Discretization errors seem pretty under control and the perturbative regime is achieved only above around 3 GeV (provided that a dimension-two gluon condensate is accounted).

•Different estimates of the pure Yang-Mills Lambda QCD parameter agree fairly well and the job for 2 and 2+1+1 dynamical flavors is in progress.